Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/35-1.2.1.4-d+e-x-^m-f+g-x-^n-a+b-x+c-x^2-^p

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [958]. This is test number [35].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.90 (957)	0.10 (1)
Mathematica	98.33 (942)	1.67 (16)
Maple	75.99 (728)	24.01 (230)
Fricas	68.79 (659)	31.21 (299)
Giac	49.58 (475)	50.42 (483)
Maxima	32.57 (312)	67.43 (646)
Mupad	28.81 (276)	71.19 (682)
Sympy	27.66 (265)	72.34 (693)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

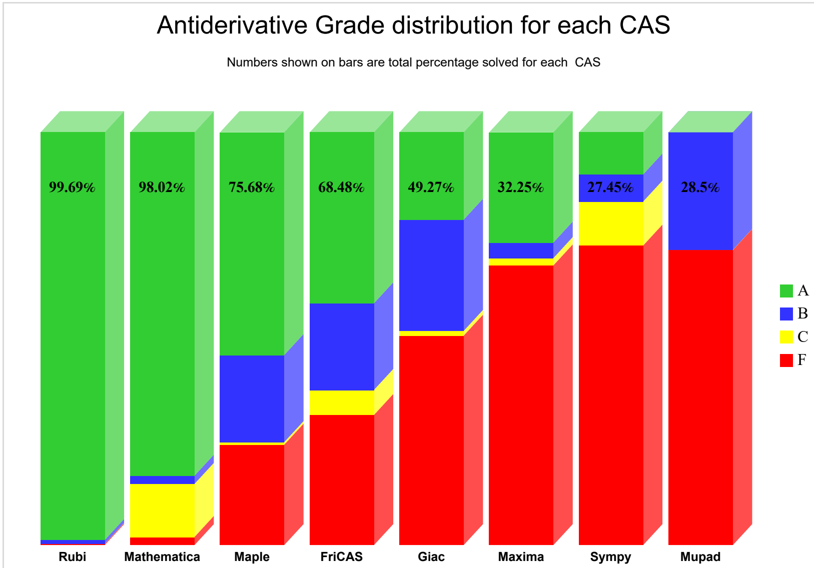
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

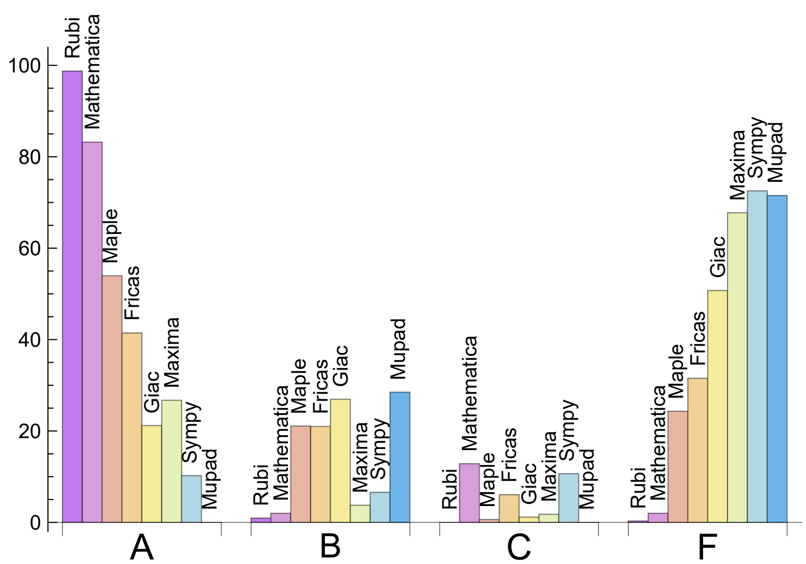
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.599	1.983	0.000	0.418
Mathematica	83.194	1.983	12.839	1.983
Maple	53.967	21.086	0.626	24.322
Fricas	41.441	20.981	6.054	31.524
Maxima	26.722	3.758	1.775	67.745
Giac	21.190	26.931	1.148	50.731
Sympy	10.230	6.576	10.647	72.547
Mupad	0.000	28.497	0.000	71.503

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	1	100.00	0.00	0.00
Mathematica	16	100.00	0.00	0.00
Maple	230	99.57	0.43	0.00
Fricas	299	77.26	22.74	0.00
Giac	483	83.64	4.35	12.01
Maxima	646	83.90	0.46	15.63
Mupad	682	0.00	100.00	0.00
Sympy	693	72.87	25.25	1.88

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.25
Rubi	0.58
Giac	0.58
Maple	0.75
Mathematica	3.33
Fricas	3.55
Sympy	7.19
Mupad	12.09

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	199.82	1.33	150.00	1.21
Rubi	260.84	1.09	183.00	1.01
Mathematica	526.65	1.25	152.00	0.93
Giac	764.04	3.13	303.00	1.98
Fricas	834.10	3.19	251.00	1.72
Maple	1160.25	3.07	211.00	1.26
Sympy	1676.32	8.08	313.00	2.27
Mupad	2029.94	5.61	161.00	1.28

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

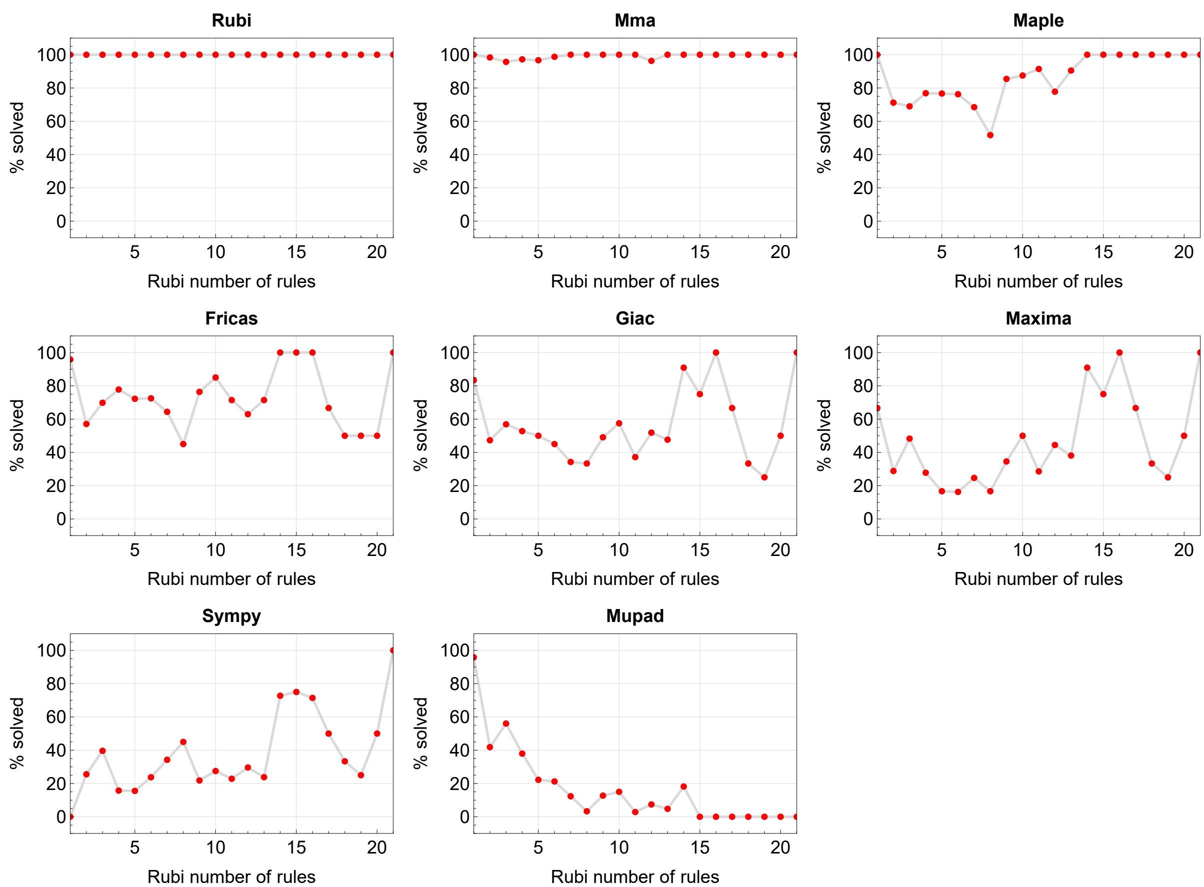


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

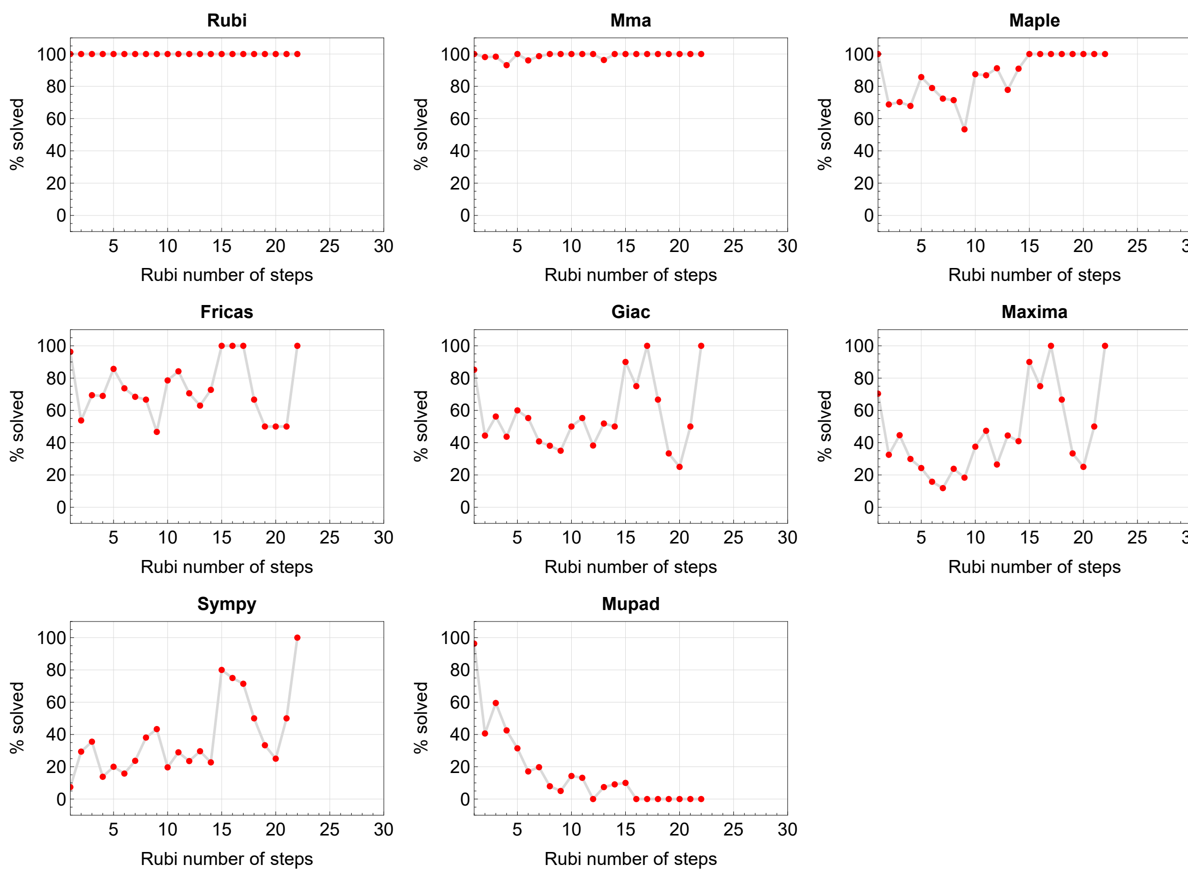


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

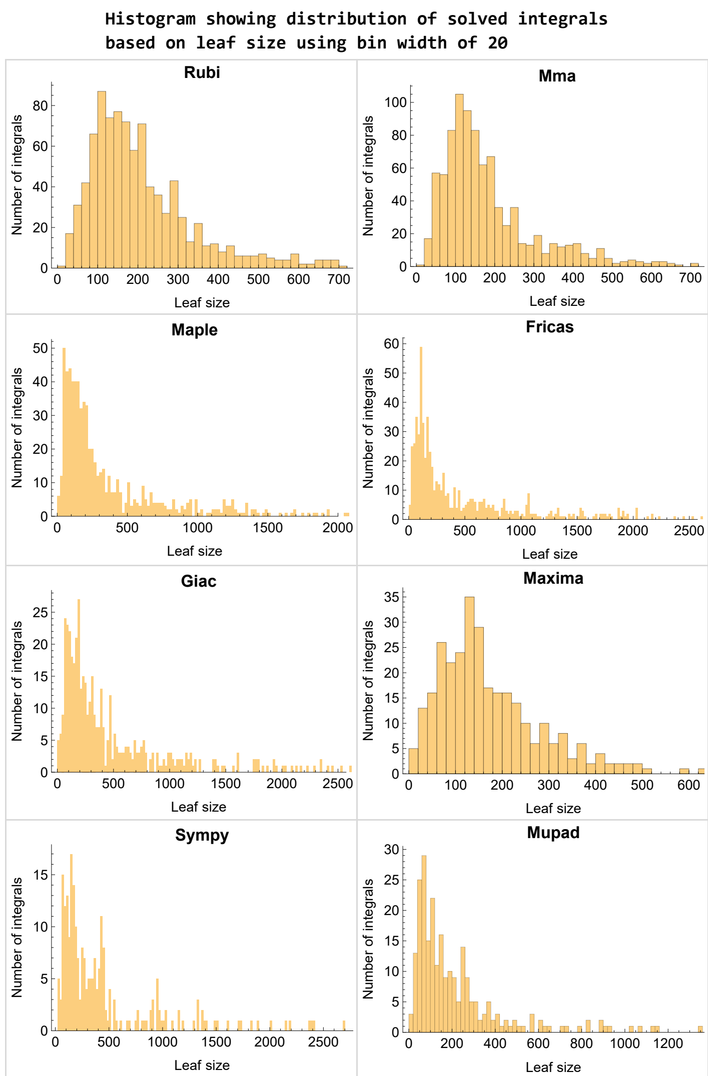


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

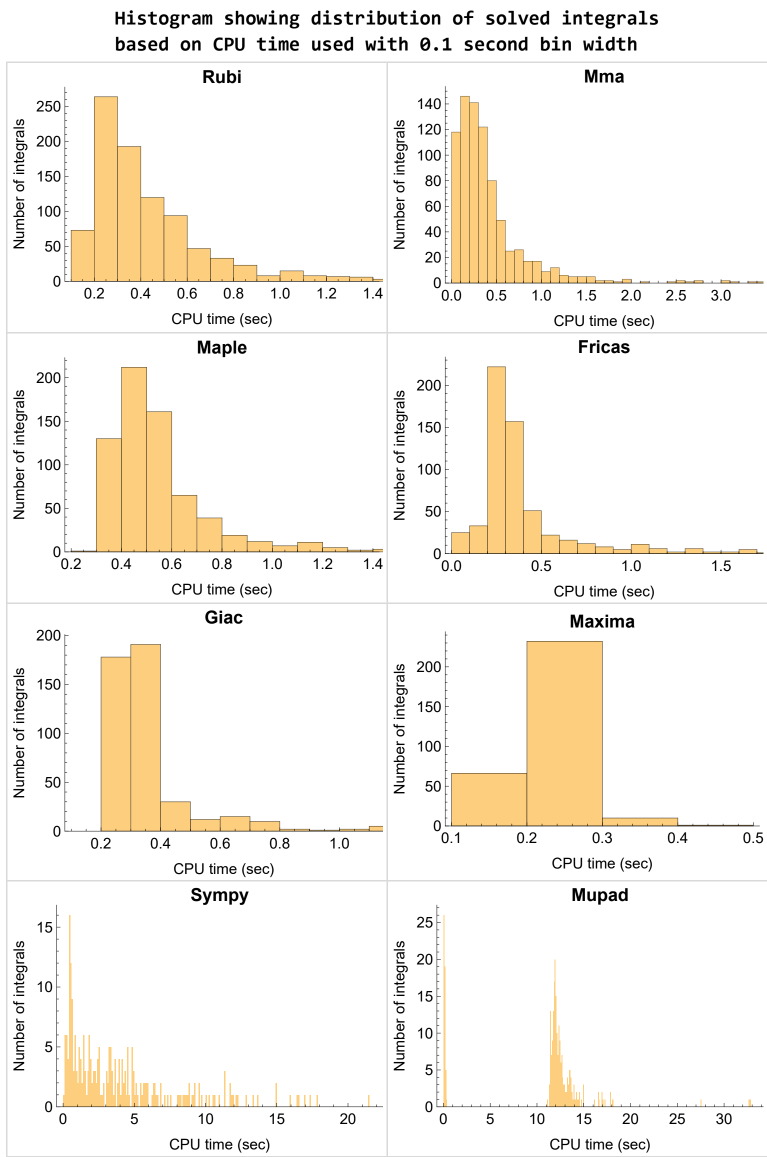


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

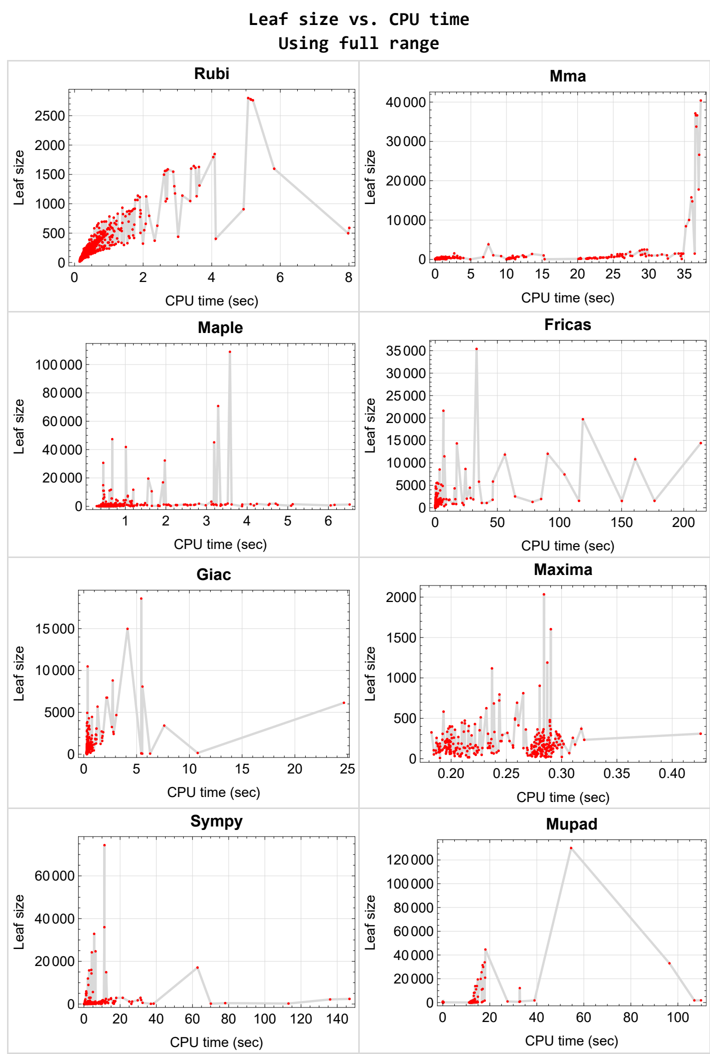


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{948, 952, 957\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{622, 623, 624, 627, 628, 629, 630, 631, 633, 634, 635, 636, 637, 640, 641, 642, 643, 644, 645, 646, 649, 650, 651, 652, 653, 654, 919\}$

Mathematica $\{267, 275, 276, 314, 418, 422, 429, 656, 802, 899, 907, 915\}$

Maple $\{796, 800, 801, 850, 853, 918, 919\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the **Timofeev** test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

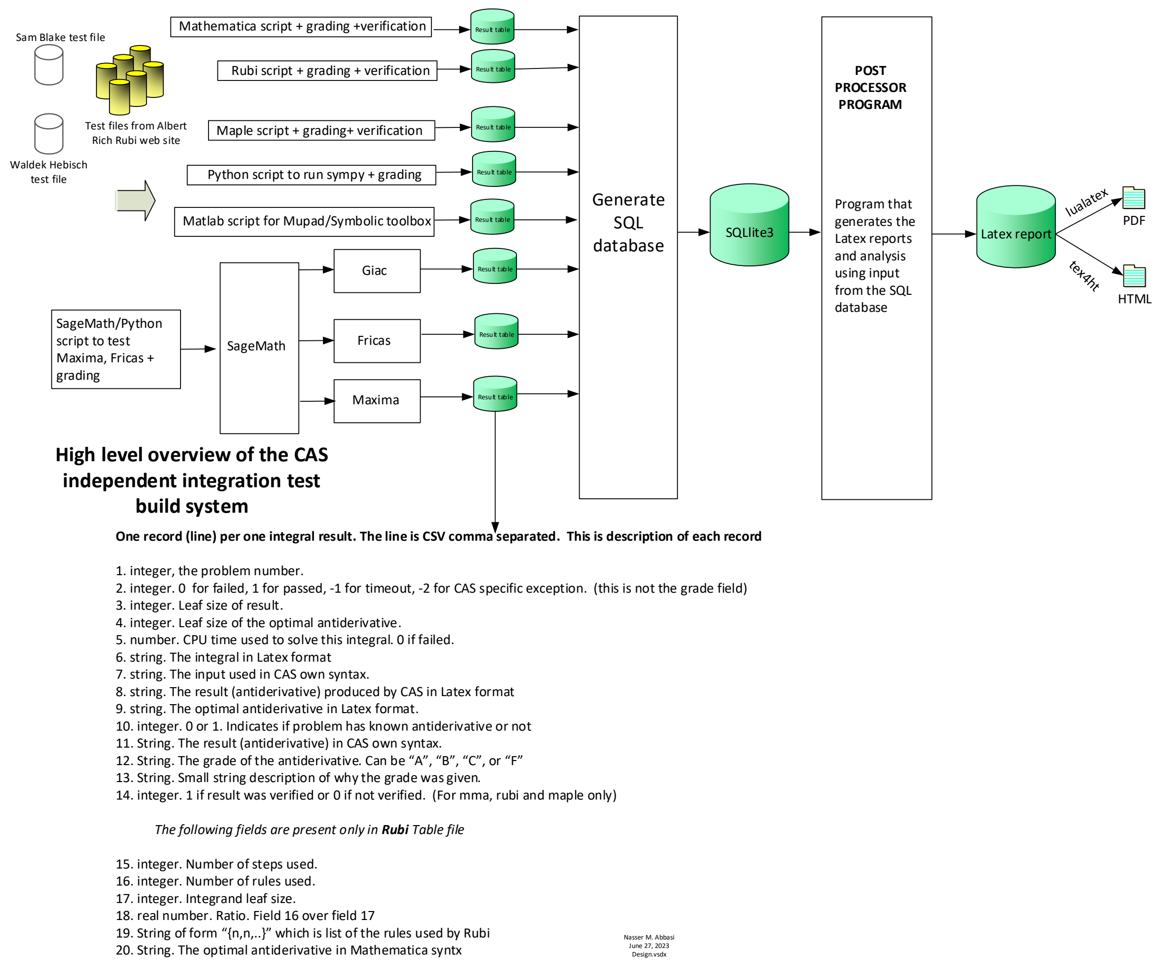
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 629, 630, 631, 632, 636, 637, 645, 646, 648, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 958 }

B grade { 627, 628, 633, 634, 635, 638, 639, 640, 641, 642, 643, 644, 647, 649, 650, 651, 652, 653, 654 }

C grade { }

F normal fail { 626 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 269, 270, 271, 272, 273, 278, 279, 280, 281, 282, 283, 286, 287, 288, 289, 290, 291, 292, 293, 296, 297, 298, 299, 300, 302, 303, 306, 307, 308, 309, 310, 311, 312, 313, 314, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 409, 410, 411, 412, 413, 414, 415, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 437, 438, 439, 440, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 490, 493, 497, 500, 504, 507, 511, 514, 518, 521, 524, 526, 527, 530, 531, 532, 535, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 607, 619, 620, 621, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 797, 798, 799, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 919, 920, 921, 922, 923, 924, 925, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 947, 951, 953, 956 }

B grade { 263, 268, 274, 277, 284, 285, 294, 295, 301, 304, 305, 360, 441, 442, 656, 802, 907, 918, 926 }

C grade { 267, 275, 276, 315, 489, 491, 492, 494, 495, 496, 498, 499, 501, 502, 503, 505, 506, 508, 509, 510, 512, 513, 515, 516, 517, 519, 520, 522, 523, 525, 528, 529, 533, 534, 536, 537, 538, 539, 540, 587, 605, 606, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 795, 796, 800, 801, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 958 }

F normal fail { 379, 380, 408, 416, 434, 435, 436, 541, 803, 804, 945, 946, 949, 950, 954, 955 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 134, 135, 139, 140, 141, 142, 143, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 177, 178, 179, 180, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 209, 211, 212, 213, 214, 215, 218, 219, 222, 224, 315, 316, 317, 318, 319, 323, 324, 325, 326, 327, 328, 332, 333, 341, 350, 352, 353, 439, 440, 448, 449, 459, 460, 471, 472, 473, 474, 475, 476, 480, 481, 482, 483, 484, 487, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 582, 583, 584, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 604, 619, 626, 627, 628, 629, 630, 634, 635, 636, 640, 641, 642, 643, 645, 648, 649, 650, 651, 653, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 679, 680, 681, 682, 683, 684, 685, 686, 687, 689, 690, 691, 692, 693, 694, 695, 696, 697, 700, 701, 702, 703, 704, 708, 712, 713, 714, 715, 716, 717, 718, 719, 721, 722, 723, 724, 725, 726, 728, 729, 730, 731, 732, 734, 735, 736, 737, 738, 739, 740, 741, 743, 744, 745, 746, 747, 748, 749, 750, 753, 757, 758, 759, 760, 768, 769, 770, 771, 780, 783, 784, 785, 786, 787, 788, 792, 793, 808, 814, 815, 816, 817, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 840, 841, 848, 849, 854, 855, 856, 857, 862, 863, 864, 865, 866, 867, 869, 870, 871, 872, 873, 875, 876, 889, 890, 891, 892, 893, 894, 897, 898, 899, 900, 901, 904, 905, 906, 907, 908, 909, 910, 912, 913, 914, 915, 917, 919, 958 }

B grade { 45, 71, 85, 89, 97, 108, 129, 133, 136, 137, 138, 144, 145, 163, 171, 176, 181, 185, 191, 194, 204, 210, 216, 217, 220, 221, 223, 225, 320, 321, 322, 329, 330, 331, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 349, 351, 355, 356, 357, 359, 360, 361, 437, 438, 441, 442, 443, 444, 445, 446, 447, 450, 451, 452, 453, 454, 455, 456, 457, 458, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 477, 478, 479, 485, 486, 488, 579, 580, 581, 585, 586, 587, 603, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 620, 621, 622, 623, 624, 625, 631, 632, 633, 637, 638, 639, 644, 646, 647, 652, 654, 678, 688, 698, 699, 705, 706, 707, 709, 710, 711, 720, 727, 733, 742, 751, 752, 754, 755, 756, 789, 790, 791, 797, 798, 805, 806, 807, 818, 833, 834, 835, 836, 837, 838, 839, 842, 843, 844, 845, 846, 847, 850, 851, 852, 853, 858, 859, 860, 861, 868, 874, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 895, 896, 902, 903, 911, 916, 918, 920, 921, 925, 926 }

C grade { 794, 795, 796, 799, 800, 801 }

F normal fail { 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 541, 542, 543, 544, 545, 546, 547, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 781, 782, 802, 803, 804, 809, 810, 811, 812, 813, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956 }

F(-1) timedout fail { 499 }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 28, 29, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 133, 134, 135, 136, 137, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 222, 224, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 331, 332, 333, 341, 348, 349, 350, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 490, 493, 497, 500, 504, 507, 511, 514, 518, 521, 524, 548, 549, 550, 551, 552, 557, 558, 559, 560, 562, 569, 570, 572, 573, 583, 584, 589, 590, 591, 592, 593, 596, 597, 598, 599, 603, 604, 620, 621, 656, 657, 658, 659, 660, 661, 665, 666, 667, 668, 672, 673, 674, 679, 680, 681, 682, 683, 684, 689, 690, 691, 692, 693, 694, 695, 700, 701, 702, 703, 705, 706, 707, 712, 713, 714, 715, 720, 721, 722, 727, 728, 733, 734, 735, 736, 737, 742, 743, 744, 745, 746, 751, 752, 753, 754, 755, 756, 769, 770, 771, 780, 781, 783, 784, 785, 786, 787, 788, 792, 793, 794, 797, 798, 814, 815, 816, 819, 820, 821, 822, 823, 826, 827, 828, 829, 834, 835, 836, 837, 840, 842, 843, 844, 845, 857, 865 }

B grade { 18, 20, 21, 22, 23, 24, 25, 26, 27, 30, 31, 131, 132, 138, 139, 140, 141, 142, 143, 144, 148, 149, 216, 221, 223, 225, 330, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 351, 352, 353, 355, 356, 357, 359, 360, 361, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 553, 554, 555, 556, 561, 563, 564, 565, 566, 567, 568, 571, 574, 575, 576, 577, 578, 579, 580, 581, 582, 585, 586, 587, 588, 594, 595, 600, 601, 602, 607, 608, 609, 611, 612, 613, 615, 616, 618, 619, 662, 663, 664, 669, 670, 671, 675, 676, 677, 678, 685, 686, 687, 688, 696, 697, 698, 699, 704, 708, 709, 710, 711, 716, 717, 718, 719, 723, 724, 725, 726, 729, 730, 731, 732, 738, 739, 740, 741, 747, 748, 749, 750, 757, 758, 759, 760, 768, 789, 790, 791, 795, 796, 799, 800, 805, 806, 807, 808, 824, 825, 830, 831, 832, 833, 838, 839, 841, 846, 847, 848, 849, 851, 852, 873, 874, 875, 880, 881, 882, 920, 921, 925, 926 }

C grade { 315, 489, 491, 492, 494, 495, 496, 498, 499, 501, 502, 503, 505, 506, 508, 509, 510, 512, 513, 515, 516, 517, 519, 520, 522, 523, 622, 623, 624, 625, 629, 630, 631, 632, 636, 637, 638, 639, 645, 646, 647, 648, 886, 887, 888, 889, 893, 894, 895, 896, 900, 901, 902, 903, 909, 910, 911, 912 }

F normal fail { 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 541, 542, 543, 544, 545, 546, 547, 655, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 782, 802, 803, 804, 809, 810, 811, 812, 813, 919, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 958 }

F(-1) timedout fail { 605, 606, 610, 614, 617, 626, 627, 628, 633, 634, 635, 640, 641, 642, 643, 644, 649, 650, 651, 652, 653, 654, 801, 817, 818, 850, 853, 854, 855, 856, 858, 859, 860, 861, 862, 863, 864, 866, 867, 868, 869, 870, 871, 872, 876, 877, 878, 879, 883, 884, 885, 890, 891, 892, 897, 898, 899, 904, 905, 906, 907, 908, 913, 914, 915, 916, 917, 918 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 137, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 179, 180, 181, 182, 183, 184, 202, 203, 220, 224, 321, 330, 337, 338, 351, 352, 353, 355, 356, 357, 490, 497, 504, 511, 518, 524, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 584, 589, 590, 591, 592, 596, 597, 598, 599, 657, 658, 659, 660, 665, 666, 667, 668, 672, 673, 674, 675, 679, 680, 681, 682, 683, 689, 690, 691, 692, 693, 700, 701, 702, 703, 704, 768, 769, 770, 771, 780, 781, 783, 784, 785, 786, 787, 792, 793, 794, 797, 798, 799, 807, 808, 814, 815, 816, 819, 820, 821, 822, 826, 827, 828, 829 }

B grade { 11, 19, 20, 21, 22, 44, 45, 79, 83, 84, 85, 136, 138, 192, 193, 211, 212, 213, 214, 215, 222, 336, 359, 360, 361, 579, 580, 581, 582, 583, 805, 806, 920, 921, 925, 926 }

C grade { 102, 103, 104, 105, 106, 107, 157, 158, 159, 160, 161, 162, 198, 199, 200, 201, 604 }

F normal fail { 98, 99, 100, 101, 124, 125, 126, 133, 134, 135, 144, 145, 146, 147, 154, 155, 156, 176, 177, 178, 185, 186, 187, 191, 194, 195, 196, 197, 204, 205, 206, 207, 208, 209, 210, 216, 217, 218, 219, 221, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 322, 323, 324, 325, 331, 332, 333, 339, 340, 341, 348, 349, 350, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 442, 443, 444, 445, 451, 452, 453, 454, 455, 456, 462, 463, 464, 465, 466, 467, 468, 469, 474, 475, 476, 483, 484, 485, 486, 489, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 519, 520, 521, 522, 523, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 661, 662, 663, 664, 669, 670, 671, 676, 677, 678, 684, 685, 686, 687, 688, 694, 695, 696, 697, 698, 699, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 782, 788, 789, 790, 791, 795, 796, 800, 801, 802, 803, 804, 809, 810, 811, 812, 813, 833, 850, 851, 852, 853, 859, 860, 861, 867, 868, 875, 876, 877, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 958 }

F(-1) timedout fail { 188, 189, 190 }

F(-2) exception fail { 316, 317, 318, 319, 320, 326, 327, 328, 329, 334, 335, 342, 343, 344, 345, 346, 347, 437, 438, 439, 440, 441, 446, 447, 448, 449, 450, 457, 458, 459, 460, 461, 470, 471, 472, 473, 477, 478, 479, 480, 481, 482, 487, 488, 585, 586, 587, 588, 593, 594, 595, 600, 601, 602, 603, 621, 817, 818, 823, 824, 825, 830, 831, 832, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 854, 855, 856, 857, 858, 862, 863, 864, 865, 866, 869, 870, 871, 872, 873, 874, 878, 879, 880, 881, 882 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 16, 17, 32, 33, 34, 35, 36, 37, 38, 54, 55, 56, 57, 58, 59, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 83, 84, 85, 86, 87, 88, 92, 93, 94, 95, 96, 97, 102, 103, 104, 105, 106, 107, 108, 109, 117, 119, 120, 121, 122, 123, 124, 151, 152, 153, 154, 157, 158, 159, 161, 162, 179, 180, 181, 182, 183, 184, 188, 189, 190, 191, 198, 199, 200, 201, 202, 203, 204, 219, 315, 322, 323, 324, 330, 332, 333, 340, 341, 437, 438, 439, 440, 442, 446, 447, 448, 449, 451, 457, 458, 459, 460, 462, 470, 471, 504, 524, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 584, 588, 589, 590, 591, 592, 593, 594, 595, 597, 598, 599, 600, 601, 602, 603, 604, 621, 658, 659, 660, 661, 666, 667, 668, 673, 674, 715, 771, 780, 786, 787, 792, 793, 794, 814, 815, 816, 817, 820, 821, 822, 823, 824, 828, 829, 830, 831, 833, 834, 835, 836, 837, 838, 842, 845, 874 }

B grade { 9, 10, 11, 12, 13, 14, 15, 39, 40, 41, 42, 43, 60, 61, 62, 63, 64, 75, 76, 77, 78, 79, 80, 81, 82, 89, 90, 91, 98, 99, 100, 101, 110, 111, 112, 113, 114, 115, 116, 118, 125, 126, 156, 160, 185, 186, 187, 192, 193, 194, 195, 196, 197, 205, 206, 207, 208, 209, 220, 221, 222, 223, 224, 325, 336, 337, 338, 351, 352, 353, 355, 356, 357, 359, 360, 361, 443, 444, 445, 452, 453, 454, 455, 456, 463, 464, 465, 466, 467, 468, 469, 490, 497, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 579, 580, 581, 582, 583, 585, 587, 596, 618, 619, 620, 657, 662, 663, 664, 665, 669, 670, 671, 672, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 768, 769, 770, 783, 784, 785, 788, 789, 790, 791, 795, 797, 798, 799, 805, 806, 807, 808, 818, 819, 825, 826, 827, 832, 839, 840, 841, 843, 844, 846, 847, 848, 849, 860, 861, 877, 880, 881, 882, 885, 920, 921, 925, 926 }

C grade { 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178 }

F normal fail { 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 212, 213, 214, 215, 216, 217, 226, 227, 228, 229, 230, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 342, 343, 344, 346, 348, 349, 350, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 541, 542, 543, 544, 545, 546, 547, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 761, 762, 763, 764, 765, 766, 772, 773, 774, 775, 776, 777, 778, 779, 781, 782, 802, 803, 804, 809, 810, 811, 812, 813, 859, 876, 884, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 958 }

F(-1) timedout fail { 586, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 767, 796, 800, 801, 850, 851, 852, 853, 867 }

F(-2) exception fail { 150, 155, 163, 164, 165, 166, 171, 210, 211, 218, 225, 231, 316, 317, 318, 319, 320, 321, 326, 327, 328, 329, 331, 334, 335, 339, 345, 347, 441, 450, 461, 472, 473, 474, 475, 476, 605, 606, 854, 855, 856, 857, 858, 862, 863, 864, 865, 866, 868, 869, 870, 871, 872, 873, 875, 878, 879, 883 }

2.1.7 Mupad

A grade { }

B grade { 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 46, 47, 48, 49, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 86, 87, 88, 117, 118, 123, 130, 131, 132, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 154, 155, 156, 172, 173, 174, 175, 182, 183, 184, 192, 193, 210, 211, 212, 213, 214, 215, 220, 222, 224, 245, 315, 351, 352, 353, 355, 356, 357, 359, 360, 361, 386, 473, 480, 481, 482, 487, 488, 490, 497, 504, 511, 518, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 582, 583, 584, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 618, 619, 620, 621, 657, 658, 659, 660, 665, 666, 667, 668, 672, 673, 674, 675, 679, 680, 681, 682, 683, 689, 690, 691, 692, 693, 700, 701, 702, 703, 704, 716, 717, 718, 719, 723, 724, 725, 726, 729, 730, 731, 732, 738, 739, 740, 741, 747, 748, 749, 750, 757, 758, 759, 760, 768, 769, 770, 771, 780, 783, 784, 785, 786, 787, 792, 793, 794, 795, 805, 806, 807, 808, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 836, 837, 840, 841, 844, 845, 848, 849, 920, 921, 925, 926 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 10, 11, 19, 20, 21, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 50, 51, 52, 53, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 133, 134, 135, 136, 137, 138, 144, 145, 146, 147, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 176, 177, 178, 179, 180, 181, 185, 186, 187, 188, 189, 190, 191, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 216, 217, 218, 219, 221, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 474, 475, 476, 477, 478, 479, 483, 484, 485, 486, 489, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 519, 520, 521, 522, 523, 541, 542, 543, 544, 545, 546, 547, 579, 580, 581, 585, 586, 587, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 661, 662, 663, 664, 669, 670, 671, 676, 677, 678, 684, 685, 686, 687, 688, 694, 695, 696, 697, 698, 699, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 720, 721, 722, 727, 728, 733, 734, 735, 736, 737, 742, 743, 744, 745, 746, 751, 752, 753, 754, 755, 756, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 781, 782, 788, 789, 790, 791, 796, 797, 798, 799, 800, 801, 802, 803, 804, 809, 810, 811, 812, 813, 835, 838, 839, 842, 843, 846, 847, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 958 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 16, 25, 32, 33, 34, 35, 36, 37, 54, 55, 56, 57, 58, 62, 65, 66, 67, 68, 69, 70, 102, 103, 104, 105, 106, 107, 117, 157, 158, 159, 160, 161, 162, 222, 244, 245, 253, 254, 255, 262, 264, 265, 385, 386, 394, 395, 396, 401, 403, 405, 406, 448, 449, 524, 548, 549, 550, 551, 557, 558, 559, 560, 561, 562, 566, 569, 570, 571, 572, 573, 576, 577, 578, 588, 590, 591, 592, 593, 596, 597, 598, 599, 600, 820, 821, 822, 823, 826, 827, 828, 829, 830 }

B grade { 19, 21, 23, 240, 241, 242, 243, 249, 250, 251, 252, 258, 259, 260, 261, 263, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 381, 382, 383, 384, 390, 391, 392, 393, 399, 400, 402, 404, 552, 553, 554, 555, 556, 563, 564, 565, 567, 568, 574, 575, 589, 805, 806, 807, 808, 814, 819, 920, 921, 925, 926 }

C grade { 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 20, 22, 24, 26, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 59, 60, 61, 63, 64, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 163, 164, 165, 166, 167, 168, 169, 170, 220, 224, 226, 227, 228, 229, 230, 231, 235, 236, 246, 247, 248, 256, 257, 266, 268, 269, 270, 271, 272, 273, 274, 306, 307, 308, 309, 310, 313, 377, 378, 387, 388, 389, 397, 398, 407, 430, 431, 432, 433 }

F normal fail { 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 223, 225, 233, 234, 237, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 363, 364, 365, 366, 367, 368, 375, 411, 412, 413, 419, 420, 421, 428, 437, 438, 439, 440, 441, 442, 443, 450, 451, 461, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 526, 527, 528, 529, 530, 531, 532, 537, 538, 543, 544, 545, 546, 579, 580, 581, 582, 583, 584, 585, 586, 587, 594, 603, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 666, 667, 668, 679, 680, 681, 682, 683, 684, 685, 689, 690, 691, 692, 693, 694, 704, 713, 714, 715, 716, 717, 722, 723, 724, 734, 735, 736, 737, 738, 743, 744, 745, 763, 764, 783, 784, 785, 786, 787, 788, 789, 790, 795, 796, 797, 798, 800, 809, 811, 812, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 862, 863, 864, 865, 866, 870, 871, 872, 873, 874, 875, 877, 878, 879, 880, 881, 882, 883, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 922, 924, 927, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 945, 946, 947, 949, 950, 951, 958 }

F(-1) timedout fail { 267, 369, 370, 371, 372, 373, 374, 376, 379, 380, 408, 409, 410, 414, 415, 416, 417, 418, 422, 423, 424, 425, 426, 427, 429, 434, 435, 436, 444, 445, 446, 447, 452, 453, 454, 455, 456, 457, 458, 459, 460, 462, 463, 464, 465, 466, 467, 468, 469, 487, 488, 525, 533, 534, 535, 536, 539, 540, 541, 547, 595, 601, 602, 604, 635, 651, 665, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 686, 687, 688, 695, 696, 697, 698, 699, 700, 701, 702, 703, 705, 706, 707, 708, 709, 710, 711, 712, 718, 719, 720, 721, 725, 726, 727, 728, 729, 730, 731, 732, 733, 739, 740, 741, 742, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 765, 766, 768, 769, 770, 771, 773, 775, 776, 777, 778, 779, 780, 781, 782, 791, 792, 793, 794, 799, 801, 802, 803, 804, 815, 816, 817, 818, 824, 825, 831, 832, 861, 867, 868, 869, 876, 884, 885, 908, 954, 955, 956, 957 }

F(-2) exception fail { 232, 238, 239, 542, 767, 772, 774, 810, 813, 923, 928, 944, 953 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	147	103	97	116	95	146	83	0
N.S.	1	1.11	0.78	0.73	0.88	0.72	1.11	0.63	0.00
time (sec)	N/A	0.247	0.285	0.460	0.280	0.391	0.461	0.297	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	246	147	141	189	138	202	130	0
N.S.	1	1.22	0.73	0.70	0.94	0.69	1.00	0.65	0.00
time (sec)	N/A	0.356	0.434	0.369	0.277	0.335	0.565	0.313	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	211	136	130	164	127	189	118	0
N.S.	1	1.23	0.79	0.76	0.95	0.74	1.10	0.69	0.00
time (sec)	N/A	0.312	0.395	0.352	0.283	0.329	0.529	0.303	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	176	125	119	139	116	173	107	0
N.S.	1	1.11	0.79	0.75	0.87	0.73	1.09	0.67	0.00
time (sec)	N/A	0.275	0.361	0.359	0.287	0.336	0.531	0.292	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	140	114	108	114	105	158	94	0
N.S.	1	1.21	0.98	0.93	0.98	0.91	1.36	0.81	0.00
time (sec)	N/A	0.233	0.335	0.342	0.280	0.323	0.533	0.294	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	140	114	108	114	105	158	94	0
N.S.	1	1.21	0.98	0.93	0.98	0.91	1.36	0.81	0.00
time (sec)	N/A	0.233	0.003	0.297	0.290	0.354	0.551	0.307	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	118	142	157	137	107	400	113	107
N.S.	1	1.04	1.26	1.39	1.21	0.95	3.54	1.00	0.95
time (sec)	N/A	0.270	0.384	0.336	0.287	0.319	5.955	0.308	11.738

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	116	143	165	143	124	389	178	114
N.S.	1	0.99	1.22	1.41	1.22	1.06	3.32	1.52	0.97
time (sec)	N/A	0.273	0.329	0.364	0.289	0.397	2.579	0.293	12.364

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	108	148	150	172	133	444	244	120
N.S.	1	0.89	1.22	1.24	1.42	1.10	3.67	2.02	0.99
time (sec)	N/A	0.281	0.369	0.362	0.284	0.336	3.222	0.287	12.526

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	118	149	137	196	129	457	285	0
N.S.	1	0.98	1.24	1.14	1.63	1.08	3.81	2.38	0.00
time (sec)	N/A	0.281	0.304	0.368	0.293	0.352	3.460	0.282	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	124	151	125	222	119	541	327	0
N.S.	1	1.05	1.28	1.06	1.88	1.01	4.58	2.77	0.00
time (sec)	N/A	0.285	0.281	0.383	0.283	0.306	4.588	0.297	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	113	122	107	155	98	774	388	93
N.S.	1	1.05	1.13	0.99	1.44	0.91	7.17	3.59	0.86
time (sec)	N/A	0.228	0.278	0.394	0.290	0.385	4.208	0.287	13.170

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	148	142	121	180	109	918	463	118
N.S.	1	1.03	0.99	0.85	1.26	0.76	6.42	3.24	0.83
time (sec)	N/A	0.256	0.293	0.402	0.279	0.313	8.065	0.295	13.786

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	183	153	132	205	120	1037	518	192
N.S.	1	1.06	0.89	0.77	1.19	0.70	6.03	3.01	1.12
time (sec)	N/A	0.298	0.355	0.419	0.275	0.274	8.683	0.290	14.487

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	218	164	143	230	131	1159	463	212
N.S.	1	1.08	0.82	0.71	1.14	0.65	5.77	2.30	1.05
time (sec)	N/A	0.335	0.325	0.395	0.284	0.299	25.087	0.294	14.951

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	116	83	75	93	72	129	64	112
N.S.	1	1.13	0.81	0.73	0.90	0.70	1.25	0.62	1.09
time (sec)	N/A	0.232	0.188	0.358	0.281	0.272	0.431	0.301	11.929

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	79	75	99	90	87	134	82	87
N.S.	1	1.08	1.03	1.36	1.23	1.19	1.84	1.12	1.19
time (sec)	N/A	0.198	0.205	0.362	0.281	0.268	3.265	0.287	11.780

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	63	59	55	88	104	231	0	55
N.S.	1	1.09	1.02	0.95	1.52	1.79	3.98	0.00	0.95
time (sec)	N/A	0.181	0.217	0.353	0.200	0.265	3.441	0.000	11.493

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	204	140	251	324	278	2004	0	0
N.S.	1	1.27	0.87	1.56	2.01	1.73	12.45	0.00	0.00
time (sec)	N/A	0.597	0.493	0.437	0.283	0.298	13.692	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	168	127	220	290	263	1821	0	0
N.S.	1	1.14	0.86	1.50	1.97	1.79	12.39	0.00	0.00
time (sec)	N/A	0.450	0.436	0.415	0.290	0.286	10.507	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	141	115	189	262	247	1739	0	0
N.S.	1	1.16	0.94	1.55	2.15	2.02	14.25	0.00	0.00
time (sec)	N/A	0.398	0.409	0.378	0.288	0.302	10.203	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	102	82	77	159	171	418	0	78
N.S.	1	1.21	0.98	0.92	1.89	2.04	4.98	0.00	0.93
time (sec)	N/A	0.296	0.277	0.369	0.199	0.338	7.364	0.000	11.854

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	97	82	77	134	172	337	0	78
N.S.	1	1.08	0.91	0.86	1.49	1.91	3.74	0.00	0.87
time (sec)	N/A	0.262	0.268	0.360	0.197	0.260	6.831	0.000	11.597

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	95	82	77	112	173	513	0	78
N.S.	1	1.01	0.87	0.82	1.19	1.84	5.46	0.00	0.83
time (sec)	N/A	0.196	0.276	0.352	0.190	0.267	7.106	0.000	11.441

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	85	82	77	87	172	432	0	78
N.S.	1	1.02	0.99	0.93	1.05	2.07	5.20	0.00	0.94
time (sec)	N/A	0.195	0.271	0.346	0.199	0.279	6.599	0.000	11.531

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	88	82	77	80	171	604	0	78
N.S.	1	1.10	1.02	0.96	1.00	2.14	7.55	0.00	0.98
time (sec)	N/A	0.188	0.009	0.339	0.189	0.269	7.511	0.000	11.456

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	130	129	182	157	244	2378	0	127
N.S.	1	1.11	1.10	1.56	1.34	2.09	20.32	0.00	1.09
time (sec)	N/A	0.281	0.353	0.348	0.188	0.278	14.974	0.000	11.973

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	161	134	213	189	270	2404	0	141
N.S.	1	1.05	0.88	1.39	1.24	1.76	15.71	0.00	0.92
time (sec)	N/A	0.497	0.407	0.412	0.206	0.287	12.159	0.000	12.182

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	202	147	244	221	291	2691	0	181
N.S.	1	1.10	0.80	1.33	1.20	1.58	14.62	0.00	0.98
time (sec)	N/A	0.667	0.468	0.388	0.197	0.329	14.955	0.000	12.280

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	127	104	99	135	239	903	0	164
N.S.	1	1.05	0.86	0.82	1.12	1.98	7.46	0.00	1.36
time (sec)	N/A	0.217	0.360	0.365	0.194	0.407	8.458	0.000	11.539

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	159	126	121	158	305	1401	0	202
N.S.	1	1.07	0.85	0.82	1.07	2.06	9.47	0.00	1.36
time (sec)	N/A	0.257	0.467	0.356	0.222	0.500	16.452	0.000	11.617

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	58	59	90	63	66	119	70	84
N.S.	1	1.07	1.09	1.67	1.17	1.22	2.20	1.30	1.56
time (sec)	N/A	0.180	0.184	0.374	0.274	0.277	3.399	0.293	0.090

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	215	114	108	165	105	182	96	0
N.S.	1	1.24	0.66	0.62	0.95	0.61	1.05	0.55	0.00
time (sec)	N/A	0.368	0.325	0.392	0.293	0.289	0.473	0.291	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	175	103	97	140	94	168	84	0
N.S.	1	1.22	0.72	0.67	0.97	0.65	1.17	0.58	0.00
time (sec)	N/A	0.327	0.275	0.378	0.300	0.265	0.459	0.292	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	145	92	86	115	83	155	73	0
N.S.	1	1.26	0.80	0.75	1.00	0.72	1.35	0.63	0.00
time (sec)	N/A	0.284	0.246	0.380	0.280	0.295	0.421	0.298	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	105	80	73	89	71	134	58	0
N.S.	1	1.27	0.96	0.88	1.07	0.86	1.61	0.70	0.00
time (sec)	N/A	0.232	0.220	0.391	0.283	0.250	0.454	0.281	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	81	69	60	62	60	112	45	0
N.S.	1	0.98	0.83	0.72	0.75	0.72	1.35	0.54	0.00
time (sec)	N/A	0.186	0.017	0.360	0.279	0.279	0.471	0.285	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	67	100	91	75	73	144	78	0
N.S.	1	1.02	1.52	1.38	1.14	1.11	2.18	1.18	0.00
time (sec)	N/A	0.251	0.173	0.349	0.286	0.291	2.289	0.291	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	112	93	78	79	168	126	0
N.S.	1	1.00	1.65	1.37	1.15	1.16	2.47	1.85	0.00
time (sec)	N/A	0.239	0.181	0.385	0.278	0.270	1.564	0.287	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	81	116	72	83	63	214	188	0
N.S.	1	1.01	1.45	0.90	1.04	0.79	2.68	2.35	0.00
time (sec)	N/A	0.232	0.237	0.371	0.277	0.279	2.416	0.284	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	111	99	86	108	74	303	255	0
N.S.	1	1.04	0.93	0.80	1.01	0.69	2.83	2.38	0.00
time (sec)	N/A	0.275	0.234	0.355	0.279	0.426	2.165	0.285	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	151	104	99	133	87	449	329	0
N.S.	1	1.08	0.74	0.71	0.95	0.62	3.21	2.35	0.00
time (sec)	N/A	0.314	0.280	0.395	0.280	0.358	4.328	0.314	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	181	131	110	158	98	510	385	0
N.S.	1	1.07	0.78	0.65	0.93	0.58	3.02	2.28	0.00
time (sec)	N/A	0.354	0.250	0.405	0.275	0.339	3.964	0.292	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	169	126	238	288	188	0	0	0
N.S.	1	1.18	0.88	1.66	2.01	1.31	0.00	0.00	0.00
time (sec)	N/A	0.504	0.427	0.418	0.284	0.356	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	141	104	324	310	172	0	0	0
N.S.	1	1.17	0.86	2.68	2.56	1.42	0.00	0.00	0.00
time (sec)	N/A	0.419	0.396	0.409	0.426	0.343	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	100	70	65	155	116	0	0	66
N.S.	1	1.03	0.72	0.67	1.60	1.20	0.00	0.00	0.68
time (sec)	N/A	0.322	0.303	0.382	0.273	0.404	0.000	0.000	11.604

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	92	70	66	131	117	0	0	67
N.S.	1	1.06	0.80	0.76	1.51	1.34	0.00	0.00	0.77
time (sec)	N/A	0.210	0.334	0.378	0.265	0.337	0.000	0.000	11.400

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	94	69	64	109	117	0	0	65
N.S.	1	1.06	0.78	0.72	1.22	1.31	0.00	0.00	0.73
time (sec)	N/A	0.202	0.329	0.386	0.269	0.570	0.000	0.000	11.333

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	82	70	65	78	116	0	0	66
N.S.	1	1.06	0.91	0.84	1.01	1.51	0.00	0.00	0.86
time (sec)	N/A	0.188	0.012	0.345	0.241	0.267	0.000	0.000	11.494

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	130	107	202	154	169	0	0	0
N.S.	1	1.11	0.91	1.73	1.32	1.44	0.00	0.00	0.00
time (sec)	N/A	0.288	0.481	0.361	0.238	0.264	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	164	135	250	187	195	0	0	0
N.S.	1	1.13	0.93	1.72	1.29	1.34	0.00	0.00	0.00
time (sec)	N/A	0.507	0.424	0.382	0.209	0.368	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	194	136	257	218	216	0	0	0
N.S.	1	1.07	0.75	1.41	1.20	1.19	0.00	0.00	0.00
time (sec)	N/A	0.648	0.473	0.398	0.199	0.267	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	236	161	270	243	227	0	0	0
N.S.	1	1.13	0.77	1.29	1.16	1.09	0.00	0.00	0.00
time (sec)	N/A	0.867	0.457	0.424	0.196	0.328	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	101	58	42	70	50	73	34	36
N.S.	1	1.25	0.72	0.52	0.86	0.62	0.90	0.42	0.44
time (sec)	N/A	0.237	0.140	0.367	0.307	0.273	0.210	0.278	11.175

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	85	53	37	56	45	60	30	31
N.S.	1	1.35	0.84	0.59	0.89	0.71	0.95	0.48	0.49
time (sec)	N/A	0.207	0.122	0.392	0.277	0.269	0.156	0.286	0.031

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	55	46	28	40	38	37	21	22
N.S.	1	1.34	1.12	0.68	0.98	0.93	0.90	0.51	0.54
time (sec)	N/A	0.189	0.096	0.355	0.269	0.274	0.111	0.282	0.029

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	39	41	25	28	33	27	19	21
N.S.	1	0.98	1.02	0.62	0.70	0.82	0.68	0.48	0.52
time (sec)	N/A	0.158	0.097	0.342	0.277	0.280	0.082	0.289	0.031

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	52	29	41	46	31	34	32
N.S.	1	1.00	1.62	0.91	1.28	1.44	0.97	1.06	1.00
time (sec)	N/A	0.181	0.086	0.378	0.271	0.284	2.515	0.277	11.438

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	57	30	42	53	51	55	35
N.S.	1	1.00	1.73	0.91	1.27	1.61	1.55	1.67	1.06
time (sec)	N/A	0.184	0.086	0.352	0.278	0.262	1.871	0.280	0.082

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	54	48	39	54	43	116	91	47
N.S.	1	1.06	0.94	0.76	1.06	0.84	2.27	1.78	0.92
time (sec)	N/A	0.191	0.090	0.371	0.277	0.295	3.098	0.294	11.488

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	70	47	44	68	48	128	125	67
N.S.	1	1.04	0.70	0.66	1.01	0.72	1.91	1.87	1.00
time (sec)	N/A	0.212	0.087	0.370	0.269	0.269	3.378	0.306	0.034

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	100	58	49	82	53	223	163	77
N.S.	1	1.12	0.65	0.55	0.92	0.60	2.51	1.83	0.87
time (sec)	N/A	0.239	0.095	0.398	0.276	0.265	5.407	0.288	0.033

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	116	63	54	96	58	201	199	90
N.S.	1	1.08	0.59	0.50	0.90	0.54	1.88	1.86	0.84
time (sec)	N/A	0.269	0.102	0.373	0.289	0.255	5.687	0.284	0.034

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	150	127	116	171	111	544	325	0
N.S.	1	1.12	0.95	0.87	1.28	0.83	4.06	2.43	0.00
time (sec)	N/A	0.434	0.431	0.404	0.288	0.279	4.416	0.298	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	380	202	196	282	194	303	188	0
N.S.	1	1.23	0.65	0.63	0.91	0.63	0.98	0.61	0.00
time (sec)	N/A	0.650	0.784	0.486	0.281	0.280	0.794	0.309	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	345	191	185	257	183	286	177	0
N.S.	1	1.23	0.68	0.66	0.91	0.65	1.02	0.63	0.00
time (sec)	N/A	0.600	0.678	0.434	0.284	0.272	0.685	0.301	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	310	180	174	232	172	274	165	0
N.S.	1	1.23	0.71	0.69	0.92	0.68	1.09	0.65	0.00
time (sec)	N/A	0.547	0.656	0.437	0.287	0.312	0.677	0.290	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	275	169	163	207	161	262	154	0
N.S.	1	1.23	0.76	0.73	0.93	0.72	1.17	0.69	0.00
time (sec)	N/A	0.504	0.622	0.444	0.280	0.291	0.641	0.297	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	240	158	152	182	150	241	142	0
N.S.	1	1.04	0.69	0.66	0.79	0.65	1.05	0.62	0.00
time (sec)	N/A	0.427	0.582	0.434	0.287	0.372	0.615	0.305	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	206	155	138	145	139	206	127	0
N.S.	1	1.10	0.82	0.73	0.77	0.74	1.10	0.68	0.00
time (sec)	N/A	0.295	0.399	0.404	0.275	0.259	0.621	0.293	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	216	186	353	217	151	954	161	0
N.S.	1	1.14	0.98	1.86	1.14	0.79	5.02	0.85	0.00
time (sec)	N/A	0.498	0.597	0.375	0.282	0.276	8.890	0.306	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	212	193	265	231	167	887	224	0
N.S.	1	1.10	1.00	1.37	1.20	0.87	4.60	1.16	0.00
time (sec)	N/A	0.499	0.590	0.430	0.280	0.276	3.247	0.297	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	219	203	175	241	179	887	293	0
N.S.	1	1.06	0.98	0.85	1.16	0.86	4.29	1.42	0.00
time (sec)	N/A	0.498	0.609	0.397	0.279	0.278	3.957	0.295	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	222	189	238	238	179	843	346	0
N.S.	1	1.06	0.90	1.13	1.13	0.85	4.01	1.65	0.00
time (sec)	N/A	0.501	0.577	0.401	0.277	0.272	4.179	0.295	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	208	198	223	262	180	959	409	0
N.S.	1	1.00	0.95	1.07	1.25	0.86	4.59	1.96	0.00
time (sec)	N/A	0.513	0.616	0.426	0.283	0.284	5.954	0.302	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	219	195	210	290	180	1182	460	0
N.S.	1	1.01	0.90	0.97	1.34	0.83	5.47	2.13	0.00
time (sec)	N/A	0.518	0.542	0.463	0.298	0.291	6.314	0.297	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	224	196	196	315	179	1380	526	0
N.S.	1	1.05	0.92	0.92	1.47	0.84	6.45	2.46	0.00
time (sec)	N/A	0.510	0.543	0.454	0.290	0.286	10.646	0.310	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	224	193	173	338	173	1513	542	0
N.S.	1	1.09	0.94	0.84	1.64	0.84	7.34	2.63	0.00
time (sec)	N/A	0.511	0.504	0.519	0.283	0.269	11.745	0.305	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	230	198	170	364	163	1719	584	0
N.S.	1	1.13	0.97	0.83	1.78	0.80	8.43	2.86	0.00
time (sec)	N/A	0.515	0.523	0.490	0.284	0.313	29.730	0.311	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	214	169	151	247	142	1889	648	0
N.S.	1	1.14	0.90	0.81	1.32	0.76	10.10	3.47	0.00
time (sec)	N/A	0.419	0.489	0.584	0.281	0.315	31.652	0.317	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	249	186	165	272	153	2159	731	0
N.S.	1	1.11	0.83	0.73	1.21	0.68	9.60	3.25	0.00
time (sec)	N/A	0.466	0.524	0.546	0.276	0.331	136.085	0.311	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	284	189	176	297	164	2397	778	0
N.S.	1	1.12	0.74	0.69	1.17	0.65	9.44	3.06	0.00
time (sec)	N/A	0.508	0.556	0.745	0.278	0.352	146.794	0.317	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	203	111	207	317	192	0	233	0
N.S.	1	1.17	0.64	1.19	1.82	1.10	0.00	1.34	0.00
time (sec)	N/A	0.685	0.548	0.464	0.279	0.281	0.000	0.303	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	164	98	195	336	177	0	211	0
N.S.	1	1.15	0.69	1.37	2.37	1.25	0.00	1.49	0.00
time (sec)	N/A	0.554	0.387	0.496	0.295	0.267	0.000	0.301	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	135	86	377	308	161	0	186	0
N.S.	1	1.14	0.73	3.19	2.61	1.36	0.00	1.58	0.00
time (sec)	N/A	0.393	0.366	0.400	0.296	0.281	0.000	0.300	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	101	53	50	154	106	0	106	49
N.S.	1	1.09	0.57	0.54	1.66	1.14	0.00	1.14	0.53
time (sec)	N/A	0.231	0.308	0.415	0.195	0.260	0.000	0.311	11.568

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	91	50	47	128	104	0	137	46
N.S.	1	1.06	0.58	0.55	1.49	1.21	0.00	1.59	0.53
time (sec)	N/A	0.203	0.293	0.395	0.197	0.328	0.000	0.308	11.725

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	111	53	50	101	106	0	165	49
N.S.	1	1.08	0.51	0.49	0.98	1.03	0.00	1.60	0.48
time (sec)	N/A	0.221	0.011	0.381	0.186	0.278	0.000	0.317	11.685

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	124	100	305	152	158	0	212	0
N.S.	1	1.09	0.88	2.68	1.33	1.39	0.00	1.86	0.00
time (sec)	N/A	0.290	0.415	0.380	0.198	0.279	0.000	0.307	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	155	117	208	184	184	0	304	0
N.S.	1	1.07	0.81	1.43	1.27	1.27	0.00	2.10	0.00
time (sec)	N/A	0.510	0.410	0.431	0.202	0.286	0.000	0.299	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	201	132	214	216	205	0	381	0
N.S.	1	1.10	0.73	1.18	1.19	1.13	0.00	2.09	0.00
time (sec)	N/A	0.698	0.444	0.408	0.195	0.287	0.000	0.314	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	186	103	97	125	95	0	88	0
N.S.	1	1.27	0.70	0.66	0.85	0.65	0.00	0.60	0.00
time (sec)	N/A	0.351	0.246	0.406	0.292	0.270	0.000	0.281	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	151	92	86	101	83	0	76	0
N.S.	1	1.28	0.78	0.73	0.86	0.70	0.00	0.64	0.00
time (sec)	N/A	0.306	0.216	0.388	0.275	0.276	0.000	0.282	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	116	83	75	77	73	0	64	0
N.S.	1	1.35	0.97	0.87	0.90	0.85	0.00	0.74	0.00
time (sec)	N/A	0.256	0.167	0.382	0.274	0.265	0.000	0.285	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	81	70	64	56	60	0	52	0
N.S.	1	1.31	1.13	1.03	0.90	0.97	0.00	0.84	0.00
time (sec)	N/A	0.191	0.161	0.402	0.275	0.291	0.000	0.289	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	63	49	31	52	0	36	0
N.S.	1	1.00	1.37	1.07	0.67	1.13	0.00	0.78	0.00
time (sec)	N/A	0.173	0.113	0.368	0.284	0.271	0.000	0.276	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	78	141	56	54	0	60	0
N.S.	1	1.00	1.70	3.07	1.22	1.17	0.00	1.30	0.00
time (sec)	N/A	0.213	0.130	0.376	0.282	0.279	0.000	0.297	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	73	63	0	50	0	115	0
N.S.	1	1.00	1.43	1.24	0.00	0.98	0.00	2.25	0.00
time (sec)	N/A	0.207	0.125	0.392	0.000	0.268	0.000	0.282	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	86	93	75	0	63	0	193	0
N.S.	1	1.05	1.13	0.91	0.00	0.77	0.00	2.35	0.00
time (sec)	N/A	0.245	0.148	0.395	0.000	0.273	0.000	0.284	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	122	109	88	0	75	0	255	0
N.S.	1	1.07	0.96	0.77	0.00	0.66	0.00	2.24	0.00
time (sec)	N/A	0.282	0.164	0.430	0.000	0.273	0.000	0.285	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	157	120	99	0	86	0	329	0
N.S.	1	1.10	0.84	0.69	0.00	0.60	0.00	2.30	0.00
time (sec)	N/A	0.317	0.180	0.395	0.000	0.263	0.000	0.283	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	147	103	97	174	94	167	83	0
N.S.	1	1.30	0.91	0.86	1.54	0.83	1.48	0.73	0.00
time (sec)	N/A	0.283	0.250	0.379	0.286	0.351	1.166	0.290	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	246	147	141	246	139	427	132	0
N.S.	1	1.22	0.73	0.70	1.22	0.69	2.12	0.66	0.00
time (sec)	N/A	0.393	0.419	0.404	0.294	0.288	1.470	0.305	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	211	136	130	221	127	400	120	0
N.S.	1	1.23	0.79	0.76	1.28	0.74	2.33	0.70	0.00
time (sec)	N/A	0.337	0.369	0.355	0.300	0.296	1.375	0.311	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	176	125	119	198	117	371	107	0
N.S.	1	1.26	0.89	0.85	1.41	0.84	2.65	0.76	0.00
time (sec)	N/A	0.305	0.345	0.387	0.289	0.332	1.421	0.312	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	140	114	108	176	105	343	96	0
N.S.	1	1.21	0.98	0.93	1.52	0.91	2.96	0.83	0.00
time (sec)	N/A	0.245	0.319	0.393	0.312	0.318	1.416	0.285	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	107	111	94	109	95	313	80	0
N.S.	1	1.07	1.11	0.94	1.09	0.95	3.13	0.80	0.00
time (sec)	N/A	0.202	0.226	0.382	0.291	0.292	1.301	0.300	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	118	142	297	137	107	400	114	0
N.S.	1	1.04	1.26	2.63	1.21	0.95	3.54	1.01	0.00
time (sec)	N/A	0.277	0.351	0.436	0.280	0.262	5.771	0.314	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	140	164	145	123	389	177	0
N.S.	1	1.00	1.22	1.43	1.26	1.07	3.38	1.54	0.00
time (sec)	N/A	0.301	0.356	0.413	0.288	0.291	3.157	0.304	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	111	149	150	150	135	444	245	0
N.S.	1	0.92	1.23	1.24	1.24	1.12	3.67	2.02	0.00
time (sec)	N/A	0.290	0.362	0.411	0.283	0.276	3.856	0.295	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	118	146	136	144	130	457	284	0
N.S.	1	0.98	1.22	1.13	1.20	1.08	3.81	2.37	0.00
time (sec)	N/A	0.290	0.389	0.435	0.284	0.279	4.118	0.288	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	124	151	126	171	119	541	328	0
N.S.	1	1.04	1.27	1.06	1.44	1.00	4.55	2.76	0.00
time (sec)	N/A	0.292	0.324	0.458	0.273	0.271	5.160	0.296	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	113	122	107	153	97	774	388	0
N.S.	1	1.05	1.13	0.99	1.42	0.90	7.17	3.59	0.00
time (sec)	N/A	0.231	0.365	0.465	0.277	0.268	5.043	0.296	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	148	142	121	178	108	918	463	0
N.S.	1	1.03	0.99	0.85	1.24	0.76	6.42	3.24	0.00
time (sec)	N/A	0.267	0.358	0.542	0.284	0.311	9.032	0.294	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	183	153	132	203	119	1037	518	0
N.S.	1	1.06	0.89	0.77	1.18	0.69	6.03	3.01	0.00
time (sec)	N/A	0.317	0.415	0.572	0.280	0.277	9.722	0.291	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	218	164	143	228	130	1159	463	0
N.S.	1	1.08	0.82	0.71	1.13	0.65	5.77	2.30	0.00
time (sec)	N/A	0.349	0.398	0.632	0.287	0.303	26.641	0.300	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	36	37	25	28	31	29	19	20
N.S.	1	1.33	1.37	0.93	1.04	1.15	1.07	0.70	0.74
time (sec)	N/A	0.163	0.083	0.353	0.281	0.256	1.112	0.284	0.039

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	73	87	68	74	170	125	74
N.S.	1	1.00	1.43	1.71	1.33	1.45	3.33	2.45	1.45
time (sec)	N/A	0.224	0.134	0.371	0.275	0.271	2.683	0.289	0.050

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	153	100	120	113	112	0	108	0
N.S.	1	1.30	0.85	1.02	0.96	0.95	0.00	0.92	0.00
time (sec)	N/A	0.391	0.262	0.355	0.283	0.268	0.000	0.308	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	110	89	108	86	101	0	96	0
N.S.	1	1.21	0.98	1.19	0.95	1.11	0.00	1.05	0.00
time (sec)	N/A	0.298	0.249	0.417	0.292	0.263	0.000	0.287	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	81	74	97	63	85	0	83	0
N.S.	1	1.05	0.96	1.26	0.82	1.10	0.00	1.08	0.00
time (sec)	N/A	0.213	0.198	0.386	0.286	0.266	0.000	0.305	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	65	74	40	67	0	61	0
N.S.	1	1.00	1.25	1.42	0.77	1.29	0.00	1.17	0.00
time (sec)	N/A	0.190	0.159	0.397	0.282	0.262	0.000	0.296	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	30	35	0	41	29
N.S.	1	1.00	1.00	0.94	0.97	1.13	0.00	1.32	0.94
time (sec)	N/A	0.152	0.004	0.396	0.273	0.258	0.000	0.280	11.555

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	76	88	0	62	0	87	0
N.S.	1	1.00	1.41	1.63	0.00	1.15	0.00	1.61	0.00
time (sec)	N/A	0.200	0.176	0.398	0.000	0.268	0.000	0.290	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	85	88	108	0	88	0	177	0
N.S.	1	1.05	1.09	1.33	0.00	1.09	0.00	2.19	0.00
time (sec)	N/A	0.250	0.216	0.387	0.000	0.412	0.000	0.303	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	117	107	117	0	113	0	256	0
N.S.	1	1.04	0.95	1.04	0.00	1.00	0.00	2.27	0.00
time (sec)	N/A	0.356	0.236	0.414	0.000	0.257	0.000	0.292	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	163	120	202	151	190	0	0	0
N.S.	1	1.27	0.94	1.58	1.18	1.48	0.00	0.00	0.00
time (sec)	N/A	0.370	0.348	0.408	0.290	0.272	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	125	105	186	124	175	0	0	0
N.S.	1	1.11	0.93	1.65	1.10	1.55	0.00	0.00	0.00
time (sec)	N/A	0.262	0.312	0.408	0.291	0.261	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	95	92	204	99	157	0	0	0
N.S.	1	1.07	1.03	2.29	1.11	1.76	0.00	0.00	0.00
time (sec)	N/A	0.225	0.272	0.381	0.287	0.259	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	48	86	103	0	0	56
N.S.	1	1.00	1.00	0.80	1.43	1.72	0.00	0.00	0.93
time (sec)	N/A	0.180	0.215	0.376	0.203	0.259	0.000	0.000	12.312

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	52	56	44	67	101	0	0	52
N.S.	1	0.90	0.97	0.76	1.16	1.74	0.00	0.00	0.90
time (sec)	N/A	0.172	0.192	0.387	0.210	0.284	0.000	0.000	12.273

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	60	46	65	102	0	0	56
N.S.	1	1.00	1.03	0.79	1.12	1.76	0.00	0.00	0.97
time (sec)	N/A	0.164	0.221	0.379	0.200	0.336	0.000	0.000	11.693

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	93	106	171	0	155	0	0	0
N.S.	1	1.06	1.20	1.94	0.00	1.76	0.00	0.00	0.00
time (sec)	N/A	0.244	0.301	0.374	0.000	0.274	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	128	124	198	0	181	0	0	0
N.S.	1	1.07	1.03	1.65	0.00	1.51	0.00	0.00	0.00
time (sec)	N/A	0.282	0.308	0.400	0.000	0.298	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	164	138	206	0	201	0	0	0
N.S.	1	1.08	0.91	1.36	0.00	1.32	0.00	0.00	0.00
time (sec)	N/A	0.402	0.335	0.400	0.000	0.287	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	207	140	295	289	274	0	0	0
N.S.	1	1.28	0.86	1.82	1.78	1.69	0.00	0.00	0.00
time (sec)	N/A	0.521	0.553	0.444	0.297	0.324	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	172	130	281	259	258	0	0	0
N.S.	1	1.16	0.88	1.90	1.75	1.74	0.00	0.00	0.00
time (sec)	N/A	0.387	0.468	0.427	0.310	0.282	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	144	115	450	234	241	0	0	0
N.S.	1	1.18	0.94	3.69	1.92	1.98	0.00	0.00	0.00
time (sec)	N/A	0.340	0.417	0.392	0.320	0.337	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	92	82	70	134	168	0	0	78
N.S.	1	1.08	0.96	0.82	1.58	1.98	0.00	0.00	0.92
time (sec)	N/A	0.228	0.312	0.432	0.213	0.275	0.000	0.000	11.743

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	97	82	70	110	171	0	0	78
N.S.	1	1.07	0.90	0.77	1.21	1.88	0.00	0.00	0.86
time (sec)	N/A	0.224	0.280	0.379	0.204	0.279	0.000	0.000	12.042

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	98	82	70	110	170	0	0	78
N.S.	1	1.03	0.86	0.74	1.16	1.79	0.00	0.00	0.82
time (sec)	N/A	0.211	0.289	0.374	0.205	0.290	0.000	0.000	11.854

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	84	82	70	90	171	0	0	78
N.S.	1	0.99	0.96	0.82	1.06	2.01	0.00	0.00	0.92
time (sec)	N/A	0.195	0.277	0.372	0.203	0.306	0.000	0.000	11.794

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	90	82	70	85	168	0	0	78
N.S.	1	1.10	1.00	0.85	1.04	2.05	0.00	0.00	0.95
time (sec)	N/A	0.193	0.011	0.388	0.196	0.289	0.000	0.000	11.710

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	132	118	255	0	237	0	0	0
N.S.	1	1.11	0.99	2.14	0.00	1.99	0.00	0.00	0.00
time (sec)	N/A	0.294	0.468	0.379	0.000	0.303	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	163	145	291	0	265	0	0	0
N.S.	1	1.06	0.94	1.89	0.00	1.72	0.00	0.00	0.00
time (sec)	N/A	0.442	0.430	0.442	0.000	0.409	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	204	160	296	0	286	0	0	0
N.S.	1	1.10	0.86	1.59	0.00	1.54	0.00	0.00	0.00
time (sec)	N/A	0.581	0.452	0.415	0.000	0.369	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	239	171	312	0	297	0	0	0
N.S.	1	1.11	0.80	1.45	0.00	1.38	0.00	0.00	0.00
time (sec)	N/A	0.742	0.489	0.438	0.000	0.361	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	129	104	92	133	239	0	0	161
N.S.	1	1.09	0.88	0.78	1.13	2.03	0.00	0.00	1.36
time (sec)	N/A	0.236	0.374	0.378	0.228	0.318	0.000	0.000	11.923

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	130	104	92	133	238	0	0	161
N.S.	1	1.06	0.85	0.75	1.08	1.93	0.00	0.00	1.31
time (sec)	N/A	0.224	0.386	0.384	0.220	0.351	0.000	0.000	11.764

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	83	69	97	68	75	0	0	116
N.S.	1	1.26	1.05	1.47	1.03	1.14	0.00	0.00	1.76
time (sec)	N/A	0.266	0.188	0.367	0.291	0.276	0.000	0.000	0.067

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	59	59	84	52	66	0	70	84
N.S.	1	1.07	1.07	1.53	0.95	1.20	0.00	1.27	1.53
time (sec)	N/A	0.188	0.166	0.367	0.286	0.279	0.000	0.302	0.067

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	53	65	33	58	0	52	57
N.S.	1	1.00	1.56	1.91	0.97	1.71	0.00	1.53	1.68
time (sec)	N/A	0.164	0.116	0.349	0.285	0.254	0.000	0.318	11.445

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	22	23	28	0	34	23
N.S.	1	1.00	1.00	0.85	0.88	1.08	0.00	1.31	0.88
time (sec)	N/A	0.146	0.094	0.349	0.276	0.265	0.000	0.299	11.445

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	45	58	0	52	0	74	58
N.S.	1	1.00	1.10	1.41	0.00	1.27	0.00	1.80	1.41
time (sec)	N/A	0.186	0.118	0.357	0.000	0.246	0.000	0.296	11.467

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	62	57	73	0	76	0	0	81
N.S.	1	0.97	0.89	1.14	0.00	1.19	0.00	0.00	1.27
time (sec)	N/A	0.211	0.144	0.369	0.000	0.278	0.000	0.000	11.474

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	91	72	94	0	97	0	213	105
N.S.	1	1.01	0.80	1.04	0.00	1.08	0.00	2.37	1.17
time (sec)	N/A	0.311	0.173	0.407	0.000	0.255	0.000	0.291	11.438

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	281	155	141	299	138	299	344	0
N.S.	1	1.23	0.68	0.62	1.31	0.60	1.31	1.50	0.00
time (sec)	N/A	0.442	0.269	0.421	0.298	0.271	2.140	0.339	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	246	136	130	275	128	340	312	0
N.S.	1	1.23	0.68	0.65	1.38	0.64	1.70	1.56	0.00
time (sec)	N/A	0.400	0.400	0.402	0.299	0.364	2.025	0.340	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	209	133	119	251	116	255	280	0
N.S.	1	1.22	0.78	0.70	1.47	0.68	1.49	1.64	0.00
time (sec)	N/A	0.368	0.254	0.394	0.297	0.293	1.959	0.331	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	176	114	108	230	106	298	248	0
N.S.	1	1.24	0.80	0.76	1.62	0.75	2.10	1.75	0.00
time (sec)	N/A	0.331	0.332	0.384	0.289	0.288	1.894	0.330	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	141	111	97	167	94	216	216	0
N.S.	1	1.04	0.82	0.71	1.23	0.69	1.59	1.59	0.00
time (sec)	N/A	0.267	0.179	0.388	0.282	0.315	1.832	0.330	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	113	92	83	119	84	253	184	0
N.S.	1	1.05	0.85	0.77	1.10	0.78	2.34	1.70	0.00
time (sec)	N/A	0.212	0.281	0.399	0.285	0.285	1.656	0.319	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	97	125	544	116	95	270	0	0
N.S.	1	1.01	1.30	5.67	1.21	0.99	2.81	0.00	0.00
time (sec)	N/A	0.292	0.297	0.393	0.281	0.270	4.817	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	104	130	140	126	111	330	0	0
N.S.	1	0.99	1.24	1.33	1.20	1.06	3.14	0.00	0.00
time (sec)	N/A	0.290	0.282	0.428	0.273	0.289	3.483	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	106	124	116	123	119	347	0	0
N.S.	1	0.96	1.13	1.05	1.12	1.08	3.15	0.00	0.00
time (sec)	N/A	0.291	0.318	0.441	0.285	0.292	4.326	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	106	138	113	146	106	338	0	0
N.S.	1	1.04	1.35	1.11	1.43	1.04	3.31	0.00	0.00
time (sec)	N/A	0.298	0.289	0.465	0.284	0.286	3.925	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	111	114	96	130	86	422	246	0
N.S.	1	1.03	1.06	0.89	1.20	0.80	3.91	2.28	0.00
time (sec)	N/A	0.256	0.293	0.455	0.287	0.264	5.176	0.323	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	146	131	110	155	97	660	275	0
N.S.	1	1.04	0.94	0.79	1.11	0.69	4.71	1.96	0.00
time (sec)	N/A	0.296	0.268	0.542	0.276	0.256	4.879	0.329	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	181	123	121	180	108	808	304	0
N.S.	1	1.07	0.73	0.72	1.07	0.64	4.78	1.80	0.00
time (sec)	N/A	0.327	0.416	0.590	0.302	0.274	9.278	0.332	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	214	145	132	205	119	835	333	0
N.S.	1	1.08	0.73	0.67	1.04	0.60	4.22	1.68	0.00
time (sec)	N/A	0.381	0.371	0.707	0.296	0.333	8.258	0.346	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	143	104	363	170	171	0	0	0
N.S.	1	1.16	0.85	2.95	1.38	1.39	0.00	0.00	0.00
time (sec)	N/A	0.454	0.417	0.405	0.297	0.294	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	101	70	65	157	116	0	191	66
N.S.	1	1.02	0.71	0.66	1.59	1.17	0.00	1.93	0.67
time (sec)	N/A	0.350	0.308	0.388	0.235	0.284	0.000	0.328	11.482

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	94	70	65	136	118	0	191	66
N.S.	1	1.06	0.79	0.73	1.53	1.33	0.00	2.15	0.74
time (sec)	N/A	0.233	0.325	0.382	0.220	0.271	0.000	0.338	11.352

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	96	69	64	138	116	0	177	65
N.S.	1	1.05	0.76	0.70	1.52	1.27	0.00	1.95	0.71
time (sec)	N/A	0.203	0.327	0.378	0.211	0.272	0.000	0.305	11.808

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	99	70	66	136	115	0	193	66
N.S.	1	1.09	0.77	0.73	1.49	1.26	0.00	2.12	0.73
time (sec)	N/A	0.202	0.323	0.384	0.219	0.273	0.000	0.319	11.850

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	131	107	330	0	168	0	284	0
N.S.	1	1.11	0.91	2.80	0.00	1.42	0.00	2.41	0.00
time (sec)	N/A	0.304	0.470	0.412	0.000	0.259	0.000	0.322	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	166	135	244	0	194	0	324	0
N.S.	1	1.14	0.92	1.67	0.00	1.33	0.00	2.22	0.00
time (sec)	N/A	0.534	0.436	0.429	0.000	0.276	0.000	0.322	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	195	136	251	0	215	0	360	0
N.S.	1	1.07	0.74	1.37	0.00	1.17	0.00	1.97	0.00
time (sec)	N/A	0.704	0.486	0.449	0.000	0.274	0.000	0.348	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	206	112	199	185	190	0	233	0
N.S.	1	1.16	0.63	1.12	1.05	1.07	0.00	1.32	0.00
time (sec)	N/A	0.778	0.469	0.467	0.302	0.289	0.000	0.308	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	166	99	187	160	174	0	212	0
N.S.	1	1.14	0.68	1.28	1.10	1.19	0.00	1.45	0.00
time (sec)	N/A	0.593	0.392	0.459	0.294	0.300	0.000	0.308	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	138	87	319	136	157	0	185	0
N.S.	1	1.15	0.72	2.66	1.13	1.31	0.00	1.54	0.00
time (sec)	N/A	0.428	0.338	0.427	0.296	0.278	0.000	0.315	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	135	52	49	125	104	0	106	48
N.S.	1	1.42	0.55	0.52	1.32	1.09	0.00	1.12	0.51
time (sec)	N/A	0.259	0.288	0.398	0.292	0.256	0.000	0.325	11.686

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	105	49	46	129	100	0	137	45
N.S.	1	1.08	0.51	0.47	1.33	1.03	0.00	1.41	0.46
time (sec)	N/A	0.210	0.277	0.399	0.296	0.271	0.000	0.321	11.833

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	108	52	49	128	104	0	165	48
N.S.	1	1.08	0.52	0.49	1.28	1.04	0.00	1.65	0.48
time (sec)	N/A	0.206	0.007	0.409	0.287	0.277	0.000	0.310	11.829

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	125	99	332	0	153	0	212	0
N.S.	1	1.09	0.86	2.89	0.00	1.33	0.00	1.84	0.00
time (sec)	N/A	0.309	0.395	0.411	0.000	0.286	0.000	0.329	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	158	115	199	0	181	0	304	0
N.S.	1	1.08	0.79	1.36	0.00	1.24	0.00	2.08	0.00
time (sec)	N/A	0.541	0.458	0.443	0.000	0.260	0.000	0.311	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	202	130	205	0	202	0	382	0
N.S.	1	1.10	0.71	1.12	0.00	1.10	0.00	2.09	0.00
time (sec)	N/A	0.704	0.446	0.448	0.000	0.274	0.000	0.332	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	236	130	209	0	200	0	244	0
N.S.	1	1.16	0.64	1.02	0.00	0.98	0.00	1.20	0.00
time (sec)	N/A	0.955	0.411	0.445	0.000	0.284	0.000	0.327	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	197	112	199	0	190	0	233	0
N.S.	1	1.23	0.70	1.24	0.00	1.19	0.00	1.46	0.00
time (sec)	N/A	0.647	0.418	0.409	0.000	0.276	0.000	0.307	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	154	106	186	0	174	0	211	0
N.S.	1	1.04	0.72	1.26	0.00	1.18	0.00	1.43	0.00
time (sec)	N/A	0.428	0.365	0.430	0.000	0.290	0.000	0.311	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	87	268	0	157	0	186	0
N.S.	1	1.00	0.76	2.33	0.00	1.37	0.00	1.62	0.00
time (sec)	N/A	0.276	0.335	0.413	0.000	0.288	0.000	0.308	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	52	42	125	102	0	137	46
N.S.	1	1.00	0.81	0.66	1.95	1.59	0.00	2.14	0.72
time (sec)	N/A	0.182	0.248	0.409	0.189	0.273	0.000	0.299	11.793

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	51	43	123	104	0	165	47
N.S.	1	1.00	0.76	0.64	1.84	1.55	0.00	2.46	0.70
time (sec)	N/A	0.175	0.317	0.402	0.198	0.269	0.000	0.324	11.676

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	114	88	415	0	153	0	212	0
N.S.	1	1.04	0.80	3.77	0.00	1.39	0.00	1.93	0.00
time (sec)	N/A	0.381	0.477	0.445	0.000	0.266	0.000	0.301	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	155	115	199	0	181	0	304	0
N.S.	1	1.08	0.80	1.39	0.00	1.27	0.00	2.13	0.00
time (sec)	N/A	0.522	0.475	0.461	0.000	0.264	0.000	0.309	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	206	116	205	0	202	0	382	0
N.S.	1	1.13	0.63	1.12	0.00	1.10	0.00	2.09	0.00
time (sec)	N/A	0.712	0.500	0.447	0.000	0.309	0.000	0.311	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	235	141	216	0	213	0	444	0
N.S.	1	1.12	0.67	1.03	0.00	1.01	0.00	2.11	0.00
time (sec)	N/A	0.859	0.551	0.483	0.000	0.305	0.000	0.305	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	298	151	164	478	156	0	151	0
N.S.	1	1.18	0.60	0.65	1.90	0.62	0.00	0.60	0.00
time (sec)	N/A	1.057	0.352	0.464	0.289	0.279	0.000	0.312	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	266	132	153	456	146	0	140	0
N.S.	1	1.19	0.59	0.68	2.04	0.65	0.00	0.62	0.00
time (sec)	N/A	0.867	0.449	0.455	0.289	0.279	0.000	0.307	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	218	129	142	407	134	0	126	0
N.S.	1	1.14	0.67	0.74	2.12	0.70	0.00	0.66	0.00
time (sec)	N/A	0.683	0.319	0.462	0.289	0.263	0.000	0.328	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	192	110	131	363	124	0	117	0
N.S.	1	1.05	0.60	0.72	1.99	0.68	0.00	0.64	0.00
time (sec)	N/A	0.521	0.375	0.447	0.278	0.275	0.000	0.328	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	147	107	119	235	111	0	103	0
N.S.	1	1.13	0.82	0.92	1.81	0.85	0.00	0.79	0.00
time (sec)	N/A	0.382	0.279	0.465	0.277	0.288	0.000	0.300	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	106	89	106	134	99	0	86	0
N.S.	1	0.94	0.79	0.94	1.19	0.88	0.00	0.76	0.00
time (sec)	N/A	0.288	0.326	0.435	0.269	0.263	0.000	0.305	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	91	115	1192	0	111	0	116	0
N.S.	1	1.02	1.29	13.39	0.00	1.25	0.00	1.30	0.00
time (sec)	N/A	0.351	0.381	0.477	0.000	0.271	0.000	0.304	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	95	126	133	0	127	0	189	0
N.S.	1	1.01	1.34	1.41	0.00	1.35	0.00	2.01	0.00
time (sec)	N/A	0.359	0.353	0.529	0.000	0.285	0.000	0.305	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	112	107	115	0	112	0	252	0
N.S.	1	1.02	0.97	1.05	0.00	1.02	0.00	2.29	0.00
time (sec)	N/A	0.344	0.335	0.595	0.000	0.277	0.000	0.319	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	144	117	131	0	123	0	318	0
N.S.	1	1.05	0.85	0.96	0.00	0.90	0.00	2.32	0.00
time (sec)	N/A	0.483	0.394	0.703	0.000	0.297	0.000	0.313	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	187	116	144	0	136	0	392	0
N.S.	1	1.10	0.68	0.85	0.00	0.80	0.00	2.31	0.00
time (sec)	N/A	0.647	0.446	0.742	0.000	0.266	0.000	0.330	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	216	141	155	0	147	0	448	0
N.S.	1	1.10	0.72	0.79	0.00	0.75	0.00	2.29	0.00
time (sec)	N/A	0.859	0.449	0.867	0.000	0.275	0.000	0.312	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	70	245	0	126	0	0	220
N.S.	1	1.00	0.74	2.58	0.00	1.33	0.00	0.00	2.32
time (sec)	N/A	0.272	0.256	0.378	0.000	0.278	0.000	0.000	11.455

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	125	50	44	153	102	0	0	287
N.S.	1	1.42	0.57	0.50	1.74	1.16	0.00	0.00	3.26
time (sec)	N/A	0.253	0.273	0.400	0.195	0.260	0.000	0.000	0.059

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	316	137	132	399	316	0	0	252
N.S.	1	1.51	0.66	0.63	1.91	1.51	0.00	0.00	1.21
time (sec)	N/A	0.520	0.631	0.411	0.201	0.561	0.000	0.000	12.079

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	278	137	132	401	317	0	0	252
N.S.	1	1.33	0.66	0.63	1.92	1.52	0.00	0.00	1.21
time (sec)	N/A	0.369	0.675	0.425	0.199	0.614	0.000	0.000	11.993

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	242	137	132	405	316	0	0	252
N.S.	1	1.15	0.65	0.63	1.92	1.50	0.00	0.00	1.19
time (sec)	N/A	0.317	0.593	0.403	0.216	0.677	0.000	0.000	11.916

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	245	137	132	393	314	0	0	242
N.S.	1	1.20	0.67	0.64	1.92	1.53	0.00	0.00	1.18
time (sec)	N/A	0.316	0.019	0.418	0.199	0.733	0.000	0.000	11.903

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	267	173	1328	0	432	0	0	0
N.S.	1	1.14	0.74	5.68	0.00	1.85	0.00	0.00	0.00
time (sec)	N/A	0.586	0.968	0.438	0.000	0.758	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	300	205	520	0	458	0	0	0
N.S.	1	1.11	0.76	1.92	0.00	1.69	0.00	0.00	0.00
time (sec)	N/A	1.152	0.921	0.434	0.000	1.116	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	107	81	95	0	217	0	0	0
N.S.	1	1.05	0.79	0.93	0.00	2.13	0.00	0.00	0.00
time (sec)	N/A	0.273	0.336	0.362	0.000	0.295	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	47	58	0	110	0	57	0
N.S.	1	1.00	1.21	1.49	0.00	2.82	0.00	1.46	0.00
time (sec)	N/A	0.177	0.238	0.363	0.000	0.281	0.000	0.283	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	50	57	48	92	82	65	38
N.S.	1	1.00	1.43	1.63	1.37	2.63	2.34	1.86	1.09
time (sec)	N/A	0.155	0.116	0.352	0.272	0.266	0.861	5.535	11.910

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	76	0	199	0	95	0
N.S.	1	1.00	1.00	2.17	0.00	5.69	0.00	2.71	0.00
time (sec)	N/A	0.167	1.518	0.389	0.000	0.310	0.000	5.441	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	48	41	68	90	29	65	36
N.S.	1	1.00	1.41	1.21	2.00	2.65	0.85	1.91	1.06
time (sec)	N/A	0.150	0.118	0.346	0.274	0.283	0.820	6.284	12.049

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	86	0	208	0	89	0
N.S.	1	1.00	1.00	2.53	0.00	6.12	0.00	2.62	0.00
time (sec)	N/A	0.165	1.500	0.361	0.000	0.291	0.000	5.451	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	65	66	82	111	148	141	54
N.S.	1	1.00	1.03	1.05	1.30	1.76	2.35	2.24	0.86
time (sec)	N/A	0.167	0.159	0.351	0.274	0.268	1.730	10.789	11.608

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	49	92	0	221	0	0	0
N.S.	1	1.00	0.78	1.46	0.00	3.51	0.00	0.00	0.00
time (sec)	N/A	0.183	1.401	0.364	0.000	0.331	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	250	275	199	0	0	0	502	0	0
N.S.	1	1.10	0.80	0.00	0.00	0.00	2.01	0.00	0.00
time (sec)	N/A	0.543	0.877	0.000	0.000	0.000	15.984	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	206	212	174	0	0	0	432	0	0
N.S.	1	1.03	0.84	0.00	0.00	0.00	2.10	0.00	0.00
time (sec)	N/A	0.338	0.741	0.000	0.000	0.000	11.942	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	162	162	121	0	0	0	366	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	2.26	0.00	0.00
time (sec)	N/A	0.258	0.629	0.000	0.000	0.000	8.752	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	78	0	0	0	61	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.76	0.00	0.00
time (sec)	N/A	0.192	0.338	0.000	0.000	0.000	4.297	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	122	0	0	0	243	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	1.49	0.00	0.00
time (sec)	N/A	0.277	0.654	0.000	0.000	0.000	10.034	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	204	207	173	0	0	0	182	0	0
N.S.	1	1.01	0.85	0.00	0.00	0.00	0.89	0.00	0.00
time (sec)	N/A	0.340	0.642	0.000	0.000	0.000	38.527	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	250	262	245	0	0	0	0	0	0
N.S.	1	1.05	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.548	0.741	0.000	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	213	212	199	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.327	1.097	0.000	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	216	219	174	0	0	0	0	0	0
N.S.	1	1.01	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.333	0.876	0.000	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	162	121	0	0	0	116	0	0
N.S.	1	1.31	0.98	0.00	0.00	0.00	0.94	0.00	0.00
time (sec)	N/A	0.260	0.783	0.000	0.000	0.000	26.222	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	78	0	0	0	60	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.75	0.00	0.00
time (sec)	N/A	0.185	0.449	0.000	0.000	0.000	4.554	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	122	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.280	1.112	0.000	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	220	176	0	0	0	0	0	0
N.S.	1	1.01	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.342	1.323	0.000	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	214	214	200	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.353	1.590	0.000	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	150	132	0	0	0	972	0	0
N.S.	1	1.01	0.89	0.00	0.00	0.00	6.57	0.00	0.00
time (sec)	N/A	0.288	0.221	0.000	0.000	0.000	1.955	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	150	129	0	0	0	972	0	0
N.S.	1	1.02	0.88	0.00	0.00	0.00	6.61	0.00	0.00
time (sec)	N/A	0.287	0.209	0.000	0.000	0.000	1.868	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	106	0	0	0	382	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	3.18	0.00	0.00
time (sec)	N/A	0.262	0.183	0.000	0.000	0.000	1.590	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	120	103	0	0	0	382	0	0
N.S.	1	1.01	0.87	0.00	0.00	0.00	3.21	0.00	0.00
time (sec)	N/A	0.258	0.180	0.000	0.000	0.000	1.444	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	85	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.96	0.00	0.00
time (sec)	N/A	0.200	0.142	0.000	0.000	0.000	1.190	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	83	0	0	0	82	0	78
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.99	0.00	0.94
time (sec)	N/A	0.189	0.155	0.000	0.000	0.000	1.132	0.000	13.295

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	104	104	0	0	0	78	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.75	0.00	0.00
time (sec)	N/A	0.219	0.160	0.000	0.000	0.000	3.088	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	108	108	0	0	0	82	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.76	0.00	0.00
time (sec)	N/A	0.231	0.176	0.000	0.000	0.000	1.787	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	110	106	0	0	0	83	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.75	0.00	0.00
time (sec)	N/A	0.228	0.185	0.000	0.000	0.000	1.871	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	178	178	159	0	0	0	2924	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	16.43	0.00	0.00
time (sec)	N/A	0.323	0.272	0.000	0.000	0.000	2.899	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	185	188	186	0	0	0	1015	0	0
N.S.	1	1.02	1.01	0.00	0.00	0.00	5.49	0.00	0.00
time (sec)	N/A	0.333	0.289	0.000	0.000	0.000	2.390	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	150	138	0	0	0	1328	0	0
N.S.	1	1.01	0.93	0.00	0.00	0.00	8.91	0.00	0.00
time (sec)	N/A	0.301	0.244	0.000	0.000	0.000	2.132	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	158	168	0	0	0	425	0	0
N.S.	1	1.02	1.08	0.00	0.00	0.00	2.74	0.00	0.00
time (sec)	N/A	0.308	0.247	0.000	0.000	0.000	1.949	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	114	110	0	0	0	440	0	0
N.S.	1	0.97	0.93	0.00	0.00	0.00	3.73	0.00	0.00
time (sec)	N/A	0.230	0.179	0.000	0.000	0.000	1.577	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	134	0	0	0	124	0	0
N.S.	1	1.00	1.89	0.00	0.00	0.00	1.75	0.00	0.00
time (sec)	N/A	0.188	0.206	0.000	0.000	0.000	1.446	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	129	103	0	0	0	136	0	0
N.S.	1	1.01	0.80	0.00	0.00	0.00	1.06	0.00	0.00
time (sec)	N/A	0.272	0.196	0.000	0.000	0.000	3.044	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	128	153	0	0	0	116	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.91	0.00	0.00
time (sec)	N/A	0.250	0.239	0.000	0.000	0.000	2.303	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	140	131	0	0	0	138	0	0
N.S.	1	1.01	0.94	0.00	0.00	0.00	0.99	0.00	0.00
time (sec)	N/A	0.259	0.224	0.000	0.000	0.000	2.583	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	222	223	205	0	0	0	2966	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	13.36	0.00	0.00
time (sec)	N/A	0.386	0.421	0.000	0.000	0.000	3.684	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	218	219	219	0	0	0	2966	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	13.61	0.00	0.00
time (sec)	N/A	0.378	0.411	0.000	0.000	0.000	3.361	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	193	194	187	0	0	0	1370	0	0
N.S.	1	1.01	0.97	0.00	0.00	0.00	7.10	0.00	0.00
time (sec)	N/A	0.367	0.336	0.000	0.000	0.000	2.788	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	189	191	187	0	0	0	1370	0	0
N.S.	1	1.01	0.99	0.00	0.00	0.00	7.25	0.00	0.00
time (sec)	N/A	0.360	0.320	0.000	0.000	0.000	2.555	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	116	159	0	0	0	479	0	0
N.S.	1	1.00	1.37	0.00	0.00	0.00	4.13	0.00	0.00
time (sec)	N/A	0.235	0.345	0.000	0.000	0.000	2.085	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	155	0	0	0	476	0	0
N.S.	1	1.00	2.12	0.00	0.00	0.00	6.52	0.00	0.00
time (sec)	N/A	0.185	0.288	0.000	0.000	0.000	1.847	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	171	169	0	0	0	178	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	1.04	0.00	0.00
time (sec)	N/A	0.294	0.276	0.000	0.000	0.000	3.810	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	159	159	158	0	0	0	177	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	1.11	0.00	0.00
time (sec)	N/A	0.279	0.267	0.000	0.000	0.000	2.489	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	167	182	0	0	0	175	0	0
N.S.	1	1.01	1.10	0.00	0.00	0.00	1.05	0.00	0.00
time (sec)	N/A	0.291	0.280	0.000	0.000	0.000	3.167	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	148	150	66	0	0	0	0	0	0
N.S.	1	1.01	0.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.304	0.332	0.000	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	245	0	0	0	17065	0	0
N.S.	1	1.00	2.02	0.00	0.00	0.00	141.03	0.00	0.00
time (sec)	N/A	0.285	0.405	0.000	0.000	0.000	62.784	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	120	198	0	0	0	14895	0	0
N.S.	1	1.01	1.66	0.00	0.00	0.00	125.17	0.00	0.00
time (sec)	N/A	0.276	0.337	0.000	0.000	0.000	12.294	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	147	0	0	0	440	0	0
N.S.	1	1.00	1.63	0.00	0.00	0.00	4.89	0.00	0.00
time (sec)	N/A	0.204	0.235	0.000	0.000	0.000	4.637	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	318	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	4.36	0.00	0.00
time (sec)	N/A	0.190	0.183	0.000	0.000	0.000	3.232	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	104	151	0	0	0	348	0	0
N.S.	1	1.00	1.45	0.00	0.00	0.00	3.35	0.00	0.00
time (sec)	N/A	0.238	0.250	0.000	0.000	0.000	3.395	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	167	0	0	0	434	0	0
N.S.	1	1.00	1.58	0.00	0.00	0.00	4.09	0.00	0.00
time (sec)	N/A	0.240	0.313	0.000	0.000	0.000	4.907	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	108	219	0	0	0	484	0	0
N.S.	1	1.00	2.03	0.00	0.00	0.00	4.48	0.00	0.00
time (sec)	N/A	0.241	0.586	0.000	0.000	0.000	13.398	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	179	180	66	0	0	0	0	0	0
N.S.	1	1.01	0.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.350	0.339	0.000	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	184	188	66	0	0	0	0	0	0
N.S.	1	1.02	0.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.379	0.343	0.000	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	152	332	0	0	0	0	0	0
N.S.	1	1.01	2.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.328	0.387	0.000	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	158	177	0	0	0	0	0	0
N.S.	1	1.01	1.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.352	0.305	0.000	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	118	102	0	0	0	0	0	0
N.S.	1	1.03	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.228	0.214	0.000	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.189	0.215	0.000	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	132	201	0	0	0	0	0	0
N.S.	1	1.03	1.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.266	0.282	0.000	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	137	223	0	0	0	0	0	0
N.S.	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.279	0.552	0.000	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	283	0	0	0	0	0	0
N.S.	1	1.00	1.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.300	0.730	0.000	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	145	334	0	0	0	0	0	0
N.S.	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.315	0.813	0.000	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	150	389	0	0	0	0	0	0
N.S.	1	1.03	2.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.293	0.903	0.000	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	220	221	245	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.436	0.450	0.000	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	195	202	0	0	0	0	0	0
N.S.	1	1.01	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.392	0.379	0.000	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	157	165	130	0	0	0	0	0	0
N.S.	1	1.05	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.310	0.284	0.000	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	102	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.228	0.243	0.000	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.192	0.237	0.000	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	175	177	328	0	0	0	0	0	0
N.S.	1	1.01	1.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.353	0.369	0.000	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	168	280	0	0	0	0	0	0
N.S.	1	1.01	1.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.338	0.507	0.000	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	173	173	341	0	0	0	0	0	0
N.S.	1	1.00	1.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.326	0.668	0.000	0.000	0.000	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	179	393	0	0	0	0	0	0
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.351	0.625	0.000	0.000	0.000	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	174	179	446	0	0	0	0	0	0
N.S.	1	1.03	2.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.345	0.714	0.000	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	265	273	231	0	0	0	0	0	0
N.S.	1	1.03	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.565	0.528	0.000	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	211	221	156	0	0	0	0	0	0
N.S.	1	1.05	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.493	0.393	0.000	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	173	130	0	0	0	0	0	0
N.S.	1	1.06	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.322	0.362	0.000	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	102	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.234	0.278	0.000	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.199	0.279	0.000	0.000	0.000	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	204	207	417	0	0	0	0	0	0
N.S.	1	1.01	2.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.450	0.424	0.000	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	207	207	337	0	0	0	0	0	0
N.S.	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.417	0.585	0.000	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	211	207	399	0	0	0	0	0	0
N.S.	1	0.98	1.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.420	0.789	0.000	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	210	213	452	0	0	0	0	0	0
N.S.	1	1.01	2.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.423	0.778	0.000	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	216	221	505	0	0	0	0	0	0
N.S.	1	1.02	2.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.438	0.898	0.000	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	264	292	194	0	0	0	257	0	0
N.S.	1	1.11	0.73	0.00	0.00	0.00	0.97	0.00	0.00
time (sec)	N/A	0.564	0.208	0.000	0.000	0.000	9.153	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	206	211	169	0	0	0	189	0	0
N.S.	1	1.02	0.82	0.00	0.00	0.00	0.92	0.00	0.00
time (sec)	N/A	0.327	0.146	0.000	0.000	0.000	5.872	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	153	153	116	0	0	0	121	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.79	0.00	0.00
time (sec)	N/A	0.249	0.104	0.000	0.000	0.000	3.591	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	73	0	0	0	61	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.81	0.00	0.00
time (sec)	N/A	0.187	0.069	0.000	0.000	0.000	1.292	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	124	0	0	0	372	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	2.28	0.00	0.00
time (sec)	N/A	0.282	0.128	0.000	0.000	0.000	5.226	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	214	224	180	0	0	0	0	0	0
N.S.	1	1.05	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.351	0.162	0.000	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	275	314	206	0	0	0	0	0	0
N.S.	1	1.14	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.627	0.234	0.000	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	77	0	0	0	328	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	3.69	0.00	0.00
time (sec)	N/A	0.223	0.119	0.000	0.000	0.000	4.563	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	90	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.250	0.238	0.000	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	215	68	167	0	72	0	167	201
N.S.	1	1.00	0.32	0.78	0.00	0.34	0.00	0.78	0.94
time (sec)	N/A	0.492	0.681	0.675	0.000	0.267	0.000	0.637	0.119

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	279	230	312	0	1104	0	0	0
N.S.	1	1.09	0.90	1.22	0.00	4.33	0.00	0.00	0.00
time (sec)	N/A	0.859	0.658	0.443	0.000	4.061	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	225	199	266	0	963	0	0	0
N.S.	1	1.07	0.94	1.26	0.00	4.56	0.00	0.00	0.00
time (sec)	N/A	0.552	0.443	0.437	0.000	4.133	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	162	161	227	0	776	0	0	0
N.S.	1	1.06	1.05	1.48	0.00	5.07	0.00	0.00	0.00
time (sec)	N/A	0.346	0.366	0.421	0.000	0.632	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	134	133	203	0	684	0	0	0
N.S.	1	1.06	1.05	1.60	0.00	5.39	0.00	0.00	0.00
time (sec)	N/A	0.298	0.297	0.421	0.000	0.607	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	108	110	184	0	574	0	0	0
N.S.	1	1.05	1.07	1.79	0.00	5.57	0.00	0.00	0.00
time (sec)	N/A	0.261	0.015	0.422	0.000	0.464	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	122	137	305	122	1316	0	0	0
N.S.	1	1.05	1.18	2.63	1.05	11.34	0.00	0.00	0.00
time (sec)	N/A	0.326	0.219	0.375	0.208	1.014	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	129	195	0	599	0	146	0
N.S.	1	1.00	1.23	1.86	0.00	5.70	0.00	1.39	0.00
time (sec)	N/A	0.339	0.364	0.385	0.000	0.484	0.000	0.285	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	144	212	0	726	0	230	0
N.S.	1	1.00	0.90	1.32	0.00	4.54	0.00	1.44	0.00
time (sec)	N/A	0.383	0.445	0.443	0.000	0.461	0.000	0.302	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	173	241	0	824	0	313	0
N.S.	1	1.00	0.91	1.26	0.00	4.31	0.00	1.64	0.00
time (sec)	N/A	0.412	0.606	0.454	0.000	0.451	0.000	0.292	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	215	282	0	1007	0	605	0
N.S.	1	1.00	0.78	1.03	0.00	3.68	0.00	2.21	0.00
time (sec)	N/A	0.525	0.757	0.450	0.000	0.507	0.000	0.301	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	212	161	222	0	1060	0	0	0
N.S.	1	1.09	0.83	1.14	0.00	5.44	0.00	0.00	0.00
time (sec)	N/A	0.728	0.512	0.443	0.000	2.393	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	159	137	201	0	924	0	0	0
N.S.	1	1.05	0.90	1.32	0.00	6.08	0.00	0.00	0.00
time (sec)	N/A	0.448	0.382	0.425	0.000	2.406	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	111	128	172	0	745	0	0	0
N.S.	1	1.02	1.17	1.58	0.00	6.83	0.00	0.00	0.00
time (sec)	N/A	0.264	0.445	0.419	0.000	0.471	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	98	151	0	631	0	0	0
N.S.	1	1.00	1.14	1.76	0.00	7.34	0.00	0.00	0.00
time (sec)	N/A	0.226	0.240	0.411	0.000	0.453	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	64	127	60	211	0	60	0
N.S.	1	1.00	1.19	2.35	1.11	3.91	0.00	1.11	0.00
time (sec)	N/A	0.177	0.004	0.382	0.199	0.341	0.000	0.282	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	104	158	0	634	0	0	0
N.S.	1	1.00	1.21	1.84	0.00	7.37	0.00	0.00	0.00
time (sec)	N/A	0.269	0.241	0.398	0.000	0.407	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	128	180	0	767	0	143	0
N.S.	1	1.00	1.15	1.62	0.00	6.91	0.00	1.29	0.00
time (sec)	N/A	0.283	0.329	0.425	0.000	0.438	0.000	0.282	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	198	209	0	956	0	240	0
N.S.	1	1.00	1.18	1.24	0.00	5.69	0.00	1.43	0.00
time (sec)	N/A	0.335	0.480	0.435	0.000	0.571	0.000	0.287	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	157	178	351	0	1525	0	0	0
N.S.	1	1.08	1.22	2.40	0.00	10.45	0.00	0.00	0.00
time (sec)	N/A	0.460	0.806	0.471	0.000	3.477	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	129	137	397	0	1323	0	0	0
N.S.	1	1.05	1.11	3.23	0.00	10.76	0.00	0.00	0.00
time (sec)	N/A	0.319	0.528	0.425	0.000	2.923	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	109	355	179	455	0	181	0
N.S.	1	1.00	1.15	3.74	1.88	4.79	0.00	1.91	0.00
time (sec)	N/A	0.237	0.470	0.417	0.217	0.358	0.000	0.282	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	99	335	156	425	0	168	0
N.S.	1	1.00	1.12	3.81	1.77	4.83	0.00	1.91	0.00
time (sec)	N/A	0.225	0.376	0.404	0.215	0.332	0.000	0.301	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	104	315	131	456	0	180	0
N.S.	1	1.00	1.11	3.35	1.39	4.85	0.00	1.91	0.00
time (sec)	N/A	0.230	0.045	0.348	0.204	0.321	0.000	0.281	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	144	363	0	1325	0	0	0
N.S.	1	1.00	0.98	2.47	0.00	9.01	0.00	0.00	0.00
time (sec)	N/A	0.341	0.703	0.368	0.000	0.471	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	178	361	0	1556	0	273	0
N.S.	1	1.00	0.92	1.86	0.00	8.02	0.00	1.41	0.00
time (sec)	N/A	0.365	0.688	0.388	0.000	0.501	0.000	0.280	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	217	407	0	1943	0	365	0
N.S.	1	1.00	0.79	1.47	0.00	7.04	0.00	1.32	0.00
time (sec)	N/A	0.453	0.765	0.453	0.000	0.696	0.000	0.303	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	316	251	440	0	2025	0	0	0
N.S.	1	1.30	1.03	1.80	0.00	8.30	0.00	0.00	0.00
time (sec)	N/A	1.190	1.136	0.481	0.000	25.370	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	256	223	420	0	1786	0	0	0
N.S.	1	1.25	1.09	2.06	0.00	8.75	0.00	0.00	0.00
time (sec)	N/A	0.795	1.461	0.452	0.000	46.200	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	196	173	386	0	1449	0	0	0
N.S.	1	1.22	1.08	2.41	0.00	9.06	0.00	0.00	0.00
time (sec)	N/A	0.509	0.796	0.443	0.000	5.640	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	169	158	369	0	1260	0	0	0
N.S.	1	1.23	1.15	2.69	0.00	9.20	0.00	0.00	0.00
time (sec)	N/A	0.326	0.851	0.424	0.000	5.491	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	100	344	0	382	0	0	0
N.S.	1	1.00	1.11	3.82	0.00	4.24	0.00	0.00	0.00
time (sec)	N/A	0.217	0.404	0.415	0.000	0.450	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	101	215	0	381	0	0	0
N.S.	1	1.00	1.11	2.36	0.00	4.19	0.00	0.00	0.00
time (sec)	N/A	0.214	0.037	0.398	0.000	0.453	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	153	376	0	1261	0	0	0
N.S.	1	1.00	0.85	2.10	0.00	7.04	0.00	0.00	0.00
time (sec)	N/A	0.356	0.707	0.379	0.000	0.817	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	183	395	0	1512	0	0	0
N.S.	1	1.00	0.86	1.86	0.00	7.13	0.00	0.00	0.00
time (sec)	N/A	0.381	0.913	0.392	0.000	0.671	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	224	424	0	1867	0	0	0
N.S.	1	1.00	0.84	1.58	0.00	6.97	0.00	0.00	0.00
time (sec)	N/A	0.453	1.126	0.419	0.000	1.020	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	114	320	210	368	4134	624	363
N.S.	1	1.00	0.84	2.37	1.56	2.73	30.62	4.62	2.69
time (sec)	N/A	0.285	0.107	0.368	0.200	0.301	1.258	0.289	11.633

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	109	195	146	250	2181	410	255
N.S.	1	1.00	1.07	1.91	1.43	2.45	21.38	4.02	2.50
time (sec)	N/A	0.256	0.109	0.402	0.199	0.286	0.783	0.283	11.563

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	65	100	89	148	952	237	163
N.S.	1	1.00	0.93	1.43	1.27	2.11	13.60	3.39	2.33
time (sec)	N/A	0.214	0.058	0.388	0.190	0.308	0.495	0.269	11.522

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	64	0	0	0	279	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	3.62	0.00	0.00
time (sec)	N/A	0.220	0.064	0.000	0.000	0.000	2.287	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	199	754	447	1027	14317	1750	932
N.S.	1	1.00	0.86	3.25	1.93	4.43	61.71	7.54	4.02
time (sec)	N/A	0.409	0.171	0.475	0.210	0.319	4.186	0.299	11.960

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	323	601	335	757	8940	1266	723
N.S.	1	1.00	1.75	3.25	1.81	4.09	48.32	6.84	3.91
time (sec)	N/A	0.349	0.344	0.431	0.212	0.299	2.449	0.300	11.735

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	160	420	235	519	5097	851	496
N.S.	1	1.00	1.14	3.00	1.68	3.71	36.41	6.08	3.54
time (sec)	N/A	0.287	0.162	0.420	0.192	0.306	1.436	0.277	11.625

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	132	0	0	0	1608	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	10.86	0.00	0.00
time (sec)	N/A	0.299	0.141	0.000	0.000	0.000	3.316	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	302	2232	795	2165	35984	3713	1796
N.S.	1	1.00	0.88	6.51	2.32	6.31	104.91	10.83	5.24
time (sec)	N/A	0.523	0.220	0.453	0.244	0.311	11.304	0.297	12.660

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	709	1214	625	1675	24687	2851	1459
N.S.	1	1.00	2.51	4.30	2.22	5.94	87.54	10.11	5.17
time (sec)	N/A	0.469	0.874	0.480	0.232	0.308	6.425	0.309	12.260

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	347	958	472	1244	15990	2085	1144
N.S.	1	1.00	1.56	4.30	2.12	5.58	71.70	9.35	5.13
time (sec)	N/A	0.383	0.387	0.443	0.212	0.270	4.078	0.279	12.107

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	246	246	226	0	0	0	5622	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	22.85	0.00	0.00
time (sec)	N/A	0.409	0.189	0.000	0.000	0.000	4.956	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	250	276	217	0	0	0	0	0	0
N.S.	1	1.10	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.757	0.401	0.000	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	209	232	168	0	0	0	0	0	0
N.S.	1	1.11	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.513	0.212	0.000	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	195	170	0	0	0	0	0	0
N.S.	1	1.01	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.339	0.146	0.000	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	151	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.280	0.083	0.000	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	145	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.275	0.087	0.000	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	207	207	189	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.353	0.152	0.000	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	207	207	167	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.366	0.246	0.000	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	332	342	413	0	0	0	0	0	0
N.S.	1	1.03	1.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.663	0.550	0.000	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	292	247	0	0	0	0	0	0
N.S.	1	0.98	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.501	0.589	0.000	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	306	295	403	0	0	0	0	0	0
N.S.	1	0.96	1.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.492	0.426	0.000	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	279	263	230	0	0	0	0	0	0
N.S.	1	0.94	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.416	0.271	0.000	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	304	299	253	0	0	0	0	0	0
N.S.	1	0.98	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.490	0.251	0.000	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	489	484	391	0	0	0	0	0	0
N.S.	1	0.99	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.751	0.594	0.000	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	513	513	437	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.815	0.537	0.000	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	399	378	275	0	0	0	128	0	0
N.S.	1	0.95	0.69	0.00	0.00	0.00	0.32	0.00	0.00
time (sec)	N/A	0.732	0.130	0.000	0.000	0.000	16.572	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	164	157	113	0	0	0	80	0	0
N.S.	1	0.96	0.69	0.00	0.00	0.00	0.49	0.00	0.00
time (sec)	N/A	0.281	0.075	0.000	0.000	0.000	6.299	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.346	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	295	295	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.546	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	122	112	0	0	0	950	0	0
N.S.	1	0.98	0.90	0.00	0.00	0.00	7.60	0.00	0.00
time (sec)	N/A	0.271	0.138	0.000	0.000	0.000	12.803	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	122	112	0	0	0	950	0	0
N.S.	1	0.98	0.90	0.00	0.00	0.00	7.60	0.00	0.00
time (sec)	N/A	0.273	0.122	0.000	0.000	0.000	8.511	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	99	87	0	0	0	364	0	0
N.S.	1	0.99	0.87	0.00	0.00	0.00	3.64	0.00	0.00
time (sec)	N/A	0.243	0.118	0.000	0.000	0.000	6.896	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	99	87	0	0	0	364	0	0
N.S.	1	0.99	0.87	0.00	0.00	0.00	3.64	0.00	0.00
time (sec)	N/A	0.248	0.106	0.000	0.000	0.000	4.564	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	71	0	0	0	65	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.87	0.00	0.00
time (sec)	N/A	0.190	0.073	0.000	0.000	0.000	3.694	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	98	0	0	0	61	0	65
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.87	0.00	0.93
time (sec)	N/A	0.180	0.108	0.000	0.000	0.000	2.451	0.000	12.309

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	0	0	65	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.74	0.00	0.00
time (sec)	N/A	0.204	0.097	0.000	0.000	0.000	3.697	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	0	68	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.75	0.00	0.00
time (sec)	N/A	0.213	0.105	0.000	0.000	0.000	3.952	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	89	0	0	0	70	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.76	0.00	0.00
time (sec)	N/A	0.216	0.101	0.000	0.000	0.000	5.504	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	188	182	205	0	0	0	2883	0	0
N.S.	1	0.97	1.09	0.00	0.00	0.00	15.34	0.00	0.00
time (sec)	N/A	0.347	0.266	0.000	0.000	0.000	17.848	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	177	167	156	0	0	0	986	0	0
N.S.	1	0.94	0.88	0.00	0.00	0.00	5.57	0.00	0.00
time (sec)	N/A	0.332	0.253	0.000	0.000	0.000	16.936	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	146	152	0	0	0	1294	0	0
N.S.	1	0.98	1.02	0.00	0.00	0.00	8.68	0.00	0.00
time (sec)	N/A	0.305	0.196	0.000	0.000	0.000	9.547	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	144	139	0	0	0	400	0	0
N.S.	1	0.95	0.91	0.00	0.00	0.00	2.63	0.00	0.00
time (sec)	N/A	0.309	0.216	0.000	0.000	0.000	8.843	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	113	112	184	0	0	0	408	0	0
N.S.	1	0.99	1.63	0.00	0.00	0.00	3.61	0.00	0.00
time (sec)	N/A	0.269	0.200	0.000	0.000	0.000	4.841	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	126	133	0	0	0	97	0	0
N.S.	1	0.95	1.00	0.00	0.00	0.00	0.73	0.00	0.00
time (sec)	N/A	0.260	0.174	0.000	0.000	0.000	4.566	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	101	0	0	0	109	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.92	0.00	0.00
time (sec)	N/A	0.244	0.167	0.000	0.000	0.000	4.186	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	127	134	0	0	0	95	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.75	0.00	0.00
time (sec)	N/A	0.255	0.192	0.000	0.000	0.000	4.985	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	128	119	0	0	0	117	0	0
N.S.	1	1.01	0.94	0.00	0.00	0.00	0.92	0.00	0.00
time (sec)	N/A	0.252	0.164	0.000	0.000	0.000	6.649	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	247	233	249	0	0	0	2919	0	0
N.S.	1	0.94	1.01	0.00	0.00	0.00	11.82	0.00	0.00
time (sec)	N/A	0.429	0.344	0.000	0.000	0.000	31.281	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	249	234	249	0	0	0	2919	0	0
N.S.	1	0.94	1.00	0.00	0.00	0.00	11.72	0.00	0.00
time (sec)	N/A	0.428	0.315	0.000	0.000	0.000	21.453	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	207	197	196	0	0	0	1329	0	0
N.S.	1	0.95	0.95	0.00	0.00	0.00	6.42	0.00	0.00
time (sec)	N/A	0.374	0.260	0.000	0.000	0.000	17.322	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	210	198	196	0	0	0	1329	0	0
N.S.	1	0.94	0.93	0.00	0.00	0.00	6.33	0.00	0.00
time (sec)	N/A	0.380	0.233	0.000	0.000	0.000	11.369	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	158	228	0	0	0	440	0	0
N.S.	1	0.95	1.37	0.00	0.00	0.00	2.63	0.00	0.00
time (sec)	N/A	0.339	0.264	0.000	0.000	0.000	9.271	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	185	223	0	0	0	437	0	0
N.S.	1	1.05	1.27	0.00	0.00	0.00	2.48	0.00	0.00
time (sec)	N/A	0.339	0.262	0.000	0.000	0.000	5.711	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	166	170	0	0	0	144	0	0
N.S.	1	0.97	0.99	0.00	0.00	0.00	0.84	0.00	0.00
time (sec)	N/A	0.302	0.239	0.000	0.000	0.000	6.361	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	159	159	154	0	0	0	143	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.90	0.00	0.00
time (sec)	N/A	0.286	0.236	0.000	0.000	0.000	5.470	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	168	169	174	0	0	0	148	0	0
N.S.	1	1.01	1.04	0.00	0.00	0.00	0.88	0.00	0.00
time (sec)	N/A	0.292	0.252	0.000	0.000	0.000	8.174	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	199	198	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.404	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	260	0	0	0	0	0	0
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.304	0.328	0.000	0.000	0.000	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	161	162	227	0	0	0	0	0	0
N.S.	1	1.01	1.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	0.267	0.000	0.000	0.000	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	173	131	172	0	0	0	0	0	0
N.S.	1	0.76	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.279	0.232	0.000	0.000	0.000	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	125	131	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.254	0.130	0.000	0.000	0.000	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	177	170	0	0	0	0	0	0
N.S.	1	1.01	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.302	0.265	0.000	0.000	0.000	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	178	178	214	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.322	0.401	0.000	0.000	0.000	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	213	218	256	0	0	0	0	0	0
N.S.	1	1.02	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.383	0.457	0.000	0.000	0.000	0.000	0.000	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	392	389	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.010	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	321	309	343	0	0	0	0	0	0
N.S.	1	0.96	1.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.645	0.588	0.000	0.000	0.000	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	281	267	300	0	0	0	0	0	0
N.S.	1	0.95	1.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.447	0.420	0.000	0.000	0.000	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	273	249	223	0	0	0	0	0	0
N.S.	1	0.91	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.405	0.241	0.000	0.000	0.000	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	244	191	141	0	0	0	0	0	0
N.S.	1	0.78	0.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.363	0.212	0.000	0.000	0.000	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	368	306	303	0	0	0	0	0	0
N.S.	1	0.83	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.574	0.412	0.000	0.000	0.000	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	421	296	342	0	0	0	0	0	0
N.S.	1	0.70	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.569	0.601	0.000	0.000	0.000	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	449	433	462	0	0	0	0	0	0
N.S.	1	0.96	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.095	0.895	0.000	0.000	0.000	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	416	387	436	0	0	0	0	0	0
N.S.	1	0.93	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.741	0.665	0.000	0.000	0.000	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	396	364	290	0	0	0	0	0	0
N.S.	1	0.92	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.587	0.407	0.000	0.000	0.000	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	336	326	229	0	0	0	0	0	0
N.S.	1	0.97	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.525	0.312	0.000	0.000	0.000	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	322	322	142	0	0	0	0	0	0
N.S.	1	1.00	0.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.562	0.269	0.000	0.000	0.000	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	700	464	434	0	0	0	0	0	0
N.S.	1	0.66	0.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.813	0.691	0.000	0.000	0.000	0.000	0.000	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	754	460	478	0	0	0	0	0	0
N.S.	1	0.61	0.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.814	0.773	0.000	0.000	0.000	0.000	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	276	291	182	0	0	0	230	0	0
N.S.	1	1.05	0.66	0.00	0.00	0.00	0.83	0.00	0.00
time (sec)	N/A	0.642	0.217	0.000	0.000	0.000	113.065	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	205	214	158	0	0	0	168	0	0
N.S.	1	1.04	0.77	0.00	0.00	0.00	0.82	0.00	0.00
time (sec)	N/A	0.345	0.106	0.000	0.000	0.000	70.148	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	106	0	0	0	107	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.79	0.00	0.00
time (sec)	N/A	0.238	0.059	0.000	0.000	0.000	37.017	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	64	0	0	0	54	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.82	0.00	0.00
time (sec)	N/A	0.179	0.026	0.000	0.000	0.000	9.519	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	157	164	0	0	0	0	0	0	0
N.S.	1	1.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.316	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	238	242	0	0	0	0	0	0	0
N.S.	1	1.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.448	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	321	322	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.524	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	369	304	955	0	678	0	305	0
N.S.	1	1.07	0.88	2.77	0.00	1.97	0.00	0.88	0.00
time (sec)	N/A	0.642	11.128	0.681	0.000	0.380	0.000	0.363	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	259	245	512	0	536	0	224	0
N.S.	1	1.03	0.98	2.04	0.00	2.14	0.00	0.89	0.00
time (sec)	N/A	0.458	10.564	0.655	0.000	0.327	0.000	0.370	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	173	181	291	0	418	0	164	0
N.S.	1	0.84	0.87	1.41	0.00	2.02	0.00	0.79	0.00
time (sec)	N/A	0.317	0.773	0.569	0.000	0.311	0.000	0.338	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	112	131	0	337	0	114	0
N.S.	1	1.00	0.85	1.00	0.00	2.57	0.00	0.87	0.00
time (sec)	N/A	0.253	0.061	0.545	0.000	0.311	0.000	0.342	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	162	476	308	0	947	0	0	0
N.S.	1	0.96	2.83	1.83	0.00	5.64	0.00	0.00	0.00
time (sec)	N/A	0.326	1.413	0.533	0.000	0.429	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	841	708	0	355	0	220	0
N.S.	1	1.00	6.14	5.17	0.00	2.59	0.00	1.61	0.00
time (sec)	N/A	0.310	9.249	0.720	0.000	0.359	0.000	0.327	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	211	162	1353	0	442	0	505	0
N.S.	1	1.04	0.80	6.70	0.00	2.19	0.00	2.50	0.00
time (sec)	N/A	0.448	10.119	0.759	0.000	0.630	0.000	0.314	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	307	210	2059	0	558	0	912	0
N.S.	1	1.07	0.73	7.20	0.00	1.95	0.00	3.19	0.00
time (sec)	N/A	0.586	10.171	1.155	0.000	1.230	0.000	0.333	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	421	273	3471	0	702	0	1483	0
N.S.	1	1.08	0.70	8.92	0.00	1.80	0.00	3.81	0.00
time (sec)	N/A	0.779	10.243	1.121	0.000	8.549	0.000	0.338	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	446	385	1426	0	1044	0	513	0
N.S.	1	0.99	0.86	3.18	0.00	2.33	0.00	1.14	0.00
time (sec)	N/A	0.702	1.239	0.636	0.000	0.343	0.000	0.367	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	347	303	797	0	846	0	405	0
N.S.	1	0.99	0.86	2.26	0.00	2.40	0.00	1.15	0.00
time (sec)	N/A	0.536	0.732	0.644	0.000	0.323	0.000	0.350	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	255	236	483	0	676	1093	310	0
N.S.	1	0.86	0.80	1.64	0.00	2.29	3.71	1.05	0.00
time (sec)	N/A	0.388	0.518	0.549	0.000	0.311	10.979	0.355	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	215	180	230	0	532	751	229	0
N.S.	1	1.07	0.90	1.14	0.00	2.65	3.74	1.14	0.00
time (sec)	N/A	0.328	0.071	0.590	0.000	0.329	3.611	0.347	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	257	232	607	0	1327	0	0	0
N.S.	1	1.02	0.92	2.42	0.00	5.29	0.00	0.00	0.00
time (sec)	N/A	0.499	0.532	0.638	0.000	2.598	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	213	1300	0	1221	0	339	0
N.S.	1	1.00	0.89	5.42	0.00	5.09	0.00	1.41	0.00
time (sec)	N/A	0.483	0.398	0.721	0.000	1.327	0.000	0.374	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	263	243	2438	0	1375	0	626	0
N.S.	1	1.03	0.95	9.52	0.00	5.37	0.00	2.45	0.00
time (sec)	N/A	0.522	0.504	0.767	0.000	1.435	0.000	0.442	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	225	201	4330	0	558	0	1005	0
N.S.	1	1.07	0.95	20.52	0.00	2.64	0.00	4.76	0.00
time (sec)	N/A	0.406	0.343	1.011	0.000	0.956	0.000	0.341	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	304	247	7421	0	704	0	1618	0
N.S.	1	1.03	0.84	25.16	0.00	2.39	0.00	5.48	0.00
time (sec)	N/A	0.531	0.529	1.062	0.000	4.941	0.000	0.366	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	402	317	10575	0	872	0	2352	0
N.S.	1	1.02	0.80	26.77	0.00	2.21	0.00	5.95	0.00
time (sec)	N/A	0.695	0.791	1.651	0.000	14.930	0.000	0.403	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	498	505	402	16883	0	1072	0	3251	0
N.S.	1	1.01	0.81	33.90	0.00	2.15	0.00	6.53	0.00
time (sec)	N/A	0.891	1.092	1.928	0.000	37.471	0.000	0.484	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	574	543	549	1895	0	1524	0	784	0
N.S.	1	0.95	0.96	3.30	0.00	2.66	0.00	1.37	0.00
time (sec)	N/A	0.835	1.396	0.658	0.000	0.510	0.000	0.376	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	452	434	479	1080	0	1272	0	648	0
N.S.	1	0.96	1.06	2.39	0.00	2.81	0.00	1.43	0.00
time (sec)	N/A	0.643	1.200	0.662	0.000	0.671	0.000	0.376	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	342	388	673	0	1046	0	526	0
N.S.	1	0.90	1.02	1.77	0.00	2.75	0.00	1.38	0.00
time (sec)	N/A	0.485	0.890	0.547	0.000	0.360	0.000	0.381	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	302	295	327	0	844	0	415	0
N.S.	1	1.10	1.08	1.19	0.00	3.08	0.00	1.51	0.00
time (sec)	N/A	0.419	0.204	0.561	0.000	0.322	0.000	0.377	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	414	342	997	0	1873	0	0	0
N.S.	1	1.05	0.87	2.53	0.00	4.75	0.00	0.00	0.00
time (sec)	N/A	0.745	0.948	0.595	0.000	30.623	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	361	309	2076	0	1717	0	473	0
N.S.	1	1.03	0.88	5.90	0.00	4.88	0.00	1.34	0.00
time (sec)	N/A	0.723	1.038	0.746	0.000	10.887	0.000	0.409	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	339	342	287	3893	0	1569	0	746	0
N.S.	1	1.01	0.85	11.48	0.00	4.63	0.00	2.20	0.00
time (sec)	N/A	0.667	0.864	0.812	0.000	4.423	0.000	0.458	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	375	316	6850	0	1741	0	1195	0
N.S.	1	1.01	0.85	18.46	0.00	4.69	0.00	3.22	0.00
time (sec)	N/A	0.745	1.024	1.060	0.000	5.776	0.000	0.498	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	426	358	11685	0	1917	0	1750	0
N.S.	1	1.05	0.89	28.92	0.00	4.75	0.00	4.33	0.00
time (sec)	N/A	0.776	1.041	1.197	0.000	16.145	0.000	0.680	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	317	271	19539	0	872	0	2449	0
N.S.	1	1.10	0.94	67.61	0.00	3.02	0.00	8.47	0.00
time (sec)	N/A	0.518	0.711	1.566	0.000	13.959	0.000	0.395	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	396	404	32291	0	1072	0	3388	0
N.S.	1	1.03	1.05	83.66	0.00	2.78	0.00	8.78	0.00
time (sec)	N/A	0.666	0.912	1.973	0.000	41.437	0.000	0.510	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	500	494	497	45106	0	1300	0	4452	0
N.S.	1	0.99	0.99	90.21	0.00	2.60	0.00	8.90	0.00
time (sec)	N/A	0.830	1.217	3.187	0.000	78.301	0.000	0.775	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	628	608	572	70736	0	1550	0	5681	0
N.S.	1	0.97	0.91	112.64	0.00	2.47	0.00	9.05	0.00
time (sec)	N/A	1.035	1.539	3.286	0.000	176.453	0.000	1.301	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	274	277	514	0	758	0	228	0
N.S.	1	1.01	1.02	1.90	0.00	2.80	0.00	0.84	0.00
time (sec)	N/A	0.545	0.438	0.686	0.000	0.665	0.000	0.369	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	202	201	259	0	586	0	182	0
N.S.	1	1.04	1.03	1.33	0.00	3.01	0.00	0.93	0.00
time (sec)	N/A	0.345	0.277	0.697	0.000	0.402	0.000	0.366	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	130	131	0	443	0	0	0
N.S.	1	1.00	0.94	0.94	0.00	3.19	0.00	0.00	0.00
time (sec)	N/A	0.271	0.190	0.556	0.000	0.392	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	42	50	0	59	0	0	50
N.S.	1	1.00	0.81	0.96	0.00	1.13	0.00	0.00	0.96
time (sec)	N/A	0.181	0.010	0.575	0.000	0.340	0.000	0.000	11.710

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	142	131	136	0	454	0	0	0
N.S.	1	0.99	0.92	0.95	0.00	3.17	0.00	0.00	0.00
time (sec)	N/A	0.302	0.174	0.586	0.000	0.511	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	206	201	270	0	610	0	0	0
N.S.	1	0.90	0.88	1.18	0.00	2.66	0.00	0.00	0.00
time (sec)	N/A	0.384	0.260	0.687	0.000	0.972	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	281	283	545	0	792	0	0	0
N.S.	1	0.85	0.86	1.66	0.00	2.41	0.00	0.00	0.00
time (sec)	N/A	0.635	0.431	0.687	0.000	2.251	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	515	511	388	1923	0	2120	0	0	0
N.S.	1	0.99	0.75	3.73	0.00	4.12	0.00	0.00	0.00
time (sec)	N/A	0.795	0.833	0.869	0.000	4.284	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	438	431	300	1112	0	1782	0	0	0
N.S.	1	0.98	0.68	2.54	0.00	4.07	0.00	0.00	0.00
time (sec)	N/A	0.676	0.531	0.862	0.000	1.669	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	309	218	639	0	1466	0	0	0
N.S.	1	1.04	0.73	2.15	0.00	4.94	0.00	0.00	0.00
time (sec)	N/A	0.477	0.319	0.694	0.000	2.061	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	95	145	0	308	0	0	1071
N.S.	1	1.00	0.75	1.15	0.00	2.44	0.00	0.00	8.50
time (sec)	N/A	0.296	0.136	0.658	0.000	1.676	0.000	0.000	12.581

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	100	149	0	314	0	0	499
N.S.	1	1.00	0.72	1.08	0.00	2.28	0.00	0.00	3.62
time (sec)	N/A	0.276	0.132	0.536	0.000	1.866	0.000	0.000	12.477

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	95	138	0	306	0	0	120
N.S.	1	1.00	0.79	1.14	0.00	2.53	0.00	0.00	0.99
time (sec)	N/A	0.236	0.017	0.574	0.000	2.119	0.000	0.000	12.275

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	303	217	360	0	1476	0	0	0
N.S.	1	1.12	0.80	1.33	0.00	5.45	0.00	0.00	0.00
time (sec)	N/A	0.478	0.334	0.593	0.000	5.464	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	437	303	716	0	1812	0	0	0
N.S.	1	1.11	0.77	1.82	0.00	4.60	0.00	0.00	0.00
time (sec)	N/A	0.711	0.526	0.744	0.000	11.466	0.000	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	522	565	390	1354	0	2162	0	0	0
N.S.	1	1.08	0.75	2.59	0.00	4.14	0.00	0.00	0.00
time (sec)	N/A	0.946	0.783	0.773	0.000	28.523	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	664	708	493	2409	0	2526	0	0	0
N.S.	1	1.07	0.74	3.63	0.00	3.80	0.00	0.00	0.00
time (sec)	N/A	1.210	0.977	1.145	0.000	64.197	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	265	235	366	0	820	0	0	3099
N.S.	1	1.02	0.91	1.41	0.00	3.17	0.00	0.00	11.97
time (sec)	N/A	0.447	0.219	0.684	0.000	16.877	0.000	0.000	13.602

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	353	440	663	0	1540	0	0	11469
N.S.	1	1.04	1.29	1.94	0.00	4.52	0.00	0.00	33.63
time (sec)	N/A	0.528	0.346	0.657	0.000	115.684	0.000	0.000	16.999

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	221	169	0	33	0	0	0
N.S.	1	1.00	1.30	0.99	0.00	0.19	0.00	0.00	0.00
time (sec)	N/A	0.271	32.665	0.684	0.000	0.091	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	22	22	0	67	22
N.S.	1	1.00	1.00	0.78	0.96	0.96	0.00	2.91	0.96
time (sec)	N/A	0.156	10.025	0.581	0.286	0.317	0.000	0.299	11.905

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	289	347	216	0	30	0	0	0
N.S.	1	0.98	1.18	0.73	0.00	0.10	0.00	0.00	0.00
time (sec)	N/A	0.364	20.404	0.660	0.000	0.179	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	149	169	157	0	25	0	0	0
N.S.	1	1.03	1.17	1.09	0.00	0.17	0.00	0.00	0.00
time (sec)	N/A	0.240	20.351	0.637	0.000	0.081	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	56	48	43	0	60	0	0	0
N.S.	1	0.85	0.73	0.65	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.203	15.354	0.549	0.000	0.264	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	349	216	0	32	0	0	0
N.S.	1	1.00	1.22	0.75	0.00	0.11	0.00	0.00	0.00
time (sec)	N/A	0.369	10.297	0.634	0.000	0.077	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	151	185	159	0	32	0	0	0
N.S.	1	1.03	1.27	1.09	0.00	0.22	0.00	0.00	0.00
time (sec)	N/A	0.248	20.309	0.654	0.000	0.073	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	191	235	176	0	38	0	0	0
N.S.	1	0.95	1.17	0.88	0.00	0.19	0.00	0.00	0.00
time (sec)	N/A	0.279	21.557	0.711	0.000	0.081	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	27	27	0	173	25
N.S.	1	1.00	1.00	0.78	1.17	1.17	0.00	7.52	1.09
time (sec)	N/A	0.166	10.029	0.565	0.288	0.267	0.000	0.314	0.120

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	310	244	230	0	38	0	0	0
N.S.	1	0.95	0.75	0.71	0.00	0.12	0.00	0.00	0.00
time (sec)	N/A	0.393	10.333	0.645	0.000	0.077	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F(-1)	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	173	168	176	0	0	33	0	0	0
N.S.	1	0.97	1.02	0.00	0.00	0.19	0.00	0.00	0.00
time (sec)	N/A	0.256	20.484	180.000	0.000	0.074	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	69	53	57	0	65	0	0	0
N.S.	1	0.73	0.56	0.61	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.202	10.057	0.783	0.000	0.264	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	308	244	230	0	39	0	0	0
N.S.	1	0.95	0.76	0.71	0.00	0.12	0.00	0.00	0.00
time (sec)	N/A	0.405	10.373	0.664	0.000	0.127	0.000	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	170	192	173	0	38	0	0	0
N.S.	1	0.97	1.10	0.99	0.00	0.22	0.00	0.00	0.00
time (sec)	N/A	0.269	10.290	0.672	0.000	0.097	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	149	169	157	0	25	0	0	0
N.S.	1	1.05	1.19	1.11	0.00	0.18	0.00	0.00	0.00
time (sec)	N/A	0.239	21.426	0.681	0.000	0.180	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	22	17	0	18	9
N.S.	1	1.00	1.00	0.78	0.96	0.74	0.00	0.78	0.39
time (sec)	N/A	0.163	10.020	0.619	0.300	0.267	0.000	0.302	0.150

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	268	375	202	0	9	0	0	0
N.S.	1	1.06	1.48	0.80	0.00	0.04	0.00	0.00	0.00
time (sec)	N/A	0.344	21.738	0.683	0.000	0.079	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	148	137	0	6	0	0	0
N.S.	1	1.00	1.35	1.25	0.00	0.05	0.00	0.00	0.00
time (sec)	N/A	0.210	20.117	0.609	0.000	0.077	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	29	33	0	43	0	0	0
N.S.	1	1.00	0.69	0.79	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.191	4.921	0.583	0.000	0.278	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	287	400	215	0	31	0	0	0
N.S.	1	1.02	1.42	0.76	0.00	0.11	0.00	0.00	0.00
time (sec)	N/A	0.375	10.424	0.638	0.000	0.075	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	151	171	159	0	30	0	0	0
N.S.	1	1.03	1.17	1.09	0.00	0.21	0.00	0.00	0.00
time (sec)	N/A	0.247	10.415	0.660	0.000	0.091	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	149	161	157	0	38	0	0	0
N.S.	1	1.09	1.18	1.15	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.236	21.379	0.759	0.000	0.100	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	24	0	0	17
N.S.	1	1.00	1.00	0.78	0.74	1.04	0.00	0.00	0.74
time (sec)	N/A	0.165	10.023	0.571	0.286	0.251	0.000	0.000	12.328

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	288	402	216	0	42	0	0	0
N.S.	1	1.02	1.43	0.77	0.00	0.15	0.00	0.00	0.00
time (sec)	N/A	0.364	10.418	0.741	0.000	0.076	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	149	216	157	0	37	0	0	0
N.S.	1	1.09	1.58	1.15	0.00	0.27	0.00	0.00	0.00
time (sec)	N/A	0.242	20.267	0.715	0.000	0.121	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	56	81	43	0	78	0	0	0
N.S.	1	0.85	1.23	0.65	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.195	11.231	0.666	0.000	0.345	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	308	409	228	0	47	0	0	0
N.S.	1	0.97	1.29	0.72	0.00	0.15	0.00	0.00	0.00
time (sec)	N/A	0.391	10.489	0.750	0.000	0.119	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	172	170	169	0	48	0	0	0
N.S.	1	1.01	1.00	0.99	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.271	10.258	0.675	0.000	0.102	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	178	167	0	56	0	0	0
N.S.	1	1.00	1.06	0.99	0.00	0.33	0.00	0.00	0.00
time (sec)	N/A	0.260	21.354	0.722	0.000	0.081	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	24	29	0	0	82
N.S.	1	1.00	1.00	0.78	1.04	1.26	0.00	0.00	3.57
time (sec)	N/A	0.169	10.037	0.664	0.290	0.263	0.000	0.000	12.444

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	309	409	228	0	61	0	0	0
N.S.	1	0.97	1.29	0.72	0.00	0.19	0.00	0.00	0.00
time (sec)	N/A	0.418	10.484	0.855	0.000	0.078	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	178	167	0	56	0	0	0
N.S.	1	1.00	1.06	0.99	0.00	0.33	0.00	0.00	0.00
time (sec)	N/A	0.255	20.335	0.731	0.000	0.078	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	69	95	60	0	101	0	0	0
N.S.	1	0.72	0.99	0.62	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.201	10.095	0.700	0.000	0.261	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	329	414	240	0	62	0	0	0
N.S.	1	0.94	1.19	0.69	0.00	0.18	0.00	0.00	0.00
time (sec)	N/A	0.427	10.542	0.710	0.000	0.082	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	193	183	179	0	63	0	0	0
N.S.	1	0.95	0.90	0.88	0.00	0.31	0.00	0.00	0.00
time (sec)	N/A	0.292	10.426	0.681	0.000	0.080	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	102	78	68	75	134	88	71	84
N.S.	1	1.05	0.80	0.70	0.77	1.38	0.91	0.73	0.87
time (sec)	N/A	0.269	0.039	0.763	0.278	0.290	0.108	0.269	0.129

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	498	627	594	0	5507	0	1200	13879
N.S.	1	1.02	1.28	1.21	0.00	11.24	0.00	2.45	28.32
time (sec)	N/A	7.953	1.957	0.826	0.000	0.987	0.000	0.379	14.523

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	404	465	504	0	4245	0	1074	11143
N.S.	1	1.24	1.43	1.55	0.00	13.02	0.00	3.29	34.18
time (sec)	N/A	4.161	1.368	0.740	0.000	0.947	0.000	0.376	14.182

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	321	375	425	0	2966	0	897	8171
N.S.	1	1.02	1.19	1.34	0.00	9.39	0.00	2.84	25.86
time (sec)	N/A	2.018	0.987	0.674	0.000	0.498	0.000	0.365	13.250

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	293	341	326	0	1721	0	758	5664
N.S.	1	1.02	1.19	1.14	0.00	6.00	0.00	2.64	19.74
time (sec)	N/A	1.068	0.949	0.684	0.000	0.404	0.000	0.344	13.175

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	233	252	222	0	715	0	445	709
N.S.	1	1.18	1.27	1.12	0.00	3.61	0.00	2.25	3.58
time (sec)	N/A	0.370	0.100	0.553	0.000	0.517	0.000	0.317	12.289

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	283	266	291	0	2446	0	719	10894
N.S.	1	1.03	0.97	1.06	0.00	8.89	0.00	2.61	39.61
time (sec)	N/A	0.892	0.796	0.578	0.000	0.538	0.000	0.319	16.579

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	371	337	373	0	4860	0	776	19887
N.S.	1	1.01	0.92	1.01	0.00	13.21	0.00	2.11	54.04
time (sec)	N/A	2.404	1.231	0.646	0.000	6.027	0.000	0.343	16.142

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	531	550	433	489	0	7425	0	1043	33838
N.S.	1	1.04	0.82	0.92	0.00	13.98	0.00	1.96	63.73
time (sec)	N/A	2.073	1.966	0.719	0.000	104.003	0.000	0.361	17.846

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	650	659	901	762	0	14340	0	1612	31485
N.S.	1	1.01	1.39	1.17	0.00	22.06	0.00	2.48	48.44
time (sec)	N/A	2.191	3.094	0.904	0.000	17.368	0.000	0.404	16.934

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	589	755	680	0	11459	0	1397	25497
N.S.	1	1.01	1.30	1.17	0.00	19.72	0.00	2.40	43.88
time (sec)	N/A	8.091	2.509	0.829	0.000	7.340	0.000	0.402	16.511

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	441	447	537	542	0	8530	0	1195	19465
N.S.	1	1.01	1.22	1.23	0.00	19.34	0.00	2.71	44.14
time (sec)	N/A	1.571	1.717	0.736	0.000	3.548	0.000	0.372	14.904

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	453	386	493	473	0	5572	0	986	13841
N.S.	1	0.85	1.09	1.04	0.00	12.30	0.00	2.18	30.55
time (sec)	N/A	0.882	1.753	0.704	0.000	1.653	0.000	0.362	13.800

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	300	364	344	0	2770	0	797	8334
N.S.	1	0.93	1.13	1.07	0.00	8.60	0.00	2.48	25.88
time (sec)	N/A	0.598	0.275	0.638	0.000	1.111	0.000	0.331	13.556

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	348	371	370	0	5167	0	833	20897
N.S.	1	1.02	1.09	1.09	0.00	15.20	0.00	2.45	61.46
time (sec)	N/A	1.095	1.304	0.628	0.000	4.767	0.000	0.333	18.016

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	417	416	409	0	8653	0	890	29890
N.S.	1	1.03	1.03	1.01	0.00	21.47	0.00	2.21	74.17
time (sec)	N/A	1.752	1.587	0.740	0.000	24.310	0.000	0.359	17.202

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	607	626	560	560	0	14417	0	1129	44649
N.S.	1	1.03	0.92	0.92	0.00	23.75	0.00	1.86	73.56
time (sec)	N/A	2.464	2.736	0.913	0.000	213.706	0.000	0.383	18.134

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	201	201	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.501	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	290	290	320	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.705	0.903	0.000	0.000	0.000	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	237	237	289	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.483	0.403	0.000	0.000	0.000	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	198	198	183	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.371	0.207	0.000	0.000	0.000	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	191	191	163	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.397	0.183	0.000	0.000	0.000	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	242	242	207	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.493	0.349	0.000	0.000	0.000	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	296	296	246	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.574	0.403	0.000	0.000	0.000	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	134	155	175	176	150	190	351
N.S.	1	1.00	0.95	1.10	1.24	1.25	1.06	1.35	2.49
time (sec)	N/A	0.366	0.054	0.424	0.198	0.380	0.278	0.269	0.116

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	103	122	138	139	109	149	197
N.S.	1	1.00	0.94	1.12	1.27	1.28	1.00	1.37	1.81
time (sec)	N/A	0.307	0.033	0.440	0.196	0.364	0.225	0.265	12.025

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	73	88	97	98	70	105	127
N.S.	1	1.00	1.12	1.35	1.49	1.51	1.08	1.62	1.95
time (sec)	N/A	0.229	0.024	0.422	0.223	0.568	0.187	0.275	0.075

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	43	59	63	64	46	63	65
N.S.	1	1.00	0.86	1.18	1.26	1.28	0.92	1.26	1.30
time (sec)	N/A	0.204	0.016	0.377	0.195	0.280	0.130	0.285	11.930

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	65	55	82	82	76	112	84	81
N.S.	1	1.05	0.89	1.32	1.32	1.23	1.81	1.35	1.31
time (sec)	N/A	0.233	0.019	0.402	0.199	0.286	0.286	0.269	0.162

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	82	112	113	165	182	115	109
N.S.	1	1.00	0.95	1.30	1.31	1.92	2.12	1.34	1.27
time (sec)	N/A	0.271	0.032	0.467	0.192	0.303	0.474	0.283	12.020

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	138	149	271	185	149	100
N.S.	1	1.00	1.00	1.59	1.71	3.11	2.13	1.71	1.15
time (sec)	N/A	0.267	0.053	0.453	0.189	0.307	0.453	0.281	0.138

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	122	177	206	400	248	186	152
N.S.	1	1.00	1.08	1.57	1.82	3.54	2.19	1.65	1.35
time (sec)	N/A	0.301	0.041	0.467	0.199	0.278	0.587	0.285	12.078

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	142	199	236	511	282	213	180
N.S.	1	1.00	1.02	1.43	1.70	3.68	2.03	1.53	1.29
time (sec)	N/A	0.321	0.056	0.471	0.196	0.296	0.680	0.300	0.154

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	226	259	258	328	250	273	1029
N.S.	1	1.00	1.04	1.19	1.18	1.50	1.15	1.25	4.72
time (sec)	N/A	0.491	0.081	0.433	0.187	0.295	0.510	0.334	11.989

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	185	217	218	288	199	231	565
N.S.	1	1.00	1.05	1.23	1.23	1.63	1.12	1.31	3.19
time (sec)	N/A	0.432	0.074	0.440	0.197	0.328	0.460	0.362	11.853

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	154	177	182	251	162	191	316
N.S.	1	1.00	1.05	1.21	1.25	1.72	1.11	1.31	2.16
time (sec)	N/A	0.361	0.057	0.374	0.188	0.290	0.390	0.628	0.097

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	115	133	141	206	119	148	185
N.S.	1	1.00	1.07	1.24	1.32	1.93	1.11	1.38	1.73
time (sec)	N/A	0.304	0.057	0.425	0.196	0.278	0.326	0.270	0.073

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	83	93	104	157	94	108	116
N.S.	1	1.00	1.06	1.19	1.33	2.01	1.21	1.38	1.49
time (sec)	N/A	0.263	0.040	0.431	0.190	0.270	0.279	0.278	12.071

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	46	63	69	95	61	68	72
N.S.	1	1.00	0.92	1.26	1.38	1.90	1.22	1.36	1.44
time (sec)	N/A	0.234	0.030	0.424	0.187	0.270	0.187	0.265	11.898

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	91	112	114	168	182	117	111
N.S.	1	1.00	1.06	1.30	1.33	1.95	2.12	1.36	1.29
time (sec)	N/A	0.268	0.031	0.450	0.189	0.282	0.471	0.265	11.883

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	89	85	109	111	155	156	114	115
N.S.	1	1.20	1.15	1.47	1.50	2.09	2.11	1.54	1.55
time (sec)	N/A	0.278	0.027	0.431	0.196	0.258	0.333	0.278	12.052

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	139	180	212	417	279	201	198
N.S.	1	1.00	1.15	1.49	1.75	3.45	2.31	1.66	1.64
time (sec)	N/A	0.312	0.064	0.460	0.195	0.350	0.579	0.288	0.151

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	171	162	197	337	241	227	148
N.S.	1	1.00	1.17	1.11	1.35	2.31	1.65	1.55	1.01
time (sec)	N/A	0.352	0.064	0.458	0.195	0.366	0.628	0.273	12.059

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	195	241	298	648	376	264	274
N.S.	1	1.00	1.10	1.35	1.67	3.64	2.11	1.48	1.54
time (sec)	N/A	0.399	0.091	0.415	0.200	0.351	0.804	0.280	11.801

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	229	278	342	693	427	308	314
N.S.	1	1.00	1.09	1.32	1.63	3.30	2.03	1.47	1.50
time (sec)	N/A	0.463	0.118	0.477	0.216	0.344	0.935	0.294	0.236

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	193	216	227	336	219	226	375
N.S.	1	1.00	1.08	1.21	1.27	1.88	1.22	1.26	2.09
time (sec)	N/A	0.426	0.063	0.428	0.198	0.624	0.689	0.277	0.145

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	157	174	188	294	178	185	240
N.S.	1	1.00	1.05	1.17	1.26	1.97	1.19	1.24	1.61
time (sec)	N/A	0.389	0.056	0.441	0.190	0.355	0.599	0.274	0.111

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	118	133	149	241	151	143	161
N.S.	1	1.00	1.00	1.13	1.26	2.04	1.28	1.21	1.36
time (sec)	N/A	0.323	0.057	0.424	0.195	0.362	0.530	0.268	11.943

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	93	96	105	159	102	94	107
N.S.	1	1.00	1.15	1.19	1.30	1.96	1.26	1.16	1.32
time (sec)	N/A	0.273	0.026	0.427	0.184	0.320	0.417	0.271	11.744

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	49	69	81	100	83	72	80
N.S.	1	1.00	0.80	1.13	1.33	1.64	1.36	1.18	1.31
time (sec)	N/A	0.223	0.018	0.460	0.189	0.264	0.252	0.290	0.073

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	90	152	150	271	185	143	103
N.S.	1	1.00	1.02	1.73	1.70	3.08	2.10	1.62	1.17
time (sec)	N/A	0.269	0.053	0.414	0.199	0.272	0.456	0.270	0.140

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	140	183	211	417	277	200	198
N.S.	1	1.00	1.15	1.50	1.73	3.42	2.27	1.64	1.62
time (sec)	N/A	0.316	0.065	0.440	0.194	0.289	0.578	0.270	11.811

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	158	110	135	152	252	144	146	114
N.S.	1	1.24	0.87	1.06	1.20	1.98	1.13	1.15	0.90
time (sec)	N/A	0.383	0.030	0.430	0.190	0.274	0.454	0.276	0.107

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	197	245	308	662	321	277	249
N.S.	1	1.00	1.05	1.30	1.64	3.52	1.71	1.47	1.32
time (sec)	N/A	0.407	0.097	0.467	0.192	0.309	0.822	0.279	11.801

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	244	286	359	793	372	335	296
N.S.	1	1.00	1.04	1.22	1.53	3.37	1.58	1.43	1.26
time (sec)	N/A	0.481	0.107	0.494	0.204	0.353	0.926	0.269	11.944

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	298	301	604	1603	807	0	969	0
N.S.	1	1.11	1.12	2.25	5.96	3.00	0.00	3.60	0.00
time (sec)	N/A	1.446	3.378	1.138	0.290	0.329	0.000	0.315	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	236	240	481	1190	624	0	757	0
N.S.	1	1.10	1.12	2.24	5.53	2.90	0.00	3.52	0.00
time (sec)	N/A	1.026	1.131	0.895	0.287	0.402	0.000	0.306	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	202	157	688	903	454	0	560	0
N.S.	1	1.10	0.86	3.76	4.93	2.48	0.00	3.06	0.00
time (sec)	N/A	0.656	0.895	0.647	0.280	0.334	0.000	0.305	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	156	105	126	583	279	0	370	125
N.S.	1	1.08	0.72	0.87	4.02	1.92	0.00	2.55	0.86
time (sec)	N/A	0.331	0.562	0.641	0.193	0.311	0.000	0.304	12.125

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	113	77	80	373	183	0	276	79
N.S.	1	0.97	0.66	0.68	3.19	1.56	0.00	2.36	0.68
time (sec)	N/A	0.224	0.475	0.551	0.198	0.316	0.000	0.306	12.721

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	111	53	50	101	106	0	165	49
N.S.	1	1.08	0.51	0.49	0.98	1.03	0.00	1.60	0.48
time (sec)	N/A	0.230	0.010	0.441	0.199	0.299	0.000	0.294	12.659

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	324	225	1667	0	1767	0	642	0
N.S.	1	1.34	0.93	6.89	0.00	7.30	0.00	2.65	0.00
time (sec)	N/A	0.603	10.298	0.520	0.000	0.374	0.000	0.306	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	328	341	3289	0	3305	0	0	0
N.S.	1	1.05	1.10	10.58	0.00	10.63	0.00	0.00	0.00
time (sec)	N/A	1.527	10.429	0.525	0.000	0.753	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	398	438	387	6396	0	5361	0	1401	0
N.S.	1	1.10	0.97	16.07	0.00	13.47	0.00	3.52	0.00
time (sec)	N/A	3.005	10.993	0.484	0.000	2.536	0.000	0.430	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	113	118	111	0	499	155	101	124
N.S.	1	1.01	1.05	0.99	0.00	4.46	1.38	0.90	1.11
time (sec)	N/A	0.345	0.224	0.473	0.000	0.295	5.872	0.295	0.252

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	282	243	326	324	502	380	222
N.S.	1	1.00	1.18	1.01	1.36	1.35	2.09	1.58	0.92
time (sec)	N/A	0.424	0.176	0.460	0.182	0.287	0.998	0.282	0.127

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	177	155	197	197	311	243	159
N.S.	1	1.00	1.01	0.89	1.13	1.13	1.78	1.39	0.91
time (sec)	N/A	0.348	0.114	0.451	0.187	0.281	0.857	0.278	12.185

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	94	79	104	100	165	132	100
N.S.	1	1.00	0.83	0.70	0.92	0.88	1.46	1.17	0.88
time (sec)	N/A	0.280	0.065	0.424	0.192	0.268	0.717	0.276	0.076

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	44	40	53	40	70	53	44
N.S.	1	1.00	0.72	0.66	0.87	0.66	1.15	0.87	0.72
time (sec)	N/A	0.201	0.033	0.402	0.184	0.277	0.352	0.281	11.828

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	106	92	82	0	297	143	109	107
N.S.	1	1.02	0.88	0.79	0.00	2.86	1.38	1.05	1.03
time (sec)	N/A	0.293	0.194	0.684	0.000	0.290	1.789	0.268	0.112

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	164	133	138	0	539	0	151	128
N.S.	1	1.34	1.09	1.13	0.00	4.42	0.00	1.24	1.05
time (sec)	N/A	0.338	0.388	0.475	0.000	0.297	0.000	0.279	11.969

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	221	166	166	0	896	0	285	224
N.S.	1	1.24	0.93	0.93	0.00	5.03	0.00	1.60	1.26
time (sec)	N/A	0.394	0.636	0.500	0.000	0.297	0.000	0.284	12.013

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	278	247	334	333	420	465	292
N.S.	1	1.00	1.17	1.04	1.40	1.40	1.76	1.95	1.23
time (sec)	N/A	0.404	0.196	0.478	0.196	0.283	11.707	0.278	0.091

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	177	154	205	206	264	277	199
N.S.	1	1.00	1.02	0.89	1.18	1.19	1.53	1.60	1.15
time (sec)	N/A	0.337	0.143	0.477	0.197	0.260	4.815	0.269	11.824

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	92	86	112	110	150	137	111
N.S.	1	1.00	0.83	0.77	1.01	0.99	1.35	1.23	1.00
time (sec)	N/A	0.262	0.067	0.428	0.199	0.273	1.969	0.281	0.078

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	43	39	54	49	75	56	44
N.S.	1	1.00	0.73	0.66	0.92	0.83	1.27	0.95	0.75
time (sec)	N/A	0.202	0.035	0.416	0.209	0.279	0.626	0.279	0.054

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	114	112	0	492	151	114	141
N.S.	1	1.00	1.02	1.00	0.00	4.39	1.35	1.02	1.26
time (sec)	N/A	0.321	0.302	0.477	0.000	0.282	4.858	0.282	12.195

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	172	148	152	0	906	0	228	187
N.S.	1	1.19	1.03	1.06	0.00	6.29	0.00	1.58	1.30
time (sec)	N/A	0.387	0.475	0.480	0.000	0.316	0.000	0.277	12.675

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	260	230	230	0	1539	0	368	310
N.S.	1	1.21	1.07	1.07	0.00	7.19	0.00	1.72	1.45
time (sec)	N/A	0.503	0.907	0.538	0.000	0.343	0.000	0.275	12.682

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	150	123	306	0	336	0	165	569
N.S.	1	1.02	0.84	2.08	0.00	2.29	0.00	1.12	3.87
time (sec)	N/A	0.323	0.350	0.422	0.000	0.322	0.000	0.319	32.606

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	9	12	0	12	16
N.S.	1	1.00	1.00	0.81	0.56	0.75	0.00	0.75	1.00
time (sec)	N/A	0.146	0.040	0.442	0.190	0.277	0.000	0.281	12.481

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	409	438	2385	0	0	0	0	0
N.S.	1	1.00	1.07	5.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.269	1.310	0.450	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	341	363	1499	0	0	0	0	0
N.S.	1	1.00	1.06	4.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.026	0.900	0.454	0.000	0.000	0.000	0.000	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	229	1387	0	1921	0	0	0
N.S.	1	1.00	0.95	5.78	0.00	8.00	0.00	0.00	0.00
time (sec)	N/A	0.473	10.305	0.405	0.000	6.811	0.000	0.000	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	340	361	5383	0	5816	0	0	0
N.S.	1	0.97	1.03	15.34	0.00	16.57	0.00	0.00	0.00
time (sec)	N/A	0.947	1.191	0.460	0.000	35.163	0.000	0.000	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	613	582	422	14861	0	10812	0	0	0
N.S.	1	0.95	0.69	24.24	0.00	17.64	0.00	0.00	0.00
time (sec)	N/A	1.307	1.650	0.456	0.000	160.903	0.000	0.000	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	363	2336	0	0	0	0	0
N.S.	1	1.00	1.08	6.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.978	1.017	0.455	0.000	0.000	0.000	0.000	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	577	1383	0	1913	0	0	0
N.S.	1	1.00	2.40	5.76	0.00	7.97	0.00	0.00	0.00
time (sec)	N/A	0.447	6.746	0.460	0.000	8.914	0.000	0.000	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	286	1415	0	4325	0	0	0
N.S.	1	1.00	1.24	6.15	0.00	18.80	0.00	0.00	0.00
time (sec)	N/A	0.425	0.870	0.451	0.000	15.586	0.000	0.000	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	383	10977	0	11846	0	0	0
N.S.	1	1.00	1.08	31.01	0.00	33.46	0.00	0.00	0.00
time (sec)	N/A	0.614	0.965	0.472	0.000	56.032	0.000	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	625	574	1049	8264	0	0	0	0	0
N.S.	1	0.92	1.68	13.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.303	8.148	0.480	0.000	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	340	361	5383	0	5844	0	0	0
N.S.	1	0.97	1.03	15.34	0.00	16.65	0.00	0.00	0.00
time (sec)	N/A	0.827	1.266	0.462	0.000	46.421	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	383	10977	0	12028	0	0	0
N.S.	1	1.00	1.08	31.01	0.00	33.98	0.00	0.00	0.00
time (sec)	N/A	0.707	0.929	0.458	0.000	90.510	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	549	543	477	30648	0	0	0	0	0
N.S.	1	0.99	0.87	55.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.180	1.809	0.458	0.000	0.000	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	59	305	0	129	0	375	1610
N.S.	1	1.00	0.91	4.69	0.00	1.98	0.00	5.77	24.77
time (sec)	N/A	0.186	0.075	0.431	0.000	0.303	0.000	0.785	17.832

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	87	86	82	0	193	0	266	164
N.S.	1	1.09	1.08	1.02	0.00	2.41	0.00	3.32	2.05
time (sec)	N/A	0.210	0.329	0.500	0.000	0.318	0.000	0.295	0.066

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	130	233	0	318	0	208	148
N.S.	1	1.00	1.21	2.18	0.00	2.97	0.00	1.94	1.38
time (sec)	N/A	0.300	0.542	0.561	0.000	0.392	0.000	0.300	0.298

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	112	130	233	0	318	0	131	148
N.S.	1	1.05	1.21	2.18	0.00	2.97	0.00	1.22	1.38
time (sec)	N/A	0.284	0.393	0.470	0.000	0.385	0.000	0.285	0.130

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	851	1311	1045	1824	0	765	0	0	0
N.S.	1	1.54	1.23	2.14	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	3.638	30.019	2.815	0.000	0.108	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	635	1041	809	1142	0	510	0	0	0
N.S.	1	1.64	1.27	1.80	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	2.629	25.889	1.397	0.000	0.093	0.000	0.000	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	434	784	610	794	0	343	0	0	0
N.S.	1	1.81	1.41	1.83	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	1.067	24.391	1.444	0.000	0.107	0.000	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	693	521	602	0	229	0	0	0
N.S.	1	1.91	1.44	1.66	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.809	23.158	0.927	0.000	0.089	0.000	0.000	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	683	0	1216	922	0	0	0	0	0
N.S.	1	0.00	1.78	1.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	29.085	2.612	0.000	0.000	0.000	0.000	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	650	1547	1331	895	0	0	0	0	0
N.S.	1	2.38	2.05	1.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.971	26.771	0.516	0.000	0.000	0.000	0.000	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1205	2763	2526	1161	0	0	0	0	0
N.S.	1	2.29	2.10	0.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.360	29.362	0.891	0.000	0.000	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	666	1085	872	1156	0	578	0	0	0
N.S.	1	1.63	1.31	1.74	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	2.704	27.664	2.444	0.000	0.180	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	508	878	665	848	0	409	0	0	0
N.S.	1	1.73	1.31	1.67	0.00	0.81	0.00	0.00	0.00
time (sec)	N/A	1.621	25.165	2.276	0.000	0.106	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	698	545	678	0	268	0	0	0
N.S.	1	1.92	1.50	1.86	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.889	24.317	1.075	0.000	0.104	0.000	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	639	456	584	0	208	0	0	0
N.S.	1	1.98	1.42	1.81	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.668	22.495	0.977	0.000	0.097	0.000	0.000	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	473	1496	1096	833	0	0	0	0	0
N.S.	1	3.16	2.32	1.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.519	24.768	0.784	0.000	0.000	0.000	0.000	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	694	1559	1336	921	0	0	0	0	0
N.S.	1	2.25	1.93	1.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.732	26.289	1.467	0.000	0.000	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1241	2774	2197	1196	0	0	0	0	0
N.S.	1	2.24	1.77	0.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.308	28.723	2.721	0.000	0.000	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	531	916	777	882	0	445	0	0	0
N.S.	1	1.73	1.46	1.66	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	1.804	25.522	3.372	0.000	0.114	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	764	591	700	0	300	0	0	0
N.S.	1	1.86	1.44	1.71	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	1.161	25.092	2.094	0.000	0.105	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	667	464	602	0	227	0	0	0
N.S.	1	2.02	1.40	1.82	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.705	23.524	1.286	0.000	0.101	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	583	294	396	0	173	0	0	0
N.S.	1	4.29	2.16	2.91	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.588	20.378	0.440	0.000	0.104	0.000	0.000	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	1112	300	439	0	0	0	0	0
N.S.	1	3.49	0.94	1.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.893	21.358	1.170	0.000	0.000	0.000	0.000	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	698	1567	1330	922	0	0	0	0	0
N.S.	1	2.24	1.91	1.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.701	25.587	2.246	0.000	0.000	0.000	0.000	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1246	2782	2450	1224	0	0	0	0	0
N.S.	1	2.23	1.97	0.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.154	29.038	3.125	0.000	0.000	0.000	0.000	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	600	1848	1440	948	0	0	0	0	0
N.S.	1	3.08	2.40	1.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.034	29.463	4.530	0.000	0.000	0.000	0.000	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	469	1299	927	852	0	0	0	0	0
N.S.	1	2.77	1.98	1.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.842	10.873	1.373	0.000	0.000	0.000	0.000	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	831	619	711	0	319	0	0	0
N.S.	1	1.82	1.35	1.56	0.00	0.70	0.00	0.00	0.00
time (sec)	N/A	1.314	24.724	2.980	0.000	0.119	0.000	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	697	473	614	0	246	0	0	0
N.S.	1	1.96	1.33	1.72	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.894	23.650	2.535	0.000	0.133	0.000	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	614	439	520	0	180	0	0	0
N.S.	1	2.13	1.52	1.81	0.00	0.62	0.00	0.00	0.00
time (sec)	N/A	0.647	22.289	1.220	0.000	0.132	0.000	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	233	186	200	0	66	0	0	0
N.S.	1	1.71	1.37	1.47	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.269	21.200	0.949	0.000	0.124	0.000	0.000	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	867	311	235	0	0	0	0	0
N.S.	1	5.19	1.86	1.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.560	22.363	1.845	0.000	0.000	0.000	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	746	1586	1349	995	0	0	0	0	0
N.S.	1	2.13	1.81	1.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.760	25.970	2.599	0.000	0.000	0.000	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1257	2800	2491	1192	0	0	0	0	0
N.S.	1	2.23	1.98	0.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.120	29.749	3.459	0.000	0.000	0.000	0.000	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	1176	468	929	0	0	0	0	0
N.S.	1	3.04	1.21	2.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.928	23.301	2.527	0.000	0.000	0.000	0.000	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	818	1795	1917	1079	0	0	0	0	0
N.S.	1	2.19	2.34	1.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.985	27.374	3.177	0.000	0.000	0.000	0.000	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	826	261	215	0	0	0	0	0
N.S.	1	7.51	2.37	1.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.643	21.998	1.809	0.000	0.000	0.000	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	454	344	401	0	0	0	0	0
N.S.	1	1.00	0.76	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.481	24.183	5.083	0.000	0.000	0.000	0.000	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	107	58	0	4	0	0	0
N.S.	1	1.00	2.06	1.12	0.00	0.08	0.00	0.00	0.00
time (sec)	N/A	0.182	34.621	1.125	0.000	0.086	0.000	0.000	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	275	136	170	218	193	0	613	218
N.S.	1	1.02	0.51	0.63	0.81	0.72	0.00	2.28	0.81
time (sec)	N/A	0.527	0.129	0.509	0.245	0.302	0.000	0.309	12.607

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	199	89	98	133	123	0	350	142
N.S.	1	1.00	0.44	0.49	0.66	0.62	0.00	1.75	0.71
time (sec)	N/A	0.391	0.072	0.533	0.235	0.312	0.000	0.288	12.504

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	123	53	49	65	71	0	165	88
N.S.	1	0.98	0.42	0.39	0.52	0.57	0.00	1.32	0.70
time (sec)	N/A	0.261	0.050	0.524	0.239	0.275	0.000	0.286	12.352

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	35	32	18	49	0	68	54
N.S.	1	1.00	0.76	0.70	0.39	1.07	0.00	1.48	1.17
time (sec)	N/A	0.166	0.013	0.529	0.216	0.284	0.000	0.279	12.361

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	93	77	0	252	0	126	0
N.S.	1	1.00	1.16	0.96	0.00	3.15	0.00	1.58	0.00
time (sec)	N/A	0.258	0.074	0.537	0.000	0.306	0.000	0.285	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	136	158	0	703	0	395	0
N.S.	1	1.00	0.97	1.13	0.00	5.02	0.00	2.82	0.00
time (sec)	N/A	0.361	0.243	0.548	0.000	0.308	0.000	0.369	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	220	163	275	0	1283	0	844	0
N.S.	1	1.03	0.77	1.29	0.00	6.02	0.00	3.96	0.00
time (sec)	N/A	0.464	0.293	0.526	0.000	0.352	0.000	0.454	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	300	191	440	0	2027	0	1460	0
N.S.	1	1.07	0.68	1.57	0.00	7.24	0.00	5.21	0.00
time (sec)	N/A	0.581	0.463	0.534	0.000	0.673	0.000	0.656	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	262	134	179	165	216	0	513	252
N.S.	1	1.02	0.52	0.70	0.64	0.84	0.00	2.00	0.98
time (sec)	N/A	0.502	0.104	0.529	0.256	0.278	0.000	0.313	12.727

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	186	88	108	98	147	0	298	178
N.S.	1	1.03	0.49	0.60	0.54	0.81	0.00	1.65	0.98
time (sec)	N/A	0.366	0.068	0.527	0.243	0.273	0.000	0.302	12.583

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	51	58	48	96	0	138	118
N.S.	1	1.00	0.34	0.39	0.32	0.64	0.00	0.92	0.79
time (sec)	N/A	0.315	0.047	0.533	0.228	0.285	0.000	0.292	12.387

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	35	42	18	74	0	70	82
N.S.	1	1.00	0.76	0.91	0.39	1.61	0.00	1.52	1.78
time (sec)	N/A	0.168	0.008	0.527	0.216	0.308	0.000	0.287	12.376

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	109	118	0	553	0	281	0
N.S.	1	1.00	0.82	0.89	0.00	4.16	0.00	2.11	0.00
time (sec)	N/A	0.343	0.101	0.545	0.000	0.294	0.000	0.385	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	215	141	215	0	1067	0	753	0
N.S.	1	1.06	0.70	1.06	0.00	5.28	0.00	3.73	0.00
time (sec)	N/A	0.449	0.354	0.510	0.000	0.331	0.000	0.496	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	295	185	369	0	1863	0	1390	0
N.S.	1	1.08	0.68	1.35	0.00	6.80	0.00	5.07	0.00
time (sec)	N/A	0.602	0.527	0.552	0.000	0.486	0.000	0.697	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	251	131	179	219	251	0	524	278
N.S.	1	1.05	0.55	0.75	0.92	1.05	0.00	2.19	1.16
time (sec)	N/A	0.479	0.116	0.543	0.253	0.415	0.000	0.343	12.938

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	217	87	108	138	180	0	303	206
N.S.	1	1.03	0.41	0.51	0.65	0.85	0.00	1.44	0.98
time (sec)	N/A	0.411	0.079	0.568	0.256	0.370	0.000	0.315	12.730

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	52	58	73	129	0	163	149
N.S.	1	1.00	0.34	0.38	0.47	0.84	0.00	1.06	0.97
time (sec)	N/A	0.316	0.054	0.533	0.237	0.310	0.000	0.303	12.512

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	37	42	28	107	0	97	110
N.S.	1	1.00	0.77	0.88	0.58	2.23	0.00	2.02	2.29
time (sec)	N/A	0.172	0.018	0.536	0.209	0.270	0.000	0.294	12.472

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	203	129	209	0	1015	0	683	0
N.S.	1	1.08	0.69	1.11	0.00	5.40	0.00	3.63	0.00
time (sec)	N/A	0.465	0.200	0.554	0.000	0.387	0.000	0.440	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	294	180	414	0	1907	0	1404	0
N.S.	1	1.10	0.67	1.54	0.00	7.12	0.00	5.24	0.00
time (sec)	N/A	0.590	0.417	0.543	0.000	0.625	0.000	0.643	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	374	240	660	0	2935	0	2347	0
N.S.	1	1.09	0.70	1.93	0.00	8.58	0.00	6.86	0.00
time (sec)	N/A	0.731	0.743	0.543	0.000	1.308	0.000	1.139	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	351	195	273	320	375	0	1107	347
N.S.	1	1.04	0.58	0.81	0.95	1.12	0.00	3.29	1.03
time (sec)	N/A	0.672	0.158	0.536	0.251	0.277	0.000	0.324	12.361

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	275	136	178	218	264	0	762	242
N.S.	1	1.02	0.51	0.66	0.81	0.98	0.00	2.83	0.90
time (sec)	N/A	0.533	0.113	0.541	0.246	0.308	0.000	0.304	12.292

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	199	90	106	133	173	0	476	157
N.S.	1	1.00	0.45	0.53	0.66	0.86	0.00	2.38	0.78
time (sec)	N/A	0.394	0.070	0.534	0.233	0.369	0.000	0.300	12.085

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	123	54	57	65	102	0	253	93
N.S.	1	0.98	0.43	0.46	0.52	0.82	0.00	2.02	0.74
time (sec)	N/A	0.268	0.043	0.529	0.221	0.371	0.000	0.281	11.983

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	37	40	18	57	0	91	49
N.S.	1	1.00	0.77	0.83	0.38	1.19	0.00	1.90	1.02
time (sec)	N/A	0.173	0.016	0.484	0.215	0.366	0.000	0.279	11.983

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	114	143	0	318	0	255	0
N.S.	1	1.00	0.92	1.15	0.00	2.56	0.00	2.06	0.00
time (sec)	N/A	0.349	0.105	0.549	0.000	0.380	0.000	0.364	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	110	151	0	562	0	315	0
N.S.	1	1.00	0.83	1.14	0.00	4.26	0.00	2.39	0.00
time (sec)	N/A	0.332	0.242	0.553	0.000	0.439	0.000	0.355	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	202	165	275	0	1056	0	662	0
N.S.	1	0.98	0.80	1.33	0.00	5.10	0.00	3.20	0.00
time (sec)	N/A	0.447	0.468	0.537	0.000	0.401	0.000	0.486	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	282	202	443	0	1732	0	1234	0
N.S.	1	1.02	0.73	1.60	0.00	6.25	0.00	4.45	0.00
time (sec)	N/A	0.579	0.717	0.534	0.000	0.803	0.000	0.663	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	362	234	686	0	2610	0	1955	0
N.S.	1	1.04	0.67	1.98	0.00	7.52	0.00	5.63	0.00
time (sec)	N/A	0.731	1.059	0.538	0.000	1.346	0.000	1.733	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	351	195	275	413	472	0	2535	445
N.S.	1	1.04	0.58	0.82	1.23	1.40	0.00	7.54	1.32
time (sec)	N/A	0.690	0.201	0.545	0.261	0.349	0.000	0.384	12.561

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	275	137	180	294	340	0	1778	310
N.S.	1	1.02	0.51	0.67	1.09	1.26	0.00	6.61	1.15
time (sec)	N/A	0.528	0.153	0.515	0.246	0.316	0.000	0.358	12.428

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	199	90	108	192	230	0	1145	206
N.S.	1	1.00	0.45	0.54	0.96	1.15	0.00	5.72	1.03
time (sec)	N/A	0.396	0.101	0.543	0.234	0.302	0.000	0.325	12.126

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	123	54	59	107	137	0	632	109
N.S.	1	0.98	0.43	0.47	0.86	1.10	0.00	5.06	0.87
time (sec)	N/A	0.271	0.064	0.575	0.225	0.296	0.000	0.298	11.946

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	37	42	43	74	0	254	62
N.S.	1	1.00	0.77	0.88	0.90	1.54	0.00	5.29	1.29
time (sec)	N/A	0.172	0.023	0.559	0.214	0.286	0.000	0.287	12.110

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	185	132	253	0	408	0	506	0
N.S.	1	1.03	0.74	1.41	0.00	2.28	0.00	2.83	0.00
time (sec)	N/A	0.476	0.186	0.562	0.000	0.351	0.000	0.469	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	184	144	296	0	444	0	605	0
N.S.	1	1.03	0.81	1.66	0.00	2.49	0.00	3.40	0.00
time (sec)	N/A	0.448	0.333	0.553	0.000	0.331	0.000	0.424	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	194	135	266	0	840	0	581	0
N.S.	1	0.99	0.69	1.36	0.00	4.31	0.00	2.98	0.00
time (sec)	N/A	0.437	0.462	0.550	0.000	0.343	0.000	0.476	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	264	201	443	0	1434	0	1087	0
N.S.	1	1.00	0.76	1.67	0.00	5.41	0.00	4.10	0.00
time (sec)	N/A	0.573	0.731	0.565	0.000	0.437	0.000	0.688	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	344	240	655	0	2238	0	1785	0
N.S.	1	1.03	0.72	1.96	0.00	6.68	0.00	5.33	0.00
time (sec)	N/A	0.705	1.103	0.573	0.000	0.800	0.000	1.208	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	424	302	945	0	3204	0	2659	0
N.S.	1	1.05	0.75	2.33	0.00	7.91	0.00	6.57	0.00
time (sec)	N/A	0.828	2.194	0.582	0.000	2.278	0.000	2.799	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	351	205	275	498	567	0	4287	523
N.S.	1	1.04	0.61	0.82	1.48	1.69	0.00	12.76	1.56
time (sec)	N/A	0.682	0.206	0.585	0.258	0.320	0.000	0.483	12.826

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	275	147	180	362	416	0	3058	379
N.S.	1	1.02	0.55	0.67	1.35	1.55	0.00	11.37	1.41
time (sec)	N/A	0.538	0.148	0.573	0.268	0.314	0.000	0.414	12.926

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	199	100	108	243	284	0	2010	259
N.S.	1	1.00	0.50	0.54	1.22	1.42	0.00	10.05	1.30
time (sec)	N/A	0.419	0.099	0.564	0.236	0.400	0.000	0.376	12.675

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	123	64	59	141	173	0	1147	134
N.S.	1	0.98	0.51	0.47	1.13	1.38	0.00	9.18	1.07
time (sec)	N/A	0.269	0.071	0.514	0.224	0.407	0.000	0.322	12.433

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	37	42	60	91	0	477	79
N.S.	1	1.00	0.77	0.88	1.25	1.90	0.00	9.94	1.65
time (sec)	N/A	0.174	0.028	0.536	0.206	0.410	0.000	0.299	12.416

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	246	168	421	0	587	0	853	0
N.S.	1	1.04	0.71	1.78	0.00	2.49	0.00	3.61	0.00
time (sec)	N/A	0.593	0.216	0.576	0.000	0.409	0.000	0.637	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	245	183	513	0	672	0	1025	0
N.S.	1	1.04	0.78	2.18	0.00	2.86	0.00	4.36	0.00
time (sec)	N/A	0.566	0.445	0.602	0.000	0.433	0.000	0.602	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	189	516	0	683	0	1038	0
N.S.	1	1.00	0.77	2.10	0.00	2.78	0.00	4.22	0.00
time (sec)	N/A	0.556	0.534	0.557	0.000	0.551	0.000	0.583	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	256	171	431	0	1140	0	940	0
N.S.	1	1.01	0.68	1.70	0.00	4.51	0.00	3.72	0.00
time (sec)	N/A	0.546	0.776	0.562	0.000	0.818	0.000	0.707	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	326	244	655	0	1862	0	1574	0
N.S.	1	1.01	0.76	2.03	0.00	5.76	0.00	4.87	0.00
time (sec)	N/A	0.668	1.212	0.551	0.000	0.548	0.000	1.746	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	406	301	914	0	2750	0	2407	0
N.S.	1	1.03	0.77	2.33	0.00	7.00	0.00	6.12	0.00
time (sec)	N/A	0.820	1.689	0.566	0.000	1.368	0.000	2.828	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	486	370	1251	0	3872	0	3412	0
N.S.	1	1.05	0.80	2.70	0.00	8.36	0.00	7.37	0.00
time (sec)	N/A	0.960	2.720	0.579	0.000	3.973	0.000	7.620	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	325	191	501	0	841	0	989	0
N.S.	1	1.04	0.61	1.60	0.00	2.69	0.00	3.16	0.00
time (sec)	N/A	0.657	0.396	0.571	0.000	1.023	0.000	0.644	0.000

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	247	156	318	0	655	0	633	0
N.S.	1	1.01	0.64	1.30	0.00	2.68	0.00	2.59	0.00
time (sec)	N/A	0.509	0.314	0.576	0.000	0.798	0.000	0.520	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	138	191	0	521	0	380	0
N.S.	1	1.00	0.82	1.13	0.00	3.08	0.00	2.25	0.00
time (sec)	N/A	0.377	0.249	0.627	0.000	0.750	0.000	0.435	0.000

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	94	102	0	343	0	161	0
N.S.	1	1.00	0.90	0.97	0.00	3.27	0.00	1.53	0.00
time (sec)	N/A	0.250	0.137	0.569	0.000	0.746	0.000	0.370	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	50	45	0	114	0	194	100
N.S.	1	1.00	0.82	0.74	0.00	1.87	0.00	3.18	1.64
time (sec)	N/A	0.216	0.046	0.531	0.000	0.345	0.000	0.306	13.992

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	69	61	0	288	0	676	147
N.S.	1	1.00	0.53	0.47	0.00	2.23	0.00	5.24	1.14
time (sec)	N/A	0.316	0.086	0.574	0.000	0.435	0.000	0.353	14.283

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	211	105	111	0	572	0	1612	242
N.S.	1	1.07	0.53	0.56	0.00	2.89	0.00	8.14	1.22
time (sec)	N/A	0.446	0.126	0.556	0.000	1.041	0.000	0.423	14.539

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	293	152	183	0	953	0	2971	357
N.S.	1	1.10	0.57	0.69	0.00	3.57	0.00	11.13	1.34
time (sec)	N/A	0.568	0.182	0.572	0.000	2.747	0.000	0.547	14.094

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	312	183	638	0	971	0	834	0
N.S.	1	1.04	0.61	2.12	0.00	3.23	0.00	2.77	0.00
time (sec)	N/A	0.625	0.462	0.550	0.000	1.018	0.000	0.627	0.000

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	234	142	386	0	725	0	527	0
N.S.	1	1.03	0.63	1.70	0.00	3.19	0.00	2.32	0.00
time (sec)	N/A	0.491	0.302	0.549	0.000	0.745	0.000	0.517	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	117	200	0	569	0	408	0
N.S.	1	1.00	0.73	1.24	0.00	3.53	0.00	2.53	0.00
time (sec)	N/A	0.354	0.168	0.515	0.000	0.690	0.000	0.472	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	50	55	0	125	0	218	147
N.S.	1	1.00	0.82	0.90	0.00	2.05	0.00	3.57	2.41
time (sec)	N/A	0.221	0.040	0.546	0.000	0.293	0.000	0.315	13.249

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	64	70	0	325	0	793	151
N.S.	1	1.00	0.52	0.56	0.00	2.62	0.00	6.40	1.22
time (sec)	N/A	0.323	0.093	0.559	0.000	0.348	0.000	0.380	13.591

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	206	105	120	0	649	0	2618	268
N.S.	1	1.07	0.55	0.62	0.00	3.38	0.00	13.64	1.40
time (sec)	N/A	0.439	0.130	0.555	0.000	0.420	0.000	1.841	13.978

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	288	150	192	0	1062	0	6137	414
N.S.	1	1.10	0.57	0.73	0.00	4.05	0.00	23.42	1.58
time (sec)	N/A	0.564	0.174	0.556	0.000	1.397	0.000	24.647	14.396

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	303	188	642	0	1055	0	1105	0
N.S.	1	1.05	0.65	2.22	0.00	3.65	0.00	3.82	0.00
time (sec)	N/A	0.602	0.422	0.547	0.000	0.892	0.000	0.855	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	227	134	333	0	755	0	685	0
N.S.	1	1.04	0.61	1.52	0.00	3.45	0.00	3.13	0.00
time (sec)	N/A	0.463	0.263	0.550	0.000	0.782	0.000	0.651	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	52	55	0	193	0	316	169
N.S.	1	1.00	0.83	0.87	0.00	3.06	0.00	5.02	2.68
time (sec)	N/A	0.217	0.044	0.500	0.000	0.320	0.000	0.446	12.871

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	67	72	0	318	0	552	246
N.S.	1	1.00	0.52	0.56	0.00	2.48	0.00	4.31	1.92
time (sec)	N/A	0.328	0.095	0.543	0.000	0.439	0.000	0.353	13.620

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	205	103	121	0	667	0	1923	255
N.S.	1	1.06	0.53	0.62	0.00	3.44	0.00	9.91	1.31
time (sec)	N/A	0.435	0.133	0.568	0.000	0.534	0.000	0.496	13.938

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	285	152	191	0	1065	0	4668	416
N.S.	1	1.10	0.58	0.73	0.00	4.10	0.00	17.95	1.60
time (sec)	N/A	0.570	0.188	0.588	0.000	1.032	0.000	3.080	14.247

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	397	235	732	0	1065	0	6752	0
N.S.	1	1.03	0.61	1.90	0.00	2.77	0.00	17.54	0.00
time (sec)	N/A	0.769	0.523	0.569	0.000	2.538	0.000	2.147	0.000

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	319	188	504	0	847	0	3056	0
N.S.	1	1.02	0.60	1.61	0.00	2.71	0.00	9.76	0.00
time (sec)	N/A	0.628	0.427	0.575	0.000	1.381	0.000	1.196	0.000

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	162	319	0	657	0	995	0
N.S.	1	1.00	0.67	1.32	0.00	2.73	0.00	4.13	0.00
time (sec)	N/A	0.481	0.387	0.524	0.000	1.293	0.000	0.650	0.000

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	144	188	0	516	0	409	0
N.S.	1	1.00	0.86	1.13	0.00	3.09	0.00	2.45	0.00
time (sec)	N/A	0.361	0.263	0.547	0.000	0.764	0.000	0.465	0.000

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	133	187	0	521	0	426	0
N.S.	1	1.00	0.84	1.18	0.00	3.30	0.00	2.70	0.00
time (sec)	N/A	0.343	0.187	0.568	0.000	0.724	0.000	0.520	0.000

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	52	53	0	169	0	311	136
N.S.	1	1.00	0.83	0.84	0.00	2.68	0.00	4.94	2.16
time (sec)	N/A	0.211	0.046	0.595	0.000	0.306	0.000	0.513	12.994

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	69	70	0	402	0	762	187
N.S.	1	1.00	0.53	0.54	0.00	3.12	0.00	5.91	1.45
time (sec)	N/A	0.326	0.101	0.577	0.000	0.351	0.000	0.841	13.348

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	211	105	119	0	748	0	1426	289
N.S.	1	1.07	0.53	0.60	0.00	3.78	0.00	7.20	1.46
time (sec)	N/A	0.435	0.136	0.568	0.000	0.555	0.000	0.792	13.574

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	293	152	191	0	1179	0	2280	409
N.S.	1	1.10	0.57	0.72	0.00	4.42	0.00	8.54	1.53
time (sec)	N/A	0.576	0.218	0.562	0.000	1.086	0.000	1.184	13.456

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	391	244	732	0	1059	0	8807	0
N.S.	1	1.02	0.64	1.92	0.00	2.77	0.00	23.05	0.00
time (sec)	N/A	0.780	0.631	0.544	0.000	1.639	0.000	2.739	0.000

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	313	199	504	0	847	0	3057	0
N.S.	1	1.01	0.64	1.63	0.00	2.73	0.00	9.86	0.00
time (sec)	N/A	0.611	0.517	0.566	0.000	1.006	0.000	1.206	0.000

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	239	165	315	0	651	0	709	0
N.S.	1	1.00	0.69	1.32	0.00	2.74	0.00	2.98	0.00
time (sec)	N/A	0.493	0.401	0.563	0.000	0.804	0.000	0.551	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	227	158	373	0	663	0	559	0
N.S.	1	1.02	0.71	1.68	0.00	2.99	0.00	2.52	0.00
time (sec)	N/A	0.472	0.374	0.594	0.000	1.150	0.000	0.591	0.000

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	219	152	321	0	685	0	686	0
N.S.	1	1.02	0.71	1.50	0.00	3.20	0.00	3.21	0.00
time (sec)	N/A	0.452	0.309	0.575	0.000	1.107	0.000	0.724	0.000

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	52	55	0	232	0	446	232
N.S.	1	1.00	0.83	0.87	0.00	3.68	0.00	7.08	3.68
time (sec)	N/A	0.217	0.061	0.579	0.000	0.417	0.000	0.953	12.483

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	69	99	0	526	0	1001	247
N.S.	1	1.00	0.53	0.77	0.00	4.08	0.00	7.76	1.91
time (sec)	N/A	0.326	0.123	0.544	0.000	0.498	0.000	0.740	12.656

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	211	105	169	0	918	0	1760	377
N.S.	1	1.07	0.53	0.85	0.00	4.64	0.00	8.89	1.90
time (sec)	N/A	0.440	0.181	0.575	0.000	1.032	0.000	1.113	13.496

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	293	152	260	0	1420	0	2715	519
N.S.	1	1.10	0.57	0.97	0.00	5.32	0.00	10.17	1.94
time (sec)	N/A	0.584	0.249	0.596	0.000	1.444	0.000	1.693	13.888

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	463	303	1005	0	1331	0	18597	0
N.S.	1	1.03	0.68	2.24	0.00	2.97	0.00	41.51	0.00
time (sec)	N/A	0.910	0.815	0.620	0.000	3.552	0.000	5.441	0.000

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	385	244	732	0	1065	0	6756	0
N.S.	1	1.02	0.65	1.95	0.00	2.83	0.00	17.97	0.00
time (sec)	N/A	0.763	0.690	0.593	0.000	1.735	0.000	2.203	0.000

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	311	189	498	0	837	0	1098	0
N.S.	1	1.02	0.62	1.64	0.00	2.75	0.00	3.61	0.00
time (sec)	N/A	0.641	0.566	0.578	0.000	1.030	0.000	0.695	0.000

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	299	199	625	0	915	0	894	0
N.S.	1	1.02	0.68	2.13	0.00	3.11	0.00	3.04	0.00
time (sec)	N/A	0.612	0.453	0.572	0.000	0.823	0.000	0.768	0.000

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	291	187	628	0	973	0	1178	0
N.S.	1	1.02	0.66	2.21	0.00	3.43	0.00	4.15	0.00
time (sec)	N/A	0.588	0.459	0.566	0.000	0.785	0.000	1.010	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	280	188	501	0	933	0	1224	0
N.S.	1	1.02	0.69	1.83	0.00	3.41	0.00	4.47	0.00
time (sec)	N/A	0.584	0.355	0.582	0.000	1.180	0.000	1.591	0.000

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	52	63	0	299	0	602	325
N.S.	1	1.00	0.83	1.00	0.00	4.75	0.00	9.56	5.16
time (sec)	N/A	0.223	0.075	0.579	0.000	0.442	0.000	0.778	12.820

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	69	99	0	639	0	1275	315
N.S.	1	1.00	0.53	0.77	0.00	4.95	0.00	9.88	2.44
time (sec)	N/A	0.331	0.174	0.575	0.000	0.599	0.000	1.163	13.083

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	211	115	169	0	1101	0	2167	465
N.S.	1	1.07	0.58	0.85	0.00	5.56	0.00	10.94	2.35
time (sec)	N/A	0.456	0.190	0.559	0.000	1.082	0.000	1.772	13.494

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	293	162	260	0	1648	0	3241	627
N.S.	1	1.10	0.61	0.97	0.00	6.17	0.00	12.14	2.35
time (sec)	N/A	0.576	0.261	0.565	0.000	1.568	0.000	2.660	13.617

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	122	100	0	0	0	0	0	0
N.S.	1	1.17	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.284	0.274	0.000	0.000	0.000	0.000	0.000	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	110	98	0	0	0	0	0	0
N.S.	1	1.06	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.268	0.189	0.000	0.000	0.000	0.000	0.000	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	118	98	0	0	0	0	0	0
N.S.	1	1.13	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.263	0.131	0.000	0.000	0.000	0.000	0.000	0.000

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	120	100	0	0	0	0	0	0
N.S.	1	1.15	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.262	0.131	0.000	0.000	0.000	0.000	0.000	0.000

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	122	100	0	0	0	0	0	0
N.S.	1	1.17	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	0.195	0.000	0.000	0.000	0.000	0.000	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	122	110	0	0	0	0	0	0
N.S.	1	1.17	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	0.189	0.000	0.000	0.000	0.000	0.000	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	107	95	0	0	0	0	0	0
N.S.	1	1.04	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.260	0.077	0.000	0.000	0.000	0.000	0.000	0.000

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	324	134	527	331	705	0	1926	615
N.S.	1	0.94	0.39	1.54	0.97	2.06	0.00	5.62	1.79
time (sec)	N/A	0.563	0.275	4.168	0.239	0.331	0.000	0.493	12.306

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	237	131	235	193	350	0	929	327
N.S.	1	0.96	0.53	0.96	0.78	1.42	0.00	3.78	1.33
time (sec)	N/A	0.418	0.196	2.121	0.236	0.307	0.000	0.537	12.132

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	67	89	94	145	0	347	139
N.S.	1	1.00	0.45	0.59	0.63	0.97	0.00	2.31	0.93
time (sec)	N/A	0.282	0.108	1.208	0.215	0.288	0.000	0.300	12.062

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	42	57	33	57	0	81	57
N.S.	1	1.00	0.78	1.06	0.61	1.06	0.00	1.50	1.06
time (sec)	N/A	0.179	0.010	0.783	0.203	0.292	0.000	0.300	11.773

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	82	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.236	0.070	0.000	0.000	0.000	0.000	0.000	0.000

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	84	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.238	0.068	0.000	0.000	0.000	0.000	0.000	0.000

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	88	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.244	0.072	0.000	0.000	0.000	0.000	0.000	0.000

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	93	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.253	0.162	0.000	0.000	0.000	0.000	0.000	0.000

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	93	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.253	0.117	0.000	0.000	0.000	0.000	0.000	0.000

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	91	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.244	0.133	0.000	0.000	0.000	0.000	0.000	0.000

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	91	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.245	0.179	0.000	0.000	0.000	0.000	0.000	0.000

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	93	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.247	0.217	0.000	0.000	0.000	0.000	0.000	0.000

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	52	64	49	66	0	106	63
N.S.	1	1.00	0.80	0.98	0.75	1.02	0.00	1.63	0.97
time (sec)	N/A	0.207	0.013	3.875	0.222	0.293	0.000	0.302	12.029

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	81	64	0	32	35	0	0	0
N.S.	1	1.04	0.82	0.00	0.41	0.45	0.00	0.00	0.00
time (sec)	N/A	0.309	0.025	0.000	0.207	0.291	0.000	0.000	0.000

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	213	222	145	0	0	0	0	0	0
N.S.	1	1.04	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.423	0.247	0.000	0.000	0.000	0.000	0.000	0.000

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	501	438	380	623	693	597	0	1789	653
N.S.	1	0.87	0.76	1.24	1.38	1.19	0.00	3.57	1.30
time (sec)	N/A	0.871	0.306	0.547	0.260	0.311	0.000	0.366	12.765

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	362	264	407	484	408	0	1185	438
N.S.	1	0.88	0.64	0.99	1.17	0.99	0.00	2.88	1.06
time (sec)	N/A	0.716	0.201	0.550	0.258	0.305	0.000	0.338	12.400

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	286	169	237	309	256	0	704	279
N.S.	1	0.89	0.53	0.74	0.96	0.80	0.00	2.19	0.87
time (sec)	N/A	0.551	0.133	0.549	0.233	0.308	0.000	0.318	12.290

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	189	96	113	168	141	0	346	152
N.S.	1	0.90	0.46	0.54	0.80	0.67	0.00	1.66	0.73
time (sec)	N/A	0.360	0.079	0.536	0.222	0.301	0.000	0.308	11.996

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	110	54	51	65	73	0	142	85
N.S.	1	1.01	0.50	0.47	0.60	0.67	0.00	1.30	0.78
time (sec)	N/A	0.241	0.021	0.521	0.221	0.294	0.000	0.280	11.914

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	140	153	0	511	0	284	0
N.S.	1	1.00	1.01	1.10	0.00	3.68	0.00	2.04	0.00
time (sec)	N/A	0.368	0.156	0.552	0.000	0.302	0.000	0.409	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	154	337	0	896	0	469	0
N.S.	1	1.00	0.91	1.98	0.00	5.27	0.00	2.76	0.00
time (sec)	N/A	0.414	0.444	0.560	0.000	0.318	0.000	0.406	0.000

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	252	200	663	0	1704	0	1071	0
N.S.	1	0.97	0.77	2.54	0.00	6.53	0.00	4.10	0.00
time (sec)	N/A	0.521	0.828	0.583	0.000	0.351	0.000	0.527	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	332	279	1132	0	2736	0	1879	0
N.S.	1	0.95	0.79	3.23	0.00	7.79	0.00	5.35	0.00
time (sec)	N/A	0.659	1.031	0.557	0.000	0.959	0.000	0.742	0.000

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	351	247	334	365	251	0	353	1768
N.S.	1	1.08	0.76	1.03	1.13	0.77	0.00	1.09	5.46
time (sec)	N/A	1.268	0.905	0.603	0.290	0.318	0.000	0.314	39.014

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	177	132	199	171	134	0	159	897
N.S.	1	1.07	0.80	1.20	1.03	0.81	0.00	0.96	5.40
time (sec)	N/A	0.574	0.523	0.532	0.292	0.335	0.000	0.288	27.554

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	65	64	117	57	67	0	60	232
N.S.	1	1.03	1.02	1.86	0.90	1.06	0.00	0.95	3.68
time (sec)	N/A	0.242	0.019	0.459	0.286	0.301	0.000	0.283	16.600

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	154	1759	0	4313	0	681	33018
N.S.	1	1.00	0.55	6.24	0.00	15.29	0.00	2.41	117.09
time (sec)	N/A	0.514	0.308	0.629	0.000	0.428	0.000	0.419	96.416

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	571	564	1548	41837	0	35403	0	0	0
N.S.	1	0.99	2.71	73.27	0.00	62.00	0.00	0.00	0.00
time (sec)	N/A	1.028	2.668	1.015	0.000	33.260	0.000	0.000	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	294	244	567	371	376	0	736	0
N.S.	1	1.07	0.88	2.05	1.34	1.36	0.00	2.67	0.00
time (sec)	N/A	1.019	0.937	0.615	0.318	0.308	0.000	0.357	0.000

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	140	144	346	176	204	0	391	0
N.S.	1	1.04	1.07	2.56	1.30	1.51	0.00	2.90	0.00
time (sec)	N/A	0.419	0.513	0.622	0.299	0.293	0.000	0.316	0.000

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	57	151	61	101	0	186	0
N.S.	1	1.00	1.42	3.78	1.52	2.52	0.00	4.65	0.00
time (sec)	N/A	0.215	0.331	0.453	0.281	0.306	0.000	0.295	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	443	425	456	11142	0	21628	0	0	0
N.S.	1	0.96	1.03	25.15	0.00	48.82	0.00	0.00	0.00
time (sec)	N/A	0.720	0.562	0.613	0.000	6.566	0.000	0.000	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	939	907	3830	108969	0	0	0	0	0
N.S.	1	0.97	4.08	116.05	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.942	7.473	3.576	0.000	0.000	0.000	0.000	0.000

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	167	0	0	0	0	0	0
N.S.	1	1.00	3.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.195	0.247	0.000	0.000	0.000	0.000	0.000	0.000

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.232	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	249	1786	811	2032	24206	3830	1943
N.S.	1	1.00	0.91	6.49	2.95	7.39	88.02	13.93	7.07
time (sec)	N/A	0.512	0.301	0.577	0.265	0.353	4.357	0.333	13.019

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	187	1029	512	1122	11946	2133	1133
N.S.	1	1.00	0.90	4.95	2.46	5.39	57.43	10.25	5.45
time (sec)	N/A	0.411	0.206	0.556	0.227	0.386	2.211	0.313	12.353

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	130	449	289	549	4952	1008	572
N.S.	1	1.00	0.89	3.08	1.98	3.76	33.92	6.90	3.92
time (sec)	N/A	0.313	0.156	0.500	0.219	0.331	1.157	0.284	11.958

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	73	147	135	218	1489	366	211
N.S.	1	1.00	0.87	1.75	1.61	2.60	17.73	4.36	2.51
time (sec)	N/A	0.236	0.114	0.469	0.234	0.316	0.581	0.273	12.083

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	114	93	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.284	0.184	0.000	0.000	0.000	0.000	0.000	0.000

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	83	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.247	0.144	0.000	0.000	0.000	0.000	0.000	0.000

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	193	203	106	0	0	0	0	0	0
N.S.	1	1.05	0.55	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.356	0.155	0.000	0.000	0.000	0.000	0.000	0.000

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	197	212	106	0	0	0	0	0	0
N.S.	1	1.08	0.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.355	0.176	0.000	0.000	0.000	0.000	0.000	0.000

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	231	223	179	0	0	0	0	0	0
N.S.	1	0.97	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.399	0.181	0.000	0.000	0.000	0.000	0.000	0.000

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	85	84	87	99	420	89	84
N.S.	1	1.00	1.02	1.01	1.05	1.19	5.06	1.07	1.01
time (sec)	N/A	0.284	0.035	0.484	0.202	0.295	78.093	0.266	12.098

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	177	262	255	313	0	289	266
N.S.	1	1.00	0.96	1.42	1.39	1.70	0.00	1.57	1.45
time (sec)	N/A	0.486	0.096	0.618	0.201	0.541	0.000	0.270	12.142

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	531	531	476	837	721	736	0	953	794
N.S.	1	1.00	0.90	1.58	1.36	1.39	0.00	1.79	1.50
time (sec)	N/A	1.141	0.260	0.645	0.244	2.796	0.000	0.304	13.435

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	246	244	0	0	0	388	12173
N.S.	1	1.00	1.00	0.99	0.00	0.00	0.00	1.58	49.48
time (sec)	N/A	0.598	0.209	0.796	0.000	0.000	0.000	0.292	32.761

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	644	644	710	2228	0	0	0	3400	130035
N.S.	1	1.00	1.10	3.46	0.00	0.00	0.00	5.28	201.92
time (sec)	N/A	1.724	1.428	1.488	0.000	0.000	0.000	0.301	54.584

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	412	285	429	429	695	568	283
N.S.	1	1.00	1.44	0.99	1.49	1.49	2.42	1.98	0.99
time (sec)	N/A	0.509	0.293	0.526	0.222	0.565	1.226	0.279	0.161

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	256	192	261	260	427	363	204
N.S.	1	1.00	1.21	0.91	1.23	1.23	2.01	1.71	0.96
time (sec)	N/A	0.403	0.193	0.530	0.203	0.475	1.049	0.271	0.089

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	131	103	129	125	209	196	125
N.S.	1	1.00	0.96	0.75	0.94	0.91	1.53	1.43	0.91
time (sec)	N/A	0.294	0.101	0.496	0.217	0.504	0.849	0.282	0.085

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	54	46	77	54	102	77	58
N.S.	1	1.00	0.74	0.63	1.05	0.74	1.40	1.05	0.79
time (sec)	N/A	0.219	0.039	0.477	0.224	0.470	0.451	0.277	11.826

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	117	104	93	0	341	160	130	117
N.S.	1	1.01	0.90	0.80	0.00	2.94	1.38	1.12	1.01
time (sec)	N/A	0.316	0.151	0.542	0.000	0.447	2.578	0.282	0.148

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	187	150	152	0	637	0	176	146
N.S.	1	1.34	1.07	1.09	0.00	4.55	0.00	1.26	1.04
time (sec)	N/A	0.360	0.432	0.552	0.000	0.494	0.000	0.279	0.247

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	253	203	201	0	1096	0	383	270
N.S.	1	1.23	0.99	0.98	0.00	5.32	0.00	1.86	1.31
time (sec)	N/A	0.420	0.696	0.566	0.000	0.494	0.000	0.283	0.295

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	406	303	437	438	568	687	394
N.S.	1	1.00	1.42	1.06	1.53	1.54	1.99	2.41	1.38
time (sec)	N/A	0.481	0.325	0.570	0.212	0.418	32.418	0.311	0.121

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	252	196	269	269	350	407	270
N.S.	1	1.00	1.20	0.93	1.28	1.28	1.67	1.94	1.29
time (sec)	N/A	0.379	0.217	0.534	0.207	0.375	11.823	0.301	11.856

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	128	107	137	135	182	195	147
N.S.	1	1.00	0.95	0.79	1.01	1.00	1.35	1.44	1.09
time (sec)	N/A	0.292	0.103	0.487	0.206	0.399	3.246	0.301	11.750

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	54	47	66	63	94	74	58
N.S.	1	1.00	0.76	0.66	0.93	0.89	1.32	1.04	0.82
time (sec)	N/A	0.209	0.047	0.475	0.203	0.389	1.046	0.270	0.063

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	124	122	0	540	172	124	162
N.S.	1	1.00	1.02	1.00	0.00	4.43	1.41	1.02	1.33
time (sec)	N/A	0.356	0.220	0.539	0.000	0.426	6.446	0.282	11.826

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	193	176	175	0	1088	0	286	218
N.S.	1	1.17	1.07	1.06	0.00	6.59	0.00	1.73	1.32
time (sec)	N/A	0.439	0.583	0.563	0.000	0.497	0.000	0.297	0.315

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	294	297	279	0	1883	0	472	363
N.S.	1	1.19	1.20	1.12	0.00	7.59	0.00	1.90	1.46
time (sec)	N/A	0.577	1.170	0.652	0.000	0.591	0.000	0.296	12.175

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	102	231	0	201	0	16	916
N.S.	1	1.00	1.12	2.54	0.00	2.21	0.00	0.18	10.07
time (sec)	N/A	0.409	0.297	0.602	0.000	0.426	0.000	0.295	14.728

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	167	141	425	0	380	0	190	833
N.S.	1	1.02	0.86	2.59	0.00	2.32	0.00	1.16	5.08
time (sec)	N/A	0.305	0.359	0.480	0.000	0.512	0.000	0.307	17.077

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	250	289	1207	0	852	0	480	0
N.S.	1	0.75	0.87	3.62	0.00	2.56	0.00	1.44	0.00
time (sec)	N/A	0.352	0.820	0.483	0.000	0.617	0.000	0.323	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	208	217	763	0	576	0	313	1832
N.S.	1	0.85	0.88	3.10	0.00	2.34	0.00	1.27	7.45
time (sec)	N/A	0.334	3.547	0.486	0.000	0.536	0.000	0.301	109.822

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	167	141	425	0	380	0	190	833
N.S.	1	1.02	0.86	2.59	0.00	2.32	0.00	1.16	5.08
time (sec)	N/A	0.276	0.056	0.479	0.000	0.476	0.000	0.305	0.003

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	132	131	697	0	588	0	195	0
N.S.	1	1.02	1.02	5.40	0.00	4.56	0.00	1.51	0.00
time (sec)	N/A	0.279	0.340	0.475	0.000	1.513	0.000	0.343	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	180	145	773	0	792	0	491	0
N.S.	1	1.12	0.91	4.83	0.00	4.95	0.00	3.07	0.00
time (sec)	N/A	0.330	0.225	0.477	0.000	3.310	0.000	0.384	0.000

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	210	177	210	0	353	0	1175	260
N.S.	1	1.06	0.89	1.06	0.00	1.78	0.00	5.93	1.31
time (sec)	N/A	0.362	0.185	0.497	0.000	7.211	0.000	0.419	13.213

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	258	301	427	0	641	0	2035	452
N.S.	1	0.92	1.07	1.52	0.00	2.28	0.00	7.24	1.61
time (sec)	N/A	0.399	0.346	0.479	0.000	22.889	0.000	0.524	13.496

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	221	167	834	0	580	0	263	0
N.S.	1	0.89	0.67	3.35	0.00	2.33	0.00	1.06	0.00
time (sec)	N/A	0.367	0.441	0.473	0.000	0.754	0.000	0.337	0.000

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	201	189	571	0	546	0	739	0
N.S.	1	0.84	0.79	2.38	0.00	2.28	0.00	3.08	0.00
time (sec)	N/A	0.321	0.455	0.444	0.000	0.422	0.000	0.370	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	157	163	392	0	414	0	446	1797
N.S.	1	0.89	0.93	2.23	0.00	2.35	0.00	2.53	10.21
time (sec)	N/A	0.308	10.344	0.474	0.000	0.410	0.000	0.339	107.002

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	124	103	247	0	308	0	146	893
N.S.	1	1.02	0.84	2.02	0.00	2.52	0.00	1.20	7.32
time (sec)	N/A	0.256	0.262	0.485	0.000	0.408	0.000	0.292	32.857

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	111	122	438	0	463	0	197	0
N.S.	1	1.03	1.13	4.06	0.00	4.29	0.00	1.82	0.00
time (sec)	N/A	0.249	0.262	0.483	0.000	0.389	0.000	0.322	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	137	99	601	0	665	0	220	0
N.S.	1	1.18	0.85	5.18	0.00	5.73	0.00	1.90	0.00
time (sec)	N/A	0.277	0.175	0.483	0.000	0.635	0.000	0.355	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	148	115	122	0	293	0	359	268
N.S.	1	1.11	0.86	0.92	0.00	2.20	0.00	2.70	2.02
time (sec)	N/A	0.286	0.131	0.477	0.000	0.898	0.000	0.380	13.362

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	196	185	207	0	487	0	560	389
N.S.	1	1.04	0.98	1.10	0.00	2.58	0.00	2.96	2.06
time (sec)	N/A	0.306	0.165	0.478	0.000	1.657	0.000	0.444	13.532

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	417	417	473	11686	0	0	0	0	0
N.S.	1	1.00	1.13	28.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.617	3.558	0.648	0.000	0.000	0.000	0.000	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	266	5482	0	4471	0	0	0
N.S.	1	1.00	0.93	19.24	0.00	15.69	0.00	0.00	0.00
time (sec)	N/A	0.604	10.698	0.672	0.000	27.867	0.000	0.000	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	410	5507	0	19727	0	0	0
N.S.	1	1.00	1.43	19.19	0.00	68.74	0.00	0.00	0.00
time (sec)	N/A	0.518	2.524	0.662	0.000	118.801	0.000	0.000	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	429	429	543	47351	0	0	0	0	0
N.S.	1	1.00	1.27	110.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.003	3.468	0.679	0.000	0.000	0.000	0.000	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	532	562	513	879	0	0	0	0	0
N.S.	1	1.06	0.96	1.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.693	3.040	0.891	0.000	0.000	0.000	0.000	0.000

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	348	316	570	0	0	0	0	0
N.S.	1	1.07	0.97	1.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.817	1.423	0.806	0.000	0.000	0.000	0.000	0.000

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	229	212	335	0	0	0	0	0
N.S.	1	1.05	0.97	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.561	0.878	0.723	0.000	0.000	0.000	0.000	0.000

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	158	175	238	0	992	0	0	0
N.S.	1	1.04	1.15	1.57	0.00	6.53	0.00	0.00	0.00
time (sec)	N/A	0.343	0.140	0.694	0.000	1.114	0.000	0.000	0.000

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	240	212	673	0	0	0	0	0
N.S.	1	1.05	0.93	2.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.570	0.634	0.765	0.000	0.000	0.000	0.000	0.000

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	490	228	1359	0	0	0	0	0
N.S.	1	1.00	0.47	2.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.861	0.903	0.905	0.000	0.000	0.000	0.000	0.000

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	673	673	609	2512	0	0	0	1824	0
N.S.	1	1.00	0.90	3.73	0.00	0.00	0.00	2.71	0.00
time (sec)	N/A	1.074	10.904	0.991	0.000	0.000	0.000	1.071	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	933	933	858	3777	0	0	0	8076	0
N.S.	1	1.00	0.92	4.05	0.00	0.00	0.00	8.66	0.00
time (sec)	N/A	1.423	12.448	1.162	0.000	0.000	0.000	5.559	0.000

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1098	1139	743	1409	0	0	0	0	0
N.S.	1	1.04	0.68	1.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.181	11.510	0.968	0.000	0.000	0.000	0.000	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	662	696	536	998	0	0	0	0	0
N.S.	1	1.05	0.81	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.596	10.879	0.891	0.000	0.000	0.000	0.000	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	441	464	427	735	0	0	0	0	0
N.S.	1	1.05	0.97	1.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.138	2.481	0.727	0.000	0.000	0.000	0.000	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	271	247	383	0	1523	0	0	0
N.S.	1	1.08	0.98	1.52	0.00	6.04	0.00	0.00	0.00
time (sec)	N/A	0.594	0.110	0.677	0.000	150.148	0.000	0.000	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	491	453	323	599	0	0	0	0	0
N.S.	1	0.92	0.66	1.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.075	10.719	0.822	0.000	0.000	0.000	0.000	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	787	787	357	900	0	0	0	0	0
N.S.	1	1.00	0.45	1.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.490	10.930	0.921	0.000	0.000	0.000	0.000	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1066	1066	1036	4242	0	0	0	0	0
N.S.	1	1.00	0.97	3.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.795	12.294	0.961	0.000	0.000	0.000	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	886	834	647	1248	0	0	0	0	0
N.S.	1	0.94	0.73	1.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.949	11.704	0.893	0.000	0.000	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	431	461	380	556	0	0	0	0	0
N.S.	1	1.07	0.88	1.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.734	1.975	1.095	0.000	0.000	0.000	0.000	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	287	247	355	0	0	0	0	0
N.S.	1	1.06	0.91	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.887	1.146	0.891	0.000	0.000	0.000	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	186	184	259	0	0	0	0	0
N.S.	1	1.06	1.05	1.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.415	0.695	0.802	0.000	0.000	0.000	0.000	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	142	199	0	1071	0	0	0
N.S.	1	1.00	1.08	1.52	0.00	8.18	0.00	0.00	0.00
time (sec)	N/A	0.277	0.541	0.710	0.000	21.849	0.000	0.000	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	92	157	0	343	0	71	0
N.S.	1	1.00	1.16	1.99	0.00	4.34	0.00	0.90	0.00
time (sec)	N/A	0.197	0.010	0.658	0.000	0.327	0.000	0.294	0.000

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	207	327	0	1952	0	0	0
N.S.	1	1.00	1.14	1.80	0.00	10.73	0.00	0.00	0.00
time (sec)	N/A	0.407	0.738	0.782	0.000	85.212	0.000	0.000	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	256	609	0	0	0	0	0
N.S.	1	1.00	0.75	1.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.610	10.645	0.876	0.000	0.000	0.000	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	587	587	549	1185	0	0	0	2239	0
N.S.	1	1.00	0.94	2.02	0.00	0.00	0.00	3.81	0.00
time (sec)	N/A	0.956	11.309	0.908	0.000	0.000	0.000	0.772	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	496	536	587	1026	0	0	0	0	0
N.S.	1	1.08	1.18	2.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.402	12.463	1.035	0.000	0.000	0.000	0.000	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	366	354	740	0	0	0	0	0
N.S.	1	1.03	0.99	2.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.729	3.950	0.836	0.000	0.000	0.000	0.000	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	244	555	0	2023	0	773	0
N.S.	1	1.00	1.02	2.31	0.00	8.43	0.00	3.22	0.00
time (sec)	N/A	0.413	1.148	0.773	0.000	3.325	0.000	0.286	0.000

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	198	445	0	1663	0	583	0
N.S.	1	1.00	1.06	2.38	0.00	8.89	0.00	3.12	0.00
time (sec)	N/A	0.333	0.779	0.700	0.000	2.947	0.000	0.286	0.000

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	190	400	0	1349	0	461	0
N.S.	1	1.00	1.23	2.58	0.00	8.70	0.00	2.97	0.00
time (sec)	N/A	0.289	0.158	0.688	0.000	0.604	0.000	0.278	0.000

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	347	815	0	0	0	0	0
N.S.	1	1.00	0.99	2.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.668	3.109	0.774	0.000	0.000	0.000	0.000	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	642	642	623	1481	0	0	0	0	0
N.S.	1	1.00	0.97	2.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.062	12.751	0.948	0.000	0.000	0.000	0.000	0.000

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1064	1064	1013	2685	0	0	0	14979	0
N.S.	1	1.00	0.95	2.52	0.00	0.00	0.00	14.08	0.00
time (sec)	N/A	1.833	15.227	1.136	0.000	0.000	0.000	4.142	0.000

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1551	1597	26600	3254	0	1741	0	0	0
N.S.	1	1.03	17.15	2.10	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	5.935	37.075	3.119	0.000	0.153	0.000	0.000	0.000

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1015	1047	15781	1936	0	1132	0	0	0
N.S.	1	1.03	15.55	1.91	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	3.389	35.974	3.522	0.000	0.136	0.000	0.000	0.000

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	652	671	8432	1229	0	726	0	0	0
N.S.	1	1.03	12.93	1.88	0.00	1.11	0.00	0.00	0.00
time (sec)	N/A	1.149	35.210	1.947	0.000	0.121	0.000	0.000	0.000

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	513	522	1052	892	0	481	0	0	0
N.S.	1	1.02	2.05	1.74	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.721	32.546	2.029	0.000	0.107	0.000	0.000	0.000

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	764	890	1470	1245	0	0	0	0	0
N.S.	1	1.16	1.92	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.883	36.442	3.880	0.000	0.000	0.000	0.000	0.000

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	743	874	1473	1190	0	0	0	0	0
N.S.	1	1.18	1.98	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.675	34.833	0.828	0.000	0.000	0.000	0.000	0.000

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1034	1599	33765	1593	0	0	0	0	0
N.S.	1	1.55	32.65	1.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.499	36.667	1.293	0.000	0.000	0.000	0.000	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1098	1128	17771	1845	0	1241	0	0	0
N.S.	1	1.03	16.18	1.68	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	3.625	36.997	4.703	0.000	0.123	0.000	0.000	0.000

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	755	776	10030	1272	0	828	0	0	0
N.S.	1	1.03	13.28	1.68	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	1.960	35.649	2.481	0.000	0.141	0.000	0.000	0.000

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	519	525	792	955	0	557	0	0	0
N.S.	1	1.01	1.53	1.84	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.741	29.596	2.695	0.000	0.107	0.000	0.000	0.000

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	444	447	936	811	0	417	0	0	0
N.S.	1	1.01	2.11	1.83	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.548	27.416	1.276	0.000	0.098	0.000	0.000	0.000

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	700	824	1261	1114	0	0	0	0	0
N.S.	1	1.18	1.80	1.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.406	31.833	1.158	0.000	0.000	0.000	0.000	0.000

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	736	869	1471	1208	0	0	0	0	0
N.S.	1	1.18	2.00	1.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.665	33.660	2.063	0.000	0.000	0.000	0.000	0.000

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1049	1613	36617	1634	0	0	0	0	0
N.S.	1	1.54	34.91	1.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.554	36.623	3.160	0.000	0.000	0.000	0.000	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	774	795	1402	1283	0	879	0	0	0
N.S.	1	1.03	1.81	1.66	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	2.155	34.650	3.617	0.000	0.190	0.000	0.000	0.000

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	567	581	1002	985	0	610	0	0	0
N.S.	1	1.02	1.77	1.74	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	1.107	31.192	4.750	0.000	0.153	0.000	0.000	0.000

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	452	455	638	821	0	448	0	0	0
N.S.	1	1.01	1.41	1.82	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.623	26.338	1.536	0.000	0.148	0.000	0.000	0.000

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	365	746	0	359	0	0	0
N.S.	1	1.00	1.94	3.97	0.00	1.91	0.00	0.00	0.00
time (sec)	N/A	0.262	21.598	0.635	0.000	0.158	0.000	0.000	0.000

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	467	589	379	674	0	0	0	0	0
N.S.	1	1.26	0.81	1.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.114	23.143	2.269	0.000	0.000	0.000	0.000	0.000

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	994	882	1502	1229	0	0	0	0	0
N.S.	1	0.89	1.51	1.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.667	34.192	3.474	0.000	0.000	0.000	0.000	0.000

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1786	1627	36634	1698	0	0	0	0	0
N.S.	1	0.91	20.51	0.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.650	36.792	4.239	0.000	0.000	0.000	0.000	0.000

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	675	675	1385	1122	0	0	0	0	0
N.S.	1	1.00	2.05	1.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.476	13.596	2.003	0.000	0.000	0.000	0.000	0.000

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1138	1138	37137	1245	0	0	0	0	0
N.S.	1	1.00	32.63	1.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.014	36.529	6.523	0.000	0.000	0.000	0.000	0.000

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	631	648	855	985	0	633	0	0	0
N.S.	1	1.03	1.35	1.56	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	1.198	34.486	6.151	0.000	0.136	0.000	0.000	0.000

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	479	486	981	834	0	471	0	0	0
N.S.	1	1.01	2.05	1.74	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.746	30.397	3.249	0.000	0.108	0.000	0.000	0.000

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	814	746	0	367	0	0	0
N.S.	1	1.00	2.07	1.90	0.00	0.93	0.00	0.00	0.00
time (sec)	N/A	0.470	25.686	1.810	0.000	0.099	0.000	0.000	0.000

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	308	287	0	128	0	0	0
N.S.	1	1.00	1.63	1.52	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.271	22.451	1.655	0.000	0.089	0.000	0.000	0.000

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	402	499	330	0	0	0	0	0
N.S.	1	1.44	1.78	1.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.759	24.038	3.344	0.000	0.000	0.000	0.000	0.000

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1037	909	1513	1347	0	0	0	0	0
N.S.	1	0.88	1.46	1.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.707	34.534	3.352	0.000	0.000	0.000	0.000	0.000

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1114	1643	40396	1686	0	0	0	0	0
N.S.	1	1.47	36.26	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.538	37.301	4.552	0.000	0.000	0.000	0.000	0.000

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	553	553	950	1264	0	0	0	0	0
N.S.	1	1.00	1.72	2.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.357	26.224	3.229	0.000	0.000	0.000	0.000	0.000

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1125	1125	14762	1505	0	0	0	0	0
N.S.	1	1.00	13.12	1.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.105	36.106	4.085	0.000	0.000	0.000	0.000	0.000

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	475	475	1118	1463	0	0	0	0	0
N.S.	1	1.00	2.35	3.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.592	28.868	5.121	0.000	0.000	0.000	0.000	0.000

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	588	588	375	601	0	0	0	0	0
N.S.	1	1.00	0.64	1.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.719	26.553	6.056	0.000	0.000	0.000	0.000	0.000

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	198	1249	684	1381	15757	2735	1354
N.S.	1	1.00	0.90	5.68	3.11	6.28	71.62	12.43	6.15
time (sec)	N/A	0.419	0.332	0.555	0.239	0.331	2.870	0.289	13.395

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	180	503	352	613	5930	1156	602
N.S.	1	1.00	1.25	3.49	2.44	4.26	41.18	8.03	4.18
time (sec)	N/A	0.317	0.316	0.487	0.226	0.295	1.270	0.283	13.538

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	111	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.299	0.218	0.000	0.000	0.000	0.000	0.000	0.000

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	157	174	134	0	0	0	0	0	0
N.S.	1	1.11	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.334	0.237	0.000	0.000	0.000	0.000	0.000	0.000

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	245	250	157	0	0	0	0	0	0
N.S.	1	1.02	0.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.429	0.282	0.000	0.000	0.000	0.000	0.000	0.000

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	525	525	492	4653	2034	4747	74400	10486	4871
N.S.	1	1.00	0.94	8.86	3.87	9.04	141.71	19.97	9.28
time (sec)	N/A	0.899	1.145	0.776	0.284	0.376	11.368	0.361	14.951

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	655	2149	1118	2368	32864	4936	2307
N.S.	1	1.00	2.11	6.91	3.59	7.61	105.67	15.87	7.42
time (sec)	N/A	0.578	1.106	0.643	0.237	0.339	5.649	0.324	13.680

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	287	287	265	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.574	0.564	0.000	0.000	0.000	0.000	0.000	0.000

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	298	334	261	0	0	0	0	0	0
N.S.	1	1.12	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.179	1.022	0.000	0.000	0.000	0.000	0.000	0.000

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	461	504	257	0	0	0	0	0	0
N.S.	1	1.09	0.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.876	1.182	0.000	0.000	0.000	0.000	0.000	0.000

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	183	183	151	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.376	0.289	0.000	0.000	0.000	0.000	0.000	0.000

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	165	165	117	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.302	0.146	0.000	0.000	0.000	0.000	0.000	0.000

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	91	0	0	0	0	0	0
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.289	0.126	0.000	0.000	0.000	0.000	0.000	0.000

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	89	0	0	0	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.261	0.116	0.000	0.000	0.000	0.000	0.000	0.000

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	94	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	0.143	0.000	0.000	0.000	0.000	0.000	0.000

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	164	164	110	0	0	0	0	0	0
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.320	0.356	0.000	0.000	0.000	0.000	0.000	0.000

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	199	199	152	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.343	0.171	0.000	0.000	0.000	0.000	0.000	0.000

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	202	207	251	0	0	0	0	0	0
N.S.	1	1.02	1.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.446	0.321	0.000	0.000	0.000	0.000	0.000	0.000

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	181	186	252	0	0	0	0	0	0
N.S.	1	1.03	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.378	0.321	0.000	0.000	0.000	0.000	0.000	0.000

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	180	156	0	0	0	0	0	0
N.S.	1	1.01	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.364	0.316	0.000	0.000	0.000	0.000	0.000	0.000

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	178	149	0	0	0	0	0	0
N.S.	1	0.99	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.344	0.200	0.000	0.000	0.000	0.000	0.000	0.000

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	177	178	150	0	0	0	0	0	0
N.S.	1	1.01	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.349	0.173	0.000	0.000	0.000	0.000	0.000	0.000

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	340	340	274	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.548	0.537	0.000	0.000	0.000	0.000	0.000	0.000

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	376	376	287	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.579	0.517	0.000	0.000	0.000	0.000	0.000	0.000

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	237	240	171	0	0	0	0	0	0
N.S.	1	1.01	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.476	0.501	0.000	0.000	0.000	0.000	0.000	0.000

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	509	506	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.764	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	388	388	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.446	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	189	189	207	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.266	0.095	0.000	0.000	0.000	0.000	0.000	0.000

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	29	26	29	29
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.90	1.00	1.00
time (sec)	N/A	0.170	2.084	0.223	0.261	0.287	1.413	0.298	12.564

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	502	505	0	0	0	0	0	0	0
N.S.	1	1.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.687	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	388	388	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.434	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	189	189	207	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	0.060	0.000	0.000	0.000	0.000	0.000	0.000

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	54	26	29	29
N.S.	1	1.00	1.07	0.93	1.00	1.86	0.90	1.00	1.00
time (sec)	N/A	0.163	5.769	0.245	0.249	0.292	1.820	0.321	73.048

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	265	255	187	0	0	0	0	0	0
N.S.	1	0.96	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.407	0.225	0.000	0.000	0.000	0.000	0.000	0.000

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	525	518	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.707	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	384	384	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.435	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	187	187	205	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.262	0.168	0.000	0.000	0.000	0.000	0.000	0.000

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	29	0	29	29
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.00	1.07	1.07
time (sec)	N/A	0.164	2.068	0.228	0.246	0.293	0.000	0.291	12.514

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	107	188	148	0	0	0	0	0
N.S.	1	1.20	2.11	1.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.346	22.740	1.178	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [65] had the largest ratio of [.7777779999999999969]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	9	8	1.11	25	0.320
2	A	14	13	1.22	25	0.520
3	A	12	11	1.23	25	0.440
4	A	10	9	1.11	25	0.360
5	A	8	7	1.21	23	0.304
6	A	8	7	1.21	23	0.304
7	A	9	8	1.04	25	0.320
8	A	10	9	0.99	25	0.360
9	A	11	10	0.89	25	0.400
10	A	10	9	0.98	25	0.360
11	A	11	10	1.05	25	0.400
12	A	7	6	1.05	25	0.240
13	A	10	9	1.03	25	0.360
14	A	13	12	1.06	25	0.480
15	A	15	14	1.08	25	0.560
16	A	8	7	1.13	25	0.280
17	A	5	4	1.08	25	0.160
18	A	3	3	1.09	25	0.120
19	A	11	10	1.27	25	0.400
20	A	8	7	1.14	25	0.280
21	A	7	6	1.16	25	0.240
22	A	4	4	1.21	25	0.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	4	4	1.08	25	0.160
24	A	4	4	1.01	25	0.160
25	A	4	4	1.02	23	0.174
26	A	3	3	1.10	22	0.136
27	A	10	9	1.11	25	0.360
28	A	11	10	1.05	25	0.400
29	A	14	13	1.10	25	0.520
30	A	5	5	1.05	25	0.200
31	A	6	6	1.07	25	0.240
32	A	3	3	1.07	24	0.125
33	A	15	14	1.24	27	0.519
34	A	13	12	1.22	27	0.444
35	A	11	10	1.26	27	0.370
36	A	9	8	1.27	25	0.320
37	A	5	4	0.98	24	0.167
38	A	10	9	1.02	27	0.333
39	A	10	9	1.00	27	0.333
40	A	8	7	1.01	27	0.259
41	A	10	9	1.04	27	0.333
42	A	14	13	1.08	27	0.481
43	A	15	14	1.07	27	0.519
44	A	8	7	1.18	27	0.259
45	A	7	6	1.17	27	0.222
46	A	4	4	1.03	27	0.148
47	A	4	4	1.06	27	0.148
48	A	4	4	1.06	25	0.160
49	A	3	3	1.06	24	0.125
50	A	11	10	1.11	27	0.370
51	A	11	10	1.13	27	0.370
52	A	14	13	1.07	27	0.481
53	A	15	14	1.13	27	0.519
54	A	9	9	1.25	20	0.450
55	A	6	6	1.35	20	0.300
56	A	6	6	1.34	18	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	3	3	0.98	17	0.176
58	A	8	7	1.00	20	0.350
59	A	8	7	1.00	20	0.350
60	A	7	6	1.06	20	0.300
61	A	9	8	1.04	20	0.400
62	A	11	10	1.12	20	0.500
63	A	13	12	1.08	20	0.600
64	A	13	12	1.12	27	0.444
65	A	22	21	1.23	27	0.778
66	A	19	18	1.23	27	0.667
67	A	18	17	1.23	27	0.630
68	A	16	15	1.23	27	0.556
69	A	13	12	1.04	25	0.480
70	A	9	8	1.10	24	0.333
71	A	15	14	1.14	27	0.519
72	A	15	14	1.10	27	0.519
73	A	17	16	1.06	27	0.593
74	A	16	15	1.06	27	0.556
75	A	17	16	1.00	27	0.593
76	A	17	16	1.01	27	0.593
77	A	18	17	1.05	27	0.630
78	A	16	15	1.09	27	0.556
79	A	17	16	1.13	27	0.593
80	A	13	12	1.14	27	0.444
81	A	15	14	1.11	27	0.519
82	A	19	18	1.12	27	0.667
83	A	11	10	1.17	27	0.370
84	A	9	8	1.15	27	0.296
85	A	8	7	1.14	27	0.259
86	A	4	4	1.09	27	0.148
87	A	4	4	1.06	25	0.160
88	A	4	4	1.08	24	0.167
89	A	11	10	1.09	27	0.370
90	A	11	10	1.07	27	0.370

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	14	13	1.10	27	0.481
92	A	17	16	1.27	27	0.593
93	A	14	13	1.28	27	0.481
94	A	11	10	1.35	27	0.370
95	A	5	4	1.31	25	0.160
96	A	4	3	1.00	24	0.125
97	A	8	7	1.00	27	0.259
98	A	6	5	1.00	27	0.185
99	A	8	7	1.05	27	0.259
100	A	10	9	1.07	27	0.333
101	A	12	11	1.10	27	0.407
102	A	12	11	1.30	27	0.407
103	A	18	17	1.22	27	0.630
104	A	15	14	1.23	27	0.519
105	A	13	12	1.26	27	0.444
106	A	10	9	1.21	25	0.360
107	A	6	5	1.07	24	0.208
108	A	10	9	1.04	27	0.333
109	A	12	11	1.00	27	0.407
110	A	12	11	0.92	27	0.407
111	A	11	10	0.98	27	0.370
112	A	12	11	1.04	27	0.407
113	A	8	7	1.05	27	0.259
114	A	10	9	1.03	27	0.333
115	A	12	11	1.06	27	0.407
116	A	14	13	1.08	27	0.481
117	A	3	3	1.33	18	0.167
118	A	8	7	1.00	26	0.269
119	A	10	9	1.30	27	0.333
120	A	9	8	1.21	27	0.296
121	A	5	4	1.05	27	0.148
122	A	5	4	1.00	25	0.160
123	A	1	1	1.00	24	0.042
124	A	7	6	1.00	27	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
125	A	8	7	1.05	27	0.259
126	A	9	8	1.04	27	0.296
127	A	10	9	1.27	27	0.333
128	A	7	6	1.11	27	0.222
129	A	6	5	1.07	27	0.185
130	A	2	2	1.00	27	0.074
131	A	2	2	0.90	25	0.080
132	A	2	2	1.00	24	0.083
133	A	8	7	1.06	27	0.259
134	A	9	8	1.07	27	0.296
135	A	11	10	1.08	27	0.370
136	A	12	11	1.28	27	0.407
137	A	8	7	1.16	27	0.259
138	A	8	7	1.18	27	0.259
139	A	5	4	1.08	27	0.148
140	A	4	4	1.07	27	0.148
141	A	3	3	1.03	27	0.111
142	A	3	3	0.99	25	0.120
143	A	3	3	1.10	24	0.125
144	A	10	9	1.11	27	0.333
145	A	11	10	1.06	27	0.370
146	A	13	12	1.10	27	0.444
147	A	14	13	1.11	27	0.481
148	A	5	5	1.09	27	0.185
149	A	4	4	1.06	27	0.148
150	A	7	7	1.26	25	0.280
151	A	3	3	1.07	25	0.120
152	A	3	3	1.00	23	0.130
153	A	1	1	1.00	22	0.045
154	A	6	5	1.00	26	0.192
155	A	7	6	0.97	26	0.231
156	A	10	9	1.01	26	0.346
157	A	21	20	1.23	27	0.741
158	A	20	19	1.23	27	0.704

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
159	A	17	16	1.22	27	0.593
160	A	14	13	1.24	27	0.481
161	A	11	10	1.04	25	0.400
162	A	7	6	1.05	24	0.250
163	A	12	11	1.01	27	0.407
164	A	11	10	0.99	27	0.370
165	A	11	10	0.96	27	0.370
166	A	12	11	1.04	27	0.407
167	A	9	8	1.03	27	0.296
168	A	11	10	1.04	27	0.370
169	A	13	12	1.07	27	0.444
170	A	15	14	1.08	27	0.519
171	A	8	7	1.16	27	0.259
172	A	6	6	1.02	27	0.222
173	A	5	5	1.06	27	0.185
174	A	3	3	1.05	25	0.120
175	A	3	3	1.09	24	0.125
176	A	12	11	1.11	27	0.407
177	A	12	11	1.14	27	0.407
178	A	14	13	1.07	27	0.481
179	A	13	12	1.16	27	0.444
180	A	10	9	1.14	27	0.333
181	A	10	9	1.15	27	0.333
182	A	6	6	1.42	27	0.222
183	A	3	3	1.08	25	0.120
184	A	3	3	1.08	24	0.125
185	A	12	11	1.09	27	0.407
186	A	12	11	1.08	27	0.407
187	A	14	13	1.10	27	0.481
188	A	14	13	1.16	27	0.481
189	A	10	9	1.23	27	0.333
190	A	4	4	1.04	27	0.148
191	A	2	2	1.00	27	0.074
192	A	2	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	2	2	1.00	24	0.083
194	A	11	10	1.04	27	0.370
195	A	12	11	1.08	27	0.407
196	A	14	13	1.13	27	0.481
197	A	15	14	1.12	27	0.519
198	A	18	17	1.18	27	0.630
199	A	17	16	1.19	27	0.593
200	A	13	12	1.14	27	0.444
201	A	13	12	1.05	27	0.444
202	A	9	8	1.13	25	0.320
203	A	9	8	0.94	24	0.333
204	A	11	10	1.02	27	0.370
205	A	10	9	1.01	27	0.333
206	A	8	7	1.02	27	0.259
207	A	9	8	1.05	27	0.296
208	A	10	9	1.10	27	0.333
209	A	13	12	1.10	27	0.444
210	A	2	2	1.00	26	0.077
211	A	5	5	1.42	26	0.192
212	A	11	11	1.51	27	0.407
213	A	10	10	1.33	27	0.370
214	A	7	7	1.15	25	0.280
215	A	7	7	1.20	24	0.292
216	A	19	18	1.14	27	0.667
217	A	20	19	1.11	27	0.704
218	A	5	4	1.05	29	0.138
219	A	3	2	1.00	29	0.069
220	A	4	3	1.00	16	0.188
221	A	5	4	1.00	29	0.138
222	A	4	3	1.00	15	0.200
223	A	5	4	1.00	30	0.133
224	A	5	4	1.00	16	0.250
225	A	6	5	1.00	29	0.172
226	A	8	8	1.10	29	0.276

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
227	A	6	6	1.03	29	0.207
228	A	3	3	1.00	27	0.111
229	A	2	2	1.00	22	0.091
230	A	4	4	1.00	29	0.138
231	A	7	7	1.01	29	0.241
232	A	9	9	1.05	29	0.310
233	A	6	6	1.00	29	0.207
234	A	6	6	1.01	29	0.207
235	A	3	3	1.31	27	0.111
236	A	2	2	1.00	22	0.091
237	A	4	4	1.00	29	0.138
238	A	7	7	1.01	29	0.241
239	A	7	7	1.00	29	0.241
240	A	7	6	1.01	23	0.261
241	A	7	6	1.02	23	0.261
242	A	7	6	1.00	23	0.261
243	A	7	6	1.01	23	0.261
244	A	4	4	1.00	21	0.190
245	A	3	3	1.00	20	0.150
246	A	6	5	1.00	23	0.217
247	A	6	5	1.00	23	0.217
248	A	6	5	1.00	23	0.217
249	A	8	7	1.00	25	0.280
250	A	9	8	1.02	25	0.320
251	A	8	7	1.01	25	0.280
252	A	9	8	1.02	25	0.320
253	A	3	3	0.97	23	0.130
254	A	2	2	1.00	22	0.091
255	A	8	7	1.01	25	0.280
256	A	8	7	1.00	25	0.280
257	A	8	7	1.01	25	0.280
258	A	9	8	1.00	25	0.320
259	A	9	8	1.00	25	0.320
260	A	9	8	1.01	25	0.320

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
261	A	9	8	1.01	25	0.320
262	A	3	3	1.00	23	0.130
263	A	2	2	1.00	22	0.091
264	A	9	8	1.00	25	0.320
265	A	9	8	1.00	25	0.320
266	A	9	8	1.01	25	0.320
267	A	8	7	1.01	25	0.280
268	A	8	7	1.00	25	0.280
269	A	8	7	1.01	25	0.280
270	A	3	3	1.00	23	0.130
271	A	2	2	1.00	22	0.091
272	A	7	6	1.00	25	0.240
273	A	7	6	1.00	25	0.240
274	A	7	6	1.00	25	0.240
275	A	9	8	1.01	25	0.320
276	A	10	9	1.02	25	0.360
277	A	9	8	1.01	25	0.320
278	A	10	9	1.01	25	0.360
279	A	3	3	1.03	23	0.130
280	A	2	2	1.00	22	0.091
281	A	9	8	1.03	25	0.320
282	A	9	8	1.00	25	0.320
283	A	9	8	1.00	25	0.320
284	A	9	8	1.00	25	0.320
285	A	9	8	1.03	25	0.320
286	A	11	10	1.00	25	0.400
287	A	10	9	1.01	25	0.360
288	A	5	5	1.05	25	0.200
289	A	3	3	1.00	23	0.130
290	A	2	2	1.00	22	0.091
291	A	10	9	1.01	25	0.360
292	A	11	10	1.01	25	0.400
293	A	10	9	1.00	25	0.360
294	A	11	10	1.00	25	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
295	A	10	9	1.03	25	0.360
296	A	13	12	1.03	25	0.480
297	A	7	7	1.05	25	0.280
298	A	6	6	1.06	25	0.240
299	A	3	3	1.00	23	0.130
300	A	2	2	1.00	22	0.091
301	A	9	8	1.01	25	0.320
302	A	13	12	1.00	25	0.480
303	A	12	11	0.98	25	0.440
304	A	13	12	1.01	25	0.480
305	A	12	11	1.02	25	0.440
306	A	7	7	1.11	27	0.259
307	A	5	5	1.02	27	0.185
308	A	3	3	1.00	25	0.120
309	A	2	2	1.00	20	0.100
310	A	4	4	1.00	27	0.148
311	A	6	6	1.05	27	0.222
312	A	7	7	1.14	27	0.259
313	A	3	3	1.00	25	0.120
314	A	4	4	1.00	27	0.148
315	A	11	10	1.00	16	0.625
316	A	14	13	1.09	22	0.591
317	A	12	11	1.07	22	0.500
318	A	10	9	1.06	22	0.409
319	A	8	7	1.06	20	0.350
320	A	7	6	1.05	19	0.316
321	A	10	9	1.05	22	0.409
322	A	2	2	1.00	22	0.091
323	A	2	2	1.00	22	0.091
324	A	2	2	1.00	22	0.091
325	A	2	2	1.00	22	0.091
326	A	12	11	1.09	22	0.500
327	A	10	9	1.05	22	0.409
328	A	9	8	1.02	22	0.364

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	6	5	1.00	20	0.250
330	A	3	2	1.00	19	0.105
331	A	2	2	1.00	22	0.091
332	A	2	2	1.00	22	0.091
333	A	2	2	1.00	22	0.091
334	A	10	9	1.08	22	0.409
335	A	9	8	1.05	22	0.364
336	A	6	5	1.00	22	0.227
337	A	6	5	1.00	20	0.250
338	A	6	5	1.00	19	0.263
339	A	2	2	1.00	22	0.091
340	A	2	2	1.00	22	0.091
341	A	2	2	1.00	22	0.091
342	A	13	12	1.30	22	0.545
343	A	10	9	1.25	22	0.409
344	A	10	9	1.22	22	0.409
345	A	8	7	1.23	22	0.318
346	A	4	3	1.00	20	0.150
347	A	4	3	1.00	19	0.158
348	A	2	2	1.00	22	0.091
349	A	2	2	1.00	22	0.091
350	A	2	2	1.00	22	0.091
351	A	2	2	1.00	18	0.111
352	A	2	2	1.00	16	0.125
353	A	2	2	1.00	15	0.133
354	A	2	2	1.00	18	0.111
355	A	2	2	1.00	20	0.100
356	A	2	2	1.00	18	0.111
357	A	2	2	1.00	17	0.118
358	A	2	2	1.00	20	0.100
359	A	2	2	1.00	20	0.100
360	A	2	2	1.00	18	0.111
361	A	2	2	1.00	17	0.118
362	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
363	A	4	4	1.10	20	0.200
364	A	4	4	1.11	20	0.200
365	A	5	5	1.01	20	0.250
366	A	2	2	1.00	18	0.111
367	A	2	2	1.00	17	0.118
368	A	2	2	1.00	20	0.100
369	A	2	2	1.00	20	0.100
370	A	3	3	1.03	20	0.150
371	A	4	4	0.98	20	0.200
372	A	5	5	0.96	20	0.250
373	A	4	4	0.94	18	0.222
374	A	4	4	0.98	17	0.235
375	A	2	2	0.99	20	0.100
376	A	2	2	1.00	20	0.100
377	A	8	8	0.95	22	0.364
378	A	5	5	0.96	20	0.250
379	A	2	2	1.00	22	0.091
380	A	2	2	1.00	22	0.091
381	A	7	6	0.98	18	0.333
382	A	7	6	0.98	18	0.333
383	A	7	6	0.99	18	0.333
384	A	7	6	0.99	18	0.333
385	A	4	4	1.00	16	0.250
386	A	3	3	1.00	15	0.200
387	A	6	5	1.00	18	0.278
388	A	6	5	1.00	18	0.278
389	A	6	5	1.00	18	0.278
390	A	8	7	0.97	20	0.350
391	A	9	8	0.94	20	0.400
392	A	8	7	0.98	20	0.350
393	A	9	8	0.95	20	0.400
394	A	8	7	0.99	18	0.389
395	A	5	5	0.95	17	0.294
396	A	8	7	1.00	20	0.350

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
397	A	8	7	1.00	20	0.350
398	A	8	7	1.01	20	0.350
399	A	9	8	0.94	20	0.400
400	A	9	8	0.94	20	0.400
401	A	9	8	0.95	20	0.400
402	A	9	8	0.94	20	0.400
403	A	9	8	0.95	18	0.444
404	A	5	5	1.05	17	0.294
405	A	9	8	0.97	20	0.400
406	A	9	8	1.00	20	0.400
407	A	9	8	1.01	20	0.400
408	A	7	6	0.99	20	0.300
409	A	7	6	1.00	20	0.300
410	A	7	6	1.01	20	0.300
411	A	6	5	0.76	18	0.278
412	A	6	5	1.00	17	0.294
413	A	8	7	1.01	20	0.350
414	A	8	7	1.00	20	0.350
415	A	10	9	1.02	20	0.450
416	A	13	12	0.99	20	0.600
417	A	13	12	0.96	20	0.600
418	A	11	10	0.95	20	0.500
419	A	11	10	0.91	18	0.556
420	A	2	2	0.78	17	0.118
421	A	2	2	0.83	20	0.100
422	A	2	2	0.70	20	0.100
423	A	14	13	0.96	20	0.650
424	A	13	12	0.93	20	0.600
425	A	12	11	0.92	20	0.550
426	A	14	13	0.97	18	0.722
427	A	2	2	1.00	17	0.118
428	A	2	2	0.66	20	0.100
429	A	2	2	0.61	20	0.100
430	A	7	7	1.05	22	0.318

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
431	A	5	5	1.04	22	0.227
432	A	3	3	1.00	20	0.150
433	A	2	2	1.00	15	0.133
434	A	4	4	1.04	22	0.182
435	A	3	3	1.02	22	0.136
436	A	3	3	1.00	22	0.136
437	A	9	8	1.07	40	0.200
438	A	7	6	1.03	40	0.150
439	A	5	4	0.84	38	0.105
440	A	4	3	1.00	37	0.081
441	A	8	7	0.96	40	0.175
442	A	5	4	1.00	40	0.100
443	A	7	6	1.04	40	0.150
444	A	9	8	1.07	40	0.200
445	A	11	10	1.08	40	0.250
446	A	10	9	0.99	40	0.225
447	A	8	7	0.99	40	0.175
448	A	6	5	0.86	38	0.132
449	A	5	4	1.07	37	0.108
450	A	9	8	1.02	40	0.200
451	A	9	8	1.00	40	0.200
452	A	9	8	1.03	40	0.200
453	A	6	5	1.07	40	0.125
454	A	8	7	1.03	40	0.175
455	A	10	9	1.02	40	0.225
456	A	12	11	1.01	40	0.275
457	A	11	10	0.95	40	0.250
458	A	9	8	0.96	40	0.200
459	A	7	6	0.90	38	0.158
460	A	6	5	1.10	37	0.135
461	A	11	10	1.05	40	0.250
462	A	11	10	1.03	40	0.250
463	A	11	10	1.01	40	0.250
464	A	11	10	1.01	40	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
465	A	11	10	1.05	40	0.250
466	A	7	6	1.10	40	0.150
467	A	9	8	1.03	40	0.200
468	A	11	10	0.99	40	0.250
469	A	13	12	0.97	40	0.300
470	A	8	7	1.01	40	0.175
471	A	5	4	1.04	40	0.100
472	A	5	4	1.00	38	0.105
473	A	1	1	1.00	37	0.027
474	A	6	5	0.99	40	0.125
475	A	7	6	0.90	40	0.150
476	A	8	7	0.85	40	0.175
477	A	8	7	0.99	40	0.175
478	A	8	7	0.98	40	0.175
479	A	6	5	1.04	40	0.125
480	A	2	2	1.00	40	0.050
481	A	2	2	1.00	38	0.053
482	A	2	2	1.00	37	0.054
483	A	7	6	1.12	40	0.150
484	A	8	7	1.11	40	0.175
485	A	10	9	1.08	40	0.225
486	A	12	11	1.07	40	0.275
487	A	4	4	1.02	40	0.100
488	A	5	5	1.04	40	0.125
489	A	4	4	1.00	23	0.174
490	A	1	1	1.00	23	0.043
491	A	5	5	0.98	21	0.238
492	A	3	3	1.03	20	0.150
493	A	6	5	0.85	23	0.217
494	A	5	5	1.00	23	0.217
495	A	3	3	1.03	23	0.130
496	A	5	5	0.95	23	0.217
497	A	1	1	1.00	23	0.043
498	A	6	6	0.95	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
499	A	4	4	0.97	20	0.200
500	A	7	6	0.73	23	0.261
501	A	6	6	0.95	23	0.261
502	A	4	4	0.97	23	0.174
503	A	3	3	1.05	23	0.130
504	A	1	1	1.00	23	0.043
505	A	4	4	1.06	21	0.190
506	A	2	2	1.00	20	0.100
507	A	5	4	1.00	23	0.174
508	A	5	5	1.02	23	0.217
509	A	3	3	1.03	23	0.130
510	A	3	3	1.09	23	0.130
511	A	1	1	1.00	23	0.043
512	A	5	5	1.02	21	0.238
513	A	3	3	1.09	20	0.150
514	A	6	5	0.85	23	0.217
515	A	6	6	0.97	23	0.261
516	A	4	4	1.01	23	0.174
517	A	4	4	1.00	23	0.174
518	A	1	1	1.00	23	0.043
519	A	6	6	0.97	21	0.286
520	A	4	4	1.00	20	0.200
521	A	7	6	0.72	23	0.261
522	A	7	7	0.94	23	0.304
523	A	5	5	0.95	23	0.217
524	A	4	4	1.05	19	0.211
525	A	3	2	1.02	25	0.080
526	A	3	2	1.24	25	0.080
527	A	3	2	1.02	25	0.080
528	A	6	5	1.02	23	0.217
529	A	4	3	1.18	22	0.136
530	A	3	2	1.03	25	0.080
531	A	3	2	1.01	25	0.080
532	A	3	2	1.04	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
533	A	3	2	1.01	25	0.080
534	A	3	2	1.01	25	0.080
535	A	3	2	1.01	25	0.080
536	A	7	6	0.85	23	0.261
537	A	7	6	0.93	22	0.273
538	A	3	2	1.02	25	0.080
539	A	3	2	1.03	25	0.080
540	A	3	2	1.03	25	0.080
541	A	2	2	1.00	23	0.087
542	A	2	2	1.00	23	0.087
543	A	2	2	1.00	23	0.087
544	A	2	2	1.00	21	0.095
545	A	2	2	1.00	20	0.100
546	A	2	2	1.00	23	0.087
547	A	2	2	1.00	23	0.087
548	A	3	3	1.00	29	0.103
549	A	3	3	1.00	29	0.103
550	A	3	3	1.00	29	0.103
551	A	3	3	1.00	27	0.111
552	A	2	2	1.05	22	0.091
553	A	3	3	1.00	29	0.103
554	A	3	3	1.00	29	0.103
555	A	3	3	1.00	29	0.103
556	A	3	3	1.00	29	0.103
557	A	3	3	1.00	29	0.103
558	A	3	3	1.00	29	0.103
559	A	3	3	1.00	29	0.103
560	A	3	3	1.00	29	0.103
561	A	3	3	1.00	29	0.103
562	A	3	3	1.00	29	0.103
563	A	3	3	1.00	27	0.111
564	A	2	2	1.20	22	0.091
565	A	3	3	1.00	29	0.103
566	A	3	3	1.00	29	0.103

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
567	A	3	3	1.00	29	0.103
568	A	3	3	1.00	29	0.103
569	A	3	3	1.00	29	0.103
570	A	3	3	1.00	29	0.103
571	A	3	3	1.00	29	0.103
572	A	3	3	1.00	29	0.103
573	A	3	3	1.00	29	0.103
574	A	3	3	1.00	29	0.103
575	A	3	3	1.00	27	0.111
576	A	2	2	1.24	22	0.091
577	A	3	3	1.00	29	0.103
578	A	3	3	1.00	29	0.103
579	A	13	12	1.11	31	0.387
580	A	10	9	1.10	31	0.290
581	A	10	9	1.10	31	0.290
582	A	5	5	1.08	31	0.161
583	A	3	3	0.97	29	0.103
584	A	4	4	1.08	24	0.167
585	A	10	9	1.34	31	0.290
586	A	8	7	1.05	31	0.226
587	A	10	9	1.10	31	0.290
588	A	4	3	1.01	24	0.125
589	A	2	2	1.00	24	0.083
590	A	2	2	1.00	24	0.083
591	A	2	2	1.00	22	0.091
592	A	2	2	1.00	17	0.118
593	A	5	4	1.02	24	0.167
594	A	6	5	1.34	24	0.208
595	A	7	6	1.24	24	0.250
596	A	2	2	1.00	24	0.083
597	A	2	2	1.00	24	0.083
598	A	2	2	1.00	22	0.091
599	A	2	2	1.00	17	0.118
600	A	5	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
601	A	7	6	1.19	24	0.250
602	A	10	9	1.21	24	0.375
603	A	6	5	1.02	26	0.192
604	A	1	1	1.00	22	0.045
605	A	7	6	1.00	28	0.214
606	A	6	5	1.00	28	0.179
607	A	2	2	1.00	28	0.071
608	A	4	4	0.97	28	0.143
609	A	5	5	0.95	28	0.179
610	A	2	2	1.00	28	0.071
611	A	2	2	1.00	28	0.071
612	A	2	2	1.00	28	0.071
613	A	2	2	1.00	28	0.071
614	A	7	6	0.92	28	0.214
615	A	4	4	0.97	28	0.143
616	A	2	2	1.00	28	0.071
617	A	2	2	0.99	28	0.071
618	A	4	3	1.00	20	0.150
619	A	7	7	1.09	26	0.269
620	A	2	2	1.00	30	0.067
621	A	8	7	1.05	29	0.241
622	A	16	15	1.54	28	0.536
623	A	12	11	1.64	28	0.393
624	A	10	9	1.81	26	0.346
625	A	9	8	1.91	21	0.381
626	F	0	0	N/A	0.000	N/A
627	B	13	12	2.38	28	0.429
628	B	19	18	2.29	28	0.643
629	A	13	12	1.63	28	0.429
630	A	12	11	1.73	28	0.393
631	A	8	7	1.92	26	0.269
632	A	7	6	1.98	21	0.286
633	B	13	12	3.16	28	0.429

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
634	B	14	13	2.25	28	0.464
635	B	20	19	2.24	28	0.679
636	A	12	11	1.73	28	0.393
637	A	10	9	1.86	28	0.321
638	B	7	6	2.02	26	0.231
639	B	5	4	4.29	21	0.190
640	B	9	8	3.49	28	0.286
641	B	14	13	2.24	28	0.464
642	B	21	20	2.23	28	0.714
643	B	2	2	3.08	28	0.071
644	B	2	2	2.77	28	0.071
645	A	10	9	1.82	28	0.321
646	A	8	7	1.96	28	0.250
647	B	5	4	2.13	26	0.154
648	A	3	2	1.71	21	0.095
649	B	6	5	5.19	28	0.179
650	B	14	13	2.13	28	0.464
651	B	19	18	2.23	28	0.643
652	B	2	2	3.04	28	0.071
653	B	2	2	2.19	28	0.071
654	B	6	5	7.51	28	0.179
655	A	3	2	1.00	30	0.067
656	A	4	4	1.00	26	0.154
657	A	4	4	1.02	46	0.087
658	A	3	3	1.00	46	0.065
659	A	2	2	0.98	44	0.045
660	A	1	1	1.00	39	0.026
661	A	3	2	1.00	46	0.043
662	A	4	3	1.00	46	0.065
663	A	5	4	1.03	46	0.087
664	A	6	5	1.07	46	0.109
665	A	4	4	1.02	46	0.087
666	A	3	3	1.03	46	0.065
667	A	2	2	1.00	44	0.045

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
668	A	1	1	1.00	39	0.026
669	A	4	3	1.00	46	0.065
670	A	5	4	1.06	46	0.087
671	A	6	5	1.08	46	0.109
672	A	4	4	1.05	46	0.087
673	A	3	3	1.03	46	0.065
674	A	2	2	1.00	44	0.045
675	A	1	1	1.00	39	0.026
676	A	5	4	1.08	46	0.087
677	A	6	5	1.10	46	0.109
678	A	7	6	1.09	46	0.130
679	A	5	5	1.04	46	0.109
680	A	4	4	1.02	46	0.087
681	A	3	3	1.00	46	0.065
682	A	2	2	0.98	44	0.045
683	A	1	1	1.00	39	0.026
684	A	4	3	1.00	46	0.065
685	A	4	3	1.00	46	0.065
686	A	5	4	0.98	46	0.087
687	A	6	5	1.02	46	0.109
688	A	7	6	1.04	46	0.130
689	A	5	5	1.04	46	0.109
690	A	4	4	1.02	46	0.087
691	A	3	3	1.00	46	0.065
692	A	2	2	0.98	44	0.045
693	A	1	1	1.00	39	0.026
694	A	5	4	1.03	46	0.087
695	A	5	4	1.03	46	0.087
696	A	5	4	0.99	46	0.087
697	A	6	5	1.00	46	0.109
698	A	7	6	1.03	46	0.130
699	A	8	7	1.05	46	0.152
700	A	5	5	1.04	46	0.109
701	A	4	4	1.02	46	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
702	A	3	3	1.00	46	0.065
703	A	2	2	0.98	44	0.045
704	A	1	1	1.00	39	0.026
705	A	6	5	1.04	46	0.109
706	A	6	5	1.04	46	0.109
707	A	6	5	1.00	46	0.109
708	A	6	5	1.01	46	0.109
709	A	7	6	1.01	46	0.130
710	A	8	7	1.03	46	0.152
711	A	9	8	1.05	46	0.174
712	A	7	6	1.04	48	0.125
713	A	6	5	1.01	48	0.104
714	A	5	4	1.00	48	0.083
715	A	4	3	1.00	48	0.062
716	A	1	1	1.00	48	0.021
717	A	2	2	1.00	48	0.042
718	A	3	3	1.07	48	0.062
719	A	4	4	1.10	48	0.083
720	A	7	6	1.04	48	0.125
721	A	6	5	1.03	48	0.104
722	A	5	4	1.00	48	0.083
723	A	1	1	1.00	48	0.021
724	A	2	2	1.00	48	0.042
725	A	3	3	1.07	48	0.062
726	A	4	4	1.10	48	0.083
727	A	7	6	1.05	48	0.125
728	A	6	5	1.04	48	0.104
729	A	1	1	1.00	48	0.021
730	A	2	2	1.00	48	0.042
731	A	3	3	1.06	48	0.062
732	A	4	4	1.10	48	0.083
733	A	8	7	1.03	48	0.146
734	A	7	6	1.02	48	0.125
735	A	6	5	1.00	48	0.104

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
736	A	5	4	1.00	48	0.083
737	A	5	4	1.00	48	0.083
738	A	1	1	1.00	48	0.021
739	A	2	2	1.00	48	0.042
740	A	3	3	1.07	48	0.062
741	A	4	4	1.10	48	0.083
742	A	8	7	1.02	48	0.146
743	A	7	6	1.01	48	0.125
744	A	6	5	1.00	48	0.104
745	A	6	5	1.02	48	0.104
746	A	6	5	1.02	48	0.104
747	A	1	1	1.00	48	0.021
748	A	2	2	1.00	48	0.042
749	A	3	3	1.07	48	0.062
750	A	4	4	1.10	48	0.083
751	A	9	8	1.03	48	0.167
752	A	8	7	1.02	48	0.146
753	A	7	6	1.02	48	0.125
754	A	7	6	1.02	48	0.125
755	A	7	6	1.02	48	0.125
756	A	7	6	1.02	48	0.125
757	A	1	1	1.00	48	0.021
758	A	2	2	1.00	48	0.042
759	A	3	3	1.07	48	0.062
760	A	4	4	1.10	48	0.083
761	A	3	3	1.17	46	0.065
762	A	3	3	1.06	46	0.065
763	A	3	3	1.13	46	0.065
764	A	3	3	1.15	46	0.065
765	A	3	3	1.17	46	0.065
766	A	3	3	1.17	46	0.065
767	A	3	3	1.04	44	0.068
768	A	4	4	0.94	44	0.091
769	A	3	3	0.96	44	0.068

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
770	A	2	2	1.00	42	0.048
771	A	1	1	1.00	37	0.027
772	A	2	2	1.00	44	0.045
773	A	2	2	1.00	44	0.045
774	A	2	2	1.00	44	0.045
775	A	3	3	1.00	46	0.065
776	A	3	3	1.00	46	0.065
777	A	3	3	1.00	46	0.065
778	A	3	3	1.00	46	0.065
779	A	3	3	1.00	46	0.065
780	A	1	1	1.00	47	0.021
781	A	3	3	1.04	73	0.041
782	A	4	4	1.04	46	0.087
783	A	6	6	0.87	46	0.130
784	A	5	5	0.88	46	0.109
785	A	4	4	0.89	46	0.087
786	A	3	3	0.90	44	0.068
787	A	2	2	1.01	39	0.051
788	A	4	3	1.00	46	0.065
789	A	4	3	1.00	46	0.065
790	A	5	4	0.97	46	0.087
791	A	6	5	0.95	46	0.109
792	A	14	14	1.08	32	0.438
793	A	10	10	1.07	32	0.312
794	A	5	5	1.03	30	0.167
795	A	5	4	1.00	32	0.125
796	A	7	6	0.99	32	0.188
797	A	10	10	1.07	32	0.312
798	A	6	6	1.04	32	0.188
799	A	4	4	1.00	30	0.133
800	A	7	6	0.96	32	0.188
801	A	9	8	0.97	32	0.250
802	A	3	3	1.00	25	0.120
803	A	4	4	1.00	25	0.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
804	A	4	4	1.00	28	0.143
805	A	2	2	1.00	28	0.071
806	A	2	2	1.00	28	0.071
807	A	2	2	1.00	26	0.077
808	A	2	2	1.00	21	0.095
809	A	2	2	1.00	28	0.071
810	A	2	2	1.00	28	0.071
811	A	3	3	1.05	28	0.107
812	A	3	3	1.08	28	0.107
813	A	5	5	0.97	28	0.179
814	A	2	2	1.00	25	0.080
815	A	2	2	1.00	27	0.074
816	A	2	2	1.00	27	0.074
817	A	2	2	1.00	27	0.074
818	A	2	2	1.00	27	0.074
819	A	2	2	1.00	27	0.074
820	A	2	2	1.00	27	0.074
821	A	2	2	1.00	25	0.080
822	A	2	2	1.00	20	0.100
823	A	5	4	1.01	27	0.148
824	A	5	4	1.34	27	0.148
825	A	7	6	1.23	27	0.222
826	A	2	2	1.00	27	0.074
827	A	2	2	1.00	27	0.074
828	A	2	2	1.00	25	0.080
829	A	2	2	1.00	20	0.100
830	A	5	4	1.00	27	0.148
831	A	7	6	1.17	27	0.222
832	A	10	9	1.19	27	0.333
833	A	5	5	1.00	25	0.200
834	A	6	5	1.02	29	0.172
835	A	8	7	0.75	29	0.241
836	A	7	6	0.85	29	0.207
837	A	6	5	1.02	29	0.172

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
838	A	6	5	1.02	29	0.172
839	A	6	5	1.12	29	0.172
840	A	4	4	1.06	29	0.138
841	A	5	5	0.92	29	0.172
842	A	7	6	0.89	29	0.207
843	A	8	7	0.84	38	0.184
844	A	7	6	0.89	38	0.158
845	A	6	5	1.02	38	0.132
846	A	6	5	1.03	38	0.132
847	A	6	5	1.18	38	0.132
848	A	4	4	1.11	38	0.105
849	A	4	4	1.04	38	0.105
850	A	2	2	1.00	31	0.065
851	A	2	2	1.00	31	0.065
852	A	2	2	1.00	31	0.065
853	A	2	2	1.00	31	0.065
854	A	12	11	1.06	29	0.379
855	A	10	9	1.07	29	0.310
856	A	8	7	1.05	27	0.259
857	A	7	6	1.04	22	0.273
858	A	9	8	1.05	29	0.276
859	A	2	2	1.00	29	0.069
860	A	2	2	1.00	29	0.069
861	A	2	2	1.00	29	0.069
862	A	14	13	1.04	29	0.448
863	A	12	11	1.05	29	0.379
864	A	10	9	1.05	27	0.333
865	A	9	8	1.08	22	0.364
866	A	10	9	0.92	29	0.310
867	A	2	2	1.00	29	0.069
868	A	2	2	1.00	29	0.069
869	A	12	11	0.94	29	0.379
870	A	12	11	1.07	29	0.379
871	A	10	9	1.06	29	0.310

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
872	A	8	7	1.06	29	0.241
873	A	6	5	1.00	27	0.185
874	A	3	2	1.00	22	0.091
875	A	2	2	1.00	29	0.069
876	A	2	2	1.00	29	0.069
877	A	2	2	1.00	29	0.069
878	A	10	9	1.08	29	0.310
879	A	8	7	1.03	29	0.241
880	A	5	4	1.00	29	0.138
881	A	5	4	1.00	27	0.148
882	A	5	4	1.00	22	0.182
883	A	2	2	1.00	29	0.069
884	A	2	2	1.00	29	0.069
885	A	2	2	1.00	29	0.069
886	A	14	13	1.03	31	0.419
887	A	12	11	1.03	31	0.355
888	A	10	9	1.03	29	0.310
889	A	8	7	1.02	24	0.292
890	A	12	11	1.16	31	0.355
891	A	12	11	1.18	31	0.355
892	A	18	17	1.55	31	0.548
893	A	12	11	1.03	31	0.355
894	A	10	9	1.03	31	0.290
895	A	7	6	1.01	29	0.207
896	A	6	5	1.01	24	0.208
897	A	11	10	1.18	31	0.323
898	A	12	11	1.18	31	0.355
899	A	19	18	1.54	31	0.581
900	A	11	10	1.03	31	0.323
901	A	9	8	1.02	31	0.258
902	A	7	6	1.01	29	0.207
903	A	3	2	1.00	24	0.083
904	A	9	8	1.26	31	0.258
905	A	13	12	0.89	31	0.387

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
906	A	20	19	0.91	31	0.613
907	A	2	2	1.00	31	0.065
908	A	2	2	1.00	31	0.065
909	A	8	7	1.03	31	0.226
910	A	7	6	1.01	31	0.194
911	A	5	4	1.00	29	0.138
912	A	3	2	1.00	24	0.083
913	A	6	5	1.44	31	0.161
914	A	13	12	0.88	31	0.387
915	A	18	17	1.47	31	0.548
916	A	2	2	1.00	31	0.065
917	A	2	2	1.00	31	0.065
918	A	1	1	1.00	33	0.030
919	A	3	2	1.00	33	0.061
920	A	2	2	1.00	25	0.080
921	A	2	2	1.00	23	0.087
922	A	2	2	1.00	25	0.080
923	A	4	4	1.11	25	0.160
924	A	4	4	1.02	25	0.160
925	A	2	2	1.00	27	0.074
926	A	2	2	1.00	25	0.080
927	A	2	2	1.00	27	0.074
928	A	3	3	1.12	27	0.111
929	A	4	4	1.09	27	0.148
930	A	2	2	1.00	27	0.074
931	A	2	2	1.00	27	0.074
932	A	2	2	1.00	27	0.074
933	A	2	2	1.00	25	0.080
934	A	2	2	1.00	20	0.100
935	A	2	2	1.00	27	0.074
936	A	2	2	1.00	27	0.074
937	A	4	4	1.02	27	0.148
938	A	4	4	1.03	27	0.148
939	A	4	4	1.01	27	0.148

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
940	A	4	4	0.99	25	0.160
941	A	4	4	1.01	20	0.200
942	A	2	2	1.00	27	0.074
943	A	2	2	1.00	27	0.074
944	A	5	5	1.01	27	0.185
945	A	6	5	0.99	29	0.172
946	A	4	3	1.00	27	0.111
947	A	3	2	1.00	22	0.091
948	N/A	1	0	1.00	29	0.000
949	A	6	5	1.01	29	0.172
950	A	4	3	1.00	27	0.111
951	A	3	2	1.00	22	0.091
952	N/A	1	0	1.00	29	0.000
953	A	4	4	0.96	25	0.160
954	A	6	5	0.99	27	0.185
955	A	4	3	1.00	25	0.120
956	A	3	2	1.00	20	0.100
957	N/A	1	0	1.00	27	0.000
958	A	7	6	1.20	27	0.222

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx$

3.1.1 Optimal result

Integrand size = 25, antiderivative size = 132

$$\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx=\frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{d^2(d^2-e^2x^2)^{3/2}}{3e^3}-\frac{dx(d^2-e^2x^2)^{3/2}}{4e^2}+\frac{(d^2-e^2x^2)^{5/2}}{5e^3}+\frac{d^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

output
```
-1/3*d^2*(-e^2*x^2+d^2)^(3/2)/e^3-1/4*d*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/5*(-e
^2*x^2+d^2)^(5/2)/e^3+1/8*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/8*d^3
*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

3.1.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.78

$$\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx$$
$$=\frac{\sqrt{d^2-e^2x^2}(-16d^4-15d^3ex-8d^2e^2x^2+30de^3x^3+24e^4x^4)-30d^5\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{120e^3}$$

input
```
Integrate[x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2],x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-16*d^4 - 15*d^3*e*x - 8*d^2*e^2*x^2 + 30*d*e^3*x^3
+ 24*e^4*x^4) - 30*d^5*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(1
20*e^3)
```

3.1.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {533, 27, 533, 27, 455, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx$$

$$\Big\downarrow \text{ 533}$$

$$\frac{\int dex(2d+5ex)\sqrt{d^2-e^2x^2}dx}{5e^2}-\frac{x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\int x(2d+5ex)\sqrt{d^2-e^2x^2}dx}{5e}-\frac{x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}$$

$$\Big\downarrow \text{ 533}$$

$$\frac{d\left(\frac{\int de(5d+8ex)\sqrt{d^2-e^2x^2}dx}{4e^2}-\frac{5x\left(d^2-e^2x^2\right)^{3/2}}{4e}\right)}{5e}-\frac{x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\left(\frac{d\int(5d+8ex)\sqrt{d^2-e^2x^2}dx}{4e}-\frac{5x\left(d^2-e^2x^2\right)^{3/2}}{4e}\right)}{5e}-\frac{x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{d\left(\frac{d\left(5d\int\sqrt{d^2-e^2x^2}dx-\frac{8\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)}{4e}-\frac{5x\left(d^2-e^2x^2\right)^{3/2}}{4e}\right)}{5e}-\frac{x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{d\left(\frac{d\left(5d\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)-\frac{8\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)}{4e}-\frac{5x\left(d^2-e^2x^2\right)^{3/2}}{4e}\right)}{5e}-\frac{x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{d\left(\frac{d\left(5d\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)-\frac{8\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)}{4e}-\frac{5x\left(d^2-e^2x^2\right)^{3/2}}{4e}\right)}{5e}-\frac{x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{d\left(\frac{d\left(5d\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)-\frac{8\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)}{4e}-\frac{5x\left(d^2-e^2x^2\right)^{3/2}}{4e}\right)}{5e}-\frac{x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}$$

input
```
Int[x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/5*(x^2*(d^2 - e^2*x^2)^(3/2))/e + (d*((-5*x*(d^2 - e^2*x^2)^(3/2))/(4*e
) + (d*((-8*(d^2 - e^2*x^2)^(3/2))/(3*e) + 5*d*((x*Sqrt[d^2 - e^2*x^2])/2
+ (d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e))))/(4*e)))/(5*e)
```

3.1.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.1.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{\left(-24 e^{4} x^{4}-30 d \,e^{3} x^{3}+8 d^{2} e^{2} x^{2}+15 d^{3} e x +16 d^{4}\right) \sqrt{-e^{2} x^{2}+d^{2}}}{120 e^{3}}+\frac{d^{5} \arctan \left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{8 e^{2} \sqrt{e^{2}}}$
default	$e \left(-\frac{x^{2} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{5 e^{2}}-\frac{2 d^{2} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{15 e^{4}}\right)+d \left(-\frac{x \left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{4 e^{2}}+\frac{d^{2} \left(\frac{x \sqrt{-e^{2} x^{2}+d^{2}}}{2}+\frac{d^{2} \arctan \left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{2 \sqrt{e^{2}}}\right)}{4 e^{2}}\right)$

input
```
int(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120*(-24*e^4*x^4-30*d*e^3*x^3+8*d^2*e^2*x^2+15*d^3*e*x+16*d^4)/e^3*(-e^
2*x^2+d^2)^(1/2)+1/8*d^5/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^
2)^(1/2))
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.72

$$\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx = -\frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(24\,e^4x^4+30\,de^3x^3-8\,d^2e^2x^2-15\,d^3ex-16\,d^4\right)\sqrt{-e^2x^2+d^2}}{120\,e^3}$$

input
```
integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/120*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (24*e^4*x^4 + 3
0*d*e^3*x^3 - 8*d^2*e^2*x^2 - 15*d^3*e*x - 16*d^4)*sqrt(-e^2*x^2 + d^2))/e
^3
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

$$\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx$$
$$=\begin{cases}\frac{d^5\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{8e^2}+\sqrt{d^2-e^2x^2}\left(-\frac{2d^4}{15e^3}-\frac{d^3x}{8e^2}-\frac{d^2x^2}{15e}+\frac{dx^3}{4}+\frac{ex^4}{5}\right) & \text{for } e^2\neq 0\\ \left(\frac{dx^3}{3}+\frac{ex^4}{4}\right)\sqrt{d^2} & \text{otherwise}\end{cases}$$

input
```
integrate(x**2*(e*x+d)*(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Piecewise((d**5*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*
x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(8*e*
*2) + sqrt(d**2 - e**2*x**2)*(-2*d**4/(15*e**3) - d**3*x/(8*e**2) - d**2*x
**2/(15*e) + d*x**3/4 + e*x**4/5), Ne(e**2, 0)), ((d*x**3/3 + e*x**4/4)*sq
rt(d**2), True))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.88

$$\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx = \frac{d^5 \arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\sqrt{e^2}e^2} + \frac{\sqrt{-e^2x^2+d^2}d^3x}{8\,e^2} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}x^2}{5\,e} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}dx}{4\,e^2} - \frac{2\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2}{15\,e^3}$$

input
```
integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
1/8*d^5*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) + 1/8*sqrt(-e^2*x^2 +
d^2)*d^3*x/e^2 - 1/5*(-e^2*x^2 + d^2)^(3/2)*x^2/e - 1/4*(-e^2*x^2 + d^2)^(
3/2)*d*x/e^2 - 2/15*(-e^2*x^2 + d^2)^(3/2)*d^2/e^3
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.63

$$\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx = \frac{d^5 \arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,e^2|e|} + \frac{1}{120}\sqrt{-e^2x^2+d^2}\left(\left(2\left(3\,(4\,ex+5\,d)x - \frac{4\,d^2}{e}\right)x - \frac{15\,d^3}{e^2}\right)x - \frac{16\,d^4}{e^3}\right)$$

input
```
integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*d^5*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) + 1/120*sqrt(-e^2*x^2 + d
^2)*((2*(3*(4*e*x + 5*d)*x - 4*d^2/e)*x - 15*d^3/e^2)*x - 16*d^4/e^3)
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx=\int x^2\,\sqrt{d^2-e^2\,x^2}\,(d+e\,x)\;dx$$

input
```
int(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x),x)
```

output
```
int(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x), x)
```

3.2 $\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

3.2.1 Optimal result

Integrand size = 25, antiderivative size = 201

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4}-\frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3}-\frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2}-\frac{x^4(d^2-e^2x^2)^{5/2}}{9e}-\frac{d^3(128d+315ex)\left(d^2-e^2x^2\right)^{5/2}}{5040e^5}+\frac{3d^9\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}$$

output
```
1/64*d^5*x*(-e^2*x^2+d^2)^(3/2)/e^4-4/63*d^2*x^2*(-e^2*x^2+d^2)^(5/2)/e^3-
1/8*d*x^3*(-e^2*x^2+d^2)^(5/2)/e^2-1/9*x^4*(-e^2*x^2+d^2)^(5/2)/e-1/5040*d
^3*(315*e*x+128*d)*(-e^2*x^2+d^2)^(5/2)/e^5+3/128*d^9*arctan(e*x/(-e^2*x^2
+d^2)^(1/2))/e^5+3/128*d^7*x*(-e^2*x^2+d^2)^(1/2)/e^4
```

3.2.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.73

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=-\frac{\sqrt{d^2-e^2x^2}(1024d^8+945d^7ex+512d^6e^2x^2+630d^5e^3x^3+384d^4e^4x^4-7560d^3e^5x^5-6400d^2e^6x^6+5040}{40320e^5}$$

input
```
Integrate[x^4*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

output
```
-1/40320*(Sqrt[d^2 - e^2*x^2]*(1024*d^8 + 945*d^7*e*x + 512*d^6*e^2*x^2 +
630*d^5*e^3*x^3 + 384*d^4*e^4*x^4 - 7560*d^3*e^5*x^5 - 6400*d^2*e^6*x^6 +
5040*d*e^7*x^7 + 4480*e^8*x^8) + 1890*d^9*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d
^2 - e^2*x^2])])/e^5
```

3.2.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.22, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {533, 27, 533, 27, 533, 27, 533, 27, 455, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}dx$$

$$\Big\downarrow \text{533}$$

$$\frac{\int dex^3(4d+9ex)\left(d^2-e^2x^2\right)^{3/2}dx}{9e^2}-\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int x^3(4d+9ex)\left(d^2-e^2x^2\right)^{3/2}dx}{9e}-\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}$$

$$\Big\downarrow \text{533}$$

$$\frac{d\left(\frac{\int dex^2(27d+32ex)\left(d^2-e^2x^2\right)^{3/2}dx}{8e^2}-\frac{9x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}\right)}{9e}-\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\left(\frac{d\int x^2(27d+32ex)\left(d^2-e^2x^2\right)^{3/2}dx}{8e}-\frac{9x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}\right)}{9e}-\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}$$

$$\Big\downarrow \text{533}$$

$$\frac{d\left(\frac{d\left(\frac{\int dex(64d+189ex)\left(d^2-e^2x^2\right)^{3/2}dx}{7e^2}-\frac{32x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}\right)}{8e}-\frac{9x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}\right)}{9e}-\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}$$

$$\Big\downarrow \; 27$$

$$\frac{d\left(\frac{d\left(\frac{d\int x(64d+189ex)\left(d^2-e^2x^2\right)^{3/2}dx}{7e}-\frac{32x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}\right)}{8e}-\frac{9x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}\right)}{9e}-\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}$$

$$\Big\downarrow \; 533$$

$$\frac{d\left(\frac{d\left(\frac{d\left(\frac{\int 3de(63d+128ex)\left(d^2-e^2x^2\right)^{3/2}dx}{6e^2}-\frac{63x\left(d^2-e^2x^2\right)^{5/2}}{2e}\right)}{7e}-\frac{32x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}\right)}{8e}-\frac{9x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}\right)}{9e}-\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}$$

$$\Big\downarrow \; 27$$

$$\frac{d\left(\frac{d\left(\frac{d\left(\frac{d\int (63d+128ex)\left(d^2-e^2x^2\right)^{3/2}dx}{2e}-\frac{63x\left(d^2-e^2x^2\right)^{5/2}}{2e}\right)}{7e}-\frac{32x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}\right)}{8e}-\frac{9x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}\right)}{9e}-\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}$$

$$\Big\downarrow \; 455$$

3.2. $\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

$$\frac{d\left(\frac{d\left(\frac{d\left(\frac{d\left(63d\int \left(d^2-e^2x^2\right)^{3/2}dx-\frac{128\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{2e}-\frac{63x\left(d^2-e^2x^2\right)^{5/2}}{2e}\right)}{7e}-\frac{32x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}\right)}{8e}-\frac{9x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}\right)}{9e}-\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}$$

$$\Big\downarrow \; 211$$

$$\frac{d\left(\frac{d\left(\frac{d\left(\frac{d\left(63d\left(\frac{3}{4}d^2\int \sqrt{d^2-e^2x^2}dx+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{128\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{2e}-\frac{63x\left(d^2-e^2x^2\right)^{5/2}}{2e}\right)}{7e}-\frac{32x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}\right)}{8e}-\frac{9x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}\right)}{9e}-\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}$$

$$\Big\downarrow \; 211$$

3.2. $\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

$$\frac{d\left(\frac{d\left(\frac{d\left(\frac{d\left(63d\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{128\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{2e}-\frac{63x\left(d^2-e^2x^2\right)^{5/2}}{2e}\right)}{7e}-\frac{32x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}\right)}{8e}-\right)}{9e}$$

$$\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}$$

$\downarrow$ 224

$$\frac{d\left(\frac{d\left(\frac{d\left(\frac{d\left(63d\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{128\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{2e}-\frac{63x\left(d^2-e^2x^2\right)^{5/2}}{2e}\right)}{7e}-\frac{32x^2\left(d^2-e^2\right.}{7e}}{8e}}{9e}$$

$$\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}$$

$\downarrow$ 216

3.2. $\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

$$\frac{d\left(\frac{d\left(\frac{d\left(\frac{d\left(63d\left(\frac{3}{4}d^2\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{128\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{2e}-\frac{63x\left(d^2-e^2x^2\right)^{5/2}}{2e}\right)}{7e}-\frac{32x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}}{8e}}{9e}$$

$$\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}$$

input
```
Int[x^4*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

output
```
-1/9*(x^4*(d^2 - e^2*x^2)^(5/2))/e + (d*((-9*x^3*(d^2 - e^2*x^2)^(5/2))/(8
*e) + (d*((-32*x^2*(d^2 - e^2*x^2)^(5/2))/(7*e) + (d*((-63*x*(d^2 - e^2*x^
2)^(5/2))/(2*e) + (d*((-128*(d^2 - e^2*x^2)^(5/2))/(5*e) + 63*d*((x*(d^2 -
 e^2*x^2)^(3/2))/4 + (3*d^2*((x*Sqrt[d^2 - e^2*x^2])/2 + (d^2*ArcTan[(e*x)
/Sqrt[d^2 - e^2*x^2]])/(2*e)))/4)))/(2*e)))/(7*e)))/(8*e)))/(9*e)
```

3.2.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.2.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{\left(4480e^{8}x^{8}+5040d\,e^{7}x^{7}-6400d^{2}e^{6}x^{6}-7560d^{3}e^{5}x^{5}+384d^{4}x^{4}e^{4}+630d^{5}e^{3}x^{3}+512d^{6}e^{2}x^{2}+945d^{7}ex+1024d^{8}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{40320e^{5}}+\frac{3d^{9}}{}$
default	$e\left(-\frac{x^{4}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{9e^{2}}+\frac{4d^{2}\left(-\frac{x^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{7e^{2}}-\frac{2d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{35e^{4}}\right)}{9e^{2}}\right)+d\left(-\frac{x^{3}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{8e^{2}}+\frac{3d^{2}\left(-\frac{x\left(-e^{2}x^{2}+d^{2}\right)}{6e^{2}}\right.}{}\right.$

input
```
int(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/40320*(4480*e^8*x^8+5040*d*e^7*x^7-6400*d^2*e^6*x^6-7560*d^3*e^5*x^5+38
4*d^4*e^4*x^4+630*d^5*e^3*x^3+512*d^6*e^2*x^2+945*d^7*e*x+1024*d^8)/e^5*(-
e^2*x^2+d^2)^(1/2)+3/128*d^9/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^
2+d^2)^(1/2))
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.69

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx =$$

$$-\frac{1890\,d^9\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(4480\,e^8x^8+5040\,de^7x^7-6400\,d^2e^6x^6-7560\,d^3e^5x^5+384\,d^4e^4x^4+63\right.}{40320\,e^5}$$

input
```
integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/40320*(1890*d^9*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (4480*e^8*x
^8 + 5040*d*e^7*x^7 - 6400*d^2*e^6*x^6 - 7560*d^3*e^5*x^5 + 384*d^4*e^4*x^
4 + 630*d^5*e^3*x^3 + 512*d^6*e^2*x^2 + 945*d^7*e*x + 1024*d^8)*sqrt(-e^2*
x^2 + d^2))/e^5
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\begin{cases}\frac{3d^9\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{128e^4}+\sqrt{d^2-e^2x^2}\left(-\frac{8d^8}{315e^5}-\frac{3d^7x}{128e^4}-\frac{4d^6x^2}{315e^3}-\frac{d^5x^3}{64e^2}\right. \\ \left(\frac{dx^5}{5}+\frac{ex^6}{6}\right)\left(d^2\right)^{\frac{3}{2}}\end{cases}$$

input
```
integrate(x**4*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Piecewise((3*d**9*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**
2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(12
8*e**4) + sqrt(d**2 - e**2*x**2)*(-8*d**8/(315*e**5) - 3*d**7*x/(128*e**4)
 - 4*d**6*x**2/(315*e**3) - d**5*x**3/(64*e**2) - d**4*x**4/(105*e) + 3*d*
*3*x**5/16 + 10*d**2*e*x**6/63 - d*e**2*x**7/8 - e**3*x**8/9), Ne(e**2, 0)
), ((d*x**5/5 + e*x**6/6)*(d**2)**(3/2), True))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.94

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^4}{9\,e}+\frac{3\,d^9\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{128\,\sqrt{e^2}e^4}+\frac{3\,\sqrt{-e^2x^2+d^2}d^7x}{128\,e^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx^3}{8\,e^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5x}{64\,e^4}-\frac{4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2x^2}{63\,e^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3x}{16\,e^4}-\frac{8\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4}{315\,e^5}$$

input
```
integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/9*(-e^2*x^2 + d^2)^(5/2)*x^4/e + 3/128*d^9*arcsin(e^2*x/(d*sqrt(e^2)))/
(sqrt(e^2)*e^4) + 3/128*sqrt(-e^2*x^2 + d^2)*d^7*x/e^4 - 1/8*(-e^2*x^2 + d
^2)^(5/2)*d*x^3/e^2 + 1/64*(-e^2*x^2 + d^2)^(3/2)*d^5*x/e^4 - 4/63*(-e^2*x
^2 + d^2)^(5/2)*d^2*x^2/e^3 - 1/16*(-e^2*x^2 + d^2)^(5/2)*d^3*x/e^4 - 8/31
5*(-e^2*x^2 + d^2)^(5/2)*d^4/e^5
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.65

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{3\,d^9\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{128\,e^4|e|}$$
$$-\frac{1}{40320}\sqrt{-e^2x^2+d^2}\left(\frac{1024\,d^8}{e^5}+\left(\frac{945\,d^7}{e^4}+2\left(\frac{256\,d^6}{e^3}+\left(\frac{315\,d^5}{e^2}+4\left(\frac{48\,d^4}{e}-5\left(189\,d^3+2\left(80\,d^2e-7\right.\right.\right.\right.\right.\right.\right.$$

input
```
integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
3/128*d^9*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^4*abs(e)) - 1/40320*sqrt(-e^2*x^2
 + d^2)*(1024*d^8/e^5 + (945*d^7/e^4 + 2*(256*d^6/e^3 + (315*d^5/e^2 + 4*(
48*d^4/e - 5*(189*d^3 + 2*(80*d^2*e - 7*(8*e^3*x + 9*d*e^2)*x)*x)*x)*x)*x)
*x)*x)
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\int x^4\left(d^2-e^2\,x^2\right)^{3/2}(d+e\,x)\,dx$$

input
```
int(x^4*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
```

output
```
int(x^4*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

3.3 $\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

3.3.1 Optimal result

Integrand size = 25, antiderivative size = 172

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}+\frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3}-\frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2}-\frac{x^3(d^2-e^2x^2)^{5/2}}{8e}-\frac{d^2(32d+35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4}+\frac{3d^8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^4}$$

output
```
1/64*d^4*x*(-e^2*x^2+d^2)^(3/2)/e^3-1/7*d*x^2*(-e^2*x^2+d^2)^(5/2)/e^2-1/8
*x^3*(-e^2*x^2+d^2)^(5/2)/e-1/560*d^2*(35*e*x+32*d)*(-e^2*x^2+d^2)^(5/2)/e
^4+3/128*d^8*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4+3/128*d^6*x*(-e^2*x^2+d^
2)^(1/2)/e^3
```

3.3.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.79

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=-\frac{\sqrt{d^2-e^2x^2}(256d^7+105d^6ex+128d^5e^2x^2+70d^4e^3x^3-1024d^3e^4x^4-840d^2e^5x^5+640de^6x^6+560e^7x^7)}{4480e^4}$$

input
```
Integrate[x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

output
```
-1/4480*(Sqrt[d^2 - e^2*x^2]*(256*d^7 + 105*d^6*e*x + 128*d^5*e^2*x^2 + 70
*d^4*e^3*x^3 - 1024*d^3*e^4*x^4 - 840*d^2*e^5*x^5 + 640*d*e^6*x^6 + 560*e^
7*x^7) + 210*d^8*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^4
```

3.3.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.23, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {533, 27, 533, 27, 533, 27, 455, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}dx$$

$$\Big\downarrow \ 533$$

$$\frac{\int dex^2(3d+8ex)\left(d^2-e^2x^2\right)^{3/2}dx}{8e^2}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \ 27$$

$$\frac{d\int x^2(3d+8ex)\left(d^2-e^2x^2\right)^{3/2}dx}{8e}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \ 533$$

$$\frac{d\left(\frac{\int dex(16d+21ex)\left(d^2-e^2x^2\right)^{3/2}dx}{7e^2}-\frac{8x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}\right)}{8e}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \ 27$$

$$\frac{d\left(\frac{d\int x(16d+21ex)\left(d^2-e^2x^2\right)^{3/2}dx}{7e}-\frac{8x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}\right)}{8e}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \ 533$$

$$\frac{d\left(\frac{d\left(\frac{\int 3de(7d+32ex)\left(d^2-e^2x^2\right)^{3/2}dx}{6e^2}-\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{2e}\right)}{7e}-\frac{8x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}\right)}{8e}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \; 27$$

$$\frac{d\left(\frac{d\left(\frac{d\int (7d+32ex)\left(d^2-e^2x^2\right)^{3/2}dx}{2e}-\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{2e}\right)}{7e}-\frac{8x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}\right)}{8e}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \; 455$$

$$\frac{d\left(\frac{d\left(\frac{d\left(7d\int \left(d^2-e^2x^2\right)^{3/2}dx-\frac{32\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{2e}-\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{2e}\right)}{7e}-\frac{8x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}\right)}{8e}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \; 211$$

$$\frac{d\left(\frac{d\left(\frac{d\left(7d\left(\frac{3}{4}d^2\int \sqrt{d^2-e^2x^2}dx+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{32\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{2e}-\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{2e}\right)}{7e}-\frac{8x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}\right)}{8e}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \; 211$$

$$\frac{d\left(\frac{d\left(\frac{d\left(7d\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{32\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{2e}-\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{2e}\right)}{7e}-\frac{8x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}\right)}{8e}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \; 224$$

3.3. $\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

$$\frac{d\left(\frac{d\left(\frac{d\left(7d\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{32\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{2e}-\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{2e}\right)}{7e}-\frac{8x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}\right)}{8e}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}$$

$\Big\downarrow$ 216

$$\frac{d\left(\frac{d\left(\frac{d\left(7d\left(\frac{3}{4}d^2\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{32\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{2e}-\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{2e}\right)}{7e}-\frac{8x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}\right)}{8e}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}$$

input
```
Int[x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

output
```
-1/8*(x^3*(d^2 - e^2*x^2)^(5/2))/e + (d*((-8*x^2*(d^2 - e^2*x^2)^(5/2))/(7
*e) + (d*((-7*x*(d^2 - e^2*x^2)^(5/2))/(2*e) + (d*((-32*(d^2 - e^2*x^2)^(5
/2))/(5*e) + 7*d*((x*(d^2 - e^2*x^2)^(3/2))/4 + (3*d^2*((x*Sqrt[d^2 - e^2*
x^2])/2 + (d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)))/4))/(2*e)))/(7*
e)))/(8*e)
```

3.3. $\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

3.3.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.3.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{(560e^7x^7+640d\,e^6x^6-840d^2e^5x^5-1024d^3e^4x^4+70d^4e^3x^3+128d^5e^2x^2+105d^6ex+256d^7)\sqrt{-e^2x^2+d^2}}{4480e^4}+\frac{3d^8\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+}}\right.}{128e^3\sqrt{e^2}}$
default	$e\left(-\frac{x^3(-e^2x^2+d^2)^{\frac{5}{2}}}{8e^2}+\frac{3d^2\left(-\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6e^2}+\frac{d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6e^2}\right)}{8e^2}\right)+$

input
```
int(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4480*(560*e^7*x^7+640*d*e^6*x^6-840*d^2*e^5*x^5-1024*d^3*e^4*x^4+70*d^4
*e^3*x^3+128*d^5*e^2*x^2+105*d^6*e*x+256*d^7)/e^4*(-e^2*x^2+d^2)^(1/2)+3/1
28*d^8/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.74

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx =$$

$$-\frac{210\,d^8\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(560\,e^7x^7+640\,de^6x^6-840\,d^2e^5x^5-1024\,d^3e^4x^4+70\,d^4e^3x^3+128\,d^5e^2}{4480\,e^4}$$

input
```
integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/4480*(210*d^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (560*e^7*x^7
+ 640*d*e^6*x^6 - 840*d^2*e^5*x^5 - 1024*d^3*e^4*x^4 + 70*d^4*e^3*x^3 + 12
8*d^5*e^2*x^2 + 105*d^6*e*x + 256*d^7)*sqrt(-e^2*x^2 + d^2))/e^4
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.10

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\begin{cases}\frac{3d^8\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{128e^3}+\sqrt{d^2-e^2x^2}\left(-\frac{2d^7}{35e^4}-\frac{3d^6x}{128e^3}-\frac{d^5x^2}{35e^2}-\frac{d^4x^3}{64e}+\right. \\ \left(\frac{dx^4}{4}+\frac{ex^5}{5}\right)\left(d^2\right)^{\frac{3}{2}}\end{cases}$$

input
```
integrate(x**3*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Piecewise((3*d**8*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**
2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(12
8*e**3) + sqrt(d**2 - e**2*x**2)*(-2*d**7/(35*e**4) - 3*d**6*x/(128*e**3)
- d**5*x**2/(35*e**2) - d**4*x**3/(64*e) + 8*d**3*x**4/35 + 3*d**2*e*x**5/
16 - d*e**2*x**6/7 - e**3*x**7/8), Ne(e**2, 0)), ((d*x**4/4 + e*x**5/5)*(d
**2)**(3/2), True))
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.95

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{3\,d^8\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{128\,\sqrt{e^2}e^3}+\frac{3\,\sqrt{-e^2x^2+d^2}d^6x}{128\,e^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^3}{8\,e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4x}{64\,e^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx^2}{7\,e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2x}{16\,e^3}-\frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}{35\,e^4}$$

input
```
integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
3/128*d^8*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^3) + 3/128*sqrt(-e^2*x^
2 + d^2)*d^6*x/e^3 - 1/8*(-e^2*x^2 + d^2)^(5/2)*x^3/e + 1/64*(-e^2*x^2 + d
^2)^(3/2)*d^4*x/e^3 - 1/7*(-e^2*x^2 + d^2)^(5/2)*d*x^2/e^2 - 1/16*(-e^2*x^
2 + d^2)^(5/2)*d^2*x/e^3 - 2/35*(-e^2*x^2 + d^2)^(5/2)*d^3/e^4
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.69

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{3\,d^8\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{128\,e^3|e|}$$
$$-\frac{1}{4480}\sqrt{-e^2x^2+d^2}\left(\frac{256\,d^7}{e^4}+\left(\frac{105\,d^6}{e^3}+2\left(\frac{64\,d^5}{e^2}+\left(\frac{35\,d^4}{e}-4\left(128\,d^3+5\left(21\,d^2e-2\left(7\,e^3x+8\,de^2\right)x\right)\right.\right.\right.\right.\right.$$

input
```
integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
3/128*d^8*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^3*abs(e)) - 1/4480*sqrt(-e^2*x^2
+ d^2)*(256*d^7/e^4 + (105*d^6/e^3 + 2*(64*d^5/e^2 + (35*d^4/e - 4*(128*d^
3 + 5*(21*d^2*e - 2*(7*e^3*x + 8*d*e^2)*x)*x)*x)*x)*x)*x)
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\int x^3\,\left(d^2-e^2\,x^2\right)^{3/2}\,(d+e\,x)\;dx$$

input
```
int(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
```

output
```
int(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

3.4 $\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

3.4.1 Optimal result

Integrand size = 25, antiderivative size = 159

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2}-\frac{d^2(d^2-e^2x^2)^{5/2}}{5e^3}-\frac{dx(d^2-e^2x^2)^{5/2}}{6e^2}+\frac{(d^2-e^2x^2)^{7/2}}{7e^3}+\frac{d^7\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

output
```
1/24*d^3*x*(-e^2*x^2+d^2)^(3/2)/e^2-1/5*d^2*(-e^2*x^2+d^2)^(5/2)/e^3-1/6*d
*x*(-e^2*x^2+d^2)^(5/2)/e^2+1/7*(-e^2*x^2+d^2)^(7/2)/e^3+1/16*d^7*arctan(e
*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/16*d^5*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

3.4.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.79

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=$$
$$-\frac{\sqrt{d^2-e^2x^2}(96d^6+105d^5ex+48d^4e^2x^2-490d^3e^3x^3-384d^2e^4x^4+280de^5x^5+240e^6x^6)+210d^7\arctan}{1680e^3}$$

input
```
Integrate[x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

output
```
-1/1680*(Sqrt[d^2 - e^2*x^2]*(96*d^6 + 105*d^5*e*x + 48*d^4*e^2*x^2 - 490*
d^3*e^3*x^3 - 384*d^2*e^4*x^4 + 280*d*e^5*x^5 + 240*e^6*x^6) + 210*d^7*Arc
Tan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^3
```

3.4.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {533, 27, 533, 27, 455, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}dx$$

$$\Big\downarrow \text{533}$$

$$\frac{\int dex(2d+7ex)\left(d^2-e^2x^2\right)^{3/2}dx}{7e^2}-\frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int x(2d+7ex)\left(d^2-e^2x^2\right)^{3/2}dx}{7e}-\frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}$$

$$\Big\downarrow \text{533}$$

$$\frac{d\left(\frac{\int de(7d+12ex)\left(d^2-e^2x^2\right)^{3/2}dx}{6e^2}-\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{6e}\right)}{7e}-\frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\left(\frac{d\int(7d+12ex)\left(d^2-e^2x^2\right)^{3/2}dx}{6e}-\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{6e}\right)}{7e}-\frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}$$

$$\Big\downarrow \text{455}$$

$$\frac{d\left(\frac{d\left(7d\int\left(d^2-e^2x^2\right)^{3/2}dx-\frac{12\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}-\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{6e}\right)}{7e}-\frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}$$

$$\Big\downarrow \text{211}$$

$$\frac{d\left(\frac{d\left(7d\left(\frac{3}{4}d^2\int\sqrt{d^2-e^2x^2}dx+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{12\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}-\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{6e}\right)}{7e}-\frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}$$

$\downarrow$ 211

$$\frac{d\left(\frac{d\left(7d\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{12\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}-\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{6e}\right)}{7e}-\frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}$$

$\downarrow$ 224

$$\frac{d\left(\frac{d\left(7d\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{12\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}-\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{6e}\right)}{7e}-\frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}$$

$\downarrow$ 216

$$\frac{d\left(\frac{d\left(7d\left(\frac{3}{4}d^2\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{12\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}-\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{6e}\right)}{7e}-\frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}$$

input
```
Int[x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

output
```
-1/7*(x^2*(d^2 - e^2*x^2)^(5/2))/e + (d*((-7*x*(d^2 - e^2*x^2)^(5/2))/(6*e
) + (d*((-12*(d^2 - e^2*x^2)^(5/2))/(5*e) + 7*d*((x*(d^2 - e^2*x^2)^(3/2))
/4 + (3*d^2*((x*Sqrt[d^2 - e^2*x^2])/2 + (d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*
x^2]])/(2*e)))/4)))/(6*e)))/(7*e)
```

3.4. $\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

3.4.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.4.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{(240e^6x^6+280d\,e^5x^5-384d^2e^4x^4-490d^3x^3e^3+48d^4e^2x^2+105d^5ex+96d^6)\sqrt{-e^2x^2+d^2}}{1680e^3}+\frac{d^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16e^2\sqrt{e^2}}$
default	$e\left(-\frac{x^2(-e^2x^2+d^2)^{\frac{5}{2}}}{7e^2}-\frac{2d^2(-e^2x^2+d^2)^{\frac{5}{2}}}{35e^4}\right)+d\left(-\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6e^2}+\frac{d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan}{}\right.}{4}\right.}{6e^2}\right.$

input
```
int(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1680*(240*e^6*x^6+280*d*e^5*x^5-384*d^2*e^4*x^4-490*d^3*e^3*x^3+48*d^4*
e^2*x^2+105*d^5*e*x+96*d^6)/e^3*(-e^2*x^2+d^2)^(1/2)+1/16*d^7/e^2/(e^2)^(1
/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.73

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx =$$

$$-\frac{210\,d^7\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(240\,e^6x^6+280\,de^5x^5-384\,d^2e^4x^4-490\,d^3e^3x^3+48\,d^4e^2x^2+105\,d^5ex}{1680\,e^3}$$

input
```
integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/1680*(210*d^7*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (240*e^6*x^6
+ 280*d*e^5*x^5 - 384*d^2*e^4*x^4 - 490*d^3*e^3*x^3 + 48*d^4*e^2*x^2 + 105
*d^5*e*x + 96*d^6)*sqrt(-e^2*x^2 + d^2))/e^3
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.09

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = \begin{cases} \frac{d^7\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{16e^2} + \sqrt{d^2-e^2x^2}\left(-\frac{2d^6}{35e^3}-\frac{d^5x}{16e^2}-\frac{d^4x^2}{35e}+\frac{7d^3x^3}{24}+\right. \\ \left(\frac{dx^3}{3}+\frac{ex^4}{4}\right)\left(d^2\right)^{\frac{3}{2}} \end{cases}$$

input
```
integrate(x**2*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Piecewise((d**7*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*
x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(16*e
**2) + sqrt(d**2 - e**2*x**2)*(-2*d**6/(35*e**3) - d**5*x/(16*e**2) - d**4
*x**2/(35*e) + 7*d**3*x**3/24 + 8*d**2*e*x**4/35 - d*e**2*x**5/6 - e**3*x*
*6/7), Ne(e**2, 0)), ((d*x**3/3 + e*x**4/4)*(d**2)**(3/2), True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = \frac{d^7\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{16\sqrt{e^2}e^2} + \frac{\sqrt{-e^2x^2+d^2}d^5x}{16\,e^2} + \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3x}{24\,e^2} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^2}{7\,e} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx}{6\,e^2} - \frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}{35\,e^3}$$

input
```
integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
1/16*d^7*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) + 1/16*sqrt(-e^2*x^2
+ d^2)*d^5*x/e^2 + 1/24*(-e^2*x^2 + d^2)^(3/2)*d^3*x/e^2 - 1/7*(-e^2*x^2 +
 d^2)^(5/2)*x^2/e - 1/6*(-e^2*x^2 + d^2)^(5/2)*d*x/e^2 - 2/35*(-e^2*x^2 +
d^2)^(5/2)*d^2/e^3
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.67

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{d^7\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{16\,e^2|e|}$$
$$-\frac{1}{1680}\sqrt{-e^2x^2+d^2}\left(\frac{96\,d^6}{e^3}+\left(\frac{105\,d^5}{e^2}+2\left(\frac{24\,d^4}{e}-\left(245\,d^3+4\left(48\,d^2e-5\left(6\,e^3x+7\,de^2\right)x\right)x\right)x\right)x\right)x\right)$$

input
```
integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
1/16*d^7*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) - 1/1680*sqrt(-e^2*x^2 +
 d^2)*(96*d^6/e^3 + (105*d^5/e^2 + 2*(24*d^4/e - (245*d^3 + 4*(48*d^2*e -
5*(6*e^3*x + 7*d*e^2)*x)*x)*x)*x)*x)
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\int x^2\left(d^2-e^2\,x^2\right)^{3/2}(d+e\,x)\,dx$$

input
```
int(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
```

output
```
int(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

3.5 $\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

3.5.1 Optimal result

Integrand size = 23, antiderivative size = 116

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

output
```
1/24*d^2*x*(-e^2*x^2+d^2)^(3/2)/e-1/30*(5*e*x+6*d)*(-e^2*x^2+d^2)^(5/2)/e^
2+1/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2+1/16*d^4*x*(-e^2*x^2+d^2)^
(1/2)/e
```

3.5.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=-\frac{\sqrt{d^2-e^2x^2}(48d^5+15d^4ex-96d^3e^2x^2-70d^2e^3x^3+48de^4x^4+40e^5x^5)+30d^6\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{240e^2}$$

input
```
Integrate[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

output
```
-1/240*(Sqrt[d^2 - e^2*x^2]*(48*d^5 + 15*d^4*e*x - 96*d^3*e^2*x^2 - 70*d^2
*e^3*x^3 + 48*d*e^4*x^4 + 40*e^5*x^5) + 30*d^6*ArcTan[(e*x)/(Sqrt[d^2] - S
qrt[d^2 - e^2*x^2])])/e^2
```

3.5.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {533, 27, 455, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}dx$$

$$\Big\downarrow \text{533}$$

$$\frac{\int de(d+6ex)\left(d^2-e^2x^2\right)^{3/2}dx}{6e^2}-\frac{x\left(d^2-e^2x^2\right)^{5/2}}{6e}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int (d+6ex)\left(d^2-e^2x^2\right)^{3/2}dx}{6e}-\frac{x\left(d^2-e^2x^2\right)^{5/2}}{6e}$$

$$\Big\downarrow \text{455}$$

$$\frac{d\left(d\int \left(d^2-e^2x^2\right)^{3/2}dx-\frac{6\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}-\frac{x\left(d^2-e^2x^2\right)^{5/2}}{6e}$$

$$\Big\downarrow \text{211}$$

$$\frac{d\left(d\left(\frac{3}{4}d^2\int \sqrt{d^2-e^2x^2}dx+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{6\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}-\frac{x\left(d^2-e^2x^2\right)^{5/2}}{6e}$$

$$\Big\downarrow \text{211}$$

$$\frac{d\left(d\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{6\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}-\frac{x\left(d^2-e^2x^2\right)^{5/2}}{6e}$$

$$\Big\downarrow \text{224}$$

$$\frac{d\left(d\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{6\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}-\frac{x\left(d^2-e^2x^2\right)^{5/2}}{6e}$$

$$\Big\downarrow \text{216}$$

$$\frac{d\left(d\left(\frac{3}{4}d^2\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{6\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}-\frac{x\left(d^2-e^2x^2\right)^{5/2}}{6e}$$

input
```
Int[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

output
```
-1/6*(x*(d^2 - e^2*x^2)^(5/2))/e + (d*((-6*(d^2 - e^2*x^2)^(5/2))/(5*e) +
d*((x*(d^2 - e^2*x^2)^(3/2))/4 + (3*d^2*((x*Sqrt[d^2 - e^2*x^2])/2 + (d^2*
ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)))/4)))/(6*e)
```

3.5.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.5.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{\left(40 e^{5} x^{5}+48 d\,e^{4} x^{4}-70 d^{2} e^{3} x^{3}-96 d^{3} e^{2} x^{2}+15 d^{4} e x +48 d^{5}\right) \sqrt{-e^{2} x^{2}+d^{2}}}{240 e^{2}}+\frac{d^{6} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{16 e \sqrt{e^{2}}}$	108
default	$e \left(-\frac{x \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{6 e^{2}}+\frac{d^{2} \left(\frac{x \left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3 d^{2} \left(\frac{x \sqrt{-e^{2} x^{2}+d^{2}}}{2}+\frac{d^{2} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{2 \sqrt{e^{2}}}\right)}{4}\right)}{6 e^{2}}\right)-\frac{d \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{5 e^{2}}$	126

input
```
int(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/240*(40*e^5*x^5+48*d*e^4*x^4-70*d^2*e^3*x^3-96*d^3*e^2*x^2+15*d^4*e*x+4
8*d^5)/e^2*(-e^2*x^2+d^2)^(1/2)+1/16*d^6/e/(e^2)^(1/2)*arctan((e^2)^(1/2)*
x/(-e^2*x^2+d^2)^(1/2))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx =$$

$$-\frac{30\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(40\,e^5x^5+48\,de^4x^4-70\,d^2e^3x^3-96\,d^3e^2x^2+15\,d^4ex+48\,d^5\right)\sqrt{-e^2x^2+}}{240\,e^2}$$

input
```
integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/240*(30*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (40*e^5*x^5 + 4
8*d*e^4*x^4 - 70*d^2*e^3*x^3 - 96*d^3*e^2*x^2 + 15*d^4*e*x + 48*d^5)*sqrt(
-e^2*x^2 + d^2))/e^2
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.36

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = \begin{cases} \frac{d^6\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{16e} + \sqrt{d^2-e^2x^2}\left(-\frac{d^5}{5e^2}-\frac{d^4x}{16e}+\frac{2d^3x^2}{5}+\frac{7d^2ex^3}{24}-\right. \\ \left(\frac{dx^2}{2}+\frac{ex^3}{3}\right)\left(d^2\right)^{\frac{3}{2}} \end{cases}$$

input
```
integrate(x*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Piecewise((d**6*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*
x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(16*e
) + sqrt(d**2 - e**2*x**2)*(-d**5/(5*e**2) - d**4*x/(16*e) + 2*d**3*x**2/5
 + 7*d**2*e*x**3/24 - d*e**2*x**4/5 - e**3*x**5/6), Ne(e**2, 0)), ((d*x**2
/2 + e*x**3/3)*(d**2)**(3/2), True))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{d^6\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{16\sqrt{e^2}e}+\frac{\sqrt{-e^2x^2+d^2}d^4x}{16\,e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2x}{24\,e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}{6\,e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}{5\,e^2}$$

input
```
integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
1/16*d^6*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e) + 1/16*sqrt(-e^2*x^2 +
d^2)*d^4*x/e + 1/24*(-e^2*x^2 + d^2)^(3/2)*d^2*x/e - 1/6*(-e^2*x^2 + d^2)^
(5/2)*x/e - 1/5*(-e^2*x^2 + d^2)^(5/2)*d/e^2
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.81

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{d^6\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{16\,e|e|}-\frac{1}{240}\sqrt{-e^2x^2+d^2}\left(\frac{48\,d^5}{e^2}+\left(\frac{15\,d^4}{e}-2\left(48\,d^3+\left(35\,d^2e-4\left(5\,e^3x+6\,de^2\right)x\right)x\right)x\right)x\right)$$

input
```
integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
1/16*d^6*arcsin(e*x/d)*sgn(d)*sgn(e)/(e*abs(e)) - 1/240*sqrt(-e^2*x^2 + d^
2)*(48*d^5/e^2 + (15*d^4/e - 2*(48*d^3 + (35*d^2*e - 4*(5*e^3*x + 6*d*e^2)
*x)*x)*x)*x)
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\int x\,\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)\,dx$$

input
```
int(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
```

output
```
int(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

3.6 $\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

3.6.1 Optimal result

Integrand size = 23, antiderivative size = 116

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

output
```
1/24*d^2*x*(-e^2*x^2+d^2)^(3/2)/e-1/30*(5*e*x+6*d)*(-e^2*x^2+d^2)^(5/2)/e^
2+1/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2+1/16*d^4*x*(-e^2*x^2+d^2)^
(1/2)/e
```

3.6.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=-\frac{\sqrt{d^2-e^2x^2}(48d^5+15d^4ex-96d^3e^2x^2-70d^2e^3x^3+48de^4x^4+40e^5x^5)+30d^6\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{240e^2}$$

input
```
Integrate[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

output
```
-1/240*(Sqrt[d^2 - e^2*x^2]*(48*d^5 + 15*d^4*e*x - 96*d^3*e^2*x^2 - 70*d^2
*e^3*x^3 + 48*d*e^4*x^4 + 40*e^5*x^5) + 30*d^6*ArcTan[(e*x)/(Sqrt[d^2] - S
qrt[d^2 - e^2*x^2])])/e^2
```

3.6.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {533, 27, 455, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}dx$$

$$\Big\downarrow \text{533}$$

$$\frac{\int de(d+6ex)\left(d^2-e^2x^2\right)^{3/2}dx}{6e^2}-\frac{x\left(d^2-e^2x^2\right)^{5/2}}{6e}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int (d+6ex)\left(d^2-e^2x^2\right)^{3/2}dx}{6e}-\frac{x\left(d^2-e^2x^2\right)^{5/2}}{6e}$$

$$\Big\downarrow \text{455}$$

$$\frac{d\left(d\int \left(d^2-e^2x^2\right)^{3/2}dx-\frac{6\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}-\frac{x\left(d^2-e^2x^2\right)^{5/2}}{6e}$$

$$\Big\downarrow \text{211}$$

$$\frac{d\left(d\left(\frac{3}{4}d^2\int \sqrt{d^2-e^2x^2}dx+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{6\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}-\frac{x\left(d^2-e^2x^2\right)^{5/2}}{6e}$$

$$\Big\downarrow \text{211}$$

$$\frac{d\left(d\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{6\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}-\frac{x\left(d^2-e^2x^2\right)^{5/2}}{6e}$$

$$\Big\downarrow \text{224}$$

$$\frac{d\left(d\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{6\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}-\frac{x\left(d^2-e^2x^2\right)^{5/2}}{6e}$$

$$\Big\downarrow \text{216}$$

$$\frac{d\left(d\left(\frac{3}{4}d^2\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{6\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}-\frac{x\left(d^2-e^2x^2\right)^{5/2}}{6e}$$

input
```
Int[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

output
```
-1/6*(x*(d^2 - e^2*x^2)^(5/2))/e + (d*((-6*(d^2 - e^2*x^2)^(5/2))/(5*e) +
d*((x*(d^2 - e^2*x^2)^(3/2))/4 + (3*d^2*((x*Sqrt[d^2 - e^2*x^2])/2 + (d^2*
ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)))/4)))/(6*e)
```

3.6.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.6.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{\left(40 e^{5} x^{5}+48 d\, e^{4} x^{4}-70 d^{2} e^{3} x^{3}-96 d^{3} e^{2} x^{2}+15 d^{4} e x +48 d^{5}\right) \sqrt{-e^{2} x^{2}+d^{2}}}{240 e^{2}}+\frac{d^{6} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{16 e \sqrt{e^{2}}}$	108
default	$e \left(-\frac{x \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{6 e^{2}}+\frac{d^{2} \left(\frac{x \left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3 d^{2} \left(\frac{x \sqrt{-e^{2} x^{2}+d^{2}}}{2}+\frac{d^{2} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{2 \sqrt{e^{2}}}\right)}{4}\right)}{6 e^{2}}\right)-\frac{d \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{5 e^{2}}$	126

input
```
int(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/240*(40*e^5*x^5+48*d*e^4*x^4-70*d^2*e^3*x^3-96*d^3*e^2*x^2+15*d^4*e*x+4
8*d^5)/e^2*(-e^2*x^2+d^2)^(1/2)+1/16*d^6/e/(e^2)^(1/2)*arctan((e^2)^(1/2)*
x/(-e^2*x^2+d^2)^(1/2))
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = -\frac{30\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(40\,e^5x^5+48\,de^4x^4-70\,d^2e^3x^3-96\,d^3e^2x^2+15\,d^4ex+48\,d^5\right)\sqrt{-e^2x^2+}}{240\,e^2}$$

input
```
integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/240*(30*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (40*e^5*x^5 + 4
8*d*e^4*x^4 - 70*d^2*e^3*x^3 - 96*d^3*e^2*x^2 + 15*d^4*e*x + 48*d^5)*sqrt(
-e^2*x^2 + d^2))/e^2
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.36

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = \begin{cases} \frac{d^6\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{16e}+\sqrt{d^2-e^2x^2}\left(-\frac{d^5}{5e^2}-\frac{d^4x}{16e}+\frac{2d^3x^2}{5}+\frac{7d^2ex^3}{24}-\right. \\ \left(\frac{dx^2}{2}+\frac{ex^3}{3}\right)\left(d^2\right)^{\frac{3}{2}} \end{cases}$$

input
```
integrate(x*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Piecewise((d**6*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*
x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(16*e
) + sqrt(d**2 - e**2*x**2)*(-d**5/(5*e**2) - d**4*x/(16*e) + 2*d**3*x**2/5
 + 7*d**2*e*x**3/24 - d*e**2*x**4/5 - e**3*x**5/6), Ne(e**2, 0)), ((d*x**2
/2 + e*x**3/3)*(d**2)**(3/2), True))
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{d^6\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{16\sqrt{e^2}e}+\frac{\sqrt{-e^2x^2+d^2}d^4x}{16\,e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2x}{24\,e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}{6\,e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}{5\,e^2}$$

input
```
integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
1/16*d^6*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e) + 1/16*sqrt(-e^2*x^2 +
d^2)*d^4*x/e + 1/24*(-e^2*x^2 + d^2)^(3/2)*d^2*x/e - 1/6*(-e^2*x^2 + d^2)^
(5/2)*x/e - 1/5*(-e^2*x^2 + d^2)^(5/2)*d/e^2
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.81

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{d^6\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{16\,e|e|}-\frac{1}{240}\sqrt{-e^2x^2+d^2}\left(\frac{48\,d^5}{e^2}+\left(\frac{15\,d^4}{e}-2\left(48\,d^3+\left(35\,d^2e-4\left(5\,e^3x+6\,de^2\right)x\right)x\right)x\right)x\right)$$

input
```
integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
1/16*d^6*arcsin(e*x/d)*sgn(d)*sgn(e)/(e*abs(e)) - 1/240*sqrt(-e^2*x^2 + d^
2)*(48*d^5/e^2 + (15*d^4/e - 2*(48*d^3 + (35*d^2*e - 4*(5*e^3*x + 6*d*e^2)
*x)*x)*x)*x)
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\int x\left(d^2-e^2\,x^2\right)^{3/2}(d+e\,x)\,dx$$

input
```
int(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
```

output
```
int(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

3.7 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx$

3.7.1 Optimal result

Integrand size = 25, antiderivative size = 113

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx = \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{3}{8}d^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^4\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
1/12*(3*e*x+4*d)*(-e^2*x^2+d^2)^(3/2)+3/8*d^4*arctan(e*x/(-e^2*x^2+d^2)^(1
/2))-d^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/8*d^2*(3*e*x+8*d)*(-e^2*x^2+d^2
)^(1/2)
```

3.7.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx = \frac{1}{24}\sqrt{d^2-e^2x^2}\left(32d^3+15d^2ex-8de^2x^2-6e^3x^3\right) - \frac{3}{4}d^4\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) - d^3\sqrt{d^2}\log(x) + d^3\sqrt{d^2}\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)$$

input
```
Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(32*d^3 + 15*d^2*e*x - 8*d*e^2*x^2 - 6*e^3*x^3))/24 -
 (3*d^4*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/4 - d^3*Sqrt[d^2]
*Log[x] + d^3*Sqrt[d^2]*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]]
```

3.7.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {535, 535, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{535}$$

$$\frac{1}{4}d^2\int \frac{(4d+3ex)\sqrt{d^2-e^2x^2}}{x}dx+\frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{535}$$

$$\frac{1}{4}d^2\left(\frac{1}{2}d^2\int \frac{8d+3ex}{x\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}(8d+3ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{538}$$

$$\frac{1}{4}d^2\left(\frac{1}{2}d^2\left(3e\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+8d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx\right)+\frac{1}{2}(8d+3ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{4}d^2\left(\frac{1}{2}d^2\left(8d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx+3e\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)+\frac{1}{2}(8d+3ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}d^2\left(\frac{1}{2}d^2\left(8d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx+3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{1}{2}(8d+3ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{4}d^2\left(\frac{1}{2}d^2\left(4d\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2+3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{1}{2}(8d+3ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow 73$$

$$\frac{1}{4}d^2\left(\frac{1}{2}d^2\left(3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{8d\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}\right)+\frac{1}{2}(8d+3ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow 221$$

$$\frac{1}{4}d^2\left(\frac{1}{2}d^2\left(3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-8\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)+\frac{1}{2}(8d+3ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2}$$

input
```
Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x,x]
```

output
```
((4*d + 3*e*x)*(d^2 - e^2*x^2)^(3/2))/12 + (d^2*(((8*d + 3*e*x)*Sqrt[d^2 -
 e^2*x^2])/2 + (d^2*(3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 8*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]))/2))/4
```

3.7.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

3.7.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.39

method	result
default	$e\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)}{4}\right)+d\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{3}+d^{2}\left(\sqrt{-e^{2}x^{2}+d^{2}}-\right.\right.$

input
```
int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
e*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/
(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))+d*(1/3*(-e^2*x^2+
d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/
2)*(-e^2*x^2+d^2)^(1/2))/x)))
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx = -\frac{3}{4}\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + d^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - \frac{1}{24}\left(6\,e^3x^3+8\,de^2x^2-15\,d^2ex-32\,d^3\right)\sqrt{-e^2x^2+d^2}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
-3/4*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d^4*log(-(d - sqrt(-e
^2*x^2 + d^2))/x) - 1/24*(6*e^3*x^3 + 8*d*e^2*x^2 - 15*d^2*e*x - 32*d^3)*s
qrt(-e^2*x^2 + d^2)
```

3.7.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.95 (sec) , antiderivative size = 400, normalized size of antiderivative = 3.54

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx = d^3\left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d\operatorname{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id\operatorname{asin}\left(\frac{d}{ex}\right) + \frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases}\right) + d^2e\left(\begin{cases} \frac{d^2\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{d^2-e^2x^2}}{2} & \text{for } e^2\neq 0 \\ x\sqrt{d^2} & \text{otherwise} \end{cases}\right) - de^2\left(\begin{cases} -\frac{d^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^2\sqrt{d^2-e^2x^2}}{3} & \text{for } e^2\neq 0 \\ \frac{x^2\sqrt{d^2}}{2} & \text{otherwise} \end{cases}\right) - e^3\left(\begin{cases} \frac{d^4\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{8e^2} - \frac{d^2x\sqrt{d^2-e^2x^2}}{8e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4} & \text{for } e^2\neq 0 \\ \frac{x^3\sqrt{d^2}}{3} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x,x)
```

output
```
d**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
 e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x
*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2
*x**2) + 1), True)) + d**2*e*Piecewise((d**2*Piecewise((log(-2*e**2*x + 2*
sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/s
qrt(-e**2*x**2), True))/2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2, 0)), (x*s
qrt(d**2), True)) - d*e**2*Piecewise((-d**2*sqrt(d**2 - e**2*x**2)/(3*e**2
) + x**2*sqrt(d**2 - e**2*x**2)/3, Ne(e**2, 0)), (x**2*sqrt(d**2)/2, True)
) - e**3*Piecewise((d**4*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**
2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), Tru
e))/(8*e**2) - d**2*x*sqrt(d**2 - e**2*x**2)/(8*e**2) + x**3*sqrt(d**2 - e
**2*x**2)/4, Ne(e**2, 0)), (x**3*sqrt(d**2)/3, True))
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx = \frac{3\,d^4e\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\,\sqrt{e^2}} - d^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right) + \frac{3}{8}\,\sqrt{-e^2x^2+d^2}d^2ex + \sqrt{-e^2x^2+d^2}d^3 + \frac{1}{4}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}ex + \frac{1}{3}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
3/8*d^4*e*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - d^4*log(2*d^2/abs(x) + 2
*sqrt(-e^2*x^2 + d^2)*d/abs(x)) + 3/8*sqrt(-e^2*x^2 + d^2)*d^2*e*x + sqrt(
-e^2*x^2 + d^2)*d^3 + 1/4*(-e^2*x^2 + d^2)^(3/2)*e*x + 1/3*(-e^2*x^2 + d^2
)^(3/2)*d
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx = \frac{3\,d^4e\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,|e|} - \frac{d^4e\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{|e|} + \frac{1}{24}\,\sqrt{-e^2x^2+d^2}\left(32\,d^3+\left(15\,d^2e-2\,\left(3\,e^3x+4\,de^2\right)x\right)x\right)$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x, algorithm="giac")
```

output
```
3/8*d^4*e*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - d^4*e*log(1/2*abs(-2*d*e -
2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/24*sqrt(-e^2*x^2 +
 d^2)*(32*d^3 + (15*d^2*e - 2*(3*e^3*x + 4*d*e^2)*x)*x)
```

3.7.9 Mupad [B] (verification not implemented)

Time = 11.74 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx = \frac{d\left(d^2-e^2\,x^2\right)^{3/2}}{3} - d^4\,\operatorname{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right) + d^3\,\sqrt{d^2-e^2\,x^2} + \frac{e\,x\left(d^2-e^2\,x^2\right)^{3/2}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ \frac{e^2\,x^2}{d^2}\right)}{\left(1-\frac{e^2\,x^2}{d^2}\right)^{3/2}}$$

input
```
int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x,x)
```

output
```
(d*(d^2 - e^2*x^2)^(3/2))/3 - d^4*atanh((d^2 - e^2*x^2)^(1/2)/d) + d^3*(d^
2 - e^2*x^2)^(1/2) + (e*x*(d^2 - e^2*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2
, (e^2*x^2)/d^2))/(1 - (e^2*x^2)/d^2)^(3/2)
```

3.8 $\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^2}\,dx$

3.8.1 Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^2}\,dx = \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2} - \frac{(3d-ex)(d^2-e^2x^2)^{3/2}}{3x} - \frac{3}{2}d^3e\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^3e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
-1/3*(-e*x+3*d)*(-e^2*x^2+d^2)^(3/2)/x-3/2*d^3*e*arctan(e*x/(-e^2*x^2+d^2)
^(1/2))-d^3*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/2*d*e*(-3*e*x+2*d)*(-e^2*x
^2+d^2)^(1/2)
```

3.8.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^2}\,dx = \frac{\sqrt{d^2-e^2x^2}(-6d^3+8d^2ex-3de^2x^2-2e^3x^3)}{6x} + 2d^3e\,\text{arctanh}\left(\frac{\sqrt{-e^2}x}{d} - \frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{3}{2}d^3\sqrt{-e^2}\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)$$

input
```
Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^2,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-6*d^3 + 8*d^2*e*x - 3*d*e^2*x^2 - 2*e^3*x^3))/(6*x)
 + 2*d^3*e*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] - (3*d^3*Sqrt
[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/2
```

3.8.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {536, 535, 27, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{536}$$

$$\int \frac{\left(d^2e-3de^2x\right)\sqrt{d^2-e^2x^2}}{x}dx-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}$$

$$\Big\downarrow \text{535}$$

$$\frac{1}{2}d^2\int \frac{de(2d-3ex)}{x\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}d^3e\int \frac{2d-3ex}{x\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}$$

$$\Big\downarrow \text{538}$$

$$\frac{1}{2}d^3e\left(2d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-3e\int \frac{1}{\sqrt{d^2-e^2x^2}}dx\right)+\frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}d^3e\left(2d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-3e\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)+\frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}d^3e\left(2d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}$$

$\Big\downarrow$ 243

$$\frac{1}{2}d^3e\left(d\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}$$

$\Big\downarrow$ 73

$$\frac{1}{2}d^3e\left(-\frac{2d\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}-3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}$$

$\Big\downarrow$ 221

$$\frac{1}{2}d^3e\left(-3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)+\frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}$$

input
```
Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^2,x]
```

output
```
(d*e*(2*d - 3*e*x)*Sqrt[d^2 - e^2*x^2])/2 - ((3*d - e*x)*(d^2 - e^2*x^2)^(
3/2))/(3*x) + (d^3*e*(-3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 2*ArcTanh[Sqr
t[d^2 - e^2*x^2]/d]))/2
```

3.8. $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}\,dx$

3.8.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

3.8.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{d^{3}\sqrt{-e^{2}x^{2}+d^{2}}}{x}-\frac{e^{3}x^{2}\sqrt{-e^{2}x^{2}+d^{2}}}{3}+\frac{4e\,d^{2}\sqrt{-e^{2}x^{2}+d^{2}}}{3}-\frac{e\,d^{4}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}}-\frac{3d^{3}e^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}$
default	$e\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{3}+d^{2}\left(\sqrt{-e^{2}x^{2}+d^{2}}-\frac{d^{2}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}}\right)\right)+d\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{d^{2}x}-\frac{4e^{2}\left(\frac{x\left(-e^{2}x\right.}{}\right.}{}\right.$

input
```
int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-d^3*(-e^2*x^2+d^2)^(1/2)/x-1/3*e^3*x^2*(-e^2*x^2+d^2)^(1/2)+4/3*e*d^2*(-e
^2*x^2+d^2)^(1/2)-e*d^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)
^(1/2))/x)-3/2*d^3*e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/
2))-1/2*d*e^2*x*(-e^2*x^2+d^2)^(1/2)
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}\,dx=\frac{18\,d^3ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+6\,d^3ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+8\,d^3ex-(2}{6\,x}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
1/6*(18*d^3*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 6*d^3*e*x*log(
-(d - sqrt(-e^2*x^2 + d^2))/x) + 8*d^3*e*x - (2*e^3*x^3 + 3*d*e^2*x^2 - 8*
d^2*e*x + 6*d^3)*sqrt(-e^2*x^2 + d^2))/x
```

3.8.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.58 (sec) , antiderivative size = 389, normalized size of antiderivative = 3.32

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}\,dx = d^3\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}} + ie\operatorname{acosh}\left(\frac{ex}{d}\right) - \frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - e\operatorname{asin}\left(\frac{ex}{d}\right) + \frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$
$$+ d^2e\left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d\operatorname{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id\operatorname{asin}\left(\frac{d}{ex}\right) + \frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases}\right)$$
$$- de^2\left(\begin{cases} \frac{d^2\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for}\ d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{d^2-e^2x^2}}{2} & \text{for}\ e^2\neq 0 \\ x\sqrt{d^2} & \text{otherwise} \end{cases}\right)$$
$$- e^3\left(\begin{cases} -\frac{d^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^2\sqrt{d^2-e^2x^2}}{3} & \text{for}\ e^2\neq 0 \\ \frac{x^2\sqrt{d^2}}{2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**2,x)
```

output
```
d**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e
**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt
(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)
), True)) + d**2*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*ac
osh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1),
 (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sq
rt(-d**2/(e**2*x**2) + 1), True)) - d*e**2*Piecewise((d**2*Piecewise((log(
-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)
), (x*log(x)/sqrt(-e**2*x**2), True))/2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e
**2, 0)), (x*sqrt(d**2), True)) - e**3*Piecewise((-d**2*sqrt(d**2 - e**2*x
**2)/(3*e**2) + x**2*sqrt(d**2 - e**2*x**2)/3, Ne(e**2, 0)), (x**2*sqrt(d*
*2)/2, True))
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}\,dx = -\frac{3\,d^3e^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\,\sqrt{e^2}} - d^3e\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{3}{2}\,\sqrt{-e^2x^2+d^2}de^2x + \sqrt{-e^2x^2+d^2}d^2e + \frac{1}{3}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e - \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}{x}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
-3/2*d^3*e^2*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - d^3*e*log(2*d^2/abs(x
) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) - 3/2*sqrt(-e^2*x^2 + d^2)*d*e^2*x +
sqrt(-e^2*x^2 + d^2)*d^2*e + 1/3*(-e^2*x^2 + d^2)^(3/2)*e - (-e^2*x^2 + d^
2)^(3/2)*d/x
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.52

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^2}\,dx = -\frac{3\,d^3e^2\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,|e|} + \frac{d^3e^4x}{2\left(de+\sqrt{-e^2x^2+d^2}|e|\right)|e|} - \frac{d^3e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{|e|} - \frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^3}{2\,x|e|} + \frac{1}{6}\sqrt{-e^2x^2+d^2}\left(8\,d^2e-\left(2\,e^3x+3\,de^2\right)x\right)$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
-3/2*d^3*e^2*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/2*d^3*e^4*x/((d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))*abs(e)) - d^3*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-
e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) - 1/2*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))*d^3/(x*abs(e)) + 1/6*sqrt(-e^2*x^2 + d^2)*(8*d^2*e - (2*e^3*x
+ 3*d*e^2)*x)
```

3.8.9 Mupad [B] (verification not implemented)

Time = 12.36 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^2}\,dx = \frac{e\,(d^2-e^2\,x^2)^{3/2}}{3} + d^2\,e\,\sqrt{d^2-e^2\,x^2} - d^3\,e\,\operatorname{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right) - \frac{d^3\,\sqrt{d^2-e^2\,x^2}\;{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ \frac{e^2\,x^2}{d^2}\right)}{x\,\sqrt{1-\frac{e^2\,x^2}{d^2}}}$$

input
```
int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^2,x)
```

output
```
(e*(d^2 - e^2*x^2)^(3/2))/3 + d^2*e*(d^2 - e^2*x^2)^(1/2) - d^3*e*atanh((d
^2 - e^2*x^2)^(1/2)/d) - (d^3*(d^2 - e^2*x^2)^(1/2)*hypergeom([-3/2, -1/2]
, 1/2, (e^2*x^2)/d^2))/(x*(1 - (e^2*x^2)/d^2)^(1/2))
```

3.9 $\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^3} dx$

3.9.1 Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^3}\,dx = -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d-ex)(d^2-e^2x^2)^{3/2}}{2x^2} - \frac{3}{2}d^2e^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{3}{2}d^2e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
-1/2*(-e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2-3/2*d^2*e^2*arctan(e*x/(-e^2*x^2+d^
2)^(1/2))+3/2*d^2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)-3/2*d*e*(e*x+d)*(-e^
2*x^2+d^2)^(1/2)/x
```

3.9.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^3}\,dx = \frac{1}{2}\left(-\frac{\sqrt{d^2-e^2x^2}(d^3+2d^2ex+2de^2x^2+e^3x^3)}{x^2} + 6d^2e^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) + 3d\sqrt{d^2}e^2\log(x) - 3d\sqrt{d^2}e^2\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)\right)$$

input
```
Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^3,x]
```

output
```
(-((Sqrt[d^2 - e^2*x^2]*(d^3 + 2*d^2*e*x + 2*d*e^2*x^2 + e^3*x^3))/x^2) +
6*d^2*e^2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] + 3*d*Sqrt[d^2]*
e^2*Log[x] - 3*d*Sqrt[d^2]*e^2*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/2
```

3.9.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {537, 25, 535, 27, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{537}$$

$$\frac{3}{2}e^2\int -\frac{(d+2ex)\sqrt{d^2-e^2x^2}}{x}dx-\frac{(d+2ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3}{2}e^2\int \frac{(d+2ex)\sqrt{d^2-e^2x^2}}{x}dx-\frac{(d+2ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{535}$$

$$-\frac{3}{2}e^2\left(\frac{1}{2}d^2\int \frac{2(d+ex)}{x\sqrt{d^2-e^2x^2}}dx+(d+ex)\sqrt{d^2-e^2x^2}\right)-\frac{(d+2ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{2}e^2\left(d^2\int \frac{d+ex}{x\sqrt{d^2-e^2x^2}}dx+(d+ex)\sqrt{d^2-e^2x^2}\right)-\frac{(d+2ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{538}$$

$$-\frac{3}{2}e^2\left(d^2\left(e\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx\right)+(d+ex)\sqrt{d^2-e^2x^2}\right)-\frac{(d+2ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{224}$$

$$-\frac{3}{2}e^2\left(d^2\left(d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx+e\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)+(d+ex)\sqrt{d^2-e^2x^2}\right)-\frac{(d+2ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{3}{2}e^2\left(d^2\left(d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx+\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+(d+ex)\sqrt{d^2-e^2x^2}\right)-\frac{(d+2ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{3}{2}e^2\left(d^2\left(\frac{1}{2}d\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2+\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+(d+ex)\sqrt{d^2-e^2x^2}\right)-\frac{(d+2ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{3}{2}e^2\left(d^2\left(\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{d\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}\right)+(d+ex)\sqrt{d^2-e^2x^2}\right)-\frac{(d+2ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{3}{2}e^2\left(d^2\left(\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)+(d+ex)\sqrt{d^2-e^2x^2}\right)-\frac{(d+2ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

input
```
Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^3,x]
```

output
```
-1/2*((d + 2*e*x)*(d^2 - e^2*x^2)^(3/2))/x^2 - (3*e^2*((d + e*x)*Sqrt[d^2
- e^2*x^2] + d^2*(ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - ArcTanh[Sqrt[d^2 - e
^2*x^2]/d])))/2
```

3.9. $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}\,dx$

3.9.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

3.9. $\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^3}\,dx$

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

3.9.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{d^{2}\sqrt{-e^{2}x^{2}+d^{2}}\,(2ex+d)}{2x^{2}}-\frac{3e^{3}d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}-\frac{e^{3}x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{3e^{2}d^{3}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{2\sqrt{d^{2}}}-d\,e^{2}\sqrt{}$
default	$d\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{2d^{2}x^{2}}-\frac{3e^{2}\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{3}+d^{2}\left(\sqrt{-e^{2}x^{2}+d^{2}}-\frac{d^{2}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}}\right)\right)}{2d^{2}}\right)+e\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{d^{2}x}\right.$

input
```
int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*d^2*(-e^2*x^2+d^2)^(1/2)*(2*e*x+d)/x^2-3/2*e^3*d^2/(e^2)^(1/2)*arctan
((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/2*e^3*x*(-e^2*x^2+d^2)^(1/2)+3/2*e^
2*d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-d*e^2*(
-e^2*x^2+d^2)^(1/2)
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}\,dx = \frac{6\,d^2e^2x^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-3\,d^2e^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-2\,d^2e^2x^2}{2\,x^2}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(6*d^2*e^2*x^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 3*d^2*e^2*x
^2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 2*d^2*e^2*x^2 - (e^3*x^3 + 2*d*e^2
*x^2 + 2*d^2*e*x + d^3)*sqrt(-e^2*x^2 + d^2))/x^2
```

3.9.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.22 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.67

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}\,dx = d^3\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right)$$
$$+d^2e\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$
$$-de^2\left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases}\right)$$
$$-e^3\left(\begin{cases} \frac{d^2\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for}\ d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{2}+\frac{x\sqrt{d^2-e^2x^2}}{2} & \text{for}\ e^2\neq 0 \\ x\sqrt{d^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**3,x)
```

output
```
d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/
(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2
) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*
d), True)) + d**2*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*aco
sh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 -
e**2*x**2/d**2)), True)) - d*e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**
2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e*
*2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e
*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) - e**3*Piecewise((d**2*Pi
ecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2)
, Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/2 + x*sqrt(d**2 - e**2*
x**2)/2, Ne(e**2, 0)), (x*sqrt(d**2), True))
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.42

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}\,dx = -\frac{3\,d^2e^3\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}} + \frac{3}{2}\,d^2e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{3}{2}\sqrt{-e^2x^2+d^2}e^3x - \frac{3}{2}\sqrt{-e^2x^2+d^2}de^2 - \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}{2\,d} - \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}{x} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{2\,dx^2}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
-3/2*d^2*e^3*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 3/2*d^2*e^2*log(2*d^2
/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) - 3/2*sqrt(-e^2*x^2 + d^2)*e^3*
x - 3/2*sqrt(-e^2*x^2 + d^2)*d*e^2 - 1/2*(-e^2*x^2 + d^2)^(3/2)*e^2/d - (-
e^2*x^2 + d^2)^(3/2)*e/x - 1/2*(-e^2*x^2 + d^2)^(5/2)/(d*x^2)
```

3.9.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(106) = 212$.

Time = 0.29 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.02

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}\,dx = -\frac{3\,d^2e^3\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,|e|} + \frac{3\,d^2e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,|e|} + \frac{\left(d^2e^3+\frac{4\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2e}{x}\right)e^4x^2}{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2|e|} - \frac{1}{2}\left(e^3x+2\,de^2\right)\sqrt{-e^2x^2+d^2} - \frac{\frac{4\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2e|e|}{x}+\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^2|e|}{ex^2}}{8\,e^2}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
-3/2*d^2*e^3*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 3/2*d^2*e^3*log(1/2*abs(
-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/8*(d^2*e^
3 + 4*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^2*e/x)*e^4*x^2/((d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))^2*abs(e)) - 1/2*(e^3*x + 2*d*e^2)*sqrt(-e^2*x^2 + d^
2) - 1/8*(4*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^2*e*abs(e)/x + (d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))^2*d^2*abs(e)/(e*x^2))/e^2
```

3.9.9 Mupad [B] (verification not implemented)

Time = 12.53 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}\,dx = \frac{3\,d^2\,e^2\,\operatorname{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{2} - \frac{d^3\,\sqrt{d^2-e^2\,x^2}}{2\,x^2} - d\,e^2\,\sqrt{d^2-e^2\,x^2} - \frac{e\,\left(d^2-e^2\,x^2\right)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ \frac{e^2\,x^2}{d^2}\right)}{x\,\left(1-\frac{e^2\,x^2}{d^2}\right)^{3/2}}$$

input
```
int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^3,x)
```

output
```
(3*d^2*e^2*atanh((d^2 - e^2*x^2)^(1/2)/d))/2 - (d^3*(d^2 - e^2*x^2)^(1/2))
/(2*x^2) - d*e^2*(d^2 - e^2*x^2)^(1/2) - (e*(d^2 - e^2*x^2)^(3/2)*hypergeo
m([-3/2, -1/2], 1/2, (e^2*x^2)/d^2))/(x*(1 - (e^2*x^2)/d^2)^(3/2))
```

3.10 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx$

3.10.1 Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx = \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + de^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{3}{2}de^3\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
-1/6*(3*e*x+2*d)*(-e^2*x^2+d^2)^(3/2)/x^3+d*e^3*arctan(e*x/(-e^2*x^2+d^2)^
(1/2))+3/2*d*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/2*e^2*(-3*e*x+2*d)*(-e^
2*x^2+d^2)^(1/2)/x
```

3.10.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx = \frac{\sqrt{d^2-e^2x^2}\left(-2d^3-3d^2ex+8de^2x^2-6e^3x^3\right)}{6x^3} - 2de^3\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) + \frac{3}{2}\sqrt{d^2}e^3\log(x) - \frac{3}{2}\sqrt{d^2}e^3\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)$$

input
```
Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^4,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-2*d^3 - 3*d^2*e*x + 8*d*e^2*x^2 - 6*e^3*x^3))/(6*x^
3) - 2*d*e^3*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] + (3*Sqrt[d^2
]*e^3*Log[x])/2 - (3*Sqrt[d^2]*e^3*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/2
```

3.10.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {537, 25, 536, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{537}$$

$$\frac{1}{2}e^2\int -\frac{(2d+3ex)\sqrt{d^2-e^2x^2}}{x^2}dx-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}e^2\int \frac{(2d+3ex)\sqrt{d^2-e^2x^2}}{x^2}dx-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}$$

$$\Big\downarrow \text{536}$$

$$-\frac{1}{2}e^2\left(\int \frac{3d^2e-2de^2x}{x\sqrt{d^2-e^2x^2}}dx-\frac{(2d-3ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}$$

$$\Big\downarrow \text{538}$$

$$-\frac{1}{2}e^2\left(3d^2e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-2de^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{(2d-3ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}$$

$$\Big\downarrow \text{224}$$

$$-\frac{1}{2}e^2\left(3d^2e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-2de^2\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{(2d-3ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{2}e^2\left(3d^2e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-2de\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{(2d-3ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}$$

$\Big\downarrow$ 243

$$-\frac{1}{2}e^2\left(\frac{3}{2}d^2e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-2de\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{(2d-3ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}$$

$\Big\downarrow$ 73

$$-\frac{1}{2}e^2\left(-\frac{3d^2\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-2de\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{(2d-3ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}$$

$\Big\downarrow$ 221

$$-\frac{1}{2}e^2\left(-2de\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-3de\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)-\frac{\sqrt{d^2-e^2x^2}(2d-3ex)}{x}\right)-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}$$

input
```
Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^4,x]
```

output
```
-1/6*((2*d + 3*e*x)*(d^2 - e^2*x^2)^(3/2))/x^3 - (e^2*(-(((2*d - 3*e*x)*Sq
rt[d^2 - e^2*x^2])/x) - 2*d*e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 3*d*e*Ar
cTanh[Sqrt[d^2 - e^2*x^2]/d]))/2
```

3.10. $\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^4}\,dx$

3.10.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

3.10.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.14

method	result
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, d \left(-8 e^{2} x^{2}+3 d e x+2 d^{2}\right)}{6 x^{3}}+\frac{e^{4} d \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}}-e^{3} \sqrt{-e^{2} x^{2}+d^{2}}+\frac{3 e^{3} d^{2} \ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{2 \sqrt{d^{2}}}$
default	$e \left(-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{2 d^{2} x^{2}}-\frac{3 e^{2} \left(\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{3}+d^{2} \left(\sqrt{-e^{2} x^{2}+d^{2}}-\frac{d^{2} \ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}}\right)\right)}{2 d^{2}}\right)+d \left(-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{3 d^{2} x^{3}}\right.$

input
```
int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-e^2*x^2+d^2)^(1/2)*d*(-8*e^2*x^2+3*d*e*x+2*d^2)/x^3+e^4*d/(e^2)^(1/
2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-e^3*(-e^2*x^2+d^2)^(1/2)+3/2
*e^3*d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx =$$
$$-\frac{12\,de^3x^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+9\,de^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+6\,de^3x^3+\left(6\,e^3x^3-8\,de^2x^2+3\,d^2ex+\right.}{6\,x^3}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(12*d*e^3*x^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 9*d*e^3*x^3
*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 6*d*e^3*x^3 + (6*e^3*x^3 - 8*d*e^2*x
^2 + 3*d^2*e*x + 2*d^3)*sqrt(-e^2*x^2 + d^2))/x^3
```

3.10.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.46 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.81

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx = d^3\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right)$$
$$+d^2e\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right)$$
$$-de^2\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$
$$-e^3\left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**4,x)
```

output
```
d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e
**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**
2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)
) + d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(
e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**
2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x
))/(2*d), True)) - d*e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I
*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d*
*2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqr
t(1 - e**2*x**2/d**2)), True)) - e**3*Piecewise(((d**2/(e*x*sqrt(d**2/(e**2
*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2
/(e**2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(
d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True))
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.63

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx = \frac{de^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} + \frac{3}{2}\,de^3\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right) + \frac{\sqrt{-e^2x^2+d^2}e^4x}{d} - \frac{3}{2}\,\sqrt{-e^2x^2+d^2}e^3 - \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}{2\,d^2} + \frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}{3\,dx} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{2\,d^2x^2} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{3\,dx^3}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
d*e^4*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 3/2*d*e^3*log(2*d^2/abs(x) +
 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) + sqrt(-e^2*x^2 + d^2)*e^4*x/d - 3/2*sqr
t(-e^2*x^2 + d^2)*e^3 - 1/2*(-e^2*x^2 + d^2)^(3/2)*e^3/d^2 + 2/3*(-e^2*x^2
 + d^2)^(3/2)*e^2/(d*x) - 1/2*(-e^2*x^2 + d^2)^(5/2)*e/(d^2*x^2) - 1/3*(-e
^2*x^2 + d^2)^(5/2)/(d*x^3)
```

3.10.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(106) = 212$.

Time = 0.28 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.38

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx = \frac{de^4\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} + \frac{\left(de^4 + \frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)de^2}{x} - \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d}{x^2}\right)e^6x^3}{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3|e|} + \frac{3\,de^4\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,|e|} - \sqrt{-e^2x^2+d^2}e^3 + \frac{\frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)de^4}{x} - \frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2de^2}{x^2} - \frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d}{x^3}}{24\,e^2|e|}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
d*e^4*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/24*(d*e^4 + 3*(d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))*d*e^2/x - 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d/
x^2)*e^6*x^3/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*abs(e)) + 3/2*d*e^4*lo
g(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) - s
qrt(-e^2*x^2 + d^2)*e^3 + 1/24*(15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d*e
^4/x - 3*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d*e^2/x^2 - (d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^3*d/x^3)/(e^2*abs(e))
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx = \int \frac{\left(d^2-e^2\,x^2\right)^{3/2}\,(d+e\,x)}{x^4}\,dx$$

input
```
int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^4,x)
```

output
```
int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^4, x)
```

3.11 $\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^5}\,dx$

3.11.1 Optimal result

Integrand size = 25, antiderivative size = 118

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^5}\,dx=\frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d+4ex)(d^2-e^2x^2)^{3/2}}{12x^4}+e^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3}{8}e^4\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
-1/12*(4*e*x+3*d)*(-e^2*x^2+d^2)^(3/2)/x^4+e^4*arctan(e*x/(-e^2*x^2+d^2)^(
1/2))-3/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/8*e^2*(8*e*x+3*d)*(-e^2*x^
2+d^2)^(1/2)/x^2
```

3.11.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^5}\,dx=\frac{1}{24}\left(\frac{\sqrt{d^2-e^2x^2}(-6d^3-8d^2ex+15de^2x^2+32e^3x^3)}{x^4}-48e^4\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)-\frac{9\sqrt{d^2}e^4\log(x)}{d}+\frac{9\sqrt{d^2}e^4\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{d}\right)$$

input
```
Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^5,x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(-6*d^3 - 8*d^2*e*x + 15*d*e^2*x^2 + 32*e^3*x^3))/x^
4 - 48*e^4*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] - (9*Sqrt[d^2]*
e^4*Log[x])/d + (9*Sqrt[d^2]*e^4*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/d)/
24
```

3.11.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {537, 25, 537, 25, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{537}$$

$$\frac{1}{4}e^2\int -\frac{(3d+4ex)\sqrt{d^2-e^2x^2}}{x^3}dx-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{4}e^2\int \frac{(3d+4ex)\sqrt{d^2-e^2x^2}}{x^3}dx-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}$$

$$\Big\downarrow \text{537}$$

$$-\frac{1}{4}e^2\left(\frac{1}{2}e^2\int -\frac{3d+8ex}{x\sqrt{d^2-e^2x^2}}dx-\frac{(3d+8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{4}e^2\left(-\frac{1}{2}e^2\int \frac{3d+8ex}{x\sqrt{d^2-e^2x^2}}dx-\frac{(3d+8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}$$

$$\Big\downarrow \text{538}$$

$$-\frac{1}{4}e^2\left(-\frac{1}{2}e^2\left(8e\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+3d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx\right)-\frac{(3d+8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}$$

$$\Big\downarrow \; 224$$

$$-\frac{1}{4}e^2\left(-\frac{1}{2}e^2\left(3d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx+8e\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)-\frac{(3d+8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}$$

$$\Big\downarrow \; 216$$

$$-\frac{1}{4}e^2\left(-\frac{1}{2}e^2\left(3d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx+8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{(3d+8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}$$

$$\Big\downarrow \; 243$$

$$-\frac{1}{4}e^2\left(-\frac{1}{2}e^2\left(\frac{3}{2}d\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2+8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{(3d+8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}$$

$$\Big\downarrow \; 73$$

$$-\frac{1}{4}e^2\left(-\frac{1}{2}e^2\left(8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3d\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}\right)-\frac{(3d+8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}$$

$$\Big\downarrow \; 221$$

$$-\frac{1}{4}e^2\left(-\frac{1}{2}e^2\left(8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-3\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)-\frac{(3d+8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}$$

input
```
Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^5,x]
```

output
```
-1/12*((3*d + 4*e*x)*(d^2 - e^2*x^2)^(3/2))/x^4 - (e^2*(-1/2*((3*d + 8*e*x
)*Sqrt[d^2 - e^2*x^2])/x^2 - (e^2*(8*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 3
*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]))/2))/4
```

3.11. $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx$

3.11.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

3.11. $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx$

3.11.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.06

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-32e^3x^3-15d\,e^2x^2+8d^2ex+6d^3)}{24x^4}+\frac{e^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}-\frac{3e^4d\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}}$
default	$d\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{4d^2x^4}-\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{2d^2x^2}-\frac{3e^2\left(\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)\right)}{2d^2}\right)}{4d^2}\right)+e$

input
```
int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-e^2*x^2+d^2)^(1/2)*(-32*e^3*x^3-15*d*e^2*x^2+8*d^2*e*x+6*d^3)/x^4+
e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-3/8*e^4*d/(d^2)
^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)\,(d^2-e^2x^2)^{3/2}}{x^5}\,dx =$$

$$-\frac{48\,e^4x^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-9\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-(32\,e^3x^3+15\,de^2x^2-8\,d^2ex-6\,d^3)\sqrt{-e^2}}{24\,x^4}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
-1/24*(48*e^4*x^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 9*e^4*x^4*lo
g(-(d - sqrt(-e^2*x^2 + d^2))/x) - (32*e^3*x^3 + 15*d*e^2*x^2 - 8*d^2*e*x
- 6*d^3)*sqrt(-e^2*x^2 + d^2))/x^4
```

3.11.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.59 (sec) , antiderivative size = 541, normalized size of antiderivative = 4.58

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx = d^3\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise} \end{cases}\right.$$

$$+ d^2e\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right)$$

$$- de^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x} + \frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right)$$

$$- e^3\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}} + ie\operatorname{acosh}\left(\frac{ex}{d}\right) - \frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - e\operatorname{asin}\left(\frac{ex}{d}\right) + \frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**5,x)
```

output
```
d**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*
sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) +
 e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x*
*5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1
)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/
(8*d**3), True)) + d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2
) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1),
(-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2)
 + 1)/(3*d**2), True)) - d*e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(
2*x) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e
*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1))
 - I*e**2*asin(d/(e*x))/(2*d), True)) - e**3*Piecewise((I*d/(x*sqrt(-1 + e
**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)
), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x
/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.11.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(104) = 208$.

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.88

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx = \frac{e^5\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} - \frac{3}{8}e^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) + \frac{\sqrt{-e^2x^2+d^2}e^5x}{d^2} + \frac{3\sqrt{-e^2x^2+d^2}e^4}{8\,d} + \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}{8\,d^3} + \frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}{3\,d^2x} + \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}{8\,d^3x^2} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{3\,d^2x^3} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{4\,dx^4}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
e^5*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 3/8*e^4*log(2*d^2/abs(x) + 2*s
qrt(-e^2*x^2 + d^2)*d/abs(x)) + sqrt(-e^2*x^2 + d^2)*e^5*x/d^2 + 3/8*sqrt(
-e^2*x^2 + d^2)*e^4/d + 1/8*(-e^2*x^2 + d^2)^(3/2)*e^4/d^3 + 2/3*(-e^2*x^2
 + d^2)^(3/2)*e^3/(d^2*x) + 1/8*(-e^2*x^2 + d^2)^(5/2)*e^2/(d^3*x^2) - 1/3
*(-e^2*x^2 + d^2)^(5/2)*e/(d^2*x^3) - 1/4*(-e^2*x^2 + d^2)^(5/2)/(d*x^4)
```

3.11.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(104) = 208$.

Time = 0.30 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.77

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx = \frac{\left(3\,e^5+\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^3}{x}-\frac{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e}{x^2}-\frac{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{ex^3}\right)}{192\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4|e|} + \frac{e^5\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} - \frac{3\,e^5\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{8\,|e|} + \frac{\frac{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^5|e|}{x}+\frac{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^3|e|}{x^2}-\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3e|e|}{x^3}-\frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4|e|}{ex^4}}{192\,e^4}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x, algorithm="giac")
```

output
```
1/192*(3*e^5 + 8*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^3/x - 24*(d*e + sqr
t(-e^2*x^2 + d^2)*abs(e))^2*e/x^2 - 120*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e)
)^3/(e*x^3))*e^8*x^4/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*abs(e)) + e^5*
arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 3/8*e^5*log(1/2*abs(-2*d*e - 2*sqrt(-
e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/192*(120*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))*e^5*abs(e)/x + 24*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2
*e^3*abs(e)/x^2 - 8*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*e*abs(e)/x^3 - 3
*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*abs(e)/(e*x^4))/e^4
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx = \int \frac{\left(d^2-e^2\,x^2\right)^{3/2}\,(d+e\,x)}{x^5}\,dx$$

input
```
int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^5,x)
```

output
```
int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^5, x)
```

3.12 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx$

3.12.1 Optimal result

Integrand size = 25, antiderivative size = 108

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx=\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}-\frac{3e^5\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

output
```
-1/4*e*(-e^2*x^2+d^2)^(3/2)/x^4-1/5*(-e^2*x^2+d^2)^(5/2)/d/x^5-3/8*e^5*arc
tanh((-e^2*x^2+d^2)^(1/2)/d)/d+3/8*e^3*(-e^2*x^2+d^2)^(1/2)/x^2
```

3.12.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx=\frac{1}{40}\left(\frac{\sqrt{d^2-e^2x^2}\left(-8d^4-10d^3ex+16d^2e^2x^2+25de^3x^3-8e^4x^4\right)}{dx^5}-\frac{15e^5\log(x)}{\sqrt{d^2}}+\frac{15e^5\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{\sqrt{d^2}}\right)$$

input
```
Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^6,x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(-8*d^4 - 10*d^3*e*x + 16*d^2*e^2*x^2 + 25*d*e^3*x^3
 - 8*e^4*x^4))/(d*x^5) - (15*e^5*Log[x])/Sqrt[d^2] + (15*e^5*Log[Sqrt[d^2]
 - Sqrt[d^2 - e^2*x^2]])/Sqrt[d^2])/40
```

3.12.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {534, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}dx$$

$$\Big\downarrow \text{534}$$

$$e\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}dx-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}e\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^6}dx^2-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}e\left(-\frac{3}{4}e^2\int \frac{\sqrt{d^2-e^2x^2}}{x^4}dx^2-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}e\left(-\frac{3}{4}e^2\left(-\frac{1}{2}e^2\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}e\left(-\frac{3}{4}e^2\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}e\left(-\frac{3}{4}e^2\left(\frac{e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}$$

input
```
Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^6,x]
```

output
```
-1/5*(d^2 - e^2*x^2)^(5/2)/(d*x^5) + (e*(-1/2*(d^2 - e^2*x^2)^(3/2)/x^4 -
(3*e^2*(-(Sqrt[d^2 - e^2*x^2]/x^2) + (e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/
d))/4))/2
```

3.12.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

3.12.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(8e^4x^4-25d\,e^3x^3-16d^2e^2x^2+10d^3ex+8d^4)}{40x^5d}-\frac{3e^5\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}}$
default	$e\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{4d^2x^4}-\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{2d^2x^2}-\frac{3e^2\left(\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)\right)}{2d^2}\right)}{4d^2}\right)-$

input

```
int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output

```
-1/40*(-e^2*x^2+d^2)^(1/2)*(8*e^4*x^4-25*d*e^3*x^3-16*d^2*e^2*x^2+10*d^3*e
*x+8*d^4)/x^5/d-3/8*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)
^(1/2))/x)
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx=\frac{15\,e^5x^5\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-(8\,e^4x^4-25\,de^3x^3-16\,d^2e^2x^2+10\,d^3ex+8}{40\,dx^5}$$

input

```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x, algorithm="fricas")
```

output

```
1/40*(15*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (8*e^4*x^4 - 25*d*e^
3*x^3 - 16*d^2*e^2*x^2 + 10*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/(d*x^5)
```

3.12.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.21 (sec) , antiderivative size = 774, normalized size of antiderivative = 7.17

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx = d^3\left(\begin{cases} \frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} - \frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} + \frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7} - \frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} \\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} - \frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} + \frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7} - \frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} \end{cases}\right)$$

$$+ d^2e\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise} \end{cases}\right)$$

$$- de^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right)$$

$$- e^3\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x} + \frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**6,x)
```

output
```
d**3*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**
2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e*
*2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**
3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d
*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-
15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15
*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**
5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3
*x**5 + 15*d*e**2*x**7), True)) + d**2*e*Piecewise((-d**2/(4*e*x**5*sqrt(d
**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*
d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**
2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*
e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2
*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) - d*e**2*Piecewise((-
e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3
*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x
**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) - e**3*Piecewis
e((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/(2*d), Abs(d*
*2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e
/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d), True))
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx = -\frac{3\,e^5\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{8\,d}+\frac{3\,\sqrt{-e^2x^2+d^2}e^5}{8\,d^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5}{8\,d^4}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}{8\,d^4x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{4\,d^2x^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5\,dx^5}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
-3/8*e^5*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d + 3/8*sqrt(
-e^2*x^2 + d^2)*e^5/d^2 + 1/8*(-e^2*x^2 + d^2)^(3/2)*e^5/d^4 + 1/8*(-e^2*x
^2 + d^2)^(5/2)*e^3/(d^4*x^2) - 1/4*(-e^2*x^2 + d^2)^(5/2)*e/(d^2*x^4) - 1
/5*(-e^2*x^2 + d^2)^(5/2)/(d*x^5)
```

3.12.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(92) = 184$.

Time = 0.29 (sec) , antiderivative size = 388, normalized size of antiderivative = 3.59

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^6}\,dx = \frac{\left(2\,e^6 + \frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^4}{x} - \frac{10\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^2}{x^2} - \frac{40\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{x^3} + \cdots\right.}{320\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^5 d|e|}$$
$$- \frac{3\,e^6\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{8\,d|e|}$$
$$- \frac{\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^4e^8}{x} - \frac{40\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^4e^6}{x^2} - \frac{10\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^4e^4}{x^3} + \frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^4e^2}{x^4} + \frac{2\left(de-\cdots\right.}{\cdots}}{320\,d^5e^4|e|}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x, algorithm="giac")
```

output
```
1/320*(2*e^6 + 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^4/x - 10*(d*e + sqr
t(-e^2*x^2 + d^2)*abs(e))^2*e^2/x^2 - 40*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e
))^3/x^3 + 20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^2*x^4))*e^10*x^5/((
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d*abs(e)) - 3/8*e^6*log(1/2*abs(-2*d*
e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d*abs(e)) - 1/320*(20*(d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^4*e^8/x - 40*(d*e + sqrt(-e^2*x^2 + d^
2)*abs(e))^2*d^4*e^6/x^2 - 10*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^4*e^
4/x^3 + 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^4*e^2/x^4 + 2*(d*e + sqr
t(-e^2*x^2 + d^2)*abs(e))^5*d^4/x^5)/(d^5*e^4*abs(e))
```

3.12.9 Mupad [B] (verification not implemented)

Time = 13.17 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^6}\,dx = \frac{3\,d^2\,e\,\sqrt{d^2-e^2\,x^2}}{8\,x^4} - \frac{(d^2-e^2\,x^2)^{5/2}}{5\,d\,x^5} - \frac{3\,e^5\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{8\,d} - \frac{5\,e\,(d^2-e^2\,x^2)^{3/2}}{8\,x^4}$$

input
```
int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^6,x)
```

output
```
(3*d^2*e*(d^2 - e^2*x^2)^(1/2))/(8*x^4) - (d^2 - e^2*x^2)^(5/2)/(5*d*x^5)
- (3*e^5*atanh((d^2 - e^2*x^2)^(1/2)/d))/(8*d) - (5*e*(d^2 - e^2*x^2)^(3/2
))/(8*x^4)
```

3.12. $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx$

3.13 $\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^7} dx$

3.13.1 Optimal result

Integrand size = 25, antiderivative size = 143

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^7} dx = \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2} - \frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4} - \frac{(d^2-e^2x^2)^{5/2}}{6dx^6} - \frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5} - \frac{e^6\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}$$

output
```
-1/24*e^2*(-e^2*x^2+d^2)^(3/2)/d/x^4-1/6*(-e^2*x^2+d^2)^(5/2)/d/x^6-1/5*e*
(-e^2*x^2+d^2)^(5/2)/d^2/x^5-1/16*e^6*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2+
1/16*e^4*(-e^2*x^2+d^2)^(1/2)/d/x^2
```

3.13.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^7} dx = \frac{\sqrt{d^2-e^2x^2}(-40d^5-48d^4ex+70d^3e^2x^2+96d^2e^3x^3-15de^4x^4-48e^5x^5)}{240d^2x^6} - \frac{\sqrt{d^2}e^6\log(x)}{16d^3} + \frac{\sqrt{d^2}e^6\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{16d^3}$$

input
```
Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^7,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-40*d^5 - 48*d^4*e*x + 70*d^3*e^2*x^2 + 96*d^2*e^3*x
^3 - 15*d*e^4*x^4 - 48*e^5*x^5))/(240*d^2*x^6) - (Sqrt[d^2]*e^6*Log[x])/(1
6*d^3) + (Sqrt[d^2]*e^6*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(16*d^3)
```

3.13.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {539, 25, 27, 534, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{539}$$

$$-\frac{\int -\frac{de(6d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}dx}{6d^2}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{de(6d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}dx}{6d^2}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int \frac{(6d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}dx}{6d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}$$

$$\Big\downarrow \text{534}$$

$$\frac{e\left(e\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}dx-\frac{6\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{6d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}$$

$$\Big\downarrow \text{243}$$

$$\frac{e\left(\frac{1}{2}e\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^6}dx^2-\frac{6\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{6d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}$$

$$\Big\downarrow \text{51}$$

$$\frac{e\left(\frac{1}{2}e\left(-\frac{3}{4}e^2\int \frac{\sqrt{d^2-e^2x^2}}{x^4}dx^2-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{6\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{6d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}$$

$$\Big\downarrow \text{51}$$

$$\frac{e\left(\frac{1}{2}e\left(-\frac{3}{4}e^2\left(-\frac{1}{2}e^2\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{6\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{6d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}$$

$$\Big\downarrow \text{73}$$

$$\frac{e\left(\frac{1}{2}e\left(-\frac{3}{4}e^2\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{6\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{6d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}$$

$$\Big\downarrow \text{221}$$

$$\frac{e\left(\frac{1}{2}e\left(-\frac{3}{4}e^2\left(\frac{e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{6\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{6d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}$$

input
```
Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^7,x]
```

output
```
-1/6*(d^2 - e^2*x^2)^(5/2)/(d*x^6) + (e*((-6*(d^2 - e^2*x^2)^(5/2))/(5*d*x
^5) + (e*(-1/2*(d^2 - e^2*x^2)^(3/2)/x^4 - (3*e^2*(-(Sqrt[d^2 - e^2*x^2]/x
^2) + (e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/4))/2))/(6*d)
```

3.13.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

3.13. $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx$

3.13.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(48e^5x^5+15d\,e^4x^4-96d^2e^3x^3-70d^3e^2x^2+48d^4ex+40d^5)}{240x^6d^2}-\frac{e^6\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{16d\sqrt{d^2}}$
default	$d\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{6d^2x^6}+\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{4d^2x^4}-\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{2d^2x^2}-\frac{3e^2\left(\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2}}{x}\right.}{\sqrt{d^2}}\right.\right.}{2d^2}\right.}{4d^2}\right.}{6d^2}\right.$

input
```
int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/240*(-e^2*x^2+d^2)^(1/2)*(48*e^5*x^5+15*d*e^4*x^4-96*d^2*e^3*x^3-70*d^3
*e^2*x^2+48*d^4*e*x+40*d^5)/x^6/d^2-1/16/d*e^6/(d^2)^(1/2)*ln((2*d^2+2*(d^
2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^7}\,dx = \frac{15\,e^6x^6\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - (48\,e^5x^5 + 15\,de^4x^4 - 96\,d^2e^3x^3 - 70\,d^3e^2x^2 \cdots}{240\,d^2x^6}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
1/240*(15*e^6*x^6*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (48*e^5*x^5 + 15*d*
e^4*x^4 - 96*d^2*e^3*x^3 - 70*d^3*e^2*x^2 + 48*d^4*e*x + 40*d^5)*sqrt(-e^2
*x^2 + d^2))/(d^2*x^6)
```

3.13.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.07 (sec) , antiderivative size = 918, normalized size of antiderivative = 6.42

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^7}\,dx = d^3\left(\begin{cases} -\frac{d^2}{6ex^7\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{5e}{24x^5\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^3}{48d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e^5}{16d^4x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^6\operatorname{acosh}(\cdots}{16d^5\cdots} \\ \frac{id^2}{6ex^7\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{5ie}{24x^5\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^3}{48d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^5}{16d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^6\,\mathrm{a}\cdots}{\cdots} \end{cases}\right.$$

$$+d^2e\left(\begin{cases} \frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} - \frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} + \frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7} - \frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} - \frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} + \frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7} - \frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise} \end{cases}\right)$$

$$-de^2\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise} \end{cases}\right)$$

$$-e^3\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**7,x)
```

output
```
d**3*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5
*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) -
1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(1
6*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x*
*2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*
x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**
2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) + d**2*e*Piecewise((3*I*
d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2
*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*
x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**
4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**
2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e*
*2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**
2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x*
*7), True)) - d*e**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)
) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e*
*2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1),
 (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2
/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*...
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx = -\frac{e^6\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{16\,d^2} + \frac{\sqrt{-e^2x^2+d^2}e^6}{16\,d^3} + \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^6}{48\,d^5} + \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}{48\,d^5x^2} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}{24\,d^3x^4} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{5\,d^2x^5} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6\,dx^6}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
-1/16*e^6*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^2 + 1/16*s
qrt(-e^2*x^2 + d^2)*e^6/d^3 + 1/48*(-e^2*x^2 + d^2)^(3/2)*e^6/d^5 + 1/48*(
-e^2*x^2 + d^2)^(5/2)*e^4/(d^5*x^2) - 1/24*(-e^2*x^2 + d^2)^(5/2)*e^2/(d^3
*x^4) - 1/5*(-e^2*x^2 + d^2)^(5/2)*e/(d^2*x^5) - 1/6*(-e^2*x^2 + d^2)^(5/2
)/(d*x^6)
```

3.13.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(123) = 246$.

Time = 0.30 (sec) , antiderivative size = 463, normalized size of antiderivative = 3.24

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx = \frac{\left(5\,e^7 + \frac{12\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^5}{x} - \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^3}{x^2} - \frac{60\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3e}{x^3} \cdots\right.}{1920\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^{\cdots}} - \frac{e^7\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{16\,d^2|e|} - \frac{\frac{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{10}e^9|e|}{x} - \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^{10}e^7|e|}{x^2} - \frac{60\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^{10}e^5|e|}{x^3} - \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^{10}\cdots}{x^4}\cdots}{1920\,d^{12}e^6}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x, algorithm="giac")
```

output
```
1/1920*(5*e^7 + 12*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^5/x - 15*(d*e + s
qrt(-e^2*x^2 + d^2)*abs(e))^2*e^3/x^2 - 60*(d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))^3*e/x^3 - 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e*x^4) + 120*(d*e
 + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^3*x^5))*e^12*x^6/((d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))^6*d^2*abs(e)) - 1/16*e^7*log(1/2*abs(-2*d*e - 2*sqrt(-e^2
*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^2*abs(e)) - 1/1920*(120*(d*e + sqrt(-
e^2*x^2 + d^2)*abs(e))*d^10*e^9*abs(e)/x - 15*(d*e + sqrt(-e^2*x^2 + d^2)*
abs(e))^2*d^10*e^7*abs(e)/x^2 - 60*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d
^10*e^5*abs(e)/x^3 - 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^10*e^3*abs
(e)/x^4 + 12*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d^10*e*abs(e)/x^5 + 5*(
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6*d^10*abs(e)/(e*x^6))/(d^12*e^6)
```

3.13.9 Mupad [B] (verification not implemented)

Time = 13.79 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx = \frac{d^3\sqrt{d^2-e^2\,x^2}}{16\,x^6} - \frac{d\left(d^2-e^2\,x^2\right)^{3/2}}{6\,x^6} - \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{16\,d\,x^6} - \frac{e\left(d^2-e^2\,x^2\right)^{5/2}}{5\,d^2\,x^5} + \frac{e^6\,\mathrm{atan}\left(\frac{\sqrt{d^2-e^2\,x^2}\,1\mathrm{i}}{d}\right)\,1\mathrm{i}}{16\,d^2}$$

input
```
int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^7,x)
```

output
```
(d^3*(d^2 - e^2*x^2)^(1/2))/(16*x^6) - (d*(d^2 - e^2*x^2)^(3/2))/(6*x^6) -
 (d^2 - e^2*x^2)^(5/2)/(16*d*x^6) + (e^6*atan(((d^2 - e^2*x^2)^(1/2)*1i)/d
)*1i)/(16*d^2) - (e*(d^2 - e^2*x^2)^(5/2))/(5*d^2*x^5)
```

3.13. $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx$

3.14 $\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^8}\,dx$

3.14.1 Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^8}\,dx = \frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4} - \frac{(d^2-e^2x^2)^{5/2}}{7dx^7} - \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5} - \frac{e^7\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

output
```
-1/24*e^3*(-e^2*x^2+d^2)^(3/2)/d^2/x^4-1/7*(-e^2*x^2+d^2)^(5/2)/d/x^7-1/6*
e*(-e^2*x^2+d^2)^(5/2)/d^2/x^6-2/35*e^2*(-e^2*x^2+d^2)^(5/2)/d^3/x^5-1/16*
e^7*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3+1/16*e^5*(-e^2*x^2+d^2)^(1/2)/d^2/
x^2
```

3.14.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^8}\,dx = \frac{\sqrt{d^2-e^2x^2}(-240d^6-280d^5ex+384d^4e^2x^2+490d^3e^3x^3-48d^2e^4x^4-105}{1680d^3x^7} - \frac{\sqrt{d^2}e^7\log(x)}{16d^4} + \frac{\sqrt{d^2}e^7\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{16d^4}$$

input
```
Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^8,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-240*d^6 - 280*d^5*e*x + 384*d^4*e^2*x^2 + 490*d^3*e
^3*x^3 - 48*d^2*e^4*x^4 - 105*d*e^5*x^5 - 96*e^6*x^6))/(1680*d^3*x^7) - (S
qrt[d^2]*e^7*Log[x])/(16*d^4) + (Sqrt[d^2]*e^7*Log[Sqrt[d^2] - Sqrt[d^2 -
e^2*x^2]])/(16*d^4)
```

3.14.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {539, 25, 27, 539, 25, 27, 534, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}dx$$

$$\Big\downarrow \text{539}$$

$$-\frac{\int -\frac{de(7d+2ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}dx}{7d^2}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{de(7d+2ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}dx}{7d^2}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int \frac{(7d+2ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}dx}{7d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{539}$$

$$\frac{e\left(-\frac{\int -\frac{de(12d+7ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}dx}{6d^2}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}\right)}{7d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{25}$$

$$\frac{e\left(\frac{\int \frac{de(12d+7ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}dx}{6d^2}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}\right)}{7d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\left(\frac{e\int\frac{(12d+7ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}dx}{6d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}\right)}{7d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{534}$$

$$\frac{e\left(\frac{e\left(7e\int\frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}dx-\frac{12\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{6d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}\right)}{7d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{243}$$

$$\frac{e\left(\frac{e\left(\frac{7}{2}e\int\frac{\left(d^2-e^2x^2\right)^{3/2}}{x^6}dx^2-\frac{12\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{6d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}\right)}{7d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{51}$$

$$\frac{e\left(\frac{e\left(\frac{7}{2}e\left(-\frac{3}{4}e^2\int\frac{\sqrt{d^2-e^2x^2}}{x^4}dx^2-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{12\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{6d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}\right)}{7d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{51}$$

$$\frac{e\left(\frac{e\left(\frac{7}{2}e\left(-\frac{3}{4}e^2\left(-\frac{1}{2}e^2\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{12\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{6d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}\right)}{7d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{73}$$

$$\frac{e\left(\frac{e\left(\frac{7}{2}e\left(-\frac{3}{4}e^2\left(\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{12\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{6d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}\right)}{7d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}$$

3.14. $\int\frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}dx$

$$\Big\downarrow \; 221$$

$$\frac{e\left(\frac{e\left(\frac{7}{2}e\left(-\frac{3}{4}e^2\left(\frac{e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{12\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{6d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}\right)}{7d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}$$

input
```
Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^8,x]
```

output
```
-1/7*(d^2 - e^2*x^2)^(5/2)/(d*x^7) + (e*((-7*(d^2 - e^2*x^2)^(5/2))/(6*d*x
^6) + (e*((-12*(d^2 - e^2*x^2)^(5/2))/(5*d*x^5) + (7*e*(-1/2*(d^2 - e^2*x^
2)^(3/2)/x^4 - (3*e^2*(-(Sqrt[d^2 - e^2*x^2]/x^2) + (e^2*ArcTanh[Sqrt[d^2
- e^2*x^2]/d])/d))/4))/2))/(6*d)))/(7*d)
```

3.14.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

3.14.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(96e^6x^6+105d\,e^5x^5+48d^2e^4x^4-490d^3x^3e^3-384d^4e^2x^2+280d^5ex+240d^6)}{1680x^7d^3}-\frac{e^7\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{16d^2\sqrt{d^2}}$
default	$e\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{6d^2x^6}+\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{4d^2x^4}-\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{2d^2x^2}-\frac{3e^2\left(\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2}}{x}\right.}{\sqrt{d^2}}\right.\right.}{2d^2}\right.}{4d^2}\right.}{6d^2}\right.$

input
```
int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/1680*(-e^2*x^2+d^2)^(1/2)*(96*e^6*x^6+105*d*e^5*x^5+48*d^2*e^4*x^4-490*
d^3*e^3*x^3-384*d^4*e^2*x^2+280*d^5*e*x+240*d^6)/x^7/d^3-1/16/d^2*e^7/(d^2
)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.70

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx = \frac{105\,e^7x^7\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-(96\,e^6x^6+105\,de^5x^5+48\,d^2e^4x^4-490\,d^3e^3}{1680\,d^3x^7}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
1/1680*(105*e^7*x^7*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (96*e^6*x^6 + 105
*d*e^5*x^5 + 48*d^2*e^4*x^4 - 490*d^3*e^3*x^3 - 384*d^4*e^2*x^2 + 280*d^5*
e*x + 240*d^6)*sqrt(-e^2*x^2 + d^2))/(d^3*x^7)
```

3.14.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.68 (sec) , antiderivative size = 1037, normalized size of antiderivative = 6.03

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**8,x)
```

output
```
d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e
**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**
4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**
2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(
e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105
*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) + d*
*2*e*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5
*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) -
1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(1
6*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x*
*2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*
x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**
2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - d*e**2*Piecewise((3*I*
d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2
*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*
x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**
4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**
2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e*
*2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*...
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx = -\frac{e^7\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{16\,d^3}+\frac{\sqrt{-e^2x^2+d^2}e^7}{16\,d^4} + \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^7}{48\,d^6}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5}{48\,d^6x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}{24\,d^4x^4} - \frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}{35\,d^3x^5}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{6\,d^2x^6}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{7\,dx^7}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
-1/16*e^7*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 + 1/16*s
qrt(-e^2*x^2 + d^2)*e^7/d^4 + 1/48*(-e^2*x^2 + d^2)^(3/2)*e^7/d^6 + 1/48*(
-e^2*x^2 + d^2)^(5/2)*e^5/(d^6*x^2) - 1/24*(-e^2*x^2 + d^2)^(5/2)*e^3/(d^4
*x^4) - 2/35*(-e^2*x^2 + d^2)^(5/2)*e^2/(d^3*x^5) - 1/6*(-e^2*x^2 + d^2)^(
5/2)*e/(d^2*x^6) - 1/7*(-e^2*x^2 + d^2)^(5/2)/(d*x^7)
```

3.14.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(148) = 296$.

Time = 0.29 (sec) , antiderivative size = 518, normalized size of antiderivative = 3.01

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx = \frac{\left(15\,e^8+\frac{35\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^6}{x}-\frac{21\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^4}{x^2}-\frac{105\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{x^3}\right.}{13440\left(de+\sqrt{-\right.} - \frac{e^8\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{16\,d^3|e|} - \frac{\frac{315\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{18}e^{12}}{x}-\frac{105\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^{18}e^{10}}{x^2}-\frac{105\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^{18}e^8}{x^3}-\frac{105\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^{18}e}{x^4}}{13440\,d^{21}e^6|e|}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x, algorithm="giac")
```

output
```
1/13440*(15*e^8 + 35*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^6/x - 21*(d*e +
 sqrt(-e^2*x^2 + d^2)*abs(e))^2*e^4/x^2 - 105*(d*e + sqrt(-e^2*x^2 + d^2)*
abs(e))^3*e^2/x^3 - 105*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/x^4 - 105*(d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^2*x^5) + 315*(d*e + sqrt(-e^2*x^2 +
 d^2)*abs(e))^6/(e^4*x^6))*e^14*x^7/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7
*d^3*abs(e)) - 1/16*e^8*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e)
)/(e^2*abs(x)))/(d^3*abs(e)) - 1/13440*(315*(d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))*d^18*e^12/x - 105*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^18*e^10/x^
2 - 105*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^18*e^8/x^3 - 105*(d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))^4*d^18*e^6/x^4 - 21*(d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))^5*d^18*e^4/x^5 + 35*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6*d^18*e^
2/x^6 + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7*d^18/x^7)/(d^21*e^6*abs(e
))
```

3.14.9 Mupad [B] (verification not implemented)

Time = 14.49 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx = \frac{8\,d\,e^2\,\sqrt{d^2-e^2\,x^2}}{35\,x^5} - \frac{d^3\,\sqrt{d^2-e^2\,x^2}}{7\,x^7} - \frac{e^4\,\sqrt{d^2-e^2\,x^2}}{35\,d\,x^3} - \frac{2\,e^6\,\sqrt{d^2-e^2\,x^2}}{35\,d^3\,x} - \frac{e\,\left(d^2-e^2\,x^2\right)^{3/2}}{6\,x^6} + \frac{d^2\,e\,\sqrt{d^2-e^2\,x^2}}{16\,x^6} - \frac{e\,\left(d^2-e^2\,x^2\right)^{5/2}}{16\,d^2\,x^6} + \frac{e^7\,\mathrm{atan}\left(\frac{\sqrt{d^2-e^2\,x^2}\,1\mathrm{i}}{d}\right)\,1\mathrm{i}}{16\,d^3}$$

input
```
int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^8,x)
```

output
```
(e^7*atan(((d^2 - e^2*x^2)^(1/2)*1i)/d)*1i)/(16*d^3) - (d^3*(d^2 - e^2*x^2
)^(1/2))/(7*x^7) - (e*(d^2 - e^2*x^2)^(3/2))/(6*x^6) - (e^4*(d^2 - e^2*x^2
)^(1/2))/(35*d*x^3) - (2*e^6*(d^2 - e^2*x^2)^(1/2))/(35*d^3*x) + (8*d*e^2*
(d^2 - e^2*x^2)^(1/2))/(35*x^5) + (d^2*e*(d^2 - e^2*x^2)^(1/2))/(16*x^6) -
 (e*(d^2 - e^2*x^2)^(5/2))/(16*d^2*x^6)
```

3.15 $\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^9}\,dx$

3.15.1 Optimal result

Integrand size = 25, antiderivative size = 201

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx = \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4} - \frac{(d^2-e^2x^2)^{5/2}}{8dx^8} - \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} - \frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5} - \frac{3e^8\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d^4}$$

output
```
-1/64*e^4*(-e^2*x^2+d^2)^(3/2)/d^3/x^4-1/8*(-e^2*x^2+d^2)^(5/2)/d/x^8-1/7*
e*(-e^2*x^2+d^2)^(5/2)/d^2/x^7-1/16*e^2*(-e^2*x^2+d^2)^(5/2)/d^3/x^6-2/35*
e^3*(-e^2*x^2+d^2)^(5/2)/d^4/x^5-3/128*e^8*arctanh((-e^2*x^2+d^2)^(1/2)/d)
/d^4+3/128*e^6*(-e^2*x^2+d^2)^(1/2)/d^3/x^2
```

3.15.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx = \frac{\sqrt{d^2-e^2x^2}(-560d^7-640d^6ex+840d^5e^2x^2+1024d^4e^3x^3-70d^3e^4x^4-12}{4480d^4x^8} - \frac{3\sqrt{d^2}e^8\log(x)}{128d^5} + \frac{3\sqrt{d^2}e^8\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{128d^5}$$

input
```
Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^9,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-560*d^7 - 640*d^6*e*x + 840*d^5*e^2*x^2 + 1024*d^4*
e^3*x^3 - 70*d^3*e^4*x^4 - 128*d^2*e^5*x^5 - 105*d*e^6*x^6 - 256*e^7*x^7))
/(4480*d^4*x^8) - (3*Sqrt[d^2]*e^8*Log[x])/(128*d^5) + (3*Sqrt[d^2]*e^8*Lo
g[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(128*d^5)
```

3.15.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {539, 25, 27, 539, 25, 27, 539, 27, 534, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}dx$$

$$\Big\downarrow \text{539}$$

$$-\frac{\int -\frac{de(8d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}dx}{8d^2}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{de(8d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}dx}{8d^2}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int \frac{(8d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}dx}{8d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \text{539}$$

$$\frac{e\left(-\frac{\int -\frac{de(21d+16ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}dx}{7d^2}-\frac{8\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}\right)}{8d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \text{25}$$

$$\frac{e\left(\frac{\int \frac{de(21d+16ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}dx}{7d^2}-\frac{8\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}\right)}{8d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\left(\frac{e\int \frac{(21d+16ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}dx}{7d}-\frac{8\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}\right)}{8d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \text{ 539}$$

$$\frac{e\left(\frac{e\left(-\frac{\int -\frac{3de(32d+7ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}dx}{6d^2}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{2dx^6}\right)}{7d}-\frac{8\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}\right)}{8d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e\left(\frac{e\left(\frac{e\int \frac{(32d+7ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}dx}{2d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{2dx^6}\right)}{7d}-\frac{8\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}\right)}{8d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \text{ 534}$$

$$\frac{e\left(\frac{e\left(\frac{e\left(7e\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}dx-\frac{32\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{2d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{2dx^6}\right)}{7d}-\frac{8\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}\right)}{8d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{e\left(\frac{e\left(\frac{e\left(\frac{7}{2}e\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^6}dx^2-\frac{32\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{2d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{2dx^6}\right)}{7d}-\frac{8\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}\right)}{8d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \text{ 51}$$

3.15. $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx$

$$\frac{e\left(\frac{e\left(\frac{e\left(\frac{7}{2}e\left(-\frac{3}{4}e^2\int\frac{\sqrt{d^2-e^2x^2}}{x^4}dx^2-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{32\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{2d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{2dx^6}\right)}{7d}-\frac{8\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}\right)}{8d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}$$

$\Big\downarrow$ 51

$$\frac{e\left(\frac{e\left(\frac{e\left(\frac{7}{2}e\left(-\frac{3}{4}e^2\left(-\frac{1}{2}e^2\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{32\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{2d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{2dx^6}\right)}{7d}-\frac{8\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}\right)}{8d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}$$

$\Big\downarrow$ 73

$$\frac{e\left(\frac{e\left(\frac{e\left(\frac{7}{2}e\left(-\frac{3}{4}e^2\left(\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{32\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{2d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{2dx^6}\right)}{7d}-\frac{8\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}\right)}{8d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}$$

$\Big\downarrow$ 221

3.15. $\int\frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx$

$$\frac{e\left(\frac{e\left(\frac{e\left(\frac{7}{2}e\left(-\frac{3}{4}e^2\left(\frac{e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{32\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{2d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{2dx^6}\right)}{7d}-\frac{8\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}\right)}{8d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}$$

input
```
Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^9,x]
```

output
```
-1/8*(d^2 - e^2*x^2)^(5/2)/(d*x^8) + (e*((-8*(d^2 - e^2*x^2)^(5/2))/(7*d*x
^7) + (e*((-7*(d^2 - e^2*x^2)^(5/2))/(2*d*x^6) + (e*((-32*(d^2 - e^2*x^2)^
(5/2))/(5*d*x^5) + (7*e*(-1/2*(d^2 - e^2*x^2)^(3/2)/x^4 - (3*e^2*(-(Sqrt[d
^2 - e^2*x^2]/x^2) + (e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/4))/2))/(2*d
)))/(7*d)))/(8*d)
```

3.15.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

3.15.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,\left(256e^{7}x^{7}+105d\,e^{6}x^{6}+128d^{2}e^{5}x^{5}+70d^{3}e^{4}x^{4}-1024d^{4}e^{3}x^{3}-840d^{5}e^{2}x^{2}+640d^{6}ex+560d^{7}\right)}{4480x^{8}d^{4}}-\frac{3e^{8}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-}}{x}\right.}{128d^{3}\sqrt{d^{2}}}$
default	$e\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{7d^{2}x^{7}}-\frac{2e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{35d^{4}x^{5}}\right)+d\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{8d^{2}x^{8}}+\frac{3e^{2}\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{6d^{2}x^{6}}+\frac{e^{2}\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{4d^{2}x^{4}}-\frac{e^{2}\left(-\frac{\left(-\right.}{}\right.}{}\right)}{}\right)}{}\right.$

3.15. $\int\frac{(d+ex)\left(d^{2}-e^{2}x^{2}\right)^{3/2}}{x^{9}}\,dx$

input
```
int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/4480*(-e^2*x^2+d^2)^(1/2)*(256*e^7*x^7+105*d*e^6*x^6+128*d^2*e^5*x^5+70
*d^3*e^4*x^4-1024*d^4*e^3*x^3-840*d^5*e^2*x^2+640*d^6*e*x+560*d^7)/x^8/d^4
-3/128/d^3*e^8/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x
)
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.65

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx = \frac{105\,e^8x^8\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(256\,e^7x^7+105\,de^6x^6+128\,d^2e^5x^5+70\,d^3e\right.}{4480\,d^4x^8}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
1/4480*(105*e^8*x^8*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (256*e^7*x^7 + 10
5*d*e^6*x^6 + 128*d^2*e^5*x^5 + 70*d^3*e^4*x^4 - 1024*d^4*e^3*x^3 - 840*d^
5*e^2*x^2 + 640*d^6*e*x + 560*d^7)*sqrt(-e^2*x^2 + d^2))/(d^4*x^8)
```

3.15.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 25.09 (sec) , antiderivative size = 1159, normalized size of antiderivative = 5.77

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**9,x)
```

output
```
d**3*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)) + 7*e/(48*x**7
*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**2/(e**2*x**2) -
 1)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5*e**7/(128*d**
6*x*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*d**7), Abs(d*
*2/(e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I
*e/(48*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**5*sqrt(-d**
2/(e**2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x**2) + 1))
 + 5*I*e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(e*
x))/(128*d**7), True)) + d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(
7*x**6) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**
2/(e**2*x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(10
5*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*
x**6) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(
-d**2/(e**2*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) +
 1)/(105*d**6), True)) - d*e**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2
*x**2) - 1)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x*
*3*sqrt(d**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1
)) + e**6*acosh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6
*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**
2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(...
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx = -\frac{3\,e^8\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{128\,d^4} + \frac{3\sqrt{-e^2x^2+d^2}e^8}{128\,d^5} + \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^8}{128\,d^7} + \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}{128\,d^7x^2} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}{64\,d^5x^4} - \frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}{35\,d^4x^5} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}{16\,d^3x^6} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{7\,d^2x^7} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{8\,dx^8}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
-3/128*e^8*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^4 + 3/128
*sqrt(-e^2*x^2 + d^2)*e^8/d^5 + 1/128*(-e^2*x^2 + d^2)^(3/2)*e^8/d^7 + 1/1
28*(-e^2*x^2 + d^2)^(5/2)*e^6/(d^7*x^2) - 1/64*(-e^2*x^2 + d^2)^(5/2)*e^4/
(d^5*x^4) - 2/35*(-e^2*x^2 + d^2)^(5/2)*e^3/(d^4*x^5) - 1/16*(-e^2*x^2 + d
^2)^(5/2)*e^2/(d^3*x^6) - 1/7*(-e^2*x^2 + d^2)^(5/2)*e/(d^2*x^7) - 1/8*(-e
^2*x^2 + d^2)^(5/2)/(d*x^8)
```

3.15.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(173) = 346$.

Time = 0.29 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.30

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx = \frac{\left(35\,e^9 + \frac{80\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^7}{x} - \frac{112\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3e^3}{x^3} - \frac{280\left(de+\sqrt{-e^2x^2+d^2}|e|\cdots\right.}{x^4}\cdots}{71680\left(de+\sqrt{-e^2x^2+d^2}|\cdots\right.}$$
$$-\frac{3\,e^9\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{128\,d^4|e|}$$
$$-\frac{\frac{1680\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{28}e^{13}|e|}{x} - \frac{560\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^{28}e^9|e|}{x^3} - \frac{280\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^{28}e^7|e|}{x^4} - \frac{112\left(de+\sqrt{-e^2x^2+d^2}|e\cdots\right.}{x^5}\cdots}{71680\,d^{32}e^8}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x, algorithm="giac")
```

output
```
1/71680*(35*e^9 + 80*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^7/x - 112*(d*e
+ sqrt(-e^2*x^2 + d^2)*abs(e))^3*e^3/x^3 - 280*(d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))^4*e/x^4 - 560*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e*x^5) + 168
0*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7/(e^5*x^7))*e^16*x^8/((d*e + sqrt(-
e^2*x^2 + d^2)*abs(e))^8*d^4*abs(e)) - 3/128*e^9*log(1/2*abs(-2*d*e - 2*sq
rt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^4*abs(e)) - 1/71680*(1680*(d*e
 + sqrt(-e^2*x^2 + d^2)*abs(e))*d^28*e^13*abs(e)/x - 560*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^3*d^28*e^9*abs(e)/x^3 - 280*(d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))^4*d^28*e^7*abs(e)/x^4 - 112*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5
*d^28*e^5*abs(e)/x^5 + 80*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7*d^28*e*abs
(e)/x^7 + 35*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8*d^28*abs(e)/(e*x^8))/(d
^32*e^8)
```

3.15.9 Mupad [B] (verification not implemented)

Time = 14.95 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)\,(d^2-e^2x^2)^{3/2}}{x^9}\,dx = \frac{3\,d^3\,\sqrt{d^2-e^2\,x^2}}{128\,x^8} - \frac{11\,d\,(d^2-e^2\,x^2)^{3/2}}{128\,x^8} - \frac{11\,(d^2-e^2\,x^2)^{5/2}}{128\,d\,x^8} + \frac{3\,(d^2-e^2\,x^2)^{7/2}}{128\,d^3\,x^8} + \frac{8\,e^3\,\sqrt{d^2-e^2\,x^2}}{35\,x^5} - \frac{e^5\,\sqrt{d^2-e^2\,x^2}}{35\,d^2\,x^3} - \frac{2\,e^7\,\sqrt{d^2-e^2\,x^2}}{35\,d^4\,x} - \frac{d^2\,e\,\sqrt{d^2-e^2\,x^2}}{7\,x^7} + \frac{e^8\,\mathrm{atan}\left(\frac{\sqrt{d^2-e^2\,x^2}\,1\mathrm{i}}{d}\right)\,3\mathrm{i}}{128\,d^4}$$

input
```
int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^9,x)
```

output
```
(3*d^3*(d^2 - e^2*x^2)^(1/2))/(128*x^8) - (11*d*(d^2 - e^2*x^2)^(3/2))/(12
8*x^8) - (11*(d^2 - e^2*x^2)^(5/2))/(128*d*x^8) + (3*(d^2 - e^2*x^2)^(7/2)
)/(128*d^3*x^8) + (8*e^3*(d^2 - e^2*x^2)^(1/2))/(35*x^5) + (e^8*atan(((d^2
 - e^2*x^2)^(1/2)*1i)/d)*3i)/(128*d^4) - (e^5*(d^2 - e^2*x^2)^(1/2))/(35*d
^2*x^3) - (2*e^7*(d^2 - e^2*x^2)^(1/2))/(35*d^4*x) - (d^2*e*(d^2 - e^2*x^2
)^(1/2))/(7*x^7)
```

3.16 $\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx$

3.16.1 Optimal result

Integrand size = 25, antiderivative size = 103

$$\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{d^2\sqrt{d^2-e^2x^2}}{e^3}-\frac{dx\sqrt{d^2-e^2x^2}}{2e^2}+\frac{(d^2-e^2x^2)^{3/2}}{3e^3}+\frac{d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

output
```
1/3*(-e^2*x^2+d^2)^(3/2)/e^3+1/2*d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-
d^2*(-e^2*x^2+d^2)^(1/2)/e^3-1/2*d*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

3.16.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.81

$$\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx = \frac{(-4d^2-3dex-2e^2x^2)\sqrt{d^2-e^2x^2}}{6e^3}-\frac{d^3\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

input
```
Integrate[(x^2*(d + e*x))/Sqrt[d^2 - e^2*x^2],x]
```

output
```
((-4*d^2 - 3*d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(6*e^3) - (d^3*ArcTan
[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^3
```

3.16.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {533, 27, 533, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{533}$$

$$\frac{\int \frac{dex(2d+3ex)}{\sqrt{d^2-e^2x^2}}dx}{3e^2}-\frac{x^2\sqrt{d^2-e^2x^2}}{3e}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int \frac{x(2d+3ex)}{\sqrt{d^2-e^2x^2}}dx}{3e}-\frac{x^2\sqrt{d^2-e^2x^2}}{3e}$$

$$\Big\downarrow \text{533}$$

$$\frac{d\left(\frac{\int \frac{de(3d+4ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{3x\sqrt{d^2-e^2x^2}}{2e}\right)}{3e}-\frac{x^2\sqrt{d^2-e^2x^2}}{3e}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\left(\frac{d\int \frac{3d+4ex}{\sqrt{d^2-e^2x^2}}dx}{2e}-\frac{3x\sqrt{d^2-e^2x^2}}{2e}\right)}{3e}-\frac{x^2\sqrt{d^2-e^2x^2}}{3e}$$

$$\Big\downarrow \text{455}$$

$$\frac{d\left(\frac{d\left(3d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{4\sqrt{d^2-e^2x^2}}{e}\right)}{2e}-\frac{3x\sqrt{d^2-e^2x^2}}{2e}\right)}{3e}-\frac{x^2\sqrt{d^2-e^2x^2}}{3e}$$

$$\Big\downarrow \text{224}$$

$$\frac{d\left(\frac{d\left(3d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{4\sqrt{d^2-e^2x^2}}{e}\right)}{2e}-\frac{3x\sqrt{d^2-e^2x^2}}{2e}\right)}{3e}-\frac{x^2\sqrt{d^2-e^2x^2}}{3e}$$

$$\Big\downarrow \text{216}$$

$$\frac{d\left(\frac{d\left(\frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{4\sqrt{d^2-e^2x^2}}{e}\right)}{2e}-\frac{3x\sqrt{d^2-e^2x^2}}{2e}\right)}{3e}-\frac{x^2\sqrt{d^2-e^2x^2}}{3e}$$

input
```
Int[(x^2*(d + e*x))/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/3*(x^2*Sqrt[d^2 - e^2*x^2])/e + (d*((-3*x*Sqrt[d^2 - e^2*x^2])/(2*e) +
(d*((-4*Sqrt[d^2 - e^2*x^2])/e + (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
))/(2*e)))/(3*e)
```

3.16.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.16.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.73

method	result	size
risch	$-\frac{(2e^{2}x^{2}+3dex+4d^{2})\sqrt{-e^{2}x^{2}+d^{2}}}{6e^{3}}+\frac{d^{3}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2e^{2}\sqrt{e^{2}}}$	75
default	$e\left(-\frac{x^{2}\sqrt{-e^{2}x^{2}+d^{2}}}{3e^{2}}-\frac{2d^{2}\sqrt{-e^{2}x^{2}+d^{2}}}{3e^{4}}\right)+d\left(-\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2e^{2}}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2e^{2}\sqrt{e^{2}}}\right)$	107

input
```
int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*e^2*x^2+3*d*e*x+4*d^2)/e^3*(-e^2*x^2+d^2)^(1/2)+1/2*d^3/e^2/(e^2)^
(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.70

$$\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{6\,d^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(2\,e^2x^2+3\,dex+4\,d^2)\sqrt{-e^2x^2+d^2}}{6\,e^3}$$

input
```
integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(6*d^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (2*e^2*x^2 + 3*d*e
*x + 4*d^2)*sqrt(-e^2*x^2 + d^2))/e^3
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.25

$$\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= \begin{cases} \frac{d^3\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{2e^2} + \sqrt{d^2-e^2x^2}\left(-\frac{2d^2}{3e^3}-\frac{dx}{2e^2}-\frac{x^2}{3e}\right) & \text{for } e^2\neq 0 \\ \frac{\frac{dx^3}{3}+\frac{ex^4}{4}}{\sqrt{d^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Piecewise((d**3*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*
x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(2*e*
*2) + sqrt(d**2 - e**2*x**2)*(-2*d**2/(3*e**3) - d*x/(2*e**2) - x**2/(3*e)
), Ne(e**2, 0)), ((d*x**3/3 + e*x**4/4)/sqrt(d**2), True))
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.90

$$\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}x^2}{3\,e} + \frac{d^3\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}e^2} - \frac{\sqrt{-e^2x^2+d^2}dx}{2\,e^2} - \frac{2\sqrt{-e^2x^2+d^2}d^2}{3\,e^3}$$

input
```
integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(-e^2*x^2 + d^2)*x^2/e + 1/2*d^3*arcsin(e^2*x/(d*sqrt(e^2)))/(sqr
t(e^2)*e^2) - 1/2*sqrt(-e^2*x^2 + d^2)*d*x/e^2 - 2/3*sqrt(-e^2*x^2 + d^2)*
d^2/e^3
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.62

$$\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx = \frac{d^3 \arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,e^2|e|} - \frac{1}{6}\sqrt{-e^2x^2+d^2}\left(x\left(\frac{2\,x}{e}+\frac{3\,d}{e^2}\right)+\frac{4\,d^2}{e^3}\right)$$

input
```
integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*d^3*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) - 1/6*sqrt(-e^2*x^2 + d^2
)*(x*(2*x/e + 3*d/e^2) + 4*d^2/e^3)
```

3.16.9 Mupad [B] (verification not implemented)

Time = 11.93 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09

$$\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx = \begin{cases} \frac{d\,x^3}{3\,\sqrt{d^2}} & \text{if } e=0 \\ -\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2+e^2\,x^2\right)}{3\,e^3} - \frac{d^3\,\ln\left(2\,x\,\sqrt{-e^2}+2\,\sqrt{d^2-e^2\,x^2}\right)}{2\,{\left(-e^2\right)}^{3/2}} - \frac{d\,x\,\sqrt{d^2-e^2\,x^2}}{2\,e^2} & \text{if } e\neq 0 \end{cases}$$

input
```
int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(1/2),x)
```

output
```
piecewise(e == 0, (d*x^3)/(3*(d^2)^(1/2)), e ~= 0, - ((d^2 - e^2*x^2)^(1/2
)*(2*d^2 + e^2*x^2))/(3*e^3) - (d^3*log(2*x*(-e^2)^(1/2) + 2*(d^2 - e^2*x^
2)^(1/2)))/(2*(-e^2)^(3/2)) - (d*x*(d^2 - e^2*x^2)^(1/2))/(2*e^2))
```

3.17 $\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{3/2}}\,dx$

3.17.1 Optimal result

Integrand size = 25, antiderivative size = 73

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{3/2}}\,dx=\frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

output
```
-d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+d*(e*x+d)/e^3/(-e^2*x^2+d^2)^(1/2)
+(-e^2*x^2+d^2)^(1/2)/e^3
```

3.17.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{3/2}}\,dx=\frac{(-2d+ex)\sqrt{d^2-e^2x^2}}{e^3(-d+ex)}+\frac{2d\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

input
```
Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(3/2),x]
```

output
```
((-2*d + e*x)*Sqrt[d^2 - e^2*x^2])/(e^3*(-d + e*x)) + (2*d*ArcTan[(e*x)/(S
qrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^3
```

3.17.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {527, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{527}$$

$$\frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}}-\frac{\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}dx}{e^2}$$

$$\Big\downarrow \text{455}$$

$$\frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{e}}{e^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}}-\frac{d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{e}}{e^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}}-\frac{\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{\sqrt{d^2-e^2x^2}}{e}}{e^2}$$

input
```
Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(3/2),x]
```

output
```
(d*(d + e*x))/(e^3*Sqrt[d^2 - e^2*x^2]) - (-(Sqrt[d^2 - e^2*x^2]/e) + (d*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e)/e^2
```

3.17.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 527
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_S
ymbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a +
b*x^2])), x] + Simp[1/(b*d^(m - 2))   Int[(1/Sqrt[a + b*x^2])*ExpandToSum[(
2^(n - 1)*c^(m + n - 1) - d^m*x^m*(c + d*x)^(n - 1))/(c - d*x), x], x], x]
/; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && EqQ[b*c^2 + a*d^2,
 0]
```

3.17.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

method	result	size
risch	$\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{e^{3}}-\frac{d \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{e^{2} \sqrt{e^{2}}}-\frac{d \sqrt{-\left(x-\frac{d}{e}\right)^{2} e^{2}-2 d e \left(x-\frac{d}{e}\right)}}{e^{4} \left(x-\frac{d}{e}\right)}$	99
default	$e \left(-\frac{x^{2}}{e^{2} \sqrt{-e^{2} x^{2}+d^{2}}}+\frac{2 d^{2}}{e^{4} \sqrt{-e^{2} x^{2}+d^{2}}}\right)+d \left(\frac{x}{e^{2} \sqrt{-e^{2} x^{2}+d^{2}}}-\frac{\arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{e^{2} \sqrt{e^{2}}}\right)$	103

input
```
int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-e^2*x^2+d^2)^(1/2)/e^3-d/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+
d^2)^(1/2))-d/e^4/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.19

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{3/2}}\,dx = \frac{2\,dex-2\,d^2+2\,(dex-d^2)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}(ex-2\,d)}{e^4x-de^3}$$

input
```
integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
(2*d*e*x - 2*d^2 + 2*(d*e*x - d^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x
)) + sqrt(-e^2*x^2 + d^2)*(e*x - 2*d))/(e^4*x - d*e^3)
```

3.17.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.84

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{3/2}}\,dx = d\left(\begin{cases} \frac{i\,\mathrm{acosh}\left(\frac{ex}{d}\right)}{e^3}-\frac{ix}{de^2\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ -\frac{\mathrm{asin}\left(\frac{ex}{d}\right)}{e^3}+\frac{x}{de^2\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e\left(\begin{cases} \frac{2d^2}{e^4\sqrt{d^2-e^2x^2}}-\frac{x^2}{e^2\sqrt{d^2-e^2x^2}} & \text{for } e\neq 0 \\ \frac{x^4}{4(d^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
d*Piecewise((I*acosh(e*x/d)/e**3 - I*x/(d*e**2*sqrt(-1 + e**2*x**2/d**2)),
 Abs(e**2*x**2/d**2) > 1), (-asin(e*x/d)/e**3 + x/(d*e**2*sqrt(1 - e**2*x*
*2/d**2)), True)) + e*Piecewise((2*d**2/(e**4*sqrt(d**2 - e**2*x**2)) - x*
*2/(e**2*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)**(3/2)), True
))
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.23

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{3/2}}\,dx = -\frac{x^2}{\sqrt{-e^2x^2+d^2}e}+\frac{dx}{\sqrt{-e^2x^2+d^2}e^2}-\frac{d\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}e^2}+\frac{2\,d^2}{\sqrt{-e^2x^2+d^2}e^3}$$

input
```
integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
-x^2/(sqrt(-e^2*x^2 + d^2)*e) + d*x/(sqrt(-e^2*x^2 + d^2)*e^2) - d*arcsin(
e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) + 2*d^2/(sqrt(-e^2*x^2 + d^2)*e^3)
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{3/2}}\,dx = -\frac{d\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{e^2|e|}+\frac{\sqrt{-e^2x^2+d^2}}{e^3}+\frac{2\,d}{e^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)|e|}$$

input
```
integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
-d*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) + sqrt(-e^2*x^2 + d^2)/e^3 + 2
*d/(e^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)*abs(e))
```

3.17.9 Mupad [B] (verification not implemented)

Time = 11.78 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.19

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{3/2}}\,dx = \frac{2\,d^2-e^2\,x^2}{e^3\,\sqrt{d^2-e^2\,x^2}}+\frac{d\,\ln\left(x\,\sqrt{-e^2}+\sqrt{d^2-e^2\,x^2}\right)}{(-e^2)^{3/2}}+\frac{d\,x}{e^2\,\sqrt{d^2-e^2\,x^2}}$$

input
```
int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(3/2),x)
```

output
```
(2*d^2 - e^2*x^2)/(e^3*(d^2 - e^2*x^2)^(1/2)) + (d*log(x*(-e^2)^(1/2) + (d
^2 - e^2*x^2)^(1/2)))/(-e^2)^(3/2) + (d*x)/(e^2*(d^2 - e^2*x^2)^(1/2))
```

3.18 $\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{5/2}}\,dx$

3.18.1 Optimal result

Integrand size = 25, antiderivative size = 58

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{x^2(d+ex)}{3de\,(d^2-e^2x^2)^{3/2}} - \frac{2}{3e^3\sqrt{d^2-e^2x^2}}$$

output
```
1/3*x^2*(e*x+d)/d/e/(-e^2*x^2+d^2)^(3/2)-2/3/e^3/(-e^2*x^2+d^2)^(1/2)
```

3.18.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-2d^2+2dex+e^2x^2)}{3de^3(d-ex)^2(d+ex)}$$

input
```
Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(5/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-2*d^2 + 2*d*e*x + e^2*x^2))/(3*d*e^3*(d - e*x)^2*(d
 + e*x))
```

3.18.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {529, 27, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 529$$

$$\frac{d(d+ex)}{3e^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int \frac{d(d+3ex)}{e^2(d^2-e^2x^2)^{3/2}}dx}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{d(d+ex)}{3e^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int \frac{d+3ex}{(d^2-e^2x^2)^{3/2}}dx}{3e^2}$$

$$\Big\downarrow \; 453$$

$$\frac{d(d+ex)}{3e^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{3d+ex}{3de^3\sqrt{d^2-e^2x^2}}$$

input
```
Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(5/2),x]
```

output
```
(d*(d + e*x))/(3*e^3*(d^2 - e^2*x^2)^(3/2)) - (3*d + e*x)/(3*d*e^3*Sqrt[d^
2 - e^2*x^2])
```

3.18.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

3.18.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

method	result	size
gosper	$-\frac{(-ex+d)(ex+d)^2(-e^2x^2-2dex+2d^2)}{3d\,e^3(-e^2x^2+d^2)^{\frac{5}{2}}}$	55
trager	$-\frac{(-e^2x^2-2dex+2d^2)\sqrt{-e^2x^2+d^2}}{3d\,e^3(-ex+d)^2(ex+d)}$	57
default	$e\left(\frac{x^2}{e^2(-e^2x^2+d^2)^{\frac{3}{2}}}-\frac{2d^2}{3e^4(-e^2x^2+d^2)^{\frac{3}{2}}}\right)+d\left(\frac{x}{2e^2(-e^2x^2+d^2)^{\frac{3}{2}}}-\frac{d^2\left(\frac{x}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{2e^2}\right)$	120

input
```
int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-e*x+d)*(e*x+d)^2*(-e^2*x^2-2*d*e*x+2*d^2)/d/e^3/(-e^2*x^2+d^2)^(5/2
)
```

3.18.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(50) = 100.

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.79

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{5/2}}\,dx = -\frac{2\,e^3x^3-2\,de^2x^2-2\,d^2ex+2\,d^3-(e^2x^2+2\,dex-2\,d^2)\sqrt{-e^2x^2+d^2}}{3\,(de^6x^3-d^2e^5x^2-d^3e^4x+d^4e^3)}$$

input
```
integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(2*e^3*x^3 - 2*d*e^2*x^2 - 2*d^2*e*x + 2*d^3 - (e^2*x^2 + 2*d*e*x - 2
*d^2)*sqrt(-e^2*x^2 + d^2))/(d*e^6*x^3 - d^2*e^5*x^2 - d^3*e^4*x + d^4*e^3
)
```

3.18.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.44 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.98

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{5/2}}\,dx = d\left(\begin{cases} \frac{ix^3}{-3d^5\sqrt{-1+\frac{e^2x^2}{d^2}}+3d^3e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ -\frac{x^3}{-3d^5\sqrt{1-\frac{e^2x^2}{d^2}}+3d^3e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e\left(\begin{cases} \frac{2d^2}{-3d^2e^4\sqrt{d^2-e^2x^2}+3e^6x^2\sqrt{d^2-e^2x^2}} - \frac{3e^2x^2}{-3d^2e^4\sqrt{d^2-e^2x^2}+3e^6x^2\sqrt{d^2-e^2x^2}} & \text{for } e\neq 0 \\ \frac{x^4}{4(d^2)^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

output
```
d*Piecewise((I*x**3/(-3*d**5*sqrt(-1 + e**2*x**2/d**2) + 3*d**3*e**2*x**2*
sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-x**3/(-3*d**5*sqrt
(1 - e**2*x**2/d**2) + 3*d**3*e**2*x**2*sqrt(1 - e**2*x**2/d**2)), True))
+ e*Piecewise((2*d**2/(-3*d**2*e**4*sqrt(d**2 - e**2*x**2) + 3*e**6*x**2*s
qrt(d**2 - e**2*x**2)) - 3*e**2*x**2/(-3*d**2*e**4*sqrt(d**2 - e**2*x**2)
+ 3*e**6*x**2*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)**(5/2)),
 True))
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.52

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e} + \frac{dx}{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^3} - \frac{x}{3\,\sqrt{-e^2x^2+d^2}de^2}$$

input
```
integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
x^2/((-e^2*x^2 + d^2)^(3/2)*e) + 1/3*d*x/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2/
3*d^2/((-e^2*x^2 + d^2)^(3/2)*e^3) - 1/3*x/(sqrt(-e^2*x^2 + d^2)*d*e^2)
```

3.18.8 Giac [F]

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{(ex+d)x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)*x^2/(-e^2*x^2 + d^2)^(5/2), x)
```

3.18.9 Mupad [B] (verification not implemented)

Time = 11.49 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,(-2\,d^2+2\,d\,e\,x+e^2\,x^2)}{3\,d\,e^3\,(d+e\,x)\,(d-e\,x)^2}$$

input
```
int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(5/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(e^2*x^2 - 2*d^2 + 2*d*e*x))/(3*d*e^3*(d + e*x)*(d
- e*x)^2)
```

3.19 $\int \frac{x^7(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx$

3.19.1 Optimal result

Integrand size = 25, antiderivative size = 161

$$\int \frac{x^7(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^6(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8} - \frac{7d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8}$$

output
```
1/5*x^6*(e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^4*(7*e*x+6*d)/e^4/(-e^2*x^
2+d^2)^(3/2)-7/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^8+1/15*x^2*(35*e*x
+24*d)/e^6/(-e^2*x^2+d^2)^(1/2)+1/10*(35*e*x+32*d)*(-e^2*x^2+d^2)^(1/2)/e^
8
```

3.19.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.87

$$\int \frac{x^7(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(96d^6+9d^5ex-249d^4e^2x^2+4d^3e^3x^3+176d^2e^4x^4-15de^5x^5-15e^6x^6)}{(d-ex)^3(d+ex)^2} + 210d^2\arctan\left(\frac{ex}{\sqrt{d^2-\sqrt{d^2-}}}\right.}{30e^8}$$

input
```
Integrate[(x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(96*d^6 + 9*d^5*e*x - 249*d^4*e^2*x^2 + 4*d^3*e^3*x^
3 + 176*d^2*e^4*x^4 - 15*d*e^5*x^5 - 15*e^6*x^6))/((d - e*x)^3*(d + e*x)^2
) + 210*d^2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(30*e^8)
```

3.19.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.27, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {529, 2345, 2345, 27, 2346, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7(d+ex)}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{529}$$

$$\frac{d^6(d+ex)}{5e^8(d^2-e^2x^2)^{5/2}}-\frac{\int \frac{\frac{d^7}{e^7}+\frac{5xd^6}{e^6}+\frac{5x^2d^5}{e^5}+\frac{5x^3d^4}{e^4}+\frac{5x^4d^3}{e^3}+\frac{5x^5d^2}{e^2}+\frac{5x^6d}{e}}{(d^2-e^2x^2)^{5/2}}dx}{5d}$$

$$\Big\downarrow \text{2345}$$

$$\frac{d^6(d+ex)}{5e^8(d^2-e^2x^2)^{5/2}}-\frac{\frac{d^5(15d+16ex)}{3e^8(d^2-e^2x^2)^{3/2}}-\frac{\int \frac{\frac{13d^7}{e^7}+\frac{30xd^6}{e^6}+\frac{30x^2d^5}{e^5}+\frac{15x^3d^4}{e^4}+\frac{15x^4d^3}{e^3}}{(d^2-e^2x^2)^{3/2}}dx}{3d^2}}{5d}$$

$$\Big\downarrow \text{2345}$$

$$\frac{d^6(d+ex)}{5e^8(d^2-e^2x^2)^{5/2}}-\frac{\frac{d^5(15d+16ex)}{3e^8(d^2-e^2x^2)^{3/2}}-\frac{\frac{d^5(45d+58ex)}{e^8\sqrt{d^2-e^2x^2}}-\frac{\int \frac{15\left(\frac{3d^7}{e^7}+\frac{xd^6}{e^6}+\frac{x^2d^5}{e^5}\right)}{\sqrt{d^2-e^2x^2}}dx}{d^2}}{3d^2}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^6(d+ex)}{5e^8(d^2-e^2x^2)^{5/2}}-\frac{\frac{d^5(15d+16ex)}{3e^8(d^2-e^2x^2)^{3/2}}-\frac{\frac{d^5(45d+58ex)}{e^8\sqrt{d^2-e^2x^2}}-\frac{15\int \frac{\frac{3d^7}{e^7}+\frac{xd^6}{e^6}+\frac{x^2d^5}{e^5}}{\sqrt{d^2-e^2x^2}}dx}{d^2}}{3d^2}}{5d}$$

$$\Big\downarrow \text{2346}$$

$$\frac{d^6(d+ex)}{5e^8\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{d^5(15d+16ex)}{3e^8(d^2-e^2x^2)^{3/2}}-\frac{\frac{d^5(45d+58ex)}{e^8\sqrt{d^2-e^2x^2}}-\frac{15\left(-\frac{\int -\frac{d^6(7d+2ex)}{e^5\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{d^5x\sqrt{d^2-e^2x^2}}{2e^7}\right)}{d^2}}{3d^2}}{5d}$$

$$\Big\downarrow \; 25$$

$$\frac{d^6(d+ex)}{5e^8\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{d^5(15d+16ex)}{3e^8(d^2-e^2x^2)^{3/2}}-\frac{\frac{d^5(45d+58ex)}{e^8\sqrt{d^2-e^2x^2}}-\frac{15\left(\frac{\int \frac{d^6(7d+2ex)}{e^5\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{d^5x\sqrt{d^2-e^2x^2}}{2e^7}\right)}{d^2}}{3d^2}}{5d}$$

$$\Big\downarrow \; 27$$

$$\frac{d^6(d+ex)}{5e^8\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{d^5(15d+16ex)}{3e^8(d^2-e^2x^2)^{3/2}}-\frac{\frac{d^5(45d+58ex)}{e^8\sqrt{d^2-e^2x^2}}-\frac{15\left(\frac{d^6\int \frac{7d+2ex}{\sqrt{d^2-e^2x^2}}dx}{2e^7}-\frac{d^5x\sqrt{d^2-e^2x^2}}{2e^7}\right)}{d^2}}{3d^2}}{5d}$$

$$\Big\downarrow \; 455$$

$$\frac{d^6(d+ex)}{5e^8\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{d^5(15d+16ex)}{3e^8(d^2-e^2x^2)^{3/2}}-\frac{\frac{d^5(45d+58ex)}{e^8\sqrt{d^2-e^2x^2}}-\frac{15\left(\frac{d^6\left(7d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{2\sqrt{d^2-e^2x^2}}{e}\right)}{2e^7}-\frac{d^5x\sqrt{d^2-e^2x^2}}{2e^7}\right)}{d^2}}{3d^2}}{5d}$$

$$\Big\downarrow \; 224$$

$$\frac{d^6(d+ex)}{5e^8\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{d^5(15d+16ex)}{3e^8(d^2-e^2x^2)^{3/2}}-\frac{\frac{d^5(45d+58ex)}{e^8\sqrt{d^2-e^2x^2}}-\frac{15\left(\frac{d^6\left(7d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{2\sqrt{d^2-e^2x^2}}{e}\right)}{2e^7}-\frac{d^5x\sqrt{d^2-e^2x^2}}{2e^7}\right)}{d^2}}{3d^2}}{5d}$$

$$\Big\downarrow \; 216$$

3.19. $\int \frac{x^7(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx$

$$\frac{d^6(d+ex)}{5e^8\left(d^2-e^2x^2\right)^{5/2}} - \frac{\frac{d^5(15d+16ex)}{3e^8(d^2-e^2x^2)^{3/2}} - \frac{\frac{d^5(45d+58ex)}{e^8\sqrt{d^2-e^2x^2}} - \frac{15\left(\frac{d^6\left(\frac{7d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e} - \frac{2\sqrt{d^2-e^2x^2}}{e}\right)}{2e^7} - \frac{d^5x\sqrt{d^2-e^2x^2}}{2e^7}\right)}{d^2}}{3d^2}}{5d}$$

input
```
Int[(x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(d^6*(d + e*x))/(5*e^8*(d^2 - e^2*x^2)^(5/2)) - ((d^5*(15*d + 16*e*x))/(3*
e^8*(d^2 - e^2*x^2)^(3/2)) - ((d^5*(45*d + 58*e*x))/(e^8*Sqrt[d^2 - e^2*x^
2]) - (15*(-1/2*(d^5*x*Sqrt[d^2 - e^2*x^2])/e^7 + (d^6*((-2*Sqrt[d^2 - e^2
*x^2])/e + (7*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/(2*e^7)))/d^2)/(3*d
^2))/(5*d)
```

3.19.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.19.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.56

method	result
default	$e\left(-\frac{x^7}{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{7d^2\left(\frac{x^5}{5e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{\frac{x^3}{3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e^2}}{e^2}\right)}{2e^2}\right)+d\left(-\frac{}{e^2}\right.$
risch	$\frac{(ex+2d)\sqrt{-e^2x^2+d^2}}{2e^8}-\frac{7d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^7\sqrt{e^2}}-\frac{7d^3\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{15e^{10}\left(x-\frac{d}{e}\right)^2}-\frac{773d^2\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{240e^9\left(x-\frac{d}{e}\right)}+$

input
```
int(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
e*(-1/2*x^7/e^2/(-e^2*x^2+d^2)^(5/2)+7/2*d^2/e^2*(1/5*x^5/e^2/(-e^2*x^2+d^
2)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)-1/e^2*(x/e^2/(-e^2*x^2+d^
2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))+
d*(-x^6/e^2/(-e^2*x^2+d^2)^(5/2)+6*d^2/e^2*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4
*d^2/e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/
2))))
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.73

$$\int \frac{x^7(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{96\,d^2e^5x^5-96\,d^3e^4x^4-192\,d^4e^3x^3+192\,d^5e^2x^2+96\,d^6ex-96\,d^7+210\,(d^2e^5x^5-d}{}$$

input
```
integrate(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/30*(96*d^2*e^5*x^5 - 96*d^3*e^4*x^4 - 192*d^4*e^3*x^3 + 192*d^5*e^2*x^2
+ 96*d^6*e*x - 96*d^7 + 210*(d^2*e^5*x^5 - d^3*e^4*x^4 - 2*d^4*e^3*x^3 + 2
*d^5*e^2*x^2 + d^6*e*x - d^7)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) +
(15*e^6*x^6 + 15*d*e^5*x^5 - 176*d^2*e^4*x^4 - 4*d^3*e^3*x^3 + 249*d^4*e^2
*x^2 - 9*d^5*e*x - 96*d^6)*sqrt(-e^2*x^2 + d^2))/(e^13*x^5 - d*e^12*x^4 -
2*d^2*e^11*x^3 + 2*d^3*e^10*x^2 + d^4*e^9*x - d^5*e^8)
```

3.19.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(144) = 288$.

Time = 13.69 (sec) , antiderivative size = 2004, normalized size of antiderivative = 12.45

$$\int \frac{x^7(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**7*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
d*Piecewise((16*d**6/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x
**2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) - 40*d**
4*e**2*x**2/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(
d**2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) + 30*d**2*e**4*x*
*4/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**2 - e*
*2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) - 5*e**6*x**6/(5*d**4*e**8
*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**2 - e**2*x**2) + 5*e*
*12*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**8/(8*(d**2)**(7/2)), True
)) + e*Piecewise((210*I*d**7*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(60*d*
*5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**11*x**2*sqrt(-1 + e**2*x**
2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) - 105*pi*d**7*sqrt(-1
 + e**2*x**2/d**2)/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**1
1*x**2*sqrt(-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**
2)) - 210*I*d**6*e*x/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e*
*11*x**2*sqrt(-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d
**2)) - 420*I*d**5*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(60*d*
*5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**11*x**2*sqrt(-1 + e**2*x**
2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) + 210*pi*d**5*e**2*x*
*2*sqrt(-1 + e**2*x**2/d**2)/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120
*d**3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + ...
```

3.19.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(141) = 282$.

Time = 0.28 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.01

$$\int \frac{x^7(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{x^7}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{7\,d^2x\left(\frac{15\,x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right)}{30\,e} - \frac{dx^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{7\,d^2x\left(\frac{3\,x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right)}{6\,e^3} + \frac{6\,d^3x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4} - \frac{8\,d^5x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6} + \frac{16\,d^7}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^8} + \frac{14\,d^4x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^7} - \frac{49\,d^2x}{30\sqrt{-e^2x^2+d^2}\,e^7} - \frac{7\,d^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}\,e^7}$$

input
```
integrate(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
-1/2*x^7/((-e^2*x^2 + d^2)^(5/2)*e) + 7/30*d^2*x*(15*x^4/((-e^2*x^2 + d^2)
^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 +
 d^2)^(5/2)*e^6))/e - d*x^6/((-e^2*x^2 + d^2)^(5/2)*e^2) - 7/6*d^2*x*(3*x^
2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e^3 +
 6*d^3*x^4/((-e^2*x^2 + d^2)^(5/2)*e^4) - 8*d^5*x^2/((-e^2*x^2 + d^2)^(5/2
)*e^6) + 16/5*d^7/((-e^2*x^2 + d^2)^(5/2)*e^8) + 14/15*d^4*x/((-e^2*x^2 +
d^2)^(3/2)*e^7) - 49/30*d^2*x/(sqrt(-e^2*x^2 + d^2)*e^7) - 7/2*d^2*arcsin(
e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^7)
```

3.19.8 Giac [F]

$$\int \frac{x^7(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)x^7}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)*x^7/(-e^2*x^2 + d^2)^(7/2), x)
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^7\,(d+e\,x)}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

input
```
int((x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
int((x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2), x)
```

3.20 $\int \frac{x^6(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx$

3.20.1 Optimal result

Integrand size = 25, antiderivative size = 147

$$\int \frac{x^6(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{x^5(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}}-\frac{x^3(5d+6ex)}{15e^4(d^2-e^2x^2)^{3/2}}+\frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}}+\frac{16\sqrt{d^2-e^2x^2}}{5e^7}-\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}$$

output
```
1/5*x^5*(e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^3*(6*e*x+5*d)/e^4/(-e^2*x^
2+d^2)^(3/2)-d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^7+1/5*x*(8*e*x+5*d)/e^6/
(-e^2*x^2+d^2)^(1/2)+16/5*(-e^2*x^2+d^2)^(1/2)/e^7
```

3.20.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.86

$$\int \frac{x^6(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{\frac{\sqrt{d^2-e^2x^2}(48d^5-33d^4ex-87d^3e^2x^2+52d^2e^3x^3+38de^4x^4-15e^5x^5)}{(d-ex)^3(d+ex)^2}+30d\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{15e^7}$$

input
```
Integrate[(x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(48*d^5 - 33*d^4*e*x - 87*d^3*e^2*x^2 + 52*d^2*e^3*x
^3 + 38*d*e^4*x^4 - 15*e^5*x^5))/((d - e*x)^3*(d + e*x)^2) + 30*d*ArcTan[(
e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(15*e^7)
```

3.20.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {529, 2345, 2345, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(d+ex)}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{ 529}$$

$$\frac{d^5(d+ex)}{5e^7(d^2-e^2x^2)^{5/2}}-\frac{\int \frac{\frac{d^6}{e^6}+\frac{5xd^5}{e^5}+\frac{5x^2d^4}{e^4}+\frac{5x^3d^3}{e^3}+\frac{5x^4d^2}{e^2}+\frac{5x^5d}{e}}{(d^2-e^2x^2)^{5/2}}dx}{5d}$$

$$\Big\downarrow \text{ 2345}$$

$$\frac{d^5(d+ex)}{5e^7(d^2-e^2x^2)^{5/2}}-\frac{\frac{d^4(15d+11ex)}{3e^7(d^2-e^2x^2)^{3/2}}-\frac{\int \frac{\frac{8d^6}{e^6}+\frac{30xd^5}{e^5}+\frac{15x^2d^4}{e^4}+\frac{15x^3d^3}{e^3}}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}}{5d}$$

$$\Big\downarrow \text{ 2345}$$

$$\frac{d^5(d+ex)}{5e^7(d^2-e^2x^2)^{5/2}}-\frac{\frac{d^4(15d+11ex)}{3e^7(d^2-e^2x^2)^{3/2}}-\frac{\frac{d^4(45d+23ex)}{e^7\sqrt{d^2-e^2x^2}}-\frac{\int \frac{15d^5(d+ex)}{e^6\sqrt{d^2-e^2x^2}}dx}{d^2}}{3d^2}}{5d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d^5(d+ex)}{5e^7(d^2-e^2x^2)^{5/2}}-\frac{\frac{d^4(15d+11ex)}{3e^7(d^2-e^2x^2)^{3/2}}-\frac{\frac{d^4(45d+23ex)}{e^7\sqrt{d^2-e^2x^2}}-\frac{15d^3\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}dx}{e^6}}{3d^2}}{5d}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{d^5(d+ex)}{5e^7(d^2-e^2x^2)^{5/2}}-\frac{\frac{d^4(15d+11ex)}{3e^7(d^2-e^2x^2)^{3/2}}-\frac{\frac{d^4(45d+23ex)}{e^7\sqrt{d^2-e^2x^2}}-\frac{15d^3\left(d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{e}\right)}{e^6}}{3d^2}}{5d}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{d^5(d+ex)}{5e^7\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{d^4(15d+11ex)}{3e^7(d^2-e^2x^2)^{3/2}}-\frac{\frac{d^4(45d+23ex)}{e^7\sqrt{d^2-e^2x^2}}-\frac{15d^3\left(d\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{e}\right)}{e^6}}{3d^2}}{5d}$$

$$\Big\downarrow \text{216}$$

$$\frac{d^5(d+ex)}{5e^7\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{d^4(15d+11ex)}{3e^7(d^2-e^2x^2)^{3/2}}-\frac{\frac{d^4(45d+23ex)}{e^7\sqrt{d^2-e^2x^2}}-\frac{15d^3\left(\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{\sqrt{d^2-e^2x^2}}{e}\right)}{e^6}}{3d^2}}{5d}$$

input
```
Int[(x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(d^5*(d + e*x))/(5*e^7*(d^2 - e^2*x^2)^(5/2)) - ((d^4*(15*d + 11*e*x))/(3*
e^7*(d^2 - e^2*x^2)^(3/2)) - ((d^4*(45*d + 23*e*x))/(e^7*Sqrt[d^2 - e^2*x^
2]) - (15*d^3*(-(Sqrt[d^2 - e^2*x^2]/e) + (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x
^2]])/e))/e^6)/(3*d^2))/(5*d)
```

3.20.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.20.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.50

method	result
default	$e\left(-\frac{x^{6}}{e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{6d^{2}\left(\frac{x^{4}}{e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{4d^{2}\left(\frac{x^{2}}{3e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{2d^{2}}{15e^{4}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}\right)}{e^{2}}\right)}{e^{2}}\right)+d\left(\frac{x^{5}}{5e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}\right.$
risch	$\frac{\sqrt{-e^{2}x^{2}+d^{2}}}{e^{7}}-\frac{d\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{e^{6}\sqrt{e^{2}}}-\frac{d^{2}\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{24e^{9}\left(x+\frac{d}{e}\right)^{2}}+\frac{25d\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{48e^{8}\left(x+\frac{d}{e}\right)}-\frac{23d^{2}\sqrt{-\left(x-\right.}}{60e^{9}}$

input
```
int(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
e*(-x^6/e^2/(-e^2*x^2+d^2)^(5/2)+6*d^2/e^2*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4
*d^2/e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/
2))))+d*(1/5*x^5/e^2/(-e^2*x^2+d^2)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2
)^(3/2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(
1/2)*x/(-e^2*x^2+d^2)^(1/2)))))
```

3.20.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(129) = 258.

Time = 0.29 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.79

$$\int \frac{x^6(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{48\,de^5x^5 - 48\,d^2e^4x^4 - 96\,d^3e^3x^3 + 96\,d^4e^2x^2 + 48\,d^5ex - 48\,d^6 + 30\,(de^5x^5 - d^2e^4x^4}{15}$$

input
```
integrate(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(48*d*e^5*x^5 - 48*d^2*e^4*x^4 - 96*d^3*e^3*x^3 + 96*d^4*e^2*x^2 + 48
*d^5*e*x - 48*d^6 + 30*(d*e^5*x^5 - d^2*e^4*x^4 - 2*d^3*e^3*x^3 + 2*d^4*e^
2*x^2 + d^5*e*x - d^6)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (15*e^5
*x^5 - 38*d*e^4*x^4 - 52*d^2*e^3*x^3 + 87*d^3*e^2*x^2 + 33*d^4*e*x - 48*d^
5)*sqrt(-e^2*x^2 + d^2))/(e^12*x^5 - d*e^11*x^4 - 2*d^2*e^10*x^3 + 2*d^3*e
^9*x^2 + d^4*e^8*x - d^5*e^7)
```

3.20.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.51 (sec) , antiderivative size = 1821, normalized size of antiderivative = 12.39

$$\int \frac{x^6(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**6*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
d*Piecewise((30*I*d**5*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(30*d**5*e**
7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2)
+ 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 15*pi*d**5*sqrt(-1 + e**2*x
**2/d**2)/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt
(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 30*I*
d**4*e*x/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(
-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 60*I*d
**3*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(30*d**5*e**7*sqrt(-1
 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) + 30*d*e*
*11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 30*pi*d**3*e**2*x**2*sqrt(-1 + e**2*
x**2/d**2)/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqr
t(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 70*I
*d**2*e**3*x**3/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**
2*sqrt(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) +
 30*I*d*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(30*d**5*e**7*sqr
t(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) + 30*
d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 15*pi*d*e**4*x**4*sqrt(-1 + e**2
*x**2/d**2)/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sq
rt(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 46*
I*e**5*x**5/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2...
```

3.20.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(129) = 258$.

Time = 0.29 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.97

$$\int \frac{x^6(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{1}{15}\,dx\left(\frac{15\,x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right) - \frac{x^6}{(-e^2x^2+d^2)^{\frac{5}{2}}e} - \frac{dx\left(\frac{3\,x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right)}{3\,e^2} + \frac{6\,d^2x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^3} - \frac{8\,d^4x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^5} + \frac{16\,d^6}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^7} + \frac{4\,d^3x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}e^6} - \frac{7\,dx}{15\,\sqrt{-e^2x^2+d^2}e^6} - \frac{d\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}e^6}$$

input
```
integrate(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/15*d*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^
2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - x^6/((-e^2*x^2 + d^2
)^(5/2)*e) - 1/3*d*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^
2 + d^2)^(3/2)*e^4))/e^2 + 6*d^2*x^4/((-e^2*x^2 + d^2)^(5/2)*e^3) - 8*d^4*
x^2/((-e^2*x^2 + d^2)^(5/2)*e^5) + 16/5*d^6/((-e^2*x^2 + d^2)^(5/2)*e^7) +
 4/15*d^3*x/((-e^2*x^2 + d^2)^(3/2)*e^6) - 7/15*d*x/(sqrt(-e^2*x^2 + d^2)*
e^6) - d*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^6)
```

3.20.8 Giac [F]

$$\int \frac{x^6(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)x^6}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)*x^6/(-e^2*x^2 + d^2)^(7/2), x)
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^6\,(d+e\,x)}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

input
```
int((x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
int((x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2), x)
```

3.21 $\int \frac{x^5(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx$

3.21.1 Optimal result

Integrand size = 25, antiderivative size = 122

$$\int \frac{x^5(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^4(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x^2(4d+5ex)}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

output
```
1/5*x^4*(e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^2*(5*e*x+4*d)/e^4/(-e^2*x^
2+d^2)^(3/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+1/15*(15*e*x+8*d)/e^6/(-
e^2*x^2+d^2)^(1/2)
```

3.21.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.94

$$\int \frac{x^5(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(8d^4+7d^3ex-27d^2e^2x^2-8de^3x^3+23e^4x^4)}{(d-ex)^3(d+ex)^2}+30\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{15e^6}$$

input
```
Integrate[(x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(8*d^4 + 7*d^3*e*x - 27*d^2*e^2*x^2 - 8*d*e^3*x^3 +
23*e^4*x^4))/((d - e*x)^3*(d + e*x)^2) + 30*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt
[d^2 - e^2*x^2]])])/(15*e^6)
```

3.21.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {529, 2345, 2345, 27, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{529}$$

$$\frac{d^4(d+ex)}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{\frac{d^5}{e^5}+\frac{5xd^4}{e^4}+\frac{5x^2d^3}{e^3}+\frac{5x^3d^2}{e^2}+\frac{5x^4d}{e}}{(d^2-e^2x^2)^{5/2}}dx}{5d}$$

$$\Big\downarrow \text{2345}$$

$$\frac{d^4(d+ex)}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{d^3(10d+11ex)}{3e^6(d^2-e^2x^2)^{3/2}}-\frac{\int \frac{\frac{8d^5}{e^5}+\frac{15xd^4}{e^4}+\frac{15x^2d^3}{e^3}}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}}{5d}$$

$$\Big\downarrow \text{2345}$$

$$\frac{d^4(d+ex)}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{d^3(10d+11ex)}{3e^6(d^2-e^2x^2)^{3/2}}-\frac{\frac{d^3(15d+23ex)}{e^6\sqrt{d^2-e^2x^2}}-\frac{\int \frac{15d^5}{e^5\sqrt{d^2-e^2x^2}}dx}{d^2}}{3d^2}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^4(d+ex)}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{d^3(10d+11ex)}{3e^6(d^2-e^2x^2)^{3/2}}-\frac{\frac{d^3(15d+23ex)}{e^6\sqrt{d^2-e^2x^2}}-\frac{15d^3\int \frac{1}{\sqrt{d^2-e^2x^2}}dx}{e^5}}{3d^2}}{5d}$$

$$\Big\downarrow \text{224}$$

$$\frac{d^4(d+ex)}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{d^3(10d+11ex)}{3e^6(d^2-e^2x^2)^{3/2}}-\frac{\frac{d^3(15d+23ex)}{e^6\sqrt{d^2-e^2x^2}}-\frac{15d^3\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}}{e^5}}{3d^2}}{5d}$$

$$\Big\downarrow \text{216}$$

$$\frac{d^4(d+ex)}{5e^6(d^2-e^2x^2)^{5/2}}-\frac{\frac{d^3(10d+11ex)}{3e^6(d^2-e^2x^2)^{3/2}}-\frac{\frac{d^3(15d+23ex)}{e^6\sqrt{d^2-e^2x^2}}-\frac{15d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}}{3d^2}}{5d}$$

input
```
Int[(x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(d^4*(d + e*x))/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - ((d^3*(10*d + 11*e*x))/(3*
e^6*(d^2 - e^2*x^2)^(3/2)) - ((d^3*(15*d + 23*e*x))/(e^6*Sqrt[d^2 - e^2*x^
2]) - (15*d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^6)/(3*d^2))/(5*d)
```

3.21.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.21.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.55

method	result
default	$e\left(\frac{x^5}{5e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{\frac{x^3}{3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e^2}}{e^2}\right)+d\left(\frac{x^4}{e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{4d^2\left(\frac{}{3e^2\left(-e^2\right.}\right.}{}\right.$

input
```
int(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
e*(1/5*x^5/e^2/(-e^2*x^2+d^2)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2
)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x
/(-e^2*x^2+d^2)^(1/2)))))+d*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*d^2/e^2*(1/3*x
^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2)))
```

3.21.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(108) = 216.

Time = 0.30 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.02

$$\int \frac{x^5(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{8\,e^5x^5-8\,de^4x^4-16\,d^2e^3x^3+16\,d^3e^2x^2+8\,d^4ex-8\,d^5+30\,(e^5x^5-de^4x^4-2\,d^2e^3}{15\,(e^{11}x^5-de}$$

input
```
integrate(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(8*e^5*x^5 - 8*d*e^4*x^4 - 16*d^2*e^3*x^3 + 16*d^3*e^2*x^2 + 8*d^4*e*
x - 8*d^5 + 30*(e^5*x^5 - d*e^4*x^4 - 2*d^2*e^3*x^3 + 2*d^3*e^2*x^2 + d^4*
e*x - d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (23*e^4*x^4 - 8*d*e
^3*x^3 - 27*d^2*e^2*x^2 + 7*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/(e^11*x
^5 - d*e^10*x^4 - 2*d^2*e^9*x^3 + 2*d^3*e^8*x^2 + d^4*e^7*x - d^5*e^6)
```

3.21.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(104) = 208$.

Time = 10.20 (sec) , antiderivative size = 1739, normalized size of antiderivative = 14.25

$$\int \frac{x^5(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**5*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
d*Piecewise((8*d**4/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x*
*2*sqrt(d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)) - 20*d**
2*e**2*x**2/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**2*sqrt(
d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)) + 15*e**4*x**4/(
15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**2*sqrt(d**2 - e**2*x
**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**6/(6*(d**2)**
(7/2)), True)) + e*Piecewise((30*I*d**5*sqrt(-1 + e**2*x**2/d**2)*acosh(e*
x/d)/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 +
 e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 15*pi*d**5
*sqrt(-1 + e**2*x**2/d**2)/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d*
*3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x*
*2/d**2)) - 30*I*d**4*e*x/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**
3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**
2/d**2)) - 60*I*d**3*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(30*
d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**
2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 30*pi*d**3*e**2*x**
2*sqrt(-1 + e**2*x**2/d**2)/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d
**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x
**2/d**2)) + 70*I*d**2*e**3*x**3/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) -
 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 ...
```

3.21.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(108) = 216$.

Time = 0.29 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.15

$$\int \frac{x^5(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{1}{15}\,ex\left(\frac{15\,x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right) - \frac{x\left(\frac{3\,x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right)}{3\,e} + \frac{dx^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{4\,d^3x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^5}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^6} + \frac{4\,d^2x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}e^5} - \frac{7\,x}{15\,\sqrt{-e^2x^2+d^2}e^5} - \frac{\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}e^5}$$

input
```
integrate(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/15*e*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^
2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 1/3*x*(3*x^2/((-e^2*
x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e + d*x^4/((-e
^2*x^2 + d^2)^(5/2)*e^2) - 4/3*d^3*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8/15
*d^5/((-e^2*x^2 + d^2)^(5/2)*e^6) + 4/15*d^2*x/((-e^2*x^2 + d^2)^(3/2)*e^5
) - 7/15*x/(sqrt(-e^2*x^2 + d^2)*e^5) - arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(
e^2)*e^5)
```

3.21.8 Giac [F]

$$\int \frac{x^5(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)x^5}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)*x^5/(-e^2*x^2 + d^2)^(7/2), x)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^5\,(d+e\,x)}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

input
```
int((x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
int((x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2), x)
```

3.22 $\int \frac{x^4(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx$

3.22.1 Optimal result

Integrand size = 25, antiderivative size = 84

$$\int \frac{x^4(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^4(d+ex)}{5de\,(d^2-e^2x^2)^{5/2}} - \frac{4d^2}{15e^5\,(d^2-e^2x^2)^{3/2}} + \frac{4}{5e^5\sqrt{d^2-e^2x^2}}$$

output
```
1/5*x^4*(e*x+d)/d/e/(-e^2*x^2+d^2)^(5/2)-4/15*d^2/e^5/(-e^2*x^2+d^2)^(3/2)
+4/5/e^5/(-e^2*x^2+d^2)^(1/2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

$$\int \frac{x^4(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(8d^4-8d^3ex-12d^2e^2x^2+12de^3x^3+3e^4x^4)}{15de^5(d-ex)^3(d+ex)^2}$$

input
```
Integrate[(x^4*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(8*d^4 - 8*d^3*e*x - 12*d^2*e^2*x^2 + 12*d*e^3*x^3 +
3*e^4*x^4))/(15*d*e^5*(d - e*x)^3*(d + e*x)^2)
```

3.22.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {529, 2345, 27, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d+ex)}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \ 529$$

$$\frac{d^3(d+ex)}{5e^5(d^2-e^2x^2)^{5/2}}-\frac{\int \frac{\frac{d^4}{e^4}+\frac{5xd^3}{e^3}+\frac{5x^2d^2}{e^2}+\frac{5x^3d}{e}}{(d^2-e^2x^2)^{5/2}}dx}{5d}$$

$$\Big\downarrow \ 2345$$

$$\frac{d^3(d+ex)}{5e^5(d^2-e^2x^2)^{5/2}}-\frac{\frac{2d^2(5d+3ex)}{3e^5(d^2-e^2x^2)^{3/2}}-\frac{\int \frac{3d^3(d+5ex)}{e^4(d^2-e^2x^2)^{3/2}}dx}{3d^2}}{5d}$$

$$\Big\downarrow \ 27$$

$$\frac{d^3(d+ex)}{5e^5(d^2-e^2x^2)^{5/2}}-\frac{\frac{2d^2(5d+3ex)}{3e^5(d^2-e^2x^2)^{3/2}}-\frac{d\int \frac{d+5ex}{(d^2-e^2x^2)^{3/2}}dx}{e^4}}{5d}$$

$$\Big\downarrow \ 453$$

$$\frac{d^3(d+ex)}{5e^5(d^2-e^2x^2)^{5/2}}-\frac{\frac{2d^2(5d+3ex)}{3e^5(d^2-e^2x^2)^{3/2}}-\frac{5d+ex}{e^5\sqrt{d^2-e^2x^2}}}{5d}$$

input
```
Int[(x^4*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(d^3*(d + e*x))/(5*e^5*(d^2 - e^2*x^2)^(5/2)) - ((2*d^2*(5*d + 3*e*x))/(3*
e^5*(d^2 - e^2*x^2)^(3/2)) - (5*d + e*x)/(e^5*Sqrt[d^2 - e^2*x^2]))/(5*d)
```

3.22.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.22.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

method	result
gosper	$\frac{(-ex+d)(ex+d)^2(3e^4x^4+12d\,e^3x^3-12d^2e^2x^2-8d^3ex+8d^4)}{15d\,e^5(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$\frac{(3e^4x^4+12d\,e^3x^3-12d^2e^2x^2-8d^3ex+8d^4)\sqrt{-e^2x^2+d^2}}{15e^5d(-ex+d)^3(ex+d)^2}$
default	$e\left(\frac{x^4}{e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{4d^2\left(\frac{x^2}{3e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{2d^2}{15e^4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}\right)}{e^2}\right)+d\left(\frac{x^3}{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{3d^2\left(\frac{x}{4e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\right.}{}\right.$

input
```
int(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(-e*x+d)*(e*x+d)^2*(3*e^4*x^4+12*d*e^3*x^3-12*d^2*e^2*x^2-8*d^3*e*x+8
*d^4)/d/e^5/(-e^2*x^2+d^2)^(7/2)
```

3.22.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(72) = 144$.

Time = 0.34 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.04

$$\int \frac{x^4(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{8\,e^5x^5-8\,de^4x^4-16\,d^2e^3x^3+16\,d^3e^2x^2+8\,d^4ex-8\,d^5-(3\,e^4x^4+12\,de^3x^3-12\,d}{15\,(de^{10}x^5-d^2e^9x^4-2\,d^3e^8x^3+2\,d^4e^7x^2+d^5e^6x-d}$$

input
```
integrate(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(8*e^5*x^5 - 8*d*e^4*x^4 - 16*d^2*e^3*x^3 + 16*d^3*e^2*x^2 + 8*d^4*e*
x - 8*d^5 - (3*e^4*x^4 + 12*d*e^3*x^3 - 12*d^2*e^2*x^2 - 8*d^3*e*x + 8*d^4
)*sqrt(-e^2*x^2 + d^2))/(d*e^10*x^5 - d^2*e^9*x^4 - 2*d^3*e^8*x^3 + 2*d^4*
e^7*x^2 + d^5*e^6*x - d^6*e^5)
```

3.22.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.36 (sec) , antiderivative size = 418, normalized size of antiderivative = 4.98

$$\int \frac{x^4(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = d\left(\begin{cases} -\frac{ix^5}{5d^7\sqrt{-1+\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{x^5}{5d^7\sqrt{1-\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$

$$+e\left(\begin{cases} \frac{8d^4}{15d^4e^6\sqrt{d^2-e^2x^2}-30d^2e^8x^2\sqrt{d^2-e^2x^2}+15e^{10}x^4\sqrt{d^2-e^2x^2}} - \frac{20d^2e^2x^2}{15d^4e^6\sqrt{d^2-e^2x^2}-30d^2e^8x^2\sqrt{d^2-e^2x^2}+15e^{10}x^4\sqrt{d^2-e^2x^2}} + \frac{}{15d^4e^6 \cdots} \\ \frac{x^6}{6(d^2)^{\frac{7}{2}}} \end{cases}\right.$$

input
```
integrate(x**4*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
d*Piecewise((-I*x**5/(5*d**7*sqrt(-1 + e**2*x**2/d**2) - 10*d**5*e**2*x**2
*sqrt(-1 + e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)),
Abs(e**2*x**2/d**2) > 1), (x**5/(5*d**7*sqrt(1 - e**2*x**2/d**2) - 10*d**5
*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt(1 - e**2*x**2/
d**2)), True)) + e*Piecewise((8*d**4/(15*d**4*e**6*sqrt(d**2 - e**2*x**2)
- 30*d**2*e**8*x**2*sqrt(d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**
2*x**2)) - 20*d**2*e**2*x**2/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**
2*e**8*x**2*sqrt(d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2))
 + 15*e**4*x**4/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**2*s
qrt(d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)),
(x**6/(6*(d**2)**(7/2)), True))
```

3.22.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(72) = 144.

Time = 0.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.89

$$\int \frac{x^4(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{dx^3}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{4\,d^2x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e^3} - \frac{3\,d^3x}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^5} + \frac{dx}{10\,(-e^2x^2+d^2)^{\frac{3}{2}}e^4} + \frac{x}{5\sqrt{-e^2x^2+d^2}\,de^4}$$

input
```
integrate(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
x^4/((-e^2*x^2 + d^2)^(5/2)*e) + 1/2*d*x^3/((-e^2*x^2 + d^2)^(5/2)*e^2) -
4/3*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^3) - 3/10*d^3*x/((-e^2*x^2 + d^2)^(5
/2)*e^4) + 8/15*d^4/((-e^2*x^2 + d^2)^(5/2)*e^5) + 1/10*d*x/((-e^2*x^2 + d
^2)^(3/2)*e^4) + 1/5*x/(sqrt(-e^2*x^2 + d^2)*d*e^4)
```

3.22.8 Giac [F]

$$\int \frac{x^4(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)x^4}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)*x^4/(-e^2*x^2 + d^2)^(7/2), x)
```

3.22.9 Mupad [B] (verification not implemented)

Time = 11.85 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93

$$\int \frac{x^4(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,(8\,d^4-8\,d^3\,e\,x-12\,d^2\,e^2\,x^2+12\,d\,e^3\,x^3+3\,e^4\,x^4)}{15\,d\,e^5\,(d+e\,x)^2\,(d-e\,x)^3}$$

input
```
int((x^4*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(8*d^4 + 3*e^4*x^4 + 12*d*e^3*x^3 - 12*d^2*e^2*x^2
- 8*d^3*e*x))/(15*d*e^5*(d + e*x)^2*(d - e*x)^3)
```

3.23 $\int \frac{x^3(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx$

3.23.1 Optimal result

Integrand size = 25, antiderivative size = 90

$$\int \frac{x^3(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{x^2(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}}-\frac{2d+3ex}{15e^4(d^2-e^2x^2)^{3/2}}+\frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

output
```
1/5*x^2*(e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)+1/15*(-3*e*x-2*d)/e^4/(-e^2*x^2+d
^2)^(3/2)+1/5*x/d^2/e^3/(-e^2*x^2+d^2)^(1/2)
```

3.23.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^3(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}(-2d^4+2d^3ex+3d^2e^2x^2-3de^3x^3+3e^4x^4)}{15d^2e^4(d-ex)^3(d+ex)^2}$$

input
```
Integrate[(x^3*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-2*d^4 + 2*d^3*e*x + 3*d^2*e^2*x^2 - 3*d*e^3*x^3 + 3
*e^4*x^4))/(15*d^2*e^4*(d - e*x)^3*(d + e*x)^2)
```

3.23.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {529, 2345, 27, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex)}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{529}$$

$$\frac{d^2(d+ex)}{5e^4(d^2-e^2x^2)^{5/2}}-\frac{\int \frac{\frac{d^3}{e^3}+\frac{5xd^2}{e^2}+\frac{5x^2d}{e}}{(d^2-e^2x^2)^{5/2}}dx}{5d}$$

$$\Big\downarrow \text{2345}$$

$$\frac{d^2(d+ex)}{5e^4(d^2-e^2x^2)^{5/2}}-\frac{\frac{d(5d+6ex)}{3e^4(d^2-e^2x^2)^{3/2}}-\frac{\int \frac{3d^3}{e^3\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2(d+ex)}{5e^4(d^2-e^2x^2)^{5/2}}-\frac{\frac{d(5d+6ex)}{3e^4(d^2-e^2x^2)^{3/2}}-\frac{d\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{e^3}}{5d}$$

$$\Big\downarrow \text{208}$$

$$\frac{d^2(d+ex)}{5e^4(d^2-e^2x^2)^{5/2}}-\frac{\frac{d(5d+6ex)}{3e^4(d^2-e^2x^2)^{3/2}}-\frac{x}{de^3\sqrt{d^2-e^2x^2}}}{5d}$$

input
```
Int[(x^3*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(d^2*(d + e*x))/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - ((d*(5*d + 6*e*x))/(3*e^4*
(d^2 - e^2*x^2)^(3/2)) - x/(d*e^3*Sqrt[d^2 - e^2*x^2]))/(5*d)
```

3.23.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.23.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.86

method	result
gosper	$-\frac{(-ex+d)(ex+d)^2(-3e^4x^4+3d\,e^3x^3-3d^2e^2x^2-2d^3ex+2d^4)}{15d^2e^4(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$-\frac{(-3e^4x^4+3d\,e^3x^3-3d^2e^2x^2-2d^3ex+2d^4)\sqrt{-e^2x^2+d^2}}{15d^2e^4(-ex+d)^3(ex+d)^2}$
default	$e\left(\frac{x^3}{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{3d^2\left(\frac{x}{4e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{4e^2}\right)}{2e^2}\right)+d\Big(\frac{}{3e}$

input
```
int(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-e*x+d)*(e*x+d)^2*(-3*e^4*x^4+3*d*e^3*x^3-3*d^2*e^2*x^2-2*d^3*e*x+2
*d^4)/d^2/e^4/(-e^2*x^2+d^2)^(7/2)
```

3.23.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(78) = 156$.

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.91

$$\int \frac{x^3(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx =$$
$$-\frac{2\,e^5x^5-2\,de^4x^4-4\,d^2e^3x^3+4\,d^3e^2x^2+2\,d^4ex-2\,d^5+(3\,e^4x^4-3\,de^3x^3+3\,d^2e^2x^2+2\,d^3ex-2\,d^4)\sqrt{-}}{15\,(d^2e^9x^5-d^3e^8x^4-2\,d^4e^7x^3+2\,d^5e^6x^2+d^6e^5x-d^7e^4)}$$

input
```
integrate(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/15*(2*e^5*x^5 - 2*d*e^4*x^4 - 4*d^2*e^3*x^3 + 4*d^3*e^2*x^2 + 2*d^4*e*x
 - 2*d^5 + (3*e^4*x^4 - 3*d*e^3*x^3 + 3*d^2*e^2*x^2 + 2*d^3*e*x - 2*d^4)*s
qrt(-e^2*x^2 + d^2))/(d^2*e^9*x^5 - d^3*e^8*x^4 - 2*d^4*e^7*x^3 + 2*d^5*e^
6*x^2 + d^6*e^5*x - d^7*e^4)
```

3.23.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(78) = 156.

Time = 6.83 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.74

$$\int \frac{x^3(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = d\left(\begin{cases} -\frac{2d^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15e^8x^4\sqrt{d^2-e^2x^2}} + \frac{5e^2x^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+\cdots} \\ \frac{x^4}{4(d^2)^{\frac{7}{2}}} \end{cases}\right.$$

$$+ e\left(\begin{cases} -\frac{ix^5}{5d^7\sqrt{-1+\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{x^5}{5d^7\sqrt{1-\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(x**3*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
d*Piecewise((-2*d**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 30*d**2*e**6*x
**2*sqrt(d**2 - e**2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**2)) + 5*e**2
*x**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 30*d**2*e**6*x**2*sqrt(d**2 -
 e**2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d
**2)**(7/2)), True)) + e*Piecewise((-I*x**5/(5*d**7*sqrt(-1 + e**2*x**2/d*
*2) - 10*d**5*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt(
-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (x**5/(5*d**7*sqrt(1 - e*
*2*x**2/d**2) - 10*d**5*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 5*d**3*e**4*x
**4*sqrt(1 - e**2*x**2/d**2)), True))
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.49

$$\int \frac{x^3(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^3}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{dx^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{3\,d^2x}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}e^3} - \frac{2\,d^3}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{x}{10\,(-e^2x^2+d^2)^{\frac{3}{2}}e^3} + \frac{x}{5\,\sqrt{-e^2x^2+d^2}d^2e^3}$$

input
```
integrate(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/2*x^3/((-e^2*x^2 + d^2)^(5/2)*e) + 1/3*d*x^2/((-e^2*x^2 + d^2)^(5/2)*e^2
) - 3/10*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^3) - 2/15*d^3/((-e^2*x^2 + d^2)^(
5/2)*e^4) + 1/10*x/((-e^2*x^2 + d^2)^(3/2)*e^3) + 1/5*x/(sqrt(-e^2*x^2 + d
^2)*d^2*e^3)
```

3.23.8 Giac [F]

$$\int \frac{x^3(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)x^3}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)*x^3/(-e^2*x^2 + d^2)^(7/2), x)
```

3.23.9 Mupad [B] (verification not implemented)

Time = 11.60 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

$$\int \frac{x^3(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^4+2\,d^3\,e\,x+3\,d^2\,e^2\,x^2-3\,d\,e^3\,x^3+3\,e^4\,x^4\right)}{15\,d^2\,e^4\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^3}$$

input
```
int((x^3*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(3*e^4*x^4 - 2*d^4 - 3*d*e^3*x^3 + 3*d^2*e^2*x^2 +
2*d^3*e*x))/(15*d^2*e^4*(d + e*x)^2*(d - e*x)^3)
```

3.24 $\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx$

3.24.1 Optimal result

Integrand size = 25, antiderivative size = 94

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^2(d+ex)}{5de\,(d^2-e^2x^2)^{5/2}} - \frac{2(d-ex)}{15de^3\,(d^2-e^2x^2)^{3/2}} - \frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

output
```
1/5*x^2*(e*x+d)/d/e/(-e^2*x^2+d^2)^(5/2)-2/15*(-e*x+d)/d/e^3/(-e^2*x^2+d^2
)^(3/2)-2/15*x/d^3/e^2/(-e^2*x^2+d^2)^(1/2)
```

3.24.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-2d^4+2d^3ex+3d^2e^2x^2+2de^3x^3-2e^4x^4)}{15d^3e^3(d-ex)^3(d+ex)^2}$$

input
```
Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-2*d^4 + 2*d^3*e*x + 3*d^2*e^2*x^2 + 2*d*e^3*x^3 - 2
*e^4*x^4))/(15*d^3*e^3*(d - e*x)^3*(d + e*x)^2)
```

3.24.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {529, 27, 454, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \; 529$$

$$\frac{d(d+ex)}{5e^3(d^2-e^2x^2)^{5/2}}-\frac{\int \frac{d(d+5ex)}{e^2(d^2-e^2x^2)^{5/2}}dx}{5d}$$

$$\Big\downarrow \; 27$$

$$\frac{d(d+ex)}{5e^3(d^2-e^2x^2)^{5/2}}-\frac{\int \frac{d+5ex}{(d^2-e^2x^2)^{5/2}}dx}{5e^2}$$

$$\Big\downarrow \; 454$$

$$\frac{d(d+ex)}{5e^3(d^2-e^2x^2)^{5/2}}-\frac{\frac{2\int \frac{1}{(d^2-e^2x^2)^{3/2}}dx}{3d}+\frac{5d+ex}{3de(d^2-e^2x^2)^{3/2}}}{5e^2}$$

$$\Big\downarrow \; 208$$

$$\frac{d(d+ex)}{5e^3(d^2-e^2x^2)^{5/2}}-\frac{\frac{5d+ex}{3de(d^2-e^2x^2)^{3/2}}+\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}}{5e^2}$$

input
```
Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(d*(d + e*x))/(5*e^3*(d^2 - e^2*x^2)^(5/2)) - ((5*d + e*x)/(3*d*e*(d^2 - e
^2*x^2)^(3/2)) + (2*x)/(3*d^3*Sqrt[d^2 - e^2*x^2]))/(5*e^2)
```

3.24.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

3.24.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.82

method	result
gosper	$-\frac{(-ex+d)(ex+d)^{2}\left(2e^{4}x^{4}-2d\,e^{3}x^{3}-3d^{2}e^{2}x^{2}-2d^{3}ex+2d^{4}\right)}{15d^{3}e^{3}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}$
trager	$-\frac{\left(2e^{4}x^{4}-2d\,e^{3}x^{3}-3d^{2}e^{2}x^{2}-2d^{3}ex+2d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{15d^{3}e^{3}(-ex+d)^{3}(ex+d)^{2}}$
default	$e\left(\frac{x^{2}}{3e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{2d^{2}}{15e^{4}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}\right)+d\left(\frac{x}{4e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{d^{2}\left(\frac{x}{5d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{\cdots}{15d^{4}\sqrt{-\cdots}}}{d^{2}}\cdots\right.}{4e^{2}}\cdots\right.$

input
```
int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-e*x+d)*(e*x+d)^2*(2*e^4*x^4-2*d*e^3*x^3-3*d^2*e^2*x^2-2*d^3*e*x+2*
d^4)/d^3/e^3/(-e^2*x^2+d^2)^(7/2)
```

3.24.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(83) = 166.

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.84

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{2\,e^5x^5 - 2\,de^4x^4 - 4\,d^2e^3x^3 + 4\,d^3e^2x^2 + 2\,d^4ex - 2\,d^5 - (2\,e^4x^4 - 2\,de^3x^3 - 3\,d^2e^2x^2 - 2\,d^3ex + 2\,d^4)\sqrt{-}}{15\,(d^3e^8x^5 - d^4e^7x^4 - 2\,d^5e^6x^3 + 2\,d^6e^5x^2 + d^7e^4x - d^8e^3)}$$

input
```
integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/15*(2*e^5*x^5 - 2*d*e^4*x^4 - 4*d^2*e^3*x^3 + 4*d^3*e^2*x^2 + 2*d^4*e*x
 - 2*d^5 - (2*e^4*x^4 - 2*d*e^3*x^3 - 3*d^2*e^2*x^2 - 2*d^3*e*x + 2*d^4)*s
qrt(-e^2*x^2 + d^2))/(d^3*e^8*x^5 - d^4*e^7*x^4 - 2*d^5*e^6*x^3 + 2*d^6*e^
5*x^2 + d^7*e^4*x - d^8*e^3)
```

3.24.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.11 (sec) , antiderivative size = 513, normalized size of antiderivative = 5.46

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = d\left(\left(\begin{cases} -\frac{5id^2x^3}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{2ie^2x^5}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\cdots}} \\ \frac{5d^2x^3}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{2e^2x^5}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4\cdots} \end{cases}\right.\right.$$
$$+e\left(\left(\begin{cases} -\frac{2d^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15e^8x^4\sqrt{d^2-e^2x^2}} + \frac{5e^2x^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15e^8x^4\sqrt{d^2-e^2x^2}} & \text{for } e \neq \cdots \\ \frac{x^4}{4(d^2)^{\frac{7}{2}}} & \text{other}\cdots \end{cases}\right.\right.$$

input
```
integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
d*Piecewise((-5*I*d**2*x**3/(15*d**9*sqrt(-1 + e**2*x**2/d**2) - 30*d**7*e
**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(-1 + e**2*x**2
/d**2)) + 2*I*e**2*x**5/(15*d**9*sqrt(-1 + e**2*x**2/d**2) - 30*d**7*e**2*
x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(-1 + e**2*x**2/d**
2)), Abs(e**2*x**2/d**2) > 1), (5*d**2*x**3/(15*d**9*sqrt(1 - e**2*x**2/d*
*2) - 30*d**7*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(
1 - e**2*x**2/d**2)) - 2*e**2*x**5/(15*d**9*sqrt(1 - e**2*x**2/d**2) - 30*
d**7*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(1 - e**2*
x**2/d**2)), True)) + e*Piecewise((-2*d**2/(15*d**4*e**4*sqrt(d**2 - e**2*
x**2) - 30*d**2*e**6*x**2*sqrt(d**2 - e**2*x**2) + 15*e**8*x**4*sqrt(d**2
- e**2*x**2)) + 5*e**2*x**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 30*d**2
*e**6*x**2*sqrt(d**2 - e**2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**2)),
Ne(e, 0)), (x**4/(4*(d**2)**(7/2)), True))
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.19

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{dx}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{2\,d^2}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^3} - \frac{x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}de^2} - \frac{2\,x}{15\,\sqrt{-e^2x^2+d^2}d^3e^2}$$

input
```
integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/3*x^2/((-e^2*x^2 + d^2)^(5/2)*e) + 1/5*d*x/((-e^2*x^2 + d^2)^(5/2)*e^2)
- 2/15*d^2/((-e^2*x^2 + d^2)^(5/2)*e^3) - 1/15*x/((-e^2*x^2 + d^2)^(3/2)*d
*e^2) - 2/15*x/(sqrt(-e^2*x^2 + d^2)*d^3*e^2)
```

3.24.8 Giac [F]

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)x^2}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)*x^2/(-e^2*x^2 + d^2)^(7/2), x)
```

3.24.9 Mupad [B] (verification not implemented)

Time = 11.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^4+2\,d^3\,e\,x+3\,d^2\,e^2\,x^2+2\,d\,e^3\,x^3-2\,e^4\,x^4\right)}{15\,d^3\,e^3\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^3}$$

input
```
int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(2*d*e^3*x^3 - 2*e^4*x^4 - 2*d^4 + 3*d^2*e^2*x^2 +
2*d^3*e*x))/(15*d^3*e^3*(d + e*x)^2*(d - e*x)^3)
```

3.25 $\int \frac{x(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx$

3.25.1 Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{x(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{d+ex}{5e^2(d^2-e^2x^2)^{5/2}}-\frac{x}{15d^2e(d^2-e^2x^2)^{3/2}}-\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

output
```
1/5*(e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x/d^2/e/(-e^2*x^2+d^2)^(3/2)-2/1
5*x/d^4/e/(-e^2*x^2+d^2)^(1/2)
```

3.25.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int \frac{x(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}(3d^4-3d^3ex+3d^2e^2x^2+2de^3x^3-2e^4x^4)}{15d^4e^2(d-ex)^3(d+ex)^2}$$

input
```
Integrate[(x*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(3*d^4 - 3*d^3*e*x + 3*d^2*e^2*x^2 + 2*d*e^3*x^3 - 2*
e^4*x^4))/(15*d^4*e^2*(d - e*x)^3*(d + e*x)^2)
```

3.25.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {530, 27, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex)}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{530}$$

$$\frac{d+ex}{5e^2(d^2-e^2x^2)^{5/2}}-\frac{\int \frac{d^2}{e(d^2-e^2x^2)^{5/2}}dx}{5d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{d+ex}{5e^2(d^2-e^2x^2)^{5/2}}-\frac{\int \frac{1}{(d^2-e^2x^2)^{5/2}}dx}{5e}$$

$$\Big\downarrow \text{209}$$

$$\frac{d+ex}{5e^2(d^2-e^2x^2)^{5/2}}-\frac{\frac{2\int \frac{1}{(d^2-e^2x^2)^{3/2}}dx}{3d^2}+\frac{x}{3d^2(d^2-e^2x^2)^{3/2}}}{5e}$$

$$\Big\downarrow \text{208}$$

$$\frac{d+ex}{5e^2(d^2-e^2x^2)^{5/2}}-\frac{\frac{x}{3d^2(d^2-e^2x^2)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}}{5e}$$

input
```
Int[(x*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(d + e*x)/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x/(3*d^2*(d^2 - e^2*x^2)^(3/2))
 + (2*x)/(3*d^4*Sqrt[d^2 - e^2*x^2]))/(5*e)
```

3.25.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

3.25.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{(-ex+d)(ex+d)^2\left(-2e^4x^4+2d\,e^3x^3+3d^2e^2x^2-3d^3ex+3d^4\right)}{15d^4e^2\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}$	77
trager	$\frac{\left(-2e^4x^4+2d\,e^3x^3+3d^2e^2x^2-3d^3ex+3d^4\right)\sqrt{-e^2x^2+d^2}}{15d^4(-ex+d)^3(ex+d)^2e^2}$	79
default	$e\left(\frac{x}{4e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{4e^2}\right)+\frac{d}{5e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}$	120

input
```
int(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(-e*x+d)*(e*x+d)^2*(-2*e^4*x^4+2*d*e^3*x^3+3*d^2*e^2*x^2-3*d^3*e*x+3*
d^4)/d^4/e^2/(-e^2*x^2+d^2)^(7/2)
```

3.25.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(71) = 142$.

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.07

$$\int \frac{x(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{3\,e^5x^5 - 3\,de^4x^4 - 6\,d^2e^3x^3 + 6\,d^3e^2x^2 + 3\,d^4ex - 3\,d^5 + (2\,e^4x^4 - 2\,de^3x^3 - 3\,d^2e^2x^2}{15\,(d^4e^7x^5 - d^5e^6x^4 - 2\,d^6e^5x^3 + 2\,d^7e^4x^2 + d^8e^3x - d^9e}$$

input
```
integrate(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(3*e^5*x^5 - 3*d*e^4*x^4 - 6*d^2*e^3*x^3 + 6*d^3*e^2*x^2 + 3*d^4*e*x
- 3*d^5 + (2*e^4*x^4 - 2*d*e^3*x^3 - 3*d^2*e^2*x^2 + 3*d^3*e*x - 3*d^4)*sq
rt(-e^2*x^2 + d^2))/(d^4*e^7*x^5 - d^5*e^6*x^4 - 2*d^6*e^5*x^3 + 2*d^7*e^4
*x^2 + d^8*e^3*x - d^9*e^2)
```

3.25.6 Sympy [A] (verification not implemented)

Time = 6.60 (sec) , antiderivative size = 432, normalized size of antiderivative = 5.20

$$\int \frac{x(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = d\left(\begin{cases} \frac{1}{5d^4e^2\sqrt{d^2-e^2x^2} - 10d^2e^4x^2\sqrt{d^2-e^2x^2} + 5e^6x^4\sqrt{d^2-e^2x^2}} & \text{for}\ e \neq 0 \\ \frac{x^2}{2(d^2)^{\frac{7}{2}}} & \text{otherwise} \end{cases}\right)$$
$$+e\left(\begin{cases} -\frac{5id^2x^3}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}} - 30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}} + 15d^5e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{2ie^2x^5}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}} - 30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}} + 15d^5e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} \\ \frac{5d^2x^3}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}} - 30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}} + 15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{2e^2x^5}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}} - 30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}} + 15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} \end{cases}\right.$$

input
```
integrate(x*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
d*Piecewise((1/(5*d**4*e**2*sqrt(d**2 - e**2*x**2) - 10*d**2*e**4*x**2*sqr
t(d**2 - e**2*x**2) + 5*e**6*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**
2/(2*(d**2)**(7/2)), True)) + e*Piecewise((-5*I*d**2*x**3/(15*d**9*sqrt(-1
 + e**2*x**2/d**2) - 30*d**7*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5
*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)) + 2*I*e**2*x**5/(15*d**9*sqrt(-1 + e
**2*x**2/d**2) - 30*d**7*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**
4*x**4*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**2*x**3/
(15*d**9*sqrt(1 - e**2*x**2/d**2) - 30*d**7*e**2*x**2*sqrt(1 - e**2*x**2/d
**2) + 15*d**5*e**4*x**4*sqrt(1 - e**2*x**2/d**2)) - 2*e**2*x**5/(15*d**9*
sqrt(1 - e**2*x**2/d**2) - 30*d**7*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15
*d**5*e**4*x**4*sqrt(1 - e**2*x**2/d**2)), True))
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{x(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{d}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e} - \frac{2\,x}{15\,\sqrt{-e^2x^2+d^2}d^4e}$$

input
```
integrate(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/5*x/((-e^2*x^2 + d^2)^(5/2)*e) + 1/5*d/((-e^2*x^2 + d^2)^(5/2)*e^2) - 1/
15*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e) - 2/15*x/(sqrt(-e^2*x^2 + d^2)*d^4*e)
```

3.25.8 Giac [F]

$$\int \frac{x(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)x}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)*x/(-e^2*x^2 + d^2)^(7/2), x)
```

3.25.9 Mupad [B] (verification not implemented)

Time = 11.53 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{x(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,(3\,d^4-3\,d^3\,e\,x+3\,d^2\,e^2\,x^2+2\,d\,e^3\,x^3-2\,e^4\,x^4)}{15\,d^4\,e^2\,(d+e\,x)^2\,(d-e\,x)^3}$$

input
```
int((x*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(3*d^4 - 2*e^4*x^4 + 2*d*e^3*x^3 + 3*d^2*e^2*x^2 -
3*d^3*e*x))/(15*d^4*e^2*(d + e*x)^2*(d - e*x)^3)
```

3.26 $\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{7/2}} dx$

3.26.1 Optimal result

Integrand size = 22, antiderivative size = 80

$$\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{7/2}} dx = \frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}} + \frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{8x}{15d^5\sqrt{d^2-e^2x^2}}$$

output
```
1/5*(e*x+d)/d/e/(-e^2*x^2+d^2)^(5/2)+4/15*x/d^3/(-e^2*x^2+d^2)^(3/2)+8/15*
x/d^5/(-e^2*x^2+d^2)^(1/2)
```

3.26.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{7/2}} dx = \frac{\sqrt{d^2-e^2x^2}(3d^4+12d^3ex-12d^2e^2x^2-8de^3x^3+8e^4x^4)}{15d^5e(d-ex)^3(d+ex)^2}$$

input
```
Integrate[(d + e*x)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(3*d^4 + 12*d^3*e*x - 12*d^2*e^2*x^2 - 8*d*e^3*x^3 +
8*e^4*x^4))/(15*d^5*e*(d - e*x)^3*(d + e*x)^2)
```

3.26.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {454, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$
$$\Big\downarrow \text{ 454}$$
$$\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d}+\frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}$$
$$\Big\downarrow \text{ 209}$$
$$\frac{4\left(\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)}{5d}+\frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}$$
$$\Big\downarrow \text{ 208}$$
$$\frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4\left(\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d}$$

input
```
Int[(d + e*x)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(d + e*x)/(5*d*e*(d^2 - e^2*x^2)^(5/2)) + (4*(x/(3*d^2*(d^2 - e^2*x^2)^(3/
2)) + (2*x)/(3*d^4*Sqrt[d^2 - e^2*x^2])))/(5*d)
```

3.26.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

3.26.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{(-ex+d)(ex+d)^2(8e^4x^4-8d\,e^3x^3-12d^2e^2x^2+12d^3ex+3d^4)}{15d^5e(-e^2x^2+d^2)^{\frac{7}{2}}}$	77
trager	$\frac{(8e^4x^4-8d\,e^3x^3-12d^2e^2x^2+12d^3ex+3d^4)\sqrt{-e^2x^2+d^2}}{15d^5(-ex+d)^3(ex+d)^2e}$	79
default	$d\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+\frac{1}{5e(-e^2x^2+d^2)^{\frac{5}{2}}}$	90

input
```
int((e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(-e*x+d)*(e*x+d)^2*(8*e^4*x^4-8*d*e^3*x^3-12*d^2*e^2*x^2+12*d^3*e*x+3
*d^4)/d^5/e/(-e^2*x^2+d^2)^(7/2)
```

3.26.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(68) = 136.

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.14

$$\int \frac{d+ex}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{3\,e^5x^5 - 3\,de^4x^4 - 6\,d^2e^3x^3 + 6\,d^3e^2x^2 + 3\,d^4ex - 3\,d^5 - (8\,e^4x^4 - 8\,de^3x^3 - 12\,d^2e^2x}{15\,(d^5e^6x^5 - d^6e^5x^4 - 2\,d^7e^4x^3 + 2\,d^8e^3x^2 + d^9e^2x - d^1}$$

input
```
integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(3*e^5*x^5 - 3*d*e^4*x^4 - 6*d^2*e^3*x^3 + 6*d^3*e^2*x^2 + 3*d^4*e*x
- 3*d^5 - (8*e^4*x^4 - 8*d*e^3*x^3 - 12*d^2*e^2*x^2 + 12*d^3*e*x + 3*d^4)*
sqrt(-e^2*x^2 + d^2))/(d^5*e^6*x^5 - d^6*e^5*x^4 - 2*d^7*e^4*x^3 + 2*d^8*e
^3*x^2 + d^9*e^2*x - d^10*e)
```

3.26.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.51 (sec) , antiderivative size = 604, normalized size of antiderivative = 7.55

$$\int \frac{d+ex}{(d^2-e^2x^2)^{7/2}}\,dx = d\left(\left\{\begin{array}{l} -\frac{15id^4x}{15d^{11}\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^7e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{20id^2e^2x}{15d^{11}\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{-1+}} \\ \frac{15d^4x}{15d^{11}\sqrt{1-\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^7e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{20d^2e^2x^3}{15d^{11}\sqrt{1-\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^7} \end{array}\right.\right.$$

$$+e\left(\left\{\begin{array}{ll} \frac{1}{5d^4e^2\sqrt{d^2-e^2x^2}-10d^2e^4x^2\sqrt{d^2-e^2x^2}+5e^6x^4\sqrt{d^2-e^2x^2}} & \text{for } e\neq 0 \\ \frac{x^2}{2(d^2)^{\frac{7}{2}}} & \text{otherwise} \end{array}\right.\right)$$

input
```
integrate((e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
d*Piecewise((-15*I*d**4*x/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e*
*2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/
d**2)) + 20*I*d**2*e**2*x**3/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9
*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x*
*2/d**2)) - 8*I*e**4*x**5/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e*
*2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/
d**2)), Abs(e**2*x**2/d**2) > 1), (15*d**4*x/(15*d**11*sqrt(1 - e**2*x**2/
d**2) - 30*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqr
t(1 - e**2*x**2/d**2)) - 20*d**2*e**2*x**3/(15*d**11*sqrt(1 - e**2*x**2/d*
*2) - 30*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(
1 - e**2*x**2/d**2)) + 8*e**4*x**5/(15*d**11*sqrt(1 - e**2*x**2/d**2) - 30
*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e**2
*x**2/d**2)), True)) + e*Piecewise((1/(5*d**4*e**2*sqrt(d**2 - e**2*x**2)
- 10*d**2*e**4*x**2*sqrt(d**2 - e**2*x**2) + 5*e**6*x**4*sqrt(d**2 - e**2*
x**2)), Ne(e, 0)), (x**2/(2*(d**2)**(7/2)), True))
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}d} + \frac{1}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{4\,x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d^3} + \frac{8\,x}{15\,\sqrt{-e^2x^2+d^2}d^5}$$

input
```
integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/5*x/((-e^2*x^2 + d^2)^(5/2)*d) + 1/5/((-e^2*x^2 + d^2)^(5/2)*e) + 4/15*x
/((-e^2*x^2 + d^2)^(3/2)*d^3) + 8/15*x/(sqrt(-e^2*x^2 + d^2)*d^5)
```

3.26.8 Giac [F]

$$\int \frac{d+ex}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{ex+d}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)/(-e^2*x^2 + d^2)^(7/2), x)
```

3.26.9 Mupad [B] (verification not implemented)

Time = 11.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{d+ex}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,(3\,d^4+12\,d^3\,e\,x-12\,d^2\,e^2\,x^2-8\,d\,e^3\,x^3+8\,e^4\,x^4)}{15\,d^5\,e\,(d+e\,x)^2\,(d-e\,x)^3}$$

input
```
int((d + e*x)/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(3*d^4 + 8*e^4*x^4 - 8*d*e^3*x^3 - 12*d^2*e^2*x^2 +
 12*d^3*e*x))/(15*d^5*e*(d + e*x)^2*(d - e*x)^3)
```

3.27 $\int \frac{d+ex}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx$

3.27.1 Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \frac{d+ex}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

output
```
1/5*(e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)+1/15*(4*e*x+5*d)/d^4/(-e^2*x^2+d^2)^(
3/2)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/15*(8*e*x+15*d)/d^6/(-e^2*x^2+d
^2)^(1/2)
```

3.27.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.10

$$\int \frac{d+ex}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}\left(23d^4-8d^3ex-27d^2e^2x^2+7de^3x^3+8e^4x^4\right)}{(d-ex)^3(d+ex)^2} - 15\sqrt{d^2}\log(x) + 15\sqrt{d^2}\log\left(\sqrt{d^2}-\sqrt{d^2}\right.}{15d^7}$$

input
```
Integrate[(d + e*x)/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
((d*Sqrt[d^2 - e^2*x^2]*(23*d^4 - 8*d^3*e*x - 27*d^2*e^2*x^2 + 7*d*e^3*x^3
 + 8*e^4*x^4))/((d - e*x)^3*(d + e*x)^2) - 15*Sqrt[d^2]*Log[x] + 15*Sqrt[d
^2]*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(15*d^7)
```

3.27.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {532, 25, 532, 25, 532, 27, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x\left(d^2-e^2x^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{532}$$

$$\frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int -\frac{5d+4ex}{x(d^2-e^2x^2)^{5/2}}dx}{5d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5d+4ex}{x(d^2-e^2x^2)^{5/2}}dx}{5d^2}+\frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{\frac{5d+4ex}{3d^2(d^2-e^2x^2)^{3/2}}-\frac{\int -\frac{15d+8ex}{x\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}}{5d^2}+\frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{15d+8ex}{x\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{5d+4ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{\frac{\frac{15d+8ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{15d}{x\sqrt{d^2-e^2x^2}}dx}{d^2}}{3d^2}+\frac{5d+4ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{15\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx}{d}+\frac{15d+8ex}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{5d+4ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{\frac{15\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2}{2d}+\frac{15d+8ex}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{5d+4ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{\frac{15d+8ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{15\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{de^2}}{3d^2}+\frac{5d+4ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{15d+8ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{15\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}}{3d^2}+\frac{5d+4ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[(d + e*x)/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(d + e*x)/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + ((5*d + 4*e*x)/(3*d^2*(d^2 - e^2
*x^2)^(3/2)) + ((15*d + 8*e*x)/(d^2*Sqrt[d^2 - e^2*x^2]) - (15*ArcTanh[Sqr
t[d^2 - e^2*x^2]/d])/d^2)/(3*d^2))/(5*d^2)
```

3.27.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

3.27.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.56

method	result
default	$e\left(\frac{x}{5d^{2}(-e^{2}x^{2}+d^{2})^{\frac{5}{2}}}+\frac{\frac{4x}{15d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{8x}{15d^{4}\sqrt{-e^{2}x^{2}+d^{2}}}}{d^{2}}\right)+d\left(\frac{1}{5d^{2}(-e^{2}x^{2}+d^{2})^{\frac{5}{2}}}+\frac{\frac{1}{3d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{\frac{1}{d^{2}\sqrt{-e^{2}x^{2}+d}}}{}}{d^{2}}\right.$

input
```
int((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
e*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+
2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))+d*(1/5/d^2/(-e^2*x^2+d^2)^(5/2)+1/d^2*(1/
3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(
1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))
```

3.27.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(103) = 206$.

Time = 0.28 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.09

$$\int \frac{d+ex}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{23\,e^5x^5 - 23\,de^4x^4 - 46\,d^2e^3x^3 + 46\,d^3e^2x^2 + 23\,d^4ex - 23\,d^5 + 15\,(e^5x^5 - de^4x^4 -}{15\,(d^6e^5x^5}$$

input
```
integrate((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(23*e^5*x^5 - 23*d*e^4*x^4 - 46*d^2*e^3*x^3 + 46*d^3*e^2*x^2 + 23*d^4
*e*x - 23*d^5 + 15*(e^5*x^5 - d*e^4*x^4 - 2*d^2*e^3*x^3 + 2*d^3*e^2*x^2 +
d^4*e*x - d^5)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (8*e^4*x^4 + 7*d*e^3*x
^3 - 27*d^2*e^2*x^2 - 8*d^3*e*x + 23*d^4)*sqrt(-e^2*x^2 + d^2))/(d^6*e^5*x
^5 - d^7*e^4*x^4 - 2*d^8*e^3*x^3 + 2*d^9*e^2*x^2 + d^10*e*x - d^11)
```

3.27.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.97 (sec) , antiderivative size = 2378, normalized size of antiderivative = 20.32

$$\int \frac{d+ex}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/x/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
d*Piecewise((-46*I*d**6*sqrt(-1 + e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e*
*2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 15*d**6*log(e**2*x**2/d
**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x*
*6) + 30*d**6*log(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x*
*4 + 30*d**7*e**6*x**6) - 30*I*d**6*asin(d/(e*x))/(-30*d**13 + 90*d**11*e*
*2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 70*I*d**4*e**2*x**2*sqr
t(-1 + e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4
 + 30*d**7*e**6*x**6) + 45*d**4*e**2*x**2*log(e**2*x**2/d**2)/(-30*d**13 +
 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 90*d**4*e**
2*x**2*log(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30
*d**7*e**6*x**6) + 90*I*d**4*e**2*x**2*asin(d/(e*x))/(-30*d**13 + 90*d**11
*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 30*I*d**2*e**4*x**4*
sqrt(-1 + e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x
**4 + 30*d**7*e**6*x**6) - 45*d**2*e**4*x**4*log(e**2*x**2/d**2)/(-30*d**1
3 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 90*d**2*
e**4*x**4*log(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 +
 30*d**7*e**6*x**6) - 90*I*d**2*e**4*x**4*asin(d/(e*x))/(-30*d**13 + 90*d*
*11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 15*e**6*x**6*log(
e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d
**7*e**6*x**6) - 30*e**6*x**6*log(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**...
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.34

$$\int \frac{d+ex}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{ex}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}+\frac{1}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}+\frac{4\,ex}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4}+\frac{1}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3}+\frac{8\,ex}{15\sqrt{-e^2x^2+d^2}d^6}-\frac{\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d^6}+\frac{1}{\sqrt{-e^2x^2+d^2}d^5}$$

input
```
integrate((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/5*e*x/((-e^2*x^2 + d^2)^(5/2)*d^2) + 1/5/((-e^2*x^2 + d^2)^(5/2)*d) + 4/
15*e*x/((-e^2*x^2 + d^2)^(3/2)*d^4) + 1/3/((-e^2*x^2 + d^2)^(3/2)*d^3) + 8
/15*e*x/(sqrt(-e^2*x^2 + d^2)*d^6) - log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 +
d^2)*d/abs(x))/d^6 + 1/(sqrt(-e^2*x^2 + d^2)*d^5)
```

3.27.8 Giac [F]

$$\int \frac{d+ex}{x(d^2-e^2x^2)^{7/2}}\,dx=\int \frac{ex+d}{(-e^2x^2+d^2)^{\frac{7}{2}}x}\,dx$$

input
```
integrate((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)/((-e^2*x^2 + d^2)^(7/2)*x), x)
```

3.27.9 Mupad [B] (verification not implemented)

Time = 11.97 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{d+ex}{x(d^2-e^2x^2)^{7/2}}\,dx=\frac{\frac{d^2-e^2\,x^2}{3\,d^3}+\frac{\left(d^2-e^2\,x^2\right)^2}{d^5}+\frac{1}{5\,d}}{\left(d^2-e^2\,x^2\right)^{5/2}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{d^6}+\frac{e\,x\,\left(15\,d^4-20\,d^2\,e^2\,x^2+8\,e^4\,x^4\right)}{15\,d^6\,\left(d^2-e^2\,x^2\right)^{5/2}}$$

input
```
int((d + e*x)/(x*(d^2 - e^2*x^2)^(7/2)),x)
```

output
```
((d^2 - e^2*x^2)/(3*d^3) + (d^2 - e^2*x^2)^2/d^5 + 1/(5*d))/(d^2 - e^2*x^2
)^(5/2) - atanh((d^2 - e^2*x^2)^(1/2)/d)/d^6 + (e*x*(15*d^4 + 8*e^4*x^4 -
20*d^2*e^2*x^2))/(15*d^6*(d^2 - e^2*x^2)^(5/2))
```

3.28 $\int \frac{d+ex}{x^2(d^2-e^2x^2)^{7/2}}\,dx$

3.28.1 Optimal result

Integrand size = 25, antiderivative size = 153

$$\int \frac{d+ex}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}}+\frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}-\frac{e\,\mathrm{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}$$

output
```
1/5*(e*x+d)/d^2/x/(-e^2*x^2+d^2)^(5/2)+1/15*(5*e*x+6*d)/d^4/x/(-e^2*x^2+d^
2)^(3/2)-e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^7+1/5*(5*e*x+8*d)/d^6/x/(-e^2
*x^2+d^2)^(1/2)-16/5*(-e^2*x^2+d^2)^(1/2)/d^7/x
```

3.28.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.88

$$\int \frac{d+ex}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{\frac{\sqrt{d^2-e^2x^2}\left(15d^5-38d^4ex-52d^3e^2x^2+87d^2e^3x^3+33de^4x^4-48e^5x^5\right)}{x(-d+ex)^3(d+ex)^2}+30e\,\mathrm{arctanh}\left(\frac{\sqrt{-e^2}x-\sqrt{d^2-e^2x^2}}{d}\right)}{15d^7}$$

input
```
Integrate[(d + e*x)/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(15*d^5 - 38*d^4*e*x - 52*d^3*e^2*x^2 + 87*d^2*e^3*x
^3 + 33*d*e^4*x^4 - 48*e^5*x^5))/(x*(-d + e*x)^3*(d + e*x)^2) + 30*e*ArcTa
nh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(15*d^7)
```

3.28.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {532, 25, 2336, 27, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x^2 (d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{ 532}$$

$$\frac{e(d+ex)}{5d^3 (d^2-e^2x^2)^{5/2}}-\frac{\int -\frac{\frac{4e^2x^2}{d}+5ex+5d}{x^2(d^2-e^2x^2)^{5/2}}dx}{5d^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\frac{4e^2x^2}{d}+5ex+5d}{x^2(d^2-e^2x^2)^{5/2}}dx}{5d^2}+\frac{e(d+ex)}{5d^3 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{ 2336}$$

$$\frac{\frac{e(5d+9ex)}{3d^3(d^2-e^2x^2)^{3/2}}-\frac{\int -\frac{3\left(\frac{6e^2x^2}{d}+5ex+5d\right)}{x^2\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}}{5d^2}+\frac{e(d+ex)}{5d^3 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{\frac{6e^2x^2}{d}+5ex+5d}{x^2\left(d^2-e^2x^2\right)^{3/2}}dx}{d^2}+\frac{e(5d+9ex)}{3d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{e(d+ex)}{5d^3 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{ 2336}$$

$$\frac{\frac{\frac{e(5d+11ex)}{d^3\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{5(d+ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{d^2}}{d^2}+\frac{e(5d+9ex)}{3d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{e(d+ex)}{5d^3 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{5\int \frac{d+ex}{x^2\sqrt{d^2-e^2x^2}}dx}{d^2}+\frac{e(5d+11ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e(5d+9ex)}{3d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{e(d+ex)}{5d^3 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \; 534$$

$$\frac{\frac{\frac{5\left(e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)}{d^2}+\frac{e(5d+11ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e(5d+9ex)}{3d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{e(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{\frac{5\left(\frac{1}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)}{d^2}+\frac{e(5d+11ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e(5d+9ex)}{3d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{e(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{5\left(-\frac{\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)}{d^2}+\frac{e(5d+11ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e(5d+9ex)}{3d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{e(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{5\left(-\frac{e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)}{d^2}+\frac{e(5d+11ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e(5d+9ex)}{3d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{e(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[(d + e*x)/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(e*(d + e*x))/(5*d^3*(d^2 - e^2*x^2)^(5/2)) + ((e*(5*d + 9*e*x))/(3*d^3*(d
^2 - e^2*x^2)^(3/2)) + ((e*(5*d + 11*e*x))/(d^3*Sqrt[d^2 - e^2*x^2]) + (5*
(-(Sqrt[d^2 - e^2*x^2]/(d*x)) - (e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/d^2
)/d^2)/(5*d^2)
```

3.28. $\int \frac{d+ex}{x^2(d^2-e^2x^2)^{7/2}}\,dx$

3.28.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.28.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.39

method	result
default	$e\left(\frac{1}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{1}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{\frac{1}{d^2\sqrt{-e^2x^2+d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}}}{d^2}}{d^2}\right)+d\left(-\frac{1}{d^2x(-e^2x^2+d^2)^{\frac{5}{2}}}+\right.$
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^7x}-\frac{e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^6\sqrt{d^2}}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{24d^6e\left(x+\frac{d}{e}\right)^2}-\frac{23\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{48d^7\left(x+\frac{d}{e}\right)}+\frac{17\sqrt{-(x}}{6}$

input
```
int((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
e*(1/5/d^2/(-e^2*x^2+d^2)^(5/2)+1/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*
(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^
2*x^2+d^2)^(1/2))/x))))+d*(-1/d^2/x/(-e^2*x^2+d^2)^(5/2)+6*e^2/d^2*(1/5*x/
d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4
/(-e^2*x^2+d^2)^(1/2))))
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.76

$$\int \frac{d+ex}{x^2 (d^2-e^2x^2)^{7/2}}\,dx = \frac{23\,e^6x^6 - 23\,de^5x^5 - 46\,d^2e^4x^4 + 46\,d^3e^3x^3 + 23\,d^4e^2x^2 - 23\,d^5ex + 15\,(e^6x^6 - de}{}$$

input
```
integrate((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(23*e^6*x^6 - 23*d*e^5*x^5 - 46*d^2*e^4*x^4 + 46*d^3*e^3*x^3 + 23*d^4
*e^2*x^2 - 23*d^5*e*x + 15*(e^6*x^6 - d*e^5*x^5 - 2*d^2*e^4*x^4 + 2*d^3*e^
3*x^3 + d^4*e^2*x^2 - d^5*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (48*e^
5*x^5 - 33*d*e^4*x^4 - 87*d^2*e^3*x^3 + 52*d^3*e^2*x^2 + 38*d^4*e*x - 15*d
^5)*sqrt(-e^2*x^2 + d^2))/(d^7*e^5*x^6 - d^8*e^4*x^5 - 2*d^9*e^3*x^4 + 2*d
^10*e^2*x^3 + d^11*e*x^2 - d^12*x)
```

3.28.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.16 (sec) , antiderivative size = 2404, normalized size of antiderivative = 15.71

$$\int \frac{d+ex}{x^2 (d^2-e^2x^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/x**2/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
d*Piecewise((5*d**6*e*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2
*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 30*d**4*e**3*x**2*sqrt(d*
*2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 +
5*d**8*e**6*x**6) + 40*d**2*e**5*x**4*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14
 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 16*e**7*x
**6*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e
**4*x**4 + 5*d**8*e**6*x**6), Abs(d**2/(e**2*x**2)) > 1), (5*I*d**6*e*sqrt
(-d**2/(e**2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**
4 + 5*d**8*e**6*x**6) - 30*I*d**4*e**3*x**2*sqrt(-d**2/(e**2*x**2) + 1)/(-
5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) + 40
*I*d**2*e**5*x**4*sqrt(-d**2/(e**2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x*
*2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 16*I*e**7*x**6*sqrt(-d**2/(e
**2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**
8*e**6*x**6), True)) + e*Piecewise((-46*I*d**6*sqrt(-1 + e**2*x**2/d**2)/(
-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) -
15*d**6*log(e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4
*x**4 + 30*d**7*e**6*x**6) + 30*d**6*log(e*x/d)/(-30*d**13 + 90*d**11*e**2
*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 30*I*d**6*asin(d/(e*x))/(
-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) +
70*I*d**4*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**...
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.24

$$\int \frac{d+ex}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{6\,e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3} + \frac{e}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2} + \frac{8\,e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5} + \frac{e}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4} - \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx} + \frac{16\,e^2x}{5\sqrt{-e^2x^2+d^2}d^7} - \frac{e\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d^7} + \frac{e}{\sqrt{-e^2x^2+d^2}d^6}$$

input
```
integrate((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
6/5*e^2*x/((-e^2*x^2 + d^2)^(5/2)*d^3) + 1/5*e/((-e^2*x^2 + d^2)^(5/2)*d^2
) + 8/5*e^2*x/((-e^2*x^2 + d^2)^(3/2)*d^5) + 1/3*e/((-e^2*x^2 + d^2)^(3/2)
*d^4) - 1/((-e^2*x^2 + d^2)^(5/2)*d*x) + 16/5*e^2*x/(sqrt(-e^2*x^2 + d^2)*
d^7) - e*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^7 + e/(sqrt
(-e^2*x^2 + d^2)*d^6)
```

3.28.8 Giac [F]

$$\int \frac{d+ex}{x^2(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{ex+d}{(-e^2x^2+d^2)^{\frac{7}{2}}x^2}\,dx$$

input
```
integrate((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)/((-e^2*x^2 + d^2)^(7/2)*x^2), x)
```

3.28.9 Mupad [B] (verification not implemented)

Time = 12.18 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.92

$$\int \frac{d+ex}{x^2(d^2-e^2x^2)^{7/2}}\,dx = \frac{\frac{e}{5\,d^2}+\frac{e\,(d^2-e^2\,x^2)^2}{d^6}+\frac{e\,(d^2-e^2\,x^2)}{3\,d^4}}{(d^2-e^2\,x^2)^{5/2}} - \frac{e\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{d^7} - \frac{d^6-6\,d^4\,e^2\,x^2+8\,d^2\,e^4\,x^4-\frac{16\,e^6\,x^6}{5}}{d^7\,x\,(d^2-e^2\,x^2)^{5/2}}$$

input
```
int((d + e*x)/(x^2*(d^2 - e^2*x^2)^(7/2)),x)
```

output
```
(e/(5*d^2) + (e*(d^2 - e^2*x^2)^2)/d^6 + (e*(d^2 - e^2*x^2))/(3*d^4))/(d^2
 - e^2*x^2)^(5/2) - (e*atanh((d^2 - e^2*x^2)^(1/2)/d))/d^7 - (d^6 - (16*e^
6*x^6)/5 - 6*d^4*e^2*x^2 + 8*d^2*e^4*x^4)/(d^7*x*(d^2 - e^2*x^2)^(5/2))
```

3.29 $\int \frac{d+ex}{x^3(d^2-e^2x^2)^{7/2}}\,dx$

3.29.1 Optimal result

Integrand size = 25, antiderivative size = 184

$$\int \frac{d+ex}{x^3(d^2-e^2x^2)^{7/2}}\,dx = \frac{d+ex}{5d^2x^2(d^2-e^2x^2)^{5/2}} + \frac{7d+6ex}{15d^4x^2(d^2-e^2x^2)^{3/2}} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} - \frac{16e\sqrt{d^2-e^2x^2}}{5d^8x} - \frac{7e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8}$$

output
```
1/5*(e*x+d)/d^2/x^2/(-e^2*x^2+d^2)^(5/2)+1/15*(6*e*x+7*d)/d^4/x^2/(-e^2*x^
2+d^2)^(3/2)-7/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^8+1/15*(24*e*x+35*d
)/d^6/x^2/(-e^2*x^2+d^2)^(1/2)-7/2*(-e^2*x^2+d^2)^(1/2)/d^7/x^2-16/5*e*(-e
^2*x^2+d^2)^(1/2)/d^8/x
```

3.29.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.80

$$\int \frac{d+ex}{x^3(d^2-e^2x^2)^{7/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(15d^6+15d^5ex-176d^4e^2x^2-4d^3e^3x^3+249d^2e^4x^4-9de^5x^5-96e^6x^6)}{x^2(-d+ex)^3(d+ex)^2} + 210e^2\operatorname{arctanh}\left(\frac{\sqrt{-e^2x}}{\ }\right.}{30d^8}$$

input
```
Integrate[(d + e*x)/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(15*d^6 + 15*d^5*e*x - 176*d^4*e^2*x^2 - 4*d^3*e^3*x
^3 + 249*d^2*e^4*x^4 - 9*d*e^5*x^5 - 96*e^6*x^6))/(x^2*(-d + e*x)^3*(d + e
*x)^2) + 210*e^2*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(30*d^8)
```

3.29.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {532, 25, 2336, 27, 2336, 27, 2338, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x^3\left(d^2-e^2x^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{532}$$

$$\frac{e^2(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int -\frac{\frac{4e^3x^3}{d^2}+\frac{5e^2x^2}{d}+5ex+5d}{x^3\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{4e^3x^3}{d^2}+\frac{5e^2x^2}{d}+5ex+5d}{x^3\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}+\frac{e^2(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{e^2(10d+9ex)}{3d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int -\frac{3\left(\frac{6e^3x^3}{d^2}+\frac{10e^2x^2}{d}+5ex+5d\right)}{x^3\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}}{5d^2}+\frac{e^2(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\frac{6e^3x^3}{d^2}+\frac{10e^2x^2}{d}+5ex+5d}{x^3\left(d^2-e^2x^2\right)^{3/2}}dx}{d^2}+\frac{e^2(10d+9ex)}{3d^4\left(d^2-e^2x^2\right)^{3/2}}}{5d^2}+\frac{e^2(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{\frac{e^2(15d+11ex)}{d^4\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{5\left(\frac{3e^2x^2}{d}+ex+d\right)}{x^3\sqrt{d^2-e^2x^2}}dx}{d^2}}{d^2}+\frac{e^2(10d+9ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{e^2(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{5\int \frac{\frac{3e^2x^2}{d}+ex+d}{x^3\sqrt{d^2-e^2x^2}}dx}{d^2}+\frac{e^2(15d+11ex)}{d^4\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(10d+9ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{e^2(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \ 2338$$

$$\frac{\frac{\frac{5\left(-\frac{\int -\frac{de(2d+7ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{d^2}+\frac{e^2(15d+11ex)}{d^4\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(10d+9ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{e^2(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\frac{5\left(\frac{\int \frac{de(2d+7ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{d^2}+\frac{e^2(15d+11ex)}{d^4\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(10d+9ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{e^2(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{5\left(\frac{e\int \frac{2d+7ex}{x^2\sqrt{d^2-e^2x^2}}dx}{2d}-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{d^2}+\frac{e^2(15d+11ex)}{d^4\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(10d+9ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{e^2(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \ 534$$

$$\frac{\frac{\frac{5\left(\frac{e\left(7e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{2\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{d^2}+\frac{e^2(15d+11ex)}{d^4\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(10d+9ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{e^2(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \ 243$$

$$\frac{\frac{\frac{5\left(\frac{e\left(\frac{7}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{2\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{d^2}+\frac{e^2(15d+11ex)}{d^4\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(10d+9ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{e^2(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

3.29. $\int \frac{d+ex}{x^3(d^2-e^2x^2)^{7/2}}\,dx$

$$\downarrow 73$$

$$\frac{\frac{\frac{5\left(\frac{e\left(-\frac{7\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{2\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{d^2}+\frac{e^2(15d+11ex)}{d^4\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(10d+9ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{e^2(d+ex)}{5d^4(d^2-e^2x^2)^{5/2}}$$

$$\downarrow 221$$

$$\frac{\frac{\frac{5\left(\frac{e\left(-\frac{7e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{2\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{d^2}+\frac{e^2(15d+11ex)}{d^4\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(10d+9ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{e^2(d+ex)}{5d^4(d^2-e^2x^2)^{5/2}}$$

input
```
Int[(d + e*x)/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(e^2*(d + e*x))/(5*d^4*(d^2 - e^2*x^2)^(5/2)) + ((e^2*(10*d + 9*e*x))/(3*d
^4*(d^2 - e^2*x^2)^(3/2)) + ((e^2*(15*d + 11*e*x))/(d^4*Sqrt[d^2 - e^2*x^2
]) + (5*(-1/2*Sqrt[d^2 - e^2*x^2]/(d*x^2) + (e*((-2*Sqrt[d^2 - e^2*x^2])/(
d*x) - (7*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/(2*d)))/d^2)/d^2)/(5*d^2)
```

3.29.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.29. $\int \frac{d+ex}{x^3(d^2-e^2x^2)^{7/2}}\,dx$

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.29.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.33

method	result
default	$d\left(-\frac{1}{2d^2x^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{7e^2\left(\frac{1}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{1}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{\frac{1}{d^2\sqrt{-e^2x^2+d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}}}{d^2}}{d^2}\right)}{2d^2}\right)+$
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(2ex+d)}{2d^8x^2}-\frac{7e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^7\sqrt{d^2}}+\frac{29e\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{48d^8\left(x+\frac{d}{e}\right)}+\frac{11\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{30d^7\left(x-\frac{d}{e}\right)^2}$

input
```
int((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
d*(-1/2/d^2/x^2/(-e^2*x^2+d^2)^(5/2)+7/2*e^2/d^2*(1/5/d^2/(-e^2*x^2+d^2)^(
5/2)+1/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)
-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))+e*
(-1/d^2/x/(-e^2*x^2+d^2)^(5/2)+6*e^2/d^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4
/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))))
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.58

$$\int \frac{d+ex}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{116\,e^7x^7-116\,de^6x^6-232\,d^2e^5x^5+232\,d^3e^4x^4+116\,d^4e^3x^3-116\,d^5e^2x^2+105\,(}{}$$

input
```
integrate((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/30*(116*e^7*x^7 - 116*d*e^6*x^6 - 232*d^2*e^5*x^5 + 232*d^3*e^4*x^4 + 11
6*d^4*e^3*x^3 - 116*d^5*e^2*x^2 + 105*(e^7*x^7 - d*e^6*x^6 - 2*d^2*e^5*x^5
 + 2*d^3*e^4*x^4 + d^4*e^3*x^3 - d^5*e^2*x^2)*log(-(d - sqrt(-e^2*x^2 + d^
2))/x) - (96*e^6*x^6 + 9*d*e^5*x^5 - 249*d^2*e^4*x^4 + 4*d^3*e^3*x^3 + 176
*d^4*e^2*x^2 - 15*d^5*e*x - 15*d^6)*sqrt(-e^2*x^2 + d^2))/(d^8*e^5*x^7 - d
^9*e^4*x^6 - 2*d^10*e^3*x^5 + 2*d^11*e^2*x^4 + d^12*e*x^3 - d^13*x^2)
```

3.29.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.95 (sec) , antiderivative size = 2691, normalized size of antiderivative = 14.62

$$\int \frac{d+ex}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/x**3/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
d*Piecewise((30*I*d**8*sqrt(-1 + e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**
13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 322*I*d**6*e**2*
x**2*sqrt(-1 + e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180
*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 105*d**6*e**2*x**2*log(e**2*x**2/d
**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9
*e**6*x**8) + 210*d**6*e**2*x**2*log(e*x/d)/(-60*d**15*x**2 + 180*d**13*e*
*2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 210*I*d**6*e**2*x**2*
asin(d/(e*x))/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6
+ 60*d**9*e**6*x**8) + 490*I*d**4*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-60
*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**
8) + 315*d**4*e**4*x**4*log(e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e*
*2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 630*d**4*e**4*x**4*lo
g(e*x/d)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*
d**9*e**6*x**8) + 630*I*d**4*e**4*x**4*asin(d/(e*x))/(-60*d**15*x**2 + 180
*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 210*I*d**2*e
**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 -
 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 315*d**2*e**6*x**6*log(e**2*x*
*2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*
d**9*e**6*x**8) + 630*d**2*e**6*x**6*log(e*x/d)/(-60*d**15*x**2 + 180*d**1
3*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 630*I*d**2*e**...
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.20

$$\int \frac{d+ex}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{6\,e^3x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4} + \frac{7\,e^2}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3} + \frac{8\,e^3x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^6} + \frac{7\,e^2}{6\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5} - \frac{e}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2x} + \frac{16\,e^3x}{5\sqrt{-e^2x^2+d^2}d^8} - \frac{7\,e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{2\,d^8} + \frac{7\,e^2}{2\sqrt{-e^2x^2+d^2}d^7} - \frac{1}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx^2}$$

input
```
integrate((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
6/5*e^3*x/((-e^2*x^2 + d^2)^(5/2)*d^4) + 7/10*e^2/((-e^2*x^2 + d^2)^(5/2)*
d^3) + 8/5*e^3*x/((-e^2*x^2 + d^2)^(3/2)*d^6) + 7/6*e^2/((-e^2*x^2 + d^2)^
(3/2)*d^5) - e/((-e^2*x^2 + d^2)^(5/2)*d^2*x) + 16/5*e^3*x/(sqrt(-e^2*x^2
+ d^2)*d^8) - 7/2*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/
d^8 + 7/2*e^2/(sqrt(-e^2*x^2 + d^2)*d^7) - 1/2/((-e^2*x^2 + d^2)^(5/2)*d*x
^2)
```

3.29.8 Giac [F]

$$\int \frac{d+ex}{x^3 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{ex+d}{(-e^2x^2+d^2)^{\frac{7}{2}}x^3}\,dx$$

input
```
integrate((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)/((-e^2*x^2 + d^2)^(7/2)*x^3), x)
```

3.29.9 Mupad [B] (verification not implemented)

Time = 12.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{d+ex}{x^3 (d^2-e^2x^2)^{7/2}}\,dx &= \frac{161\,e^2}{30\,d^3\,(d^2-e^2\,x^2)^{5/2}} - \frac{1}{2\,d\,x^2\,(d^2-e^2\,x^2)^{5/2}} \\ &\quad - \frac{7\,e^2\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{2\,d^8} - \frac{49\,e^4\,x^2}{6\,d^5\,(d^2-e^2\,x^2)^{5/2}} + \frac{7\,e^6\,x^4}{2\,d^7\,(d^2-e^2\,x^2)^{5/2}} \\ &\quad - \frac{e\,(5\,d^6-30\,d^4\,e^2\,x^2+40\,d^2\,e^4\,x^4-16\,e^6\,x^6)}{5\,d^8\,x\,(d^2-e^2\,x^2)^{5/2}}\end{aligned}$$

input
```
int((d + e*x)/(x^3*(d^2 - e^2*x^2)^(7/2)),x)
```

output
```
(161*e^2)/(30*d^3*(d^2 - e^2*x^2)^(5/2)) - 1/(2*d*x^2*(d^2 - e^2*x^2)^(5/2
)) - (7*e^2*atanh((d^2 - e^2*x^2)^(1/2)/d))/(2*d^8) - (49*e^4*x^2)/(6*d^5*
(d^2 - e^2*x^2)^(5/2)) + (7*e^6*x^4)/(2*d^7*(d^2 - e^2*x^2)^(5/2)) - (e*(5
*d^6 - 16*e^6*x^6 - 30*d^4*e^2*x^2 + 40*d^2*e^4*x^4))/(5*d^8*x*(d^2 - e^2*
x^2)^(5/2))
```

3.30 $\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{9/2}}\,dx$

3.30.1 Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{9/2}}\,dx = \frac{x^2(d+ex)}{7de\,(d^2-e^2x^2)^{7/2}} - \frac{2(d-2ex)}{35de^3\,(d^2-e^2x^2)^{5/2}} - \frac{4x}{105d^3e^2\,(d^2-e^2x^2)^{3/2}} - \frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}$$

output
```
1/7*x^2*(e*x+d)/d/e/(-e^2*x^2+d^2)^(7/2)-2/35*(-2*e*x+d)/d/e^3/(-e^2*x^2+d
^2)^(5/2)-4/105*x/d^3/e^2/(-e^2*x^2+d^2)^(3/2)-8/105*x/d^5/e^2/(-e^2*x^2+d
^2)^(1/2)
```

3.30.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.86

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{9/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-6d^6+6d^5ex+15d^4e^2x^2+20d^3e^3x^3-20d^2e^4x^4-8de^5x^5+8e^6x^6)}{105d^5e^3(d-ex)^4(d+ex)^3}$$

input
```
Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(9/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-6*d^6 + 6*d^5*e*x + 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3
 - 20*d^2*e^4*x^4 - 8*d*e^5*x^5 + 8*e^6*x^6))/(105*d^5*e^3*(d - e*x)^4*(d
+ e*x)^3)
```

3.30.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {529, 27, 454, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{9/2}}dx$$

$$\Big\downarrow \ 529$$

$$\frac{d(d+ex)}{7e^3(d^2-e^2x^2)^{7/2}}-\frac{\int \frac{d(d+7ex)}{e^2(d^2-e^2x^2)^{7/2}}dx}{7d}$$

$$\Big\downarrow \ 27$$

$$\frac{d(d+ex)}{7e^3(d^2-e^2x^2)^{7/2}}-\frac{\int \frac{d+7ex}{(d^2-e^2x^2)^{7/2}}dx}{7e^2}$$

$$\Big\downarrow \ 454$$

$$\frac{d(d+ex)}{7e^3(d^2-e^2x^2)^{7/2}}-\frac{\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d}+\frac{7d+ex}{5de(d^2-e^2x^2)^{5/2}}}{7e^2}$$

$$\Big\downarrow \ 209$$

$$\frac{d(d+ex)}{7e^3(d^2-e^2x^2)^{7/2}}-\frac{\frac{4\left(\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)}{5d}+\frac{7d+ex}{5de(d^2-e^2x^2)^{5/2}}}{7e^2}$$

$$\Big\downarrow \ 208$$

$$\frac{d(d+ex)}{7e^3(d^2-e^2x^2)^{7/2}}-\frac{\frac{7d+ex}{5de(d^2-e^2x^2)^{5/2}}+\frac{4\left(\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d}}{7e^2}$$

input
```
Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(9/2),x]
```

output
```
(d*(d + e*x))/(7*e^3*(d^2 - e^2*x^2)^(7/2)) - ((7*d + e*x)/(5*d*e*(d^2 - e
^2*x^2)^(5/2)) + (4*(x/(3*d^2*(d^2 - e^2*x^2)^(3/2)) + (2*x)/(3*d^4*Sqrt[d
^2 - e^2*x^2])))/(5*d))/(7*e^2)
```

3.30.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

3.30.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.82

method	result
gosper	$-\frac{(-ex+d)(ex+d)^2(-8e^6x^6+8d\,e^5x^5+20d^2e^4x^4-20d^3x^3e^3-15d^4e^2x^2-6d^5ex+6d^6)}{105d^5e^3(-e^2x^2+d^2)^{\frac{9}{2}}}$
trager	$-\frac{(-8e^6x^6+8d\,e^5x^5+20d^2e^4x^4-20d^3x^3e^3-15d^4e^2x^2-6d^5ex+6d^6)\sqrt{-e^2x^2+d^2}}{105d^5(-ex+d)^4(ex+d)^3e^3}$
default	$e\left(\frac{x^2}{5e^2(-e^2x^2+d^2)^{\frac{7}{2}}}-\frac{2d^2}{35e^4(-e^2x^2+d^2)^{\frac{7}{2}}}\right)+d\left(\frac{x}{6e^2(-e^2x^2+d^2)^{\frac{7}{2}}}-\frac{d^2\left(\frac{x}{7d^2\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}+\frac{6x}{35d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{6\left(\frac{}{15d^2}\right.}{}\right.}{6e^2}\right.$

input
```
int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105*(-e*x+d)*(e*x+d)^2*(-8*e^6*x^6+8*d*e^5*x^5+20*d^2*e^4*x^4-20*d^3*e^
3*x^3-15*d^4*e^2*x^2-6*d^5*e*x+6*d^6)/d^5/e^3/(-e^2*x^2+d^2)^(9/2)
```

3.30.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(107) = 214.

Time = 0.41 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.98

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{9/2}}\,dx =$$
$$-\frac{6\,e^7x^7-6\,de^6x^6-18\,d^2e^5x^5+18\,d^3e^4x^4+18\,d^4e^3x^3-18\,d^5e^2x^2-6\,d^6ex+6\,d^7-(8\,e^6x^6-8\,de^5x^5-20}{105\,(d^5e^{10}x^7-d^6e^9x^6-3\,d^7e^8x^5+3\,d^8e^7x^4+3\,d^9e^6x^3-3\,d^{10}e^5}$$

input
```
integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x, algorithm="fricas")
```

output
```
-1/105*(6*e^7*x^7 - 6*d*e^6*x^6 - 18*d^2*e^5*x^5 + 18*d^3*e^4*x^4 + 18*d^4
*e^3*x^3 - 18*d^5*e^2*x^2 - 6*d^6*e*x + 6*d^7 - (8*e^6*x^6 - 8*d*e^5*x^5 -
 20*d^2*e^4*x^4 + 20*d^3*e^3*x^3 + 15*d^4*e^2*x^2 + 6*d^5*e*x - 6*d^6)*sqr
t(-e^2*x^2 + d^2))/(d^5*e^10*x^7 - d^6*e^9*x^6 - 3*d^7*e^8*x^5 + 3*d^8*e^7
*x^4 + 3*d^9*e^6*x^3 - 3*d^10*e^5*x^2 - d^11*e^4*x + d^12*e^3)
```

3.30.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.46 (sec) , antiderivative size = 903, normalized size of antiderivative = 7.46

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{9/2}}\,dx = d\left(\left\{\begin{array}{l} \frac{35id^4x^3}{-105d^{13}\sqrt{-1+\frac{e^2x^2}{d^2}}+315d^{11}e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}-315d^9e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}+105d^7e^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{}{-105d^1} \\ -\frac{35d^4x^3}{-105d^{13}\sqrt{1-\frac{e^2x^2}{d^2}}+315d^{11}e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}-315d^9e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}+105d^7e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{}{-105d^{13}\sqrt{1}} \end{array}\right.\right.$$
$$+e\left(\left\{\begin{array}{l} \frac{2d^2}{-35d^6e^4\sqrt{d^2-e^2x^2}+105d^4e^6x^2\sqrt{d^2-e^2x^2}-105d^2e^8x^4\sqrt{d^2-e^2x^2}+35e^{10}x^6\sqrt{d^2-e^2x^2}} - \frac{7e^2x^2}{-35d^6e^4\sqrt{d^2-e^2x^2}+105d^4e^6x^2\sqrt{d^2-e^2x^2}-10} \\ \frac{x^4}{4(d^2)^{\frac{9}{2}}} \end{array}\right.\right.$$

input
```
integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(9/2),x)
```

output
```
d*Piecewise((35*I*d**4*x**3/(-105*d**13*sqrt(-1 + e**2*x**2/d**2) + 315*d*
*11*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) - 315*d**9*e**4*x**4*sqrt(-1 + e**
2*x**2/d**2) + 105*d**7*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)) - 28*I*d**2*e
**2*x**5/(-105*d**13*sqrt(-1 + e**2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(
-1 + e**2*x**2/d**2) - 315*d**9*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) + 105*
d**7*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)) + 8*I*e**4*x**7/(-105*d**13*sqrt
(-1 + e**2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) - 31
5*d**9*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) + 105*d**7*e**6*x**6*sqrt(-1 +
e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-35*d**4*x**3/(-105*d**13*sqr
t(1 - e**2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(1 - e**2*x**2/d**2) - 315
*d**9*e**4*x**4*sqrt(1 - e**2*x**2/d**2) + 105*d**7*e**6*x**6*sqrt(1 - e**
2*x**2/d**2)) + 28*d**2*e**2*x**5/(-105*d**13*sqrt(1 - e**2*x**2/d**2) + 3
15*d**11*e**2*x**2*sqrt(1 - e**2*x**2/d**2) - 315*d**9*e**4*x**4*sqrt(1 -
e**2*x**2/d**2) + 105*d**7*e**6*x**6*sqrt(1 - e**2*x**2/d**2)) - 8*e**4*x*
*7/(-105*d**13*sqrt(1 - e**2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(1 - e**
2*x**2/d**2) - 315*d**9*e**4*x**4*sqrt(1 - e**2*x**2/d**2) + 105*d**7*e**6
*x**6*sqrt(1 - e**2*x**2/d**2)), True)) + e*Piecewise((2*d**2/(-35*d**6*e*
*4*sqrt(d**2 - e**2*x**2) + 105*d**4*e**6*x**2*sqrt(d**2 - e**2*x**2) - 10
5*d**2*e**8*x**4*sqrt(d**2 - e**2*x**2) + 35*e**10*x**6*sqrt(d**2 - e**2*x
**2)) - 7*e**2*x**2/(-35*d**6*e**4*sqrt(d**2 - e**2*x**2) + 105*d**4*e*...
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.12

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{9/2}}\,dx = \frac{x^2}{5\,(-e^2x^2+d^2)^{\frac{7}{2}}e} + \frac{dx}{7\,(-e^2x^2+d^2)^{\frac{7}{2}}e^2} - \frac{2\,d^2}{35\,(-e^2x^2+d^2)^{\frac{7}{2}}e^3} - \frac{x}{35\,(-e^2x^2+d^2)^{\frac{5}{2}}de^2} - \frac{4\,x}{105\,(-e^2x^2+d^2)^{\frac{3}{2}}d^3e^2} - \frac{8\,x}{105\,\sqrt{-e^2x^2+d^2}d^5e^2}$$

input
```
integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x, algorithm="maxima")
```

output
```
1/5*x^2/((-e^2*x^2 + d^2)^(7/2)*e) + 1/7*d*x/((-e^2*x^2 + d^2)^(7/2)*e^2)
- 2/35*d^2/((-e^2*x^2 + d^2)^(7/2)*e^3) - 1/35*x/((-e^2*x^2 + d^2)^(5/2)*d
*e^2) - 4/105*x/((-e^2*x^2 + d^2)^(3/2)*d^3*e^2) - 8/105*x/(sqrt(-e^2*x^2
+ d^2)*d^5*e^2)
```

3.30.8 Giac [F]

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{9/2}}\,dx = \int \frac{(ex+d)x^2}{(-e^2x^2+d^2)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)*x^2/(-e^2*x^2 + d^2)^(9/2), x)
```

3.30.9 Mupad [B] (verification not implemented)

Time = 11.54 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.36

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{9/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}}{56\,d^2\,e^3\,(d-e\,x)^4} - \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{2}{35\,e^3}-\frac{3\,x}{70\,d\,e^2}\right)}{(d+e\,x)^3\,(d-e\,x)^3} - \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1}{56\,d^2\,e^3}+\frac{4\,x}{105\,d^3\,e^2}\right)}{(d+e\,x)^2\,(d-e\,x)^2} - \frac{8\,x\,\sqrt{d^2-e^2\,x^2}}{105\,d^5\,e^2\,(d+e\,x)\,(d-e\,x)}$$

input
```
int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(9/2),x)
```

output
```
(d^2 - e^2*x^2)^(1/2)/(56*d^2*e^3*(d - e*x)^4) - ((d^2 - e^2*x^2)^(1/2)*(2
/(35*e^3) - (3*x)/(70*d*e^2)))/((d + e*x)^3*(d - e*x)^3) - ((d^2 - e^2*x^2
)^(1/2)*(1/(56*d^2*e^3) + (4*x)/(105*d^3*e^2)))/((d + e*x)^2*(d - e*x)^2)
- (8*x*(d^2 - e^2*x^2)^(1/2))/(105*d^5*e^2*(d + e*x)*(d - e*x))
```

3.31 $\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{11/2}}\,dx$

3.31.1 Optimal result

Integrand size = 25, antiderivative size = 148

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{x^2(d+ex)}{9de\,(d^2-e^2x^2)^{9/2}} - \frac{2(d-3ex)}{63de^3\,(d^2-e^2x^2)^{7/2}} - \frac{2x}{105d^3e^2\,(d^2-e^2x^2)^{5/2}} - \frac{8x}{315d^5e^2\,(d^2-e^2x^2)^{3/2}} - \frac{16x}{315d^7e^2\sqrt{d^2-e^2x^2}}$$

output
```
1/9*x^2*(e*x+d)/d/e/(-e^2*x^2+d^2)^(9/2)-2/63*(-3*e*x+d)/d/e^3/(-e^2*x^2+d
^2)^(7/2)-2/105*x/d^3/e^2/(-e^2*x^2+d^2)^(5/2)-8/315*x/d^5/e^2/(-e^2*x^2+d
^2)^(3/2)-16/315*x/d^7/e^2/(-e^2*x^2+d^2)^(1/2)
```

3.31.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-10d^8+10d^7ex+35d^6e^2x^2+70d^5e^3x^3-70d^4e^4x^4-56d^3e^5x^5+56d^2}{315d^7e^3(d-ex)^5(d+ex)^4}$$

input
```
Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-10*d^8 + 10*d^7*e*x + 35*d^6*e^2*x^2 + 70*d^5*e^3*x
^3 - 70*d^4*e^4*x^4 - 56*d^3*e^5*x^5 + 56*d^2*e^6*x^6 + 16*d*e^7*x^7 - 16*
e^8*x^8))/(315*d^7*e^3*(d - e*x)^5*(d + e*x)^4)
```

3.31.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {529, 27, 454, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{11/2}}dx$$

$$\Big\downarrow \ 529$$

$$\frac{d(d+ex)}{9e^3(d^2-e^2x^2)^{9/2}}-\frac{\int \frac{d(d+9ex)}{e^2(d^2-e^2x^2)^{9/2}}dx}{9d}$$

$$\Big\downarrow \ 27$$

$$\frac{d(d+ex)}{9e^3(d^2-e^2x^2)^{9/2}}-\frac{\int \frac{d+9ex}{(d^2-e^2x^2)^{9/2}}dx}{9e^2}$$

$$\Big\downarrow \ 454$$

$$\frac{d(d+ex)}{9e^3(d^2-e^2x^2)^{9/2}}-\frac{\frac{6\int \frac{1}{\left(d^2-e^2x^2\right)^{7/2}}dx}{7d}+\frac{9d+ex}{7de(d^2-e^2x^2)^{7/2}}}{9e^2}$$

$$\Big\downarrow \ 209$$

$$\frac{d(d+ex)}{9e^3(d^2-e^2x^2)^{9/2}}-\frac{\frac{6\left(\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d}+\frac{9d+ex}{7de(d^2-e^2x^2)^{7/2}}}{9e^2}$$

$$\Big\downarrow \ 209$$

$$\frac{d(d+ex)}{9e^3(d^2-e^2x^2)^{9/2}}-\frac{\frac{6\left(\frac{4\left(\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d}+\frac{9d+ex}{7de(d^2-e^2x^2)^{7/2}}}{9e^2}$$

$$\Big\downarrow \ 208$$

$$\frac{d(d+ex)}{9e^3\left(d^2-e^2x^2\right)^{9/2}}-\frac{\frac{9d+ex}{7de(d^2-e^2x^2)^{7/2}}+\frac{6\left(\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{4\left(\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d^2}\right)}{7d}}{9e^2}$$

input
```
Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
(d*(d + e*x))/(9*e^3*(d^2 - e^2*x^2)^(9/2)) - ((9*d + e*x)/(7*d*e*(d^2 - e
^2*x^2)^(7/2)) + (6*(x/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (4*(x/(3*d^2*(d^2 -
 e^2*x^2)^(3/2)) + (2*x)/(3*d^4*Sqrt[d^2 - e^2*x^2])))/(5*d^2)))/(7*d))/(9
*e^2)
```

3.31.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

3.31.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.82

method	result
gosper	$-\frac{(-ex+d)(ex+d)^{2}\left(16e^{8}x^{8}-16d\,e^{7}x^{7}-56d^{2}e^{6}x^{6}+56d^{3}e^{5}x^{5}+70d^{4}x^{4}e^{4}-70d^{5}e^{3}x^{3}-35d^{6}e^{2}x^{2}-10d^{7}ex+10d^{8}\right)}{315d^{7}e^{3}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{11}{2}}}$
trager	$-\frac{\left(16e^{8}x^{8}-16d\,e^{7}x^{7}-56d^{2}e^{6}x^{6}+56d^{3}e^{5}x^{5}+70d^{4}x^{4}e^{4}-70d^{5}e^{3}x^{3}-35d^{6}e^{2}x^{2}-10d^{7}ex+10d^{8}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{315d^{7}(-ex+d)^{5}(ex+d)^{4}e^{3}}$
default	$e\left(\frac{x^{2}}{7e^{2}(-e^{2}x^{2}+d^{2})^{\frac{9}{2}}}-\frac{2d^{2}}{63e^{4}(-e^{2}x^{2}+d^{2})^{\frac{9}{2}}}\right)+d\left(\frac{x}{8e^{2}(-e^{2}x^{2}+d^{2})^{\frac{9}{2}}}-\frac{d^{2}\left(\frac{x}{9d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{9}{2}}}+\frac{\frac{8x}{63d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}+\frac{8\left(\frac{}{35d^{2}}\right.}{}}{}\right.}{}\right.$

input
```
int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-1/315*(-e*x+d)*(e*x+d)^2*(16*e^8*x^8-16*d*e^7*x^7-56*d^2*e^6*x^6+56*d^3*e
^5*x^5+70*d^4*e^4*x^4-70*d^5*e^3*x^3-35*d^6*e^2*x^2-10*d^7*e*x+10*d^8)/d^7
/e^3/(-e^2*x^2+d^2)^(11/2)
```

3.31.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(130) = 260.

Time = 0.50 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.06

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{11/2}}\,dx =$$
$$-\frac{10\,e^9x^9 - 10\,de^8x^8 - 40\,d^2e^7x^7 + 40\,d^3e^6x^6 + 60\,d^4e^5x^5 - 60\,d^5e^4x^4 - 40\,d^6e^3x^3 + 40\,d^7e^2x^2 + 10\,d^8ex - \cdots}{315\,(d^7e^{12}x^9 - d^8e^{11}x^8 - 4\,d^9e^{10}x^7 + 4\,d^{10}e^9x^6 + \cdots}$$

input
```
integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="fricas")
```

output
```
-1/315*(10*e^9*x^9 - 10*d*e^8*x^8 - 40*d^2*e^7*x^7 + 40*d^3*e^6*x^6 + 60*d
^4*e^5*x^5 - 60*d^5*e^4*x^4 - 40*d^6*e^3*x^3 + 40*d^7*e^2*x^2 + 10*d^8*e*x
 - 10*d^9 - (16*e^8*x^8 - 16*d*e^7*x^7 - 56*d^2*e^6*x^6 + 56*d^3*e^5*x^5 +
 70*d^4*e^4*x^4 - 70*d^5*e^3*x^3 - 35*d^6*e^2*x^2 - 10*d^7*e*x + 10*d^8)*s
qrt(-e^2*x^2 + d^2))/(d^7*e^12*x^9 - d^8*e^11*x^8 - 4*d^9*e^10*x^7 + 4*d^1
0*e^9*x^6 + 6*d^11*e^8*x^5 - 6*d^12*e^7*x^4 - 4*d^13*e^6*x^3 + 4*d^14*e^5*
x^2 + d^15*e^4*x - d^16*e^3)
```

3.31.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 16.45 (sec) , antiderivative size = 1401, normalized size of antiderivative = 9.47

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(11/2),x)
```

output
```
d*Piecewise((-105*I*d**6*x**3/(315*d**17*sqrt(-1 + e**2*x**2/d**2) - 1260*
d**15*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(-1 +
 e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 315*d*
*9*e**8*x**8*sqrt(-1 + e**2*x**2/d**2)) + 126*I*d**4*e**2*x**5/(315*d**17*
sqrt(-1 + e**2*x**2/d**2) - 1260*d**15*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)
 + 1890*d**13*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) - 1260*d**11*e**6*x**6*s
qrt(-1 + e**2*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(-1 + e**2*x**2/d**2)) -
 72*I*d**2*e**4*x**7/(315*d**17*sqrt(-1 + e**2*x**2/d**2) - 1260*d**15*e**
2*x**2*sqrt(-1 + e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(-1 + e**2*x**
2/d**2) - 1260*d**11*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 315*d**9*e**8*x
**8*sqrt(-1 + e**2*x**2/d**2)) + 16*I*e**6*x**9/(315*d**17*sqrt(-1 + e**2*
x**2/d**2) - 1260*d**15*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 1890*d**13*e
**4*x**4*sqrt(-1 + e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(-1 + e**2*x
**2/d**2) + 315*d**9*e**8*x**8*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d
**2) > 1), (105*d**6*x**3/(315*d**17*sqrt(1 - e**2*x**2/d**2) - 1260*d**15
*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(1 - e**2*x
**2/d**2) - 1260*d**11*e**6*x**6*sqrt(1 - e**2*x**2/d**2) + 315*d**9*e**8*
x**8*sqrt(1 - e**2*x**2/d**2)) - 126*d**4*e**2*x**5/(315*d**17*sqrt(1 - e*
*2*x**2/d**2) - 1260*d**15*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 1890*d**13
*e**4*x**4*sqrt(1 - e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(1 - e**...
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.07

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{x^2}{7\,(-e^2x^2+d^2)^{\frac{9}{2}}e} + \frac{dx}{9\,(-e^2x^2+d^2)^{\frac{9}{2}}e^2} - \frac{2\,d^2}{63\,(-e^2x^2+d^2)^{\frac{9}{2}}e^3} - \frac{x}{63\,(-e^2x^2+d^2)^{\frac{7}{2}}de^2} - \frac{2\,x}{105\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3e^2} - \frac{8\,x}{315\,(-e^2x^2+d^2)^{\frac{3}{2}}d^5e^2} - \frac{16\,x}{315\,\sqrt{-e^2x^2+d^2}d^7e^2}$$

input
```
integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="maxima")
```

output
```
1/7*x^2/((-e^2*x^2 + d^2)^(9/2)*e) + 1/9*d*x/((-e^2*x^2 + d^2)^(9/2)*e^2)
- 2/63*d^2/((-e^2*x^2 + d^2)^(9/2)*e^3) - 1/63*x/((-e^2*x^2 + d^2)^(7/2)*d
*e^2) - 2/105*x/((-e^2*x^2 + d^2)^(5/2)*d^3*e^2) - 8/315*x/((-e^2*x^2 + d^
2)^(3/2)*d^5*e^2) - 16/315*x/(sqrt(-e^2*x^2 + d^2)*d^7*e^2)
```

3.31.8 Giac [F]

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(ex+d)x^2}{(-e^2x^2+d^2)^{\frac{11}{2}}}\,dx$$

input
```
integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)*x^2/(-e^2*x^2 + d^2)^(11/2), x)
```

3.31.9 Mupad [B] (verification not implemented)

Time = 11.62 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.36

$$\begin{aligned}\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{11/2}}\,dx &= \frac{\sqrt{d^2-e^2\,x^2}}{144\,d^3\,e^3\,(d-e\,x)^5} \\ &- \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1}{252\,e^3}-\frac{17\,x}{252\,d\,e^2}\right)}{(d+e\,x)^4\,(d-e\,x)^4} - \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{5}{144\,d^2\,e^3}+\frac{131\,x}{5040\,d^3\,e^2}\right)}{(d+e\,x)^3\,(d-e\,x)^3} \\ &- \frac{8\,x\,\sqrt{d^2-e^2\,x^2}}{315\,d^5\,e^2\,(d+e\,x)^2\,(d-e\,x)^2} - \frac{16\,x\,\sqrt{d^2-e^2\,x^2}}{315\,d^7\,e^2\,(d+e\,x)\,(d-e\,x)}\end{aligned}$$

input
```
int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(11/2),x)
```

output
```
(d^2 - e^2*x^2)^(1/2)/(144*d^3*e^3*(d - e*x)^5) - ((d^2 - e^2*x^2)^(1/2)*(
1/(252*e^3) - (17*x)/(252*d*e^2)))/((d + e*x)^4*(d - e*x)^4) - ((d^2 - e^2
*x^2)^(1/2)*(5/(144*d^2*e^3) + (131*x)/(5040*d^3*e^2)))/((d + e*x)^3*(d -
e*x)^3) - (8*x*(d^2 - e^2*x^2)^(1/2))/(315*d^5*e^2*(d + e*x)^2*(d - e*x)^2
) - (16*x*(d^2 - e^2*x^2)^(1/2))/(315*d^7*e^2*(d + e*x)*(d - e*x))
```

3.32 $\int \frac{x^2(1-ax)}{(1-a^2x^2)^{3/2}}\,dx$

3.32.1 Optimal result

Integrand size = 24, antiderivative size = 54

$$\int \frac{x^2(1-ax)}{(1-a^2x^2)^{3/2}}\,dx = -\frac{1-ax}{a^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{a^3} - \frac{\arcsin(ax)}{a^3}$$

output
```
-arcsin(a*x)/a^3+(a*x-1)/a^3/(-a^2*x^2+1)^(1/2)-(-a^2*x^2+1)^(1/2)/a^3
```

3.32.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \frac{x^2(1-ax)}{(1-a^2x^2)^{3/2}}\,dx = \frac{(-2-ax)\sqrt{1-a^2x^2}}{a^3(1+ax)} - \frac{2\arctan\left(\frac{ax}{-1+\sqrt{1-a^2x^2}}\right)}{a^3}$$

input
```
Integrate[(x^2*(1 - a*x))/(1 - a^2*x^2)^(3/2),x]
```

output
```
((-2 - a*x)*Sqrt[1 - a^2*x^2])/(a^3*(1 + a*x)) - (2*ArcTan[(a*x)/(-1 + Sqr
t[1 - a^2*x^2])])/a^3
```

3.32.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {527, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(1-ax)}{(1-a^2x^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{527}$$

$$-\frac{\int \frac{1-ax}{\sqrt{1-a^2x^2}}dx}{a^2}-\frac{1-ax}{a^3\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{455}$$

$$-\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}}{a}}{a^2}-\frac{1-ax}{a^3\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{\frac{\sqrt{1-a^2x^2}}{a}+\frac{\arcsin(ax)}{a}}{a^2}-\frac{1-ax}{a^3\sqrt{1-a^2x^2}}$$

input
```
Int[(x^2*(1 - a*x))/(1 - a^2*x^2)^(3/2),x]
```

output
```
-((1 - a*x)/(a^3*Sqrt[1 - a^2*x^2])) - (Sqrt[1 - a^2*x^2]/a + ArcSin[a*x]/
a)/a^2
```

3.32.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 527
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_S
ymbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a +
b*x^2])), x] + Simp[1/(b*d^(m - 2))   Int[(1/Sqrt[a + b*x^2])*ExpandToSum[(
2^(n - 1)*c^(m + n - 1) - d^m*x^m*(c + d*x)^(n - 1))/(c - d*x), x], x], x]
/; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && EqQ[b*c^2 + a*d^2,
 0]
```

3.32.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.67

method	result	size
default	$-a\left(-\frac{x^{2}}{a^{2}\sqrt{-a^{2}x^{2}+1}}+\frac{2}{a^{4}\sqrt{-a^{2}x^{2}+1}}\right)+\frac{x}{a^{2}\sqrt{-a^{2}x^{2}+1}}-\frac{\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{a^{2}\sqrt{a^{2}}}$	90
risch	$\frac{a^{2}x^{2}-1}{a^{3}\sqrt{-a^{2}x^{2}+1}}-\frac{\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{a^{2}\sqrt{a^{2}}}-\frac{\sqrt{-\left(x+\frac{1}{a}\right)^{2}a^{2}+2\left(x+\frac{1}{a}\right)a}}{a^{4}\left(x+\frac{1}{a}\right)}$	92
meijerg	$-\frac{-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(-4a^{2}x^{2}+8\right)}{4\sqrt{-a^{2}x^{2}+1}}}{a^{3}\sqrt{\pi}}-\frac{\frac{\sqrt{\pi}\,x\left(-a^{2}\right)^{\frac{3}{2}}}{a^{2}\sqrt{-a^{2}x^{2}+1}}-\frac{\sqrt{\pi}\left(-a^{2}\right)^{\frac{3}{2}}\arcsin\left(ax\right)}{a^{3}}}{a^{2}\sqrt{\pi}\,\sqrt{-a^{2}}}$	105

input
```
int(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-a*(-x^2/a^2/(-a^2*x^2+1)^(1/2)+2/a^4/(-a^2*x^2+1)^(1/2))+x/a^2/(-a^2*x^2+
1)^(1/2)-1/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

$$\int \frac{x^2(1-ax)}{(1-a^2x^2)^{3/2}}\,dx = -\frac{2\,ax-2\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}(ax+2)+2}{a^4x+a^3}$$

input
```
integrate(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
-(2*a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x
^2 + 1)*(a*x + 2) + 2)/(a^4*x + a^3)
```

3.32.6 Sympy [A] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.20

$$\int \frac{x^2(1-ax)}{(1-a^2x^2)^{3/2}}\,dx = -a\left(\begin{cases} \frac{a^2x^2\sqrt{-a^2x^2+1}}{a^6x^2-a^4} - \frac{2\sqrt{-a^2x^2+1}}{a^6x^2-a^4} & \text{for } a \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{ix}{a^2\sqrt{a^2x^2-1}} + \frac{i\operatorname{acosh}(ax)}{a^3} & \text{for } |a^2x^2| > 1 \\ \frac{x}{a^2\sqrt{-a^2x^2+1}} - \frac{\operatorname{asin}(ax)}{a^3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(-a*x+1)/(-a**2*x**2+1)**(3/2),x)
```

output
```
-a*Piecewise((a**2*x**2*sqrt(-a**2*x**2 + 1)/(a**6*x**2 - a**4) - 2*sqrt(-
a**2*x**2 + 1)/(a**6*x**2 - a**4), Ne(a, 0)), (x**4/4, True)) + Piecewise(
(-I*x/(a**2*sqrt(a**2*x**2 - 1)) + I*acosh(a*x)/a**3, Abs(a**2*x**2) > 1),
 (x/(a**2*sqrt(-a**2*x**2 + 1)) - asin(a*x)/a**3, True))
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int \frac{x^2(1-ax)}{(1-a^2x^2)^{3/2}}\,dx = \frac{x^2}{\sqrt{-a^2x^2+1}a} + \frac{x}{\sqrt{-a^2x^2+1}a^2} - \frac{\arcsin(ax)}{a^3} - \frac{2}{\sqrt{-a^2x^2+1}a^3}$$

input
```
integrate(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
x^2/(sqrt(-a^2*x^2 + 1)*a) + x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a*x)/a^3
- 2/(sqrt(-a^2*x^2 + 1)*a^3)
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{x^2(1-ax)}{(1-a^2x^2)^{3/2}}\,dx = -\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{a^2|a|} - \frac{\sqrt{-a^2x^2+1}}{a^3} + \frac{2}{a^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

input
```
integrate(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
-arcsin(a*x)*sgn(a)/(a^2*abs(a)) - sqrt(-a^2*x^2 + 1)/a^3 + 2/(a^2*((sqrt(
-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

3.32.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.56

$$\int \frac{x^2(1-ax)}{(1-a^2x^2)^{3/2}}\,dx = \frac{\sqrt{1-a^2\,x^2}}{\left(a\,\sqrt{-a^2}+a^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}} - \frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{a^2\,\sqrt{-a^2}} - \frac{\sqrt{1-a^2\,x^2}}{a^3}$$

input
```
int(-(x^2*(a*x - 1))/(1 - a^2*x^2)^(3/2),x)
```

output
```
(1 - a^2*x^2)^(1/2)/((a*(-a^2)^(1/2) + a^2*x*(-a^2)^(1/2))*(-a^2)^(1/2)) -
 asinh(x*(-a^2)^(1/2))/(a^2*(-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/a^3
```

3.33 $\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

3.33.1 Optimal result

Integrand size = 27, antiderivative size = 173

$$\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3}-\frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2}-\frac{2dx^4\sqrt{d^2-e^2x^2}}{5e}-\frac{1}{6}x^5\sqrt{d^2-e^2x^2}-\frac{d^4(256d+165ex)\sqrt{d^2-e^2x^2}}{240e^5}+\frac{11d^6\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^5}$$

output
```
11/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5-8/15*d^3*x^2*(-e^2*x^2+d^2)
^(1/2)/e^3-11/24*d^2*x^3*(-e^2*x^2+d^2)^(1/2)/e^2-2/5*d*x^4*(-e^2*x^2+d^2)
^(1/2)/e-1/6*x^5*(-e^2*x^2+d^2)^(1/2)-1/240*d^4*(165*e*x+256*d)*(-e^2*x^2+
d^2)^(1/2)/e^5
```

3.33.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.66

$$\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}(256d^5+165d^4ex+128d^3e^2x^2+110d^2e^3x^3+96de^4x^4+40e^5x^5)+330d^6\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-\cdots}}\right.}{240e^5}$$

input
```
Integrate[(x^4*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/240*(Sqrt[d^2 - e^2*x^2]*(256*d^5 + 165*d^4*e*x + 128*d^3*e^2*x^2 + 110
*d^2*e^3*x^3 + 96*d*e^4*x^4 + 40*e^5*x^5) + 330*d^6*ArcTan[(e*x)/(Sqrt[d^2
] - Sqrt[d^2 - e^2*x^2])])/e^5
```

3.33.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.24, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.519, Rules used = {541, 25, 27, 533, 27, 533, 27, 533, 27, 533, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{541}$$

$$-\frac{\int -\frac{de^2x^4(11d+12ex)}{\sqrt{d^2-e^2x^2}}dx}{6e^2}-\frac{1}{6}x^5\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{de^2x^4(11d+12ex)}{\sqrt{d^2-e^2x^2}}dx}{6e^2}-\frac{1}{6}x^5\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}d\int \frac{x^4(11d+12ex)}{\sqrt{d^2-e^2x^2}}dx-\frac{1}{6}x^5\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{6}d\left(\frac{\int \frac{dex^3(48d+55ex)}{\sqrt{d^2-e^2x^2}}dx}{5e^2}-\frac{12x^4\sqrt{d^2-e^2x^2}}{5e}\right)-\frac{1}{6}x^5\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}d\left(\frac{d\int \frac{x^3(48d+55ex)}{\sqrt{d^2-e^2x^2}}dx}{5e}-\frac{12x^4\sqrt{d^2-e^2x^2}}{5e}\right)-\frac{1}{6}x^5\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{6}d\left(\frac{d\left(\frac{\int \frac{3dex^2(55d+64ex)}{\sqrt{d^2-e^2x^2}}dx}{4e^2}-\frac{55x^3\sqrt{d^2-e^2x^2}}{4e}\right)}{5e}-\frac{12x^4\sqrt{d^2-e^2x^2}}{5e}\right)-\frac{1}{6}x^5\sqrt{d^2-e^2x^2}$$

$\downarrow$ 27

$$\frac{1}{6}d\left(\frac{d\left(\frac{3d\int \frac{x^2(55d+64ex)}{\sqrt{d^2-e^2x^2}}dx}{4e}-\frac{55x^3\sqrt{d^2-e^2x^2}}{4e}\right)}{5e}-\frac{12x^4\sqrt{d^2-e^2x^2}}{5e}\right)-\frac{1}{6}x^5\sqrt{d^2-e^2x^2}$$

$\downarrow$ 533

$$\frac{1}{6}d\left(\frac{d\left(\frac{3d\left(\frac{\int \frac{dex(128d+165ex)}{\sqrt{d^2-e^2x^2}}dx}{3e^2}-\frac{64x^2\sqrt{d^2-e^2x^2}}{3e}\right)}{4e}-\frac{55x^3\sqrt{d^2-e^2x^2}}{4e}\right)}{5e}-\frac{12x^4\sqrt{d^2-e^2x^2}}{5e}\right)-\frac{1}{6}x^5\sqrt{d^2-e^2x^2}$$

$\downarrow$ 27

$$\frac{1}{6}d\left(\frac{d\left(\frac{3d\left(\frac{d\int \frac{x(128d+165ex)}{\sqrt{d^2-e^2x^2}}dx}{3e}-\frac{64x^2\sqrt{d^2-e^2x^2}}{3e}\right)}{4e}-\frac{55x^3\sqrt{d^2-e^2x^2}}{4e}\right)}{5e}-\frac{12x^4\sqrt{d^2-e^2x^2}}{5e}\right)-\frac{1}{6}x^5\sqrt{d^2-e^2x^2}$$

$\downarrow$ 533

3.33. $\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

$$\frac{1}{6}d\left(\frac{d\left(\frac{3d\left(\frac{d\left(\frac{\int\frac{de(165d+256ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{165x\sqrt{d^2-e^2x^2}}{2e}\right)}{3e}-\frac{64x^2\sqrt{d^2-e^2x^2}}{3e}\right)}{4e}-\frac{55x^3\sqrt{d^2-e^2x^2}}{4e}\right)}{5e}-\frac{12x^4\sqrt{d^2-e^2x^2}}{5e}\right)-\frac{1}{6}x^5\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{6}d\left(\frac{d\left(\frac{3d\left(\frac{d\left(\frac{d\int\frac{165d+256ex}{\sqrt{d^2-e^2x^2}}dx}{2e}-\frac{165x\sqrt{d^2-e^2x^2}}{2e}\right)}{3e}-\frac{64x^2\sqrt{d^2-e^2x^2}}{3e}\right)}{4e}-\frac{55x^3\sqrt{d^2-e^2x^2}}{4e}\right)}{5e}-\frac{12x^4\sqrt{d^2-e^2x^2}}{5e}\right)-\frac{1}{6}x^5\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \; 455$$

3.33. $\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

$$\frac{1}{6}d\left(\frac{d\left(\frac{3d\left(\frac{d\left(\frac{d\left(165d\int\frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{256\sqrt{d^2-e^2x^2}}{e}\right)}{2e}-\frac{165x\sqrt{d^2-e^2x^2}}{2e}\right)}{3e}-\frac{64x^2\sqrt{d^2-e^2x^2}}{3e}\right)}{4e}-\frac{55x^3\sqrt{d^2-e^2x^2}}{4e}\right)}{5e}-\frac{12x^4\sqrt{d^2-e^2x^2}}{5e}\right)$$

$$\frac{1}{6}x^5\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \; 224$$

3.33. $\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

$$\frac{1}{6}d\left(\frac{d\left(\frac{3d\left(\frac{d\left(\frac{d\left(165d\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{256\sqrt{d^2-e^2x^2}}{e}\right)}{2e}-\frac{165x\sqrt{d^2-e^2x^2}}{2e}\right)}{3e}-\frac{64x^2\sqrt{d^2-e^2x^2}}{3e}\right)}{4e}-\frac{55x^3\sqrt{d^2-e^2x^2}}{4e}\right)}{5e}-\frac{12x^4\sqrt{d^2}}{5e}\right.$$

$$\frac{1}{6}x^5\sqrt{d^2-e^2x^2}$$

$\downarrow$ 216

3.33. $\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

$$\frac{1}{6}d\left(\frac{d\left(\frac{3d\left(\frac{d\left(\frac{d\left(\frac{165d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{256\sqrt{d^2-e^2x^2}}{e}\right)}{2e}-\frac{165x\sqrt{d^2-e^2x^2}}{2e}\right)}{3e}-\frac{64x^2\sqrt{d^2-e^2x^2}}{3e}\right)}{4e}-\frac{55x^3\sqrt{d^2-e^2x^2}}{4e}\right)}{5e}-\frac{12x^4\sqrt{d^2-e^2}}{5e}\right.$$

$$\frac{1}{6}x^5\sqrt{d^2-e^2x^2}$$

input
```
Int[(x^4*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

3.33. $\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

output
```
-1/6*(x^5*Sqrt[d^2 - e^2*x^2]) + (d*((-12*x^4*Sqrt[d^2 - e^2*x^2])/(5*e) +
 (d*((-55*x^3*Sqrt[d^2 - e^2*x^2])/(4*e) + (3*d*((-64*x^2*Sqrt[d^2 - e^2*x
^2])/(3*e) + (d*((-165*x*Sqrt[d^2 - e^2*x^2])/(2*e) + (d*((-256*Sqrt[d^2 -
 e^2*x^2])/e + (165*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/(2*e)))/(3*e)
))/(4*e)))/(5*e)))/6
```

3.33.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.33. $\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}} \, dx$

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

3.33.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.62

method	result
risch	$-\frac{\left(40e^{5}x^{5}+96d\,e^{4}x^{4}+110d^{2}e^{3}x^{3}+128d^{3}e^{2}x^{2}+165d^{4}ex+256d^{5}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{240e^{5}}+\frac{11d^{6}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{16e^{4}\sqrt{e^{2}}}$
default	$e^{2}\left(-\frac{x^{5}\sqrt{-e^{2}x^{2}+d^{2}}}{6e^{2}}+\frac{5d^{2}\left(-\frac{x^{3}\sqrt{-e^{2}x^{2}+d^{2}}}{4e^{2}}+\frac{3d^{2}\left(-\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2e^{2}}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2e^{2}\sqrt{e^{2}}}\right)}{4e^{2}}\right)}{6e^{2}}\right)+d^{2}\left(-\frac{x^{3}\sqrt{-e^{2}x^{2}+d^{2}}}{4e^{2}}\right.$

input
```
int(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/240*(40*e^5*x^5+96*d*e^4*x^4+110*d^2*e^3*x^3+128*d^3*e^2*x^2+165*d^4*e*
x+256*d^5)/e^5*(-e^2*x^2+d^2)^(1/2)+11/16*d^6/e^4/(e^2)^(1/2)*arctan((e^2)
^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.61

$$\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx =$$
$$-\frac{330\,d^{6}\arctan\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{ex}\right)+\left(40\,e^{5}x^{5}+96\,de^{4}x^{4}+110\,d^{2}e^{3}x^{3}+128\,d^{3}e^{2}x^{2}+165\,d^{4}ex+256\,d^{5}\right)\sqrt{}}{240\,e^{5}}$$

input
```
integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/240*(330*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (40*e^5*x^5 +
96*d*e^4*x^4 + 110*d^2*e^3*x^3 + 128*d^3*e^2*x^2 + 165*d^4*e*x + 256*d^5)*
sqrt(-e^2*x^2 + d^2))/e^5
```

3.33.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05

$$\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \begin{cases} \frac{11d^6\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{16e^4} + \sqrt{d^2-e^2x^2}\left(-\frac{16d^5}{15e^5}-\frac{11d^4x}{16e^4}-\frac{8d^3x^2}{15e^3}-\frac{11d^2x^3}{24e^2}-\frac{2dx^4}{5e}-\frac{x^5}{6}\right. \\ \frac{\frac{d^2x^5}{5}+\frac{dex^6}{3}+\frac{e^2x^7}{7}}{\sqrt{d^2}} \end{cases}$$

input
```
integrate(x**4*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Piecewise((11*d**6*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e*
*2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(1
6*e**4) + sqrt(d**2 - e**2*x**2)*(-16*d**5/(15*e**5) - 11*d**4*x/(16*e**4)
 - 8*d**3*x**2/(15*e**3) - 11*d**2*x**3/(24*e**2) - 2*d*x**4/(5*e) - x**5/
6), Ne(e**2, 0)), ((d**2*x**5/5 + d*e*x**6/3 + e**2*x**7/7)/sqrt(d**2), Tr
ue))
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.95

$$\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{6}\sqrt{-e^2x^2+d^2}x^5 - \frac{2\sqrt{-e^2x^2+d^2}dx^4}{5\,e} - \frac{11\sqrt{-e^2x^2+d^2}d^2x^3}{24\,e^2} - \frac{8\sqrt{-e^2x^2+d^2}d^3x^2}{15\,e^3} + \frac{11\,d^6\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{16\sqrt{e^2}e^4} - \frac{11\sqrt{-e^2x^2+d^2}d^4x}{16\,e^4} - \frac{16\sqrt{-e^2x^2+d^2}d^5}{15\,e^5}$$

input
```
integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*sqrt(-e^2*x^2 + d^2)*x^5 - 2/5*sqrt(-e^2*x^2 + d^2)*d*x^4/e - 11/24*s
qrt(-e^2*x^2 + d^2)*d^2*x^3/e^2 - 8/15*sqrt(-e^2*x^2 + d^2)*d^3*x^2/e^3 +
11/16*d^6*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^4) - 11/16*sqrt(-e^2*x^
2 + d^2)*d^4*x/e^4 - 16/15*sqrt(-e^2*x^2 + d^2)*d^5/e^5
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.55

$$\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \frac{11\,d^6\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{16\,e^4|e|} - \frac{1}{240}\sqrt{-e^2x^2+d^2}\left(\left(2\left(\left(4\left(5\,x+\frac{12\,d}{e}\right)x+\frac{55\,d^2}{e^2}\right)x+\frac{64\,d^3}{e^3}\right)x+\frac{165\,d^4}{e^4}\right)x+\frac{256\,d^5}{e^5}\right)$$

input
```
integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
11/16*d^6*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^4*abs(e)) - 1/240*sqrt(-e^2*x^2 +
 d^2)*((2*((4*(5*x + 12*d/e)*x + 55*d^2/e^2)*x + 64*d^3/e^3)*x + 165*d^4/e
^4)*x + 256*d^5/e^5)
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^4\,{\left(d+e\,x\right)}^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

input
```
int((x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2),x)
```

output
```
int((x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2), x)
```

3.34 $\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}} \, dx$

3.34.1 Optimal result

Integrand size = 27, antiderivative size = 144

$$\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}} \, dx = -\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4} + \frac{3d^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^4}$$

output
```
3/4*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4-3/5*d^2*x^2*(-e^2*x^2+d^2)^(1
/2)/e^2-1/2*d*x^3*(-e^2*x^2+d^2)^(1/2)/e-1/5*x^4*(-e^2*x^2+d^2)^(1/2)-3/20
*d^3*(5*e*x+8*d)*(-e^2*x^2+d^2)^(1/2)/e^4
```

3.34.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.72

$$\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}} \, dx = -\frac{\sqrt{d^2-e^2x^2}(24d^4+15d^3ex+12d^2e^2x^2+10de^3x^3+4e^4x^4)+30d^5\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{20e^4}$$

input
```
Integrate[(x^3*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/20*(Sqrt[d^2 - e^2*x^2]*(24*d^4 + 15*d^3*e*x + 12*d^2*e^2*x^2 + 10*d*e^
3*x^3 + 4*e^4*x^4) + 30*d^5*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])
])/e^4
```

3.34.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {541, 25, 27, 533, 27, 533, 27, 533, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{541}$$

$$-\frac{\int -\frac{de^2x^3(9d+10ex)}{\sqrt{d^2-e^2x^2}}dx}{5e^2}-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{de^2x^3(9d+10ex)}{\sqrt{d^2-e^2x^2}}dx}{5e^2}-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}d\int \frac{x^3(9d+10ex)}{\sqrt{d^2-e^2x^2}}dx-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{5}d\left(\frac{\int \frac{6dex^2(5d+6ex)}{\sqrt{d^2-e^2x^2}}dx}{4e^2}-\frac{5x^3\sqrt{d^2-e^2x^2}}{2e}\right)-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}d\left(\frac{3d\int \frac{x^2(5d+6ex)}{\sqrt{d^2-e^2x^2}}dx}{2e}-\frac{5x^3\sqrt{d^2-e^2x^2}}{2e}\right)-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{5}d\left(\frac{3d\left(\frac{\int \frac{3dex(4d+5ex)}{\sqrt{d^2-e^2x^2}}dx}{3e^2}-\frac{2x^2\sqrt{d^2-e^2x^2}}{e}\right)}{2e}-\frac{5x^3\sqrt{d^2-e^2x^2}}{2e}\right)-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}d\left(\frac{3d\left(\frac{d\int \frac{x(4d+5ex)}{\sqrt{d^2-e^2x^2}}dx}{e}-\frac{2x^2\sqrt{d^2-e^2x^2}}{e}\right)}{2e}-\frac{5x^3\sqrt{d^2-e^2x^2}}{2e}\right)-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{5}d\left(\frac{3d\left(\frac{d\left(\frac{\int \frac{de(5d+8ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{5x\sqrt{d^2-e^2x^2}}{2e}\right)}{e}-\frac{2x^2\sqrt{d^2-e^2x^2}}{e}\right)}{2e}-\frac{5x^3\sqrt{d^2-e^2x^2}}{2e}\right)-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}d\left(\frac{3d\left(\frac{d\left(\frac{d\int \frac{5d+8ex}{\sqrt{d^2-e^2x^2}}dx}{2e}-\frac{5x\sqrt{d^2-e^2x^2}}{2e}\right)}{e}-\frac{2x^2\sqrt{d^2-e^2x^2}}{e}\right)}{2e}-\frac{5x^3\sqrt{d^2-e^2x^2}}{2e}\right)-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{455}$$

3.34. $\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

$$\frac{1}{5}d\left(\frac{3d\left(\frac{d\left(\frac{d\left(5d\int\frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{8\sqrt{d^2-e^2x^2}}{e}\right)}{2e}-\frac{5x\sqrt{d^2-e^2x^2}}{2e}\right)}{e}-\frac{2x^2\sqrt{d^2-e^2x^2}}{e}\right)}{2e}-\frac{5x^3\sqrt{d^2-e^2x^2}}{2e}\right)-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{5}d\left(\frac{3d\left(\frac{d\left(\frac{d\left(5d\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{8\sqrt{d^2-e^2x^2}}{e}\right)}{2e}-\frac{5x\sqrt{d^2-e^2x^2}}{2e}\right)}{e}-\frac{2x^2\sqrt{d^2-e^2x^2}}{e}\right)}{2e}-\frac{5x^3\sqrt{d^2-e^2x^2}}{2e}\right)-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{216}$$

3.34. $\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

$$\frac{1}{5}d\left(\frac{3d\left(\frac{d\left(\frac{d\left(\frac{5d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{8\sqrt{d^2-e^2x^2}}{e}\right)}{2e}-\frac{5x\sqrt{d^2-e^2x^2}}{2e}\right)}{e}-\frac{2x^2\sqrt{d^2-e^2x^2}}{e}\right)}{2e}-\frac{5x^3\sqrt{d^2-e^2x^2}}{2e}\right)-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}$$

input
```
Int[(x^3*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/5*(x^4*Sqrt[d^2 - e^2*x^2]) + (d*((-5*x^3*Sqrt[d^2 - e^2*x^2])/(2*e) +
(3*d*((-2*x^2*Sqrt[d^2 - e^2*x^2])/e + (d*((-5*x*Sqrt[d^2 - e^2*x^2])/(2*e
) + (d*((-8*Sqrt[d^2 - e^2*x^2])/e + (5*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/e))/(2*e)))/e))/(2*e)))/5
```

3.34.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

3.34.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.67

method	result
risch	$-\frac{\left(4 e^{4} x^{4}+10 d\, e^{3} x^{3}+12 d^{2} e^{2} x^{2}+15 d^{3} e x +24 d^{4}\right) \sqrt{-e^{2} x^{2}+d^{2}}}{20 e^{4}}+\frac{3 d^{5} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{4 e^{3} \sqrt{e^{2}}}$
default	$e^{2} \left(-\frac{x^{4} \sqrt{-e^{2} x^{2}+d^{2}}}{5 e^{2}}+\frac{4 d^{2} \left(-\frac{x^{2} \sqrt{-e^{2} x^{2}+d^{2}}}{3 e^{2}}-\frac{2 d^{2} \sqrt{-e^{2} x^{2}+d^{2}}}{3 e^{4}}\right)}{5 e^{2}}\right)+d^{2} \left(-\frac{x^{2} \sqrt{-e^{2} x^{2}+d^{2}}}{3 e^{2}}-\frac{2 d^{2} \sqrt{-e^{2} x^{2}+d^{2}}}{3 e^{4}}\right)+2 d e$

input
```
int(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*(4*e^4*x^4+10*d*e^3*x^3+12*d^2*e^2*x^2+15*d^3*e*x+24*d^4)/e^4*(-e^2*
x^2+d^2)^(1/2)+3/4*d^5/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)
^(1/2))
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.65

$$\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(4\,e^4x^4+10\,de^3x^3+12\,d^2e^2x^2+15\,d^3ex+24\,d^4\right)\sqrt{-e^2x^2+d^2}}{20\,e^4}$$

input
```
integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/20*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (4*e^4*x^4 + 10*
d*e^3*x^3 + 12*d^2*e^2*x^2 + 15*d^3*e*x + 24*d^4)*sqrt(-e^2*x^2 + d^2))/e^
4
```

3.34.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.17

$$\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \begin{cases} \frac{3d^5\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{4e^3}+\sqrt{d^2-e^2x^2}\left(-\frac{6d^4}{5e^4}-\frac{3d^3x}{4e^3}-\frac{3d^2x^2}{5e^2}-\frac{dx^3}{2e}-\frac{x^4}{5}\right) & \text{for } e^2\neq \\ \frac{\frac{d^2x^4}{4}+\frac{2dex^5}{5}+\frac{e^2x^6}{6}}{\sqrt{d^2}} & \text{otherwis} \end{cases}$$

input
```
integrate(x**3*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Piecewise((3*d**5*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**
2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(4*
e**3) + sqrt(d**2 - e**2*x**2)*(-6*d**4/(5*e**4) - 3*d**3*x/(4*e**3) - 3*d
**2*x**2/(5*e**2) - d*x**3/(2*e) - x**4/5), Ne(e**2, 0)), ((d**2*x**4/4 +
2*d*e*x**5/5 + e**2*x**6/6)/sqrt(d**2), True))
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.97

$$\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{5}\sqrt{-e^2x^2+d^2}x^4 - \frac{\sqrt{-e^2x^2+d^2}dx^3}{2\,e} - \frac{3\sqrt{-e^2x^2+d^2}d^2x^2}{5\,e^2} + \frac{3\,d^5\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{4\sqrt{e^2}e^3} - \frac{3\sqrt{-e^2x^2+d^2}d^3x}{4\,e^3} - \frac{6\sqrt{-e^2x^2+d^2}d^4}{5\,e^4}$$

input
```
integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/5*sqrt(-e^2*x^2 + d^2)*x^4 - 1/2*sqrt(-e^2*x^2 + d^2)*d*x^3/e - 3/5*sqr
t(-e^2*x^2 + d^2)*d^2*x^2/e^2 + 3/4*d^5*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(
e^2)*e^3) - 3/4*sqrt(-e^2*x^2 + d^2)*d^3*x/e^3 - 6/5*sqrt(-e^2*x^2 + d^2)*
d^4/e^4
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.58

$$\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \frac{3\,d^5\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{4\,e^3|e|} - \frac{1}{20}\sqrt{-e^2x^2+d^2}\left(\left(2\left(\left(2\,x+\frac{5\,d}{e}\right)x+\frac{6\,d^2}{e^2}\right)x+\frac{15\,d^3}{e^3}\right)x+\frac{24\,d^4}{e^4}\right)$$

input
```
integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
3/4*d^5*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^3*abs(e)) - 1/20*sqrt(-e^2*x^2 + d^
2)*((2*((2*x + 5*d/e)*x + 6*d^2/e^2)*x + 15*d^3/e^3)*x + 24*d^4/e^4)
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx=\int \frac{x^3\,{\left(d+e\,x\right)}^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

input
```
int((x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2),x)
```

output
```
int((x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2), x)
```

3.35 $\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

3.35.1 Optimal result

Integrand size = 27, antiderivative size = 115

$$\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e} - \frac{1}{4}x^3\sqrt{d^2-e^2x^2} - \frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3} + \frac{7d^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

output
```
7/8*d^4*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-2/3*d*x^2*(-e^2*x^2+d^2)^(1/2
)/e-1/4*x^3*(-e^2*x^2+d^2)^(1/2)-1/24*d^2*(21*e*x+32*d)*(-e^2*x^2+d^2)^(1/
2)/e^3
```

3.35.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.80

$$\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}(32d^3+21d^2ex+16de^2x^2+6e^3x^3)+42d^4\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{24e^3}$$

input
```
Integrate[(x^2*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/24*(Sqrt[d^2 - e^2*x^2]*(32*d^3 + 21*d^2*e*x + 16*d*e^2*x^2 + 6*e^3*x^3
) + 42*d^4*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^3
```

3.35.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.26, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {541, 25, 27, 533, 27, 533, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{ 541}$$

$$-\frac{\int -\frac{de^2x^2(7d+8ex)}{\sqrt{d^2-e^2x^2}}dx}{4e^2}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{de^2x^2(7d+8ex)}{\sqrt{d^2-e^2x^2}}dx}{4e^2}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}d\int \frac{x^2(7d+8ex)}{\sqrt{d^2-e^2x^2}}dx-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{ 533}$$

$$\frac{1}{4}d\left(\frac{\int \frac{dex(16d+21ex)}{\sqrt{d^2-e^2x^2}}dx}{3e^2}-\frac{8x^2\sqrt{d^2-e^2x^2}}{3e}\right)-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}d\left(\frac{d\int \frac{x(16d+21ex)}{\sqrt{d^2-e^2x^2}}dx}{3e}-\frac{8x^2\sqrt{d^2-e^2x^2}}{3e}\right)-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{ 533}$$

$$\frac{1}{4}d\left(\frac{d\left(\frac{\int \frac{de(21d+32ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{21x\sqrt{d^2-e^2x^2}}{2e}\right)}{3e}-\frac{8x^2\sqrt{d^2-e^2x^2}}{3e}\right)-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}d\left(\frac{d\left(\frac{d\int \frac{21d+32ex}{\sqrt{d^2-e^2x^2}}dx}{2e}-\frac{21x\sqrt{d^2-e^2x^2}}{2e}\right)}{3e}-\frac{8x^2\sqrt{d^2-e^2x^2}}{3e}\right)-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}$$

$\Big\downarrow$ 455

$$\frac{1}{4}d\left(\frac{d\left(\frac{d\left(21d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{32\sqrt{d^2-e^2x^2}}{e}\right)}{2e}-\frac{21x\sqrt{d^2-e^2x^2}}{2e}\right)}{3e}-\frac{8x^2\sqrt{d^2-e^2x^2}}{3e}\right)-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}$$

$\Big\downarrow$ 224

$$\frac{1}{4}d\left(\frac{d\left(\frac{d\left(21d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{32\sqrt{d^2-e^2x^2}}{e}\right)}{2e}-\frac{21x\sqrt{d^2-e^2x^2}}{2e}\right)}{3e}-\frac{8x^2\sqrt{d^2-e^2x^2}}{3e}\right)-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}$$

$\Big\downarrow$ 216

$$\frac{1}{4}d\left(\frac{d\left(\frac{d\left(\frac{21d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{32\sqrt{d^2-e^2x^2}}{e}\right)}{2e}-\frac{21x\sqrt{d^2-e^2x^2}}{2e}\right)}{3e}-\frac{8x^2\sqrt{d^2-e^2x^2}}{3e}\right)-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}$$

input
```
Int[(x^2*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/4*(x^3*Sqrt[d^2 - e^2*x^2]) + (d*((-8*x^2*Sqrt[d^2 - e^2*x^2])/(3*e) +
(d*((-21*x*Sqrt[d^2 - e^2*x^2])/(2*e) + (d*((-32*Sqrt[d^2 - e^2*x^2])/e +
(21*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/(2*e)))/(3*e)))/4
```

3.35. $\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

3.35.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

3.35. $\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

3.35.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{(6e^3x^3+16d\,e^2x^2+21d^2ex+32d^3)\sqrt{-e^2x^2+d^2}}{24e^3}+\frac{7d^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8e^2\sqrt{e^2}}$
default	$e^2\left(-\frac{x^3\sqrt{-e^2x^2+d^2}}{4e^2}+\frac{3d^2\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}\right)}{4e^2}\right)+d^2\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+}}\right.}{2e^2\sqrt{e^2}}\right.$

input
```
int(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(6*e^3*x^3+16*d*e^2*x^2+21*d^2*e*x+32*d^3)/e^3*(-e^2*x^2+d^2)^(1/2)+
7/8*d^4/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{42\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(6\,e^3x^3+16\,de^2x^2+21\,d^2ex+32\,d^3)\sqrt{-e^2x^2+d^2}}{24\,e^3}$$

input
```
integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(42*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (6*e^3*x^3 + 16*
d*e^2*x^2 + 21*d^2*e*x + 32*d^3)*sqrt(-e^2*x^2 + d^2))/e^3
```

3.35.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.35

$$\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \begin{cases} \frac{7d^4\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{8e^2} + \sqrt{d^2-e^2x^2}\left(-\frac{4d^3}{3e^3}-\frac{7d^2x}{8e^2}-\frac{2dx^2}{3e}-\frac{x^3}{4}\right) & \text{for } e^2\neq 0 \\ \frac{\frac{d^2x^3}{3}+\frac{dex^4}{2}+\frac{e^2x^5}{5}}{\sqrt{d^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Piecewise((7*d**4*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**
2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(8*
e**2) + sqrt(d**2 - e**2*x**2)*(-4*d**3/(3*e**3) - 7*d**2*x/(8*e**2) - 2*d
*x**2/(3*e) - x**3/4), Ne(e**2, 0)), ((d**2*x**3/3 + d*e*x**4/2 + e**2*x**
5/5)/sqrt(d**2), True))
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{4}\sqrt{-e^2x^2+d^2}x^3 - \frac{2\sqrt{-e^2x^2+d^2}dx^2}{3e} + \frac{7d^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\sqrt{e^2}e^2} - \frac{7\sqrt{-e^2x^2+d^2}d^2x}{8e^2} - \frac{4\sqrt{-e^2x^2+d^2}d^3}{3e^3}$$

input
```
integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(-e^2*x^2 + d^2)*x^3 - 2/3*sqrt(-e^2*x^2 + d^2)*d*x^2/e + 7/8*d^4
*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) - 7/8*sqrt(-e^2*x^2 + d^2)*d^
2*x/e^2 - 4/3*sqrt(-e^2*x^2 + d^2)*d^3/e^3
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.63

$$\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \frac{7\,d^4 \arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,e^2|e|} - \frac{1}{24}\sqrt{-e^2x^2+d^2}\left(\left(2\left(3\,x+\frac{8\,d}{e}\right)x+\frac{21\,d^2}{e^2}\right)x+\frac{32\,d^3}{e^3}\right)$$

input
```
integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
7/8*d^4*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) - 1/24*sqrt(-e^2*x^2 + d^
2)*((2*(3*x + 8*d/e)*x + 21*d^2/e^2)*x + 32*d^3/e^3)
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^2\,{(d+e\,x)}^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

input
```
int((x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2),x)
```

output
```
int((x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2), x)
```

3.36 $\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

3.36.1 Optimal result

Integrand size = 25, antiderivative size = 83

$$\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{3}x^2\sqrt{d^2-e^2x^2} - \frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2} + \frac{d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

output
```
d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2-1/3*x^2*(-e^2*x^2+d^2)^(1/2)-1/3*
d*(3*e*x+5*d)*(-e^2*x^2+d^2)^(1/2)/e^2
```

3.36.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}(5d^2+3dex+e^2x^2)+6d^3\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{3e^2}$$

input
```
Integrate[(x*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/3*(Sqrt[d^2 - e^2*x^2]*(5*d^2 + 3*d*e*x + e^2*x^2) + 6*d^3*ArcTan[(e*x)
/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^2
```

3.36.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {541, 25, 27, 533, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$$

$$\Big\downarrow \text{541}$$

$$-\frac{\int -\frac{de^2x(5d+6ex)}{\sqrt{d^2-e^2x^2}}dx}{3e^2}-\frac{1}{3}x^2\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{de^2x(5d+6ex)}{\sqrt{d^2-e^2x^2}}dx}{3e^2}-\frac{1}{3}x^2\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}d\int \frac{x(5d+6ex)}{\sqrt{d^2-e^2x^2}}dx-\frac{1}{3}x^2\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{3}d\left(\frac{\int \frac{2de(3d+5ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{3x\sqrt{d^2-e^2x^2}}{e}\right)-\frac{1}{3}x^2\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}d\left(\frac{d\int \frac{3d+5ex}{\sqrt{d^2-e^2x^2}}dx}{e}-\frac{3x\sqrt{d^2-e^2x^2}}{e}\right)-\frac{1}{3}x^2\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{3}d\left(\frac{d\left(3d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{5\sqrt{d^2-e^2x^2}}{e}\right)}{e}-\frac{3x\sqrt{d^2-e^2x^2}}{e}\right)-\frac{1}{3}x^2\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{3}d\left(\frac{d\left(3d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{5\sqrt{d^2-e^2x^2}}{e}\right)}{e}-\frac{3x\sqrt{d^2-e^2x^2}}{e}\right)-\frac{1}{3}x^2\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{3}d\left(\frac{d\left(\frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{5\sqrt{d^2-e^2x^2}}{e}\right)}{e}-\frac{3x\sqrt{d^2-e^2x^2}}{e}\right)-\frac{1}{3}x^2\sqrt{d^2-e^2x^2}$$

input
```
Int[(x*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/3*(x^2*Sqrt[d^2 - e^2*x^2]) + (d*((-3*x*Sqrt[d^2 - e^2*x^2])/e + (d*((-
5*Sqrt[d^2 - e^2*x^2])/e + (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/e))
/3
```

3.36.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

3.36.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

method	result	s
risch	$-\frac{(e^2x^2+3dex+5d^2)\sqrt{-e^2x^2+d^2}}{3e^2}+\frac{d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e\sqrt{e^2}}$	7
default	$e^2\left(-\frac{x^2\sqrt{-e^2x^2+d^2}}{3e^2}-\frac{2d^2\sqrt{-e^2x^2+d^2}}{3e^4}\right)-\frac{d^2\sqrt{-e^2x^2+d^2}}{e^2}+2de\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}\right)$	1

input
```
int(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(e^2*x^2+3*d*e*x+5*d^2)/e^2*(-e^2*x^2+d^2)^(1/2)+d^3/e/(e^2)^(1/2)*ar
ctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.86

$$\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{6\,d^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(e^2x^2+3\,dex+5\,d^2)\sqrt{-e^2x^2+d^2}}{3\,e^2}$$

input
```
integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(6*d^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (e^2*x^2 + 3*d*e*x
 + 5*d^2)*sqrt(-e^2*x^2 + d^2))/e^2
```

3.36.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.61

$$\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \begin{cases} \frac{d^3\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{e} + \sqrt{d^2-e^2x^2}\left(-\frac{5d^2}{3e^2}-\frac{dx}{e}-\frac{x^2}{3}\right) & \text{for } e^2\neq 0 \\ \frac{\frac{d^2x^2}{2}+\frac{2dex^3}{3}+\frac{e^2x^4}{4}}{\sqrt{d^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Piecewise((d**3*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*
x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/e + s
qrt(d**2 - e**2*x**2)*(-5*d**2/(3*e**2) - d*x/e - x**2/3), Ne(e**2, 0)), (
(d**2*x**2/2 + 2*d*e*x**3/3 + e**2*x**4/4)/sqrt(d**2), True))
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07

$$\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{3}\sqrt{-e^2x^2+d^2}x^2 + \frac{d^3\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}e} - \frac{\sqrt{-e^2x^2+d^2}dx}{e} - \frac{5\sqrt{-e^2x^2+d^2}d^2}{3e^2}$$

input
```
integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(-e^2*x^2 + d^2)*x^2 + d^3*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)
*e) - sqrt(-e^2*x^2 + d^2)*d*x/e - 5/3*sqrt(-e^2*x^2 + d^2)*d^2/e^2
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \frac{d^3 \arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e|e|} - \frac{1}{3}\sqrt{-e^2x^2+d^2}\left(\left(x+\frac{3\,d}{e}\right)x+\frac{5\,d^2}{e^2}\right)$$

input
```
integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
d^3*arcsin(e*x/d)*sgn(d)*sgn(e)/(e*abs(e)) - 1/3*sqrt(-e^2*x^2 + d^2)*((x
+ 3*d/e)*x + 5*d^2/e^2)
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x\,{\left(d+e\,x\right)}^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

input
```
int((x*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2),x)
```

output
```
int((x*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2), x)
```

3.37 $\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

3.37.1 Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx=-\frac{3d\sqrt{d^2-e^2x^2}}{2e}-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}+\frac{3d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

output
```
3/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e-3/2*d*(-e^2*x^2+d^2)^(1/2)/e-1/
2*(e*x+d)*(-e^2*x^2+d^2)^(1/2)/e
```

3.37.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx=-\frac{(4d+ex)\sqrt{d^2-e^2x^2}+6d^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{2e}$$

input
```
Integrate[(d + e*x)^2/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/2*((4*d + e*x)*Sqrt[d^2 - e^2*x^2] + 6*d^2*ArcTan[(e*x)/(Sqrt[d^2] - Sq
rt[d^2 - e^2*x^2])])/e
```

3.37.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {469, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{ 469}$$

$$\frac{3}{2}d\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}dx-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{3}{2}d\left(d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{e}\right)-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{3}{2}d\left(d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{e}\right)-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{3}{2}d\left(\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{\sqrt{d^2-e^2x^2}}{e}\right)-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}$$

input
```
Int[(d + e*x)^2/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/2*((d + e*x)*Sqrt[d^2 - e^2*x^2])/e + (3*d*(-(Sqrt[d^2 - e^2*x^2]/e) +
(d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/2
```

3.37.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

3.37.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{(ex+4d)\sqrt{-e^2x^2+d^2}}{2e}+\frac{3d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}$	60
default	$\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+e^2\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}\right)-\frac{2d\sqrt{-e^2x^2+d^2}}{e}$	113

input
```
int((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(e*x+4*d)/e*(-e^2*x^2+d^2)^(1/2)+3/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/
2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{6\,d^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}(ex+4\,d)}{2\,e}$$

input
```
integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(6*d^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + sqrt(-e^2*x^2 + d^
2)*(e*x + 4*d))/e
```

3.37.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \begin{cases} \frac{3d^2\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{2}+\sqrt{d^2-e^2x^2}\left(-\frac{2d}{e}-\frac{x}{2}\right) & \text{for } e^2\neq 0\\ \frac{\begin{cases} d^2x & \text{for } e=0\\ \frac{(d+ex)^3}{3e} & \text{otherwise}\end{cases}}{\sqrt{d^2}} & \text{otherwise}\end{cases}$$

input
```
integrate((e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Piecewise((3*d**2*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**
2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/2 +
 sqrt(d**2 - e**2*x**2)*(-2*d/e - x/2), Ne(e**2, 0)), (Piecewise((d**2*x,
Eq(e, 0)), ((d + e*x)**3/(3*e), True))/sqrt(d**2), True))
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \frac{3\,d^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\,\sqrt{e^2}} - \frac{1}{2}\,\sqrt{-e^2x^2+d^2}x - \frac{2\,\sqrt{-e^2x^2+d^2}d}{e}$$

input
```
integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
3/2*d^2*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 1/2*sqrt(-e^2*x^2 + d^2)*x
 - 2*sqrt(-e^2*x^2 + d^2)*d/e
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \frac{3\,d^2\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,|e|} - \frac{1}{2}\,\sqrt{-e^2x^2+d^2}\left(x+\frac{4\,d}{e}\right)$$

input
```
integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
3/2*d^2*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 1/2*sqrt(-e^2*x^2 + d^2)*(x +
 4*d/e)
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \int \frac{{(d+e\,x)}^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

input
```
int((d + e*x)^2/(d^2 - e^2*x^2)^(1/2),x)
```

output
```
int((d + e*x)^2/(d^2 - e^2*x^2)^(1/2), x)
```

3.38 $\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx$

3.38.1 Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx = -\sqrt{d^2-e^2x^2}+2d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
2*d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-d*arctanh((-e^2*x^2+d^2)^(1/2)/d)-(-e
^2*x^2+d^2)^(1/2)
```

3.38.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.52

$$\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx = -\sqrt{d^2-e^2x^2}-4d\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) -\sqrt{d^2}\log(x)+\sqrt{d^2}\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)$$

input
```
Integrate[(d + e*x)^2/(x*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-Sqrt[d^2 - e^2*x^2] - 4*d*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])]
 - Sqrt[d^2]*Log[x] + Sqrt[d^2]*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]]
```

3.38.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {541, 25, 27, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx$$

$$\Big\downarrow \text{541}$$

$$-\frac{\int -\frac{de^2(d+2ex)}{x\sqrt{d^2-e^2x^2}}dx}{e^2}-\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{de^2(d+2ex)}{x\sqrt{d^2-e^2x^2}}dx}{e^2}-\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{27}$$

$$d\int \frac{d+2ex}{x\sqrt{d^2-e^2x^2}}dx-\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{538}$$

$$d\left(2e\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx\right)-\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{224}$$

$$d\left(d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx+2e\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)-\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{216}$$

$$d\left(d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx+2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{243}$$

$$d\left(\frac{1}{2}d\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2+2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{73}$$

$$d\left(2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{d\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}\right)-\sqrt{d^2-e^2x^2}$$

$\downarrow$ 221

$$d\left(2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)-\sqrt{d^2-e^2x^2}$$

input
```
Int[(d + e*x)^2/(x*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-Sqrt[d^2 - e^2*x^2] + d*(2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - ArcTanh[Sq
rt[d^2 - e^2*x^2]/d])
```

3.38.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

3.38.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.38

method	result	size
default	$-\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}+\frac{2de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}$	91

input
```
int((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^
2)^(1/2))/x)+2*d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx = -4\,d\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + d\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - \sqrt{-e^2x^2+d^2}$$

input
```
integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-4*d*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d*log(-(d - sqrt(-e^2*x^2
 + d^2))/x) - sqrt(-e^2*x^2 + d^2)
```

3.38.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.18

$$\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx = d^2\left(\begin{cases} -\frac{\operatorname{acosh}\left(\frac{d}{ex}\right)}{d} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{i\operatorname{asin}\left(\frac{d}{ex}\right)}{d} & \text{otherwise} \end{cases}\right) + 2de\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \wedge e^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{for } e^2\neq 0 \\ \frac{x}{\sqrt{d^2}} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{for } e^2\neq 0 \\ \frac{x^2}{2\sqrt{d^2}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x+d)**2/x/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
d**2*Piecewise((-acosh(d/(e*x))/d, Abs(d**2/(e**2*x**2)) > 1), (I*asin(d/(
e*x))/d, True)) + 2*d*e*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2
 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0) & Ne(e**2, 0)), (x*log(x)/sqrt(-e*
*2*x**2), Ne(e**2, 0)), (x/sqrt(d**2), True)) + e**2*Piecewise((-sqrt(d**2
 - e**2*x**2)/e**2, Ne(e**2, 0)), (x**2/(2*sqrt(d**2)), True))
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx = \frac{2\,de\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} - d\log\left(\frac{2\,d^2}{|x|} + \frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \sqrt{-e^2x^2+d^2}$$

input
```
integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
2*d*e*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - d*log(2*d^2/abs(x) + 2*sqrt(
-e^2*x^2 + d^2)*d/abs(x)) - sqrt(-e^2*x^2 + d^2)
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx = \frac{2\,de\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} - \frac{de\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{|e|} - \sqrt{-e^2x^2+d^2}$$

input
```
integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
2*d*e*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - d*e*log(1/2*abs(-2*d*e - 2*sqrt
(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) - sqrt(-e^2*x^2 + d^2)
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d+e\,x)^2}{x\,\sqrt{d^2-e^2\,x^2}}\,dx$$

input
```
int((d + e*x)^2/(x*(d^2 - e^2*x^2)^(1/2)),x)
```

output
```
int((d + e*x)^2/(x*(d^2 - e^2*x^2)^(1/2)), x)
```

3.39 $\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx$

3.39.1 Optimal result

Integrand size = 27, antiderivative size = 68

$$\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{x} + e\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - 2e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
e*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-2*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)-(-e
^2*x^2+d^2)^(1/2)/x
```

3.39.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.65

$$\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{x} - 2e\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) - \frac{2\sqrt{d^2}e\log(x)}{d} + \frac{2\sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{d}$$

input
```
Integrate[(d + e*x)^2/(x^2*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-(Sqrt[d^2 - e^2*x^2]/x) - 2*e*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^
2])] - (2*Sqrt[d^2]*e*Log[x])/d + (2*Sqrt[d^2]*e*Log[Sqrt[d^2] - Sqrt[d^2
- e^2*x^2]])/d
```

3.39.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {540, 25, 27, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx$$

$$\Big\downarrow \text{ 540}$$

$$-\frac{\int -\frac{d^2e(2d+ex)}{x\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{\sqrt{d^2-e^2x^2}}{x}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{d^2e(2d+ex)}{x\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{\sqrt{d^2-e^2x^2}}{x}$$

$$\Big\downarrow \text{ 27}$$

$$e\int \frac{2d+ex}{x\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{x}$$

$$\Big\downarrow \text{ 538}$$

$$e\left(e\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+2d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx\right)-\frac{\sqrt{d^2-e^2x^2}}{x}$$

$$\Big\downarrow \text{ 224}$$

$$e\left(2d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx+e\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)-\frac{\sqrt{d^2-e^2x^2}}{x}$$

$$\Big\downarrow \text{ 216}$$

$$e\left(2d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx+\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{\sqrt{d^2-e^2x^2}}{x}$$

$$\Big\downarrow \text{ 243}$$

$$e\left(d\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2+\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{\sqrt{d^2-e^2x^2}}{x}$$

$$\Big\downarrow \text{ 73}$$

$$e\left(\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{2d\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}\right)-\frac{\sqrt{d^2-e^2x^2}}{x}$$

$$\Big\downarrow \ 221$$

$$e\left(\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)-\frac{\sqrt{d^2-e^2x^2}}{x}$$

input
```
Int[(d + e*x)^2/(x^2*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-(Sqrt[d^2 - e^2*x^2]/x) + e*(ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 2*ArcTan
h[Sqrt[d^2 - e^2*x^2]/d])
```

3.39.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

3.39.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.37

method	result	size
default	$\frac{e^{2} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}}-\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{x}-\frac{2 d e \ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}}$	93
risch	$\frac{e^{2} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}}-\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{x}-\frac{2 d e \ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}}$	93

input
```
int((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-(-e^2*x^2+d^2)^
(1/2)/x-2*d*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx$$
$$= -\frac{2\,ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - 2\,ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + \sqrt{-e^2x^2+d^2}}{x}$$

input
```
integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-(2*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 2*e*x*log(-(d - sqrt(-
e^2*x^2 + d^2))/x) + sqrt(-e^2*x^2 + d^2))/x
```

3.39.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.47

$$\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx = d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{d^2} & \text{otherwise} \end{cases}\right) + 2de\left(\begin{cases} -\frac{\operatorname{acosh}\left(\frac{d}{ex}\right)}{d} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{i\operatorname{asin}\left(\frac{d}{ex}\right)}{d} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2 \neq 0 \wedge e^2 \neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{for } e^2 \neq 0 \\ \frac{x}{\sqrt{d^2}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x+d)**2/x**2/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/d**2, Abs(d**2/(e**2*x**2))
> 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/d**2, True)) + 2*d*e*Piecewise((-a
cosh(d/(e*x))/d, Abs(d**2/(e**2*x**2)) > 1), (I*asin(d/(e*x))/d, True)) +
e**2*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt
(-e**2), Ne(d**2, 0) & Ne(e**2, 0)), (x*log(x)/sqrt(-e**2*x**2), Ne(e**2,
0)), (x/sqrt(d**2), True))
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx = \frac{e^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} - 2\,e\log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{\sqrt{-e^2x^2+d^2}}{x}$$

input
```
integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
e^2*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 2*e*log(2*d^2/abs(x) + 2*sqrt(
-e^2*x^2 + d^2)*d/abs(x)) - sqrt(-e^2*x^2 + d^2)/x
```

3.39.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(62) = 124.

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.85

$$\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx = \frac{e^2\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} + \frac{e^4x}{2\left(de+\sqrt{-e^2x^2+d^2}|e|\right)|e|} - \frac{2\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{|e|} - \frac{de+\sqrt{-e^2x^2+d^2}|e|}{2\,x|e|}$$

input
```
integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
e^2*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/2*e^4*x/((d*e + sqrt(-e^2*x^2 +
 d^2)*abs(e))*abs(e)) - 2*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*
abs(e))/(e^2*abs(x)))/abs(e) - 1/2*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(x*
abs(e))
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx$$
$$= \begin{cases} \frac{e^2 \ln\left(x\sqrt{-e^2}+\sqrt{d^2-e^2 x^2}\right)}{\sqrt{-e^2}} - \frac{\sqrt{d^2-e^2 x^2}}{x} - \frac{2 d e \ln\left(\frac{\sqrt{d^2}+\sqrt{d^2-e^2 x^2}}{x}\right)}{\sqrt{d^2}} & \text{if } e^2 < 0 \\ \int \frac{e^2}{\sqrt{d^2-e^2 x^2}} + \frac{d^2}{x^2\sqrt{d^2-e^2 x^2}} + \frac{2 d e}{x\sqrt{d^2-e^2 x^2}}\,dx & \text{if } \neg e^2 < 0 \end{cases}$$

input
```
int((d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(1/2)),x)
```

output
```
piecewise(e^2 < 0, - (d^2 - e^2*x^2)^(1/2)/x + (e^2*log(x*(-e^2)^(1/2) + (
d^2 - e^2*x^2)^(1/2)))/(-e^2)^(1/2) - (2*d*e*log(((d^2)^(1/2) + (d^2 - e^2
*x^2)^(1/2))/x))/(d^2)^(1/2), ~e^2 < 0, int(e^2/(d^2 - e^2*x^2)^(1/2) + d^
2/(x^2*(d^2 - e^2*x^2)^(1/2)) + (2*d*e)/(x*(d^2 - e^2*x^2)^(1/2)), x))
```

3.40 $\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx$

3.40.1 Optimal result

Integrand size = 27, antiderivative size = 80

$$\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{dx}-\frac{3e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

output
```
-3/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d-1/2*(-e^2*x^2+d^2)^(1/2)/x^2-2*
e*(-e^2*x^2+d^2)^(1/2)/d/x
```

3.40.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.45

$$\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{(d+4ex)\sqrt{d^2-e^2x^2}-3e^2x^2\log\left(d\left(-d-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)\right)+3e^2x^2\log\left(d-\sqrt{-e^2}x+\sqrt{d^2-e\right.}}{2dx^2}$$

input
```
Integrate[(d + e*x)^2/(x^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-1/2*((d + 4*e*x)*Sqrt[d^2 - e^2*x^2] - 3*e^2*x^2*Log[d*(-d - Sqrt[-e^2]*x
 + Sqrt[d^2 - e^2*x^2])] + 3*e^2*x^2*Log[d - Sqrt[-e^2]*x + Sqrt[d^2 - e^2
*x^2]])/(d*x^2)
```

3.40.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {540, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{540}$$

$$-\frac{\int -\frac{d^2e(4d+3ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d^2e(4d+3ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}e\int \frac{4d+3ex}{x^2\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{2x^2}$$

$$\Big\downarrow \text{534}$$

$$\frac{1}{2}e\left(3e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{4\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{\sqrt{d^2-e^2x^2}}{2x^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}e\left(\frac{3}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{4\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{\sqrt{d^2-e^2x^2}}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}e\left(-\frac{3\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{4\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{\sqrt{d^2-e^2x^2}}{2x^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}e\left(-\frac{3e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{4\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{\sqrt{d^2-e^2x^2}}{2x^2}$$

input
```
Int[(d + e*x)^2/(x^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-1/2*Sqrt[d^2 - e^2*x^2]/x^2 + (e*((-4*Sqrt[d^2 - e^2*x^2])/(d*x) - (3*e*A
rcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/2
```

3.40.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

3.40.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

method	result	size
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(4ex+d)}{2d\,x^2}-\frac{3e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}$	72
default	$-\frac{e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}+d^2\left(-\frac{\sqrt{-e^2x^2+d^2}}{2d^2x^2}-\frac{e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^2\sqrt{d^2}}\right)-\frac{2e\sqrt{-e^2x^2+d^2}}{dx}$	139

input
```
int((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e^2*x^2+d^2)^(1/2)*(4*e*x+d)/d/x^2-3/2*e^2/(d^2)^(1/2)*ln((2*d^2+2*
(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx=\frac{3\,e^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\sqrt{-e^2x^2+d^2}(4\,ex+d)}{2\,dx^2}$$

input
```
integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(3*e^2*x^2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - sqrt(-e^2*x^2 + d^2)*(
4*e*x + d))/(d*x^2)
```

3.40.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.42 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.68

$$\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx = d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2d^2x} - \frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{i}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie}{2d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d^3} & \text{otherwise} \end{cases}\right) + 2de\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{d^2} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} -\frac{\operatorname{acosh}\left(\frac{d}{ex}\right)}{d} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{i\operatorname{asin}\left(\frac{d}{ex}\right)}{d} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x+d)**2/x**3/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*d**2*x) - e**2*acosh(d/(e
*x))/(2*d**3), Abs(d**2/(e**2*x**2)) > 1), (I/(2*e*x**3*sqrt(-d**2/(e**2*x
**2) + 1)) - I*e/(2*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**2*asin(d/(e
*x))/(2*d**3), True)) + 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/d**
2, Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/d**2, Tru
e)) + e**2*Piecewise((-acosh(d/(e*x))/d, Abs(d**2/(e**2*x**2)) > 1), (I*as
in(d/(e*x))/d, True))
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{3\,e^2\log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{2\,d} - \frac{2\,\sqrt{-e^2x^2+d^2}e}{dx} - \frac{\sqrt{-e^2x^2+d^2}}{2\,x^2}$$

input
```
integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-3/2*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 2*sqrt(-e
^2*x^2 + d^2)*e/(d*x) - 1/2*sqrt(-e^2*x^2 + d^2)/x^2
```

3.40.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(70) = 140$.

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.35

$$\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\left(e^3 + \frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e}{x}\right)e^4x^2}{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d|e|} - \frac{3\,e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,d|e|} - \frac{\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)de|e|}{x} + \frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d|e|}{ex^2}}{8\,d^2e^2}$$

input
```
integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*(e^3 + 8*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e/x)*e^4*x^2/((d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^2*d*abs(e)) - 3/2*e^3*log(1/2*abs(-2*d*e - 2*sqrt
(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d*abs(e)) - 1/8*(8*(d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))*d*e*abs(e)/x + (d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*
d*abs(e)/(e*x^2))/(d^2*e^2)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d+e\,x)^2}{x^3\,\sqrt{d^2-e^2\,x^2}}\,dx$$

input
```
int((d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(1/2)),x)
```

output
```
int((d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(1/2)), x)
```

3.41 $\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx$

3.41.1 Optimal result

Integrand size = 27, antiderivative size = 107

$$\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx=-\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}-\frac{e^3\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

output
```
-e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2-1/3*(-e^2*x^2+d^2)^(1/2)/x^3-e*(-
e^2*x^2+d^2)^(1/2)/d/x^2-5/3*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x
```

3.41.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx =-\frac{\frac{d\sqrt{d^2-e^2x^2}(d^2+3dex+5e^2x^2)}{x^3}+3\sqrt{d^2}e^3\log(x)-3\sqrt{d^2}e^3\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{3d^3}$$

input
```
Integrate[(d + e*x)^2/(x^4*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-1/3*((d*Sqrt[d^2 - e^2*x^2]*(d^2 + 3*d*e*x + 5*e^2*x^2))/x^3 + 3*Sqrt[d^2
]*e^3*Log[x] - 3*Sqrt[d^2]*e^3*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/d^3
```

3.41.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {540, 25, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{540}$$

$$-\frac{\int -\frac{d^2e(6d+5ex)}{x^3\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d^2e(6d+5ex)}{x^3\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}e\int \frac{6d+5ex}{x^3\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{3x^3}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{3}e\left(-\frac{\int -\frac{2de(5d+3ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{3\sqrt{d^2-e^2x^2}}{dx^2}\right)-\frac{\sqrt{d^2-e^2x^2}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}e\left(\frac{e\int \frac{5d+3ex}{x^2\sqrt{d^2-e^2x^2}}dx}{d}-\frac{3\sqrt{d^2-e^2x^2}}{dx^2}\right)-\frac{\sqrt{d^2-e^2x^2}}{3x^3}$$

$$\Big\downarrow \text{534}$$

$$\frac{1}{3}e\left(\frac{e\left(3e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{5\sqrt{d^2-e^2x^2}}{dx}\right)}{d}-\frac{3\sqrt{d^2-e^2x^2}}{dx^2}\right)-\frac{\sqrt{d^2-e^2x^2}}{3x^3}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{3}e\left(\frac{e\left(\frac{3}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{5\sqrt{d^2-e^2x^2}}{dx}\right)}{d}-\frac{3\sqrt{d^2-e^2x^2}}{dx^2}\right)-\frac{\sqrt{d^2-e^2x^2}}{3x^3}$$

$$\downarrow 73$$

$$\frac{1}{3}e\left(\frac{e\left(-\frac{3\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{5\sqrt{d^2-e^2x^2}}{dx}\right)}{d}-\frac{3\sqrt{d^2-e^2x^2}}{dx^2}\right)-\frac{\sqrt{d^2-e^2x^2}}{3x^3}$$

$$\downarrow 221$$

$$\frac{1}{3}e\left(\frac{e\left(-\frac{3e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{5\sqrt{d^2-e^2x^2}}{dx}\right)}{d}-\frac{3\sqrt{d^2-e^2x^2}}{dx^2}\right)-\frac{\sqrt{d^2-e^2x^2}}{3x^3}$$

input
```
Int[(d + e*x)^2/(x^4*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-1/3*Sqrt[d^2 - e^2*x^2]/x^3 + (e*((-3*Sqrt[d^2 - e^2*x^2])/(d*x^2) + (e*(
(-5*Sqrt[d^2 - e^2*x^2])/(d*x) - (3*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/
d))/3
```

3.41.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

3.41.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(5e^2x^2+3dex+d^2)}{3x^3d^2}-\frac{e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d\sqrt{d^2}}$
default	$-\frac{e^2\sqrt{-e^2x^2+d^2}}{d^2x}+d^2\left(-\frac{\sqrt{-e^2x^2+d^2}}{3d^2x^3}-\frac{2e^2\sqrt{-e^2x^2+d^2}}{3d^4x}\right)+2de\left(-\frac{\sqrt{-e^2x^2+d^2}}{2d^2x^2}-\frac{e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^2\sqrt{d^2}}\right)$

input
```
int((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-e^2*x^2+d^2)^(1/2)*(5*e^2*x^2+3*d*e*x+d^2)/x^3/d^2-1/d*e^3/(d^2)^(1
/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx = \frac{3\,e^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-(5\,e^2x^2+3\,dex+d^2)\sqrt{-e^2x^2+d^2}}{3\,d^2x^3}$$

input
```
integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(3*e^3*x^3*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (5*e^2*x^2 + 3*d*e*x +
 d^2)*sqrt(-e^2*x^2 + d^2))/(d^2*x^3)
```

3.41.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.17 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.83

$$\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx = d^2\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2x^2}-\frac{2e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^4} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2x^2}-\frac{2ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^4} & \text{otherwise}\end{cases}\right) + 2de\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2d^2x}-\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{i}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d^3} & \text{otherwise}\end{cases}\right) + e^2\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{d^2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((e*x+d)**2/x**4/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2*x**2) - 2*e**3*sqrt(
d**2/(e**2*x**2) - 1)/(3*d**4), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d*
*2/(e**2*x**2) + 1)/(3*d**2*x**2) - 2*I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(
3*d**4), True)) + 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*d**2*x
) - e**2*acosh(d/(e*x))/(2*d**3), Abs(d**2/(e**2*x**2)) > 1), (I/(2*e*x**3
*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*d**2*x*sqrt(-d**2/(e**2*x**2) + 1))
 + I*e**2*asin(d/(e*x))/(2*d**3), True)) + e**2*Piecewise((-e*sqrt(d**2/(e
**2*x**2) - 1)/d**2, Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x*
*2) + 1)/d**2, True))
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx = -\frac{e^3\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d^2} - \frac{5\,\sqrt{-e^2x^2+d^2}e^2}{3\,d^2x} - \frac{\sqrt{-e^2x^2+d^2}e}{dx^2} - \frac{\sqrt{-e^2x^2+d^2}}{3\,x^3}$$

input
```
integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-e^3*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^2 - 5/3*sqrt(-e
^2*x^2 + d^2)*e^2/(d^2*x) - sqrt(-e^2*x^2 + d^2)*e/(d*x^2) - 1/3*sqrt(-e^2
*x^2 + d^2)/x^3
```

3.41.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(95) = 190.

Time = 0.29 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.38

$$\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx = \frac{\left(e^4+\frac{6\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^2}{x}+\frac{21\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{x^2}\right)e^6x^3}{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^2|e|} - \frac{e^4\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d^2|e|} - \frac{\frac{21\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^4e^4}{x}+\frac{6\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^4e^2}{x^2}+\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^4}{x^3}}{24\,d^6e^2|e|}$$

input
```
integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
1/24*(e^4 + 6*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^2/x + 21*(d*e + sqrt(-
e^2*x^2 + d^2)*abs(e))^2/x^2)*e^6*x^3/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
^3*d^2*abs(e)) - e^4*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(
e^2*abs(x)))/(d^2*abs(e)) - 1/24*(21*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d
^4*e^4/x + 6*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^4*e^2/x^2 + (d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))^3*d^4/x^3)/(d^6*e^2*abs(e))
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d+e\,x)^2}{x^4\,\sqrt{d^2-e^2\,x^2}}\,dx$$

input
```
int((d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(1/2)),x)
```

output
```
int((d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(1/2)), x)
```

3.42 $\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx$

3.42.1 Optimal result

Integrand size = 27, antiderivative size = 140

$$\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{4x^4}-\frac{2e\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} -\frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x}-\frac{7e^4\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}$$

output
```
-7/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3-1/4*(-e^2*x^2+d^2)^(1/2)/x^4-
2/3*e*(-e^2*x^2+d^2)^(1/2)/d/x^3-7/8*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x^2-4/3*
e^3*(-e^2*x^2+d^2)^(1/2)/d^3/x
```

3.42.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.74

$$\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-6d^3-16d^2ex-21de^2x^2-32e^3x^3)}{24d^3x^4} + \frac{7e^4\operatorname{arctanh}\left(\frac{\sqrt{-e^2}x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^3}$$

input
```
Integrate[(d + e*x)^2/(x^5*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-6*d^3 - 16*d^2*e*x - 21*d*e^2*x^2 - 32*e^3*x^3))/(2
4*d^3*x^4) + (7*e^4*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/(4*
d^3)
```

3.42.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {540, 25, 27, 539, 25, 27, 539, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{540}$$

$$-\frac{\int -\frac{d^2e(8d+7ex)}{x^4\sqrt{d^2-e^2x^2}}dx}{4d^2}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d^2e(8d+7ex)}{x^4\sqrt{d^2-e^2x^2}}dx}{4d^2}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}e\int \frac{8d+7ex}{x^4\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{4x^4}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{4}e\left(-\frac{\int -\frac{de(21d+16ex)}{x^3\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{8\sqrt{d^2-e^2x^2}}{3dx^3}\right)-\frac{\sqrt{d^2-e^2x^2}}{4x^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}e\left(\frac{\int \frac{de(21d+16ex)}{x^3\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{8\sqrt{d^2-e^2x^2}}{3dx^3}\right)-\frac{\sqrt{d^2-e^2x^2}}{4x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}e\left(\frac{e\int \frac{21d+16ex}{x^3\sqrt{d^2-e^2x^2}}dx}{3d}-\frac{8\sqrt{d^2-e^2x^2}}{3dx^3}\right)-\frac{\sqrt{d^2-e^2x^2}}{4x^4}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{4}e\left(\frac{e\left(-\frac{\int -\frac{de(32d+21ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{21\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{3d}-\frac{8\sqrt{d^2-e^2x^2}}{3dx^3}\right)-\frac{\sqrt{d^2-e^2x^2}}{4x^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}e\left(\frac{e\left(\frac{\int \frac{de(32d+21ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{21\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{3d}-\frac{8\sqrt{d^2-e^2x^2}}{3dx^3}\right)-\frac{\sqrt{d^2-e^2x^2}}{4x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}e\left(\frac{e\left(\frac{e\int \frac{32d+21ex}{x^2\sqrt{d^2-e^2x^2}}dx}{2d}-\frac{21\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{3d}-\frac{8\sqrt{d^2-e^2x^2}}{3dx^3}\right)-\frac{\sqrt{d^2-e^2x^2}}{4x^4}$$

$$\Big\downarrow \text{534}$$

$$\frac{1}{4}e\left(\frac{e\left(\frac{e\left(21e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{32\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{21\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{3d}-\frac{8\sqrt{d^2-e^2x^2}}{3dx^3}\right)-\frac{\sqrt{d^2-e^2x^2}}{4x^4}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{4}e\left(\frac{e\left(\frac{e\left(\frac{21}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{32\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{21\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{3d}-\frac{8\sqrt{d^2-e^2x^2}}{3dx^3}\right)-\frac{\sqrt{d^2-e^2x^2}}{4x^4}$$

$$\Big\downarrow \text{73}$$

3.42. $\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx$

$$\frac{1}{4}e\left(\frac{e\left(\frac{e\left(-\frac{21\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{32\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{21\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{3d}-\frac{8\sqrt{d^2-e^2x^2}}{3dx^3}\right)-\frac{\sqrt{d^2-e^2x^2}}{4x^4}$$

$\Big\downarrow$ 221

$$\frac{1}{4}e\left(\frac{e\left(\frac{e\left(-\frac{21e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{32\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{21\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{3d}-\frac{8\sqrt{d^2-e^2x^2}}{3dx^3}\right)-\frac{\sqrt{d^2-e^2x^2}}{4x^4}$$

input
```
Int[(d + e*x)^2/(x^5*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-1/4*Sqrt[d^2 - e^2*x^2]/x^4 + (e*((-8*Sqrt[d^2 - e^2*x^2])/(3*d*x^3) + (e
*((-21*Sqrt[d^2 - e^2*x^2])/(2*d*x^2) + (e*((-32*Sqrt[d^2 - e^2*x^2])/(d*x
) - (21*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/(2*d)))/(3*d)))/4
```

3.42.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

3.42. $\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx$

3.42.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(32e^3x^3+21d\,e^2x^2+16d^2ex+6d^3)}{24d^3x^4}-\frac{7e^4\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8d^2\sqrt{d^2}}$
default	$d^2\left(-\frac{\sqrt{-e^2x^2+d^2}}{4d^2x^4}+\frac{3e^2\left(-\frac{\sqrt{-e^2x^2+d^2}}{2d^2x^2}-\frac{e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^2\sqrt{d^2}}\right)}{4d^2}\right)+e^2\left(-\frac{\sqrt{-e^2x^2+d^2}}{2d^2x^2}-\frac{e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{}}{x}\right.}{2d^2\sqrt{d^2}}\right.$

input
```
int((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-e^2*x^2+d^2)^(1/2)*(32*e^3*x^3+21*d*e^2*x^2+16*d^2*e*x+6*d^3)/d^3/
x^4-7/8/d^2*e^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/
x)
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.62

$$\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx = \frac{21\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-(32\,e^3x^3+21\,de^2x^2+16\,d^2ex+6\,d^3)\sqrt{-e^2x^2+d^2}}{24\,d^3x^4}$$

input
```
integrate((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
1/24*(21*e^4*x^4*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (32*e^3*x^3 + 21*d*e
^2*x^2 + 16*d^2*e*x + 6*d^3)*sqrt(-e^2*x^2 + d^2))/(d^3*x^4)
```

3.42.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.33 (sec) , antiderivative size = 449, normalized size of antiderivative = 3.21

$$\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx$$
$$= d^2\left(\begin{cases} -\frac{1}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e}{8d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e^3}{8d^4x\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{3e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^5} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{i}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie}{8d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie^3}{8d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{3ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^5} & \text{otherwise} \end{cases}\right)$$
$$+ 2de\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2x^2} - \frac{2e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^4} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2x^2} - \frac{2ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^4} & \text{otherwise} \end{cases}\right)$$
$$+ e^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2d^2x} - \frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{i}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie}{2d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d^3} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x+d)**2/x**5/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
d**2*Piecewise((-1/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) - e/(8*d**2*x**3*
sqrt(d**2/(e**2*x**2) - 1)) + 3*e**3/(8*d**4*x*sqrt(d**2/(e**2*x**2) - 1))
 - 3*e**4*acosh(d/(e*x))/(8*d**5), Abs(d**2/(e**2*x**2)) > 1), (I/(4*e*x**
5*sqrt(-d**2/(e**2*x**2) + 1)) + I*e/(8*d**2*x**3*sqrt(-d**2/(e**2*x**2) +
 1)) - 3*I*e**3/(8*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) + 3*I*e**4*asin(d/(
e*x))/(8*d**5), True)) + 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3
*d**2*x**2) - 2*e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**4), Abs(d**2/(e**2*x
**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2*x**2) - 2*I*e**3*sqr
t(-d**2/(e**2*x**2) + 1)/(3*d**4), True)) + e**2*Piecewise((-e*sqrt(d**2/(
e**2*x**2) - 1)/(2*d**2*x) - e**2*acosh(d/(e*x))/(2*d**3), Abs(d**2/(e**2*
x**2)) > 1), (I/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*d**2*x*sqr
t(-d**2/(e**2*x**2) + 1)) + I*e**2*asin(d/(e*x))/(2*d**3), True))
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx = -\frac{7\,e^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{8\,d^3} - \frac{4\,\sqrt{-e^2x^2+d^2}e^3}{3\,d^3x} - \frac{7\,\sqrt{-e^2x^2+d^2}e^2}{8\,d^2x^2} - \frac{2\,\sqrt{-e^2x^2+d^2}e}{3\,dx^3} - \frac{\sqrt{-e^2x^2+d^2}}{4\,x^4}$$

input

```
integrate((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output

```
-7/8*e^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 4/3*sqr
t(-e^2*x^2 + d^2)*e^3/(d^3*x) - 7/8*sqrt(-e^2*x^2 + d^2)*e^2/(d^2*x^2) - 2
/3*sqrt(-e^2*x^2 + d^2)*e/(d*x^3) - 1/4*sqrt(-e^2*x^2 + d^2)/x^4
```

3.42.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(120) = 240$.

Time = 0.31 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.35

$$\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx = \frac{\left(3\,e^5+\frac{16\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^3}{x}+\frac{48\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e}{x^2}+\frac{144\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{ex^3}\right)e^8x^4}{192\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^3|e|} - \frac{7\,e^5\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{8\,d^3|e|} - \frac{\frac{144\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^9e^5|e|}{x}+\frac{48\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^9e^3|e|}{x^2}+\frac{16\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^9e|e|}{x^3}+\frac{3\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^9|e|}{ex^4}}{192\,d^{12}e^4}$$

input

```
integrate((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
1/192*(3*e^5 + 16*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^3/x + 48*(d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))^2*e/x^2 + 144*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e
))^3/(e*x^3))*e^8*x^4/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^3*abs(e)) -
 7/8*e^5*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))
/(d^3*abs(e)) - 1/192*(144*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^9*e^5*abs
(e)/x + 48*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^9*e^3*abs(e)/x^2 + 16*(
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^9*e*abs(e)/x^3 + 3*(d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))^4*d^9*abs(e)/(e*x^4))/(d^12*e^4)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d+e\,x)^2}{x^5\,\sqrt{d^2-e^2\,x^2}}\,dx$$

input
```
int((d + e*x)^2/(x^5*(d^2 - e^2*x^2)^(1/2)),x)
```

output
```
int((d + e*x)^2/(x^5*(d^2 - e^2*x^2)^(1/2)), x)
```

3.43 $\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx$

3.43.1 Optimal result

Integrand size = 27, antiderivative size = 169

$$\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{e\sqrt{d^2-e^2x^2}}{2dx^4}-\frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}-\frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2}-\frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x}-\frac{3e^5\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^4}$$

output
```
-3/4*e^5*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4-1/5*(-e^2*x^2+d^2)^(1/2)/x^5-
1/2*e*(-e^2*x^2+d^2)^(1/2)/d/x^4-3/5*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x^3-3/4*
e^3*(-e^2*x^2+d^2)^(1/2)/d^3/x^2-6/5*e^4*(-e^2*x^2+d^2)^(1/2)/d^4/x
```

3.43.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-4d^4-10d^3ex-12d^2e^2x^2-15de^3x^3-24e^4x^4)}{20d^4x^5}-\frac{3\sqrt{d^2}e^5\log(x)}{4d^5}+\frac{3\sqrt{d^2}e^5\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{4d^5}$$

input
```
Integrate[(d + e*x)^2/(x^6*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-4*d^4 - 10*d^3*e*x - 12*d^2*e^2*x^2 - 15*d*e^3*x^3
- 24*e^4*x^4))/(20*d^4*x^5) - (3*Sqrt[d^2]*e^5*Log[x])/(4*d^5) + (3*Sqrt[d
^2]*e^5*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(4*d^5)
```

3.43.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {540, 25, 27, 539, 27, 539, 27, 539, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{ 540}$$

$$-\frac{\int -\frac{d^2e(10d+9ex)}{x^5\sqrt{d^2-e^2x^2}}dx}{5d^2}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{d^2e(10d+9ex)}{x^5\sqrt{d^2-e^2x^2}}dx}{5d^2}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}e\int \frac{10d+9ex}{x^5\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{5x^5}$$

$$\Big\downarrow \text{ 539}$$

$$\frac{1}{5}e\left(-\frac{\int -\frac{6de(6d+5ex)}{x^4\sqrt{d^2-e^2x^2}}dx}{4d^2}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^4}\right)-\frac{\sqrt{d^2-e^2x^2}}{5x^5}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}e\left(\frac{3e\int \frac{6d+5ex}{x^4\sqrt{d^2-e^2x^2}}dx}{2d}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^4}\right)-\frac{\sqrt{d^2-e^2x^2}}{5x^5}$$

$$\Big\downarrow \text{ 539}$$

$$\frac{1}{5}e\left(\frac{3e\left(-\frac{\int -\frac{3de(5d+4ex)}{x^3\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{2\sqrt{d^2-e^2x^2}}{dx^3}\right)}{2d}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^4}\right)-\frac{\sqrt{d^2-e^2x^2}}{5x^5}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{5}e\left(\frac{3e\left(\frac{e\int \frac{5d+4ex}{x^3\sqrt{d^2-e^2x^2}}dx}{d}-\frac{2\sqrt{d^2-e^2x^2}}{dx^3}\right)}{2d}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^4}\right)-\frac{\sqrt{d^2-e^2x^2}}{5x^5}$$

$$\Big\downarrow \ 539$$

$$\frac{1}{5}e\left(\frac{3e\left(\frac{e\left(-\frac{\int -\frac{de(8d+5ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{d}-\frac{2\sqrt{d^2-e^2x^2}}{dx^3}\right)}{2d}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^4}\right)-\frac{\sqrt{d^2-e^2x^2}}{5x^5}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{5}e\left(\frac{3e\left(\frac{e\left(\frac{\int \frac{de(8d+5ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{d}-\frac{2\sqrt{d^2-e^2x^2}}{dx^3}\right)}{2d}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^4}\right)-\frac{\sqrt{d^2-e^2x^2}}{5x^5}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{5}e\left(\frac{3e\left(\frac{e\left(\frac{e\int \frac{8d+5ex}{x^2\sqrt{d^2-e^2x^2}}dx}{2d}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{d}-\frac{2\sqrt{d^2-e^2x^2}}{dx^3}\right)}{2d}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^4}\right)-\frac{\sqrt{d^2-e^2x^2}}{5x^5}$$

$$\Big\downarrow \ 534$$

3.43. $\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx$

$$\frac{1}{5}e\left(\frac{3e\left(\frac{e\left(\frac{e\left(5e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{8\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{d}-\frac{2\sqrt{d^2-e^2x^2}}{dx^3}\right)}{2d}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^4}\right)-\frac{\sqrt{d^2-e^2x^2}}{5x^5}$$

$\downarrow$ 243

$$\frac{1}{5}e\left(\frac{3e\left(\frac{e\left(\frac{e\left(\frac{5}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{8\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{d}-\frac{2\sqrt{d^2-e^2x^2}}{dx^3}\right)}{2d}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^4}\right)-\frac{\sqrt{d^2-e^2x^2}}{5x^5}$$

$\downarrow$ 73

3.43. $\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx$

$$\frac{1}{5}e\left(\frac{3e\left(\frac{e\left(\frac{e\left(-\frac{5\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{8\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{d}-\frac{2\sqrt{d^2-e^2x^2}}{dx^3}\right)}{2d}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^4}\right)-\frac{\sqrt{d^2-e^2x^2}}{5x^5}$$

$$\Big\downarrow \; 221$$

$$\frac{1}{5}e\left(\frac{3e\left(\frac{e\left(\frac{e\left(-\frac{5e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{8\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{d}-\frac{2\sqrt{d^2-e^2x^2}}{dx^3}\right)}{2d}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^4}\right)-\frac{\sqrt{d^2-e^2x^2}}{5x^5}$$

3.43. $\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx$

input
```
Int[(d + e*x)^2/(x^6*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-1/5*Sqrt[d^2 - e^2*x^2]/x^5 + (e*((-5*Sqrt[d^2 - e^2*x^2])/(2*d*x^4) + (3
*e*((-2*Sqrt[d^2 - e^2*x^2])/(d*x^3) + (e*((-5*Sqrt[d^2 - e^2*x^2])/(2*d*x
^2) + (e*((-8*Sqrt[d^2 - e^2*x^2])/(d*x) - (5*e*ArcTanh[Sqrt[d^2 - e^2*x^2
]/d])/d))/(2*d)))/d))/(2*d)))/5
```

3.43.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

3.43.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.65

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(24e^4x^4+15d\,e^3x^3+12d^2e^2x^2+10d^3ex+4d^4)}{20x^5d^4}-\frac{3e^5\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{4d^3\sqrt{d^2}}$
default	$d^2\left(-\frac{\sqrt{-e^2x^2+d^2}}{5d^2x^5}+\frac{4e^2\left(-\frac{\sqrt{-e^2x^2+d^2}}{3d^2x^3}-\frac{2e^2\sqrt{-e^2x^2+d^2}}{3d^4x}\right)}{5d^2}\right)+e^2\left(-\frac{\sqrt{-e^2x^2+d^2}}{3d^2x^3}-\frac{2e^2\sqrt{-e^2x^2+d^2}}{3d^4x}\right)+2de\Bigg(-\sqrt{\ }$

input
```
int((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*(-e^2*x^2+d^2)^(1/2)*(24*e^4*x^4+15*d*e^3*x^3+12*d^2*e^2*x^2+10*d^3*
e*x+4*d^4)/x^5/d^4-3/4/d^3*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x
^2+d^2)^(1/2))/x)
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.58

$$\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx = \frac{15\,e^5x^5\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - (24\,e^4x^4 + 15\,de^3x^3 + 12\,d^2e^2x^2 + 10\,d^3ex + 4\,d^4)\sqrt{-e^2x^2+d^2}}{20\,d^4x^5}$$

input
```
integrate((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
1/20*(15*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (24*e^4*x^4 + 15*d*e
^3*x^3 + 12*d^2*e^2*x^2 + 10*d^3*e*x + 4*d^4)*sqrt(-e^2*x^2 + d^2))/(d^4*x
^5)
```

3.43.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.96 (sec) , antiderivative size = 510, normalized size of antiderivative = 3.02

$$\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx = d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{5d^2x^4} - \frac{4e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{15d^4x^2} - \frac{8e^5\sqrt{\frac{d^2}{e^2x^2}-1}}{15d^6} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{5d^2x^4} - \frac{4ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{15d^4x^2} - \frac{8ie^5\sqrt{-\frac{d^2}{e^2x^2}+1}}{15d^6} & \text{otherwise} \end{cases}\right) + 2de\left(\begin{cases} -\frac{1}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e}{8d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e^3}{8d^4x\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{3e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^5} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{i}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie}{8d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie^3}{8d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{3ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^5} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2x^2} - \frac{2e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^4} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2x^2} - \frac{2ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^4} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x+d)**2/x**6/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(5*d**2*x**4) - 4*e**3*sqrt(
d**2/(e**2*x**2) - 1)/(15*d**4*x**2) - 8*e**5*sqrt(d**2/(e**2*x**2) - 1)/(
15*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(5
*d**2*x**4) - 4*I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(15*d**4*x**2) - 8*I*e*
*5*sqrt(-d**2/(e**2*x**2) + 1)/(15*d**6), True)) + 2*d*e*Piecewise((-1/(4*
e*x**5*sqrt(d**2/(e**2*x**2) - 1)) - e/(8*d**2*x**3*sqrt(d**2/(e**2*x**2)
- 1)) + 3*e**3/(8*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) - 3*e**4*acosh(d/(e*x
))/(8*d**5), Abs(d**2/(e**2*x**2)) > 1), (I/(4*e*x**5*sqrt(-d**2/(e**2*x**
2) + 1)) + I*e/(8*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e**3/(8*d**
4*x*sqrt(-d**2/(e**2*x**2) + 1)) + 3*I*e**4*asin(d/(e*x))/(8*d**5), True))
 + e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2*x**2) - 2*e**3*sq
rt(d**2/(e**2*x**2) - 1)/(3*d**4), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(
-d**2/(e**2*x**2) + 1)/(3*d**2*x**2) - 2*I*e**3*sqrt(-d**2/(e**2*x**2) + 1
)/(3*d**4), True))
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx = -\frac{3\,e^5\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{4\,d^4}-\frac{6\,\sqrt{-e^2x^2+d^2}e^4}{5\,d^4x}-\frac{3\,\sqrt{-e^2x^2+d^2}e^3}{4\,d^3x^2}-\frac{3\,\sqrt{-e^2x^2+d^2}e^2}{5\,d^2x^3}-\frac{\sqrt{-e^2x^2+d^2}e}{2\,dx^4}-\frac{\sqrt{-e^2x^2+d^2}}{5\,x^5}$$

input
```
integrate((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-3/4*e^5*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^4 - 6/5*sqr
t(-e^2*x^2 + d^2)*e^4/(d^4*x) - 3/4*sqrt(-e^2*x^2 + d^2)*e^3/(d^3*x^2) - 3
/5*sqrt(-e^2*x^2 + d^2)*e^2/(d^2*x^3) - 1/2*sqrt(-e^2*x^2 + d^2)*e/(d*x^4)
 - 1/5*sqrt(-e^2*x^2 + d^2)/x^5
```

3.43.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(145) = 290$.

Time = 0.29 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.28

$$\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{\left(e^6 + \frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^4}{x} + \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^2}{x^2} + \frac{40\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{x^3} + \frac{110\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^2x^4}\right)e^{10}x^5}{160\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^5d^4|e|}$$
$$- \frac{3\,e^6\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{4\,d^4|e|}$$
$$- \frac{\frac{110\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{16}e^8}{x} + \frac{40\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^{16}e^6}{x^2} + \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^{16}e^4}{x^3} + \frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^{16}e^2}{x^4} + \cdots}{160\,d^{20}e^4|e|}$$

input
```
integrate((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
1/160*(e^6 + 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^4/x + 15*(d*e + sqrt(
-e^2*x^2 + d^2)*abs(e))^2*e^2/x^2 + 40*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
^3/x^3 + 110*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^2*x^4))*e^10*x^5/((d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d^4*abs(e)) - 3/4*e^6*log(1/2*abs(-2*d
*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^4*abs(e)) - 1/160*(11
0*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^16*e^8/x + 40*(d*e + sqrt(-e^2*x^2
 + d^2)*abs(e))^2*d^16*e^6/x^2 + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*
d^16*e^4/x^3 + 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^16*e^2/x^4 + (d*e
 + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d^16/x^5)/(d^20*e^4*abs(e))
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d+e\,x)^2}{x^6\,\sqrt{d^2-e^2\,x^2}}\,dx$$

input
```
int((d + e*x)^2/(x^6*(d^2 - e^2*x^2)^(1/2)),x)
```

output
```
int((d + e*x)^2/(x^6*(d^2 - e^2*x^2)^(1/2)), x)
```

3.44 $\int \frac{x^5(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$

3.44.1 Optimal result

Integrand size = 27, antiderivative size = 143

$$\int \frac{x^5(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{d^4(d+ex)^2}{5e^6(d^2-e^2x^2)^{5/2}}-\frac{22d^3(d+ex)}{15e^6(d^2-e^2x^2)^{3/2}}+\frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^6}-\frac{2d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

output
```
1/5*d^4*(e*x+d)^2/e^6/(-e^2*x^2+d^2)^(5/2)-22/15*d^3*(e*x+d)/e^6/(-e^2*x^2
+d^2)^(3/2)-2*d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+2/15*d*(23*e*x+30*d)/
e^6/(-e^2*x^2+d^2)^(1/2)+(-e^2*x^2+d^2)^(1/2)/e^6
```

3.44.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.88

$$\int \frac{x^5(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}(-56d^4+82d^3ex+32d^2e^2x^2-76de^3x^3+15e^4x^4)}{15e^6(-d+ex)^3(d+ex)}+\frac{2d(-e^2)^{3/2}\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{e^9}$$

input
```
Integrate[(x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-56*d^4 + 82*d^3*e*x + 32*d^2*e^2*x^2 - 76*d*e^3*x^3
 + 15*e^4*x^4))/(15*e^6*(-d + e*x)^3*(d + e*x)) + (2*d*(-e^2)^(3/2)*Log[-(
Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e^9
```

3.44.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {529, 2166, 2345, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{529}$$

$$\frac{d^4(d+ex)^2}{5e^6(d^2-e^2x^2)^{5/2}}-\frac{\int \frac{(d+ex)\left(\frac{2d^5}{e^5}+\frac{5xd^4}{e^4}+\frac{5x^2d^3}{e^3}+\frac{5x^3d^2}{e^2}+\frac{5x^4d}{e}\right)}{(d^2-e^2x^2)^{5/2}}dx}{5d}$$

$$\Big\downarrow \text{2166}$$

$$\frac{d^4(d+ex)^2}{5e^6(d^2-e^2x^2)^{5/2}}-\frac{\frac{22d^4(d+ex)}{3e^6(d^2-e^2x^2)^{3/2}}-\frac{\int \frac{\frac{16d^5}{e^5}+\frac{45xd^4}{e^4}+\frac{30x^2d^3}{e^3}+\frac{15x^3d^2}{e^2}}{(d^2-e^2x^2)^{3/2}}dx}{3d}}{5d}$$

$$\Big\downarrow \text{2345}$$

$$\frac{d^4(d+ex)^2}{5e^6(d^2-e^2x^2)^{5/2}}-\frac{\frac{22d^4(d+ex)}{3e^6(d^2-e^2x^2)^{3/2}}-\frac{\frac{2d^3(30d+23ex)}{e^6\sqrt{d^2-e^2x^2}}-\frac{\int \frac{15d^4(2d+ex)}{e^5\sqrt{d^2-e^2x^2}}dx}{d^2}}{3d}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^4(d+ex)^2}{5e^6(d^2-e^2x^2)^{5/2}}-\frac{\frac{22d^4(d+ex)}{3e^6(d^2-e^2x^2)^{3/2}}-\frac{\frac{2d^3(30d+23ex)}{e^6\sqrt{d^2-e^2x^2}}-\frac{15d^2\int \frac{2d+ex}{\sqrt{d^2-e^2x^2}}dx}{e^5}}{3d}}{5d}$$

$$\Big\downarrow \text{455}$$

$$\frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{22d^4(d+ex)}{3e^6(d^2-e^2x^2)^{3/2}}-\frac{\frac{2d^3(30d+23ex)}{e^6\sqrt{d^2-e^2x^2}}-\frac{15d^2\left(2d\int\frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{e}\right)}{e^5}}{3d}}{5d}$$

$\downarrow$ 224

$$\frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{22d^4(d+ex)}{3e^6(d^2-e^2x^2)^{3/2}}-\frac{\frac{2d^3(30d+23ex)}{e^6\sqrt{d^2-e^2x^2}}-\frac{15d^2\left(2d\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{e}\right)}{e^5}}{3d}}{5d}$$

$\downarrow$ 216

$$\frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{22d^4(d+ex)}{3e^6(d^2-e^2x^2)^{3/2}}-\frac{\frac{2d^3(30d+23ex)}{e^6\sqrt{d^2-e^2x^2}}-\frac{15d^2\left(\frac{2d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{\sqrt{d^2-e^2x^2}}{e}\right)}{e^5}}{3d}}{5d}$$

input
```
Int[(x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(d^4*(d + e*x)^2)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - ((22*d^4*(d + e*x))/(3*e
^6*(d^2 - e^2*x^2)^(3/2)) - ((2*d^3*(30*d + 23*e*x))/(e^6*Sqrt[d^2 - e^2*x
^2]) - (15*d^2*(-(Sqrt[d^2 - e^2*x^2]/e) + (2*d*ArcTan[(e*x)/Sqrt[d^2 - e^
2*x^2]])/e))/e^5)/(3*d))/(5*d)
```

3.44.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.44. $\int \frac{x^5(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.44.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.66

method	result
risch	$\frac{\sqrt{-e^2x^2+d^2}}{e^6}-\frac{2d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^5\sqrt{e^2}}+\frac{d\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{8e^7\left(x+\frac{d}{e}\right)}-\frac{41d^2\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{60e^8\left(x-\frac{d}{e}\right)^2}-\frac{383d\sqrt{-\left(x-\right.}}{120}$
default	$e^2\left(-\frac{x^6}{e^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{6d^2\left(\frac{x^4}{e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{4d^2\left(\frac{x^2}{3e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{2d^2}{15e^4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}\right)}{e^2}\right)}{e^2}\right)+d^2\left(\frac{x^4}{e^2(-e^2x^2+d^2)^{\frac{5}{2}}}\right.$

input
```
int(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(-e^2*x^2+d^2)^(1/2)/e^6-2*d/e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^
2+d^2)^(1/2))+1/8*d/e^7/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-41/60
*d^2/e^8/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-383/120*d/e^7/(x-d
/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-1/10*d^3/e^9/(x-d/e)^3*(-(x-d/e)^
2*e^2-2*d*e*(x-d/e))^(1/2)
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.31

$$\int \frac{x^5(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{56\,de^4x^4-112\,d^2e^3x^3+112\,d^4ex-56\,d^5+60\,(de^4x^4-2\,d^2e^3x^3+2\,d^4ex-d^5)\,\mathrm{arcta}}{15\,(e^{10}x^4-2\,de^9x}$$

input
```
integrate(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(56*d*e^4*x^4 - 112*d^2*e^3*x^3 + 112*d^4*e*x - 56*d^5 + 60*(d*e^4*x^
4 - 2*d^2*e^3*x^3 + 2*d^4*e*x - d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e
*x)) + (15*e^4*x^4 - 76*d*e^3*x^3 + 32*d^2*e^2*x^2 + 82*d^3*e*x - 56*d^4)*
sqrt(-e^2*x^2 + d^2))/(e^10*x^4 - 2*d*e^9*x^3 + 2*d^3*e^7*x - d^4*e^6)
```

3.44.6 Sympy [F]

$$\int \frac{x^5(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^5(d+ex)^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x**5*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral(x**5*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.44.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(127) = 254$.

Time = 0.28 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.01

$$\int \frac{x^5(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2}{15}\,dex\left(\frac{15\,x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right) - \frac{x^6}{(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{2\,dx\left(\frac{3\,x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right)}{3\,e} + \frac{7\,d^2x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{28\,d^4x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{56\,d^6}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^6} + \frac{8\,d^3x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}e^5} - \frac{14\,dx}{15\,\sqrt{-e^2x^2+d^2}\,e^5} - \frac{2\,d\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}e^5}$$

input
```
integrate(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
2/15*d*e*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 +
d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - x^6/(-e^2*x^2 + d^
2)^(5/2) - 2/3*d*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2
+ d^2)^(3/2)*e^4))/e + 7*d^2*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 28/3*d^4*x
^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 56/15*d^6/((-e^2*x^2 + d^2)^(5/2)*e^6) +
 8/15*d^3*x/((-e^2*x^2 + d^2)^(3/2)*e^5) - 14/15*d*x/(sqrt(-e^2*x^2 + d^2)
*e^5) - 2*d*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^5)
```

3.44.8 Giac [F]

$$\int \frac{x^5(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2x^5}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*x^5/(-e^2*x^2 + d^2)^(7/2), x)
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^5\,(d+e\,x)^2}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

input
```
int((x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
int((x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2), x)
```

3.45 $\int \frac{x^4(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$

3.45.1 Optimal result

Integrand size = 27, antiderivative size = 121

$$\int \frac{x^4(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d^3(d+ex)^2}{5e^5(d^2-e^2x^2)^{5/2}} - \frac{17d^2(d+ex)}{15e^5(d^2-e^2x^2)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

output
```
1/5*d^3*(e*x+d)^2/e^5/(-e^2*x^2+d^2)^(5/2)-17/15*d^2*(e*x+d)/e^5/(-e^2*x^2
+d^2)^(3/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5+2/15*(13*e*x+15*d)/e^5/(-
e^2*x^2+d^2)^(1/2)
```

3.45.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.86

$$\int \frac{x^4(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(16d^3-17d^2ex-22de^2x^2+26e^3x^3)}{(d-ex)^3(d+ex)} + 30\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{15e^5}$$

input
```
Integrate[(x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(16*d^3 - 17*d^2*e*x - 22*d*e^2*x^2 + 26*e^3*x^3))/(
(d - e*x)^3*(d + e*x)) + 30*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])
])/(15*e^5)
```

3.45.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {529, 2166, 2345, 27, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{529}$$

$$\frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{(d+ex)\left(\frac{2d^4}{e^4}+\frac{5xd^3}{e^3}+\frac{5x^2d^2}{e^2}+\frac{5x^3d}{e}\right)}{(d^2-e^2x^2)^{5/2}}dx}{5d}$$

$$\Big\downarrow \text{2166}$$

$$\frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{17d^3(d+ex)}{3e^5(d^2-e^2x^2)^{3/2}}-\frac{\int \frac{\frac{11d^4}{e^4}+\frac{30xd^3}{e^3}+\frac{15x^2d^2}{e^2}}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d}}{5d}$$

$$\Big\downarrow \text{2345}$$

$$\frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{17d^3(d+ex)}{3e^5(d^2-e^2x^2)^{3/2}}-\frac{\frac{2d^2(15d+13ex)}{e^5\sqrt{d^2-e^2x^2}}-\frac{\int \frac{15d^4}{e^4\sqrt{d^2-e^2x^2}}dx}{d^2}}{3d}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{17d^3(d+ex)}{3e^5(d^2-e^2x^2)^{3/2}}-\frac{\frac{2d^2(15d+13ex)}{e^5\sqrt{d^2-e^2x^2}}-\frac{15d^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx}{e^4}}{3d}}{5d}$$

$$\Big\downarrow \text{224}$$

$$\frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{17d^3(d+ex)}{3e^5(d^2-e^2x^2)^{3/2}}-\frac{\frac{2d^2(15d+13ex)}{e^5\sqrt{d^2-e^2x^2}}-\frac{15d^2\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}}{e^4}}{3d}}{5d}$$

$$\Big\downarrow \text{216}$$

$$\frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{17d^3(d+ex)}{3e^5(d^2-e^2x^2)^{3/2}}-\frac{\frac{2d^2(15d+13ex)}{e^5\sqrt{d^2-e^2x^2}}-\frac{15d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}}{3d}}{5d}$$

input
```
Int[(x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(d^3*(d + e*x)^2)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) - ((17*d^3*(d + e*x))/(3*e
^5*(d^2 - e^2*x^2)^(3/2)) - ((2*d^2*(15*d + 13*e*x))/(e^5*Sqrt[d^2 - e^2*x
^2]) - (15*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^5)/(3*d))/(5*d)
```

3.45.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.45.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(107) = 214$.

Time = 0.41 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.68

method	result
default	$e^{2}\left(\frac{x^{5}}{5e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{\frac{x^{3}}{3e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}-\frac{\frac{x}{e^{2}\sqrt{-e^{2}x^{2}+d^{2}}}-\frac{\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{e^{2}\sqrt{e^{2}}}}{e^{2}}}{e^{2}}\right)+d^{2}\left(\frac{x^{3}}{2e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{3d^{2}\left(\frac{\ }{4e^{2}(\ }\right.}{}\right.$

input
```
int(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
e^2*(1/5*x^5/e^2/(-e^2*x^2+d^2)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3
/2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)
*x/(-e^2*x^2+d^2)^(1/2)))))+d^2*(1/2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)-3/2*d^2/
e^2*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^
(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/
2)))))+2*d*e*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*d^2/e^2*(1/3*x^2/e^2/(-e^2*x^
2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2)))
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.42

$$\int \frac{x^4(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{16\,e^4x^4-32\,de^3x^3+32\,d^3ex-16\,d^4+30\,(e^4x^4-2\,de^3x^3+2\,d^3ex-d^4)\arctan\left(-\frac{d}{}\right.}{15\,(e^9x^4-2\,de^8x^3+2\,d^3e^6}$$

input
```
integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(16*e^4*x^4 - 32*d*e^3*x^3 + 32*d^3*e*x - 16*d^4 + 30*(e^4*x^4 - 2*d*
e^3*x^3 + 2*d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (26
*e^3*x^3 - 22*d*e^2*x^2 - 17*d^2*e*x + 16*d^3)*sqrt(-e^2*x^2 + d^2))/(e^9*
x^4 - 2*d*e^8*x^3 + 2*d^3*e^6*x - d^4*e^5)
```

3.45.6 Sympy [F]

$$\int \frac{x^4(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^4(d+ex)^2}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

input
```
integrate(x**4*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral(x**4*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.45.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(107) = 214$.

Time = 0.43 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.56

$$\int \frac{x^4(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{1}{15}e^2x\left(\frac{15\,x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right) - \frac{1}{3}x\left(\frac{3\,x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right) + \frac{2\,dx^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{d^2x^3}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{8\,d^3x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e^3} - \frac{3\,d^4x}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{16\,d^5}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^5} + \frac{11\,d^2x}{30\,(-e^2x^2+d^2)^{\frac{3}{2}}e^4} - \frac{4\,x}{15\,\sqrt{-e^2x^2+d^2}e^4} - \frac{\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}e^4}$$

input
```
integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/15*e^2*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 +
d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 1/3*x*(3*x^2/((-e^
2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4)) + 2*d*x^4/((
-e^2*x^2 + d^2)^(5/2)*e) + 1/2*d^2*x^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 8/3*
d^3*x^2/((-e^2*x^2 + d^2)^(5/2)*e^3) - 3/10*d^4*x/((-e^2*x^2 + d^2)^(5/2)*
e^4) + 16/15*d^5/((-e^2*x^2 + d^2)^(5/2)*e^5) + 11/30*d^2*x/((-e^2*x^2 + d
^2)^(3/2)*e^4) - 4/15*x/(sqrt(-e^2*x^2 + d^2)*e^4) - arcsin(e^2*x/(d*sqrt(
e^2)))/(sqrt(e^2)*e^4)
```

3.45.8 Giac [F]

$$\int \frac{x^4(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2x^4}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*x^4/(-e^2*x^2 + d^2)^(7/2), x)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^4\,(d+e\,x)^2}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

input
```
int((x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
int((x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2), x)
```

3.46 $\int \frac{x^3(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$

3.46.1 Optimal result

Integrand size = 27, antiderivative size = 97

$$\int \frac{x^3(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d^2(d+ex)^2}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{4d(d+ex)}{5e^4(d^2-e^2x^2)^{3/2}} + \frac{5d+2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

output
```
1/5*d^2*(e*x+d)^2/e^4/(-e^2*x^2+d^2)^(5/2)-4/5*d*(e*x+d)/e^4/(-e^2*x^2+d^2
)^(3/2)+1/5*(2*e*x+5*d)/d/e^4/(-e^2*x^2+d^2)^(1/2)
```

3.46.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.72

$$\int \frac{x^3(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(2d^3-4d^2ex+de^2x^2+2e^3x^3)}{5de^4(d-ex)^3(d+ex)}$$

input
```
Integrate[(x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(2*d^3 - 4*d^2*e*x + d*e^2*x^2 + 2*e^3*x^3))/(5*d*e^4
*(d - e*x)^3*(d + e*x))
```

3.46.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {529, 2166, 27, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{529}$$

$$\frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{(d+ex)\left(\frac{2d^3}{e^3}+\frac{5xd^2}{e^2}+\frac{5x^2d}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d}$$

$$\Big\downarrow \text{2166}$$

$$\frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{4d^2(d+ex)}{e^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int \frac{3d^2(2d+5ex)}{e^3\left(d^2-e^2x^2\right)^{3/2}}dx}{3d}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{4d^2(d+ex)}{e^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{d\int \frac{2d+5ex}{\left(d^2-e^2x^2\right)^{3/2}}dx}{e^3}}{5d}$$

$$\Big\downarrow \text{453}$$

$$\frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{4d^2(d+ex)}{e^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{5d+2ex}{e^4\sqrt{d^2-e^2x^2}}}{5d}$$

input
```
Int[(x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(d^2*(d + e*x)^2)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - ((4*d^2*(d + e*x))/(e^4*
(d^2 - e^2*x^2)^(3/2)) - (5*d + 2*e*x)/(e^4*Sqrt[d^2 - e^2*x^2]))/(5*d)
```

3.46.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

3.46.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.67

method	result
gosper	$\frac{(-ex+d)(ex+d)^{3}\left(2e^{3}x^{3}+d\,e^{2}x^{2}-4d^{2}ex+2d^{3}\right)}{5d\,e^{4}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}$
trager	$\frac{\left(2e^{3}x^{3}+d\,e^{2}x^{2}-4d^{2}ex+2d^{3}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{5d\,e^{4}(-ex+d)^{3}(ex+d)}$
default	$e^{2}\left(\frac{x^{4}}{e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{4d^{2}\left(\frac{x^{2}}{3e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{2d^{2}}{15e^{4}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}\right)}{e^{2}}\right)+d^{2}\left(\frac{x^{2}}{3e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{2d^{2}}{15e^{4}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}\right)$

input
```
int(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(-e*x+d)*(e*x+d)^3*(2*e^3*x^3+d*e^2*x^2-4*d^2*e*x+2*d^3)/d/e^4/(-e^2*x
^2+d^2)^(7/2)
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.20

$$\int \frac{x^3(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2\,e^4x^4-4\,de^3x^3+4\,d^3ex-2\,d^4-\left(2\,e^3x^3+de^2x^2-4\,d^2ex+2\,d^3\right)\sqrt{-e^2x^2+d^2}}{5\left(de^8x^4-2\,d^2e^7x^3+2\,d^4e^5x-d^5e^4\right)}$$

input
```
integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/5*(2*e^4*x^4 - 4*d*e^3*x^3 + 4*d^3*e*x - 2*d^4 - (2*e^3*x^3 + d*e^2*x^2
- 4*d^2*e*x + 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d*e^8*x^4 - 2*d^2*e^7*x^3 + 2*
d^4*e^5*x - d^5*e^4)
```

3.46.6 Sympy [F]

$$\int \frac{x^3(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^3(d+ex)^2}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

input
```
integrate(x**3*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral(x**3*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.60

$$\int \frac{x^3(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{dx^3}{(-e^2x^2+d^2)^{\frac{5}{2}}e} - \frac{d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{3\,d^3x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^3} + \frac{2\,d^4}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{dx}{5\,(-e^2x^2+d^2)^{\frac{3}{2}}e^3} + \frac{2\,x}{5\,\sqrt{-e^2x^2+d^2}de^3}$$

input
```
integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
x^4/(-e^2*x^2 + d^2)^(5/2) + d*x^3/((-e^2*x^2 + d^2)^(5/2)*e) - d^2*x^2/((
-e^2*x^2 + d^2)^(5/2)*e^2) - 3/5*d^3*x/((-e^2*x^2 + d^2)^(5/2)*e^3) + 2/5*
d^4/((-e^2*x^2 + d^2)^(5/2)*e^4) + 1/5*d*x/((-e^2*x^2 + d^2)^(3/2)*e^3) +
2/5*x/(sqrt(-e^2*x^2 + d^2)*d*e^3)
```

3.46.8 Giac [F]

$$\int \frac{x^3(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2x^3}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*x^3/(-e^2*x^2 + d^2)^(7/2), x)
```

3.46.9 Mupad [B] (verification not implemented)

Time = 11.60 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.68

$$\int \frac{x^3(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^3-4\,d^2\,e\,x+d\,e^2\,x^2+2\,e^3\,x^3\right)}{5\,d\,e^4\,\left(d+e\,x\right)\,{\left(d-e\,x\right)}^3}$$

input
```
int((x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(2*d^3 + 2*e^3*x^3 + d*e^2*x^2 - 4*d^2*e*x))/(5*d*e
^4*(d + e*x)*(d - e*x)^3)
```

3.47 $\int \frac{x^2(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$

3.47.1 Optimal result

Integrand size = 27, antiderivative size = 87

$$\int \frac{x^2(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d(d+ex)^2}{5e^3\,(d^2-e^2x^2)^{5/2}} - \frac{7(d+ex)}{15e^3\,(d^2-e^2x^2)^{3/2}} + \frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}$$

output
```
1/5*d*(e*x+d)^2/e^3/(-e^2*x^2+d^2)^(5/2)-7/15*(e*x+d)/e^3/(-e^2*x^2+d^2)^(
3/2)+1/15*x/d^2/e^2/(-e^2*x^2+d^2)^(1/2)
```

3.47.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \frac{x^2(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-4d^3+8d^2ex-2de^2x^2+e^3x^3)}{15d^2e^3(d-ex)^3(d+ex)}$$

input
```
Integrate[(x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-4*d^3 + 8*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3))/(15*d^2
*e^3*(d - e*x)^3*(d + e*x))
```

3.47.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {529, 27, 669, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{ 529}$$

$$\frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{d(d+ex)(2d+5ex)}{e^2\left(d^2-e^2x^2\right)^{5/2}}dx}{5d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{(d+ex)(2d+5ex)}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5e^2}$$

$$\Big\downarrow \text{ 669}$$

$$\frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{7(d+ex)}{3e(d^2-e^2x^2)^{3/2}}-\frac{1}{3}\int \frac{1}{(d^2-e^2x^2)^{3/2}}dx}{5e^2}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{7(d+ex)}{3e(d^2-e^2x^2)^{3/2}}-\frac{x}{3d^2\sqrt{d^2-e^2x^2}}}{5e^2}$$

input
```
Int[(x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(d*(d + e*x)^2)/(5*e^3*(d^2 - e^2*x^2)^(5/2)) - ((7*(d + e*x))/(3*e*(d^2 -
 e^2*x^2)^(3/2)) - x/(3*d^2*Sqrt[d^2 - e^2*x^2]))/(5*e^2)
```

3.47.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 669
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*
(p + 1))), x] - Simp[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1)))
Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

3.47.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.76

method	result
gosper	$-\frac{(-ex+d)(ex+d)^3(-e^3x^3+2d\,e^2x^2-8d^2ex+4d^3)}{15d^2e^3(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$-\frac{(-e^3x^3+2d\,e^2x^2-8d^2ex+4d^3)\sqrt{-e^2x^2+d^2}}{15d^2e^3(-ex+d)^3(ex+d)}$
default	$e^2\left(\frac{x^3}{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{3d^2\left(\frac{x}{4e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2(-e^2x^2+d^2)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{4e^2}\right)}{2e^2}\right)+d^2\Big($

input
```
int(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-e*x+d)*(e*x+d)^3*(-e^3*x^3+2*d*e^2*x^2-8*d^2*e*x+4*d^3)/d^2/e^3/(-
e^2*x^2+d^2)^(7/2)
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.34

$$\int \frac{x^2(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{4\,e^4x^4-8\,de^3x^3+8\,d^3ex-4\,d^4+(e^3x^3-2\,de^2x^2+8\,d^2ex-4\,d^3)\sqrt{-e^2x^2+d^2}}{15\,(d^2e^7x^4-2\,d^3e^6x^3+2\,d^5e^4x-d^6e^3)}$$

input
```
integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/15*(4*e^4*x^4 - 8*d*e^3*x^3 + 8*d^3*e*x - 4*d^4 + (e^3*x^3 - 2*d*e^2*x^
2 + 8*d^2*e*x - 4*d^3)*sqrt(-e^2*x^2 + d^2))/(d^2*e^7*x^4 - 2*d^3*e^6*x^3
+ 2*d^5*e^4*x - d^6*e^3)
```

3.47.6 Sympy [F]

$$\int \frac{x^2(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^2(d+ex)^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x**2*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral(x**2*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.51

$$\int \frac{x^2(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{2\,dx^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} - \frac{d^2x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{4\,d^3}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3} + \frac{x}{30\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2} + \frac{x}{15\sqrt{-e^2x^2+d^2}d^2e^2}$$

input
```
integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/2*x^3/(-e^2*x^2 + d^2)^(5/2) + 2/3*d*x^2/((-e^2*x^2 + d^2)^(5/2)*e) - 1/
10*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^2) - 4/15*d^3/((-e^2*x^2 + d^2)^(5/2)*e
^3) + 1/30*x/((-e^2*x^2 + d^2)^(3/2)*e^2) + 1/15*x/(sqrt(-e^2*x^2 + d^2)*d
^2*e^2)
```

3.47.8 Giac [F]

$$\int \frac{x^2(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*x^2/(-e^2*x^2 + d^2)^(7/2), x)
```

3.47.9 Mupad [B] (verification not implemented)

Time = 11.40 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.77

$$\int \frac{x^2(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}\,\left(4\,d^3-8\,d^2\,e\,x+2\,d\,e^2\,x^2-e^3\,x^3\right)}{15\,d^2\,e^3\,\left(d+e\,x\right)\,{\left(d-e\,x\right)}^3}$$

input
```
int((x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
-((d^2 - e^2*x^2)^(1/2)*(4*d^3 - e^3*x^3 + 2*d*e^2*x^2 - 8*d^2*e*x))/(15*d
^2*e^3*(d + e*x)*(d - e*x)^3)
```

3.48 $\int \frac{x(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$

3.48.1 Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{x(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{(d+ex)^2}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2(d+ex)}{15de^2(d^2-e^2x^2)^{3/2}} - \frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

output
```
1/5*(e*x+d)^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*(e*x+d)/d/e^2/(-e^2*x^2+d^2)^(
3/2)-4/15*x/d^3/e/(-e^2*x^2+d^2)^(1/2)
```

3.48.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int \frac{x(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(d^3-2d^2ex+8de^2x^2-4e^3x^3)}{15d^3e^2(d-ex)^3(d+ex)}$$

input
```
Integrate[(x*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(d^3 - 2*d^2*e*x + 8*d*e^2*x^2 - 4*e^3*x^3))/(15*d^3*
e^2*(d - e*x)^3*(d + e*x))
```

3.48.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {531, 27, 454, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \ 531$$

$$\frac{\int -\frac{2d^2e(d+ex)}{(d^2-e^2x^2)^{5/2}}dx}{5d^2e^2}+\frac{(d+ex)^2}{5e^2(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{(d+ex)^2}{5e^2(d^2-e^2x^2)^{5/2}}-\frac{2\int \frac{d+ex}{(d^2-e^2x^2)^{5/2}}dx}{5e}$$

$$\Big\downarrow \ 454$$

$$\frac{(d+ex)^2}{5e^2(d^2-e^2x^2)^{5/2}}-\frac{2\left(\frac{2\int \frac{1}{(d^2-e^2x^2)^{3/2}}dx}{3d}+\frac{d+ex}{3de(d^2-e^2x^2)^{3/2}}\right)}{5e}$$

$$\Big\downarrow \ 208$$

$$\frac{(d+ex)^2}{5e^2(d^2-e^2x^2)^{5/2}}-\frac{2\left(\frac{d+ex}{3de(d^2-e^2x^2)^{3/2}}+\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}\right)}{5e}$$

input
```
Int[(x*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(d + e*x)^2/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*((d + e*x)/(3*d*e*(d^2 - e^
2*x^2)^(3/2)) + (2*x)/(3*d^3*Sqrt[d^2 - e^2*x^2])))/(5*e)
```

3.48.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 531
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m, a + b*x^2, x], e = Coeff[Polynomi
alRemainder[x^m, a + b*x^2, x], x, 0], f = Coeff[PolynomialRemainder[x^m, a
 + b*x^2, x], x, 1]}, Simp[(c + d*x)^n*(a*f - b*e*x)*((a + b*x^2)^(p + 1)/(
2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b
*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*(c + d*x)*Qx - a*d*f*n + b*c*e*(2*p
 + 3) + b*d*e*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGt
Q[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && GtQ[n, 1] && IntegerQ[2*p]
```

3.48.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

method	result
gosper	$\frac{(-ex+d)(ex+d)^3(-4e^3x^3+8d\,e^2x^2-2d^2ex+d^3)}{15d^3e^2(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$\frac{(-4e^3x^3+8d\,e^2x^2-2d^2ex+d^3)\sqrt{-e^2x^2+d^2}}{15d^3(-ex+d)^3e^2(ex+d)}$
default	$e^2\left(\frac{x^2}{3e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{2d^2}{15e^4(-e^2x^2+d^2)^{\frac{5}{2}}}\right)+\frac{d^2}{5e^2(-e^2x^2+d^2)^{\frac{5}{2}}}+2de\left(\frac{x}{4e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\right.}{}\right.$

input
```
int(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(-e*x+d)*(e*x+d)^3*(-4*e^3*x^3+8*d*e^2*x^2-2*d^2*e*x+d^3)/d^3/e^2/(-e
^2*x^2+d^2)^(7/2)
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.31

$$\int \frac{x(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{e^4x^4-2\,de^3x^3+2\,d^3ex-d^4+(4\,e^3x^3-8\,de^2x^2+2\,d^2ex-d^3)\sqrt{-e^2x^2+d^2}}{15\,(d^3e^6x^4-2\,d^4e^5x^3+2\,d^6e^3x-d^7e^2)}$$

input
```
integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(e^4*x^4 - 2*d*e^3*x^3 + 2*d^3*e*x - d^4 + (4*e^3*x^3 - 8*d*e^2*x^2 +
 2*d^2*e*x - d^3)*sqrt(-e^2*x^2 + d^2))/(d^3*e^6*x^4 - 2*d^4*e^5*x^3 + 2*d
^6*e^3*x - d^7*e^2)
```

3.48.6 Sympy [F]

$$\int \frac{x(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x(d+ex)^2}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

input
```
integrate(x*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral(x*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.22

$$\int \frac{x(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{2\,dx}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{d^2}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{2\,x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}de} - \frac{4\,x}{15\,\sqrt{-e^2x^2+d^2}d^3e}$$

input
```
integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/3*x^2/(-e^2*x^2 + d^2)^(5/2) + 2/5*d*x/((-e^2*x^2 + d^2)^(5/2)*e) + 1/15
*d^2/((-e^2*x^2 + d^2)^(5/2)*e^2) - 2/15*x/((-e^2*x^2 + d^2)^(3/2)*d*e) -
4/15*x/(sqrt(-e^2*x^2 + d^2)*d^3*e)
```

3.48.8 Giac [F]

$$\int \frac{x(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2x}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*x/(-e^2*x^2 + d^2)^(7/2), x)
```

3.48.9 Mupad [B] (verification not implemented)

Time = 11.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.73

$$\int \frac{x(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,(d^3-2\,d^2\,e\,x+8\,d\,e^2\,x^2-4\,e^3\,x^3)}{15\,d^3\,e^2\,(d+e\,x)\,(d-e\,x)^3}$$

input
```
int((x*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(d^3 - 4*e^3*x^3 + 8*d*e^2*x^2 - 2*d^2*e*x))/(15*d^
3*e^2*(d + e*x)*(d - e*x)^3)
```

3.49 $\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$

3.49.1 Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2(d+ex)}{5e\,(d^2-e^2x^2)^{5/2}} + \frac{x}{5d^2\,(d^2-e^2x^2)^{3/2}} + \frac{2x}{5d^4\sqrt{d^2-e^2x^2}}$$

output
```
2/5*(e*x+d)/e/(-e^2*x^2+d^2)^(5/2)+1/5*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/5*x/d^
4/(-e^2*x^2+d^2)^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(2d^3+d^2ex-4de^2x^2+2e^3x^3)}{5d^4e(d-ex)^3(d+ex)}$$

input
```
Integrate[(d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(2*d^3 + d^2*e*x - 4*d*e^2*x^2 + 2*e^3*x^3))/(5*d^4*e
*(d - e*x)^3*(d + e*x))
```

3.49.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {457, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}} \, dx$$

$$\Big\downarrow \text{457}$$

$$\frac{3}{5}\int \frac{1}{(d^2-e^2x^2)^{5/2}}dx+\frac{2(d+ex)}{5e\,(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{3}{5}\left(\frac{2\int \frac{1}{(d^2-e^2x^2)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\,(d^2-e^2x^2)^{3/2}}\right)+\frac{2(d+ex)}{5e\,(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{2(d+ex)}{5e\,(d^2-e^2x^2)^{5/2}}+\frac{3}{5}\left(\frac{x}{3d^2\,(d^2-e^2x^2)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)$$

input
```
Int[(d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(2*(d + e*x))/(5*e*(d^2 - e^2*x^2)^(5/2)) + (3*(x/(3*d^2*(d^2 - e^2*x^2)^(
3/2)) + (2*x)/(3*d^4*Sqrt[d^2 - e^2*x^2])))/5
```

3.49.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

3.49.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

method	result
gosper	$\frac{(-ex+d)(ex+d)^3(2e^3x^3-4d\,e^2x^2+d^2ex+2d^3)}{5d^4e(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$\frac{(2e^3x^3-4d\,e^2x^2+d^2ex+2d^3)\sqrt{-e^2x^2+d^2}}{5d^4(-ex+d)^3e(ex+d)}$
default	$d^2\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+e^2\left(\frac{x}{4e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{}{15d^2(}\right.}{}\right.$

input
```
int((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(-e*x+d)*(e*x+d)^3*(2*e^3*x^3-4*d*e^2*x^2+d^2*e*x+2*d^3)/d^4/e/(-e^2*x
^2+d^2)^(7/2)
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2\,e^4x^4-4\,de^3x^3+4\,d^3ex-2\,d^4-(2\,e^3x^3-4\,de^2x^2+d^2ex+2\,d^3)\sqrt{-e^2x^2+d^2}}{5\,(d^4e^5x^4-2\,d^5e^4x^3+2\,d^7e^2x-d^8e)}$$

input
```
integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/5*(2*e^4*x^4 - 4*d*e^3*x^3 + 4*d^3*e*x - 2*d^4 - (2*e^3*x^3 - 4*d*e^2*x^
2 + d^2*e*x + 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d^4*e^5*x^4 - 2*d^5*e^4*x^3 +
2*d^7*e^2*x - d^8*e)
```

3.49.6 Sympy [F]

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^2}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2x}{5(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{2d}{5(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{x}{5(-e^2x^2+d^2)^{\frac{3}{2}}d^2} + \frac{2x}{5\sqrt{-e^2x^2+d^2}d^4}$$

input
```
integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
2/5*x/(-e^2*x^2 + d^2)^(5/2) + 2/5*d/((-e^2*x^2 + d^2)^(5/2)*e) + 1/5*x/((
-e^2*x^2 + d^2)^(3/2)*d^2) + 2/5*x/(sqrt(-e^2*x^2 + d^2)*d^4)
```

3.49.8 Giac [F]

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2/(-e^2*x^2 + d^2)^(7/2), x)
```

3.49.9 Mupad [B] (verification not implemented)

Time = 11.49 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^3+d^2\,e\,x-4\,d\,e^2\,x^2+2\,e^3\,x^3\right)}{5\,d^4\,e\,\left(d+e\,x\right)\,\left(d-e\,x\right)^3}$$

input
```
int((d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(2*d^3 + 2*e^3*x^3 - 4*d*e^2*x^2 + d^2*e*x))/(5*d^4
*e*(d + e*x)*(d - e*x)^3)
```

3.50 $\int \frac{(d+ex)^2}{x(d^2-e^2x^2)^{7/2}}\,dx$

3.50.1 Optimal result

Integrand size = 27, antiderivative size = 117

$$\int \frac{(d+ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

output
```
2/5*(e*x+d)/d/(-e^2*x^2+d^2)^(5/2)+1/15*(8*e*x+5*d)/d^3/(-e^2*x^2+d^2)^(3/
2)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5+1/15*(16*e*x+15*d)/d^5/(-e^2*x^2+d^
2)^(1/2)
```

3.50.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{\frac{\sqrt{d^2-e^2x^2}\left(26d^3-22d^2ex-17de^2x^2+16e^3x^3\right)}{(d-ex)^3(d+ex)}+30\operatorname{arctanh}\left(\frac{\sqrt{-e^2}x-\sqrt{d^2-e^2x^2}}{d}\right)}{15d^5}$$

input
```
Integrate[(d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(26*d^3 - 22*d^2*e*x - 17*d*e^2*x^2 + 16*e^3*x^3))/(
(d - e*x)^3*(d + e*x)) + 30*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d
])/(15*d^5)
```

3.50.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {532, 25, 27, 532, 25, 532, 27, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{x(d^2-e^2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \textbf{532}$$

$$\frac{2(d+ex)}{5d(d^2-e^2x^2)^{5/2}}-\frac{\int -\frac{d(5d+8ex)}{x(d^2-e^2x^2)^{5/2}}dx}{5d^2}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int \frac{d(5d+8ex)}{x(d^2-e^2x^2)^{5/2}}dx}{5d^2}+\frac{2(d+ex)}{5d(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{5d+8ex}{x(d^2-e^2x^2)^{5/2}}dx}{5d}+\frac{2(d+ex)}{5d(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \textbf{532}$$

$$\frac{\frac{5d+8ex}{3d^2(d^2-e^2x^2)^{3/2}}-\frac{\int -\frac{15d+16ex}{x\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}}{5d}+\frac{2(d+ex)}{5d(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\frac{\int \frac{15d+16ex}{x\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{5d+8ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d}+\frac{2(d+ex)}{5d(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \textbf{532}$$

$$\frac{\frac{\frac{15d+16ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{15d}{x\sqrt{d^2-e^2x^2}}dx}{d^2}}{3d^2}+\frac{5d+8ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d}+\frac{2(d+ex)}{5d(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{\frac{15\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx}{d}+\frac{15d+16ex}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{5d+8ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d}+\frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\frac{\frac{15\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2}{2d}+\frac{15d+16ex}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{5d+8ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d}+\frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\frac{\frac{15d+16ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{15\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{de^2}}{3d^2}+\frac{5d+8ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d}+\frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{\frac{15d+16ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{15\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}}{3d^2}+\frac{5d+8ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d}+\frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[(d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(2*(d + e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + ((5*d + 8*e*x)/(3*d^2*(d^2 - e
^2*x^2)^(3/2)) + ((15*d + 16*e*x)/(d^2*Sqrt[d^2 - e^2*x^2]) - (15*ArcTanh[
Sqrt[d^2 - e^2*x^2]/d])/d^2)/(3*d^2))/(5*d)
```

3.50.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

3.50.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.73

method	result
default	$\frac{1}{5(-e^2x^2+d^2)^{\frac{5}{2}}}+d^2\left(\frac{1}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{1}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{\frac{1}{d^2\sqrt{-e^2x^2+d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}}}{d^2}}{d^2}\right)+2de\left(\frac{}{5d^2}\right.$

input
```
int((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/(-e^2*x^2+d^2)^(5/2)+d^2*(1/5/d^2/(-e^2*x^2+d^2)^(5/2)+1/d^2*(1/3/d^2/
(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*l
n((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))+2*d*e*(1/5*x/d^2/(-e^2*
x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2
+d^2)^(1/2)))
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)^2}{x\,(d^2-e^2x^2)^{7/2}}\,dx = \frac{26\,e^4x^4-52\,de^3x^3+52\,d^3ex-26\,d^4+15\,(e^4x^4-2\,de^3x^3+2\,d^3ex-d^4)\log\left(-\frac{d-\cdots}{}\right.}{15\,(d^5e^4x^4-2\,d^6e^3x^3+2\,d\cdots}$$

input
```
integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(26*e^4*x^4 - 52*d*e^3*x^3 + 52*d^3*e*x - 26*d^4 + 15*(e^4*x^4 - 2*d*
e^3*x^3 + 2*d^3*e*x - d^4)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (16*e^3*x^
3 - 17*d*e^2*x^2 - 22*d^2*e*x + 26*d^3)*sqrt(-e^2*x^2 + d^2))/(d^5*e^4*x^4
 - 2*d^6*e^3*x^3 + 2*d^8*e*x - d^9)
```

3.50.6 Sympy [F]

$$\int \frac{(d+ex)^2}{x\,(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^2}{x\,(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/x/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**2/(x*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.32

$$\int \frac{(d+ex)^2}{x(d^2-e^2x^2)^{7/2}}\,dx = \frac{2\,ex}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}d} + \frac{2}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{8\,ex}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d^3} + \frac{1}{3\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2} + \frac{16\,ex}{15\,\sqrt{-e^2x^2+d^2}d^5} - \frac{\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d^5} + \frac{1}{\sqrt{-e^2x^2+d^2}d^4}$$

input
```
integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
2/5*e*x/((-e^2*x^2 + d^2)^(5/2)*d) + 2/5/(-e^2*x^2 + d^2)^(5/2) + 8/15*e*x
/((-e^2*x^2 + d^2)^(3/2)*d^3) + 1/3/((-e^2*x^2 + d^2)^(3/2)*d^2) + 16/15*e
*x/(sqrt(-e^2*x^2 + d^2)*d^5) - log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*
d/abs(x))/d^5 + 1/(sqrt(-e^2*x^2 + d^2)*d^4)
```

3.50.8 Giac [F]

$$\int \frac{(d+ex)^2}{x(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2}{(-e^2x^2+d^2)^{\frac{7}{2}}x}\,dx$$

input
```
integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2/((-e^2*x^2 + d^2)^(7/2)*x), x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+e\,x)^2}{x\,(d^2-e^2\,x^2)^{7/2}}\,dx$$

input
```
int((d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)),x)
```

output
```
int((d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)), x)
```

3.51 $\int \frac{(d+ex)^2}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$

3.51.1 Optimal result

Integrand size = 27, antiderivative size = 145

$$\int \frac{(d+ex)^2}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} - \frac{2e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

output
```
2/5*e*(e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)+1/15*e*(13*e*x+10*d)/d^4/(-e^2*x^2+
d^2)^(3/2)-2*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/15*e*(41*e*x+30*d)/d^
6/(-e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^6/x
```

3.51.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^2}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}\left(15d^4-76d^3ex+32d^2e^2x^2+82de^3x^3-56e^4x^4\right)}{x(-d+ex)^3(d+ex)} - 30\sqrt{d^2}e\log(x) + 30\sqrt{d^2}e\log\left(\sqrt{d^2} -\right.}{15d^7}$$

input
```
Integrate[(d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
((d*Sqrt[d^2 - e^2*x^2]*(15*d^4 - 76*d^3*e*x + 32*d^2*e^2*x^2 + 82*d*e^3*x
^3 - 56*e^4*x^4))/(x*(-d + e*x)^3*(d + e*x)) - 30*Sqrt[d^2]*e*Log[x] + 30*
Sqrt[d^2]*e*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(15*d^7)
```

3.51.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {532, 25, 2336, 25, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{x^2 (d^2-e^2x^2)^{7/2}} \, dx$$

$$\Big\downarrow \text{ 532}$$

$$\frac{2e(d+ex)}{5d^2 (d^2-e^2x^2)^{5/2}}-\frac{\int -\frac{5d^2+10exd+8e^2x^2}{x^2 (d^2-e^2x^2)^{5/2}}dx}{5d^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{5d^2+10exd+8e^2x^2}{x^2 (d^2-e^2x^2)^{5/2}}dx}{5d^2}+\frac{2e(d+ex)}{5d^2 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{ 2336}$$

$$\frac{\frac{e(10d+13ex)}{3d^2(d^2-e^2x^2)^{3/2}}-\frac{\int -\frac{15d^2+30exd+26e^2x^2}{x^2 \left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}}{5d^2}+\frac{2e(d+ex)}{5d^2 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{15d^2+30exd+26e^2x^2}{x^2 \left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{e(10d+13ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e(d+ex)}{5d^2 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{ 2336}$$

$$\frac{\frac{\frac{e(30d+41ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{15d(d+2ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{d^2}}{3d^2}+\frac{e(10d+13ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e(d+ex)}{5d^2 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{15\int \frac{d+2ex}{x^2\sqrt{d^2-e^2x^2}}dx}{d}+\frac{e(30d+41ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e(10d+13ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e(d+ex)}{5d^2 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow 534$$

$$\frac{\frac{\frac{15\left(2e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)}{d}+\frac{e(30d+41ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e(10d+13ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e(d+ex)}{5d^2\,(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow 243$$

$$\frac{\frac{\frac{15\left(e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)}{d}+\frac{e(30d+41ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e(10d+13ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e(d+ex)}{5d^2\,(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow 73$$

$$\frac{\frac{\frac{15\left(-\frac{2\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)}{d}+\frac{e(30d+41ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e(10d+13ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e(d+ex)}{5d^2\,(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow 221$$

$$\frac{\frac{\frac{15\left(-\frac{2e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)}{d}+\frac{e(30d+41ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e(10d+13ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e(d+ex)}{5d^2\,(d^2-e^2x^2)^{5/2}}$$

input
```
Int[(d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(2*e*(d + e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + ((e*(10*d + 13*e*x))/(3*d^
2*(d^2 - e^2*x^2)^(3/2)) + ((e*(30*d + 41*e*x))/(d^2*Sqrt[d^2 - e^2*x^2])
+ (15*(-(Sqrt[d^2 - e^2*x^2]/(d*x)) - (2*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])
/d))/d)/(3*d^2))/(5*d^2)
```

3.51. $\int \frac{(d+ex)^2}{x^2(d^2-e^2x^2)^{7/2}}\,dx$

3.51.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

3.51. $\int \frac{(d+ex)^2}{x^2(d^2-e^2x^2)^{7/2}}\,dx$

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.51.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.72

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^6 x}-\frac{2e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^5\sqrt{d^2}}+\frac{29\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{60d^5e\left(x-\frac{d}{e}\right)^2}-\frac{313\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{120d^6\left(x-\frac{d}{e}\right)}-\sqrt{-$
default	$e^2\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+d^2\left(-\frac{1}{d^2x(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{6e^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{}{15}\right.}{}\right.$

input
```
int((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-(-e^2*x^2+d^2)^(1/2)/d^6/x-2/d^5*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-
e^2*x^2+d^2)^(1/2))/x)+29/60/d^5/e/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e)
)^(1/2)-313/120/d^6/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-1/8/d^6/(
x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/10/d^4/e^2/(x-d/e)^3*(-(x-d/
e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.34

$$\int \frac{(d+ex)^2}{x^2 (d^2-e^2x^2)^{7/2}}\,dx = \frac{46\,e^5x^5 - 92\,de^4x^4 + 92\,d^3e^2x^2 - 46\,d^4ex + 30\,(e^5x^5 - 2\,de^4x^4 + 2\,d^3e^2x^2 - d^4ex)}{15\,(d^6e^4x^5 - 2\,d^7}$$

input
```
integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(46*e^5*x^5 - 92*d*e^4*x^4 + 92*d^3*e^2*x^2 - 46*d^4*e*x + 30*(e^5*x^
5 - 2*d*e^4*x^4 + 2*d^3*e^2*x^2 - d^4*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))
/x) - (56*e^4*x^4 - 82*d*e^3*x^3 - 32*d^2*e^2*x^2 + 76*d^3*e*x - 15*d^4)*s
qrt(-e^2*x^2 + d^2))/(d^6*e^4*x^5 - 2*d^7*e^3*x^4 + 2*d^9*e*x^2 - d^10*x)
```

3.51.6 Sympy [F]

$$\int \frac{(d+ex)^2}{x^2 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^2}{x^2 \left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/x**2/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**2/(x**2*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.29

$$\begin{aligned}\int \frac{(d+ex)^2}{x^2 (d^2-e^2x^2)^{7/2}}\,dx &= \frac{7\,e^2x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2} + \frac{2\,e}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}d} \\ &+ \frac{28\,e^2x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d^4} + \frac{2\,e}{3\,(-e^2x^2+d^2)^{\frac{3}{2}}d^3} - \frac{1}{(-e^2x^2+d^2)^{\frac{5}{2}}x} \\ &+ \frac{56\,e^2x}{15\,\sqrt{-e^2x^2+d^2}d^6} - \frac{2\,e\log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d^6} + \frac{2\,e}{\sqrt{-e^2x^2+d^2}d^5}\end{aligned}$$

input
```
integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
7/5*e^2*x/((-e^2*x^2 + d^2)^(5/2)*d^2) + 2/5*e/((-e^2*x^2 + d^2)^(5/2)*d)
+ 28/15*e^2*x/((-e^2*x^2 + d^2)^(3/2)*d^4) + 2/3*e/((-e^2*x^2 + d^2)^(3/2)
*d^3) - 1/((-e^2*x^2 + d^2)^(5/2)*x) + 56/15*e^2*x/(sqrt(-e^2*x^2 + d^2)*d
^6) - 2*e*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^6 + 2*e/(s
qrt(-e^2*x^2 + d^2)*d^5)
```

3.51.8 Giac [F]

$$\int \frac{(d+ex)^2}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{\left(ex+d\right)^2}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}x^2}\,dx$$

input
```
integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2/((-e^2*x^2 + d^2)^(7/2)*x^2), x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{\left(d+e\,x\right)^2}{x^2\,\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

input
```
int((d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)),x)
```

output
```
int((d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)), x)
```

3.52 $\int \frac{(d+ex)^2}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$

3.52.1 Optimal result

Integrand size = 27, antiderivative size = 182

$$\int \frac{(d+ex)^2}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{2e\sqrt{d^2-e^2x^2}}{d^7x} - \frac{9e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}$$

output
```
2/5*e^2*(e*x+d)/d^3/(-e^2*x^2+d^2)^(5/2)+1/5*e^2*(6*e*x+5*d)/d^5/(-e^2*x^2
+d^2)^(3/2)-9/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^7+2/5*e^2*(11*e*x+10
*d)/d^7/(-e^2*x^2+d^2)^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/d^6/x^2-2*e*(-e^2*x^
2+d^2)^(1/2)/d^7/x
```

3.52.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)^2}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}\left(5d^5+10d^4ex-94d^3e^2x^2+58d^2e^3x^3+83de^4x^4-64e^5x^5\right)}{x^2(-d+ex)^3(d+ex)} + 90e^2\operatorname{arctanh}\left(\frac{\sqrt{-e^2}x-\sqrt{d^2-e^2x^2}}{d}\right)}{10d^7}$$

input
```
Integrate[(d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(5*d^5 + 10*d^4*e*x - 94*d^3*e^2*x^2 + 58*d^2*e^3*x^
3 + 83*d*e^4*x^4 - 64*e^5*x^5))/(x^2*(-d + e*x)^3*(d + e*x)) + 90*e^2*ArcT
anh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(10*d^7)
```

3.52.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {532, 25, 2336, 27, 2336, 27, 2338, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{x^3 (d^2-e^2x^2)^{7/2}} \, dx$$

$$\Big\downarrow \text{532}$$

$$\frac{2e^2(d+ex)}{5d^3 (d^2-e^2x^2)^{5/2}}-\frac{\int -\frac{\frac{8e^3x^3}{d}+10e^2x^2+10dex+5d^2}{x^3 (d^2-e^2x^2)^{5/2}}dx}{5d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{8e^3x^3}{d}+10e^2x^2+10dex+5d^2}{x^3 (d^2-e^2x^2)^{5/2}}dx}{5d^2}+\frac{2e^2(d+ex)}{5d^3 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{e^2(5d+6ex)}{d^3(d^2-e^2x^2)^{3/2}}-\frac{\int -\frac{3\left(\frac{12e^3x^3}{d}+15e^2x^2+10dex+5d^2\right)}{x^3 \left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}}{5d^2}+\frac{2e^2(d+ex)}{5d^3 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\frac{12e^3x^3}{d}+15e^2x^2+10dex+5d^2}{x^3 \left(d^2-e^2x^2\right)^{3/2}}dx}{d^2}+\frac{e^2(5d+6ex)}{d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^2(d+ex)}{5d^3 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{\frac{2e^2(10d+11ex)}{d^3\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{5\left(d^2+2exd+4e^2x^2\right)}{x^3\sqrt{d^2-e^2x^2}}dx}{d^2}}{d^2}+\frac{e^2(5d+6ex)}{d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{5\int \frac{d^2+2exd+4e^2x^2}{x^3\sqrt{d^2-e^2x^2}}dx}{d^2}+\frac{2e^2(10d+11ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(5d+6ex)}{d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 2338$$

$$\frac{\frac{\frac{5\left(-\frac{\int -\frac{d^2e(4d+9ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{2e^2(10d+11ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(5d+6ex)}{d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{5\left(\frac{\int \frac{d^2e(4d+9ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{2e^2(10d+11ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(5d+6ex)}{d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{5\left(\frac{1}{2}e\int \frac{4d+9ex}{x^2\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{2e^2(10d+11ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(5d+6ex)}{d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 534$$

$$\frac{\frac{\frac{5\left(\frac{1}{2}e\left(9e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{4\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{2e^2(10d+11ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(5d+6ex)}{d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{\frac{5\left(\frac{1}{2}e\left(\frac{9}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{4\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{2e^2(10d+11ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(5d+6ex)}{d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 73$$

3.52. $\int \frac{(d+ex)^2}{x^3(d^2-e^2x^2)^{7/2}}\,dx$

$$\frac{\frac{\frac{5\left(\frac{1}{2}e\left(-\frac{9\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{4\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{2e^2(10d+11ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(5d+6ex)}{d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^2(d+ex)}{5d^3(d^2-e^2x^2)^{5/2}}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{5\left(\frac{1}{2}e\left(-\frac{9e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{4\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{2e^2(10d+11ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(5d+6ex)}{d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^2(d+ex)}{5d^3(d^2-e^2x^2)^{5/2}}$$

input
```
Int[(d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(2*e^2*(d + e*x))/(5*d^3*(d^2 - e^2*x^2)^(5/2)) + ((e^2*(5*d + 6*e*x))/(d^
3*(d^2 - e^2*x^2)^(3/2)) + ((2*e^2*(10*d + 11*e*x))/(d^3*Sqrt[d^2 - e^2*x^
2]) + (5*(-1/2*Sqrt[d^2 - e^2*x^2]/x^2 + (e*((-4*Sqrt[d^2 - e^2*x^2])/(d*x
) - (9*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/2))/d^2)/d^2)/(5*d^2)
```

3.52.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.52. $\int \frac{(d+ex)^2}{x^3(d^2-e^2x^2)^{7/2}}\,dx$

3.52.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,(4ex+d)}{2d^{7}x^{2}}-\frac{9e^{2}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{2d^{6}\sqrt{d^{2}}}+\frac{e\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{8d^{7}\left(x+\frac{d}{e}\right)}-\frac{\sqrt{-\left(x-\frac{d}{e}\right)^{2}e^{2}-2de\left(x-\frac{d}{e}\right)}}{10d^{5}e\left(x-\frac{d}{e}\right)^{3}}+$
default	$e^{2}\left(\frac{1}{5d^{2}(-e^{2}x^{2}+d^{2})^{\frac{5}{2}}}+\frac{\frac{1}{3d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{\frac{1}{d^{2}\sqrt{-e^{2}x^{2}+d^{2}}}-\frac{\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{d^{2}\sqrt{d^{2}}}}{d^{2}}}{d^{2}}\right)+d^{2}\left(-\frac{1}{2d^{2}x^{2}(-e^{2}x^{2}+d^{2})^{\frac{5}{2}}}+\right.$

input
```
int((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e^2*x^2+d^2)^(1/2)*(4*e*x+d)/d^7/x^2-9/2/d^6*e^2/(d^2)^(1/2)*ln((2*
d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/8/d^7*e/(x+d/e)*(-(x+d/e)^2*e
^2+2*d*e*(x+d/e))^(1/2)-1/10/d^5/e/(x-d/e)^3*(-(x-d/e)^2*e^2-2*d*e*(x-d/e)
)^(1/2)+13/20/d^6/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-181/40/d^
7*e/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex)^2}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{54\,e^6x^6-108\,de^5x^5+108\,d^3e^3x^3-54\,d^4e^2x^2+45\left(e^6x^6-2\,de^5x^5+2\,d^3e^3x^3-d\right.}{10\left(d^7e^4x\right.}$$

input
```
integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/10*(54*e^6*x^6 - 108*d*e^5*x^5 + 108*d^3*e^3*x^3 - 54*d^4*e^2*x^2 + 45*(
e^6*x^6 - 2*d*e^5*x^5 + 2*d^3*e^3*x^3 - d^4*e^2*x^2)*log(-(d - sqrt(-e^2*x
^2 + d^2))/x) - (64*e^5*x^5 - 83*d*e^4*x^4 - 58*d^2*e^3*x^3 + 94*d^3*e^2*x
^2 - 10*d^4*e*x - 5*d^5)*sqrt(-e^2*x^2 + d^2))/(d^7*e^4*x^6 - 2*d^8*e^3*x^
5 + 2*d^10*e*x^3 - d^11*x^2)
```

3.52.6 Sympy [F]

$$\int \frac{(d+ex)^2}{x^3 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^2}{x^3 \left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/x**3/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**2/(x**3*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \frac{(d+ex)^2}{x^3 (d^2-e^2x^2)^{7/2}}\,dx &= \frac{12\,e^3x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3} + \frac{9\,e^2}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2} \\ &+ \frac{16\,e^3x}{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^5} + \frac{3\,e^2}{2\,(-e^2x^2+d^2)^{\frac{3}{2}}d^4} - \frac{2\,e}{(-e^2x^2+d^2)^{\frac{5}{2}}dx} + \frac{32\,e^3x}{5\,\sqrt{-e^2x^2+d^2}d^7} \\ &- \frac{9\,e^2\log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{2\,d^7} + \frac{9\,e^2}{2\,\sqrt{-e^2x^2+d^2}d^6} - \frac{1}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}x^2}\end{aligned}$$

input
```
integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
12/5*e^3*x/((-e^2*x^2 + d^2)^(5/2)*d^3) + 9/10*e^2/((-e^2*x^2 + d^2)^(5/2)
*d^2) + 16/5*e^3*x/((-e^2*x^2 + d^2)^(3/2)*d^5) + 3/2*e^2/((-e^2*x^2 + d^2
)^(3/2)*d^4) - 2*e/((-e^2*x^2 + d^2)^(5/2)*d*x) + 32/5*e^3*x/(sqrt(-e^2*x^
2 + d^2)*d^7) - 9/2*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)
)/d^7 + 9/2*e^2/(sqrt(-e^2*x^2 + d^2)*d^6) - 1/2/((-e^2*x^2 + d^2)^(5/2)*x
^2)
```

3.52.8 Giac [F]

$$\int \frac{(d+ex)^2}{x^3 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2}{(-e^2x^2+d^2)^{\frac{7}{2}}x^3}\,dx$$

input
```
integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2/((-e^2*x^2 + d^2)^(7/2)*x^3), x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x^3 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+e\,x)^2}{x^3\,(d^2-e^2\,x^2)^{7/2}}\,dx$$

input
```
int((d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)),x)
```

output
```
int((d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)), x)
```

3.53 $\int \frac{(d+ex)^2}{x^4(d^2-e^2x^2)^{7/2}}\,dx$

3.53.1 Optimal result

Integrand size = 27, antiderivative size = 209

$$\int \frac{(d+ex)^2}{x^4\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3d^6x^3}-\frac{e\sqrt{d^2-e^2x^2}}{d^7x^2}-\frac{14e^2\sqrt{d^2-e^2x^2}}{3d^8x}-\frac{7e^3\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^8}$$

output
```
2/5*e^3*(e*x+d)/d^4/(-e^2*x^2+d^2)^(5/2)+1/15*e^3*(23*e*x+20*d)/d^6/(-e^2*
x^2+d^2)^(3/2)-7*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^8+2/15*e^3*(53*e*x+
45*d)/d^8/(-e^2*x^2+d^2)^(1/2)-1/3*(-e^2*x^2+d^2)^(1/2)/d^6/x^3-e*(-e^2*x^
2+d^2)^(1/2)/d^7/x^2-14/3*e^2*(-e^2*x^2+d^2)^(1/2)/d^8/x
```

3.53.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex)^2}{x^4\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{\frac{d\sqrt{d^2-e^2x^2}\left(5d^6+5d^5ex+40d^4e^2x^2-246d^3e^3x^3+122d^2e^4x^4+247de^5x^5-176e^6x^6\right)}{x^3(-d+ex)^3(d+ex)}-105\sqrt{d^2}e^3\log(x)+}{15d^9}$$

input
```
Integrate[(d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
((d*Sqrt[d^2 - e^2*x^2]*(5*d^6 + 5*d^5*e*x + 40*d^4*e^2*x^2 - 246*d^3*e^3*
x^3 + 122*d^2*e^4*x^4 + 247*d*e^5*x^5 - 176*e^6*x^6))/(x^3*(-d + e*x)^3*(d
 + e*x)) - 105*Sqrt[d^2]*e^3*Log[x] + 105*Sqrt[d^2]*e^3*Log[Sqrt[d^2] - Sq
rt[d^2 - e^2*x^2]])/(15*d^9)
```

3.53.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.13, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.519, Rules used = {532, 25, 2336, 25, 2336, 27, 2338, 27, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{x^4 \left(d^2-e^2x^2\right)^{7/2}} \, dx$$

$$\Big\downarrow \text{532}$$

$$\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int -\frac{\frac{8e^4x^4}{d^2}+\frac{10e^3x^3}{d}+10e^2x^2+10dex+5d^2}{x^4\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{8e^4x^4}{d^2}+\frac{10e^3x^3}{d}+10e^2x^2+10dex+5d^2}{x^4\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}+\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{e^3(20d+23ex)}{3d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int -\frac{\frac{46e^4x^4}{d^2}+\frac{60e^3x^3}{d}+45e^2x^2+30dex+15d^2}{x^4\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}}{5d^2}+\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\frac{46e^4x^4}{d^2}+\frac{60e^3x^3}{d}+45e^2x^2+30dex+15d^2}{x^4\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{e^3(20d+23ex)}{3d^4\left(d^2-e^2x^2\right)^{3/2}}}{5d^2}+\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{\frac{2e^3(45d+53ex)}{d^4\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{15\left(\frac{6e^3x^3}{d}+4e^2x^2+2dex+d^2\right)}{x^4\sqrt{d^2-e^2x^2}}dx}{d^2}}{3d^2}+\frac{e^3(20d+23ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{15\int \frac{\frac{6e^3x^3}{d}+4e^2x^2+2dex+d^2}{x^4\sqrt{d^2-e^2x^2}}dx}{d^2}+\frac{2e^3(45d+53ex)}{d^4\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^3(20d+23ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

$\big\downarrow$ 2338

$$\frac{\frac{\frac{15\left(-\frac{\int -\frac{2\left(3ed^3+7e^2xd^2+9e^3x^2d\right)}{x^3\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}\right)}{d^2}+\frac{2e^3(45d+53ex)}{d^4\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^3(20d+23ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{15\left(\frac{2\int \frac{3ed^3+7e^2xd^2+9e^3x^2d}{x^3\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}\right)}{d^2}+\frac{2e^3(45d+53ex)}{d^4\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^3(20d+23ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

$\big\downarrow$ 2338

$$\frac{\frac{\frac{15\left(\frac{2\left(-\frac{\int -\frac{7d^3e^2(2d+3ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{3de\sqrt{d^2-e^2x^2}}{2x^2}\right)}{3d^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}\right)}{d^2}+\frac{2e^3(45d+53ex)}{d^4\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^3(20d+23ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{15\left(\frac{2\left(\frac{7}{2}de^2\int \frac{2d+3ex}{x^2\sqrt{d^2-e^2x^2}}dx-\frac{3de\sqrt{d^2-e^2x^2}}{2x^2}\right)}{3d^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}\right)}{d^2}+\frac{2e^3(45d+53ex)}{d^4\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^3(20d+23ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

$\big\downarrow$ 534

3.53. $\int \frac{(d+ex)^2}{x^4(d^2-e^2x^2)^{7/2}}\,dx$

$$\frac{\frac{\frac{15\left(\frac{2\left(\frac{7}{2}de^2\left(3e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{2\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{3de\sqrt{d^2-e^2x^2}}{2x^2}\right)}{3d^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}\right)}{d^2}+\frac{2e^3(45d+53ex)}{d^4\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^3(20d+23ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{\frac{15\left(\frac{2\left(\frac{7}{2}de^2\left(\frac{3}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{2\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{3de\sqrt{d^2-e^2x^2}}{2x^2}\right)}{3d^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}\right)}{d^2}+\frac{2e^3(45d+53ex)}{d^4\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^3(20d+23ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{15\left(\frac{2\left(\frac{7}{2}de^2\left(-\frac{3\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{2\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{3de\sqrt{d^2-e^2x^2}}{2x^2}\right)}{3d^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}\right)}{d^2}+\frac{2e^3(45d+53ex)}{d^4\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^3(20d+23ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{15\left(\frac{2\left(\frac{7}{2}de^2\left(-\frac{3e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{2\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{3de\sqrt{d^2-e^2x^2}}{2x^2}\right)}{3d^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}\right)}{d^2}+\frac{2e^3(45d+53ex)}{d^4\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^3(20d+23ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[(d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)),x]
```

3.53. $\int \frac{(d+ex)^2}{x^4(d^2-e^2x^2)^{7/2}}\,dx$

output
```
(2*e^3*(d + e*x))/(5*d^4*(d^2 - e^2*x^2)^(5/2)) + ((e^3*(20*d + 23*e*x))/(
3*d^4*(d^2 - e^2*x^2)^(3/2)) + ((2*e^3*(45*d + 53*e*x))/(d^4*Sqrt[d^2 - e^
2*x^2]) + (15*(-1/3*Sqrt[d^2 - e^2*x^2]/x^3 + (2*((-3*d*e*Sqrt[d^2 - e^2*x
^2])/(2*x^2) + (7*d*e^2*((-2*Sqrt[d^2 - e^2*x^2])/(d*x) - (3*e*ArcTanh[Sqr
t[d^2 - e^2*x^2]/d])/d))/2))/(3*d^2)))/d^2)/(3*d^2))/(5*d^2)
```

3.53.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.53.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.29

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(14e^2x^2+3dex+d^2)}{3d^8x^3}-\frac{7e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^7\sqrt{d^2}}-\frac{e^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{8d^8\left(x+\frac{d}{e}\right)}-\frac{833e^2\sqrt{-\left(x-\frac{d}{e}\right)^2e}}{120d^8\left(x-\right.}$
default	$e^2\left(-\frac{1}{d^2x(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{6e^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{d^2}\right)+d^2\left(-\frac{1}{3d^2x^3(-e^2x^2+d^2)^{\frac{5}{2}}}\right.$

input
```
int((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-e^2*x^2+d^2)^(1/2)*(14*e^2*x^2+3*d*e*x+d^2)/d^8/x^3-7*e^3/d^7/(d^2)
^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/8*e^2/d^8/(x+d/e
)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-833/120*e^2/d^8/(x-d/e)*(-(x-d/e)^2
*e^2-2*d*e*(x-d/e))^(1/2)+49/60/d^7*e/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d
/e))^(1/2)-1/10/d^6/(x-d/e)^3*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^2}{x^4 (d^2-e^2x^2)^{7/2}}\,dx = \frac{116\,e^7x^7 - 232\,de^6x^6 + 232\,d^3e^4x^4 - 116\,d^4e^3x^3 + 105\,(e^7x^7 - 2\,de^6x^6 + 2\,d^3e^4x^4}{}$$

input
```
integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(116*e^7*x^7 - 232*d*e^6*x^6 + 232*d^3*e^4*x^4 - 116*d^4*e^3*x^3 + 10
5*(e^7*x^7 - 2*d*e^6*x^6 + 2*d^3*e^4*x^4 - d^4*e^3*x^3)*log(-(d - sqrt(-e^
2*x^2 + d^2))/x) - (176*e^6*x^6 - 247*d*e^5*x^5 - 122*d^2*e^4*x^4 + 246*d^
3*e^3*x^3 - 40*d^4*e^2*x^2 - 5*d^5*e*x - 5*d^6)*sqrt(-e^2*x^2 + d^2))/(d^8
*e^4*x^7 - 2*d^9*e^3*x^6 + 2*d^11*e*x^4 - d^12*x^3)
```

3.53.6 Sympy [F]

$$\int \frac{(d+ex)^2}{x^4 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^2}{x^4 \left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/x**4/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**2/(x**4*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)^2}{x^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{22\,e^4x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}d^4} + \frac{7\,e^3}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3} + \frac{88\,e^4x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d^6} + \frac{7\,e^3}{3\,(-e^2x^2+d^2)^{\frac{3}{2}}d^5} - \frac{11\,e^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2x} + \frac{176\,e^4x}{15\,\sqrt{-e^2x^2+d^2}d^8} - \frac{7\,e^3\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d^8} + \frac{7\,e^3}{\sqrt{-e^2x^2+d^2}d^7} - \frac{e}{(-e^2x^2+d^2)^{\frac{5}{2}}dx^2} - \frac{1}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}x^3}$$

input
```
integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
22/5*e^4*x/((-e^2*x^2 + d^2)^(5/2)*d^4) + 7/5*e^3/((-e^2*x^2 + d^2)^(5/2)*
d^3) + 88/15*e^4*x/((-e^2*x^2 + d^2)^(3/2)*d^6) + 7/3*e^3/((-e^2*x^2 + d^2
)^(3/2)*d^5) - 11/3*e^2/((-e^2*x^2 + d^2)^(5/2)*d^2*x) + 176/15*e^4*x/(sqr
t(-e^2*x^2 + d^2)*d^8) - 7*e^3*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d
/abs(x))/d^8 + 7*e^3/(sqrt(-e^2*x^2 + d^2)*d^7) - e/((-e^2*x^2 + d^2)^(5/2
)*d*x^2) - 1/3/((-e^2*x^2 + d^2)^(5/2)*x^3)
```

3.53.8 Giac [F]

$$\int \frac{(d+ex)^2}{x^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{(ex+d)^2}{(-e^2x^2+d^2)^{\frac{7}{2}}x^4}\,dx$$

input
```
integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2/((-e^2*x^2 + d^2)^(7/2)*x^4), x)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{\left(d+e\,x\right)^2}{x^4\,\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

input
```
int((d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)),x)
```

output
```
int((d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)), x)
```

3.54 $\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx$

3.54.1 Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{3}{5}x^2\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{5}x^4\sqrt{1-x^2} -\frac{3}{20}(8+5x)\sqrt{1-x^2}+\frac{3\arcsin(x)}{4}$$

output
```
3/4*arcsin(x)-3/5*x^2*(-x^2+1)^(1/2)-1/2*x^3*(-x^2+1)^(1/2)-1/5*x^4*(-x^2+
1)^(1/2)-3/20*(8+5*x)*(-x^2+1)^(1/2)
```

3.54.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx = \frac{1}{20}\sqrt{1-x^2}\left(-24-15x-12x^2-10x^3-4x^4\right)+\frac{3}{2}\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

input
```
Integrate[(x^3*(1 + x)^2)/Sqrt[1 - x^2],x]
```

output
```
(Sqrt[1 - x^2]*(-24 - 15*x - 12*x^2 - 10*x^3 - 4*x^4))/20 + (3*ArcTan[x/(-
1 + Sqrt[1 - x^2])])/2
```

3.54.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.25, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {541, 25, 533, 27, 533, 27, 533, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(x+1)^2}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 541}$$

$$-\frac{1}{5}\int -\frac{x^3(10x+9)}{\sqrt{1-x^2}}dx-\frac{1}{5}\sqrt{1-x^2}x^4$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{5}\int \frac{x^3(10x+9)}{\sqrt{1-x^2}}dx-\frac{1}{5}x^4\sqrt{1-x^2}$$

$$\Big\downarrow \text{ 533}$$

$$\frac{1}{5}\left(\frac{1}{4}\int \frac{6x^2(6x+5)}{\sqrt{1-x^2}}dx-\frac{5}{2}x^3\sqrt{1-x^2}\right)-\frac{1}{5}x^4\sqrt{1-x^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\left(\frac{3}{2}\int \frac{x^2(6x+5)}{\sqrt{1-x^2}}dx-\frac{5}{2}x^3\sqrt{1-x^2}\right)-\frac{1}{5}x^4\sqrt{1-x^2}$$

$$\Big\downarrow \text{ 533}$$

$$\frac{1}{5}\left(\frac{3}{2}\left(\frac{1}{3}\int \frac{3x(5x+4)}{\sqrt{1-x^2}}dx-2x^2\sqrt{1-x^2}\right)-\frac{5}{2}x^3\sqrt{1-x^2}\right)-\frac{1}{5}x^4\sqrt{1-x^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\left(\frac{3}{2}\left(\int \frac{x(5x+4)}{\sqrt{1-x^2}}dx-2x^2\sqrt{1-x^2}\right)-\frac{5}{2}x^3\sqrt{1-x^2}\right)-\frac{1}{5}x^4\sqrt{1-x^2}$$

$$\Big\downarrow \text{ 533}$$

$$\frac{1}{5}\left(\frac{3}{2}\left(\frac{1}{2}\int \frac{8x+5}{\sqrt{1-x^2}}dx-2\sqrt{1-x^2}x^2-\frac{5}{2}\sqrt{1-x^2}x\right)-\frac{5}{2}x^3\sqrt{1-x^2}\right)-\frac{1}{5}x^4\sqrt{1-x^2}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{1}{5}\left(\frac{3}{2}\left(\frac{1}{2}\left(5\int \frac{1}{\sqrt{1-x^2}}dx-8\sqrt{1-x^2}\right)-2\sqrt{1-x^2}x^2-\frac{5}{2}\sqrt{1-x^2}x\right)-\frac{5}{2}x^3\sqrt{1-x^2}\right)-\frac{1}{5}x^4\sqrt{1-x^2}$$

$$\downarrow 223$$

$$\frac{1}{5}\left(\frac{3}{2}\left(\frac{1}{2}\left(5\arcsin(x)-8\sqrt{1-x^2}\right)-2\sqrt{1-x^2}x^2-\frac{5}{2}\sqrt{1-x^2}x\right)-\frac{5}{2}x^3\sqrt{1-x^2}\right)-\frac{1}{5}x^4\sqrt{1-x^2}$$

input
```
Int[(x^3*(1 + x)^2)/Sqrt[1 - x^2],x]
```

output
```
-1/5*(x^4*Sqrt[1 - x^2]) + ((-5*x^3*Sqrt[1 - x^2])/2 + (3*((-5*x*x*Sqrt[1 -
x^2])/2 - 2*x^2*Sqrt[1 - x^2] + (-8*Sqrt[1 - x^2] + 5*ArcSin[x])/2))/2)/5
```

3.54.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

3.54.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

method	result	size
risch	$\frac{(4x^4+10x^3+12x^2+15x+24)(x^2-1)}{20\sqrt{-x^2+1}}+\frac{3\arcsin(x)}{4}$	42
trager	$\left(-\frac{1}{5}x^4-\frac{1}{2}x^3-\frac{3}{5}x^2-\frac{3}{4}x-\frac{6}{5}\right)\sqrt{-x^2+1}+\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{4}$	59
default	$-\frac{x^4\sqrt{-x^2+1}}{5}-\frac{3x^2\sqrt{-x^2+1}}{5}-\frac{6\sqrt{-x^2+1}}{5}-\frac{x^3\sqrt{-x^2+1}}{2}-\frac{3x\sqrt{-x^2+1}}{4}+\frac{3\arcsin(x)}{4}$	71
meijerg	$\frac{\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(4x^2+8\right)\sqrt{-x^2+1}}{6}}{2\sqrt{\pi}}-\frac{i\left(-\frac{i\sqrt{\pi}\,x\left(10x^2+15\right)\sqrt{-x^2+1}}{20}+\frac{3i\sqrt{\pi}\,\arcsin(x)}{4}\right)}{\sqrt{\pi}}-\frac{-\frac{16\sqrt{\pi}}{15}+\frac{\sqrt{\pi}\left(6x^4+8x^2+16\right)\sqrt{-x^2+1}}{15}}{2\sqrt{\pi}}$	109

input
```
int(x^3*(1+x)^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*(4*x^4+10*x^3+12*x^2+15*x+24)*(x^2-1)/(-x^2+1)^(1/2)+3/4*arcsin(x)
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{20}\left(4\,x^4+10\,x^3+12\,x^2+15\,x+24\right)\sqrt{-x^2+1} - \frac{3}{2}\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate(x^3*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/20*(4*x^4 + 10*x^3 + 12*x^2 + 15*x + 24)*sqrt(-x^2 + 1) - 3/2*arctan((s
qrt(-x^2 + 1) - 1)/x)
```

3.54.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{x^4\sqrt{1-x^2}}{5} - \frac{x^3\sqrt{1-x^2}}{2} - \frac{3x^2\sqrt{1-x^2}}{5} - \frac{3x\sqrt{1-x^2}}{4} - \frac{6\sqrt{1-x^2}}{5} + \frac{3\operatorname{asin}(x)}{4}$$

input
```
integrate(x**3*(1+x)**2/(-x**2+1)**(1/2),x)
```

output
```
-x**4*sqrt(1 - x**2)/5 - x**3*sqrt(1 - x**2)/2 - 3*x**2*sqrt(1 - x**2)/5 -
 3*x*sqrt(1 - x**2)/4 - 6*sqrt(1 - x**2)/5 + 3*asin(x)/4
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{5}\sqrt{-x^2+1}x^4 - \frac{1}{2}\sqrt{-x^2+1}x^3 - \frac{3}{5}\sqrt{-x^2+1}x^2 - \frac{3}{4}\sqrt{-x^2+1}x - \frac{6}{5}\sqrt{-x^2+1} + \frac{3}{4}\arcsin(x)$$

input
```
integrate(x^3*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/5*sqrt(-x^2 + 1)*x^4 - 1/2*sqrt(-x^2 + 1)*x^3 - 3/5*sqrt(-x^2 + 1)*x^2
- 3/4*sqrt(-x^2 + 1)*x - 6/5*sqrt(-x^2 + 1) + 3/4*arcsin(x)
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.42

$$\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{20}\left((2\left((2x+5)x+6\right)x+15)x+24\right)\sqrt{-x^2+1} + \frac{3}{4}\arcsin(x)$$

input
```
integrate(x^3*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/20*((2*((2*x + 5)*x + 6)*x + 15)*x + 24)*sqrt(-x^2 + 1) + 3/4*arcsin(x)
```

3.54.9 Mupad [B] (verification not implemented)

Time = 11.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.44

$$\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx = \frac{3\,\mathrm{asin}(x)}{4} - \sqrt{1-x^2}\left(\frac{x^4}{5}+\frac{x^3}{2}+\frac{3\,x^2}{5}+\frac{3\,x}{4}+\frac{6}{5}\right)$$

input
```
int((x^3*(x + 1)^2)/(1 - x^2)^(1/2),x)
```

output
```
(3*asin(x))/4 - (1 - x^2)^(1/2)*((3*x)/4 + (3*x^2)/5 + x^3/2 + x^4/5 + 6/5
)
```

3.55 $\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}} \, dx$

3.55.1 Optimal result

Integrand size = 20, antiderivative size = 63

$$\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}} \, dx = -\frac{2}{3}x^2\sqrt{1-x^2} - \frac{1}{4}x^3\sqrt{1-x^2} - \frac{1}{24}(32+21x)\sqrt{1-x^2} + \frac{7\arcsin(x)}{8}$$

output
```
7/8*arcsin(x)-2/3*x^2*(-x^2+1)^(1/2)-1/4*x^3*(-x^2+1)^(1/2)-1/24*(32+21*x)
*(-x^2+1)^(1/2)
```

3.55.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}} \, dx = \frac{1}{24}\sqrt{1-x^2}\left(-32-21x-16x^2-6x^3\right) + \frac{7}{4}\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

input
```
Integrate[(x^2*(1 + x)^2)/Sqrt[1 - x^2],x]
```

output
```
(Sqrt[1 - x^2]*(-32 - 21*x - 16*x^2 - 6*x^3))/24 + (7*ArcTan[x/(-1 + Sqrt[
1 - x^2])])/4
```

3.55.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {541, 25, 533, 533, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(x+1)^2}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 541}$$

$$-\frac{1}{4}\int -\frac{x^2(8x+7)}{\sqrt{1-x^2}}dx-\frac{1}{4}\sqrt{1-x^2}x^3$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{4}\int \frac{x^2(8x+7)}{\sqrt{1-x^2}}dx-\frac{1}{4}x^3\sqrt{1-x^2}$$

$$\Big\downarrow \text{ 533}$$

$$\frac{1}{4}\left(\frac{1}{3}\int \frac{x(21x+16)}{\sqrt{1-x^2}}dx-\frac{8}{3}x^2\sqrt{1-x^2}\right)-\frac{1}{4}x^3\sqrt{1-x^2}$$

$$\Big\downarrow \text{ 533}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{32x+21}{\sqrt{1-x^2}}dx-\frac{21}{2}x\sqrt{1-x^2}\right)-\frac{8}{3}x^2\sqrt{1-x^2}\right)-\frac{1}{4}x^3\sqrt{1-x^2}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(21\int \frac{1}{\sqrt{1-x^2}}dx-32\sqrt{1-x^2}\right)-\frac{21}{2}x\sqrt{1-x^2}\right)-\frac{8}{3}x^2\sqrt{1-x^2}\right)-\frac{1}{4}x^3\sqrt{1-x^2}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(21\arcsin(x)-32\sqrt{1-x^2}\right)-\frac{21}{2}x\sqrt{1-x^2}\right)-\frac{8}{3}x^2\sqrt{1-x^2}\right)-\frac{1}{4}x^3\sqrt{1-x^2}$$

input
```
Int[(x^2*(1 + x)^2)/Sqrt[1 - x^2],x]
```

output
```
-1/4*(x^3*Sqrt[1 - x^2]) + ((-8*x^2*Sqrt[1 - x^2])/3 + ((-21*x*Sqrt[1 - x^
2])/2 + (-32*Sqrt[1 - x^2] + 21*ArcSin[x])/2)/3)/4
```

3.55.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

3.55.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.59

method	result	size
risch	$\frac{(6x^3+16x^2+21x+32)(x^2-1)}{24\sqrt{-x^2+1}}+\frac{7\arcsin(x)}{8}$	37
trager	$\left(-\frac{1}{4}x^3-\frac{2}{3}x^2-\frac{7}{8}x-\frac{4}{3}\right)\sqrt{-x^2+1}+\frac{7\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{8}$	54
default	$-\frac{x^3\sqrt{-x^2+1}}{4}-\frac{7x\sqrt{-x^2+1}}{8}+\frac{7\arcsin(x)}{8}-\frac{2x^2\sqrt{-x^2+1}}{3}-\frac{4\sqrt{-x^2+1}}{3}$	57
meijerg	$\frac{i\left(i\sqrt{\pi}\,x\sqrt{-x^2+1}-i\sqrt{\pi}\,\arcsin(x)\right)}{2\sqrt{\pi}}+\frac{\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\,(4x^2+8)\sqrt{-x^2+1}}{6}}{\sqrt{\pi}}-\frac{i\left(-\frac{i\sqrt{\pi}\,x(10x^2+15)\sqrt{-x^2+1}}{20}+\frac{3i\sqrt{\pi}\,\arcsin(x)}{4}\right)}{2\sqrt{\pi}}$	102

input
```
int(x^2*(1+x)^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*x^3+16*x^2+21*x+32)*(x^2-1)/(-x^2+1)^(1/2)+7/8*arcsin(x)
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.71

$$\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{24}\left(6\,x^3+16\,x^2+21\,x+32\right)\sqrt{-x^2+1}-\frac{7}{4}\,\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate(x^2*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(6*x^3 + 16*x^2 + 21*x + 32)*sqrt(-x^2 + 1) - 7/4*arctan((sqrt(-x^2
+ 1) - 1)/x)
```

3.55.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{x^3\sqrt{1-x^2}}{4}-\frac{2x^2\sqrt{1-x^2}}{3}-\frac{7x\sqrt{1-x^2}}{8}-\frac{4\sqrt{1-x^2}}{3}+\frac{7\,\mathrm{asin}\,(x)}{8}$$

input
```
integrate(x**2*(1+x)**2/(-x**2+1)**(1/2),x)
```

output
```
-x**3*sqrt(1 - x**2)/4 - 2*x**2*sqrt(1 - x**2)/3 - 7*x*sqrt(1 - x**2)/8 -
4*sqrt(1 - x**2)/3 + 7*asin(x)/8
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{4}\sqrt{-x^2+1}x^3 - \frac{2}{3}\sqrt{-x^2+1}x^2 - \frac{7}{8}\sqrt{-x^2+1}x - \frac{4}{3}\sqrt{-x^2+1} + \frac{7}{8}\arcsin{(x)}$$

input
```
integrate(x^2*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(-x^2 + 1)*x^3 - 2/3*sqrt(-x^2 + 1)*x^2 - 7/8*sqrt(-x^2 + 1)*x -
4/3*sqrt(-x^2 + 1) + 7/8*arcsin(x)
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.48

$$\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{24}\left((2\,(3\,x+8)x+21)x+32\right)\sqrt{-x^2+1} + \frac{7}{8}\arcsin{(x)}$$

input
```
integrate(x^2*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/24*((2*(3*x + 8)*x + 21)*x + 32)*sqrt(-x^2 + 1) + 7/8*arcsin(x)
```

3.55.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.49

$$\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}}\,dx = \frac{7\,\mathrm{asin}(x)}{8} - \sqrt{1-x^2}\left(\frac{x^3}{4} + \frac{2\,x^2}{3} + \frac{7\,x}{8} + \frac{4}{3}\right)$$

input
```
int((x^2*(x + 1)^2)/(1 - x^2)^(1/2),x)
```

output
```
(7*asin(x))/8 - (1 - x^2)^(1/2)*((7*x)/8 + (2*x^2)/3 + x^3/4 + 4/3)
```

3.56 $\int \frac{x(1+x)^2}{\sqrt{1-x^2}}\,dx$

3.56.1 Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{x(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{3}x^2\sqrt{1-x^2}-\frac{1}{3}(5+3x)\sqrt{1-x^2}+\arcsin(x)$$

output
```
arcsin(x)-1/3*x^2*(-x^2+1)^(1/2)-1/3*(5+3*x)*(-x^2+1)^(1/2)
```

3.56.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{x(1+x)^2}{\sqrt{1-x^2}}\,dx = \frac{1}{3}\sqrt{1-x^2}\left(-5-3x-x^2\right)+2\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

input
```
Integrate[(x*(1 + x)^2)/Sqrt[1 - x^2],x]
```

output
```
(Sqrt[1 - x^2]*(-5 - 3*x - x^2))/3 + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

3.56.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {541, 25, 533, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(x+1)^2}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{541}$$

$$-\frac{1}{3}\int -\frac{x(6x+5)}{\sqrt{1-x^2}}dx-\frac{1}{3}\sqrt{1-x^2}x^2$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{x(6x+5)}{\sqrt{1-x^2}}dx-\frac{1}{3}x^2\sqrt{1-x^2}$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{2(5x+3)}{\sqrt{1-x^2}}dx-3x\sqrt{1-x^2}\right)-\frac{1}{3}x^2\sqrt{1-x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\int \frac{5x+3}{\sqrt{1-x^2}}dx-3x\sqrt{1-x^2}\right)-\frac{1}{3}x^2\sqrt{1-x^2}$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{3}\left(3\int \frac{1}{\sqrt{1-x^2}}dx-3\sqrt{1-x^2}x-5\sqrt{1-x^2}\right)-\frac{1}{3}x^2\sqrt{1-x^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{3}\left(3\arcsin(x)-3\sqrt{1-x^2}x-5\sqrt{1-x^2}\right)-\frac{1}{3}x^2\sqrt{1-x^2}$$

input
```
Int[(x*(1 + x)^2)/Sqrt[1 - x^2],x]
```

output
```
-1/3*(x^2*Sqrt[1 - x^2]) + (-5*Sqrt[1 - x^2] - 3*x*Sqrt[1 - x^2] + 3*ArcSi
n[x])/3
```

3.56.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

3.56. $\int \frac{x(1+x)^2}{\sqrt{1-x^2}}\,dx$

3.56.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

method	result	size
risch	$\frac{(x^2+3x+5)(x^2-1)}{3\sqrt{-x^2+1}}+\arcsin(x)$	28
default	$-\frac{5\sqrt{-x^2+1}}{3}-\frac{x^2\sqrt{-x^2+1}}{3}-x\sqrt{-x^2+1}+\arcsin(x)$	41
trager	$\left(-\frac{1}{3}x^2-x-\frac{5}{3}\right)\sqrt{-x^2+1}+\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)$	48
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\sqrt{-x^2+1}}{2\sqrt{\pi}}+\frac{i\left(i\sqrt{\pi}\,x\sqrt{-x^2+1}-i\sqrt{\pi}\arcsin(x)\right)}{\sqrt{\pi}}+\frac{\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(4x^2+8\right)\sqrt{-x^2+1}}{6}}{2\sqrt{\pi}}$	90

input
```
int(x*(1+x)^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(x^2+3*x+5)*(x^2-1)/(-x^2+1)^(1/2)+arcsin(x)
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{x(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{3}\left(x^2+3x+5\right)\sqrt{-x^2+1}-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate(x*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(x^2 + 3*x + 5)*sqrt(-x^2 + 1) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

3.56.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{x(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{x^2\sqrt{1-x^2}}{3}-x\sqrt{1-x^2}-\frac{5\sqrt{1-x^2}}{3}+\operatorname{asin}(x)$$

input
```
integrate(x*(1+x)**2/(-x**2+1)**(1/2),x)
```

output
```
-x**2*sqrt(1 - x**2)/3 - x*sqrt(1 - x**2) - 5*sqrt(1 - x**2)/3 + asin(x)
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{x(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{3}\sqrt{-x^2+1}x^2 - \sqrt{-x^2+1}x - \frac{5}{3}\sqrt{-x^2+1} + \arcsin(x)$$

input
```
integrate(x*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(-x^2 + 1)*x^2 - sqrt(-x^2 + 1)*x - 5/3*sqrt(-x^2 + 1) + arcsin(x
)
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.51

$$\int \frac{x(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{3}\left((x+3)x+5\right)\sqrt{-x^2+1} + \arcsin(x)$$

input
```
integrate(x*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/3*((x + 3)*x + 5)*sqrt(-x^2 + 1) + arcsin(x)
```

3.56.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.54

$$\int \frac{x(1+x)^2}{\sqrt{1-x^2}}\,dx = \operatorname{asin}(x) - \sqrt{1-x^2}\left(\frac{x^2}{3} + x + \frac{5}{3}\right)$$

input
```
int((x*(x + 1)^2)/(1 - x^2)^(1/2),x)
```

output
```
asin(x) - (1 - x^2)^(1/2)*(x + x^2/3 + 5/3)
```

3.57 $\int \frac{(1+x)^2}{\sqrt{1-x^2}}\,dx$

3.57.1 Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \frac{(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{3}{2}\sqrt{1-x^2}-\frac{1}{2}(1+x)\sqrt{1-x^2}+\frac{3\arcsin(x)}{2}$$

output
```
3/2*arcsin(x)-3/2*(-x^2+1)^(1/2)-1/2*(1+x)*(-x^2+1)^(1/2)
```

3.57.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{(1+x)^2}{\sqrt{1-x^2}}\,dx = \frac{1}{2}(-4-x)\sqrt{1-x^2}-3\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[(1 + x)^2/Sqrt[1 - x^2],x]
```

output
```
((-4 - x)*Sqrt[1 - x^2])/2 - 3*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

3.57.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {469, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^2}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{469}$$

$$\frac{3}{2}\int \frac{x+1}{\sqrt{1-x^2}}dx-\frac{1}{2}(x+1)\sqrt{1-x^2}$$

$$\Big\downarrow \text{455}$$

$$\frac{3}{2}\left(\int \frac{1}{\sqrt{1-x^2}}dx-\sqrt{1-x^2}\right)-\frac{1}{2}(x+1)\sqrt{1-x^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{3}{2}\left(\arcsin(x)-\sqrt{1-x^2}\right)-\frac{1}{2}(x+1)\sqrt{1-x^2}$$

input
```
Int[(1 + x)^2/Sqrt[1 - x^2],x]
```

output
```
-1/2*((1 + x)*Sqrt[1 - x^2]) + (3*(-Sqrt[1 - x^2] + ArcSin[x]))/2
```

3.57.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

3.57.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.62

method	result	size
risch	$\frac{(4+x)(x^2-1)}{2\sqrt{-x^2+1}}+\frac{3\arcsin(x)}{2}$	25
default	$\frac{3\arcsin(x)}{2}-\frac{x\sqrt{-x^2+1}}{2}-2\sqrt{-x^2+1}$	29
trager	$\left(-2-\frac{x}{2}\right)\sqrt{-x^2+1}+\frac{3\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}$	44
meijerg	$\arcsin(x)-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^2+1}}{\sqrt{\pi}}+\frac{i\left(i\sqrt{\pi}\,x\sqrt{-x^2+1}-i\sqrt{\pi}\,\arcsin(x)\right)}{2\sqrt{\pi}}$	60

input
```
int((1+x)^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(4+x)*(x^2-1)/(-x^2+1)^(1/2)+3/2*arcsin(x)
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{(1+x)^2}{\sqrt{1-x^2}}\,dx=-\frac{1}{2}\sqrt{-x^2+1}(x+4)-3\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-x^2 + 1)*(x + 4) - 3*arctan((sqrt(-x^2 + 1) - 1)/x)
```

3.57.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.68

$$\int \frac{(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{x\sqrt{1-x^2}}{2} - 2\sqrt{1-x^2} + \frac{3\operatorname{asin}(x)}{2}$$

input
```
integrate((1+x)**2/(-x**2+1)**(1/2),x)
```

output
```
-x*sqrt(1 - x**2)/2 - 2*sqrt(1 - x**2) + 3*asin(x)/2
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int \frac{(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\sqrt{-x^2+1}x - 2\sqrt{-x^2+1} + \frac{3}{2}\arcsin(x)$$

input
```
integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(-x^2 + 1)*x - 2*sqrt(-x^2 + 1) + 3/2*arcsin(x)
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.48

$$\int \frac{(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\sqrt{-x^2+1}(x+4) + \frac{3}{2}\arcsin(x)$$

input
```
integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-x^2 + 1)*(x + 4) + 3/2*arcsin(x)
```

3.57.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

$$\int \frac{(1+x)^2}{\sqrt{1-x^2}}\,dx = \frac{3\,\mathrm{asin}(x)}{2} - \left(\frac{x}{2}+2\right)\sqrt{1-x^2}$$

input
```
int((x + 1)^2/(1 - x^2)^(1/2),x)
```

output
```
(3*asin(x))/2 - (x/2 + 2)*(1 - x^2)^(1/2)
```

3.58 $\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx$

3.58.1 Optimal result

Integrand size = 20, antiderivative size = 32

$$\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2}+2\arcsin(x)-\operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

output
```
2*arcsin(x)-arctanh((-x^2+1)^(1/2))-(-x^2+1)^(1/2)
```

3.58.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2}+4\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)-\log(x)+\log\left(-1+\sqrt{1-x^2}\right)$$

input
```
Integrate[(1 + x)^2/(x*Sqrt[1 - x^2]),x]
```

output
```
-Sqrt[1 - x^2] + 4*ArcTan[x/(-1 + Sqrt[1 - x^2])] - Log[x] + Log[-1 + Sqrt
[1 - x^2]]
```

3.58.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {541, 25, 538, 223, 243, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^2}{x\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 541}$$

$$-\int -\frac{2x+1}{x\sqrt{1-x^2}}dx-\sqrt{1-x^2}$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{2x+1}{x\sqrt{1-x^2}}dx-\sqrt{1-x^2}$$

$$\Big\downarrow \text{ 538}$$

$$2\int \frac{1}{\sqrt{1-x^2}}dx+\int \frac{1}{x\sqrt{1-x^2}}dx-\sqrt{1-x^2}$$

$$\Big\downarrow \text{ 223}$$

$$\int \frac{1}{x\sqrt{1-x^2}}dx+2\arcsin(x)-\sqrt{1-x^2}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{1-x^2}}dx^2+2\arcsin(x)-\sqrt{1-x^2}$$

$$\Big\downarrow \text{ 73}$$

$$-\int \frac{1}{1-x^4}d\sqrt{1-x^2}+2\arcsin(x)-\sqrt{1-x^2}$$

$$\Big\downarrow \text{ 219}$$

$$2\arcsin(x)-\text{arctanh}\left(\sqrt{1-x^2}\right)-\sqrt{1-x^2}$$

input
```
Int[(1 + x)^2/(x*Sqrt[1 - x^2]),x]
```

output
```
-Sqrt[1 - x^2] + 2*ArcSin[x] - ArcTanh[Sqrt[1 - x^2]]
```

3.58.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

3.58.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	si
default	$2\arcsin\left(x\right)-\sqrt{-x^{2}+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)$	29
trager	$-\sqrt{-x^{2}+1}+\ln\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)-2\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}+1\right)x+\sqrt{-x^{2}+1}\right)$	56
meijerg	$\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^{2}+1}}{2}\right)+\left(-2\ln\left(2\right)+2\ln\left(x\right)+i\pi\right)\sqrt{\pi}}{2\sqrt{\pi}}+2\arcsin\left(x\right)-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^{2}+1}}{2\sqrt{\pi}}$	73

input
```
int((1+x)^2/x/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arcsin(x)-(-x^2+1)^(1/2)-arctanh(1/(-x^2+1)^(1/2))
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.44

$$\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx = -\sqrt{-x^2+1}-4\,\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate((1+x)^2/x/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-x^2 + 1) - 4*arctan((sqrt(-x^2 + 1) - 1)/x) + log((sqrt(-x^2 + 1) -
 1)/x)
```

3.58.6 Sympy [A] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2}+\begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for } \frac{1}{|x^2|}>1 \\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases}+2\operatorname{asin}(x)$$

input
```
integrate((1+x)**2/x/(-x**2+1)**(1/2),x)
```

output
```
-sqrt(1 - x**2) + Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x),
True)) + 2*asin(x)
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx = -\sqrt{-x^2+1} + 2\arcsin(x) - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

input
```
integrate((1+x)^2/x/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-x^2 + 1) + 2*arcsin(x) - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx = -\sqrt{-x^2+1} + 2\arcsin(x) + \log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

input
```
integrate((1+x)^2/x/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(-x^2 + 1) + 2*arcsin(x) + log(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

3.58.9 Mupad [B] (verification not implemented)

Time = 11.44 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx = 2\,\mathrm{asin}(x) + \ln\left(\sqrt{\frac{1}{x^2}-1} - \sqrt{\frac{1}{x^2}}\right) - \sqrt{1-x^2}$$

input
```
int((x + 1)^2/(x*(1 - x^2)^(1/2)),x)
```

output
```
2*asin(x) + log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - (1 - x^2)^(1/2)
```

3.59 $\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx$

3.59.1 Optimal result

Integrand size = 20, antiderivative size = 33

$$\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx=-\frac{\sqrt{1-x^2}}{x}+\arcsin(x)-2\text{arctanh}\left(\sqrt{1-x^2}\right)$$

output
```
arcsin(x)-2*arctanh((-x^2+1)^(1/2))-(-x^2+1)^(1/2)/x
```

3.59.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

$$\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx=-\frac{\sqrt{1-x^2}}{x}+2\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)-2\log(x)+2\log\left(-1+\sqrt{1-x^2}\right)$$

input
```
Integrate[(1 + x)^2/(x^2*Sqrt[1 - x^2]),x]
```

output
```
-(Sqrt[1 - x^2]/x) + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])] - 2*Log[x] + 2*Log[-
1 + Sqrt[1 - x^2]]
```

3.59.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {540, 25, 538, 223, 243, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^2}{x^2\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 540}$$

$$-\int -\frac{x+2}{x\sqrt{1-x^2}}dx-\frac{\sqrt{1-x^2}}{x}$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{x+2}{x\sqrt{1-x^2}}dx-\frac{\sqrt{1-x^2}}{x}$$

$$\Big\downarrow \text{ 538}$$

$$\int \frac{1}{\sqrt{1-x^2}}dx+2\int \frac{1}{x\sqrt{1-x^2}}dx-\frac{\sqrt{1-x^2}}{x}$$

$$\Big\downarrow \text{ 223}$$

$$2\int \frac{1}{x\sqrt{1-x^2}}dx+\arcsin(x)-\frac{\sqrt{1-x^2}}{x}$$

$$\Big\downarrow \text{ 243}$$

$$\int \frac{1}{x^2\sqrt{1-x^2}}dx^2+\arcsin(x)-\frac{\sqrt{1-x^2}}{x}$$

$$\Big\downarrow \text{ 73}$$

$$-2\int \frac{1}{1-x^4}d\sqrt{1-x^2}+\arcsin(x)-\frac{\sqrt{1-x^2}}{x}$$

$$\Big\downarrow \text{ 219}$$

$$\arcsin(x)-2\text{arctanh}\left(\sqrt{1-x^2}\right)-\frac{\sqrt{1-x^2}}{x}$$

input
```
Int[(1 + x)^2/(x^2*Sqrt[1 - x^2]),x]
```

output
```
-(Sqrt[1 - x^2]/x) + ArcSin[x] - 2*ArcTanh[Sqrt[1 - x^2]]
```

3.59.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

3.59.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
default	$\arcsin\left(x\right)-\frac{\sqrt{-x^{2}+1}}{x}-2\ \operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)$	30
risch	$\frac{x^{2}-1}{x\sqrt{-x^{2}+1}}+\arcsin\left(x\right)-2\ \operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)$	34
meijerg	$-\frac{\sqrt{-x^{2}+1}}{x}+\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^{2}+1}}{2}\right)+\left(-2\ln\left(2\right)+2\ln\left(x\right)+i\pi\right)\sqrt{\pi}}{\sqrt{\pi}}+\arcsin\left(x\right)$	59
trager	$-\frac{\sqrt{-x^{2}+1}}{x}+2\ln\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)-\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}+1\right)x+\sqrt{-x^{2}+1}\right)$	61

input
```
int((1+x)^2/x^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(x)-(-x^2+1)^(1/2)/x-2*arctanh(1/(-x^2+1)^(1/2))
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.61

$$\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx=-\frac{2\,x\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-2\,x\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+\sqrt{-x^2+1}}{x}$$

input
```
integrate((1+x)^2/x^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-(2*x*arctan((sqrt(-x^2 + 1) - 1)/x) - 2*x*log((sqrt(-x^2 + 1) - 1)/x) + s
qrt(-x^2 + 1))/x
```

3.59.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx=\begin{cases}-\frac{i\sqrt{x^2-1}}{x} & \text{for}\ |x^2|>1\\ -\frac{\sqrt{1-x^2}}{x} & \text{otherwise}\end{cases}+2\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for}\ \frac{1}{|x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise}\end{cases}\right)+\operatorname{asin}(x)$$

input
```
integrate((1+x)**2/x**2/(-x**2+1)**(1/2),x)
```

output
```
Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x, True))
 + 2*Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True)) + asin
(x)
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx = -\frac{\sqrt{-x^2+1}}{x} + \arcsin(x) - 2\log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

input
```
integrate((1+x)^2/x^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-x^2 + 1)/x + arcsin(x) - 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.67

$$\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx = \frac{x}{2\left(\sqrt{-x^2+1}-1\right)} - \frac{\sqrt{-x^2+1}-1}{2x} + \arcsin(x) + 2\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

input
```
integrate((1+x)^2/x^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*x/(sqrt(-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)/x + arcsin(x) + 2*lo
g(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

3.59.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx = \operatorname{asin}(x) + 2\,\ln\left(\sqrt{\frac{1}{x^2}-1} - \sqrt{\frac{1}{x^2}}\right) - \frac{\sqrt{1-x^2}}{x}$$

input
```
int((x + 1)^2/(x^2*(1 - x^2)^(1/2)),x)
```

output
```
asin(x) + 2*log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - (1 - x^2)^(1/2)/x
```

3.60 $\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx$

3.60.1 Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx=-\frac{\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x}-\frac{3}{2}\text{arctanh}\left(\sqrt{1-x^2}\right)$$

output
```
-3/2*arctanh((-x^2+1)^(1/2))-1/2*(-x^2+1)^(1/2)/x^2-2*(-x^2+1)^(1/2)/x
```

3.60.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx=\frac{(-1-4x)\sqrt{1-x^2}}{2x^2}-\frac{3\log(x)}{2}+\frac{3}{2}\log\left(-1+\sqrt{1-x^2}\right)$$

input
```
Integrate[(1 + x)^2/(x^3*Sqrt[1 - x^2]),x]
```

output
```
((-1 - 4*x)*Sqrt[1 - x^2])/(2*x^2) - (3*Log[x])/2 + (3*Log[-1 + Sqrt[1 - x
^2]])/2
```

3.60.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {540, 25, 534, 243, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^2}{x^3\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{540}$$

$$-\frac{1}{2}\int -\frac{3x+4}{x^2\sqrt{1-x^2}}dx-\frac{\sqrt{1-x^2}}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{3x+4}{x^2\sqrt{1-x^2}}dx-\frac{\sqrt{1-x^2}}{2x^2}$$

$$\Big\downarrow \text{534}$$

$$\frac{1}{2}\left(3\int \frac{1}{x\sqrt{1-x^2}}dx-\frac{4\sqrt{1-x^2}}{x}\right)-\frac{\sqrt{1-x^2}}{2x^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(\frac{3}{2}\int \frac{1}{x^2\sqrt{1-x^2}}dx^2-\frac{4\sqrt{1-x^2}}{x}\right)-\frac{\sqrt{1-x^2}}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-3\int \frac{1}{1-x^4}d\sqrt{1-x^2}-\frac{4\sqrt{1-x^2}}{x}\right)-\frac{\sqrt{1-x^2}}{2x^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-3\text{arctanh}\left(\sqrt{1-x^2}\right)-\frac{4\sqrt{1-x^2}}{x}\right)-\frac{\sqrt{1-x^2}}{2x^2}$$

input
```
Int[(1 + x)^2/(x^3*Sqrt[1 - x^2]),x]
```

output
```
-1/2*Sqrt[1 - x^2]/x^2 + ((-4*Sqrt[1 - x^2])/x - 3*ArcTanh[Sqrt[1 - x^2]])
/2
```

3.60.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

3.60. $\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx$

3.60.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result
trager	$-\frac{(1+4x)\sqrt{-x^2+1}}{2x^2}+\frac{3\ln\left(\frac{\sqrt{-x^2+1}-1}{x}\right)}{2}$
risch	$\frac{4x^3+x^2-4x-1}{2x^2\sqrt{-x^2+1}}-\frac{3\,\text{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{2}$
default	$-\frac{3\,\text{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{2}-\frac{\sqrt{-x^2+1}}{2x^2}-\frac{2\sqrt{-x^2+1}}{x}$
meijerg	$-\frac{-\frac{\sqrt{\pi}\left(-4x^2+8\right)}{8x^2}+\frac{\sqrt{\pi}\sqrt{-x^2+1}}{x^2}+\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{-x^2+1}}{2}\right)-\frac{(1-2\ln(2)+2\ln(x)+i\pi)\sqrt{\pi}}{2}+\frac{\sqrt{\pi}}{x^2}}{2\sqrt{\pi}}-\frac{2\sqrt{-x^2+1}}{x}+\frac{-2\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{-x^2+1}}{2}\right.}{}$

input
```
int((1+x)^2/x^3/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(1+4*x)/x^2*(-x^2+1)^(1/2)+3/2*ln(((-x^2+1)^(1/2)-1)/x)
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx=\frac{3\,x^2\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\sqrt{-x^2+1}(4\,x+1)}{2\,x^2}$$

input
```
integrate((1+x)^2/x^3/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(3*x^2*log((sqrt(-x^2 + 1) - 1)/x) - sqrt(-x^2 + 1)*(4*x + 1))/x^2
```

3.60.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.27

$$\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx = 2\left(\begin{cases} -\frac{i\sqrt{x^2-1}}{x} & \text{for } |x^2|>1 \\ -\frac{\sqrt{1-x^2}}{x} & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{\operatorname{acosh}\left(\frac{1}{x}\right)}{2} + \frac{1}{2x\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{2x^3\sqrt{-1+\frac{1}{x^2}}} & \text{for } \frac{1}{|x^2|}>1 \\ \frac{i\operatorname{asin}\left(\frac{1}{x}\right)}{2} - \frac{i\sqrt{1-\frac{1}{x^2}}}{2x} & \text{otherwise} \end{cases} + \begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for } \frac{1}{|x^2|}>1 \\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((1+x)**2/x**3/(-x**2+1)**(1/2),x)
```

output
```
2*Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x, True
)) + Piecewise((-acosh(1/x)/2 + 1/(2*x*sqrt(-1 + x**(-2))) - 1/(2*x**3*sqr
t(-1 + x**(-2))), 1/Abs(x**2) > 1), (I*asin(1/x)/2 - I*sqrt(1 - 1/x**2)/(2
*x), True)) + Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True
))
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx = -\frac{2\sqrt{-x^2+1}}{x} - \frac{\sqrt{-x^2+1}}{2x^2} - \frac{3}{2}\log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

input
```
integrate((1+x)^2/x^3/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(-x^2 + 1)/x - 1/2*sqrt(-x^2 + 1)/x^2 - 3/2*log(2*sqrt(-x^2 + 1)/ab
s(x) + 2/abs(x))
```

3.60.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(41) = 82.

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.78

$$\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx = \frac{x^2\left(\frac{8\left(\sqrt{-x^2+1}-1\right)}{x}-1\right)}{8\left(\sqrt{-x^2+1}-1\right)^2} - \frac{\sqrt{-x^2+1}-1}{x} + \frac{\left(\sqrt{-x^2+1}-1\right)^2}{8\,x^2} + \frac{3}{2}\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

input
```
integrate((1+x)^2/x^3/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/8*x^2*(8*(sqrt(-x^2 + 1) - 1)/x - 1)/(sqrt(-x^2 + 1) - 1)^2 - (sqrt(-x^2
 + 1) - 1)/x + 1/8*(sqrt(-x^2 + 1) - 1)^2/x^2 + 3/2*log(-(sqrt(-x^2 + 1) -
 1)/abs(x))
```

3.60.9 Mupad [B] (verification not implemented)

Time = 11.49 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx = \frac{3\,\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)}{2} - \frac{2\sqrt{1-x^2}}{x} - \frac{\sqrt{1-x^2}}{2\,x^2}$$

input
```
int((x + 1)^2/(x^3*(1 - x^2)^(1/2)),x)
```

output
```
(3*log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)))/2 - (2*(1 - x^2)^(1/2))/x - (1
- x^2)^(1/2)/(2*x^2)
```

3.61 $\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx$

3.61.1 Optimal result

Integrand size = 20, antiderivative size = 67

$$\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx = -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}-\frac{5\sqrt{1-x^2}}{3x}-\operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

output
```
-arctanh((-x^2+1)^(1/2))-1/3*(-x^2+1)^(1/2)/x^3-(-x^2+1)^(1/2)/x^2-5/3*(-x
^2+1)^(1/2)/x
```

3.61.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx = \frac{(-1-3x-5x^2)\sqrt{1-x^2}}{3x^3}-\log(x)+\log\left(-1+\sqrt{1-x^2}\right)$$

input
```
Integrate[(1 + x)^2/(x^4*Sqrt[1 - x^2]),x]
```

output
```
((-1 - 3*x - 5*x^2)*Sqrt[1 - x^2])/(3*x^3) - Log[x] + Log[-1 + Sqrt[1 - x^
2]]
```

3.61.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {540, 25, 539, 27, 534, 243, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^2}{x^4\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{540}$$

$$-\frac{1}{3}\int -\frac{5x+6}{x^3\sqrt{1-x^2}}dx-\frac{\sqrt{1-x^2}}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{5x+6}{x^3\sqrt{1-x^2}}dx-\frac{\sqrt{1-x^2}}{3x^3}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{3}\left(-\frac{1}{2}\int -\frac{2(3x+5)}{x^2\sqrt{1-x^2}}dx-\frac{3\sqrt{1-x^2}}{x^2}\right)-\frac{\sqrt{1-x^2}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\int \frac{3x+5}{x^2\sqrt{1-x^2}}dx-\frac{3\sqrt{1-x^2}}{x^2}\right)-\frac{\sqrt{1-x^2}}{3x^3}$$

$$\Big\downarrow \text{534}$$

$$\frac{1}{3}\left(3\int \frac{1}{x\sqrt{1-x^2}}dx-\frac{5\sqrt{1-x^2}}{x}-\frac{3\sqrt{1-x^2}}{x^2}\right)-\frac{\sqrt{1-x^2}}{3x^3}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2\sqrt{1-x^2}}dx^2-\frac{5\sqrt{1-x^2}}{x}-\frac{3\sqrt{1-x^2}}{x^2}\right)-\frac{\sqrt{1-x^2}}{3x^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-3\int \frac{1}{1-x^4}d\sqrt{1-x^2}-\frac{5\sqrt{1-x^2}}{x}-\frac{3\sqrt{1-x^2}}{x^2}\right)-\frac{\sqrt{1-x^2}}{3x^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(-3\text{arctanh}\left(\sqrt{1-x^2}\right)-\frac{5\sqrt{1-x^2}}{x}-\frac{3\sqrt{1-x^2}}{x^2}\right)-\frac{\sqrt{1-x^2}}{3x^3}$$

input
```
Int[(1 + x)^2/(x^4*Sqrt[1 - x^2]),x]
```

output
```
-1/3*Sqrt[1 - x^2]/x^3 + ((-3*Sqrt[1 - x^2])/x^2 - (5*Sqrt[1 - x^2])/x - 3
*ArcTanh[Sqrt[1 - x^2]])/3
```

3.61.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

3.61.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

method	result	siz
trager	$-\frac{(5x^2+3x+1)\sqrt{-x^2+1}}{3x^3}-\ln\left(\frac{\sqrt{-x^2+1}+1}{x}\right)$	44
risch	$\frac{5x^4+3x^3-4x^2-3x-1}{3x^3\sqrt{-x^2+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$	48
default	$-\frac{5\sqrt{-x^2+1}}{3x}-\frac{\sqrt{-x^2+1}}{3x^3}-\frac{\sqrt{-x^2+1}}{x^2}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$	56
meijerg	$-\frac{(2x^2+1)\sqrt{-x^2+1}}{3x^3}-\frac{-\frac{\sqrt{\pi}\,(-4x^2+8)}{8x^2}+\frac{\sqrt{\pi}\,\sqrt{-x^2+1}}{x^2}+\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^2+1}}{2}\right)-\frac{(1-2\ln(2)+2\ln(x)+i\pi)\sqrt{\pi}}{2}+\frac{\sqrt{\pi}}{x^2}}{\sqrt{\pi}}-\frac{\sqrt{-x^2+1}}{x}$	11

input
```
int((1+x)^2/x^4/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(5*x^2+3*x+1)/x^3*(-x^2+1)^(1/2)-ln(((-x^2+1)^(1/2)+1)/x)
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx = \frac{3\,x^3\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-(5\,x^2+3\,x+1)\sqrt{-x^2+1}}{3\,x^3}$$

input
```
integrate((1+x)^2/x^4/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(3*x^3*log((sqrt(-x^2 + 1) - 1)/x) - (5*x^2 + 3*x + 1)*sqrt(-x^2 + 1))
/x^3
```

3.61.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.38 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.91

$$\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx = \begin{cases} -\frac{\sqrt{1-x^2}}{x}-\frac{(1-x^2)^{\frac{3}{2}}}{3x^3} & \text{for } x>-1\wedge x<1 \end{cases} + \begin{cases} -\frac{i\sqrt{x^2-1}}{x} & \text{for } |x^2|>1 \\ -\frac{\sqrt{1-x^2}}{x} & \text{otherwise} \end{cases} + 2\left(\begin{cases} -\frac{\operatorname{acosh}\left(\frac{1}{x}\right)}{2}+\frac{1}{2x\sqrt{-1+\frac{1}{x^2}}}-\frac{1}{2x^3\sqrt{-1+\frac{1}{x^2}}} & \text{for } \frac{1}{|x^2|}>1 \\ \frac{i\operatorname{asin}\left(\frac{1}{x}\right)}{2}-\frac{i\sqrt{1-\frac{1}{x^2}}}{2x} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((1+x)**2/x**4/(-x**2+1)**(1/2),x)
```

output
```
Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x <
 1))) + Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x
, True)) + 2*Piecewise((-acosh(1/x)/2 + 1/(2*x*sqrt(-1 + x**(-2))) - 1/(2*
x**3*sqrt(-1 + x**(-2))), 1/Abs(x**2) > 1), (I*asin(1/x)/2 - I*sqrt(1 - 1/
x**2)/(2*x), True))
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx = -\frac{5\sqrt{-x^2+1}}{3\,x} - \frac{\sqrt{-x^2+1}}{x^2} - \frac{\sqrt{-x^2+1}}{3\,x^3} - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

input
```
integrate((1+x)^2/x^4/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-5/3*sqrt(-x^2 + 1)/x - sqrt(-x^2 + 1)/x^2 - 1/3*sqrt(-x^2 + 1)/x^3 - log(
2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

3.61.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(55) = 110.

Time = 0.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.87

$$\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx = -\frac{x^3\left(\frac{6\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{21\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 1\right)}{24\left(\sqrt{-x^2+1}-1\right)^3} - \frac{7\left(\sqrt{-x^2+1}-1\right)}{8\,x} + \frac{\left(\sqrt{-x^2+1}-1\right)^2}{4\,x^2} - \frac{\left(\sqrt{-x^2+1}-1\right)^3}{24\,x^3} + \log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

input
```
integrate((1+x)^2/x^4/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/24*x^3*(6*(sqrt(-x^2 + 1) - 1)/x - 21*(sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(
sqrt(-x^2 + 1) - 1)^3 - 7/8*(sqrt(-x^2 + 1) - 1)/x + 1/4*(sqrt(-x^2 + 1) -
 1)^2/x^2 - 1/24*(sqrt(-x^2 + 1) - 1)^3/x^3 + log(-(sqrt(-x^2 + 1) - 1)/ab
s(x))
```

3.61.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx = \ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)-\sqrt{1-x^2}\left(\frac{2}{3\,x}+\frac{1}{3\,x^3}\right)-\frac{\sqrt{1-x^2}}{x}-\frac{\sqrt{1-x^2}}{x^2}$$

input
```
int((x + 1)^2/(x^4*(1 - x^2)^(1/2)),x)
```

output
```
log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - (1 - x^2)^(1/2)*(2/(3*x) + 1/(3*x
^3)) - (1 - x^2)^(1/2)/x - (1 - x^2)^(1/2)/x^2
```

3.62 $\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx$

3.62.1 Optimal result

Integrand size = 20, antiderivative size = 89

$$\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx=-\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{4\sqrt{1-x^2}}{3x}-\frac{7}{8}\text{arctanh}\left(\sqrt{1-x^2}\right)$$

output
```
-7/8*arctanh((-x^2+1)^(1/2))-1/4*(-x^2+1)^(1/2)/x^4-2/3*(-x^2+1)^(1/2)/x^3
-7/8*(-x^2+1)^(1/2)/x^2-4/3*(-x^2+1)^(1/2)/x
```

3.62.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.65

$$\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx=\frac{\sqrt{1-x^2}(-6-16x-21x^2-32x^3)}{24x^4}-\frac{7\log(x)}{8}+\frac{7}{8}\log\left(-1+\sqrt{1-x^2}\right)$$

input
```
Integrate[(1 + x)^2/(x^5*Sqrt[1 - x^2]),x]
```

output
```
(Sqrt[1 - x^2]*(-6 - 16*x - 21*x^2 - 32*x^3))/(24*x^4) - (7*Log[x])/8 + (7
*Log[-1 + Sqrt[1 - x^2]])/8
```

3.62.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {540, 25, 539, 25, 539, 25, 534, 243, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^2}{x^5\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 540}$$

$$-\frac{1}{4}\int -\frac{7x+8}{x^4\sqrt{1-x^2}}dx-\frac{\sqrt{1-x^2}}{4x^4}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{4}\int \frac{7x+8}{x^4\sqrt{1-x^2}}dx-\frac{\sqrt{1-x^2}}{4x^4}$$

$$\Big\downarrow \text{ 539}$$

$$\frac{1}{4}\left(-\frac{1}{3}\int -\frac{16x+21}{x^3\sqrt{1-x^2}}dx-\frac{8\sqrt{1-x^2}}{3x^3}\right)-\frac{\sqrt{1-x^2}}{4x^4}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{4}\left(\frac{1}{3}\int \frac{16x+21}{x^3\sqrt{1-x^2}}dx-\frac{8\sqrt{1-x^2}}{3x^3}\right)-\frac{\sqrt{1-x^2}}{4x^4}$$

$$\Big\downarrow \text{ 539}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(-\frac{1}{2}\int -\frac{21x+32}{x^2\sqrt{1-x^2}}dx-\frac{21\sqrt{1-x^2}}{2x^2}\right)-\frac{8\sqrt{1-x^2}}{3x^3}\right)-\frac{\sqrt{1-x^2}}{4x^4}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{21x+32}{x^2\sqrt{1-x^2}}dx-\frac{21\sqrt{1-x^2}}{2x^2}\right)-\frac{8\sqrt{1-x^2}}{3x^3}\right)-\frac{\sqrt{1-x^2}}{4x^4}$$

$$\Big\downarrow \text{ 534}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(21\int \frac{1}{x\sqrt{1-x^2}}dx-\frac{32\sqrt{1-x^2}}{x}\right)-\frac{21\sqrt{1-x^2}}{2x^2}\right)-\frac{8\sqrt{1-x^2}}{3x^3}\right)-\frac{\sqrt{1-x^2}}{4x^4}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{21}{2}\int\frac{1}{x^2\sqrt{1-x^2}}dx^2-\frac{32\sqrt{1-x^2}}{x}\right)-\frac{21\sqrt{1-x^2}}{2x^2}\right)-\frac{8\sqrt{1-x^2}}{3x^3}\right)-\frac{\sqrt{1-x^2}}{4x^4}$$

$$\Big\downarrow 73$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(-21\int\frac{1}{1-x^4}d\sqrt{1-x^2}-\frac{32\sqrt{1-x^2}}{x}\right)-\frac{21\sqrt{1-x^2}}{2x^2}\right)-\frac{8\sqrt{1-x^2}}{3x^3}\right)-\frac{\sqrt{1-x^2}}{4x^4}$$

$$\Big\downarrow 219$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(-21\text{arctanh}\left(\sqrt{1-x^2}\right)-\frac{32\sqrt{1-x^2}}{x}\right)-\frac{21\sqrt{1-x^2}}{2x^2}\right)-\frac{8\sqrt{1-x^2}}{3x^3}\right)-\frac{\sqrt{1-x^2}}{4x^4}$$

input
```
Int[(1 + x)^2/(x^5*Sqrt[1 - x^2]),x]
```

output
```
-1/4*Sqrt[1 - x^2]/x^4 + ((-8*Sqrt[1 - x^2])/(3*x^3) + ((-21*Sqrt[1 - x^2]
)/(2*x^2) + ((-32*Sqrt[1 - x^2])/x - 21*ArcTanh[Sqrt[1 - x^2]])/2)/3)/4
```

3.62.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

3.62.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.55

method	result
trager	$-\frac{\left(32x^3+21x^2+16x+6\right)\sqrt{-x^2+1}}{24x^4}-\frac{7\ln\left(\frac{\sqrt{-x^2+1}+1}{x}\right)}{8}$
risch	$\frac{32x^5+21x^4-16x^3-15x^2-16x-6}{24x^4\sqrt{-x^2+1}}-\frac{7\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{8}$
default	$-\frac{\sqrt{-x^2+1}}{4x^4}-\frac{7\sqrt{-x^2+1}}{8x^2}-\frac{7\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{8}-\frac{2\sqrt{-x^2+1}}{3x^3}-\frac{4\sqrt{-x^2+1}}{3x}$
meijerg	$\frac{\frac{\sqrt{\pi}\left(-7x^4+8x^2+8\right)}{16x^4}-\frac{\sqrt{\pi}\left(12x^2+8\right)\sqrt{-x^2+1}}{16x^4}-\frac{3\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^2+1}}{2}\right)}{4}+\frac{3\left(\frac{7}{6}-2\ln(2)+2\ln(x)+i\pi\right)\sqrt{\pi}}{8}-\frac{\sqrt{\pi}}{2x^4}-\frac{\sqrt{\pi}}{2x^2}}{2\sqrt{\pi}}-\frac{2\left(2x^2+1\right)\sqrt{-x^2+1}}{3x^3}$

input
```
int((1+x)^2/x^5/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(32*x^3+21*x^2+16*x+6)/x^4*(-x^2+1)^(1/2)-7/8*ln(((-x^2+1)^(1/2)+1)/
x)
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx = \frac{21\,x^4\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - (32\,x^3+21\,x^2+16\,x+6)\sqrt{-x^2+1}}{24\,x^4}$$

input
```
integrate((1+x)^2/x^5/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/24*(21*x^4*log((sqrt(-x^2 + 1) - 1)/x) - (32*x^3 + 21*x^2 + 16*x + 6)*sq
rt(-x^2 + 1))/x^4
```

3.62.6 Sympy [A] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.51

$$\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx = 2\left(\begin{cases} -\frac{\sqrt{1-x^2}}{x} - \frac{(1-x^2)^{\frac{3}{2}}}{3x^3} & \text{for}\ x > -1 \wedge x < 1 \end{cases}\right) + \begin{cases} -\frac{\operatorname{acosh}\left(\frac{1}{x}\right)}{2} + \frac{1}{2x\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{2x^3\sqrt{-1+\frac{1}{x^2}}} & \text{for}\ \frac{1}{|x^2|} > 1 \\ \frac{i\operatorname{asin}\left(\frac{1}{x}\right)}{2} - \frac{i\sqrt{1-\frac{1}{x^2}}}{2x} & \text{otherwise} \end{cases} + \begin{cases} -\frac{3\operatorname{acosh}\left(\frac{1}{x}\right)}{8} + \frac{3}{8x\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{8x^3\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{4x^5\sqrt{-1+\frac{1}{x^2}}} & \text{for}\ \frac{1}{|x^2|} > 1 \\ \frac{3i\operatorname{asin}\left(\frac{1}{x}\right)}{8} - \frac{3i}{8x\sqrt{1-\frac{1}{x^2}}} + \frac{i}{8x^3\sqrt{1-\frac{1}{x^2}}} + \frac{i}{4x^5\sqrt{1-\frac{1}{x^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((1+x)**2/x**5/(-x**2+1)**(1/2),x)
```

output
```
2*Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x
 < 1))) + Piecewise((-acosh(1/x)/2 + 1/(2*x*sqrt(-1 + x**(-2))) - 1/(2*x**
3*sqrt(-1 + x**(-2))), 1/Abs(x**2) > 1), (I*asin(1/x)/2 - I*sqrt(1 - 1/x**
2)/(2*x), True)) + Piecewise((-3*acosh(1/x)/8 + 3/(8*x*sqrt(-1 + x**(-2)))
 - 1/(8*x**3*sqrt(-1 + x**(-2))) - 1/(4*x**5*sqrt(-1 + x**(-2))), 1/Abs(x*
*2) > 1), (3*I*asin(1/x)/8 - 3*I/(8*x*sqrt(1 - 1/x**2)) + I/(8*x**3*sqrt(1
 - 1/x**2)) + I/(4*x**5*sqrt(1 - 1/x**2)), True))
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx = -\frac{4\sqrt{-x^2+1}}{3\,x} - \frac{7\sqrt{-x^2+1}}{8\,x^2} - \frac{2\sqrt{-x^2+1}}{3\,x^3} - \frac{\sqrt{-x^2+1}}{4\,x^4} - \frac{7}{8}\log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

input
```
integrate((1+x)^2/x^5/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-4/3*sqrt(-x^2 + 1)/x - 7/8*sqrt(-x^2 + 1)/x^2 - 2/3*sqrt(-x^2 + 1)/x^3 -
1/4*sqrt(-x^2 + 1)/x^4 - 7/8*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

3.62.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(69) = 138.

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.83

$$\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx = \frac{x^4\left(\frac{16\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{48\left(\sqrt{-x^2+1}-1\right)^2}{x^2} + \frac{144\left(\sqrt{-x^2+1}-1\right)^3}{x^3} - 3\right)}{192\left(\sqrt{-x^2+1}-1\right)^4} - \frac{3\left(\sqrt{-x^2+1}-1\right)}{4\,x} + \frac{\left(\sqrt{-x^2+1}-1\right)^2}{4\,x^2} - \frac{\left(\sqrt{-x^2+1}-1\right)^3}{12\,x^3} + \frac{\left(\sqrt{-x^2+1}-1\right)^4}{64\,x^4} + \frac{7}{8}\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

input
```
integrate((1+x)^2/x^5/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/192*x^4*(16*(sqrt(-x^2 + 1) - 1)/x - 48*(sqrt(-x^2 + 1) - 1)^2/x^2 + 144
*(sqrt(-x^2 + 1) - 1)^3/x^3 - 3)/(sqrt(-x^2 + 1) - 1)^4 - 3/4*(sqrt(-x^2 +
 1) - 1)/x + 1/4*(sqrt(-x^2 + 1) - 1)^2/x^2 - 1/12*(sqrt(-x^2 + 1) - 1)^3/
x^3 + 1/64*(sqrt(-x^2 + 1) - 1)^4/x^4 + 7/8*log(-(sqrt(-x^2 + 1) - 1)/abs(
x))
```

3.62.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.87

$$\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx = \frac{7\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)}{8} - \sqrt{1-x^2}\left(\frac{4}{3\,x}+\frac{2}{3\,x^3}\right) - \sqrt{1-x^2}\left(\frac{3}{8\,x^2}+\frac{1}{4\,x^4}\right) - \frac{\sqrt{1-x^2}}{2\,x^2}$$

input
```
int((x + 1)^2/(x^5*(1 - x^2)^(1/2)),x)
```

output
```
(7*log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)))/8 - (1 - x^2)^(1/2)*(4/(3*x) +
2/(3*x^3)) - (1 - x^2)^(1/2)*(3/(8*x^2) + 1/(4*x^4)) - (1 - x^2)^(1/2)/(2*
x^2)
```

3.63 $\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx$

3.63.1 Optimal result

Integrand size = 20, antiderivative size = 107

$$\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx=-\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{6\sqrt{1-x^2}}{5x}-\frac{3}{4}\operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

output
```
-3/4*arctanh((-x^2+1)^(1/2))-1/5*(-x^2+1)^(1/2)/x^5-1/2*(-x^2+1)^(1/2)/x^4
-3/5*(-x^2+1)^(1/2)/x^3-3/4*(-x^2+1)^(1/2)/x^2-6/5*(-x^2+1)^(1/2)/x
```

3.63.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.59

$$\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx=\frac{\sqrt{1-x^2}(-4-10x-12x^2-15x^3-24x^4)}{20x^5}-\frac{3\log(x)}{4}+\frac{3}{4}\log\left(-1+\sqrt{1-x^2}\right)$$

input
```
Integrate[(1 + x)^2/(x^6*Sqrt[1 - x^2]),x]
```

output
```
(Sqrt[1 - x^2]*(-4 - 10*x - 12*x^2 - 15*x^3 - 24*x^4))/(20*x^5) - (3*Log[x
])/4 + (3*Log[-1 + Sqrt[1 - x^2]])/4
```

3.63.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {540, 25, 539, 27, 539, 27, 539, 25, 534, 243, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^2}{x^6\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{540}$$

$$-\frac{1}{5}\int -\frac{9x+10}{x^5\sqrt{1-x^2}}dx-\frac{\sqrt{1-x^2}}{5x^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\int \frac{9x+10}{x^5\sqrt{1-x^2}}dx-\frac{\sqrt{1-x^2}}{5x^5}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{5}\left(-\frac{1}{4}\int -\frac{6(5x+6)}{x^4\sqrt{1-x^2}}dx-\frac{5\sqrt{1-x^2}}{2x^4}\right)-\frac{\sqrt{1-x^2}}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{3}{2}\int \frac{5x+6}{x^4\sqrt{1-x^2}}dx-\frac{5\sqrt{1-x^2}}{2x^4}\right)-\frac{\sqrt{1-x^2}}{5x^5}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{5}\left(\frac{3}{2}\left(-\frac{1}{3}\int -\frac{3(4x+5)}{x^3\sqrt{1-x^2}}dx-\frac{2\sqrt{1-x^2}}{x^3}\right)-\frac{5\sqrt{1-x^2}}{2x^4}\right)-\frac{\sqrt{1-x^2}}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{3}{2}\left(\int \frac{4x+5}{x^3\sqrt{1-x^2}}dx-\frac{2\sqrt{1-x^2}}{x^3}\right)-\frac{5\sqrt{1-x^2}}{2x^4}\right)-\frac{\sqrt{1-x^2}}{5x^5}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{5}\left(\frac{3}{2}\left(-\frac{1}{2}\int -\frac{5x+8}{x^2\sqrt{1-x^2}}dx-\frac{5\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x^3}\right)-\frac{5\sqrt{1-x^2}}{2x^4}\right)-\frac{\sqrt{1-x^2}}{5x^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\left(\frac{3}{2}\left(\frac{1}{2}\int \frac{5x+8}{x^2\sqrt{1-x^2}}dx-\frac{5\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x^3}\right)-\frac{5\sqrt{1-x^2}}{2x^4}\right)-\frac{\sqrt{1-x^2}}{5x^5}$$

$$\Big\downarrow \text{534}$$

$$\frac{1}{5}\left(\frac{3}{2}\left(\frac{1}{2}\left(5\int \frac{1}{x\sqrt{1-x^2}}dx-\frac{8\sqrt{1-x^2}}{x}\right)-\frac{5\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x^3}\right)-\frac{5\sqrt{1-x^2}}{2x^4}\right)-\frac{\sqrt{1-x^2}}{5x^5}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{5}\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{5}{2}\int \frac{1}{x^2\sqrt{1-x^2}}dx^2-\frac{8\sqrt{1-x^2}}{x}\right)-\frac{5\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x^3}\right)-\frac{5\sqrt{1-x^2}}{2x^4}\right)-\frac{\sqrt{1-x^2}}{5x^5}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{5}\left(\frac{3}{2}\left(\frac{1}{2}\left(-5\int \frac{1}{1-x^4}d\sqrt{1-x^2}-\frac{8\sqrt{1-x^2}}{x}\right)-\frac{5\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x^3}\right)-\frac{5\sqrt{1-x^2}}{2x^4}\right)-\frac{\sqrt{1-x^2}}{5x^5}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{5}\left(\frac{3}{2}\left(\frac{1}{2}\left(-5\text{arctanh}\left(\sqrt{1-x^2}\right)-\frac{8\sqrt{1-x^2}}{x}\right)-\frac{5\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x^3}\right)-\frac{5\sqrt{1-x^2}}{2x^4}\right)-\frac{\sqrt{1-x^2}}{5x^5}$$

input
```
Int[(1 + x)^2/(x^6*Sqrt[1 - x^2]),x]
```

output
```
-1/5*Sqrt[1 - x^2]/x^5 + ((-5*Sqrt[1 - x^2])/(2*x^4) + (3*((-2*Sqrt[1 - x^
2])/x^3 - (5*Sqrt[1 - x^2])/(2*x^2) + ((-8*Sqrt[1 - x^2])/x - 5*ArcTanh[Sq
rt[1 - x^2]])/2))/2)/5
```

3.63.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

3.63. $\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx$

3.63.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.50

method	result
trager	$-\frac{(24x^4+15x^3+12x^2+10x+4)\sqrt{-x^2+1}}{20x^5}+\frac{3\ln\left(\frac{\sqrt{-x^2+1}-1}{x}\right)}{4}$
risch	$\frac{24x^6+15x^5-12x^4-5x^3-8x^2-10x-4}{20x^5\sqrt{-x^2+1}}-\frac{3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{4}$
default	$-\frac{\sqrt{-x^2+1}}{5x^5}-\frac{3\sqrt{-x^2+1}}{5x^3}-\frac{6\sqrt{-x^2+1}}{5x}-\frac{\sqrt{-x^2+1}}{2x^4}-\frac{3\sqrt{-x^2+1}}{4x^2}-\frac{3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{4}$
meijerg	$-\frac{\left(\frac{8}{3}x^4+\frac{4}{3}x^2+1\right)\sqrt{-x^2+1}}{5x^5}+\frac{\frac{\sqrt{\pi}\left(-7x^4+8x^2+8\right)}{16x^4}-\frac{\sqrt{\pi}\left(12x^2+8\right)\sqrt{-x^2+1}}{16x^4}-\frac{3\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^2+1}}{2}\right)}{4}+\frac{3\left(\frac{7}{6}-2\ln(2)+2\ln(x)+i\pi\right)\sqrt{\pi}}{8}-\frac{\sqrt{\pi}}{2x^4}}{\sqrt{\pi}}$

input
```
int((1+x)^2/x^6/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*(24*x^4+15*x^3+12*x^2+10*x+4)/x^5*(-x^2+1)^(1/2)+3/4*ln(((-x^2+1)^(1
/2)-1)/x)
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

$$\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx = \frac{15\,x^5\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-(24\,x^4+15\,x^3+12\,x^2+10\,x+4)\sqrt{-x^2+1}}{20\,x^5}$$

input
```
integrate((1+x)^2/x^6/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/20*(15*x^5*log((sqrt(-x^2 + 1) - 1)/x) - (24*x^4 + 15*x^3 + 12*x^2 + 10*
x + 4)*sqrt(-x^2 + 1))/x^5
```

3.63.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.69 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.88

$$\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx = \begin{cases} -\frac{\sqrt{1-x^2}}{x} - \frac{(1-x^2)^{\frac{3}{2}}}{3x^3} & \text{for } x > -1 \wedge x < 1 \end{cases} + \begin{cases} -\frac{\sqrt{1-x^2}}{x} - \frac{2(1-x^2)^{\frac{3}{2}}}{3x^3} - \frac{(1-x^2)^{\frac{5}{2}}}{5x^5} & \text{for } x > -1 \wedge x < 1 \end{cases} + 2\left(\begin{cases} -\frac{3\operatorname{acosh}\left(\frac{1}{x}\right)}{8} + \frac{3}{8x\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{8x^3\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{4x^5\sqrt{-1+\frac{1}{x^2}}} & \text{for } \frac{1}{|x^2|} > 1 \\ \frac{3i\operatorname{asin}\left(\frac{1}{x}\right)}{8} - \frac{3i}{8x\sqrt{1-\frac{1}{x^2}}} + \frac{i}{8x^3\sqrt{1-\frac{1}{x^2}}} + \frac{i}{4x^5\sqrt{1-\frac{1}{x^2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((1+x)**2/x**6/(-x**2+1)**(1/2),x)
```

output
```
Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x <
 1))) + Piecewise((-sqrt(1 - x**2)/x - 2*(1 - x**2)**(3/2)/(3*x**3) - (1 -
 x**2)**(5/2)/(5*x**5), (x > -1) & (x < 1))) + 2*Piecewise((-3*acosh(1/x)/
8 + 3/(8*x*sqrt(-1 + x**(-2))) - 1/(8*x**3*sqrt(-1 + x**(-2))) - 1/(4*x**5
*sqrt(-1 + x**(-2))), 1/Abs(x**2) > 1), (3*I*asin(1/x)/8 - 3*I/(8*x*sqrt(1
 - 1/x**2)) + I/(8*x**3*sqrt(1 - 1/x**2)) + I/(4*x**5*sqrt(1 - 1/x**2)), T
rue))
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

$$\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx = -\frac{6\sqrt{-x^2+1}}{5\,x} - \frac{3\sqrt{-x^2+1}}{4\,x^2} - \frac{3\sqrt{-x^2+1}}{5\,x^3} - \frac{\sqrt{-x^2+1}}{2\,x^4} - \frac{\sqrt{-x^2+1}}{5\,x^5} - \frac{3}{4}\log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

input
```
integrate((1+x)^2/x^6/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-6/5*sqrt(-x^2 + 1)/x - 3/4*sqrt(-x^2 + 1)/x^2 - 3/5*sqrt(-x^2 + 1)/x^3 -
1/2*sqrt(-x^2 + 1)/x^4 - 1/5*sqrt(-x^2 + 1)/x^5 - 3/4*log(2*sqrt(-x^2 + 1)
/abs(x) + 2/abs(x))
```

3.63.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(83) = 166$.

Time = 0.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.86

$$\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx = -\frac{x^5\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{15\left(\sqrt{-x^2+1}-1\right)^2}{x^2} + \frac{40\left(\sqrt{-x^2+1}-1\right)^3}{x^3} - \frac{110\left(\sqrt{-x^2+1}-1\right)^4}{x^4} - 1\right)}{160\left(\sqrt{-x^2+1}-1\right)^5} - \frac{11\left(\sqrt{-x^2+1}-1\right)}{16\,x} + \frac{\left(\sqrt{-x^2+1}-1\right)^2}{4\,x^2} - \frac{3\left(\sqrt{-x^2+1}-1\right)^3}{32\,x^3} + \frac{\left(\sqrt{-x^2+1}-1\right)^4}{32\,x^4} - \frac{\left(\sqrt{-x^2+1}-1\right)^5}{160\,x^5} + \frac{3}{4}\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

input
```
integrate((1+x)^2/x^6/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/160*x^5*(5*(sqrt(-x^2 + 1) - 1)/x - 15*(sqrt(-x^2 + 1) - 1)^2/x^2 + 40*
(sqrt(-x^2 + 1) - 1)^3/x^3 - 110*(sqrt(-x^2 + 1) - 1)^4/x^4 - 1)/(sqrt(-x^
2 + 1) - 1)^5 - 11/16*(sqrt(-x^2 + 1) - 1)/x + 1/4*(sqrt(-x^2 + 1) - 1)^2/
x^2 - 3/32*(sqrt(-x^2 + 1) - 1)^3/x^3 + 1/32*(sqrt(-x^2 + 1) - 1)^4/x^4 -
1/160*(sqrt(-x^2 + 1) - 1)^5/x^5 + 3/4*log(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

3.63.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.84

$$\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx = \frac{3\ln\left(\sqrt{\frac{1}{x^2}-1} - \sqrt{\frac{1}{x^2}}\right)}{4} - \sqrt{1-x^2}\left(\frac{2}{3\,x} + \frac{1}{3\,x^3}\right) - \sqrt{1-x^2}\left(\frac{3}{4\,x^2} + \frac{1}{2\,x^4}\right) - \sqrt{1-x^2}\left(\frac{8}{15\,x} + \frac{4}{15\,x^3} + \frac{1}{5\,x^5}\right)$$

input
```
int((x + 1)^2/(x^6*(1 - x^2)^(1/2)),x)
```

output
```
(3*log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)))/4 - (1 - x^2)^(1/2)*(2/(3*x) +
1/(3*x^3)) - (1 - x^2)^(1/2)*(3/(4*x^2) + 1/(2*x^4)) - (1 - x^2)^(1/2)*(8/
(15*x) + 4/(15*x^3) + 1/(5*x^5))
```

3.64 $\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx$

3.64.1 Optimal result

Integrand size = 27, antiderivative size = 134

$$\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx=-\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{d(d^2-e^2x^2)^{3/2}}{4x^4}-\frac{e(d^2-e^2x^2)^{3/2}}{x^3}-e^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{13}{8}e^4\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
-1/4*d*(-e^2*x^2+d^2)^(3/2)/x^4-e*(-e^2*x^2+d^2)^(3/2)/x^3-e^4*arctan(e*x/
(-e^2*x^2+d^2)^(1/2))+13/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)-1/8*e^2*(8*
e*x+13*d)*(-e^2*x^2+d^2)^(1/2)/x^2
```

3.64.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx=-\frac{d\sqrt{d^2-e^2x^2}(2d^2+8dex+11e^2x^2)}{8x^4}-\frac{13}{4}e^4\text{arctanh}\left(\frac{\sqrt{-e^2}x-\sqrt{d^2-e^2x^2}}{d}\right)+e\left(-e^2\right)^{3/2}\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)$$

input
```
Integrate[((d + e*x)^3*Sqrt[d^2 - e^2*x^2])/x^5,x]
```

output
```
-1/8*(d*Sqrt[d^2 - e^2*x^2]*(2*d^2 + 8*d*e*x + 11*e^2*x^2))/x^4 - (13*e^4*
ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/4 + e*(-e^2)^(3/2)*Log[-(
Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]
```

3.64.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {540, 25, 2338, 27, 537, 25, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx$$

$$\Big\downarrow \text{540}$$

$$-\frac{\int -\frac{\sqrt{d^2-e^2x^2}\left(12ed^4+13e^2xd^3+4e^3x^2d^2\right)}{x^4}dx}{4d^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{d^2-e^2x^2}\left(12ed^4+13e^2xd^3+4e^3x^2d^2\right)}{x^4}dx}{4d^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int -\frac{3d^4e^2(13d+4ex)\sqrt{d^2-e^2x^2}}{x^3}dx}{3d^2}-\frac{4d^2e\left(d^2-e^2x^2\right)^{3/2}}{x^3}}{4d^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2e^2\int \frac{(13d+4ex)\sqrt{d^2-e^2x^2}}{x^3}dx-\frac{4d^2e\left(d^2-e^2x^2\right)^{3/2}}{x^3}}{4d^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}$$

$$\Big\downarrow \text{537}$$

$$\frac{d^2e^2\left(\frac{1}{2}e^2\int -\frac{13d+8ex}{x\sqrt{d^2-e^2x^2}}dx-\frac{(13d+8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{4d^2e\left(d^2-e^2x^2\right)^{3/2}}{x^3}}{4d^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{d^2e^2\left(-\frac{1}{2}e^2\int \frac{13d+8ex}{x\sqrt{d^2-e^2x^2}}dx-\frac{(13d+8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{4d^2e\left(d^2-e^2x^2\right)^{3/2}}{x^3}}{4d^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}$$

$$\Big\downarrow \; 538$$

$$\frac{d^2e^2\left(-\frac{1}{2}e^2\left(8e\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+13d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx\right)-\frac{(13d+8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{4d^2e\left(d^2-e^2x^2\right)^{3/2}}{x^3}}{4d^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}$$

$$\Big\downarrow \; 224$$

$$\frac{d^2e^2\left(-\frac{1}{2}e^2\left(13d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx+8e\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)-\frac{(13d+8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{4d^2e\left(d^2-e^2x^2\right)^{3/2}}{x^3}}{4d^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}$$

$$\Big\downarrow \; 216$$

$$\frac{d^2e^2\left(-\frac{1}{2}e^2\left(13d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx+8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{(13d+8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{4d^2e\left(d^2-e^2x^2\right)^{3/2}}{x^3}}{4d^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}$$

$$\Big\downarrow \; 243$$

$$\frac{d^2e^2\left(-\frac{1}{2}e^2\left(\frac{13}{2}d\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2+8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{(13d+8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{4d^2e\left(d^2-e^2x^2\right)^{3/2}}{x^3}}{4d^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}$$

$$\Big\downarrow \; 73$$

$$\frac{d^2e^2\left(-\frac{1}{2}e^2\left(8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{13d\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}\right)-\frac{(13d+8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{4d^2e\left(d^2-e^2x^2\right)^{3/2}}{x^3}}{4d^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}$$

$$\Big\downarrow \; 221$$

3.64. $\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx$

$$\frac{d^2e^2\left(-\frac{1}{2}e^2\left(8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-13\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)-\frac{(13d+8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{4d^2e(d^2-e^2x^2)^{3/2}}{x^3}}{4d^2}-\frac{d(d^2-e^2x^2)^{3/2}}{4x^4}$$

input
```
Int[((d + e*x)^3*Sqrt[d^2 - e^2*x^2])/x^5,x]
```

output
```
-1/4*(d*(d^2 - e^2*x^2)^(3/2))/x^4 + ((-4*d^2*e*(d^2 - e^2*x^2)^(3/2))/x^3
 + d^2*e^2*(-1/2*((13*d + 8*e*x)*Sqrt[d^2 - e^2*x^2])/x^2 - (e^2*(8*ArcTan
[(e*x)/Sqrt[d^2 - e^2*x^2]] - 13*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]))/2))/(4*d
^2)
```

3.64.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.64. $\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx$

3.64.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\,\left(11 e^{2} x^{2}+8 d e x+2 d^{2}\right) d}{8 x^{4}}-\frac{e^{5} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}}+\frac{13 e^{4} d \ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{8 \sqrt{d^{2}}}$
default	$d^{3} \left(-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{4 d^{2} x^{4}}+\frac{e^{2} \left(-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{2 d^{2} x^{2}}-\frac{e^{2} \left(\sqrt{-e^{2} x^{2}+d^{2}}-\frac{d^{2} \ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}}\right)}{2 d^{2}}\right)}{4 d^{2}}\right)+e^{3} \left(-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{d^{2} x}\right.$

input
```
int((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-e^2*x^2+d^2)^(1/2)*(11*e^2*x^2+8*d*e*x+2*d^2)*d/x^4-e^5/(e^2)^(1/2)
*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+13/8*e^4*d/(d^2)^(1/2)*ln((2*d
^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx$$
$$=\frac{16\,e^4x^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-13\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-(11\,de^2x^2+8\,d^2ex+2\,d^3)\sqrt{-e^2x^2+d^2}}{8\,x^4}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
1/8*(16*e^4*x^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 13*e^4*x^4*log
(-(d - sqrt(-e^2*x^2 + d^2))/x) - (11*d*e^2*x^2 + 8*d^2*e*x + 2*d^3)*sqrt(
-e^2*x^2 + d^2))/x^4
```

3.64.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.42 (sec) , antiderivative size = 544, normalized size of antiderivative = 4.06

$$\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx$$
$$= d^3\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise} \end{cases}\right)$$
$$+ 3d^2e\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right)$$
$$+ 3de^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x} + \frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right)$$
$$+ e^3\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}} + ie\operatorname{acosh}\left(\frac{ex}{d}\right) - \frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - e\operatorname{asin}\left(\frac{ex}{d}\right) + \frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x+d)**3*(-e**2*x**2+d**2)**(1/2)/x**5,x)
```

output
```
d**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*
sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) +
 e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x*
*5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1
)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/
(8*d**3), True)) + 3*d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x*
*2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1)
, (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**
2) + 1)/(3*d**2), True)) + 3*d*e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) -
1)/(2*x) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/
(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) +
 1)) - I*e**2*asin(d/(e*x))/(2*d), True)) + e**3*Piecewise((I*d/(x*sqrt(-1
 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d
**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin
(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx = -\frac{e^5\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} + \frac{13}{8}\,e^4\log\left(\frac{2\,d^2}{|x|} + \frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{13\sqrt{-e^2x^2+d^2}e^4}{8\,d} - \frac{\sqrt{-e^2x^2+d^2}e^3}{x} - \frac{13\,(-e^2x^2+d^2)^{\frac{3}{2}}e^2}{8\,dx^2} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e}{x^3} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d}{4\,x^4}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-e^5*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 13/8*e^4*log(2*d^2/abs(x) + 2
*sqrt(-e^2*x^2 + d^2)*d/abs(x)) - 13/8*sqrt(-e^2*x^2 + d^2)*e^4/d - sqrt(-
e^2*x^2 + d^2)*e^3/x - 13/8*(-e^2*x^2 + d^2)^(3/2)*e^2/(d*x^2) - (-e^2*x^2
 + d^2)^(3/2)*e/x^3 - 1/4*(-e^2*x^2 + d^2)^(3/2)*d/x^4
```

3.64.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(118) = 236.

Time = 0.30 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.43

$$\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx$$
$$= \frac{\left(e^5+\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^3}{x}+\frac{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e}{x^2}+\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{ex^3}\right)e^8x^4}{64\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4|e|}$$
$$-\frac{e^5\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|}+\frac{13\,e^5\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{8\,|e|}$$
$$-\frac{\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^5|e|}{x}+\frac{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^3|e|}{x^2}+\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3e|e|}{x^3}+\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4|e|}{ex^4}}{64\,e^4}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x, algorithm="giac")
```

output
```
1/64*(e^5 + 8*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^3/x + 24*(d*e + sqrt(-
e^2*x^2 + d^2)*abs(e))^2*e/x^2 + 8*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/(
e*x^3))*e^8*x^4/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*abs(e)) - e^5*arcsi
n(e*x/d)*sgn(d)*sgn(e)/abs(e) + 13/8*e^5*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*
x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) - 1/64*(8*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))*e^5*abs(e)/x + 24*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*e^3*ab
s(e)/x^2 + 8*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*e*abs(e)/x^3 + (d*e + s
qrt(-e^2*x^2 + d^2)*abs(e))^4*abs(e)/(e*x^4))/e^4
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx = \int \frac{\sqrt{d^2-e^2\,x^2}\,(d+e\,x)^3}{x^5}\,dx$$

input
```
int(((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3)/x^5,x)
```

output
```
int(((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3)/x^5, x)
```

3.65 $\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

3.65.1 Optimal result

Integrand size = 27, antiderivative size = 310

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5}+\frac{35d^{10}x(d^2-e^2x^2)^{3/2}}{3072e^5}+\frac{7d^8x(d^2-e^2x^2)^{5/2}}{768e^5}-\frac{124d^5x^2(d^2-e^2x^2)^{7/2}}{1287e^4}-\frac{7d^4x^3(d^2-e^2x^2)^{7/2}}{48e^3}-\frac{31d^3x^4(d^2-e^2x^2)^{7/2}}{143e^2}-\frac{7d^2x^5(d^2-e^2x^2)^{7/2}}{24e}-\frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}-\frac{d^6(31744d+63063ex)\left(d^2-e^2x^2\right)^{7/2}}{1153152e^6}+\frac{35d^{14}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2048e^6}$$

output
```
35/3072*d^10*x*(-e^2*x^2+d^2)^(3/2)/e^5+7/768*d^8*x*(-e^2*x^2+d^2)^(5/2)/e
^5-124/1287*d^5*x^2*(-e^2*x^2+d^2)^(7/2)/e^4-7/48*d^4*x^3*(-e^2*x^2+d^2)^(
7/2)/e^3-31/143*d^3*x^4*(-e^2*x^2+d^2)^(7/2)/e^2-7/24*d^2*x^5*(-e^2*x^2+d^
2)^(7/2)/e-3/13*d*x^6*(-e^2*x^2+d^2)^(7/2)-1/14*e*x^7*(-e^2*x^2+d^2)^(7/2)
-1/1153152*d^6*(63063*e*x+31744*d)*(-e^2*x^2+d^2)^(7/2)/e^6+35/2048*d^14*a
rctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+35/2048*d^12*x*(-e^2*x^2+d^2)^(1/2)/e^
5
```

3.65.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.65

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=$$

$$-\frac{\sqrt{d^2-e^2x^2}(507904d^{13}+315315d^{12}ex+253952d^{11}e^2x^2+210210d^{10}e^3x^3+190464d^9e^4x^4+168168d^8e^5x^5}{}$$

input
```
Integrate[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

output
```
-1/18450432*(Sqrt[d^2 - e^2*x^2]*(507904*d^13 + 315315*d^12*e*x + 253952*d
^11*e^2*x^2 + 210210*d^10*e^3*x^3 + 190464*d^9*e^4*x^4 + 168168*d^8*e^5*x^
5 - 2916352*d^7*e^6*x^6 - 7763184*d^6*e^7*x^7 - 2551808*d^5*e^8*x^8 + 9499
776*d^4*e^9*x^9 + 8773632*d^3*e^10*x^10 - 1427712*d^2*e^11*x^11 - 4257792*
d*e^12*x^12 - 1317888*e^13*x^13) + 630630*d^14*ArcTan[(e*x)/(Sqrt[d^2] - S
qrt[d^2 - e^2*x^2])])/e^6
```

3.65.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.23, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {541, 27, 2340, 25, 27, 533, 27, 533, 27, 533, 27, 533, 27, 533, 27, 455, 211, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$$

$$\Big\downarrow \text{541}$$

$$-\frac{\int -7x^5\left(d^2-e^2x^2\right)^{5/2}\left(6dx^2e^4+7d^2xe^3+2d^3e^2\right)dx}{14e^2}-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int x^5\left(d^2-e^2x^2\right)^{5/2}\left(6dx^2e^4+7d^2xe^3+2d^3e^2\right)dx}{2e^2}-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{2340}$$

$$\frac{-\frac{\int -d^2e^4x^5(62d+91ex)\left(d^2-e^2x^2\right)^{5/2}dx}{13e^2}-\frac{6}{13}de^2x^6\left(d^2-e^2x^2\right)^{7/2}}{2e^2}-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int d^2e^4x^5(62d+91ex)\left(d^2-e^2x^2\right)^{5/2}dx}{13e^2}-\frac{6}{13}de^2x^6\left(d^2-e^2x^2\right)^{7/2}}{2e^2}-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{13}d^2e^2\int x^5(62d+91ex)\left(d^2-e^2x^2\right)^{5/2}dx-\frac{6}{13}de^2x^6\left(d^2-e^2x^2\right)^{7/2}}{2e^2}-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{533}$$

$$\frac{\frac{1}{13}d^2e^2\left(\frac{\int dex^4(455d+744ex)\left(d^2-e^2x^2\right)^{5/2}dx}{12e^2}-\frac{91x^5\left(d^2-e^2x^2\right)^{7/2}}{12e}\right)-\frac{6}{13}de^2x^6\left(d^2-e^2x^2\right)^{7/2}}{2e^2}-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{13}d^2e^2\left(\frac{d\int x^4(455d+744ex)\left(d^2-e^2x^2\right)^{5/2}dx}{12e}-\frac{91x^5\left(d^2-e^2x^2\right)^{7/2}}{12e}\right)-\frac{6}{13}de^2x^6\left(d^2-e^2x^2\right)^{7/2}}{2e^2}-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{533}$$

$$\frac{\frac{1}{13}d^2e^2\left(\frac{d\left(\frac{\int dex^3(2976d+5005ex)\left(d^2-e^2x^2\right)^{5/2}dx}{11e^2}-\frac{744x^4\left(d^2-e^2x^2\right)^{7/2}}{11e}\right)}{12e}-\frac{91x^5\left(d^2-e^2x^2\right)^{7/2}}{12e}\right)-\frac{6}{13}de^2x^6\left(d^2-e^2x^2\right)^{7/2}}{2e^2}-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{13}d^2e^2\left(\frac{d\left(\frac{d\int x^3(2976d+5005ex)\left(d^2-e^2x^2\right)^{5/2}dx}{11e}-\frac{744x^4\left(d^2-e^2x^2\right)^{7/2}}{11e}\right)}{12e}-\frac{91x^5\left(d^2-e^2x^2\right)^{7/2}}{12e}\right)-\frac{6}{13}de^2x^6\left(d^2-e^2x^2\right)^{7/2}}{2e^2}-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{533}$$

3.65. $\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}dx$

$$\frac{\frac{1}{13}d^2e^2\left(\frac{d\left(\frac{d\left(\frac{\int 15dex^2(1001d+1984ex)\left(d^2-e^2x^2\right)^{5/2}dx}{10e^2}-\frac{1001x^3\left(d^2-e^2x^2\right)^{7/2}}{2e}\right)}{11e}-\frac{744x^4\left(d^2-e^2x^2\right)^{7/2}}{11e}\right)}{12e}-\frac{91x^5\left(d^2-e^2x^2\right)^{7/2}}{12e}\right)-\frac{6}{13}de^2x^6}{2e^2}$$

$$\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{13}d^2e^2\left(\frac{d\left(\frac{d\left(\frac{3d\int x^2(1001d+1984ex)\left(d^2-e^2x^2\right)^{5/2}dx}{2e}-\frac{1001x^3\left(d^2-e^2x^2\right)^{7/2}}{2e}\right)}{11e}-\frac{744x^4\left(d^2-e^2x^2\right)^{7/2}}{11e}\right)}{12e}-\frac{91x^5\left(d^2-e^2x^2\right)^{7/2}}{12e}\right)-\frac{6}{13}de^2x^6(}{2e^2}$$

$$\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \; 533$$

$$\frac{\frac{1}{13}d^2e^2\left(\frac{d\left(\frac{d\left(\frac{3d\left(\frac{\int dex(3968d+9009ex)\left(d^2-e^2x^2\right)^{5/2}dx}{9e^2}-\frac{1984x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)}{2e}-\frac{1001x^3\left(d^2-e^2x^2\right)^{7/2}}{2e}\right)}{11e}-\frac{744x^4\left(d^2-e^2x^2\right)^{7/2}}{11e}\right)}{12e}-\frac{91x^5\left(d^2-e\right.}{12e}}{2e^2}$$

$$\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \; 27$$

3.65. $\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\frac{\frac{1}{13}d^2e^2\left(\frac{d\left(\frac{d\left(\frac{3d\left(\frac{d\int x(3968d+9009ex)\left(d^2-e^2x^2\right)^{5/2}dx}{9e}-\frac{1984x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)}{2e}-\frac{1001x^3\left(d^2-e^2x^2\right)^{7/2}}{2e}\right)}{11e}-\frac{744x^4\left(d^2-e^2x^2\right)^{7/2}}{11e}\right)}{12e}-\frac{91x^5\left(d^2-e^{2}\right.}{12e}\right.}{2e^2}$$

$$\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{533}$$

3.65. $\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\frac{\frac{1}{13}d^2e^2\left(\frac{d\left(\frac{d\left(\frac{3d\left(\frac{d\left(\frac{\int de(9009d+31744ex)\left(d^2-e^2x^2\right)^{5/2}dx}{8e^2}-\frac{9009x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)}{9e}-\frac{1984x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)}{2e}-\frac{1001x^3\left(d^2-e^2x^2\right)^{7/2}}{2e}\right)}{11e}-\frac{744x^4\left(d^2-e\ldots\right.}{11e}\ldots}{12e}\ldots\right.}{2e^2}$$

$$\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 27

3.65. $\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\frac{\frac{1}{13}d^2e^2\left(\frac{d\left(\frac{d\left(\frac{3d\left(\frac{d\left(\frac{d\int(9009d+31744ex)\left(d^2-e^2x^2\right)^{5/2}dx}{8e}-\frac{9009x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)}{9e}-\frac{1984x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)}{2e}-\frac{1001x^3\left(d^2-e^2x^2\right)^{7/2}}{2e}\right)}{11e}-\frac{744x^4\left(d^2-e^2\right.}{11e}}{12e}\right.}{2e^2}$$

$$\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 455

3.65. $\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\frac{1}{13}d^2e^2\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{3d\left(\frac{d\left(\frac{d\left(9009d\int\left(d^2-e^2x^2\right)^{5/2}dx-\frac{31744\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-\frac{9009x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)}{9e}-\frac{1984x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)}{2e}-\frac{1001x^3\left(d^2-e^2x^2\right)^{7/2}}{2e}\right)}{11e}\right.}{12e}\right.$$

3.65. $\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\downarrow \text{211}$$

3.65. $\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\frac{1}{13}d^2e^2 \left(\frac{d \left(\frac{d \left(\frac{3d \left(\frac{d \left(\frac{d \left(9009d \left(\frac{5}{6}d^2 \int \left(d^2-e^2x^2\right)^{3/2} dx + \frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right) - \frac{31744\left(d^2-e^2x^2\right)^{7/2}}{7e} \right)}{8e} - \frac{9009x\left(d^2-e^2x^2\right)^{7/2}}{8e} \right)}{9e} - \frac{1984x^2\left(d^2-e^2x^2\right)^{7/2}}{9e} \right)}{2e}}{11e}}{12e} \right.$$

3.65. $\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\Bigg\downarrow \; 211$$

3.65. $\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\frac{1}{13}d^2e^2\left(\frac{d\left(\frac{d\left(\frac{3d\left(\frac{d\left(\frac{d\left(9009d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\int\sqrt{d^2-e^2x^2}dx+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{31744\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-\frac{9009x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)}{9e}-\cdots}{2e}\cdots}{11e}\cdots}{12e}\cdots\right.$$

3.65. $\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\Big\downarrow \text{211}$$

3.65. $\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\frac{1}{13}d^2e^2\left(\frac{d\left(\frac{d\left(\frac{3d\left(\frac{d\left(9009d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{31744\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-\frac{90\cdots}{\cdots}\cdots}{9e}\cdots}{2e}\cdots}{11e}\cdots}{12e}\cdots\right.$$

3.65. $\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$\downarrow$ 224

3.65. $\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

3.65.

$$\frac{1}{13}d^2e^2\left(\frac{\left(d\,\frac{\left(d\,\frac{\left(3d\,\frac{\left(d\,\frac{d\left(9009d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}\,d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{31744\left(d^2-e^2\cdots\right.}{7e}\cdots}{8e}\cdots\right.}{9e}\cdots\right.}{2e}\cdots\right.}{11e}\cdots\right.}{12e}\cdots\right.$$

$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

↓ 216

3.65. $\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\frac{1}{13}d^2e^2\left(\frac{d\left(\frac{d\left(\frac{3d\left(\frac{d\left(\frac{d\left(9009d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{31744\left(d^2-e^2x^2\right)^{7/2}}{7e}\cdots}{8e}\cdots}{9e}\cdots}{2e}\cdots}{11e}\cdots}{12e}\cdots$$

3.65. $\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

input
```
Int[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

output
```
-1/14*(e*x^7*(d^2 - e^2*x^2)^(7/2)) + ((-6*d*e^2*x^6*(d^2 - e^2*x^2)^(7/2)
)/13 + (d^2*e^2*((-91*x^5*(d^2 - e^2*x^2)^(7/2))/(12*e) + (d*((-744*x^4*(d
^2 - e^2*x^2)^(7/2))/(11*e) + (d*((-1001*x^3*(d^2 - e^2*x^2)^(7/2))/(2*e)
+ (3*d*((-1984*x^2*(d^2 - e^2*x^2)^(7/2))/(9*e) + (d*((-9009*x*(d^2 - e^2*
x^2)^(7/2))/(8*e) + (d*((-31744*(d^2 - e^2*x^2)^(7/2))/(7*e) + 9009*d*((x*
(d^2 - e^2*x^2)^(5/2))/6 + (5*d^2*((x*(d^2 - e^2*x^2)^(3/2))/4 + (3*d^2*((
x*Sqrt[d^2 - e^2*x^2])/2 + (d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)))
/4))/6)))/(8*e)))/(9*e)))/(2*e)))/(11*e)))/(12*e)))/13)/(2*e^2)
```

3.65.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

3.65.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{\left(-1317888e^{13}x^{13}-4257792d\,e^{12}x^{12}-1427712d^{2}e^{11}x^{11}+8773632d^{3}e^{10}x^{10}+9499776d^{4}e^{9}x^{9}-2551808d^{5}e^{8}x^{8}-7763184d^{6}e^{7}x^{7}-2\right.}{18450432e}$ $d^{2}\left(-\frac{x^{5}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{12e^{2}}+\frac{5d^{2}\left(-\frac{x^{3}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{10e^{2}}+\frac{3d^{2}\left(-\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{8e^{2}}+\frac{d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}+\frac{5d^{2}(\ldots)}{}\right)}{}\right)}{}\right)}{12e^{2}}\right.$

3.65. $\int x^{5}(d+ex)^{3}\left(d^{2}-e^{2}x^{2}\right)^{5/2}\,dx$

input
```
int(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/18450432*(-1317888*e^13*x^13-4257792*d*e^12*x^12-1427712*d^2*e^11*x^11+
8773632*d^3*e^10*x^10+9499776*d^4*e^9*x^9-2551808*d^5*e^8*x^8-7763184*d^6*
e^7*x^7-2916352*d^7*e^6*x^6+168168*d^8*e^5*x^5+190464*d^9*e^4*x^4+210210*d
^10*e^3*x^3+253952*d^11*e^2*x^2+315315*d^12*e*x+507904*d^13)/e^6*(-e^2*x^2
+d^2)^(1/2)+35/2048*d^14/e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^
2)^(1/2))
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.63

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx =$$

$$-\frac{630630\,d^{14}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(1317888\,e^{13}x^{13}+4257792\,de^{12}x^{12}+1427712\,d^2e^{11}x^{11}-8773632\,d^3\right.}{}$$

input
```
integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/18450432*(630630*d^14*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (1317
888*e^13*x^13 + 4257792*d*e^12*x^12 + 1427712*d^2*e^11*x^11 - 8773632*d^3*
e^10*x^10 - 9499776*d^4*e^9*x^9 + 2551808*d^5*e^8*x^8 + 7763184*d^6*e^7*x^
7 + 2916352*d^7*e^6*x^6 - 168168*d^8*e^5*x^5 - 190464*d^9*e^4*x^4 - 210210
*d^10*e^3*x^3 - 253952*d^11*e^2*x^2 - 315315*d^12*e*x - 507904*d^13)*sqrt(
-e^2*x^2 + d^2))/e^6
```

3.65.6 Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.98

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = \begin{cases}\frac{35d^{14}\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{2048e^5}+\sqrt{d^2-e^2x^2}\left(-\frac{248d^{13}}{9009e^6}-\frac{35d^{12}x}{2048e^5}-\frac{124d^{11}x^2}{9009e^4}-\right. \\ \left(\frac{d^3x^6}{6}+\frac{3d^2ex^7}{7}+\frac{3de^2x^8}{8}+\frac{e^3x^9}{9}\right)\left(d^2\right)^{\frac{5}{2}}\end{cases}$$

input
```
integrate(x**5*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Piecewise((35*d**14*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e
**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(
2048*e**5) + sqrt(d**2 - e**2*x**2)*(-248*d**13/(9009*e**6) - 35*d**12*x/(
2048*e**5) - 124*d**11*x**2/(9009*e**4) - 35*d**10*x**3/(3072*e**3) - 31*d
**9*x**4/(3003*e**2) - 7*d**8*x**5/(768*e) + 1424*d**7*x**6/9009 + 377*d**
6*e*x**7/896 + 178*d**5*e**2*x**8/1287 - 173*d**4*e**3*x**9/336 - 68*d**3*
e**4*x**10/143 + 13*d**2*e**5*x**11/168 + 3*d*e**6*x**12/13 + e**7*x**13/1
4), Ne(e**2, 0)), ((d**3*x**6/6 + 3*d**2*e*x**7/7 + 3*d*e**2*x**8/8 + e**3
*x**9/9)*(d**2)**(5/2), True))
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.91

$$
\begin{aligned}
\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx &= -\frac{1}{14}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^7 \\
&- \frac{3}{13}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^6 - \frac{7\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x^5}{24\,e} \\
&+ \frac{35\,d^{14}\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2048\sqrt{e^2}e^5} + \frac{35\sqrt{-e^2x^2+d^2}d^{12}x}{2048\,e^5} - \frac{31\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3x^4}{143\,e^2} \\
&+ \frac{35\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^{10}x}{3072\,e^5} - \frac{7\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^4x^3}{48\,e^3} + \frac{7\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^8x}{768\,e^5} \\
&- \frac{124\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^5x^2}{1287\,e^4} - \frac{7\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^6x}{128\,e^5} - \frac{248\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^7}{9009\,e^6}
\end{aligned}
$$

input
```
integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/14*(-e^2*x^2 + d^2)^(7/2)*e*x^7 - 3/13*(-e^2*x^2 + d^2)^(7/2)*d*x^6 - 7
/24*(-e^2*x^2 + d^2)^(7/2)*d^2*x^5/e + 35/2048*d^14*arcsin(e^2*x/(d*sqrt(e
^2)))/(sqrt(e^2)*e^5) + 35/2048*sqrt(-e^2*x^2 + d^2)*d^12*x/e^5 - 31/143*(
-e^2*x^2 + d^2)^(7/2)*d^3*x^4/e^2 + 35/3072*(-e^2*x^2 + d^2)^(3/2)*d^10*x/
e^5 - 7/48*(-e^2*x^2 + d^2)^(7/2)*d^4*x^3/e^3 + 7/768*(-e^2*x^2 + d^2)^(5/
2)*d^8*x/e^5 - 124/1287*(-e^2*x^2 + d^2)^(7/2)*d^5*x^2/e^4 - 7/128*(-e^2*x
^2 + d^2)^(7/2)*d^6*x/e^5 - 248/9009*(-e^2*x^2 + d^2)^(7/2)*d^7/e^6
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.61

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{35\,d^{14}\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2048\,e^5|e|}$$
$$-\frac{1}{18450432}\left(\frac{507904\,d^{13}}{e^6}+\left(\frac{315315\,d^{12}}{e^5}+2\left(\frac{126976\,d^{11}}{e^4}+\left(\frac{105105\,d^{10}}{e^3}+4\left(\frac{23808\,d^9}{e^2}+\left(\frac{21021\,d^8}{e}-2\right.\right.\right.\right.\right.\right.$$

input
```
integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
35/2048*d^14*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^5*abs(e)) - 1/18450432*(507904
*d^13/e^6 + (315315*d^12/e^5 + 2*(126976*d^11/e^4 + (105105*d^10/e^3 + 4*(
23808*d^9/e^2 + (21021*d^8/e - 2*(182272*d^7 + (485199*d^6*e + 8*(19936*d^
5*e^2 - 3*(24739*d^4*e^3 + 2*(11424*d^3*e^4 - 11*(169*d^2*e^5 + 12*(13*e^7
*x + 42*d*e^6)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*sqrt(-e^2*x^2 + d^2)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\int x^5\,\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3\,dx$$

input
```
int(x^5*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

output
```
int(x^5*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

3.66 $\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

3.66.1 Optimal result

Integrand size = 27, antiderivative size = 281

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4}+\frac{9d^9x(d^2-e^2x^2)^{3/2}}{512e^4}+\frac{9d^7x(d^2-e^2x^2)^{5/2}}{640e^4}-\frac{20d^4x^2(d^2-e^2x^2)^{7/2}}{143e^3}-\frac{9d^3x^3(d^2-e^2x^2)^{7/2}}{40e^2}-\frac{45d^2x^4(d^2-e^2x^2)^{7/2}}{143e}-\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}-\frac{d^5(12800d+27027ex)\left(d^2-e^2x^2\right)^{7/2}}{320320e^5}+\frac{27d^{13}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{1024e^5}$$

output
```
9/512*d^9*x*(-e^2*x^2+d^2)^(3/2)/e^4+9/640*d^7*x*(-e^2*x^2+d^2)^(5/2)/e^4-
20/143*d^4*x^2*(-e^2*x^2+d^2)^(7/2)/e^3-9/40*d^3*x^3*(-e^2*x^2+d^2)^(7/2)/
e^2-45/143*d^2*x^4*(-e^2*x^2+d^2)^(7/2)/e-1/4*d*x^5*(-e^2*x^2+d^2)^(7/2)-1
/13*e*x^6*(-e^2*x^2+d^2)^(7/2)-1/320320*d^5*(27027*e*x+12800*d)*(-e^2*x^2+
d^2)^(7/2)/e^5+27/1024*d^13*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5+27/1024*d
^11*x*(-e^2*x^2+d^2)^(1/2)/e^4
```

3.66.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.68

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{\sqrt{d^2-e^2x^2}(-204800d^{12}-135135d^{11}ex-102400d^{10}e^2x^2-90090d^9e^3x^3-76800d^8e^4x^4+\cdots}{}$$

input
```
Integrate[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-204800*d^12 - 135135*d^11*e*x - 102400*d^10*e^2*x^2
 - 90090*d^9*e^3*x^3 - 76800*d^8*e^4*x^4 + 952952*d^7*e^5*x^5 + 2498560*d^
6*e^6*x^6 + 816816*d^5*e^7*x^7 - 2938880*d^4*e^8*x^8 - 2690688*d^3*e^9*x^9
 + 430080*d^2*e^10*x^10 + 1281280*d*e^11*x^11 + 394240*e^12*x^12) - 270270
*d^13*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(5125120*e^5)
```

3.66.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.23, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {541, 25, 2340, 27, 533, 27, 533, 27, 533, 27, 533, 27, 455, 211, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}dx$$

$$\Big\downarrow \text{541}$$

$$-\frac{\int -x^4\left(d^2-e^2x^2\right)^{5/2}\left(39dx^2e^4+45d^2xe^3+13d^3e^2\right)dx}{13e^2}-\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int x^4\left(d^2-e^2x^2\right)^{5/2}\left(39dx^2e^4+45d^2xe^3+13d^3e^2\right)dx}{13e^2}-\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{2340}$$

$$\frac{-\frac{\int -27d^2e^4x^4(13d+20ex)\left(d^2-e^2x^2\right)^{5/2}dx}{12e^2}-\frac{13}{4}de^2x^5\left(d^2-e^2x^2\right)^{7/2}}{13e^2}-\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{9}{4}d^2e^2\int x^4(13d+20ex)\left(d^2-e^2x^2\right)^{5/2}dx-\frac{13}{4}de^2x^5\left(d^2-e^2x^2\right)^{7/2}}{13e^2}-\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{533}$$

$$\frac{\frac{9}{4}d^2e^2\left(\frac{\int dex^3(80d+143ex)\left(d^2-e^2x^2\right)^{5/2}dx}{11e^2}-\frac{20x^4\left(d^2-e^2x^2\right)^{7/2}}{11e}\right)-\frac{13}{4}de^2x^5\left(d^2-e^2x^2\right)^{7/2}}{13e^2}-\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{9}{4}d^2e^2\left(\frac{d\int x^3(80d+143ex)\left(d^2-e^2x^2\right)^{5/2}dx}{11e}-\frac{20x^4\left(d^2-e^2x^2\right)^{7/2}}{11e}\right)-\frac{13}{4}de^2x^5\left(d^2-e^2x^2\right)^{7/2}}{13e^2}-\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{533}$$

$$\frac{\frac{9}{4}d^2e^2\left(\frac{d\left(\frac{\int dex^2(429d+800ex)\left(d^2-e^2x^2\right)^{5/2}dx}{10e^2}-\frac{143x^3\left(d^2-e^2x^2\right)^{7/2}}{10e}\right)}{11e}-\frac{20x^4\left(d^2-e^2x^2\right)^{7/2}}{11e}\right)-\frac{13}{4}de^2x^5\left(d^2-e^2x^2\right)^{7/2}}{13e^2}-\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{9}{4}d^2e^2\left(\frac{d\left(\frac{d\int x^2(429d+800ex)\left(d^2-e^2x^2\right)^{5/2}dx}{10e}-\frac{143x^3\left(d^2-e^2x^2\right)^{7/2}}{10e}\right)}{11e}-\frac{20x^4\left(d^2-e^2x^2\right)^{7/2}}{11e}\right)-\frac{13}{4}de^2x^5\left(d^2-e^2x^2\right)^{7/2}}{13e^2}-\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{533}$$

3.66. $\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\frac{9}{4}d^2e^2\left(\frac{d\left(\frac{d\left(\frac{\int dex(1600d+3861ex)\left(d^2-e^2x^2\right)^{5/2}dx}{9e^2}-\frac{800x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)}{10e}-\frac{143x^3\left(d^2-e^2x^2\right)^{7/2}}{10e}\right)}{11e}-\frac{20x^4\left(d^2-e^2x^2\right)^{7/2}}{11e}\right)-\frac{13}{4}de^2x^5\left(d^2 \cdots\right.$$

$$13e^2$$

$$\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 27

$$\frac{9}{4}d^2e^2\left(\frac{d\left(\frac{d\left(\frac{d\int x(1600d+3861ex)\left(d^2-e^2x^2\right)^{5/2}dx}{9e}-\frac{800x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)}{10e}-\frac{143x^3\left(d^2-e^2x^2\right)^{7/2}}{10e}\right)}{11e}-\frac{20x^4\left(d^2-e^2x^2\right)^{7/2}}{11e}\right)-\frac{13}{4}de^2x^5\left(d^2 \cdots\right.$$

$$13e^2$$

$$\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 533

$$\frac{9}{4}d^2e^2\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{\int de(3861d+12800ex)\left(d^2-e^2x^2\right)^{5/2}dx}{8e^2}-\frac{3861x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)}{9e}-\frac{800x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)}{10e}-\frac{143x^3\left(d^2-e^2x^2\right)^{7/2}}{10e}\right)}{11e}-\frac{20x^4\left(d^2-e^2x^2\right) \cdots}{11e}\right.$$

$$13e^2$$

$$\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 27

3.66. $\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\frac{\frac{9}{4}d^2e^2\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{d\int(3861d+12800ex)\left(d^2-e^2x^2\right)^{5/2}dx}{8e}-\frac{3861x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)}{9e}-\frac{800x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)}{10e}-\frac{143x^3\left(d^2-e^2x^2\right)^{7/2}}{10e}\right)}{11e}-\frac{20x^4\left(d^2-e^2x^2\right)^{7/2}}{11e}\right)}{13e^2}$$

$$\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 455

3.66. $\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\frac{\frac{9}{4}d^2e^2\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{d\left(3861d\int\left(d^2-e^2x^2\right)^{5/2}dx-\frac{12800\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-\frac{3861x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)}{9e}-\frac{800x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)}{10e}-\frac{143x^3\left(d^2-e^2x^2\right)^{7/2}}{10e}\right)}{11e}-\frac{20}{}\cdots}{13e^2}$$

$$\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 211

3.66. $\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\frac{\frac{9}{4}d^2e^2\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{d\left(3861d\left(\frac{5}{6}d^2\int\left(d^2-e^2x^2\right)^{3/2}dx+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{12800\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-\frac{3861x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)}{9e}-\frac{800x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)}{10e}-\frac{143x\cdots}{\cdots}}{11e}\cdots\right)}{13e^2}$$

$$\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 211

3.66. $\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\frac{\frac{9}{4}d^2e^2\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{d\left(3861d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\int\sqrt{d^2-e^2x^2}dx+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{12800\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-\frac{3861x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)}{9e}-\frac{800x^2}{\cdots}}{10e}}{11e}\cdots\right)}{13e^2}$$

$$\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 211

3.66. $\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\frac{\frac{9}{4}d^2e^2\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{d\left(3861d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{12800\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-\frac{3861x\left(d\right.\cdots}{\cdots}}{9e}}{10e}}{11e}\right.\cdots}{13e^2}$$

$$\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 224

3.66. $\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\frac{\frac{9}{4}d^2e^2\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{d\left(3861d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{12800\left(d^2-e^2x^2\right)^{7/}}{7e}\right.}{8e}\right.}{9e}\right.}{10e}\right.}{11e}\right.}{13e^2}$$

$$\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}$$

$$\downarrow 216$$

3.66. $\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\frac{\frac{9}{4}d^2e^2\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{d\left(3861d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{12800\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-\frac{38\cdots}{\cdots}\cdots}{9e}\cdots}{10e}\cdots}{11e}\cdots\right.}{13e^2}\cdots$$

$$\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}$$

input
```
Int[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

3.66. $\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

output
```
-1/13*(e*x^6*(d^2 - e^2*x^2)^(7/2)) + ((-13*d*e^2*x^5*(d^2 - e^2*x^2)^(7/2
))/4 + (9*d^2*e^2*((-20*x^4*(d^2 - e^2*x^2)^(7/2))/(11*e) + (d*((-143*x^3*
(d^2 - e^2*x^2)^(7/2))/(10*e) + (d*((-800*x^2*(d^2 - e^2*x^2)^(7/2))/(9*e)
 + (d*((-3861*x*(d^2 - e^2*x^2)^(7/2))/(8*e) + (d*((-12800*(d^2 - e^2*x^2)
^(7/2))/(7*e) + 3861*d*((x*(d^2 - e^2*x^2)^(5/2))/6 + (5*d^2*((x*(d^2 - e^
2*x^2)^(3/2))/4 + (3*d^2*((x*Sqrt[d^2 - e^2*x^2])/2 + (d^2*ArcTan[(e*x)/Sq
rt[d^2 - e^2*x^2]])/(2*e)))/4))/6)))/(8*e)))/(9*e)))/(10*e)))/(11*e)))/4)/
(13*e^2)
```

3.66.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

3.66.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{(-394240e^{12}x^{12}-1281280d\,e^{11}x^{11}-430080d^2e^{10}x^{10}+2690688d^3e^9x^9+2938880d^4e^8x^8-816816d^5e^7x^7-2498560d^6e^6x^6-952952}{5125120e^5}$
default	$e^3\left(-\frac{x^6(-e^2x^2+d^2)^{\frac{7}{2}}}{13e^2}+\frac{6d^2\left(-\frac{x^4(-e^2x^2+d^2)^{\frac{7}{2}}}{11e^2}+\frac{4d^2\left(-\frac{x^2(-e^2x^2+d^2)^{\frac{7}{2}}}{9e^2}-\frac{2d^2(-e^2x^2+d^2)^{\frac{7}{2}}}{63e^4}\right)}{11e^2}\right)}{13e^2}\right)+d^3\left(-\frac{x^3(-e^2x^2+}{10e^2}\right.$

3.66. $\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

input
```
int(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5125120*(-394240*e^12*x^12-1281280*d*e^11*x^11-430080*d^2*e^10*x^10+269
0688*d^3*e^9*x^9+2938880*d^4*e^8*x^8-816816*d^5*e^7*x^7-2498560*d^6*e^6*x^
6-952952*d^7*e^5*x^5+76800*d^8*e^4*x^4+90090*d^9*e^3*x^3+102400*d^10*e^2*x
^2+135135*d^11*e*x+204800*d^12)/e^5*(-e^2*x^2+d^2)^(1/2)+27/1024*d^13/e^4/
(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.65

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx =$$

$$-\frac{270270\,d^{13}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(394240\,e^{12}x^{12}+1281280\,de^{11}x^{11}+430080\,d^2e^{10}x^{10}-2690688\,d^3e^9\right.}{}$$

input
```
integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/5125120*(270270*d^13*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (39424
0*e^12*x^12 + 1281280*d*e^11*x^11 + 430080*d^2*e^10*x^10 - 2690688*d^3*e^9
*x^9 - 2938880*d^4*e^8*x^8 + 816816*d^5*e^7*x^7 + 2498560*d^6*e^6*x^6 + 95
2952*d^7*e^5*x^5 - 76800*d^8*e^4*x^4 - 90090*d^9*e^3*x^3 - 102400*d^10*e^2
*x^2 - 135135*d^11*e*x - 204800*d^12)*sqrt(-e^2*x^2 + d^2))/e^5
```

3.66.6 Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.02

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = \begin{cases} \frac{27d^{13}\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{1024e^4}+\sqrt{d^2-e^2x^2}\left(-\frac{40d^{12}}{1001e^5}-\frac{27d^{11}x}{1024e^4}-\frac{20d^{10}x^2}{1001e^3}-\right. \\ \left(\frac{d^3x^5}{5}+\frac{d^2ex^6}{2}+\frac{3de^2x^7}{7}+\frac{e^3x^8}{8}\right)\left(d^2\right)^{\frac{5}{2}} \end{cases}$$

input
```
integrate(x**4*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Piecewise((27*d**13*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e
**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(
1024*e**4) + sqrt(d**2 - e**2*x**2)*(-40*d**12/(1001*e**5) - 27*d**11*x/(1
024*e**4) - 20*d**10*x**2/(1001*e**3) - 9*d**9*x**3/(512*e**2) - 15*d**8*x
**4/(1001*e) + 119*d**7*x**5/640 + 488*d**6*e*x**6/1001 + 51*d**5*e**2*x**
7/320 - 82*d**4*e**3*x**8/143 - 21*d**3*e**4*x**9/40 + 12*d**2*e**5*x**10/
143 + d*e**6*x**11/4 + e**7*x**12/13), Ne(e**2, 0)), ((d**3*x**5/5 + d**2*
e*x**6/2 + 3*d*e**2*x**7/7 + e**3*x**8/8)*(d**2)**(5/2), True))
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.91

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{1}{13}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^6-\frac{1}{4}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^5 + \frac{27\,d^{13}\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{1024\,\sqrt{e^2}e^4}+\frac{27\,\sqrt{-e^2x^2+d^2}d^{11}x}{1024\,e^4}-\frac{45\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x^4}{143\,e} + \frac{9\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^9x}{512\,e^4}-\frac{9\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3x^3}{40\,e^2}+\frac{9\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^7x}{640\,e^4} - \frac{20\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^4x^2}{143\,e^3}-\frac{27\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^5x}{320\,e^4}-\frac{40\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^6}{1001\,e^5}$$

input
```
integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/13*(-e^2*x^2 + d^2)^(7/2)*e*x^6 - 1/4*(-e^2*x^2 + d^2)^(7/2)*d*x^5 + 27
/1024*d^13*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^4) + 27/1024*sqrt(-e^2
*x^2 + d^2)*d^11*x/e^4 - 45/143*(-e^2*x^2 + d^2)^(7/2)*d^2*x^4/e + 9/512*(
-e^2*x^2 + d^2)^(3/2)*d^9*x/e^4 - 9/40*(-e^2*x^2 + d^2)^(7/2)*d^3*x^3/e^2
+ 9/640*(-e^2*x^2 + d^2)^(5/2)*d^7*x/e^4 - 20/143*(-e^2*x^2 + d^2)^(7/2)*d
^4*x^2/e^3 - 27/320*(-e^2*x^2 + d^2)^(7/2)*d^5*x/e^4 - 40/1001*(-e^2*x^2 +
 d^2)^(7/2)*d^6/e^5
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.63

$$\int x^4(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} \, dx = \frac{27\,d^{13}\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{1024\,e^4|e|}$$
$$-\frac{1}{5125120}\left(\frac{204800\,d^{12}}{e^5}+\left(\frac{135135\,d^{11}}{e^4}+2\left(\frac{51200\,d^{10}}{e^3}+\left(\frac{45045\,d^9}{e^2}+4\left(\frac{9600\,d^8}{e}-\left(119119\,d^7+2\left(1561\right.\right.\right.\right.\right.\right.\right.$$

input
```
integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
27/1024*d^13*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^4*abs(e)) - 1/5125120*(204800*
d^12/e^5 + (135135*d^11/e^4 + 2*(51200*d^10/e^3 + (45045*d^9/e^2 + 4*(9600
*d^8/e - (119119*d^7 + 2*(156160*d^6*e + 7*(7293*d^5*e^2 - 8*(3280*d^4*e^3
 + (3003*d^3*e^4 - 10*(48*d^2*e^5 + 11*(4*e^7*x + 13*d*e^6)*x)*x)*x)*x)*x)
*x)*x)*x)*x)*x)*x)*x)*sqrt(-e^2*x^2 + d^2)
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} \, dx = \int x^4\,\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3 \,dx$$

input
```
int(x^4*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

output
```
int(x^4*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

3.67 $\int x^3(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx$

3.67.1 Optimal result

Integrand size = 27, antiderivative size = 252

$$\int x^3(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx = \frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3}+\frac{41d^8x(d^2-e^2x^2)^{3/2}}{1536e^3}+\frac{41d^6x(d^2-e^2x^2)^{5/2}}{1920e^3}-\frac{23d^3x^2(d^2-e^2x^2)^{7/2}}{99e^2}-\frac{41d^2x^3(d^2-e^2x^2)^{7/2}}{120e}-\frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}-\frac{d^4(14720d+28413ex)\left(d^2-e^2x^2\right)^{7/2}}{221760e^4}+\frac{41d^{12}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{1024e^4}$$

output
```
41/1536*d^8*x*(-e^2*x^2+d^2)^(3/2)/e^3+41/1920*d^6*x*(-e^2*x^2+d^2)^(5/2)/
e^3-23/99*d^3*x^2*(-e^2*x^2+d^2)^(7/2)/e^2-41/120*d^2*x^3*(-e^2*x^2+d^2)^(
7/2)/e-3/11*d*x^4*(-e^2*x^2+d^2)^(7/2)-1/12*e*x^5*(-e^2*x^2+d^2)^(7/2)-1/2
21760*d^4*(28413*e*x+14720*d)*(-e^2*x^2+d^2)^(7/2)/e^4+41/1024*d^12*arctan
(e*x/(-e^2*x^2+d^2)^(1/2))/e^4+41/1024*d^10*x*(-e^2*x^2+d^2)^(1/2)/e^3
```

3.67.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.71

$$\int x^3(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx = \frac{\sqrt{d^2-e^2x^2}(-235520d^{11}-142065d^{10}ex-117760d^9e^2x^2-94710d^8e^3x^3+798720d^7e^4x^4+\cdots}{}$$

input
```
Integrate[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-235520*d^11 - 142065*d^10*e*x - 117760*d^9*e^2*x^2
- 94710*d^8*e^3*x^3 + 798720*d^7*e^4*x^4 + 2053128*d^6*e^5*x^5 + 665600*d^
5*e^6*x^6 - 2295216*d^4*e^7*x^7 - 2078720*d^3*e^8*x^8 + 325248*d^2*e^9*x^9
 + 967680*d*e^10*x^10 + 295680*e^11*x^11) - 284130*d^12*ArcTan[(e*x)/(Sqrt
[d^2] - Sqrt[d^2 - e^2*x^2])])/(3548160*e^4)
```

3.67.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.23, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.630, Rules used = {541, 25, 2340, 25, 27, 533, 27, 533, 27, 533, 27, 455, 211, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}dx$$

$$\Big\downarrow \text{ 541}$$

$$-\frac{\int -x^3\left(d^2-e^2x^2\right)^{5/2}\left(36dx^2e^4+41d^2xe^3+12d^3e^2\right)dx}{12e^2}-\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int x^3\left(d^2-e^2x^2\right)^{5/2}\left(36dx^2e^4+41d^2xe^3+12d^3e^2\right)dx}{12e^2}-\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{ 2340}$$

$$\frac{-\frac{\int -d^2e^4x^3(276d+451ex)\left(d^2-e^2x^2\right)^{5/2}dx}{11e^2}-\frac{36}{11}de^2x^4\left(d^2-e^2x^2\right)^{7/2}}{12e^2}-\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int d^2e^4x^3(276d+451ex)\left(d^2-e^2x^2\right)^{5/2}dx}{11e^2}-\frac{36}{11}de^2x^4\left(d^2-e^2x^2\right)^{7/2}}{12e^2}-\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{1}{11}d^2e^2\int x^3(276d+451ex)\left(d^2-e^2x^2\right)^{5/2}dx-\frac{36}{11}de^2x^4\left(d^2-e^2x^2\right)^{7/2}}{12e^2}-\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{ 533}$$

$$\frac{\frac{1}{11}d^2e^2\left(\frac{\int 3dex^2(451d+920ex)\left(d^2-e^2x^2\right)^{5/2}dx}{10e^2}-\frac{451x^3\left(d^2-e^2x^2\right)^{7/2}}{10e}\right)-\frac{36}{11}de^2x^4\left(d^2-e^2x^2\right)^{7/2}}{12e^2}-\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{1}{11}d^2e^2\left(\frac{3d\int x^2(451d+920ex)\left(d^2-e^2x^2\right)^{5/2}dx}{10e}-\frac{451x^3\left(d^2-e^2x^2\right)^{7/2}}{10e}\right)-\frac{36}{11}de^2x^4\left(d^2-e^2x^2\right)^{7/2}}{12e^2}-\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{ 533}$$

$$\frac{\frac{1}{11}d^2e^2\left(\frac{3d\left(\frac{\int dex(1840d+4059ex)\left(d^2-e^2x^2\right)^{5/2}dx}{9e^2}-\frac{920x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)}{10e}-\frac{451x^3\left(d^2-e^2x^2\right)^{7/2}}{10e}\right)-\frac{36}{11}de^2x^4\left(d^2-e^2x^2\right)^{7/2}}{12e^2}-\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{1}{11}d^2e^2\left(\frac{3d\left(\frac{d\int x(1840d+4059ex)\left(d^2-e^2x^2\right)^{5/2}dx}{9e}-\frac{920x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)}{10e}-\frac{451x^3\left(d^2-e^2x^2\right)^{7/2}}{10e}\right)-\frac{36}{11}de^2x^4\left(d^2-e^2x^2\right)^{7/2}}{12e^2}-\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{ 533}$$

$$\frac{\frac{1}{11}d^2e^2\left(\frac{3d\left(\frac{d\left(\frac{\int de(4059d+14720ex)\left(d^2-e^2x^2\right)^{5/2}dx}{8e^2}-\frac{4059x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)}{9e}-\frac{920x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)}{10e}-\frac{451x^3\left(d^2-e^2x^2\right)^{7/2}}{10e}\right)-\frac{36}{11}de^2x^4(d}{12e^2}$$

$$\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{ 27}$$

3.67. $\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\frac{\frac{1}{11}d^2e^2\left(\frac{3d\left(\frac{d\left(\frac{d\int(4059d+14720ex)\left(d^2-e^2x^2\right)^{5/2}dx}{8e}-\frac{4059x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)}{9e}-\frac{920x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)}{10e}-\frac{451x^3\left(d^2-e^2x^2\right)^{7/2}}{10e}\right)-\frac{36}{11}de^2x^4\left(d^2\right.}{12e^2}$$

$$\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 455

$$\frac{\frac{1}{11}d^2e^2\left(\frac{3d\left(\frac{d\left(\frac{d\left(4059d\int\left(d^2-e^2x^2\right)^{5/2}dx-\frac{14720\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-\frac{4059x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)}{9e}-\frac{920x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)}{10e}-\frac{451x^3\left(d^2-e^2x^2\right)^{7/2}}{10e}\right)-}{12e^2}$$

$$\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 211

3.67. $\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\frac{\frac{1}{11}d^2e^2\left(\frac{3d\left(\frac{d\left(\frac{d\left(4059d\left(\frac{5}{6}d^2\int\left(d^2-e^2x^2\right)^{3/2}dx+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{14720\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-\frac{4059x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)}{9e}-\frac{920x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)}{10e}-\frac{45\ldots}{}\right.}{12e^2}$$

$$\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{1}{11}d^2e^2\left(\frac{3d\left(\frac{d\left(\frac{d\left(4059d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\int\sqrt{d^2-e^2x^2}dx+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{14720\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-\frac{4059x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)}{9e}-\frac{920x^2(\ldots}{}\right.}{10e}\right.}{12e^2}$$

$$\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \; 211$$

3.67. $\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\frac{\frac{1}{11}d^2e^2\left(\frac{3d\left(\frac{d\left(\frac{d\left(4059d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{14720\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-\frac{4059x\left(d\cdots\right.}{\cdots}}{9e}\cdots}{10e}\cdots\right.}{12e^2}\cdots$$

$$\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 224

$$\frac{\frac{1}{11}d^2e^2\left(\frac{3d\left(\frac{d\left(\frac{d\left(4059d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{14720\left(d^2-e^2x^2\right)^{7/2}}{7e}\cdots\right.}{8e}\cdots}{9e}\cdots}{10e}\cdots\right.}{12e^2}\cdots$$

$$\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 216

3.67. $\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\frac{\frac{1}{11}d^2e^2\left(\frac{3d\left(\frac{d\left(\frac{d\left(4059d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{14720\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-405}{9e}}{10e}\right.}{12e^2}$$

$$\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}$$

input
```
Int[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

output
```
-1/12*(e*x^5*(d^2 - e^2*x^2)^(7/2)) + ((-36*d*e^2*x^4*(d^2 - e^2*x^2)^(7/2
))/11 + (d^2*e^2*((-451*x^3*(d^2 - e^2*x^2)^(7/2))/(10*e) + (3*d*((-920*x^
2*(d^2 - e^2*x^2)^(7/2))/(9*e) + (d*((-4059*x*(d^2 - e^2*x^2)^(7/2))/(8*e)
 + (d*((-14720*(d^2 - e^2*x^2)^(7/2))/(7*e) + 4059*d*((x*(d^2 - e^2*x^2)^(
5/2))/6 + (5*d^2*((x*(d^2 - e^2*x^2)^(3/2))/4 + (3*d^2*((x*Sqrt[d^2 - e^2*
x^2])/2 + (d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)))/4))/6)))/(8*e)))
/(9*e)))/(10*e)))/11)/(12*e^2)
```

3.67.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

3.67. $\int x^3(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} \, dx$

3.67.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{\left(-295680e^{11}x^{11}-967680d\,e^{10}x^{10}-325248d^2e^9x^9+2078720d^3e^8x^8+2295216d^4e^7x^7-665600d^5e^6x^6-2053128d^6e^5x^5-798720d^7e \cdots\right.}{3548160e^4}$
default	$e^3\left(-\frac{x^5\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{12e^2}+\frac{5d^2\left(-\frac{x^3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{10e^2}+\frac{3d^2\left(-\frac{x\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{8e^2}+\frac{d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x \cdots}{}\right)}{}\right)}{}\right)}{8e^2}\right)}{10e^2}\right)}{12e^2}\cdots\right.$

3.67. $\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

input
```
int(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3548160*(-295680*e^11*x^11-967680*d*e^10*x^10-325248*d^2*e^9*x^9+207872
0*d^3*e^8*x^8+2295216*d^4*e^7*x^7-665600*d^5*e^6*x^6-2053128*d^6*e^5*x^5-7
98720*d^7*e^4*x^4+94710*d^8*e^3*x^3+117760*d^9*e^2*x^2+142065*d^10*e*x+235
520*d^11)/e^4*(-e^2*x^2+d^2)^(1/2)+41/1024*d^12/e^3/(e^2)^(1/2)*arctan((e^
2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.68

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx =$$
$$-\frac{284130\,d^{12}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(295680\,e^{11}x^{11}+967680\,de^{10}x^{10}+325248\,d^2e^9x^9-2078720\,d^3e^8x^8}{}$$

input
```
integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/3548160*(284130*d^12*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (29568
0*e^11*x^11 + 967680*d*e^10*x^10 + 325248*d^2*e^9*x^9 - 2078720*d^3*e^8*x^
8 - 2295216*d^4*e^7*x^7 + 665600*d^5*e^6*x^6 + 2053128*d^6*e^5*x^5 + 79872
0*d^7*e^4*x^4 - 94710*d^8*e^3*x^3 - 117760*d^9*e^2*x^2 - 142065*d^10*e*x -
 235520*d^11)*sqrt(-e^2*x^2 + d^2))/e^4
```

3.67.6 Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.09

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = \begin{cases} \frac{41d^{12}\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for}\ d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{1024e^3} + \sqrt{d^2-e^2x^2}\left(-\frac{46d^{11}}{693e^4}-\frac{41d^{10}x}{1024e^3}-\frac{23d^9x^2}{693e^2}-\right. \\ \left(\frac{d^3x^4}{4}+\frac{3d^2ex^5}{5}+\frac{de^2x^6}{2}+\frac{e^3x^7}{7}\right)\left(d^2\right)^{\frac{5}{2}} \end{cases}$$

input
```
integrate(x**3*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Piecewise((41*d**12*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e
**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(
1024*e**3) + sqrt(d**2 - e**2*x**2)*(-46*d**11/(693*e**4) - 41*d**10*x/(10
24*e**3) - 23*d**9*x**2/(693*e**2) - 41*d**8*x**3/(1536*e) + 52*d**7*x**4/
231 + 1111*d**6*e*x**5/1920 + 130*d**5*e**2*x**6/693 - 207*d**4*e**3*x**7/
320 - 58*d**3*e**4*x**8/99 + 11*d**2*e**5*x**9/120 + 3*d*e**6*x**10/11 + e
**7*x**11/12), Ne(e**2, 0)), ((d**3*x**4/4 + 3*d**2*e*x**5/5 + d*e**2*x**6
/2 + e**3*x**7/7)*(d**2)**(5/2), True))
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.92

$$\int x^3(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx = -\frac{1}{12} \left(-e^2x^2+d^2\right)^{\frac{7}{2}} ex^5 + \frac{41 d^{12} \arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{1024 \sqrt{e^2}e^3} + \frac{41 \sqrt{-e^2x^2+d^2}d^{10}x}{1024 e^3} - \frac{3}{11} \left(-e^2x^2+d^2\right)^{\frac{7}{2}} dx^4 + \frac{41 \left(-e^2x^2+d^2\right)^{\frac{3}{2}} d^8x}{1536 e^3} - \frac{41 \left(-e^2x^2+d^2\right)^{\frac{7}{2}} d^2x^3}{120 e} + \frac{41 \left(-e^2x^2+d^2\right)^{\frac{5}{2}} d^6x}{1920 e^3} - \frac{23 \left(-e^2x^2+d^2\right)^{\frac{7}{2}} d^3x^2}{99 e^2} - \frac{41 \left(-e^2x^2+d^2\right)^{\frac{7}{2}} d^4x}{320 e^3} - \frac{46 \left(-e^2x^2+d^2\right)^{\frac{7}{2}} d^5}{693 e^4}$$

input
```
integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/12*(-e^2*x^2 + d^2)^(7/2)*e*x^5 + 41/1024*d^12*arcsin(e^2*x/(d*sqrt(e^2
)))/(sqrt(e^2)*e^3) + 41/1024*sqrt(-e^2*x^2 + d^2)*d^10*x/e^3 - 3/11*(-e^2
*x^2 + d^2)^(7/2)*d*x^4 + 41/1536*(-e^2*x^2 + d^2)^(3/2)*d^8*x/e^3 - 41/12
0*(-e^2*x^2 + d^2)^(7/2)*d^2*x^3/e + 41/1920*(-e^2*x^2 + d^2)^(5/2)*d^6*x/
e^3 - 23/99*(-e^2*x^2 + d^2)^(7/2)*d^3*x^2/e^2 - 41/320*(-e^2*x^2 + d^2)^(
7/2)*d^4*x/e^3 - 46/693*(-e^2*x^2 + d^2)^(7/2)*d^5/e^4
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.65

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{41\,d^{12}\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{1024\,e^3|e|}$$
$$-\frac{1}{3548160}\left(\frac{235520\,d^{11}}{e^4}+\left(\frac{142065\,d^{10}}{e^3}+2\left(\frac{58880\,d^9}{e^2}+\left(\frac{47355\,d^8}{e}-4\left(99840\,d^7+\left(256641\,d^6e+2\left(4160\right.\right.\right.\right.\right.\right.\right.$$

input
```
integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
41/1024*d^12*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^3*abs(e)) - 1/3548160*(235520*
d^11/e^4 + (142065*d^10/e^3 + 2*(58880*d^9/e^2 + (47355*d^8/e - 4*(99840*d
^7 + (256641*d^6*e + 2*(41600*d^5*e^2 - 7*(20493*d^4*e^3 + 8*(2320*d^3*e^4
 - 3*(121*d^2*e^5 + 10*(11*e^7*x + 36*d*e^6)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)
*sqrt(-e^2*x^2 + d^2)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\int x^3\left(d^2-e^2x^2\right)^{5/2}(d+e\,x)^3\,dx$$

input
```
int(x^3*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

output
```
int(x^3*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

3.68 $\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

3.68.1 Optimal result

Integrand size = 27, antiderivative size = 223

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2}+\frac{19d^7x(d^2-e^2x^2)^{3/2}}{384e^2}+\frac{19d^5x(d^2-e^2x^2)^{5/2}}{480e^2}-\frac{37d^2x^2(d^2-e^2x^2)^{7/2}}{99e}-\frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}-\frac{d^3(5920d+13167ex)\left(d^2-e^2x^2\right)^{7/2}}{55440e^3}+\frac{19d^{11}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e^3}$$

output
```
19/384*d^7*x*(-e^2*x^2+d^2)^(3/2)/e^2+19/480*d^5*x*(-e^2*x^2+d^2)^(5/2)/e^
2-37/99*d^2*x^2*(-e^2*x^2+d^2)^(7/2)/e-3/10*d*x^3*(-e^2*x^2+d^2)^(7/2)-1/1
1*e*x^4*(-e^2*x^2+d^2)^(7/2)-1/55440*d^3*(13167*e*x+5920*d)*(-e^2*x^2+d^2)
^(7/2)/e^3+19/256*d^11*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+19/256*d^9*x*(
-e^2*x^2+d^2)^(1/2)/e^2
```

3.68.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.76

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{\sqrt{d^2-e^2x^2}(-94720d^{10}-65835d^9ex-47360d^8e^2x^2+251790d^7e^3x^3+629760d^6e^4x^4+201\cdots}{\cdots}$$

input
```
Integrate[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-94720*d^10 - 65835*d^9*e*x - 47360*d^8*e^2*x^2 + 25
1790*d^7*e^3*x^3 + 629760*d^6*e^4*x^4 + 201432*d^5*e^5*x^5 - 657920*d^4*e^
6*x^6 - 587664*d^3*e^7*x^7 + 89600*d^2*e^8*x^8 + 266112*d*e^9*x^9 + 80640*
e^10*x^10) - 131670*d^11*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/
(887040*e^3)
```

3.68.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.23, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {541, 25, 2340, 25, 27, 533, 27, 533, 27, 455, 211, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}dx$$

$$\Big\downarrow \text{541}$$

$$-\frac{\int -x^2\left(d^2-e^2x^2\right)^{5/2}\left(33dx^2e^4+37d^2xe^3+11d^3e^2\right)dx}{11e^2}-\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int x^2\left(d^2-e^2x^2\right)^{5/2}\left(33dx^2e^4+37d^2xe^3+11d^3e^2\right)dx}{11e^2}-\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{2340}$$

$$\frac{-\frac{\int -d^2e^4x^2(209d+370ex)\left(d^2-e^2x^2\right)^{5/2}dx}{10e^2}-\frac{33}{10}de^2x^3\left(d^2-e^2x^2\right)^{7/2}}{11e^2}-\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int d^2e^4x^2(209d+370ex)\left(d^2-e^2x^2\right)^{5/2}dx}{10e^2}-\frac{33}{10}de^2x^3\left(d^2-e^2x^2\right)^{7/2}}{11e^2}-\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{10}d^2e^2\int x^2(209d+370ex)\left(d^2-e^2x^2\right)^{5/2}dx-\frac{33}{10}de^2x^3\left(d^2-e^2x^2\right)^{7/2}}{11e^2}-\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{533}$$

$$\frac{\frac{1}{10}d^2e^2\left(\frac{\int dex(740d+1881ex)\left(d^2-e^2x^2\right)^{5/2}dx}{9e^2}-\frac{370x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)-\frac{33}{10}de^2x^3\left(d^2-e^2x^2\right)^{7/2}}{11e^2}-\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{10}d^2e^2\left(\frac{d\int x(740d+1881ex)\left(d^2-e^2x^2\right)^{5/2}dx}{9e}-\frac{370x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)-\frac{33}{10}de^2x^3\left(d^2-e^2x^2\right)^{7/2}}{11e^2}-\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \; 533$$

$$\frac{\frac{1}{10}d^2e^2\left(\frac{d\left(\frac{\int de(1881d+5920ex)\left(d^2-e^2x^2\right)^{5/2}dx}{8e^2}-\frac{1881x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)}{9e}-\frac{370x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)-\frac{33}{10}de^2x^3\left(d^2-e^2x^2\right)^{7/2}}{11e^2}-\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{10}d^2e^2\left(\frac{d\left(\frac{d\int (1881d+5920ex)\left(d^2-e^2x^2\right)^{5/2}dx}{8e}-\frac{1881x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)}{9e}-\frac{370x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)-\frac{33}{10}de^2x^3\left(d^2-e^2x^2\right)^{7/2}}{11e^2}-\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \; 455$$

$$\frac{\frac{1}{10}d^2e^2\left(\frac{d\left(\frac{d\left(1881d\int \left(d^2-e^2x^2\right)^{5/2}dx-\frac{5920\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-\frac{1881x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)}{9e}-\frac{370x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)-\frac{33}{10}de^2x^3\left(d^2-e^2x^2\right)^{7/2}}{11e^2}-\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \; 211$$

3.68. $\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\frac{\frac{1}{10}d^2e^2\left(\frac{d\left(\frac{d\left(1881d\left(\frac{5}{6}d^2\int\left(d^2-e^2x^2\right)^{3/2}dx+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{5920\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-\frac{1881x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)}{9e}-\frac{370x^2\left(d^2-e^2x^2\right)^{7/2}}{9e}\right)-\frac{33}{10}}{11e^2}$$

$$\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 211

$$\frac{\frac{1}{10}d^2e^2\left(\frac{d\left(\frac{d\left(1881d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\int\sqrt{d^2-e^2x^2}dx+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{5920\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-\frac{1881x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)}{9e}-\frac{370x^2\left(d^2\right.}{}\right.}{11e^2}$$

$$\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 211

$$\frac{\frac{1}{10}d^2e^2\left(\frac{d\left(\frac{d\left(1881d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{5920\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-\frac{1881x\left(d^2-e^2\right.}{8e}\right.}{9e}\right.}{11e^2}$$

$$\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 224

3.68. $\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\frac{\frac{1}{10}d^2e^2\left(\frac{d\left(\frac{d\left(1881d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{5920\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-\cdots}{9e}\cdots\right.}{11e^2}\cdots$$

$$\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{1}{10}d^2e^2\left(\frac{d\left(\frac{d\left(1881d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{5920\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-\frac{1881x\left(d\cdots\right.}{\cdots}}{9e}\cdots\right.}{11e^2}\cdots$$

$$\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}$$

input
```
Int[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

output
```
-1/11*(e*x^4*(d^2 - e^2*x^2)^(7/2)) + ((-33*d*e^2*x^3*(d^2 - e^2*x^2)^(7/2
))/10 + (d^2*e^2*((-370*x^2*(d^2 - e^2*x^2)^(7/2))/(9*e) + (d*((-1881*x*(d
^2 - e^2*x^2)^(7/2))/(8*e) + (d*((-5920*(d^2 - e^2*x^2)^(7/2))/(7*e) + 188
1*d*((x*(d^2 - e^2*x^2)^(5/2))/6 + (5*d^2*((x*(d^2 - e^2*x^2)^(3/2))/4 + (
3*d^2*((x*Sqrt[d^2 - e^2*x^2])/2 + (d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])
/(2*e)))/4))/6)))/(8*e)))/(9*e)))/10)/(11*e^2)
```

3.68. $\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

3.68.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

3.68. $\int x^2(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx$

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

3.68.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{\left(-80640e^{10}x^{10}-266112d\,e^{9}x^{9}-89600d^{2}e^{8}x^{8}+587664d^{3}e^{7}x^{7}+657920d^{4}e^{6}x^{6}-201432d^{5}e^{5}x^{5}-629760d^{6}e^{4}x^{4}-251790d^{7}e^{3}x^{3}+47\right.}{887040e^{3}}$
default	$e^{3}\left(-\frac{x^{4}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{11e^{2}}+\frac{4d^{2}\left(-\frac{x^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{9e^{2}}-\frac{2d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{63e^{4}}\right)}{11e^{2}}\right)+d^{3}\left(-\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{8e^{2}}+\frac{d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}\right.}{}\right.$

input
```
int(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

3.68. $\int x^{2}(d+ex)^{3}\left(d^{2}-e^{2}x^{2}\right)^{5/2}\,dx$

output
```
-1/887040*(-80640*e^10*x^10-266112*d*e^9*x^9-89600*d^2*e^8*x^8+587664*d^3*
e^7*x^7+657920*d^4*e^6*x^6-201432*d^5*e^5*x^5-629760*d^6*e^4*x^4-251790*d^
7*e^3*x^3+47360*d^8*e^2*x^2+65835*d^9*e*x+94720*d^10)/e^3*(-e^2*x^2+d^2)^(
1/2)+19/256*d^11/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)
)
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.72

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx =$$

$$-\frac{131670\,d^{11}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(80640\,e^{10}x^{10}+266112\,de^9x^9+89600\,d^2e^8x^8-587664\,d^3e^7x^7-657\right.}{}$$

input
```
integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/887040*(131670*d^11*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (80640*
e^10*x^10 + 266112*d*e^9*x^9 + 89600*d^2*e^8*x^8 - 587664*d^3*e^7*x^7 - 65
7920*d^4*e^6*x^6 + 201432*d^5*e^5*x^5 + 629760*d^6*e^4*x^4 + 251790*d^7*e^
3*x^3 - 47360*d^8*e^2*x^2 - 65835*d^9*e*x - 94720*d^10)*sqrt(-e^2*x^2 + d^
2))/e^3
```

3.68.6 Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.17

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = \begin{cases} \frac{19d^{11}\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{256e^2}+\sqrt{d^2-e^2x^2}\left(-\frac{74d^{10}}{693e^3}-\frac{19d^9x}{256e^2}-\frac{37d^8x^2}{693e}+\frac{10}{}\right. \\ \left(\frac{d^3x^3}{3}+\frac{3d^2ex^4}{4}+\frac{3de^2x^5}{5}+\frac{e^3x^6}{6}\right)\left(d^2\right)^{\frac{5}{2}} \end{cases}$$

input
```
integrate(x**2*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Piecewise((19*d**11*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e
**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(
256*e**2) + sqrt(d**2 - e**2*x**2)*(-74*d**10/(693*e**3) - 19*d**9*x/(256*
e**2) - 37*d**8*x**2/(693*e) + 109*d**7*x**3/384 + 164*d**6*e*x**4/231 + 1
09*d**5*e**2*x**5/480 - 514*d**4*e**3*x**6/693 - 53*d**3*e**4*x**7/80 + 10
*d**2*e**5*x**8/99 + 3*d*e**6*x**9/10 + e**7*x**10/11), Ne(e**2, 0)), ((d*
*3*x**3/3 + 3*d**2*e*x**4/4 + 3*d*e**2*x**5/5 + e**3*x**6/6)*(d**2)**(5/2)
, True))
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.93

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = \frac{19\,d^{11}\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{256\,\sqrt{e^2}e^2} + \frac{19\,\sqrt{-e^2x^2+d^2}d^9x}{256\,e^2} - \frac{1}{11}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^4 + \frac{19\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^7x}{384\,e^2} - \frac{3}{10}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^3 + \frac{19\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^5x}{480\,e^2} - \frac{37\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x^2}{99\,e} - \frac{19\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3x}{80\,e^2} - \frac{74\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^4}{693\,e^3}$$

input
```
integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
19/256*d^11*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) + 19/256*sqrt(-e^2
*x^2 + d^2)*d^9*x/e^2 - 1/11*(-e^2*x^2 + d^2)^(7/2)*e*x^4 + 19/384*(-e^2*x
^2 + d^2)^(3/2)*d^7*x/e^2 - 3/10*(-e^2*x^2 + d^2)^(7/2)*d*x^3 + 19/480*(-e
^2*x^2 + d^2)^(5/2)*d^5*x/e^2 - 37/99*(-e^2*x^2 + d^2)^(7/2)*d^2*x^2/e - 1
9/80*(-e^2*x^2 + d^2)^(7/2)*d^3*x/e^2 - 74/693*(-e^2*x^2 + d^2)^(7/2)*d^4/
e^3
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.69

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{19\,d^{11}\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{256\,e^2|e|}$$
$$-\frac{1}{887040}\left(\frac{94720\,d^{10}}{e^3}+\left(\frac{65835\,d^9}{e^2}+2\left(\frac{23680\,d^8}{e}-\left(125895\,d^7+4\left(78720\,d^6e+\left(25179\,d^5e^2-2\left(41120\,d^4\right.\right.\right.\right.\right.\right.\right.$$

input
```
integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
19/256*d^11*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) - 1/887040*(94720*d^1
0/e^3 + (65835*d^9/e^2 + 2*(23680*d^8/e - (125895*d^7 + 4*(78720*d^6*e + (
25179*d^5*e^2 - 2*(41120*d^4*e^3 + 7*(5247*d^3*e^4 - 8*(100*d^2*e^5 + 9*(1
0*e^7*x + 33*d*e^6)*x)*x)*x)*x)*x)*x)*x)*x)*x)*sqrt(-e^2*x^2 + d^2)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\int x^2\left(d^2-e^2x^2\right)^{5/2}(d+e\,x)^3\,dx$$

input
```
int(x^2*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

output
```
int(x^2*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

3.69 $\int x(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx$

3.69.1 Optimal result

Integrand size = 25, antiderivative size = 230

$$\int x(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx = \frac{33d^8x\sqrt{d^2-e^2x^2}}{256e}+\frac{11d^6x(d^2-e^2x^2)^{3/2}}{128e} +\frac{11d^4x(d^2-e^2x^2)^{5/2}}{160e}-\frac{33d^3(d^2-e^2x^2)^{7/2}}{560e^2}-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2} -\frac{d(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{30e^2}-\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}{10e^2}+\frac{33d^{10}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e^2}$$

output
```
11/128*d^6*x*(-e^2*x^2+d^2)^(3/2)/e+11/160*d^4*x*(-e^2*x^2+d^2)^(5/2)/e-33
/560*d^3*(-e^2*x^2+d^2)^(7/2)/e^2-11/240*d^2*(e*x+d)*(-e^2*x^2+d^2)^(7/2)/
e^2-1/30*d*(e*x+d)^2*(-e^2*x^2+d^2)^(7/2)/e^2-1/10*(e*x+d)^3*(-e^2*x^2+d^2
)^(7/2)/e^2+33/256*d^10*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2+33/256*d^8*x*
(-e^2*x^2+d^2)^(1/2)/e
```

3.69.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.69

$$\int x(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx = \frac{\sqrt{d^2-e^2x^2}(-6400d^9-3465d^8ex+10240d^7e^2x^2+24570d^6e^3x^3+7680d^5e^4x^4-23352d^4e^5}{268}$$

input
```
Integrate[x*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-6400*d^9 - 3465*d^8*e*x + 10240*d^7*e^2*x^2 + 24570
*d^6*e^3*x^3 + 7680*d^5*e^4*x^4 - 23352*d^4*e^5*x^5 - 20480*d^3*e^6*x^6 +
3024*d^2*e^7*x^7 + 8960*d*e^8*x^8 + 2688*e^9*x^9) - 6930*d^10*ArcTan[(e*x)
/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(26880*e^2)
```

3.69.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {541, 25, 2340, 27, 533, 27, 455, 211, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}dx$$

$$\Big\downarrow \text{541}$$

$$-\frac{\int -x\left(d^2-e^2x^2\right)^{5/2}\left(30dx^2e^4+33d^2xe^3+10d^3e^2\right)dx}{10e^2}-\frac{1}{10}ex^3\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int x\left(d^2-e^2x^2\right)^{5/2}\left(30dx^2e^4+33d^2xe^3+10d^3e^2\right)dx}{10e^2}-\frac{1}{10}ex^3\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{2340}$$

$$\frac{-\frac{\int -3d^2e^4x(50d+99ex)\left(d^2-e^2x^2\right)^{5/2}dx}{9e^2}-\frac{10}{3}de^2x^2\left(d^2-e^2x^2\right)^{7/2}}{10e^2}-\frac{1}{10}ex^3\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{3}d^2e^2\int x(50d+99ex)\left(d^2-e^2x^2\right)^{5/2}dx-\frac{10}{3}de^2x^2\left(d^2-e^2x^2\right)^{7/2}}{10e^2}-\frac{1}{10}ex^3\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{533}$$

$$\frac{\frac{1}{3}d^2e^2\left(\frac{\int de(99d+400ex)\left(d^2-e^2x^2\right)^{5/2}dx}{8e^2}-\frac{99x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)-\frac{10}{3}de^2x^2\left(d^2-e^2x^2\right)^{7/2}}{10e^2}-\frac{1}{10}ex^3\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow 27$$

$$\frac{\frac{1}{3}d^2e^2\left(\frac{d\int (99d+400ex)\left(d^2-e^2x^2\right)^{5/2}dx}{8e}-\frac{99x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)-\frac{10}{3}de^2x^2\left(d^2-e^2x^2\right)^{7/2}}{10e^2}-\frac{1}{10}ex^3\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow 455$$

$$\frac{\frac{1}{3}d^2e^2\left(\frac{d\left(99d\int \left(d^2-e^2x^2\right)^{5/2}dx-\frac{400\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-\frac{99x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)-\frac{10}{3}de^2x^2\left(d^2-e^2x^2\right)^{7/2}}{10e^2}-\frac{1}{10}ex^3\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow 211$$

$$\frac{\frac{1}{3}d^2e^2\left(\frac{d\left(99d\left(\frac{5}{6}d^2\int \left(d^2-e^2x^2\right)^{3/2}dx+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{400\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-\frac{99x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)-\frac{10}{3}de^2x^2\left(d^2-e^2x^2\right)^{7/2}}{10e^2}-\frac{1}{10}ex^3\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow 211$$

$$\frac{\frac{1}{3}d^2e^2\left(\frac{d\left(99d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\int \sqrt{d^2-e^2x^2}dx+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{400\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-\frac{99x\left(d^2-e^2x^2\right)^{7/2}}{8e}\right)-\frac{10}{3}de^2x^2}{10e^2}-\frac{1}{10}ex^3\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow 211$$

$$\frac{\frac{1}{3}d^2e^2\left(\frac{d\left(99d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{400\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)}{8e}-\frac{99x\left(d^2-e^2\right.}{8e}}{10e^2}-\frac{1}{10}ex^3\left(d^2-e^2x^2\right)^{7/2}$$

3.69. $\int x(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$\big\downarrow$ 224

$$\frac{\frac{1}{3}d^2e^2\left(\frac{d\left(99d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x(d^2-e^2x^2)^{3/2}\right)+\frac{1}{6}x(d^2-e^2x^2)^{5/2}\right)-\frac{400(d^2-e^2x^2)^{7/2}}{7e}\right)}{8e}-\right.}{10e^2}$$

$$\frac{1}{10}ex^3(d^2-e^2x^2)^{7/2}$$

$\big\downarrow$ 216

$$\frac{\frac{1}{3}d^2e^2\left(\frac{d\left(99d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x(d^2-e^2x^2)^{3/2}\right)+\frac{1}{6}x(d^2-e^2x^2)^{5/2}\right)-\frac{400(d^2-e^2x^2)^{7/2}}{7e}\right)}{8e}-\frac{99x(d}{}\right.}{10e^2}$$

$$\frac{1}{10}ex^3(d^2-e^2x^2)^{7/2}$$

input
```
Int[x*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

output
```
-1/10*(e*x^3*(d^2 - e^2*x^2)^(7/2)) + ((-10*d*e^2*x^2*(d^2 - e^2*x^2)^(7/2
))/3 + (d^2*e^2*((-99*x*(d^2 - e^2*x^2)^(7/2))/(8*e) + (d*((-400*(d^2 - e^
2*x^2)^(7/2))/(7*e) + 99*d*((x*(d^2 - e^2*x^2)^(5/2))/6 + (5*d^2*((x*(d^2
- e^2*x^2)^(3/2))/4 + (3*d^2*((x*Sqrt[d^2 - e^2*x^2])/2 + (d^2*ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]])/(2*e)))/4))/6)))/(8*e)))/3)/(10*e^2)
```

3.69.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

3.69. $\int x(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} \, dx$

3.69.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{(-2688e^9x^9-8960d\,e^8x^8-3024d^2e^7x^7+20480d^3e^6x^6+23352d^4e^5x^5-7680d^5e^4x^4-24570d^6e^3x^3-10240x^2d^7e^2+3465x\,d^8e+6400}{26880e^2}$
default	$e^3\left(-\frac{x^3(-e^2x^2+d^2)^{\frac{7}{2}}}{10e^2}+\frac{3d^2\left(-\frac{x\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{8e^2}+\frac{d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{}{\sqrt{}}\right.}{2\sqrt{}}\right.}{4}\right.}{6}\right.}{8e^2}\right.}{10e^2}\right.$

input
```
int(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/26880*(-2688*e^9*x^9-8960*d*e^8*x^8-3024*d^2*e^7*x^7+20480*d^3*e^6*x^6+
23352*d^4*e^5*x^5-7680*d^5*e^4*x^4-24570*d^6*e^3*x^3-10240*d^7*e^2*x^2+346
5*d^8*e*x+6400*d^9)/e^2*(-e^2*x^2+d^2)^(1/2)+33/256*d^10/e/(e^2)^(1/2)*arc
tan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.65

$$\int x(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx =$$
$$-\frac{6930\,d^{10}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(2688\,e^9x^9+8960\,de^8x^8+3024\,d^2e^7x^7-20480\,d^3e^6x^6-23352\,d^4e^5x^5\right.}{26880\,e^2}$$

input
```
integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/26880*(6930*d^10*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (2688*e^9*
x^9 + 8960*d*e^8*x^8 + 3024*d^2*e^7*x^7 - 20480*d^3*e^6*x^6 - 23352*d^4*e^
5*x^5 + 7680*d^5*e^4*x^4 + 24570*d^6*e^3*x^3 + 10240*d^7*e^2*x^2 - 3465*d^
8*e*x - 6400*d^9)*sqrt(-e^2*x^2 + d^2))/e^2
```

3.69.6 Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.05

$$\int x(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = \begin{cases} \frac{33d^{10}\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{256e}+\sqrt{d^2-e^2x^2}\left(-\frac{5d^9}{21e^2}-\frac{33d^8x}{256e}+\frac{8d^7x^2}{21}+\frac{117d}{1}\right. \\ \left(\frac{d^3x^2}{2}+d^2ex^3+\frac{3de^2x^4}{4}+\frac{e^3x^5}{5}\right)\left(d^2\right)^{\frac{5}{2}} \end{cases}$$

input
```
integrate(x*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Piecewise((33*d**10*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e
**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(
256*e) + sqrt(d**2 - e**2*x**2)*(-5*d**9/(21*e**2) - 33*d**8*x/(256*e) + 8
*d**7*x**2/21 + 117*d**6*e*x**3/128 + 2*d**5*e**2*x**4/7 - 139*d**4*e**3*x
**5/160 - 16*d**3*e**4*x**6/21 + 9*d**2*e**5*x**7/80 + d*e**6*x**8/3 + e**
7*x**9/10), Ne(e**2, 0)), ((d**3*x**2/2 + d**2*e*x**3 + 3*d*e**2*x**4/4 +
e**3*x**5/5)*(d**2)**(5/2), True))
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.79

$$\int x(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{33\,d^{10}\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{256\,\sqrt{e^2}e}+\frac{33\,\sqrt{-e^2x^2+d^2}d^8x}{256\,e}+\frac{11\,(-e^2x^2+d^2)^{\frac{3}{2}}d^6x}{128\,e}-\frac{1}{10}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^3+\frac{11\,(-e^2x^2+d^2)^{\frac{5}{2}}d^4x}{160\,e}-\frac{1}{3}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^2-\frac{33\,(-e^2x^2+d^2)^{\frac{7}{2}}d^2x}{80\,e}-\frac{5\,(-e^2x^2+d^2)^{\frac{7}{2}}d^3}{21\,e^2}$$

input
```
integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
33/256*d^10*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e) + 33/256*sqrt(-e^2*x
^2 + d^2)*d^8*x/e + 11/128*(-e^2*x^2 + d^2)^(3/2)*d^6*x/e - 1/10*(-e^2*x^2
 + d^2)^(7/2)*e*x^3 + 11/160*(-e^2*x^2 + d^2)^(5/2)*d^4*x/e - 1/3*(-e^2*x^
2 + d^2)^(7/2)*d*x^2 - 33/80*(-e^2*x^2 + d^2)^(7/2)*d^2*x/e - 5/21*(-e^2*x
^2 + d^2)^(7/2)*d^3/e^2
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.62

$$\int x(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{33\,d^{10}\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{256\,e|e|}-\frac{1}{26880}\left(\frac{6400\,d^9}{e^2}+\left(\frac{3465\,d^8}{e}-2\left(5120\,d^7+\left(12285\,d^6e+4\left(960\,d^5e^2-\left(2919\,d^4e^3+2\left(1280\,d^3e^4-7\left(27\right.\right.\right.\right.\right.\right.\right.\right.$$

input
```
integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
33/256*d^10*arcsin(e*x/d)*sgn(d)*sgn(e)/(e*abs(e)) - 1/26880*(6400*d^9/e^2
 + (3465*d^8/e - 2*(5120*d^7 + (12285*d^6*e + 4*(960*d^5*e^2 - (2919*d^4*e
^3 + 2*(1280*d^3*e^4 - 7*(27*d^2*e^5 + 8*(3*e^7*x + 10*d*e^6)*x)*x)*x)*x)*
x)*x)*x)*x)*sqrt(-e^2*x^2 + d^2)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int x(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx = \int x \left(d^2-e^2 x^2\right)^{5/2} (d+e x)^3 dx$$

input
```
int(x*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

output
```
int(x*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

3.70 $\int (d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx$

3.70.1 Optimal result

Integrand size = 24, antiderivative size = 188

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx = \frac{55}{128}d^7x\sqrt{d^2-e^2x^2} + \frac{55}{192}d^5x\left(d^2-e^2x^2\right)^{3/2} + \frac{11}{48}d^3x\left(d^2-e^2x^2\right)^{5/2} - \frac{11d^2\left(d^2-e^2x^2\right)^{7/2}}{56e} - \frac{11d(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{72e} - \frac{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{9e} + \frac{55d^9\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e}$$

output
```
55/192*d^5*x*(-e^2*x^2+d^2)^(3/2)+11/48*d^3*x*(-e^2*x^2+d^2)^(5/2)-11/56*d
^2*(-e^2*x^2+d^2)^(7/2)/e-11/72*d*(e*x+d)*(-e^2*x^2+d^2)^(7/2)/e-1/9*(e*x+
d)^2*(-e^2*x^2+d^2)^(7/2)/e+55/128*d^9*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e+
55/128*d^7*x*(-e^2*x^2+d^2)^(1/2)
```

3.70.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.82

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx = \frac{\sqrt{d^2-e^2x^2}\left(-3712d^8+4599d^7ex+10240d^6e^2x^2+3066d^5e^3x^3-8448d^4e^4x^4-7224d^3e^5x^5\right.}{8064e} - \frac{55d^9\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{128\sqrt{-e^2}}$$

input
```
Integrate[(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-3712*d^8 + 4599*d^7*e*x + 10240*d^6*e^2*x^2 + 3066*
d^5*e^3*x^3 - 8448*d^4*e^4*x^4 - 7224*d^3*e^5*x^5 + 1024*d^2*e^6*x^6 + 302
4*d*e^7*x^7 + 896*e^8*x^8))/(8064*e) - (55*d^9*Log[-(Sqrt[-e^2]*x) + Sqrt[
d^2 - e^2*x^2]])/(128*Sqrt[-e^2])
```

3.70.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {469, 469, 455, 211, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx$$

$$\Big\downarrow \ 469$$

$$\frac{11}{9}d\int (d+ex)^2 \left(d^2-e^2x^2\right)^{5/2} dx-\frac{(d+ex)^2 \left(d^2-e^2x^2\right)^{7/2}}{9e}$$

$$\Big\downarrow \ 469$$

$$\frac{11}{9}d\left(\frac{9}{8}d\int (d+ex) \left(d^2-e^2x^2\right)^{5/2} dx-\frac{(d+ex) \left(d^2-e^2x^2\right)^{7/2}}{8e}\right)-\frac{(d+ex)^2 \left(d^2-e^2x^2\right)^{7/2}}{9e}$$

$$\Big\downarrow \ 455$$

$$\frac{11}{9}d\left(\frac{9}{8}d\left(d\int \left(d^2-e^2x^2\right)^{5/2} dx-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)-\frac{(d+ex) \left(d^2-e^2x^2\right)^{7/2}}{8e}\right)-\frac{(d+ex)^2 \left(d^2-e^2x^2\right)^{7/2}}{9e}$$

$$\Big\downarrow \ 211$$

$$\frac{11}{9}d\left(\frac{9}{8}d\left(d\left(\frac{5}{6}d^2\int \left(d^2-e^2x^2\right)^{3/2} dx+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)-\frac{(d+ex) \left(d^2-e^2x^2\right)^{7/2}}{8e}\right)-\frac{(d+ex)^2 \left(d^2-e^2x^2\right)^{7/2}}{9e}$$

$$\Big\downarrow \ 211$$

$$\frac{11}{9}d\left(\frac{9}{8}d\left(d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\int\sqrt{d^2-e^2x^2}dx+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e}\right)-\frac{(d+\right.$$

$$\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{9e}$$

$\downarrow$ 211

$$\frac{11}{9}d\left(\frac{9}{8}d\left(d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)-\right.\right.$$

$$\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{9e}$$

$\downarrow$ 224

$$\frac{11}{9}d\left(\frac{9}{8}d\left(d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2\right.\right.\right.\right.$$

$$\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{9e}$$

$\downarrow$ 216

$$\frac{11}{9}d\left(\frac{9}{8}d\left(d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)\right.\right.$$

$$\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{9e}$$

input
```
Int[(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

output
```
-1/9*((d + e*x)^2*(d^2 - e^2*x^2)^(7/2))/e + (11*d*(-1/8*((d + e*x)*(d^2 -
 e^2*x^2)^(7/2))/e + (9*d*(-1/7*(d^2 - e^2*x^2)^(7/2)/e + d*((x*(d^2 - e^2
*x^2)^(5/2))/6 + (5*d^2*((x*(d^2 - e^2*x^2)^(3/2))/4 + (3*d^2*((x*Sqrt[d^2
 - e^2*x^2])/2 + (d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)))/4))/6)))/
8))/9
```

3.70. $\int(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

3.70.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

3.70.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{\left(-896e^{8}x^{8}-3024d\,e^{7}x^{7}-1024d^{2}e^{6}x^{6}+7224d^{3}e^{5}x^{5}+8448d^{4}x^{4}e^{4}-3066d^{5}e^{3}x^{3}-10240d^{6}e^{2}x^{2}-4599d^{7}ex+3712d^{8}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{8064e}$
default	$d^{3}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}+\frac{5d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)}{4}\right)}{6}\right)+e^{3}\left(-\frac{x^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{9e^{2}}\right.$

input
```
int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8064*(-896*e^8*x^8-3024*d*e^7*x^7-1024*d^2*e^6*x^6+7224*d^3*e^5*x^5+844
8*d^4*e^4*x^4-3066*d^5*e^3*x^3-10240*d^6*e^2*x^2-4599*d^7*e*x+3712*d^8)/e*
(-e^2*x^2+d^2)^(1/2)+55/128*d^9/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2
+d^2)^(1/2))
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.74

$$\int (d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx =$$

$$-\frac{6930\,d^{9}\arctan\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{ex}\right)-\left(896\,e^{8}x^{8}+3024\,de^{7}x^{7}+1024\,d^{2}e^{6}x^{6}-7224\,d^{3}e^{5}x^{5}-8448\,d^{4}e^{4}x^{4}+30\right.}{8064\,e}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/8064*(6930*d^9*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (896*e^8*x^8
 + 3024*d*e^7*x^7 + 1024*d^2*e^6*x^6 - 7224*d^3*e^5*x^5 - 8448*d^4*e^4*x^4
 + 3066*d^5*e^3*x^3 + 10240*d^6*e^2*x^2 + 4599*d^7*e*x - 3712*d^8)*sqrt(-e
^2*x^2 + d^2))/e
```

3.70.6 Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.10

$$\int (d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\begin{cases}\frac{55d^9\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{128}+\sqrt{d^2-e^2x^2}\left(-\frac{29d^8}{63e}+\frac{73d^7x}{128}+\frac{80d^6ex^2}{63}+\frac{73c}{}\right. \\ \left(d^2\right)^{\frac{5}{2}}\left(\begin{cases}d^3x & \text{for } e=0\\ \frac{(d+ex)^4}{4e} & \text{otherwise}\end{cases}\right)\end{cases}$$

input
```
integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Piecewise((55*d**9*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e*
*2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/12
8 + sqrt(d**2 - e**2*x**2)*(-29*d**8/(63*e) + 73*d**7*x/128 + 80*d**6*e*x*
*2/63 + 73*d**5*e**2*x**3/192 - 22*d**4*e**3*x**4/21 - 43*d**3*e**4*x**5/4
8 + 8*d**2*e**5*x**6/63 + 3*d*e**6*x**7/8 + e**7*x**8/9), Ne(e**2, 0)), ((
d**2)**(5/2)*Piecewise((d**3*x, Eq(e, 0)), ((d + e*x)**4/(4*e), True)), Tr
ue))
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.77

$$\begin{aligned}\int (d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx &= \frac{55\,d^9\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{128\sqrt{e^2}}\\ &+\frac{55}{128}\sqrt{-e^2x^2+d^2}d^7x+\frac{55}{192}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5x+\frac{11}{48}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3x\\ &-\frac{1}{9}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^2-\frac{3}{8}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx-\frac{29\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2}{63\,e}\end{aligned}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
55/128*d^9*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 55/128*sqrt(-e^2*x^2 +
d^2)*d^7*x + 55/192*(-e^2*x^2 + d^2)^(3/2)*d^5*x + 11/48*(-e^2*x^2 + d^2)^
(5/2)*d^3*x - 1/9*(-e^2*x^2 + d^2)^(7/2)*e*x^2 - 3/8*(-e^2*x^2 + d^2)^(7/2
)*d*x - 29/63*(-e^2*x^2 + d^2)^(7/2)*d^2/e
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.68

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx = \frac{55\, d^9 \arcsin\left(\frac{ex}{d}\right) \operatorname{sgn}(d) \operatorname{sgn}(e)}{128\, |e|}$$
$$-\frac{1}{8064}\left(\frac{3712\, d^8}{e} - \left(4599\, d^7 + 2\left(5120\, d^6 e + \left(1533\, d^5 e^2 - 4\left(1056\, d^4 e^3 + \left(903\, d^3 e^4 - 2\left(64\, d^2 e^5 + 7\left(8\, e^7 x \right.\right.\right.\right.\right.\right.\right.$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
55/128*d^9*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 1/8064*(3712*d^8/e - (4599
*d^7 + 2*(5120*d^6*e + (1533*d^5*e^2 - 4*(1056*d^4*e^3 + (903*d^3*e^4 - 2*
(64*d^2*e^5 + 7*(8*e^7*x + 27*d*e^6)*x)*x)*x)*x)*x)*x)*x)*sqrt(-e^2*x^2 +
d^2)
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx = \int \left(d^2-e^2\,x^2\right)^{5/2} (d+e\,x)^3\, dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

3.71 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x}\,dx$

3.71.1 Optimal result

Integrand size = 27, antiderivative size = 190

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x}\,dx = \frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2}$$
$$+\frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2}$$
$$-\frac{3}{7}d\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}+\frac{125}{128}d^8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^8\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
1/192*d^4*(125*e*x+64*d)*(-e^2*x^2+d^2)^(3/2)+1/240*d^2*(125*e*x+48*d)*(-e
^2*x^2+d^2)^(5/2)-3/7*d*(-e^2*x^2+d^2)^(7/2)-1/8*e*x*(-e^2*x^2+d^2)^(7/2)+
125/128*d^8*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-d^8*arctanh((-e^2*x^2+d^2)^(1
/2)/d)+1/128*d^6*(125*e*x+128*d)*(-e^2*x^2+d^2)^(1/2)
```

3.71.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x}\,dx = \frac{\sqrt{d^2-e^2x^2}(14848d^7+27195d^6ex+7424d^5e^2x^2-17710d^4e^3x^3-14592d^3}{13440}$$
$$-\frac{125}{64}d^8\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)-d^7\sqrt{d^2}\log(x)+d^7\sqrt{d^2}\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)$$

input
```
Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(14848*d^7 + 27195*d^6*e*x + 7424*d^5*e^2*x^2 - 17710
*d^4*e^3*x^3 - 14592*d^3*e^4*x^4 + 1960*d^2*e^5*x^5 + 5760*d*e^6*x^6 + 168
0*e^7*x^7))/13440 - (125*d^8*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]
)])/64 - d^7*Sqrt[d^2]*Log[x] + d^7*Sqrt[d^2]*Log[Sqrt[d^2] - Sqrt[d^2 - e
^2*x^2]]
```

3.71.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.14, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {541, 25, 2340, 27, 535, 535, 27, 535, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x} \, dx$$

$$\Big\downarrow \text{541}$$

$$-\frac{\int -\frac{\left(d^2-e^2x^2\right)^{5/2}\left(24dx^2e^4+25d^2xe^3+8d^3e^2\right)}{x}dx}{8e^2}-\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(24dx^2e^4+25d^2xe^3+8d^3e^2\right)}{x}dx}{8e^2}-\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{2340}$$

$$\frac{-\frac{\int -\frac{7d^2e^4(8d+25ex)\left(d^2-e^2x^2\right)^{5/2}}{x}dx}{7e^2}-\frac{24}{7}de^2\left(d^2-e^2x^2\right)^{7/2}}{8e^2}-\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2e^2\int \frac{(8d+25ex)\left(d^2-e^2x^2\right)^{5/2}}{x}dx-\frac{24}{7}de^2\left(d^2-e^2x^2\right)^{7/2}}{8e^2}-\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}$$

$$\Big\downarrow \text{535}$$

$$\frac{d^2e^2\left(\frac{1}{6}d^2\int\frac{(48d+125ex)(d^2-e^2x^2)^{3/2}}{x}dx+\frac{1}{30}(48d+125ex)\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{24}{7}de^2\left(d^2-e^2x^2\right)^{7/2}}{8e^2}-\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 535

$$\frac{d^2e^2\left(\frac{1}{6}d^2\left(\frac{1}{4}d^2\int\frac{3(64d+125ex)\sqrt{d^2-e^2x^2}}{x}dx+\frac{1}{4}(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{30}(48d+125ex)\left(d^2-e^2x^2\right)^{5/2}\right)-}{8e^2}\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 27

$$\frac{d^2e^2\left(\frac{1}{6}d^2\left(\frac{3}{4}d^2\int\frac{(64d+125ex)\sqrt{d^2-e^2x^2}}{x}dx+\frac{1}{4}(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{30}(48d+125ex)\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{2}{}}{8e^2}\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 535

$$\frac{d^2e^2\left(\frac{1}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{128d+125ex}{x\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}(128d+125ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{30}(48d-}{8e^2}\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 538

$$\frac{d^2e^2\left(\frac{1}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\left(125e\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+128d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx\right)+\frac{1}{2}(128d+125ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}(64d+125ex}{8e^2}\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 224

$$\frac{d^2e^2\left(\frac{1}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\left(128d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx+125e\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)+\frac{1}{2}(128d+125ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}(6}{8e^2}\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 216

3.71. $\int\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x}dx$

$$\frac{d^2e^2\left(\frac{1}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\left(128d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx+125\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{1}{2}(128d+125ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}(64d+12\right.\right.}{8e^2}$$

$$\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 243

$$\frac{d^2e^2\left(\frac{1}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\left(64d\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2+125\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{1}{2}(128d+125ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}(64d+1\right.\right.}{8e^2}$$

$$\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 73

$$\frac{d^2e^2\left(\frac{1}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\left(125\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{128d\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}\right)+\frac{1}{2}(128d+125ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}(64\right.\right.}{8e^2}$$

$$\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}$$

$\downarrow$ 221

$$\frac{d^2e^2\left(\frac{1}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\left(125\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-128\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)+\frac{1}{2}(128d+125ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}(64d\right.\right.}{8e^2}$$

$$\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}$$

input
```
Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x,x]
```

output
```
-1/8*(e*x*(d^2 - e^2*x^2)^(7/2)) + ((-24*d*e^2*(d^2 - e^2*x^2)^(7/2))/7 +
d^2*e^2*(((48*d + 125*e*x)*(d^2 - e^2*x^2)^(5/2))/30 + (d^2*(((64*d + 125*
e*x)*(d^2 - e^2*x^2)^(3/2))/4 + (3*d^2*(((128*d + 125*e*x)*Sqrt[d^2 - e^2*
x^2])/2 + (d^2*(125*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 128*ArcTanh[Sqrt[d
^2 - e^2*x^2]/d]))/2))/4))/6))/(8*e^2)
```

3.71. $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x}\,dx$

3.71.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

3.71.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(164) = 328$.

Time = 0.38 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.86

method	result
default	$e^{3}\left(-\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{8e^{2}}+\frac{d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}+\frac{5d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)}{4}\right)}{6}\right)}{8e^{2}}\right)+$

input
```
int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
e^3*(-1/8*x*(-e^2*x^2+d^2)^(7/2)/e^2+1/8*d^2/e^2*(1/6*x*(-e^2*x^2+d^2)^(5/
2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)
+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))+d^3*(1
/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^
(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))+
3*d^2*e*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/
4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x
/(-e^2*x^2+d^2)^(1/2)))))-3/7*d*(-e^2*x^2+d^2)^(7/2)
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x} dx =$$
$$-\frac{125}{64} d^8 \arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + d^8 \log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)$$
$$+\frac{1}{13440} \left(1680 e^7x^7 + 5760 de^6x^6 + 1960 d^2e^5x^5 - 14592 d^3e^4x^4 - 17710 d^4e^3x^3 + 7424 d^5e^2x^2 + 27195 d^6ex\right.$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x, algorithm="fricas")
```

output
```
-125/64*d^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d^8*log(-(d - sqrt
(-e^2*x^2 + d^2))/x) + 1/13440*(1680*e^7*x^7 + 5760*d*e^6*x^6 + 1960*d^2*e
^5*x^5 - 14592*d^3*e^4*x^4 - 17710*d^4*e^3*x^3 + 7424*d^5*e^2*x^2 + 27195*
d^6*e*x + 14848*d^7)*sqrt(-e^2*x^2 + d^2)
```

3.71.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.89 (sec) , antiderivative size = 954, normalized size of antiderivative = 5.02

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x} dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x,x)
```

output
```
d**7*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
 e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x
*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2
*x**2) + 1), True)) + 3*d**6*e*Piecewise((d**2*Piecewise((log(-2*e**2*x +
2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)
/sqrt(-e**2*x**2), True))/2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2, 0)), (x
*sqrt(d**2), True)) + d**5*e**2*Piecewise((-d**2*sqrt(d**2 - e**2*x**2)/(3
*e**2) + x**2*sqrt(d**2 - e**2*x**2)/3, Ne(e**2, 0)), (x**2*sqrt(d**2)/2,
True)) - 5*d**4*e**3*Piecewise((d**4*Piecewise((log(-2*e**2*x + 2*sqrt(-e*
*2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**
2*x**2), True))/(8*e**2) - d**2*x*sqrt(d**2 - e**2*x**2)/(8*e**2) + x**3*s
qrt(d**2 - e**2*x**2)/4, Ne(e**2, 0)), (x**3*sqrt(d**2)/3, True)) - 5*d**3
*e**4*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt
(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e**2, 0))
, (x**4*sqrt(d**2)/4, True)) + d**2*e**5*Piecewise((d**6*Piecewise((log(-2
*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)),
 (x*log(x)/sqrt(-e**2*x**2), True))/(16*e**4) - d**4*x*sqrt(d**2 - e**2*x*
*2)/(16*e**4) - d**2*x**3*sqrt(d**2 - e**2*x**2)/(24*e**2) + x**5*sqrt(d**
2 - e**2*x**2)/6, Ne(e**2, 0)), (x**5*sqrt(d**2)/5, True)) + 3*d*e**6*Piec
ewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**...
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x} \, dx = \frac{125 \, d^8 e \arcsin\left(\frac{e^2 x}{d\sqrt{e^2}}\right)}{128 \sqrt{e^2}} - d^8 \log\left(\frac{2 \, d^2}{|x|} + \frac{2 \sqrt{-e^2x^2+d^2} d}{|x|}\right) + \frac{125}{128} \sqrt{-e^2x^2+d^2} d^6 e x + \sqrt{-e^2x^2+d^2} d^7 + \frac{125}{192} \left(-e^2x^2+d^2\right)^{\frac{3}{2}} d^4 e x + \frac{1}{3} \left(-e^2x^2+d^2\right)^{\frac{3}{2}} d^5 + \frac{25}{48} \left(-e^2x^2+d^2\right)^{\frac{5}{2}} d^2 e x + \frac{1}{5} \left(-e^2x^2+d^2\right)^{\frac{5}{2}} d^3 - \frac{1}{8} \left(-e^2x^2+d^2\right)^{\frac{7}{2}} e x - \frac{3}{7} \left(-e^2x^2+d^2\right)^{\frac{7}{2}} d$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x, algorithm="maxima")
```

output
```
125/128*d^8*e*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - d^8*log(2*d^2/abs(x)
 + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) + 125/128*sqrt(-e^2*x^2 + d^2)*d^6*e*x
 + sqrt(-e^2*x^2 + d^2)*d^7 + 125/192*(-e^2*x^2 + d^2)^(3/2)*d^4*e*x + 1/3
*(-e^2*x^2 + d^2)^(3/2)*d^5 + 25/48*(-e^2*x^2 + d^2)^(5/2)*d^2*e*x + 1/5*(
-e^2*x^2 + d^2)^(5/2)*d^3 - 1/8*(-e^2*x^2 + d^2)^(7/2)*e*x - 3/7*(-e^2*x^2
 + d^2)^(7/2)*d
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x} dx = \frac{125 d^8 e \arcsin\left(\frac{ex}{d}\right) \mathrm{sgn}(d) \mathrm{sgn}(e)}{128 |e|} - \frac{d^8 e \log\left(\frac{\left|-2 de-2 \sqrt{-e^2x^2+d^2}|e|\right|}{2 e^2|x|}\right)}{|e|} + \frac{1}{13440} (14848 d^7 + (27195 d^6 e + 2 (3712 d^5 e^2 - (8855 d^4 e^3 + 4 (1824 d^3 e^4 - 5 (49 d^2 e^5 + 6 (7 e^7 x + 24 de^6)$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x, algorithm="giac")
```

output
```
125/128*d^8*e*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - d^8*e*log(1/2*abs(-2*d*
e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/13440*(14848*d
^7 + (27195*d^6*e + 2*(3712*d^5*e^2 - (8855*d^4*e^3 + 4*(1824*d^3*e^4 - 5*
(49*d^2*e^5 + 6*(7*e^7*x + 24*d*e^6)*x)*x)*x)*x)*x)*x)*sqrt(-e^2*x^2 + d^2
)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x} dx = \int \frac{(d^2-e^2 x^2)^{5/2} (d+e x)^3}{x} dx$$

input
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x,x)
```

output
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x, x)
```

3.72 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^2} dx$

3.72.1 Optimal result

Integrand size = 27, antiderivative size = 193

$$\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^2} dx = \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2} - \frac{d(d^2-e^2x^2)^{7/2}}{x} - \frac{15}{16}d^7e\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - 3d^7e\,\mathrm{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
1/8*d^3*e*(-5*e*x+8*d)*(-e^2*x^2+d^2)^(3/2)+1/10*d*e*(-5*e*x+6*d)*(-e^2*x^
2+d^2)^(5/2)-1/7*e*(-e^2*x^2+d^2)^(7/2)-d*(-e^2*x^2+d^2)^(7/2)/x-15/16*d^7
*e*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-3*d^7*e*arctanh((-e^2*x^2+d^2)^(1/2)/d
)+3/16*d^5*e*(-5*e*x+16*d)*(-e^2*x^2+d^2)^(1/2)
```

3.72.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^2} dx = \frac{\sqrt{d^2-e^2x^2}(-560d^7+2496d^6ex+525d^5e^2x^2-992d^4e^3x^3-770d^3e^4x^4+9}{560x} + \frac{15}{8}d^7e\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) - 3d^6\sqrt{d^2}e\log(x) + 3d^6\sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)$$

input
```
Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^2,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-560*d^7 + 2496*d^6*e*x + 525*d^5*e^2*x^2 - 992*d^4*
e^3*x^3 - 770*d^3*e^4*x^4 + 96*d^2*e^5*x^5 + 280*d*e^6*x^6 + 80*e^7*x^7))/
(560*x) + (15*d^7*e*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/8 - 3
*d^6*Sqrt[d^2]*e*Log[x] + 3*d^6*Sqrt[d^2]*e*Log[Sqrt[d^2] - Sqrt[d^2 - e^2
*x^2]]
```

3.72.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {540, 25, 2340, 27, 535, 535, 27, 535, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^2} \, dx$$

$$\Big\downarrow \text{540}$$

$$-\frac{\int -\frac{\left(d^2-e^2x^2\right)^{5/2}\left(3ed^4-3e^2xd^3+e^3x^2d^2\right)}{x}dx}{d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(3ed^4-3e^2xd^3+e^3x^2d^2\right)}{x}dx}{d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x}$$

$$\Big\downarrow \text{2340}$$

$$\frac{-\frac{\int -\frac{21d^3e^3(d-ex)\left(d^2-e^2x^2\right)^{5/2}}{x}dx}{7e^2}-\frac{1}{7}d^2e\left(d^2-e^2x^2\right)^{7/2}}{d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{3d^3e\int \frac{(d-ex)\left(d^2-e^2x^2\right)^{5/2}}{x}dx-\frac{1}{7}d^2e\left(d^2-e^2x^2\right)^{7/2}}{d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x}$$

$$\Big\downarrow \text{535}$$

$$\frac{3d^3e\left(\frac{1}{6}d^2\int\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{3/2}}{x}dx+\frac{1}{30}(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{1}{7}d^2e\left(d^2-e^2x^2\right)^{7/2}}{d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x}$$

$\downarrow$ 535

$$\frac{3d^3e\left(\frac{1}{6}d^2\left(\frac{1}{4}d^2\int\frac{3(8d-5ex)\sqrt{d^2-e^2x^2}}{x}dx+\frac{1}{4}(8d-5ex)\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{30}(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{1}{7}d^2e\left(d^2-\right.}{d^2}$$
$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x}$$

$\downarrow$ 27

$$\frac{3d^3e\left(\frac{1}{6}d^2\left(\frac{3}{4}d^2\int\frac{(8d-5ex)\sqrt{d^2-e^2x^2}}{x}dx+\frac{1}{4}(8d-5ex)\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{30}(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{1}{7}d^2e\left(d^2-\right.}{d^2}$$
$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x}$$

$\downarrow$ 535

$$\frac{3d^3e\left(\frac{1}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{16d-5ex}{x\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}(16d-5ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}(8d-5ex)\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{30}(6d-5ex)\left(\right.}{d^2}$$
$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x}$$

$\downarrow$ 538

$$\frac{3d^3e\left(\frac{1}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\left(16d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx-5e\int\frac{1}{\sqrt{d^2-e^2x^2}}dx\right)+\frac{1}{2}(16d-5ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}(8d-5ex)\left(d^2-e^2\right.}{d^2}$$
$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x}$$

$\downarrow$ 224

$$\frac{3d^3e\left(\frac{1}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\left(16d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx-5e\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)+\frac{1}{2}(16d-5ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}(8d-5ex\right.}{d^2}$$
$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x}$$

$\downarrow$ 216

3.72. $\int\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^2}dx$

$$\frac{3d^3e\left(\frac{1}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\left(16d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx-5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{1}{2}(16d-5ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}(8d-5ex)\left(d^2-\right.\right.\right.}{d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x}$$

$\big\downarrow$ 243

$$\frac{3d^3e\left(\frac{1}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\left(8d\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{1}{2}(16d-5ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}(8d-5ex)\left(d^2\right.\right.\right.}{d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x}$$

$\big\downarrow$ 73

$$\frac{3d^3e\left(\frac{1}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\left(-\frac{16d\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}-5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{1}{2}(16d-5ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}(8d-5\right.\right.}{d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x}$$

$\big\downarrow$ 221

$$\frac{3d^3e\left(\frac{1}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\left(-5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-16\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)+\frac{1}{2}(16d-5ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}(8d-5ex)\right.\right.}{d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x}$$

input
```
Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^2,x]
```

output
```
-((d*(d^2 - e^2*x^2)^(7/2))/x) + (-1/7*(d^2*e*(d^2 - e^2*x^2)^(7/2)) + 3*d
^3*e*(((6*d - 5*e*x)*(d^2 - e^2*x^2)^(5/2))/30 + (d^2*(((8*d - 5*e*x)*(d^2
 - e^2*x^2)^(3/2))/4 + (3*d^2*(((16*d - 5*e*x)*Sqrt[d^2 - e^2*x^2])/2 + (d
^2*(-5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 16*ArcTanh[Sqrt[d^2 - e^2*x^2]/
d]))/2))/4))/6))/d^2
```

3.72. $\int\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^2}dx$

3.72.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

---

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

3.72.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{d^7\sqrt{-e^2x^2+d^2}}{x}+\frac{e^7x^6\sqrt{-e^2x^2+d^2}}{7}+\frac{6e^5d^2x^4\sqrt{-e^2x^2+d^2}}{35}-\frac{62e^3d^4x^2\sqrt{-e^2x^2+d^2}}{35}+\frac{156e\,d^6\sqrt{-e^2x^2+d^2}}{35}-\frac{3e\,d^8\ln\left(\ldots\right.}{\ldots}$
default	$-\frac{e\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{7}+d^3\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{d^2x}-\frac{6e^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\ldots\right)}{2\sqrt{e^2}\ldots}\right)}{4}\ldots\right)}{6}\ldots\right)}{d^2}\ldots\right.$

input
```
int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-d^7*(-e^2*x^2+d^2)^(1/2)/x+1/7*e^7*x^6*(-e^2*x^2+d^2)^(1/2)+6/35*e^5*d^2*
x^4*(-e^2*x^2+d^2)^(1/2)-62/35*e^3*d^4*x^2*(-e^2*x^2+d^2)^(1/2)+156/35*e*d
^6*(-e^2*x^2+d^2)^(1/2)-3*e*d^8/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*
x^2+d^2)^(1/2))/x)+1/2*d*e^6*x^5*(-e^2*x^2+d^2)^(1/2)-11/8*d^3*e^4*x^3*(-e
^2*x^2+d^2)^(1/2)+15/16*d^5*e^2*x*(-e^2*x^2+d^2)^(1/2)-15/16*d^7*e^2/(e^2)
^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^2}\,dx = \frac{1050\,d^7ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+1680\,d^7ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+2496}{}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
1/560*(1050*d^7*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 1680*d^7*e
*x*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 2496*d^7*e*x + (80*e^7*x^7 + 280*d
*e^6*x^6 + 96*d^2*e^5*x^5 - 770*d^3*e^4*x^4 - 992*d^4*e^3*x^3 + 525*d^5*e^
2*x^2 + 2496*d^6*e*x - 560*d^7)*sqrt(-e^2*x^2 + d^2))/x
```

3.72.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.25 (sec) , antiderivative size = 887, normalized size of antiderivative = 4.60

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**2,x)
```

output
```
d**7*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e
**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt
(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)
), True)) + 3*d**6*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*
acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1
), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/
sqrt(-d**2/(e**2*x**2) + 1), True)) + d**5*e**2*Piecewise((d**2*Piecewise(
(log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**
2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/2 + x*sqrt(d**2 - e**2*x**2)/2,
 Ne(e**2, 0)), (x*sqrt(d**2), True)) - 5*d**4*e**3*Piecewise((-d**2*sqrt(d
**2 - e**2*x**2)/(3*e**2) + x**2*sqrt(d**2 - e**2*x**2)/3, Ne(e**2, 0)), (
x**2*sqrt(d**2)/2, True)) - 5*d**3*e**4*Piecewise((d**4*Piecewise((log(-2*
e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)),
(x*log(x)/sqrt(-e**2*x**2), True))/(8*e**2) - d**2*x*sqrt(d**2 - e**2*x**2
)/(8*e**2) + x**3*sqrt(d**2 - e**2*x**2)/4, Ne(e**2, 0)), (x**3*sqrt(d**2)
/3, True)) + d**2*e**5*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4)
 - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2
)/5, Ne(e**2, 0)), (x**4*sqrt(d**2)/4, True)) + 3*d*e**6*Piecewise((d**6*P
iecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2
), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(16*e**4) - d**4*x**...
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^2}\,dx = -\frac{15\,d^7e^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{16\sqrt{e^2}} - 3\,d^7e\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{15}{16}\sqrt{-e^2x^2+d^2}d^5e^2x + 3\sqrt{-e^2x^2+d^2}d^6e - \frac{5}{8}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3e^2x + \left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4e + \frac{1}{2}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^2x + \frac{3}{5}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2e - \frac{1}{7}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}{x}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
-15/16*d^7*e^2*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 3*d^7*e*log(2*d^2/a
bs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) - 15/16*sqrt(-e^2*x^2 + d^2)*d^5*
e^2*x + 3*sqrt(-e^2*x^2 + d^2)*d^6*e - 5/8*(-e^2*x^2 + d^2)^(3/2)*d^3*e^2*
x + (-e^2*x^2 + d^2)^(3/2)*d^4*e + 1/2*(-e^2*x^2 + d^2)^(5/2)*d*e^2*x + 3/
5*(-e^2*x^2 + d^2)^(5/2)*d^2*e - 1/7*(-e^2*x^2 + d^2)^(7/2)*e - (-e^2*x^2
+ d^2)^(5/2)*d^3/x
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^2}\,dx = -\frac{15\,d^7e^2\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{16\,|e|} + \frac{d^7e^4x}{2\left(de+\sqrt{-e^2x^2+d^2}|e|\right)|e|} - \frac{3\,d^7e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{|e|} - \frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^7}{2\,x|e|} + \frac{1}{560}\left(2496\,d^6e+\left(525\,d^5e^2-2\left(496\,d^4e^3+\left(385\,d^3e^4-4\left(12\,d^2e^5+5\left(2\,e^7x+7\,de^6\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-e^2x^2}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x, algorithm="giac")
```

output
```
-15/16*d^7*e^2*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/2*d^7*e^4*x/((d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))*abs(e)) - 3*d^7*e^2*log(1/2*abs(-2*d*e - 2*sq
rt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) - 1/2*(d*e + sqrt(-e^2*x^2
 + d^2)*abs(e))*d^7/(x*abs(e)) + 1/560*(2496*d^6*e + (525*d^5*e^2 - 2*(496
*d^4*e^3 + (385*d^3*e^4 - 4*(12*d^2*e^5 + 5*(2*e^7*x + 7*d*e^6)*x)*x)*x)*x
)*x)*sqrt(-e^2*x^2 + d^2)
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^2}\,dx = \int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x^2}\,dx$$

input
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^2,x)
```

output
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^2, x)
```

3.73 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^3}\,dx$

3.73.1 Optimal result

Integrand size = 27, antiderivative size = 207

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^3}\,dx = \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{d(d^2-e^2x^2)^{7/2}}{2x^2} - \frac{3e(d^2-e^2x^2)^{7/2}}{x} - \frac{85}{16}d^6e^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{2}d^6e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
1/24*d^2*e^2*(-85*e*x+4*d)*(-e^2*x^2+d^2)^(3/2)+1/30*e^2*(-85*e*x+3*d)*(-e
^2*x^2+d^2)^(5/2)-1/2*d*(-e^2*x^2+d^2)^(7/2)/x^2-3*e*(-e^2*x^2+d^2)^(7/2)/
x-85/16*d^6*e^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-1/2*d^6*e^2*arctanh((-e^2
*x^2+d^2)^(1/2)/d)+1/16*d^4*e^2*(-85*e*x+8*d)*(-e^2*x^2+d^2)^(1/2)
```

3.73.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^3}\,dx = \frac{\sqrt{d^2-e^2x^2}(-120d^7-720d^6ex+544d^5e^2x^2-645d^4e^3x^3-448d^3e^4x^4+5}{240x^2} + \frac{85}{8}d^6e^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) - \frac{1}{2}d^5\sqrt{d^2}e^2\log(x) + \frac{1}{2}d^5\sqrt{d^2}e^2\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)$$

input
```
Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^3,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-120*d^7 - 720*d^6*e*x + 544*d^5*e^2*x^2 - 645*d^4*e
^3*x^3 - 448*d^3*e^4*x^4 + 50*d^2*e^5*x^5 + 144*d*e^6*x^6 + 40*e^7*x^7))/(
240*x^2) + (85*d^6*e^2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/8
- (d^5*Sqrt[d^2]*e^2*Log[x])/2 + (d^5*Sqrt[d^2]*e^2*Log[Sqrt[d^2] - Sqrt[d
^2 - e^2*x^2]])/2
```

3.73.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.06, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.593, Rules used = {540, 25, 2338, 25, 27, 535, 27, 535, 27, 535, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^3} \, dx$$

$$\Big\downarrow \text{540}$$

$$-\frac{\int -\frac{\left(d^2-e^2x^2\right)^{5/2}\left(6ed^4+e^2xd^3+2e^3x^2d^2\right)}{x^2}dx}{2d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(6ed^4+e^2xd^3+2e^3x^2d^2\right)}{x^2}dx}{2d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int -\frac{d^4e^2(d-34ex)\left(d^2-e^2x^2\right)^{5/2}}{x}dx}{d^2}-\frac{6d^2e\left(d^2-e^2x^2\right)^{7/2}}{x}}{2d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{d^4e^2(d-34ex)\left(d^2-e^2x^2\right)^{5/2}}{x}dx}{d^2}-\frac{6d^2e\left(d^2-e^2x^2\right)^{7/2}}{x}}{2d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2e^2\int \frac{(d-34ex)\left(d^2-e^2x^2\right)^{5/2}}{x}dx-\frac{6d^2e\left(d^2-e^2x^2\right)^{7/2}}{x}}{2d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}$$

$$\Big\downarrow \text{535}$$

$$\frac{d^2e^2\left(\frac{1}{6}d^2\int\frac{2(3d-85ex)(d^2-e^2x^2)^{3/2}}{x}dx+\frac{1}{15}(3d-85ex)\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{6d^2e(d^2-e^2x^2)^{7/2}}{x}}{2d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2e^2\left(\frac{1}{3}d^2\int\frac{(3d-85ex)(d^2-e^2x^2)^{3/2}}{x}dx+\frac{1}{15}(3d-85ex)\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{6d^2e(d^2-e^2x^2)^{7/2}}{x}}{2d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}$$

$$\Big\downarrow \text{535}$$

$$\frac{d^2e^2\left(\frac{1}{3}d^2\left(\frac{1}{4}d^2\int\frac{3(4d-85ex)\sqrt{d^2-e^2x^2}}{x}dx+\frac{1}{4}(4d-85ex)\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{15}(3d-85ex)\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{6d^2e(d^2}{}}{2d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2e^2\left(\frac{1}{3}d^2\left(\frac{3}{4}d^2\int\frac{(4d-85ex)\sqrt{d^2-e^2x^2}}{x}dx+\frac{1}{4}(4d-85ex)\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{15}(3d-85ex)\left(d^2-e^2x^2\right)^{5/2}\right)-\frac{6d^2e(d^2-}{x}}{2d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}$$

$$\Big\downarrow \text{535}$$

$$\frac{d^2e^2\left(\frac{1}{3}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{8d-85ex}{x\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}(8d-85ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}(4d-85ex)\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{15}(3d-85ex)}{2d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}$$

$$\Big\downarrow \text{538}$$

$$\frac{d^2e^2\left(\frac{1}{3}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\left(8d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx-85e\int\frac{1}{\sqrt{d^2-e^2x^2}}dx\right)+\frac{1}{2}(8d-85ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}(4d-85ex)\left(d^2-e^2\right.}{2d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}$$

$$\Big\downarrow \text{224}$$

3.73. $\int\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^3}\,dx$

$$\frac{d^2e^2\left(\frac{1}{3}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\left(8d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx-85e\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)+\frac{1}{2}(8d-85ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}(4d-85e}{2d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}$$

$\downarrow$ 216

$$\frac{d^2e^2\left(\frac{1}{3}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\left(8d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx-85\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{1}{2}(8d-85ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}(4d-85ex)\left(d^2\right.}{2d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}$$

$\downarrow$ 243

$$\frac{d^2e^2\left(\frac{1}{3}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\left(4d\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-85\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{1}{2}(8d-85ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}(4d-85ex)\left(\right.}{2d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}$$

$\downarrow$ 73

$$\frac{d^2e^2\left(\frac{1}{3}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\left(-\frac{8d\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}-85\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{1}{2}(8d-85ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}(4d-8}{2d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}$$

$\downarrow$ 221

$$\frac{d^2e^2\left(\frac{1}{3}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\left(-85\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-8\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)+\frac{1}{2}(8d-85ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}(4d-85ex}{2d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}$$

input
```
Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^3,x]
```

3.73. $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^3}\,dx$

output
```
-1/2*(d*(d^2 - e^2*x^2)^(7/2))/x^2 + ((-6*d^2*e*(d^2 - e^2*x^2)^(7/2))/x +
 d^2*e^2*(((3*d - 85*e*x)*(d^2 - e^2*x^2)^(5/2))/15 + (d^2*(((4*d - 85*e*x
)*(d^2 - e^2*x^2)^(3/2))/4 + (3*d^2*(((8*d - 85*e*x)*Sqrt[d^2 - e^2*x^2])/
2 + (d^2*(-85*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 8*ArcTanh[Sqrt[d^2 - e^2
*x^2]/d]))/2))/4))/3))/(2*d^2)
```

3.73.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.73.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.85

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-40e^7x^7-144d\,e^6x^6-50d^2e^5x^5+448d^3e^4x^4+645d^4e^3x^3-544d^5e^2x^2+720d^6ex+120d^7)}{240x^2}-\frac{85d^6e^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16\sqrt{e^2}}$
default	$e^3\left(\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6}\right)+d^3\left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{2d^2x^2}-\right.$

input
```
int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/240*(-e^2*x^2+d^2)^(1/2)*(-40*e^7*x^7-144*d*e^6*x^6-50*d^2*e^5*x^5+448*
d^3*e^4*x^4+645*d^4*e^3*x^3-544*d^5*e^2*x^2+720*d^6*e*x+120*d^7)/x^2-85/16
*d^6*e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/2*d^7*e^
2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)^3\,(d^2-e^2x^2)^{5/2}}{x^3}\,dx = \frac{2550\,d^6e^2x^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+120\,d^6e^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+54}{}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x, algorithm="fricas")
```

output
```
1/240*(2550*d^6*e^2*x^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 120*d^
6*e^2*x^2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 544*d^6*e^2*x^2 + (40*e^7*x
^7 + 144*d*e^6*x^6 + 50*d^2*e^5*x^5 - 448*d^3*e^4*x^4 - 645*d^4*e^3*x^3 +
544*d^5*e^2*x^2 - 720*d^6*e*x - 120*d^7)*sqrt(-e^2*x^2 + d^2))/x^2
```

3.73.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.96 (sec) , antiderivative size = 887, normalized size of antiderivative = 4.29

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^3} dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**3,x)
```

output
```
d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/
(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2
) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*
d), True)) + 3*d**6*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*a
cosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
> 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1
- e**2*x**2/d**2)), True)) + d**5*e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**
2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**
2/(e**2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin
(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) - 5*d**4*e**3*Piecew
ise((d**2*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))
/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/2 + x*sqrt(
d**2 - e**2*x**2)/2, Ne(e**2, 0)), (x*sqrt(d**2), True)) - 5*d**3*e**4*Pie
cewise((-d**2*sqrt(d**2 - e**2*x**2)/(3*e**2) + x**2*sqrt(d**2 - e**2*x**2
)/3, Ne(e**2, 0)), (x**2*sqrt(d**2)/2, True)) + d**2*e**5*Piecewise((d**4*
Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**
2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(8*e**2) - d**2*x*sqr
t(d**2 - e**2*x**2)/(8*e**2) + x**3*sqrt(d**2 - e**2*x**2)/4, Ne(e**2, 0))
, (x**3*sqrt(d**2)/3, True)) + 3*d*e**6*Piecewise((-2*d**4*sqrt(d**2 - e**
2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*s...
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^3}\,dx = -\frac{85\,d^6e^3\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{16\,\sqrt{e^2}} - \frac{1}{2}\,d^6e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{85}{16}\,\sqrt{-e^2x^2+d^2}d^4e^3x + \frac{1}{2}\,\sqrt{-e^2x^2+d^2}d^5e^2 - \frac{85}{24}\,\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e^3x + \frac{1}{6}\,\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3e^2 + \frac{1}{6}\,\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3x + \frac{1}{10}\,\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^2 - \frac{3\,\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2e}{x} - \frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{2\,x^2}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x, algorithm="maxima")
```

output
```
-85/16*d^6*e^3*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 1/2*d^6*e^2*log(2*d
^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) - 85/16*sqrt(-e^2*x^2 + d^2)*
d^4*e^3*x + 1/2*sqrt(-e^2*x^2 + d^2)*d^5*e^2 - 85/24*(-e^2*x^2 + d^2)^(3/2
)*d^2*e^3*x + 1/6*(-e^2*x^2 + d^2)^(3/2)*d^3*e^2 + 1/6*(-e^2*x^2 + d^2)^(5
/2)*e^3*x + 1/10*(-e^2*x^2 + d^2)^(5/2)*d*e^2 - 3*(-e^2*x^2 + d^2)^(5/2)*d
^2*e/x - 1/2*(-e^2*x^2 + d^2)^(7/2)*d/x^2
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.42

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^3}\,dx = -\frac{85\,d^6e^3\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{16\,|e|} - \frac{d^6e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,|e|} + \frac{\left(d^6e^3+\frac{12\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^6e}{x}\right)e^4x^2}{8\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2|e|} + \frac{1}{240}\,\left(544\,d^5e^2-\left(645\,d^4e^3+2\,\left(224\,d^3e^4-\left(25\,d^2e^5+4\,\left(5\,e^7x+18\,de^6\right)x\right)x\right)x\right)x\right)\sqrt{-e^2x^2+d^2} - \frac{\frac{12\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^6e|e|}{x}+\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^6|e|}{ex^2}}{8\,e^2}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x, algorithm="giac")
```

output
```
-85/16*d^6*e^3*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 1/2*d^6*e^3*log(1/2*ab
s(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/8*(d^6*
e^3 + 12*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^6*e/x)*e^4*x^2/((d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^2*abs(e)) + 1/240*(544*d^5*e^2 - (645*d^4*e^3 + 2
*(224*d^3*e^4 - (25*d^2*e^5 + 4*(5*e^7*x + 18*d*e^6)*x)*x)*x)*x)*sqrt(-e^2
*x^2 + d^2) - 1/8*(12*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^6*e*abs(e)/x +
 (d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^6*abs(e)/(e*x^2))/e^2
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^3}\,dx=\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(d+e\,x\right)}^3}{x^3}\,dx$$

input
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^3,x)
```

output
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^3, x)
```

3.74 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^4}\,dx$

3.74.1 Optimal result

Integrand size = 27, antiderivative size = 210

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^4}\,dx = -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+39ex)\left(d^2-e^2x^2\right)^{5/2}}{30x} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{2x^2} - \frac{25}{8}d^5e^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{13}{2}d^5e^3\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
-1/12*d*e^3*(25*e*x+26*d)*(-e^2*x^2+d^2)^(3/2)-1/30*e^2*(39*e*x+50*d)*(-e^
2*x^2+d^2)^(5/2)/x-1/3*d*(-e^2*x^2+d^2)^(7/2)/x^3-3/2*e*(-e^2*x^2+d^2)^(7/
2)/x^2-25/8*d^5*e^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))+13/2*d^5*e^3*arctanh(
(-e^2*x^2+d^2)^(1/2)/d)-1/8*d^3*e^3*(25*e*x+52*d)*(-e^2*x^2+d^2)^(1/2)
```

3.74.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^4}\,dx = \frac{\sqrt{d^2-e^2x^2}\left(-40d^7-180d^6ex-80d^5e^2x^2-656d^4e^3x^3-345d^3e^4x^4+32d\right.}{120x^3} - 13d^5e^3\operatorname{arctanh}\left(\frac{\sqrt{-e^2}x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right) + \frac{25}{8}d^5\left(-e^2\right)^{3/2}\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)$$

input
```
Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^4,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-40*d^7 - 180*d^6*e*x - 80*d^5*e^2*x^2 - 656*d^4*e^3
*x^3 - 345*d^3*e^4*x^4 + 32*d^2*e^5*x^5 + 90*d*e^6*x^6 + 24*e^7*x^7))/(120
*x^3) - 13*d^5*e^3*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] + (25
*d^5*(-e^2)^(3/2)*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/8
```

3.74.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {540, 25, 2338, 25, 27, 536, 535, 27, 535, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^4} \, dx$$

$$\Big\downarrow \text{540}$$

$$-\frac{\int -\frac{\left(d^2-e^2x^2\right)^{5/2}\left(9ed^4+5e^2xd^3+3e^3x^2d^2\right)}{x^3}dx}{3d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(9ed^4+5e^2xd^3+3e^3x^2d^2\right)}{x^3}dx}{3d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int -\frac{d^4e^2(10d-39ex)\left(d^2-e^2x^2\right)^{5/2}}{x^2}dx}{2d^2}-\frac{9d^2e\left(d^2-e^2x^2\right)^{7/2}}{2x^2}}{3d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{d^4e^2(10d-39ex)\left(d^2-e^2x^2\right)^{5/2}}{x^2}dx}{2d^2}-\frac{9d^2e\left(d^2-e^2x^2\right)^{7/2}}{2x^2}}{3d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}d^2e^2\int \frac{(10d-39ex)\left(d^2-e^2x^2\right)^{5/2}}{x^2}dx-\frac{9d^2e\left(d^2-e^2x^2\right)^{7/2}}{2x^2}}{3d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3}$$

$$\Big\downarrow \text{536}$$

$$\frac{\frac{1}{2}d^2e^2\left(\int \frac{(-39ed^2-50e^2xd)(d^2-e^2x^2)^{3/2}}{x}dx-\frac{(50d+39ex)(d^2-e^2x^2)^{5/2}}{5x}\right)-\frac{9d^2e(d^2-e^2x^2)^{7/2}}{2x^2}}{3d^2}-\frac{d(d^2-e^2x^2)^{7/2}}{3x^3}$$

$$\Big\downarrow \text{535}$$

$$\frac{\frac{1}{2}d^2e^2\left(\frac{1}{4}d^2\int -\frac{6de(26d+25ex)\sqrt{d^2-e^2x^2}}{x}dx-\frac{(50d+39ex)(d^2-e^2x^2)^{5/2}}{5x}-\frac{1}{2}de(26d+25ex)\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{9d^2e(d^2-e^2x^2}{2x^2}}{3d^2}$$

$$\frac{d(d^2-e^2x^2)^{7/2}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}d^2e^2\left(-\frac{3}{2}d^3e\int \frac{(26d+25ex)\sqrt{d^2-e^2x^2}}{x}dx-\frac{1}{2}de(26d+25ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{(50d+39ex)(d^2-e^2x^2)^{5/2}}{5x}\right)-\frac{9d^2e(d^2-e^2x^2)^{7}}{2x^2}}{3d^2}$$

$$\frac{d(d^2-e^2x^2)^{7/2}}{3x^3}$$

$$\Big\downarrow \text{535}$$

$$\frac{\frac{1}{2}d^2e^2\left(-\frac{3}{2}d^3e\left(\frac{1}{2}d^2\int \frac{52d+25ex}{x\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}(52d+25ex)\sqrt{d^2-e^2x^2}\right)-\frac{1}{2}de(26d+25ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{(50d+39ex)(d}{5x}\right.}{3d^2}$$

$$\frac{d(d^2-e^2x^2)^{7/2}}{3x^3}$$

$$\Big\downarrow \text{538}$$

$$\frac{\frac{1}{2}d^2e^2\left(-\frac{3}{2}d^3e\left(\frac{1}{2}d^2\left(25e\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+52d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx\right)+\frac{1}{2}(52d+25ex)\sqrt{d^2-e^2x^2}\right)-\frac{1}{2}de(26d+25ex)\,(d\right.}{3d^2}$$

$$\frac{d(d^2-e^2x^2)^{7/2}}{3x^3}$$

$$\Big\downarrow \text{224}$$

3.74. $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^4}\,dx$

$$\frac{\frac{\frac{1}{2}d^2e^2\left(-\frac{3}{2}d^3e\left(\frac{1}{2}d^2\left(52d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx+25e\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)+\frac{1}{2}(52d+25ex)\sqrt{d^2-e^2x^2}\right)-\frac{1}{2}de(26d+}{3d^2}}{\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3}}$$

$\downarrow$ 216

$$\frac{\frac{\frac{1}{2}d^2e^2\left(-\frac{3}{2}d^3e\left(\frac{1}{2}d^2\left(52d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx+25\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{1}{2}(52d+25ex)\sqrt{d^2-e^2x^2}\right)-\frac{1}{2}de(26d+25ex}{3d^2}}{\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3}}$$

$\downarrow$ 243

$$\frac{\frac{\frac{1}{2}d^2e^2\left(-\frac{3}{2}d^3e\left(\frac{1}{2}d^2\left(26d\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2+25\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{1}{2}(52d+25ex)\sqrt{d^2-e^2x^2}\right)-\frac{1}{2}de(26d+25}{3d^2}}{\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3}}$$

$\downarrow$ 73

$$\frac{\frac{\frac{1}{2}d^2e^2\left(-\frac{3}{2}d^3e\left(\frac{1}{2}d^2\left(25\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{52d\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}\right)+\frac{1}{2}(52d+25ex)\sqrt{d^2-e^2x^2}\right)-\frac{1}{2}de(26d+}{3d^2}}{\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3}}$$

$\downarrow$ 221

$$\frac{\frac{\frac{1}{2}d^2e^2\left(-\frac{3}{2}d^3e\left(\frac{1}{2}d^2\left(25\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-52\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)+\frac{1}{2}(52d+25ex)\sqrt{d^2-e^2x^2}\right)-\frac{1}{2}de(26d+2}{3d^2}}{\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3}}$$

input
```
Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^4,x]
```

3.74. $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^4}\,dx$

output
```
-1/3*(d*(d^2 - e^2*x^2)^(7/2))/x^3 + ((-9*d^2*e*(d^2 - e^2*x^2)^(7/2))/(2*
x^2) + (d^2*e^2*(-1/2*(d*e*(26*d + 25*e*x)*(d^2 - e^2*x^2)^(3/2)) - ((50*d
 + 39*e*x)*(d^2 - e^2*x^2)^(5/2))/(5*x) - (3*d^3*e*(((52*d + 25*e*x)*Sqrt[
d^2 - e^2*x^2])/2 + (d^2*(25*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 52*ArcTan
h[Sqrt[d^2 - e^2*x^2]/d]))/2))/2))/2)/(3*d^2)
```

3.74.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.74. $\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^4} \, dx$

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.74. $\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^4} \, dx$

3.74.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{d^5\sqrt{-e^2x^2+d^2}\,(4e^2x^2+9dex+2d^2)}{6x^3}+\frac{e^7x^4\sqrt{-e^2x^2+d^2}}{5}+\frac{4e^5d^2x^2\sqrt{-e^2x^2+d^2}}{15}-\frac{82e^3d^4\sqrt{-e^2x^2+d^2}}{15}-\frac{25e^4d^5\arctan\left(\right.}{8\sqrt{e^{}}}$
default	$e^3\left(\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{5}+d^2\left(\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)\right)\right)+d^3\left(-\frac{(-}{}\right.$

input
```
int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*d^5*(-e^2*x^2+d^2)^(1/2)*(4*e^2*x^2+9*d*e*x+2*d^2)/x^3+1/5*e^7*x^4*(-
e^2*x^2+d^2)^(1/2)+4/15*e^5*d^2*x^2*(-e^2*x^2+d^2)^(1/2)-82/15*e^3*d^4*(-e
^2*x^2+d^2)^(1/2)-25/8*e^4*d^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+
d^2)^(1/2))+13/2*e^3*d^6/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2
)^(1/2))/x)+3/4*e^6*d*x^3*(-e^2*x^2+d^2)^(1/2)-23/8*e^4*d^3*x*(-e^2*x^2+d^
2)^(1/2)
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^4}\,dx = \frac{750\,d^5e^3x^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - 780\,d^5e^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - 656$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x, algorithm="fricas")
```

output
```
1/120*(750*d^5*e^3*x^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 780*d^5
*e^3*x^3*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 656*d^5*e^3*x^3 + (24*e^7*x^
7 + 90*d*e^6*x^6 + 32*d^2*e^5*x^5 - 345*d^3*e^4*x^4 - 656*d^4*e^3*x^3 - 80
*d^5*e^2*x^2 - 180*d^6*e*x - 40*d^7)*sqrt(-e^2*x^2 + d^2))/x^3
```

3.74.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.18 (sec) , antiderivative size = 843, normalized size of antiderivative = 4.01

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**4,x)
```

output
```
d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e
**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**
2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)
) + 3*d**6*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d
/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e
**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e
*x))/(2*d), True)) + d**5*e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)
) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x*
*2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(
d*sqrt(1 - e**2*x**2/d**2)), True)) - 5*d**4*e**3*Piecewise((d**2/(e*x*sqr
t(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) -
1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1))
 + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) - 5*d**3*
e**4*Piecewise((d**2*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 -
e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/
2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2, 0)), (x*sqrt(d**2), True)) + d**2
*e**5*Piecewise((-d**2*sqrt(d**2 - e**2*x**2)/(3*e**2) + x**2*sqrt(d**2 -
e**2*x**2)/3, Ne(e**2, 0)), (x**2*sqrt(d**2)/2, True)) + 3*d*e**6*Piecewis
e((d**4*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/s
qrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(8*e**2) -...
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^4} dx = -\frac{25 d^5 e^4 \arcsin\left(\frac{e^2 x}{d\sqrt{e^2}}\right)}{8\sqrt{e^2}} + \frac{13}{2} d^5 e^3 \log\left(\frac{2 d^2}{|x|} + \frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{25}{8}\sqrt{-e^2x^2+d^2}d^3e^4x - \frac{13}{2}\sqrt{-e^2x^2+d^2}d^4e^3 - \frac{25}{12}\left(-e^2x^2+d^2\right)^{\frac{3}{2}} de^4x - \frac{13}{6}\left(-e^2x^2+d^2\right)^{\frac{3}{2}} d^2e^3 - \frac{13}{10}\left(-e^2x^2+d^2\right)^{\frac{5}{2}} e^3 - \frac{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}} de^2}{3x} - \frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}} e}{2x^2} - \frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}} d}{3x^3}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x, algorithm="maxima")
```

output
```
-25/8*d^5*e^4*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 13/2*d^5*e^3*log(2*d
^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) - 25/8*sqrt(-e^2*x^2 + d^2)*d
^3*e^4*x - 13/2*sqrt(-e^2*x^2 + d^2)*d^4*e^3 - 25/12*(-e^2*x^2 + d^2)^(3/2
)*d*e^4*x - 13/6*(-e^2*x^2 + d^2)^(3/2)*d^2*e^3 - 13/10*(-e^2*x^2 + d^2)^(
5/2)*e^3 - 5/3*(-e^2*x^2 + d^2)^(5/2)*d*e^2/x - 3/2*(-e^2*x^2 + d^2)^(7/2)
*e/x^2 - 1/3*(-e^2*x^2 + d^2)^(7/2)*d/x^3
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.65

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^4} dx = -\frac{25 d^5 e^4 \arcsin\left(\frac{ex}{d}\right) \operatorname{sgn}(d) \operatorname{sgn}(e)}{8 |e|} + \frac{13 d^5 e^4 \log\left(\frac{\left|-2 de-2 \sqrt{-e^2x^2+d^2}|e|\right|}{2 e^2|x|}\right)}{2 |e|} + \frac{\left(d^5e^4 + \frac{9\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^5e^2}{x} + \frac{9\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d^5}{x^2}\right)e^6x^3}{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3|e|} - \frac{1}{120}\left(656 d^4e^3 + \left(345 d^3e^4 - 2\left(16 d^2e^5 + 3\left(4 e^7x + 15 de^6\right)x\right)x\right)x\right)\sqrt{-e^2x^2+d^2} - \frac{\frac{9\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^5e^4}{x} + \frac{9\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d^5e^2}{x^2} + \frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3 d^5}{x^3}}{24 e^2|e|}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x, algorithm="giac")
```

output
```
-25/8*d^5*e^4*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 13/2*d^5*e^4*log(1/2*ab
s(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/24*(d^5
*e^4 + 9*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^5*e^2/x + 9*(d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^2*d^5/x^2)*e^6*x^3/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e
))^3*abs(e)) - 1/120*(656*d^4*e^3 + (345*d^3*e^4 - 2*(16*d^2*e^5 + 3*(4*e^
7*x + 15*d*e^6)*x)*x)*x)*sqrt(-e^2*x^2 + d^2) - 1/24*(9*(d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))*d^5*e^4/x + 9*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^5*
e^2/x^2 + (d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^5/x^3)/(e^2*abs(e))
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^4} \, dx = \int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x^4} \, dx$$

input
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^4,x)
```

output
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^4, x)
```

3.75 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^5}\,dx$

3.75.1 Optimal result

Integrand size = 27, antiderivative size = 209

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^5}\,dx=-\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2}+\frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x}-\frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2}-\frac{d(d^2-e^2x^2)^{7/2}}{4x^4}-\frac{e(d^2-e^2x^2)^{7/2}}{x^3}+\frac{45}{8}d^4e^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{45}{8}d^4e^4\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
15/8*d*e^3*(-e*x+2*d)*(-e^2*x^2+d^2)^(3/2)/x-3/8*e^2*(2*e*x+3*d)*(-e^2*x^2
+d^2)^(5/2)/x^2-1/4*d*(-e^2*x^2+d^2)^(7/2)/x^4-e*(-e^2*x^2+d^2)^(7/2)/x^3+
45/8*d^4*e^4*arctan(e*x/(-e^2*x^2+d^2)^(1/2))+45/8*d^4*e^4*arctanh((-e^2*x
^2+d^2)^(1/2)/d)-45/8*d^2*e^4*(-e*x+d)*(-e^2*x^2+d^2)^(1/2)
```

3.75.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^5}\,dx=\frac{1}{8}\left(\frac{\sqrt{d^2-e^2x^2}(-2d^7-8d^6ex-3d^5e^2x^2+48d^4e^3x^3-48d^3e^4x^4+3d^2e^5x^5}{x^4}-90d^4e^4\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)+45d^3\sqrt{d^2}e^4\log(x)-45d^3\sqrt{d^2}e^4\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)\right)$$

input
```
Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^5,x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(-2*d^7 - 8*d^6*e*x - 3*d^5*e^2*x^2 + 48*d^4*e^3*x^3
 - 48*d^3*e^4*x^4 + 3*d^2*e^5*x^5 + 8*d*e^6*x^6 + 2*e^7*x^7))/x^4 - 90*d^4
*e^4*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] + 45*d^3*Sqrt[d^2]*e^
4*Log[x] - 45*d^3*Sqrt[d^2]*e^4*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/8
```

3.75.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$, Rules used = {540, 25, 2338, 27, 537, 25, 535, 27, 535, 27, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^5} \, dx$$

$$\Big\downarrow \text{540}$$

$$-\frac{\int -\frac{\left(d^2-e^2x^2\right)^{5/2}\left(12ed^4+9e^2xd^3+4e^3x^2d^2\right)}{x^4}dx}{4d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(12ed^4+9e^2xd^3+4e^3x^2d^2\right)}{x^4}dx}{4d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int -\frac{9d^4e^2(3d-4ex)\left(d^2-e^2x^2\right)^{5/2}}{x^3}dx}{3d^2}-\frac{4d^2e\left(d^2-e^2x^2\right)^{7/2}}{x^3}}{4d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{3d^2e^2\int \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{5/2}}{x^3}dx-\frac{4d^2e\left(d^2-e^2x^2\right)^{7/2}}{x^3}}{4d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4}$$

$$\Big\downarrow \text{537}$$

$$\frac{3d^2e^2\left(\frac{5}{2}e^2\int -\frac{(3d-8ex)\left(d^2-e^2x^2\right)^{3/2}}{x}dx-\frac{(3d-8ex)\left(d^2-e^2x^2\right)^{5/2}}{2x^2}\right)-\frac{4d^2e\left(d^2-e^2x^2\right)^{7/2}}{x^3}}{4d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4}$$

$$\downarrow 25$$

$$\frac{3d^2e^2\left(-\frac{5}{2}e^2\int \frac{(3d-8ex)(d^2-e^2x^2)^{3/2}}{x}dx-\frac{(3d-8ex)(d^2-e^2x^2)^{5/2}}{2x^2}\right)-\frac{4d^2e(d^2-e^2x^2)^{7/2}}{x^3}}{4d^2}-\frac{d(d^2-e^2x^2)^{7/2}}{4x^4}$$

$$\downarrow 535$$

$$\frac{3d^2e^2\left(-\frac{5}{2}e^2\left(\frac{1}{4}d^2\int \frac{12(d-2ex)\sqrt{d^2-e^2x^2}}{x}dx+(d-2ex)\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{(3d-8ex)(d^2-e^2x^2)^{5/2}}{2x^2}\right)-\frac{4d^2e(d^2-e^2x^2)^{7/2}}{x^3}}{4d^2}-\frac{d(d^2-e^2x^2)^{7/2}}{4x^4}$$

$$\downarrow 27$$

$$\frac{3d^2e^2\left(-\frac{5}{2}e^2\left(3d^2\int \frac{(d-2ex)\sqrt{d^2-e^2x^2}}{x}dx+(d-2ex)\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{(3d-8ex)(d^2-e^2x^2)^{5/2}}{2x^2}\right)-\frac{4d^2e(d^2-e^2x^2)^{7/2}}{x^3}}{4d^2}-\frac{d(d^2-e^2x^2)^{7/2}}{4x^4}$$

$$\downarrow 535$$

$$\frac{3d^2e^2\left(-\frac{5}{2}e^2\left(3d^2\left(\frac{1}{2}d^2\int \frac{2(d-ex)}{x\sqrt{d^2-e^2x^2}}dx+(d-ex)\sqrt{d^2-e^2x^2}\right)+(d-2ex)\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{(3d-8ex)(d^2-e^2x^2)^{5/2}}{2x^2}\right)}{4d^2}$$

$$\frac{d(d^2-e^2x^2)^{7/2}}{4x^4}$$

$$\downarrow 27$$

$$\frac{3d^2e^2\left(-\frac{5}{2}e^2\left(3d^2\left(d^2\int \frac{d-ex}{x\sqrt{d^2-e^2x^2}}dx+(d-ex)\sqrt{d^2-e^2x^2}\right)+(d-2ex)\left(d^2-e^2x^2\right)^{3/2}\right)-\frac{(3d-8ex)(d^2-e^2x^2)^{5/2}}{2x^2}\right)}{4d^2}$$

$$\frac{d(d^2-e^2x^2)^{7/2}}{4x^4}$$

$$\downarrow 538$$

$$\frac{3d^2e^2\left(-\frac{5}{2}e^2\left(3d^2\left(d^2\left(d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-e\int \frac{1}{\sqrt{d^2-e^2x^2}}dx\right)+(d-ex)\sqrt{d^2-e^2x^2}\right)+(d-2ex)\left(d^2-e^2x^2\right)^{3/2}\right)-}{4d^2}$$

$$\frac{d(d^2-e^2x^2)^{7/2}}{4x^4}$$

$$\downarrow 224$$

3.75. $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^5}\,dx$

$$\frac{3d^2e^2\left(-\frac{5}{2}e^2\left(3d^2\left(d^2\left(d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx-e\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)+(d-ex)\sqrt{d^2-e^2x^2}\right)+(d-2ex)\left(d^2-e^2\right.\right.\right.}{4d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4}$$

$\downarrow$ 216

$$\frac{3d^2e^2\left(-\frac{5}{2}e^2\left(3d^2\left(d^2\left(d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+(d-ex)\sqrt{d^2-e^2x^2}\right)+(d-2ex)\left(d^2-e^2x^2\right)^{3/2}\right.\right.}{4d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4}$$

$\downarrow$ 243

$$\frac{3d^2e^2\left(-\frac{5}{2}e^2\left(3d^2\left(d^2\left(\frac{1}{2}d\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+(d-ex)\sqrt{d^2-e^2x^2}\right)+(d-2ex)\left(d^2-e^2x^2\right)\right.\right.}{4d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4}$$

$\downarrow$ 73

$$\frac{3d^2e^2\left(-\frac{5}{2}e^2\left(3d^2\left(d^2\left(-\frac{d\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+(d-ex)\sqrt{d^2-e^2x^2}\right)+(d-2ex)\left(d^2-\right.\right.\right.}{4d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4}$$

$\downarrow$ 221

$$\frac{3d^2e^2\left(-\frac{5}{2}e^2\left(3d^2\left(d^2\left(-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)+(d-ex)\sqrt{d^2-e^2x^2}\right)+(d-2ex)\left(d^2-e^2x\right.\right.\right.}{4d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4}$$

input
```
Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^5,x]
```

3.75. $\int\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^5}dx$

output
```
-1/4*(d*(d^2 - e^2*x^2)^(7/2))/x^4 + ((-4*d^2*e*(d^2 - e^2*x^2)^(7/2))/x^3
 + 3*d^2*e^2*(-1/2*((3*d - 8*e*x)*(d^2 - e^2*x^2)^(5/2))/x^2 - (5*e^2*((d
- 2*e*x)*(d^2 - e^2*x^2)^(3/2) + 3*d^2*((d - e*x)*Sqrt[d^2 - e^2*x^2] + d^
2*(-ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - ArcTanh[Sqrt[d^2 - e^2*x^2]/d])))
/2))/(4*d^2)
```

3.75.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.75. $\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^5} \, dx$

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.75. $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^5}\,dx$

3.75.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{d^4\sqrt{-e^2x^2+d^2}\,(-48e^3x^3+3d\,e^2x^2+8d^2ex+2d^3)}{8x^4}+\frac{e^7x^3\sqrt{-e^2x^2+d^2}}{4}+\frac{3e^5d^2x\sqrt{-e^2x^2+d^2}}{8}+\frac{45e^5d^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}$
default	$d^3\left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{4d^2x^4}-\frac{3e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{2d^2x^2}-\frac{5e^2\left(\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{5}+d^2\left(\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2}}{x}\right)}{\sqrt{d^2}}\right.\right.\right.}{2d^2}\right.}{4d^2}\right.$

input
```
int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8*d^4*(-e^2*x^2+d^2)^(1/2)*(-48*e^3*x^3+3*d*e^2*x^2+8*d^2*e*x+2*d^3)/x^
4+1/4*e^7*x^3*(-e^2*x^2+d^2)^(1/2)+3/8*e^5*d^2*x*(-e^2*x^2+d^2)^(1/2)+45/8
*e^5*d^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+45/8*e^4*d
^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+e^6*d*x^2*
(-e^2*x^2+d^2)^(1/2)-6*e^4*d^3*(-e^2*x^2+d^2)^(1/2)
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^5}\,dx =$$

$$-\frac{90\,d^4e^4x^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+45\,d^4e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+48\,d^4e^4x^4-(2\,e^7x^7+8\,de^6x^6+3\,d^2e}{8\,x^4}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x, algorithm="fricas")
```

output
```
-1/8*(90*d^4*e^4*x^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 45*d^4*e^
4*x^4*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 48*d^4*e^4*x^4 - (2*e^7*x^7 + 8
*d*e^6*x^6 + 3*d^2*e^5*x^5 - 48*d^3*e^4*x^4 + 48*d^4*e^3*x^3 - 3*d^5*e^2*x
^2 - 8*d^6*e*x - 2*d^7)*sqrt(-e^2*x^2 + d^2))/x^4
```

3.75.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.95 (sec) , antiderivative size = 959, normalized size of antiderivative = 4.59

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**5,x)
```

output
```
d**7*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*
sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) +
 e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x*
*5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1
)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/
(8*d**3), True)) + 3*d**6*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x*
*2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1)
, (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**
2) + 1)/(3*d**2), True)) + d**5*e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) -
 1)/(2*x) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2
/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2)
+ 1)) - I*e**2*asin(d/(e*x))/(2*d), True)) - 5*d**4*e**3*Piecewise((I*d/(x
*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**
2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2))
- e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True)) - 5*d**3*e**
4*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*
x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*sq
rt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x*
*2) + 1), True)) + d**2*e**5*Piecewise((d**2*Piecewise((log(-2*e**2*x + 2*
sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x...
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^5}\,dx = \frac{45\,d^4e^5\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\,\sqrt{e^2}} + \frac{45}{8}\,d^4e^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right) + \frac{45}{8}\,\sqrt{-e^2x^2+d^2}d^2e^5x - \frac{45}{8}\,\sqrt{-e^2x^2+d^2}d^3e^4 + \frac{15}{4}\,\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5x - \frac{15}{8}\,\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^4 - \frac{9\,\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}{8\,d} + \frac{3\,\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}{x} - \frac{9\,\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{8\,dx^2} - \frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{x^3} - \frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{4\,x^4}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x, algorithm="maxima")
```

output
```
45/8*d^4*e^5*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 45/8*d^4*e^4*log(2*d^
2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) + 45/8*sqrt(-e^2*x^2 + d^2)*d^
2*e^5*x - 45/8*sqrt(-e^2*x^2 + d^2)*d^3*e^4 + 15/4*(-e^2*x^2 + d^2)^(3/2)*
e^5*x - 15/8*(-e^2*x^2 + d^2)^(3/2)*d*e^4 - 9/8*(-e^2*x^2 + d^2)^(5/2)*e^4
/d + 3*(-e^2*x^2 + d^2)^(5/2)*e^3/x - 9/8*(-e^2*x^2 + d^2)^(7/2)*e^2/(d*x^
2) - (-e^2*x^2 + d^2)^(7/2)*e/x^3 - 1/4*(-e^2*x^2 + d^2)^(7/2)*d/x^4
```

3.75.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(183) = 366$.

Time = 0.30 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.96

$$\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^5}\,dx = \frac{45\,d^4e^5\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,|e|}$$
$$+\frac{45\,d^4e^5\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{8\,|e|}$$
$$+\frac{\left(d^4e^5+\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^4e^3}{x}+\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^4e}{x^2}-\frac{184\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^4}{ex^3}\right)e^8x^4}{64\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4|e|}$$
$$-\frac{1}{8}\left(48\,d^3e^4-\left(3\,d^2e^5+2\left(e^7x+4\,de^6\right)x\right)x\right)\sqrt{-e^2x^2+d^2}$$
$$+\frac{\frac{184\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^4e^5|e|}{x}-\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^4e^3|e|}{x^2}-\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^4e|e|}{x^3}-\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^4|e|}{ex^4}}{64\,e^4}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x, algorithm="giac")
```

output
```
45/8*d^4*e^5*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 45/8*d^4*e^5*log(1/2*abs
(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/64*(d^4*
e^5 + 8*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^4*e^3/x + 8*(d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))^2*d^4*e/x^2 - 184*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^
3*d^4/(e*x^3))*e^8*x^4/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*abs(e)) - 1/
8*(48*d^3*e^4 - (3*d^2*e^5 + 2*(e^7*x + 4*d*e^6)*x)*x)*sqrt(-e^2*x^2 + d^2
) + 1/64*(184*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^4*e^5*abs(e)/x - 8*(d*
e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^4*e^3*abs(e)/x^2 - 8*(d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))^3*d^4*e*abs(e)/x^3 - (d*e + sqrt(-e^2*x^2 + d^2)*abs(e
))^4*d^4*abs(e)/(e*x^4))/e^4
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^5} \, dx = \int \frac{(d^2-e^2\,x^2)^{5/2}\,(d+e\,x)^3}{x^5} \, dx$$

input
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^5,x)
```

output
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^5, x)
```

3.76 $\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^6} \, dx$

3.76.1 Optimal result

Integrand size = 27, antiderivative size = 216

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^6} \, dx = \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)(d^2-e^2x^2)^{3/2}}{24x^2} - \frac{e^2(52d+25ex)(d^2-e^2x^2)^{5/2}}{60x^3} - \frac{d(d^2-e^2x^2)^{7/2}}{5x^5} - \frac{3e(d^2-e^2x^2)^{7/2}}{4x^4} + \frac{13}{2}d^3e^5 \arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{25}{8}d^3e^5 \operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
1/24*d*e^3*(-52*e*x+25*d)*(-e^2*x^2+d^2)^(3/2)/x^2-1/60*e^2*(25*e*x+52*d)*
(-e^2*x^2+d^2)^(5/2)/x^3-1/5*d*(-e^2*x^2+d^2)^(7/2)/x^5-3/4*e*(-e^2*x^2+d^
2)^(7/2)/x^4+13/2*d^3*e^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-25/8*d^3*e^5*ar
ctanh((-e^2*x^2+d^2)^(1/2)/d)+1/8*d^2*e^4*(25*e*x+52*d)*(-e^2*x^2+d^2)^(1/
2)/x
```

3.76.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^6} \, dx = \frac{\sqrt{d^2-e^2x^2}(-24d^7 - 90d^6ex - 32d^5e^2x^2 + 345d^4e^3x^3 + 656d^3e^4x^4 + 80d^2e}{120x^5} - 13d^3e^5 \arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) - \frac{25}{8}\left(d^2\right)^{3/2}e^5\log(x) + \frac{25}{8}\left(d^2\right)^{3/2}e^5\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)$$

input
```
Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^6,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-24*d^7 - 90*d^6*e*x - 32*d^5*e^2*x^2 + 345*d^4*e^3*
x^3 + 656*d^3*e^4*x^4 + 80*d^2*e^5*x^5 + 180*d*e^6*x^6 + 40*e^7*x^7))/(120
*x^5) - 13*d^3*e^5*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] - (25*(
d^2)^(3/2)*e^5*Log[x])/8 + (25*(d^2)^(3/2)*e^5*Log[Sqrt[d^2] - Sqrt[d^2 -
e^2*x^2]])/8
```

3.76.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.01, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$, Rules used = {540, 25, 2338, 25, 27, 537, 25, 536, 535, 27, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^6} \, dx$$

$$\Big\downarrow \ 540$$

$$-\frac{\int -\frac{\left(d^2-e^2x^2\right)^{5/2}\left(15ed^4+13e^2xd^3+5e^3x^2d^2\right)}{x^5}dx}{5d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(15ed^4+13e^2xd^3+5e^3x^2d^2\right)}{x^5}dx}{5d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5}$$

$$\Big\downarrow \ 2338$$

$$\frac{-\frac{\int -\frac{d^4e^2(52d-25ex)\left(d^2-e^2x^2\right)^{5/2}}{x^4}dx}{4d^2}-\frac{15d^2e\left(d^2-e^2x^2\right)^{7/2}}{4x^4}}{5d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{d^4e^2(52d-25ex)\left(d^2-e^2x^2\right)^{5/2}}{x^4}dx}{4d^2}-\frac{15d^2e\left(d^2-e^2x^2\right)^{7/2}}{4x^4}}{5d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{4}d^2e^2\int \frac{(52d-25ex)\left(d^2-e^2x^2\right)^{5/2}}{x^4}dx-\frac{15d^2e\left(d^2-e^2x^2\right)^{7/2}}{4x^4}}{5d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5}$$

$$\Big\downarrow \; 537$$

$$\frac{\frac{1}{4}d^2e^2\left(\frac{5}{6}e^2\int -\frac{(104d-75ex)(d^2-e^2x^2)^{3/2}}{x^2}dx-\frac{(104d-75ex)(d^2-e^2x^2)^{5/2}}{6x^3}\right)-\frac{15d^2e(d^2-e^2x^2)^{7/2}}{4x^4}}{5d^2}-\frac{d(d^2-e^2x^2)^{7/2}}{5x^5}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{4}d^2e^2\left(-\frac{5}{6}e^2\int \frac{(104d-75ex)(d^2-e^2x^2)^{3/2}}{x^2}dx-\frac{(104d-75ex)(d^2-e^2x^2)^{5/2}}{6x^3}\right)-\frac{15d^2e(d^2-e^2x^2)^{7/2}}{4x^4}}{5d^2}-\frac{d(d^2-e^2x^2)^{7/2}}{5x^5}$$

$$\Big\downarrow \; 536$$

$$\frac{\frac{1}{4}d^2e^2\left(-\frac{5}{6}e^2\left(\int \frac{(-75ed^2-312e^2xd)\sqrt{d^2-e^2x^2}}{x}dx-\frac{(104d+25ex)(d^2-e^2x^2)^{3/2}}{x}\right)-\frac{(104d-75ex)(d^2-e^2x^2)^{5/2}}{6x^3}\right)-\frac{15d^2e(d^2-e^2x^2)^{7}}{4x^4}}{5d^2}$$

$$\frac{d(d^2-e^2x^2)^{7/2}}{5x^5}$$

$$\Big\downarrow \; 535$$

$$\frac{\frac{1}{4}d^2e^2\left(-\frac{5}{6}e^2\left(\frac{1}{2}d^2\int -\frac{6de(25d+52ex)}{x\sqrt{d^2-e^2x^2}}dx-3de(25d+52ex)\sqrt{d^2-e^2x^2}-\frac{(104d+25ex)(d^2-e^2x^2)^{3/2}}{x}\right)-\frac{(104d-75ex)(d^2-e}{6x^3}}{5d^2}$$

$$\frac{d(d^2-e^2x^2)^{7/2}}{5x^5}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{4}d^2e^2\left(-\frac{5}{6}e^2\left(-3d^3e\int \frac{25d+52ex}{x\sqrt{d^2-e^2x^2}}dx-3de(25d+52ex)\sqrt{d^2-e^2x^2}-\frac{(104d+25ex)(d^2-e^2x^2)^{3/2}}{x}\right)-\frac{(104d-75ex)(d^2-e^2x}{6x^3}}{5d^2}$$

$$\frac{d(d^2-e^2x^2)^{7/2}}{5x^5}$$

$$\Big\downarrow \; 538$$

$$\frac{\frac{1}{4}d^2e^2\left(-\frac{5}{6}e^2\left(-3d^3e\left(52e\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+25d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx\right)-3de(25d+52ex)\sqrt{d^2-e^2x^2}-\frac{(104d+25ex)(d^2-e^2x}{x}}{5d^2}$$

$$\frac{d(d^2-e^2x^2)^{7/2}}{5x^5}$$

3.76. $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^6}\,dx$

$\Big\downarrow$ 224

$$\frac{\frac{1}{4}d^2e^2\left(-\frac{5}{6}e^2\left(-3d^3e\left(25d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx+52e\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)-3de(25d+52ex)\sqrt{d^2-e^2x^2}-\frac{(104d+25}{}\right.\right.}{5d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5}$$

$\Big\downarrow$ 216

$$\frac{\frac{1}{4}d^2e^2\left(-\frac{5}{6}e^2\left(-3d^3e\left(25d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx+52\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-3de(25d+52ex)\sqrt{d^2-e^2x^2}-\frac{(104d+25ex)(d^2}{x}\right.\right.}{5d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5}$$

$\Big\downarrow$ 243

$$\frac{\frac{1}{4}d^2e^2\left(-\frac{5}{6}e^2\left(-3d^3e\left(\frac{25}{2}d\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2+52\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-3de(25d+52ex)\sqrt{d^2-e^2x^2}-\frac{(104d+25ex)(}{x}\right.\right.}{5d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5}$$

$\Big\downarrow$ 73

$$\frac{\frac{1}{4}d^2e^2\left(-\frac{5}{6}e^2\left(-3d^3e\left(52\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{25d\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}\right)-3de(25d+52ex)\sqrt{d^2-e^2x^2}-\frac{(104d+25e}{}\right.\right.}{5d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5}$$

$\Big\downarrow$ 221

$$\frac{\frac{1}{4}d^2e^2\left(-\frac{5}{6}e^2\left(-3d^3e\left(52\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-25\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)-3de(25d+52ex)\sqrt{d^2-e^2x^2}-\frac{(104d+25ex}{}\right.\right.}{5d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5}$$

input
```
Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^6,x]
```

3.76. $\int\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^6}\,dx$

output
```
-1/5*(d*(d^2 - e^2*x^2)^(7/2))/x^5 + ((-15*d^2*e*(d^2 - e^2*x^2)^(7/2))/(4
*x^4) + (d^2*e^2*(-1/6*((104*d - 75*e*x)*(d^2 - e^2*x^2)^(5/2))/x^3 - (5*e
^2*(-3*d*e*(25*d + 52*e*x)*Sqrt[d^2 - e^2*x^2] - ((104*d + 25*e*x)*(d^2 -
e^2*x^2)^(3/2))/x - 3*d^3*e*(52*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 25*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])))/6))/4)/(5*d^2)
```

3.76.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.76. $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^6}\,dx$

3.76.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{d^3\sqrt{-e^2x^2+d^2}\,(-656e^4x^4-345d\,e^3x^3+32d^2e^2x^2+90d^3ex+24d^4)}{120x^5}+\frac{e^7x^2\sqrt{-e^2x^2+d^2}}{3}+\frac{2e^5d^2\sqrt{-e^2x^2+d^2}}{3}+\frac{13e^6d^3\arctan}{2}$
default	$d^3\left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{5d^2x^5}-\frac{2e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{3d^2x^3}-\frac{4e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{d^2x}-\frac{6e^2\left(\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2}}{2}\right.}{}\right.}{}\right)}{d^2}\right)}{3d^2}\right)}{5d^2}\right.$

3.76. $\int \frac{(d+ex)^3\,(d^2-e^2x^2)^{5/2}}{x^6}\,dx$

input
```
int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/120*d^3*(-e^2*x^2+d^2)^(1/2)*(-656*e^4*x^4-345*d*e^3*x^3+32*d^2*e^2*x^2
+90*d^3*e*x+24*d^4)/x^5+1/3*e^7*x^2*(-e^2*x^2+d^2)^(1/2)+2/3*e^5*d^2*(-e^2
*x^2+d^2)^(1/2)+13/2*e^6*d^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^
2)^(1/2))-25/8*e^5*d^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^
(1/2))/x)+3/2*e^6*d*x*(-e^2*x^2+d^2)^(1/2)
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^6}\,dx =$$
$$-\frac{1560\,d^3e^5x^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - 375\,d^3e^5x^5\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - 80\,d^3e^5x^5 - (40\,e^7x^7 + 180\,de^6x^6}{120\,x^5}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x, algorithm="fricas")
```

output
```
-1/120*(1560*d^3*e^5*x^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 375*d
^3*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 80*d^3*e^5*x^5 - (40*e^7*x
^7 + 180*d*e^6*x^6 + 80*d^2*e^5*x^5 + 656*d^3*e^4*x^4 + 345*d^4*e^3*x^3 -
32*d^5*e^2*x^2 - 90*d^6*e*x - 24*d^7)*sqrt(-e^2*x^2 + d^2))/x^5
```

3.76.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.31 (sec) , antiderivative size = 1182, normalized size of antiderivative = 5.47

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**6,x)
```

output
```
d**7*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**
2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e*
*2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**
3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d
*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-
15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15
*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**
5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3
*x**5 + 15*d*e**2*x**7), True)) + 3*d**6*e*Piecewise((-d**2/(4*e*x**5*sqrt
(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(
8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d
**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*
I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e*
*2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) + d**5*e**2*Piecewi
se((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) -
1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)
/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) - 5*d**4*e
**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/(
2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2)
 + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(...
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.34

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^6}\,dx = \frac{13\,d^3e^6\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}} - \frac{25}{8}\,d^3e^5\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) + \frac{13}{2}\sqrt{-e^2x^2+d^2}de^6x + \frac{25}{8}\sqrt{-e^2x^2+d^2}d^2e^5 + \frac{13\,(-e^2x^2+d^2)^{\frac{3}{2}}e^6x}{3\,d} + \frac{25}{24}\,(-e^2x^2+d^2)^{\frac{3}{2}}e^5 + \frac{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^5}{8\,d^2} + \frac{52\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4}{15\,dx} + \frac{5\,(-e^2x^2+d^2)^{\frac{7}{2}}e^3}{8\,d^2x^2} - \frac{13\,(-e^2x^2+d^2)^{\frac{7}{2}}e^2}{15\,dx^3} - \frac{3\,(-e^2x^2+d^2)^{\frac{7}{2}}e}{4\,x^4} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}d}{5\,x^5}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x, algorithm="maxima")
```

output
```
13/2*d^3*e^6*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 25/8*d^3*e^5*log(2*d^
2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) + 13/2*sqrt(-e^2*x^2 + d^2)*d*
e^6*x + 25/8*sqrt(-e^2*x^2 + d^2)*d^2*e^5 + 13/3*(-e^2*x^2 + d^2)^(3/2)*e^
6*x/d + 25/24*(-e^2*x^2 + d^2)^(3/2)*e^5 + 5/8*(-e^2*x^2 + d^2)^(5/2)*e^5/
d^2 + 52/15*(-e^2*x^2 + d^2)^(5/2)*e^4/(d*x) + 5/8*(-e^2*x^2 + d^2)^(7/2)*
e^3/(d^2*x^2) - 13/15*(-e^2*x^2 + d^2)^(7/2)*e^2/(d*x^3) - 3/4*(-e^2*x^2 +
 d^2)^(7/2)*e/x^4 - 1/5*(-e^2*x^2 + d^2)^(7/2)*d/x^5
```

3.76.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 460 vs. $2(188) = 376$.

Time = 0.30 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.13

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^6} dx = \frac{13 d^3 e^6 \arcsin\left(\frac{ex}{d}\right) \operatorname{sgn}(d) \operatorname{sgn}(e)}{2|e|}$$
$$+ \frac{\left(6 d^3 e^6 + \frac{45 \left(de+\sqrt{-e^2x^2+d^2}|e|\right) d^3 e^4}{x} + \frac{50 \left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d^3 e^2}{x^2} - \frac{600 \left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3 d^3}{x^3} - \frac{2580 \left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2 x^4}\right.}{960 \left(de + \sqrt{-e^2x^2+d^2}|e|\right)^5 |e|}$$
$$- \frac{25 d^3 e^6 \log\left(\frac{\left|-2 de - 2\sqrt{-e^2x^2+d^2}|e|\right|}{2 e^2 |x|}\right)}{8|e|} + \frac{1}{6}\left(4 d^2 e^5 + \left(2 e^7 x + 9 de^6\right) x\right)\sqrt{-e^2x^2+d^2}$$
$$+ \frac{\frac{2580 \left(de+\sqrt{-e^2x^2+d^2}|e|\right) d^3 e^8}{x} + \frac{600 \left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d^3 e^6}{x^2} - \frac{50 \left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3 d^3 e^4}{x^3} - \frac{45 \left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4 d^3 e^2}{x^4} - \frac{6}{}}{960 e^4 |e|}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x, algorithm="giac")
```

output
```
13/2*d^3*e^6*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/960*(6*d^3*e^6 + 45*(d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^3*e^4/x + 50*(d*e + sqrt(-e^2*x^2 + d^
2)*abs(e))^2*d^3*e^2/x^2 - 600*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^3/x
^3 - 2580*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^3/(e^2*x^4))*e^10*x^5/((
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*abs(e)) - 25/8*d^3*e^6*log(1/2*abs(-2
*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/6*(4*d^2*e^
5 + (2*e^7*x + 9*d*e^6)*x)*sqrt(-e^2*x^2 + d^2) + 1/960*(2580*(d*e + sqrt(
-e^2*x^2 + d^2)*abs(e))*d^3*e^8/x + 600*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e)
)^2*d^3*e^6/x^2 - 50*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^3*e^4/x^3 - 4
5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^3*e^2/x^4 - 6*(d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))^5*d^3/x^5)/(e^4*abs(e))
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^6}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}\,(d+e\,x)^3}{x^6}\,dx$$

input
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^6,x)
```

output
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^6, x)
```

3.77 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^7}\,dx$

3.77.1 Optimal result

Integrand size = 27, antiderivative size = 214

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^7}\,dx = -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)\left(d^2-e^2x^2\right)^{5/2}}{120x^4} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{5x^5} - \frac{1}{2}d^2e^6\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{85}{16}d^2e^6\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
1/48*d*e^3*(85*e*x+8*d)*(-e^2*x^2+d^2)^(3/2)/x^3-1/120*e^2*(12*e*x+85*d)*(
-e^2*x^2+d^2)^(5/2)/x^4-1/6*d*(-e^2*x^2+d^2)^(7/2)/x^6-3/5*e*(-e^2*x^2+d^2
)^(7/2)/x^5-1/2*d^2*e^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-85/16*d^2*e^6*arc
tanh((-e^2*x^2+d^2)^(1/2)/d)-1/16*d*e^5*(-85*e*x+8*d)*(-e^2*x^2+d^2)^(1/2)
/x
```

3.77.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^7}\,dx = \frac{\sqrt{d^2-e^2x^2}(-40d^7-144d^6ex-50d^5e^2x^2+448d^4e^3x^3+645d^3e^4x^4-544}{240x^6} + d^2e^6\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) - \frac{85}{16}d\sqrt{d^2}e^6\log(x) + \frac{85}{16}d\sqrt{d^2}e^6\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)$$

input
```
Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^7,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-40*d^7 - 144*d^6*e*x - 50*d^5*e^2*x^2 + 448*d^4*e^3
*x^3 + 645*d^3*e^4*x^4 - 544*d^2*e^5*x^5 + 720*d*e^6*x^6 + 120*e^7*x^7))/(
240*x^6) + d^2*e^6*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] - (85*d
*Sqrt[d^2]*e^6*Log[x])/16 + (85*d*Sqrt[d^2]*e^6*Log[Sqrt[d^2] - Sqrt[d^2 -
 e^2*x^2]])/16
```

3.77.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.05, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.630$, Rules used = {540, 25, 2338, 25, 27, 537, 27, 537, 25, 535, 27, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^7} \, dx$$

$$\Big\downarrow \text{540}$$

$$-\frac{\int -\frac{\left(d^2-e^2x^2\right)^{5/2}\left(18ed^4+17e^2xd^3+6e^3x^2d^2\right)}{x^6}dx}{6d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(18ed^4+17e^2xd^3+6e^3x^2d^2\right)}{x^6}dx}{6d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int -\frac{d^4e^2(85d-6ex)\left(d^2-e^2x^2\right)^{5/2}}{x^5}dx}{5d^2}-\frac{18d^2e\left(d^2-e^2x^2\right)^{7/2}}{5x^5}}{6d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{d^4e^2(85d-6ex)\left(d^2-e^2x^2\right)^{5/2}}{x^5}dx}{5d^2}-\frac{18d^2e\left(d^2-e^2x^2\right)^{7/2}}{5x^5}}{6d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{5}d^2e^2\int \frac{(85d-6ex)\left(d^2-e^2x^2\right)^{5/2}}{x^5}dx-\frac{18d^2e\left(d^2-e^2x^2\right)^{7/2}}{5x^5}}{6d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6}$$

$$\Big\downarrow \ 537$$

$$\frac{\frac{1}{5}d^2e^2\left(\frac{5}{12}e^2\int -\frac{3(85d-8ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}dx-\frac{(85d-8ex)\left(d^2-e^2x^2\right)^{5/2}}{4x^4}\right)-\frac{18d^2e\left(d^2-e^2x^2\right)^{7/2}}{5x^5}}{6d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{5}d^2e^2\left(-\frac{5}{4}e^2\int \frac{(85d-8ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}dx-\frac{(85d-8ex)\left(d^2-e^2x^2\right)^{5/2}}{4x^4}\right)-\frac{18d^2e\left(d^2-e^2x^2\right)^{7/2}}{5x^5}}{6d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6}$$

$$\Big\downarrow \ 537$$

$$\frac{\frac{1}{5}d^2e^2\left(-\frac{5}{4}e^2\left(\frac{3}{2}e^2\int -\frac{(85d-16ex)\sqrt{d^2-e^2x^2}}{x}dx-\frac{(85d-16ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}\right)-\frac{(85d-8ex)\left(d^2-e^2x^2\right)^{5/2}}{4x^4}\right)-\frac{18d^2e\left(d^2-e^2x^2\right)^{7/2}}{5x^5}}{6d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{1}{5}d^2e^2\left(-\frac{5}{4}e^2\left(-\frac{3}{2}e^2\int \frac{(85d-16ex)\sqrt{d^2-e^2x^2}}{x}dx-\frac{(85d-16ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}\right)-\frac{(85d-8ex)\left(d^2-e^2x^2\right)^{5/2}}{4x^4}\right)-\frac{18d^2e\left(d^2-e^2x^2\right)^{7/2}}{5x^5}}{6d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6}$$

$$\Big\downarrow \ 535$$

$$\frac{\frac{1}{5}d^2e^2\left(-\frac{5}{4}e^2\left(-\frac{3}{2}e^2\left(\frac{1}{2}d^2\int \frac{2(85d-8ex)}{x\sqrt{d^2-e^2x^2}}dx+(85d-8ex)\sqrt{d^2-e^2x^2}\right)-\frac{(85d-16ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}\right)-\frac{(85d-8ex)\left(d^2-e^2x^2\right)}{4x^4}\right.}{6d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{5}d^2e^2\left(-\frac{5}{4}e^2\left(-\frac{3}{2}e^2\left(d^2\int \frac{85d-8ex}{x\sqrt{d^2-e^2x^2}}dx+(85d-8ex)\sqrt{d^2-e^2x^2}\right)-\frac{(85d-16ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}\right)-\frac{(85d-8ex)\left(d^2-e^2x^2\right)^{5/}}{4x^4}\right.}{6d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6}$$

3.77. $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^7}\,dx$

$$\Big\downarrow \ 538$$

$$\frac{\frac{1}{5}d^2e^2\left(-\frac{5}{4}e^2\left(-\frac{3}{2}e^2\left(d^2\left(85d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-8e\int \frac{1}{\sqrt{d^2-e^2x^2}}dx\right)+(85d-8ex)\sqrt{d^2-e^2x^2}\right)-\frac{(85d-16ex)(d^2-e^2x^2)^3}{2x^2}\right.\right.}{6d^2}$$

$$\frac{d(d^2-e^2x^2)^{7/2}}{6x^6}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{1}{5}d^2e^2\left(-\frac{5}{4}e^2\left(-\frac{3}{2}e^2\left(d^2\left(85d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-8e\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)+(85d-8ex)\sqrt{d^2-e^2x^2}\right)-\frac{(85d-16ex)}{}\right.\right.}{6d^2}$$

$$\frac{d(d^2-e^2x^2)^{7/2}}{6x^6}$$

$$\Big\downarrow \ 216$$

$$\frac{\frac{1}{5}d^2e^2\left(-\frac{5}{4}e^2\left(-\frac{3}{2}e^2\left(d^2\left(85d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+(85d-8ex)\sqrt{d^2-e^2x^2}\right)-\frac{(85d-16ex)(d^2-e}{2x^2}\right.\right.}{6d^2}$$

$$\frac{d(d^2-e^2x^2)^{7/2}}{6x^6}$$

$$\Big\downarrow \ 243$$

$$\frac{\frac{1}{5}d^2e^2\left(-\frac{5}{4}e^2\left(-\frac{3}{2}e^2\left(d^2\left(\frac{85}{2}d\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+(85d-8ex)\sqrt{d^2-e^2x^2}\right)-\frac{(85d-16ex)(d^2-}{2x^2}\right.\right.}{6d^2}$$

$$\frac{d(d^2-e^2x^2)^{7/2}}{6x^6}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{1}{5}d^2e^2\left(-\frac{5}{4}e^2\left(-\frac{3}{2}e^2\left(d^2\left(-\frac{85d\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}-8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+(85d-8ex)\sqrt{d^2-e^2x^2}\right)-\frac{(85d-16e}{}\right.\right.}{6d^2}$$

$$\frac{d(d^2-e^2x^2)^{7/2}}{6x^6}$$

$$\Big\downarrow \ 221$$

3.77. $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^7}\,dx$

$$\frac{\frac{\frac{1}{5}d^2e^2\left(-\frac{5}{4}e^2\left(-\frac{3}{2}e^2\left(d^2\left(-8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-85\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)+(85d-8ex)\sqrt{d^2-e^2x^2}\right)-\frac{(85d-16ex)}{}\cdots}{6d^2}}{\frac{d(d^2-e^2x^2)^{7/2}}{6x^6}}$$

input
```
Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^7,x]
```

output
```
-1/6*(d*(d^2 - e^2*x^2)^(7/2))/x^6 + ((-18*d^2*e*(d^2 - e^2*x^2)^(7/2))/(5
*x^5) + (d^2*e^2*(-1/4*((85*d - 8*e*x)*(d^2 - e^2*x^2)^(5/2))/x^4 - (5*e^2
*(-1/2*((85*d - 16*e*x)*(d^2 - e^2*x^2)^(3/2))/x^2 - (3*e^2*((85*d - 8*e*x
)*Sqrt[d^2 - e^2*x^2] + d^2*(-8*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 85*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])))/2))/4))/5)/(6*d^2)
```

3.77.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.77. $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^7}\,dx$

3.77.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{d^{2}\sqrt{-e^{2}x^{2}+d^{2}}\left(544e^{5}x^{5}-645d\,e^{4}x^{4}-448d^{2}e^{3}x^{3}+50d^{3}e^{2}x^{2}+144d^{4}ex+40d^{5}\right)}{240x^{6}}+\frac{e^{7}x\sqrt{-e^{2}x^{2}+d^{2}}}{2}-\frac{e^{7}d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}$
default	$d^{3}\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{6d^{2}x^{6}}-\frac{e^{2}\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{4d^{2}x^{4}}-\frac{3e^{2}\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{2d^{2}x^{2}}-\frac{5e^{2}\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{5}+d^{2}\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{3}+d^{2}\left(\sqrt{-e^{2}x^{2}+d^{2}}-\right.\right.\right.}{2d^{2}}\right.}{4d^{2}}\right.}{6d^{2}}\right.$

3.77. $\int \frac{(d+ex)^{3}\left(d^{2}-e^{2}x^{2}\right)^{5/2}}{x^{7}}\,dx$

input
```
int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/240*d^2*(-e^2*x^2+d^2)^(1/2)*(544*e^5*x^5-645*d*e^4*x^4-448*d^2*e^3*x^3
+50*d^3*e^2*x^2+144*d^4*e*x+40*d^5)/x^6+1/2*e^7*x*(-e^2*x^2+d^2)^(1/2)-1/2
*e^7*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-85/16*e^6*
d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+3*e^6*d*(
-e^2*x^2+d^2)^(1/2)
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^7}\,dx = \frac{240\,d^2e^6x^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+1275\,d^2e^6x^6\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+72$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x, algorithm="fricas")
```

output
```
1/240*(240*d^2*e^6*x^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 1275*d^
2*e^6*x^6*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 720*d^2*e^6*x^6 + (120*e^7*
x^7 + 720*d*e^6*x^6 - 544*d^2*e^5*x^5 + 645*d^3*e^4*x^4 + 448*d^4*e^3*x^3
- 50*d^5*e^2*x^2 - 144*d^6*e*x - 40*d^7)*sqrt(-e^2*x^2 + d^2))/x^6
```

3.77.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.65 (sec) , antiderivative size = 1380, normalized size of antiderivative = 6.45

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^7}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**7,x)
```

output
```
d**7*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5
*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) -
1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(1
6*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x*
*2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*
x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**
2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) + 3*d**6*e*Piecewise((3*
I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e*
*2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**
6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e
**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e
**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*
e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e*
*2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e
**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*
x**7), True)) + d**5*e**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2)
 - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**
2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2))
> 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(
-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1))...
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.47

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^7}\,dx = -\frac{d^2e^7 \arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}} - \frac{85}{16}d^2e^6\log\left(\frac{2d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{1}{2}\sqrt{-e^2x^2+d^2}e^7x + \frac{85}{16}\sqrt{-e^2x^2+d^2}de^6 - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^7x}{3d^2} + \frac{85(-e^2x^2+d^2)^{\frac{3}{2}}e^6}{48d} + \frac{17(-e^2x^2+d^2)^{\frac{5}{2}}e^6}{16d^3} - \frac{4(-e^2x^2+d^2)^{\frac{5}{2}}e^5}{15d^2x} + \frac{17(-e^2x^2+d^2)^{\frac{7}{2}}e^4}{16d^3x^2} + \frac{(-e^2x^2+d^2)^{\frac{7}{2}}e^3}{15d^2x^3} - \frac{17(-e^2x^2+d^2)^{\frac{7}{2}}e^2}{24dx^4} - \frac{3(-e^2x^2+d^2)^{\frac{7}{2}}e}{5x^5} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}d}{6x^6}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x, algorithm="maxima")
```

output
```
-1/2*d^2*e^7*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 85/16*d^2*e^6*log(2*d
^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) - 1/2*sqrt(-e^2*x^2 + d^2)*e^
7*x + 85/16*sqrt(-e^2*x^2 + d^2)*d*e^6 - 1/3*(-e^2*x^2 + d^2)^(3/2)*e^7*x/
d^2 + 85/48*(-e^2*x^2 + d^2)^(3/2)*e^6/d + 17/16*(-e^2*x^2 + d^2)^(5/2)*e^
6/d^3 - 4/15*(-e^2*x^2 + d^2)^(5/2)*e^5/(d^2*x) + 17/16*(-e^2*x^2 + d^2)^(
7/2)*e^4/(d^3*x^2) + 1/15*(-e^2*x^2 + d^2)^(7/2)*e^3/(d^2*x^3) - 17/24*(-e
^2*x^2 + d^2)^(7/2)*e^2/(d*x^4) - 3/5*(-e^2*x^2 + d^2)^(7/2)*e/x^5 - 1/6*(
-e^2*x^2 + d^2)^(7/2)*d/x^6
```

3.77.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(186) = 372$.

Time = 0.31 (sec) , antiderivative size = 526, normalized size of antiderivative = 2.46

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^7}\,dx = \frac{\left(5\,d^2e^7 + \frac{36\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2e^5}{x} + \frac{45\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d^2e^3}{x^2} - \frac{340\left(de+\sqrt{-e^2x^2\cdots}\right.}{x^3}\cdots\right.}{1920\left(de+\sqrt{-e^2x^2\cdots}\right.}$$
$$- \frac{d^2e^7\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,|e|}$$
$$- \frac{85\,d^2e^7\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{16\,|e|} + \frac{1}{2}\left(e^7x+6\,de^6\right)\sqrt{-e^2x^2+d^2}$$
$$- \frac{\frac{1800\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2e^9|e|}{x} - \frac{1215\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d^2e^7|e|}{x^2} - \frac{340\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3 d^2e^5|e|}{x^3} + \frac{45\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4 d\cdots}{x^4}\cdots}{1920\,e^6}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x, algorithm="giac")
```

output
```
1/1920*(5*d^2*e^7 + 36*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^2*e^5/x + 45*
(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^2*e^3/x^2 - 340*(d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))^3*d^2*e/x^3 - 1215*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4
*d^2/(e*x^4) + 1800*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d^2/(e^3*x^5))*e
^12*x^6/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6*abs(e)) - 1/2*d^2*e^7*arcsi
n(e*x/d)*sgn(d)*sgn(e)/abs(e) - 85/16*d^2*e^7*log(1/2*abs(-2*d*e - 2*sqrt(
-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/2*(e^7*x + 6*d*e^6)*sqrt(
-e^2*x^2 + d^2) - 1/1920*(1800*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^2*e^9
*abs(e)/x - 1215*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^2*e^7*abs(e)/x^2
- 340*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^2*e^5*abs(e)/x^3 + 45*(d*e +
 sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^2*e^3*abs(e)/x^4 + 36*(d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))^5*d^2*e*abs(e)/x^5 + 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e
))^6*d^2*abs(e)/(e*x^6))/e^6
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^7} \, dx = \int \frac{(d^2-e^2\,x^2)^{5/2}\,(d+e\,x)^3}{x^7} \, dx$$

input
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^7,x)
```

output
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^7, x)
```

3.78 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^8}\,dx$

3.78.1 Optimal result

Integrand size = 27, antiderivative size = 206

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^8}\,dx = -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{2x^6} - 3de^7\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{15}{16}de^7\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
1/16*e^4*(5*e*x+16*d)*(-e^2*x^2+d^2)^(3/2)/x^3-1/40*e^2*(5*e*x+24*d)*(-e^2
*x^2+d^2)^(5/2)/x^5-1/7*d*(-e^2*x^2+d^2)^(7/2)/x^7-1/2*e*(-e^2*x^2+d^2)^(7
/2)/x^6-3*d*e^7*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-15/16*d*e^7*arctanh((-e^2
*x^2+d^2)^(1/2)/d)-3/16*e^6*(-5*e*x+16*d)*(-e^2*x^2+d^2)^(1/2)/x
```

3.78.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^8}\,dx = \frac{\sqrt{d^2-e^2x^2}\left(-80d^7-280d^6ex-96d^5e^2x^2+770d^4e^3x^3+992d^3e^4x^4-525d\right.}{560x^7} + 6de^7\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) - \frac{15}{16}\sqrt{d^2}e^7\log(x) + \frac{15}{16}\sqrt{d^2}e^7\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)$$

input
```
Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^8,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-80*d^7 - 280*d^6*e*x - 96*d^5*e^2*x^2 + 770*d^4*e^3
*x^3 + 992*d^3*e^4*x^4 - 525*d^2*e^5*x^5 - 2496*d*e^6*x^6 + 560*e^7*x^7))/
(560*x^7) + 6*d*e^7*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] - (15*
Sqrt[d^2]*e^7*Log[x])/16 + (15*Sqrt[d^2]*e^7*Log[Sqrt[d^2] - Sqrt[d^2 - e^
2*x^2]])/16
```

3.78.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {540, 27, 2338, 27, 537, 25, 537, 27, 536, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^8} \, dx$$

$$\Big\downarrow \ 540$$

$$-\frac{\int -\frac{7\left(d^2-e^2x^2\right)^{5/2}\left(3ed^4+3e^2xd^3+e^3x^2d^2\right)}{x^7}dx}{7d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(3ed^4+3e^2xd^3+e^3x^2d^2\right)}{x^7}dx}{d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7}$$

$$\Big\downarrow \ 2338$$

$$\frac{-\frac{\int -\frac{3d^4e^2(6d+ex)\left(d^2-e^2x^2\right)^{5/2}}{x^6}dx}{6d^2}-\frac{d^2e\left(d^2-e^2x^2\right)^{7/2}}{2x^6}}{d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{2}d^2e^2\int \frac{(6d+ex)\left(d^2-e^2x^2\right)^{5/2}}{x^6}dx-\frac{d^2e\left(d^2-e^2x^2\right)^{7/2}}{2x^6}}{d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7}$$

$$\Big\downarrow \ 537$$

$$\frac{\frac{1}{2}d^2e^2\left(\frac{1}{4}e^2\int -\frac{(24d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}dx-\frac{(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{20x^5}\right)-\frac{d^2e\left(d^2-e^2x^2\right)^{7/2}}{2x^6}}{d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}d^2e^2\left(-\frac{1}{4}e^2\int \frac{(24d+5ex)(d^2-e^2x^2)^{3/2}}{x^4}dx-\frac{(24d+5ex)(d^2-e^2x^2)^{5/2}}{20x^5}\right)-\frac{d^2e(d^2-e^2x^2)^{7/2}}{2x^6}}{d^2}-\frac{d(d^2-e^2x^2)^{7/2}}{7x^7}$$

$$\Big\downarrow \text{537}$$

$$\frac{\frac{1}{2}d^2e^2\left(-\frac{1}{4}e^2\left(\frac{1}{2}e^2\int -\frac{3(16d+5ex)\sqrt{d^2-e^2x^2}}{x^2}dx-\frac{(16d+5ex)(d^2-e^2x^2)^{3/2}}{2x^3}\right)-\frac{(24d+5ex)(d^2-e^2x^2)^{5/2}}{20x^5}\right)-\frac{d^2e(d^2-e^2x^2)^{7/2}}{2x^6}}{d^2}-\frac{d(d^2-e^2x^2)^{7/2}}{7x^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}d^2e^2\left(-\frac{1}{4}e^2\left(-\frac{3}{2}e^2\int \frac{(16d+5ex)\sqrt{d^2-e^2x^2}}{x^2}dx-\frac{(16d+5ex)(d^2-e^2x^2)^{3/2}}{2x^3}\right)-\frac{(24d+5ex)(d^2-e^2x^2)^{5/2}}{20x^5}\right)-\frac{d^2e(d^2-e^2x^2)^{7/2}}{2x^6}}{d^2}-\frac{d(d^2-e^2x^2)^{7/2}}{7x^7}$$

$$\Big\downarrow \text{536}$$

$$\frac{\frac{1}{2}d^2e^2\left(-\frac{1}{4}e^2\left(-\frac{3}{2}e^2\left(\int \frac{5d^2e-16de^2x}{x\sqrt{d^2-e^2x^2}}dx-\frac{(16d-5ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{(16d+5ex)(d^2-e^2x^2)^{3/2}}{2x^3}\right)-\frac{(24d+5ex)(d^2-e^2x^2)^{5/2}}{20x^5}\right)-d^2e}{d^2}$$

$$\frac{d(d^2-e^2x^2)^{7/2}}{7x^7}$$

$$\Big\downarrow \text{538}$$

$$\frac{\frac{1}{2}d^2e^2\left(-\frac{1}{4}e^2\left(-\frac{3}{2}e^2\left(5d^2e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-16de^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{(16d-5ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{(16d+5ex)(d^2-e^2x^2)^{3/2}}{2x^3}\right)-}{d^2}$$

$$\frac{d(d^2-e^2x^2)^{7/2}}{7x^7}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{2}d^2e^2\left(-\frac{1}{4}e^2\left(-\frac{3}{2}e^2\left(5d^2e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-16de^2\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{(16d-5ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{(16d+5ex)(d^2-e^2x^2}{2x^3}\right.\right.}{d^2}$$

$$\frac{d(d^2-e^2x^2)^{7/2}}{7x^7}$$

$$\Big\downarrow \text{216}$$

3.78. $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^8}\,dx$

$$\frac{\frac{1}{2}d^2e^2\left(-\frac{1}{4}e^2\left(-\frac{3}{2}e^2\left(5d^2e\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx-16de\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{(16d-5ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^3}\right)\right.}{d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7}$$

$$\Big\downarrow \ 243$$

$$\frac{\frac{1}{2}d^2e^2\left(-\frac{1}{4}e^2\left(-\frac{3}{2}e^2\left(\frac{5}{2}d^2e\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-16de\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{(16d-5ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^3}\right.\right.}{d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{1}{2}d^2e^2\left(-\frac{1}{4}e^2\left(-\frac{3}{2}e^2\left(-\frac{5d^2\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-16de\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{(16d-5ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{(16d+5ex)\left(d^2-e^2x^2\right.}{2x^3}\right.\right.}{d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{1}{2}d^2e^2\left(-\frac{1}{4}e^2\left(-\frac{3}{2}e^2\left(-16de\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-5de\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)-\frac{\sqrt{d^2-e^2x^2}(16d-5ex)}{x}\right)-\frac{(16d+5ex)\left(d^2-e^2x\right.}{2x^3}\right.\right.}{d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7}$$

input
```
Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^8,x]
```

output
```
-1/7*(d*(d^2 - e^2*x^2)^(7/2))/x^7 + (-1/2*(d^2*e*(d^2 - e^2*x^2)^(7/2))/x
^6 + (d^2*e^2*(-1/20*((24*d + 5*e*x)*(d^2 - e^2*x^2)^(5/2))/x^5 - (e^2*(-1
/2*((16*d + 5*e*x)*(d^2 - e^2*x^2)^(3/2))/x^3 - (3*e^2*(-(((16*d - 5*e*x)*
Sqrt[d^2 - e^2*x^2])/x) - 16*d*e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 5*d*e
*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]))/2))/4))/2)/d^2
```

3.78. $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^8}\,dx$

3.78.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

3.78. $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^8}\,dx$

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.78.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,\left(-560e^{7}x^{7}+2496d\,e^{6}x^{6}+525d^{2}e^{5}x^{5}-992d^{3}e^{4}x^{4}-770d^{4}e^{3}x^{3}+96d^{5}e^{2}x^{2}+280d^{6}ex+80d^{7}\right)}{560x^{7}}-\frac{3d\,e^{8}\arctan\left(\frac{\sqrt{e^{2}}}{\sqrt{-e^{2}x^{2}}}\right.}{\sqrt{e^{2}}}$
default	$e^{3}\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{4d^{2}x^{4}}-\frac{3e^{2}\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{2d^{2}x^{2}}-\frac{5e^{2}\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{5}+d^{2}\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{3}+d^{2}\left(\sqrt{-e^{2}x^{2}+d^{2}}-\frac{d^{2}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}}}{x}\right.}{\sqrt{d^{2}}}\right.\right.\right.}{2d^{2}}\right.}{4d^{2}}\right.$

3.78. $\int \frac{(d+ex)^{3}\left(d^{2}-e^{2}x^{2}\right)^{5/2}}{x^{8}}\,dx$

input
```
int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/560*(-e^2*x^2+d^2)^(1/2)*(-560*e^7*x^7+2496*d*e^6*x^6+525*d^2*e^5*x^5-9
92*d^3*e^4*x^4-770*d^4*e^3*x^3+96*d^5*e^2*x^2+280*d^6*e*x+80*d^7)/x^7-3*d*
e^8/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-15/16*d^2*e^7/(
d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^8} \, dx = \frac{3360\, de^7x^7 \arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + 525\, de^7x^7 \log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + 560}{}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x, algorithm="fricas")
```

output
```
1/560*(3360*d*e^7*x^7*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 525*d*e^
7*x^7*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 560*d*e^7*x^7 + (560*e^7*x^7 -
2496*d*e^6*x^6 - 525*d^2*e^5*x^5 + 992*d^3*e^4*x^4 + 770*d^4*e^3*x^3 - 96*
d^5*e^2*x^2 - 280*d^6*e*x - 80*d^7)*sqrt(-e^2*x^2 + d^2))/x^7
```

3.78.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.75 (sec) , antiderivative size = 1513, normalized size of antiderivative = 7.34

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^8} \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**8,x)
```

output
```
d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e
**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**
4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**
2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(
e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105
*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) + 3*
d**6*e*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x*
*5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2)
- 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/
(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*
x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**
2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x
**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) + d**5*e**2*Piecewise(
(3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d
*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*
e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) -
I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Ab
s(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 +
15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15
*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*...
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^8}\,dx = -\frac{3\,de^8 \arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} - \frac{15}{16}\,de^7\log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{3\,\sqrt{-e^2x^2+d^2}e^8x}{d} + \frac{15}{16}\,\sqrt{-e^2x^2+d^2}e^7 - \frac{2\,(-e^2x^2+d^2)^{\frac{3}{2}}e^8x}{d^3} + \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}e^7}{16\,d^2} + \frac{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e^7}{16\,d^4} - \frac{8\,(-e^2x^2+d^2)^{\frac{5}{2}}e^6}{5\,d^3x} + \frac{3\,(-e^2x^2+d^2)^{\frac{7}{2}}e^5}{16\,d^4x^2} + \frac{2\,(-e^2x^2+d^2)^{\frac{7}{2}}e^4}{5\,d^3x^3} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}e^3}{8\,d^2x^4} - \frac{3\,(-e^2x^2+d^2)^{\frac{7}{2}}e^2}{5\,dx^5} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}e}{2\,x^6} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}d}{7\,x^7}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x, algorithm="maxima")
```

output
```
-3*d*e^8*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 15/16*d*e^7*log(2*d^2/abs
(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) - 3*sqrt(-e^2*x^2 + d^2)*e^8*x/d +
15/16*sqrt(-e^2*x^2 + d^2)*e^7 - 2*(-e^2*x^2 + d^2)^(3/2)*e^8*x/d^3 + 5/16
*(-e^2*x^2 + d^2)^(3/2)*e^7/d^2 + 3/16*(-e^2*x^2 + d^2)^(5/2)*e^7/d^4 - 8/
5*(-e^2*x^2 + d^2)^(5/2)*e^6/(d^3*x) + 3/16*(-e^2*x^2 + d^2)^(7/2)*e^5/(d^
4*x^2) + 2/5*(-e^2*x^2 + d^2)^(7/2)*e^4/(d^3*x^3) - 1/8*(-e^2*x^2 + d^2)^(
7/2)*e^3/(d^2*x^4) - 3/5*(-e^2*x^2 + d^2)^(7/2)*e^2/(d*x^5) - 1/2*(-e^2*x^
2 + d^2)^(7/2)*e/x^6 - 1/7*(-e^2*x^2 + d^2)^(7/2)*d/x^7
```

3.78.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(180) = 360$.

Time = 0.31 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.63

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^8}\,dx = \frac{\left(5\,de^8 + \frac{35\left(de+\sqrt{-e^2x^2+d^2}|e|\right)de^6}{x} + \frac{49\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 de^4}{x^2} - \frac{245\left(de+\sqrt{-e^2x^2+d^2}\right.}{x^3}\right.}{4480\,(de+} - \frac{3\,de^8\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} - \frac{15\,de^8\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{16\,|e|} + \sqrt{-e^2x^2+d^2}\,e^7 - \frac{\frac{9065\left(de+\sqrt{-e^2x^2+d^2}|e|\right)de^{12}}{x} + \frac{455\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 de^{10}}{x^2} - \frac{875\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3 de^8}{x^3} - \frac{245\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4 de^6}{x^4} + \frac{49}{}}{4480\,e^6|e|}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x, algorithm="giac")
```

output
```
1/4480*(5*d*e^8 + 35*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d*e^6/x + 49*(d*e
 + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d*e^4/x^2 - 245*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))^3*d*e^2/x^3 - 875*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d/x^4
+ 455*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d/(e^2*x^5) + 9065*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^6*d/(e^4*x^6))*e^14*x^7/((d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))^7*abs(e)) - 3*d*e^8*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 15/16
*d*e^8*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/a
bs(e) + sqrt(-e^2*x^2 + d^2)*e^7 - 1/4480*(9065*(d*e + sqrt(-e^2*x^2 + d^2
)*abs(e))*d*e^12/x + 455*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d*e^10/x^2
- 875*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d*e^8/x^3 - 245*(d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))^4*d*e^6/x^4 + 49*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
^5*d*e^4/x^5 + 35*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6*d*e^2/x^6 + 5*(d*e
 + sqrt(-e^2*x^2 + d^2)*abs(e))^7*d/x^7)/(e^6*abs(e))
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^8} \, dx = \int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x^8} \, dx$$

input
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^8,x)
```

output
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^8, x)
```

3.79 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^9}\,dx$

3.79.1 Optimal result

Integrand size = 27, antiderivative size = 204

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^9}\,dx = -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4} - \frac{e^2(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{240x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{7x^7} - e^8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{125}{128}e^8\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
1/192*e^4*(64*e*x+125*d)*(-e^2*x^2+d^2)^(3/2)/x^4-1/240*e^2*(48*e*x+125*d)
*(-e^2*x^2+d^2)^(5/2)/x^6-1/8*d*(-e^2*x^2+d^2)^(7/2)/x^8-3/7*e*(-e^2*x^2+d
^2)^(7/2)/x^7-e^8*arctan(e*x/(-e^2*x^2+d^2)^(1/2))+125/128*e^8*arctanh((-e
^2*x^2+d^2)^(1/2)/d)-1/128*e^6*(128*e*x+125*d)*(-e^2*x^2+d^2)^(1/2)/x^2
```

3.79.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^9}\,dx = \frac{\sqrt{d^2-e^2x^2}(-1680d^7-5760d^6ex-1960d^5e^2x^2+14592d^4e^3x^3+17710d^3e}{13440x^8} + 2e^8\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) + \frac{125\sqrt{d^2}e^8\log(x)}{128d} - \frac{125\sqrt{d^2}e^8\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{128d}$$

input
```
Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^9,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-1680*d^7 - 5760*d^6*e*x - 1960*d^5*e^2*x^2 + 14592*
d^4*e^3*x^3 + 17710*d^3*e^4*x^4 - 7424*d^2*e^5*x^5 - 27195*d*e^6*x^6 - 148
48*e^7*x^7))/(13440*x^8) + 2*e^8*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*
x^2])] + (125*Sqrt[d^2]*e^8*Log[x])/(128*d) - (125*Sqrt[d^2]*e^8*Log[Sqrt[
d^2] - Sqrt[d^2 - e^2*x^2]])/(128*d)
```

3.79.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.13, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$, Rules used = {540, 25, 2338, 27, 537, 25, 537, 27, 537, 25, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^9} \, dx$$

$$\Big\downarrow \text{540}$$

$$-\frac{\int -\frac{\left(d^2-e^2x^2\right)^{5/2}\left(24ed^4+25e^2xd^3+8e^3x^2d^2\right)}{x^8}dx}{8d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(24ed^4+25e^2xd^3+8e^3x^2d^2\right)}{x^8}dx}{8d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int -\frac{7d^4e^2(25d+8ex)\left(d^2-e^2x^2\right)^{5/2}}{x^7}dx}{7d^2}-\frac{24d^2e\left(d^2-e^2x^2\right)^{7/2}}{7x^7}}{8d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2e^2\int \frac{(25d+8ex)\left(d^2-e^2x^2\right)^{5/2}}{x^7}dx-\frac{24d^2e\left(d^2-e^2x^2\right)^{7/2}}{7x^7}}{8d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8}$$

$$\Big\downarrow \text{537}$$

$$\frac{d^2e^2\left(\frac{1}{6}e^2\int -\frac{(125d+48ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}dx-\frac{(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{30x^6}\right)-\frac{24d^2e\left(d^2-e^2x^2\right)^{7/2}}{7x^7}}{8d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8}$$

$$\Big\downarrow \; 25$$

$$\frac{d^2e^2\left(-\frac{1}{6}e^2\int \frac{(125d+48ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}dx-\frac{(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{30x^6}\right)-\frac{24d^2e\left(d^2-e^2x^2\right)^{7/2}}{7x^7}}{8d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8}$$

$$\Big\downarrow \; 537$$

$$\frac{d^2e^2\left(-\frac{1}{6}e^2\left(\frac{1}{4}e^2\int -\frac{3(125d+64ex)\sqrt{d^2-e^2x^2}}{x^3}dx-\frac{(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{4x^4}\right)-\frac{(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{30x^6}\right)-\frac{24d^2e\left(d^2-e^2x^2\right)^{7}}{7x^7}}{8d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8}$$

$$\Big\downarrow \; 27$$

$$\frac{d^2e^2\left(-\frac{1}{6}e^2\left(-\frac{3}{4}e^2\int \frac{(125d+64ex)\sqrt{d^2-e^2x^2}}{x^3}dx-\frac{(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{4x^4}\right)-\frac{(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{30x^6}\right)-\frac{24d^2e\left(d^2-e^2x^2\right)^{7/}}{7x^7}}{8d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8}$$

$$\Big\downarrow \; 537$$

$$\frac{d^2e^2\left(-\frac{1}{6}e^2\left(-\frac{3}{4}e^2\left(\frac{1}{2}e^2\int -\frac{125d+128ex}{x\sqrt{d^2-e^2x^2}}dx-\frac{(125d+128ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{4x^4}\right)-\frac{(125d+48ex)\left(d^2-e^2x^2\right)}{30x^6}}{8d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8}$$

$$\Big\downarrow \; 25$$

$$\frac{d^2e^2\left(-\frac{1}{6}e^2\left(-\frac{3}{4}e^2\left(-\frac{1}{2}e^2\int \frac{125d+128ex}{x\sqrt{d^2-e^2x^2}}dx-\frac{(125d+128ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{4x^4}\right)-\frac{(125d+48ex)\left(d^2-e^2x^2\right)}{30x^6}}{8d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8}$$

$$\Big\downarrow \; 538$$

3.79. $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^9}\,dx$

$$\frac{d^2e^2\left(-\frac{1}{6}e^2\left(-\frac{3}{4}e^2\left(-\frac{1}{2}e^2\left(128e\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+125d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx\right)-\frac{(125d+128ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(125d+64ex)(d^2-e^2x}{4x^4}}{8d^2}\frac{d(d^2-e^2x^2)^{7/2}}{8x^8}$$

$$\downarrow 224$$

$$\frac{d^2e^2\left(-\frac{1}{6}e^2\left(-\frac{3}{4}e^2\left(-\frac{1}{2}e^2\left(125d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx+128e\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)-\frac{(125d+128ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(125d+6}{}}{8d^2}\frac{d(d^2-e^2x^2)^{7/2}}{8x^8}$$

$$\downarrow 216$$

$$\frac{d^2e^2\left(-\frac{1}{6}e^2\left(-\frac{3}{4}e^2\left(-\frac{1}{2}e^2\left(125d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx+128\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{(125d+128ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(125d+64ex)(d^2}{4x^4}}{8d^2}\frac{d(d^2-e^2x^2)^{7/2}}{8x^8}$$

$$\downarrow 243$$

$$\frac{d^2e^2\left(-\frac{1}{6}e^2\left(-\frac{3}{4}e^2\left(-\frac{1}{2}e^2\left(\frac{125}{2}d\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2+128\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{(125d+128ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(125d+64ex)(}{4x}}{8d^2}\frac{d(d^2-e^2x^2)^{7/2}}{8x^8}$$

$$\downarrow 73$$

$$\frac{d^2e^2\left(-\frac{1}{6}e^2\left(-\frac{3}{4}e^2\left(-\frac{1}{2}e^2\left(128\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{125d\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}\right)-\frac{(125d+128ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(125d+64}{}}{8d^2}\frac{d(d^2-e^2x^2)^{7/2}}{8x^8}$$

$$\downarrow 221$$

$$\frac{d^2e^2\left(-\frac{1}{6}e^2\left(-\frac{3}{4}e^2\left(-\frac{1}{2}e^2\left(128\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-125\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)-\frac{(125d+128ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(125d+64ex}{}}{8d^2}\frac{d(d^2-e^2x^2)^{7/2}}{8x^8}$$

3.79. $\int\frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^9}\,dx$

input
```
Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^9,x]
```

output
```
-1/8*(d*(d^2 - e^2*x^2)^(7/2))/x^8 + ((-24*d^2*e*(d^2 - e^2*x^2)^(7/2))/(7
*x^7) + d^2*e^2*(-1/30*((125*d + 48*e*x)*(d^2 - e^2*x^2)^(5/2))/x^6 - (e^2
*(-1/4*((125*d + 64*e*x)*(d^2 - e^2*x^2)^(3/2))/x^4 - (3*e^2*(-1/2*((125*d
 + 128*e*x)*Sqrt[d^2 - e^2*x^2])/x^2 - (e^2*(128*ArcTan[(e*x)/Sqrt[d^2 - e
^2*x^2]] - 125*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]))/2))/4))/6))/(8*d^2)
```

3.79.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.79.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(14848e^7x^7+27195d\,e^6x^6+7424d^2e^5x^5-17710d^3e^4x^4-14592d^4e^3x^3+1960d^5e^2x^2+5760d^6ex+1680d^7)}{13440x^8}-\frac{e^9\,\mathrm{arcta}}{}$
default	$e^3\left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{5d^2x^5}-\frac{2e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{3d^2x^3}-\frac{4e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{d^2x}-\frac{6e^2\left(\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2}}{2}\right.}{}\right.}{}\right.}{d^2}\right.}{3d^2}\right.}{5d^2}\right.$

3.79. $\int\frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^9}\,dx$

input
```
int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/13440*(-e^2*x^2+d^2)^(1/2)*(14848*e^7*x^7+27195*d*e^6*x^6+7424*d^2*e^5*
x^5-17710*d^3*e^4*x^4-14592*d^4*e^3*x^3+1960*d^5*e^2*x^2+5760*d^6*e*x+1680
*d^7)/x^8-e^9/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+125/1
28*e^8*d/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^9}\,dx = \frac{26880\,e^8x^8\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-13125\,e^8x^8\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-(14$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x, algorithm="fricas")
```

output
```
1/13440*(26880*e^8*x^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 13125*e
^8*x^8*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (14848*e^7*x^7 + 27195*d*e^6*x
^6 + 7424*d^2*e^5*x^5 - 17710*d^3*e^4*x^4 - 14592*d^4*e^3*x^3 + 1960*d^5*e
^2*x^2 + 5760*d^6*e*x + 1680*d^7)*sqrt(-e^2*x^2 + d^2))/x^8
```

3.79.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 29.73 (sec) , antiderivative size = 1719, normalized size of antiderivative = 8.43

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^9}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**9,x)
```

output
```
d**7*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)) + 7*e/(48*x**7
*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**2/(e**2*x**2) -
 1)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5*e**7/(128*d**
6*x*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*d**7), Abs(d*
*2/(e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I
*e/(48*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**5*sqrt(-d**
2/(e**2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x**2) + 1))
 + 5*I*e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(e*
x))/(128*d**7), True)) + 3*d**6*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)
/(7*x**6) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d
**2/(e**2*x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(
105*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(
7*x**6) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqr
t(-d**2/(e**2*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2)
 + 1)/(105*d**6), True)) + d**5*e**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/
(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d*
*2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2
) - 1)) + e**6*acosh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d*
*2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**
2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e...
```

3.79.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(178) = 356$.

Time = 0.28 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.78

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^9}\,dx = -\frac{e^9 \arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} + \frac{125}{128}\,e^8 \log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{\sqrt{-e^2x^2+d^2}e^9x}{d^2} - \frac{125\,\sqrt{-e^2x^2+d^2}e^8}{128\,d} - \frac{2\,(-e^2x^2+d^2)^{\frac{3}{2}}e^9x}{3\,d^4} - \frac{125\,(-e^2x^2+d^2)^{\frac{3}{2}}e^8}{384\,d^3} - \frac{25\,(-e^2x^2+d^2)^{\frac{5}{2}}e^8}{128\,d^5} - \frac{8\,(-e^2x^2+d^2)^{\frac{5}{2}}e^7}{15\,d^4x} - \frac{25\,(-e^2x^2+d^2)^{\frac{7}{2}}e^6}{128\,d^5x^2} + \frac{2\,(-e^2x^2+d^2)^{\frac{7}{2}}e^5}{15\,d^4x^3} + \frac{25\,(-e^2x^2+d^2)^{\frac{7}{2}}e^4}{192\,d^3x^4} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}e^3}{5\,d^2x^5} - \frac{25\,(-e^2x^2+d^2)^{\frac{7}{2}}e^2}{48\,dx^6} - \frac{3\,(-e^2x^2+d^2)^{\frac{7}{2}}e}{7\,x^7} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}d}{8\,x^8}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x, algorithm="maxima")
```

output
```
-e^9*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 125/128*e^8*log(2*d^2/abs(x)
+ 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) - sqrt(-e^2*x^2 + d^2)*e^9*x/d^2 - 125/
128*sqrt(-e^2*x^2 + d^2)*e^8/d - 2/3*(-e^2*x^2 + d^2)^(3/2)*e^9*x/d^4 - 12
5/384*(-e^2*x^2 + d^2)^(3/2)*e^8/d^3 - 25/128*(-e^2*x^2 + d^2)^(5/2)*e^8/d
^5 - 8/15*(-e^2*x^2 + d^2)^(5/2)*e^7/(d^4*x) - 25/128*(-e^2*x^2 + d^2)^(7/
2)*e^6/(d^5*x^2) + 2/15*(-e^2*x^2 + d^2)^(7/2)*e^5/(d^4*x^3) + 25/192*(-e^
2*x^2 + d^2)^(7/2)*e^4/(d^3*x^4) - 1/5*(-e^2*x^2 + d^2)^(7/2)*e^3/(d^2*x^5
) - 25/48*(-e^2*x^2 + d^2)^(7/2)*e^2/(d*x^6) - 3/7*(-e^2*x^2 + d^2)^(7/2)*
e/x^7 - 1/8*(-e^2*x^2 + d^2)^(7/2)*d/x^8
```

3.79.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(178) = 356$.

Time = 0.31 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.86

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^9}\,dx = \frac{\left(105\,e^9 + \frac{720\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^7}{x} + \frac{1120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^5}{x^2} - \frac{3696\left(de+\sqrt{-e^2x^2\cdots}\right.}{x^3}\cdots}{\cdots}$$

$$- \frac{e^9 \arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} + \frac{125\,e^9 \log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{128\,|e|}$$

$$- \frac{\frac{122640\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^{13}|e|}{x} + \frac{77280\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^{11}|e|}{x^2} - \frac{560\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3e^9|e|}{x^3} - \frac{14280\left(de+\sqrt{-e^2x^2+d^2}|e|\right)\cdots}{x^4}\cdots}{215\cdots}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x, algorithm="giac")
```

output
```
1/215040*(105*e^9 + 720*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^7/x + 1120*(
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*e^5/x^2 - 3696*(d*e + sqrt(-e^2*x^2 +
 d^2)*abs(e))^3*e^3/x^3 - 14280*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*e/x^
4 - 560*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e*x^5) + 77280*(d*e + sqrt(
-e^2*x^2 + d^2)*abs(e))^6/(e^3*x^6) + 122640*(d*e + sqrt(-e^2*x^2 + d^2)*a
bs(e))^7/(e^5*x^7))*e^16*x^8/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8*abs(e)
) - e^9*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 125/128*e^9*log(1/2*abs(-2*d*
e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) - 1/215040*(122640
*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^13*abs(e)/x + 77280*(d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^2*e^11*abs(e)/x^2 - 560*(d*e + sqrt(-e^2*x^2 + d^2)*a
bs(e))^3*e^9*abs(e)/x^3 - 14280*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*e^7*
abs(e)/x^4 - 3696*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*e^5*abs(e)/x^5 + 1
120*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6*e^3*abs(e)/x^6 + 720*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^7*e*abs(e)/x^7 + 105*(d*e + sqrt(-e^2*x^2 + d^2)*
abs(e))^8*abs(e)/(e*x^8))/e^8
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^9} \, dx = \int \frac{(d^2-e^2\,x^2)^{5/2}\,(d+e\,x)^3}{x^9} \, dx$$

input
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^9,x)
```

output
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^9, x)
```

3.80 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^{10}}\,dx$

3.80.1 Optimal result

Integrand size = 27, antiderivative size = 187

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{10}}\,dx = -\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2} + \frac{55e^5(d^2-e^2x^2)^{3/2}}{192x^4} - \frac{11e^3(d^2-e^2x^2)^{5/2}}{48x^6} - \frac{d(d^2-e^2x^2)^{7/2}}{9x^9} - \frac{3e(d^2-e^2x^2)^{7/2}}{8x^8} - \frac{29e^2(d^2-e^2x^2)^{7/2}}{63dx^7} + \frac{55e^9\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d}$$

output
```
55/192*e^5*(-e^2*x^2+d^2)^(3/2)/x^4-11/48*e^3*(-e^2*x^2+d^2)^(5/2)/x^6-1/9
*d*(-e^2*x^2+d^2)^(7/2)/x^9-3/8*e*(-e^2*x^2+d^2)^(7/2)/x^8-29/63*e^2*(-e^2
*x^2+d^2)^(7/2)/d/x^7+55/128*e^9*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d-55/128*
e^7*(-e^2*x^2+d^2)^(1/2)/x^2
```

3.80.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{10}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-896d^8-3024d^7ex-1024d^6e^2x^2+7224d^5e^3x^3+8448d^4e^4x^4}{8064dx^9} + \frac{55e^9\log(x)}{128\sqrt{d^2}} - \frac{55e^9\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{128\sqrt{d^2}}$$

input
```
Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^10,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-896*d^8 - 3024*d^7*e*x - 1024*d^6*e^2*x^2 + 7224*d^
5*e^3*x^3 + 8448*d^4*e^4*x^4 - 3066*d^3*e^5*x^5 - 10240*d^2*e^6*x^6 - 4599
*d*e^7*x^7 + 3712*e^8*x^8))/(8064*d*x^9) + (55*e^9*Log[x])/(128*Sqrt[d^2])
 - (55*e^9*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(128*Sqrt[d^2])
```

3.80.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {540, 25, 2338, 25, 27, 534, 243, 51, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^{10}} \, dx$$

$$\Big\downarrow \text{540}$$

$$-\frac{\int -\frac{\left(d^2-e^2x^2\right)^{5/2}\left(27ed^4+29e^2xd^3+9e^3x^2d^2\right)}{x^9}dx}{9d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(27ed^4+29e^2xd^3+9e^3x^2d^2\right)}{x^9}dx}{9d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int -\frac{d^4e^2(232d+99ex)\left(d^2-e^2x^2\right)^{5/2}}{x^8}dx}{8d^2}-\frac{27d^2e\left(d^2-e^2x^2\right)^{7/2}}{8x^8}}{9d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{d^4e^2(232d+99ex)\left(d^2-e^2x^2\right)^{5/2}}{x^8}dx}{8d^2}-\frac{27d^2e\left(d^2-e^2x^2\right)^{7/2}}{8x^8}}{9d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{8}d^2e^2\int \frac{(232d+99ex)\left(d^2-e^2x^2\right)^{5/2}}{x^8}dx-\frac{27d^2e\left(d^2-e^2x^2\right)^{7/2}}{8x^8}}{9d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}$$

$$\Big\downarrow \text{534}$$

$$\frac{\frac{1}{8}d^2e^2\left(99e\int \frac{(d^2-e^2x^2)^{5/2}}{x^7}dx-\frac{232(d^2-e^2x^2)^{7/2}}{7dx^7}\right)-\frac{27d^2e(d^2-e^2x^2)^{7/2}}{8x^8}}{9d^2}-\frac{d(d^2-e^2x^2)^{7/2}}{9x^9}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{8}d^2e^2\left(\frac{99}{2}e\int \frac{(d^2-e^2x^2)^{5/2}}{x^8}dx^2-\frac{232(d^2-e^2x^2)^{7/2}}{7dx^7}\right)-\frac{27d^2e(d^2-e^2x^2)^{7/2}}{8x^8}}{9d^2}-\frac{d(d^2-e^2x^2)^{7/2}}{9x^9}$$

$$\Big\downarrow \text{51}$$

$$\frac{\frac{1}{8}d^2e^2\left(\frac{99}{2}e\left(-\frac{5}{6}e^2\int \frac{(d^2-e^2x^2)^{3/2}}{x^6}dx^2-\frac{(d^2-e^2x^2)^{5/2}}{3x^6}\right)-\frac{232(d^2-e^2x^2)^{7/2}}{7dx^7}\right)-\frac{27d^2e(d^2-e^2x^2)^{7/2}}{8x^8}}{9d^2}-\frac{d(d^2-e^2x^2)^{7/2}}{9x^9}$$

$$\Big\downarrow \text{51}$$

$$\frac{\frac{1}{8}d^2e^2\left(\frac{99}{2}e\left(-\frac{5}{6}e^2\left(-\frac{3}{4}e^2\int \frac{\sqrt{d^2-e^2x^2}}{x^4}dx^2-\frac{(d^2-e^2x^2)^{3/2}}{2x^4}\right)-\frac{(d^2-e^2x^2)^{5/2}}{3x^6}\right)-\frac{232(d^2-e^2x^2)^{7/2}}{7dx^7}\right)-\frac{27d^2e(d^2-e^2x^2)^{7/2}}{8x^8}}{9d^2}-\frac{d(d^2-e^2x^2)^{7/2}}{9x^9}$$

$$\Big\downarrow \text{51}$$

$$\frac{\frac{1}{8}d^2e^2\left(\frac{99}{2}e\left(-\frac{5}{6}e^2\left(-\frac{3}{4}e^2\left(-\frac{1}{2}e^2\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{(d^2-e^2x^2)^{3/2}}{2x^4}\right)-\frac{(d^2-e^2x^2)^{5/2}}{3x^6}\right)-\frac{232(d^2-e^2x^2)^{7/2}}{7dx^7}\right)}{9d^2}$$

$$\frac{d(d^2-e^2x^2)^{7/2}}{9x^9}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{1}{8}d^2e^2\left(\frac{99}{2}e\left(-\frac{5}{6}e^2\left(-\frac{3}{4}e^2\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{(d^2-e^2x^2)^{3/2}}{2x^4}\right)-\frac{(d^2-e^2x^2)^{5/2}}{3x^6}\right)-\frac{232(d^2-e^2x^2)^{7/2}}{7dx^7}\right)}{9d^2}$$

$$\frac{d(d^2-e^2x^2)^{7/2}}{9x^9}$$

$$\Big\downarrow \text{221}$$

3.80. $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^{10}}\,dx$

$$\frac{\frac{1}{8}d^2e^2\left(\frac{99}{2}e\left(-\frac{5}{6}e^2\left(-\frac{3}{4}e^2\left(\frac{e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{(d^2-e^2x^2)^{3/2}}{2x^4}\right)-\frac{(d^2-e^2x^2)^{5/2}}{3x^6}\right)-\frac{232(d^2-e^2x^2)^{7/2}}{7dx^7}}{9d^2}$$

$$\frac{d(d^2-e^2x^2)^{7/2}}{9x^9}$$

input
```
Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^10,x]
```

output
```
-1/9*(d*(d^2 - e^2*x^2)^(7/2))/x^9 + ((-27*d^2*e*(d^2 - e^2*x^2)^(7/2))/(8
*x^8) + (d^2*e^2*((-232*(d^2 - e^2*x^2)^(7/2))/(7*d*x^7) + (99*e*(-1/3*(d^
2 - e^2*x^2)^(5/2)/x^6 - (5*e^2*(-1/2*(d^2 - e^2*x^2)^(3/2)/x^4 - (3*e^2*(
-(Sqrt[d^2 - e^2*x^2]/x^2) + (e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/4))/
6))/2))/8)/(9*d^2)
```

3.80.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.80.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,\left(-3712e^{8}x^{8}+4599d\,e^{7}x^{7}+10240d^{2}e^{6}x^{6}+3066d^{3}e^{5}x^{5}-8448d^{4}x^{4}e^{4}-7224d^{5}e^{3}x^{3}+1024d^{6}e^{2}x^{2}+3024d^{7}ex+896d^{8}\right)}{8064x^{9}d}$
default	$e^{3}\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{6d^{2}x^{6}}-\frac{e^{2}\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{4d^{2}x^{4}}-\frac{3e^{2}\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{2d^{2}x^{2}}-\frac{5e^{2}\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{5}+d^{2}\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{3}+d^{2}\left(\sqrt{-e^{2}x^{2}+d^{2}}-\right.\right.\right.}{2d^{2}}\right.}{4d^{2}}\right.}{6d^{2}}\right.$

3.80. $\int \frac{(d+ex)^{3}\left(d^{2}-e^{2}x^{2}\right)^{5/2}}{x^{10}}\,dx$

input
```
int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/8064*(-e^2*x^2+d^2)^(1/2)*(-3712*e^8*x^8+4599*d*e^7*x^7+10240*d^2*e^6*x
^6+3066*d^3*e^5*x^5-8448*d^4*e^4*x^4-7224*d^5*e^3*x^3+1024*d^6*e^2*x^2+302
4*d^7*e*x+896*d^8)/x^9/d+55/128*e^9/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-
e^2*x^2+d^2)^(1/2))/x)
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^{10}}\,dx = -\frac{3465\,e^9x^9\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - (3712\,e^8x^8 - 4599\,de^7x^7 - 10240\,d^2e^6x^6 - 3066\,d^3e^5x^5 + 8448\,d^4e^4x^4 + 7}{8064\,dx^9}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x, algorithm="fricas")
```

output
```
-1/8064*(3465*e^9*x^9*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (3712*e^8*x^8 -
 4599*d*e^7*x^7 - 10240*d^2*e^6*x^6 - 3066*d^3*e^5*x^5 + 8448*d^4*e^4*x^4
+ 7224*d^5*e^3*x^3 - 1024*d^6*e^2*x^2 - 3024*d^7*e*x - 896*d^8)*sqrt(-e^2*
x^2 + d^2))/(d*x^9)
```

3.80.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 31.65 (sec) , antiderivative size = 1889, normalized size of antiderivative = 10.10

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^{10}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**10,x)
```

output
```
d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(9*x**8) + e**3*sqrt(d**2/(e
**2*x**2) - 1)/(63*d**2*x**6) + 2*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**
4*x**4) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(315*d**6*x**2) + 16*e**9*sqrt
(d**2/(e**2*x**2) - 1)/(315*d**8), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(
-d**2/(e**2*x**2) + 1)/(9*x**8) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(63*d
**2*x**6) + 2*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**4) + 8*I*e**
7*sqrt(-d**2/(e**2*x**2) + 1)/(315*d**6*x**2) + 16*I*e**9*sqrt(-d**2/(e**2
*x**2) + 1)/(315*d**8), True)) + 3*d**6*e*Piecewise((-d**2/(8*e*x**9*sqrt(
d**2/(e**2*x**2) - 1)) + 7*e/(48*x**7*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(
192*d**2*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**5/(384*d**4*x**3*sqrt(d**
2/(e**2*x**2) - 1)) - 5*e**7/(128*d**6*x*sqrt(d**2/(e**2*x**2) - 1)) + 5*e
**8*acosh(d/(e*x))/(128*d**7), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(8*e*x*
*9*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e/(48*x**7*sqrt(-d**2/(e**2*x**2) +
1)) - I*e**3/(192*d**2*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**5/(384*d
**4*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 5*I*e**7/(128*d**6*x*sqrt(-d**2/(e
**2*x**2) + 1)) - 5*I*e**8*asin(d/(e*x))/(128*d**7), True)) + d**5*e**2*Pi
ecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e**2*x**
2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**2)
 + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2)) > 1
), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2...
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.32

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{10}}\,dx = \frac{55\,e^9 \log\left(\frac{2\,d^2}{|x|} + \frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{128\,d} - \frac{55\sqrt{-e^2x^2+d^2}e^9}{128\,d^2} - \frac{55\,(-e^2x^2+d^2)^{\frac{3}{2}}e^9}{384\,d^4} - \frac{11\,(-e^2x^2+d^2)^{\frac{5}{2}}e^9}{128\,d^6} - \frac{11\,(-e^2x^2+d^2)^{\frac{7}{2}}e^7}{128\,d^6x^2} + \frac{11\,(-e^2x^2+d^2)^{\frac{7}{2}}e^5}{192\,d^4x^4} - \frac{11\,(-e^2x^2+d^2)^{\frac{7}{2}}e^3}{48\,d^2x^6} - \frac{29\,(-e^2x^2+d^2)^{\frac{7}{2}}e^2}{63\,dx^7} - \frac{3\,(-e^2x^2+d^2)^{\frac{7}{2}}e}{8\,x^8} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}d}{9\,x^9}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x, algorithm="maxima")
```

output
```
55/128*e^9*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 55/128*
sqrt(-e^2*x^2 + d^2)*e^9/d^2 - 55/384*(-e^2*x^2 + d^2)^(3/2)*e^9/d^4 - 11/
128*(-e^2*x^2 + d^2)^(5/2)*e^9/d^6 - 11/128*(-e^2*x^2 + d^2)^(7/2)*e^7/(d^
6*x^2) + 11/192*(-e^2*x^2 + d^2)^(7/2)*e^5/(d^4*x^4) - 11/48*(-e^2*x^2 + d
^2)^(7/2)*e^3/(d^2*x^6) - 29/63*(-e^2*x^2 + d^2)^(7/2)*e^2/(d*x^7) - 3/8*(
-e^2*x^2 + d^2)^(7/2)*e/x^8 - 1/9*(-e^2*x^2 + d^2)^(7/2)*d/x^9
```

3.80.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(159) = 318$.

Time = 0.32 (sec) , antiderivative size = 648, normalized size of antiderivative = 3.47

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{10}}\,dx = \frac{\left(28\,e^{10} + \frac{189\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^8}{x} + \frac{324\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 e^6}{x^2} - \frac{672\left(de+\sqrt{-e^2x^2+d\right.}}{x^3}\right.}{}$$

$$+ \frac{55\,e^{10}\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{128\,d|e|}$$

$$+ \frac{\frac{16632\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^8e^{16}}{x} - \frac{18144\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d^8e^{14}}{x^2} - \frac{9744\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3 d^8e^{12}}{x^3} + \frac{1512\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{x^4}}{}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x, algorithm="giac")
```

output
```
1/129024*(28*e^10 + 189*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^8/x + 324*(d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*e^6/x^2 - 672*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))^3*e^4/x^3 - 3024*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*e^2/x^4
 - 1512*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/x^5 + 9744*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^6/(e^2*x^6) + 18144*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
^7/(e^4*x^7) - 16632*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8/(e^6*x^8))*e^18
*x^9/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^9*d*abs(e)) + 55/128*e^10*log(1/
2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d*abs(e)) + 1
/129024*(16632*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^8*e^16/x - 18144*(d*e
 + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^8*e^14/x^2 - 9744*(d*e + sqrt(-e^2*x^2
 + d^2)*abs(e))^3*d^8*e^12/x^3 + 1512*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^
4*d^8*e^10/x^4 + 3024*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d^8*e^8/x^5 +
672*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6*d^8*e^6/x^6 - 324*(d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))^7*d^8*e^4/x^7 - 189*(d*e + sqrt(-e^2*x^2 + d^2)*abs(
e))^8*d^8*e^2/x^8 - 28*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^9*d^8/x^9)/(d^9
*e^8*abs(e))
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{10}}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}\,(d+e\,x)^3}{x^{10}}\,dx$$

input
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^10,x)
```

output
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^10, x)
```

3.81 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^{11}}\,dx$

3.81.1 Optimal result

Integrand size = 27, antiderivative size = 225

$$\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^{11}}\,dx = -\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2}+\frac{11e^6(d^2-e^2x^2)^{3/2}}{128dx^4} - \frac{11e^4(d^2-e^2x^2)^{5/2}}{160dx^6}-\frac{d(d^2-e^2x^2)^{7/2}}{10x^{10}}-\frac{e(d^2-e^2x^2)^{7/2}}{3x^9} - \frac{33e^2(d^2-e^2x^2)^{7/2}}{80dx^8}-\frac{5e^3(d^2-e^2x^2)^{7/2}}{21d^2x^7}+\frac{33e^{10}\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{256d^2}$$

output
```
11/128*e^6*(-e^2*x^2+d^2)^(3/2)/d/x^4-11/160*e^4*(-e^2*x^2+d^2)^(5/2)/d/x^
6-1/10*d*(-e^2*x^2+d^2)^(7/2)/x^10-1/3*e*(-e^2*x^2+d^2)^(7/2)/x^9-33/80*e^
2*(-e^2*x^2+d^2)^(7/2)/d/x^8-5/21*e^3*(-e^2*x^2+d^2)^(7/2)/d^2/x^7+33/256*
e^10*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2-33/256*e^8*(-e^2*x^2+d^2)^(1/2)/d
/x^2
```

3.81.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^{11}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-2688d^9-8960d^8ex-3024d^7e^2x^2+20480d^6e^3x^3+23352d^5e}{26880} + \frac{33\sqrt{d^2}e^{10}\log(x)}{256d^3}-\frac{33\sqrt{d^2}e^{10}\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{256d^3}$$

input
```
Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^11,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-2688*d^9 - 8960*d^8*e*x - 3024*d^7*e^2*x^2 + 20480*
d^6*e^3*x^3 + 23352*d^5*e^4*x^4 - 7680*d^4*e^5*x^5 - 24570*d^3*e^6*x^6 - 1
0240*d^2*e^7*x^7 + 3465*d*e^8*x^8 + 6400*e^9*x^9))/(26880*d^2*x^10) + (33*
Sqrt[d^2]*e^10*Log[x])/(256*d^3) - (33*Sqrt[d^2]*e^10*Log[Sqrt[d^2] - Sqrt
[d^2 - e^2*x^2]])/(256*d^3)
```

3.81.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.11, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {540, 25, 2338, 27, 539, 25, 27, 534, 243, 51, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^{11}} \, dx$$

$$\Big\downarrow \; 540$$

$$-\frac{\int -\frac{\left(d^2-e^2x^2\right)^{5/2}\left(30ed^4+33e^2xd^3+10e^3x^2d^2\right)}{x^{10}}dx}{10d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(30ed^4+33e^2xd^3+10e^3x^2d^2\right)}{x^{10}}dx}{10d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}$$

$$\Big\downarrow \; 2338$$

$$\frac{-\frac{\int -\frac{3d^4e^2(99d+50ex)\left(d^2-e^2x^2\right)^{5/2}}{x^9}dx}{9d^2}-\frac{10d^2e\left(d^2-e^2x^2\right)^{7/2}}{3x^9}}{10d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{3}d^2e^2\int \frac{(99d+50ex)\left(d^2-e^2x^2\right)^{5/2}}{x^9}dx-\frac{10d^2e\left(d^2-e^2x^2\right)^{7/2}}{3x^9}}{10d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}$$

$$\Big\downarrow \; 539$$

$$\frac{\frac{1}{3}d^2e^2\left(-\frac{\int -\frac{de(400d+99ex)\left(d^2-e^2x^2\right)^{5/2}}{x^8}dx}{8d^2}-\frac{99\left(d^2-e^2x^2\right)^{7/2}}{8dx^8}\right)-\frac{10d^2e\left(d^2-e^2x^2\right)^{7/2}}{3x^9}}{10d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{1}{3}d^2e^2\left(\frac{\int \frac{de(400d+99ex)\left(d^2-e^2x^2\right)^{5/2}}{x^8}dx}{8d^2}-\frac{99\left(d^2-e^2x^2\right)^{7/2}}{8dx^8}\right)-\frac{10d^2e\left(d^2-e^2x^2\right)^{7/2}}{3x^9}}{10d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{3}d^2e^2\left(\frac{e\int \frac{(400d+99ex)\left(d^2-e^2x^2\right)^{5/2}}{x^8}dx}{8d}-\frac{99\left(d^2-e^2x^2\right)^{7/2}}{8dx^8}\right)-\frac{10d^2e\left(d^2-e^2x^2\right)^{7/2}}{3x^9}}{10d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}$$

$$\Big\downarrow \ 534$$

$$\frac{\frac{1}{3}d^2e^2\left(\frac{e\left(99e\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^7}dx-\frac{400\left(d^2-e^2x^2\right)^{7/2}}{7dx^7}\right)}{8d}-\frac{99\left(d^2-e^2x^2\right)^{7/2}}{8dx^8}\right)-\frac{10d^2e\left(d^2-e^2x^2\right)^{7/2}}{3x^9}}{10d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}$$

$$\Big\downarrow \ 243$$

$$\frac{\frac{1}{3}d^2e^2\left(\frac{e\left(\frac{99}{2}e\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^8}dx^2-\frac{400\left(d^2-e^2x^2\right)^{7/2}}{7dx^7}\right)}{8d}-\frac{99\left(d^2-e^2x^2\right)^{7/2}}{8dx^8}\right)-\frac{10d^2e\left(d^2-e^2x^2\right)^{7/2}}{3x^9}}{10d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}$$

$$\Big\downarrow \ 51$$

$$\frac{\frac{1}{3}d^2e^2\left(\frac{e\left(\frac{99}{2}e\left(-\frac{5}{6}e^2\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^6}dx^2-\frac{\left(d^2-e^2x^2\right)^{5/2}}{3x^6}\right)-\frac{400\left(d^2-e^2x^2\right)^{7/2}}{7dx^7}\right)}{8d}-\frac{99\left(d^2-e^2x^2\right)^{7/2}}{8dx^8}\right)-\frac{10d^2e\left(d^2-e^2x^2\right)^{7/2}}{3x^9}}{10d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}$$

3.81. $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{11}}\,dx$

$$\big\downarrow \; 51$$

$$\frac{\frac{1}{3}d^2e^2\left(\frac{e\left(\frac{99}{2}e\left(-\frac{5}{6}e^2\left(-\frac{3}{4}e^2\int\frac{\sqrt{d^2-e^2x^2}}{x^4}dx^2-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{\left(d^2-e^2x^2\right)^{5/2}}{3x^6}\right)-\frac{400\left(d^2-e^2x^2\right)^{7/2}}{7dx^7}\right)}{8d}-\frac{99\left(d^2-e^2x^2\right)^{7/2}}{8dx^8}\right)-\frac{10d^2e(d^2-}{3x}}{10d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}$$

$$\big\downarrow \; 51$$

$$\frac{\frac{1}{3}d^2e^2\left(\frac{e\left(\frac{99}{2}e\left(-\frac{5}{6}e^2\left(-\frac{3}{4}e^2\left(-\frac{1}{2}e^2\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{\left(d^2-e^2x^2\right)^{5/2}}{3x^6}\right)-\frac{400\left(d^2-e^2x^2\right)^{7/2}}{7dx^7}\right)}{8d}-\frac{99(d^2-e^2}{8dx}}{10d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}$$

$$\big\downarrow \; 73$$

$$\frac{\frac{1}{3}d^2e^2\left(\frac{e\left(\frac{99}{2}e\left(-\frac{5}{6}e^2\left(-\frac{3}{4}e^2\left(\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{\left(d^2-e^2x^2\right)^{5/2}}{3x^6}\right)-\frac{400\left(d^2-e^2x^2\right)^{7/2}}{7dx^7}\right)}{8d}-\frac{99(d^2-e^2x^2}{8dx^8}}{10d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}$$

$$\big\downarrow \; 221$$

$$\frac{\frac{1}{3}d^2e^2\left(\frac{e\left(\frac{99}{2}e\left(-\frac{5}{6}e^2\left(-\frac{3}{4}e^2\left(\frac{e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{\left(d^2-e^2x^2\right)^{5/2}}{3x^6}\right)-\frac{400\left(d^2-e^2x^2\right)^{7/2}}{7dx^7}\right)}{8d}-\frac{99(d^2-}{8}}{10d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}$$

input
```
Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^11,x]
```

3.81. $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{11}}\,dx$

output
```
-1/10*(d*(d^2 - e^2*x^2)^(7/2))/x^10 + ((-10*d^2*e*(d^2 - e^2*x^2)^(7/2))/
(3*x^9) + (d^2*e^2*((-99*(d^2 - e^2*x^2)^(7/2))/(8*d*x^8) + (e*((-400*(d^2
 - e^2*x^2)^(7/2))/(7*d*x^7) + (99*e*(-1/3*(d^2 - e^2*x^2)^(5/2)/x^6 - (5*
e^2*(-1/2*(d^2 - e^2*x^2)^(3/2)/x^4 - (3*e^2*(-(Sqrt[d^2 - e^2*x^2]/x^2) +
 (e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/4))/6))/2))/(8*d)))/3)/(10*d^2)
```

3.81.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.81.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-6400e^9x^9-3465d\,e^8x^8+10240d^2e^7x^7+24570d^3e^6x^6+7680d^4e^5x^5-23352d^5e^4x^4-20480d^6e^3x^3+3024x^2d^7e^2+89}{26880d^2x^{10}}$
default	$-\frac{e^3(-e^2x^2+d^2)^{\frac{7}{2}}}{7d^2x^7}+d^3\left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{10d^2x^{10}}+\frac{3e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{8d^2x^8}+\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{6d^2x^6}-\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{4d^2x^4}-\frac{3e^2\left(-\frac{(-e^2}{2}\right.}{}\right.}{}\right.}{}\right.}{}\right.$

3.81. $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^{11}}\,dx$

input
```
int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/26880*(-e^2*x^2+d^2)^(1/2)*(-6400*e^9*x^9-3465*d*e^8*x^8+10240*d^2*e^7*
x^7+24570*d^3*e^6*x^6+7680*d^4*e^5*x^5-23352*d^5*e^4*x^4-20480*d^6*e^3*x^3
+3024*d^7*e^2*x^2+8960*d^8*e*x+2688*d^9)/d^2/x^10+33/256/d*e^10/(d^2)^(1/2
)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{11}} \, dx =$$
$$-\frac{3465 \, e^{10} x^{10} \log \left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - (6400 \, e^9 x^9 + 3465 \, d e^8 x^8 - 10240 \, d^2 e^7 x^7 - 24570 \, d^3 e^6 x^6 - 7680 \, d^4 e^5 x^5}{26880 \, d^2 x^{10}}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x, algorithm="fricas")
```

output
```
-1/26880*(3465*e^10*x^10*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (6400*e^9*x^
9 + 3465*d*e^8*x^8 - 10240*d^2*e^7*x^7 - 24570*d^3*e^6*x^6 - 7680*d^4*e^5*
x^5 + 23352*d^5*e^4*x^4 + 20480*d^6*e^3*x^3 - 3024*d^7*e^2*x^2 - 8960*d^8*
e*x - 2688*d^9)*sqrt(-e^2*x^2 + d^2))/(d^2*x^10)
```

3.81.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 136.08 (sec) , antiderivative size = 2159, normalized size of antiderivative = 9.60

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{11}} \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**11,x)
```

output
```
d**7*Piecewise((-d**2/(10*e*x**11*sqrt(d**2/(e**2*x**2) - 1)) + 9*e/(80*x*
*9*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(480*d**2*x**7*sqrt(d**2/(e**2*x**2)
 - 1)) + 7*e**5/(1920*d**4*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 7*e**7/(768*
d**6*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 7*e**9/(256*d**8*x*sqrt(d**2/(e**2
*x**2) - 1)) + 7*e**10*acosh(d/(e*x))/(256*d**9), Abs(d**2/(e**2*x**2)) >
1), (I*d**2/(10*e*x**11*sqrt(-d**2/(e**2*x**2) + 1)) - 9*I*e/(80*x**9*sqrt
(-d**2/(e**2*x**2) + 1)) - I*e**3/(480*d**2*x**7*sqrt(-d**2/(e**2*x**2) +
1)) - 7*I*e**5/(1920*d**4*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e**7/(76
8*d**6*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 7*I*e**9/(256*d**8*x*sqrt(-d**2
/(e**2*x**2) + 1)) - 7*I*e**10*asin(d/(e*x))/(256*d**9), True)) + 3*d**6*e
*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(9*x**8) + e**3*sqrt(d**2/(e**2*
x**2) - 1)/(63*d**2*x**6) + 2*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x*
*4) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(315*d**6*x**2) + 16*e**9*sqrt(d**
2/(e**2*x**2) - 1)/(315*d**8), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**
2/(e**2*x**2) + 1)/(9*x**8) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(63*d**2*
x**6) + 2*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**4) + 8*I*e**7*sq
rt(-d**2/(e**2*x**2) + 1)/(315*d**6*x**2) + 16*I*e**9*sqrt(-d**2/(e**2*x**
2) + 1)/(315*d**8), True)) + d**5*e**2*Piecewise((-d**2/(8*e*x**9*sqrt(d**
2/(e**2*x**2) - 1)) + 7*e/(48*x**7*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192
*d**2*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**5/(384*d**4*x**3*sqrt(d**...
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{11}}\,dx = \frac{33\,e^{10}\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{256\,d^2} - \frac{33\,\sqrt{-e^2x^2+d^2}e^{10}}{256\,d^3} - \frac{11\,(-e^2x^2+d^2)^{\frac{3}{2}}e^{10}}{256\,d^5} - \frac{33\,(-e^2x^2+d^2)^{\frac{5}{2}}e^{10}}{1280\,d^7} - \frac{33\,(-e^2x^2+d^2)^{\frac{7}{2}}e^8}{1280\,d^7x^2} + \frac{11\,(-e^2x^2+d^2)^{\frac{7}{2}}e^6}{640\,d^5x^4} - \frac{11\,(-e^2x^2+d^2)^{\frac{7}{2}}e^4}{160\,d^3x^6} - \frac{5\,(-e^2x^2+d^2)^{\frac{7}{2}}e^3}{21\,d^2x^7} - \frac{33\,(-e^2x^2+d^2)^{\frac{7}{2}}e^2}{80\,dx^8} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}e}{3\,x^9} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}d}{10\,x^{10}}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x, algorithm="maxima")
```

output
```
33/256*e^10*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^2 - 33/2
56*sqrt(-e^2*x^2 + d^2)*e^10/d^3 - 11/256*(-e^2*x^2 + d^2)^(3/2)*e^10/d^5
- 33/1280*(-e^2*x^2 + d^2)^(5/2)*e^10/d^7 - 33/1280*(-e^2*x^2 + d^2)^(7/2)
*e^8/(d^7*x^2) + 11/640*(-e^2*x^2 + d^2)^(7/2)*e^6/(d^5*x^4) - 11/160*(-e^
2*x^2 + d^2)^(7/2)*e^4/(d^3*x^6) - 5/21*(-e^2*x^2 + d^2)^(7/2)*e^3/(d^2*x^
7) - 33/80*(-e^2*x^2 + d^2)^(7/2)*e^2/(d*x^8) - 1/3*(-e^2*x^2 + d^2)^(7/2)
*e/x^9 - 1/10*(-e^2*x^2 + d^2)^(7/2)*d/x^10
```

3.81.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(193) = 386$.

Time = 0.31 (sec) , antiderivative size = 731, normalized size of antiderivative = 3.25

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{11}}\,dx = \frac{\left(42\,e^{11} + \frac{280\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^9}{x} + \frac{525\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 e^7}{x^2} - \frac{600\left(de+\sqrt{-e^2x^2+d\right.}}{x^3}}{}$$
$$+ \frac{33\,e^{11}\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{256\,d^2|e|}$$
$$+ \frac{\frac{31920\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{18}e^{17}|e|}{x} - \frac{10500\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d^{18}e^{15}|e|}{x^2} - \frac{16800\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3 d^{18}e^{13}|e|}{x^3} - \frac{5880\left(de+\sqrt{-e^2\right.}}{}}{}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x, algorithm="giac")
```

output
```
1/430080*(42*e^11 + 280*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^9/x + 525*(d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*e^7/x^2 - 600*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))^3*e^5/x^3 - 3570*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*e^3/x^4
 - 3360*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*e/x^5 + 5880*(d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^6/(e*x^6) + 16800*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
^7/(e^3*x^7) + 10500*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8/(e^5*x^8) - 319
20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^9/(e^7*x^9))*e^20*x^10/((d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^10*d^2*abs(e)) + 33/256*e^11*log(1/2*abs(-2*d*e -
 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^2*abs(e)) + 1/430080*(319
20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^18*e^17*abs(e)/x - 10500*(d*e + s
qrt(-e^2*x^2 + d^2)*abs(e))^2*d^18*e^15*abs(e)/x^2 - 16800*(d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^3*d^18*e^13*abs(e)/x^3 - 5880*(d*e + sqrt(-e^2*x^2 +
d^2)*abs(e))^4*d^18*e^11*abs(e)/x^4 + 3360*(d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))^5*d^18*e^9*abs(e)/x^5 + 3570*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6*d^
18*e^7*abs(e)/x^6 + 600*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7*d^18*e^5*abs
(e)/x^7 - 525*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8*d^18*e^3*abs(e)/x^8 -
280*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^9*d^18*e*abs(e)/x^9 - 42*(d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))^10*d^18*abs(e)/(e*x^10))/(d^20*e^10)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{11}}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}\,(d+e\,x)^3}{x^{11}}\,dx$$

input
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^11,x)
```

output
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^11, x)
```

3.82 $\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{12}} \, dx$

3.82.1 Optimal result

Integrand size = 27, antiderivative size = 254

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{12}} \, dx = -\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2} + \frac{19e^7(d^2-e^2x^2)^{3/2}}{384d^2x^4} - \frac{19e^5(d^2-e^2x^2)^{5/2}}{480d^2x^6} - \frac{d(d^2-e^2x^2)^{7/2}}{11x^{11}} - \frac{3e(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{37e^2(d^2-e^2x^2)^{7/2}}{99dx^9} - \frac{19e^3(d^2-e^2x^2)^{7/2}}{80d^2x^8} - \frac{74e^4(d^2-e^2x^2)^{7/2}}{693d^3x^7} + \frac{19e^{11}\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{256d^3}$$

output
```
19/384*e^7*(-e^2*x^2+d^2)^(3/2)/d^2/x^4-19/480*e^5*(-e^2*x^2+d^2)^(5/2)/d^
2/x^6-1/11*d*(-e^2*x^2+d^2)^(7/2)/x^11-3/10*e*(-e^2*x^2+d^2)^(7/2)/x^10-37
/99*e^2*(-e^2*x^2+d^2)^(7/2)/d/x^9-19/80*e^3*(-e^2*x^2+d^2)^(7/2)/d^2/x^8-
74/693*e^4*(-e^2*x^2+d^2)^(7/2)/d^3/x^7+19/256*e^11*arctanh((-e^2*x^2+d^2)
^(1/2)/d)/d^3-19/256*e^9*(-e^2*x^2+d^2)^(1/2)/d^2/x^2
```

3.82.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.74

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{12}} \, dx = \frac{\frac{d\sqrt{d^2-e^2x^2}(-80640d^{10}-266112d^9ex-89600d^8e^2x^2+587664d^7e^3x^3+657920d^6e^4x^4-201432d^5e^5x^5-}{x^{11}}}{}$$

input
```
Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^12,x]
```

output
```
((d*Sqrt[d^2 - e^2*x^2]*(-80640*d^10 - 266112*d^9*e*x - 89600*d^8*e^2*x^2
+ 587664*d^7*e^3*x^3 + 657920*d^6*e^4*x^4 - 201432*d^5*e^5*x^5 - 629760*d^
4*e^6*x^6 - 251790*d^3*e^7*x^7 + 47360*d^2*e^8*x^8 + 65835*d*e^9*x^9 + 947
20*e^10*x^10))/x^11 + 65835*Sqrt[d^2]*e^11*Log[x] - 65835*Sqrt[d^2]*e^11*L
og[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(887040*d^4)
```

3.82.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.12, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {540, 25, 2338, 25, 27, 539, 25, 27, 539, 25, 27, 534, 243, 51, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^{12}} \, dx$$

$$\Big\downarrow \; 540$$

$$-\frac{\int -\frac{\left(d^2-e^2x^2\right)^{5/2}\left(33ed^4+37e^2xd^3+11e^3x^2d^2\right)}{x^{11}}dx}{11d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(33ed^4+37e^2xd^3+11e^3x^2d^2\right)}{x^{11}}dx}{11d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}$$

$$\Big\downarrow \; 2338$$

$$\frac{-\frac{\int -\frac{d^4e^2(370d+209ex)\left(d^2-e^2x^2\right)^{5/2}}{x^{10}}dx}{10d^2}-\frac{33d^2e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}}{11d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{d^4e^2(370d+209ex)\left(d^2-e^2x^2\right)^{5/2}}{x^{10}}dx}{10d^2}-\frac{33d^2e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}}{11d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{10}d^2e^2\int \frac{(370d+209ex)\left(d^2-e^2x^2\right)^{5/2}}{x^{10}}dx-\frac{33d^2e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}}{11d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}$$

$$\Big\downarrow 539$$

$$\frac{\frac{1}{10}d^2e^2\left(-\frac{\int -\frac{de(1881d+740ex)\left(d^2-e^2x^2\right)^{5/2}}{x^9}dx}{9d^2}-\frac{370\left(d^2-e^2x^2\right)^{7/2}}{9dx^9}\right)-\frac{33d^2e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}}{11d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}$$

$$\Big\downarrow 25$$

$$\frac{\frac{1}{10}d^2e^2\left(\frac{\int \frac{de(1881d+740ex)\left(d^2-e^2x^2\right)^{5/2}}{x^9}dx}{9d^2}-\frac{370\left(d^2-e^2x^2\right)^{7/2}}{9dx^9}\right)-\frac{33d^2e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}}{11d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}$$

$$\Big\downarrow 27$$

$$\frac{\frac{1}{10}d^2e^2\left(\frac{e\int \frac{(1881d+740ex)\left(d^2-e^2x^2\right)^{5/2}}{x^9}dx}{9d}-\frac{370\left(d^2-e^2x^2\right)^{7/2}}{9dx^9}\right)-\frac{33d^2e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}}{11d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}$$

$$\Big\downarrow 539$$

$$\frac{\frac{1}{10}d^2e^2\left(\frac{e\left(-\frac{\int -\frac{de(5920d+1881ex)\left(d^2-e^2x^2\right)^{5/2}}{x^8}dx}{8d^2}-\frac{1881\left(d^2-e^2x^2\right)^{7/2}}{8dx^8}\right)}{9d}-\frac{370\left(d^2-e^2x^2\right)^{7/2}}{9dx^9}\right)-\frac{33d^2e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}}{11d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}$$

$$\Big\downarrow 25$$

$$\frac{\frac{1}{10}d^2e^2\left(\frac{e\left(\frac{\int \frac{de(5920d+1881ex)\left(d^2-e^2x^2\right)^{5/2}}{x^8}dx}{8d^2}-\frac{1881\left(d^2-e^2x^2\right)^{7/2}}{8dx^8}\right)}{9d}-\frac{370\left(d^2-e^2x^2\right)^{7/2}}{9dx^9}\right)-\frac{33d^2e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}}{11d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}$$

$$\Big\downarrow 27$$

3.82. $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{12}}\,dx$

$$\frac{\frac{1}{10}d^2e^2\left(\frac{e\left(\frac{e\int\frac{(5920d+1881ex)\left(d^2-e^2x^2\right)^{5/2}}{x^8}dx}{8d}-\frac{1881\left(d^2-e^2x^2\right)^{7/2}}{8dx^8}\right)}{9d}-\frac{370\left(d^2-e^2x^2\right)^{7/2}}{9dx^9}\right)-\frac{33d^2e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}}{11d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}$$

$$\Big\downarrow \text{ 534}$$

$$\frac{\frac{1}{10}d^2e^2\left(\frac{e\left(\frac{e\left(1881e\int\frac{\left(d^2-e^2x^2\right)^{5/2}}{x^7}dx-\frac{5920\left(d^2-e^2x^2\right)^{7/2}}{7dx^7}\right)}{8d}-\frac{1881\left(d^2-e^2x^2\right)^{7/2}}{8dx^8}\right)}{9d}-\frac{370\left(d^2-e^2x^2\right)^{7/2}}{9dx^9}\right)-\frac{33d^2e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}}{11d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\frac{1}{10}d^2e^2\left(\frac{e\left(\frac{e\left(\frac{1881}{2}e\int\frac{\left(d^2-e^2x^2\right)^{5/2}}{x^8}dx^2-\frac{5920\left(d^2-e^2x^2\right)^{7/2}}{7dx^7}\right)}{8d}-\frac{1881\left(d^2-e^2x^2\right)^{7/2}}{8dx^8}\right)}{9d}-\frac{370\left(d^2-e^2x^2\right)^{7/2}}{9dx^9}\right)-\frac{33d^2e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}}{11d^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}$$

$$\Big\downarrow \text{ 51}$$

3.82. $\int\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{12}}\,dx$

$$\frac{\frac{1}{10}d^2e^2\left(\frac{e\left(\frac{e\left(\frac{1881}{2}e\left(-\frac{5}{6}e^2\int\frac{\left(d^2-e^2x^2\right)^{3/2}}{x^6}dx^2-\frac{\left(d^2-e^2x^2\right)^{5/2}}{3x^6}\right)-\frac{5920\left(d^2-e^2x^2\right)^{7/2}}{7dx^7}\right)}{8d}-\frac{1881\left(d^2-e^2x^2\right)^{7/2}}{8dx^8}\right)}{9d}-\frac{370\left(d^2-e^2x^2\right)^{7/2}}{9dx^9}\right)-\frac{33d^2}{}}{11d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}$$

$\downarrow$ 51

$$\frac{\frac{1}{10}d^2e^2\left(\frac{e\left(\frac{e\left(\frac{1881}{2}e\left(-\frac{5}{6}e^2\left(-\frac{3}{4}e^2\int\frac{\sqrt{d^2-e^2x^2}}{x^4}dx^2-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{\left(d^2-e^2x^2\right)^{5/2}}{3x^6}\right)-\frac{5920\left(d^2-e^2x^2\right)^{7/2}}{7dx^7}\right)}{8d}-\frac{1881\left(d^2-e^2x^2\right)^{7/2}}{8dx^8}\right)}{9d}-\frac{370(d^2-}{9d}\right.}{11d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}$$

$\downarrow$ 51

$$\frac{\frac{1}{10}d^2e^2\left(\frac{e\left(\frac{e\left(\frac{1881}{2}e\left(-\frac{5}{6}e^2\left(-\frac{3}{4}e^2\left(-\frac{1}{2}e^2\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{\left(d^2-e^2x^2\right)^{5/2}}{3x^6}\right)-\frac{5920\left(d^2-e^2x^2\right)^{7/2}}{7dx^7}\right)}{8d}-\frac{1881\left(d^2-e\right.}{8dx}\right.}{9d}\right.}{11d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}$$

$\downarrow$ 73

3.82. $\int\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{12}}dx$

$$\frac{\frac{1}{10}d^2e^2\left(\frac{e\left(\frac{e\left(\frac{1881}{2}e\left(-\frac{5}{6}e^2\left(-\frac{3}{4}e^2\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{\left(d^2-e^2x^2\right)^{5/2}}{3x^6}\right)-\frac{5920\left(d^2-e^2x^2\right)^{7/2}}{7dx^7}\right)}{8d}-\frac{1881\left(d^2-e^2x\right.}{8dx^8}}{9d}\right.}{11d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{1}{10}d^2e^2\left(\frac{e\left(\frac{e\left(\frac{1881}{2}e\left(-\frac{5}{6}e^2\left(-\frac{3}{4}e^2\left(\frac{e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{\left(d^2-e^2x^2\right)^{5/2}}{3x^6}\right)-\frac{5920\left(d^2-e^2x^2\right)^{7/2}}{7dx^7}\right)}{8d}-\frac{1881\left(d\right.}{}}{9d}\right.}{11d^2}$$

$$\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}$$

input
```
Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^12,x]
```

output
```
-1/11*(d*(d^2 - e^2*x^2)^(7/2))/x^11 + ((-33*d^2*e*(d^2 - e^2*x^2)^(7/2))/
(10*x^10) + (d^2*e^2*((-370*(d^2 - e^2*x^2)^(7/2))/(9*d*x^9) + (e*((-1881*
(d^2 - e^2*x^2)^(7/2))/(8*d*x^8) + (e*((-5920*(d^2 - e^2*x^2)^(7/2))/(7*d*
x^7) + (1881*e*(-1/3*(d^2 - e^2*x^2)^(5/2)/x^6 - (5*e^2*(-1/2*(d^2 - e^2*x
^2)^(3/2)/x^4 - (3*e^2*(-(Sqrt[d^2 - e^2*x^2]/x^2) + (e^2*ArcTanh[Sqrt[d^2
 - e^2*x^2]/d])/d))/4))/6))/2))/(8*d)))/(9*d)))/10)/(11*d^2)
```

3.82. $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{12}}\,dx$

3.82.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.82.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-94720e^{10}x^{10}-65835d\,e^9x^9-47360d^2e^8x^8+251790d^3e^7x^7+629760d^4e^6x^6+201432d^5e^5x^5-657920d^6e^4x^4-58766}{887040x^{11}d^3}$
default	$e^3\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{8d^2x^8}+\frac{e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{6d^2x^6}-\frac{e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{4d^2x^4}-\frac{3e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{2d^2x^2}-\frac{5e^2\left(\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5}+d^2\left(\frac{\left(-e^2x^2+d^2\right.}{3}\right.\right.}{}\right)}{4d}\right)}{6d^2}\right)}{8d^2}\right.$

3.82. $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{12}}\,dx$

input
```
int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/887040*(-e^2*x^2+d^2)^(1/2)*(-94720*e^10*x^10-65835*d*e^9*x^9-47360*d^2
*e^8*x^8+251790*d^3*e^7*x^7+629760*d^4*e^6*x^6+201432*d^5*e^5*x^5-657920*d
^6*e^4*x^4-587664*d^7*e^3*x^3+89600*d^8*e^2*x^2+266112*d^9*e*x+80640*d^10)
/x^11/d^3+19/256/d^2*e^11/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^
2)^(1/2))/x)
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.65

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{12}}\,dx =$$

$$-\frac{65835\,e^{11}x^{11}\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-(94720\,e^{10}x^{10}+65835\,de^9x^9+47360\,d^2e^8x^8-251790\,d^3e^7x^7-62976}{8}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x, algorithm="fricas")
```

output
```
-1/887040*(65835*e^11*x^11*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (94720*e^1
0*x^10 + 65835*d*e^9*x^9 + 47360*d^2*e^8*x^8 - 251790*d^3*e^7*x^7 - 629760
*d^4*e^6*x^6 - 201432*d^5*e^5*x^5 + 657920*d^6*e^4*x^4 + 587664*d^7*e^3*x^
3 - 89600*d^8*e^2*x^2 - 266112*d^9*e*x - 80640*d^10)*sqrt(-e^2*x^2 + d^2))
/(d^3*x^11)
```

3.82.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 146.79 (sec) , antiderivative size = 2397, normalized size of antiderivative = 9.44

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{12}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**12,x)
```

output
```
d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(11*x**10) + e**3*sqrt(d**2/
(e**2*x**2) - 1)/(99*d**2*x**8) + 8*e**5*sqrt(d**2/(e**2*x**2) - 1)/(693*d
**4*x**6) + 16*e**7*sqrt(d**2/(e**2*x**2) - 1)/(1155*d**6*x**4) + 64*e**9*
sqrt(d**2/(e**2*x**2) - 1)/(3465*d**8*x**2) + 128*e**11*sqrt(d**2/(e**2*x*
*2) - 1)/(3465*d**10), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*
x**2) + 1)/(11*x**10) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(99*d**2*x**8)
+ 8*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(693*d**4*x**6) + 16*I*e**7*sqrt(-d
**2/(e**2*x**2) + 1)/(1155*d**6*x**4) + 64*I*e**9*sqrt(-d**2/(e**2*x**2) +
 1)/(3465*d**8*x**2) + 128*I*e**11*sqrt(-d**2/(e**2*x**2) + 1)/(3465*d**10
), True)) + 3*d**6*e*Piecewise((-d**2/(10*e*x**11*sqrt(d**2/(e**2*x**2) -
1)) + 9*e/(80*x**9*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(480*d**2*x**7*sqrt(
d**2/(e**2*x**2) - 1)) + 7*e**5/(1920*d**4*x**5*sqrt(d**2/(e**2*x**2) - 1)
) + 7*e**7/(768*d**6*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 7*e**9/(256*d**8*x
*sqrt(d**2/(e**2*x**2) - 1)) + 7*e**10*acosh(d/(e*x))/(256*d**9), Abs(d**2
/(e**2*x**2)) > 1), (I*d**2/(10*e*x**11*sqrt(-d**2/(e**2*x**2) + 1)) - 9*I
*e/(80*x**9*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(480*d**2*x**7*sqrt(-d**
2/(e**2*x**2) + 1)) - 7*I*e**5/(1920*d**4*x**5*sqrt(-d**2/(e**2*x**2) + 1)
) - 7*I*e**7/(768*d**6*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 7*I*e**9/(256*d
**8*x*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e**10*asin(d/(e*x))/(256*d**9), T
rue)) + d**5*e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(9*x**8) + e...
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{12}}\,dx = \frac{19\,e^{11}\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{256\,d^3} - \frac{19\sqrt{-e^2x^2+d^2}e^{11}}{256\,d^4} - \frac{19\,(-e^2x^2+d^2)^{\frac{3}{2}}e^{11}}{768\,d^6} - \frac{19\,(-e^2x^2+d^2)^{\frac{5}{2}}e^{11}}{1280\,d^8} - \frac{19\,(-e^2x^2+d^2)^{\frac{7}{2}}e^9}{1280\,d^8x^2} + \frac{19\,(-e^2x^2+d^2)^{\frac{7}{2}}e^7}{1920\,d^6x^4} - \frac{19\,(-e^2x^2+d^2)^{\frac{7}{2}}e^5}{480\,d^4x^6} - \frac{74\,(-e^2x^2+d^2)^{\frac{7}{2}}e^4}{693\,d^3x^7} - \frac{19\,(-e^2x^2+d^2)^{\frac{7}{2}}e^3}{80\,d^2x^8} - \frac{37\,(-e^2x^2+d^2)^{\frac{7}{2}}e^2}{99\,dx^9} - \frac{3\,(-e^2x^2+d^2)^{\frac{7}{2}}e}{10\,x^{10}} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}d}{11\,x^{11}}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x, algorithm="maxima")
```

output
```
19/256*e^11*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 19/2
56*sqrt(-e^2*x^2 + d^2)*e^11/d^4 - 19/768*(-e^2*x^2 + d^2)^(3/2)*e^11/d^6
- 19/1280*(-e^2*x^2 + d^2)^(5/2)*e^11/d^8 - 19/1280*(-e^2*x^2 + d^2)^(7/2)
*e^9/(d^8*x^2) + 19/1920*(-e^2*x^2 + d^2)^(7/2)*e^7/(d^6*x^4) - 19/480*(-e
^2*x^2 + d^2)^(7/2)*e^5/(d^4*x^6) - 74/693*(-e^2*x^2 + d^2)^(7/2)*e^4/(d^3
*x^7) - 19/80*(-e^2*x^2 + d^2)^(7/2)*e^3/(d^2*x^8) - 37/99*(-e^2*x^2 + d^2
)^(7/2)*e^2/(d*x^9) - 3/10*(-e^2*x^2 + d^2)^(7/2)*e/x^10 - 1/11*(-e^2*x^2
+ d^2)^(7/2)*d/x^11
```

3.82.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 778 vs. $2(218) = 436$.

Time = 0.32 (sec) , antiderivative size = 778, normalized size of antiderivative = 3.06

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{12}}\,dx = \frac{\left(630\,e^{12} + \frac{4158\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^{10}}{x} + \frac{8470\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 e^8}{x^2} - \frac{3465\left(de+\sqrt{-e^2\right.}}{x^3}}{} + \frac{19\,e^{12}\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{256\,d^3|e|} + \frac{\frac{568260\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{30}e^{20}}{x} - \frac{152460\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d^{30}e^{18}}{x^2} - \frac{244860\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3 d^{30}e^{16}}{x^3} - \frac{138600\left(de+\sqrt{-e^2x\right.}}{x^4}}{}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x, algorithm="giac")
```

output
```
1/14192640*(630*e^12 + 4158*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^10/x + 8
470*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*e^8/x^2 - 3465*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^3*e^6/x^3 - 40590*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4
*e^4/x^4 - 57750*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*e^2/x^5 + 6930*(d*e
 + sqrt(-e^2*x^2 + d^2)*abs(e))^6/x^6 + 138600*(d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))^7/(e^2*x^7) + 244860*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8/(e^4*x
^8) + 152460*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^9/(e^6*x^9) - 568260*(d*e
 + sqrt(-e^2*x^2 + d^2)*abs(e))^10/(e^8*x^10))*e^22*x^11/((d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))^11*d^3*abs(e)) + 19/256*e^12*log(1/2*abs(-2*d*e - 2*sq
rt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^3*abs(e)) + 1/14192640*(568260
*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^30*e^20/x - 152460*(d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))^2*d^30*e^18/x^2 - 244860*(d*e + sqrt(-e^2*x^2 + d^2)*a
bs(e))^3*d^30*e^16/x^3 - 138600*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^30
*e^14/x^4 - 6930*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d^30*e^12/x^5 + 577
50*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6*d^30*e^10/x^6 + 40590*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^7*d^30*e^8/x^7 + 3465*(d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))^8*d^30*e^6/x^8 - 8470*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^9*d^30*
e^4/x^9 - 4158*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^10*d^30*e^2/x^10 - 630*
(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^11*d^30/x^11)/(d^33*e^10*abs(e))
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{12}} \, dx = \int \frac{(d^2-e^2\,x^2)^{5/2}\,(d+e\,x)^3}{x^{12}} \, dx$$

input
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^12,x)
```

output
```
int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^12, x)
```

3.83 $\int \frac{x^5(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$

3.83.1 Optimal result

Integrand size = 27, antiderivative size = 174

$$\int \frac{x^5(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d^4(d+ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} + \frac{x\sqrt{d^2-e^2x^2}}{2e^5} - \frac{13d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

output
```
1/5*d^4*(e*x+d)^3/e^6/(-e^2*x^2+d^2)^(5/2)-23/15*d^3*(e*x+d)^2/e^6/(-e^2*x
^2+d^2)^(3/2)-13/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+127/15*d^2*(e*
x+d)/e^6/(-e^2*x^2+d^2)^(1/2)+3*d*(-e^2*x^2+d^2)^(1/2)/e^6+1/2*x*(-e^2*x^2
+d^2)^(1/2)/e^5
```

3.83.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.64

$$\int \frac{x^5(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(304d^4-717d^3ex+479d^2e^2x^2-45de^3x^3-15e^4x^4)}{(d-ex)^3} + 390d^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{30e^6}$$

input
```
Integrate[(x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(304*d^4 - 717*d^3*e*x + 479*d^2*e^2*x^2 - 45*d*e^3*
x^3 - 15*e^4*x^4))/(d - e*x)^3 + 390*d^2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^
2 - e^2*x^2])])/(30*e^6)
```

3.83.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {529, 2166, 2166, 27, 2346, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(d+ex)^3}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{529}$$

$$\frac{d^4(d+ex)^3}{5e^6(d^2-e^2x^2)^{5/2}}-\frac{\int \frac{(d+ex)^2\left(\frac{3d^5}{e^5}+\frac{5xd^4}{e^4}+\frac{5x^2d^3}{e^3}+\frac{5x^3d^2}{e^2}+\frac{5x^4d}{e}\right)}{(d^2-e^2x^2)^{5/2}}dx}{5d}$$

$$\Big\downarrow \text{2166}$$

$$\frac{d^4(d+ex)^3}{5e^6(d^2-e^2x^2)^{5/2}}-\frac{\frac{23d^4(d+ex)^2}{3e^6(d^2-e^2x^2)^{3/2}}-\frac{\int \frac{(d+ex)\left(\frac{37d^5}{e^5}+\frac{45xd^4}{e^4}+\frac{30x^2d^3}{e^3}+\frac{15x^3d^2}{e^2}\right)}{(d^2-e^2x^2)^{3/2}}dx}{3d}}{5d}$$

$$\Big\downarrow \text{2166}$$

$$\frac{d^4(d+ex)^3}{5e^6(d^2-e^2x^2)^{5/2}}-\frac{\frac{23d^4(d+ex)^2}{3e^6(d^2-e^2x^2)^{3/2}}-\frac{\frac{127d^4(d+ex)}{e^6\sqrt{d^2-e^2x^2}}-\frac{\int \frac{15\left(\frac{6d^5}{e^5}+\frac{3xd^4}{e^4}+\frac{x^2d^3}{e^3}\right)}{\sqrt{d^2-e^2x^2}}dx}{d}}{3d}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^4(d+ex)^3}{5e^6(d^2-e^2x^2)^{5/2}}-\frac{\frac{23d^4(d+ex)^2}{3e^6(d^2-e^2x^2)^{3/2}}-\frac{\frac{127d^4(d+ex)}{e^6\sqrt{d^2-e^2x^2}}-\frac{15\int \frac{\frac{6d^5}{e^5}+\frac{3xd^4}{e^4}+\frac{x^2d^3}{e^3}}{\sqrt{d^2-e^2x^2}}dx}{d}}{3d}}{5d}$$

$$\Big\downarrow \text{2346}$$

$$\frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{23d^4(d+ex)^2}{3e^6(d^2-e^2x^2)^{3/2}}-\frac{\frac{127d^4(d+ex)}{e^6\sqrt{d^2-e^2x^2}}-\frac{15\left(-\frac{\int -\frac{d^4(13d+6ex)}{e^3\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{d^3x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d}}{3d}}{5d}$$

$\Big\downarrow$ 25

$$\frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{23d^4(d+ex)^2}{3e^6(d^2-e^2x^2)^{3/2}}-\frac{\frac{127d^4(d+ex)}{e^6\sqrt{d^2-e^2x^2}}-\frac{15\left(\frac{\int \frac{d^4(13d+6ex)}{e^3\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{d^3x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d}}{3d}}{5d}$$

$\Big\downarrow$ 27

$$\frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{23d^4(d+ex)^2}{3e^6(d^2-e^2x^2)^{3/2}}-\frac{\frac{127d^4(d+ex)}{e^6\sqrt{d^2-e^2x^2}}-\frac{15\left(\frac{d^4\int \frac{13d+6ex}{\sqrt{d^2-e^2x^2}}dx}{2e^5}-\frac{d^3x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d}}{3d}}{5d}$$

$\Big\downarrow$ 455

$$\frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{23d^4(d+ex)^2}{3e^6(d^2-e^2x^2)^{3/2}}-\frac{\frac{127d^4(d+ex)}{e^6\sqrt{d^2-e^2x^2}}-\frac{15\left(\frac{d^4\left(13d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{6\sqrt{d^2-e^2x^2}}{e}\right)}{2e^5}-\frac{d^3x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d}}{3d}}{5d}$$

$\Big\downarrow$ 224

$$\frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{23d^4(d+ex)^2}{3e^6(d^2-e^2x^2)^{3/2}}-\frac{\frac{127d^4(d+ex)}{e^6\sqrt{d^2-e^2x^2}}-\frac{15\left(\frac{d^4\left(13d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{6\sqrt{d^2-e^2x^2}}{e}\right)}{2e^5}-\frac{d^3x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d}}{3d}}{5d}$$

$\Big\downarrow$ 216

3.83. $\int \frac{x^5(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$

$$\frac{\frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{23d^4(d+ex)^2}{3e^6(d^2-e^2x^2)^{3/2}}-\frac{\frac{127d^4(d+ex)}{e^6\sqrt{d^2-e^2x^2}}-\frac{15\left(\frac{d^4\left(\frac{13d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{6\sqrt{d^2-e^2x^2}}{e}\right)}{2e^5}-\frac{d^3x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d}}{3d}}{5d}}{}$$

input
```
Int[(x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(d^4*(d + e*x)^3)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - ((23*d^4*(d + e*x)^2)/(3
*e^6*(d^2 - e^2*x^2)^(3/2)) - ((127*d^4*(d + e*x))/(e^6*Sqrt[d^2 - e^2*x^2
]) - (15*(-1/2*(d^3*x*Sqrt[d^2 - e^2*x^2])/e^5 + (d^4*((-6*Sqrt[d^2 - e^2*
x^2])/e + (13*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/(2*e^5)))/d)/(3*d))
/(5*d)
```

3.83.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.83.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.19

method	result
risch	$\frac{(ex+6d)\sqrt{-e^2x^2+d^2}}{2e^6}-\frac{13d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^5\sqrt{e^2}}-\frac{d^4\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{5e^9\left(x-\frac{d}{e}\right)^3}-\frac{23d^3\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{15e^8\left(x-\frac{d}{e}\right)^2}-$
default	$e^3\left(-\frac{x^7}{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{7d^2\left(\frac{x^5}{5e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{\frac{x^3}{3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e^2}}{e^2}\right)}{2e^2}\right)+d^3\Bigg(\frac{}{e^2}$

input
```
int(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(e*x+6*d)/e^6*(-e^2*x^2+d^2)^(1/2)-13/2*d^2/e^5/(e^2)^(1/2)*arctan((e^
2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/5*d^4/e^9/(x-d/e)^3*(-(x-d/e)^2*e^2-2*d
*e*(x-d/e))^(1/2)-23/15*d^3/e^8/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(
1/2)-127/15*d^2/e^7/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.10

$$\int \frac{x^5(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{304\,d^2e^3x^3-912\,d^3e^2x^2+912\,d^4ex-304\,d^5+390\,(d^2e^3x^3-3\,d^3e^2x^2+3\,d^4ex-d^5)}{30\,(e^9x^3-3\,}$$

input
```
integrate(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/30*(304*d^2*e^3*x^3 - 912*d^3*e^2*x^2 + 912*d^4*e*x - 304*d^5 + 390*(d^2
*e^3*x^3 - 3*d^3*e^2*x^2 + 3*d^4*e*x - d^5)*arctan(-(d - sqrt(-e^2*x^2 + d
^2))/(e*x)) + (15*e^4*x^4 + 45*d*e^3*x^3 - 479*d^2*e^2*x^2 + 717*d^3*e*x -
 304*d^4)*sqrt(-e^2*x^2 + d^2))/(e^9*x^3 - 3*d*e^8*x^2 + 3*d^2*e^7*x - d^3
*e^6)
```

3.83.6 Sympy [F]

$$\int \frac{x^5(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^5(d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

input
```
integrate(x**5*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral(x**5*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.83.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(152) = 304$.

Time = 0.28 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.82

$$\begin{aligned}\int \frac{x^5(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = &-\frac{ex^7}{2(-e^2x^2+d^2)^{\frac{5}{2}}} \\ &+\frac{13}{30}d^2ex\left(\frac{15x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2}-\frac{20d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4}+\frac{8d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right) \\ &-\frac{3dx^6}{(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{13d^2x\left(\frac{3x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2}-\frac{2d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right)}{6e} \\ &+\frac{19d^3x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2}-\frac{76d^5x^2}{3(-e^2x^2+d^2)^{\frac{5}{2}}e^4}+\frac{152d^7}{15(-e^2x^2+d^2)^{\frac{5}{2}}e^6} \\ &+\frac{26d^4x}{15(-e^2x^2+d^2)^{\frac{3}{2}}e^5}-\frac{91d^2x}{30\sqrt{-e^2x^2+d^2}e^5}-\frac{13d^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}e^5}\end{aligned}$$

input
```
integrate(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
-1/2*e*x^7/(-e^2*x^2 + d^2)^(5/2) + 13/30*d^2*e*x*(15*x^4/((-e^2*x^2 + d^2
)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2
+ d^2)^(5/2)*e^6)) - 3*d*x^6/(-e^2*x^2 + d^2)^(5/2) - 13/6*d^2*x*(3*x^2/((
-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e + 19*d^
3*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 76/3*d^5*x^2/((-e^2*x^2 + d^2)^(5/2)*
e^4) + 152/15*d^7/((-e^2*x^2 + d^2)^(5/2)*e^6) + 26/15*d^4*x/((-e^2*x^2 +
d^2)^(3/2)*e^5) - 91/30*d^2*x/(sqrt(-e^2*x^2 + d^2)*e^5) - 13/2*d^2*arcsin
(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^5)
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.34

$$\int \frac{x^5(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{1}{2}\sqrt{-e^2x^2+d^2}\left(\frac{x}{e^5}+\frac{6\,d}{e^6}\right)-\frac{13\,d^2\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,e^5|e|}$$
$$+\frac{2\left(107\,d^2-\frac{445\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2}{e^2x}+\frac{665\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^2}{e^4x^2}-\frac{405\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^2}{e^6x^3}+\frac{90\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4 d}{e^8x^4}\right.}{15\,e^5\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^5|e|}$$

input
```
integrate(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-e^2*x^2 + d^2)*(x/e^5 + 6*d/e^6) - 13/2*d^2*arcsin(e*x/d)*sgn(d)
*sgn(e)/(e^5*abs(e)) + 2/15*(107*d^2 - 445*(d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))*d^2/(e^2*x) + 665*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^2/(e^4*x^2)
 - 405*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^2/(e^6*x^3) + 90*(d*e + sqr
t(-e^2*x^2 + d^2)*abs(e))^4*d^2/(e^8*x^4))/(e^5*((d*e + sqrt(-e^2*x^2 + d^
2)*abs(e))/(e^2*x) - 1)^5*abs(e))
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^5\,(d+e\,x)^3}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

input
```
int((x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
int((x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

3.84 $\int \frac{x^4(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$

3.84.1 Optimal result

Integrand size = 27, antiderivative size = 142

$$\int \frac{x^4(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d^3(d+ex)^3}{5e^5(d^2-e^2x^2)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5(d^2-e^2x^2)^{3/2}} + \frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

output
```
1/5*d^3*(e*x+d)^3/e^5/(-e^2*x^2+d^2)^(5/2)-6/5*d^2*(e*x+d)^2/e^5/(-e^2*x^2
+d^2)^(3/2)-3*d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5+24/5*d*(e*x+d)/e^5/(-
e^2*x^2+d^2)^(1/2)+(-e^2*x^2+d^2)^(1/2)/e^5
```

3.84.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.69

$$\int \frac{x^4(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(24d^3-57d^2ex+39de^2x^2-5e^3x^3)}{(d-ex)^3} + 30d\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{5e^5}$$

input
```
Integrate[(x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(24*d^3 - 57*d^2*e*x + 39*d*e^2*x^2 - 5*e^3*x^3))/(d
 - e*x)^3 + 30*d*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(5*e^5)
```

3.84.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {529, 2166, 27, 2166, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d+ex)^3}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{529}$$

$$\frac{d^3(d+ex)^3}{5e^5(d^2-e^2x^2)^{5/2}}-\frac{\int \frac{(d+ex)^2\left(\frac{3d^4}{e^4}+\frac{5xd^3}{e^3}+\frac{5x^2d^2}{e^2}+\frac{5x^3d}{e}\right)}{(d^2-e^2x^2)^{5/2}}dx}{5d}$$

$$\Big\downarrow \text{2166}$$

$$\frac{d^3(d+ex)^3}{5e^5(d^2-e^2x^2)^{5/2}}-\frac{\frac{6d^3(d+ex)^2}{e^5(d^2-e^2x^2)^{3/2}}-\frac{\int \frac{3(d+ex)\left(\frac{9d^4}{e^4}+\frac{10xd^3}{e^3}+\frac{5x^2d^2}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3(d+ex)^3}{5e^5(d^2-e^2x^2)^{5/2}}-\frac{\frac{6d^3(d+ex)^2}{e^5(d^2-e^2x^2)^{3/2}}-\frac{\int \frac{(d+ex)\left(\frac{9d^4}{e^4}+\frac{10xd^3}{e^3}+\frac{5x^2d^2}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}dx}{d}}{5d}$$

$$\Big\downarrow \text{2166}$$

$$\frac{d^3(d+ex)^3}{5e^5(d^2-e^2x^2)^{5/2}}-\frac{\frac{6d^3(d+ex)^2}{e^5(d^2-e^2x^2)^{3/2}}-\frac{\frac{24d^3(d+ex)}{e^5\sqrt{d^2-e^2x^2}}-\frac{\int \frac{5d^3(3d+ex)}{e^4\sqrt{d^2-e^2x^2}}dx}{d}}{d}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3(d+ex)^3}{5e^5(d^2-e^2x^2)^{5/2}}-\frac{\frac{6d^3(d+ex)^2}{e^5(d^2-e^2x^2)^{3/2}}-\frac{\frac{24d^3(d+ex)}{e^5\sqrt{d^2-e^2x^2}}-\frac{5d^2\int \frac{3d+ex}{\sqrt{d^2-e^2x^2}}dx}{e^4}}{d}}{5d}$$

$$\Big\downarrow \text{455}$$

$$\frac{d^3(d+ex)^3}{5e^5(d^2-e^2x^2)^{5/2}}-\frac{\frac{6d^3(d+ex)^2}{e^5(d^2-e^2x^2)^{3/2}}-\frac{\frac{24d^3(d+ex)}{e^5\sqrt{d^2-e^2x^2}}-\frac{5d^2\left(3d\int\frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{e}\right)}{e^4}}{d}}{5d}$$

$\downarrow$ 224

$$\frac{d^3(d+ex)^3}{5e^5(d^2-e^2x^2)^{5/2}}-\frac{\frac{6d^3(d+ex)^2}{e^5(d^2-e^2x^2)^{3/2}}-\frac{\frac{24d^3(d+ex)}{e^5\sqrt{d^2-e^2x^2}}-\frac{5d^2\left(3d\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{e}\right)}{e^4}}{d}}{5d}$$

$\downarrow$ 216

$$\frac{d^3(d+ex)^3}{5e^5(d^2-e^2x^2)^{5/2}}-\frac{\frac{6d^3(d+ex)^2}{e^5(d^2-e^2x^2)^{3/2}}-\frac{\frac{24d^3(d+ex)}{e^5\sqrt{d^2-e^2x^2}}-\frac{5d^2\left(\frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{\sqrt{d^2-e^2x^2}}{e}\right)}{e^4}}{d}}{5d}$$

input
```
Int[(x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(d^3*(d + e*x)^3)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) - ((6*d^3*(d + e*x)^2)/(e^
5*(d^2 - e^2*x^2)^(3/2)) - ((24*d^3*(d + e*x))/(e^5*Sqrt[d^2 - e^2*x^2]) -
 (5*d^2*(-(Sqrt[d^2 - e^2*x^2]/e) + (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]
)/e))/e^4)/d)/(5*d)
```

3.84.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.84. $\int \frac{x^4(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

3.84.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.37

method	result
risch	$\frac{\sqrt{-e^2x^2+d^2}}{e^5}-\frac{3d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^4\sqrt{e^2}}-\frac{d^3\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{5e^8\left(x-\frac{d}{e}\right)^3}-\frac{6d^2\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{5e^7\left(x-\frac{d}{e}\right)^2}-\frac{24d\sqrt{-\left(x-\right.}}{5e}$
default	$e^3\left(-\frac{x^6}{e^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{6d^2\left(\frac{x^4}{e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{4d^2\left(\frac{x^2}{3e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{2d^2}{15e^4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}\right)}{e^2}\right)}{e^2}\right)+d^3\left(\frac{x^3}{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}}\right.$

input
```
int(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(-e^2*x^2+d^2)^(1/2)/e^5-3*d/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^
2+d^2)^(1/2))-1/5*d^3/e^8/(x-d/e)^3*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-6
/5*d^2/e^7/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-24/5*d/e^6/(x-d/
e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.25

$$\int \frac{x^4(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{24\,de^3x^3 - 72\,d^2e^2x^2 + 72\,d^3ex - 24\,d^4 + 30\,(de^3x^3 - 3\,d^2e^2x^2 + 3\,d^3ex - d^4)\arctan}{5\,(e^8x^3 - 3\,de^7x^2 + 3\,d^2e}$$

input
```
integrate(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/5*(24*d*e^3*x^3 - 72*d^2*e^2*x^2 + 72*d^3*e*x - 24*d^4 + 30*(d*e^3*x^3 -
 3*d^2*e^2*x^2 + 3*d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)
) + (5*e^3*x^3 - 39*d*e^2*x^2 + 57*d^2*e*x - 24*d^3)*sqrt(-e^2*x^2 + d^2))
/(e^8*x^3 - 3*d*e^7*x^2 + 3*d^2*e^6*x - d^3*e^5)
```

3.84.6 Sympy [F]

$$\int \frac{x^4(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^4(d+ex)^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x**4*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral(x**4*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.84.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(126) = 252$.

Time = 0.29 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.37

$$\int \frac{x^4(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{1}{5}\,de^2x\left(\frac{15\,x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right) - \frac{ex^6}{(-e^2x^2+d^2)^{\frac{5}{2}}} - dx\left(\frac{3\,x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right) + \frac{9\,d^2x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{d^3x^3}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{12\,d^4x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^3} - \frac{3\,d^5x}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{24\,d^6}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^5} + \frac{9\,d^3x}{10\,(-e^2x^2+d^2)^{\frac{3}{2}}e^4} - \frac{6\,dx}{5\,\sqrt{-e^2x^2+d^2}e^4} - \frac{3\,d\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}e^4}$$

input
```
integrate(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/5*d*e^2*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 +
 d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - e*x^6/(-e^2*x^2 +
 d^2)^(5/2) - d*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 +
 d^2)^(3/2)*e^4)) + 9*d^2*x^4/((-e^2*x^2 + d^2)^(5/2)*e) + 1/2*d^3*x^3/((-
e^2*x^2 + d^2)^(5/2)*e^2) - 12*d^4*x^2/((-e^2*x^2 + d^2)^(5/2)*e^3) - 3/10
*d^5*x/((-e^2*x^2 + d^2)^(5/2)*e^4) + 24/5*d^6/((-e^2*x^2 + d^2)^(5/2)*e^5
) + 9/10*d^3*x/((-e^2*x^2 + d^2)^(3/2)*e^4) - 6/5*d*x/(sqrt(-e^2*x^2 + d^2
)*e^4) - 3*d*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^4)
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.49

$$\int \frac{x^4(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{3\,d\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^4|e|} + \frac{\sqrt{-e^2x^2+d^2}}{e^5} + \frac{2\left(19\,d - \frac{80\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d}{e^2x} + \frac{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d}{e^4x^2} - \frac{70\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3 d}{e^6x^3} + \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4 d}{e^8x^4}\right)}{5\,e^4\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x} - 1\right)^5|e|}$$

input
```
integrate(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
-3*d*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^4*abs(e)) + sqrt(-e^2*x^2 + d^2)/e^5 +
 2/5*(19*d - 80*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d/(e^2*x) + 120*(d*e +
 sqrt(-e^2*x^2 + d^2)*abs(e))^2*d/(e^4*x^2) - 70*(d*e + sqrt(-e^2*x^2 + d^
2)*abs(e))^3*d/(e^6*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d/(e^8
*x^4))/(e^4*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^5*abs(e))
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^4\,(d+e\,x)^3}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

input
```
int((x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
int((x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

3.85 $\int \frac{x^3(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$

3.85.1 Optimal result

Integrand size = 27, antiderivative size = 118

$$\int \frac{x^3(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d^2(d+ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{13d(d+ex)^2}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}} - \frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

output
```
1/5*d^2*(e*x+d)^3/e^4/(-e^2*x^2+d^2)^(5/2)-13/15*d*(e*x+d)^2/e^4/(-e^2*x^2
+d^2)^(3/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4+32/15*(e*x+d)/e^4/(-e^2*x
^2+d^2)^(1/2)
```

3.85.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.73

$$\int \frac{x^3(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(22d^2-51dex+32e^2x^2)}{(d-ex)^3} + 30\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{15e^4}$$

input
```
Integrate[(x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(22*d^2 - 51*d*e*x + 32*e^2*x^2))/(d - e*x)^3 + 30*A
rcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(15*e^4)
```

3.85.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {529, 2166, 27, 665, 27, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex)^3}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{529}$$

$$\frac{d^2(d+ex)^3}{5e^4(d^2-e^2x^2)^{5/2}}-\frac{\int \frac{(d+ex)^2\left(\frac{3d^3}{e^3}+\frac{5xd^2}{e^2}+\frac{5x^2d}{e}\right)}{(d^2-e^2x^2)^{5/2}}dx}{5d}$$

$$\Big\downarrow \text{2166}$$

$$\frac{d^2(d+ex)^3}{5e^4(d^2-e^2x^2)^{5/2}}-\frac{\frac{13d^2(d+ex)^2}{3e^4(d^2-e^2x^2)^{3/2}}-\frac{\int \frac{d^2(d+ex)(17d+15ex)}{e^3\left(d^2-e^2x^2\right)^{3/2}}dx}{3d}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2(d+ex)^3}{5e^4(d^2-e^2x^2)^{5/2}}-\frac{\frac{13d^2(d+ex)^2}{3e^4(d^2-e^2x^2)^{3/2}}-\frac{d\int \frac{(d+ex)(17d+15ex)}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3e^3}}{5d}$$

$$\Big\downarrow \text{665}$$

$$\frac{d^2(d+ex)^3}{5e^4(d^2-e^2x^2)^{5/2}}-\frac{\frac{13d^2(d+ex)^2}{3e^4(d^2-e^2x^2)^{3/2}}-\frac{d\left(\frac{32(d+ex)}{e\sqrt{d^2-e^2x^2}}-\frac{\int \frac{15e}{\sqrt{d^2-e^2x^2}}dx}{e}\right)}{3e^3}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2(d+ex)^3}{5e^4(d^2-e^2x^2)^{5/2}}-\frac{\frac{13d^2(d+ex)^2}{3e^4(d^2-e^2x^2)^{3/2}}-\frac{d\left(\frac{32(d+ex)}{e\sqrt{d^2-e^2x^2}}-15\int \frac{1}{\sqrt{d^2-e^2x^2}}dx\right)}{3e^3}}{5d}$$

$$\Big\downarrow \text{224}$$

$$\frac{d^2(d+ex)^3}{5e^4(d^2-e^2x^2)^{5/2}}-\frac{\frac{13d^2(d+ex)^2}{3e^4(d^2-e^2x^2)^{3/2}}-\frac{d\left(\frac{32(d+ex)}{e\sqrt{d^2-e^2x^2}}-15\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{3e^3}}{5d}$$

$$\Big\downarrow \text{216}$$

$$\frac{d^2(d+ex)^3}{5e^4(d^2-e^2x^2)^{5/2}}-\frac{\frac{13d^2(d+ex)^2}{3e^4(d^2-e^2x^2)^{3/2}}-\frac{d\left(\frac{32(d+ex)}{e\sqrt{d^2-e^2x^2}}-\frac{15\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}\right)}{3e^3}}{5d}$$

input
```
Int[(x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(d^2*(d + e*x)^3)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - ((13*d^2*(d + e*x)^2)/(3
*e^4*(d^2 - e^2*x^2)^(3/2)) - (d*((32*(d + e*x))/(e*Sqrt[d^2 - e^2*x^2]) -
 (15*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/(3*e^3))/(5*d)
```

3.85.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 665
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2)^(3/2), x_Symbol] :> Simp[(-2^(m - 1))*d^(m - 2)*(e*f + d*g)^n*((d +
e*x)/(c*e^(n - 1)*Sqrt[a + c*x^2])), x] + Simp[1/(c*e^(n - 2))   Int[Expand
ToSum[(2^(m - 1)*d^(m - 1)*(e*f + d*g)^n - e^n*(d + e*x)^(m - 1)*(f + g*x)^
n)/(d - e*x), x]/Sqrt[a + c*x^2], x], x] /; FreeQ[{a, c, d, e, f, g}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

3.85.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(104) = 208$.

Time = 0.40 (sec) , antiderivative size = 377, normalized size of antiderivative = 3.19

method	result
default	$e^3\left(\frac{x^5}{5e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{\frac{x^3}{3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e^2}}{e^2}\right)+d^3\left(\frac{x^2}{3e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{2d}{15e^4(-e^2x}\right.$

input
```
int(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
e^3*(1/5*x^5/e^2/(-e^2*x^2+d^2)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3
/2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)
*x/(-e^2*x^2+d^2)^(1/2)))))+d^3*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2
/e^4/(-e^2*x^2+d^2)^(5/2))+3*d*e^2*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*d^2/e^2
*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2)))+3*d
^2*e*(1/2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)-3/2*d^2/e^2*(1/4*x/e^2/(-e^2*x^2+d^
2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-
e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))))
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.36

$$\int \frac{x^3(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{22\,e^3x^3-66\,de^2x^2+66\,d^2ex-22\,d^3+30\left(e^3x^3-3\,de^2x^2+3\,d^2ex-d^3\right)\arctan\left(-\frac{d}{\ldots}\right.}{15\left(e^7x^3-3\,de^6x^2+3\,d^2e^5x-d^3\ldots\right.}$$

input
```
integrate(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(22*e^3*x^3 - 66*d*e^2*x^2 + 66*d^2*e*x - 22*d^3 + 30*(e^3*x^3 - 3*d*
e^2*x^2 + 3*d^2*e*x - d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (32
*e^2*x^2 - 51*d*e*x + 22*d^2)*sqrt(-e^2*x^2 + d^2))/(e^7*x^3 - 3*d*e^6*x^2
 + 3*d^2*e^5*x - d^3*e^4)
```

3.85.6 Sympy [F]

$$\int \frac{x^3(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{x^3(d+ex)^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x**3*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral(x**3*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.85.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(104) = 208$.

Time = 0.30 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.61

$$\int \frac{x^3(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{1}{15}e^3x\left(\frac{15\,x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right) - \frac{1}{3}ex\left(\frac{3\,x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right) + \frac{3\,dx^4}{(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{3\,d^2x^3}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}e} - \frac{11\,d^3x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{9\,d^4x}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}e^3} + \frac{22\,d^5}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{17\,d^2x}{30\,(-e^2x^2+d^2)^{\frac{3}{2}}e^3} + \frac{2\,x}{15\,\sqrt{-e^2x^2+d^2}e^3} - \frac{\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}e^3}$$

input
```
integrate(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/15*e^3*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 +
d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 1/3*e*x*(3*x^2/((-
e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4)) + 3*d*x^4/
(-e^2*x^2 + d^2)^(5/2) + 3/2*d^2*x^3/((-e^2*x^2 + d^2)^(5/2)*e) - 11/3*d^3
*x^2/((-e^2*x^2 + d^2)^(5/2)*e^2) - 9/10*d^4*x/((-e^2*x^2 + d^2)^(5/2)*e^3
) + 22/15*d^5/((-e^2*x^2 + d^2)^(5/2)*e^4) + 17/30*d^2*x/((-e^2*x^2 + d^2)
^(3/2)*e^3) + 2/15*x/(sqrt(-e^2*x^2 + d^2)*e^3) - arcsin(e^2*x/(d*sqrt(e^2
)))/(sqrt(e^2)*e^3)
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.58

$$\int \frac{x^3(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^3|e|} - \frac{2\left(\frac{95\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} - \frac{145\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{75\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} - \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4} - 22\right)}{15\,e^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x} - 1\right)^5|e|}$$

input
```
integrate(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
-arcsin(e*x/d)*sgn(d)*sgn(e)/(e^3*abs(e)) - 2/15*(95*(d*e + sqrt(-e^2*x^2
+ d^2)*abs(e))/(e^2*x) - 145*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e^4*x^
2) + 75*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/(e^6*x^3) - 15*(d*e + sqrt(-
e^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) - 22)/(e^3*((d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))/(e^2*x) - 1)^5*abs(e))
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^3\,(d+e\,x)^3}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

input
```
int((x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
int((x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

3.86 $\int \frac{x^2(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$

3.86.1 Optimal result

Integrand size = 27, antiderivative size = 93

$$\int \frac{x^2(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d(d+ex)^3}{5e^3(d^2-e^2x^2)^{5/2}} - \frac{8(d+ex)^2}{15e^3(d^2-e^2x^2)^{3/2}} + \frac{7(d+ex)}{15de^3\sqrt{d^2-e^2x^2}}$$

output
```
1/5*d*(e*x+d)^3/e^3/(-e^2*x^2+d^2)^(5/2)-8/15*(e*x+d)^2/e^3/(-e^2*x^2+d^2)
^(3/2)+7/15*(e*x+d)/d/e^3/(-e^2*x^2+d^2)^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.57

$$\int \frac{x^2(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(2d^2-6dex+7e^2x^2)}{15de^3(d-ex)^3}$$

input
```
Integrate[(x^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(2*d^2 - 6*d*e*x + 7*e^2*x^2))/(15*d*e^3*(d - e*x)^3)
```

3.86.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {529, 27, 669, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \ 529$$

$$\frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{d(d+ex)^2(3d+5ex)}{e^2(d^2-e^2x^2)^{5/2}}dx}{5d}$$

$$\Big\downarrow \ 27$$

$$\frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{(d+ex)^2(3d+5ex)}{(d^2-e^2x^2)^{5/2}}dx}{5e^2}$$

$$\Big\downarrow \ 669$$

$$\frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{8(d+ex)^2}{3e(d^2-e^2x^2)^{3/2}}-\frac{7}{3}\int \frac{d+ex}{(d^2-e^2x^2)^{3/2}}dx}{5e^2}$$

$$\Big\downarrow \ 453$$

$$\frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{\frac{8(d+ex)^2}{3e(d^2-e^2x^2)^{3/2}}-\frac{7(d+ex)}{3de\sqrt{d^2-e^2x^2}}}{5e^2}$$

input
```
Int[(x^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(d*(d + e*x)^3)/(5*e^3*(d^2 - e^2*x^2)^(5/2)) - ((8*(d + e*x)^2)/(3*e*(d^2
 - e^2*x^2)^(3/2)) - (7*(d + e*x))/(3*d*e*Sqrt[d^2 - e^2*x^2]))/(5*e^2)
```

3.86.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 669
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*
(p + 1))), x] - Simp[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1)))
Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

3.86.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

method	result
trager	$\frac{(7e^2x^2-6dex+2d^2)\sqrt{-e^2x^2+d^2}}{15d\,e^3(-ex+d)^3}$
gosper	$\frac{(-ex+d)(ex+d)^4(7e^2x^2-6dex+2d^2)}{15d\,e^3(-e^2x^2+d^2)^{\frac{7}{2}}}$
default	$e^3\left(\frac{x^4}{e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{4d^2\left(\frac{x^2}{3e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{2d^2}{15e^4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}\right)}{e^2}\right)+d^3\left(\frac{x}{4e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}\right.}{}\right.$

input
```
int(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(7*e^2*x^2-6*d*e*x+2*d^2)/d/e^3/(-e*x+d)^3*(-e^2*x^2+d^2)^(1/2)
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.14

$$\int \frac{x^2(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2\,e^3x^3-6\,de^2x^2+6\,d^2ex-2\,d^3-(7\,e^2x^2-6\,dex+2\,d^2)\sqrt{-e^2x^2+d^2}}{15\,(de^6x^3-3\,d^2e^5x^2+3\,d^3e^4x-d^4e^3)}$$

input
```
integrate(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(2*e^3*x^3 - 6*d*e^2*x^2 + 6*d^2*e*x - 2*d^3 - (7*e^2*x^2 - 6*d*e*x +
 2*d^2)*sqrt(-e^2*x^2 + d^2))/(d*e^6*x^3 - 3*d^2*e^5*x^2 + 3*d^3*e^4*x - d
^4*e^3)
```

3.86.6 Sympy [F]

$$\int \frac{x^2(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^2(d+ex)^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x**2*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral(x**2*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.66

$$\int \frac{x^2(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{ex^4}{(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{3\,dx^3}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{d^2x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e} - \frac{7\,d^3x}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} + \frac{2\,d^4}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^3} + \frac{7\,dx}{30\,(-e^2x^2+d^2)^{\frac{3}{2}}e^2} + \frac{7\,x}{15\,\sqrt{-e^2x^2+d^2}de^2}$$

input
```
integrate(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
e*x^4/(-e^2*x^2 + d^2)^(5/2) + 3/2*d*x^3/(-e^2*x^2 + d^2)^(5/2) - 1/3*d^2*
x^2/((-e^2*x^2 + d^2)^(5/2)*e) - 7/10*d^3*x/((-e^2*x^2 + d^2)^(5/2)*e^2) +
 2/15*d^4/((-e^2*x^2 + d^2)^(5/2)*e^3) + 7/30*d*x/((-e^2*x^2 + d^2)^(3/2)*
e^2) + 7/15*x/(sqrt(-e^2*x^2 + d^2)*d*e^2)
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.14

$$\int \frac{x^2(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{4\left(\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} - \frac{10\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} - 1\right)}{15\,de^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x} - 1\right)^5|e|}$$

input
```
integrate(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
-4/15*(5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 10*(d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))^2/(e^4*x^2) - 1)/(d*e^2*((d*e + sqrt(-e^2*x^2 + d^2)*a
bs(e))/(e^2*x) - 1)^5*abs(e))
```

3.86.9 Mupad [B] (verification not implemented)

Time = 11.57 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

$$\int \frac{x^2(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2-6\,d\,e\,x+7\,e^2\,x^2\right)}{15\,d\,e^3\,{\left(d-e\,x\right)}^3}$$

input
```
int((x^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(2*d^2 + 7*e^2*x^2 - 6*d*e*x))/(15*d*e^3*(d - e*x)^
3)
```

3.87 $\int \frac{x(d+ex)^3}{(d^2-e^2x^2)^{7/2}} \, dx$

3.87.1 Optimal result

Integrand size = 25, antiderivative size = 86

$$\int \frac{x(d+ex)^3}{(d^2-e^2x^2)^{7/2}} \, dx = \frac{(d+ex)^3}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2(d+ex)}{5e^2(d^2-e^2x^2)^{3/2}} - \frac{x}{5d^2e\sqrt{d^2-e^2x^2}}$$

output
```
1/5*(e*x+d)^3/e^2/(-e^2*x^2+d^2)^(5/2)-2/5*(e*x+d)/e^2/(-e^2*x^2+d^2)^(3/2
)-1/5*x/d^2/e/(-e^2*x^2+d^2)^(1/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.58

$$\int \frac{x(d+ex)^3}{(d^2-e^2x^2)^{7/2}} \, dx = -\frac{\sqrt{d^2-e^2x^2}(d^2-3dex+e^2x^2)}{5d^2e^2(d-ex)^3}$$

input
```
Integrate[(x*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
-1/5*(Sqrt[d^2 - e^2*x^2]*(d^2 - 3*d*e*x + e^2*x^2))/(d^2*e^2*(d - e*x)^3)
```

3.87.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {531, 27, 457, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex)^3}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{531}$$

$$\frac{\int -\frac{3d^2e(d+ex)^2}{(d^2-e^2x^2)^{5/2}}dx}{5d^2e^2}+\frac{(d+ex)^3}{5e^2(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(d+ex)^3}{5e^2(d^2-e^2x^2)^{5/2}}-\frac{3\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{5/2}}dx}{5e}$$

$$\Big\downarrow \text{457}$$

$$\frac{(d+ex)^3}{5e^2(d^2-e^2x^2)^{5/2}}-\frac{3\left(\frac{1}{3}\int \frac{1}{(d^2-e^2x^2)^{3/2}}dx+\frac{2(d+ex)}{3e(d^2-e^2x^2)^{3/2}}\right)}{5e}$$

$$\Big\downarrow \text{208}$$

$$\frac{(d+ex)^3}{5e^2(d^2-e^2x^2)^{5/2}}-\frac{3\left(\frac{x}{3d^2\sqrt{d^2-e^2x^2}}+\frac{2(d+ex)}{3e(d^2-e^2x^2)^{3/2}}\right)}{5e}$$

input
```
Int[(x*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(d + e*x)^3/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (3*((2*(d + e*x))/(3*e*(d^2 -
e^2*x^2)^(3/2)) + x/(3*d^2*Sqrt[d^2 - e^2*x^2])))/(5*e)
```

3.87.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 531
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m, a + b*x^2, x], e = Coeff[Polynomi
alRemainder[x^m, a + b*x^2, x], x, 0], f = Coeff[PolynomialRemainder[x^m, a
 + b*x^2, x], x, 1]}, Simp[(c + d*x)^n*(a*f - b*e*x)*((a + b*x^2)^(p + 1)/(
2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b
*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*(c + d*x)*Qx - a*d*f*n + b*c*e*(2*p
 + 3) + b*d*e*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGt
Q[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && GtQ[n, 1] && IntegerQ[2*p]
```

3.87.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.55

method	result
trager	$-\frac{(e^2x^2-3dex+d^2)\sqrt{-e^2x^2+d^2}}{5d^2(-ex+d)^3e^2}$
gosper	$-\frac{(-ex+d)(ex+d)^4(e^2x^2-3dex+d^2)}{5d^2e^2(-e^2x^2+d^2)^{\frac{7}{2}}}$
default	$e^3\left(\frac{x^3}{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{3d^2\left(\frac{x}{4e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{4e^2}\right)}{2e^2}\right)+\frac{\cdots}{5e^2(-\cdots}$

input
```
int(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(e^2*x^2-3*d*e*x+d^2)/d^2/(-e*x+d)^3/e^2*(-e^2*x^2+d^2)^(1/2)
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.21

$$\int \frac{x(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{e^3x^3-3\,de^2x^2+3\,d^2ex-d^3-(e^2x^2-3\,dex+d^2)\sqrt{-e^2x^2+d^2}}{5\left(d^2e^5x^3-3\,d^3e^4x^2+3\,d^4e^3x-d^5e^2\right)}$$

input
```
integrate(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/5*(e^3*x^3 - 3*d*e^2*x^2 + 3*d^2*e*x - d^3 - (e^2*x^2 - 3*d*e*x + d^2)*
sqrt(-e^2*x^2 + d^2))/(d^2*e^5*x^3 - 3*d^3*e^4*x^2 + 3*d^4*e^3*x - d^5*e^2
)
```

3.87.6 Sympy [F]

$$\int \frac{x(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x(d+ex)^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral(x*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.49

$$\int \frac{x(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{ex^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{dx^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{3\,d^2x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} - \frac{d^3}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{x}{10\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e} - \frac{x}{5\sqrt{-e^2x^2+d^2}d^2e}$$

input
```
integrate(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/2*e*x^3/(-e^2*x^2 + d^2)^(5/2) + d*x^2/(-e^2*x^2 + d^2)^(5/2) + 3/10*d^2
*x/((-e^2*x^2 + d^2)^(5/2)*e) - 1/5*d^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 1/1
0*x/((-e^2*x^2 + d^2)^(3/2)*e) - 1/5*x/(sqrt(-e^2*x^2 + d^2)*d^2*e)
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.59

$$\int \frac{x(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2\left(\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} - \frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} - 1\right)}{5\,d^2e\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^5|e|}$$

input
```
integrate(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
2/5*(5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 5*(d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))^2/(e^4*x^2) + 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/(e^
6*x^3) - 1)/(d^2*e*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^5*abs
(e))
```

3.87.9 Mupad [B] (verification not implemented)

Time = 11.73 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.53

$$\int \frac{x(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}\,(d^2-3\,d\,e\,x+e^2\,x^2)}{5\,d^2\,e^2\,(d-e\,x)^3}$$

input
```
int((x*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
-((d^2 - e^2*x^2)^(1/2)*(d^2 + e^2*x^2 - 3*d*e*x))/(5*d^2*e^2*(d - e*x)^3)
```

3.88 $\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$

3.88.1 Optimal result

Integrand size = 24, antiderivative size = 103

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

output
```
1/5*(-e^2*x^2+d^2)^(1/2)/d/e/(-e*x+d)^3+2/15*(-e^2*x^2+d^2)^(1/2)/d^2/e/(-
e*x+d)^2+2/15*(-e^2*x^2+d^2)^(1/2)/d^3/e/(-e*x+d)
```

3.88.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.51

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}(7d^2-6dex+2e^2x^2)}{15d^3e(d-ex)^3}$$

input
```
Integrate[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(7*d^2 - 6*d*e*x + 2*e^2*x^2))/(15*d^3*e*(d - e*x)^3)
```

3.88.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {464, 461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{464}$$

$$\int \frac{1}{(d-ex)^3\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{461}$$

$$\frac{2\int \frac{1}{(d-ex)^2\sqrt{d^2-e^2x^2}}dx}{5d}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}$$

$$\Big\downarrow \text{461}$$

$$\frac{2\left(\frac{\int \frac{1}{(d-ex)\sqrt{d^2-e^2x^2}}dx}{3d}+\frac{\sqrt{d^2-e^2x^2}}{3de(d-ex)^2}\right)}{5d}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}$$

$$\Big\downarrow \text{460}$$

$$\frac{2\left(\frac{\sqrt{d^2-e^2x^2}}{3d^2e(d-ex)}+\frac{\sqrt{d^2-e^2x^2}}{3de(d-ex)^2}\right)}{5d}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}$$

input
```
Int[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
Sqrt[d^2 - e^2*x^2]/(5*d*e*(d - e*x)^3) + (2*(Sqrt[d^2 - e^2*x^2]/(3*d*e*(
d - e*x)^2) + Sqrt[d^2 - e^2*x^2]/(3*d^2*e*(d - e*x))))/(5*d)
```

3.88.3.1 Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 464
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
a + b*x^2)^(n + p)/(a/c + b*(x/d))^n, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c^2 + a*d^2, 0] && IntegerQ[n] && RationalQ[p] && (LtQ[0, -n, p] || LtQ[p,
 -n, 0]) && NeQ[n, 2] && NeQ[n, -1]
```

3.88.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.49

method	result
trager	$\frac{(2e^2x^2-6dex+7d^2)\sqrt{-e^2x^2+d^2}}{15d^3(-ex+d)^3e}$
gosper	$\frac{(-ex+d)(ex+d)^4(2e^2x^2-6dex+7d^2)}{15d^3e(-e^2x^2+d^2)^{\frac{7}{2}}}$
default	$d^3\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+e^3\left(\frac{x^2}{3e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{2d^2}{15e^4(-e^2x^2+d^2)^{\frac{5}{2}}}\right)+3d\,e$

input
```
int((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(2*e^2*x^2-6*d*e*x+7*d^2)/d^3/(-e*x+d)^3/e*(-e^2*x^2+d^2)^(1/2)
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{7\,e^3x^3-21\,de^2x^2+21\,d^2ex-7\,d^3-(2\,e^2x^2-6\,dex+7\,d^2)\sqrt{-e^2x^2+d^2}}{15\,(d^3e^4x^3-3\,d^4e^3x^2+3\,d^5e^2x-d^6e)}$$

input
```
integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(7*e^3*x^3 - 21*d*e^2*x^2 + 21*d^2*e*x - 7*d^3 - (2*e^2*x^2 - 6*d*e*x
 + 7*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^4*x^3 - 3*d^4*e^3*x^2 + 3*d^5*e^2*x
 - d^6*e)
```

3.88.6 Sympy [F]

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^3}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{ex^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{4\,dx}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{7\,d^2}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e}+\frac{x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d}+\frac{2\,x}{15\,\sqrt{-e^2x^2+d^2}d^3}$$

input
```
integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/3*e*x^2/(-e^2*x^2 + d^2)^(5/2) + 4/5*d*x/(-e^2*x^2 + d^2)^(5/2) + 7/15*d
^2/((-e^2*x^2 + d^2)^(5/2)*e) + 1/15*x/((-e^2*x^2 + d^2)^(3/2)*d) + 2/15*x
/(sqrt(-e^2*x^2 + d^2)*d^3)
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{2\left(\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} - \frac{40\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{30\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} - \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4} - 7\right)}{15\,d^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x} - 1\right)^5|e|}$$

input
```
integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
-2/15*(20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 40*(d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^2/(e^4*x^2) + 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^
3/(e^6*x^3) - 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) - 7)/(d^3
*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^5*abs(e))
```

3.88.9 Mupad [B] (verification not implemented)

Time = 11.68 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.48

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(7\,d^2-6\,d\,e\,x+2\,e^2\,x^2\right)}{15\,d^3\,e\,\left(d-e\,x\right)^3}$$

input
```
int((d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(7*d^2 + 2*e^2*x^2 - 6*d*e*x))/(15*d^3*e*(d - e*x)^
3)
```

3.89 $\int \frac{(d+ex)^3}{x(d^2-e^2x^2)^{7/2}}\,dx$

3.89.1 Optimal result

Integrand size = 27, antiderivative size = 114

$$\int \frac{(d+ex)^3}{x(d^2-e^2x^2)^{7/2}}\,dx = \frac{4(d+ex)}{5(d^2-e^2x^2)^{5/2}} + \frac{5d+11ex}{15d^2(d^2-e^2x^2)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

output
```
4/5*(e*x+d)/(-e^2*x^2+d^2)^(5/2)+1/15*(11*e*x+5*d)/d^2/(-e^2*x^2+d^2)^(3/2
)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4+1/15*(22*e*x+15*d)/d^4/(-e^2*x^2+d^2
)^(1/2)
```

3.89.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^3}{x(d^2-e^2x^2)^{7/2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}(32d^2-51dex+22e^2x^2)}{(d-ex)^3} - 15\sqrt{d^2}\log(x) + 15\sqrt{d^2}\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{15d^5}$$

input
```
Integrate[(d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
((d*Sqrt[d^2 - e^2*x^2]*(32*d^2 - 51*d*e*x + 22*e^2*x^2))/(d - e*x)^3 - 15
*Sqrt[d^2]*Log[x] + 15*Sqrt[d^2]*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(15
*d^5)
```

3.89.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {532, 25, 27, 532, 25, 532, 27, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{532}$$

$$\frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int -\frac{d^2(5d+11ex)}{x\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d^2(5d+11ex)}{x\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}+\frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{5d+11ex}{x\left(d^2-e^2x^2\right)^{5/2}}dx+\frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{1}{5}\left(\frac{5d+11ex}{3d^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int -\frac{15d+22ex}{x\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}\right)+\frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\left(\frac{\int \frac{15d+22ex}{x\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{5d+11ex}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)+\frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{1}{5}\left(\frac{\frac{15d+22ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{15d}{x\sqrt{d^2-e^2x^2}}dx}{d^2}}{3d^2}+\frac{5d+11ex}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)+\frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{\frac{15\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx}{d}+\frac{15d+22ex}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{5d+11ex}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)+\frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{5}\left(\frac{\frac{15\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2}{2d}+\frac{15d+22ex}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{5d+11ex}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)+\frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{5}\left(\frac{\frac{15d+22ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{15\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{de^2}}{3d^2}+\frac{5d+11ex}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)+\frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{5}\left(\frac{\frac{15d+22ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{15\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}}{3d^2}+\frac{5d+11ex}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)+\frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[(d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(4*(d + e*x))/(5*(d^2 - e^2*x^2)^(5/2)) + ((5*d + 11*e*x)/(3*d^2*(d^2 - e^
2*x^2)^(3/2)) + ((15*d + 22*e*x)/(d^2*Sqrt[d^2 - e^2*x^2]) - (15*ArcTanh[S
qrt[d^2 - e^2*x^2]/d])/d^2)/(3*d^2))/5
```

3.89.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

3.89.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(100) = 200$.

Time = 0.38 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.68

method	result
default	$e^3\left(\frac{x}{4e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{4e^2}\right)+d^3\left(\frac{1}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\right.$

input
```
int((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
e^3*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^
(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/
2))))+d^3*(1/5/d^2/(-e^2*x^2+d^2)^(5/2)+1/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2
)+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1
/2)*(-e^2*x^2+d^2)^(1/2))/x))))+3*d^2*e*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/
5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))+3/5
*d/(-e^2*x^2+d^2)^(5/2)
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.39

$$\int \frac{(d+ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{32\,e^3x^3-96\,de^2x^2+96\,d^2ex-32\,d^3+15\left(e^3x^3-3\,de^2x^2+3\,d^2ex-d^3\right)\log\left(-\frac{d-\cdots}{}\right.}{15\left(d^4e^3x^3-3\,d^5e^2x^2+3\,d^6ex-\cdots\right.}$$

input
```
integrate((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(32*e^3*x^3 - 96*d*e^2*x^2 + 96*d^2*e*x - 32*d^3 + 15*(e^3*x^3 - 3*d*
e^2*x^2 + 3*d^2*e*x - d^3)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (22*e^2*x^
2 - 51*d*e*x + 32*d^2)*sqrt(-e^2*x^2 + d^2))/(d^4*e^3*x^3 - 3*d^5*e^2*x^2
+ 3*d^6*e*x - d^7)
```

3.89.6 Sympy [F]

$$\int \frac{(d+ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{(d+ex)^3}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/x/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**3/(x*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.33

$$\int \frac{(d+ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4\,ex}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{4\,d}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{11\,ex}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2} + \frac{1}{3\,(-e^2x^2+d^2)^{\frac{3}{2}}d} + \frac{22\,ex}{15\,\sqrt{-e^2x^2+d^2}d^4} - \frac{\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d^4} + \frac{1}{\sqrt{-e^2x^2+d^2}d^3}$$

input
```
integrate((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
4/5*e*x/(-e^2*x^2 + d^2)^(5/2) + 4/5*d/(-e^2*x^2 + d^2)^(5/2) + 11/15*e*x/
((-e^2*x^2 + d^2)^(3/2)*d^2) + 1/3/((-e^2*x^2 + d^2)^(3/2)*d) + 22/15*e*x/
(sqrt(-e^2*x^2 + d^2)*d^4) - log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/a
bs(x))/d^4 + 1/(sqrt(-e^2*x^2 + d^2)*d^3)
```

3.89.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(100) = 200.

Time = 0.31 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.86

$$\int \frac{(d+ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = -\frac{e\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d^4|e|} + \frac{2\left(32\,e-\frac{115\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{ex}+\frac{185\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^3x^2}-\frac{135\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^5x^3}+\frac{45\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^7x^4}\right)}{15\,d^4\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^5|e|}$$

input
```
integrate((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
-e*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^4*
abs(e)) + 2/15*(32*e - 115*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e*x) + 185
*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e^3*x^2) - 135*(d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))^3/(e^5*x^3) + 45*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e
^7*x^4))/(d^4*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^5*abs(e))
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx=\int \frac{{\left(d+e\,x\right)}^3}{x\,{\left(d^2-e^2\,x^2\right)}^{7/2}}\,dx$$

input
```
int((d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)),x)
```

output
```
int((d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)), x)
```

3.90 $\int \frac{(d+ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$

3.90.1 Optimal result

Integrand size = 27, antiderivative size = 145

$$\int \frac{(d+ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}-\frac{3e\,\mathrm{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

output
```
4/5*e*(e*x+d)/d/(-e^2*x^2+d^2)^(5/2)+1/5*e*(7*e*x+5*d)/d^3/(-e^2*x^2+d^2)^
(3/2)-3*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5+1/5*e*(19*e*x+15*d)/d^5/(-e^
2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^5/x
```

3.90.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}\left(5d^3-39d^2ex+57de^2x^2-24e^3x^3\right)}{x(-d+ex)^3}-15\sqrt{d^2}e\log(x)+15\sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e\right.}{5d^6}$$

input
```
Integrate[(d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
((d*Sqrt[d^2 - e^2*x^2]*(5*d^3 - 39*d^2*e*x + 57*d*e^2*x^2 - 24*e^3*x^3))/
(x*(-d + e*x)^3) - 15*Sqrt[d^2]*e*Log[x] + 15*Sqrt[d^2]*e*Log[Sqrt[d^2] -
Sqrt[d^2 - e^2*x^2]])/(5*d^6)
```

3.90.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {532, 25, 2336, 27, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{x^2 \left(d^2-e^2x^2\right)^{7/2}} \, dx$$

$$\Big\downarrow \text{532}$$

$$\frac{4e(d+ex)}{5d \left(d^2-e^2x^2\right)^{5/2}}-\frac{\int -\frac{5d^3+15exd^2+16e^2x^2d}{x^2 \left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5d^3+15exd^2+16e^2x^2d}{x^2 \left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}+\frac{4e(d+ex)}{5d \left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{e(5d+7ex)}{d(d^2-e^2x^2)^{3/2}}-\frac{\int -\frac{3\left(5d^3+15exd^2+14e^2x^2d\right)}{x^2 \left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}}{5d^2}+\frac{4e(d+ex)}{5d \left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{5d^3+15exd^2+14e^2x^2d}{x^2 \left(d^2-e^2x^2\right)^{3/2}}dx}{d^2}+\frac{e(5d+7ex)}{d(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e(d+ex)}{5d \left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{\frac{e(15d+19ex)}{d\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{5d^2(d+3ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{d^2}}{d^2}+\frac{e(5d+7ex)}{d(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e(d+ex)}{5d \left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{5\int \frac{d+3ex}{x^2\sqrt{d^2-e^2x^2}}dx+\frac{e(15d+19ex)}{d\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e(5d+7ex)}{d(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e(d+ex)}{5d \left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{534}$$

$$\frac{\frac{5\left(3e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)+\frac{e(15d+19ex)}{d\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e(5d+7ex)}{d(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{5\left(\frac{3}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)+\frac{e(15d+19ex)}{d\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e(5d+7ex)}{d(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{5\left(-\frac{3\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)+\frac{e(15d+19ex)}{d\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e(5d+7ex)}{d(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{5\left(-\frac{3e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)+\frac{e(15d+19ex)}{d\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e(5d+7ex)}{d(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[(d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(4*e*(d + e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + ((e*(5*d + 7*e*x))/(d*(d^2 -
 e^2*x^2)^(3/2)) + ((e*(15*d + 19*e*x))/(d*Sqrt[d^2 - e^2*x^2]) + 5*(-(Sqr
t[d^2 - e^2*x^2]/(d*x)) - (3*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/d^2)/(5
*d^2)
```

3.90.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.90. $\int \frac{(d+ex)^3}{x^2(d^2-e^2x^2)^{7/2}}\,dx$

3.90.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.43

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^5x}-\frac{3e\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{d^4\sqrt{d^2}}+\frac{4\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{5d^4e\left(x-\frac{d}{e}\right)^2}-\frac{19\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{5d^5\left(x-\frac{d}{e}\right)}-\frac{\sqrt{-(x}}{}$
default	$\frac{e}{5(-e^2x^2+d^2)^{\frac{5}{2}}}+d^3\left(-\frac{1}{d^2x(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{6e^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{d^2}\right)+3d\,e^2\Bigg($

input
```
int((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-(-e^2*x^2+d^2)^(1/2)/d^5/x-3/d^4*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-
e^2*x^2+d^2)^(1/2))/x)+4/5/d^4/e/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^
(1/2)-19/5/d^5/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-1/5/d^3/e^2/(x
-d/e)^3*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{24\,e^4x^4-72\,de^3x^3+72\,d^2e^2x^2-24\,d^3ex+15\left(e^4x^4-3\,de^3x^3+3\,d^2e^2x^2-d^3ex\right)}{5\left(d^5e^3x^4-3\,d^6e^2x^3+\right.}$$

input
```
integrate((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/5*(24*e^4*x^4 - 72*d*e^3*x^3 + 72*d^2*e^2*x^2 - 24*d^3*e*x + 15*(e^4*x^4
 - 3*d*e^3*x^3 + 3*d^2*e^2*x^2 - d^3*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/
x) - (24*e^3*x^3 - 57*d*e^2*x^2 + 39*d^2*e*x - 5*d^3)*sqrt(-e^2*x^2 + d^2)
)/(d^5*e^3*x^4 - 3*d^6*e^2*x^3 + 3*d^7*e*x^2 - d^8*x)
```

3.90.6 Sympy [F]

$$\int \frac{(d+ex)^3}{x^2 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^3}{x^2 \left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/x**2/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**3/(x**2*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex)^3}{x^2 (d^2-e^2x^2)^{7/2}}\,dx = \frac{9\,e^2x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}d} + \frac{4\,e}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{12\,e^2x}{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^3} + \frac{e}{(-e^2x^2+d^2)^{\frac{3}{2}}d^2} - \frac{d}{(-e^2x^2+d^2)^{\frac{5}{2}}x} + \frac{24\,e^2x}{5\,\sqrt{-e^2x^2+d^2}d^5} - \frac{3\,e\log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d^5} + \frac{3\,e}{\sqrt{-e^2x^2+d^2}d^4}$$

input
```
integrate((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
9/5*e^2*x/((-e^2*x^2 + d^2)^(5/2)*d) + 4/5*e/(-e^2*x^2 + d^2)^(5/2) + 12/5
*e^2*x/((-e^2*x^2 + d^2)^(3/2)*d^3) + e/((-e^2*x^2 + d^2)^(3/2)*d^2) - d/(
(-e^2*x^2 + d^2)^(5/2)*x) + 24/5*e^2*x/(sqrt(-e^2*x^2 + d^2)*d^5) - 3*e*lo
g(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^5 + 3*e/(sqrt(-e^2*x^2
 + d^2)*d^4)
```

3.90.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(129) = 258$.

Time = 0.30 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.10

$$\int \frac{(d+ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = -\frac{3\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d^5|e|} - \frac{de+\sqrt{-e^2x^2+d^2}|e|}{2\,d^5x|e|}$$
$$-\frac{\left(5\,e^2 - \frac{121\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{x} + \frac{410\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^2x^2} - \frac{610\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^4x^3} + \frac{425\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^6x^4} - \frac{125\left(d\right.}{}\right.}{10\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^5\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^5|e|}$$

input
```
integrate((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
-3*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(
d^5*abs(e)) - 1/2*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(d^5*x*abs(e)) - 1/1
0*(5*e^2 - 121*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/x + 410*(d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^2/(e^2*x^2) - 610*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
^3/(e^4*x^3) + 425*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^6*x^4) - 125*(
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^8*x^5))*e^2*x/((d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))*d^5*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^5*a
bs(e))
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{\left(d+e\,x\right)^3}{x^2\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

input
```
int((d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)),x)
```

output
```
int((d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)), x)
```

3.91 $\int \frac{(d+ex)^3}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$

3.91.1 Optimal result

Integrand size = 27, antiderivative size = 182

$$\int \frac{(d+ex)^3}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - \frac{3e\sqrt{d^2-e^2x^2}}{d^6x} - \frac{13e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}$$

output
```
4/5*e^2*(e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)+1/15*e^2*(31*e*x+25*d)/d^4/(-e^2*
x^2+d^2)^(3/2)-13/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/15*e^2*(107*
e*x+90*d)/d^6/(-e^2*x^2+d^2)^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/d^5/x^2-3*e*(-
e^2*x^2+d^2)^(1/2)/d^6/x
```

3.91.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.73

$$\int \frac{(d+ex)^3}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}\left(15d^4+45d^3ex-479d^2e^2x^2+717de^3x^3-304e^4x^4\right)}{x^2(-d+ex)^3} - 195\sqrt{d^2}e^2\log(x) + 195\sqrt{d^2}e^2\log}{30d^7}$$

input
```
Integrate[(d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
((d*Sqrt[d^2 - e^2*x^2]*(15*d^4 + 45*d^3*e*x - 479*d^2*e^2*x^2 + 717*d*e^3
*x^3 - 304*e^4*x^4))/(x^2*(-d + e*x)^3) - 195*Sqrt[d^2]*e^2*Log[x] + 195*S
qrt[d^2]*e^2*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(30*d^7)
```

3.91.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {532, 25, 2336, 25, 2336, 27, 2338, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{x^3 (d^2-e^2x^2)^{7/2}} \, dx$$

$$\Big\downarrow \text{532}$$

$$\frac{4e^2(d+ex)}{5d^2 (d^2-e^2x^2)^{5/2}}-\frac{\int -\frac{5d^3+15exd^2+20e^2x^2d+16e^3x^3}{x^3 (d^2-e^2x^2)^{5/2}}dx}{5d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5d^3+15exd^2+20e^2x^2d+16e^3x^3}{x^3 (d^2-e^2x^2)^{5/2}}dx}{5d^2}+\frac{4e^2(d+ex)}{5d^2 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{e^2(25d+31ex)}{3d^2(d^2-e^2x^2)^{3/2}}-\frac{\int -\frac{15d^3+45exd^2+75e^2x^2d+62e^3x^3}{x^3 \left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}}{5d^2}+\frac{4e^2(d+ex)}{5d^2 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{15d^3+45exd^2+75e^2x^2d+62e^3x^3}{x^3 \left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{e^2(25d+31ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e^2(d+ex)}{5d^2 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{\frac{e^2(90d+107ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{15\left(d^3+3exd^2+6e^2x^2d\right)}{x^3\sqrt{d^2-e^2x^2}}dx}{d^2}}{3d^2}+\frac{e^2(25d+31ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e^2(d+ex)}{5d^2 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{15\int \frac{d^3+3exd^2+6e^2x^2d}{x^3\sqrt{d^2-e^2x^2}}dx}{d^2}+\frac{e^2(90d+107ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^2(25d+31ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 2338$$

$$\frac{\frac{\frac{15\left(-\frac{\int -\frac{d^3e(6d+13ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{d\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{e^2(90d+107ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^2(25d+31ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{15\left(\frac{\int \frac{d^3e(6d+13ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{d\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{e^2(90d+107ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^2(25d+31ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{15\left(\frac{1}{2}de\int \frac{6d+13ex}{x^2\sqrt{d^2-e^2x^2}}dx-\frac{d\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{e^2(90d+107ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^2(25d+31ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 534$$

$$\frac{\frac{\frac{15\left(\frac{1}{2}de\left(13e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{6\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{d\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{e^2(90d+107ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^2(25d+31ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{\frac{15\left(\frac{1}{2}de\left(\frac{13}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{6\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{d\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{e^2(90d+107ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^2(25d+31ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 73$$

3.91. $\int \frac{(d+ex)^3}{x^3(d^2-e^2x^2)^{7/2}}\,dx$

$$\frac{\frac{\frac{15\left(\frac{1}{2}de\left(-\frac{13\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{6\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{d\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{e^2(90d+107ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^2(25d+31ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow 221$$

$$\frac{\frac{\frac{15\left(\frac{1}{2}de\left(-\frac{13e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{6\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{d\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{e^2(90d+107ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^2(25d+31ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[(d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(4*e^2*(d + e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + ((e^2*(25*d + 31*e*x))/(
3*d^2*(d^2 - e^2*x^2)^(3/2)) + ((e^2*(90*d + 107*e*x))/(d^2*Sqrt[d^2 - e^2
*x^2]) + (15*(-1/2*(d*Sqrt[d^2 - e^2*x^2])/x^2 + (d*e*((-6*Sqrt[d^2 - e^2*
x^2])/(d*x) - (13*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/2))/d^2)/(3*d^2))/
(5*d^2)
```

3.91.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.91. $\int \frac{(d+ex)^3}{x^3(d^2-e^2x^2)^{7/2}}\,dx$

3.91.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(6ex+d)}{2d^6x^2}-\frac{13e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^5\sqrt{d^2}}-\frac{107e\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{15d^6\left(x-\frac{d}{e}\right)}+\frac{17\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\right.}}{15d^5\left(x-\frac{d}{e}\right)^2}$
default	$e^3\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+d^3\left(-\frac{1}{2d^2x^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{7e^2\left(\frac{1}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\right.}{}\right.$

input
```
int((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e^2*x^2+d^2)^(1/2)*(6*e*x+d)/d^6/x^2-13/2/d^5*e^2/(d^2)^(1/2)*ln((2
*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-107/15/d^6*e/(x-d/e)*(-(x-d/e)
^2*e^2-2*d*e*(x-d/e))^(1/2)+17/15/d^5/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d
/e))^(1/2)-1/5/d^4/e/(x-d/e)^3*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^3}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{254\,e^5x^5-762\,de^4x^4+762\,d^2e^3x^3-254\,d^3e^2x^2+195\left(e^5x^5-3\,de^4x^4+3\,d^2e^3x^3\right.}{30\left(d^6e^3x^5\right.}$$

input
```
integrate((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/30*(254*e^5*x^5 - 762*d*e^4*x^4 + 762*d^2*e^3*x^3 - 254*d^3*e^2*x^2 + 19
5*(e^5*x^5 - 3*d*e^4*x^4 + 3*d^2*e^3*x^3 - d^3*e^2*x^2)*log(-(d - sqrt(-e^
2*x^2 + d^2))/x) - (304*e^4*x^4 - 717*d*e^3*x^3 + 479*d^2*e^2*x^2 - 45*d^3
*e*x - 15*d^4)*sqrt(-e^2*x^2 + d^2))/(d^6*e^3*x^5 - 3*d^7*e^2*x^4 + 3*d^8*
e*x^3 - d^9*x^2)
```

3.91.6 Sympy [F]

$$\int \frac{(d+ex)^3}{x^3(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^3}{x^3\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/x**3/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**3/(x**3*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{(d+ex)^3}{x^3(d^2-e^2x^2)^{7/2}}\,dx &= \frac{19\,e^3x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2} + \frac{13\,e^2}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}d} \\ &+ \frac{76\,e^3x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d^4} + \frac{13\,e^2}{6\,(-e^2x^2+d^2)^{\frac{3}{2}}d^3} - \frac{3\,e}{(-e^2x^2+d^2)^{\frac{5}{2}}x} + \frac{152\,e^3x}{15\,\sqrt{-e^2x^2+d^2}d^6} \\ &- \frac{13\,e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{2\,d^6} + \frac{13\,e^2}{2\,\sqrt{-e^2x^2+d^2}d^5} - \frac{d}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}x^2}\end{aligned}$$

input
```
integrate((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
19/5*e^3*x/((-e^2*x^2 + d^2)^(5/2)*d^2) + 13/10*e^2/((-e^2*x^2 + d^2)^(5/2
)*d) + 76/15*e^3*x/((-e^2*x^2 + d^2)^(3/2)*d^4) + 13/6*e^2/((-e^2*x^2 + d^
2)^(3/2)*d^3) - 3*e/((-e^2*x^2 + d^2)^(5/2)*x) + 152/15*e^3*x/(sqrt(-e^2*x
^2 + d^2)*d^6) - 13/2*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(
x))/d^6 + 13/2*e^2/(sqrt(-e^2*x^2 + d^2)*d^5) - 1/2*d/((-e^2*x^2 + d^2)^(5
/2)*x^2)
```

3.91.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(160) = 320$.

Time = 0.31 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.09

$$\int \frac{(d+ex)^3}{x^3(d^2-e^2x^2)^{7/2}}\,dx = -\frac{13\,e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,d^6|e|}$$
$$-\frac{\left(15\,e^3+\frac{105\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e}{x}-\frac{2782\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{ex^2}+\frac{9410\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^3x^3}-\frac{13645\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^5x^4}+\right.}{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^6\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^5|e}$$
$$-\frac{\frac{12\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^6e|e|}{x}+\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^6|e|}{ex^2}}{8\,d^{12}e^2}$$

input
```
integrate((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
-13/2*e^3*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x))
)/(d^6*abs(e)) - 1/120*(15*e^3 + 105*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e
/x - 2782*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e*x^2) + 9410*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^3/(e^3*x^3) - 13645*(d*e + sqrt(-e^2*x^2 + d^2)*a
bs(e))^4/(e^5*x^4) + 9285*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^7*x^5)
- 2580*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6/(e^9*x^6))*e^4*x^2/((d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))^2*d^6*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2
*x) - 1)^5*abs(e)) - 1/8*(12*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^6*e*abs
(e)/x + (d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^6*abs(e)/(e*x^2))/(d^12*e^
2)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{x^3(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+e\,x)^3}{x^3\,(d^2-e^2\,x^2)^{7/2}}\,dx$$

input
```
int((d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)),x)
```

output
```
int((d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)), x)
```

3.92 $\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

3.92.1 Optimal result

Integrand size = 27, antiderivative size = 147

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3}-\frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2}+\frac{x^4\sqrt{d^2-e^2x^2}}{5e}+\frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5}+\frac{3d^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^5}$$

output
```
3/8*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5+4/15*d^2*x^2*(-e^2*x^2+d^2)^(
1/2)/e^3-1/4*d*x^3*(-e^2*x^2+d^2)^(1/2)/e^2+1/5*x^4*(-e^2*x^2+d^2)^(1/2)/e
+1/120*d^3*(-45*e*x+64*d)*(-e^2*x^2+d^2)^(1/2)/e^5
```

3.92.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.70

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\frac{\sqrt{d^2-e^2x^2}(64d^4-45d^3ex+32d^2e^2x^2-30de^3x^3+24e^4x^4)-90d^5\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{120e^5}$$

input
```
Integrate[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(64*d^4 - 45*d^3*e*x + 32*d^2*e^2*x^2 - 30*d*e^3*x^3
+ 24*e^4*x^4) - 90*d^5*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(1
20*e^5)
```

3.92.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.27, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$, Rules used = {566, 533, 25, 27, 533, 25, 27, 533, 25, 27, 533, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}dx$$

$$\Big\downarrow \text{566}$$

$$\int \frac{x^4(d-ex)}{\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{533}$$

$$\frac{\int -\frac{dex^3(4d-5ex)}{\sqrt{d^2-e^2x^2}}dx}{5e^2}+\frac{x^4\sqrt{d^2-e^2x^2}}{5e}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^4\sqrt{d^2-e^2x^2}}{5e}-\frac{\int \frac{dex^3(4d-5ex)}{\sqrt{d^2-e^2x^2}}dx}{5e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4\sqrt{d^2-e^2x^2}}{5e}-\frac{d\int \frac{x^3(4d-5ex)}{\sqrt{d^2-e^2x^2}}dx}{5e}$$

$$\Big\downarrow \text{533}$$

$$\frac{x^4\sqrt{d^2-e^2x^2}}{5e}-\frac{d\left(\frac{\int -\frac{dex^2(15d-16ex)}{\sqrt{d^2-e^2x^2}}dx}{4e^2}+\frac{5x^3\sqrt{d^2-e^2x^2}}{4e}\right)}{5e}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^4\sqrt{d^2-e^2x^2}}{5e}-\frac{d\left(\frac{5x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{\int \frac{dex^2(15d-16ex)}{\sqrt{d^2-e^2x^2}}dx}{4e^2}\right)}{5e}$$

$$\Big\downarrow \; 27$$

$$\frac{x^4\sqrt{d^2-e^2x^2}}{5e}-\frac{d\left(\frac{5x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d\int\frac{x^2(15d-16ex)}{\sqrt{d^2-e^2x^2}}dx}{4e}\right)}{5e}$$

$$\Big\downarrow \; 533$$

$$\frac{x^4\sqrt{d^2-e^2x^2}}{5e}-\frac{d\left(\frac{5x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d\left(\frac{\int-\frac{dex(32d-45ex)}{\sqrt{d^2-e^2x^2}}dx}{3e^2}+\frac{16x^2\sqrt{d^2-e^2x^2}}{3e}\right)}{4e}\right)}{5e}$$

$$\Big\downarrow \; 25$$

$$\frac{x^4\sqrt{d^2-e^2x^2}}{5e}-\frac{d\left(\frac{5x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d\left(\frac{16x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{\int\frac{dex(32d-45ex)}{\sqrt{d^2-e^2x^2}}dx}{3e^2}\right)}{4e}\right)}{5e}$$

$$\Big\downarrow \; 27$$

$$\frac{x^4\sqrt{d^2-e^2x^2}}{5e}-\frac{d\left(\frac{5x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d\left(\frac{16x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{d\int\frac{x(32d-45ex)}{\sqrt{d^2-e^2x^2}}dx}{3e}\right)}{4e}\right)}{5e}$$

$$\Big\downarrow \; 533$$

$$\frac{x^4\sqrt{d^2-e^2x^2}}{5e}-\frac{d\left(\frac{5x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d\left(\frac{16x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{d\left(\frac{\int-\frac{de(45d-64ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}+\frac{45x\sqrt{d^2-e^2x^2}}{2e}\right)}{3e}\right)}{4e}\right)}{5e}$$

$$\Big\downarrow \; 25$$

3.92. $\int\frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

$$\frac{x^4\sqrt{d^2-e^2x^2}}{5e}-\frac{d\left(\frac{5x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d\left(\frac{16x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{d\left(\frac{45x\sqrt{d^2-e^2x^2}}{2e}-\frac{\int\frac{de(45d-64ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}\right)}{3e}\right)}{4e}\right)}{5e}$$

$\downarrow$ 27

$$\frac{x^4\sqrt{d^2-e^2x^2}}{5e}-\frac{d\left(\frac{5x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d\left(\frac{16x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{d\left(\frac{45x\sqrt{d^2-e^2x^2}}{2e}-\frac{d\int\frac{45d-64ex}{\sqrt{d^2-e^2x^2}}dx}{2e}\right)}{3e}\right)}{4e}\right)}{5e}$$

$\downarrow$ 455

$$\frac{x^4\sqrt{d^2-e^2x^2}}{5e}-\frac{d\left(\frac{5x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d\left(\frac{16x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{d\left(\frac{45x\sqrt{d^2-e^2x^2}}{2e}-\frac{d\left(45d\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{64\sqrt{d^2-e^2x^2}}{e}\right)}{2e}\right)}{3e}\right)}{4e}\right)}{5e}$$

$\downarrow$ 224

3.92. $\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

$$\frac{\frac{x^4\sqrt{d^2-e^2x^2}}{5e}-\frac{d\left(\frac{5x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d\left(\frac{16x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{d\left(\frac{45x\sqrt{d^2-e^2x^2}}{2e}-\frac{d\left(45d\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{64\sqrt{d^2-e^2x^2}}{e}\right)}{2e}\right)}{3e}\right)}{4e}\right)}{5e}}{}$$

$\Big\downarrow$ 216

$$\frac{x^4\sqrt{d^2-e^2x^2}}{5e}-\frac{d\left(\frac{5x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d\left(\frac{16x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{d\left(\frac{45x\sqrt{d^2-e^2x^2}}{2e}-\frac{d\left(\frac{45d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{64\sqrt{d^2-e^2x^2}}{e}\right)}{2e}\right)}{3e}\right)}{4e}\right)}{5e}$$

input
```
Int[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

3.92. $\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

output
```
(x^4*Sqrt[d^2 - e^2*x^2])/(5*e) - (d*((5*x^3*Sqrt[d^2 - e^2*x^2])/(4*e) -
(d*((16*x^2*Sqrt[d^2 - e^2*x^2])/(3*e) - (d*((45*x*Sqrt[d^2 - e^2*x^2])/(2
*e) - (d*((64*Sqrt[d^2 - e^2*x^2])/e + (45*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x
^2]])/e))/(2*e)))/(3*e)))/(4*e)))/(5*e)
```

3.92.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 566
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Int[x^m*(a/c + b*(x/d))*(a + b*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0]
```

3.92.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.66

method	result
risch	$\frac{(24e^4x^4-30d\,e^3x^3+32d^2e^2x^2-45d^3ex+64d^4)\sqrt{-e^2x^2+d^2}}{120e^5}+\frac{3d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8e^4\sqrt{e^2}}$
default	$\frac{-\frac{x^2(-e^2x^2+d^2)^{\frac{3}{2}}}{5e^2}-\frac{2d^2(-e^2x^2+d^2)^{\frac{3}{2}}}{15e^4}}{e}-\frac{d^2(-e^2x^2+d^2)^{\frac{3}{2}}}{3e^5}-\frac{d^3\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{e^4}-\frac{d\left(-\frac{x(-e^2x^2+d}{4e^2}\right.}{}$

input
```
int(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/120*(24*e^4*x^4-30*d*e^3*x^3+32*d^2*e^2*x^2-45*d^3*e*x+64*d^4)/e^5*(-e^2
*x^2+d^2)^(1/2)+3/8*d^5/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2
)^(1/2))
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.65

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{90\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(24\,e^4x^4-30\,de^3x^3+32\,d^2e^2x^2-45\,d^3ex+64\,d^4)\sqrt{-e^2x^2+d^2}}{120\,e^5}$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
-1/120*(90*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (24*e^4*x^4 - 3
0*d*e^3*x^3 + 32*d^2*e^2*x^2 - 45*d^3*e*x + 64*d^4)*sqrt(-e^2*x^2 + d^2))/
e^5
```

3.92.6 Sympy [F]

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\int \frac{x^4\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{d+ex}\,dx$$

input
```
integrate(x**4*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

output
```
Integral(x**4*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.85

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\frac{3\,d^5\arcsin\left(\frac{ex}{d}\right)}{8\,e^5}-\frac{5\,\sqrt{-e^2x^2+d^2}d^3x}{8\,e^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}x^2}{5\,e^3}+\frac{\sqrt{-e^2x^2+d^2}d^4}{e^5}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}dx}{4\,e^4}-\frac{7\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}{15\,e^5}$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
3/8*d^5*arcsin(e*x/d)/e^5 - 5/8*sqrt(-e^2*x^2 + d^2)*d^3*x/e^4 - 1/5*(-e^2
*x^2 + d^2)^(3/2)*x^2/e^3 + sqrt(-e^2*x^2 + d^2)*d^4/e^5 + 1/4*(-e^2*x^2 +
 d^2)^(3/2)*d*x/e^4 - 7/15*(-e^2*x^2 + d^2)^(3/2)*d^2/e^5
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.60

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\frac{3\,d^5\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{8\,e^4|e|}+\frac{1}{120}\,\sqrt{-e^2x^2+d^2}\left(\left(2\left(3\,x\left(\frac{4\,x}{e}-\frac{5\,d}{e^2}\right)+\frac{16\,d^2}{e^3}\right)x-\frac{45\,d^3}{e^4}\right)x+\frac{64\,d^4}{e^5}\right)$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
3/8*d^5*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^4*abs(e)) + 1/120*sqrt(-e^2*x^2 + d
^2)*((2*(3*x*(4*x/e - 5*d/e^2) + 16*d^2/e^3)*x - 45*d^3/e^4)*x + 64*d^4/e^
5)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\int \frac{x^4\,\sqrt{d^2-e^2\,x^2}}{d+e\,x}\,dx$$

input
```
int((x^4*(d^2 - e^2*x^2)^(1/2))/(d + e*x),x)
```

output
```
int((x^4*(d^2 - e^2*x^2)^(1/2))/(d + e*x), x)
```

3.93 $\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

3.93.1 Optimal result

Integrand size = 27, antiderivative size = 118

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2}+\frac{x^3\sqrt{d^2-e^2x^2}}{4e} - \frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4}-\frac{3d^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4}$$

output
```
-3/8*d^4*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4-1/3*d*x^2*(-e^2*x^2+d^2)^(1/
2)/e^2+1/4*x^3*(-e^2*x^2+d^2)^(1/2)/e-1/24*d^2*(-9*e*x+16*d)*(-e^2*x^2+d^2
)^(1/2)/e^4
```

3.93.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.78

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{\sqrt{d^2-e^2x^2}(-16d^3+9d^2ex-8de^2x^2+6e^3x^3)+18d^4\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{24e^4}$$

input
```
Integrate[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-16*d^3 + 9*d^2*e*x - 8*d*e^2*x^2 + 6*e^3*x^3) + 18*
d^4*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(24*e^4)
```

3.93.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.28, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {566, 533, 25, 27, 533, 25, 27, 533, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}dx$$

$$\Big\downarrow \text{566}$$

$$\int \frac{x^3(d-ex)}{\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{533}$$

$$\frac{\int -\frac{dex^2(3d-4ex)}{\sqrt{d^2-e^2x^2}}dx}{4e^2}+\frac{x^3\sqrt{d^2-e^2x^2}}{4e}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{\int \frac{dex^2(3d-4ex)}{\sqrt{d^2-e^2x^2}}dx}{4e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d\int \frac{x^2(3d-4ex)}{\sqrt{d^2-e^2x^2}}dx}{4e}$$

$$\Big\downarrow \text{533}$$

$$\frac{x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d\left(\frac{\int -\frac{dex(8d-9ex)}{\sqrt{d^2-e^2x^2}}dx}{3e^2}+\frac{4x^2\sqrt{d^2-e^2x^2}}{3e}\right)}{4e}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d\left(\frac{4x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{\int \frac{dex(8d-9ex)}{\sqrt{d^2-e^2x^2}}dx}{3e^2}\right)}{4e}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d\left(\frac{4x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{d\int \frac{x(8d-9ex)}{\sqrt{d^2-e^2x^2}}dx}{3e}\right)}{4e}$$

$$\Big\downarrow \text{533}$$

$$\frac{x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d\left(\frac{4x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{d\left(\frac{\int -\frac{de(9d-16ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}+\frac{9x\sqrt{d^2-e^2x^2}}{2e}\right)}{3e}\right)}{4e}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d\left(\frac{4x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{d\left(\frac{9x\sqrt{d^2-e^2x^2}}{2e}-\frac{\int \frac{de(9d-16ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}\right)}{3e}\right)}{4e}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d\left(\frac{4x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{d\left(\frac{9x\sqrt{d^2-e^2x^2}}{2e}-\frac{d\int \frac{9d-16ex}{\sqrt{d^2-e^2x^2}}dx}{2e}\right)}{3e}\right)}{4e}$$

$$\Big\downarrow \text{455}$$

$$\frac{x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d\left(\frac{4x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{d\left(\frac{9x\sqrt{d^2-e^2x^2}}{2e}-\frac{d\left(9d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{16\sqrt{d^2-e^2x^2}}{e}\right)}{2e}\right)}{3e}\right)}{4e}$$

$$\Big\downarrow \text{224}$$

$$\frac{x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d\left(\frac{4x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{d\left(\frac{9x\sqrt{d^2-e^2x^2}}{2e}-\frac{d\left(9d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{16\sqrt{d^2-e^2x^2}}{e}\right)}{2e}\right)}{3e}\right)}{4e}$$

$$\Big\downarrow \text{216}$$

3.93. $\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

$$\frac{x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d\left(\frac{4x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{d\left(\frac{9x\sqrt{d^2-e^2x^2}}{2e}-\frac{d\left(\frac{9d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{16\sqrt{d^2-e^2x^2}}{e}\right)}{2e}\right)}{3e}\right)}{4e}$$

input
```
Int[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

output
```
(x^3*Sqrt[d^2 - e^2*x^2])/(4*e) - (d*((4*x^2*Sqrt[d^2 - e^2*x^2])/(3*e) -
(d*((9*x*Sqrt[d^2 - e^2*x^2])/(2*e) - (d*((16*Sqrt[d^2 - e^2*x^2])/e + (9*
d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/(2*e)))/(3*e)))/(4*e)
```

3.93.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 566
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Int[x^m*(a/c + b*(x/d))*(a + b*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0]
```

3.93.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{\left(-6e^{3}x^{3}+8d\,e^{2}x^{2}-9d^{2}ex+16d^{3}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{24e^{4}}-\frac{3d^{4}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{8e^{3}\sqrt{e^{2}}}$
default	$\frac{-\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4e^{2}}+\frac{d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)}{4e^{2}}}{e}+\frac{d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)}{e^{3}}+\frac{d(-e^{2}x^{2}+d}{3e^{4}}$

input
```
int(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-6*e^3*x^3+8*d*e^2*x^2-9*d^2*e*x+16*d^3)/e^4*(-e^2*x^2+d^2)^(1/2)-3
/8*d^4/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.70

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx$$
$$=\frac{18\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(6\,e^3x^3-8\,de^2x^2+9\,d^2ex-16\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,e^4}$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
1/24*(18*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (6*e^3*x^3 - 8*d*
e^2*x^2 + 9*d^2*e*x - 16*d^3)*sqrt(-e^2*x^2 + d^2))/e^4
```

3.93.6 Sympy [F]

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \int \frac{x^3\sqrt{-(-d+ex)(d+ex)}}{d+ex}\,dx$$

input
```
integrate(x**3*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

output
```
Integral(x**3*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{3\,d^4\arcsin\left(\frac{ex}{d}\right)}{8\,e^4} + \frac{5\,\sqrt{-e^2x^2+d^2}d^2x}{8\,e^3} - \frac{\sqrt{-e^2x^2+d^2}d^3}{e^4} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}x}{4\,e^3} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d}{3\,e^4}$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
-3/8*d^4*arcsin(e*x/d)/e^4 + 5/8*sqrt(-e^2*x^2 + d^2)*d^2*x/e^3 - sqrt(-e^
2*x^2 + d^2)*d^3/e^4 - 1/4*(-e^2*x^2 + d^2)^(3/2)*x/e^3 + 1/3*(-e^2*x^2 +
d^2)^(3/2)*d/e^4
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.64

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{3\,d^4\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,e^3|e|} + \frac{1}{24}\sqrt{-e^2x^2+d^2}\left(\left(2\,x\left(\frac{3\,x}{e}-\frac{4\,d}{e^2}\right)+\frac{9\,d^2}{e^3}\right)x-\frac{16\,d^3}{e^4}\right)$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
-3/8*d^4*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^3*abs(e)) + 1/24*sqrt(-e^2*x^2 + d
^2)*((2*x*(3*x/e - 4*d/e^2) + 9*d^2/e^3)*x - 16*d^3/e^4)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \int \frac{x^3\,\sqrt{d^2-e^2\,x^2}}{d+e\,x}\,dx$$

input
```
int((x^3*(d^2 - e^2*x^2)^(1/2))/(d + e*x),x)
```

output
```
int((x^3*(d^2 - e^2*x^2)^(1/2))/(d + e*x), x)
```

3.94 $\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

3.94.1 Optimal result

Integrand size = 27, antiderivative size = 86

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

output
```
-1/3*(-e^2*x^2+d^2)^(3/2)/e^3+1/2*d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3
+1/2*d*(-e*x+2*d)*(-e^2*x^2+d^2)^(1/2)/e^3
```

3.94.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\frac{\sqrt{d^2-e^2x^2}\left(4d^2-3dex+2e^2x^2\right)}{6e^3}-\frac{d^3\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

input
```
Integrate[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(4*d^2 - 3*d*e*x + 2*e^2*x^2))/(6*e^3) - (d^3*ArcTan[
(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^3
```

3.94.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.35, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {566, 533, 25, 27, 533, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx$$

$$\Big\downarrow \text{566}$$

$$\int \frac{x^2(d-ex)}{\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{533}$$

$$\frac{\int -\frac{dex(2d-3ex)}{\sqrt{d^2-e^2x^2}}dx}{3e^2}+\frac{x^2\sqrt{d^2-e^2x^2}}{3e}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{\int \frac{dex(2d-3ex)}{\sqrt{d^2-e^2x^2}}dx}{3e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{d\int \frac{x(2d-3ex)}{\sqrt{d^2-e^2x^2}}dx}{3e}$$

$$\Big\downarrow \text{533}$$

$$\frac{x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{d\left(\frac{\int -\frac{de(3d-4ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}+\frac{3x\sqrt{d^2-e^2x^2}}{2e}\right)}{3e}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{d\left(\frac{3x\sqrt{d^2-e^2x^2}}{2e}-\frac{\int \frac{de(3d-4ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}\right)}{3e}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{d\left(\frac{3x\sqrt{d^2-e^2x^2}}{2e}-\frac{d\int \frac{3d-4ex}{\sqrt{d^2-e^2x^2}}dx}{2e}\right)}{3e}$$

$$\Big\downarrow \text{455}$$

$$\frac{x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{d\left(\frac{3x\sqrt{d^2-e^2x^2}}{2e}-\frac{d\left(3d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{4\sqrt{d^2-e^2x^2}}{e}\right)}{2e}\right)}{3e}$$

$$\Big\downarrow \text{224}$$

$$\frac{x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{d\left(\frac{3x\sqrt{d^2-e^2x^2}}{2e}-\frac{d\left(3d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{4\sqrt{d^2-e^2x^2}}{e}\right)}{2e}\right)}{3e}$$

$$\Big\downarrow \text{216}$$

$$\frac{x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{d\left(\frac{3x\sqrt{d^2-e^2x^2}}{2e}-\frac{d\left(\frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{4\sqrt{d^2-e^2x^2}}{e}\right)}{2e}\right)}{3e}$$

input
```
Int[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

output
```
(x^2*Sqrt[d^2 - e^2*x^2])/(3*e) - (d*((3*x*Sqrt[d^2 - e^2*x^2])/(2*e) - (d
*((4*Sqrt[d^2 - e^2*x^2])/e + (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/
(2*e)))/(3*e)
```

3.94.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 566
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Int[x^m*(a/c + b*(x/d))*(a + b*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0]
```

3.94.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

method	result
risch	$\frac{\left(2e^2x^2-3dex+4d^2\right)\sqrt{-e^2x^2+d^2}}{6e^3}+\frac{d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}$
default	$-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{3e^3}-\frac{d\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{e^2}+\frac{d^2\left(\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2}}\right)}{\sqrt{e^2}}\right)}{e^3}$

input
```
int(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*e^2*x^2-3*d*e*x+4*d^2)/e^3*(-e^2*x^2+d^2)^(1/2)+1/2*d^3/e^2/(e^2)^(
1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.85

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{6\,d^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(2\,e^2x^2-3\,dex+4\,d^2\right)\sqrt{-e^2x^2+d^2}}{6\,e^3}$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
-1/6*(6*d^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (2*e^2*x^2 - 3*d*e
*x + 4*d^2)*sqrt(-e^2*x^2 + d^2))/e^3
```

3.94.6 Sympy [F]

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \int \frac{x^2\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{d+ex}\,dx$$

input
```
integrate(x**2*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

output
```
Integral(x**2*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{d^3\arcsin\left(\frac{ex}{d}\right)}{2\,e^3}-\frac{\sqrt{-e^2x^2+d^2}dx}{2\,e^2}+\frac{\sqrt{-e^2x^2+d^2}d^2}{e^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{3\,e^3}$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
1/2*d^3*arcsin(e*x/d)/e^3 - 1/2*sqrt(-e^2*x^2 + d^2)*d*x/e^2 + sqrt(-e^2*x
^2 + d^2)*d^2/e^3 - 1/3*(-e^2*x^2 + d^2)^(3/2)/e^3
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.74

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{d^3\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,e^2|e|} + \frac{1}{6}\sqrt{-e^2x^2+d^2}\left(x\left(\frac{2\,x}{e}-\frac{3\,d}{e^2}\right)+\frac{4\,d^2}{e^3}\right)$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
1/2*d^3*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) + 1/6*sqrt(-e^2*x^2 + d^2
)*(x*(2*x/e - 3*d/e^2) + 4*d^2/e^3)
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \int \frac{x^2\,\sqrt{d^2-e^2\,x^2}}{d+e\,x}\,dx$$

input
```
int((x^2*(d^2 - e^2*x^2)^(1/2))/(d + e*x),x)
```

output
```
int((x^2*(d^2 - e^2*x^2)^(1/2))/(d + e*x), x)
```

3.95 $\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

3.95.1 Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx=-\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2}-\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}$$

output
```
-1/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2-1/2*(-e*x+2*d)*(-e^2*x^2+d^2
)^(1/2)/e^2
```

3.95.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\frac{(-2d+ex)\sqrt{d^2-e^2x^2}}{2e^2}+\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

input
```
Integrate[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

output
```
((-2*d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*e^2) + (d^2*ArcTan[(e*x)/(Sqrt[d^2]
- Sqrt[d^2 - e^2*x^2])])/e^2
```

3.95.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {565, 262, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}dx$$

$$\Big\downarrow \text{565}$$

$$-e\int \frac{x^2}{\sqrt{d^2-e^2x^2}}dx-\frac{d\sqrt{d^2-e^2x^2}}{e^2}$$

$$\Big\downarrow \text{262}$$

$$-e\left(\frac{d^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{x\sqrt{d^2-e^2x^2}}{2e^2}\right)-\frac{d\sqrt{d^2-e^2x^2}}{e^2}$$

$$\Big\downarrow \text{224}$$

$$-e\left(\frac{d^2\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}}{2e^2}-\frac{x\sqrt{d^2-e^2x^2}}{2e^2}\right)-\frac{d\sqrt{d^2-e^2x^2}}{e^2}$$

$$\Big\downarrow \text{216}$$

$$-e\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}-\frac{x\sqrt{d^2-e^2x^2}}{2e^2}\right)-\frac{d\sqrt{d^2-e^2x^2}}{e^2}$$

input
```
Int[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

output
```
-((d*Sqrt[d^2 - e^2*x^2])/e^2) - e*(-1/2*(x*Sqrt[d^2 - e^2*x^2])/e^2 + (d^
2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3))
```

3.95.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 565
```
Int[((x_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :> Sim
p[a*((a + b*x^2)^p/(2*b*c*p)), x] + Simp[b/d   Int[x^2*(a + b*x^2)^(p - 1),
 x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0]
```

3.95.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

method	result	size
risch	$-\frac{(-ex+2d)\sqrt{-e^2x^2+d^2}}{2e^2}-\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e\sqrt{e^2}}$	64
default	$\frac{\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}}{e}-\frac{d\left(\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}\right)}{e^2}$	135

input
```
int(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e*x+2*d)*(-e^2*x^2+d^2)^(1/2)/e^2-1/2*d^2/e/(e^2)^(1/2)*arctan((e^2
)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{2\,d^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}(ex-2\,d)}{2\,e^2}$$

input
```
integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
1/2*(2*d^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + sqrt(-e^2*x^2 + d^2
)*(e*x - 2*d))/e^2
```

3.95.6 Sympy [F]

$$\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \int \frac{x\sqrt{-(-d+ex)\,(d+ex)}}{d+ex}\,dx$$

input
```
integrate(x*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

output
```
Integral(x*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{d^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^2}+\frac{\sqrt{-e^2x^2+d^2}x}{2\,e}-\frac{\sqrt{-e^2x^2+d^2}d}{e^2}$$

input
```
integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
-1/2*d^2*arcsin(e*x/d)/e^2 + 1/2*sqrt(-e^2*x^2 + d^2)*x/e - sqrt(-e^2*x^2
+ d^2)*d/e^2
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{d^2\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,e|e|} + \frac{1}{2}\sqrt{-e^2x^2+d^2}\left(\frac{x}{e}-\frac{2\,d}{e^2}\right)$$

input
```
integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
-1/2*d^2*arcsin(e*x/d)*sgn(d)*sgn(e)/(e*abs(e)) + 1/2*sqrt(-e^2*x^2 + d^2)
*(x/e - 2*d/e^2)
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \int \frac{x\,\sqrt{d^2-e^2\,x^2}}{d+e\,x}\,dx$$

input
```
int((x*(d^2 - e^2*x^2)^(1/2))/(d + e*x),x)
```

output
```
int((x*(d^2 - e^2*x^2)^(1/2))/(d + e*x), x)
```

3.96 $\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

3.96.1 Optimal result

Integrand size = 24, antiderivative size = 46

$$\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\frac{\sqrt{d^2-e^2x^2}}{e}+\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

output
```
d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e+(-e^2*x^2+d^2)^(1/2)/e
```

3.96.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\frac{\sqrt{d^2-e^2x^2}}{e}-\frac{d\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^2]/(d + e*x),x]
```

output
```
Sqrt[d^2 - e^2*x^2]/e - (d*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/Sqr
t[-e^2]
```

3.96.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {466, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}dx$$

$$\Big\downarrow \text{466}$$

$$d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{\sqrt{d^2-e^2x^2}}{e}$$

$$\Big\downarrow \text{224}$$

$$d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e}$$

$$\Big\downarrow \text{216}$$

$$\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{\sqrt{d^2-e^2x^2}}{e}$$

input
```
Int[Sqrt[d^2 - e^2*x^2]/(d + e*x),x]
```

output
```
Sqrt[d^2 - e^2*x^2]/e + (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
```

3.96.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

3.96.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

method	result	size
risch	$\frac{\sqrt{-e^2x^2+d^2}}{e}+\frac{d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}$	49
default	$\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}}{e}$	78

input
```
int((-e^2*x^2+d^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
(-e^2*x^2+d^2)^(1/2)/e+d/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(
1/2))
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int\frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx=-\frac{2\,d\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\sqrt{-e^2x^2+d^2}}{e}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
-(2*d*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - sqrt(-e^2*x^2 + d^2))/e
```

3.96.6 Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\int \frac{\sqrt{-(-d+ex)(d+ex)}}{d+ex}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

output
```
Integral(sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\frac{d\arcsin\left(\frac{ex}{d}\right)}{e}+\frac{\sqrt{-e^2x^2+d^2}}{e}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
d*arcsin(e*x/d)/e + sqrt(-e^2*x^2 + d^2)/e
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\frac{d\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|}+\frac{\sqrt{-e^2x^2+d^2}}{e}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
d*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + sqrt(-e^2*x^2 + d^2)/e
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \int \frac{\sqrt{d^2-e^2\,x^2}}{d+e\,x}\,dx$$

input
```
int((d^2 - e^2*x^2)^(1/2)/(d + e*x),x)
```

output
```
int((d^2 - e^2*x^2)^(1/2)/(d + e*x), x)
```

3.97 $\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)}\,dx$

3.97.1 Optimal result

Integrand size = 27, antiderivative size = 46

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)}\,dx = -\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
-arctan(e*x/(-e^2*x^2+d^2)^(1/2))-arctanh((-e^2*x^2+d^2)^(1/2)/d)
```

3.97.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)}\,dx = 2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) + \frac{\sqrt{d^2}\left(-\log(x)+\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)\right)}{d}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)),x]
```

output
```
2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] + (Sqrt[d^2]*(-Log[x] +
Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]]))/d
```

3.97.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {566, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)}dx$$

$$\Big\downarrow \text{566}$$

$$\int \frac{d-ex}{x\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{538}$$

$$d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-e\int \frac{1}{\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{224}$$

$$d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-e\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{216}$$

$$d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}d\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)$$

$$\Big\downarrow \text{73}$$

$$-\frac{d\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)$$

$$\Big\downarrow \text{221}$$

$$-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

input
```
Int[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)),x]
```

output
```
-ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

3.97.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 566
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Int[x^m*(a/c + b*(x/d))*(a + b*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0]
```

3.97.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(42) = 84.

Time = 0.38 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.07

method	result	size
default	$\frac{\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}}{d}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}}{d}$	141

input
```
int((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/d*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^
2+d^2)^(1/2))/x))-1/d*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2
)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)}\,dx = 2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x, algorithm="fricas")
```

output
```
2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + log(-(d - sqrt(-e^2*x^2 + d^
2))/x)
```

3.97.6 Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)}\,dx = \int \frac{\sqrt{-(-d+ex)(d+ex)}}{x(d+ex)}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**(1/2)/x/(e*x+d),x)
```

output
```
Integral(sqrt(-(-d + e*x)*(d + e*x))/(x*(d + e*x)), x)
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)}\,dx = -\frac{e\left(\frac{d\arcsin\left(\frac{ex}{d}\right)}{e}+\frac{d\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{e}\right)}{d}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x, algorithm="maxima")
```

output
```
-e*(d*arcsin(e*x/d)/e + d*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(
x))/e)/d
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)}\,dx = -\frac{e\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|}-\frac{e\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{|e|}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x, algorithm="giac")
```

output
```
-e*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - e*log(1/2*abs(-2*d*e - 2*sqrt(-e^2
*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e)
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)}\,dx = \int \frac{\sqrt{d^2-e^2\,x^2}}{x\,\left(d+e\,x\right)}\,dx$$

input
```
int((d^2 - e^2*x^2)^(1/2)/(x*(d + e*x)),x)
```

output
```
int((d^2 - e^2*x^2)^(1/2)/(x*(d + e*x)), x)
```

3.98 $\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx$

3.98.1 Optimal result

Integrand size = 27, antiderivative size = 51

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx=-\frac{\sqrt{d^2-e^2x^2}}{dx}+\frac{e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}$$

output
```
e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d-(-e^2*x^2+d^2)^(1/2)/d/x
```

3.98.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx=-\frac{\sqrt{d^2-e^2x^2}}{dx}+\frac{e\log(x)}{\sqrt{d^2}}-\frac{e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{\sqrt{d^2}}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)),x]
```

output
```
-(Sqrt[d^2 - e^2*x^2]/(d*x)) + (e*Log[x])/Sqrt[d^2] - (e*Log[Sqrt[d^2] - S
qrt[d^2 - e^2*x^2]])/Sqrt[d^2]
```

3.98.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {566, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}dx$$

$$\Big\downarrow \text{566}$$

$$\int \frac{d-ex}{x^2\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{534}$$

$$-e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{dx}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{dx}$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{\sqrt{d^2-e^2x^2}}{dx}$$

$$\Big\downarrow \text{221}$$

$$\frac{e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{dx}$$

input
```
Int[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)),x]
```

output
```
-(Sqrt[d^2 - e^2*x^2]/(d*x)) + (e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d
```

3.98.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 566
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Int[x^m*(a/c + b*(x/d))*(a + b*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0]
```

3.98.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{dx}+\frac{e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}$
default	$\frac{-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{d^2x}-\frac{2e^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{d^2}}{d}-\frac{e\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)}{d^2}+\frac{e\left(\sqrt{-\right.}{}$

input
```
int((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-(-e^2*x^2+d^2)^(1/2)/d/x+e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+
d^2)^(1/2))/x)
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx = -\frac{ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\sqrt{-e^2x^2+d^2}}{dx}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x, algorithm="fricas")
```

output
```
-(e*x*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + sqrt(-e^2*x^2 + d^2))/(d*x)
```

3.98.6 Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx = \int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x^2\left(d+ex\right)}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**(1/2)/x**2/(e*x+d),x)
```

output
```
Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**2*(d + e*x)), x)
```

3.98.7 Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx = \int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)x^2}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^2), x)
```

3.98.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(47) = 94$.

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx = \frac{e^4x}{2\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d|e|} + \frac{e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d|e|} - \frac{de+\sqrt{-e^2x^2+d^2}|e|}{2\,dx|e|}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x, algorithm="giac")
```

output
```
1/2*e^4*x/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d*abs(e)) + e^2*log(1/2*abs
(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d*abs(e)) - 1/2*(d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(d*x*abs(e))
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx = \int \frac{\sqrt{d^2-e^2\,x^2}}{x^2\,\left(d+e\,x\right)}\,dx$$

input
```
int((d^2 - e^2*x^2)^(1/2)/(x^2*(d + e*x)),x)
```

output
```
int((d^2 - e^2*x^2)^(1/2)/(x^2*(d + e*x)), x)
```

3.99 $\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx$

3.99.1 Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx=-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}+\frac{e\sqrt{d^2-e^2x^2}}{d^2x}-\frac{e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^2}$$

output
```
-1/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2-1/2*(-e^2*x^2+d^2)^(1/2)/d/x^
2+e*(-e^2*x^2+d^2)^(1/2)/d^2/x
```

3.99.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx=-\frac{d(d-2ex)\sqrt{d^2-e^2x^2}+\sqrt{d^2}e^2x^2\log(x)-\sqrt{d^2}e^2x^2\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{2d^3x^2}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)),x]
```

output
```
-1/2*(d*(d - 2*e*x)*Sqrt[d^2 - e^2*x^2] + Sqrt[d^2]*e^2*x^2*Log[x] - Sqrt[
d^2]*e^2*x^2*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(d^3*x^2)
```

3.99.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {566, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx$$

$$\Big\downarrow \text{566}$$

$$\int \frac{d-ex}{x^3\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{539}$$

$$-\frac{\int \frac{de(2d-ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e\int \frac{2d-ex}{x^2\sqrt{d^2-e^2x^2}}dx}{2d}-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}$$

$$\Big\downarrow \text{534}$$

$$-\frac{e\left(-e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{2\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}$$

$$\Big\downarrow \text{243}$$

$$-\frac{e\left(-\frac{1}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{2\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}$$

$$\Big\downarrow \text{73}$$

$$-\frac{e\left(\frac{\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{2\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{e\left(\frac{e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{2\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}$$

input
```
Int[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)),x]
```

output
```
-1/2*Sqrt[d^2 - e^2*x^2]/(d*x^2) - (e*((-2*Sqrt[d^2 - e^2*x^2])/(d*x) + (e
*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/(2*d)
```

3.99.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 566
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Int[x^m*(a/c + b*(x/d))*(a + b*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0]
```

3.99.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-2ex+d)}{2d^2x^2}-\frac{e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d\sqrt{d^2}}$
default	$\frac{-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{2d^2x^2}-\frac{e^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)}{2d^2}}{d}+\frac{e^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)}{d^3}-$ …

input
```
int((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e^2*x^2+d^2)^(1/2)*(-2*e*x+d)/d^2/x^2-1/2/d*e^2/(d^2)^(1/2)*ln((2*d
^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx = \frac{e^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\sqrt{-e^2x^2+d^2}(2\,ex-d)}{2\,d^2x^2}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x, algorithm="fricas")
```

output
```
1/2*(e^2*x^2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + sqrt(-e^2*x^2 + d^2)*(2*
e*x - d))/(d^2*x^2)
```

3.99.6 Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx=\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x^3\left(d+ex\right)}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**(1/2)/x**3/(e*x+d),x)
```

output
```
Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**3*(d + e*x)), x)
```

3.99.7 Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx=\int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)x^3}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^3), x)
```

3.99.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(72) = 144$.

Time = 0.28 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.35

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx=\frac{\left(e^3-\frac{4\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e}{x}\right)e^4x^2}{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^2|e|}-\frac{e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,d^2|e|}+\frac{\frac{4\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2e|e|}{x}-\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^2|e|}{ex^2}}{8\,d^4e^2}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x, algorithm="giac")
```

output
```
1/8*(e^3 - 4*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e/x)*e^4*x^2/((d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^2*d^2*abs(e)) - 1/2*e^3*log(1/2*abs(-2*d*e - 2*sq
rt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^2*abs(e)) + 1/8*(4*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))*d^2*e*abs(e)/x - (d*e + sqrt(-e^2*x^2 + d^2)*abs(
e))^2*d^2*abs(e)/(e*x^2))/(d^4*e^2)
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx=\int \frac{\sqrt{d^2-e^2\,x^2}}{x^3\,\left(d+e\,x\right)}\,dx$$

input
```
int((d^2 - e^2*x^2)^(1/2)/(x^3*(d + e*x)),x)
```

output
```
int((d^2 - e^2*x^2)^(1/2)/(x^3*(d + e*x)), x)
```

3.100 $\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx$

3.100.1 Optimal result

Integrand size = 27, antiderivative size = 114

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{3dx^3}+\frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2}-\frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x}+\frac{e^3\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3}$$

output
```
1/2*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3-1/3*(-e^2*x^2+d^2)^(1/2)/d/x^3
+1/2*e*(-e^2*x^2+d^2)^(1/2)/d^2/x^2-2/3*e^2*(-e^2*x^2+d^2)^(1/2)/d^3/x
```

3.100.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx = \frac{(-2d^2+3dex-4e^2x^2)\sqrt{d^2-e^2x^2}}{6d^3x^3}+\frac{\sqrt{d^2}e^3\log(x)}{2d^4}-\frac{\sqrt{d^2}e^3\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{2d^4}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)),x]
```

output
```
((-2*d^2 + 3*d*e*x - 4*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(6*d^3*x^3) + (Sqrt[d
^2]*e^3*Log[x])/(2*d^4) - (Sqrt[d^2]*e^3*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^
2]])/(2*d^4)
```

3.100.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {566, 539, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}dx$$

$$\Big\downarrow \text{ 566}$$

$$\int \frac{d-ex}{x^4\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{ 539}$$

$$-\frac{\int \frac{de(3d-2ex)}{x^3\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{\sqrt{d^2-e^2x^2}}{3dx^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{e\int \frac{3d-2ex}{x^3\sqrt{d^2-e^2x^2}}dx}{3d}-\frac{\sqrt{d^2-e^2x^2}}{3dx^3}$$

$$\Big\downarrow \text{ 539}$$

$$-\frac{e\left(-\frac{\int \frac{de(4d-3ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{3\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{3d}-\frac{\sqrt{d^2-e^2x^2}}{3dx^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{e\left(-\frac{e\int \frac{4d-3ex}{x^2\sqrt{d^2-e^2x^2}}dx}{2d}-\frac{3\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{3d}-\frac{\sqrt{d^2-e^2x^2}}{3dx^3}$$

$$\Big\downarrow \text{ 534}$$

$$-\frac{e\left(-\frac{e\left(-3e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{4\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{3\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{3d}-\frac{\sqrt{d^2-e^2x^2}}{3dx^3}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{e\left(-\frac{e\left(-\frac{3}{2}e\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{4\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{3\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{3d}-\frac{\sqrt{d^2-e^2x^2}}{3dx^3}$$

$$\Big\downarrow \text{73}$$

$$-\frac{e\left(-\frac{e\left(\frac{3\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{4\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{3\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{3d}-\frac{\sqrt{d^2-e^2x^2}}{3dx^3}$$

$$\Big\downarrow \text{221}$$

$$-\frac{e\left(-\frac{e\left(\frac{3e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{4\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{3\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{3d}-\frac{\sqrt{d^2-e^2x^2}}{3dx^3}$$

input
```
Int[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)),x]
```

output
```
-1/3*Sqrt[d^2 - e^2*x^2]/(d*x^3) - (e*((-3*Sqrt[d^2 - e^2*x^2])/(2*d*x^2)
- (e*((-4*Sqrt[d^2 - e^2*x^2])/(d*x) + (3*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
)/d))/(2*d)))/(3*d)
```

3.100.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 566
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Int[x^m*(a/c + b*(x/d))*(a + b*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0]
```

3.100.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(4e^2x^2-3dex+2d^2)}{6d^3x^3}+\frac{e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^2\sqrt{d^2}}$
default	$-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3d^3x^3}+\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{d^2x}-\frac{2e^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{d^2}\right)}{d^3}-\frac{e^3\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d}\ldots}{\ldots}\right)}{\sqrt{\ldots}}\right)}{d^4}$

input
```
int((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-e^2*x^2+d^2)^(1/2)*(4*e^2*x^2-3*d*e*x+2*d^2)/d^3/x^3+1/2/d^2*e^3/(d
^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx = -\frac{3\,e^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(4\,e^2x^2-3\,dex+2\,d^2)\sqrt{-e^2x^2+d^2}}{6\,d^3x^3}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x, algorithm="fricas")
```

output
```
-1/6*(3*e^3*x^3*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (4*e^2*x^2 - 3*d*e*x
+ 2*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*x^3)
```

3.100.6 Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx = \int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x^4\left(d+ex\right)}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**(1/2)/x**4/(e*x+d),x)
```

output
```
Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**4*(d + e*x)), x)
```

3.100.7 Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx = \int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)x^4}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^4), x)
```

3.100.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(98) = 196.

Time = 0.29 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.24

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx = \frac{\left(e^4 - \frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^2}{x} + \frac{9\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{x^2}\right)e^6x^3}{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3 d^3|e|} + \frac{e^4\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,d^3|e|} - \frac{\frac{9\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^6e^4}{x} - \frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^6e^2}{x^2} + \frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^6}{x^3}}{24\,d^9e^2|e|}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x, algorithm="giac")
```

output
```
1/24*(e^4 - 3*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^2/x + 9*(d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))^2/x^2)*e^6*x^3/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^
3*d^3*abs(e)) + 1/2*e^4*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e)
)/(e^2*abs(x)))/(d^3*abs(e)) - 1/24*(9*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
*d^6*e^4/x - 3*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^6*e^2/x^2 + (d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^6/x^3)/(d^9*e^2*abs(e))
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx = \int \frac{\sqrt{d^2-e^2\,x^2}}{x^4\,\left(d+e\,x\right)}\,dx$$

input
```
int((d^2 - e^2*x^2)^(1/2)/(x^4*(d + e*x)),x)
```

output
```
int((d^2 - e^2*x^2)^(1/2)/(x^4*(d + e*x)), x)
```

3.101 $\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx$

3.101.1 Optimal result

Integrand size = 27, antiderivative size = 143

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{4dx^4}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2}+\frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x}-\frac{3e^4\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}$$

output
```
-3/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4-1/4*(-e^2*x^2+d^2)^(1/2)/d/x^
4+1/3*e*(-e^2*x^2+d^2)^(1/2)/d^2/x^3-3/8*e^2*(-e^2*x^2+d^2)^(1/2)/d^3/x^2+
2/3*e^3*(-e^2*x^2+d^2)^(1/2)/d^4/x
```

3.101.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx = \frac{\sqrt{d^2-e^2x^2}(-6d^3+8d^2ex-9de^2x^2+16e^3x^3)}{24d^4x^4}-\frac{3\sqrt{d^2}e^4\log(x)}{8d^5}+\frac{3\sqrt{d^2}e^4\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{8d^5}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^2]/(x^5*(d + e*x)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-6*d^3 + 8*d^2*e*x - 9*d*e^2*x^2 + 16*e^3*x^3))/(24*
d^4*x^4) - (3*Sqrt[d^2]*e^4*Log[x])/(8*d^5) + (3*Sqrt[d^2]*e^4*Log[Sqrt[d^
2] - Sqrt[d^2 - e^2*x^2]])/(8*d^5)
```

3.101.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {566, 539, 27, 539, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}dx$$

$$\Big\downarrow \text{566}$$

$$\int \frac{d-ex}{x^5\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{539}$$

$$-\frac{\int \frac{de(4d-3ex)}{x^4\sqrt{d^2-e^2x^2}}dx}{4d^2}-\frac{\sqrt{d^2-e^2x^2}}{4dx^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e\int \frac{4d-3ex}{x^4\sqrt{d^2-e^2x^2}}dx}{4d}-\frac{\sqrt{d^2-e^2x^2}}{4dx^4}$$

$$\Big\downarrow \text{539}$$

$$-\frac{e\left(-\frac{\int \frac{de(9d-8ex)}{x^3\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{4\sqrt{d^2-e^2x^2}}{3dx^3}\right)}{4d}-\frac{\sqrt{d^2-e^2x^2}}{4dx^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e\left(-\frac{e\int \frac{9d-8ex}{x^3\sqrt{d^2-e^2x^2}}dx}{3d}-\frac{4\sqrt{d^2-e^2x^2}}{3dx^3}\right)}{4d}-\frac{\sqrt{d^2-e^2x^2}}{4dx^4}$$

$$\Big\downarrow \text{539}$$

$$-\frac{e\left(-\frac{e\left(-\frac{\int \frac{de(16d-9ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{9\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{3d}-\frac{4\sqrt{d^2-e^2x^2}}{3dx^3}\right)}{4d}-\frac{\sqrt{d^2-e^2x^2}}{4dx^4}$$

$\downarrow$ 27

$$-\frac{e\left(-\frac{e\left(-\frac{e\int \frac{16d-9ex}{x^2\sqrt{d^2-e^2x^2}}dx}{2d}-\frac{9\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{3d}-\frac{4\sqrt{d^2-e^2x^2}}{3dx^3}\right)}{4d}-\frac{\sqrt{d^2-e^2x^2}}{4dx^4}$$

$\downarrow$ 534

$$-\frac{e\left(-\frac{e\left(-\frac{e\left(-9e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{16\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{9\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{3d}-\frac{4\sqrt{d^2-e^2x^2}}{3dx^3}\right)}{4d}-\frac{\sqrt{d^2-e^2x^2}}{4dx^4}$$

$\downarrow$ 243

$$-\frac{e\left(-\frac{e\left(-\frac{e\left(-\frac{9}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{16\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{9\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{3d}-\frac{4\sqrt{d^2-e^2x^2}}{3dx^3}\right)}{4d}-\frac{\sqrt{d^2-e^2x^2}}{4dx^4}$$

$\downarrow$ 73

$$-\frac{e\left(-\frac{e\left(-\frac{e\left(\frac{9\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{16\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{9\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{3d}-\frac{4\sqrt{d^2-e^2x^2}}{3dx^3}\right)}{4d}-\frac{\sqrt{d^2-e^2x^2}}{4dx^4}$$

$\downarrow$ 221

3.101. $\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx$

$$-\frac{e\left(-\frac{e\left(-\frac{e\left(\frac{9e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{16\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{9\sqrt{d^2-e^2x^2}}{2dx^2}\right)}{3d}-\frac{4\sqrt{d^2-e^2x^2}}{3dx^3}\right)}{4d}-\frac{\sqrt{d^2-e^2x^2}}{4dx^4}$$

input
```
Int[Sqrt[d^2 - e^2*x^2]/(x^5*(d + e*x)),x]
```

output
```
-1/4*Sqrt[d^2 - e^2*x^2]/(d*x^4) - (e*((-4*Sqrt[d^2 - e^2*x^2])/(3*d*x^3)
- (e*((-9*Sqrt[d^2 - e^2*x^2])/(2*d*x^2) - (e*((-16*Sqrt[d^2 - e^2*x^2])/(
d*x) + (9*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/(2*d)))/(3*d)))/(4*d)
```

3.101.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 566
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Int[x^m*(a/c + b*(x/d))*(a + b*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0]
```

3.101.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,\left(-16e^3x^3+9d\,e^2x^2-8d^2ex+6d^3\right)}{24d^4x^4}-\frac{3e^4\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8d^3\sqrt{d^2}}$
default	$\frac{-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4d^2x^4}+\frac{e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{2d^2x^2}-\frac{e^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)}{2d^2}\right)}{4d^2}}{d}+\frac{e^4\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(2d^2+2\sqrt{d^2}\right.}{\sqrt{d}}\right.}{d^5}$

input
```
int((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-e^2*x^2+d^2)^(1/2)*(-16*e^3*x^3+9*d*e^2*x^2-8*d^2*e*x+6*d^3)/d^4/x
^4-3/8/d^3*e^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x
)
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx = \frac{9\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(16\,e^3x^3-9\,de^2x^2+8\,d^2ex-6\,d^3)\sqrt{-e^2x^2+d^2}}{24\,d^4x^4}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x, algorithm="fricas")
```

output
```
1/24*(9*e^4*x^4*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (16*e^3*x^3 - 9*d*e^2
*x^2 + 8*d^2*e*x - 6*d^3)*sqrt(-e^2*x^2 + d^2))/(d^4*x^4)
```

3.101.6 Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx = \int \frac{\sqrt{-(-d+ex)(d+ex)}}{x^5(d+ex)}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**(1/2)/x**5/(e*x+d),x)
```

output
```
Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**5*(d + e*x)), x)
```

3.101.7 Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx = \int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)x^5}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^5), x)
```

3.101.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(123) = 246$.

Time = 0.28 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.30

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx$$
$$= \frac{\left(3\,e^5 - \frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^3}{x} + \frac{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 e}{x^2} - \frac{72\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{ex^3}\right)e^8x^4}{192\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4 d^4|e|}$$
$$- \frac{3\,e^5\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{8\,d^4|e|}$$
$$+ \frac{\frac{72\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{12}e^5|e|}{x} - \frac{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d^{12}e^3|e|}{x^2} + \frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3 d^{12}e|e|}{x^3} - \frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4 d^{12}|}{ex^4}}{192\,d^{16}e^4}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x, algorithm="giac")
```

output
```
1/192*(3*e^5 - 8*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^3/x + 24*(d*e + sqr
t(-e^2*x^2 + d^2)*abs(e))^2*e/x^2 - 72*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
^3/(e*x^3))*e^8*x^4/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^4*abs(e)) - 3
/8*e^5*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(
d^4*abs(e)) + 1/192*(72*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^12*e^5*abs(e
)/x - 24*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^12*e^3*abs(e)/x^2 + 8*(d*
e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^12*e*abs(e)/x^3 - 3*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^4*d^12*abs(e)/(e*x^4))/(d^16*e^4)
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx = \int \frac{\sqrt{d^2-e^2\,x^2}}{x^5\,(d+e\,x)}\,dx$$

input
```
int((d^2 - e^2*x^2)^(1/2)/(x^5*(d + e*x)),x)
```

output
```
int((d^2 - e^2*x^2)^(1/2)/(x^5*(d + e*x)), x)
```

3.102 $\int \frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex}\,dx$

3.102.1 Optimal result

Integrand size = 27, antiderivative size = 113

$$\int \frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex}\,dx = \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2} + \frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3} + \frac{d^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

output
```
1/12*d*(-3*e*x+4*d)*(-e^2*x^2+d^2)^(3/2)/e^3-1/5*(-e^2*x^2+d^2)^(5/2)/e^3+
1/8*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/8*d^3*x*(-e^2*x^2+d^2)^(1/2
)/e^2
```

3.102.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int \frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex}\,dx = \frac{\sqrt{d^2-e^2x^2}(16d^4-15d^3ex+8d^2e^2x^2+30de^3x^3-24e^4x^4)-30d^5\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{120e^3}$$

input
```
Integrate[(x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(16*d^4 - 15*d^3*e*x + 8*d^2*e^2*x^2 + 30*d*e^3*x^3 -
 24*e^4*x^4) - 30*d^5*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(12
0*e^3)
```

3.102.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.30, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {562, 533, 25, 27, 533, 25, 27, 455, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(d^2-e^2 x^2\right)^{3/2}}{d+e x} \, dx$$

$$\Big\downarrow \text{562}$$

$$\int x^2 (d-e x) \sqrt{d^2-e^2 x^2}dx$$

$$\Big\downarrow \text{533}$$

$$\frac{\int -d e x (2 d-5 e x) \sqrt{d^2-e^2 x^2}dx}{5 e^2}+\frac{x^2 \left(d^2-e^2 x^2\right)^{3/2}}{5 e}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^2 \left(d^2-e^2 x^2\right)^{3/2}}{5 e}-\frac{\int d e x (2 d-5 e x) \sqrt{d^2-e^2 x^2}dx}{5 e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2 \left(d^2-e^2 x^2\right)^{3/2}}{5 e}-\frac{d \int x (2 d-5 e x) \sqrt{d^2-e^2 x^2}dx}{5 e}$$

$$\Big\downarrow \text{533}$$

$$\frac{x^2 \left(d^2-e^2 x^2\right)^{3/2}}{5 e}-\frac{d \left(\frac{\int -d e (5 d-8 e x) \sqrt{d^2-e^2 x^2}dx}{4 e^2}+\frac{5 x \left(d^2-e^2 x^2\right)^{3/2}}{4 e}\right)}{5 e}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^2 \left(d^2-e^2 x^2\right)^{3/2}}{5 e}-\frac{d \left(\frac{5 x \left(d^2-e^2 x^2\right)^{3/2}}{4 e}-\frac{\int d e (5 d-8 e x) \sqrt{d^2-e^2 x^2}dx}{4 e^2}\right)}{5 e}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2 \left(d^2-e^2 x^2\right)^{3/2}}{5 e}-\frac{d \left(\frac{5 x \left(d^2-e^2 x^2\right)^{3/2}}{4 e}-\frac{d \int (5 d-8 e x) \sqrt{d^2-e^2 x^2}dx}{4 e}\right)}{5 e}$$

$$\Big\downarrow \text{455}$$

$$\frac{x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{5x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\left(5d\int\sqrt{d^2-e^2x^2}dx+\frac{8\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)}{4e}\right)}{5e}$$

$\downarrow$ 211

$$\frac{x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{5x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\left(5d\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{8\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)}{4e}\right)}{5e}$$

$\downarrow$ 224

$$\frac{x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{5x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\left(5d\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{8\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)}{4e}\right)}{5e}$$

$\downarrow$ 216

$$\frac{x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{5x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\left(5d\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{8\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)}{4e}\right)}{5e}$$

input
```
Int[(x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x),x]
```

output
```
(x^2*(d^2 - e^2*x^2)^(3/2))/(5*e) - (d*((5*x*(d^2 - e^2*x^2)^(3/2))/(4*e)
- (d*((8*(d^2 - e^2*x^2)^(3/2))/(3*e) + 5*d*((x*Sqrt[d^2 - e^2*x^2])/2 + (
d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e))))/(4*e)))/(5*e)
```

3.102. $\int \frac{x^2\left(d^2-e^2x^2\right)^{3/2}}{d+ex}\,dx$

3.102.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 562
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[c^(2*n)/a^n   Int[x^m*((a + b*x^2)^(n + p)/(c - d*x)^n), x], x]
/; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[m, 0] && ILtQ[n,
 0] && IGtQ[n + p + 1/2, 0]
```

3.102. $\int \frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex}\,dx$

3.102.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

method	result
risch	$\frac{(-24e^4x^4+30d e^3x^3+8d^2e^2x^2-15d^3ex+16d^4)\sqrt{-e^2x^2+d^2}}{120e^3}+\frac{d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8e^2\sqrt{e^2}}$
default	$-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{5e^3}-\frac{d\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{e^2}+\frac{d^2\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3}+d \cdots\right.}{\cdots}$

input
```
int(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/120*(-24*e^4*x^4+30*d*e^3*x^3+8*d^2*e^2*x^2-15*d^3*e*x+16*d^4)/e^3*(-e^2
*x^2+d^2)^(1/2)+1/8*d^5/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2
)^(1/2))
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

$$\int \frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex}\,dx = -\frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(24\,e^4x^4-30\,de^3x^3-8\,d^2e^2x^2+15\,d^3ex-16\,d^4)\sqrt{-e^2x^2+d^2}}{120\,e^3}$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x, algorithm="fricas")
```

output
```
-1/120*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (24*e^4*x^4 - 3
0*d*e^3*x^3 - 8*d^2*e^2*x^2 + 15*d^3*e*x - 16*d^4)*sqrt(-e^2*x^2 + d^2))/e
^3
```

3.102.6 Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.48

$$\int \frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex}\,dx = d\left(\begin{cases} \frac{d^4\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{8e^2}+\sqrt{d^2-e^2x^2}\left(-\frac{d^2x}{8e^2}+\frac{x^3}{4}\right) & \text{for } e^2\neq 0 \\ \frac{x^3\sqrt{d^2}}{3} & \text{otherwise}\end{cases}\right) - e\left(\begin{cases} \sqrt{d^2-e^2x^2}\left(-\frac{2d^4}{15e^4}-\frac{d^2x^2}{15e^2}+\frac{x^4}{5}\right) & \text{for } e^2\neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**2*(-e**2*x**2+d**2)**(3/2)/(e*x+d),x)
```

output
```
d*Piecewise((d**4*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**
2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(8*
e**2) + sqrt(d**2 - e**2*x**2)*(-d**2*x/(8*e**2) + x**3/4), Ne(e**2, 0)),
(x**3*sqrt(d**2)/3, True)) - e*Piecewise((sqrt(d**2 - e**2*x**2)*(-2*d**4/
(15*e**4) - d**2*x**2/(15*e**2) + x**4/5), Ne(e**2, 0)), (x**4*sqrt(d**2)/
4, True))
```

3.102.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.54

$$\int \frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex}\,dx = -\frac{i\,d^5\arcsin\left(\frac{ex}{d}+2\right)}{2\,e^3} - \frac{3\,d^5\arcsin\left(\frac{ex}{d}\right)}{8\,e^3} + \frac{\sqrt{e^2x^2+4\,dex+3\,d^2}d^3x}{2\,e^2} - \frac{3\sqrt{-e^2x^2+d^2}d^3x}{8\,e^2} + \frac{\sqrt{e^2x^2+4\,dex+3\,d^2}d^4}{e^3} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}dx}{4\,e^2} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d^2}{3\,e^3} - \frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{5\,e^3}$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x, algorithm="maxima")
```

output
```
-1/2*I*d^5*arcsin(e*x/d + 2)/e^3 - 3/8*d^5*arcsin(e*x/d)/e^3 + 1/2*sqrt(e^
2*x^2 + 4*d*e*x + 3*d^2)*d^3*x/e^2 - 3/8*sqrt(-e^2*x^2 + d^2)*d^3*x/e^2 +
sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^4/e^3 - 1/4*(-e^2*x^2 + d^2)^(3/2)*d*x/e
^2 + 1/3*(-e^2*x^2 + d^2)^(3/2)*d^2/e^3 - 1/5*(-e^2*x^2 + d^2)^(5/2)/e^3
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.73

$$\int \frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex}\,dx = \frac{d^5\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,e^2|e|} - \frac{1}{120}\sqrt{-e^2x^2+d^2}\left(\left(2\left(3\,(4\,ex-5\,d)x-\frac{4\,d^2}{e}\right)x+\frac{15\,d^3}{e^2}\right)x-\frac{16\,d^4}{e^3}\right)$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x, algorithm="giac")
```

output
```
1/8*d^5*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) - 1/120*sqrt(-e^2*x^2 + d
^2)*((2*(3*(4*e*x - 5*d)*x - 4*d^2/e)*x + 15*d^3/e^2)*x - 16*d^4/e^3)
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex}\,dx = \int \frac{x^2\,(d^2-e^2\,x^2)^{3/2}}{d+e\,x}\,dx$$

input
```
int((x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x),x)
```

output
```
int((x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x), x)
```

3.103 $\int \frac{x^4 (d^2-e^2x^2)^{5/2}}{d+ex} \, dx$

3.103.1 Optimal result

Integrand size = 27, antiderivative size = 201

$$\int \frac{x^4 (d^2-e^2x^2)^{5/2}}{d+ex} \, dx = \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4} + \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3}-\frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2}+\frac{x^4(d^2-e^2x^2)^{5/2}}{9e} + \frac{d^3(128d-315ex)\left(d^2-e^2x^2\right)^{5/2}}{5040e^5}+\frac{3d^9\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}$$

output
```
1/64*d^5*x*(-e^2*x^2+d^2)^(3/2)/e^4+4/63*d^2*x^2*(-e^2*x^2+d^2)^(5/2)/e^3-
1/8*d*x^3*(-e^2*x^2+d^2)^(5/2)/e^2+1/9*x^4*(-e^2*x^2+d^2)^(5/2)/e+1/5040*d
^3*(-315*e*x+128*d)*(-e^2*x^2+d^2)^(5/2)/e^5+3/128*d^9*arctan(e*x/(-e^2*x^
2+d^2)^(1/2))/e^5+3/128*d^7*x*(-e^2*x^2+d^2)^(1/2)/e^4
```

3.103.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.73

$$\int \frac{x^4 (d^2-e^2x^2)^{5/2}}{d+ex} \, dx = \frac{\sqrt{d^2-e^2x^2}(1024d^8-945d^7ex+512d^6e^2x^2-630d^5e^3x^3+384d^4e^4x^4+7560d^3e^5x}{40320e^5}$$

input
```
Integrate[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(1024*d^8 - 945*d^7*e*x + 512*d^6*e^2*x^2 - 630*d^5*e
^3*x^3 + 384*d^4*e^4*x^4 + 7560*d^3*e^5*x^5 - 6400*d^2*e^6*x^6 - 5040*d*e^
7*x^7 + 4480*e^8*x^8) - 1890*d^9*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*
x^2])])/(40320*e^5)
```

3.103.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.22, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.630$, Rules used = {562, 533, 25, 27, 533, 25, 27, 533, 25, 27, 533, 27, 455, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(d^2-e^2 x^2\right)^{5/2}}{d+e x} \, dx$$

$$\Big\downarrow \ 562$$

$$\int x^4 (d-e x) \left(d^2-e^2 x^2\right)^{3/2} dx$$

$$\Big\downarrow \ 533$$

$$\frac{\int -d e x^3 (4 d-9 e x) \left(d^2-e^2 x^2\right)^{3/2} dx}{9 e^2}+\frac{x^4 \left(d^2-e^2 x^2\right)^{5/2}}{9 e}$$

$$\Big\downarrow \ 25$$

$$\frac{x^4 \left(d^2-e^2 x^2\right)^{5/2}}{9 e}-\frac{\int d e x^3 (4 d-9 e x) \left(d^2-e^2 x^2\right)^{3/2} dx}{9 e^2}$$

$$\Big\downarrow \ 27$$

$$\frac{x^4 \left(d^2-e^2 x^2\right)^{5/2}}{9 e}-\frac{d \int x^3 (4 d-9 e x) \left(d^2-e^2 x^2\right)^{3/2} dx}{9 e}$$

$$\Big\downarrow \ 533$$

$$\frac{x^4 \left(d^2-e^2 x^2\right)^{5/2}}{9 e}-\frac{d \left(\frac{\int -d e x^2 (27 d-32 e x) \left(d^2-e^2 x^2\right)^{3/2} dx}{8 e^2}+\frac{9 x^3 \left(d^2-e^2 x^2\right)^{5/2}}{8 e}\right)}{9 e}$$

$$\Big\downarrow \ 25$$

$$\frac{x^4 \left(d^2-e^2 x^2\right)^{5/2}}{9 e}-\frac{d \left(\frac{9 x^3 \left(d^2-e^2 x^2\right)^{5/2}}{8 e}-\frac{\int d e x^2 (27 d-32 e x) \left(d^2-e^2 x^2\right)^{3/2} dx}{8 e^2}\right)}{9 e}$$

$$\Big\downarrow \ 27$$

$$\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}-\frac{d\left(\frac{9x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d\int x^2(27d-32ex)\left(d^2-e^2x^2\right)^{3/2}dx}{8e}\right)}{9e}$$

$$\Big\downarrow \ 533$$

$$\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}-\frac{d\left(\frac{9x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d\left(\frac{\int -dex(64d-189ex)\left(d^2-e^2x^2\right)^{3/2}dx}{7e^2}+\frac{32x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}\right)}{8e}\right)}{9e}$$

$$\Big\downarrow \ 25$$

$$\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}-\frac{d\left(\frac{9x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d\left(\frac{32x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}-\frac{\int dex(64d-189ex)\left(d^2-e^2x^2\right)^{3/2}dx}{7e^2}\right)}{8e}\right)}{9e}$$

$$\Big\downarrow \ 27$$

$$\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}-\frac{d\left(\frac{9x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d\left(\frac{32x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}-\frac{d\int x(64d-189ex)\left(d^2-e^2x^2\right)^{3/2}dx}{7e}\right)}{8e}\right)}{9e}$$

$$\Big\downarrow \ 533$$

$$\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}-\frac{d\left(\frac{9x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d\left(\frac{32x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}-\frac{d\left(\frac{\int -3de(63d-128ex)\left(d^2-e^2x^2\right)^{3/2}dx}{6e^2}+\frac{63x\left(d^2-e^2x^2\right)^{5/2}}{2e}\right)}{7e}\right)}{8e}\right)}{9e}$$

$$\Big\downarrow \ 27$$

3.103. $\int \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx$

$$\frac{\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}-d\left(\frac{9x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d\left(\frac{32x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}-\frac{d\left(\frac{63x\left(d^2-e^2x^2\right)^{5/2}}{2e}-\frac{d\int(63d-128ex)\left(d^2-e^2x^2\right)^{3/2}dx}{2e}\right)}{7e}\right)}{8e}\right)}{9e}$$

$$\Big\downarrow \; 455$$

$$\frac{\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}-d\left(\frac{9x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d\left(\frac{32x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}-\frac{d\left(\frac{63x\left(d^2-e^2x^2\right)^{5/2}}{2e}-\frac{d\left(63d\int\left(d^2-e^2x^2\right)^{3/2}dx+\frac{128\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{2e}\right)}{7e}\right)}{8e}\right)}{9e}$$

$$\Big\downarrow \; 211$$

3.103. $\int \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx$

$$\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}-\frac{d\left(\frac{9x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d\left(\frac{32x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}-\frac{d\left(\frac{63x\left(d^2-e^2x^2\right)^{5/2}}{2e}-\frac{d\left(63d\left(\frac{3}{4}d^2\int\sqrt{d^2-e^2x^2}dx+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{128\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{2e}\right)}{7e}\right)}{8e}\right)}{9e}$$

$\downarrow$ 211

$$\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}-\frac{d\left(\frac{9x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d\left(\frac{32x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}-\frac{d\left(\frac{63x\left(d^2-e^2x^2\right)^{5/2}}{2e}-\frac{d\left(63d\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{128(\cdots}{\cdots}\right.}{2e}\right.}{7e}\right.}{8e}\right.}{9e}$$

$\downarrow$ 224

3.103. $\int \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx$

$$\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}-\frac{d\left(\frac{9x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d\left(\frac{32x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}-\frac{d\left(\frac{63x\left(d^2-e^2x^2\right)^{5/2}}{2e}-\frac{d\left(63d\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right.\right.\right.}{2e}\right.}{7e}\right.}{8e}\right.}{9e}$$

$\downarrow$ 216

$$\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}-\frac{d\left(\frac{9x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d\left(\frac{32x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}-\frac{d\left(\frac{63x\left(d^2-e^2x^2\right)^{5/2}}{2e}-\frac{d\left(63d\left(\frac{3}{4}d^2\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\right.}{2e}\right.}{7e}\right.}{8e}\right.}{9e}$$

input
```
Int[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

3.103. $\int\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx$

output
```
(x^4*(d^2 - e^2*x^2)^(5/2))/(9*e) - (d*((9*x^3*(d^2 - e^2*x^2)^(5/2))/(8*e
) - (d*((32*x^2*(d^2 - e^2*x^2)^(5/2))/(7*e) - (d*((63*x*(d^2 - e^2*x^2)^(
5/2))/(2*e) - (d*((128*(d^2 - e^2*x^2)^(5/2))/(5*e) + 63*d*((x*(d^2 - e^2*
x^2)^(3/2))/4 + (3*d^2*((x*Sqrt[d^2 - e^2*x^2])/2 + (d^2*ArcTan[(e*x)/Sqrt
[d^2 - e^2*x^2]])/(2*e)))/4)))/(2*e)))/(7*e)))/(8*e)))/(9*e)
```

3.103.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 562
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[c^(2*n)/a^n   Int[x^m*((a + b*x^2)^(n + p)/(c - d*x)^n), x], x]
/; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[m, 0] && ILtQ[n,
 0] && IGtQ[n + p + 1/2, 0]
```

3.103.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.70

method	result
risch	$\frac{\left(4480e^{8}x^{8}-5040d\,e^{7}x^{7}-6400d^{2}e^{6}x^{6}+7560d^{3}e^{5}x^{5}+384d^{4}x^{4}e^{4}-630d^{5}e^{3}x^{3}+512d^{6}e^{2}x^{2}-945d^{7}ex+1024d^{8}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{40320e^{5}}+\frac{3d^{9}\,\mathrm{arc}}{}$
default	$\frac{-\frac{x^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{9e^{2}}-\frac{2d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{63e^{4}}}{e}-\frac{d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{7e^{5}}-\frac{d^{3}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}+\frac{5d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}}{}\right)}{}\right)}{6}\right)}{e^{4}}$

input
```
int(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/40320*(4480*e^8*x^8-5040*d*e^7*x^7-6400*d^2*e^6*x^6+7560*d^3*e^5*x^5+384
*d^4*e^4*x^4-630*d^5*e^3*x^3+512*d^6*e^2*x^2-945*d^7*e*x+1024*d^8)/e^5*(-e
^2*x^2+d^2)^(1/2)+3/128*d^9/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2
+d^2)^(1/2))
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.69

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{d+ex}\,dx =$$
$$-\frac{1890\,d^9\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(4480\,e^8x^8-5040\,de^7x^7-6400\,d^2e^6x^6+7560\,d^3e^5x^5+384\,d^4e^4x^4-63}{40320\,e^5}$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

output
```
-1/40320*(1890*d^9*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (4480*e^8*x
^8 - 5040*d*e^7*x^7 - 6400*d^2*e^6*x^6 + 7560*d^3*e^5*x^5 + 384*d^4*e^4*x^
4 - 630*d^5*e^3*x^3 + 512*d^6*e^2*x^2 - 945*d^7*e*x + 1024*d^8)*sqrt(-e^2*
x^2 + d^2))/e^5
```

3.103.6 Sympy [A] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.12

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = d^3\left(\begin{cases}\frac{d^6\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{16e^4}+\sqrt{d^2-e^2x^2}\left(-\frac{d^4x}{16e^4}-\frac{d^2x^3}{24e^2}+\cdots\right. \\ \frac{x^5\sqrt{d^2}}{5}\end{cases}\right.$$
$$-d^2e\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{8d^6}{105e^6}-\frac{4d^4x^2}{105e^4}-\frac{d^2x^4}{35e^2}+\frac{x^6}{7}\right) & \text{for } e^2\neq 0\\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise}\end{cases}\right)$$
$$-de^2\left(\begin{cases}\frac{5d^8\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{128e^6}+\sqrt{d^2-e^2x^2}\left(-\frac{5d^6x}{128e^6}-\frac{5d^4x^3}{192e^4}-\frac{d^2x^5}{48e^2}+\frac{x^7}{8}\right) & \text{for } e^2\neq \\ \frac{x^7\sqrt{d^2}}{7} & \text{otherwi}\end{cases}\right.$$
$$+e^3\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{16d^8}{315e^8}-\frac{8d^6x^2}{315e^6}-\frac{2d^4x^4}{105e^4}-\frac{d^2x^6}{63e^2}+\frac{x^8}{9}\right) & \text{for } e^2\neq 0\\ \frac{x^8\sqrt{d^2}}{8} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**4*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

output
```
d**3*Piecewise((d**6*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 -
e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/
(16*e**4) + sqrt(d**2 - e**2*x**2)*(-d**4*x/(16*e**4) - d**2*x**3/(24*e**2
) + x**5/6), Ne(e**2, 0)), (x**5*sqrt(d**2)/5, True)) - d**2*e*Piecewise((
sqrt(d**2 - e**2*x**2)*(-8*d**6/(105*e**6) - 4*d**4*x**2/(105*e**4) - d**2
*x**4/(35*e**2) + x**6/7), Ne(e**2, 0)), (x**6*sqrt(d**2)/6, True)) - d*e*
*2*Piecewise((5*d**8*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 -
e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/
(128*e**6) + sqrt(d**2 - e**2*x**2)*(-5*d**6*x/(128*e**6) - 5*d**4*x**3/(1
92*e**4) - d**2*x**5/(48*e**2) + x**7/8), Ne(e**2, 0)), (x**7*sqrt(d**2)/7
, True)) + e**3*Piecewise((sqrt(d**2 - e**2*x**2)*(-16*d**8/(315*e**8) - 8
*d**6*x**2/(315*e**6) - 2*d**4*x**4/(105*e**4) - d**2*x**6/(63*e**2) + x**
8/9), Ne(e**2, 0)), (x**8*sqrt(d**2)/8, True))
```

3.103.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.22

$$\begin{aligned}\int \frac{x^4(d^2-e^2x^2)^{5/2}}{d+ex}\,dx =& -\frac{3i\,d^9\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^5} - \frac{45\,d^9\arcsin\left(\frac{ex}{d}\right)}{128\,e^5} \\ &+ \frac{3\sqrt{e^2x^2+4\,dex+3\,d^2}d^7x}{8\,e^4} - \frac{45\sqrt{-e^2x^2+d^2}d^7x}{128\,e^4} + \frac{3\sqrt{e^2x^2+4\,dex+3\,d^2}d^8}{4\,e^5} \\ &+ \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d^5x}{64\,e^4} - \frac{3\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3x}{16\,e^4} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}x^2}{9\,e^3} \\ &+ \frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^4}{5\,e^5} + \frac{(-e^2x^2+d^2)^{\frac{7}{2}}dx}{8\,e^4} - \frac{11\,(-e^2x^2+d^2)^{\frac{7}{2}}d^2}{63\,e^5}\end{aligned}$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

output
```
-3/8*I*d^9*arcsin(e*x/d + 2)/e^5 - 45/128*d^9*arcsin(e*x/d)/e^5 + 3/8*sqrt
(e^2*x^2 + 4*d*e*x + 3*d^2)*d^7*x/e^4 - 45/128*sqrt(-e^2*x^2 + d^2)*d^7*x/
e^4 + 3/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^8/e^5 + 1/64*(-e^2*x^2 + d^2)^
(3/2)*d^5*x/e^4 - 3/16*(-e^2*x^2 + d^2)^(5/2)*d^3*x/e^4 - 1/9*(-e^2*x^2 +
d^2)^(7/2)*x^2/e^3 + 1/5*(-e^2*x^2 + d^2)^(5/2)*d^4/e^5 + 1/8*(-e^2*x^2 +
d^2)^(7/2)*d*x/e^4 - 11/63*(-e^2*x^2 + d^2)^(7/2)*d^2/e^5
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.66

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{3\,d^9\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{128\,e^4|e|}$$
$$+\frac{1}{40320}\sqrt{-e^2x^2+d^2}\left(\frac{1024\,d^8}{e^5}-\left(\frac{945\,d^7}{e^4}-2\left(\frac{256\,d^6}{e^3}-\left(\frac{315\,d^5}{e^2}-4\left(\frac{48\,d^4}{e}+5\left(189\,d^3-2\left(80\,d^2e-7\right.\right.\right.\right.\right.\right.\right.$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

output
```
3/128*d^9*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^4*abs(e)) + 1/40320*sqrt(-e^2*x^2
 + d^2)*(1024*d^8/e^5 - (945*d^7/e^4 - 2*(256*d^6/e^3 - (315*d^5/e^2 - 4*(
48*d^4/e + 5*(189*d^3 - 2*(80*d^2*e - 7*(8*e^3*x - 9*d*e^2)*x)*x)*x)*x)*x)
*x)*x)
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \int \frac{x^4\left(d^2-e^2\,x^2\right)^{5/2}}{d+e\,x}\,dx$$

input
```
int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x)
```

output
```
int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x), x)
```

3.104 $\int \frac{x^3(d^2-e^2x^2)^{5/2}}{d+ex}\,dx$

3.104.1 Optimal result

Integrand size = 27, antiderivative size = 172

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = -\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} - \frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3} - \frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} + \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} - \frac{d^2(32d-35ex)\,(d^2-e^2x^2)^{5/2}}{560e^4} - \frac{3d^8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^4}$$

output
```
-1/64*d^4*x*(-e^2*x^2+d^2)^(3/2)/e^3-1/7*d*x^2*(-e^2*x^2+d^2)^(5/2)/e^2+1/
8*x^3*(-e^2*x^2+d^2)^(5/2)/e-1/560*d^2*(-35*e*x+32*d)*(-e^2*x^2+d^2)^(5/2)
/e^4-3/128*d^8*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4-3/128*d^6*x*(-e^2*x^2+
d^2)^(1/2)/e^3
```

3.104.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.79

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{\sqrt{d^2-e^2x^2}(-256d^7+105d^6ex-128d^5e^2x^2+70d^4e^3x^3+1024d^3e^4x^4-840d^2e^5x}{4480e^4}$$

input
```
Integrate[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-256*d^7 + 105*d^6*e*x - 128*d^5*e^2*x^2 + 70*d^4*e^
3*x^3 + 1024*d^3*e^4*x^4 - 840*d^2*e^5*x^5 - 640*d*e^6*x^6 + 560*e^7*x^7)
+ 210*d^8*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(4480*e^4)
```

3.104.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.23, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.519, Rules used = {562, 533, 25, 27, 533, 25, 27, 533, 27, 455, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(d^2-e^2 x^2\right)^{5/2}}{d+e x} \, dx$$

$$\Big\downarrow \text{562}$$

$$\int x^3 (d-e x) \left(d^2-e^2 x^2\right)^{3/2} \, dx$$

$$\Big\downarrow \text{533}$$

$$\frac{\int -d e x^2 (3 d-8 e x) \left(d^2-e^2 x^2\right)^{3/2} \, dx}{8 e^2}+\frac{x^3 \left(d^2-e^2 x^2\right)^{5/2}}{8 e}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^3 \left(d^2-e^2 x^2\right)^{5/2}}{8 e}-\frac{\int d e x^2 (3 d-8 e x) \left(d^2-e^2 x^2\right)^{3/2} \, dx}{8 e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3 \left(d^2-e^2 x^2\right)^{5/2}}{8 e}-\frac{d \int x^2 (3 d-8 e x) \left(d^2-e^2 x^2\right)^{3/2} \, dx}{8 e}$$

$$\Big\downarrow \text{533}$$

$$\frac{x^3 \left(d^2-e^2 x^2\right)^{5/2}}{8 e}-\frac{d \left(\frac{\int -d e x (16 d-21 e x) \left(d^2-e^2 x^2\right)^{3/2} dx}{7 e^2}+\frac{8 x^2 \left(d^2-e^2 x^2\right)^{5/2}}{7 e}\right)}{8 e}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^3 \left(d^2-e^2 x^2\right)^{5/2}}{8 e}-\frac{d \left(\frac{8 x^2 \left(d^2-e^2 x^2\right)^{5/2}}{7 e}-\frac{\int d e x (16 d-21 e x) \left(d^2-e^2 x^2\right)^{3/2} dx}{7 e^2}\right)}{8 e}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d\left(\frac{8x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}-\frac{d\int x(16d-21ex)\left(d^2-e^2x^2\right)^{3/2}dx}{7e}\right)}{8e}$$

$$\Big\downarrow \text{533}$$

$$\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d\left(\frac{8x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}-\frac{d\left(\frac{\int -3de(7d-32ex)\left(d^2-e^2x^2\right)^{3/2}dx}{6e^2}+\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{2e}\right)}{7e}\right)}{8e}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d\left(\frac{8x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}-\frac{d\left(\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{2e}-\frac{d\int (7d-32ex)\left(d^2-e^2x^2\right)^{3/2}dx}{2e}\right)}{7e}\right)}{8e}$$

$$\Big\downarrow \text{455}$$

$$\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d\left(\frac{8x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}-\frac{d\left(\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{2e}-\frac{d\left(7d\int \left(d^2-e^2x^2\right)^{3/2}dx+\frac{32\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{2e}\right)}{7e}\right)}{8e}$$

$$\Big\downarrow \text{211}$$

$$\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d\left(\frac{8x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}-\frac{d\left(\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{2e}-\frac{d\left(7d\left(\frac{3}{4}d^2\int \sqrt{d^2-e^2x^2}dx+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{32\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{2e}\right)}{7e}\right)}{8e}$$

$$\Big\downarrow \text{211}$$

3.104. $\int \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx$

$$\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d\left(\frac{8x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}-\frac{d\left(\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{2e}-\frac{d\left(7d\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{32\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{2e}\right)}{7e}\right)}{8e}$$

$$\Big\downarrow \text{224}$$

$$\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d\left(\frac{8x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}-\frac{d\left(\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{2e}-\frac{d\left(7d\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{32\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{2e}\right)}{7e}\right)}{8e}$$

$$\Big\downarrow \text{216}$$

$$\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d\left(\frac{8x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}-\frac{d\left(\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{2e}-\frac{d\left(7d\left(\frac{3}{4}d^2\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{32\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{2e}\right)}{7e}\right)}{8e}$$

input
```
Int[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

3.104. $\int \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx$

output
```
(x^3*(d^2 - e^2*x^2)^(5/2))/(8*e) - (d*((8*x^2*(d^2 - e^2*x^2)^(5/2))/(7*e
) - (d*((7*x*(d^2 - e^2*x^2)^(5/2))/(2*e) - (d*((32*(d^2 - e^2*x^2)^(5/2))
/(5*e) + 7*d*((x*(d^2 - e^2*x^2)^(3/2))/4 + (3*d^2*((x*Sqrt[d^2 - e^2*x^2]
)/2 + (d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)))/4)))/(2*e)))/(7*e)))
/(8*e)
```

3.104.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.104. $\int \frac{x^3(d^2-e^2x^2)^{5/2}}{d+ex}\,dx$

rule 562
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[c^(2*n)/a^n   Int[x^m*((a + b*x^2)^(n + p)/(c - d*x)^n), x], x]
/; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[m, 0] && ILtQ[n,
 0] && IGtQ[n + p + 1/2, 0]
```

3.104.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\left(-560e^7x^7+640d\,e^6x^6+840d^2e^5x^5-1024d^3e^4x^4-70d^4e^3x^3+128d^5e^2x^2-105d^6ex+256d^7\right)\sqrt{-e^2x^2+d^2}}{4480e^4}-\frac{3d^8\arctan\left(\frac{\sqrt{e^2}}{\sqrt{-e^2x^2}}\right)}{128e^3\sqrt{e^2}}$
default	$\frac{-\frac{x\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{8e^2}+\frac{d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6}\right)}{8e^2}}{e}+\frac{d^2\left(\frac{x\left(-e^2x^2\right.}{6}\right.}{}$

input
```
int(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4480*(-560*e^7*x^7+640*d*e^6*x^6+840*d^2*e^5*x^5-1024*d^3*e^4*x^4-70*d^
4*e^3*x^3+128*d^5*e^2*x^2-105*d^6*e*x+256*d^7)/e^4*(-e^2*x^2+d^2)^(1/2)-3/
128*d^8/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.74

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{210\,d^8\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(560\,e^7x^7-640\,de^6x^6-840\,d^2e^5x^5+1024\,d^3e^4x^4\right.}{4480\,e^4}$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

output
```
1/4480*(210*d^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (560*e^7*x^7 -
 640*d*e^6*x^6 - 840*d^2*e^5*x^5 + 1024*d^3*e^4*x^4 + 70*d^4*e^3*x^3 - 128
*d^5*e^2*x^2 + 105*d^6*e*x - 256*d^7)*sqrt(-e^2*x^2 + d^2))/e^4
```

3.104.6 Sympy [A] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.33

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = d^3\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{2d^4}{15e^4}-\frac{d^2x^2}{15e^2}+\frac{x^4}{5}\right) & \text{for } e^2\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)$$
$$-d^2e\left(\begin{cases}\frac{d^6\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{16e^4}+\sqrt{d^2-e^2x^2}\left(-\frac{d^4x}{16e^4}-\frac{d^2x^3}{24e^2}+\frac{x^5}{6}\right) & \text{for } e^2\neq 0\\ \frac{x^5\sqrt{d^2}}{5} & \text{otherwise}\end{cases}\right)$$
$$-de^2\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{8d^6}{105e^6}-\frac{4d^4x^2}{105e^4}-\frac{d^2x^4}{35e^2}+\frac{x^6}{7}\right) & \text{for } e^2\neq 0\\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise}\end{cases}\right)$$
$$+e^3\left(\begin{cases}\frac{5d^8\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{128e^6}+\sqrt{d^2-e^2x^2}\left(-\frac{5d^6x}{128e^6}-\frac{5d^4x^3}{192e^4}-\frac{d^2x^5}{48e^2}+\frac{x^7}{8}\right) & \text{for } e^2\neq 0\\ \frac{x^7\sqrt{d^2}}{7} & \text{otherwise}\end{cases}\right.$$

input
```
integrate(x**3*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

output
```
d**3*Piecewise((sqrt(d**2 - e**2*x**2)*(-2*d**4/(15*e**4) - d**2*x**2/(15*
e**2) + x**4/5), Ne(e**2, 0)), (x**4*sqrt(d**2)/4, True)) - d**2*e*Piecewi
se((d**6*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/
sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(16*e**4) +
sqrt(d**2 - e**2*x**2)*(-d**4*x/(16*e**4) - d**2*x**3/(24*e**2) + x**5/6),
 Ne(e**2, 0)), (x**5*sqrt(d**2)/5, True)) - d*e**2*Piecewise((sqrt(d**2 -
e**2*x**2)*(-8*d**6/(105*e**6) - 4*d**4*x**2/(105*e**4) - d**2*x**4/(35*e*
*2) + x**6/7), Ne(e**2, 0)), (x**6*sqrt(d**2)/6, True)) + e**3*Piecewise((
5*d**8*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sq
rt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(128*e**6) + s
qrt(d**2 - e**2*x**2)*(-5*d**6*x/(128*e**6) - 5*d**4*x**3/(192*e**4) - d**
2*x**5/(48*e**2) + x**7/8), Ne(e**2, 0)), (x**7*sqrt(d**2)/7, True))
```

3.104.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.28

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{3i\,d^8\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^4} + \frac{45\,d^8\arcsin\left(\frac{ex}{d}\right)}{128\,e^4} - \frac{3\sqrt{e^2x^2+4\,dex+3\,d^2}d^6x}{8\,e^3} + \frac{45\sqrt{-e^2x^2+d^2}d^6x}{128\,e^3} - \frac{3\sqrt{e^2x^2+4\,dex+3\,d^2}d^7}{4\,e^4} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d^4x}{64\,e^3} + \frac{3\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2x}{16\,e^3} - \frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^3}{5\,e^4} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}x}{8\,e^3} + \frac{(-e^2x^2+d^2)^{\frac{7}{2}}d}{7\,e^4}$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

output
```
3/8*I*d^8*arcsin(e*x/d + 2)/e^4 + 45/128*d^8*arcsin(e*x/d)/e^4 - 3/8*sqrt(
e^2*x^2 + 4*d*e*x + 3*d^2)*d^6*x/e^3 + 45/128*sqrt(-e^2*x^2 + d^2)*d^6*x/e
^3 - 3/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^7/e^4 - 1/64*(-e^2*x^2 + d^2)^(
3/2)*d^4*x/e^3 + 3/16*(-e^2*x^2 + d^2)^(5/2)*d^2*x/e^3 - 1/5*(-e^2*x^2 + d
^2)^(5/2)*d^3/e^4 - 1/8*(-e^2*x^2 + d^2)^(7/2)*x/e^3 + 1/7*(-e^2*x^2 + d^2
)^(7/2)*d/e^4
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.70

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = -\frac{3\,d^8\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{128\,e^3|e|}$$

$$-\frac{1}{4480}\sqrt{-e^2x^2+d^2}\left(\frac{256\,d^7}{e^4}-\left(\frac{105\,d^6}{e^3}-2\left(\frac{64\,d^5}{e^2}-\left(\frac{35\,d^4}{e}+4\left(128\,d^3-5\left(21\,d^2e-2\left(7\,e^3x-8\,de^2\right)x\right.\right.\right.\right.\right.\right.$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

output
```
-3/128*d^8*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^3*abs(e)) - 1/4480*sqrt(-e^2*x^2
 + d^2)*(256*d^7/e^4 - (105*d^6/e^3 - 2*(64*d^5/e^2 - (35*d^4/e + 4*(128*d
^3 - 5*(21*d^2*e - 2*(7*e^3*x - 8*d*e^2)*x)*x)*x)*x)*x)*x)
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \int \frac{x^3\,\left(d^2-e^2\,x^2\right)^{5/2}}{d+e\,x}\,dx$$

input
```
int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x)
```

output
```
int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x), x)
```

3.105 $\int \frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx$

3.105.1 Optimal result

Integrand size = 27, antiderivative size = 140

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{d+ex}\,dx=\frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2}+\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{(d^2-e^2x^2)^{7/2}}{7e^3}+\frac{d^7\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

output
```
1/24*d^3*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/30*d*(-5*e*x+6*d)*(-e^2*x^2+d^2)^(5/
2)/e^3-1/7*(-e^2*x^2+d^2)^(7/2)/e^3+1/16*d^7*arctan(e*x/(-e^2*x^2+d^2)^(1/
2))/e^3+1/16*d^5*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

3.105.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.89

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{d+ex}\,dx=\frac{\sqrt{d^2-e^2x^2}(96d^6-105d^5ex+48d^4e^2x^2+490d^3e^3x^3-384d^2e^4x^4-280de^5x^5+2}{1680e^3}$$

input
```
Integrate[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(96*d^6 - 105*d^5*e*x + 48*d^4*e^2*x^2 + 490*d^3*e^3*
x^3 - 384*d^2*e^4*x^4 - 280*d*e^5*x^5 + 240*e^6*x^6) - 210*d^7*ArcTan[(e*x
)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(1680*e^3)
```

3.105.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.26, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {562, 533, 25, 27, 533, 25, 27, 455, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(d^2-e^2 x^2\right)^{5/2}}{d+e x} \, dx$$

$$\Big\downarrow \text{562}$$

$$\int x^2 (d-e x) \left(d^2-e^2 x^2\right)^{3/2}dx$$

$$\Big\downarrow \text{533}$$

$$\frac{\int -d e x (2 d-7 e x) \left(d^2-e^2 x^2\right)^{3/2}dx}{7 e^2}+\frac{x^2 \left(d^2-e^2 x^2\right)^{5/2}}{7 e}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^2 \left(d^2-e^2 x^2\right)^{5/2}}{7 e}-\frac{\int d e x (2 d-7 e x) \left(d^2-e^2 x^2\right)^{3/2}dx}{7 e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2 \left(d^2-e^2 x^2\right)^{5/2}}{7 e}-\frac{d \int x (2 d-7 e x) \left(d^2-e^2 x^2\right)^{3/2}dx}{7 e}$$

$$\Big\downarrow \text{533}$$

$$\frac{x^2 \left(d^2-e^2 x^2\right)^{5/2}}{7 e}-\frac{d \left(\frac{\int -d e (7 d-12 e x) \left(d^2-e^2 x^2\right)^{3/2}dx}{6 e^2}+\frac{7 x \left(d^2-e^2 x^2\right)^{5/2}}{6 e}\right)}{7 e}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^2 \left(d^2-e^2 x^2\right)^{5/2}}{7 e}-\frac{d \left(\frac{7 x \left(d^2-e^2 x^2\right)^{5/2}}{6 e}-\frac{\int d e (7 d-12 e x) \left(d^2-e^2 x^2\right)^{3/2}dx}{6 e^2}\right)}{7 e}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2 \left(d^2-e^2 x^2\right)^{5/2}}{7 e}-\frac{d \left(\frac{7 x \left(d^2-e^2 x^2\right)^{5/2}}{6 e}-\frac{d \int (7 d-12 e x) \left(d^2-e^2 x^2\right)^{3/2}dx}{6 e}\right)}{7 e}$$

$$\Big\downarrow \text{455}$$

$$\frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}-\frac{d\left(\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{6e}-\frac{d\left(7d\int\left(d^2-e^2x^2\right)^{3/2}dx+\frac{12\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}\right)}{7e}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}-\frac{d\left(\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{6e}-\frac{d\left(7d\left(\frac{3}{4}d^2\int\sqrt{d^2-e^2x^2}dx+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{12\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}\right)}{7e}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}-\frac{d\left(\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{6e}-\frac{d\left(7d\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{12\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}\right)}{7e}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}-\frac{d\left(\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{6e}-\frac{d\left(7d\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{12\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}\right)}{7e}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{7e}-\frac{d\left(\frac{7x\left(d^2-e^2x^2\right)^{5/2}}{6e}-\frac{d\left(7d\left(\frac{3}{4}d^2\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{12\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}\right)}{7e}$$

input
```
Int[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

3.105. $\int \frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx$

output
```
(x^2*(d^2 - e^2*x^2)^(5/2))/(7*e) - (d*((7*x*(d^2 - e^2*x^2)^(5/2))/(6*e)
- (d*((12*(d^2 - e^2*x^2)^(5/2))/(5*e) + 7*d*((x*(d^2 - e^2*x^2)^(3/2))/4
+ (3*d^2*((x*Sqrt[d^2 - e^2*x^2])/2 + (d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2
]])/(2*e)))/4)))/(6*e)))/(7*e)
```

3.105.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.105. $\int \frac{x^2(d^2-e^2x^2)^{5/2}}{d+ex}\,dx$

rule 562
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[c^(2*n)/a^n   Int[x^m*((a + b*x^2)^(n + p)/(c - d*x)^n), x], x]
/; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[m, 0] && ILtQ[n,
 0] && IGtQ[n + p + 1/2, 0]
```

3.105.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.85

method	result
risch	$\frac{(240e^6x^6-280d\,e^5x^5-384d^2e^4x^4+490d^3x^3e^3+48d^4e^2x^2-105d^5ex+96d^6)\sqrt{-e^2x^2+d^2}}{1680e^3}+\frac{d^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16e^2\sqrt{e^2}}$
default	$-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{7e^3}-\frac{d\left(\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6}\right)}{e^2}+ \frac{d^2\left(\left(-(x+\right.\right.}{}$

input
```
int(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/1680*(240*e^6*x^6-280*d*e^5*x^5-384*d^2*e^4*x^4+490*d^3*e^3*x^3+48*d^4*e
^2*x^2-105*d^5*e*x+96*d^6)/e^3*(-e^2*x^2+d^2)^(1/2)+1/16*d^7/e^2/(e^2)^(1/
2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.84

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{d+ex}\,dx =$$
$$-\frac{210\,d^7\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(240\,e^6x^6-280\,de^5x^5-384\,d^2e^4x^4+490\,d^3e^3x^3+48\,d^4e^2x^2-105\,d^5ex}{1680\,e^3}$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

output
```
-1/1680*(210*d^7*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (240*e^6*x^6
- 280*d*e^5*x^5 - 384*d^2*e^4*x^4 + 490*d^3*e^3*x^3 + 48*d^4*e^2*x^2 - 105
*d^5*e*x + 96*d^6)*sqrt(-e^2*x^2 + d^2))/e^3
```

3.105.6 Sympy [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.65

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = d^3\left(\begin{cases}\frac{d^4\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{8e^2}+\sqrt{d^2-e^2x^2}\left(-\frac{d^2x}{8e^2}+\frac{x^3}{4}\right) & \text{for } e^2\neq 0\\ \frac{x^3\sqrt{d^2}}{3} & \text{otherwise}\end{cases}\right) - d^2e\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{2d^4}{15e^4}-\frac{d^2x^2}{15e^2}+\frac{x^4}{5}\right) & \text{for } e^2\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right) - de^2\left(\begin{cases}\frac{d^6\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{16e^4}+\sqrt{d^2-e^2x^2}\left(-\frac{d^4x}{16e^4}-\frac{d^2x^3}{24e^2}+\frac{x^5}{6}\right) & \text{for } e^2\neq 0\\ \frac{x^5\sqrt{d^2}}{5} & \text{otherwise}\end{cases}\right) + e^3\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{8d^6}{105e^6}-\frac{4d^4x^2}{105e^4}-\frac{d^2x^4}{35e^2}+\frac{x^6}{7}\right) & \text{for } e^2\neq 0\\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**2*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

output
```
d**3*Piecewise((d**4*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 -
e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/
(8*e**2) + sqrt(d**2 - e**2*x**2)*(-d**2*x/(8*e**2) + x**3/4), Ne(e**2, 0)
), (x**3*sqrt(d**2)/3, True)) - d**2*e*Piecewise((sqrt(d**2 - e**2*x**2)*(
-2*d**4/(15*e**4) - d**2*x**2/(15*e**2) + x**4/5), Ne(e**2, 0)), (x**4*sqr
t(d**2)/4, True)) - d*e**2*Piecewise((d**6*Piecewise((log(-2*e**2*x + 2*sq
rt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqr
t(-e**2*x**2), True))/(16*e**4) + sqrt(d**2 - e**2*x**2)*(-d**4*x/(16*e**4
) - d**2*x**3/(24*e**2) + x**5/6), Ne(e**2, 0)), (x**5*sqrt(d**2)/5, True)
) + e**3*Piecewise((sqrt(d**2 - e**2*x**2)*(-8*d**6/(105*e**6) - 4*d**4*x*
*2/(105*e**4) - d**2*x**4/(35*e**2) + x**6/7), Ne(e**2, 0)), (x**6*sqrt(d*
*2)/6, True))
```

3.105.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.41

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = -\frac{3i\,d^7\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^3} - \frac{5\,d^7\arcsin\left(\frac{ex}{d}\right)}{16\,e^3} + \frac{3\sqrt{e^2x^2+4\,dex+3\,d^2}d^5x}{8\,e^2} - \frac{5\sqrt{-e^2x^2+d^2}d^5x}{16\,e^2} + \frac{3\sqrt{e^2x^2+4\,dex+3\,d^2}d^6}{4\,e^3} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d^3x}{24\,e^2} - \frac{(-e^2x^2+d^2)^{\frac{5}{2}}dx}{6\,e^2} + \frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^2}{5\,e^3} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{7\,e^3}$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

output
```
-3/8*I*d^7*arcsin(e*x/d + 2)/e^3 - 5/16*d^7*arcsin(e*x/d)/e^3 + 3/8*sqrt(e
^2*x^2 + 4*d*e*x + 3*d^2)*d^5*x/e^2 - 5/16*sqrt(-e^2*x^2 + d^2)*d^5*x/e^2
+ 3/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^6/e^3 + 1/24*(-e^2*x^2 + d^2)^(3/2
)*d^3*x/e^2 - 1/6*(-e^2*x^2 + d^2)^(5/2)*d*x/e^2 + 1/5*(-e^2*x^2 + d^2)^(5
/2)*d^2/e^3 - 1/7*(-e^2*x^2 + d^2)^(7/2)/e^3
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{d^7\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{16\,e^2|e|}$$
$$+\frac{1}{1680}\sqrt{-e^2x^2+d^2}\left(\frac{96\,d^6}{e^3}-\left(\frac{105\,d^5}{e^2}-2\left(\frac{24\,d^4}{e}+\left(245\,d^3-4\left(48\,d^2e-5\left(6\,e^3x-7\,de^2\right)x\right)x\right)x\right)x\right)x\right)$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

output
```
1/16*d^7*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) + 1/1680*sqrt(-e^2*x^2 +
 d^2)*(96*d^6/e^3 - (105*d^5/e^2 - 2*(24*d^4/e + (245*d^3 - 4*(48*d^2*e -
5*(6*e^3*x - 7*d*e^2)*x)*x)*x)*x)*x)
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \int \frac{x^2\,(d^2-e^2\,x^2)^{5/2}}{d+e\,x}\,dx$$

input
```
int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x)
```

output
```
int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x), x)
```

3.106 $\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx$

3.106.1 Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx = -\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}-\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e} - \frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}-\frac{d^6\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

output
```
-1/24*d^2*x*(-e^2*x^2+d^2)^(3/2)/e-1/30*(-5*e*x+6*d)*(-e^2*x^2+d^2)^(5/2)/
e^2-1/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2-1/16*d^4*x*(-e^2*x^2+d^2
)^(1/2)/e
```

3.106.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98

$$\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx = \frac{\sqrt{d^2-e^2x^2}\left(-48d^5+15d^4ex+96d^3e^2x^2-70d^2e^3x^3-48de^4x^4+40e^5x^5\right)+30d^6\,\mathrm{a}}{240e^2}$$

input
```
Integrate[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-48*d^5 + 15*d^4*e*x + 96*d^3*e^2*x^2 - 70*d^2*e^3*x
^3 - 48*d*e^4*x^4 + 40*e^5*x^5) + 30*d^6*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^
2 - e^2*x^2])])/(240*e^2)
```

3.106.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {562, 533, 25, 27, 455, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{d+ex}dx$$

$$\Big\downarrow \text{562}$$

$$\int x(d-ex)\left(d^2-e^2x^2\right)^{3/2}dx$$

$$\Big\downarrow \text{533}$$

$$\frac{\int -de(d-6ex)\left(d^2-e^2x^2\right)^{3/2}dx}{6e^2}+\frac{x\left(d^2-e^2x^2\right)^{5/2}}{6e}$$

$$\Big\downarrow \text{25}$$

$$\frac{x\left(d^2-e^2x^2\right)^{5/2}}{6e}-\frac{\int de(d-6ex)\left(d^2-e^2x^2\right)^{3/2}dx}{6e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\left(d^2-e^2x^2\right)^{5/2}}{6e}-\frac{d\int (d-6ex)\left(d^2-e^2x^2\right)^{3/2}dx}{6e}$$

$$\Big\downarrow \text{455}$$

$$\frac{x\left(d^2-e^2x^2\right)^{5/2}}{6e}-\frac{d\left(d\int \left(d^2-e^2x^2\right)^{3/2}dx+\frac{6\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}$$

$$\Big\downarrow \text{211}$$

$$\frac{x\left(d^2-e^2x^2\right)^{5/2}}{6e}-\frac{d\left(d\left(\frac{3}{4}d^2\int \sqrt{d^2-e^2x^2}dx+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{6\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}$$

$$\Big\downarrow \text{211}$$

$$\frac{x\left(d^2-e^2x^2\right)^{5/2}}{6e}-$$
$$\frac{d\left(d\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{6\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)}{6e}$$

$$\Big\downarrow \text{224}$$

$$\frac{x(d^2-e^2x^2)^{5/2}}{6e}-\frac{d\left(d\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x(d^2-e^2x^2)^{3/2}\right)+\frac{6(d^2-e^2x^2)^{5/2}}{5e}\right)}{6e}$$

$$\Big\downarrow \text{216}$$

$$\frac{x(d^2-e^2x^2)^{5/2}}{6e}-\frac{d\left(d\left(\frac{3}{4}d^2\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x(d^2-e^2x^2)^{3/2}\right)+\frac{6(d^2-e^2x^2)^{5/2}}{5e}\right)}{6e}$$

input
```
Int[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

output
```
(x*(d^2 - e^2*x^2)^(5/2))/(6*e) - (d*((6*(d^2 - e^2*x^2)^(5/2))/(5*e) + d*
((x*(d^2 - e^2*x^2)^(3/2))/4 + (3*d^2*((x*Sqrt[d^2 - e^2*x^2])/2 + (d^2*Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)))/4)))/(6*e)
```

3.106.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.106. $\int \frac{x(d^2-e^2x^2)^{5/2}}{d+ex}\,dx$

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 562
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[c^(2*n)/a^n   Int[x^m*((a + b*x^2)^(n + p)/(c - d*x)^n), x], x]
/; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[m, 0] && ILtQ[n,
 0] && IGtQ[n + p + 1/2, 0]
```

3.106.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{\left(-40e^5x^5+48d\,e^4x^4+70d^2e^3x^3-96d^3e^2x^2-15d^4ex+48d^5\right)\sqrt{-e^2x^2+d^2}}{240e^2}-\frac{d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16e\sqrt{e^2}}$
default	$\frac{\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6}}{e}-\frac{d\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+de\left(\right.\right.}{}$

input
```
int(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/240*(-40*e^5*x^5+48*d*e^4*x^4+70*d^2*e^3*x^3-96*d^3*e^2*x^2-15*d^4*e*x+
48*d^5)/e^2*(-e^2*x^2+d^2)^(1/2)-1/16*d^6/e/(e^2)^(1/2)*arctan((e^2)^(1/2)
*x/(-e^2*x^2+d^2)^(1/2))
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{30\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(40\,e^5x^5-48\,de^4x^4-70\,d^2e^3x^3+96\,d^3e^2x^2+15\,d^4}{240\,e^2}$$

input
```
integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

output
```
1/240*(30*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (40*e^5*x^5 - 48
*d*e^4*x^4 - 70*d^2*e^3*x^3 + 96*d^3*e^2*x^2 + 15*d^4*e*x - 48*d^5)*sqrt(-
e^2*x^2 + d^2))/e^2
```

3.106.6 Sympy [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.96

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = d^3\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{d^2}{3e^2}+\frac{x^2}{3}\right) & \text{for } e^2\neq 0\\ \frac{x^2\sqrt{d^2}}{2} & \text{otherwise}\end{cases}\right)$$
$$-d^2e\left(\begin{cases}\frac{d^4\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{8e^2}+\sqrt{d^2-e^2x^2}\left(-\frac{d^2x}{8e^2}+\frac{x^3}{4}\right) & \text{for } e^2\neq 0\\ \frac{x^3\sqrt{d^2}}{3} & \text{otherwise}\end{cases}\right)$$
$$-de^2\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{2d^4}{15e^4}-\frac{d^2x^2}{15e^2}+\frac{x^4}{5}\right) & \text{for } e^2\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)$$
$$+e^3\left(\begin{cases}\frac{d^6\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{16e^4}+\sqrt{d^2-e^2x^2}\left(-\frac{d^4x}{16e^4}-\frac{d^2x^3}{24e^2}+\frac{x^5}{6}\right) & \text{for } e^2\neq 0\\ \frac{x^5\sqrt{d^2}}{5} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

output
```
d**3*Piecewise((sqrt(d**2 - e**2*x**2)*(-d**2/(3*e**2) + x**2/3), Ne(e**2,
 0)), (x**2*sqrt(d**2)/2, True)) - d**2*e*Piecewise((d**4*Piecewise((log(-
2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0))
, (x*log(x)/sqrt(-e**2*x**2), True))/(8*e**2) + sqrt(d**2 - e**2*x**2)*(-d
**2*x/(8*e**2) + x**3/4), Ne(e**2, 0)), (x**3*sqrt(d**2)/3, True)) - d*e**
2*Piecewise((sqrt(d**2 - e**2*x**2)*(-2*d**4/(15*e**4) - d**2*x**2/(15*e**
2) + x**4/5), Ne(e**2, 0)), (x**4*sqrt(d**2)/4, True)) + e**3*Piecewise((d
**6*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(
-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(16*e**4) + sqrt(
d**2 - e**2*x**2)*(-d**4*x/(16*e**4) - d**2*x**3/(24*e**2) + x**5/6), Ne(e
**2, 0)), (x**5*sqrt(d**2)/5, True))
```

3.106.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.52

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{3i\,d^6\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^2}+\frac{5\,d^6\arcsin\left(\frac{ex}{d}\right)}{16\,e^2} - \frac{3\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^4x}{8\,e}+\frac{5\,\sqrt{-e^2x^2+d^2}d^4x}{16\,e}-\frac{3\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^5}{4\,e^2} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d^2x}{24\,e}+\frac{(-e^2x^2+d^2)^{\frac{5}{2}}x}{6\,e}-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}d}{5\,e^2}$$

input
```
integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

output
```
3/8*I*d^6*arcsin(e*x/d + 2)/e^2 + 5/16*d^6*arcsin(e*x/d)/e^2 - 3/8*sqrt(e^
2*x^2 + 4*d*e*x + 3*d^2)*d^4*x/e + 5/16*sqrt(-e^2*x^2 + d^2)*d^4*x/e - 3/4
*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^5/e^2 - 1/24*(-e^2*x^2 + d^2)^(3/2)*d^2
*x/e + 1/6*(-e^2*x^2 + d^2)^(5/2)*x/e - 1/5*(-e^2*x^2 + d^2)^(5/2)*d/e^2
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = -\frac{d^6\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{16\,e|e|} - \frac{1}{240}\sqrt{-e^2x^2+d^2}\left(\frac{48\,d^5}{e^2}-\left(\frac{15\,d^4}{e}+2\left(48\,d^3-\left(35\,d^2e-4\left(5\,e^3x-6\,de^2\right)x\right)x\right)x\right)x\right)$$

input
```
integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

output
```
-1/16*d^6*arcsin(e*x/d)*sgn(d)*sgn(e)/(e*abs(e)) - 1/240*sqrt(-e^2*x^2 + d
^2)*(48*d^5/e^2 - (15*d^4/e + 2*(48*d^3 - (35*d^2*e - 4*(5*e^3*x - 6*d*e^2
)*x)*x)*x)*x)
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \int \frac{x\,(d^2-e^2\,x^2)^{5/2}}{d+e\,x}\,dx$$

input
```
int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x)
```

output
```
int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x), x)
```

3.107 $\int \frac{(d^2-e^2x^2)^{5/2}}{d+ex} dx$

3.107.1 Optimal result

Integrand size = 24, antiderivative size = 100

$$\int \frac{(d^2-e^2x^2)^{5/2}}{d+ex} dx = \frac{3}{8}d^3x\sqrt{d^2-e^2x^2} + \frac{1}{4}dx(d^2-e^2x^2)^{3/2} + \frac{(d^2-e^2x^2)^{5/2}}{5e} + \frac{3d^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

output
```
1/4*d*x*(-e^2*x^2+d^2)^(3/2)+1/5*(-e^2*x^2+d^2)^(5/2)/e+3/8*d^5*arctan(e*x
/(-e^2*x^2+d^2)^(1/2))/e+3/8*d^3*x*(-e^2*x^2+d^2)^(1/2)
```

3.107.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.11

$$\int \frac{(d^2-e^2x^2)^{5/2}}{d+ex} dx = \frac{\sqrt{d^2-e^2x^2}(8d^4+25d^3ex-16d^2e^2x^2-10de^3x^3+8e^4x^4)}{40e} - \frac{3d^5\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{8\sqrt{-e^2}}$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(d + e*x),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(8*d^4 + 25*d^3*e*x - 16*d^2*e^2*x^2 - 10*d*e^3*x^3 +
 8*e^4*x^4))/(40*e) - (3*d^5*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(
8*Sqrt[-e^2])
```

3.107.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {466, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{d+ex}dx$$

$$\Big\downarrow \text{466}$$

$$d\int \left(d^2-e^2x^2\right)^{3/2}dx+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e}$$

$$\Big\downarrow \text{211}$$

$$d\left(\frac{3}{4}d^2\int \sqrt{d^2-e^2x^2}dx+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e}$$

$$\Big\downarrow \text{211}$$

$$d\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e}$$

$$\Big\downarrow \text{224}$$

$$d\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e}$$

$$\Big\downarrow \text{216}$$

$$d\left(\frac{3}{4}d^2\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(d + e*x),x]
```

output
```
(d^2 - e^2*x^2)^(5/2)/(5*e) + d*((x*(d^2 - e^2*x^2)^(3/2))/4 + (3*d^2*((x*
Sqrt[d^2 - e^2*x^2])/2 + (d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)))/4
)
```

3.107.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

3.107.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

method	result
risch	$\frac{(8e^4x^4-10d\,e^3x^3-16d^2e^2x^2+25d^3ex+8d^4)\sqrt{-e^2x^2+d^2}}{40e}+\frac{3d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}$
default	$\frac{\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+de\left(-\frac{\left(-2\left(x+\frac{d}{e}\right)e^2+2de\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8e^2}+\frac{3d^2\left(-\frac{\left(-2\left(x+\frac{d}{e}\right)e^2+2de\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\right.}}{4e^2}\right.}{4}\right.}{e}$

input
```
int((-e^2*x^2+d^2)^(5/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/40*(8*e^4*x^4-10*d*e^3*x^3-16*d^2*e^2*x^2+25*d^3*e*x+8*d^4)/e*(-e^2*x^2+
d^2)^(1/2)+3/8*d^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int \frac{(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = -\frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(8\,e^4x^4-10\,de^3x^3-16\,d^2e^2x^2+25\,d^3ex+8\,d^4)\sqrt{-e^2x^2+d^2}}{40\,e}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

output
```
-1/40*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (8*e^4*x^4 - 10*
d*e^3*x^3 - 16*d^2*e^2*x^2 + 25*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/e
```

3.107.6 Sympy [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.13

$$\int \frac{(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = d^3\left(\begin{cases} \frac{d^2\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{d^2-e^2x^2}}{2} & \text{for } e^2\neq 0 \\ x\sqrt{d^2} & \text{otherwise} \end{cases}\right)$$
$$- d^2e\left(\begin{cases} \sqrt{d^2-e^2x^2}\left(-\frac{d^2}{3e^2}+\frac{x^2}{3}\right) & \text{for } e^2\neq 0 \\ \frac{x^2\sqrt{d^2}}{2} & \text{otherwise} \end{cases}\right)$$
$$-de^2\left(\begin{cases} \frac{d^4\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{8e^2} + \sqrt{d^2-e^2x^2}\left(-\frac{d^2x}{8e^2}+\frac{x^3}{4}\right) & \text{for } e^2\neq 0 \\ \frac{x^3\sqrt{d^2}}{3} & \text{otherwise} \end{cases}\right)$$
$$+ e^3\left(\begin{cases} \sqrt{d^2-e^2x^2}\left(-\frac{2d^4}{15e^4}-\frac{d^2x^2}{15e^2}+\frac{x^4}{5}\right) & \text{for } e^2\neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

output
```
d**3*Piecewise((d**2*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 -
e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/
2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2, 0)), (x*sqrt(d**2), True)) - d**2
*e*Piecewise((sqrt(d**2 - e**2*x**2)*(-d**2/(3*e**2) + x**2/3), Ne(e**2, 0
)), (x**2*sqrt(d**2)/2, True)) - d*e**2*Piecewise((d**4*Piecewise((log(-2*
e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)),
(x*log(x)/sqrt(-e**2*x**2), True))/(8*e**2) + sqrt(d**2 - e**2*x**2)*(-d**
2*x/(8*e**2) + x**3/4), Ne(e**2, 0)), (x**3*sqrt(d**2)/3, True)) + e**3*Pi
ecewise((sqrt(d**2 - e**2*x**2)*(-2*d**4/(15*e**4) - d**2*x**2/(15*e**2) +
 x**4/5), Ne(e**2, 0)), (x**4*sqrt(d**2)/4, True))
```

3.107.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.09

$$\int \frac{(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = -\frac{3i\,d^5\arcsin\left(\frac{ex}{d}+2\right)}{8\,e} + \frac{3}{8}\sqrt{e^2x^2+4\,dex+3\,d^2}d^3x + \frac{3\sqrt{e^2x^2+4\,dex+3\,d^2}d^4}{4\,e} + \frac{1}{4}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}dx + \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5\,e}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

output
```
-3/8*I*d^5*arcsin(e*x/d + 2)/e + 3/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3*x
 + 3/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^4/e + 1/4*(-e^2*x^2 + d^2)^(3/2)*
d*x + 1/5*(-e^2*x^2 + d^2)^(5/2)/e
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int \frac{(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{3\,d^5\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,|e|} + \frac{1}{40}\sqrt{-e^2x^2+d^2}\left(\frac{8\,d^4}{e} + \left(25\,d^3-2\left(8\,d^2e-\left(4\,e^3x-5\,de^2\right)x\right)x\right)x\right)$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

output
```
3/8*d^5*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/40*sqrt(-e^2*x^2 + d^2)*(8*
d^4/e + (25*d^3 - 2*(8*d^2*e - (4*e^3*x - 5*d*e^2)*x)*x)*x)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx=\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{d+e\,x}\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(d + e*x),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(d + e*x), x)
```

3.108 $\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)}\,dx$

3.108.1 Optimal result

Integrand size = 27, antiderivative size = 113

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)}\,dx = \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2}+\frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} -\frac{3}{8}d^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^4\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
1/12*(-3*e*x+4*d)*(-e^2*x^2+d^2)^(3/2)-3/8*d^4*arctan(e*x/(-e^2*x^2+d^2)^(
1/2))-d^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/8*d^2*(-3*e*x+8*d)*(-e^2*x^2+d
^2)^(1/2)
```

3.108.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.26

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)}\,dx = \frac{1}{24}\sqrt{d^2-e^2x^2}\left(32d^3-15d^2ex-8de^2x^2+6e^3x^3\right) +\frac{3}{4}d^4\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)-d^3\sqrt{d^2}\log(x)+d^3\sqrt{d^2}\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(32*d^3 - 15*d^2*e*x - 8*d*e^2*x^2 + 6*e^3*x^3))/24 +
 (3*d^4*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/4 - d^3*Sqrt[d^2]
*Log[x] + d^3*Sqrt[d^2]*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]]
```

3.108.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {566, 535, 535, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x(d+ex)}\,dx$$

$$\Big\downarrow \text{566}$$

$$\int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{535}$$

$$\frac{1}{4}d^2\int \frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{x}dx+\frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{535}$$

$$\frac{1}{4}d^2\left(\frac{1}{2}d^2\int \frac{8d-3ex}{x\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}(8d-3ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{538}$$

$$\frac{1}{4}d^2\left(\frac{1}{2}d^2\left(8d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-3e\int \frac{1}{\sqrt{d^2-e^2x^2}}dx\right)+\frac{1}{2}(8d-3ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{4}d^2\left(\frac{1}{2}d^2\left(8d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-3e\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)+\frac{1}{2}(8d-3ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}d^2\left(\frac{1}{2}d^2\left(8d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx-3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{1}{2}(8d-3ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \; 243$$

$$\frac{1}{4}d^2\left(\frac{1}{2}d^2\left(4d\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{1}{2}(8d-3ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \; 73$$

$$\frac{1}{4}d^2\left(\frac{1}{2}d^2\left(-\frac{8d\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}-3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{1}{2}(8d-3ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \; 221$$

$$\frac{1}{4}d^2\left(\frac{1}{2}d^2\left(-3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-8\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)+\frac{1}{2}(8d-3ex)\sqrt{d^2-e^2x^2}\right)+\frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)),x]
```

output
```
((4*d - 3*e*x)*(d^2 - e^2*x^2)^(3/2))/12 + (d^2*(((8*d - 3*e*x)*Sqrt[d^2 -
 e^2*x^2])/2 + (d^2*(-3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 8*ArcTanh[Sqrt
[d^2 - e^2*x^2]/d]))/2))/4
```

3.108.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 566
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Int[x^m*(a/c + b*(x/d))*(a + b*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0]
```

3.108.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(99) = 198$.

Time = 0.44 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.63

method	result
default	$\frac{\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5}+d^2\left(\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)\right)}{d}-\frac{\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+de\left(-\right.}{}$

input
```
int((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2
+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/
x))))-1/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*(x+d/e)*
e^2+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*(x+d
/e)*e^2+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2
)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)}\,dx = \frac{3}{4}\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + d^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + \frac{1}{24}\left(6\,e^3x^3-8\,de^2x^2-15\,d^2ex+32\,d^3\right)\sqrt{-e^2x^2+d^2}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x, algorithm="fricas")
```

output
```
3/4*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d^4*log(-(d - sqrt(-e^
2*x^2 + d^2))/x) + 1/24*(6*e^3*x^3 - 8*d*e^2*x^2 - 15*d^2*e*x + 32*d^3)*sq
rt(-e^2*x^2 + d^2)
```

3.108.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.77 (sec) , antiderivative size = 400, normalized size of antiderivative = 3.54

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)}\,dx = d^3\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d\operatorname{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id\operatorname{asin}\left(\frac{d}{ex}\right) + \frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)$$
$$- d^2e\left(\begin{cases}\frac{d^2\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for}\ d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{2} + \frac{x\sqrt{d^2-e^2x^2}}{2} & \text{for}\ e^2\neq 0\\ x\sqrt{d^2} & \text{otherwise}\end{cases}\right)$$
$$- de^2\left(\begin{cases}-\frac{d^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^2\sqrt{d^2-e^2x^2}}{3} & \text{for}\ e^2\neq 0\\ \frac{x^2\sqrt{d^2}}{2} & \text{otherwise}\end{cases}\right)$$
$$+ e^3\left(\begin{cases}\frac{d^4\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for}\ d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{8e^2} - \frac{d^2x\sqrt{d^2-e^2x^2}}{8e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4} & \text{for}\ e^2\neq 0\\ \frac{x^3\sqrt{d^2}}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/x/(e*x+d),x)
```

output
```
d**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
 e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x
*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2
*x**2) + 1), True)) - d**2*e*Piecewise((d**2*Piecewise((log(-2*e**2*x + 2*
sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/s
qrt(-e**2*x**2), True))/2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2, 0)), (x*s
qrt(d**2), True)) - d*e**2*Piecewise((-d**2*sqrt(d**2 - e**2*x**2)/(3*e**2
) + x**2*sqrt(d**2 - e**2*x**2)/3, Ne(e**2, 0)), (x**2*sqrt(d**2)/2, True)
) + e**3*Piecewise((d**4*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**
2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), Tru
e))/(8*e**2) - d**2*x*sqrt(d**2 - e**2*x**2)/(8*e**2) + x**3*sqrt(d**2 - e
**2*x**2)/4, Ne(e**2, 0)), (x**3*sqrt(d**2)/3, True))
```

3.108.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.21

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)}\,dx = -\frac{3\,d^4e\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\sqrt{e^2}} - d^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{3}{8}\sqrt{-e^2x^2+d^2}d^2ex + \sqrt{-e^2x^2+d^2}d^3 - \frac{1}{4}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}ex + \frac{1}{3}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x, algorithm="maxima")
```

output
```
-3/8*d^4*e*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - d^4*log(2*d^2/abs(x) +
2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) - 3/8*sqrt(-e^2*x^2 + d^2)*d^2*e*x + sqrt
(-e^2*x^2 + d^2)*d^3 - 1/4*(-e^2*x^2 + d^2)^(3/2)*e*x + 1/3*(-e^2*x^2 + d^
2)^(3/2)*d
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)}\,dx = -\frac{3\,d^4e\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,|e|} - \frac{d^4e\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{|e|} + \frac{1}{24}\sqrt{-e^2x^2+d^2}\left(32\,d^3-\left(15\,d^2e-2\left(3\,e^3x-4\,de^2\right)x\right)x\right)$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x, algorithm="giac")
```

output
```
-3/8*d^4*e*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - d^4*e*log(1/2*abs(-2*d*e -
 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/24*sqrt(-e^2*x^2
+ d^2)*(32*d^3 - (15*d^2*e - 2*(3*e^3*x - 4*d*e^2)*x)*x)
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}}{x\,\left(d+e\,x\right)}\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)), x)
```

3.109 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)}\,dx$

3.109.1 Optimal result

Integrand size = 27, antiderivative size = 115

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)}\,dx = -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2}-\frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} -\frac{3}{2}d^3e\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+d^3e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
-1/3*(e*x+3*d)*(-e^2*x^2+d^2)^(3/2)/x-3/2*d^3*e*arctan(e*x/(-e^2*x^2+d^2)^
(1/2))+d^3*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)-1/2*d*e*(3*e*x+2*d)*(-e^2*x^2
+d^2)^(1/2)
```

3.109.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.22

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)}\,dx = \frac{\sqrt{d^2-e^2x^2}(-6d^3-8d^2ex-3de^2x^2+2e^3x^3)}{6x} +3d^3e\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) +\left(d^2\right)^{3/2}e\log(x)-\left(d^2\right)^{3/2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-6*d^3 - 8*d^2*e*x - 3*d*e^2*x^2 + 2*e^3*x^3))/(6*x)
 + 3*d^3*e*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] + (d^2)^(3/2)*e
*Log[x] - (d^2)^(3/2)*e*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]]
```

3.109.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {566, 536, 535, 25, 27, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^2(d+ex)}dx$$

$$\Big\downarrow \text{566}$$

$$\int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{536}$$

$$\int \frac{\left(-ed^2-3e^2xd\right)\sqrt{d^2-e^2x^2}}{x}dx-\frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}$$

$$\Big\downarrow \text{535}$$

$$\frac{1}{2}d^2\int -\frac{de(2d+3ex)}{x\sqrt{d^2-e^2x^2}}dx-\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2}-\frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}d^2\int \frac{de(2d+3ex)}{x\sqrt{d^2-e^2x^2}}dx-\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2}-\frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}d^3e\int \frac{2d+3ex}{x\sqrt{d^2-e^2x^2}}dx-\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2}-\frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}$$

$$\Big\downarrow \text{538}$$

$$-\frac{1}{2}d^3e\left(3e\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+2d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx\right)-\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2}-\frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}$$

$\Big\downarrow$ 224

$$-\frac{1}{2}d^3e\left(2d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx+3e\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)-\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2}-\frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}$$

$\Big\downarrow$ 216

$$-\frac{1}{2}d^3e\left(2d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx+3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2}-\frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}$$

$\Big\downarrow$ 243

$$-\frac{1}{2}d^3e\left(d\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2+3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2}-\frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}$$

$\Big\downarrow$ 73

$$-\frac{1}{2}d^3e\left(3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{2d\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}\right)-\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2}-\frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}$$

$\Big\downarrow$ 221

$$-\frac{1}{2}d^3e\left(3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)-\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2}-\frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

output
```
-1/2*(d*e*(2*d + 3*e*x)*Sqrt[d^2 - e^2*x^2]) - ((3*d + e*x)*(d^2 - e^2*x^2
)^(3/2))/(3*x) - (d^3*e*(3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 2*ArcTanh[S
qrt[d^2 - e^2*x^2]/d]))/2
```

3.109. $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^2(d+ex)}\,dx$

3.109.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

3.109. $\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)}\,dx$

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 566
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Int[x^m*(a/c + b*(x/d))*(a + b*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0]
```

3.109.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.43

method	result
risch	$-\frac{d^{3}\sqrt{-e^{2}x^{2}+d^{2}}}{x}+\frac{e^{3}x^{2}\sqrt{-e^{2}x^{2}+d^{2}}}{3}-\frac{4e\,d^{2}\sqrt{-e^{2}x^{2}+d^{2}}}{3}+\frac{e\,d^{4}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}}-\frac{3d^{3}e^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}$
default	$\frac{-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{d^{2}x}-\frac{6e^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}+\frac{5d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)}{4}\right)}{6}\right)}{d^{2}}}{d}-\frac{e\left(\frac{\left(-e^{2}x^{2}+d\right.}{5}\right.}{}$

input
```
int((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-d^3*(-e^2*x^2+d^2)^(1/2)/x+1/3*e^3*x^2*(-e^2*x^2+d^2)^(1/2)-4/3*e*d^2*(-e
^2*x^2+d^2)^(1/2)+e*d^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)
^(1/2))/x)-3/2*d^3*e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/
2))-1/2*d*e^2*x*(-e^2*x^2+d^2)^(1/2)
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)}\,dx = \frac{18\,d^3ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-6\,d^3ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-8\,d^3ex+(2\,e^3x^3-3}{6\,x}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x, algorithm="fricas")
```

output
```
1/6*(18*d^3*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 6*d^3*e*x*log(
-(d - sqrt(-e^2*x^2 + d^2))/x) - 8*d^3*e*x + (2*e^3*x^3 - 3*d*e^2*x^2 - 8*
d^2*e*x - 6*d^3)*sqrt(-e^2*x^2 + d^2))/x
```

3.109.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.16 (sec) , antiderivative size = 389, normalized size of antiderivative = 3.38

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)}\,dx = d^3\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$
$$-d^2e\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)$$
$$-de^2\left(\begin{cases}\frac{d^2\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for}\ d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{d^2-e^2x^2}}{2} & \text{for}\ e^2\neq 0\\ x\sqrt{d^2} & \text{otherwise}\end{cases}\right)$$
$$+e^3\left(\begin{cases}-\frac{d^2\sqrt{d^2-e^2x^2}}{3e^2}+\frac{x^2\sqrt{d^2-e^2x^2}}{3} & \text{for}\ e^2\neq 0\\ \frac{x^2\sqrt{d^2}}{2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/x**2/(e*x+d),x)
```

output
```
d**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e
**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt
(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)
), True)) - d**2*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*ac
osh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1),
 (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sq
rt(-d**2/(e**2*x**2) + 1), True)) - d*e**2*Piecewise((d**2*Piecewise((log(
-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)
), (x*log(x)/sqrt(-e**2*x**2), True))/2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e
**2, 0)), (x*sqrt(d**2), True)) + e**3*Piecewise((-d**2*sqrt(d**2 - e**2*x
**2)/(3*e**2) + x**2*sqrt(d**2 - e**2*x**2)/3, Ne(e**2, 0)), (x**2*sqrt(d*
*2)/2, True))
```

3.109.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.26

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)}\,dx = -\frac{3\,d^3e^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\,\sqrt{e^2}} + d^3e\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{1}{2}\,\sqrt{-e^2x^2+d^2}de^2x - \sqrt{-e^2x^2+d^2}d^2e - \frac{1}{3}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e - \frac{\sqrt{-e^2x^2+d^2}d^3}{x}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x, algorithm="maxima")
```

output
```
-3/2*d^3*e^2*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + d^3*e*log(2*d^2/abs(x
) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) - 1/2*sqrt(-e^2*x^2 + d^2)*d*e^2*x -
sqrt(-e^2*x^2 + d^2)*d^2*e - 1/3*(-e^2*x^2 + d^2)^(3/2)*e - sqrt(-e^2*x^2
+ d^2)*d^3/x
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.54

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)}\,dx = -\frac{3\,d^3e^2\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,|e|} + \frac{d^3e^4x}{2\left(de+\sqrt{-e^2x^2+d^2}|e|\right)|e|} + \frac{d^3e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{|e|} - \frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^3}{2\,x|e|} - \frac{1}{6}\sqrt{-e^2x^2+d^2}\left(8\,d^2e-\left(2\,e^3x-3\,de^2\right)x\right)$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x, algorithm="giac")
```

output
```
-3/2*d^3*e^2*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/2*d^3*e^4*x/((d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))*abs(e)) + d^3*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-
e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) - 1/2*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))*d^3/(x*abs(e)) - 1/6*sqrt(-e^2*x^2 + d^2)*(8*d^2*e - (2*e^3*x
- 3*d*e^2)*x)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)}\,dx = \int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^2\,\left(d+e\,x\right)}\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)), x)
```

3.110 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)} dx$

3.110.1 Optimal result

Integrand size = 27, antiderivative size = 121

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)} dx = \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{2x^2} + \frac{3}{2}d^2e^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{3}{2}d^2e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
-1/2*(e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2+3/2*d^2*e^2*arctan(e*x/(-e^2*x^2+d^2
)^(1/2))+3/2*d^2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)+3/2*d*e*(-e*x+d)*(-e^
2*x^2+d^2)^(1/2)/x
```

3.110.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.23

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)} dx = \frac{1}{2}\left(\frac{\sqrt{d^2-e^2x^2}(-d^3+2d^2ex-2de^2x^2+e^3x^3)}{x^2} - 6d^2e^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) + 3d\sqrt{d^2}e^2\log(x) - 3d\sqrt{d^2}e^2\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)\right)$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(-d^3 + 2*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3))/x^2 - 6*
d^2*e^2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] + 3*d*Sqrt[d^2]*e^
2*Log[x] - 3*d*Sqrt[d^2]*e^2*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/2
```

3.110.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {566, 537, 25, 535, 27, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^3(d+ex)}dx$$

$$\Big\downarrow \text{566}$$

$$\int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{537}$$

$$\frac{3}{2}e^2\int -\frac{(d-2ex)\sqrt{d^2-e^2x^2}}{x}dx-\frac{(d-2ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3}{2}e^2\int \frac{(d-2ex)\sqrt{d^2-e^2x^2}}{x}dx-\frac{(d-2ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{535}$$

$$-\frac{3}{2}e^2\left(\frac{1}{2}d^2\int \frac{2(d-ex)}{x\sqrt{d^2-e^2x^2}}dx+(d-ex)\sqrt{d^2-e^2x^2}\right)-\frac{(d-2ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{2}e^2\left(d^2\int \frac{d-ex}{x\sqrt{d^2-e^2x^2}}dx+(d-ex)\sqrt{d^2-e^2x^2}\right)-\frac{(d-2ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{538}$$

$$-\frac{3}{2}e^2\left(d^2\left(d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-e\int \frac{1}{\sqrt{d^2-e^2x^2}}dx\right)+(d-ex)\sqrt{d^2-e^2x^2}\right)-\frac{(d-2ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

$\big\downarrow$ 224

$$-\frac{3}{2}e^2\left(d^2\left(d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-e\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)+(d-ex)\sqrt{d^2-e^2x^2}\right)-\frac{(d-2ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

$\big\downarrow$ 216

$$-\frac{3}{2}e^2\left(d^2\left(d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+(d-ex)\sqrt{d^2-e^2x^2}\right)-\frac{(d-2ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

$\big\downarrow$ 243

$$-\frac{3}{2}e^2\left(d^2\left(\frac{1}{2}d\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+(d-ex)\sqrt{d^2-e^2x^2}\right)-\frac{(d-2ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

$\big\downarrow$ 73

$$-\frac{3}{2}e^2\left(d^2\left(-\frac{d\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+(d-ex)\sqrt{d^2-e^2x^2}\right)-\frac{(d-2ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

$\big\downarrow$ 221

$$-\frac{3}{2}e^2\left(d^2\left(-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)+(d-ex)\sqrt{d^2-e^2x^2}\right)-\frac{(d-2ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

output
```
-1/2*((d - 2*e*x)*(d^2 - e^2*x^2)^(3/2))/x^2 - (3*e^2*((d - e*x)*Sqrt[d^2
- e^2*x^2] + d^2*(-ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - ArcTanh[Sqrt[d^2 -
e^2*x^2]/d])))/2
```

3.110. $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^3(d+ex)}\,dx$

3.110.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

3.110. $\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)}\,dx$

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 566
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Int[x^m*(a/c + b*(x/d))*(a + b*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0]
```

3.110.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{d^{2}\sqrt{-e^{2}x^{2}+d^{2}}\,(-2ex+d)}{2x^{2}}+\frac{3e^{3}d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}+\frac{e^{3}x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{3e^{2}d^{3}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{2\sqrt{d^{2}}}-d\,e^{2}\sqrt{\ }$
default	$\frac{-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{2d^{2}x^{2}}-\frac{5e^{2}\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{5}+d^{2}\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{3}+d^{2}\left(\sqrt{-e^{2}x^{2}+d^{2}}-\frac{d^{2}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}}\right)\right)\right)}{2d^{2}}}{d}+\frac{e^{2}\left(\frac{\left(-e^{2}x^{2}+d\right.}{5}\right.}{}$

input
```
int((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d^2*(-e^2*x^2+d^2)^(1/2)*(-2*e*x+d)/x^2+3/2*e^3*d^2/(e^2)^(1/2)*arcta
n((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/2*e^3*x*(-e^2*x^2+d^2)^(1/2)+3/2*e
^2*d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-d*e^2*
(-e^2*x^2+d^2)^(1/2)
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.12

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)}\,dx =$$
$$-\frac{6\,d^2e^2x^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+3\,d^2e^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+2\,d^2e^2x^2-(e^3x^3-2\,de^2x^2+2\,d^2ex-}{2\,x^2}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x, algorithm="fricas")
```

output
```
-1/2*(6*d^2*e^2*x^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 3*d^2*e^2*
x^2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 2*d^2*e^2*x^2 - (e^3*x^3 - 2*d*e^
2*x^2 + 2*d^2*e*x - d^3)*sqrt(-e^2*x^2 + d^2))/x^2
```

3.110.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.86 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.67

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)}\,dx = d^3\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)$$

$$-d^2e\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

$$-de^2\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)$$

$$+e^3\left(\begin{cases}\frac{d^2\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{d^2-e^2x^2}}{2} & \text{for } e^2\neq 0 \\ x\sqrt{d^2} & \text{otherwise}\end{cases}\right)$$

input

```
integrate((-e**2*x**2+d**2)**(5/2)/x**3/(e*x+d),x)
```

output

```
d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/
(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2
) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*
d), True)) - d**2*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*aco
sh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 -
e**2*x**2/d**2)), True)) - d*e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**
2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e*
*2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e
*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) + e**3*Piecewise((d**2*Pi
ecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2)
, Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/2 + x*sqrt(d**2 - e**2*
x**2)/2, Ne(e**2, 0)), (x*sqrt(d**2), True))
```

3.110. $\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)}\,dx$

3.110.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.24

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)}\,dx = \frac{3\,d^2e^3\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\,\sqrt{e^2}} + \frac{3}{2}\,d^2e^2\log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right) + \frac{1}{2}\,\sqrt{-e^2x^2+d^2}e^3x - \frac{3}{2}\,\sqrt{-e^2x^2+d^2}de^2 + \frac{\sqrt{-e^2x^2+d^2}d^2e}{x} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d}{2\,x^2}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x, algorithm="maxima")
```

output
```
3/2*d^2*e^3*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 3/2*d^2*e^2*log(2*d^2/
abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) + 1/2*sqrt(-e^2*x^2 + d^2)*e^3*x
 - 3/2*sqrt(-e^2*x^2 + d^2)*d*e^2 + sqrt(-e^2*x^2 + d^2)*d^2*e/x - 1/2*(-e
^2*x^2 + d^2)^(3/2)*d/x^2
```

3.110.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(106) = 212$.

Time = 0.30 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.02

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)}\,dx = \frac{3\,d^2e^3\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,|e|} + \frac{3\,d^2e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,|e|} + \frac{\left(d^2e^3 - \frac{4\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2e}{x}\right)e^4x^2}{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2|e|} + \frac{1}{2}\left(e^3x-2\,de^2\right)\sqrt{-e^2x^2+d^2} + \frac{\frac{4\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2e|e|}{x} - \frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^2|e|}{ex^2}}{8\,e^2}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x, algorithm="giac")
```

output
```
3/2*d^2*e^3*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 3/2*d^2*e^3*log(1/2*abs(-
2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/8*(d^2*e^3
 - 4*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^2*e/x)*e^4*x^2/((d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^2*abs(e)) + 1/2*(e^3*x - 2*d*e^2)*sqrt(-e^2*x^2 + d^2
) + 1/8*(4*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^2*e*abs(e)/x - (d*e + sqr
t(-e^2*x^2 + d^2)*abs(e))^2*d^2*abs(e)/(e*x^2))/e^2
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^3(d+ex)}\,dx=\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x^3\,\left(d+e\,x\right)} \,d x$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)), x)
```

3.111 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)}\,dx$

3.111.1 Optimal result

Integrand size = 27, antiderivative size = 120

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)}\,dx = \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + de^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{3}{2}de^3\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
-1/6*(-3*e*x+2*d)*(-e^2*x^2+d^2)^(3/2)/x^3+d*e^3*arctan(e*x/(-e^2*x^2+d^2)
^(1/2))-3/2*d*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/2*e^2*(3*e*x+2*d)*(-e^
2*x^2+d^2)^(1/2)/x
```

3.111.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.22

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)}\,dx = \frac{1}{6}\left(\frac{\sqrt{d^2-e^2x^2}(-2d^3+3d^2ex+8de^2x^2+6e^3x^3)}{x^3} - 12de^3\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) - 9\sqrt{d^2}e^3\log(x) + 9\sqrt{d^2}e^3\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)\right)$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(-2*d^3 + 3*d^2*e*x + 8*d*e^2*x^2 + 6*e^3*x^3))/x^3
- 12*d*e^3*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] - 9*Sqrt[d^2]*e
^3*Log[x] + 9*Sqrt[d^2]*e^3*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/6
```

3.111.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {566, 537, 25, 536, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)}dx$$

$$\Big\downarrow \text{566}$$

$$\int \frac{(d-ex)(d^2-e^2x^2)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{537}$$

$$\frac{1}{2}e^2\int -\frac{(2d-3ex)\sqrt{d^2-e^2x^2}}{x^2}dx-\frac{(2d-3ex)(d^2-e^2x^2)^{3/2}}{6x^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}e^2\int \frac{(2d-3ex)\sqrt{d^2-e^2x^2}}{x^2}dx-\frac{(2d-3ex)(d^2-e^2x^2)^{3/2}}{6x^3}$$

$$\Big\downarrow \text{536}$$

$$-\frac{1}{2}e^2\left(\int \frac{-3ed^2-2e^2xd}{x\sqrt{d^2-e^2x^2}}dx-\frac{(2d+3ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{(2d-3ex)(d^2-e^2x^2)^{3/2}}{6x^3}$$

$$\Big\downarrow \text{538}$$

$$-\frac{1}{2}e^2\left(-3d^2e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-2de^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{(2d+3ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{(2d-3ex)(d^2-e^2x^2)^{3/2}}{6x^3}$$

$$\Big\downarrow \text{224}$$

$$-\frac{1}{2}e^2\left(-3d^2e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-2de^2\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{(2d+3ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{2}e^2\left(-3d^2e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-2de\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{(2d+3ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}e^2\left(-\frac{3}{2}d^2e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-2de\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{(2d+3ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}$$

$$\Big\downarrow \text{73}$$

$$-\frac{1}{2}e^2\left(\frac{3d^2\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-2de\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{(2d+3ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}$$

$$\Big\downarrow \text{221}$$

$$-\frac{1}{2}e^2\left(-2de\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+3de\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)-\frac{\sqrt{d^2-e^2x^2}(2d+3ex)}{x}\right)-\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

output
```
-1/6*((2*d - 3*e*x)*(d^2 - e^2*x^2)^(3/2))/x^3 - (e^2*(-(((2*d + 3*e*x)*Sq
rt[d^2 - e^2*x^2])/x) - 2*d*e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + 3*d*e*Ar
cTanh[Sqrt[d^2 - e^2*x^2]/d]))/2
```

3.111. $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^4(d+ex)}\,dx$

3.111.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

3.111. $\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)}\,dx$

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 566
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Int[x^m*(a/c + b*(x/d))*(a + b*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0]
```

3.111.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,d\left(-8e^2x^2-3dex+2d^2\right)}{6x^3}+\frac{e^4d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+e^3\sqrt{-e^2x^2+d^2}-\frac{3e^3d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\cdots\right)}{2\sqrt{d^2}}\cdots$
default	$\frac{-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{3d^2x^3}-\frac{4e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{d^2x}-\frac{6e^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\cdots\right)}{6}\cdots\right)}{d^2}\cdots\right)}{3d^2}\cdots}{d}$

input
```
int((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-e^2*x^2+d^2)^(1/2)*d*(-8*e^2*x^2-3*d*e*x+2*d^2)/x^3+e^4*d/(e^2)^(1/
2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+e^3*(-e^2*x^2+d^2)^(1/2)-3/2
*e^3*d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.08

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)}\,dx =$$
$$-\frac{12\,de^3x^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-9\,de^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-6\,de^3x^3-\left(6\,e^3x^3+8\,de^2x^2+3\,d^2ex-2\right.}{6\,x^3}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x, algorithm="fricas")
```

output
```
-1/6*(12*d*e^3*x^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 9*d*e^3*x^3
*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 6*d*e^3*x^3 - (6*e^3*x^3 + 8*d*e^2*x
^2 + 3*d^2*e*x - 2*d^3)*sqrt(-e^2*x^2 + d^2))/x^3
```

3.111.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.12 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.81

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)}\,dx = d^3\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right) - d^2e\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right) - de^2\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e^3\left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/x**4/(e*x+d),x)
```

output
```
d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e
**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**
2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)
) - d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(
e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**
2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x
))/(2*d), True)) - d*e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I
*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d*
*2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqr
t(1 - e**2*x**2/d**2)), True)) + e**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2
*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2
/(e**2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(
d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True))
```

3.111. $\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)}\,dx$

3.111.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.20

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)}\,dx = \frac{de^4 \arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} - \frac{3}{2}\,de^3\log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right) + \frac{3}{2}\,\sqrt{-e^2x^2+d^2}e^3 + \frac{\sqrt{-e^2x^2+d^2}de^2}{x} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e}{2\,x^2} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d}{3\,x^3}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x, algorithm="maxima")
```

output
```
d*e^4*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 3/2*d*e^3*log(2*d^2/abs(x) +
 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) + 3/2*sqrt(-e^2*x^2 + d^2)*e^3 + sqrt(-e
^2*x^2 + d^2)*d*e^2/x + 1/2*(-e^2*x^2 + d^2)^(3/2)*e/x^2 - 1/3*(-e^2*x^2 +
 d^2)^(3/2)*d/x^3
```

3.111.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(106) = 212.

Time = 0.29 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.37

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)}\,dx = \frac{de^4\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} + \frac{\left(de^4 - \frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)de^2}{x} - \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d}{x^2}\right)e^6x^3}{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3|e|} - \frac{3\,de^4\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,|e|} + \sqrt{-e^2x^2+d^2}e^3 + \frac{\frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)de^4}{x} + \frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2de^2}{x^2} - \frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d}{x^3}}{24\,e^2|e|}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x, algorithm="giac")
```

output
```
d*e^4*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/24*(d*e^4 - 3*(d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))*d*e^2/x - 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d/
x^2)*e^6*x^3/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*abs(e)) - 3/2*d*e^4*lo
g(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + s
qrt(-e^2*x^2 + d^2)*e^3 + 1/24*(15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d*e
^4/x + 3*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d*e^2/x^2 - (d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^3*d/x^3)/(e^2*abs(e))
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{x^4(d + ex)}\, dx = \int \frac{(d^2 - e^2\,x^2)^{5/2}}{x^4\,(d + e\,x)}\, dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)), x)
```

3.112 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)}\,dx$

3.112.1 Optimal result

Integrand size = 27, antiderivative size = 119

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)}\,dx = \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} - e^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{3}{8}e^4\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
-1/12*(-4*e*x+3*d)*(-e^2*x^2+d^2)^(3/2)/x^4-e^4*arctan(e*x/(-e^2*x^2+d^2)^
(1/2))-3/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/8*e^2*(-8*e*x+3*d)*(-e^2*
x^2+d^2)^(1/2)/x^2
```

3.112.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.27

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)}\,dx = \frac{1}{24}\left(\frac{\sqrt{d^2-e^2x^2}(-6d^3+8d^2ex+15de^2x^2-32e^3x^3)}{x^4} + 48e^4\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) - \frac{9\sqrt{d^2}e^4\log(x)}{d} + \frac{9\sqrt{d^2}e^4\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{d}\right)$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(-6*d^3 + 8*d^2*e*x + 15*d*e^2*x^2 - 32*e^3*x^3))/x^
4 + 48*e^4*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] - (9*Sqrt[d^2]*
e^4*Log[x])/d + (9*Sqrt[d^2]*e^4*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/d)/
24
```

3.112.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {566, 537, 25, 537, 25, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^5(d+ex)}dx$$

$$\Big\downarrow \text{566}$$

$$\int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{537}$$

$$\frac{1}{4}e^2\int -\frac{(3d-4ex)\sqrt{d^2-e^2x^2}}{x^3}dx-\frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{4}e^2\int \frac{(3d-4ex)\sqrt{d^2-e^2x^2}}{x^3}dx-\frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}$$

$$\Big\downarrow \text{537}$$

$$-\frac{1}{4}e^2\left(\frac{1}{2}e^2\int -\frac{3d-8ex}{x\sqrt{d^2-e^2x^2}}dx-\frac{(3d-8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{4}e^2\left(-\frac{1}{2}e^2\int \frac{3d-8ex}{x\sqrt{d^2-e^2x^2}}dx-\frac{(3d-8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}$$

$$\Big\downarrow \text{538}$$

$$-\frac{1}{4}e^2\left(-\frac{1}{2}e^2\left(3d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx-8e\int\frac{1}{\sqrt{d^2-e^2x^2}}dx\right)-\frac{(3d-8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}$$

$$\Big\downarrow \text{224}$$

$$-\frac{1}{4}e^2\left(-\frac{1}{2}e^2\left(3d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx-8e\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)-\frac{(3d-8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{4}e^2\left(-\frac{1}{2}e^2\left(3d\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx-8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{(3d-8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{4}e^2\left(-\frac{1}{2}e^2\left(\frac{3}{2}d\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{(3d-8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}$$

$$\Big\downarrow \text{73}$$

$$-\frac{1}{4}e^2\left(-\frac{1}{2}e^2\left(-\frac{3d\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}-8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{(3d-8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}$$

$$\Big\downarrow \text{221}$$

$$-\frac{1}{4}e^2\left(-\frac{1}{2}e^2\left(-8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-3\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)-\frac{(3d-8ex)\sqrt{d^2-e^2x^2}}{2x^2}\right)-\frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

3.112. $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^5(d+ex)}\,dx$

output
```
-1/12*((3*d - 4*e*x)*(d^2 - e^2*x^2)^(3/2))/x^4 - (e^2*(-1/2*((3*d - 8*e*x
)*Sqrt[d^2 - e^2*x^2])/x^2 - (e^2*(-8*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] -
3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]))/2))/4
```

3.112.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 566
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Int[x^m*(a/c + b*(x/d))*(a + b*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0]
```

3.112.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.06

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(32e^3x^3-15d\,e^2x^2-8d^2ex+6d^3)}{24x^4}-\frac{e^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}-\frac{3e^4d\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}}$
default	$\frac{-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{4d^2x^4}-\frac{3e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{2d^2x^2}-\frac{5e^2\left(\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5}+d^2\left(\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)\right)\right)}{2d^2}\right)}{4d^2}}{d}$

input

```
int((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x,method=_RETURNVERBOSE)
```

3.112. $\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)}\,dx$

output
```
-1/24*(-e^2*x^2+d^2)^(1/2)*(32*e^3*x^3-15*d*e^2*x^2-8*d^2*e*x+6*d^3)/x^4-e
^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-3/8*e^4*d/(d^2)^
(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^5(d+ex)}\,dx = \frac{48\,e^4x^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+9\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(32\,e^3x^3-15\,de^2x^2-\right.}{24\,x^4}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x, algorithm="fricas")
```

output
```
1/24*(48*e^4*x^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 9*e^4*x^4*log
(-(d - sqrt(-e^2*x^2 + d^2))/x) - (32*e^3*x^3 - 15*d*e^2*x^2 - 8*d^2*e*x +
 6*d^3)*sqrt(-e^2*x^2 + d^2))/x^4
```

3.112.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.16 (sec) , antiderivative size = 541, normalized size of antiderivative = 4.55

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^5(d+ex)}\,dx = d^3\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise}\end{cases}\right)$$
$$-d^2e\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise}\end{cases}\right)$$
$$-de^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)$$
$$+e^3\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/x**5/(e*x+d),x)
```

output
```
d**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*
sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) +
 e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x*
*5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1
)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/
(8*d**3), True)) - d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2
) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1),
(-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2)
 + 1)/(3*d**2), True)) - d*e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(
2*x) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e
*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1))
 - I*e**2*asin(d/(e*x))/(2*d), True)) + e**3*Piecewise((I*d/(x*sqrt(-1 + e
**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)
), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x
/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.112.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.44

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)}\,dx = -\frac{e^5\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} - \frac{3}{8}e^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) + \frac{3\sqrt{-e^2x^2+d^2}e^4}{8\,d} - \frac{\sqrt{-e^2x^2+d^2}e^3}{x} + \frac{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^2}{8\,dx^2} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e}{3\,x^3} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d}{4\,x^4}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x, algorithm="maxima")
```

output
```
-e^5*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 3/8*e^4*log(2*d^2/abs(x) + 2*
sqrt(-e^2*x^2 + d^2)*d/abs(x)) + 3/8*sqrt(-e^2*x^2 + d^2)*e^4/d - sqrt(-e^
2*x^2 + d^2)*e^3/x + 3/8*(-e^2*x^2 + d^2)^(3/2)*e^2/(d*x^2) + 1/3*(-e^2*x^
2 + d^2)^(3/2)*e/x^3 - 1/4*(-e^2*x^2 + d^2)^(3/2)*d/x^4
```

3.112.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(105) = 210$.

Time = 0.30 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.76

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)}\,dx = \frac{\left(3\,e^5 - \frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^3}{x} - \frac{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 e}{x^2} + \frac{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{ex^3}\right)e^8x^4}{192\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4|e|} - \frac{e^5\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{|e|} - \frac{3\,e^5\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{8\,|e|} - \frac{\frac{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^5|e|}{x} - \frac{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^3|e|}{x^2} - \frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3e|e|}{x^3} + \frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4|e|}{ex^4}}{192\,e^4}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x, algorithm="giac")
```

output
```
1/192*(3*e^5 - 8*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^3/x - 24*(d*e + sqr
t(-e^2*x^2 + d^2)*abs(e))^2*e/x^2 + 120*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e)
)^3/(e*x^3))*e^8*x^4/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*abs(e)) - e^5*
arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 3/8*e^5*log(1/2*abs(-2*d*e - 2*sqrt(-
e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) - 1/192*(120*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))*e^5*abs(e)/x - 24*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2
*e^3*abs(e)/x^2 - 8*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*e*abs(e)/x^3 + 3
*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*abs(e)/(e*x^4))/e^4
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}}{x^5\,(d+e\,x)}\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)), x)
```

3.113 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)}\,dx$

3.113.1 Optimal result

Integrand size = 27, antiderivative size = 108

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)}\,dx = -\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}+\frac{e(d^2-e^2x^2)^{3/2}}{4x^4}-\frac{(d^2-e^2x^2)^{5/2}}{5dx^5}+\frac{3e^5\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

output
```
1/4*e*(-e^2*x^2+d^2)^(3/2)/x^4-1/5*(-e^2*x^2+d^2)^(5/2)/d/x^5+3/8*e^5*arct
anh((-e^2*x^2+d^2)^(1/2)/d)/d-3/8*e^3*(-e^2*x^2+d^2)^(1/2)/x^2
```

3.113.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.13

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)}\,dx = \frac{1}{40}\left(\frac{\sqrt{d^2-e^2x^2}(-8d^4+10d^3ex+16d^2e^2x^2-25de^3x^3-8e^4x^4)}{dx^5}+\frac{15e^5\log(x)}{\sqrt{d^2}}-\frac{15e^5\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{\sqrt{d^2}}\right)$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(-8*d^4 + 10*d^3*e*x + 16*d^2*e^2*x^2 - 25*d*e^3*x^3
 - 8*e^4*x^4))/(d*x^5) + (15*e^5*Log[x])/Sqrt[d^2] - (15*e^5*Log[Sqrt[d^2]
 - Sqrt[d^2 - e^2*x^2]])/Sqrt[d^2])/40
```

3.113.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {566, 534, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)}dx$$

$$\Big\downarrow \text{566}$$

$$\int \frac{(d-ex)(d^2-e^2x^2)^{3/2}}{x^6}dx$$

$$\Big\downarrow \text{534}$$

$$-e\int \frac{(d^2-e^2x^2)^{3/2}}{x^5}dx-\frac{(d^2-e^2x^2)^{5/2}}{5dx^5}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}e\int \frac{(d^2-e^2x^2)^{3/2}}{x^6}dx^2-\frac{(d^2-e^2x^2)^{5/2}}{5dx^5}$$

$$\Big\downarrow \text{51}$$

$$-\frac{1}{2}e\left(-\frac{3}{4}e^2\int \frac{\sqrt{d^2-e^2x^2}}{x^4}dx^2-\frac{(d^2-e^2x^2)^{3/2}}{2x^4}\right)-\frac{(d^2-e^2x^2)^{5/2}}{5dx^5}$$

$$\Big\downarrow \text{51}$$

$$-\frac{1}{2}e\left(-\frac{3}{4}e^2\left(-\frac{1}{2}e^2\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{(d^2-e^2x^2)^{3/2}}{2x^4}\right)-\frac{(d^2-e^2x^2)^{5/2}}{5dx^5}$$

$$\Big\downarrow \text{73}$$

$$-\frac{1}{2}e\left(-\frac{3}{4}e^2\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{(d^2-e^2x^2)^{3/2}}{2x^4}\right)-\frac{(d^2-e^2x^2)^{5/2}}{5dx^5}$$

$$\Big\downarrow \text{221}$$

$$-\frac{1}{2}e\left(-\frac{3}{4}e^2\left(\frac{e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{(d^2-e^2x^2)^{3/2}}{2x^4}\right)-\frac{(d^2-e^2x^2)^{5/2}}{5dx^5}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

output
```
-1/5*(d^2 - e^2*x^2)^(5/2)/(d*x^5) - (e*(-1/2*(d^2 - e^2*x^2)^(3/2)/x^4 -
(3*e^2*(-(Sqrt[d^2 - e^2*x^2]/x^2) + (e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/
d))/4))/2
```

3.113.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 566
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Int[x^m*(a/c + b*(x/d))*(a + b*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0]
```

3.113.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

method	result	size
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(8e^4x^4+25d\,e^3x^3-16d^2e^2x^2-10d^3ex+8d^4)}{40x^5d}+\frac{3e^5\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}}$	107
default	Expression too large to display	1100

input
```
int((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/40*(-e^2*x^2+d^2)^(1/2)*(8*e^4*x^4+25*d*e^3*x^3-16*d^2*e^2*x^2-10*d^3*e
*x+8*d^4)/x^5/d+3/8*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)
^(1/2))/x)
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)}\,dx = -\frac{15\,e^5x^5\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(8\,e^4x^4+25\,de^3x^3-16\,d^2e^2x^2-10\,d^3ex+8\,d^4)\sqrt{-e^2x^2+d^2}}{40\,dx^5}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x, algorithm="fricas")
```

output
```
-1/40*(15*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (8*e^4*x^4 + 25*d*e
^3*x^3 - 16*d^2*e^2*x^2 - 10*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/(d*x^5
)
```

3.113.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.04 (sec) , antiderivative size = 774, normalized size of antiderivative = 7.17

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)}\,dx = d^3\left(\begin{cases} \frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} - \frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} + \frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7} - \frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for } \left|\frac{e^2x^2}{d^2}\right| \\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} - \frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} + \frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7} - \frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwis} \end{cases}\right)$$

$$- d^2e\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise} \end{cases}\right)$$

$$- de^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right)$$

$$+ e^3\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x} + \frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/x**6/(e*x+d),x)
```

output
```
d**3*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**
2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e*
*2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**
3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d
*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-
15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15
*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**
5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3
*x**5 + 15*d*e**2*x**7), True)) - d**2*e*Piecewise((-d**2/(4*e*x**5*sqrt(d
**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*
d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**
2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*
e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2
*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) - d*e**2*Piecewise((-
e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3
*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x
**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) + e**3*Piecewis
e((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/(2*d), Abs(d*
*2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e
/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d), True))
```

3.113.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.42

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)}\,dx = \frac{3\,e^5\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{8\,d} - \frac{3\,\sqrt{-e^2x^2+d^2}e^5}{8\,d^2} - \frac{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^3}{8\,d^2x^2} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^2}{5\,dx^3} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e}{4\,x^4} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d}{5\,x^5}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x, algorithm="maxima")
```

output
```
3/8*e^5*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 3/8*sqrt(-
e^2*x^2 + d^2)*e^5/d^2 - 3/8*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^2*x^2) + 1/5*(-
e^2*x^2 + d^2)^(3/2)*e^2/(d*x^3) + 1/4*(-e^2*x^2 + d^2)^(3/2)*e/x^4 - 1/5*
(-e^2*x^2 + d^2)^(3/2)*d/x^5
```

3.113.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(92) = 184$.

Time = 0.30 (sec) , antiderivative size = 388, normalized size of antiderivative = 3.59

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)}\,dx = \frac{\left(2e^6 - \frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^4}{x} - \frac{10\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^2}{x^2} + \frac{40\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{x^3} + \frac{20\left(de+\sqrt{\right.}}{}\right.}{320\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^5 d|e|} + \frac{3e^6\log\left(\frac{\left|-2de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2e^2|x|}\right)}{8d|e|} - \frac{\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^4e^8}{x} + \frac{40\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^4e^6}{x^2} - \frac{10\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^4e^4}{x^3} - \frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^4e^2}{x^4} + \frac{2\left(de-\right.}{}}{320d^5e^4|e|}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x, algorithm="giac")
```

output
```
1/320*(2*e^6 - 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^4/x - 10*(d*e + sqr
t(-e^2*x^2 + d^2)*abs(e))^2*e^2/x^2 + 40*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e
))^3/x^3 + 20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^2*x^4))*e^10*x^5/((
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d*abs(e)) + 3/8*e^6*log(1/2*abs(-2*d*
e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d*abs(e)) - 1/320*(20*(d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^4*e^8/x + 40*(d*e + sqrt(-e^2*x^2 + d^
2)*abs(e))^2*d^4*e^6/x^2 - 10*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^4*e^
4/x^3 - 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^4*e^2/x^4 + 2*(d*e + sqr
t(-e^2*x^2 + d^2)*abs(e))^5*d^4/x^5)/(d^5*e^4*abs(e))
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}}{x^6\,(d+e\,x)}\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)), x)
```

3.114 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx$

3.114.1 Optimal result

Integrand size = 27, antiderivative size = 143

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx = \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2} - \frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4} - \frac{(d^2-e^2x^2)^{5/2}}{6dx^6} + \frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5} - \frac{e^6\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}$$

output
```
-1/24*e^2*(-e^2*x^2+d^2)^(3/2)/d/x^4-1/6*(-e^2*x^2+d^2)^(5/2)/d/x^6+1/5*e*
(-e^2*x^2+d^2)^(5/2)/d^2/x^5-1/16*e^6*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2+
1/16*e^4*(-e^2*x^2+d^2)^(1/2)/d/x^2
```

3.114.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx = \frac{\sqrt{d^2-e^2x^2}(-40d^5+48d^4ex+70d^3e^2x^2-96d^2e^3x^3-15de^4x^4+48e^5x^5)}{240d^2x^6} - \frac{\sqrt{d^2}e^6\log(x)}{16d^3} + \frac{\sqrt{d^2}e^6\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{16d^3}$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-40*d^5 + 48*d^4*e*x + 70*d^3*e^2*x^2 - 96*d^2*e^3*x
^3 - 15*d*e^4*x^4 + 48*e^5*x^5))/(240*d^2*x^6) - (Sqrt[d^2]*e^6*Log[x])/(1
6*d^3) + (Sqrt[d^2]*e^6*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(16*d^3)
```

3.114.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {566, 539, 27, 534, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}dx$$

$$\Big\downarrow \text{566}$$

$$\int \frac{(d-ex)(d^2-e^2x^2)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{539}$$

$$-\frac{\int \frac{de(6d-ex)(d^2-e^2x^2)^{3/2}}{x^6}dx}{6d^2}-\frac{(d^2-e^2x^2)^{5/2}}{6dx^6}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e\int \frac{(6d-ex)(d^2-e^2x^2)^{3/2}}{x^6}dx}{6d}-\frac{(d^2-e^2x^2)^{5/2}}{6dx^6}$$

$$\Big\downarrow \text{534}$$

$$-\frac{e\left(-e\int \frac{(d^2-e^2x^2)^{3/2}}{x^5}dx-\frac{6(d^2-e^2x^2)^{5/2}}{5dx^5}\right)}{6d}-\frac{(d^2-e^2x^2)^{5/2}}{6dx^6}$$

$$\Big\downarrow \text{243}$$

$$-\frac{e\left(-\frac{1}{2}e\int \frac{(d^2-e^2x^2)^{3/2}}{x^6}dx^2-\frac{6(d^2-e^2x^2)^{5/2}}{5dx^5}\right)}{6d}-\frac{(d^2-e^2x^2)^{5/2}}{6dx^6}$$

$$\Big\downarrow \text{51}$$

$$-\frac{e\left(-\frac{1}{2}e\left(-\frac{3}{4}e^2\int \frac{\sqrt{d^2-e^2x^2}}{x^4}dx^2-\frac{(d^2-e^2x^2)^{3/2}}{2x^4}\right)-\frac{6(d^2-e^2x^2)^{5/2}}{5dx^5}\right)}{6d}-\frac{(d^2-e^2x^2)^{5/2}}{6dx^6}$$

$$\Big\downarrow \; 51$$

$$-\frac{e\left(-\frac{1}{2}e\left(-\frac{3}{4}e^2\left(-\frac{1}{2}e^2\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{(d^2-e^2x^2)^{3/2}}{2x^4}\right)-\frac{6(d^2-e^2x^2)^{5/2}}{5dx^5}\right)}{6d}-\frac{(d^2-e^2x^2)^{5/2}}{6dx^6}$$

$$\Big\downarrow \; 73$$

$$-\frac{e\left(-\frac{1}{2}e\left(-\frac{3}{4}e^2\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{(d^2-e^2x^2)^{3/2}}{2x^4}\right)-\frac{6(d^2-e^2x^2)^{5/2}}{5dx^5}\right)}{6d}-\frac{(d^2-e^2x^2)^{5/2}}{6dx^6}$$

$$\Big\downarrow \; 221$$

$$-\frac{e\left(-\frac{1}{2}e\left(-\frac{3}{4}e^2\left(\frac{e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{(d^2-e^2x^2)^{3/2}}{2x^4}\right)-\frac{6(d^2-e^2x^2)^{5/2}}{5dx^5}\right)}{6d}-\frac{(d^2-e^2x^2)^{5/2}}{6dx^6}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

output
```
-1/6*(d^2 - e^2*x^2)^(5/2)/(d*x^6) - (e*((-6*(d^2 - e^2*x^2)^(5/2))/(5*d*x
^5) - (e*(-1/2*(d^2 - e^2*x^2)^(3/2)/x^4 - (3*e^2*(-(Sqrt[d^2 - e^2*x^2]/x
^2) + (e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/4))/2))/(6*d)
```

3.114.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 566
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Int[x^m*(a/c + b*(x/d))*(a + b*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0]
```

3.114.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

method	result	size
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, \left(-48 e^{5} x^{5}+15 d \,e^{4} x^{4}+96 d^{2} e^{3} x^{3}-70 d^{3} e^{2} x^{2}-48 d^{4} e x +40 d^{5}\right)}{240 x^{6} d^{2}}-\frac{e^{6} \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{16 d \sqrt{d^{2}}}$	121
default	Expression too large to display	1300

3.114. $\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx$

input
```
int((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/240*(-e^2*x^2+d^2)^(1/2)*(-48*e^5*x^5+15*d*e^4*x^4+96*d^2*e^3*x^3-70*d^
3*e^2*x^2-48*d^4*e*x+40*d^5)/x^6/d^2-1/16/d*e^6/(d^2)^(1/2)*ln((2*d^2+2*(d
^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.76

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx = \frac{15\,e^6x^6\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(48\,e^5x^5-15\,de^4x^4-96\,d^2e^3x^3+70\,d^3e^2x^2+48\,d^4ex}{240\,d^2x^6}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x, algorithm="fricas")
```

output
```
1/240*(15*e^6*x^6*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (48*e^5*x^5 - 15*d*
e^4*x^4 - 96*d^2*e^3*x^3 + 70*d^3*e^2*x^2 + 48*d^4*e*x - 40*d^5)*sqrt(-e^2
*x^2 + d^2))/(d^2*x^6)
```

3.114.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.03 (sec) , antiderivative size = 918, normalized size of antiderivative = 6.42

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx = d^3\left(\begin{cases} -\frac{d^2}{6ex^7\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{5e}{24x^5\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^3}{48d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e^5}{16d^4x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^6\operatorname{acosh}\left(\frac{d}{ex}\right)}{16d^5} \\ \frac{id^2}{6ex^7\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{5ie}{24x^5\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^3}{48d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^5}{16d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^6\operatorname{asin}\left(\frac{d}{ex}\right)}{16d^5} \end{cases}\right.$$

$$-d^2e\left(\begin{cases} \frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} - \frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} + \frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7} - \frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} - \frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} + \frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7} - \frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise} \end{cases}\right)$$

$$-de^2\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise} \end{cases}\right)$$

$$+e^3\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/x**7/(e*x+d),x)
```

3.114. $\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx$

output
```
d**3*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5
*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) -
1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(1
6*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x*
*2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*
x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**
2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - d**2*e*Piecewise((3*I*
d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2
*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*
x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**
4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**
2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e*
*2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**
2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x*
*7), True)) - d*e**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)
) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e*
*2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1),
 (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2
/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*...
```

3.114.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.24

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx = -\frac{e^6\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{16\,d^2} + \frac{\sqrt{-e^2x^2+d^2}e^6}{16\,d^3} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}{16\,d^3x^2} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^3}{5\,d^2x^3} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^2}{8\,dx^4} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e}{5\,x^5} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d}{6\,x^6}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x, algorithm="maxima")
```

output
```
-1/16*e^6*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^2 + 1/16*s
qrt(-e^2*x^2 + d^2)*e^6/d^3 + 1/16*(-e^2*x^2 + d^2)^(3/2)*e^4/(d^3*x^2) -
1/5*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^2*x^3) + 1/8*(-e^2*x^2 + d^2)^(3/2)*e^2/
(d*x^4) + 1/5*(-e^2*x^2 + d^2)^(3/2)*e/x^5 - 1/6*(-e^2*x^2 + d^2)^(3/2)*d/
x^6
```

3.114.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(123) = 246$.

Time = 0.29 (sec) , antiderivative size = 463, normalized size of antiderivative = 3.24

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx = \frac{\left(5\,e^7 - \frac{12\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^5}{x} - \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^3}{x^2} + \frac{60\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3e}{x^3} - \frac{15\left(de+\right.}{}\right.}{1920\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^6d^2|e|}$$
$$- \frac{e^7\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{16\,d^2|e|}$$
$$+ \frac{\frac{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{10}e^9|e|}{x} + \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^{10}e^7|e|}{x^2} - \frac{60\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^{10}e^5|e|}{x^3} + \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^{10}}{x^4}}{1920\,d^{12}e^6}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x, algorithm="giac")
```

output
```
1/1920*(5*e^7 - 12*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^5/x - 15*(d*e + s
qrt(-e^2*x^2 + d^2)*abs(e))^2*e^3/x^2 + 60*(d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))^3*e/x^3 - 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e*x^4) - 120*(d*e
 + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^3*x^5))*e^12*x^6/((d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))^6*d^2*abs(e)) - 1/16*e^7*log(1/2*abs(-2*d*e - 2*sqrt(-e^2
*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^2*abs(e)) + 1/1920*(120*(d*e + sqrt(-
e^2*x^2 + d^2)*abs(e))*d^10*e^9*abs(e)/x + 15*(d*e + sqrt(-e^2*x^2 + d^2)*
abs(e))^2*d^10*e^7*abs(e)/x^2 - 60*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d
^10*e^5*abs(e)/x^3 + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^10*e^3*abs
(e)/x^4 + 12*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d^10*e*abs(e)/x^5 - 5*(
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6*d^10*abs(e)/(e*x^6))/(d^12*e^6)
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}}{x^7\,(d+e\,x)}\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)), x)
```

3.115 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)}\,dx$

3.115.1 Optimal result

Integrand size = 27, antiderivative size = 172

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)}\,dx = -\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} + \frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4} - \frac{(d^2-e^2x^2)^{5/2}}{7dx^7} + \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5} + \frac{e^7\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

output
```
1/24*e^3*(-e^2*x^2+d^2)^(3/2)/d^2/x^4-1/7*(-e^2*x^2+d^2)^(5/2)/d/x^7+1/6*e
*(-e^2*x^2+d^2)^(5/2)/d^2/x^6-2/35*e^2*(-e^2*x^2+d^2)^(5/2)/d^3/x^5+1/16*e
^7*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3-1/16*e^5*(-e^2*x^2+d^2)^(1/2)/d^2/x
^2
```

3.115.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.89

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)}\,dx = \frac{\sqrt{d^2-e^2x^2}(-240d^6+280d^5ex+384d^4e^2x^2-490d^3e^3x^3-48d^2e^4x^4+105de^5x^5-9}{1680d^3x^7} + \frac{\sqrt{d^2}e^7\log(x)}{16d^4} - \frac{\sqrt{d^2}e^7\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{16d^4}$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-240*d^6 + 280*d^5*e*x + 384*d^4*e^2*x^2 - 490*d^3*e
^3*x^3 - 48*d^2*e^4*x^4 + 105*d*e^5*x^5 - 96*e^6*x^6))/(1680*d^3*x^7) + (S
qrt[d^2]*e^7*Log[x])/(16*d^4) - (Sqrt[d^2]*e^7*Log[Sqrt[d^2] - Sqrt[d^2 -
e^2*x^2]])/(16*d^4)
```

3.115.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {566, 539, 27, 539, 27, 534, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)}dx$$

$$\Big\downarrow \text{566}$$

$$\int \frac{(d-ex)(d^2-e^2x^2)^{3/2}}{x^8}dx$$

$$\Big\downarrow \text{539}$$

$$-\frac{\int \frac{de(7d-2ex)(d^2-e^2x^2)^{3/2}}{x^7}dx}{7d^2}-\frac{(d^2-e^2x^2)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e\int \frac{(7d-2ex)(d^2-e^2x^2)^{3/2}}{x^7}dx}{7d}-\frac{(d^2-e^2x^2)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{539}$$

$$-\frac{e\left(-\frac{\int \frac{de(12d-7ex)(d^2-e^2x^2)^{3/2}}{x^6}dx}{6d^2}-\frac{7(d^2-e^2x^2)^{5/2}}{6dx^6}\right)}{7d}-\frac{(d^2-e^2x^2)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e\left(-\frac{e\int \frac{(12d-7ex)(d^2-e^2x^2)^{3/2}}{x^6}dx}{6d}-\frac{7(d^2-e^2x^2)^{5/2}}{6dx^6}\right)}{7d}-\frac{(d^2-e^2x^2)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{534}$$

$$-\frac{e\left(-\frac{e\left(-7e\int\frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}dx-\frac{12\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{6d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}\right)}{7d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{e\left(-\frac{e\left(-\frac{7}{2}e\int\frac{\left(d^2-e^2x^2\right)^{3/2}}{x^6}dx^2-\frac{12\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{6d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}\right)}{7d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{ 51}$$

$$-\frac{e\left(-\frac{e\left(-\frac{7}{2}e\left(-\frac{3}{4}e^2\int\frac{\sqrt{d^2-e^2x^2}}{x^4}dx^2-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{12\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{6d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}\right)}{7d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{ 51}$$

$$-\frac{e\left(-\frac{e\left(-\frac{7}{2}e\left(-\frac{3}{4}e^2\left(-\frac{1}{2}e^2\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{12\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{6d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}\right)}{7d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{e\left(-\frac{e\left(-\frac{7}{2}e\left(-\frac{3}{4}e^2\left(\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{12\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{6d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}\right)}{7d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{ 221}$$

3.115. $\int\frac{\left(d^2-e^2x^2\right)^{5/2}}{x^8(d+ex)}dx$

$$-\frac{e\left(-\frac{e\left(-\frac{7}{2}e\left(-\frac{3}{4}e^2\left(\frac{e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{12\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{6d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}\right)}{7d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

output
```
-1/7*(d^2 - e^2*x^2)^(5/2)/(d*x^7) - (e*((-7*(d^2 - e^2*x^2)^(5/2))/(6*d*x
^6) - (e*((-12*(d^2 - e^2*x^2)^(5/2))/(5*d*x^5) - (7*e*(-1/2*(d^2 - e^2*x^
2)^(3/2)/x^4 - (3*e^2*(-(Sqrt[d^2 - e^2*x^2]/x^2) + (e^2*ArcTanh[Sqrt[d^2
- e^2*x^2]/d])/d))/4))/2))/(6*d)))/(7*d)
```

3.115.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 566
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Int[x^m*(a/c + b*(x/d))*(a + b*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0]
```

3.115.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.77

method	result	si
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, \left(96 e^{6} x^{6}-105 d\, e^{5} x^{5}+48 d^{2} e^{4} x^{4}+490 d^{3} x^{3} e^{3}-384 d^{4} e^{2} x^{2}-280 d^{5} e x +240 d^{6}\right)}{1680 x^{7} d^{3}}+\frac{e^{7} \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{16 d^{2} \sqrt{d^{2}}}$	13
default	Expression too large to display	13

input
```
int((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/1680*(-e^2*x^2+d^2)^(1/2)*(96*e^6*x^6-105*d*e^5*x^5+48*d^2*e^4*x^4+490*
d^3*e^3*x^3-384*d^4*e^2*x^2-280*d^5*e*x+240*d^6)/x^7/d^3+1/16/d^2*e^7/(d^2
)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.69

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)}\,dx =$$
$$-\frac{105\,e^7x^7\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(96\,e^6x^6-105\,de^5x^5+48\,d^2e^4x^4+490\,d^3e^3x^3-384\,d^4e^2x^2-280\,d^5ex+\right.}{1680\,d^3x^7}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x, algorithm="fricas")
```

output
```
-1/1680*(105*e^7*x^7*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (96*e^6*x^6 - 10
5*d*e^5*x^5 + 48*d^2*e^4*x^4 + 490*d^3*e^3*x^3 - 384*d^4*e^2*x^2 - 280*d^5
*e*x + 240*d^6)*sqrt(-e^2*x^2 + d^2))/(d^3*x^7)
```

3.115.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.72 (sec) , antiderivative size = 1037, normalized size of antiderivative = 6.03

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/x**8/(e*x+d),x)
```

output
```
d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e
**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**
4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**
2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(
e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105
*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) - d*
*2*e*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5
*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) -
1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(1
6*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x*
*2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*
x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**
2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - d*e**2*Piecewise((3*I*
d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2
*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*
x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**
4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**
2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e*
*2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*...
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.18

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)}\,dx = \frac{e^7\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{16\,d^3} - \frac{\sqrt{-e^2x^2+d^2}e^7}{16\,d^4} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^5}{16\,d^4x^2} + \frac{2\,(-e^2x^2+d^2)^{\frac{3}{2}}e^4}{35\,d^3x^3} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^3}{8\,d^2x^4} + \frac{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^2}{35\,dx^5} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e}{6\,x^6} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d}{7\,x^7}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x, algorithm="maxima")
```

output
```
1/16*e^7*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 1/16*sq
rt(-e^2*x^2 + d^2)*e^7/d^4 - 1/16*(-e^2*x^2 + d^2)^(3/2)*e^5/(d^4*x^2) + 2
/35*(-e^2*x^2 + d^2)^(3/2)*e^4/(d^3*x^3) - 1/8*(-e^2*x^2 + d^2)^(3/2)*e^3/
(d^2*x^4) + 3/35*(-e^2*x^2 + d^2)^(3/2)*e^2/(d*x^5) + 1/6*(-e^2*x^2 + d^2)
^(3/2)*e/x^6 - 1/7*(-e^2*x^2 + d^2)^(3/2)*d/x^7
```

3.115.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(148) = 296$.

Time = 0.29 (sec) , antiderivative size = 518, normalized size of antiderivative = 3.01

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)}\,dx = \frac{\left(15\,e^8 - \frac{35\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^6}{x} - \frac{21\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^4}{x^2} + \frac{105\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3e^2}{x^3} - \frac{105 \cdots}{\cdots}\right.}{13440\left(de+\sqrt{-e^2x^2+d^2}\cdots\right.} + \frac{e^8\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{16\,d^3|e|} - \frac{\frac{315\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{18}e^{12}}{x} + \frac{105\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^{18}e^{10}}{x^2} - \frac{105\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^{18}e^8}{x^3} + \frac{105\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^{18}e\cdots}{x^4}\cdots}{13440\,d^{21}e^6|e|}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x, algorithm="giac")
```

output
```
1/13440*(15*e^8 - 35*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^6/x - 21*(d*e +
 sqrt(-e^2*x^2 + d^2)*abs(e))^2*e^4/x^2 + 105*(d*e + sqrt(-e^2*x^2 + d^2)*
abs(e))^3*e^2/x^3 - 105*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/x^4 + 105*(d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^2*x^5) + 315*(d*e + sqrt(-e^2*x^2 +
 d^2)*abs(e))^6/(e^4*x^6))*e^14*x^7/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7
*d^3*abs(e)) + 1/16*e^8*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e)
)/(e^2*abs(x)))/(d^3*abs(e)) - 1/13440*(315*(d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))*d^18*e^12/x + 105*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^18*e^10/x^
2 - 105*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^18*e^8/x^3 + 105*(d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))^4*d^18*e^6/x^4 - 21*(d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))^5*d^18*e^4/x^5 - 35*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6*d^18*e^
2/x^6 + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7*d^18/x^7)/(d^21*e^6*abs(e
))
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}}{x^8\,(d+e\,x)}\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)), x)
```

3.115. $\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)}\,dx$

3.116 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^9(d+ex)}\,dx$

3.116.1 Optimal result

Integrand size = 27, antiderivative size = 201

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^9(d+ex)}\,dx=\frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2}-\frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4}-\frac{(d^2-e^2x^2)^{5/2}}{8dx^8}+\frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7}-\frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6}+\frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5}-\frac{3e^8\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d^4}$$

output
```
-1/64*e^4*(-e^2*x^2+d^2)^(3/2)/d^3/x^4-1/8*(-e^2*x^2+d^2)^(5/2)/d/x^8+1/7*
e*(-e^2*x^2+d^2)^(5/2)/d^2/x^7-1/16*e^2*(-e^2*x^2+d^2)^(5/2)/d^3/x^6+2/35*
e^3*(-e^2*x^2+d^2)^(5/2)/d^4/x^5-3/128*e^8*arctanh((-e^2*x^2+d^2)^(1/2)/d)
/d^4+3/128*e^6*(-e^2*x^2+d^2)^(1/2)/d^3/x^2
```

3.116.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.82

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^9(d+ex)}\,dx=\frac{\sqrt{d^2-e^2x^2}(-560d^7+640d^6ex+840d^5e^2x^2-1024d^4e^3x^3-70d^3e^4x^4+128d^2e^5x^5 -}{4480d^4x^8}-\frac{3\sqrt{d^2}e^8\log(x)}{128d^5}+\frac{3\sqrt{d^2}e^8\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{128d^5}$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-560*d^7 + 640*d^6*e*x + 840*d^5*e^2*x^2 - 1024*d^4*
e^3*x^3 - 70*d^3*e^4*x^4 + 128*d^2*e^5*x^5 - 105*d*e^6*x^6 + 256*e^7*x^7))
/(4480*d^4*x^8) - (3*Sqrt[d^2]*e^8*Log[x])/(128*d^5) + (3*Sqrt[d^2]*e^8*Lo
g[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(128*d^5)
```

3.116.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {566, 539, 27, 539, 27, 539, 27, 534, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^9(d+ex)}dx$$

$$\Big\downarrow \text{566}$$

$$\int \frac{(d-ex)(d^2-e^2x^2)^{3/2}}{x^9}dx$$

$$\Big\downarrow \text{539}$$

$$-\frac{\int \frac{de(8d-3ex)(d^2-e^2x^2)^{3/2}}{x^8}dx}{8d^2}-\frac{(d^2-e^2x^2)^{5/2}}{8dx^8}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e\int \frac{(8d-3ex)(d^2-e^2x^2)^{3/2}}{x^8}dx}{8d}-\frac{(d^2-e^2x^2)^{5/2}}{8dx^8}$$

$$\Big\downarrow \text{539}$$

$$-\frac{e\left(-\frac{\int \frac{de(21d-16ex)(d^2-e^2x^2)^{3/2}}{x^7}dx}{7d^2}-\frac{8(d^2-e^2x^2)^{5/2}}{7dx^7}\right)}{8d}-\frac{(d^2-e^2x^2)^{5/2}}{8dx^8}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e\left(-\frac{e\int \frac{(21d-16ex)(d^2-e^2x^2)^{3/2}}{x^7}dx}{7d}-\frac{8(d^2-e^2x^2)^{5/2}}{7dx^7}\right)}{8d}-\frac{(d^2-e^2x^2)^{5/2}}{8dx^8}$$

$$\Big\downarrow \text{539}$$

$$-\frac{e\left(-\frac{e\left(-\frac{\int \frac{3de(32d-7ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}dx}{6d^2}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{2dx^6}\right)}{7d}-\frac{8\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}\right)}{8d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e\left(-\frac{e\left(-\frac{e\int \frac{(32d-7ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}dx}{2d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{2dx^6}\right)}{7d}-\frac{8\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}\right)}{8d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \text{534}$$

$$-\frac{e\left(-\frac{e\left(-\frac{e\left(-7e\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}dx-\frac{32\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{2d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{2dx^6}\right)}{7d}-\frac{8\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}\right)}{8d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \text{243}$$

$$-\frac{e\left(-\frac{e\left(-\frac{e\left(-\frac{7}{2}e\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^6}dx^2-\frac{32\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{2d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{2dx^6}\right)}{7d}-\frac{8\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}\right)}{8d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \text{51}$$

3.116. $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^9(d+ex)}\,dx$

$$-\frac{e\left(-\frac{e\left(-\frac{e\left(-\frac{7}{2}e\left(-\frac{3}{4}e^2\int\frac{\sqrt{d^2-e^2x^2}}{x^4}dx^2-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{32\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{2d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{2dx^6}\right)}{7d}-\frac{8\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}\right)}{8d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \; 51$$

$$-\frac{e\left(-\frac{e\left(-\frac{e\left(-\frac{7}{2}e\left(-\frac{3}{4}e^2\left(-\frac{1}{2}e^2\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{32\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{2d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{2dx^6}\right)}{7d}-\frac{8\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}\right)}{8d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \; 73$$

$$-\frac{e\left(-\frac{e\left(-\frac{e\left(-\frac{7}{2}e\left(-\frac{3}{4}e^2\left(\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^4}\right)-\frac{32\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}\right)}{2d}-\frac{7\left(d^2-e^2x^2\right)^{5/2}}{2dx^6}\right)}{7d}-\frac{8\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}\right)}{8d}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \; 221$$

3.116. $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^9(d+ex)}\,dx$

$$-\frac{e\left(-\frac{e\left(-\frac{e\left(-\frac{7}{2}e\left(-\frac{3}{4}e^2\left(\frac{e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{(d^2-e^2x^2)^{3/2}}{2x^4}\right)-\frac{32(d^2-e^2x^2)^{5/2}}{5dx^5}\right)}{2d}-\frac{7(d^2-e^2x^2)^{5/2}}{2dx^6}\right)}{7d}-\frac{8(d^2-e^2x^2)^{5/}}{7dx^7}\right)}{8d}$$

$$\frac{(d^2-e^2x^2)^{5/2}}{8dx^8}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

output
```
-1/8*(d^2 - e^2*x^2)^(5/2)/(d*x^8) - (e*((-8*(d^2 - e^2*x^2)^(5/2))/(7*d*x
^7) - (e*((-7*(d^2 - e^2*x^2)^(5/2))/(2*d*x^6) - (e*((-32*(d^2 - e^2*x^2)^
(5/2))/(5*d*x^5) - (7*e*(-1/2*(d^2 - e^2*x^2)^(3/2)/x^4 - (3*e^2*(-(Sqrt[d
^2 - e^2*x^2]/x^2) + (e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/4))/2))/(2*d
)))/(7*d)))/(8*d)
```

3.116.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 566
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Int[x^m*(a/c + b*(x/d))*(a + b*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0]
```

3.116.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-256e^7x^7+105d\,e^6x^6-128d^2e^5x^5+70d^3e^4x^4+1024d^4e^3x^3-840d^5e^2x^2-640d^6ex+560d^7)}{4480x^8d^4} - \frac{3e^8\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{}}{x}\right.}{128d^3\sqrt{d}}$
default	Expression too large to display

input
```
int((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4480*(-e^2*x^2+d^2)^(1/2)*(-256*e^7*x^7+105*d*e^6*x^6-128*d^2*e^5*x^5+7
0*d^3*e^4*x^4+1024*d^4*e^3*x^3-840*d^5*e^2*x^2-640*d^6*e*x+560*d^7)/x^8/d^
4-3/128/d^3*e^8/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/
x)
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.65

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^9(d+ex)}\,dx = \frac{105\,e^8x^8\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(256\,e^7x^7-105\,de^6x^6+128\,d^2e^5x^5-70\,d^3e^4x^4-10}{4480\,d^4x^8}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x, algorithm="fricas")
```

output
```
1/4480*(105*e^8*x^8*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (256*e^7*x^7 - 10
5*d*e^6*x^6 + 128*d^2*e^5*x^5 - 70*d^3*e^4*x^4 - 1024*d^4*e^3*x^3 + 840*d^
5*e^2*x^2 + 640*d^6*e*x - 560*d^7)*sqrt(-e^2*x^2 + d^2))/(d^4*x^8)
```

3.116.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 26.64 (sec) , antiderivative size = 1159, normalized size of antiderivative = 5.77

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^9(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/x**9/(e*x+d),x)
```

output
```
d**3*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)) + 7*e/(48*x**7
*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**2/(e**2*x**2) -
 1)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5*e**7/(128*d**
6*x*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*d**7), Abs(d*
*2/(e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I
*e/(48*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**5*sqrt(-d**
2/(e**2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x**2) + 1))
 + 5*I*e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(e*
x))/(128*d**7), True)) - d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(
7*x**6) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**
2/(e**2*x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(10
5*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*
x**6) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(
-d**2/(e**2*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) +
 1)/(105*d**6), True)) - d*e**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2
*x**2) - 1)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x*
*3*sqrt(d**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1
)) + e**6*acosh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6
*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**
2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(...
```

3.116.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.13

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^9(d+ex)}\,dx = -\frac{3\,e^8\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{128\,d^4} + \frac{3\,\sqrt{-e^2x^2+d^2}e^8}{128\,d^5} + \frac{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^6}{128\,d^5x^2} - \frac{2\,(-e^2x^2+d^2)^{\frac{3}{2}}e^5}{35\,d^4x^3} + \frac{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^4}{64\,d^3x^4} - \frac{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^3}{35\,d^2x^5} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^2}{16\,dx^6} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e}{7\,x^7} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d}{8\,x^8}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x, algorithm="maxima")
```

output
```
-3/128*e^8*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^4 + 3/128
*sqrt(-e^2*x^2 + d^2)*e^8/d^5 + 3/128*(-e^2*x^2 + d^2)^(3/2)*e^6/(d^5*x^2)
 - 2/35*(-e^2*x^2 + d^2)^(3/2)*e^5/(d^4*x^3) + 3/64*(-e^2*x^2 + d^2)^(3/2)
*e^4/(d^3*x^4) - 3/35*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^2*x^5) + 1/16*(-e^2*x^
2 + d^2)^(3/2)*e^2/(d*x^6) + 1/7*(-e^2*x^2 + d^2)^(3/2)*e/x^7 - 1/8*(-e^2*
x^2 + d^2)^(3/2)*d/x^8
```

3.116.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(173) = 346$.

Time = 0.30 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.30

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^9(d+ex)}\,dx = \frac{\left(35\,e^9 - \frac{80\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^7}{x} + \frac{112\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3e^3}{x^3} - \frac{280\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4e}{x^4} + \frac{560}{\cdots}\right.}{71680\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^8d^4|e|}$$

$$-\frac{3\,e^9\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{128\,d^4|e|}$$

$$+\frac{\frac{1680\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{28}e^{13}|e|}{x} - \frac{560\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^{28}e^9|e|}{x^3} + \frac{280\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^{28}e^7|e|}{x^4} - \frac{112\left(de+\sqrt{-e^2x^2+d^2}|e|\cdots\right.}{x^5}}{71680\,d^{32}e^8}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x, algorithm="giac")
```

output
```
1/71680*(35*e^9 - 80*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^7/x + 112*(d*e
+ sqrt(-e^2*x^2 + d^2)*abs(e))^3*e^3/x^3 - 280*(d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))^4*e/x^4 + 560*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e*x^5) - 168
0*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7/(e^5*x^7))*e^16*x^8/((d*e + sqrt(-
e^2*x^2 + d^2)*abs(e))^8*d^4*abs(e)) - 3/128*e^9*log(1/2*abs(-2*d*e - 2*sq
rt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^4*abs(e)) + 1/71680*(1680*(d*e
 + sqrt(-e^2*x^2 + d^2)*abs(e))*d^28*e^13*abs(e)/x - 560*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^3*d^28*e^9*abs(e)/x^3 + 280*(d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))^4*d^28*e^7*abs(e)/x^4 - 112*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5
*d^28*e^5*abs(e)/x^5 + 80*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7*d^28*e*abs
(e)/x^7 - 35*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8*d^28*abs(e)/(e*x^8))/(d
^32*e^8)
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{x^9(d + ex)}\,dx = \int \frac{(d^2 - e^2\,x^2)^{5/2}}{x^9\,(d + e\,x)}\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)), x)
```

3.117 $\int \frac{x\sqrt{1-x^2}}{1+x} \, dx$

3.117.1 Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{x\sqrt{1-x^2}}{1+x} \, dx = -\frac{1}{2}(2-x)\sqrt{1-x^2} - \frac{\arcsin(x)}{2}$$

output
```
-1/2*arcsin(x)-1/2*(2-x)*(-x^2+1)^(1/2)
```

3.117.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{x\sqrt{1-x^2}}{1+x} \, dx = \frac{1}{2}(-2+x)\sqrt{1-x^2} + \arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[(x*Sqrt[1 - x^2])/(1 + x),x]
```

output
```
((-2 + x)*Sqrt[1 - x^2])/2 + ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

3.117.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {565, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{1-x^2}}{x+1}dx$$
$$\Big\downarrow \text{565}$$
$$-\int \frac{x^2}{\sqrt{1-x^2}}dx-\sqrt{1-x^2}$$
$$\Big\downarrow \text{262}$$
$$-\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x-\sqrt{1-x^2}$$
$$\Big\downarrow \text{223}$$
$$-\frac{\arcsin(x)}{2}+\frac{1}{2}\sqrt{1-x^2}x-\sqrt{1-x^2}$$

input
```
Int[(x*Sqrt[1 - x^2])/(1 + x),x]
```

output
```
-Sqrt[1 - x^2] + (x*Sqrt[1 - x^2])/2 - ArcSin[x]/2
```

3.117.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 565
```
Int[((x_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :> Sim
p[a*((a + b*x^2)^p/(2*b*c*p)), x] + Simp[b/d   Int[x^2*(a + b*x^2)^(p - 1),
 x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0]
```

3.117.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{(-2+x)(x^2-1)}{2\sqrt{-x^2+1}}-\frac{\arcsin(x)}{2}$	25
default	$\frac{x\sqrt{-x^2+1}}{2}-\frac{\arcsin(x)}{2}-\sqrt{-\left(1+x\right)^2+2+2x}$	34
trager	$\left(-1+\frac{x}{2}\right)\sqrt{-x^2+1}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}$	45

input
```
int(x*(-x^2+1)^(1/2)/(1+x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2+x)*(x^2-1)/(-x^2+1)^(1/2)-1/2*arcsin(x)
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{x\sqrt{1-x^2}}{1+x}\,dx = \frac{1}{2}\sqrt{-x^2+1}(x-2)+\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate(x*(-x^2+1)^(1/2)/(1+x),x, algorithm="fricas")
```

output
```
1/2*sqrt(-x^2 + 1)*(x - 2) + arctan((sqrt(-x^2 + 1) - 1)/x)
```

3.117.6 Sympy [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x\sqrt{1-x^2}}{1+x}\,dx = \begin{cases} \frac{x\sqrt{1-x^2}}{2} - \sqrt{1-x^2} - \frac{\operatorname{asin}(x)}{2} & \text{for } x > -1 \wedge x < 1 \end{cases}$$

input
```
integrate(x*(-x**2+1)**(1/2)/(1+x),x)
```

output
```
Piecewise((x*sqrt(1 - x**2)/2 - sqrt(1 - x**2) - asin(x)/2, (x > -1) & (x
< 1)))
```

3.117.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{x\sqrt{1-x^2}}{1+x}\,dx = \frac{1}{2}\sqrt{-x^2+1}x - \sqrt{-x^2+1} - \frac{1}{2}\arcsin(x)$$

input
```
integrate(x*(-x^2+1)^(1/2)/(1+x),x, algorithm="maxima")
```

output
```
1/2*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1) - 1/2*arcsin(x)
```

3.117.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x\sqrt{1-x^2}}{1+x}\,dx = \frac{1}{2}\sqrt{-x^2+1}(x-2) - \frac{1}{2}\arcsin(x)$$

input
```
integrate(x*(-x^2+1)^(1/2)/(1+x),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 1)*(x - 2) - 1/2*arcsin(x)
```

3.117.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x\sqrt{1-x^2}}{1+x}\,dx = \left(\frac{x}{2}-1\right)\sqrt{1-x^2}-\frac{\operatorname{asin}(x)}{2}$$

input
```
int((x*(1 - x^2)^(1/2))/(x + 1),x)
```

output
```
(x/2 - 1)*(1 - x^2)^(1/2) - asin(x)/2
```

3.118 $\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)}\,dx$

3.118.1 Optimal result

Integrand size = 26, antiderivative size = 51

$$\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)}\,dx = -\frac{(1-ax)\sqrt{1-a^2x^2}}{x} - a\arcsin(ax) - a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

output
```
-a*arcsin(a*x)-a*arctanh((-a^2*x^2+1)^(1/2))-(-a*x+1)*(-a^2*x^2+1)^(1/2)/x
```

3.118.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.43

$$\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)}\,dx = \frac{(-1+ax)\sqrt{1-a^2x^2}}{x} - 2a\arctan\left(\frac{ax}{-1+\sqrt{1-a^2x^2}}\right) - a\log(x) + a\log\left(-1+\sqrt{1-a^2x^2}\right)$$

input
```
Integrate[(1 - a^2*x^2)^(3/2)/(x^2*(1 - a*x)),x]
```

output
```
((-1 + a*x)*Sqrt[1 - a^2*x^2])/x - 2*a*ArcTan[(a*x)/(-1 + Sqrt[1 - a^2*x^2
])] - a*Log[x] + a*Log[-1 + Sqrt[1 - a^2*x^2]]
```

3.118.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {566, 536, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)}\,dx$$

$$\Big\downarrow \text{566}$$

$$\int \frac{(ax+1)\sqrt{1-a^2x^2}}{x^2}dx$$

$$\Big\downarrow \text{536}$$

$$\int \frac{a-a^2x}{x\sqrt{1-a^2x^2}}dx-\frac{(1-ax)\sqrt{1-a^2x^2}}{x}$$

$$\Big\downarrow \text{538}$$

$$a^2\left(-\int \frac{1}{\sqrt{1-a^2x^2}}dx\right)+a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{(1-ax)\sqrt{1-a^2x^2}}{x}$$

$$\Big\downarrow \text{223}$$

$$a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}(1-ax)}{x}-a\arcsin(ax)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}(1-ax)}{x}-a\arcsin(ax)$$

$$\Big\downarrow \text{73}$$

$$-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{\sqrt{1-a^2x^2}(1-ax)}{x}-a\arcsin(ax)$$

$$\Big\downarrow \text{221}$$

$$-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}(1-ax)}{x}-a\arcsin(ax)$$

input
```
Int[(1 - a^2*x^2)^(3/2)/(x^2*(1 - a*x)),x]
```

3.118. $\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)}\,dx$

output
```
-(((1 - a*x)*Sqrt[1 - a^2*x^2])/x) - a*ArcSin[a*x] - a*ArcTanh[Sqrt[1 - a^
2*x^2]]
```

3.118.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 566
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Int[x^m*(a/c + b*(x/d))*(a + b*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0]
```

3.118. $\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)}\,dx$

3.118.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

method	result
risch	$\frac{a^2x^2-1}{x\sqrt{-a^2x^2+1}}-\frac{a^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+a\sqrt{-a^2x^2+1}-a\ \operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$
default	$-\frac{(-a^2x^2+1)^{\frac{5}{2}}}{x}-4a^2\left(\frac{x(-a^2x^2+1)^{\frac{3}{2}}}{4}+\frac{3x\sqrt{-a^2x^2+1}}{8}+\frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{8\sqrt{a^2}}\right)+a\left(\frac{(-a^2x^2+1)^{\frac{3}{2}}}{3}+\sqrt{-a^2x^2}\right.$

input
```
int((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x,method=_RETURNVERBOSE)
```

output
```
(a^2*x^2-1)/x/(-a^2*x^2+1)^(1/2)-a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^
2*x^2+1)^(1/2))+a*(-a^2*x^2+1)^(1/2)-a*arctanh(1/(-a^2*x^2+1)^(1/2))
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.45

$$\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)}\,dx=\frac{2\,ax\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+ax\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+ax+\sqrt{-a^2x^2+1}(ax-1)}{x}$$

input
```
integrate((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x, algorithm="fricas")
```

output
```
(2*a*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + a*x*log((sqrt(-a^2*x^2 + 1
) - 1)/x) + a*x + sqrt(-a^2*x^2 + 1)*(a*x - 1))/x
```

3.118.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.68 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.33

$$\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)}\,dx = a\left(\begin{cases} i\sqrt{a^2x^2-1}-\log(ax)+\frac{\log(a^2x^2)}{2}+i\,\text{asin}\left(\frac{1}{ax}\right) & \text{for } |a^2x^2|>1 \\ \sqrt{-a^2x^2+1}+\frac{\log(a^2x^2)}{2}-\log\left(\sqrt{-a^2x^2+1}+1\right) & \text{otherwise}\end{cases}\right) + \begin{cases} -\frac{ia^2x}{\sqrt{a^2x^2-1}}+ia\,\text{acosh}(ax)+\frac{i}{x\sqrt{a^2x^2-1}} & \text{for } |a^2x^2|>1 \\ \frac{a^2x}{\sqrt{-a^2x^2+1}}-a\,\text{asin}(ax)-\frac{1}{x\sqrt{-a^2x^2+1}} & \text{otherwise}\end{cases}$$

input
```
integrate((-a**2*x**2+1)**(3/2)/x**2/(-a*x+1),x)
```

output
```
a*Piecewise((I*sqrt(a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*asin(
1/(a*x)), Abs(a**2*x**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)/2 -
log(sqrt(-a**2*x**2 + 1) + 1), True)) + Piecewise((-I*a**2*x/sqrt(a**2*x**
2 - 1) + I*a*acosh(a*x) + I/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**2) > 1),
(a**2*x/sqrt(-a**2*x**2 + 1) - a*asin(a*x) - 1/(x*sqrt(-a**2*x**2 + 1)), T
rue))
```

3.118.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.33

$$\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)}\,dx = -a\arcsin(ax) - a\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\sqrt{-a^2x^2+1}a-\frac{\sqrt{-a^2x^2+1}}{x}$$

input
```
integrate((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x, algorithm="maxima")
```

output
```
-a*arcsin(a*x) - a*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-a^2
*x^2 + 1)*a - sqrt(-a^2*x^2 + 1)/x
```

3.118.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(45) = 90.

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.45

$$\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)}\,dx = \frac{a^4x}{2\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|} - \frac{a^2\arcsin(ax)\,\mathrm{sgn}(a)}{|a|} - \frac{a^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{|a|} + \sqrt{-a^2x^2+1}a - \frac{\sqrt{-a^2x^2+1}|a|+a}{2x|a|}$$

input
```
integrate((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x, algorithm="giac")
```

output
```
1/2*a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - a^2*arcsin(a*x)*sgn(a
)/abs(a) - a^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)
))/abs(a) + sqrt(-a^2*x^2 + 1)*a - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(x*
abs(a))
```

3.118.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.45

$$\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)}\,dx = a\sqrt{1-a^2x^2} - \frac{\sqrt{1-a^2x^2}}{x} - \frac{a^2\,\mathrm{asinh}\left(x\sqrt{-a^2}\right)}{\sqrt{-a^2}} + a\,\mathrm{atan}\left(\sqrt{1-a^2x^2}\,1\mathrm{i}\right)\,1\mathrm{i}$$

input
```
int(-(1 - a^2*x^2)^(3/2)/(x^2*(a*x - 1)),x)
```

output
```
a*atan((1 - a^2*x^2)^(1/2)*1i)*1i + a*(1 - a^2*x^2)^(1/2) - (1 - a^2*x^2)^
(1/2)/x - (a^2*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2)
```

3.119 $\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

3.119.1 Optimal result

Integrand size = 27, antiderivative size = 118

$$\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}}-\frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3}-\frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5}-\frac{3d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

output
```
-3/2*d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5+x^3*(-e*x+d)/e^2/(-e^2*x^2+d
^2)^(1/2)-4/3*x^2*(-e^2*x^2+d^2)^(1/2)/e^3-1/6*d*(-9*e*x+16*d)*(-e^2*x^2+d
^2)^(1/2)/e^5
```

3.119.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-16d^3-7d^2ex+de^2x^2-2e^3x^3)}{6e^5(d+ex)}+\frac{3d^3\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

input
```
Integrate[x^4/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-16*d^3 - 7*d^2*e*x + d*e^2*x^2 - 2*e^3*x^3))/(6*e^5
*(d + e*x)) + (3*d^3*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^5
```

3.119.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.30, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {563, 2346, 25, 2346, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{563}$$

$$-\frac{\int \frac{d^3-exd^2+e^2x^2d-e^3x^3}{\sqrt{d^2-e^2x^2}}dx}{e^4}-\frac{d^3\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$$\Big\downarrow \text{2346}$$

$$-\frac{\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}-\frac{\int -\frac{3dx^2e^4-5d^2xe^3+3d^3e^2}{\sqrt{d^2-e^2x^2}}dx}{3e^2}}{e^4}-\frac{d^3\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\int \frac{3dx^2e^4-5d^2xe^3+3d^3e^2}{\sqrt{d^2-e^2x^2}}dx}{3e^2}+\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}}{e^4}-\frac{d^3\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$$\Big\downarrow \text{2346}$$

$$-\frac{\frac{-\frac{\int -\frac{d^2e^4(9d-10ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{3}{2}de^2x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}}{e^4}-\frac{d^3\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\frac{\int \frac{d^2e^4(9d-10ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{3}{2}de^2x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}}{e^4}-\frac{d^3\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\frac{1}{2}d^2e^2\int \frac{9d-10ex}{\sqrt{d^2-e^2x^2}}dx-\frac{3}{2}de^2x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}}{e^4}-\frac{d^3\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$$\Big\downarrow \text{455}$$

$$-\frac{\frac{\frac{1}{2}d^2e^2\left(9d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{10\sqrt{d^2-e^2x^2}}{e}\right)-\frac{3}{2}de^2x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}}{e^4}-\frac{d^3\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\frac{\frac{1}{2}d^2e^2\left(9d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{10\sqrt{d^2-e^2x^2}}{e}\right)-\frac{3}{2}de^2x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}}{e^4}-\frac{d^3\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\frac{\frac{1}{2}d^2e^2\left(\frac{9d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{10\sqrt{d^2-e^2x^2}}{e}\right)-\frac{3}{2}de^2x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}}{e^4}-\frac{d^3\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

input
```
Int[x^4/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-((d^3*Sqrt[d^2 - e^2*x^2])/(e^5*(d + e*x))) - ((e*x^2*Sqrt[d^2 - e^2*x^2]
)/3 + ((-3*d*e^2*x*Sqrt[d^2 - e^2*x^2])/2 + (d^2*e^2*((10*Sqrt[d^2 - e^2*x
^2])/e + (9*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/2)/(3*e^2))/e^4
```

3.119.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

3.119.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{\left(2e^{2}x^{2}-3dex+10d^{2}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{6e^{5}}-\frac{3d^{3}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2e^{4}\sqrt{e^{2}}}-\frac{d^{3}\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{e^{6}\left(x+\frac{d}{e}\right)}$
default	$\frac{-\frac{x^{2}\sqrt{-e^{2}x^{2}+d^{2}}}{3e^{2}}-\frac{2d^{2}\sqrt{-e^{2}x^{2}+d^{2}}}{3e^{4}}}{e}-\frac{d^{2}\sqrt{-e^{2}x^{2}+d^{2}}}{e^{5}}-\frac{d^{3}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{e^{4}\sqrt{e^{2}}}-\frac{d\left(-\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2e^{2}}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2e^{2}\sqrt{e^{2}}}\right.}{e^{2}}$

input
```
int(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*e^2*x^2-3*d*e*x+10*d^2)/e^5*(-e^2*x^2+d^2)^(1/2)-3/2*d^3/e^4/(e^2)
^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-d^3/e^6/(x+d/e)*(-(x+d/e
)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.95

$$\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{16\,d^3ex+16\,d^4-18\,(d^3ex+d^4)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(2\,e^3x^3-de^2x^2+7\,d^2ex+16\,d^3)\sqrt{-e^2x^2+}}{6\,(e^6x+de^5)}$$

input
```
integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(16*d^3*e*x + 16*d^4 - 18*(d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^2
+ d^2))/(e*x)) + (2*e^3*x^3 - d*e^2*x^2 + 7*d^2*e*x + 16*d^3)*sqrt(-e^2*x^
2 + d^2))/(e^6*x + d*e^5)
```

3.119.6 Sympy [F]

$$\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^4}{\sqrt{-(-d+ex)\,(d+ex)}\,(d+ex)}\,dx$$

input
```
integrate(x**4/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(x**4/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.119.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}d^3}{e^6x+de^5} - \frac{\sqrt{-e^2x^2+d^2}x^2}{3\,e^3} - \frac{3\,d^3\arcsin\left(\frac{ex}{d}\right)}{2\,e^5} + \frac{\sqrt{-e^2x^2+d^2}dx}{2\,e^4} - \frac{5\sqrt{-e^2x^2+d^2}d^2}{3\,e^5}$$

input
```
integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-e^2*x^2 + d^2)*d^3/(e^6*x + d*e^5) - 1/3*sqrt(-e^2*x^2 + d^2)*x^2/e
^3 - 3/2*d^3*arcsin(e*x/d)/e^5 + 1/2*sqrt(-e^2*x^2 + d^2)*d*x/e^4 - 5/3*sq
rt(-e^2*x^2 + d^2)*d^2/e^5
```

3.119.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{6}\sqrt{-e^2x^2+d^2}\left(x\left(\frac{2\,x}{e^3}-\frac{3\,d}{e^4}\right)+\frac{10\,d^2}{e^5}\right) - \frac{3\,d^3\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,e^4|e|} + \frac{2\,d^3}{e^4\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

input
```
integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
-1/6*sqrt(-e^2*x^2 + d^2)*(x*(2*x/e^3 - 3*d/e^4) + 10*d^2/e^5) - 3/2*d^3*a
rcsin(e*x/d)*sgn(d)*sgn(e)/(e^4*abs(e)) + 2*d^3/(e^4*((d*e + sqrt(-e^2*x^2
 + d^2)*abs(e))/(e^2*x) + 1)*abs(e))
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^4}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

input
```
int(x^4/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int(x^4/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.120 $\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

3.120.1 Optimal result

Integrand size = 27, antiderivative size = 91

$$\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx=\frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}}+\frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4}+\frac{3d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}$$

output
```
3/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4+x^2*(-e*x+d)/e^2/(-e^2*x^2+d^
2)^(1/2)+1/2*(-3*e*x+4*d)*(-e^2*x^2+d^2)^(1/2)/e^4
```

3.120.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98

$$\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx=\frac{\sqrt{d^2-e^2x^2}(4d^2+dex-e^2x^2)}{2e^4(d+ex)}-\frac{3d^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

input
```
Integrate[x^3/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(4*d^2 + d*e*x - e^2*x^2))/(2*e^4*(d + e*x)) - (3*d^2
*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^4
```

3.120.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {563, 25, 2346, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{563}$$

$$\frac{d^2\sqrt{d^2-e^2x^2}}{e^4(d+ex)}-\frac{\int -\frac{d^2-exd+e^2x^2}{\sqrt{d^2-e^2x^2}}dx}{e^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d^2-exd+e^2x^2}{\sqrt{d^2-e^2x^2}}dx}{e^3}+\frac{d^2\sqrt{d^2-e^2x^2}}{e^4(d+ex)}$$

$$\Big\downarrow \text{2346}$$

$$\frac{-\frac{\int -\frac{de^2(3d-2ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{1}{2}x\sqrt{d^2-e^2x^2}}{e^3}+\frac{d^2\sqrt{d^2-e^2x^2}}{e^4(d+ex)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{de^2(3d-2ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{1}{2}x\sqrt{d^2-e^2x^2}}{e^3}+\frac{d^2\sqrt{d^2-e^2x^2}}{e^4(d+ex)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}d\int \frac{3d-2ex}{\sqrt{d^2-e^2x^2}}dx-\frac{1}{2}x\sqrt{d^2-e^2x^2}}{e^3}+\frac{d^2\sqrt{d^2-e^2x^2}}{e^4(d+ex)}$$

$$\Big\downarrow \text{455}$$

$$\frac{\frac{1}{2}d\left(3d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{2\sqrt{d^2-e^2x^2}}{e}\right)-\frac{1}{2}x\sqrt{d^2-e^2x^2}}{e^3}+\frac{d^2\sqrt{d^2-e^2x^2}}{e^4(d+ex)}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{2}d\left(3d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{2\sqrt{d^2-e^2x^2}}{e}\right)-\frac{1}{2}x\sqrt{d^2-e^2x^2}}{e^3}+\frac{d^2\sqrt{d^2-e^2x^2}}{e^4(d+ex)}$$

$$\downarrow 216$$

$$\frac{\frac{1}{2}d\left(\frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{2\sqrt{d^2-e^2x^2}}{e}\right)-\frac{1}{2}x\sqrt{d^2-e^2x^2}}{e^3}+\frac{d^2\sqrt{d^2-e^2x^2}}{e^4(d+ex)}$$

input
```
Int[x^3/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
(d^2*Sqrt[d^2 - e^2*x^2])/(e^4*(d + e*x)) + (-1/2*(x*Sqrt[d^2 - e^2*x^2])
+ (d*((2*Sqrt[d^2 - e^2*x^2])/e + (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/
e))/2)/e^3
```

3.120.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.120.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

method	result	siz
risch	$\frac{(-ex+2d)\sqrt{-e^2x^2+d^2}}{2e^4}+\frac{3d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^3\sqrt{e^2}}+\frac{d^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^5\left(x+\frac{d}{e}\right)}$	10
default	$\frac{-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}}{e}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^3\sqrt{e^2}}+\frac{d\sqrt{-e^2x^2+d^2}}{e^4}+\frac{d^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^5\left(x+\frac{d}{e}\right)}$	15

input
```
int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-e*x+2*d)/e^4*(-e^2*x^2+d^2)^(1/2)+3/2*d^2/e^3/(e^2)^(1/2)*arctan((e^
2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+d^2/e^5/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+
d/e))^(1/2)
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{4\,d^2ex+4\,d^3-6\,(d^2ex+d^3)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(e^2x^2-dex-4\,d^2)\sqrt{-e^2x^2+d^2}}{2\,(e^5x+de^4)}$$

input
```
integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(4*d^2*e*x + 4*d^3 - 6*(d^2*e*x + d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^
2))/(e*x)) - (e^2*x^2 - d*e*x - 4*d^2)*sqrt(-e^2*x^2 + d^2))/(e^5*x + d*e^
4)
```

3.120.6 Sympy [F]

$$\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^3}{\sqrt{-(-d+ex)\,(d+ex)}\,(d+ex)}\,dx$$

input
```
integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(x**3/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.120.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{-e^2x^2+d^2}d^2}{e^5x+de^4} + \frac{3\,d^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^4} - \frac{\sqrt{-e^2x^2+d^2}x}{2\,e^3} + \frac{\sqrt{-e^2x^2+d^2}d}{e^4}$$

input
```
integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(-e^2*x^2 + d^2)*d^2/(e^5*x + d*e^4) + 3/2*d^2*arcsin(e*x/d)/e^4 - 1/2
*sqrt(-e^2*x^2 + d^2)*x/e^3 + sqrt(-e^2*x^2 + d^2)*d/e^4
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{2}\sqrt{-e^2x^2+d^2}\left(\frac{x}{e^3}-\frac{2\,d}{e^4}\right) + \frac{3\,d^2\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{2\,e^3|e|} - \frac{2\,d^2}{e^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

input
```
integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-e^2*x^2 + d^2)*(x/e^3 - 2*d/e^4) + 3/2*d^2*arcsin(e*x/d)*sgn(d)
*sgn(e)/(e^3*abs(e)) - 2*d^2/(e^3*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^
2*x) + 1)*abs(e))
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^3}{\sqrt{d^2-e^2\,x^2}\,(d+e\,x)}\,dx$$

input
```
int(x^3/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int(x^3/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.121 $\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}} \, dx$

3.121.1 Optimal result

Integrand size = 27, antiderivative size = 77

$$\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}} \, dx = -\frac{\sqrt{d^2-e^2x^2}}{e^3} - \frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

output
```
-d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-(-e^2*x^2+d^2)^(1/2)/e^3-d*(-e^2*x
^2+d^2)^(1/2)/e^3/(e*x+d)
```

3.121.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}} \, dx = \frac{(-2d-ex)\sqrt{d^2-e^2x^2}}{e^3(d+ex)} + \frac{2d\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

input
```
Integrate[x^2/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
((-2*d - e*x)*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) + (2*d*ArcTan[(e*x)/(Sq
rt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^3
```

3.121.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {563, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \ 563$$

$$-\frac{\int \frac{d-ex}{\sqrt{d^2-e^2x^2}}dx}{e^2}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}$$

$$\Big\downarrow \ 455$$

$$-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{\sqrt{d^2-e^2x^2}}{e}}{e^2}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}$$

$$\Big\downarrow \ 224$$

$$-\frac{d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e}}{e^2}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}$$

$$\Big\downarrow \ 216$$

$$-\frac{\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{\sqrt{d^2-e^2x^2}}{e}}{e^2}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}$$

input
```
Int[x^2/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-((d*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x))) - (Sqrt[d^2 - e^2*x^2]/e + (d*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e)/e^2
```

3.121.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

3.121.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.26

method	result	size
default	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{e^{3}}-\frac{d \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{e^{2} \sqrt{e^{2}}}-\frac{d \sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{e^{4} \left(x+\frac{d}{e}\right)}$	97
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{e^{3}}-\frac{d \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{e^{2} \sqrt{e^{2}}}-\frac{d \sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{e^{4} \left(x+\frac{d}{e}\right)}$	97

input
```
int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-e^2*x^2+d^2)^(1/2)/e^3-d/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2
+d^2)^(1/2))-d/e^4/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{2\,dex+2\,d^2-2\,(dex+d^2)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}(ex+2\,d)}{e^4x+de^3}$$

input
```
integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-(2*d*e*x + 2*d^2 - 2*(d*e*x + d^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*
x)) + sqrt(-e^2*x^2 + d^2)*(e*x + 2*d))/(e^4*x + d*e^3)
```

3.121.6 Sympy [F]

$$\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^2}{\sqrt{-(-d+ex)(d+ex)}(d+ex)}\,dx$$

input
```
integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.121.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}d}{e^4x+de^3} - \frac{d\arcsin\left(\frac{ex}{d}\right)}{e^3} - \frac{\sqrt{-e^2x^2+d^2}}{e^3}$$

input
```
integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-e^2*x^2 + d^2)*d/(e^4*x + d*e^3) - d*arcsin(e*x/d)/e^3 - sqrt(-e^2*
x^2 + d^2)/e^3
```

3.121.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{d\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^2|e|} - \frac{\sqrt{-e^2x^2+d^2}}{e^3} + \frac{2\,d}{e^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

input
```
integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
-d*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) - sqrt(-e^2*x^2 + d^2)/e^3 + 2
*d/(e^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e))
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^2}{\sqrt{d^2-e^2\,x^2}\,(d+e\,x)}\,dx$$

input
```
int(x^2/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int(x^2/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.122 $\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

3.122.1 Optimal result

Integrand size = 25, antiderivative size = 52

$$\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)} + \frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

output
```
arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2+(-e^2*x^2+d^2)^(1/2)/e^2/(e*x+d)
```

3.122.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.25

$$\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)} - \frac{2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

input
```
Integrate[x/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
Sqrt[d^2 - e^2*x^2]/(e^2*(d + e*x)) - (2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^
2 - e^2*x^2])])/e^2
```

3.122.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {563, 25, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{ 563}$$

$$\frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}-\frac{\int -\frac{1}{\sqrt{d^2-e^2x^2}}dx}{e}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}dx}{e}+\frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}}{e}+\frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}+\frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}$$

input
```
Int[x/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
Sqrt[d^2 - e^2*x^2]/(e^2*(d + e*x)) + ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^
2
```

3.122.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

3.122.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.42

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e\sqrt{e^2}}+\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^3\left(x+\frac{d}{e}\right)}$	74

input
```
int(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/e^3/(x+d/e)*(
-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.29

$$\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{ex - 2\,(ex+d)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + d + \sqrt{-e^2x^2+d^2}}{e^3x + de^2}$$

input
```
integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
(e*x - 2*(e*x + d)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d + sqrt(-e
^2*x^2 + d^2))/(e^3*x + d*e^2)
```

3.122.6 Sympy [F]

$$\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)}\,dx$$

input
```
integrate(x/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(x/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.122.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{-e^2x^2+d^2}}{e^3x+de^2} + \frac{\arcsin\left(\frac{ex}{d}\right)}{e^2}$$

input
```
integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(-e^2*x^2 + d^2)/(e^3*x + d*e^2) + arcsin(e*x/d)/e^2
```

3.122.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

$$\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e|e|} - \frac{2}{e\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

input
```
integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
arcsin(e*x/d)*sgn(d)*sgn(e)/(e*abs(e)) - 2/(e*((d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))/(e^2*x) + 1)*abs(e))
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x}{\sqrt{d^2-e^2\,x^2}\,(d+e\,x)}\,dx$$

input
```
int(x/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int(x/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.123 $\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

3.123.1 Optimal result

Integrand size = 24, antiderivative size = 31

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

output
```
-(-e^2*x^2+d^2)^(1/2)/d/e/(e*x+d)
```

3.123.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

input
```
Integrate[1/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-(Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)))
```

3.123.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$$

$$\Big\downarrow \text{460}$$

$$-\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

input
```
Int[1/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-(Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)))
```

3.123.3.1 Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

3.123.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{-ex+d}{de\sqrt{-e^2x^2+d^2}}$	29
trager	$-\frac{\sqrt{-e^2x^2+d^2}}{de(ex+d)}$	30
default	$-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^2d\left(x+\frac{d}{e}\right)}$	46

input
```
int(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-e*x+d)/d/e/(-e^2*x^2+d^2)^(1/2)
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{ex+d+\sqrt{-e^2x^2+d^2}}{de^2x+d^2e}$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-(e*x + d + sqrt(-e^2*x^2 + d^2))/(d*e^2*x + d^2*e)
```

3.123.6 Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-(-d+ex)(d+ex)}\,(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.123.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}}{de^2x+d^2e}$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-e^2*x^2 + d^2)/(d*e^2*x + d^2*e)
```

3.123.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{2}{d\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
2/(d*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e))
```

3.123.9 Mupad [B] (verification not implemented)

Time = 11.55 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}}{d\,e\,\left(d+e\,x\right)}$$

input
```
int(1/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

output
```
-(d^2 - e^2*x^2)^(1/2)/(d*e*(d + e*x))
```

3.124 $\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}} \, dx$

3.124.1 Optimal result

Integrand size = 27, antiderivative size = 54

$$\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}} \, dx = \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

output
```
-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2+(-e^2*x^2+d^2)^(1/2)/d^2/(e*x+d)
```

3.124.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.41

$$\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}} \, dx = \frac{\frac{d\sqrt{d^2-e^2x^2}}{d+ex} - \sqrt{d^2}\log(x) + \sqrt{d^2}\log\left(\sqrt{d^2} - \sqrt{d^2-e^2x^2}\right)}{d^3}$$

input
```
Integrate[1/(x*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
((d*Sqrt[d^2 - e^2*x^2])/(d + e*x) - Sqrt[d^2]*Log[x] + Sqrt[d^2]*Log[Sqrt
[d^2] - Sqrt[d^2 - e^2*x^2]])/d^3
```

3.124.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {564, 25, 27, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx$$
$$\Big\downarrow \text{564}$$
$$\frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}-\int -\frac{1}{dx\sqrt{d^2-e^2x^2}}dx$$
$$\Big\downarrow \text{25}$$
$$\int \frac{1}{dx\sqrt{d^2-e^2x^2}}dx+\frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx}{d}+\frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}$$
$$\Big\downarrow \text{243}$$
$$\frac{\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2}{2d}+\frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}$$
$$\Big\downarrow \text{73}$$
$$\frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}-\frac{\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{de^2}$$
$$\Big\downarrow \text{221}$$
$$\frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}-\frac{\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

input
```
Int[1/(x*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
Sqrt[d^2 - e^2*x^2]/(d^2*(d + e*x)) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]/d^2
```

3.124.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

3.124.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.63

method	result	size
default	$-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d\sqrt{d^2}}+\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e\,d^2\left(x+\frac{d}{e}\right)}$	88

input
```
int(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/e/d^2/
(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

3.124.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.15

$$\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{ex+(ex+d)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+d+\sqrt{-e^2x^2+d^2}}{d^2ex+d^3}$$

input
```
integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
(e*x + (e*x + d)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + d + sqrt(-e^2*x^2 +
d^2))/(d^2*e*x + d^3)
```

3.124.6 Sympy [F]

$$\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)}\,dx$$

input
```
integrate(1/x/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.124.7 Maxima [F]

$$\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-e^2x^2+d^2}(ex+d)x}\,dx$$

input
```
integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)*x), x)
```

3.124.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.61

$$\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{e\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d^2|e|} - \frac{2\,e}{d^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

input
```
integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
-e*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^2*
abs(e)) - 2*e/(d^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e
))
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x\,\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

input
```
int(1/(x*(d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int(1/(x*(d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.125 $\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

3.125.1 Optimal result

Integrand size = 27, antiderivative size = 81

$$\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{2\sqrt{d^2-e^2x^2}}{d^3x}+\frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)}+\frac{e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

output
```
e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3-2*(-e^2*x^2+d^2)^(1/2)/d^3/x+(-e^2*x
^2+d^2)^(1/2)/d^2/x/(e*x+d)
```

3.125.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{-\frac{d(d+2ex)\sqrt{d^2-e^2x^2}}{x(d+ex)}+\sqrt{d^2}e\log(x)-\sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{d^4}$$

input
```
Integrate[1/(x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
(-((d*(d + 2*e*x)*Sqrt[d^2 - e^2*x^2])/(x*(d + e*x))) + Sqrt[d^2]*e*Log[x]
 - Sqrt[d^2]*e*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/d^4
```

3.125.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {564, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{564}$$

$$-\int -\frac{d-ex}{d^2x^2\sqrt{d^2-e^2x^2}}dx-\frac{e\sqrt{d^2-e^2x^2}}{d^3(d+ex)}$$

$$\Big\downarrow \text{25}$$

$$\int \frac{d-ex}{d^2x^2\sqrt{d^2-e^2x^2}}dx-\frac{e\sqrt{d^2-e^2x^2}}{d^3(d+ex)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{d-ex}{x^2\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{e\sqrt{d^2-e^2x^2}}{d^3(d+ex)}$$

$$\Big\downarrow \text{534}$$

$$\frac{-e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{dx}}{d^2}-\frac{e\sqrt{d^2-e^2x^2}}{d^3(d+ex)}$$

$$\Big\downarrow \text{243}$$

$$\frac{-\frac{1}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{dx}}{d^2}-\frac{e\sqrt{d^2-e^2x^2}}{d^3(d+ex)}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{\sqrt{d^2-e^2x^2}}{dx}}{d^2}-\frac{e\sqrt{d^2-e^2x^2}}{d^3(d+ex)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{dx}}{d^2}-\frac{e\sqrt{d^2-e^2x^2}}{d^3(d+ex)}$$

input
```
Int[1/(x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-((e*Sqrt[d^2 - e^2*x^2])/(d^3*(d + e*x))) + (-(Sqrt[d^2 - e^2*x^2]/(d*x))
 + (e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d)/d^2
```

3.125.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

3.125.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.33

method	result	size
default	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{d^{3} x}+\frac{e \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{d^{2} \sqrt{d^{2}}}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{d^{3} \left(x+\frac{d}{e}\right)}$	108
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{d^{3} x}+\frac{e \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{d^{2} \sqrt{d^{2}}}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{d^{3} \left(x+\frac{d}{e}\right)}$	108

input
```
int(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-e^2*x^2+d^2)^(1/2)/d^3/x+e/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^
2*x^2+d^2)^(1/2))/x)-1/d^3/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{e^2x^2+dex+(e^2x^2+dex)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\sqrt{-e^2x^2+d^2}(2\,ex+d)}{d^3ex^2+d^4x}$$

input
```
integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-(e^2*x^2 + d*e*x + (e^2*x^2 + d*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) +
 sqrt(-e^2*x^2 + d^2)*(2*e*x + d))/(d^3*e*x^2 + d^4*x)
```

3.125.6 Sympy [F]

$$\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x^2\sqrt{-(-d+ex)(d+ex)}\,(d+ex)}\,dx$$

input
```
integrate(1/x**2/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.125.7 Maxima [F]

$$\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-e^2x^2+d^2}(ex+d)x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)*x^2), x)
```

3.125.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(75) = 150.

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.19

$$\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d^3|e|} + \frac{\left(e^2+\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{x}\right)e^2x}{2\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|} - \frac{de+\sqrt{-e^2x^2+d^2}|e|}{2\,d^3x|e|}$$

input
```
integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
e^2*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^3
*abs(e)) + 1/2*(e^2 + 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/x)*e^2*x/((d*e
 + sqrt(-e^2*x^2 + d^2)*abs(e))*d^3*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(
e^2*x) + 1)*abs(e)) - 1/2*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(d^3*x*abs(e
))
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x^2\,\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

input
```
int(1/(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int(1/(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.126 $\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

3.126.1 Optimal result

Integrand size = 27, antiderivative size = 113

$$\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2}+\frac{2e\sqrt{d^2-e^2x^2}}{d^4x}+\frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)}-\frac{3e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}$$

output
```
-3/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4-3/2*(-e^2*x^2+d^2)^(1/2)/d^3/
x^2+2*e*(-e^2*x^2+d^2)^(1/2)/d^4/x+(-e^2*x^2+d^2)^(1/2)/d^2/x^2/(e*x+d)
```

3.126.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}(-d^2+dex+4e^2x^2)}{x^2(d+ex)}-3\sqrt{d^2}e^2\log(x)+3\sqrt{d^2}e^2\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{2d^5}$$

input
```
Integrate[1/(x^3*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
((d*Sqrt[d^2 - e^2*x^2]*(-d^2 + d*e*x + 4*e^2*x^2))/(x^2*(d + e*x)) - 3*Sq
rt[d^2]*e^2*Log[x] + 3*Sqrt[d^2]*e^2*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])
/(2*d^5)
```

3.126.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {564, 25, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{564}$$

$$\frac{e^2\sqrt{d^2-e^2x^2}}{d^4(d+ex)}-\int -\frac{\frac{e^2x^2}{d^3}-\frac{ex}{d^2}+\frac{1}{d}}{x^3\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\frac{e^2x^2}{d^3}-\frac{ex}{d^2}+\frac{1}{d}}{x^3\sqrt{d^2-e^2x^2}}dx+\frac{e^2\sqrt{d^2-e^2x^2}}{d^4(d+ex)}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int \frac{e(2d-3ex)}{dx^2\sqrt{d^2-e^2x^2}}dx}{2d^2}+\frac{e^2\sqrt{d^2-e^2x^2}}{d^4(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{2d^3x^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e\int \frac{2d-3ex}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^3}+\frac{e^2\sqrt{d^2-e^2x^2}}{d^4(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{2d^3x^2}$$

$$\Big\downarrow \text{534}$$

$$-\frac{e\left(-3e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{2\sqrt{d^2-e^2x^2}}{dx}\right)}{2d^3}+\frac{e^2\sqrt{d^2-e^2x^2}}{d^4(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{2d^3x^2}$$

$$\Big\downarrow \text{243}$$

$$-\frac{e\left(-\frac{3}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{2\sqrt{d^2-e^2x^2}}{dx}\right)}{2d^3}+\frac{e^2\sqrt{d^2-e^2x^2}}{d^4(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{2d^3x^2}$$

$$\Big\downarrow \text{73}$$

$$-\frac{e\left(\frac{3\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{2\sqrt{d^2-e^2x^2}}{dx}\right)}{2d^3}+\frac{e^2\sqrt{d^2-e^2x^2}}{d^4(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{2d^3x^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{e\left(\frac{3e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{2\sqrt{d^2-e^2x^2}}{dx}\right)}{2d^3}+\frac{e^2\sqrt{d^2-e^2x^2}}{d^4(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{2d^3x^2}$$

input
```
Int[1/(x^3*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-1/2*Sqrt[d^2 - e^2*x^2]/(d^3*x^2) + (e^2*Sqrt[d^2 - e^2*x^2])/(d^4*(d + e
*x)) - (e*((-2*Sqrt[d^2 - e^2*x^2])/(d*x) + (3*e*ArcTanh[Sqrt[d^2 - e^2*x^
2]/d])/d))/(2*d^3)
```

3.126.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.126.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-2ex+d)}{2d^4x^2}-\frac{3e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^3\sqrt{d^2}}+\frac{e\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{d^4\left(x+\frac{d}{e}\right)}$
default	$\frac{-\frac{\sqrt{-e^2x^2+d^2}}{2d^2x^2}-\frac{e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^2\sqrt{d^2}}}{d}-\frac{e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^3\sqrt{d^2}}+\frac{e\sqrt{-e^2x^2+d^2}}{d^4x}+\frac{e\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\right.}}{d^4\left(x+\frac{d}{e}\right)}$

input
```
int(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e^2*x^2+d^2)^(1/2)*(-2*e*x+d)/d^4/x^2-3/2*e^2/d^3/(d^2)^(1/2)*ln((2
*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+e/d^4/(x+d/e)*(-(x+d/e)^2*e^2+
2*d*e*(x+d/e))^(1/2)
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{2\,e^3x^3+2\,de^2x^2+3\,(e^3x^3+de^2x^2)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(4\,e^2x^2+dex-d^2)\sqrt{-e^2x^2+d^2}}{2\,(d^4ex^3+d^5x^2)}$$

input
```
integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*e^3*x^3 + 2*d*e^2*x^2 + 3*(e^3*x^3 + d*e^2*x^2)*log(-(d - sqrt(-e^2
*x^2 + d^2))/x) + (4*e^2*x^2 + d*e*x - d^2)*sqrt(-e^2*x^2 + d^2))/(d^4*e*x
^3 + d^5*x^2)
```

3.126.6 Sympy [F]

$$\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x^3\sqrt{-(-d+ex)(d+ex)}\,(d+ex)}\,dx$$

input
```
integrate(1/x**3/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.126.7 Maxima [F]

$$\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-e^2x^2+d^2}(ex+d)x^3}\,dx$$

input
```
integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)*x^3), x)
```

3.126.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(101) = 202$.

Time = 0.29 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.27

$$\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\left(e^3 - \frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e}{x} - \frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{ex^2}\right)e^4x^2}{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d^4\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|} - \frac{3\,e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,d^4|e|} + \frac{\frac{4\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^4e|e|}{x} - \frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d^4|e|}{ex^2}}{8\,d^8e^2}$$

input
```
integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*(e^3 - 3*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e/x - 20*(d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))^2/(e*x^2))*e^4*x^2/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e)
)^2*d^4*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e)) - 3/2*e^
3*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^4*a
bs(e)) + 1/8*(4*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^4*e*abs(e)/x - (d*e
+ sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^4*abs(e)/(e*x^2))/(d^8*e^2)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x^3\,\sqrt{d^2-e^2\,x^2}\,(d+e\,x)}\,dx$$

input
```
int(1/(x^3*(d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int(1/(x^3*(d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.127 $\int \frac{x^5}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx$

3.127.1 Optimal result

Integrand size = 27, antiderivative size = 128

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}} - \frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6} - \frac{5d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

output
```
1/3*x^4*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(3/2)-5/2*d^2*arctan(e*x/(-e^2*x^2+d^2
)^(1/2))/e^6-1/3*x^2*(-5*e*x+4*d)/e^4/(-e^2*x^2+d^2)^(1/2)-1/6*(-15*e*x+16
*d)*(-e^2*x^2+d^2)^(1/2)/e^6
```

3.127.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}\left(16d^4+d^3ex-23d^2e^2x^2-3de^3x^3+3e^4x^4\right)}{6e^6(-d+ex)(d+ex)^2} + \frac{5d^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

input
```
Integrate[x^5/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(16*d^4 + d^3*e*x - 23*d^2*e^2*x^2 - 3*d*e^3*x^3 + 3*
e^4*x^4))/(6*e^6*(-d + e*x)*(d + e*x)^2) + (5*d^2*ArcTan[(e*x)/(Sqrt[d^2]
- Sqrt[d^2 - e^2*x^2])])/e^6
```

3.127.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {568, 530, 25, 2346, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(d+ex)(d^2-e^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{568}$$

$$\frac{x^4}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\int \frac{x^3(4d-5ex)}{(d^2-e^2x^2)^{3/2}}dx}{3e^2}$$

$$\Big\downarrow \text{530}$$

$$\frac{x^4}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\frac{d^2(4d-5ex)}{e^4\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{\frac{5d^4}{e^3}-\frac{4xd^3}{e^2}+\frac{5x^2d^2}{e}}{\sqrt{d^2-e^2x^2}}dx}{d^2}}{3e^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^4}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\frac{\int \frac{\frac{5d^4}{e^3}-\frac{4xd^3}{e^2}+\frac{5x^2d^2}{e}}{\sqrt{d^2-e^2x^2}}dx}{d^2}+\frac{d^2(4d-5ex)}{e^4\sqrt{d^2-e^2x^2}}}{3e^2}$$

$$\Big\downarrow \text{2346}$$

$$\frac{x^4}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\frac{-\frac{\int -\frac{d^3(15d-8ex)}{e\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{5d^2x\sqrt{d^2-e^2x^2}}{2e^3}}{d^2}+\frac{d^2(4d-5ex)}{e^4\sqrt{d^2-e^2x^2}}}{3e^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^4}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\frac{\frac{\int \frac{d^3(15d-8ex)}{e\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{5d^2x\sqrt{d^2-e^2x^2}}{2e^3}}{d^2}+\frac{d^2(4d-5ex)}{e^4\sqrt{d^2-e^2x^2}}}{3e^2}$$

$$\Big\downarrow \; 27$$

$$\frac{x^4}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\frac{\frac{d^3\int\frac{15d-8ex}{\sqrt{d^2-e^2x^2}}dx}{2e^3}-\frac{5d^2x\sqrt{d^2-e^2x^2}}{2e^3}}{d^2}+\frac{d^2(4d-5ex)}{e^4\sqrt{d^2-e^2x^2}}}{3e^2}$$

$$\Big\downarrow \; 455$$

$$\frac{x^4}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\frac{\frac{d^3\left(15d\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{8\sqrt{d^2-e^2x^2}}{e}\right)}{2e^3}-\frac{5d^2x\sqrt{d^2-e^2x^2}}{2e^3}}{d^2}+\frac{d^2(4d-5ex)}{e^4\sqrt{d^2-e^2x^2}}}{3e^2}$$

$$\Big\downarrow \; 224$$

$$\frac{x^4}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\frac{\frac{d^3\left(15d\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{8\sqrt{d^2-e^2x^2}}{e}\right)}{2e^3}-\frac{5d^2x\sqrt{d^2-e^2x^2}}{2e^3}}{d^2}+\frac{d^2(4d-5ex)}{e^4\sqrt{d^2-e^2x^2}}}{3e^2}$$

$$\Big\downarrow \; 216$$

$$\frac{x^4}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\frac{\frac{d^3\left(\frac{15d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{8\sqrt{d^2-e^2x^2}}{e}\right)}{2e^3}-\frac{5d^2x\sqrt{d^2-e^2x^2}}{2e^3}}{d^2}+\frac{d^2(4d-5ex)}{e^4\sqrt{d^2-e^2x^2}}}{3e^2}$$

input
```
Int[x^5/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
x^4/(3*e^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]) - ((d^2*(4*d - 5*e*x))/(e^4*Sqrt
[d^2 - e^2*x^2]) + ((-5*d^2*x*Sqrt[d^2 - e^2*x^2])/(2*e^3) + (d^3*((8*Sqrt
[d^2 - e^2*x^2])/e + (15*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/(2*e^3))
/d^2)/(3*e^2)
```

3.127.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 568
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[x^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*p*(c + d*x))), x] + Simp[1/(2*d^
2*p)   Int[x^(m - 2)*(a + b*x^2)^p*(c*(m - 1) - d*m*x), x], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[m, 1] && LtQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.127. $\int \frac{x^5}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx$

3.127.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.58

method	result
risch	$-\frac{(-ex+2d)\sqrt{-e^2x^2+d^2}}{2e^6}-\frac{5d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^5\sqrt{e^2}}+\frac{d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{6e^8\left(x+\frac{d}{e}\right)^2}-\frac{25d^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{12e^7\left(x+\frac{d}{e}\right)}-$
default	$\frac{-\frac{x^3}{2e^2\sqrt{-e^2x^2+d^2}}+\frac{3d^2\left(\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}\right)}{2e^2}}{e}+\frac{d^2x}{e^5\sqrt{-e^2x^2+d^2}}+\frac{d^2\left(\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}\right)}{e^3}$

input
```
int(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e*x+2*d)/e^6*(-e^2*x^2+d^2)^(1/2)-5/2*d^2/e^5/(e^2)^(1/2)*arctan((e
^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/6*d^3/e^8/(x+d/e)^2*(-(x+d/e)^2*e^2+2*
d*e*(x+d/e))^(1/2)-25/12*d^2/e^7/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1
/2)-1/4*d^2/e^7/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

3.127.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.48

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx =$$
$$-\frac{16\,d^2e^3x^3+16\,d^3e^2x^2-16\,d^4ex-16\,d^5-30\left(d^2e^3x^3+d^3e^2x^2-d^4ex-d^5\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(3}{6\left(e^9x^3+de^8x^2-d^2e^7x-d^3e^6\right)}$$

input
```
integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(16*d^2*e^3*x^3 + 16*d^3*e^2*x^2 - 16*d^4*e*x - 16*d^5 - 30*(d^2*e^3*
x^3 + d^3*e^2*x^2 - d^4*e*x - d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x
)) - (3*e^4*x^4 - 3*d*e^3*x^3 - 23*d^2*e^2*x^2 + d^3*e*x + 16*d^4)*sqrt(-e
^2*x^2 + d^2))/(e^9*x^3 + d*e^8*x^2 - d^2*e^7*x - d^3*e^6)
```

3.127.6 Sympy [F]

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^5}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate(x**5/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral(x**5/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.127.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.18

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{d^4}{3\left(\sqrt{-e^2x^2+d^2}e^7x+\sqrt{-e^2x^2+d^2}de^6\right)} - \frac{x^3}{2\sqrt{-e^2x^2+d^2}e^3} + \frac{dx^2}{\sqrt{-e^2x^2+d^2}e^4} + \frac{17\,d^2x}{6\sqrt{-e^2x^2+d^2}e^5} - \frac{5\,d^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^6} - \frac{3\,d^3}{\sqrt{-e^2x^2+d^2}e^6}$$

input
```
integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
1/3*d^4/(sqrt(-e^2*x^2 + d^2)*e^7*x + sqrt(-e^2*x^2 + d^2)*d*e^6) - 1/2*x^
3/(sqrt(-e^2*x^2 + d^2)*e^3) + d*x^2/(sqrt(-e^2*x^2 + d^2)*e^4) + 17/6*d^2
*x/(sqrt(-e^2*x^2 + d^2)*e^5) - 5/2*d^2*arcsin(e*x/d)/e^6 - 3*d^3/(sqrt(-e
^2*x^2 + d^2)*e^6)
```

3.127.8 Giac [F]

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^5}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}\left(ex+d\right)}\,dx$$

input
```
integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^5/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)), x)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=\int \frac{x^5}{\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

input
```
int(x^5/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

output
```
int(x^5/((d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

3.128 $\int \frac{x^4}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx$

3.128.1 Optimal result

Integrand size = 27, antiderivative size = 113

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}}+\frac{8\sqrt{d^2-e^2x^2}}{3e^5}+\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

output
```
1/3*x^3*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(3/2)+d*arctan(e*x/(-e^2*x^2+d^2)^(1/2
))/e^5-1/3*x*(-4*e*x+3*d)/e^4/(-e^2*x^2+d^2)^(1/2)+8/3*(-e^2*x^2+d^2)^(1/2
)/e^5
```

3.128.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{\frac{\sqrt{d^2-e^2x^2}\left(8d^3+5d^2ex-7de^2x^2-3e^3x^3\right)}{(d-ex)(d+ex)^2}-6d\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{3e^5}$$

input
```
Integrate[x^4/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(8*d^3 + 5*d^2*e*x - 7*d*e^2*x^2 - 3*e^3*x^3))/((d -
 e*x)*(d + e*x)^2) - 6*d*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/
(3*e^5)
```

3.128.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {568, 530, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \ 568$$

$$\frac{x^3}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\int \frac{x^2(3d-4ex)}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3e^2}$$

$$\Big\downarrow \ 530$$

$$\frac{x^3}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{-\frac{\int \frac{d^2(3d-4ex)}{e^2\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{d(4d-3ex)}{e^3\sqrt{d^2-e^2x^2}}}{3e^2}$$

$$\Big\downarrow \ 27$$

$$\frac{x^3}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{-\frac{\int \frac{3d-4ex}{\sqrt{d^2-e^2x^2}}dx}{e^2}-\frac{d(4d-3ex)}{e^3\sqrt{d^2-e^2x^2}}}{3e^2}$$

$$\Big\downarrow \ 455$$

$$\frac{x^3}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{-\frac{3d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{4\sqrt{d^2-e^2x^2}}{e}}{e^2}-\frac{d(4d-3ex)}{e^3\sqrt{d^2-e^2x^2}}}{3e^2}$$

$$\Big\downarrow \ 224$$

$$\frac{x^3}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{-\frac{3d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{4\sqrt{d^2-e^2x^2}}{e}}{e^2}-\frac{d(4d-3ex)}{e^3\sqrt{d^2-e^2x^2}}}{3e^2}$$

$$\Big\downarrow \ 216$$

$$\frac{x^3}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{-\frac{\frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{4\sqrt{d^2-e^2x^2}}{e}}{e^2}-\frac{d(4d-3ex)}{e^3\sqrt{d^2-e^2x^2}}}{3e^2}$$

input
```
Int[x^4/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
x^3/(3*e^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]) - (-((d*(4*d - 3*e*x))/(e^3*Sqrt
[d^2 - e^2*x^2])) - ((4*Sqrt[d^2 - e^2*x^2])/e + (3*d*ArcTan[(e*x)/Sqrt[d^
2 - e^2*x^2]])/e)/e^2)/(3*e^2)
```

3.128.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 568
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[x^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*p*(c + d*x))), x] + Simp[1/(2*d^
2*p)   Int[x^(m - 2)*(a + b*x^2)^p*(c*(m - 1) - d*m*x), x], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[m, 1] && LtQ[p, -1]
```

3.128.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.65

method	result
risch	$\frac{\sqrt{-e^2x^2+d^2}}{e^5}+\frac{d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^4\sqrt{e^2}}+\frac{19d\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{12e^6\left(x+\frac{d}{e}\right)}-\frac{d\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{4e^6\left(x-\frac{d}{e}\right)}-\frac{d^2\sqrt{-\left(x+\frac{d}{e}\right)^2}}{6e^7\left(x\right.}$
default	$\frac{-\frac{x^2}{e^2\sqrt{-e^2x^2+d^2}}+\frac{2d^2}{e^4\sqrt{-e^2x^2+d^2}}}{e}+\frac{d^2}{e^5\sqrt{-e^2x^2+d^2}}-\frac{dx}{e^4\sqrt{-e^2x^2+d^2}}-\frac{d\left(\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}\right)}{e^2}+\frac{d^4\left(-\right.}{3}$

input
```
int(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-e^2*x^2+d^2)^(1/2)/e^5+d/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+
d^2)^(1/2))+19/12*d/e^6/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/4*d
/e^6/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-1/6*d^2/e^7/(x+d/e)^2*(-
(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

3.128.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.55

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{8\,de^3x^3+8\,d^2e^2x^2-8\,d^3ex-8\,d^4-6\left(de^3x^3+d^2e^2x^2-d^3ex-d^4\right)\arctan}{3\left(e^8x^3+de^7x^2-d^2\right.}$$

input
```
integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(8*d*e^3*x^3 + 8*d^2*e^2*x^2 - 8*d^3*e*x - 8*d^4 - 6*(d*e^3*x^3 + d^2*
e^2*x^2 - d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (3*e^
3*x^3 + 7*d*e^2*x^2 - 5*d^2*e*x - 8*d^3)*sqrt(-e^2*x^2 + d^2))/(e^8*x^3 +
d*e^7*x^2 - d^2*e^6*x - d^3*e^5)
```

3.128.6 Sympy [F]

$$\int \frac{x^4}{(d+ex)\,(d^2-e^2x^2)^{3/2}}\,dx = \int \frac{x^4}{\left(-(-d+ex)\,(d+ex)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(x**4/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral(x**4/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.128.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.10

$$\int \frac{x^4}{(d+ex)\,(d^2-e^2x^2)^{3/2}}\,dx = -\frac{d^3}{3\left(\sqrt{-e^2x^2+d^2}e^6x+\sqrt{-e^2x^2+d^2}de^5\right)} - \frac{x^2}{\sqrt{-e^2x^2+d^2}e^3} - \frac{4\,dx}{3\,\sqrt{-e^2x^2+d^2}e^4} + \frac{d\arcsin\left(\frac{ex}{d}\right)}{e^5} + \frac{3\,d^2}{\sqrt{-e^2x^2+d^2}e^5}$$

input
```
integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/3*d^3/(sqrt(-e^2*x^2 + d^2)*e^6*x + sqrt(-e^2*x^2 + d^2)*d*e^5) - x^2/(
sqrt(-e^2*x^2 + d^2)*e^3) - 4/3*d*x/(sqrt(-e^2*x^2 + d^2)*e^4) + d*arcsin(
e*x/d)/e^5 + 3*d^2/(sqrt(-e^2*x^2 + d^2)*e^5)
```

3.128.8 Giac [F]

$$\int \frac{x^4}{(d+ex)\,(d^2-e^2x^2)^{3/2}}\,dx = \int \frac{x^4}{(-e^2x^2+d^2)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)), x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=\int \frac{x^4}{\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

input
```
int(x^4/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

output
```
int(x^4/((d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

3.129 $\int \frac{x^3}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx$

3.129.1 Optimal result

Integrand size = 27, antiderivative size = 89

$$\int \frac{x^3}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx=\frac{x^2(d-ex)}{3e^2(d^2-e^2x^2)^{3/2}}-\frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}}-\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

output
```
1/3*x^2*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(3/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))
/e^4+1/3*(3*e*x-2*d)/e^4/(-e^2*x^2+d^2)^(1/2)
```

3.129.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{x^3}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx=\frac{\frac{\sqrt{d^2-e^2x^2}(-2d^2+dex+4e^2x^2)}{(d-ex)(d+ex)^2}+6\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{3e^4}$$

input
```
Integrate[x^3/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(-2*d^2 + d*e*x + 4*e^2*x^2))/((d - e*x)*(d + e*x)^2
) + 6*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(3*e^4)
```

3.129.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {568, 530, 27, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{568}$$

$$\frac{x^2}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\int \frac{x(2d-3ex)}{(d^2-e^2x^2)^{3/2}}dx}{3e^2}$$

$$\Big\downarrow \text{530}$$

$$\frac{x^2}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\frac{2d-3ex}{e^2\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{3d^2}{e\sqrt{d^2-e^2x^2}}dx}{d^2}}{3e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\frac{3\int \frac{1}{\sqrt{d^2-e^2x^2}}dx}{e}+\frac{2d-3ex}{e^2\sqrt{d^2-e^2x^2}}}{3e^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{x^2}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\frac{3\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}}{e}+\frac{2d-3ex}{e^2\sqrt{d^2-e^2x^2}}}{3e^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{x^2}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\frac{3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}+\frac{2d-3ex}{e^2\sqrt{d^2-e^2x^2}}}{3e^2}$$

input
```
Int[x^3/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
x^2/(3*e^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]) - ((2*d - 3*e*x)/(e^2*Sqrt[d^2 -
 e^2*x^2]) + (3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^2)/(3*e^2)
```

3.129.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 568
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[x^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*p*(c + d*x))), x] + Simp[1/(2*d^
2*p)   Int[x^(m - 2)*(a + b*x^2)^p*(c*(m - 1) - d*m*x), x], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[m, 1] && LtQ[p, -1]
```

3.129.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(79) = 158$.

Time = 0.38 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.29

method	result
default	$\frac{\frac{x}{e^{2}\sqrt{-e^{2}x^{2}+d^{2}}}-\frac{\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{e^{2}\sqrt{e^{2}}}}{e}+\frac{x}{\sqrt{-e^{2}x^{2}+d^{2}}\,e^{3}}-\frac{d}{e^{4}\sqrt{-e^{2}x^{2}+d^{2}}}-\frac{d^{3}\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}-\frac{}{3e\,d^{3}\sqrt{}}\right.}{e^{4}}$

input
```
int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e
^2*x^2+d^2)^(1/2)))+1/(-e^2*x^2+d^2)^(1/2)/e^3*x-d/e^4/(-e^2*x^2+d^2)^(1/2
)-d^3/e^4*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3
*(-2*(x+d/e)*e^2+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.76

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = -\frac{2\,e^3x^3+2\,de^2x^2-2\,d^2ex-2\,d^3-6\left(e^3x^3+de^2x^2-d^2ex-d^3\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(4\,e^2x^2+dex\right.}{3\left(e^7x^3+de^6x^2-d^2e^5x-d^3e^4\right)}$$

input
```
integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(2*e^3*x^3 + 2*d*e^2*x^2 - 2*d^2*e*x - 2*d^3 - 6*(e^3*x^3 + d*e^2*x^2
 - d^2*e*x - d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (4*e^2*x^2 +
 d*e*x - 2*d^2)*sqrt(-e^2*x^2 + d^2))/(e^7*x^3 + d*e^6*x^2 - d^2*e^5*x - d
^3*e^4)
```

3.129.6 Sympy [F]

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral(x**3/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.129.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{d^2}{3\left(\sqrt{-e^2x^2+d^2}e^5x+\sqrt{-e^2x^2+d^2}de^4\right)} + \frac{4\,x}{3\,\sqrt{-e^2x^2+d^2}e^3} - \frac{\arcsin\left(\frac{ex}{d}\right)}{e^4} - \frac{d}{\sqrt{-e^2x^2+d^2}e^4}$$

input
```
integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
1/3*d^2/(sqrt(-e^2*x^2 + d^2)*e^5*x + sqrt(-e^2*x^2 + d^2)*d*e^4) + 4/3*x/
(sqrt(-e^2*x^2 + d^2)*e^3) - arcsin(e*x/d)/e^4 - d/(sqrt(-e^2*x^2 + d^2)*e
^4)
```

3.129.8 Giac [F]

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^3}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)), x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^3}{\left(d^2-e^2\,x^2\right)^{3/2}\,(d+e\,x)}\,dx$$

input
```
int(x^3/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

output
```
int(x^3/((d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

3.130 $\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}} dx$

3.130.1 Optimal result

Integrand size = 27, antiderivative size = 60

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}} dx = \frac{2}{3e^3\sqrt{d^2-e^2x^2}} - \frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

output
```
2/3/e^3/(-e^2*x^2+d^2)^(1/2)-1/3*x^2/d/e/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

3.130.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}} dx = \frac{\sqrt{d^2-e^2x^2}(2d^2+2dex-e^2x^2)}{3de^3(d-ex)(d+ex)^2}$$

input
```
Integrate[x^2/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(2*d^2 + 2*d*e*x - e^2*x^2))/(3*d*e^3*(d - e*x)*(d +
e*x)^2)
```

3.130.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {567, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{567}$$

$$\frac{2\int \frac{x}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3e}-\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{241}$$

$$\frac{2}{3e^3\sqrt{d^2-e^2x^2}}-\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

input
```
Int[x^2/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
2/(3*e^3*Sqrt[d^2 - e^2*x^2]) - x^2/(3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

3.130.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 567
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[c*x^m*((a + b*x^2)^(p + 1)/(2*a*d*p*(c + d*x))), x] - Simp[m/(2*d*p)
   Int[x^(m - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
^2 + a*d^2, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[m + 2*p + 1, 0]
```

3.130.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{(-ex+d)(-e^2x^2+2dex+2d^2)}{3d\,e^3(-e^2x^2+d^2)^{\frac{3}{2}}}$	48
trager	$\frac{(-e^2x^2+2dex+2d^2)\sqrt{-e^2x^2+d^2}}{3d\,e^3(ex+d)^2(-ex+d)}$	57
default	$\frac{1}{e^3\sqrt{-e^2x^2+d^2}}-\frac{x}{d\,e^2\sqrt{-e^2x^2+d^2}}+\frac{d^2\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\frac{-2\left(x+\frac{d}{e}\right)e^2+2de}{3e\,d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{e^3}$	149

input
```
int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-e*x+d)*(-e^2*x^2+2*d*e*x+2*d^2)/d/e^3/(-e^2*x^2+d^2)^(3/2)
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.72

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{2\,e^3x^3+2\,de^2x^2-2\,d^2ex-2\,d^3+(e^2x^2-2\,dex-2\,d^2)\sqrt{-e^2x^2+d^2}}{3\,(de^6x^3+d^2e^5x^2-d^3e^4x-d^4e^3)}$$

input
```
integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*e^3*x^3 + 2*d*e^2*x^2 - 2*d^2*e*x - 2*d^3 + (e^2*x^2 - 2*d*e*x - 2*
d^2)*sqrt(-e^2*x^2 + d^2))/(d*e^6*x^3 + d^2*e^5*x^2 - d^3*e^4*x - d^4*e^3)
```

3.130.6 Sympy [F]

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=\int \frac{x^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral(x**2/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = -\frac{d}{3\left(\sqrt{-e^2x^2+d^2}e^4x+\sqrt{-e^2x^2+d^2}de^3\right)} - \frac{x}{3\sqrt{-e^2x^2+d^2}de^2} + \frac{1}{\sqrt{-e^2x^2+d^2}e^3}$$

input
```
integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/3*d/(sqrt(-e^2*x^2 + d^2)*e^4*x + sqrt(-e^2*x^2 + d^2)*d*e^3) - 1/3*x/(
sqrt(-e^2*x^2 + d^2)*d*e^2) + 1/(sqrt(-e^2*x^2 + d^2)*e^3)
```

3.130.8 Giac [F]

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)), x)
```

3.130.9 Mupad [B] (verification not implemented)

Time = 12.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2+2\,d\,e\,x-e^2\,x^2\right)}{3\,d\,e^3\,{\left(d+e\,x\right)}^2\,\left(d-e\,x\right)}$$

input
```
int(x^2/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(2*d^2 - e^2*x^2 + 2*d*e*x))/(3*d*e^3*(d + e*x)^2*(
d - e*x))
```

3.131 $\int \frac{x}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx$

3.131.1 Optimal result

Integrand size = 25, antiderivative size = 58

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{x}{3d^2e\sqrt{d^2-e^2x^2}}+\frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}$$

output
```
1/3*x/d^2/e/(-e^2*x^2+d^2)^(1/2)+1/3/e^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

3.131.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}(d^2+dex+e^2x^2)}{3d^2e^2(d-ex)(d+ex)^2}$$

input
```
Integrate[x/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(d^2 + d*e*x + e^2*x^2))/(3*d^2*e^2*(d - e*x)*(d + e*
x)^2)
```

3.131.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {565, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 565}$$

$$\frac{d}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-e\int \frac{x^2}{\left(d^2-e^2x^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 242}$$

$$\frac{d}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{ex^3}{3d^2\left(d^2-e^2x^2\right)^{3/2}}$$

input
```
Int[x/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
d/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (e*x^3)/(3*d^2*(d^2 - e^2*x^2)^(3/2))
```

3.131.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 565
```
Int[((x_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :> Sim
p[a*((a + b*x^2)^p/(2*b*c*p)), x] + Simp[b/d   Int[x^2*(a + b*x^2)^(p - 1),
 x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0]
```

3.131.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{(-ex+d)(e^2x^2+dex+d^2)}{3d^2e^2(-e^2x^2+d^2)^{\frac{3}{2}}}$	44
trager	$\frac{(e^2x^2+dex+d^2)\sqrt{-e^2x^2+d^2}}{3d^2e^2(ex+d)^2(-ex+d)}$	53
default	$\frac{x}{d^2e\sqrt{-e^2x^2+d^2}}-\frac{d\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\frac{-2\left(x+\frac{d}{e}\right)e^2+2de}{3e\,d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{e^2}$	129

input
```
int(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-e*x+d)*(e^2*x^2+d*e*x+d^2)/d^2/e^2/(-e^2*x^2+d^2)^(3/2)
```

3.131.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(50) = 100.

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.74

$$\int \frac{x}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx = \frac{e^3x^3+de^2x^2-d^2ex-d^3-(e^2x^2+dex+d^2)\sqrt{-e^2x^2+d^2}}{3(d^2e^5x^3+d^3e^4x^2-d^4e^3x-d^5e^2)}$$

input
```
integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(e^3*x^3 + d*e^2*x^2 - d^2*e*x - d^3 - (e^2*x^2 + d*e*x + d^2)*sqrt(-e
^2*x^2 + d^2))/(d^2*e^5*x^3 + d^3*e^4*x^2 - d^4*e^3*x - d^5*e^2)
```

3.131.6 Sympy [F]

$$\int \frac{x}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx = \int \frac{x}{(-(-d+ex)(d+ex))^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(x/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral(x/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{1}{3\left(\sqrt{-e^2x^2+d^2}e^3x+\sqrt{-e^2x^2+d^2}de^2\right)}+\frac{x}{3\sqrt{-e^2x^2+d^2}d^2e}$$

input
```
integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
1/3/(sqrt(-e^2*x^2 + d^2)*e^3*x + sqrt(-e^2*x^2 + d^2)*d*e^2) + 1/3*x/(sqr
t(-e^2*x^2 + d^2)*d^2*e)
```

3.131.8 Giac [F]

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)), x)
```

3.131.9 Mupad [B] (verification not implemented)

Time = 12.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(d^2+d\,e\,x+e^2\,x^2\right)}{3\,d^2\,e^2\,{\left(d+e\,x\right)}^2\,\left(d-e\,x\right)}$$

input
```
int(x/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(d^2 + e^2*x^2 + d*e*x))/(3*d^2*e^2*(d + e*x)^2*(d
- e*x))
```

3.132 $\int \frac{1}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx$

3.132.1 Optimal result

Integrand size = 24, antiderivative size = 58

$$\int \frac{1}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx = \frac{2x}{3d^3\sqrt{d^2-e^2x^2}} - \frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

output
```
2/3*x/d^3/(-e^2*x^2+d^2)^(1/2)-1/3/d/e/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

3.132.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \frac{1}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-d^2+2dex+2e^2x^2)}{3d^3e(d-ex)(d+ex)^2}$$

input
```
Integrate[1/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-d^2 + 2*d*e*x + 2*e^2*x^2))/(3*d^3*e*(d - e*x)*(d +
 e*x)^2)
```

3.132.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {470, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{470}$$

$$\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

input
```
Int[1/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
(2*x)/(3*d^3*Sqrt[d^2 - e^2*x^2]) - 1/(3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2]
)
```

3.132.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

3.132.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

method	result	size
gosper	$-\frac{(-ex+d)(-2e^2x^2-2dex+d^2)}{3d^3e(-e^2x^2+d^2)^{\frac{3}{2}}}$	46
trager	$-\frac{(-2e^2x^2-2dex+d^2)\sqrt{-e^2x^2+d^2}}{3d^3(ex+d)^2e(-ex+d)}$	55
default	$\frac{-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\frac{-2\left(x+\frac{d}{e}\right)e^2+2de}{3e\,d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}}{e}$	104

input
```
int(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-e*x+d)*(-2*e^2*x^2-2*d*e*x+d^2)/d^3/e/(-e^2*x^2+d^2)^(3/2)
```

3.132.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(50) = 100.

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.76

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = -\frac{e^3x^3+de^2x^2-d^2ex-d^3+\left(2\,e^2x^2+2\,dex-d^2\right)\sqrt{-e^2x^2+d^2}}{3\left(d^3e^4x^3+d^4e^3x^2-d^5e^2x-d^6e\right)}$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(e^3*x^3 + d*e^2*x^2 - d^2*e*x - d^3 + (2*e^2*x^2 + 2*d*e*x - d^2)*sq
rt(-e^2*x^2 + d^2))/(d^3*e^4*x^3 + d^4*e^3*x^2 - d^5*e^2*x - d^6*e)
```

3.132.6 Sympy [F]

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(-(-d+ex)(d+ex)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral(1/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = -\frac{1}{3\left(\sqrt{-e^2x^2+d^2}de^2x+\sqrt{-e^2x^2+d^2}d^2e\right)}+\frac{2\,x}{3\,\sqrt{-e^2x^2+d^2}d^3}$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/3/(sqrt(-e^2*x^2 + d^2)*d*e^2*x + sqrt(-e^2*x^2 + d^2)*d^2*e) + 2/3*x/(
sqrt(-e^2*x^2 + d^2)*d^3)
```

3.132.8 Giac [F]

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)), x)
```

3.132.9 Mupad [B] (verification not implemented)

Time = 11.69 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(-d^2+2\,d\,e\,x+2\,e^2\,x^2\right)}{3\,d^3\,e\,{\left(d+e\,x\right)}^2\,\left(d-e\,x\right)}$$

input
```
int(1/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(2*e^2*x^2 - d^2 + 2*d*e*x))/(3*d^3*e*(d + e*x)^2*(
d - e*x))
```

3.133 $\int \frac{1}{x(d+ex)(d^2-e^2x^2)^{3/2}}\,dx$

3.133.1 Optimal result

Integrand size = 27, antiderivative size = 88

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

output
```
-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4+1/3*(-2*e*x+3*d)/d^4/(-e^2*x^2+d^2)^(
1/2)+1/3/d^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

3.133.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.20

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{\frac{d(4d^2+dex-2e^2x^2)\sqrt{d^2-e^2x^2}}{(d-ex)(d+ex)^2} - 3\sqrt{d^2}\log(x) + 3\sqrt{d^2}\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{3d^5}$$

input
```
Integrate[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
((d*(4*d^2 + d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/((d - e*x)*(d + e*x)^
2) - 3*Sqrt[d^2]*Log[x] + 3*Sqrt[d^2]*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]]
)/(3*d^5)
```

3.133.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {569, 25, 532, 27, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(d+ex)(d^2-e^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{569}$$

$$\frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{3d-2ex}{x(d^2-e^2x^2)^{3/2}}dx}{3d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3d-2ex}{x(d^2-e^2x^2)^{3/2}}dx}{3d^2}+\frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{\frac{3d-2ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{3d}{x\sqrt{d^2-e^2x^2}}dx}{d^2}}{3d^2}+\frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx}{d}+\frac{3d-2ex}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{3\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2}{2d}+\frac{3d-2ex}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{3d-2ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{3\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{de^2}}{3d^2}+\frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{3d-2ex}{d^2\sqrt{d^2-e^2x^2}} - \frac{3\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}}{3d^2} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}}$$

input
```
Int[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
1/(3*d^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]) + ((3*d - 2*e*x)/(d^2*Sqrt[d^2 - e
^2*x^2]) - (3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^2)/(3*d^2)
```

3.133.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 569
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[(-x^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*p*(c + d*x))), x] + Simp[1/(2
*c^2*p)   Int[x^m*(a + b*x^2)^p*(c*(m + 2*p + 1) - d*(m + 2*p + 2)*x), x],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[m + 2*p, 0]
```

3.133.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(78) = 156$.

Time = 0.37 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.94

method	result	size
default	$\frac{\frac{1}{d^{2}\sqrt{-e^{2}x^{2}+d^{2}}}-\frac{\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{d^{2}\sqrt{d^{2}}}}{d}-\frac{-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}-\frac{-2\left(x+\frac{d}{e}\right)e^{2}+2de}{3e\,d^{3}\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}}{d}$	171

input
```
int(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*
(-e^2*x^2+d^2)^(1/2))/x))-1/d*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d
/e))^(1/2)-1/3/e/d^3*(-2*(x+d/e)*e^2+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))
^(1/2))
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.76

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{4\,e^3x^3+4\,de^2x^2-4\,d^2ex-4\,d^3+3\left(e^3x^3+de^2x^2-d^2ex-d^3\right)\log\left(-\frac{d-\sqrt{}}{}\right.}{3\left(d^4e^3x^3+d^5e^2x^2-d^6ex-d\right.}$$

input
```
integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(4*e^3*x^3 + 4*d*e^2*x^2 - 4*d^2*e*x - 4*d^3 + 3*(e^3*x^3 + d*e^2*x^2
- d^2*e*x - d^3)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (2*e^2*x^2 - d*e*x -
 4*d^2)*sqrt(-e^2*x^2 + d^2))/(d^4*e^3*x^3 + d^5*e^2*x^2 - d^6*e*x - d^7)
```

3.133.6 Sympy [F]

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate(1/x/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral(1/(x*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.133.7 Maxima [F]

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)x}\,dx$$

input
```
integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x), x)
```

3.133.8 Giac [F]

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)x}\,dx$$

input
```
integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x), x)
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{x\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

input
```
int(1/(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

output
```
int(1/(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

3.134 $\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

3.134.1 Optimal result

Integrand size = 27, antiderivative size = 120

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} + \frac{e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

output
```
e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5+1/3*(-3*e*x+4*d)/d^4/x/(-e^2*x^2+d^2
)^(1/2)+1/3/d^2/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2)-8/3*(-e^2*x^2+d^2)^(1/2)/d^
5/x
```

3.134.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}\left(3d^3+7d^2ex-5de^2x^2-8e^3x^3\right)}{x(-d+ex)(d+ex)^2} + 3\sqrt{d^2}e\log(x) - 3\sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2\right.}}{3d^6}$$

input
```
Integrate[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
((d*Sqrt[d^2 - e^2*x^2]*(3*d^3 + 7*d^2*e*x - 5*d*e^2*x^2 - 8*e^3*x^3))/(x*
(-d + e*x)*(d + e*x)^2) + 3*Sqrt[d^2]*e*Log[x] - 3*Sqrt[d^2]*e*Log[Sqrt[d^
2] - Sqrt[d^2 - e^2*x^2]])/(3*d^6)
```

3.134.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {569, 25, 532, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{569}$$

$$\frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{4d-3ex}{x^2\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4d-3ex}{x^2\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{-\frac{\int -\frac{4d-3ex}{x^2\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{e(3d-4ex)}{d^3\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{4d-3ex}{x^2\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{e(3d-4ex)}{d^3\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{534}$$

$$\frac{\frac{-3e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{4\sqrt{d^2-e^2x^2}}{dx}}{d^2}-\frac{e(3d-4ex)}{d^3\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{-\frac{3}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{4\sqrt{d^2-e^2x^2}}{dx}}{d^2}-\frac{e(3d-4ex)}{d^3\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{\frac{3\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{4\sqrt{d^2-e^2x^2}}{dx}}{d^2}-\frac{e(3d-4ex)}{d^3\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\frac{3e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{4\sqrt{d^2-e^2x^2}}{dx}}{d^2}-\frac{e(3d-4ex)}{d^3\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}}$$

input
```
Int[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
1/(3*d^2*x*(d + e*x)*Sqrt[d^2 - e^2*x^2]) + (-((e*(3*d - 4*e*x))/(d^3*Sqrt
[d^2 - e^2*x^2])) + ((-4*Sqrt[d^2 - e^2*x^2])/(d*x) + (3*e*ArcTanh[Sqrt[d^
2 - e^2*x^2]/d])/d)/d^2)/(3*d^2)
```

3.134.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 569
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[(-x^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*p*(c + d*x))), x] + Simp[1/(2
*c^2*p)   Int[x^m*(a + b*x^2)^p*(c*(m + 2*p + 1) - d*(m + 2*p + 2)*x), x],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[m + 2*p, 0]
```

3.134.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.65

method	result
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{d^{5} x}+\frac{e \ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{d^{4} \sqrt{d^{2}}}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{6 d^{4} e \left(x+\frac{d}{e}\right)^{2}}-\frac{17 \sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{12 d^{5} \left(x+\frac{d}{e}\right)}-\cdots$
default	$\frac{-\frac{1}{d^{2} x \sqrt{-e^{2} x^{2}+d^{2}}}+\frac{2 e^{2} x}{d^{4} \sqrt{-e^{2} x^{2}+d^{2}}}}{d}-\frac{e \left(\frac{1}{d^{2} \sqrt{-e^{2} x^{2}+d^{2}}}-\frac{\ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{d^{2} \sqrt{d^{2}}}\right)}{d^{2}}+\frac{e \left(-\frac{1}{3 d e \left(x+\frac{d}{e}\right) \sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}\cdots\right)}{d^{2}}$

input
```
int(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-e^2*x^2+d^2)^(1/2)/d^5/x+1/d^4*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-
e^2*x^2+d^2)^(1/2))/x)-1/6/d^4/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^
(1/2)-17/12/d^5/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/4/d^5/(x-d/
e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.51

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx =$$
$$-\frac{4\,e^4x^4+4\,de^3x^3-4\,d^2e^2x^2-4\,d^3ex+3\left(e^4x^4+de^3x^3-d^2e^2x^2-d^3ex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(8\,e^3x^3+\right.}{3\left(d^5e^3x^4+d^6e^2x^3-d^7ex^2-d^8x\right)}$$

input
```
integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(4*e^4*x^4 + 4*d*e^3*x^3 - 4*d^2*e^2*x^2 - 4*d^3*e*x + 3*(e^4*x^4 + d
*e^3*x^3 - d^2*e^2*x^2 - d^3*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (8*
e^3*x^3 + 5*d*e^2*x^2 - 7*d^2*e*x - 3*d^3)*sqrt(-e^2*x^2 + d^2))/(d^5*e^3*
x^4 + d^6*e^2*x^3 - d^7*e*x^2 - d^8*x)
```

3.134.6 Sympy [F]

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{x^2\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate(1/x**2/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.134.7 Maxima [F]

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x^2), x)
```

3.134.8 Giac [F]

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{(-e^2x^2+d^2)^{\frac{3}{2}}(ex+d)x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x^2), x)
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{x^2\left(d^2-e^2\,x^2\right)^{3/2}\,(d+e\,x)}\,dx$$

input
```
int(1/(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

output
```
int(1/(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

3.135 $\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

3.135.1 Optimal result

Integrand size = 27, antiderivative size = 152

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}}+\frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} -\frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{8e\sqrt{d^2-e^2x^2}}{3d^6x}-\frac{5e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}$$

output
```
-5/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/3*(-4*e*x+5*d)/d^4/x^2/(-e^
2*x^2+d^2)^(1/2)+1/3/d^2/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2)-5/2*(-e^2*x^2+d^
2)^(1/2)/d^5/x^2+8/3*e*(-e^2*x^2+d^2)^(1/2)/d^6/x
```

3.135.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}\left(3d^4-3d^3ex-23d^2e^2x^2+de^3x^3+16e^4x^4\right)}{x^2(-d+ex)(d+ex)^2}-15\sqrt{d^2}e^2\log(x)+15\sqrt{d^2}e^2\log}{6d^7}$$

input
```
Integrate[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
((d*Sqrt[d^2 - e^2*x^2]*(3*d^4 - 3*d^3*e*x - 23*d^2*e^2*x^2 + d*e^3*x^3 +
16*e^4*x^4))/(x^2*(-d + e*x)*(d + e*x)^2) - 15*Sqrt[d^2]*e^2*Log[x] + 15*S
qrt[d^2]*e^2*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(6*d^7)
```

3.135.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {569, 25, 532, 25, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{569}$$

$$\frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{5d-4ex}{x^3\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5d-4ex}{x^3\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{\frac{e^2(5d-4ex)}{d^4\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{\frac{5e^2x^2}{d}-4ex+5d}{x^3\sqrt{d^2-e^2x^2}}dx}{d^2}}{3d^2}+\frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\frac{5e^2x^2}{d}-4ex+5d}{x^3\sqrt{d^2-e^2x^2}}dx}{d^2}+\frac{e^2(5d-4ex)}{d^4\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{2338}$$

$$\frac{\frac{-\frac{\int \frac{de(8d-15ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^2}}{d^2}+\frac{e^2(5d-4ex)}{d^4\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{-\frac{e\int \frac{8d-15ex}{x^2\sqrt{d^2-e^2x^2}}dx}{2d}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^2}}{d^2}+\frac{e^2(5d-4ex)}{d^4\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{534}$$

$$\frac{-\frac{\frac{e\left(-15e\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{8\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^2}}{d^2}+\frac{e^2(5d-4ex)}{d^4\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{-\frac{\frac{e\left(-\frac{15}{2}e\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{8\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^2}}{d^2}+\frac{e^2(5d-4ex)}{d^4\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{-\frac{\frac{e\left(\frac{15\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{8\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^2}}{d^2}+\frac{e^2(5d-4ex)}{d^4\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{\frac{e\left(\frac{15e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{8\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{5\sqrt{d^2-e^2x^2}}{2dx^2}}{d^2}+\frac{e^2(5d-4ex)}{d^4\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}}$$

input
```
Int[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
1/(3*d^2*x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]) + ((e^2*(5*d - 4*e*x))/(d^4*Sq
rt[d^2 - e^2*x^2]) + ((-5*Sqrt[d^2 - e^2*x^2])/(2*d*x^2) - (e*((-8*Sqrt[d^
2 - e^2*x^2])/(d*x) + (15*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/(2*d))/d^2
)/(3*d^2)
```

3.135.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 569
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[(-x^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*p*(c + d*x))), x] + Simp[1/(2
*c^2*p)   Int[x^m*(a + b*x^2)^p*(c*(m + 2*p + 1) - d*(m + 2*p + 2)*x), x],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[m + 2*p, 0]
```

3.135. $\int \frac{1}{x^3(d+ex)(d^2-e^2x^2)^{3/2}}\,dx$

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.135.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.36

method	result
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\,(-2ex+d)}{2 d^{6} x^{2}}-\frac{5 e^{2} \ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\,\sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{2 d^{5} \sqrt{d^{2}}}+\frac{23 e \sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{12 d^{6} \left(x+\frac{d}{e}\right)}-\frac{e \sqrt{-\left(x-\frac{d}{e}\right)^{2} e^{2}-2 d e \left(x-\frac{d}{e}\right.}}{4 d^{6} \left(x-\frac{d}{e}\right)}$
default	$\frac{-\frac{1}{2 d^{2} x^{2} \sqrt{-e^{2} x^{2}+d^{2}}}+\frac{3 e^{2} \left(\frac{1}{d^{2} \sqrt{-e^{2} x^{2}+d^{2}}}-\frac{\ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\,\sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{d^{2} \sqrt{d^{2}}}\right)}{2 d^{2}}}{d}+\frac{e^{2} \left(\frac{1}{d^{2} \sqrt{-e^{2} x^{2}+d^{2}}}-\frac{\ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\,\sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{d^{2} \sqrt{d^{2}}}\right)}{d^{3}}-$

input
```
int(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e^2*x^2+d^2)^(1/2)*(-2*e*x+d)/d^6/x^2-5/2/d^5*e^2/(d^2)^(1/2)*ln((2
*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+23/12/d^6*e/(x+d/e)*(-(x+d/e)^
2*e^2+2*d*e*(x+d/e))^(1/2)-1/4/d^6*e/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e)
)^(1/2)+1/6/d^5/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^{3}(d+ex)\left(d^{2}-e^{2}x^{2}\right)^{3/2}}\,dx=\frac{14\,e^{5}x^{5}+14\,de^{4}x^{4}-14\,d^{2}e^{3}x^{3}-14\,d^{3}e^{2}x^{2}+15\left(e^{5}x^{5}+de^{4}x^{4}-d^{2}e^{3}x^{3}-\right.}{6\left(d^{6}e^{3}x^{5}\right.}$$

input
```
integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(14*e^5*x^5 + 14*d*e^4*x^4 - 14*d^2*e^3*x^3 - 14*d^3*e^2*x^2 + 15*(e^5
*x^5 + d*e^4*x^4 - d^2*e^3*x^3 - d^3*e^2*x^2)*log(-(d - sqrt(-e^2*x^2 + d^
2))/x) + (16*e^4*x^4 + d*e^3*x^3 - 23*d^2*e^2*x^2 - 3*d^3*e*x + 3*d^4)*sqr
t(-e^2*x^2 + d^2))/(d^6*e^3*x^5 + d^7*e^2*x^4 - d^8*e*x^3 - d^9*x^2)
```

3.135.6 Sympy [F]

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{x^3\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate(1/x**3/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral(1/(x**3*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.135.7 Maxima [F]

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)x^3}\,dx$$

input
```
integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x^3), x)
```

3.135.8 Giac [F]

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)x^3}\,dx$$

input
```
integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x^3), x)
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=\int \frac{1}{x^3\,\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)} \,dx$$

input
```
int(1/(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

output
```
int(1/(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

3.136 $\int \frac{x^7}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

3.136.1 Optimal result

Integrand size = 27, antiderivative size = 162

$$\int \frac{x^7}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{(32d-35ex)\sqrt{d^2-e^2x^2}}{10e^8}+\frac{7d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8}$$

output
```
1/5*x^6*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^4*(-7*e*x+6*d)/e^4/(-e^2*
x^2+d^2)^(3/2)+7/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^8+1/15*x^2*(-35*
e*x+24*d)/e^6/(-e^2*x^2+d^2)^(1/2)+1/10*(-35*e*x+32*d)*(-e^2*x^2+d^2)^(1/2
)/e^8
```

3.136.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.86

$$\int \frac{x^7}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\frac{\frac{\sqrt{d^2-e^2x^2}\left(96d^6-9d^5ex-249d^4e^2x^2-4d^3e^3x^3+176d^2e^4x^4+15de^5x^5-15e^6x^6\right)}{(d-ex)^2(d+ex)^3}-210d^2\arctan\left(\right.}{30e^8}$$

input
```
Integrate[x^7/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(96*d^6 - 9*d^5*e*x - 249*d^4*e^2*x^2 - 4*d^3*e^3*x^
3 + 176*d^2*e^4*x^4 + 15*d*e^5*x^5 - 15*e^6*x^6))/((d - e*x)^2*(d + e*x)^3
) - 210*d^2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(30*e^8)
```

3.136.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.28, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {568, 530, 25, 2345, 27, 2346, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(d+ex)(d^2-e^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{568}$$

$$\frac{x^6}{5e^2(d+ex)(d^2-e^2x^2)^{3/2}}-\frac{\int \frac{x^5(6d-7ex)}{(d^2-e^2x^2)^{5/2}}dx}{5e^2}$$

$$\Big\downarrow \text{530}$$

$$\frac{x^6}{5e^2(d+ex)(d^2-e^2x^2)^{3/2}}-\frac{\frac{d^4(6d-7ex)}{3e^6(d^2-e^2x^2)^{3/2}}-\frac{\int -\frac{\frac{7d^6}{e^5}-\frac{18xd^5}{e^4}+\frac{21x^2d^4}{e^3}-\frac{18x^3d^3}{e^2}+\frac{21x^4d^2}{e}}{(d^2-e^2x^2)^{3/2}}dx}{3d^2}}{5e^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^6}{5e^2(d+ex)(d^2-e^2x^2)^{3/2}}-\frac{\frac{\int \frac{\frac{7d^6}{e^5}-\frac{18xd^5}{e^4}+\frac{21x^2d^4}{e^3}-\frac{18x^3d^3}{e^2}+\frac{21x^4d^2}{e}}{(d^2-e^2x^2)^{3/2}}dx}{3d^2}+\frac{d^4(6d-7ex)}{3e^6(d^2-e^2x^2)^{3/2}}}{5e^2}$$

$$\Big\downarrow \text{2345}$$

$$\frac{x^6}{5e^2(d+ex)(d^2-e^2x^2)^{3/2}}-\frac{\frac{-\frac{\int \frac{3\left(\frac{14d^6}{e^5}-\frac{6xd^5}{e^4}+\frac{7x^2d^4}{e^3}\right)}{\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{d^4(36d-49ex)}{e^6\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{d^4(6d-7ex)}{3e^6(d^2-e^2x^2)^{3/2}}}{5e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^6}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\frac{-\frac{3\int\frac{\frac{14d^6}{e^5}-\frac{6xd^5}{e^4}+\frac{7x^2d^4}{e^3}}{\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{d^4(36d-49ex)}{e^6\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{d^4(6d-7ex)}{3e^6\left(d^2-e^2x^2\right)^{3/2}}}{5e^2}$$

$$\Big\downarrow \ 2346$$

$$\frac{x^6}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\frac{-\frac{3\left(-\frac{\int -\frac{d^5(35d-12ex)}{e^3\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{7d^4x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d^2}-\frac{d^4(36d-49ex)}{e^6\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{d^4(6d-7ex)}{3e^6\left(d^2-e^2x^2\right)^{3/2}}}{5e^2}$$

$$\Big\downarrow \ 25$$

$$\frac{x^6}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\frac{-\frac{3\left(\frac{\int \frac{d^5(35d-12ex)}{e^3\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{7d^4x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d^2}-\frac{d^4(36d-49ex)}{e^6\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{d^4(6d-7ex)}{3e^6\left(d^2-e^2x^2\right)^{3/2}}}{5e^2}$$

$$\Big\downarrow \ 27$$

$$\frac{x^6}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\frac{-\frac{3\left(\frac{d^5\int \frac{35d-12ex}{\sqrt{d^2-e^2x^2}}dx}{2e^5}-\frac{7d^4x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d^2}-\frac{d^4(36d-49ex)}{e^6\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{d^4(6d-7ex)}{3e^6\left(d^2-e^2x^2\right)^{3/2}}}{5e^2}$$

$$\Big\downarrow \ 455$$

$$\frac{x^6}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\frac{-\frac{3\left(\frac{d^5\left(35d\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{12\sqrt{d^2-e^2x^2}}{e}\right)}{2e^5}-\frac{7d^4x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d^2}-\frac{d^4(36d-49ex)}{e^6\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{d^4(6d-7ex)}{3e^6\left(d^2-e^2x^2\right)^{3/2}}}{5e^2}$$

$$\Big\downarrow \ 224$$

$$\frac{x^6}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\frac{-\frac{3\left(\frac{d^5\left(35d\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{12\sqrt{d^2-e^2x^2}}{e}\right)}{2e^5}-\frac{7d^4x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d^2}-\frac{d^4(36d-49ex)}{e^6\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{d^4(6d-7ex)}{3e^6\left(d^2-e^2x^2\right)^{3/2}}}{5e^2}$$

3.136. $\int \frac{x^7}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

$$\Big\downarrow \text{216}$$

$$\frac{x^6}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\frac{-\frac{3\left(\frac{d^5\left(\frac{35d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{12\sqrt{d^2-e^2x^2}}{e}\right)}{2e^5}-\frac{7d^4x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d^2}-\frac{d^4(36d-49ex)}{e^6\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{d^4(6d-7ex)}{3e^6\left(d^2-e^2x^2\right)^{3/2}}}{5e^2}$$

input
```
Int[x^7/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
x^6/(5*e^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2)) - ((d^4*(6*d - 7*e*x))/(3*e^6*
(d^2 - e^2*x^2)^(3/2)) + (-((d^4*(36*d - 49*e*x))/(e^6*Sqrt[d^2 - e^2*x^2]
)) - (3*((-7*d^4*x*Sqrt[d^2 - e^2*x^2])/(2*e^5) + (d^5*((12*Sqrt[d^2 - e^2
*x^2])/e + (35*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/(2*e^5)))/d^2)/(3*
d^2))/(5*e^2)
```

3.136.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 568
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[x^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*p*(c + d*x))), x] + Simp[1/(2*d^
2*p)   Int[x^(m - 2)*(a + b*x^2)^p*(c*(m - 1) - d*m*x), x], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[m, 1] && LtQ[p, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.136.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(142) = 284$.

Time = 0.44 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.82

method	result
risch	$\frac{(-ex+2d)\sqrt{-e^2x^2+d^2}}{2e^8}+\frac{7d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^7\sqrt{e^2}}-\frac{7d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15e^{10}\left(x+\frac{d}{e}\right)^2}+\frac{773d^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{240e^9\left(x+\frac{d}{e}\right)}+$
default	$\frac{-\frac{x^5}{2e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{5d^2\left(\frac{x^3}{3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e^2}\right)}{2e^2}}{e}+\frac{d^6\left(\frac{x}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{e^7}$

input
```
int(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-e*x+2*d)/e^8*(-e^2*x^2+d^2)^(1/2)+7/2*d^2/e^7/(e^2)^(1/2)*arctan((e^
2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-7/15*d^3/e^10/(x+d/e)^2*(-(x+d/e)^2*e^2+2
*d*e*(x+d/e))^(1/2)+773/240*d^2/e^9/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))
^(1/2)+1/24*d^3/e^10/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)+31/48*
d^2/e^9/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)+1/20*d^4/e^11/(x+d/e)
^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.69

$$\int \frac{x^7}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\frac{96\,d^2e^5x^5+96\,d^3e^4x^4-192\,d^4e^3x^3-192\,d^5e^2x^2+96\,d^6ex+96\,d^7-210\,(d}{}$$

input
```
integrate(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
1/30*(96*d^2*e^5*x^5 + 96*d^3*e^4*x^4 - 192*d^4*e^3*x^3 - 192*d^5*e^2*x^2
+ 96*d^6*e*x + 96*d^7 - 210*(d^2*e^5*x^5 + d^3*e^4*x^4 - 2*d^4*e^3*x^3 - 2
*d^5*e^2*x^2 + d^6*e*x + d^7)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) -
(15*e^6*x^6 - 15*d*e^5*x^5 - 176*d^2*e^4*x^4 + 4*d^3*e^3*x^3 + 249*d^4*e^2
*x^2 + 9*d^5*e*x - 96*d^6)*sqrt(-e^2*x^2 + d^2))/(e^13*x^5 + d*e^12*x^4 -
2*d^2*e^11*x^3 - 2*d^3*e^10*x^2 + d^4*e^9*x + d^5*e^8)
```

3.136.6 Sympy [F]

$$\int \frac{x^7}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^7}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate(x**7/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Integral(x**7/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.136.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(143) = 286$.

Time = 0.30 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.78

$$\int \frac{x^7}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{d^6}{5\left(\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^9x+\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^8\right)} - \frac{x^5}{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3} + \frac{dx^4}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4} + \frac{25\,d^2x^3}{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5} - \frac{65\,d^3x^2}{6\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^6} - \frac{164\,d^4x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^7} - \frac{7\,dx^2}{6\sqrt{-e^2x^2+d^2}e^6} + \frac{53\,d^5}{6\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^8} + \frac{229\,d^2x}{30\sqrt{-e^2x^2+d^2}e^7} + \frac{7\,d^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^8} - \frac{14\,d^3}{3\sqrt{-e^2x^2+d^2}e^8} - \frac{7\sqrt{-e^2x^2+d^2}d}{6\,e^8}$$

input
```
integrate(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
1/5*d^6/((-e^2*x^2 + d^2)^(3/2)*e^9*x + (-e^2*x^2 + d^2)^(3/2)*d*e^8) - 1/
2*x^5/((-e^2*x^2 + d^2)^(3/2)*e^3) + d*x^4/((-e^2*x^2 + d^2)^(3/2)*e^4) +
25/2*d^2*x^3/((-e^2*x^2 + d^2)^(3/2)*e^5) - 65/6*d^3*x^2/((-e^2*x^2 + d^2)
^(3/2)*e^6) - 164/15*d^4*x/((-e^2*x^2 + d^2)^(3/2)*e^7) - 7/6*d*x^2/(sqrt(
-e^2*x^2 + d^2)*e^6) + 53/6*d^5/((-e^2*x^2 + d^2)^(3/2)*e^8) + 229/30*d^2*
x/(sqrt(-e^2*x^2 + d^2)*e^7) + 7/2*d^2*arcsin(e*x/d)/e^8 - 14/3*d^3/(sqrt(
-e^2*x^2 + d^2)*e^8) - 7/6*sqrt(-e^2*x^2 + d^2)*d/e^8
```

3.136.8 Giac [F]

$$\int \frac{x^7}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^7}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)}\,dx$$

input
```
integrate(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^7/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)), x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^7}{\left(d^2-e^2\,x^2\right)^{5/2}\,(d+e\,x)}\,dx$$

input
```
int(x^7/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

output
```
int(x^7/((d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

3.137 $\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

3.137.1 Optimal result

Integrand size = 27, antiderivative size = 148

$$\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}}-\frac{16\sqrt{d^2-e^2x^2}}{5e^7}-\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}$$

output
```
1/5*x^5*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^3*(-6*e*x+5*d)/e^4/(-e^2*
x^2+d^2)^(3/2)-d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^7+1/5*x*(-8*e*x+5*d)/e
^6/(-e^2*x^2+d^2)^(1/2)-16/5*(-e^2*x^2+d^2)^(1/2)/e^7
```

3.137.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.88

$$\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}\left(-48d^5-33d^4ex+87d^3e^2x^2+52d^2e^3x^3-38de^4x^4-15e^5x^5\right)}{15e^7(-d+ex)^2(d+ex)^3}+\frac{2d\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^7}$$

input
```
Integrate[x^6/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-48*d^5 - 33*d^4*e*x + 87*d^3*e^2*x^2 + 52*d^2*e^3*x
^3 - 38*d*e^4*x^4 - 15*e^5*x^5))/(15*e^7*(-d + e*x)^2*(d + e*x)^3) + (2*d*
ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^7
```

3.137.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {568, 530, 2345, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(d+ex)\,(d^2-e^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 568$$

$$\frac{x^5}{5e^2(d+ex)\,(d^2-e^2x^2)^{3/2}}-\frac{\int \frac{x^4(5d-6ex)}{(d^2-e^2x^2)^{5/2}}dx}{5e^2}$$

$$\Big\downarrow \ 530$$

$$\frac{x^5}{5e^2(d+ex)\,(d^2-e^2x^2)^{3/2}}-\frac{-\frac{\int \frac{\frac{5d^5}{e^4}-\frac{18xd^4}{e^3}+\frac{15x^2d^3}{e^2}-\frac{18x^3d^2}{e}}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}-\frac{d^3(6d-5ex)}{3e^5(d^2-e^2x^2)^{3/2}}}{5e^2}$$

$$\Big\downarrow \ 2345$$

$$\frac{x^5}{5e^2(d+ex)\,(d^2-e^2x^2)^{3/2}}-\frac{-\frac{-\frac{\int \frac{3d^4(5d-6ex)}{e^4\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{4d^3(9d-5ex)}{e^5\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{d^3(6d-5ex)}{3e^5(d^2-e^2x^2)^{3/2}}}{5e^2}$$

$$\Big\downarrow \ 27$$

$$\frac{x^5}{5e^2(d+ex)\,(d^2-e^2x^2)^{3/2}}-\frac{-\frac{-\frac{3d^2\int \frac{5d-6ex}{\sqrt{d^2-e^2x^2}}dx}{e^4}-\frac{4d^3(9d-5ex)}{e^5\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{d^3(6d-5ex)}{3e^5(d^2-e^2x^2)^{3/2}}}{5e^2}$$

$$\Big\downarrow \ 455$$

$$\frac{x^5}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{-\frac{-\frac{3d^2\left(5d\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{6\sqrt{d^2-e^2x^2}}{e}\right)}{e^4}-\frac{4d^3(9d-5ex)}{e^5\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{d^3(6d-5ex)}{3e^5\left(d^2-e^2x^2\right)^{3/2}}}{5e^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{x^5}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{-\frac{-\frac{3d^2\left(5d\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{6\sqrt{d^2-e^2x^2}}{e}\right)}{e^4}-\frac{4d^3(9d-5ex)}{e^5\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{d^3(6d-5ex)}{3e^5\left(d^2-e^2x^2\right)^{3/2}}}{5e^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{x^5}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{-\frac{-\frac{3d^2\left(\frac{5d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{6\sqrt{d^2-e^2x^2}}{e}\right)}{e^4}-\frac{4d^3(9d-5ex)}{e^5\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{d^3(6d-5ex)}{3e^5\left(d^2-e^2x^2\right)^{3/2}}}{5e^2}$$

input
```
Int[x^6/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
x^5/(5*e^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2)) - (-1/3*(d^3*(6*d - 5*e*x))/(e
^5*(d^2 - e^2*x^2)^(3/2)) - ((-4*d^3*(9*d - 5*e*x))/(e^5*Sqrt[d^2 - e^2*x^
2]) - (3*d^2*((6*Sqrt[d^2 - e^2*x^2])/e + (5*d*ArcTan[(e*x)/Sqrt[d^2 - e^2
*x^2]])/e))/e^4)/(3*d^2))/(5*e^2)
```

3.137.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 568
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[x^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*p*(c + d*x))), x] + Simp[1/(2*d^
2*p)   Int[x^(m - 2)*(a + b*x^2)^p*(c*(m - 1) - d*m*x), x], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[m, 1] && LtQ[p, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.137.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(130) = 260.

Time = 0.43 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.90

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{e^7}-\frac{d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^6\sqrt{e^2}}+\frac{23d^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{60e^9\left(x+\frac{d}{e}\right)^2}-\frac{493d\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{240e^8\left(x+\frac{d}{e}\right)}+\frac{d^2\sqrt{-(\cdots}}{}$
default	$\frac{-\frac{x^4}{e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{4d^2\left(\frac{x^2}{e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{2d^2}{3e^4\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}\right)}{e^2}}{e}+\frac{d^2\left(\frac{x^2}{e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{2d^2}{3e^4\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}\right)}{e^3}+\frac{d^4}{3e^7(-e^2x^2+d^2)\cdots}$

input
```
int(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(-e^2*x^2+d^2)^(1/2)/e^7-d/e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2
+d^2)^(1/2))+23/60*d^2/e^9/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-
493/240*d/e^8/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/24*d^2/e^9/(x
-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)+25/48*d/e^8/(x-d/e)*(-(x-d/e)
^2*e^2-2*d*e*(x-d/e))^(1/2)-1/20*d^3/e^10/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*
(x+d/e))^(1/2)
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.74

$$\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx =$$

$$-\frac{48\,de^5x^5+48\,d^2e^4x^4-96\,d^3e^3x^3-96\,d^4e^2x^2+48\,d^5ex+48\,d^6-30\left(de^5x^5+d^2e^4x^4-2\,d^3e^3x^3-2\,d^4e^2x\cdots\right.}{15\left(e^{12}x^5+de^{11}x^4-2\cdots\right.}$$

input
```
integrate(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/15*(48*d*e^5*x^5 + 48*d^2*e^4*x^4 - 96*d^3*e^3*x^3 - 96*d^4*e^2*x^2 + 4
8*d^5*e*x + 48*d^6 - 30*(d*e^5*x^5 + d^2*e^4*x^4 - 2*d^3*e^3*x^3 - 2*d^4*e
^2*x^2 + d^5*e*x + d^6)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (15*e^
5*x^5 + 38*d*e^4*x^4 - 52*d^2*e^3*x^3 - 87*d^3*e^2*x^2 + 33*d^4*e*x + 48*d
^5)*sqrt(-e^2*x^2 + d^2))/(e^12*x^5 + d*e^11*x^4 - 2*d^2*e^10*x^3 - 2*d^3*
e^9*x^2 + d^4*e^8*x + d^5*e^7)
```

3.137.6 Sympy [F]

$$\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^6}{\left(-(-d+ex)(d+ex)\right)^{\frac{5}{2}}(d+ex)}\,dx$$

input
```
integrate(x**6/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Integral(x**6/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.75

$$\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = -\frac{d^5}{5\left(\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^8x+\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^7\right)} - \frac{x^4}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3} - \frac{5\,dx^3}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4} + \frac{20\,d^2x^2}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5} + \frac{64\,d^3x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^6} + \frac{x^2}{3\sqrt{-e^2x^2+d^2}\,e^5} - \frac{14\,d^4}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^7} - \frac{52\,dx}{15\sqrt{-e^2x^2+d^2}\,e^6} - \frac{d\arcsin\left(\frac{ex}{d}\right)}{e^7} + \frac{4\,d^2}{3\sqrt{-e^2x^2+d^2}\,e^7} + \frac{\sqrt{-e^2x^2+d^2}}{3\,e^7}$$

input
```
integrate(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/5*d^5/((-e^2*x^2 + d^2)^(3/2)*e^8*x + (-e^2*x^2 + d^2)^(3/2)*d*e^7) - x
^4/((-e^2*x^2 + d^2)^(3/2)*e^3) - 5*d*x^3/((-e^2*x^2 + d^2)^(3/2)*e^4) + 2
0/3*d^2*x^2/((-e^2*x^2 + d^2)^(3/2)*e^5) + 64/15*d^3*x/((-e^2*x^2 + d^2)^(
3/2)*e^6) + 1/3*x^2/(sqrt(-e^2*x^2 + d^2)*e^5) - 14/3*d^4/((-e^2*x^2 + d^2
)^(3/2)*e^7) - 52/15*d*x/(sqrt(-e^2*x^2 + d^2)*e^6) - d*arcsin(e*x/d)/e^7
+ 4/3*d^2/(sqrt(-e^2*x^2 + d^2)*e^7) + 1/3*sqrt(-e^2*x^2 + d^2)/e^7
```

3.137.8 Giac [F]

$$\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)}\,dx$$

input
```
integrate(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^6/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)), x)
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^6}{\left(d^2-e^2\,x^2\right)^{5/2}\,(d+e\,x)}\,dx$$

input
```
int(x^6/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

output
```
int(x^6/((d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

3.138 $\int \frac{x^5}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

3.138.1 Optimal result

Integrand size = 27, antiderivative size = 122

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

output
```
1/5*x^4*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^2*(-5*e*x+4*d)/e^4/(-e^2*
x^2+d^2)^(3/2)+arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+1/15*(-15*e*x+8*d)/e^6
/(-e^2*x^2+d^2)^(1/2)
```

3.138.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}\left(8d^4-7d^3ex-27d^2e^2x^2+8de^3x^3+23e^4x^4\right)}{(d-ex)^2(d+ex)^3} - 30\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{15e^6}$$

input
```
Integrate[x^5/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(8*d^4 - 7*d^3*e*x - 27*d^2*e^2*x^2 + 8*d*e^3*x^3 +
23*e^4*x^4))/((d - e*x)^2*(d + e*x)^3) - 30*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt
[d^2 - e^2*x^2]])])/(15*e^6)
```

3.138.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {568, 530, 25, 2345, 27, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(d+ex)(d^2-e^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{568}$$

$$\frac{x^4}{5e^2(d+ex)(d^2-e^2x^2)^{3/2}}-\frac{\int \frac{x^3(4d-5ex)}{(d^2-e^2x^2)^{5/2}}dx}{5e^2}$$

$$\Big\downarrow \text{530}$$

$$\frac{x^4}{5e^2(d+ex)(d^2-e^2x^2)^{3/2}}-\frac{\frac{d^2(4d-5ex)}{3e^4(d^2-e^2x^2)^{3/2}}-\frac{\int -\frac{\frac{5d^4}{e^3}-\frac{12xd^3}{e^2}+\frac{15x^2d^2}{e}}{(d^2-e^2x^2)^{3/2}}dx}{3d^2}}{5e^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^4}{5e^2(d+ex)(d^2-e^2x^2)^{3/2}}-\frac{\frac{\int \frac{\frac{5d^4}{e^3}-\frac{12xd^3}{e^2}+\frac{15x^2d^2}{e}}{(d^2-e^2x^2)^{3/2}}dx}{3d^2}+\frac{d^2(4d-5ex)}{3e^4(d^2-e^2x^2)^{3/2}}}{5e^2}$$

$$\Big\downarrow \text{2345}$$

$$\frac{x^4}{5e^2(d+ex)(d^2-e^2x^2)^{3/2}}-\frac{\frac{-\frac{\int \frac{15d^4}{e^3\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{4d^2(3d-5ex)}{e^4\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{d^2(4d-5ex)}{3e^4(d^2-e^2x^2)^{3/2}}}{5e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4}{5e^2(d+ex)(d^2-e^2x^2)^{3/2}}-\frac{\frac{-\frac{15d^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx}{e^3}-\frac{4d^2(3d-5ex)}{e^4\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{d^2(4d-5ex)}{3e^4(d^2-e^2x^2)^{3/2}}}{5e^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{x^4}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\frac{-\frac{15d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}}{e^3}-\frac{4d^2(3d-5ex)}{e^4\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{d^2(4d-5ex)}{3e^4(d^2-e^2x^2)^{3/2}}}{5e^2}$$

$\downarrow$ 216

$$\frac{x^4}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\frac{-\frac{15d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}-\frac{4d^2(3d-5ex)}{e^4\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{d^2(4d-5ex)}{3e^4(d^2-e^2x^2)^{3/2}}}{5e^2}$$

input
```
Int[x^5/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
x^4/(5*e^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2)) - ((d^2*(4*d - 5*e*x))/(3*e^4*
(d^2 - e^2*x^2)^(3/2)) + ((-4*d^2*(3*d - 5*e*x))/(e^4*Sqrt[d^2 - e^2*x^2])
 - (15*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^4)/(3*d^2))/(5*e^2)
```

3.138.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.138. $\int\frac{x^5}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 568
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[x^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*p*(c + d*x))), x] + Simp[1/(2*d^
2*p)   Int[x^(m - 2)*(a + b*x^2)^p*(c*(m - 1) - d*m*x), x], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[m, 1] && LtQ[p, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.138.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(108) = 216$.

Time = 0.39 (sec) , antiderivative size = 450, normalized size of antiderivative = 3.69

method	result
default	$\frac{\frac{x^3}{3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e^2}}{e}+\frac{d^4\left(\frac{x}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{e^5}+\frac{d^2\left(\frac{x}{2e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}\right.}{}$

input
```
int(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/e*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/
e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))+d^4/e^5*(1/3*
x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))+d^2/e^3*(1/2*x/
e^2/(-e^2*x^2+d^2)^(3/2)-1/2*d^2/e^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x
/d^4/(-e^2*x^2+d^2)^(1/2)))-d/e^2*(x^2/e^2/(-e^2*x^2+d^2)^(3/2)-2/3*d^2/e^
4/(-e^2*x^2+d^2)^(3/2))-1/3*d^3/e^6/(-e^2*x^2+d^2)^(3/2)-d^5/e^6*(-1/5/d/e
/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+4/5*e/d*(-1/6*(-2*(x+d/e)*e^
2+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-2*(x+d
/e)*e^2+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
```

3.138.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(109) = 218$.

Time = 0.34 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.98

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{8\,e^5x^5+8\,de^4x^4-16\,d^2e^3x^3-16\,d^3e^2x^2+8\,d^4ex+8\,d^5-30\,(e^5x^5+de^4x^4}{15\,(e}$$

input
```
integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(8*e^5*x^5 + 8*d*e^4*x^4 - 16*d^2*e^3*x^3 - 16*d^3*e^2*x^2 + 8*d^4*e*
x + 8*d^5 - 30*(e^5*x^5 + d*e^4*x^4 - 2*d^2*e^3*x^3 - 2*d^3*e^2*x^2 + d^4*
e*x + d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (23*e^4*x^4 + 8*d*e
^3*x^3 - 27*d^2*e^2*x^2 - 7*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/(e^11*x
^5 + d*e^10*x^4 - 2*d^2*e^9*x^3 - 2*d^3*e^8*x^2 + d^4*e^7*x + d^5*e^6)
```

3.138.6 Sympy [F]

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^5}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate(x**5/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Integral(x**5/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.138.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(109) = 218$.

Time = 0.32 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.92

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{d^4}{5\left((-e^2x^2+d^2)^{\frac{3}{2}}e^7x+(-e^2x^2+d^2)^{\frac{3}{2}}de^6\right)} + \frac{x^3}{(-e^2x^2+d^2)^{\frac{3}{2}}e^3} - \frac{8\,dx^2}{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^4} - \frac{4\,d^2x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}e^5} - \frac{x^2}{3\sqrt{-e^2x^2+d^2}de^4} + \frac{2\,d^3}{(-e^2x^2+d^2)^{\frac{3}{2}}e^6} - \frac{8\,x}{15\sqrt{-e^2x^2+d^2}e^5} + \frac{\arcsin\left(\frac{ex}{d}\right)}{e^6} - \frac{4\,d}{3\sqrt{-e^2x^2+d^2}e^6} - \frac{\sqrt{-e^2x^2+d^2}}{3\,de^6}$$

input
```
integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
1/5*d^4/((-e^2*x^2 + d^2)^(3/2)*e^7*x + (-e^2*x^2 + d^2)^(3/2)*d*e^6) + x^
3/((-e^2*x^2 + d^2)^(3/2)*e^3) - 8/3*d*x^2/((-e^2*x^2 + d^2)^(3/2)*e^4) -
4/15*d^2*x/((-e^2*x^2 + d^2)^(3/2)*e^5) - 1/3*x^2/(sqrt(-e^2*x^2 + d^2)*d*
e^4) + 2*d^3/((-e^2*x^2 + d^2)^(3/2)*e^6) - 8/15*x/(sqrt(-e^2*x^2 + d^2)*e
^5) + arcsin(e*x/d)/e^6 - 4/3*d/(sqrt(-e^2*x^2 + d^2)*e^6) - 1/3*sqrt(-e^2
*x^2 + d^2)/(d*e^6)
```

3.138.8 Giac [F]

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^5}{(-e^2x^2+d^2)^{\frac{5}{2}}(ex+d)}\,dx$$

input
```
integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^5/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)), x)
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex)\,(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{x^5}{(d^2-e^2\,x^2)^{5/2}\,(d+e\,x)}\,dx$$

input
```
int(x^5/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

output
```
int(x^5/((d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

3.139 $\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

3.139.1 Optimal result

Integrand size = 27, antiderivative size = 85

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = -\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{4}{5e^5\sqrt{d^2-e^2x^2}}$$

output
```
-1/5*x^4*(-e*x+d)/d/e/(-e^2*x^2+d^2)^(5/2)+4/15*d^2/e^5/(-e^2*x^2+d^2)^(3/
2)-4/5/e^5/(-e^2*x^2+d^2)^(1/2)
```

3.139.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-8d^4-8d^3ex+12d^2e^2x^2+12de^3x^3-3e^4x^4)}{15de^5(d-ex)^2(d+ex)^3}$$

input
```
Integrate[x^4/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-8*d^4 - 8*d^3*e*x + 12*d^2*e^2*x^2 + 12*d*e^3*x^3 -
 3*e^4*x^4))/(15*d*e^5*(d - e*x)^2*(d + e*x)^3)
```

3.139.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {567, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(d+ex)(d^2-e^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{567}$$

$$\frac{4\int \frac{x^3}{(d^2-e^2x^2)^{5/2}}dx}{5e}-\frac{x^4}{5de(d+ex)(d^2-e^2x^2)^{3/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{2\int \frac{x^2}{(d^2-e^2x^2)^{5/2}}dx^2}{5e}-\frac{x^4}{5de(d+ex)(d^2-e^2x^2)^{3/2}}$$

$$\Big\downarrow \text{53}$$

$$\frac{2\int \left(\frac{d^2}{e^2(d^2-e^2x^2)^{5/2}}-\frac{1}{e^2(d^2-e^2x^2)^{3/2}}\right)dx^2}{5e}-\frac{x^4}{5de(d+ex)(d^2-e^2x^2)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{2d^2}{3e^4(d^2-e^2x^2)^{3/2}}-\frac{2}{e^4\sqrt{d^2-e^2x^2}}\right)}{5e}-\frac{x^4}{5de(d+ex)(d^2-e^2x^2)^{3/2}}$$

input
```
Int[x^4/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
-1/5*x^4/(d*e*(d + e*x)*(d^2 - e^2*x^2)^(3/2)) + (2*((2*d^2)/(3*e^4*(d^2 -
 e^2*x^2)^(3/2)) - 2/(e^4*Sqrt[d^2 - e^2*x^2])))/(5*e)
```

3.139.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 567
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[c*x^m*((a + b*x^2)^(p + 1)/(2*a*d*p*(c + d*x))), x] - Simp[m/(2*d*p)
   Int[x^(m - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
^2 + a*d^2, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[m + 2*p + 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.139.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

method	result
gosper	$-\frac{(-ex+d)\left(3e^4x^4-12d\,e^3x^3-12d^2e^2x^2+8d^3ex+8d^4\right)}{15d\,e^5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}$
trager	$-\frac{\left(3e^4x^4-12d\,e^3x^3-12d^2e^2x^2+8d^3ex+8d^4\right)\sqrt{-e^2x^2+d^2}}{15e^5d(ex+d)^3(-ex+d)^2}$
default	$\frac{\frac{x^2}{e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{2d^2}{3e^4\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}}{e}+\frac{d^2}{3e^5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{d^3\left(\frac{x}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{e^4}-\frac{d\left(\frac{x}{2e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}\right.}{}$

input
```
int(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-e*x+d)*(3*e^4*x^4-12*d*e^3*x^3-12*d^2*e^2*x^2+8*d^3*e*x+8*d^4)/d/e
^5/(-e^2*x^2+d^2)^(5/2)
```

3.139.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(74) = 148$.

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.98

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx =$$
$$-\frac{8\,e^5x^5+8\,de^4x^4-16\,d^2e^3x^3-16\,d^3e^2x^2+8\,d^4ex+8\,d^5+(3\,e^4x^4-12\,de^3x^3-12\,d^2e^2x^2+8\,d^3ex+8\,d^4}{15\left(de^{10}x^5+d^2e^9x^4-2\,d^3e^8x^3-2\,d^4e^7x^2+d^5e^6x+d^6e^5\right)}$$

input
```
integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/15*(8*e^5*x^5 + 8*d*e^4*x^4 - 16*d^2*e^3*x^3 - 16*d^3*e^2*x^2 + 8*d^4*e
*x + 8*d^5 + (3*e^4*x^4 - 12*d*e^3*x^3 - 12*d^2*e^2*x^2 + 8*d^3*e*x + 8*d^
4)*sqrt(-e^2*x^2 + d^2))/(d*e^10*x^5 + d^2*e^9*x^4 - 2*d^3*e^8*x^3 - 2*d^4
*e^7*x^2 + d^5*e^6*x + d^6*e^5)
```

3.139.6 Sympy [F]

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^4}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate(x**4/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Integral(x**4/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.139.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.58

$$\int \frac{x^4}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx = -\frac{d^3}{5\left((-e^2x^2+d^2)^{\frac{3}{2}}e^6x+(-e^2x^2+d^2)^{\frac{3}{2}}de^5\right)} + \frac{x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^3} - \frac{2\,dx}{5\,(-e^2x^2+d^2)^{\frac{3}{2}}e^4} - \frac{d^2}{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^5} + \frac{x}{5\,\sqrt{-e^2x^2+d^2}de^4}$$

input
```
integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/5*d^3/((-e^2*x^2 + d^2)^(3/2)*e^6*x + (-e^2*x^2 + d^2)^(3/2)*d*e^5) + x
^2/((-e^2*x^2 + d^2)^(3/2)*e^3) - 2/5*d*x/((-e^2*x^2 + d^2)^(3/2)*e^4) - 1
/3*d^2/((-e^2*x^2 + d^2)^(3/2)*e^5) + 1/5*x/(sqrt(-e^2*x^2 + d^2)*d*e^4)
```

3.139.8 Giac [F]

$$\int \frac{x^4}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}(ex+d)}\,dx$$

input
```
integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^4/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)), x)
```

3.139.9 Mupad [B] (verification not implemented)

Time = 11.74 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}\,(8\,d^4+8\,d^3\,e\,x-12\,d^2\,e^2\,x^2-12\,d\,e^3\,x^3+3\,e^4\,x^4)}{15\,d\,e^5\,(d+e\,x)^3\,(d-e\,x)^2}$$

input
```
int(x^4/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

output
```
-((d^2 - e^2*x^2)^(1/2)*(8*d^4 + 3*e^4*x^4 - 12*d*e^3*x^3 - 12*d^2*e^2*x^2
 + 8*d^3*e*x))/(15*d*e^5*(d + e*x)^3*(d - e*x)^2)
```

3.140 $\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

3.140.1 Optimal result

Integrand size = 27, antiderivative size = 91

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2d-3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

output
```
1/5*x^2*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)+1/15*(3*e*x-2*d)/e^4/(-e^2*x^2+d
^2)^(3/2)-1/5*x/d^2/e^3/(-e^2*x^2+d^2)^(1/2)
```

3.140.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}(-2d^4-2d^3ex+3d^2e^2x^2+3de^3x^3+3e^4x^4)}{15d^2e^4(d-ex)^2(d+ex)^3}$$

input
```
Integrate[x^3/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-2*d^4 - 2*d^3*e*x + 3*d^2*e^2*x^2 + 3*d*e^3*x^3 + 3
*e^4*x^4))/(15*d^2*e^4*(d - e*x)^2*(d + e*x)^3)
```

3.140.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {568, 530, 27, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{568}$$

$$\frac{x^2}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int \frac{x(2d-3ex)}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5e^2}$$

$$\Big\downarrow \text{530}$$

$$\frac{x^2}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\frac{2d-3ex}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int -\frac{3d^2}{e\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}}{5e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{e}+\frac{2d-3ex}{3e^2\left(d^2-e^2x^2\right)^{3/2}}}{5e^2}$$

$$\Big\downarrow \text{208}$$

$$\frac{x^2}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\frac{x}{d^2e\sqrt{d^2-e^2x^2}}+\frac{2d-3ex}{3e^2\left(d^2-e^2x^2\right)^{3/2}}}{5e^2}$$

input
```
Int[x^3/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
x^2/(5*e^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2)) - ((2*d - 3*e*x)/(3*e^2*(d^2 -
 e^2*x^2)^(3/2)) + x/(d^2*e*Sqrt[d^2 - e^2*x^2]))/(5*e^2)
```

3.140.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 568
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[x^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*p*(c + d*x))), x] + Simp[1/(2*d^
2*p)   Int[x^(m - 2)*(a + b*x^2)^p*(c*(m - 1) - d*m*x), x], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[m, 1] && LtQ[p, -1]
```

3.140.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

method	result
gosper	$-\frac{(-ex+d)\left(-3e^4x^4-3d\,e^3x^3-3d^2e^2x^2+2d^3ex+2d^4\right)}{15d^2e^4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}$
trager	$-\frac{\left(-3e^4x^4-3d\,e^3x^3-3d^2e^2x^2+2d^3ex+2d^4\right)\sqrt{-e^2x^2+d^2}}{15d^2e^4(ex+d)^3(-ex+d)^2}$
default	$\frac{\frac{x}{2e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{d^2\left(\frac{x}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{2e^2}}{e}+\frac{d^2\left(\frac{x}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{e^3}-\frac{d}{3e^4\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-$

input
```
int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-e*x+d)*(-3*e^4*x^4-3*d*e^3*x^3-3*d^2*e^2*x^2+2*d^3*e*x+2*d^4)/d^2/
e^4/(-e^2*x^2+d^2)^(5/2)
```

3.140.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(80) = 160$.

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.88

$$\int \frac{x^3}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx =$$
$$-\frac{2\,e^5x^5+2\,de^4x^4-4\,d^2e^3x^3-4\,d^3e^2x^2+2\,d^4ex+2\,d^5-(3\,e^4x^4+3\,de^3x^3+3\,d^2e^2x^2-2\,d^3ex-2\,d^4)\sqrt{-}}{15\,(d^2e^9x^5+d^3e^8x^4-2\,d^4e^7x^3-2\,d^5e^6x^2+d^6e^5x+d^7e^4)}$$

input
```
integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/15*(2*e^5*x^5 + 2*d*e^4*x^4 - 4*d^2*e^3*x^3 - 4*d^3*e^2*x^2 + 2*d^4*e*x
 + 2*d^5 - (3*e^4*x^4 + 3*d*e^3*x^3 + 3*d^2*e^2*x^2 - 2*d^3*e*x - 2*d^4)*s
qrt(-e^2*x^2 + d^2))/(d^2*e^9*x^5 + d^3*e^8*x^4 - 2*d^4*e^7*x^3 - 2*d^5*e^
6*x^2 + d^6*e^5*x + d^7*e^4)
```

3.140.6 Sympy [F]

$$\int \frac{x^3}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{x^3}{(-(-d+ex)(d+ex))^{\frac{5}{2}}(d+ex)}\,dx$$

input
```
integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Integral(x**3/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.21

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{d^2}{5\left((-e^2x^2+d^2)^{\frac{3}{2}}e^5x+(-e^2x^2+d^2)^{\frac{3}{2}}de^4\right)} + \frac{2\,x}{5\,(-e^2x^2+d^2)^{\frac{3}{2}}e^3} - \frac{d}{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^4} - \frac{x}{5\,\sqrt{-e^2x^2+d^2}d^2e^3}$$

input
```
integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
1/5*d^2/((-e^2*x^2 + d^2)^(3/2)*e^5*x + (-e^2*x^2 + d^2)^(3/2)*d*e^4) + 2/
5*x/((-e^2*x^2 + d^2)^(3/2)*e^3) - 1/3*d/((-e^2*x^2 + d^2)^(3/2)*e^4) - 1/
5*x/(sqrt(-e^2*x^2 + d^2)*d^2*e^3)
```

3.140.8 Giac [F]

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^3}{(-e^2x^2+d^2)^{\frac{5}{2}}(ex+d)}\,dx$$

input
```
integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^3/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)), x)
```

3.140.9 Mupad [B] (verification not implemented)

Time = 12.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^4-2\,d^3\,e\,x+3\,d^2\,e^2\,x^2+3\,d\,e^3\,x^3+3\,e^4\,x^4\right)}{15\,d^2\,e^4\,{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^2}$$

input
```
int(x^3/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(3*e^4*x^4 - 2*d^4 + 3*d*e^3*x^3 + 3*d^2*e^2*x^2 -
2*d^3*e*x))/(15*d^2*e^4*(d + e*x)^3*(d - e*x)^2)
```

3.141 $\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

3.141.1 Optimal result

Integrand size = 27, antiderivative size = 95

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = -\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(d+ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

output
```
-1/5*x^2/d/e/(e*x+d)/(-e^2*x^2+d^2)^(3/2)+2/15*(e*x+d)/d/e^3/(-e^2*x^2+d^2
)^(3/2)-2/15*x/d^3/e^2/(-e^2*x^2+d^2)^(1/2)
```

3.141.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(2d^4+2d^3ex-3d^2e^2x^2+2de^3x^3+2e^4x^4)}{15d^3e^3(d-ex)^2(d+ex)^3}$$

input
```
Integrate[x^2/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(2*d^4 + 2*d^3*e*x - 3*d^2*e^2*x^2 + 2*d*e^3*x^3 + 2*
e^4*x^4))/(15*d^3*e^3*(d - e*x)^2*(d + e*x)^3)
```

3.141.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {568, 454, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{568}$$

$$\frac{x}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int \frac{d-2ex}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5e^2}$$

$$\Big\downarrow \text{454}$$

$$\frac{x}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d}-\frac{2d-ex}{3de\left(d^2-e^2x^2\right)^{3/2}}}{5e^2}$$

$$\Big\downarrow \text{208}$$

$$\frac{x}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}-\frac{2d-ex}{3de\left(d^2-e^2x^2\right)^{3/2}}}{5e^2}$$

input
```
Int[x^2/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
x/(5*e^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2)) - (-1/3*(2*d - e*x)/(d*e*(d^2 -
e^2*x^2)^(3/2)) + (2*x)/(3*d^3*Sqrt[d^2 - e^2*x^2]))/(5*e^2)
```

3.141.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 568
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[x^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*p*(c + d*x))), x] + Simp[1/(2*d^
2*p)   Int[x^(m - 2)*(a + b*x^2)^p*(c*(m - 1) - d*m*x), x], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[m, 1] && LtQ[p, -1]
```

3.141.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

method	result
gosper	$\frac{(-ex+d)\left(2e^4x^4+2d\,e^3x^3-3d^2e^2x^2+2d^3ex+2d^4\right)}{15d^3e^3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}$
trager	$\frac{\left(2e^4x^4+2d\,e^3x^3-3d^2e^2x^2+2d^3ex+2d^4\right)\sqrt{-e^2x^2+d^2}}{15d^3e^3(ex+d)^3(-ex+d)^2}$
default	$\frac{1}{3e^3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{d\left(\frac{x}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{e^2}+\frac{d^2\left(-\frac{1}{5de\left(x+\frac{d}{e}\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{4e\left(-\frac{-2(\cdots}{6d^2e^2\left(-\left(x+\cdots\right.\right.}\cdots\right.}{\cdots}\cdots\right.}{e^3}$

input
```
int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(-e*x+d)*(2*e^4*x^4+2*d*e^3*x^3-3*d^2*e^2*x^2+2*d^3*e*x+2*d^4)/d^3/e^
3/(-e^2*x^2+d^2)^(5/2)
```

3.141.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(83) = 166.

Time = 0.29 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.79

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{2\,e^5x^5+2\,de^4x^4-4\,d^2e^3x^3-4\,d^3e^2x^2+2\,d^4ex+2\,d^5+\left(2\,e^4x^4+2\,de^3x^3-\right.}{15\left(d^3e^8x^5+d^4e^7x^4-2\,d^5e^6x^3-2\,d^6e^5x^2+d^7e\right.}$$

input
```
integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(2*e^5*x^5 + 2*d*e^4*x^4 - 4*d^2*e^3*x^3 - 4*d^3*e^2*x^2 + 2*d^4*e*x
+ 2*d^5 + (2*e^4*x^4 + 2*d*e^3*x^3 - 3*d^2*e^2*x^2 + 2*d^3*e*x + 2*d^4)*sq
rt(-e^2*x^2 + d^2))/(d^3*e^8*x^5 + d^4*e^7*x^4 - 2*d^5*e^6*x^3 - 2*d^6*e^5
*x^2 + d^7*e^4*x + d^8*e^3)
```

3.141.6 Sympy [F]

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Integral(x**2/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.141.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.16

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = -\frac{d}{5\left(\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4x+\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^3\right)} - \frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^2} + \frac{1}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3} - \frac{2\,x}{15\sqrt{-e^2x^2+d^2}d^3e^2}$$

input
```
integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/5*d/((-e^2*x^2 + d^2)^(3/2)*e^4*x + (-e^2*x^2 + d^2)^(3/2)*d*e^3) - 1/1
5*x/((-e^2*x^2 + d^2)^(3/2)*d*e^2) + 1/3/((-e^2*x^2 + d^2)^(3/2)*e^3) - 2/
15*x/(sqrt(-e^2*x^2 + d^2)*d^3*e^2)
```

3.141.8 Giac [F]

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\int \frac{x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)}\,dx$$

input
```
integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^2/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)), x)
```

3.141.9 Mupad [B] (verification not implemented)

Time = 11.85 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^4+2\,d^3\,e\,x-3\,d^2\,e^2\,x^2+2\,d\,e^3\,x^3+2\,e^4\,x^4\right)}{15\,d^3\,e^3\,{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^2}$$

input
```
int(x^2/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(2*d^4 + 2*e^4*x^4 + 2*d*e^3*x^3 - 3*d^2*e^2*x^2 +
2*d^3*e*x))/(15*d^3*e^3*(d + e*x)^3*(d - e*x)^2)
```

3.142 $\int \frac{x}{(d+ex)(d^2-e^2x^2)^{5/2}} dx$

3.142.1 Optimal result

Integrand size = 25, antiderivative size = 85

$$\int \frac{x}{(d+ex)(d^2-e^2x^2)^{5/2}} dx = \frac{x}{15d^2e(d^2-e^2x^2)^{3/2}} + \frac{1}{5e^2(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

output
```
1/15*x/d^2/e/(-e^2*x^2+d^2)^(3/2)+1/5/e^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2)+2/1
5*x/d^4/e/(-e^2*x^2+d^2)^(1/2)
```

3.142.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int \frac{x}{(d+ex)(d^2-e^2x^2)^{5/2}} dx = \frac{\sqrt{d^2-e^2x^2}(3d^4+3d^3ex+3d^2e^2x^2-2de^3x^3-2e^4x^4)}{15d^4e^2(d-ex)^2(d+ex)^3}$$

input
```
Integrate[x/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(3*d^4 + 3*d^3*e*x + 3*d^2*e^2*x^2 - 2*d*e^3*x^3 - 2*
e^4*x^4))/(15*d^4*e^2*(d - e*x)^2*(d + e*x)^3)
```

3.142.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {565, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex)(d^2-e^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{565}$$

$$\frac{d}{5e^2(d^2-e^2x^2)^{5/2}}-e\int \frac{x^2}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{245}$$

$$\frac{d}{5e^2(d^2-e^2x^2)^{5/2}}-e\left(\frac{x^3}{3d^2(d^2-e^2x^2)^{5/2}}-\frac{2e^2\int \frac{x^4}{(d^2-e^2x^2)^{7/2}}dx}{3d^2}\right)$$

$$\Big\downarrow \text{242}$$

$$\frac{d}{5e^2(d^2-e^2x^2)^{5/2}}-e\left(\frac{x^3}{3d^2(d^2-e^2x^2)^{5/2}}-\frac{2e^2x^5}{15d^4(d^2-e^2x^2)^{5/2}}\right)$$

input
```
Int[x/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
d/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - e*(x^3/(3*d^2*(d^2 - e^2*x^2)^(5/2)) - (
2*e^2*x^5)/(15*d^4*(d^2 - e^2*x^2)^(5/2)))
```

3.142.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 565
```
Int[((x_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :> Sim
p[a*((a + b*x^2)^p/(2*b*c*p)), x] + Simp[b/d   Int[x^2*(a + b*x^2)^(p - 1),
 x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0]
```

3.142.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

method	result
gosper	$\frac{(-ex+d)(-2e^4x^4-2d\,e^3x^3+3d^2e^2x^2+3d^3ex+3d^4)}{15d^4e^2(-e^2x^2+d^2)^{\frac{5}{2}}}$
trager	$\frac{(-2e^4x^4-2d\,e^3x^3+3d^2e^2x^2+3d^3ex+3d^4)\sqrt{-e^2x^2+d^2}}{15d^4(ex+d)^3(-ex+d)^2e^2}$
default	$\frac{\frac{x}{3d^2(-e^2x^2+d^2)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}}{e}-\frac{d\left(-\frac{1}{5de\left(x+\frac{d}{e}\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{4e\left(-\frac{-2\left(x+\frac{d}{e}\right)e^2+2de}{6d^2e^2\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}-\frac{\dots}{3e^2d^4\dots}\right)}{5d}\dots\right)}{e^2}$

input
```
int(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(-e*x+d)*(-2*e^4*x^4-2*d*e^3*x^3+3*d^2*e^2*x^2+3*d^3*e*x+3*d^4)/d^4/e
^2/(-e^2*x^2+d^2)^(5/2)
```

3.142.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(73) = 146$.

Time = 0.31 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.01

$$\int \frac{x}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx = \frac{3\,e^5x^5+3\,de^4x^4-6\,d^2e^3x^3-6\,d^3e^2x^2+3\,d^4ex+3\,d^5-(2\,e^4x^4+2\,de^3x^3-}{15\,(d^4e^7x^5+d^5e^6x^4-2\,d^6e^5x^3-2\,d^7e^4x^2+d^8e}$$

input
```
integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(3*e^5*x^5 + 3*d*e^4*x^4 - 6*d^2*e^3*x^3 - 6*d^3*e^2*x^2 + 3*d^4*e*x
+ 3*d^5 - (2*e^4*x^4 + 2*d*e^3*x^3 - 3*d^2*e^2*x^2 - 3*d^3*e*x - 3*d^4)*sq
rt(-e^2*x^2 + d^2))/(d^4*e^7*x^5 + d^5*e^6*x^4 - 2*d^6*e^5*x^3 - 2*d^7*e^4
*x^2 + d^8*e^3*x + d^9*e^2)
```

3.142.6 Sympy [F]

$$\int \frac{x}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{x}{\left(-(-d+ex)(d+ex)\right)^{\frac{5}{2}}(d+ex)}\,dx$$

input
```
integrate(x/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Integral(x/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.142.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06

$$\int \frac{x}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx = \frac{1}{5\left((-e^2x^2+d^2)^{\frac{3}{2}}e^3x+(-e^2x^2+d^2)^{\frac{3}{2}}de^2\right)} + \frac{x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2e} + \frac{2\,x}{15\,\sqrt{-e^2x^2+d^2}\,d^4e}$$

input
```
integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
1/5/((-e^2*x^2 + d^2)^(3/2)*e^3*x + (-e^2*x^2 + d^2)^(3/2)*d*e^2) + 1/15*x
/((-e^2*x^2 + d^2)^(3/2)*d^2*e) + 2/15*x/(sqrt(-e^2*x^2 + d^2)*d^4*e)
```

3.142.8 Giac [F]

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)}\,dx$$

input
```
integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(x/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)), x)
```

3.142.9 Mupad [B] (verification not implemented)

Time = 11.79 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(3\,d^4+3\,d^3\,e\,x+3\,d^2\,e^2\,x^2-2\,d\,e^3\,x^3-2\,e^4\,x^4\right)}{15\,d^4\,e^2\,\left(d+e\,x\right)^3\,\left(d-e\,x\right)^2}$$

input
```
int(x/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(3*d^4 - 2*e^4*x^4 - 2*d*e^3*x^3 + 3*d^2*e^2*x^2 +
3*d^3*e*x))/(15*d^4*e^2*(d + e*x)^3*(d - e*x)^2)
```

3.143 $\int \frac{1}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

3.143.1 Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8x}{15d^5\sqrt{d^2-e^2x^2}}$$

output
```
4/15*x/d^3/(-e^2*x^2+d^2)^(3/2)-1/5/d/e/(e*x+d)/(-e^2*x^2+d^2)^(3/2)+8/15*
x/d^5/(-e^2*x^2+d^2)^(1/2)
```

3.143.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-3d^4+12d^3ex+12d^2e^2x^2-8de^3x^3-8e^4x^4)}{15d^5e(d-ex)^2(d+ex)^3}$$

input
```
Integrate[1/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-3*d^4 + 12*d^3*e*x + 12*d^2*e^2*x^2 - 8*d*e^3*x^3 -
 8*e^4*x^4))/(15*d^5*e*(d - e*x)^2*(d + e*x)^3)
```

3.143.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {470, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}dx$$
$$\Big\downarrow \ 470$$
$$\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$
$$\Big\downarrow \ 209$$
$$\frac{4\left(\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)}{5d}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$
$$\Big\downarrow \ 208$$
$$\frac{4\left(\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

input
```
Int[1/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
-1/5*1/(d*e*(d + e*x)*(d^2 - e^2*x^2)^(3/2)) + (4*(x/(3*d^2*(d^2 - e^2*x^2
)^(3/2)) + (2*x)/(3*d^4*Sqrt[d^2 - e^2*x^2])))/(5*d)
```

3.143.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

3.143.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

method	result	size
gosper	$-\frac{(-ex+d)\left(8e^4x^4+8d\,e^3x^3-12d^2e^2x^2-12d^3ex+3d^4\right)}{15d^5e\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}$	70
trager	$-\frac{\left(8e^4x^4+8d\,e^3x^3-12d^2e^2x^2-12d^3ex+3d^4\right)\sqrt{-e^2x^2+d^2}}{15d^5(ex+d)^3(-ex+d)^2e}$	79
default	$\frac{-\frac{1}{5de\left(x+\frac{d}{e}\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{4e\left(-\frac{-2\left(x+\frac{d}{e}\right)e^2+2de}{6d^2e^2\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}-\frac{-2\left(x+\frac{d}{e}\right)e^2+2de}{3e^2d^4\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{5d}}{e}$	164

input
```
int(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-e*x+d)*(8*e^4*x^4+8*d*e^3*x^3-12*d^2*e^2*x^2-12*d^3*e*x+3*d^4)/d^5
/e/(-e^2*x^2+d^2)^(5/2)
```

3.143.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(70) = 140.

Time = 0.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.05

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx =$$
$$-\frac{3\,e^5x^5+3\,de^4x^4-6\,d^2e^3x^3-6\,d^3e^2x^2+3\,d^4ex+3\,d^5+\left(8\,e^4x^4+8\,de^3x^3-12\,d^2e^2x^2-12\,d^3ex+3\,d^4\right)\sqrt{}}{15\left(d^5e^6x^5+d^6e^5x^4-2\,d^7e^4x^3-2\,d^8e^3x^2+d^9e^2x+d^{10}e\right)}$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/15*(3*e^5*x^5 + 3*d*e^4*x^4 - 6*d^2*e^3*x^3 - 6*d^3*e^2*x^2 + 3*d^4*e*x
 + 3*d^5 + (8*e^4*x^4 + 8*d*e^3*x^3 - 12*d^2*e^2*x^2 - 12*d^3*e*x + 3*d^4)
*sqrt(-e^2*x^2 + d^2))/(d^5*e^6*x^5 + d^6*e^5*x^4 - 2*d^7*e^4*x^3 - 2*d^8*
e^3*x^2 + d^9*e^2*x + d^10*e)
```

3.143.6 Sympy [F]

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\int \frac{1}{\left(-(-d+ex)(d+ex)\right)^{\frac{5}{2}}(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Integral(1/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.143.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=-\frac{1}{5\left((-e^2x^2+d^2)^{\frac{3}{2}}de^2x+(-e^2x^2+d^2)^{\frac{3}{2}}d^2e\right)}+\frac{4\,x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d^3}+\frac{8\,x}{15\,\sqrt{-e^2x^2+d^2}d^5}$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/5/((-e^2*x^2 + d^2)^(3/2)*d*e^2*x + (-e^2*x^2 + d^2)^(3/2)*d^2*e) + 4/1
5*x/((-e^2*x^2 + d^2)^(3/2)*d^3) + 8/15*x/(sqrt(-e^2*x^2 + d^2)*d^5)
```

3.143.8 Giac [F]

$$\int \frac{1}{(d+ex)\,(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{1}{(-e^2x^2+d^2)^{\frac{5}{2}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)), x)
```

3.143.9 Mupad [B] (verification not implemented)

Time = 11.71 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{1}{(d+ex)\,(d^2-e^2x^2)^{5/2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}\,(3\,d^4-12\,d^3\,e\,x-12\,d^2\,e^2\,x^2+8\,d\,e^3\,x^3+8\,e^4\,x^4)}{15\,d^5\,e\,(d+e\,x)^3\,(d-e\,x)^2}$$

input
```
int(1/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

output
```
-((d^2 - e^2*x^2)^(1/2)*(3*d^4 + 8*e^4*x^4 + 8*d*e^3*x^3 - 12*d^2*e^2*x^2
- 12*d^3*e*x))/(15*d^5*e*(d + e*x)^3*(d - e*x)^2)
```

3.144 $\int \frac{1}{x(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

3.144.1 Optimal result

Integrand size = 27, antiderivative size = 119

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

output
```
1/15*(-4*e*x+5*d)/d^4/(-e^2*x^2+d^2)^(3/2)+1/5/d^2/(e*x+d)/(-e^2*x^2+d^2)^
(3/2)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/15*(-8*e*x+15*d)/d^6/(-e^2*x^2
+d^2)^(1/2)
```

3.144.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\frac{\frac{\sqrt{d^2-e^2x^2}(23d^4+8d^3ex-27d^2e^2x^2-7de^3x^3+8e^4x^4)}{(d-ex)^2(d+ex)^3}+30\operatorname{arctanh}\left(\frac{\sqrt{-e^2}x-\sqrt{d^2-e^2x^2}}{d}\right)}{15d^6}$$

input
```
Integrate[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(23*d^4 + 8*d^3*e*x - 27*d^2*e^2*x^2 - 7*d*e^3*x^3 +
 8*e^4*x^4))/((d - e*x)^2*(d + e*x)^3) + 30*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d
^2 - e^2*x^2])/d])/(15*d^6)
```

3.144.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {569, 25, 532, 25, 532, 27, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{569}$$

$$\frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int -\frac{5d-4ex}{x(d^2-e^2x^2)^{5/2}}dx}{5d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5d-4ex}{x(d^2-e^2x^2)^{5/2}}dx}{5d^2}+\frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{\frac{5d-4ex}{3d^2(d^2-e^2x^2)^{3/2}}-\frac{\int -\frac{15d-8ex}{x\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}}{5d^2}+\frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{15d-8ex}{x\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{5d-4ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{\frac{\frac{15d-8ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{15d}{x\sqrt{d^2-e^2x^2}}dx}{d^2}}{3d^2}+\frac{5d-4ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{15\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx}{d}+\frac{15d-8ex}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{5d-4ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{\frac{15\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2}{2d}+\frac{15d-8ex}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{5d-4ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{15d-8ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{15\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{de^2}}{3d^2}+\frac{5d-4ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{15d-8ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{15\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}}{3d^2}+\frac{5d-4ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

input
```
Int[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
1/(5*d^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2)) + ((5*d - 4*e*x)/(3*d^2*(d^2 - e
^2*x^2)^(3/2)) + ((15*d - 8*e*x)/(d^2*Sqrt[d^2 - e^2*x^2]) - (15*ArcTanh[S
qrt[d^2 - e^2*x^2]/d])/d^2)/(3*d^2))/(5*d^2)
```

3.144.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 569
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[(-x^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*p*(c + d*x))), x] + Simp[1/(2
*c^2*p)   Int[x^m*(a + b*x^2)^p*(c*(m + 2*p + 1) - d*(m + 2*p + 2)*x), x],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[m + 2*p, 0]
```

3.144.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(105) = 210$.

Time = 0.38 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.14

method	result
default	$\frac{\frac{1}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{\frac{1}{d^2\sqrt{-e^2x^2+d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}}}{d^2}}{d}-\frac{-\frac{1}{5de\left(x+\frac{d}{e}\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{4e\left(-\frac{-2\left(x+\right.}{6d^2e^2\left(-\left(x+\frac{d}{e}\right)\right.}\right.}{}}{d}$

input
```
int(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/
(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))-1/d*(-1/5/d
/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+4/5*e/d*(-1/6*(-2*(x+d/e)*
e^2+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-2*(x
+d/e)*e^2+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
```

3.144.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(105) = 210$.

Time = 0.30 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.99

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{23\,e^5x^5+23\,de^4x^4-46\,d^2e^3x^3-46\,d^3e^2x^2+23\,d^4ex+23\,d^5+15\,(e^5x^5+}{15}$$

input
```
integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(23*e^5*x^5 + 23*d*e^4*x^4 - 46*d^2*e^3*x^3 - 46*d^3*e^2*x^2 + 23*d^4
*e*x + 23*d^5 + 15*(e^5*x^5 + d*e^4*x^4 - 2*d^2*e^3*x^3 - 2*d^3*e^2*x^2 +
d^4*e*x + d^5)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (8*e^4*x^4 - 7*d*e^3*x
^3 - 27*d^2*e^2*x^2 + 8*d^3*e*x + 23*d^4)*sqrt(-e^2*x^2 + d^2))/(d^6*e^5*x
^5 + d^7*e^4*x^4 - 2*d^8*e^3*x^3 - 2*d^9*e^2*x^2 + d^10*e*x + d^11)
```

3.144.6 Sympy [F]

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate(1/x/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Integral(1/(x*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.144.7 Maxima [F]

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x}\,dx$$

input
```
integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x), x)
```

3.144.8 Giac [F]

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x}\,dx$$

input
```
integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x), x)
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\int \frac{1}{x\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)}\,dx$$

input
```
int(1/(x*(d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

output
```
int(1/(x*(d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

3.145 $\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

3.145.1 Optimal result

Integrand size = 27, antiderivative size = 154

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}+\frac{e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}$$

output
```
1/15*(-5*e*x+6*d)/d^4/x/(-e^2*x^2+d^2)^(3/2)+1/5/d^2/x/(e*x+d)/(-e^2*x^2+d
^2)^(3/2)+e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^7+1/5*(-5*e*x+8*d)/d^6/x/(-e
^2*x^2+d^2)^(1/2)-16/5*(-e^2*x^2+d^2)^(1/2)/d^7/x
```

3.145.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=-\frac{\frac{d\sqrt{d^2-e^2x^2}\left(15d^5+38d^4ex-52d^3e^2x^2-87d^2e^3x^3+33de^4x^4+48e^5x^5\right)}{x(d-ex)^2(d+ex)^3}-15\sqrt{d^2}e\log(x)+15\sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{15d^8}$$

input
```
Integrate[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
-1/15*((d*Sqrt[d^2 - e^2*x^2]*(15*d^5 + 38*d^4*e*x - 52*d^3*e^2*x^2 - 87*d
^2*e^3*x^3 + 33*d*e^4*x^4 + 48*e^5*x^5))/(x*(d - e*x)^2*(d + e*x)^3) - 15*
Sqrt[d^2]*e*Log[x] + 15*Sqrt[d^2]*e*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/
d^8
```

3.145.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {569, 25, 532, 27, 2336, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(d+ex)(d^2-e^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{569}$$

$$\frac{1}{5d^2x(d+ex)(d^2-e^2x^2)^{3/2}}-\frac{\int -\frac{6d-5ex}{x^2(d^2-e^2x^2)^{5/2}}dx}{5d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{6d-5ex}{x^2(d^2-e^2x^2)^{5/2}}dx}{5d^2}+\frac{1}{5d^2x(d+ex)(d^2-e^2x^2)^{3/2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{-\frac{\int -\frac{3\left(\frac{4e^2x^2}{d}-5ex+6d\right)}{x^2\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}-\frac{e(5d-6ex)}{3d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x(d+ex)(d^2-e^2x^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\frac{4e^2x^2}{d}-5ex+6d}{x^2\left(d^2-e^2x^2\right)^{3/2}}dx}{d^2}-\frac{e(5d-6ex)}{3d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x(d+ex)(d^2-e^2x^2)^{3/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{-\frac{\int -\frac{6d-5ex}{x^2\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{5e(d-2ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}-\frac{e(5d-6ex)}{3d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x(d+ex)(d^2-e^2x^2)^{3/2}}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{6d-5ex}{x^2\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{5e(d-2ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}-\frac{e(5d-6ex)}{3d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$\Big\downarrow$ 534

$$\frac{\frac{\frac{-5e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{6\sqrt{d^2-e^2x^2}}{dx}}{d^2}-\frac{5e(d-2ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}-\frac{e(5d-6ex)}{3d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$\Big\downarrow$ 243

$$\frac{\frac{\frac{-\frac{5}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{6\sqrt{d^2-e^2x^2}}{dx}}{d^2}-\frac{5e(d-2ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}-\frac{e(5d-6ex)}{3d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$\Big\downarrow$ 73

$$\frac{\frac{\frac{\frac{5\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{6\sqrt{d^2-e^2x^2}}{dx}}{d^2}-\frac{5e(d-2ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}-\frac{e(5d-6ex)}{3d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{\frac{5e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{6\sqrt{d^2-e^2x^2}}{dx}}{d^2}-\frac{5e(d-2ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}-\frac{e(5d-6ex)}{3d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

input
```
Int[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
1/(5*d^2*x*(d + e*x)*(d^2 - e^2*x^2)^(3/2)) + (-1/3*(e*(5*d - 6*e*x))/(d^3
*(d^2 - e^2*x^2)^(3/2)) + ((-5*e*(d - 2*e*x))/(d^3*Sqrt[d^2 - e^2*x^2]) +
((-6*Sqrt[d^2 - e^2*x^2])/(d*x) + (5*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d)/
d^2)/d^2)/(5*d^2)
```

3.145. $\int \frac{1}{x^2(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

3.145.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 569
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[(-x^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*p*(c + d*x))), x] + Simp[1/(2
*c^2*p)   Int[x^m*(a + b*x^2)^p*(c*(m + 2*p + 1) - d*(m + 2*p + 2)*x), x],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[m + 2*p, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.145.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(136) = 272$.

Time = 0.44 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.89

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^7x}+\frac{e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^6\sqrt{d^2}}-\frac{17\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{60d^6e\left(x+\frac{d}{e}\right)^2}-\frac{413\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{240d^7\left(x+\frac{d}{e}\right)}+\frac{\sqrt{-(}}{}$
default	$\frac{-\frac{1}{d^2x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{4e^2\left(\frac{x}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{d^2}}{d}-\frac{e\left(\frac{1}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{\frac{1}{d^2\sqrt{-e^2x^2+d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+}}{x}\right)}{d^2\sqrt{d^2}}}{d^2}\right.}{d^2}$

input
```
int(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(-e^2*x^2+d^2)^(1/2)/d^7/x+1/d^6*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-
e^2*x^2+d^2)^(1/2))/x)-17/60/d^6/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e)
)^(1/2)-413/240/d^7/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/24/d^6/
e/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-23/48/d^7/(x-d/e)*(-(x-d/
e)^2*e^2-2*d*e*(x-d/e))^(1/2)-1/20/d^5/e^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e
*(x+d/e))^(1/2)
```

3.145.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.72

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx =$$

$$-\frac{23\,e^6x^6+23\,de^5x^5-46\,d^2e^4x^4-46\,d^3e^3x^3+23\,d^4e^2x^2+23\,d^5ex+15\left(e^6x^6+de^5x^5-2\,d^2e^4x^4-2\,d^3e^3x\right.}{15\left(d^7e^5x^6+d^8e^4x^5-2\right.}$$

input
```
integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/15*(23*e^6*x^6 + 23*d*e^5*x^5 - 46*d^2*e^4*x^4 - 46*d^3*e^3*x^3 + 23*d^
4*e^2*x^2 + 23*d^5*e*x + 15*(e^6*x^6 + d*e^5*x^5 - 2*d^2*e^4*x^4 - 2*d^3*e
^3*x^3 + d^4*e^2*x^2 + d^5*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (48*e
^5*x^5 + 33*d*e^4*x^4 - 87*d^2*e^3*x^3 - 52*d^3*e^2*x^2 + 38*d^4*e*x + 15*
d^5)*sqrt(-e^2*x^2 + d^2))/(d^7*e^5*x^6 + d^8*e^4*x^5 - 2*d^9*e^3*x^4 - 2*
d^10*e^2*x^3 + d^11*e*x^2 + d^12*x)
```

3.145.6 Sympy [F]

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{x^2\left(-(-d+ex)(d+ex)\right)^{\frac{5}{2}}(d+ex)}\,dx$$

input
```
integrate(1/x**2/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.145.7 Maxima [F]

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^2), x)
```

3.145.8 Giac [F]

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^2), x)
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\int \frac{1}{x^2\left(d^2-e^2x^2\right)^{5/2}(d+e\,x)}\,dx$$

input
```
int(1/(x^2*(d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

output
```
int(1/(x^2*(d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

3.146 $\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

3.146.1 Optimal result

Integrand size = 27, antiderivative size = 186

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} + \frac{16e\sqrt{d^2-e^2x^2}}{5d^8x} - \frac{7e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8}$$

output
```
1/15*(-6*e*x+7*d)/d^4/x^2/(-e^2*x^2+d^2)^(3/2)+1/5/d^2/x^2/(e*x+d)/(-e^2*x
^2+d^2)^(3/2)-7/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^8+1/15*(-24*e*x+35
*d)/d^6/x^2/(-e^2*x^2+d^2)^(1/2)-7/2*(-e^2*x^2+d^2)^(1/2)/d^7/x^2+16/5*e*(
-e^2*x^2+d^2)^(1/2)/d^8/x
```

3.146.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}\left(-15d^6+15d^5ex+176d^4e^2x^2-4d^3e^3x^3-249d^2e^4x^4-9de^5x^5+96e^6x^6\right)}{x^2(d-ex)^2(d+ex)^3} - 105\sqrt{d^2}e^2}{30d^9}$$

input
```
Integrate[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
((d*Sqrt[d^2 - e^2*x^2]*(-15*d^6 + 15*d^5*e*x + 176*d^4*e^2*x^2 - 4*d^3*e^
3*x^3 - 249*d^2*e^4*x^4 - 9*d*e^5*x^5 + 96*e^6*x^6))/(x^2*(d - e*x)^2*(d +
 e*x)^3) - 105*Sqrt[d^2]*e^2*Log[x] + 105*Sqrt[d^2]*e^2*Log[Sqrt[d^2] - Sq
rt[d^2 - e^2*x^2]])/(30*d^9)
```

3.146.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {569, 25, 532, 27, 2336, 25, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{569}$$

$$\frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int -\frac{7d-6ex}{x^3\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{7d-6ex}{x^3\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}+\frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{\frac{e^2(7d-6ex)}{3d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int -\frac{3\left(-\frac{4e^3x^3}{d^2}+\frac{7e^2x^2}{d}-6ex+7d\right)}{x^3\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}}{5d^2}+\frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{-\frac{4e^3x^3}{d^2}+\frac{7e^2x^2}{d}-6ex+7d}{x^3\left(d^2-e^2x^2\right)^{3/2}}dx}{d^2}+\frac{e^2(7d-6ex)}{3d^4\left(d^2-e^2x^2\right)^{3/2}}}{5d^2}+\frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{\frac{2e^2(7d-5ex)}{d^4\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{\frac{14e^2x^2}{d}-6ex+7d}{x^3\sqrt{d^2-e^2x^2}}dx}{d^2}}{d^2}+\frac{e^2(7d-6ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int \frac{\frac{14e^2x^2}{d}-6ex+7d}{x^3\sqrt{d^2-e^2x^2}}dx}{d^2}+\frac{2e^2(7d-5ex)}{d^4\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(7d-6ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 2338$$

$$\frac{\frac{\frac{-\frac{\int \frac{de(12d-35ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{7\sqrt{d^2-e^2x^2}}{2dx^2}}{d^2}+\frac{2e^2(7d-5ex)}{d^4\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(7d-6ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{-\frac{e\int \frac{12d-35ex}{x^2\sqrt{d^2-e^2x^2}}dx}{2d}-\frac{7\sqrt{d^2-e^2x^2}}{2dx^2}}{d^2}+\frac{2e^2(7d-5ex)}{d^4\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(7d-6ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 534$$

$$\frac{\frac{\frac{-\frac{e\left(-35e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{12\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{7\sqrt{d^2-e^2x^2}}{2dx^2}}{d^2}+\frac{2e^2(7d-5ex)}{d^4\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(7d-6ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{\frac{-\frac{e\left(-\frac{35}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{12\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{7\sqrt{d^2-e^2x^2}}{2dx^2}}{d^2}+\frac{2e^2(7d-5ex)}{d^4\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(7d-6ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 73$$

3.146. $\int \frac{1}{x^3(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

$$\frac{\frac{\frac{-\frac{e\left(\frac{35\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{12\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{7\sqrt{d^2-e^2x^2}}{2dx^2}}{d^2}+\frac{2e^2(7d-5ex)}{d^4\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(7d-6ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{-\frac{e\left(\frac{35e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{12\sqrt{d^2-e^2x^2}}{dx}\right)}{2d}-\frac{7\sqrt{d^2-e^2x^2}}{2dx^2}}{d^2}+\frac{2e^2(7d-5ex)}{d^4\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(7d-6ex)}{3d^4(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

input
```
Int[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
1/(5*d^2*x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2)) + ((e^2*(7*d - 6*e*x))/(3*d^
4*(d^2 - e^2*x^2)^(3/2)) + ((2*e^2*(7*d - 5*e*x))/(d^4*Sqrt[d^2 - e^2*x^2]
) + ((-7*Sqrt[d^2 - e^2*x^2])/(2*d*x^2) - (e*((-12*Sqrt[d^2 - e^2*x^2])/(d
*x) + (35*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/(2*d))/d^2)/d^2)/(5*d^2)
```

3.146.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 569
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[(-x^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*p*(c + d*x))), x] + Simp[1/(2
*c^2*p)   Int[x^m*(a + b*x^2)^p*(c*(m + 2*p + 1) - d*(m + 2*p + 2)*x), x],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[m + 2*p, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.146. $\int \frac{1}{x^3(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.146.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.59

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-2ex+d)}{2d^8x^2}-\frac{7e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^7\sqrt{d^2}}-\frac{29e\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{48d^8\left(x-\frac{d}{e}\right)}+\frac{11\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{30d^7\left(x+\frac{d}{e}\right)^2}$
default	$\frac{-\frac{1}{2d^2x^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{5e^2\left(\frac{1}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{\frac{1}{d^2\sqrt{-e^2x^2+d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}}}{d^2}\right)}{2d^2}}{d}+\frac{e^2\left(\frac{1}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{\frac{1}{d^2\sqrt{-e^2}}}{}\right.}{}$

input
```
int(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e^2*x^2+d^2)^(1/2)*(-2*e*x+d)/d^8/x^2-7/2/d^7*e^2/(d^2)^(1/2)*ln((2
*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-29/48/d^8*e/(x-d/e)*(-(x-d/e)^
2*e^2-2*d*e*(x-d/e))^(1/2)+11/30/d^7/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/
e))^(1/2)+673/240/d^8*e/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/20/
d^6/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/24/d^7/(x-d/e)^2*(-
(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{116\,e^7x^7+116\,de^6x^6-232\,d^2e^5x^5-232\,d^3e^4x^4+116\,d^4e^3x^3+116\,d^5e^2x}{}$$

input
```
integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
1/30*(116*e^7*x^7 + 116*d*e^6*x^6 - 232*d^2*e^5*x^5 - 232*d^3*e^4*x^4 + 11
6*d^4*e^3*x^3 + 116*d^5*e^2*x^2 + 105*(e^7*x^7 + d*e^6*x^6 - 2*d^2*e^5*x^5
 - 2*d^3*e^4*x^4 + d^4*e^3*x^3 + d^5*e^2*x^2)*log(-(d - sqrt(-e^2*x^2 + d^
2))/x) + (96*e^6*x^6 - 9*d*e^5*x^5 - 249*d^2*e^4*x^4 - 4*d^3*e^3*x^3 + 176
*d^4*e^2*x^2 + 15*d^5*e*x - 15*d^6)*sqrt(-e^2*x^2 + d^2))/(d^8*e^5*x^7 + d
^9*e^4*x^6 - 2*d^10*e^3*x^5 - 2*d^11*e^2*x^4 + d^12*e*x^3 + d^13*x^2)
```

3.146.6 Sympy [F]

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{x^3\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate(1/x**3/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Integral(1/(x**3*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.146.7 Maxima [F]

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x^3}\,dx$$

input
```
integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^3), x)
```

3.146.8 Giac [F]

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{(-e^2x^2+d^2)^{\frac{5}{2}}(ex+d)x^3}\,dx$$

input
```
integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^3), x)
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{x^3\left(d^2-e^2x^2\right)^{5/2}(d+e\,x)}\,dx$$

input
```
int(1/(x^3*(d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

output
```
int(1/(x^3*(d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

3.147 $\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

3.147.1 Optimal result

Integrand size = 27, antiderivative size = 215

$$\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} - \frac{64\sqrt{d^2-e^2x^2}}{15d^7x^3} + \frac{7e\sqrt{d^2-e^2x^2}}{2d^8x^2} - \frac{128e^2\sqrt{d^2-e^2x^2}}{15d^9x} + \frac{7e^3\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^9}$$

output
```
1/15*(-7*e*x+8*d)/d^4/x^3/(-e^2*x^2+d^2)^(3/2)+1/5/d^2/x^3/(e*x+d)/(-e^2*x
^2+d^2)^(3/2)+7/2*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^9+1/15*(-35*e*x+48
*d)/d^6/x^3/(-e^2*x^2+d^2)^(1/2)-64/15*(-e^2*x^2+d^2)^(1/2)/d^7/x^3+7/2*e*
(-e^2*x^2+d^2)^(1/2)/d^8/x^2-128/15*e^2*(-e^2*x^2+d^2)^(1/2)/d^9/x
```

3.147.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = -\frac{\frac{d\sqrt{d^2-e^2x^2}\left(10d^7-5d^6ex+75d^5e^2x^2+236d^4e^3x^3-244d^3e^4x^4-489d^2e^5x^5+151de^6x^6+256e^7x^7\right)}{x^3(d-ex)^2(d+ex)^3} - 105\sqrt{d^2}e^3\log(x) + 105\sqrt{d^2}e^3\log}{30d^{10}}$$

input
```
Integrate[1/(x^4*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
-1/30*((d*Sqrt[d^2 - e^2*x^2]*(10*d^7 - 5*d^6*e*x + 75*d^5*e^2*x^2 + 236*d
^4*e^3*x^3 - 244*d^3*e^4*x^4 - 489*d^2*e^5*x^5 + 151*d*e^6*x^6 + 256*e^7*x
^7))/(x^3*(d - e*x)^2*(d + e*x)^3) - 105*Sqrt[d^2]*e^3*Log[x] + 105*Sqrt[d
^2]*e^3*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/d^10
```

3.147.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {569, 25, 532, 25, 2336, 27, 2338, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (d+e x) \left(d^2-e^2 x^2\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{569}$$

$$\frac{1}{5 d^2 x^3 (d+e x) \left(d^2-e^2 x^2\right)^{3/2}}-\frac{\int -\frac{8 d-7 e x}{x^4 \left(d^2-e^2 x^2\right)^{5/2}}dx}{5 d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{8 d-7 e x}{x^4 \left(d^2-e^2 x^2\right)^{5/2}}dx}{5 d^2}+\frac{1}{5 d^2 x^3 (d+e x) \left(d^2-e^2 x^2\right)^{3/2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{-\frac{\int -\frac{\frac{16 e^4 x^4}{d^3}-\frac{21 e^3 x^3}{d^2}+\frac{24 e^2 x^2}{d}-21 e x+24 d}{x^4 \left(d^2-e^2 x^2\right)^{3/2}}dx}{3 d^2}-\frac{e^3 (7 d-8 e x)}{3 d^5 \left(d^2-e^2 x^2\right)^{3/2}}}{5 d^2}+\frac{1}{5 d^2 x^3 (d+e x) \left(d^2-e^2 x^2\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\frac{16 e^4 x^4}{d^3}-\frac{21 e^3 x^3}{d^2}+\frac{24 e^2 x^2}{d}-21 e x+24 d}{x^4 \left(d^2-e^2 x^2\right)^{3/2}}dx}{3 d^2}-\frac{e^3 (7 d-8 e x)}{3 d^5 \left(d^2-e^2 x^2\right)^{3/2}}}{5 d^2}+\frac{1}{5 d^2 x^3 (d+e x) \left(d^2-e^2 x^2\right)^{3/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{-\frac{\int -\frac{3\left(-\frac{14e^3x^3}{d^2}+\frac{16e^2x^2}{d}-7ex+8d\right)}{x^4\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{2e^3(21d-32ex)}{d^5\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{e^3(7d-8ex)}{3d^5(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$\downarrow$ 27

$$\frac{\frac{\frac{3\int \frac{-\frac{14e^3x^3}{d^2}+\frac{16e^2x^2}{d}-7ex+8d}{x^4\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{2e^3(21d-32ex)}{d^5\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{e^3(7d-8ex)}{3d^5(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$\downarrow$ 2338

$$\frac{\frac{3\left(-\frac{\int \frac{42x^2e^3-64dxe^2+21d^2e}{x^3\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{8\sqrt{d^2-e^2x^2}}{3dx^3}\right)}{d^2}-\frac{2e^3(21d-32ex)}{d^5\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{e^3(7d-8ex)}{3d^5(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$\downarrow$ 2338

$$\frac{\frac{3\left(-\frac{-\frac{\int \frac{d^2e^2(128d-105ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{21e\sqrt{d^2-e^2x^2}}{2x^2}}{3d^2}-\frac{8\sqrt{d^2-e^2x^2}}{3dx^3}\right)}{d^2}-\frac{2e^3(21d-32ex)}{d^5\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{e^3(7d-8ex)}{3d^5(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$\downarrow$ 27

$$\frac{\frac{3\left(-\frac{-\frac{1}{2}e^2\int \frac{128d-105ex}{x^2\sqrt{d^2-e^2x^2}}dx-\frac{21e\sqrt{d^2-e^2x^2}}{2x^2}}{3d^2}-\frac{8\sqrt{d^2-e^2x^2}}{3dx^3}\right)}{d^2}-\frac{2e^3(21d-32ex)}{d^5\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{e^3(7d-8ex)}{3d^5(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$\downarrow$ 534

$$\frac{\frac{3\left(-\frac{-\frac{1}{2}e^2\left(-105e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{128\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{21e\sqrt{d^2-e^2x^2}}{2x^2}}{3d^2}-\frac{8\sqrt{d^2-e^2x^2}}{3dx^3}\right)}{d^2}-\frac{2e^3(21d-32ex)}{d^5\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{e^3(7d-8ex)}{3d^5(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

3.147. $\int \frac{1}{x^4(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

$$\Big\downarrow \ 243$$

$$\frac{\frac{3\left(-\frac{-\frac{1}{2}e^2\left(-\frac{105}{2}e\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{128\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{21e\sqrt{d^2-e^2x^2}}{2x^2}}{3d^2}-\frac{8\sqrt{d^2-e^2x^2}}{3dx^3}\right)}{d^2}-\frac{2e^3(21d-32ex)}{d^5\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{e^3(7d-8ex)}{3d^5(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x^3(d+ex)(d^2-e^2x^2)^{3/2}}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{3\left(-\frac{-\frac{1}{2}e^2\left(\frac{105\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{128\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{21e\sqrt{d^2-e^2x^2}}{2x^2}}{3d^2}-\frac{8\sqrt{d^2-e^2x^2}}{3dx^3}\right)}{d^2}-\frac{2e^3(21d-32ex)}{d^5\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{e^3(7d-8ex)}{3d^5(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x^3(d+ex)(d^2-e^2x^2)^{3/2}}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{3\left(-\frac{-\frac{1}{2}e^2\left(\frac{105e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{128\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{21e\sqrt{d^2-e^2x^2}}{2x^2}}{3d^2}-\frac{8\sqrt{d^2-e^2x^2}}{3dx^3}\right)}{d^2}-\frac{2e^3(21d-32ex)}{d^5\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{e^3(7d-8ex)}{3d^5(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{1}{5d^2x^3(d+ex)(d^2-e^2x^2)^{3/2}}$$

input
```
Int[1/(x^4*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
1/(5*d^2*x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2)) + (-1/3*(e^3*(7*d - 8*e*x))/
(d^5*(d^2 - e^2*x^2)^(3/2)) + ((-2*e^3*(21*d - 32*e*x))/(d^5*Sqrt[d^2 - e^
2*x^2]) + (3*((-8*Sqrt[d^2 - e^2*x^2])/(3*d*x^3) - ((-21*e*Sqrt[d^2 - e^2*
x^2])/(2*x^2) - (e^2*((-128*Sqrt[d^2 - e^2*x^2])/(d*x) + (105*e*ArcTanh[Sq
rt[d^2 - e^2*x^2]/d])/d))/2)/(3*d^2)))/d^2)/(3*d^2))/(5*d^2)
```

3.147. $\int \frac{1}{x^4(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

3.147.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 569
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[(-x^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*p*(c + d*x))), x] + Simp[1/(2
*c^2*p)   Int[x^m*(a + b*x^2)^p*(c*(m + 2*p + 1) - d*(m + 2*p + 2)*x), x],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[m + 2*p, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.147.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.45

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(22e^2x^2-3dex+2d^2)}{6d^9x^3}+\frac{7e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^8\sqrt{d^2}}-\frac{331e^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{80d^9\left(x+\frac{d}{e}\right)}-\frac{35e^2\sqrt{-\left(x-\frac{d}{e}\cdots\right.}}{48d^9\cdots}$
default	$\frac{-\frac{1}{3d^2x^3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2e^2\left(-\frac{1}{d^2x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{4e^2\left(\frac{x}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{d^2}\right)}{d^2}}{d}+\frac{e^2\left(-\frac{1}{d^2x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{4e^2\left(\cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-e^2*x^2+d^2)^(1/2)*(22*e^2*x^2-3*d*e*x+2*d^2)/d^9/x^3+7/2*e^3/d^8/(
d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-331/80*e^2/d^9
/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-35/48*e^2/d^9/(x-d/e)*(-(x-d
/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-9/20/d^8*e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*
(x+d/e))^(1/2)-1/20/d^7/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2
4/d^8*e/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.38

$$\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx =$$

$$-\frac{116\,e^8x^8+116\,de^7x^7-232\,d^2e^6x^6-232\,d^3e^5x^5+116\,d^4e^4x^4+116\,d^5e^3x^3+105\left(e^8x^8+de^7x^7-2\,d^2e^6x^6 \cdots\right.}{30\,(\cdots}$$

input
```
integrate(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/30*(116*e^8*x^8 + 116*d*e^7*x^7 - 232*d^2*e^6*x^6 - 232*d^3*e^5*x^5 + 1
16*d^4*e^4*x^4 + 116*d^5*e^3*x^3 + 105*(e^8*x^8 + d*e^7*x^7 - 2*d^2*e^6*x^
6 - 2*d^3*e^5*x^5 + d^4*e^4*x^4 + d^5*e^3*x^3)*log(-(d - sqrt(-e^2*x^2 + d
^2))/x) + (256*e^7*x^7 + 151*d*e^6*x^6 - 489*d^2*e^5*x^5 - 244*d^3*e^4*x^4
 + 236*d^4*e^3*x^3 + 75*d^5*e^2*x^2 - 5*d^6*e*x + 10*d^7)*sqrt(-e^2*x^2 +
d^2))/(d^9*e^5*x^8 + d^10*e^4*x^7 - 2*d^11*e^3*x^6 - 2*d^12*e^2*x^5 + d^13
*e*x^4 + d^14*x^3)
```

3.147.6 Sympy [F]

$$\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{x^4\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate(1/x**4/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Integral(1/(x**4*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.147.7 Maxima [F]

$$\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x^4}\,dx$$

input
```
integrate(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^4), x)
```

3.147.8 Giac [F]

$$\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x^4}\,dx$$

input
```
integrate(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^4), x)
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{x^4\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)}\,dx$$

input
```
int(1/(x^4*(d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

output
```
int(1/(x^4*(d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

3.148 $\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx$

3.148.1 Optimal result

Integrand size = 27, antiderivative size = 118

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}} - \frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{x}{35d^2e^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2x}{35d^4e^3\sqrt{d^2-e^2x^2}}$$

output
```
1/7*x^2*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(7/2)+1/35*(3*e*x-2*d)/e^4/(-e^2*x^2+d
^2)^(5/2)-1/35*x/d^2/e^3/(-e^2*x^2+d^2)^(3/2)-2/35*x/d^4/e^3/(-e^2*x^2+d^2
)^(1/2)
```

3.148.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}\left(-2d^6-2d^5ex+5d^4e^2x^2+5d^3e^3x^3+5d^2e^4x^4-2de^5x^5-2e^6x^6\right)}{35d^4e^4(d-ex)^3(d+ex)^4}$$

input
```
Integrate[x^3/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-2*d^6 - 2*d^5*e*x + 5*d^4*e^2*x^2 + 5*d^3*e^3*x^3 +
 5*d^2*e^4*x^4 - 2*d*e^5*x^5 - 2*e^6*x^6))/(35*d^4*e^4*(d - e*x)^3*(d + e*
x)^4)
```

3.148.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {568, 530, 27, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex)(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{568}$$

$$\frac{x^2}{7e^2(d+ex)(d^2-e^2x^2)^{5/2}}-\frac{\int \frac{x(2d-3ex)}{(d^2-e^2x^2)^{7/2}}dx}{7e^2}$$

$$\Big\downarrow \text{530}$$

$$\frac{x^2}{7e^2(d+ex)(d^2-e^2x^2)^{5/2}}-\frac{\frac{2d-3ex}{5e^2(d^2-e^2x^2)^{5/2}}-\frac{\int -\frac{3d^2}{e\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}}{7e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2}{7e^2(d+ex)(d^2-e^2x^2)^{5/2}}-\frac{\frac{3\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5e}+\frac{2d-3ex}{5e^2(d^2-e^2x^2)^{5/2}}}{7e^2}$$

$$\Big\downarrow \text{209}$$

$$\frac{x^2}{7e^2(d+ex)(d^2-e^2x^2)^{5/2}}-\frac{\frac{3\left(\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)}{5e}+\frac{2d-3ex}{5e^2(d^2-e^2x^2)^{5/2}}}{7e^2}$$

$$\Big\downarrow \text{208}$$

$$\frac{x^2}{7e^2(d+ex)(d^2-e^2x^2)^{5/2}}-\frac{\frac{2d-3ex}{5e^2(d^2-e^2x^2)^{5/2}}+\frac{3\left(\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5e}}{7e^2}$$

input
```
Int[x^3/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
x^2/(7*e^2*(d + e*x)*(d^2 - e^2*x^2)^(5/2)) - ((2*d - 3*e*x)/(5*e^2*(d^2 -
 e^2*x^2)^(5/2)) + (3*(x/(3*d^2*(d^2 - e^2*x^2)^(3/2)) + (2*x)/(3*d^4*Sqrt
[d^2 - e^2*x^2])))/(5*e))/(7*e^2)
```

3.148.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 568
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[x^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*p*(c + d*x))), x] + Simp[1/(2*d^
2*p)   Int[x^(m - 2)*(a + b*x^2)^p*(c*(m - 1) - d*m*x), x], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[m, 1] && LtQ[p, -1]
```

3.148.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.78

method	result
gosper	$-\frac{(-ex+d)(2e^6x^6+2d\,e^5x^5-5d^2e^4x^4-5d^3x^3e^3-5d^4e^2x^2+2d^5ex+2d^6)}{35d^4e^4(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$-\frac{(2e^6x^6+2d\,e^5x^5-5d^2e^4x^4-5d^3x^3e^3-5d^4e^2x^2+2d^5ex+2d^6)\sqrt{-e^2x^2+d^2}}{35d^4e^4(ex+d)^4(-ex+d)^3}$
default	$\frac{\frac{x}{4e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2(-e^2x^2+d^2)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{4e^2}}{e}+\frac{d^2\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2(-e^2x^2+d^2)^{\frac{3}{2}}}+}{d^2}\right.}{e^3}$

input
```
int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*(-e*x+d)*(2*e^6*x^6+2*d*e^5*x^5-5*d^2*e^4*x^4-5*d^3*e^3*x^3-5*d^4*e^
2*x^2+2*d^5*e*x+2*d^6)/d^4/e^4/(-e^2*x^2+d^2)^(7/2)
```

3.148.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(103) = 206$.

Time = 0.32 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.03

$$\int \frac{x^3}{(d+ex)\,(d^2-e^2x^2)^{7/2}}\,dx =$$
$$-\frac{2\,e^7x^7+2\,de^6x^6-6\,d^2e^5x^5-6\,d^3e^4x^4+6\,d^4e^3x^3+6\,d^5e^2x^2-2\,d^6ex-2\,d^7-(2\,e^6x^6+2\,de^5x^5-5\,d^2e^4x}{35\,(d^4e^{11}x^7+d^5e^{10}x^6-3\,d^6e^9x^5-3\,d^7e^8x^4+3\,d^8e^7x^3+3\,d^9e^6x^2-}$$

input
```
integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/35*(2*e^7*x^7 + 2*d*e^6*x^6 - 6*d^2*e^5*x^5 - 6*d^3*e^4*x^4 + 6*d^4*e^3
*x^3 + 6*d^5*e^2*x^2 - 2*d^6*e*x - 2*d^7 - (2*e^6*x^6 + 2*d*e^5*x^5 - 5*d^
2*e^4*x^4 - 5*d^3*e^3*x^3 - 5*d^4*e^2*x^2 + 2*d^5*e*x + 2*d^6)*sqrt(-e^2*x
^2 + d^2))/(d^4*e^11*x^7 + d^5*e^10*x^6 - 3*d^6*e^9*x^5 - 3*d^7*e^8*x^4 +
3*d^8*e^7*x^3 + 3*d^9*e^6*x^2 - d^10*e^5*x - d^11*e^4)
```

3.148.6 Sympy [F]

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral(x**3/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)), x)
```

3.148.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.13

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d^2}{7\left(\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5x+\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^4\right)} + \frac{8\,x}{35\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3} - \frac{d}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4} - \frac{x}{35\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e^3} - \frac{2\,x}{35\sqrt{-e^2x^2+d^2}\,d^4e^3}$$

input
```
integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/7*d^2/((-e^2*x^2 + d^2)^(5/2)*e^5*x + (-e^2*x^2 + d^2)^(5/2)*d*e^4) + 8/
35*x/((-e^2*x^2 + d^2)^(5/2)*e^3) - 1/5*d/((-e^2*x^2 + d^2)^(5/2)*e^4) - 1
/35*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e^3) - 2/35*x/(sqrt(-e^2*x^2 + d^2)*d^4*
e^3)
```

3.148.8 Giac [F]

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{x^3}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}(ex+d)}\,dx$$

input
```
integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate(x^3/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)), x)
```

3.148.9 Mupad [B] (verification not implemented)

Time = 11.92 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.36

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}}{56\,d\,e^4\,(d+e\,x)^4} - \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1}{56\,d\,e^4}+\frac{x}{35\,d^2\,e^3}\right)}{(d+e\,x)^2\,(d-e\,x)^2} - \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{2\,d}{35\,e^4}-\frac{11\,x}{70\,e^3}\right)}{(d+e\,x)^3\,(d-e\,x)^3} - \frac{2\,x\,\sqrt{d^2-e^2\,x^2}}{35\,d^4\,e^3\,(d+e\,x)\,(d-e\,x)}$$

input
```
int(x^3/((d^2 - e^2*x^2)^(7/2)*(d + e*x)),x)
```

output
```
(d^2 - e^2*x^2)^(1/2)/(56*d*e^4*(d + e*x)^4) - ((d^2 - e^2*x^2)^(1/2)*(1/(
56*d*e^4) + x/(35*d^2*e^3)))/((d + e*x)^2*(d - e*x)^2) - ((d^2 - e^2*x^2)^
(1/2)*((2*d)/(35*e^4) - (11*x)/(70*e^3)))/((d + e*x)^3*(d - e*x)^3) - (2*x
*(d^2 - e^2*x^2)^(1/2))/(35*d^4*e^3*(d + e*x)*(d - e*x))
```

3.149 $\int \frac{x^2}{(d+ex)(d^2-e^2x^2)^{7/2}}\,dx$

3.149.1 Optimal result

Integrand size = 27, antiderivative size = 123

$$\int \frac{x^2}{(d+ex)(d^2-e^2x^2)^{7/2}}\,dx = -\frac{x^2}{7de(d+ex)(d^2-e^2x^2)^{5/2}} + \frac{2(d+2ex)}{35de^3(d^2-e^2x^2)^{5/2}} - \frac{4x}{105d^3e^2(d^2-e^2x^2)^{3/2}} - \frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}$$

output
```
-1/7*x^2/d/e/(e*x+d)/(-e^2*x^2+d^2)^(5/2)+2/35*(2*e*x+d)/d/e^3/(-e^2*x^2+d
^2)^(5/2)-4/105*x/d^3/e^2/(-e^2*x^2+d^2)^(3/2)-8/105*x/d^5/e^2/(-e^2*x^2+d
^2)^(1/2)
```

3.149.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{(d+ex)(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(6d^6+6d^5ex-15d^4e^2x^2+20d^3e^3x^3+20d^2e^4x^4-8de^5x^5-8e^6}{105d^5e^3(d-ex)^3(d+ex)^4}$$

input
```
Integrate[x^2/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(6*d^6 + 6*d^5*e*x - 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3
+ 20*d^2*e^4*x^4 - 8*d*e^5*x^5 - 8*e^6*x^6))/(105*d^5*e^3*(d - e*x)^3*(d +
 e*x)^4)
```

3.149.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {568, 454, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex)(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{568}$$

$$\frac{x}{7e^2(d+ex)(d^2-e^2x^2)^{5/2}}-\frac{\int \frac{d-2ex}{(d^2-e^2x^2)^{7/2}}dx}{7e^2}$$

$$\Big\downarrow \text{454}$$

$$\frac{x}{7e^2(d+ex)(d^2-e^2x^2)^{5/2}}-\frac{\frac{4\int \frac{1}{(d^2-e^2x^2)^{5/2}}dx}{5d}-\frac{2d-ex}{5de(d^2-e^2x^2)^{5/2}}}{7e^2}$$

$$\Big\downarrow \text{209}$$

$$\frac{x}{7e^2(d+ex)(d^2-e^2x^2)^{5/2}}-\frac{\frac{4\left(\frac{2\int \frac{1}{(d^2-e^2x^2)^{3/2}}dx}{3d^2}+\frac{x}{3d^2(d^2-e^2x^2)^{3/2}}\right)}{5d}-\frac{2d-ex}{5de(d^2-e^2x^2)^{5/2}}}{7e^2}$$

$$\Big\downarrow \text{208}$$

$$\frac{x}{7e^2(d+ex)(d^2-e^2x^2)^{5/2}}-\frac{\frac{4\left(\frac{x}{3d^2(d^2-e^2x^2)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d}-\frac{2d-ex}{5de(d^2-e^2x^2)^{5/2}}}{7e^2}$$

input
```
Int[x^2/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
x/(7*e^2*(d + e*x)*(d^2 - e^2*x^2)^(5/2)) - (-1/5*(2*d - e*x)/(d*e*(d^2 -
e^2*x^2)^(5/2)) + (4*(x/(3*d^2*(d^2 - e^2*x^2)^(3/2)) + (2*x)/(3*d^4*Sqrt[
d^2 - e^2*x^2])))/(5*d))/(7*e^2)
```

3.149.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 568
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[x^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*p*(c + d*x))), x] + Simp[1/(2*d^
2*p)   Int[x^(m - 2)*(a + b*x^2)^p*(c*(m - 1) - d*m*x), x], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[m, 1] && LtQ[p, -1]
```

3.149.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.75

method	result
gosper	$\frac{(-ex+d)\left(-8e^6x^6-8d\,e^5x^5+20d^2e^4x^4+20d^3x^3e^3-15d^4e^2x^2+6d^5ex+6d^6\right)}{105d^5e^3(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$\frac{\left(-8e^6x^6-8d\,e^5x^5+20d^2e^4x^4+20d^3x^3e^3-15d^4e^2x^2+6d^5ex+6d^6\right)\sqrt{-e^2x^2+d^2}}{105d^5(ex+d)^4(-ex+d)^3e^3}$
default	$\frac{1}{5e^3(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{d\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{e^2}+\frac{d^2\left(-\frac{1}{7de\left(x+\frac{d}{e}\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)}\right.}{}$

input
```
int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(-e*x+d)*(-8*e^6*x^6-8*d*e^5*x^5+20*d^2*e^4*x^4+20*d^3*e^3*x^3-15*d^
4*e^2*x^2+6*d^5*e*x+6*d^6)/d^5/e^3/(-e^2*x^2+d^2)^(7/2)
```

3.149.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(107) = 214$.

Time = 0.35 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.93

$$\int \frac{x^2}{(d+ex)\,(d^2-e^2x^2)^{7/2}}\,dx = \frac{6\,e^7x^7+6\,de^6x^6-18\,d^2e^5x^5-18\,d^3e^4x^4+18\,d^4e^3x^3+18\,d^5e^2x^2-6\,d^6ex-}{105\,(d^5e^{10}x^7+d^6e^9x^6-3\,d^7e^8x^5-}$$

input
```
integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/105*(6*e^7*x^7 + 6*d*e^6*x^6 - 18*d^2*e^5*x^5 - 18*d^3*e^4*x^4 + 18*d^4*
e^3*x^3 + 18*d^5*e^2*x^2 - 6*d^6*e*x - 6*d^7 + (8*e^6*x^6 + 8*d*e^5*x^5 -
20*d^2*e^4*x^4 - 20*d^3*e^3*x^3 + 15*d^4*e^2*x^2 - 6*d^5*e*x - 6*d^6)*sqrt
(-e^2*x^2 + d^2))/(d^5*e^10*x^7 + d^6*e^9*x^6 - 3*d^7*e^8*x^5 - 3*d^8*e^7*
x^4 + 3*d^9*e^6*x^3 + 3*d^10*e^5*x^2 - d^11*e^4*x - d^12*e^3)
```

3.149.6 Sympy [F]

$$\int \frac{x^2}{(d+ex)\,(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral(x**2/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)), x)
```

3.149.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = -\frac{d}{7\left(\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4x+\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^3\right)} - \frac{x}{35\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^2} + \frac{1}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3} - \frac{4\,x}{105\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3e^2} - \frac{8\,x}{105\,\sqrt{-e^2x^2+d^2}d^5e^2}$$

input
```
integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
-1/7*d/((-e^2*x^2 + d^2)^(5/2)*e^4*x + (-e^2*x^2 + d^2)^(5/2)*d*e^3) - 1/3
5*x/((-e^2*x^2 + d^2)^(5/2)*d*e^2) + 1/5/((-e^2*x^2 + d^2)^(5/2)*e^3) - 4/
105*x/((-e^2*x^2 + d^2)^(3/2)*d^3*e^2) - 8/105*x/(sqrt(-e^2*x^2 + d^2)*d^5
*e^2)
```

3.149.8 Giac [F]

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{x^2}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}(ex+d)}\,dx$$

input
```
integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate(x^2/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)), x)
```

3.149.9 Mupad [B] (verification not implemented)

Time = 11.76 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.31

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1}{56\,d^2\,e^3}-\frac{4\,x}{105\,d^3\,e^2}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2} + \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{2}{35\,e^3}+\frac{3\,x}{70\,d\,e^2}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3} - \frac{\sqrt{d^2-e^2\,x^2}}{56\,d^2\,e^3\,{\left(d+e\,x\right)}^4} - \frac{8\,x\,\sqrt{d^2-e^2\,x^2}}{105\,d^5\,e^2\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

input
```
int(x^2/((d^2 - e^2*x^2)^(7/2)*(d + e*x)),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(1/(56*d^2*e^3) - (4*x)/(105*d^3*e^2)))/((d + e*x)^
2*(d - e*x)^2) + ((d^2 - e^2*x^2)^(1/2)*(2/(35*e^3) + (3*x)/(70*d*e^2)))/(
(d + e*x)^3*(d - e*x)^3) - (d^2 - e^2*x^2)^(1/2)/(56*d^2*e^3*(d + e*x)^4)
- (8*x*(d^2 - e^2*x^2)^(1/2))/(105*d^5*e^2*(d + e*x)*(d - e*x))
```

3.149. $\int \frac{x^2}{(d+ex)(d^2-e^2x^2)^{7/2}}\,dx$

3.150 $\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx$

3.150.1 Optimal result

Integrand size = 25, antiderivative size = 66

$$\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx=\frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}}+\frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4}+\frac{3\arcsin(ax)}{2a^4}$$

output
```
3/2*arcsin(a*x)/a^4+x^2*(-a*x+1)/a^2/(-a^2*x^2+1)^(1/2)+1/2*(-3*a*x+4)*(-a
^2*x^2+1)^(1/2)/a^4
```

3.150.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx=\frac{\sqrt{1-a^2x^2}(4+ax-a^2x^2)}{2a^4(1+ax)}+\frac{3\arctan\left(\frac{ax}{-1+\sqrt{1-a^2x^2}}\right)}{a^4}$$

input
```
Integrate[x^3/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

output
```
(Sqrt[1 - a^2*x^2]*(4 + a*x - a^2*x^2))/(2*a^4*(1 + a*x)) + (3*ArcTan[(a*x
)/(-1 + Sqrt[1 - a^2*x^2])])/a^4
```

3.150.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {563, 25, 2346, 25, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax+1)\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{563}$$

$$\frac{\sqrt{1-a^2x^2}}{a^4(ax+1)}-\frac{\int -\frac{a^2x^2-ax+1}{\sqrt{1-a^2x^2}}dx}{a^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a^2x^2-ax+1}{\sqrt{1-a^2x^2}}dx}{a^3}+\frac{\sqrt{1-a^2x^2}}{a^4(ax+1)}$$

$$\Big\downarrow \text{2346}$$

$$\frac{-\frac{\int -\frac{a^2(3-2ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{1}{2}x\sqrt{1-a^2x^2}}{a^3}+\frac{\sqrt{1-a^2x^2}}{a^4(ax+1)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{a^2(3-2ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{1}{2}x\sqrt{1-a^2x^2}}{a^3}+\frac{\sqrt{1-a^2x^2}}{a^4(ax+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}\int \frac{3-2ax}{\sqrt{1-a^2x^2}}dx-\frac{1}{2}x\sqrt{1-a^2x^2}}{a^3}+\frac{\sqrt{1-a^2x^2}}{a^4(ax+1)}$$

$$\Big\downarrow \text{455}$$

$$\frac{\frac{1}{2}\left(3\int \frac{1}{\sqrt{1-a^2x^2}}dx+\frac{2\sqrt{1-a^2x^2}}{a}\right)-\frac{1}{2}x\sqrt{1-a^2x^2}}{a^3}+\frac{\sqrt{1-a^2x^2}}{a^4(ax+1)}$$

$$\Big\downarrow \text{223}$$

$$\frac{\sqrt{1-a^2x^2}}{a^4(ax+1)}+\frac{\frac{1}{2}\left(\frac{2\sqrt{1-a^2x^2}}{a}+\frac{3\arcsin(ax)}{a}\right)-\frac{1}{2}x\sqrt{1-a^2x^2}}{a^3}$$

input
```
Int[x^3/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

output
```
Sqrt[1 - a^2*x^2]/(a^4*(1 + a*x)) + (-1/2*(x*Sqrt[1 - a^2*x^2]) + ((2*Sqrt
[1 - a^2*x^2])/a + (3*ArcSin[a*x])/a)/2)/a^3
```

3.150.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.150.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.47

method	result	size
risch	$\frac{(ax-2)\left(a^{2}x^{2}-1\right)}{2a^{4}\sqrt{-a^{2}x^{2}+1}}+\frac{3\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{2a^{3}\sqrt{a^{2}}}+\frac{\sqrt{-\left(x+\frac{1}{a}\right)^{2}a^{2}+2\left(x+\frac{1}{a}\right)a}}{a^{5}\left(x+\frac{1}{a}\right)}$	97
default	$\frac{\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{a^{3}\sqrt{a^{2}}}+\frac{-\frac{x\sqrt{-a^{2}x^{2}+1}}{2a^{2}}+\frac{\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{2a^{2}\sqrt{a^{2}}}}{a}+\frac{\sqrt{-a^{2}x^{2}+1}}{a^{4}}+\frac{\sqrt{-\left(x+\frac{1}{a}\right)^{2}a^{2}+2\left(x+\frac{1}{a}\right)a}}{a^{5}\left(x+\frac{1}{a}\right)}$	134

input
```
int(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x-2)*(a^2*x^2-1)/a^4/(-a^2*x^2+1)^(1/2)+3/2/a^3/(a^2)^(1/2)*arctan(
(a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/a^5/(x+1/a)*(-(x+1/a)^2*a^2+2*(x+1/a)*
a)^(1/2)
```

3.150.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{4\,ax-6\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-(a^2x^2-ax-4)\sqrt{-a^2x^2+1}+4}{2\,(a^5x+a^4)}$$

input
```
integrate(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(4*a*x - 6*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (a^2*x^2
 - a*x - 4)*sqrt(-a^2*x^2 + 1) + 4)/(a^5*x + a^4)
```

3.150.6 Sympy [F]

$$\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3}{\sqrt{-(ax-1)(ax+1)}\,(ax+1)}\,dx$$

input
```
integrate(x**3/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**3/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

3.150.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{-a^2x^2+1}}{a^5x+a^4} - \frac{\sqrt{-a^2x^2+1}x}{2\,a^3} + \frac{3\arcsin(ax)}{2\,a^4} + \frac{\sqrt{-a^2x^2+1}}{a^4}$$

input
```
integrate(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(-a^2*x^2 + 1)/(a^5*x + a^4) - 1/2*sqrt(-a^2*x^2 + 1)*x/a^3 + 3/2*arcs
in(a*x)/a^4 + sqrt(-a^2*x^2 + 1)/a^4
```

3.150.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.150.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.76

$$\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,a^3\,\sqrt{-a^2}} - \frac{\left(\frac{1}{a^2\,\sqrt{-a^2}}+\frac{x\,\sqrt{-a^2}}{2\,a^3}\right)\,\sqrt{1-a^2\,x^2}}{\sqrt{-a^2}} - \frac{\sqrt{1-a^2\,x^2}}{a^3\,\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}$$

input
```
int(x^3/((1 - a^2*x^2)^(1/2)*(a*x + 1)),x)
```

output
```
(3*asinh(x*(-a^2)^(1/2)))/(2*a^3*(-a^2)^(1/2)) - ((1/(a^2*(-a^2)^(1/2)) +
(x*(-a^2)^(1/2))/(2*a^3))*(1 - a^2*x^2)^(1/2))/(-a^2)^(1/2) - (1 - a^2*x^2
)^(1/2)/(a^3*(x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

3.151 $\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx$

3.151.1 Optimal result

Integrand size = 25, antiderivative size = 55

$$\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}}{a^3} - \frac{\sqrt{1-a^2x^2}}{a^3(1+ax)} - \frac{\arcsin(ax)}{a^3}$$

output
```
-arcsin(a*x)/a^3-(-a^2*x^2+1)^(1/2)/a^3-(-a^2*x^2+1)^(1/2)/a^3/(a*x+1)
```

3.151.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{(-2-ax)\sqrt{1-a^2x^2}}{a^3(1+ax)} - \frac{2\arctan\left(\frac{ax}{-1+\sqrt{1-a^2x^2}}\right)}{a^3}$$

input
```
Integrate[x^2/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

output
```
((-2 - a*x)*Sqrt[1 - a^2*x^2])/(a^3*(1 + a*x)) - (2*ArcTan[(a*x)/(-1 + Sqr
t[1 - a^2*x^2])])/a^3
```

3.151.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {563, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax+1)\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{563}$$

$$-\frac{\int \frac{1-ax}{\sqrt{1-a^2x^2}}dx}{a^2}-\frac{\sqrt{1-a^2x^2}}{a^3(ax+1)}$$

$$\Big\downarrow \text{455}$$

$$-\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}}{a}}{a^2}-\frac{\sqrt{1-a^2x^2}}{a^3(ax+1)}$$

$$\Big\downarrow \text{223}$$

$$-\frac{\frac{\sqrt{1-a^2x^2}}{a}+\frac{\arcsin(ax)}{a}}{a^2}-\frac{\sqrt{1-a^2x^2}}{a^3(ax+1)}$$

input
```
Int[x^2/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

output
```
-(Sqrt[1 - a^2*x^2]/(a^3*(1 + a*x))) - (Sqrt[1 - a^2*x^2]/a + ArcSin[a*x]/
a)/a^2
```

3.151.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

3.151.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.53

method	result	size
default	$-\frac{\sqrt{-a^{2}x^{2}+1}}{a^{3}}-\frac{\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{a^{2}\sqrt{a^{2}}}-\frac{\sqrt{-\left(x+\frac{1}{a}\right)^{2}a^{2}+2\left(x+\frac{1}{a}\right)a}}{a^{4}\left(x+\frac{1}{a}\right)}$	84
risch	$\frac{a^{2}x^{2}-1}{a^{3}\sqrt{-a^{2}x^{2}+1}}-\frac{\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{a^{2}\sqrt{a^{2}}}-\frac{\sqrt{-\left(x+\frac{1}{a}\right)^{2}a^{2}+2\left(x+\frac{1}{a}\right)a}}{a^{4}\left(x+\frac{1}{a}\right)}$	92

input
```
int(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-a^2*x^2+1)^(1/2)/a^3-1/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1
)^(1/2))-1/a^4/(x+1/a)*(-(x+1/a)^2*a^2+2*(x+1/a)*a)^(1/2)
```

3.151.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx=-\frac{2\,ax-2\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}(ax+2)+2}{a^4x+a^3}$$

input
```
integrate(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-(2*a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x
^2 + 1)*(a*x + 2) + 2)/(a^4*x + a^3)
```

3.151.6 Sympy [F]

$$\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2}{\sqrt{-(ax-1)(ax+1)}(ax+1)}\,dx$$

input
```
integrate(x**2/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

3.151.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{-a^2x^2+1}}{a^4x+a^3} - \frac{\arcsin(ax)}{a^3} - \frac{\sqrt{-a^2x^2+1}}{a^3}$$

input
```
integrate(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-a^2*x^2 + 1)/(a^4*x + a^3) - arcsin(a*x)/a^3 - sqrt(-a^2*x^2 + 1)/a
^3
```

3.151.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{a^2|a|} - \frac{\sqrt{-a^2x^2+1}}{a^3} + \frac{2}{a^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

input
```
integrate(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-arcsin(a*x)*sgn(a)/(a^2*abs(a)) - sqrt(-a^2*x^2 + 1)/a^3 + 2/(a^2*((sqrt(
-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

3.151.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.53

$$\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{1-a^2\,x^2}}{\left(a\,\sqrt{-a^2}+a^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}} - \frac{\operatorname{asinh}\left(x\,\sqrt{-a^2}\right)}{a^2\,\sqrt{-a^2}} - \frac{\sqrt{1-a^2\,x^2}}{a^3}$$

input
```
int(x^2/((1 - a^2*x^2)^(1/2)*(a*x + 1)),x)
```

output
```
(1 - a^2*x^2)^(1/2)/((a*(-a^2)^(1/2) + a^2*x*(-a^2)^(1/2))*(-a^2)^(1/2)) -
 asinh(x*(-a^2)^(1/2))/(a^2*(-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/a^3
```

3.152 $\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx$

3.152.1 Optimal result

Integrand size = 23, antiderivative size = 34

$$\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{1-a^2x^2}}{a^2(1+ax)}+\frac{\arcsin(ax)}{a^2}$$

output
```
arcsin(a*x)/a^2+(-a^2*x^2+1)^(1/2)/a^2/(a*x+1)
```

3.152.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{1-a^2x^2}}{a^2(1+ax)}+\frac{2\arctan\left(\frac{ax}{-1+\sqrt{1-a^2x^2}}\right)}{a^2}$$

input
```
Integrate[x/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

output
```
Sqrt[1 - a^2*x^2]/(a^2*(1 + a*x)) + (2*ArcTan[(a*x)/(-1 + Sqrt[1 - a^2*x^2
])])/a^2
```

3.152.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {563, 25, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax+1)\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{563}$$

$$\frac{\sqrt{1-a^2x^2}}{a^2(ax+1)}-\frac{\int -\frac{1}{\sqrt{1-a^2x^2}}dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{a}+\frac{\sqrt{1-a^2x^2}}{a^2(ax+1)}$$

$$\Big\downarrow \text{223}$$

$$\frac{\arcsin(ax)}{a^2}+\frac{\sqrt{1-a^2x^2}}{a^2(ax+1)}$$

input
```
Int[x/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

output
```
Sqrt[1 - a^2*x^2]/(a^2*(1 + a*x)) + ArcSin[a*x]/a^2
```

3.152.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

3.152.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{a\sqrt{a^2}}+\frac{\sqrt{-\left(x+\frac{1}{a}\right)^2a^2+2\left(x+\frac{1}{a}\right)a}}{a^3\left(x+\frac{1}{a}\right)}$	65

input
```
int(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/a^3/(x+1/a)*(-(
x+1/a)^2*a^2+2*(x+1/a)*a)^(1/2)
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.71

$$\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx=\frac{ax-2\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}+1}{a^3x+a^2}$$

input
```
integrate(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
(a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2
+ 1) + 1)/(a^3*x + a^2)
```

3.152.6 Sympy [F]

$$\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \int \frac{x}{\sqrt{-(ax-1)(ax+1)}\,(ax+1)}\,dx$$

input
```
integrate(x/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

3.152.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{-a^2x^2+1}}{a^3x+a^2} + \frac{\arcsin(ax)}{a^2}$$

input
```
integrate(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(-a^2*x^2 + 1)/(a^3*x + a^2) + arcsin(a*x)/a^2
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.53

$$\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{\arcsin(ax)\,\mathrm{sgn}(a)}{a|a|} - \frac{2}{a\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

input
```
integrate(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
arcsin(a*x)*sgn(a)/(a*abs(a)) - 2/(a*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2
*x) + 1)*abs(a))
```

3.152.9 Mupad [B] (verification not implemented)

Time = 11.45 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{1}{a^2\sqrt{1-a^2\,x^2}} - \frac{x}{a\sqrt{1-a^2\,x^2}} - \frac{\operatorname{asinh}\left(x\sqrt{-a^2}\right)\,\sqrt{-a^2}}{a^3}$$

input
```
int(x/((1 - a^2*x^2)^(1/2)*(a*x + 1)),x)
```

output
```
1/(a^2*(1 - a^2*x^2)^(1/2)) - x/(a*(1 - a^2*x^2)^(1/2)) - (asinh(x*(-a^2)^
(1/2))*(-a^2)^(1/2))/a^3
```

3.153 $\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx$

3.153.1 Optimal result

Integrand size = 22, antiderivative size = 26

$$\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}}{a(1+ax)}$$

output
```
-(-a^2*x^2+1)^(1/2)/a/(a*x+1)
```

3.153.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}}{a(1+ax)}$$

input
```
Integrate[1/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

output
```
-(Sqrt[1 - a^2*x^2]/(a*(1 + a*x)))
```

3.153.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+1)\sqrt{1-a^2x^2}}\,dx$$
$$\Big\downarrow \text{460}$$
$$-\frac{\sqrt{1-a^2x^2}}{a(ax+1)}$$

input
```
Int[1/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

output
```
-(Sqrt[1 - a^2*x^2]/(a*(1 + a*x)))
```

3.153.3.1 Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

3.153.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

method	result	size
gosper	$\frac{ax-1}{a\sqrt{-a^2x^2+1}}$	22
trager	$-\frac{\sqrt{-a^2x^2+1}}{a(ax+1)}$	25
default	$-\frac{\sqrt{-\left(x+\frac{1}{a}\right)^2a^2+2\left(x+\frac{1}{a}\right)a}}{a^2\left(x+\frac{1}{a}\right)}$	36

input
```
int(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)/a/(-a^2*x^2+1)^(1/2)
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx = -\frac{ax+\sqrt{-a^2x^2+1}+1}{a^2x+a}$$

input
```
integrate(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-(a*x + sqrt(-a^2*x^2 + 1) + 1)/(a^2*x + a)
```

3.153.6 Sympy [F]

$$\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \int \frac{1}{\sqrt{-(ax-1)(ax+1)}(ax+1)}\,dx$$

input
```
integrate(1/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{-a^2x^2+1}}{a^2x+a}$$

input
```
integrate(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-a^2*x^2 + 1)/(a^2*x + a)
```

3.153.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{2}{\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

input
```
integrate(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
2/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

3.153.9 Mupad [B] (verification not implemented)

Time = 11.44 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2\,x^2}}{x\,a^2+a}$$

input
```
int(1/((1 - a^2*x^2)^(1/2)*(a*x + 1)),x)
```

output
```
-(1 - a^2*x^2)^(1/2)/(a + a^2*x)
```

3.154 $\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx$

3.154.1 Optimal result

Integrand size = 26, antiderivative size = 41

$$\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{1-a^2x^2}}{1-ax} - \operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

output
```
-arctanh((-a^2*x^2+1)^(1/2))+(-a^2*x^2+1)^(1/2)/(-a*x+1)
```

3.154.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}}{-1+ax} - \log(x) + \log\left(-1+\sqrt{1-a^2x^2}\right)$$

input
```
Integrate[1/(x*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

output
```
-(Sqrt[1 - a^2*x^2]/(-1 + a*x)) - Log[x] + Log[-1 + Sqrt[1 - a^2*x^2]]
```

3.154.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {564, 25, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{564}$$

$$\frac{\sqrt{1-a^2x^2}}{1-ax}-\int -\frac{1}{x\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{1}{x\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}}{1-ax}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2+\frac{\sqrt{1-a^2x^2}}{1-ax}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{1-a^2x^2}}{1-ax}-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{1-a^2x^2}}{1-ax}-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

input
```
Int[1/(x*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

output
```
Sqrt[1 - a^2*x^2]/(1 - a*x) - ArcTanh[Sqrt[1 - a^2*x^2]]
```

3.154.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

3.154.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

method	result	size
default	$-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{a\left(x-\frac{1}{a}\right)}$	58

input
```
int(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arctanh(1/(-a^2*x^2+1)^(1/2))-1/a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1
/2)
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27

$$\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx = \frac{ax + (ax-1)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \sqrt{-a^2x^2+1} - 1}{ax-1}$$

input
```
integrate(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
(a*x + (a*x - 1)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^2*x^2 + 1) - 1)
/(a*x - 1)
```

3.154.6 Sympy [F]

$$\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx = -\int \frac{1}{ax^2\sqrt{-a^2x^2+1} - x\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(1/x/(-a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

output
```
-Integral(1/(a*x**2*sqrt(-a**2*x**2 + 1) - x*sqrt(-a**2*x**2 + 1)), x)
```

3.154.7 Maxima [F]

$$\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx = \int -\frac{1}{\sqrt{-a^2x^2+1}(ax-1)x}\,dx$$

input
```
integrate(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x), x)
```

3.154.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.80

$$\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx = -\frac{a\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{|a|} + \frac{2a}{\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}$$

input
```
integrate(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) +
2*a/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

3.154.9 Mupad [B] (verification not implemented)

Time = 11.47 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx = \frac{a\sqrt{1-a^2\,x^2}}{\sqrt{-a^2}\,\left(\frac{a}{\sqrt{-a^2}}+x\sqrt{-a^2}\right)} - \operatorname{atanh}\left(\sqrt{1-a^2\,x^2}\right)$$

input
```
int(-1/(x*(1 - a^2*x^2)^(1/2)*(a*x - 1)),x)
```

output
```
(a*(1 - a^2*x^2)^(1/2))/((-a^2)^(1/2)*(a/(-a^2)^(1/2) + x*(-a^2)^(1/2))) -
 atanh((1 - a^2*x^2)^(1/2))
```

3.155 $\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx$

3.155.1 Optimal result

Integrand size = 26, antiderivative size = 64

$$\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx=-\frac{2\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x(1-ax)}-a\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

output
```
-a*arctanh((-a^2*x^2+1)^(1/2))-2*(-a^2*x^2+1)^(1/2)/x+(-a^2*x^2+1)^(1/2)/x
/(-a*x+1)
```

3.155.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx=-\frac{(-1+2ax)\sqrt{1-a^2x^2}}{x(-1+ax)}-a\log(x)+a\log\left(-1+\sqrt{1-a^2x^2}\right)$$

input
```
Integrate[1/(x^2*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

output
```
-(((-1 + 2*a*x)*Sqrt[1 - a^2*x^2])/(x*(-1 + a*x))) - a*Log[x] + a*Log[-1 +
 Sqrt[1 - a^2*x^2]]
```

3.155.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {564, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{564}$$

$$\frac{a\sqrt{1-a^2x^2}}{1-ax}-\int -\frac{ax+1}{x^2\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{ax+1}{x^2\sqrt{1-a^2x^2}}dx+\frac{a\sqrt{1-a^2x^2}}{1-ax}$$

$$\Big\downarrow \text{534}$$

$$a\int \frac{1}{x\sqrt{1-a^2x^2}}dx+\frac{a\sqrt{1-a^2x^2}}{1-ax}-\frac{\sqrt{1-a^2x^2}}{x}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2+\frac{a\sqrt{1-a^2x^2}}{1-ax}-\frac{\sqrt{1-a^2x^2}}{x}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}+\frac{a\sqrt{1-a^2x^2}}{1-ax}-\frac{\sqrt{1-a^2x^2}}{x}$$

$$\Big\downarrow \text{221}$$

$$a\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)+\frac{a\sqrt{1-a^2x^2}}{1-ax}-\frac{\sqrt{1-a^2x^2}}{x}$$

input
```
Int[1/(x^2*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

output
```
-(Sqrt[1 - a^2*x^2]/x) + (a*Sqrt[1 - a^2*x^2])/(1 - a*x) - a*ArcTanh[Sqrt[
1 - a^2*x^2]]
```

3.155.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

3.155. $\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx$

3.155.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{\sqrt{-a^2x^2+1}}{x}-a\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}$	73
risch	$\frac{a^2x^2-1}{x\sqrt{-a^2x^2+1}}-a\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{a\left(x-\frac{1}{a}\right)}\right)$	84

input
```
int(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-a^2*x^2+1)^(1/2)/x-a*arctanh(1/(-a^2*x^2+1)^(1/2))-1/(x-1/a)*(-a^2*(x-1
/a)^2-2*a*(x-1/a))^(1/2)
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx = \frac{a^2x^2-ax+(a^2x^2-ax)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\sqrt{-a^2x^2+1}(2\,ax-1)}{ax^2-x}$$

input
```
integrate(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
(a^2*x^2 - a*x + (a^2*x^2 - a*x)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a
^2*x^2 + 1)*(2*a*x - 1))/(a*x^2 - x)
```

3.155.6 Sympy [F]

$$\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx = -\int \frac{1}{ax^3\sqrt{-a^2x^2+1}-x^2\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(1/x**2/(-a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

output
```
-Integral(1/(a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x)
```

3.155.7 Maxima [F]

$$\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx = \int -\frac{1}{\sqrt{-a^2x^2+1}(ax-1)x^2}\,dx$$

input
```
integrate(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x^2), x)
```

3.155.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.155.9 Mupad [B] (verification not implemented)

Time = 11.47 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx = \frac{a^2\sqrt{1-a^2x^2}}{\left(x\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\sqrt{-a^2}} - \frac{\sqrt{1-a^2x^2}}{x} - a\,\mathrm{atanh}\left(\sqrt{1-a^2x^2}\right)$$

input
```
int(-1/(x^2*(1 - a^2*x^2)^(1/2)*(a*x - 1)),x)
```

output
```
(a^2*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
 - (1 - a^2*x^2)^(1/2)/x - a*atanh((1 - a^2*x^2)^(1/2))
```

3.156 $\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx$

3.156.1 Optimal result

Integrand size = 26, antiderivative size = 90

$$\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx = -\frac{3\sqrt{1-a^2x^2}}{2x^2}-\frac{2a\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x^2(1-ax)}-\frac{3}{2}a^2\text{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

output
```
-3/2*a^2*arctanh((-a^2*x^2+1)^(1/2))-3/2*(-a^2*x^2+1)^(1/2)/x^2-2*a*(-a^2*
x^2+1)^(1/2)/x+(-a^2*x^2+1)^(1/2)/x^2/(-a*x+1)
```

3.156.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx = \frac{1}{2}\left(\frac{(1+ax-4a^2x^2)\sqrt{1-a^2x^2}}{x^2(-1+ax)}-3a^2\log(x)+3a^2\log\left(-1+\sqrt{1-a^2x^2}\right)\right)$$

input
```
Integrate[1/(x^3*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

output
```
(((1 + a*x - 4*a^2*x^2)*Sqrt[1 - a^2*x^2])/(x^2*(-1 + a*x)) - 3*a^2*Log[x]
 + 3*a^2*Log[-1 + Sqrt[1 - a^2*x^2]])/2
```

3.156.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {564, 25, 2338, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{564}$$

$$\frac{a^2\sqrt{1-a^2x^2}}{1-ax}-\int -\frac{a^2x^2+ax+1}{x^3\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{a^2x^2+ax+1}{x^3\sqrt{1-a^2x^2}}dx+\frac{a^2\sqrt{1-a^2x^2}}{1-ax}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{1}{2}\int -\frac{a(3ax+2)}{x^2\sqrt{1-a^2x^2}}dx+\frac{a^2\sqrt{1-a^2x^2}}{1-ax}-\frac{\sqrt{1-a^2x^2}}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{a(3ax+2)}{x^2\sqrt{1-a^2x^2}}dx+\frac{a^2\sqrt{1-a^2x^2}}{1-ax}-\frac{\sqrt{1-a^2x^2}}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}a\int \frac{3ax+2}{x^2\sqrt{1-a^2x^2}}dx+\frac{a^2\sqrt{1-a^2x^2}}{1-ax}-\frac{\sqrt{1-a^2x^2}}{2x^2}$$

$$\Big\downarrow \text{534}$$

$$\frac{1}{2}a\left(3a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{2\sqrt{1-a^2x^2}}{x}\right)+\frac{a^2\sqrt{1-a^2x^2}}{1-ax}-\frac{\sqrt{1-a^2x^2}}{2x^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\left(\frac{3}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{2\sqrt{1-a^2x^2}}{x}\right)+\frac{a^2\sqrt{1-a^2x^2}}{1-ax}-\frac{\sqrt{1-a^2x^2}}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(-\frac{3\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{2\sqrt{1-a^2x^2}}{x}\right)+\frac{a^2\sqrt{1-a^2x^2}}{1-ax}-\frac{\sqrt{1-a^2x^2}}{2x^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}a\left(-3a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x}\right)+\frac{a^2\sqrt{1-a^2x^2}}{1-ax}-\frac{\sqrt{1-a^2x^2}}{2x^2}$$

input
```
Int[1/(x^3*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

output
```
-1/2*Sqrt[1 - a^2*x^2]/x^2 + (a^2*Sqrt[1 - a^2*x^2])/(1 - a*x) + (a*((-2*S
qrt[1 - a^2*x^2])/x - 3*a*ArcTanh[Sqrt[1 - a^2*x^2]]))/2
```

3.156.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.156. $\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx$

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.156.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.04

method	result	size
default	$-\frac{\sqrt{-a^2x^2+1}}{2x^2}-\frac{3a^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}-\frac{a\sqrt{-a^2x^2+1}}{x}-\frac{a\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}$	94
risch	$\frac{2a^3x^3+a^2x^2-2ax-1}{2x^2\sqrt{-a^2x^2+1}}+\frac{a^2\left(-3\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{a\left(x-\frac{1}{a}\right)}\right)}{2}$	102

input
```
int(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-a^2*x^2+1)^(1/2)/x^2-3/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))-a*(-a^2*
x^2+1)^(1/2)/x-a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

3.156.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx = \frac{2\,a^3x^3-2\,a^2x^2+3\left(a^3x^3-a^2x^2\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(4\,a^2x^2-ax-1\right)\sqrt{-a^2x^2+1}}{2\left(ax^3-x^2\right)}$$

input
```
integrate(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*a^3*x^3 - 2*a^2*x^2 + 3*(a^3*x^3 - a^2*x^2)*log((sqrt(-a^2*x^2 + 1)
 - 1)/x) - (4*a^2*x^2 - a*x - 1)*sqrt(-a^2*x^2 + 1))/(a*x^3 - x^2)
```

3.156.6 Sympy [F]

$$\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx = -\int \frac{1}{ax^4\sqrt{-a^2x^2+1}-x^3\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(1/x**3/(-a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

output
```
-Integral(1/(a*x**4*sqrt(-a**2*x**2 + 1) - x**3*sqrt(-a**2*x**2 + 1)), x)
```

3.156.7 Maxima [F]

$$\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx = \int -\frac{1}{\sqrt{-a^2x^2+1}(ax-1)x^3}\,dx$$

input
```
integrate(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x^3), x)
```

3.156.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(78) = 156$.

Time = 0.29 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.37

$$\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\left(a^3 + \frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)a}{x} - \frac{20\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{ax^2}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}|a|+a\right)^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|} - \frac{3\,a^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|} - \frac{\frac{4\left(\sqrt{-a^2x^2+1}|a|+a\right)a|a|}{x} + \frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)^2|a|}{ax^2}}{8\,a^2}$$

input
```
integrate(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/8*(a^3 + 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x - 20*(sqrt(-a^2*x^2 + 1)
*abs(a) + a)^2/(a*x^2))*a^4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*((sqrt(
-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a)) - 3/2*a^3*log(1/2*abs(-2*sq
rt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/8*(4*(sqrt(-a^2*x^
2 + 1)*abs(a) + a)*a*abs(a)/x + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*abs(a)/(
a*x^2))/a^2
```

3.156.9 Mupad [B] (verification not implemented)

Time = 11.44 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx = \frac{a^3\sqrt{1-a^2\,x^2}}{\left(x\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\sqrt{-a^2}} - \frac{a\sqrt{1-a^2\,x^2}}{x} - \frac{\sqrt{1-a^2\,x^2}}{2\,x^2} + \frac{a^2\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,3\mathrm{i}}{2}$$

input
```
int(-1/(x^3*(1 - a^2*x^2)^(1/2)*(a*x - 1)),x)
```

output
```
(a^2*atan((1 - a^2*x^2)^(1/2)*1i)*3i)/2 - (1 - a^2*x^2)^(1/2)/(2*x^2) - (a
*(1 - a^2*x^2)^(1/2))/x + (a^3*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) - (-a
^2)^(1/2)/a)*(-a^2)^(1/2))
```

3.157 $\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

3.157.1 Optimal result

Integrand size = 27, antiderivative size = 229

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = -\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5} - \frac{4d^4x^2(d^2-e^2x^2)^{3/2}}{21e^4} + \frac{5d^3x^3(d^2-e^2x^2)^{3/2}}{24e^3} - \frac{5d^2x^4(d^2-e^2x^2)^{3/2}}{21e^2} + \frac{dx^5(d^2-e^2x^2)^{3/2}}{4e} - \frac{1}{9}x^6(d^2-e^2x^2)^{3/2} - \frac{d^5(256d-315ex)(d^2-e^2x^2)^{3/2}}{2016e^6} - \frac{5d^9\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{64e^6}$$

output
```
-4/21*d^4*x^2*(-e^2*x^2+d^2)^(3/2)/e^4+5/24*d^3*x^3*(-e^2*x^2+d^2)^(3/2)/e
^3-5/21*d^2*x^4*(-e^2*x^2+d^2)^(3/2)/e^2+1/4*d*x^5*(-e^2*x^2+d^2)^(3/2)/e-
1/9*x^6*(-e^2*x^2+d^2)^(3/2)-1/2016*d^5*(-315*e*x+256*d)*(-e^2*x^2+d^2)^(3
/2)/e^6-5/64*d^9*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6-5/64*d^7*x*(-e^2*x^2
+d^2)^(1/2)/e^5
```

3.157.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.68

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{e\sqrt{d^2-e^2x^2}(-512d^8+315d^7ex-256d^6e^2x^2+210d^5e^3x^3-192d^4e^4x^4+168d^3e^5}{40}$$

input
```
Integrate[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

output
```
(e*Sqrt[d^2 - e^2*x^2]*(-512*d^8 + 315*d^7*e*x - 256*d^6*e^2*x^2 + 210*d^5
*e^3*x^3 - 192*d^4*e^4*x^4 + 168*d^3*e^5*x^5 + 512*d^2*e^6*x^6 - 1008*d*e^
7*x^7 + 448*e^8*x^8) - 315*d^9*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 -
 e^2*x^2]])/(4032*e^7)
```

3.157.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.23, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.741$, Rules used = {562, 541, 27, 533, 27, 533, 25, 27, 533, 27, 533, 25, 27, 533, 25, 27, 455, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}dx$$

$$\Big\downarrow \text{562}$$

$$\int x^5(d-ex)^2\sqrt{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{541}$$

$$-\frac{\int -3de^2x^5(5d-6ex)\sqrt{d^2-e^2x^2}dx}{9e^2}-\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}d\int x^5(5d-6ex)\sqrt{d^2-e^2x^2}dx-\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{3}d\left(\frac{\int -10dex^4(3d-4ex)\sqrt{d^2-e^2x^2}dx}{8e^2}+\frac{3x^5\left(d^2-e^2x^2\right)^{3/2}}{4e}\right)-\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}d\left(\frac{3x^5\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{5d\int x^4(3d-4ex)\sqrt{d^2-e^2x^2}dx}{4e}\right)-\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \ 533$$

$$\frac{1}{3}d\left(\frac{3x^5\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{5d\left(\frac{\int -dex^3(16d-21ex)\sqrt{d^2-e^2x^2}dx}{7e^2}+\frac{4x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}\right)}{4e}\right)-\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{3}d\left(\frac{3x^5\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{5d\left(\frac{4x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{\int dex^3(16d-21ex)\sqrt{d^2-e^2x^2}dx}{7e^2}\right)}{4e}\right)-\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}d\left(\frac{3x^5\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{5d\left(\frac{4x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\int x^3(16d-21ex)\sqrt{d^2-e^2x^2}dx}{7e}\right)}{4e}\right)-\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \ 533$$

$$\frac{1}{3}d\left(\frac{3x^5\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{5d\left(\frac{4x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{\int -3dex^2(21d-32ex)\sqrt{d^2-e^2x^2}dx}{6e^2}+\frac{7x^3\left(d^2-e^2x^2\right)^{3/2}}{2e}\right)}{7e}\right)}{4e}\right)-\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \ 27$$

3.157. $\int \frac{x^5\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

$$\frac{1}{3}d\left(\frac{3x^5\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{5d\left(\frac{4x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{7x^3\left(d^2-e^2x^2\right)^{3/2}}{2e}-\frac{d\int x^2(21d-32ex)\sqrt{d^2-e^2x^2}dx}{2e}\right)}{7e}\right)}{4e}\right)-\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \ 533$$

$$\frac{1}{3}d\left(\frac{3x^5\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{5d\left(\frac{4x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{7x^3\left(d^2-e^2x^2\right)^{3/2}}{2e}-\frac{d\left(\frac{\int -dex(64d-105ex)\sqrt{d^2-e^2x^2}dx}{5e^2}+\frac{32x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}\right)}{2e}\right)}{7e}\right)}{4e}\right)$$

$$\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \ 25$$

3.157. $\int \frac{x^5\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

$$\frac{1}{3}d\left(\frac{3x^5\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{5d\left(\frac{4x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{7x^3\left(d^2-e^2x^2\right)^{3/2}}{2e}-\frac{d\left(\frac{32x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{\int dex(64d-105ex)\sqrt{d^2-e^2x^2}dx}{5e^2}\right)}{2e}\right)}{7e}\right)}{4e}\right)$$

$$\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}d\left(\frac{3x^5\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{5d\left(\frac{4x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{7x^3\left(d^2-e^2x^2\right)^{3/2}}{2e}-\frac{d\left(\frac{32x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\int x(64d-105ex)\sqrt{d^2-e^2x^2}dx}{5e}\right)}{2e}\right)}{7e}\right)}{4e}\right)$$

$$\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \; 533$$

3.157. $\int \frac{x^5\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

$$\frac{1}{3}d\left(\frac{3x^5\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{5d\left(\frac{4x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{7x^3\left(d^2-e^2x^2\right)^{3/2}}{2e}-\frac{d\left(\frac{32x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{\int -de(105d-256ex)\sqrt{d^2-e^2x^2}dx}{4e^2}+10\cdots\right.}{5e}\right.}{2e}\right.}{7e}\right.}{4e}\right.$$

$$\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \; 25$$

3.157. $\int \frac{x^5\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

$$\frac{1}{3}d\left(\frac{3x^5\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{5d\left(\frac{4x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{7x^3\left(d^2-e^2x^2\right)^{3/2}}{2e}-\frac{d\left(\frac{32x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{105x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{\int de(105d-256}{4}\right.}{5e}\right.}{2e}\right.}{7e}\right.}{4e}\right.$$

$$\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \; 27$$

3.157. $\int \frac{x^5\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

$$\frac{1}{3}d\left(\frac{3x^5\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{5d\left(\frac{4x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{7x^3\left(d^2-e^2x^2\right)^{3/2}}{2e}-\frac{d\left(\frac{32x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{105x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\int(105d-256e\ldots}{4\ldots}\right.}{5e}\right.}{2e}\right.}{7e}\right.}{4e}\right.$$

$$\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}$$

$$\downarrow \text{455}$$

3.157. $\int \frac{x^5\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

$$\frac{1}{3}d\left(\frac{3x^5\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{5d\left(\frac{4x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{7x^3\left(d^2-e^2x^2\right)^{3/2}}{2e}-\frac{d\left(\frac{32x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{105x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\left(105d\int\sqrt{d^2-}\right.}{}\right.}{5e}\right.}{2e}\right.}{7e}\right.}{4e}\right.$$

3.157. $\int \frac{x^5\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

$\downarrow$ 211

3.157. $\int \frac{x^5 (d^2-e^2x^2)^{5/2}}{(d+ex)^2} \, dx$

$$\frac{1}{3}d\left(\frac{3x^5\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{5d\left(\frac{4x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{7x^3\left(d^2-e^2x^2\right)^{3/2}}{2e}-\frac{d\left(\frac{32x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{105x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\left(105d\left(\frac{1}{2}d^2\int\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{2e}\cdots\right.}{7e}\cdots\right.}{4e}\cdots$$

3.157. $\int \frac{x^5\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

$$\downarrow 224$$

3.157. $\int \frac{x^5 \left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

$$\frac{1}{3}d\left(\frac{3x^5\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{5d\left(\frac{4x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{7x^3\left(d^2-e^2x^2\right)^{3/2}}{2e}-\frac{d\left(\frac{32x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{105x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\left(105d\left(\frac{1}{2}d^2\int \cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{7e}\right.}{4e}\right.$$

3.157. $\int \frac{x^5\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

$\downarrow$ 216

3.157. $\int \frac{x^5 \left(d^2-e^2 x^2\right)^{5/2}}{(d+e x)^2} \, dx$

$$\frac{1}{3}d\left(\frac{3x^5\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{5d\left(\frac{4x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{7x^3\left(d^2-e^2x^2\right)^{3/2}}{2e}-\frac{d\left(\frac{32x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{105x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\left(105d\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{2e\cdots}\right.}{7e\cdots}\right.}{4e\cdots}\right.$$

3.157. $\int \frac{x^5\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

input
```
Int[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

output
```
-1/9*(x^6*(d^2 - e^2*x^2)^(3/2)) + (d*((3*x^5*(d^2 - e^2*x^2)^(3/2))/(4*e)
 - (5*d*((4*x^4*(d^2 - e^2*x^2)^(3/2))/(7*e) - (d*((7*x^3*(d^2 - e^2*x^2)^
(3/2))/(2*e) - (d*((32*x^2*(d^2 - e^2*x^2)^(3/2))/(5*e) - (d*((105*x*(d^2
- e^2*x^2)^(3/2))/(4*e) - (d*((256*(d^2 - e^2*x^2)^(3/2))/(3*e) + 105*d*((
x*Sqrt[d^2 - e^2*x^2])/2 + (d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)))
)/(4*e)))/(5*e)))/(2*e)))/(7*e)))/(4*e)))/3
```

3.157.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 562
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[c^(2*n)/a^n   Int[x^m*((a + b*x^2)^(n + p)/(c - d*x)^n), x], x]
/; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[m, 0] && ILtQ[n,
 0] && IGtQ[n + p + 1/2, 0]
```

3.157.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.62

method	result
risch	$-\frac{\left(-448e^{8}x^{8}+1008d\,e^{7}x^{7}-512d^{2}e^{6}x^{6}-168d^{3}e^{5}x^{5}+192d^{4}x^{4}e^{4}-210d^{5}e^{3}x^{3}+256d^{6}e^{2}x^{2}-315d^{7}ex+512d^{8}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{4032e^{6}}-\frac{5d^{9}\text{arc}\cdots}{}$
default	$\frac{-\frac{x^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{9e^{2}}-\frac{2d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{63e^{4}}}{e^{2}}-\frac{4d^{3}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}+\frac{5d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\cdots\right)}{4}\cdots\right)}{6}\cdots\right)}{e^{5}}\cdots$

input
```
int(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4032*(-448*e^8*x^8+1008*d*e^7*x^7-512*d^2*e^6*x^6-168*d^3*e^5*x^5+192*d
^4*e^4*x^4-210*d^5*e^3*x^3+256*d^6*e^2*x^2-315*d^7*e*x+512*d^8)/e^6*(-e^2*
x^2+d^2)^(1/2)-5/64*d^9/e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2
)^(1/2))
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.60

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{630\,d^9\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(448\,e^8x^8-1008\,de^7x^7+512\,d^2e^6x^6+168\,d^3e^5x^5\cdots\right.}{4032\,e^6\cdots}$$

input
```
integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/4032*(630*d^9*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (448*e^8*x^8 -
 1008*d*e^7*x^7 + 512*d^2*e^6*x^6 + 168*d^3*e^5*x^5 - 192*d^4*e^4*x^4 + 21
0*d^5*e^3*x^3 - 256*d^6*e^2*x^2 + 315*d^7*e*x - 512*d^8)*sqrt(-e^2*x^2 + d
^2))/e^6
```

3.157.6 Sympy [A] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.31

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = d^2\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{8d^6}{105e^6}-\frac{4d^4x^2}{105e^4}-\frac{d^2x^4}{35e^2}+\frac{x^6}{7}\right) & \text{for } e^2\neq 0\\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise}\end{cases}\right) - 2de\left(\begin{cases}\frac{5d^8\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{128e^6}+\sqrt{d^2-e^2x^2}\left(-\frac{5d^6x}{128e^6}-\frac{5d^4x^3}{192e^4}-\frac{d^2x^5}{48e^2}+\frac{x^7}{8}\right) & \text{for } e^2\neq 0\\ \frac{x^7\sqrt{d^2}}{7} & \text{otherwise}\end{cases}\right) + e^2\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{16d^8}{315e^8}-\frac{8d^6x^2}{315e^6}-\frac{2d^4x^4}{105e^4}-\frac{d^2x^6}{63e^2}+\frac{x^8}{9}\right) & \text{for } e^2\neq 0\\ \frac{x^8\sqrt{d^2}}{8} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**5*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

output
```
d**2*Piecewise((sqrt(d**2 - e**2*x**2)*(-8*d**6/(105*e**6) - 4*d**4*x**2/(
105*e**4) - d**2*x**4/(35*e**2) + x**6/7), Ne(e**2, 0)), (x**6*sqrt(d**2)/
6, True)) - 2*d*e*Piecewise((5*d**8*Piecewise((log(-2*e**2*x + 2*sqrt(-e**
2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2
*x**2), True))/(128*e**6) + sqrt(d**2 - e**2*x**2)*(-5*d**6*x/(128*e**6) -
 5*d**4*x**3/(192*e**4) - d**2*x**5/(48*e**2) + x**7/8), Ne(e**2, 0)), (x*
*7*sqrt(d**2)/7, True)) + e**2*Piecewise((sqrt(d**2 - e**2*x**2)*(-16*d**8
/(315*e**8) - 8*d**6*x**2/(315*e**6) - 2*d**4*x**4/(105*e**4) - d**2*x**6/
(63*e**2) + x**8/9), Ne(e**2, 0)), (x**8*sqrt(d**2)/8, True))
```

3.157.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.31

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = -\frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^5}{4\,(e^7x+de^6)} - \frac{5i\,d^9\arcsin\left(\frac{ex}{d}+2\right)}{4\,e^6} - \frac{85\,d^9\arcsin\left(\frac{ex}{d}\right)}{64\,e^6} + \frac{5\sqrt{e^2x^2+4\,dex+3\,d^2}d^7x}{4\,e^5} - \frac{85\sqrt{-e^2x^2+d^2}d^7x}{64\,e^5} + \frac{5\sqrt{e^2x^2+4\,dex+3\,d^2}d^8}{2\,e^6} + \frac{35\,(-e^2x^2+d^2)^{\frac{3}{2}}d^5x}{96\,e^5} - \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^6}{12\,e^6} - \frac{17\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3x}{24\,e^5} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}x^2}{9\,e^4} + \frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^4}{e^6} + \frac{(-e^2x^2+d^2)^{\frac{7}{2}}dx}{4\,e^5} - \frac{29\,(-e^2x^2+d^2)^{\frac{7}{2}}d^2}{63\,e^6}$$

input
```
integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-1/4*(-e^2*x^2 + d^2)^(5/2)*d^5/(e^7*x + d*e^6) - 5/4*I*d^9*arcsin(e*x/d +
 2)/e^6 - 85/64*d^9*arcsin(e*x/d)/e^6 + 5/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2
)*d^7*x/e^5 - 85/64*sqrt(-e^2*x^2 + d^2)*d^7*x/e^5 + 5/2*sqrt(e^2*x^2 + 4*
d*e*x + 3*d^2)*d^8/e^6 + 35/96*(-e^2*x^2 + d^2)^(3/2)*d^5*x/e^5 - 5/12*(-e
^2*x^2 + d^2)^(3/2)*d^6/e^6 - 17/24*(-e^2*x^2 + d^2)^(5/2)*d^3*x/e^5 - 1/9
*(-e^2*x^2 + d^2)^(7/2)*x^2/e^4 + (-e^2*x^2 + d^2)^(5/2)*d^4/e^6 + 1/4*(-e
^2*x^2 + d^2)^(7/2)*d*x/e^5 - 29/63*(-e^2*x^2 + d^2)^(7/2)*d^2/e^6
```

3.157.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.50

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{\left(161280\,d^{10}e^{10}\arctan\left(\sqrt{\frac{2\,d}{ex+d}-1}\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e) + \frac{\left(315\,d^{10}e^{10}\left(\frac{2\,d}{ex+d}-1\right)^{\frac{17}{2}}\operatorname{sgn}\left(\frac{1}{ex+\cdots}\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

output
```
1/1032192*(161280*d^10*e^10*arctan(sqrt(2*d/(e*x + d) - 1))*sgn(1/(e*x + d
))*sgn(e) + (315*d^10*e^10*(2*d/(e*x + d) - 1)^(17/2)*sgn(1/(e*x + d))*sgn
(e) - 18774*d^10*e^10*(2*d/(e*x + d) - 1)^(15/2)*sgn(1/(e*x + d))*sgn(e) +
 10458*d^10*e^10*(2*d/(e*x + d) - 1)^(13/2)*sgn(1/(e*x + d))*sgn(e) - 6895
8*d^10*e^10*(2*d/(e*x + d) - 1)^(11/2)*sgn(1/(e*x + d))*sgn(e) - 8192*d^10
*e^10*(2*d/(e*x + d) - 1)^(9/2)*sgn(1/(e*x + d))*sgn(e) - 32418*d^10*e^10*
(2*d/(e*x + d) - 1)^(7/2)*sgn(1/(e*x + d))*sgn(e) - 10458*d^10*e^10*(2*d/(
e*x + d) - 1)^(5/2)*sgn(1/(e*x + d))*sgn(e) - 2730*d^10*e^10*(2*d/(e*x + d
) - 1)^(3/2)*sgn(1/(e*x + d))*sgn(e) - 315*d^10*e^10*sqrt(2*d/(e*x + d) -
1)*sgn(1/(e*x + d))*sgn(e))*(e*x + d)^9/d^9)*abs(e)/(d*e^17)
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \int \frac{x^5\,(d^2-e^2\,x^2)^{5/2}}{(d+e\,x)^2}\,dx$$

input
```
int((x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

output
```
int((x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

3.158 $\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

3.158.1 Optimal result

Integrand size = 27, antiderivative size = 200

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{8d^3x^2(d^2-e^2x^2)^{3/2}}{35e^3} - \frac{13d^2x^3(d^2-e^2x^2)^{3/2}}{48e^2}+\frac{2dx^4(d^2-e^2x^2)^{3/2}}{7e} - \frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2}+\frac{d^4(1024d-1365ex)\left(d^2-e^2x^2\right)^{3/2}}{6720e^5}+\frac{13d^8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}$$

output
```
8/35*d^3*x^2*(-e^2*x^2+d^2)^(3/2)/e^3-13/48*d^2*x^3*(-e^2*x^2+d^2)^(3/2)/e
^2+2/7*d*x^4*(-e^2*x^2+d^2)^(3/2)/e-1/8*x^5*(-e^2*x^2+d^2)^(3/2)+1/6720*d^
4*(-1365*e*x+1024*d)*(-e^2*x^2+d^2)^(3/2)/e^5+13/128*d^8*arctan(e*x/(-e^2*
x^2+d^2)^(1/2))/e^5+13/128*d^6*x*(-e^2*x^2+d^2)^(1/2)/e^4
```

3.158.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.68

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{\sqrt{d^2-e^2x^2}(2048d^7-1365d^6ex+1024d^5e^2x^2-910d^4e^3x^3+768d^3e^4x^4+1960d^2e}{13440e^5}$$

input
```
Integrate[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(2048*d^7 - 1365*d^6*e*x + 1024*d^5*e^2*x^2 - 910*d^4
*e^3*x^3 + 768*d^3*e^4*x^4 + 1960*d^2*e^5*x^5 - 3840*d*e^6*x^6 + 1680*e^7*
x^7) - 2730*d^8*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(13440*e^
5)
```

3.158.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.23, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.704$, Rules used = {562, 541, 25, 27, 533, 25, 27, 533, 27, 533, 25, 27, 533, 25, 27, 455, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(d^2-e^2 x^2\right)^{5/2}}{(d+e x)^2} \, dx$$

$$\Big\downarrow \text{562}$$

$$\int x^4 (d-e x)^2 \sqrt{d^2-e^2 x^2}dx$$

$$\Big\downarrow \text{541}$$

$$-\frac{\int -d e^2 x^4 (13 d-16 e x) \sqrt{d^2-e^2 x^2}dx}{8 e^2}-\frac{1}{8} x^5 \left(d^2-e^2 x^2\right)^{3/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int d e^2 x^4 (13 d-16 e x) \sqrt{d^2-e^2 x^2}dx}{8 e^2}-\frac{1}{8} x^5 \left(d^2-e^2 x^2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8} d \int x^4 (13 d-16 e x) \sqrt{d^2-e^2 x^2}dx-\frac{1}{8} x^5 \left(d^2-e^2 x^2\right)^{3/2}$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{8} d \left(\frac{\int -d e x^3 (64 d-91 e x) \sqrt{d^2-e^2 x^2}dx}{7 e^2}+\frac{16 x^4 \left(d^2-e^2 x^2\right)^{3/2}}{7 e}\right)-\frac{1}{8} x^5 \left(d^2-e^2 x^2\right)^{3/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{8} d \left(\frac{16 x^4 \left(d^2-e^2 x^2\right)^{3/2}}{7 e}-\frac{\int d e x^3 (64 d-91 e x) \sqrt{d^2-e^2 x^2}dx}{7 e^2}\right)-\frac{1}{8} x^5 \left(d^2-e^2 x^2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}d\left(\frac{16x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\int x^3(64d-91ex)\sqrt{d^2-e^2x^2}dx}{7e}\right)-\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{8}d\left(\frac{16x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{\int -3dex^2(91d-128ex)\sqrt{d^2-e^2x^2}dx}{6e^2}+\frac{91x^3\left(d^2-e^2x^2\right)^{3/2}}{6e}\right)}{7e}\right)-\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}d\left(\frac{16x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{91x^3\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{d\int x^2(91d-128ex)\sqrt{d^2-e^2x^2}dx}{2e}\right)}{7e}\right)-\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{8}d\left(\frac{16x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{91x^3\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{d\left(\frac{\int -dex(256d-455ex)\sqrt{d^2-e^2x^2}dx}{5e^2}+\frac{128x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}\right)}{2e}\right)}{7e}\right)-\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{8}d\left(\frac{16x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{91x^3\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{d\left(\frac{128x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{\int dex(256d-455ex)\sqrt{d^2-e^2x^2}dx}{5e^2}\right)}{2e}\right)}{7e}\right)-\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2}$$

3.158. $\int \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

$$\Big\downarrow \ \color{blue}{27}$$

$$\frac{1}{8}d\left(\frac{16x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{91x^3\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{d\left(\frac{128x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\int x(256d-455ex)\sqrt{d^2-e^2x^2}dx}{5e}\right)}{2e}\right)}{7e}\right)-$$

$$\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \ \color{blue}{533}$$

$$\frac{1}{8}d\left(\frac{16x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{91x^3\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{d\left(\frac{128x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{\int -de(455d-1024ex)\sqrt{d^2-e^2x^2}dx}{4e^2}+\frac{455x\left(d^2-e^2x^2\right)^{3/2}}{4e}\right)}{5e}\right.}{2e}\right.}{7e}\right.$$

$$\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \ \color{blue}{25}$$

3.158. $\int \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

$$\frac{1}{8}d\left(\frac{16x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{91x^3\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{d\left(\frac{128x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{455x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{\int de(455d-1024ex)\sqrt{d^2-e^2x^2}dx}{4e^2}\right)}{5e}\right)}{2e}\right)}{7e}\right)$$

$$\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2}$$

$\downarrow$ 27

$$\frac{1}{8}d\left(\frac{16x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{91x^3\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{d\left(\frac{128x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{455x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\int(455d-1024ex)\sqrt{d^2-e^2x^2}dx}{4e}\right)}{5e}\right)}{2e}\right)}{7e}\right)$$

$$\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2}$$

$\downarrow$ 455

3.158. $\int \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

$$\frac{1}{8}d\left(\frac{16x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{91x^3\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{d\left(\frac{128x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{455x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\left(455d\int\sqrt{d^2-e^2x^2}dx+\frac{1024\left(d^2-e^2\cdots\right.}{3e}\cdots\right)}{4e}\cdots\right)}{5e}\cdots\right)}{2e}\cdots\right)}{7e}\cdots\right)$$

$$\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \; 211$$

3.158. $\int \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

$$\frac{1}{8}d\left(\frac{16x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{91x^3\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{d\left(\frac{128x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{455x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\left(455d\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{\cdots}\right.\right.}{4e}\right.}{5e}\right.}{2e}\right.}{7e}\right.$$

$$\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2}$$

$\downarrow$ 224

3.158. $\int\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

$$\frac{1}{8}d\left(\frac{16x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{91x^3\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{d\left(\frac{128x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{455x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\left(455d\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-}}\right.\right.}{}\right.}{5e}\right.}{2e}\right.}{7e}\right.$$

$$\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2}$$

$\downarrow$ 216

3.158. $\int \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

$$\frac{1}{8}d\left(\frac{16x^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{d\left(\frac{91x^3\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{d\left(\frac{128x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{455x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\left(455d\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}\cdots\right.\right.}{\cdots}\right.}{5e}\right.}{2e}\right.}{7e}\right.$$

$$\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2}$$

input

```
Int[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

3.158. $\int \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

output
```
-1/8*(x^5*(d^2 - e^2*x^2)^(3/2)) + (d*((16*x^4*(d^2 - e^2*x^2)^(3/2))/(7*e
) - (d*((91*x^3*(d^2 - e^2*x^2)^(3/2))/(6*e) - (d*((128*x^2*(d^2 - e^2*x^2
)^(3/2))/(5*e) - (d*((455*x*(d^2 - e^2*x^2)^(3/2))/(4*e) - (d*((1024*(d^2
- e^2*x^2)^(3/2))/(3*e) + 455*d*((x*Sqrt[d^2 - e^2*x^2])/2 + (d^2*ArcTan[(
e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e))))/(4*e)))/(5*e)))/(2*e)))/(7*e)))/8
```

3.158.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.158. $\int \frac{x^4 (d^2-e^2x^2)^{5/2}}{(d+ex)^2} \, dx$

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 562
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[c^(2*n)/a^n   Int[x^m*((a + b*x^2)^(n + p)/(c - d*x)^n), x], x]
/; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[m, 0] && ILtQ[n,
 0] && IGtQ[n + p + 1/2, 0]
```

3.158.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.65

method	result
risch	$\frac{\left(1680e^{7}x^{7}-3840d\,e^{6}x^{6}+1960d^{2}e^{5}x^{5}+768d^{3}e^{4}x^{4}-910d^{4}e^{3}x^{3}+1024d^{5}e^{2}x^{2}-1365d^{6}ex+2048d^{7}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{13440e^{5}}+\frac{13d^{8}\arctan\left(\cdots\right)}{128e^{4}\sqrt{e\cdots}}$
default	$\frac{-\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{8e^{2}}+\frac{d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}+\frac{5d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)}{4}\right)}{6}\right)}{8e^{2}}}{e^{2}}+\frac{3d^{2}\left(\frac{x\left(-e^{2}x\cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/13440*(1680*e^7*x^7-3840*d*e^6*x^6+1960*d^2*e^5*x^5+768*d^3*e^4*x^4-910*
d^4*e^3*x^3+1024*d^5*e^2*x^2-1365*d^6*e*x+2048*d^7)/e^5*(-e^2*x^2+d^2)^(1/
2)+13/128*d^8/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.64

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = -\frac{2730\,d^8\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(1680\,e^7x^7-3840\,de^6x^6+1960\,d^2e^5x^5+768\,d^3e^4x^4-910\,d^4e^3x^3+10}{13440\,e^5}$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/13440*(2730*d^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (1680*e^7*x
^7 - 3840*d*e^6*x^6 + 1960*d^2*e^5*x^5 + 768*d^3*e^4*x^4 - 910*d^4*e^3*x^3
 + 1024*d^5*e^2*x^2 - 1365*d^6*e*x + 2048*d^7)*sqrt(-e^2*x^2 + d^2))/e^5
```

3.158.6 Sympy [A] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.70

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = d^2\left(\begin{cases}\frac{d^6\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{16e^4}+\sqrt{d^2-e^2x^2}\left(-\frac{d^4x}{16e^4}-\frac{d^2x^3}{24e^2}+\frac{x}{6}\right. \\ \frac{x^5\sqrt{d^2}}{5}\end{cases}\right.$$
$$-2de\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{8d^6}{105e^6}-\frac{4d^4x^2}{105e^4}-\frac{d^2x^4}{35e^2}+\frac{x^6}{7}\right) & \text{for } e^2\neq 0\\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise}\end{cases}\right)$$
$$+e^2\left(\begin{cases}\frac{5d^8\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{128e^6}+\sqrt{d^2-e^2x^2}\left(-\frac{5d^6x}{128e^6}-\frac{5d^4x^3}{192e^4}-\frac{d^2x^5}{48e^2}+\frac{x^7}{8}\right) & \text{for } e^2\neq 0\\ \frac{x^7\sqrt{d^2}}{7} & \text{otherwise}\end{cases}\right.$$

input
```
integrate(x**4*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

output
```
d**2*Piecewise((d**6*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 -
e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/
(16*e**4) + sqrt(d**2 - e**2*x**2)*(-d**4*x/(16*e**4) - d**2*x**3/(24*e**2
) + x**5/6), Ne(e**2, 0)), (x**5*sqrt(d**2)/5, True)) - 2*d*e*Piecewise((s
qrt(d**2 - e**2*x**2)*(-8*d**6/(105*e**6) - 4*d**4*x**2/(105*e**4) - d**2*
x**4/(35*e**2) + x**6/7), Ne(e**2, 0)), (x**6*sqrt(d**2)/6, True)) + e**2*
Piecewise((5*d**8*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**
2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(12
8*e**6) + sqrt(d**2 - e**2*x**2)*(-5*d**6*x/(128*e**6) - 5*d**4*x**3/(192*
e**4) - d**2*x**5/(48*e**2) + x**7/8), Ne(e**2, 0)), (x**7*sqrt(d**2)/7, T
rue))
```

3.158.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.38

$$\begin{aligned}\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx &= \frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^4}{4\,(e^6x+de^5)} + \frac{7i\,d^8\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^5} \\ &+ \frac{125\,d^8\arcsin\left(\frac{ex}{d}\right)}{128\,e^5} - \frac{7\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^6x}{8\,e^4} + \frac{125\,\sqrt{-e^2x^2+d^2}d^6x}{128\,e^4} \\ &- \frac{7\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^7}{4\,e^5} - \frac{67\,(-e^2x^2+d^2)^{\frac{3}{2}}d^4x}{192\,e^4} + \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^5}{12\,e^5} \\ &+ \frac{25\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2x}{48\,e^4} - \frac{4\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3}{5\,e^5} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}x}{8\,e^4} + \frac{2\,(-e^2x^2+d^2)^{\frac{7}{2}}d}{7\,e^5}\end{aligned}$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
1/4*(-e^2*x^2 + d^2)^(5/2)*d^4/(e^6*x + d*e^5) + 7/8*I*d^8*arcsin(e*x/d +
2)/e^5 + 125/128*d^8*arcsin(e*x/d)/e^5 - 7/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^
2)*d^6*x/e^4 + 125/128*sqrt(-e^2*x^2 + d^2)*d^6*x/e^4 - 7/4*sqrt(e^2*x^2 +
 4*d*e*x + 3*d^2)*d^7/e^5 - 67/192*(-e^2*x^2 + d^2)^(3/2)*d^4*x/e^4 + 5/12
*(-e^2*x^2 + d^2)^(3/2)*d^5/e^5 + 25/48*(-e^2*x^2 + d^2)^(5/2)*d^2*x/e^4 -
 4/5*(-e^2*x^2 + d^2)^(5/2)*d^3/e^5 - 1/8*(-e^2*x^2 + d^2)^(7/2)*x/e^4 + 2
/7*(-e^2*x^2 + d^2)^(7/2)*d/e^5
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.56

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx =$$

$$-\frac{\left(349440\,d^9e^9\arctan\left(\sqrt{\frac{2\,d}{ex+d}-1}\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)+\frac{\left(1365\,d^9e^9\left(\frac{2\,d}{ex+d}-1\right)^{\frac{15}{2}}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)-61215\,d^9e^9\left(\frac{2\,d}{ex+d}-1\right)^{\frac{1}{}}\right.}{}\right.}{}$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

output
```
-1/1720320*(349440*d^9*e^9*arctan(sqrt(2*d/(e*x + d) - 1))*sgn(1/(e*x + d)
)*sgn(e) + (1365*d^9*e^9*(2*d/(e*x + d) - 1)^(15/2)*sgn(1/(e*x + d))*sgn(e
) - 61215*d^9*e^9*(2*d/(e*x + d) - 1)^(13/2)*sgn(1/(e*x + d))*sgn(e) + 205
17*d^9*e^9*(2*d/(e*x + d) - 1)^(11/2)*sgn(1/(e*x + d))*sgn(e) - 141159*d^9
*e^9*(2*d/(e*x + d) - 1)^(9/2)*sgn(1/(e*x + d))*sgn(e) - 34969*d^9*e^9*(2*
d/(e*x + d) - 1)^(7/2)*sgn(1/(e*x + d))*sgn(e) - 34853*d^9*e^9*(2*d/(e*x +
 d) - 1)^(5/2)*sgn(1/(e*x + d))*sgn(e) - 10465*d^9*e^9*(2*d/(e*x + d) - 1)
^(3/2)*sgn(1/(e*x + d))*sgn(e) - 1365*d^9*e^9*sqrt(2*d/(e*x + d) - 1)*sgn(
1/(e*x + d))*sgn(e))*(e*x + d)^8/d^8)*abs(e)/(d*e^15)
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \int \frac{x^4\,\left(d^2-e^2\,x^2\right)^{5/2}}{\left(d+e\,x\right)^2}\,dx$$

input
```
int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

output
```
int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

3.159 $\int \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

3.159.1 Optimal result

Integrand size = 27, antiderivative size = 171

$$\int \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = -\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3}-\frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2}+\frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}-\frac{d^3(88d-105ex)\left(d^2-e^2x^2\right)^{3/2}}{420e^4}-\frac{d^7\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4}$$

output
```
-11/35*d^2*x^2*(-e^2*x^2+d^2)^(3/2)/e^2+1/3*d*x^3*(-e^2*x^2+d^2)^(3/2)/e-1
/7*x^4*(-e^2*x^2+d^2)^(3/2)-1/420*d^3*(-105*e*x+88*d)*(-e^2*x^2+d^2)^(3/2)
/e^4-1/8*d^7*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4-1/8*d^5*x*(-e^2*x^2+d^2)
^(1/2)/e^3
```

3.159.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.78

$$\int \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = \frac{e\sqrt{d^2-e^2x^2}\left(-176d^6+105d^5ex-88d^4e^2x^2+70d^3e^3x^3+144d^2e^4x^4-280de^5x^5 \right.}{840e^5}$$

input
```
Integrate[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

output
```
(e*Sqrt[d^2 - e^2*x^2]*(-176*d^6 + 105*d^5*e*x - 88*d^4*e^2*x^2 + 70*d^3*e
^3*x^3 + 144*d^2*e^4*x^4 - 280*d*e^5*x^5 + 120*e^6*x^6) - 105*d^7*Sqrt[-e^
2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(840*e^5)
```

3.159.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.22, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$, Rules used = {562, 541, 25, 27, 533, 27, 533, 25, 27, 533, 25, 27, 455, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(d^2-e^2 x^2\right)^{5/2}}{(d+e x)^2} \, dx$$

$$\Big\downarrow \ 562$$

$$\int x^3 (d-e x)^2 \sqrt{d^2-e^2 x^2}dx$$

$$\Big\downarrow \ 541$$

$$-\frac{\int -d e^2 x^3 (11 d-14 e x) \sqrt{d^2-e^2 x^2}dx}{7 e^2}-\frac{1}{7} x^4 \left(d^2-e^2 x^2\right)^{3/2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int d e^2 x^3 (11 d-14 e x) \sqrt{d^2-e^2 x^2}dx}{7 e^2}-\frac{1}{7} x^4 \left(d^2-e^2 x^2\right)^{3/2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{7} d \int x^3 (11 d-14 e x) \sqrt{d^2-e^2 x^2}dx-\frac{1}{7} x^4 \left(d^2-e^2 x^2\right)^{3/2}$$

$$\Big\downarrow \ 533$$

$$\frac{1}{7} d \left(\frac{\int -6 d e x^2 (7 d-11 e x) \sqrt{d^2-e^2 x^2}dx}{6 e^2}+\frac{7 x^3 \left(d^2-e^2 x^2\right)^{3/2}}{3 e}\right)-\frac{1}{7} x^4 \left(d^2-e^2 x^2\right)^{3/2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{7} d \left(\frac{7 x^3 \left(d^2-e^2 x^2\right)^{3/2}}{3 e}-\frac{d \int x^2 (7 d-11 e x) \sqrt{d^2-e^2 x^2}dx}{e}\right)-\frac{1}{7} x^4 \left(d^2-e^2 x^2\right)^{3/2}$$

$$\Big\downarrow \ 533$$

$$\frac{1}{7}d\left(\frac{7x^3\left(d^2-e^2x^2\right)^{3/2}}{3e}-\frac{d\left(\frac{\int -dex(22d-35ex)\sqrt{d^2-e^2x^2}dx}{5e^2}+\frac{11x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}\right)}{e}\right)-\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{7}d\left(\frac{7x^3\left(d^2-e^2x^2\right)^{3/2}}{3e}-\frac{d\left(\frac{11x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{\int dex(22d-35ex)\sqrt{d^2-e^2x^2}dx}{5e^2}\right)}{e}\right)-\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}d\left(\frac{7x^3\left(d^2-e^2x^2\right)^{3/2}}{3e}-\frac{d\left(\frac{11x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\int x(22d-35ex)\sqrt{d^2-e^2x^2}dx}{5e}\right)}{e}\right)-\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{7}d\left(\frac{7x^3\left(d^2-e^2x^2\right)^{3/2}}{3e}-\frac{d\left(\frac{11x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{\int -de(35d-88ex)\sqrt{d^2-e^2x^2}dx}{4e^2}+\frac{35x\left(d^2-e^2x^2\right)^{3/2}}{4e}\right)}{5e}\right)}{e}\right)-\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{7}d\left(\frac{7x^3\left(d^2-e^2x^2\right)^{3/2}}{3e}-\frac{d\left(\frac{11x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{35x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{\int de(35d-88ex)\sqrt{d^2-e^2x^2}dx}{4e^2}\right)}{5e}\right)}{e}\right)-\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

3.159. $\int \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

$$\frac{1}{7}d\left(\frac{7x^3\left(d^2-e^2x^2\right)^{3/2}}{3e}-\frac{d\left(\frac{11x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{35x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\int(35d-88ex)\sqrt{d^2-e^2x^2}dx}{4e}\right)}{5e}\right)}{e}\right)-\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}$$

$\downarrow$ 455

$$\frac{1}{7}d\left(\frac{7x^3\left(d^2-e^2x^2\right)^{3/2}}{3e}-\frac{d\left(\frac{11x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{35x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\left(35d\int\sqrt{d^2-e^2x^2}dx+\frac{88\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)}{4e}\right)}{5e}\right)}{e}\right)-\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}$$

$\downarrow$ 211

3.159. $\int\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

$$\frac{1}{7}d\left(\frac{7x^3\left(d^2-e^2x^2\right)^{3/2}}{3e}-\frac{d\left(\frac{11x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{35x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\left(35d\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{88\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)}{4e}\right)}{5e}\right)}{e}\right)$$

$$\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}$$

$\downarrow$ 224

$$\frac{1}{7}d\left(\frac{7x^3\left(d^2-e^2x^2\right)^{3/2}}{3e}-\frac{d\left(\frac{11x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{35x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\left(35d\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{88\left(d^2\right.}{}\right)}{4e}\right)}{5e}\right)}{e}\right)$$

$$\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}$$

$\downarrow$ 216

3.159. $\int\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

$$\frac{1}{7}d\left(\frac{7x^3\left(d^2-e^2x^2\right)^{3/2}}{3e}-\frac{d\left(\frac{11x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{35x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\left(35d\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{88\left(d^2-e^2x^2\right)}{3e}}{4e}\right)}{5e}\right)}{e}\right.$$

$$\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}$$

input
```
Int[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

output
```
-1/7*(x^4*(d^2 - e^2*x^2)^(3/2)) + (d*((7*x^3*(d^2 - e^2*x^2)^(3/2))/(3*e)
 - (d*((11*x^2*(d^2 - e^2*x^2)^(3/2))/(5*e) - (d*((35*x*(d^2 - e^2*x^2)^(3
/2))/(4*e) - (d*((88*(d^2 - e^2*x^2)^(3/2))/(3*e) + 35*d*((x*Sqrt[d^2 - e^
2*x^2])/2 + (d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e))))/(4*e)))/(5*e)
))/e))/7
```

3.159.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 562
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[c^(2*n)/a^n   Int[x^m*((a + b*x^2)^(n + p)/(c - d*x)^n), x], x]
/; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[m, 0] && ILtQ[n,
 0] && IGtQ[n + p + 1/2, 0]
```

3.159. $\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

3.159.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{(-120e^6x^6+280d\,e^5x^5-144d^2e^4x^4-70d^3x^3e^3+88d^4e^2x^2-105d^5ex+176d^6)\sqrt{-e^2x^2+d^2}}{840e^4}-\frac{d^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8e^3\sqrt{e^2}}$
default	$-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{7e^4}-\frac{2d\left(\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6}\right)}{e^3}-\frac{d^3\left(\frac{(-(x}{\ldots}\right.}{\ldots}$

input
```
int(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/840*(-120*e^6*x^6+280*d*e^5*x^5-144*d^2*e^4*x^4-70*d^3*e^3*x^3+88*d^4*e
^2*x^2-105*d^5*e*x+176*d^6)/e^4*(-e^2*x^2+d^2)^(1/2)-1/8*d^7/e^3/(e^2)^(1/
2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.68

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{210\,d^7\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(120\,e^6x^6-280\,de^5x^5+144\,d^2e^4x^4+70\,d^3e^3x^3-\ldots}{840\,e^4}$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/840*(210*d^7*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (120*e^6*x^6 -
280*d*e^5*x^5 + 144*d^2*e^4*x^4 + 70*d^3*e^3*x^3 - 88*d^4*e^2*x^2 + 105*d^
5*e*x - 176*d^6)*sqrt(-e^2*x^2 + d^2))/e^4
```

3.159.6 Sympy [A] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.49

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = d^2\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{2d^4}{15e^4}-\frac{d^2x^2}{15e^2}+\frac{x^4}{5}\right) & \text{for } e^2\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right) - 2de\left(\begin{cases}\frac{d^6\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{16e^4}+\sqrt{d^2-e^2x^2}\left(-\frac{d^4x}{16e^4}-\frac{d^2x^3}{24e^2}+\frac{x^5}{6}\right) & \text{for } e^2\neq 0\\ \frac{x^5\sqrt{d^2}}{5} & \text{otherwise}\end{cases}\right) + e^2\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{8d^6}{105e^6}-\frac{4d^4x^2}{105e^4}-\frac{d^2x^4}{35e^2}+\frac{x^6}{7}\right) & \text{for } e^2\neq 0\\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**3*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

output
```
d**2*Piecewise((sqrt(d**2 - e**2*x**2)*(-2*d**4/(15*e**4) - d**2*x**2/(15*
e**2) + x**4/5), Ne(e**2, 0)), (x**4*sqrt(d**2)/4, True)) - 2*d*e*Piecewis
e((d**6*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/s
qrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(16*e**4) + s
qrt(d**2 - e**2*x**2)*(-d**4*x/(16*e**4) - d**2*x**3/(24*e**2) + x**5/6),
Ne(e**2, 0)), (x**5*sqrt(d**2)/5, True)) + e**2*Piecewise((sqrt(d**2 - e**
2*x**2)*(-8*d**6/(105*e**6) - 4*d**4*x**2/(105*e**4) - d**2*x**4/(35*e**2)
 + x**6/7), Ne(e**2, 0)), (x**6*sqrt(d**2)/6, True))
```

3.159.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.47

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = -\frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^3}{4\,(e^5x+de^4)} - \frac{i\,d^7\arcsin\left(\frac{ex}{d}+2\right)}{2\,e^4} - \frac{5\,d^7\arcsin\left(\frac{ex}{d}\right)}{8\,e^4} + \frac{\sqrt{e^2x^2+4\,dex+3\,d^2}d^5x}{2\,e^3} - \frac{5\sqrt{-e^2x^2+d^2}d^5x}{8\,e^3} + \frac{\sqrt{e^2x^2+4\,dex+3\,d^2}d^6}{e^4} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d^3x}{3\,e^3} - \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^4}{12\,e^4} - \frac{(-e^2x^2+d^2)^{\frac{5}{2}}dx}{3\,e^3} + \frac{3\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2}{5\,e^4} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{7\,e^4}$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-1/4*(-e^2*x^2 + d^2)^(5/2)*d^3/(e^5*x + d*e^4) - 1/2*I*d^7*arcsin(e*x/d +
 2)/e^4 - 5/8*d^7*arcsin(e*x/d)/e^4 + 1/2*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*
d^5*x/e^3 - 5/8*sqrt(-e^2*x^2 + d^2)*d^5*x/e^3 + sqrt(e^2*x^2 + 4*d*e*x +
3*d^2)*d^6/e^4 + 1/3*(-e^2*x^2 + d^2)^(3/2)*d^3*x/e^3 - 5/12*(-e^2*x^2 + d
^2)^(3/2)*d^4/e^4 - 1/3*(-e^2*x^2 + d^2)^(5/2)*d*x/e^3 + 3/5*(-e^2*x^2 + d
^2)^(5/2)*d^2/e^4 - 1/7*(-e^2*x^2 + d^2)^(7/2)/e^4
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.64

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{\left(13440\,d^8e^8\arctan\left(\sqrt{\frac{2\,d}{ex+d}-1}\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e) + \frac{\left(105\,d^8e^8\left(\frac{2\,d}{ex+d}-1\right)^{\frac{13}{2}}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sg}\right.}{}\right.}{}$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

output
```
1/53760*(13440*d^8*e^8*arctan(sqrt(2*d/(e*x + d) - 1))*sgn(1/(e*x + d))*sg
n(e) + (105*d^8*e^8*(2*d/(e*x + d) - 1)^(13/2)*sgn(1/(e*x + d))*sgn(e) - 3
780*d^8*e^8*(2*d/(e*x + d) - 1)^(11/2)*sgn(1/(e*x + d))*sgn(e) + 189*d^8*e
^8*(2*d/(e*x + d) - 1)^(9/2)*sgn(1/(e*x + d))*sgn(e) - 4992*d^8*e^8*(2*d/(
e*x + d) - 1)^(7/2)*sgn(1/(e*x + d))*sgn(e) - 1981*d^8*e^8*(2*d/(e*x + d)
- 1)^(5/2)*sgn(1/(e*x + d))*sgn(e) - 700*d^8*e^8*(2*d/(e*x + d) - 1)^(3/2)
*sgn(1/(e*x + d))*sgn(e) - 105*d^8*e^8*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x
+ d))*sgn(e))*(e*x + d)^7/d^7)*abs(e)/(d*e^13)
```

3.159.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \int \frac{x^3\,(d^2-e^2\,x^2)^{5/2}}{(d+e\,x)^2}\,dx$$

input
```
int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

output
```
int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

3.160 $\int \frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

3.160.1 Optimal result

Integrand size = 27, antiderivative size = 142

$$\int \frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx=\frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}+\frac{d^2(32d-45ex)\left(d^2-e^2x^2\right)^{3/2}}{120e^3}+\frac{3d^6\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

output
```
2/5*d*x^2*(-e^2*x^2+d^2)^(3/2)/e-1/6*x^3*(-e^2*x^2+d^2)^(3/2)+1/120*d^2*(-
45*e*x+32*d)*(-e^2*x^2+d^2)^(3/2)/e^3+3/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(
1/2))/e^3+3/16*d^4*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

3.160.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.80

$$\int \frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx=\frac{\sqrt{d^2-e^2x^2}\left(64d^5-45d^4ex+32d^3e^2x^2+50d^2e^3x^3-96de^4x^4+40e^5x^5\right)-90d^6\,\mathrm{ar}}{240e^3}$$

input
```
Integrate[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(64*d^5 - 45*d^4*e*x + 32*d^3*e^2*x^2 + 50*d^2*e^3*x^
3 - 96*d*e^4*x^4 + 40*e^5*x^5) - 90*d^6*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2
 - e^2*x^2])])/(240*e^3)
```

3.160.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.24, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {562, 541, 27, 533, 25, 27, 533, 25, 27, 455, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(d^2-e^2 x^2\right)^{5/2}}{(d+e x)^2} \, dx$$

$$\Big\downarrow \text{562}$$

$$\int x^2 (d-e x)^2 \sqrt{d^2-e^2 x^2}dx$$

$$\Big\downarrow \text{541}$$

$$-\frac{\int -3 d e^2 x^2 (3 d-4 e x) \sqrt{d^2-e^2 x^2}dx}{6 e^2}-\frac{1}{6} x^3 \left(d^2-e^2 x^2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} d \int x^2 (3 d-4 e x) \sqrt{d^2-e^2 x^2}dx-\frac{1}{6} x^3 \left(d^2-e^2 x^2\right)^{3/2}$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{2} d \left(\frac{\int -d e x (8 d-15 e x) \sqrt{d^2-e^2 x^2}dx}{5 e^2}+\frac{4 x^2 \left(d^2-e^2 x^2\right)^{3/2}}{5 e}\right)-\frac{1}{6} x^3 \left(d^2-e^2 x^2\right)^{3/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} d \left(\frac{4 x^2 \left(d^2-e^2 x^2\right)^{3/2}}{5 e}-\frac{\int d e x (8 d-15 e x) \sqrt{d^2-e^2 x^2}dx}{5 e^2}\right)-\frac{1}{6} x^3 \left(d^2-e^2 x^2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} d \left(\frac{4 x^2 \left(d^2-e^2 x^2\right)^{3/2}}{5 e}-\frac{d \int x (8 d-15 e x) \sqrt{d^2-e^2 x^2}dx}{5 e}\right)-\frac{1}{6} x^3 \left(d^2-e^2 x^2\right)^{3/2}$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{2} d \left(\frac{4 x^2 \left(d^2-e^2 x^2\right)^{3/2}}{5 e}-\frac{d \left(\frac{\int -d e (15 d-32 e x) \sqrt{d^2-e^2 x^2}dx}{4 e^2}+\frac{15 x \left(d^2-e^2 x^2\right)^{3/2}}{4 e}\right)}{5 e}\right)-\frac{1}{6} x^3 \left(d^2-e^2 x^2\right)^{3/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}d\left(\frac{4x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{15x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{\int de(15d-32ex)\sqrt{d^2-e^2x^2}dx}{4e^2}\right)}{5e}\right)-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}$$

$\downarrow$ 27

$$\frac{1}{2}d\left(\frac{4x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{15x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\int (15d-32ex)\sqrt{d^2-e^2x^2}dx}{4e}\right)}{5e}\right)-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}$$

$\downarrow$ 455

$$\frac{1}{2}d\left(\frac{4x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{15x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\left(15d\int \sqrt{d^2-e^2x^2}dx+\frac{32\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)}{4e}\right)}{5e}\right)-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}$$

$\downarrow$ 211

$$\frac{1}{2}d\left(\frac{4x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{15x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\left(15d\left(\frac{1}{2}d^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{32\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)}{4e}\right)}{5e}\right)-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}$$

$\downarrow$ 224

3.160. $\int \frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

$$\frac{1}{2}d\left(\frac{4x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{15x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\left(15d\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{32\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)}{4e}\right)}{5e}\right)-$$

$$\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}d\left(\frac{4x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{d\left(\frac{15x\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{d\left(15d\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{32\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)}{4e}\right)}{5e}\right)-$$

$$\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}$$

input
```
Int[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

output
```
-1/6*(x^3*(d^2 - e^2*x^2)^(3/2)) + (d*((4*x^2*(d^2 - e^2*x^2)^(3/2))/(5*e)
 - (d*((15*x*(d^2 - e^2*x^2)^(3/2))/(4*e) - (d*((32*(d^2 - e^2*x^2)^(3/2))
/(3*e) + 15*d*((x*Sqrt[d^2 - e^2*x^2])/2 + (d^2*ArcTan[(e*x)/Sqrt[d^2 - e^
2*x^2]])/(2*e))))/(4*e)))/(5*e)))/2
```

3.160.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.160. $\int \frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 562
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[c^(2*n)/a^n   Int[x^m*((a + b*x^2)^(n + p)/(c - d*x)^n), x], x]
/; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[m, 0] && ILtQ[n,
 0] && IGtQ[n + p + 1/2, 0]
```

3.160. $\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

3.160.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.76

method	result
risch	$\frac{(40e^5x^5-96d\,e^4x^4+50d^2e^3x^3+32d^3e^2x^2-45d^4ex+64d^5)\sqrt{-e^2x^2+d^2}}{240e^3}+\frac{3d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16e^2\sqrt{e^2}}$
default	$\frac{\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6}}{e^2}+\frac{d^2\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{3de\left(x+\frac{d}{e}\right)^2}+\frac{5e\left(\ldots\right.}{}\right.}{}$

input
```
int(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/240*(40*e^5*x^5-96*d*e^4*x^4+50*d^2*e^3*x^3+32*d^3*e^2*x^2-45*d^4*e*x+64
*d^5)/e^3*(-e^2*x^2+d^2)^(1/2)+3/16*d^6/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)
*x/(-e^2*x^2+d^2)^(1/2))
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.75

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx =$$

$$-\frac{90\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(40\,e^5x^5-96\,de^4x^4+50\,d^2e^3x^3+32\,d^3e^2x^2-45\,d^4ex+64\,d^5)\sqrt{-e^2x^2+\ldots}}{240\,e^3}$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/240*(90*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (40*e^5*x^5 - 9
6*d*e^4*x^4 + 50*d^2*e^3*x^3 + 32*d^3*e^2*x^2 - 45*d^4*e*x + 64*d^5)*sqrt(
-e^2*x^2 + d^2))/e^3
```

3.160.6 Sympy [A] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.10

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = d^2\left(\begin{cases} \frac{d^4\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{8e^2} + \sqrt{d^2-e^2x^2}\left(-\frac{d^2x}{8e^2}+\frac{x^3}{4}\right) & \text{for } e^2\neq 0 \\ \frac{x^3\sqrt{d^2}}{3} & \text{otherwise} \end{cases}\right) - 2de\left(\begin{cases} \sqrt{d^2-e^2x^2}\left(-\frac{2d^4}{15e^4}-\frac{d^2x^2}{15e^2}+\frac{x^4}{5}\right) & \text{for } e^2\neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} \frac{d^6\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{16e^4} + \sqrt{d^2-e^2x^2}\left(-\frac{d^4x}{16e^4}-\frac{d^2x^3}{24e^2}+\frac{x^5}{6}\right) & \text{for } e^2\neq 0 \\ \frac{x^5\sqrt{d^2}}{5} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(x**2*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

output
```
d**2*Piecewise((d**4*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 -
e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/
(8*e**2) + sqrt(d**2 - e**2*x**2)*(-d**2*x/(8*e**2) + x**3/4), Ne(e**2, 0)
), (x**3*sqrt(d**2)/3, True)) - 2*d*e*Piecewise((sqrt(d**2 - e**2*x**2)*(-
2*d**4/(15*e**4) - d**2*x**2/(15*e**2) + x**4/5), Ne(e**2, 0)), (x**4*sqrt
(d**2)/4, True)) + e**2*Piecewise((d**6*Piecewise((log(-2*e**2*x + 2*sqrt(
-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-
e**2*x**2), True))/(16*e**4) + sqrt(d**2 - e**2*x**2)*(-d**4*x/(16*e**4) -
 d**2*x**3/(24*e**2) + x**5/6), Ne(e**2, 0)), (x**5*sqrt(d**2)/5, True))
```

3.160.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.62

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{i\,d^6\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^3} + \frac{5\,d^6\arcsin\left(\frac{ex}{d}\right)}{16\,e^3} + \frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^2}{4\,(e^4x+de^3)} - \frac{\sqrt{e^2x^2+4\,dex+3\,d^2}d^4x}{8\,e^2} + \frac{5\,\sqrt{-e^2x^2+d^2}d^4x}{16\,e^2} - \frac{\sqrt{e^2x^2+4\,dex+3\,d^2}d^5}{4\,e^3} - \frac{7\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2x}{24\,e^2} + \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^3}{12\,e^3} + \frac{(-e^2x^2+d^2)^{\frac{5}{2}}x}{6\,e^2} - \frac{2\,(-e^2x^2+d^2)^{\frac{5}{2}}d}{5\,e^3}$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
1/8*I*d^6*arcsin(e*x/d + 2)/e^3 + 5/16*d^6*arcsin(e*x/d)/e^3 + 1/4*(-e^2*x
^2 + d^2)^(5/2)*d^2/(e^4*x + d*e^3) - 1/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*
d^4*x/e^2 + 5/16*sqrt(-e^2*x^2 + d^2)*d^4*x/e^2 - 1/4*sqrt(e^2*x^2 + 4*d*e
*x + 3*d^2)*d^5/e^3 - 7/24*(-e^2*x^2 + d^2)^(3/2)*d^2*x/e^2 + 5/12*(-e^2*x
^2 + d^2)^(3/2)*d^3/e^3 + 1/6*(-e^2*x^2 + d^2)^(5/2)*x/e^2 - 2/5*(-e^2*x^2
 + d^2)^(5/2)*d/e^3
```

3.160.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(122) = 244$.

Time = 0.33 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.75

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = -\frac{\left(2880\,d^7e^7\arctan\left(\sqrt{\frac{2\,d}{ex+d}-1}\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e) + \frac{\left(45\,d^7e^7\left(\frac{2\,d}{ex+d}-1\right)^{\frac{11}{2}}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e) - 1025\,d^7e^7\left(\frac{2\,d}{ex+d}-1\right)^{\frac{9}{2}}\operatorname{sgn}\left(\frac{}{e}\right.}{}\right.}{}$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

output
```
-1/7680*(2880*d^7*e^7*arctan(sqrt(2*d/(e*x + d) - 1))*sgn(1/(e*x + d))*sgn
(e) + (45*d^7*e^7*(2*d/(e*x + d) - 1)^(11/2)*sgn(1/(e*x + d))*sgn(e) - 102
5*d^7*e^7*(2*d/(e*x + d) - 1)^(9/2)*sgn(1/(e*x + d))*sgn(e) - 174*d^7*e^7*
(2*d/(e*x + d) - 1)^(7/2)*sgn(1/(e*x + d))*sgn(e) - 594*d^7*e^7*(2*d/(e*x
+ d) - 1)^(5/2)*sgn(1/(e*x + d))*sgn(e) - 255*d^7*e^7*(2*d/(e*x + d) - 1)^
(3/2)*sgn(1/(e*x + d))*sgn(e) - 45*d^7*e^7*sqrt(2*d/(e*x + d) - 1)*sgn(1/(
e*x + d))*sgn(e))*(e*x + d)^6/d^6)*abs(e)/(d*e^11)
```

3.160.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \int \frac{x^2\,(d^2-e^2\,x^2)^{5/2}}{(d+e\,x)^2}\,dx$$

input
```
int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

output
```
int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

3.161 $\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

3.161.1 Optimal result

Integrand size = 25, antiderivative size = 136

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = -\frac{d^3x\sqrt{d^2-e^2x^2}}{4e}-\frac{dx(d^2-e^2x^2)^{3/2}}{6e} -\frac{2(d^2-e^2x^2)^{5/2}}{15e^2}-\frac{(d^2-e^2x^2)^{7/2}}{3e^2(d+ex)^2}-\frac{d^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^2}$$

output
```
-1/6*d*x*(-e^2*x^2+d^2)^(3/2)/e-2/15*(-e^2*x^2+d^2)^(5/2)/e^2-1/3*(-e^2*x^
2+d^2)^(7/2)/e^2/(e*x+d)^2-1/4*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2-1/
4*d^3*x*(-e^2*x^2+d^2)^(1/2)/e
```

3.161.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.82

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{e\sqrt{d^2-e^2x^2}(-28d^4+15d^3ex+16d^2e^2x^2-30de^3x^3+12e^4x^4)-15d^5\sqrt{-e^2}\log(-$$

input
```
Integrate[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

output
```
(e*Sqrt[d^2 - e^2*x^2]*(-28*d^4 + 15*d^3*e*x + 16*d^2*e^2*x^2 - 30*d*e^3*x
^3 + 12*e^4*x^4) - 15*d^5*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*
x^2]])/(60*e^3)
```

3.161.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {562, 541, 25, 27, 533, 27, 455, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}dx$$

$$\Big\downarrow \text{562}$$

$$\int x(d-ex)^2\sqrt{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{541}$$

$$-\frac{\int -de^2x(7d-10ex)\sqrt{d^2-e^2x^2}dx}{5e^2}-\frac{1}{5}x^2\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int de^2x(7d-10ex)\sqrt{d^2-e^2x^2}dx}{5e^2}-\frac{1}{5}x^2\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}d\int x(7d-10ex)\sqrt{d^2-e^2x^2}dx-\frac{1}{5}x^2\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{5}d\left(\frac{\int -2de(5d-14ex)\sqrt{d^2-e^2x^2}dx}{4e^2}+\frac{5x\left(d^2-e^2x^2\right)^{3/2}}{2e}\right)-\frac{1}{5}x^2\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}d\left(\frac{5x\left(d^2-e^2x^2\right)^{3/2}}{2e}-\frac{d\int (5d-14ex)\sqrt{d^2-e^2x^2}dx}{2e}\right)-\frac{1}{5}x^2\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{5}d\left(\frac{5x\left(d^2-e^2x^2\right)^{3/2}}{2e}-\frac{d\left(5d\int \sqrt{d^2-e^2x^2}dx+\frac{14\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)}{2e}\right)-\frac{1}{5}x^2\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{5}d\left(\frac{5x(d^2-e^2x^2)^{3/2}}{2e}-\frac{d\left(5d\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{14(d^2-e^2x^2)^{3/2}}{3e}\right)}{2e}\right)-\frac{1}{5}x^2(d^2-e^2x^2)^{3/2}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{5}d\left(\frac{5x(d^2-e^2x^2)^{3/2}}{2e}-\frac{d\left(5d\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{14(d^2-e^2x^2)^{3/2}}{3e}\right)}{2e}\right)-\frac{1}{5}x^2(d^2-e^2x^2)^{3/2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{5}d\left(\frac{5x(d^2-e^2x^2)^{3/2}}{2e}-\frac{d\left(5d\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{14(d^2-e^2x^2)^{3/2}}{3e}\right)}{2e}\right)-\frac{1}{5}x^2(d^2-e^2x^2)^{3/2}$$

input
```
Int[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

output
```
-1/5*(x^2*(d^2 - e^2*x^2)^(3/2)) + (d*((5*x*(d^2 - e^2*x^2)^(3/2))/(2*e) -
 (d*((14*(d^2 - e^2*x^2)^(3/2))/(3*e) + 5*d*((x*Sqrt[d^2 - e^2*x^2])/2 + (
d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e))))/(2*e)))/5
```

3.161.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.161. $\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 562
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[c^(2*n)/a^n   Int[x^m*((a + b*x^2)^(n + p)/(c - d*x)^n), x], x]
/; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[m, 0] && ILtQ[n,
 0] && IGtQ[n + p + 1/2, 0]
```

3.161. $\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

3.161.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{(-12e^4x^4+30d\,e^3x^3-16d^2e^2x^2-15d^3ex+28d^4)\sqrt{-e^2x^2+d^2}}{60e^2}-\frac{d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{4e\sqrt{e^2}}$
default	$\frac{\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+de\left(-\frac{\left(-2\left(x+\frac{d}{e}\right)e^2+2de\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8e^2}+\frac{3d^2\left(-\frac{\left(-2\left(x+\frac{d}{e}\right)e^2+2de\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\right.}}{4e^2}\right.}{4}\right.}{e^2}$

input
```
int(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/60*(-12*e^4*x^4+30*d*e^3*x^3-16*d^2*e^2*x^2-15*d^3*e*x+28*d^4)/e^2*(-e^
2*x^2+d^2)^(1/2)-1/4*d^5/e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)
^(1/2))
```

3.161.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.69

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(12\,e^4x^4-30\,de^3x^3+16\,d^2e^2x^2+15\,d^3ex-28\,d^4)}{60\,e^2}$$

input
```
integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/60*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (12*e^4*x^4 - 30*
d*e^3*x^3 + 16*d^2*e^2*x^2 + 15*d^3*e*x - 28*d^4)*sqrt(-e^2*x^2 + d^2))/e^
2
```

3.161.6 Sympy [A] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.59

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = d^2\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{d^2}{3e^2}+\frac{x^2}{3}\right) & \text{for } e^2\neq 0\\ \frac{x^2\sqrt{d^2}}{2} & \text{otherwise}\end{cases}\right) - 2de\left(\begin{cases}\frac{d^4\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{8e^2}+\sqrt{d^2-e^2x^2}\left(-\frac{d^2x}{8e^2}+\frac{x^3}{4}\right) & \text{for } e^2\neq 0\\ \frac{x^3\sqrt{d^2}}{3} & \text{otherwise}\end{cases}\right) + e^2\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{2d^4}{15e^4}-\frac{d^2x^2}{15e^2}+\frac{x^4}{5}\right) & \text{for } e^2\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

output
```
d**2*Piecewise((sqrt(d**2 - e**2*x**2)*(-d**2/(3*e**2) + x**2/3), Ne(e**2,
 0)), (x**2*sqrt(d**2)/2, True)) - 2*d*e*Piecewise((d**4*Piecewise((log(-2
*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)),
 (x*log(x)/sqrt(-e**2*x**2), True))/(8*e**2) + sqrt(d**2 - e**2*x**2)*(-d*
*2*x/(8*e**2) + x**3/4), Ne(e**2, 0)), (x**3*sqrt(d**2)/3, True)) + e**2*P
iecewise((sqrt(d**2 - e**2*x**2)*(-2*d**4/(15*e**4) - d**2*x**2/(15*e**2)
+ x**4/5), Ne(e**2, 0)), (x**4*sqrt(d**2)/4, True))
```

3.161.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.23

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{i\,d^5\arcsin\left(\frac{ex}{d}+2\right)}{4\,e^2} - \frac{\sqrt{e^2x^2+4\,dex+3\,d^2}d^3x}{4\,e} - \frac{(-e^2x^2+d^2)^{\frac{5}{2}}d}{4\,(e^3x+de^2)} - \frac{\sqrt{e^2x^2+4\,dex+3\,d^2}d^4}{2\,e^2} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}dx}{4\,e} - \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2}{12\,e^2} + \frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{5\,e^2}$$

input
```
integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
1/4*I*d^5*arcsin(e*x/d + 2)/e^2 - 1/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3*
x/e - 1/4*(-e^2*x^2 + d^2)^(5/2)*d/(e^3*x + d*e^2) - 1/2*sqrt(e^2*x^2 + 4*
d*e*x + 3*d^2)*d^4/e^2 + 1/4*(-e^2*x^2 + d^2)^(3/2)*d*x/e - 5/12*(-e^2*x^2
 + d^2)^(3/2)*d^2/e^2 + 1/5*(-e^2*x^2 + d^2)^(5/2)/e^2
```

3.161.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.59

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{\left(480\,d^6e^6\arctan\left(\sqrt{\frac{2\,d}{ex+d}-1}\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e) + \frac{\left(15\,d^6e^6\left(\frac{2\,d}{ex+d}-1\right)^{\frac{9}{2}}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)-2\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

output
```
1/960*(480*d^6*e^6*arctan(sqrt(2*d/(e*x + d) - 1))*sgn(1/(e*x + d))*sgn(e)
 + (15*d^6*e^6*(2*d/(e*x + d) - 1)^(9/2)*sgn(1/(e*x + d))*sgn(e) - 250*d^6
*e^6*(2*d/(e*x + d) - 1)^(7/2)*sgn(1/(e*x + d))*sgn(e) - 128*d^6*e^6*(2*d/
(e*x + d) - 1)^(5/2)*sgn(1/(e*x + d))*sgn(e) - 70*d^6*e^6*(2*d/(e*x + d) -
 1)^(3/2)*sgn(1/(e*x + d))*sgn(e) - 15*d^6*e^6*sqrt(2*d/(e*x + d) - 1)*sgn
(1/(e*x + d))*sgn(e))*(e*x + d)^5/d^5)*abs(e)/(d*e^9)
```

3.161.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \int \frac{x\,(d^2-e^2\,x^2)^{5/2}}{(d+e\,x)^2}\,dx$$

input
```
int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

output
```
int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

3.162 $\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

3.162.1 Optimal result

Integrand size = 24, antiderivative size = 108

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{5}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{5d(d^2-e^2x^2)^{3/2}}{12e}+\frac{(d-ex)(d^2-e^2x^2)^{3/2}}{4e}+\frac{5d^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

output
```
5/12*d*(-e^2*x^2+d^2)^(3/2)/e+1/4*(-e*x+d)*(-e^2*x^2+d^2)^(3/2)/e+5/8*d^4*
arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e+5/8*d^2*x*(-e^2*x^2+d^2)^(1/2)
```

3.162.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{\sqrt{d^2-e^2x^2}(16d^3+9d^2ex-16de^2x^2+6e^3x^3)-30d^4\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{24e}$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^2,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(16*d^3 + 9*d^2*e*x - 16*d*e^2*x^2 + 6*e^3*x^3) - 30*
d^4*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(24*e)
```

3.162.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {464, 469, 455, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}dx$$

$$\Big\downarrow \text{464}$$

$$\int (d-ex)^2\sqrt{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{469}$$

$$\frac{5}{4}d\int (d-ex)\sqrt{d^2-e^2x^2}dx+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}$$

$$\Big\downarrow \text{455}$$

$$\frac{5}{4}d\left(d\int \sqrt{d^2-e^2x^2}dx+\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{4}d\left(d\left(\frac{1}{2}d^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}$$

$$\Big\downarrow \text{224}$$

$$\frac{5}{4}d\left(d\left(\frac{1}{2}d^2\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}$$

$$\Big\downarrow \text{216}$$

$$\frac{5}{4}d\left(d\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^2,x]
```

output
```
((d - e*x)*(d^2 - e^2*x^2)^(3/2))/(4*e) + (5*d*((d^2 - e^2*x^2)^(3/2)/(3*e
) + d*((x*Sqrt[d^2 - e^2*x^2])/2 + (d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])
/(2*e))))/4
```

3.162.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 464
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
a + b*x^2)^(n + p)/(a/c + b*(x/d))^n, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c^2 + a*d^2, 0] && IntegerQ[n] && RationalQ[p] && (LtQ[0, -n, p] || LtQ[p,
 -n, 0]) && NeQ[n, 2] && NeQ[n, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

3.162.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.77

method	result
risch	$\frac{(6e^3x^3-16d\,e^2x^2+9d^2ex+16d^3)\sqrt{-e^2x^2+d^2}}{24e}+\frac{5d^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}$
default	$\frac{\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{3de\left(x+\frac{d}{e}\right)^2}+\frac{5e\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+de\left(-\frac{\left(-2\left(x+\frac{d}{e}\right)e^2+2de\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8e^2}+\frac{3d^2\left(-\frac{\left(-2\left(x+\cdots\right.\right.}{\cdots}\cdots\right)}{\cdots}\right)\right)}{3d}\cdots}{e^2}$

input
```
int((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*e^3*x^3-16*d*e^2*x^2+9*d^2*e*x+16*d^3)/e*(-e^2*x^2+d^2)^(1/2)+5/8*
d^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.78

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = -\frac{30\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(6\,e^3x^3-16\,de^2x^2+9\,d^2ex+16\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,e}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/24*(30*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (6*e^3*x^3 - 16*
d*e^2*x^2 + 9*d^2*e*x + 16*d^3)*sqrt(-e^2*x^2 + d^2))/e
```

3.162.6 Sympy [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.34

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = d^2\left(\begin{cases} \frac{d^2\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{2} + \frac{x\sqrt{d^2-e^2x^2}}{2} & \text{for } e^2\neq 0 \\ x\sqrt{d^2} & \text{otherwise}\end{cases}\right)$$
$$-2de\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{d^2}{3e^2}+\frac{x^2}{3}\right) & \text{for } e^2\neq 0 \\ \frac{x^2\sqrt{d^2}}{2} & \text{otherwise}\end{cases}\right)$$
$$+e^2\left(\begin{cases} \frac{d^4\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{8e^2} + \sqrt{d^2-e^2x^2}\left(-\frac{d^2x}{8e^2}+\frac{x^3}{4}\right) & \text{for } e^2\neq 0 \\ \frac{x^3\sqrt{d^2}}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

output
```
d**2*Piecewise((d**2*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 -
e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/
2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2, 0)), (x*sqrt(d**2), True)) - 2*d*
e*Piecewise((sqrt(d**2 - e**2*x**2)*(-d**2/(3*e**2) + x**2/3), Ne(e**2, 0)
), (x**2*sqrt(d**2)/2, True)) + e**2*Piecewise((d**4*Piecewise((log(-2*e**
2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*
log(x)/sqrt(-e**2*x**2), True))/(8*e**2) + sqrt(d**2 - e**2*x**2)*(-d**2*x
/(8*e**2) + x**3/4), Ne(e**2, 0)), (x**3*sqrt(d**2)/3, True))
```

3.162.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = -\frac{5i\,d^4\arcsin\left(\frac{ex}{d}+2\right)}{8\,e} + \frac{5}{8}\sqrt{e^2x^2+4\,dex+3\,d^2}d^2x$$
$$+\frac{5\sqrt{e^2x^2+4\,dex+3\,d^2}d^3}{4\,e} + \frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{4\,(e^2x+de)} + \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d}{12\,e}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-5/8*I*d^4*arcsin(e*x/d + 2)/e + 5/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^2*x
 + 5/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3/e + 1/4*(-e^2*x^2 + d^2)^(5/2)/
(e^2*x + d*e) + 5/12*(-e^2*x^2 + d^2)^(3/2)*d/e
```

3.162.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.70

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = -\frac{\left(240\,d^5e^5\arctan\left(\sqrt{\frac{2\,d}{ex+d}-1}\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)+\frac{\left(15\,d^5e^5\left(\frac{2\,d}{ex+d}-1\right)^{\frac{7}{2}}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)-73\,d^5e^5\left(\frac{2\,d}{ex+d}-1\right)^{\frac{5}{2}}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\right.}{}\right.}{192\,de^7}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

output
```
-1/192*(240*d^5*e^5*arctan(sqrt(2*d/(e*x + d) - 1))*sgn(1/(e*x + d))*sgn(e
) + (15*d^5*e^5*(2*d/(e*x + d) - 1)^(7/2)*sgn(1/(e*x + d))*sgn(e) - 73*d^5
*e^5*(2*d/(e*x + d) - 1)^(5/2)*sgn(1/(e*x + d))*sgn(e) - 55*d^5*e^5*(2*d/(
e*x + d) - 1)^(3/2)*sgn(1/(e*x + d))*sgn(e) - 15*d^5*e^5*sqrt(2*d/(e*x + d
) - 1)*sgn(1/(e*x + d))*sgn(e))*(e*x + d)^4/d^4)*abs(e)/(d*e^7)
```

3.162.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}}{(d+e\,x)^2}\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(d + e*x)^2,x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(d + e*x)^2, x)
```

3.163 $\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2}\,dx$

3.163.1 Optimal result

Integrand size = 27, antiderivative size = 96

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2}\,dx = d(d-ex)\sqrt{d^2-e^2x^2}-\frac{1}{3}(d^2-e^2x^2)^{3/2} - d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^3\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
-1/3*(-e^2*x^2+d^2)^(3/2)-d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-d^3*arctanh
((-e^2*x^2+d^2)^(1/2)/d)+d*(-e*x+d)*(-e^2*x^2+d^2)^(1/2)
```

3.163.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.30

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2}\,dx = \frac{1}{3}\sqrt{d^2-e^2x^2}(2d^2-3dex+e^2x^2) + 2d^3\operatorname{arctanh}\left(\frac{\sqrt{-e^2}x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{d^3e\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}}$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(2*d^2 - 3*d*e*x + e^2*x^2))/3 + 2*d^3*ArcTanh[(Sqrt[
-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] + (d^3*e*Log[-(Sqrt[-e^2]*x) + Sqrt[d^
2 - e^2*x^2]])/Sqrt[-e^2]
```

3.163.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {570, 541, 27, 535, 27, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x(d+ex)^2}dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x}dx$$

$$\Big\downarrow \text{541}$$

$$-\frac{\int -\frac{3de^2(d-2ex)\sqrt{d^2-e^2x^2}}{x}dx}{3e^2}-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$d\int \frac{(d-2ex)\sqrt{d^2-e^2x^2}}{x}dx-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{535}$$

$$d\left(\frac{1}{2}d^2\int \frac{2(d-ex)}{x\sqrt{d^2-e^2x^2}}dx+(d-ex)\sqrt{d^2-e^2x^2}\right)-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$d\left(d^2\int \frac{d-ex}{x\sqrt{d^2-e^2x^2}}dx+(d-ex)\sqrt{d^2-e^2x^2}\right)-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{538}$$

$$d\left(d^2\left(d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-e\int \frac{1}{\sqrt{d^2-e^2x^2}}dx\right)+(d-ex)\sqrt{d^2-e^2x^2}\right)-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \text{224}$$

$$d\left(d^2\left(d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-e\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)+(d-ex)\sqrt{d^2-e^2x^2}\right)-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}$$

$\Big\downarrow$ 216

$$d\left(d^2\left(d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+(d-ex)\sqrt{d^2-e^2x^2}\right)-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}$$

$\Big\downarrow$ 243

$$d\left(d^2\left(\frac{1}{2}d\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+(d-ex)\sqrt{d^2-e^2x^2}\right)-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}$$

$\Big\downarrow$ 73

$$d\left(d^2\left(-\frac{d\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+(d-ex)\sqrt{d^2-e^2x^2}\right)-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}$$

$\Big\downarrow$ 221

$$d\left(d^2\left(-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)+(d-ex)\sqrt{d^2-e^2x^2}\right)-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2),x]
```

output
```
-1/3*(d^2 - e^2*x^2)^(3/2) + d*((d - e*x)*Sqrt[d^2 - e^2*x^2] + d^2*(-ArcT
an[(e*x)/Sqrt[d^2 - e^2*x^2]] - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]))
```

3.163. $\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2}\,dx$

3.163.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

3.163. $\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2}\,dx$

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.163.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(86) = 172$.

Time = 0.39 (sec) , antiderivative size = 544, normalized size of antiderivative = 5.67

method	result
default	$\frac{\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5}+d^2\left(\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)\right)}{d^2}-\frac{\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{3de\left(x+\frac{d}{e}\right)^2}+\frac{5e\left(\cdots\right.}{\cdots}}{\cdots}$

input
```
int((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x
^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2)
)/x))))-1/e/d*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*
e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*(x+d/e)*e^2+2*
d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*(x+d/e)*e^
2+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arct
an((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))-1/d^2*(1/5*(-(x
+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*(x+d/e)*e^2+2*d*e)/e^2*(-(x
+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*(x+d/e)*e^2+2*d*e)/e^2*
(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2
)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.99

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2}\,dx = 2\,d^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + d^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + \frac{1}{3}\left(e^2x^2-3\,dex+2\,d^2\right)\sqrt{-e^2x^2+d^2}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x, algorithm="fricas")
```

output
```
2*d^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d^3*log(-(d - sqrt(-e^2*
x^2 + d^2))/x) + 1/3*(e^2*x^2 - 3*d*e*x + 2*d^2)*sqrt(-e^2*x^2 + d^2)
```

3.163.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.82 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.81

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2}\,dx = d^2\left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d\operatorname{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id\operatorname{asin}\left(\frac{d}{ex}\right) + \frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases}\right) - 2de\left(\begin{cases} \frac{d^2\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for}\ d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{d^2-e^2x^2}}{2} & \text{for}\ e^2\neq 0 \\ x\sqrt{d^2} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} -\frac{d^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^2\sqrt{d^2-e^2x^2}}{3} & \text{for}\ e^2\neq 0 \\ \frac{x^2\sqrt{d^2}}{2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/x/(e*x+d)**2,x)
```

output
```
d**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
 e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x
*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2
*x**2) + 1), True)) - 2*d*e*Piecewise((d**2*Piecewise((log(-2*e**2*x + 2*s
qrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sq
rt(-e**2*x**2), True))/2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2, 0)), (x*sq
rt(d**2), True)) + e**2*Piecewise((-d**2*sqrt(d**2 - e**2*x**2)/(3*e**2) +
 x**2*sqrt(d**2 - e**2*x**2)/3, Ne(e**2, 0)), (x**2*sqrt(d**2)/2, True))
```

3.163.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.21

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2}\,dx = -\frac{d^3e\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} - d^3\log\left(\frac{2\,d^2}{|x|} + \frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \sqrt{-e^2x^2+d^2}dex + \sqrt{-e^2x^2+d^2}d^2 - \frac{1}{3}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-d^3*e*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - d^3*log(2*d^2/abs(x) + 2*sq
rt(-e^2*x^2 + d^2)*d/abs(x)) - sqrt(-e^2*x^2 + d^2)*d*e*x + sqrt(-e^2*x^2
+ d^2)*d^2 - 1/3*(-e^2*x^2 + d^2)^(3/2)
```

3.163.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.163.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}}{x\,(d+e\,x)^2}\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2), x)
```

3.164 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2}\,dx$

3.164.1 Optimal result

Integrand size = 27, antiderivative size = 105

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2}\,dx = -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{(d^2-e^2x^2)^{3/2}}{x} - \frac{1}{2}d^2e\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + 2d^2e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
-(-e^2*x^2+d^2)^(3/2)/x-1/2*d^2*e*arctan(e*x/(-e^2*x^2+d^2)^(1/2))+2*d^2*e
*arctanh((-e^2*x^2+d^2)^(1/2)/d)-1/2*e*(e*x+4*d)*(-e^2*x^2+d^2)^(1/2)
```

3.164.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.24

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2}\,dx = \frac{\sqrt{d^2-e^2x^2}(-2d^2-4dex+e^2x^2)}{2x} + d^2e\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) + 2d\sqrt{d^2}e\log(x) - 2d\sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-2*d^2 - 4*d*e*x + e^2*x^2))/(2*x) + d^2*e*ArcTan[(e
*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] + 2*d*Sqrt[d^2]*e*Log[x] - 2*d*Sqrt
[d^2]*e*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]]
```

3.164.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {570, 540, 27, 535, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^2(d+ex)^2}dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^2}dx$$

$$\Big\downarrow \text{540}$$

$$-\frac{\int \frac{d^2e(2d+ex)\sqrt{d^2-e^2x^2}}{x}dx}{d^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{x}$$

$$\Big\downarrow \text{27}$$

$$-e\int \frac{(2d+ex)\sqrt{d^2-e^2x^2}}{x}dx-\frac{\left(d^2-e^2x^2\right)^{3/2}}{x}$$

$$\Big\downarrow \text{535}$$

$$-e\left(\frac{1}{2}d^2\int \frac{4d+ex}{x\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}(4d+ex)\sqrt{d^2-e^2x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{x}$$

$$\Big\downarrow \text{538}$$

$$-e\left(\frac{1}{2}d^2\left(e\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+4d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx\right)+\frac{1}{2}(4d+ex)\sqrt{d^2-e^2x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{x}$$

$$\Big\downarrow \text{224}$$

$$-e\left(\frac{1}{2}d^2\left(4d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx+e\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)+\frac{1}{2}(4d+ex)\sqrt{d^2-e^2x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{x}$$

$$\Big\downarrow \; 216$$

$$-e\left(\frac{1}{2}d^2\left(4d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx+\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{1}{2}(4d+ex)\sqrt{d^2-e^2x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{x}$$

$$\Big\downarrow \; 243$$

$$-e\left(\frac{1}{2}d^2\left(2d\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2+\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{1}{2}(4d+ex)\sqrt{d^2-e^2x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{x}$$

$$\Big\downarrow \; 73$$

$$-e\left(\frac{1}{2}d^2\left(\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{4d\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}\right)+\frac{1}{2}(4d+ex)\sqrt{d^2-e^2x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{x}$$

$$\Big\downarrow \; 221$$

$$-e\left(\frac{1}{2}d^2\left(\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-4\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)+\frac{1}{2}(4d+ex)\sqrt{d^2-e^2x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{x}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2),x]
```

output
```
-((d^2 - e^2*x^2)^(3/2)/x) - e*(((4*d + e*x)*Sqrt[d^2 - e^2*x^2])/2 + (d^2
*(ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]))/2
)
```

3.164. $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^2(d+ex)^2}\,dx$

3.164.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

3.164. $\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2} dx$

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.164.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{d^2\sqrt{-e^2x^2+d^2}}{x}+\frac{\sqrt{-e^2x^2+d^2}\,e^2x}{2}-\frac{e^2d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}+\frac{2e\,d^3\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}-2ed\sqrt{-e^2x^2+}$
default	$\frac{-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{d^2x}-\frac{6e^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6}\right)}{d^2}}{d^2}-\frac{2e\left(\frac{\left(-e^2x^2+\right.}{5}\right.}{}$

input
```
int((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-d^2*(-e^2*x^2+d^2)^(1/2)/x+1/2*(-e^2*x^2+d^2)^(1/2)*e^2*x-1/2*e^2*d^2/(e^
2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+2*e*d^3/(d^2)^(1/2)*ln
((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-2*e*d*(-e^2*x^2+d^2)^(1/2)
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.06

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2}\,dx = \frac{2\,d^2ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-4\,d^2ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-4\,d^2ex+(e^2x^2-4\,de\ldots}{2\,x}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/2*(2*d^2*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 4*d^2*e*x*log(-
(d - sqrt(-e^2*x^2 + d^2))/x) - 4*d^2*e*x + (e^2*x^2 - 4*d*e*x - 2*d^2)*sq
rt(-e^2*x^2 + d^2))/x
```

3.164.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.48 (sec) , antiderivative size = 330, normalized size of antiderivative = 3.14

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2}\,dx = d^2\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$
$$-2de\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)$$
$$+e^2\left(\begin{cases}\frac{d^2\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{d^2-e^2x^2}}{2} & \text{for } e^2\neq 0\\ x\sqrt{d^2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/x**2/(e*x+d)**2,x)
```

output
```
d**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e
**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt
(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)
), True)) - 2*d*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*aco
sh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1),
(-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqr
t(-d**2/(e**2*x**2) + 1), True)) + e**2*Piecewise((d**2*Piecewise((log(-2*
e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)),
(x*log(x)/sqrt(-e**2*x**2), True))/2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2
, 0)), (x*sqrt(d**2), True))
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.20

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2}\,dx = -\frac{d^2e^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}} + 2\,d^2e\log\left(\frac{2\,d^2}{|x|} + \frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) + \frac{1}{2}\sqrt{-e^2x^2+d^2}e^2x - 2\sqrt{-e^2x^2+d^2}de - \frac{\sqrt{-e^2x^2+d^2}d^2}{x}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-1/2*d^2*e^2*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 2*d^2*e*log(2*d^2/abs
(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) + 1/2*sqrt(-e^2*x^2 + d^2)*e^2*x -
2*sqrt(-e^2*x^2 + d^2)*d*e - sqrt(-e^2*x^2 + d^2)*d^2/x
```

3.164.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.164.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2}\,dx = \int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x^2\,{\left(d+e\,x\right)}^2}\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2), x)
```

3.165 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^2}\,dx$

3.165.1 Optimal result

Integrand size = 27, antiderivative size = 110

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^2}\,dx = \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d^2-e^2x^2)^{3/2}}{2x^2} + 2de^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{2}de^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
-1/2*(-e^2*x^2+d^2)^(3/2)/x^2+2*d*e^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-1/2
*d*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/2*e*(e*x+4*d)*(-e^2*x^2+d^2)^(1/2
)/x
```

3.165.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.13

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^2}\,dx = -\frac{(d^2-4dex-2e^2x^2)\sqrt{d^2-e^2x^2}}{2x^2} + de^2\text{arctanh}\left(\frac{\sqrt{-e^2}x-\sqrt{d^2-e^2x^2}}{d}\right) + 2de\sqrt{-e^2}\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2),x]
```

output
```
-1/2*((d^2 - 4*d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/x^2 + d*e^2*ArcTanh
[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d] + 2*d*e*Sqrt[-e^2]*Log[-(Sqrt[-e^
2]*x) + Sqrt[d^2 - e^2*x^2]]
```

3.165.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {570, 540, 27, 536, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^2}dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^3}dx$$

$$\Big\downarrow \text{540}$$

$$-\frac{\int \frac{d^2e(4d-ex)\sqrt{d^2-e^2x^2}}{x^2}dx}{2d^2}-\frac{(d^2-e^2x^2)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}e\int \frac{(4d-ex)\sqrt{d^2-e^2x^2}}{x^2}dx-\frac{(d^2-e^2x^2)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{536}$$

$$-\frac{1}{2}e\left(\int \frac{-ed^2-4e^2xd}{x\sqrt{d^2-e^2x^2}}dx-\frac{(4d+ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{(d^2-e^2x^2)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{538}$$

$$-\frac{1}{2}e\left(d^2(-e)\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-4de^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{(4d+ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{(d^2-e^2x^2)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{224}$$

$$-\frac{1}{2}e\left(d^2(-e)\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx-4de^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{(4d+ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

$\Big\downarrow$ 216

$$-\frac{1}{2}e\left(d^2(-e)\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx-4de\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{(4d+ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

$\Big\downarrow$ 243

$$-\frac{1}{2}e\left(-\frac{1}{2}d^2e\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-4de\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{(4d+ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

$\Big\downarrow$ 73

$$-\frac{1}{2}e\left(\frac{d^2\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-4de\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{(4d+ex)\sqrt{d^2-e^2x^2}}{x}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

$\Big\downarrow$ 221

$$-\frac{1}{2}e\left(-4de\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+de\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)-\frac{\sqrt{d^2-e^2x^2}(4d+ex)}{x}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2),x]
```

output
```
-1/2*(d^2 - e^2*x^2)^(3/2)/x^2 - (e*(-(((4*d + e*x)*Sqrt[d^2 - e^2*x^2])/x
) - 4*d*e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + d*e*ArcTanh[Sqrt[d^2 - e^2*x
^2]/d]))/2
```

3.165. $\int\frac{\left(d^2-e^2x^2\right)^{5/2}}{x^3(d+ex)^2}\,dx$

3.165.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

3.165. $\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^2} dx$

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.165.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, \left(-2 e^{2} x^{2}-4 d e x +d^{2}\right)}{2 x^{2}}+\frac{2 d \,e^{3} \arctan \left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}}-\frac{d^{2} e^{2} \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{2 \sqrt{d^{2}}}$
default	$\frac{-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}}}{2 d^{2} x^{2}}-\frac{5 e^{2} \left(\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{5}+d^{2} \left(\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{3}+d^{2} \left(\sqrt{-e^{2} x^{2}+d^{2}}-\frac{d^{2} \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}}\right)\right)\right)}{2 d^{2}}}{d^{2}}+\frac{3 e^{2} \left(\frac{\left(-e^{2} x^{2}+\right.}{5}\right.}{}$

input
```
int((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

3.165. $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^3(d+ex)^2}\,dx$

output
```
-1/2*(-e^2*x^2+d^2)^(1/2)*(-2*e^2*x^2-4*d*e*x+d^2)/x^2+2*d*e^3/(e^2)^(1/2)
*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/2*d^2*e^2/(d^2)^(1/2)*ln((2*
d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.08

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^2}\,dx = -\frac{8\,de^2x^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-de^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-2\,de^2x^2-(2\,e^2x^2+4\,dex-d^2)\sqrt{-e^2x^2+}}{2\,x^2}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/2*(8*d*e^2*x^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - d*e^2*x^2*lo
g(-(d - sqrt(-e^2*x^2 + d^2))/x) - 2*d*e^2*x^2 - (2*e^2*x^2 + 4*d*e*x - d^
2)*sqrt(-e^2*x^2 + d^2))/x^2
```

3.165.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.33 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.15

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^2}\,dx = d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right) - 2de\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/x**3/(e*x+d)**2,x)
```

output
```
d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/
(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2
) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*
d), True)) - 2*d*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acos
h(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1
), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e
**2*x**2/d**2)), True)) + e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2)
- 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*
x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)
) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True))
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.12

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^2}\,dx = \frac{2\,de^3\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} - \frac{1}{2}\,de^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right) + \frac{1}{2}\,\sqrt{-e^2x^2+d^2}e^2 + \frac{2\,\sqrt{-e^2x^2+d^2}de}{x} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{2\,x^2}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x, algorithm="maxima")
```

output
```
2*d*e^3*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 1/2*d*e^2*log(2*d^2/abs(x)
 + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) + 1/2*sqrt(-e^2*x^2 + d^2)*e^2 + 2*sqr
t(-e^2*x^2 + d^2)*d*e/x - 1/2*(-e^2*x^2 + d^2)^(3/2)/x^2
```

3.165.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.165.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{x^3(d + ex)^2}\, dx = \int \frac{(d^2 - e^2\,x^2)^{5/2}}{x^3\,(d + e\,x)^2}\, dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2), x)
```

3.166 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^2}\,dx$

3.166.1 Optimal result

Integrand size = 27, antiderivative size = 102

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^2}\,dx = \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2} - \frac{(d^2-e^2x^2)^{3/2}}{3x^3} - e^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - e^3\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
-1/3*(-e^2*x^2+d^2)^(3/2)/x^3-e^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-e^3*arc
tanh((-e^2*x^2+d^2)^(1/2)/d)+e*(-e*x+d)*(-e^2*x^2+d^2)^(1/2)/x^2
```

3.166.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.35

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^2}\,dx = \frac{(-d^2+3dex-2e^2x^2)\sqrt{d^2-e^2x^2}}{3x^3} + 2e^3\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) - \frac{\sqrt{d^2}e^3\log(x)}{d} + \frac{\sqrt{d^2}e^3\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{d}$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2),x]
```

output
```
((-d^2 + 3*d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(3*x^3) + 2*e^3*ArcTan[
(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] - (Sqrt[d^2]*e^3*Log[x])/d + (Sqr
t[d^2]*e^3*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/d
```

3.166.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {570, 540, 27, 537, 27, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^2}dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^4}dx$$

$$\Big\downarrow \text{540}$$

$$-\frac{\int \frac{3d^2e(2d-ex)\sqrt{d^2-e^2x^2}}{x^3}dx}{3d^2}-\frac{(d^2-e^2x^2)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$-e\int \frac{(2d-ex)\sqrt{d^2-e^2x^2}}{x^3}dx-\frac{(d^2-e^2x^2)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{537}$$

$$-e\left(\frac{1}{2}e^2\int -\frac{2(d-ex)}{x\sqrt{d^2-e^2x^2}}dx-\frac{(d-ex)\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{(d^2-e^2x^2)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$-e\left(e^2\left(-\int \frac{d-ex}{x\sqrt{d^2-e^2x^2}}dx\right)-\frac{(d-ex)\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{(d^2-e^2x^2)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{538}$$

$$-e\left(-\left(e^2\left(d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-e\int \frac{1}{\sqrt{d^2-e^2x^2}}dx\right)\right)-\frac{(d-ex)\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}$$

$\Big\downarrow$ 224

$$-e\left(-\left(e^2\left(d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-e\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{(d-ex)\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}$$

$\Big\downarrow$ 216

$$-e\left(-\left(e^2\left(d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)\right)-\frac{(d-ex)\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}$$

$\Big\downarrow$ 243

$$-e\left(-\left(e^2\left(\frac{1}{2}d\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)\right)-\frac{(d-ex)\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}$$

$\Big\downarrow$ 73

$$-e\left(-\left(e^2\left(-\frac{d\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)\right)-\frac{(d-ex)\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}$$

$\Big\downarrow$ 221

$$-e\left(-\left(e^2\left(-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)\right)-\frac{(d-ex)\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2),x]
```

3.166. $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^4(d+ex)^2}\,dx$

output
```
-1/3*(d^2 - e^2*x^2)^(3/2)/x^3 - e*(-(((d - e*x)*Sqrt[d^2 - e^2*x^2])/x^2)
 - e^2*(-ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - ArcTanh[Sqrt[d^2 - e^2*x^2]/d
]))
```

3.166.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

3.166. $\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^2}\,dx$

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.166.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.11

method	result	size
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, \left(2 e^{2} x^{2}-3 d e x +d^{2}\right)}{3 x^{3}}-\frac{e^{4} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}}-\frac{e^{3} d \ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}}$	113
default	Expression too large to display	983

input
```
int((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-e^2*x^2+d^2)^(1/2)*(2*e^2*x^2-3*d*e*x+d^2)/x^3-e^4/(e^2)^(1/2)*arct
an((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-e^3*d/(d^2)^(1/2)*ln((2*d^2+2*(d^2)
^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^2}\,dx = \frac{6\,e^3x^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+3\,e^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-(2\,e^2x^2-3\,dex+d^2)\sqrt{\ }}{3\,x^3}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/3*(6*e^3*x^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 3*e^3*x^3*log(-
(d - sqrt(-e^2*x^2 + d^2))/x) - (2*e^2*x^2 - 3*d*e*x + d^2)*sqrt(-e^2*x^2
+ d^2))/x^3
```

3.166.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.93 (sec) , antiderivative size = 338, normalized size of antiderivative = 3.31

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^2}\,dx = d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right) - 2de\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/x**4/(e*x+d)**2,x)
```

output
```
d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e
**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**
2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)
) - 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e
*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2
*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x)
)/(2*d), True)) + e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*
acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
 > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1
 - e**2*x**2/d**2)), True))
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.43

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^2}\,dx = -\frac{e^4 \arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} - e^3 \log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right) + \frac{\sqrt{-e^2x^2+d^2}e^3}{d} - \frac{\sqrt{-e^2x^2+d^2}e^2}{x} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e}{dx^2} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3\,x^3}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-e^4*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - e^3*log(2*d^2/abs(x) + 2*sqrt
(-e^2*x^2 + d^2)*d/abs(x)) + sqrt(-e^2*x^2 + d^2)*e^3/d - sqrt(-e^2*x^2 +
d^2)*e^2/x + (-e^2*x^2 + d^2)^(3/2)*e/(d*x^2) - 1/3*(-e^2*x^2 + d^2)^(3/2)
/x^3
```

3.166.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^2}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: abs(
sageVARe)*(1/3*(12*sageVARe^2*sqrt(2*sageVARd*sageVARe*(sageVARe*sageVARx+
sageVARd)^-1/sageVARe-1)*(2*sageVARd*sageVARe*(sageVARe*sageVARx+sageVARd)
^-1/sageVARe-1)^2*s
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^2}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}}{x^4\,(d+e\,x)^2}\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2), x)
```

3.167 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^2}\,dx$

3.167.1 Optimal result

Integrand size = 27, antiderivative size = 108

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^2}\,dx = -\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(d^2-e^2x^2)^{3/2}}{4x^4} + \frac{2e(d^2-e^2x^2)^{3/2}}{3dx^3} + \frac{5e^4\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

output
```
-1/4*(-e^2*x^2+d^2)^(3/2)/x^4+2/3*e*(-e^2*x^2+d^2)^(3/2)/d/x^3+5/8*e^4*arc
tanh((-e^2*x^2+d^2)^(1/2)/d)/d-5/8*e^2*(-e^2*x^2+d^2)^(1/2)/x^2
```

3.167.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.06

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^2}\,dx = \frac{\sqrt{d^2-e^2x^2}(-6d^3+16d^2ex-9de^2x^2-16e^3x^3)}{24dx^4} + \frac{5e^4\log(x)}{8\sqrt{d^2}} - \frac{5e^4\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{8\sqrt{d^2}}$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-6*d^3 + 16*d^2*e*x - 9*d*e^2*x^2 - 16*e^3*x^3))/(24
*d*x^4) + (5*e^4*Log[x])/(8*Sqrt[d^2]) - (5*e^4*Log[Sqrt[d^2] - Sqrt[d^2 -
 e^2*x^2]])/(8*Sqrt[d^2])
```

3.167.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {570, 540, 27, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^5(d+ex)^2}dx$$

$$\Big\downarrow \; 570$$

$$\int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^5}dx$$

$$\Big\downarrow \; 540$$

$$-\frac{\int \frac{d^2e(8d-5ex)\sqrt{d^2-e^2x^2}}{x^4}dx}{4d^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{4}e\int \frac{(8d-5ex)\sqrt{d^2-e^2x^2}}{x^4}dx-\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}$$

$$\Big\downarrow \; 534$$

$$-\frac{1}{4}e\left(-5e\int \frac{\sqrt{d^2-e^2x^2}}{x^3}dx-\frac{8\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}$$

$$\Big\downarrow \; 243$$

$$-\frac{1}{4}e\left(-\frac{5}{2}e\int \frac{\sqrt{d^2-e^2x^2}}{x^4}dx^2-\frac{8\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}$$

$$\Big\downarrow \; 51$$

$$-\frac{1}{4}e\left(-\frac{5}{2}e\left(-\frac{1}{2}e^2\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{8\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}$$

$$\Big\downarrow \; 73$$

$$-\frac{1}{4}e\left(-\frac{5}{2}e\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{8\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}$$

$$\Big\downarrow \; 221$$

$$-\frac{1}{4}e\left(-\frac{5}{2}e\left(\frac{e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{8\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2),x]
```

output
```
-1/4*(d^2 - e^2*x^2)^(3/2)/x^4 - (e*((-8*(d^2 - e^2*x^2)^(3/2))/(3*d*x^3)
- (5*e*(-(Sqrt[d^2 - e^2*x^2]/x^2) + (e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/
d))/2))/4
```

3.167.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.167.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, \left(16 e^{3} x^{3}+9 d\, e^{2} x^{2}-16 d^{2} e x +6 d^{3}\right)}{24 x^{4} d}+\frac{5 e^{4} \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{8 \sqrt{d^{2}}}$	96
default	Expression too large to display	1153

input
```
int((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-e^2*x^2+d^2)^(1/2)*(16*e^3*x^3+9*d*e^2*x^2-16*d^2*e*x+6*d^3)/x^4/d
+5/8*e^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^5(d+ex)^2}\,dx = -\frac{15\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(16\,e^3x^3+9\,de^2x^2-16\,d^2ex+6\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,dx^4}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/24*(15*e^4*x^4*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (16*e^3*x^3 + 9*d*e
^2*x^2 - 16*d^2*e*x + 6*d^3)*sqrt(-e^2*x^2 + d^2))/(d*x^4)
```

3.167.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.18 (sec) , antiderivative size = 422, normalized size of antiderivative = 3.91

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^5(d+ex)^2}\,dx = d^2\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise} \end{cases}\right) - 2de\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/x**5/(e*x+d)**2,x)
```

output
```
d**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*
sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) +
 e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x*
*5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1
)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/
(8*d**3), True)) - 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2)
 + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (
-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2)
+ 1)/(3*d**2), True)) + e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x
) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x*
*3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) -
I*e**2*asin(d/(e*x))/(2*d), True))
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.20

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^2}\,dx = \frac{5\,e^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{8\,d} - \frac{5\,\sqrt{-e^2x^2+d^2}e^4}{8\,d^2} - \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}e^2}{8\,d^2x^2} + \frac{2\,(-e^2x^2+d^2)^{\frac{3}{2}}e}{3\,dx^3} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{4\,x^4}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x, algorithm="maxima")
```

output
```
5/8*e^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 5/8*sqrt(-
e^2*x^2 + d^2)*e^4/d^2 - 5/8*(-e^2*x^2 + d^2)^(3/2)*e^2/(d^2*x^2) + 2/3*(-
e^2*x^2 + d^2)^(3/2)*e/(d*x^3) - 1/4*(-e^2*x^2 + d^2)^(3/2)/x^4
```

3.167.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.28

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^2}\,dx = \frac{1}{192}\left(\frac{120\,e^3\log\left(\sqrt{\frac{2\,d}{ex+d}-1}+1\right)\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)}{d} - \frac{120\,e^3\log\left(\left|\sqrt{\frac{2\,d}{ex+d}-1}-1\right|\right)}{d}\right.$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x, algorithm="giac")
```

output
```
1/192*(120*e^3*log(sqrt(2*d/(e*x + d) - 1) + 1)*sgn(1/(e*x + d))*sgn(e)/d
- 120*e^3*log(abs(sqrt(2*d/(e*x + d) - 1) - 1))*sgn(1/(e*x + d))*sgn(e)/d
+ 4*(15*e^3*log(2) - 30*e^3*log(I + 1) + 32*I*e^3)*sgn(1/(e*x + d))*sgn(e)
/d - (15*e^3*(2*d/(e*x + d) - 1)^(7/2)*sgn(1/(e*x + d))*sgn(e) + 73*e^3*(2
*d/(e*x + d) - 1)^(5/2)*sgn(1/(e*x + d))*sgn(e) - 55*e^3*(2*d/(e*x + d) -
1)^(3/2)*sgn(1/(e*x + d))*sgn(e) + 15*e^3*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e
*x + d))*sgn(e))/(d*(d/(e*x + d) - 1)^4))*abs(e)
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^2}\,dx = \int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^5\,\left(d+e\,x\right)^2}\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2), x)
```

3.168 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx$

3.168.1 Optimal result

Integrand size = 27, antiderivative size = 140

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx = \frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2} - \frac{(d^2-e^2x^2)^{3/2}}{5x^5} + \frac{e(d^2-e^2x^2)^{3/2}}{2dx^4} - \frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3} - \frac{e^5\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^2}$$

output
```
-1/5*(-e^2*x^2+d^2)^(3/2)/x^5+1/2*e*(-e^2*x^2+d^2)^(3/2)/d/x^4-7/15*e^2*(-
e^2*x^2+d^2)^(3/2)/d^2/x^3-1/4*e^5*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2+1/4
*e^3*(-e^2*x^2+d^2)^(1/2)/d/x^2
```

3.168.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx = \frac{\sqrt{d^2-e^2x^2}(-12d^4+30d^3ex-16d^2e^2x^2-15de^3x^3+28e^4x^4)}{60d^2x^5} - \frac{\sqrt{d^2}e^5\log(x)}{4d^3} + \frac{\sqrt{d^2}e^5\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{4d^3}$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-12*d^4 + 30*d^3*e*x - 16*d^2*e^2*x^2 - 15*d*e^3*x^3
 + 28*e^4*x^4))/(60*d^2*x^5) - (Sqrt[d^2]*e^5*Log[x])/(4*d^3) + (Sqrt[d^2]
*e^5*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(4*d^3)
```

3.168.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {570, 540, 27, 539, 27, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^6(d+ex)^2}dx$$

$$\Big\downarrow \text{ 570}$$

$$\int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^6}dx$$

$$\Big\downarrow \text{ 540}$$

$$-\frac{\int \frac{d^2e(10d-7ex)\sqrt{d^2-e^2x^2}}{x^5}dx}{5d^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{5}e\int \frac{(10d-7ex)\sqrt{d^2-e^2x^2}}{x^5}dx-\frac{\left(d^2-e^2x^2\right)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{ 539}$$

$$-\frac{1}{5}e\left(-\frac{\int \frac{2de(14d-5ex)\sqrt{d^2-e^2x^2}}{x^4}dx}{4d^2}-\frac{5\left(d^2-e^2x^2\right)^{3/2}}{2dx^4}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{5}e\left(-\frac{e\int \frac{(14d-5ex)\sqrt{d^2-e^2x^2}}{x^4}dx}{2d}-\frac{5\left(d^2-e^2x^2\right)^{3/2}}{2dx^4}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{ 534}$$

$$-\frac{1}{5}e\left(-\frac{e\left(-5e\int\frac{\sqrt{d^2-e^2x^2}}{x^3}dx-\frac{14(d^2-e^2x^2)^{3/2}}{3dx^3}\right)}{2d}-\frac{5(d^2-e^2x^2)^{3/2}}{2dx^4}\right)-\frac{(d^2-e^2x^2)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{1}{5}e\left(-\frac{e\left(-\frac{5}{2}e\int\frac{\sqrt{d^2-e^2x^2}}{x^4}dx^2-\frac{14(d^2-e^2x^2)^{3/2}}{3dx^3}\right)}{2d}-\frac{5(d^2-e^2x^2)^{3/2}}{2dx^4}\right)-\frac{(d^2-e^2x^2)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{ 51}$$

$$-\frac{1}{5}e\left(-\frac{e\left(-\frac{5}{2}e\left(-\frac{1}{2}e^2\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{14(d^2-e^2x^2)^{3/2}}{3dx^3}\right)}{2d}-\frac{5(d^2-e^2x^2)^{3/2}}{2dx^4}\right)-\frac{(d^2-e^2x^2)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{1}{5}e\left(-\frac{e\left(-\frac{5}{2}e\left(\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{14(d^2-e^2x^2)^{3/2}}{3dx^3}\right)}{2d}-\frac{5(d^2-e^2x^2)^{3/2}}{2dx^4}\right)-\frac{(d^2-e^2x^2)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{1}{5}e\left(-\frac{e\left(-\frac{5}{2}e\left(\frac{e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{14(d^2-e^2x^2)^{3/2}}{3dx^3}\right)}{2d}-\frac{5(d^2-e^2x^2)^{3/2}}{2dx^4}\right)-\frac{(d^2-e^2x^2)^{3/2}}{5x^5}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2),x]
```

output
```
-1/5*(d^2 - e^2*x^2)^(3/2)/x^5 - (e*((-5*(d^2 - e^2*x^2)^(3/2))/(2*d*x^4)
- (e*((-14*(d^2 - e^2*x^2)^(3/2))/(3*d*x^3) - (5*e*(-(Sqrt[d^2 - e^2*x^2]/
x^2) + (e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/2))/(2*d)))/5
```

3.168. $\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx$

3.168.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

3.168. $\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx$

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.168.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\,\left(-28 e^{4} x^{4}+15 d\,e^{3} x^{3}+16 d^{2} e^{2} x^{2}-30 d^{3} e x +12 d^{4}\right)}{60 x^{5} d^{2}}-\frac{e^{5} \ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\,\sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{4 d \sqrt{d^{2}}}$	110
default	Expression too large to display	1349

input
```
int((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/60*(-e^2*x^2+d^2)^(1/2)*(-28*e^4*x^4+15*d*e^3*x^3+16*d^2*e^2*x^2-30*d^3
*e*x+12*d^4)/x^5/d^2-1/4/d*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x
^2+d^2)^(1/2))/x)
```

3.168.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.69

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx = \frac{15\,e^5x^5\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(28\,e^4x^4-15\,de^3x^3-16\,d^2e^2x^2+30\,d^3ex-12\,d^4)\sqrt{-}}{60\,d^2x^5}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/60*(15*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (28*e^4*x^4 - 15*d*e
^3*x^3 - 16*d^2*e^2*x^2 + 30*d^3*e*x - 12*d^4)*sqrt(-e^2*x^2 + d^2))/(d^2*
x^5)
```

3.168.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.88 (sec) , antiderivative size = 660, normalized size of antiderivative = 4.71

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx = d^2\left(\begin{cases} \frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} - \frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} + \frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7} - \frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for } \left|\frac{e^2x^2}{d^2}\right| \\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} - \frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} + \frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7} - \frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwis} \end{cases}\right.$$

$$- 2de\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise} \end{cases}\right)$$

$$+ e^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/x**6/(e*x+d)**2,x)
```

output
```
d**2*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**
2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e*
*2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**
3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d
*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-
15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15
*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**
5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3
*x**5 + 15*d*e**2*x**7), True)) - 2*d*e*Piecewise((-d**2/(4*e*x**5*sqrt(d*
*2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d
**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2
/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e
/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*
x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) + e**2*Piecewise((-e*s
qrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d*
*2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2
) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True))
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.11

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx = -\frac{e^5\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{4\,d^2}+\frac{\sqrt{-e^2x^2+d^2}e^5}{4\,d^3} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^3}{4\,d^3x^2}-\frac{7\,(-e^2x^2+d^2)^{\frac{3}{2}}e^2}{15\,d^2x^3}+\frac{(-e^2x^2+d^2)^{\frac{3}{2}}e}{2\,dx^4}-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{5\,x^5}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-1/4*e^5*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^2 + 1/4*sqr
t(-e^2*x^2 + d^2)*e^5/d^3 + 1/4*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^3*x^2) - 7/1
5*(-e^2*x^2 + d^2)^(3/2)*e^2/(d^2*x^3) + 1/2*(-e^2*x^2 + d^2)^(3/2)*e/(d*x
^4) - 1/5*(-e^2*x^2 + d^2)^(3/2)/x^5
```

3.168.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.96

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx =$$
$$-\frac{1}{960}\left(\frac{240\,e^4\log\left(\sqrt{\frac{2\,d}{ex+d}-1}+1\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)}{d^2}-\frac{240\,e^4\log\left(\left|\sqrt{\frac{2\,d}{ex+d}-1}-1\right|\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)}{d^2}+\right.$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x, algorithm="giac")
```

output
```
-1/960*(240*e^4*log(sqrt(2*d/(e*x + d) - 1) + 1)*sgn(1/(e*x + d))*sgn(e)/d
^2 - 240*e^4*log(abs(sqrt(2*d/(e*x + d) - 1) - 1))*sgn(1/(e*x + d))*sgn(e)
/d^2 + 8*(15*e^4*log(2) - 30*e^4*log(I + 1) + 56*I*e^4)*sgn(1/(e*x + d))*s
gn(e)/d^2 - (15*e^4*(2*d/(e*x + d) - 1)^(9/2)*sgn(1/(e*x + d))*sgn(e) + 25
0*e^4*(2*d/(e*x + d) - 1)^(7/2)*sgn(1/(e*x + d))*sgn(e) - 128*e^4*(2*d/(e*
x + d) - 1)^(5/2)*sgn(1/(e*x + d))*sgn(e) + 70*e^4*(2*d/(e*x + d) - 1)^(3/
2)*sgn(1/(e*x + d))*sgn(e) - 15*e^4*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d
))*sgn(e))/(d^2*(d/(e*x + d) - 1)^5))*abs(e)
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}}{x^6\,(d+e\,x)^2}\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2), x)
```

3.169 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^7(d+ex)^2}\,dx$

3.169.1 Optimal result

Integrand size = 27, antiderivative size = 169

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^7(d+ex)^2}\,dx = -\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5} - \frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4} + \frac{4e^3\left(d^2-e^2x^2\right)^{3/2}}{15d^3x^3} + \frac{3e^6\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

output
```
-1/6*(-e^2*x^2+d^2)^(3/2)/x^6+2/5*e*(-e^2*x^2+d^2)^(3/2)/d/x^5-3/8*e^2*(-e
^2*x^2+d^2)^(3/2)/d^2/x^4+4/15*e^3*(-e^2*x^2+d^2)^(3/2)/d^3/x^3+3/16*e^6*a
rctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3-3/16*e^4*(-e^2*x^2+d^2)^(1/2)/d^2/x^2
```

3.169.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.73

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^7(d+ex)^2}\,dx = -\frac{\sqrt{d^2-e^2x^2}(40d^5-96d^4ex+50d^3e^2x^2+32d^2e^3x^3-45de^4x^4+64e^5x^5)+90e^6x^6\text{arctanh}\left(\frac{\sqrt{-e^2}x-\sqrt{d^2-e^2x^2}}{d}\right.}{240d^3x^6}$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2),x]
```

output
```
-1/240*(Sqrt[d^2 - e^2*x^2]*(40*d^5 - 96*d^4*e*x + 50*d^3*e^2*x^2 + 32*d^2
*e^3*x^3 - 45*d*e^4*x^4 + 64*e^5*x^5) + 90*e^6*x^6*ArcTanh[(Sqrt[-e^2]*x -
 Sqrt[d^2 - e^2*x^2])/d])/(d^3*x^6)
```

3.169.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {570, 540, 27, 539, 27, 539, 27, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^7(d+ex)^2}dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^7}dx$$

$$\Big\downarrow \text{540}$$

$$-\frac{\int \frac{3d^2e(4d-3ex)\sqrt{d^2-e^2x^2}}{x^6}dx}{6d^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}e\int \frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{x^6}dx-\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6}$$

$$\Big\downarrow \text{539}$$

$$-\frac{1}{2}e\left(-\frac{\int \frac{de(15d-8ex)\sqrt{d^2-e^2x^2}}{x^5}dx}{5d^2}-\frac{4\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}e\left(-\frac{e\int \frac{(15d-8ex)\sqrt{d^2-e^2x^2}}{x^5}dx}{5d}-\frac{4\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6}$$

$$\Big\downarrow \text{539}$$

$$-\frac{1}{2}e\left(-\frac{e\left(-\frac{\int \frac{de(32d-15ex)\sqrt{d^2-e^2x^2}}{x^4}dx}{4d^2}-\frac{15\left(d^2-e^2x^2\right)^{3/2}}{4dx^4}\right)}{5d}-\frac{4\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}e\left(-\frac{e\left(-\frac{e\int \frac{(32d-15ex)\sqrt{d^2-e^2x^2}}{x^4}dx}{4d}-\frac{15\left(d^2-e^2x^2\right)^{3/2}}{4dx^4}\right)}{5d}-\frac{4\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6}$$

$$\Big\downarrow \text{534}$$

$$-\frac{1}{2}e\left(-\frac{e\left(-\frac{e\left(-15e\int \frac{\sqrt{d^2-e^2x^2}}{x^3}dx-\frac{32\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}\right)}{4d}-\frac{15\left(d^2-e^2x^2\right)^{3/2}}{4dx^4}\right)}{5d}-\frac{4\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}e\left(-\frac{e\left(-\frac{e\left(-\frac{15}{2}e\int \frac{\sqrt{d^2-e^2x^2}}{x^4}dx^2-\frac{32\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}\right)}{4d}-\frac{15\left(d^2-e^2x^2\right)^{3/2}}{4dx^4}\right)}{5d}-\frac{4\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6}$$

$$\Big\downarrow \text{51}$$

3.169. $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^7(d+ex)^2}\,dx$

$$-\frac{1}{2}e\left(-\frac{e\left(-\frac{e\left(-\frac{15}{2}e\left(-\frac{1}{2}e^2\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{32\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}\right)}{4d}-\frac{15\left(d^2-e^2x^2\right)^{3/2}}{4dx^4}\right)}{5d}-\frac{4\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6}$$

$$\Big\downarrow \; 73$$

$$-\frac{1}{2}e\left(-\frac{e\left(-\frac{e\left(-\frac{15}{2}e\left(\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{32\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}\right)}{4d}-\frac{15\left(d^2-e^2x^2\right)^{3/2}}{4dx^4}\right)}{5d}-\frac{4\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6}$$

$$\Big\downarrow \; 221$$

$$-\frac{1}{2}e\left(-\frac{e\left(-\frac{e\left(-\frac{15}{2}e\left(\frac{e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{32\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}\right)}{4d}-\frac{15\left(d^2-e^2x^2\right)^{3/2}}{4dx^4}\right)}{5d}-\frac{4\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2),x]
```

3.169. $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^7(d+ex)^2}\,dx$

output
```
-1/6*(d^2 - e^2*x^2)^(3/2)/x^6 - (e*((-4*(d^2 - e^2*x^2)^(3/2))/(5*d*x^5)
- (e*((-15*(d^2 - e^2*x^2)^(3/2))/(4*d*x^4) - (e*((-32*(d^2 - e^2*x^2)^(3/
2))/(3*d*x^3) - (15*e*(-(Sqrt[d^2 - e^2*x^2]/x^2) + (e^2*ArcTanh[Sqrt[d^2
- e^2*x^2]/d])/d))/2))/(4*d)))/(5*d)))/2
```

3.169.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.169.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, \left(64 e^{5} x^{5}-45 d\, e^{4} x^{4}+32 d^{2} e^{3} x^{3}+50 d^{3} e^{2} x^{2}-96 d^{4} e x +40 d^{5}\right)}{240 d^{3} x^{6}}+\frac{3 e^{6} \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{16 d^{2} \sqrt{d^{2}}}$	121
default	Expression too large to display	1550

input
```
int((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/240*(-e^2*x^2+d^2)^(1/2)*(64*e^5*x^5-45*d*e^4*x^4+32*d^2*e^3*x^3+50*d^3
*e^2*x^2-96*d^4*e*x+40*d^5)/d^3/x^6+3/16/d^2*e^6/(d^2)^(1/2)*ln((2*d^2+2*(
d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.169.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.64

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)^2}\,dx = -\frac{45\,e^6x^6\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(64\,e^5x^5-45\,de^4x^4+32\,d^2e^3x^3+50\,d^3e^2x^2-96\,d^4ex+40\,d^5\right)\sqrt{-e^2x^2+d^2}}{240\,d^3x^6}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/240*(45*e^6*x^6*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (64*e^5*x^5 - 45*d
*e^4*x^4 + 32*d^2*e^3*x^3 + 50*d^3*e^2*x^2 - 96*d^4*e*x + 40*d^5)*sqrt(-e^
2*x^2 + d^2))/(d^3*x^6)
```

3.169.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.28 (sec) , antiderivative size = 808, normalized size of antiderivative = 4.78

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)^2}\,dx = d^2\left(\begin{cases} -\frac{d^2}{6ex^7\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{5e}{24x^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^3}{48d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^5}{16d^4x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^6\operatorname{acosh}\left(\frac{d}{ex}\right)}{16d^5} \\ \frac{id^2}{6ex^7\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{5ie}{24x^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^3}{48d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^5}{16d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^6\operatorname{asin}\left(\frac{d}{ex}\right)}{16d^5} \end{cases}\right. -2de\left(\begin{cases} \frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/x**7/(e*x+d)**2,x)
```

output
```
d**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5
*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) -
1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(1
6*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x*
*2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*
x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**
2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - 2*d*e*Piecewise((3*I*d
**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*
x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x
**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4
*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2
*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**
2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*
x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2
*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**
7), True)) + e**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) +
 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*
x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I
*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e
**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**...
```

3.169.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.07

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)^2}\,dx = \frac{3\,e^6\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{16\,d^3} - \frac{3\sqrt{-e^2x^2+d^2}e^6}{16\,d^4} - \frac{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^4}{16\,d^4x^2} + \frac{4\,(-e^2x^2+d^2)^{\frac{3}{2}}e^3}{15\,d^3x^3} - \frac{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^2}{8\,d^2x^4} + \frac{2\,(-e^2x^2+d^2)^{\frac{3}{2}}e}{5\,dx^5} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{6\,x^6}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x, algorithm="maxima")
```

output
```
3/16*e^6*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 3/16*sq
rt(-e^2*x^2 + d^2)*e^6/d^4 - 3/16*(-e^2*x^2 + d^2)^(3/2)*e^4/(d^4*x^2) + 4
/15*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^3*x^3) - 3/8*(-e^2*x^2 + d^2)^(3/2)*e^2/
(d^2*x^4) + 2/5*(-e^2*x^2 + d^2)^(3/2)*e/(d*x^5) - 1/6*(-e^2*x^2 + d^2)^(3
/2)/x^6
```

3.169.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.80

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)^2}\,dx = \frac{1}{7680}\left(\frac{1440\,e^5\log\left(\sqrt{\frac{2\,d}{ex+d}-1}+1\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)}{d^3} - \frac{1440\,e^5\log\left(\left|\sqrt{\frac{2\,d}{ex+d}-1}-\right.\right.}{d^3}\right.$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x, algorithm="giac")
```

output
```
1/7680*(1440*e^5*log(sqrt(2*d/(e*x + d) - 1) + 1)*sgn(1/(e*x + d))*sgn(e)/
d^3 - 1440*e^5*log(abs(sqrt(2*d/(e*x + d) - 1) - 1))*sgn(1/(e*x + d))*sgn(
e)/d^3 + 16*(45*e^5*log(2) - 90*e^5*log(I + 1) + 128*I*e^5)*sgn(1/(e*x + d
))*sgn(e)/d^3 - (45*e^5*(2*d/(e*x + d) - 1)^(11/2)*sgn(1/(e*x + d))*sgn(e)
 + 1025*e^5*(2*d/(e*x + d) - 1)^(9/2)*sgn(1/(e*x + d))*sgn(e) - 174*e^5*(2
*d/(e*x + d) - 1)^(7/2)*sgn(1/(e*x + d))*sgn(e) + 594*e^5*(2*d/(e*x + d) -
 1)^(5/2)*sgn(1/(e*x + d))*sgn(e) - 255*e^5*(2*d/(e*x + d) - 1)^(3/2)*sgn(
1/(e*x + d))*sgn(e) + 45*e^5*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))*sgn(
e))/(d^3*(d/(e*x + d) - 1)^6))*abs(e)
```

3.169.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)^2}\,dx = \int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^7\,\left(d+e\,x\right)^2}\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2), x)
```

3.170 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^8(d+ex)^2}\,dx$

3.170.1 Optimal result

Integrand size = 27, antiderivative size = 198

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^8(d+ex)^2}\,dx=\frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}+\frac{e\left(d^2-e^2x^2\right)^{3/2}}{3dx^6}-\frac{11e^2\left(d^2-e^2x^2\right)^{3/2}}{35d^2x^5}+\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{4d^3x^4}-\frac{22e^4\left(d^2-e^2x^2\right)^{3/2}}{105d^4x^3}-\frac{e^7\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}$$

output
```
-1/7*(-e^2*x^2+d^2)^(3/2)/x^7+1/3*e*(-e^2*x^2+d^2)^(3/2)/d/x^6-11/35*e^2*(
-e^2*x^2+d^2)^(3/2)/d^2/x^5+1/4*e^3*(-e^2*x^2+d^2)^(3/2)/d^3/x^4-22/105*e^
4*(-e^2*x^2+d^2)^(3/2)/d^4/x^3-1/8*e^7*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4
+1/8*e^5*(-e^2*x^2+d^2)^(1/2)/d^3/x^2
```

3.170.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.73

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^8(d+ex)^2}\,dx=\frac{\frac{d\sqrt{d^2-e^2x^2}\left(-120d^6+280d^5ex-144d^4e^2x^2-70d^3e^3x^3+88d^2e^4x^4-105de^5x^5+176e^6x^6\right)}{x^7}-105\sqrt{d^2}e^7\log(x)}{840d^5}$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2),x]
```

output
```
((d*Sqrt[d^2 - e^2*x^2]*(-120*d^6 + 280*d^5*e*x - 144*d^4*e^2*x^2 - 70*d^3
*e^3*x^3 + 88*d^2*e^4*x^4 - 105*d*e^5*x^5 + 176*e^6*x^6))/x^7 - 105*Sqrt[d
^2]*e^7*Log[x] + 105*Sqrt[d^2]*e^7*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(
840*d^5)
```

3.170.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {570, 540, 27, 539, 27, 539, 27, 539, 27, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^8(d+ex)^2}dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^8}dx$$

$$\Big\downarrow \text{540}$$

$$-\frac{\int \frac{d^2e(14d-11ex)\sqrt{d^2-e^2x^2}}{x^7}dx}{7d^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{7}e\int \frac{(14d-11ex)\sqrt{d^2-e^2x^2}}{x^7}dx-\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}$$

$$\Big\downarrow \text{539}$$

$$-\frac{1}{7}e\left(-\frac{\int \frac{6de(11d-7ex)\sqrt{d^2-e^2x^2}}{x^6}dx}{6d^2}-\frac{7\left(d^2-e^2x^2\right)^{3/2}}{3dx^6}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{7}e\left(-\frac{e\int \frac{(11d-7ex)\sqrt{d^2-e^2x^2}}{x^6}dx}{d}-\frac{7\left(d^2-e^2x^2\right)^{3/2}}{3dx^6}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}$$

$$\Big\downarrow \text{539}$$

$$-\frac{1}{7}e\left(-\frac{e\left(-\frac{\int \frac{de(35d-22ex)\sqrt{d^2-e^2x^2}}{x^5}dx}{5d^2}-\frac{11\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}\right)}{d}-\frac{7\left(d^2-e^2x^2\right)^{3/2}}{3dx^6}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{7}e\left(-\frac{e\left(-\frac{e\int \frac{(35d-22ex)\sqrt{d^2-e^2x^2}}{x^5}dx}{5d}-\frac{11\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}\right)}{d}-\frac{7\left(d^2-e^2x^2\right)^{3/2}}{3dx^6}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}$$

$$\Big\downarrow \; 539$$

$$-\frac{1}{7}e\left(-\frac{e\left(-\frac{e\left(-\frac{\int \frac{de(88d-35ex)\sqrt{d^2-e^2x^2}}{x^4}dx}{4d^2}-\frac{35\left(d^2-e^2x^2\right)^{3/2}}{4dx^4}\right)}{5d}-\frac{11\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}\right)}{d}-\frac{7\left(d^2-e^2x^2\right)^{3/2}}{3dx^6}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{7}e\left(-\frac{e\left(-\frac{e\left(-\frac{e\int \frac{(88d-35ex)\sqrt{d^2-e^2x^2}}{x^4}dx}{4d}-\frac{35\left(d^2-e^2x^2\right)^{3/2}}{4dx^4}\right)}{5d}-\frac{11\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}\right)}{d}-\frac{7\left(d^2-e^2x^2\right)^{3/2}}{3dx^6}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}$$

$$\Big\downarrow \; 534$$

3.170. $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^8(d+ex)^2}\,dx$

$$-\frac{1}{7}e\left(-\frac{e\left(-\frac{e\left(-\frac{e\left(-35e\int\frac{\sqrt{d^2-e^2x^2}}{x^3}dx-\frac{88\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}\right)}{4d}-\frac{35\left(d^2-e^2x^2\right)^{3/2}}{4dx^4}\right)}{5d}-\frac{11\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}\right)}{d}-\frac{7\left(d^2-e^2x^2\right)^{3/2}}{3dx^6}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}$$

$\Big\downarrow$ 243

$$-\frac{1}{7}e\left(-\frac{e\left(-\frac{e\left(-\frac{e\left(-\frac{35}{2}e\int\frac{\sqrt{d^2-e^2x^2}}{x^4}dx^2-\frac{88\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}\right)}{4d}-\frac{35\left(d^2-e^2x^2\right)^{3/2}}{4dx^4}\right)}{5d}-\frac{11\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}\right)}{d}-\frac{7\left(d^2-e^2x^2\right)^{3/2}}{3dx^6}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}$$

$\Big\downarrow$ 51

3.170. $\int\frac{\left(d^2-e^2x^2\right)^{5/2}}{x^8(d+ex)^2}\,dx$

$$-\frac{1}{7}e\left(-\frac{e\left(-\frac{e\left(-\frac{e\left(-\frac{35}{2}e\left(-\frac{1}{2}e^2\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{88\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}\right)}{4d}-\frac{35\left(d^2-e^2x^2\right)^{3/2}}{4dx^4}\right)}{5d}-\frac{11\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}\right)}{d}-\frac{7\left(d^2-\cdots\right.}{\cdots}\right.$$

$$\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}$$

$$\Big\downarrow \; 73$$

$$-\frac{1}{7}e\left(-\frac{e\left(-\frac{e\left(-\frac{e\left(-\frac{35}{2}e\left(\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{88\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}\right)}{4d}-\frac{35\left(d^2-e^2x^2\right)^{3/2}}{4dx^4}\right)}{5d}-\frac{11\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}\right)}{d}-\frac{7\left(d^2-\cdots\right.}{3d\cdots}\right.$$

$$\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}$$

$$\Big\downarrow \; 221$$

3.170. $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^8(d+ex)^2}\,dx$

$$-\frac{1}{7}e\left(-\frac{e\left(-\frac{e\left(-\frac{e\left(-\frac{35}{2}e\left(\frac{e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{x^2}\right)-\frac{88\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}\right)}{4d}-\frac{35\left(d^2-e^2x^2\right)^{3/2}}{4dx^4}\right)}{5d}-\frac{11\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}\right)}{d}-\frac{7(d\right.$$

$$\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2),x]
```

output
```
-1/7*(d^2 - e^2*x^2)^(3/2)/x^7 - (e*((-7*(d^2 - e^2*x^2)^(3/2))/(3*d*x^6)
- (e*((-11*(d^2 - e^2*x^2)^(3/2))/(5*d*x^5) - (e*((-35*(d^2 - e^2*x^2)^(3/
2))/(4*d*x^4) - (e*((-88*(d^2 - e^2*x^2)^(3/2))/(3*d*x^3) - (35*e*(-(Sqrt[
d^2 - e^2*x^2]/x^2) + (e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/2))/(4*d)))
/(5*d)))/d))/7
```

3.170.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.170. $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^8(d+ex)^2}\,dx$

3.170.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.67

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-176e^6x^6+105d\,e^5x^5-88d^2e^4x^4+70d^3x^3e^3+144d^4e^2x^2-280d^5ex+120d^6)}{840x^7d^4}-\frac{e^7\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8d^3\sqrt{d^2}}$
default	Expression too large to display

input
```
int((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/840*(-e^2*x^2+d^2)^(1/2)*(-176*e^6*x^6+105*d*e^5*x^5-88*d^2*e^4*x^4+70*
d^3*e^3*x^3+144*d^4*e^2*x^2-280*d^5*e*x+120*d^6)/x^7/d^4-1/8/d^3*e^7/(d^2)
^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

3.170.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.60

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)^2}\,dx = \frac{105\,e^7x^7\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(176\,e^6x^6-105\,de^5x^5+88\,d^2e^4x^4-70\,d^3e^3x^3-144}{840\,d^4x^7}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/840*(105*e^7*x^7*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (176*e^6*x^6 - 105
*d*e^5*x^5 + 88*d^2*e^4*x^4 - 70*d^3*e^3*x^3 - 144*d^4*e^2*x^2 + 280*d^5*e
*x - 120*d^6)*sqrt(-e^2*x^2 + d^2))/(d^4*x^7)
```

3.170.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.26 (sec) , antiderivative size = 835, normalized size of antiderivative = 4.22

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)^2}\,dx = d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{7x^6}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{35d^2x^4}+\frac{4e^5\sqrt{\frac{d^2}{e^2x^2}-1}}{105d^4x^2}+\frac{8e^7\sqrt{\frac{d^2}{e^2x^2}-1}}{105d^6} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{7x^6}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{35d^2x^4}+\frac{4ie^5\sqrt{-\frac{d^2}{e^2x^2}+1}}{105d^4x^2}+\frac{8ie^7\sqrt{-\frac{d^2}{e^2x^2}+1}}{105d^6} & \text{otherwise}\end{cases}\right)$$
$$-2de\left(\begin{cases} -\frac{d^2}{6ex^7\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{5e}{24x^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^3}{48d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^5}{16d^4x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^6\operatorname{acosh}\left(\frac{d}{ex}\right)}{16d^5} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{6ex^7\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{5ie}{24x^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^3}{48d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^5}{16d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^6\operatorname{asin}\left(\frac{d}{ex}\right)}{16d^5} & \text{otherwise}\end{cases}\right)$$
$$+e^2\left(\begin{cases} \frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/x**8/(e*x+d)**2,x)
```

output
```
d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e
**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**
4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**
2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(
e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105
*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) - 2*
d*e*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1
)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16
*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**
2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x
**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2
) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) + e**2*Piecewise((3*I*d**
3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x*
*2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**
6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x
**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x
**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*
x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x*
*7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**...
```

3.170. $\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)^2}\,dx$

3.170.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.04

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)^2}\,dx = -\frac{e^7\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{8\,d^4}+\frac{\sqrt{-e^2x^2+d^2}e^7}{8\,d^5} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^5}{8\,d^5x^2}-\frac{22\,(-e^2x^2+d^2)^{\frac{3}{2}}e^4}{105\,d^4x^3}+\frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^3}{4\,d^3x^4} - \frac{11\,(-e^2x^2+d^2)^{\frac{3}{2}}e^2}{35\,d^2x^5}+\frac{(-e^2x^2+d^2)^{\frac{3}{2}}e}{3\,dx^6}-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{7\,x^7}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-1/8*e^7*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^4 + 1/8*sqr
t(-e^2*x^2 + d^2)*e^7/d^5 + 1/8*(-e^2*x^2 + d^2)^(3/2)*e^5/(d^5*x^2) - 22/
105*(-e^2*x^2 + d^2)^(3/2)*e^4/(d^4*x^3) + 1/4*(-e^2*x^2 + d^2)^(3/2)*e^3/
(d^3*x^4) - 11/35*(-e^2*x^2 + d^2)^(3/2)*e^2/(d^2*x^5) + 1/3*(-e^2*x^2 + d
^2)^(3/2)*e/(d*x^6) - 1/7*(-e^2*x^2 + d^2)^(3/2)/x^7
```

3.170.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.68

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)^2}\,dx = -\frac{1}{53760}\left(\frac{6720\,e^6\log\left(\sqrt{\frac{2\,d}{ex+d}-1}+1\right)\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)}{d^4}-\frac{6720\,e^6\log\left(\left|\sqrt{\frac{2\,d}{ex+d}-1}-1\right|\right)\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e}{d^4}\right.$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x, algorithm="giac")
```

output
```
-1/53760*(6720*e^6*log(sqrt(2*d/(e*x + d) - 1) + 1)*sgn(1/(e*x + d))*sgn(e
)/d^4 - 6720*e^6*log(abs(sqrt(2*d/(e*x + d) - 1) - 1))*sgn(1/(e*x + d))*sg
n(e)/d^4 + 32*(105*e^6*log(2) - 210*e^6*log(I + 1) + 352*I*e^6)*sgn(1/(e*x
 + d))*sgn(e)/d^4 - (105*e^6*(2*d/(e*x + d) - 1)^(13/2)*sgn(1/(e*x + d))*s
gn(e) + 3780*e^6*(2*d/(e*x + d) - 1)^(11/2)*sgn(1/(e*x + d))*sgn(e) + 189*
e^6*(2*d/(e*x + d) - 1)^(9/2)*sgn(1/(e*x + d))*sgn(e) + 4992*e^6*(2*d/(e*x
 + d) - 1)^(7/2)*sgn(1/(e*x + d))*sgn(e) - 1981*e^6*(2*d/(e*x + d) - 1)^(5
/2)*sgn(1/(e*x + d))*sgn(e) + 700*e^6*(2*d/(e*x + d) - 1)^(3/2)*sgn(1/(e*x
 + d))*sgn(e) - 105*e^6*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))*sgn(e))/(
d^4*(d/(e*x + d) - 1)^7))*abs(e)
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)^2}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}}{x^8\,(d+e\,x)^2}\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2), x)
```

3.171 $\int \frac{x^4}{(d+ex)^2 (d^2-e^2x^2)^{3/2}} \, dx$

3.171.1 Optimal result

Integrand size = 27, antiderivative size = 123

$$\int \frac{x^4}{(d+ex)^2 (d^2-e^2x^2)^{3/2}} \, dx = -\frac{d^3(d-ex)^2}{5e^5 (d^2-e^2x^2)^{5/2}} + \frac{17d^2(d-ex)}{15e^5 (d^2-e^2x^2)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

output
```
-1/5*d^3*(-e*x+d)^2/e^5/(-e^2*x^2+d^2)^(5/2)+17/15*d^2*(-e*x+d)/e^5/(-e^2*
x^2+d^2)^(3/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5-2/15*(-13*e*x+15*d)/e^
5/(-e^2*x^2+d^2)^(1/2)
```

3.171.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{(d+ex)^2 (d^2-e^2x^2)^{3/2}} \, dx = \frac{\frac{\sqrt{d^2-e^2x^2}(-16d^3-17d^2ex+22de^2x^2+26e^3x^3)}{(d-ex)(d+ex)^3} + 30\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{15e^5}$$

input
```
Integrate[x^4/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(-16*d^3 - 17*d^2*e*x + 22*d*e^2*x^2 + 26*e^3*x^3))/
((d - e*x)*(d + e*x)^3) + 30*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]
)])/(15*e^5)
```

3.171.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {570, 529, 2166, 2345, 27, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(d+ex)^2 (d^2-e^2x^2)^{3/2}} \, dx$$

$$\Big\downarrow \ \textbf{570}$$

$$\int \frac{x^4(d-ex)^2}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \ \textbf{529}$$

$$-\frac{\int \frac{(d-ex)\left(\frac{2d^4}{e^4}-\frac{5xd^3}{e^3}+\frac{5x^2d^2}{e^2}-\frac{5x^3d}{e}\right)}{(d^2-e^2x^2)^{5/2}}dx}{5d}-\frac{d^3(d-ex)^2}{5e^5(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \ \textbf{2166}$$

$$-\frac{-\frac{\int \frac{\frac{11d^4}{e^4}-\frac{30xd^3}{e^3}+\frac{15x^2d^2}{e^2}}{(d^2-e^2x^2)^{3/2}}dx}{3d}-\frac{17d^3(d-ex)}{3e^5(d^2-e^2x^2)^{3/2}}}{5d}-\frac{d^3(d-ex)^2}{5e^5(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \ \textbf{2345}$$

$$-\frac{-\frac{-\frac{\int \frac{15d^4}{e^4\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{2d^2(15d-13ex)}{e^5\sqrt{d^2-e^2x^2}}}{3d}-\frac{17d^3(d-ex)}{3e^5(d^2-e^2x^2)^{3/2}}}{5d}-\frac{d^3(d-ex)^2}{5e^5(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \ \textbf{27}$$

$$-\frac{-\frac{-\frac{15d^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx}{e^4}-\frac{2d^2(15d-13ex)}{e^5\sqrt{d^2-e^2x^2}}}{3d}-\frac{17d^3(d-ex)}{3e^5(d^2-e^2x^2)^{3/2}}}{5d}-\frac{d^3(d-ex)^2}{5e^5(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \ \textbf{224}$$

$$-\frac{-\frac{-\frac{15d^2\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}}{e^4}-\frac{2d^2(15d-13ex)}{e^5\sqrt{d^2-e^2x^2}}}{3d}-\frac{17d^3(d-ex)}{3e^5(d^2-e^2x^2)^{3/2}}}{5d}-\frac{d^3(d-ex)^2}{5e^5(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow 216$$

$$-\frac{-\frac{-\frac{15d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}-\frac{2d^2(15d-13ex)}{e^5\sqrt{d^2-e^2x^2}}}{3d}-\frac{17d^3(d-ex)}{3e^5(d^2-e^2x^2)^{3/2}}}{5d}-\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[x^4/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
-1/5*(d^3*(d - e*x)^2)/(e^5*(d^2 - e^2*x^2)^(5/2)) - ((-17*d^3*(d - e*x))/
(3*e^5*(d^2 - e^2*x^2)^(3/2)) - ((-2*d^2*(15*d - 13*e*x))/(e^5*Sqrt[d^2 -
e^2*x^2]) - (15*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^5)/(3*d))/(5*d)
```

3.171.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.171.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(109) = 218$.

Time = 0.40 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.95

method	result
default	$\frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e^2}+\frac{3x}{\sqrt{-e^2x^2+d^2}\,e^4}-\frac{2d}{e^5\sqrt{-e^2x^2+d^2}}+\frac{d^4\left(-\frac{1}{5de\left(x+\frac{d}{e}\right)^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}+\frac{3e\left(-\right.}{}\right.}{}$

input
```
int(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(
-e^2*x^2+d^2)^(1/2)))+3/(-e^2*x^2+d^2)^(1/2)/e^4*x-2*d/e^5/(-e^2*x^2+d^2)^
(1/2)+d^4/e^6*(-1/5/d/e/(x+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+3/5
*e/d*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2*
(x+d/e)*e^2+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))-4/e^5*d^3*(-1/3/
d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2*(x+d/e)*e^2
+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.39

$$\int \frac{x^4}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx =$$
$$-\frac{16\,e^4x^4+32\,de^3x^3-32\,d^3ex-16\,d^4-30\,(e^4x^4+2\,de^3x^3-2\,d^3ex-d^4)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(26\,e^3}{15\,(e^9x^4+2\,de^8x^3-2\,d^3e^6x-d^4e^5)}$$

input
```
integrate(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/15*(16*e^4*x^4 + 32*d*e^3*x^3 - 32*d^3*e*x - 16*d^4 - 30*(e^4*x^4 + 2*d
*e^3*x^3 - 2*d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (2
6*e^3*x^3 + 22*d*e^2*x^2 - 17*d^2*e*x - 16*d^3)*sqrt(-e^2*x^2 + d^2))/(e^9
*x^4 + 2*d*e^8*x^3 - 2*d^3*e^6*x - d^4*e^5)
```

3.171.6 Sympy [F]

$$\int \frac{x^4}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx = \int \frac{x^4}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

input
```
integrate(x**4/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral(x**4/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.38

$$\int \frac{x^4}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx =$$
$$-\frac{d^3}{5\left(\sqrt{-e^2x^2+d^2}e^7x^2+2\sqrt{-e^2x^2+d^2}de^6x+\sqrt{-e^2x^2+d^2}d^2e^5\right)}$$
$$+\frac{17\,d^2}{15\left(\sqrt{-e^2x^2+d^2}e^6x+\sqrt{-e^2x^2+d^2}de^5\right)}$$
$$+\frac{26\,x}{15\sqrt{-e^2x^2+d^2}e^4}-\frac{\arcsin\left(\frac{ex}{d}\right)}{e^5}-\frac{2\,d}{\sqrt{-e^2x^2+d^2}e^5}$$

input
```
integrate(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/5*d^3/(sqrt(-e^2*x^2 + d^2)*e^7*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d*e^6*x +
sqrt(-e^2*x^2 + d^2)*d^2*e^5) + 17/15*d^2/(sqrt(-e^2*x^2 + d^2)*e^6*x + sq
rt(-e^2*x^2 + d^2)*d*e^5) + 26/15*x/(sqrt(-e^2*x^2 + d^2)*e^4) - arcsin(e*
x/d)/e^5 - 2*d/(sqrt(-e^2*x^2 + d^2)*e^5)
```

3.171.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: 1/ab
s(sageVARe)*(1/32768*(-20480/3*sageVARe^16*sqrt(2*sageVARd*sageVARe*(sageV
ARe*sageVARx+sageVARd)^-1/sageVARe-1)*(2*sageVARd*sageVARe*(sageVARe*sageV
ARx+sageVARd)^-1/sa
```

3.171.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \int \frac{x^4}{(d^2-e^2\,x^2)^{3/2}\,(d+e\,x)^2}\,dx$$

input
```
int(x^4/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

output
```
int(x^4/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2), x)
```

3.172 $\int \frac{x^3}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

3.172.1 Optimal result

Integrand size = 27, antiderivative size = 99

$$\int \frac{x^3}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx=\frac{d^2(d-ex)^2}{5e^4(d^2-e^2x^2)^{5/2}}-\frac{4d(d-ex)}{5e^4(d^2-e^2x^2)^{3/2}}+\frac{5d-2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

output
```
1/5*d^2*(-e*x+d)^2/e^4/(-e^2*x^2+d^2)^(5/2)-4/5*d*(-e*x+d)/e^4/(-e^2*x^2+d
^2)^(3/2)+1/5*(-2*e*x+5*d)/d/e^4/(-e^2*x^2+d^2)^(1/2)
```

3.172.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}(2d^3+4d^2ex+de^2x^2-2e^3x^3)}{5de^4(d-ex)(d+ex)^3}$$

input
```
Integrate[x^3/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(2*d^3 + 4*d^2*e*x + d*e^2*x^2 - 2*e^3*x^3))/(5*d*e^4
*(d - e*x)*(d + e*x)^3)
```

3.172.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {570, 529, 25, 2166, 27, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex)^2 (d^2-e^2x^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{x^3(d-ex)^2}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{529}$$

$$\frac{d^2(d-ex)^2}{5e^4(d^2-e^2x^2)^{5/2}}-\frac{\int -\frac{(d-ex)\left(\frac{2d^3}{e^3}-\frac{5xd^2}{e^2}+\frac{5x^2d}{e}\right)}{(d^2-e^2x^2)^{5/2}}dx}{5d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(d-ex)\left(\frac{2d^3}{e^3}-\frac{5xd^2}{e^2}+\frac{5x^2d}{e}\right)}{(d^2-e^2x^2)^{5/2}}dx}{5d}+\frac{d^2(d-ex)^2}{5e^4(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{2166}$$

$$\frac{-\frac{\int \frac{3d^2(2d-5ex)}{e^3\left(d^2-e^2x^2\right)^{3/2}}dx}{3d}-\frac{4d^2(d-ex)}{e^4(d^2-e^2x^2)^{3/2}}}{5d}+\frac{d^2(d-ex)^2}{5e^4(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{d\int \frac{2d-5ex}{\left(d^2-e^2x^2\right)^{3/2}}dx}{e^3}-\frac{4d^2(d-ex)}{e^4(d^2-e^2x^2)^{3/2}}}{5d}+\frac{d^2(d-ex)^2}{5e^4(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{453}$$

$$\frac{d^2(d-ex)^2}{5e^4(d^2-e^2x^2)^{5/2}}+\frac{\frac{5d-2ex}{e^4\sqrt{d^2-e^2x^2}}-\frac{4d^2(d-ex)}{e^4(d^2-e^2x^2)^{3/2}}}{5d}$$

input
```
Int[x^3/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
(d^2*(d - e*x)^2)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) + ((-4*d^2*(d - e*x))/(e^4
*(d^2 - e^2*x^2)^(3/2)) + (5*d - 2*e*x)/(e^4*Sqrt[d^2 - e^2*x^2]))/(5*d)
```

3.172.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

3.172.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.66

method	result
gosper	$\frac{(-ex+d)\left(-2e^3x^3+d\,e^2x^2+4d^2ex+2d^3\right)}{5(ex+d)d\,e^4\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}$
trager	$\frac{\left(-2e^3x^3+d\,e^2x^2+4d^2ex+2d^3\right)\sqrt{-e^2x^2+d^2}}{5d\,e^4(ex+d)^3(-ex+d)}$
default	$\frac{1}{\sqrt{-e^2x^2+d^2}\,e^4}-\frac{2x}{d\,e^3\sqrt{-e^2x^2+d^2}}-\frac{d^3\left(-\frac{1}{5de\left(x+\frac{d}{e}\right)^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}+\frac{3e\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\cdots\right)}{5d}\cdots\right)}{e^5}$

input
```
int(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(-e*x+d)*(-2*e^3*x^3+d*e^2*x^2+4*d^2*e*x+2*d^3)/(e*x+d)/d/e^4/(-e^2*x^
2+d^2)^(3/2)
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \frac{x^3}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{2\,e^4x^4+4\,de^3x^3-4\,d^3ex-2\,d^4+\left(2\,e^3x^3-de^2x^2-4\,d^2ex-2\,d^3\right)\sqrt{-e^2x^2\cdots}}{5\left(de^8x^4+2\,d^2e^7x^3-2\,d^4e^5x-d^5e^4\right)}$$

input
```
integrate(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
1/5*(2*e^4*x^4 + 4*d*e^3*x^3 - 4*d^3*e*x - 2*d^4 + (2*e^3*x^3 - d*e^2*x^2
- 4*d^2*e*x - 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d*e^8*x^4 + 2*d^2*e^7*x^3 - 2*
d^4*e^5*x - d^5*e^4)
```

3.172.6 Sympy [F]

$$\int \frac{x^3}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx=\int \frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

input
```
integrate(x**3/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral(x**3/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.59

$$\int \frac{x^3}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{d^2}{5\left(\sqrt{-e^2x^2+d^2}e^6x^2+2\sqrt{-e^2x^2+d^2}de^5x+\sqrt{-e^2x^2+d^2}d^2e^4\right)}-\frac{4\,d}{5\left(\sqrt{-e^2x^2+d^2}e^5x+\sqrt{-e^2x^2+d^2}de^4\right)}-\frac{2\,x}{5\sqrt{-e^2x^2+d^2}de^3}+\frac{1}{\sqrt{-e^2x^2+d^2}e^4}$$

input
```
integrate(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
1/5*d^2/(sqrt(-e^2*x^2 + d^2)*e^6*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d*e^5*x + s
qrt(-e^2*x^2 + d^2)*d^2*e^4) - 4/5*d/(sqrt(-e^2*x^2 + d^2)*e^5*x + sqrt(-e
^2*x^2 + d^2)*d*e^4) - 2/5*x/(sqrt(-e^2*x^2 + d^2)*d*e^3) + 1/(sqrt(-e^2*x
^2 + d^2)*e^4)
```

3.172.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.93

$$\int \frac{x^3}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx=$$

$$-\frac{\frac{16i\,\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)}{de^3}-\frac{5}{de^3\sqrt{\frac{2\,d}{ex+d}-1}\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)}-\frac{d^4e^{12}\left(\frac{2\,d}{ex+d}-1\right)^{\frac{5}{2}}\mathrm{sgn}\left(\frac{1}{ex+d}\right)^4\mathrm{sgn}(e)^4-5\,d^4e^{12}\left(\frac{2\,d}{ex+d}-1\right)^{\frac{3}{2}}\mathrm{sgn}\left(\frac{1}{ex+d}\right)^4\mathrm{sgn}(e)^4 \cdots}{d^5e^{15}\mathrm{sgn}\left(\frac{1}{ex+d}\right)^5\mathrm{sgn}(e)^5}}{40\,|e|}$$

input
```
integrate(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
-1/40*(16*I*sgn(1/(e*x + d))*sgn(e)/(d*e^3) - 5/(d*e^3*sqrt(2*d/(e*x + d)
- 1)*sgn(1/(e*x + d))*sgn(e)) - (d^4*e^12*(2*d/(e*x + d) - 1)^(5/2)*sgn(1/
(e*x + d))^4*sgn(e)^4 - 5*d^4*e^12*(2*d/(e*x + d) - 1)^(3/2)*sgn(1/(e*x +
d))^4*sgn(e)^4 + 15*d^4*e^12*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))^4*sg
n(e)^4)/(d^5*e^15*sgn(1/(e*x + d))^5*sgn(e)^5))/abs(e)
```

3.172.9 Mupad [B] (verification not implemented)

Time = 11.48 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{(d+ex)^2\,(d^2-e^2x^2)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,(2\,d^3+4\,d^2\,e\,x+d\,e^2\,x^2-2\,e^3\,x^3)}{5\,d\,e^4\,(d+e\,x)^3\,(d-e\,x)}$$

input
```
int(x^3/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(2*d^3 - 2*e^3*x^3 + d*e^2*x^2 + 4*d^2*e*x))/(5*d*e
^4*(d + e*x)^3*(d - e*x))
```

3.173 $\int \frac{x^2}{(d+ex)^2 (d^2-e^2x^2)^{3/2}} \, dx$

3.173.1 Optimal result

Integrand size = 27, antiderivative size = 89

$$\int \frac{x^2}{(d+ex)^2 (d^2-e^2x^2)^{3/2}} \, dx = \frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}} - \frac{d}{5e^3(d+ex)^2\sqrt{d^2-e^2x^2}} + \frac{7}{15e^3(d+ex)\sqrt{d^2-e^2x^2}}$$

output
```
1/15*x/d^2/e^2/(-e^2*x^2+d^2)^(1/2)-1/5*d/e^3/(e*x+d)^2/(-e^2*x^2+d^2)^(1/
2)+7/15/e^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

3.173.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(d+ex)^2 (d^2-e^2x^2)^{3/2}} \, dx = \frac{\sqrt{d^2-e^2x^2}(4d^3+8d^2ex+2de^2x^2+e^3x^3)}{15d^2e^3(d-ex)(d+ex)^3}$$

input
```
Integrate[x^2/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(4*d^3 + 8*d^2*e*x + 2*d*e^2*x^2 + e^3*x^3))/(15*d^2*
e^3*(d - e*x)*(d + e*x)^3)
```

3.173.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {570, 529, 27, 669, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex)^2 (d^2-e^2x^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{x^2(d-ex)^2}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{529}$$

$$-\frac{\int \frac{d(2d-5ex)(d-ex)}{e^2(d^2-e^2x^2)^{5/2}}dx}{5d}-\frac{d(d-ex)^2}{5e^3 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(2d-5ex)(d-ex)}{(d^2-e^2x^2)^{5/2}}dx}{5e^2}-\frac{d(d-ex)^2}{5e^3 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{669}$$

$$-\frac{-\frac{1}{3}\int \frac{1}{(d^2-e^2x^2)^{3/2}}dx-\frac{7(d-ex)}{3e(d^2-e^2x^2)^{3/2}}}{5e^2}-\frac{d(d-ex)^2}{5e^3 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{208}$$

$$-\frac{-\frac{x}{3d^2\sqrt{d^2-e^2x^2}}-\frac{7(d-ex)}{3e(d^2-e^2x^2)^{3/2}}}{5e^2}-\frac{d(d-ex)^2}{5e^3 (d^2-e^2x^2)^{5/2}}$$

input
```
Int[x^2/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
-1/5*(d*(d - e*x)^2)/(e^3*(d^2 - e^2*x^2)^(5/2)) - ((-7*(d - e*x))/(3*e*(d
^2 - e^2*x^2)^(3/2)) - x/(3*d^2*Sqrt[d^2 - e^2*x^2]))/(5*e^2)
```

3.173.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 669
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*
(p + 1))), x] - Simp[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1)))
Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

3.173. $\int \frac{x^2}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

3.173.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.73

method	result
gosper	$\frac{(-ex+d)\left(e^{3} x^{3}+2 d \,e^{2} x^{2}+8 d^{2} e x +4 d^{3}\right)}{15 (ex+d) d^{2} e^{3} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}$
trager	$\frac{\left(e^{3} x^{3}+2 d \,e^{2} x^{2}+8 d^{2} e x +4 d^{3}\right) \sqrt{-e^{2} x^{2}+d^{2}}}{15 d^{2} e^{3} \left(e x +d \right)^{3} \left(-e x +d \right)}$
default	$\frac{x}{d^{2} e^{2} \sqrt{-e^{2} x^{2}+d^{2}}}+\frac{d^{2} \left(-\frac{1}{5 d e \left(x +\frac{d}{e}\right)^{2} \sqrt{-\left(x +\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x +\frac{d}{e}\right)}}+\frac{3 e \left(-\frac{1}{3 d e \left(x +\frac{d}{e}\right) \sqrt{-\left(x +\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x +\frac{d}{e}\right)}}-\frac{-2 \left(x +\frac{d}{e}\right) e^{2}+2 d e}{3 e \,d^{3} \sqrt{-\left(x +\frac{d}{e}\right)^{2} e^{2}+2 d e}}\right.}{5 d}\right.}{e^{4}}$

input
```
int(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(-e*x+d)*(e^3*x^3+2*d*e^2*x^2+8*d^2*e*x+4*d^3)/(e*x+d)/d^2/e^3/(-e^2*
x^2+d^2)^(3/2)
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.33

$$\int \frac{x^2}{(d+ex)^2 (d^2-e^2x^2)^{3/2}} dx = \frac{4 e^4x^4 + 8 de^3x^3 - 8 d^3ex - 4 d^4 - (e^3x^3 + 2 de^2x^2 + 8 d^2ex + 4 d^3)\sqrt{-e^2x^2}}{15 (d^2e^7x^4 + 2 d^3e^6x^3 - 2 d^5e^4x - d^6e^3)}$$

input
```
integrate(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(4*e^4*x^4 + 8*d*e^3*x^3 - 8*d^3*e*x - 4*d^4 - (e^3*x^3 + 2*d*e^2*x^2
 + 8*d^2*e*x + 4*d^3)*sqrt(-e^2*x^2 + d^2))/(d^2*e^7*x^4 + 2*d^3*e^6*x^3 -
 2*d^5*e^4*x - d^6*e^3)
```

3.173.6 Sympy [F]

$$\int \frac{x^2}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \int \frac{x^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

input
```
integrate(x**2/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral(x**2/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.173.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.53

$$\int \frac{x^2}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = -\frac{d}{5\left(\sqrt{-e^2x^2+d^2}e^5x^2+2\sqrt{-e^2x^2+d^2}de^4x+\sqrt{-e^2x^2+d^2}d^2e^3\right)} + \frac{7}{15\left(\sqrt{-e^2x^2+d^2}e^4x+\sqrt{-e^2x^2+d^2}de^3\right)} + \frac{x}{15\sqrt{-e^2x^2+d^2}d^2e^2}$$

input
```
integrate(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/5*d/(sqrt(-e^2*x^2 + d^2)*e^5*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d*e^4*x + sq
rt(-e^2*x^2 + d^2)*d^2*e^3) + 7/15/(sqrt(-e^2*x^2 + d^2)*e^4*x + sqrt(-e^2
*x^2 + d^2)*d*e^3) + 1/15*x/(sqrt(-e^2*x^2 + d^2)*d^2*e^2)
```

3.173.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.15

$$\int \frac{x^2}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = -\frac{-\frac{8i\,\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)}{d^2e^2} - \frac{15}{d^2e^2\sqrt{\frac{2d}{ex+d}-1}\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)} + \frac{3d^8e^8\left(\frac{2d}{ex+d}-1\right)^{\frac{5}{2}}\mathrm{sgn}\left(\frac{1}{ex+d}\right)^4\mathrm{sgn}(e)^4 - 5d^8e^8\left(\frac{2d}{ex+d}-1\right)^{\frac{3}{2}}\mathrm{sgn}\left(\frac{1}{ex+d}\right)^4\mathrm{sgn}(e)}{d^{10}e^{10}\mathrm{sgn}\left(\frac{1}{ex+d}\right)^5\mathrm{sgn}(e)^5}}{120\,|e|}$$

input
```
integrate(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
-1/120*(-8*I*sgn(1/(e*x + d))*sgn(e)/(d^2*e^2) - 15/(d^2*e^2*sqrt(2*d/(e*x
 + d) - 1)*sgn(1/(e*x + d))*sgn(e)) + (3*d^8*e^8*(2*d/(e*x + d) - 1)^(5/2)
*sgn(1/(e*x + d))^4*sgn(e)^4 - 5*d^8*e^8*(2*d/(e*x + d) - 1)^(3/2)*sgn(1/(
e*x + d))^4*sgn(e)^4 - 15*d^8*e^8*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))
^4*sgn(e)^4)/(d^10*e^10*sgn(1/(e*x + d))^5*sgn(e)^5))/abs(e)
```

3.173.9 Mupad [B] (verification not implemented)

Time = 11.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{(d+ex)^2 (d^2-e^2x^2)^{3/2}} \, dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(4\,d^3+8\,d^2\,e\,x+2\,d\,e^2\,x^2+e^3\,x^3\right)}{15\,d^2\,e^3\,{\left(d+e\,x\right)}^3\,\left(d-e\,x\right)}$$

input
```
int(x^2/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(4*d^3 + e^3*x^3 + 2*d*e^2*x^2 + 8*d^2*e*x))/(15*d^
2*e^3*(d + e*x)^3*(d - e*x))
```

3.174 $\int \frac{x}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

3.174.1 Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \frac{x}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx = \frac{4x}{15d^3e\sqrt{d^2-e^2x^2}} + \frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}} - \frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}}$$

output
```
4/15*x/d^3/e/(-e^2*x^2+d^2)^(1/2)+1/5/e^2/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2)-2
/15/d/e^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

3.174.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.76

$$\int \frac{x}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(d^3+2d^2ex+8de^2x^2+4e^3x^3)}{15d^3e^2(d-ex)(d+ex)^3}$$

input
```
Integrate[x/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(d^3 + 2*d^2*e*x + 8*d*e^2*x^2 + 4*e^3*x^3))/(15*d^3*
e^2*(d - e*x)*(d + e*x)^3)
```

3.174.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {571, 470, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex)^2 (d^2-e^2x^2)^{3/2}} dx$$

$$\Big\downarrow \text{571}$$

$$\frac{2\int \frac{1}{(d+ex)(d^2-e^2x^2)^{3/2}}dx}{5e}+\frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{470}$$

$$\frac{2\left(\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}\right)}{5e}+\frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}}+\frac{2\left(\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}\right)}{5e}$$

input
```
Int[x/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
1/(5*e^2*(d + e*x)^2*Sqrt[d^2 - e^2*x^2]) + (2*((2*x)/(3*d^3*Sqrt[d^2 - e^
2*x^2]) - 1/(3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])))/(5*e)
```

3.174.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

rule 571
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*(n + p + 1))), x] + Simp[n/(2*d*
(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ((LtQ[n, -1] &&  !IGtQ[n + p +
1, 0]) || (LtQ[n, 0] && LtQ[p, -1]) || EqQ[n + 2*p + 2, 0]) && NeQ[n + p +
1, 0]
```

3.174.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

method	result
gosper	$\frac{(-ex+d)\left(4e^3x^3+8d\,e^2x^2+2d^2ex+d^3\right)}{15(ex+d)d^3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}$
trager	$\frac{\left(4e^3x^3+8d\,e^2x^2+2d^2ex+d^3\right)\sqrt{-e^2x^2+d^2}}{15d^3(ex+d)^3e^2(-ex+d)}$
default	$\frac{-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\frac{-2\left(x+\frac{d}{e}\right)e^2+2de}{3e\,d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}}{e^2}-\frac{d\left(-\frac{1}{5de\left(x+\frac{d}{e}\right)^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}+\frac{3e\left(-\frac{}{3de\left(x+\frac{d}{e}\right.}\right.}{}\right.}{}$

input
```
int(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(-e*x+d)*(4*e^3*x^3+8*d*e^2*x^2+2*d^2*e*x+d^3)/(e*x+d)/d^3/e^2/(-e^2*
x^2+d^2)^(3/2)
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

$$\int \frac{x}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \frac{e^4x^4+2\,de^3x^3-2\,d^3ex-d^4-(4\,e^3x^3+8\,de^2x^2+2\,d^2ex+d^3)\sqrt{-e^2x^2+}}{15\,(d^3e^6x^4+2\,d^4e^5x^3-2\,d^6e^3x-d^7e^2)}$$

input
```
integrate(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(e^4*x^4 + 2*d*e^3*x^3 - 2*d^3*e*x - d^4 - (4*e^3*x^3 + 8*d*e^2*x^2 +
 2*d^2*e*x + d^3)*sqrt(-e^2*x^2 + d^2))/(d^3*e^6*x^4 + 2*d^4*e^5*x^3 - 2*d
^6*e^3*x - d^7*e^2)
```

3.174.6 Sympy [F]

$$\int \frac{x}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \int \frac{x}{(-(-d+ex)(d+ex))^{\frac{3}{2}}(d+ex)^2}\,dx$$

input
```
integrate(x/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral(x/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.174.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.52

$$\int \frac{x}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \frac{1}{5\left(\sqrt{-e^2x^2+d^2}e^4x^2+2\sqrt{-e^2x^2+d^2}de^3x+\sqrt{-e^2x^2+d^2}d^2e^2\right)} - \frac{2}{15\left(\sqrt{-e^2x^2+d^2}de^3x+\sqrt{-e^2x^2+d^2}d^2e^2\right)} + \frac{4\,x}{15\sqrt{-e^2x^2+d^2}d^3e}$$

input
```
integrate(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
1/5/(sqrt(-e^2*x^2 + d^2)*e^4*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d*e^3*x + sqrt(
-e^2*x^2 + d^2)*d^2*e^2) - 2/15/(sqrt(-e^2*x^2 + d^2)*d*e^3*x + sqrt(-e^2*
x^2 + d^2)*d^2*e^2) + 4/15*x/(sqrt(-e^2*x^2 + d^2)*d^3*e)
```

3.174.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.95

$$\int \frac{x}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx =$$

$$-\frac{-\frac{32i\,\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)}{d^3} - \frac{15}{d^3\sqrt{\frac{2d}{ex+d}-1}\,\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)} - \frac{3d^{12}\left(\frac{2d}{ex+d}-1\right)^{\frac{5}{2}}\mathrm{sgn}\left(\frac{1}{ex+d}\right)^4\mathrm{sgn}(e)^4+5d^{12}\left(\frac{2d}{ex+d}-1\right)^{\frac{3}{2}}\mathrm{sgn}\left(\frac{1}{ex+d}\right)^4\mathrm{sgn}(e)^4-}{d^{15}\mathrm{sgn}\left(\frac{1}{ex+d}\right)^5\mathrm{sgn}(e)^5}}{120\,e|e|}$$

input
```
integrate(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
-1/120*(-32*I*sgn(1/(e*x + d))*sgn(e)/d^3 - 15/(d^3*sqrt(2*d/(e*x + d) - 1
)*sgn(1/(e*x + d))*sgn(e)) - (3*d^12*(2*d/(e*x + d) - 1)^(5/2)*sgn(1/(e*x
+ d))^4*sgn(e)^4 + 5*d^12*(2*d/(e*x + d) - 1)^(3/2)*sgn(1/(e*x + d))^4*sgn
(e)^4 - 15*d^12*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))^4*sgn(e)^4)/(d^15
*sgn(1/(e*x + d))^5*sgn(e)^5))/(e*abs(e))
```

3.174.9 Mupad [B] (verification not implemented)

Time = 11.81 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int \frac{x}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,(d^3+2\,d^2\,e\,x+8\,d\,e^2\,x^2+4\,e^3\,x^3)}{15\,d^3\,e^2\,(d+e\,x)^3\,(d-e\,x)}$$

input
```
int(x/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(d^3 + 4*e^3*x^3 + 8*d*e^2*x^2 + 2*d^2*e*x))/(15*d^
3*e^2*(d + e*x)^3*(d - e*x))
```

3.175 $\int \frac{1}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

3.175.1 Optimal result

Integrand size = 24, antiderivative size = 91

$$\int \frac{1}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx = \frac{2x}{5d^4\sqrt{d^2-e^2x^2}} - \frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}} - \frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}}$$

output
```
2/5*x/d^4/(-e^2*x^2+d^2)^(1/2)-1/5/d/e/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2)-1/5/
d^2/e/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

3.175.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

$$\int \frac{1}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-2d^3+d^2ex+4de^2x^2+2e^3x^3)}{5d^4e(d-ex)(d+ex)^3}$$

input
```
Integrate[1/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-2*d^3 + d^2*e*x + 4*d*e^2*x^2 + 2*e^3*x^3))/(5*d^4*
e*(d - e*x)*(d + e*x)^3)
```

3.175.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {461, 470, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{461}$$

$$\frac{3\int \frac{1}{(d+ex)(d^2-e^2x^2)^{3/2}}dx}{5d}-\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{470}$$

$$\frac{3\left(\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}\right)}{5d}-\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{3\left(\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}\right)}{5d}-\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}$$

input
```
Int[1/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
-1/5*1/(d*e*(d + e*x)^2*Sqrt[d^2 - e^2*x^2]) + (3*((2*x)/(3*d^3*Sqrt[d^2 -
 e^2*x^2]) - 1/(3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])))/(5*d)
```

3.175.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

3.175.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{(-ex+d)\left(-2e^{3}x^{3}-4d\,e^{2}x^{2}-d^{2}ex+2d^{3}\right)}{5(ex+d)d^{4}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}$	66
trager	$-\frac{\left(-2e^{3}x^{3}-4d\,e^{2}x^{2}-d^{2}ex+2d^{3}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{5d^{4}(ex+d)^{3}e(-ex+d)}$	68
default	$\frac{-\frac{1}{5de\left(x+\frac{d}{e}\right)^{2}\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}+\frac{3e\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}-\frac{-2\left(x+\frac{d}{e}\right)e^{2}+2de}{3e\,d^{3}\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}\right)}{5d}}{e^{2}}$	156

input
```
int(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-e*x+d)*(-2*e^3*x^3-4*d*e^2*x^2-d^2*e*x+2*d^3)/(e*x+d)/d^4/e/(-e^2*x
^2+d^2)^(3/2)
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.26

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = -\frac{2\,e^4x^4+4\,de^3x^3-4\,d^3ex-2\,d^4+(2\,e^3x^3+4\,de^2x^2+d^2ex-2\,d^3)\sqrt{-e^2x^2+d^2}}{5\,(d^4e^5x^4+2\,d^5e^4x^3-2\,d^7e^2x-d^8e)}$$

input
```
integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/5*(2*e^4*x^4 + 4*d*e^3*x^3 - 4*d^3*e*x - 2*d^4 + (2*e^3*x^3 + 4*d*e^2*x
^2 + d^2*e*x - 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d^4*e^5*x^4 + 2*d^5*e^4*x^3 -
 2*d^7*e^2*x - d^8*e)
```

3.175.6 Sympy [F]

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \int \frac{1}{(-(-d+ex)(d+ex))^{\frac{3}{2}}(d+ex)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral(1/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.175.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.49

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = -\frac{1}{5\left(\sqrt{-e^2x^2+d^2}de^3x^2+2\sqrt{-e^2x^2+d^2}d^2e^2x+\sqrt{-e^2x^2+d^2}d^3e\right)} - \frac{1}{5\left(\sqrt{-e^2x^2+d^2}d^2e^2x+\sqrt{-e^2x^2+d^2}d^3e\right)} + \frac{2\,x}{5\sqrt{-e^2x^2+d^2}d^4}$$

input
```
integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/5/(sqrt(-e^2*x^2 + d^2)*d*e^3*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d^2*e^2*x +
sqrt(-e^2*x^2 + d^2)*d^3*e) - 1/5/(sqrt(-e^2*x^2 + d^2)*d^2*e^2*x + sqrt(-
e^2*x^2 + d^2)*d^3*e) + 2/5*x/(sqrt(-e^2*x^2 + d^2)*d^4)
```

3.175.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.12

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \frac{e^3\left(\frac{5}{d^4e^3\sqrt{\frac{2d}{ex+d}-1}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)} - \frac{d^{16}e^{12}\left(\frac{2d}{ex+d}-1\right)^{\frac{5}{2}}\operatorname{sgn}\left(\frac{1}{ex+d}\right)^4\operatorname{sgn}(e)^4+5d^{16}e^{12}\left(\frac{2d}{ex+d}-\cdots\right.}{d^{20}e^{15}\operatorname{sgn}\cdots}\right.}{40\,|e|\cdots}$$

input
```
integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
1/40*(e^3*(5/(d^4*e^3*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))*sgn(e)) - (
d^16*e^12*(2*d/(e*x + d) - 1)^(5/2)*sgn(1/(e*x + d))^4*sgn(e)^4 + 5*d^16*e
^12*(2*d/(e*x + d) - 1)^(3/2)*sgn(1/(e*x + d))^4*sgn(e)^4 + 15*d^16*e^12*s
qrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))^4*sgn(e)^4)/(d^20*e^15*sgn(1/(e*x
+ d))^5*sgn(e)^5)) + 16*I*sgn(1/(e*x + d))*sgn(e)/d^4)/abs(e)
```

3.175.9 Mupad [B] (verification not implemented)

Time = 11.85 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^3+d^2\,e\,x+4\,d\,e^2\,x^2+2\,e^3\,x^3\right)}{5\,d^4\,e\,{\left(d+e\,x\right)}^3\,\left(d-e\,x\right)}$$

input
```
int(1/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(2*e^3*x^3 - 2*d^3 + 4*d*e^2*x^2 + d^2*e*x))/(5*d^4
*e*(d + e*x)^3*(d - e*x))
```

3.176 $\int \frac{1}{x(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

3.176.1 Optimal result

Integrand size = 27, antiderivative size = 118

$$\int \frac{1}{x(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx = \frac{2(d-ex)}{5d(d^2-e^2x^2)^{5/2}} + \frac{5d-8ex}{15d^3(d^2-e^2x^2)^{3/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

output
```
2/5*(-e*x+d)/d/(-e^2*x^2+d^2)^(5/2)+1/15*(-8*e*x+5*d)/d^3/(-e^2*x^2+d^2)^(
3/2)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5+1/15*(-16*e*x+15*d)/d^5/(-e^2*x^2
+d^2)^(1/2)
```

3.176.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int \frac{1}{x(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(26d^3+22d^2ex-17de^2x^2-16e^3x^3)}{(d-ex)(d+ex)^3} + 30\operatorname{arctanh}\left(\frac{\sqrt{-e^2}x-\sqrt{d^2-e^2x^2}}{d}\right)}{15d^5}$$

input
```
Integrate[1/(x*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(26*d^3 + 22*d^2*e*x - 17*d*e^2*x^2 - 16*e^3*x^3))/(
(d - e*x)*(d + e*x)^3) + 30*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d
])/(15*d^5)
```

3.176.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {570, 532, 25, 27, 532, 25, 532, 27, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{(d-ex)^2}{x (d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{532}$$

$$\frac{2(d-ex)}{5d (d^2-e^2x^2)^{5/2}}-\frac{\int -\frac{d(5d-8ex)}{x(d^2-e^2x^2)^{5/2}}dx}{5d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d(5d-8ex)}{x(d^2-e^2x^2)^{5/2}}dx}{5d^2}+\frac{2(d-ex)}{5d (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5d-8ex}{x(d^2-e^2x^2)^{5/2}}dx}{5d}+\frac{2(d-ex)}{5d (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{\frac{5d-8ex}{3d^2(d^2-e^2x^2)^{3/2}}-\frac{\int -\frac{15d-16ex}{x\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}}{5d}+\frac{2(d-ex)}{5d (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{15d-16ex}{x\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{5d-8ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d}+\frac{2(d-ex)}{5d (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{\frac{\frac{15d-16ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{15d}{x\sqrt{d^2-e^2x^2}}dx}{d^2}}{3d^2}+\frac{5d-8ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d}+\frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{15\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx}{d}+\frac{15d-16ex}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{5d-8ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d}+\frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{\frac{15\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2}{2d}+\frac{15d-16ex}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{5d-8ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d}+\frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{15d-16ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{15\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{de^2}}{3d^2}+\frac{5d-8ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d}+\frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{15d-16ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{15\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}}{3d^2}+\frac{5d-8ex}{3d^2(d^2-e^2x^2)^{3/2}}}{5d}+\frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[1/(x*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
(2*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + ((5*d - 8*e*x)/(3*d^2*(d^2 - e
^2*x^2)^(3/2)) + ((15*d - 16*e*x)/(d^2*Sqrt[d^2 - e^2*x^2]) - (15*ArcTanh[
Sqrt[d^2 - e^2*x^2]/d])/d^2)/(3*d^2))/(5*d)
```

3.176. $\int \frac{1}{x(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

3.176.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.176. $\int \frac{1}{x(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

3.176.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(104) = 208$.

Time = 0.41 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.80

method	result
default	$\frac{\frac{1}{d^2\sqrt{-e^2x^2+d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}}}{d^2} - \frac{-\frac{1}{5de\left(x+\frac{d}{e}\right)^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}+\frac{3e\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\cdots\right.}{5d}}{ed}$

input
```
int(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2
)*(-e^2*x^2+d^2)^(1/2))/x))-1/e/d*(-1/5/d/e/(x+d/e)^2/(-(x+d/e)^2*e^2+2*d*
e*(x+d/e))^(1/2)+3/5*e/d*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^
(1/2)-1/3/e/d^3*(-2*(x+d/e)*e^2+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2
)))-1/d^2*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3
*(-2*(x+d/e)*e^2+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.42

$$\int \frac{1}{x(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{26\,e^4x^4+52\,de^3x^3-52\,d^3ex-26\,d^4+15\left(e^4x^4+2\,de^3x^3-2\,d^3ex-d^4\right)\cdots}{15\left(d^5e^4x^4+2\,d^6e\cdots\right.}$$

input
```
integrate(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(26*e^4*x^4 + 52*d*e^3*x^3 - 52*d^3*e*x - 26*d^4 + 15*(e^4*x^4 + 2*d*
e^3*x^3 - 2*d^3*e*x - d^4)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (16*e^3*x^
3 + 17*d*e^2*x^2 - 22*d^2*e*x - 26*d^3)*sqrt(-e^2*x^2 + d^2))/(d^5*e^4*x^4
 + 2*d^6*e^3*x^3 - 2*d^8*e*x - d^9)
```

3.176.6 Sympy [F]

$$\int \frac{1}{x(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

input
```
integrate(1/x/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral(1/(x*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.176.7 Maxima [F]

$$\int \frac{1}{x(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}\left(ex+d\right)^2x}\,dx$$

input
```
integrate(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2*x), x)
```

3.176.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.41

$$\int \frac{1}{x(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx =$$
$$-\frac{\left(e^4\left(\frac{120\log\left(\sqrt{\frac{2d}{ex+d}-1}+1\right)}{d^5e^4\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)} - \frac{120\log\left(\left|\sqrt{\frac{2d}{ex+d}-1}-1\right|\right)}{d^5e^4\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)} - \frac{15}{d^5e^4\sqrt{\frac{2d}{ex+d}-1}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)} - \frac{3d^{20}e^{16}\left(\frac{2d}{ex+d}-1\right)^{\frac{5}{2}}\operatorname{sgn}\left(\frac{1}{ex+d}\right)^4\operatorname{sgn}(e}{\cdots}\right.\right.}{1\cdots}$$

input
```
integrate(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
-1/120*(e^4*(120*log(sqrt(2*d/(e*x + d) - 1) + 1)/(d^5*e^4*sgn(1/(e*x + d)
)*sgn(e)) - 120*log(abs(sqrt(2*d/(e*x + d) - 1) - 1))/(d^5*e^4*sgn(1/(e*x
+ d))*sgn(e)) - 15/(d^5*e^4*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))*sgn(e
)) - (3*d^20*e^16*(2*d/(e*x + d) - 1)^(5/2)*sgn(1/(e*x + d))^4*sgn(e)^4 +
25*d^20*e^16*(2*d/(e*x + d) - 1)^(3/2)*sgn(1/(e*x + d))^4*sgn(e)^4 + 165*d
^20*e^16*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))^4*sgn(e)^4)/(d^25*e^20*s
gn(1/(e*x + d))^5*sgn(e)^5)) + 4*(15*log(2) - 30*log(I + 1) + 32*I)*sgn(1/
(e*x + d))*sgn(e)/d^5)*e/abs(e)
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)^2 (d^2-e^2x^2)^{3/2}} \, dx = \int \frac{1}{x\,(d^2-e^2\,x^2)^{3/2}\,(d+e\,x)^2} \, dx$$

input
```
int(1/(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

output
```
int(1/(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2), x)
```

3.177 $\int \frac{1}{x^2(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

3.177.1 Optimal result

Integrand size = 27, antiderivative size = 146

$$\int \frac{1}{x^2(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = -\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}+\frac{2e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

output
```
-2/5*e*(-e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)-1/15*e*(-13*e*x+10*d)/d^4/(-e^2*x
^2+d^2)^(3/2)+2*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6-1/15*e*(-41*e*x+30*d
)/d^6/(-e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^6/x
```

3.177.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}\left(15d^4+76d^3ex+32d^2e^2x^2-82de^3x^3-56e^4x^4\right)}{x(-d+ex)(d+ex)^3}+30\sqrt{d^2}e\log(x)-30\sqrt{d^2}e\,\text{lo}}{15d^7}$$

input
```
Integrate[1/(x^2*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
((d*Sqrt[d^2 - e^2*x^2]*(15*d^4 + 76*d^3*e*x + 32*d^2*e^2*x^2 - 82*d*e^3*x
^3 - 56*e^4*x^4))/(x*(-d + e*x)*(d + e*x)^3) + 30*Sqrt[d^2]*e*Log[x] - 30*
Sqrt[d^2]*e*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(15*d^7)
```

3.177.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {570, 532, 25, 2336, 25, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(d+ex)^2(d^2-e^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{(d-ex)^2}{x^2(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{532}$$

$$-\frac{\int -\frac{5d^2-10exd+8e^2x^2}{x^2(d^2-e^2x^2)^{5/2}}dx}{5d^2}-\frac{2e(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5d^2-10exd+8e^2x^2}{x^2(d^2-e^2x^2)^{5/2}}dx}{5d^2}-\frac{2e(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{-\frac{\int -\frac{15d^2-30exd+26e^2x^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}-\frac{e(10d-13ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{2e(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{15d^2-30exd+26e^2x^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}-\frac{e(10d-13ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{2e(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{-\frac{\int -\frac{15d(d-2ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{e(30d-41ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{e(10d-13ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{2e(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{15\int \frac{d-2ex}{x^2\sqrt{d^2-e^2x^2}}dx}{d}-\frac{e(30d-41ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{e(10d-13ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{534}$$

$$\frac{\frac{\frac{15\left(-2e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)}{d}-\frac{e(30d-41ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{e(10d-13ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{\frac{15\left(-e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)}{d}-\frac{e(30d-41ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{e(10d-13ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{\frac{15\left(\frac{2\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)}{d}-\frac{e(30d-41ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{e(10d-13ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{15\left(\frac{2e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)}{d}-\frac{e(30d-41ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{e(10d-13ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[1/(x^2*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
(-2*e*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (-1/3*(e*(10*d - 13*e*x))
/(d^2*(d^2 - e^2*x^2)^(3/2)) + (-((e*(30*d - 41*e*x))/(d^2*Sqrt[d^2 - e^2*
x^2])) + (15*(-(Sqrt[d^2 - e^2*x^2]/(d*x)) + (2*e*ArcTanh[Sqrt[d^2 - e^2*x
^2]/d])/d))/d)/(3*d^2))/(5*d^2)
```

3.177. $\int \frac{1}{x^2(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

3.177.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.177.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{d^{6} x}+\frac{2 e \ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{d^{5} \sqrt{d^{2}}}-\frac{29 \sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{60 d^{5} e \left(x+\frac{d}{e}\right)^{2}}-\frac{313 \sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{120 d^{6} \left(x+\frac{d}{e}\right)}-$
default	$\frac{-\frac{1}{d^{2} x \sqrt{-e^{2} x^{2}+d^{2}}}+\frac{2 e^{2} x}{d^{4} \sqrt{-e^{2} x^{2}+d^{2}}}}{d^{2}}-\frac{2 e \left(\frac{1}{d^{2} \sqrt{-e^{2} x^{2}+d^{2}}}-\frac{\ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{d^{2} \sqrt{d^{2}}}\right)}{d^{3}}+$

input
```
int(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-e^2*x^2+d^2)^(1/2)/d^6/x+2/d^5*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-
e^2*x^2+d^2)^(1/2))/x)-29/60/d^5/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e)
)^(1/2)-313/120/d^6/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/10/d^4/
e^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/8/d^6/(x-d/e)*(-(x-d/
e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

3.177.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.33

$$\int \frac{1}{x^2(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx =$$
$$-\frac{46\,e^5x^5+92\,de^4x^4-92\,d^3e^2x^2-46\,d^4ex+30\left(e^5x^5+2\,de^4x^4-2\,d^3e^2x^2-d^4ex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(}{15\left(d^6e^4x^5+2\,d^7e^3x^4-2\,d^9ex^2-d^{10}x\right)}$$

input
```
integrate(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/15*(46*e^5*x^5 + 92*d*e^4*x^4 - 92*d^3*e^2*x^2 - 46*d^4*e*x + 30*(e^5*x
^5 + 2*d*e^4*x^4 - 2*d^3*e^2*x^2 - d^4*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2)
)/x) + (56*e^4*x^4 + 82*d*e^3*x^3 - 32*d^2*e^2*x^2 - 76*d^3*e*x - 15*d^4)*
sqrt(-e^2*x^2 + d^2))/(d^6*e^4*x^5 + 2*d^7*e^3*x^4 - 2*d^9*e*x^2 - d^10*x)
```

3.177.6 Sympy [F]

$$\int \frac{1}{x^2(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{x^2\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

input
```
integrate(1/x**2/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.177.7 Maxima [F]

$$\int \frac{1}{x^2(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}\left(ex+d\right)^2x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2*x^2), x)
```

3.177.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.22

$$\int \frac{1}{x^2(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{e^7\left(\frac{240\log\left(\sqrt{\frac{2d}{ex+d}-1}+1\right)}{d^6e^5\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)}-\frac{240\log\left(\left|\sqrt{\frac{2d}{ex+d}-1}-1\right|\right)}{d^6e^5\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)}+\frac{30\left(\frac{17d}{ex+d}-9\right)}{\left(\left(\frac{2d}{ex+d}-1\right)^{\frac{3}{2}}-\sqrt{\frac{2d}{ex+d}-1}\right)d^6e^5\mathrm{s}}\right.}{}$$

input
```
integrate(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
1/120*(e^7*(240*log(sqrt(2*d/(e*x + d) - 1) + 1)/(d^6*e^5*sgn(1/(e*x + d))
*sgn(e)) - 240*log(abs(sqrt(2*d/(e*x + d) - 1) - 1))/(d^6*e^5*sgn(1/(e*x +
 d))*sgn(e)) + 30*(17*d/(e*x + d) - 9)/(((2*d/(e*x + d) - 1)^(3/2) - sqrt(
2*d/(e*x + d) - 1))*d^6*e^5*sgn(1/(e*x + d))*sgn(e)) - (3*d^24*e^20*(2*d/(
e*x + d) - 1)^(5/2)*sgn(1/(e*x + d))^4*sgn(e)^4 + 35*d^24*e^20*(2*d/(e*x +
 d) - 1)^(3/2)*sgn(1/(e*x + d))^4*sgn(e)^4 + 345*d^24*e^20*sqrt(2*d/(e*x +
 d) - 1)*sgn(1/(e*x + d))^4*sgn(e)^4)/(d^30*e^25*sgn(1/(e*x + d))^5*sgn(e)
^5)) + 8*(15*e^2*log(2) - 30*e^2*log(I + 1) + 56*I*e^2)*sgn(1/(e*x + d))*s
gn(e)/d^6)/abs(e)
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{x^2\left(d^2-e^2x^2\right)^{3/2}(d+e\,x)^2}\,dx$$

input
```
int(1/(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

output
```
int(1/(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2), x)
```

3.178 $\int \frac{1}{x^3(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

3.178.1 Optimal result

Integrand size = 27, antiderivative size = 183

$$\int \frac{1}{x^3(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx = \frac{2e^2(d-ex)}{5d^3(d^2-e^2x^2)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5(d^2-e^2x^2)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{2e\sqrt{d^2-e^2x^2}}{d^7x} - \frac{9e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}$$

output
```
2/5*e^2*(-e*x+d)/d^3/(-e^2*x^2+d^2)^(5/2)+1/5*e^2*(-6*e*x+5*d)/d^5/(-e^2*x
^2+d^2)^(3/2)-9/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^7+2/5*e^2*(-11*e*x
+10*d)/d^7/(-e^2*x^2+d^2)^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/d^6/x^2+2*e*(-e^2
*x^2+d^2)^(1/2)/d^7/x
```

3.178.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(5d^5-10d^4ex-94d^3e^2x^2-58d^2e^3x^3+83de^4x^4+64e^5x^5)}{x^2(-d+ex)(d+ex)^3} + 90e^2\operatorname{arctanh}\left(\frac{\sqrt{-e^2}x-}{}\right.}{10d^7}$$

input
```
Integrate[1/(x^3*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(5*d^5 - 10*d^4*e*x - 94*d^3*e^2*x^2 - 58*d^2*e^3*x^
3 + 83*d*e^4*x^4 + 64*e^5*x^5))/(x^2*(-d + e*x)*(d + e*x)^3) + 90*e^2*ArcT
anh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(10*d^7)
```

3.178.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {570, 532, 25, 2336, 27, 2336, 27, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (d+ex)^2 (d^2-e^2x^2)^{3/2}} \, dx$$

$$\Big\downarrow \ 570$$

$$\int \frac{(d-ex)^2}{x^3 (d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \ 532$$

$$\frac{2e^2(d-ex)}{5d^3 (d^2-e^2x^2)^{5/2}}-\frac{\int -\frac{-\frac{8e^3x^3}{d}+10e^2x^2-10dex+5d^2}{x^3(d^2-e^2x^2)^{5/2}}dx}{5d^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{-\frac{8e^3x^3}{d}+10e^2x^2-10dex+5d^2}{x^3(d^2-e^2x^2)^{5/2}}dx}{5d^2}+\frac{2e^2(d-ex)}{5d^3 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \ 2336$$

$$\frac{\frac{e^2(5d-6ex)}{d^3(d^2-e^2x^2)^{3/2}}-\frac{\int -\frac{3\left(-\frac{12e^3x^3}{d}+15e^2x^2-10dex+5d^2\right)}{x^3\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}}{5d^2}+\frac{2e^2(d-ex)}{5d^3 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{-\frac{12e^3x^3}{d}+15e^2x^2-10dex+5d^2}{x^3\left(d^2-e^2x^2\right)^{3/2}}dx}{d^2}+\frac{e^2(5d-6ex)}{d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^2(d-ex)}{5d^3 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{\frac{2e^2(10d-11ex)}{d^3\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{5(d^2-2exd+4e^2x^2)}{x^3\sqrt{d^2-e^2x^2}}dx}{d^2}}{d^2}+\frac{e^2(5d-6ex)}{d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{5\int \frac{d^2-2exd+4e^2x^2}{x^3\sqrt{d^2-e^2x^2}}dx}{d^2}+\frac{2e^2(10d-11ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(5d-6ex)}{d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{2338}$$

$$\frac{\frac{\frac{5\left(-\frac{\int \frac{d^2e(4d-9ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{2e^2(10d-11ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(5d-6ex)}{d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{5\left(-\frac{1}{2}e\int \frac{4d-9ex}{x^2\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{2e^2(10d-11ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(5d-6ex)}{d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{534}$$

$$\frac{\frac{\frac{5\left(-\frac{1}{2}e\left(-9e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{4\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{2e^2(10d-11ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(5d-6ex)}{d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{\frac{5\left(-\frac{1}{2}e\left(-\frac{9}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{4\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{2e^2(10d-11ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(5d-6ex)}{d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{\frac{5\left(-\frac{1}{2}e\left(\frac{9\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{4\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{2e^2(10d-11ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(5d-6ex)}{d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

3.178. $\displaystyle\int \frac{1}{x^3(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

$$\downarrow \text{221}$$

$$\frac{\frac{\frac{5\left(-\frac{1}{2}e\left(\frac{9e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{4\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{2e^2(10d-11ex)}{d^3\sqrt{d^2-e^2x^2}}}{d^2}+\frac{e^2(5d-6ex)}{d^3(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[1/(x^3*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
(2*e^2*(d - e*x))/(5*d^3*(d^2 - e^2*x^2)^(5/2)) + ((e^2*(5*d - 6*e*x))/(d^
3*(d^2 - e^2*x^2)^(3/2)) + ((2*e^2*(10*d - 11*e*x))/(d^3*Sqrt[d^2 - e^2*x^
2]) + (5*(-1/2*Sqrt[d^2 - e^2*x^2]/x^2 - (e*((-4*Sqrt[d^2 - e^2*x^2])/(d*x
) + (9*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/2))/d^2)/d^2)/(5*d^2)
```

3.178.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.178. $\int \frac{1}{x^3(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.178. $\int \frac{1}{x^3(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

3.178.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-4ex+d)}{2d^7x^2}-\frac{9e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^6\sqrt{d^2}}+\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{10d^5e\left(x+\frac{d}{e}\right)^3}+\frac{13\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{20d^6\left(x+\frac{d}{e}\right)^2}$
default	$\frac{-\frac{1}{2d^2x^2\sqrt{-e^2x^2+d^2}}+\frac{3e^2\left(\frac{1}{d^2\sqrt{-e^2x^2+d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}}\right)}{2d^2}}{d^2}+\frac{3e^2\left(\frac{1}{d^2\sqrt{-e^2x^2+d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}}\right)}{d^4}-$

input
```
int(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e^2*x^2+d^2)^(1/2)*(-4*e*x+d)/d^7/x^2-9/2/d^6*e^2/(d^2)^(1/2)*ln((2
*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/10/d^5/e/(x+d/e)^3*(-(x+d/e)
^2*e^2+2*d*e*(x+d/e))^(1/2)+13/20/d^6/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d
/e))^(1/2)+181/40/d^7*e/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/8/d
^7*e/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^3(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{54\,e^6x^6+108\,de^5x^5-108\,d^3e^3x^3-54\,d^4e^2x^2+45\,(e^6x^6+2\,de^5x^5-2\,d^3}{}$$

input
```
integrate(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
1/10*(54*e^6*x^6 + 108*d*e^5*x^5 - 108*d^3*e^3*x^3 - 54*d^4*e^2*x^2 + 45*(
e^6*x^6 + 2*d*e^5*x^5 - 2*d^3*e^3*x^3 - d^4*e^2*x^2)*log(-(d - sqrt(-e^2*x
^2 + d^2))/x) + (64*e^5*x^5 + 83*d*e^4*x^4 - 58*d^2*e^3*x^3 - 94*d^3*e^2*x
^2 - 10*d^4*e*x + 5*d^5)*sqrt(-e^2*x^2 + d^2))/(d^7*e^4*x^6 + 2*d^8*e^3*x^
5 - 2*d^10*e*x^3 - d^11*x^2)
```

3.178.6 Sympy [F]

$$\int \frac{1}{x^3(d+ex)^2 (d^2-e^2x^2)^{3/2}}\, dx = \int \frac{1}{x^3 \left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}} \left(d+ex\right)^2}\, dx$$

input
```
integrate(1/x**3/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral(1/(x**3*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.178.7 Maxima [F]

$$\int \frac{1}{x^3(d+ex)^2 (d^2-e^2x^2)^{3/2}}\, dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}} \left(ex+d\right)^2 x^3}\, dx$$

input
```
integrate(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2*x^3), x)
```

3.178.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.97

$$\int \frac{1}{x^3(d+ex)^2 (d^2-e^2x^2)^{3/2}}\, dx =$$
$$-\frac{e^9 \left(\frac{180 \log\left(\sqrt{\frac{2d}{ex+d}-1}+1\right)}{d^7 e^6 \mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)} - \frac{180 \log\left(\left|\sqrt{\frac{2d}{ex+d}-1}-1\right|\right)}{d^7 e^6 \mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)} - \frac{5}{d^7 e^6 \sqrt{\frac{2d}{ex+d}-1}\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)} + \frac{10\left(5\left(\frac{2d}{ex+d}-1\right)^{\frac{3}{2}} - 3\sqrt{\frac{2d}{ex+d}-1}\right)}{d^7 e^6 \left(\frac{d}{ex+d}-1\right)^2 \mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)} - \frac{d^2 \cdots}{\cdots}\right)}{\cdots}$$

input
```
integrate(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
-1/40*(e^9*(180*log(sqrt(2*d/(e*x + d) - 1) + 1)/(d^7*e^6*sgn(1/(e*x + d))
*sgn(e)) - 180*log(abs(sqrt(2*d/(e*x + d) - 1) - 1))/(d^7*e^6*sgn(1/(e*x +
 d))*sgn(e)) - 5/(d^7*e^6*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))*sgn(e))
 + 10*(5*(2*d/(e*x + d) - 1)^(3/2) - 3*sqrt(2*d/(e*x + d) - 1))/(d^7*e^6*(
d/(e*x + d) - 1)^2*sgn(1/(e*x + d))*sgn(e)) - (d^28*e^24*(2*d/(e*x + d) -
1)^(5/2)*sgn(1/(e*x + d))^4*sgn(e)^4 + 15*d^28*e^24*(2*d/(e*x + d) - 1)^(3
/2)*sgn(1/(e*x + d))^4*sgn(e)^4 + 195*d^28*e^24*sqrt(2*d/(e*x + d) - 1)*sg
n(1/(e*x + d))^4*sgn(e)^4)/(d^35*e^30*sgn(1/(e*x + d))^5*sgn(e)^5)) + 2*(4
5*e^3*log(2) - 90*e^3*log(I + 1) + 128*I*e^3)*sgn(1/(e*x + d))*sgn(e)/d^7)
/abs(e)
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)^2\,(d^2-e^2x^2)^{3/2}}\,dx=\int \frac{1}{x^3\,(d^2-e^2\,x^2)^{3/2}\,(d+e\,x)^2}\,dx$$

input
```
int(1/(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

output
```
int(1/(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2), x)
```

3.179 $\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

3.179.1 Optimal result

Integrand size = 27, antiderivative size = 177

$$\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} - \frac{x\sqrt{d^2-e^2x^2}}{2e^5} + \frac{13d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

output
```
1/5*d^4*(-e*x+d)^3/e^6/(-e^2*x^2+d^2)^(5/2)-23/15*d^3*(-e*x+d)^2/e^6/(-e^2
*x^2+d^2)^(3/2)+13/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+127/15*d^2*(
-e*x+d)/e^6/(-e^2*x^2+d^2)^(1/2)+3*d*(-e^2*x^2+d^2)^(1/2)/e^6-1/2*x*(-e^2*
x^2+d^2)^(1/2)/e^5
```

3.179.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.63

$$\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(304d^4+717d^3ex+479d^2e^2x^2+45de^3x^3-15e^4x^4)}{30e^6(d+ex)^3} - \frac{13d^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

input
```
Integrate[x^5/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(304*d^4 + 717*d^3*e*x + 479*d^2*e^2*x^2 + 45*d*e^3*x
^3 - 15*e^4*x^4))/(30*e^6*(d + e*x)^3) - (13*d^2*ArcTan[(e*x)/(Sqrt[d^2] -
 Sqrt[d^2 - e^2*x^2])])/e^6
```

3.179.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.16, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {570, 529, 25, 2166, 2166, 27, 2346, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{x^5(d-ex)^3}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{529}$$

$$\frac{d^4(d-ex)^3}{5e^6(d^2-e^2x^2)^{5/2}}-\frac{\int -\frac{(d-ex)^2\left(\frac{3d^5}{e^5}-\frac{5xd^4}{e^4}+\frac{5x^2d^3}{e^3}-\frac{5x^3d^2}{e^2}+\frac{5x^4d}{e}\right)}{(d^2-e^2x^2)^{5/2}}dx}{5d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(d-ex)^2\left(\frac{3d^5}{e^5}-\frac{5xd^4}{e^4}+\frac{5x^2d^3}{e^3}-\frac{5x^3d^2}{e^2}+\frac{5x^4d}{e}\right)}{(d^2-e^2x^2)^{5/2}}dx}{5d}+\frac{d^4(d-ex)^3}{5e^6(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{2166}$$

$$\frac{-\frac{\int \frac{(d-ex)\left(\frac{37d^5}{e^5}-\frac{45xd^4}{e^4}+\frac{30x^2d^3}{e^3}-\frac{15x^3d^2}{e^2}\right)}{(d^2-e^2x^2)^{3/2}}dx}{3d}-\frac{23d^4(d-ex)^2}{3e^6(d^2-e^2x^2)^{3/2}}}{5d}+\frac{d^4(d-ex)^3}{5e^6(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{2166}$$

$$-\frac{-\frac{-\frac{\int \frac{15\left(\frac{6d^5}{e^5}-\frac{3xd^4}{e^4}+\frac{x^2d^3}{e^3}\right)}{\sqrt{d^2-e^2x^2}}dx}{d}-\frac{127d^4(d-ex)}{e^6\sqrt{d^2-e^2x^2}}}{3d}-\frac{23d^4(d-ex)^2}{3e^6(d^2-e^2x^2)^{3/2}}}{5d}+\frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{-\frac{15\int \frac{\frac{6d^5}{e^5}-\frac{3xd^4}{e^4}+\frac{x^2d^3}{e^3}}{\sqrt{d^2-e^2x^2}}dx}{d}-\frac{127d^4(d-ex)}{e^6\sqrt{d^2-e^2x^2}}}{3d}-\frac{23d^4(d-ex)^2}{3e^6(d^2-e^2x^2)^{3/2}}}{5d}+\frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 2346$$

$$-\frac{-\frac{-\frac{15\left(-\frac{\int -\frac{d^4(13d-6ex)}{e^3\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{d^3x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d}-\frac{127d^4(d-ex)}{e^6\sqrt{d^2-e^2x^2}}}{3d}-\frac{23d^4(d-ex)^2}{3e^6(d^2-e^2x^2)^{3/2}}}{5d}+\frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{-\frac{15\left(\frac{\int \frac{d^4(13d-6ex)}{e^3\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{d^3x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d}-\frac{127d^4(d-ex)}{e^6\sqrt{d^2-e^2x^2}}}{3d}-\frac{23d^4(d-ex)^2}{3e^6(d^2-e^2x^2)^{3/2}}}{5d}+\frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{-\frac{15\left(\frac{d^4\int \frac{13d-6ex}{\sqrt{d^2-e^2x^2}}dx}{2e^5}-\frac{d^3x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d}-\frac{127d^4(d-ex)}{e^6\sqrt{d^2-e^2x^2}}}{3d}-\frac{23d^4(d-ex)^2}{3e^6(d^2-e^2x^2)^{3/2}}}{5d}+\frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 455$$

$$-\frac{-\frac{-\frac{15\left(\frac{d^4\left(13d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{6\sqrt{d^2-e^2x^2}}{e}\right)}{2e^5}-\frac{d^3x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d}-\frac{127d^4(d-ex)}{e^6\sqrt{d^2-e^2x^2}}}{3d}-\frac{23d^4(d-ex)^2}{3e^6(d^2-e^2x^2)^{3/2}}}{5d}+\frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 224$$

3.179. $\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

$$\frac{-\frac{-\frac{15\left(\frac{d^4\left(13d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{6\sqrt{d^2-e^2x^2}}{e}\right)}{2e^5}-\frac{d^3x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d}-\frac{127d^4(d-ex)}{e^6\sqrt{d^2-e^2x^2}}}{3d}-\frac{23d^4(d-ex)^2}{3e^6(d^2-e^2x^2)^{3/2}}}{5d}+\frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 216$$

$$\frac{-\frac{-\frac{15\left(\frac{d^4\left(\frac{13d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{6\sqrt{d^2-e^2x^2}}{e}\right)}{2e^5}-\frac{d^3x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d}-\frac{127d^4(d-ex)}{e^6\sqrt{d^2-e^2x^2}}}{3d}-\frac{23d^4(d-ex)^2}{3e^6(d^2-e^2x^2)^{3/2}}}{5d}+\frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[x^5/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
(d^4*(d - e*x)^3)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) + ((-23*d^4*(d - e*x)^2)/(
3*e^6*(d^2 - e^2*x^2)^(3/2)) - ((-127*d^4*(d - e*x))/(e^6*Sqrt[d^2 - e^2*x
^2]) - (15*(-1/2*(d^3*x*Sqrt[d^2 - e^2*x^2])/e^5 + (d^4*((6*Sqrt[d^2 - e^2
*x^2])/e + (13*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/(2*e^5)))/d)/(3*d)
)/(5*d)
```

3.179.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.179. $\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.179. $\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

3.179.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.12

method	result
risch	$\frac{(-ex+6d)\sqrt{-e^2x^2+d^2}}{2e^6}+\frac{13d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^5\sqrt{e^2}}+\frac{d^4\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5e^9\left(x+\frac{d}{e}\right)^3}-\frac{23d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15e^8\left(x+\frac{d}{e}\right)^2}+$
default	$\frac{-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}}{e^3}+\frac{6d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^5\sqrt{e^2}}+\frac{3d\sqrt{-e^2x^2+d^2}}{e^6}+\frac{5d^4\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^2}-\right.}{e^7}$

input
```
int(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-e*x+6*d)/e^6*(-e^2*x^2+d^2)^(1/2)+13/2*d^2/e^5/(e^2)^(1/2)*arctan((e
^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/5*d^4/e^9/(x+d/e)^3*(-(x+d/e)^2*e^2+2*
d*e*(x+d/e))^(1/2)-23/15*d^3/e^8/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^
(1/2)+127/15*d^2/e^7/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.07

$$\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$$
$$=\frac{304\,d^2e^3x^3+912\,d^3e^2x^2+912\,d^4ex+304\,d^5-390\,(d^2e^3x^3+3\,d^3e^2x^2+3\,d^4ex+d^5)\arctan\left(-\frac{d-\sqrt{-e^2x^2+}}{ex}\right.}{30\,(e^9x^3+3\,de^8x^2+3\,d^2e^7x+d^3}$$

input
```
integrate(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
1/30*(304*d^2*e^3*x^3 + 912*d^3*e^2*x^2 + 912*d^4*e*x + 304*d^5 - 390*(d^2
*e^3*x^3 + 3*d^3*e^2*x^2 + 3*d^4*e*x + d^5)*arctan(-(d - sqrt(-e^2*x^2 + d
^2))/(e*x)) - (15*e^4*x^4 - 45*d*e^3*x^3 - 479*d^2*e^2*x^2 - 717*d^3*e*x -
 304*d^4)*sqrt(-e^2*x^2 + d^2))/(e^9*x^3 + 3*d*e^8*x^2 + 3*d^2*e^7*x + d^3
*e^6)
```

3.179.6 Sympy [F]

$$\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^5}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^3}\,dx$$

input
```
integrate(x**5/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(x**5/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.179.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.05

$$\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{-e^2x^2+d^2}d^4}{5\left(e^9x^3+3\,de^8x^2+3\,d^2e^7x+d^3e^6\right)} - \frac{23\,\sqrt{-e^2x^2+d^2}d^3}{15\left(e^8x^2+2\,de^7x+d^2e^6\right)} + \frac{127\,\sqrt{-e^2x^2+d^2}d^2}{15\left(e^7x+de^6\right)} + \frac{13\,d^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^6} - \frac{\sqrt{-e^2x^2+d^2}x}{2\,e^5} + \frac{3\,\sqrt{-e^2x^2+d^2}d}{e^6}$$

input
```
integrate(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
1/5*sqrt(-e^2*x^2 + d^2)*d^4/(e^9*x^3 + 3*d*e^8*x^2 + 3*d^2*e^7*x + d^3*e^
6) - 23/15*sqrt(-e^2*x^2 + d^2)*d^3/(e^8*x^2 + 2*d*e^7*x + d^2*e^6) + 127/
15*sqrt(-e^2*x^2 + d^2)*d^2/(e^7*x + d*e^6) + 13/2*d^2*arcsin(e*x/d)/e^6 -
 1/2*sqrt(-e^2*x^2 + d^2)*x/e^5 + 3*sqrt(-e^2*x^2 + d^2)*d/e^6
```

3.179.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.32

$$\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{2}\,\sqrt{-e^2x^2+d^2}\left(\frac{x}{e^5}-\frac{6\,d}{e^6}\right) + \frac{13\,d^2\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{2\,e^5|e|} - \frac{2\left(107\,d^2 + \frac{445\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2}{e^2x} + \frac{665\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^2}{e^4x^2} + \frac{405\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^2}{e^6x^3} + \frac{90\left(de+\sqrt{-e^2x^2+d^2}|e\right.}{e^8x^4}\right.}{15\,e^5\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

input
```
integrate(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-e^2*x^2 + d^2)*(x/e^5 - 6*d/e^6) + 13/2*d^2*arcsin(e*x/d)*sgn(d
)*sgn(e)/(e^5*abs(e)) - 2/15*(107*d^2 + 445*(d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))*d^2/(e^2*x) + 665*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^2/(e^4*x^2
) + 405*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^2/(e^6*x^3) + 90*(d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))^4*d^2/(e^8*x^4))/(e^5*((d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))/(e^2*x) + 1)^5*abs(e))
```

3.179.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^5}{\sqrt{d^2-e^2\,x^2}\,{(d+e\,x)}^3}\,dx$$

input
```
int(x^5/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

output
```
int(x^5/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

3.180 $\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

3.180.1 Optimal result

Integrand size = 27, antiderivative size = 146

$$\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx=-\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{e^5}-\frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

output
```
-1/5*d^3*(-e*x+d)^3/e^5/(-e^2*x^2+d^2)^(5/2)+6/5*d^2*(-e*x+d)^2/e^5/(-e^2*
x^2+d^2)^(3/2)-3*d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5-24/5*d*(-e*x+d)/e^
5/(-e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/e^5
```

3.180.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.68

$$\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx=\frac{\sqrt{d^2-e^2x^2}(-24d^3-57d^2ex-39de^2x^2-5e^3x^3)}{5e^5(d+ex)^3}+\frac{6d\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

input
```
Integrate[x^4/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-24*d^3 - 57*d^2*e*x - 39*d*e^2*x^2 - 5*e^3*x^3))/(5
*e^5*(d + e*x)^3) + (6*d*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/
e^5
```

3.180.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {570, 529, 2166, 27, 2166, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{x^4(d-ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{529}$$

$$-\frac{\int \frac{(d-ex)^2\left(\frac{3d^4}{e^4}-\frac{5xd^3}{e^3}+\frac{5x^2d^2}{e^2}-\frac{5x^3d}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d}-\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{2166}$$

$$-\frac{-\frac{\int \frac{3(d-ex)\left(\frac{9d^4}{e^4}-\frac{10xd^3}{e^3}+\frac{5x^2d^2}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d}-\frac{6d^3(d-ex)^2}{e^5\left(d^2-e^2x^2\right)^{3/2}}}{5d}-\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{(d-ex)\left(\frac{9d^4}{e^4}-\frac{10xd^3}{e^3}+\frac{5x^2d^2}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}dx}{d}-\frac{6d^3(d-ex)^2}{e^5\left(d^2-e^2x^2\right)^{3/2}}}{5d}-\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{2166}$$

$$-\frac{-\frac{-\frac{\int \frac{5d^3(3d-ex)}{e^4\sqrt{d^2-e^2x^2}}dx}{d}-\frac{24d^3(d-ex)}{e^5\sqrt{d^2-e^2x^2}}}{d}-\frac{6d^3(d-ex)^2}{e^5\left(d^2-e^2x^2\right)^{3/2}}}{5d}-\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{-\frac{5d^2\int\frac{3d-ex}{\sqrt{d^2-e^2x^2}}dx}{e^4}-\frac{24d^3(d-ex)}{e^5\sqrt{d^2-e^2x^2}}}{d}-\frac{6d^3(d-ex)^2}{e^5(d^2-e^2x^2)^{3/2}}}{5d}-\frac{d^3(d-ex)^3}{5e^5(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \; 455$$

$$-\frac{-\frac{-\frac{5d^2\left(3d\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{\sqrt{d^2-e^2x^2}}{e}\right)}{e^4}-\frac{24d^3(d-ex)}{e^5\sqrt{d^2-e^2x^2}}}{d}-\frac{6d^3(d-ex)^2}{e^5(d^2-e^2x^2)^{3/2}}}{5d}-\frac{d^3(d-ex)^3}{5e^5(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \; 224$$

$$-\frac{-\frac{-\frac{5d^2\left(3d\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e}\right)}{e^4}-\frac{24d^3(d-ex)}{e^5\sqrt{d^2-e^2x^2}}}{d}-\frac{6d^3(d-ex)^2}{e^5(d^2-e^2x^2)^{3/2}}}{5d}-\frac{d^3(d-ex)^3}{5e^5(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \; 216$$

$$-\frac{-\frac{-\frac{5d^2\left(\frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{\sqrt{d^2-e^2x^2}}{e}\right)}{e^4}-\frac{24d^3(d-ex)}{e^5\sqrt{d^2-e^2x^2}}}{d}-\frac{6d^3(d-ex)^2}{e^5(d^2-e^2x^2)^{3/2}}}{5d}-\frac{d^3(d-ex)^3}{5e^5(d^2-e^2x^2)^{5/2}}$$

input
```
Int[x^4/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-1/5*(d^3*(d - e*x)^3)/(e^5*(d^2 - e^2*x^2)^(5/2)) - ((-6*d^3*(d - e*x)^2)
/(e^5*(d^2 - e^2*x^2)^(3/2)) - ((-24*d^3*(d - e*x))/(e^5*Sqrt[d^2 - e^2*x^
2]) - (5*d^2*(Sqrt[d^2 - e^2*x^2]/e + (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2
]])/e))/e^4)/d)/(5*d)
```

3.180. $\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

3.180.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

3.180. $\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

3.180.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{e^5}-\frac{3d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^4\sqrt{e^2}}-\frac{24d\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5e^6\left(x+\frac{d}{e}\right)}+\frac{6d^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5e^7\left(x+\frac{d}{e}\right)^2}-\frac{d^3\sqrt{-\left(\cdots\right.}}{\cdots}$
default	$-\frac{\sqrt{-e^2x^2+d^2}}{e^5}-\frac{3d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^4\sqrt{e^2}}+\frac{d^4\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5de\left(x+\frac{d}{e}\right)^3}+\frac{2e\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^2}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+\cdots}}{3d^2\left(x+\cdots\right)}\cdots\right)}{5d}\cdots\right)}{e^7}\cdots$

input
```
int(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-e^2*x^2+d^2)^(1/2)/e^5-3*d/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x
^2+d^2)^(1/2))-24/5*d/e^6/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+6/5
*d^2/e^7/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/5*d^3/e^8/(x+d/e
)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

3.180.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.19

$$\int\frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx=$$
$$-\frac{24\,de^3x^3+72\,d^2e^2x^2+72\,d^3ex+24\,d^4-30\,(de^3x^3+3\,d^2e^2x^2+3\,d^3ex+d^4)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\cdots}{5\,(e^8x^3+3\,de^7x^2+3\,d^2e^6x+d^3e^5)}$$

input
```
integrate(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/5*(24*d*e^3*x^3 + 72*d^2*e^2*x^2 + 72*d^3*e*x + 24*d^4 - 30*(d*e^3*x^3
+ 3*d^2*e^2*x^2 + 3*d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x
)) + (5*e^3*x^3 + 39*d*e^2*x^2 + 57*d^2*e*x + 24*d^3)*sqrt(-e^2*x^2 + d^2)
)/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5)
```

3.180.6 Sympy [F]

$$\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^4}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^3}\,dx$$

input
```
integrate(x**4/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(x**4/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.180.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.10

$$\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}d^3}{5\left(e^8x^3+3\,de^7x^2+3\,d^2e^6x+d^3e^5\right)} + \frac{6\sqrt{-e^2x^2+d^2}d^2}{5\left(e^7x^2+2\,de^6x+d^2e^5\right)} - \frac{24\sqrt{-e^2x^2+d^2}d}{5\left(e^6x+de^5\right)} - \frac{3\,d\arcsin\left(\frac{ex}{d}\right)}{e^5} - \frac{\sqrt{-e^2x^2+d^2}}{e^5}$$

input
```
integrate(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/5*sqrt(-e^2*x^2 + d^2)*d^3/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e
^5) + 6/5*sqrt(-e^2*x^2 + d^2)*d^2/(e^7*x^2 + 2*d*e^6*x + d^2*e^5) - 24/5*
sqrt(-e^2*x^2 + d^2)*d/(e^6*x + d*e^5) - 3*d*arcsin(e*x/d)/e^5 - sqrt(-e^2
*x^2 + d^2)/e^5
```

3.180.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.45

$$\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{3\,d\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^4|e|} - \frac{\sqrt{-e^2x^2+d^2}}{e^5} + \frac{2\left(19\,d + \frac{80\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d}{e^2x} + \frac{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d}{e^4x^2} + \frac{70\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d}{e^6x^3} + \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d}{e^8x^4}\right)}{5\,e^4\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

input
```
integrate(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
-3*d*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^4*abs(e)) - sqrt(-e^2*x^2 + d^2)/e^5 +
 2/5*(19*d + 80*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d/(e^2*x) + 120*(d*e +
 sqrt(-e^2*x^2 + d^2)*abs(e))^2*d/(e^4*x^2) + 70*(d*e + sqrt(-e^2*x^2 + d^
2)*abs(e))^3*d/(e^6*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d/(e^8
*x^4))/(e^4*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e))
```

3.180.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^4}{\sqrt{d^2-e^2\,x^2}\,(d+e\,x)^3}\,dx$$

input
```
int(x^4/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

output
```
int(x^4/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

3.181 $\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

3.181.1 Optimal result

Integrand size = 27, antiderivative size = 120

$$\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{d^2(d-ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{13d(d-ex)^2}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}} + \frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

output
```
1/5*d^2*(-e*x+d)^3/e^4/(-e^2*x^2+d^2)^(5/2)-13/15*d*(-e*x+d)^2/e^4/(-e^2*x
^2+d^2)^(3/2)+arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4+32/15*(-e*x+d)/e^4/(-e^
2*x^2+d^2)^(1/2)
```

3.181.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(22d^2+51dex+32e^2x^2)}{15e^4(d+ex)^3} - \frac{2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

input
```
Integrate[x^3/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(22*d^2 + 51*d*e*x + 32*e^2*x^2))/(15*e^4*(d + e*x)^3
) - (2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^4
```

3.181.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {570, 529, 25, 2166, 27, 665, 27, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{ 570}$$

$$\int \frac{x^3(d-ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 529}$$

$$\frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int -\frac{(d-ex)^2\left(\frac{3d^3}{e^3}-\frac{5xd^2}{e^2}+\frac{5x^2d}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(d-ex)^2\left(\frac{3d^3}{e^3}-\frac{5xd^2}{e^2}+\frac{5x^2d}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d}+\frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 2166}$$

$$\frac{-\frac{\int \frac{d^2(17d-15ex)(d-ex)}{e^3\left(d^2-e^2x^2\right)^{3/2}}dx}{3d}-\frac{13d^2(d-ex)^2}{3e^4(d^2-e^2x^2)^{3/2}}}{5d}+\frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{d\int \frac{(17d-15ex)(d-ex)}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3e^3}-\frac{13d^2(d-ex)^2}{3e^4(d^2-e^2x^2)^{3/2}}}{5d}+\frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 665}$$

$$\frac{-\frac{d\left(\frac{\int -\frac{15e}{\sqrt{d^2-e^2x^2}}dx}{e}-\frac{32(d-ex)}{e\sqrt{d^2-e^2x^2}}\right)}{3e^3}-\frac{13d^2(d-ex)^2}{3e^4(d^2-e^2x^2)^{3/2}}}{5d}+\frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{d\left(-15\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{32(d-ex)}{e\sqrt{d^2-e^2x^2}}\right)}{3e^3}-\frac{13d^2(d-ex)^2}{3e^4(d^2-e^2x^2)^{3/2}}}{5d}+\frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{-\frac{d\left(-15\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{32(d-ex)}{e\sqrt{d^2-e^2x^2}}\right)}{3e^3}-\frac{13d^2(d-ex)^2}{3e^4(d^2-e^2x^2)^{3/2}}}{5d}+\frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{-\frac{d\left(-\frac{15\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{32(d-ex)}{e\sqrt{d^2-e^2x^2}}\right)}{3e^3}-\frac{13d^2(d-ex)^2}{3e^4(d^2-e^2x^2)^{3/2}}}{5d}+\frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[x^3/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
(d^2*(d - e*x)^3)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) + ((-13*d^2*(d - e*x)^2)/(
3*e^4*(d^2 - e^2*x^2)^(3/2)) - (d*((-32*(d - e*x))/(e*Sqrt[d^2 - e^2*x^2])
 - (15*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/(3*e^3))/(5*d)
```

3.181.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 665
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2)^(3/2), x_Symbol] :> Simp[(-2^(m - 1))*d^(m - 2)*(e*f + d*g)^n*((d +
e*x)/(c*e^(n - 1)*Sqrt[a + c*x^2])), x] + Simp[1/(c*e^(n - 2))   Int[Expand
ToSum[(2^(m - 1)*d^(m - 1)*(e*f + d*g)^n - e^n*(d + e*x)^(m - 1)*(f + g*x)^
n)/(d - e*x), x]/Sqrt[a + c*x^2], x], x] /; FreeQ[{a, c, d, e, f, g}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

3.181. $\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

3.181.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(106) = 212$.

Time = 0.43 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.66

method	result
default	$\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^3\sqrt{e^2}} + \frac{3d^2\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^2} - \frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3d^2\left(x+\frac{d}{e}\right)}\right)}{e^5} + \frac{3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^5\left(x+\frac{d}{e}\right)} - \frac{d^3 \ldots}{}$

input
```
int(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+3/e^5*d^2*(-1
/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*(-(x
+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))+3/e^5/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d
/e))^(1/2)-d^3/e^6*(-1/5/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2
)+2/5*e/d*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2
/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.31

$$\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{22\,e^3x^3 + 66\,de^2x^2 + 66\,d^2ex + 22\,d^3 - 30\,(e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + (32\,e^2 \ldots}{15\,(e^7x^3 + 3\,de^6x^2 + 3\,d^2e^5x + d^3e^4)}$$

input
```
integrate(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(22*e^3*x^3 + 66*d*e^2*x^2 + 66*d^2*e*x + 22*d^3 - 30*(e^3*x^3 + 3*d*
e^2*x^2 + 3*d^2*e*x + d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (32
*e^2*x^2 + 51*d*e*x + 22*d^2)*sqrt(-e^2*x^2 + d^2))/(e^7*x^3 + 3*d*e^6*x^2
 + 3*d^2*e^5*x + d^3*e^4)
```

3.181.6 Sympy [F]

$$\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^3}{\sqrt{-(-d+ex)(d+ex)}(d+ex)^3}\,dx$$

input
```
integrate(x**3/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(x**3/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.181.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.13

$$\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{-e^2x^2+d^2}d^2}{5(e^7x^3+3de^6x^2+3d^2e^5x+d^3e^4)} - \frac{13\sqrt{-e^2x^2+d^2}d}{15(e^6x^2+2de^5x+d^2e^4)} + \frac{32\sqrt{-e^2x^2+d^2}}{15(e^5x+de^4)} + \frac{\arcsin\left(\frac{ex}{d}\right)}{e^4}$$

input
```
integrate(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
1/5*sqrt(-e^2*x^2 + d^2)*d^2/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^
4) - 13/15*sqrt(-e^2*x^2 + d^2)*d/(e^6*x^2 + 2*d*e^5*x + d^2*e^4) + 32/15*
sqrt(-e^2*x^2 + d^2)/(e^5*x + d*e^4) + arcsin(e*x/d)/e^4
```

3.181.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.54

$$\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^3|e|} - \frac{2\left(\frac{95\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} + \frac{145\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{75\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} + \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4} + 22\right)}{15e^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

input
```
integrate(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
arcsin(e*x/d)*sgn(d)*sgn(e)/(e^3*abs(e)) - 2/15*(95*(d*e + sqrt(-e^2*x^2 +
 d^2)*abs(e))/(e^2*x) + 145*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e^4*x^2
) + 75*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/(e^6*x^3) + 15*(d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) + 22)/(e^3*((d*e + sqrt(-e^2*x^2 + d^2)*
abs(e))/(e^2*x) + 1)^5*abs(e))
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^3}{\sqrt{d^2-e^2\,x^2}\,{\left(d+e\,x\right)}^3}\,dx$$

input
```
int(x^3/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

output
```
int(x^3/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

3.182 $\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

3.182.1 Optimal result

Integrand size = 27, antiderivative size = 95

$$\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{8\sqrt{d^2-e^2x^2}}{15e^3(d+ex)^2}-\frac{7\sqrt{d^2-e^2x^2}}{15de^3(d+ex)}$$

output
```
-1/5*d*(-e^2*x^2+d^2)^(1/2)/e^3/(e*x+d)^3+8/15*(-e^2*x^2+d^2)^(1/2)/e^3/(e
*x+d)^2-7/15*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)
```

3.182.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.55

$$\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{(-2d^2-6dex-7e^2x^2)\sqrt{d^2-e^2x^2}}{15de^3(d+ex)^3}$$

input
```
Integrate[x^2/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
((-2*d^2 - 6*d*e*x - 7*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(15*d*e^3*(d + e*x)^3
)
```

3.182.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.42, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {581, 25, 27, 671, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \; 581$$

$$\frac{\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}-\frac{\int -\frac{d(2d+ex)}{(d+ex)^3\sqrt{d^2-e^2x^2}}dx}{e^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{d(2d+ex)}{(d+ex)^3\sqrt{d^2-e^2x^2}}dx}{e^2}+\frac{\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{d\int \frac{2d+ex}{(d+ex)^3\sqrt{d^2-e^2x^2}}dx}{e^2}+\frac{\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}$$

$$\Big\downarrow \; 671$$

$$\frac{d\left(\frac{7}{5}\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{5e(d+ex)^3}\right)}{e^2}+\frac{\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}$$

$$\Big\downarrow \; 461$$

$$\frac{d\left(\frac{7}{5}\left(\frac{\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}dx}{3d}-\frac{\sqrt{d^2-e^2x^2}}{3de(d+ex)^2}\right)-\frac{\sqrt{d^2-e^2x^2}}{5e(d+ex)^3}\right)}{e^2}+\frac{\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}$$

$$\Big\downarrow \; 460$$

$$\frac{d\left(\frac{7}{5}\left(-\frac{\sqrt{d^2-e^2x^2}}{3d^2e(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{3de(d+ex)^2}\right)-\frac{\sqrt{d^2-e^2x^2}}{5e(d+ex)^3}\right)}{e^2}+\frac{\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}$$

input
```
Int[x^2/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
Sqrt[d^2 - e^2*x^2]/(e^3*(d + e*x)^2) + (d*(-1/5*Sqrt[d^2 - e^2*x^2]/(e*(d
 + e*x)^3) + (7*(-1/3*Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)^2) - Sqrt[d^2 - e
^2*x^2]/(3*d^2*e*(d + e*x))))/5))/e^2
```

3.182.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

3.182. $\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

3.182.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.52

method	result
trager	$-\frac{(7e^2x^2+6dex+2d^2)\sqrt{-e^2x^2+d^2}}{15d\,e^3(ex+d)^3}$
gosper	$-\frac{(-ex+d)(7e^2x^2+6dex+2d^2)}{15(ex+d)^2d\,e^3\sqrt{-e^2x^2+d^2}}$
default	$-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^4d\left(x+\frac{d}{e}\right)}+\frac{d^2\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5de\left(x+\frac{d}{e}\right)^3}+\frac{2e\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^2}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3d^2\left(x+\frac{d}{e}\right)}\right)}{5d}\right)}{e^5}$

input
```
int(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(7*e^2*x^2+6*d*e*x+2*d^2)/d/e^3/(e*x+d)^3*(-e^2*x^2+d^2)^(1/2)
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{2\,e^3x^3+6\,de^2x^2+6\,d^2ex+2\,d^3+(7\,e^2x^2+6\,dex+2\,d^2)\sqrt{-e^2x^2+d^2}}{15\,(de^6x^3+3\,d^2e^5x^2+3\,d^3e^4x+d^4e^3)}$$

input
```
integrate(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(2*e^3*x^3 + 6*d*e^2*x^2 + 6*d^2*e*x + 2*d^3 + (7*e^2*x^2 + 6*d*e*x
+ 2*d^2)*sqrt(-e^2*x^2 + d^2))/(d*e^6*x^3 + 3*d^2*e^5*x^2 + 3*d^3*e^4*x +
d^4*e^3)
```

3.182.6 Sympy [F]

$$\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^2}{\sqrt{-(-d+ex)(d+ex)}(d+ex)^3}\,dx$$

input
```
integrate(x**2/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.182.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.32

$$\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}d}{5\,(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3)} + \frac{8\,\sqrt{-e^2x^2+d^2}}{15\,(e^5x^2+2\,de^4x+d^2e^3)} - \frac{7\,\sqrt{-e^2x^2+d^2}}{15\,(de^4x+d^2e^3)}$$

input
```
integrate(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/5*sqrt(-e^2*x^2 + d^2)*d/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3
) + 8/15*sqrt(-e^2*x^2 + d^2)/(e^5*x^2 + 2*d*e^4*x + d^2*e^3) - 7/15*sqrt(
-e^2*x^2 + d^2)/(d*e^4*x + d^2*e^3)
```

3.182.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{4\left(\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x}+\frac{10\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2}+1\right)}{15\,de^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

input
```
integrate(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
4/15*(5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 10*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^2/(e^4*x^2) + 1)/(d*e^2*((d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))/(e^2*x) + 1)^5*abs(e))
```

3.182.9 Mupad [B] (verification not implemented)

Time = 11.69 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.51

$$\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2+6\,d\,e\,x+7\,e^2\,x^2\right)}{15\,d\,e^3\,{\left(d+e\,x\right)}^3}$$

input
```
int(x^2/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

output
```
-((d^2 - e^2*x^2)^(1/2)*(2*d^2 + 7*e^2*x^2 + 6*d*e*x))/(15*d*e^3*(d + e*x)
^3)
```

3.183 $\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

3.183.1 Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx=\frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3}-\frac{\sqrt{d^2-e^2x^2}}{5de^2(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}}{5d^2e^2(d+ex)}$$

output
```
1/5*(-e^2*x^2+d^2)^(1/2)/e^2/(e*x+d)^3-1/5*(-e^2*x^2+d^2)^(1/2)/d/e^2/(e*x
+d)^2-1/5*(-e^2*x^2+d^2)^(1/2)/d^2/e^2/(e*x+d)
```

3.183.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.51

$$\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx=-\frac{\sqrt{d^2-e^2x^2}(d^2+3dex+e^2x^2)}{5d^2e^2(d+ex)^3}$$

input
```
Integrate[x/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-1/5*(Sqrt[d^2 - e^2*x^2]*(d^2 + 3*d*e*x + e^2*x^2))/(d^2*e^2*(d + e*x)^3)
```

3.183.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {571, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{571}$$

$$\frac{3\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}dx}{5e}+\frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3}$$

$$\Big\downarrow \text{461}$$

$$\frac{3\left(\frac{\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}dx}{3d}-\frac{\sqrt{d^2-e^2x^2}}{3de(d+ex)^2}\right)}{5e}+\frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3}$$

$$\Big\downarrow \text{460}$$

$$\frac{3\left(-\frac{\sqrt{d^2-e^2x^2}}{3d^2e(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{3de(d+ex)^2}\right)}{5e}+\frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3}$$

input
```
Int[x/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
Sqrt[d^2 - e^2*x^2]/(5*e^2*(d + e*x)^3) + (3*(-1/3*Sqrt[d^2 - e^2*x^2]/(d*
e*(d + e*x)^2) - Sqrt[d^2 - e^2*x^2]/(3*d^2*e*(d + e*x))))/(5*e)
```

3.183.3.1 Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 571
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*(n + p + 1))), x] + Simp[n/(2*d*
(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ((LtQ[n, -1] &&  !IGtQ[n + p +
1, 0]) || (LtQ[n, 0] && LtQ[p, -1]) || EqQ[n + 2*p + 2, 0]) && NeQ[n + p +
1, 0]
```

3.183.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.47

method	result
trager	$-\frac{(e^2x^2+3dex+d^2)\sqrt{-e^2x^2+d^2}}{5d^2(ex+d)^3e^2}$
gosper	$-\frac{(-ex+d)(e^2x^2+3dex+d^2)}{5(ex+d)^2d^2e^2\sqrt{-e^2x^2+d^2}}$
default	$\frac{-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^2}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3d^2\left(x+\frac{d}{e}\right)}}{e^3}-\frac{d\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5de\left(x+\frac{d}{e}\right)^3}+\frac{2e\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^2}-\frac{\sqrt{-(x+}}{}\right.}{5d}\right.}{e^4}$

input
```
int(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(e^2*x^2+3*d*e*x+d^2)/d^2/(e*x+d)^3/e^2*(-e^2*x^2+d^2)^(1/2)
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3+(e^2x^2+3\,dex+d^2)\sqrt{-e^2x^2+d^2}}{5\,(d^2e^5x^3+3\,d^3e^4x^2+3\,d^4e^3x+d^5e^2)}$$

input
```
integrate(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/5*(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3 + (e^2*x^2 + 3*d*e*x + d^2)*
sqrt(-e^2*x^2 + d^2))/(d^2*e^5*x^3 + 3*d^3*e^4*x^2 + 3*d^4*e^3*x + d^5*e^2
)
```

3.183.6 Sympy [F]

$$\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x}{\sqrt{-(-d+ex)(d+ex)}\,(d+ex)^3}\,dx$$

input
```
integrate(x/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(x/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.183.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.33

$$\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{-e^2x^2+d^2}}{5\,(e^5x^3+3\,de^4x^2+3\,d^2e^3x+d^3e^2)} - \frac{\sqrt{-e^2x^2+d^2}}{5\,(de^4x^2+2\,d^2e^3x+d^3e^2)} - \frac{\sqrt{-e^2x^2+d^2}}{5\,(d^2e^3x+d^3e^2)}$$

input
```
integrate(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
1/5*sqrt(-e^2*x^2 + d^2)/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*e^2) -
 1/5*sqrt(-e^2*x^2 + d^2)/(d*e^4*x^2 + 2*d^2*e^3*x + d^3*e^2) - 1/5*sqrt(-
e^2*x^2 + d^2)/(d^2*e^3*x + d^3*e^2)
```

3.183.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.41

$$\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{2\left(\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x}+\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2}+\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3}+1\right)}{5\,d^2e\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

input
```
integrate(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
2/5*(5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 5*(d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))^2/(e^4*x^2) + 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/(e^
6*x^3) + 1)/(d^2*e*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs
(e))
```

3.183.9 Mupad [B] (verification not implemented)

Time = 11.83 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.46

$$\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}\,\left(d^2+3\,d\,e\,x+e^2\,x^2\right)}{5\,d^2\,e^2\,{\left(d+e\,x\right)}^3}$$

input
```
int(x/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

output
```
-((d^2 - e^2*x^2)^(1/2)*(d^2 + e^2*x^2 + 3*d*e*x))/(5*d^2*e^2*(d + e*x)^3)
```

3.184 $\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

3.184.1 Optimal result

Integrand size = 24, antiderivative size = 100

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}-\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)}$$

output
```
-1/5*(-e^2*x^2+d^2)^(1/2)/d/e/(e*x+d)^3-2/15*(-e^2*x^2+d^2)^(1/2)/d^2/e/(e
*x+d)^2-2/15*(-e^2*x^2+d^2)^(1/2)/d^3/e/(e*x+d)
```

3.184.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.52

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{(-7d^2-6dex-2e^2x^2)\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)^3}$$

input
```
Integrate[1/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
((-7*d^2 - 6*d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d + e*x)^3
)
```

3.184.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{461}$$

$$\frac{2\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}dx}{5d}-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}$$

$$\Big\downarrow \text{461}$$

$$\frac{2\left(\frac{\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}dx}{3d}-\frac{\sqrt{d^2-e^2x^2}}{3de(d+ex)^2}\right)}{5d}-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}$$

$$\Big\downarrow \text{460}$$

$$\frac{2\left(-\frac{\sqrt{d^2-e^2x^2}}{3d^2e(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{3de(d+ex)^2}\right)}{5d}-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}$$

input
```
Int[1/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-1/5*Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)^3) + (2*(-1/3*Sqrt[d^2 - e^2*x^2]/
(d*e*(d + e*x)^2) - Sqrt[d^2 - e^2*x^2]/(3*d^2*e*(d + e*x))))/(5*d)
```

3.184.3.1 Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

3.184.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.49

method	result	size
trager	$-\frac{\left(2 e^{2} x^{2}+6 d e x+7 d^{2}\right) \sqrt{-e^{2} x^{2}+d^{2}}}{15 d^{3} \left(e x+d\right)^{3} e}$	49
gosper	$-\frac{\left(-e x+d\right) \left(2 e^{2} x^{2}+6 d e x+7 d^{2}\right)}{15 \left(e x+d\right)^{2} d^{3} e \sqrt{-e^{2} x^{2}+d^{2}}}$	55
default	$\frac{-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{5 d e \left(x+\frac{d}{e}\right)^{3}}+\frac{2 e \left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{3 d e \left(x+\frac{d}{e}\right)^{2}}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{3 d^{2} \left(x+\frac{d}{e}\right)}\right)}{5 d}}{e^{3}}$	145

input
```
int(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(2*e^2*x^2+6*d*e*x+7*d^2)/d^3/(e*x+d)^3/e*(-e^2*x^2+d^2)^(1/2)
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{7\,e^3x^3+21\,de^2x^2+21\,d^2ex+7\,d^3+\left(2\,e^2x^2+6\,dex+7\,d^2\right)\sqrt{-e^2x^2+d^2}}{15\,\left(d^3e^4x^3+3\,d^4e^3x^2+3\,d^5e^2x+d^6e\right)}$$

input
```
integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(7*e^3*x^3 + 21*d*e^2*x^2 + 21*d^2*e*x + 7*d^3 + (2*e^2*x^2 + 6*d*e*
x + 7*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^4*x^3 + 3*d^4*e^3*x^2 + 3*d^5*e^2*
x + d^6*e)
```

3.184.6 Sympy [F]

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^3}\,dx$$

input
```
integrate(1/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.184.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.28

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}}{5\left(de^4x^3+3\,d^2e^3x^2+3\,d^3e^2x+d^4e\right)} - \frac{2\sqrt{-e^2x^2+d^2}}{15\left(d^2e^3x^2+2\,d^3e^2x+d^4e\right)} - \frac{2\sqrt{-e^2x^2+d^2}}{15\left(d^3e^2x+d^4e\right)}$$

input
```
integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/5*sqrt(-e^2*x^2 + d^2)/(d*e^4*x^3 + 3*d^2*e^3*x^2 + 3*d^3*e^2*x + d^4*e
) - 2/15*sqrt(-e^2*x^2 + d^2)/(d^2*e^3*x^2 + 2*d^3*e^2*x + d^4*e) - 2/15*s
qrt(-e^2*x^2 + d^2)/(d^3*e^2*x + d^4*e)
```

3.184.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.65

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{2\left(\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} + \frac{40\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{30\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} + \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4} + 7\right)}{15\,d^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

input
```
integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
2/15*(20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 40*(d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))^2/(e^4*x^2) + 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3
/(e^6*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) + 7)/(d^3*
((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e))
```

3.184.9 Mupad [B] (verification not implemented)

Time = 11.83 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.48

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}\,\left(7\,d^2+6\,d\,e\,x+2\,e^2\,x^2\right)}{15\,d^3\,e\,{\left(d+e\,x\right)}^3}$$

input
```
int(1/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

output
```
-((d^2 - e^2*x^2)^(1/2)*(7*d^2 + 2*e^2*x^2 + 6*d*e*x))/(15*d^3*e*(d + e*x)
^3)
```

3.185 $\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

3.185.1 Optimal result

Integrand size = 27, antiderivative size = 115

$$\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

output
```
4/5*(-e*x+d)/(-e^2*x^2+d^2)^(5/2)+1/15*(-11*e*x+5*d)/d^2/(-e^2*x^2+d^2)^(3
/2)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4+1/15*(-22*e*x+15*d)/d^4/(-e^2*x^2+
d^2)^(1/2)
```

3.185.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.86

$$\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}\left(32d^2+51dex+22e^2x^2\right)}{(d+ex)^3} - 15\sqrt{d^2}\log(x) + 15\sqrt{d^2}\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{15d^5}$$

input
```
Integrate[1/(x*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
((d*Sqrt[d^2 - e^2*x^2]*(32*d^2 + 51*d*e*x + 22*e^2*x^2))/(d + e*x)^3 - 15
*Sqrt[d^2]*Log[x] + 15*Sqrt[d^2]*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(15
*d^5)
```

3.185.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {570, 532, 25, 27, 532, 25, 532, 27, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{(d-ex)^3}{x(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{532}$$

$$\frac{4(d-ex)}{5(d^2-e^2x^2)^{5/2}}-\frac{\int -\frac{d^2(5d-11ex)}{x(d^2-e^2x^2)^{5/2}}dx}{5d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d^2(5d-11ex)}{x(d^2-e^2x^2)^{5/2}}dx}{5d^2}+\frac{4(d-ex)}{5(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{5d-11ex}{x(d^2-e^2x^2)^{5/2}}dx+\frac{4(d-ex)}{5(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{1}{5}\left(\frac{5d-11ex}{3d^2(d^2-e^2x^2)^{3/2}}-\frac{\int -\frac{15d-22ex}{x(d^2-e^2x^2)^{3/2}}dx}{3d^2}\right)+\frac{4(d-ex)}{5(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\left(\frac{\int \frac{15d-22ex}{x(d^2-e^2x^2)^{3/2}}dx}{3d^2}+\frac{5d-11ex}{3d^2(d^2-e^2x^2)^{3/2}}\right)+\frac{4(d-ex)}{5(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{1}{5}\left(\frac{\frac{15d-22ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{15d}{x\sqrt{d^2-e^2x^2}}dx}{d^2}}{3d^2}+\frac{5d-11ex}{3d^2(d^2-e^2x^2)^{3/2}}\right)+\frac{4(d-ex)}{5(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{\frac{15\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx}{d}+\frac{15d-22ex}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{5d-11ex}{3d^2(d^2-e^2x^2)^{3/2}}\right)+\frac{4(d-ex)}{5(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{5}\left(\frac{\frac{15\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2}{2d}+\frac{15d-22ex}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{5d-11ex}{3d^2(d^2-e^2x^2)^{3/2}}\right)+\frac{4(d-ex)}{5(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{5}\left(\frac{\frac{15d-22ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{15\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{de^2}}{3d^2}+\frac{5d-11ex}{3d^2(d^2-e^2x^2)^{3/2}}\right)+\frac{4(d-ex)}{5(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{5}\left(\frac{\frac{15d-22ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{15\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}}{3d^2}+\frac{5d-11ex}{3d^2(d^2-e^2x^2)^{3/2}}\right)+\frac{4(d-ex)}{5(d^2-e^2x^2)^{5/2}}$$

input
```
Int[1/(x*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
(4*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) + ((5*d - 11*e*x)/(3*d^2*(d^2 - e^
2*x^2)^(3/2)) + ((15*d - 22*e*x)/(d^2*Sqrt[d^2 - e^2*x^2]) - (15*ArcTanh[S
qrt[d^2 - e^2*x^2]/d])/d^2)/(3*d^2))/5
```

3.185. $\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

3.185.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.185. $\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

3.185.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(101) = 202$.

Time = 0.41 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.89

method	result
default	$-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{d^3\sqrt{d^2}} - \frac{-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^2} - \frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3d^2\left(x+\frac{d}{e}\right)}}{e\,d^2} + \frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e\,d^4\left(x+\frac{d}{e}\right)} - \frac{-\sqrt{}}{}$

input
```
int(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/e/d^
2*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)
*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))+1/e/d^4/(x+d/e)*(-(x+d/e)^2*e^2+2*d
*e*(x+d/e))^(1/2)-1/e^2/d*(-1/5/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e
))^(1/2)+2/5*e/d*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-
1/3/d^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
```

3.185.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.33

$$\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{32\,e^3x^3+96\,de^2x^2+96\,d^2ex+32\,d^3+15\,(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(22\,e^2x^2}{15\,(d^4e^3x^3+3\,d^5e^2x^2+3\,d^6ex+d^7)}$$

input
```
integrate(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(32*e^3*x^3 + 96*d*e^2*x^2 + 96*d^2*e*x + 32*d^3 + 15*(e^3*x^3 + 3*d*
e^2*x^2 + 3*d^2*e*x + d^3)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (22*e^2*x^
2 + 51*d*e*x + 32*d^2)*sqrt(-e^2*x^2 + d^2))/(d^4*e^3*x^3 + 3*d^5*e^2*x^2
+ 3*d^6*e*x + d^7)
```

3.185.6 Sympy [F]

$$\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x\sqrt{-(-d+ex)(d+ex)}(d+ex)^3}\,dx$$

input
```
integrate(1/x/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.185.7 Maxima [F]

$$\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-e^2x^2+d^2}(ex+d)^3x}\,dx$$

input
```
integrate(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3*x), x)
```

3.185.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(102) = 204$.

Time = 0.33 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.84

$$\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{e\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d^4|e|} - \frac{2\left(32\,e+\frac{115\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{ex}+\frac{185\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^3x^2}+\frac{135\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^5x^3}+\frac{45\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^7x^4}\right)}{15\,d^4\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

input
```
integrate(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
-e*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^4*
abs(e)) - 2/15*(32*e + 115*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e*x) + 185
*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e^3*x^2) + 135*(d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))^3/(e^5*x^3) + 45*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e
^7*x^4))/(d^4*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e))
```

3.185.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x\,\sqrt{d^2-e^2\,x^2}\,(d+e\,x)^3}\,dx$$

input
```
int(1/(x*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

output
```
int(1/(x*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

3.186 $\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

3.186.1 Optimal result

Integrand size = 27, antiderivative size = 146

$$\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}+\frac{3e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

output
```
-4/5*e*(-e*x+d)/d/(-e^2*x^2+d^2)^(5/2)-1/5*e*(-7*e*x+5*d)/d^3/(-e^2*x^2+d^
2)^(3/2)+3*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5-1/5*e*(-19*e*x+15*d)/d^5/
(-e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^5/x
```

3.186.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\frac{d\sqrt{d^2-e^2x^2}\left(5d^3+39d^2ex+57de^2x^2+24e^3x^3\right)}{x(d+ex)^3}-15\sqrt{d^2}e\log(x)+15\sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{5d^6}$$

input
```
Integrate[1/(x^2*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-1/5*((d*Sqrt[d^2 - e^2*x^2]*(5*d^3 + 39*d^2*e*x + 57*d*e^2*x^2 + 24*e^3*x
^3))/(x*(d + e*x)^3) - 15*Sqrt[d^2]*e*Log[x] + 15*Sqrt[d^2]*e*Log[Sqrt[d^2
] - Sqrt[d^2 - e^2*x^2]])/d^6
```

3.186.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {570, 532, 25, 2336, 27, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{(d-ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{532}$$

$$-\frac{\int -\frac{5d^3-15exd^2+16e^2x^2d}{x^2(d^2-e^2x^2)^{5/2}}dx}{5d^2}-\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5d^3-15exd^2+16e^2x^2d}{x^2(d^2-e^2x^2)^{5/2}}dx}{5d^2}-\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{-\frac{\int -\frac{3\left(5d^3-15exd^2+14e^2x^2d\right)}{x^2\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}-\frac{e(5d-7ex)}{d(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{5d^3-15exd^2+14e^2x^2d}{x^2\left(d^2-e^2x^2\right)^{3/2}}dx}{d^2}-\frac{e(5d-7ex)}{d(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{-\frac{\int -\frac{5d^2(d-3ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{e(15d-19ex)}{d\sqrt{d^2-e^2x^2}}}{d^2}-\frac{e(5d-7ex)}{d(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{5\int \frac{d-3ex}{x^2\sqrt{d^2-e^2x^2}}dx-\frac{e(15d-19ex)}{d\sqrt{d^2-e^2x^2}}}{d^2}-\frac{e(5d-7ex)}{d(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{534}$$

$$\frac{\frac{5\left(-3e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{e(15d-19ex)}{d\sqrt{d^2-e^2x^2}}}{d^2}-\frac{e(5d-7ex)}{d(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{5\left(-\frac{3}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{e(15d-19ex)}{d\sqrt{d^2-e^2x^2}}}{d^2}-\frac{e(5d-7ex)}{d(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{5\left(\frac{3\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{e(15d-19ex)}{d\sqrt{d^2-e^2x^2}}}{d^2}-\frac{e(5d-7ex)}{d(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{5\left(\frac{3e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{e(15d-19ex)}{d\sqrt{d^2-e^2x^2}}}{d^2}-\frac{e(5d-7ex)}{d(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[1/(x^2*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
(-4*e*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (-((e*(5*d - 7*e*x))/(d*(d^
2 - e^2*x^2)^(3/2))) + (-((e*(15*d - 19*e*x))/(d*Sqrt[d^2 - e^2*x^2])) + 5
*(-(Sqrt[d^2 - e^2*x^2]/(d*x)) + (3*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/
d^2)/(5*d^2)
```

3.186. $\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

3.186.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

3.186. $\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.186.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.36

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^5x}+\frac{3e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^4\sqrt{d^2}}-\frac{4\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5d^4e\left(x+\frac{d}{e}\right)^2}-\frac{19\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5d^5\left(x+\frac{d}{e}\right)}-\sqrt{-\left(x\right.}$
default	$-\frac{\sqrt{-e^2x^2+d^2}}{d^5x}+\frac{3e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^4\sqrt{d^2}}+\frac{-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5de\left(x+\frac{d}{e}\right)^3}+\frac{2e\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^2}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+}}{3d^2\left(x+\right.}\right.}{5d}}{e\,d^2}$

input
```
int(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-e^2*x^2+d^2)^(1/2)/d^5/x+3/d^4*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-
e^2*x^2+d^2)^(1/2))/x)-4/5/d^4/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^
(1/2)-19/5/d^5/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/5/d^3/e^2/(x
+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

3.186.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{24\,e^4x^4+72\,de^3x^3+72\,d^2e^2x^2+24\,d^3ex+15\,(e^4x^4+3\,de^3x^3+3\,d^2e^2x^2+d^3ex)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)}{5\,(d^5e^3x^4+3\,d^6e^2x^3+3\,d^7ex^2+d^8x)}$$

input
```
integrate(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/5*(24*e^4*x^4 + 72*d*e^3*x^3 + 72*d^2*e^2*x^2 + 24*d^3*e*x + 15*(e^4*x^
4 + 3*d*e^3*x^3 + 3*d^2*e^2*x^2 + d^3*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))
/x) + (24*e^3*x^3 + 57*d*e^2*x^2 + 39*d^2*e*x + 5*d^3)*sqrt(-e^2*x^2 + d^2
))/(d^5*e^3*x^4 + 3*d^6*e^2*x^3 + 3*d^7*e*x^2 + d^8*x)
```

3.186.6 Sympy [F]

$$\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x^2\sqrt{-(-d+ex)\,(d+ex)}\,(d+ex)^3}\,dx$$

input
```
integrate(1/x**2/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.186.7 Maxima [F]

$$\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-e^2x^2+d^2}(ex+d)^3x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3*x^2), x)
```

3.186.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(131) = 262$.

Time = 0.31 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.08

$$\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{3\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d^5|e|} - \frac{de+\sqrt{-e^2x^2+d^2}|e|}{2\,d^5x|e|} + \frac{\left(5\,e^2 + \frac{121\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{x} + \frac{410\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^2x^2} + \frac{610\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^4x^3} + \frac{425\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^6x^4} + 125\right.}{10\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^5\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

input
```
integrate(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
3*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d
^5*abs(e)) - 1/2*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(d^5*x*abs(e)) + 1/10
*(5*e^2 + 121*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/x + 410*(d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))^2/(e^2*x^2) + 610*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^
3/(e^4*x^3) + 425*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^6*x^4) + 125*(d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^8*x^5))*e^2*x/((d*e + sqrt(-e^2*x^2
 + d^2)*abs(e))*d^5*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*ab
s(e))
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x^2\,\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)^3}\,dx$$

input
```
int(1/(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

output
```
int(1/(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

3.187 $\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

3.187.1 Optimal result

Integrand size = 27, antiderivative size = 183

$$\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{4e^2(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}}+\frac{e^2(25d-31ex)}{15d^4(d^2-e^2x^2)^{3/2}}+\frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{13e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}$$

output
```
4/5*e^2*(-e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)+1/15*e^2*(-31*e*x+25*d)/d^4/(-e^
2*x^2+d^2)^(3/2)-13/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/15*e^2*(-1
07*e*x+90*d)/d^6/(-e^2*x^2+d^2)^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/d^5/x^2+3*e
*(-e^2*x^2+d^2)^(1/2)/d^6/x
```

3.187.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}(-15d^4+45d^3ex+479d^2e^2x^2+717de^3x^3+304e^4x^4)}{x^2(d+ex)^3}-195\sqrt{d^2}e^2\log(x)+195\sqrt{d^2}e^2\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{30d^7}$$

input
```
Integrate[1/(x^3*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
((d*Sqrt[d^2 - e^2*x^2]*(-15*d^4 + 45*d^3*e*x + 479*d^2*e^2*x^2 + 717*d*e^
3*x^3 + 304*e^4*x^4))/(x^2*(d + e*x)^3) - 195*Sqrt[d^2]*e^2*Log[x] + 195*S
qrt[d^2]*e^2*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(30*d^7)
```

3.187.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {570, 532, 25, 2336, 25, 2336, 27, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{ 570}$$

$$\int \frac{(d-ex)^3}{x^3\left(d^2-e^2x^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 532}$$

$$\frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int -\frac{5d^3-15exd^2+20e^2x^2d-16e^3x^3}{x^3\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{5d^3-15exd^2+20e^2x^2d-16e^3x^3}{x^3\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}+\frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 2336}$$

$$\frac{\frac{e^2(25d-31ex)}{3d^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int -\frac{15d^3-45exd^2+75e^2x^2d-62e^3x^3}{x^3\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}}{5d^2}+\frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{15d^3-45exd^2+75e^2x^2d-62e^3x^3}{x^3\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{e^2(25d-31ex)}{3d^2\left(d^2-e^2x^2\right)^{3/2}}}{5d^2}+\frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 2336}$$

$$\frac{\frac{\frac{e^2(90d-107ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{15\left(d^3-3exd^2+6e^2x^2d\right)}{x^3\sqrt{d^2-e^2x^2}}dx}{d^2}}{3d^2}+\frac{e^2(25d-31ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{15\int \frac{d^3-3exd^2+6e^2x^2d}{x^3\sqrt{d^2-e^2x^2}}dx}{d^2}+\frac{e^2(90d-107ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^2(25d-31ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 2338$$

$$\frac{\frac{\frac{15\left(-\frac{\int \frac{d^3e(6d-13ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{d\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{e^2(90d-107ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^2(25d-31ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{15\left(-\frac{1}{2}de\int \frac{6d-13ex}{x^2\sqrt{d^2-e^2x^2}}dx-\frac{d\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{e^2(90d-107ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^2(25d-31ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 534$$

$$\frac{\frac{\frac{15\left(-\frac{1}{2}de\left(-13e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{6\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{d\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{e^2(90d-107ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^2(25d-31ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{\frac{15\left(-\frac{1}{2}de\left(-\frac{13}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{6\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{d\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{e^2(90d-107ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^2(25d-31ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 73$$

3.187. $\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

$$\frac{\frac{15\left(-\frac{1}{2}de\left(\frac{13\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{6\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{d\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{e^2(90d-107ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^2(25d-31ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{15\left(-\frac{1}{2}de\left(\frac{13e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{6\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{d\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{e^2(90d-107ex)}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{e^2(25d-31ex)}{3d^2(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[1/(x^3*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
(4*e^2*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + ((e^2*(25*d - 31*e*x))/(
3*d^2*(d^2 - e^2*x^2)^(3/2)) + ((e^2*(90*d - 107*e*x))/(d^2*Sqrt[d^2 - e^2
*x^2]) + (15*(-1/2*(d*Sqrt[d^2 - e^2*x^2])/x^2 - (d*e*((-6*Sqrt[d^2 - e^2*
x^2])/(d*x) + (13*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/2))/d^2)/(3*d^2))/
(5*d^2)
```

3.187.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.187. $\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.187.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-6ex+d)}{2d^6x^2}-\frac{13e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^5\sqrt{d^2}}+\frac{107e\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15d^6\left(x+\frac{d}{e}\right)}+\frac{17\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x\right.}}{15d^5\left(x+\frac{d}{e}\right)^2}$
default	$\frac{-\frac{\sqrt{-e^2x^2+d^2}}{2d^2x^2}-\frac{e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^2\sqrt{d^2}}}{d^3}-\frac{6e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^5\sqrt{d^2}}+\frac{3e\sqrt{-e^2x^2+d^2}}{d^6x}-\frac{3e\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de}}{3de\left(x+\frac{d}{e}\right)^2}\right.}{}$

input
```
int(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e^2*x^2+d^2)^(1/2)*(-6*e*x+d)/d^6/x^2-13/2/d^5*e^2/(d^2)^(1/2)*ln((
2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+107/15/d^6*e/(x+d/e)*(-(x+d/e
)^2*e^2+2*d*e*(x+d/e))^(1/2)+17/15/d^5/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+
d/e))^(1/2)+1/5/d^4/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

3.187.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{254\,e^5x^5+762\,de^4x^4+762\,d^2e^3x^3+254\,d^3e^2x^2+195\,(e^5x^5+3\,de^4x^4+3\,d^2e^3x^3+d^3e^2x^2)\log\left(-\frac{d-\sqrt{-e^2}}{x}\right.}{30\,(d^6e^3x^5+3\,d^7e^2x^4+3\,d^8ex^3+}$$

input
```
integrate(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
1/30*(254*e^5*x^5 + 762*d*e^4*x^4 + 762*d^2*e^3*x^3 + 254*d^3*e^2*x^2 + 19
5*(e^5*x^5 + 3*d*e^4*x^4 + 3*d^2*e^3*x^3 + d^3*e^2*x^2)*log(-(d - sqrt(-e^
2*x^2 + d^2))/x) + (304*e^4*x^4 + 717*d*e^3*x^3 + 479*d^2*e^2*x^2 + 45*d^3
*e*x - 15*d^4)*sqrt(-e^2*x^2 + d^2))/(d^6*e^3*x^5 + 3*d^7*e^2*x^4 + 3*d^8*
e*x^3 + d^9*x^2)
```

3.187.6 Sympy [F]

$$\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x^3\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^3}\,dx$$

input
```
integrate(1/x**3/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.187.7 Maxima [F]

$$\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-e^2x^2+d^2}\left(ex+d\right)^3x^3}\,dx$$

input
```
integrate(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3*x^3), x)
```

3.187.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(162) = 324$.

Time = 0.33 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.09

$$\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{13\,e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,d^6|e|}$$
$$+\frac{\left(15\,e^3-\frac{105\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e}{x}-\frac{2782\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{ex^2}-\frac{9410\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^3x^3}-\frac{13645\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^5x^4}-\right.}{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^6\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^{\cdots}}$$
$$+\frac{\frac{12\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^6e|e|}{x}-\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^6|e|}{ex^2}}{8\,d^{12}e^2}$$

input
```
integrate(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
-13/2*e^3*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x))
)/(d^6*abs(e)) + 1/120*(15*e^3 - 105*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e
/x - 2782*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e*x^2) - 9410*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^3/(e^3*x^3) - 13645*(d*e + sqrt(-e^2*x^2 + d^2)*a
bs(e))^4/(e^5*x^4) - 9285*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^7*x^5)
- 2580*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6/(e^9*x^6))*e^4*x^2/((d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))^2*d^6*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2
*x) + 1)^5*abs(e)) + 1/8*(12*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^6*e*abs
(e)/x - (d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^6*abs(e)/(e*x^2))/(d^12*e^
2)
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x^3\,\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)^3}\,dx$$

input
```
int(1/(x^3*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

output
```
int(1/(x^3*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

3.188 $\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

3.188.1 Optimal result

Integrand size = 27, antiderivative size = 204

$$\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{d^4(d-ex)^4}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6(d^2-e^2x^2)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} + \frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6} - \frac{2dx\sqrt{d^2-e^2x^2}}{e^5} + \frac{x^2\sqrt{d^2-e^2x^2}}{3e^4} + \frac{18d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

output
```
1/5*d^4*(-e*x+d)^4/e^6/(-e^2*x^2+d^2)^(5/2)-8/5*d^3*(-e*x+d)^3/e^6/(-e^2*x
^2+d^2)^(3/2)+18*d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+10*d^2*(-e*x+d)^
2/e^6/(-e^2*x^2+d^2)^(1/2)+59/3*d^2*(-e^2*x^2+d^2)^(1/2)/e^6-2*d*x*(-e^2*x
^2+d^2)^(1/2)/e^5+1/3*x^2*(-e^2*x^2+d^2)^(1/2)/e^4
```

3.188.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.64

$$\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{\sqrt{d^2-e^2x^2}(424d^5+1002d^4ex+674d^3e^2x^2+70d^2e^3x^3-15de^4x^4+5e^5x^5)}{15e^6(d+ex)^3} + \frac{18d^3\sqrt{-e^2}\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{e^7}$$

input
```
Integrate[(x^5*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(424*d^5 + 1002*d^4*e*x + 674*d^3*e^2*x^2 + 70*d^2*e^
3*x^3 - 15*d*e^4*x^4 + 5*e^5*x^5))/(15*e^6*(d + e*x)^3) + (18*d^3*Sqrt[-e^
2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e^7
```

3.188.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.16, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {570, 529, 25, 2166, 27, 2166, 2346, 25, 2346, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{x^5(d-ex)^4}{\left(d^2-e^2x^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{529}$$

$$\frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int -\frac{(d-ex)^3\left(\frac{4d^5}{e^5}-\frac{5xd^4}{e^4}+\frac{5x^2d^3}{e^3}-\frac{5x^3d^2}{e^2}+\frac{5x^4d}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(d-ex)^3\left(\frac{4d^5}{e^5}-\frac{5xd^4}{e^4}+\frac{5x^2d^3}{e^3}-\frac{5x^3d^2}{e^2}+\frac{5x^4d}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d}+\frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{2166}$$

$$\frac{-\frac{\int \frac{15(d-ex)^2\left(\frac{4d^5}{e^5}-\frac{3xd^4}{e^4}+\frac{2x^2d^3}{e^3}-\frac{x^3d^2}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d}-\frac{8d^4(d-ex)^3}{e^6\left(d^2-e^2x^2\right)^{3/2}}}{5d}+\frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{5\int \frac{(d-ex)^2\left(\frac{4d^5}{e^5}-\frac{3xd^4}{e^4}+\frac{2x^2d^3}{e^3}-\frac{x^3d^2}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}dx}{d}-\frac{8d^4(d-ex)^3}{e^6\left(d^2-e^2x^2\right)^{3/2}}}{5d}+\frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 2166$$

$$\frac{-\frac{5\left(-\frac{\int \frac{(d-ex)\left(\frac{16d^5}{e^5}-\frac{3xd^4}{e^4}+\frac{x^2d^3}{e^3}\right)}{\sqrt{d^2-e^2x^2}}dx}{d}-\frac{10d^4(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}}\right)}{d}-\frac{8d^4(d-ex)^3}{e^6\left(d^2-e^2x^2\right)^{3/2}}}{5d}+\frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 2346$$

$$\frac{-\frac{5\left(-\frac{-\frac{d^3x^2\sqrt{d^2-e^2x^2}}{3e^4}-\frac{\int -\frac{\frac{48d^6}{e^3}-\frac{59xd^5}{e^2}+\frac{12x^2d^4}{e}}{\sqrt{d^2-e^2x^2}}dx}{3e^2}}{d}-\frac{10d^4(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}}\right)}{d}-\frac{8d^4(d-ex)^3}{e^6\left(d^2-e^2x^2\right)^{3/2}}}{5d}+\frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{5\left(-\frac{\frac{\int \frac{\frac{48d^6}{e^3}-\frac{59xd^5}{e^2}+\frac{12x^2d^4}{e}}{\sqrt{d^2-e^2x^2}}dx}{3e^2}+\frac{d^3x^2\sqrt{d^2-e^2x^2}}{3e^4}}{d}-\frac{10d^4(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}}\right)}{d}-\frac{8d^4(d-ex)^3}{e^6\left(d^2-e^2x^2\right)^{3/2}}}{5d}+\frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 2346$$

$$\frac{-\frac{5\left(-\frac{-\frac{-\frac{\int -\frac{2d^5(54d-59ex)}{e\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{6d^4x\sqrt{d^2-e^2x^2}}{e^3}}{3e^2}+\frac{d^3x^2\sqrt{d^2-e^2x^2}}{3e^4}}{d}-\frac{10d^4(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}}\right)}{d}-\frac{8d^4(d-ex)^3}{e^6\left(d^2-e^2x^2\right)^{3/2}}}{5d}+\frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

3.188. $\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

$$\frac{-\frac{5\left(-\frac{\frac{\frac{d^5\int\frac{54d-59ex}{\sqrt{d^2-e^2x^2}}dx}{e^3}-\frac{6d^4x\sqrt{d^2-e^2x^2}}{e^3}}{3e^2}+\frac{d^3x^2\sqrt{d^2-e^2x^2}}{3e^4}}{d}-\frac{10d^4(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}}\right)}{d}-\frac{8d^4(d-ex)^3}{e^6(d^2-e^2x^2)^{3/2}}}{5d}+\frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}}$$

$\Big\downarrow$ 455

$$\frac{-\frac{5\left(-\frac{\frac{\frac{d^5\left(54d\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{59\sqrt{d^2-e^2x^2}}{e}\right)}{e^3}-\frac{6d^4x\sqrt{d^2-e^2x^2}}{e^3}}{3e^2}+\frac{d^3x^2\sqrt{d^2-e^2x^2}}{3e^4}}{d}-\frac{10d^4(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}}\right)}{d}-\frac{8d^4(d-ex)^3}{e^6(d^2-e^2x^2)^{3/2}}}{5d}+\frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}}$$

$\Big\downarrow$ 224

$$\frac{-\frac{5\left(-\frac{\frac{\frac{d^5\left(54d\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{59\sqrt{d^2-e^2x^2}}{e}\right)}{e^3}-\frac{6d^4x\sqrt{d^2-e^2x^2}}{e^3}}{3e^2}+\frac{d^3x^2\sqrt{d^2-e^2x^2}}{3e^4}}{d}-\frac{10d^4(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}}\right)}{d}-\frac{8d^4(d-ex)^3}{e^6(d^2-e^2x^2)^{3/2}}}{5d}+\frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}}$$

$\Big\downarrow$ 216

$$\frac{-\frac{5\left(-\frac{\frac{\frac{d^5\left(\frac{54d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{59\sqrt{d^2-e^2x^2}}{e}\right)}{e^3}-\frac{6d^4x\sqrt{d^2-e^2x^2}}{e^3}}{3e^2}+\frac{d^3x^2\sqrt{d^2-e^2x^2}}{3e^4}}{d}-\frac{10d^4(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}}\right)}{d}-\frac{8d^4(d-ex)^3}{e^6(d^2-e^2x^2)^{3/2}}}{5d}+\frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[(x^5*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

3.188. $\int\frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

output
```
(d^4*(d - e*x)^4)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) + ((-8*d^4*(d - e*x)^3)/(e
^6*(d^2 - e^2*x^2)^(3/2)) - (5*((-10*d^4*(d - e*x)^2)/(e^6*Sqrt[d^2 - e^2*
x^2]) - ((d^3*x^2*Sqrt[d^2 - e^2*x^2])/(3*e^4) + ((-6*d^4*x*Sqrt[d^2 - e^2
*x^2])/e^3 + (d^5*((59*Sqrt[d^2 - e^2*x^2])/e + (54*d*ArcTan[(e*x)/Sqrt[d^
2 - e^2*x^2]])/e))/e^3)/(3*e^2))/d))/d)/(5*d)
```

3.188.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

3.188. $\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.188.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.02

method	result
risch	$\frac{(e^2x^2-6dex+29d^2)\sqrt{-e^2x^2+d^2}}{3e^6}+\frac{18d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^5\sqrt{e^2}}+\frac{2d^5\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5e^9\left(x+\frac{d}{e}\right)^3}-\frac{17d^4\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de}}{5e^8\left(x+\frac{d}{e}\right)^2}$
default	$-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3e^6}-\frac{4d\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{e^5}-\frac{d^5\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5de\left(x+\frac{d}{e}\right)^4}-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)}{15d^2\left(x+\frac{d}{e}\right)^3}\right)}{e^9}$

input
```
int(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*(e^2*x^2-6*d*e*x+29*d^2)/e^6*(-e^2*x^2+d^2)^(1/2)+18/e^5*d^3/(e^2)^(1/
2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+2/5/e^9*d^5/(x+d/e)^3*(-(x+d
/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-17/5/e^8*d^4/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*
e*(x+d/e))^(1/2)+108/5/e^7*d^3/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2
)
```

3.188.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.98

$$\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{424\,d^3e^3x^3+1272\,d^4e^2x^2+1272\,d^5ex+424\,d^6-540\,(d^3e^3x^3+3\,d^4e^2x^2+3\,d^5ex+d^6)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\ldots}{15\,(e^9x^3+3\,de^8x^2+3\,d^2e^7x+\ldots)}$$

input
```
integrate(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
1/15*(424*d^3*e^3*x^3 + 1272*d^4*e^2*x^2 + 1272*d^5*e*x + 424*d^6 - 540*(d
^3*e^3*x^3 + 3*d^4*e^2*x^2 + 3*d^5*e*x + d^6)*arctan(-(d - sqrt(-e^2*x^2 +
 d^2))/(e*x)) + (5*e^5*x^5 - 15*d*e^4*x^4 + 70*d^2*e^3*x^3 + 674*d^3*e^2*x
^2 + 1002*d^4*e*x + 424*d^5)*sqrt(-e^2*x^2 + d^2))/(e^9*x^3 + 3*d*e^8*x^2
+ 3*d^2*e^7*x + d^3*e^6)
```

3.188.6 Sympy [F]

$$\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{x^5\sqrt{-(-d+ex)(d+ex)}}{(d+ex)^4}\,dx$$

input
```
integrate(x**5*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

output
```
Integral(x**5*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

3.188.7 Maxima [F(-1)]

Timed out.

$$\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \text{Timed out}$$

input
```
integrate(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
Timed out
```

3.188.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.20

$$\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$$
$$= \frac{1}{3}\sqrt{-e^2x^2+d^2}\left(x\left(\frac{x}{e^4}-\frac{6\,d}{e^5}\right)+\frac{29\,d^2}{e^6}\right)+\frac{18\,d^3\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^5|e|}$$
$$-\frac{2\left(93\,d^3+\frac{385\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^3}{e^2x}+\frac{575\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^3}{e^4x^2}+\frac{355\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^3}{e^6x^3}+\frac{80\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^8x^4}\right.}{5\,e^5\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

input
```
integrate(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

output
```
1/3*sqrt(-e^2*x^2 + d^2)*(x*(x/e^4 - 6*d/e^5) + 29*d^2/e^6) + 18*d^3*arcsi
n(e*x/d)*sgn(d)*sgn(e)/(e^5*abs(e)) - 2/5*(93*d^3 + 385*(d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))*d^3/(e^2*x) + 575*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*
d^3/(e^4*x^2) + 355*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^3/(e^6*x^3) +
80*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^3/(e^8*x^4))/(e^5*((d*e + sqrt(
-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e))
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{x^5\,\sqrt{d^2-e^2\,x^2}}{(d+e\,x)^4}\,dx$$

input
```
int((x^5*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
```

output
```
int((x^5*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4, x)
```

3.189 $\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

3.189.1 Optimal result

Integrand size = 27, antiderivative size = 160

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = -\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}}-\frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5}-\frac{19d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

output
```
-1/5*d^3*(-e*x+d)^4/e^5/(-e^2*x^2+d^2)^(5/2)+19/15*d^2*(-e*x+d)^3/e^5/(-e^
2*x^2+d^2)^(3/2)-19/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5-6*d*(-e*x+d
)^2/e^5/(-e^2*x^2+d^2)^(1/2)-1/2*(-e*x+20*d)*(-e^2*x^2+d^2)^(1/2)/e^5
```

3.189.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.70

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{\sqrt{d^2-e^2x^2}\left(-448d^4-1059d^3ex-713d^2e^2x^2-75de^3x^3+15e^4x^4\right)}{30e^5(d+ex)^3}+\frac{19d^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

input
```
Integrate[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-448*d^4 - 1059*d^3*e*x - 713*d^2*e^2*x^2 - 75*d*e^3
*x^3 + 15*e^4*x^4))/(30*e^5*(d + e*x)^3) + (19*d^2*ArcTan[(e*x)/(Sqrt[d^2]
 - Sqrt[d^2 - e^2*x^2])])/e^5
```

3.189.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {570, 529, 2166, 27, 2166, 27, 676, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{x^4(d-ex)^4}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{529}$$

$$-\frac{\int \frac{(d-ex)^3\left(\frac{4d^4}{e^4}-\frac{5xd^3}{e^3}+\frac{5x^2d^2}{e^2}-\frac{5x^3d}{e}\right)}{(d^2-e^2x^2)^{5/2}}dx}{5d}-\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{2166}$$

$$-\frac{-\frac{\int \frac{15(d-ex)^2\left(\frac{3d^4}{e^4}-\frac{2xd^3}{e^3}+\frac{x^2d^2}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d}-\frac{19d^3(d-ex)^3}{3e^5(d^2-e^2x^2)^{3/2}}}{5d}-\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{5\int \frac{(d-ex)^2\left(\frac{3d^4}{e^4}-\frac{2xd^3}{e^3}+\frac{x^2d^2}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}dx}{d}-\frac{19d^3(d-ex)^3}{3e^5(d^2-e^2x^2)^{3/2}}}{5d}-\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{2166}$$

$$-\frac{-\frac{5\left(-\frac{\int \frac{d^3(d-ex)(9d-ex)}{e^4\sqrt{d^2-e^2x^2}}dx}{d}-\frac{6d^3(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}}\right)}{d}-\frac{19d^3(d-ex)^3}{3e^5(d^2-e^2x^2)^{3/2}}}{5d}-\frac{d^3(d-ex)^4}{5e^5(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{5\left(-\frac{d^2\int \frac{(d-ex)(9d-ex)}{\sqrt{d^2-e^2x^2}}dx}{e^4}-\frac{6d^3(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}}\right)}{d}-\frac{19d^3(d-ex)^3}{3e^5(d^2-e^2x^2)^{3/2}}}{5d}-\frac{d^3(d-ex)^4}{5e^5(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \; 676$$

$$-\frac{-\frac{5\left(-\frac{d^2\left(\frac{19}{2}d^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{10d\sqrt{d^2-e^2x^2}}{e}-\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)}{e^4}-\frac{6d^3(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}}\right)}{d}-\frac{19d^3(d-ex)^3}{3e^5(d^2-e^2x^2)^{3/2}}}{5d}-\frac{d^3(d-ex)^4}{5e^5(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \; 224$$

$$-\frac{-\frac{5\left(-\frac{d^2\left(\frac{19}{2}d^2\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{10d\sqrt{d^2-e^2x^2}}{e}-\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)}{e^4}-\frac{6d^3(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}}\right)}{d}-\frac{19d^3(d-ex)^3}{3e^5(d^2-e^2x^2)^{3/2}}}{5d}-\frac{d^3(d-ex)^4}{5e^5(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \; 216$$

$$-\frac{-\frac{5\left(-\frac{d^2\left(\frac{19d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{10d\sqrt{d^2-e^2x^2}}{e}-\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)}{e^4}-\frac{6d^3(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}}\right)}{d}-\frac{19d^3(d-ex)^3}{3e^5(d^2-e^2x^2)^{3/2}}}{5d}-\frac{d^3(d-ex)^4}{5e^5(d^2-e^2x^2)^{5/2}}$$

input
```
Int[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

3.189. $\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

output
```
-1/5*(d^3*(d - e*x)^4)/(e^5*(d^2 - e^2*x^2)^(5/2)) - ((-19*d^3*(d - e*x)^3
)/(3*e^5*(d^2 - e^2*x^2)^(3/2)) - (5*((-6*d^3*(d - e*x)^2)/(e^5*Sqrt[d^2 -
 e^2*x^2]) - (d^2*((10*d*Sqrt[d^2 - e^2*x^2])/e - (x*Sqrt[d^2 - e^2*x^2])/
2 + (19*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)))/e^4))/d)/(5*d)
```

3.189.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

3.189. $\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

3.189.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{(-ex+8d)\sqrt{-e^2x^2+d^2}}{2e^5}-\frac{19d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^4\sqrt{e^2}}-\frac{2d^4\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5e^8\left(x+\frac{d}{e}\right)^3}+\frac{41d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15e^7\left(x+\frac{d}{e}\right)^2}$
default	$\frac{\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}}{e^4}+\frac{d^4\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5de\left(x+\frac{d}{e}\right)^4}-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{15d^2\left(x+\frac{d}{e}\right)^3}\right)}{e^8}+\frac{6d^2\left(-\frac{\left(-\left(x+\frac{d}{e}\right.\right.}{}\right.}{}$

input
```
int(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e*x+8*d)/e^5*(-e^2*x^2+d^2)^(1/2)-19/2/e^4*d^2/(e^2)^(1/2)*arctan((
e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-2/5/e^8*d^4/(x+d/e)^3*(-(x+d/e)^2*e^2+2
*d*e*(x+d/e))^(1/2)+41/15/e^7*d^3/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))
^(1/2)-199/15/e^6*d^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

3.189.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.19

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx =$$
$$-\frac{448\,d^2e^3x^3+1344\,d^3e^2x^2+1344\,d^4ex+448\,d^5-570\,(d^2e^3x^3+3\,d^3e^2x^2+3\,d^4ex+d^5)\arctan\left(-\frac{d-\sqrt{-}}{}\right.}{30\,(e^8x^3+3\,de^7x^2+3\,d^2e^6x}$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
-1/30*(448*d^2*e^3*x^3 + 1344*d^3*e^2*x^2 + 1344*d^4*e*x + 448*d^5 - 570*(
d^2*e^3*x^3 + 3*d^3*e^2*x^2 + 3*d^4*e*x + d^5)*arctan(-(d - sqrt(-e^2*x^2
+ d^2))/(e*x)) - (15*e^4*x^4 - 75*d*e^3*x^3 - 713*d^2*e^2*x^2 - 1059*d^3*e
*x - 448*d^4)*sqrt(-e^2*x^2 + d^2))/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x +
 d^3*e^5)
```

3.189.6 Sympy [F]

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{x^4\sqrt{-(-d+ex)\,(d+ex)}}{(d+ex)^4}\,dx$$

input
```
integrate(x**4*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

output
```
Integral(x**4*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

3.189.7 Maxima [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
Timed out
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.46

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{1}{2}\sqrt{-e^2x^2+d^2}\left(\frac{x}{e^4}-\frac{8\,d}{e^5}\right)-\frac{19\,d^2\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{2\,e^4|e|}$$
$$+\frac{2\left(164\,d^2+\frac{685\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2}{e^2x}+\frac{1025\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^2}{e^4x^2}+\frac{615\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^2}{e^6x^3}+\frac{135\left(de+\sqrt{-e^2x^2+d^2}\right.}{e^8x^4}\right.}{15\,e^4\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

output
```
1/2*sqrt(-e^2*x^2 + d^2)*(x/e^4 - 8*d/e^5) - 19/2*d^2*arcsin(e*x/d)*sgn(d)
*sgn(e)/(e^4*abs(e)) + 2/15*(164*d^2 + 685*(d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))*d^2/(e^2*x) + 1025*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^2/(e^4*x^2
) + 615*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^2/(e^6*x^3) + 135*(d*e + s
qrt(-e^2*x^2 + d^2)*abs(e))^4*d^2/(e^8*x^4))/(e^4*((d*e + sqrt(-e^2*x^2 +
d^2)*abs(e))/(e^2*x) + 1)^5*abs(e))
```

3.189.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{x^4\,\sqrt{d^2-e^2\,x^2}}{\left(d+e\,x\right)^4}\,dx$$

input
```
int((x^4*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
```

output
```
int((x^4*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4, x)
```

3.190 $\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

3.190.1 Optimal result

Integrand size = 27, antiderivative size = 148

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)} + \frac{d^2(d^2-e^2x^2)^{3/2}}{5e^4(d+ex)^4} - \frac{14d(d^2-e^2x^2)^{3/2}}{15e^4(d+ex)^3} - \frac{(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^2} + \frac{4d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

output
```
1/5*d^2*(-e^2*x^2+d^2)^(3/2)/e^4/(e*x+d)^4-14/15*d*(-e^2*x^2+d^2)^(3/2)/e^
4/(e*x+d)^3-(-e^2*x^2+d^2)^(3/2)/e^4/(e*x+d)^2+4*d*arctan(e*x/(-e^2*x^2+d^
2)^(1/2))/e^4+8*d*(-e^2*x^2+d^2)^(1/2)/e^4/(e*x+d)
```

3.190.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.72

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{\sqrt{d^2-e^2x^2}(94d^3+222d^2ex+149de^2x^2+15e^3x^3)}{15e^4(d+ex)^3} + \frac{4d\sqrt{-e^2}\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{e^5}$$

input
```
Integrate[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(94*d^3 + 222*d^2*e*x + 149*d*e^2*x^2 + 15*e^3*x^3))/
(15*e^4*(d + e*x)^3) + (4*d*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^
2*x^2]])/e^5
```

3.190.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {581, 25, 2168, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}dx$$

$$\Big\downarrow \text{ 581}$$

$$\frac{\int -\frac{\sqrt{d^2-e^2x^2}(2d^3+5exd^2+4e^2x^2d)}{(d+ex)^4}dx}{e^3}-\frac{(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{\sqrt{d^2-e^2x^2}(2d^3+5exd^2+4e^2x^2d)}{(d+ex)^4}dx}{e^3}-\frac{(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^2}$$

$$\Big\downarrow \text{ 2168}$$

$$-\frac{\int \left(\frac{\sqrt{d^2-e^2x^2}d^3}{(d+ex)^4}-\frac{3\sqrt{d^2-e^2x^2}d^2}{(d+ex)^3}+\frac{4\sqrt{d^2-e^2x^2}d}{(d+ex)^2}\right)dx}{e^3}-\frac{(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{-\frac{4d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{d^2(d^2-e^2x^2)^{3/2}}{5e(d+ex)^4}+\frac{14d(d^2-e^2x^2)^{3/2}}{15e(d+ex)^3}-\frac{8d\sqrt{d^2-e^2x^2}}{e(d+ex)}}{e^3}-\frac{(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^2}$$

input
```
Int[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

output
```
-((d^2 - e^2*x^2)^(3/2)/(e^4*(d + e*x)^2)) - ((-8*d*Sqrt[d^2 - e^2*x^2])/(
e*(d + e*x)) - (d^2*(d^2 - e^2*x^2)^(3/2))/(5*e*(d + e*x)^4) + (14*d*(d^2
- e^2*x^2)^(3/2))/(15*e*(d + e*x)^3) - (4*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^
2]])/e)/e^3
```

3.190.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2168
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + b*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, b,
d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

3.190.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.26

method	result
risch	$\frac{\sqrt{-e^2x^2+d^2}}{e^4}+\frac{4d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^3\sqrt{e^2}}+\frac{104d\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15e^5\left(x+\frac{d}{e}\right)}-\frac{31d^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15e^6\left(x+\frac{d}{e}\right)^2}+\frac{2d^3\sqrt{-(}}{}$
default	$\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}}{e^4}-\frac{d^3\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5de\left(x+\frac{d}{e}\right)^4}-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right.\right.}{15d^2\left(x+\frac{d}{e}\right)^3}\right.}{e^7}$

input
```
int(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
1/e^4*(-e^2*x^2+d^2)^(1/2)+4*d/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*
x^2+d^2)^(1/2))+104/15*d/e^5/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-
31/15/e^6*d^2/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+2/5/e^7*d^3/(
x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

3.190.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.18

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$$
$$= \frac{94\,de^3x^3+282\,d^2e^2x^2+282\,d^3ex+94\,d^4-120\,(de^3x^3+3\,d^2e^2x^2+3\,d^3ex+d^4)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)}{15\,(e^7x^3+3\,de^6x^2+3\,d^2e^5x+d^3e^4)}$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
1/15*(94*d*e^3*x^3 + 282*d^2*e^2*x^2 + 282*d^3*e*x + 94*d^4 - 120*(d*e^3*x
^3 + 3*d^2*e^2*x^2 + 3*d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(
e*x)) + (15*e^3*x^3 + 149*d*e^2*x^2 + 222*d^2*e*x + 94*d^3)*sqrt(-e^2*x^2
+ d^2))/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^4)
```

3.190.6 Sympy [F]

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{x^3\sqrt{-(-d+ex)(d+ex)}}{(d+ex)^4}\,dx$$

input
```
integrate(x**3*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

output
```
Integral(x**3*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

3.190.7 Maxima [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
Timed out
```

3.190.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.43

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{4\,d\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^3|e|} + \frac{\sqrt{-e^2x^2+d^2}}{e^4} - \frac{2\left(79\,d + \frac{335\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d}{e^2x} + \frac{505\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d}{e^4x^2} + \frac{285\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3 d}{e^6x^3} + \frac{60\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4 d}{e^8x^4}\right)}{15\,e^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

output
```
4*d*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^3*abs(e)) + sqrt(-e^2*x^2 + d^2)/e^4 -
2/15*(79*d + 335*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d/(e^2*x) + 505*(d*e
+ sqrt(-e^2*x^2 + d^2)*abs(e))^2*d/(e^4*x^2) + 285*(d*e + sqrt(-e^2*x^2 +
d^2)*abs(e))^3*d/(e^6*x^3) + 60*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d/(e
^8*x^4))/(e^3*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e))
```

3.190.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{x^3\,\sqrt{d^2-e^2\,x^2}}{{(d+e\,x)}^4}\,dx$$

input
```
int((x^3*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
```

output
```
int((x^3*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4, x)
```

3.191 $\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

3.191.1 Optimal result

Integrand size = 27, antiderivative size = 115

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = -\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{d(d^2-e^2x^2)^{3/2}}{5e^3(d+ex)^4} + \frac{3(d^2-e^2x^2)^{3/2}}{5e^3(d+ex)^3} - \frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

output
```
-1/5*d*(-e^2*x^2+d^2)^(3/2)/e^3/(e*x+d)^4+3/5*(-e^2*x^2+d^2)^(3/2)/e^3/(e*
x+d)^3-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-2*(-e^2*x^2+d^2)^(1/2)/e^3/(e*
x+d)
```

3.191.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.76

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{(-8d^2-19dex-13e^2x^2)\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3} + \frac{2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

input
```
Integrate[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

output
```
((-8*d^2 - 19*d*e*x - 13*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(5*e^3*(d + e*x)^3)
 + (2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^3
```

3.191.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {582, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$$

$$\Big\downarrow \text{582}$$

$$\int \left(\frac{d^2\sqrt{d^2-e^2x^2}}{e^2(d+ex)^4}-\frac{2d\sqrt{d^2-e^2x^2}}{e^2(d+ex)^3}+\frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}+\frac{3\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^3}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^4}-\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)}$$

input
```
Int[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

output
```
(-2*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - (d*(d^2 - e^2*x^2)^(3/2))/(5*e^
3*(d + e*x)^4) + (3*(d^2 - e^2*x^2)^(3/2))/(5*e^3*(d + e*x)^3) - ArcTan[(e
*x)/Sqrt[d^2 - e^2*x^2]]/e^3
```

3.191.3.1 Defintions of rubi rules used

rule 582
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[x^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IntegerQ[2*p] && IntegerQ[m]
&& ILtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.191.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(103) = 206$.

Time = 0.41 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.33

method	result
default	$\frac{-\frac{\left(-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{de\left(x+\frac{d}{e}\right)^2}-\frac{e\left(\sqrt{-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}\right)}{d}}{e^4}+\frac{d^2\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x \right.\right.}{5de\left(x+\frac{d}{e}\right)^4}\right.}{}$

input
```
int(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
1/e^4*(-1/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-e/d*((-(x+d/e
)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e
)^2*e^2+2*d*e*(x+d/e))^(1/2))))+d^2/e^6*(-1/5/d/e/(x+d/e)^4*(-(x+d/e)^2*e^
2+2*d*e*(x+d/e))^(3/2)-1/15/d^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(
3/2))+2/3/e^6/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)
```

3.191.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.37

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx =$$
$$-\frac{8\,e^3x^3+24\,de^2x^2+24\,d^2ex+8\,d^3-10\,(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(13\,e^2 \ldots}{5\,(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3)}$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
-1/5*(8*e^3*x^3 + 24*d*e^2*x^2 + 24*d^2*e*x + 8*d^3 - 10*(e^3*x^3 + 3*d*e^
2*x^2 + 3*d^2*e*x + d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (13*e
^2*x^2 + 19*d*e*x + 8*d^2)*sqrt(-e^2*x^2 + d^2))/(e^6*x^3 + 3*d*e^5*x^2 +
3*d^2*e^4*x + d^3*e^3)
```

3.191.6 Sympy [F]

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{x^2\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{\left(d+ex\right)^4}\,dx$$

input
```
integrate(x**2*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

output
```
Integral(x**2*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

3.191.7 Maxima [F]

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{\sqrt{-e^2x^2+d^2}x^2}{\left(ex+d\right)^4}\,dx$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^2 + d^2)*x^2/(e*x + d)^4, x)
```

3.191.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.62

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = -\frac{\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^2|e|} + \frac{2\left(\frac{35\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x}+\frac{55\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2}+\frac{25\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3}+\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4}+8\right)}{5\,e^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

output
```
-arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) + 2/5*(35*(d*e + sqrt(-e^2*x^2 +
 d^2)*abs(e))/(e^2*x) + 55*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e^4*x^2)
 + 25*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/(e^6*x^3) + 5*(d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))^4/(e^8*x^4) + 8)/(e^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))/(e^2*x) + 1)^5*abs(e))
```

3.191.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{x^2\,\sqrt{d^2-e^2\,x^2}}{{(d+e\,x)}^4}\,dx$$

input
```
int((x^2*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
```

output
```
int((x^2*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4, x)
```

3.192 $\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

3.192.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{\left(d^2-e^2x^2\right)^{3/2}}{5e^2(d+ex)^4} - \frac{4\left(d^2-e^2x^2\right)^{3/2}}{15de^2(d+ex)^3}$$

output
```
1/5*(-e^2*x^2+d^2)^(3/2)/e^2/(e*x+d)^4-4/15*(-e^2*x^2+d^2)^(3/2)/d/e^2/(e*
x+d)^3
```

3.192.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{\sqrt{d^2-e^2x^2}\left(-d^2-3dex+4e^2x^2\right)}{15de^2(d+ex)^3}$$

input
```
Integrate[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-d^2 - 3*d*e*x + 4*e^2*x^2))/(15*d*e^2*(d + e*x)^3)
```

3.192.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {571, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4}dx$$

$$\Big\downarrow \text{ 571}$$

$$\frac{4\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}dx}{5e}+\frac{\left(d^2-e^2x^2\right)^{3/2}}{5e^2(d+ex)^4}$$

$$\Big\downarrow \text{ 460}$$

$$\frac{\left(d^2-e^2x^2\right)^{3/2}}{5e^2(d+ex)^4}-\frac{4\left(d^2-e^2x^2\right)^{3/2}}{15de^2(d+ex)^3}$$

input
```
Int[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

output
```
(d^2 - e^2*x^2)^(3/2)/(5*e^2*(d + e*x)^4) - (4*(d^2 - e^2*x^2)^(3/2))/(15*
d*e^2*(d + e*x)^3)
```

3.192.3.1 Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 571
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*(n + p + 1))), x] + Simp[n/(2*d*
(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ((LtQ[n, -1] &&  !IGtQ[n + p +
1, 0]) || (LtQ[n, 0] && LtQ[p, -1]) || EqQ[n + 2*p + 2, 0]) && NeQ[n + p +
1, 0]
```

3.192.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{(-ex+d)(4ex+d)\sqrt{-e^2x^2+d^2}}{15(ex+d)^3 d\, e^2}$	42
trager	$-\frac{(-4e^2x^2+3dex+d^2)\sqrt{-e^2x^2+d^2}}{15d(ex+d)^3 e^2}$	47
default	$-\frac{\left(-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3e^5 d\left(x+\frac{d}{e}\right)^3}-\frac{d\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5de\left(x+\frac{d}{e}\right)^4}-\frac{\left(-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{15d^2\left(x+\frac{d}{e}\right)^3}\right)}{e^5}$	141

input
```
int(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-e*x+d)*(4*e*x+d)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3/d/e^2
```

3.192.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.59

$$\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = -\frac{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3-(4\,e^2x^2-3\,dex-d^2)\sqrt{-e^2x^2+d^2}}{15\,(de^5x^3+3\,d^2e^4x^2+3\,d^3e^3x+d^4e^2)}$$

input
```
integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
-1/15*(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3 - (4*e^2*x^2 - 3*d*e*x - d^
2)*sqrt(-e^2*x^2 + d^2))/(d*e^5*x^3 + 3*d^2*e^4*x^2 + 3*d^3*e^3*x + d^4*e^
2)
```

3.192.6 Sympy [F]

$$\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx=\int \frac{x\sqrt{-(-d+ex)(d+ex)}}{(d+ex)^4}\,dx$$

input
```
integrate(x*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

output
```
Integral(x*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

3.192.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(56) = 112.

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.95

$$\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx=\frac{2\sqrt{-e^2x^2+d^2}d}{5(e^5x^3+3de^4x^2+3d^2e^3x+d^3e^2)}-\frac{11\sqrt{-e^2x^2+d^2}}{15(e^4x^2+2de^3x+d^2e^2)}+\frac{4\sqrt{-e^2x^2+d^2}}{15(de^3x+d^2e^2)}$$

input
```
integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
2/5*sqrt(-e^2*x^2 + d^2)*d/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*e^2)
 - 11/15*sqrt(-e^2*x^2 + d^2)/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) + 4/15*sqrt(
-e^2*x^2 + d^2)/(d*e^3*x + d^2*e^2)
```

3.192.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(56) = 112.

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.14

$$\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx=\frac{2\left(\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x}-\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2}+\frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3}+1\right)}{15\,de\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

input
```
integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

output
```
2/15*(5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 5*(d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))^2/(e^4*x^2) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/(
e^6*x^3) + 1)/(d*e*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs
(e))
```

3.192.9 Mupad [B] (verification not implemented)

Time = 11.79 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}\,\left(d^2+3\,d\,e\,x-4\,e^2\,x^2\right)}{15\,d\,e^2\,{\left(d+e\,x\right)}^3}$$

input
```
int((x*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
```

output
```
-((d^2 - e^2*x^2)^(1/2)*(d^2 - 4*e^2*x^2 + 3*d*e*x))/(15*d*e^2*(d + e*x)^3
)
```

3.193 $\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4} dx$

3.193.1 Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4} dx = -\frac{(d^2-e^2x^2)^{3/2}}{5de(d+ex)^4} - \frac{(d^2-e^2x^2)^{3/2}}{15d^2e(d+ex)^3}$$

output
```
-1/5*(-e^2*x^2+d^2)^(3/2)/d/e/(e*x+d)^4-1/15*(-e^2*x^2+d^2)^(3/2)/d^2/e/(e
*x+d)^3
```

3.193.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4} dx = \frac{\sqrt{d^2-e^2x^2}(-4d^2+3dex+e^2x^2)}{15d^2e(d+ex)^3}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^2]/(d + e*x)^4,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-4*d^2 + 3*d*e*x + e^2*x^2))/(15*d^2*e*(d + e*x)^3)
```

3.193.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$$

$$\Big\downarrow \ 461$$

$$\frac{\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}dx}{5d}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{5de(d+ex)^4}$$

$$\Big\downarrow \ 460$$

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}}{15d^2e(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{5de(d+ex)^4}$$

input
```
Int[Sqrt[d^2 - e^2*x^2]/(d + e*x)^4,x]
```

output
```
-1/5*(d^2 - e^2*x^2)^(3/2)/(d*e*(d + e*x)^4) - (d^2 - e^2*x^2)^(3/2)/(15*d
^2*e*(d + e*x)^3)
```

3.193.3.1 Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

3.193.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{(-ex+d)(ex+4d)\sqrt{-e^{2}x^{2}+d^{2}}}{15(ex+d)^{3}d^{2}e}$	43
trager	$-\frac{(-e^{2}x^{2}-3dex+4d^{2})\sqrt{-e^{2}x^{2}+d^{2}}}{15d^{2}(ex+d)^{3}e}$	49
default	$\frac{-\frac{\left(-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5de\left(x+\frac{d}{e}\right)^{4}}-\frac{\left(-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{15d^{2}\left(x+\frac{d}{e}\right)^{3}}}{e^{4}}$	93

input
```
int((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-e*x+d)*(e*x+4*d)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3/d^2/e
```

3.193.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = -\frac{4\,e^3x^3+12\,de^2x^2+12\,d^2ex+4\,d^3-(e^2x^2+3\,dex-4\,d^2)\sqrt{-e^2x^2+d^2}}{15\,(d^2e^4x^3+3\,d^3e^3x^2+3\,d^4e^2x+d^5e)}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
-1/15*(4*e^3*x^3 + 12*d*e^2*x^2 + 12*d^2*e*x + 4*d^3 - (e^2*x^2 + 3*d*e*x
- 4*d^2)*sqrt(-e^2*x^2 + d^2))/(d^2*e^4*x^3 + 3*d^3*e^3*x^2 + 3*d^4*e^2*x
+ d^5*e)
```

3.193.6 Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx=\int \frac{\sqrt{-(-d+ex)(d+ex)}}{(d+ex)^4}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

output
```
Integral(sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

3.193.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(59) = 118$.

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx=-\frac{2\sqrt{-e^2x^2+d^2}}{5\,(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e)}+\frac{\sqrt{-e^2x^2+d^2}}{15\,(de^3x^2+2\,d^2e^2x+d^3e)}+\frac{\sqrt{-e^2x^2+d^2}}{15\,(d^2e^2x+d^3e)}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
-2/5*sqrt(-e^2*x^2 + d^2)/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e) +
1/15*sqrt(-e^2*x^2 + d^2)/(d*e^3*x^2 + 2*d^2*e^2*x + d^3*e) + 1/15*sqrt(-e
^2*x^2 + d^2)/(d^2*e^2*x + d^3*e)
```

3.193.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(59) = 118$.

Time = 0.32 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.46

$$\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx=\frac{2\left(\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x}+\frac{25\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2}+\frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3}+\frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4}+4\right)}{15\,d^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

output
```
2/15*(5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 25*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^2/(e^4*x^2) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/
(e^6*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) + 4)/(d^2*(
(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e))
```

3.193.9 Mupad [B] (verification not implemented)

Time = 11.68 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx=\frac{\sqrt{d^2-e^2\,x^2}\,\left(-4\,d^2+3\,d\,e\,x+e^2\,x^2\right)}{15\,d^2\,e\,{\left(d+e\,x\right)}^3}$$

input
```
int((d^2 - e^2*x^2)^(1/2)/(d + e*x)^4,x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(e^2*x^2 - 4*d^2 + 3*d*e*x))/(15*d^2*e*(d + e*x)^3)
```

3.194 $\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx$

3.194.1 Optimal result

Integrand size = 27, antiderivative size = 110

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx=\frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}-\frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

output
```
8/5*d*(-e*x+d)/(-e^2*x^2+d^2)^(5/2)-4/5*e*x/d/(-e^2*x^2+d^2)^(3/2)-arctanh
((-e^2*x^2+d^2)^(1/2)/d)/d^3+1/5*(-8*e*x+5*d)/d^3/(-e^2*x^2+d^2)^(1/2)
```

3.194.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx=\frac{\frac{\sqrt{d^2-e^2x^2}(13d^2+19dex+8e^2x^2)}{(d+ex)^3}+10\operatorname{arctanh}\left(\frac{\sqrt{-e^2}x-\sqrt{d^2-e^2x^2}}{d}\right)}{5d^3}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)^4),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(13*d^2 + 19*d*e*x + 8*e^2*x^2))/(d + e*x)^3 + 10*Ar
cTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(5*d^3)
```

3.194.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {570, 532, 25, 2336, 27, 532, 27, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx$$

$$\Big\downarrow \ 570$$

$$\int \frac{(d-ex)^4}{x\left(d^2-e^2x^2\right)^{7/2}}dx$$

$$\Big\downarrow \ 532$$

$$\frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int -\frac{5d^4-12exd^3-5e^2x^2d^2}{x\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{5d^4-12exd^3-5e^2x^2d^2}{x\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}+\frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \ 2336$$

$$\frac{-\frac{\int -\frac{3d^3(5d-8ex)}{x\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}-\frac{4dex}{\left(d^2-e^2x^2\right)^{3/2}}}{5d^2}+\frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{d\int \frac{5d-8ex}{x\left(d^2-e^2x^2\right)^{3/2}}dx-\frac{4dex}{\left(d^2-e^2x^2\right)^{3/2}}}{5d^2}+\frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \ 532$$

$$\frac{d\left(\frac{5d-8ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{5d}{x\sqrt{d^2-e^2x^2}}dx}{d^2}\right)-\frac{4dex}{\left(d^2-e^2x^2\right)^{3/2}}}{5d^2}+\frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{d\left(\frac{5\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx}{d}+\frac{5d-8ex}{d^2\sqrt{d^2-e^2x^2}}\right)-\frac{4dex}{(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{d\left(\frac{5\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2}{2d}+\frac{5d-8ex}{d^2\sqrt{d^2-e^2x^2}}\right)-\frac{4dex}{(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{d\left(\frac{5d-8ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{5\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{de^2}\right)-\frac{4dex}{(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{d\left(\frac{5d-8ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{5\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}\right)-\frac{4dex}{(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)^4),x]
```

output
```
(8*d*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) + ((-4*d*e*x)/(d^2 - e^2*x^2)^(3
/2) + d*((5*d - 8*e*x)/(d^2*Sqrt[d^2 - e^2*x^2]) - (5*ArcTanh[Sqrt[d^2 - e
^2*x^2]/d])/d^2))/(5*d^2)
```

3.194.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.194. $\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4} \, dx$

3.194.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(96) = 192$.

Time = 0.44 (sec) , antiderivative size = 415, normalized size of antiderivative = 3.77

method	result
default	$\frac{\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}}{d^4}-\frac{-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5de\left(x+\frac{d}{e}\right)^4}-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{15d^2\left(x+\frac{d}{e}\right)^3}}{e^3d}-\frac{-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+\right.}{de\left(x+\right.}}{}$

input
```
int((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d^4*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*
x^2+d^2)^(1/2))/x))-1/e^3/d*(-1/5/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d
/e))^(3/2)-1/15/d^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2))-1/e/d^
3*(-1/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-e/d*((-(x+d/e)^2*
e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*
e^2+2*d*e*(x+d/e))^(1/2))))-1/d^4*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*
e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))+
1/3/e^3/d^3/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)
```

3.194.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx$$
$$=\frac{13\,e^3x^3+39\,de^2x^2+39\,d^2ex+13\,d^3+5\,(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(8\,e^2x^2+}{5\,(d^3e^3x^3+3\,d^4e^2x^2+3\,d^5ex+d^6)}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x, algorithm="fricas")
```

output
```
1/5*(13*e^3*x^3 + 39*d*e^2*x^2 + 39*d^2*e*x + 13*d^3 + 5*(e^3*x^3 + 3*d*e^
2*x^2 + 3*d^2*e*x + d^3)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (8*e^2*x^2 +
 19*d*e*x + 13*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^3*x^3 + 3*d^4*e^2*x^2 + 3
*d^5*e*x + d^6)
```

3.194.6 Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx=\int \frac{\sqrt{-(-d+ex)(d+ex)}}{x(d+ex)^4}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**(1/2)/x/(e*x+d)**4,x)
```

output
```
Integral(sqrt(-(-d + e*x)*(d + e*x))/(x*(d + e*x)**4), x)
```

3.194.7 Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx=\int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)^4x}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x), x)
```

3.194.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(97) = 194$.

Time = 0.30 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.93

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx=-\frac{e\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d^3|e|}-\frac{2\left(13\,e+\frac{45\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{ex}+\frac{75\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^3x^2}+\frac{55\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^5x^3}+\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^7x^4}\right)}{5\,d^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x, algorithm="giac")
```

output
```
-e*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^3*
abs(e)) - 2/5*(13*e + 45*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e*x) + 75*(d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e^3*x^2) + 55*(d*e + sqrt(-e^2*x^2 +
d^2)*abs(e))^3/(e^5*x^3) + 20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^7*x
^4))/(d^3*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e))
```

3.194.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx = \int \frac{\sqrt{d^2-e^2\,x^2}}{x\,{\left(d+e\,x\right)}^4}\,dx$$

input
```
int((d^2 - e^2*x^2)^(1/2)/(x*(d + e*x)^4),x)
```

output
```
int((d^2 - e^2*x^2)^(1/2)/(x*(d + e*x)^4), x)
```

3.195 $\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx$

3.195.1 Optimal result

Integrand size = 27, antiderivative size = 143

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx = -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}-\frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} -\frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^4x}+\frac{4e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

output
```
-8/5*e*(-e*x+d)/(-e^2*x^2+d^2)^(5/2)-4/15*e*(-8*e*x+5*d)/d^2/(-e^2*x^2+d^2
)^(3/2)+4*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4-1/15*e*(-79*e*x+60*d)/d^4/
(-e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^4/x
```

3.195.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx = -\frac{\frac{d\sqrt{d^2-e^2x^2}(15d^3+149d^2ex+222de^2x^2+94e^3x^3)}{x(d+ex)^3}-60\sqrt{d^2}e\log(x)+60\sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{15d^5}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)^4),x]
```

output
```
-1/15*((d*Sqrt[d^2 - e^2*x^2]*(15*d^3 + 149*d^2*e*x + 222*d*e^2*x^2 + 94*e
^3*x^3))/(x*(d + e*x)^3) - 60*Sqrt[d^2]*e*Log[x] + 60*Sqrt[d^2]*e*Log[Sqrt
[d^2] - Sqrt[d^2 - e^2*x^2]])/d^5
```

3.195.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {570, 532, 25, 2336, 25, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{(d-ex)^4}{x^2\left(d^2-e^2x^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{532}$$

$$-\frac{\int -\frac{5d^4-20exd^3+27e^2x^2d^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}-\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5d^4-20exd^3+27e^2x^2d^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}-\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{-\frac{\int -\frac{15d^4-60exd^3+64e^2x^2d^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}-\frac{4e(5d-8ex)}{3\left(d^2-e^2x^2\right)^{3/2}}}{5d^2}-\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{15d^4-60exd^3+64e^2x^2d^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}-\frac{4e(5d-8ex)}{3\left(d^2-e^2x^2\right)^{3/2}}}{5d^2}-\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{-\frac{\int -\frac{15d^3(d-4ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{e(60d-79ex)}{\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{4e(5d-8ex)}{3(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{15d\int \frac{d-4ex}{x^2\sqrt{d^2-e^2x^2}}dx-\frac{e(60d-79ex)}{\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{4e(5d-8ex)}{3(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{534}$$

$$\frac{\frac{15d\left(-4e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{e(60d-79ex)}{\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{4e(5d-8ex)}{3(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{15d\left(-2e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{e(60d-79ex)}{\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{4e(5d-8ex)}{3(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{15d\left(\frac{4\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{e(60d-79ex)}{\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{4e(5d-8ex)}{3(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{15d\left(\frac{4e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{e(60d-79ex)}{\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{4e(5d-8ex)}{3(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)^4),x]
```

output
```
(-8*e*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) + ((-4*e*(5*d - 8*e*x))/(3*(d^2
 - e^2*x^2)^(3/2)) + (-((e*(60*d - 79*e*x))/Sqrt[d^2 - e^2*x^2]) + 15*d*(-
(Sqrt[d^2 - e^2*x^2]/(d*x)) + (4*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/(3*
d^2))/(5*d^2)
```

3.195. $\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx$

3.195.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.195.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.39

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^4x}+\frac{4e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^3\sqrt{d^2}}-\frac{19\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15e\,d^3\left(x+\frac{d}{e}\right)^2}-\frac{79\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15d^4\left(x+\frac{d}{e}\right)}-\frac{2\sqrt{-}}{}$
default	$\frac{-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{d^2x}-\frac{2e^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{d^2}}{d^4}-\frac{4e\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)}{d^5}+\frac{-\frac{(-(}{}}{}$

input
```
int((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-(-e^2*x^2+d^2)^(1/2)/d^4/x+4*e/d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-
e^2*x^2+d^2)^(1/2))/x)-19/15/e/d^3/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e)
)^(1/2)-79/15/d^4/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-2/5/e^2/d^2
/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

3.195.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx =$$
$$-\frac{104\,e^4x^4+312\,de^3x^3+312\,d^2e^2x^2+104\,d^3ex+60\,(e^4x^4+3\,de^3x^3+3\,d^2e^2x^2+d^3ex)\log\left(-\frac{d-\sqrt{-e^2x^2+\cdots}}{x}\right.}{15\,(d^4e^3x^4+3\,d^5e^2x^3+3\,d^6ex^2+d^7x)}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x, algorithm="fricas")
```

output
```
-1/15*(104*e^4*x^4 + 312*d*e^3*x^3 + 312*d^2*e^2*x^2 + 104*d^3*e*x + 60*(e
^4*x^4 + 3*d*e^3*x^3 + 3*d^2*e^2*x^2 + d^3*e*x)*log(-(d - sqrt(-e^2*x^2 +
d^2))/x) + (94*e^3*x^3 + 222*d*e^2*x^2 + 149*d^2*e*x + 15*d^3)*sqrt(-e^2*x
^2 + d^2))/(d^4*e^3*x^4 + 3*d^5*e^2*x^3 + 3*d^6*e*x^2 + d^7*x)
```

3.195.6 Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx = \int \frac{\sqrt{-(-d+ex)\,(d+ex)}}{x^2\,(d+ex)^4}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**(1/2)/x**2/(e*x+d)**4,x)
```

output
```
Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**2*(d + e*x)**4), x)
```

3.195.7 Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx = \int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)^4x^2}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x^2), x)
```

3.195.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(128) = 256$.

Time = 0.31 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.13

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx = \frac{4\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d^4|e|} - \frac{de+\sqrt{-e^2x^2+d^2}|e|}{2\,d^4x|e|} + \frac{\left(15\,e^2 + \frac{491\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{x} + \frac{1690\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^2x^2} + \frac{2570\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^4x^3} + \frac{1815\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^6x^4} + \right.}{30\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^4\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x, algorithm="giac")
```

output
```
4*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d
^4*abs(e)) - 1/2*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(d^4*x*abs(e)) + 1/30
*(15*e^2 + 491*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/x + 1690*(d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))^2/(e^2*x^2) + 2570*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e
))^3/(e^4*x^3) + 1815*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^6*x^4) + 55
5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^8*x^5))*e^2*x/((d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))*d^4*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^
5*abs(e))
```

3.195.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx = \int \frac{\sqrt{d^2-e^2\,x^2}}{x^2\,{\left(d+e\,x\right)}^4}\,dx$$

input
```
int((d^2 - e^2*x^2)^(1/2)/(x^2*(d + e*x)^4),x)
```

output
```
int((d^2 - e^2*x^2)^(1/2)/(x^2*(d + e*x)^4), x)
```

3.196 $\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx$

3.196.1 Optimal result

Integrand size = 27, antiderivative size = 183

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx=\frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{2d^4x^2}+\frac{4e\sqrt{d^2-e^2x^2}}{d^5x}-\frac{19e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^5}$$

output
```
8/5*e^2*(-e*x+d)/d/(-e^2*x^2+d^2)^(5/2)+4/15*e^2*(-13*e*x+10*d)/d^3/(-e^2*
x^2+d^2)^(3/2)-19/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5+1/15*e^2*(-164
*e*x+135*d)/d^5/(-e^2*x^2+d^2)^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/d^4/x^2+4*e*
(-e^2*x^2+d^2)^(1/2)/d^5/x
```

3.196.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx=\frac{\frac{\sqrt{d^2-e^2x^2}\left(-15d^4+75d^3ex+713d^2e^2x^2+1059de^3x^3+448e^4x^4\right)}{x^2(d+ex)^3}+570e^2\text{arctanh}\left(\frac{\sqrt{-e^2}x-\sqrt{d^2-e^2x^2}}{d}\right)}{30d^5}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)^4),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(-15*d^4 + 75*d^3*e*x + 713*d^2*e^2*x^2 + 1059*d*e^3
*x^3 + 448*e^4*x^4))/(x^2*(d + e*x)^3) + 570*e^2*ArcTanh[(Sqrt[-e^2]*x - S
qrt[d^2 - e^2*x^2])/d])/(30*d^5)
```

3.196.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {570, 532, 25, 2336, 25, 2336, 27, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{(d-ex)^4}{x^3\left(d^2-e^2x^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{532}$$

$$\frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int -\frac{5d^4-20exd^3+35e^2x^2d^2-32e^3x^3d}{x^3\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5d^4-20exd^3+35e^2x^2d^2-32e^3x^3d}{x^3\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}+\frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{4e^2(10d-13ex)}{3d\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int -\frac{15d^4-60exd^3+120e^2x^2d^2-104e^3x^3d}{x^3\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}}{5d^2}+\frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{15d^4-60exd^3+120e^2x^2d^2-104e^3x^3d}{x^3\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{4e^2(10d-13ex)}{3d\left(d^2-e^2x^2\right)^{3/2}}}{5d^2}+\frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{\frac{e^2(135d-164ex)}{d\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{15\left(d^4-4exd^3+9e^2x^2d^2\right)}{x^3\sqrt{d^2-e^2x^2}}dx}{d^2}}{3d^2}+\frac{4e^2(10d-13ex)}{3d(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{15\int \frac{d^4-4exd^3+9e^2x^2d^2}{x^3\sqrt{d^2-e^2x^2}}dx}{d^2}+\frac{e^2(135d-164ex)}{d\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{4e^2(10d-13ex)}{3d(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$\Big\downarrow$ 2338

$$\frac{\frac{\frac{15\left(-\frac{\int \frac{d^4e(8d-19ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{d^2\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{e^2(135d-164ex)}{d\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{4e^2(10d-13ex)}{3d(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{15\left(-\frac{1}{2}d^2e\int \frac{8d-19ex}{x^2\sqrt{d^2-e^2x^2}}dx-\frac{d^2\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{e^2(135d-164ex)}{d\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{4e^2(10d-13ex)}{3d(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$\Big\downarrow$ 534

$$\frac{\frac{\frac{15\left(-\frac{1}{2}d^2e\left(-19e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{8\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{d^2\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{e^2(135d-164ex)}{d\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{4e^2(10d-13ex)}{3d(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$\Big\downarrow$ 243

$$\frac{\frac{\frac{15\left(-\frac{1}{2}d^2e\left(-\frac{19}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{8\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{d^2\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{e^2(135d-164ex)}{d\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{4e^2(10d-13ex)}{3d(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}$$

$\Big\downarrow$ 73

3.196. $\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx$

$$\frac{\frac{\frac{15\left(-\frac{1}{2}d^2e\left(\frac{19\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{8\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{d^2\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{e^2(135d-164ex)}{d\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{4e^2(10d-13ex)}{3d(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{8e^2(d-ex)}{5d\,(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{15\left(-\frac{1}{2}d^2e\left(\frac{19e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{8\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{d^2\sqrt{d^2-e^2x^2}}{2x^2}\right)}{d^2}+\frac{e^2(135d-164ex)}{d\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{4e^2(10d-13ex)}{3d(d^2-e^2x^2)^{3/2}}}{5d^2}+\frac{8e^2(d-ex)}{5d\,(d^2-e^2x^2)^{5/2}}$$

input
```
Int[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)^4),x]
```

output
```
(8*e^2*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + ((4*e^2*(10*d - 13*e*x))/(
3*d*(d^2 - e^2*x^2)^(3/2)) + ((e^2*(135*d - 164*e*x))/(d*Sqrt[d^2 - e^2*x^
2]) + (15*(-1/2*(d^2*Sqrt[d^2 - e^2*x^2])/x^2 - (d^2*e*((-8*Sqrt[d^2 - e^2
*x^2])/(d*x) + (19*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/2))/d^2)/(3*d^2))
/(5*d^2)
```

3.196.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.196. $\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx$

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.196.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-8ex+d)}{2d^5x^2}-\frac{19e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^4\sqrt{d^2}}+\frac{164e\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15d^5\left(x+\frac{d}{e}\right)}+\frac{2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\cdots\right.}}{5e\,d^3\left(x+\frac{d}{e}\right)^3}$
default	$\frac{-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{2d^2x^2}-\frac{e^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)}{2d^2}}{d^4}-\frac{4e\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{d^2x}-\frac{2e^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e\cdots}}{\sqrt{-e^2\cdots}}\right)}{2\sqrt{e^2}}\right)}{d^2}\cdots\right.}{d^5}$

input
```
int((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e^2*x^2+d^2)^(1/2)*(-8*e*x+d)/d^5/x^2-19/2/d^4*e^2/(d^2)^(1/2)*ln((
2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+164/15/d^5*e/(x+d/e)*(-(x+d/e
)^2*e^2+2*d*e*(x+d/e))^(1/2)+2/5/e/d^3/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+
d/e))^(1/2)+29/15/d^4/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

3.196.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx = \frac{398\,e^5x^5+1194\,de^4x^4+1194\,d^2e^3x^3+398\,d^3e^2x^2+285\left(e^5x^5+3\,de^4x^4+3\,d^2e^3x^3+d^3e^2x^2\right)\log\left(-\frac{d-\sqrt{-}}{}\right.}{30\left(d^5e^3x^5+3\,d^6e^2x^4+3\,d^7ex^3\right.}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x, algorithm="fricas")
```

output
```
1/30*(398*e^5*x^5 + 1194*d*e^4*x^4 + 1194*d^2*e^3*x^3 + 398*d^3*e^2*x^2 +
285*(e^5*x^5 + 3*d*e^4*x^4 + 3*d^2*e^3*x^3 + d^3*e^2*x^2)*log(-(d - sqrt(-
e^2*x^2 + d^2))/x) + (448*e^4*x^4 + 1059*d*e^3*x^3 + 713*d^2*e^2*x^2 + 75*
d^3*e*x - 15*d^4)*sqrt(-e^2*x^2 + d^2))/(d^5*e^3*x^5 + 3*d^6*e^2*x^4 + 3*d
^7*e*x^3 + d^8*x^2)
```

3.196.6 Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx = \int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x^3\left(d+ex\right)^4}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**(1/2)/x**3/(e*x+d)**4,x)
```

output
```
Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**3*(d + e*x)**4), x)
```

3.196.7 Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx = \int \frac{\sqrt{-e^2x^2+d^2}}{\left(ex+d\right)^4x^3}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x^3), x)
```

3.196.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(162) = 324$.

Time = 0.31 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.09

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx = -\frac{19\,e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,d^5|e|} + \frac{\left(15\,e^3-\frac{165\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e}{x}-\frac{4234\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{ex^2}-\frac{14330\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^3x^3}-\frac{20965\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^5x^4}\right.}{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^5\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right.} + \frac{\frac{16\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^5e|e|}{x}-\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^5|e|}{ex^2}}{8\,d^{10}e^2}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x, algorithm="giac")
```

output
```
-19/2*e^3*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x))
)/(d^5*abs(e)) + 1/120*(15*e^3 - 165*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e
/x - 4234*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e*x^2) - 14330*(d*e + sqr
t(-e^2*x^2 + d^2)*abs(e))^3/(e^3*x^3) - 20965*(d*e + sqrt(-e^2*x^2 + d^2)*
abs(e))^4/(e^5*x^4) - 14385*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^7*x^5
) - 4080*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6/(e^9*x^6))*e^4*x^2/((d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^5*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e
^2*x) + 1)^5*abs(e)) + 1/8*(16*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^5*e*a
bs(e)/x - (d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^5*abs(e)/(e*x^2))/(d^10*
e^2)
```

3.196.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx = \int \frac{\sqrt{d^2-e^2\,x^2}}{x^3\,{\left(d+e\,x\right)}^4}\,dx$$

input
```
int((d^2 - e^2*x^2)^(1/2)/(x^3*(d + e*x)^4),x)
```

output
```
int((d^2 - e^2*x^2)^(1/2)/(x^3*(d + e*x)^4), x)
```

3.197 $\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx$

3.197.1 Optimal result

Integrand size = 27, antiderivative size = 210

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx=-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3d^4x^3}+\frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}-\frac{29e^2\sqrt{d^2-e^2x^2}}{3d^6x}+\frac{18e^3\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

output
```
-8/5*e^3*(-e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)-4/5*e^3*(-6*e*x+5*d)/d^4/(-e^2*
x^2+d^2)^(3/2)+18*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6-1/5*e^3*(-93*e*x
+80*d)/d^6/(-e^2*x^2+d^2)^(1/2)-1/3*(-e^2*x^2+d^2)^(1/2)/d^4/x^3+2*e*(-e^2
*x^2+d^2)^(1/2)/d^5/x^2-29/3*e^2*(-e^2*x^2+d^2)^(1/2)/d^6/x
```

3.197.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx = -\frac{\frac{d\sqrt{d^2-e^2x^2}\left(5d^5-15d^4ex+70d^3e^2x^2+674d^2e^3x^3+1002de^4x^4+424e^5x^5\right)}{x^3(d+ex)^3}-270\sqrt{d^2}e^3\log(x)+270\sqrt{d^2}e^3\log\left(\sqrt{d^2}-\sqrt{d^2}\right.}{15d^7}$$

input
```
Integrate[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)^4),x]
```

output
```
-1/15*((d*Sqrt[d^2 - e^2*x^2]*(5*d^5 - 15*d^4*e*x + 70*d^3*e^2*x^2 + 674*d
^2*e^3*x^3 + 1002*d*e^4*x^4 + 424*e^5*x^5))/(x^3*(d + e*x)^3) - 270*Sqrt[d
^2]*e^3*Log[x] + 270*Sqrt[d^2]*e^3*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/d
^7
```

3.197.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.519, Rules used = {570, 532, 25, 2336, 27, 2336, 27, 2338, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{(d-ex)^4}{x^4\left(d^2-e^2x^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{532}$$

$$-\frac{\int -\frac{5d^4-20exd^3+35e^2x^2d^2-40e^3x^3d+32e^4x^4}{x^4\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5d^4-20exd^3+35e^2x^2d^2-40e^3x^3d+32e^4x^4}{x^4\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{-\frac{\int -\frac{3\left(5d^4-20exd^3+40e^2x^2d^2-60e^3x^3d+48e^4x^4\right)}{x^4\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}-\frac{4e^3(5d-6ex)}{d^2\left(d^2-e^2x^2\right)^{3/2}}}{5d^2}-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{5d^4-20exd^3+40e^2x^2d^2-60e^3x^3d+48e^4x^4}{x^4(d^2-e^2x^2)^{3/2}}dx}{d^2}-\frac{4e^3(5d-6ex)}{d^2(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{8e^3(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \; 2336$$

$$\frac{\frac{-\frac{\int -\frac{5(d^4-4exd^3+9e^2x^2d^2-16e^3x^3d)}{x^4\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{e^3(80d-93ex)}{d^2\sqrt{d^2-e^2x^2}}}{d^2}-\frac{4e^3(5d-6ex)}{d^2(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{8e^3(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{5\int \frac{d^4-4exd^3+9e^2x^2d^2-16e^3x^3d}{x^4\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{e^3(80d-93ex)}{d^2\sqrt{d^2-e^2x^2}}}{d^2}-\frac{4e^3(5d-6ex)}{d^2(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{8e^3(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \; 2338$$

$$\frac{\frac{\frac{5\left(-\frac{\int \frac{12ed^5-29e^2xd^4+48e^3x^2d^3}{x^3\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{d^2\sqrt{d^2-e^2x^2}}{3x^3}\right)}{d^2}-\frac{e^3(80d-93ex)}{d^2\sqrt{d^2-e^2x^2}}}{d^2}-\frac{4e^3(5d-6ex)}{d^2(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{8e^3(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \; 2338$$

$$\frac{\frac{\frac{5\left(-\frac{-\frac{\int \frac{2d^5e^2(29d-54ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{6d^3e\sqrt{d^2-e^2x^2}}{x^2}}{3d^2}-\frac{d^2\sqrt{d^2-e^2x^2}}{3x^3}\right)}{d^2}-\frac{e^3(80d-93ex)}{d^2\sqrt{d^2-e^2x^2}}}{d^2}-\frac{4e^3(5d-6ex)}{d^2(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{8e^3(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{5\left(-\frac{d^3(-e^2)\int \frac{29d-54ex}{x^2\sqrt{d^2-e^2x^2}}dx-\frac{6d^3e\sqrt{d^2-e^2x^2}}{x^2}}{3d^2}-\frac{d^2\sqrt{d^2-e^2x^2}}{3x^3}\right)}{d^2}-\frac{e^3(80d-93ex)}{d^2\sqrt{d^2-e^2x^2}}}{d^2}-\frac{4e^3(5d-6ex)}{d^2(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{8e^3(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \; 534$$

3.197. $\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx$

$$\frac{\frac{\frac{5\left(-\frac{d^3\left(-e^2\right)\left(-54e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{29\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{6d^3e\sqrt{d^2-e^2x^2}}{x^2}}{3d^2}-\frac{d^2\sqrt{d^2-e^2x^2}}{3x^3}\right)}{d^2}-\frac{e^3(80d-93ex)}{d^2\sqrt{d^2-e^2x^2}}}{d^2}-\frac{4e^3(5d-6ex)}{d^2(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{\frac{5\left(-\frac{d^3\left(-e^2\right)\left(-27e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{29\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{6d^3e\sqrt{d^2-e^2x^2}}{x^2}}{3d^2}-\frac{d^2\sqrt{d^2-e^2x^2}}{3x^3}\right)}{d^2}-\frac{e^3(80d-93ex)}{d^2\sqrt{d^2-e^2x^2}}}{d^2}-\frac{4e^3(5d-6ex)}{d^2(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{5\left(-\frac{d^3\left(-e^2\right)\left(\frac{54\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{29\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{6d^3e\sqrt{d^2-e^2x^2}}{x^2}}{3d^2}-\frac{d^2\sqrt{d^2-e^2x^2}}{3x^3}\right)}{d^2}-\frac{e^3(80d-93ex)}{d^2\sqrt{d^2-e^2x^2}}}{d^2}-\frac{4e^3(5d-6ex)}{d^2(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{5\left(-\frac{d^3\left(-e^2\right)\left(\frac{54e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{29\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{6d^3e\sqrt{d^2-e^2x^2}}{x^2}}{3d^2}-\frac{d^2\sqrt{d^2-e^2x^2}}{3x^3}\right)}{d^2}-\frac{e^3(80d-93ex)}{d^2\sqrt{d^2-e^2x^2}}}{d^2}-\frac{4e^3(5d-6ex)}{d^2(d^2-e^2x^2)^{3/2}}}{5d^2}-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}$$

input

```
Int[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)^4),x]
```

3.197. $\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx$

output
```
(-8*e^3*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + ((-4*e^3*(5*d - 6*e*x))
/(d^2*(d^2 - e^2*x^2)^(3/2)) + (-((e^3*(80*d - 93*e*x))/(d^2*Sqrt[d^2 - e^
2*x^2])) + (5*(-1/3*(d^2*Sqrt[d^2 - e^2*x^2])/x^3 - ((-6*d^3*e*Sqrt[d^2 -
e^2*x^2])/x^2 - d^3*e^2*((-29*Sqrt[d^2 - e^2*x^2])/(d*x) + (54*e*ArcTanh[S
qrt[d^2 - e^2*x^2]/d])/d))/(3*d^2)))/d^2)/d^2)/(5*d^2)
```

3.197.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

3.197. $\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx$

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.197.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(29e^2x^2-6dex+d^2)}{3d^6x^3}+\frac{18e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^5\sqrt{d^2}}-\frac{93e^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5d^6\left(x+\frac{d}{e}\right)}-\frac{13e\sqrt{-\left(x+\frac{d}{e}\right)^2}}{5d^5\left(x\right.}$
default	$-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3d^6x^3}-\frac{4e\left(-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{2d^2x^2}-\frac{e^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)}{2d^2}\right)}{d^5}-\frac{20e^3\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(2d^2\right.}{}\right.}{d^7}$

input

```
int((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output

```
-1/3*(-e^2*x^2+d^2)^(1/2)*(29*e^2*x^2-6*d*e*x+d^2)/d^6/x^3+18/d^5*e^3/(d^2
)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-93/5/d^6*e^2/(x+d
/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-13/5/d^5*e/(x+d/e)^2*(-(x+d/e)^2*
e^2+2*d*e*(x+d/e))^(1/2)-2/5/d^4/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^
(1/2)
```

3.197.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx =$$

$$-\frac{324\,e^6x^6+972\,de^5x^5+972\,d^2e^4x^4+324\,d^3e^3x^3+270\,(e^6x^6+3\,de^5x^5+3\,d^2e^4x^4+d^3e^3x^3)\log\left(-\frac{d-\sqrt{}}{}\right.}{15\,(d^6e^3x^6+3\,d^7e^2x^5+3}$$

input

```
integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x, algorithm="fricas")
```

output

```
-1/15*(324*e^6*x^6 + 972*d*e^5*x^5 + 972*d^2*e^4*x^4 + 324*d^3*e^3*x^3 + 2
70*(e^6*x^6 + 3*d*e^5*x^5 + 3*d^2*e^4*x^4 + d^3*e^3*x^3)*log(-(d - sqrt(-e
^2*x^2 + d^2))/x) + (424*e^5*x^5 + 1002*d*e^4*x^4 + 674*d^2*e^3*x^3 + 70*d
^3*e^2*x^2 - 15*d^4*e*x + 5*d^5)*sqrt(-e^2*x^2 + d^2))/(d^6*e^3*x^6 + 3*d^
7*e^2*x^5 + 3*d^8*e*x^4 + d^9*x^3)
```

3.197.6 Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx=\int \frac{\sqrt{-(-d+ex)(d+ex)}}{x^4(d+ex)^4}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**(1/2)/x**4/(e*x+d)**4,x)
```

output
```
Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**4*(d + e*x)**4), x)
```

3.197.7 Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx=\int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)^4x^4}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x^4), x)
```

3.197.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(187)=374$.

Time = 0.31 (sec), antiderivative size = 444, normalized size of antiderivative = 2.11

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx=\frac{18\,e^4\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d^6|e|}+\frac{\left(5\,e^4-\frac{35\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^2}{x}+\frac{335\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{x^2}+\frac{7559\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^2x^3}+\frac{25195\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^4x^4}+\cdots\right.}{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^6\left(\frac{de+\sqrt{-e^2x\cdots}}{e^2x\cdots}\right.}-\frac{\frac{117\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{12}e^4}{x}-\frac{12\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^{12}e^2}{x^2}+\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^{12}}{x^3}}{24\,d^{18}e^2|e|}$$

input
```
integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x, algorithm="giac")
```

output
```
18*e^4*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(
d^6*abs(e)) + 1/120*(5*e^4 - 35*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^2/x
+ 335*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/x^2 + 7559*(d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))^3/(e^2*x^3) + 25195*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4
/(e^4*x^4) + 36035*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^6*x^5) + 24225
*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6/(e^8*x^6) + 6585*(d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))^7/(e^10*x^7))*e^6*x^3/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e
))^3*d^6*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e)) - 1/2
4*(117*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^12*e^4/x - 12*(d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^2*d^12*e^2/x^2 + (d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^
3*d^12/x^3)/(d^18*e^2*abs(e))
```

3.197.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx=\int \frac{\sqrt{d^2-e^2\,x^2}}{x^4\,{(d+e\,x)}^4}\,dx$$

input
```
int((d^2 - e^2*x^2)^(1/2)/(x^4*(d + e*x)^4),x)
```

output
```
int((d^2 - e^2*x^2)^(1/2)/(x^4*(d + e*x)^4), x)
```

3.198 $\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

3.198.1 Optimal result

Integrand size = 27, antiderivative size = 252

$$\begin{aligned}\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx &= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}}+\frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6}-\frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5}\\&+\frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4}-\frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3}+\frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2}\\&-\frac{2dx^5\sqrt{d^2-e^2x^2}}{3e}+\frac{1}{7}x^6\sqrt{d^2-e^2x^2}+\frac{65d^7\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^6}\end{aligned}$$

output
```
65/4*d^7*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+d^4*(-e*x+d)^4/e^6/(-e^2*x^2
+d^2)^(1/2)+515/21*d^6*(-e^2*x^2+d^2)^(1/2)/e^6-49/4*d^5*x*(-e^2*x^2+d^2)^
(1/2)/e^5+121/21*d^4*x^2*(-e^2*x^2+d^2)^(1/2)/e^4-17/6*d^3*x^3*(-e^2*x^2+d
^2)^(1/2)/e^3+11/7*d^2*x^4*(-e^2*x^2+d^2)^(1/2)/e^2-2/3*d*x^5*(-e^2*x^2+d^
2)^(1/2)/e+1/7*x^6*(-e^2*x^2+d^2)^(1/2)
```

3.198.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.60

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{\frac{e\sqrt{d^2-e^2x^2}(2144d^7+779d^6ex-293d^5e^2x^2+162d^4e^3x^3-106d^3e^4x^4+76d^2e^5x^5-44de^6x^6+12e^7x^7)}{d+ex}+1365d^7}{84e^7}$$

input
```
Integrate[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

output
```
((e*Sqrt[d^2 - e^2*x^2]*(2144*d^7 + 779*d^6*e*x - 293*d^5*e^2*x^2 + 162*d^
4*e^3*x^3 - 106*d^3*e^4*x^4 + 76*d^2*e^5*x^5 - 44*d*e^6*x^6 + 12*e^7*x^7))
/(d + e*x) + 1365*d^7*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]
])/(84*e^7)
```

3.198.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.18, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.630, Rules used = {563, 2346, 25, 2346, 27, 2346, 27, 2346, 27, 2346, 25, 2346, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}dx$$

$$\Big\downarrow \text{563}$$

$$\frac{\int \frac{8d^7-8exd^6+8e^2x^2d^5-8e^3x^3d^4+8e^4x^4d^3-7e^5x^5d^2+4e^6x^6d-e^7x^7}{\sqrt{d^2-e^2x^2}}dx}{e^5}+\frac{8d^7\sqrt{d^2-e^2x^2}}{e^6(d+ex)}$$

$$\Big\downarrow \text{2346}$$

$$\frac{\frac{1}{7}e^5x^6\sqrt{d^2-e^2x^2}-\frac{\int -\frac{28dx^6e^8-55d^2x^5e^7+56d^3x^4e^6-56d^4x^3e^5+56d^5x^2e^4-56d^6xe^3+56d^7e^2}{\sqrt{d^2-e^2x^2}}dx}{7e^2}}{e^5}+\frac{8d^7\sqrt{d^2-e^2x^2}}{e^6(d+ex)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{28dx^6e^8-55d^2x^5e^7+56d^3x^4e^6-56d^4x^3e^5+56d^5x^2e^4-56d^6xe^3+56d^7e^2}{\sqrt{d^2-e^2x^2}}dx}{7e^2}+\frac{1}{7}e^5x^6\sqrt{d^2-e^2x^2}}{e^5}+\frac{8d^7\sqrt{d^2-e^2x^2}}{e^6(d+ex)}$$

$$\Big\downarrow \text{2346}$$

$$\frac{\frac{-\frac{\int -\frac{2\left(-165d^2x^5e^9+238d^3x^4e^8-168d^4x^3e^7+168d^5x^2e^6-168d^6xe^5+168d^7e^4\right)}{\sqrt{d^2-e^2x^2}}dx}{6e^2}-\frac{14}{3}de^6x^5\sqrt{d^2-e^2x^2}}{7e^2}+\frac{1}{7}e^5x^6\sqrt{d^2-e^2x^2}}{e^5}+\frac{8d^7\sqrt{d^2-e^2x^2}}{e^6(d+ex)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{-165d^2x^5e^9+238d^3x^4e^8-168d^4x^3e^7+168d^5x^2e^6-168d^6xe^5+168d^7e^4}{\sqrt{d^2-e^2x^2}}dx}{3e^2}-\frac{14}{3}de^6x^5\sqrt{d^2-e^2x^2}}{7e^2}+\frac{1}{7}e^5x^6\sqrt{d^2-e^2x^2}}{e^5}+\frac{8d^7\sqrt{d^2-e^2x^2}}{e^6(d+ex)}$$

$$\Big\downarrow \; 2346$$

$$\frac{\frac{\frac{33d^2e^7x^4\sqrt{d^2-e^2x^2}-\frac{\int -\frac{10\left(119d^3x^4e^{10}-150d^4x^3e^9+84d^5x^2e^8-84d^6xe^7+84d^7e^6\right)}{\sqrt{d^2-e^2x^2}}dx}{5e^2}}{3e^2}-\frac{14}{3}de^6x^5\sqrt{d^2-e^2x^2}}{7e^2}+\frac{1}{7}e^5x^6\sqrt{d^2-e^2x^2}}{e^5}+\frac{8d^7\sqrt{d^2-e^2x^2}}{e^6(d+ex)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\frac{2\int \frac{119d^3x^4e^{10}-150d^4x^3e^9+84d^5x^2e^8-84d^6xe^7+84d^7e^6}{\sqrt{d^2-e^2x^2}}dx}{e^2}+33d^2e^7x^4\sqrt{d^2-e^2x^2}}{3e^2}-\frac{14}{3}de^6x^5\sqrt{d^2-e^2x^2}}{7e^2}+\frac{1}{7}e^5x^6\sqrt{d^2-e^2x^2}}{e^5}+\frac{8d^7\sqrt{d^2-e^2x^2}}{e^6(d+ex)}$$

$$\Big\downarrow \; 2346$$

$$\frac{\frac{\frac{\frac{2\left(-\frac{\int -\frac{3\left(-200d^4x^3e^{11}+231d^5x^2e^{10}-112d^6xe^9+112d^7e^8\right)}{\sqrt{d^2-e^2x^2}}dx}{4e^2}-\frac{119}{4}d^3e^8x^3\sqrt{d^2-e^2x^2}\right)}{e^2}+33d^2e^7x^4\sqrt{d^2-e^2x^2}}{3e^2}-\frac{14}{3}de^6x^5\sqrt{d^2-e^2x^2}}{7e^2}+\frac{1}{7}e^5x^6\sqrt{d^2-e^2x^2}}{e^5}+\frac{8d^7\sqrt{d^2-e^2x^2}}{e^6(d+ex)}$$

$$\Big\downarrow \; 27$$

3.198. $\int \frac{x^5\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx$

$$\frac{\frac{\frac{\frac{2\left(\frac{3\int \frac{-200d^4x^3e^{11}+231d^5x^2e^{10}-112d^6xe^9+112d^7e^8}{\sqrt{d^2-e^2x^2}}dx}{4e^2}-\frac{119}{4}d^3e^8x^3\sqrt{d^2-e^2x^2}\right)}{e^2}+33d^2e^7x^4\sqrt{d^2-e^2x^2}}{3e^2}-\frac{14}{3}de^6x^5\sqrt{d^2-e^2x^2}}{7e^2}+\frac{1}{7}e^5x^6\sqrt{d^2-e^2x^2}}{e^5}$$

$$\frac{8d^7\sqrt{d^2-e^2x^2}}{e^6(d+ex)}$$

$\downarrow$ 2346

$$\frac{\frac{\frac{\frac{2\left(\frac{3\left(\frac{200}{3}d^4e^9x^2\sqrt{d^2-e^2x^2}-\frac{\int -\frac{693d^5x^2e^{12}-736d^6xe^{11}+336d^7e^{10}}{\sqrt{d^2-e^2x^2}}dx}{3e^2}\right)}{4e^2}-\frac{119}{4}d^3e^8x^3\sqrt{d^2-e^2x^2}\right)}{e^2}+33d^2e^7x^4\sqrt{d^2-e^2x^2}}{3e^2}-\frac{14}{3}de^6x^5\sqrt{d^2-e^2x^2}}{7e^2}+\frac{1}{7}e^5x^{\cdots}}{e^5}$$

$$\frac{8d^7\sqrt{d^2-e^2x^2}}{e^6(d+ex)}$$

$\downarrow$ 25

$$\frac{\frac{\frac{\frac{2\left(\frac{3\left(\frac{\int \frac{693d^5x^2e^{12}-736d^6xe^{11}+336d^7e^{10}}{\sqrt{d^2-e^2x^2}}dx}{3e^2}+\frac{200}{3}d^4e^9x^2\sqrt{d^2-e^2x^2}\right)}{4e^2}-\frac{119}{4}d^3e^8x^3\sqrt{d^2-e^2x^2}\right)}{e^2}+33d^2e^7x^4\sqrt{d^2-e^2x^2}}{3e^2}-\frac{14}{3}de^6x^5\sqrt{d^2-e^2x^2}}{7e^2}+\frac{1}{7}e^5x^6\cdots}{e^5}$$

$$\frac{8d^7\sqrt{d^2-e^2x^2}}{e^6(d+ex)}$$

$\downarrow$ 2346

$$\frac{\frac{\frac{\frac{2\left(\frac{3\left(\frac{-\frac{\int -\frac{d^6e^{12}(1365d-1472ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{693}{2}d^5e^{10}x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{200}{3}d^4e^9x^2\sqrt{d^2-e^2x^2}\right)}{4e^2}-\frac{119}{4}d^3e^8x^3\sqrt{d^2-e^2x^2}\right)}{e^2}+33d^2e^7x^4\sqrt{d^2-e^2x^2}}{3e^2}-\frac{14}{3}de^6x^5\sqrt{d^2-e^2x^{\cdots}}}{7e^2}}{e^5}$$

$$\frac{8d^7\sqrt{d^2-e^2x^2}}{e^6(d+ex)}$$

3.198. $\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{2\left(\frac{3\left(\frac{\frac{\int \frac{d^6e^{12}(1365d-1472ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{693}{2}d^5e^{10}x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{200}{3}d^4e^9x^2\sqrt{d^2-e^2x^2}\right)}{4e^2}-\frac{119}{4}d^3e^8x^3\sqrt{d^2-e^2x^2}\right)}{e^2}+33d^2e^7x^4\sqrt{d^2-e^2x^2}}{3e^2}-\frac{14}{3}de^6x^5\sqrt{d^2-e^2x^2}}{7e^2}-}{e^5}$$

$$\frac{8d^7\sqrt{d^2-e^2x^2}}{e^6(d+ex)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{2\left(\frac{3\left(\frac{\frac{1}{2}d^6e^{10}\int \frac{1365d-1472ex}{\sqrt{d^2-e^2x^2}}dx-\frac{693}{2}d^5e^{10}x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{200}{3}d^4e^9x^2\sqrt{d^2-e^2x^2}\right)}{4e^2}-\frac{119}{4}d^3e^8x^3\sqrt{d^2-e^2x^2}\right)}{e^2}+33d^2e^7x^4\sqrt{d^2-e^2x^2}}{3e^2}-\frac{14}{3}de^6x^5\sqrt{d^2-e^2x^2}}{7e^2}-}{e^5}$$

$$\frac{8d^7\sqrt{d^2-e^2x^2}}{e^6(d+ex)}$$

$$\Big\downarrow \; 455$$

$$\frac{\frac{\frac{2\left(\frac{3\left(\frac{\frac{1}{2}d^6e^{10}\left(1365d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1472\sqrt{d^2-e^2x^2}}{e}\right)-\frac{693}{2}d^5e^{10}x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{200}{3}d^4e^9x^2\sqrt{d^2-e^2x^2}\right)}{4e^2}-\frac{119}{4}d^3e^8x^3\sqrt{d^2-e^2x^2}\right)}{e^2}+33d^2e^7x^4\sqrt{d^2-e^2x^2}-}{3e^2}}{7e^2}}{e^5}$$

$$\frac{8d^7\sqrt{d^2-e^2x^2}}{e^6(d+ex)}$$

$$\Big\downarrow \; 224$$

3.198. $\int \frac{x^5\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx$

$$\frac{\frac{\frac{\frac{2\left(\frac{3\left(\frac{\frac{1}{2}d^6e^{10}\left(1365d\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1472\sqrt{d^2-e^2x^2}}{e}\right)-\frac{693}{2}d^5e^{10}x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{200}{3}d^4e^9x^2\sqrt{d^2-e^2x^2}\right)}{4e^2}-\frac{119}{4}d^3e^8x^3\sqrt{d^2-e^2x^2}\right)}{e^2}+33d^2e^7x^4\ldots}{3e^2}}{7e^2}}{e^5}$$

$$\frac{8d^7\sqrt{d^2-e^2x^2}}{e^6(d+ex)}$$

$\downarrow$ 216

$$\frac{\frac{\frac{\frac{2\left(\frac{3\left(\frac{\frac{1}{2}d^6e^{10}\left(\frac{1365d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{1472\sqrt{d^2-e^2x^2}}{e}\right)-\frac{693}{2}d^5e^{10}x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{200}{3}d^4e^9x^2\sqrt{d^2-e^2x^2}\right)}{4e^2}-\frac{119}{4}d^3e^8x^3\sqrt{d^2-e^2x^2}\right)}{e^2}+33d^2e^7x^4\sqrt{d^2-e\ldots}}{3e^2}}{7e^2}}{e^5}$$

$$\frac{8d^7\sqrt{d^2-e^2x^2}}{e^6(d+ex)}$$

input
```
Int[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

output
```
(8*d^7*Sqrt[d^2 - e^2*x^2])/(e^6*(d + e*x)) + ((e^5*x^6*Sqrt[d^2 - e^2*x^2
])/7 + ((-14*d*e^6*x^5*Sqrt[d^2 - e^2*x^2])/3 + (33*d^2*e^7*x^4*Sqrt[d^2 -
 e^2*x^2] + (2*((-119*d^3*e^8*x^3*Sqrt[d^2 - e^2*x^2])/4 + (3*((200*d^4*e^
9*x^2*Sqrt[d^2 - e^2*x^2])/3 + ((-693*d^5*e^10*x*Sqrt[d^2 - e^2*x^2])/2 +
(d^6*e^10*((1472*Sqrt[d^2 - e^2*x^2])/e + (1365*d*ArcTan[(e*x)/Sqrt[d^2 -
e^2*x^2]])/e))/2)/(3*e^2)))/(4*e^2)))/e^2)/(3*e^2))/(7*e^2))/e^5
```

3.198. $\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

3.198.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.198. $\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4} dx$

3.198.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.65

method	result
risch	$\frac{(12e^6x^6-56d\,e^5x^5+132d^2e^4x^4-238d^3x^3e^3+400d^4e^2x^2-693d^5ex+1472d^6)\sqrt{-e^2x^2+d^2}}{84e^6}+\frac{65d^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{4e^5\sqrt{e^2}}+\frac{8d^7\sqrt{}}{}$
default	Expression too large to display

input
```
int(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
1/84*(12*e^6*x^6-56*d*e^5*x^5+132*d^2*e^4*x^4-238*d^3*e^3*x^3+400*d^4*e^2*
x^2-693*d^5*e*x+1472*d^6)/e^6*(-e^2*x^2+d^2)^(1/2)+65/4*d^7/e^5/(e^2)^(1/2
)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+8*d^7/e^7/(x+d/e)*(-(x+d/e)^2
*e^2+2*d*e*(x+d/e))^(1/2)
```

3.198.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.62

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx=\frac{2144\,d^7ex+2144\,d^8-2730\,(d^7ex+d^8)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(12\,e^7x^7-44\,de}{84}$$

input
```
integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
1/84*(2144*d^7*e*x + 2144*d^8 - 2730*(d^7*e*x + d^8)*arctan(-(d - sqrt(-e^
2*x^2 + d^2))/(e*x)) + (12*e^7*x^7 - 44*d*e^6*x^6 + 76*d^2*e^5*x^5 - 106*d
^3*e^4*x^4 + 162*d^4*e^3*x^3 - 293*d^5*e^2*x^2 + 779*d^6*e*x + 2144*d^7)*s
qrt(-e^2*x^2 + d^2))/(e^7*x + d*e^6)
```

3.198.6 Sympy [F]

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^5\left(-(-d+ex)\,(d+ex)\right)^{\frac{5}{2}}}{(d+ex)^4}\,dx$$

input
```
integrate(x**5*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

output
```
Integral(x**5*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

3.198.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.90

$$\begin{aligned}\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx &= -\frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^5}{2\,(e^9x^3+3\,de^8x^2+3\,d^2e^7x+d^3e^6)} \\ &- \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^6}{2\,(e^8x^2+2\,de^7x+d^2e^6)} + \frac{15\,\sqrt{-e^2x^2+d^2}d^7}{e^7x+de^6} + \frac{5\,(-e^2x^2+d^2)^{\frac{5}{2}}d^4}{3\,(e^8x^2+2\,de^7x+d^2e^6)} \\ &+ \frac{25\,(-e^2x^2+d^2)^{\frac{3}{2}}d^5}{6\,(e^7x+de^6)} - \frac{5\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3}{2\,(e^7x+de^6)} + \frac{5i\,d^7\arcsin\left(\frac{ex}{d}+2\right)}{2\,e^6} \\ &+ \frac{75\,d^7\arcsin\left(\frac{ex}{d}\right)}{4\,e^6} - \frac{5\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^5x}{2\,e^5} - \frac{5\,\sqrt{-e^2x^2+d^2}d^5x}{4\,e^5} \\ &- \frac{5\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^6}{e^6} + \frac{25\,\sqrt{-e^2x^2+d^2}d^6}{2\,e^6} + \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^3x}{3\,e^5} \\ &- \frac{25\,(-e^2x^2+d^2)^{\frac{3}{2}}d^4}{6\,e^6} - \frac{2\,(-e^2x^2+d^2)^{\frac{5}{2}}dx}{3\,e^5} + \frac{2\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2}{e^6} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{7\,e^6}\end{aligned}$$

input
```
integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
-1/2*(-e^2*x^2 + d^2)^(5/2)*d^5/(e^9*x^3 + 3*d*e^8*x^2 + 3*d^2*e^7*x + d^3
*e^6) - 5/2*(-e^2*x^2 + d^2)^(3/2)*d^6/(e^8*x^2 + 2*d*e^7*x + d^2*e^6) + 1
5*sqrt(-e^2*x^2 + d^2)*d^7/(e^7*x + d*e^6) + 5/3*(-e^2*x^2 + d^2)^(5/2)*d^
4/(e^8*x^2 + 2*d*e^7*x + d^2*e^6) + 25/6*(-e^2*x^2 + d^2)^(3/2)*d^5/(e^7*x
 + d*e^6) - 5/2*(-e^2*x^2 + d^2)^(5/2)*d^3/(e^7*x + d*e^6) + 5/2*I*d^7*arc
sin(e*x/d + 2)/e^6 + 75/4*d^7*arcsin(e*x/d)/e^6 - 5/2*sqrt(e^2*x^2 + 4*d*e
*x + 3*d^2)*d^5*x/e^5 - 5/4*sqrt(-e^2*x^2 + d^2)*d^5*x/e^5 - 5*sqrt(e^2*x^
2 + 4*d*e*x + 3*d^2)*d^6/e^6 + 25/2*sqrt(-e^2*x^2 + d^2)*d^6/e^6 + 5/3*(-e
^2*x^2 + d^2)^(3/2)*d^3*x/e^5 - 25/6*(-e^2*x^2 + d^2)^(3/2)*d^4/e^6 - 2/3*
(-e^2*x^2 + d^2)^(5/2)*d*x/e^5 + 2*(-e^2*x^2 + d^2)^(5/2)*d^2/e^6 - 1/7*(-
e^2*x^2 + d^2)^(7/2)/e^6
```

3.198.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.60

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{65\,d^7\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{4\,e^5|e|} + \frac{1}{84}\sqrt{-e^2x^2+d^2}\left(\left(2\left(\left(2\left(\left(3\,x-\frac{14\,d}{e}\right)x+\frac{33\,d^2}{e^2}\right)x-\frac{119\,d^3}{e^3}\right)x+\frac{200\,d^4}{e^4}\right)x-\frac{693\,d^5}{e^5}\right)x+\frac{1472\,d^6}{e^6}\right) - \frac{16\,d^7}{e^5\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

input
```
integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

output
```
65/4*d^7*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^5*abs(e)) + 1/84*sqrt(-e^2*x^2 + d
^2)*((2*((2*((3*x - 14*d/e)*x + 33*d^2/e^2)*x - 119*d^3/e^3)*x + 200*d^4/e
^4)*x - 693*d^5/e^5)*x + 1472*d^6/e^6) - 16*d^7/(e^5*((d*e + sqrt(-e^2*x^2
 + d^2)*abs(e))/(e^2*x) + 1)*abs(e))
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^5\,(d^2-e^2\,x^2)^{5/2}}{(d+e\,x)^4}\,dx$$

input
```
int((x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
```

output
```
int((x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

3.199 $\int \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx$

3.199.1 Optimal result

Integrand size = 27, antiderivative size = 224

$$\int \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx=-\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}}-\frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5}+\frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4}-\frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3}+\frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2}-\frac{4dx^4\sqrt{d^2-e^2x^2}}{5e}+\frac{1}{6}x^5\sqrt{d^2-e^2x^2}-\frac{239d^6\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^5}$$

output
```
-239/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5-d^3*(-e*x+d)^4/e^5/(-e^2*
x^2+d^2)^(1/2)-337/15*d^5*(-e^2*x^2+d^2)^(1/2)/e^5+175/16*d^4*x*(-e^2*x^2+
d^2)^(1/2)/e^4-71/15*d^3*x^2*(-e^2*x^2+d^2)^(1/2)/e^3+47/24*d^2*x^3*(-e^2*
x^2+d^2)^(1/2)/e^2-4/5*d*x^4*(-e^2*x^2+d^2)^(1/2)/e+1/6*x^5*(-e^2*x^2+d^2)
^(1/2)
```

3.199.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.59

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(-5632d^6-2047d^5ex+769d^4e^2x^2-426d^3e^3x^3+278d^2e^4x^4-152de^5x^5+40e^6x^6)}{d+ex} + 7170d^6 \arctan}{240e^5}$$

input
```
Integrate[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(-5632*d^6 - 2047*d^5*e*x + 769*d^4*e^2*x^2 - 426*d^
3*e^3*x^3 + 278*d^2*e^4*x^4 - 152*d*e^5*x^5 + 40*e^6*x^6))/(d + e*x) + 717
0*d^6*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(240*e^5)
```

3.199.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.19, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$, Rules used = {563, 25, 2346, 25, 2346, 25, 2346, 27, 2346, 25, 2346, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx$$

$$\Big\downarrow \ 563$$

$$\frac{\int -\frac{8d^6-8exd^5+8e^2x^2d^4-8e^3x^3d^3+7e^4x^4d^2-4e^5x^5d+e^6x^6}{\sqrt{d^2-e^2x^2}}dx}{e^4}-\frac{8d^6\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{8d^6-8exd^5+8e^2x^2d^4-8e^3x^3d^3+7e^4x^4d^2-4e^5x^5d+e^6x^6}{\sqrt{d^2-e^2x^2}}dx}{e^4}-\frac{8d^6\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$$\Big\downarrow \ 2346$$

$$-\frac{-\frac{\int -\frac{-24dx^5e^7+47d^2x^4e^6-48d^3x^3e^5+48d^4x^2e^4-48d^5xe^3+48d^6e^2}{\sqrt{d^2-e^2x^2}}dx}{6e^2}-\frac{1}{6}e^4x^5\sqrt{d^2-e^2x^2}}{e^4}-\frac{8d^6\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{\int \frac{-24dx^5e^7+47d^2x^4e^6-48d^3x^3e^5+48d^4x^2e^4-48d^5xe^3+48d^6e^2}{\sqrt{d^2-e^2x^2}}dx}{6e^2}-\frac{1}{6}e^4x^5\sqrt{d^2-e^2x^2}}{e^4}-\frac{8d^6\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$$\Big\downarrow \text{ 2346}$$

$$-\frac{\frac{\frac{24}{5}de^5x^4\sqrt{d^2-e^2x^2}-\frac{\int -\frac{235d^2x^4e^8-336d^3x^3e^7+240d^4x^2e^6-240d^5xe^5+240d^6e^4}{\sqrt{d^2-e^2x^2}}dx}{5e^2}}{6e^2}-\frac{1}{6}e^4x^5\sqrt{d^2-e^2x^2}}{e^4}-\frac{8d^6\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{\frac{\int \frac{235d^2x^4e^8-336d^3x^3e^7+240d^4x^2e^6-240d^5xe^5+240d^6e^4}{\sqrt{d^2-e^2x^2}}dx}{5e^2}+\frac{24}{5}de^5x^4\sqrt{d^2-e^2x^2}}{6e^2}-\frac{1}{6}e^4x^5\sqrt{d^2-e^2x^2}}{e^4}-\frac{8d^6\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$$\Big\downarrow \text{ 2346}$$

$$-\frac{\frac{\frac{-\frac{\int -\frac{3\left(-448d^3x^3e^9+555d^4x^2e^8-320d^5xe^7+320d^6e^6\right)}{\sqrt{d^2-e^2x^2}}dx}{4e^2}-\frac{235}{4}d^2e^6x^3\sqrt{d^2-e^2x^2}}{5e^2}+\frac{24}{5}de^5x^4\sqrt{d^2-e^2x^2}}{6e^2}-\frac{1}{6}e^4x^5\sqrt{d^2-e^2x^2}}{e^4}-\frac{8d^6\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{\frac{\frac{3\int \frac{-448d^3x^3e^9+555d^4x^2e^8-320d^5xe^7+320d^6e^6}{\sqrt{d^2-e^2x^2}}dx}{4e^2}-\frac{235}{4}d^2e^6x^3\sqrt{d^2-e^2x^2}}{5e^2}+\frac{24}{5}de^5x^4\sqrt{d^2-e^2x^2}}{6e^2}-\frac{1}{6}e^4x^5\sqrt{d^2-e^2x^2}}{e^4}-\frac{8d^6\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$$\Big\downarrow \text{ 2346}$$

$$-\frac{\frac{\frac{\frac{3\left(\frac{448}{3}d^3e^7x^2\sqrt{d^2-e^2x^2}-\frac{\int -\frac{1665d^4x^2e^{10}-1856d^5xe^9+960d^6e^8}{\sqrt{d^2-e^2x^2}}dx}{3e^2}\right)}{4e^2}-\frac{235}{4}d^2e^6x^3\sqrt{d^2-e^2x^2}}{5e^2}+\frac{24}{5}de^5x^4\sqrt{d^2-e^2x^2}}{6e^2}-\frac{1}{6}e^4x^5\sqrt{d^2-e^2x^2}}{e^4}-\frac{8d^6\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$$\Big\downarrow \text{ 25}$$

3.199. $\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

$$-\frac{\frac{\frac{\frac{3\left(\frac{\int \frac{1665d^4x^2e^{10}-1856d^5xe^9+960d^6e^8}{\sqrt{d^2-e^2x^2}}dx}{3e^2}+\frac{448}{3}d^3e^7x^2\sqrt{d^2-e^2x^2}\right)}{4e^2}-\frac{235}{4}d^2e^6x^3\sqrt{d^2-e^2x^2}}{5e^2}+\frac{24}{5}de^5x^4\sqrt{d^2-e^2x^2}}{6e^2}-\frac{1}{6}e^4x^5\sqrt{d^2-e^2x^2}}{e^4}-\frac{8d^6\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$\Big\downarrow$ 2346

$$-\frac{\frac{\frac{\frac{3\left(\frac{-\frac{\int -\frac{d^5e^{10}(3585d-3712ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{1665}{2}d^4e^8x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{448}{3}d^3e^7x^2\sqrt{d^2-e^2x^2}\right)}{4e^2}-\frac{235}{4}d^2e^6x^3\sqrt{d^2-e^2x^2}}{5e^2}+\frac{24}{5}de^5x^4\sqrt{d^2-e^2x^2}}{6e^2}-\frac{1}{6}e^4x^5\sqrt{d^2-e^2x^2}}{e^4}-\frac{8d^6\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\frac{\frac{3\left(\frac{\frac{\int \frac{d^5e^{10}(3585d-3712ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{1665}{2}d^4e^8x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{448}{3}d^3e^7x^2\sqrt{d^2-e^2x^2}\right)}{4e^2}-\frac{235}{4}d^2e^6x^3\sqrt{d^2-e^2x^2}}{5e^2}+\frac{24}{5}de^5x^4\sqrt{d^2-e^2x^2}}{6e^2}-\frac{1}{6}e^4x^5\sqrt{d^2-e^2x^2}}{e^4}-\frac{8d^6\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\frac{\frac{3\left(\frac{\frac{1}{2}d^5e^8\int \frac{3585d-3712ex}{\sqrt{d^2-e^2x^2}}dx-\frac{1665}{2}d^4e^8x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{448}{3}d^3e^7x^2\sqrt{d^2-e^2x^2}\right)}{4e^2}-\frac{235}{4}d^2e^6x^3\sqrt{d^2-e^2x^2}}{5e^2}+\frac{24}{5}de^5x^4\sqrt{d^2-e^2x^2}}{6e^2}-\frac{1}{6}e^4x^5\sqrt{d^2-e^2x^2}}{e^4}-\frac{8d^6\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$\Big\downarrow$ 455

3.199. $\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

$$-\frac{\frac{\frac{3\left(\frac{\frac{1}{2}d^5e^8\left(3585d\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{3712\sqrt{d^2-e^2x^2}}{e}\right)-\frac{1665}{2}d^4e^8x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{448}{3}d^3e^7x^2\sqrt{d^2-e^2x^2}\right)}{4e^2}-\frac{235}{4}d^2e^6x^3\sqrt{d^2-e^2x^2}}{5e^2}+\frac{24}{5}de^5x^4\sqrt{d^2-e^2x^2}}{6e^2}-}{e^4}$$

$$\frac{8d^6\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$$\Big\downarrow \; 224$$

$$-\frac{\frac{\frac{3\left(\frac{\frac{1}{2}d^5e^8\left(3585d\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{3712\sqrt{d^2-e^2x^2}}{e}\right)-\frac{1665}{2}d^4e^8x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{448}{3}d^3e^7x^2\sqrt{d^2-e^2x^2}\right)}{4e^2}-\frac{235}{4}d^2e^6x^3\sqrt{d^2-e^2x^2}}{5e^2}+\frac{24}{5}de^5x^4\sqrt{d^2}}{6e^2}}{e^4}$$

$$\frac{8d^6\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

$$\Big\downarrow \; 216$$

$$-\frac{\frac{\frac{3\left(\frac{\frac{1}{2}d^5e^8\left(\frac{3585d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{3712\sqrt{d^2-e^2x^2}}{e}\right)-\frac{1665}{2}d^4e^8x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{448}{3}d^3e^7x^2\sqrt{d^2-e^2x^2}\right)}{4e^2}-\frac{235}{4}d^2e^6x^3\sqrt{d^2-e^2x^2}}{5e^2}+\frac{24}{5}de^5x^4\sqrt{d^2-e^2x}}{6e^2}}{e^4}$$

$$\frac{8d^6\sqrt{d^2-e^2x^2}}{e^5(d+ex)}$$

input
```
Int[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

output
```
(-8*d^6*Sqrt[d^2 - e^2*x^2])/(e^5*(d + e*x)) - (-1/6*(e^4*x^5*Sqrt[d^2 - e
^2*x^2]) + ((24*d*e^5*x^4*Sqrt[d^2 - e^2*x^2])/5 + ((-235*d^2*e^6*x^3*Sqrt
[d^2 - e^2*x^2])/4 + (3*((448*d^3*e^7*x^2*Sqrt[d^2 - e^2*x^2])/3 + ((-1665
*d^4*e^8*x*Sqrt[d^2 - e^2*x^2])/2 + (d^5*e^8*((3712*Sqrt[d^2 - e^2*x^2])/e
 + (3585*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/2)/(3*e^2)))/(4*e^2))/(5
*e^2))/(6*e^2))/e^4
```

3.199. $\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

3.199.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.199. $\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

3.199.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{(-40e^5x^5+192d\,e^4x^4-470d^2e^3x^3+896d^3e^2x^2-1665d^4ex+3712d^5)\sqrt{-e^2x^2+d^2}}{240e^5}-\frac{239d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16e^4\sqrt{e^2}}-\frac{8d^6\sqrt{-\left(x\right.}}{}$
default	Expression too large to display

input
```
int(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/240*(-40*e^5*x^5+192*d*e^4*x^4-470*d^2*e^3*x^3+896*d^3*e^2*x^2-1665*d^4
*e*x+3712*d^5)/e^5*(-e^2*x^2+d^2)^(1/2)-239/16*d^6/e^4/(e^2)^(1/2)*arctan(
(e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-8*d^6/e^6/(x+d/e)*(-(x+d/e)^2*e^2+2*d*
e*(x+d/e))^(1/2)
```

3.199.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.65

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx =$$
$$-\frac{5632\,d^6ex+5632\,d^7-7170\,(d^6ex+d^7)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(40\,e^6x^6-152\,de^5x^5+278\,d^2e^4x^4-42}{240\,(e^6x+de^5)}$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
-1/240*(5632*d^6*e*x + 5632*d^7 - 7170*(d^6*e*x + d^7)*arctan(-(d - sqrt(-
e^2*x^2 + d^2))/(e*x)) - (40*e^6*x^6 - 152*d*e^5*x^5 + 278*d^2*e^4*x^4 - 4
26*d^3*e^3*x^3 + 769*d^4*e^2*x^2 - 2047*d^5*e*x - 5632*d^6)*sqrt(-e^2*x^2
+ d^2))/(e^6*x + d*e^5)
```

3.199.6 Sympy [F]

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^4(-(-d+ex)\,(d+ex))^{\frac{5}{2}}}{(d+ex)^4}\,dx$$

input
```
integrate(x**4*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

output
```
Integral(x**4*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

3.199.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.04

$$\begin{aligned}\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx &= \frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^4}{2\,(e^8x^3+3\,de^7x^2+3\,d^2e^6x+d^3e^5)} \\ &+ \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^5}{2\,(e^7x^2+2\,de^6x+d^2e^5)} - \frac{15\,\sqrt{-e^2x^2+d^2}d^6}{e^6x+de^5} - \frac{4\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3}{3\,(e^7x^2+2\,de^6x+d^2e^5)} \\ &- \frac{10\,(-e^2x^2+d^2)^{\frac{3}{2}}d^4}{3\,(e^6x+de^5)} + \frac{3\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2}{2\,(e^6x+de^5)} - \frac{9i\,d^6\arcsin\left(\frac{ex}{d}+2\right)}{4\,e^5} \\ &- \frac{275\,d^6\arcsin\left(\frac{ex}{d}\right)}{16\,e^5} + \frac{9\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^4x}{4\,e^4} + \frac{5\,\sqrt{-e^2x^2+d^2}d^4x}{16\,e^4} \\ &+ \frac{9\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^5}{2\,e^5} - \frac{10\,\sqrt{-e^2x^2+d^2}d^5}{e^5} - \frac{19\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2x}{24\,e^4} \\ &+ \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^3}{2\,e^5} + \frac{(-e^2x^2+d^2)^{\frac{5}{2}}x}{6\,e^4} - \frac{4\,(-e^2x^2+d^2)^{\frac{5}{2}}d}{5\,e^5}\end{aligned}$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
1/2*(-e^2*x^2 + d^2)^(5/2)*d^4/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*
e^5) + 5/2*(-e^2*x^2 + d^2)^(3/2)*d^5/(e^7*x^2 + 2*d*e^6*x + d^2*e^5) - 15
*sqrt(-e^2*x^2 + d^2)*d^6/(e^6*x + d*e^5) - 4/3*(-e^2*x^2 + d^2)^(5/2)*d^3
/(e^7*x^2 + 2*d*e^6*x + d^2*e^5) - 10/3*(-e^2*x^2 + d^2)^(3/2)*d^4/(e^6*x
+ d*e^5) + 3/2*(-e^2*x^2 + d^2)^(5/2)*d^2/(e^6*x + d*e^5) - 9/4*I*d^6*arcs
in(e*x/d + 2)/e^5 - 275/16*d^6*arcsin(e*x/d)/e^5 + 9/4*sqrt(e^2*x^2 + 4*d*
e*x + 3*d^2)*d^4*x/e^4 + 5/16*sqrt(-e^2*x^2 + d^2)*d^4*x/e^4 + 9/2*sqrt(e^
2*x^2 + 4*d*e*x + 3*d^2)*d^5/e^5 - 10*sqrt(-e^2*x^2 + d^2)*d^5/e^5 - 19/24
*(-e^2*x^2 + d^2)^(3/2)*d^2*x/e^4 + 5/2*(-e^2*x^2 + d^2)^(3/2)*d^3/e^5 + 1
/6*(-e^2*x^2 + d^2)^(5/2)*x/e^4 - 4/5*(-e^2*x^2 + d^2)^(5/2)*d/e^5
```

3.199.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.62

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = -\frac{239\,d^6\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{16\,e^4|e|} + \frac{1}{240}\sqrt{-e^2x^2+d^2}\left(\left(2\left(\left(4\left(5\,x-\frac{24\,d}{e}\right)x+\frac{235\,d^2}{e^2}\right)x-\frac{448\,d^3}{e^3}\right)x+\frac{1665\,d^4}{e^4}\right)x-\frac{3712\,d^5}{e^5}\right) + \frac{16\,d^6}{e^4\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

output
```
-239/16*d^6*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^4*abs(e)) + 1/240*sqrt(-e^2*x^2
 + d^2)*((2*((4*(5*x - 24*d/e)*x + 235*d^2/e^2)*x - 448*d^3/e^3)*x + 1665*
d^4/e^4)*x - 3712*d^5/e^5) + 16*d^6/(e^4*((d*e + sqrt(-e^2*x^2 + d^2)*abs(
e))/(e^2*x) + 1)*abs(e))
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^4\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{{\left(d+e\,x\right)}^4}\,dx$$

input
```
int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
```

output
```
int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

3.199. $\int \frac{x^4 (d^2-e^2x^2)^{5/2}}{(d+ex)^4} \, dx$

3.200 $\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

3.200.1 Optimal result

Integrand size = 27, antiderivative size = 192

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} + \frac{27d^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}$$

output
```
27/2*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4+d^2*(-e*x+d)^4/e^4/(-e^2*x^2
+d^2)^(1/2)+101/5*d^4*(-e^2*x^2+d^2)^(1/2)/e^4-19/2*d^3*x*(-e^2*x^2+d^2)^(
1/2)/e^3+18/5*d^2*x^2*(-e^2*x^2+d^2)^(1/2)/e^2-d*x^3*(-e^2*x^2+d^2)^(1/2)/
e+1/5*x^4*(-e^2*x^2+d^2)^(1/2)
```

3.200.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.67

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{\frac{e\sqrt{d^2-e^2x^2}\left(212d^5+77d^4ex-29d^3e^2x^2+16d^2e^3x^3-8de^4x^4+2e^5x^5\right)}{d+ex} + 135d^5\sqrt{-e^2}\log\left(-\sqrt{-e^2}x+\right.}{10e^5}$$

input
```
Integrate[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

output
```
((e*Sqrt[d^2 - e^2*x^2]*(212*d^5 + 77*d^4*e*x - 29*d^3*e^2*x^2 + 16*d^2*e^
3*x^3 - 8*d*e^4*x^4 + 2*e^5*x^5))/(d + e*x) + 135*d^5*Sqrt[-e^2]*Log[-(Sqr
t[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(10*e^5)
```

3.200.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {563, 2346, 25, 2346, 27, 2346, 27, 2346, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(d^2-e^2 x^2\right)^{5/2}}{(d+e x)^4} \, dx$$

$$\Big\downarrow \ 563$$

$$\frac{\int \frac{8 d^5-8 e x d^4+8 e^2 x^2 d^3-7 e^3 x^3 d^2+4 e^4 x^4 d-e^5 x^5}{\sqrt{d^2-e^2 x^2}}dx}{e^3}+\frac{8 d^5 \sqrt{d^2-e^2 x^2}}{e^4 (d+e x)}$$

$$\Big\downarrow \ 2346$$

$$\frac{\frac{1}{5} e^3 x^4 \sqrt{d^2-e^2 x^2}-\frac{\int -\frac{20 d x^4 e^6-39 d^2 x^3 e^5+40 d^3 x^2 e^4-40 d^4 x e^3+40 d^5 e^2}{\sqrt{d^2-e^2 x^2}}dx}{5 e^2}}{e^3}+\frac{8 d^5 \sqrt{d^2-e^2 x^2}}{e^4 (d+e x)}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{20 d x^4 e^6-39 d^2 x^3 e^5+40 d^3 x^2 e^4-40 d^4 x e^3+40 d^5 e^2}{\sqrt{d^2-e^2 x^2}}dx}{5 e^2}+\frac{1}{5} e^3 x^4 \sqrt{d^2-e^2 x^2}}{e^3}+\frac{8 d^5 \sqrt{d^2-e^2 x^2}}{e^4 (d+e x)}$$

$$\Big\downarrow \ 2346$$

$$\frac{\frac{-\frac{\int -\frac{4 \left(-39 d^2 x^3 e^7+55 d^3 x^2 e^6-40 d^4 x e^5+40 d^5 e^4\right)}{\sqrt{d^2-e^2 x^2}}dx}{4 e^2}-5 d e^4 x^3 \sqrt{d^2-e^2 x^2}}{5 e^2}+\frac{1}{5} e^3 x^4 \sqrt{d^2-e^2 x^2}}{e^3}+\frac{8 d^5 \sqrt{d^2-e^2 x^2}}{e^4 (d+e x)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\int \frac{-39 d^2 x^3 e^7+55 d^3 x^2 e^6-40 d^4 x e^5+40 d^5 e^4}{\sqrt{d^2-e^2 x^2}}dx}{e^2}-5 d e^4 x^3 \sqrt{d^2-e^2 x^2}}{5 e^2}+\frac{1}{5} e^3 x^4 \sqrt{d^2-e^2 x^2}}{e^3}+\frac{8 d^5 \sqrt{d^2-e^2 x^2}}{e^4 (d+e x)}$$

$$\Big\downarrow \ 2346$$

$$\frac{\frac{\frac{13d^2e^5x^2\sqrt{d^2-e^2x^2}-\frac{\int -\frac{3\left(55d^3x^2e^8-66d^4xe^7+40d^5e^6\right)}{\sqrt{d^2-e^2x^2}}dx}{3e^2}}{e^2}-5de^4x^3\sqrt{d^2-e^2x^2}}{5e^2}+\frac{1}{5}e^3x^4\sqrt{d^2-e^2x^2}}{e^3}+\frac{8d^5\sqrt{d^2-e^2x^2}}{e^4(d+ex)}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{\frac{\int \frac{55d^3x^2e^8-66d^4xe^7+40d^5e^6}{\sqrt{d^2-e^2x^2}}dx}{e^2}+13d^2e^5x^2\sqrt{d^2-e^2x^2}}{e^2}-5de^4x^3\sqrt{d^2-e^2x^2}}{5e^2}+\frac{1}{5}e^3x^4\sqrt{d^2-e^2x^2}}{e^3}+\frac{8d^5\sqrt{d^2-e^2x^2}}{e^4(d+ex)}$$

$\big\downarrow$ 2346

$$\frac{\frac{\frac{\frac{-\frac{\int -\frac{3d^4e^8(45d-44ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{55}{2}d^3e^6x\sqrt{d^2-e^2x^2}}{e^2}+13d^2e^5x^2\sqrt{d^2-e^2x^2}}{e^2}-5de^4x^3\sqrt{d^2-e^2x^2}}{5e^2}+\frac{1}{5}e^3x^4\sqrt{d^2-e^2x^2}}{e^3}+\frac{8d^5\sqrt{d^2-e^2x^2}}{e^4(d+ex)}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{\frac{\frac{3}{2}d^4e^6\int \frac{45d-44ex}{\sqrt{d^2-e^2x^2}}dx-\frac{55}{2}d^3e^6x\sqrt{d^2-e^2x^2}}{e^2}+13d^2e^5x^2\sqrt{d^2-e^2x^2}}{e^2}-5de^4x^3\sqrt{d^2-e^2x^2}}{5e^2}+\frac{1}{5}e^3x^4\sqrt{d^2-e^2x^2}}{e^3}+\frac{8d^5\sqrt{d^2-e^2x^2}}{e^4(d+ex)}$$

$\big\downarrow$ 455

$$\frac{\frac{\frac{\frac{\frac{3}{2}d^4e^6\left(45d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{44\sqrt{d^2-e^2x^2}}{e}\right)-\frac{55}{2}d^3e^6x\sqrt{d^2-e^2x^2}}{e^2}+13d^2e^5x^2\sqrt{d^2-e^2x^2}}{e^2}-5de^4x^3\sqrt{d^2-e^2x^2}}{5e^2}+\frac{1}{5}e^3x^4\sqrt{d^2-e^2x^2}}{e^3}+\frac{8d^5\sqrt{d^2-e^2x^2}}{e^4(d+ex)}$$

$\big\downarrow$ 224

$$\frac{\frac{\frac{\frac{\frac{3}{2}d^4e^6\left(45d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{44\sqrt{d^2-e^2x^2}}{e}\right)-\frac{55}{2}d^3e^6x\sqrt{d^2-e^2x^2}}{e^2}+13d^2e^5x^2\sqrt{d^2-e^2x^2}}{e^2}-5de^4x^3\sqrt{d^2-e^2x^2}}{5e^2}+\frac{1}{5}e^3x^4\sqrt{d^2-e^2x^2}}{e^3}+\frac{8d^5\sqrt{d^2-e^2x^2}}{e^4(d+ex)}$$

$\big\downarrow$ 216

3.200. $\int \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}dx$

$$\frac{\frac{\frac{\frac{\frac{3}{2}d^4e^6\left(\frac{45d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{44\sqrt{d^2-e^2x^2}}{e}\right)-\frac{55}{2}d^3e^6x\sqrt{d^2-e^2x^2}}{e^2}+13d^2e^5x^2\sqrt{d^2-e^2x^2}}{e^2}-5de^4x^3\sqrt{d^2-e^2x^2}}{5e^2}+\frac{1}{5}e^3x^4\sqrt{d^2-e^2x^2}}{e^3}+\frac{8d^5\sqrt{d^2-e^2x^2}}{e^4(d+ex)}$$

input
```
Int[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

output
```
(8*d^5*Sqrt[d^2 - e^2*x^2])/(e^4*(d + e*x)) + ((e^3*x^4*Sqrt[d^2 - e^2*x^2
])/5 + (-5*d*e^4*x^3*Sqrt[d^2 - e^2*x^2] + (13*d^2*e^5*x^2*Sqrt[d^2 - e^2*
x^2] + ((-55*d^3*e^6*x*Sqrt[d^2 - e^2*x^2])/2 + (3*d^4*e^6*((44*Sqrt[d^2 -
 e^2*x^2])/e + (45*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/2)/e^2)/e^2)/(
5*e^2))/e^3
```

3.200.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.200.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.74

method	result
risch	$\frac{\left(2 e^{4} x^{4}-10 d\,e^{3} x^{3}+26 d^{2} e^{2} x^{2}-55 d^{3} e x +132 d^{4}\right) \sqrt{-e^{2} x^{2}+d^{2}}}{10 e^{4}}+\frac{27 d^{5} \arctan \left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{2 e^{3} \sqrt{e^{2}}}+\frac{8 d^{5} \sqrt{-\left(x +\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x +\frac{d}{e}\right)}}{e^{5} \left(x +\frac{d}{e}\right)}$
default	Expression too large to display

input
```
int(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
1/10*(2*e^4*x^4-10*d*e^3*x^3+26*d^2*e^2*x^2-55*d^3*e*x+132*d^4)/e^4*(-e^2*
x^2+d^2)^(1/2)+27/2*d^5/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2
)^(1/2))+8*d^5/e^5/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

3.200.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.70

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{212\,d^5ex+212\,d^6-270\,(d^5ex+d^6)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(2\,e^5x^5-8\,de^4x^4+}{10\,(e^5x+de^4)}$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
1/10*(212*d^5*e*x + 212*d^6 - 270*(d^5*e*x + d^6)*arctan(-(d - sqrt(-e^2*x
^2 + d^2))/(e*x)) + (2*e^5*x^5 - 8*d*e^4*x^4 + 16*d^2*e^3*x^3 - 29*d^3*e^2
*x^2 + 77*d^4*e*x + 212*d^5)*sqrt(-e^2*x^2 + d^2))/(e^5*x + d*e^4)
```

3.200.6 Sympy [F]

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx=\int \frac{x^3(-(-d+ex)\,(d+ex))^{\frac{5}{2}}}{(d+ex)^4}\,dx$$

input
```
integrate(x**3*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

output
```
Integral(x**3*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

3.200.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.12

$$\begin{aligned}\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx=&-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^3}{2\,(e^7x^3+3\,de^6x^2+3\,d^2e^5x+d^3e^4)}\\&-\frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^4}{2\,(e^6x^2+2\,de^5x+d^2e^4)}+\frac{15\,\sqrt{-e^2x^2+d^2}d^5}{e^5x+de^4}+\frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^2}{e^6x^2+2\,de^5x+d^2e^4}\\&+\frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^3}{2\,(e^5x+de^4)}-\frac{3\,(-e^2x^2+d^2)^{\frac{5}{2}}d}{4\,(e^5x+de^4)}+\frac{3i\,d^5\arcsin\left(\frac{ex}{d}+2\right)}{2\,e^4}\\&+\frac{15\,d^5\arcsin\left(\frac{ex}{d}\right)}{e^4}-\frac{3\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^3x}{2\,e^3}-\frac{3\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^4}{e^4}\\&+\frac{15\,\sqrt{-e^2x^2+d^2}d^4}{2\,e^4}+\frac{(-e^2x^2+d^2)^{\frac{3}{2}}dx}{4\,e^3}-\frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2}{4\,e^4}+\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{5\,e^4}\end{aligned}$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
-1/2*(-e^2*x^2 + d^2)^(5/2)*d^3/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3
*e^4) - 5/2*(-e^2*x^2 + d^2)^(3/2)*d^4/(e^6*x^2 + 2*d*e^5*x + d^2*e^4) + 1
5*sqrt(-e^2*x^2 + d^2)*d^5/(e^5*x + d*e^4) + (-e^2*x^2 + d^2)^(5/2)*d^2/(e
^6*x^2 + 2*d*e^5*x + d^2*e^4) + 5/2*(-e^2*x^2 + d^2)^(3/2)*d^3/(e^5*x + d*
e^4) - 3/4*(-e^2*x^2 + d^2)^(5/2)*d/(e^5*x + d*e^4) + 3/2*I*d^5*arcsin(e*x
/d + 2)/e^4 + 15*d^5*arcsin(e*x/d)/e^4 - 3/2*sqrt(e^2*x^2 + 4*d*e*x + 3*d^
2)*d^3*x/e^3 - 3*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^4/e^4 + 15/2*sqrt(-e^2*
x^2 + d^2)*d^4/e^4 + 1/4*(-e^2*x^2 + d^2)^(3/2)*d*x/e^3 - 5/4*(-e^2*x^2 +
d^2)^(3/2)*d^2/e^4 + 1/5*(-e^2*x^2 + d^2)^(5/2)/e^4
```

3.200.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.66

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{27\,d^5\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,e^3|e|} + \frac{1}{10}\sqrt{-e^2x^2+d^2}\left(\left(2\left(\left(x-\frac{5\,d}{e}\right)x+\frac{13\,d^2}{e^2}\right)x-\frac{55\,d^3}{e^3}\right)x+\frac{132\,d^4}{e^4}\right) - \frac{16\,d^5}{e^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

output
```
27/2*d^5*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^3*abs(e)) + 1/10*sqrt(-e^2*x^2 + d
^2)*((2*((x - 5*d/e)*x + 13*d^2/e^2)*x - 55*d^3/e^3)*x + 132*d^4/e^4) - 16
*d^5/(e^3*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e))
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^3\,\left(d^2-e^2\,x^2\right)^{5/2}}{\left(d+e\,x\right)^4}\,dx$$

input
```
int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
```

output
```
int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

3.201 $\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

3.201.1 Optimal result

Integrand size = 27, antiderivative size = 182

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}}-\frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3}-\frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3}-\frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3}-\frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3}-\frac{95d^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

output
```
-95/8*d^4*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-d*(-e*x+d)^4/e^3/(-e^2*x^2+
d^2)^(1/2)-95/8*d^3*(-e^2*x^2+d^2)^(1/2)/e^3-95/24*d^2*(-e*x+d)*(-e^2*x^2+
d^2)^(1/2)/e^3-19/12*d*(-e*x+d)^2*(-e^2*x^2+d^2)^(1/2)/e^3-1/4*(-e*x+d)^3*
(-e^2*x^2+d^2)^(1/2)/e^3
```

3.201.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.60

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(-448d^4-163d^3ex+61d^2e^2x^2-26de^3x^3+6e^4x^4)}{d+ex}+570d^4\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{24e^3}$$

input
```
Integrate[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(-448*d^4 - 163*d^3*e*x + 61*d^2*e^2*x^2 - 26*d*e^3*
x^3 + 6*e^4*x^4))/(d + e*x) + 570*d^4*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 -
 e^2*x^2])])/(24*e^3)
```

3.201.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {563, 25, 2346, 25, 2346, 25, 2346, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(d^2-e^2 x^2\right)^{5/2}}{(d+e x)^4} \, dx$$

$$\Big\downarrow \text{563}$$

$$\frac{\int -\frac{8 d^4-8 e x d^3+7 e^2 x^2 d^2-4 e^3 x^3 d+e^4 x^4}{\sqrt{d^2-e^2 x^2}}dx}{e^2}-\frac{8 d^4 \sqrt{d^2-e^2 x^2}}{e^3 (d+e x)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{8 d^4-8 e x d^3+7 e^2 x^2 d^2-4 e^3 x^3 d+e^4 x^4}{\sqrt{d^2-e^2 x^2}}dx}{e^2}-\frac{8 d^4 \sqrt{d^2-e^2 x^2}}{e^3 (d+e x)}$$

$$\Big\downarrow \text{2346}$$

$$-\frac{-\frac{\int -\frac{-16 d x^3 e^5+31 d^2 x^2 e^4-32 d^3 x e^3+32 d^4 e^2}{\sqrt{d^2-e^2 x^2}}dx}{4 e^2}-\frac{1}{4} e^2 x^3 \sqrt{d^2-e^2 x^2}}{e^2}-\frac{8 d^4 \sqrt{d^2-e^2 x^2}}{e^3 (d+e x)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\int \frac{-16 d x^3 e^5+31 d^2 x^2 e^4-32 d^3 x e^3+32 d^4 e^2}{\sqrt{d^2-e^2 x^2}}dx}{4 e^2}-\frac{1}{4} e^2 x^3 \sqrt{d^2-e^2 x^2}}{e^2}-\frac{8 d^4 \sqrt{d^2-e^2 x^2}}{e^3 (d+e x)}$$

$$\Big\downarrow \text{2346}$$

$$-\frac{\frac{\frac{16}{3} d e^3 x^2 \sqrt{d^2-e^2 x^2}-\frac{\int -\frac{93 d^2 x^2 e^6-128 d^3 x e^5+96 d^4 e^4}{\sqrt{d^2-e^2 x^2}}dx}{3 e^2}}{4 e^2}-\frac{1}{4} e^2 x^3 \sqrt{d^2-e^2 x^2}}{e^2}-\frac{8 d^4 \sqrt{d^2-e^2 x^2}}{e^3 (d+e x)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\frac{\int \frac{93d^2x^2e^6-128d^3xe^5+96d^4e^4}{\sqrt{d^2-e^2x^2}}dx}{3e^2}+\frac{16}{3}de^3x^2\sqrt{d^2-e^2x^2}}{4e^2}-\frac{1}{4}e^2x^3\sqrt{d^2-e^2x^2}}{e^2}-\frac{8d^4\sqrt{d^2-e^2x^2}}{e^3(d+ex)}$$

$\Big\downarrow$ 2346

$$-\frac{\frac{\frac{-\frac{\int -\frac{d^3e^6(285d-256ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{93}{2}d^2e^4x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{16}{3}de^3x^2\sqrt{d^2-e^2x^2}}{4e^2}-\frac{1}{4}e^2x^3\sqrt{d^2-e^2x^2}}{e^2}-\frac{8d^4\sqrt{d^2-e^2x^2}}{e^3(d+ex)}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\frac{\frac{\int \frac{d^3e^6(285d-256ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{93}{2}d^2e^4x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{16}{3}de^3x^2\sqrt{d^2-e^2x^2}}{4e^2}-\frac{1}{4}e^2x^3\sqrt{d^2-e^2x^2}}{e^2}-\frac{8d^4\sqrt{d^2-e^2x^2}}{e^3(d+ex)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\frac{\frac{1}{2}d^3e^4\int \frac{285d-256ex}{\sqrt{d^2-e^2x^2}}dx-\frac{93}{2}d^2e^4x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{16}{3}de^3x^2\sqrt{d^2-e^2x^2}}{4e^2}-\frac{1}{4}e^2x^3\sqrt{d^2-e^2x^2}}{e^2}-\frac{8d^4\sqrt{d^2-e^2x^2}}{e^3(d+ex)}$$

$\Big\downarrow$ 455

$$-\frac{\frac{\frac{\frac{1}{2}d^3e^4\left(285d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{256\sqrt{d^2-e^2x^2}}{e}\right)-\frac{93}{2}d^2e^4x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{16}{3}de^3x^2\sqrt{d^2-e^2x^2}}{4e^2}-\frac{1}{4}e^2x^3\sqrt{d^2-e^2x^2}}{e^2}-\frac{8d^4\sqrt{d^2-e^2x^2}}{e^3(d+ex)}$$

$\Big\downarrow$ 224

$$-\frac{\frac{\frac{\frac{1}{2}d^3e^4\left(285d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{256\sqrt{d^2-e^2x^2}}{e}\right)-\frac{93}{2}d^2e^4x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{16}{3}de^3x^2\sqrt{d^2-e^2x^2}}{4e^2}-\frac{1}{4}e^2x^3\sqrt{d^2-e^2x^2}}{e^2}-\frac{8d^4\sqrt{d^2-e^2x^2}}{e^3(d+ex)}$$

$\Big\downarrow$ 216

$$-\frac{\frac{\frac{\frac{1}{2}d^3e^4\left(\frac{285d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{256\sqrt{d^2-e^2x^2}}{e}\right)-\frac{93}{2}d^2e^4x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{16}{3}de^3x^2\sqrt{d^2-e^2x^2}}{4e^2}-\frac{1}{4}e^2x^3\sqrt{d^2-e^2x^2}}{e^2}-\frac{8d^4\sqrt{d^2-e^2x^2}}{e^3(d+ex)}$$

3.201. $\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}dx$

input
```
Int[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

output
```
(-8*d^4*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - (-1/4*(e^2*x^3*Sqrt[d^2 - e
^2*x^2]) + ((16*d*e^3*x^2*Sqrt[d^2 - e^2*x^2])/3 + ((-93*d^2*e^4*x*Sqrt[d^
2 - e^2*x^2])/2 + (d^3*e^4*((256*Sqrt[d^2 - e^2*x^2])/e + (285*d*ArcTan[(e
*x)/Sqrt[d^2 - e^2*x^2]])/e))/2)/(3*e^2))/(4*e^2))/e^2
```

3.201.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

3.201. $\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.201.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{\left(-6e^{3}x^{3}+32d\,e^{2}x^{2}-93d^{2}ex+256d^{3}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{24e^{3}}-\frac{95d^{4}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{8e^{2}\sqrt{e^{2}}}-\frac{8d^{4}\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{e^{4}\left(x+\frac{d}{e}\right)}$	131
default	Expression too large to display	890

input
```
int(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-6*e^3*x^3+32*d*e^2*x^2-93*d^2*e*x+256*d^3)/e^3*(-e^2*x^2+d^2)^(1/2
)-95/8*d^4/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-8*d^
4/e^4/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

3.201.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.68

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx =$$
$$-\frac{448\,d^4ex+448\,d^5-570\,(d^4ex+d^5)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(6\,e^4x^4-26\,de^3x^3+61\,d^2e^2x^2-163\,d^3ex}{24\,(e^4x+de^3)}$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
-1/24*(448*d^4*e*x + 448*d^5 - 570*(d^4*e*x + d^5)*arctan(-(d - sqrt(-e^2*
x^2 + d^2))/(e*x)) - (6*e^4*x^4 - 26*d*e^3*x^3 + 61*d^2*e^2*x^2 - 163*d^3*
e*x - 448*d^4)*sqrt(-e^2*x^2 + d^2))/(e^4*x + d*e^3)
```

3.201.6 Sympy [F]

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^2(-(-d+ex)\,(d+ex))^{\frac{5}{2}}}{(d+ex)^4}\,dx$$

input
```
integrate(x**2*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

output
```
Integral(x**2*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

3.201.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.99

$$\begin{aligned}\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx &= \frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^2}{2\,(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3)} \\ &+ \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^3}{2\,(e^5x^2+2\,de^4x+d^2e^3)} - \frac{15\,\sqrt{-e^2x^2+d^2}d^4}{e^4x+de^3} \\ &- \frac{2\,(-e^2x^2+d^2)^{\frac{5}{2}}d}{3\,(e^5x^2+2\,de^4x+d^2e^3)} - \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2}{3\,(e^4x+de^3)} - \frac{5i\,d^4\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^3} \\ &- \frac{25\,d^4\arcsin\left(\frac{ex}{d}\right)}{2\,e^3} + \frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{4\,(e^4x+de^3)} + \frac{5\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^2x}{8\,e^2} \\ &+ \frac{5\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^3}{4\,e^3} - \frac{5\,\sqrt{-e^2x^2+d^2}d^3}{e^3} + \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d}{12\,e^3}\end{aligned}$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
1/2*(-e^2*x^2 + d^2)^(5/2)*d^2/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*
e^3) + 5/2*(-e^2*x^2 + d^2)^(3/2)*d^3/(e^5*x^2 + 2*d*e^4*x + d^2*e^3) - 15
*sqrt(-e^2*x^2 + d^2)*d^4/(e^4*x + d*e^3) - 2/3*(-e^2*x^2 + d^2)^(5/2)*d/(
e^5*x^2 + 2*d*e^4*x + d^2*e^3) - 5/3*(-e^2*x^2 + d^2)^(3/2)*d^2/(e^4*x + d
*e^3) - 5/8*I*d^4*arcsin(e*x/d + 2)/e^3 - 25/2*d^4*arcsin(e*x/d)/e^3 + 1/4
*(-e^2*x^2 + d^2)^(5/2)/(e^4*x + d*e^3) + 5/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d
^2)*d^2*x/e^2 + 5/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3/e^3 - 5*sqrt(-e^2*
x^2 + d^2)*d^3/e^3 + 5/12*(-e^2*x^2 + d^2)^(3/2)*d/e^3
```

3.201.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.64

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = -\frac{95\,d^4\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,e^2|e|} + \frac{1}{24}\sqrt{-e^2x^2+d^2}\left(\left(2\left(3\,x-\frac{16\,d}{e}\right)x+\frac{93\,d^2}{e^2}\right)x-\frac{256\,d^3}{e^3}\right) + \frac{16\,d^4}{e^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

output
```
-95/8*d^4*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) + 1/24*sqrt(-e^2*x^2 +
d^2)*((2*(3*x - 16*d/e)*x + 93*d^2/e^2)*x - 256*d^3/e^3) + 16*d^4/(e^2*((d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e))
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^2\,(d^2-e^2\,x^2)^{5/2}}{(d+e\,x)^4}\,dx$$

input
```
int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
```

output
```
int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

3.202 $\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

3.202.1 Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{10dx\sqrt{d^2-e^2x^2}}{e}+\frac{20(d^2-e^2x^2)^{3/2}}{3e^2}+\frac{8(d^2-e^2x^2)^{5/2}}{e^2(d+ex)^2}+\frac{(d^2-e^2x^2)^{7/2}}{e^2(d+ex)^4}+\frac{10d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

output
```
20/3*(-e^2*x^2+d^2)^(3/2)/e^2+8*(-e^2*x^2+d^2)^(5/2)/e^2/(e*x+d)^2+(-e^2*x
^2+d^2)^(7/2)/e^2/(e*x+d)^4+10*d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2+10
*d*x*(-e^2*x^2+d^2)^(1/2)/e
```

3.202.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.82

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{\sqrt{d^2-e^2x^2}(47d^3+17d^2ex-5de^2x^2+e^3x^3)}{3e^2(d+ex)}+\frac{10d^3\sqrt{-e^2}\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{e^3}$$

input
```
Integrate[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(47*d^3 + 17*d^2*e*x - 5*d*e^2*x^2 + e^3*x^3))/(3*e^2
*(d + e*x)) + (10*d^3*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]
])/e^3
```

3.202.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {563, 2346, 25, 2346, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}dx$$

$$\Big\downarrow \text{563}$$

$$\frac{\int \frac{8d^3-7exd^2+4e^2x^2d-e^3x^3}{\sqrt{d^2-e^2x^2}}dx}{e}+\frac{8d^3\sqrt{d^2-e^2x^2}}{e^2(d+ex)}$$

$$\Big\downarrow \text{2346}$$

$$\frac{\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}-\frac{\int -\frac{12dx^2e^4-23d^2xe^3+24d^3e^2}{\sqrt{d^2-e^2x^2}}dx}{3e^2}}{e}+\frac{8d^3\sqrt{d^2-e^2x^2}}{e^2(d+ex)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{12dx^2e^4-23d^2xe^3+24d^3e^2}{\sqrt{d^2-e^2x^2}}dx}{3e^2}+\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}}{e}+\frac{8d^3\sqrt{d^2-e^2x^2}}{e^2(d+ex)}$$

$$\Big\downarrow \text{2346}$$

$$\frac{\frac{-\frac{\int -\frac{2d^2e^4(30d-23ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-6de^2x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}}{e}+\frac{8d^3\sqrt{d^2-e^2x^2}}{e^2(d+ex)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{d^2e^2\int \frac{30d-23ex}{\sqrt{d^2-e^2x^2}}dx-6de^2x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}}{e}+\frac{8d^3\sqrt{d^2-e^2x^2}}{e^2(d+ex)}$$

$$\Big\downarrow \text{455}$$

$$\frac{\frac{d^2e^2\left(30d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{23\sqrt{d^2-e^2x^2}}{e}\right)-6de^2x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}}{e}+\frac{8d^3\sqrt{d^2-e^2x^2}}{e^2(d+ex)}$$

$$\Big\downarrow \textbf{224}$$

$$\frac{\frac{d^2e^2\left(30d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{23\sqrt{d^2-e^2x^2}}{e}\right)-6de^2x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}}{e}+\frac{8d^3\sqrt{d^2-e^2x^2}}{e^2(d+ex)}$$

$$\Big\downarrow \textbf{216}$$

$$\frac{\frac{d^2e^2\left(\frac{30d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{23\sqrt{d^2-e^2x^2}}{e}\right)-6de^2x\sqrt{d^2-e^2x^2}}{3e^2}+\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}}{e}+\frac{8d^3\sqrt{d^2-e^2x^2}}{e^2(d+ex)}$$

input
```
Int[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

output
```
(8*d^3*Sqrt[d^2 - e^2*x^2])/(e^2*(d + e*x)) + ((e*x^2*Sqrt[d^2 - e^2*x^2])
/3 + (-6*d*e^2*x*Sqrt[d^2 - e^2*x^2] + d^2*e^2*((23*Sqrt[d^2 - e^2*x^2])/e
 + (30*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/(3*e^2))/e
```

3.202.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.202. $\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.202.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

method	result
risch	$\frac{(e^2x^2-6dex+23d^2)\sqrt{-e^2x^2+d^2}}{3e^2}+\frac{10d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e\sqrt{e^2}}+\frac{8d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^3\left(x+\frac{d}{e}\right)}$

3.202. $\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx$

input
```
int(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*(e^2*x^2-6*d*e*x+23*d^2)/e^2*(-e^2*x^2+d^2)^(1/2)+10*d^3/e/(e^2)^(1/2)
*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+8*d^3/e^3/(x+d/e)*(-(x+d/e)^2*
e^2+2*d*e*(x+d/e))^(1/2)
```

3.202.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{47\,d^3ex+47\,d^4-60\,(d^3ex+d^4)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(e^3x^3-5\,de^2x^2+17\,d^2e}{3\,(e^3x+de^2)}$$

input
```
integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
1/3*(47*d^3*e*x + 47*d^4 - 60*(d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^2 +
 d^2))/(e*x)) + (e^3*x^3 - 5*d*e^2*x^2 + 17*d^2*e*x + 47*d^3)*sqrt(-e^2*x^
2 + d^2))/(e^3*x + d*e^2)
```

3.202.6 Sympy [F]

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x(-(-d+ex)\,(d+ex))^{\frac{5}{2}}}{(d+ex)^4}\,dx$$

input
```
integrate(x*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

output
```
Integral(x*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

3.202.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.81

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = -\frac{(-e^2x^2+d^2)^{\frac{5}{2}}d}{2\,(e^5x^3+3\,de^4x^2+3\,d^2e^3x+d^3e^2)} - \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2}{2\,(e^4x^2+2\,de^3x+d^2e^2)} + \frac{15\,\sqrt{-e^2x^2+d^2}d^3}{e^3x+de^2} + \frac{10\,d^3\arcsin\left(\frac{ex}{d}\right)}{e^2} + \frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{3\,(e^4x^2+2\,de^3x+d^2e^2)} + \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d}{6\,(e^3x+de^2)} + \frac{5\,\sqrt{-e^2x^2+d^2}d^2}{2\,e^2}$$

input
```
integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
-1/2*(-e^2*x^2 + d^2)^(5/2)*d/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*e
^2) - 5/2*(-e^2*x^2 + d^2)^(3/2)*d^2/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) + 15*
sqrt(-e^2*x^2 + d^2)*d^3/(e^3*x + d*e^2) + 10*d^3*arcsin(e*x/d)/e^2 + 1/3*
(-e^2*x^2 + d^2)^(5/2)/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) + 5/6*(-e^2*x^2 + d
^2)^(3/2)*d/(e^3*x + d*e^2) + 5/2*sqrt(-e^2*x^2 + d^2)*d^2/e^2
```

3.202.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.79

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{10\,d^3\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{e|e|} + \frac{1}{3}\sqrt{-e^2x^2+d^2}\left(\left(x-\frac{6\,d}{e}\right)x+\frac{23\,d^2}{e^2}\right) - \frac{16\,d^3}{e\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

input
```
integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

output
```
10*d^3*arcsin(e*x/d)*sgn(d)*sgn(e)/(e*abs(e)) + 1/3*sqrt(-e^2*x^2 + d^2)*(
(x - 6*d/e)*x + 23*d^2/e^2) - 16*d^3/(e*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e
))/(e^2*x) + 1)*abs(e))
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x\,(d^2-e^2\,x^2)^{5/2}}{(d+e\,x)^4}\,dx$$

input
```
int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
```

output
```
int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

3.203 $\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

3.203.1 Optimal result

Integrand size = 24, antiderivative size = 113

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = -\frac{15d\sqrt{d^2-e^2x^2}}{2e} - \frac{5(d^2-e^2x^2)^{3/2}}{2e(d+ex)} - \frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3} - \frac{15d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

output
```
-5/2*(-e^2*x^2+d^2)^(3/2)/e/(e*x+d)-2*(-e^2*x^2+d^2)^(5/2)/e/(e*x+d)^3-15/
2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e-15/2*d*(-e^2*x^2+d^2)^(1/2)/e
```

3.203.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.79

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{\sqrt{d^2-e^2x^2}(-24d^2-7dex+e^2x^2)}{2e(d+ex)} + \frac{15d^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e}$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^4,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-24*d^2 - 7*d*e*x + e^2*x^2))/(2*e*(d + e*x)) + (15*
d^2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e
```

3.203.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {463, 25, 2346, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^4}dx$$

$$\Big\downarrow \text{ 463}$$

$$\int -\frac{7d^2-4exd+e^2x^2}{\sqrt{d^2-e^2x^2}}dx-\frac{8d^2\sqrt{d^2-e^2x^2}}{e(d+ex)}$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{7d^2-4exd+e^2x^2}{\sqrt{d^2-e^2x^2}}dx-\frac{8d^2\sqrt{d^2-e^2x^2}}{e(d+ex)}$$

$$\Big\downarrow \text{ 2346}$$

$$\frac{\int -\frac{de^2(15d-8ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{8d^2\sqrt{d^2-e^2x^2}}{e(d+ex)}+\frac{1}{2}x\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{de^2(15d-8ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{8d^2\sqrt{d^2-e^2x^2}}{e(d+ex)}+\frac{1}{2}x\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{2}d\int \frac{15d-8ex}{\sqrt{d^2-e^2x^2}}dx-\frac{8d^2\sqrt{d^2-e^2x^2}}{e(d+ex)}+\frac{1}{2}x\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{ 455}$$

$$-\frac{1}{2}d\left(15d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{8\sqrt{d^2-e^2x^2}}{e}\right)-\frac{8d^2\sqrt{d^2-e^2x^2}}{e(d+ex)}+\frac{1}{2}x\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{ 224}$$

$$-\frac{1}{2}d\left(15d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{8\sqrt{d^2-e^2x^2}}{e}\right)-\frac{8d^2\sqrt{d^2-e^2x^2}}{e(d+ex)}+\frac{1}{2}x\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{1}{2}d\left(\frac{15d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{8\sqrt{d^2-e^2x^2}}{e}\right)-\frac{8d^2\sqrt{d^2-e^2x^2}}{e(d+ex)}+\frac{1}{2}x\sqrt{d^2-e^2x^2}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^4,x]
```

output
```
(x*Sqrt[d^2 - e^2*x^2])/2 - (8*d^2*Sqrt[d^2 - e^2*x^2])/(e*(d + e*x)) - (d
*((8*Sqrt[d^2 - e^2*x^2])/e + (15*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))
/2
```

3.203.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 463
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(-c)^(-n - 2))*d^(2*n + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b^(n + 2)*(c + d*x
))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(1/Sqrt[a + b*x^2])*ExpandToSum[
(2^(-n - 1)*(-c)^(-n - 1) - (-c + d*x)^(-n - 1))/(c + d*x), x], x], x] /; F
reeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[n, 0] && EqQ[n + p,
-3/2]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.203.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{(-ex+8d)\sqrt{-e^2x^2+d^2}}{2e}-\frac{15d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}-\frac{8d^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^2\left(x+\frac{d}{e}\right)}$
default	$-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{de\left(x+\frac{d}{e}\right)^4}-\frac{3e\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{de\left(x+\frac{d}{e}\right)^3}+\frac{4e\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{3de\left(x+\frac{d}{e}\right)^2}+\frac{5e\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}\right.}{}\right.}{}\right.}{}$

input
```
int((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

3.203. $\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

output
```
-1/2*(-e*x+8*d)/e*(-e^2*x^2+d^2)^(1/2)-15/2*d^2/(e^2)^(1/2)*arctan((e^2)^(
1/2)*x/(-e^2*x^2+d^2)^(1/2))-8*d^2/e^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/
e))^(1/2)
```

3.203.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{(d + ex)^4} dx =$$
$$-\frac{24 d^2ex + 24 d^3 - 30 (d^2ex + d^3) \arctan \left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - (e^2x^2 - 7 dex - 24 d^2)\sqrt{-e^2x^2 + d^2}}{2 (e^2x + de)}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
-1/2*(24*d^2*e*x + 24*d^3 - 30*(d^2*e*x + d^3)*arctan(-(d - sqrt(-e^2*x^2
+ d^2))/(e*x)) - (e^2*x^2 - 7*d*e*x - 24*d^2)*sqrt(-e^2*x^2 + d^2))/(e^2*x
 + d*e)
```

3.203.6 Sympy [F]

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{(d + ex)^4} dx = \int \frac{(-(-d + ex)(d + ex))^{\frac{5}{2}}}{(d + ex)^4} dx$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

3.203.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.19

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{(d+ex)^4} \, dx = -\frac{15\, d^2 \arcsin\left(\frac{ex}{d}\right)}{2\, e} + \frac{(-e^2x^2 + d^2)^{\frac{5}{2}}}{2\, (e^4x^3 + 3\, de^3x^2 + 3\, d^2e^2x + d^3e)} + \frac{5\, (-e^2x^2 + d^2)^{\frac{3}{2}} d}{2\, (e^3x^2 + 2\, de^2x + d^2e)} - \frac{15\, \sqrt{-e^2x^2 + d^2} d^2}{e^2x + de}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
-15/2*d^2*arcsin(e*x/d)/e + 1/2*(-e^2*x^2 + d^2)^(5/2)/(e^4*x^3 + 3*d*e^3*
x^2 + 3*d^2*e^2*x + d^3*e) + 5/2*(-e^2*x^2 + d^2)^(3/2)*d/(e^3*x^2 + 2*d*e
^2*x + d^2*e) - 15*sqrt(-e^2*x^2 + d^2)*d^2/(e^2*x + d*e)
```

3.203.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.76

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{(d+ex)^4} \, dx = -\frac{15\, d^2 \arcsin\left(\frac{ex}{d}\right) \operatorname{sgn}(d) \operatorname{sgn}(e)}{2\, |e|} + \frac{1}{2}\sqrt{-e^2x^2 + d^2}\left(x - \frac{8\, d}{e}\right) + \frac{16\, d^2}{\left(\frac{de + \sqrt{-e^2x^2 + d^2}|e|}{e^2x} + 1\right)|e|}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

output
```
-15/2*d^2*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/2*sqrt(-e^2*x^2 + d^2)*(x
 - 8*d/e) + 16*d^2/(((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(
e))
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{(d + ex)^4}\, dx = \int \frac{(d^2 - e^2\,x^2)^{5/2}}{(d + e\,x)^4}\, dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(d + e*x)^4,x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(d + e*x)^4, x)
```

3.204 $\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^4}\,dx$

3.204.1 Optimal result

Integrand size = 27, antiderivative size = 89

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^4}\,dx = \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}}+\sqrt{d^2-e^2x^2}+4d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
4*d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-d*arctanh((-e^2*x^2+d^2)^(1/2)/d)+8*d
*(-e*x+d)/(-e^2*x^2+d^2)^(1/2)+(-e^2*x^2+d^2)^(1/2)
```

3.204.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.29

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^4}\,dx = \frac{(9d+ex)\sqrt{d^2-e^2x^2}}{d+ex}+2d\,\text{arctanh}\left(\frac{\sqrt{-e^2}x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)-\frac{4de\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}}$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4),x]
```

output
```
((9*d + e*x)*Sqrt[d^2 - e^2*x^2])/(d + e*x) + 2*d*ArcTanh[(Sqrt[-e^2]*x)/d
 - Sqrt[d^2 - e^2*x^2]/d] - (4*d*e*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^
2]])/Sqrt[-e^2]
```

3.204.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {564, 2340, 25, 27, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x(d+ex)^4}dx$$

$$\Big\downarrow \text{564}$$

$$\int \frac{d^2+4exd-e^2x^2}{x\sqrt{d^2-e^2x^2}}dx+\frac{8d\sqrt{d^2-e^2x^2}}{d+ex}$$

$$\Big\downarrow \text{2340}$$

$$-\frac{\int -\frac{de^2(d+4ex)}{x\sqrt{d^2-e^2x^2}}dx}{e^2}+\frac{8d\sqrt{d^2-e^2x^2}}{d+ex}+\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{de^2(d+4ex)}{x\sqrt{d^2-e^2x^2}}dx}{e^2}+\frac{8d\sqrt{d^2-e^2x^2}}{d+ex}+\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{27}$$

$$d\int \frac{d+4ex}{x\sqrt{d^2-e^2x^2}}dx+\frac{8d\sqrt{d^2-e^2x^2}}{d+ex}+\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{538}$$

$$d\left(4e\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx\right)+\frac{8d\sqrt{d^2-e^2x^2}}{d+ex}+\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{224}$$

$$d\left(d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx+4e\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)+\frac{8d\sqrt{d^2-e^2x^2}}{d+ex}+\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{216}$$

$$d\left(d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx+4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{8d\sqrt{d^2-e^2x^2}}{d+ex}+\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{243}$$

$$d\left(\frac{1}{2}d\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2+4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{8d\sqrt{d^2-e^2x^2}}{d+ex}+\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{73}$$

$$d\left(4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{d\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}\right)+\frac{8d\sqrt{d^2-e^2x^2}}{d+ex}+\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{221}$$

$$d\left(4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)+\frac{8d\sqrt{d^2-e^2x^2}}{d+ex}+\sqrt{d^2-e^2x^2}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4),x]
```

output
```
Sqrt[d^2 - e^2*x^2] + (8*d*Sqrt[d^2 - e^2*x^2])/(d + e*x) + d*(4*ArcTan[(e
*x)/Sqrt[d^2 - e^2*x^2]] - ArcTanh[Sqrt[d^2 - e^2*x^2]/d])
```

3.204.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.204. $\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^4}\,dx$

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

3.204. $\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^4}\,dx$

3.204.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1191 vs. $2(81) = 162$.

Time = 0.48 (sec) , antiderivative size = 1192, normalized size of antiderivative = 13.39

method	result	size
default	Expression too large to display	1192

input
```
int((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d^4*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x
^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2)
)/x))))-1/e^3/d*(-1/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)-3*e
/d*(1/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+4*e/d*(1/3/d/e/(x
+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+
2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*(x+d/e)*e^2+2*d*e)/e^2*(-(x+d/e)^2*e^2+
2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*(x+d/e)*e^2+2*d*e)/e^2*(-(x+d/e)^2*
e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e
)^2*e^2+2*d*e*(x+d/e))^(1/2)))))))-1/e/d^3*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2
*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2
)+d*e*(-1/8*(-2*(x+d/e)*e^2+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2
)+3/4*d^2*(-1/4*(-2*(x+d/e)*e^2+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^
(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/
e))^(1/2))))))-1/d^4*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(
-2*(x+d/e)*e^2+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1
/4*(-2*(x+d/e)*e^2+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2
/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))
-1/e^2/d^2*(1/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+4*e/d*(1/
3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e
)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*(x+d/e)*e^2+2*d*e)/e^2*(-(x+...
```

3.204.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.25

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^4}\,dx = \frac{9\,dex + 9\,d^2 - 8\,(dex + d^2)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + (dex + d^2)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)}{ex + d}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x, algorithm="fricas")
```

output
```
(9*d*e*x + 9*d^2 - 8*(d*e*x + d^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x
)) + (d*e*x + d^2)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + sqrt(-e^2*x^2 + d^
2)*(e*x + 9*d))/(e*x + d)
```

3.204.6 Sympy [F]

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^4}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^{\frac{5}{2}}}{x\,(d+ex)^4}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/x/(e*x+d)**4,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x*(d + e*x)**4), x)
```

3.204.7 Maxima [F]

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{(ex+d)^4x}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x), x)
```

3.204.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.30

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^4}\,dx = \frac{4\,de\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} - \frac{de\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{|e|} + \sqrt{-e^2x^2+d^2} - \frac{16\,de}{\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x, algorithm="giac")
```

output
```
4*d*e*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - d*e*log(1/2*abs(-2*d*e - 2*sqrt
(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + sqrt(-e^2*x^2 + d^2) - 16*
d*e/(((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e))
```

3.204.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{x(d + ex)^4} \, dx = \int \frac{(d^2 - e^2\,x^2)^{5/2}}{x\,(d + e\,x)^4} \, dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4), x)
```

3.205 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^4}\,dx$

3.205.1 Optimal result

Integrand size = 27, antiderivative size = 94

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^4}\,dx = -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{x} - e\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + 4e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

output
```
-e*arctan(e*x/(-e^2*x^2+d^2)^(1/2))+4*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)-8*
e*(-e*x+d)/(-e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/x
```

3.205.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.34

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^4}\,dx = \frac{(-d-9ex)\sqrt{d^2-e^2x^2}}{x(d+ex)} + 2e\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) + \frac{4\sqrt{d^2}e\log(x)}{d} - \frac{4\sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{d}$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4),x]
```

output
```
((-d - 9*e*x)*Sqrt[d^2 - e^2*x^2])/(x*(d + e*x)) + 2*e*ArcTan[(e*x)/(Sqrt[
d^2] - Sqrt[d^2 - e^2*x^2])] + (4*Sqrt[d^2]*e*Log[x])/d - (4*Sqrt[d^2]*e*L
og[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/d
```

3.205.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {564, 2338, 27, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^4}dx$$

$$\Big\downarrow \text{564}$$

$$\int \frac{d^2-4exd-e^2x^2}{x^2\sqrt{d^2-e^2x^2}}dx-\frac{8e\sqrt{d^2-e^2x^2}}{d+ex}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int \frac{d^2e(4d+ex)}{x\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{8e\sqrt{d^2-e^2x^2}}{d+ex}-\frac{\sqrt{d^2-e^2x^2}}{x}$$

$$\Big\downarrow \text{27}$$

$$e\left(-\int \frac{4d+ex}{x\sqrt{d^2-e^2x^2}}dx\right)-\frac{8e\sqrt{d^2-e^2x^2}}{d+ex}-\frac{\sqrt{d^2-e^2x^2}}{x}$$

$$\Big\downarrow \text{538}$$

$$-\left(e\left(e\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+4d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx\right)\right)-\frac{8e\sqrt{d^2-e^2x^2}}{d+ex}-\frac{\sqrt{d^2-e^2x^2}}{x}$$

$$\Big\downarrow \text{224}$$

$$-\left(e\left(4d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx+e\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{8e\sqrt{d^2-e^2x^2}}{d+ex}-\frac{\sqrt{d^2-e^2x^2}}{x}$$

$$\Big\downarrow \text{216}$$

$$-\left(e\left(4d\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx+\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)\right)-\frac{8e\sqrt{d^2-e^2x^2}}{d+ex}-\frac{\sqrt{d^2-e^2x^2}}{x}$$

$$\Big\downarrow \text{243}$$

$$-\left(e\left(2d\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2+\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)\right)-\frac{8e\sqrt{d^2-e^2x^2}}{d+ex}-\frac{\sqrt{d^2-e^2x^2}}{x}$$

$$\Big\downarrow \text{73}$$

$$-\left(e\left(\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{4d\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e^2}\right)\right)-\frac{8e\sqrt{d^2-e^2x^2}}{d+ex}-\frac{\sqrt{d^2-e^2x^2}}{x}$$

$$\Big\downarrow \text{221}$$

$$-\left(e\left(\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-4\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)\right)-\frac{8e\sqrt{d^2-e^2x^2}}{d+ex}-\frac{\sqrt{d^2-e^2x^2}}{x}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4),x]
```

output
```
-(Sqrt[d^2 - e^2*x^2]/x) - (8*e*Sqrt[d^2 - e^2*x^2])/(d + e*x) - e*(ArcTan
[(e*x)/Sqrt[d^2 - e^2*x^2]] - 4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])
```

3.205.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.205. $\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^4}\,dx$

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.205.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.41

method	result	size
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{x}-\frac{e^{2} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}}+\frac{4 d e \ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}}-\frac{8 \sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{x+\frac{d}{e}}$	133
default	Expression too large to display	1322

input
```
int((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

3.205. $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^2(d+ex)^4}\,dx$

output
```
-(-e^2*x^2+d^2)^(1/2)/x-e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2
)^(1/2))+4*d*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x
)-8/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

3.205.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.35

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^4}\,dx =$$
$$-\frac{8\,e^2x^2+8\,dex-2\,(e^2x^2+dex)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+4\,(e^2x^2+dex)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\sqrt{-e^2x^2}}{ex^2+dx}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x, algorithm="fricas")
```

output
```
-(8*e^2*x^2 + 8*d*e*x - 2*(e^2*x^2 + d*e*x)*arctan(-(d - sqrt(-e^2*x^2 + d
^2))/(e*x)) + 4*(e^2*x^2 + d*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + sqr
t(-e^2*x^2 + d^2)*(9*e*x + d))/(e*x^2 + d*x)
```

3.205.6 Sympy [F]

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^4}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^{\frac{5}{2}}}{x^2\,(d+ex)^4}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/x**2/(e*x+d)**4,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**2*(d + e*x)**4), x)
```

3.205.7 Maxima [F]

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{(ex+d)^4x^2}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^2), x)
```

3.205.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(87) = 174.

Time = 0.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.01

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^4}\,dx = -\frac{e^2\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} + \frac{4\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{|e|} + \frac{\left(e^2+\frac{33\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{x}\right)e^2x}{2\left(de+\sqrt{-e^2x^2+d^2}|e|\right)\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|} - \frac{de+\sqrt{-e^2x^2+d^2}|e|}{2\,x|e|}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x, algorithm="giac")
```

output
```
-e^2*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 4*e^2*log(1/2*abs(-2*d*e - 2*sqr
t(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/2*(e^2 + 33*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))/x)*e^2*x/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*((d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e)) - 1/2*(d*e + sqrt(-
e^2*x^2 + d^2)*abs(e))/(x*abs(e))
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{x^2(d+ex)^4}\,dx = \int \frac{(d^2 - e^2\,x^2)^{5/2}}{x^2\,(d+e\,x)^4}\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4), x)
```

3.206 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^4}\,dx$

3.206.1 Optimal result

Integrand size = 27, antiderivative size = 110

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^4}\,dx=\frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}+\frac{4e\sqrt{d^2-e^2x^2}}{dx}-\frac{15e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

output
```
-15/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d+8*e^2*(-e*x+d)/d/(-e^2*x^2+d^2
)^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/x^2+4*e*(-e^2*x^2+d^2)^(1/2)/d/x
```

3.206.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.97

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^4}\,dx=\frac{1}{2}\left(\frac{\sqrt{d^2-e^2x^2}(-d^2+7dex+24e^2x^2)}{dx^2(d+ex)}-\frac{15e^2\log(x)}{\sqrt{d^2}}+\frac{15e^2\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{\sqrt{d^2}}\right)$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(-d^2 + 7*d*e*x + 24*e^2*x^2))/(d*x^2*(d + e*x)) - (
15*e^2*Log[x])/Sqrt[d^2] + (15*e^2*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/S
qrt[d^2])/2
```

3.206.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {564, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^4}dx$$

$$\Big\downarrow \text{564}$$

$$\int \frac{d^2-4exd+7e^2x^2}{x^3\sqrt{d^2-e^2x^2}}dx+\frac{8e^2\sqrt{d^2-e^2x^2}}{d(d+ex)}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int \frac{d^2e(8d-15ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}+\frac{8e^2\sqrt{d^2-e^2x^2}}{d(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}e\int \frac{8d-15ex}{x^2\sqrt{d^2-e^2x^2}}dx+\frac{8e^2\sqrt{d^2-e^2x^2}}{d(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}$$

$$\Big\downarrow \text{534}$$

$$-\frac{1}{2}e\left(-15e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{8\sqrt{d^2-e^2x^2}}{dx}\right)+\frac{8e^2\sqrt{d^2-e^2x^2}}{d(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}e\left(-\frac{15}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{8\sqrt{d^2-e^2x^2}}{dx}\right)+\frac{8e^2\sqrt{d^2-e^2x^2}}{d(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$-\frac{1}{2}e\left(\frac{15\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{8\sqrt{d^2-e^2x^2}}{dx}\right)+\frac{8e^2\sqrt{d^2-e^2x^2}}{d(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{1}{2}e\left(\frac{15e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{8\sqrt{d^2-e^2x^2}}{dx}\right)+\frac{8e^2\sqrt{d^2-e^2x^2}}{d(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4),x]
```

output
```
-1/2*Sqrt[d^2 - e^2*x^2]/x^2 + (8*e^2*Sqrt[d^2 - e^2*x^2])/(d*(d + e*x)) -
 (e*((-8*Sqrt[d^2 - e^2*x^2])/(d*x) + (15*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
)/d))/2
```

3.206.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.206.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

method	result	size
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\,\left(-8 e x+d\right)}{2 d\,x^{2}}-\frac{15 e^{2} \ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\,\sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{2 \sqrt{d^{2}}}+\frac{8 e \sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{d \left(x+\frac{d}{e}\right)}$	115
default	Expression too large to display	1461

input
```
int((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e^2*x^2+d^2)^(1/2)*(-8*e*x+d)/d/x^2-15/2*e^2/(d^2)^(1/2)*ln((2*d^2+
2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+8*e/d/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e
*(x+d/e))^(1/2)
```

3.206.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.02

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^4}\,dx = \frac{16\,e^3x^3+16\,de^2x^2+15\,(e^3x^3+de^2x^2)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(24\,e^2x^2+7\,dex-d^2)}{2\,(dex^3+d^2x^2)}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x, algorithm="fricas")
```

output
```
1/2*(16*e^3*x^3 + 16*d*e^2*x^2 + 15*(e^3*x^3 + d*e^2*x^2)*log(-(d - sqrt(-
e^2*x^2 + d^2))/x) + (24*e^2*x^2 + 7*d*e*x - d^2)*sqrt(-e^2*x^2 + d^2))/(d
*e*x^3 + d^2*x^2)
```

3.206.6 Sympy [F]

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^4}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^{\frac{5}{2}}}{x^3\,(d+ex)^4}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/x**3/(e*x+d)**4,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**3*(d + e*x)**4), x)
```

3.206.7 Maxima [F]

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{(ex+d)^4x^3}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^3), x)
```

3.206.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(99) = 198$.

Time = 0.32 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.29

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^4}\,dx = \frac{\left(e^3 - \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e}{x} - \frac{144\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{ex^2}\right)e^4x^2}{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|} - \frac{15\,e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,d|e|} + \frac{\frac{16\left(de+\sqrt{-e^2x^2+d^2}|e|\right)de|e|}{x} - \frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d|e|}{ex^2}}{8\,d^2e^2}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x, algorithm="giac")
```

output
```
1/8*(e^3 - 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e/x - 144*(d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))^2/(e*x^2))*e^4*x^2/((d*e + sqrt(-e^2*x^2 + d^2)*abs(
e))^2*d*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e)) - 15/2*e
^3*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d*ab
s(e)) + 1/8*(16*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d*e*abs(e)/x - (d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))^2*d*abs(e)/(e*x^2))/(d^2*e^2)
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{x^3(d+ex)^4}\,dx = \int \frac{(d^2 - e^2\,x^2)^{5/2}}{x^3\,(d+e\,x)^4}\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4), x)
```

3.207 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^4}\,dx$

3.207.1 Optimal result

Integrand size = 27, antiderivative size = 137

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^4}\,dx = -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}+\frac{2e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x}+\frac{10e^3\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

output
```
10*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2-8*e^3*(-e*x+d)/d^2/(-e^2*x^2+d^
2)^(1/2)-1/3*(-e^2*x^2+d^2)^(1/2)/x^3+2*e*(-e^2*x^2+d^2)^(1/2)/d/x^2-23/3*
e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x
```

3.207.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^4}\,dx = -\frac{\frac{d\sqrt{d^2-e^2x^2}(d^3-5d^2ex+17de^2x^2+47e^3x^3)}{x^3(d+ex)}-30\sqrt{d^2}e^3\log(x)+30\sqrt{d^2}e^3\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{3d^3}$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4),x]
```

output
```
-1/3*((d*Sqrt[d^2 - e^2*x^2]*(d^3 - 5*d^2*e*x + 17*d*e^2*x^2 + 47*e^3*x^3)
)/(x^3*(d + e*x)) - 30*Sqrt[d^2]*e^3*Log[x] + 30*Sqrt[d^2]*e^3*Log[Sqrt[d^
2] - Sqrt[d^2 - e^2*x^2]])/d^3
```

3.207.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {564, 2338, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^4}\,dx$$

$$\Big\downarrow \text{564}$$

$$\int \frac{-\frac{8e^3x^3}{d}+7e^2x^2-4dex+d^2}{x^4\sqrt{d^2-e^2x^2}}dx-\frac{8e^3\sqrt{d^2-e^2x^2}}{d^2(d+ex)}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int \frac{12ed^3-23e^2xd^2+24e^3x^2d}{x^3\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{8e^3\sqrt{d^2-e^2x^2}}{d^2(d+ex)}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{-\frac{\int \frac{2d^3e^2(23d-30ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{6de\sqrt{d^2-e^2x^2}}{x^2}}{3d^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{8e^3\sqrt{d^2-e^2x^2}}{d^2(d+ex)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-de^2\int \frac{23d-30ex}{x^2\sqrt{d^2-e^2x^2}}dx-\frac{6de\sqrt{d^2-e^2x^2}}{x^2}}{3d^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{8e^3\sqrt{d^2-e^2x^2}}{d^2(d+ex)}$$

$$\Big\downarrow \text{534}$$

$$-\frac{-de^2\left(-30e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{23\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{6de\sqrt{d^2-e^2x^2}}{x^2}}{3d^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{8e^3\sqrt{d^2-e^2x^2}}{d^2(d+ex)}$$

$$\Big\downarrow \text{243}$$

$$-\frac{-de^2\left(-15e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{23\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{6de\sqrt{d^2-e^2x^2}}{x^2}}{3d^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{8e^3\sqrt{d^2-e^2x^2}}{d^2(d+ex)}$$

$$\downarrow 73$$

$$-\frac{-de^2\left(\frac{30\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{23\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{6de\sqrt{d^2-e^2x^2}}{x^2}}{3d^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{8e^3\sqrt{d^2-e^2x^2}}{d^2(d+ex)}$$

$$\downarrow 221$$

$$-\frac{-de^2\left(\frac{30e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{23\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{6de\sqrt{d^2-e^2x^2}}{x^2}}{3d^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{8e^3\sqrt{d^2-e^2x^2}}{d^2(d+ex)}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4),x]
```

output
```
-1/3*Sqrt[d^2 - e^2*x^2]/x^3 - (8*e^3*Sqrt[d^2 - e^2*x^2])/(d^2*(d + e*x))
 - ((-6*d*e*Sqrt[d^2 - e^2*x^2])/x^2 - d*e^2*((-23*Sqrt[d^2 - e^2*x^2])/(d
*x) + (30*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/(3*d^2)
```

3.207.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.207. $\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^4}\,dx$

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.207.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

method	result	size
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\,\left(23 e^{2} x^{2}-6 d e x+d^{2}\right)}{3 x^{3} d^{2}}+\frac{10 e^{3} \ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\,\sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{d \sqrt{d^{2}}}-\frac{8 e^{2} \sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{d^{2} \left(x+\frac{d}{e}\right)}$	131
default	Expression too large to display	1626

input
```
int((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-e^2*x^2+d^2)^(1/2)*(23*e^2*x^2-6*d*e*x+d^2)/x^3/d^2+10*e^3/d/(d^2)^
(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-8*e^2/d^2/(x+d/e)*(
-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

3.207.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^4}\,dx =$$

$$-\frac{24\,e^4x^4+24\,de^3x^3+30\left(e^4x^4+de^3x^3\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(47\,e^3x^3+17\,de^2x^2-5\,d^2ex+d^3\right)\sqrt{-e^2x^2}}{3\left(d^2ex^4+d^3x^3\right)}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x, algorithm="fricas")
```

output
```
-1/3*(24*e^4*x^4 + 24*d*e^3*x^3 + 30*(e^4*x^4 + d*e^3*x^3)*log(-(d - sqrt(
-e^2*x^2 + d^2))/x) + (47*e^3*x^3 + 17*d*e^2*x^2 - 5*d^2*e*x + d^3)*sqrt(-
e^2*x^2 + d^2))/(d^2*e*x^4 + d^3*x^3)
```

3.207.6 Sympy [F]

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^4}\,dx = \int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{x^4\left(d+ex\right)^4}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/x**4/(e*x+d)**4,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**4*(d + e*x)**4), x)
```

3.207.7 Maxima [F]

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^4}\,dx = \int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{\left(ex+d\right)^4x^4}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^4), x)
```

3.207.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(124) = 248.

Time = 0.31 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.32

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^4}\,dx = \frac{\left(e^4 - \frac{11\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^2}{x} + \frac{81\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{x^2} + \frac{477\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^2x^3}\right)e^6x^3}{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3 d^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|} + \frac{10\,e^4\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d^2|e|} - \frac{\frac{93\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^4e^4}{x} - \frac{12\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^4e^2}{x^2} + \frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^4}{x^3}}{24\,d^6e^2|e|}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x, algorithm="giac")
```

output
```
1/24*(e^4 - 11*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^2/x + 81*(d*e + sqrt(
-e^2*x^2 + d^2)*abs(e))^2/x^2 + 477*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/
(e^2*x^3))*e^6*x^3/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^2*((d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e)) + 10*e^4*log(1/2*abs(-2*d*e
- 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^2*abs(e)) - 1/24*(93*(d*
e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^4*e^4/x - 12*(d*e + sqrt(-e^2*x^2 + d^2
)*abs(e))^2*d^4*e^2/x^2 + (d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^4/x^3)/(
d^6*e^2*abs(e))
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^4}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}}{x^4\,(d+e\,x)^4}\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4), x)
```

3.208 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4}\,dx$

3.208.1 Optimal result

Integrand size = 27, antiderivative size = 170

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4}\,dx=\frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}+\frac{4e\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}+\frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x}-\frac{95e^4\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}$$

output
```
-95/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3+8*e^4*(-e*x+d)/d^3/(-e^2*x^2
+d^2)^(1/2)-1/4*(-e^2*x^2+d^2)^(1/2)/x^4+4/3*e*(-e^2*x^2+d^2)^(1/2)/d/x^3-
31/8*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x^2+32/3*e^3*(-e^2*x^2+d^2)^(1/2)/d^3/x
```

3.208.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.68

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4}\,dx=\frac{\frac{\sqrt{d^2-e^2x^2}(-6d^4+26d^3ex-61d^2e^2x^2+163de^3x^3+448e^4x^4)}{x^4(d+ex)}+570e^4\text{arctanh}\left(\frac{\sqrt{-e^2}x-\sqrt{d^2-e^2x^2}}{d}\right)}{24d^3}$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(-6*d^4 + 26*d^3*e*x - 61*d^2*e^2*x^2 + 163*d*e^3*x^
3 + 448*e^4*x^4))/(x^4*(d + e*x)) + 570*e^4*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d
^2 - e^2*x^2])/d])/(24*d^3)
```

3.208.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {564, 2338, 2338, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4}\,dx$$

$$\Big\downarrow \text{564}$$

$$\int \frac{\frac{8e^4x^4}{d^2}-\frac{8e^3x^3}{d}+7e^2x^2-4dex+d^2}{x^5\sqrt{d^2-e^2x^2}}dx+\frac{8e^4\sqrt{d^2-e^2x^2}}{d^3(d+ex)}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int \frac{-32x^3e^4+32dx^2e^3-31d^2xe^2+16d^3e}{x^4\sqrt{d^2-e^2x^2}}dx}{4d^2}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}+\frac{8e^4\sqrt{d^2-e^2x^2}}{d^3(d+ex)}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{-\frac{\int \frac{93e^2d^4-128e^3xd^3+96e^4x^2d^2}{x^3\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{16de\sqrt{d^2-e^2x^2}}{3x^3}}{4d^2}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}+\frac{8e^4\sqrt{d^2-e^2x^2}}{d^3(d+ex)}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{-\frac{-\frac{\int \frac{d^4e^3(256d-285ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{93d^2e^2\sqrt{d^2-e^2x^2}}{2x^2}}{3d^2}-\frac{16de\sqrt{d^2-e^2x^2}}{3x^3}}{4d^2}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}+\frac{8e^4\sqrt{d^2-e^2x^2}}{d^3(d+ex)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{-\frac{1}{2}d^2e^3\int \frac{256d-285ex}{x^2\sqrt{d^2-e^2x^2}}dx-\frac{93d^2e^2\sqrt{d^2-e^2x^2}}{2x^2}}{3d^2}-\frac{16de\sqrt{d^2-e^2x^2}}{3x^3}}{4d^2}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}+\frac{8e^4\sqrt{d^2-e^2x^2}}{d^3(d+ex)}$$

$$\Big\downarrow \text{534}$$

$$-\frac{-\frac{-\frac{1}{2}d^2e^3\left(-285e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{256\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{93d^2e^2\sqrt{d^2-e^2x^2}}{2x^2}}{3d^2}-\frac{16de\sqrt{d^2-e^2x^2}}{3x^3}}{4d^2}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}+\frac{8e^4\sqrt{d^2-e^2x^2}}{d^3(d+ex)}$$

$$\Big\downarrow \text{243}$$

$$-\frac{-\frac{-\frac{1}{2}d^2e^3\left(-\frac{285}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{256\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{93d^2e^2\sqrt{d^2-e^2x^2}}{2x^2}}{3d^2}-\frac{16de\sqrt{d^2-e^2x^2}}{3x^3}}{4d^2}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}+\frac{8e^4\sqrt{d^2-e^2x^2}}{d^3(d+ex)}$$

$\Big\downarrow$ 73

$$-\frac{-\frac{-\frac{1}{2}d^2e^3\left(\frac{285\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{256\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{93d^2e^2\sqrt{d^2-e^2x^2}}{2x^2}}{3d^2}-\frac{16de\sqrt{d^2-e^2x^2}}{3x^3}}{4d^2}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}+\frac{8e^4\sqrt{d^2-e^2x^2}}{d^3(d+ex)}$$

$\Big\downarrow$ 221

$$-\frac{-\frac{-\frac{1}{2}d^2e^3\left(\frac{285e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{256\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{93d^2e^2\sqrt{d^2-e^2x^2}}{2x^2}}{3d^2}-\frac{16de\sqrt{d^2-e^2x^2}}{3x^3}}{4d^2}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}+\frac{8e^4\sqrt{d^2-e^2x^2}}{d^3(d+ex)}$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4),x]
```

output
```
-1/4*Sqrt[d^2 - e^2*x^2]/x^4 + (8*e^4*Sqrt[d^2 - e^2*x^2])/(d^3*(d + e*x))
 - ((-16*d*e*Sqrt[d^2 - e^2*x^2])/(3*x^3) - ((-93*d^2*e^2*Sqrt[d^2 - e^2*x
^2])/(2*x^2) - (d^2*e^3*((-256*Sqrt[d^2 - e^2*x^2])/(d*x) + (285*e*ArcTanh
[Sqrt[d^2 - e^2*x^2]/d])/d))/2)/(3*d^2))/(4*d^2)
```

3.208.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.208.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.85

method	result	s
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, \left(-256 e^{3} x^{3}+93 d\, e^{2} x^{2}-32 d^{2} e x+6 d^{3}\right)}{24 d^{3} x^{4}}-\frac{95 e^{4} \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{8 d^{2} \sqrt{d^{2}}}+\frac{8 e^{3} \sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{d^{3} \left(x+\frac{d}{e}\right)}$	1
default	Expression too large to display	1

input
```
int((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-e^2*x^2+d^2)^(1/2)*(-256*e^3*x^3+93*d*e^2*x^2-32*d^2*e*x+6*d^3)/d^
3/x^4-95/8/d^2*e^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2
))/x)+8/d^3*e^3/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

3.208.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.80

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4}\,dx = \frac{192\,e^5x^5+192\,de^4x^4+285\,(e^5x^5+de^4x^4)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(448\,e^4x^4+163\,de^3}{24\,(d^3ex^5+d^4x^4)}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x, algorithm="fricas")
```

output
```
1/24*(192*e^5*x^5 + 192*d*e^4*x^4 + 285*(e^5*x^5 + d*e^4*x^4)*log(-(d - sq
rt(-e^2*x^2 + d^2))/x) + (448*e^4*x^4 + 163*d*e^3*x^3 - 61*d^2*e^2*x^2 + 2
6*d^3*e*x - 6*d^4)*sqrt(-e^2*x^2 + d^2))/(d^3*e*x^5 + d^4*x^4)
```

3.208.6 Sympy [F]

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^{\frac{5}{2}}}{x^5\,(d+ex)^4}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/x**5/(e*x+d)**4,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**5*(d + e*x)**4), x)
```

3.208.7 Maxima [F]

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4}\,dx=\int \frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{(ex+d)^4x^5}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^5), x)
```

3.208.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(149) = 298$.

Time = 0.33 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.31

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4}\,dx=\frac{\left(3e^5-\frac{29\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^3}{x}+\frac{160\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e}{x^2}-\frac{864\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{ex^3}-\frac{4128\left(d\right.}{}\right.}{192\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}-\frac{95e^5\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2e^2|x|}\right)}{8d^3|e|}+\frac{\frac{1056\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^9e^5|e|}{x}-\frac{192\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^9e^3|e|}{x^2}+\frac{32\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^9e|e|}{x^3}-\frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^9|e|}{ex^4}}{192d^{12}e^4}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x, algorithm="giac")
```

output
```
1/192*(3*e^5 - 29*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^3/x + 160*(d*e + s
qrt(-e^2*x^2 + d^2)*abs(e))^2*e/x^2 - 864*(d*e + sqrt(-e^2*x^2 + d^2)*abs(
e))^3/(e*x^3) - 4128*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^3*x^4))*e^8*
x^4/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^3*((d*e + sqrt(-e^2*x^2 + d^2
)*abs(e))/(e^2*x) + 1)*abs(e)) - 95/8*e^5*log(1/2*abs(-2*d*e - 2*sqrt(-e^2
*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^3*abs(e)) + 1/192*(1056*(d*e + sqrt(-
e^2*x^2 + d^2)*abs(e))*d^9*e^5*abs(e)/x - 192*(d*e + sqrt(-e^2*x^2 + d^2)*
abs(e))^2*d^9*e^3*abs(e)/x^2 + 32*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^
9*e*abs(e)/x^3 - 3*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^9*abs(e)/(e*x^4
))/(d^12*e^4)
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4}\,dx=\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x^5\,{\left(d+e\,x\right)}^4}\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4), x)
```

3.209 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^4}\,dx$

3.209.1 Optimal result

Integrand size = 27, antiderivative size = 196

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^4}\,dx = -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2} - \frac{66e^4\sqrt{d^2-e^2x^2}}{5d^4x} + \frac{27e^5\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}$$

output
```
27/2*e^5*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4-8*e^5*(-e*x+d)/d^4/(-e^2*x^2+
d^2)^(1/2)-1/5*(-e^2*x^2+d^2)^(1/2)/x^5+e*(-e^2*x^2+d^2)^(1/2)/d/x^4-13/5*
e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x^3+11/2*e^3*(-e^2*x^2+d^2)^(1/2)/d^3/x^2-66/
5*e^4*(-e^2*x^2+d^2)^(1/2)/d^4/x
```

3.209.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.72

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^4}\,dx = -\frac{\frac{d\sqrt{d^2-e^2x^2}(2d^5-8d^4ex+16d^3e^2x^2-29d^2e^3x^3+77de^4x^4+212e^5x^5)}{x^5(d+ex)} - 135\sqrt{d^2}e^5\log(x) + 135\sqrt{d^2}e^5\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right.}{10d^5}$$

input
```
Integrate[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4),x]
```

output
```
-1/10*((d*Sqrt[d^2 - e^2*x^2]*(2*d^5 - 8*d^4*e*x + 16*d^3*e^2*x^2 - 29*d^2
*e^3*x^3 + 77*d*e^4*x^4 + 212*e^5*x^5))/(x^5*(d + e*x)) - 135*Sqrt[d^2]*e^
5*Log[x] + 135*Sqrt[d^2]*e^5*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/d^5
```

3.209.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {564, 2338, 2338, 27, 2338, 27, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^6(d+ex)^4}dx$$

$$\Big\downarrow \text{564}$$

$$\int \frac{-\frac{8e^5x^5}{d^3}+\frac{8e^4x^4}{d^2}-\frac{8e^3x^3}{d}+7e^2x^2-4dex+d^2}{x^6\sqrt{d^2-e^2x^2}}dx-\frac{8e^5\sqrt{d^2-e^2x^2}}{d^4(d+ex)}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int \frac{\frac{40x^4e^5}{d}-40x^3e^4+40dx^2e^3-39d^2xe^2+20d^3e}{x^5\sqrt{d^2-e^2x^2}}dx}{5d^2}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{8e^5\sqrt{d^2-e^2x^2}}{d^4(d+ex)}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{-\frac{\int \frac{4\left(-40dx^3e^5+40d^2x^2e^4-55d^3xe^3+39d^4e^2\right)}{x^4\sqrt{d^2-e^2x^2}}dx}{4d^2}-\frac{5de\sqrt{d^2-e^2x^2}}{x^4}}{5d^2}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{8e^5\sqrt{d^2-e^2x^2}}{d^4(d+ex)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{-40dx^3e^5+40d^2x^2e^4-55d^3xe^3+39d^4e^2}{x^4\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{5de\sqrt{d^2-e^2x^2}}{x^4}}{5d^2}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{8e^5\sqrt{d^2-e^2x^2}}{d^4(d+ex)}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{-\frac{-\frac{\int \frac{3\left(55e^3d^5-66e^4xd^4+40e^5x^2d^3\right)}{x^3\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{13d^2e^2\sqrt{d^2-e^2x^2}}{x^3}}{d^2}-\frac{5de\sqrt{d^2-e^2x^2}}{x^4}}{5d^2}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{8e^5\sqrt{d^2-e^2x^2}}{d^4(d+ex)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{-\frac{-\frac{\int \frac{55e^3d^5-66e^4xd^4+40e^5x^2d^3}{x^3\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{13d^2e^2\sqrt{d^2-e^2x^2}}{x^3}}{d^2}-\frac{5de\sqrt{d^2-e^2x^2}}{x^4}}{5d^2}}{}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{8e^5\sqrt{d^2-e^2x^2}}{d^4(d+ex)}$$

$$\Big\downarrow \; 2338$$

$$-\frac{-\frac{-\frac{-\frac{\int \frac{3d^5e^4(44d-45ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{2d^2}-\frac{55d^3e^3\sqrt{d^2-e^2x^2}}{2x^2}}{d^2}-\frac{13d^2e^2\sqrt{d^2-e^2x^2}}{x^3}}{d^2}-\frac{5de\sqrt{d^2-e^2x^2}}{x^4}}{5d^2}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{8e^5\sqrt{d^2-e^2x^2}}{d^4(d+ex)}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{-\frac{-\frac{3}{2}d^3e^4\int \frac{44d-45ex}{x^2\sqrt{d^2-e^2x^2}}dx-\frac{55d^3e^3\sqrt{d^2-e^2x^2}}{2x^2}}{d^2}-\frac{13d^2e^2\sqrt{d^2-e^2x^2}}{x^3}}{d^2}-\frac{5de\sqrt{d^2-e^2x^2}}{x^4}}{5d^2}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{8e^5\sqrt{d^2-e^2x^2}}{d^4(d+ex)}$$

$$\Big\downarrow \; 534$$

$$-\frac{-\frac{-\frac{-\frac{3}{2}d^3e^4\left(-45e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{44\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{55d^3e^3\sqrt{d^2-e^2x^2}}{2x^2}}{d^2}-\frac{13d^2e^2\sqrt{d^2-e^2x^2}}{x^3}}{d^2}-\frac{5de\sqrt{d^2-e^2x^2}}{x^4}}{5d^2}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{8e^5\sqrt{d^2-e^2x^2}}{d^4(d+ex)}$$

$$\Big\downarrow \; 243$$

$$-\frac{-\frac{-\frac{-\frac{3}{2}d^3e^4\left(-\frac{45}{2}e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{44\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{55d^3e^3\sqrt{d^2-e^2x^2}}{2x^2}}{d^2}-\frac{13d^2e^2\sqrt{d^2-e^2x^2}}{x^3}}{d^2}-\frac{5de\sqrt{d^2-e^2x^2}}{x^4}}{5d^2}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{8e^5\sqrt{d^2-e^2x^2}}{d^4(d+ex)}$$

$$\Big\downarrow \; 73$$

$$-\frac{-\frac{-\frac{-\frac{3}{2}d^3e^4\left(\frac{45\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{44\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{55d^3e^3\sqrt{d^2-e^2x^2}}{2x^2}}{d^2}-\frac{13d^2e^2\sqrt{d^2-e^2x^2}}{x^3}}{d^2}-\frac{5de\sqrt{d^2-e^2x^2}}{x^4}}{5d^2}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{8e^5\sqrt{d^2-e^2x^2}}{d^4(d+ex)}$$

$$\Big\downarrow \; 221$$

3.209. $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^6(d+ex)^4}\,dx$

$$
-\frac{-\frac{-\frac{-\frac{3}{2}d^3e^4\left(\frac{45e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{44\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{55d^3e^3\sqrt{d^2-e^2x^2}}{2x^2}}{d^2}-\frac{13d^2e^2\sqrt{d^2-e^2x^2}}{x^3}}{d^2}-\frac{5de\sqrt{d^2-e^2x^2}}{x^4}}{5d^2}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{8e^5\sqrt{d^2-e^2x^2}}{d^4(d+ex)}
$$

input
```
Int[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4),x]
```

output
```
-1/5*Sqrt[d^2 - e^2*x^2]/x^5 - (8*e^5*Sqrt[d^2 - e^2*x^2])/(d^4*(d + e*x))
 - ((-5*d*e*Sqrt[d^2 - e^2*x^2])/x^4 - ((-13*d^2*e^2*Sqrt[d^2 - e^2*x^2])/
x^3 - ((-55*d^3*e^3*Sqrt[d^2 - e^2*x^2])/(2*x^2) - (3*d^3*e^4*((-44*Sqrt[d
^2 - e^2*x^2])/(d*x) + (45*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/2)/d^2)/d
^2)/(5*d^2)
```

3.209.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.209.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(132e^4x^4-55d\,e^3x^3+26d^2e^2x^2-10d^3ex+2d^4)}{10x^5d^4}+\frac{27e^5\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^3\sqrt{d^2}}-\frac{8e^4\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x\right.}}{d^4\left(x+\frac{d}{e}\right)}$
default	Expression too large to display

input
```
int((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/10*(-e^2*x^2+d^2)^(1/2)*(132*e^4*x^4-55*d*e^3*x^3+26*d^2*e^2*x^2-10*d^3
*e*x+2*d^4)/x^5/d^4+27/2/d^3*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2
*x^2+d^2)^(1/2))/x)-8/d^4*e^4/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

3.209.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.75

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^4}\,dx =$$
$$-\frac{80\,e^6x^6+80\,de^5x^5+135\,(e^6x^6+de^5x^5)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(212\,e^5x^5+77\,de^4x^4-29\,d^2e^3x^3+16\,d^3e^2}{10\,(d^4ex^6+d^5x^5)}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x, algorithm="fricas")
```

output
```
-1/10*(80*e^6*x^6 + 80*d*e^5*x^5 + 135*(e^6*x^6 + d*e^5*x^5)*log(-(d - sqr
t(-e^2*x^2 + d^2))/x) + (212*e^5*x^5 + 77*d*e^4*x^4 - 29*d^2*e^3*x^3 + 16*
d^3*e^2*x^2 - 8*d^4*e*x + 2*d^5)*sqrt(-e^2*x^2 + d^2))/(d^4*e*x^6 + d^5*x^
5)
```

3.209.6 Sympy [F]

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^4}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^{\frac{5}{2}}}{x^6\,(d+ex)^4}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**(5/2)/x**6/(e*x+d)**4,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**6*(d + e*x)**4), x)
```

3.209.7 Maxima [F]

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{(ex+d)^4x^6}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^6), x)
```

3.209.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(173) = 346$.

Time = 0.31 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.29

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^4}\,dx = \frac{\left(e^6 - \frac{9\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^4}{x} + \frac{45\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^2}{x^2} - \frac{185\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{x^3} + \frac{870\left(de+\sqrt{\cdots}\right.}{\cdots}\right.}{160\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^5d^4\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+\cdots\right.}$$
$$+\frac{27\,e^6\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,d^4|e|}$$
$$-\frac{\frac{1110\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{16}e^8}{x} - \frac{240\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^{16}e^6}{x^2} + \frac{55\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^{16}e^4}{x^3} - \frac{10\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^{16}e^2}{x^4} \cdots}{160\,d^{20}e^4|e|}$$

input
```
integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x, algorithm="giac")
```

output
```
1/160*(e^6 - 9*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^4/x + 45*(d*e + sqrt(
-e^2*x^2 + d^2)*abs(e))^2*e^2/x^2 - 185*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e)
)^3/x^3 + 870*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^2*x^4) + 3670*(d*e
+ sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^4*x^5))*e^10*x^5/((d*e + sqrt(-e^2*x^2
 + d^2)*abs(e))^5*d^4*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*ab
s(e)) + 27/2*e^6*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*
abs(x)))/(d^4*abs(e)) - 1/160*(1110*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^
16*e^8/x - 240*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^16*e^6/x^2 + 55*(d*
e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^16*e^4/x^3 - 10*(d*e + sqrt(-e^2*x^2
+ d^2)*abs(e))^4*d^16*e^2/x^4 + (d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d^16
/x^5)/(d^20*e^4*abs(e))
```

3.209.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^4}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}}{x^6\,(d+e\,x)^4}\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4), x)
```

3.210 $\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4} \, dx$

3.210.1 Optimal result

Integrand size = 26, antiderivative size = 95

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4} \, dx = \frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)} + \frac{(1-a^2x^2)^{3/2}}{5a^3(1-ax)^4} - \frac{3(1-a^2x^2)^{3/2}}{5a^3(1-ax)^3} - \frac{\arcsin(ax)}{a^3}$$

output
```
1/5*(-a^2*x^2+1)^(3/2)/a^3/(-a*x+1)^4-3/5*(-a^2*x^2+1)^(3/2)/a^3/(-a*x+1)^
3-arcsin(a*x)/a^3+2*(-a^2*x^2+1)^(1/2)/a^3/(-a*x+1)
```

3.210.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4} \, dx = \frac{(-8+19ax-13a^2x^2)\sqrt{1-a^2x^2}}{5a^3(-1+ax)^3} - \frac{2\arctan\left(\frac{ax}{-1+\sqrt{1-a^2x^2}}\right)}{a^3}$$

input
```
Integrate[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^4,x]
```

output
```
((-8 + 19*a*x - 13*a^2*x^2)*Sqrt[1 - a^2*x^2])/(5*a^3*(-1 + a*x)^3) - (2*A
rcTan[(a*x)/(-1 + Sqrt[1 - a^2*x^2])])/a^3
```

3.210.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {582, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx$$

$$\Big\downarrow \; 582$$

$$\int \left(\frac{\sqrt{1-a^2x^2}}{a^2(ax-1)^2}+\frac{2\sqrt{1-a^2x^2}}{a^2(ax-1)^3}+\frac{\sqrt{1-a^2x^2}}{a^2(ax-1)^4}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\arcsin(ax)}{a^3}-\frac{3\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^4}+\frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)}$$

input
```
Int[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^4,x]
```

output
```
(2*Sqrt[1 - a^2*x^2])/(a^3*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(5*a^3*(1 - a*
x)^4) - (3*(1 - a^2*x^2)^(3/2))/(5*a^3*(1 - a*x)^3) - ArcSin[a*x]/a^3
```

3.210.3.1 Defintions of rubi rules used

rule 582
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[x^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IntegerQ[2*p] && IntegerQ[m]
&& ILtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.210.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(85) = 170$.

Time = 0.38 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.58

method	result
default	$\frac{\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{a\left(x-\frac{1}{a}\right)^2}+a\left(\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}-\frac{a\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}\right)}{\sqrt{a^2}}\right)}{a^4}+\frac{\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{5a\left(x-\frac{1}{a}\right)^4}}{}$

input
```
int(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)+a*((-a^2*(x-1/a)^2
-2*a*(x-1/a))^(1/2)-a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a
*(x-1/a))^(1/2))))+1/a^6*(1/5/a/(x-1/a)^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/
2)-1/15/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2))+2/3/a^6/(x-1/a)^3*(-
a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)
```

3.210.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.33

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx$$
$$=\frac{8\,a^3x^3-24\,a^2x^2+24\,ax+10\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(13\,a^2x^2-19\,ax+8\right)\sqrt{-}}{5\left(a^6x^3-3\,a^5x^2+3\,a^4x-a^3\right)}$$

input
```
integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x, algorithm="fricas")
```

output
```
1/5*(8*a^3*x^3 - 24*a^2*x^2 + 24*a*x + 10*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1
)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (13*a^2*x^2 - 19*a*x + 8)*sqrt(
-a^2*x^2 + 1) - 8)/(a^6*x^3 - 3*a^5*x^2 + 3*a^4*x - a^3)
```

3.210.6 Sympy [F]

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx = \int \frac{x^2\sqrt{-(ax-1)(ax+1)}}{(ax-1)^4}\,dx$$

input
```
integrate(x**2*(-a**2*x**2+1)**(1/2)/(-a*x+1)**4,x)
```

output
```
Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))/(a*x - 1)**4, x)
```

3.210.7 Maxima [F]

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx = \int \frac{\sqrt{-a^2x^2+1}x^2}{(ax-1)^4}\,dx$$

input
```
integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*x^2/(a*x - 1)^4, x)
```

3.210.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.210.9 Mupad [B] (verification not implemented)

Time = 11.46 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.32

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx = \frac{4\,a^2\,\sqrt{1-a^2\,x^2}}{15\,\left(a^7\,x^2-2\,a^6\,x+a^5\right)}-\frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{a^2\,\sqrt{-a^2}}-\frac{2\,\sqrt{1-a^2\,x^2}}{5\,\sqrt{-a^2}\,\left(a\,\sqrt{-a^2}-3\,a^2\,x\,\sqrt{-a^2}+3\,a^3\,x^2\,\sqrt{-a^2}-a^4\,x^3\,\sqrt{-a^2}\right)}-\frac{13\,\sqrt{1-a^2\,x^2}}{5\,\left(a\,\sqrt{-a^2}-a^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-\frac{5\,\sqrt{1-a^2\,x^2}}{3\,\left(a^5\,x^2-2\,a^4\,x+a^3\right)}$$

input
```
int((x^2*(1 - a^2*x^2)^(1/2))/(a*x - 1)^4,x)
```

output
```
(4*a^2*(1 - a^2*x^2)^(1/2))/(15*(a^5 - 2*a^6*x + a^7*x^2)) - asinh(x*(-a^2
)^(1/2))/(a^2*(-a^2)^(1/2)) - (2*(1 - a^2*x^2)^(1/2))/(5*(-a^2)^(1/2)*(a*(
-a^2)^(1/2) - 3*a^2*x*(-a^2)^(1/2) + 3*a^3*x^2*(-a^2)^(1/2) - a^4*x^3*(-a^
2)^(1/2))) - (13*(1 - a^2*x^2)^(1/2))/(5*(a*(-a^2)^(1/2) - a^2*x*(-a^2)^(1
/2))*(-a^2)^(1/2)) - (5*(1 - a^2*x^2)^(1/2))/(3*(a^3 - 2*a^4*x + a^5*x^2))
```

3.211 $\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx$

3.211.1 Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx=\frac{(1-a^2x^2)^{3/2}}{7a^3(1-ax)^5}-\frac{12(1-a^2x^2)^{3/2}}{35a^3(1-ax)^4}+\frac{23(1-a^2x^2)^{3/2}}{105a^3(1-ax)^3}$$

output
```
1/7*(-a^2*x^2+1)^(3/2)/a^3/(-a*x+1)^5-12/35*(-a^2*x^2+1)^(3/2)/a^3/(-a*x+1
)^4+23/105*(-a^2*x^2+1)^(3/2)/a^3/(-a*x+1)^3
```

3.211.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.57

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx=\frac{\sqrt{1-a^2x^2}(2-8ax+13a^2x^2+23a^3x^3)}{105a^3(-1+ax)^4}$$

input
```
Integrate[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^5,x]
```

output
```
(Sqrt[1 - a^2*x^2]*(2 - 8*a*x + 13*a^2*x^2 + 23*a^3*x^3))/(105*a^3*(-1 + a
*x)^4)
```

3.211.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.42, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {581, 25, 671, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx$$

$$\Big\downarrow \ 581$$

$$-\frac{\int -\frac{(4-3ax)\sqrt{1-a^2x^2}}{(1-ax)^5}dx}{a^2}-\frac{\left(1-a^2x^2\right)^{3/2}}{a^3(1-ax)^4}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(4-3ax)\sqrt{1-a^2x^2}}{(1-ax)^5}dx}{a^2}-\frac{\left(1-a^2x^2\right)^{3/2}}{a^3(1-ax)^4}$$

$$\Big\downarrow \ 671$$

$$\frac{\frac{23}{7}\int \frac{\sqrt{1-a^2x^2}}{(1-ax)^4}dx+\frac{\left(1-a^2x^2\right)^{3/2}}{7a(1-ax)^5}}{a^2}-\frac{\left(1-a^2x^2\right)^{3/2}}{a^3(1-ax)^4}$$

$$\Big\downarrow \ 461$$

$$\frac{\frac{23}{7}\left(\frac{1}{5}\int \frac{\sqrt{1-a^2x^2}}{(1-ax)^3}dx+\frac{\left(1-a^2x^2\right)^{3/2}}{5a(1-ax)^4}\right)+\frac{\left(1-a^2x^2\right)^{3/2}}{7a(1-ax)^5}}{a^2}-\frac{\left(1-a^2x^2\right)^{3/2}}{a^3(1-ax)^4}$$

$$\Big\downarrow \ 460$$

$$\frac{\frac{\left(1-a^2x^2\right)^{3/2}}{7a(1-ax)^5}+\frac{23}{7}\left(\frac{\left(1-a^2x^2\right)^{3/2}}{15a(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a(1-ax)^4}\right)}{a^2}-\frac{\left(1-a^2x^2\right)^{3/2}}{a^3(1-ax)^4}$$

input
```
Int[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^5,x]
```

output
```
-((1 - a^2*x^2)^(3/2)/(a^3*(1 - a*x)^4)) + ((1 - a^2*x^2)^(3/2)/(7*a*(1 -
a*x)^5) + (23*((1 - a^2*x^2)^(3/2)/(5*a*(1 - a*x)^4) + (1 - a^2*x^2)^(3/2)
/(15*a*(1 - a*x)^3)))/7)/a^2
```

3.211.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

3.211.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.50

method	result
gosper	$\frac{\sqrt{-a^2x^2+1}\,(23a^2x^2-10ax+2)(ax+1)}{105(ax-1)^4a^3}$
trager	$\frac{(23a^3x^3+13a^2x^2-8ax+2)\sqrt{-a^2x^2+1}}{105(ax-1)^4a^3}$
default	$-\frac{\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{7a\left(x-\frac{1}{a}\right)^5}-\frac{2a\left(\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{5a\left(x-\frac{1}{a}\right)^4}-\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{15\left(x-\frac{1}{a}\right)^3}\right)}{7}}{a^7}-\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{3a^6\left(x-\frac{1}{a}\right)^3}-$

input
```
int(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/105*(-a^2*x^2+1)^(1/2)*(23*a^2*x^2-10*a*x+2)*(a*x+1)/(a*x-1)^4/a^3
```

3.211.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.16

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx = \frac{2\,a^4x^4-8\,a^3x^3+12\,a^2x^2-8\,ax+(23\,a^3x^3+13\,a^2x^2-8\,ax+2)\sqrt{-a^2x^2+1}+2}{105\,(a^7x^4-4\,a^6x^3+6\,a^5x^2-4\,a^4x+a^3)}$$

input
```
integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x, algorithm="fricas")
```

output
```
1/105*(2*a^4*x^4 - 8*a^3*x^3 + 12*a^2*x^2 - 8*a*x + (23*a^3*x^3 + 13*a^2*x
^2 - 8*a*x + 2)*sqrt(-a^2*x^2 + 1) + 2)/(a^7*x^4 - 4*a^6*x^3 + 6*a^5*x^2 -
 4*a^4*x + a^3)
```

3.211.6 Sympy [F]

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx = -\int \frac{x^2\sqrt{-a^2x^2+1}}{a^5x^5-5a^4x^4+10a^3x^3-10a^2x^2+5ax-1}\,dx$$

input
```
integrate(x**2*(-a**2*x**2+1)**(1/2)/(-a*x+1)**5,x)
```

output
```
-Integral(x**2*sqrt(-a**2*x**2 + 1)/(a**5*x**5 - 5*a**4*x**4 + 10*a**3*x**
3 - 10*a**2*x**2 + 5*a*x - 1), x)
```

3.211.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(73) = 146$.

Time = 0.19 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.74

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx = \frac{2\sqrt{-a^2x^2+1}}{7\,(a^7x^4-4\,a^6x^3+6\,a^5x^2-4\,a^4x+a^3)} + \frac{29\sqrt{-a^2x^2+1}}{35\,(a^6x^3-3\,a^5x^2+3\,a^4x-a^3)} + \frac{82\sqrt{-a^2x^2+1}}{105\,(a^5x^2-2\,a^4x+a^3)} + \frac{23\sqrt{-a^2x^2+1}}{105\,(a^4x-a^3)}$$

input
```
integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x, algorithm="maxima")
```

output
```
2/7*sqrt(-a^2*x^2 + 1)/(a^7*x^4 - 4*a^6*x^3 + 6*a^5*x^2 - 4*a^4*x + a^3) +
 29/35*sqrt(-a^2*x^2 + 1)/(a^6*x^3 - 3*a^5*x^2 + 3*a^4*x - a^3) + 82/105*s
qrt(-a^2*x^2 + 1)/(a^5*x^2 - 2*a^4*x + a^3) + 23/105*sqrt(-a^2*x^2 + 1)/(a
^4*x - a^3)
```

3.211.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.211.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.26

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx = \frac{2\sqrt{1-a^2x^2}}{7\,(a^7x^4-4a^6x^3+6a^5x^2-4a^4x+a^3)} + \frac{4\sqrt{1-a^2x^2}}{3\,(a^5x^2-2a^4x+a^3)} + \frac{4a\sqrt{1-a^2x^2}}{35\,(a^6x^2-2a^5x+a^4)} + \frac{29\sqrt{1-a^2x^2}}{35\sqrt{-a^2}\,\left(a\sqrt{-a^2}-3a^2x\sqrt{-a^2}+3a^3x^2\sqrt{-a^2}-a^4x^3\sqrt{-a^2}\right)} + \frac{23\sqrt{1-a^2x^2}}{105\,\left(a\sqrt{-a^2}-a^2x\sqrt{-a^2}\right)\sqrt{-a^2}} - \frac{2a^2\sqrt{1-a^2x^2}}{3\,(a^7x^2-2a^6x+a^5)}$$

input
```
int(-(x^2*(1 - a^2*x^2)^(1/2))/(a*x - 1)^5,x)
```

output
```
(2*(1 - a^2*x^2)^(1/2))/(7*(a^3 - 4*a^4*x + 6*a^5*x^2 - 4*a^6*x^3 + a^7*x^
4)) + (4*(1 - a^2*x^2)^(1/2))/(3*(a^3 - 2*a^4*x + a^5*x^2)) + (4*a*(1 - a^
2*x^2)^(1/2))/(35*(a^4 - 2*a^5*x + a^6*x^2)) + (29*(1 - a^2*x^2)^(1/2))/(3
5*(-a^2)^(1/2)*(a*(-a^2)^(1/2) - 3*a^2*x*(-a^2)^(1/2) + 3*a^3*x^2*(-a^2)^(
1/2) - a^4*x^3*(-a^2)^(1/2))) + (23*(1 - a^2*x^2)^(1/2))/(105*(a*(-a^2)^(1
/2) - a^2*x*(-a^2)^(1/2))*(-a^2)^(1/2)) - (2*a^2*(1 - a^2*x^2)^(1/2))/(3*(
a^5 - 2*a^6*x + a^7*x^2))
```

3.212 $\int \frac{x^3}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

3.212.1 Optimal result

Integrand size = 27, antiderivative size = 209

$$\int \frac{x^3}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx = -\frac{24x}{5005d^3e^3(d^2-e^2x^2)^{5/2}} + \frac{d^2}{13e^4(d+ex)^4(d^2-e^2x^2)^{5/2}} - \frac{30d}{143e^4(d+ex)^3(d^2-e^2x^2)^{5/2}} + \frac{21}{143e^4(d+ex)^2(d^2-e^2x^2)^{5/2}} + \frac{4}{1001de^4(d+ex)(d^2-e^2x^2)^{5/2}} - \frac{32x}{5005d^5e^3(d^2-e^2x^2)^{3/2}} - \frac{64x}{5005d^7e^3\sqrt{d^2-e^2x^2}}$$

output
```
-24/5005*x/d^3/e^3/(-e^2*x^2+d^2)^(5/2)+1/13*d^2/e^4/(e*x+d)^4/(-e^2*x^2+d
^2)^(5/2)-30/143*d/e^4/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2)+21/143/e^4/(e*x+d)^2
/(-e^2*x^2+d^2)^(5/2)+4/1001/d/e^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2)-32/5005*x/
d^5/e^3/(-e^2*x^2+d^2)^(3/2)-64/5005*x/d^7/e^3/(-e^2*x^2+d^2)^(1/2)
```

3.212.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.66

$$\int \frac{x^3}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(90d^9+360d^8ex+315d^7e^2x^2-540d^6e^3x^3+160d^5e^4x^4+776d}{5005d^7e^4(d-ex)^3(d+e}$$

input
```
Integrate[x^3/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(90*d^9 + 360*d^8*e*x + 315*d^7*e^2*x^2 - 540*d^6*e^3
*x^3 + 160*d^5*e^4*x^4 + 776*d^4*e^5*x^5 + 384*d^3*e^6*x^6 - 224*d^2*e^7*x
^7 - 256*d*e^8*x^8 - 64*e^9*x^9))/(5005*d^7*e^4*(d - e*x)^3*(d + e*x)^7)
```

3.212.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.51, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {581, 25, 2170, 27, 671, 461, 461, 470, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{581}$$

$$\frac{1}{7e^4(d+ex)^2 (d^2-e^2x^2)^{5/2}}-\frac{\int -\frac{2d^3-3exd^2-12e^2x^2d}{(d+ex)^4(d^2-e^2x^2)^{7/2}}dx}{7e^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2d^3-3exd^2-12e^2x^2d}{(d+ex)^4(d^2-e^2x^2)^{7/2}}dx}{7e^3}+\frac{1}{7e^4(d+ex)^2 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{2170}$$

$$\frac{\frac{\int -\frac{4d^2e^4(5d-9ex)}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}dx}{8e^4}-\frac{3d}{2e(d+ex)^3(d^2-e^2x^2)^{5/2}}}{7e^3}+\frac{1}{7e^4(d+ex)^2 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{1}{2}d^2\int \frac{5d-9ex}{(d+ex)^4(d^2-e^2x^2)^{7/2}}dx-\frac{3d}{2e(d+ex)^3(d^2-e^2x^2)^{5/2}}}{7e^3}+\frac{1}{7e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{671}$$

$$\frac{-\frac{1}{2}d^2\left(\frac{9}{13}\int \frac{1}{(d+ex)^3(d^2-e^2x^2)^{7/2}}dx-\frac{14}{13e(d+ex)^4(d^2-e^2x^2)^{5/2}}\right)-\frac{3d}{2e(d+ex)^3(d^2-e^2x^2)^{5/2}}}{7e^3}+\frac{1}{7e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{461}$$

$$\frac{-\frac{1}{2}d^2\left(\frac{9}{13}\left(\frac{8\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}dx}{11d}-\frac{1}{11de(d+ex)^3(d^2-e^2x^2)^{5/2}}\right)-\frac{14}{13e(d+ex)^4(d^2-e^2x^2)^{5/2}}\right)-\frac{3d}{2e(d+ex)^3(d^2-e^2x^2)^{5/2}}}{7e^3}+\frac{1}{7e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{461}$$

$$\frac{-\frac{1}{2}d^2\left(\frac{9}{13}\left(\frac{8\left(\frac{7\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}dx}{9d}-\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{11d}-\frac{1}{11de(d+ex)^3(d^2-e^2x^2)^{5/2}}\right)-\frac{14}{13e(d+ex)^4(d^2-e^2x^2)^{5/2}}\right)-}{7e^3}$$

$$\frac{1}{7e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{470}$$

$$\frac{-\frac{1}{2}d^2\left(\frac{9}{13}\left(\frac{8\left(\frac{7\left(\frac{6\int \frac{1}{\left(d^2-e^2x^2\right)^{7/2}}dx}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\right)}{9d}-\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{11d}-\frac{1}{11de(d+ex)^3(d^2-e^2x^2)^{5/2}}\right)-\frac{}{13e(d+e}\right)}{7e^3}$$

$$\frac{1}{7e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}$$

3.212. $\int \frac{x^3}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

$\downarrow$ 209

$$-\frac{\frac{1}{2}d^2\left(\frac{9}{13}\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{4\int\frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\right)}{9d}-\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{11d}-\frac{1}{11de(d+ex)^3(d^2-e}\right.\right.}{7e^3}$$

$$\frac{1}{7e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}$$

$\downarrow$ 209

3.212. $\int \frac{x^3}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

$$-\frac{1}{2}d^2\left(\frac{9}{13}\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{4\left(\frac{2\int\frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2}+\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\right)}{9d}-\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{11d}\right)\right)$$

$$7e^3$$

$$\frac{1}{7e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}$$

3.212. $\int \frac{x^3}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

$$\downarrow 208$$

$$\frac{1}{7e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}+$$

$$-\frac{3d}{2e(d+ex)^3(d^2-e^2x^2)^{5/2}}-\frac{1}{2}d^2\left(\frac{9}{13}\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{4\left(\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d^2}\right)}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\right)}{9d}-\frac{}{9de}\cdots}{11d}\cdots\right.\right.$$

$$\overline{\qquad 7e^3 \qquad}$$

input
```
Int[x^3/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

3.212. $\int \frac{x^3}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

output
```
1/(7*e^4*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) + ((-3*d)/(2*e*(d + e*x)^3*(d^
2 - e^2*x^2)^(5/2)) - (d^2*(-14/(13*e*(d + e*x)^4*(d^2 - e^2*x^2)^(5/2)) +
 (9*(-1/11*1/(d*e*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) + (8*(-1/9*1/(d*e*(d
+ e*x)^2*(d^2 - e^2*x^2)^(5/2)) + (7*(-1/7*1/(d*e*(d + e*x)*(d^2 - e^2*x^2
)^(5/2)) + (6*(x/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (4*(x/(3*d^2*(d^2 - e^2*x
^2)^(3/2)) + (2*x)/(3*d^4*Sqrt[d^2 - e^2*x^2])))/(5*d^2)))/(7*d)))/(9*d)))
/(11*d)))/13))/2)/(7*e^3)
```

3.212.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

3.212. $\int \frac{x^3}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 2170
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
 p + q)*(d + e*x)^(q - 2)*(a*e - b*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
 0]] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2,
0] &&  !IGtQ[m, 0]
```

3.212.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.63

method	result	s
gosper	$\frac{(-ex+d)\left(-64e^9x^9-256d\,e^8x^8-224d^2e^7x^7+384d^3e^6x^6+776d^4e^5x^5+160d^5e^4x^4-540d^6e^3x^3+315x^2d^7e^2+360x\,d^8e+90d^9\right)}{5005(ex+d)^3d^7e^4\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}$	1
trager	$\frac{\left(-64e^9x^9-256d\,e^8x^8-224d^2e^7x^7+384d^3e^6x^6+776d^4e^5x^5+160d^5e^4x^4-540d^6e^3x^3+315x^2d^7e^2+360x\,d^8e+90d^9\right)\sqrt{-e^2x^2+d^2}}{5005d^7(ex+d)^7e^4(-ex+d)^3}$	1
default	Expression too large to display	1

input
```
int(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/5005*(-e*x+d)*(-64*e^9*x^9-256*d*e^8*x^8-224*d^2*e^7*x^7+384*d^3*e^6*x^6
+776*d^4*e^5*x^5+160*d^5*e^4*x^4-540*d^6*e^3*x^3+315*d^7*e^2*x^2+360*d^8*e
*x+90*d^9)/(e*x+d)^3/d^7/e^4/(-e^2*x^2+d^2)^(7/2)
```

3.212.5 Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.51

$$\int \frac{x^3}{(d+ex)^4 (d^2-e^2x^2)^{7/2}} dx = \frac{90 e^{10}x^{10} + 360 de^9x^9 + 270 d^2e^8x^8 - 720 d^3e^7x^7 - 1260 d^4e^6x^6 + 1260 d^6e^4}{5005}$$

input
```
integrate(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/5005*(90*e^10*x^10 + 360*d*e^9*x^9 + 270*d^2*e^8*x^8 - 720*d^3*e^7*x^7 -
 1260*d^4*e^6*x^6 + 1260*d^6*e^4*x^4 + 720*d^7*e^3*x^3 - 270*d^8*e^2*x^2 -
 360*d^9*e*x - 90*d^10 + (64*e^9*x^9 + 256*d*e^8*x^8 + 224*d^2*e^7*x^7 - 3
84*d^3*e^6*x^6 - 776*d^4*e^5*x^5 - 160*d^5*e^4*x^4 + 540*d^6*e^3*x^3 - 315
*d^7*e^2*x^2 - 360*d^8*e*x - 90*d^9)*sqrt(-e^2*x^2 + d^2))/(d^7*e^14*x^10
+ 4*d^8*e^13*x^9 + 3*d^9*e^12*x^8 - 8*d^10*e^11*x^7 - 14*d^11*e^10*x^6 + 1
4*d^13*e^8*x^4 + 8*d^14*e^7*x^3 - 3*d^15*e^6*x^2 - 4*d^16*e^5*x - d^17*e^4
)
```

3.212.6 Sympy [F]

$$\int \frac{x^3}{(d+ex)^4 (d^2-e^2x^2)^{7/2}} dx = \int \frac{x^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}} (d+ex)^4} dx$$

input
```
integrate(x**3/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral(x**3/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

3.212.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(181) = 362$.

Time = 0.20 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.91

$$\int \frac{x^3}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \frac{d^2}{13\left((-e^2x^2+d^2)^{\frac{5}{2}}e^8x^4 + 4(-e^2x^2+d^2)^{\frac{5}{2}}de^7x^3 + 6(-e^2x^2+d^2)^{\frac{5}{2}}d^2e^6x^2 + \ldots\right)}$$
$$- \frac{30\,d}{143\left((-e^2x^2+d^2)^{\frac{5}{2}}e^7x^3 + 3(-e^2x^2+d^2)^{\frac{5}{2}}de^6x^2 + 3(-e^2x^2+d^2)^{\frac{5}{2}}d^2e^5x + (-e^2x^2+d^2)^{\frac{5}{2}}d^3e^4\right)}$$
$$+ \frac{21}{143\left((-e^2x^2+d^2)^{\frac{5}{2}}e^6x^2 + 2(-e^2x^2+d^2)^{\frac{5}{2}}de^5x + (-e^2x^2+d^2)^{\frac{5}{2}}d^2e^4\right)}$$
$$+ \frac{4}{1001\left((-e^2x^2+d^2)^{\frac{5}{2}}de^5x + (-e^2x^2+d^2)^{\frac{5}{2}}d^2e^4\right)} - \frac{24\,x}{5005(-e^2x^2+d^2)^{\frac{5}{2}}d^3e^3}$$
$$- \frac{32\,x}{5005(-e^2x^2+d^2)^{\frac{3}{2}}d^5e^3} - \frac{64\,x}{5005\sqrt{-e^2x^2+d^2}\,d^7e^3}$$

input
```
integrate(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/13*d^2/((-e^2*x^2 + d^2)^(5/2)*e^8*x^4 + 4*(-e^2*x^2 + d^2)^(5/2)*d*e^7*
x^3 + 6*(-e^2*x^2 + d^2)^(5/2)*d^2*e^6*x^2 + 4*(-e^2*x^2 + d^2)^(5/2)*d^3*
e^5*x + (-e^2*x^2 + d^2)^(5/2)*d^4*e^4) - 30/143*d/((-e^2*x^2 + d^2)^(5/2)
*e^7*x^3 + 3*(-e^2*x^2 + d^2)^(5/2)*d*e^6*x^2 + 3*(-e^2*x^2 + d^2)^(5/2)*d
^2*e^5*x + (-e^2*x^2 + d^2)^(5/2)*d^3*e^4) + 21/143/((-e^2*x^2 + d^2)^(5/2
)*e^6*x^2 + 2*(-e^2*x^2 + d^2)^(5/2)*d*e^5*x + (-e^2*x^2 + d^2)^(5/2)*d^2*
e^4) + 4/1001/((-e^2*x^2 + d^2)^(5/2)*d*e^5*x + (-e^2*x^2 + d^2)^(5/2)*d^2
*e^4) - 24/5005*x/((-e^2*x^2 + d^2)^(5/2)*d^3*e^3) - 32/5005*x/((-e^2*x^2
+ d^2)^(3/2)*d^5*e^3) - 64/5005*x/(sqrt(-e^2*x^2 + d^2)*d^7*e^3)
```

3.212.8 Giac [F]

$$\int \frac{x^3}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^3}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^4}\,dx$$

input
```
integrate(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate(x^3/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4), x)
```

3.212.9 Mupad [B] (verification not implemented)

Time = 12.08 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.21

$$\int \frac{x^3}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{107}{4004\,d^2\,e^4}-\frac{1139\,x}{80080\,d^3\,e^3}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3} - \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{23}{32032\,d^4\,e^4}+\frac{32\,x}{5005\,d^5\,e^3}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2} + \frac{\sqrt{d^2-e^2\,x^2}}{104\,d\,e^4\,{\left(d+e\,x\right)}^7} - \frac{27\,\sqrt{d^2-e^2\,x^2}}{2288\,d^2\,e^4\,{\left(d+e\,x\right)}^6} - \frac{15\,\sqrt{d^2-e^2\,x^2}}{2288\,d^3\,e^4\,{\left(d+e\,x\right)}^5} + \frac{23\,\sqrt{d^2-e^2\,x^2}}{32032\,d^4\,e^4\,{\left(d+e\,x\right)}^4} - \frac{64\,x\,\sqrt{d^2-e^2\,x^2}}{5005\,d^7\,e^3\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

input
```
int(x^3/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(107/(4004*d^2*e^4) - (1139*x)/(80080*d^3*e^3)))/((
d + e*x)^3*(d - e*x)^3) - ((d^2 - e^2*x^2)^(1/2)*(23/(32032*d^4*e^4) + (32
*x)/(5005*d^5*e^3)))/((d + e*x)^2*(d - e*x)^2) + (d^2 - e^2*x^2)^(1/2)/(10
4*d*e^4*(d + e*x)^7) - (27*(d^2 - e^2*x^2)^(1/2))/(2288*d^2*e^4*(d + e*x)^
6) - (15*(d^2 - e^2*x^2)^(1/2))/(2288*d^3*e^4*(d + e*x)^5) + (23*(d^2 - e^
2*x^2)^(1/2))/(32032*d^4*e^4*(d + e*x)^4) - (64*x*(d^2 - e^2*x^2)^(1/2))/(
5005*d^7*e^3*(d + e*x)*(d - e*x))
```

3.213 $\int \frac{x^2}{(d+ex)^4 (d^2-e^2x^2)^{7/2}} \, dx$

3.213.1 Optimal result

Integrand size = 27, antiderivative size = 209

$$\int \frac{x^2}{(d+ex)^4 (d^2-e^2x^2)^{7/2}} \, dx = \frac{14x}{2145d^4e^2 (d^2-e^2x^2)^{5/2}} - \frac{d}{13e^3(d+ex)^4 (d^2-e^2x^2)^{5/2}} + \frac{17}{143e^3(d+ex)^3 (d^2-e^2x^2)^{5/2}} - \frac{7}{1287de^3(d+ex)^2 (d^2-e^2x^2)^{5/2}} - \frac{7}{1287d^2e^3(d+ex) (d^2-e^2x^2)^{5/2}} + \frac{56x}{6435d^6e^2 (d^2-e^2x^2)^{3/2}} + \frac{112x}{6435d^8e^2\sqrt{d^2-e^2x^2}}$$

output
```
14/2145*x/d^4/e^2/(-e^2*x^2+d^2)^(5/2)-1/13*d/e^3/(e*x+d)^4/(-e^2*x^2+d^2)
^(5/2)+17/143/e^3/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2)-7/1287/d/e^3/(e*x+d)^2/(-
e^2*x^2+d^2)^(5/2)-7/1287/d^2/e^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2)+56/6435*x/d
^6/e^2/(-e^2*x^2+d^2)^(3/2)+112/6435*x/d^8/e^2/(-e^2*x^2+d^2)^(1/2)
```

3.213.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.66

$$\int \frac{x^2}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(200d^9+800d^8ex+700d^7e^2x^2+945d^6e^3x^3-280d^5e^4x^4-135}{6435d^8e^3(d-ex)^3(d+}$$

input
```
Integrate[x^2/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(200*d^9 + 800*d^8*e*x + 700*d^7*e^2*x^2 + 945*d^6*e^
3*x^3 - 280*d^5*e^4*x^4 - 1358*d^4*e^5*x^5 - 672*d^3*e^6*x^6 + 392*d^2*e^7
*x^7 + 448*d*e^8*x^8 + 112*e^9*x^9))/(6435*d^8*e^3*(d - e*x)^3*(d + e*x)^7
)
```

3.213.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.33, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {581, 25, 27, 671, 461, 461, 470, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{ 581}$$

$$\frac{1}{8e^3(d+ex)^3(d^2-e^2x^2)^{5/2}} - \frac{\int -\frac{d(3d-5ex)}{(d+ex)^4(d^2-e^2x^2)^{7/2}}dx}{8e^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{d(3d-5ex)}{(d+ex)^4(d^2-e^2x^2)^{7/2}}dx}{8e^2} + \frac{1}{8e^3(d+ex)^3(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\int \frac{3d-5ex}{(d+ex)^4(d^2-e^2x^2)^{7/2}}dx}{8e^2} + \frac{1}{8e^3(d+ex)^3(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{ 671}$$

$$\frac{d\left(\frac{7}{13}\int \frac{1}{(d+ex)^3(d^2-e^2x^2)^{7/2}}dx-\frac{8}{13e(d+ex)^4(d^2-e^2x^2)^{5/2}}\right)}{8e^2}+\frac{1}{8e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 461$$

$$\frac{d\left(\frac{7}{13}\left(\frac{8\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}dx}{11d}-\frac{1}{11de(d+ex)^3(d^2-e^2x^2)^{5/2}}\right)-\frac{8}{13e(d+ex)^4(d^2-e^2x^2)^{5/2}}\right)}{8e^2}+\frac{1}{8e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 461$$

$$\frac{d\left(\frac{7}{13}\left(\frac{8\left(\frac{7\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}dx}{9d}-\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{11d}-\frac{1}{11de(d+ex)^3(d^2-e^2x^2)^{5/2}}\right)-\frac{8}{13e(d+ex)^4(d^2-e^2x^2)^{5/2}}\right)}{8e^2}+\frac{1}{8e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 470$$

$$\frac{d\left(\frac{7}{13}\left(\frac{8\left(\frac{7\left(\frac{6\int \frac{1}{\left(d^2-e^2x^2\right)^{7/2}}dx}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\right)}{9d}-\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{11d}-\frac{1}{11de(d+ex)^3(d^2-e^2x^2)^{5/2}}\right)-\frac{8}{13e(d+ex)^4(d^2-e^2x^2)^{5/2}}\right)}{8e^2}+\frac{1}{8e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 209$$

3.213. $\int \frac{x^2}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

$$\frac{d\left(\frac{7}{13}\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\right)}{9d}-\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{11d}-\frac{1}{11de(d+ex)^3(d^2-e^2x^2)^{5/2}}\right)\right)}{8e^2}$$

$$\frac{1}{8e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}$$

$\downarrow$ 209

3.213. $\int \frac{x^2}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

$$\frac{d\left(\frac{7}{13}\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{4\left(\frac{2\int\frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\right)}{9d}-\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{11d}-\right.\right.}{8e^2}$$

$$\frac{1}{8e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}$$

3.213. $\int\frac{x^2}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

$$\downarrow 208$$

$$\frac{1}{8e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{d\left(\frac{7}{13}\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{4\left(\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d^2}\right)}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\right)}{9d}-\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{11d}-\frac{1}{11de(d \ldots}\right)\ldots\right)}{8e^2}$$

input
```
Int[x^2/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
1/(8*e^3*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) + (d*(-8/(13*e*(d + e*x)^4*(d^
2 - e^2*x^2)^(5/2)) + (7*(-1/11*1/(d*e*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2))
+ (8*(-1/9*1/(d*e*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) + (7*(-1/7*1/(d*e*(d
+ e*x)*(d^2 - e^2*x^2)^(5/2)) + (6*(x/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (4*(
x/(3*d^2*(d^2 - e^2*x^2)^(3/2)) + (2*x)/(3*d^4*Sqrt[d^2 - e^2*x^2])))/(5*d
^2)))/(7*d)))/(9*d)))/(11*d)))/13))/(8*e^2)
```

3.213. $\int \frac{x^2}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

3.213.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

3.213. $\int \frac{x^2}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

3.213.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.63

method	result
gosper	$\frac{(-ex+d)\left(112e^{9}x^{9}+448d\,e^{8}x^{8}+392d^{2}e^{7}x^{7}-672d^{3}e^{6}x^{6}-1358d^{4}e^{5}x^{5}-280d^{5}e^{4}x^{4}+945d^{6}e^{3}x^{3}+700x^{2}d^{7}e^{2}+800x\,d^{8}e+200d^{9}\right)}{6435(ex+d)^{3}d^{8}e^{3}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}$
trager	$\frac{\left(112e^{9}x^{9}+448d\,e^{8}x^{8}+392d^{2}e^{7}x^{7}-672d^{3}e^{6}x^{6}-1358d^{4}e^{5}x^{5}-280d^{5}e^{4}x^{4}+945d^{6}e^{3}x^{3}+700x^{2}d^{7}e^{2}+800x\,d^{8}e+200d^{9}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{6435d^{8}(ex+d)^{7}(-ex+d)^{3}e^{3}}$
default	Expression too large to display

input
```
int(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/6435*(-e*x+d)*(112*e^9*x^9+448*d*e^8*x^8+392*d^2*e^7*x^7-672*d^3*e^6*x^6
-1358*d^4*e^5*x^5-280*d^5*e^4*x^4+945*d^6*e^3*x^3+700*d^7*e^2*x^2+800*d^8*
e*x+200*d^9)/(e*x+d)^3/d^8/e^3/(-e^2*x^2+d^2)^(7/2)
```

3.213.5 Fricas [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.52

$$\int \frac{x^2}{(d+ex)^4\,(d^2-e^2x^2)^{7/2}}\,dx = \frac{200\,e^{10}x^{10}+800\,de^{9}x^{9}+600\,d^{2}e^{8}x^{8}-1600\,d^{3}e^{7}x^{7}-2800\,d^{4}e^{6}x^{6}+2800\,d^{6}\cdots}{6\cdots}$$

input
```
integrate(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/6435*(200*e^10*x^10 + 800*d*e^9*x^9 + 600*d^2*e^8*x^8 - 1600*d^3*e^7*x^7
 - 2800*d^4*e^6*x^6 + 2800*d^6*e^4*x^4 + 1600*d^7*e^3*x^3 - 600*d^8*e^2*x^
2 - 800*d^9*e*x - 200*d^10 - (112*e^9*x^9 + 448*d*e^8*x^8 + 392*d^2*e^7*x^
7 - 672*d^3*e^6*x^6 - 1358*d^4*e^5*x^5 - 280*d^5*e^4*x^4 + 945*d^6*e^3*x^3
 + 700*d^7*e^2*x^2 + 800*d^8*e*x + 200*d^9)*sqrt(-e^2*x^2 + d^2))/(d^8*e^1
3*x^10 + 4*d^9*e^12*x^9 + 3*d^10*e^11*x^8 - 8*d^11*e^10*x^7 - 14*d^12*e^9*
x^6 + 14*d^14*e^7*x^4 + 8*d^15*e^6*x^3 - 3*d^16*e^5*x^2 - 4*d^17*e^4*x - d
^18*e^3)
```

3.213.6 Sympy [F]

$$\int \frac{x^2}{(d+ex)^4\,(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^4}\,dx$$

input
```
integrate(x**2/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral(x**2/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

3.213.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(181) = 362$.

Time = 0.20 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.92

$$\int \frac{x^2}{(d+ex)^4\,(d^2-e^2x^2)^{7/2}}\,dx =$$

$$-\frac{d}{13\left(\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^7x^4+4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^6x^3+6\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2e^5x^2+4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3e^4x+(-e^2\right.}$$
$$+\frac{17}{143\left(\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6x^3+3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^5x^2+3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2e^4x+\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3e^3\right)}$$
$$-\frac{7}{1287\left(\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^5x^2+2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2e^4x+\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3e^3\right)}$$
$$-\frac{7}{1287\left(\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2e^4x+\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3e^3\right)}+\frac{14\,x}{2145\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4e^2}$$
$$+\frac{56\,x}{6435\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^6e^2}+\frac{112\,x}{6435\sqrt{-e^2x^2+d^2}\,d^8e^2}$$

input
```
integrate(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
-1/13*d/((-e^2*x^2 + d^2)^(5/2)*e^7*x^4 + 4*(-e^2*x^2 + d^2)^(5/2)*d*e^6*x
^3 + 6*(-e^2*x^2 + d^2)^(5/2)*d^2*e^5*x^2 + 4*(-e^2*x^2 + d^2)^(5/2)*d^3*e
^4*x + (-e^2*x^2 + d^2)^(5/2)*d^4*e^3) + 17/143/((-e^2*x^2 + d^2)^(5/2)*e^
6*x^3 + 3*(-e^2*x^2 + d^2)^(5/2)*d*e^5*x^2 + 3*(-e^2*x^2 + d^2)^(5/2)*d^2*
e^4*x + (-e^2*x^2 + d^2)^(5/2)*d^3*e^3) - 7/1287/((-e^2*x^2 + d^2)^(5/2)*d
*e^5*x^2 + 2*(-e^2*x^2 + d^2)^(5/2)*d^2*e^4*x + (-e^2*x^2 + d^2)^(5/2)*d^3
*e^3) - 7/1287/((-e^2*x^2 + d^2)^(5/2)*d^2*e^4*x + (-e^2*x^2 + d^2)^(5/2)*
d^3*e^3) + 14/2145*x/((-e^2*x^2 + d^2)^(5/2)*d^4*e^2) + 56/6435*x/((-e^2*x
^2 + d^2)^(3/2)*d^6*e^2) + 112/6435*x/(sqrt(-e^2*x^2 + d^2)*d^8*e^2)
```

3.213.8 Giac [F]

$$\int \frac{x^2}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^2}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^4}\,dx$$

input
```
integrate(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate(x^2/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4), x)
```

3.213.9 Mupad [B] (verification not implemented)

Time = 11.99 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{227}{6864\,d^3\,e^3}-\frac{353\,x}{17160\,d^4\,e^2}\right)}{(d+e\,x)^3\,(d-e\,x)^3} - \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{353}{41184\,d^5\,e^3}-\frac{56\,x}{6435\,d^6\,e^2}\right)}{(d+e\,x)^2\,(d-e\,x)^2} - \frac{\sqrt{d^2-e^2\,x^2}}{104\,d^2\,e^3\,(d+e\,x)^7} + \frac{\sqrt{d^2-e^2\,x^2}}{2288\,d^3\,e^3\,(d+e\,x)^6} + \frac{37\,\sqrt{d^2-e^2\,x^2}}{5148\,d^4\,e^3\,(d+e\,x)^5} + \frac{353\,\sqrt{d^2-e^2\,x^2}}{41184\,d^5\,e^3\,(d+e\,x)^4} + \frac{112\,x\,\sqrt{d^2-e^2\,x^2}}{6435\,d^8\,e^2\,(d+e\,x)\,(d-e\,x)}$$

input
```
int(x^2/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(227/(6864*d^3*e^3) - (353*x)/(17160*d^4*e^2)))/((d
 + e*x)^3*(d - e*x)^3) - ((d^2 - e^2*x^2)^(1/2)*(353/(41184*d^5*e^3) - (56
*x)/(6435*d^6*e^2)))/((d + e*x)^2*(d - e*x)^2) - (d^2 - e^2*x^2)^(1/2)/(10
4*d^2*e^3*(d + e*x)^7) + (d^2 - e^2*x^2)^(1/2)/(2288*d^3*e^3*(d + e*x)^6)
+ (37*(d^2 - e^2*x^2)^(1/2))/(5148*d^4*e^3*(d + e*x)^5) + (353*(d^2 - e^2*
x^2)^(1/2))/(41184*d^5*e^3*(d + e*x)^4) + (112*x*(d^2 - e^2*x^2)^(1/2))/(6
435*d^8*e^2*(d + e*x)*(d - e*x))
```

3.213. $\int \frac{x^2}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

3.214 $\int \frac{x}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

3.214.1 Optimal result

Integrand size = 25, antiderivative size = 211

$$\int \frac{x}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{64x}{2145d^5e\left(d^2-e^2x^2\right)^{5/2}}+\frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{32}{1287d^2e^2(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{32}{1287d^3e^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}+\frac{256x}{6435d^7e\left(d^2-e^2x^2\right)^{3/2}}+\frac{512x}{6435d^9e\sqrt{d^2-e^2x^2}}$$

output
```
64/2145*x/d^5/e/(-e^2*x^2+d^2)^(5/2)+1/13/e^2/(e*x+d)^4/(-e^2*x^2+d^2)^(5/
2)-4/143/d/e^2/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2)-32/1287/d^2/e^2/(e*x+d)^2/(-
e^2*x^2+d^2)^(5/2)-32/1287/d^3/e^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2)+256/6435*x
/d^7/e/(-e^2*x^2+d^2)^(3/2)+512/6435*x/d^9/e/(-e^2*x^2+d^2)^(1/2)
```

3.214.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.65

$$\int \frac{x}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}(-5d^9-20d^8ex+3200d^7e^2x^2+4320d^6e^3x^3-1280d^5e^4x^4-62}{6435d^9e^2(d-ex)^3(}$$

input
```
Integrate[x/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-5*d^9 - 20*d^8*e*x + 3200*d^7*e^2*x^2 + 4320*d^6*e^
3*x^3 - 1280*d^5*e^4*x^4 - 6208*d^4*e^5*x^5 - 3072*d^3*e^6*x^6 + 1792*d^2*
e^7*x^7 + 2048*d*e^8*x^8 + 512*e^9*x^9))/(6435*d^9*e^2*(d - e*x)^3*(d + e*
x)^7)
```

3.214.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {571, 461, 461, 470, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{ 571}$$

$$\frac{4\int \frac{1}{(d+ex)^3(d^2-e^2x^2)^{7/2}}dx}{13e}+\frac{1}{13e^2(d+ex)^4 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{ 461}$$

$$\frac{4\left(\frac{8\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}dx}{11d}-\frac{1}{11de(d+ex)^3(d^2-e^2x^2)^{5/2}}\right)}{13e}+\frac{1}{13e^2(d+ex)^4 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{ 461}$$

$$\frac{4\left(\frac{8\left(\frac{7\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}dx}{9d}-\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{11d}-\frac{1}{11de(d+ex)^3(d^2-e^2x^2)^{5/2}}\right)}{13e}+\frac{1}{13e^2(d+ex)^4 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{ 470}$$

$$\frac{4\left(\frac{8\left(\frac{7\left(\frac{6\int \frac{1}{\left(d^2-e^2x^2\right)^{7/2}}dx}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\right)}{9d}-\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{11d}-\frac{1}{11de(d+ex)^3(d^2-e^2x^2)^{5/2}}\right)}{13e}+\frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 209$$

$$\frac{4\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\right)}{9d}-\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{11d}-\frac{1}{11de(d+ex)^3(d^2-e^2x^2)^{5/2}}\right)}{13e}\frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 209$$

3.214. $\int \frac{x}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

$$\frac{4\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{4\left(\frac{2\int\frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\right)}{9d}-\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{11d}-\frac{}{11de(\cdots}\right.}{13e}$$

$$\frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}$$

3.214. $\int \frac{x}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

$$\Big\downarrow \; 208$$

$$\frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{4\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{4\left(\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d^2}\right)}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\right)}{9d}-\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{11d}-\frac{1}{11de(d+ex)}\right)}{13e}$$

input
```
Int[x/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
1/(13*e^2*(d + e*x)^4*(d^2 - e^2*x^2)^(5/2)) + (4*(-1/11*1/(d*e*(d + e*x)^
3*(d^2 - e^2*x^2)^(5/2)) + (8*(-1/9*1/(d*e*(d + e*x)^2*(d^2 - e^2*x^2)^(5/
2)) + (7*(-1/7*1/(d*e*(d + e*x)*(d^2 - e^2*x^2)^(5/2)) + (6*(x/(5*d^2*(d^2
 - e^2*x^2)^(5/2)) + (4*(x/(3*d^2*(d^2 - e^2*x^2)^(3/2)) + (2*x)/(3*d^4*Sq
rt[d^2 - e^2*x^2])))/(5*d^2)))/(7*d)))/(9*d)))/(11*d)))/(13*e)
```

3.214. $\int \frac{x}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

3.214.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

rule 571
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*(n + p + 1))), x] + Simp[n/(2*d*
(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ((LtQ[n, -1] &&  !IGtQ[n + p +
1, 0]) || (LtQ[n, 0] && LtQ[p, -1]) || EqQ[n + 2*p + 2, 0]) && NeQ[n + p +
1, 0]
```

3.214. $\int \frac{x}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

3.214.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.63

method	result
gosper	$-\frac{(-ex+d)\left(-512e^9x^9-2048d\,e^8x^8-1792d^2e^7x^7+3072d^3e^6x^6+6208d^4e^5x^5+1280d^5e^4x^4-4320d^6e^3x^3-3200x^2d^7e^2+20x\,d^8e+5d^9\right.}{6435(ex+d)^3d^9e^2\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}$
trager	$-\frac{\left(-512e^9x^9-2048d\,e^8x^8-1792d^2e^7x^7+3072d^3e^6x^6+6208d^4e^5x^5+1280d^5e^4x^4-4320d^6e^3x^3-3200x^2d^7e^2+20x\,d^8e+5d^9\right)\sqrt{-e^2x^2}}{6435d^9(ex+d)^7(-ex+d)^3e^2}$

3.214. $\int \frac{x}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

input
```
int(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6435*(-e*x+d)*(-512*e^9*x^9-2048*d*e^8*x^8-1792*d^2*e^7*x^7+3072*d^3*e^
6*x^6+6208*d^4*e^5*x^5+1280*d^5*e^4*x^4-4320*d^6*e^3*x^3-3200*d^7*e^2*x^2+
20*d^8*e*x+5*d^9)/(e*x+d)^3/d^9/e^2/(-e^2*x^2+d^2)^(7/2)
```

3.214.5 Fricas [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.50

$$\int \frac{x}{(d+ex)^4 (d^2-e^2x^2)^{7/2}} dx =$$
$$-\frac{5 e^{10}x^{10} + 20 de^9x^9 + 15 d^2e^8x^8 - 40 d^3e^7x^7 - 70 d^4e^6x^6 + 70 d^6e^4x^4 + 40 d^7e^3x^3 - 15 d^8e^2x^2 - 20 d^9ex -}{6435 (d^9e^{12}x^{10} + 4 d^{10}e^{11}x^9 + 3 d^{11}e^{10}x}$$

input
```
integrate(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/6435*(5*e^10*x^10 + 20*d*e^9*x^9 + 15*d^2*e^8*x^8 - 40*d^3*e^7*x^7 - 70
*d^4*e^6*x^6 + 70*d^6*e^4*x^4 + 40*d^7*e^3*x^3 - 15*d^8*e^2*x^2 - 20*d^9*e
*x - 5*d^10 + (512*e^9*x^9 + 2048*d*e^8*x^8 + 1792*d^2*e^7*x^7 - 3072*d^3*
e^6*x^6 - 6208*d^4*e^5*x^5 - 1280*d^5*e^4*x^4 + 4320*d^6*e^3*x^3 + 3200*d^
7*e^2*x^2 - 20*d^8*e*x - 5*d^9)*sqrt(-e^2*x^2 + d^2))/(d^9*e^12*x^10 + 4*d
^10*e^11*x^9 + 3*d^11*e^10*x^8 - 8*d^12*e^9*x^7 - 14*d^13*e^8*x^6 + 14*d^1
5*e^6*x^4 + 8*d^16*e^5*x^3 - 3*d^17*e^4*x^2 - 4*d^18*e^3*x - d^19*e^2)
```

3.214.6 Sympy [F]

$$\int \frac{x}{(d+ex)^4 (d^2-e^2x^2)^{7/2}} dx = \int \frac{x}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^4} dx$$

input
```
integrate(x/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral(x/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

3.214.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(183) = 366$.

Time = 0.22 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.92

$$\int \frac{x}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \frac{1}{13\left((-e^2x^2+d^2)^{\frac{5}{2}}e^6x^4 + 4(-e^2x^2+d^2)^{\frac{5}{2}}de^5x^3 + 6(-e^2x^2+d^2)^{\frac{5}{2}}d^2e^4x^2 +\right.}$$

$$-\frac{4}{143\left((-e^2x^2+d^2)^{\frac{5}{2}}de^5x^3 + 3(-e^2x^2+d^2)^{\frac{5}{2}}d^2e^4x^2 + 3(-e^2x^2+d^2)^{\frac{5}{2}}d^3e^3x + (-e^2x^2+d^2)^{\frac{5}{2}}d^4e^2\right)}$$

$$-\frac{32}{1287\left((-e^2x^2+d^2)^{\frac{5}{2}}d^2e^4x^2 + 2(-e^2x^2+d^2)^{\frac{5}{2}}d^3e^3x + (-e^2x^2+d^2)^{\frac{5}{2}}d^4e^2\right)}$$

$$-\frac{32}{1287\left((-e^2x^2+d^2)^{\frac{5}{2}}d^3e^3x + (-e^2x^2+d^2)^{\frac{5}{2}}d^4e^2\right)} + \frac{64\,x}{2145\,(-e^2x^2+d^2)^{\frac{5}{2}}d^5e}$$

$$+\frac{256\,x}{6435\,(-e^2x^2+d^2)^{\frac{3}{2}}d^7e} + \frac{512\,x}{6435\,\sqrt{-e^2x^2+d^2}d^9e}$$

input
```
integrate(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/13/((-e^2*x^2 + d^2)^(5/2)*e^6*x^4 + 4*(-e^2*x^2 + d^2)^(5/2)*d*e^5*x^3
+ 6*(-e^2*x^2 + d^2)^(5/2)*d^2*e^4*x^2 + 4*(-e^2*x^2 + d^2)^(5/2)*d^3*e^3*
x + (-e^2*x^2 + d^2)^(5/2)*d^4*e^2) - 4/143/((-e^2*x^2 + d^2)^(5/2)*d*e^5*
x^3 + 3*(-e^2*x^2 + d^2)^(5/2)*d^2*e^4*x^2 + 3*(-e^2*x^2 + d^2)^(5/2)*d^3*
e^3*x + (-e^2*x^2 + d^2)^(5/2)*d^4*e^2) - 32/1287/((-e^2*x^2 + d^2)^(5/2)*
d^2*e^4*x^2 + 2*(-e^2*x^2 + d^2)^(5/2)*d^3*e^3*x + (-e^2*x^2 + d^2)^(5/2)*
d^4*e^2) - 32/1287/((-e^2*x^2 + d^2)^(5/2)*d^3*e^3*x + (-e^2*x^2 + d^2)^(5
/2)*d^4*e^2) + 64/2145*x/((-e^2*x^2 + d^2)^(5/2)*d^5*e) + 256/6435*x/((-e^
2*x^2 + d^2)^(3/2)*d^7*e) + 512/6435*x/(sqrt(-e^2*x^2 + d^2)*d^9*e)
```

3.214.8 Giac [F]

$$\int \frac{x}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^4}\,dx$$

input
```
integrate(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate(x/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4), x)
```

3.214.9 Mupad [B] (verification not implemented)

Time = 11.92 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.19

$$\int \frac{x}{(d+ex)^4\,(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{41}{41184\,d^6\,e^2}+\frac{256\,x}{6435\,d^7\,e}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2} - \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{47}{1716\,d^4\,e^2}-\frac{1369\,x}{34320\,d^5\,e}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3} + \frac{\sqrt{d^2-e^2\,x^2}}{104\,d^3\,e^2\,{\left(d+e\,x\right)}^7} + \frac{25\,\sqrt{d^2-e^2\,x^2}}{2288\,d^4\,e^2\,{\left(d+e\,x\right)}^6} + \frac{125\,\sqrt{d^2-e^2\,x^2}}{20592\,d^5\,e^2\,{\left(d+e\,x\right)}^5} - \frac{41\,\sqrt{d^2-e^2\,x^2}}{41184\,d^6\,e^2\,{\left(d+e\,x\right)}^4} + \frac{512\,x\,\sqrt{d^2-e^2\,x^2}}{6435\,d^9\,e\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

input
```
int(x/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(41/(41184*d^6*e^2) + (256*x)/(6435*d^7*e)))/((d +
e*x)^2*(d - e*x)^2) - ((d^2 - e^2*x^2)^(1/2)*(47/(1716*d^4*e^2) - (1369*x)
/(34320*d^5*e)))/((d + e*x)^3*(d - e*x)^3) + (d^2 - e^2*x^2)^(1/2)/(104*d^
3*e^2*(d + e*x)^7) + (25*(d^2 - e^2*x^2)^(1/2))/(2288*d^4*e^2*(d + e*x)^6)
 + (125*(d^2 - e^2*x^2)^(1/2))/(20592*d^5*e^2*(d + e*x)^5) - (41*(d^2 - e^
2*x^2)^(1/2))/(41184*d^6*e^2*(d + e*x)^4) + (512*x*(d^2 - e^2*x^2)^(1/2))/
(6435*d^9*e*(d + e*x)*(d - e*x))
```

3.215 $\int \frac{1}{(d+ex)^4 \left(d^2-e^2x^2\right)^{7/2}} \, dx$

3.215.1 Optimal result

Integrand size = 24, antiderivative size = 205

$$\int \frac{1}{(d+ex)^4 \left(d^2-e^2x^2\right)^{7/2}} \, dx = \frac{48x}{715d^6 \left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{13de(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}} - \frac{8}{143d^3e(d+ex)^2 \left(d^2-e^2x^2\right)^{5/2}} - \frac{8}{143d^4e(d+ex) \left(d^2-e^2x^2\right)^{5/2}} + \frac{64x}{715d^8 \left(d^2-e^2x^2\right)^{3/2}} + \frac{128x}{715d^{10}\sqrt{d^2-e^2x^2}}$$

output
```
48/715*x/d^6/(-e^2*x^2+d^2)^(5/2)-1/13/d/e/(e*x+d)^4/(-e^2*x^2+d^2)^(5/2)-
9/143/d^2/e/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2)-8/143/d^3/e/(e*x+d)^2/(-e^2*x^2
+d^2)^(5/2)-8/143/d^4/e/(e*x+d)/(-e^2*x^2+d^2)^(5/2)+64/715*x/d^8/(-e^2*x^
2+d^2)^(3/2)+128/715*x/d^10/(-e^2*x^2+d^2)^(1/2)
```

3.215.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.67

$$\int \frac{1}{(d+ex)^4 \left(d^2-e^2x^2\right)^{7/2}} \, dx = \frac{\sqrt{d^2-e^2x^2}(-180d^9-5d^8ex+800d^7e^2x^2+1080d^6e^3x^3-320d^5e^4x^4-15}{715d^{10}e(d-ex)^3(d+}$$

input
```
Integrate[1/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-180*d^9 - 5*d^8*e*x + 800*d^7*e^2*x^2 + 1080*d^6*e^
3*x^3 - 320*d^5*e^4*x^4 - 1552*d^4*e^5*x^5 - 768*d^3*e^6*x^6 + 448*d^2*e^7
*x^7 + 512*d*e^8*x^8 + 128*e^9*x^9))/(715*d^10*e*(d - e*x)^3*(d + e*x)^7)
```

3.215.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {461, 461, 461, 470, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^4 \left(d^2-e^2x^2\right)^{7/2}} \, dx$$

$$\Big\downarrow \; 461$$

$$\frac{9 \int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{7/2}}dx}{13d} - \frac{1}{13de(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 461$$

$$\frac{9 \left(\frac{8 \int \frac{1}{(d+ex)^2 \left(d^2-e^2x^2\right)^{7/2}}dx}{11d} - \frac{1}{11de(d+ex)^3 (d^2-e^2x^2)^{5/2}}\right)}{13d} - \frac{1}{13de(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 461$$

$$\frac{9 \left(\frac{8 \left(\frac{7 \int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}dx}{9d} - \frac{1}{9de(d+ex)^2 \left(d^2-e^2x^2\right)^{5/2}}\right)}{11d} - \frac{1}{11de(d+ex)^3 (d^2-e^2x^2)^{5/2}}\right)}{13d} - \frac{1}{13de(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 470$$

$$\frac{9\left(\frac{8\left(\frac{7\left(\frac{6\int\frac{1}{\left(d^2-e^2x^2\right)^{7/2}}dx}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\right)}{9d}-\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{11d}-\frac{1}{11de(d+ex)^3(d^2-e^2x^2)^{5/2}}\right)}{13d}-\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{9\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{4\int\frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\right)}{9d}-\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{11d}-\frac{1}{11de(d+ex)^3(d^2-e^2x^2)^{5/2}}\right)}{13d}-\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{209}$$

3.215. $\int \frac{1}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

$$\frac{9\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{4\left(\frac{2\int\frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\right)}{9d}-\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{11d}-\frac{}{11de(}\right)}{13d}$$

$$\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}$$

3.215. $\int\frac{1}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

$$\Big\downarrow \text{208}$$

$$\frac{9\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{4\left(\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d^2}\right)}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\right)}{9d}-\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{11d}-\frac{}{11de(d+ex)}}{13d}$$

$$\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[1/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
-1/13*1/(d*e*(d + e*x)^4*(d^2 - e^2*x^2)^(5/2)) + (9*(-1/11*1/(d*e*(d + e*
x)^3*(d^2 - e^2*x^2)^(5/2)) + (8*(-1/9*1/(d*e*(d + e*x)^2*(d^2 - e^2*x^2)^
(5/2)) + (7*(-1/7*1/(d*e*(d + e*x)*(d^2 - e^2*x^2)^(5/2)) + (6*(x/(5*d^2*(
d^2 - e^2*x^2)^(5/2)) + (4*(x/(3*d^2*(d^2 - e^2*x^2)^(3/2)) + (2*x)/(3*d^4
*Sqrt[d^2 - e^2*x^2])))/(5*d^2)))/(7*d)))/(9*d)))/(11*d)))/(13*d)
```

3.215. $\int \frac{1}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

3.215.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

3.215.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.64

method	result
gosper	$-\frac{(-ex+d)(-128e^9x^9-512d\,e^8x^8-448d^2e^7x^7+768d^3e^6x^6+1552d^4e^5x^5+320d^5e^4x^4-1080d^6e^3x^3-800x^2d^7e^2+5x\,d^8e+180d^9)}{715(ex+d)^3d^{10}e(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$-\frac{(-128e^9x^9-512d\,e^8x^8-448d^2e^7x^7+768d^3e^6x^6+1552d^4e^5x^5+320d^5e^4x^4-1080d^6e^3x^3-800x^2d^7e^2+5x\,d^8e+180d^9)\sqrt{-e^2x^2+d^2}}{715d^{10}(ex+d)^7(-ex+d)^3e}$
default	$-\frac{1}{13de\left(x+\frac{d}{e}\right)^4\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}+\frac{9e\left(-\frac{1}{11de\left(x+\frac{d}{e}\right)^3\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}+\frac{8e\left(-\frac{1}{9de\left(x+\frac{d}{e}\right)^2\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right.\right.}\right.}{}\right.}{}$

input
```
int(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

3.215. $\int \frac{1}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

output
```
-1/715*(-e*x+d)*(-128*e^9*x^9-512*d*e^8*x^8-448*d^2*e^7*x^7+768*d^3*e^6*x^
6+1552*d^4*e^5*x^5+320*d^5*e^4*x^4-1080*d^6*e^3*x^3-800*d^7*e^2*x^2+5*d^8*
e*x+180*d^9)/(e*x+d)^3/d^10/e/(-e^2*x^2+d^2)^(7/2)
```

3.215.5 Fricas [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.53

$$\int \frac{1}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\, dx =$$
$$-\frac{180\, e^{10}x^{10} + 720\, de^9x^9 + 540\, d^2e^8x^8 - 1440\, d^3e^7x^7 - 2520\, d^4e^6x^6 + 2520\, d^6e^4x^4 + 1440\, d^7e^3x^3 - 540\, d^8e^2}{715\, (d^{10}e^{11}x^{10} + 4\, d^{11}e^{10}x^9 + 3}$$

input
```
integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/715*(180*e^10*x^10 + 720*d*e^9*x^9 + 540*d^2*e^8*x^8 - 1440*d^3*e^7*x^7
 - 2520*d^4*e^6*x^6 + 2520*d^6*e^4*x^4 + 1440*d^7*e^3*x^3 - 540*d^8*e^2*x^
2 - 720*d^9*e*x - 180*d^10 + (128*e^9*x^9 + 512*d*e^8*x^8 + 448*d^2*e^7*x^
7 - 768*d^3*e^6*x^6 - 1552*d^4*e^5*x^5 - 320*d^5*e^4*x^4 + 1080*d^6*e^3*x^
3 + 800*d^7*e^2*x^2 - 5*d^8*e*x - 180*d^9)*sqrt(-e^2*x^2 + d^2))/(d^10*e^1
1*x^10 + 4*d^11*e^10*x^9 + 3*d^12*e^9*x^8 - 8*d^13*e^8*x^7 - 14*d^14*e^7*x
^6 + 14*d^16*e^5*x^4 + 8*d^17*e^4*x^3 - 3*d^18*e^3*x^2 - 4*d^19*e^2*x - d^
20*e)
```

3.215.6 Sympy [F]

$$\int \frac{1}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\, dx = \int \frac{1}{(-(-d+ex)(d+ex))^{\frac{7}{2}} (d+ex)^4}\, dx$$

input
```
integrate(1/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral(1/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

3.215.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(177) = 354$.

Time = 0.20 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.92

$$\int \frac{1}{(d+ex)^4 (d^2-e^2x^2)^{7/2}} \, dx =$$

$$-\frac{1}{13\left((-e^2x^2+d^2)^{\frac{5}{2}}de^5x^4+4\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2e^4x^3+6\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3e^3x^2+4\,(-e^2x^2+d^2)^{\frac{5}{2}}d^4e^2x+(-\right.}$$

$$-\frac{9}{143\left((-e^2x^2+d^2)^{\frac{5}{2}}d^2e^4x^3+3\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3e^3x^2+3\,(-e^2x^2+d^2)^{\frac{5}{2}}d^4e^2x+(-e^2x^2+d^2)^{\frac{5}{2}}d^5e\right)}$$

$$-\frac{8}{143\left((-e^2x^2+d^2)^{\frac{5}{2}}d^3e^3x^2+2\,(-e^2x^2+d^2)^{\frac{5}{2}}d^4e^2x+(-e^2x^2+d^2)^{\frac{5}{2}}d^5e\right)}$$

$$-\frac{8}{143\left((-e^2x^2+d^2)^{\frac{5}{2}}d^4e^2x+(-e^2x^2+d^2)^{\frac{5}{2}}d^5e\right)}$$

$$+\frac{48\,x}{715\,(-e^2x^2+d^2)^{\frac{5}{2}}d^6}+\frac{64\,x}{715\,(-e^2x^2+d^2)^{\frac{3}{2}}d^8}+\frac{128\,x}{715\,\sqrt{-e^2x^2+d^2}d^{10}}$$

input
```
integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
-1/13/((-e^2*x^2 + d^2)^(5/2)*d*e^5*x^4 + 4*(-e^2*x^2 + d^2)^(5/2)*d^2*e^4
*x^3 + 6*(-e^2*x^2 + d^2)^(5/2)*d^3*e^3*x^2 + 4*(-e^2*x^2 + d^2)^(5/2)*d^4
*e^2*x + (-e^2*x^2 + d^2)^(5/2)*d^5*e) - 9/143/((-e^2*x^2 + d^2)^(5/2)*d^2
*e^4*x^3 + 3*(-e^2*x^2 + d^2)^(5/2)*d^3*e^3*x^2 + 3*(-e^2*x^2 + d^2)^(5/2)
*d^4*e^2*x + (-e^2*x^2 + d^2)^(5/2)*d^5*e) - 8/143/((-e^2*x^2 + d^2)^(5/2)
*d^3*e^3*x^2 + 2*(-e^2*x^2 + d^2)^(5/2)*d^4*e^2*x + (-e^2*x^2 + d^2)^(5/2)
*d^5*e) - 8/143/((-e^2*x^2 + d^2)^(5/2)*d^4*e^2*x + (-e^2*x^2 + d^2)^(5/2)
*d^5*e) + 48/715*x/((-e^2*x^2 + d^2)^(5/2)*d^6) + 64/715*x/((-e^2*x^2 + d^
2)^(3/2)*d^8) + 128/715*x/(sqrt(-e^2*x^2 + d^2)*d^10)
```

3.215.8 Giac [F]

$$\int \frac{1}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\, dx = \int \frac{1}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^4}\, dx$$

input
```
integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4), x)
```

3.215.9 Mupad [B] (verification not implemented)

Time = 11.90 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.18

$$\int \frac{1}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\, dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{64\,x}{715\,d^8}+\frac{189}{4576\,d^7\,e}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2} + \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1139\,x}{5720\,d^6}-\frac{427}{2288\,d^5\,e}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3} - \frac{\sqrt{d^2-e^2\,x^2}}{104\,d^4\,e\,{\left(d+e\,x\right)}^7} - \frac{51\,\sqrt{d^2-e^2\,x^2}}{2288\,d^5\,e\,{\left(d+e\,x\right)}^6} - \frac{19\,\sqrt{d^2-e^2\,x^2}}{572\,d^6\,e\,{\left(d+e\,x\right)}^5} - \frac{189\,\sqrt{d^2-e^2\,x^2}}{4576\,d^7\,e\,{\left(d+e\,x\right)}^4} + \frac{128\,x\,\sqrt{d^2-e^2\,x^2}}{715\,d^{10}\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

input
```
int(1/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*((64*x)/(715*d^8) + 189/(4576*d^7*e)))/((d + e*x)^2
*(d - e*x)^2) + ((d^2 - e^2*x^2)^(1/2)*((1139*x)/(5720*d^6) - 427/(2288*d^
5*e)))/((d + e*x)^3*(d - e*x)^3) - (d^2 - e^2*x^2)^(1/2)/(104*d^4*e*(d + e
*x)^7) - (51*(d^2 - e^2*x^2)^(1/2))/(2288*d^5*e*(d + e*x)^6) - (19*(d^2 -
e^2*x^2)^(1/2))/(572*d^6*e*(d + e*x)^5) - (189*(d^2 - e^2*x^2)^(1/2))/(457
6*d^7*e*(d + e*x)^4) + (128*x*(d^2 - e^2*x^2)^(1/2))/(715*d^10*(d + e*x)*(
d - e*x))
```

3.216 $\int \frac{1}{x(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

3.216.1 Optimal result

Integrand size = 27, antiderivative size = 234

$$\int \frac{1}{x(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx = \frac{8d(d-ex)}{13(d^2-e^2x^2)^{13/2}} - \frac{4ex}{13d(d^2-e^2x^2)^{11/2}} + \frac{13d-40ex}{117d^3(d^2-e^2x^2)^{9/2}} + \frac{117d-320ex}{819d^5(d^2-e^2x^2)^{7/2}} + \frac{273d-640ex}{1365d^7(d^2-e^2x^2)^{5/2}} + \frac{273d-512ex}{819d^9(d^2-e^2x^2)^{3/2}} + \frac{819d-1024ex}{819d^{11}\sqrt{d^2-e^2x^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{11}}$$

output
```
8/13*d*(-e*x+d)/(-e^2*x^2+d^2)^(13/2)-4/13*e*x/d/(-e^2*x^2+d^2)^(11/2)+1/1
17*(-40*e*x+13*d)/d^3/(-e^2*x^2+d^2)^(9/2)+1/819*(-320*e*x+117*d)/d^5/(-e^
2*x^2+d^2)^(7/2)+1/1365*(-640*e*x+273*d)/d^7/(-e^2*x^2+d^2)^(5/2)+1/819*(-
512*e*x+273*d)/d^9/(-e^2*x^2+d^2)^(3/2)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^
11+1/819*(-1024*e*x+819*d)/d^11/(-e^2*x^2+d^2)^(1/2)
```

3.216.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.74

$$\int \frac{1}{x(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(9839d^9+22976d^8ex-4466d^7e^2x^2-56304d^6e^3x^3-34156d^5e^4x^4+40240d^4e^5x^5+45735d^3e}{(d-ex)^3(d+ex)^7}}{4095d^{11}}$$

input
```
Integrate[1/(x*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(9839*d^9 + 22976*d^8*e*x - 4466*d^7*e^2*x^2 - 56304
*d^6*e^3*x^3 - 34156*d^5*e^4*x^4 + 40240*d^4*e^5*x^5 + 45735*d^3*e^6*x^6 -
 1540*d^2*e^7*x^7 - 16385*d*e^8*x^8 - 5120*e^9*x^9))/((d - e*x)^3*(d + e*x
)^7) + 8190*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(4095*d^11)
```

3.216.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.14, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {570, 532, 25, 2336, 27, 532, 25, 532, 27, 532, 27, 532, 25, 532, 27, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \; 570$$

$$\int \frac{(d-ex)^4}{x\left(d^2-e^2x^2\right)^{15/2}}dx$$

$$\Big\downarrow \; 532$$

$$\frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}-\frac{\int -\frac{13d^4-44exd^3-13e^2x^2d^2}{x(d^2-e^2x^2)^{13/2}}dx}{13d^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{13d^4-44exd^3-13e^2x^2d^2}{x(d^2-e^2x^2)^{13/2}}dx}{13d^2}+\frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$$\Big\downarrow \; 2336$$

$$\frac{-\frac{\int -\frac{11d^3(13d-40ex)}{x\left(d^2-e^2x^2\right)^{11/2}}dx}{11d^2}-\frac{4dex}{(d^2-e^2x^2)^{11/2}}}{13d^2}+\frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\int \frac{13d-40ex}{x(d^2-e^2x^2)^{11/2}}dx-\frac{4dex}{(d^2-e^2x^2)^{11/2}}}{13d^2}+\frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$$\Big\downarrow \text{ 532}$$

$$\frac{d\left(\frac{13d-40ex}{9d^2(d^2-e^2x^2)^{9/2}}-\frac{\int -\frac{117d-320ex}{x\left(d^2-e^2x^2\right)^{9/2}}dx}{9d^2}\right)-\frac{4dex}{(d^2-e^2x^2)^{11/2}}}{13d^2}+\frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{d\left(\frac{\int \frac{117d-320ex}{x\left(d^2-e^2x^2\right)^{9/2}}dx}{9d^2}+\frac{13d-40ex}{9d^2(d^2-e^2x^2)^{9/2}}\right)-\frac{4dex}{(d^2-e^2x^2)^{11/2}}}{13d^2}+\frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$$\Big\downarrow \text{ 532}$$

$$\frac{d\left(\frac{\frac{117d-320ex}{7d^2\left(d^2-e^2x^2\right)^{7/2}}-\frac{\int -\frac{3(273d-640ex)}{x\left(d^2-e^2x^2\right)^{7/2}}dx}{7d^2}}{9d^2}+\frac{13d-40ex}{9d^2(d^2-e^2x^2)^{9/2}}\right)-\frac{4dex}{(d^2-e^2x^2)^{11/2}}}{13d^2}+\frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\left(\frac{\frac{3\int \frac{273d-640ex}{x\left(d^2-e^2x^2\right)^{7/2}}dx}{7d^2}+\frac{117d-320ex}{7d^2\left(d^2-e^2x^2\right)^{7/2}}}{9d^2}+\frac{13d-40ex}{9d^2(d^2-e^2x^2)^{9/2}}\right)-\frac{4dex}{(d^2-e^2x^2)^{11/2}}}{13d^2}+\frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$$\Big\downarrow \text{ 532}$$

3.216. $\int \frac{1}{x(d+ex)^4(d^2-e^2x^2)^{7/2}}dx$

$$\frac{d\left(\frac{\frac{3\left(\frac{273d-640ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int -\frac{5(273d-512ex)}{x\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}\right)}{7d^2}+\frac{117d-320ex}{7d^2\left(d^2-e^2x^2\right)^{7/2}}}{9d^2}+\frac{13d-40ex}{9d^2(d^2-e^2x^2)^{9/2}}\right)-\frac{4dex}{(d^2-e^2x^2)^{11/2}}}{13d^2}+\frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{d\left(\frac{\frac{3\left(\frac{\int \frac{273d-512ex}{x\left(d^2-e^2x^2\right)^{5/2}}dx}{d^2}+\frac{273d-640ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}+\frac{117d-320ex}{7d^2\left(d^2-e^2x^2\right)^{7/2}}}{9d^2}+\frac{13d-40ex}{9d^2(d^2-e^2x^2)^{9/2}}\right)-\frac{4dex}{(d^2-e^2x^2)^{11/2}}}{13d^2}+\frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$$\Big\downarrow \; 532$$

$$\frac{d\left(\frac{\frac{3\left(\frac{\frac{273d-512ex}{3d^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int -\frac{819d-1024ex}{x\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}}{d^2}+\frac{273d-640ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}+\frac{117d-320ex}{7d^2\left(d^2-e^2x^2\right)^{7/2}}}{9d^2}+\frac{13d-40ex}{9d^2(d^2-e^2x^2)^{9/2}}\right)-\frac{4dex}{(d^2-e^2x^2)^{11/2}}}{13d^2}+\frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$$\Big\downarrow \; 25$$

3.216. $\int \frac{1}{x(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

$$\frac{d\left(\frac{\frac{3\left(\frac{\frac{\int \frac{819d-1024ex}{x\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{273d-512ex}{3d^2\left(d^2-e^2x^2\right)^{3/2}}}{d^2}+\frac{273d-640ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}+\frac{117d-320ex}{7d^2\left(d^2-e^2x^2\right)^{7/2}}}{9d^2}+\frac{13d-40ex}{9d^2(d^2-e^2x^2)^{9/2}}\right)-\frac{4dex}{(d^2-e^2x^2)^{11/2}}}{13d^2}+\frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$$\Big\downarrow \; 532$$

$$\frac{d\left(\frac{\frac{3\left(\frac{\frac{\frac{819d-1024ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{819d}{x\sqrt{d^2-e^2x^2}}dx}{d^2}}{3d^2}+\frac{273d-512ex}{3d^2\left(d^2-e^2x^2\right)^{3/2}}}{d^2}+\frac{273d-640ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}+\frac{117d-320ex}{7d^2\left(d^2-e^2x^2\right)^{7/2}}}{9d^2}+\frac{13d-40ex}{9d^2(d^2-e^2x^2)^{9/2}}\right)-\frac{4dex}{(d^2-e^2x^2)^{11/2}}}{13d^2}+\frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$$\Big\downarrow \; 27$$

3.216. $\int \frac{1}{x(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

$$\frac{d\left(\frac{\frac{3\left(\frac{\frac{\frac{819\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx}{d}+\frac{819d-1024ex}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{273d-512ex}{3d^2\left(d^2-e^2x^2\right)^{3/2}}}{d^2}+\frac{273d-640ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}+\frac{117d-320ex}{7d^2\left(d^2-e^2x^2\right)^{7/2}}}{9d^2}+\frac{13d-40ex}{9d^2(d^2-e^2x^2)^{9/2}}\right)-\frac{4dex}{(d^2-e^2x^2)^{11/}}}{13d^2}$$

$$\frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$\downarrow$ 243

$$\frac{d\left(\frac{\frac{3\left(\frac{\frac{\frac{819\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2}{2d}+\frac{819d-1024ex}{d^2\sqrt{d^2-e^2x^2}}}{3d^2}+\frac{273d-512ex}{3d^2\left(d^2-e^2x^2\right)^{3/2}}}{d^2}+\frac{273d-640ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}+\frac{117d-320ex}{7d^2\left(d^2-e^2x^2\right)^{7/2}}}{9d^2}+\frac{13d-40ex}{9d^2(d^2-e^2x^2)^{9/2}}\right)-\frac{4dex}{(d^2-e^2x^2)^{1}}}{13d^2}$$

$$\frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$\downarrow$ 73

3.216. $\int \frac{1}{x(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

$$d\left(\frac{\frac{3\left(\frac{\frac{\frac{819d-1024ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{819\int\frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{de^2}}{3d^2}+\frac{273d-512ex}{3d^2\left(d^2-e^2x^2\right)^{3/2}}}{d^2}+\frac{273d-640ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}+\frac{117d-320ex}{7d^2\left(d^2-e^2x^2\right)^{7/2}}}{9d^2}+\frac{13d-40ex}{9d^2(d^2-e^2x^2)^{9/2}}\right)-\frac{4de}{(d^2-e^2x}$$

$$13d^2$$

$$\frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$\downarrow$ 221

$$d\left(\frac{\frac{3\left(\frac{\frac{\frac{819d-1024ex}{d^2\sqrt{d^2-e^2x^2}}-\frac{819\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}}{3d^2}+\frac{273d-512ex}{3d^2\left(d^2-e^2x^2\right)^{3/2}}}{d^2}+\frac{273d-640ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}+\frac{117d-320ex}{7d^2\left(d^2-e^2x^2\right)^{7/2}}}{9d^2}+\frac{13d-40ex}{9d^2(d^2-e^2x^2)^{9/2}}\right)-\frac{4}{(d^2-e}$$

$$13d^2$$

$$\frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

input
```
Int[1/(x*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

3.216. $\int \frac{1}{x(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

output
```
(8*d*(d - e*x))/(13*(d^2 - e^2*x^2)^(13/2)) + ((-4*d*e*x)/(d^2 - e^2*x^2)^
(11/2) + d*((13*d - 40*e*x)/(9*d^2*(d^2 - e^2*x^2)^(9/2)) + ((117*d - 320*
e*x)/(7*d^2*(d^2 - e^2*x^2)^(7/2)) + (3*((273*d - 640*e*x)/(5*d^2*(d^2 - e
^2*x^2)^(5/2)) + ((273*d - 512*e*x)/(3*d^2*(d^2 - e^2*x^2)^(3/2)) + ((819*
d - 1024*e*x)/(d^2*Sqrt[d^2 - e^2*x^2]) - (819*ArcTanh[Sqrt[d^2 - e^2*x^2]
/d])/d^2)/(3*d^2))/d^2))/(7*d^2))/(9*d^2)))/(13*d^2)
```

3.216.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.216.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1327 vs. $2(204) = 408$.

Time = 0.44 (sec) , antiderivative size = 1328, normalized size of antiderivative = 5.68

method	result	size
default	Expression too large to display	1328

input
```
int(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^4*(1/5/d^2/(-e^2*x^2+d^2)^(5/2)+1/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/
d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*
(-e^2*x^2+d^2)^(1/2))/x))))-1/e^3/d*(-1/13/d/e/(x+d/e)^4/(-(x+d/e)^2*e^2+2
*d*e*(x+d/e))^(5/2)+9/13*e/d*(-1/11/d/e/(x+d/e)^3/(-(x+d/e)^2*e^2+2*d*e*(x
+d/e))^(5/2)+8/11*e/d*(-1/9/d/e/(x+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(
5/2)+7/9*e/d*(-1/7/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+6/7*e/
d*(-1/10*(-2*(x+d/e)*e^2+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/
2)+4/5/d^2*(-1/6*(-2*(x+d/e)*e^2+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d
/e))^(3/2)-1/3/e^2/d^4*(-2*(x+d/e)*e^2+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e
))^(1/2)))))))-1/e/d^3*(-1/9/d/e/(x+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^
(5/2)+7/9*e/d*(-1/7/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+6/7*e
/d*(-1/10*(-2*(x+d/e)*e^2+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5
/2)+4/5/d^2*(-1/6*(-2*(x+d/e)*e^2+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+
d/e))^(3/2)-1/3/e^2/d^4*(-2*(x+d/e)*e^2+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/
e))^(1/2)))))-1/d^4*(-1/7/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)
+6/7*e/d*(-1/10*(-2*(x+d/e)*e^2+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/
e))^(5/2)+4/5/d^2*(-1/6*(-2*(x+d/e)*e^2+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d
*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-2*(x+d/e)*e^2+2*d*e)/(-(x+d/e)^2*e^2+2*d*e
*(x+d/e))^(1/2))))-1/e^2/d^2*(-1/11/d/e/(x+d/e)^3/(-(x+d/e)^2*e^2+2*d*e*(x
+d/e))^(5/2)+8/11*e/d*(-1/9/d/e/(x+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e)...
```

3.216.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(205) = 410$.

Time = 0.76 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.85

$$\int \frac{1}{x(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \frac{9839\,e^{10}x^{10} + 39356\,de^9x^9 + 29517\,d^2e^8x^8 - 78712\,d^3e^7x^7 - 137746\,d^4e^6x}{}$$

input
```
integrate(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/4095*(9839*e^10*x^10 + 39356*d*e^9*x^9 + 29517*d^2*e^8*x^8 - 78712*d^3*e
^7*x^7 - 137746*d^4*e^6*x^6 + 137746*d^6*e^4*x^4 + 78712*d^7*e^3*x^3 - 295
17*d^8*e^2*x^2 - 39356*d^9*e*x - 9839*d^10 + 4095*(e^10*x^10 + 4*d*e^9*x^9
 + 3*d^2*e^8*x^8 - 8*d^3*e^7*x^7 - 14*d^4*e^6*x^6 + 14*d^6*e^4*x^4 + 8*d^7
*e^3*x^3 - 3*d^8*e^2*x^2 - 4*d^9*e*x - d^10)*log(-(d - sqrt(-e^2*x^2 + d^2
))/x) + (5120*e^9*x^9 + 16385*d*e^8*x^8 + 1540*d^2*e^7*x^7 - 45735*d^3*e^6
*x^6 - 40240*d^4*e^5*x^5 + 34156*d^5*e^4*x^4 + 56304*d^6*e^3*x^3 + 4466*d^
7*e^2*x^2 - 22976*d^8*e*x - 9839*d^9)*sqrt(-e^2*x^2 + d^2))/(d^11*e^10*x^1
0 + 4*d^12*e^9*x^9 + 3*d^13*e^8*x^8 - 8*d^14*e^7*x^7 - 14*d^15*e^6*x^6 + 1
4*d^17*e^4*x^4 + 8*d^18*e^3*x^3 - 3*d^19*e^2*x^2 - 4*d^20*e*x - d^21)
```

3.216.6 Sympy [F]

$$\int \frac{1}{x(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^4}\,dx$$

input
```
integrate(1/x/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral(1/(x*(-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

3.216.7 Maxima [F]

$$\int \frac{1}{x(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}\left(ex+d\right)^4x}\,dx$$

input
```
integrate(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4*x), x)
```

3.216.8 Giac [F]

$$\int \frac{1}{x(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{1}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^4 x}\,dx$$

input
```
integrate(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4*x), x)
```

3.216.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{1}{x\,(d^2-e^2\,x^2)^{7/2}\,(d+e\,x)^4}\,dx$$

input
```
int(1/(x*(d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

output
```
int(1/(x*(d^2 - e^2*x^2)^(7/2)*(d + e*x)^4), x)
```

3.217 $\int \frac{1}{x^2(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

3.217.1 Optimal result

Integrand size = 27, antiderivative size = 271

$$\begin{aligned}\int \frac{1}{x^2(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx = & -\frac{8e(d-ex)}{13(d^2-e^2x^2)^{13/2}} \\ & -\frac{4e(13d-24ex)}{143d^2(d^2-e^2x^2)^{11/2}}-\frac{e(572d-1103ex)}{1287d^4(d^2-e^2x^2)^{9/2}}-\frac{e(5148d-10111ex)}{9009d^6(d^2-e^2x^2)^{7/2}} \\ & -\frac{e(12012d-23225ex)}{15015d^8(d^2-e^2x^2)^{5/2}}-\frac{e(12012d-21583ex)}{9009d^{10}(d^2-e^2x^2)^{3/2}} \\ & -\frac{e(36036d-52175ex)}{9009d^{12}\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^{12}x}+\frac{4e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{12}}\end{aligned}$$

output
```
-8/13*e*(-e*x+d)/(-e^2*x^2+d^2)^(13/2)-4/143*e*(-24*e*x+13*d)/d^2/(-e^2*x^
2+d^2)^(11/2)-1/1287*e*(-1103*e*x+572*d)/d^4/(-e^2*x^2+d^2)^(9/2)-1/9009*e
*(-10111*e*x+5148*d)/d^6/(-e^2*x^2+d^2)^(7/2)-1/15015*e*(-23225*e*x+12012*
d)/d^8/(-e^2*x^2+d^2)^(5/2)-1/9009*e*(-21583*e*x+12012*d)/d^10/(-e^2*x^2+d
^2)^(3/2)+4*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^12-1/9009*e*(-52175*e*x+36
036*d)/d^12/(-e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^12/x
```

3.217.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^2(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(45045d^{10}+546316d^9ex+1014094d^8e^2x^2-700504d^7e^3x^3-}{}$$
$$+\frac{4\sqrt{d^2}e\log(x)}{d^{13}}-\frac{4\sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{d^{13}}$$

input
```
Integrate[1/(x^2*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(45045*d^10 + 546316*d^9*e*x + 1014094*d^8*e^2*x^2 -
700504*d^7*e^3*x^3 - 3157776*d^6*e^4*x^4 - 1301264*d^5*e^5*x^5 + 2748320*d
^4*e^6*x^6 + 2496180*d^3*e^7*x^7 - 350000*d^2*e^8*x^8 - 1043500*d*e^9*x^9
- 305920*e^10*x^10))/(45045*d^12*x*(-d + e*x)^3*(d + e*x)^7) + (4*Sqrt[d^2
]*e*Log[x])/d^13 - (4*Sqrt[d^2]*e*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/d^
13
```

3.217.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.11, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.704$, Rules used = {570, 532, 25, 2336, 25, 2336, 25, 2336, 27, 2336, 27, 2336, 25, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{(d-ex)^4}{x^2 (d^2-e^2x^2)^{15/2}}dx$$

$$\Big\downarrow \text{532}$$

$$-\frac{\int -\frac{13d^4-52exd^3+83e^2x^2d^2}{x^2(d^2-e^2x^2)^{13/2}}dx}{13d^2}-\frac{8e(d-ex)}{13(d^2-e^2x^2)^{13/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{13d^4-52exd^3+83e^2x^2d^2}{x^2(d^2-e^2x^2)^{13/2}}dx}{13d^2}-\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$\big\downarrow$ 2336

$$\frac{-\frac{\int -\frac{143d^4-572exd^3+960e^2x^2d^2}{x^2\left(d^2-e^2x^2\right)^{11/2}}dx}{11d^2}-\frac{4e(13d-24ex)}{11(d^2-e^2x^2)^{11/2}}}{13d^2}-\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$\big\downarrow$ 25

$$\frac{\frac{\int \frac{143d^4-572exd^3+960e^2x^2d^2}{x^2\left(d^2-e^2x^2\right)^{11/2}}dx}{11d^2}-\frac{4e(13d-24ex)}{11(d^2-e^2x^2)^{11/2}}}{13d^2}-\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$\big\downarrow$ 2336

$$\frac{\frac{-\frac{\int -\frac{1287d^4-5148exd^3+8824e^2x^2d^2}{x^2\left(d^2-e^2x^2\right)^{9/2}}dx}{9d^2}-\frac{e(572d-1103ex)}{9\left(d^2-e^2x^2\right)^{9/2}}}{11d^2}-\frac{4e(13d-24ex)}{11(d^2-e^2x^2)^{11/2}}}{13d^2}-\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$\big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{1287d^4-5148exd^3+8824e^2x^2d^2}{x^2\left(d^2-e^2x^2\right)^{9/2}}dx}{9d^2}-\frac{e(572d-1103ex)}{9\left(d^2-e^2x^2\right)^{9/2}}}{11d^2}-\frac{4e(13d-24ex)}{11(d^2-e^2x^2)^{11/2}}}{13d^2}-\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$\big\downarrow$ 2336

$$\frac{\frac{\frac{-\frac{\int -\frac{3\left(3003d^4-12012exd^3+20222e^2x^2d^2\right)}{x^2\left(d^2-e^2x^2\right)^{7/2}}dx}{7d^2}-\frac{e(5148d-10111ex)}{7\left(d^2-e^2x^2\right)^{7/2}}}{9d^2}-\frac{e(572d-1103ex)}{9\left(d^2-e^2x^2\right)^{9/2}}}{11d^2}-\frac{4e(13d-24ex)}{11(d^2-e^2x^2)^{11/2}}}{13d^2}-\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{\frac{3\int \frac{3003d^4-12012exd^3+20222e^2x^2d^2}{x^2\left(d^2-e^2x^2\right)^{7/2}}dx}{7d^2}-\frac{e(5148d-10111ex)}{7\left(d^2-e^2x^2\right)^{7/2}}}{9d^2}-\frac{e(572d-1103ex)}{9\left(d^2-e^2x^2\right)^{9/2}}}{11d^2}-\frac{4e(13d-24ex)}{11(d^2-e^2x^2)^{11/2}}}{13d^2}-\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

3.217. $\int \frac{1}{x^2(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

$$\Big\downarrow \; 2336$$

$$\frac{\frac{\frac{\frac{3\left(-\frac{\int -\frac{5\left(3003d^4-12012exd^3+18580e^2x^2d^2\right)}{x^2\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}-\frac{e(12012d-23225ex)}{5\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}-\frac{e(5148d-10111ex)}{7\left(d^2-e^2x^2\right)^{7/2}}}{9d^2}-\frac{e(572d-1103ex)}{9\left(d^2-e^2x^2\right)^{9/2}}}{11d^2}-\frac{4e(13d-24ex)}{11(d^2-e^2x^2)^{11/2}}}{13d^2}-\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\frac{3\left(\frac{\int \frac{3003d^4-12012exd^3+18580e^2x^2d^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}dx}{d^2}-\frac{e(12012d-23225ex)}{5\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}-\frac{e(5148d-10111ex)}{7\left(d^2-e^2x^2\right)^{7/2}}}{9d^2}-\frac{e(572d-1103ex)}{9\left(d^2-e^2x^2\right)^{9/2}}}{11d^2}-\frac{4e(13d-24ex)}{11(d^2-e^2x^2)^{11/2}}}{13d^2}-\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$$\Big\downarrow \; 2336$$

$$\frac{\frac{\frac{\frac{3\left(\frac{-\frac{\int -\frac{9009d^4-36036exd^3+43166e^2x^2d^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}-\frac{e(12012d-21583ex)}{3\left(d^2-e^2x^2\right)^{3/2}}}{d^2}-\frac{e(12012d-23225ex)}{5\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}-\frac{e(5148d-10111ex)}{7\left(d^2-e^2x^2\right)^{7/2}}}{9d^2}-\frac{e(572d-1103ex)}{9\left(d^2-e^2x^2\right)^{9/2}}}{11d^2}-\frac{4e(13d-24ex)}{11(d^2-e^2x^2)^{11/2}}}{13d^2}-\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$$\Big\downarrow \; 25$$

3.217. $\displaystyle\int \frac{1}{x^2(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

$$\frac{\frac{\frac{\frac{3\left(\frac{\frac{\int \frac{9009d^4-36036exd^3+43166e^2x^2d^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}-\frac{e(12012d-21583ex)}{3\left(d^2-e^2x^2\right)^{3/2}}}{d^2}-\frac{e(12012d-23225ex)}{5\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}-\frac{e(5148d-10111ex)}{7\left(d^2-e^2x^2\right)^{7/2}}}{9d^2}-\frac{e(572d-1103ex)}{9\left(d^2-e^2x^2\right)^{9/2}}}{11d^2}-\frac{4e(13d-24ex)}{11(d^2-e^2x^2)^{11/2}}}{13d^2}-\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$$\Big\downarrow \; 2336$$

$$\frac{\frac{\frac{\frac{3\left(\frac{\frac{-\frac{\int -\frac{9009d^3(d-4ex)}{x^2\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{e(36036d-52175ex)}{\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{e(12012d-21583ex)}{3\left(d^2-e^2x^2\right)^{3/2}}}{d^2}-\frac{e(12012d-23225ex)}{5\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}-\frac{e(5148d-10111ex)}{7\left(d^2-e^2x^2\right)^{7/2}}}{9d^2}-\frac{e(572d-1103ex)}{9\left(d^2-e^2x^2\right)^{9/2}}}{11d^2}-\frac{4e(13d-24ex)}{11(d^2-e^2x^2)^{11/2}}}{13d^2}-\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\frac{3\left(\frac{\frac{9009d\int \frac{d-4ex}{x^2\sqrt{d^2-e^2x^2}}dx-\frac{e(36036d-52175ex)}{\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{e(12012d-21583ex)}{3\left(d^2-e^2x^2\right)^{3/2}}}{d^2}-\frac{e(12012d-23225ex)}{5\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}-\frac{e(5148d-10111ex)}{7\left(d^2-e^2x^2\right)^{7/2}}}{9d^2}-\frac{e(572d-1103ex)}{9\left(d^2-e^2x^2\right)^{9/2}}}{11d^2}-\frac{4e(13d-24ex)}{11(d^2-e^2x^2)^{11/2}}}{13d^2}-\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$$\Big\downarrow \; 534$$

3.217. $\int \frac{1}{x^2(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

$$\frac{\frac{\frac{3\left(\frac{\frac{9009d\left(-4e\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{e(36036d-52175ex)}{\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{e(12012d-21583ex)}{3\left(d^2-e^2x^2\right)^{3/2}}}{d^2}-\frac{e(12012d-23225ex)}{5\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}-\frac{e(5148d-10111ex)}{7\left(d^2-e^2x^2\right)^{7/2}}}{9d^2}-\frac{e(572d-1103ex)}{9\left(d^2-e^2x^2\right)^{9/2}}}{11d^2}-\ldots}{13d^2}$$

$$\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{\frac{3\left(\frac{\frac{9009d\left(-2e\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx^2-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{e(36036d-52175ex)}{\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{e(12012d-21583ex)}{3\left(d^2-e^2x^2\right)^{3/2}}}{d^2}-\frac{e(12012d-23225ex)}{5\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}-\frac{e(5148d-10111ex)}{7\left(d^2-e^2x^2\right)^{7/2}}}{9d^2}-\frac{e(572d-1103ex)}{9\left(d^2-e^2x^2\right)^{9/2}}}{11d^2}-\ldots}{13d^2}$$

$$\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{3\left(\frac{\frac{9009d\left(\frac{4\int \frac{1}{\frac{d^2}{e^2}-\frac{x^4}{e^2}}d\sqrt{d^2-e^2x^2}}{e}-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{e(36036d-52175ex)}{\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{e(12012d-21583ex)}{3\left(d^2-e^2x^2\right)^{3/2}}}{d^2}-\frac{e(12012d-23225ex)}{5\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}-\frac{e(5148d-10111ex)}{7\left(d^2-e^2x^2\right)^{7/2}}}{9d^2}-\frac{e(572d-1103ex)}{9\left(d^2-e^2x^2\right)^{9/2}}}{11d^2}-\ldots}{13d^2}$$

$$\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

$$\Big\downarrow \; 221$$

3.217. $\int \frac{1}{x^2(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

$$\frac{\frac{\frac{\frac{3\left(\frac{\frac{\frac{9009d\left(\frac{4e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{dx}\right)-\frac{e(36036d-52175ex)}{\sqrt{d^2-e^2x^2}}}{3d^2}-\frac{e(12012d-21583ex)}{3\left(d^2-e^2x^2\right)^{3/2}}}{d^2}-\frac{e(12012d-23225ex)}{5\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}-\frac{e(5148d-10111ex)}{7\left(d^2-e^2x^2\right)^{7/2}}}{9d^2}-\frac{e(572d-1103\ldots}{9\left(d^2-e^2x^2\right)\ldots}}{11d^2}}{13d^2}$$

$$\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}$$

input
```
Int[1/(x^2*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(-8*e*(d - e*x))/(13*(d^2 - e^2*x^2)^(13/2)) + ((-4*e*(13*d - 24*e*x))/(11
*(d^2 - e^2*x^2)^(11/2)) + (-1/9*(e*(572*d - 1103*e*x))/(d^2 - e^2*x^2)^(9
/2) + (-1/7*(e*(5148*d - 10111*e*x))/(d^2 - e^2*x^2)^(7/2) + (3*(-1/5*(e*(
12012*d - 23225*e*x))/(d^2 - e^2*x^2)^(5/2) + (-1/3*(e*(12012*d - 21583*e*
x))/(d^2 - e^2*x^2)^(3/2) + (-((e*(36036*d - 52175*e*x))/Sqrt[d^2 - e^2*x^
2]) + 9009*d*(-(Sqrt[d^2 - e^2*x^2]/(d*x)) + (4*e*ArcTanh[Sqrt[d^2 - e^2*x
^2]/d])/d))/(3*d^2))/d^2))/(7*d^2))/(9*d^2))/(11*d^2))/(13*d^2)
```

3.217.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.217. $\int \frac{1}{x^2(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

3.217.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(239) = 478$.

Time = 0.43 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.92

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^{12}x}-\frac{1257577\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{1921920d^{10}e^2\left(x+\frac{d}{e}\right)^3}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{104d^6e^6\left(x+\frac{d}{e}\right)^7}-\frac{103\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{2288d^7e^5\left(x+\frac{d}{e}\right)^6}-$
default	Expression too large to display

input
```
int(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-(-e^2*x^2+d^2)^(1/2)/d^12/x-1257577/1921920/d^10/e^2/(x+d/e)^3*(-(x+d/e)^
2*e^2+2*d*e*(x+d/e))^(1/2)-1/104/d^6/e^6/(x+d/e)^7*(-(x+d/e)^2*e^2+2*d*e*(
x+d/e))^(1/2)-103/2288/d^7/e^5/(x+d/e)^6*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1
/2)-665/5148/d^8/e^4/(x+d/e)^5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-86917/
288288/d^9/e^3/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-17417683/115
31520/d^11/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+59/3840/d^11/e
/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-569/3840/d^12/(x-d/e)*(-(x
-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-65075293/11531520/d^12/(x+d/e)*(-(x+d/e)^
2*e^2+2*d*e*(x+d/e))^(1/2)+4/d^11*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-
e^2*x^2+d^2)^(1/2))/x)-1/640/d^10/e^2/(x-d/e)^3*(-(x-d/e)^2*e^2-2*d*e*(x-d
/e))^(1/2)
```

3.217.5 Fricas [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.69

$$\int \frac{1}{x^2(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx =$$
$$-\frac{366136\,e^{11}x^{11}+1464544\,de^{10}x^{10}+1098408\,d^2e^9x^9-2929088\,d^3e^8x^8-5125904\,d^4e^7x^7+5125904\,d^6e^5x^5}{}$$

input
```
integrate(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/45045*(366136*e^11*x^11 + 1464544*d*e^10*x^10 + 1098408*d^2*e^9*x^9 - 2
929088*d^3*e^8*x^8 - 5125904*d^4*e^7*x^7 + 5125904*d^6*e^5*x^5 + 2929088*d
^7*e^4*x^4 - 1098408*d^8*e^3*x^3 - 1464544*d^9*e^2*x^2 - 366136*d^10*e*x +
 180180*(e^11*x^11 + 4*d*e^10*x^10 + 3*d^2*e^9*x^9 - 8*d^3*e^8*x^8 - 14*d^
4*e^7*x^7 + 14*d^6*e^5*x^5 + 8*d^7*e^4*x^4 - 3*d^8*e^3*x^3 - 4*d^9*e^2*x^2
 - d^10*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (305920*e^10*x^10 + 1043
500*d*e^9*x^9 + 350000*d^2*e^8*x^8 - 2496180*d^3*e^7*x^7 - 2748320*d^4*e^6
*x^6 + 1301264*d^5*e^5*x^5 + 3157776*d^6*e^4*x^4 + 700504*d^7*e^3*x^3 - 10
14094*d^8*e^2*x^2 - 546316*d^9*e*x - 45045*d^10)*sqrt(-e^2*x^2 + d^2))/(d^
12*e^10*x^11 + 4*d^13*e^9*x^10 + 3*d^14*e^8*x^9 - 8*d^15*e^7*x^8 - 14*d^16
*e^6*x^7 + 14*d^18*e^4*x^5 + 8*d^19*e^3*x^4 - 3*d^20*e^2*x^3 - 4*d^21*e*x^
2 - d^22*x)
```

3.217.6 Sympy [F]

$$\int \frac{1}{x^2(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{1}{x^2 \left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}} \left(d+ex\right)^4}\,dx$$

input
```
integrate(1/x**2/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

3.217.7 Maxima [F]

$$\int \frac{1}{x^2(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{1}{(-e^2x^2+d^2)^{\frac{7}{2}} (ex+d)^4 x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4*x^2), x)
```

3.217.8 Giac [F]

$$\int \frac{1}{x^2(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{1}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^4x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4*x^2), x)
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{1}{x^2\,\left(d^2-e^2\,x^2\right)^{7/2}\,\left(d+e\,x\right)^4}\,dx$$

input
```
int(1/(x^2*(d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

output
```
int(1/(x^2*(d^2 - e^2*x^2)^(7/2)*(d + e*x)^4), x)
```

3.218 $\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}\,dx$

3.218.1 Optimal result

Integrand size = 29, antiderivative size = 102

$$\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}\,dx = -\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}} - \frac{c^2(1-a^2x^2)^{3/2}}{x(c-acx)^{3/2}} + a\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

output
```
-c^2*(-a^2*x^2+1)^(3/2)/x/(-a*c*x+c)^(3/2)+a*arctanh(c^(1/2)*(-a^2*x^2+1)^
(1/2)/(-a*c*x+c)^(1/2))*c^(1/2)-a*c*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2)
```

3.218.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}\,dx = \frac{\sqrt{c-acx}\left((1+2ax)\sqrt{1-a^2x^2} + ax\sqrt{-1+ax}\arctan\left(\frac{\sqrt{-1+ax}}{\sqrt{1-a^2x^2}}\right)\right)}{x(-1+ax)}$$

input
```
Integrate[(Sqrt[c - a*c*x]*Sqrt[1 - a^2*x^2])/x^2,x]
```

output
```
(Sqrt[c - a*c*x]*((1 + 2*a*x)*Sqrt[1 - a^2*x^2] + a*x*Sqrt[-1 + a*x]*ArcTa
n[Sqrt[-1 + a*x]/Sqrt[1 - a^2*x^2]]))/(x*(-1 + a*x))
```

3.218.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {580, 576, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-a^2x^2}\sqrt{c-acx}}{x^2}dx$$

$$\Big\downarrow \text{580}$$

$$-\frac{1}{2}ac\int \frac{\sqrt{1-a^2x^2}}{x\sqrt{c-acx}}dx-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}}$$

$$\Big\downarrow \text{576}$$

$$-\frac{1}{2}ac\left(\frac{\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}dx}{c}+\frac{2\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}}$$

$$\Big\downarrow \text{573}$$

$$-\frac{1}{2}ac\left(\frac{2\sqrt{1-a^2x^2}}{\sqrt{c-acx}}-2\int \frac{1}{1-\frac{c(1-a^2x^2)}{c-acx}}d\frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{1}{2}ac\left(\frac{2\sqrt{1-a^2x^2}}{\sqrt{c-acx}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)}{\sqrt{c}}\right)-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}}$$

input
```
Int[(Sqrt[c - a*c*x]*Sqrt[1 - a^2*x^2])/x^2,x]
```

output
```
-((c^2*(1 - a^2*x^2)^(3/2))/(x*(c - a*c*x)^(3/2))) - (a*c*((2*Sqrt[1 - a^2
*x^2])/Sqrt[c - a*c*x] - (2*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/Sqrt[c - a
*c*x]])/Sqrt[c]))/2
```

3.218.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 576
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-(e*x)^(m + 1))*(c + d*x)^n*((a + b*x^2)^p/(e*(n - m - 1
))), x] - Simp[b*c*(n/(d^2*(n - m - 1)))   Int[(e*x)^m*(c + d*x)^(n + 1)*(a
 + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c^2 + a*
d^2, 0] && EqQ[n + p, 0] && GtQ[p, 0] && NeQ[m - n + 1, 0] &&  !IGtQ[m, 0]
&&  !(IntegerQ[m + p] && LtQ[m + p + 2, 0]) && RationalQ[m]
```

rule 580
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 2)*((a + b*x^2)^(p +
1)/(b*e*(m + 1))), x] + Simp[d*((2*m + p + 3)/(e*(m + 1)))   Int[(e*x)^(m +
 1)*(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p},
 x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p - 1, 0] && LtQ[m, -1] && IntegerQ
[p + 1/2]
```

3.218.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\left(-\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}}{\sqrt{c}}\right)acx+2ax\sqrt{c(ax+1)}\,\sqrt{c}+\sqrt{c(ax+1)}\,\sqrt{c}\right)\sqrt{-c(ax-1)}\,\sqrt{-a^2x^2+1}}{(ax-1)\sqrt{c(ax+1)}\,x\sqrt{c}}$	95
risch	$\frac{(2a^2x^2+3ax+1)\sqrt{-\frac{\left(-a^2x^2+1\right)c}{ax-1}}\,(ax-1)c}{x\sqrt{c(ax+1)}\,\sqrt{-a^2x^2+1}\,\sqrt{-c(ax-1)}}-\frac{a\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{acx+c}}{\sqrt{c}}\right)\sqrt{-\frac{\left(-a^2x^2+1\right)c}{ax-1}}\,(ax-1)}{\sqrt{-a^2x^2+1}\,\sqrt{-c(ax-1)}}$	147

input
```
int((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

3.218. $\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}\,dx$

output
```
(-arctanh((c*(a*x+1))^(1/2)/c^(1/2))*a*c*x+2*a*x*(c*(a*x+1))^(1/2)*c^(1/2)
+(c*(a*x+1))^(1/2)*c^(1/2))*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/
(c*(a*x+1))^(1/2)/x/c^(1/2)
```

3.218.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.13

$$\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}\,dx = \left[\frac{(a^2x^2-ax)\sqrt{c}\log\left(-\frac{a^2cx^2+acx-2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right)+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}(2ax+1)}{2(ax^2-x)}, \frac{(a^2x^2-\right.$$

input
```
integrate((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*((a^2*x^2 - a*x)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a^2*x^2 +
1)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a^2*x^2 + 1)*sqr
t(-a*c*x + c)*(2*a*x + 1))/(a*x^2 - x), ((a^2*x^2 - a*x)*sqrt(-c)*arctan(s
qrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*x
^2 + 1)*sqrt(-a*c*x + c)*(2*a*x + 1))/(a*x^2 - x)]
```

3.218.6 Sympy [F]

$$\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}\,dx = \int \frac{\sqrt{-c(ax-1)}\sqrt{-(ax-1)(ax+1)}}{x^2}\,dx$$

input
```
integrate((-a*c*x+c)**(1/2)*(-a**2*x**2+1)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/x**2, x)
```

3.218.7 Maxima [F]

$$\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}\,dx = \int \frac{\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{x^2}\,dx$$

input
```
integrate((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/x^2, x)
```

3.218.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}\,dx = \int \frac{\sqrt{1-a^2\,x^2}\,\sqrt{c-a\,c\,x}}{x^2}\,dx$$

input
```
int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/x^2,x)
```

output
```
int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/x^2, x)
```

3.219 $\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx$

3.219.1 Optimal result

Integrand size = 29, antiderivative size = 39

$$\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx = -2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

output
```
-2*arctanh(c^(1/2)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2))*c^(1/2)
```

3.219.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx = -\frac{2\sqrt{c-acx}\arctan\left(\frac{\sqrt{-1+ax}}{\sqrt{1-a^2x^2}}\right)}{\sqrt{-1+ax}}$$

input
```
Integrate[Sqrt[c - a*c*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

output
```
(-2*Sqrt[c - a*c*x]*ArcTan[Sqrt[-1 + a*x]/Sqrt[1 - a^2*x^2]])/Sqrt[-1 + a*
x]
```

3.219.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{573}$$

$$-2c\int \frac{1}{1-\frac{c(1-a^2x^2)}{c-acx}}d\frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}$$

$$\Big\downarrow \text{219}$$

$$-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

input
```
Int[Sqrt[c - a*c*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

output
```
-2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x]]
```

3.219.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

3.219.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.49

method	result	size
default	$\frac{2\sqrt{-c(ax-1)}\,\sqrt{-a^2x^2+1}\,\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}}{\sqrt{c}}\right)}{(ax-1)\sqrt{c(ax+1)}}$	58

input
```
int((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/(c*(a*x+1))^(1/2)*c^(1/2)*
arctanh((c*(a*x+1))^(1/2)/c^(1/2))
```

3.219.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.82

$$\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx = \left[\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2\,c}{ax^2-x}\right),\right.$$
$$\left.-2\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{-c}}{a^2cx^2-c}\right)\right]$$

input
```
integrate((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
[sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*s
qrt(c) - 2*c)/(a*x^2 - x)), -2*sqrt(-c)*arctan(sqrt(-a^2*x^2 + 1)*sqrt(-a*
c*x + c)*sqrt(-c)/(a^2*c*x^2 - c))]
```

3.219.6 Sympy [F]

$$\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\sqrt{-c\,(ax-1)}}{x\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

input
```
integrate((-a*c*x+c)**(1/2)/x/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1))/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.219.7 Maxima [F]

$$\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\sqrt{-acx+c}}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a*c*x + c)/(sqrt(-a^2*x^2 + 1)*x), x)
```

3.219.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx = -\frac{2\,c^3\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}-\frac{\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}\right)}{|c|}$$

input
```
integrate((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-2*c^3*(arctan(sqrt(2)*sqrt(c)/sqrt(-c))/(sqrt(-c)*c) - arctan(sqrt(a*c*x
+ c)/sqrt(-c))/(sqrt(-c)*c))/abs(c)
```

3.219.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\sqrt{c-a\,c\,x}}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((c - a*c*x)^(1/2)/(x*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int((c - a*c*x)^(1/2)/(x*(1 - a^2*x^2)^(1/2)), x)
```

3.220 $\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx$

3.220.1 Optimal result

Integrand size = 16, antiderivative size = 35

$$\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx = \sqrt{x}\sqrt{1-ax}+\frac{\arcsin\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

output
```
arcsin(a^(1/2)*x^(1/2))/a^(1/2)+x^(1/2)*(-a*x+1)^(1/2)
```

3.220.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx = \sqrt{x}\sqrt{1-ax}+\frac{2\arctan\left(\frac{\sqrt{a}\sqrt{x}}{-1+\sqrt{1-ax}}\right)}{\sqrt{a}}$$

input
```
Integrate[Sqrt[1 - a*x]/Sqrt[x],x]
```

output
```
Sqrt[x]*Sqrt[1 - a*x] + (2*ArcTan[(Sqrt[a]*Sqrt[x])/(-1 + Sqrt[1 - a*x])])
/Sqrt[a]
```

3.220.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {60, 63, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1-ax}}dx+\sqrt{x}\sqrt{1-ax}$$

$$\Big\downarrow \text{63}$$

$$\int \frac{1}{\sqrt{1-ax}}d\sqrt{x}+\sqrt{x}\sqrt{1-ax}$$

$$\Big\downarrow \text{223}$$

$$\frac{\arcsin\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}+\sqrt{x}\sqrt{1-ax}$$

input
```
Int[Sqrt[1 - a*x]/Sqrt[x],x]
```

output
```
Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

3.220.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

3.220.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(25) = 50.

Time = 0.35 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.63

method	result	size
meijerg	$\frac{\sqrt{-a}\left(-2\sqrt{\pi}\,\sqrt{x}\,\sqrt{-a}\,\sqrt{-ax+1}-\frac{2\sqrt{\pi}\,\sqrt{-a}\,\arcsin\left(\sqrt{a}\,\sqrt{x}\right)}{\sqrt{a}}\right)}{2\sqrt{\pi}\,a}$	57
default	$\sqrt{x}\,\sqrt{-ax+1}+\frac{\sqrt{(-ax+1)x}\,\arctan\left(\frac{\sqrt{a}\left(x-\frac{1}{2a}\right)}{\sqrt{-a\,x^{2}+x}}\right)}{2\sqrt{-ax+1}\,\sqrt{x}\,\sqrt{a}}$	62
risch	$-\frac{\sqrt{x}\,(ax-1)\sqrt{(-ax+1)x}}{\sqrt{-x(ax-1)}\,\sqrt{-ax+1}}+\frac{\sqrt{(-ax+1)x}\,\arctan\left(\frac{\sqrt{a}\left(x-\frac{1}{2a}\right)}{\sqrt{-a\,x^{2}+x}}\right)}{2\sqrt{-ax+1}\,\sqrt{x}\,\sqrt{a}}$	88

input
```
int((-a*x+1)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-a)^(1/2)/Pi^(1/2)/a*(-2*Pi^(1/2)*x^(1/2)*(-a)^(1/2)*(-a*x+1)^(1/2)-2
*Pi^(1/2)*(-a)^(1/2)/a^(1/2)*arcsin(a^(1/2)*x^(1/2)))
```

3.220.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.63

$$\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx$$
$$=\left[\frac{2\sqrt{-ax+1}a\sqrt{x}-\sqrt{-a}\log\left(-2\,ax+2\sqrt{-ax+1}\sqrt{-a}\sqrt{x}+1\right)}{2\,a},\frac{\sqrt{-ax+1}a\sqrt{x}-\sqrt{a}\arctan\left(\frac{\sqrt{-ax+1}}{\sqrt{a}\sqrt{x}}\right.}{a}\right.$$

input
```
integrate((-a*x+1)^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*sqrt(-a*x + 1)*a*sqrt(x) - sqrt(-a)*log(-2*a*x + 2*sqrt(-a*x + 1)*
sqrt(-a)*sqrt(x) + 1))/a, (sqrt(-a*x + 1)*a*sqrt(x) - sqrt(a)*arctan(sqrt(
-a*x + 1)/(sqrt(a)*sqrt(x))))/a]
```

3.220.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx = \begin{cases} i\sqrt{x}\sqrt{ax-1} - \frac{i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}} & \text{for } |ax| > 1 \\ -\frac{ax^{\frac{3}{2}}}{\sqrt{-ax+1}} + \frac{\sqrt{x}}{\sqrt{-ax+1}} + \frac{\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((-a*x+1)**(1/2)/x**(1/2),x)
```

output
```
Piecewise((I*sqrt(x)*sqrt(a*x - 1) - I*acosh(sqrt(a)*sqrt(x))/sqrt(a), Abs
(a*x) > 1), (-a*x**(3/2)/sqrt(-a*x + 1) + sqrt(x)/sqrt(-a*x + 1) + asin(sq
rt(a)*sqrt(x))/sqrt(a), True))
```

3.220.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx = -\frac{\arctan\left(\frac{\sqrt{-ax+1}}{\sqrt{a}\sqrt{x}}\right)}{\sqrt{a}} + \frac{\sqrt{-ax+1}}{\left(a - \frac{ax-1}{x}\right)\sqrt{x}}$$

input
```
integrate((-a*x+1)^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
-arctan(sqrt(-a*x + 1)/(sqrt(a)*sqrt(x)))/sqrt(a) + sqrt(-a*x + 1)/((a - (
a*x - 1)/x)*sqrt(x))
```

3.220.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(25) = 50$.

Time = 5.54 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx = \frac{a\left(\frac{\log\left(\left|-\sqrt{-ax+1}\sqrt{-a}+\sqrt{(ax-1)a+a}\right|\right)}{\sqrt{-a}} + \frac{\sqrt{(ax-1)a+a}\sqrt{-ax+1}}{a}\right)}{|a|}$$

input
```
integrate((-a*x+1)^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
a*(log(abs(-sqrt(-a*x + 1)*sqrt(-a) + sqrt((a*x - 1)*a + a)))/sqrt(-a) + s
qrt((a*x - 1)*a + a)*sqrt(-a*x + 1)/a)/abs(a)
```

3.220.9 Mupad [B] (verification not implemented)

Time = 11.91 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx = \sqrt{x}\,\sqrt{1-a\,x} + \frac{2\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{1-a\,x}-1}\right)}{\sqrt{a}}$$

input
```
int((1 - a*x)^(1/2)/x^(1/2),x)
```

output
```
x^(1/2)*(1 - a*x)^(1/2) + (2*atan((a^(1/2)*x^(1/2))/((1 - a*x)^(1/2) - 1))
)/a^(1/2)
```

3.221 $\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx$

3.221.1 Optimal result

Integrand size = 29, antiderivative size = 35

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx=\sqrt{x}\sqrt{1-ax}+\frac{\arcsin\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

output
```
arcsin(a^(1/2)*x^(1/2))/a^(1/2)+x^(1/2)*(-a*x+1)^(1/2)
```

3.221.2 Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx=\sqrt{x}\sqrt{1-ax}+\frac{\arcsin\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

input
```
Integrate[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 + a*x]),x]
```

output
```
Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

3.221.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {516, 60, 63, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{ax+1}}dx$$

$$\Big\downarrow \text{516}$$

$$\int \frac{\sqrt{1-ax}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1-ax}}dx+\sqrt{x}\sqrt{1-ax}$$

$$\Big\downarrow \text{63}$$

$$\int \frac{1}{\sqrt{1-ax}}d\sqrt{x}+\sqrt{x}\sqrt{1-ax}$$

$$\Big\downarrow \text{223}$$

$$\frac{\arcsin\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}+\sqrt{x}\sqrt{1-ax}$$

input
```
Int[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 + a*x]),x]
```

output
```
Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

3.221.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 516
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[n]))
```

3.221.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(25) = 50.

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.17

method	result	size
default	$\frac{\sqrt{-a^{2}x^{2}+1}\,\sqrt{x}\,\left(2\sqrt{a}\,\sqrt{-x(ax-1)}+\arctan\left(\frac{2ax-1}{2\sqrt{a}\,\sqrt{-x(ax-1)}}\right)\right)}{2\sqrt{ax+1}\,\sqrt{-x(ax-1)}\,\sqrt{a}}$	76
risch	$-\frac{\sqrt{x}\,(ax-1)\sqrt{\frac{x\left(-a^{2}x^{2}+1\right)}{ax+1}}\,\sqrt{ax+1}}{\sqrt{-x(ax-1)}\,\sqrt{-a^{2}x^{2}+1}}+\frac{\arctan\left(\frac{\sqrt{a}\,\left(x-\frac{1}{2a}\right)}{\sqrt{-a\,x^{2}+x}}\right)\sqrt{\frac{x\left(-a^{2}x^{2}+1\right)}{ax+1}}\,\sqrt{ax+1}}{2\sqrt{a}\,\sqrt{x}\,\sqrt{-a^{2}x^{2}+1}}$	132

input
```
int((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-a^2*x^2+1)^(1/2)*x^(1/2)/(a*x+1)^(1/2)*(2*a^(1/2)*(-x*(a*x-1))^(1/2)
+arctan(1/2/a^(1/2)*(2*a*x-1)/(-x*(a*x-1))^(1/2)))/(-x*(a*x-1))^(1/2)/a^(1
/2)
```

3.221.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(25) = 50.

Time = 0.31 (sec) , antiderivative size = 199, normalized size of antiderivative = 5.69

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx$$
$$= \left[\frac{4\sqrt{-a^2x^2+1}\sqrt{ax+1}a\sqrt{x}-(ax+1)\sqrt{-a}\log\left(-\frac{8a^3x^3-4\sqrt{-a^2x^2+1}(2ax-1)\sqrt{ax+1}\sqrt{-a}\sqrt{x}-7ax+1}{ax+1}\right)}{4(a^2x+a)}, \frac{2\sqrt{-a^2x}\cdots}{\cdots}\right.$$

input
```
integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(4*sqrt(-a^2*x^2 + 1)*sqrt(a*x + 1)*a*sqrt(x) - (a*x + 1)*sqrt(-a)*lo
g(-(8*a^3*x^3 - 4*sqrt(-a^2*x^2 + 1)*(2*a*x - 1)*sqrt(a*x + 1)*sqrt(-a)*sq
rt(x) - 7*a*x + 1)/(a*x + 1)))/(a^2*x + a), 1/2*(2*sqrt(-a^2*x^2 + 1)*sqrt
(a*x + 1)*a*sqrt(x) - (a*x + 1)*sqrt(a)*arctan(2*sqrt(-a^2*x^2 + 1)*sqrt(a
*x + 1)*sqrt(a)*sqrt(x)/(2*a^2*x^2 + a*x - 1)))/(a^2*x + a)]
```

3.221.6 Sympy [F]

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx = \int \frac{\sqrt{-(ax-1)(ax+1)}}{\sqrt{x}\sqrt{ax+1}}\,dx$$

input
```
integrate((-a**2*x**2+1)**(1/2)/x**(1/2)/(a*x+1)**(1/2),x)
```

output
```
Integral(sqrt(-(a*x - 1)*(a*x + 1))/(sqrt(x)*sqrt(a*x + 1)), x)
```

3.221.7 Maxima [F]

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx = \int \frac{\sqrt{-a^2x^2+1}}{\sqrt{ax+1}\sqrt{x}}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)/(sqrt(a*x + 1)*sqrt(x)), x)
```

3.221.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(25) = 50$.

Time = 5.44 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.71

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx = \frac{a\left(\frac{\sqrt{2}-\log\left(\left|-\sqrt{2}\sqrt{-a}+\sqrt{-a}\right|\right)}{\sqrt{-a}}+\frac{\log\left(\left|-\sqrt{-ax+1}\sqrt{-a}+\sqrt{(ax-1)a+a}\right|\right)}{\sqrt{-a}}+\frac{\sqrt{(ax-1)a+a}\sqrt{-ax+1}}{a}\right)}{|a|}$$

input
```
integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x, algorithm="giac")
```

output
```
a*((sqrt(2) - log(abs(-sqrt(2)*sqrt(-a) + sqrt(-a))))/sqrt(-a) + log(abs(-
sqrt(-a*x + 1)*sqrt(-a) + sqrt((a*x - 1)*a + a)))/sqrt(-a) + sqrt((a*x - 1
)*a + a)*sqrt(-a*x + 1)/a)/abs(a)
```

3.221.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx = \int \frac{\sqrt{1-a^2\,x^2}}{\sqrt{x}\,\sqrt{a\,x+1}}\,dx$$

input
```
int((1 - a^2*x^2)^(1/2)/(x^(1/2)*(a*x + 1)^(1/2)),x)
```

output
```
int((1 - a^2*x^2)^(1/2)/(x^(1/2)*(a*x + 1)^(1/2)), x)
```

3.222 $\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx$

3.222.1 Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx = \sqrt{x}\sqrt{1+ax}+\frac{\operatorname{arcsinh}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

output
```
arcsinh(a^(1/2)*x^(1/2))/a^(1/2)+x^(1/2)*(a*x+1)^(1/2)
```

3.222.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx = \sqrt{x}\sqrt{1+ax}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{-1+\sqrt{1+ax}}\right)}{\sqrt{a}}$$

input
```
Integrate[Sqrt[1 + a*x]/Sqrt[x],x]
```

output
```
Sqrt[x]*Sqrt[1 + a*x] + (2*ArcTanh[(Sqrt[a]*Sqrt[x])/(-1 + Sqrt[1 + a*x])]
)/Sqrt[a]
```

3.222.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+1}}{\sqrt{x}}dx$$

$$\Big\downarrow \ 60$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{ax+1}}dx+\sqrt{x}\sqrt{ax+1}$$

$$\Big\downarrow \ 63$$

$$\int \frac{1}{\sqrt{ax+1}}d\sqrt{x}+\sqrt{x}\sqrt{ax+1}$$

$$\Big\downarrow \ 222$$

$$\frac{\text{arcsinh}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}+\sqrt{x}\sqrt{ax+1}$$

input
```
Int[Sqrt[1 + a*x]/Sqrt[x],x]
```

output
```
Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

3.222.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

3.222.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

method	result	size
meijerg	$-\frac{-2\sqrt{\pi}\,\sqrt{a}\,\sqrt{x}\,\sqrt{ax+1}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(\sqrt{a}\,\sqrt{x}\right)}{2\sqrt{a}\,\sqrt{\pi}}$	41
default	$\sqrt{x}\,\sqrt{ax+1}+\frac{\sqrt{(ax+1)x}\,\ln\left(\frac{\frac{1}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+x}\right)}{2\sqrt{ax+1}\,\sqrt{x}\,\sqrt{a}}$	57
risch	$\sqrt{x}\,\sqrt{ax+1}+\frac{\sqrt{(ax+1)x}\,\ln\left(\frac{\frac{1}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+x}\right)}{2\sqrt{ax+1}\,\sqrt{x}\,\sqrt{a}}$	57

input
```
int((a*x+1)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^(1/2)/Pi^(1/2)*(-2*Pi^(1/2)*a^(1/2)*x^(1/2)*(a*x+1)^(1/2)-2*Pi^(1/2
)*arcsinh(a^(1/2)*x^(1/2)))
```

3.222.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.65

$$\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx = \left[\frac{2\,\sqrt{ax+1}a\sqrt{x}+\sqrt{a}\log\left(2\,ax+2\,\sqrt{ax+1}\sqrt{a}\sqrt{x}+1\right)}{2\,a}, \frac{\sqrt{ax+1}a\sqrt{x}-\sqrt{-a}\arctan\left(\frac{\sqrt{ax+1}\sqrt{-a}}{a\sqrt{x}}\right)}{a}\right]$$

input
```
integrate((a*x+1)^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*sqrt(a*x + 1)*a*sqrt(x) + sqrt(a)*log(2*a*x + 2*sqrt(a*x + 1)*sqrt
(a)*sqrt(x) + 1))/a, (sqrt(a*x + 1)*a*sqrt(x) - sqrt(-a)*arctan(sqrt(a*x +
 1)*sqrt(-a)/(a*sqrt(x))))/a]
```

3.222.6 Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx = \sqrt{x}\sqrt{ax+1} + \frac{\operatorname{asinh}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

input
```
integrate((a*x+1)**(1/2)/x**(1/2),x)
```

output
```
sqrt(x)*sqrt(a*x + 1) + asinh(sqrt(a)*sqrt(x))/sqrt(a)
```

3.222.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(24) = 48.

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx = -\frac{\log\left(-\frac{\sqrt{a}-\frac{\sqrt{ax+1}}{\sqrt{x}}}{\sqrt{a}+\frac{\sqrt{ax+1}}{\sqrt{x}}}\right)}{2\sqrt{a}} - \frac{\sqrt{ax+1}}{\left(a-\frac{ax+1}{x}\right)\sqrt{x}}$$

input
```
integrate((a*x+1)^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
-1/2*log(-(sqrt(a) - sqrt(a*x + 1)/sqrt(x))/(sqrt(a) + sqrt(a*x + 1)/sqrt(
x)))/sqrt(a) - sqrt(a*x + 1)/((a - (a*x + 1)/x)*sqrt(x))
```

3.222.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(24) = 48.

Time = 6.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx = -\frac{a\left(\frac{\log\left(\left|-\sqrt{ax+1}\sqrt{a}+\sqrt{(ax+1)a-a}\right|\right)}{\sqrt{a}} - \frac{\sqrt{(ax+1)a-a}\sqrt{ax+1}}{a}\right)}{|a|}$$

input
```
integrate((a*x+1)^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
-a*(log(abs(-sqrt(a*x + 1)*sqrt(a) + sqrt((a*x + 1)*a - a)))/sqrt(a) - sqr
t((a*x + 1)*a - a)*sqrt(a*x + 1)/a)/abs(a)
```

3.222.9 Mupad [B] (verification not implemented)

Time = 12.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx=\sqrt{x}\,\sqrt{a\,x+1}+\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{a\,x+1}-1}\right)}{\sqrt{a}}$$

input
```
int((a*x + 1)^(1/2)/x^(1/2),x)
```

output
```
x^(1/2)*(a*x + 1)^(1/2) + (2*atanh((a^(1/2)*x^(1/2))/((a*x + 1)^(1/2) - 1)
))/a^(1/2)
```

3.223 $\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx$

3.223.1 Optimal result

Integrand size = 30, antiderivative size = 34

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx = \sqrt{x}\sqrt{1+ax} + \frac{\operatorname{arcsinh}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

output
```
arcsinh(a^(1/2)*x^(1/2))/a^(1/2)+x^(1/2)*(a*x+1)^(1/2)
```

3.223.2 Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx = \sqrt{x}\sqrt{1+ax} + \frac{\operatorname{arcsinh}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

input
```
Integrate[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 - a*x]),x]
```

output
```
Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

3.223.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {516, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}dx$$
$$\Big\downarrow \text{516}$$
$$\int \frac{\sqrt{ax+1}}{\sqrt{x}}dx$$
$$\Big\downarrow \text{60}$$
$$\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{ax+1}}dx+\sqrt{x}\sqrt{ax+1}$$
$$\Big\downarrow \text{63}$$
$$\int \frac{1}{\sqrt{ax+1}}d\sqrt{x}+\sqrt{x}\sqrt{ax+1}$$
$$\Big\downarrow \text{222}$$
$$\frac{\text{arcsinh}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}+\sqrt{x}\sqrt{ax+1}$$

input
```
Int[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 - a*x]),x]
```

output
```
Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

3.223.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 516
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[n]))
```

3.223.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(24) = 48$.

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.53

method	result	size
default	$-\frac{\sqrt{-a^{2}x^{2}+1}\,\sqrt{x}\,\sqrt{-ax+1}\left(2\sqrt{(ax+1)x}\,\sqrt{a}+\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2(ax-1)\sqrt{(ax+1)x}\,\sqrt{a}}$	86
risch	$-\frac{(ax+1)\sqrt{x}\,\sqrt{\frac{\left(-a^{2}x^{2}+1\right)x(-ax+1)}{(ax-1)^{2}}}\,(ax-1)}{\sqrt{(ax+1)x}\,\sqrt{-a^{2}x^{2}+1}\,\sqrt{-ax+1}}-\frac{\ln\left(\frac{\frac{1}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+x}\right)\sqrt{\frac{\left(-a^{2}x^{2}+1\right)x(-ax+1)}{(ax-1)^{2}}}\,(ax-1)}{2\sqrt{a}\,\sqrt{-a^{2}x^{2}+1}\,\sqrt{x}\,\sqrt{-ax+1}}$	153

input
```
int((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-a^2*x^2+1)^(1/2)*x^(1/2)*(-a*x+1)^(1/2)*(2*((a*x+1)*x)^(1/2)*a^(1/2
)+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a*x-1)/((a*x+1)*
x)^(1/2)/a^(1/2)
```

3.223.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(24) = 48.

Time = 0.29 (sec) , antiderivative size = 208, normalized size of antiderivative = 6.12

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx$$
$$= \left[-\frac{4\sqrt{-a^2x^2+1}\sqrt{-ax+1}a\sqrt{x}-(ax-1)\sqrt{a}\log\left(-\frac{8\,a^3x^3-4\sqrt{-a^2x^2+1}(2\,ax+1)\sqrt{-ax+1}\sqrt{a}\sqrt{x}-7\,ax-1}{ax-1}\right)}{4\,(a^2x-a)},\right.$$
$$\left.-\frac{2\sqrt{-a^2x^2+1}\sqrt{-ax+1}a\sqrt{x}-(ax-1)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a^2x^2+1}\sqrt{-ax+1}\sqrt{-a}\sqrt{x}}{2\,a^2x^2-ax-1}\right)}{2\,(a^2x-a)}\right]$$

input
```
integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*(4*sqrt(-a^2*x^2 + 1)*sqrt(-a*x + 1)*a*sqrt(x) - (a*x - 1)*sqrt(a)*l
og(-(8*a^3*x^3 - 4*sqrt(-a^2*x^2 + 1)*(2*a*x + 1)*sqrt(-a*x + 1)*sqrt(a)*s
qrt(x) - 7*a*x - 1)/(a*x - 1)))/(a^2*x - a), -1/2*(2*sqrt(-a^2*x^2 + 1)*sq
rt(-a*x + 1)*a*sqrt(x) - (a*x - 1)*sqrt(-a)*arctan(2*sqrt(-a^2*x^2 + 1)*sq
rt(-a*x + 1)*sqrt(-a)*sqrt(x)/(2*a^2*x^2 - a*x - 1)))/(a^2*x - a)]
```

3.223.6 Sympy [F]

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx = \int \frac{\sqrt{-(ax-1)(ax+1)}}{\sqrt{x}\sqrt{-ax+1}}\,dx$$

input
```
integrate((-a**2*x**2+1)**(1/2)/x**(1/2)/(-a*x+1)**(1/2),x)
```

output
```
Integral(sqrt(-(a*x - 1)*(a*x + 1))/(sqrt(x)*sqrt(-a*x + 1)), x)
```

3.223.7 Maxima [F]

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx = \int \frac{\sqrt{-a^2x^2+1}}{\sqrt{-ax+1}\sqrt{x}}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)/(sqrt(-a*x + 1)*sqrt(x)), x)
```

3.223.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(24) = 48$.

Time = 5.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.62

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx = -\frac{a\left(\frac{\sqrt{2}-\log\left(\left|-\sqrt{2}\sqrt{a}+\sqrt{a}\right|\right)}{\sqrt{a}}+\frac{\log\left(\left|-\sqrt{ax+1}\sqrt{a}+\sqrt{(ax+1)a-a}\right|\right)}{\sqrt{a}}-\frac{\sqrt{(ax+1)a-a}\sqrt{ax+1}}{a}\right)}{|a|}$$

input
```
integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

output
```
-a*((sqrt(2) - log(abs(-sqrt(2)*sqrt(a) + sqrt(a))))/sqrt(a) + log(abs(-sq
rt(a*x + 1)*sqrt(a) + sqrt((a*x + 1)*a - a)))/sqrt(a) - sqrt((a*x + 1)*a -
 a)*sqrt(a*x + 1)/a)/abs(a)
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx = \int \frac{\sqrt{1-a^2\,x^2}}{\sqrt{x}\,\sqrt{1-a\,x}}\,dx$$

input
```
int((1 - a^2*x^2)^(1/2)/(x^(1/2)*(1 - a*x)^(1/2)),x)
```

output
```
int((1 - a^2*x^2)^(1/2)/(x^(1/2)*(1 - a*x)^(1/2)), x)
```

3.224 $\int \sqrt{x}\sqrt{1-ax}\,dx$

3.224.1 Optimal result

Integrand size = 16, antiderivative size = 63

$$\int \sqrt{x}\sqrt{1-ax}\,dx=-\frac{\sqrt{x}\sqrt{1-ax}}{4a}+\frac{1}{2}x^{3/2}\sqrt{1-ax}+\frac{\arcsin\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}$$

output
```
1/4*arcsin(a^(1/2)*x^(1/2))/a^(3/2)+1/2*x^(3/2)*(-a*x+1)^(1/2)-1/4*x^(1/2)
*(-a*x+1)^(1/2)/a
```

3.224.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \sqrt{x}\sqrt{1-ax}\,dx=\frac{\sqrt{a}\sqrt{x}\sqrt{1-ax}(-1+2ax)+2\arctan\left(\frac{\sqrt{a}\sqrt{x}}{-1+\sqrt{1-ax}}\right)}{4a^{3/2}}$$

input
```
Integrate[Sqrt[x]*Sqrt[1 - a*x],x]
```

output
```
(Sqrt[a]*Sqrt[x]*Sqrt[1 - a*x]*(-1 + 2*a*x) + 2*ArcTan[(Sqrt[a]*Sqrt[x])/(
-1 + Sqrt[1 - a*x])])/(4*a^(3/2))
```

3.224.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {60, 60, 63, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\sqrt{1-ax}\,dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{1-ax}}dx+\frac{1}{2}x^{3/2}\sqrt{1-ax}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{4}\left(\frac{\int \frac{1}{\sqrt{x}\sqrt{1-ax}}dx}{2a}-\frac{\sqrt{x}\sqrt{1-ax}}{a}\right)+\frac{1}{2}x^{3/2}\sqrt{1-ax}$$

$$\Big\downarrow \text{ 63}$$

$$\frac{1}{4}\left(\frac{\int \frac{1}{\sqrt{1-ax}}d\sqrt{x}}{a}-\frac{\sqrt{x}\sqrt{1-ax}}{a}\right)+\frac{1}{2}x^{3/2}\sqrt{1-ax}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{1}{4}\left(\frac{\arcsin\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}}-\frac{\sqrt{x}\sqrt{1-ax}}{a}\right)+\frac{1}{2}x^{3/2}\sqrt{1-ax}$$

input
```
Int[Sqrt[x]*Sqrt[1 - a*x],x]
```

output
```
(x^(3/2)*Sqrt[1 - a*x])/2 + (-((Sqrt[x]*Sqrt[1 - a*x])/a) + ArcSin[Sqrt[a]
*Sqrt[x]]/a^(3/2))/4
```

3.224.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

3.224.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

method	result	size
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,(-a)^{\frac{3}{2}}(-6ax+3)\sqrt{-ax+1}}{6a}-\frac{\sqrt{\pi}\,(-a)^{\frac{3}{2}}\arcsin\left(\sqrt{a}\,\sqrt{x}\right)}{2a^{\frac{3}{2}}}}{2\sqrt{-a}\,\sqrt{\pi}\,a}$	66
default	$-\frac{\sqrt{x}\,(-ax+1)^{\frac{3}{2}}}{2a}+\frac{\sqrt{x}\,\sqrt{-ax+1}+\frac{\sqrt{(-ax+1)x}\,\arctan\left(\frac{\sqrt{a}\,\left(x-\frac{1}{2a}\right)}{\sqrt{-a\,x^{2}+x}}\right)}{2\sqrt{-ax+1}\,\sqrt{x}\,\sqrt{a}}}{4a}$	84
risch	$-\frac{(2ax-1)\sqrt{x}\,(ax-1)\sqrt{(-ax+1)x}}{4a\sqrt{-x(ax-1)}\,\sqrt{-ax+1}}+\frac{\arctan\left(\frac{\sqrt{a}\,\left(x-\frac{1}{2a}\right)}{\sqrt{-a\,x^{2}+x}}\right)\sqrt{(-ax+1)x}}{8a^{\frac{3}{2}}\sqrt{x}\,\sqrt{-ax+1}}$	97

input
```
int(x^(1/2)*(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(-a)^(1/2)/Pi^(1/2)/a*(1/6*Pi^(1/2)*x^(1/2)*(-a)^(3/2)*(-6*a*x+3)/a*(-
a*x+1)^(1/2)-1/2*Pi^(1/2)*(-a)^(3/2)/a^(3/2)*arcsin(a^(1/2)*x^(1/2)))
```

3.224.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.76

$$\int \sqrt{x}\sqrt{1-ax}\,dx$$
$$= \left[\frac{2\,(2\,a^2x-a)\sqrt{-ax+1}\sqrt{x}-\sqrt{-a}\log\left(-2\,ax+2\,\sqrt{-ax+1}\sqrt{-a}\sqrt{x}+1\right)}{8\,a^2}, \frac{(2\,a^2x-a)\sqrt{-ax+1}\sqrt{x}-}{4\,a^2}\right.$$

input
```
integrate(x^(1/2)*(-a*x+1)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(2*(2*a^2*x - a)*sqrt(-a*x + 1)*sqrt(x) - sqrt(-a)*log(-2*a*x + 2*sqr
t(-a*x + 1)*sqrt(-a)*sqrt(x) + 1))/a^2, 1/4*((2*a^2*x - a)*sqrt(-a*x + 1)*
sqrt(x) - sqrt(a)*arctan(sqrt(-a*x + 1)/(sqrt(a)*sqrt(x))))/a^2]
```

3.224.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.35

$$\int \sqrt{x}\sqrt{1-ax}\,dx = \begin{cases} \frac{iax^{\frac{5}{2}}}{2\sqrt{ax-1}} - \frac{3ix^{\frac{3}{2}}}{4\sqrt{ax-1}} + \frac{i\sqrt{x}}{4a\sqrt{ax-1}} - \frac{i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{4a^{\frac{3}{2}}} & \text{for } |ax|>1 \\ -\frac{ax^{\frac{5}{2}}}{2\sqrt{-ax+1}} + \frac{3x^{\frac{3}{2}}}{4\sqrt{-ax+1}} - \frac{\sqrt{x}}{4a\sqrt{-ax+1}} + \frac{\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1/2)*(-a*x+1)**(1/2),x)
```

output
```
Piecewise((I*a*x**(5/2)/(2*sqrt(a*x - 1)) - 3*I*x**(3/2)/(4*sqrt(a*x - 1))
 + I*sqrt(x)/(4*a*sqrt(a*x - 1)) - I*acosh(sqrt(a)*sqrt(x))/(4*a**(3/2)),
Abs(a*x) > 1), (-a*x**(5/2)/(2*sqrt(-a*x + 1)) + 3*x**(3/2)/(4*sqrt(-a*x +
 1)) - sqrt(x)/(4*a*sqrt(-a*x + 1)) + asin(sqrt(a)*sqrt(x))/(4*a**(3/2)),
True))
```

3.224.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.30

$$\int \sqrt{x}\sqrt{1-ax}\,dx = \frac{\frac{\sqrt{-ax+1}a}{\sqrt{x}} - \frac{(-ax+1)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{4\left(a^3 - \frac{2(ax-1)a^2}{x} + \frac{(ax-1)^2a}{x^2}\right)} - \frac{\arctan\left(\frac{\sqrt{-ax+1}}{\sqrt{a}\sqrt{x}}\right)}{4\,a^{\frac{3}{2}}}$$

input
```
integrate(x^(1/2)*(-a*x+1)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(sqrt(-a*x + 1)*a/sqrt(x) - (-a*x + 1)^(3/2)/x^(3/2))/(a^3 - 2*(a*x -
1)*a^2/x + (a*x - 1)^2*a/x^2) - 1/4*arctan(sqrt(-a*x + 1)/(sqrt(a)*sqrt(x)
))/a^(3/2)
```

3.224.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(43) = 86.

Time = 10.79 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.24

$$\int \sqrt{x}\sqrt{1-ax}\,dx$$
$$= \frac{\frac{\left(\sqrt{(ax-1)a+a}(2\,ax+3)\sqrt{-ax+1} - \frac{3\,a\log\left(\left|-\sqrt{-ax+1}\sqrt{-a}+\sqrt{(ax-1)a+a}\right|\right)}{\sqrt{-a}}\right)|a|}{a^2} + \frac{4\left(\frac{a\log\left(\left|-\sqrt{-ax+1}\sqrt{-a}+\sqrt{(ax-1)a+a}\right|\right)}{\sqrt{-a}} - \sqrt{(ax-1)a+a}\sqrt{-ax}\right.}{a^2}}{4\,a}$$

input
```
integrate(x^(1/2)*(-a*x+1)^(1/2),x, algorithm="giac")
```

output
```
1/4*((sqrt((a*x - 1)*a + a)*(2*a*x + 3)*sqrt(-a*x + 1) - 3*a*log(abs(-sqrt
(-a*x + 1)*sqrt(-a) + sqrt((a*x - 1)*a + a)))/sqrt(-a))*abs(a)/a^2 + 4*(a*
log(abs(-sqrt(-a*x + 1)*sqrt(-a) + sqrt((a*x - 1)*a + a)))/sqrt(-a) - sqrt
((a*x - 1)*a + a)*sqrt(-a*x + 1))*abs(a)/a^2)/a
```

3.224.9 Mupad [B] (verification not implemented)

Time = 11.61 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \sqrt{x}\sqrt{1-ax}\,dx = \sqrt{x}\left(\frac{x}{2}-\frac{1}{4\,a}\right)\,\sqrt{1-a\,x}-\frac{\ln\left(2\,\sqrt{-a}\,\sqrt{x}\,\sqrt{1-a\,x}-2\,a\,x+1\right)}{8\,{\left(-a\right)}^{3/2}}$$

input
```
int(x^(1/2)*(1 - a*x)^(1/2),x)
```

output
```
x^(1/2)*(x/2 - 1/(4*a))*(1 - a*x)^(1/2) - log(2*(-a)^(1/2)*x^(1/2)*(1 - a*
x)^(1/2) - 2*a*x + 1)/(8*(-a)^(3/2))
```

3.225 $\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx$

3.225.1 Optimal result

Integrand size = 29, antiderivative size = 63

$$\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx = -\frac{\sqrt{x}\sqrt{1-ax}}{4a}+\frac{1}{2}x^{3/2}\sqrt{1-ax}+\frac{\arcsin\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}$$

output
```
1/4*arcsin(a^(1/2)*x^(1/2))/a^(3/2)+1/2*x^(3/2)*(-a*x+1)^(1/2)-1/4*x^(1/2)
*(-a*x+1)^(1/2)/a
```

3.225.2 Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx = \frac{\sqrt{a}\sqrt{x}\sqrt{1-ax}(-1+2ax)+\arcsin\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}$$

input
```
Integrate[(Sqrt[x]*Sqrt[1 - a^2*x^2])/Sqrt[1 + a*x],x]
```

output
```
(Sqrt[a]*Sqrt[x]*Sqrt[1 - a*x]*(-1 + 2*a*x) + ArcSin[Sqrt[a]*Sqrt[x]])/(4*
a^(3/2))
```

3.225.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {516, 60, 60, 63, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{ax+1}}dx$$

$$\Big\downarrow \text{516}$$

$$\int \sqrt{x}\sqrt{1-ax}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{1-ax}}dx+\frac{1}{2}x^{3/2}\sqrt{1-ax}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\frac{\int \frac{1}{\sqrt{x}\sqrt{1-ax}}dx}{2a}-\frac{\sqrt{x}\sqrt{1-ax}}{a}\right)+\frac{1}{2}x^{3/2}\sqrt{1-ax}$$

$$\Big\downarrow \text{63}$$

$$\frac{1}{4}\left(\frac{\int \frac{1}{\sqrt{1-ax}}d\sqrt{x}}{a}-\frac{\sqrt{x}\sqrt{1-ax}}{a}\right)+\frac{1}{2}x^{3/2}\sqrt{1-ax}$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{4}\left(\frac{\arcsin\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}}-\frac{\sqrt{x}\sqrt{1-ax}}{a}\right)+\frac{1}{2}x^{3/2}\sqrt{1-ax}$$

input
```
Int[(Sqrt[x]*Sqrt[1 - a^2*x^2])/Sqrt[1 + a*x],x]
```

output
```
(x^(3/2)*Sqrt[1 - a*x])/2 + (-((Sqrt[x]*Sqrt[1 - a*x])/a) + ArcSin[Sqrt[a]
*Sqrt[x]]/a^(3/2))/4
```

3.225.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 516
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[n]))
```

3.225.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(43) = 86$.

Time = 0.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.46

method	result	size
default	$\frac{\sqrt{x}\,\sqrt{-a^{2} x^{2}+1}\,\left(4 a^{\frac{3}{2}} x \sqrt{-x \left(a x -1\right)}-2 \sqrt{a}\,\sqrt{-x \left(a x -1\right)}+\arctan\left(\frac{2 a x -1}{2 \sqrt{a}\,\sqrt{-x \left(a x -1\right)}}\right)\right)}{8 a^{\frac{3}{2}} \sqrt{a x +1}\,\sqrt{-x \left(a x -1\right)}}$	92
risch	$-\frac{\left(2 a x -1\right) \sqrt{x}\,\left(a x -1\right) \sqrt{\frac{x \left(-a^{2} x^{2}+1\right)}{a x +1}}\,\sqrt{a x +1}}{4 a \sqrt{-x \left(a x -1\right)}\,\sqrt{-a^{2} x^{2}+1}}+\frac{\arctan\left(\frac{\sqrt{a}\,\left(x -\frac{1}{2 a}\right)}{\sqrt{-a\,x^{2}+x}}\right) \sqrt{\frac{x \left(-a^{2} x^{2}+1\right)}{a x +1}}\,\sqrt{a x +1}}{8 a^{\frac{3}{2}} \sqrt{x}\,\sqrt{-a^{2} x^{2}+1}}$	141

input
```
int(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x^(1/2)*(-a^2*x^2+1)^(1/2)/a^(3/2)*(4*a^(3/2)*x*(-x*(a*x-1))^(1/2)-2*a
^(1/2)*(-x*(a*x-1))^(1/2)+arctan(1/2/a^(1/2)*(2*a*x-1)/(-x*(a*x-1))^(1/2))
)/(a*x+1)^(1/2)/(-x*(a*x-1))^(1/2)
```

3.225.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(43) = 86$.

Time = 0.33 (sec) , antiderivative size = 221, normalized size of antiderivative = 3.51

$$\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx = \left[\frac{4\sqrt{-a^2x^2+1}(2\,a^2x-a)\sqrt{ax+1}\sqrt{x}-(ax+1)\sqrt{-a}\log\left(-\frac{8\,a^3x^3-4\,\sqrt{-a^2x^2+1}(2\,ax-1)\sqrt{ax+1}\sqrt{-a}\sqrt{x}-7\,ax+1}{ax+1}\right)}{16\,(a^3x+a^2)}\right.$$

input
```
integrate(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(4*sqrt(-a^2*x^2 + 1)*(2*a^2*x - a)*sqrt(a*x + 1)*sqrt(x) - (a*x + 1
)*sqrt(-a)*log(-(8*a^3*x^3 - 4*sqrt(-a^2*x^2 + 1)*(2*a*x - 1)*sqrt(a*x + 1
)*sqrt(-a)*sqrt(x) - 7*a*x + 1)/(a*x + 1)))/(a^3*x + a^2), 1/8*(2*sqrt(-a^
2*x^2 + 1)*(2*a^2*x - a)*sqrt(a*x + 1)*sqrt(x) - (a*x + 1)*sqrt(a)*arctan(
2*sqrt(-a^2*x^2 + 1)*sqrt(a*x + 1)*sqrt(a)*sqrt(x)/(2*a^2*x^2 + a*x - 1)))
/(a^3*x + a^2)]
```

3.225.6 Sympy [F]

$$\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx = \int \frac{\sqrt{x}\sqrt{-(ax-1)(ax+1)}}{\sqrt{ax+1}}\,dx$$

input
```
integrate(x**(1/2)*(-a**2*x**2+1)**(1/2)/(a*x+1)**(1/2),x)
```

output
```
Integral(sqrt(x)*sqrt(-(a*x - 1)*(a*x + 1))/sqrt(a*x + 1), x)
```

3.225.7 Maxima [F]

$$\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx = \int \frac{\sqrt{-a^2x^2+1}\sqrt{x}}{\sqrt{ax+1}}\,dx$$

input
```
integrate(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*sqrt(x)/sqrt(a*x + 1), x)
```

3.225.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.225.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx = \int \frac{\sqrt{x}\,\sqrt{1-a^2\,x^2}}{\sqrt{a\,x+1}}\,dx$$

input
```
int((x^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1)^(1/2),x)
```

output
```
int((x^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1)^(1/2), x)
```

3.226 $\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

3.226.1 Optimal result

Integrand size = 29, antiderivative size = 250

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)}-\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)}+\frac{d^7(11+4m)(gx)^{1+m}\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{g(1+m)(8+m)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^6e(29+4m)(gx)^{2+m}\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{g^2(2+m)(9+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

output
```
-3*d*(g*x)^(1+m)*(-e^2*x^2+d^2)^(7/2)/g/(8+m)-e*(g*x)^(2+m)*(-e^2*x^2+d^2)
^(7/2)/g^2/(9+m)+d^7*(11+4*m)*(g*x)^(1+m)*hypergeom([-5/2, 1/2+1/2*m],[3/2
+1/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g/(1+m)/(8+m)/(1-e^2*x^2/d^2)^(1
/2)+d^6*e*(29+4*m)*(g*x)^(2+m)*hypergeom([-5/2, 1+1/2*m],[2+1/2*m],e^2*x^2
/d^2)*(-e^2*x^2+d^2)^(1/2)/g^2/(2+m)/(9+m)/(1-e^2*x^2/d^2)^(1/2)
```

3.226.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.80

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = \frac{d^4x(gx)^m\sqrt{d^2-e^2x^2}\left(\frac{d^3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{1+m}+ex\left(\frac{3d^2\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{2+m}{2},\frac{4}{}\right.}{2+m}\right.\right.}{\sqrt{1-}}$$

input
```
Integrate[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

output
```
(d^4*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*((d^3*Hypergeometric2F1[-5/2, (1 + m)/2
, (3 + m)/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((3*d^2*Hypergeometric2F1[-5/2,
 (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2
F1[-5/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) + (e*x*Hypergeometr
ic2F1[-5/2, (4 + m)/2, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/Sqrt[1 - (e^
2*x^2)/d^2]
```

3.226.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {559, 25, 2340, 25, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3\left(d^2-e^2x^2\right)^{5/2}(gx)^m\,dx$$

$$\Big\downarrow \text{559}$$

$$-\frac{\int -(gx)^m\left(d^2-e^2x^2\right)^{5/2}\left(3d(m+9)x^2e^4+d^2(4m+29)xe^3+d^3(m+9)e^2\right)dx}{e^2(m+9)}-\frac{e\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+2}}{g^2(m+9)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int (gx)^m \left(d^2-e^2x^2\right)^{5/2} \left(3d(m+9)x^2e^4+d^2(4m+29)xe^3+d^3(m+9)e^2\right)dx}{e^2(m+9)}-\frac{e\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+2}}{g^2(m+9)}$$

$$\Big\downarrow \; 2340$$

$$\frac{-\frac{\int -d^2e^4(gx)^m(d(m+9)(4m+11)+e(m+8)(4m+29)x)\left(d^2-e^2x^2\right)^{5/2}dx}{e^2(m+8)}-\frac{3de^2(m+9)\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+1}}{g(m+8)}}{e^2(m+9)}-\frac{e\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+2}}{g^2(m+9)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int d^2e^4(gx)^m(d(m+9)(4m+11)+e(m+8)(4m+29)x)\left(d^2-e^2x^2\right)^{5/2}dx}{e^2(m+8)}-\frac{3de^2(m+9)\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+1}}{g(m+8)}}{e^2(m+9)}-\frac{e\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+2}}{g^2(m+9)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{d^2e^2\int (gx)^m(d(m+9)(4m+11)+e(m+8)(4m+29)x)\left(d^2-e^2x^2\right)^{5/2}dx}{m+8}-\frac{3de^2(m+9)\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+1}}{g(m+8)}}{e^2(m+9)}-\frac{e\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+2}}{g^2(m+9)}$$

$$\Big\downarrow \; 557$$

$$\frac{\frac{d^2e^2\left(d(m+9)(4m+11)\int (gx)^m\left(d^2-e^2x^2\right)^{5/2}dx+\frac{e(m+8)(4m+29)\int (gx)^{m+1}\left(d^2-e^2x^2\right)^{5/2}dx}{g}\right)}{m+8}-\frac{3de^2(m+9)\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+1}}{g(m+8)}}{e^2(m+9)}-\frac{e\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+2}}{g^2(m+9)}$$

$$\Big\downarrow \; 279$$

$$\frac{\frac{d^2e^2\left(\frac{d^5(m+9)(4m+11)\sqrt{d^2-e^2x^2}\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}dx}{\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^4e(m+8)(4m+29)\sqrt{d^2-e^2x^2}\int (gx)^{m+1}\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}dx}{g\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{m+8}-\frac{3de^2(m+9)\left(d^2-e^2x^2\right)^{7/2}(}{g(m+8)}}{e^2(m+9)}$$

$$\frac{e\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+2}}{g^2(m+9)}$$

3.226. $\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

$$\Big\downarrow 278$$

$$\frac{\frac{d^2e^2\left(\frac{d^5(m+9)(4m+11)\sqrt{d^2-e^2x^2}(gx)^{m+1}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^4e(m+8)(4m+29)\sqrt{d^2-e^2x^2}(gx)^{m+2}\text{Hypergeometric2F1}\left(-\frac{5}{2},m\right.}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}}{m+8}}{e^2(m+9)}$$

$$\frac{e\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+2}}{g^2(m+9)}$$

input
```
Int[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

output
```
-((e*(g*x)^(2 + m)*(d^2 - e^2*x^2)^(7/2))/(g^2*(9 + m))) + ((-3*d*e^2*(9 +
 m)*(g*x)^(1 + m)*(d^2 - e^2*x^2)^(7/2))/(g*(8 + m)) + (d^2*e^2*((d^5*(9 +
 m)*(11 + 4*m)*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (
1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]) +
 (d^4*e*(8 + m)*(29 + 4*m)*(g*x)^(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometri
c2F1[-5/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^
2*x^2)/d^2])))/(8 + m))/(e^2*(9 + m))
```

3.226.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 559
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[d^n*(e*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*e^(n - 1)*(
m + n + 2*p + 1))), x] + Simp[1/(b*(m + n + 2*p + 1))   Int[(e*x)^m*(a + b*
x^2)^p*ExpandToSum[b*(m + n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1
)*x^n - a*d^n*(m + n - 1)*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && IGtQ[n, 1] &&  !IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

3.226.4 Maple [F]

$$\int (gx)^m (ex+d)^3 \left(-e^2x^2+d^2\right)^{\frac{5}{2}} dx$$

input
```
int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

output
```
int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

3.226.5 Fricas [F]

$$\int (gx)^m(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx = \int \left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)^3(gx)^m dx$$

input
```
integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
integral((e^7*x^7 + 3*d*e^6*x^6 + d^2*e^5*x^5 - 5*d^3*e^4*x^4 - 5*d^4*e^3*
x^3 + d^5*e^2*x^2 + 3*d^6*e*x + d^7)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)
```

3.226.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.98 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.01

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{d^8g^mx^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\right|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{3d^7eg^mx^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\right|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}+\frac{d^6e^2g^mx^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\right|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}-\frac{5d^5e^3g^mx^{m+4}\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\right|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+3\right)}-\frac{5d^4e^4g^mx^{m+5}\Gamma\left(\frac{m}{2}+\frac{5}{2}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{5}{2}\\ \frac{m}{2}+\frac{7}{2}\end{matrix}\right|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}+\frac{d^3e^5g^mx^{m+6}\Gamma\left(\frac{m}{2}+3\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{2}+3\\ \frac{m}{2}+4\end{matrix}\right|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+4\right)}+\frac{3d^2e^6g^mx^{m+7}\Gamma\left(\frac{m}{2}+\frac{7}{2}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{7}{2}\\ \frac{m}{2}+\frac{9}{2}\end{matrix}\right|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{9}{2}\right)}+\frac{de^7g^mx^{m+8}\Gamma\left(\frac{m}{2}+4\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{2}+4\\ \frac{m}{2}+5\end{matrix}\right|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+5\right)}$$

input
```
integrate((g*x)**m*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

3.226. $\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

output
```
d**8*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,
), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + 3*d**7*e*g**m*
x**(m + 2)*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp
_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 2)) + d**6*e**2*g**m*x**(m + 3)*gamma(
m/2 + 3/2)*hyper((-1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*
pi)/d**2)/(2*gamma(m/2 + 5/2)) - 5*d**5*e**3*g**m*x**(m + 4)*gamma(m/2 + 2
)*hyper((-1/2, m/2 + 2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*
gamma(m/2 + 3)) - 5*d**4*e**4*g**m*x**(m + 5)*gamma(m/2 + 5/2)*hyper((-1/2
, m/2 + 5/2), (m/2 + 7/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2
 + 7/2)) + d**3*e**5*g**m*x**(m + 6)*gamma(m/2 + 3)*hyper((-1/2, m/2 + 3),
 (m/2 + 4,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 4)) + 3*d**2
*e**6*g**m*x**(m + 7)*gamma(m/2 + 7/2)*hyper((-1/2, m/2 + 7/2), (m/2 + 9/2
,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 9/2)) + d*e**7*g**m*x
**(m + 8)*gamma(m/2 + 4)*hyper((-1/2, m/2 + 4), (m/2 + 5,), e**2*x**2*exp_
polar(2*I*pi)/d**2)/(2*gamma(m/2 + 5))
```

3.226.7 Maxima [F]

$$\int (gx)^m (d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} \, dx = \int \left(-e^2x^2+d^2\right)^{\frac{5}{2}} (ex+d)^3 (gx)^m \, dx$$

input
```
integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3*(g*x)^m, x)
```

3.226.8 Giac [F]

$$\int (gx)^m (d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} \, dx = \int \left(-e^2x^2+d^2\right)^{\frac{5}{2}} (ex+d)^3 (gx)^m \, dx$$

input
```
integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3*(g*x)^m, x)
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\int \left(d^2-e^2\,x^2\right)^{5/2}\left(g\,x\right)^m\left(d+e\,x\right)^3\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x)^3,x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x)^3, x)
```

3.227 $\int (gx)^m (d+ex)^2 \left(d^2-e^2x^2\right)^{5/2}\,dx$

3.227.1 Optimal result

Integrand size = 29, antiderivative size = 206

$$\int (gx)^m (d+ex)^2 \left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)} + \frac{d^6(9+2m)(gx)^{1+m}\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{g(1+m)(8+m)\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{2d^5e(gx)^{2+m}\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{g^2(2+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

output
```
-(g*x)^(1+m)*(-e^2*x^2+d^2)^(7/2)/g/(8+m)+d^6*(9+2*m)*(g*x)^(1+m)*hypergeo
m([-5/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g/(1+m)/
(8+m)/(1-e^2*x^2/d^2)^(1/2)+2*d^5*e*(g*x)^(2+m)*hypergeom([-5/2, 1+1/2*m],
[2+1/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g^2/(2+m)/(1-e^2*x^2/d^2)^(1/2
)
```

3.227.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.84

$$\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}\,dx = \frac{d^4x(gx)^m\sqrt{d^2-e^2x^2}\left(d^2(6+5m+m^2)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)+e(1+\right.}{(1+}$$

input
```
Integrate[(g*x)^m*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2),x]
```

output
```
(d^4*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[
-5/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*(2*d*(3 + m)*Hype
rgeometric2F1[-5/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2] + e*(2 + m)*x*Hyp
ergeometric2F1[-5/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])))/((1 + m)*(2 +
 m)*(3 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

3.227.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {559, 25, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2\left(d^2-e^2x^2\right)^{5/2}(gx)^m\,dx$$

$$\Big\downarrow \text{559}$$

$$-\frac{\int -de^2(gx)^m(d(2m+9)+2e(m+8)x)\left(d^2-e^2x^2\right)^{5/2}dx}{e^2(m+8)}-\frac{\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+1}}{g(m+8)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int de^2(gx)^m(d(2m+9)+2e(m+8)x)\left(d^2-e^2x^2\right)^{5/2}dx}{e^2(m+8)}-\frac{\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+1}}{g(m+8)}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int (gx)^m(d(2m+9)+2e(m+8)x)\left(d^2-e^2x^2\right)^{5/2}dx}{m+8}-\frac{\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+1}}{g(m+8)}$$

$$\Big\downarrow \text{557}$$

$$\frac{d\left(d(2m+9)\int (gx)^m\left(d^2-e^2x^2\right)^{5/2}dx+\frac{2e(m+8)\int (gx)^{m+1}\left(d^2-e^2x^2\right)^{5/2}dx}{g}\right)}{m+8}-\frac{\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+1}}{g(m+8)}$$

$$\Big\downarrow \text{279}$$

$$\frac{d\left(\frac{d^5(2m+9)\sqrt{d^2-e^2x^2}\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}dx}{\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{2d^4e(m+8)\sqrt{d^2-e^2x^2}\int (gx)^{m+1}\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}dx}{g\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{m+8}-\frac{\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+1}}{g(m+8)}$$

$$\Big\downarrow \text{278}$$

$$\frac{d\left(\frac{d^5(2m+9)\sqrt{d^2-e^2x^2}(gx)^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{2d^4e(m+8)\sqrt{d^2-e^2x^2}(gx)^{m+2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m+2}{2},\right.}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}\right.}{m+8}$$
$$\frac{\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+1}}{g(m+8)}$$

input
```
Int[(g*x)^m*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2),x]
```

output
```
-(((g*x)^(1 + m)*(d^2 - e^2*x^2)^(7/2))/(g*(8 + m))) + (d*((d^5*(9 + 2*m)*
(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (3 +
m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]) + (2*d^4*e*(8 +
m)*(g*x)^(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (2 + m)/2, (4
 + m)/2, (e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])))/(8 + m)
```

3.227.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 559
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[d^n*(e*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*e^(n - 1)*(
m + n + 2*p + 1))), x] + Simp[1/(b*(m + n + 2*p + 1))   Int[(e*x)^m*(a + b*
x^2)^p*ExpandToSum[b*(m + n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1
)*x^n - a*d^n*(m + n - 1)*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && IGtQ[n, 1] &&  !IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

3.227.4 Maple [F]

$$\int (gx)^m (ex+d)^2 \left(-e^2x^2+d^2\right)^{\frac{5}{2}} dx$$

input
```
int((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x)
```

output
```
int((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x)
```

3.227.5 Fricas [F]

$$\int (gx)^m (d+ex)^2 \left(d^2-e^2x^2\right)^{5/2} \, dx = \int \left(-e^2x^2+d^2\right)^{\frac{5}{2}} (ex+d)^2 (gx)^m \, dx$$

input
```
integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
integral((e^6*x^6 + 2*d*e^5*x^5 - d^2*e^4*x^4 - 4*d^3*e^3*x^3 - d^4*e^2*x^
2 + 2*d^5*e*x + d^6)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)
```

3.227.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.94 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.10

$$\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}\,dx = \frac{d^7g^mx^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{d^6eg^mx^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+2\right)} - \frac{d^5e^2g^mx^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} - \frac{2d^4e^3g^mx^{m+4}\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+3\right)} - \frac{d^3e^4g^mx^{m+5}\Gamma\left(\frac{m}{2}+\frac{5}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{5}{2}\\ \frac{m}{2}+\frac{7}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)} + \frac{d^2e^5g^mx^{m+6}\Gamma\left(\frac{m}{2}+3\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+3\\ \frac{m}{2}+4\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+4\right)} + \frac{de^6g^mx^{m+7}\Gamma\left(\frac{m}{2}+\frac{7}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{7}{2}\\ \frac{m}{2}+\frac{9}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{9}{2}\right)}$$

input
```
integrate((g*x)**m*(e*x+d)**2*(-e**2*x**2+d**2)**(5/2),x)
```

3.227. $\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}\,dx$

output
```
d**7*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,
), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + d**6*e*g**m*x*
*(m + 2)*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_p
olar(2*I*pi)/d**2)/gamma(m/2 + 2) - d**5*e**2*g**m*x**(m + 3)*gamma(m/2 +
3/2)*hyper((-1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/d*
*2)/(2*gamma(m/2 + 5/2)) - 2*d**4*e**3*g**m*x**(m + 4)*gamma(m/2 + 2)*hype
r((-1/2, m/2 + 2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*pi)/d**2)/gamma(m/2
 + 3) - d**3*e**4*g**m*x**(m + 5)*gamma(m/2 + 5/2)*hyper((-1/2, m/2 + 5/2)
, (m/2 + 7/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 7/2)) + d
**2*e**5*g**m*x**(m + 6)*gamma(m/2 + 3)*hyper((-1/2, m/2 + 3), (m/2 + 4,),
 e**2*x**2*exp_polar(2*I*pi)/d**2)/gamma(m/2 + 4) + d*e**6*g**m*x**(m + 7)
*gamma(m/2 + 7/2)*hyper((-1/2, m/2 + 7/2), (m/2 + 9/2,), e**2*x**2*exp_pol
ar(2*I*pi)/d**2)/(2*gamma(m/2 + 9/2))
```

3.227.7 Maxima [F]

$$\int (gx)^m (d+ex)^2 \left(d^2-e^2x^2\right)^{5/2} \, dx = \int \left(-e^2x^2+d^2\right)^{\frac{5}{2}} (ex+d)^2 (gx)^m \, dx$$

input
```
integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^2*(g*x)^m, x)
```

3.227.8 Giac [F]

$$\int (gx)^m (d+ex)^2 \left(d^2-e^2x^2\right)^{5/2} \, dx = \int \left(-e^2x^2+d^2\right)^{\frac{5}{2}} (ex+d)^2 (gx)^m \, dx$$

input
```
integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^2*(g*x)^m, x)
```

3.227.9 Mupad [F(-1)]

Timed out.

$$\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}\,dx=\int \left(d^2-e^2\,x^2\right)^{5/2}\,(g\,x)^m\,(d+e\,x)^2\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x)^2,x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x)^2, x)
```

3.228 $\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx$

3.228.1 Optimal result

Integrand size = 27, antiderivative size = 162

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{d^5(gx)^{1+m}\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{g(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^4e(gx)^{2+m}\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{g^2(2+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

output
```
d^5*(g*x)^(1+m)*hypergeom([-5/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(-e^2
*x^2+d^2)^(1/2)/g/(1+m)/(1-e^2*x^2/d^2)^(1/2)+d^4*e*(g*x)^(2+m)*hypergeom(
[-5/2, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g^2/(2+m)/(1-e
^2*x^2/d^2)^(1/2)
```

3.228.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.75

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx = \frac{d^4x(gx)^m\sqrt{d^2-e^2x^2}\left(d(2+m)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)+e(1+m)x\,\text{Hy}\right.}{(1+m)(2+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

input
```
Integrate[(g*x)^m*(d + e*x)*(d^2 - e^2*x^2)^(5/2),x]
```

output
```
(d^4*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*(d*(2 + m)*Hypergeometric2F1[-5/2, (1 +
 m)/2, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*Hypergeometric2F1[-5/2, (2
+ m)/2, (4 + m)/2, (e^2*x^2)/d^2]))/((1 + m)*(2 + m)*Sqrt[1 - (e^2*x^2)/d^
2])
```

3.228.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(d^2-e^2x^2\right)^{5/2}(gx)^m\,dx$$

$$\Big\downarrow \text{557}$$

$$d\int (gx)^m\left(d^2-e^2x^2\right)^{5/2}dx+\frac{e\int (gx)^{m+1}\left(d^2-e^2x^2\right)^{5/2}dx}{g}$$

$$\Big\downarrow \text{279}$$

$$\frac{d^5\sqrt{d^2-e^2x^2}\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}dx}{\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^4e\sqrt{d^2-e^2x^2}\int (gx)^{m+1}\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}dx}{g\sqrt{1-\frac{e^2x^2}{d^2}}}$$

$$\Big\downarrow \text{278}$$

$$\frac{d^5\sqrt{d^2-e^2x^2}(gx)^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^4e\sqrt{d^2-e^2x^2}(gx)^{m+2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

input
```
Int[(g*x)^m*(d + e*x)*(d^2 - e^2*x^2)^(5/2),x]
```

output
```
(d^5*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2,
(3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]) + (d^4*e*(g
*x)^(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (2 + m)/2, (4 + m)
/2, (e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

3.228.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

3.228.4 Maple [F]

$$\int (gx)^m (ex+d) \left(-e^2x^2+d^2\right)^{\frac{5}{2}} dx$$

input
```
int((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x)
```

output
```
int((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x)
```

3.228.5 Fricas [F]

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2} dx = \int \left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)(gx)^m dx$$

input
```
integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
integral((e^5*x^5 + d*e^4*x^4 - 2*d^2*e^3*x^3 - 2*d^3*e^2*x^2 + d^4*e*x +
d^5)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)
```

3.228.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.75 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.26

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx = \frac{d^6g^mx^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{d^5eg^mx^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+2\right)} - \frac{d^4e^2g^mx^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} - \frac{d^3e^3g^mx^{m+4}\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+3\right)} + \frac{d^2e^4g^mx^{m+5}\Gamma\left(\frac{m}{2}+\frac{5}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{5}{2}\\ \frac{m}{2}+\frac{7}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)} + \frac{de^5g^mx^{m+6}\Gamma\left(\frac{m}{2}+3\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+3\\ \frac{m}{2}+4\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+4\right)}$$

input
```
integrate((g*x)**m*(e*x+d)*(-e**2*x**2+d**2)**(5/2),x)
```

output
```
d**6*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,
), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + d**5*e*g**m*x*
*(m + 2)*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_p
olar(2*I*pi)/d**2)/(2*gamma(m/2 + 2)) - d**4*e**2*g**m*x**(m + 3)*gamma(m/
2 + 3/2)*hyper((-1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi
)/d**2)/gamma(m/2 + 5/2) - d**3*e**3*g**m*x**(m + 4)*gamma(m/2 + 2)*hyper(
(-1/2, m/2 + 2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*pi)/d**2)/gamma(m/2 +
 3) + d**2*e**4*g**m*x**(m + 5)*gamma(m/2 + 5/2)*hyper((-1/2, m/2 + 5/2),
(m/2 + 7/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 7/2)) + d*e
**5*g**m*x**(m + 6)*gamma(m/2 + 3)*hyper((-1/2, m/2 + 3), (m/2 + 4,), e**2
*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 4))
```

3.228. $\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx$

3.228.7 Maxima [F]

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx=\int \left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)(gx)^m\,dx$$

input
```
integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*(g*x)^m, x)
```

3.228.8 Giac [F]

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx=\int \left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)(gx)^m\,dx$$

input
```
integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*(g*x)^m, x)
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx=\int \left(d^2-e^2\,x^2\right)^{5/2}\,(g\,x)^m\,(d+e\,x)\,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x),x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x), x)
```

3.229 $\int (gx)^m \left(d^2-e^2x^2\right)^{5/2} dx$

3.229.1 Optimal result

Integrand size = 22, antiderivative size = 80

$$\int (gx)^m \left(d^2-e^2x^2\right)^{5/2} dx=\frac{d^4(gx)^{1+m}\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{g(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

output
```
d^4*(g*x)^(1+m)*hypergeom([-5/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(-e^2
*x^2+d^2)^(1/2)/g/(1+m)/(1-e^2*x^2/d^2)^(1/2)
```

3.229.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int (gx)^m \left(d^2-e^2x^2\right)^{5/2} dx=\frac{d^4x(gx)^m\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{2},1+\frac{1+m}{2},\frac{e^2x^2}{d^2}\right)}{(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

input
```
Integrate[(g*x)^m*(d^2 - e^2*x^2)^(5/2),x]
```

output
```
(d^4*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, 1 +
(1 + m)/2, (e^2*x^2)/d^2])/((1 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

3.229.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d^2-e^2x^2\right)^{5/2} (gx)^m \, dx$$
$$\Big\downarrow \ 279$$
$$\frac{d^4\sqrt{d^2-e^2x^2}\int (gx)^m \left(1-\frac{e^2x^2}{d^2}\right)^{5/2}dx}{\sqrt{1-\frac{e^2x^2}{d^2}}}$$
$$\Big\downarrow \ 278$$
$$\frac{d^4\sqrt{d^2-e^2x^2}(gx)^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

input
```
Int[(g*x)^m*(d^2 - e^2*x^2)^(5/2),x]
```

output
```
(d^4*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2,
(3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

3.229.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

3.229.4 Maple [F]

$$\int (gx)^m \left(-e^2x^2+d^2\right)^{\frac{5}{2}} dx$$

input
```
int((g*x)^m*(-e^2*x^2+d^2)^(5/2),x)
```

output
```
int((g*x)^m*(-e^2*x^2+d^2)^(5/2),x)
```

3.229.5 Fricas [F]

$$\int (gx)^m \left(d^2-e^2x^2\right)^{5/2} dx = \int \left(-e^2x^2+d^2\right)^{\frac{5}{2}} (gx)^m dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
integral((e^4*x^4 - 2*d^2*e^2*x^2 + d^4)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)
```

3.229.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int (gx)^m \left(d^2-e^2x^2\right)^{5/2} dx = \frac{d^5 g^m x^{m+1} \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

input
```
integrate((g*x)**m*(-e**2*x**2+d**2)**(5/2),x)
```

output
```
d**5*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-5/2, m/2 + 1/2), (m/2 + 3/2,
), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2))
```

3.229.7 Maxima [F]

$$\int (gx)^m \left(d^2-e^2x^2\right)^{5/2} dx = \int \left(-e^2x^2+d^2\right)^{\frac{5}{2}} (gx)^m dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m, x)
```

3.229.8 Giac [F]

$$\int (gx)^m \left(d^2-e^2x^2\right)^{5/2} dx = \int \left(-e^2x^2+d^2\right)^{\frac{5}{2}} (gx)^m dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m, x)
```

3.229.9 Mupad [F(-1)]

Timed out.

$$\int (gx)^m \left(d^2-e^2x^2\right)^{5/2} dx = \int \left(d^2-e^2\,x^2\right)^{5/2} (g\,x)^m \,dx$$

input
```
int((d^2 - e^2*x^2)^(5/2)*(g*x)^m,x)
```

output
```
int((d^2 - e^2*x^2)^(5/2)*(g*x)^m, x)
```

3.230 $\int \frac{(gx)^m\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx$

3.230.1 Optimal result

Integrand size = 29, antiderivative size = 163

$$\int \frac{(gx)^m\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx=\frac{d^3(gx)^{1+m}\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{g(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{d^2e(gx)^{2+m}\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{g^2(2+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

output
```
d^3*(g*x)^(1+m)*hypergeom([-3/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(-e^2
*x^2+d^2)^(1/2)/g/(1+m)/(1-e^2*x^2/d^2)^(1/2)-d^2*e*(g*x)^(2+m)*hypergeom(
[-3/2, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g^2/(2+m)/(1-e
^2*x^2/d^2)^(1/2)
```

3.230.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.75

$$\int \frac{(gx)^m\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx=\frac{d^2x(gx)^m\sqrt{d^2-e^2x^2}\left(-e(1+m)x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1+\frac{m}{2},2+\frac{m}{2},\frac{e^2}{d}\right.\right.}{(1+m)(2+m)\sqrt{1-}}$$

input
```
Integrate[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

output
```
(d^2*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*(-(e*(1 + m)*x*Hypergeometric2F1[-3/2,
1 + m/2, 2 + m/2, (e^2*x^2)/d^2]) + d*(2 + m)*Hypergeometric2F1[-3/2, (1 +
 m)/2, (3 + m)/2, (e^2*x^2)/d^2]))/((1 + m)*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2
])
```

3.230.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {583, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^{5/2}(gx)^m}{d+ex}dx$$

$$\Big\downarrow \text{583}$$

$$\int (d-ex)\left(d^2-e^2x^2\right)^{3/2}(gx)^m dx$$

$$\Big\downarrow \text{557}$$

$$d\int (gx)^m\left(d^2-e^2x^2\right)^{3/2}dx-\frac{e\int (gx)^{m+1}\left(d^2-e^2x^2\right)^{3/2}dx}{g}$$

$$\Big\downarrow \text{279}$$

$$\frac{d^3\sqrt{d^2-e^2x^2}\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^{3/2}dx}{\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{d^2e\sqrt{d^2-e^2x^2}\int (gx)^{m+1}\left(1-\frac{e^2x^2}{d^2}\right)^{3/2}dx}{g\sqrt{1-\frac{e^2x^2}{d^2}}}$$

$$\Big\downarrow \text{278}$$

$$\frac{d^3\sqrt{d^2-e^2x^2}(gx)^{m+1}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{d^2e\sqrt{d^2-e^2x^2}(gx)^{m+2}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+2}{2},\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

input
```
Int[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

output
```
(d^3*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-3/2, (1 + m)/2,
(3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]) - (d^2*e*(g
*x)^(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-3/2, (2 + m)/2, (4 + m)
/2, (e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

3.230.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 583
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, 0]
```

3.230.4 Maple [F]

$$\int \frac{(gx)^m (-e^2x^2+d^2)^{\frac{5}{2}}}{ex+d}dx$$

input
```
int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

output
```
int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

3.230.5 Fricas [F]

$$\int \frac{(gx)^m (d^2 - e^2x^2)^{5/2}}{d + ex}\,dx = \int \frac{(-e^2x^2 + d^2)^{\frac{5}{2}}(gx)^m}{ex + d}\,dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

output
```
integral((e^3*x^3 - d*e^2*x^2 - d^2*e*x + d^3)*sqrt(-e^2*x^2 + d^2)*(g*x)^
m, x)
```

3.230.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.03 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.49

$$\int \frac{(gx)^m (d^2 - e^2x^2)^{5/2}}{d + ex}\,dx = \frac{d^4 g^m x^{m+1} \Gamma\left(\frac{m}{2} + \frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2} + \frac{1}{2} \\ \frac{m}{2} + \frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} - \frac{d^3 e g^m x^{m+2} \Gamma\left(\frac{m}{2} + 1\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2} + 1 \\ \frac{m}{2} + 2 \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2} + 2\right)} - \frac{d^2 e^2 g^m x^{m+3} \Gamma\left(\frac{m}{2} + \frac{3}{2}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2} + \frac{3}{2} \\ \frac{m}{2} + \frac{5}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2} + \frac{5}{2}\right)} + \frac{d e^3 g^m x^{m+4} \Gamma\left(\frac{m}{2} + 2\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2} + 2 \\ \frac{m}{2} + 3 \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2} + 3\right)}$$

input
```
integrate((g*x)**m*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

output
```
d**4*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,
), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) - d**3*e*g**m*x*
*(m + 2)*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_p
olar(2*I*pi)/d**2)/(2*gamma(m/2 + 2)) - d**2*e**2*g**m*x**(m + 3)*gamma(m/
2 + 3/2)*hyper((-1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi
)/d**2)/(2*gamma(m/2 + 5/2)) + d*e**3*g**m*x**(m + 4)*gamma(m/2 + 2)*hyper
((-1/2, m/2 + 2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m
/2 + 3))
```

3.230.7 Maxima [F]

$$\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^{\frac{5}{2}}(gx)^m}{ex+d}\,dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d), x)
```

3.230.8 Giac [F]

$$\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^{\frac{5}{2}}(gx)^m}{ex+d}\,dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d), x)
```

3.230.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m (d^2 - e^2x^2)^{5/2}}{d + ex}\,dx = \int \frac{(d^2 - e^2\,x^2)^{5/2}\,(g\,x)^m}{d + e\,x}\,dx$$

input
```
int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x),x)
```

output
```
int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x), x)
```

3.231 $\int \frac{(gx)^m \left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2} dx$

3.231.1 Optimal result

Integrand size = 29, antiderivative size = 204

$$\int \frac{(gx)^m \left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2} dx = -\frac{(gx)^{1+m} \left(d^2-e^2x^2\right)^{3/2}}{g(4+m)} + \frac{d^2(5+2m)(gx)^{1+m}\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{g(1+m)(4+m)\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{2de(gx)^{2+m}\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{g^2(2+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

output
```
-(g*x)^(1+m)*(-e^2*x^2+d^2)^(3/2)/g/(4+m)+d^2*(5+2*m)*(g*x)^(1+m)*hypergeo
m([-1/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g/(1+m)/
(4+m)/(1-e^2*x^2/d^2)^(1/2)-2*d*e*(g*x)^(2+m)*hypergeom([-1/2, 1+1/2*m],[2
+1/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g^2/(2+m)/(1-e^2*x^2/d^2)^(1/2)
```

3.231.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.85

$$\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{(d+ex)^2} dx = \frac{x(gx)^m\sqrt{d^2-e^2x^2}\left(d^2(6+5m+m^2)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)\right.}{}$$

input
```
Integrate[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

output
```
(x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[-1/2
, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] - e*(1 + m)*x*(2*d*(3 + m)*Hypergeo
metric2F1[-1/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2] - e*(2 + m)*x*Hyperge
ometric2F1[-1/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])))/((1 + m)*(2 + m)*
(3 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

3.231.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {570, 559, 25, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^{5/2}(gx)^m}{(d+ex)^2}dx$$

$$\Big\downarrow \text{570}$$

$$\int (d-ex)^2\sqrt{d^2-e^2x^2}(gx)^m dx$$

$$\Big\downarrow \text{559}$$

$$-\frac{\int -de^2(gx)^m(d(2m+5)-2e(m+4)x)\sqrt{d^2-e^2x^2}dx}{e^2(m+4)}-\frac{(d^2-e^2x^2)^{3/2}(gx)^{m+1}}{g(m+4)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int de^2(gx)^m(d(2m+5)-2e(m+4)x)\sqrt{d^2-e^2x^2}dx}{e^2(m+4)}-\frac{(d^2-e^2x^2)^{3/2}(gx)^{m+1}}{g(m+4)}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int (gx)^m(d(2m+5)-2e(m+4)x)\sqrt{d^2-e^2x^2}dx}{m+4}-\frac{\left(d^2-e^2x^2\right)^{3/2}(gx)^{m+1}}{g(m+4)}$$

$$\Big\downarrow \text{557}$$

$$\frac{d\left(d(2m+5)\int (gx)^m\sqrt{d^2-e^2x^2}dx-\frac{2e(m+4)\int (gx)^{m+1}\sqrt{d^2-e^2x^2}dx}{g}\right)}{m+4}-\frac{\left(d^2-e^2x^2\right)^{3/2}(gx)^{m+1}}{g(m+4)}$$

$$\Big\downarrow \text{279}$$

$$\frac{d\left(\frac{d(2m+5)\sqrt{d^2-e^2x^2}\int (gx)^m\sqrt{1-\frac{e^2x^2}{d^2}}dx}{\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{2e(m+4)\sqrt{d^2-e^2x^2}\int (gx)^{m+1}\sqrt{1-\frac{e^2x^2}{d^2}}dx}{g\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{m+4}-\frac{\left(d^2-e^2x^2\right)^{3/2}(gx)^{m+1}}{g(m+4)}$$

$$\Big\downarrow \text{278}$$

$$\frac{d\left(\frac{d(2m+5)\sqrt{d^2-e^2x^2}(gx)^{m+1}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{2e(m+4)\sqrt{d^2-e^2x^2}(gx)^{m+2}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+2}{2},\frac{m+}{2}\right.}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}}{m+4}$$

$$\frac{\left(d^2-e^2x^2\right)^{3/2}(gx)^{m+1}}{g(m+4)}$$

input
```
Int[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

output
```
-(((g*x)^(1 + m)*(d^2 - e^2*x^2)^(3/2))/(g*(4 + m))) + (d*((d*(5 + 2*m)*(g
*x)^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)
/2, (e^2*x^2)/d^2])/(g*(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]) - (2*e*(4 + m)*(g*
x)^(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-1/2, (2 + m)/2, (4 + m)/
2, (e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])))/(4 + m)
```

3.231.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.231. $\int \frac{(gx)^m\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}dx$

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 559
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[d^n*(e*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*e^(n - 1)*(
m + n + 2*p + 1))), x] + Simp[1/(b*(m + n + 2*p + 1))   Int[(e*x)^m*(a + b*
x^2)^p*ExpandToSum[b*(m + n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1
)*x^n - a*d^n*(m + n - 1)*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && IGtQ[n, 1] && !IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] && !(IGtQ[m, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

3.231.4 Maple [F]

$$\int \frac{(gx)^m\,(-e^2x^2+d^2)^{\frac{5}{2}}}{(ex+d)^2}dx$$

input
```
int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

output
```
int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

3.231.5 Fricas [F]

$$\int \frac{(gx)^m (d^2 - e^2x^2)^{5/2}}{(d + ex)^2} \, dx = \int \frac{(-e^2x^2 + d^2)^{\frac{5}{2}} (gx)^m}{(ex + d)^2} \, dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((e^2*x^2 - 2*d*e*x + d^2)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)
```

3.231.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 38.53 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.89

$$\int \frac{(gx)^m (d^2 - e^2x^2)^{5/2}}{(d + ex)^2} \, dx = \frac{d^3 g^m x^{m+1} \Gamma\left(\frac{m}{2} + \frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2} + \frac{1}{2} \\ \frac{m}{2} + \frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} - \frac{d^2 e g^m x^{m+2} \Gamma\left(\frac{m}{2} + 1\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2} + 1 \\ \frac{m}{2} + 2 \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2} + 2\right)} + \frac{d e^2 g^m x^{m+3} \Gamma\left(\frac{m}{2} + \frac{3}{2}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2} + \frac{3}{2} \\ \frac{m}{2} + \frac{5}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2} + \frac{5}{2}\right)}$$

input
```
integrate((g*x)**m*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

output
```
d**3*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,
), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) - d**2*e*g**m*x*
*(m + 2)*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_p
olar(2*I*pi)/d**2)/gamma(m/2 + 2) + d*e**2*g**m*x**(m + 3)*gamma(m/2 + 3/2
)*hyper((-1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)
/(2*gamma(m/2 + 5/2))
```

3.231.7 Maxima [F]

$$\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^{\frac{5}{2}}(gx)^m}{(ex+d)^2}\,dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d)^2, x)
```

3.231.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,6,1,0,0]%%%}+%%%{-3,[0,4,1,0,0]%%%}+%%%{-3,[0,2,1,0,
0]%%%}+%%
```

3.231.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}\,(g\,x)^m}{(d+e\,x)^2}\,dx$$

input
```
int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x)^2,x)
```

output
```
int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x)^2, x)
```

3.232 $\int \frac{(gx)^m\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^3}\,dx$

3.232.1 Optimal result

Integrand size = 29, antiderivative size = 250

$$\int \frac{(gx)^m\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^3}\,dx = -\frac{3d(gx)^{1+m}\sqrt{d^2-e^2x^2}}{g(2+m)}+\frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)} + \frac{d^3(5+4m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{g(1+m)(2+m)\sqrt{d^2-e^2x^2}} - \frac{d^2e(11+4m)(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{g^2(2+m)(3+m)\sqrt{d^2-e^2x^2}}$$

output
```
-3*d*(g*x)^(1+m)*(-e^2*x^2+d^2)^(1/2)/g/(2+m)+e*(g*x)^(2+m)*(-e^2*x^2+d^2)
^(1/2)/g^2/(3+m)+d^3*(5+4*m)*(g*x)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1
/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/g/(1+m)/(2+m)/(-e^2*x^2+d^2)^(1/2
)-d^2*e*(11+4*m)*(g*x)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],e^2*x^2/d^
2)*(1-e^2*x^2/d^2)^(1/2)/g^2/(2+m)/(3+m)/(-e^2*x^2+d^2)^(1/2)
```

3.232.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.98

$$\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{(d+ex)^3}\,dx = \frac{x(gx)^m\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}\left(d^3(24+26m+9m^2+m^3)\,\text{Hypergeometric2F1}\right.}{}$$

input
```
Integrate[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^3,x]
```

output
```
(x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*Sqrt[1 - (e^2*x^2)/d^2]*(d^3*(24 + 26*m + 9
*m^2 + m^3)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] -
e*(1 + m)*x*(3*d^2*(12 + 7*m + m^2)*Hypergeometric2F1[1/2, (2 + m)/2, (4 +
 m)/2, (e^2*x^2)/d^2] + e*(2 + m)*x*(-3*d*(4 + m)*Hypergeometric2F1[1/2, (
3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2] + e*(3 + m)*x*Hypergeometric2F1[1/2, (
4 + m)/2, (6 + m)/2, (e^2*x^2)/d^2]))))/((1 + m)*(2 + m)*(3 + m)*(4 + m)*(
d - e*x)*(d + e*x))
```

3.232.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {570, 559, 25, 2340, 25, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^{5/2}(gx)^m}{(d+ex)^3}\,dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{(d-ex)^3(gx)^m}{\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{559}$$

$$\frac{e\sqrt{d^2-e^2x^2}(gx)^{m+2}}{g^2(m+3)} - \frac{\int -\frac{(gx)^m(3d(m+3)x^2e^4-d^2(4m+11)xe^3+d^3(m+3)e^2)}{\sqrt{d^2-e^2x^2}}dx}{e^2(m+3)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(gx)^m\left(3d(m+3)x^2e^4-d^2(4m+11)xe^3+d^3(m+3)e^2\right)}{\sqrt{d^2-e^2x^2}}dx}{e^2(m+3)}+\frac{e\sqrt{d^2-e^2x^2}(gx)^{m+2}}{g^2(m+3)}$$

$\downarrow$ 2340

$$\frac{-\frac{\int -\frac{d^2e^4(gx)^m(d(m+3)(4m+5)-e(m+2)(4m+11)x)}{\sqrt{d^2-e^2x^2}}dx}{e^2(m+2)}-\frac{3de^2(m+3)\sqrt{d^2-e^2x^2}(gx)^{m+1}}{g(m+2)}}{e^2(m+3)}+\frac{e\sqrt{d^2-e^2x^2}(gx)^{m+2}}{g^2(m+3)}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{d^2e^4(gx)^m(d(m+3)(4m+5)-e(m+2)(4m+11)x)}{\sqrt{d^2-e^2x^2}}dx}{e^2(m+2)}-\frac{3de^2(m+3)\sqrt{d^2-e^2x^2}(gx)^{m+1}}{g(m+2)}}{e^2(m+3)}+\frac{e\sqrt{d^2-e^2x^2}(gx)^{m+2}}{g^2(m+3)}$$

$\downarrow$ 27

$$\frac{\frac{d^2e^2\int \frac{(gx)^m(d(m+3)(4m+5)-e(m+2)(4m+11)x)}{\sqrt{d^2-e^2x^2}}dx}{m+2}-\frac{3de^2(m+3)\sqrt{d^2-e^2x^2}(gx)^{m+1}}{g(m+2)}}{e^2(m+3)}+\frac{e\sqrt{d^2-e^2x^2}(gx)^{m+2}}{g^2(m+3)}$$

$\downarrow$ 557

$$\frac{\frac{d^2e^2\left(d(m+3)(4m+5)\int \frac{(gx)^m}{\sqrt{d^2-e^2x^2}}dx-\frac{e(m+2)(4m+11)\int \frac{(gx)^{m+1}}{\sqrt{d^2-e^2x^2}}dx}{g}\right)}{m+2}-\frac{3de^2(m+3)\sqrt{d^2-e^2x^2}(gx)^{m+1}}{g(m+2)}}{e^2(m+3)}+\frac{e\sqrt{d^2-e^2x^2}(gx)^{m+2}}{g^2(m+3)}$$

$\downarrow$ 279

$$\frac{\frac{d^2e^2\left(\frac{d(m+3)(4m+5)\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^m}{\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{\sqrt{d^2-e^2x^2}}-\frac{e(m+2)(4m+11)\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^{m+1}}{\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{g\sqrt{d^2-e^2x^2}}\right)}{m+2}-\frac{3de^2(m+3)\sqrt{d^2-e^2x^2}(gx)^{m+1}}{g(m+2)}}{e^2(m+3)}+\frac{e\sqrt{d^2-e^2x^2}(gx)^{m+2}}{g^2(m+3)}$$

$\downarrow$ 278

$$\frac{\frac{d^2e^2\left(\frac{d(m+3)(4m+5)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{d^2-e^2x^2}}-\frac{e(4m+11)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{g^2\sqrt{d^2-e^2x^2}}\right)}{m+2}}{e^2(m+3)}$$

$$\frac{e\sqrt{d^2-e^2x^2}(gx)^{m+2}}{g^2(m+3)}$$

3.232. $\int \frac{(gx)^m\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^3}\,dx$

input
```
Int[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^3,x]
```

output
```
(e*(g*x)^(2 + m)*Sqrt[d^2 - e^2*x^2])/(g^2*(3 + m)) + ((-3*d*e^2*(3 + m)*(
g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2])/(g*(2 + m)) + (d^2*e^2*((d*(3 + m)*(5 +
4*m)*(g*x)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*Sqrt[d^2 - e^2*x^2]) - (e*(11 + 4
*m)*(g*x)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[1/2, (2 + m)/2
, (4 + m)/2, (e^2*x^2)/d^2])/(g^2*Sqrt[d^2 - e^2*x^2])))/(2 + m))/(e^2*(3
+ m))
```

3.232.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 559
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[d^n*(e*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*e^(n - 1)*(
m + n + 2*p + 1))), x] + Simp[1/(b*(m + n + 2*p + 1))   Int[(e*x)^m*(a + b*
x^2)^p*ExpandToSum[b*(m + n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1
)*x^n - a*d^n*(m + n - 1)*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && IGtQ[n, 1] &&  !IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

3.232.4 Maple [F]

$$\int \frac{(gx)^m\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{(ex+d)^3}dx$$

input
```
int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x)
```

output
```
int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x)
```

3.232.5 Fricas [F]

$$\int \frac{(gx)^m \left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^3}\,dx=\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(gx)^m}{(ex+d)^3}\,dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((e^2*x^2 - 2*d*e*x + d^2)*sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e*x + d),
 x)
```

3.232.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(gx)^m \left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^3}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((g*x)**m*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.232.7 Maxima [F]

$$\int \frac{(gx)^m \left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^3}\,dx=\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(gx)^m}{(ex+d)^3}\,dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d)^3, x)
```

3.232.8 Giac [F]

$$\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{(d+ex)^3}\,dx=\int \frac{(-e^2x^2+d^2)^{\frac{5}{2}}(gx)^m}{(ex+d)^3}\,dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d)^3, x)
```

3.232.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{(d+ex)^3}\,dx=\int \frac{(d^2-e^2\,x^2)^{5/2}\,(g\,x)^m}{(d+e\,x)^3}\,dx$$

input
```
int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x)^3,x)
```

output
```
int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x)^3, x)
```

3.233 $\int \frac{(gx)^m(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

3.233.1 Optimal result

Integrand size = 29, antiderivative size = 213

$$\int \frac{(gx)^m(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4(gx)^{1+m}(d+ex)}{5g\left(d^2-e^2x^2\right)^{5/2}} + \frac{(1-4m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{5d^3g(1+m)\sqrt{d^2-e^2x^2}} + \frac{e(7-4m)(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{5d^4g^2(2+m)\sqrt{d^2-e^2x^2}}$$

output
```
4/5*(g*x)^(1+m)*(e*x+d)/g/(-e^2*x^2+d^2)^(5/2)+1/5*(1-4*m)*(g*x)^(1+m)*hyp
ergeom([5/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^3
/g/(1+m)/(-e^2*x^2+d^2)^(1/2)+1/5*e*(7-4*m)*(g*x)^(2+m)*hypergeom([5/2, 1+
1/2*m],[2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^4/g^2/(2+m)/(-e^2*x^
2+d^2)^(1/2)
```

3.233.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.93

$$\int \frac{(gx)^m(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x(gx)^m\sqrt{1-\frac{e^2x^2}{d^2}}\left(\frac{d^3\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{1+m}+ex\left(\frac{3d^2\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{2+m}{2}\right.}{2+m}\right.\right.}{d^6}$$

input
```
Integrate[((g*x)^m*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*((d^3*Hypergeometric2F1[7/2, (1 + m)/2,
 (3 + m)/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((3*d^2*Hypergeometric2F1[7/2, (
2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1
[7/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) + (e*x*Hypergeometric2
F1[7/2, (4 + m)/2, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/(d^6*Sqrt[d^2 -
e^2*x^2])
```

3.233.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {558, 25, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3(gx)^m}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{558}$$

$$\frac{4(d+ex)(gx)^{m+1}}{5g\,(d^2-e^2x^2)^{5/2}}-\frac{\int -\frac{d^2(gx)^m(d(1-4m)+e(7-4m)x)}{(d^2-e^2x^2)^{5/2}}dx}{5d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d^2(gx)^m(d(1-4m)+e(7-4m)x)}{(d^2-e^2x^2)^{5/2}}dx}{5d^2}+\frac{4(d+ex)(gx)^{m+1}}{5g\,(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{(gx)^m(d(1-4m)+e(7-4m)x)}{(d^2-e^2x^2)^{5/2}}dx+\frac{4(d+ex)(gx)^{m+1}}{5g\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{557}$$

$$\frac{1}{5}\left(d(1-4m)\int \frac{(gx)^m}{(d^2-e^2x^2)^{5/2}}dx+\frac{e(7-4m)\int \frac{(gx)^{m+1}}{(d^2-e^2x^2)^{5/2}}dx}{g}\right)+\frac{4(d+ex)(gx)^{m+1}}{5g\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{279}$$

$$\frac{1}{5}\left(\frac{e(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^{m+1}}{\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}}dx}{d^4g\sqrt{d^2-e^2x^2}}+\frac{(1-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}}dx}{d^3\sqrt{d^2-e^2x^2}}\right)+$$
$$\frac{4(d+ex)(gx)^{m+1}}{5g\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{278}$$

$$\frac{4(d+ex)(gx)^{m+1}}{5g\left(d^2-e^2x^2\right)^{5/2}}+$$
$$\frac{1}{5}\left(\frac{e(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{d^4g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{(1-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,\text{Hyperg}}{d^3g(m+1)\sqrt{}}\right.$$

input
```
Int[((g*x)^m*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(4*(g*x)^(1 + m)*(d + e*x))/(5*g*(d^2 - e^2*x^2)^(5/2)) + (((1 - 4*m)*(g*x
)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m
)/2, (e^2*x^2)/d^2])/(d^3*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) + (e*(7 - 4*m)*(g
*x)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (2 + m)/2, (4 +
 m)/2, (e^2*x^2)/d^2])/(d^4*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2]))/5
```

3.233.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 558
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, a + b*x^2, x], f =
 Coeff[PolynomialRemainder[(c + d*x)^n, a + b*x^2, x], x, 0], g = Coeff[Pol
ynomialRemainder[(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(-(e*x)^(m + 1))*
(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*e*(p + 1))), x] + Simp[1/(2*a*(p + 1))
  Int[(e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + f*(m + 2*p +
 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] && IGt
Q[n, 1] &&  !IntegerQ[m] && LtQ[p, -1]
```

3.233.4 Maple [F]

$$\int \frac{(gx)^m (ex+d)^3}{(-e^2x^2+d^2)^{\frac{7}{2}}}dx$$

input
```
int((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

output
```
int((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

3.233.5 Fricas [F]

$$\int \frac{(gx)^m(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^3(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^5*x^5 - 3*d*e^4*x^4 + 2*d^2*e^3*x
^3 + 2*d^3*e^2*x^2 - 3*d^4*e*x + d^5), x)
```

3.233.6 Sympy [F]

$$\int \frac{(gx)^m(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(gx)^m\,(d+ex)^3}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x)**m*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((g*x)**m*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.233.7 Maxima [F]

$$\int \frac{(gx)^m(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^3(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

3.233.8 Giac [F]

$$\int \frac{(gx)^m(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^3(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(g\,x)^m\,(d+e\,x)^3}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

input
```
int(((g*x)^m*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
int(((g*x)^m*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

3.234 $\int \frac{(gx)^m(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$

3.234.1 Optimal result

Integrand size = 29, antiderivative size = 216

$$\int \frac{(gx)^m(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2(gx)^{1+m}(d+ex)}{5dg\,(d^2-e^2x^2)^{5/2}} + \frac{(3-2m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{5d^4g(1+m)\sqrt{d^2-e^2x^2}} + \frac{2e(3-m)(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{5d^5g^2(2+m)\sqrt{d^2-e^2x^2}}$$

output
```
2/5*(g*x)^(1+m)*(e*x+d)/d/g/(-e^2*x^2+d^2)^(5/2)+1/5*(3-2*m)*(g*x)^(1+m)*h
ypergeom([5/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d
^4/g/(1+m)/(-e^2*x^2+d^2)^(1/2)+2/5*e*(3-m)*(g*x)^(2+m)*hypergeom([5/2, 1+
1/2*m],[2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^5/g^2/(2+m)/(-e^2*x^
2+d^2)^(1/2)
```

3.234.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.81

$$\int \frac{(gx)^m(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x(gx)^m\sqrt{1-\frac{e^2x^2}{d^2}}\left(d^2(6+5m+m^2)\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)+e(1\right.}{d^6}$$

input
```
Integrate[((g*x)^m*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[
7/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*(2*d*(3 + m)*Hyper
geometric2F1[7/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2] + e*(2 + m)*x*Hyper
geometric2F1[7/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])))/(d^6*(1 + m)*(2
+ m)*(3 + m)*Sqrt[d^2 - e^2*x^2])
```

3.234.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {558, 25, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2(gx)^m}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \ 558$$

$$\frac{2(d+ex)(gx)^{m+1}}{5dg\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int -\frac{d(gx)^m(d(3-2m)+2e(3-m)x)}{(d^2-e^2x^2)^{5/2}}dx}{5d^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{d(gx)^m(d(3-2m)+2e(3-m)x)}{(d^2-e^2x^2)^{5/2}}dx}{5d^2}+\frac{2(d+ex)(gx)^{m+1}}{5dg\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(gx)^m(d(3-2m)+2e(3-m)x)}{(d^2-e^2x^2)^{5/2}}dx}{5d}+\frac{2(d+ex)(gx)^{m+1}}{5dg\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \textbf{557}$$

$$\frac{d(3-2m)\int \frac{(gx)^m}{(d^2-e^2x^2)^{5/2}}dx+\frac{2e(3-m)\int \frac{(gx)^{m+1}}{(d^2-e^2x^2)^{5/2}}dx}{g}}{5d}+\frac{2(d+ex)(gx)^{m+1}}{5dg\,(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \textbf{279}$$

$$\frac{\frac{2e(3-m)\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^{m+1}}{\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}}dx}{d^4g\sqrt{d^2-e^2x^2}}+\frac{(3-2m)\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}}dx}{d^3\sqrt{d^2-e^2x^2}}}{5d}+\frac{2(d+ex)(gx)^{m+1}}{5dg\,(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \textbf{278}$$

$$\frac{2(d+ex)(gx)^{m+1}}{5dg\,(d^2-e^2x^2)^{5/2}}+$$
$$\frac{\frac{2e(3-m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{d^4g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{(3-2m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{d^3g(m+1)\sqrt{d^2-e^2x^2}}}{5d}$$

input
```
Int[((g*x)^m*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(2*(g*x)^(1 + m)*(d + e*x))/(5*d*g*(d^2 - e^2*x^2)^(5/2)) + (((3 - 2*m)*(g
*x)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (1 + m)/2, (3 +
 m)/2, (e^2*x^2)/d^2])/(d^3*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) + (2*e*(3 - m)*
(g*x)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (2 + m)/2, (4
 + m)/2, (e^2*x^2)/d^2])/(d^4*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2]))/(5*d)
```

3.234.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

3.234. $\int \frac{(gx)^m(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 558
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, a + b*x^2, x], f =
 Coeff[PolynomialRemainder[(c + d*x)^n, a + b*x^2, x], x, 0], g = Coeff[Pol
ynomialRemainder[(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(-(e*x)^(m + 1))*
(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*e*(p + 1))), x] + Simp[1/(2*a*(p + 1))
  Int[(e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + f*(m + 2*p +
 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] && IGt
Q[n, 1] &&  !IntegerQ[m] && LtQ[p, -1]
```

3.234.4 Maple [F]

$$\int \frac{(gx)^m (ex+d)^2}{(-e^2x^2+d^2)^{\frac{7}{2}}}dx$$

input
```
int((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

output
```
int((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

3.234.5 Fricas [F]

$$\int \frac{(gx)^m(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^6*x^6 - 2*d*e^5*x^5 - d^2*e^4*x^4
 + 4*d^3*e^3*x^3 - d^4*e^2*x^2 - 2*d^5*e*x + d^6), x)
```

3.234.6 Sympy [F]

$$\int \frac{(gx)^m(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(gx)^m\,(d+ex)^2}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x)**m*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((g*x)**m*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.234.7 Maxima [F]

$$\int \frac{(gx)^m(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

3.234.8 Giac [F]

$$\int \frac{(gx)^m(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(g\,x)^m\,(d+e\,x)^2}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

input
```
int(((g*x)^m*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
int(((g*x)^m*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2), x)
```

3.235 $\int \frac{(gx)^m(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx$

3.235.1 Optimal result

Integrand size = 27, antiderivative size = 124

$$\int \frac{(gx)^m(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{(gx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-4+m),\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{dg(1+m)\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(gx)^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-3+m),\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{d^2g^2(2+m)\left(d^2-e^2x^2\right)^{5/2}}$$

output
```
(g*x)^(1+m)*hypergeom([1, -2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)/d/g/(1+m)/(-e
^2*x^2+d^2)^(5/2)+e*(g*x)^(2+m)*hypergeom([1, -3/2+1/2*m],[2+1/2*m],e^2*x^
2/d^2)/d^2/g^2/(2+m)/(-e^2*x^2+d^2)^(5/2)
```

3.235.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int \frac{(gx)^m(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x(gx)^m\sqrt{1-\frac{e^2x^2}{d^2}}\left(d(2+m)\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)+e(1+m)x\,\text{H}\right.}{d^6(1+m)(2+m)\sqrt{d^2-e^2x^2}}$$

input
```
Integrate[((g*x)^m*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*(d*(2 + m)*Hypergeometric2F1[7/2, (1 +
m)/2, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*Hypergeometric2F1[7/2, (2 +
m)/2, (4 + m)/2, (e^2*x^2)/d^2]))/(d^6*(1 + m)*(2 + m)*Sqrt[d^2 - e^2*x^2]
)
```

3.235.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.31, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)(gx)^m}{(d^2-e^2x^2)^{7/2}}\,dx$$
$$\Big\downarrow \text{557}$$
$$d\int \frac{(gx)^m}{(d^2-e^2x^2)^{7/2}}dx+\frac{e\int \frac{(gx)^{m+1}}{(d^2-e^2x^2)^{7/2}}dx}{g}$$
$$\Big\downarrow \text{279}$$
$$\frac{e\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^{m+1}}{\left(1-\frac{e^2x^2}{d^2}\right)^{7/2}}dx}{d^6g\sqrt{d^2-e^2x^2}}+\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{7/2}}dx}{d^5\sqrt{d^2-e^2x^2}}$$
$$\Big\downarrow \text{278}$$
$$\frac{e\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{m+2}{2},\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{d^6g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{d^5g(m+1)\sqrt{d^2-e^2x^2}}$$

input
```
Int[((g*x)^m*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
((g*x)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[7/2, (1 + m)/2, (
3 + m)/2, (e^2*x^2)/d^2])/(d^5*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) + (e*(g*x)^(
2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[7/2, (2 + m)/2, (4 + m)/2
, (e^2*x^2)/d^2])/(d^6*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

3.235.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

3.235.4 Maple [F]

$$\int \frac{(gx)^m (ex+d)}{(-e^2x^2+d^2)^{\frac{7}{2}}}dx$$

input
```
int((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

output
```
int((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

3.235.5 Fricas [F]

$$\int \frac{(gx)^m(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^7*x^7 - d*e^6*x^6 - 3*d^2*e^5*x^5
 + 3*d^3*e^4*x^4 + 3*d^4*e^3*x^3 - 3*d^5*e^2*x^2 - d^6*e*x + d^7), x)
```

3.235.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 26.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int \frac{(gx)^m(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{g^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}\frac{7}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2d^6\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{eg^m x^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{7}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2d^7\Gamma\left(\frac{m}{2}+2\right)}$$

input
```
integrate((g*x)**m*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((7/2, m/2 + 1/2), (m/2 + 3/2,), e**
2*x**2*exp_polar(2*I*pi)/d**2)/(2*d**6*gamma(m/2 + 3/2)) + e*g**m*x**(m +
2)*gamma(m/2 + 1)*hyper((7/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_polar(2*
I*pi)/d**2)/(2*d**7*gamma(m/2 + 2))
```

3.235.7 Maxima [F]

$$\int \frac{(gx)^m(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

3.235.8 Giac [F]

$$\int \frac{(gx)^m(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(g\,x)^m\,(d+e\,x)}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

input
```
int(((g*x)^m*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
int(((g*x)^m*(d + e*x))/(d^2 - e^2*x^2)^(7/2), x)
```

3.236 $\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

3.236.1 Optimal result

Integrand size = 22, antiderivative size = 80

$$\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{d^6g(1+m)\sqrt{d^2-e^2x^2}}$$

output
```
(g*x)^(1+m)*hypergeom([7/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2
/d^2)^(1/2)/d^6/g/(1+m)/(-e^2*x^2+d^2)^(1/2)
```

3.236.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{x(gx)^m\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{1+m}{2},1+\frac{1+m}{2},\frac{e^2x^2}{d^2}\right)}{d^6(1+m)\sqrt{d^2-e^2x^2}}$$

input
```
Integrate[(g*x)^m/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[7/2, (1 + m)/2, 1 + (
1 + m)/2, (e^2*x^2)/d^2])/(d^6*(1 + m)*Sqrt[d^2 - e^2*x^2])
```

3.236.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(gx)^m}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{7/2}}dx}{d^6\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{278}$$

$$\frac{\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{d^6g(m+1)\sqrt{d^2-e^2x^2}}$$

input
```
Int[(g*x)^m/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
((g*x)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[7/2, (1 + m)/2, (
3 + m)/2, (e^2*x^2)/d^2])/(d^6*g*(1 + m)*Sqrt[d^2 - e^2*x^2])
```

3.236.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

3.236.4 Maple [F]

$$\int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}dx$$

input
```
int((g*x)^m/(-e^2*x^2+d^2)^(7/2),x)
```

output
```
int((g*x)^m/(-e^2*x^2+d^2)^(7/2),x)
```

3.236.5 Fricas [F]

$$\int \frac{(gx)^m}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x)^m/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^8*x^8 - 4*d^2*e^6*x^6 + 6*d^4*e^4
*x^4 - 4*d^6*e^2*x^2 + d^8), x)
```

3.236.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.55 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int \frac{(gx)^m}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{g^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}\frac{7}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2d^7\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

input
```
integrate((g*x)**m/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((7/2, m/2 + 1/2), (m/2 + 3/2,), e**
2*x**2*exp_polar(2*I*pi)/d**2)/(2*d**7*gamma(m/2 + 3/2))
```

3.236.7 Maxima [F]

$$\int \frac{(gx)^m}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x)^m/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
integrate((g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

3.236.8 Giac [F]

$$\int \frac{(gx)^m}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x)^m/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

3.236.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(g\,x)^m}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

input
```
int((g*x)^m/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
int((g*x)^m/(d^2 - e^2*x^2)^(7/2), x)
```

3.237 $\int \frac{(gx)^m}{(d+ex)(d^2-e^2x^2)^{7/2}}\,dx$

3.237.1 Optimal result

Integrand size = 29, antiderivative size = 163

$$\int \frac{(gx)^m}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{9}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{d^7g(1+m)\sqrt{d^2-e^2x^2}} - \frac{e(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{9}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{d^8g^2(2+m)\sqrt{d^2-e^2x^2}}$$

output
```
(g*x)^(1+m)*hypergeom([9/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2
/d^2)^(1/2)/d^7/g/(1+m)/(-e^2*x^2+d^2)^(1/2)-e*(g*x)^(2+m)*hypergeom([9/2,
 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^8/g^2/(2+m)/(-e^2
*x^2+d^2)^(1/2)
```

3.237.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.75

$$\int \frac{(gx)^m}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{x(gx)^m\sqrt{1-\frac{e^2x^2}{d^2}}\left(-e(1+m)x\,\text{Hypergeometric2F1}\left(\frac{9}{2},1+\frac{m}{2},2+\frac{m}{2},\frac{e^2x^2}{d^2}\right)\right.}{d^8(1+m)(2+m)\sqrt{d^2-}}$$

input
```
Integrate[(g*x)^m/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*(-(e*(1 + m)*x*Hypergeometric2F1[9/2, 1
 + m/2, 2 + m/2, (e^2*x^2)/d^2]) + d*(2 + m)*Hypergeometric2F1[9/2, (1 + m
)/2, (3 + m)/2, (e^2*x^2)/d^2]))/(d^8*(1 + m)*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

3.237.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {583, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(gx)^m}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{583}$$

$$\int \frac{(d-ex)(gx)^m}{\left(d^2-e^2x^2\right)^{9/2}}dx$$

$$\Big\downarrow \text{557}$$

$$d\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{9/2}}dx-\frac{e\int \frac{(gx)^{m+1}}{\left(d^2-e^2x^2\right)^{9/2}}dx}{g}$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{9/2}}dx}{d^7\sqrt{d^2-e^2x^2}}-\frac{e\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^{m+1}}{\left(1-\frac{e^2x^2}{d^2}\right)^{9/2}}dx}{d^8g\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{278}$$

$$\frac{\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\text{Hypergeometric2F1}\left(\frac{9}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{d^7g(m+1)\sqrt{d^2-e^2x^2}}-\frac{e\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\text{Hypergeometric2F1}\left(\frac{9}{2},\frac{m+2}{2},\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{d^8g^2(m+2)\sqrt{d^2-e^2x^2}}$$

input
```
Int[(g*x)^m/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
((g*x)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (1 + m)/2, (
3 + m)/2, (e^2*x^2)/d^2])/(d^7*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) - (e*(g*x)^(
2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (2 + m)/2, (4 + m)/2
, (e^2*x^2)/d^2])/(d^8*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

3.237.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 583
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, 0]
```

3.237.4 Maple [F]

$$\int \frac{(gx)^m}{(ex+d)\,(-e^2x^2+d^2)^{\frac{7}{2}}}dx$$

input
```
int((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

output
```
int((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

3.237.5 Fricas [F]

$$\int \frac{(gx)^m}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)}\,dx$$

input
```
integrate((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^9*x^9 + d*e^8*x^8 - 4*d^2*e^7*x^7
 - 4*d^3*e^6*x^6 + 6*d^4*e^5*x^5 + 6*d^5*e^4*x^4 - 4*d^6*e^3*x^3 - 4*d^7*e
^2*x^2 + d^8*e*x + d^9), x)
```

3.237.6 Sympy [F]

$$\int \frac{(gx)^m}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{(gx)^m}{(-(-d+ex)(d+ex))^{\frac{7}{2}}(d+ex)}\,dx$$

input
```
integrate((g*x)**m/(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((g*x)**m/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)), x)
```

3.237.7 Maxima [F]

$$\int \frac{(gx)^m}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)}\,dx$$

input
```
integrate((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)), x)
```

3.237.8 Giac [F]

$$\int \frac{(gx)^m}{(d+ex)(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)}\,dx$$

input
```
integrate((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)), x)
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m}{(d+ex)(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(g\,x)^m}{(d^2-e^2\,x^2)^{7/2}\,(d+e\,x)}\,dx$$

input
```
int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)),x)
```

output
```
int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)), x)
```

3.238 $\int \frac{(gx)^m}{(d+ex)^2(d^2-e^2x^2)^{7/2}}\,dx$

3.238.1 Optimal result

Integrand size = 29, antiderivative size = 217

$$\int \frac{(gx)^m}{(d+ex)^2(d^2-e^2x^2)^{7/2}}\,dx = \frac{2(gx)^{1+m}(d-ex)}{9dg(d^2-e^2x^2)^{9/2}} + \frac{(7-2m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{9}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{9d^8g(1+m)\sqrt{d^2-e^2x^2}} - \frac{2e(7-m)(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{9}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{9d^9g^2(2+m)\sqrt{d^2-e^2x^2}}$$

output
```
2/9*(g*x)^(1+m)*(-e*x+d)/d/g/(-e^2*x^2+d^2)^(9/2)+1/9*(7-2*m)*(g*x)^(1+m)*
hypergeom([9/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/
d^8/g/(1+m)/(-e^2*x^2+d^2)^(1/2)-2/9*e*(7-m)*(g*x)^(2+m)*hypergeom([9/2, 1
+1/2*m],[2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^9/g^2/(2+m)/(-e^2*x
^2+d^2)^(1/2)
```

3.238.2 Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.81

$$\int \frac{(gx)^m}{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{x(gx)^m\sqrt{1-\frac{e^2x^2}{d^2}}\left(d^2(6+5m+m^2)\,\text{Hypergeometric2F1}\left(\frac{11}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2}{d}\right.\right.}{}$$

input
```
Integrate[(g*x)^m/((d + e*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[
11/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] - e*(1 + m)*x*(2*d*(3 + m)*Hype
rgeometric2F1[11/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2] - e*(2 + m)*x*Hyp
ergeometric2F1[11/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])))/(d^10*(1 + m)
*(2 + m)*(3 + m)*Sqrt[d^2 - e^2*x^2])
```

3.238.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {570, 558, 25, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(gx)^m}{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{(d-ex)^2(gx)^m}{\left(d^2-e^2x^2\right)^{11/2}}dx$$

$$\Big\downarrow \text{558}$$

$$\frac{2(d-ex)(gx)^{m+1}}{9dg\left(d^2-e^2x^2\right)^{9/2}}-\frac{\int -\frac{d(gx)^m(d(7-2m)-2e(7-m)x)}{\left(d^2-e^2x^2\right)^{9/2}}dx}{9d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d(gx)^m(d(7-2m)-2e(7-m)x)}{\left(d^2-e^2x^2\right)^{9/2}}dx}{9d^2}+\frac{2(d-ex)(gx)^{m+1}}{9dg\left(d^2-e^2x^2\right)^{9/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(gx)^m(d(7-2m)-2e(7-m)x)}{(d^2-e^2x^2)^{9/2}}dx}{9d}+\frac{2(d-ex)(gx)^{m+1}}{9dg\left(d^2-e^2x^2\right)^{9/2}}$$

$$\Big\downarrow \; 557$$

$$\frac{d(7-2m)\int \frac{(gx)^m}{(d^2-e^2x^2)^{9/2}}dx-\frac{2e(7-m)\int \frac{(gx)^{m+1}}{\left(d^2-e^2x^2\right)^{9/2}}dx}{g}}{9d}+\frac{2(d-ex)(gx)^{m+1}}{9dg\left(d^2-e^2x^2\right)^{9/2}}$$

$$\Big\downarrow \; 279$$

$$\frac{\frac{(7-2m)\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{9/2}}dx}{d^7\sqrt{d^2-e^2x^2}}-\frac{2e(7-m)\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^{m+1}}{\left(1-\frac{e^2x^2}{d^2}\right)^{9/2}}dx}{d^8g\sqrt{d^2-e^2x^2}}}{9d}+\frac{2(d-ex)(gx)^{m+1}}{9dg\left(d^2-e^2x^2\right)^{9/2}}$$

$$\Big\downarrow \; 278$$

$$\frac{2(d-ex)(gx)^{m+1}}{9dg\left(d^2-e^2x^2\right)^{9/2}}+\frac{\frac{(7-2m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\text{Hypergeometric2F1}\left(\frac{9}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{d^7g(m+1)\sqrt{d^2-e^2x^2}}-\frac{2e(7-m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\text{Hypergeometric2F1}\left(\frac{9}{2},\frac{m+2}{2},\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{d^8g^2(m+2)\sqrt{d^2-e^2x^2}}}{9d}$$

input
```
Int[(g*x)^m/((d + e*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(2*(g*x)^(1 + m)*(d - e*x))/(9*d*g*(d^2 - e^2*x^2)^(9/2)) + (((7 - 2*m)*(g
*x)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (1 + m)/2, (3 +
 m)/2, (e^2*x^2)/d^2])/(d^7*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) - (2*e*(7 - m)*
(g*x)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (2 + m)/2, (4
 + m)/2, (e^2*x^2)/d^2])/(d^8*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2]))/(9*d)
```

3.238.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.238. $\int \frac{(gx)^m}{(d+ex)^2(d^2-e^2x^2)^{7/2}}\,dx$

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 558
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, a + b*x^2, x], f =
 Coeff[PolynomialRemainder[(c + d*x)^n, a + b*x^2, x], x, 0], g = Coeff[Pol
ynomialRemainder[(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(-(e*x)^(m + 1))*
(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*e*(p + 1))), x] + Simp[1/(2*a*(p + 1))
  Int[(e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + f*(m + 2*p +
 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] && IGt
Q[n, 1] &&  !IntegerQ[m] && LtQ[p, -1]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.238.4 Maple [F]

$$\int \frac{(gx)^m}{(ex+d)^2\,(-e^2x^2+d^2)^{\frac{7}{2}}}dx$$

input
```
int((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

output
```
int((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

3.238.5 Fricas [F]

$$\int \frac{(gx)^m}{(d+ex)^2 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^2}\,dx$$

input
```
integrate((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^10*x^10 + 2*d*e^9*x^9 - 3*d^2*e^8
*x^8 - 8*d^3*e^7*x^7 + 2*d^4*e^6*x^6 + 12*d^5*e^5*x^5 + 2*d^6*e^4*x^4 - 8*
d^7*e^3*x^3 - 3*d^8*e^2*x^2 + 2*d^9*e*x + d^10), x)
```

3.238.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(gx)^m}{(d+ex)^2 (d^2-e^2x^2)^{7/2}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((g*x)**m/(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.238.7 Maxima [F]

$$\int \frac{(gx)^m}{(d+ex)^2 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^2}\,dx$$

input
```
integrate((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^2), x)
```

3.238.8 Giac [F]

$$\int \frac{(gx)^m}{(d+ex)^2 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^2}\,dx$$

input
```
integrate((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^2), x)
```

3.238.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m}{(d+ex)^2 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(g\,x)^m}{(d^2-e^2\,x^2)^{7/2}\,(d+e\,x)^2}\,dx$$

input
```
int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^2),x)
```

output
```
int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^2), x)
```

3.239 $\int \frac{(gx)^m}{(d+ex)^3 (d^2-e^2x^2)^{7/2}}\,dx$

3.239.1 Optimal result

Integrand size = 29, antiderivative size = 214

$$\begin{aligned}\int \frac{(gx)^m}{(d+ex)^3\,(d^2-e^2x^2)^{7/2}}\,dx &= \frac{4(gx)^{1+m}(d-ex)}{11g\,(d^2-e^2x^2)^{11/2}} \\ &+ \frac{(7-4m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{11}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{11d^9g(1+m)\sqrt{d^2-e^2x^2}} \\ &- \frac{e(25-4m)(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{11}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{11d^{10}g^2(2+m)\sqrt{d^2-e^2x^2}}\end{aligned}$$

output
```
4/11*(g*x)^(1+m)*(-e*x+d)/g/(-e^2*x^2+d^2)^(11/2)+1/11*(7-4*m)*(g*x)^(1+m)
*hypergeom([11/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2
)/d^9/g/(1+m)/(-e^2*x^2+d^2)^(1/2)-1/11*e*(25-4*m)*(g*x)^(2+m)*hypergeom([
11/2, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^10/g^2/(2+m)
/(-e^2*x^2+d^2)^(1/2)
```

3.239.2 Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.93

$$\int \frac{(gx)^m}{(d+ex)^3 (d^2-e^2x^2)^{7/2}}\,dx = \frac{x(gx)^m\sqrt{1-\frac{e^2x^2}{d^2}}\left(\frac{d^3\,\text{Hypergeometric2F1}\left(\frac{13}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{1+m}+ex\left(-\frac{3d^2\,\text{Hypergeome}}{}\right.\right.}{}$$

input
```
Integrate[(g*x)^m/((d + e*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*((d^3*Hypergeometric2F1[13/2, (1 + m)/2
, (3 + m)/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((-3*d^2*Hypergeometric2F1[13/2
, (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric
2F1[13/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) - (e*x*Hypergeomet
ric2F1[13/2, (4 + m)/2, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/(d^12*Sqrt[
d^2 - e^2*x^2])
```

3.239.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {570, 558, 25, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(gx)^m}{(d+ex)^3 (d^2-e^2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \ 570$$

$$\int \frac{(d-ex)^3(gx)^m}{(d^2-e^2x^2)^{13/2}}dx$$

$$\Big\downarrow \ 558$$

$$\frac{4(d-ex)(gx)^{m+1}}{11g\,(d^2-e^2x^2)^{11/2}}-\frac{\int -\frac{d^2(gx)^m(d(7-4m)-e(25-4m)x)}{(d^2-e^2x^2)^{11/2}}dx}{11d^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{d^2(gx)^m(d(7-4m)-e(25-4m)x)}{(d^2-e^2x^2)^{11/2}}dx}{11d^2}+\frac{4(d-ex)(gx)^{m+1}}{11g\left(d^2-e^2x^2\right)^{11/2}}$$

$\big\downarrow$ 27

$$\frac{1}{11}\int \frac{(gx)^m(d(7-4m)-e(25-4m)x)}{(d^2-e^2x^2)^{11/2}}dx+\frac{4(d-ex)(gx)^{m+1}}{11g\left(d^2-e^2x^2\right)^{11/2}}$$

$\big\downarrow$ 557

$$\frac{1}{11}\left(d(7-4m)\int \frac{(gx)^m}{(d^2-e^2x^2)^{11/2}}dx-\frac{e(25-4m)\int \frac{(gx)^{m+1}}{(d^2-e^2x^2)^{11/2}}dx}{g}\right)+\frac{4(d-ex)(gx)^{m+1}}{11g\left(d^2-e^2x^2\right)^{11/2}}$$

$\big\downarrow$ 279

$$\frac{1}{11}\left(\frac{(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{11/2}}dx}{d^9\sqrt{d^2-e^2x^2}}-\frac{e(25-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^{m+1}}{\left(1-\frac{e^2x^2}{d^2}\right)^{11/2}}dx}{d^{10}g\sqrt{d^2-e^2x^2}}\right)+$$
$$\frac{4(d-ex)(gx)^{m+1}}{11g\left(d^2-e^2x^2\right)^{11/2}}$$

$\big\downarrow$ 278

$$\frac{4(d-ex)(gx)^{m+1}}{11g\left(d^2-e^2x^2\right)^{11/2}}+$$
$$\frac{1}{11}\left(\frac{(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\text{Hypergeometric2F1}\left(\frac{11}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{d^9g(m+1)\sqrt{d^2-e^2x^2}}-\frac{e(25-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\text{Hyp}}{d^{10}g^2(m+}\right.$$

input
```
Int[(g*x)^m/((d + e*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(4*(g*x)^(1 + m)*(d - e*x))/(11*g*(d^2 - e^2*x^2)^(11/2)) + (((7 - 4*m)*(g
*x)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[11/2, (1 + m)/2, (3
+ m)/2, (e^2*x^2)/d^2])/(d^9*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) - (e*(25 - 4*m
)*(g*x)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[11/2, (2 + m)/2,
 (4 + m)/2, (e^2*x^2)/d^2])/(d^10*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2]))/11
```

3.239. $\int \frac{(gx)^m}{(d+ex)^3(d^2-e^2x^2)^{7/2}}\,dx$

3.239.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 558
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, a + b*x^2, x], f =
 Coeff[PolynomialRemainder[(c + d*x)^n, a + b*x^2, x], x, 0], g = Coeff[Pol
ynomialRemainder[(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(-(e*x)^(m + 1))*
(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*e*(p + 1))), x] + Simp[1/(2*a*(p + 1))
  Int[(e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + f*(m + 2*p +
 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] && IGt
Q[n, 1] &&  !IntegerQ[m] && LtQ[p, -1]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.239. $\int \frac{(gx)^m}{(d+ex)^3(d^2-e^2x^2)^{7/2}}\,dx$

3.239.4 Maple [F]

$$\int \frac{(gx)^m}{(ex+d)^3 (-e^2x^2+d^2)^{\frac{7}{2}}}dx$$

input
```
int((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

output
```
int((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

3.239.5 Fricas [F]

$$\int \frac{(gx)^m}{(d+ex)^3 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^3}\,dx$$

input
```
integrate((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^11*x^11 + 3*d*e^10*x^10 - d^2*e^9
*x^9 - 11*d^3*e^8*x^8 - 6*d^4*e^7*x^7 + 14*d^5*e^6*x^6 + 14*d^6*e^5*x^5 -
6*d^7*e^4*x^4 - 11*d^8*e^3*x^3 - d^9*e^2*x^2 + 3*d^10*e*x + d^11), x)
```

3.239.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(gx)^m}{(d+ex)^3 (d^2-e^2x^2)^{7/2}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((g*x)**m/(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.239.7 Maxima [F]

$$\int \frac{(gx)^m}{(d+ex)^3(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^3}\,dx$$

input
```
integrate((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^3), x)
```

3.239.8 Giac [F]

$$\int \frac{(gx)^m}{(d+ex)^3(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^3}\,dx$$

input
```
integrate((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^3), x)
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m}{(d+ex)^3(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(g\,x)^m}{(d^2-e^2\,x^2)^{7/2}\,(d+e\,x)^3}\,dx$$

input
```
int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^3),x)
```

output
```
int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^3), x)
```

3.240 $\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

3.240.1 Optimal result

Integrand size = 23, antiderivative size = 148

$$\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx=-\frac{d^5\left(d^2-e^2x^2\right)^{1+p}}{2e^6(1+p)}+\frac{d^3\left(d^2-e^2x^2\right)^{2+p}}{e^6(2+p)}-\frac{d\left(d^2-e^2x^2\right)^{3+p}}{2e^6(3+p)}+\frac{1}{7}ex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)$$

output
```
-1/2*d^5*(-e^2*x^2+d^2)^(p+1)/e^6/(p+1)+d^3*(-e^2*x^2+d^2)^(2+p)/e^6/(2+p)
-1/2*d*(-e^2*x^2+d^2)^(3+p)/e^6/(3+p)+1/7*e*x^7*(-e^2*x^2+d^2)^p*hypergeom
([7/2, -p],[9/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
```

3.240.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.89

$$\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx$$
$$=\frac{\left(d^2-e^2x^2\right)^p\left(-\frac{7d\left(d^2-e^2x^2\right)\left(2d^4+2d^2e^2(1+p)x^2+e^4\left(2+3p+p^2\right)x^4\right)}{(1+p)(2+p)(3+p)}+2e^7x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p\right.\right.}{14e^6}$$

input
```
Integrate[x^5*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

output
```
((d^2 - e^2*x^2)^p*((-7*d*(d^2 - e^2*x^2)*(2*d^4 + 2*d^2*e^2*(1 + p)*x^2 +
 e^4*(2 + 3*p + p^2)*x^4))/((1 + p)*(2 + p)*(3 + p)) + (2*e^7*x^7*Hypergeo
metric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/(14*e^6)
```

3.240.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {542, 243, 53, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(d+ex)\left(d^2-e^2x^2\right)^p dx$$

$$\Big\downarrow \text{542}$$

$$e\int x^6\left(d^2-e^2x^2\right)^p dx+d\int x^5\left(d^2-e^2x^2\right)^p dx$$

$$\Big\downarrow \text{243}$$

$$e\int x^6\left(d^2-e^2x^2\right)^p dx+\frac{1}{2}d\int x^4\left(d^2-e^2x^2\right)^p dx^2$$

$$\Big\downarrow \text{53}$$

$$e\int x^6\left(d^2-e^2x^2\right)^p dx+\frac{1}{2}d\int \left(\frac{d^4\left(d^2-e^2x^2\right)^p}{e^4}-\frac{2d^2\left(d^2-e^2x^2\right)^{p+1}}{e^4}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{e^4}\right)dx^2$$

$$\Big\downarrow \text{279}$$

$$e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int x^6\left(1-\frac{e^2x^2}{d^2}\right)^p dx+\frac{1}{2}d\int \left(\frac{d^4\left(d^2-e^2x^2\right)^p}{e^4}-\frac{2d^2\left(d^2-e^2x^2\right)^{p+1}}{e^4}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{e^4}\right)dx^2$$

$$\Big\downarrow \text{278}$$

$$\frac{1}{2}d\int \left(\frac{d^4\left(d^2-e^2x^2\right)^p}{e^4}-\frac{2d^2\left(d^2-e^2x^2\right)^{p+1}}{e^4}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{e^4}\right)dx^2+\frac{1}{7}ex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}ex^7(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)+\frac{1}{2}d\left(\frac{2d^2(d^2-e^2x^2)^{p+2}}{e^6(p+2)}-\frac{(d^2-e^2x^2)^{p+3}}{e^6(p+3)}-\frac{d^4(d^2-e^2x^2)^{p+1}}{e^6(p+1)}\right)$$

input
```
Int[x^5*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

output
```
(d*(-((d^4*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p))) + (2*d^2*(d^2 - e^2*x^2
)^(2 + p))/(e^6*(2 + p)) - (d^2 - e^2*x^2)^(3 + p)/(e^6*(3 + p))))/2 + (e*
x^7*(d^2 - e^2*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(7*(
1 - (e^2*x^2)/d^2)^p)
```

3.240.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

3.240. $\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.240.4 Maple [F]

$$\int x^5(ex+d)\left(-e^2x^2+d^2\right)^p dx$$

input
```
int(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

output
```
int(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

3.240.5 Fricas [F]

$$\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx = \int (ex+d)\left(-e^2x^2+d^2\right)^p x^5\,dx$$

input
```
integrate(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((e*x^6 + d*x^5)*(-e^2*x^2 + d^2)^p, x)
```

3.240.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(121) = 242$.

Time = 1.95 (sec) , antiderivative size = 972, normalized size of antiderivative = 6.57

$$\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx$$
$$= d\left(\begin{cases} \frac{x^6(d^2)^p}{6} \\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} - \frac{2d^4\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} - \frac{3d^4}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} + \frac{4d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} + \frac{4d^2e^2x^2\log\left(\frac{d}{e}\right.}{4d^4e^6-8d^2e^8x^2+} \\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2} - \frac{2d^4\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2} - \frac{2d^4}{-2d^2e^6+2e^8x^2} + \frac{2d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2} + \frac{2d^2e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2} + \frac{e^4x^4}{-2d^2e^6+2e^8x^2} \\ -\frac{d^4\log\left(-\frac{d}{e}+x\right)}{2e^6} - \frac{d^4\log\left(\frac{d}{e}+x\right)}{2e^6} - \frac{d^2x^2}{2e^4} - \frac{x^4}{4e^2} \\ -\frac{2d^6(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} - \frac{2d^4e^2px^2(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} - \frac{d^2e^4p^2x^4(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} - \frac{d^2e^4px^4(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} + \frac{}{2e} \end{cases}\right.$$
$$+ \frac{d^{2p}ex^7{}_2F_1\left(\begin{matrix}\frac{7}{2},-p\\ \frac{9}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{7}$$

input
```
integrate(x**5*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

output
```
d*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/(4*d**4*e
**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d**4*e**6
- 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2*e**8*x**
2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e
**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6 - 8*
d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d**2*e*
*8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*
e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d**2
*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*e*
*6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d
**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2
*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**
8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-
d/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4
/(4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e*
*6*p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/
(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(
d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) -
d**2*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e*
*6*p + 12*e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 1...
```

3.240. $\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

3.240.7 Maxima [F]

$$\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p x^5\,dx$$

input
```
integrate(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
e*integrate(x^6*e^(p*log(e*x + d) + p*log(-e*x + d)), x) + 1/2*((p^2 + 3*p
 + 2)*e^6*x^6 - (p^2 + p)*d^2*e^4*x^4 - 2*d^4*e^2*p*x^2 - 2*d^6)*(-e^2*x^2
 + d^2)^p*d/((p^3 + 6*p^2 + 11*p + 6)*e^6)
```

3.240.8 Giac [F]

$$\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p x^5\,dx$$

input
```
integrate(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^5, x)
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int x^5\left(d^2-e^2x^2\right)^p(d+e\,x)\,dx$$

input
```
int(x^5*(d^2 - e^2*x^2)^p*(d + e*x),x)
```

output
```
int(x^5*(d^2 - e^2*x^2)^p*(d + e*x), x)
```

3.241 $\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

3.241.1 Optimal result

Integrand size = 23, antiderivative size = 147

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx=-\frac{d^4\left(d^2-e^2x^2\right)^{1+p}}{2e^5(1+p)}+\frac{d^2\left(d^2-e^2x^2\right)^{2+p}}{e^5(2+p)}-\frac{\left(d^2-e^2x^2\right)^{3+p}}{2e^5(3+p)}+\frac{1}{5}dx^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)$$

output
```
-1/2*d^4*(-e^2*x^2+d^2)^(p+1)/e^5/(p+1)+d^2*(-e^2*x^2+d^2)^(2+p)/e^5/(2+p)
-1/2*(-e^2*x^2+d^2)^(3+p)/e^5/(3+p)+1/5*d*x^5*(-e^2*x^2+d^2)^p*hypergeom([
5/2, -p],[7/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
```

3.241.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.88

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\frac{1}{10}\left(d^2-e^2x^2\right)^p\left(-\frac{5\left(d^2-e^2x^2\right)\left(2d^4+2d^2e^2(1+p)x^2+e^4\left(2+3p+p^2\right)x^4\right)}{e^5(1+p)(2+p)(3+p)}+2dx^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)\right)$$

input
```
Integrate[x^4*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

output
```
((d^2 - e^2*x^2)^p*((-5*(d^2 - e^2*x^2)*(2*d^4 + 2*d^2*e^2*(1 + p)*x^2 + e
^4*(2 + 3*p + p^2)*x^4))/(e^5*(1 + p)*(2 + p)*(3 + p)) + (2*d*x^5*Hypergeo
metric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/10
```

3.241.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {542, 243, 53, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx$$

$$\Big\downarrow \text{542}$$

$$e\int x^5\left(d^2-e^2x^2\right)^pdx+d\int x^4\left(d^2-e^2x^2\right)^pdx$$

$$\Big\downarrow \text{243}$$

$$d\int x^4\left(d^2-e^2x^2\right)^pdx+\frac{1}{2}e\int x^4\left(d^2-e^2x^2\right)^pdx^2$$

$$\Big\downarrow \text{53}$$

$$d\int x^4\left(d^2-e^2x^2\right)^pdx+\frac{1}{2}e\int\left(\frac{d^4\left(d^2-e^2x^2\right)^p}{e^4}-\frac{2d^2\left(d^2-e^2x^2\right)^{p+1}}{e^4}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{e^4}\right)dx^2$$

$$\Big\downarrow \text{279}$$

$$d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^pdx+\frac{1}{2}e\int\left(\frac{d^4\left(d^2-e^2x^2\right)^p}{e^4}-\frac{2d^2\left(d^2-e^2x^2\right)^{p+1}}{e^4}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{e^4}\right)dx^2$$

$$\Big\downarrow \text{278}$$

$$\frac{1}{2}e\int\left(\frac{d^4\left(d^2-e^2x^2\right)^p}{e^4}-\frac{2d^2\left(d^2-e^2x^2\right)^{p+1}}{e^4}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{e^4}\right)dx^2+\frac{1}{5}dx^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}dx^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)+\frac{1}{2}e\left(\frac{2d^2(d^2-e^2x^2)^{p+2}}{e^6(p+2)}-\frac{(d^2-e^2x^2)^{p+3}}{e^6(p+3)}-\frac{d^4(d^2-e^2x^2)^{p+1}}{e^6(p+1)}\right)$$

input
```
Int[x^4*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

output
```
(e*(-((d^4*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p))) + (2*d^2*(d^2 - e^2*x^2
)^(2 + p))/(e^6*(2 + p)) - (d^2 - e^2*x^2)^(3 + p)/(e^6*(3 + p))))/2 + (d*
x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(
1 - (e^2*x^2)/d^2)^p)
```

3.241.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

3.241. $\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.241.4 Maple [F]

$$\int x^4(ex+d)\left(-e^2x^2+d^2\right)^p dx$$

input
```
int(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

output
```
int(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

3.241.5 Fricas [F]

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p x^4\,dx$$

input
```
integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((e*x^5 + d*x^4)*(-e^2*x^2 + d^2)^p, x)
```

3.241.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(119) = 238$.

Time = 1.87 (sec) , antiderivative size = 972, normalized size of antiderivative = 6.61

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\frac{dd^{2p}x^5{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{5}$$
$$+e\left(\begin{cases}\frac{x^6(d^2)^p}{6}\\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}-\frac{2d^4\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}-\frac{3d^4}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}+\frac{4d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}+\frac{4d^2e^2x^2\log}{4d^4e^6-8d^2e^8x}\\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}-\frac{2d^4\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}-\frac{2d^4}{-2d^2e^6+2e^8x^2}+\frac{2d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}+\frac{2d^2e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}+\frac{e^4x^4}{-2d^2e^6+2e^8x^2}\\ -\frac{d^4\log\left(-\frac{d}{e}+x\right)}{2e^6}-\frac{d^4\log\left(\frac{d}{e}+x\right)}{2e^6}-\frac{d^2x^2}{2e^4}-\frac{x^4}{4e^2}\\ -\frac{2d^6(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{2d^4e^2px^2(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{d^2e^4p^2x^4(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{d^2e^4px^4(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}+\end{cases}\right.$$

input
```
integrate(x**4*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

output
```
d*d**(2*p)*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)
/5 + e*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/(4*d
**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d**4*
e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2*e**
8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d
**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6
 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d*
*2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*
d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8
*d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d*
*2*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2)
- 2*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2
*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 +
2*e**8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*
log(-d/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) -
 x**4/(4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 +
12*e**6*p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2
)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x
**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**
6) - d**2*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2...
```

3.241. $\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

3.241.7 Maxima [F]

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p x^4\,dx$$

input
```
integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^4, x)
```

3.241.8 Giac [F]

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p x^4\,dx$$

input
```
integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^4, x)
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int x^4\left(d^2-e^2x^2\right)^p(d+e\,x)\,dx$$

input
```
int(x^4*(d^2 - e^2*x^2)^p*(d + e*x),x)
```

output
```
int(x^4*(d^2 - e^2*x^2)^p*(d + e*x), x)
```

3.242 $\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

3.242.1 Optimal result

Integrand size = 23, antiderivative size = 120

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx=-\frac{d^3\left(d^2-e^2x^2\right)^{1+p}}{2e^4(1+p)}+\frac{d\left(d^2-e^2x^2\right)^{2+p}}{2e^4(2+p)}+\frac{1}{5}ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)$$

output
```
-1/2*d^3*(-e^2*x^2+d^2)^(p+1)/e^4/(p+1)+1/2*d*(-e^2*x^2+d^2)^(2+p)/e^4/(2+
p)+1/5*e*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, -p],[7/2],e^2*x^2/d^2)/((1-e
^2*x^2/d^2)^p)
```

3.242.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\frac{\left(d^2-e^2x^2\right)^p\left(-\frac{5d(d^2-e^2x^2)(d^2+e^2(1+p)x^2)}{(1+p)(2+p)}+2e^5x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)\right)}{10e^4}$$

input
```
Integrate[x^3*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

output
```
((d^2 - e^2*x^2)^p*((-5*d*(d^2 - e^2*x^2)*(d^2 + e^2*(1 + p)*x^2))/((1 + p
)*(2 + p)) + (2*e^5*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1
 - (e^2*x^2)/d^2)^p))/(10*e^4)
```

3.242.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {542, 243, 53, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx$$

$$\Big\downarrow \text{542}$$

$$e\int x^4\left(d^2-e^2x^2\right)^p\,dx+d\int x^3\left(d^2-e^2x^2\right)^p\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}d\int x^2\left(d^2-e^2x^2\right)^p\,dx^2+e\int x^4\left(d^2-e^2x^2\right)^p\,dx$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}d\int\left(\frac{d^2\left(d^2-e^2x^2\right)^p}{e^2}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{e^2}\right)dx^2+e\int x^4\left(d^2-e^2x^2\right)^p\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{1}{2}d\int\left(\frac{d^2\left(d^2-e^2x^2\right)^p}{e^2}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{e^2}\right)dx^2+e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx$$

$$\Big\downarrow \text{278}$$

$$\frac{1}{2}d\int\left(\frac{d^2\left(d^2-e^2x^2\right)^p}{e^2}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{e^2}\right)dx^2+\frac{1}{5}ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}ex^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)+\frac{1}{2}d\left(\frac{(d^2-e^2x^2)^{p+2}}{e^4(p+2)}-\frac{d^2(d^2-e^2x^2)^{p+1}}{e^4(p+1)}\right)$$

input
```
Int[x^3*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

output
```
(d*(-((d^2*(d^2 - e^2*x^2)^(1 + p))/(e^4*(1 + p))) + (d^2 - e^2*x^2)^(2 +
p)/(e^4*(2 + p))))/2 + (e*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p,
 7/2, (e^2*x^2)/d^2])/(5*(1 - (e^2*x^2)/d^2)^p)
```

3.242.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.242.4 Maple [F]

$$\int x^3(ex+d)\left(-e^2x^2+d^2\right)^p dx$$

input
```
int(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

output
```
int(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

3.242.5 Fricas [F]

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p x^3\,dx$$

input
```
integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((e*x^4 + d*x^3)*(-e^2*x^2 + d^2)^p, x)
```

3.242.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(97) = 194.

Time = 1.59 (sec) , antiderivative size = 382, normalized size of antiderivative = 3.18

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx$$
$$=d\left(\begin{cases}\frac{x^4(d^2)^p}{4} & \text{for } e=0\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} & \text{for } p=-2\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4}-\frac{x^2}{2e^2} & \text{for } p=-1\\ -\frac{d^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}-\frac{d^2e^2px^2(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4px^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4x^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} & \text{otherwise}\end{cases}\right)$$
$$+\frac{d^{2p}ex^5\,{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{5}$$

input
```
integrate(x**3*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

output
```
d*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2*e*
*4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2/
(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e
**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p, -2)
), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2*e*
*2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*
e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4
*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**
4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4), Tr
ue)) + d**(2*p)*e*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi
)/d**2)/5
```

3.242.7 Maxima [F]

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int(ex+d)\left(-e^2x^2+d^2\right)^px^3\,dx$$

input
```
integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
e*integrate(x^4*e^(p*log(e*x + d) + p*log(-e*x + d)), x) + 1/2*(e^4*(p + 1
)*x^4 - d^2*e^2*p*x^2 - d^4)*(-e^2*x^2 + d^2)^p*d/((p^2 + 3*p + 2)*e^4)
```

3.242.8 Giac [F]

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int(ex+d)\left(-e^2x^2+d^2\right)^px^3\,dx$$

input
```
integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^3, x)
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int x^3\left(d^2-e^2x^2\right)^p(d+e\,x)\,dx$$

input
```
int(x^3*(d^2 - e^2*x^2)^p*(d + e*x),x)
```

output
```
int(x^3*(d^2 - e^2*x^2)^p*(d + e*x), x)
```

3.243 $\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

3.243.1 Optimal result

Integrand size = 23, antiderivative size = 119

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx=-\frac{d^2(d^2-e^2x^2)^{1+p}}{2e^3(1+p)}+\frac{(d^2-e^2x^2)^{2+p}}{2e^3(2+p)}+\frac{1}{3}dx^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)$$

output
```
-1/2*d^2*(-e^2*x^2+d^2)^(p+1)/e^3/(p+1)+1/2*(-e^2*x^2+d^2)^(2+p)/e^3/(2+p)
+1/3*d*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/2, -p],[5/2],e^2*x^2/d^2)/((1-e^2
*x^2/d^2)^p)
```

3.243.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\frac{1}{6}\left(d^2-e^2x^2\right)^p\left(-\frac{3(d^2-e^2x^2)\left(d^2+e^2(1+p)x^2\right)}{e^3(1+p)(2+p)}+2dx^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)\right)$$

input
```
Integrate[x^2*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

output
```
((d^2 - e^2*x^2)^p*((-3*(d^2 - e^2*x^2)*(d^2 + e^2*(1 + p)*x^2))/(e^3*(1 +
 p)*(2 + p)) + (2*d*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(1
 - (e^2*x^2)/d^2)^p))/6
```

3.243.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {542, 243, 53, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx$$

$$\Big\downarrow \text{ 542}$$

$$d\int x^2\left(d^2-e^2x^2\right)^p dx+e\int x^3\left(d^2-e^2x^2\right)^p dx$$

$$\Big\downarrow \text{ 243}$$

$$d\int x^2\left(d^2-e^2x^2\right)^p dx+\frac{1}{2}e\int x^2\left(d^2-e^2x^2\right)^p dx^2$$

$$\Big\downarrow \text{ 53}$$

$$d\int x^2\left(d^2-e^2x^2\right)^p dx+\frac{1}{2}e\int\left(\frac{d^2\left(d^2-e^2x^2\right)^p}{e^2}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{e^2}\right)dx^2$$

$$\Big\downarrow \text{ 279}$$

$$d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^p dx+\frac{1}{2}e\int\left(\frac{d^2\left(d^2-e^2x^2\right)^p}{e^2}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{e^2}\right)dx^2$$

$$\Big\downarrow \text{ 278}$$

$$\frac{1}{2}e\int\left(\frac{d^2\left(d^2-e^2x^2\right)^p}{e^2}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{e^2}\right)dx^2+\frac{1}{3}dx^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}dx^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)+\frac{1}{2}e\left(\frac{(d^2-e^2x^2)^{p+2}}{e^4(p+2)}-\frac{d^2(d^2-e^2x^2)^{p+1}}{e^4(p+1)}\right)$$

input
```
Int[x^2*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

output
```
(e*(-((d^2*(d^2 - e^2*x^2)^(1 + p))/(e^4*(1 + p))) + (d^2 - e^2*x^2)^(2 +
p)/(e^4*(2 + p))))/2 + (d*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p,
 5/2, (e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)
```

3.243.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.243.4 Maple [F]

$$\int x^2(ex+d)\left(-e^2x^2+d^2\right)^p dx$$

input
```
int(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

output
```
int(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

3.243.5 Fricas [F]

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx = \int (ex+d)\left(-e^2x^2+d^2\right)^p x^2\,dx$$

input
```
integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((e*x^3 + d*x^2)*(-e^2*x^2 + d^2)^p, x)
```

3.243.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(95) = 190.

Time = 1.44 (sec) , antiderivative size = 382, normalized size of antiderivative = 3.21

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx = \frac{dd^{2p}x^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3} + e\left(\begin{cases}\frac{x^4(d^2)^p}{4} & \text{for } e=0\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} & \text{for } p=-2\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4}-\frac{x^2}{2e^2} & \text{for } p=-1\\ -\frac{d^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}-\frac{d^2e^2px^2(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4px^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4x^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**2*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

output
```
d*d**(2*p)*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)
/3 + e*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d*
*2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) -
d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4
+ 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p
, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/
(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p
 + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*
p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p +
4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4
), True))
```

3.243.7 Maxima [F]

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p x^2\,dx$$

input
```
integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^2, x)
```

3.243.8 Giac [F]

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p x^2\,dx$$

input
```
integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^2, x)
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int x^2\,\left(d^2-e^2\,x^2\right)^p\,(d+e\,x)\,dx$$

input
```
int(x^2*(d^2 - e^2*x^2)^p*(d + e*x),x)
```

output
```
int(x^2*(d^2 - e^2*x^2)^p*(d + e*x), x)
```

3.244 $\int x(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

3.244.1 Optimal result

Integrand size = 21, antiderivative size = 89

$$\int x(d+ex)\left(d^2-e^2x^2\right)^p\,dx=-\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^2(1+p)}+\frac{1}{3}ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)$$

output
```
-1/2*d*(-e^2*x^2+d^2)^(p+1)/e^2/(p+1)+1/3*e*x^3*(-e^2*x^2+d^2)^p*hypergeom
([3/2, -p],[5/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
```

3.244.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int x(d+ex)\left(d^2-e^2x^2\right)^p\,dx=-\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^2(1+p)}+\frac{1}{3}ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)$$

input
```
Integrate[x*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

output
```
-1/2*(d*(d^2 - e^2*x^2)^(1 + p))/(e^2*(1 + p)) + (e*x^3*(d^2 - e^2*x^2)^p*
Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)
```

3.244.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {542, 241, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex)\left(d^2-e^2x^2\right)^p\,dx$$

$$\Big\downarrow \text{542}$$

$$d\int x\left(d^2-e^2x^2\right)^p\,dx+e\int x^2\left(d^2-e^2x^2\right)^p\,dx$$

$$\Big\downarrow \text{241}$$

$$e\int x^2\left(d^2-e^2x^2\right)^p\,dx-\frac{d\left(d^2-e^2x^2\right)^{p+1}}{2e^2(p+1)}$$

$$\Big\downarrow \text{279}$$

$$e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx-\frac{d\left(d^2-e^2x^2\right)^{p+1}}{2e^2(p+1)}$$

$$\Big\downarrow \text{278}$$

$$\frac{1}{3}ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)-\frac{d\left(d^2-e^2x^2\right)^{p+1}}{2e^2(p+1)}$$

input
```
Int[x*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

output
```
-1/2*(d*(d^2 - e^2*x^2)^(1 + p))/(e^2*(1 + p)) + (e*x^3*(d^2 - e^2*x^2)^p*
Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)
```

3.244.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

3.244.4 Maple [F]

$$\int x(ex+d)\left(-e^2x^2+d^2\right)^p dx$$

input
```
int(x*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

output
```
int(x*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

3.244.5 Fricas [F]

$$\int x(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p x\,dx$$

input
```
integrate(x*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((e*x^2 + d*x)*(-e^2*x^2 + d^2)^p, x)
```

3.244.6 Sympy [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

$$\int x(d+ex)\left(d^2-e^2x^2\right)^p\,dx = d\left(\begin{cases}\frac{x^2(d^2)^p}{2} & \text{for } e^2=0\\ -\frac{\begin{cases}\frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(d^2-e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right) + \frac{d^{2p}ex^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}$$

input
```
integrate(x*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

output
```
d*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x**
2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2),
True)) + d**(2*p)*e*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*
pi)/d**2)/3
```

3.244.7 Maxima [F]

$$\int x(d+ex)\left(d^2-e^2x^2\right)^p\,dx = \int (ex+d)\left(-e^2x^2+d^2\right)^p x\,dx$$

input
```
integrate(x*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
e*integrate(x^2*e^(p*log(e*x + d) + p*log(-e*x + d)), x) - 1/2*(-e^2*x^2 +
 d^2)^(p + 1)*d/(e^2*(p + 1))
```

3.244.8 Giac [F]

$$\int x(d+ex) \left(d^2-e^2x^2\right)^p \, dx = \int (ex+d)\left(-e^2x^2+d^2\right)^p x \, dx$$

input
```
integrate(x*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x, x)
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int x(d+ex) \left(d^2-e^2x^2\right)^p \, dx = \int x \left(d^2-e^2\,x^2\right)^p (d+e\,x) \, dx$$

input
```
int(x*(d^2 - e^2*x^2)^p*(d + e*x),x)
```

output
```
int(x*(d^2 - e^2*x^2)^p*(d + e*x), x)
```

3.245 $\int (d+ex)\left(d^2-e^2x^2\right)^p\,dx$

3.245.1 Optimal result

Integrand size = 20, antiderivative size = 83

$$\int (d+ex)\left(d^2-e^2x^2\right)^p\,dx=-\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e(1+p)}+dx\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)$$

output
```
-1/2*(-e^2*x^2+d^2)^(p+1)/e/(p+1)+d*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, -p]
,[3/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
```

3.245.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int (d+ex)\left(d^2-e^2x^2\right)^p\,dx=-\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e(1+p)}+dx\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)$$

input
```
Integrate[(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

output
```
-1/2*(d^2 - e^2*x^2)^(1 + p)/(e*(1 + p)) + (d*x*(d^2 - e^2*x^2)^p*Hypergeo
metric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p
```

3.245.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {455, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(d^2-e^2x^2\right)^p\,dx$$

$$\Big\downarrow \ 455$$

$$d\int \left(d^2-e^2x^2\right)^p dx-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e(p+1)}$$

$$\Big\downarrow \ 238$$

$$d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int \left(1-\frac{e^2x^2}{d^2}\right)^p dx-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e(p+1)}$$

$$\Big\downarrow \ 237$$

$$dx\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e(p+1)}$$

input
```
Int[(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

output
```
-1/2*(d^2 - e^2*x^2)^(1 + p)/(e*(1 + p)) + (d*x*(d^2 - e^2*x^2)^p*Hypergeo
metric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p
```

3.245.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

3.245.4 Maple [F]

$$\int (ex+d)\left(-e^2x^2+d^2\right)^p dx$$

input
```
int((e*x+d)*(-e^2*x^2+d^2)^p,x)
```

output
```
int((e*x+d)*(-e^2*x^2+d^2)^p,x)
```

3.245.5 Fricas [F]

$$\int (d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p dx$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((e*x + d)*(-e^2*x^2 + d^2)^p, x)
```

3.245.6 Sympy [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int (d+ex)\left(d^2-e^2x^2\right)^p\,dx = dd^{2p}x_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right) + e\left(\begin{cases}\frac{x^2(d^2)^p}{2} & \text{for } e^2=0\\ -\frac{\begin{cases}\frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(d^2-e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((e*x+d)*(-e**2*x**2+d**2)**p,x)
```

output
```
d*d**(2*p)*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2) +
e*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x**
2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2),
True))
```

3.245.7 Maxima [F]

$$\int (d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p\,dx$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)*(-e^2*x^2 + d^2)^p, x)
```

3.245.8 Giac [F]

$$\int (d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p\,dx$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(-e^2*x^2 + d^2)^p, x)
```

3.245.9 Mupad [B] (verification not implemented)

Time = 13.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int (d+ex)\left(d^2-e^2x^2\right)^p\,dx=\frac{d\,x\,\left(d^2-e^2\,x^2\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ \frac{e^2\,x^2}{d^2}\right)}{\left(1-\frac{e^2\,x^2}{d^2}\right)^p}-\frac{\left(d^2-e^2\,x^2\right)^{p+1}}{2\,e\,\left(p+1\right)}$$

input
```
int((d^2 - e^2*x^2)^p*(d + e*x),x)
```

output
```
(d*x*(d^2 - e^2*x^2)^p*hypergeom([1/2, -p], 3/2, (e^2*x^2)/d^2))/(1 - (e^2
*x^2)/d^2)^p - (d^2 - e^2*x^2)^(p + 1)/(2*e*(p + 1))
```

3.245. $\int (d + ex) (d^2 - e^2x^2)^p \, dx$

3.246 $\int \frac{(d+ex)(d^2-e^2x^2)^p}{x}\,dx$

3.246.1 Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \frac{(d+ex)(d^2-e^2x^2)^p}{x}\,dx = ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right) - \frac{(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{2d(1+p)}$$

output
```
e*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],[3/2],e^2*x^2/d^2)/((1-e^2*x^2/d^
2)^p)-1/2*(-e^2*x^2+d^2)^(p+1)*hypergeom([1, p+1],[2+p],1-e^2*x^2/d^2)/d/(
p+1)
```

3.246.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)(d^2-e^2x^2)^p}{x}\,dx = ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right) - \frac{(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{2d(1+p)}$$

input
```
Integrate[((d + e*x)*(d^2 - e^2*x^2)^p)/x,x]
```

output
```
(e*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1
- (e^2*x^2)/d^2)^p - ((d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p,
2 + p, 1 - (e^2*x^2)/d^2])/(2*d*(1 + p))
```

3.246.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {542, 238, 237, 243, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)(d^2-e^2x^2)^p}{x}dx$$

$$\Big\downarrow \text{542}$$

$$e\int (d^2-e^2x^2)^p dx + d\int \frac{(d^2-e^2x^2)^p}{x}dx$$

$$\Big\downarrow \text{238}$$

$$e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int \left(1-\frac{e^2x^2}{d^2}\right)^p dx + d\int \frac{(d^2-e^2x^2)^p}{x}dx$$

$$\Big\downarrow \text{237}$$

$$d\int \frac{(d^2-e^2x^2)^p}{x}dx + ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}d\int \frac{(d^2-e^2x^2)^p}{x^2}dx^2 + ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)$$

$$\Big\downarrow \text{75}$$

$$ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right) - \frac{(d^2-e^2x^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{2d(p+1)}$$

input
```
Int[((d + e*x)*(d^2 - e^2*x^2)^p)/x,x]
```

output
```
(e*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1
- (e^2*x^2)/d^2)^p - ((d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p,
2 + p, 1 - (e^2*x^2)/d^2])/(2*d*(1 + p))
```

3.246.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

3.246. $\int \frac{(d+ex)(d^2-e^2x^2)^p}{x}\,dx$

3.246.4 Maple [F]

$$\int \frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x}dx$$

input
```
int((e*x+d)*(-e^2*x^2+d^2)^p/x,x)
```

output
```
int((e*x+d)*(-e^2*x^2+d^2)^p/x,x)
```

3.246.5 Fricas [F]

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x}\,dx = \int \frac{(ex+d)(-e^2x^2+d^2)^p}{x}\,dx$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^p/x,x, algorithm="fricas")
```

output
```
integral((e*x + d)*(-e^2*x^2 + d^2)^p/x, x)
```

3.246.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x}\,dx = -\frac{de^{2p}x^{2p}e^{i\pi p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma(1-p)} + d^{2p}ex\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)$$

input
```
integrate((e*x+d)*(-e**2*x**2+d**2)**p/x,x)
```

output
```
-d*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**2/
(e**2*x**2))/(2*gamma(1 - p)) + d**(2*p)*e*x*hyper((1/2, -p), (3/2,), e**2
*x**2*exp_polar(2*I*pi)/d**2)
```

3.246.7 Maxima [F]

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x}\,dx = \int \frac{(ex+d)(-e^2x^2+d^2)^p}{x}\,dx$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^p/x,x, algorithm="maxima")
```

output
```
integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x, x)
```

3.246.8 Giac [F]

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x}\,dx = \int \frac{(ex+d)(-e^2x^2+d^2)^p}{x}\,dx$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^p/x,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x, x)
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x}\,dx = \int \frac{\left(d^2-e^2\,x^2\right)^p\,(d+e\,x)}{x}\,dx$$

input
```
int(((d^2 - e^2*x^2)^p*(d + e*x))/x,x)
```

output
```
int(((d^2 - e^2*x^2)^p*(d + e*x))/x, x)
```

3.247 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^2}\,dx$

3.247.1 Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^2}\,dx = -\frac{d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x} - \frac{e\left(d^2-e^2x^2\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{2d^2(1+p)}$$

output
```
-d*(-e^2*x^2+d^2)^p*hypergeom([-1/2, -p],[1/2],e^2*x^2/d^2)/x/((1-e^2*x^2/
d^2)^p)-1/2*e*(-e^2*x^2+d^2)^(p+1)*hypergeom([1, p+1],[2+p],1-e^2*x^2/d^2)
/d^2/(p+1)
```

3.247.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^2}\,dx = -\frac{d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x} - \frac{e\left(d^2-e^2x^2\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{2d^2(1+p)}$$

input
```
Integrate[((d + e*x)*(d^2 - e^2*x^2)^p)/x^2,x]
```

output
```
-((d*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(x
*(1 - (e^2*x^2)/d^2)^p)) - (e*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1,
 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*(1 + p))
```

3.247.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {542, 243, 75, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^2}dx$$

$$\Big\downarrow \text{542}$$

$$d\int \frac{\left(d^2-e^2x^2\right)^p}{x^2}dx+e\int \frac{\left(d^2-e^2x^2\right)^p}{x}dx$$

$$\Big\downarrow \text{243}$$

$$d\int \frac{\left(d^2-e^2x^2\right)^p}{x^2}dx+\frac{1}{2}e\int \frac{\left(d^2-e^2x^2\right)^p}{x^2}dx^2$$

$$\Big\downarrow \text{75}$$

$$d\int \frac{\left(d^2-e^2x^2\right)^p}{x^2}dx-\frac{e\left(d^2-e^2x^2\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{2d^2(p+1)}$$

$$\Big\downarrow \text{279}$$

$$d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^p}{x^2}dx-\frac{e\left(d^2-e^2x^2\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{2d^2(p+1)}$$

$$\Big\downarrow \text{278}$$

$$-\frac{d(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}-\frac{e(d^2-e^2x^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{2d^2(p+1)}$$

input
```
Int[((d + e*x)*(d^2 - e^2*x^2)^p)/x^2,x]
```

output
```
-((d*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(x
*(1 - (e^2*x^2)/d^2)^p)) - (e*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1,
 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*(1 + p))
```

3.247.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

3.247.4 Maple [F]

$$\int \frac{(ex+d)\left(-e^{2}x^{2}+d^{2}\right)^{p}}{x^{2}}dx$$

input
```
int((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x)
```

output
```
int((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x)
```

3.247.5 Fricas [F]

$$\int \frac{(d+ex)\left(d^{2}-e^{2}x^{2}\right)^{p}}{x^{2}}\,dx = \int \frac{(ex+d)(-e^{2}x^{2}+d^{2})^{p}}{x^{2}}\,dx$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x, algorithm="fricas")
```

output
```
integral((e*x + d)*(-e^2*x^2 + d^2)^p/x^2, x)
```

3.247.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.79 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)\left(d^{2}-e^{2}x^{2}\right)^{p}}{x^{2}}\,dx = -\frac{dd^{2p}{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-p\\ \frac{1}{2}\end{matrix}\middle|\frac{e^{2}x^{2}e^{2i\pi}}{d^{2}}\right)}{x} - \frac{ee^{2p}x^{2p}e^{i\pi p}\Gamma(-p)\,{}_{2}F_{1}\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{d^{2}}{e^{2}x^{2}}\right)}{2\Gamma\left(1-p\right)}$$

input
```
integrate((e*x+d)*(-e**2*x**2+d**2)**p/x**2,x)
```

output
```
-d*d**(2*p)*hyper((-1/2, -p), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/x
- e*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**2
/(e**2*x**2))/(2*gamma(1 - p))
```

3.247.7 Maxima [F]

$$\int \frac{(d+ex)\,(d^2-e^2x^2)^p}{x^2}\,dx = \int \frac{(ex+d)(-e^2x^2+d^2)^p}{x^2}\,dx$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^2, x)
```

3.247.8 Giac [F]

$$\int \frac{(d+ex)\,(d^2-e^2x^2)^p}{x^2}\,dx = \int \frac{(ex+d)(-e^2x^2+d^2)^p}{x^2}\,dx$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^2, x)
```

3.247.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\,(d^2-e^2x^2)^p}{x^2}\,dx = \int \frac{(d^2-e^2\,x^2)^p\,(d+e\,x)}{x^2}\,dx$$

input
```
int(((d^2 - e^2*x^2)^p*(d + e*x))/x^2,x)
```

output
```
int(((d^2 - e^2*x^2)^p*(d + e*x))/x^2, x)
```

3.248 $\int \frac{(d+ex)(d^2-e^2x^2)^p}{x^3} dx$

3.248.1 Optimal result

Integrand size = 23, antiderivative size = 110

$$\int \frac{(d+ex)(d^2-e^2x^2)^p}{x^3} dx = -\frac{e(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x} - \frac{e^2(d^2-e^2x^2)^{1+p} \text{Hypergeometric2F1}\left(2,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{2d^3(1+p)}$$

output
```
-e*(-e^2*x^2+d^2)^p*hypergeom([-1/2, -p],[1/2],e^2*x^2/d^2)/x/((1-e^2*x^2/
d^2)^p)-1/2*e^2*(-e^2*x^2+d^2)^(p+1)*hypergeom([2, p+1],[2+p],1-e^2*x^2/d^
2)/d^3/(p+1)
```

3.248.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)(d^2-e^2x^2)^p}{x^3}\,dx$$
$$=\frac{1}{2}e\left(d^2-e^2x^2\right)^p\left(-\frac{2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}+\frac{e(-d^2+e^2x^2)\,\text{Hypergeometric2F1}\left(2,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{d^3(1+p)}\right)$$

input
```
Integrate[((d + e*x)*(d^2 - e^2*x^2)^p)/x^3,x]
```

output
```
(e*(d^2 - e^2*x^2)^p*((-2*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])
/(x*(1 - (e^2*x^2)/d^2)^p) + (e*(-d^2 + e^2*x^2)*Hypergeometric2F1[2, 1 +
p, 2 + p, 1 - (e^2*x^2)/d^2])/(d^3*(1 + p))))/2
```

3.248.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {542, 243, 75, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)(d^2-e^2x^2)^p}{x^3}\,dx$$
$$\Big\downarrow \; 542$$
$$e\int \frac{(d^2-e^2x^2)^p}{x^2}dx+d\int \frac{(d^2-e^2x^2)^p}{x^3}dx$$
$$\Big\downarrow \; 243$$
$$e\int \frac{(d^2-e^2x^2)^p}{x^2}dx+\frac{1}{2}d\int \frac{(d^2-e^2x^2)^p}{x^4}dx^2$$
$$\Big\downarrow \; 75$$

$$e\int \frac{(d^2-e^2x^2)^p}{x^2}dx-\frac{e^2(d^2-e^2x^2)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{2d^3(p+1)}$$

$$\Big\downarrow \text{279}$$

$$e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^p}{x^2}dx-\frac{e^2(d^2-e^2x^2)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{2d^3(p+1)}$$

$$\Big\downarrow \text{278}$$

$$-\frac{e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}-\frac{e^2(d^2-e^2x^2)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{2d^3(p+1)}$$

input
```
Int[((d + e*x)*(d^2 - e^2*x^2)^p)/x^3,x]
```

output
```
-((e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(x
*(1 - (e^2*x^2)/d^2)^p)) - (e^2*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[
2, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d^3*(1 + p))
```

3.248.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

3.248.4 Maple [F]

$$\int \frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x^3}dx$$

input
```
int((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x)
```

output
```
int((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x)
```

3.248.5 Fricas [F]

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^3}\,dx = \int \frac{(ex+d)(-e^2x^2+d^2)^p}{x^3}\,dx$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x, algorithm="fricas")
```

output
```
integral((e*x + d)*(-e^2*x^2 + d^2)^p/x^3, x)
```

3.248.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^3}\,dx = -\frac{de^{2p}x^{2p-2}e^{i\pi p}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}-p,1-p\\2-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma(2-p)} - \frac{d^{2p}e\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\\frac{1}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x}$$

input
```
integrate((e*x+d)*(-e**2*x**2+d**2)**p/x**3,x)
```

output
```
-d*e**(2*p)*x**(2*p - 2)*exp(I*pi*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 -
p,), d**2/(e**2*x**2))/(2*gamma(2 - p)) - d**(2*p)*e*hyper((-1/2, -p), (1/
2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/x
```

3.248.7 Maxima [F]

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^3}\,dx = \int \frac{(ex+d)(-e^2x^2+d^2)^p}{x^3}\,dx$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x, algorithm="maxima")
```

output
```
integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^3, x)
```

3.248.8 Giac [F]

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^3}\,dx = \int \frac{(ex+d)(-e^2x^2+d^2)^p}{x^3}\,dx$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^3, x)
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\,(d^2-e^2x^2)^p}{x^3}\,dx=\int \frac{(d^2-e^2\,x^2)^p\,(d+e\,x)}{x^3}\,dx$$

input
```
int(((d^2 - e^2*x^2)^p*(d + e*x))/x^3,x)
```

output
```
int(((d^2 - e^2*x^2)^p*(d + e*x))/x^3, x)
```

3.249 $\int x^5(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx$

3.249.1 Optimal result

Integrand size = 25, antiderivative size = 178

$$\int x^5(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = -\frac{d^6\left(d^2-e^2x^2\right)^{1+p}}{e^6(1+p)}+\frac{5d^4\left(d^2-e^2x^2\right)^{2+p}}{2e^6(2+p)}-\frac{2d^2\left(d^2-e^2x^2\right)^{3+p}}{e^6(3+p)}+\frac{\left(d^2-e^2x^2\right)^{4+p}}{2e^6(4+p)}+\frac{2}{7}dex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)$$

output
```
-d^6*(-e^2*x^2+d^2)^(p+1)/e^6/(p+1)+5/2*d^4*(-e^2*x^2+d^2)^(2+p)/e^6/(2+p)
-2*d^2*(-e^2*x^2+d^2)^(3+p)/e^6/(3+p)+1/2*(-e^2*x^2+d^2)^(4+p)/e^6/(4+p)+2
/7*d*e*x^7*(-e^2*x^2+d^2)^p*hypergeom([7/2, -p],[9/2],e^2*x^2/d^2)/((1-e^2
*x^2/d^2)^p)
```

3.249.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.89

$$\int x^5(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = \frac{\left(d^2-e^2x^2\right)^p\left(-\frac{14d^6\left(d^2-e^2x^2\right)}{1+p}+\frac{35d^4\left(d^2-e^2x^2\right)^2}{2+p}-\frac{28d^2\left(d^2-e^2x^2\right)^3}{3+p}+\frac{7\left(d^2-e^2x^2\right)^4}{4+p}+4de^7x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hyperge}\right.}{14e^6}$$

input
```
Integrate[x^5*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

output
```
((d^2 - e^2*x^2)^p*((-14*d^6*(d^2 - e^2*x^2))/(1 + p) + (35*d^4*(d^2 - e^2
*x^2)^2)/(2 + p) - (28*d^2*(d^2 - e^2*x^2)^3)/(3 + p) + (7*(d^2 - e^2*x^2)
^4)/(4 + p) + (4*d*e^7*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])
/(1 - (e^2*x^2)/d^2)^p))/(14*e^6)
```

3.249.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {543, 27, 279, 278, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(d+ex)^2\left(d^2-e^2x^2\right)^p dx$$

$$\Big\downarrow \text{543}$$

$$\int 2dex^6\left(d^2-e^2x^2\right)^p dx+\int x^5\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx$$

$$\Big\downarrow \text{27}$$

$$2de\int x^6\left(d^2-e^2x^2\right)^p dx+\int x^5\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx$$

$$\Big\downarrow \text{279}$$

$$2de\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int x^6\left(1-\frac{e^2x^2}{d^2}\right)^p dx+\int x^5\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx$$

$$\Big\downarrow \text{278}$$

$$\int x^5\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx+\frac{2}{7}dex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int x^4\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx^2+\frac{2}{7}dex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{2d^6(d^2-e^2x^2)^p}{e^4}-\frac{5d^4(d^2-e^2x^2)^{p+1}}{e^4}+\frac{4d^2(d^2-e^2x^2)^{p+2}}{e^4}-\frac{(d^2-e^2x^2)^{p+3}}{e^4}\right)dx^2+\frac{2}{7}dex^7(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}dex^7(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)+\frac{1}{2}\left(-\frac{4d^2(d^2-e^2x^2)^{p+3}}{e^6(p+3)}+\frac{(d^2-e^2x^2)^{p+4}}{e^6(p+4)}-\frac{2d^6(d^2-e^2x^2)^{p+1}}{e^6(p+1)}+\frac{5d^4(d^2-e^2x^2)^{p+2}}{e^6(p+2)}\right)$$

input
```
Int[x^5*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

output
```
((-2*d^6*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p)) + (5*d^4*(d^2 - e^2*x^2)^(
2 + p))/(e^6*(2 + p)) - (4*d^2*(d^2 - e^2*x^2)^(3 + p))/(e^6*(3 + p)) + (d
^2 - e^2*x^2)^(4 + p)/(e^6*(4 + p)))/2 + (2*d*e*x^7*(d^2 - e^2*x^2)^p*Hype
rgeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(7*(1 - (e^2*x^2)/d^2)^p)
```

3.249.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

3.249. $\int x^5(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.249.4 Maple [F]

$$\int x^5(ex+d)^2\left(-e^2x^2+d^2\right)^p dx$$

input
```
int(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

output
```
int(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

3.249.5 Fricas [F]

$$\int x^5(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^2(-e^2x^2+d^2)^p x^5\,dx$$

input
```
integrate(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((e^2*x^7 + 2*d*e*x^6 + d^2*x^5)*(-e^2*x^2 + d^2)^p, x)
```

3.249.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 938 vs. 2(150) = 300.

Time = 2.90 (sec) , antiderivative size = 2924, normalized size of antiderivative = 16.43

$$\int x^5(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx = \text{Too large to display}$$

input
```
integrate(x**5*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

output
```
d**2*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/(4*d**
4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d**4*e*
*6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2*e**8*
x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d**
2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6 -
 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d**2
*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d*
*2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d
**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2
*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) -
2*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d
**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*
e**8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*lo
g(-d/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x
**4/(4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12
*e**6*p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)*
*p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**
4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6)
 - d**2*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22
*e**6*p + 12*e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 ...
```

3.249.7 Maxima [F]

$$\int x^5(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^2\left(-e^2x^2+d^2\right)^p x^5\,dx$$

input
```
integrate(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
1/2*((p^2 + 3*p + 2)*e^6*x^6 - (p^2 + p)*d^2*e^4*x^4 - 2*d^4*e^2*p*x^2 - 2
*d^6)*(-e^2*x^2 + d^2)^p*d^2/((p^3 + 6*p^2 + 11*p + 6)*e^6) + integrate((e
^2*x^7 + 2*d*e*x^6)*e^(p*log(e*x + d) + p*log(-e*x + d)), x)
```

3.249.8 Giac [F]

$$\int x^5(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^2\left(-e^2x^2+d^2\right)^p x^5\,dx$$

input
```
integrate(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^5, x)
```

3.249.9 Mupad [F(-1)]

Timed out.

$$\int x^5(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int x^5\,\left(d^2-e^2\,x^2\right)^p\,(d+e\,x)^2\,dx$$

input
```
int(x^5*(d^2 - e^2*x^2)^p*(d + e*x)^2,x)
```

output
```
int(x^5*(d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

3.250 $\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$

3.250.1 Optimal result

Integrand size = 25, antiderivative size = 185

$$\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx = -\frac{d^5\left(d^2-e^2x^2\right)^{1+p}}{e^5(1+p)}-\frac{x^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{2d^3\left(d^2-e^2x^2\right)^{2+p}}{e^5(2+p)}-\frac{d\left(d^2-e^2x^2\right)^{3+p}}{e^5(3+p)}+\frac{2d^2(6+p)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5(7+2p)}$$

output
```
-d^5*(-e^2*x^2+d^2)^(p+1)/e^5/(p+1)-x^5*(-e^2*x^2+d^2)^(p+1)/(7+2*p)+2*d^3
*(-e^2*x^2+d^2)^(2+p)/e^5/(2+p)-d*(-e^2*x^2+d^2)^(3+p)/e^5/(3+p)+2/5*d^2*(
6+p)*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, -p],[7/2],e^2*x^2/d^2)/(7+2*p)/(
(1-e^2*x^2/d^2)^p)
```

3.250.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.01

$$\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\frac{1}{35}\left(d^2-e^2x^2\right)^p\left(-\frac{35d^5(d^2-e^2x^2)}{e^5(1+p)}+\frac{70d^3(d^2-e^2x^2)^2}{e^5(2+p)}-\frac{35d(d^2-e^2x^2)^3}{e^5(3+p)}+7d^2x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)+5e^2x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)\right)$$

input
```
Integrate[x^4*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

output
```
((d^2 - e^2*x^2)^p*((-35*d^5*(d^2 - e^2*x^2))/(e^5*(1 + p)) + (70*d^3*(d^2
 - e^2*x^2)^2)/(e^5*(2 + p)) - (35*d*(d^2 - e^2*x^2)^3)/(e^5*(3 + p)) + (7
*d^2*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^
2)^p + (5*e^2*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1 - (e^
2*x^2)/d^2)^p))/35
```

3.250.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {543, 27, 243, 53, 363, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$$

$$\Big\downarrow \text{ 543}$$

$$\int 2dex^5\left(d^2-e^2x^2\right)^p\,dx+\int x^4\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx$$

$$\Big\downarrow \text{ 27}$$

$$2de\int x^5\left(d^2-e^2x^2\right)^p dx+\int x^4\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx$$

$$\Big\downarrow \text{243}$$

$$de\int x^4\left(d^2-e^2x^2\right)^p dx^2+\int x^4\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx$$

$$\Big\downarrow \text{53}$$

$$\int x^4\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx+ de\int\left(\frac{d^4\left(d^2-e^2x^2\right)^p}{e^4}-\frac{2d^2\left(d^2-e^2x^2\right)^{p+1}}{e^4}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{e^4}\right)dx^2$$

$$\Big\downarrow \text{363}$$

$$\frac{2d^2(p+6)\int x^4\left(d^2-e^2x^2\right)^p dx}{2p+7}+ de\int\left(\frac{d^4\left(d^2-e^2x^2\right)^p}{e^4}-\frac{2d^2\left(d^2-e^2x^2\right)^{p+1}}{e^4}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{e^4}\right)dx^2-\frac{x^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}$$

$$\Big\downarrow \text{279}$$

$$\frac{2d^2(p+6)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^p dx}{2p+7}+ de\int\left(\frac{d^4\left(d^2-e^2x^2\right)^p}{e^4}-\frac{2d^2\left(d^2-e^2x^2\right)^{p+1}}{e^4}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{e^4}\right)dx^2-\frac{x^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}$$

$$\Big\downarrow \text{278}$$

$$de\int\left(\frac{d^4\left(d^2-e^2x^2\right)^p}{e^4}-\frac{2d^2\left(d^2-e^2x^2\right)^{p+1}}{e^4}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{e^4}\right)dx^2+ \frac{2d^2(p+6)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5(2p+7)}-\frac{x^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2d^2(p+6)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5(2p+7)}- \frac{x^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}+de\left(\frac{2d^2\left(d^2-e^2x^2\right)^{p+2}}{e^6(p+2)}-\frac{\left(d^2-e^2x^2\right)^{p+3}}{e^6(p+3)}-\frac{d^4\left(d^2-e^2x^2\right)^{p+1}}{e^6(p+1)}\right)$$

input
```
Int[x^4*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

3.250. $\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$

output
```
-((x^5*(d^2 - e^2*x^2)^(1 + p))/(7 + 2*p)) + d*e*(-((d^4*(d^2 - e^2*x^2)^(
1 + p))/(e^6*(1 + p))) + (2*d^2*(d^2 - e^2*x^2)^(2 + p))/(e^6*(2 + p)) - (
d^2 - e^2*x^2)^(3 + p)/(e^6*(3 + p))) + (2*d^2*(6 + p)*x^5*(d^2 - e^2*x^2)
^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(7 + 2*p)*(1 - (e^2*
x^2)/d^2)^p)
```

3.250.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.250.4 Maple [F]

$$\int x^4(ex+d)^2\left(-e^2x^2+d^2\right)^p dx$$

input
```
int(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

output
```
int(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

3.250.5 Fricas [F]

$$\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx = \int (ex+d)^2(-e^2x^2+d^2)^p x^4\,dx$$

input
```
integrate(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((e^2*x^6 + 2*d*e*x^5 + d^2*x^4)*(-e^2*x^2 + d^2)^p, x)
```

3.250.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 940 vs. $2(153) = 306$.

Time = 2.39 (sec) , antiderivative size = 1015, normalized size of antiderivative = 5.49

$$\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx = \frac{d^2d^{2p}x^5{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{5} + 2de\left(\begin{cases}\frac{x^6(d^2)^p}{6} \\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}-\frac{2d^4\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}-\frac{3d^4}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}+\frac{4d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}+\frac{4d^2e^2x^2}{4d^4e^6-8d^2} \\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}-\frac{2d^4\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}-\frac{2d^4}{-2d^2e^6+2e^8x^2}+\frac{2d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}+\frac{2d^2e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}+\frac{e^4x^4}{-2d^2e^6+2e^8x^2} \\ -\frac{d^4\log\left(-\frac{d}{e}+x\right)}{2e^6}-\frac{d^4\log\left(\frac{d}{e}+x\right)}{2e^6}-\frac{d^2x^2}{2e^4}-\frac{x^4}{4e^2} \\ -\frac{2d^6(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{2d^4e^2px^2(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{d^2e^4p^2x^4(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{d^2e^4px^4(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}\end{cases}\right) + \frac{d^{2p}e^2x^7{}_2F_1\left(\begin{matrix}\frac{7}{2},-p\\ \frac{9}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{7}$$

input
```
integrate(x**4*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

3.250. $\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$

output
```
d**2*d**(2*p)*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d*
*2)/5 + 2*d*e*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e +
x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(
4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d
**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**
6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d*
*4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6
 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e*
*6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e
**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)
/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8
*x**2) - 2*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e +
 x)/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*
e**6 + 2*e**8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)),
(-d**4*log(-d/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*
e**4) - x**4/(4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*
p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e*
*2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4
*p**2*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p +
 12*e**6) - d**2*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e*...
```

3.250.7 Maxima [F]

$$\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int(ex+d)^2\left(-e^2x^2+d^2\right)^px^4\,dx$$

input
```
integrate(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^4, x)
```

3.250.8 Giac [F]

$$\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^2\left(-e^2x^2+d^2\right)^p x^4\,dx$$

input
```
integrate(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^4, x)
```

3.250.9 Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int x^4\left(d^2-e^2x^2\right)^p(d+e\,x)^2\,dx$$

input
```
int(x^4*(d^2 - e^2*x^2)^p*(d + e*x)^2,x)
```

output
```
int(x^4*(d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

3.251 $\int x^3(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$

3.251.1 Optimal result

Integrand size = 25, antiderivative size = 149

$$\int x^3(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=-\frac{d^4\left(d^2-e^2x^2\right)^{1+p}}{e^4(1+p)}+\frac{3d^2\left(d^2-e^2x^2\right)^{2+p}}{2e^4(2+p)}-\frac{\left(d^2-e^2x^2\right)^{3+p}}{2e^4(3+p)}+\frac{2}{5}dex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)$$

output
```
-d^4*(-e^2*x^2+d^2)^(p+1)/e^4/(p+1)+3/2*d^2*(-e^2*x^2+d^2)^(2+p)/e^4/(2+p)
-1/2*(-e^2*x^2+d^2)^(3+p)/e^4/(3+p)+2/5*d*e*x^5*(-e^2*x^2+d^2)^p*hypergeom
([5/2, -p],[7/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
```

3.251.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.93

$$\int x^3(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$$
$$=\frac{\left(d^2-e^2x^2\right)^p\left(-\frac{5\left(d^2-e^2x^2\right)\left(d^4(5+p)+d^2e^2\left(5+6p+p^2\right)x^2+e^4\left(2+3p+p^2\right)x^4\right)}{(1+p)(2+p)(3+p)}+4de^5x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\right.}{10e^4}$$

input
```
Integrate[x^3*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

output
```
((d^2 - e^2*x^2)^p*((-5*(d^2 - e^2*x^2)*(d^4*(5 + p) + d^2*e^2*(5 + 6*p +
p^2)*x^2 + e^4*(2 + 3*p + p^2)*x^4))/((1 + p)*(2 + p)*(3 + p)) + (4*d*e^5*
x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p)
)/(10*e^4)
```

3.251.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {543, 27, 279, 278, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$$

$$\Big\downarrow \; 543$$

$$\int 2dex^4\left(d^2-e^2x^2\right)^p\,dx+\int x^3\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx$$

$$\Big\downarrow \; 27$$

$$2de\int x^4\left(d^2-e^2x^2\right)^p\,dx+\int x^3\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx$$

$$\Big\downarrow \; 279$$

$$2de\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^p dx+\int x^3\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx$$

$$\Big\downarrow \; 278$$

$$\int x^3\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx+\frac{2}{5}dex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int x^2\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx^2+\frac{2}{5}dex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{2d^4(d^2-e^2x^2)^p}{e^2}-\frac{3d^2(d^2-e^2x^2)^{p+1}}{e^2}+\frac{(d^2-e^2x^2)^{p+2}}{e^2}\right)dx^2+\frac{2}{5}dex^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}dex^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)+\frac{1}{2}\left(\frac{3d^2(d^2-e^2x^2)^{p+2}}{e^4(p+2)}-\frac{(d^2-e^2x^2)^{p+3}}{e^4(p+3)}-\frac{2d^4(d^2-e^2x^2)^{p+1}}{e^4(p+1)}\right)$$

input
```
Int[x^3*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

output
```
((-2*d^4*(d^2 - e^2*x^2)^(1 + p))/(e^4*(1 + p)) + (3*d^2*(d^2 - e^2*x^2)^(
2 + p))/(e^4*(2 + p)) - (d^2 - e^2*x^2)^(3 + p)/(e^4*(3 + p)))/2 + (2*d*e*
x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(
1 - (e^2*x^2)/d^2)^p)
```

3.251.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

3.251. $\int x^3(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.251.4 Maple [F]

$$\int x^3(ex+d)^2\left(-e^2x^2+d^2\right)^p dx$$

input
```
int(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

output
```
int(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

3.251.5 Fricas [F]

$$\int x^3(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^2(-e^2x^2+d^2)^p x^3\,dx$$

input
```
integrate(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((e^2*x^5 + 2*d*e*x^4 + d^2*x^3)*(-e^2*x^2 + d^2)^p, x)
```

3.251.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(124) = 248.

Time = 2.13 (sec) , antiderivative size = 1328, normalized size of antiderivative = 8.91

$$\int x^3(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\text{Too large to display}$$

input
```
integrate(x**3*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

output
```
d**2*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2
*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d*
*2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 +
2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p,
-2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2
*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p +
 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p
+ 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*
e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4),
 True)) + 2*d*d**(2*p)*e*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar
(2*I*pi)/d**2)/5 + e**2*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*l
og(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(
d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*
e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(
4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e
+ x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4
*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/
(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)
/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log
(-d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*...
```

3.251.7 Maxima [F]

$$\int x^3(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^2\left(-e^2x^2+d^2\right)^p x^3\,dx$$

input
```
integrate(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
1/2*(e^4*(p + 1)*x^4 - d^2*e^2*p*x^2 - d^4)*(-e^2*x^2 + d^2)^p*d^2/((p^2 +
 3*p + 2)*e^4) + integrate((e^2*x^5 + 2*d*e*x^4)*e^(p*log(e*x + d) + p*log
(-e*x + d)), x)
```

3.251.8 Giac [F]

$$\int x^3(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^2\left(-e^2x^2+d^2\right)^p x^3\,dx$$

input
```
integrate(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^3, x)
```

3.251.9 Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int x^3\left(d^2-e^2x^2\right)^p(d+e\,x)^2\,dx$$

input
```
int(x^3*(d^2 - e^2*x^2)^p*(d + e*x)^2,x)
```

output
```
int(x^3*(d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

3.252 $\int x^2(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx$

3.252.1 Optimal result

Integrand size = 25, antiderivative size = 155

$$\int x^2(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = -\frac{d^3\left(d^2-e^2x^2\right)^{1+p}}{e^3(1+p)} - \frac{x^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p} + \frac{d\left(d^2-e^2x^2\right)^{2+p}}{e^3(2+p)} + \frac{2d^2(4+p)x^3\left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3(5+2p)}$$

output
```
-d^3*(-e^2*x^2+d^2)^(p+1)/e^3/(p+1)-x^3*(-e^2*x^2+d^2)^(p+1)/(5+2*p)+d*(-e
^2*x^2+d^2)^(2+p)/e^3/(2+p)+2/3*d^2*(4+p)*x^3*(-e^2*x^2+d^2)^p*hypergeom([
3/2, -p],[5/2],e^2*x^2/d^2)/(5+2*p)/((1-e^2*x^2/d^2)^p)
```

3.252.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.08

$$\int x^2(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = \frac{\left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \left(-15d\left(d^2-e^2x^2\right)\left(1-\frac{e^2x^2}{d^2}\right)^p \left(d^2+e^2(1+p)x^2\right)+5d^2e^3\left(2+3p+p^2\right)x^3 \text{Hyp}\right.}{15e^3(1+p)(2+}$$

input
```
Integrate[x^2*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

output
```
((d^2 - e^2*x^2)^p*(-15*d*(d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p*(d^2 + e^2
*(1 + p)*x^2) + 5*d^2*e^3*(2 + 3*p + p^2)*x^3*Hypergeometric2F1[3/2, -p, 5
/2, (e^2*x^2)/d^2] + 3*e^5*(2 + 3*p + p^2)*x^5*Hypergeometric2F1[5/2, -p,
7/2, (e^2*x^2)/d^2]))/(15*e^3*(1 + p)*(2 + p)*(1 - (e^2*x^2)/d^2)^p)
```

3.252.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {543, 27, 243, 53, 363, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+ex)^2\left(d^2-e^2x^2\right)^p dx$$

$$\Big\downarrow \text{ 543}$$

$$\int x^2\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx+\int 2dex^3\left(d^2-e^2x^2\right)^p dx$$

$$\Big\downarrow \text{ 27}$$

$$\int x^2\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx+2de\int x^3\left(d^2-e^2x^2\right)^p dx$$

$$\Big\downarrow \text{ 243}$$

$$de\int x^2\left(d^2-e^2x^2\right)^p dx^2+\int x^2\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx$$

$$\Big\downarrow \text{ 53}$$

$$\int x^2\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx+de\int\left(\frac{d^2\left(d^2-e^2x^2\right)^p}{e^2}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{e^2}\right)dx^2$$

$$\Big\downarrow \text{ 363}$$

$$\frac{2d^2(p+4)\int x^2\left(d^2-e^2x^2\right)^p dx}{2p+5}+de\int\left(\frac{d^2\left(d^2-e^2x^2\right)^p}{e^2}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{e^2}\right)dx^2-\frac{x^3\left(d^2-e^2x^2\right)^{p+1}}{2p+5}$$

$$\Big\downarrow \text{ 279}$$

$$\frac{2d^2(p+4)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^p dx}{2p+5}+$$
$$de\int\left(\frac{d^2\left(d^2-e^2x^2\right)^p}{e^2}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{e^2}\right)dx^2-\frac{x^3\left(d^2-e^2x^2\right)^{p+1}}{2p+5}$$

$$\Big\downarrow \; 278$$

$$de\int\left(\frac{d^2\left(d^2-e^2x^2\right)^p}{e^2}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{e^2}\right)dx^2+$$
$$\frac{2d^2(p+4)x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3(2p+5)}-\frac{x^3\left(d^2-e^2x^2\right)^{p+1}}{2p+5}$$

$$\Big\downarrow \; 2009$$

$$\frac{2d^2(p+4)x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3(2p+5)}-$$
$$\frac{x^3\left(d^2-e^2x^2\right)^{p+1}}{2p+5}+de\left(\frac{\left(d^2-e^2x^2\right)^{p+2}}{e^4(p+2)}-\frac{d^2\left(d^2-e^2x^2\right)^{p+1}}{e^4(p+1)}\right)$$

input
```
Int[x^2*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

output
```
-((x^3*(d^2 - e^2*x^2)^(1 + p))/(5 + 2*p)) + d*e*(-((d^2*(d^2 - e^2*x^2)^(
1 + p))/(e^4*(1 + p))) + (d^2 - e^2*x^2)^(2 + p)/(e^4*(2 + p))) + (2*d^2*(
4 + p)*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2
])/(3*(5 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

3.252.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

3.252. $\int x^2(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.252. $\int x^2(d+ex)^2 \left(d^2-e^2x^2\right)^p \,dx$

3.252.4 Maple [F]

$$\int x^2(ex+d)^2\left(-e^2x^2+d^2\right)^p dx$$

input
```
int(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

output
```
int(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

3.252.5 Fricas [F]

$$\int x^2(d+ex)^2\left(d^2-e^2x^2\right)^p dx = \int (ex+d)^2\left(-e^2x^2+d^2\right)^p x^2 dx$$

input
```
integrate(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((e^2*x^4 + 2*d*e*x^3 + d^2*x^2)*(-e^2*x^2 + d^2)^p, x)
```

3.252.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(128) = 256$.

Time = 1.95 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.74

$$\int x^2(d+ex)^2\left(d^2-e^2x^2\right)^p dx = \frac{d^2d^{2p}x^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3} + 2de\left(\begin{cases}\frac{x^4(d^2)^p}{4} & \text{for } e=0\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} & \text{for } p=-2\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4} - \frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4} - \frac{x^2}{2e^2} & \text{for } p=-1\\ -\frac{d^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} - \frac{d^2e^2px^2(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4px^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4x^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} & \text{otherwise}\end{cases}\right) + \frac{d^{2p}e^2x^5{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{5}$$

input
```
integrate(x**2*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

output
```
d**2*d**(2*p)*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d*
*2)/3 + 2*d*e*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)
/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x
**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**
2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2
), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4)
- x**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6
*e**4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 +
6*e**4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e*
*4*p + 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p +
 4*e**4), True)) + d**(2*p)*e**2*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*e
xp_polar(2*I*pi)/d**2)/5
```

3.252.7 Maxima [F]

$$\int x^2(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^2\left(-e^2x^2+d^2\right)^p x^2\,dx$$

input
```
integrate(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^2, x)
```

3.252.8 Giac [F]

$$\int x^2(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^2\left(-e^2x^2+d^2\right)^p x^2\,dx$$

input
```
integrate(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^2, x)
```

3.252.9 Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = \int x^2 \left(d^2-e^2 x^2\right)^p (d+e x)^2 \, dx$$

input
```
int(x^2*(d^2 - e^2*x^2)^p*(d + e*x)^2,x)
```

output
```
int(x^2*(d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

3.253 $\int x(d+ex)^2 \left(d^2-e^2x^2\right)^p dx$

3.253.1 Optimal result

Integrand size = 23, antiderivative size = 118

$$\int x(d+ex)^2 \left(d^2-e^2x^2\right)^p dx = -\frac{d^2(d^2-e^2x^2)^{1+p}}{e^2(1+p)}+\frac{(d^2-e^2x^2)^{2+p}}{2e^2(2+p)}+\frac{2}{3}dex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)$$

output
```
-d^2*(-e^2*x^2+d^2)^(p+1)/e^2/(p+1)+1/2*(-e^2*x^2+d^2)^(2+p)/e^2/(2+p)+2/3
*d*e*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/2, -p],[5/2],e^2*x^2/d^2)/((1-e^2*x
^2/d^2)^p)
```

3.253.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.93

$$\int x(d+ex)^2 \left(d^2-e^2x^2\right)^p dx = \frac{(d^2-e^2x^2)^p\left(-\frac{3(d^2-e^2x^2)(d^2(3+p)+e^2(1+p)x^2)}{(1+p)(2+p)}+4de^3x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)\right)}{6e^2}$$

input
```
Integrate[x*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

output
```
((d^2 - e^2*x^2)^p*((-3*(d^2 - e^2*x^2)*(d^2*(3 + p) + e^2*(1 + p)*x^2))/(
(1 + p)*(2 + p)) + (4*d*e^3*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/
d^2])/(1 - (e^2*x^2)/d^2)^p))/(6*e^2)
```

3.253.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {572, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx$$

$$\Big\downarrow \; 572$$

$$\frac{d\int (d+ex)^2 \left(d^2-e^2x^2\right)^p dx}{e(p+2)}-\frac{(d+ex)^2 \left(d^2-e^2x^2\right)^{p+1}}{2e^2(p+2)}$$

$$\Big\downarrow \; 473$$

$$\frac{d^2(d-ex)^{-p-1}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\int (d-ex)^p\left(\frac{ex}{d}+1\right)^{p+2}dx}{e(p+2)}-\frac{(d+ex)^2 \left(d^2-e^2x^2\right)^{p+1}}{2e^2(p+2)}$$

$$\Big\downarrow \; 79$$

$$-\frac{d^2 2^{p+2}\left(d^2-e^2x^2\right)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}\text{Hypergeometric2F1}\left(-p-2,p+1,p+2,\frac{d-ex}{2d}\right)}{e^2(p+1)(p+2)}-\frac{(d+ex)^2 \left(d^2-e^2x^2\right)^{p+1}}{2e^2(p+2)}$$

input
```
Int[x*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

output
```
-1/2*((d + e*x)^2*(d^2 - e^2*x^2)^(1 + p))/(e^2*(2 + p)) - (2^(2 + p)*d^2*
(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[-2 - p, 1
 + p, 2 + p, (d - e*x)/(2*d)])/(e^2*(1 + p)*(2 + p))
```

3.253.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 572
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 2))), x] + Simp[c*(n/(d
*(n + 2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && NeQ[n + 2*p + 2, 0]
```

3.253.4 Maple [F]

$$\int x(ex+d)^2 \left(-e^2x^2+d^2\right)^p dx$$

input
```
int(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

output
```
int(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

3.253.5 Fricas [F]

$$\int x(d+ex)^2 \left(d^2-e^2x^2\right)^p dx = \int (ex+d)^2 (-e^2x^2+d^2)^p x \, dx$$

input
```
integrate(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((e^2*x^3 + 2*d*e*x^2 + d^2*x)*(-e^2*x^2 + d^2)^p, x)
```

3.253.6 Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 440, normalized size of antiderivative = 3.73

$$\int x(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$$

$$= d^2\left(\begin{cases}\frac{x^2(d^2)^p}{2} & \text{for } e^2=0\\ -\frac{\begin{cases}\frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(d^2-e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right)+\frac{2dd^{2p}ex^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}$$

$$+e^2\left(\begin{cases}\frac{x^4(d^2)^p}{4} & \text{for } e=0\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} & \text{for } p=-2\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4}-\frac{x^2}{2e^2} & \text{for } p=-1\\ -\frac{d^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}-\frac{d^2e^2px^2(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4px^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4x^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

output
```
d**2*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*
x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2
), True)) + 2*d*d**(2*p)*e*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_pol
ar(2*I*pi)/d**2)/3 + e**2*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*l
og(-d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**
4 + 2*e**6*x**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e
+ x)/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 +
 2*e**6*x**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e +
x)/(2*e**4) - x**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e
**4*p**2 + 6*e**4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*
e**4*p**2 + 6*e**4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4
*p**2 + 6*e**4*p + 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2
+ 6*e**4*p + 4*e**4), True))
```

3.253.7 Maxima [F]

$$\int x(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = \int (ex+d)^2 \left(-e^2x^2+d^2\right)^p x \, dx$$

input
```
integrate(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
-1/2*(-e^2*x^2 + d^2)^(p + 1)*d^2/(e^2*(p + 1)) + integrate((e^2*x^3 + 2*d
*e*x^2)*e^(p*log(e*x + d) + p*log(-e*x + d)), x)
```

3.253.8 Giac [F]

$$\int x(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = \int (ex+d)^2 \left(-e^2x^2+d^2\right)^p x \, dx$$

input
```
integrate(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x, x)
```

3.253.9 Mupad [F(-1)]

Timed out.

$$\int x(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = \int x \left(d^2-e^2\,x^2\right)^p \left(d+e\,x\right)^2 \, dx$$

input
```
int(x*(d^2 - e^2*x^2)^p*(d + e*x)^2,x)
```

output
```
int(x*(d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

3.254 $\int (d+ex)^2 \left(d^2-e^2x^2\right)^p \,dx$

3.254.1 Optimal result

Integrand size = 22, antiderivative size = 71

$$\int (d+ex)^2 \left(d^2-e^2x^2\right)^p \,dx = -\frac{2^{2+p} d \left(1+\frac{ex}{d}\right)^{-1-p} \left(d^2-e^2x^2\right)^{1+p} \text{Hypergeometric2F1}\left(-2-p,1+p,2+p,\frac{d-ex}{2d}\right)}{e(1+p)}$$

output
```
-2^(2+p)*d*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(p+1)*hypergeom([p+1, -2-p],[2+
p],1/2*(-e*x+d)/d)/e/(p+1)
```

3.254.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.89

$$\int (d+ex)^2 \left(d^2-e^2x^2\right)^p \,dx = \frac{\left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \left(-3d(d^2-e^2x^2)\left(1-\frac{e^2x^2}{d^2}\right)^p + 3d^2e(1+p)x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2 \ldots}{d^2 \ldots}\right.\right.}{3e(1+p)}$$

input
```
Integrate[(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

output
```
((d^2 - e^2*x^2)^p*(-3*d*(d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p + 3*d^2*e*(
1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + e^3*(1 + p)*x^3*
Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2]))/(3*e*(1 + p)*(1 - (e^2*x^
2)/d^2)^p)
```

3.254.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx$$

$$\Big\downarrow \ 473$$

$$d(d-ex)^{-p-1} \left(\frac{ex}{d}+1\right)^{-p-1} \left(d^2-e^2x^2\right)^{p+1} \int (d-ex)^p \left(\frac{ex}{d}+1\right)^{p+2} dx$$

$$\Big\downarrow \ 79$$

$$-\frac{d2^{p+2} \left(\frac{ex}{d}+1\right)^{-p-1} \left(d^2-e^2x^2\right)^{p+1} \text{Hypergeometric2F1}\left(-p-2,p+1,p+2,\frac{d-ex}{2d}\right)}{e(p+1)}$$

input
```
Int[(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

output
```
-((2^(2 + p)*d*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometr
ic2F1[-2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(e*(1 + p)))
```

3.254.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

3.254.4 Maple [F]

$$\int (ex+d)^2 \left(-e^2x^2+d^2\right)^p dx$$

input
```
int((e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

output
```
int((e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

3.254.5 Fricas [F]

$$\int (d+ex)^2 \left(d^2-e^2x^2\right)^p dx = \int (ex+d)^2 \left(-e^2x^2+d^2\right)^p dx$$

input
```
integrate((e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((e^2*x^2 + 2*d*e*x + d^2)*(-e^2*x^2 + d^2)^p, x)
```

3.254.6 Sympy [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.75

$$\int (d+ex)^2 (d^2-e^2x^2)^p \, dx = d^2 d^{2p} x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix} \middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right) + 2de \left(\begin{cases} \frac{x^2(d^2)^p}{2} & \text{for } e^2 = 0 \\ -\frac{\begin{cases} \frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(d^2-e^2x^2) & \text{otherwise} \end{cases}}{2e^2} & \text{otherwise} \end{cases}\right) + \frac{d^{2p}e^2x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix} \middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}$$

input
```
integrate((e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

output
```
d**2*d**(2*p)*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)
 + 2*d*e*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e
**2*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*
e**2), True)) + d**(2*p)*e**2*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_
polar(2*I*pi)/d**2)/3
```

3.254.7 Maxima [F]

$$\int (d+ex)^2 (d^2-e^2x^2)^p \, dx = \int (ex+d)^2(-e^2x^2+d^2)^p \, dx$$

input
```
integrate((e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p, x)
```

3.254.8 Giac [F]

$$\int (d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = \int (ex+d)^2 \left(-e^2x^2+d^2\right)^p \, dx$$

input
```
integrate((e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p, x)
```

3.254.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = \int \left(d^2-e^2\,x^2\right)^p \, (d+e\,x)^2 \, dx$$

input
```
int((d^2 - e^2*x^2)^p*(d + e*x)^2,x)
```

output
```
int((d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

3.255 $\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x} dx$

3.255.1 Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x} dx = -\frac{(d^2-e^2x^2)^{1+p}}{2(1+p)} + 2dex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right) - \frac{(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{2(1+p)}$$

output
```
-1/2*(-e^2*x^2+d^2)^(p+1)/(p+1)+2*d*e*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, -
p],[3/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)-1/2*(-e^2*x^2+d^2)^(p+1)*hyperge
om([1, p+1],[2+p],1-e^2*x^2/d^2)/(p+1)
```

3.255.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x} \, dx = \frac{1}{2}(d^2-e^2x^2)^p \left(4dex\left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right) - \frac{(d^2-e^2x^2)\left(1+\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)\right)}{1+p}\right)$$

input
```
Integrate[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x,x]
```

output
```
((d^2 - e^2*x^2)^p*((4*d*e*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2
])/(1 - (e^2*x^2)/d^2)^p - ((d^2 - e^2*x^2)*(1 + Hypergeometric2F1[1, 1 +
p, 2 + p, 1 - (e^2*x^2)/d^2]))/(1 + p)))/2
```

3.255.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {543, 27, 238, 237, 354, 90, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x} \, dx$$

$$\Big\downarrow \; 543$$

$$\int 2de(d^2-e^2x^2)^p dx + \int \frac{(d^2-e^2x^2)^p (d^2+e^2x^2)}{x} dx$$

$$\Big\downarrow \; 27$$

$$2de \int (d^2-e^2x^2)^p dx + \int \frac{(d^2-e^2x^2)^p (d^2+e^2x^2)}{x} dx$$

$$\Big\downarrow \; 238$$

$$2de\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int\left(1-\frac{e^2x^2}{d^2}\right)^p dx+\int\frac{\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)}{x}dx$$

$$\Big\downarrow \text{237}$$

$$\int\frac{\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)}{x}dx+ 2dex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int\frac{\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)}{x^2}dx^2+ 2dex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(d^2\int\frac{\left(d^2-e^2x^2\right)^p}{x^2}dx^2-\frac{\left(d^2-e^2x^2\right)^{p+1}}{p+1}\right)+ 2dex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)$$

$$\Big\downarrow \text{75}$$

$$2dex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)+ \frac{1}{2}\left(-\frac{\left(d^2-e^2x^2\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{p+1}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{p+1}\right)$$

input
```
Int[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x,x]
```

output
```
(2*d*e*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])
/(1 - (e^2*x^2)/d^2)^p + (-((d^2 - e^2*x^2)^(1 + p)/(1 + p)) - ((d^2 - e^2
*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(1 +
p))/2
```

3.255. $\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x}\,dx$

3.255.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

3.255. $\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x}\,dx$

3.255.4 Maple [F]

$$\int \frac{(ex+d)^2 (-e^2x^2+d^2)^p}{x}dx$$

input
```
int((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x)
```

output
```
int((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x)
```

3.255.5 Fricas [F]

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x}\,dx = \int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x}\,dx$$

input
```
integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x, algorithm="fricas")
```

output
```
integral((e^2*x^2 + 2*d*e*x + d^2)*(-e^2*x^2 + d^2)^p/x, x)
```

3.255.6 Sympy [A] (verification not implemented)

Time = 3.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x}\,dx = -\frac{d^2e^{2p}x^{2p}e^{i\pi p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma(1-p)} + 2dd^{2p}ex\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right) + e^2\left(\begin{cases}\frac{x^2(d^2)^p}{2} & \text{for } e^2=0\\ -\frac{\begin{cases}\frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log(d^2-e^2x^2) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((e*x+d)**2*(-e**2*x**2+d**2)**p/x,x)
```

output
```
-d**2*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d*
*2/(e**2*x**2))/(2*gamma(1 - p)) + 2*d*d**(2*p)*e*x*hyper((1/2, -p), (3/2,
), e**2*x**2*exp_polar(2*I*pi)/d**2) + e**2*Piecewise((x**2*(d**2)**p/2, E
q(e**2, 0)), (-Piecewise(((d**2 - e**2*x**2)**(p + 1)/(p + 1), Ne(p, -1)),
 (log(d**2 - e**2*x**2), True))/(2*e**2), True))
```

3.255.7 Maxima [F]

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x} \, dx = \int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x} \, dx$$

input
```
integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x, x)
```

3.255.8 Giac [F]

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x} \, dx = \int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x} \, dx$$

input
```
integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x, x)
```

3.255.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x} \, dx = \int \frac{(d^2-e^2\,x^2)^p\,(d+e\,x)^2}{x} \, dx$$

input
```
int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x,x)
```

output
```
int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x, x)
```

3.256 $\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^2}\,dx$

3.256.1 Optimal result

Integrand size = 25, antiderivative size = 128

$$\begin{aligned}\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^2}\,dx &= -\frac{(d^2-e^2x^2)^{1+p}}{x} \\ &\quad - 2e^2px(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right) \\ &\quad - \frac{e(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{d(1+p)}\end{aligned}$$

output
```
-(-e^2*x^2+d^2)^(p+1)/x-2*e^2*p*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],[3/
2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)-e*(-e^2*x^2+d^2)^(p+1)*hypergeom([1, p
+1],[2+p],1-e^2*x^2/d^2)/d/(p+1)
```

3.256.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^2}\,dx = \frac{(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(-d^3(1+p)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)+ex\Big(de(1+p)x\,\text{Hypergeo}\right.}{d(1+}$$

input
```
Integrate[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x^2,x]
```

output
```
((d^2 - e^2*x^2)^p*(-(d^3*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^
2)/d^2]) + e*x*(d*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^
2] - (d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p*Hypergeometric2F1[1, 1 + p, 2 +
 p, 1 - (e^2*x^2)/d^2])))/(d*(1 + p)*x*(1 - (e^2*x^2)/d^2)^p)
```

3.256.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {543, 27, 243, 75, 359, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2 \left(d^2-e^2x^2\right)^p}{x^2} \, dx$$

$$\Big\downarrow \text{543}$$

$$\int \frac{2de\left(d^2-e^2x^2\right)^p}{x}dx+\int \frac{\left(d^2-e^2x^2\right)^p \left(d^2+e^2x^2\right)}{x^2}dx$$

$$\Big\downarrow \text{27}$$

$$2de\int \frac{\left(d^2-e^2x^2\right)^p}{x}dx+\int \frac{\left(d^2-e^2x^2\right)^p \left(d^2+e^2x^2\right)}{x^2}dx$$

$$\Big\downarrow \text{243}$$

$$de\int \frac{\left(d^2-e^2x^2\right)^p}{x^2}dx^2+\int \frac{\left(d^2-e^2x^2\right)^p \left(d^2+e^2x^2\right)}{x^2}dx$$

$$\Big\downarrow \text{75}$$

$$\int \frac{\left(d^2-e^2x^2\right)^p \left(d^2+e^2x^2\right)}{x^2}dx-\frac{e\left(d^2-e^2x^2\right)^{p+1} \text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{d(p+1)}$$

$$\Big\downarrow \text{359}$$

$$-2e^2p\int \left(d^2-e^2x^2\right)^p dx-\frac{e\left(d^2-e^2x^2\right)^{p+1} \text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{d(p+1)}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{x}$$

$$\Big\downarrow \text{238}$$

$$-2e^2p(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int\left(1-\frac{e^2x^2}{d^2}\right)^p dx-\frac{e(d^2-e^2x^2)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{d(p+1)}-\frac{(d^2-e^2x^2)^{p+1}}{x}$$

$$\Big\downarrow \text{237}$$

$$-2e^2px(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)-\frac{e(d^2-e^2x^2)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{d(p+1)}-\frac{(d^2-e^2x^2)^{p+1}}{x}$$

input
```
Int[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x^2,x]
```

output
```
-((d^2 - e^2*x^2)^(1 + p)/x) - (2*e^2*p*x*(d^2 - e^2*x^2)^p*Hypergeometric
2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - (e*(d^2 - e^2*x^
2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(d*(1 +
p))
```

3.256.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

3.256. $\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^2}\,dx$

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && !ILtQ[p, -1]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] && !IntegerQ[2*p]
&& !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

3.256.4 Maple [F]

$$\int \frac{(ex+d)^2 (-e^2x^2+d^2)^p}{x^2} dx$$

input
```
int((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x)
```

output
```
int((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x)
```

3.256.5 Fricas [F]

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x^2} \, dx = \int \frac{(ex+d)^2 (-e^2x^2+d^2)^p}{x^2} \, dx$$

input
```
integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x, algorithm="fricas")
```

output
```
integral((e^2*x^2 + 2*d*e*x + d^2)*(-e^2*x^2 + d^2)^p/x^2, x)
```

3.256.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.30 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x^2} \, dx = -\frac{d^2 d^{2p} {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix} \middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x} - \frac{dee^{2p}x^{2p}e^{i\pi p}\Gamma(-p)\, {}_2F_1\left(\begin{matrix} -p, -p \\ 1-p \end{matrix} \middle| \frac{d^2}{e^2x^2}\right)}{\Gamma(1-p)} + d^{2p}e^2x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix} \middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)$$

input
```
integrate((e*x+d)**2*(-e**2*x**2+d**2)**p/x**2,x)
```

output
```
-d**2*d**(2*p)*hyper((-1/2, -p), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)
/x - d*e*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,),
 d**2/(e**2*x**2))/gamma(1 - p) + d**(2*p)*e**2*x*hyper((1/2, -p), (3/2,),
 e**2*x**2*exp_polar(2*I*pi)/d**2)
```

3.256.7 Maxima [F]

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x^2}\,dx = \int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x^2}\,dx$$

input
```
integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x^2, x)
```

3.256.8 Giac [F]

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x^2}\,dx = \int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x^2}\,dx$$

input
```
integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x^2, x)
```

3.256.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x^2}\,dx = \int \frac{(d^2-e^2\,x^2)^p\,(d+e\,x)^2}{x^2}\,dx$$

input
```
int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x^2,x)
```

output
```
int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x^2, x)
```

3.257 $\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^3}\,dx$

3.257.1 Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^3}\,dx = -\frac{(d^2-e^2x^2)^{1+p}}{2x^2} - \frac{2de(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x} - \frac{e^2(1-p)(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{2d^2(1+p)}$$

output
```
-1/2*(-e^2*x^2+d^2)^(p+1)/x^2-2*d*e*(-e^2*x^2+d^2)^p*hypergeom([-1/2, -p],
[1/2],e^2*x^2/d^2)/x/((1-e^2*x^2/d^2)^p)-1/2*e^2*(1-p)*(-e^2*x^2+d^2)^(p+1
)*hypergeom([1, p+1],[2+p],1-e^2*x^2/d^2)/d^2/(p+1)
```

3.257.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^3}\,dx = \frac{e(d^2-e^2x^2)^p\left(-\frac{4d^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}+\frac{e(-d^2+e^2x^2)\left(\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)+\right.}{1+p}\right.}{2d^2}$$

input
```
Integrate[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x^3,x]
```

output
```
(e*(d^2 - e^2*x^2)^p*((-4*d^3*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d
^2])/(x*(1 - (e^2*x^2)/d^2)^p) + (e*(-d^2 + e^2*x^2)*(Hypergeometric2F1[1,
 1 + p, 2 + p, 1 - (e^2*x^2)/d^2] + Hypergeometric2F1[2, 1 + p, 2 + p, 1 -
 (e^2*x^2)/d^2]))/(1 + p)))/(2*d^2)
```

3.257.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {543, 27, 279, 278, 354, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x^3} \, dx$$

$$\Big\downarrow \text{543}$$

$$\int \frac{2de(d^2-e^2x^2)^p}{x^2}dx+\int \frac{(d^2-e^2x^2)^p (d^2+e^2x^2)}{x^3}dx$$

$$\Big\downarrow \text{27}$$

$$2de\int \frac{(d^2-e^2x^2)^p}{x^2}dx+\int \frac{(d^2-e^2x^2)^p (d^2+e^2x^2)}{x^3}dx$$

$$\Big\downarrow \text{279}$$

$$2de(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^p}{x^2}dx+\int \frac{(d^2-e^2x^2)^p (d^2+e^2x^2)}{x^3}dx$$

$$\Big\downarrow \text{278}$$

$$\int \frac{(d^2-e^2x^2)^p (d^2+e^2x^2)}{x^3}dx-\frac{2de(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(d^2-e^2x^2)^p (d^2+e^2x^2)}{x^4}dx^2 - \frac{2de(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}$$

$$\Big\downarrow \; 87$$

$$\frac{1}{2}\left(e^2(1-p)\int \frac{(d^2-e^2x^2)^p}{x^2}dx^2 - \frac{(d^2-e^2x^2)^{p+1}}{x^2}\right) - \frac{2de(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}$$

$$\Big\downarrow \; 75$$

$$\frac{1}{2}\left(-\frac{e^2(1-p)(d^2-e^2x^2)^{p+1} \text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{d^2(p+1)} - \frac{(d^2-e^2x^2)^{p+1}}{x^2}\right) - \frac{2de(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}$$

input
```
Int[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x^3,x]
```

output
```
(-2*d*e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])
/(x*(1 - (e^2*x^2)/d^2)^p) + (-((d^2 - e^2*x^2)^(1 + p)/x^2) - (e^2*(1 - p
)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)
/d^2])/(d^2*(1 + p)))/2
```

3.257.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

3.257. $\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^3}\,dx$

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

3.257.4 Maple [F]

$$\int \frac{(ex+d)^2 \left(-e^2x^2+d^2\right)^p}{x^3}dx$$

input
```
int((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x)
```

output
```
int((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x)
```

3.257.5 Fricas [F]

$$\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^3}\,dx = \int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x^3}\,dx$$

input
```
integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x, algorithm="fricas")
```

output
```
integral((e^2*x^2 + 2*d*e*x + d^2)*(-e^2*x^2 + d^2)^p/x^3, x)
```

3.257.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.58 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^3}\,dx = -\frac{d^2e^{2p}x^{2p-2}e^{i\pi p}\Gamma(1-p)\,{}_2F_1\left(\left.\begin{matrix}-p,1-p\\2-p\end{matrix}\right|\frac{d^2}{e^2x^2}\right)}{2\Gamma(2-p)} - \frac{2dd^{2p}e\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-p\\\frac{1}{2}\end{matrix}\right|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x} - \frac{e^2e^{2p}x^{2p}e^{i\pi p}\Gamma(-p)\,{}_2F_1\left(\left.\begin{matrix}-p,-p\\1-p\end{matrix}\right|\frac{d^2}{e^2x^2}\right)}{2\Gamma(1-p)}$$

input
```
integrate((e*x+d)**2*(-e**2*x**2+d**2)**p/x**3,x)
```

output
```
-d**2*e**(2*p)*x**(2*p - 2)*exp(I*pi*p)*gamma(1 - p)*hyper((-p, 1 - p), (2
 - p,), d**2/(e**2*x**2))/(2*gamma(2 - p)) - 2*d*d**(2*p)*e*hyper((-1/2, -
p), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/x - e**2*e**(2*p)*x**(2*p)*e
xp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**2/(e**2*x**2))/(2*gamma(
1 - p))
```

3.257.7 Maxima [F]

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x^3}\,dx = \int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x^3}\,dx$$

input
```
integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x^3, x)
```

3.257.8 Giac [F]

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x^3}\,dx = \int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x^3}\,dx$$

input
```
integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x^3, x)
```

3.257.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x^3}\,dx = \int \frac{(d^2-e^2\,x^2)^p\,(d+e\,x)^2}{x^3}\,dx$$

input
```
int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x^3,x)
```

output
```
int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x^3, x)
```

3.258 $\int x^5(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx$

3.258.1 Optimal result

Integrand size = 25, antiderivative size = 222

$$\int x^5(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = -\frac{2d^7(d^2-e^2x^2)^{1+p}}{e^6(1+p)} - \frac{ex^7(d^2-e^2x^2)^{1+p}}{9+2p} + \frac{11d^5(d^2-e^2x^2)^{2+p}}{2e^6(2+p)} - \frac{5d^3(d^2-e^2x^2)^{3+p}}{e^6(3+p)} + \frac{3d(d^2-e^2x^2)^{4+p}}{2e^6(4+p)} + \frac{2d^2e(17+3p)x^7(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)}{7(9+2p)}$$

output
```
-2*d^7*(-e^2*x^2+d^2)^(p+1)/e^6/(p+1)-e*x^7*(-e^2*x^2+d^2)^(p+1)/(9+2*p)+1
1/2*d^5*(-e^2*x^2+d^2)^(2+p)/e^6/(2+p)-5*d^3*(-e^2*x^2+d^2)^(3+p)/e^6/(3+p
)+3/2*d*(-e^2*x^2+d^2)^(4+p)/e^6/(4+p)+2/7*d^2*e*(17+3*p)*x^7*(-e^2*x^2+d^
2)^p*hypergeom([7/2, -p],[9/2],e^2*x^2/d^2)/(9+2*p)/((1-e^2*x^2/d^2)^p)
```

3.258.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.92

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$$
$$=\frac{\left(d^2-e^2x^2\right)^p\left(-\frac{252d^7(d^2-e^2x^2)}{1+p}+\frac{693d^5(d^2-e^2x^2)^2}{2+p}+\frac{189d(d^2-e^2x^2)^4}{4+p}-\frac{630(d^3-de^2x^2)^3}{3+p}+54d^2e^7x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hy}\right.}{126e^6}$$

input
```
Integrate[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

output
```
((d^2 - e^2*x^2)^p*((-252*d^7*(d^2 - e^2*x^2))/(1 + p) + (693*d^5*(d^2 - e
^2*x^2)^2)/(2 + p) + (189*d*(d^2 - e^2*x^2)^4)/(4 + p) - (630*(d^3 - d*e^2
*x^2)^3)/(3 + p) + (54*d^2*e^7*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^
2)/d^2])/(1 - (e^2*x^2)/d^2)^p + (14*e^9*x^9*Hypergeometric2F1[9/2, -p, 11
/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/(126*e^6)
```

3.258.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {543, 354, 27, 86, 363, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$$
$$\Big\downarrow \text{543}$$
$$\int x^6\left(d^2-e^2x^2\right)^p\left(x^2e^3+3d^2e\right)dx+\int x^5\left(d^2-e^2x^2\right)^p\left(d^3+3e^2x^2d\right)dx$$
$$\Big\downarrow \text{354}$$
$$\frac{1}{2}\int dx^4\left(d^2-e^2x^2\right)^p\left(d^2+3e^2x^2\right)dx^2+\int x^6\left(d^2-e^2x^2\right)^p\left(x^2e^3+3d^2e\right)dx$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{2}d\int x^4\left(d^2-e^2x^2\right)^p\left(d^2+3e^2x^2\right)dx^2+\int x^6\left(d^2-e^2x^2\right)^p\left(x^2e^3+3d^2e\right)dx$$

$$\Big\downarrow \text{86}$$

$$\int x^6 \left(d^2-e^2x^2\right)^p \left(x^2e^3+3d^2e\right)dx+\frac{1}{2}d\int \left(\frac{4d^6\left(d^2-e^2x^2\right)^p}{e^4}-\frac{11d^4\left(d^2-e^2x^2\right)^{p+1}}{e^4}+\frac{10d^2\left(d^2-e^2x^2\right)^{p+2}}{e^4}-\frac{3\left(d^2-e^2x^2\right)^{p+3}}{e^4}\right)dx^2$$

$$\Big\downarrow \text{363}$$

$$\frac{2d^2e(3p+17)\int x^6\left(d^2-e^2x^2\right)^p dx}{2p+9}+\frac{1}{2}d\int \left(\frac{4d^6\left(d^2-e^2x^2\right)^p}{e^4}-\frac{11d^4\left(d^2-e^2x^2\right)^{p+1}}{e^4}+\frac{10d^2\left(d^2-e^2x^2\right)^{p+2}}{e^4}-\frac{3\left(d^2-e^2x^2\right)^{p+3}}{e^4}\right)dx^2-\frac{ex^7\left(d^2-e^2x^2\right)^{p+1}}{2p+9}$$

$$\Big\downarrow \text{279}$$

$$\frac{2d^2e(3p+17)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int x^6\left(1-\frac{e^2x^2}{d^2}\right)^p dx}{2p+9}+\frac{1}{2}d\int \left(\frac{4d^6\left(d^2-e^2x^2\right)^p}{e^4}-\frac{11d^4\left(d^2-e^2x^2\right)^{p+1}}{e^4}+\frac{10d^2\left(d^2-e^2x^2\right)^{p+2}}{e^4}-\frac{3\left(d^2-e^2x^2\right)^{p+3}}{e^4}\right)dx^2-\frac{ex^7\left(d^2-e^2x^2\right)^{p+1}}{2p+9}$$

$$\Big\downarrow \text{278}$$

$$\frac{1}{2}d\int \left(\frac{4d^6\left(d^2-e^2x^2\right)^p}{e^4}-\frac{11d^4\left(d^2-e^2x^2\right)^{p+1}}{e^4}+\frac{10d^2\left(d^2-e^2x^2\right)^{p+2}}{e^4}-\frac{3\left(d^2-e^2x^2\right)^{p+3}}{e^4}\right)dx^2+\frac{2d^2e(3p+17)x^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)}{7(2p+9)}-\frac{ex^7\left(d^2-e^2x^2\right)^{p+1}}{2p+9}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2d^2e(3p+17)x^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)}{7(2p+9)}-\frac{ex^7\left(d^2-e^2x^2\right)^{p+1}}{2p+9}+\frac{1}{2}d\left(-\frac{10d^2\left(d^2-e^2x^2\right)^{p+3}}{e^6(p+3)}+\frac{3\left(d^2-e^2x^2\right)^{p+4}}{e^6(p+4)}-\frac{4d^6\left(d^2-e^2x^2\right)^{p+1}}{e^6(p+1)}+\frac{11d^4\left(d^2-e^2x^2\right)^{p+2}}{e^6(p+2)}\right)$$

3.258. $\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$

input
```
Int[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

output
```
-((e*x^7*(d^2 - e^2*x^2)^(1 + p))/(9 + 2*p)) + (d*((-4*d^6*(d^2 - e^2*x^2)
^(1 + p))/(e^6*(1 + p)) + (11*d^4*(d^2 - e^2*x^2)^(2 + p))/(e^6*(2 + p)) -
 (10*d^2*(d^2 - e^2*x^2)^(3 + p))/(e^6*(3 + p)) + (3*(d^2 - e^2*x^2)^(4 +
p))/(e^6*(4 + p))))/2 + (2*d^2*e*(17 + 3*p)*x^7*(d^2 - e^2*x^2)^p*Hypergeo
metric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(7*(9 + 2*p)*(1 - (e^2*x^2)/d^2)^p
)
```

3.258.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.258.4 Maple [F]

$$\int x^5(ex+d)^3 \left(-e^2x^2+d^2\right)^p dx$$

input
```
int(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

output
```
int(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

3.258.5 Fricas [F]

$$\int x^5(d+ex)^3 \left(d^2-e^2x^2\right)^p dx = \int (ex+d)^3 \left(-e^2x^2+d^2\right)^p x^5 dx$$

input
```
integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((e^3*x^8 + 3*d*e^2*x^7 + 3*d^2*e*x^6 + d^3*x^5)*(-e^2*x^2 + d^2)^
p, x)
```

3.258.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 938 vs. $2(190) = 380$.

Time = 3.68 (sec) , antiderivative size = 2966, normalized size of antiderivative = 13.36

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx = \text{Too large to display}$$

input
```
integrate(x**5*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

output
```
d**3*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/(4*d**
4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d**4*e*
*6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2*e**8*
x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d**
2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6 -
 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d**2
*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d*
*2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d
**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2
*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) -
2*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d
**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*
e**8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*lo
g(-d/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x
**4/(4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12
*e**6*p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)*
*p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**
4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6)
 - d**2*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22
*e**6*p + 12*e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 ...
```

3.258.7 Maxima [F]

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx = \int (ex+d)^3(-e^2x^2+d^2)^p x^5\,dx$$

input
```
integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
1/2*((p^2 + 3*p + 2)*e^6*x^6 - (p^2 + p)*d^2*e^4*x^4 - 2*d^4*e^2*p*x^2 - 2
*d^6)*(-e^2*x^2 + d^2)^p*d^3/((p^3 + 6*p^2 + 11*p + 6)*e^6) + integrate((e
^3*x^8 + 3*d*e^2*x^7 + 3*d^2*e*x^6)*e^(p*log(e*x + d) + p*log(-e*x + d)),
x)
```

3.258.8 Giac [F]

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^3\left(-e^2x^2+d^2\right)^p x^5\,dx$$

input
```
integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^5, x)
```

3.258.9 Mupad [F(-1)]

Timed out.

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int x^5\left(d^2-e^2\,x^2\right)^p(d+e\,x)^3\,dx$$

input
```
int(x^5*(d^2 - e^2*x^2)^p*(d + e*x)^3,x)
```

output
```
int(x^5*(d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

3.259 $\int x^4(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx$

3.259.1 Optimal result

Integrand size = 25, antiderivative size = 218

$$\begin{aligned}\int x^4(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx &= -\frac{2d^6(d^2-e^2x^2)^{1+p}}{e^5(1+p)}-\frac{3dx^5(d^2-e^2x^2)^{1+p}}{7+2p} \\ &\quad+\frac{9d^4(d^2-e^2x^2)^{2+p}}{2e^5(2+p)}-\frac{3d^2(d^2-e^2x^2)^{3+p}}{e^5(3+p)}+\frac{(d^2-e^2x^2)^{4+p}}{2e^5(4+p)} \\ &\quad+\frac{2d^3(11+p)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5(7+2p)}\end{aligned}$$

output
```
-2*d^6*(-e^2*x^2+d^2)^(p+1)/e^5/(p+1)-3*d*x^5*(-e^2*x^2+d^2)^(p+1)/(7+2*p)
+9/2*d^4*(-e^2*x^2+d^2)^(2+p)/e^5/(2+p)-3*d^2*(-e^2*x^2+d^2)^(3+p)/e^5/(3+
p)+1/2*(-e^2*x^2+d^2)^(4+p)/e^5/(4+p)+2/5*d^3*(11+p)*x^5*(-e^2*x^2+d^2)^p*
hypergeom([5/2, -p],[7/2],e^2*x^2/d^2)/(7+2*p)/((1-e^2*x^2/d^2)^p)
```

3.259.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\frac{1}{70}(d^2-e^2x^2)^p\left(-\frac{35(d^2-e^2x^2)\left(6d^6(5+p)+6d^4e^2(5+6p+p^2)\,x^2+3d^2e^4(10+17p+8p^2+p^3)\,x^4+e^6(6+\right.}{e^5(1+p)(2+p)(3+p)(4+p)}\right.$$
$$+14d^3x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)$$
$$\left.+30de^2x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)\right)$$

input
```
Integrate[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

output
```
((d^2 - e^2*x^2)^p*((-35*(d^2 - e^2*x^2)*(6*d^6*(5 + p) + 6*d^4*e^2*(5 + 6
*p + p^2)*x^2 + 3*d^2*e^4*(10 + 17*p + 8*p^2 + p^3)*x^4 + e^6*(6 + 11*p +
6*p^2 + p^3)*x^6))/(e^5*(1 + p)*(2 + p)*(3 + p)*(4 + p)) + (14*d^3*x^5*Hyp
ergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p + (30*d
*e^2*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^
2)^p))/70
```

3.259.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {543, 354, 27, 86, 363, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$$
$$\Big\downarrow \text{543}$$
$$\int x^5\left(d^2-e^2x^2\right)^p\left(x^2e^3+3d^2e\right)dx+\int x^4\left(d^2-e^2x^2\right)^p\left(d^3+3e^2x^2d\right)dx$$
$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int ex^4\left(d^2-e^2x^2\right)^p\left(3d^2+e^2x^2\right)dx^2+\int x^4\left(d^2-e^2x^2\right)^p\left(d^3+3e^2x^2d\right)dx$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}e\int x^4\left(d^2-e^2x^2\right)^p\left(3d^2+e^2x^2\right)dx^2+\int x^4\left(d^2-e^2x^2\right)^p\left(d^3+3e^2x^2d\right)dx$$

$$\Big\downarrow \; 86$$

$$\int x^4\left(d^2-e^2x^2\right)^p\left(d^3+3e^2x^2d\right)dx+$$
$$\frac{1}{2}e\int\left(\frac{4d^6\left(d^2-e^2x^2\right)^p}{e^4}-\frac{9d^4\left(d^2-e^2x^2\right)^{p+1}}{e^4}+\frac{6d^2\left(d^2-e^2x^2\right)^{p+2}}{e^4}-\frac{\left(d^2-e^2x^2\right)^{p+3}}{e^4}\right)dx^2$$

$$\Big\downarrow \; 363$$

$$\frac{2d^3(p+11)\int x^4\left(d^2-e^2x^2\right)^p dx}{2p+7}+$$
$$\frac{1}{2}e\int\left(\frac{4d^6\left(d^2-e^2x^2\right)^p}{e^4}-\frac{9d^4\left(d^2-e^2x^2\right)^{p+1}}{e^4}+\frac{6d^2\left(d^2-e^2x^2\right)^{p+2}}{e^4}-\frac{\left(d^2-e^2x^2\right)^{p+3}}{e^4}\right)dx^2-$$
$$\frac{3dx^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}$$

$$\Big\downarrow \; 279$$

$$\frac{2d^3(p+11)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^p dx}{2p+7}+$$
$$\frac{1}{2}e\int\left(\frac{4d^6\left(d^2-e^2x^2\right)^p}{e^4}-\frac{9d^4\left(d^2-e^2x^2\right)^{p+1}}{e^4}+\frac{6d^2\left(d^2-e^2x^2\right)^{p+2}}{e^4}-\frac{\left(d^2-e^2x^2\right)^{p+3}}{e^4}\right)dx^2-$$
$$\frac{3dx^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}$$

$$\Big\downarrow \; 278$$

$$\frac{1}{2}e\int\left(\frac{4d^6\left(d^2-e^2x^2\right)^p}{e^4}-\frac{9d^4\left(d^2-e^2x^2\right)^{p+1}}{e^4}+\frac{6d^2\left(d^2-e^2x^2\right)^{p+2}}{e^4}-\frac{\left(d^2-e^2x^2\right)^{p+3}}{e^4}\right)dx^2-$$
$$\frac{3dx^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}+$$
$$\frac{2d^3(p+11)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5(2p+7)}$$

$$\Big\downarrow \; 2009$$

3.259. $\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^p dx$

$$-\frac{3dx^5(d^2-e^2x^2)^{p+1}}{2p+7}+$$
$$\frac{2d^3(p+11)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5(2p+7)}+$$
$$\frac{1}{2}e\left(-\frac{6d^2(d^2-e^2x^2)^{p+3}}{e^6(p+3)}+\frac{(d^2-e^2x^2)^{p+4}}{e^6(p+4)}-\frac{4d^6(d^2-e^2x^2)^{p+1}}{e^6(p+1)}+\frac{9d^4(d^2-e^2x^2)^{p+2}}{e^6(p+2)}\right)$$

input
```
Int[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

output
```
(-3*d*x^5*(d^2 - e^2*x^2)^(1 + p))/(7 + 2*p) + (e*((-4*d^6*(d^2 - e^2*x^2)
^(1 + p))/(e^6*(1 + p)) + (9*d^4*(d^2 - e^2*x^2)^(2 + p))/(e^6*(2 + p)) -
(6*d^2*(d^2 - e^2*x^2)^(3 + p))/(e^6*(3 + p)) + (d^2 - e^2*x^2)^(4 + p)/(e
^6*(4 + p))))/2 + (2*d^3*(11 + p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[
5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(7 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

3.259.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.259.4 Maple [F]

$$\int x^4(ex+d)^3\left(-e^2x^2+d^2\right)^p dx$$

input
```
int(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

output
```
int(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

3.259.5 Fricas [F]

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^3\left(-e^2x^2+d^2\right)^p x^4\,dx$$

input
```
integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((e^3*x^7 + 3*d*e^2*x^6 + 3*d^2*e*x^5 + d^3*x^4)*(-e^2*x^2 + d^2)^
p, x)
```

3.259.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1945 vs. $2(185) = 370$.

Time = 3.36 (sec) , antiderivative size = 2966, normalized size of antiderivative = 13.61

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\text{Too large to display}$$

input
```
integrate(x**4*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

output
```
d**3*d**(2*p)*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d*
*2)/5 + 3*d**2*e*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e
 + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x
)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 -
8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*
e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4
*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e
**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4
*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**
4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e +
 x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e
**8*x**2) - 2*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/
e + x)/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d*
*2*e**6 + 2*e**8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)
), (-d**4*log(-d/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/
(2*e**4) - x**4/(4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e*
*6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 -
 e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e
**4*p**2*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*
p + 12*e**6) - d**2*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12...
```

3.259.7 Maxima [F]

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^3\left(-e^2x^2+d^2\right)^p x^4\,dx$$

input
```
integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^4, x)
```

3.259.8 Giac [F]

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^3\left(-e^2x^2+d^2\right)^p x^4\,dx$$

input
```
integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^4, x)
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int x^4\left(d^2-e^2x^2\right)^p(d+e\,x)^3\,dx$$

input
```
int(x^4*(d^2 - e^2*x^2)^p*(d + e*x)^3,x)
```

output
```
int(x^4*(d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

3.260 $\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$

3.260.1 Optimal result

Integrand size = 25, antiderivative size = 193

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx = -\frac{2d^5(d^2-e^2x^2)^{1+p}}{e^4(1+p)}-\frac{ex^5(d^2-e^2x^2)^{1+p}}{7+2p}+\frac{7d^3(d^2-e^2x^2)^{2+p}}{2e^4(2+p)}-\frac{3d(d^2-e^2x^2)^{3+p}}{2e^4(3+p)}+\frac{2d^2e(13+3p)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5(7+2p)}$$

output
```
-2*d^5*(-e^2*x^2+d^2)^(p+1)/e^4/(p+1)-e*x^5*(-e^2*x^2+d^2)^(p+1)/(7+2*p)+7
/2*d^3*(-e^2*x^2+d^2)^(2+p)/e^4/(2+p)-3/2*d*(-e^2*x^2+d^2)^(3+p)/e^4/(3+p)
+2/5*d^2*e*(13+3*p)*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, -p],[7/2],e^2*x^2
/d^2)/(7+2*p)/((1-e^2*x^2/d^2)^p)
```

3.260.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.97

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx = \frac{(d^2-e^2x^2)^p\left(-\frac{35d(d^2-e^2x^2)(d^4(9+p)+d^2e^2(9+10p+p^2)x^2+3e^4(2+3p+p^2)x^4)}{(1+p)(2+p)(3+p)}+42d^2e^5x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometr}\right.}{70e^4}$$

input
```
Integrate[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

output
```
((d^2 - e^2*x^2)^p*((-35*d*(d^2 - e^2*x^2)*(d^4*(9 + p) + d^2*e^2*(9 + 10*
p + p^2)*x^2 + 3*e^4*(2 + 3*p + p^2)*x^4))/((1 + p)*(2 + p)*(3 + p)) + (42
*d^2*e^5*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2
)/d^2)^p + (10*e^7*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1
- (e^2*x^2)/d^2)^p))/(70*e^4)
```

3.260.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {543, 354, 27, 86, 363, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$$

$$\Big\downarrow \; 543$$

$$\int x^4\left(d^2-e^2x^2\right)^p\left(x^2e^3+3d^2e\right)dx+\int x^3\left(d^2-e^2x^2\right)^p\left(d^3+3e^2x^2d\right)dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int dx^2\left(d^2-e^2x^2\right)^p\left(d^2+3e^2x^2\right)dx^2+\int x^4\left(d^2-e^2x^2\right)^p\left(x^2e^3+3d^2e\right)dx$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}d\int x^2\left(d^2-e^2x^2\right)^p\left(d^2+3e^2x^2\right)dx^2+\int x^4\left(d^2-e^2x^2\right)^p\left(x^2e^3+3d^2e\right)dx$$

$$\Big\downarrow \; 86$$

$$\int x^4\left(d^2-e^2x^2\right)^p\left(x^2e^3+3d^2e\right)dx+\frac{1}{2}d\int\left(\frac{4d^4\left(d^2-e^2x^2\right)^p}{e^2}-\frac{7d^2\left(d^2-e^2x^2\right)^{p+1}}{e^2}+\frac{3\left(d^2-e^2x^2\right)^{p+2}}{e^2}\right)dx^2$$

$$\Big\downarrow \; 363$$

$$\frac{2d^2e(3p+13)\int x^4\left(d^2-e^2x^2\right)^p dx}{2p+7}+\frac{1}{2}d\int\left(\frac{4d^4\left(d^2-e^2x^2\right)^p}{e^2}-\frac{7d^2\left(d^2-e^2x^2\right)^{p+1}}{e^2}+\frac{3\left(d^2-e^2x^2\right)^{p+2}}{e^2}\right)dx^2-\frac{ex^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}$$

$$\Big\downarrow \text{279}$$

$$\frac{2d^2e(3p+13)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^p dx}{2p+7}+\frac{1}{2}d\int\left(\frac{4d^4\left(d^2-e^2x^2\right)^p}{e^2}-\frac{7d^2\left(d^2-e^2x^2\right)^{p+1}}{e^2}+\frac{3\left(d^2-e^2x^2\right)^{p+2}}{e^2}\right)dx^2-\frac{ex^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}$$

$$\Big\downarrow \text{278}$$

$$\frac{1}{2}d\int\left(\frac{4d^4\left(d^2-e^2x^2\right)^p}{e^2}-\frac{7d^2\left(d^2-e^2x^2\right)^{p+1}}{e^2}+\frac{3\left(d^2-e^2x^2\right)^{p+2}}{e^2}\right)dx^2+\frac{2d^2e(3p+13)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5(2p+7)}-\frac{ex^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2d^2e(3p+13)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5(2p+7)}-\frac{ex^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}+\frac{1}{2}d\left(\frac{7d^2\left(d^2-e^2x^2\right)^{p+2}}{e^4(p+2)}-\frac{3\left(d^2-e^2x^2\right)^{p+3}}{e^4(p+3)}-\frac{4d^4\left(d^2-e^2x^2\right)^{p+1}}{e^4(p+1)}\right)$$

input
```
Int[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

output
```
-((e*x^5*(d^2 - e^2*x^2)^(1 + p))/(7 + 2*p)) + (d*((-4*d^4*(d^2 - e^2*x^2)
^(1 + p))/(e^4*(1 + p)) + (7*d^2*(d^2 - e^2*x^2)^(2 + p))/(e^4*(2 + p)) -
(3*(d^2 - e^2*x^2)^(3 + p))/(e^4*(3 + p))))/2 + (2*d^2*e*(13 + 3*p)*x^5*(d
^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(7 + 2*
p)*(1 - (e^2*x^2)/d^2)^p)
```

3.260. $\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$

3.260.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.260. $\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.260.4 Maple [F]

$$\int x^3(ex+d)^3\left(-e^2x^2+d^2\right)^p dx$$

input
```
int(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

output
```
int(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

3.260.5 Fricas [F]

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx = \int (ex+d)^3\left(-e^2x^2+d^2\right)^p x^3\,dx$$

input
```
integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((e^3*x^6 + 3*d*e^2*x^5 + 3*d^2*e*x^4 + d^3*x^3)*(-e^2*x^2 + d^2)^
p, x)
```

3.260.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(165) = 330$.

Time = 2.79 (sec) , antiderivative size = 1370, normalized size of antiderivative = 7.10

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx = \text{Too large to display}$$

input
```
integrate(x**3*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

output
```
d**3*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2
*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d*
*2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 +
2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p,
-2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2
*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p +
 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p
+ 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*
e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4),
 True)) + 3*d**2*d**(2*p)*e*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_po
lar(2*I*pi)/d**2)/5 + 3*d*e**2*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2
*d**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d*
*4*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(
4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e
 + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*l
og(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*
x**2/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/
e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d
/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d
**4*log(-d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-...
```

3.260.7 Maxima [F]

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx = \int (ex+d)^3\left(-e^2x^2+d^2\right)^p x^3\,dx$$

input
```
integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
1/2*(e^4*(p + 1)*x^4 - d^2*e^2*p*x^2 - d^4)*(-e^2*x^2 + d^2)^p*d^3/((p^2 +
 3*p + 2)*e^4) + integrate((e^3*x^6 + 3*d*e^2*x^5 + 3*d^2*e*x^4)*e^(p*log(
e*x + d) + p*log(-e*x + d)), x)
```

3.260.8 Giac [F]

$$\int x^3(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \int (ex+d)^3 \left(-e^2x^2+d^2\right)^p x^3 \, dx$$

input
```
integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^3, x)
```

3.260.9 Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \int x^3 \left(d^2-e^2\,x^2\right)^p \left(d+e\,x\right)^3 \, dx$$

input
```
int(x^3*(d^2 - e^2*x^2)^p*(d + e*x)^3,x)
```

output
```
int(x^3*(d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

3.261 $\int x^2(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx$

3.261.1 Optimal result

Integrand size = 25, antiderivative size = 189

$$\int x^2(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = -\frac{2d^4\left(d^2-e^2x^2\right)^{1+p}}{e^3(1+p)} - \frac{3dx^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p} + \frac{5d^2\left(d^2-e^2x^2\right)^{2+p}}{2e^3(2+p)} - \frac{\left(d^2-e^2x^2\right)^{3+p}}{2e^3(3+p)} + \frac{2d^3(7+p)x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3(5+2p)}$$

output
```
-2*d^4*(-e^2*x^2+d^2)^(p+1)/e^3/(p+1)-3*d*x^3*(-e^2*x^2+d^2)^(p+1)/(5+2*p)
+5/2*d^2*(-e^2*x^2+d^2)^(2+p)/e^3/(2+p)-1/2*(-e^2*x^2+d^2)^(3+p)/e^3/(3+p)
+2/3*d^3*(7+p)*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/2, -p],[5/2],e^2*x^2/d^2)
/(5+2*p)/((1-e^2*x^2/d^2)^p)
```

3.261.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.99

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$$
$$=\frac{1}{30}\left(d^2-e^2x^2\right)^p\left(-\frac{15(d^2-e^2x^2)\left(d^4(11+3p)+d^2e^2(11+14p+3p^2)\,x^2+e^4(2+3p+p^2)\,x^4\right)}{e^3(1+p)(2+p)(3+p)}+10d^3x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)+18de^2x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)\right)$$

input
```
Integrate[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

output
```
((d^2 - e^2*x^2)^p*((-15*(d^2 - e^2*x^2)*(d^4*(11 + 3*p) + d^2*e^2*(11 + 1
4*p + 3*p^2)*x^2 + e^4*(2 + 3*p + p^2)*x^4))/(e^3*(1 + p)*(2 + p)*(3 + p))
 + (10*d^3*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(1 - (e^2*x
^2)/d^2)^p + (18*d*e^2*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])
/(1 - (e^2*x^2)/d^2)^p))/30
```

3.261.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {543, 354, 27, 86, 363, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$$
$$\Big\downarrow \text{ 543}$$
$$\int x^3\left(d^2-e^2x^2\right)^p\left(x^2e^3+3d^2e\right)dx+\int x^2\left(d^2-e^2x^2\right)^p\left(d^3+3e^2x^2d\right)dx$$
$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int ex^2\left(d^2-e^2x^2\right)^p\left(3d^2+e^2x^2\right)dx^2+\int x^2\left(d^2-e^2x^2\right)^p\left(d^3+3e^2x^2d\right)dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}e\int x^2\left(d^2-e^2x^2\right)^p\left(3d^2+e^2x^2\right)dx^2+\int x^2\left(d^2-e^2x^2\right)^p\left(d^3+3e^2x^2d\right)dx$$

$$\Big\downarrow \ 86$$

$$\frac{1}{2}e\int\left(\frac{4d^4\left(d^2-e^2x^2\right)^p}{e^2}-\frac{5d^2\left(d^2-e^2x^2\right)^{p+1}}{e^2}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{e^2}\right)dx^2+\int x^2\left(d^2-e^2x^2\right)^p\left(d^3+3e^2x^2d\right)dx$$

$$\Big\downarrow \ 363$$

$$\frac{1}{2}e\int\left(\frac{4d^4\left(d^2-e^2x^2\right)^p}{e^2}-\frac{5d^2\left(d^2-e^2x^2\right)^{p+1}}{e^2}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{e^2}\right)dx^2+\frac{2d^3(p+7)\int x^2\left(d^2-e^2x^2\right)^p dx}{2p+5}-\frac{3dx^3\left(d^2-e^2x^2\right)^{p+1}}{2p+5}$$

$$\Big\downarrow \ 279$$

$$\frac{1}{2}e\int\left(\frac{4d^4\left(d^2-e^2x^2\right)^p}{e^2}-\frac{5d^2\left(d^2-e^2x^2\right)^{p+1}}{e^2}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{e^2}\right)dx^2+\frac{2d^3(p+7)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^p dx}{2p+5}-\frac{3dx^3\left(d^2-e^2x^2\right)^{p+1}}{2p+5}$$

$$\Big\downarrow \ 278$$

$$\frac{1}{2}e\int\left(\frac{4d^4\left(d^2-e^2x^2\right)^p}{e^2}-\frac{5d^2\left(d^2-e^2x^2\right)^{p+1}}{e^2}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{e^2}\right)dx^2-\frac{3dx^3\left(d^2-e^2x^2\right)^{p+1}}{2p+5}+\frac{2d^3(p+7)x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3(2p+5)}$$

$$\Big\downarrow \ 2009$$

$$-\frac{3dx^3\left(d^2-e^2x^2\right)^{p+1}}{2p+5}+\frac{1}{2}e\left(\frac{5d^2\left(d^2-e^2x^2\right)^{p+2}}{e^4(p+2)}-\frac{\left(d^2-e^2x^2\right)^{p+3}}{e^4(p+3)}-\frac{4d^4\left(d^2-e^2x^2\right)^{p+1}}{e^4(p+1)}\right)+\frac{2d^3(p+7)x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3(2p+5)}$$

input
```
Int[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

3.261. $\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^p dx$

output
```
(-3*d*x^3*(d^2 - e^2*x^2)^(1 + p))/(5 + 2*p) + (e*((-4*d^4*(d^2 - e^2*x^2)
^(1 + p))/(e^4*(1 + p)) + (5*d^2*(d^2 - e^2*x^2)^(2 + p))/(e^4*(2 + p)) -
(d^2 - e^2*x^2)^(3 + p)/(e^4*(3 + p))))/2 + (2*d^3*(7 + p)*x^3*(d^2 - e^2*
x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(5 + 2*p)*(1 - (
e^2*x^2)/d^2)^p)
```

3.261.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.261.4 Maple [F]

$$\int x^2(ex+d)^3\left(-e^2x^2+d^2\right)^p dx$$

input
```
int(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

output
```
int(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

3.261.5 Fricas [F]

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^p\, dx=\int (ex+d)^3\left(-e^2x^2+d^2\right)^p x^2\, dx$$

input
```
integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*e*x^3 + d^3*x^2)*(-e^2*x^2 + d^2)^
p, x)
```

3.261.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 938 vs. $2(160) = 320$.

Time = 2.55 (sec) , antiderivative size = 1370, normalized size of antiderivative = 7.25

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

output
```
d**3*d**(2*p)*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d*
*2)/3 + 3*d**2*e*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e +
 x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**
6*x**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*
d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x
**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**
4) - x**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2
+ 6*e**4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2
 + 6*e**4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6
*e**4*p + 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*
p + 4*e**4), True)) + 3*d*d**(2*p)*e**2*x**5*hyper((5/2, -p), (7/2,), e**2
*x**2*exp_polar(2*I*pi)/d**2)/5 + e**3*Piecewise((x**6*(d**2)**p/6, Eq(e,
0)), (-2*d**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4
) - 2*d**4*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) -
3*d**4/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*
log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**
2*x**2*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d*
*2*e**2*x**2/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4
*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x*
*4*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, ...
```

3.261.7 Maxima [F]

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx = \int (ex+d)^3\left(-e^2x^2+d^2\right)^p x^2\,dx$$

input
```
integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^2, x)
```

3.261.8 Giac [F]

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^3\left(-e^2x^2+d^2\right)^p x^2\,dx$$

input
```
integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^2, x)
```

3.261.9 Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int x^2\left(d^2-e^2\,x^2\right)^p(d+e\,x)^3\,dx$$

input
```
int(x^2*(d^2 - e^2*x^2)^p*(d + e*x)^3,x)
```

output
```
int(x^2*(d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

3.262 $\int x(d+ex)^3 (d^2-e^2x^2)^p \, dx$

3.262.1 Optimal result

Integrand size = 23, antiderivative size = 116

$$\int x(d+ex)^3 (d^2-e^2x^2)^p \, dx = -\frac{(d+ex)^3 (d^2-e^2x^2)^{1+p}}{e^2(5+2p)} - \frac{3\ 2^{3+p}d^3 \left(1+\frac{ex}{d}\right)^{-1-p} (d^2-e^2x^2)^{1+p} \text{Hypergeometric2F1}\left(-3-p,1+p,2+p,\frac{d-ex}{2d}\right)}{e^2(1+p)(5+2p)}$$

output
```
-(e*x+d)^3*(-e^2*x^2+d^2)^(p+1)/e^2/(5+2*p)-3*2^(3+p)*d^3*(1+e*x/d)^(-1-p)
*(-e^2*x^2+d^2)^(p+1)*hypergeom([p+1, -3-p],[2+p],1/2*(-e*x+d)/d)/e^2/(2*p
^2+7*p+5)
```

3.262.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.37

$$\int x(d+ex)^3 (d^2-e^2x^2)^p \, dx = \frac{(d^2-e^2x^2)^p \left(-\frac{5d(d^2-e^2x^2)(d^2(5+p)+3e^2(1+p)x^2)}{(1+p)(2+p)} + 10d^2e^3x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)\right.}{10e^2}$$

input
```
Integrate[x*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

output
```
((d^2 - e^2*x^2)^p*((-5*d*(d^2 - e^2*x^2)*(d^2*(5 + p) + 3*e^2*(1 + p)*x^2
))/((1 + p)*(2 + p)) + (10*d^2*e^3*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^
2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p + (2*e^5*x^5*Hypergeometric2F1[5/2, -p,
 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/(10*e^2)
```

3.262.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {572, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$$

$$\Big\downarrow \text{572}$$

$$\frac{3d\int (d+ex)^3\left(d^2-e^2x^2\right)^p dx}{e(2p+5)}-\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{p+1}}{e^2(2p+5)}$$

$$\Big\downarrow \text{473}$$

$$\frac{3d^3(d-ex)^{-p-1}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\int (d-ex)^p\left(\frac{ex}{d}+1\right)^{p+3}dx}{e(2p+5)}-\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{p+1}}{e^2(2p+5)}$$

$$\Big\downarrow \text{79}$$

$$-\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{p+1}}{e^2(2p+5)}-\frac{3d^3 2^{p+3}\left(d^2-e^2x^2\right)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}\text{Hypergeometric2F1}\left(-p-3,p+1,p+2,\frac{d-ex}{2d}\right)}{e^2(p+1)(2p+5)}$$

input
```
Int[x*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

output
```
-(((d + e*x)^3*(d^2 - e^2*x^2)^(1 + p))/(e^2*(5 + 2*p))) - (3*2^(3 + p)*d^
3*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[-3 - p,
 1 + p, 2 + p, (d - e*x)/(2*d)])/(e^2*(1 + p)*(5 + 2*p))
```

3.262.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 572
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 2))), x] + Simp[c*(n/(d
*(n + 2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && NeQ[n + 2*p + 2, 0]
```

3.262.4 Maple [F]

$$\int x(ex+d)^3\left(-e^2x^2+d^2\right)^p dx$$

input
```
int(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

output
```
int(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

3.262.5 Fricas [F]

$$\int x(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \int (ex+d)^3(-e^2x^2+d^2)^p x \, dx$$

input
```
integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e*x^2 + d^3*x)*(-e^2*x^2 + d^2)^p,
 x)
```

3.262.6 Sympy [A] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 479, normalized size of antiderivative = 4.13

$$\int x(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx$$
$$= d^3\left(\begin{cases} \frac{x^2(d^2)^p}{2} & \text{for } e^2=0 \\ -\frac{\begin{cases} \frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p\neq -1 \\ \log\left(d^2-e^2x^2\right) & \text{otherwise} \end{cases}}{2e^2} & \text{otherwise} \end{cases}\right) + d^2 d^{2p} e x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)$$
$$+ 3de^2\left(\begin{cases} \frac{x^4(d^2)^p}{4} & \text{for } e=0 \\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} & \text{for } p=-2 \\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4} - \frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4} - \frac{x^2}{2e^2} & \text{for } p=-1 \\ -\frac{d^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} - \frac{d^2e^2px^2(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4px^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4x^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} & \text{otherwise} \end{cases}\right)$$
$$+ \frac{d^{2p}e^3x^5 {}_2F_1\left(\begin{matrix} \frac{5}{2}, -p \\ \frac{7}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{5}$$

input
```
integrate(x*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

output
```
d**3*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*
x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2
), True)) + d**2*d**(2*p)*e*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_po
lar(2*I*pi)/d**2) + 3*d*e**2*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**
2*log(-d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*
e**4 + 2*e**6*x**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d
/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**
4 + 2*e**6*x**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e
 + x)/(2*e**4) - x**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(
2*e**4*p**2 + 6*e**4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/
(2*e**4*p**2 + 6*e**4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e
**4*p**2 + 6*e**4*p + 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p*
*2 + 6*e**4*p + 4*e**4), True)) + d**(2*p)*e**3*x**5*hyper((5/2, -p), (7/2
,), e**2*x**2*exp_polar(2*I*pi)/d**2)/5
```

3.262.7 Maxima [F]

$$\int x(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^3(-e^2x^2+d^2)^p x\,dx$$

input
```
integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
-1/2*(-e^2*x^2 + d^2)^(p + 1)*d^3/(e^2*(p + 1)) + integrate((e^3*x^4 + 3*d
*e^2*x^3 + 3*d^2*e*x^2)*e^(p*log(e*x + d) + p*log(-e*x + d)), x)
```

3.262.8 Giac [F]

$$\int x(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^3(-e^2x^2+d^2)^p x\,dx$$

input
```
integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x, x)
```

3.262.9 Mupad [F(-1)]

Timed out.

$$\int x(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int x\left(d^2-e^2\,x^2\right)^p\left(d+e\,x\right)^3\,dx$$

input
```
int(x*(d^2 - e^2*x^2)^p*(d + e*x)^3,x)
```

output
```
int(x*(d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

3.263 $\int (d+ex)^3 (d^2-e^2x^2)^p \, dx$

3.263.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int (d+ex)^3 (d^2-e^2x^2)^p \, dx = -\frac{2^{3+p} d^2 \left(1+\frac{ex}{d}\right)^{-1-p} (d^2-e^2x^2)^{1+p} \text{Hypergeometric2F1}\left(-3-p,1+p,2+p,\frac{d-ex}{2d}\right)}{e(1+p)}$$

output
```
-2^(3+p)*d^2*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(p+1)*hypergeom([p+1, -3-p],[
2+p],1/2*(-e*x+d)/d)/e/(p+1)
```

3.263.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 155 vs. 2(73) = 146.

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.12

$$\int (d+ex)^3 (d^2-e^2x^2)^p \, dx = \frac{1}{2}(d^2-e^2x^2)^p \left(\frac{(-d^2+e^2x^2)(d^2(7+3p)+e^2(1+p)x^2)}{e(1+p)(2+p)} + 2d^3x\left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right) + 2de^2x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)\right)$$

input
```
Integrate[(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

output
```
((d^2 - e^2*x^2)^p*(((-d^2 + e^2*x^2)*(d^2*(7 + 3*p) + e^2*(1 + p)*x^2))/(
e*(1 + p)*(2 + p)) + (2*d^3*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^
2])/(1 - (e^2*x^2)/d^2)^p + (2*d*e^2*x^3*Hypergeometric2F1[3/2, -p, 5/2, (
e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/2
```

3.263.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx$$

$$\Big\downarrow \text{ 473}$$

$$d^2(d-ex)^{-p-1}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\int (d-ex)^p\left(\frac{ex}{d}+1\right)^{p+3}dx$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{d^2 2^{p+3}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\text{Hypergeometric2F1}\left(-p-3,p+1,p+2,\frac{d-ex}{2d}\right)}{e(p+1)}$$

input
```
Int[(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

output
```
-((2^(3 + p)*d^2*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeome
tric2F1[-3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(e*(1 + p)))
```

3.263.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

3.263.4 Maple [F]

$$\int (ex+d)^3 \left(-e^2x^2+d^2\right)^p dx$$

input
```
int((e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

output
```
int((e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

3.263.5 Fricas [F]

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \int (ex+d)^3 \left(-e^2x^2+d^2\right)^p dx$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(-e^2*x^2 + d^2)^p, x)
```

3.263.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(58) = 116$.

Time = 1.85 (sec) , antiderivative size = 476, normalized size of antiderivative = 6.52

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = d^3 d^{2p} x\, {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right) + 3d^2e\left(\begin{cases} \frac{x^2(d^2)^p}{2} & \text{for } e^2=0 \\ -\frac{\begin{cases} \frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(d^2-e^2x^2) & \text{otherwise} \end{cases}}{2e^2} & \text{otherwise} \end{cases}\right) + dd^{2p}e^2x^3\, {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right) + e^3\left(\begin{cases} \frac{x^4(d^2)^p}{4} & \text{for } e=0 \\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} & \text{for } p=-2 \\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4} - \frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4} - \frac{x^2}{2e^2} & \text{for } p=-1 \\ -\frac{d^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} - \frac{d^2e^2px^2(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4px^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4x^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

output
```
d**3*d**(2*p)*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)
 + 3*d**2*e*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2
- e**2*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/
(2*e**2), True)) + d*d**(2*p)*e**2*x**3*hyper((3/2, -p), (5/2,), e**2*x**2
*exp_polar(2*I*pi)/d**2) + e**3*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-
d**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d*
*2*e**4 + 2*e**6*x**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log
(-d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*
e**4 + 2*e**6*x**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(
d/e + x)/(2*e**4) - x**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**
p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)*
*p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(
2*e**4*p**2 + 6*e**4*p + 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4
*p**2 + 6*e**4*p + 4*e**4), True))
```

3.263.7 Maxima [F]

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \int (ex+d)^3 \left(-e^2x^2+d^2\right)^p \, dx$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p, x)
```

3.263.8 Giac [F]

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \int (ex+d)^3 \left(-e^2x^2+d^2\right)^p \, dx$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p, x)
```

3.263.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \int \left(d^2-e^2\,x^2\right)^p \, (d+e\,x)^3 \, dx$$

input
```
int((d^2 - e^2*x^2)^p*(d + e*x)^3,x)
```

output
```
int((d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

3.264 $\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x}\,dx$

3.264.1 Optimal result

Integrand size = 25, antiderivative size = 171

$$\begin{aligned}
&\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x}\,dx\\
&= -\frac{3d(d^2-e^2x^2)^{1+p}}{2(1+p)} - \frac{ex(d^2-e^2x^2)^{1+p}}{3+2p}\\
&\quad + \frac{2d^2e(5+3p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{3+2p}\\
&\quad - \frac{d(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{2(1+p)}
\end{aligned}$$

output
```
-3/2*d*(-e^2*x^2+d^2)^(p+1)/(p+1)-e*x*(-e^2*x^2+d^2)^(p+1)/(3+2*p)+2*d^2*e
*(5+3*p)*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],[3/2],e^2*x^2/d^2)/(3+2*p)
/((1-e^2*x^2/d^2)^p)-1/2*d*(-e^2*x^2+d^2)^(p+1)*hypergeom([1, p+1],[2+p],1
-e^2*x^2/d^2)/(p+1)
```

3.264.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x}\,dx$$
$$=\frac{1}{6}(d^2-e^2x^2)^p\left(-\frac{9d(d^2-e^2x^2)}{1+p}\right.$$
$$+18d^2ex\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)$$
$$-\frac{3d(d^2-e^2x^2)\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{1+p}$$
$$\left.+2e^3x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)\right)$$

input
```
Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x,x]
```

output
```
((d^2 - e^2*x^2)^p*((-9*d*(d^2 - e^2*x^2))/(1 + p) + (18*d^2*e*x*Hypergeom
etric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - (3*d*(d^2 -
 e^2*x^2)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(1 + p) +
 (2*e^3*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)
/d^2)^p))/6
```

3.264.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {543, 299, 238, 237, 354, 27, 90, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x}\,dx$$
$$\Big\downarrow \; 543$$
$$\int (d^2-e^2x^2)^p\,(x^2e^3+3d^2e)\,dx+\int \frac{(d^2-e^2x^2)^p\,(d^3+3e^2x^2d)}{x}dx$$

$$\Big\downarrow \text{299}$$

$$\frac{2d^2e(3p+5)\int (d^2-e^2x^2)^p\,dx}{2p+3}+\int \frac{(d^2-e^2x^2)^p\,(d^3+3e^2x^2d)}{x}dx-\frac{ex(d^2-e^2x^2)^{p+1}}{2p+3}$$

$$\Big\downarrow \text{238}$$

$$\frac{2d^2e(3p+5)\,(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int \left(1-\frac{e^2x^2}{d^2}\right)^p dx}{2p+3}+\int \frac{(d^2-e^2x^2)^p\,(d^3+3e^2x^2d)}{x}dx-\frac{ex(d^2-e^2x^2)^{p+1}}{2p+3}$$

$$\Big\downarrow \text{237}$$

$$\int \frac{(d^2-e^2x^2)^p\,(d^3+3e^2x^2d)}{x}dx+\frac{2d^2e(3p+5)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{2p+3}-\frac{ex(d^2-e^2x^2)^{p+1}}{2p+3}$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{d(d^2-e^2x^2)^p\,(d^2+3e^2x^2)}{x^2}dx^2+\frac{2d^2e(3p+5)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{2p+3}-\frac{ex(d^2-e^2x^2)^{p+1}}{2p+3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}d\int \frac{(d^2-e^2x^2)^p\,(d^2+3e^2x^2)}{x^2}dx^2+\frac{2d^2e(3p+5)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{2p+3}-\frac{ex(d^2-e^2x^2)^{p+1}}{2p+3}$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}d\left(d^2\int \frac{(d^2-e^2x^2)^p}{x^2}dx^2-\frac{3(d^2-e^2x^2)^{p+1}}{p+1}\right)+\frac{2d^2e(3p+5)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{2p+3}-\frac{ex(d^2-e^2x^2)^{p+1}}{2p+3}$$

$$\Big\downarrow \text{75}$$

3.264. $\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x}\,dx$

$$\frac{2d^2e(3p+5)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{2p+3}+\frac{1}{2}d\left(-\frac{(d^2-e^2x^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{p+1}-\frac{3(d^2-e^2x^2)^{p+1}}{p+1}\right)-\frac{ex(d^2-e^2x^2)^{p+1}}{2p+3}$$

input
```
Int[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x,x]
```

output
```
-((e*x*(d^2 - e^2*x^2)^(1 + p))/(3 + 2*p)) + (2*d^2*e*(5 + 3*p)*x*(d^2 - e
^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/((3 + 2*p)*(1 -
(e^2*x^2)/d^2)^p) + (d*((-3*(d^2 - e^2*x^2)^(1 + p))/(1 + p) - ((d^2 - e^2
*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(1 +
p)))/2
```

3.264.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] && !IntegerQ[2*p]
&& !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

3.264.4 Maple [F]

$$\int \frac{(ex+d)^3 (-e^2x^2+d^2)^p}{x} dx$$

input
```
int((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x)
```

output
```
int((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x)
```

3.264.5 Fricas [F]

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x} \, dx = \int \frac{(ex+d)^3 (-e^2x^2+d^2)^p}{x} \, dx$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x, algorithm="fricas")
```

output
```
integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(-e^2*x^2 + d^2)^p/x, x
)
```

3.264.6 Sympy [A] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x} \, dx = -\frac{d^3 e^{2p} x^{2p} e^{i\pi p} \Gamma(-p) \, {}_2F_1\left(\begin{matrix} -p, -p \\ 1-p \end{matrix} \middle| \frac{d^2}{e^2x^2}\right)}{2\Gamma(1-p)} + 3d^2 d^{2p} e x \, {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix} \middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right) + 3de^2 \left(\begin{cases} \frac{x^2 (d^2)^p}{2} & \text{for } e^2 = 0 \\ -\frac{\begin{cases} \frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(d^2-e^2x^2) & \text{otherwise} \end{cases}}{2e^2} & \text{otherwise} \end{cases}\right) + \frac{d^{2p} e^3 x^3 \, {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix} \middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}$$

input
```
integrate((e*x+d)**3*(-e**2*x**2+d**2)**p/x,x)
```

output
```
-d**3*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d*
*2/(e**2*x**2))/(2*gamma(1 - p)) + 3*d**2*d**(2*p)*e*x*hyper((1/2, -p), (3
/2,), e**2*x**2*exp_polar(2*I*pi)/d**2) + 3*d*e**2*Piecewise((x**2*(d**2)*
*p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x**2)**(p + 1)/(p + 1), Ne(p
, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2), True)) + d**(2*p)*e**3*x*
*3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/3
```

3.264.7 Maxima [F]

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x}\,dx = \int \frac{(ex+d)^3(-e^2x^2+d^2)^p}{x}\,dx$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x, x)
```

3.264.8 Giac [F]

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x}\,dx = \int \frac{(ex+d)^3(-e^2x^2+d^2)^p}{x}\,dx$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x, x)
```

3.264.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x}\,dx = \int \frac{(d^2-e^2\,x^2)^p\,(d+e\,x)^3}{x}\,dx$$

input
```
int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x,x)
```

output
```
int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x, x)
```

3.265 $\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x^2}\,dx$

3.265.1 Optimal result

Integrand size = 25, antiderivative size = 159

$$\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x^2}\,dx = -\frac{e(d^2-e^2x^2)^{1+p}}{2(1+p)}-\frac{d(d^2-e^2x^2)^{1+p}}{x} + 2de^2(1-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right) - \frac{3e(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{2(1+p)}$$

output
```
-1/2*e*(-e^2*x^2+d^2)^(p+1)/(p+1)-d*(-e^2*x^2+d^2)^(p+1)/x+2*d*e^2*(1-p)*x
*(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],[3/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^
p)-3/2*e*(-e^2*x^2+d^2)^(p+1)*hypergeom([1, p+1],[2+p],1-e^2*x^2/d^2)/(p+1
)
```

3.265.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^2} \, dx$$
$$= \frac{(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \left(-2d^3(1+p)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)+ex\left(6de(1+p)x\,\text{Hyperg}\right.\right.}{2}$$

input
```
Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x^2,x]
```

output
```
((d^2 - e^2*x^2)^p*(-2*d^3*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x
^2)/d^2] + e*x*(6*d*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/
d^2] - (d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p*(1 + 3*Hypergeometric2F1[1, 1
 + p, 2 + p, 1 - (e^2*x^2)/d^2]))))/(2*(1 + p)*x*(1 - (e^2*x^2)/d^2)^p)
```

3.265.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {543, 354, 27, 90, 75, 359, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^2} \, dx$$
$$\Big\downarrow \text{543}$$
$$\int \frac{(d^2-e^2x^2)^p (x^2e^3+3d^2e)}{x}dx+\int \frac{(d^2-e^2x^2)^p (d^3+3e^2x^2d)}{x^2}dx$$
$$\Big\downarrow \text{354}$$
$$\frac{1}{2}\int \frac{e(d^2-e^2x^2)^p (3d^2+e^2x^2)}{x^2}dx^2+\int \frac{(d^2-e^2x^2)^p (d^3+3e^2x^2d)}{x^2}dx$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{2}e\int \frac{(d^2-e^2x^2)^p (3d^2+e^2x^2)}{x^2}dx^2+\int \frac{(d^2-e^2x^2)^p (d^3+3e^2x^2d)}{x^2}dx$$
$$\Big\downarrow \text{90}$$

$$\frac{1}{2}e\left(3d^2\int\frac{\left(d^2-e^2x^2\right)^p}{x^2}dx^2-\frac{\left(d^2-e^2x^2\right)^{p+1}}{p+1}\right)+\int\frac{\left(d^2-e^2x^2\right)^p\left(d^3+3e^2x^2d\right)}{x^2}dx$$

$$\Big\downarrow \text{75}$$

$$\int\frac{\left(d^2-e^2x^2\right)^p\left(d^3+3e^2x^2d\right)}{x^2}dx+\frac{1}{2}e\left(-\frac{3\left(d^2-e^2x^2\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{p+1}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{p+1}\right)$$

$$\Big\downarrow \text{359}$$

$$2de^2(1-p)\int\left(d^2-e^2x^2\right)^p dx+\frac{1}{2}e\left(-\frac{3\left(d^2-e^2x^2\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{p+1}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{p+1}\right)-\frac{d\left(d^2-e^2x^2\right)^{p+1}}{x}$$

$$\Big\downarrow \text{238}$$

$$2de^2(1-p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int\left(1-\frac{e^2x^2}{d^2}\right)^p dx+\frac{1}{2}e\left(-\frac{3\left(d^2-e^2x^2\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{p+1}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{p+1}\right)-\frac{d\left(d^2-e^2x^2\right)^{p+1}}{x}$$

$$\Big\downarrow \text{237}$$

$$2de^2(1-p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)+\frac{1}{2}e\left(-\frac{3\left(d^2-e^2x^2\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{p+1}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{p+1}\right)-\frac{d\left(d^2-e^2x^2\right)^{p+1}}{x}$$

input
```
Int[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x^2,x]
```

3.265. $\int\frac{(d+ex)^3\left(d^2-e^2x^2\right)^p}{x^2}dx$

output
```
-((d*(d^2 - e^2*x^2)^(1 + p))/x) + (2*d*e^2*(1 - p)*x*(d^2 - e^2*x^2)^p*Hy
pergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p + (e*(
-((d^2 - e^2*x^2)^(1 + p)/(1 + p)) - (3*(d^2 - e^2*x^2)^(1 + p)*Hypergeome
tric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(1 + p)))/2
```

3.265.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.265. $\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x^2} dx$

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

3.265.4 Maple [F]

$$\int \frac{(ex+d)^3 \left(-e^2x^2+d^2\right)^p}{x^2}dx$$

input
```
int((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x)
```

output
```
int((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x)
```

3.265.5 Fricas [F]

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^2} dx = \int \frac{(ex+d)^3(-e^2x^2+d^2)^p}{x^2} dx$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x, algorithm="fricas")
```

output
```
integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(-e^2*x^2 + d^2)^p/x^2,
 x)
```

3.265.6 Sympy [A] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^2}\,dx = -\frac{d^3 d^{2p} {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x} - \frac{3d^2ee^{2p}x^{2p}e^{i\pi p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix} -p, -p \\ 1-p \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{2\Gamma(1-p)} + 3dd^{2p}e^2x\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right) + e^3\left(\begin{cases} \frac{x^2(d^2)^p}{2} & \text{for } e^2=0 \\ -\frac{\begin{cases} \frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(d^2-e^2x^2) & \text{otherwise} \end{cases}}{2e^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x+d)**3*(-e**2*x**2+d**2)**p/x**2,x)
```

output
```
-d**3*d**(2*p)*hyper((-1/2, -p), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)
/x - 3*d**2*e*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 -
 p,), d**2/(e**2*x**2))/(2*gamma(1 - p)) + 3*d*d**(2*p)*e**2*x*hyper((1/2,
 -p), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2) + e**3*Piecewise((x**2*(d*
*2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x**2)**(p + 1)/(p + 1),
Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2), True))
```

3.265.7 Maxima [F]

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^2}\,dx = \int \frac{(ex+d)^3(-e^2x^2+d^2)^p}{x^2}\,dx$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x^2, x)
```

3.265.8 Giac [F]

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^2}\,dx = \int \frac{(ex+d)^3(-e^2x^2+d^2)^p}{x^2}\,dx$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x^2, x)
```

3.265.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^2}\,dx = \int \frac{(d^2-e^2\,x^2)^p\,(d+e\,x)^3}{x^2}\,dx$$

input
```
int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x^2,x)
```

output
```
int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x^2, x)
```

3.266 $\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x^3}\,dx$

3.266.1 Optimal result

Integrand size = 25, antiderivative size = 166

$$\begin{aligned}\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x^3}\,dx &= -\frac{d(d^2-e^2x^2)^{1+p}}{2x^2}-\frac{3e(d^2-e^2x^2)^{1+p}}{x}\\ &\quad -2e^3(1+3p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)\\ &\quad -\frac{e^2(3-p)(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{2d(1+p)}\end{aligned}$$

output
```
-1/2*d*(-e^2*x^2+d^2)^(p+1)/x^2-3*e*(-e^2*x^2+d^2)^(p+1)/x-2*e^3*(1+3*p)*x
*(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],[3/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^
p)-1/2*e^2*(3-p)*(-e^2*x^2+d^2)^(p+1)*hypergeom([1, p+1],[2+p],1-e^2*x^2/d
^2)/d/(p+1)
```

3.266.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^3} dx$$
$$= \frac{e(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \left(-6d^3(1+p) \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)+ex\left(2de(1+p)x \text{ Hyper}\right.\right.}{}$$

input
```
Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x^3,x]
```

output
```
(e*(d^2 - e^2*x^2)^p*(-6*d^3*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, (e^2
*x^2)/d^2] + e*x*(2*d*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2
)/d^2] - (d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p*(3*Hypergeometric2F1[1, 1 +
 p, 2 + p, 1 - (e^2*x^2)/d^2] + Hypergeometric2F1[2, 1 + p, 2 + p, 1 - (e^
2*x^2)/d^2]))))/(2*d*(1 + p)*x*(1 - (e^2*x^2)/d^2)^p)
```

3.266.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {543, 354, 27, 87, 75, 359, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^3} dx$$
$$\Big\downarrow \text{ 543}$$
$$\int \frac{(d^2-e^2x^2)^p (x^2e^3+3d^2e)}{x^2}dx+\int \frac{(d^2-e^2x^2)^p (d^3+3e^2x^2d)}{x^3}dx$$
$$\Big\downarrow \text{ 354}$$
$$\frac{1}{2}\int \frac{d(d^2-e^2x^2)^p (d^2+3e^2x^2)}{x^4}dx^2+\int \frac{(d^2-e^2x^2)^p (x^2e^3+3d^2e)}{x^2}dx$$
$$\Big\downarrow \text{ 27}$$
$$\frac{1}{2}d\int \frac{(d^2-e^2x^2)^p (d^2+3e^2x^2)}{x^4}dx^2+\int \frac{(d^2-e^2x^2)^p (x^2e^3+3d^2e)}{x^2}dx$$

$$\Big\downarrow \; 87$$

$$\frac{1}{2}d\left(e^2(3-p)\int \frac{(d^2-e^2x^2)^p}{x^2}dx^2-\frac{(d^2-e^2x^2)^{p+1}}{x^2}\right)+\int \frac{(d^2-e^2x^2)^p\,(x^2e^3+3d^2e)}{x^2}dx$$

$$\Big\downarrow \; 75$$

$$\int \frac{(d^2-e^2x^2)^p\,(x^2e^3+3d^2e)}{x^2}dx+\frac{1}{2}d\left(-\frac{e^2(3-p)\,(d^2-e^2x^2)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{d^2(p+1)}-\frac{(d^2-e^2x^2)^{p+1}}{x^2}\right)$$

$$\Big\downarrow \; 359$$

$$-2e^3(3p+1)\int (d^2-e^2x^2)^p\,dx+\frac{1}{2}d\left(-\frac{e^2(3-p)\,(d^2-e^2x^2)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{d^2(p+1)}-\frac{(d^2-e^2x^2)^{p+1}}{x^2}\right)-\frac{3e(d^2-e^2x^2)^{p+1}}{x}$$

$$\Big\downarrow \; 238$$

$$-2e^3(3p+1)\,(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int \left(1-\frac{e^2x^2}{d^2}\right)^p dx+\frac{1}{2}d\left(-\frac{e^2(3-p)\,(d^2-e^2x^2)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{d^2(p+1)}-\frac{(d^2-e^2x^2)^{p+1}}{x^2}\right)-\frac{3e(d^2-e^2x^2)^{p+1}}{x}$$

$$\Big\downarrow \; 237$$

$$\frac{1}{2}d\left(-\frac{e^2(3-p)\,(d^2-e^2x^2)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{d^2(p+1)}-\frac{(d^2-e^2x^2)^{p+1}}{x^2}\right)-\frac{3e(d^2-e^2x^2)^{p+1}}{x}-2e^3(3p+1)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)$$

input
```
Int[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x^3,x]
```

3.266. $\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x^3}\,dx$

output
```
(-3*e*(d^2 - e^2*x^2)^(1 + p))/x - (2*e^3*(1 + 3*p)*x*(d^2 - e^2*x^2)^p*Hy
pergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p + (d*(
-((d^2 - e^2*x^2)^(1 + p)/x^2) - (e^2*(3 - p)*(d^2 - e^2*x^2)^(1 + p)*Hype
rgeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(d^2*(1 + p))))/2
```

3.266.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.266. $\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x^3} dx$

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

3.266.4 Maple [F]

$$\int \frac{(ex+d)^3 (-e^2x^2+d^2)^p}{x^3}dx$$

input
```
int((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x)
```

output
```
int((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x)
```

3.266.5 Fricas [F]

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^3} dx = \int \frac{(ex+d)^3(-e^2x^2+d^2)^p}{x^3} dx$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x, algorithm="fricas")
```

output
```
integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(-e^2*x^2 + d^2)^p/x^3,
 x)
```

3.266.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.17 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^3}\,dx = -\frac{d^3 e^{2p} x^{2p-2} e^{i\pi p}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}-p, 1-p\\ 2-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma(2-p)} - \frac{3d^2 d^{2p} e\,{}_2F_1\left(\begin{matrix}-\frac{1}{2}, -p\\ \frac{1}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x} - \frac{3de^2 e^{2p} x^{2p} e^{i\pi p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p, -p\\ 1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma(1-p)} + d^{2p} e^3 x\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, -p\\ \frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)$$

input
```
integrate((e*x+d)**3*(-e**2*x**2+d**2)**p/x**3,x)
```

output
```
-d**3*e**(2*p)*x**(2*p - 2)*exp(I*pi*p)*gamma(1 - p)*hyper((-p, 1 - p), (2
 - p,), d**2/(e**2*x**2))/(2*gamma(2 - p)) - 3*d**2*d**(2*p)*e*hyper((-1/2
, -p), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/x - 3*d*e**2*e**(2*p)*x**
(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**2/(e**2*x**2))/(2
*gamma(1 - p)) + d**(2*p)*e**3*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_po
lar(2*I*pi)/d**2)
```

3.266.7 Maxima [F]

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^3}\,dx = \int \frac{(ex+d)^3(-e^2x^2+d^2)^p}{x^3}\,dx$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x^3, x)
```

3.266.8 Giac [F]

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^3}\,dx = \int \frac{(ex+d)^3(-e^2x^2+d^2)^p}{x^3}\,dx$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x^3, x)
```

3.266.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^3}\,dx = \int \frac{(d^2-e^2\,x^2)^p\,(d+e\,x)^3}{x^3}\,dx$$

input
```
int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x^3,x)
```

output
```
int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x^3, x)
```

3.267 $\int \frac{x^4(d^2-e^2x^2)^p}{d+ex}\,dx$

3.267.1 Optimal result

Integrand size = 25, antiderivative size = 148

$$\int \frac{x^4(d^2-e^2x^2)^p}{d+ex}\,dx = \frac{d^4(d^2-e^2x^2)^p}{2e^5p} - \frac{d^2(d^2-e^2x^2)^{1+p}}{e^5(1+p)} + \frac{(d^2-e^2x^2)^{2+p}}{2e^5(2+p)} + \frac{x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},1-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d}$$

output
```
1/2*d^4*(-e^2*x^2+d^2)^p/e^5/p-d^2*(-e^2*x^2+d^2)^(p+1)/e^5/(p+1)+1/2*(-e^
2*x^2+d^2)^(2+p)/e^5/(2+p)+1/5*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, 1-p],[
7/2],e^2*x^2/d^2)/d/((1-e^2*x^2/d^2)^p)
```

3.267.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.45

$$\int \frac{x^4(d^2-e^2x^2)^p}{d+ex}\,dx = \frac{x^5(d-ex)^p(d+ex)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{AppellF1}\left(5,-p,1-p,6,\frac{ex}{d},-\frac{ex}{d}\right)}{5d}$$

input
```
Integrate[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

output
```
(x^5*(d - e*x)^p*(d + e*x)^p*AppellF1[5, -p, 1 - p, 6, (e*x)/d, -((e*x)/d)
])/(5*d*(1 - (e^2*x^2)/d^2)^p)
```

3.267.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {583, 542, 243, 53, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d^2-e^2x^2)^p}{d+ex}dx$$

$$\Big\downarrow \text{583}$$

$$\int x^4(d-ex)\left(d^2-e^2x^2\right)^{p-1}dx$$

$$\Big\downarrow \text{542}$$

$$d\int x^4\left(d^2-e^2x^2\right)^{p-1}dx-e\int x^5\left(d^2-e^2x^2\right)^{p-1}dx$$

$$\Big\downarrow \text{243}$$

$$d\int x^4\left(d^2-e^2x^2\right)^{p-1}dx-\frac{1}{2}e\int x^4\left(d^2-e^2x^2\right)^{p-1}dx^2$$

$$\Big\downarrow \text{53}$$

$$d\int x^4\left(d^2-e^2x^2\right)^{p-1}dx-\frac{1}{2}e\int\left(\frac{d^4\left(d^2-e^2x^2\right)^{p-1}}{e^4}-\frac{2d^2\left(d^2-e^2x^2\right)^p}{e^4}+\frac{\left(d^2-e^2x^2\right)^{p+1}}{e^4}\right)dx^2$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^{p-1}dx}{d}-\frac{1}{2}e\int\left(\frac{d^4\left(d^2-e^2x^2\right)^{p-1}}{e^4}-\frac{2d^2\left(d^2-e^2x^2\right)^p}{e^4}+\frac{\left(d^2-e^2x^2\right)^{p+1}}{e^4}\right)dx^2$$

$$\Big\downarrow \text{278}$$

$$\frac{x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},1-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d}-\frac{1}{2}e\int\left(\frac{d^4\left(d^2-e^2x^2\right)^{p-1}}{e^4}-\frac{2d^2\left(d^2-e^2x^2\right)^p}{e^4}+\frac{\left(d^2-e^2x^2\right)^{p+1}}{e^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},1-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d}-\frac{1}{2}e\left(\frac{2d^2\left(d^2-e^2x^2\right)^{p+1}}{e^6(p+1)}-\frac{\left(d^2-e^2x^2\right)^{p+2}}{e^6(p+2)}-\frac{d^4\left(d^2-e^2x^2\right)^p}{e^6p}\right)$$

input
```
Int[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

output
```
-1/2*(e*(-((d^4*(d^2 - e^2*x^2)^p)/(e^6*p)) + (2*d^2*(d^2 - e^2*x^2)^(1 +
p))/(e^6*(1 + p)) - (d^2 - e^2*x^2)^(2 + p)/(e^6*(2 + p)))) + (x^5*(d^2 -
e^2*x^2)^p*Hypergeometric2F1[5/2, 1 - p, 7/2, (e^2*x^2)/d^2])/(5*d*(1 - (e
^2*x^2)/d^2)^p)
```

3.267.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

rule 583
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.267.4 Maple [F]

$$\int \frac{x^4(-e^2x^2+d^2)^p}{ex+d}dx$$

input
```
int(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

output
```
int(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

3.267.5 Fricas [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^p x^4}{ex+d}\,dx$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*x^4/(e*x + d), x)
```

3.267.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d^2-e^2x^2)^p}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d),x)
```

output
```
Timed out
```

3.267.7 Maxima [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^p x^4}{ex+d}\,dx$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d), x)
```

3.267.8 Giac [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^p x^4}{ex+d}\,dx$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d), x)
```

3.267.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d^2 - e^2x^2)^p}{d + ex}\, dx = \int \frac{x^4\,(d^2 - e^2\,x^2)^p}{d + e\,x}\, dx$$

input
```
int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x),x)
```

output
```
int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x), x)
```

3.268 $\int \frac{x^3(d^2-e^2x^2)^p}{d+ex}\,dx$

3.268.1 Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \frac{x^3(d^2-e^2x^2)^p}{d+ex}\,dx = -\frac{d^3(d^2-e^2x^2)^p}{2e^4p}+\frac{d(d^2-e^2x^2)^{1+p}}{2e^4(1+p)} - \frac{ex^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},1-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^2}$$

output
```
-1/2*d^3*(-e^2*x^2+d^2)^p/e^4/p+1/2*d*(-e^2*x^2+d^2)^(p+1)/e^4/(p+1)-1/5*e
*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, 1-p],[7/2],e^2*x^2/d^2)/d^2/((1-e^2*
x^2/d^2)^p)
```

3.268.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 245 vs. 2(121) = 242.

Time = 0.41 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.02

$$\int \frac{x^3(d^2-e^2x^2)^p}{d+ex}\,dx = \frac{\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(6d^2e(1+p)x\left(1+\frac{ex}{d}\right)^p\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)+2e^3(\right.}{}$$

input
```
Integrate[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

output
```
((d^2 - e^2*x^2)^p*(6*d^2*e*(1 + p)*x*(1 + (e*x)/d)^p*Hypergeometric2F1[1/
2, -p, 3/2, (e^2*x^2)/d^2] + 2*e^3*(1 + p)*x^3*(1 + (e*x)/d)^p*Hypergeomet
ric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2] + 3*d*((1 + (e*x)/d)^p*(-(e^2*x^2*(1 -
 (e^2*x^2)/d^2)^p) + d^2*(-1 + (1 - (e^2*x^2)/d^2)^p)) + d*(d - e*x)*(2 -
(2*e^2*x^2)/d^2)^p*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]
)))/(6*e^4*(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

3.268.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {583, 542, 243, 53, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (d^2-e^2x^2)^p}{d+ex} \, dx$$

$$\Big\downarrow \text{ 583}$$

$$\int x^3 (d-ex) \left(d^2-e^2x^2\right)^{p-1} dx$$

$$\Big\downarrow \text{ 542}$$

$$d \int x^3 \left(d^2-e^2x^2\right)^{p-1} dx - e \int x^4 \left(d^2-e^2x^2\right)^{p-1} dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} d \int x^2 \left(d^2-e^2x^2\right)^{p-1} dx^2 - e \int x^4 \left(d^2-e^2x^2\right)^{p-1} dx$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2} d \int \left(\frac{d^2 \left(d^2-e^2x^2\right)^{p-1}}{e^2} - \frac{\left(d^2-e^2x^2\right)^p}{e^2}\right) dx^2 - e \int x^4 \left(d^2-e^2x^2\right)^{p-1} dx$$

$$\Big\downarrow \text{ 279}$$

$$\frac{1}{2}d\int \left(\frac{d^2(d^2-e^2x^2)^{p-1}}{e^2}-\frac{(d^2-e^2x^2)^p}{e^2}\right)dx^2-\frac{e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^{p-1}dx}{d^2}$$

$$\Big\downarrow \text{278}$$

$$\frac{1}{2}d\int \left(\frac{d^2(d^2-e^2x^2)^{p-1}}{e^2}-\frac{(d^2-e^2x^2)^p}{e^2}\right)dx^2-\frac{ex^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},1-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}d\left(\frac{(d^2-e^2x^2)^{p+1}}{e^4(p+1)}-\frac{d^2(d^2-e^2x^2)^p}{e^4p}\right)-\frac{ex^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},1-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^2}$$

input
```
Int[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

output
```
(d*(-((d^2*(d^2 - e^2*x^2)^p)/(e^4*p)) + (d^2 - e^2*x^2)^(1 + p)/(e^4*(1 +
 p))))/2 - (e*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 1 - p, 7/2, (e^
2*x^2)/d^2])/(5*d^2*(1 - (e^2*x^2)/d^2)^p)
```

3.268.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.268. $\int \frac{x^3(d^2-e^2x^2)^p}{d+ex}\,dx$

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

rule 583
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.268.4 Maple [F]

$$\int \frac{x^3(-e^2x^2+d^2)^p}{ex+d}dx$$

input
```
int(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

output
```
int(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

3.268.5 Fricas [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{d+ex}\,dx=\int \frac{(-e^2x^2+d^2)^p x^3}{ex+d}\,dx$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*x^3/(e*x + d), x)
```

3.268.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 62.78 (sec) , antiderivative size = 17065, normalized size of antiderivative = 141.03

$$\int \frac{x^3(d^2-e^2x^2)^p}{d+ex}\,dx=\text{Too large to display}$$

input
```
integrate(x**3*(-e**2*x**2+d**2)**p/(e*x+d),x)
```

output
```
Piecewise((-3*0**p*d**5*d**(2*p + 3)*p*log(d**2/(e**2*x**2))*gamma(-p - 1/
2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4
*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p
 + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 3*0**p*d**5*d**(2
*p + 3)*p*log(d**2/(e**2*x**2) - 1)*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*
e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p
+ 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*
gamma(-p - 1/2)*gamma(p + 1)) + 6*0**p*d**5*d**(2*p + 3)*p*acoth(d/(e*x))*
gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1)
- 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p -
 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 3*0*
*p*d**5*d**(2*p + 3)*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*gamma(p + 1)/(-
6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*g
amma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**
6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 3*0**p*d**5*d**(2*p + 3)*log(d**2/(
e**2*x**2) - 1)*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/
2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p
*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamm
a(p + 1)) + 6*0**p*d**5*d**(2*p + 3)*acoth(d/(e*x))*gamma(-p - 1/2)*gamma(
p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma...
```

3.268.7 Maxima [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^p x^3}{ex+d}\,dx$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d), x)
```

3.268.8 Giac [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^p x^3}{ex+d}\,dx$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d), x)
```

3.268.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{x^3\,(d^2-e^2\,x^2)^p}{d+e\,x}\,dx$$

input
```
int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x),x)
```

output
```
int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x), x)
```

3.269 $\int \frac{x^2(d^2-e^2x^2)^p}{d+ex}\,dx$

3.269.1 Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{x^2(d^2-e^2x^2)^p}{d+ex}\,dx=\frac{d^2(d^2-e^2x^2)^p}{2e^3p}-\frac{(d^2-e^2x^2)^{1+p}}{2e^3(1+p)}+\frac{x^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},1-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3d}$$

output
```
1/2*d^2*(-e^2*x^2+d^2)^p/e^3/p-1/2*(-e^2*x^2+d^2)^(p+1)/e^3/(p+1)+1/3*x^3*
(-e^2*x^2+d^2)^p*hypergeom([3/2, 1-p],[5/2],e^2*x^2/d^2)/d/((1-e^2*x^2/d^2
)^p)
```

3.269.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.66

$$\int \frac{x^2(d^2-e^2x^2)^p}{d+ex}\,dx=-\frac{\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(\left(1+\frac{ex}{d}\right)^p\left(-e^2x^2\left(1-\frac{e^2x^2}{d^2}\right)^p+d^2\left(-1+\left(1-\frac{e^2x^2}{d^2}\right)^p\right)\right)+2de(\cdots}{\cdots}$$

input
```
Integrate[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

output
```
-1/2*((d^2 - e^2*x^2)^p*((1 + (e*x)/d)^p*(-(e^2*x^2*(1 - (e^2*x^2)/d^2)^p)
 + d^2*(-1 + (1 - (e^2*x^2)/d^2)^p)) + 2*d*e*(1 + p)*x*(1 + (e*x)/d)^p*Hyp
ergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + d*(d - e*x)*(2 - (2*e^2*x^2)
/d^2)^p*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(e^3*(1
+ p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

3.269.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {583, 542, 243, 53, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d^2-e^2x^2)^p}{d+ex}\,dx$$

$$\Big\downarrow \text{ 583}$$

$$\int x^2(d-ex)\left(d^2-e^2x^2\right)^{p-1}dx$$

$$\Big\downarrow \text{ 542}$$

$$d\int x^2\left(d^2-e^2x^2\right)^{p-1}dx-e\int x^3\left(d^2-e^2x^2\right)^{p-1}dx$$

$$\Big\downarrow \text{ 243}$$

$$d\int x^2\left(d^2-e^2x^2\right)^{p-1}dx-\frac{1}{2}e\int x^2\left(d^2-e^2x^2\right)^{p-1}dx^2$$

$$\Big\downarrow \text{ 53}$$

$$d\int x^2\left(d^2-e^2x^2\right)^{p-1}dx-\frac{1}{2}e\int\left(\frac{d^2\left(d^2-e^2x^2\right)^{p-1}}{e^2}-\frac{\left(d^2-e^2x^2\right)^p}{e^2}\right)dx^2$$

$$\Big\downarrow \text{ 279}$$

$$\frac{\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^{p-1}dx}{d}-\frac{1}{2}e\int\left(\frac{d^2\left(d^2-e^2x^2\right)^{p-1}}{e^2}-\frac{\left(d^2-e^2x^2\right)^p}{e^2}\right)dx^2$$

$$\Big\downarrow \text{ 278}$$

$$\frac{x^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},1-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3d}-\frac{1}{2}e\int\left(\frac{d^2(d^2-e^2x^2)^{p-1}}{e^2}-\frac{(d^2-e^2x^2)^p}{e^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},1-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3d}-\frac{1}{2}e\left(\frac{(d^2-e^2x^2)^{p+1}}{e^4(p+1)}-\frac{d^2(d^2-e^2x^2)^p}{e^4p}\right)$$

input
```
Int[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

output
```
-1/2*(e*(-((d^2*(d^2 - e^2*x^2)^p)/(e^4*p)) + (d^2 - e^2*x^2)^(1 + p)/(e^4
*(1 + p)))) + (x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, 1 - p, 5/2, (e
^2*x^2)/d^2])/(3*d*(1 - (e^2*x^2)/d^2)^p)
```

3.269.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

3.269. $\int \frac{x^2(d^2-e^2x^2)^p}{d+ex}\,dx$

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

rule 583
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.269.4 Maple [F]

$$\int \frac{x^2(-e^2x^2+d^2)^p}{ex+d}dx$$

input
```
int(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

output
```
int(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

3.269.5 Fricas [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^p x^2}{ex+d}\,dx$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*x^2/(e*x + d), x)
```

3.269.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.29 (sec) , antiderivative size = 14895, normalized size of antiderivative = 125.17

$$\int \frac{x^2(d^2-e^2x^2)^p}{d+ex}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d),x)
```

output
```
Piecewise((0**p*d**4*d**(2*p + 2)*p*log(d**2/(e**2*x**2))*gamma(1/2 - p)*g
amma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamm
a(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) +
 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - 0**p*d**4*d**(2*p + 2)*p*
log(d**2/(e**2*x**2) - 1)*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamm
a(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2
*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)
*gamma(p + 1)) - 2*0**p*d**4*d**(2*p + 2)*p*acoth(d/(e*x))*gamma(1/2 - p)*
gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gam
ma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1)
+ 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + 0**p*d**4*d**(2*p + 2)*l
og(d**2/(e**2*x**2))*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2
 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5
*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamm
a(p + 1)) - 0**p*d**4*d**(2*p + 2)*log(d**2/(e**2*x**2) - 1)*gamma(1/2 - p
)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*g
amma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1
) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - 2*0**p*d**4*d**(2*p +
2)*acoth(d/(e*x))*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 -
p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5...
```

3.269.7 Maxima [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^p x^2}{ex+d}\,dx$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d), x)
```

3.269.8 Giac [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^p x^2}{ex+d}\,dx$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d), x)
```

3.269.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{x^2\,(d^2-e^2\,x^2)^p}{d+e\,x}\,dx$$

input
```
int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x),x)
```

output
```
int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x), x)
```

3.270 $\int \frac{x(d^2-e^2x^2)^p}{d+ex}\,dx$

3.270.1 Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \frac{x(d^2-e^2x^2)^p}{d+ex}\,dx = -\frac{d(d^2-e^2x^2)^p}{2e^2p} - \frac{ex^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},1-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3d^2}$$

output
```
-1/2*d*(-e^2*x^2+d^2)^p/e^2/p-1/3*e*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/2, 1
-p],[5/2],e^2*x^2/d^2)/d^2/((1-e^2*x^2/d^2)^p)
```

3.270.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.63

$$\int \frac{x(d^2-e^2x^2)^p}{d+ex}\,dx = \frac{2^{-1+p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(2e(1+p)x\left(\frac{1}{2}+\frac{ex}{2d}\right)^p\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)+\left(\right.\right.}{e^2(1+p)}$$

input
```
Integrate[(x*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

output
```
(2^(-1 + p)*(d^2 - e^2*x^2)^p*(2*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hyperge
ometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^2)^p*
Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(e^2*(1 + p)*(1
+ (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

3.270.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {565, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d^2-e^2x^2)^p}{d+ex}\,dx$$

$$\Big\downarrow \text{565}$$

$$-e\int x^2(d^2-e^2x^2)^{p-1}\,dx-\frac{d(d^2-e^2x^2)^p}{2e^2p}$$

$$\Big\downarrow \text{279}$$

$$-\frac{e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^{p-1}dx}{d^2}-\frac{d(d^2-e^2x^2)^p}{2e^2p}$$

$$\Big\downarrow \text{278}$$

$$-\frac{ex^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},1-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3d^2}-\frac{d(d^2-e^2x^2)^p}{2e^2p}$$

input
```
Int[(x*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

output
```
-1/2*(d*(d^2 - e^2*x^2)^p)/(e^2*p) - (e*x^3*(d^2 - e^2*x^2)^p*Hypergeometr
ic2F1[3/2, 1 - p, 5/2, (e^2*x^2)/d^2])/(3*d^2*(1 - (e^2*x^2)/d^2)^p)
```

3.270.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 565
```
Int[((x_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :> Sim
p[a*((a + b*x^2)^p/(2*b*c*p)), x] + Simp[b/d   Int[x^2*(a + b*x^2)^(p - 1),
 x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0]
```

3.270.4 Maple [F]

$$\int \frac{x(-e^2x^2+d^2)^p}{ex+d}dx$$

input
```
int(x*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

output
```
int(x*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

3.270.5 Fricas [F]

$$\int \frac{x(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^p x}{ex+d}\,dx$$

input
```
integrate(x*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*x/(e*x + d), x)
```

3.270.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.64 (sec) , antiderivative size = 440, normalized size of antiderivative = 4.89

$$\int \frac{x(d^2-e^2x^2)^p}{d+ex}\,dx$$

$$= \begin{cases} \frac{0^p d^{2p+1}\log\left(\frac{d^2}{e^2x^2}\right)}{2e^2} - \frac{0^p d^{2p+1}\log\left(\frac{d^2}{e^2x^2}-1\right)}{2e^2} - \frac{0^p d^{2p+1}\operatorname{acoth}\left(\frac{d}{ex}\right)}{e^2} + \frac{0^p d^{2p+1}x}{de} - \frac{e^{2p-1}px^{2p+1}e^{i\pi p}\Gamma(p)\Gamma\left(-p-\frac{1}{2}\right){}_2F_1\left(\begin{matrix}1-p,-\\ \frac{1}{2}-\end{matrix}\right.}{2\Gamma\left(\frac{1}{2}-p\right)\Gamma(p+1)} \\ \frac{0^p d^{2p+1}\log\left(\frac{d^2}{e^2x^2}\right)}{2e^2} - \frac{0^p d^{2p+1}\log\left(-\frac{d^2}{e^2x^2}+1\right)}{2e^2} - \frac{0^p d^{2p+1}\operatorname{atanh}\left(\frac{d}{ex}\right)}{e^2} + \frac{0^p d^{2p+1}x}{de} - \frac{e^{2p-1}px^{2p+1}e^{i\pi p}\Gamma(p)\Gamma\left(-p-\frac{1}{2}\right){}_2F_1\left(\begin{matrix}1-p,\\ \frac{1}{2}\end{matrix}\right.}{2\Gamma\left(\frac{1}{2}-p\right)\Gamma(p+1)} \end{cases}$$

input
```
integrate(x*(-e**2*x**2+d**2)**p/(e*x+d),x)
```

output
```
Piecewise((0**p*d**(2*p + 1)*log(d**2/(e**2*x**2))/(2*e**2) - 0**p*d**(2*p
 + 1)*log(d**2/(e**2*x**2) - 1)/(2*e**2) - 0**p*d**(2*p + 1)*acoth(d/(e*x)
)/e**2 + 0**p*d**(2*p + 1)*x/(d*e) - e**(2*p - 1)*p*x**(2*p + 1)*exp(I*pi*
p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**
2*x**2))/(2*gamma(1/2 - p)*gamma(p + 1)) - d**(2*p + 1)*x**2*gamma(p)*gamm
a(1 - p)*hyper((2, 1, 1 - p), (2, 2), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2
*d**2*gamma(-p)*gamma(p + 1)), Abs(d**2/(e**2*x**2)) > 1), (0**p*d**(2*p +
 1)*log(d**2/(e**2*x**2))/(2*e**2) - 0**p*d**(2*p + 1)*log(-d**2/(e**2*x**
2) + 1)/(2*e**2) - 0**p*d**(2*p + 1)*atanh(d/(e*x))/e**2 + 0**p*d**(2*p +
1)*x/(d*e) - e**(2*p - 1)*p*x**(2*p + 1)*exp(I*pi*p)*gamma(p)*gamma(-p - 1
/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(2*gamma(1/2 -
p)*gamma(p + 1)) - d**(2*p + 1)*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1
- p), (2, 2), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*d**2*gamma(-p)*gamma(p
+ 1)), True))
```

3.270.7 Maxima [F]

$$\int \frac{x(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^p x}{ex+d}\,dx$$

input
```
integrate(x*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x/(e*x + d), x)
```

3.270.8 Giac [F]

$$\int \frac{x(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^p x}{ex+d}\,dx$$

input
```
integrate(x*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x/(e*x + d), x)
```

3.270.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{x\,(d^2-e^2\,x^2)^p}{d+e\,x}\,dx$$

input
```
int((x*(d^2 - e^2*x^2)^p)/(d + e*x),x)
```

output
```
int((x*(d^2 - e^2*x^2)^p)/(d + e*x), x)
```

3.271 $\int \frac{(d^2-e^2x^2)^p}{d+ex} \, dx$

3.271.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{(d^2-e^2x^2)^p}{d+ex} \, dx = -\frac{2^{-1+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(1-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^2e(1+p)}$$

output
```
-2^(-1+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(p+1)*hypergeom([1-p, p+1],[2+p]
,1/2*(-e*x+d)/d)/d^2/e/(p+1)
```

3.271.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{(d^2-e^2x^2)^p}{d+ex} \, dx = -\frac{2^{-1+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(1-p,1+p,2+p,\frac{d-ex}{2d}\right)}{de(1+p)}$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(d + e*x),x]
```

output
```
-((2^(-1 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1 - p, 1 + p,
2 + p, (d - e*x)/(2*d)])/(d*e*(1 + p)*(1 + (e*x)/d)^p))
```

3.271.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^p}{d+ex} \, dx$$

$$\Big\downarrow \text{473}$$

$$\frac{(d-ex)^{-p-1} \left(\frac{ex}{d}+1\right)^{-p-1} (d^2-e^2x^2)^{p+1} \int (d-ex)^p \left(\frac{ex}{d}+1\right)^{p-1} dx}{d^2}$$

$$\Big\downarrow \text{79}$$

$$-\frac{2^{p-1} \left(\frac{ex}{d}+1\right)^{-p-1} (d^2-e^2x^2)^{p+1} \text{Hypergeometric2F1}\left(1-p,p+1,p+2,\frac{d-ex}{2d}\right)}{d^2e(p+1)}$$

input
```
Int[(d^2 - e^2*x^2)^p/(d + e*x),x]
```

output
```
-((2^(-1 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometri
c2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^2*e*(1 + p)))
```

3.271.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

3.271.4 Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{ex+d}dx$$

input
```
int((-e^2*x^2+d^2)^p/(e*x+d),x)
```

output
```
int((-e^2*x^2+d^2)^p/(e*x+d),x)
```

3.271.5 Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^p}{ex+d}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e*x + d), x)
```

3.271.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.23 (sec) , antiderivative size = 318, normalized size of antiderivative = 4.36

$$\int \frac{(d^2-e^2x^2)^p}{d+ex}\,dx = \begin{cases} \frac{0^p\log\left(-1+\frac{e^2x^2}{d^2}\right)}{2e} + \frac{0^p\operatorname{acoth}\left(\frac{ex}{d}\right)}{e} + \frac{de^{2p-2}px^{2p-1}e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right){}_2F_1\left(\begin{matrix}1-p,\frac{1}{2}-p\\ \frac{3}{2}-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)} + \frac{d^{2p}ex^2\Gamma(p)\Gamma(1-p){}_3F_2\left(\begin{matrix}2,1,1\\ 2,2\end{matrix}\right.}{2d^2\Gamma(-p)\Gamma(p+1)} \\ \frac{0^p\log\left(1-\frac{e^2x^2}{d^2}\right)}{2e} + \frac{0^p\operatorname{atanh}\left(\frac{ex}{d}\right)}{e} + \frac{de^{2p-2}px^{2p-1}e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right){}_2F_1\left(\begin{matrix}1-p,\frac{1}{2}-p\\ \frac{3}{2}-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)} + \frac{d^{2p}ex^2\Gamma(p)\Gamma(1-p){}_3F_2\left(\begin{matrix}2,1,1-\\ 2,2\end{matrix}\right.}{2d^2\Gamma(-p)\Gamma(p+1)} \end{cases}$$

input
```
integrate((-e**2*x**2+d**2)**p/(e*x+d),x)
```

output
```
Piecewise((0**p*log(-1 + e**2*x**2/d**2)/(2*e) + 0**p*acoth(e*x/d)/e + d*e
**(2*p - 2)*p*x**(2*p - 1)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 -
p, 1/2 - p), (3/2 - p,), d**2/(e**2*x**2))/(2*gamma(3/2 - p)*gamma(p + 1))
 + d**(2*p)*e*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), e**2
*x**2*exp_polar(2*I*pi)/d**2)/(2*d**2*gamma(-p)*gamma(p + 1)), Abs(e**2*x*
*2/d**2) > 1), (0**p*log(1 - e**2*x**2/d**2)/(2*e) + 0**p*atanh(e*x/d)/e +
 d*e**(2*p - 2)*p*x**(2*p - 1)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((
1 - p, 1/2 - p), (3/2 - p,), d**2/(e**2*x**2))/(2*gamma(3/2 - p)*gamma(p +
 1)) + d**(2*p)*e*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2),
e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*d**2*gamma(-p)*gamma(p + 1)), True))
```

3.271.7 Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^p}{d + ex} \, dx = \int \frac{(-e^2x^2 + d^2)^p}{ex + d} \, dx$$

input
```
integrate((-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/(e*x + d), x)
```

3.271.8 Giac [F]

$$\int \frac{(d^2 - e^2x^2)^p}{d + ex} \, dx = \int \frac{(-e^2x^2 + d^2)^p}{ex + d} \, dx$$

input
```
integrate((-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/(e*x + d), x)
```

3.271.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{d+ex}\,dx=\int \frac{{\left(d^2-e^2\,x^2\right)}^p}{d+e\,x}\,dx$$

input
```
int((d^2 - e^2*x^2)^p/(d + e*x),x)
```

output
```
int((d^2 - e^2*x^2)^p/(d + e*x), x)
```

3.272 $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)}\,dx$

3.272.1 Optimal result

Integrand size = 25, antiderivative size = 104

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)}\,dx = -\frac{ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},1-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^2} - \frac{(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(1,p,1+p,1-\frac{e^2x^2}{d^2}\right)}{2dp}$$

output
```
-e*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 1-p],[3/2],e^2*x^2/d^2)/d^2/((1-e^2*
x^2/d^2)^p)-1/2*(-e^2*x^2+d^2)^p*hypergeom([1, p],[p+1],1-e^2*x^2/d^2)/d/p
```

3.272.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.45

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)}\,dx = \frac{2^{-1+p}\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(p\left(1-\frac{d^2}{e^2x^2}\right)^p(d-ex)\,\text{Hypergeometric2F1}\left(1-p,1+p,2+\right.\right.}{d^2p(1+p)}$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(x*(d + e*x)),x]
```

output
```
(2^(-1 + p)*(d^2 - e^2*x^2)^p*(p*(1 - d^2/(e^2*x^2))^p*(d - e*x)*Hypergeom
etric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + d*(1 + p)*(1/2 + (e*x)/(2
*d))^p*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)]))/(d^2*p*(1 + p)*(1
 - d^2/(e^2*x^2))^p*(1 + (e*x)/d)^p)
```

3.272.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {583, 542, 238, 237, 243, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)}dx$$

$$\Big\downarrow \; 583$$

$$\int \frac{(d-ex)\left(d^2-e^2x^2\right)^{p-1}}{x}dx$$

$$\Big\downarrow \; 542$$

$$d\int \frac{(d^2-e^2x^2)^{p-1}}{x}dx-e\int (d^2-e^2x^2)^{p-1}dx$$

$$\Big\downarrow \; 238$$

$$d\int \frac{(d^2-e^2x^2)^{p-1}}{x}dx-\frac{e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int \left(1-\frac{e^2x^2}{d^2}\right)^{p-1}dx}{d^2}$$

$$\Big\downarrow \; 237$$

$$d\int \frac{(d^2-e^2x^2)^{p-1}}{x}dx-\frac{ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},1-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^2}$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}d\int \frac{(d^2-e^2x^2)^{p-1}}{x^2}dx^2-\frac{ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},1-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^2}$$

$$\Big\downarrow \; 75$$

$$-\frac{ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},1-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^2}-\frac{(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(1,p,p+1,1-\frac{e^2x^2}{d^2}\right)}{2dp}$$

input
```
Int[(d^2 - e^2*x^2)^p/(x*(d + e*x)),x]
```

output
```
-((e*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 1 - p, 3/2, (e^2*x^2)/d^2]
)/(d^2*(1 - (e^2*x^2)/d^2)^p)) - ((d^2 - e^2*x^2)^p*Hypergeometric2F1[1, p
, 1 + p, 1 - (e^2*x^2)/d^2])/(2*d*p)
```

3.272.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

rule 583
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, 0]
```

3.272.4 Maple [F]

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{x \left(e x +d \right)}d x$$

input
```
int((-e^2*x^2+d^2)^p/x/(e*x+d),x)
```

output
```
int((-e^2*x^2+d^2)^p/x/(e*x+d),x)
```

3.272.5 Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)}\,dx=\int \frac{{\left(-e^{2} x^{2} + d^{2}\right)}^{p}}{{\left(e x + d\right)} x}\,{d x}$$

input
```
integrate((-e^2*x^2+d^2)^p/x/(e*x+d),x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e*x^2 + d*x), x)
```

3.272.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.39 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.35

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)}\,dx$$
$$=\begin{cases} -\frac{0^{p}d^{2p-1}\log\left(\frac{d^{2}}{e^{2}x^{2}}-1\right)}{2}-0^{p}d^{2p-1}\operatorname{acoth}\left(\frac{d}{ex}\right)+\frac{de^{2p-2}px^{2p-2}e^{i\pi p}\Gamma(p)\Gamma(1-p){}_2F_1\left(\begin{matrix}1-p,1-p\\2-p\end{matrix}\middle|\frac{d^{2}}{e^{2}x^{2}}\right)}{2\Gamma(2-p)\Gamma(p+1)}-\frac{e^{2p-1}px^{2p-1}e^{i\pi p}\cdots}{\cdots} \\ -\frac{0^{p}d^{2p-1}\log\left(-\frac{d^{2}}{e^{2}x^{2}}+1\right)}{2}-0^{p}d^{2p-1}\operatorname{atanh}\left(\frac{d}{ex}\right)+\frac{de^{2p-2}px^{2p-2}e^{i\pi p}\Gamma(p)\Gamma(1-p){}_2F_1\left(\begin{matrix}1-p,1-p\\2-p\end{matrix}\middle|\frac{d^{2}}{e^{2}x^{2}}\right)}{2\Gamma(2-p)\Gamma(p+1)}-\frac{e^{2p-1}px^{2p-1}e\cdots}{\cdots}\end{cases}$$

input
```
integrate((-e**2*x**2+d**2)**p/x/(e*x+d),x)
```

output
```
Piecewise((-0**p*d**(2*p - 1)*log(d**2/(e**2*x**2) - 1)/2 - 0**p*d**(2*p -
 1)*acoth(d/(e*x)) + d*e**(2*p - 2)*p*x**(2*p - 2)*exp(I*pi*p)*gamma(p)*ga
mma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), d**2/(e**2*x**2))/(2*gamma(2 -
p)*gamma(p + 1)) - e**(2*p - 1)*p*x**(2*p - 1)*exp(I*pi*p)*gamma(p)*gamma(
1/2 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), d**2/(e**2*x**2))/(2*gamma(3/
2 - p)*gamma(p + 1)), Abs(d**2/(e**2*x**2)) > 1), (-0**p*d**(2*p - 1)*log(
-d**2/(e**2*x**2) + 1)/2 - 0**p*d**(2*p - 1)*atanh(d/(e*x)) + d*e**(2*p -
2)*p*x**(2*p - 2)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*hyper((1 - p, 1 - p),
(2 - p,), d**2/(e**2*x**2))/(2*gamma(2 - p)*gamma(p + 1)) - e**(2*p - 1)*p
*x**(2*p - 1)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p),
(3/2 - p,), d**2/(e**2*x**2))/(2*gamma(3/2 - p)*gamma(p + 1)), True))
```

3.272.7 Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x(d + ex)} dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)x} dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x), x)
```

3.272.8 Giac [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x(d + ex)} dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)x} dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x/(e*x+d),x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x), x)
```

3.272.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^p}{x(d+ex)}\,dx = \int \frac{(d^2 - e^2\,x^2)^p}{x\,(d+e\,x)}\,dx$$

input
```
int((d^2 - e^2*x^2)^p/(x*(d + e*x)),x)
```

output
```
int((d^2 - e^2*x^2)^p/(x*(d + e*x)), x)
```

3.273 $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)}\,dx$

3.273.1 Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)}\,dx = -\frac{(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},1-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{dx} + \frac{e(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(1,p,1+p,1-\frac{e^2x^2}{d^2}\right)}{2d^2p}$$

output
```
-(-e^2*x^2+d^2)^p*hypergeom([-1/2, 1-p],[1/2],e^2*x^2/d^2)/d/x/((1-e^2*x^2
/d^2)^p)+1/2*e*(-e^2*x^2+d^2)^p*hypergeom([1, p],[p+1],1-e^2*x^2/d^2)/d^2/
p
```

3.273.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.58

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)}\,dx = \frac{(d^2-e^2x^2)^p\left(-\frac{2d^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}+\frac{2^pe(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,1+p,2+p,\frac{d}{}\right.}{1+p}\right.}{2d^3}$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)),x]
```

output
```
((d^2 - e^2*x^2)^p*((-2*d^2*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2
])/(x*(1 - (e^2*x^2)/d^2)^p) + (2^p*e*(-d + e*x)*Hypergeometric2F1[1 - p,
1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) - (d*e*Hypergeom
etric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(2*d^3
)
```

3.273.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {583, 542, 243, 75, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)}dx$$

$$\Big\downarrow \ 583$$

$$\int \frac{(d-ex)\left(d^2-e^2x^2\right)^{p-1}}{x^2}dx$$

$$\Big\downarrow \ 542$$

$$d\int \frac{(d^2-e^2x^2)^{p-1}}{x^2}dx-e\int \frac{(d^2-e^2x^2)^{p-1}}{x}dx$$

$$\Big\downarrow \ 243$$

$$d\int \frac{(d^2-e^2x^2)^{p-1}}{x^2}dx-\frac{1}{2}e\int \frac{(d^2-e^2x^2)^{p-1}}{x^2}dx^2$$

$$\Big\downarrow \ 75$$

$$d\int \frac{(d^2-e^2x^2)^{p-1}}{x^2}dx+\frac{e\left(d^2-e^2x^2\right)^p \text{Hypergeometric2F1}\left(1,p,p+1,1-\frac{e^2x^2}{d^2}\right)}{2d^2p}$$

$$\Big\downarrow \ 279$$

$$\frac{(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int\frac{\left(1-\frac{e^2x^2}{d^2}\right)^{p-1}}{x^2}dx}{d}+\frac{e(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(1,p,p+1,1-\frac{e^2x^2}{d^2}\right)}{2d^2p}$$

$$\Big\downarrow\ 278$$

$$\frac{e(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(1,p,p+1,1-\frac{e^2x^2}{d^2}\right)}{2d^2p}-\frac{(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},1-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{dx}$$

input
```
Int[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)),x]
```

output
```
-(((d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 1 - p, 1/2, (e^2*x^2)/d^2])/(
d*x*(1 - (e^2*x^2)/d^2)^p)) + (e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1, p,
 1 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*p)
```

3.273.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

3.273. $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)}\,dx$

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

rule 583
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, 0]
```

3.273.4 Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x^2\,(ex+d)}dx$$

input
```
int((-e^2*x^2+d^2)^p/x^2/(e*x+d),x)
```

output
```
int((-e^2*x^2+d^2)^p/x^2/(e*x+d),x)
```

3.273.5 Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)x^2}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e*x^3 + d*x^2), x)
```

3.273.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.91 (sec) , antiderivative size = 434, normalized size of antiderivative = 4.09

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)}\,dx$$
$$= \begin{cases} -\frac{0^p d d^{2p-2}}{x} - \frac{0^p d^{2p-2} e \log\left(\frac{e^2x^2}{d^2}\right)}{2} + \frac{0^p d^{2p-2} e \log\left(-1+\frac{e^2x^2}{d^2}\right)}{2} + 0^p d^{2p-2} e \operatorname{acoth}\left(\frac{ex}{d}\right) + \frac{d e^{2p-2} p x^{2p-3} e^{i\pi p}\Gamma(p)\Gamma\left(\frac{3}{2}-p\right){}_2F_1\Bigg(}{2\Gamma\left(\frac{5}{2}-p\right)\Gamma(p+} \\ -\frac{0^p d d^{2p-2}}{x} - \frac{0^p d^{2p-2} e \log\left(\frac{e^2x^2}{d^2}\right)}{2} + \frac{0^p d^{2p-2} e \log\left(1-\frac{e^2x^2}{d^2}\right)}{2} + 0^p d^{2p-2} e \operatorname{atanh}\left(\frac{ex}{d}\right) + \frac{d e^{2p-2} p x^{2p-3} e^{i\pi p}\Gamma(p)\Gamma\left(\frac{3}{2}-p\right){}_2F_1\Bigg(1}{2\Gamma\left(\frac{5}{2}-p\right)\Gamma(p+1)} \end{cases}$$

input
```
integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d),x)
```

output
```
Piecewise((-0**p*d*d**(2*p - 2)/x - 0**p*d**(2*p - 2)*e*log(e**2*x**2/d**2
)/2 + 0**p*d**(2*p - 2)*e*log(-1 + e**2*x**2/d**2)/2 + 0**p*d**(2*p - 2)*e
*acoth(e*x/d) + d*e**(2*p - 2)*p*x**(2*p - 3)*exp(I*pi*p)*gamma(p)*gamma(3
/2 - p)*hyper((1 - p, 3/2 - p), (5/2 - p,), d**2/(e**2*x**2))/(2*gamma(5/2
 - p)*gamma(p + 1)) - e**(2*p - 1)*p*x**(2*p - 2)*exp(I*pi*p)*gamma(p)*gam
ma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), d**2/(e**2*x**2))/(2*gamma(2 - p
)*gamma(p + 1)), Abs(e**2*x**2/d**2) > 1), (-0**p*d*d**(2*p - 2)/x - 0**p*
d**(2*p - 2)*e*log(e**2*x**2/d**2)/2 + 0**p*d**(2*p - 2)*e*log(1 - e**2*x*
*2/d**2)/2 + 0**p*d**(2*p - 2)*e*atanh(e*x/d) + d*e**(2*p - 2)*p*x**(2*p -
 3)*exp(I*pi*p)*gamma(p)*gamma(3/2 - p)*hyper((1 - p, 3/2 - p), (5/2 - p,)
, d**2/(e**2*x**2))/(2*gamma(5/2 - p)*gamma(p + 1)) - e**(2*p - 1)*p*x**(2
*p - 2)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,),
d**2/(e**2*x**2))/(2*gamma(2 - p)*gamma(p + 1)), True))
```

3.273.7 Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)x^2}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x^2), x)
```

3.273.8 Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)x^2}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x^2), x)
```

3.273.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)}\,dx = \int \frac{(d^2-e^2\,x^2)^p}{x^2\,(d+e\,x)}\,dx$$

input
```
int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)),x)
```

output
```
int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)), x)
```

3.274 $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)}\,dx$

3.274.1 Optimal result

Integrand size = 25, antiderivative size = 108

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)}\,dx = \frac{e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},1-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{d^2x} - \frac{e^2(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(2,p,1+p,1-\frac{e^2x^2}{d^2}\right)}{2d^3p}$$

output
```
e*(-e^2*x^2+d^2)^p*hypergeom([-1/2, 1-p],[1/2],e^2*x^2/d^2)/d^2/x/((1-e^2*
x^2/d^2)^p)-1/2*e^2*(-e^2*x^2+d^2)^p*hypergeom([2, p],[p+1],1-e^2*x^2/d^2)
/d^3/p
```

3.274.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 219 vs. 2(108) = 216.

Time = 0.59 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.03

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)}\,dx = \frac{(d^2-e^2x^2)^p\left(\frac{2d^2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}+\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(\frac{d^3\,\text{Hypergeometric2F1}\left(1-p,-p,2-p,\frac{d^2}{e^2x^2}\right)}{(-1+p)x^2}\right.\right.}{2d^4}$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)),x]
```

output
```
((d^2 - e^2*x^2)^p*((2*d^2*e*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^
2])/(x*(1 - (e^2*x^2)/d^2)^p) + ((d^3*Hypergeometric2F1[1 - p, -p, 2 - p,
d^2/(e^2*x^2)])/((-1 + p)*x^2) + e^2*(((2 - (2*d^2)/(e^2*x^2))^p*(d - e*x)
*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e
*x)/d)^p) + (d*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/p))/(1 - d
^2/(e^2*x^2))^p))/(2*d^4)
```

3.274.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {583, 542, 243, 75, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)}\,dx$$

$$\Big\downarrow \; 583$$

$$\int \frac{(d-ex)\,(d^2-e^2x^2)^{p-1}}{x^3}dx$$

$$\Big\downarrow \; 542$$

$$d\int \frac{(d^2-e^2x^2)^{p-1}}{x^3}dx - e\int \frac{(d^2-e^2x^2)^{p-1}}{x^2}dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}d\int \frac{(d^2-e^2x^2)^{p-1}}{x^4}dx^2 - e\int \frac{(d^2-e^2x^2)^{p-1}}{x^2}dx$$

$$\Big\downarrow \; 75$$

$$-e\int \frac{(d^2-e^2x^2)^{p-1}}{x^2}dx - \frac{e^2(d^2-e^2x^2)^p \text{Hypergeometric2F1}\left(2,p,p+1,1-\frac{e^2x^2}{d^2}\right)}{2d^3p}$$

$$\Big\downarrow \; 279$$

$$-\frac{e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^{p-1}}{x^2}dx}{d^2}-\frac{e^2(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(2,p,p+1,1-\frac{e^2x^2}{d^2}\right)}{2d^3p}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},1-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{d^2x}-\frac{e^2(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(2,p,p+1,1-\frac{e^2x^2}{d^2}\right)}{2d^3p}$$

input
```
Int[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)),x]
```

output
```
(e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 1 - p, 1/2, (e^2*x^2)/d^2])/(
d^2*x*(1 - (e^2*x^2)/d^2)^p) - (e^2*(d^2 - e^2*x^2)^p*Hypergeometric2F1[2,
 p, 1 + p, 1 - (e^2*x^2)/d^2])/(2*d^3*p)
```

3.274.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

3.274. $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)}\,dx$

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

rule 583
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, 0]
```

3.274.4 Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x^3\,(ex+d)}dx$$

input
```
int((-e^2*x^2+d^2)^p/x^3/(e*x+d),x)
```

output
```
int((-e^2*x^2+d^2)^p/x^3/(e*x+d),x)
```

3.274.5 Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)x^3}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d),x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e*x^4 + d*x^3), x)
```

3.274.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.40 (sec) , antiderivative size = 484, normalized size of antiderivative = 4.48

$$\int \frac{(d^2 - e^2x^2)^p}{x^3(d + ex)}\,dx$$

$$= \begin{cases} -\frac{0^p d^2 d^{2p-3}}{2x^2} + \frac{0^p d d^{2p-3} e}{x} + \frac{0^p d^{2p-3} e^2 \log\left(\frac{e^2x^2}{d^2}\right)}{2} - \frac{0^p d^{2p-3} e^2 \log\left(-1+\frac{e^2x^2}{d^2}\right)}{2} - 0^p d^{2p-3} e^2 \operatorname{acoth}\left(\frac{ex}{d}\right) + \frac{d e^{2p-2} p x^{2p-4} e^{i\pi \cdots}}{\cdots} \\ -\frac{0^p d^2 d^{2p-3}}{2x^2} + \frac{0^p d d^{2p-3} e}{x} + \frac{0^p d^{2p-3} e^2 \log\left(\frac{e^2x^2}{d^2}\right)}{2} - \frac{0^p d^{2p-3} e^2 \log\left(1-\frac{e^2x^2}{d^2}\right)}{2} - 0^p d^{2p-3} e^2 \operatorname{atanh}\left(\frac{ex}{d}\right) + \frac{d e^{2p-2} p x^{2p-4} e^{i\pi p \cdots}}{\cdots} \end{cases}$$

input
```
integrate((-e**2*x**2+d**2)**p/x**3/(e*x+d),x)
```

output
```
Piecewise((-0**p*d**2*d**(2*p - 3)/(2*x**2) + 0**p*d*d**(2*p - 3)*e/x + 0*
*p*d**(2*p - 3)*e**2*log(e**2*x**2/d**2)/2 - 0**p*d**(2*p - 3)*e**2*log(-1
 + e**2*x**2/d**2)/2 - 0**p*d**(2*p - 3)*e**2*acoth(e*x/d) + d*e**(2*p - 2
)*p*x**(2*p - 4)*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((1 - p, 2 - p), (
3 - p,), d**2/(e**2*x**2))/(2*gamma(3 - p)*gamma(p + 1)) - e**(2*p - 1)*p*
x**(2*p - 3)*exp(I*pi*p)*gamma(p)*gamma(3/2 - p)*hyper((1 - p, 3/2 - p), (
5/2 - p,), d**2/(e**2*x**2))/(2*gamma(5/2 - p)*gamma(p + 1)), Abs(e**2*x**
2/d**2) > 1), (-0**p*d**2*d**(2*p - 3)/(2*x**2) + 0**p*d*d**(2*p - 3)*e/x
+ 0**p*d**(2*p - 3)*e**2*log(e**2*x**2/d**2)/2 - 0**p*d**(2*p - 3)*e**2*lo
g(1 - e**2*x**2/d**2)/2 - 0**p*d**(2*p - 3)*e**2*atanh(e*x/d) + d*e**(2*p
- 2)*p*x**(2*p - 4)*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((1 - p, 2 - p)
, (3 - p,), d**2/(e**2*x**2))/(2*gamma(3 - p)*gamma(p + 1)) - e**(2*p - 1)
*p*x**(2*p - 3)*exp(I*pi*p)*gamma(p)*gamma(3/2 - p)*hyper((1 - p, 3/2 - p)
, (5/2 - p,), d**2/(e**2*x**2))/(2*gamma(5/2 - p)*gamma(p + 1)), True))
```

3.274.7 Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^3(d + ex)} dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)x^3} dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x^3), x)
```

3.274.8 Giac [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^3(d + ex)} dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)x^3} dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d),x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x^3), x)
```

3.274.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^p}{x^3(d + ex)} dx = \int \frac{(d^2 - e^2 x^2)^p}{x^3 (d + e x)} dx$$

input
```
int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)),x)
```

output
```
int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)), x)
```

3.275 $\int \frac{x^5(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

3.275.1 Optimal result

Integrand size = 25, antiderivative size = 179

$$\int \frac{x^5(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \frac{d^6(d^2-e^2x^2)^{-1+p}}{e^6(1-p)}+\frac{5d^4(d^2-e^2x^2)^p}{2e^6p}-\frac{2d^2(d^2-e^2x^2)^{1+p}}{e^6(1+p)}+\frac{(d^2-e^2x^2)^{2+p}}{2e^6(2+p)} - \frac{2ex^7(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},2-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)}{7d^3}$$

output
```
d^6*(-e^2*x^2+d^2)^(-1+p)/e^6/(1-p)+5/2*d^4*(-e^2*x^2+d^2)^p/e^6/p-2*d^2*(
-e^2*x^2+d^2)^(p+1)/e^6/(p+1)+1/2*(-e^2*x^2+d^2)^(2+p)/e^6/(2+p)-2/7*e*x^7
*(-e^2*x^2+d^2)^p*hypergeom([7/2, 2-p],[9/2],e^2*x^2/d^2)/d^3/((1-e^2*x^2/
d^2)^p)
```

3.275.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.37

$$\int \frac{x^5(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \frac{x^6(d-ex)^p(d+ex)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{AppellF1}\left(6,-p,2-p,7,\frac{ex}{d},-\frac{ex}{d}\right)}{6d^2}$$

input
```
Integrate[(x^5*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

output
```
(x^6*(d - e*x)^p*(d + e*x)^p*AppellF1[6, -p, 2 - p, 7, (e*x)/d, -((e*x)/d)
])/(6*d^2*(1 - (e^2*x^2)/d^2)^p)
```

3.275.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {570, 543, 27, 279, 278, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (d^2-e^2x^2)^p}{(d+ex)^2} \, dx$$

$$\Big\downarrow \text{570}$$

$$\int x^5(d-ex)^2 \left(d^2-e^2x^2\right)^{p-2} dx$$

$$\Big\downarrow \text{543}$$

$$\int -2dex^6 \left(d^2-e^2x^2\right)^{p-2} dx+\int x^5 \left(d^2-e^2x^2\right)^{p-2} \left(d^2+e^2x^2\right) dx$$

$$\Big\downarrow \text{27}$$

$$\int x^5 \left(d^2-e^2x^2\right)^{p-2} \left(d^2+e^2x^2\right) dx-2de\int x^6 \left(d^2-e^2x^2\right)^{p-2} dx$$

$$\Big\downarrow \text{279}$$

$$\int x^5 \left(d^2-e^2x^2\right)^{p-2} \left(d^2+e^2x^2\right) dx-\frac{2e\left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \int x^6 \left(1-\frac{e^2x^2}{d^2}\right)^{p-2} dx}{d^3}$$

$$\Big\downarrow \text{278}$$

$$\int x^5 \left(d^2-e^2x^2\right)^{p-2} \left(d^2+e^2x^2\right) dx-\frac{2ex^7 \left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{7}{2},2-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)}{7d^3}$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int x^4\left(d^2-e^2x^2\right)^{p-2}\left(d^2+e^2x^2\right)dx^2-\frac{2ex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},2-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)}{7d^3}$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int\left(\frac{2d^6\left(d^2-e^2x^2\right)^{p-2}}{e^4}-\frac{5d^4\left(d^2-e^2x^2\right)^{p-1}}{e^4}+\frac{4d^2\left(d^2-e^2x^2\right)^p}{e^4}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{e^4}\right)dx^2-\frac{2ex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},2-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)}{7d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{4d^2\left(d^2-e^2x^2\right)^{p+1}}{e^6(p+1)}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{e^6(p+2)}+\frac{2d^6\left(d^2-e^2x^2\right)^{p-1}}{e^6(1-p)}+\frac{5d^4\left(d^2-e^2x^2\right)^p}{e^6p}\right)-\frac{2ex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},2-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)}{7d^3}$$

input
```
Int[(x^5*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

output
```
((2*d^6*(d^2 - e^2*x^2)^(-1 + p))/(e^6*(1 - p)) + (5*d^4*(d^2 - e^2*x^2)^p
)/(e^6*p) - (4*d^2*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p)) + (d^2 - e^2*x^2
)^(2 + p)/(e^6*(2 + p)))/2 - (2*e*x^7*(d^2 - e^2*x^2)^p*Hypergeometric2F1[
7/2, 2 - p, 9/2, (e^2*x^2)/d^2])/(7*d^3*(1 - (e^2*x^2)/d^2)^p)
```

3.275.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] && !IntegerQ[2*p]
&& !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] && !(IGtQ[m, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.275. $\int \frac{x^5(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

3.275.4 Maple [F]

$$\int \frac{x^5(-e^2x^2+d^2)^p}{(ex+d)^2}dx$$

input
```
int(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

output
```
int(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

3.275.5 Fricas [F]

$$\int \frac{x^5(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p x^5}{(ex+d)^2}\,dx$$

input
```
integrate(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*x^5/(e^2*x^2 + 2*d*e*x + d^2), x)
```

3.275.6 Sympy [F]

$$\int \frac{x^5(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{x^5(-(-d+ex)\,(d+ex))^p}{(d+ex)^2}\,dx$$

input
```
integrate(x**5*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

output
```
Integral(x**5*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

3.275.7 Maxima [F]

$$\int \frac{x^5(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p x^5}{(ex+d)^2}\,dx$$

input
```
integrate(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^5/(e*x + d)^2, x)
```

3.275.8 Giac [F]

$$\int \frac{x^5(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p x^5}{(ex+d)^2}\,dx$$

input
```
integrate(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^5/(e*x + d)^2, x)
```

3.275.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{x^5\,(d^2-e^2\,x^2)^p}{(d+e\,x)^2}\,dx$$

input
```
int((x^5*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x)
```

output
```
int((x^5*(d^2 - e^2*x^2)^p)/(d + e*x)^2, x)
```

3.276 $\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

3.276.1 Optimal result

Integrand size = 25, antiderivative size = 184

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = -\frac{d^5(d^2-e^2x^2)^{-1+p}}{e^5(1-p)} - \frac{x^5(d^2-e^2x^2)^{-1+p}}{3+2p} - \frac{2d^3(d^2-e^2x^2)^p}{e^5p} + \frac{d(d^2-e^2x^2)^{1+p}}{e^5(1+p)} + \frac{2(4+p)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},2-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^2(3+2p)}$$

output
```
-d^5*(-e^2*x^2+d^2)^(-1+p)/e^5/(1-p)-x^5*(-e^2*x^2+d^2)^(-1+p)/(3+2*p)-2*d
^3*(-e^2*x^2+d^2)^p/e^5/p+d*(-e^2*x^2+d^2)^(p+1)/e^5/(p+1)+2/5*(4+p)*x^5*(
-e^2*x^2+d^2)^p*hypergeom([5/2, 2-p],[7/2],e^2*x^2/d^2)/d^2/(3+2*p)/((1-e^
2*x^2/d^2)^p)
```

3.276.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.36

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \frac{x^5(d-ex)^p(d+ex)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{AppellF1}\left(5,-p,2-p,6,\frac{ex}{d},-\frac{ex}{d}\right)}{5d^2}$$

input
```
Integrate[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

output
```
(x^5*(d - e*x)^p*(d + e*x)^p*AppellF1[5, -p, 2 - p, 6, (e*x)/d, -((e*x)/d)
])/(5*d^2*(1 - (e^2*x^2)/d^2)^p)
```

3.276.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {570, 543, 27, 243, 53, 363, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(d^2-e^2 x^2\right)^p}{(d+e x)^2} \, dx$$

$$\Big\downarrow \text{ 570}$$

$$\int x^4 (d-e x)^2 \left(d^2-e^2 x^2\right)^{p-2}dx$$

$$\Big\downarrow \text{ 543}$$

$$\int -2 d e x^5 \left(d^2-e^2 x^2\right)^{p-2}dx+\int x^4 \left(d^2-e^2 x^2\right)^{p-2} \left(d^2+e^2 x^2\right)dx$$

$$\Big\downarrow \text{ 27}$$

$$\int x^4 \left(d^2-e^2 x^2\right)^{p-2} \left(d^2+e^2 x^2\right)dx-2 d e \int x^5 \left(d^2-e^2 x^2\right)^{p-2}dx$$

$$\Big\downarrow \text{ 243}$$

$$\int x^4 \left(d^2-e^2 x^2\right)^{p-2} \left(d^2+e^2 x^2\right)dx-d e \int x^4 \left(d^2-e^2 x^2\right)^{p-2}dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\int x^4 \left(d^2-e^2 x^2\right)^{p-2} \left(d^2+e^2 x^2\right)dx-$$
$$d e \int \left(\frac{d^4 \left(d^2-e^2 x^2\right)^{p-2}}{e^4}-\frac{2 d^2 \left(d^2-e^2 x^2\right)^{p-1}}{e^4}+\frac{\left(d^2-e^2 x^2\right)^p}{e^4}\right)dx^2$$

$$\Big\downarrow \text{ 363}$$

$$\frac{2d^2(p+4)\int x^4\left(d^2-e^2x^2\right)^{p-2}dx}{2p+3}-$$
$$de\int\left(\frac{d^4\left(d^2-e^2x^2\right)^{p-2}}{e^4}-\frac{2d^2\left(d^2-e^2x^2\right)^{p-1}}{e^4}+\frac{\left(d^2-e^2x^2\right)^p}{e^4}\right)dx^2-\frac{x^5\left(d^2-e^2x^2\right)^{p-1}}{2p+3}$$

$$\Big\downarrow \ 279$$

$$\frac{2(p+4)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^{p-2}dx}{d^2(2p+3)}-$$
$$de\int\left(\frac{d^4\left(d^2-e^2x^2\right)^{p-2}}{e^4}-\frac{2d^2\left(d^2-e^2x^2\right)^{p-1}}{e^4}+\frac{\left(d^2-e^2x^2\right)^p}{e^4}\right)dx^2-\frac{x^5\left(d^2-e^2x^2\right)^{p-1}}{2p+3}$$

$$\Big\downarrow \ 278$$

$$-de\int\left(\frac{d^4\left(d^2-e^2x^2\right)^{p-2}}{e^4}-\frac{2d^2\left(d^2-e^2x^2\right)^{p-1}}{e^4}+\frac{\left(d^2-e^2x^2\right)^p}{e^4}\right)dx^2+$$
$$\frac{2(p+4)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\text{Hypergeometric2F1}\left(\frac{5}{2},2-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^2(2p+3)}-\frac{x^5\left(d^2-e^2x^2\right)^{p-1}}{2p+3}$$

$$\Big\downarrow \ 2009$$

$$\frac{2(p+4)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\text{Hypergeometric2F1}\left(\frac{5}{2},2-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^2(2p+3)}-$$
$$\frac{x^5\left(d^2-e^2x^2\right)^{p-1}}{2p+3}-de\left(\frac{2d^2\left(d^2-e^2x^2\right)^p}{e^6p}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{e^6(p+1)}+\frac{d^4\left(d^2-e^2x^2\right)^{p-1}}{e^6(1-p)}\right)$$

input
```
Int[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

output
```
-((x^5*(d^2 - e^2*x^2)^(-1 + p))/(3 + 2*p)) - d*e*((d^4*(d^2 - e^2*x^2)^(-
1 + p))/(e^6*(1 - p)) + (2*d^2*(d^2 - e^2*x^2)^p)/(e^6*p) - (d^2 - e^2*x^2
)^(1 + p)/(e^6*(1 + p))) + (2*(4 + p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric
2F1[5/2, 2 - p, 7/2, (e^2*x^2)/d^2])/(5*d^2*(3 + 2*p)*(1 - (e^2*x^2)/d^2)^
p)
```

3.276. $\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

3.276.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

3.276. $\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.276.4 Maple [F]

$$\int \frac{x^4(-e^2x^2+d^2)^p}{(ex+d)^2}dx$$

input
```
int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

output
```
int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

3.276.5 Fricas [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p x^4}{(ex+d)^2}\,dx$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*x^4/(e^2*x^2 + 2*d*e*x + d^2), x)
```

3.276.6 Sympy [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{x^4(-(-d+ex)\,(d+ex))^p}{(d+ex)^2}\,dx$$

input
```
integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

output
```
Integral(x**4*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

3.276.7 Maxima [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p x^4}{(ex+d)^2}\,dx$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^2, x)
```

3.276.8 Giac [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p x^4}{(ex+d)^2}\,dx$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^2, x)
```

3.276.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{x^4\,(d^2-e^2\,x^2)^p}{(d+e\,x)^2}\,dx$$

input
```
int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x)
```

output
```
int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^2, x)
```

3.277 $\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

3.277.1 Optimal result

Integrand size = 25, antiderivative size = 150

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \frac{d^4(d^2-e^2x^2)^{-1+p}}{e^4(1-p)}+\frac{3d^2(d^2-e^2x^2)^p}{2e^4p}-\frac{(d^2-e^2x^2)^{1+p}}{2e^4(1+p)} - \frac{2ex^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},2-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^3}$$

output
```
d^4*(-e^2*x^2+d^2)^(-1+p)/e^4/(1-p)+3/2*d^2*(-e^2*x^2+d^2)^p/e^4/p-1/2*(-e
^2*x^2+d^2)^(p+1)/e^4/(p+1)-2/5*e*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, 2-p
],[7/2],e^2*x^2/d^2)/d^3/((1-e^2*x^2/d^2)^p)
```

3.277.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 332 vs. 2(150) = 300.

Time = 0.39 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.21

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \frac{2^{-2+p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(2d^2\left(\frac{1}{2}+\frac{ex}{2d}\right)^p-2d^2\left(\frac{1}{2}+\frac{ex}{2d}\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^p+2e^2x^2\left(\frac{1}{2}+\frac{ex}{2d}\right)^p\right.}{}$$

input
```
Integrate[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

output
```
(2^(-2 + p)*(d^2 - e^2*x^2)^p*(2*d^2*(1/2 + (e*x)/(2*d))^p - 2*d^2*(1/2 +
(e*x)/(2*d))^p*(1 - (e^2*x^2)/d^2)^p + 2*e^2*x^2*(1/2 + (e*x)/(2*d))^p*(1
- (e^2*x^2)/d^2)^p - 8*d*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hypergeometric2
F1[1/2, -p, 3/2, (e^2*x^2)/d^2] - 6*d*(d - e*x)*(1 - (e^2*x^2)/d^2)^p*Hype
rgeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + d^2*(1 - (e^2*x^2)/d
^2)^p*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - d*e*x*(1 -
 (e^2*x^2)/d^2)^p*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])
)/(e^4*(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

3.277.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {570, 543, 27, 279, 278, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (d^2-e^2x^2)^p}{(d+ex)^2} \, dx$$

$$\Big\downarrow \text{570}$$

$$\int x^3(d-ex)^2 \left(d^2-e^2x^2\right)^{p-2} dx$$

$$\Big\downarrow \text{543}$$

$$\int -2dex^4 \left(d^2-e^2x^2\right)^{p-2} dx+\int x^3 \left(d^2-e^2x^2\right)^{p-2} \left(d^2+e^2x^2\right) dx$$

$$\Big\downarrow \text{27}$$

$$\int x^3 \left(d^2-e^2x^2\right)^{p-2} \left(d^2+e^2x^2\right) dx-2de\int x^4 \left(d^2-e^2x^2\right)^{p-2} dx$$

$$\Big\downarrow \text{279}$$

$$\int x^3 \left(d^2-e^2x^2\right)^{p-2} \left(d^2+e^2x^2\right) dx-\frac{2e\left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \int x^4 \left(1-\frac{e^2x^2}{d^2}\right)^{p-2} dx}{d^3}$$

$$\Big\downarrow \text{278}$$

$$\int x^3 \left(d^2-e^2x^2\right)^{p-2} \left(d^2+e^2x^2\right)dx - \frac{2ex^5\left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{2},2-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^3}$$

$\Big\downarrow$ 354

$$\frac{1}{2}\int x^2 \left(d^2-e^2x^2\right)^{p-2} \left(d^2+e^2x^2\right)dx^2 - \frac{2ex^5\left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{2},2-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^3}$$

$\Big\downarrow$ 86

$$\frac{1}{2}\int \left(\frac{2d^4\left(d^2-e^2x^2\right)^{p-2}}{e^2} - \frac{3d^2\left(d^2-e^2x^2\right)^{p-1}}{e^2} + \frac{\left(d^2-e^2x^2\right)^p}{e^2}\right)dx^2 - \frac{2ex^5\left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{2},2-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^3}$$

$\Big\downarrow$ 2009

$$\frac{1}{2}\left(\frac{3d^2\left(d^2-e^2x^2\right)^p}{e^4p} - \frac{\left(d^2-e^2x^2\right)^{p+1}}{e^4(p+1)} + \frac{2d^4\left(d^2-e^2x^2\right)^{p-1}}{e^4(1-p)}\right) - \frac{2ex^5\left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{2},2-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^3}$$

input
```
Int[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

output
```
((2*d^4*(d^2 - e^2*x^2)^(-1 + p))/(e^4*(1 - p)) + (3*d^2*(d^2 - e^2*x^2)^p
)/(e^4*p) - (d^2 - e^2*x^2)^(1 + p)/(e^4*(1 + p)))/2 - (2*e*x^5*(d^2 - e^2
*x^2)^p*Hypergeometric2F1[5/2, 2 - p, 7/2, (e^2*x^2)/d^2])/(5*d^3*(1 - (e^
2*x^2)/d^2)^p)
```

3.277. $\int \frac{x^3\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx$

3.277.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.277. $\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.277.4 Maple [F]

$$\int \frac{x^3(-e^2x^2+d^2)^p}{(ex+d)^2}dx$$

input
```
int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

output
```
int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

3.277.5 Fricas [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p x^3}{(ex+d)^2}\,dx$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*x^3/(e^2*x^2 + 2*d*e*x + d^2), x)
```

3.277.6 Sympy [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{x^3(-(-d+ex)\,(d+ex))^p}{(d+ex)^2}\,dx$$

input
```
integrate(x**3*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

output
```
Integral(x**3*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

3.277.7 Maxima [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^2}\,dx=\int \frac{(-e^2x^2+d^2)^p x^3}{(ex+d)^2}\,dx$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^2, x)
```

3.277.8 Giac [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^2}\,dx=\int \frac{(-e^2x^2+d^2)^p x^3}{(ex+d)^2}\,dx$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^2, x)
```

3.277.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^2}\,dx=\int \frac{x^3\,(d^2-e^2\,x^2)^p}{(d+e\,x)^2}\,dx$$

input
```
int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x)
```

output
```
int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^2, x)
```

3.278 $\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

3.278.1 Optimal result

Integrand size = 25, antiderivative size = 156

$$\begin{aligned}\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^2}\,dx &= -\frac{d^3(d^2-e^2x^2)^{-1+p}}{e^3(1-p)}-\frac{x^3(d^2-e^2x^2)^{-1+p}}{1+2p}-\frac{d(d^2-e^2x^2)^p}{e^3p}\\ &\quad+\frac{2(2+p)x^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},2-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3d^2(1+2p)}\end{aligned}$$

output
```
-d^3*(-e^2*x^2+d^2)^(-1+p)/e^3/(1-p)-x^3*(-e^2*x^2+d^2)^(-1+p)/(1+2*p)-d*(
-e^2*x^2+d^2)^p/e^3/p+2/3*(2+p)*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/2, 2-p],
[5/2],e^2*x^2/d^2)/d^2/(1+2*p)/((1-e^2*x^2/d^2)^p)
```

3.278.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.13

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \frac{2^{-2+p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(4e(1+p)x\left(\frac{1}{2}+\frac{ex}{2d}\right)^p\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)+\right.}{}$$

input
```
Integrate[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

output
```
(2^(-2 + p)*(d^2 - e^2*x^2)^p*(4*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hyperge
ometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^2)^p*
(4*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - Hypergeometri
c2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])))/(e^3*(1 + p)*(1 + (e*x)/d)^p
*(1 - (e^2*x^2)/d^2)^p)
```

3.278.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {570, 543, 27, 243, 53, 363, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (d^2-e^2x^2)^p}{(d+ex)^2} \, dx$$

$$\Big\downarrow \text{570}$$

$$\int x^2(d-ex)^2 \left(d^2-e^2x^2\right)^{p-2}dx$$

$$\Big\downarrow \text{543}$$

$$\int x^2 \left(d^2-e^2x^2\right)^{p-2} \left(d^2+e^2x^2\right)dx+\int -2dex^3 \left(d^2-e^2x^2\right)^{p-2}dx$$

$$\Big\downarrow \text{27}$$

$$\int x^2 \left(d^2-e^2x^2\right)^{p-2} \left(d^2+e^2x^2\right)dx-2de\int x^3 \left(d^2-e^2x^2\right)^{p-2}dx$$

$$\Big\downarrow \text{243}$$

$$\int x^2 \left(d^2-e^2x^2\right)^{p-2} \left(d^2+e^2x^2\right)dx-de\int x^2 \left(d^2-e^2x^2\right)^{p-2}dx^2$$

$$\Big\downarrow \text{53}$$

$$\int x^2 \left(d^2-e^2x^2\right)^{p-2} \left(d^2+e^2x^2\right)dx-de\int \left(\frac{d^2\left(d^2-e^2x^2\right)^{p-2}}{e^2}-\frac{\left(d^2-e^2x^2\right)^{p-1}}{e^2}\right)dx^2$$

$$\Big\downarrow \text{363}$$

$$\frac{2d^2(p+2)\int x^2\left(d^2-e^2x^2\right)^{p-2}dx}{2p+1}-de\int\left(\frac{d^2\left(d^2-e^2x^2\right)^{p-2}}{e^2}-\frac{\left(d^2-e^2x^2\right)^{p-1}}{e^2}\right)dx^2-\frac{x^3\left(d^2-e^2x^2\right)^{p-1}}{2p+1}$$

$$\Big\downarrow \text{279}$$

$$\frac{2(p+2)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^{p-2}dx}{d^2(2p+1)}-de\int\left(\frac{d^2\left(d^2-e^2x^2\right)^{p-2}}{e^2}-\frac{\left(d^2-e^2x^2\right)^{p-1}}{e^2}\right)dx^2-\frac{x^3\left(d^2-e^2x^2\right)^{p-1}}{2p+1}$$

$$\Big\downarrow \text{278}$$

$$-de\int\left(\frac{d^2\left(d^2-e^2x^2\right)^{p-2}}{e^2}-\frac{\left(d^2-e^2x^2\right)^{p-1}}{e^2}\right)dx^2+\frac{2(p+2)x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\text{Hypergeometric2F1}\left(\frac{3}{2},2-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3d^2(2p+1)}-\frac{x^3\left(d^2-e^2x^2\right)^{p-1}}{2p+1}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(p+2)x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\text{Hypergeometric2F1}\left(\frac{3}{2},2-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3d^2(2p+1)}-\frac{x^3\left(d^2-e^2x^2\right)^{p-1}}{2p+1}-de\left(\frac{d^2\left(d^2-e^2x^2\right)^{p-1}}{e^4(1-p)}+\frac{\left(d^2-e^2x^2\right)^p}{e^4p}\right)$$

input
```
Int[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

output
```
-((x^3*(d^2 - e^2*x^2)^(-1 + p))/(1 + 2*p)) - d*e*((d^2*(d^2 - e^2*x^2)^(-
1 + p))/(e^4*(1 - p)) + (d^2 - e^2*x^2)^p/(e^4*p)) + (2*(2 + p)*x^3*(d^2 -
 e^2*x^2)^p*Hypergeometric2F1[3/2, 2 - p, 5/2, (e^2*x^2)/d^2])/(3*d^2*(1 +
 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

3.278. $\int \frac{x^2\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx$

3.278.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

3.278. $\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] && !(IGtQ[m, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.278.4 Maple [F]

$$\int \frac{x^2(-e^2x^2+d^2)^p}{(ex+d)^2}dx$$

input
```
int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

output
```
int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

3.278.5 Fricas [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p x^2}{(ex+d)^2}\,dx$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*x^2/(e^2*x^2 + 2*d*e*x + d^2), x)
```

3.278.6 Sympy [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{x^2(-(-d+ex)\,(d+ex))^p}{(d+ex)^2}\,dx$$

input
```
integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

output
```
Integral(x**2*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

3.278.7 Maxima [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^2}\,dx=\int \frac{(-e^2x^2+d^2)^p x^2}{(ex+d)^2}\,dx$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^2, x)
```

3.278.8 Giac [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^2}\,dx=\int \frac{(-e^2x^2+d^2)^p x^2}{(ex+d)^2}\,dx$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^2, x)
```

3.278.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^2}\,dx=\int \frac{x^2\,(d^2-e^2\,x^2)^p}{(d+e\,x)^2}\,dx$$

input
```
int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x)
```

output
```
int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^2, x)
```

3.279 $\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

3.279.1 Optimal result

Integrand size = 23, antiderivative size = 115

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \frac{(d^2-e^2x^2)^{1+p}}{2e^2(1-p)(d+ex)^2} - \frac{2^{-1+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(1-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^2e^2(1-p^2)}$$

output
```
1/2*(-e^2*x^2+d^2)^(p+1)/e^2/(1-p)/(e*x+d)^2-2^(-1+p)*(1+e*x/d)^(-1-p)*(-e
^2*x^2+d^2)^(p+1)*hypergeom([1-p, p+1],[2+p],1/2*(-e*x+d)/d)/d^2/e^2/(-p^2
+1)
```

3.279.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \frac{2^{-2+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(-2\,\text{Hypergeometric2F1}\left(1-p,1+p,2+p,\frac{d-ex}{2d}\right)+\text{Hypergeome}\right.}{de^2(1+p)}$$

input
```
Integrate[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

output
```
(2^(-2 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(-2*Hypergeometric2F1[1 - p, 1 + p
, 2 + p, (d - e*x)/(2*d)] + Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*
x)/(2*d)]))/(d*e^2*(1 + p)*(1 + (e*x)/d)^p)
```

3.279.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {571, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^2}dx$$

$$\Big\downarrow \; 571$$

$$\frac{\int \frac{(d^2-e^2x^2)^p}{d+ex}dx}{e(1-p)}+\frac{(d^2-e^2x^2)^{p+1}}{2e^2(1-p)(d+ex)^2}$$

$$\Big\downarrow \; 473$$

$$\frac{(d-ex)^{-p-1}(d^2-e^2x^2)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}\int (d-ex)^p\left(\frac{ex}{d}+1\right)^{p-1}dx}{d^2e(1-p)}+\frac{(d^2-e^2x^2)^{p+1}}{2e^2(1-p)(d+ex)^2}$$

$$\Big\downarrow \; 79$$

$$\frac{(d^2-e^2x^2)^{p+1}}{2e^2(1-p)(d+ex)^2}-\frac{2^{p-1}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\text{Hypergeometric2F1}\left(1-p,p+1,p+2,\frac{d-ex}{2d}\right)}{d^2e^2(1-p)(p+1)}$$

input
```
Int[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

output
```
(d^2 - e^2*x^2)^(1 + p)/(2*e^2*(1 - p)*(d + e*x)^2) - (2^(-1 + p)*(1 + (e*
x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1 - p, 1 + p, 2 +
 p, (d - e*x)/(2*d)])/(d^2*e^2*(1 - p)*(1 + p))
```

3.279.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 571
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*(n + p + 1))), x] + Simp[n/(2*d*
(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ((LtQ[n, -1] &&  !IGtQ[n + p +
1, 0]) || (LtQ[n, 0] && LtQ[p, -1]) || EqQ[n + 2*p + 2, 0]) && NeQ[n + p +
1, 0]
```

3.279.4 Maple [F]

$$\int \frac{x(-e^2x^2+d^2)^p}{(ex+d)^2}dx$$

input
```
int(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

output
```
int(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

3.279.5 Fricas [F]

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p x}{(ex+d)^2}\,dx$$

input
```
integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*x/(e^2*x^2 + 2*d*e*x + d^2), x)
```

3.279.6 Sympy [F]

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{x(-(-d+ex)(d+ex))^p}{(d+ex)^2}\,dx$$

input
```
integrate(x*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

output
```
Integral(x*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

3.279.7 Maxima [F]

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p x}{(ex+d)^2}\,dx$$

input
```
integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^2, x)
```

3.279.8 Giac [F]

$$\int \frac{x(d^2 - e^2x^2)^p}{(d + ex)^2}\,dx = \int \frac{(-e^2x^2 + d^2)^p x}{(ex + d)^2}\,dx$$

input
```
integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^2, x)
```

3.279.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(d^2 - e^2x^2)^p}{(d + ex)^2}\,dx = \int \frac{x\,(d^2 - e^2\,x^2)^p}{(d + e\,x)^2}\,dx$$

input
```
int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x)
```

output
```
int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^2, x)
```

3.280 $\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2} dx$

3.280.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2} dx = -\frac{2^{-2+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(2-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^3e(1+p)}$$

output
```
-2^(-2+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(p+1)*hypergeom([p+1, 2-p],[2+p]
,1/2*(-e*x+d)/d)/d^3/e/(p+1)
```

3.280.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2} dx = -\frac{2^{-2+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(2-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^2e(1+p)}$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(d + e*x)^2,x]
```

output
```
-((2^(-2 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[2 - p, 1 + p,
2 + p, (d - e*x)/(2*d)])/(d^2*e*(1 + p)*(1 + (e*x)/d)^p))
```

3.280.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx$$

$$\Big\downarrow \text{473}$$

$$\frac{(d-ex)^{-p-1}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\int (d-ex)^p\left(\frac{ex}{d}+1\right)^{p-2}dx}{d^3}$$

$$\Big\downarrow \text{79}$$

$$-\frac{2^{p-2}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\text{Hypergeometric2F1}\left(2-p,p+1,p+2,\frac{d-ex}{2d}\right)}{d^3e(p+1)}$$

input
```
Int[(d^2 - e^2*x^2)^p/(d + e*x)^2,x]
```

output
```
-((2^(-2 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometri
c2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^3*e*(1 + p)))
```

3.280.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

3.280.4 Maple [F]

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{\left(e x +d \right)^{2}}d x$$

input
```
int((-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

output
```
int((-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

3.280.5 Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2}\,dx=\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{\left(e x +d\right)^{2}}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e^2*x^2 + 2*d*e*x + d^2), x)
```

3.280.6 Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2}\,dx=\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{p}}{\left(d+ex\right)^{2}}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

3.280.7 Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/(e*x + d)^2, x)
```

3.280.8 Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/(e*x + d)^2, x)
```

3.280.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(d^2-e^2\,x^2)^p}{(d+e\,x)^2}\,dx$$

input
```
int((d^2 - e^2*x^2)^p/(d + e*x)^2,x)
```

output
```
int((d^2 - e^2*x^2)^p/(d + e*x)^2, x)
```

3.281 $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^2}\,dx$

3.281.1 Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^2}\,dx = \frac{(d^2-e^2x^2)^{-1+p}}{1-p} - \frac{2ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},2-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^3} - \frac{(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(1,p,1+p,1-\frac{e^2x^2}{d^2}\right)}{2d^2p}$$

output
```
(-e^2*x^2+d^2)^(-1+p)/(1-p)-2*e*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 2-p],[3
/2],e^2*x^2/d^2)/d^3/((1-e^2*x^2/d^2)^p)-1/2*(-e^2*x^2+d^2)^p*hypergeom([1
, p],[p+1],1-e^2*x^2/d^2)/d^2/p
```

3.281.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.57

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^2}\,dx = \frac{2^{-2+p}\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(2p\left(1-\frac{d^2}{e^2x^2}\right)^p(d-ex)\,\text{Hypergeometric2F1}\left(1-p,1+p,2-\right.\right.}{}$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(x*(d + e*x)^2),x]
```

output
```
(2^(-2 + p)*(d^2 - e^2*x^2)^p*(2*p*(1 - d^2/(e^2*x^2))^p*(d - e*x)*Hyperge
ometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + p*(1 - d^2/(e^2*x^2))^p
*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 2*d*(
1 + p)*(1/2 + (e*x)/(2*d))^p*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2
)]))/(d^3*p*(1 + p)*(1 - d^2/(e^2*x^2))^p*(1 + (e*x)/d)^p)
```

3.281.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {570, 543, 27, 238, 237, 354, 88, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^2}\,dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{p-2}}{x}dx$$

$$\Big\downarrow \text{543}$$

$$\int -2de\left(d^2-e^2x^2\right)^{p-2}dx+\int \frac{\left(d^2-e^2x^2\right)^{p-2}\left(d^2+e^2x^2\right)}{x}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\left(d^2-e^2x^2\right)^{p-2}\left(d^2+e^2x^2\right)}{x}dx-2de\int \left(d^2-e^2x^2\right)^{p-2}dx$$

$$\Big\downarrow \text{238}$$

$$\int \frac{\left(d^2-e^2x^2\right)^{p-2}\left(d^2+e^2x^2\right)}{x}dx-\frac{2e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int \left(1-\frac{e^2x^2}{d^2}\right)^{p-2}dx}{d^3}$$

$$\Big\downarrow \text{237}$$

$$\int \frac{(d^2-e^2x^2)^{p-2}(d^2+e^2x^2)}{x}dx - \frac{2ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},2-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^3}$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(d^2-e^2x^2)^{p-2}(d^2+e^2x^2)}{x^2}dx^2 - \frac{2ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},2-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^3}$$

$$\Big\downarrow \text{88}$$

$$\frac{1}{2}\left(\int \frac{(d^2-e^2x^2)^{p-1}}{x^2}dx^2 + \frac{2(d^2-e^2x^2)^{p-1}}{1-p}\right) - \frac{2ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},2-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^3}$$

$$\Big\downarrow \text{75}$$

$$\frac{1}{2}\left(\frac{2(d^2-e^2x^2)^{p-1}}{1-p} - \frac{(d^2-e^2x^2)^p\text{Hypergeometric2F1}\left(1,p,p+1,1-\frac{e^2x^2}{d^2}\right)}{d^2p}\right) - \frac{2ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},2-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^3}$$

input
```
Int[(d^2 - e^2*x^2)^p/(x*(d + e*x)^2),x]
```

output
```
(-2*e*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 2 - p, 3/2, (e^2*x^2)/d^2
])/(d^3*(1 - (e^2*x^2)/d^2)^p) + ((2*(d^2 - e^2*x^2)^(-1 + p))/(1 - p) - (
(d^2 - e^2*x^2)^p*Hypergeometric2F1[1, p, 1 + p, 1 - (e^2*x^2)/d^2])/(d^2*
p))/2
```

3.281. $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^2}\,dx$

3.281.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

3.281. $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^2}\,dx$

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.281.4 Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x\,(ex+d)^2}dx$$

input
```
int((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x)
```

output
```
int((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x)
```

3.281.5 Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2x}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e^2*x^3 + 2*d*e*x^2 + d^2*x), x)
```

3.281.6 Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^2}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^p}{x\,(d+ex)^2}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**p/x/(e*x+d)**2,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p/(x*(d + e*x)**2), x)
```

3.281.7 Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x(d + ex)^2} \, dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^2 x} \, dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x), x)
```

3.281.8 Giac [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x(d + ex)^2} \, dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^2 x} \, dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x), x)
```

3.281.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^p}{x(d + ex)^2} \, dx = \int \frac{(d^2 - e^2\,x^2)^p}{x\,(d + e\,x)^2} \, dx$$

input
```
int((d^2 - e^2*x^2)^p/(x*(d + e*x)^2),x)
```

output
```
int((d^2 - e^2*x^2)^p/(x*(d + e*x)^2), x)
```

3.282 $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^2}\,dx$

3.282.1 Optimal result

Integrand size = 25, antiderivative size = 137

$$\begin{aligned}\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^2}\,dx &= -\frac{(d^2-e^2x^2)^{-1+p}}{x} \\ &\quad+\frac{2e^2(2-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},2-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^4} \\ &\quad-\frac{e(d^2-e^2x^2)^{-1+p}\,\text{Hypergeometric2F1}\left(1,-1+p,p,1-\frac{e^2x^2}{d^2}\right)}{d(1-p)}\end{aligned}$$

output
```
-(-e^2*x^2+d^2)^(-1+p)/x+2*e^2*(2-p)*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 2-
p],[3/2],e^2*x^2/d^2)/d^4/((1-e^2*x^2/d^2)^p)-e*(-e^2*x^2+d^2)^(-1+p)*hype
rgeom([1, -1+p],[p],1-e^2*x^2/d^2)/d/(1-p)
```

3.282.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.63

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^2}\,dx$$
$$= \frac{(d^2-e^2x^2)^p\left(-\frac{4d^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}+\frac{2^{2+p}e(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,1+p,2+p\right.}{1+p}\right.}{4d}$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^2),x]
```

output
```
((d^2 - e^2*x^2)^p*((-4*d^2*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2
])/(x*(1 - (e^2*x^2)/d^2)^p) + (2^(2 + p)*e*(-d + e*x)*Hypergeometric2F1[1
 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (2^p*e*(
-d + e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p
)*(1 + (e*x)/d)^p) - (4*d*e*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)
])/(p*(1 - d^2/(e^2*x^2))^p)))/(4*d^4)
```

3.282.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {570, 543, 27, 243, 75, 359, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^2}\,dx$$

$$\Big\downarrow \ 570$$

$$\int \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{p-2}}{x^2}dx$$

$$\Big\downarrow \ 543$$

$$\int -\frac{2de\left(d^2-e^2x^2\right)^{p-2}}{x}dx+\int \frac{\left(d^2-e^2x^2\right)^{p-2}\left(d^2+e^2x^2\right)}{x^2}dx$$

$$\Big\downarrow \ 27$$

$$\int \frac{(d^2-e^2x^2)^{p-2}(d^2+e^2x^2)}{x^2}dx-2de\int \frac{(d^2-e^2x^2)^{p-2}}{x}dx$$

$$\Big\downarrow \text{243}$$

$$\int \frac{(d^2-e^2x^2)^{p-2}(d^2+e^2x^2)}{x^2}dx-de\int \frac{(d^2-e^2x^2)^{p-2}}{x^2}dx^2$$

$$\Big\downarrow \text{75}$$

$$\int \frac{(d^2-e^2x^2)^{p-2}(d^2+e^2x^2)}{x^2}dx-\frac{e(d^2-e^2x^2)^{p-1}\text{Hypergeometric2F1}\left(1,p-1,p,1-\frac{e^2x^2}{d^2}\right)}{d(1-p)}$$

$$\Big\downarrow \text{359}$$

$$2e^2(2-p)\int (d^2-e^2x^2)^{p-2}dx-\frac{e(d^2-e^2x^2)^{p-1}\text{Hypergeometric2F1}\left(1,p-1,p,1-\frac{e^2x^2}{d^2}\right)}{d(1-p)}-\frac{(d^2-e^2x^2)^{p-1}}{x}$$

$$\Big\downarrow \text{238}$$

$$\frac{2e^2(2-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\int \left(1-\frac{e^2x^2}{d^2}\right)^{p-2}dx}{d^4}-\frac{e(d^2-e^2x^2)^{p-1}\text{Hypergeometric2F1}\left(1,p-1,p,1-\frac{e^2x^2}{d^2}\right)}{d(1-p)}-\frac{(d^2-e^2x^2)^{p-1}}{x}$$

$$\Big\downarrow \text{237}$$

$$-\frac{e(d^2-e^2x^2)^{p-1}\text{Hypergeometric2F1}\left(1,p-1,p,1-\frac{e^2x^2}{d^2}\right)}{d(1-p)}-\frac{(d^2-e^2x^2)^{p-1}}{x}+\frac{2e^2(2-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\text{Hypergeometric2F1}\left(\frac{1}{2},2-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^4}$$

input
```
Int[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^2),x]
```

output
```
-((d^2 - e^2*x^2)^(-1 + p)/x) + (2*e^2*(2 - p)*x*(d^2 - e^2*x^2)^p*Hyperge
ometric2F1[1/2, 2 - p, 3/2, (e^2*x^2)/d^2])/(d^4*(1 - (e^2*x^2)/d^2)^p) -
(e*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[1, -1 + p, p, 1 - (e^2*x^2)/
d^2])/(d*(1 - p))
```

3.282. $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^2}\,dx$

3.282.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.282.4 Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x^2(ex+d)^2}dx$$

input
```
int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x)
```

output
```
int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x)
```

3.282.5 Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2x^2}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e^2*x^4 + 2*d*e*x^3 + d^2*x^2), x)
```

3.282.6 Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^2}\,dx = \int \frac{(-(-d+ex)(d+ex))^p}{x^2(d+ex)^2}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d)**2,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p/(x**2*(d + e*x)**2), x)
```

3.282.7 Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^2(d + ex)^2} dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^2 x^2} dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^2), x)
```

3.282.8 Giac [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^2(d + ex)^2} dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^2 x^2} dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^2), x)
```

3.282.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^p}{x^2(d + ex)^2} dx = \int \frac{(d^2 - e^2 x^2)^p}{x^2 (d + e x)^2} dx$$

input
```
int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^2),x)
```

output
```
int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^2), x)
```

3.283 $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^2}\,dx$

3.283.1 Optimal result

Integrand size = 25, antiderivative size = 143

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^2}\,dx = -\frac{(d^2-e^2x^2)^{-1+p}}{2x^2} + \frac{2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},2-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{d^3x} + \frac{e^2(3-p)\left(d^2-e^2x^2\right)^{-1+p}\text{Hypergeometric2F1}\left(1,-1+p,p,1-\frac{e^2x^2}{d^2}\right)}{2d^2(1-p)}$$

output
```
-1/2*(-e^2*x^2+d^2)^(-1+p)/x^2+2*e*(-e^2*x^2+d^2)^p*hypergeom([-1/2, 2-p],
[1/2],e^2*x^2/d^2)/d^3/x/((1-e^2*x^2/d^2)^p)+1/2*e^2*(3-p)*(-e^2*x^2+d^2)^
(-1+p)*hypergeom([1, -1+p],[p],1-e^2*x^2/d^2)/d^2/(1-p)
```

3.283.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.98

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^2}\,dx$$
$$= \frac{(d^2-e^2x^2)^p\left(\frac{8d^2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}+\frac{2d^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,\frac{d^2}{e^2x^2}\right)}{(-1+p)x^2}+\right.}{}$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^2),x]
```

output
```
((d^2 - e^2*x^2)^p*((8*d^2*e*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^
2])/(x*(1 - (e^2*x^2)/d^2)^p) + (2*d^3*Hypergeometric2F1[1 - p, -p, 2 - p,
 d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (3*2^(1 + p)*e^2*(
d - e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)
*(1 + (e*x)/d)^p) + (2^p*e^2*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 +
 p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (6*d*e^2*Hypergeometric2
F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(4*d^5)
```

3.283.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {570, 543, 27, 279, 278, 354, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^2}dx$$
$$\Big\downarrow \text{570}$$
$$\int \frac{(d-ex)^2\,(d^2-e^2x^2)^{p-2}}{x^3}dx$$
$$\Big\downarrow \text{543}$$
$$\int -\frac{2de(d^2-e^2x^2)^{p-2}}{x^2}dx+\int \frac{(d^2-e^2x^2)^{p-2}\,(d^2+e^2x^2)}{x^3}dx$$
$$\Big\downarrow \text{27}$$

$$\int \frac{(d^2-e^2x^2)^{p-2}(d^2+e^2x^2)}{x^3}dx - 2de\int \frac{(d^2-e^2x^2)^{p-2}}{x^2}dx$$

$$\Big\downarrow \text{279}$$

$$\int \frac{(d^2-e^2x^2)^{p-2}(d^2+e^2x^2)}{x^3}dx - \frac{2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^{p-2}}{x^2}dx}{d^3}$$

$$\Big\downarrow \text{278}$$

$$\int \frac{(d^2-e^2x^2)^{p-2}(d^2+e^2x^2)}{x^3}dx + \frac{2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},2-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{d^3x}$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(d^2-e^2x^2)^{p-2}(d^2+e^2x^2)}{x^4}dx^2 + \frac{2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},2-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{d^3x}$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(e^2(3-p)\int \frac{(d^2-e^2x^2)^{p-2}}{x^2}dx^2 - \frac{(d^2-e^2x^2)^{p-1}}{x^2}\right) + \frac{2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},2-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{d^3x}$$

$$\Big\downarrow \text{75}$$

$$\frac{1}{2}\left(\frac{e^2(3-p)(d^2-e^2x^2)^{p-1}\text{Hypergeometric2F1}\left(1,p-1,p,1-\frac{e^2x^2}{d^2}\right)}{d^2(1-p)} - \frac{(d^2-e^2x^2)^{p-1}}{x^2}\right) + \frac{2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},2-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{d^3x}$$

input
```
Int[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^2),x]
```

output
```
(2*e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 2 - p, 1/2, (e^2*x^2)/d^2])
/(d^3*x*(1 - (e^2*x^2)/d^2)^p) + (-((d^2 - e^2*x^2)^(-1 + p)/x^2) + (e^2*(
3 - p)*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[1, -1 + p, p, 1 - (e^2*x
^2)/d^2])/(d^2*(1 - p)))/2
```

3.283. $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^2}\,dx$

3.283.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.283.4 Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x^3 (ex+d)^2}dx$$

input
```
int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x)
```

output
```
int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x)
```

3.283.5 Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2x^3}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e^2*x^5 + 2*d*e*x^4 + d^2*x^3), x)
```

3.283.6 Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^2}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^p}{x^3\,(d+ex)^2}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**p/x**3/(e*x+d)**2,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p/(x**3*(d + e*x)**2), x)
```

3.283.7 Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2x^3}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^3), x)
```

3.283.8 Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2x^3}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^3), x)
```

3.283.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^p}{x^3(d+ex)^2}\,dx = \int \frac{(d^2 - e^2\,x^2)^p}{x^3\,(d+e\,x)^2}\,dx$$

input
```
int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^2),x)
```

output
```
int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^2), x)
```

3.284 $\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx$

3.284.1 Optimal result

Integrand size = 25, antiderivative size = 145

$$\begin{aligned}\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx &= -\frac{(d^2-e^2x^2)^{-1+p}}{3x^3} \\ &\quad -\frac{2e^2(4-p)\,(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},2-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^4x} \\ &\quad -\frac{e^3(d^2-e^2x^2)^{-1+p}\,\text{Hypergeometric2F1}\left(2,-1+p,p,1-\frac{e^2x^2}{d^2}\right)}{d^3(1-p)}\end{aligned}$$

output
```
-1/3*(-e^2*x^2+d^2)^(-1+p)/x^3-2/3*e^2*(4-p)*(-e^2*x^2+d^2)^p*hypergeom([-
1/2, 2-p],[1/2],e^2*x^2/d^2)/d^4/x/((1-e^2*x^2/d^2)^p)-e^3*(-e^2*x^2+d^2)^
(-1+p)*hypergeom([2, -1+p],[p],1-e^2*x^2/d^2)/d^3/(1-p)
```

3.284.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 334 vs. 2(145) = 290.

Time = 0.81 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.30

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx$$
$$= \frac{(d^2-e^2x^2)^p\left(-\frac{4d^4\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{2},-p,-\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x^3}-\frac{36d^2e^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}\right.}{}$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^2),x]
```

output
```
((d^2 - e^2*x^2)^p*((-4*d^4*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d^
2])/(x^3*(1 - (e^2*x^2)/d^2)^p) - (36*d^2*e^2*Hypergeometric2F1[-1/2, -p,
1/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) - (12*d^3*e*Hypergeometric2
F1[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2)
+ (3*2^(3 + p)*e^3*(-d + e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d -
e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^p*e^3*(-d + e*x)*Hypergeomet
ric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) -
(24*d*e^3*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^
2*x^2))^p)))/(12*d^6)
```

3.284.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {570, 543, 27, 243, 75, 359, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{(d-ex)^2\,(d^2-e^2x^2)^{p-2}}{x^4}dx$$

$$\Big\downarrow \text{543}$$

$$\int \frac{\left(d^2-e^2x^2\right)^{p-2}\left(d^2+e^2x^2\right)}{x^4}dx+\int -\frac{2de\left(d^2-e^2x^2\right)^{p-2}}{x^3}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{\left(d^2-e^2x^2\right)^{p-2}\left(d^2+e^2x^2\right)}{x^4}dx-2de\int \frac{\left(d^2-e^2x^2\right)^{p-2}}{x^3}dx$$

$$\Big\downarrow \; 243$$

$$\int \frac{\left(d^2-e^2x^2\right)^{p-2}\left(d^2+e^2x^2\right)}{x^4}dx-de\int \frac{\left(d^2-e^2x^2\right)^{p-2}}{x^4}dx^2$$

$$\Big\downarrow \; 75$$

$$\int \frac{\left(d^2-e^2x^2\right)^{p-2}\left(d^2+e^2x^2\right)}{x^4}dx-\frac{e^3\left(d^2-e^2x^2\right)^{p-1}\text{Hypergeometric2F1}\left(2,p-1,p,1-\frac{e^2x^2}{d^2}\right)}{d^3(1-p)}$$

$$\Big\downarrow \; 359$$

$$\frac{2}{3}e^2(4-p)\int \frac{\left(d^2-e^2x^2\right)^{p-2}}{x^2}dx-\frac{\left(d^2-e^2x^2\right)^{p-1}}{3x^3}-\frac{e^3\left(d^2-e^2x^2\right)^{p-1}\text{Hypergeometric2F1}\left(2,p-1,p,1-\frac{e^2x^2}{d^2}\right)}{d^3(1-p)}$$

$$\Big\downarrow \; 279$$

$$\frac{2e^2(4-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^{p-2}}{x^2}dx}{3d^4}-\frac{\left(d^2-e^2x^2\right)^{p-1}}{3x^3}-\frac{e^3\left(d^2-e^2x^2\right)^{p-1}\text{Hypergeometric2F1}\left(2,p-1,p,1-\frac{e^2x^2}{d^2}\right)}{d^3(1-p)}$$

$$\Big\downarrow \; 278$$

$$-\frac{\left(d^2-e^2x^2\right)^{p-1}}{3x^3}-\frac{2e^2(4-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\text{Hypergeometric2F1}\left(-\frac{1}{2},2-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^4x}-\frac{e^3\left(d^2-e^2x^2\right)^{p-1}\text{Hypergeometric2F1}\left(2,p-1,p,1-\frac{e^2x^2}{d^2}\right)}{d^3(1-p)}$$

input
```
Int[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^2),x]
```

3.284. $\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx$

output
```
-1/3*(d^2 - e^2*x^2)^(-1 + p)/x^3 - (2*e^2*(4 - p)*(d^2 - e^2*x^2)^p*Hyper
geometric2F1[-1/2, 2 - p, 1/2, (e^2*x^2)/d^2])/(3*d^4*x*(1 - (e^2*x^2)/d^2
)^p) - (e^3*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[2, -1 + p, p, 1 - (
e^2*x^2)/d^2])/(d^3*(1 - p))
```

3.284.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.284.4 Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x^4(ex+d)^2}dx$$

input
```
int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x)
```

output
```
int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x)
```

3.284.5 Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2x^4}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e^2*x^6 + 2*d*e*x^5 + d^2*x^4), x)
```

3.284.6 Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx=\int \frac{(-(-d+ex)\,(d+ex))^p}{x^4\,(d+ex)^2}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**p/x**4/(e*x+d)**2,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p/(x**4*(d + e*x)**2), x)
```

3.284.7 Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx=\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2x^4}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^4), x)
```

3.284.8 Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx=\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2x^4}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^4), x)
```

3.284.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx = \int \frac{(d^2-e^2\,x^2)^p}{x^4\,(d+e\,x)^2}\,dx$$

input
```
int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^2),x)
```

output
```
int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^2), x)
```

3.285 $\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^2}\,dx$

3.285.1 Optimal result

Integrand size = 25, antiderivative size = 145

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^2}\,dx = -\frac{(d^2-e^2x^2)^{-1+p}}{4x^4} + \frac{2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{2},2-p,-\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^3x^3} + \frac{e^4(5-p)\,(d^2-e^2x^2)^{-1+p}\,\text{Hypergeometric2F1}\left(2,-1+p,p,1-\frac{e^2x^2}{d^2}\right)}{4d^4(1-p)}$$

output
```
-1/4*(-e^2*x^2+d^2)^(-1+p)/x^4+2/3*e*(-e^2*x^2+d^2)^p*hypergeom([-3/2, 2-p
],[-1/2],e^2*x^2/d^2)/d^3/x^3/((1-e^2*x^2/d^2)^p)+1/4*e^4*(5-p)*(-e^2*x^2+
d^2)^(-1+p)*hypergeom([2, -1+p],[p],1-e^2*x^2/d^2)/d^4/(1-p)
```

3.285.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 389 vs. 2(145) = 290.

Time = 0.90 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.68

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^2}\,dx = \frac{(d^2-e^2x^2)^p\left(\frac{8d^4e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{2},-p,-\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x^3}+\frac{48d^2e^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}\right.}{}$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^2),x]
```

output
```
((d^2 - e^2*x^2)^p*((8*d^4*e*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d
^2])/(x^3*(1 - (e^2*x^2)/d^2)^p) + (48*d^2*e^3*Hypergeometric2F1[-1/2, -p,
 1/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) + (18*d^3*e^2*Hypergeometr
ic2F1[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^
2) + (15*2^(1 + p)*e^4*(d - e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d
 - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (6*d^5*Hypergeometric2F1[2 - p
, -p, 3 - p, d^2/(e^2*x^2)])/((-2 + p)*(1 - d^2/(e^2*x^2))^p*x^4) + (3*2^p
*e^4*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((
1 + p)*(1 + (e*x)/d)^p) + (30*d*e^4*Hypergeometric2F1[-p, -p, 1 - p, d^2/(
e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(12*d^7)
```

3.285.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {570, 543, 27, 279, 278, 354, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^2}\,dx$$

$$\Big\downarrow \text{570}$$

$$\int \frac{(d-ex)^2(d^2-e^2x^2)^{p-2}}{x^5}dx$$

$$\Big\downarrow \text{543}$$

$$\int \frac{(d^2-e^2x^2)^{p-2}(d^2+e^2x^2)}{x^5}dx+\int -\frac{2de(d^2-e^2x^2)^{p-2}}{x^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(d^2-e^2x^2)^{p-2}(d^2+e^2x^2)}{x^5}dx-2de\int \frac{(d^2-e^2x^2)^{p-2}}{x^4}dx$$

$$\Big\downarrow \text{279}$$

$$\int \frac{(d^2-e^2x^2)^{p-2}(d^2+e^2x^2)}{x^5}dx-\frac{2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^{p-2}}{x^4}dx}{d^3}$$

$$\Big\downarrow \text{278}$$

$$\int \frac{(d^2-e^2x^2)^{p-2}(d^2+e^2x^2)}{x^5}dx+\frac{2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{2},2-p,-\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^3x^3}$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(d^2-e^2x^2)^{p-2}(d^2+e^2x^2)}{x^6}dx^2+\frac{2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{2},2-p,-\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^3x^3}$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{1}{2}e^2(5-p)\int \frac{(d^2-e^2x^2)^{p-2}}{x^4}dx^2-\frac{(d^2-e^2x^2)^{p-1}}{2x^4}\right)+\frac{2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{2},2-p,-\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^3x^3}$$

$$\Big\downarrow \text{75}$$

$$\frac{1}{2}\left(\frac{e^4(5-p)(d^2-e^2x^2)^{p-1}\text{Hypergeometric2F1}\left(2,p-1,p,1-\frac{e^2x^2}{d^2}\right)}{2d^4(1-p)}-\frac{(d^2-e^2x^2)^{p-1}}{2x^4}\right)+\frac{2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{2},2-p,-\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^3x^3}$$

input
```
Int[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^2),x]
```

3.285. $\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^2}\,dx$

output
```
(2*e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-3/2, 2 - p, -1/2, (e^2*x^2)/d^2]
)/(3*d^3*x^3*(1 - (e^2*x^2)/d^2)^p) + (-1/2*(d^2 - e^2*x^2)^(-1 + p)/x^4 +
 (e^4*(5 - p)*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[2, -1 + p, p, 1 -
 (e^2*x^2)/d^2])/(2*d^4*(1 - p)))/2
```

3.285.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.285. $\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^2} \, dx$

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.285.4 Maple [F]

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{x^{5} \left(e x +d \right)^{2}}d x$$

input
```
int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x)
```

output
```
int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x)
```

3.285.5 Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^2}\,dx=\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{\left(e x+d\right)^{2} x^{5}}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e^2*x^7 + 2*d*e*x^6 + d^2*x^5), x)
```

3.285.6 Sympy [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^5(d+ex)^2}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^p}{x^5\,(d+ex)^2}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**p/x**5/(e*x+d)**2,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p/(x**5*(d + e*x)**2), x)
```

3.285.7 Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^5(d+ex)^2}\,dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex+d)^2x^5}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^5), x)
```

3.285.8 Giac [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^5(d+ex)^2}\,dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex+d)^2x^5}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^5), x)
```

3.285.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^p}{x^5(d+ex)^2}\,dx = \int \frac{(d^2 - e^2\,x^2)^p}{x^5\,(d+e\,x)^2}\,dx$$

input
```
int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^2),x)
```

output
```
int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^2), x)
```

3.286 $\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3} dx$

3.286.1 Optimal result

Integrand size = 25, antiderivative size = 220

$$\begin{aligned}\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3}\,dx &= -\frac{2d^6(d^2-e^2x^2)^{-2+p}}{e^5(2-p)}-\frac{3dx^5(d^2-e^2x^2)^{-2+p}}{1+2p}\\ &\quad+\frac{9d^4(d^2-e^2x^2)^{-1+p}}{2e^5(1-p)}+\frac{3d^2(d^2-e^2x^2)^p}{e^5p}-\frac{(d^2-e^2x^2)^{1+p}}{2e^5(1+p)}\\ &\quad+\frac{2(8+p)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},3-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^3(1+2p)}\end{aligned}$$

output
```
-2*d^6*(-e^2*x^2+d^2)^(-2+p)/e^5/(2-p)-3*d*x^5*(-e^2*x^2+d^2)^(-2+p)/(1+2*
p)+9/2*d^4*(-e^2*x^2+d^2)^(-1+p)/e^5/(1-p)+3*d^2*(-e^2*x^2+d^2)^p/e^5/p-1/
2*(-e^2*x^2+d^2)^(p+1)/e^5/(p+1)+2/5*(8+p)*x^5*(-e^2*x^2+d^2)^p*hypergeom(
[5/2, 3-p],[7/2],e^2*x^2/d^2)/d^3/(1+2*p)/((1-e^2*x^2/d^2)^p)
```

3.286.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.11

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3}\,dx =$$
$$-\frac{2^{-3+p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(24de(1+p)x\left(\frac{1}{2}+\frac{ex}{2d}\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right.\right.}{}$$

input
```
Integrate[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

output
```
-((2^(-3 + p)*(d^2 - e^2*x^2)^p*(24*d*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hy
pergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^
2)^p*(4*d*(1/2 + (e*x)/(2*d))^p + 4*e*x*(1/2 + (e*x)/(2*d))^p + 24*d*Hyper
geometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - 8*d*Hypergeometric2F1
[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + d*Hypergeometric2F1[3 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)])))/(e^5*(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^
2)^p))
```

3.286.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {570, 543, 354, 25, 27, 86, 363, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$$
$$\Big\downarrow \text{570}$$
$$\int x^4(d-ex)^3\left(d^2-e^2x^2\right)^{p-3}dx$$
$$\Big\downarrow \text{543}$$
$$\int x^5\left(d^2-e^2x^2\right)^{p-3}\left(-x^2e^3-3d^2e\right)dx+\int x^4\left(d^2-e^2x^2\right)^{p-3}\left(d^3+3e^2x^2d\right)dx$$
$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int -ex^4\left(d^2-e^2x^2\right)^{p-3}\left(3d^2+e^2x^2\right)dx^2+\int x^4\left(d^2-e^2x^2\right)^{p-3}\left(d^3+3e^2x^2d\right)dx$$

$$\Big\downarrow \ 25$$

$$\int x^4\left(d^2-e^2x^2\right)^{p-3}\left(d^3+3e^2x^2d\right)dx-\frac{1}{2}\int ex^4\left(d^2-e^2x^2\right)^{p-3}\left(3d^2+e^2x^2\right)dx^2$$

$$\Big\downarrow \ 27$$

$$\int x^4\left(d^2-e^2x^2\right)^{p-3}\left(d^3+3e^2x^2d\right)dx-\frac{1}{2}e\int x^4\left(d^2-e^2x^2\right)^{p-3}\left(3d^2+e^2x^2\right)dx^2$$

$$\Big\downarrow \ 86$$

$$\int x^4\left(d^2-e^2x^2\right)^{p-3}\left(d^3+3e^2x^2d\right)dx-$$
$$\frac{1}{2}e\int\left(\frac{4d^6\left(d^2-e^2x^2\right)^{p-3}}{e^4}-\frac{9d^4\left(d^2-e^2x^2\right)^{p-2}}{e^4}+\frac{6d^2\left(d^2-e^2x^2\right)^{p-1}}{e^4}-\frac{\left(d^2-e^2x^2\right)^p}{e^4}\right)dx^2$$

$$\Big\downarrow \ 363$$

$$\frac{2d^3(p+8)\int x^4\left(d^2-e^2x^2\right)^{p-3}dx}{2p+1}-$$
$$\frac{1}{2}e\int\left(\frac{4d^6\left(d^2-e^2x^2\right)^{p-3}}{e^4}-\frac{9d^4\left(d^2-e^2x^2\right)^{p-2}}{e^4}+\frac{6d^2\left(d^2-e^2x^2\right)^{p-1}}{e^4}-\frac{\left(d^2-e^2x^2\right)^p}{e^4}\right)dx^2-$$
$$\frac{3dx^5\left(d^2-e^2x^2\right)^{p-2}}{2p+1}$$

$$\Big\downarrow \ 279$$

$$\frac{2(p+8)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^{p-3}dx}{d^3(2p+1)}-$$
$$\frac{1}{2}e\int\left(\frac{4d^6\left(d^2-e^2x^2\right)^{p-3}}{e^4}-\frac{9d^4\left(d^2-e^2x^2\right)^{p-2}}{e^4}+\frac{6d^2\left(d^2-e^2x^2\right)^{p-1}}{e^4}-\frac{\left(d^2-e^2x^2\right)^p}{e^4}\right)dx^2-$$
$$\frac{3dx^5\left(d^2-e^2x^2\right)^{p-2}}{2p+1}$$

$$\Big\downarrow \ 278$$

$$-\frac{1}{2}e\int\left(\frac{4d^6\left(d^2-e^2x^2\right)^{p-3}}{e^4}-\frac{9d^4\left(d^2-e^2x^2\right)^{p-2}}{e^4}+\frac{6d^2\left(d^2-e^2x^2\right)^{p-1}}{e^4}-\frac{\left(d^2-e^2x^2\right)^p}{e^4}\right)dx^2-$$
$$\frac{3dx^5\left(d^2-e^2x^2\right)^{p-2}}{2p+1}+\frac{2(p+8)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\text{Hypergeometric2F1}\left(\frac{5}{2},3-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^3(2p+1)}$$

$$\Big\downarrow \ 2009$$

3.286. $\int\frac{x^4\left(d^2-e^2x^2\right)^p}{(d+ex)^3}dx$

$$-\frac{3dx^5(d^2-e^2x^2)^{p-2}}{2p+1}+\frac{2(p+8)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{5}{2},3-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^3(2p+1)}-\frac{1}{2}e\left(-\frac{6d^2(d^2-e^2x^2)^p}{e^6p}+\frac{(d^2-e^2x^2)^{p+1}}{e^6(p+1)}+\frac{4d^6(d^2-e^2x^2)^{p-2}}{e^6(2-p)}-\frac{9d^4(d^2-e^2x^2)^{p-1}}{e^6(1-p)}\right)$$

input
```
Int[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

output
```
(-3*d*x^5*(d^2 - e^2*x^2)^(-2 + p))/(1 + 2*p) - (e*((4*d^6*(d^2 - e^2*x^2)
^(-2 + p))/(e^6*(2 - p)) - (9*d^4*(d^2 - e^2*x^2)^(-1 + p))/(e^6*(1 - p))
- (6*d^2*(d^2 - e^2*x^2)^p)/(e^6*p) + (d^2 - e^2*x^2)^(1 + p)/(e^6*(1 + p)
)))/2 + (2*(8 + p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 3 - p, 7/2
, (e^2*x^2)/d^2])/(5*d^3*(1 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

3.286.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

3.286. $\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.286. $\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$

3.286.4 Maple [F]

$$\int \frac{x^4(-e^2x^2+d^2)^p}{(ex+d)^3}dx$$

input
```
int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

output
```
int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

3.286.5 Fricas [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p x^4}{(ex+d)^3}\,dx$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*x^4/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3),
 x)
```

3.286.6 Sympy [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{x^4(-(-d+ex)(d+ex))^p}{(d+ex)^3}\,dx$$

input
```
integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
```

output
```
Integral(x**4*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

3.286.7 Maxima [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p x^4}{(ex+d)^3}\,dx$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^3, x)
```

3.286.8 Giac [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p x^4}{(ex+d)^3}\,dx$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^3, x)
```

3.286.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{x^4\,(d^2-e^2\,x^2)^p}{(d+e\,x)^3}\,dx$$

input
```
int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x)
```

output
```
int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^3, x)
```

3.287 $\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$

3.287.1 Optimal result

Integrand size = 25, antiderivative size = 194

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \frac{2d^5(d^2-e^2x^2)^{-2+p}}{e^4(2-p)}+\frac{ex^5(d^2-e^2x^2)^{-2+p}}{1+2p}-\frac{7d^3(d^2-e^2x^2)^{-1+p}}{2e^4(1-p)}-\frac{3d(d^2-e^2x^2)^p}{2e^4p}-\frac{2e(4+3p)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},3-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^4(1+2p)}$$

output
```
2*d^5*(-e^2*x^2+d^2)^(-2+p)/e^4/(2-p)+e*x^5*(-e^2*x^2+d^2)^(-2+p)/(1+2*p)-
7/2*d^3*(-e^2*x^2+d^2)^(-1+p)/e^4/(1-p)-3/2*d*(-e^2*x^2+d^2)^p/e^4/p-2/5*e
*(4+3*p)*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, 3-p],[7/2],e^2*x^2/d^2)/d^4/
(1+2*p)/((1-e^2*x^2/d^2)^p)
```

3.287.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.04

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \frac{2^{-3+p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(8e(1+p)x\left(\frac{1}{2}+\frac{ex}{2d}\right)^p\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)+\right.}{}$$

input
```
Integrate[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

output
```
(2^(-3 + p)*(d^2 - e^2*x^2)^p*(8*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hyperge
ometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^2)^p*
(12*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - 6*Hypergeome
tric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + Hypergeometric2F1[3 - p, 1
 + p, 2 + p, (d - e*x)/(2*d)])))/(e^4*(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^
2)/d^2)^p)
```

3.287.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {570, 543, 354, 27, 86, 363, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (d^2-e^2x^2)^p}{(d+ex)^3} \, dx$$

$$\Big\downarrow \text{570}$$

$$\int x^3 (d-ex)^3 \left(d^2-e^2x^2\right)^{p-3} dx$$

$$\Big\downarrow \text{543}$$

$$\int x^4 \left(d^2-e^2x^2\right)^{p-3} \left(-x^2e^3-3d^2e\right) dx + \int x^3 \left(d^2-e^2x^2\right)^{p-3} \left(d^3+3e^2x^2d\right) dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int dx^2 \left(d^2-e^2x^2\right)^{p-3} \left(d^2+3e^2x^2\right) dx^2 + \int x^4 \left(d^2-e^2x^2\right)^{p-3} \left(-x^2e^3-3d^2e\right) dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}d\int x^2 \left(d^2-e^2x^2\right)^{p-3} \left(d^2+3e^2x^2\right) dx^2 + \int x^4 \left(d^2-e^2x^2\right)^{p-3} \left(-x^2e^3-3d^2e\right) dx$$

$$\Big\downarrow \text{86}$$

$$\int x^4 \left(d^2-e^2x^2\right)^{p-3} \left(-x^2e^3-3d^2e\right) dx + \frac{1}{2}d\int \left(\frac{4d^4 \left(d^2-e^2x^2\right)^{p-3}}{e^2} - \frac{7d^2 \left(d^2-e^2x^2\right)^{p-2}}{e^2} + \frac{3 \left(d^2-e^2x^2\right)^{p-1}}{e^2}\right) dx^2$$

$$\Big\downarrow \text{363}$$

$$-\frac{2d^2e(3p+4)\int x^4(d^2-e^2x^2)^{p-3}\,dx}{2p+1}+$$
$$\frac{1}{2}d\int\left(\frac{4d^4(d^2-e^2x^2)^{p-3}}{e^2}-\frac{7d^2(d^2-e^2x^2)^{p-2}}{e^2}+\frac{3(d^2-e^2x^2)^{p-1}}{e^2}\right)dx^2+\frac{ex^5(d^2-e^2x^2)^{p-2}}{2p+1}$$

$$\Big\downarrow \text{279}$$

$$\frac{1}{2}d\int\left(\frac{4d^4(d^2-e^2x^2)^{p-3}}{e^2}-\frac{7d^2(d^2-e^2x^2)^{p-2}}{e^2}+\frac{3(d^2-e^2x^2)^{p-1}}{e^2}\right)dx^2-$$
$$\frac{2e(3p+4)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^{p-3}dx}{d^4(2p+1)}+\frac{ex^5(d^2-e^2x^2)^{p-2}}{2p+1}$$

$$\Big\downarrow \text{278}$$

$$\frac{1}{2}d\int\left(\frac{4d^4(d^2-e^2x^2)^{p-3}}{e^2}-\frac{7d^2(d^2-e^2x^2)^{p-2}}{e^2}+\frac{3(d^2-e^2x^2)^{p-1}}{e^2}\right)dx^2+\frac{ex^5(d^2-e^2x^2)^{p-2}}{2p+1}-$$
$$\frac{2e(3p+4)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{5}{2},3-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^4(2p+1)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ex^5(d^2-e^2x^2)^{p-2}}{2p+1}-$$
$$\frac{2e(3p+4)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{5}{2},3-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^4(2p+1)}+$$
$$\frac{1}{2}d\left(-\frac{7d^2(d^2-e^2x^2)^{p-1}}{e^4(1-p)}-\frac{3(d^2-e^2x^2)^p}{e^4p}+\frac{4d^4(d^2-e^2x^2)^{p-2}}{e^4(2-p)}\right)$$

input
```
Int[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

output
```
(e*x^5*(d^2 - e^2*x^2)^(-2 + p))/(1 + 2*p) + (d*((4*d^4*(d^2 - e^2*x^2)^(-
2 + p))/(e^4*(2 - p)) - (7*d^2*(d^2 - e^2*x^2)^(-1 + p))/(e^4*(1 - p)) - (
3*(d^2 - e^2*x^2)^p)/(e^4*p)))/2 - (2*e*(4 + 3*p)*x^5*(d^2 - e^2*x^2)^p*Hy
pergeometric2F1[5/2, 3 - p, 7/2, (e^2*x^2)/d^2])/(5*d^4*(1 + 2*p)*(1 - (e^
2*x^2)/d^2)^p)
```

3.287. $\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$

3.287.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.287. $\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.287.4 Maple [F]

$$\int \frac{x^3(-e^2x^2+d^2)^p}{(ex+d)^3}dx$$

input
```
int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

output
```
int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

3.287.5 Fricas [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^3}\,dx=\int \frac{(-e^2x^2+d^2)^p x^3}{(ex+d)^3}\,dx$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*x^3/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3),
 x)
```

3.287.6 Sympy [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{x^3(-(-d+ex)\,(d+ex))^p}{(d+ex)^3}\,dx$$

input
```
integrate(x**3*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
```

output
```
Integral(x**3*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

3.287.7 Maxima [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p x^3}{(ex+d)^3}\,dx$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^3, x)
```

3.287.8 Giac [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p x^3}{(ex+d)^3}\,dx$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^3, x)
```

3.287.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{x^3\,(d^2-e^2\,x^2)^p}{(d+e\,x)^3}\,dx$$

input
```
int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x)
```

output
```
int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^3, x)
```

3.288 $\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^3} dx$

3.288.1 Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = -\frac{d(d^2-e^2x^2)^{1+p}}{2e^3(2-p)(d+ex)^3} - \frac{(d^2-e^2x^2)^{1+p}}{2e^3p(d+ex)^2} + \frac{2^{-3+p}(4+p)\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(2-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^2e^3(2-p)p(1+p)}$$

output
```
-1/2*d*(-e^2*x^2+d^2)^(p+1)/e^3/(2-p)/(e*x+d)^3-1/2*(-e^2*x^2+d^2)^(p+1)/e
^3/p/(e*x+d)^2+2^(-3+p)*(4+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(p+1)*hyperg
eom([p+1, 2-p],[2+p],1/2*(-e*x+d)/d)/d^2/e^3/p/(-p^2+p+2)
```

3.288.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.83

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = -\frac{2^{-3+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(4\,\text{Hypergeometric2F1}\left(1-p,1+p,2+p,\frac{d-ex}{2d}\right)-4\,\text{Hypergeon}\right.}{de^3(1+p)}$$

input
```
Integrate[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

output
```
-((2^(-3 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(4*Hypergeometric2F1[1 - p, 1 +
p, 2 + p, (d - e*x)/(2*d)] - 4*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d -
 e*x)/(2*d)] + Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(
d*e^3*(1 + p)*(1 + (e*x)/d)^p))
```

3.288.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {581, 27, 671, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(d^2-e^2 x^2\right)^p}{(d+e x)^3} \, dx$$

$$\Big\downarrow \text{581}$$

$$\frac{\int -\frac{2 d (d+e (p+1) x) \left(d^2-e^2 x^2\right)^p}{(d+e x)^3}dx}{2 e^2 p}-\frac{\left(d^2-e^2 x^2\right)^{p+1}}{2 e^3 p (d+e x)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d \int \frac{(d+e (p+1) x) \left(d^2-e^2 x^2\right)^p}{(d+e x)^3}dx}{e^2 p}-\frac{\left(d^2-e^2 x^2\right)^{p+1}}{2 e^3 p (d+e x)^2}$$

$$\Big\downarrow \text{671}$$

$$-\frac{d \left(\frac{(p+4) \int \frac{\left(d^2-e^2 x^2\right)^p}{(d+e x)^2}dx}{2 (2-p)}+\frac{p \left(d^2-e^2 x^2\right)^{p+1}}{2 e (2-p) (d+e x)^3}\right)}{e^2 p}-\frac{\left(d^2-e^2 x^2\right)^{p+1}}{2 e^3 p (d+e x)^2}$$

$$\Big\downarrow \text{473}$$

$$-\frac{d \left(\frac{(p+4) (d-e x)^{-p-1} \left(d^2-e^2 x^2\right)^{p+1} \left(\frac{e x}{d}+1\right)^{-p-1} \int (d-e x)^p \left(\frac{e x}{d}+1\right)^{p-2}dx}{2 d^3 (2-p)}+\frac{p \left(d^2-e^2 x^2\right)^{p+1}}{2 e (2-p) (d+e x)^3}\right)}{e^2 p}-\frac{\left(d^2-e^2 x^2\right)^{p+1}}{2 e^3 p (d+e x)^2}$$

$$\Big\downarrow \text{79}$$

$$-\frac{(d^2-e^2x^2)^{p+1}}{2e^3p(d+ex)^2}-\frac{d\left(\frac{p(d^2-e^2x^2)^{p+1}}{2e(2-p)(d+ex)^3}-\frac{2^{p-3}(p+4)\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\,\text{Hypergeometric2F1}\left(2-p,p+1,p+2,\frac{d-ex}{2d}\right)}{d^3e(2-p)(p+1)}\right)}{e^2p}$$

input
```
Int[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

output
```
-1/2*(d^2 - e^2*x^2)^(1 + p)/(e^3*p*(d + e*x)^2) - (d*((p*(d^2 - e^2*x^2)^
(1 + p))/(2*e*(2 - p)*(d + e*x)^3) - (2^(-3 + p)*(4 + p)*(1 + (e*x)/d)^(-1
 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d -
e*x)/(2*d)])/(d^3*e*(2 - p)*(1 + p))))/(e^2*p)
```

3.288.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

3.288.4 Maple [F]

$$\int \frac{x^2(-e^2x^2+d^2)^p}{(ex+d)^3}dx$$

input
```
int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

output
```
int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

3.288.5 Fricas [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^3}\,dx=\int \frac{(-e^2x^2+d^2)^px^2}{(ex+d)^3}\,dx$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*x^2/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3),
 x)
```

3.288.6 Sympy [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{x^2(-(-d+ex)\,(d+ex))^p}{(d+ex)^3}\,dx$$

input
```
integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
```

output
```
Integral(x**2*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

3.288.7 Maxima [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p x^2}{(ex+d)^3}\,dx$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^3, x)
```

3.288.8 Giac [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p x^2}{(ex+d)^3}\,dx$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^3, x)
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^3}\,dx=\int \frac{x^2\,{\left(d^2-e^2\,x^2\right)}^p}{{\left(d+e\,x\right)}^3} \,d x$$

input
```
int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x)
```

output
```
int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^3, x)
```

3.289 $\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$

3.289.1 Optimal result

Integrand size = 23, antiderivative size = 118

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \frac{(d^2-e^2x^2)^{1+p}}{2e^2(2-p)(d+ex)^3} - \frac{3\ 2^{-3+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\text{Hypergeometric2F1}\left(2-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^3e^2(2-p)(1+p)}$$

output
```
1/2*(-e^2*x^2+d^2)^(p+1)/e^2/(2-p)/(e*x+d)^3-3*2^(-3+p)*(1+e*x/d)^(-1-p)*(
-e^2*x^2+d^2)^(p+1)*hypergeom([p+1, 2-p],[2+p],1/2*(-e*x+d)/d)/d^3/e^2/(-p
^2+p+2)
```

3.289.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.86

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \frac{2^{-3+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(-2\,\text{Hypergeometric2F1}\left(2-p,1+p,2+p,\frac{d-ex}{2d}\right)+\text{Hypergeome}\right.}{d^2e^2(1+p)}$$

input
```
Integrate[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

output
```
(2^(-3 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(-2*Hypergeometric2F1[2 - p, 1 + p
, 2 + p, (d - e*x)/(2*d)] + Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*
x)/(2*d)]))/(d^2*e^2*(1 + p)*(1 + (e*x)/d)^p)
```

3.289.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {571, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^3}dx$$

$$\Big\downarrow \text{571}$$

$$\frac{3\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2}dx}{2e(2-p)}+\frac{(d^2-e^2x^2)^{p+1}}{2e^2(2-p)(d+ex)^3}$$

$$\Big\downarrow \text{473}$$

$$\frac{3(d-ex)^{-p-1}(d^2-e^2x^2)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}\int (d-ex)^p\left(\frac{ex}{d}+1\right)^{p-2}dx}{2d^3e(2-p)}+\frac{(d^2-e^2x^2)^{p+1}}{2e^2(2-p)(d+ex)^3}$$

$$\Big\downarrow \text{79}$$

$$\frac{(d^2-e^2x^2)^{p+1}}{2e^2(2-p)(d+ex)^3}-\frac{3\ 2^{p-3}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\text{Hypergeometric2F1}\left(2-p,p+1,p+2,\frac{d-ex}{2d}\right)}{d^3e^2(2-p)(p+1)}$$

input
```
Int[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

output
```
(d^2 - e^2*x^2)^(1 + p)/(2*e^2*(2 - p)*(d + e*x)^3) - (3*2^(-3 + p)*(1 + (
e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[2 - p, 1 + p, 2
 + p, (d - e*x)/(2*d)])/(d^3*e^2*(2 - p)*(1 + p))
```

3.289.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 571
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*(n + p + 1))), x] + Simp[n/(2*d*
(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ((LtQ[n, -1] &&  !IGtQ[n + p +
1, 0]) || (LtQ[n, 0] && LtQ[p, -1]) || EqQ[n + 2*p + 2, 0]) && NeQ[n + p +
1, 0]
```

3.289.4 Maple [F]

$$\int \frac{x(-e^2x^2+d^2)^p}{(ex+d)^3}dx$$

input
```
int(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

output
```
int(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

3.289.5 Fricas [F]

$$\int \frac{x(d^2 - e^2x^2)^p}{(d + ex)^3} \, dx = \int \frac{(-e^2x^2 + d^2)^p x}{(ex + d)^3} \, dx$$

input
```
integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*x/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x
)
```

3.289.6 Sympy [F]

$$\int \frac{x(d^2 - e^2x^2)^p}{(d + ex)^3} \, dx = \int \frac{x(-(-d + ex)\,(d + ex))^p}{(d + ex)^3} \, dx$$

input
```
integrate(x*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
```

output
```
Integral(x*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

3.289.7 Maxima [F]

$$\int \frac{x(d^2 - e^2x^2)^p}{(d + ex)^3} \, dx = \int \frac{(-e^2x^2 + d^2)^p x}{(ex + d)^3} \, dx$$

input
```
integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^3, x)
```

3.289.8 Giac [F]

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p x}{(ex+d)^3}\,dx$$

input
```
integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^3, x)
```

3.289.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{x\,(d^2-e^2\,x^2)^p}{(d+e\,x)^3}\,dx$$

input
```
int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x)
```

output
```
int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^3, x)
```

3.290 $\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$

3.290.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = -\frac{2^{-3+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(3-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^4e(1+p)}$$

output
```
-2^(-3+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(p+1)*hypergeom([p+1, 3-p],[2+p]
,1/2*(-e*x+d)/d)/d^4/e/(p+1)
```

3.290.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = -\frac{2^{-3+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(3-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^3e(1+p)}$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(d + e*x)^3,x]
```

output
```
-((2^(-3 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3 - p, 1 + p,
2 + p, (d - e*x)/(2*d)])/(d^3*e*(1 + p)*(1 + (e*x)/d)^p))
```

3.290.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$$

$$\Big\downarrow \; 473$$

$$\frac{(d-ex)^{-p-1}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\int (d-ex)^p\left(\frac{ex}{d}+1\right)^{p-3}dx}{d^4}$$

$$\Big\downarrow \; 79$$

$$-\frac{2^{p-3}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\text{Hypergeometric2F1}\left(3-p,p+1,p+2,\frac{d-ex}{2d}\right)}{d^4e(p+1)}$$

input
```
Int[(d^2 - e^2*x^2)^p/(d + e*x)^3,x]
```

output
```
-((2^(-3 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometri
c2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^4*e*(1 + p)))
```

3.290.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

3.290.4 Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3}dx$$

input
```
int((-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

output
```
int((-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

3.290.5 Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

3.290.6 Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^p}{(d+ex)^3}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**p/(e*x+d)**3,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

3.290.7 Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/(e*x + d)^3, x)
```

3.290.8 Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/(e*x + d)^3, x)
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(d^2-e^2\,x^2)^p}{(d+e\,x)^3}\,dx$$

input
```
int((d^2 - e^2*x^2)^p/(d + e*x)^3,x)
```

output
```
int((d^2 - e^2*x^2)^p/(d + e*x)^3, x)
```

3.291 $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^3} dx$

3.291.1 Optimal result

Integrand size = 25, antiderivative size = 175

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^3} dx = \frac{2d(d^2-e^2x^2)^{-2+p}}{2-p} - \frac{ex(d^2-e^2x^2)^{-2+p}}{3-2p} - \frac{2e(4-3p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},3-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^4(3-2p)} + \frac{(d^2-e^2x^2)^{-1+p}\text{Hypergeometric2F1}\left(1,-1+p,p,1-\frac{e^2x^2}{d^2}\right)}{2d(1-p)}$$

output
```
2*d*(-e^2*x^2+d^2)^(-2+p)/(2-p)-e*x*(-e^2*x^2+d^2)^(-2+p)/(3-2*p)-2*e*(4-3
*p)*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 3-p],[3/2],e^2*x^2/d^2)/d^4/(3-2*p)
/((1-e^2*x^2/d^2)^p)+1/2*(-e^2*x^2+d^2)^(-1+p)*hypergeom([1, -1+p],[p],1-e
^2*x^2/d^2)/d/(1-p)
```

3.291.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.87

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^3}\,dx$$
$$= \frac{2^{-3+p}\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(4p\left(1-\frac{d^2}{e^2x^2}\right)^p(d-ex)\,\text{Hypergeometric2F1}\left(1-p,1+p,2-\right.\right.}{}$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(x*(d + e*x)^3),x]
```

output
```
(2^(-3 + p)*(d^2 - e^2*x^2)^p*(4*p*(1 - d^2/(e^2*x^2))^p*(d - e*x)*Hyperge
ometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 2*p*(1 - d^2/(e^2*x^2))
^p*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + d*p
*(1 - d^2/(e^2*x^2))^p*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2
*d)] - e*p*(1 - d^2/(e^2*x^2))^p*x*Hypergeometric2F1[3 - p, 1 + p, 2 + p,
(d - e*x)/(2*d)] + 4*d*(1/2 + (e*x)/(2*d))^p*Hypergeometric2F1[-p, -p, 1 -
 p, d^2/(e^2*x^2)] + 4*d*p*(1/2 + (e*x)/(2*d))^p*Hypergeometric2F1[-p, -p,
 1 - p, d^2/(e^2*x^2)]))/(d^4*p*(1 + p)*(1 - d^2/(e^2*x^2))^p*(1 + (e*x)/d
)^p)
```

3.291.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {570, 543, 299, 238, 237, 354, 27, 88, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^3}\,dx$$
$$\Big\downarrow \ 570$$
$$\int \frac{(d-ex)^3(d^2-e^2x^2)^{p-3}}{x}dx$$
$$\Big\downarrow \ 543$$
$$\int (d^2-e^2x^2)^{p-3}(-x^2e^3-3d^2e)\,dx+\int \frac{(d^2-e^2x^2)^{p-3}(d^3+3e^2x^2d)}{x}dx$$

$$\Big\downarrow \text{299}$$

$$-\frac{2d^2e(4-3p)\int (d^2-e^2x^2)^{p-3}dx}{3-2p}+\int \frac{(d^2-e^2x^2)^{p-3}(d^3+3e^2x^2d)}{x}dx-\frac{ex(d^2-e^2x^2)^{p-2}}{3-2p}$$

$$\Big\downarrow \text{238}$$

$$-\frac{2e(4-3p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\int \left(1-\frac{e^2x^2}{d^2}\right)^{p-3}dx}{d^4(3-2p)}+\int \frac{(d^2-e^2x^2)^{p-3}(d^3+3e^2x^2d)}{x}dx-\frac{ex(d^2-e^2x^2)^{p-2}}{3-2p}$$

$$\Big\downarrow \text{237}$$

$$\int \frac{(d^2-e^2x^2)^{p-3}(d^3+3e^2x^2d)}{x}dx-\frac{ex(d^2-e^2x^2)^{p-2}}{3-2p}-\frac{2e(4-3p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\text{Hypergeometric2F1}\left(\frac{1}{2},3-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^4(3-2p)}$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{d(d^2-e^2x^2)^{p-3}(d^2+3e^2x^2)}{x^2}dx^2-\frac{ex(d^2-e^2x^2)^{p-2}}{3-2p}-\frac{2e(4-3p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\text{Hypergeometric2F1}\left(\frac{1}{2},3-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^4(3-2p)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}d\int \frac{(d^2-e^2x^2)^{p-3}(d^2+3e^2x^2)}{x^2}dx^2-\frac{ex(d^2-e^2x^2)^{p-2}}{3-2p}-\frac{2e(4-3p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\text{Hypergeometric2F1}\left(\frac{1}{2},3-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^4(3-2p)}$$

$$\Big\downarrow \text{88}$$

$$\frac{1}{2}d\left(\int \frac{(d^2-e^2x^2)^{p-2}}{x^2}dx^2+\frac{4(d^2-e^2x^2)^{p-2}}{2-p}\right)-\frac{ex(d^2-e^2x^2)^{p-2}}{3-2p}-\frac{2e(4-3p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\text{Hypergeometric2F1}\left(\frac{1}{2},3-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^4(3-2p)}$$

$$\Big\downarrow \text{75}$$

3.291. $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^3}\,dx$

$$\frac{1}{2}d\left(\frac{\left(d^2-e^2x^2\right)^{p-1}\text{Hypergeometric2F1}\left(1,p-1,p,1-\frac{e^2x^2}{d^2}\right)}{d^2(1-p)}+\frac{4\left(d^2-e^2x^2\right)^{p-2}}{2-p}\right)-\frac{ex\left(d^2-e^2x^2\right)^{p-2}}{3-2p}-\frac{2e(4-3p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\text{Hypergeometric2F1}\left(\frac{1}{2},3-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^4(3-2p)}$$

input
```
Int[(d^2 - e^2*x^2)^p/(x*(d + e*x)^3),x]
```

output
```
-((e*x*(d^2 - e^2*x^2)^(-2 + p))/(3 - 2*p)) - (2*e*(4 - 3*p)*x*(d^2 - e^2*
x^2)^p*Hypergeometric2F1[1/2, 3 - p, 3/2, (e^2*x^2)/d^2])/(d^4*(3 - 2*p)*(
1 - (e^2*x^2)/d^2)^p) + (d*((4*(d^2 - e^2*x^2)^(-2 + p))/(2 - p) + ((d^2 -
 e^2*x^2)^(-1 + p)*Hypergeometric2F1[1, -1 + p, p, 1 - (e^2*x^2)/d^2])/(d^
2*(1 - p))))/2
```

3.291.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

3.291. $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^3}\,dx$

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.291.4 Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x\,(ex+d)^3}dx$$

input
```
int((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x)
```

output
```
int((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x)
```

3.291.5 Fricas [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x(d + ex)^3} dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^3 x} dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e*x^2 + d^3*x),
 x)
```

3.291.6 Sympy [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x(d + ex)^3} dx = \int \frac{(-(-d + ex)(d + ex))^p}{x(d + ex)^3} dx$$

input
```
integrate((-e**2*x**2+d**2)**p/x/(e*x+d)**3,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p/(x*(d + e*x)**3), x)
```

3.291.7 Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x(d + ex)^3} dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^3 x} dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x), x)
```

3.291.8 Giac [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x(d + ex)^3} dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^3 x} dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x), x)
```

3.291.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^p}{x(d + ex)^3} dx = \int \frac{(d^2 - e^2 x^2)^p}{x (d + e x)^3} dx$$

input
```
int((d^2 - e^2*x^2)^p/(x*(d + e*x)^3),x)
```

output
```
int((d^2 - e^2*x^2)^p/(x*(d + e*x)^3), x)
```

3.292 $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^3}\,dx$

3.292.1 Optimal result

Integrand size = 25, antiderivative size = 166

$$\begin{aligned}\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^3}\,dx &= -\frac{2e(d^2-e^2x^2)^{-2+p}}{2-p}-\frac{d(d^2-e^2x^2)^{-2+p}}{x} \\ &\quad+\frac{2e^2(4-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},3-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^5} \\ &\quad-\frac{3e(d^2-e^2x^2)^{-1+p}\,\text{Hypergeometric2F1}\left(1,-1+p,p,1-\frac{e^2x^2}{d^2}\right)}{2d^2(1-p)}\end{aligned}$$

output
```
-2*e*(-e^2*x^2+d^2)^(-2+p)/(2-p)-d*(-e^2*x^2+d^2)^(-2+p)/x+2*e^2*(4-p)*x*(
-e^2*x^2+d^2)^p*hypergeom([1/2, 3-p],[3/2],e^2*x^2/d^2)/d^5/((1-e^2*x^2/d^
2)^p)-3/2*e*(-e^2*x^2+d^2)^(-1+p)*hypergeom([1, -1+p],[p],1-e^2*x^2/d^2)/d
^2/(1-p)
```

3.292.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.69

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^3}\,dx$$
$$= \frac{(d^2-e^2x^2)^p\left(-\frac{8d^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}+\frac{3\ 2^{2+p}e(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,1+p,2\right.}{1+p}\right.}{}$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^3),x]
```

output
```
((d^2 - e^2*x^2)^p*((-8*d^2*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2
])/(x*(1 - (e^2*x^2)/d^2)^p) + (3*2^(2 + p)*e*(-d + e*x)*Hypergeometric2F1
[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (2^(2
+ p)*e*(-d + e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])
/((1 + p)*(1 + (e*x)/d)^p) + (2^p*e*(-d + e*x)*Hypergeometric2F1[3 - p, 1
+ p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) - (12*d*e*Hypergeo
metric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(8*d^
5)
```

3.292.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {570, 543, 354, 25, 27, 88, 75, 359, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^3}\,dx$$
$$\Big\downarrow \text{570}$$
$$\int \frac{(d-ex)^3\left(d^2-e^2x^2\right)^{p-3}}{x^2}dx$$
$$\Big\downarrow \text{543}$$
$$\int \frac{\left(d^2-e^2x^2\right)^{p-3}\left(-x^2e^3-3d^2e\right)}{x}dx+\int \frac{\left(d^2-e^2x^2\right)^{p-3}\left(d^3+3e^2x^2d\right)}{x^2}dx$$

$$\Big\downarrow 354$$

$$\frac{1}{2}\int -\frac{e\left(d^2-e^2x^2\right)^{p-3}\left(3d^2+e^2x^2\right)}{x^2}dx^2+\int \frac{\left(d^2-e^2x^2\right)^{p-3}\left(d^3+3e^2x^2d\right)}{x^2}dx$$

$$\Big\downarrow 25$$

$$\int \frac{\left(d^2-e^2x^2\right)^{p-3}\left(d^3+3e^2x^2d\right)}{x^2}dx-\frac{1}{2}\int \frac{e\left(d^2-e^2x^2\right)^{p-3}\left(3d^2+e^2x^2\right)}{x^2}dx^2$$

$$\Big\downarrow 27$$

$$\int \frac{\left(d^2-e^2x^2\right)^{p-3}\left(d^3+3e^2x^2d\right)}{x^2}dx-\frac{1}{2}e\int \frac{\left(d^2-e^2x^2\right)^{p-3}\left(3d^2+e^2x^2\right)}{x^2}dx^2$$

$$\Big\downarrow 88$$

$$\int \frac{\left(d^2-e^2x^2\right)^{p-3}\left(d^3+3e^2x^2d\right)}{x^2}dx-\frac{1}{2}e\left(3\int \frac{\left(d^2-e^2x^2\right)^{p-2}}{x^2}dx^2+\frac{4\left(d^2-e^2x^2\right)^{p-2}}{2-p}\right)$$

$$\Big\downarrow 75$$

$$\int \frac{\left(d^2-e^2x^2\right)^{p-3}\left(d^3+3e^2x^2d\right)}{x^2}dx-\frac{1}{2}e\left(\frac{3\left(d^2-e^2x^2\right)^{p-1}\text{Hypergeometric2F1}\left(1,p-1,p,1-\frac{e^2x^2}{d^2}\right)}{d^2(1-p)}+\frac{4\left(d^2-e^2x^2\right)^{p-2}}{2-p}\right)$$

$$\Big\downarrow 359$$

$$2de^2(4-p)\int \left(d^2-e^2x^2\right)^{p-3}dx-\frac{1}{2}e\left(\frac{3\left(d^2-e^2x^2\right)^{p-1}\text{Hypergeometric2F1}\left(1,p-1,p,1-\frac{e^2x^2}{d^2}\right)}{d^2(1-p)}+\frac{4\left(d^2-e^2x^2\right)^{p-2}}{2-p}\right)-\frac{d\left(d^2-e^2x^2\right)^{p-2}}{x}$$

$$\Big\downarrow 238$$

$$\frac{2e^2(4-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\int \left(1-\frac{e^2x^2}{d^2}\right)^{p-3}dx}{d^5}-\frac{1}{2}e\left(\frac{3\left(d^2-e^2x^2\right)^{p-1}\text{Hypergeometric2F1}\left(1,p-1,p,1-\frac{e^2x^2}{d^2}\right)}{d^2(1-p)}+\frac{4\left(d^2-e^2x^2\right)^{p-2}}{2-p}\right)-\frac{d\left(d^2-e^2x^2\right)^{p-2}}{x}$$

$$\Big\downarrow 237$$

3.292. $\int \frac{\left(d^2-e^2x^2\right)^p}{x^2(d+ex)^3}\,dx$

$$-\frac{1}{2}e\left(\frac{3(d^2-e^2x^2)^{p-1}\text{Hypergeometric2F1}\left(1,p-1,p,1-\frac{e^2x^2}{d^2}\right)}{d^2(1-p)}+\frac{4(d^2-e^2x^2)^{p-2}}{2-p}\right)-\frac{d(d^2-e^2x^2)^{p-2}}{x}+\frac{2e^2(4-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\text{Hypergeometric2F1}\left(\frac{1}{2},3-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^5}$$

input
```
Int[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^3),x]
```

output
```
-((d*(d^2 - e^2*x^2)^(-2 + p))/x) + (2*e^2*(4 - p)*x*(d^2 - e^2*x^2)^p*Hyp
ergeometric2F1[1/2, 3 - p, 3/2, (e^2*x^2)/d^2])/(d^5*(1 - (e^2*x^2)/d^2)^p
) - (e*((4*(d^2 - e^2*x^2)^(-2 + p))/(2 - p) + (3*(d^2 - e^2*x^2)^(-1 + p)
*Hypergeometric2F1[1, -1 + p, p, 1 - (e^2*x^2)/d^2])/(d^2*(1 - p))))/2
```

3.292.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && !ILtQ[p, -1]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] && !IntegerQ[2*p]
&& !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] && !(IGtQ[m, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

3.292.4 Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x^2(ex+d)^3}dx$$

input
```
int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x)
```

output
```
int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x)
```

3.292.5 Fricas [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^2(d + ex)^3}\, dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^3 x^2}\, dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*e*x^3 + d^3*x^2
), x)
```

3.292.6 Sympy [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^2(d + ex)^3}\, dx = \int \frac{(-(-d + ex)\,(d + ex))^p}{x^2\,(d + ex)^3}\, dx$$

input
```
integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d)**3,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p/(x**2*(d + e*x)**3), x)
```

3.292.7 Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^2(d + ex)^3}\, dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^3 x^2}\, dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^2), x)
```

3.292.8 Giac [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^2(d + ex)^3} \, dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^3 x^2} \, dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^2), x)
```

3.292.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^p}{x^2(d + ex)^3} \, dx = \int \frac{(d^2 - e^2 x^2)^p}{x^2 (d + e x)^3} \, dx$$

input
```
int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^3),x)
```

output
```
int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^3), x)
```

3.293 $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^3}\,dx$

3.293.1 Optimal result

Integrand size = 25, antiderivative size = 173

$$\begin{aligned}
&\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^3}\,dx \\
&= -\frac{d(d^2-e^2x^2)^{-2+p}}{2x^2}+\frac{3e(d^2-e^2x^2)^{-2+p}}{x} \\
&\quad -\frac{2e^3(8-3p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},3-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^6} \\
&\quad +\frac{e^2(6-p)\,(d^2-e^2x^2)^{-2+p}\,\text{Hypergeometric2F1}\left(1,-2+p,-1+p,1-\frac{e^2x^2}{d^2}\right)}{2d(2-p)}
\end{aligned}$$

output
```
-1/2*d*(-e^2*x^2+d^2)^(-2+p)/x^2+3*e*(-e^2*x^2+d^2)^(-2+p)/x-2*e^3*(8-3*p)
*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 3-p],[3/2],e^2*x^2/d^2)/d^6/((1-e^2*x^
2/d^2)^p)+1/2*e^2*(6-p)*(-e^2*x^2+d^2)^(-2+p)*hypergeom([1, -2+p],[-1+p],1
-e^2*x^2/d^2)/d/(2-p)
```

3.293.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.97

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^3}\,dx$$
$$= \frac{(d^2-e^2x^2)^p\left(\frac{24d^2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}+\frac{4d^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,\frac{d^2}{e^2x^2}\right)}{(-1+p)x^2}+\right.}{}$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^3),x]
```

output
```
((d^2 - e^2*x^2)^p*((24*d^2*e*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d
^2])/(x*(1 - (e^2*x^2)/d^2)^p) + (4*d^3*Hypergeometric2F1[1 - p, -p, 2 - p
, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (3*2^(3 + p)*e^2*
(d - e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p
)*(1 + (e*x)/d)^p) + (3*2^(1 + p)*e^2*(d - e*x)*Hypergeometric2F1[2 - p, 1
 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (2^p*e^2*(d - e
*x)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 +
 (e*x)/d)^p) + (24*d*e^2*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/
(p*(1 - d^2/(e^2*x^2))^p)))/(8*d^6)
```

3.293.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {570, 543, 354, 27, 87, 75, 359, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^3}\,dx$$
$$\Big\downarrow \text{570}$$
$$\int \frac{(d-ex)^3\left(d^2-e^2x^2\right)^{p-3}}{x^3}dx$$
$$\Big\downarrow \text{543}$$

$$\int \frac{(d^2-e^2x^2)^{p-3}(-x^2e^3-3d^2e)}{x^2}dx+\int \frac{(d^2-e^2x^2)^{p-3}(d^3+3e^2x^2d)}{x^3}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{d(d^2-e^2x^2)^{p-3}(d^2+3e^2x^2)}{x^4}dx^2+\int \frac{(d^2-e^2x^2)^{p-3}(-x^2e^3-3d^2e)}{x^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}d\int \frac{(d^2-e^2x^2)^{p-3}(d^2+3e^2x^2)}{x^4}dx^2+\int \frac{(d^2-e^2x^2)^{p-3}(-x^2e^3-3d^2e)}{x^2}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}d\left(e^2(6-p)\int \frac{(d^2-e^2x^2)^{p-3}}{x^2}dx^2-\frac{(d^2-e^2x^2)^{p-2}}{x^2}\right)+\int \frac{(d^2-e^2x^2)^{p-3}(-x^2e^3-3d^2e)}{x^2}dx$$

$$\Big\downarrow \text{75}$$

$$\int \frac{(d^2-e^2x^2)^{p-3}(-x^2e^3-3d^2e)}{x^2}dx+\frac{1}{2}d\left(\frac{e^2(6-p)(d^2-e^2x^2)^{p-2}\text{Hypergeometric2F1}\left(1,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{d^2(2-p)}-\frac{(d^2-e^2x^2)^{p-2}}{x^2}\right)$$

$$\Big\downarrow \text{359}$$

$$-2e^3(8-3p)\int (d^2-e^2x^2)^{p-3}dx+\frac{1}{2}d\left(\frac{e^2(6-p)(d^2-e^2x^2)^{p-2}\text{Hypergeometric2F1}\left(1,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{d^2(2-p)}-\frac{(d^2-e^2x^2)^{p-2}}{x^2}\right)+\frac{3e(d^2-e^2x^2)^{p-2}}{x}$$

$$\Big\downarrow \text{238}$$

$$-\frac{2e^3(8-3p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\int \left(1-\frac{e^2x^2}{d^2}\right)^{p-3}dx}{d^6}+\frac{1}{2}d\left(\frac{e^2(6-p)(d^2-e^2x^2)^{p-2}\text{Hypergeometric2F1}\left(1,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{d^2(2-p)}-\frac{(d^2-e^2x^2)^{p-2}}{x^2}\right)+\frac{3e(d^2-e^2x^2)^{p-2}}{x}$$

$$\Big\downarrow \text{237}$$

3.293. $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^3}\,dx$

$$\frac{1}{2}d\left(\frac{e^2(6-p)\left(d^2-e^2x^2\right)^{p-2}\text{Hypergeometric2F1}\left(1,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{d^2(2-p)}-\frac{\left(d^2-e^2x^2\right)^{p-2}}{x^2}\right)+$$
$$\frac{3e\left(d^2-e^2x^2\right)^{p-2}}{x}-\frac{2e^3(8-3p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\text{Hypergeometric2F1}\left(\frac{1}{2},3-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^6}$$

input
```
Int[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^3),x]
```

output
```
(3*e*(d^2 - e^2*x^2)^(-2 + p))/x - (2*e^3*(8 - 3*p)*x*(d^2 - e^2*x^2)^p*Hy
pergeometric2F1[1/2, 3 - p, 3/2, (e^2*x^2)/d^2])/(d^6*(1 - (e^2*x^2)/d^2)^
p) + (d*(-((d^2 - e^2*x^2)^(-2 + p)/x^2) + (e^2*(6 - p)*(d^2 - e^2*x^2)^(-
2 + p)*Hypergeometric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(d^2*(2 -
p))))/2
```

3.293.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && !ILtQ[p, -1]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] && !IntegerQ[2*p]
&& !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] && !(IGtQ[m, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

3.293.4 Maple [F]

$$\int \frac{(-e^2x^2 + d^2)^p}{x^3 (ex + d)^3} dx$$

input
```
int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x)
```

output
```
int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x)
```

3.293.5 Fricas [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^3(d + ex)^3}\, dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^3 x^3}\, dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e^3*x^6 + 3*d*e^2*x^5 + 3*d^2*e*x^4 + d^3*x^3
), x)
```

3.293.6 Sympy [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^3(d + ex)^3}\, dx = \int \frac{(-(-d + ex)(d + ex))^p}{x^3 (d + ex)^3}\, dx$$

input
```
integrate((-e**2*x**2+d**2)**p/x**3/(e*x+d)**3,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p/(x**3*(d + e*x)**3), x)
```

3.293.7 Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^3(d + ex)^3}\, dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^3 x^3}\, dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^3), x)
```

3.293.8 Giac [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^3(d+ex)^3} \, dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex+d)^3 x^3} \, dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^3), x)
```

3.293.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^p}{x^3(d+ex)^3} \, dx = \int \frac{(d^2 - e^2\,x^2)^p}{x^3\,(d+e\,x)^3} \, dx$$

input
```
int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^3),x)
```

output
```
int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^3), x)
```

3.294 $\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^3}\,dx$

3.294.1 Optimal result

Integrand size = 25, antiderivative size = 179

$$\begin{aligned}\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^3}\,dx &= -\frac{d(d^2-e^2x^2)^{-2+p}}{3x^3}+\frac{3e(d^2-e^2x^2)^{-2+p}}{2x^2}\\ &\quad-\frac{2e^2(8-p)\,(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},3-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^5x}\\ &\quad-\frac{e^3(10-3p)\,(d^2-e^2x^2)^{-2+p}\,\text{Hypergeometric2F1}\left(1,-2+p,-1+p,1-\frac{e^2x^2}{d^2}\right)}{2d^2(2-p)}\end{aligned}$$

output
```
-1/3*d*(-e^2*x^2+d^2)^(-2+p)/x^3+3/2*e*(-e^2*x^2+d^2)^(-2+p)/x^2-2/3*e^2*(
8-p)*(-e^2*x^2+d^2)^p*hypergeom([-1/2, 3-p],[1/2],e^2*x^2/d^2)/d^5/x/((1-e
^2*x^2/d^2)^p)-1/2*e^3*(10-3*p)*(-e^2*x^2+d^2)^(-2+p)*hypergeom([1, -2+p],
[-1+p],1-e^2*x^2/d^2)/d^2/(2-p)
```

3.294.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 393 vs. 2(179) = 358.

Time = 0.63 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.20

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^3}\,dx$$
$$= \frac{(d^2-e^2x^2)^p\left(-\frac{8d^4\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{2},-p,-\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x^3}-\frac{144d^2e^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right.}{x}\right.}{}$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^3),x]
```

output
```
((d^2 - e^2*x^2)^p*((-8*d^4*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d^
2])/(x^3*(1 - (e^2*x^2)/d^2)^p) - (144*d^2*e^2*Hypergeometric2F1[-1/2, -p,
 1/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) - (36*d^3*e*Hypergeometric
2F1[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2)
 + (15*2^(3 + p)*e^3*(-d + e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d
- e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^(3 + p)*e^3*(-d + e*x)*Hyp
ergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/
d)^p) + (3*2^p*e^3*(-d + e*x)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d -
e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) - (120*d*e^3*Hypergeometric2F1[-p,
-p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(24*d^7)
```

3.294.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {570, 543, 354, 25, 27, 87, 75, 359, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^3}\,dx$$
$$\Big\downarrow \ 570$$
$$\int \frac{(d-ex)^3\left(d^2-e^2x^2\right)^{p-3}}{x^4}dx$$

$$\Big\downarrow \text{543}$$

$$\int \frac{(d^2-e^2x^2)^{p-3}(-x^2e^3-3d^2e)}{x^3}dx+\int \frac{(d^2-e^2x^2)^{p-3}(d^3+3e^2x^2d)}{x^4}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int -\frac{e(d^2-e^2x^2)^{p-3}(3d^2+e^2x^2)}{x^4}dx^2+\int \frac{(d^2-e^2x^2)^{p-3}(d^3+3e^2x^2d)}{x^4}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{(d^2-e^2x^2)^{p-3}(d^3+3e^2x^2d)}{x^4}dx-\frac{1}{2}\int \frac{e(d^2-e^2x^2)^{p-3}(3d^2+e^2x^2)}{x^4}dx^2$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(d^2-e^2x^2)^{p-3}(d^3+3e^2x^2d)}{x^4}dx-\frac{1}{2}e\int \frac{(d^2-e^2x^2)^{p-3}(3d^2+e^2x^2)}{x^4}dx^2$$

$$\Big\downarrow \text{87}$$

$$\int \frac{(d^2-e^2x^2)^{p-3}(d^3+3e^2x^2d)}{x^4}dx-\frac{1}{2}e\left(e^2(10-3p)\int \frac{(d^2-e^2x^2)^{p-3}}{x^2}dx^2-\frac{3(d^2-e^2x^2)^{p-2}}{x^2}\right)$$

$$\Big\downarrow \text{75}$$

$$\int \frac{(d^2-e^2x^2)^{p-3}(d^3+3e^2x^2d)}{x^4}dx-\frac{1}{2}e\left(\frac{e^2(10-3p)(d^2-e^2x^2)^{p-2}\text{Hypergeometric2F1}\left(1,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{d^2(2-p)}-\frac{3(d^2-e^2x^2)^{p-2}}{x^2}\right)$$

$$\Big\downarrow \text{359}$$

$$\frac{2}{3}de^2(8-p)\int \frac{(d^2-e^2x^2)^{p-3}}{x^2}dx-\frac{1}{2}e\left(\frac{e^2(10-3p)(d^2-e^2x^2)^{p-2}\text{Hypergeometric2F1}\left(1,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{d^2(2-p)}-\frac{3(d^2-e^2x^2)^{p-2}}{x^2}\right)-\frac{d(d^2-e^2x^2)^{p-2}}{3x^3}$$

$$\Big\downarrow \text{279}$$

3.294. $\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^3}\,dx$

$$\frac{2e^2(8-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\int\frac{\left(1-\frac{e^2x^2}{d^2}\right)^{p-3}}{x^2}dx}{3d^5}-$$
$$\frac{1}{2}e\left(\frac{e^2(10-3p)\left(d^2-e^2x^2\right)^{p-2}\text{Hypergeometric2F1}\left(1,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{d^2(2-p)}-\frac{3\left(d^2-e^2x^2\right)^{p-2}}{x^2}\right)-$$
$$\frac{d\left(d^2-e^2x^2\right)^{p-2}}{3x^3}$$

$\downarrow$ 278

$$-\frac{1}{2}e\left(\frac{e^2(10-3p)\left(d^2-e^2x^2\right)^{p-2}\text{Hypergeometric2F1}\left(1,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{d^2(2-p)}-\frac{3\left(d^2-e^2x^2\right)^{p-2}}{x^2}\right)-$$
$$\frac{d\left(d^2-e^2x^2\right)^{p-2}}{3x^3}-\frac{2e^2(8-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(-\frac{1}{2},3-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^5x}$$

input
```
Int[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^3),x]
```

output
```
-1/3*(d*(d^2 - e^2*x^2)^(-2 + p))/x^3 - (2*e^2*(8 - p)*(d^2 - e^2*x^2)^p*H
ypergeometric2F1[-1/2, 3 - p, 1/2, (e^2*x^2)/d^2])/(3*d^5*x*(1 - (e^2*x^2)
/d^2)^p) - (e*((-3*(d^2 - e^2*x^2)^(-2 + p))/x^2 + (e^2*(10 - 3*p)*(d^2 -
e^2*x^2)^(-2 + p)*Hypergeometric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])
/(d^2*(2 - p))))/2
```

3.294.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

3.294. $\int\frac{(d^2-e^2x^2)^p}{x^4(d+ex)^3}\,dx$

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

3.294. $\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^3}\,dx$

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] && !(IGtQ[m, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

3.294.4 Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x^4(ex+d)^3}dx$$

input
```
int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x)
```

output
```
int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x)
```

3.294.5 Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3x^4}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e^3*x^7 + 3*d*e^2*x^6 + 3*d^2*e*x^5 + d^3*x^4
), x)
```

3.294.6 Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^3}\,dx = \int \frac{(-(-d+ex)(d+ex))^p}{x^4(d+ex)^3}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**p/x**4/(e*x+d)**3,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p/(x**4*(d + e*x)**3), x)
```

3.294.7 Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^3}\,dx=\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3x^4}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^4), x)
```

3.294.8 Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^3}\,dx=\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3x^4}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^4), x)
```

3.294.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^3}\,dx=\int \frac{(d^2-e^2\,x^2)^p}{x^4\,(d+e\,x)^3}\,dx$$

input
```
int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^3),x)
```

output
```
int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^3), x)
```

3.295 $\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^3}\,dx$

3.295.1 Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^3}\,dx = -\frac{d(d^2-e^2x^2)^{-2+p}}{4x^4}+\frac{e(d^2-e^2x^2)^{-2+p}}{x^3} + \frac{2e^3(4-p)\,(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},3-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{d^6x} + \frac{e^4(10-p)\,(d^2-e^2x^2)^{-2+p}\,\text{Hypergeometric2F1}\left(2,-2+p,-1+p,1-\frac{e^2x^2}{d^2}\right)}{4d^3(2-p)}$$

output
```
-1/4*d*(-e^2*x^2+d^2)^(-2+p)/x^4+e*(-e^2*x^2+d^2)^(-2+p)/x^3+2*e^3*(4-p)*(
-e^2*x^2+d^2)^p*hypergeom([-1/2, 3-p],[1/2],e^2*x^2/d^2)/d^6/x/((1-e^2*x^2
/d^2)^p)+1/4*e^4*(10-p)*(-e^2*x^2+d^2)^(-2+p)*hypergeom([2, -2+p],[-1+p],1
-e^2*x^2/d^2)/d^3/(2-p)
```

3.295.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 446 vs. 2(174) = 348.

Time = 0.71 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.56

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^3}\,dx$$
$$= \frac{(d^2-e^2x^2)^p\left(\frac{8d^4e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{2},-p,-\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x^3}+\frac{80d^2e^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}\right.}{}$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^3),x]
```

output
```
((d^2 - e^2*x^2)^p*((8*d^4*e*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d
^2])/(x^3*(1 - (e^2*x^2)/d^2)^p) + (80*d^2*e^3*Hypergeometric2F1[-1/2, -p,
 1/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) + (24*d^3*e^2*Hypergeometr
ic2F1[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^
2) + (15*2^(2 + p)*e^4*(d - e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d
 - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (4*d^5*Hypergeometric2F1[2 - p
, -p, 3 - p, d^2/(e^2*x^2)])/((-2 + p)*(1 - d^2/(e^2*x^2))^p*x^4) + (5*2^(
1 + p)*e^4*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d
)])/((1 + p)*(1 + (e*x)/d)^p) + (2^p*e^4*(d - e*x)*Hypergeometric2F1[3 - p
, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (60*d*e^4*Hy
pergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))
/(8*d^8)
```

3.295.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {570, 543, 354, 27, 87, 75, 359, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^3}\,dx$$
$$\Big\downarrow \ 570$$

$$\int \frac{(d-ex)^3 \left(d^2-e^2x^2\right)^{p-3}}{x^5}dx$$

$$\Big\downarrow \text{ 543}$$

$$\int \frac{\left(d^2-e^2x^2\right)^{p-3} \left(-x^2e^3-3d^2e\right)}{x^4}dx+\int \frac{\left(d^2-e^2x^2\right)^{p-3} \left(d^3+3e^2x^2d\right)}{x^5}dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{d\left(d^2-e^2x^2\right)^{p-3} \left(d^2+3e^2x^2\right)}{x^6}dx^2+\int \frac{\left(d^2-e^2x^2\right)^{p-3} \left(-x^2e^3-3d^2e\right)}{x^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}d\int \frac{\left(d^2-e^2x^2\right)^{p-3} \left(d^2+3e^2x^2\right)}{x^6}dx^2+\int \frac{\left(d^2-e^2x^2\right)^{p-3} \left(-x^2e^3-3d^2e\right)}{x^4}dx$$

$$\Big\downarrow \text{ 87}$$

$$\frac{1}{2}d\left(\frac{1}{2}e^2(10-p)\int \frac{\left(d^2-e^2x^2\right)^{p-3}}{x^4}dx^2-\frac{\left(d^2-e^2x^2\right)^{p-2}}{2x^4}\right)+\int \frac{\left(d^2-e^2x^2\right)^{p-3} \left(-x^2e^3-3d^2e\right)}{x^4}dx$$

$$\Big\downarrow \text{ 75}$$

$$\int \frac{\left(d^2-e^2x^2\right)^{p-3} \left(-x^2e^3-3d^2e\right)}{x^4}dx+\frac{1}{2}d\left(\frac{e^4(10-p)\left(d^2-e^2x^2\right)^{p-2} \text{Hypergeometric2F1}\left(2,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{2d^4(2-p)}-\frac{\left(d^2-e^2x^2\right)^{p-2}}{2x^4}\right)$$

$$\Big\downarrow \text{ 359}$$

$$-2e^3(4-p)\int \frac{\left(d^2-e^2x^2\right)^{p-3}}{x^2}dx+\frac{e\left(d^2-e^2x^2\right)^{p-2}}{x^3}+\frac{1}{2}d\left(\frac{e^4(10-p)\left(d^2-e^2x^2\right)^{p-2} \text{Hypergeometric2F1}\left(2,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{2d^4(2-p)}-\frac{\left(d^2-e^2x^2\right)^{p-2}}{2x^4}\right)$$

$$\Big\downarrow \text{ 279}$$

$$-\frac{2e^3(4-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^{p-3}}{x^2}dx}{d^6}+\frac{e\left(d^2-e^2x^2\right)^{p-2}}{x^3}+\frac{1}{2}d\left(\frac{e^4(10-p)\left(d^2-e^2x^2\right)^{p-2} \text{Hypergeometric2F1}\left(2,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{2d^4(2-p)}-\frac{\left(d^2-e^2x^2\right)^{p-2}}{2x^4}\right)$$

$$\Big\downarrow \text{ 278}$$

3.295. $\int \frac{\left(d^2-e^2x^2\right)^p}{x^5(d+ex)^3}\,dx$

$$\frac{e\left(d^2-e^2x^2\right)^{p-2}}{x^3}+\frac{2e^3(4-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p \text{Hypergeometric2F1}\left(-\frac{1}{2},3-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{d^6x}+\frac{1}{2}d\left(\frac{e^4(10-p)\left(d^2-e^2x^2\right)^{p-2}\text{Hypergeometric2F1}\left(2,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{2d^4(2-p)}-\frac{\left(d^2-e^2x^2\right)^{p-2}}{2x^4}\right)$$

input
```
Int[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^3),x]
```

output
```
(e*(d^2 - e^2*x^2)^(-2 + p))/x^3 + (2*e^3*(4 - p)*(d^2 - e^2*x^2)^p*Hyperg
eometric2F1[-1/2, 3 - p, 1/2, (e^2*x^2)/d^2])/(d^6*x*(1 - (e^2*x^2)/d^2)^p
) + (d*(-1/2*(d^2 - e^2*x^2)^(-2 + p)/x^4 + (e^4*(10 - p)*(d^2 - e^2*x^2)^
(-2 + p)*Hypergeometric2F1[2, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2*d^4*(
2 - p))))/2
```

3.295.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

3.295. $\int \frac{\left(d^2-e^2x^2\right)^p}{x^5(d+ex)^3}\,dx$

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.295.4 Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x^5\,(ex+d)^3}dx$$

input
```
int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x)
```

output
```
int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x)
```

3.295.5 Fricas [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^5(d + ex)^3}\,dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^3 x^5}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e^3*x^8 + 3*d*e^2*x^7 + 3*d^2*e*x^6 + d^3*x^5
), x)
```

3.295.6 Sympy [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^5(d + ex)^3}\,dx = \int \frac{(-(-d + ex)\,(d + ex))^p}{x^5\,(d + ex)^3}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**p/x**5/(e*x+d)**3,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p/(x**5*(d + e*x)**3), x)
```

3.295.7 Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^5(d + ex)^3}\,dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^3 x^5}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^5), x)
```

3.295.8 Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3x^5}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^5), x)
```

3.295.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^3}\,dx = \int \frac{(d^2-e^2\,x^2)^p}{x^5\,(d+e\,x)^3}\,dx$$

input
```
int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^3),x)
```

output
```
int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^3), x)
```

3.296 $\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^4}\,dx$

3.296.1 Optimal result

Integrand size = 25, antiderivative size = 265

$$\begin{aligned}\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^4}\,dx &= -\frac{4d^7(d^2-e^2x^2)^{-3+p}}{e^5(3-p)}+\frac{d^2(13+12p)x^5(d^2-e^2x^2)^{-3+p}}{1-4p^2}-\frac{e^2x^7(d^2-e^2x^2)^{-3+p}}{1+2p}\\&\quad+\frac{10d^5(d^2-e^2x^2)^{-2+p}}{e^5(2-p)}-\frac{8d^3(d^2-e^2x^2)^{-1+p}}{e^5(1-p)}-\frac{2d(d^2-e^2x^2)^p}{e^5p}\\&\quad-\frac{4(16+15p+p^2)\,x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},4-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^4\,(1-4p^2)}\end{aligned}$$

output
```
-4*d^7*(-e^2*x^2+d^2)^(-3+p)/e^5/(3-p)+d^2*(13+12*p)*x^5*(-e^2*x^2+d^2)^(-
3+p)/(-4*p^2+1)-e^2*x^7*(-e^2*x^2+d^2)^(-3+p)/(1+2*p)+10*d^5*(-e^2*x^2+d^2
)^(-2+p)/e^5/(2-p)-8*d^3*(-e^2*x^2+d^2)^(-1+p)/e^5/(1-p)-2*d*(-e^2*x^2+d^2
)^p/e^5/p-4/5*(p^2+15*p+16)*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, 4-p],[7/2
],e^2*x^2/d^2)/d^4/(-4*p^2+1)/((1-e^2*x^2/d^2)^p)
```

3.296.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.87

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^4}\,dx$$
$$= \frac{2^{-4+p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(16e(1+p)x\left(\frac{1}{2}+\frac{ex}{2d}\right)^p\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)+\right.}{}$$

input
```
Integrate[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

output
```
(2^(-4 + p)*(d^2 - e^2*x^2)^p*(16*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hyperg
eometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^2)^p
*(32*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - 24*Hypergeo
metric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 8*Hypergeometric2F1[3 -
p, 1 + p, 2 + p, (d - e*x)/(2*d)] - Hypergeometric2F1[4 - p, 1 + p, 2 + p,
 (d - e*x)/(2*d)])))/(e^5*(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

3.296.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {570, 543, 354, 27, 86, 1590, 25, 27, 363, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^4}\,dx$$
$$\Big\downarrow \ 570$$
$$\int x^4(d-ex)^4\left(d^2-e^2x^2\right)^{p-4}dx$$
$$\Big\downarrow \ 543$$
$$\int x^4\left(d^2-e^2x^2\right)^{p-4}\left(d^4+6e^2x^2d^2+e^4x^4\right)dx+\int x^5\left(d^2-e^2x^2\right)^{p-4}\left(-4ed^3-4e^3x^2d\right)dx$$
$$\Big\downarrow \ 354$$
$$\frac{1}{2}\int -4dex^4\left(d^2-e^2x^2\right)^{p-4}\left(d^2+e^2x^2\right)dx^2+\int x^4\left(d^2-e^2x^2\right)^{p-4}\left(d^4+6e^2x^2d^2+e^4x^4\right)dx$$

$$\Big\downarrow \; 27$$

$$\int x^4 (d^2-e^2x^2)^{p-4} \left(d^4+6e^2x^2d^2+e^4x^4\right) dx - 2de \int x^4 (d^2-e^2x^2)^{p-4} \left(d^2+e^2x^2\right) dx^2$$

$$\Big\downarrow \; 86$$

$$\int x^4 (d^2-e^2x^2)^{p-4} \left(d^4+6e^2x^2d^2+e^4x^4\right) dx - 2de \int \left(\frac{2d^6 (d^2-e^2x^2)^{p-4}}{e^4} - \frac{5d^4 (d^2-e^2x^2)^{p-3}}{e^4} + \frac{4d^2 (d^2-e^2x^2)^{p-2}}{e^4} - \frac{(d^2-e^2x^2)^{p-1}}{e^4}\right) dx^2$$

$$\Big\downarrow \; 1590$$

$$-\frac{\int -d^2e^2x^4 (d^2-e^2x^2)^{p-4} \left((2p+1)d^2+e^2(12p+13)x^2\right) dx}{e^2(2p+1)} - 2de \int \left(\frac{2d^6 (d^2-e^2x^2)^{p-4}}{e^4} - \frac{5d^4 (d^2-e^2x^2)^{p-3}}{e^4} + \frac{4d^2 (d^2-e^2x^2)^{p-2}}{e^4} - \frac{(d^2-e^2x^2)^{p-1}}{e^4}\right) dx^2 - \frac{e^2x^7 (d^2-e^2x^2)^{p-3}}{2p+1}$$

$$\Big\downarrow \; 25$$

$$\frac{\int d^2e^2x^4 (d^2-e^2x^2)^{p-4} \left((2p+1)d^2+e^2(12p+13)x^2\right) dx}{e^2(2p+1)} - 2de \int \left(\frac{2d^6 (d^2-e^2x^2)^{p-4}}{e^4} - \frac{5d^4 (d^2-e^2x^2)^{p-3}}{e^4} + \frac{4d^2 (d^2-e^2x^2)^{p-2}}{e^4} - \frac{(d^2-e^2x^2)^{p-1}}{e^4}\right) dx^2 - \frac{e^2x^7 (d^2-e^2x^2)^{p-3}}{2p+1}$$

$$\Big\downarrow \; 27$$

$$\frac{d^2 \int x^4 (d^2-e^2x^2)^{p-4} \left((2p+1)d^2+e^2(12p+13)x^2\right) dx}{2p+1} - 2de \int \left(\frac{2d^6 (d^2-e^2x^2)^{p-4}}{e^4} - \frac{5d^4 (d^2-e^2x^2)^{p-3}}{e^4} + \frac{4d^2 (d^2-e^2x^2)^{p-2}}{e^4} - \frac{(d^2-e^2x^2)^{p-1}}{e^4}\right) dx^2 - \frac{e^2x^7 (d^2-e^2x^2)^{p-3}}{2p+1}$$

$$\Big\downarrow \; 363$$

3.296. $\int \frac{x^4 (d^2-e^2x^2)^p}{(d+ex)^4} \, dx$

$$\frac{d^2\left(\frac{(12p+13)x^5(d^2-e^2x^2)^{p-3}}{1-2p}-\frac{4d^2(p^2+15p+16)\int x^4(d^2-e^2x^2)^{p-4}dx}{1-2p}\right)}{2p+1}-2de\int\left(\frac{2d^6(d^2-e^2x^2)^{p-4}}{e^4}-\frac{5d^4(d^2-e^2x^2)^{p-3}}{e^4}+\frac{4d^2(d^2-e^2x^2)^{p-2}}{e^4}-\frac{(d^2-e^2x^2)^{p-1}}{e^4}\right)dx^2-\frac{e^2x^7(d^2-e^2x^2)^{p-3}}{2p+1}$$

$$\Big\downarrow \text{279}$$

$$\frac{d^2\left(\frac{(12p+13)x^5(d^2-e^2x^2)^{p-3}}{1-2p}-\frac{4(p^2+15p+16)(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^{p-4}dx}{d^6(1-2p)}\right)}{2p+1}-2de\int\left(\frac{2d^6(d^2-e^2x^2)^{p-4}}{e^4}-\frac{5d^4(d^2-e^2x^2)^{p-3}}{e^4}+\frac{4d^2(d^2-e^2x^2)^{p-2}}{e^4}-\frac{(d^2-e^2x^2)^{p-1}}{e^4}\right)dx^2-\frac{e^2x^7(d^2-e^2x^2)^{p-3}}{2p+1}$$

$$\Big\downarrow \text{278}$$

$$-2de\int\left(\frac{2d^6(d^2-e^2x^2)^{p-4}}{e^4}-\frac{5d^4(d^2-e^2x^2)^{p-3}}{e^4}+\frac{4d^2(d^2-e^2x^2)^{p-2}}{e^4}-\frac{(d^2-e^2x^2)^{p-1}}{e^4}\right)dx^2-\frac{e^2x^7(d^2-e^2x^2)^{p-3}}{2p+1}+\frac{d^2\left(\frac{(12p+13)x^5(d^2-e^2x^2)^{p-3}}{1-2p}-\frac{4(p^2+15p+16)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},4-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^6(1-2p)}\right)}{2p+1}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^2x^7(d^2-e^2x^2)^{p-3}}{2p+1}+\frac{d^2\left(\frac{(12p+13)x^5(d^2-e^2x^2)^{p-3}}{1-2p}-\frac{4(p^2+15p+16)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},4-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^6(1-2p)}\right)}{2p+1}-2de\left(\frac{4d^2(d^2-e^2x^2)^{p-1}}{e^6(1-p)}+\frac{(d^2-e^2x^2)^p}{e^6p}+\frac{2d^6(d^2-e^2x^2)^{p-3}}{e^6(3-p)}-\frac{5d^4(d^2-e^2x^2)^{p-2}}{e^6(2-p)}\right)$$

input
```
Int[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

3.296. $\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^4}\,dx$

output
```
-((e^2*x^7*(d^2 - e^2*x^2)^(-3 + p))/(1 + 2*p)) - 2*d*e*((2*d^6*(d^2 - e^2
*x^2)^(-3 + p))/(e^6*(3 - p)) - (5*d^4*(d^2 - e^2*x^2)^(-2 + p))/(e^6*(2 -
 p)) + (4*d^2*(d^2 - e^2*x^2)^(-1 + p))/(e^6*(1 - p)) + (d^2 - e^2*x^2)^p/
(e^6*p)) + (d^2*(((13 + 12*p)*x^5*(d^2 - e^2*x^2)^(-3 + p))/(1 - 2*p) - (4
*(16 + 15*p + p^2)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 4 - p, 7/2
, (e^2*x^2)/d^2])/(5*d^6*(1 - 2*p)*(1 - (e^2*x^2)/d^2)^p)))/(1 + 2*p)
```

3.296.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.296. $\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^4} dx$

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.296.4 Maple [F]

$$\int \frac{x^4(-e^2x^2+d^2)^p}{(ex+d)^4}dx$$

input
```
int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

output
```
int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

3.296.5 Fricas [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p x^4}{(ex+d)^4}\,dx$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*x^4/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4
*d^3*e*x + d^4), x)
```

3.296.6 Sympy [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{x^4(-(-d+ex)(d+ex))^p}{(d+ex)^4}\,dx$$

input
```
integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d)**4,x)
```

output
```
Integral(x**4*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

3.296.7 Maxima [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p x^4}{(ex+d)^4}\,dx$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^4, x)
```

3.296.8 Giac [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p x^4}{(ex+d)^4}\,dx$$

input
```
integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^4, x)
```

3.296.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{x^4\,(d^2-e^2\,x^2)^p}{(d+e\,x)^4}\,dx$$

input
```
int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x)
```

output
```
int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^4, x)
```

3.297 $\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^4}\,dx$

3.297.1 Optimal result

Integrand size = 25, antiderivative size = 211

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \frac{d^2(d^2-e^2x^2)^{1+p}}{2e^4(3-p)(d+ex)^4} - \frac{d(1+2p)\,(d^2-e^2x^2)^{1+p}}{e^4(1-2p)p(d+ex)^3} - \frac{(d^2-e^2x^2)^{1+p}}{2e^4p(d+ex)^2} + \frac{3\,2^{-2+p}(2+p)\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(3-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^2e^4(1-2p)(3-p)p(1+p)}$$

output
```
1/2*d^2*(-e^2*x^2+d^2)^(p+1)/e^4/(3-p)/(e*x+d)^4-d*(1+2*p)*(-e^2*x^2+d^2)^
(p+1)/e^4/(1-2*p)/p/(e*x+d)^3-1/2*(-e^2*x^2+d^2)^(p+1)/e^4/p/(e*x+d)^2+3*2
^(-2+p)*(2+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(p+1)*hypergeom([p+1, 3-p],[
2+p],1/2*(-e*x+d)/d)/d^2/e^4/p/(2*p^3-5*p^2-4*p+3)
```

3.297.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.74

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \frac{2^{-4+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(-8\,\text{Hypergeometric2F1}\left(1-p,1+p,2+p,\frac{d-ex}{2d}\right)+12\,\text{Hypergeor}\right.}{}$$

input
```
Integrate[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

output
```
(2^(-4 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(-8*Hypergeometric2F1[1 - p, 1 + p
, 2 + p, (d - e*x)/(2*d)] + 12*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d -
 e*x)/(2*d)] - 6*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] +
 Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(d*e^4*(1 + p)*
(1 + (e*x)/d)^p)
```

3.297.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {581, 27, 2170, 27, 671, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (d^2-e^2x^2)^p}{(d+ex)^4} \, dx$$

$$\Big\downarrow \text{581}$$

$$\frac{\int -\frac{2(d^2-e^2x^2)^p(d^3+e(p+2)xd^2+e^2(2p+1)x^2d)}{(d+ex)^4}dx}{2e^3p}-\frac{(d^2-e^2x^2)^{p+1}}{2e^4p(d+ex)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(d^2-e^2x^2)^p(d^3+e(p+2)xd^2+e^2(2p+1)x^2d)}{(d+ex)^4}dx}{e^3p}-\frac{(d^2-e^2x^2)^{p+1}}{2e^4p(d+ex)^2}$$

$$\Big\downarrow \text{2170}$$

$$-\frac{\frac{\int \frac{d^2e^4\left(4d(p+1)+e\left(2p^2+3p+4\right)x\right)\left(d^2-e^2x^2\right)^p}{(d+ex)^4}dx}{e^4(1-2p)}+\frac{d(2p+1)(d^2-e^2x^2)^{p+1}}{e(1-2p)(d+ex)^3}}{e^3p}-\frac{(d^2-e^2x^2)^{p+1}}{2e^4p(d+ex)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{d^2\int \frac{\left(4d(p+1)+e\left(2p^2+3p+4\right)x\right)\left(d^2-e^2x^2\right)^p}{(d+ex)^4}dx}{1-2p}+\frac{d(2p+1)(d^2-e^2x^2)^{p+1}}{e(1-2p)(d+ex)^3}}{e^3p}-\frac{(d^2-e^2x^2)^{p+1}}{2e^4p(d+ex)^2}$$

$$\Big\downarrow \text{671}$$

$$-\frac{\frac{d^2\left(\frac{6(p+2)\int \frac{\left(d^2-e^2x^2\right)^p}{(d+ex)^3}dx}{3-p}-\frac{(1-2p)p\left(d^2-e^2x^2\right)^{p+1}}{2e(3-p)(d+ex)^4}\right)}{1-2p}+\frac{d(2p+1)(d^2-e^2x^2)^{p+1}}{e(1-2p)(d+ex)^3}}{e^3p}-\frac{(d^2-e^2x^2)^{p+1}}{2e^4p(d+ex)^2}$$

$$\Big\downarrow \ 473$$

$$-\frac{\frac{d^2\left(\frac{6(p+2)(d-ex)^{-p-1}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\int(d-ex)^p\left(\frac{ex}{d}+1\right)^{p-3}dx}{d^4(3-p)}-\frac{(1-2p)p\left(d^2-e^2x^2\right)^{p+1}}{2e(3-p)(d+ex)^4}\right)}{1-2p}+\frac{d(2p+1)\left(d^2-e^2x^2\right)^{p+1}}{e(1-2p)(d+ex)^3}}{e^3p}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^4p(d+ex)^2}$$

$$\Big\downarrow \ 79$$

$$-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^4p(d+ex)^2}-\frac{\frac{d(2p+1)\left(d^2-e^2x^2\right)^{p+1}}{e(1-2p)(d+ex)^3}+\frac{d^2\left(-\frac{(1-2p)p\left(d^2-e^2x^2\right)^{p+1}}{2e(3-p)(d+ex)^4}-\frac{3\ 2^{p-2}(p+2)\left(d^2-e^2x^2\right)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}\text{Hypergeometric2F1}\left(3-p,p+1,p+2,\frac{d-ex}{2d}\right)}{d^4e(3-p)(p+1)}\right)}{1-2p}}{e^3p}$$

input
```
Int[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

output
```
-1/2*(d^2 - e^2*x^2)^(1 + p)/(e^4*p*(d + e*x)^2) - ((d*(1 + 2*p)*(d^2 - e^
2*x^2)^(1 + p))/(e*(1 - 2*p)*(d + e*x)^3) + (d^2*(-1/2*((1 - 2*p)*p*(d^2 -
 e^2*x^2)^(1 + p))/(e*(3 - p)*(d + e*x)^4) - (3*2^(-2 + p)*(2 + p)*(1 + (e
*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2
+ p, (d - e*x)/(2*d)])/(d^4*e*(3 - p)*(1 + p))))/(1 - 2*p))/(e^3*p)
```

3.297.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

3.297. $\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^4}\,dx$

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 2170
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
 p + q)*(d + e*x)^(q - 2)*(a*e - b*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
 0]] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2,
0] &&  !IGtQ[m, 0]
```

3.297. $\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^4}\,dx$

3.297.4 Maple [F]

$$\int \frac{x^3(-e^2x^2+d^2)^p}{(ex+d)^4}dx$$

input
```
int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

output
```
int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

3.297.5 Fricas [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p x^3}{(ex+d)^4}\,dx$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*x^3/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4
*d^3*e*x + d^4), x)
```

3.297.6 Sympy [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{x^3(-(-d+ex)\,(d+ex))^p}{(d+ex)^4}\,dx$$

input
```
integrate(x**3*(-e**2*x**2+d**2)**p/(e*x+d)**4,x)
```

output
```
Integral(x**3*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

3.297.7 Maxima [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p x^3}{(ex+d)^4}\,dx$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^4, x)
```

3.297.8 Giac [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p x^3}{(ex+d)^4}\,dx$$

input
```
integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^4, x)
```

3.297.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{x^3\,(d^2-e^2\,x^2)^p}{(d+e\,x)^4}\,dx$$

input
```
int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x)
```

output
```
int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^4, x)
```

3.298 $\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^4} \, dx$

3.298.1 Optimal result

Integrand size = 25, antiderivative size = 163

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^4} \, dx = -\frac{d(d^2-e^2x^2)^{1+p}}{2e^3(3-p)(d+ex)^4} + \frac{(d^2-e^2x^2)^{1+p}}{e^3(1-2p)(d+ex)^3} - \frac{2^{-3+p}(7+p)\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(3-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^3e^3(1-2p)(3-p)(1+p)}$$

output
```
-1/2*d*(-e^2*x^2+d^2)^(p+1)/e^3/(3-p)/(e*x+d)^4+(-e^2*x^2+d^2)^(p+1)/e^3/(
1-2*p)/(e*x+d)^3-2^(-3+p)*(7+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(p+1)*hype
rgeom([p+1, 3-p],[2+p],1/2*(-e*x+d)/d)/d^3/e^3/(2*p^3-5*p^2-4*p+3)
```

3.298.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.80

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^4} \, dx = -\frac{2^{-4+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(4\,\text{Hypergeometric2F1}\left(2-p,1+p,2+p,\frac{d-ex}{2d}\right)-4\,\text{Hypergeor}\right.}{d^2e^3(1+p)}$$

input
```
Integrate[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

output
```
-((2^(-4 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(4*Hypergeometric2F1[2 - p, 1 +
p, 2 + p, (d - e*x)/(2*d)] - 4*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d -
 e*x)/(2*d)] + Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(
d^2*e^3*(1 + p)*(1 + (e*x)/d)^p))
```

3.298.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {581, 25, 27, 671, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (d^2-e^2x^2)^p}{(d+ex)^4} \, dx$$

$$\Big\downarrow \text{581}$$

$$\frac{(d^2-e^2x^2)^{p+1}}{e^3(1-2p)(d+ex)^3}-\frac{\int -\frac{d(3d+2e(p+1)x)(d^2-e^2x^2)^p}{(d+ex)^4}dx}{e^2(1-2p)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d(3d+2e(p+1)x)(d^2-e^2x^2)^p}{(d+ex)^4}dx}{e^2(1-2p)}+\frac{(d^2-e^2x^2)^{p+1}}{e^3(1-2p)(d+ex)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int \frac{(3d+2e(p+1)x)(d^2-e^2x^2)^p}{(d+ex)^4}dx}{e^2(1-2p)}+\frac{(d^2-e^2x^2)^{p+1}}{e^3(1-2p)(d+ex)^3}$$

$$\Big\downarrow \text{671}$$

$$\frac{d\left(\frac{(p+7)\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3}dx}{3-p}-\frac{(1-2p)(d^2-e^2x^2)^{p+1}}{2e(3-p)(d+ex)^4}\right)}{e^2(1-2p)}+\frac{(d^2-e^2x^2)^{p+1}}{e^3(1-2p)(d+ex)^3}$$

$$\Big\downarrow \text{473}$$

$$\frac{d\left(\frac{(p+7)(d-ex)^{-p-1}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\int (d-ex)^p\left(\frac{ex}{d}+1\right)^{p-3}dx}{d^4(3-p)}-\frac{(1-2p)(d^2-e^2x^2)^{p+1}}{2e(3-p)(d+ex)^4}\right)}{e^2(1-2p)}+\frac{(d^2-e^2x^2)^{p+1}}{e^3(1-2p)(d+ex)^3}$$

$$\Big\downarrow 79$$

$$\frac{(d^2-e^2x^2)^{p+1}}{e^3(1-2p)(d+ex)^3}+\frac{d\left(-\frac{(1-2p)(d^2-e^2x^2)^{p+1}}{2e(3-p)(d+ex)^4}-\frac{2^{p-3}(p+7)(d^2-e^2x^2)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}\text{Hypergeometric2F1}\left(3-p,p+1,p+2,\frac{d-ex}{2d}\right)}{d^4e(3-p)(p+1)}\right)}{e^2(1-2p)}$$

input
```
Int[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

output
```
(d^2 - e^2*x^2)^(1 + p)/(e^3*(1 - 2*p)*(d + e*x)^3) + (d*(-1/2*((1 - 2*p)*
(d^2 - e^2*x^2)^(1 + p))/(e*(3 - p)*(d + e*x)^4) - (2^(-3 + p)*(7 + p)*(1
+ (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p
, 2 + p, (d - e*x)/(2*d)])/(d^4*e*(3 - p)*(1 + p))))/(e^2*(1 - 2*p))
```

3.298.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

3.298. $\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^4}\,dx$

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

3.298.4 Maple [F]

$$\int \frac{x^2(-e^2x^2+d^2)^p}{(ex+d)^4}dx$$

input
```
int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

output
```
int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

3.298.5 Fricas [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^4} dx = \int \frac{(-e^2x^2+d^2)^p x^2}{(ex+d)^4} dx$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*x^2/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4
*d^3*e*x + d^4), x)
```

3.298.6 Sympy [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{x^2(-(-d+ex)\,(d+ex))^p}{(d+ex)^4}\,dx$$

input
```
integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d)**4,x)
```

output
```
Integral(x**2*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

3.298.7 Maxima [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p x^2}{(ex+d)^4}\,dx$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^4, x)
```

3.298.8 Giac [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p x^2}{(ex+d)^4}\,dx$$

input
```
integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^4, x)
```

3.298.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{x^2\,(d^2-e^2\,x^2)^p}{(d+e\,x)^4}\,dx$$

input
```
int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x)
```

output
```
int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^4, x)
```

3.299 $\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^4}\,dx$

3.299.1 Optimal result

Integrand size = 23, antiderivative size = 118

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \frac{(d^2-e^2x^2)^{1+p}}{2e^2(3-p)(d+ex)^4} - \frac{2^{-2+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(3-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^4e^2(3-p)(1+p)}$$

output
```
1/2*(-e^2*x^2+d^2)^(p+1)/e^2/(3-p)/(e*x+d)^4-2^(-2+p)*(1+e*x/d)^(-1-p)*(-e
^2*x^2+d^2)^(p+1)*hypergeom([p+1, 3-p],[2+p],1/2*(-e*x+d)/d)/d^4/e^2/(-p^2
+2*p+3)
```

3.299.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.86

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \frac{2^{-4+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(-2\,\text{Hypergeometric2F1}\left(3-p,1+p,2+p,\frac{d-ex}{2d}\right)+\text{Hypergeome}\right.}{d^3e^2(1+p)}$$

input
```
Integrate[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

output
```
(2^(-4 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(-2*Hypergeometric2F1[3 - p, 1 + p
, 2 + p, (d - e*x)/(2*d)] + Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*
x)/(2*d)]))/(d^3*e^2*(1 + p)*(1 + (e*x)/d)^p)
```

3.299.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {571, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^4}dx$$
$$\Big\downarrow \text{571}$$
$$\frac{2\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3}dx}{e(3-p)}+\frac{(d^2-e^2x^2)^{p+1}}{2e^2(3-p)(d+ex)^4}$$
$$\Big\downarrow \text{473}$$
$$\frac{2(d-ex)^{-p-1}(d^2-e^2x^2)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}\int (d-ex)^p\left(\frac{ex}{d}+1\right)^{p-3}dx}{d^4e(3-p)}+\frac{(d^2-e^2x^2)^{p+1}}{2e^2(3-p)(d+ex)^4}$$
$$\Big\downarrow \text{79}$$
$$\frac{(d^2-e^2x^2)^{p+1}}{2e^2(3-p)(d+ex)^4}-\frac{2^{p-2}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\text{Hypergeometric2F1}\left(3-p,p+1,p+2,\frac{d-ex}{2d}\right)}{d^4e^2(3-p)(p+1)}$$

input
```
Int[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

output
```
(d^2 - e^2*x^2)^(1 + p)/(2*e^2*(3 - p)*(d + e*x)^4) - (2^(-2 + p)*(1 + (e*
x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 +
 p, (d - e*x)/(2*d)])/(d^4*e^2*(3 - p)*(1 + p))
```

3.299.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 571
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*(n + p + 1))), x] + Simp[n/(2*d*
(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ((LtQ[n, -1] &&  !IGtQ[n + p +
1, 0]) || (LtQ[n, 0] && LtQ[p, -1]) || EqQ[n + 2*p + 2, 0]) && NeQ[n + p +
1, 0]
```

3.299.4 Maple [F]

$$\int \frac{x(-e^2x^2+d^2)^p}{(ex+d)^4}dx$$

input
```
int(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

output
```
int(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

3.299.5 Fricas [F]

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^4}\,dx=\int \frac{(-e^2x^2+d^2)^p x}{(ex+d)^4}\,dx$$

input
```
integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*x/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d
^3*e*x + d^4), x)
```

3.299.6 Sympy [F]

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^4}\,dx=\int \frac{x(-(-d+ex)\,(d+ex))^p}{(d+ex)^4}\,dx$$

input
```
integrate(x*(-e**2*x**2+d**2)**p/(e*x+d)**4,x)
```

output
```
Integral(x*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

3.299.7 Maxima [F]

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^4}\,dx=\int \frac{(-e^2x^2+d^2)^p x}{(ex+d)^4}\,dx$$

input
```
integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^4, x)
```

3.299.8 Giac [F]

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^4}\,dx=\int \frac{(-e^2x^2+d^2)^p x}{(ex+d)^4}\,dx$$

input
```
integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^4, x)
```

3.299.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^4}\,dx=\int \frac{x\,(d^2-e^2\,x^2)^p}{(d+e\,x)^4}\,dx$$

input
```
int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x)
```

output
```
int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^4, x)
```

3.300 $\int \frac{(d^2-e^2x^2)^p}{(d+ex)^4} dx$

3.300.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^4} dx = -\frac{2^{-4+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(4-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^5e(1+p)}$$

output
```
-2^(-4+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(p+1)*hypergeom([p+1, 4-p],[2+p]
,1/2*(-e*x+d)/d)/d^5/e/(p+1)
```

3.300.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^4} dx = -\frac{2^{-4+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(4-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^4e(1+p)}$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(d + e*x)^4,x]
```

output
```
-((2^(-4 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[4 - p, 1 + p,
2 + p, (d - e*x)/(2*d)])/(d^4*e*(1 + p)*(1 + (e*x)/d)^p))
```

3.300.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^4}\,dx$$

$$\Big\downarrow \text{473}$$

$$\frac{(d-ex)^{-p-1}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\int (d-ex)^p\left(\frac{ex}{d}+1\right)^{p-4}dx}{d^5}$$

$$\Big\downarrow \text{79}$$

$$-\frac{2^{p-4}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\text{Hypergeometric2F1}\left(4-p,p+1,p+2,\frac{d-ex}{2d}\right)}{d^5e(p+1)}$$

input
```
Int[(d^2 - e^2*x^2)^p/(d + e*x)^4,x]
```

output
```
-((2^(-4 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometri
c2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^5*e*(1 + p)))
```

3.300.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

3.300.4 Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4}dx$$

input
```
int((-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

output
```
int((-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

3.300.5 Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3
*e*x + d^4), x)
```

3.300.6 Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(-(-d+ex)(d+ex))^p}{(d+ex)^4}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**p/(e*x+d)**4,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

3.300.7 Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/(e*x + d)^4, x)
```

3.300.8 Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/(e*x + d)^4, x)
```

3.300.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(d^2-e^2\,x^2)^p}{(d+e\,x)^4}\,dx$$

input
```
int((d^2 - e^2*x^2)^p/(d + e*x)^4,x)
```

output
```
int((d^2 - e^2*x^2)^p/(d + e*x)^4, x)
```

3.301 $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4}\,dx$

3.301.1 Optimal result

Integrand size = 25, antiderivative size = 204

$$\begin{aligned}\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4}\,dx &= \frac{4d^2(d^2-e^2x^2)^{-3+p}}{3-p} - \frac{4dex(d^2-e^2x^2)^{-3+p}}{5-2p} - \frac{(d^2-e^2x^2)^{-2+p}}{2(2-p)} \\ &\quad - \frac{8e(2-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^5(5-2p)} \\ &\quad + \frac{(d^2-e^2x^2)^{-2+p}\,\text{Hypergeometric2F1}\left(1,-2+p,-1+p,1-\frac{e^2x^2}{d^2}\right)}{2(2-p)}\end{aligned}$$

output
```
4*d^2*(-e^2*x^2+d^2)^(-3+p)/(3-p)-4*d*e*x*(-e^2*x^2+d^2)^(-3+p)/(5-2*p)-1/
2*(-e^2*x^2+d^2)^(-2+p)/(2-p)-8*e*(2-p)*x*(-e^2*x^2+d^2)^p*hypergeom([1/2,
 4-p],[3/2],e^2*x^2/d^2)/d^5/(5-2*p)/((1-e^2*x^2/d^2)^p)+1/2*(-e^2*x^2+d^2
)^(-2+p)*hypergeom([1, -2+p],[-1+p],1-e^2*x^2/d^2)/(2-p)
```

3.301.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 417 vs. 2(204) = 408.

Time = 0.42 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.04

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4}\,dx$$
$$=\frac{2^{-4+p}\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(8p\left(1-\frac{d^2}{e^2x^2}\right)^p(d-ex)\,\text{Hypergeometric2F1}(1-p,1+p,2-$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(x*(d + e*x)^4),x]
```

output
```
(2^(-4 + p)*(d^2 - e^2*x^2)^p*(8*p*(1 - d^2/(e^2*x^2))^p*(d - e*x)*Hyperge
ometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 4*p*(1 - d^2/(e^2*x^2))
^p*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 2*d
*p*(1 - d^2/(e^2*x^2))^p*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/
(2*d)] - 2*e*p*(1 - d^2/(e^2*x^2))^p*x*Hypergeometric2F1[3 - p, 1 + p, 2 +
 p, (d - e*x)/(2*d)] + d*p*(1 - d^2/(e^2*x^2))^p*Hypergeometric2F1[4 - p,
1 + p, 2 + p, (d - e*x)/(2*d)] - e*p*(1 - d^2/(e^2*x^2))^p*x*Hypergeometri
c2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 8*d*(1/2 + (e*x)/(2*d))^p*Hyp
ergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)] + 8*d*p*(1/2 + (e*x)/(2*d))^p
*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)]))/(d^5*p*(1 + p)*(1 - d^2
/(e^2*x^2))^p*(1 + (e*x)/d)^p)
```

3.301.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {570, 543, 299, 238, 237, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4}\,dx$$
$$\Big\downarrow \; 570$$
$$\int \frac{(d-ex)^4(d^2-e^2x^2)^{p-4}}{x}dx$$

$$\Big\downarrow \text{543}$$

$$\int \frac{(d^2-e^2x^2)^{p-4}(d^4+6e^2x^2d^2+e^4x^4)}{x}dx+\int (d^2-e^2x^2)^{p-4}(-4ed^3-4e^3x^2d)\,dx$$

$$\Big\downarrow \text{299}$$

$$\int \frac{(d^2-e^2x^2)^{p-4}(d^4+6e^2x^2d^2+e^4x^4)}{x}dx-\frac{8d^3e(2-p)\int (d^2-e^2x^2)^{p-4}dx}{5-2p}-\frac{4dex(d^2-e^2x^2)^{p-3}}{5-2p}$$

$$\Big\downarrow \text{238}$$

$$-\frac{8e(2-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\int \left(1-\frac{e^2x^2}{d^2}\right)^{p-4}dx}{d^5(5-2p)}+\int \frac{(d^2-e^2x^2)^{p-4}(d^4+6e^2x^2d^2+e^4x^4)}{x}dx-\frac{4dex(d^2-e^2x^2)^{p-3}}{5-2p}$$

$$\Big\downarrow \text{237}$$

$$\int \frac{(d^2-e^2x^2)^{p-4}(d^4+6e^2x^2d^2+e^4x^4)}{x}dx-\frac{4dex(d^2-e^2x^2)^{p-3}}{5-2p}-\frac{8e(2-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^5(5-2p)}$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{(d^2-e^2x^2)^{p-4}(d^4+6e^2x^2d^2+e^4x^4)}{x^2}dx^2-\frac{4dex(d^2-e^2x^2)^{p-3}}{5-2p}-\frac{8e(2-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^5(5-2p)}$$

$$\Big\downarrow \text{1195}$$

$$\frac{1}{2}\int \left(7d^2e^2(d^2-e^2x^2)^{p-4}+\frac{d^4(d^2-e^2x^2)^{p-4}}{x^2}-e^2(d^2-e^2x^2)^{p-3}\right)dx^2-\frac{4dex(d^2-e^2x^2)^{p-3}}{5-2p}-\frac{8e(2-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^5(5-2p)}$$

$$\Big\downarrow \text{2009}$$

3.301. $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4}\,dx$

$$\frac{1}{2}\left(\frac{d^2(d^2-e^2x^2)^{p-3}\operatorname{Hypergeometric2F1}\left(1,p-3,p-2,1-\frac{e^2x^2}{d^2}\right)}{3-p}+\frac{7d^2(d^2-e^2x^2)^{p-3}}{3-p}-\frac{(d^2-e^2x^2)^{p-2}}{2-p}\right)-\frac{4dex(d^2-e^2x^2)^{p-3}}{5-2p}-\frac{8e(2-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\operatorname{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^5(5-2p)}$$

input
```
Int[(d^2 - e^2*x^2)^p/(x*(d + e*x)^4),x]
```

output
```
(-4*d*e*x*(d^2 - e^2*x^2)^(-3 + p))/(5 - 2*p) - (8*e*(2 - p)*x*(d^2 - e^2*
x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2])/(d^5*(5 - 2*p)*(
1 - (e^2*x^2)/d^2)^p) + ((7*d^2*(d^2 - e^2*x^2)^(-3 + p))/(3 - p) - (d^2 -
 e^2*x^2)^(-2 + p)/(2 - p) + (d^2*(d^2 - e^2*x^2)^(-3 + p)*Hypergeometric2
F1[1, -3 + p, -2 + p, 1 - (e^2*x^2)/d^2])/(3 - p))/2
```

3.301.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

3.301. $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4}\,dx$

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.301.4 Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x\,(ex+d)^4}dx$$

input
```
int((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x)
```

output
```
int((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x)
```

3.301.5 Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e^4*x^5 + 4*d*e^3*x^4 + 6*d^2*e^2*x^3 + 4*d^3
*e*x^2 + d^4*x), x)
```

3.301.6 Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^p}{x\,(d+ex)^4}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**p/x/(e*x+d)**4,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p/(x*(d + e*x)**4), x)
```

3.301.7 Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x), x)
```

3.301.8 Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x), x)
```

3.301.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4}\,dx=\int \frac{{\left(d^2-e^2\,x^2\right)}^p}{x\,{\left(d+e\,x\right)}^4}\,dx$$

input
```
int((d^2 - e^2*x^2)^p/(x*(d + e*x)^4),x)
```

output
```
int((d^2 - e^2*x^2)^p/(x*(d + e*x)^4), x)
```

3.302 $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^4}\,dx$

3.302.1 Optimal result

Integrand size = 25, antiderivative size = 207

$$\begin{aligned}
&\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^4}\,dx \\
&= -\frac{4de(d^2-e^2x^2)^{-3+p}}{3-p}-\frac{d^2(d^2-e^2x^2)^{-3+p}}{x}+\frac{e^2x(d^2-e^2x^2)^{-3+p}}{5-2p} \\
&\quad+\frac{4e^2(16-9p+p^2)\,x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^6(5-2p)} \\
&\quad-\frac{2e(d^2-e^2x^2)^{-2+p}\,\text{Hypergeometric2F1}\left(1,-2+p,-1+p,1-\frac{e^2x^2}{d^2}\right)}{d(2-p)}
\end{aligned}$$

output
```
-4*d*e*(-e^2*x^2+d^2)^(-3+p)/(3-p)-d^2*(-e^2*x^2+d^2)^(-3+p)/x+e^2*x*(-e^2
*x^2+d^2)^(-3+p)/(5-2*p)+4*e^2*(p^2-9*p+16)*x*(-e^2*x^2+d^2)^p*hypergeom([
1/2, 4-p],[3/2],e^2*x^2/d^2)/d^6/(5-2*p)/((1-e^2*x^2/d^2)^p)-2*e*(-e^2*x^2
+d^2)^(-2+p)*hypergeom([1, -2+p],[-1+p],1-e^2*x^2/d^2)/d/(2-p)
```

3.302.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.63

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^4}\,dx$$
$$= \frac{(d^2-e^2x^2)^p\left(-16d^2p(1+p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)+2^{5+p}epx(-d+ex)\left(1\right.\right.}{}$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^4),x]
```

output
```
((d^2 - e^2*x^2)^p*((-16*d^2*p*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, (e
^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p + (2^(5 + p)*e*p*x*(-d + e*x)*Hypergeo
metric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(1 + (e*x)/d)^p + (3*2^(2
 + p)*e*p*x*(-d + e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2
*d)])/(1 + (e*x)/d)^p + (2^(2 + p)*e*p*x*(-d + e*x)*Hypergeometric2F1[3 -
p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(1 + (e*x)/d)^p + (2^p*e*p*x*(-d + e*x)
*Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(1 + (e*x)/d)^p
- (32*d*e*(1 + p)*x*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(1 -
d^2/(e^2*x^2))^p))/(16*d^6*p*(1 + p)*x)
```

3.302.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {570, 543, 354, 27, 88, 75, 1588, 25, 27, 299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^4}\,dx$$
$$\Big\downarrow \text{570}$$
$$\int \frac{(d-ex)^4\left(d^2-e^2x^2\right)^{p-4}}{x^2}dx$$
$$\Big\downarrow \text{543}$$
$$\int \frac{\left(d^2-e^2x^2\right)^{p-4}\left(d^4+6e^2x^2d^2+e^4x^4\right)}{x^2}dx+\int \frac{\left(d^2-e^2x^2\right)^{p-4}\left(-4ed^3-4e^3x^2d\right)}{x}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int -\frac{4de\left(d^2-e^2x^2\right)^{p-4}\left(d^2+e^2x^2\right)}{x^2}dx^2+\int \frac{\left(d^2-e^2x^2\right)^{p-4}\left(d^4+6e^2x^2d^2+e^4x^4\right)}{x^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\left(d^2-e^2x^2\right)^{p-4}\left(d^4+6e^2x^2d^2+e^4x^4\right)}{x^2}dx-2de\int \frac{\left(d^2-e^2x^2\right)^{p-4}\left(d^2+e^2x^2\right)}{x^2}dx^2$$

$$\Big\downarrow \text{88}$$

$$\int \frac{\left(d^2-e^2x^2\right)^{p-4}\left(d^4+6e^2x^2d^2+e^4x^4\right)}{x^2}dx-2de\left(\int \frac{\left(d^2-e^2x^2\right)^{p-3}}{x^2}dx^2+\frac{2\left(d^2-e^2x^2\right)^{p-3}}{3-p}\right)$$

$$\Big\downarrow \text{75}$$

$$\int \frac{\left(d^2-e^2x^2\right)^{p-4}\left(d^4+6e^2x^2d^2+e^4x^4\right)}{x^2}dx-$$
$$2de\left(\frac{\left(d^2-e^2x^2\right)^{p-2}\text{Hypergeometric2F1}\left(1,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{d^2(2-p)}+\frac{2\left(d^2-e^2x^2\right)^{p-3}}{3-p}\right)$$

$$\Big\downarrow \text{1588}$$

$$-\frac{\int -d^2e^2\left(d^2-e^2x^2\right)^{p-4}\left((13-2p)d^2+e^2x^2\right)dx}{d^2}-$$
$$2de\left(\frac{\left(d^2-e^2x^2\right)^{p-2}\text{Hypergeometric2F1}\left(1,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{d^2(2-p)}+\frac{2\left(d^2-e^2x^2\right)^{p-3}}{3-p}\right)-$$
$$\frac{d^2\left(d^2-e^2x^2\right)^{p-3}}{x}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int d^2e^2\left(d^2-e^2x^2\right)^{p-4}\left((13-2p)d^2+e^2x^2\right)dx}{d^2}-$$
$$2de\left(\frac{\left(d^2-e^2x^2\right)^{p-2}\text{Hypergeometric2F1}\left(1,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{d^2(2-p)}+\frac{2\left(d^2-e^2x^2\right)^{p-3}}{3-p}\right)-$$
$$\frac{d^2\left(d^2-e^2x^2\right)^{p-3}}{x}$$

$$\Big\downarrow \text{27}$$

3.302. $\int \frac{\left(d^2-e^2x^2\right)^p}{x^2(d+ex)^4}\,dx$

$$e^2 \int \left(d^2-e^2x^2\right)^{p-4} \left((13-2p)d^2+e^2x^2\right) dx - 2de \left(\frac{\left(d^2-e^2x^2\right)^{p-2} \text{Hypergeometric2F1}\left(1,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{d^2(2-p)}+\frac{2\left(d^2-e^2x^2\right)^{p-3}}{3-p}\right) - \frac{d^2\left(d^2-e^2x^2\right)^{p-3}}{x}$$

$\Big\downarrow$ 299

$$e^2 \left(\frac{4d^2\left(p^2-9p+16\right)\int \left(d^2-e^2x^2\right)^{p-4}dx}{5-2p}+\frac{x\left(d^2-e^2x^2\right)^{p-3}}{5-2p}\right) - 2de \left(\frac{\left(d^2-e^2x^2\right)^{p-2} \text{Hypergeometric2F1}\left(1,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{d^2(2-p)}+\frac{2\left(d^2-e^2x^2\right)^{p-3}}{3-p}\right) - \frac{d^2\left(d^2-e^2x^2\right)^{p-3}}{x}$$

$\Big\downarrow$ 238

$$e^2 \left(\frac{4\left(p^2-9p+16\right)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p \int \left(1-\frac{e^2x^2}{d^2}\right)^{p-4}dx}{d^6(5-2p)}+\frac{x\left(d^2-e^2x^2\right)^{p-3}}{5-2p}\right) - 2de \left(\frac{\left(d^2-e^2x^2\right)^{p-2} \text{Hypergeometric2F1}\left(1,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{d^2(2-p)}+\frac{2\left(d^2-e^2x^2\right)^{p-3}}{3-p}\right) - \frac{d^2\left(d^2-e^2x^2\right)^{p-3}}{x}$$

$\Big\downarrow$ 237

$$-2de \left(\frac{\left(d^2-e^2x^2\right)^{p-2} \text{Hypergeometric2F1}\left(1,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{d^2(2-p)}+\frac{2\left(d^2-e^2x^2\right)^{p-3}}{3-p}\right) - \frac{d^2\left(d^2-e^2x^2\right)^{p-3}}{x} + e^2 \left(\frac{x\left(d^2-e^2x^2\right)^{p-3}}{5-2p}+\frac{4\left(p^2-9p+16\right)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^6(5-2p)}\right)$$

input
```
Int[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^4),x]
```

3.302. $\int \frac{\left(d^2-e^2x^2\right)^p}{x^2(d+ex)^4}\,dx$

output
```
-((d^2*(d^2 - e^2*x^2)^(-3 + p))/x) + e^2*((x*(d^2 - e^2*x^2)^(-3 + p))/(5
 - 2*p) + (4*(16 - 9*p + p^2)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 4
 - p, 3/2, (e^2*x^2)/d^2])/(d^6*(5 - 2*p)*(1 - (e^2*x^2)/d^2)^p)) - 2*d*e*
((2*(d^2 - e^2*x^2)^(-3 + p))/(3 - p) + ((d^2 - e^2*x^2)^(-2 + p)*Hypergeo
metric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(d^2*(2 - p)))
```

3.302.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

3.302. $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^4}\,dx$

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

3.302. $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^4}\,dx$

3.302.4 Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x^2(ex+d)^4}dx$$

input
```
int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x)
```

output
```
int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x)
```

3.302.5 Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x^2}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e^4*x^6 + 4*d*e^3*x^5 + 6*d^2*e^2*x^4 + 4*d^3
*e*x^3 + d^4*x^2), x)
```

3.302.6 Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^4}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^p}{x^2\,(d+ex)^4}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d)**4,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p/(x**2*(d + e*x)**4), x)
```

3.302.7 Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^2(d+ex)^4} \, dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex+d)^4 x^2} \, dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^2), x)
```

3.302.8 Giac [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^2(d+ex)^4} \, dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex+d)^4 x^2} \, dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^2), x)
```

3.302.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^p}{x^2(d+ex)^4} \, dx = \int \frac{(d^2 - e^2\,x^2)^p}{x^2\,(d+e\,x)^4} \, dx$$

input
```
int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^4),x)
```

output
```
int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^4), x)
```

3.303 $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^4}\,dx$

3.303.1 Optimal result

Integrand size = 25, antiderivative size = 211

$$\begin{aligned}\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^4}\,dx &= \frac{e^2(11-p)\,(d^2-e^2x^2)^{-3+p}}{2(3-p)} - \frac{d^2(d^2-e^2x^2)^{-3+p}}{2x^2} + \frac{4de(d^2-e^2x^2)^{-3+p}}{x} \\ &- \frac{8e^3(4-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^7} \\ &+ \frac{e^2(10-p)\,(d^2-e^2x^2)^{-2+p}\,\text{Hypergeometric2F1}\left(1,-2+p,-1+p,1-\frac{e^2x^2}{d^2}\right)}{2d^2(2-p)}\end{aligned}$$

output
```
1/2*e^2*(11-p)*(-e^2*x^2+d^2)^(-3+p)/(3-p)-1/2*d^2*(-e^2*x^2+d^2)^(-3+p)/x
^2+4*d*e*(-e^2*x^2+d^2)^(-3+p)/x-8*e^3*(4-p)*x*(-e^2*x^2+d^2)^p*hypergeom(
[1/2, 4-p],[3/2],e^2*x^2/d^2)/d^7/((1-e^2*x^2/d^2)^p)+1/2*e^2*(10-p)*(-e^2
*x^2+d^2)^(-2+p)*hypergeom([1, -2+p],[-1+p],1-e^2*x^2/d^2)/d^2/(2-p)
```

3.303.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.89

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^4}\,dx$$
$$= \frac{(d^2-e^2x^2)^p\left(\frac{64d^2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}+\frac{8d^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,\frac{d^2}{e^2x^2}\right)}{(-1+p)x^2}+\cdots\right.}{\cdots}$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^4),x]
```

output
```
((d^2 - e^2*x^2)^p*((64*d^2*e*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d
^2])/(x*(1 - (e^2*x^2)/d^2)^p) + (8*d^3*Hypergeometric2F1[1 - p, -p, 2 - p
, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (5*2^(4 + p)*e^2*
(d - e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p
)*(1 + (e*x)/d)^p) + (3*2^(3 + p)*e^2*(d - e*x)*Hypergeometric2F1[2 - p, 1
 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^(1 + p)*e^
2*(d - e*x)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 +
 p)*(1 + (e*x)/d)^p) + (2^p*e^2*(d - e*x)*Hypergeometric2F1[4 - p, 1 + p,
2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (80*d*e^2*Hypergeomet
ric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(16*d^7)
```

3.303.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {570, 543, 359, 238, 237, 1578, 1193, 25, 27, 88, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^4}\,dx$$
$$\Big\downarrow \text{570}$$
$$\int \frac{(d-ex)^4(d^2-e^2x^2)^{p-4}}{x^3}dx$$
$$\Big\downarrow \text{543}$$

$$\int \frac{(d^2-e^2x^2)^{p-4}(d^4+6e^2x^2d^2+e^4x^4)}{x^3}dx+\int \frac{(d^2-e^2x^2)^{p-4}(-4ed^3-4e^3x^2d)}{x^2}dx$$

$$\Big\downarrow \ 359$$

$$-8de^3(4-p)\int (d^2-e^2x^2)^{p-4}dx+\int \frac{(d^2-e^2x^2)^{p-4}(d^4+6e^2x^2d^2+e^4x^4)}{x^3}dx+\frac{4de(d^2-e^2x^2)^{p-3}}{x}$$

$$\Big\downarrow \ 238$$

$$-\frac{8e^3(4-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\int \left(1-\frac{e^2x^2}{d^2}\right)^{p-4}dx}{d^7}+\int \frac{(d^2-e^2x^2)^{p-4}(d^4+6e^2x^2d^2+e^4x^4)}{x^3}dx+\frac{4de(d^2-e^2x^2)^{p-3}}{x}$$

$$\Big\downarrow \ 237$$

$$\int \frac{(d^2-e^2x^2)^{p-4}(d^4+6e^2x^2d^2+e^4x^4)}{x^3}dx+\frac{4de(d^2-e^2x^2)^{p-3}}{x}-\frac{8e^3(4-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^7}$$

$$\Big\downarrow \ 1578$$

$$\frac{1}{2}\int \frac{(d^2-e^2x^2)^{p-4}(d^4+6e^2x^2d^2+e^4x^4)}{x^4}dx^2+\frac{4de(d^2-e^2x^2)^{p-3}}{x}-\frac{8e^3(4-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^7}$$

$$\Big\downarrow \ 1193$$

$$\frac{1}{2}\left(-\frac{\int -\frac{d^2e^2(d^2-e^2x^2)^{p-4}((10-p)d^2+e^2x^2)}{x^2}dx^2}{d^2}-\frac{d^2(d^2-e^2x^2)^{p-3}}{x^2}\right)+\frac{4de(d^2-e^2x^2)^{p-3}}{x}-\frac{8e^3(4-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^7}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(\frac{\int \frac{d^2e^2(d^2-e^2x^2)^{p-4}((10-p)d^2+e^2x^2)}{x^2}dx^2}{d^2}-\frac{d^2(d^2-e^2x^2)^{p-3}}{x^2}\right)+\frac{4de(d^2-e^2x^2)^{p-3}}{x}-\frac{8e^3(4-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^7}$$

$$\Big\downarrow \ 27$$

3.303. $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^4}\,dx$

$$\frac{1}{2}\left(e^2\int \frac{(d^2-e^2x^2)^{p-4}\left((10-p)d^2+e^2x^2\right)}{x^2}dx^2-\frac{d^2(d^2-e^2x^2)^{p-3}}{x^2}\right)+\frac{4de(d^2-e^2x^2)^{p-3}}{x}-\frac{8e^3(4-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^7}$$

$$\Big\downarrow \; 88$$

$$\frac{1}{2}\left(e^2\left((10-p)\int \frac{(d^2-e^2x^2)^{p-3}}{x^2}dx^2+\frac{(11-p)\,(d^2-e^2x^2)^{p-3}}{3-p}\right)-\frac{d^2(d^2-e^2x^2)^{p-3}}{x^2}\right)+\frac{4de(d^2-e^2x^2)^{p-3}}{x}-\frac{8e^3(4-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^7}$$

$$\Big\downarrow \; 75$$

$$\frac{1}{2}\left(e^2\left(\frac{(10-p)\,(d^2-e^2x^2)^{p-2}\,\text{Hypergeometric2F1}\left(1,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{d^2(2-p)}+\frac{(11-p)\,(d^2-e^2x^2)^{p-3}}{3-p}\right)-d^2\right.$$
$$\frac{4de(d^2-e^2x^2)^{p-3}}{x}-\frac{8e^3(4-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^7}$$

input
```
Int[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^4),x]
```

output
```
(4*d*e*(d^2 - e^2*x^2)^(-3 + p))/x - (8*e^3*(4 - p)*x*(d^2 - e^2*x^2)^p*Hy
pergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2])/(d^7*(1 - (e^2*x^2)/d^2)^
p) + (-((d^2*(d^2 - e^2*x^2)^(-3 + p))/x^2) + e^2*(((11 - p)*(d^2 - e^2*x^
2)^(-3 + p))/(3 - p) + ((10 - p)*(d^2 - e^2*x^2)^(-2 + p)*Hypergeometric2F
1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(d^2*(2 - p))))/2
```

3.303.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

3.303. $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^4} \, dx$

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.303.4 Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x^3\,(ex+d)^4}dx$$

input
```
int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x)
```

output
```
int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x)
```

3.303.5 Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x^3}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e^4*x^7 + 4*d*e^3*x^6 + 6*d^2*e^2*x^5 + 4*d^3
*e*x^4 + d^4*x^3), x)
```

3.303.6 Sympy [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^3(d + ex)^4} \, dx = \int \frac{(-(-d + ex)(d + ex))^p}{x^3 (d + ex)^4} \, dx$$

input
```
integrate((-e**2*x**2+d**2)**p/x**3/(e*x+d)**4,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p/(x**3*(d + e*x)**4), x)
```

3.303.7 Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^3(d + ex)^4} \, dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^4 x^3} \, dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^3), x)
```

3.303.8 Giac [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^3(d + ex)^4} \, dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^4 x^3} \, dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^3), x)
```

3.303.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^p}{x^3(d + ex)^4} \, dx = \int \frac{(d^2 - e^2\,x^2)^p}{x^3\,(d + e\,x)^4} \, dx$$

input
```
int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^4),x)
```

output
```
int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^4), x)
```

3.304 $\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^4}\,dx$

3.304.1 Optimal result

Integrand size = 25, antiderivative size = 210

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^4}\,dx = -\frac{d^2(d^2-e^2x^2)^{-3+p}}{3x^3}+\frac{2de(d^2-e^2x^2)^{-3+p}}{x^2}-\frac{e^2(27-2p)\,(d^2-e^2x^2)^{-3+p}}{3x} + \frac{4e^4(48-17p+p^2)\,x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{3d^8} - \frac{2e^3(5-p)\,(d^2-e^2x^2)^{-3+p}\,\text{Hypergeometric2F1}\left(1,-3+p,-2+p,1-\frac{e^2x^2}{d^2}\right)}{d(3-p)}$$

output
```
-1/3*d^2*(-e^2*x^2+d^2)^(-3+p)/x^3+2*d*e*(-e^2*x^2+d^2)^(-3+p)/x^2-1/3*e^2
*(27-2*p)*(-e^2*x^2+d^2)^(-3+p)/x+4/3*e^4*(p^2-17*p+48)*x*(-e^2*x^2+d^2)^p
*hypergeom([1/2, 4-p],[3/2],e^2*x^2/d^2)/d^8/((1-e^2*x^2/d^2)^p)-2*e^3*(5-
p)*(-e^2*x^2+d^2)^(-3+p)*hypergeom([1, -3+p],[-2+p],1-e^2*x^2/d^2)/d/(3-p)
```

3.304.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 452 vs. 2(210) = 420.

Time = 0.78 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.15

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^4}\,dx$$
$$= \frac{(d^2-e^2x^2)^p\left(-\frac{16d^4\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{2},-p,-\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x^3}-\frac{480d^2e^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}\right.}{}$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^4),x]
```

output
```
((d^2 - e^2*x^2)^p*((-16*d^4*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d
^2])/(x^3*(1 - (e^2*x^2)/d^2)^p) - (480*d^2*e^2*Hypergeometric2F1[-1/2, -p
, 1/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) - (96*d^3*e*Hypergeometri
c2F1[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2
) + (15*2^(5 + p)*e^3*(-d + e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d
 - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (15*2^(3 + p)*e^3*(-d + e*x)*H
ypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x
)/d)^p) + (3*2^(3 + p)*e^3*(-d + e*x)*Hypergeometric2F1[3 - p, 1 + p, 2 +
p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^p*e^3*(-d + e*x)*Hyp
ergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/
d)^p) - (480*d*e^3*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1
- d^2/(e^2*x^2))^p)))/(48*d^8)
```

3.304.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {570, 543, 354, 27, 87, 75, 1588, 25, 27, 359, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^4}\,dx$$
$$\Big\downarrow \text{570}$$

$$\int \frac{(d-ex)^4 \left(d^2-e^2x^2\right)^{p-4}}{x^4}dx$$

$\Big\downarrow$ 543

$$\int \frac{\left(d^2-e^2x^2\right)^{p-4} \left(d^4+6e^2x^2d^2+e^4x^4\right)}{x^4}dx+\int \frac{\left(d^2-e^2x^2\right)^{p-4} \left(-4ed^3-4e^3x^2d\right)}{x^3}dx$$

$\Big\downarrow$ 354

$$\frac{1}{2}\int -\frac{4de\left(d^2-e^2x^2\right)^{p-4} \left(d^2+e^2x^2\right)}{x^4}dx^2+\int \frac{\left(d^2-e^2x^2\right)^{p-4} \left(d^4+6e^2x^2d^2+e^4x^4\right)}{x^4}dx$$

$\Big\downarrow$ 27

$$\int \frac{\left(d^2-e^2x^2\right)^{p-4} \left(d^4+6e^2x^2d^2+e^4x^4\right)}{x^4}dx-2de\int \frac{\left(d^2-e^2x^2\right)^{p-4} \left(d^2+e^2x^2\right)}{x^4}dx^2$$

$\Big\downarrow$ 87

$$\int \frac{\left(d^2-e^2x^2\right)^{p-4} \left(d^4+6e^2x^2d^2+e^4x^4\right)}{x^4}dx-2de\left(e^2(5-p)\int \frac{\left(d^2-e^2x^2\right)^{p-4}}{x^2}dx^2-\frac{\left(d^2-e^2x^2\right)^{p-3}}{x^2}\right)$$

$\Big\downarrow$ 75

$$\int \frac{\left(d^2-e^2x^2\right)^{p-4} \left(d^4+6e^2x^2d^2+e^4x^4\right)}{x^4}dx-2de\left(\frac{e^2(5-p)\left(d^2-e^2x^2\right)^{p-3}\text{Hypergeometric2F1}\left(1,p-3,p-2,1-\frac{e^2x^2}{d^2}\right)}{d^2(3-p)}-\frac{\left(d^2-e^2x^2\right)^{p-3}}{x^2}\right)$$

$\Big\downarrow$ 1588

$$-\frac{\int -\frac{d^2e^2\left(d^2-e^2x^2\right)^{p-4}\left((27-2p)d^2+3e^2x^2\right)}{x^2}dx}{3d^2}-2de\left(\frac{e^2(5-p)\left(d^2-e^2x^2\right)^{p-3}\text{Hypergeometric2F1}\left(1,p-3,p-2,1-\frac{e^2x^2}{d^2}\right)}{d^2(3-p)}-\frac{\left(d^2-e^2x^2\right)^{p-3}}{x^2}\right)-\frac{d^2\left(d^2-e^2x^2\right)^{p-3}}{3x^3}$$

$\Big\downarrow$ 25

3.304. $\int \frac{\left(d^2-e^2x^2\right)^p}{x^4(d+ex)^4}\,dx$

$$\frac{\int \frac{d^2e^2\left(d^2-e^2x^2\right)^{p-4}\left((27-2p)d^2+3e^2x^2\right)}{x^2}dx}{3d^2}-2de\left(\frac{e^2(5-p)\left(d^2-e^2x^2\right)^{p-3}\text{Hypergeometric2F1}\left(1,p-3,p-2,1-\frac{e^2x^2}{d^2}\right)}{d^2(3-p)}-\frac{\left(d^2-e^2x^2\right)^{p-3}}{x^2}\right)-\frac{d^2\left(d^2-e^2x^2\right)^{p-3}}{3x^3}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}e^2\int \frac{\left(d^2-e^2x^2\right)^{p-4}\left((27-2p)d^2+3e^2x^2\right)}{x^2}dx-2de\left(\frac{e^2(5-p)\left(d^2-e^2x^2\right)^{p-3}\text{Hypergeometric2F1}\left(1,p-3,p-2,1-\frac{e^2x^2}{d^2}\right)}{d^2(3-p)}-\frac{\left(d^2-e^2x^2\right)^{p-3}}{x^2}\right)-\frac{d^2\left(d^2-e^2x^2\right)^{p-3}}{3x^3}$$

$$\Big\downarrow \; 359$$

$$\frac{1}{3}e^2\left(4e^2\left(p^2-17p+48\right)\int \left(d^2-e^2x^2\right)^{p-4}dx-\frac{(27-2p)\left(d^2-e^2x^2\right)^{p-3}}{x}\right)-2de\left(\frac{e^2(5-p)\left(d^2-e^2x^2\right)^{p-3}\text{Hypergeometric2F1}\left(1,p-3,p-2,1-\frac{e^2x^2}{d^2}\right)}{d^2(3-p)}-\frac{\left(d^2-e^2x^2\right)^{p-3}}{x^2}\right)-\frac{d^2\left(d^2-e^2x^2\right)^{p-3}}{3x^3}$$

$$\Big\downarrow \; 238$$

$$\frac{1}{3}e^2\left(\frac{4e^2\left(p^2-17p+48\right)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int \left(1-\frac{e^2x^2}{d^2}\right)^{p-4}dx}{d^8}-\frac{(27-2p)\left(d^2-e^2x^2\right)^{p-3}}{x}\right)-2de\left(\frac{e^2(5-p)\left(d^2-e^2x^2\right)^{p-3}\text{Hypergeometric2F1}\left(1,p-3,p-2,1-\frac{e^2x^2}{d^2}\right)}{d^2(3-p)}-\frac{\left(d^2-e^2x^2\right)^{p-3}}{x^2}\right)-\frac{d^2\left(d^2-e^2x^2\right)^{p-3}}{3x^3}$$

$$\Big\downarrow \; 237$$

3.304. $\int \frac{\left(d^2-e^2x^2\right)^p}{x^4(d+ex)^4}\,dx$

$$-2de\left(\frac{e^2(5-p)\left(d^2-e^2x^2\right)^{p-3}\text{Hypergeometric2F1}\left(1,p-3,p-2,1-\frac{e^2x^2}{d^2}\right)}{d^2(3-p)}-\frac{\left(d^2-e^2x^2\right)^{p-3}}{x^2}\right)-\frac{d^2\left(d^2-e^2x^2\right)^{p-3}}{3x^3}+\frac{1}{3}e^2\left(\frac{4e^2\left(p^2-17p+48\right)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^8}-\frac{(27-2p)\left(d^2-e\right.}{x}\right.$$

input
```
Int[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^4),x]
```

output
```
-1/3*(d^2*(d^2 - e^2*x^2)^(-3 + p))/x^3 + (e^2*(-(((27 - 2*p)*(d^2 - e^2*x
^2)^(-3 + p))/x) + (4*e^2*(48 - 17*p + p^2)*x*(d^2 - e^2*x^2)^p*Hypergeome
tric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2])/(d^8*(1 - (e^2*x^2)/d^2)^p)))/3 -
 2*d*e*(-((d^2 - e^2*x^2)^(-3 + p)/x^2) + (e^2*(5 - p)*(d^2 - e^2*x^2)^(-3
 + p)*Hypergeometric2F1[1, -3 + p, -2 + p, 1 - (e^2*x^2)/d^2])/(d^2*(3 - p
)))
```

3.304.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.304. $\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^4}\,dx$

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

3.304.4 Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x^4\,(ex+d)^4}dx$$

input
```
int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x)
```

output
```
int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x)
```

3.304.5 Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x^4}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e^4*x^8 + 4*d*e^3*x^7 + 6*d^2*e^2*x^6 + 4*d^3
*e*x^5 + d^4*x^4), x)
```

3.304.6 Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^4}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^p}{x^4\,(d+ex)^4}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**p/x**4/(e*x+d)**4,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p/(x**4*(d + e*x)**4), x)
```

3.304.7 Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^4(d+ex)^4}\,dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex+d)^4x^4}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^4), x)
```

3.304.8 Giac [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^4(d+ex)^4}\,dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex+d)^4x^4}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^4), x)
```

3.304.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^p}{x^4(d+ex)^4}\,dx = \int \frac{(d^2 - e^2\,x^2)^p}{x^4\,(d+e\,x)^4}\,dx$$

input
```
int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^4),x)
```

output
```
int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^4), x)
```

3.305 $\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4}\,dx$

3.305.1 Optimal result

Integrand size = 25, antiderivative size = 216

$$\begin{aligned}\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4}\,dx &= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{4x^4}+\frac{4de(d^2-e^2x^2)^{-3+p}}{3x^3}-\frac{e^2(17-p)\,(d^2-e^2x^2)^{-3+p}}{4x^2}\\&+\frac{8e^3(6-p)\,(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},4-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^7x}\\&+\frac{e^4(70-21p+p^2)\,(d^2-e^2x^2)^{-3+p}\,\text{Hypergeometric2F1}\left(1,-3+p,-2+p,1-\frac{e^2x^2}{d^2}\right)}{4d^2(3-p)}\end{aligned}$$

output
```
-1/4*d^2*(-e^2*x^2+d^2)^(-3+p)/x^4+4/3*d*e*(-e^2*x^2+d^2)^(-3+p)/x^3-1/4*e
^2*(17-p)*(-e^2*x^2+d^2)^(-3+p)/x^2+8/3*e^3*(6-p)*(-e^2*x^2+d^2)^p*hyperge
om([-1/2, 4-p],[1/2],e^2*x^2/d^2)/d^7/x/((1-e^2*x^2/d^2)^p)+1/4*e^4*(p^2-2
1*p+70)*(-e^2*x^2+d^2)^(-3+p)*hypergeom([1, -3+p],[-2+p],1-e^2*x^2/d^2)/d^
2/(3-p)
```

3.305.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 505 vs. 2(216) = 432.

Time = 0.90 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.34

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4}\,dx$$
$$= \frac{(d^2-e^2x^2)^p\left(\frac{64d^4e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{2},-p,-\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x^3}+\frac{960d^2e^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}\right.}{}$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^4),x]
```

output
```
((d^2 - e^2*x^2)^p*((64*d^4*e*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/
d^2])/(x^3*(1 - (e^2*x^2)/d^2)^p) + (960*d^2*e^3*Hypergeometric2F1[-1/2, -
p, 1/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) + (240*d^3*e^2*Hypergeom
etric2F1[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p
*x^2) + (105*2^(3 + p)*e^4*(d - e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p
, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (24*d^5*Hypergeometric2F1[
2 - p, -p, 3 - p, d^2/(e^2*x^2)])/((-2 + p)*(1 - d^2/(e^2*x^2))^p*x^4) + (
45*2^(2 + p)*e^4*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x
)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (15*2^(1 + p)*e^4*(d - e*x)*Hypergeo
metric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p)
 + (3*2^p*e^4*(d - e*x)*Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(
2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (840*d*e^4*Hypergeometric2F1[-p, -p, 1
- p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(48*d^9)
```

3.305.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {570, 543, 359, 279, 278, 1578, 1193, 25, 27, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4}\,dx$$
$$\Big\downarrow \text{570}$$

$$\int \frac{(d-ex)^4 \left(d^2-e^2x^2\right)^{p-4}}{x^5}dx$$

$$\Big\downarrow \ 543$$

$$\int \frac{\left(d^2-e^2x^2\right)^{p-4} \left(d^4+6e^2x^2d^2+e^4x^4\right)}{x^5}dx+\int \frac{\left(d^2-e^2x^2\right)^{p-4} \left(-4ed^3-4e^3x^2d\right)}{x^4}dx$$

$$\Big\downarrow \ 359$$

$$-\frac{8}{3}de^3(6-p)\int \frac{\left(d^2-e^2x^2\right)^{p-4}}{x^2}dx+\int \frac{\left(d^2-e^2x^2\right)^{p-4} \left(d^4+6e^2x^2d^2+e^4x^4\right)}{x^5}dx+\frac{4de\left(d^2-e^2x^2\right)^{p-3}}{3x^3}$$

$$\Big\downarrow \ 279$$

$$-\frac{8e^3(6-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^{p-4}}{x^2}dx}{3d^7}+\int \frac{\left(d^2-e^2x^2\right)^{p-4} \left(d^4+6e^2x^2d^2+e^4x^4\right)}{x^5}dx+\frac{4de\left(d^2-e^2x^2\right)^{p-3}}{3x^3}$$

$$\Big\downarrow \ 278$$

$$\int \frac{\left(d^2-e^2x^2\right)^{p-4} \left(d^4+6e^2x^2d^2+e^4x^4\right)}{x^5}dx+\frac{4de\left(d^2-e^2x^2\right)^{p-3}}{3x^3}+\frac{8e^3(6-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p \text{Hypergeometric2F1}\left(-\frac{1}{2},4-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^7x}$$

$$\Big\downarrow \ 1578$$

$$\frac{1}{2}\int \frac{\left(d^2-e^2x^2\right)^{p-4} \left(d^4+6e^2x^2d^2+e^4x^4\right)}{x^6}dx^2+\frac{4de\left(d^2-e^2x^2\right)^{p-3}}{3x^3}+\frac{8e^3(6-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p \text{Hypergeometric2F1}\left(-\frac{1}{2},4-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^7x}$$

$$\Big\downarrow \ 1193$$

$$\frac{1}{2}\left(-\frac{\int -\frac{d^2e^2\left(d^2-e^2x^2\right)^{p-4}\left((17-p)d^2+2e^2x^2\right)}{x^4}dx^2}{2d^2}-\frac{d^2\left(d^2-e^2x^2\right)^{p-3}}{2x^4}\right)+\frac{4de\left(d^2-e^2x^2\right)^{p-3}}{3x^3}+\frac{8e^3(6-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p \text{Hypergeometric2F1}\left(-\frac{1}{2},4-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^7x}$$

$$\Big\downarrow \ 25$$

3.305. $\int \frac{\left(d^2-e^2x^2\right)^p}{x^5(d+ex)^4}\,dx$

$$\frac{1}{2}\left(\frac{\int \frac{d^2e^2(d^2-e^2x^2)^{p-4}((17-p)d^2+2e^2x^2)}{x^4}dx^2}{2d^2}-\frac{d^2(d^2-e^2x^2)^{p-3}}{2x^4}\right)+\frac{4de(d^2-e^2x^2)^{p-3}}{3x^3}+\frac{8e^3(6-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(-\frac{1}{2},4-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^7x}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(\frac{1}{2}e^2\int \frac{(d^2-e^2x^2)^{p-4}((17-p)d^2+2e^2x^2)}{x^4}dx^2-\frac{d^2(d^2-e^2x^2)^{p-3}}{2x^4}\right)+\frac{4de(d^2-e^2x^2)^{p-3}}{3x^3}+\frac{8e^3(6-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(-\frac{1}{2},4-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^7x}$$

$$\Big\downarrow \ 87$$

$$\frac{1}{2}\left(\frac{1}{2}e^2\left(e^2(p^2-21p+70)\int \frac{(d^2-e^2x^2)^{p-4}}{x^2}dx^2-\frac{(17-p)(d^2-e^2x^2)^{p-3}}{x^2}\right)-\frac{d^2(d^2-e^2x^2)^{p-3}}{2x^4}\right)+\frac{4de(d^2-e^2x^2)^{p-3}}{3x^3}+\frac{8e^3(6-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(-\frac{1}{2},4-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^7x}$$

$$\Big\downarrow \ 75$$

$$\frac{1}{2}\left(\frac{1}{2}e^2\left(\frac{e^2(p^2-21p+70)(d^2-e^2x^2)^{p-3}\,\text{Hypergeometric2F1}\left(1,p-3,p-2,1-\frac{e^2x^2}{d^2}\right)}{d^2(3-p)}-\frac{(17-p)(d^2-e^2x^2)}{x^2}\right.\right.$$
$$\frac{4de(d^2-e^2x^2)^{p-3}}{3x^3}+\frac{8e^3(6-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(-\frac{1}{2},4-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^7x}$$

input
```
Int[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^4),x]
```

output
```
(4*d*e*(d^2 - e^2*x^2)^(-3 + p))/(3*x^3) + (8*e^3*(6 - p)*(d^2 - e^2*x^2)^
p*Hypergeometric2F1[-1/2, 4 - p, 1/2, (e^2*x^2)/d^2])/(3*d^7*x*(1 - (e^2*x
^2)/d^2)^p) + (-1/2*(d^2*(d^2 - e^2*x^2)^(-3 + p))/x^4 + (e^2*(-(((17 - p)
*(d^2 - e^2*x^2)^(-3 + p))/x^2) + (e^2*(70 - 21*p + p^2)*(d^2 - e^2*x^2)^(
-3 + p)*Hypergeometric2F1[1, -3 + p, -2 + p, 1 - (e^2*x^2)/d^2])/(d^2*(3 -
 p))))/2)/2
```

3.305. $\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4}\,dx$

3.305.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.305. $\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4}\,dx$

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.305.4 Maple [F]

$$\int \frac{(-e^2x^2 + d^2)^p}{x^5 (ex + d)^4} dx$$

input
```
int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x)
```

output
```
int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x)
```

3.305.5 Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4}\,dx=\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x^5}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e^4*x^9 + 4*d*e^3*x^8 + 6*d^2*e^2*x^7 + 4*d^3
*e*x^6 + d^4*x^5), x)
```

3.305.6 Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4}\,dx=\int \frac{(-(-d+ex)\,(d+ex))^p}{x^5\,(d+ex)^4}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**p/x**5/(e*x+d)**4,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p/(x**5*(d + e*x)**4), x)
```

3.305.7 Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4}\,dx=\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x^5}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^5), x)
```

3.305.8 Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4}\,dx=\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x^5}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^5), x)
```

3.305.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4}\,dx=\int \frac{(d^2-e^2\,x^2)^p}{x^5\,(d+e\,x)^4}\,dx$$

input
```
int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^4),x)
```

output
```
int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^4), x)
```

3.306 $\int (gx)^m(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx$

3.306.1 Optimal result

Integrand size = 27, antiderivative size = 264

$$\int (gx)^m(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{1+p}}{g^2(4+m+2p)} + \frac{2d^3(3+2m+p)(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{g(1+m)(3+m+2p)} + \frac{2d^2e(7+2m+3p)(gx)^{2+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2+m}{2},-p,\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{g^2(2+m)(4+m+2p)}$$

output
```
-3*d*(g*x)^(1+m)*(-e^2*x^2+d^2)^(p+1)/g/(3+m+2*p)-e*(g*x)^(2+m)*(-e^2*x^2+
d^2)^(p+1)/g^2/(4+m+2*p)+2*d^3*(3+2*m+p)*(g*x)^(1+m)*(-e^2*x^2+d^2)^p*hype
rgeom([-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)/g/(1+m)/(3+m+2*p)/((1-e^2*x
^2/d^2)^p)+2*d^2*e*(7+2*m+3*p)*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hypergeom([-p,
 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)/g^2/(2+m)/(4+m+2*p)/((1-e^2*x^2/d^2)^p)
```

3.306.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.73

$$\int (gx)^m (d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx$$

$$= x(gx)^m \left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \left(\frac{d^3 \text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{1+m} + ex\left(\frac{3d^2 \text{Hypergeometric2F1}\left(\frac{2+m}{2},-p,\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{2+m} + ex\left(\frac{3d \text{Hypergeometric2F1}\left(\frac{3+m}{2},-p,\frac{5+m}{2},\frac{e^2x^2}{d^2}\right)}{3+m} + \frac{ex \text{Hypergeometric2F1}\left(\frac{4+m}{2},-p,\frac{6+m}{2},\frac{e^2x^2}{d^2}\right)}{4+m}\right)\right)\right)$$

input
```
Integrate[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

output
```
(x*(g*x)^m*(d^2 - e^2*x^2)^p*((d^3*Hypergeometric2F1[(1 + m)/2, -p, (3 + m
)/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((3*d^2*Hypergeometric2F1[(2 + m)/2, -p
, (4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1[(3 + m)
/2, -p, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) + (e*x*Hypergeometric2F1[(4 + m
)/2, -p, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/(1 - (e^2*x^2)/d^2)^p
```

3.306.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {559, 25, 2340, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 (gx)^m \left(d^2-e^2x^2\right)^p \, dx$$

$$\Big\downarrow \; 559$$

$$-\frac{\int -(gx)^m \left(d^2-e^2x^2\right)^p \left(3d(m+2p+4)x^2e^4+2d^2(2m+3p+7)xe^3+d^3(m+2p+4)e^2\right)dx}{e^2(m+2p+4)} - \frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p+1}}{g^2(m+2p+4)}$$

$$\Big\downarrow 25$$

$$\frac{\int (gx)^m \left(d^2-e^2x^2\right)^p \left(3d(m+2p+4)x^2e^4+2d^2(2m+3p+7)xe^3+d^3(m+2p+4)e^2\right)dx}{e^2(m+2p+4)}-\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p+1}}{g^2(m+2p+4)}$$

$$\Big\downarrow 2340$$

$$\frac{-\frac{\int -2d^2e^4(gx)^m(d(2m+p+3)(m+2p+4)+e(m+2p+3)(2m+3p+7)x)\left(d^2-e^2x^2\right)^pdx}{e^2(m+2p+3)}-\frac{3de^2(m+2p+4)(gx)^{m+1}\left(d^2-e^2x^2\right)^{p+1}}{g(m+2p+3)}}{e^2(m+2p+4)}-\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p+1}}{g^2(m+2p+4)}$$

$$\Big\downarrow 27$$

$$\frac{\frac{2d^2e^2\int (gx)^m(d(2m+p+3)(m+2p+4)+e(m+2p+3)(2m+3p+7)x)\left(d^2-e^2x^2\right)^pdx}{m+2p+3}-\frac{3de^2(m+2p+4)(gx)^{m+1}\left(d^2-e^2x^2\right)^{p+1}}{g(m+2p+3)}}{e^2(m+2p+4)}-\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p+1}}{g^2(m+2p+4)}$$

$$\Big\downarrow 557$$

$$\frac{\frac{2d^2e^2\left(d(2m+p+3)(m+2p+4)\int (gx)^m\left(d^2-e^2x^2\right)^pdx+\frac{e(m+2p+3)(2m+3p+7)\int (gx)^{m+1}\left(d^2-e^2x^2\right)^pdx}{g}\right)}{m+2p+3}-\frac{3de^2(m+2p+4)(gx)^{m+1}\left(d^2-e^2x^2\right)^{p+}}{g(m+2p+3)}}{e^2(m+2p+4)}$$
$$\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p+1}}{g^2(m+2p+4)}$$

$$\Big\downarrow 279$$

$$\frac{\frac{2d^2e^2\left(d(2m+p+3)(m+2p+4)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^pdx+\frac{e(m+2p+3)(2m+3p+7)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int (gx)^{m+1}\left(1-\frac{e^2}{d^2}\right.}{g}\right.}{m+2p+3}}{e^2(m+2p+4)}$$
$$\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p+1}}{g^2(m+2p+4)}$$

$$\Big\downarrow 278$$

3.306. $\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$

$$\frac{\frac{2d^2e^2\left(\frac{e(m+2p+3)(2m+3p+7)(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+2}{2},-p,\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{g^2(m+2)}+\frac{d(2m+p+3)(m+2p+4)(gx)^{m+1}\left(d^2-e^2x^2\right)^p}{}\right.}{m+2p+3}}{e^2(m+2p+4)}$$

$$\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p+1}}{g^2(m+2p+4)}$$

input
```
Int[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

output
```
-((e*(g*x)^(2 + m)*(d^2 - e^2*x^2)^(1 + p))/(g^2*(4 + m + 2*p))) + ((-3*d*
e^2*(4 + m + 2*p)*(g*x)^(1 + m)*(d^2 - e^2*x^2)^(1 + p))/(g*(3 + m + 2*p))
 + (2*d^2*e^2*((d*(3 + 2*m + p)*(4 + m + 2*p)*(g*x)^(1 + m)*(d^2 - e^2*x^2
)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)
*(1 - (e^2*x^2)/d^2)^p) + (e*(3 + m + 2*p)*(7 + 2*m + 3*p)*(g*x)^(2 + m)*(
d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2, (e^2*x^2)/d^2
])/(g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)))/(3 + m + 2*p))/(e^2*(4 + m + 2*p)
)
```

3.306.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 559
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[d^n*(e*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*e^(n - 1)*(
m + n + 2*p + 1))), x] + Simp[1/(b*(m + n + 2*p + 1))   Int[(e*x)^m*(a + b*
x^2)^p*ExpandToSum[b*(m + n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1
)*x^n - a*d^n*(m + n - 1)*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && IGtQ[n, 1] &&  !IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

3.306.4 Maple [F]

$$\int (gx)^m (ex+d)^3 \left(-e^2x^2+d^2\right)^p dx$$

input
```
int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

output
```
int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

3.306.5 Fricas [F]

$$\int (gx)^m(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \int (ex+d)^3(-e^2x^2+d^2)^p(gx)^m \, dx$$

input
```
integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(-e^2*x^2 + d^2)^p*(g*x
)^m, x)
```

3.306.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.15 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.97

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx = \frac{d^3d^{2p}g^mx^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{3d^2d^{2p}eg^mx^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+2\right)} + \frac{3dd^{2p}e^2g^mx^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{d^{2p}e^3g^mx^{m+4}\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+3\right)}$$

input
```
integrate((g*x)**m*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

output
```
d**3*d**(2*p)*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2
 + 3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + 3*d**2*
d**(2*p)*e*g**m*x**(m + 2)*gamma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,),
 e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 2)) + 3*d*d**(2*p)*e**2*
g**m*x**(m + 3)*gamma(m/2 + 3/2)*hyper((-p, m/2 + 3/2), (m/2 + 5/2,), e**2
*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 5/2)) + d**(2*p)*e**3*g**m*x*
*(m + 4)*gamma(m/2 + 2)*hyper((-p, m/2 + 2), (m/2 + 3,), e**2*x**2*exp_pol
ar(2*I*pi)/d**2)/(2*gamma(m/2 + 3))
```

3.306.7 Maxima [F]

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^3\left(-e^2x^2+d^2\right)^p(gx)^m\,dx$$

input
```
integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

3.306.8 Giac [F]

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^3\left(-e^2x^2+d^2\right)^p(gx)^m\,dx$$

input
```
integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

3.306.9 Mupad [F(-1)]

Timed out.

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int \left(d^2-e^2\,x^2\right)^p\,(g\,x)^m\,(d+e\,x)^3\,dx$$

input
```
int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^3,x)
```

output
```
int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^3, x)
```

3.307 $\int (gx)^m(d+ex)^2 (d^2-e^2x^2)^p \, dx$

3.307.1 Optimal result

Integrand size = 27, antiderivative size = 206

$$\int (gx)^m(d+ex)^2 (d^2-e^2x^2)^p \, dx = -\frac{(gx)^{1+m}(d^2-e^2x^2)^{1+p}}{g(3+m+2p)} + \frac{2d^2(2+m+p)(gx)^{1+m}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{g(1+m)(3+m+2p)} + \frac{2de(gx)^{2+m}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2+m}{2},-p,\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{g^2(2+m)}$$

output
```
-(g*x)^(1+m)*(-e^2*x^2+d^2)^(p+1)/g/(3+m+2*p)+2*d^2*(2+m+p)*(g*x)^(1+m)*(-
e^2*x^2+d^2)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)/g/(1+m)/
(3+m+2*p)/((1-e^2*x^2/d^2)^p)+2*d*e*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hypergeom
([-p, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)/g^2/(2+m)/((1-e^2*x^2/d^2)^p)
```

3.307.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.82

$$\int (gx)^m(d+ex)^2 (d^2-e^2x^2)^p \, dx = \frac{x(gx)^m(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2(6+5m+m^2)\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)+e(1+m\right.}{(1+m}$$

input
```
Integrate[(g*x)^m*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

output
```
(x*(g*x)^m*(d^2 - e^2*x^2)^p*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[(1 + m
)/2, -p, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*(2*d*(3 + m)*Hypergeometr
ic2F1[(2 + m)/2, -p, (4 + m)/2, (e^2*x^2)/d^2] + e*(2 + m)*x*Hypergeometri
c2F1[(3 + m)/2, -p, (5 + m)/2, (e^2*x^2)/d^2])))/((1 + m)*(2 + m)*(3 + m)*
(1 - (e^2*x^2)/d^2)^p)
```

3.307.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {559, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2(gx)^m \left(d^2-e^2x^2\right)^p \, dx$$

$$\Big\downarrow \text{559}$$

$$-\frac{\int -2de^2(gx)^m(d(m+p+2)+e(m+2p+3)x)\left(d^2-e^2x^2\right)^p dx}{e^2(m+2p+3)}-\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^{p+1}}{g(m+2p+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2d\int (gx)^m(d(m+p+2)+e(m+2p+3)x)\left(d^2-e^2x^2\right)^p dx}{m+2p+3}-\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^{p+1}}{g(m+2p+3)}$$

$$\Big\downarrow \text{557}$$

$$\frac{2d\left(d(m+p+2)\int (gx)^m\left(d^2-e^2x^2\right)^p dx+\frac{e(m+2p+3)\int (gx)^{m+1}\left(d^2-e^2x^2\right)^p dx}{g}\right)}{m+2p+3}-\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^{p+1}}{g(m+2p+3)}$$

$$\Big\downarrow \text{279}$$

$$\frac{2d\left(d(m+p+2)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int(gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^p dx+\frac{e(m+2p+3)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int(gx)^{m+1}\left(1-\right.}{g}\right.}{m+2p+3}$$

$$\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^{p+1}}{g(m+2p+3)}$$

$$\Big\downarrow 278$$

$$\frac{2d\left(\frac{e(m+2p+3)(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+2}{2},-p,\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{g^2(m+2)}+\frac{d(m+p+2)(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-}}{g(m}\right.}{m+2p+3}$$

$$\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^{p+1}}{g(m+2p+3)}$$

input
```
Int[(g*x)^m*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

output
```
-(((g*x)^(1 + m)*(d^2 - e^2*x^2)^(1 + p))/(g*(3 + m + 2*p))) + (2*d*((d*(2
 + m + p)*(g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p,
 (3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*(1 - (e^2*x^2)/d^2)^p) + (e*(3 + m
+ 2*p)*(g*x)^(2 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, -p, (4
 + m)/2, (e^2*x^2)/d^2])/(g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)))/(3 + m + 2*
p)
```

3.307.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 559
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[d^n*(e*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*e^(n - 1)*(
m + n + 2*p + 1))), x] + Simp[1/(b*(m + n + 2*p + 1))   Int[(e*x)^m*(a + b*
x^2)^p*ExpandToSum[b*(m + n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1
)*x^n - a*d^n*(m + n - 1)*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && IGtQ[n, 1] &&  !IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

3.307.4 Maple [F]

$$\int (gx)^m (ex+d)^2 \left(-e^2x^2+d^2\right)^p dx$$

input
```
int((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

output
```
int((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

3.307.5 Fricas [F]

$$\int (gx)^m(d+ex)^2 \left(d^2-e^2x^2\right)^p dx = \int (ex+d)^2(-e^2x^2+d^2)^p(gx)^m dx$$

input
```
integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((e^2*x^2 + 2*d*e*x + d^2)*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

3.307.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.87 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.92

$$\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\frac{d^2d^{2p}g^mx^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{dd^{2p}eg^mx^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+2\right)}+\frac{d^{2p}e^2g^mx^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

input
```
integrate((g*x)**m*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

output
```
d**2*d**(2*p)*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2
 + 3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + d*d**(2
*p)*e*g**m*x**(m + 2)*gamma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), e**2
*x**2*exp_polar(2*I*pi)/d**2)/gamma(m/2 + 2) + d**(2*p)*e**2*g**m*x**(m +
3)*gamma(m/2 + 3/2)*hyper((-p, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_pol
ar(2*I*pi)/d**2)/(2*gamma(m/2 + 5/2))
```

3.307.7 Maxima [F]

$$\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^2(-e^2x^2+d^2)^p(gx)^m\,dx$$

input
```
integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

3.307.8 Giac [F]

$$\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^2\left(-e^2x^2+d^2\right)^p(gx)^m\,dx$$

input
```
integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

3.307.9 Mupad [F(-1)]

Timed out.

$$\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int \left(d^2-e^2\,x^2\right)^p\,{\left(g\,x\right)}^m\,{\left(d+e\,x\right)}^2\,dx$$

input
```
int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^2,x)
```

output
```
int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^2, x)
```

3.308 $\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

3.308.1 Optimal result

Integrand size = 25, antiderivative size = 153

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^p\,dx = \frac{d(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{g(1+m)} + \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2+m}{2},-p,\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{g^2(2+m)}$$

output
```
d*(g*x)^(1+m)*(-e^2*x^2+d^2)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x
^2/d^2)/g/(1+m)/((1-e^2*x^2/d^2)^p)+e*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hyperge
om([-p, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)/g^2/(2+m)/((1-e^2*x^2/d^2)^p)
```

3.308.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.76

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^p\,dx = \frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d(2+m)\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)+e(1+m)x\,\text{Hyp}\right.}{(1+m)(2+m)}$$

input
```
Integrate[(g*x)^m*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

output
```
(x*(g*x)^m*(d^2 - e^2*x^2)^p*(d*(2 + m)*Hypergeometric2F1[(1 + m)/2, -p, (
3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*Hypergeometric2F1[(2 + m)/2, -p, (4
 + m)/2, (e^2*x^2)/d^2]))/((1 + m)*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

3.308.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)(gx)^m \left(d^2-e^2x^2\right)^p dx$$

$$\Big\downarrow \text{557}$$

$$d\int (gx)^m \left(d^2-e^2x^2\right)^p dx+\frac{e\int (gx)^{m+1}\left(d^2-e^2x^2\right)^p dx}{g}$$

$$\Big\downarrow \text{279}$$

$$d(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^p dx+\frac{e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int (gx)^{m+1}\left(1-\frac{e^2x^2}{d^2}\right)^p dx}{g}$$

$$\Big\downarrow \text{278}$$

$$\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+2}{2},-p,\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{g^2(m+2)}+\frac{d(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{g(m+1)}$$

input
```
Int[(g*x)^m*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

output
```
(d*(g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m
)/2, (e^2*x^2)/d^2])/(g*(1 + m)*(1 - (e^2*x^2)/d^2)^p) + (e*(g*x)^(2 + m)*
(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2, (e^2*x^2)/d^
2])/(g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

3.308.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

3.308.4 Maple [F]

$$\int (gx)^m (ex+d) \left(-e^2x^2+d^2\right)^p dx$$

input
```
int((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

output
```
int((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

3.308.5 Fricas [F]

$$\int (gx)^m (d+ex) \left(d^2-e^2x^2\right)^p \, dx = \int (ex+d)\left(-e^2x^2+d^2\right)^p (gx)^m \, dx$$

input
```
integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((e*x + d)*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

3.308.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.59 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

$$\int (gx)^m (d+ex) \left(d^2-e^2x^2\right)^p \, dx = \frac{dd^{2p}g^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{d^{2p}eg^m x^{m+2}\Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+1 \\ \frac{m}{2}+2 \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}$$

input
```
integrate((g*x)**m*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

output
```
d*d**(2*p)*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 +
3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + d**(2*p)*e
*g**m*x**(m + 2)*gamma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), e**2*x**2
*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 2))
```

3.308.7 Maxima [F]

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p(gx)^m\,dx$$

input
```
integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

3.308.8 Giac [F]

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p(gx)^m\,dx$$

input
```
integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

3.308.9 Mupad [F(-1)]

Timed out.

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int \left(d^2-e^2\,x^2\right)^p\,(g\,x)^m\,(d+e\,x)\,dx$$

input
```
int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x),x)
```

output
```
int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x), x)
```

3.309 $\int (gx)^m \left(d^2-e^2x^2\right)^p \, dx$

3.309.1 Optimal result

Integrand size = 20, antiderivative size = 75

$$\int (gx)^m \left(d^2-e^2x^2\right)^p \, dx = \frac{(gx)^{1+m} \left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{g(1+m)}$$

output
```
(g*x)^(1+m)*(-e^2*x^2+d^2)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2
/d^2)/g/(1+m)/((1-e^2*x^2/d^2)^p)
```

3.309.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int (gx)^m \left(d^2-e^2x^2\right)^p \, dx = \frac{x(gx)^m \left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,1+\frac{1+m}{2},\frac{e^2x^2}{d^2}\right)}{1+m}$$

input
```
Integrate[(g*x)^m*(d^2 - e^2*x^2)^p,x]
```

output
```
(x*(g*x)^m*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, 1 + (1 + m)/
2, (e^2*x^2)/d^2])/((1 + m)*(1 - (e^2*x^2)/d^2)^p)
```

3.309.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (gx)^m \left(d^2-e^2x^2\right)^p \, dx$$

$$\Big\downarrow \ 279$$

$$\left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \int (gx)^m \left(1-\frac{e^2x^2}{d^2}\right)^p dx$$

$$\Big\downarrow \ 278$$

$$\frac{(gx)^{m+1} \left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{g(m+1)}$$

input
```
Int[(g*x)^m*(d^2 - e^2*x^2)^p,x]
```

output
```
((g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/
2, (e^2*x^2)/d^2])/(g*(1 + m)*(1 - (e^2*x^2)/d^2)^p)
```

3.309.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

3.309.4 Maple [F]

$$\int (gx)^m \left(-e^2x^2+d^2\right)^p dx$$

input
```
int((g*x)^m*(-e^2*x^2+d^2)^p,x)
```

output
```
int((g*x)^m*(-e^2*x^2+d^2)^p,x)
```

3.309.5 Fricas [F]

$$\int (gx)^m \left(d^2-e^2x^2\right)^p dx = \int \left(-e^2x^2+d^2\right)^p (gx)^m dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

3.309.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int (gx)^m \left(d^2-e^2x^2\right)^p dx = \frac{d^{2p}g^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

input
```
integrate((g*x)**m*(-e**2*x**2+d**2)**p,x)
```

output
```
d**(2*p)*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/
2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2))
```

3.309.7 Maxima [F]

$$\int (gx)^m \left(d^2-e^2x^2\right)^p \, dx = \int (-e^2x^2+d^2)^p (gx)^m \, dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

3.309.8 Giac [F]

$$\int (gx)^m \left(d^2-e^2x^2\right)^p \, dx = \int (-e^2x^2+d^2)^p (gx)^m \, dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

3.309.9 Mupad [F(-1)]

Timed out.

$$\int (gx)^m \left(d^2-e^2x^2\right)^p \, dx = \int \left(d^2-e^2\,x^2\right)^p \,(g\,x)^m \, dx$$

input
```
int((d^2 - e^2*x^2)^p*(g*x)^m,x)
```

output
```
int((d^2 - e^2*x^2)^p*(g*x)^m, x)
```

3.310 $\int \frac{(gx)^m(d^2-e^2x^2)^p}{d+ex}\,dx$

3.310.1 Optimal result

Integrand size = 27, antiderivative size = 163

$$\int \frac{(gx)^m (d^2-e^2x^2)^p}{d+ex}\,dx = \frac{(gx)^{1+m} (d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{2},1-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{dg(1+m)} - \frac{e(gx)^{2+m} (d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{2+m}{2},1-p,\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{d^2g^2(2+m)}$$

output
```
(g*x)^(1+m)*(-e^2*x^2+d^2)^p*hypergeom([1-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^
2/d^2)/d/g/(1+m)/((1-e^2*x^2/d^2)^p)-e*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hyperg
eom([1-p, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)/d^2/g^2/(2+m)/((1-e^2*x^2/d^2)^p
)
```

3.310.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.76

$$\int \frac{(gx)^m (d^2-e^2x^2)^p}{d+ex}\,dx = \frac{x(gx)^m (d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \left(-e(1+m)x\,\text{Hypergeometric2F1}\left(1+\frac{m}{2},1-p,2+\frac{m}{2},\frac{e^2x^2}{d^2}\right)+d(2+\right.}{d^2(1+m)(2+m)}$$

input
```
Integrate[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

output
```
(x*(g*x)^m*(d^2 - e^2*x^2)^p*(-(e*(1 + m)*x*Hypergeometric2F1[1 + m/2, 1 -
 p, 2 + m/2, (e^2*x^2)/d^2]) + d*(2 + m)*Hypergeometric2F1[(1 + m)/2, 1 -
p, (3 + m)/2, (e^2*x^2)/d^2]))/(d^2*(1 + m)*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

3.310.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {583, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(gx)^m \left(d^2-e^2x^2\right)^p}{d+ex} \, dx$$

$$\Big\downarrow \text{583}$$

$$\int (d-ex)(gx)^m \left(d^2-e^2x^2\right)^{p-1}dx$$

$$\Big\downarrow \text{557}$$

$$d\int (gx)^m \left(d^2-e^2x^2\right)^{p-1}dx-\frac{e\int (gx)^{m+1}\left(d^2-e^2x^2\right)^{p-1}dx}{g}$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^{p-1}dx}{d}-\frac{e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int (gx)^{m+1}\left(1-\frac{e^2x^2}{d^2}\right)^{p-1}dx}{d^2g}$$

$$\Big\downarrow \text{278}$$

$$\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{2},1-p,\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{dg(m+1)}-\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+2}{2},1-p,\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{d^2g^2(m+2)}$$

input
```
Int[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

output
```
((g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, 1 - p, (3 +
m)/2, (e^2*x^2)/d^2])/(d*g*(1 + m)*(1 - (e^2*x^2)/d^2)^p) - (e*(g*x)^(2 +
m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, 1 - p, (4 + m)/2, (e^2*x
^2)/d^2])/(d^2*g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

3.310.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 583
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, 0]
```

3.310.4 Maple [F]

$$\int \frac{(gx)^m\,(-e^2x^2+d^2)^p}{ex+d}dx$$

input
```
int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

output
```
int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

3.310.5 Fricas [F]

$$\int \frac{(gx)^m (d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^p (gx)^m}{ex+d}\,dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d), x)
```

3.310.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.23 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.28

$$\begin{aligned}
&\int \frac{(gx)^m (d^2-e^2x^2)^p}{d+ex}\,dx \\
&= -\frac{0^p d^{1-m} d^{m+2p} e^{-m-1} e^{m-1} g^m m x^{m-1} \Phi\left(\frac{d^2}{e^2x^2},1,\frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)} \\
&+ \frac{0^p d^{1-m} d^{m+2p} e^{-m-1} e^{m-1} g^m x^{m-1} \Phi\left(\frac{d^2}{e^2x^2},1,\frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)} \\
&+ \frac{0^p d^{-m} d^{m+2p} e^m e^{-m-1} g^m m x^m \Phi\left(\frac{d^2}{e^2x^2},1,\frac{me^{i\pi}}{2}\right)\Gamma\left(-\frac{m}{2}\right)}{4\Gamma\left(1-\frac{m}{2}\right)} \\
&+ \frac{d e^{2p-2} g^m p x^{m+2p-1} e^{i\pi p}\Gamma(p)\Gamma\left(-\frac{m}{2}-p+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix}1-p,-\frac{m}{2}-p+\frac{1}{2}\\ -\frac{m}{2}-p+\frac{3}{2}\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma(p+1)\Gamma\left(-\frac{m}{2}-p+\frac{3}{2}\right)} \\
&- \frac{e^{2p-1} g^m p x^{m+2p} e^{i\pi p}\Gamma(p)\Gamma\left(-\frac{m}{2}-p\right) {}_2F_1\left(\begin{matrix}1-p,-\frac{m}{2}-p\\ -\frac{m}{2}-p+1\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma(p+1)\Gamma\left(-\frac{m}{2}-p+1\right)}
\end{aligned}$$

input
```
integrate((g*x)**m*(-e**2*x**2+d**2)**p/(e*x+d),x)
```

output
```
-0**p*d**(1 - m)*d**(m + 2*p)*e**(-m - 1)*e**(m - 1)*g**m*m*x**(m - 1)*ler
chphi(d**2/(e**2*x**2), 1, 1/2 - m/2)*gamma(1/2 - m/2)/(4*gamma(3/2 - m/2)
) + 0**p*d**(1 - m)*d**(m + 2*p)*e**(-m - 1)*e**(m - 1)*g**m*x**(m - 1)*le
rchphi(d**2/(e**2*x**2), 1, 1/2 - m/2)*gamma(1/2 - m/2)/(4*gamma(3/2 - m/2
)) + 0**p*d**(m + 2*p)*e**m*e**(-m - 1)*g**m*m*x**m*lerchphi(d**2/(e**2*x*
*2), 1, m*exp_polar(I*pi)/2)*gamma(-m/2)/(4*d**m*gamma(1 - m/2)) + d*e**(2
*p - 2)*g**m*p*x**(m + 2*p - 1)*exp(I*pi*p)*gamma(p)*gamma(-m/2 - p + 1/2)
*hyper((1 - p, -m/2 - p + 1/2), (-m/2 - p + 3/2,), d**2/(e**2*x**2))/(2*ga
mma(p + 1)*gamma(-m/2 - p + 3/2)) - e**(2*p - 1)*g**m*p*x**(m + 2*p)*exp(I
*pi*p)*gamma(p)*gamma(-m/2 - p)*hyper((1 - p, -m/2 - p), (-m/2 - p + 1,),
d**2/(e**2*x**2))/(2*gamma(p + 1)*gamma(-m/2 - p + 1))
```

3.310.7 Maxima [F]

$$\int \frac{(gx)^m (d^2 - e^2x^2)^p}{d + ex}\,dx = \int \frac{(-e^2x^2 + d^2)^p (gx)^m}{ex + d}\,dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d), x)
```

3.310.8 Giac [F]

$$\int \frac{(gx)^m (d^2 - e^2x^2)^p}{d + ex}\,dx = \int \frac{(-e^2x^2 + d^2)^p (gx)^m}{ex + d}\,dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d), x)
```

3.310.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m (d^2 - e^2x^2)^p}{d + ex} \, dx = \int \frac{(d^2 - e^2 x^2)^p (g x)^m}{d + e x} \, dx$$

input
```
int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x),x)
```

output
```
int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x), x)
```

3.311 $\int \frac{(gx)^m(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

3.311.1 Optimal result

Integrand size = 27, antiderivative size = 214

$$\int \frac{(gx)^m\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx=\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{-1+p}}{g(1-m-2p)}-\frac{2(m+p)(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},2-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{d^2g(1+m)(1-m-2p)}-\frac{2e(gx)^{2+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2+m}{2},2-p,\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{d^3g^2(2+m)}$$

output
```
(g*x)^(1+m)*(-e^2*x^2+d^2)^(-1+p)/g/(1-m-2*p)-2*(m+p)*(g*x)^(1+m)*(-e^2*x^
2+d^2)^p*hypergeom([2-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)/d^2/g/(1+m)/(
1-m-2*p)/((1-e^2*x^2/d^2)^p)-2*e*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hypergeom([2
-p, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)/d^3/g^2/(2+m)/((1-e^2*x^2/d^2)^p)
```

3.311.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.84

$$\int \frac{(gx)^m\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx=\frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2(6+5m+m^2)\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},2-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)-e(1+\cdots\right.}{d^4(1\cdots}$$

input
```
Integrate[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

output
```
(x*(g*x)^m*(d^2 - e^2*x^2)^p*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[(1 + m
)/2, 2 - p, (3 + m)/2, (e^2*x^2)/d^2] - e*(1 + m)*x*(2*d*(3 + m)*Hypergeom
etric2F1[(2 + m)/2, 2 - p, (4 + m)/2, (e^2*x^2)/d^2] - e*(2 + m)*x*Hyperge
ometric2F1[(3 + m)/2, 2 - p, (5 + m)/2, (e^2*x^2)/d^2])))/(d^4*(1 + m)*(2
+ m)*(3 + m)*(1 - (e^2*x^2)/d^2)^p)
```

3.311.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {570, 559, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(gx)^m \left(d^2-e^2x^2\right)^p}{(d+ex)^2} \, dx$$

$$\Big\downarrow \text{570}$$

$$\int (d-ex)^2(gx)^m \left(d^2-e^2x^2\right)^{p-2}dx$$

$$\Big\downarrow \text{559}$$

$$\frac{\int -2de^2(gx)^m(d(m+p)+e(-m-2p+1)x)\left(d^2-e^2x^2\right)^{p-2}dx}{e^2(-m-2p+1)}+\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^{p-1}}{g(-m-2p+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^{p-1}}{g(-m-2p+1)}-\frac{2d\int (gx)^m(d(m+p)+e(-m-2p+1)x)\left(d^2-e^2x^2\right)^{p-2}dx}{-m-2p+1}$$

$$\Big\downarrow \text{557}$$

$$\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^{p-1}}{g(-m-2p+1)}-\frac{2d\left(d(m+p)\int (gx)^m\left(d^2-e^2x^2\right)^{p-2}dx+\frac{e(-m-2p+1)\int (gx)^{m+1}\left(d^2-e^2x^2\right)^{p-2}dx}{g}\right)}{-m-2p+1}$$

$$\Big\downarrow \text{279}$$

$$\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^{p-1}}{g(-m-2p+1)}-\frac{2d\left(\frac{e(-m-2p+1)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int (gx)^{m+1}\left(1-\frac{e^2x^2}{d^2}\right)^{p-2}dx}{d^4g}+\frac{(m+p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^{p-2}dx}{d^3}\right)}{-m-2p+1}$$

$$\Big\downarrow \; 278$$

$$\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^{p-1}}{g(-m-2p+1)}-\frac{2d\left(\frac{e(-m-2p+1)(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+2}{2},2-p,\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{d^4g^2(m+2)}+\frac{(m+p)(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}}{d^3g(}\right.}{-m-2p+1}$$

input
```
Int[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

output
```
((g*x)^(1 + m)*(d^2 - e^2*x^2)^(-1 + p))/(g*(1 - m - 2*p)) - (2*d*(((m + p
)*(g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, 2 - p, (3 +
 m)/2, (e^2*x^2)/d^2])/(d^3*g*(1 + m)*(1 - (e^2*x^2)/d^2)^p) + (e*(1 - m -
 2*p)*(g*x)^(2 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, 2 - p,
(4 + m)/2, (e^2*x^2)/d^2])/(d^4*g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)))/(1 -
m - 2*p)
```

3.311.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

3.311. $\int \frac{(gx)^m\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx$

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 559
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[d^n*(e*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*e^(n - 1)*(
m + n + 2*p + 1))), x] + Simp[1/(b*(m + n + 2*p + 1))   Int[(e*x)^m*(a + b*
x^2)^p*ExpandToSum[b*(m + n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1
)*x^n - a*d^n*(m + n - 1)*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && IGtQ[n, 1] &&  !IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.311.4 Maple [F]

$$\int \frac{(gx)^m \left(-e^2x^2+d^2\right)^p}{(ex+d)^2}dx$$

input
```
int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

output
```
int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

3.311.5 Fricas [F]

$$\int \frac{(gx)^m (d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p(gx)^m}{(ex+d)^2}\,dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*(g*x)^m/(e^2*x^2 + 2*d*e*x + d^2), x)
```

3.311.6 Sympy [F]

$$\int \frac{(gx)^m (d^2 - e^2x^2)^p}{(d+ex)^2} \, dx = \int \frac{(gx)^m (-(-d+ex)(d+ex))^p}{(d+ex)^2} \, dx$$

input
```
integrate((g*x)**m*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

output
```
Integral((g*x)**m*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

3.311.7 Maxima [F]

$$\int \frac{(gx)^m (d^2 - e^2x^2)^p}{(d+ex)^2} \, dx = \int \frac{(-e^2x^2 + d^2)^p (gx)^m}{(ex+d)^2} \, dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d)^2, x)
```

3.311.8 Giac [F]

$$\int \frac{(gx)^m (d^2 - e^2x^2)^p}{(d+ex)^2} \, dx = \int \frac{(-e^2x^2 + d^2)^p (gx)^m}{(ex+d)^2} \, dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d)^2, x)
```

3.311.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m (d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(d^2-e^2\,x^2)^p\,(g\,x)^m}{(d+e\,x)^2}\,dx$$

input
```
int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x)^2,x)
```

output
```
int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x)^2, x)
```

3.312 $\int \frac{(gx)^m(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$

3.312.1 Optimal result

Integrand size = 27, antiderivative size = 275

$$\int \frac{(gx)^m(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \frac{3d(gx)^{1+m}(d^2-e^2x^2)^{-2+p}}{g(3-m-2p)} - \frac{e(gx)^{2+m}(d^2-e^2x^2)^{-2+p}}{g^2(2-m-2p)} - \frac{2(2m+p)(gx)^{1+m}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},3-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{d^3g(1+m)(3-m-2p)} - \frac{2e(2-2m-3p)(gx)^{2+m}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2+m}{2},3-p,\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{d^4g^2(2+m)(2-m-2p)}$$

output
```
3*d*(g*x)^(1+m)*(-e^2*x^2+d^2)^(-2+p)/g/(3-m-2*p)-e*(g*x)^(2+m)*(-e^2*x^2+
d^2)^(-2+p)/g^2/(2-m-2*p)-2*(2*m+p)*(g*x)^(1+m)*(-e^2*x^2+d^2)^p*hypergeom
([3-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)/d^3/g/(1+m)/(3-m-2*p)/((1-e^2*x
^2/d^2)^p)-2*e*(2-2*m-3*p)*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hypergeom([3-p, 1+
1/2*m],[2+1/2*m],e^2*x^2/d^2)/d^4/g^2/(2+m)/(2-m-2*p)/((1-e^2*x^2/d^2)^p)
```

3.312.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.75

$$\int \frac{(gx)^m \left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx$$
$$= \frac{x(gx)^m \left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \left(\frac{d^3 \text{Hypergeometric2F1}\left(\frac{1+m}{2},3-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{1+m}+ex\left(-\frac{3d^2 \text{Hypergeometric2F1}\left(\frac{2+m}{2},3-p\right.}{2+m}\right.\right.}{d^6}$$

input
```
Integrate[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

output
```
(x*(g*x)^m*(d^2 - e^2*x^2)^p*((d^3*Hypergeometric2F1[(1 + m)/2, 3 - p, (3
+ m)/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((-3*d^2*Hypergeometric2F1[(2 + m)/2
, 3 - p, (4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1[
(3 + m)/2, 3 - p, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) - (e*x*Hypergeometric
2F1[(4 + m)/2, 3 - p, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/(d^6*(1 - (e^
2*x^2)/d^2)^p)
```

3.312.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {570, 559, 2340, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(gx)^m \left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx$$
$$\Big\downarrow \text{570}$$
$$\int (d-ex)^3(gx)^m \left(d^2-e^2x^2\right)^{p-3}dx$$
$$\Big\downarrow \text{559}$$
$$\frac{\int (gx)^m \left(d^2-e^2x^2\right)^{p-3} \left(3d(-m-2p+2)x^2e^4-2d^2(-2m-3p+2)xe^3+d^3(-m-2p+2)e^2\right)dx}{e^2(-m-2p+2)}-\frac{e(gx)^{m+2} \left(d^2-e^2x^2\right)^{p-2}}{g^2(-m-2p+2)}$$

$$\Big\downarrow \text{2340}$$

$$\frac{\frac{\int -2d^2e^4(gx)^m(d(-m-2p+2)(2m+p)+e(-2m-3p+2)(-m-2p+3)x)(d^2-e^2x^2)^{p-3}dx}{e^2(-m-2p+3)}+\frac{3de^2(-m-2p+2)(gx)^{m+1}(d^2-e^2x^2)^{p-2}}{g(-m-2p+3)}}{e^2(-m-2p+2)}-\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p-2}}{g^2(-m-2p+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3de^2(-m-2p+2)(gx)^{m+1}(d^2-e^2x^2)^{p-2}}{g(-m-2p+3)}-\frac{2d^2e^2\int(gx)^m(d(-m-2p+2)(2m+p)+e(-2m-3p+2)(-m-2p+3)x)(d^2-e^2x^2)^{p-3}dx}{-m-2p+3}}{e^2(-m-2p+2)}-\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p-2}}{g^2(-m-2p+2)}$$

$$\Big\downarrow \text{557}$$

$$\frac{\frac{3de^2(-m-2p+2)(gx)^{m+1}(d^2-e^2x^2)^{p-2}}{g(-m-2p+3)}-\frac{2d^2e^2\left(d(-m-2p+2)(2m+p)\int(gx)^m\left(d^2-e^2x^2\right)^{p-3}dx+\frac{e(-2m-3p+2)(-m-2p+3)\int(gx)^{m+1}\left(d^2-e^2x\right.}{g}\right.}{-m-2p+3}}{e^2(-m-2p+2)}$$
$$\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p-2}}{g^2(-m-2p+2)}$$

$$\Big\downarrow \text{279}$$

$$\frac{\frac{3de^2(-m-2p+2)(gx)^{m+1}(d^2-e^2x^2)^{p-2}}{g(-m-2p+3)}-\frac{2d^2e^2\left(\frac{e(-2m-3p+2)(-m-2p+3)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\int(gx)^{m+1}\left(1-\frac{e^2x^2}{d^2}\right)^{p-3}dx}{d^6g}+\frac{(-m-2p+2)(2\right.}{-m-2p+3}}{e^2(-m-2p+2)}$$
$$\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p-2}}{g^2(-m-2p+2)}$$

$$\Big\downarrow \text{278}$$

$$\frac{\frac{3de^2(-m-2p+2)(gx)^{m+1}(d^2-e^2x^2)^{p-2}}{g(-m-2p+3)}-\frac{2d^2e^2\left(\frac{e(-2m-3p+2)(-m-2p+3)(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+2}{2},3-p,\frac{m+4}{2}\right.}{d^6g^2(m+2)}\right.}{-n}}{e^2(-m-2p+2)}$$
$$\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p-2}}{g^2(-m-2p+2)}$$

3.312. $\int \frac{(gx)^m\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx$

input
```
Int[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

output
```
-((e*(g*x)^(2 + m)*(d^2 - e^2*x^2)^(-2 + p))/(g^2*(2 - m - 2*p))) + ((3*d*
e^2*(2 - m - 2*p)*(g*x)^(1 + m)*(d^2 - e^2*x^2)^(-2 + p))/(g*(3 - m - 2*p)
) - (2*d^2*e^2*(((2 - m - 2*p)*(2*m + p)*(g*x)^(1 + m)*(d^2 - e^2*x^2)^p*H
ypergeometric2F1[(1 + m)/2, 3 - p, (3 + m)/2, (e^2*x^2)/d^2])/(d^5*g*(1 +
m)*(1 - (e^2*x^2)/d^2)^p) + (e*(2 - 2*m - 3*p)*(3 - m - 2*p)*(g*x)^(2 + m)
*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, 3 - p, (4 + m)/2, (e^2*x^2
)/d^2])/(d^6*g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)))/(3 - m - 2*p))/(e^2*(2 -
 m - 2*p))
```

3.312.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 559
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[d^n*(e*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*e^(n - 1)*(
m + n + 2*p + 1))), x] + Simp[1/(b*(m + n + 2*p + 1))   Int[(e*x)^m*(a + b*
x^2)^p*ExpandToSum[b*(m + n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1
)*x^n - a*d^n*(m + n - 1)*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && IGtQ[n, 1] &&  !IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

3.312.4 Maple [F]

$$\int \frac{(gx)^m \left(-e^2x^2+d^2\right)^p}{(ex+d)^3}dx$$

input
```
int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

output
```
int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

3.312.5 Fricas [F]

$$\int \frac{(gx)^m (d^2-e^2x^2)^p}{(d+ex)^3} dx = \int \frac{(-e^2x^2+d^2)^p (gx)^m}{(ex+d)^3} dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*(g*x)^m/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d
^3), x)
```

3.312.6 Sympy [F]

$$\int \frac{(gx)^m (d^2 - e^2x^2)^p}{(d+ex)^3} \, dx = \int \frac{(gx)^m (-(-d+ex)(d+ex))^p}{(d+ex)^3} \, dx$$

input
```
integrate((g*x)**m*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
```

output
```
Integral((g*x)**m*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

3.312.7 Maxima [F]

$$\int \frac{(gx)^m (d^2 - e^2x^2)^p}{(d+ex)^3} \, dx = \int \frac{(-e^2x^2 + d^2)^p (gx)^m}{(ex+d)^3} \, dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d)^3, x)
```

3.312.8 Giac [F]

$$\int \frac{(gx)^m (d^2 - e^2x^2)^p}{(d+ex)^3} \, dx = \int \frac{(-e^2x^2 + d^2)^p (gx)^m}{(ex+d)^3} \, dx$$

input
```
integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d)^3, x)
```

3.312.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m (d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(d^2-e^2\,x^2)^p\,(g\,x)^m}{(d+e\,x)^3}\,dx$$

input
```
int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x)^3,x)
```

output
```
int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x)^3, x)
```

3.313 $\int \frac{(gx)^m(1-a^2x^2)^p}{1+ax}\,dx$

3.313.1 Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{(gx)^m(1-a^2x^2)^p}{1+ax}\,dx = \frac{(gx)^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},1-p,\frac{3+m}{2},a^2x^2\right)}{g(1+m)} - \frac{a(gx)^{2+m}\,\text{Hypergeometric2F1}\left(\frac{2+m}{2},1-p,\frac{4+m}{2},a^2x^2\right)}{g^2(2+m)}$$

output
```
(g*x)^(1+m)*hypergeom([1-p, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/g/(1+m)-a*(g*x
)^(2+m)*hypergeom([1-p, 1+1/2*m],[2+1/2*m],a^2*x^2)/g^2/(2+m)
```

3.313.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.87

$$\int \frac{(gx)^m(1-a^2x^2)^p}{1+ax}\,dx = x(gx)^m\left(-\frac{ax\,\text{Hypergeometric2F1}\left(1+\frac{m}{2},1-p,2+\frac{m}{2},a^2x^2\right)}{2+m} + \frac{\text{Hypergeometric2F1}\left(\frac{1+m}{2},1-p,\frac{3+m}{2},a^2x^2\right)}{1+m}\right)$$

input
```
Integrate[((g*x)^m*(1 - a^2*x^2)^p)/(1 + a*x),x]
```

output
```
x*(g*x)^m*(-((a*x*Hypergeometric2F1[1 + m/2, 1 - p, 2 + m/2, a^2*x^2])/(2
+ m)) + Hypergeometric2F1[(1 + m)/2, 1 - p, (3 + m)/2, a^2*x^2]/(1 + m))
```

3.313.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {583, 557, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^p (gx)^m}{ax+1} dx$$

$$\Big\downarrow \text{583}$$

$$\int (1-ax)(1-a^2x^2)^{p-1}(gx)^m dx$$

$$\Big\downarrow \text{557}$$

$$\int (gx)^m (1-a^2x^2)^{p-1} dx - \frac{a\int (gx)^{m+1}(1-a^2x^2)^{p-1}dx}{g}$$

$$\Big\downarrow \text{278}$$

$$\frac{(gx)^{m+1}\text{Hypergeometric2F1}\left(\frac{m+1}{2},1-p,\frac{m+3}{2},a^2x^2\right)}{g(m+1)} - \frac{a(gx)^{m+2}\text{Hypergeometric2F1}\left(\frac{m+2}{2},1-p,\frac{m+4}{2},a^2x^2\right)}{g^2(m+2)}$$

input
```
Int[((g*x)^m*(1 - a^2*x^2)^p)/(1 + a*x),x]
```

output
```
((g*x)^(1 + m)*Hypergeometric2F1[(1 + m)/2, 1 - p, (3 + m)/2, a^2*x^2])/(g
*(1 + m)) - (a*(g*x)^(2 + m)*Hypergeometric2F1[(2 + m)/2, 1 - p, (4 + m)/2
, a^2*x^2])/(g^2*(2 + m))
```

3.313.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 583
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, 0]
```

3.313.4 Maple [F]

$$\int \frac{(gx)^m (-a^2x^2+1)^p}{ax+1}dx$$

input
```
int((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x)
```

output
```
int((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x)
```

3.313.5 Fricas [F]

$$\int \frac{(gx)^m (1-a^2x^2)^p}{1+ax}\,dx = \int \frac{(-a^2x^2+1)^p(gx)^m}{ax+1}\,dx$$

input
```
integrate((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x, algorithm="fricas")
```

output
```
integral((-a^2*x^2 + 1)^p*(g*x)^m/(a*x + 1), x)
```

3.313.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.56 (sec) , antiderivative size = 328, normalized size of antiderivative = 3.69

$$\int \frac{(gx)^m (1-a^2x^2)^p}{1+ax}\,dx = \frac{0^p a^m a^{-m-1} g^m m x^m \Phi\left(\frac{1}{a^2x^2},1,\frac{me^{i\pi}}{2}\right)\Gamma\left(-\frac{m}{2}\right)}{4\Gamma\left(1-\frac{m}{2}\right)} - \frac{0^p a^{-m-1} a^{m-1} g^m m x^{m-1} \Phi\left(\frac{1}{a^2x^2},1,\frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)} + \frac{0^p a^{-m-1} a^{m-1} g^m x^{m-1} \Phi\left(\frac{1}{a^2x^2},1,\frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)} + \frac{a^{2p-2} g^m p x^{m+2p-1} e^{i\pi p}\Gamma(p)\Gamma\left(-\frac{m}{2}-p+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix}1-p, -\frac{m}{2}-p+\frac{1}{2}\\ -\frac{m}{2}-p+\frac{3}{2}\end{matrix}\middle|\frac{1}{a^2x^2}\right)}{2\Gamma(p+1)\Gamma\left(-\frac{m}{2}-p+\frac{3}{2}\right)} - \frac{a^{2p-1} g^m p x^{m+2p} e^{i\pi p}\Gamma(p)\Gamma\left(-\frac{m}{2}-p\right) {}_2F_1\left(\begin{matrix}1-p, -\frac{m}{2}-p\\ -\frac{m}{2}-p+1\end{matrix}\middle|\frac{1}{a^2x^2}\right)}{2\Gamma(p+1)\Gamma\left(-\frac{m}{2}-p+1\right)}$$

input
```
integrate((g*x)**m*(-a**2*x**2+1)**p/(a*x+1),x)
```

output
```
0**p*a**m*a**(-m - 1)*g**m*m*x**m*lerchphi(1/(a**2*x**2), 1, m*exp_polar(I
*pi)/2)*gamma(-m/2)/(4*gamma(1 - m/2)) - 0**p*a**(-m - 1)*a**(m - 1)*g**m*
m*x**(m - 1)*lerchphi(1/(a**2*x**2), 1, 1/2 - m/2)*gamma(1/2 - m/2)/(4*gam
ma(3/2 - m/2)) + 0**p*a**(-m - 1)*a**(m - 1)*g**m*x**(m - 1)*lerchphi(1/(a
**2*x**2), 1, 1/2 - m/2)*gamma(1/2 - m/2)/(4*gamma(3/2 - m/2)) + a**(2*p -
 2)*g**m*p*x**(m + 2*p - 1)*exp(I*pi*p)*gamma(p)*gamma(-m/2 - p + 1/2)*hyp
er((1 - p, -m/2 - p + 1/2), (-m/2 - p + 3/2,), 1/(a**2*x**2))/(2*gamma(p +
 1)*gamma(-m/2 - p + 3/2)) - a**(2*p - 1)*g**m*p*x**(m + 2*p)*exp(I*pi*p)*
gamma(p)*gamma(-m/2 - p)*hyper((1 - p, -m/2 - p), (-m/2 - p + 1,), 1/(a**2
*x**2))/(2*gamma(p + 1)*gamma(-m/2 - p + 1))
```

3.313.7 Maxima [F]

$$\int \frac{(gx)^m (1-a^2x^2)^p}{1+ax}\,dx = \int \frac{(-a^2x^2+1)^p (gx)^m}{ax+1}\,dx$$

input
```
integrate((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((-a^2*x^2 + 1)^p*(g*x)^m/(a*x + 1), x)
```

3.313.8 Giac [F]

$$\int \frac{(gx)^m (1-a^2x^2)^p}{1+ax}\,dx = \int \frac{(-a^2x^2+1)^p (gx)^m}{ax+1}\,dx$$

input
```
integrate((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x, algorithm="giac")
```

output
```
integrate((-a^2*x^2 + 1)^p*(g*x)^m/(a*x + 1), x)
```

3.313.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m (1-a^2x^2)^p}{1+ax}\,dx = \int \frac{(g\,x)^m \left(1-a^2\,x^2\right)^p}{a\,x+1}\,dx$$

input
```
int(((g*x)^m*(1 - a^2*x^2)^p)/(a*x + 1),x)
```

output
```
int(((g*x)^m*(1 - a^2*x^2)^p)/(a*x + 1), x)
```

3.314 $\int (gx)^m(d+ex)^n \left(d^2-e^2x^2\right)^p \, dx$

3.314.1 Optimal result

Integrand size = 27, antiderivative size = 96

$$\int (gx)^m(d+ex)^n \left(d^2-e^2x^2\right)^p \, dx = \frac{(gx)^{1+m}(d+ex)^n \left(1-\frac{ex}{d}\right)^{-p} \left(1+\frac{ex}{d}\right)^{-n-p} \left(d^2-e^2x^2\right)^p \operatorname{AppellF1}\left(1+m,-p,-n-p,2+m,\frac{ex}{d},-\frac{ex}{d}\right)}{g(1+m)}$$

output
```
(g*x)^(1+m)*(e*x+d)^n*(1+e*x/d)^(-n-p)*(-e^2*x^2+d^2)^p*AppellF1(1+m,-p,-n
-p,2+m,e*x/d,-e*x/d)/g/(1+m)/((1-e*x/d)^p)
```

3.314.2 Mathematica [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int (gx)^m(d+ex)^n \left(d^2-e^2x^2\right)^p \, dx = \frac{x(gx)^m(d-ex)^p \left(\frac{d-ex}{d}\right)^{-p} (d+ex)^{n+p} \left(\frac{d+ex}{d}\right)^{-n-p} \operatorname{AppellF1}\left(1+m,-p,-n-p,2+m,\frac{ex}{d},-\frac{ex}{d}\right)}{1+m}$$

input
```
Integrate[(g*x)^m*(d + e*x)^n*(d^2 - e^2*x^2)^p,x]
```

output
```
(x*(g*x)^m*(d - e*x)^p*(d + e*x)^(n + p)*((d + e*x)/d)^(-n - p)*AppellF1[1
 + m, -p, -n - p, 2 + m, (e*x)/d, -((e*x)/d)])/((1 + m)*((d - e*x)/d)^p)
```

3.314.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {586, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (gx)^m(d+ex)^n \left(d^2-e^2x^2\right)^p \, dx$$

$$\Big\downarrow \text{586}$$

$$(d-ex)^{-p}(d+ex)^{-p} \left(d^2-e^2x^2\right)^p \int (gx)^m(d-ex)^p(d+ex)^{n+p}dx$$

$$\Big\downarrow \text{152}$$

$$(d+ex)^{-p} \left(1-\frac{ex}{d}\right)^{-p} \left(d^2-e^2x^2\right)^p \int (gx)^m(d+ex)^{n+p} \left(1-\frac{ex}{d}\right)^p dx$$

$$\Big\downarrow \text{152}$$

$$(d+ex)^n \left(1-\frac{ex}{d}\right)^{-p} \left(d^2-e^2x^2\right)^p \left(\frac{ex}{d}+1\right)^{-n-p} \int (gx)^m \left(1-\frac{ex}{d}\right)^p \left(\frac{ex}{d}+1\right)^{n+p} dx$$

$$\Big\downarrow \text{150}$$

$$\frac{(gx)^{m+1}(d+ex)^n \left(1-\frac{ex}{d}\right)^{-p} \left(d^2-e^2x^2\right)^p \left(\frac{ex}{d}+1\right)^{-n-p} \text{AppellF1}\left(m+1,-p,-n-p,m+2,\frac{ex}{d},-\frac{ex}{d}\right)}{g(m+1)}$$

input
```
Int[(g*x)^m*(d + e*x)^n*(d^2 - e^2*x^2)^p,x]
```

output
```
((g*x)^(1 + m)*(d + e*x)^n*(1 + (e*x)/d)^(-n - p)*(d^2 - e^2*x^2)^p*Appell
F1[1 + m, -p, -n - p, 2 + m, (e*x)/d, -((e*x)/d)])/(g*(1 + m)*(1 - (e*x)/d
)^p)
```

3.314.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 586
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[(a + b*x^2)^FracPart[p]/((c + d*x)^FracPart[p]*(a/c + (
b*x)/d)^FracPart[p])   Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0]
```

3.314.4 Maple [F]

$$\int (gx)^m (ex+d)^n \left(-e^2x^2+d^2\right)^p dx$$

input
```
int((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x)
```

output
```
int((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x)
```

3.314.5 Fricas [F]

$$\int (gx)^m(d+ex)^n \left(d^2-e^2x^2\right)^p \, dx = \int \left(-e^2x^2+d^2\right)^p (ex+d)^n (gx)^m \, dx$$

input
```
integrate((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*(e*x + d)^n*(g*x)^m, x)
```

3.314.6 Sympy [F]

$$\int (gx)^m(d+ex)^n \left(d^2-e^2x^2\right)^p \, dx = \int (gx)^m \left(-(-d+ex)\left(d+ex\right)\right)^p \left(d+ex\right)^n \, dx$$

input
```
integrate((g*x)**m*(e*x+d)**n*(-e**2*x**2+d**2)**p,x)
```

output
```
Integral((g*x)**m*(-(-d + e*x)*(d + e*x))**p*(d + e*x)**n, x)
```

3.314.7 Maxima [F]

$$\int (gx)^m(d+ex)^n \left(d^2-e^2x^2\right)^p \, dx = \int \left(-e^2x^2+d^2\right)^p (ex+d)^n (gx)^m \, dx$$

input
```
integrate((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*(e*x + d)^n*(g*x)^m, x)
```

3.314.8 Giac [F]

$$\int (gx)^m(d+ex)^n \left(d^2-e^2x^2\right)^p \, dx = \int \left(-e^2x^2+d^2\right)^p (ex+d)^n (gx)^m \, dx$$

input
```
integrate((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*(e*x + d)^n*(g*x)^m, x)
```

3.314.9 Mupad [F(-1)]

Timed out.

$$\int (gx)^m(d+ex)^n\left(d^2-e^2x^2\right)^p\,dx=\int \left(d^2-e^2\,x^2\right)^p\,(g\,x)^m\,(d+e\,x)^n\,dx$$

input
```
int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^n,x)
```

output
```
int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^n, x)
```

3.315 $\int \frac{x\sqrt{1+x}}{1+x^2}\,dx$

3.315.1 Optimal result

Integrand size = 16, antiderivative size = 214

$$\int \frac{x\sqrt{1+x}}{1+x^2}\,dx = 2\sqrt{1+x}+\frac{\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{2\left(1+\sqrt{2}\right)}} + \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(1+\sqrt{2}+x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right) - \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(1+\sqrt{2}+x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right)$$

output
```
2*(1+x)^(1/2)+1/4*ln(1+x+2^(1/2)-(1+x)^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(
1/2))^(1/2)-1/4*ln(1+x+2^(1/2)+(1+x)^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/
2))^(1/2)+arctan((-2*(1+x)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)
)/(2+2*2^(1/2))^(1/2)-arctan((2*(1+x)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(
1/2))^(1/2))/(2+2*2^(1/2))^(1/2)
```

3.315.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.32

$$\int \frac{x\sqrt{1+x}}{1+x^2}\,dx = 2\sqrt{1+x} - \sqrt{-1+i}\arctan\left(\sqrt{-\frac{1}{2}-\frac{i}{2}}\sqrt{1+x}\right) - \sqrt{-1-i}\arctan\left(\sqrt{-\frac{1}{2}+\frac{i}{2}}\sqrt{1+x}\right)$$

input
```
Integrate[(x*Sqrt[1 + x])/(1 + x^2),x]
```

output
```
2*Sqrt[1 + x] - Sqrt[-1 + I]*ArcTan[Sqrt[-1/2 - I/2]*Sqrt[1 + x]] - Sqrt[-
1 - I]*ArcTan[Sqrt[-1/2 + I/2]*Sqrt[1 + x]]
```

3.315.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {561, 25, 1602, 25, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{x+1}}{x^2+1}\,dx$$

$$\Big\downarrow \text{561}$$

$$2\int \frac{x(x+1)}{(x+1)^2-2(x+1)+2}d\sqrt{x+1}$$

$$\Big\downarrow \text{25}$$

$$-2\int -\frac{x(x+1)}{(x+1)^2-2(x+1)+2}d\sqrt{x+1}$$

$$\Big\downarrow \text{1602}$$

$$2\left(\int -\frac{1-x}{(x+1)^2-2(x+1)+2}d\sqrt{x+1}+\sqrt{x+1}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\sqrt{x+1}-\int \frac{1-x}{(x+1)^2-2(x+1)+2}d\sqrt{x+1}\right)$$

$$\Big\downarrow \text{1483}$$

$$2\left(-\frac{\int \frac{2\sqrt{2\left(1+\sqrt{2}\right)}-\left(2+\sqrt{2}\right)\sqrt{x+1}}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}}{4\sqrt{1+\sqrt{2}}}-\frac{\int \frac{\left(2+\sqrt{2}\right)\sqrt{x+1}+2\sqrt{2\left(1+\sqrt{2}\right)}}{x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}}{4\sqrt{1+\sqrt{2}}}+\sqrt{x+1}\right)$$

$$\Big\downarrow \text{1142}$$

$$2\left(-\frac{\sqrt{\sqrt{2}-1}\int \frac{1}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}-\frac{1}{2}\left(2+\sqrt{2}\right)\int -\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{x+1}}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}}{4\sqrt{1+\sqrt{2}}}-\frac{\sqrt{\sqrt{2}-1}\int \cdots}{\cdots}\right.$$

$$\Big\downarrow \text{25}$$

$$2\left(-\frac{\sqrt{\sqrt{2}-1}\int \frac{1}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}+\frac{1}{2}\left(2+\sqrt{2}\right)\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{x+1}}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}}{4\sqrt{1+\sqrt{2}}}-\frac{\sqrt{\sqrt{2}-1}\int \cdots}{\cdots}\right.$$

$$\Big\downarrow \text{1083}$$

$$2\left(-\frac{\frac{1}{2}\left(2+\sqrt{2}\right)\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{x+1}}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}-2\sqrt{\sqrt{2}-1}\int \frac{1}{-x+2\left(1-\sqrt{2}\right)-1}d\left(2\sqrt{x+1}-\sqrt{2\left(1+\sqrt{2}\right)}\right)}{4\sqrt{1+\sqrt{2}}}-\cdots\right.$$

$$\Big\downarrow \text{217}$$

3.315. $\int \frac{x\sqrt{1+x}}{1+x^2}\,dx$

$$2\left(-\frac{\frac{1}{2}(2+\sqrt{2})\int\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{x+1}}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}+\sqrt{2}\arctan\left(\frac{2\sqrt{x+1}-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{4\sqrt{1+\sqrt{2}}}-\frac{\frac{1}{2}(2+\sqrt{2})\int\frac{2\sqrt{x+1}+\sqrt{\ldots}}{x+\sqrt{2\left(1+\sqrt{2}\right.}\ldots}\right.$$

$\Big\downarrow$ 1103

$$2\left(-\frac{\sqrt{2}\arctan\left(\frac{2\sqrt{x+1}-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)-\frac{1}{2}(2+\sqrt{2})\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}}-\frac{\sqrt{2}\arctan\left(\frac{2\sqrt{x+\ldots}}{\ldots}\right.}{\ldots}\right.$$

input
```
Int[(x*Sqrt[1 + x])/(1 + x^2),x]
```

output
```
2*(Sqrt[1 + x] - (Sqrt[2]*ArcTan[(-Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[1 + x])/
Sqrt[2*(-1 + Sqrt[2])]] - ((2 + Sqrt[2])*Log[1 + Sqrt[2] + x - Sqrt[2*(1 +
 Sqrt[2])]*Sqrt[1 + x]])/2)/(4*Sqrt[1 + Sqrt[2]]) - (Sqrt[2]*ArcTan[(Sqrt[
2*(1 + Sqrt[2])] + 2*Sqrt[1 + x])/Sqrt[2*(-1 + Sqrt[2])]] + ((2 + Sqrt[2])
*Log[1 + Sqrt[2] + x + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]])/2)/(4*Sqrt[1 +
Sqrt[2]]))
```

3.315.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.315. $\int \frac{x\sqrt{1+x}}{1+x^2}\,dx$

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

3.315. $\int \frac{x\sqrt{1+x}}{1+x^2}\,dx$

3.315.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.78

method	result
derivativedivides	$2\sqrt{1+x}+\frac{\ln\left(1+x+\sqrt{2}-\sqrt{1+x}\,\sqrt{2+2\sqrt{2}}\right)\sqrt{2+2\sqrt{2}}}{4}-\frac{\left(\sqrt{2}-1\right)\arctan\left(\frac{2\sqrt{1+x}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}-\ln\left(1+x+\sqrt{2}+\sqrt{\right.$
default	$2\sqrt{1+x}+\frac{\ln\left(1+x+\sqrt{2}-\sqrt{1+x}\,\sqrt{2+2\sqrt{2}}\right)\sqrt{2+2\sqrt{2}}}{4}-\frac{\left(\sqrt{2}-1\right)\arctan\left(\frac{2\sqrt{1+x}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}-\ln\left(1+x+\sqrt{2}+\sqrt{\right.$
risch	$2\sqrt{1+x}+\frac{\ln\left(1+x+\sqrt{2}-\sqrt{1+x}\,\sqrt{2+2\sqrt{2}}\right)\sqrt{2+2\sqrt{2}}}{4}-\frac{\left(\sqrt{2}-1\right)\arctan\left(\frac{2\sqrt{1+x}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}-\ln\left(1+x+\sqrt{2}+\sqrt{\right.$
trager	$2\sqrt{1+x}-2\,\mathrm{RootOf}\left(128\_Z^4-16\_Z^2+1\right)\ln\left(256\,\mathrm{RootOf}\left(128\_Z^4-16\_Z^2+1\right)^5 x-16\,\mathrm{RootOf}\right.$

input
```
int(x*(1+x)^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
2*(1+x)^(1/2)+1/4*ln(1+x+2^(1/2)-(1+x)^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(
1/2))^(1/2)-(2^(1/2)-1)/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+x)^(1/2)-(2+2*2^
(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/4*ln(1+x+2^(1/2)+(1+x)^(1/2)*(2+2*2^
(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)+(-2^(1/2)+1)/(-2+2*2^(1/2))^(1/2)*arctan
((2*(1+x)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))
```

3.315.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.34

$$\int \frac{x\sqrt{1+x}}{1+x^2}\,dx = -\frac{1}{2}\sqrt{i+1}\log\left(\sqrt{i+1}+\sqrt{x+1}\right)+\frac{1}{2}\sqrt{i+1}\log\left(-\sqrt{i+1}+\sqrt{x+1}\right) -\frac{1}{2}\sqrt{-i+1}\log\left(\sqrt{-i+1}+\sqrt{x+1}\right) +\frac{1}{2}\sqrt{-i+1}\log\left(-\sqrt{-i+1}+\sqrt{x+1}\right)+2\sqrt{x+1}$$

input
```
integrate(x*(1+x)^(1/2)/(x^2+1),x, algorithm="fricas")
```

output
```
-1/2*sqrt(I + 1)*log(sqrt(I + 1) + sqrt(x + 1)) + 1/2*sqrt(I + 1)*log(-sqr
t(I + 1) + sqrt(x + 1)) - 1/2*sqrt(-I + 1)*log(sqrt(-I + 1) + sqrt(x + 1))
 + 1/2*sqrt(-I + 1)*log(-sqrt(-I + 1) + sqrt(x + 1)) + 2*sqrt(x + 1)
```

3.315.6 Sympy [F]

$$\int \frac{x\sqrt{1+x}}{1+x^2}\,dx = \int \frac{x\sqrt{x+1}}{x^2+1}\,dx$$

input
```
integrate(x*(1+x)**(1/2)/(x**2+1),x)
```

output
```
Integral(x*sqrt(x + 1)/(x**2 + 1), x)
```

3.315.7 Maxima [F]

$$\int \frac{x\sqrt{1+x}}{1+x^2}\,dx = \int \frac{\sqrt{x+1}x}{x^2+1}\,dx$$

input
```
integrate(x*(1+x)^(1/2)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(sqrt(x + 1)*x/(x^2 + 1), x)
```

3.315.8 Giac [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.78

$$\begin{aligned}\int \frac{x\sqrt{1+x}}{1+x^2}\,dx &= -\frac{1}{2}\sqrt{2\sqrt{2}-2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)\\ &\quad-\frac{1}{2}\sqrt{2\sqrt{2}-2}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)\\ &\quad-\frac{1}{4}\sqrt{2\sqrt{2}+2}\log\left(2^{\frac{1}{4}}\sqrt{x+1}\sqrt{\sqrt{2}+2}+x+\sqrt{2}+1\right)\\ &\quad+\frac{1}{4}\sqrt{2\sqrt{2}+2}\log\left(-2^{\frac{1}{4}}\sqrt{x+1}\sqrt{\sqrt{2}+2}+x+\sqrt{2}+1\right)+2\sqrt{x+1}\end{aligned}$$

input
```
integrate(x*(1+x)^(1/2)/(x^2+1),x, algorithm="giac")
```

output
```
-1/2*sqrt(2*sqrt(2) - 2)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2
*sqrt(x + 1))/sqrt(-sqrt(2) + 2)) - 1/2*sqrt(2*sqrt(2) - 2)*arctan(-1/2*2^
(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(x + 1))/sqrt(-sqrt(2) + 2)) - 1/
4*sqrt(2*sqrt(2) + 2)*log(2^(1/4)*sqrt(x + 1)*sqrt(sqrt(2) + 2) + x + sqrt
(2) + 1) + 1/4*sqrt(2*sqrt(2) + 2)*log(-2^(1/4)*sqrt(x + 1)*sqrt(sqrt(2) +
 2) + x + sqrt(2) + 1) + 2*sqrt(x + 1)
```

3.315.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.94

$$\int \frac{x\sqrt{1+x}}{1+x^2}\,dx = 2\sqrt{x+1} + \operatorname{atanh}\left(\frac{\sqrt{x+1}}{4\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}} - \frac{\sqrt{x+1}}{4\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}} + \frac{\sqrt{2}\sqrt{x+1}}{8\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}} + \frac{\sqrt{2}\sqrt{x+1}}{8\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}}\right)\left(2\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}} - 2\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}\right) - \operatorname{atanh}\left(\frac{\sqrt{x+1}}{4\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}} + \frac{\sqrt{x+1}}{4\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}} - \frac{\sqrt{2}\sqrt{x+1}}{8\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}} + \frac{\sqrt{2}\sqrt{x+1}}{8\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}}\right)\left(2\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}} + 2\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}\right)$$

input
```
int((x*(x + 1)^(1/2))/(x^2 + 1),x)
```

output
```
2*(x + 1)^(1/2) + atanh((x + 1)^(1/2)/(4*(2^(1/2)/8 + 1/8)^(1/2)) - (x + 1
)^(1/2)/(4*(1/8 - 2^(1/2)/8)^(1/2)) + (2^(1/2)*(x + 1)^(1/2))/(8*(1/8 - 2^
(1/2)/8)^(1/2)) + (2^(1/2)*(x + 1)^(1/2))/(8*(2^(1/2)/8 + 1/8)^(1/2)))*(2*
(1/8 - 2^(1/2)/8)^(1/2) - 2*(2^(1/2)/8 + 1/8)^(1/2)) - atanh((x + 1)^(1/2)
/(4*(1/8 - 2^(1/2)/8)^(1/2)) + (x + 1)^(1/2)/(4*(2^(1/2)/8 + 1/8)^(1/2)) -
 (2^(1/2)*(x + 1)^(1/2))/(8*(1/8 - 2^(1/2)/8)^(1/2)) + (2^(1/2)*(x + 1)^(1
/2))/(8*(2^(1/2)/8 + 1/8)^(1/2)))*(2*(1/8 - 2^(1/2)/8)^(1/2) + 2*(2^(1/2)/
8 + 1/8)^(1/2))
```

3.316 $\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx$

3.316.1 Optimal result

Integrand size = 22, antiderivative size = 255

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx = \frac{d(8cd^3-e(4cd^2-ae^2)\,x)\sqrt{a+cx^2}}{8ce^5}+\frac{(47cd^2-8ae^2)\,(a+cx^2)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)\,(a+cx^2)^{3/2}}{20ce^3}+\frac{(d+ex)^2\,(a+cx^2)^{3/2}}{5ce^3} - \frac{d(8c^2d^4+4acd^2e^2-a^2e^4)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^6} - \frac{d^4\sqrt{cd^2+ae^2}\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^6}$$

output
```
1/60*(-8*a*e^2+47*c*d^2)*(c*x^2+a)^(3/2)/c^2/e^3-13/20*d*(e*x+d)*(c*x^2+a)
^(3/2)/c/e^3+1/5*(e*x+d)^2*(c*x^2+a)^(3/2)/c/e^3-1/8*d*(-a^2*e^4+4*a*c*d^2
*e^2+8*c^2*d^4)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^6-d^4*arctanh
((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/e^6
+1/8*d*(8*c*d^3-e*(-a*e^2+4*c*d^2)*x)*(c*x^2+a)^(1/2)/c/e^5
```

3.316.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.90

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx$$
$$= \frac{e\sqrt{a+cx^2}(-16a^2e^4+ace^2(40d^2-15dex+8e^2x^2)+2c^2(60d^4-30d^3ex+20d^2e^2x^2-15de^3x^3+12e^4x^4)}{1}$$

input
```
Integrate[(x^4*Sqrt[a + c*x^2])/(d + e*x),x]
```

output
```
(e*Sqrt[a + c*x^2]*(-16*a^2*e^4 + a*c*e^2*(40*d^2 - 15*d*e*x + 8*e^2*x^2)
+ 2*c^2*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 - 15*d*e^3*x^3 + 12*e^4*x^4)
) + 240*c^2*d^4*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[
a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]] + 15*Sqrt[c]*d*(8*c^2*d^4 + 4*a*c*d^2*
e^2 - a^2*e^4)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(120*c^2*e^6)
```

3.316.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {604, 25, 2185, 25, 2185, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx$$
$$\Big\downarrow \; 604$$
$$\frac{\int -\frac{\sqrt{cx^2+a}\left(13cde^3x^3+e^2\left(11cd^2+2ae^2\right)x^2+de\left(3cd^2+4ae^2\right)x+2ad^2e^2\right)}{d+ex}dx}{5ce^4}+\frac{\left(a+cx^2\right)^{3/2}(d+ex)^2}{5ce^3}$$
$$\Big\downarrow \; 25$$
$$\frac{\left(a+cx^2\right)^{3/2}(d+ex)^2}{5ce^3}-\frac{\int \frac{\sqrt{cx^2+a}\left(13cde^3x^3+e^2\left(11cd^2+2ae^2\right)x^2+de\left(3cd^2+4ae^2\right)x+2ad^2e^2\right)}{d+ex}dx}{5ce^4}$$
$$\Big\downarrow \; 2185$$

$$\frac{\left(a+cx^2\right)^{3/2}(d+ex)^2}{5ce^3}-\frac{\frac{\int -\frac{\sqrt{cx^2+a}\left(5acd^2e^5+c\left(47cd^2-8ae^2\right)x^2e^5+3cd\left(9cd^2-ae^2\right)xe^4\right)}{d+ex}dx}{4ce^3}+\frac{13}{4}de\left(a+cx^2\right)^{3/2}(d+ex)}{5ce^4}$$

$\Big\downarrow$ 25

$$\frac{\left(a+cx^2\right)^{3/2}(d+ex)^2}{5ce^3}-\frac{\frac{13}{4}de\left(a+cx^2\right)^{3/2}(d+ex)-\frac{\int \frac{\sqrt{cx^2+a}\left(5acd^2e^5+c\left(47cd^2-8ae^2\right)x^2e^5+3cd\left(9cd^2-ae^2\right)xe^4\right)}{d+ex}dx}{4ce^3}}{5ce^4}$$

$\Big\downarrow$ 2185

$$\frac{\left(a+cx^2\right)^{3/2}(d+ex)^2}{5ce^3}-\frac{\frac{13}{4}de\left(a+cx^2\right)^{3/2}(d+ex)-\frac{\frac{\int \frac{15c^2de^6\left(ade-\left(4cd^2-ae^2\right)x\right)\sqrt{cx^2+a}}{d+ex}dx}{3ce^2}+\frac{1}{3}e^4\left(a+cx^2\right)^{3/2}\left(47cd^2-8ae^2\right)}{4ce^3}}{5ce^4}$$

$\Big\downarrow$ 27

$$\frac{\left(a+cx^2\right)^{3/2}(d+ex)^2}{5ce^3}-\frac{\frac{13}{4}de\left(a+cx^2\right)^{3/2}(d+ex)-\frac{5cde^4\int \frac{\left(ade-\left(4cd^2-ae^2\right)x\right)\sqrt{cx^2+a}}{d+ex}dx+\frac{1}{3}e^4\left(a+cx^2\right)^{3/2}\left(47cd^2-8ae^2\right)}{4ce^3}}{5ce^4}$$

$\Big\downarrow$ 682

$$\frac{\left(a+cx^2\right)^{3/2}(d+ex)^2}{5ce^3}-\frac{\frac{13}{4}de\left(a+cx^2\right)^{3/2}(d+ex)-\frac{5cde^4\left(\frac{\int \frac{c\left(ade\left(4cd^2+ae^2\right)-\left(8c^2d^4+4ace^2d^2-a^2e^4\right)x\right)}{(d+ex)\sqrt{cx^2+a}}dx}{2ce^2}+\frac{\sqrt{a+cx^2}\left(8cd^3-ex\left(4cd^2-ae^2\right)\right)}{2e^2}\right)+\frac{1}{3}e^4\left(a+cx^2\right)^{3/2}\left(\cdots\right.}{4ce^3}}{5ce^4}$$

$\Big\downarrow$ 27

$$\frac{\left(a+cx^2\right)^{3/2}(d+ex)^2}{5ce^3}-\frac{\frac{13}{4}de\left(a+cx^2\right)^{3/2}(d+ex)-\frac{5cde^4\left(\frac{\int \frac{ade\left(4cd^2+ae^2\right)-\left(8c^2d^4+4ace^2d^2-a^2e^4\right)x}{(d+ex)\sqrt{cx^2+a}}dx}{2e^2}+\frac{\sqrt{a+cx^2}\left(8cd^3-ex\left(4cd^2-ae^2\right)\right)}{2e^2}\right)+\frac{1}{3}e^4\left(a+cx^2\right)^{3/2}\left(47c\cdots\right.}{4ce^3}}{5ce^4}$$

$\Big\downarrow$ 719

3.316. $\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx$

$$\frac{(a+cx^2)^{3/2}(d+ex)^2}{5ce^3} -$$

$$\frac{\frac{13}{4}de(a+cx^2)^{3/2}(d+ex) - \frac{5cde^4\left(\frac{\frac{8cd^3(ae^2+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e} - \frac{(-a^2e^4+4acd^2e^2+8c^2d^4)\int \frac{1}{\sqrt{cx^2+a}}dx}{e}}{2e^2} + \frac{\sqrt{a+cx^2}\left(8cd^3-ex\left(4cd^2-a\right.\right.}{2e^2}\right.}{4ce^3}}{5ce^4}$$

$\downarrow$ 224

$$\frac{(a+cx^2)^{3/2}(d+ex)^2}{5ce^3} -$$

$$\frac{\frac{13}{4}de(a+cx^2)^{3/2}(d+ex) - \frac{5cde^4\left(\frac{\frac{8cd^3(ae^2+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e} - \frac{(-a^2e^4+4acd^2e^2+8c^2d^4)\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{e}}{2e^2} + \frac{\sqrt{a+cx^2}\left(8cd^3-ex\right.}{2e^2}\right.}{4ce^3}}{5ce^4}$$

$\downarrow$ 219

$$\frac{(a+cx^2)^{3/2}(d+ex)^2}{5ce^3} -$$

$$\frac{\frac{13}{4}de(a+cx^2)^{3/2}(d+ex) - \frac{5cde^4\left(\frac{\frac{8cd^3(ae^2+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e} - \frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(-a^2e^4+4acd^2e^2+8c^2d^4)}{\sqrt{c}e}}{2e^2} + \frac{\sqrt{a+cx^2}\left(8cd^3-ex\right.}{2e^2}\right.}{4ce^3}}{5ce^4}$$

$\downarrow$ 488

$$\frac{(a+cx^2)^{3/2}(d+ex)^2}{5ce^3} -$$

$$\frac{\frac{13}{4}de(a+cx^2)^{3/2}(d+ex) - \frac{5cde^4\left(\frac{-\frac{8cd^3(ae^2+cd^2)\int \frac{1}{cd^2+ae^2-\frac{(ae-cdx)^2}{cx^2+a}}d\frac{ae-cdx}{\sqrt{cx^2+a}}}{e} - \frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(-a^2e^4+4acd^2e^2+8c^2d^4)}{\sqrt{c}e}}{2e^2} + \frac{\sqrt{}}{}\right.}{4ce^3}}{5ce^4}$$

$\downarrow$ 219

$$\frac{(a+cx^2)^{3/2}(d+ex)^2}{5ce^3} -$$

$$\frac{\frac{13}{4}de(a+cx^2)^{3/2}(d+ex) - \frac{5cde^4\left(\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(-a^2e^4+4acd^2e^2+8c^2d^4)}{\sqrt{c}e} - \frac{8cd^3\sqrt{ae^2+cd^2}\,\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e}}{2e^2} + \frac{\sqrt{a}}{}\right.}{4ce^3}}{5ce^4}$$

3.316. $\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx$

input
```
Int[(x^4*Sqrt[a + c*x^2])/(d + e*x),x]
```

output
```
((d + e*x)^2*(a + c*x^2)^(3/2))/(5*c*e^3) - ((13*d*e*(d + e*x)*(a + c*x^2)
^(3/2))/4 - ((e^4*(47*c*d^2 - 8*a*e^2)*(a + c*x^2)^(3/2))/3 + 5*c*d*e^4*((
(8*c*d^3 - e*(4*c*d^2 - a*e^2)*x)*Sqrt[a + c*x^2])/(2*e^2) + (-(((8*c^2*d^
4 + 4*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c
]*e)) - (8*c*d^3*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a
*e^2]*Sqrt[a + c*x^2])])/e)/(2*e^2)))/(4*c*e^3))/(5*c*e^4)
```

3.316.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 604
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(b*d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*
x^2)^p*ExpandToSum[b*d^m*(m + n + 2*p + 1)*x^m - b*(m + n + 2*p + 1)*(c + d
*x)^m - (c + d*x)^(m - 2)*(a*d^2*(m + n - 1) - b*c^2*(m + n + 2*p + 1) - 2*
b*c*d*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && IGtQ[m,
 1] && NeQ[m + n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.316.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{(-24e^4c^2x^4+30d\,e^3c^2x^3-8ac\,e^4x^2-40c^2d^2e^2x^2+15acd\,e^3x+60c^2d^3ex+16a^2e^4-40ac\,d^2e^2-120c^2d^4)\sqrt{c\,x^2+a}}{120c^2e^5}+\frac{d\left(\frac{(a^2e^4-4a \cdots}{\cdots}\right)}{\cdots}$
default	$\frac{\frac{x^2(c\,x^2+a)^{\frac{3}{2}}}{5c}-\frac{2a(c\,x^2+a)^{\frac{3}{2}}}{15c^2}}{e}+\frac{d^2(c\,x^2+a)^{\frac{3}{2}}}{3e^3c}-\frac{d^3\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}\right)}{e^4}-\frac{d\left(\frac{x(c\,x^2+a)^{\frac{3}{2}}}{4c}-\frac{a\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln(x\sqrt{\cdots}}{\cdots}\right)}{4c}\cdots\right)}{e^2}$

input
```
int(x^4*(c*x^2+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/120*(-24*c^2*e^4*x^4+30*c^2*d*e^3*x^3-8*a*c*e^4*x^2-40*c^2*d^2*e^2*x^2+
15*a*c*d*e^3*x+60*c^2*d^3*e*x+16*a^2*e^4-40*a*c*d^2*e^2-120*c^2*d^4)*(c*x^
2+a)^(1/2)/c^2/e^5+1/8*d/e^5/c*((a^2*e^4-4*a*c*d^2*e^2-8*c^2*d^4)/e*ln(x*c
^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-8*d^3*(a*e^2+c*d^2)*c/e^2/((a*e^2+c*d^2)/e
^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1
/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

3.316.5 Fricas [A] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 1104, normalized size of antiderivative = 4.33

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx$$

$$= \left[\frac{120\sqrt{cd^2+ae^2}c^2d^4\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-15\,(8\,c^2d^5+4\,acd^3e^2\cdots}{\cdots}\right.$$

$$-\frac{240\sqrt{-cd^2-ae^2}c^2d^4\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)+15\,(8\,c^2d^5+4\,acd^3e^2-a^2de^4)\sqrt{c}\log\left(-2\,cx^2\cdots\right.}{\cdots}$$

$$-\frac{120\sqrt{-cd^2-ae^2}c^2d^4\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-15\,(8\,c^2d^5+4\,acd^3e^2-a^2de^4)\sqrt{-c}\arctan\left(\cdots\right.}{\cdots}$$

input
```
integrate(x^4*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
[1/240*(120*sqrt(c*d^2 + a*e^2)*c^2*d^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2
*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqr
t(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 15*(8*c^2*d^5 + 4*a*c*d^3*e^2 -
 a^2*d*e^4)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(2
4*c^2*e^5*x^4 - 30*c^2*d*e^4*x^3 + 120*c^2*d^4*e + 40*a*c*d^2*e^3 - 16*a^2
*e^5 + 8*(5*c^2*d^2*e^3 + a*c*e^5)*x^2 - 15*(4*c^2*d^3*e^2 + a*c*d*e^4)*x)
*sqrt(c*x^2 + a))/(c^2*e^6), -1/240*(240*sqrt(-c*d^2 - a*e^2)*c^2*d^4*arct
an(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 +
 (c^2*d^2 + a*c*e^2)*x^2)) + 15*(8*c^2*d^5 + 4*a*c*d^3*e^2 - a^2*d*e^4)*sq
rt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(24*c^2*e^5*x^4
- 30*c^2*d*e^4*x^3 + 120*c^2*d^4*e + 40*a*c*d^2*e^3 - 16*a^2*e^5 + 8*(5*c^
2*d^2*e^3 + a*c*e^5)*x^2 - 15*(4*c^2*d^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 +
a))/(c^2*e^6), 1/120*(60*sqrt(c*d^2 + a*e^2)*c^2*d^4*log((2*a*c*d*e*x - a*
c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d
*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 15*(8*c^2*d^5 + 4*
a*c*d^3*e^2 - a^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (24
*c^2*e^5*x^4 - 30*c^2*d*e^4*x^3 + 120*c^2*d^4*e + 40*a*c*d^2*e^3 - 16*a^2*
e^5 + 8*(5*c^2*d^2*e^3 + a*c*e^5)*x^2 - 15*(4*c^2*d^3*e^2 + a*c*d*e^4)*x)*
sqrt(c*x^2 + a))/(c^2*e^6), -1/120*(120*sqrt(-c*d^2 - a*e^2)*c^2*d^4*arcta
n(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2...
```

3.316.6 Sympy [F]

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx$$

input
```
integrate(x**4*(c*x**2+a)**(1/2)/(e*x+d),x)
```

output
```
Integral(x**4*sqrt(a + c*x**2)/(d + e*x), x)
```

3.316.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.316.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.316.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{x^4\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

input
```
int((x^4*(a + c*x^2)^(1/2))/(d + e*x),x)
```

output
```
int((x^4*(a + c*x^2)^(1/2))/(d + e*x), x)
```

3.317 $\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx$

3.317.1 Optimal result

Integrand size = 22, antiderivative size = 211

$$\begin{aligned}\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx = &-\frac{(8cd^3-e(4cd^2-ae^2)\,x)\sqrt{a+cx^2}}{8ce^4}-\frac{7d(a+cx^2)^{3/2}}{12ce^2}\\ &+\frac{(d+ex)\,(a+cx^2)^{3/2}}{4ce^2}+\frac{(8c^2d^4+4acd^2e^2-a^2e^4)\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^5}\\ &+\frac{d^3\sqrt{cd^2+ae^2}\,\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^5}\end{aligned}$$

output
```
-7/12*d*(c*x^2+a)^(3/2)/c/e^2+1/4*(e*x+d)*(c*x^2+a)^(3/2)/c/e^2+1/8*(-a^2*
e^4+4*a*c*d^2*e^2+8*c^2*d^4)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^
5+d^3*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d
^2)^(1/2)/e^5-1/8*(8*c*d^3-e*(-a*e^2+4*c*d^2)*x)*(c*x^2+a)^(1/2)/c/e^4
```

3.317.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.94

$$\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx$$

$$= \frac{\sqrt{c}e\sqrt{a+cx^2}(ae^2(-8d+3ex)+c(-24d^3+12d^2ex-8de^2x^2+6e^3x^3))-48c^{3/2}d^3\sqrt{-cd^2-ae^2}\,\arctan\Big(}{24c^{3/2}e^5}$$

input
```
Integrate[(x^3*Sqrt[a + c*x^2])/(d + e*x),x]
```

output
```
(Sqrt[c]*e*Sqrt[a + c*x^2]*(a*e^2*(-8*d + 3*e*x) + c*(-24*d^3 + 12*d^2*e*x
 - 8*d*e^2*x^2 + 6*e^3*x^3)) - 48*c^(3/2)*d^3*Sqrt[-(c*d^2) - a*e^2]*ArcTa
n[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]] - 3*(8*c
^2*d^4 + 4*a*c*d^2*e^2 - a^2*e^4)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(24
*c^(3/2)*e^5)
```

3.317.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {604, 25, 2185, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx$$

$$\Big\downarrow \text{604}$$

$$\frac{\int -\frac{\sqrt{cx^2+a}\left(7cdx^2e^2+ade^2+\left(3cd^2+ae^2\right)xe\right)}{d+ex}dx}{4ce^3}+\frac{\left(a+cx^2\right)^{3/2}(d+ex)}{4ce^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\left(a+cx^2\right)^{3/2}(d+ex)}{4ce^2}-\frac{\int \frac{\sqrt{cx^2+a}\left(7cdx^2e^2+ade^2+\left(3cd^2+ae^2\right)xe\right)}{d+ex}dx}{4ce^3}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\left(a+cx^2\right)^{3/2}(d+ex)}{4ce^2}-\frac{\frac{\int \frac{3ce^3\left(ade-\left(4cd^2-ae^2\right)x\right)\sqrt{cx^2+a}}{d+ex}dx}{3ce^2}+\frac{7}{3}de\left(a+cx^2\right)^{3/2}}{4ce^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+cx^2\right)^{3/2}(d+ex)}{4ce^2}-\frac{e\int \frac{\left(ade-\left(4cd^2-ae^2\right)x\right)\sqrt{cx^2+a}}{d+ex}dx+\frac{7}{3}de\left(a+cx^2\right)^{3/2}}{4ce^3}$$

$$\Big\downarrow \text{682}$$

$$\frac{\frac{(a+cx^2)^{3/2}(d+ex)}{4ce^2}-\frac{e\left(\frac{\int \frac{c\left(ade(4cd^2+ae^2)-(8c^2d^4+4ace^2d^2-a^2e^4)x\right)}{(d+ex)\sqrt{cx^2+a}}dx}{2ce^2}+\frac{\sqrt{a+cx^2}(8cd^3-ex(4cd^2-ae^2))}{2e^2}\right)+\frac{7}{3}de(a+cx^2)^{3/2}}{4ce^3}}{}$$

$\downarrow$ 27

$$\frac{(a+cx^2)^{3/2}(d+ex)}{4ce^2}-\frac{e\left(\frac{\int \frac{ade(4cd^2+ae^2)-(8c^2d^4+4ace^2d^2-a^2e^4)x}{(d+ex)\sqrt{cx^2+a}}dx}{2e^2}+\frac{\sqrt{a+cx^2}(8cd^3-ex(4cd^2-ae^2))}{2e^2}\right)+\frac{7}{3}de(a+cx^2)^{3/2}}{4ce^3}$$

$\downarrow$ 719

$$\frac{(a+cx^2)^{3/2}(d+ex)}{4ce^2}-\frac{e\left(\frac{\frac{8cd^3(ae^2+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{(-a^2e^4+4acd^2e^2+8c^2d^4)\int \frac{1}{\sqrt{cx^2+a}}dx}{e}}{2e^2}+\frac{\sqrt{a+cx^2}(8cd^3-ex(4cd^2-ae^2))}{2e^2}\right)+\frac{7}{3}de(a+cx^2)^{3/2}}{4ce^3}$$

$\downarrow$ 224

$$\frac{(a+cx^2)^{3/2}(d+ex)}{4ce^2}-\frac{e\left(\frac{\frac{8cd^3(ae^2+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{(-a^2e^4+4acd^2e^2+8c^2d^4)\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{e}}{2e^2}+\frac{\sqrt{a+cx^2}(8cd^3-ex(4cd^2-ae^2))}{2e^2}\right)+\frac{7}{3}de(a+cx^2)}{4ce^3}$$

$\downarrow$ 219

$$\frac{(a+cx^2)^{3/2}(d+ex)}{4ce^2}-\frac{e\left(\frac{\frac{8cd^3(ae^2+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(-a^2e^4+4acd^2e^2+8c^2d^4)}{\sqrt{c}e}}{2e^2}+\frac{\sqrt{a+cx^2}(8cd^3-ex(4cd^2-ae^2))}{2e^2}\right)+\frac{7}{3}de(a+cx^2)}{4ce^3}$$

$\downarrow$ 488

3.317. $\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx$

$$\frac{(a+cx^2)^{3/2}(d+ex)}{4ce^2} - \frac{e\left(\frac{-\frac{8cd^3(ae^2+cd^2)\int \frac{1}{cd^2+ae^2-\frac{(ae-cdx)^2}{cx^2+a}}d\frac{ae-cdx}{\sqrt{cx^2+a}}}{e}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(-a^2e^4+4acd^2e^2+8c^2d^4)}{\sqrt{c}e}}{2e^2}+\frac{\sqrt{a+cx^2}(8cd^3-ex(4cd^2-ae^2))}{2e^2}\right)+\frac{7}{3}}{4ce^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{(a+cx^2)^{3/2}(d+ex)}{4ce^2} - \frac{e\left(\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(-a^2e^4+4acd^2e^2+8c^2d^4)}{\sqrt{c}e}-\frac{8cd^3\sqrt{ae^2+cd^2}\,\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e}}{2e^2}+\frac{\sqrt{a+cx^2}(8cd^3-ex(4cd^2-ae^2))}{2e^2}\right)+\frac{7}{3}d}{4ce^3}$$

input
```
Int[(x^3*Sqrt[a + c*x^2])/(d + e*x),x]
```

output
```
((d + e*x)*(a + c*x^2)^(3/2))/(4*c*e^2) - ((7*d*e*(a + c*x^2)^(3/2))/3 + e
*(((8*c*d^3 - e*(4*c*d^2 - a*e^2)*x)*Sqrt[a + c*x^2])/(2*e^2) + (-(((8*c^2
*d^4 + 4*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqr
t[c]*e)) - (8*c*d^3*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2
+ a*e^2]*Sqrt[a + c*x^2])])/e)/(2*e^2)))/(4*c*e^3)
```

3.317.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 604
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(b*d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*
x^2)^p*ExpandToSum[b*d^m*(m + n + 2*p + 1)*x^m - b*(m + n + 2*p + 1)*(c + d
*x)^m - (c + d*x)^(m - 2)*(a*d^2*(m + n - 1) - b*c^2*(m + n + 2*p + 1) - 2*
b*c*d*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && IGtQ[m,
 1] && NeQ[m + n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.317. $\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx$

3.317.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{(-6c x^3 e^3+8cd e^2x^2-3a e^3x-12c d^2ex+8ad e^2+24c d^3)\sqrt{c x^2+a}}{24c e^4}-\frac{\frac{(a^2e^4-4ac d^2e^2-8c^2d^4)\ln(x\sqrt{c}+\sqrt{c x^2+a})}{e\sqrt{c}}-\frac{8d^3(e^2a+c d^2)c\,\mathrm{l}\ldots}{\ldots}}{\ldots}$
default	$\frac{\frac{x(c x^2+a)^{\frac{3}{2}}}{4c}-\frac{a\left(\frac{x\sqrt{c x^2+a}}{2}+\frac{a\ln(x\sqrt{c}+\sqrt{c x^2+a})}{2\sqrt{c}}\right)}{4c}}{e}+\frac{d^2\left(\frac{x\sqrt{c x^2+a}}{2}+\frac{a\ln(x\sqrt{c}+\sqrt{c x^2+a})}{2\sqrt{c}}\right)}{e^3}-\frac{d(c x^2+a)^{\frac{3}{2}}}{3c e^2}-\frac{d^3\left(\sqrt{\left(x+\frac{d}{e}\right)^2c-\ldots}\right.}{\ldots}$

input
```
int(x^3*(c*x^2+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-6*c*e^3*x^3+8*c*d*e^2*x^2-3*a*e^3*x-12*c*d^2*e*x+8*a*d*e^2+24*c*d^
3)*(c*x^2+a)^(1/2)/c/e^4-1/8/c/e^4*((a^2*e^4-4*a*c*d^2*e^2-8*c^2*d^4)/e*ln
(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-8*d^3*(a*e^2+c*d^2)*c/e^2/((a*e^2+c*d^
2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2
)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

3.317.5 Fricas [A] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 963, normalized size of antiderivative = 4.56

$$\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx$$
$$=\left[\frac{24\sqrt{cd^2+ae^2}c^2d^3\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2+2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-3\,(8\,c^2d^4+4\,acd^2e^2-\ldots}{\ldots}\right.$$

input
```
integrate(x^3*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
[1/48*(24*sqrt(c*d^2 + a*e^2)*c^2*d^3*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e
^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(
c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(8*c^2*d^4 + 4*a*c*d^2*e^2 - a^
2*e^4)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(6*c^2*
e^4*x^3 - 8*c^2*d*e^3*x^2 - 24*c^2*d^3*e - 8*a*c*d*e^3 + 3*(4*c^2*d^2*e^2
+ a*c*e^4)*x)*sqrt(c*x^2 + a))/(c^2*e^5), 1/48*(48*sqrt(-c*d^2 - a*e^2)*c^
2*d^3*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 +
 a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(8*c^2*d^4 + 4*a*c*d^2*e^2 - a^2*
e^4)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(6*c^2*e^
4*x^3 - 8*c^2*d*e^3*x^2 - 24*c^2*d^3*e - 8*a*c*d*e^3 + 3*(4*c^2*d^2*e^2 +
a*c*e^4)*x)*sqrt(c*x^2 + a))/(c^2*e^5), 1/24*(12*sqrt(c*d^2 + a*e^2)*c^2*d
^3*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*
sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^
2)) - 3*(8*c^2*d^4 + 4*a*c*d^2*e^2 - a^2*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/s
qrt(c*x^2 + a)) + (6*c^2*e^4*x^3 - 8*c^2*d*e^3*x^2 - 24*c^2*d^3*e - 8*a*c*
d*e^3 + 3*(4*c^2*d^2*e^2 + a*c*e^4)*x)*sqrt(c*x^2 + a))/(c^2*e^5), 1/24*(2
4*sqrt(-c*d^2 - a*e^2)*c^2*d^3*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(8*c^2*d
^4 + 4*a*c*d^2*e^2 - a^2*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a))
+ (6*c^2*e^4*x^3 - 8*c^2*d*e^3*x^2 - 24*c^2*d^3*e - 8*a*c*d*e^3 + 3*(4*...
```

3.317.6 Sympy [F]

$$\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx=\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx$$

input
```
integrate(x**3*(c*x**2+a)**(1/2)/(e*x+d),x)
```

output
```
Integral(x**3*sqrt(a + c*x**2)/(d + e*x), x)
```

3.317.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.317.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.317.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{x^3\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

input
```
int((x^3*(a + c*x^2)^(1/2))/(d + e*x),x)
```

output
```
int((x^3*(a + c*x^2)^(1/2))/(d + e*x), x)
```

3.318 $\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx$

3.318.1 Optimal result

Integrand size = 22, antiderivative size = 153

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx=\frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3}+\frac{(a+cx^2)^{3/2}}{3ce}-\frac{d(2cd^2+ae^2)\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}e^4}-\frac{d^2\sqrt{cd^2+ae^2}\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^4}$$

output
```
1/3*(c*x^2+a)^(3/2)/c/e-1/2*d*(a*e^2+2*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^
(1/2))/e^4/c^(1/2)-d^2*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^
(1/2))*(a*e^2+c*d^2)^(1/2)/e^4+1/2*d*(-e*x+2*d)*(c*x^2+a)^(1/2)/e^3
```

3.318.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx=\frac{e\sqrt{a+cx^2}(6cd^2+2ae^2-3cdex+2ce^2x^2)+12cd^2\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)+3\sqrt{c}d(2cd^2}{6ce^4}$$

input
```
Integrate[(x^2*Sqrt[a + c*x^2])/(d + e*x),x]
```

output
```
(e*Sqrt[a + c*x^2]*(6*c*d^2 + 2*a*e^2 - 3*c*d*e*x + 2*c*e^2*x^2) + 12*c*d^
2*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sq
rt[-(c*d^2) - a*e^2]] + 3*Sqrt[c]*d*(2*c*d^2 + a*e^2)*Log[-(Sqrt[c]*x) + S
qrt[a + c*x^2]])/(6*c*e^4)
```

3.318.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {604, 27, 591, 25, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx$$

$$\Big\downarrow \ 604$$

$$\frac{\int -\frac{3cdex\sqrt{cx^2+a}}{d+ex}dx}{3ce^2}+\frac{\left(a+cx^2\right)^{3/2}}{3ce}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(a+cx^2\right)^{3/2}}{3ce}-\frac{d\int \frac{x\sqrt{cx^2+a}}{d+ex}dx}{e}$$

$$\Big\downarrow \ 591$$

$$\frac{\left(a+cx^2\right)^{3/2}}{3ce}-\frac{d\left(\frac{\int -\frac{ade-\left(2cd^2+ae^2\right)x}{(d+ex)\sqrt{cx^2+a}}dx}{2e^2}-\frac{\sqrt{a+cx^2}(2d-ex)}{2e^2}\right)}{e}$$

$$\Big\downarrow \ 25$$

$$\frac{\left(a+cx^2\right)^{3/2}}{3ce}-\frac{d\left(-\frac{\int \frac{ade-\left(2cd^2+ae^2\right)x}{(d+ex)\sqrt{cx^2+a}}dx}{2e^2}-\frac{\sqrt{a+cx^2}(2d-ex)}{2e^2}\right)}{e}$$

$$\Big\downarrow \ 719$$

$$\frac{\left(a+cx^2\right)^{3/2}}{3ce}-\frac{d\left(-\frac{\frac{2d\left(ae^2+cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{\left(ae^2+2cd^2\right)\int \frac{1}{\sqrt{cx^2+a}}dx}{e}}{2e^2}-\frac{\sqrt{a+cx^2}(2d-ex)}{2e^2}\right)}{e}$$

$$\Big\downarrow \text{224}$$

$$\frac{(a+cx^2)^{3/2}}{3ce}-\frac{d\left(-\frac{\frac{2d\left(ae^2+cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{\left(ae^2+2cd^2\right)\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{e}}{2e^2}-\frac{\sqrt{a+cx^2}(2d-ex)}{2e^2}\right)}{e}$$

$$\Big\downarrow \text{219}$$

$$\frac{(a+cx^2)^{3/2}}{3ce}-\frac{d\left(-\frac{\frac{2d\left(ae^2+cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(ae^2+2cd^2\right)}{\sqrt{c}e}}{2e^2}-\frac{\sqrt{a+cx^2}(2d-ex)}{2e^2}\right)}{e}$$

$$\Big\downarrow \text{488}$$

$$\frac{(a+cx^2)^{3/2}}{3ce}-\frac{d\left(-\frac{-\frac{2d\left(ae^2+cd^2\right)\int \frac{1}{cd^2+ae^2-\frac{(ae-cdx)^2}{cx^2+a}}d\frac{ae-cdx}{\sqrt{cx^2+a}}}{e}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(ae^2+2cd^2\right)}{\sqrt{c}e}}{2e^2}-\frac{\sqrt{a+cx^2}(2d-ex)}{2e^2}\right)}{e}$$

$$\Big\downarrow \text{219}$$

$$\frac{(a+cx^2)^{3/2}}{3ce}-\frac{d\left(-\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(ae^2+2cd^2\right)}{\sqrt{c}e}-\frac{2d\sqrt{ae^2+cd^2}\,\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e}}{2e^2}-\frac{\sqrt{a+cx^2}(2d-ex)}{2e^2}\right)}{e}$$

input
```
Int[(x^2*Sqrt[a + c*x^2])/(d + e*x),x]
```

output
```
(a + c*x^2)^(3/2)/(3*c*e) - (d*(-1/2*((2*d - e*x)*Sqrt[a + c*x^2])/e^2 - (
-(((2*c*d^2 + a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*e)) -
(2*d*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a
 + c*x^2])])/e)/(2*e^2)))/e
```

3.318. $\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx$

3.318.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 591
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 2*p + 1)*x)/
(d^2*(n + 2*p + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 2*p + 1)*(n +
2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*Simp[a*c*d*n + (b*c^2*(2*p
 + 1) + a*d^2*(n + 2*p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &&
GtQ[p, 0] && LeQ[-1, n, 0] &&  !ILtQ[n + 2*p, 0]
```

rule 604
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(b*d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*
x^2)^p*ExpandToSum[b*d^m*(m + n + 2*p + 1)*x^m - b*(m + n + 2*p + 1)*(c + d
*x)^m - (c + d*x)^(m - 2)*(a*d^2*(m + n - 1) - b*c^2*(m + n + 2*p + 1) - 2*
b*c*d*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && IGtQ[m,
 1] && NeQ[m + n + 2*p + 1, 0] && IntegerQ[2*p]
```

3.318. $\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx$

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.318.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.48

method	result
risch	$\frac{(2c e^2x^2-3cdex+2e^2a+6c d^2)\sqrt{c x^2+a}}{6c e^3} - \frac{d\left(\frac{(e^2a+2c d^2)\ln\left(x\sqrt{c}+\sqrt{c x^2+a}\right)}{e\sqrt{c}} + \frac{2d(e^2a+c d^2)\ln\left(\frac{\frac{2e^2a+2c d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c d^2}{e^2}}}{x+}\right.}{e^2\sqrt{\frac{e^2a+c d^2}{e^2}}}\right.}{2e^3}$
default	$\frac{(c x^2+a)^{\frac{3}{2}}}{3ce} - \frac{d\left(\frac{x\sqrt{c x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c x^2+a}\right)}{2\sqrt{c}}\right)}{e^2} + \frac{d^2\left(\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c d^2}{e^2}}-\frac{\sqrt{c}\,d\ln\left(\frac{-\frac{cd}{e}+c\left(x+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{\left(x+\frac{d}{e}\right)^2}\right.}{e}\right.}{}$

input
```
int(x^2*(c*x^2+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*c*e^2*x^2-3*c*d*e*x+2*a*e^2+6*c*d^2)*(c*x^2+a)^(1/2)/c/e^3-1/2*d/e^
3*((a*e^2+2*c*d^2)/e*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)+2*d*(a*e^2+c*d^
2)/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2
*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)
^(1/2))/(x+d/e)))
```

3.318.5 Fricas [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 776, normalized size of antiderivative = 5.07

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx$$
$$= \left[\frac{6\sqrt{cd^2+ae^2}cd^2\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+3\,(2\,cd^3+ade^2)\sqrt{c}\log\left(-\right.}{12\,ce^4}\right.$$
$$-\frac{12\sqrt{-cd^2-ae^2}cd^2\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-3\,(2\,cd^3+ade^2)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}\right.}{12\,ce^4}$$
$$-\frac{6\sqrt{-cd^2-ae^2}cd^2\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-3\,(2\,cd^3+ade^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)-(2\,ce^3x^2}{6\,ce^4}$$

input
```
integrate(x^2*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
[1/12*(6*sqrt(c*d^2 + a*e^2)*c*d^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2
- (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x
^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 3*(2*c*d^3 + a*d*e^2)*sqrt(c)*log(-2
*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(2*c*e^3*x^2 - 3*c*d*e^2*x +
 6*c*d^2*e + 2*a*e^3)*sqrt(c*x^2 + a))/(c*e^4), -1/12*(12*sqrt(-c*d^2 - a*
e^2)*c*d^2*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*
d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(2*c*d^3 + a*d*e^2)*sqrt(c)*
log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(2*c*e^3*x^2 - 3*c*d*e
^2*x + 6*c*d^2*e + 2*a*e^3)*sqrt(c*x^2 + a))/(c*e^4), 1/6*(3*sqrt(c*d^2 +
a*e^2)*c*d^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2
)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*
d*e*x + d^2)) + 3*(2*c*d^3 + a*d*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^
2 + a)) + (2*c*e^3*x^2 - 3*c*d*e^2*x + 6*c*d^2*e + 2*a*e^3)*sqrt(c*x^2 + a
))/(c*e^4), -1/6*(6*sqrt(-c*d^2 - a*e^2)*c*d^2*arctan(sqrt(-c*d^2 - a*e^2)
*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^
2)) - 3*(2*c*d^3 + a*d*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) -
(2*c*e^3*x^2 - 3*c*d*e^2*x + 6*c*d^2*e + 2*a*e^3)*sqrt(c*x^2 + a))/(c*e^4)
]
```

3.318.6 Sympy [F]

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx$$

input
```
integrate(x**2*(c*x**2+a)**(1/2)/(e*x+d),x)
```

output
```
Integral(x**2*sqrt(a + c*x**2)/(d + e*x), x)
```

3.318.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.318.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.318.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx=\int \frac{x^2\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

input
```
int((x^2*(a + c*x^2)^(1/2))/(d + e*x),x)
```

output
```
int((x^2*(a + c*x^2)^(1/2))/(d + e*x), x)
```

3.319 $\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx$

3.319.1 Optimal result

Integrand size = 20, antiderivative size = 127

$$\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx = -\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2}+\frac{(2cd^2+ae^2)\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}e^3}+\frac{d\sqrt{cd^2+ae^2}\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^3}$$

output
```
1/2*(a*e^2+2*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/e^3/c^(1/2)+d*arcta
nh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/e
^3-1/2*(-e*x+2*d)*(c*x^2+a)^(1/2)/e^2
```

3.319.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05

$$\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx = \frac{e(-2d+ex)\sqrt{a+cx^2}-4d\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)-\frac{(2cd^2+ae^2)\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{\sqrt{c}}}{2e^3}$$

input
```
Integrate[(x*Sqrt[a + c*x^2])/(d + e*x),x]
```

output
```
(e*(-2*d + e*x)*Sqrt[a + c*x^2] - 4*d*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[
c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]] - ((2*c*d^2 + a*
e^2)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/(2*e^3)
```

3.319.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {591, 25, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a+cx^2}}{d+ex}dx$$

$$\Big\downarrow \text{591}$$

$$\frac{\int -\frac{ade-(2cd^2+ae^2)x}{(d+ex)\sqrt{cx^2+a}}dx}{2e^2}-\frac{\sqrt{a+cx^2}(2d-ex)}{2e^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{ade-(2cd^2+ae^2)x}{(d+ex)\sqrt{cx^2+a}}dx}{2e^2}-\frac{\sqrt{a+cx^2}(2d-ex)}{2e^2}$$

$$\Big\downarrow \text{719}$$

$$-\frac{\frac{2d(ae^2+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{(ae^2+2cd^2)\int \frac{1}{\sqrt{cx^2+a}}dx}{e}}{2e^2}-\frac{\sqrt{a+cx^2}(2d-ex)}{2e^2}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\frac{2d(ae^2+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{(ae^2+2cd^2)\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{e}}{2e^2}-\frac{\sqrt{a+cx^2}(2d-ex)}{2e^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{2d(ae^2+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(ae^2+2cd^2)}{\sqrt{c}e}}{2e^2}-\frac{\sqrt{a+cx^2}(2d-ex)}{2e^2}$$

$$\Big\downarrow \text{488}$$

$$-\frac{-\frac{2d(ae^2+cd^2)\int \frac{1}{cd^2+ae^2-\frac{(ae-cdx)^2}{cx^2+a}}d\frac{ae-cdx}{\sqrt{cx^2+a}}}{e}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(ae^2+2cd^2)}{\sqrt{c}e}}{2e^2}-\frac{\sqrt{a+cx^2}(2d-ex)}{2e^2}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(ae^2+2cd^2)}{\sqrt{c}e}-\frac{2d\sqrt{ae^2+cd^2}\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e}}{2e^2}-\frac{\sqrt{a+cx^2}(2d-ex)}{2e^2}$$

input
```
Int[(x*Sqrt[a + c*x^2])/(d + e*x),x]
```

output
```
-1/2*((2*d - e*x)*Sqrt[a + c*x^2])/e^2 - (-(((2*c*d^2 + a*e^2)*ArcTanh[(Sq
rt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*e)) - (2*d*Sqrt[c*d^2 + a*e^2]*ArcTanh
[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/e)/(2*e^2)
```

3.319.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 591
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 2*p + 1)*x)/
(d^2*(n + 2*p + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 2*p + 1)*(n +
2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*Simp[a*c*d*n + (b*c^2*(2*p
 + 1) + a*d^2*(n + 2*p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &&
GtQ[p, 0] && LeQ[-1, n, 0] &&  !ILtQ[n + 2*p, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

3.319.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.60

method	result
risch	$-\frac{(-ex+2d)\sqrt{c\,x^2+a}}{2e^2}+\frac{\frac{\left(e^2a+2c\,d^2\right)\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e\sqrt{c}}+\frac{2d\left(e^2a+c\,d^2\right)\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\right.}{e}}}{x+\frac{d}{e}}\right.}{e^2\sqrt{\frac{e^2a+c\,d^2}{e^2}}}}{2e^2}$
default	$\frac{\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}}{e}-\frac{d\left(\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}-\frac{\sqrt{c}\,d\ln\left(\frac{-\frac{cd}{e}+c\left(x+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d}{e^2}}\right.}{e}\right.}{e^2}$

input
```
int(x*(c*x^2+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e*x+2*d)*(c*x^2+a)^(1/2)/e^2+1/2/e^2*((a*e^2+2*c*d^2)/e*ln(x*c^(1/2
)+(c*x^2+a)^(1/2))/c^(1/2)+2*d*(a*e^2+c*d^2)/e^2/((a*e^2+c*d^2)/e^2)^(1/2)
*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d
/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

3.319.5 Fricas [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 684, normalized size of antiderivative = 5.39

$$\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx$$
$$=\left[\frac{2\sqrt{cd^2+ae^2}cd\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2+2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+(2\,cd^2+ae^2)\sqrt{c}\log\left(-2\,c\right.}{4\,ce^3}\right.$$

input
```
integrate(x*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(c*d^2 + a*e^2)*c*d*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (
2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2
+ a))/(e^2*x^2 + 2*d*e*x + d^2)) + (2*c*d^2 + a*e^2)*sqrt(c)*log(-2*c*x^2
- 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(c*e^2*x - 2*c*d*e)*sqrt(c*x^2 + a)
)/(c*e^3), 1/4*(4*sqrt(-c*d^2 - a*e^2)*c*d*arctan(sqrt(-c*d^2 - a*e^2)*(c*
d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2))
+ (2*c*d^2 + a*e^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a
) + 2*(c*e^2*x - 2*c*d*e)*sqrt(c*x^2 + a))/(c*e^3), 1/2*(sqrt(c*d^2 + a*e^
2)*c*d*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2
+ 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x
+ d^2)) - (2*c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) +
(c*e^2*x - 2*c*d*e)*sqrt(c*x^2 + a))/(c*e^3), 1/2*(2*sqrt(-c*d^2 - a*e^2)*
c*d*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a
^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (2*c*d^2 + a*e^2)*sqrt(-c)*arctan(sqr
t(-c)*x/sqrt(c*x^2 + a)) + (c*e^2*x - 2*c*d*e)*sqrt(c*x^2 + a))/(c*e^3)]
```

3.319.6 Sympy [F]

$$\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx$$

input
```
integrate(x*(c*x**2+a)**(1/2)/(e*x+d),x)
```

output
```
Integral(x*sqrt(a + c*x**2)/(d + e*x), x)
```

3.319.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.319.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.319.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{x\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

input
```
int((x*(a + c*x^2)^(1/2))/(d + e*x),x)
```

output
```
int((x*(a + c*x^2)^(1/2))/(d + e*x), x)
```

3.320 $\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx$

3.320.1 Optimal result

Integrand size = 19, antiderivative size = 103

$$\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx = \frac{\sqrt{a+cx^2}}{e} - \frac{\sqrt{c}d\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e^2} - \frac{\sqrt{cd^2+ae^2}\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^2}$$

output
```
-d*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/e^2-arctanh((-c*d*x+a*e)/(a*
e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/e^2+(c*x^2+a)^(1/2)/
e
```

3.320.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx = \frac{e\sqrt{a+cx^2} + 2\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right) + \sqrt{c}d\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{e^2}$$

input
```
Integrate[Sqrt[a + c*x^2]/(d + e*x),x]
```

output
```
(e*Sqrt[a + c*x^2] + 2*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) -
e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]] + Sqrt[c]*d*Log[-(Sqrt[c]*x) +
Sqrt[a + c*x^2]])/e^2
```

3.320.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {493, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx$$

$$\Big\downarrow \text{ 493}$$

$$\frac{\int \frac{ae-cdx}{(d+ex)\sqrt{cx^2+a}}dx}{e}+\frac{\sqrt{a+cx^2}}{e}$$

$$\Big\downarrow \text{ 719}$$

$$\frac{\frac{(ae^2+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{cd\int \frac{1}{\sqrt{cx^2+a}}dx}{e}}{e}+\frac{\sqrt{a+cx^2}}{e}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{(ae^2+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{cd\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{e}}{e}+\frac{\sqrt{a+cx^2}}{e}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{(ae^2+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{\sqrt{c}d\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e}}{e}+\frac{\sqrt{a+cx^2}}{e}$$

$$\Big\downarrow \text{ 488}$$

$$\frac{-\frac{(ae^2+cd^2)\int \frac{1}{cd^2+ae^2-\frac{(ae-cdx)^2}{cx^2+a}}d\frac{ae-cdx}{\sqrt{cx^2+a}}}{e}-\frac{\sqrt{c}d\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e}}{e}+\frac{\sqrt{a+cx^2}}{e}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{-\frac{\sqrt{ae^2+cd^2}\,\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e}-\frac{\sqrt{c}d\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e}}{e}+\frac{\sqrt{a+cx^2}}{e}$$

input
```
Int[Sqrt[a + c*x^2]/(d + e*x),x]
```

output
```
Sqrt[a + c*x^2]/e + (-((Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/e)
 - (Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a
+ c*x^2])])/e)/e
```

3.320.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.320.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(89) = 178$.

Time = 0.42 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.79

method	result
risch	$\frac{\sqrt{c\,x^2+a}}{e}-\frac{\frac{\sqrt{c}\,d\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e}-\frac{\left(-e^2a-c\,d^2\right)\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{e^2\sqrt{\frac{e^2a+c\,d^2}{e^2}}}}{e}$
default	$\frac{\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}-\frac{\sqrt{c}\,d\ln\left(\frac{-\frac{cd}{e}+c\left(x+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}\right)}{e}-\frac{\left(e^2a+c\,d^2\right)\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd(x\cdots}{e}}{\cdots}\right.}{\cdots}}{e}$

input
```
int((c*x^2+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+a)^(1/2)/e-1/e*(1/e*c^(1/2)*d*ln(x*c^(1/2)+(c*x^2+a)^(1/2))-(-a*e^2
-c*d^2)/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d
/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)
/e^2)^(1/2))/(x+d/e)))
```

3.320.5 Fricas [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 574, normalized size of antiderivative = 5.57

$$\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx$$
$$= \left[\frac{\sqrt{c}d\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\,\sqrt{cx^2+a}e+\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x\cdots}{e^2x^2+2\,dex\cdots}\right.}{2\,e^2}\right.$$

input
```
integrate((c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(c)*d*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*sqrt(c
*x^2 + a)*e + sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 -
 (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^
2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/e^2, 1/2*(2*sqrt(-c)*d*arctan(sqrt(-c)
*x/sqrt(c*x^2 + a)) + 2*sqrt(c*x^2 + a)*e + sqrt(c*d^2 + a*e^2)*log((2*a*c
*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 +
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/e^2, 1/2
*(sqrt(c)*d*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*sqrt(c*x^2
 + a)*e - 2*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)
*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/e^2, (sqr
t(-c)*d*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + sqrt(c*x^2 + a)*e - sqrt(-c*d
^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c
*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/e^2]
```

3.320.6 Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{a+cx^2}}{d+ex}\,dx$$

input
```
integrate((c*x**2+a)**(1/2)/(e*x+d),x)
```

output
```
Integral(sqrt(a + c*x**2)/(d + e*x), x)
```

3.320.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.320.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.320.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

input
```
int((a + c*x^2)^(1/2)/(d + e*x),x)
```

output
```
int((a + c*x^2)^(1/2)/(d + e*x), x)
```

3.321 $\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx$

3.321.1 Optimal result

Integrand size = 22, antiderivative size = 116

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx = \frac{\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e} + \frac{\sqrt{cd^2+ae^2}\,\mathrm{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{de} - \frac{\sqrt{a}\,\mathrm{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

output
```
-arctanh((c*x^2+a)^(1/2)/a^(1/2))*a^(1/2)/d+arctanh(x*c^(1/2)/(c*x^2+a)^(1
/2))*c^(1/2)/e+arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(
a*e^2+c*d^2)^(1/2)/d/e
```

3.321.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx = -\frac{2\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right) - 2\sqrt{a}e\,\mathrm{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right) + \sqrt{c}d\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{de}$$

input
```
Integrate[Sqrt[a + c*x^2]/(x*(d + e*x)),x]
```

output
```
-((2*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])
/Sqrt[-(c*d^2) - a*e^2]] - 2*Sqrt[a]*e*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2
])/Sqrt[a]] + Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(d*e))
```

3.321.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {606, 243, 73, 221, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex)}dx$$

$$\Big\downarrow \text{606}$$

$$\frac{a\int \frac{1}{x\sqrt{cx^2+a}}dx}{d}-\frac{\int \frac{ae-cdx}{(d+ex)\sqrt{cx^2+a}}dx}{d}$$

$$\Big\downarrow \text{243}$$

$$\frac{a\int \frac{1}{x^2\sqrt{cx^2+a}}dx^2}{2d}-\frac{\int \frac{ae-cdx}{(d+ex)\sqrt{cx^2+a}}dx}{d}$$

$$\Big\downarrow \text{73}$$

$$\frac{a\int \frac{1}{\frac{x^4}{c}-\frac{a}{c}}d\sqrt{cx^2+a}}{cd}-\frac{\int \frac{ae-cdx}{(d+ex)\sqrt{cx^2+a}}dx}{d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\int \frac{ae-cdx}{(d+ex)\sqrt{cx^2+a}}dx}{d}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

$$\Big\downarrow \text{719}$$

$$-\frac{\frac{(ae^2+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{cd\int \frac{1}{\sqrt{cx^2+a}}dx}{e}}{d}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\frac{(ae^2+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{cd\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{e}}{d}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{(ae^2+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{\sqrt{c}d\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e}}{d}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

$$\Big\downarrow \text{488}$$

$$-\frac{-\frac{(ae^2+cd^2)\int \frac{1}{cd^2+ae^2-\frac{(ae-cdx)^2}{cx^2+a}}d\frac{ae-cdx}{\sqrt{cx^2+a}}}{e}-\frac{\sqrt{c}d\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e}}{d}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-\frac{\sqrt{ae^2+cd^2}\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e}-\frac{\sqrt{c}d\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e}}{d}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

input
```
Int[Sqrt[a + c*x^2]/(x*(d + e*x)),x]
```

output
```
-((-((Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/e) - (Sqrt[c*d^2 + a
*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/e)/d)
- (Sqrt[a]*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d
```

3.321.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.321. $\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx$

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 606
```
Int[(((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :
> Simp[a/c   Int[(c + d*x)^(n + 1)*((a + b*x^2)^(p - 1)/x), x], x] - Simp[1
/c   Int[(c + d*x)^n*(a*d - b*c*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && GtQ[p, 0] && ILtQ[n, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.321.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(98) = 196$.

Time = 0.38 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.63

method	result
default	$\frac{\sqrt{c\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^{2}+a}}{x}\right)}{d}-\frac{\sqrt{\left(x+\frac{d}{e}\right)^{2}c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^{2}a+c\,d^{2}}{e^{2}}}-\frac{\sqrt{c}\,d\ln\left(\frac{-\frac{cd}{e}+c\left(x+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{\left(x+\frac{d}{e}\right)^{2}c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^{2}a+}{e^{2}}\right.}{e}}{}$

input
```
int((c*x^2+a)^(1/2)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

3.321. $\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx$

output
```
1/d*((c*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x))-1/d*((
(x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-1/e*c^(1/2)*d*ln((-1/
e*c*d+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(
1/2))-(a*e^2+c*d^2)/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-
2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(
a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

3.321.5 Fricas [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 1316, normalized size of antiderivative = 11.34

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)^(1/2)/x/(e*x+d),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + sqrt(a)*
e*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + sqrt(c*d^2 + a*e^2
)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*s
qrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2
)))/(d*e), -1/2*(2*sqrt(-c)*d*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - sqrt(a)
*e*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - sqrt(c*d^2 + a*e^
2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*
sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^
2)))/(d*e), 1/2*(sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a)
 + sqrt(a)*e*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*sqrt(
-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/
(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/(d*e), -1/2*(2*sqrt(-c)*d*
arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - sqrt(a)*e*log(-(c*x^2 - 2*sqrt(c*x^2
+ a)*sqrt(a) + 2*a)/x^2) - 2*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e
^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)
*x^2)))/(d*e), 1/2*(2*sqrt(-a)*e*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + sqrt(c
)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + sqrt(c*d^2 + a*e^2)*
log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqr
t(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2))
)/(d*e), -1/2*(2*sqrt(-c)*d*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - 2*sqrt...
```

3.321.6 Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx = \int \frac{\sqrt{a+cx^2}}{x\,(d+ex)}\,dx$$

input
```
integrate((c*x**2+a)**(1/2)/x/(e*x+d),x)
```

output
```
Integral(sqrt(a + c*x**2)/(x*(d + e*x)), x)
```

3.321.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx = -\frac{e\left(\frac{\sqrt{a+\frac{cd^2}{e^2}}\operatorname{arsinh}\left(\frac{2\,cdx}{e\sqrt{\frac{ac}{e^2}}|2\,ex+2\,d|}-\frac{2\,a}{\sqrt{\frac{ac}{e^2}}|2\,ex+2\,d|}\right)}{e}+\frac{\sqrt{a}\operatorname{arsinh}\left(\frac{a}{\sqrt{ac}|x|}\right)}{e}-\frac{cd\operatorname{arsinh}\left(\frac{cx}{e\sqrt{\frac{ac}{e^2}}}\right)}{e^3\sqrt{\frac{c}{e^2}}}\right)}{d}$$

input
```
integrate((c*x^2+a)^(1/2)/x/(e*x+d),x, algorithm="maxima")
```

output
```
-e*(sqrt(a + c*d^2/e^2)*arcsinh(2*c*d*x/(e*sqrt(a*c/e^2)*abs(2*e*x + 2*d))
 - 2*a/(sqrt(a*c/e^2)*abs(2*e*x + 2*d)))/e + sqrt(a)*arcsinh(a/(sqrt(a*c)*
abs(x)))/e - c*d*arcsinh(c*x/(e*sqrt(a*c/e^2)))/(e^3*sqrt(c/e^2)))/d
```

3.321.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+a)^(1/2)/x/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.321.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx = \int \frac{\sqrt{c\,x^2+a}}{x\,\left(d+e\,x\right)} \,dx$$

input
```
int((a + c*x^2)^(1/2)/(x*(d + e*x)),x)
```

output
```
int((a + c*x^2)^(1/2)/(x*(d + e*x)), x)
```

3.322 $\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx$

3.322.1 Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx = -\frac{\sqrt{a+cx^2}}{dx} - \frac{\sqrt{cd^2+ae^2}\,\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2} + \frac{\sqrt{a}\,e\,\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}$$

output
```
e*arctanh((c*x^2+a)^(1/2)/a^(1/2))*a^(1/2)/d^2-arctanh((-c*d*x+a*e)/(a*e^2
+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/d^2-(c*x^2+a)^(1/2)/d/x
```

3.322.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx = \frac{-d\sqrt{a+cx^2}+2\sqrt{-cd^2-ae^2}\,x\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,x}{\sqrt{a}(d+ex)-d\sqrt{a+cx^2}}\right)+\sqrt{a}ex\log(x)-\sqrt{a}ex\log\left(-\sqrt{a}+\sqrt{a+cx^2}\right)}{d^2x}$$

input
```
Integrate[Sqrt[a + c*x^2]/(x^2*(d + e*x)),x]
```

output
```
(-(d*Sqrt[a + c*x^2]) + 2*Sqrt[-(c*d^2) - a*e^2]*x*ArcTan[(Sqrt[-(c*d^2) -
 a*e^2]*x)/(Sqrt[a]*(d + e*x) - d*Sqrt[a + c*x^2])] + Sqrt[a]*e*x*Log[x] -
 Sqrt[a]*e*x*Log[-Sqrt[a] + Sqrt[a + c*x^2]])/(d^2*x)
```

3.322.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx$$
$$\Big\downarrow \text{617}$$
$$\int \left(\frac{e^2\sqrt{a+cx^2}}{d^2(d+ex)}-\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{\sqrt{a+cx^2}}{dx^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\sqrt{ae^2+cd^2}\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2}+\frac{\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}-\frac{\sqrt{a+cx^2}}{dx}$$

input
```
Int[Sqrt[a + c*x^2]/(x^2*(d + e*x)),x]
```

output
```
-(Sqrt[a + c*x^2]/(d*x)) - (Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqr
t[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^2 + (Sqrt[a]*e*ArcTanh[Sqrt[a + c*x^
2]/Sqrt[a]])/d^2
```

3.322.3.1 Defintions of rubi rules used

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.322.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(91) = 182$.

Time = 0.38 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.86

method	result
risch	$-\frac{\sqrt{c\,x^2+a}}{dx}+\frac{\frac{\sqrt{a}\,e\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{d}-\frac{\left(e^2a+c\,d^2\right)\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\,\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{de\sqrt{\frac{e^2a+c\,d^2}{e^2}}}}{d}$
default	$\frac{-\frac{\left(c\,x^2+a\right)^{\frac{3}{2}}}{ax}+\frac{2c\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}\right)}{a}}{d}-\frac{e\left(\sqrt{c\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)\right)}{d^2}+\frac{e\left(\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\right.}{}$

input
```
int((c*x^2+a)^(1/2)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2+a)^(1/2)/d/x+1/d*(1/d*a^(1/2)*e*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))
/x)-(a*e^2+c*d^2)/d/e/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/
e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*
e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

3.322.5 Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 599, normalized size of antiderivative = 5.70

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx$$
$$= \left[\frac{\sqrt{a}ex\log\left(-\frac{cx^2+2\sqrt{cx^2+a}\sqrt{a}+2a}{x^2}\right)+\sqrt{cd^2+ae^2}x\log\left(\frac{2acdex-acd^2-2a^2e^2-(2c^2d^2+ace^2)x^2-2\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2}}{e^2x^2+2dex+d^2}\right.}{2d^2x}\right.$$
$$-\frac{2\sqrt{-a}ex\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right)-\sqrt{cd^2+ae^2}x\log\left(\frac{2acdex-acd^2-2a^2e^2-(2c^2d^2+ace^2)x^2-2\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2dex+d^2}\right)}{2d^2x}$$
$$\left.-\frac{\sqrt{-a}ex\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right)+\sqrt{-cd^2-ae^2}x\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)+\sqrt{cx^2+a}d}{d^2x}\right]$$

input
```
integrate((c*x^2+a)^(1/2)/x^2/(e*x+d),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*e*x*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + sq
rt(c*d^2 + a*e^2)*x*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 +
a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x
^2 + 2*d*e*x + d^2)) - 2*sqrt(c*x^2 + a)*d)/(d^2*x), 1/2*(sqrt(a)*e*x*log(
-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*sqrt(-c*d^2 - a*e^2)*x
*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*
e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 2*sqrt(c*x^2 + a)*d)/(d^2*x), -1/2*(2*sq
rt(-a)*e*x*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - sqrt(c*d^2 + a*e^2)*x*log((2
*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^
2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*s
qrt(c*x^2 + a)*d)/(d^2*x), -(sqrt(-a)*e*x*arctan(sqrt(-a)/sqrt(c*x^2 + a))
 + sqrt(-c*d^2 - a*e^2)*x*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c
*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + sqrt(c*x^2 + a)
*d)/(d^2*x)]
```

3.322.6 Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx = \int \frac{\sqrt{a+cx^2}}{x^2\,(d+ex)}\,dx$$

input
```
integrate((c*x**2+a)**(1/2)/x**2/(e*x+d),x)
```

output
```
Integral(sqrt(a + c*x**2)/(x**2*(d + e*x)), x)
```

3.322.7 Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx = \int \frac{\sqrt{cx^2+a}}{(ex+d)x^2}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/x^2/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)/((e*x + d)*x^2), x)
```

3.322.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx = -\frac{2\,ae\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}d^2} + \frac{2\,a\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2-a\right)d} + \frac{2\left(cd^2+ae^2\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}d^2}$$

input
```
integrate((c*x^2+a)^(1/2)/x^2/(e*x+d),x, algorithm="giac")
```

output
```
-2*a*e*arctan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*d^2) + 2*
a*sqrt(c)/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)*d) + 2*(c*d^2 + a*e^2)*ar
ctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))/
(sqrt(-c*d^2 - a*e^2)*d^2)
```

3.322.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx = \int \frac{\sqrt{c\,x^2+a}}{x^2\,(d+e\,x)}\,dx$$

input
```
int((a + c*x^2)^(1/2)/(x^2*(d + e*x)),x)
```

output
```
int((a + c*x^2)^(1/2)/(x^2*(d + e*x)), x)
```

3.323 $\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx$

3.323.1 Optimal result

Integrand size = 22, antiderivative size = 160

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx = -\frac{\sqrt{a+cx^2}}{2dx^2}+\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{e\sqrt{cd^2+ae^2}\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3} - \frac{c\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d}-\frac{\sqrt{a}e^2\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}$$

output
```
-1/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d/a^(1/2)-e^2*arctanh((c*x^2+a)^(1
/2)/a^(1/2))*a^(1/2)/d^3+e*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2
+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/d^3-1/2*(c*x^2+a)^(1/2)/d/x^2+e*(c*x^2+a)^(
1/2)/d^2/x
```

3.323.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx = \frac{\frac{d(-d+2ex)\sqrt{a+cx^2}}{x^2}-4e\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)+\frac{2(cd^2+2ae^2)\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{2d^3}$$

input
```
Integrate[Sqrt[a + c*x^2]/(x^3*(d + e*x)),x]
```

output
```
((d*(-d + 2*e*x)*Sqrt[a + c*x^2])/x^2 - 4*e*Sqrt[-(c*d^2) - a*e^2]*ArcTan[
(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]] + (2*(c*d^
2 + 2*a*e^2)*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/Sqrt[a])/(2*d
^3)
```

3.323.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx$$

$$\Big\downarrow \text{617}$$

$$\int \left(-\frac{e^3\sqrt{a+cx^2}}{d^3(d+ex)}+\frac{e^2\sqrt{a+cx^2}}{d^3x}-\frac{e\sqrt{a+cx^2}}{d^2x^2}+\frac{\sqrt{a+cx^2}}{dx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{a}e^2\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+\frac{e\sqrt{ae^2+cd^2}\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3}-\frac{c\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d}+\frac{e\sqrt{a+cx^2}}{d^2x}-\frac{\sqrt{a+cx^2}}{2dx^2}$$

input
```
Int[Sqrt[a + c*x^2]/(x^3*(d + e*x)),x]
```

output
```
-1/2*Sqrt[a + c*x^2]/(d*x^2) + (e*Sqrt[a + c*x^2])/(d^2*x) + (e*Sqrt[c*d^2
 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^
3 - (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*Sqrt[a]*d) - (Sqrt[a]*e^2*ArcT
anh[Sqrt[a + c*x^2]/Sqrt[a]])/d^3
```

3.323.3.1 Defintions of rubi rules used

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.323.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{\sqrt{c x^2+a}\,(-2ex+d)}{2d^2x^2}-\frac{-\frac{\left(-2e^2a-c\,d^2\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{d\sqrt{a}}-\frac{2\left(e^2a+c\,d^2\right)\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-\cdots}}{x+\frac{d}{e}}\right)}{d\sqrt{\frac{e^2a+c\,d^2}{e^2}}}}{2d^2}\cdots$
default	$\frac{-\frac{\left(c\,x^2+a\right)^{\frac{3}{2}}}{2a\,x^2}+\frac{c\left(\sqrt{c\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)\right)}{2a}}{d}+\frac{e^2\left(\sqrt{c\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)\right)}{d^3}-\frac{e\left(-\frac{\left(c\,x^2+a\right)^{\frac{3}{2}}}{ax}+\frac{2c\left(x\sqrt{c\cdots}\right.}{\cdots}\right.}{\cdots}$

input
```
int((c*x^2+a)^(1/2)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*x^2+a)^(1/2)*(-2*e*x+d)/d^2/x^2-1/2/d^2*(-(-2*a*e^2-c*d^2)/d/a^(1/
2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-2*(a*e^2+c*d^2)/d/((a*e^2+c*d^2)/
e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(
1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

3.323.5 Fricas [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 726, normalized size of antiderivative = 4.54

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx$$
$$= \left[\frac{2\sqrt{cd^2+ae^2}aex^2\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2+2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+(cd^2+2\,ae^2)\sqrt{a}x^2\log}{4\,ad^3x^2}\right.$$

input
```
integrate((c*x^2+a)^(1/2)/x^3/(e*x+d),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(c*d^2 + a*e^2)*a*e*x^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2
 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*
x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2 + 2*a*e^2)*sqrt(a)*x^2*log(-
(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*a*d*e*x - a*d^2)*sqr
t(c*x^2 + a))/(a*d^3*x^2), 1/4*(4*sqrt(-c*d^2 - a*e^2)*a*e*x^2*arctan(sqrt
(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d
^2 + a*c*e^2)*x^2)) + (c*d^2 + 2*a*e^2)*sqrt(a)*x^2*log(-(c*x^2 - 2*sqrt(c
*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*a*d*e*x - a*d^2)*sqrt(c*x^2 + a))/(a*
d^3*x^2), 1/2*(sqrt(c*d^2 + a*e^2)*a*e*x^2*log((2*a*c*d*e*x - a*c*d^2 - 2*
a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*
sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2 + 2*a*e^2)*sqrt(-a)*x
^2*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (2*a*d*e*x - a*d^2)*sqrt(c*x^2 + a))
/(a*d^3*x^2), 1/2*(2*sqrt(-c*d^2 - a*e^2)*a*e*x^2*arctan(sqrt(-c*d^2 - a*e
^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)
*x^2)) + (c*d^2 + 2*a*e^2)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(c*x^2 + a)) +
 (2*a*d*e*x - a*d^2)*sqrt(c*x^2 + a))/(a*d^3*x^2)]
```

3.323.6 Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx = \int \frac{\sqrt{a+cx^2}}{x^3\,(d+ex)}\,dx$$

input
```
integrate((c*x**2+a)**(1/2)/x**3/(e*x+d),x)
```

output
```
Integral(sqrt(a + c*x**2)/(x**3*(d + e*x)), x)
```

3.323.7 Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx = \int \frac{\sqrt{cx^2+a}}{(ex+d)x^3}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/x^3/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)/((e*x + d)*x^3), x)
```

3.323.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx$$
$$= -\frac{2\,(cd^2e+ae^3)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}d^3} + \frac{(cd^2+2\,ae^2)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}d^3}$$
$$+ \frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3cd-2\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2a\sqrt{c}e+\left(\sqrt{c}x-\sqrt{cx^2+a}\right)acd+2\,a^2\sqrt{c}e}{\left(\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2-a\right)^2d^2}$$

input
```
integrate((c*x^2+a)^(1/2)/x^3/(e*x+d),x, algorithm="giac")
```

output
```
-2*(c*d^2*e + a*e^3)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)
/sqrt(-c*d^2 - a*e^2))/(sqrt(-c*d^2 - a*e^2)*d^3) + (c*d^2 + 2*a*e^2)*arct
an(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*d^3) + ((sqrt(c)*x -
 sqrt(c*x^2 + a))^3*c*d - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*sqrt(c)*e +
(sqrt(c)*x - sqrt(c*x^2 + a))*a*c*d + 2*a^2*sqrt(c)*e)/(((sqrt(c)*x - sqrt
(c*x^2 + a))^2 - a)^2*d^2)
```

3.323.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx = \int \frac{\sqrt{c\,x^2+a}}{x^3\,\left(d+e\,x\right)}\,dx$$

input
```
int((a + c*x^2)^(1/2)/(x^3*(d + e*x)),x)
```

output
```
int((a + c*x^2)^(1/2)/(x^3*(d + e*x)), x)
```

3.324 $\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx$

3.324.1 Optimal result

Integrand size = 22, antiderivative size = 191

$$\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx = \frac{e\sqrt{a+cx^2}}{2d^2x^2} - \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{\left(a+cx^2\right)^{3/2}}{3adx^3} - \frac{e^2\sqrt{cd^2+ae^2}\,\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^4} + \frac{ce\,\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d^2} + \frac{\sqrt{a}e^3\,\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^4}$$

output
```
-1/3*(c*x^2+a)^(3/2)/a/d/x^3+1/2*c*e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^2/
a^(1/2)+e^3*arctanh((c*x^2+a)^(1/2)/a^(1/2))*a^(1/2)/d^4-e^2*arctanh((-c*d
*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/d^4+1/2*e
*(c*x^2+a)^(1/2)/d^2/x^2-e^2*(c*x^2+a)^(1/2)/d^3/x
```

3.324.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx =$$

$$-\frac{\frac{d\sqrt{a+cx^2}\left(2ad^2-3adex+2cd^2x^2+6ae^2x^2\right)}{ax^3} - 12e^2\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right) + \frac{6e\left(cd^2+2ae^2\right)\text{arctanh}\left(\sqrt{\cdots}\right.}{\sqrt{a}}}{6d^4}$$

input
```
Integrate[Sqrt[a + c*x^2]/(x^4*(d + e*x)),x]
```

output
```
-1/6*((d*Sqrt[a + c*x^2]*(2*a*d^2 - 3*a*d*e*x + 2*c*d^2*x^2 + 6*a*e^2*x^2)
)/(a*x^3) - 12*e^2*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sq
rt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]] + (6*e*(c*d^2 + 2*a*e^2)*ArcTanh[(S
qrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/Sqrt[a])/d^4
```

3.324.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}dx$$
$$\Big\downarrow \text{617}$$
$$\int \left(\frac{e^4\sqrt{a+cx^2}}{d^4(d+ex)}-\frac{e^3\sqrt{a+cx^2}}{d^4x}+\frac{e^2\sqrt{a+cx^2}}{d^3x^2}-\frac{e\sqrt{a+cx^2}}{d^2x^3}+\frac{\sqrt{a+cx^2}}{dx^4}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{a}e^3\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^4}+\frac{ce\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d^2}-\frac{e^2\sqrt{ae^2+cd^2}\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^4}-\frac{e^2\sqrt{a+cx^2}}{d^3x}+\frac{e\sqrt{a+cx^2}}{2d^2x^2}-\frac{(a+cx^2)^{3/2}}{3adx^3}$$

input
```
Int[Sqrt[a + c*x^2]/(x^4*(d + e*x)),x]
```

output
```
(e*Sqrt[a + c*x^2])/(2*d^2*x^2) - (e^2*Sqrt[a + c*x^2])/(d^3*x) - (a + c*x
^2)^(3/2)/(3*a*d*x^3) - (e^2*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sq
rt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^4 + (c*e*ArcTanh[Sqrt[a + c*x^2]/Sq
rt[a]])/(2*Sqrt[a]*d^2) + (Sqrt[a]*e^3*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d
^4
```

3.324.3.1 Defintions of rubi rules used

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.324.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{\sqrt{c\,x^2+a}\,(6a\,e^2x^2+2c\,d^2x^2-3adex+2a\,d^2)}{6d^3x^3a}+\frac{e\left(-\frac{\left(-2e^2a-c\,d^2\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{d\sqrt{a}}-\frac{2\left(e^2a+c\,d^2\right)\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}\cdots}{\cdots}\right)}{\cdots}\right)}{2d^3}\cdots$
default	$-\frac{\left(c\,x^2+a\right)^{\frac{3}{2}}}{3ad\,x^3}+\frac{e^2\left(-\frac{\left(c\,x^2+a\right)^{\frac{3}{2}}}{ax}+\frac{2c\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}\right)}{a}\right)}{d^3}-\frac{e^3\left(\sqrt{c\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)\right)}{d^4}-\cdots$

input
```
int((c*x^2+a)^(1/2)/x^4/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(c*x^2+a)^(1/2)*(6*a*e^2*x^2+2*c*d^2*x^2-3*a*d*e*x+2*a*d^2)/d^3/x^3/a
+1/2*e/d^3*(-(-2*a*e^2-c*d^2)/d/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))
/x)-2*(a*e^2+c*d^2)/d/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/
e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*
e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

3.324.5 Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 824, normalized size of antiderivative = 4.31

$$\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx$$
$$= \left[\frac{6\sqrt{cd^2+ae^2}ae^2x^3\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+3\,(cd^2e+2\,ae^3)\sqrt{a}x^3}{12\,ad^4x^3}\right.$$
$$-\frac{12\sqrt{-cd^2-ae^2}ae^2x^3\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-3\,(cd^2e+2\,ae^3)\sqrt{a}x^3\log\left(-\frac{cx^2+2\,\sqrt{cx^2+a}\sqrt{a}+2}{x^2}\right.}{12\,ad^4x^3}$$
$$-\frac{6\sqrt{-cd^2-ae^2}ae^2x^3\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)+3\,(cd^2e+2\,ae^3)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right)-(3}{6\,ad^4x^3}$$

input
```
integrate((c*x^2+a)^(1/2)/x^4/(e*x+d),x, algorithm="fricas")
```

output
```
[1/12*(6*sqrt(c*d^2 + a*e^2)*a*e^2*x^3*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*
e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt
(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 3*(c*d^2*e + 2*a*e^3)*sqrt(a)*x^
3*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*a*d^2*e*x - 2
*a*d^3 - 2*(c*d^3 + 3*a*d*e^2)*x^2)*sqrt(c*x^2 + a))/(a*d^4*x^3), -1/12*(1
2*sqrt(-c*d^2 - a*e^2)*a*e^2*x^3*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)
*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(c*d^2
*e + 2*a*e^3)*sqrt(a)*x^3*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x
^2) - 2*(3*a*d^2*e*x - 2*a*d^3 - 2*(c*d^3 + 3*a*d*e^2)*x^2)*sqrt(c*x^2 + a
))/(a*d^4*x^3), 1/6*(3*sqrt(c*d^2 + a*e^2)*a*e^2*x^3*log((2*a*c*d*e*x - a*
c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d
*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(c*d^2*e + 2*a*e
^3)*sqrt(-a)*x^3*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (3*a*d^2*e*x - 2*a*d^3
 - 2*(c*d^3 + 3*a*d*e^2)*x^2)*sqrt(c*x^2 + a))/(a*d^4*x^3), -1/6*(6*sqrt(-
c*d^2 - a*e^2)*a*e^2*x^3*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*
x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + 3*(c*d^2*e + 2*a
*e^3)*sqrt(-a)*x^3*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (3*a*d^2*e*x - 2*a*d
^3 - 2*(c*d^3 + 3*a*d*e^2)*x^2)*sqrt(c*x^2 + a))/(a*d^4*x^3)]
```

3.324.6 Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx = \int \frac{\sqrt{a+cx^2}}{x^4\,(d+ex)}\,dx$$

input
```
integrate((c*x**2+a)**(1/2)/x**4/(e*x+d),x)
```

output
```
Integral(sqrt(a + c*x**2)/(x**4*(d + e*x)), x)
```

3.324.7 Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx = \int \frac{\sqrt{cx^2+a}}{(ex+d)x^4}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/x^4/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)/((e*x + d)*x^4), x)
```

3.324.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx$$
$$= \frac{2\left(cd^2e^2+ae^4\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}\,d^4} - \frac{\left(cd^2e+2\,ae^3\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,d^4}$$
$$- \frac{3\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^5 cde - 6\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^4 c^{\frac{3}{2}}d^2 - 6\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^4 a\sqrt{c}e^2 + 12\left(\sqrt{c}x-\sqrt{cx\right.}}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2-a\right)^3 d^3}$$

input
```
integrate((c*x^2+a)^(1/2)/x^4/(e*x+d),x, algorithm="giac")
```

output
```
2*(c*d^2*e^2 + a*e^4)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d
)/sqrt(-c*d^2 - a*e^2))/(sqrt(-c*d^2 - a*e^2)*d^4) - (c*d^2*e + 2*a*e^3)*a
rctan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*d^4) - 1/3*(3*(sq
rt(c)*x - sqrt(c*x^2 + a))^5*c*d*e - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(
3/2)*d^2 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*sqrt(c)*e^2 + 12*(sqrt(c)*x
 - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*e^2 - 3*(sqrt(c)*x - sqrt(c*x^2 + a))*a^
2*c*d*e - 2*a^2*c^(3/2)*d^2 - 6*a^3*sqrt(c)*e^2)/(((sqrt(c)*x - sqrt(c*x^2
 + a))^2 - a)^3*d^3)
```

3.324.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx = \int \frac{\sqrt{c\,x^2+a}}{x^4\,\left(d+e\,x\right)}\,dx$$

input
```
int((a + c*x^2)^(1/2)/(x^4*(d + e*x)),x)
```

output
```
int((a + c*x^2)^(1/2)/(x^4*(d + e*x)), x)
```

3.325 $\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)} dx$

3.325.1 Optimal result

Integrand size = 22, antiderivative size = 274

$$\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)} dx = -\frac{\sqrt{a+cx^2}}{4dx^4} - \frac{c\sqrt{a+cx^2}}{8adx^2} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e(a+cx^2)^{3/2}}{3ad^2x^3} + \frac{e^3\sqrt{cd^2+ae^2}\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^5} + \frac{c^2\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{8a^{3/2}d} - \frac{ce^2\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d^3} - \frac{\sqrt{a}e^4\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^5}$$

output
```
1/3*e*(c*x^2+a)^(3/2)/a/d^2/x^3+1/8*c^2*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a
^(3/2)/d-1/2*c*e^2*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^3/a^(1/2)-e^4*arctan
h((c*x^2+a)^(1/2)/a^(1/2))*a^(1/2)/d^5+e^3*arctanh((-c*d*x+a*e)/(a*e^2+c*d
^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/d^5-1/4*(c*x^2+a)^(1/2)/d/x
^4-1/8*c*(c*x^2+a)^(1/2)/a/d/x^2-1/2*e^2*(c*x^2+a)^(1/2)/d^3/x^2+e^3*(c*x^
2+a)^(1/2)/d^4/x
```

3.325.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx$$
$$=\frac{\sqrt{a}\Big(d\sqrt{a+cx^2}(cd^2x^2(-3d+8ex)+a(-6d^3+8d^2ex-12de^2x^2+24e^3x^3))-48ae^3\sqrt{-cd^2-ae^2}x^4\,\text{arcta}}{24a^{3/2}d^5x^4}$$

input
```
Integrate[Sqrt[a + c*x^2]/(x^5*(d + e*x)),x]
```

output
```
(Sqrt[a]*(d*Sqrt[a + c*x^2]*(c*d^2*x^2*(-3*d + 8*e*x) + a*(-6*d^3 + 8*d^2*
e*x - 12*d*e^2*x^2 + 24*e^3*x^3)) - 48*a*e^3*Sqrt[-(c*d^2) - a*e^2]*x^4*Ar
cTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]]) - 6*
(c^2*d^4 - 4*a*c*d^2*e^2 - 8*a^2*e^4)*x^4*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*
x^2])/Sqrt[a]])/(24*a^(3/2)*d^5*x^4)
```

3.325.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx$$
$$\Big\downarrow \text{617}$$
$$\int \left(-\frac{e^5\sqrt{a+cx^2}}{d^5(d+ex)}+\frac{e^4\sqrt{a+cx^2}}{d^5x}-\frac{e^3\sqrt{a+cx^2}}{d^4x^2}+\frac{e^2\sqrt{a+cx^2}}{d^3x^3}-\frac{e\sqrt{a+cx^2}}{d^2x^4}+\frac{\sqrt{a+cx^2}}{dx^5}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{c^2\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{8a^{3/2}d}-\frac{\sqrt{a}e^4\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^5}-\frac{ce^2\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d^3}+\frac{e^3\sqrt{ae^2+cd^2}\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^5}+\frac{e^3\sqrt{a+cx^2}}{d^4x}-\frac{e^2\sqrt{a+cx^2}}{2d^3x^2}+\frac{e\left(a+cx^2\right)^{3/2}}{3ad^2x^3}-\frac{c\sqrt{a+cx^2}}{8adx^2}-\frac{\sqrt{a+cx^2}}{4dx^4}$$

input
```
Int[Sqrt[a + c*x^2]/(x^5*(d + e*x)),x]
```

output
```
-1/4*Sqrt[a + c*x^2]/(d*x^4) - (c*Sqrt[a + c*x^2])/(8*a*d*x^2) - (e^2*Sqrt
[a + c*x^2])/(2*d^3*x^2) + (e^3*Sqrt[a + c*x^2])/(d^4*x) + (e*(a + c*x^2)^
(3/2))/(3*a*d^2*x^3) + (e^3*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqr
t[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^5 + (c^2*ArcTanh[Sqrt[a + c*x^2]/Sqr
t[a]])/(8*a^(3/2)*d) - (c*e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*Sqrt[a]
*d^3) - (Sqrt[a]*e^4*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^5
```

3.325.3.1 Defintions of rubi rules used

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.325.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\sqrt{c\,x^2+a}\,\left(-24a\,e^3x^3-8c\,d^2e\,x^3+12ad\,e^2x^2+3c\,d^3x^2-8a\,d^2ex+6a\,d^3\right)}{24d^4x^4a}-\frac{-\frac{\left(-8a^2e^4-4ac\,d^2e^2+c^2d^4\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{d\sqrt{a}}-\frac{8a\,e\cdots}{\cdots}}{\cdots}$
default	$\frac{-\frac{\left(c\,x^2+a\right)^{\frac{3}{2}}}{4a\,x^4}-\frac{c\left(-\frac{\left(c\,x^2+a\right)^{\frac{3}{2}}}{2a\,x^2}+\frac{c\left(\sqrt{c\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)\right)}{2a}\right)}{4a}}{d}+\frac{e^4\left(\sqrt{c\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)\right)}{d^5}+\frac{e^2\left(-\cdots\right)}{\cdots}$

input
```
int((c*x^2+a)^(1/2)/x^5/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(c*x^2+a)^(1/2)*(-24*a*e^3*x^3-8*c*d^2*e*x^3+12*a*d*e^2*x^2+3*c*d^3*
x^2-8*a*d^2*e*x+6*a*d^3)/d^4/x^4/a-1/8/d^4/a*(-(-8*a^2*e^4-4*a*c*d^2*e^2+c
^2*d^4)/d/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-8*a*e^2*(a*e^2+c*d
^2)/d/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*
((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^
(1/2))/(x+d/e)))
```

3.325.5 Fricas [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 1007, normalized size of antiderivative = 3.68

$$\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx$$
$$= \left[\frac{24\sqrt{cd^2+ae^2}a^2e^3x^4\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2+2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-3\left(c^2d^4-4\,acd^2e^2-\right.}{}\right.$$

input
```
integrate((c*x^2+a)^(1/2)/x^5/(e*x+d),x, algorithm="fricas")
```

output
```
[1/48*(24*sqrt(c*d^2 + a*e^2)*a^2*e^3*x^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a
^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(c^2*d^4 - 4*a*c*d^2*e^2 -
8*a^2*e^4)*sqrt(a)*x^4*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2)
 + 2*(8*a^2*d^3*e*x - 6*a^2*d^4 + 8*(a*c*d^3*e + 3*a^2*d*e^3)*x^3 - 3*(a*c
*d^4 + 4*a^2*d^2*e^2)*x^2)*sqrt(c*x^2 + a))/(a^2*d^5*x^4), 1/48*(48*sqrt(-
c*d^2 - a*e^2)*a^2*e^3*x^4*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(
c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(c^2*d^4 - 4
*a*c*d^2*e^2 - 8*a^2*e^4)*sqrt(a)*x^4*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt
(a) + 2*a)/x^2) + 2*(8*a^2*d^3*e*x - 6*a^2*d^4 + 8*(a*c*d^3*e + 3*a^2*d*e^
3)*x^3 - 3*(a*c*d^4 + 4*a^2*d^2*e^2)*x^2)*sqrt(c*x^2 + a))/(a^2*d^5*x^4),
1/24*(12*sqrt(c*d^2 + a*e^2)*a^2*e^3*x^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^
2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sq
rt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(c^2*d^4 - 4*a*c*d^2*e^2 - 8
*a^2*e^4)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (8*a^2*d^3*e*x -
 6*a^2*d^4 + 8*(a*c*d^3*e + 3*a^2*d*e^3)*x^3 - 3*(a*c*d^4 + 4*a^2*d^2*e^2)
*x^2)*sqrt(c*x^2 + a))/(a^2*d^5*x^4), 1/24*(24*sqrt(-c*d^2 - a*e^2)*a^2*e^
3*x^4*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 +
 a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(c^2*d^4 - 4*a*c*d^2*e^2 - 8*a^2*
e^4)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (8*a^2*d^3*e*x - 6...
```

3.325.6 Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx = \int \frac{\sqrt{a+cx^2}}{x^5\,(d+ex)}\,dx$$

input
```
integrate((c*x**2+a)**(1/2)/x**5/(e*x+d),x)
```

output
```
Integral(sqrt(a + c*x**2)/(x**5*(d + e*x)), x)
```

3.325.7 Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx = \int \frac{\sqrt{cx^2+a}}{(ex+d)x^5}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/x^5/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)/((e*x + d)*x^5), x)
```

3.325.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(229) = 458$.

Time = 0.30 (sec) , antiderivative size = 605, normalized size of antiderivative = 2.21

$$\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx = -\frac{2\,(cd^2e^3+ae^5)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}\,d^5} - \frac{(c^2d^4-4\,acd^2e^2-8\,a^2e^4)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}\,ad^5} + \frac{3\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^7c^2d^3+12\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^7acde^2-24\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^6ac^{\frac{3}{2}}d^2e-24\left(\sqrt{c}x-\right.}{}$$

input
```
integrate((c*x^2+a)^(1/2)/x^5/(e*x+d),x, algorithm="giac")
```

output
```
-2*(c*d^2*e^3 + a*e^5)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*
d)/sqrt(-c*d^2 - a*e^2))/(sqrt(-c*d^2 - a*e^2)*d^5) - 1/4*(c^2*d^4 - 4*a*c
*d^2*e^2 - 8*a^2*e^4)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqr
t(-a)*a*d^5) + 1/12*(3*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^2*d^3 + 12*(sqrt(
c)*x - sqrt(c*x^2 + a))^7*a*c*d*e^2 - 24*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a
*c^(3/2)*d^2*e - 24*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*sqrt(c)*e^3 + 21*(
sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2*d^3 - 12*(sqrt(c)*x - sqrt(c*x^2 + a)
)^5*a^2*c*d*e^2 + 24*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*d^2*e + 7
2*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*sqrt(c)*e^3 + 21*(sqrt(c)*x - sqrt(c
*x^2 + a))^3*a^2*c^2*d^3 - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c*d*e^2
- 8*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(3/2)*d^2*e - 72*(sqrt(c)*x - sq
rt(c*x^2 + a))^2*a^4*sqrt(c)*e^3 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2
*d^3 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c*d*e^2 + 8*a^4*c^(3/2)*d^2*e
+ 24*a^5*sqrt(c)*e^3)/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)^4*a*d^4)
```

3.325.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx = \int \frac{\sqrt{c\,x^2+a}}{x^5\,\left(d+e\,x\right)}\,dx$$

input
```
int((a + c*x^2)^(1/2)/(x^5*(d + e*x)),x)
```

output
```
int((a + c*x^2)^(1/2)/(x^5*(d + e*x)), x)
```

3.326 $\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx$

3.326.1 Optimal result

Integrand size = 22, antiderivative size = 195

$$\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{(11cd^2-4ae^2)\sqrt{a+cx^2}}{6c^2e^3} - \frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} - \frac{d(2cd^2-ae^2)\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4} - \frac{d^4\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^4\sqrt{cd^2+ae^2}}$$

output
```
-1/2*d*(-a*e^2+2*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^4-d^4
*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e^4/(a*e^2+c*d^
2)^(1/2)+1/6*(-4*a*e^2+11*c*d^2)*(c*x^2+a)^(1/2)/c^2/e^3-7/6*d*(e*x+d)*(c*
x^2+a)^(1/2)/c/e^3+1/3*(e*x+d)^2*(c*x^2+a)^(1/2)/c/e^3
```

3.326.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{\frac{e\sqrt{a+cx^2}(-4ae^2+c(6d^2-3dex+2e^2x^2))}{c^2} - \frac{12d^4\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}} + \frac{3d(2cd^2-ae^2)\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{c^{3/2}}}{6e^4}$$

input
```
Integrate[x^4/((d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
((e*Sqrt[a + c*x^2]*(-4*a*e^2 + c*(6*d^2 - 3*d*e*x + 2*e^2*x^2)))/c^2 - (1
2*d^4*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2
]])/Sqrt[-(c*d^2) - a*e^2] + (3*d*(2*c*d^2 - a*e^2)*Log[-(Sqrt[c]*x) + Sqr
t[a + c*x^2]])/c^(3/2))/(6*e^4)
```

3.326.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {604, 25, 2185, 25, 2185, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+cx^2}(d+ex)}dx$$

$$\Big\downarrow \text{604}$$

$$\frac{\int -\frac{7cde^3x^3+e^2(5cd^2+2ae^2)x^2+de(cd^2+4ae^2)x+2ad^2e^2}{(d+ex)\sqrt{cx^2+a}}dx}{3ce^4}+\frac{\sqrt{a+cx^2}(d+ex)^2}{3ce^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a+cx^2}(d+ex)^2}{3ce^3}-\frac{\int \frac{7cde^3x^3+e^2(5cd^2+2ae^2)x^2+de(cd^2+4ae^2)x+2ad^2e^2}{(d+ex)\sqrt{cx^2+a}}dx}{3ce^4}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\sqrt{a+cx^2}(d+ex)^2}{3ce^3}-\frac{\frac{\int -\frac{3acd^2e^5+c\left(11cd^2-4ae^2\right)x^2e^5+cd\left(5cd^2-ae^2\right)xe^4}{(d+ex)\sqrt{cx^2+a}}dx}{2ce^3}+\frac{7}{2}de\sqrt{a+cx^2}(d+ex)}{3ce^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a+cx^2}(d+ex)^2}{3ce^3}-\frac{\frac{7}{2}de\sqrt{a+cx^2}(d+ex)-\frac{\int \frac{3acd^2e^5+c\left(11cd^2-4ae^2\right)x^2e^5+cd\left(5cd^2-ae^2\right)xe^4}{(d+ex)\sqrt{cx^2+a}}dx}{2ce^3}}{3ce^4}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\sqrt{a+cx^2}(d+ex)^2}{3ce^3}-\frac{\frac{7}{2}de\sqrt{a+cx^2}(d+ex)-\frac{\frac{\int \frac{3c^2de^6\left(ade-\left(2cd^2-ae^2\right)x\right)}{(d+ex)\sqrt{cx^2+a}}dx}{ce^2}+e^4\sqrt{a+cx^2}\left(11cd^2-4ae^2\right)}{2ce^3}}{3ce^4}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a+cx^2}(d+ex)^2}{3ce^3}-\frac{\frac{7}{2}de\sqrt{a+cx^2}(d+ex)-\frac{3cde^4\int\frac{ade-\left(2cd^2-ae^2\right)x}{(d+ex)\sqrt{cx^2+a}}dx+e^4\sqrt{a+cx^2}\left(11cd^2-4ae^2\right)}{2ce^3}}{3ce^4}$$

$$\Big\downarrow \; 719$$

$$\frac{\sqrt{a+cx^2}(d+ex)^2}{3ce^3}-\frac{\frac{7}{2}de\sqrt{a+cx^2}(d+ex)-\frac{3cde^4\left(\frac{2cd^3\int\frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{\left(2cd^2-ae^2\right)\int\frac{1}{\sqrt{cx^2+a}}dx}{e}\right)+e^4\sqrt{a+cx^2}\left(11cd^2-4ae^2\right)}{2ce^3}}{3ce^4}$$

$$\Big\downarrow \; 224$$

$$\frac{\sqrt{a+cx^2}(d+ex)^2}{3ce^3}-\frac{\frac{7}{2}de\sqrt{a+cx^2}(d+ex)-\frac{3cde^4\left(\frac{2cd^3\int\frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{\left(2cd^2-ae^2\right)\int\frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{e}\right)+e^4\sqrt{a+cx^2}\left(11cd^2-4ae^2\right)}{2ce^3}}{3ce^4}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{a+cx^2}(d+ex)^2}{3ce^3}-\frac{\frac{7}{2}de\sqrt{a+cx^2}(d+ex)-\frac{3cde^4\left(\frac{2cd^3\int\frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(2cd^2-ae^2\right)}{\sqrt{c}e}\right)+e^4\sqrt{a+cx^2}\left(11cd^2-4ae^2\right)}{2ce^3}}{3ce^4}$$

$$\Big\downarrow \; 488$$

$$\frac{\sqrt{a+cx^2}(d+ex)^2}{3ce^3}-\frac{\frac{7}{2}de\sqrt{a+cx^2}(d+ex)-\frac{3cde^4\left(-\frac{2cd^3\int\frac{1}{cd^2+ae^2-\frac{(ae-cdx)^2}{cx^2+a}}d\frac{ae-cdx}{\sqrt{cx^2+a}}}{e}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(2cd^2-ae^2\right)}{\sqrt{c}e}\right)+e^4\sqrt{a+cx^2}\left(11cd^2-4ae^2\right)}{2ce^3}}{3ce^4}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{a+cx^2}(d+ex)^2}{3ce^3}-\frac{\frac{7}{2}de\sqrt{a+cx^2}(d+ex)-\frac{3cde^4\left(-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(2cd^2-ae^2\right)}{\sqrt{c}e}-\frac{2cd^3\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\sqrt{ae^2+cd^2}}\right)+e^4\sqrt{a+cx^2}\left(11cd^2-4ae^2\right)}{2ce^3}}{3ce^4}$$

3.326. $\int\frac{x^4}{(d+ex)\sqrt{a+cx^2}}dx$

input
```
Int[x^4/((d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
((d + e*x)^2*Sqrt[a + c*x^2])/(3*c*e^3) - ((7*d*e*(d + e*x)*Sqrt[a + c*x^2
])/2 - (e^4*(11*c*d^2 - 4*a*e^2)*Sqrt[a + c*x^2] + 3*c*d*e^4*(-(((2*c*d^2
- a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*e)) - (2*c*d^3*Arc
Tanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e*Sqrt[c*d^2 +
 a*e^2])))/(2*c*e^3))/(3*c*e^4)
```

3.326.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 604
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(b*d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*
x^2)^p*ExpandToSum[b*d^m*(m + n + 2*p + 1)*x^m - b*(m + n + 2*p + 1)*(c + d
*x)^m - (c + d*x)^(m - 2)*(a*d^2*(m + n - 1) - b*c^2*(m + n + 2*p + 1) - 2*
b*c*d*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && IGtQ[m,
 1] && NeQ[m + n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.326.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.14

method	result
risch	$-\frac{(-2ce^2x^2+3cdex+4e^2a-6cd^2)\sqrt{cx^2+a}}{6c^2e^3}+\frac{d\left(\frac{(e^2a-2cd^2)\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{e\sqrt{c}}-\frac{2cd^3\ln\left(\frac{\frac{2e^2a+2cd^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+cd^2}{e^2}}\sqrt{(}}{x+\frac{d}{e}}\right)}{e^2\sqrt{\frac{e^2a+cd^2}{e^2}}}\right)}{2e^3c}$
default	$\frac{\frac{x^2\sqrt{cx^2+a}}{3c}-\frac{2a\sqrt{cx^2+a}}{3c^2}}{e}+\frac{d^2\sqrt{cx^2+a}}{e^3c}-\frac{d^3\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{e^4\sqrt{c}}-\frac{d\left(\frac{x\sqrt{cx^2+a}}{2c}-\frac{a\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{2c^{\frac{3}{2}}}\right)}{e^2}-\frac{d^4\ln\left(\frac{\frac{2e^2a+2cd^2}{e^2}}{}\right)}{}$

input
```
int(x^4/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-2*c*e^2*x^2+3*c*d*e*x+4*a*e^2-6*c*d^2)*(c*x^2+a)^(1/2)/c^2/e^3+1/2*
d/e^3/c*((a*e^2-2*c*d^2)/e*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-2*c*d^3/e
^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a
*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/
2))/(x+d/e)))
```

3.326.5 Fricas [A] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 1060, normalized size of antiderivative = 5.44

$$\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx$$
$$= \left[\frac{6\sqrt{cd^2+ae^2}c^2d^4\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-3\left(2\,c^2d^5+acd^3e^2-a^2d\right.}{} \right.$$
$$-\frac{12\sqrt{-cd^2-ae^2}c^2d^4\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)+3\left(2\,c^2d^5+acd^3e^2-a^2de^4\right)\sqrt{c}\log\left(-2\,cx^2-2\right.}{12\left(c^3d^2\right.}$$
$$-\frac{6\sqrt{-cd^2-ae^2}c^2d^4\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-3\left(2\,c^2d^5+acd^3e^2-a^2de^4\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right.}{6\left(c^3d^2e^4+ac^2e^6\right.}$$

input
```
integrate(x^4/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(6*sqrt(c*d^2 + a*e^2)*c^2*d^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^
2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c
*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(2*c^2*d^5 + a*c*d^3*e^2 - a^2*d
*e^4)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(6*c^2*d
^4*e + 2*a*c*d^2*e^3 - 4*a^2*e^5 + 2*(c^2*d^2*e^3 + a*c*e^5)*x^2 - 3*(c^2*
d^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^4 + a*c^2*e^6), -1/12*
(12*sqrt(-c*d^2 - a*e^2)*c^2*d^4*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)
*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + 3*(2*c^2
*d^5 + a*c*d^3*e^2 - a^2*d*e^4)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*s
qrt(c)*x - a) - 2*(6*c^2*d^4*e + 2*a*c*d^2*e^3 - 4*a^2*e^5 + 2*(c^2*d^2*e^
3 + a*c*e^5)*x^2 - 3*(c^2*d^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^
2*e^4 + a*c^2*e^6), 1/6*(3*sqrt(c*d^2 + a*e^2)*c^2*d^4*log((2*a*c*d*e*x -
a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c
*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 3*(2*c^2*d^5 + a
*c*d^3*e^2 - a^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (6*c
^2*d^4*e + 2*a*c*d^2*e^3 - 4*a^2*e^5 + 2*(c^2*d^2*e^3 + a*c*e^5)*x^2 - 3*(
c^2*d^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^4 + a*c^2*e^6), -1
/6*(6*sqrt(-c*d^2 - a*e^2)*c^2*d^4*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*
e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(2*c
^2*d^5 + a*c*d^3*e^2 - a^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2...
```

3.326.6 Sympy [F]

$$\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{x^4}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

input
```
integrate(x**4/(e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(x**4/(sqrt(a + c*x**2)*(d + e*x)), x)
```

3.326.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.326.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.326.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{x^4}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

input

```
int(x^4/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

output

```
int(x^4/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.327 $\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx$

3.327.1 Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx = -\frac{3d\sqrt{a+cx^2}}{2ce^2}+\frac{(d+ex)\sqrt{a+cx^2}}{2ce^2}+\frac{(2cd^2-ae^2)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^3}+\frac{d^3\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^3\sqrt{cd^2+ae^2}}$$

output
```
1/2*(-a*e^2+2*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^3+d^3*ar
ctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e^3/(a*e^2+c*d^2)^
(1/2)-3/2*d*(c*x^2+a)^(1/2)/c/e^2+1/2*(e*x+d)*(c*x^2+a)^(1/2)/c/e^2
```

3.327.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{\frac{e(-2d+ex)\sqrt{a+cx^2}}{c}+\frac{4d^3\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}}+\frac{(-2cd^2+ae^2)\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{c^{3/2}}}{2e^3}$$

input
```
Integrate[x^3/((d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
((e*(-2*d + e*x)*Sqrt[a + c*x^2])/c + (4*d^3*ArcTan[(Sqrt[c]*(d + e*x) - e
*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/Sqrt[-(c*d^2) - a*e^2] + ((-2*c
*d^2 + a*e^2)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/c^(3/2))/(2*e^3)
```

3.327.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {604, 25, 2185, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+cx^2}(d+ex)}dx$$

$$\Big\downarrow \text{604}$$

$$\frac{\int -\frac{3cdx^2e^2+ade^2+(cd^2+ae^2)xe}{(d+ex)\sqrt{cx^2+a}}dx}{2ce^3}+\frac{\sqrt{a+cx^2}(d+ex)}{2ce^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a+cx^2}(d+ex)}{2ce^2}-\frac{\int \frac{3cdx^2e^2+ade^2+(cd^2+ae^2)xe}{(d+ex)\sqrt{cx^2+a}}dx}{2ce^3}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\sqrt{a+cx^2}(d+ex)}{2ce^2}-\frac{\frac{\int \frac{ce^3\left(ade-\left(2cd^2-ae^2\right)x\right)}{(d+ex)\sqrt{cx^2+a}}dx}{ce^2}+3de\sqrt{a+cx^2}}{2ce^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a+cx^2}(d+ex)}{2ce^2}-\frac{e\int \frac{ade-(2cd^2-ae^2)x}{(d+ex)\sqrt{cx^2+a}}dx+3de\sqrt{a+cx^2}}{2ce^3}$$

$$\Big\downarrow \text{719}$$

$$\frac{\sqrt{a+cx^2}(d+ex)}{2ce^2}-\frac{e\left(\frac{2cd^3\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{(2cd^2-ae^2)\int \frac{1}{\sqrt{cx^2+a}}dx}{e}\right)+3de\sqrt{a+cx^2}}{2ce^3}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{a+cx^2}(d+ex)}{2ce^2}-\frac{e\left(\frac{2cd^3\int\frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{(2cd^2-ae^2)\int\frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{e}\right)+3de\sqrt{a+cx^2}}{2ce^3}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{a+cx^2}(d+ex)}{2ce^2}-\frac{e\left(\frac{2cd^3\int\frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(2cd^2-ae^2)}{\sqrt{c}e}\right)+3de\sqrt{a+cx^2}}{2ce^3}$$

$$\Big\downarrow \; 488$$

$$\frac{\sqrt{a+cx^2}(d+ex)}{2ce^2}-\frac{e\left(-\frac{2cd^3\int\frac{1}{cd^2+ae^2-\frac{(ae-cdx)^2}{cx^2+a}}d\frac{ae-cdx}{\sqrt{cx^2+a}}}{e}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(2cd^2-ae^2)}{\sqrt{c}e}\right)+3de\sqrt{a+cx^2}}{2ce^3}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{a+cx^2}(d+ex)}{2ce^2}-\frac{e\left(-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(2cd^2-ae^2)}{\sqrt{c}e}-\frac{2cd^3\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\sqrt{ae^2+cd^2}}\right)+3de\sqrt{a+cx^2}}{2ce^3}$$

input
```
Int[x^3/((d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
((d + e*x)*Sqrt[a + c*x^2])/(2*c*e^2) - (3*d*e*Sqrt[a + c*x^2] + e*(-((((2*
c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*e)) - (2*c*d
^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e*Sqrt[c
*d^2 + a*e^2])))/(2*c*e^3)
```

3.327.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 604
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(b*d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*
x^2)^p*ExpandToSum[b*d^m*(m + n + 2*p + 1)*x^m - b*(m + n + 2*p + 1)*(c + d
*x)^m - (c + d*x)^(m - 2)*(a*d^2*(m + n - 1) - b*c^2*(m + n + 2*p + 1) - 2*
b*c*d*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && IGtQ[m,
 1] && NeQ[m + n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.327. $\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx$

3.327.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{(-ex+2d)\sqrt{c\,x^2+a}}{2c\,e^2}-\frac{\frac{\left(e^2a-2c\,d^2\right)\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e\sqrt{c}}-\frac{2c\,d^3\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a}{}}}{x+\frac{d}{e}}\right)}{e^2\sqrt{\frac{e^2a+c\,d^2}{e^2}}}}{2e^2c}$
default	$\frac{\frac{x\sqrt{c\,x^2+a}}{2c}-\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}}}{e}+\frac{d^2\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e^3\sqrt{c}}-\frac{d\sqrt{c\,x^2+a}}{c\,e^2}+\frac{d^3\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2}}{x+\frac{d}{e}}\right)}{e^4\sqrt{\frac{e^2a+c\,d^2}{e^2}}}$

input
```
int(x^3/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e*x+2*d)*(c*x^2+a)^(1/2)/c/e^2-1/2/e^2/c*((a*e^2-2*c*d^2)/e*ln(x*c^
(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-2*c*d^3/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2
*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*
c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

3.327.5 Fricas [A] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 924, normalized size of antiderivative = 6.08

$$\int\frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx$$
$$=\left[\frac{2\sqrt{cd^2+ae^2}c^2d^3\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2+2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-\left(2\,c^2d^4+acd^2e^2-a^2e^4\right.}{4\left(c^3d^2e^3+ac^2\right.}\right.$$

input
```
integrate(x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(c*d^2 + a*e^2)*c^2*d^3*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2
 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*
x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - (2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)
*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(2*c^2*d^3*e
+ 2*a*c*d*e^3 - (c^2*d^2*e^2 + a*c*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^3 +
 a*c^2*e^5), 1/4*(4*sqrt(-c*d^2 - a*e^2)*c^2*d^3*arctan(sqrt(-c*d^2 - a*e^
2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*
x^2)) - (2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(
c*x^2 + a)*sqrt(c)*x - a) - 2*(2*c^2*d^3*e + 2*a*c*d*e^3 - (c^2*d^2*e^2 +
a*c*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^3 + a*c^2*e^5), 1/2*(sqrt(c*d^2 +
a*e^2)*c^2*d^3*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e
^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 +
2*d*e*x + d^2)) - (2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)*sqrt(-c)*arctan(sqrt
(-c)*x/sqrt(c*x^2 + a)) - (2*c^2*d^3*e + 2*a*c*d*e^3 - (c^2*d^2*e^2 + a*c*
e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^3 + a*c^2*e^5), 1/2*(2*sqrt(-c*d^2 - a
*e^2)*c^2*d^3*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a
*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (2*c^2*d^4 + a*c*d^2*e^2 -
a^2*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (2*c^2*d^3*e + 2*a*
c*d*e^3 - (c^2*d^2*e^2 + a*c*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^3 + a*c^2
*e^5)]
```

3.327.6 Sympy [F]

$$\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

input
```
integrate(x**3/(e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(x**3/(sqrt(a + c*x**2)*(d + e*x)), x)
```

3.327.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.327.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.327.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{x^3}{\sqrt{c\,x^2+a}\,(d+e\,x)}\,dx$$

input
```
int(x^3/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int(x^3/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.328 $\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx$

3.328.1 Optimal result

Integrand size = 22, antiderivative size = 109

$$\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx=\frac{\sqrt{a+cx^2}}{ce}-\frac{d\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^2}-\frac{d^2\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^2\sqrt{cd^2+ae^2}}$$

output
```
-d*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/e^2/c^(1/2)-d^2*arctanh((-c*d*x+a*e)
/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e^2/(a*e^2+c*d^2)^(1/2)+(c*x^2+a)^(1
/2)/c/e
```

3.328.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx=\frac{\frac{e\sqrt{a+cx^2}}{c}-\frac{2d^2\arctan\left(\frac{\sqrt{-cd^2-ae^2}x}{\sqrt{a}(d+ex)-d\sqrt{a+cx^2}}\right)}{\sqrt{-cd^2-ae^2}}+\frac{2d\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a}-\sqrt{a+cx^2}}\right)}{\sqrt{c}}}{e^2}$$

input
```
Integrate[x^2/((d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
((e*Sqrt[a + c*x^2])/c - (2*d^2*ArcTan[(Sqrt[-(c*d^2) - a*e^2]*x)/(Sqrt[a]
*(d + e*x) - d*Sqrt[a + c*x^2])])/Sqrt[-(c*d^2) - a*e^2] + (2*d*ArcTanh[(S
qrt[c]*x)/(Sqrt[a] - Sqrt[a + c*x^2])])/Sqrt[c])/e^2
```

3.328.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {604, 25, 27, 605, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+cx^2}(d+ex)}dx$$

$$\Big\downarrow \text{604}$$

$$\frac{\int -\frac{cdex}{(d+ex)\sqrt{cx^2+a}}dx}{ce^2}+\frac{\sqrt{a+cx^2}}{ce}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a+cx^2}}{ce}-\frac{\int \frac{cdex}{(d+ex)\sqrt{cx^2+a}}dx}{ce^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a+cx^2}}{ce}-\frac{d\int \frac{x}{(d+ex)\sqrt{cx^2+a}}dx}{e}$$

$$\Big\downarrow \text{605}$$

$$\frac{\sqrt{a+cx^2}}{ce}-\frac{d\left(\frac{\int \frac{1}{\sqrt{cx^2+a}}dx}{e}-\frac{d\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}\right)}{e}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{a+cx^2}}{ce}-\frac{d\left(\frac{\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{e}-\frac{d\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}\right)}{e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a+cx^2}}{ce}-\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e}-\frac{d\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}\right)}{e}$$

$$\Big\downarrow \text{488}$$

$$\frac{\sqrt{a+cx^2}}{ce}-\frac{d\left(\frac{d\int \frac{1}{cd^2+ae^2-\frac{(ae-cdx)^2}{cx^2+a}}d\frac{ae-cdx}{\sqrt{cx^2+a}}}{e}+\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e}\right)}{e}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{a+cx^2}}{ce}-\frac{d\left(\frac{d\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\sqrt{ae^2+cd^2}}+\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e}\right)}{e}$$

input
```
Int[x^2/((d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
Sqrt[a + c*x^2]/(c*e) - (d*(ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*
e) + (d*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e*S
qrt[c*d^2 + a*e^2])))/e
```

3.328.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

3.328. $\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx$

rule 604
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(b*d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*
x^2)^p*ExpandToSum[b*d^m*(m + n + 2*p + 1)*x^m - b*(m + n + 2*p + 1)*(c + d
*x)^m - (c + d*x)^(m - 2)*(a*d^2*(m + n - 1) - b*c^2*(m + n + 2*p + 1) - 2*
b*c*d*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && IGtQ[m,
 1] && NeQ[m + n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 605
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[1/d   Int[x^(m - 1)*(a + b*x^2)^p, x], x] - Simp[c/d   Int[x^(m - 1
)*((a + b*x^2)^p/(c + d*x)), x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m,
 0] && LtQ[-1, p, 0]
```

3.328.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.58

method	result	size
default	$\frac{\sqrt{c\,x^{2}+a}}{c e}-\frac{d \ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{e^{2}\sqrt{c}}-\frac{d^{2} \ln\left(\frac{\frac{2e^{2}a+2c\,d^{2}}{e^{2}}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^{2}a+c\,d^{2}}{e^{2}}}\,\sqrt{\left(x+\frac{d}{e}\right)^{2}c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^{2}a+c\,d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{e^{3}\sqrt{\frac{e^{2}a+c\,d^{2}}{e^{2}}}}$	172
risch	$\frac{\sqrt{c\,x^{2}+a}}{c e}-\frac{d \ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{e^{2}\sqrt{c}}-\frac{d^{2} \ln\left(\frac{\frac{2e^{2}a+2c\,d^{2}}{e^{2}}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^{2}a+c\,d^{2}}{e^{2}}}\,\sqrt{\left(x+\frac{d}{e}\right)^{2}c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^{2}a+c\,d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{e^{3}\sqrt{\frac{e^{2}a+c\,d^{2}}{e^{2}}}}$	172

input
```
int(x^2/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+a)^(1/2)/c/e-d/e^2*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-d^2/e^3/((
a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+
c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(
x+d/e))
```

3.328.5 Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 745, normalized size of antiderivative = 6.83

$$\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx$$
$$= \left[\frac{\sqrt{cd^2+ae^2}cd^2\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+(cd^3+ade^2)\sqrt{c}\log\left(-2\,cx\right.}{2\,(c^2d^2e^2+ace^4)}\right.$$
$$-\frac{2\,\sqrt{-cd^2-ae^2}cd^2\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-(cd^3+ade^2)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-\right.}{2\,(c^2d^2e^2+ace^4)}$$
$$-\frac{\sqrt{-cd^2-ae^2}cd^2\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-(cd^3+ade^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)-(cd^2e+ae^3)}{c^2d^2e^2+ace^4}$$

input
```
integrate(x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(c*d^2 + a*e^2)*c*d^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (
2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2
+ a))/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^3 + a*d*e^2)*sqrt(c)*log(-2*c*x^2
+ 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/
(c^2*d^2*e^2 + a*c*e^4), -1/2*(2*sqrt(-c*d^2 - a*e^2)*c*d^2*arctan(sqrt(-c
*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2
+ a*c*e^2)*x^2)) - (c*d^3 + a*d*e^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 +
 a)*sqrt(c)*x - a) - 2*(c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2*d^2*e^2 + a
*c*e^4), 1/2*(sqrt(c*d^2 + a*e^2)*c*d^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2
*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqr
t(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(c*d^3 + a*d*e^2)*sqrt(-c)*ar
ctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + 2*(c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c
^2*d^2*e^2 + a*c*e^4), -(sqrt(-c*d^2 - a*e^2)*c*d^2*arctan(sqrt(-c*d^2 - a
*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^
2)*x^2)) - (c*d^3 + a*d*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) -
 (c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2*d^2*e^2 + a*c*e^4)]
```

3.328.6 Sympy [F]

$$\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

input
```
integrate(x**2/(e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(a + c*x**2)*(d + e*x)), x)
```

3.328.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.328.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.328.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{x^2}{\sqrt{c\,x^2+a}\,(d+e\,x)}\,dx$$

input
```
int(x^2/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int(x^2/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.329 $\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx$

3.329.1 Optimal result

Integrand size = 20, antiderivative size = 86

$$\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e} + \frac{d\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e\sqrt{cd^2+ae^2}}$$

output
```
arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/e/c^(1/2)+d*arctanh((-c*d*x+a*e)/(a*e^2
+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e/(a*e^2+c*d^2)^(1/2)
```

3.329.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.14

$$\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{\frac{2d\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}} - \frac{\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{\sqrt{c}}}{e}$$

input
```
Integrate[x/((d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
((2*d*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2
]])/Sqrt[-(c*d^2) - a*e^2] - Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]]/Sqrt[c])/
e
```

3.329.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {605, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+cx^2}(d+ex)}dx$$

$$\Big\downarrow \text{605}$$

$$\frac{\int \frac{1}{\sqrt{cx^2+a}}dx}{e}-\frac{d\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}$$

$$\Big\downarrow \text{224}$$

$$\frac{\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{e}-\frac{d\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e}-\frac{d\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}$$

$$\Big\downarrow \text{488}$$

$$\frac{d\int \frac{1}{cd^2+ae^2-\frac{(ae-cdx)^2}{cx^2+a}}d\frac{ae-cdx}{\sqrt{cx^2+a}}}{e}+\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\sqrt{ae^2+cd^2}}+\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e}$$

input
```
Int[x/((d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*e) + (d*ArcTanh[(a*e - c*d*x
)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e*Sqrt[c*d^2 + a*e^2])
```

3.329.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 605
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[1/d   Int[x^(m - 1)*(a + b*x^2)^p, x], x] - Simp[c/d   Int[x^(m - 1
)*((a + b*x^2)^p/(c + d*x)), x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m,
 0] && LtQ[-1, p, 0]
```

3.329.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(74) = 148$.

Time = 0.41 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.76

method	result	size
default	$\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{e\sqrt{c}}+\frac{d\ln\left(\frac{\frac{2e^{2}a+2c\,d^{2}}{e^{2}}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^{2}a+c\,d^{2}}{e^{2}}}\,\sqrt{\left(x+\frac{d}{e}\right)^{2}c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^{2}a+c\,d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{e^{2}\sqrt{\frac{e^{2}a+c\,d^{2}}{e^{2}}}}$	151

input
```
int(x/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/e*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)+d/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*
ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/
e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

3.329.5 Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 631, normalized size of antiderivative = 7.34

$$\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx$$
$$= \left[\frac{\sqrt{cd^2+ae^2}cd\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2+2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+(cd^2+ae^2)\sqrt{c}\log\left(-2\,cx^2\right.}{2\,(c^2d^2e+ace^3)} \right.$$

input
```
integrate(x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(c*d^2 + a*e^2)*c*d*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*
c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
a))/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2 + a*e^2)*sqrt(c)*log(-2*c*x^2 - 2*
sqrt(c*x^2 + a)*sqrt(c)*x - a))/(c^2*d^2*e + a*c*e^3), 1/2*(2*sqrt(-c*d^2
- a*e^2)*c*d*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*
c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (c*d^2 + a*e^2)*sqrt(c)*log(
-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a))/(c^2*d^2*e + a*c*e^3), 1/2*(s
qrt(c*d^2 + a*e^2)*c*d*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2
 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^
2*x^2 + 2*d*e*x + d^2)) - 2*(c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqr
t(c*x^2 + a)))/(c^2*d^2*e + a*c*e^3), (sqrt(-c*d^2 - a*e^2)*c*d*arctan(sqr
t(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*
d^2 + a*c*e^2)*x^2)) - (c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x
^2 + a)))/(c^2*d^2*e + a*c*e^3)]
```

3.329.6 Sympy [F]

$$\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{x}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

input
```
integrate(x/(e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(x/(sqrt(a + c*x**2)*(d + e*x)), x)
```

3.329.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.329.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.329.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{x}{\sqrt{c\,x^2+a}\,(d+e\,x)}\,dx$$

input
```
int(x/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int(x/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.330 $\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx$

3.330.1 Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\sqrt{cd^2+ae^2}}$$

output
```
-arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2)^(
1/2)
```

3.330.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

$$\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}}$$

input
```
Integrate[1/((d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
(-2*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]]
)/Sqrt[-(c*d^2) - a*e^2]
```

3.330.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^2}(d+ex)} \, dx$$
$$\Big\downarrow \text{488}$$
$$-\int \frac{1}{cd^2+ae^2-\frac{(ae-cdx)^2}{cx^2+a}}d\frac{ae-cdx}{\sqrt{cx^2+a}}$$
$$\Big\downarrow \text{219}$$
$$-\frac{\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}$$

input
```
Int[1/((d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
-(ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])]/Sqrt[c*d^2
+ a*e^2])
```

3.330.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

3.330.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(48) = 96$.

Time = 0.38 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.35

method	result	size
default	$-\frac{\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{e\sqrt{\frac{e^2a+c\,d^2}{e^2}}}$	127

input
```
int(1/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/e/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*(
(a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(
1/2))/(x+d/e))
```

3.330.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(49) = 98$.

Time = 0.34 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.91

$$\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx = \left[\frac{\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)}{2\,\sqrt{cd^2+ae^2}}, -\frac{\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)}{cd^2+ae^2}\right]$$

input
```
integrate(1/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 -
2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x +
d^2))/sqrt(c*d^2 + a*e^2), -sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^
2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*
x^2))/(c*d^2 + a*e^2)]
```

3.330.6 Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**2)*(d + e*x)), x)
```

3.330.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{\operatorname{arsinh}\left(\frac{cdx}{e\sqrt{\frac{ac}{e^2}}|ex+d|} - \frac{a}{\sqrt{\frac{ac}{e^2}}|ex+d|}\right)}{\sqrt{a+\frac{cd^2}{e^2}}e}$$

input
```
integrate(1/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(c*d*x/(e*sqrt(a*c/e^2)*abs(e*x + d)) - a/(sqrt(a*c/e^2)*abs(e*x +
d)))/(sqrt(a + c*d^2/e^2)*e)
```

3.330.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}}$$

input
```
integrate(1/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^
2))/sqrt(-c*d^2 - a*e^2)
```

3.330.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

input
```
int(1/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int(1/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.331 $\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx$

3.331.1 Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx=\frac{e\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\sqrt{cd^2+ae^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

output
```
-arctanh((c*x^2+a)^(1/2)/a^(1/2))/d/a^(1/2)+e*arctanh((-c*d*x+a*e)/(a*e^2+
c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d/(a*e^2+c*d^2)^(1/2)
```

3.331.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.21

$$\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx=\frac{2\left(\frac{e\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{d}$$

input
```
Integrate[1/(x*(d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
(2*((e*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^
2]])/Sqrt[-(c*d^2) - a*e^2] + ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a
]]/Sqrt[a]))/d
```

3.331.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+cx^2}(d+ex)}\,dx$$

$$\Big\downarrow \text{617}$$

$$\int \left(\frac{1}{dx\sqrt{a+cx^2}}-\frac{e}{d\sqrt{a+cx^2}(d+ex)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\sqrt{ae^2+cd^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
Int[1/(x*(d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
(e*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d*Sqrt[c
*d^2 + a*e^2]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d)
```

3.331.3.1 Defintions of rubi rules used

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.331.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(74) = 148.

Time = 0.40 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.84

method	result	size
default	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{d\sqrt{a}}+\frac{\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\,\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{d\sqrt{\frac{e^2a+c\,d^2}{e^2}}}$	158

input
```
int(1/x/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)+1/d/((a*e^2+c*d^2)/e^2)
^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)
*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

3.331.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 634, normalized size of antiderivative = 7.37

$$\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx$$
$$= \left[\frac{\sqrt{cd^2+ae^2}ae\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2+2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+(cd^2+ae^2)\sqrt{a}\log\left(-\frac{cx^2-2}{}\right.}{2\,(acd^3+a^2de^2)}\right.$$

input
```
integrate(1/x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(c*d^2 + a*e^2)*a*e*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*
c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
a))/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2 + a*e^2)*sqrt(a)*log(-(c*x^2 - 2*s
qrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*c*d^3 + a^2*d*e^2), 1/2*(2*sqrt(-c*
d^2 - a*e^2)*a*e*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)
/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (c*d^2 + a*e^2)*sqrt(a)*
log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*c*d^3 + a^2*d*e^2)
, 1/2*(sqrt(c*d^2 + a*e^2)*a*e*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2
*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
 a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(c*d^2 + a*e^2)*sqrt(-a)*arctan(sqrt(-
a)/sqrt(c*x^2 + a)))/(a*c*d^3 + a^2*d*e^2), (sqrt(-c*d^2 - a*e^2)*a*e*arct
an(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 +
 (c^2*d^2 + a*c*e^2)*x^2)) + (c*d^2 + a*e^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt
(c*x^2 + a)))/(a*c*d^3 + a^2*d*e^2)]
```

3.331.6 Sympy [F]

$$\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{1}{x\sqrt{a+cx^2}\,(d+ex)}\,dx$$

input
```
integrate(1/x/(e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + c*x**2)*(d + e*x)), x)
```

3.331.7 Maxima [F]

$$\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)x}\,dx$$

input
```
integrate(1/x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*x), x)
```

3.331.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.331.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{1}{x\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

input
```
int(1/(x*(a + c*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int(1/(x*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.332 $\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx$

3.332.1 Optimal result

Integrand size = 22, antiderivative size = 111

$$\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx=-\frac{\sqrt{a+cx^2}}{adx}-\frac{e^2\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}}+\frac{e\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}$$

output
```
e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^2/a^(1/2)-e^2*arctanh((-c*d*x+a*e)/(a
*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^2/(a*e^2+c*d^2)^(1/2)-(c*x^2+a)^(1/2)
/a/d/x
```

3.332.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx=-\frac{\frac{d\sqrt{a+cx^2}}{ax}+\frac{2e^2\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}}+\frac{2e\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{d^2}$$

input
```
Integrate[1/(x^2*(d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
-(((d*Sqrt[a + c*x^2])/(a*x) + (2*e^2*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a
 + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/Sqrt[-(c*d^2) - a*e^2] + (2*e*ArcTanh[
(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/Sqrt[a])/d^2)
```

3.332.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+cx^2}(d+ex)}\,dx$$

$$\Big\downarrow \text{617}$$

$$\int \left(\frac{e^2}{d^2\sqrt{a+cx^2}(d+ex)}-\frac{e}{d^2x\sqrt{a+cx^2}}+\frac{1}{dx^2\sqrt{a+cx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^2\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\sqrt{ae^2+cd^2}}+\frac{e\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}-\frac{\sqrt{a+cx^2}}{adx}$$

input
```
Int[1/(x^2*(d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
-(Sqrt[a + c*x^2]/(a*d*x)) - (e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^
2]*Sqrt[a + c*x^2])])/(d^2*Sqrt[c*d^2 + a*e^2]) + (e*ArcTanh[Sqrt[a + c*x^
2]/Sqrt[a]])/(Sqrt[a]*d^2)
```

3.332.3.1 Defintions of rubi rules used

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.332.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.62

method	result	size
default	$-\frac{\sqrt{c\,x^{2}+a}}{a d x}+\frac{e \ln\left(\frac{2 a+2 \sqrt{a}\, \sqrt{c\,x^{2}+a}}{x}\right)}{d^{2} \sqrt{a}}-\frac{e \ln\left(\frac{\frac{2 e^{2} a+2 c\,d^{2}}{e^{2}}-\frac{2 c d \left(x+\frac{d}{e}\right)}{e}+2 \sqrt{\frac{e^{2} a+c\,d^{2}}{e^{2}}}\, \sqrt{\left(x+\frac{d}{e}\right)^{2} c-\frac{2 c d \left(x+\frac{d}{e}\right)}{e}+\frac{e^{2} a+c\,d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{d^{2} \sqrt{\frac{e^{2} a+c\,d^{2}}{e^{2}}}}$	180
risch	$-\frac{\sqrt{c\,x^{2}+a}}{a d x}+\frac{e \ln\left(\frac{2 a+2 \sqrt{a}\, \sqrt{c\,x^{2}+a}}{x}\right)}{d^{2} \sqrt{a}}-\frac{e \ln\left(\frac{\frac{2 e^{2} a+2 c\,d^{2}}{e^{2}}-\frac{2 c d \left(x+\frac{d}{e}\right)}{e}+2 \sqrt{\frac{e^{2} a+c\,d^{2}}{e^{2}}}\, \sqrt{\left(x+\frac{d}{e}\right)^{2} c-\frac{2 c d \left(x+\frac{d}{e}\right)}{e}+\frac{e^{2} a+c\,d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{d^{2} \sqrt{\frac{e^{2} a+c\,d^{2}}{e^{2}}}}$	180

input
```
int(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2+a)^(1/2)/a/d/x+e/d^2/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)
-e/d^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2
*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)
^(1/2))/(x+d/e))
```

3.332.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 767, normalized size of antiderivative = 6.91

$$\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx$$
$$= \left[\frac{\sqrt{cd^2+ae^2}ae^2x\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+(cd^2e+ae^3)\sqrt{a}x\log\left(-\right.}{2\,(acd^4+a^2d^2e^2)x}\right.$$
$$-\frac{2\,\sqrt{-cd^2-ae^2}ae^2x\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-(cd^2e+ae^3)\sqrt{a}x\log\left(-\frac{cx^2+2\,\sqrt{cx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2}{2\,(acd^4+a^2d^2e^2)x}$$
$$-\frac{\sqrt{-cd^2-ae^2}ae^2x\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)+(cd^2e+ae^3)\sqrt{-a}x\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right)+(cd^3+ad}{(acd^4+a^2d^2e^2)x}$$

input
```
integrate(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(c*d^2 + a*e^2)*a*e^2*x*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 -
 (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^
2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2*e + a*e^3)*sqrt(a)*x*log(-(c*x
^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(c*d^3 + a*d*e^2)*sqrt(c*x^
2 + a))/((a*c*d^4 + a^2*d^2*e^2)*x), -1/2*(2*sqrt(-c*d^2 - a*e^2)*a*e^2*x*
arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e
^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (c*d^2*e + a*e^3)*sqrt(a)*x*log(-(c*x^2 +
 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(c*d^3 + a*d*e^2)*sqrt(c*x^2 +
a))/((a*c*d^4 + a^2*d^2*e^2)*x), 1/2*(sqrt(c*d^2 + a*e^2)*a*e^2*x*log((2*a
*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2
+ a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c*
d^2*e + a*e^3)*sqrt(-a)*x*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - 2*(c*d^3 + a*
d*e^2)*sqrt(c*x^2 + a))/((a*c*d^4 + a^2*d^2*e^2)*x), -(sqrt(-c*d^2 - a*e^2
)*a*e^2*x*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d
^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (c*d^2*e + a*e^3)*sqrt(-a)*x*ar
ctan(sqrt(-a)/sqrt(c*x^2 + a)) + (c*d^3 + a*d*e^2)*sqrt(c*x^2 + a))/((a*c*
d^4 + a^2*d^2*e^2)*x)]
```

3.332.6 Sympy [F]

$$\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex)}\,dx$$

input
```
integrate(1/x**2/(e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a + c*x**2)*(d + e*x)), x)
```

3.332.7 Maxima [F]

$$\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*x^2), x)
```

3.332.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx$$
$$=2c\left(\frac{e^2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}cd^2}-\frac{e\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}cd^2}+\frac{1}{\left(\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2-a\right)\sqrt{c}d}\right)$$

input
```
integrate(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
2*c*(e^2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2
 - a*e^2))/(sqrt(-c*d^2 - a*e^2)*c*d^2) - e*arctan(-(sqrt(c)*x - sqrt(c*x^
2 + a))/sqrt(-a))/(sqrt(-a)*c*d^2) + 1/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 -
 a)*sqrt(c)*d))
```

3.332.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx=\int \frac{1}{x^2\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

input
```
int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.333 $\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx$

3.333.1 Optimal result

Integrand size = 22, antiderivative size = 168

$$\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx = -\frac{\sqrt{a+cx^2}}{2adx^2}+\frac{e\sqrt{a+cx^2}}{ad^2x}+\frac{e^3\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}}+\frac{c\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}-\frac{e^2\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3}$$

output
```
1/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d-e^2*arctanh((c*x^2+a)^(1/
2)/a^(1/2))/d^3/a^(1/2)+e^3*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^
2+a)^(1/2))/d^3/(a*e^2+c*d^2)^(1/2)-1/2*(c*x^2+a)^(1/2)/a/d/x^2+e*(c*x^2+a
)^(1/2)/a/d^2/x
```

3.333.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx = \frac{\sqrt{a}\left(d(cd^2+ae^2)(-d+2ex)\sqrt{a+cx^2}-4ae^3\sqrt{-cd^2-ae^2}x^2\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)\right)-2(c^2d^4-acd}{2a^{3/2}d^3(cd^2+ae^2)x^2}$$

input
```
Integrate[1/(x^3*(d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
(Sqrt[a]*(d*(c*d^2 + a*e^2)*(-d + 2*e*x)*Sqrt[a + c*x^2] - 4*a*e^3*Sqrt[-(
c*d^2) - a*e^2]*x^2*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(
c*d^2) - a*e^2]]) - 2*(c^2*d^4 - a*c*d^2*e^2 - 2*a^2*e^4)*x^2*ArcTanh[(Sqr
t[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/(2*a^(3/2)*d^3*(c*d^2 + a*e^2)*x^2)
```

3.333.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+cx^2}(d+ex)}\,dx$$

$$\Big\downarrow \text{617}$$

$$\int \left(-\frac{e^3}{d^3\sqrt{a+cx^2}(d+ex)}+\frac{e^2}{d^3x\sqrt{a+cx^2}}-\frac{e}{d^2x^2\sqrt{a+cx^2}}+\frac{1}{dx^3\sqrt{a+cx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}-\frac{e^2\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3}+\frac{e^3\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}}+\frac{e\sqrt{a+cx^2}}{ad^2x}-\frac{\sqrt{a+cx^2}}{2adx^2}$$

input
```
Int[1/(x^3*(d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
-1/2*Sqrt[a + c*x^2]/(a*d*x^2) + (e*Sqrt[a + c*x^2])/(a*d^2*x) + (e^3*ArcT
anh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^3*Sqrt[c*d^2
+ a*e^2]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(3/2)*d) - (e^2*ArcT
anh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)
```

3.333.3.1 Defintions of rubi rules used

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.333.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{\sqrt{c\,x^{2}+a}\,(-2ex+d)}{2a\,d^{2}x^{2}}+\frac{\frac{2e^{2}a\ln\left(\frac{\frac{2e^{2}a+2c\,d^{2}}{e^{2}}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^{2}a+c\,d^{2}}{e^{2}}}\sqrt{\left(x+\frac{d}{e}\right)^{2}c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^{2}a+c\,d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{d\sqrt{\frac{e^{2}a+c\,d^{2}}{e^{2}}}}+\frac{\left(-2e^{2}a+c\,d^{2}\right)\ln\left(\frac{2a+2}{\ldots}\right.}{d\sqrt{a}}}{2a\,d^{2}}$
default	$\frac{-\frac{\sqrt{c\,x^{2}+a}}{2a\,x^{2}}+\frac{c\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}}{d}-\frac{e^{2}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^{2}+a}}{x}\right)}{d^{3}\sqrt{a}}+\frac{e\sqrt{c\,x^{2}+a}}{a\,d^{2}x}+\frac{e^{2}\ln\left(\frac{\frac{2e^{2}a+2c\,d^{2}}{e^{2}}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^{2}a+c\,d^{2}}{e^{2}}}\sqrt{(\ldots}}{x+\frac{d}{e}}\right.}{d^{3}\sqrt{\frac{e^{2}a+c\,d^{2}}{e^{2}}}}$

input
```
int(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*x^2+a)^(1/2)*(-2*e*x+d)/a/d^2/x^2+1/2/a/d^2*(2*e^2*a/d/((a*e^2+c*d
^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^
2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))+1
/d*(-2*a*e^2+c*d^2)/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x))
```

3.333.5 Fricas [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 956, normalized size of antiderivative = 5.69

$$\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx$$
$$=\left[\frac{2\sqrt{cd^2+ae^2}a^2e^3x^2\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2+2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-(c^2d^4-acd^2e^2-2\,a^2\ldots}{4\,(a^2cd^5+a^3d^3e^2)x\ldots}\right.$$

input
```
integrate(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(c*d^2 + a*e^2)*a^2*e^3*x^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2
*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqr
t(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - (c^2*d^4 - a*c*d^2*e^2 - 2*a^2*
e^4)*sqrt(a)*x^2*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(
a*c*d^4 + a^2*d^2*e^2 - 2*(a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*x^2 + a))/((a^
2*c*d^5 + a^3*d^3*e^2)*x^2), 1/4*(4*sqrt(-c*d^2 - a*e^2)*a^2*e^3*x^2*arcta
n(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 +
(c^2*d^2 + a*c*e^2)*x^2)) - (c^2*d^4 - a*c*d^2*e^2 - 2*a^2*e^4)*sqrt(a)*x^
2*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a*c*d^4 + a^2*d
^2*e^2 - 2*(a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c*d^5 + a^3*d
^3*e^2)*x^2), 1/2*(sqrt(c*d^2 + a*e^2)*a^2*e^3*x^2*log((2*a*c*d*e*x - a*c*
d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x
 - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - (c^2*d^4 - a*c*d^2*e
^2 - 2*a^2*e^4)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (a*c*d^4 +
 a^2*d^2*e^2 - 2*(a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c*d^5 +
 a^3*d^3*e^2)*x^2), 1/2*(2*sqrt(-c*d^2 - a*e^2)*a^2*e^3*x^2*arctan(sqrt(-c
*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2
+ a*c*e^2)*x^2)) - (c^2*d^4 - a*c*d^2*e^2 - 2*a^2*e^4)*sqrt(-a)*x^2*arctan
(sqrt(-a)/sqrt(c*x^2 + a)) - (a*c*d^4 + a^2*d^2*e^2 - 2*(a*c*d^3*e + a^2*d
*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c*d^5 + a^3*d^3*e^2)*x^2)]
```

3.333.6 Sympy [F]

$$\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex)}\,dx$$

input
```
integrate(1/x**3/(e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(a + c*x**2)*(d + e*x)), x)
```

3.333.7 Maxima [F]

$$\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)x^3}\,dx$$

input
```
integrate(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*x^3), x)
```

3.333.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.43

$$\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx = -c^{\frac{3}{2}}\left(\frac{2\,e^3\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}c^{\frac{3}{2}}d^3}+\frac{(cd^2-2\,ae^2)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}ac^{\frac{3}{2}}d^3}-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3\cdots}{\cdots}\right.$$

input
```
integrate(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-c^(3/2)*(2*e^3*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt
(-c*d^2 - a*e^2))/(sqrt(-c*d^2 - a*e^2)*c^(3/2)*d^3) + (c*d^2 - 2*a*e^2)*a
rctan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*a*c^(3/2)*d^3) -
((sqrt(c)*x - sqrt(c*x^2 + a))^3*sqrt(c)*d - 2*(sqrt(c)*x - sqrt(c*x^2 + a
))^2*a*e + (sqrt(c)*x - sqrt(c*x^2 + a))*a*sqrt(c)*d + 2*a^2*e)/(((sqrt(c)
*x - sqrt(c*x^2 + a))^2 - a)^2*a*c*d^2))
```

3.333.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{1}{x^3\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

input
```
int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.334 $\int \frac{x^4}{(d+ex)(a+cx^2)^{3/2}} \, dx$

3.334.1 Optimal result

Integrand size = 22, antiderivative size = 146

$$\int \frac{x^4}{(d+ex)(a+cx^2)^{3/2}} \, dx = \frac{a(ae+cdx)}{c^2(cd^2+ae^2)\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2 e} - \frac{d\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^2} - \frac{d^4\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^2(cd^2+ae^2)^{3/2}}$$

output
```
-d*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^2-d^4*arctanh((-c*d*x+a*e)
/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e^2/(a*e^2+c*d^2)^(3/2)+a*(c*d*x+a*e
)/c^2/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)+(c*x^2+a)^(1/2)/c^2/e
```

3.334.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{(d+ex)(a+cx^2)^{3/2}} \, dx = \frac{\frac{e(2a^2e^2+c^2d^2x^2+ac(d^2+dex+e^2x^2))}{c^2(cd^2+ae^2)\sqrt{a+cx^2}} + \frac{2d^4\arctan\left(\frac{\sqrt{-cd^2-ae^2}x}{\sqrt{a}(d+ex)-d\sqrt{a+cx^2}}\right)}{(-cd^2-ae^2)^{3/2}} + \frac{2d\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a}-\sqrt{a+\cdots}}\right)}{c^{3/2}}}{e^2}$$

input
```
Integrate[x^4/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

output
```
((e*(2*a^2*e^2 + c^2*d^2*x^2 + a*c*(d^2 + d*e*x + e^2*x^2)))/(c^2*(c*d^2 +
 a*e^2)*Sqrt[a + c*x^2]) + (2*d^4*ArcTan[(Sqrt[-(c*d^2) - a*e^2]*x)/(Sqrt[
a]*(d + e*x) - d*Sqrt[a + c*x^2])])/(-(c*d^2) - a*e^2)^(3/2) + (2*d*ArcTan
h[(Sqrt[c]*x)/(Sqrt[a] - Sqrt[a + c*x^2])])/c^(3/2))/e^2
```

3.334.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {601, 27, 2185, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+cx^2)^{3/2}(d+ex)}dx$$

$$\Big\downarrow \text{601}$$

$$\frac{a(ae+cdx)}{c^2\sqrt{a+cx^2}(ae^2+cd^2)}-\frac{\int \frac{a\left(\frac{ad^2}{cd^2+ae^2}-x^2\right)}{c(d+ex)\sqrt{cx^2+a}}dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(ae+cdx)}{c^2\sqrt{a+cx^2}(ae^2+cd^2)}-\frac{\int \frac{\frac{ad^2}{cd^2+ae^2}-x^2}{(d+ex)\sqrt{cx^2+a}}dx}{c}$$

$$\Big\downarrow \text{2185}$$

$$\frac{a(ae+cdx)}{c^2\sqrt{a+cx^2}(ae^2+cd^2)}-\frac{\frac{\int \frac{cde\left(\frac{ade}{cd^2+ae^2}+x\right)}{(d+ex)\sqrt{cx^2+a}}dx}{ce^2}-\frac{\sqrt{a+cx^2}}{ce}}{c}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(ae+cdx)}{c^2\sqrt{a+cx^2}(ae^2+cd^2)}-\frac{\frac{d\int \frac{\frac{ade}{cd^2+ae^2}+x}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{\sqrt{a+cx^2}}{ce}}{c}$$

$$\Big\downarrow \text{719}$$

$$\frac{a(ae+cdx)}{c^2\sqrt{a+cx^2}(ae^2+cd^2)}-\frac{\frac{d\left(\frac{\int \frac{1}{\sqrt{cx^2+a}}dx}{e}-\frac{cd^3\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e\left(ae^2+cd^2\right)}\right)}{e}-\frac{\sqrt{a+cx^2}}{ce}}{c}$$

$$\Big\downarrow \text{224}$$

$$\frac{a(ae+cdx)}{c^2\sqrt{a+cx^2}\,(ae^2+cd^2)}-\frac{\frac{d\left(\frac{\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{e}-\frac{cd^3\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e\left(ae^2+cd^2\right)}\right)}{e}-\frac{\sqrt{a+cx^2}}{ce}}{c}$$

$$\Big\downarrow \text{219}$$

$$\frac{a(ae+cdx)}{c^2\sqrt{a+cx^2}\,(ae^2+cd^2)}-\frac{\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e}-\frac{cd^3\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e\left(ae^2+cd^2\right)}\right)}{e}-\frac{\sqrt{a+cx^2}}{ce}}{c}$$

$$\Big\downarrow \text{488}$$

$$\frac{a(ae+cdx)}{c^2\sqrt{a+cx^2}\,(ae^2+cd^2)}-\frac{\frac{d\left(\frac{cd^3\int \frac{1}{cd^2+ae^2-\frac{(ae-cdx)^2}{cx^2+a}}d\frac{ae-cdx}{\sqrt{cx^2+a}}}{e\left(ae^2+cd^2\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e}\right)}{e}-\frac{\sqrt{a+cx^2}}{ce}}{c}$$

$$\Big\downarrow \text{219}$$

$$\frac{a(ae+cdx)}{c^2\sqrt{a+cx^2}\,(ae^2+cd^2)}-\frac{\frac{d\left(\frac{cd^3\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\left(ae^2+cd^2\right)^{3/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e}\right)}{e}-\frac{\sqrt{a+cx^2}}{ce}}{c}$$

input
```
Int[x^4/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

output
```
(a*(a*e + c*d*x))/(c^2*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (-(Sqrt[a + c*x^
2]/(c*e)) + (d*(ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*e) + (c*d^3*
ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e*(c*d^2 +
a*e^2)^(3/2))))/e)/c
```

3.334. $\int \frac{x^4}{(d+ex)(a+cx^2)^{3/2}}\,dx$

3.334.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.334.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(130) = 260$.

Time = 0.47 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.40

method	result
risch	$\frac{\sqrt{c\,x^2+a}}{c^2 e}-\frac{d\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{c^{\frac{3}{2}}e^2}+\frac{c\,d^4\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{e^3\left(e\sqrt{-ac}+cd\right)\left(e\sqrt{-ac}-cd\right)\sqrt{\frac{e^2a+c\,d^2}{e^2}}}+\frac{a\sqrt{\left(x \cdots\right.}}{2\cdots}$
default	$\frac{\frac{x^2}{c\sqrt{c\,x^2+a}}+\frac{2a}{c^2\sqrt{c\,x^2+a}}}{e}-\frac{d^2}{e^3c\sqrt{c\,x^2+a}}-\frac{d^3x}{e^4a\sqrt{c\,x^2+a}}-\frac{d\left(-\frac{x}{c\sqrt{c\,x^2+a}}+\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{c^{\frac{3}{2}}}\right)}{e^2}+\frac{d^4\left(\frac{e^2}{\left(e^2a+c\,d^2\right)\sqrt{\left(x+\frac{d}{e}\right)^2c-\cdots}}\cdots\right)}{\cdots}$

input
```
int(x^4/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+a)^(1/2)/c^2/e-1/c^(3/2)/e^2*d*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+c/e^3*
d^4/(e*(-a*c)^(1/2)+c*d)/(e*(-a*c)^(1/2)-c*d)/((a*e^2+c*d^2)/e^2)^(1/2)*ln
((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)
^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))+1/2/c^2*a/(e*(-a*c
)^(1/2)+c*d)/(x-(-a*c)^(1/2)/c)*((x-(-a*c)^(1/2)/c)^2*c+2*(-a*c)^(1/2)*(x-
(-a*c)^(1/2)/c))^(1/2)-1/2/c^2*a/(e*(-a*c)^(1/2)-c*d)/(x+(-a*c)^(1/2)/c)*(
(x+(-a*c)^(1/2)/c)^2*c-2*(-a*c)^(1/2)*(x+(-a*c)^(1/2)/c))^(1/2)
```

3.334.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(131) = 262$.

Time = 3.48 (sec) , antiderivative size = 1525, normalized size of antiderivative = 10.45

$$\int \frac{x^4}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4 + (c^3*d^5 + 2*a*c^2*d^3*e^
2 + a^2*c*d*e^4)*x^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x -
 a) + (c^3*d^4*x^2 + a*c^2*d^4)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c
*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*
x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a*c^2*d^4*e + 3*
a^2*c*d^2*e^3 + 2*a^3*e^5 + (c^3*d^4*e + 2*a*c^2*d^2*e^3 + a^2*c*e^5)*x^2
+ (a*c^2*d^3*e^2 + a^2*c*d*e^4)*x)*sqrt(c*x^2 + a))/(a*c^4*d^4*e^2 + 2*a^2
*c^3*d^2*e^4 + a^3*c^2*e^6 + (c^5*d^4*e^2 + 2*a*c^4*d^2*e^4 + a^2*c^3*e^6)
*x^2), -1/2*(2*(c^3*d^4*x^2 + a*c^2*d^4)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(
-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^
2 + a*c*e^2)*x^2)) - (a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4 + (c^3*d^5 +
 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 +
 a)*sqrt(c)*x - a) - 2*(a*c^2*d^4*e + 3*a^2*c*d^2*e^3 + 2*a^3*e^5 + (c^3*d
^4*e + 2*a*c^2*d^2*e^3 + a^2*c*e^5)*x^2 + (a*c^2*d^3*e^2 + a^2*c*d*e^4)*x)
*sqrt(c*x^2 + a))/(a*c^4*d^4*e^2 + 2*a^2*c^3*d^2*e^4 + a^3*c^2*e^6 + (c^5*
d^4*e^2 + 2*a*c^4*d^2*e^4 + a^2*c^3*e^6)*x^2), 1/2*(2*(a*c^2*d^5 + 2*a^2*c
*d^3*e^2 + a^3*d*e^4 + (c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2)*sqrt
(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (c^3*d^4*x^2 + a*c^2*d^4)*sqrt(c
*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^
2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 ...
```

3.334.6 Sympy [F]

$$\int \frac{x^4}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \frac{x^4}{\left(a+cx^2\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(x**4/(e*x+d)/(c*x**2+a)**(3/2),x)
```

output
```
Integral(x**4/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.334.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.334.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.334.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \frac{x^4}{\left(c\,x^2+a\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

input
```
int(x^4/((a + c*x^2)^(3/2)*(d + e*x)),x)
```

output
```
int(x^4/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.335 $\int \frac{x^3}{(d+ex)(a+cx^2)^{3/2}}\,dx$

3.335.1 Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \frac{x^3}{(d+ex)(a+cx^2)^{3/2}}\,dx = \frac{a(d-ex)}{c(cd^2+ae^2)\sqrt{a+cx^2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e} + \frac{d^3\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e(cd^2+ae^2)^{3/2}}$$

output
```
arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e+d^3*arctanh((-c*d*x+a*e)/(a*e
^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e/(a*e^2+c*d^2)^(3/2)+a*(-e*x+d)/c/(a*e^2
+c*d^2)/(c*x^2+a)^(1/2)
```

3.335.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{(d+ex)(a+cx^2)^{3/2}}\,dx = \frac{a(d-ex)}{c(cd^2+ae^2)\sqrt{a+cx^2}} - \frac{2d^3\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{e(-cd^2-ae^2)^{3/2}} - \frac{\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{c^{3/2}e}$$

input
```
Integrate[x^3/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

output
```
(a*(d - e*x))/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (2*d^3*ArcTan[(Sqrt[c]
*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(e*(-(c*d^2) - a*
e^2)^(3/2)) - Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]]/(c^(3/2)*e)
```

3.335.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {601, 25, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex)}dx$$

$$\Big\downarrow \text{601}$$

$$\frac{a(d-ex)}{c\sqrt{a+cx^2}(ae^2+cd^2)}-\frac{\int -\frac{a\left(\frac{ade}{cd^2+ae^2}+x\right)}{c(d+ex)\sqrt{cx^2+a}}dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a\left(\frac{ade}{cd^2+ae^2}+x\right)}{c(d+ex)\sqrt{cx^2+a}}dx}{a}+\frac{a(d-ex)}{c\sqrt{a+cx^2}(ae^2+cd^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\frac{ade}{cd^2+ae^2}+x}{(d+ex)\sqrt{cx^2+a}}dx}{c}+\frac{a(d-ex)}{c\sqrt{a+cx^2}(ae^2+cd^2)}$$

$$\Big\downarrow \text{719}$$

$$\frac{\frac{\int \frac{1}{\sqrt{cx^2+a}}dx}{e}-\frac{cd^3\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e(ae^2+cd^2)}}{c}+\frac{a(d-ex)}{c\sqrt{a+cx^2}(ae^2+cd^2)}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{e}-\frac{cd^3\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e(ae^2+cd^2)}}{c}+\frac{a(d-ex)}{c\sqrt{a+cx^2}(ae^2+cd^2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e}-\frac{cd^3\int\frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e(ae^2+cd^2)}}{c}+\frac{a(d-ex)}{c\sqrt{a+cx^2}\,(ae^2+cd^2)}$$

$$\Big\downarrow \text{488}$$

$$\frac{\frac{cd^3\int\frac{1}{cd^2+ae^2-\frac{(ae-cdx)^2}{cx^2+a}}d\frac{ae-cdx}{\sqrt{cx^2+a}}}{e(ae^2+cd^2)}+\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e}}{c}+\frac{a(d-ex)}{c\sqrt{a+cx^2}\,(ae^2+cd^2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{cd^3\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e(ae^2+cd^2)^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e}}{c}+\frac{a(d-ex)}{c\sqrt{a+cx^2}\,(ae^2+cd^2)}$$

input
```
Int[x^3/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

output
```
(a*(d - e*x))/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + (ArcTanh[(Sqrt[c]*x)/S
qrt[a + c*x^2]]/(Sqrt[c]*e) + (c*d^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a
*e^2]*Sqrt[a + c*x^2])])/(e*(c*d^2 + a*e^2)^(3/2)))/c
```

3.335.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.335. $\int \frac{x^3}{(d+ex)(a+cx^2)^{3/2}}\,dx$

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.335.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(109) = 218$.

Time = 0.42 (sec) , antiderivative size = 397, normalized size of antiderivative = 3.23

method	result
default	$\frac{-\frac{x}{c\sqrt{c\,x^2+a}}+\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{c^{\frac{3}{2}}}}{e}+\frac{d^2 x}{e^3 a\sqrt{c\,x^2+a}}+\frac{d}{e^2 c\sqrt{c\,x^2+a}}-\frac{d^3\left(\frac{e^2}{\left(e^2 a+c\,d^2\right)\sqrt{\left(x+\frac{d}{e}\right)^2 c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2 a+c\,d^2}{e^2}}}+\frac{}{\left(e^2 a+c\,d^2\right)}\right.}{}$

input
```
int(x^3/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e*(-x/c/(c*x^2+a)^(1/2)+1/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))+d^2/e^3
*x/a/(c*x^2+a)^(1/2)+d/e^2/c/(c*x^2+a)^(1/2)-d^3/e^4*(1/(a*e^2+c*d^2)*e^2/
((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)+2*e*c*d/(a*e^2+c*d^2
)*(2*c*(x+d/e)-2/e*c*d)/(4*c*(a*e^2+c*d^2)/e^2-4/e^2*c^2*d^2)/((x+d/e)^2*c
-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-1/(a*e^2+c*d^2)*e^2/((a*e^2+c*d^
2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2
)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

3.335.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(112) = 224$.

Time = 2.92 (sec) , antiderivative size = 1323, normalized size of antiderivative = 10.76

$$\int \frac{x^3}{(d+ex)(a+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2
+ a^2*c*e^4)*x^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a)
+ (c^3*d^3*x^2 + a*c^2*d^3)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2
 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x -
a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a*c^2*d^3*e + a^2*c*
d*e^3 - (a*c^2*d^2*e^2 + a^2*c*e^4)*x)*sqrt(c*x^2 + a))/(a*c^4*d^4*e + 2*a
^2*c^3*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2*a*c^4*d^2*e^3 + a^2*c^3*e^5)
*x^2), 1/2*(2*(c^3*d^3*x^2 + a*c^2*d^3)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-
c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2
 + a*c*e^2)*x^2)) + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*
a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*s
qrt(c)*x - a) + 2*(a*c^2*d^3*e + a^2*c*d*e^3 - (a*c^2*d^2*e^2 + a^2*c*e^4)
*x)*sqrt(c*x^2 + a))/(a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 + (c^5
*d^4*e + 2*a*c^4*d^2*e^3 + a^2*c^3*e^5)*x^2), -1/2*(2*(a*c^2*d^4 + 2*a^2*c
*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(-c)
*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c^3*d^3*x^2 + a*c^2*d^3)*sqrt(c*d^2
 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x
^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e
*x + d^2)) - 2*(a*c^2*d^3*e + a^2*c*d*e^3 - (a*c^2*d^2*e^2 + a^2*c*e^4)*x)
*sqrt(c*x^2 + a))/(a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 + (c^5...
```

3.335.6 Sympy [F]

$$\int \frac{x^3}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \frac{x^3}{\left(a+cx^2\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(x**3/(e*x+d)/(c*x**2+a)**(3/2),x)
```

output
```
Integral(x**3/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.335.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.335.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx=\int \frac{x^3}{\left(c\,x^2+a\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

input
```
int(x^3/((a + c*x^2)^(3/2)*(d + e*x)),x)
```

output
```
int(x^3/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.336 $\int \frac{x^2}{(d+ex)(a+cx^2)^{3/2}}\,dx$

3.336.1 Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \frac{x^2}{(d+ex)(a+cx^2)^{3/2}}\,dx = -\frac{ae+cdx}{c(cd^2+ae^2)\sqrt{a+cx^2}} - \frac{d^2\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{(cd^2+ae^2)^{3/2}}$$

output
```
-d^2*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^
2)^(3/2)+(-c*d*x-a*e)/c/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)
```

3.336.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.15

$$\int \frac{x^2}{(d+ex)(a+cx^2)^{3/2}}\,dx = \frac{-ae-cdx}{c(cd^2+ae^2)\sqrt{a+cx^2}} + \frac{2d^2\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,x}{\sqrt{a}(d+ex)-d\sqrt{a+cx^2}}\right)}{(-cd^2-ae^2)^{3/2}}$$

input
```
Integrate[x^2/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

output
```
(-(a*e) - c*d*x)/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + (2*d^2*ArcTan[(Sqrt
[-(c*d^2) - a*e^2]*x)/(Sqrt[a]*(d + e*x) - d*Sqrt[a + c*x^2])])/(-(c*d^2)
- a*e^2)^(3/2)
```

3.336.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {601, 25, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+cx^2)^{3/2}(d+ex)}dx$$

$$\Big\downarrow \ 601$$

$$-\frac{\int -\frac{ad^2}{(cd^2+ae^2)(d+ex)\sqrt{cx^2+a}}dx}{a}-\frac{ae+cdx}{c\sqrt{a+cx^2}(ae^2+cd^2)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{ad^2}{(cd^2+ae^2)(d+ex)\sqrt{cx^2+a}}dx}{a}-\frac{ae+cdx}{c\sqrt{a+cx^2}(ae^2+cd^2)}$$

$$\Big\downarrow \ 27$$

$$\frac{d^2\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{ae^2+cd^2}-\frac{ae+cdx}{c\sqrt{a+cx^2}(ae^2+cd^2)}$$

$$\Big\downarrow \ 488$$

$$-\frac{d^2\int \frac{1}{cd^2+ae^2-\frac{(ae-cdx)^2}{cx^2+a}}d\frac{ae-cdx}{\sqrt{cx^2+a}}}{ae^2+cd^2}-\frac{ae+cdx}{c\sqrt{a+cx^2}(ae^2+cd^2)}$$

$$\Big\downarrow \ 219$$

$$-\frac{d^2\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}}-\frac{ae+cdx}{c\sqrt{a+cx^2}(ae^2+cd^2)}$$

input
```
Int[x^2/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

output
```
-((a*e + c*d*x)/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2])) - (d^2*ArcTanh[(a*e -
 c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

3.336.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

3.336.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(88) = 176$.

Time = 0.42 (sec) , antiderivative size = 355, normalized size of antiderivative = 3.74

method	result
default	$-\frac{1}{ec\sqrt{c\,x^2+a}}-\frac{dx}{e^2a\sqrt{c\,x^2+a}}+\frac{d^2\left(\frac{e^2}{\left(e^2a+c\,d^2\right)\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}+\frac{2ecd\left(2c\left(x+\frac{d}{e}\right)-\frac{2cd}{e}\right)}{\left(e^2a+c\,d^2\right)\left(\frac{4c\left(e^2a+c\,d^2\right)}{e^2}-\frac{4c^2d^2}{e^2}\right)\sqrt{\left(x+\frac{d}{e}\right)^2c}}\right.}{}$

3.336. $\int \frac{x^2}{(d+ex)(a+cx^2)^{3/2}}\,dx$

input
```
int(x^2/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/e/c/(c*x^2+a)^(1/2)-d/e^2*x/a/(c*x^2+a)^(1/2)+d^2/e^3*(1/(a*e^2+c*d^2)*
e^2/((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)+2*e*c*d/(a*e^2+c
*d^2)*(2*c*(x+d/e)-2/e*c*d)/(4*c*(a*e^2+c*d^2)/e^2-4/e^2*c^2*d^2)/((x+d/e)
^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-1/(a*e^2+c*d^2)*e^2/((a*e^2+
c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)
/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)
))
```

3.336.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(88) = 176$.

Time = 0.36 (sec) , antiderivative size = 455, normalized size of antiderivative = 4.79

$$\int \frac{x^2}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \left[\frac{(c^2d^2x^2+acd^2)\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2-2\,\sqrt{cd^2+ae^2}(cdx\cdots}{e^2x^2+2\,dex+d^2}\right.}{2\left(ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4+(c^4d^4+\cdots\right.}\right.$$

$$-\frac{(c^2d^2x^2+acd^2)\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)+(acd^2e+a^2e^3+(c^2d^3+acde^2)x)\sqrt{cx^2+\cdots}}{ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4+(c^4d^4+2\,ac^3d^2e^2+a^2c^2e^4)x^2}$$

input
```
integrate(x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((c^2*d^2*x^2 + a*c*d^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d
^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x
- a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(a*c*d^2*e + a^2*e^
3 + (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a*c^3*d^4 + 2*a^2*c^2*d^2*e
^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^2), -((c^2*d^
2*x^2 + a*c*d^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x -
 a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (a*
c*d^2*e + a^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a*c^3*d^4 +
 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)
*x^2)]
```

3.336.6 Sympy [F]

$$\int \frac{x^2}{(d+ex)(a+cx^2)^{3/2}}\,dx = \int \frac{x^2}{(a+cx^2)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(x**2/(e*x+d)/(c*x**2+a)**(3/2),x)
```

output
```
Integral(x**2/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.336.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(88) = 176$.

Time = 0.22 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.88

$$\begin{aligned}\int \frac{x^2}{(d+ex)(a+cx^2)^{3/2}}\,dx &= \frac{cd^3x}{\sqrt{cx^2+a}acd^2e^2+\sqrt{cx^2+a}a^2e^4} \\ &+ \frac{d^2}{\sqrt{cx^2+a}cd^2e+\sqrt{cx^2+a}ae^3} - \frac{dx}{\sqrt{cx^2+a}ae^2} \\ &+ \frac{d^2\operatorname{arsinh}\left(\frac{cdx}{e\sqrt{\frac{ac}{e^2}}|ex+d|}-\frac{a}{\sqrt{\frac{ac}{e^2}}|ex+d|}\right)}{\left(a+\frac{cd^2}{e^2}\right)^{\frac{3}{2}}e^3} - \frac{1}{\sqrt{cx^2+a}ce}\end{aligned}$$

input
```
integrate(x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
c*d^3*x/(sqrt(c*x^2 + a)*a*c*d^2*e^2 + sqrt(c*x^2 + a)*a^2*e^4) + d^2/(sqr
t(c*x^2 + a)*c*d^2*e + sqrt(c*x^2 + a)*a*e^3) - d*x/(sqrt(c*x^2 + a)*a*e^2
) + d^2*arcsinh(c*d*x/(e*sqrt(a*c/e^2)*abs(e*x + d)) - a/(sqrt(a*c/e^2)*ab
s(e*x + d)))/((a + c*d^2/e^2)^(3/2)*e^3) - 1/(sqrt(c*x^2 + a)*c*e)
```

3.336.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(88) = 176$.

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.91

$$\int \frac{x^2}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = -\frac{2\,d^2\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{(cd^2+ae^2)\sqrt{-cd^2-ae^2}} - \frac{\frac{(c^2d^3+acde^2)x}{c^3d^4+2\,ac^2d^2e^2+a^2ce^4}+\frac{acd^2e+a^2e^3}{c^3d^4+2\,ac^2d^2e^2+a^2ce^4}}{\sqrt{cx^2+a}}$$

input
```
integrate(x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-2*d^2*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 -
a*e^2))/((c*d^2 + a*e^2)*sqrt(-c*d^2 - a*e^2)) - ((c^2*d^3 + a*c*d*e^2)*x/
(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4) + (a*c*d^2*e + a^2*e^3)/(c^3*d^4 +
 2*a*c^2*d^2*e^2 + a^2*c*e^4))/sqrt(c*x^2 + a)
```

3.336.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \frac{x^2}{\left(c\,x^2+a\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

input
```
int(x^2/((a + c*x^2)^(3/2)*(d + e*x)),x)
```

output
```
int(x^2/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.337 $\int \frac{x}{(d+ex)(a+cx^2)^{3/2}}\,dx$

3.337.1 Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{x}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx=-\frac{d-ex}{(cd^2+ae^2)\sqrt{a+cx^2}}+\frac{de\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{(cd^2+ae^2)^{3/2}}$$

output
```
d*e*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2
)^(3/2)+(e*x-d)/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)
```

3.337.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.12

$$\int \frac{x}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx=\frac{-d+ex}{(cd^2+ae^2)\sqrt{a+cx^2}}-\frac{2de\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}}$$

input
```
Integrate[x/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

output
```
(-d + e*x)/((c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (2*d*e*ArcTan[(Sqrt[c]*(d +
 e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^2)^(3/
2)
```

3.337.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {593, 25, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+cx^2)^{3/2}(d+ex)}dx$$

$$\Big\downarrow \text{ 593}$$

$$\frac{e\int -\frac{d}{(d+ex)\sqrt{cx^2+a}}dx}{ae^2+cd^2}-\frac{d-ex}{\sqrt{a+cx^2}(ae^2+cd^2)}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{e\int \frac{d}{(d+ex)\sqrt{cx^2+a}}dx}{ae^2+cd^2}-\frac{d-ex}{\sqrt{a+cx^2}(ae^2+cd^2)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{de\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{ae^2+cd^2}-\frac{d-ex}{\sqrt{a+cx^2}(ae^2+cd^2)}$$

$$\Big\downarrow \text{ 488}$$

$$\frac{de\int \frac{1}{cd^2+ae^2-\frac{(ae-cdx)^2}{cx^2+a}}d\frac{ae-cdx}{\sqrt{cx^2+a}}}{ae^2+cd^2}-\frac{d-ex}{\sqrt{a+cx^2}(ae^2+cd^2)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{de\,\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}}-\frac{d-ex}{\sqrt{a+cx^2}(ae^2+cd^2)}$$

input
```
Int[x/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

output
```
-((d - e*x)/((c*d^2 + a*e^2)*Sqrt[a + c*x^2])) + (d*e*ArcTanh[(a*e - c*d*x
)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

3.337.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

3.337.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(80) = 160$.

Time = 0.40 (sec) , antiderivative size = 335, normalized size of antiderivative = 3.81

method	result
default	$\frac{x}{ea\sqrt{c\,x^{2}+a}}-\frac{d\left(\frac{e^{2}}{\left(e^{2}a+c\,d^{2}\right)\sqrt{\left(x+\frac{d}{e}\right)^{2}c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^{2}a+c\,d^{2}}{e^{2}}}}+\frac{2ecd\left(2c\left(x+\frac{d}{e}\right)-\frac{2cd}{e}\right)}{\left(e^{2}a+c\,d^{2}\right)\left(\frac{4c\left(e^{2}a+c\,d^{2}\right)}{e^{2}}-\frac{4c^{2}d^{2}}{e^{2}}\right)\sqrt{\left(x+\frac{d}{e}\right)^{2}c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^{2}a+c\,d^{2}}{e^{2}}}}\right.}{e^{2}}$

input
```
int(x/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e*x/a/(c*x^2+a)^(1/2)-d/e^2*(1/(a*e^2+c*d^2)*e^2/((x+d/e)^2*c-2/e*c*d*(x
+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)+2*e*c*d/(a*e^2+c*d^2)*(2*c*(x+d/e)-2/e*c*d)
/(4*c*(a*e^2+c*d^2)/e^2-4/e^2*c^2*d^2)/((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2
+c*d^2)/e^2)^(1/2)-1/(a*e^2+c*d^2)*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*
e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/
e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

3.337.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(81) = 162$.

Time = 0.33 (sec) , antiderivative size = 425, normalized size of antiderivative = 4.83

$$\int \frac{x}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \left[\frac{(cdex^2+ade)\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2+2\,\sqrt{cd^2+ae^2}(cdx-a}{e^2x^2+2\,dex+d^2}\right.}{2\,(ac^2d^4+2\,a^2cd^2e^2+a^3e^4+(c^3d^4+2\,ac}\right.$$

input
```
integrate(x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((c*d*e*x^2 + a*d*e)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 -
 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*
e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c*d^3 + a*d*e^2 - (c*d
^2*e + a*e^3)*x)*sqrt(c*x^2 + a))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 +
 (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2), ((c*d*e*x^2 + a*d*e)*sqrt(-
c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(
a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (c*d^3 + a*d*e^2 - (c*d^2*
e + a*e^3)*x)*sqrt(c*x^2 + a))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c
^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)]
```

3.337.6 Sympy [F]

$$\int \frac{x}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \frac{x}{\left(a+cx^2\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(x/(e*x+d)/(c*x**2+a)**(3/2),x)
```

output
```
Integral(x/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.337.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.77

$$\int \frac{x}{(d+ex)\,(a+cx^2)^{3/2}}\,dx = -\frac{cd^2x}{\sqrt{cx^2+a}acd^2e+\sqrt{cx^2+a}a^2e^3} - \frac{d}{\sqrt{cx^2+a}cd^2+\sqrt{cx^2+a}ae^2} + \frac{x}{\sqrt{cx^2+a}ae} - \frac{d\,\text{arsinh}\left(\frac{cdx}{e\sqrt{\frac{ac}{e^2}}|ex+d|} - \frac{a}{\sqrt{\frac{ac}{e^2}}|ex+d|}\right)}{\left(a+\frac{cd^2}{e^2}\right)^{\frac{3}{2}}e^2}$$

input
```
integrate(x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-c*d^2*x/(sqrt(c*x^2 + a)*a*c*d^2*e + sqrt(c*x^2 + a)*a^2*e^3) - d/(sqrt(c
*x^2 + a)*c*d^2 + sqrt(c*x^2 + a)*a*e^2) + x/(sqrt(c*x^2 + a)*a*e) - d*arc
sinh(c*d*x/(e*sqrt(a*c/e^2)*abs(e*x + d)) - a/(sqrt(a*c/e^2)*abs(e*x + d))
)/((a + c*d^2/e^2)^(3/2)*e^2)
```

3.337.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(81) = 162$.

Time = 0.30 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.91

$$\int \frac{x}{(d+ex)\,(a+cx^2)^{3/2}}\,dx = \frac{2\,de\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{(cd^2+ae^2)\sqrt{-cd^2-ae^2}} + \frac{\frac{(cd^2e+ae^3)x}{c^2d^4+2\,acd^2e^2+a^2e^4} - \frac{cd^3+ade^2}{c^2d^4+2\,acd^2e^2+a^2e^4}}{\sqrt{cx^2+a}}$$

input
```
integrate(x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
2*d*e*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a
*e^2))/((c*d^2 + a*e^2)*sqrt(-c*d^2 - a*e^2)) + ((c*d^2*e + a*e^3)*x/(c^2*
d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - (c*d^3 + a*d*e^2)/(c^2*d^4 + 2*a*c*d^2*e^
2 + a^2*e^4))/sqrt(c*x^2 + a)
```

3.337.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx=\int \frac{x}{\left(c\,x^2+a\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

input
```
int(x/((a + c*x^2)^(3/2)*(d + e*x)),x)
```

output
```
int(x/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.338 $\int \frac{1}{(d+ex)(a+cx^2)^{3/2}}\,dx$

3.338.1 Optimal result

Integrand size = 19, antiderivative size = 94

$$\int \frac{1}{(d+ex)\,(a+cx^2)^{3/2}}\,dx = \frac{ae+cdx}{a\,(cd^2+ae^2)\sqrt{a+cx^2}} - \frac{e^2\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{(cd^2+ae^2)^{3/2}}$$

output
```
-e^2*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^
2)^(3/2)+(c*d*x+a*e)/a/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)
```

3.338.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)\,(a+cx^2)^{3/2}}\,dx = \frac{ae+cdx}{a\,(cd^2+ae^2)\sqrt{a+cx^2}} + \frac{2e^2\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}}$$

input
```
Integrate[1/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

output
```
(a*e + c*d*x)/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + (2*e^2*ArcTan[(Sqrt[c]
*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^2
)^(3/2)
```

3.338.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {496, 25, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)^{3/2}(d+ex)}dx$$

$$\Big\downarrow \ 496$$

$$\frac{ae+cdx}{a\sqrt{a+cx^2}(ae^2+cd^2)}-\frac{\int -\frac{ae^2}{(d+ex)\sqrt{cx^2+a}}dx}{a(ae^2+cd^2)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{ae^2}{(d+ex)\sqrt{cx^2+a}}dx}{a(ae^2+cd^2)}+\frac{ae+cdx}{a\sqrt{a+cx^2}(ae^2+cd^2)}$$

$$\Big\downarrow \ 27$$

$$\frac{e^2\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{ae^2+cd^2}+\frac{ae+cdx}{a\sqrt{a+cx^2}(ae^2+cd^2)}$$

$$\Big\downarrow \ 488$$

$$\frac{ae+cdx}{a\sqrt{a+cx^2}(ae^2+cd^2)}-\frac{e^2\int \frac{1}{cd^2+ae^2-\frac{(ae-cdx)^2}{cx^2+a}}d\frac{ae-cdx}{\sqrt{cx^2+a}}}{ae^2+cd^2}$$

$$\Big\downarrow \ 219$$

$$\frac{ae+cdx}{a\sqrt{a+cx^2}(ae^2+cd^2)}-\frac{e^2\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}}$$

input
```
Int[1/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

output
```
(a*e + c*d*x)/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (e^2*ArcTanh[(a*e - c*
d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

3.338.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

3.338.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(86) = 172$.

Time = 0.35 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.35

method	result
default	$\frac{\frac{e^2}{\left(e^2a+c\,d^2\right)\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}+\frac{2ecd\left(2c\left(x+\frac{d}{e}\right)-\frac{2cd}{e}\right)}{\left(e^2a+c\,d^2\right)\left(\frac{4c\left(e^2a+c\,d^2\right)}{e^2}-\frac{4c^2d^2}{e^2}\right)\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}-\frac{e^2\ln\left(\frac{\frac{2e^2a+2c}{e^2}}{}\right.}{}}{e}$

input
```
int(1/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e*(1/(a*e^2+c*d^2)*e^2/((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(
1/2)+2*e*c*d/(a*e^2+c*d^2)*(2*c*(x+d/e)-2/e*c*d)/(4*c*(a*e^2+c*d^2)/e^2-4/
e^2*c^2*d^2)/((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-1/(a*e^
2+c*d^2)*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+
d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2
)/e^2)^(1/2))/(x+d/e)))
```

3.338.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(87) = 174$.

Time = 0.32 (sec) , antiderivative size = 456, normalized size of antiderivative = 4.85

$$\int \frac{1}{(d+ex)(a+cx^2)^{3/2}}\,dx = \left[\frac{(ace^2x^2+a^2e^2)\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2-2\,\sqrt{cd^2+ae^2}(cdx\cdots}{e^2x^2+2\,dex+d^2}\right.}{2\,(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4+(ac^3d^4+\cdots}\right.$$

$$-\frac{(ace^2x^2+a^2e^2)\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-(acd^2e+a^2e^3+(c^2d^3+acde^2)x)\sqrt{cx^2+\cdots}}{a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4+(ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4)x^2}$$

input
```
integrate(1/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((a*c*e^2*x^2 + a^2*e^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d
^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x
- a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a*c*d^2*e + a^2*e^
3 + (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e
^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^2), -((a*c*e^
2*x^2 + a^2*e^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x -
 a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (a*
c*d^2*e + a^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a^2*c^2*d^4
 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)
*x^2)]
```

3.338.6 Sympy [F]

$$\int \frac{1}{(d+ex)\,(a+cx^2)^{3/2}}\,dx = \int \frac{1}{(a+cx^2)^{\frac{3}{2}}\,(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x**2+a)**(3/2),x)
```

output
```
Integral(1/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.338.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.39

$$\int \frac{1}{(d+ex)\,(a+cx^2)^{3/2}}\,dx = \frac{cdx}{\sqrt{cx^2+a}acd^2+\sqrt{cx^2+a}a^2e^2} + \frac{1}{\frac{\sqrt{cx^2+a}cd^2}{e}+\sqrt{cx^2+a}ae} + \frac{\operatorname{arsinh}\left(\frac{cdx}{e\sqrt{\frac{ac}{e^2}}|ex+d|}-\frac{a}{\sqrt{\frac{ac}{e^2}}|ex+d|}\right)}{\left(a+\frac{cd^2}{e^2}\right)^{\frac{3}{2}}e}$$

input
```
integrate(1/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
c*d*x/(sqrt(c*x^2 + a)*a*c*d^2 + sqrt(c*x^2 + a)*a^2*e^2) + 1/(sqrt(c*x^2
+ a)*c*d^2/e + sqrt(c*x^2 + a)*a*e) + arcsinh(c*d*x/(e*sqrt(a*c/e^2)*abs(e
*x + d)) - a/(sqrt(a*c/e^2)*abs(e*x + d)))/((a + c*d^2/e^2)^(3/2)*e)
```

3.338.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(87) = 174.

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.91

$$\int \frac{1}{(d+ex)\,(a+cx^2)^{3/2}}\,dx = -\frac{2\,e^2\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{(cd^2+ae^2)\sqrt{-cd^2-ae^2}} + \frac{\frac{(c^2d^3+acde^2)x}{ac^2d^4+2\,a^2cd^2e^2+a^3e^4}+\frac{acd^2e+a^2e^3}{ac^2d^4+2\,a^2cd^2e^2+a^3e^4}}{\sqrt{cx^2+a}}$$

input
```
integrate(1/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-2*e^2*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 -
a*e^2))/((c*d^2 + a*e^2)*sqrt(-c*d^2 - a*e^2)) + ((c^2*d^3 + a*c*d*e^2)*x/
(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4) + (a*c*d^2*e + a^2*e^3)/(a*c^2*d^4
 + 2*a^2*c*d^2*e^2 + a^3*e^4))/sqrt(c*x^2 + a)
```

3.338.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\,(a+cx^2)^{3/2}}\,dx = \int \frac{1}{(c\,x^2+a)^{3/2}\,(d+e\,x)}\,dx$$

input
```
int(1/((a + c*x^2)^(3/2)*(d + e*x)),x)
```

output
```
int(1/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.339 $\int \frac{1}{x(d+ex)(a+cx^2)^{3/2}}\,dx$

3.339.1 Optimal result

Integrand size = 22, antiderivative size = 147

$$\int \frac{1}{x(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \frac{1}{ad\sqrt{a+cx^2}} - \frac{e(ae+cdx)}{ad\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{e^3\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\left(cd^2+ae^2\right)^{3/2}} - \frac{\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}$$

output
```
e^3*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d/(a*e^2+c*d
^2)^(3/2)-arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d+1/a/d/(c*x^2+a)^(1/2)
-e*(c*d*x+a*e)/a/d/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)
```

3.339.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.98

$$\int \frac{1}{x(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \frac{c(d-ex)}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{2e^3\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{d\left(-cd^2-ae^2\right)^{3/2}} + \frac{2\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}$$

input
```
Integrate[1/(x*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

output
```
(c*(d - e*x))/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (2*e^3*ArcTan[(Sqrt[c]
*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(d*(-(c*d^2) - a*
e^2)^(3/2)) + (2*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/(a^(3/2)*
d)
```

3.339.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+cx^2\right)^{3/2}(d+ex)}dx$$

$$\Big\downarrow \text{617}$$

$$\int \left(\frac{1}{dx\left(a+cx^2\right)^{3/2}}-\frac{e}{d\left(a+cx^2\right)^{3/2}(d+ex)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{e^3\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\left(ae^2+cd^2\right)^{3/2}}-\frac{e(ae+cdx)}{ad\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{1}{ad\sqrt{a+cx^2}}$$

input
```
Int[1/(x*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

output
```
1/(a*d*Sqrt[a + c*x^2]) - (e*(a*e + c*d*x))/(a*d*(c*d^2 + a*e^2)*Sqrt[a +
c*x^2]) + (e^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])
])/(d*(c*d^2 + a*e^2)^(3/2)) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(a^(3/2)*d
)
```

3.339.3.1 Defintions of rubi rules used

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.339.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(131) = 262$.

Time = 0.37 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.47

method	result
default	$\frac{\frac{1}{a\sqrt{c\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}}{d}-\frac{\frac{e^2}{\left(e^2a+c\,d^2\right)\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}+\frac{2ecd\left(2c\left(x+\frac{d}{e}\right)-\frac{2cd}{e}\right)}{\left(e^2a+c\,d^2\right)\left(\frac{4c\left(e^2a+c\,d^2\right)}{e^2}-\frac{4c^2d^2}{e^2}\right)\sqrt{\left(x+\frac{d}{e}\right)^2c-}}$

input
```
int(1/x/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a/(c*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x))-
1/d*(1/(a*e^2+c*d^2)*e^2/((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(
1/2)+2*e*c*d/(a*e^2+c*d^2)*(2*c*(x+d/e)-2/e*c*d)/(4*c*(a*e^2+c*d^2)/e^2-4/
e^2*c^2*d^2)/((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-1/(a*e^
2+c*d^2)*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+
d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2
)/e^2)^(1/2))/(x+d/e)))
```

3.339.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(132) = 264$.

Time = 0.47 (sec) , antiderivative size = 1325, normalized size of antiderivative = 9.01

$$\int \frac{1}{x(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((a^2*c*e^3*x^2 + a^3*e^3)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c
*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*
x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (a*c^2*d^4 + 2*a^2*
c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(a)
*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(a*c^2*d^4 + a^2*
c*d^2*e^2 - (a*c^2*d^3*e + a^2*c*d*e^3)*x)*sqrt(c*x^2 + a))/(a^3*c^2*d^5 +
 2*a^4*c*d^3*e^2 + a^5*d*e^4 + (a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*
e^4)*x^2), 1/2*(2*(a^2*c*e^3*x^2 + a^3*e^3)*sqrt(-c*d^2 - a*e^2)*arctan(sq
rt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2
*d^2 + a*c*e^2)*x^2)) + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4
+ 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a
)*sqrt(a) + 2*a)/x^2) + 2*(a*c^2*d^4 + a^2*c*d^2*e^2 - (a*c^2*d^3*e + a^2*
c*d*e^3)*x)*sqrt(c*x^2 + a))/(a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4 +
(a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^2), 1/2*(2*(a*c^2*d^4 +
2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*s
qrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (a^2*c*e^3*x^2 + a^3*e^3)*sqrt(
c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e
^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 +
2*d*e*x + d^2)) + 2*(a*c^2*d^4 + a^2*c*d^2*e^2 - (a*c^2*d^3*e + a^2*c*d*e^
3)*x)*sqrt(c*x^2 + a))/(a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4 + (a^...
```

3.339.6 Sympy [F]

$$\int \frac{1}{x(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \frac{1}{x\left(a+cx^2\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(1/x/(e*x+d)/(c*x**2+a)**(3/2),x)
```

output
```
Integral(1/(x*(a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.339.7 Maxima [F]

$$\int \frac{1}{x(d+ex)\left(a+cx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^2+a\right)^{\frac{3}{2}}(ex+d)x}\,dx$$

input
```
integrate(1/x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)*x), x)
```

3.339.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(d+ex)\left(a+cx^2\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.339.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)\left(a+cx^2\right)^{3/2}}\,dx=\int \frac{1}{x\left(c\,x^2+a\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

input
```
int(1/(x*(a + c*x^2)^(3/2)*(d + e*x)),x)
```

output
```
int(1/(x*(a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.340 $\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^{3/2}}\,dx$

3.340.1 Optimal result

Integrand size = 22, antiderivative size = 194

$$\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = -\frac{e}{ad^2\sqrt{a+cx^2}}-\frac{1}{adx\sqrt{a+cx^2}}-\frac{2cx}{a^2d\sqrt{a+cx^2}}+\frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{e^4\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\left(cd^2+ae^2\right)^{3/2}}+\frac{e\,\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}$$

output
```
-e^4*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^2/(a*e^2+
c*d^2)^(3/2)+e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d^2-e/a/d^2/(c*x^2
+a)^(1/2)-1/a/d/x/(c*x^2+a)^(1/2)-2*c*x/a^2/d/(c*x^2+a)^(1/2)+e^2*(c*d*x+a
*e)/a/d^2/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)
```

3.340.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \frac{-\frac{d\left(a^2e^2+2c^2d^2x^2+ac\left(d^2+dex+e^2x^2\right)\right)}{a^2(cd^2+ae^2)x\sqrt{a+cx^2}}+\frac{2e^4\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}}-\frac{2e\,\text{arctanh}\left(\frac{\sqrt{c}x}{}\right.}{a^{3/2}}}{d^2}$$

input
```
Integrate[1/(x^2*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

output
```
(-((d*(a^2*e^2 + 2*c^2*d^2*x^2 + a*c*(d^2 + d*e*x + e^2*x^2)))/(a^2*(c*d^2
 + a*e^2)*x*Sqrt[a + c*x^2])) + (2*e^4*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[
a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^2)^(3/2) - (2*e*ArcTa
nh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/a^(3/2))/d^2
```

3.340.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+cx^2)^{3/2} (d+ex)} dx$$
$$\Big\downarrow \text{617}$$
$$\int \left(\frac{e^2}{d^2 (a+cx^2)^{3/2} (d+ex)} - \frac{e}{d^2 x (a+cx^2)^{3/2}} + \frac{1}{dx^2 (a+cx^2)^{3/2}}\right) dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{e\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2} - \frac{2cx}{a^2 d\sqrt{a+cx^2}} - \frac{e^4\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2 (ae^2+cd^2)^{3/2}} + \frac{e^2(ae+cdx)}{ad^2\sqrt{a+cx^2}(ae^2+cd^2)} - \frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}}$$

input
```
Int[1/(x^2*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

output
```
-(e/(a*d^2*Sqrt[a + c*x^2])) - 1/(a*d*x*Sqrt[a + c*x^2]) - (2*c*x)/(a^2*d*
Sqrt[a + c*x^2]) + (e^2*(a*e + c*d*x))/(a*d^2*(c*d^2 + a*e^2)*Sqrt[a + c*x
^2]) - (e^4*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/
(d^2*(c*d^2 + a*e^2)^(3/2)) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2
)*d^2)
```

3.340.3.1 Defintions of rubi rules used

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.340.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(174) = 348$.

Time = 0.39 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.86

method	result
risch	$-\frac{\sqrt{c x^2+a}}{a^2 d x}+\frac{e \ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c x^2+a}}{x}\right)}{a^{\frac{3}{2}} d^2}-\frac{c\sqrt{\left(x-\frac{\sqrt{-ac}}{c}\right)^2 c+2\sqrt{-ac}\left(x-\frac{\sqrt{-ac}}{c}\right)}}{2a^2\left(e\sqrt{-ac}+cd\right)\left(x-\frac{\sqrt{-ac}}{c}\right)}+\frac{c\sqrt{\left(x+\frac{\sqrt{-ac}}{c}\right)^2 c-2\sqrt{-ac}\left(x+\frac{\sqrt{-ac}}{c}\right)}}{2a^2\left(e\sqrt{-ac}-cd\right)\left(x+\frac{\sqrt{-ac}}{c}\right)}+$
default	$\frac{-\frac{1}{ax\sqrt{c x^2+a}}-\frac{2cx}{a^2\sqrt{c x^2+a}}}{d}-\frac{e\left(\frac{1}{a\sqrt{c x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{d^2}+\frac{e\left(\frac{e^2}{\left(e^2 a+c d^2\right)\sqrt{\left(x+\frac{d}{e}\right)^2 c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2 a+c d^2}{e^2}}}+\right.}{}$

input
```
int(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2/d*(c*x^2+a)^(1/2)/x+1/a^(3/2)/d^2*e*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/
2))/x)-1/2/a^2*c/(e*(-a*c)^(1/2)+c*d)/(x-(-a*c)^(1/2)/c)*((x-(-a*c)^(1/2)/
c)^2*c+2*(-a*c)^(1/2)*(x-(-a*c)^(1/2)/c))^(1/2)+1/2/a^2*c/(e*(-a*c)^(1/2)-
c*d)/(x+(-a*c)^(1/2)/c)*((x+(-a*c)^(1/2)/c)^2*c-2*(-a*c)^(1/2)*(x+(-a*c)^(
1/2)/c))^(1/2)+1/d^2*c*e^3/(e*(-a*c)^(1/2)+c*d)/(e*(-a*c)^(1/2)-c*d)/((a*e
^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d
^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d
/e))
```

3.340.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(175) = 350$.

Time = 0.50 (sec) , antiderivative size = 1556, normalized size of antiderivative = 8.02

$$\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((a^2*c*e^4*x^3 + a^3*e^4*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a
*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*
d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + ((c^3*d^4*e + 2*a
*c^2*d^2*e^3 + a^2*c*e^5)*x^3 + (a*c^2*d^4*e + 2*a^2*c*d^2*e^3 + a^3*e^5)*
x)*sqrt(a)*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a*c^2*
d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4 + (2*c^3*d^5 + 3*a*c^2*d^3*e^2 + a^2*c*d
*e^4)*x^2 + (a*c^2*d^4*e + a^2*c*d^2*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c^3*d^
6 + 2*a^3*c^2*d^4*e^2 + a^4*c*d^2*e^4)*x^3 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^
2 + a^5*d^2*e^4)*x), -1/2*(2*(a^2*c*e^4*x^3 + a^3*e^4*x)*sqrt(-c*d^2 - a*e
^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a
^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - ((c^3*d^4*e + 2*a*c^2*d^2*e^3 + a^2*c
*e^5)*x^3 + (a*c^2*d^4*e + 2*a^2*c*d^2*e^3 + a^3*e^5)*x)*sqrt(a)*log(-(c*x
^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(a*c^2*d^5 + 2*a^2*c*d^3*e^
2 + a^3*d*e^4 + (2*c^3*d^5 + 3*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2 + (a*c^2*d
^4*e + a^2*c*d^2*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c^3*d^6 + 2*a^3*c^2*d^4*e^
2 + a^4*c*d^2*e^4)*x^3 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x),
 -1/2*(2*((c^3*d^4*e + 2*a*c^2*d^2*e^3 + a^2*c*e^5)*x^3 + (a*c^2*d^4*e + 2
*a^2*c*d^2*e^3 + a^3*e^5)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (
a^2*c*e^4*x^3 + a^3*e^4*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2
- 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x ...
```

3.340.6 Sympy [F]

$$\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \frac{1}{x^2\left(a+cx^2\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(1/x**2/(e*x+d)/(c*x**2+a)**(3/2),x)
```

output
```
Integral(1/(x**2*(a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.340.7 Maxima [F]

$$\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \frac{1}{(cx^2+a)^{\frac{3}{2}}(ex+d)x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)*x^2), x)
```

3.340.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.41

$$\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = -\frac{2\,e^4\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{(cd^4+ad^2e^2)\sqrt{-cd^2-ae^2}} - \frac{\frac{(ac^3d^3+a^2c^2de^2)x}{a^3c^2d^4+2\,a^4cd^2e^2+a^5e^4}+\frac{a^2c^2d^2e+a^3ce^3}{a^3c^2d^4+2\,a^4cd^2e^2+a^5e^4}}{\sqrt{cx^2+a}} - \frac{2\,e\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}ad^2} + \frac{2\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2-a\right)ad}$$

input
```
integrate(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-2*e^4*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 -
a*e^2))/((c*d^4 + a*d^2*e^2)*sqrt(-c*d^2 - a*e^2)) - ((a*c^3*d^3 + a^2*c^2
*d*e^2)*x/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4) + (a^2*c^2*d^2*e + a^3
*c*e^3)/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))/sqrt(c*x^2 + a) - 2*e*a
rctan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*a*d^2) + 2*sqrt(c
)/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)*a*d)
```

3.340.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \frac{1}{x^2\left(c\,x^2+a\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

input
```
int(1/(x^2*(a + c*x^2)^(3/2)*(d + e*x)),x)
```

output
```
int(1/(x^2*(a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.341 $\int \frac{1}{x^3(d+ex)(a+cx^2)^{3/2}}\,dx$

3.341.1 Optimal result

Integrand size = 22, antiderivative size = 276

$$\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = -\frac{3c}{2a^2d\sqrt{a+cx^2}}+\frac{e^2}{ad^3\sqrt{a+cx^2}} - \frac{1}{2adx^2\sqrt{a+cx^2}}+\frac{e}{ad^2x\sqrt{a+cx^2}}+\frac{2cex}{a^2d^2\sqrt{a+cx^2}}-\frac{e^3(ae+cdx)}{ad^3\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{e^5\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\left(cd^2+ae^2\right)^{3/2}}+\frac{3c\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{5/2}d}-\frac{e^2\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^3}$$

output
```
e^5*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^3/(a*e^2+c
*d^2)^(3/2)+3/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(5/2)/d-e^2*arctanh((
c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d^3-3/2*c/a^2/d/(c*x^2+a)^(1/2)+e^2/a/d^3/
(c*x^2+a)^(1/2)-1/2/a/d/x^2/(c*x^2+a)^(1/2)+e/a/d^2/x/(c*x^2+a)^(1/2)+2*c*
e*x/a^2/d^2/(c*x^2+a)^(1/2)-e^3*(c*d*x+a*e)/a/d^3/(a*e^2+c*d^2)/(c*x^2+a)^
(1/2)
```

3.341.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = -\frac{\frac{d\left(c^2d^2x^2(3d-4ex)+a^2e^2(d-2ex)+ac\left(d^3-2d^2ex+de^2x^2-2e^3x^3\right)\right)}{a^2(cd^2+ae^2)x^2\sqrt{a+cx^2}}+\frac{4e^5\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}}+\frac{2(3cd^2-2ae^2)\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{5/2}}}{2d^3}$$

input
```
Integrate[1/(x^3*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

output
```
-1/2*((d*(c^2*d^2*x^2*(3*d - 4*e*x) + a^2*e^2*(d - 2*e*x) + a*c*(d^3 - 2*d
^2*e*x + d*e^2*x^2 - 2*e^3*x^3)))/(a^2*(c*d^2 + a*e^2)*x^2*Sqrt[a + c*x^2]
) + (4*e^5*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) -
a*e^2]])/(-(c*d^2) - a*e^2)^(3/2) + (2*(3*c*d^2 - 2*a*e^2)*ArcTanh[(Sqrt[c
]*x - Sqrt[a + c*x^2])/Sqrt[a]])/a^(5/2))/d^3
```

3.341.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+cx^2\right)^{3/2}(d+ex)}\,dx$$

$$\Big\downarrow \; 617$$

$$\int \left(-\frac{e^3}{d^3\left(a+cx^2\right)^{3/2}(d+ex)}+\frac{e^2}{d^3x\left(a+cx^2\right)^{3/2}}-\frac{e}{d^2x^2\left(a+cx^2\right)^{3/2}}+\frac{1}{dx^3\left(a+cx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{e^2\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^3}+\frac{3c\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{5/2}d}+\frac{2cex}{a^2d^2\sqrt{a+cx^2}}-\frac{3c}{2a^2d\sqrt{a+cx^2}}+\frac{e^5\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3\left(ae^2+cd^2\right)^{3/2}}+\frac{e^2}{ad^3\sqrt{a+cx^2}}+\frac{e}{ad^2x\sqrt{a+cx^2}}-\frac{e^3(ae+cdx)}{ad^3\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{1}{2adx^2\sqrt{a+cx^2}}$$

input
```
Int[1/(x^3*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

output
```
(-3*c)/(2*a^2*d*Sqrt[a + c*x^2]) + e^2/(a*d^3*Sqrt[a + c*x^2]) - 1/(2*a*d*
x^2*Sqrt[a + c*x^2]) + e/(a*d^2*x*Sqrt[a + c*x^2]) + (2*c*e*x)/(a^2*d^2*Sq
rt[a + c*x^2]) - (e^3*(a*e + c*d*x))/(a*d^3*(c*d^2 + a*e^2)*Sqrt[a + c*x^2
]) + (e^5*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d
^3*(c*d^2 + a*e^2)^(3/2)) + (3*c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(5
/2)*d) - (e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^3)
```

3.341.3.1 Defintions of rubi rules used

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.341. $\int \frac{1}{x^3(d+ex)(a+cx^2)^{3/2}}\,dx$

3.341.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.47

method	result
risch	$-\frac{\sqrt{c\,x^2+a}\,(-2ex+d)}{2a^2d^2x^2}-\frac{-\frac{\left(-2e^2a+3c\,d^2\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{d\sqrt{a}}+\frac{c^2d^2\sqrt{\left(x+\frac{\sqrt{-ac}}{c}\right)^2c-2\sqrt{-ac}\left(x+\frac{\sqrt{-ac}}{c}\right)}}{(e\sqrt{-ac}-cd)\sqrt{-ac}\left(x+\frac{\sqrt{-ac}}{c}\right)}+\frac{c^2d^2\sqrt{\left(x-\frac{\sqrt{-ac}}{c}\right)^2c+2}}{(e\sqrt{-ac}+cd)\sqrt{-ac}}}{2a^2}$
default	$\frac{-\frac{1}{2a\,x^2\sqrt{c\,x^2+a}}-\frac{3c\left(\frac{1}{a\sqrt{c\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}}{d}+\frac{e^2\left(\frac{1}{a\sqrt{c\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{d^3}-\frac{e\left(-\frac{1}{ax\sqrt{c\,x^2+a}}-\frac{2cx}{a^2\sqrt{c\,x^2}}\right.}{d^2}$

input
```
int(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*x^2+a)^(1/2)*(-2*e*x+d)/a^2/d^2/x^2-1/2/a^2/d^2*(-(-2*a*e^2+3*c*d^
2)/d/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)+c^2*d^2/(e*(-a*c)^(1/2)
-c*d)/(-a*c)^(1/2)/(x+(-a*c)^(1/2)/c)*((x+(-a*c)^(1/2)/c)^2*c-2*(-a*c)^(1/
2)*(x+(-a*c)^(1/2)/c))^(1/2)+c^2*d^2/(e*(-a*c)^(1/2)+c*d)/(-a*c)^(1/2)/(x-
(-a*c)^(1/2)/c)*((x-(-a*c)^(1/2)/c)^2*c+2*(-a*c)^(1/2)*(x-(-a*c)^(1/2)/c))
^(1/2)+2*c*a^2*e^4/(e*(-a*c)^(1/2)+c*d)/(e*(-a*c)^(1/2)-c*d)/d/((a*e^2+c*d
^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^
2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

3.341.5 Fricas [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 1943, normalized size of antiderivative = 7.04

$$\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(2*(a^3*c*e^5*x^4 + a^4*e^5*x^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x
 - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)
*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - ((3*c^4*d^6 +
 4*a*c^3*d^4*e^2 - a^2*c^2*d^2*e^4 - 2*a^3*c*e^6)*x^4 + (3*a*c^3*d^6 + 4*a
^2*c^2*d^4*e^2 - a^3*c*d^2*e^4 - 2*a^4*e^6)*x^2)*sqrt(a)*log(-(c*x^2 - 2*s
qrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a^2*c^2*d^6 + 2*a^3*c*d^4*e^2 + a^
4*d^2*e^4 - 2*(2*a*c^3*d^5*e + 3*a^2*c^2*d^3*e^3 + a^3*c*d*e^5)*x^3 + (3*a
*c^3*d^6 + 4*a^2*c^2*d^4*e^2 + a^3*c*d^2*e^4)*x^2 - 2*(a^2*c^2*d^5*e + 2*a
^3*c*d^3*e^3 + a^4*d*e^5)*x)*sqrt(c*x^2 + a))/((a^3*c^3*d^7 + 2*a^4*c^2*d^
5*e^2 + a^5*c*d^3*e^4)*x^4 + (a^4*c^2*d^7 + 2*a^5*c*d^5*e^2 + a^6*d^3*e^4)
*x^2), 1/4*(4*(a^3*c*e^5*x^4 + a^4*e^5*x^2)*sqrt(-c*d^2 - a*e^2)*arctan(sq
rt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2
*d^2 + a*c*e^2)*x^2)) - ((3*c^4*d^6 + 4*a*c^3*d^4*e^2 - a^2*c^2*d^2*e^4 -
2*a^3*c*e^6)*x^4 + (3*a*c^3*d^6 + 4*a^2*c^2*d^4*e^2 - a^3*c*d^2*e^4 - 2*a^
4*e^6)*x^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) -
2*(a^2*c^2*d^6 + 2*a^3*c*d^4*e^2 + a^4*d^2*e^4 - 2*(2*a*c^3*d^5*e + 3*a^2*
c^2*d^3*e^3 + a^3*c*d*e^5)*x^3 + (3*a*c^3*d^6 + 4*a^2*c^2*d^4*e^2 + a^3*c*
d^2*e^4)*x^2 - 2*(a^2*c^2*d^5*e + 2*a^3*c*d^3*e^3 + a^4*d*e^5)*x)*sqrt(c*x
^2 + a))/((a^3*c^3*d^7 + 2*a^4*c^2*d^5*e^2 + a^5*c*d^3*e^4)*x^4 + (a^4*c^2
*d^7 + 2*a^5*c*d^5*e^2 + a^6*d^3*e^4)*x^2), -1/2*(((3*c^4*d^6 + 4*a*c^3...
```

3.341.6 Sympy [F]

$$\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \frac{1}{x^3\left(a+cx^2\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(1/x**3/(e*x+d)/(c*x**2+a)**(3/2),x)
```

output
```
Integral(1/(x**3*(a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.341.7 Maxima [F]

$$\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \frac{1}{(cx^2+a)^{\frac{3}{2}}(ex+d)x^3}\,dx$$

input
```
integrate(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)*x^3), x)
```

3.341.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.32

$$\begin{aligned}\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = &-\frac{2\,e^5\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{(cd^5+ad^3e^2)\sqrt{-cd^2-ae^2}} \\ &+ \frac{\frac{\left(a^2c^3d^2e+a^3c^2e^3\right)x}{a^4c^2d^4+2\,a^5cd^2e^2+a^6e^4} - \frac{a^2c^3d^3+a^3c^2de^2}{a^4c^2d^4+2\,a^5cd^2e^2+a^6e^4}}{\sqrt{cx^2+a}} - \frac{\left(3\,cd^2-2\,ae^2\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2d^3} \\ &+ \frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3cd-2\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2a\sqrt{c}e+\left(\sqrt{c}x-\sqrt{cx^2+a}\right)acd+2\,a^2\sqrt{c}e}{\left(\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2-a\right)^2a^2d^2}\end{aligned}$$

input
```
integrate(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-2*e^5*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 -
 a*e^2))/((c*d^5 + a*d^3*e^2)*sqrt(-c*d^2 - a*e^2)) + ((a^2*c^3*d^2*e + a^
3*c^2*e^3)*x/(a^4*c^2*d^4 + 2*a^5*c*d^2*e^2 + a^6*e^4) - (a^2*c^3*d^3 + a^
3*c^2*d*e^2)/(a^4*c^2*d^4 + 2*a^5*c*d^2*e^2 + a^6*e^4))/sqrt(c*x^2 + a) -
(3*c*d^2 - 2*a*e^2)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(
-a)*a^2*d^3) + ((sqrt(c)*x - sqrt(c*x^2 + a))^3*c*d - 2*(sqrt(c)*x - sqrt(
c*x^2 + a))^2*a*sqrt(c)*e + (sqrt(c)*x - sqrt(c*x^2 + a))*a*c*d + 2*a^2*sq
rt(c)*e)/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)^2*a^2*d^2)
```

3.341.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \frac{1}{x^3\,\left(c\,x^2+a\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

input
```
int(1/(x^3*(a + c*x^2)^(3/2)*(d + e*x)),x)
```

output
```
int(1/(x^3*(a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.342 $\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

3.342.1 Optimal result

Integrand size = 22, antiderivative size = 244

$$\begin{aligned}\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx &= \frac{(13cd^2-2ae^2)\sqrt{a+cx^2}}{3c^2e^4}+\frac{d^5\sqrt{a+cx^2}}{e^4(cd^2+ae^2)(d+ex)} \\ &\quad -\frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4}+\frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^4} \\ &\quad -\frac{d(4cd^2-ae^2)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^5} \\ &\quad -\frac{d^4(4cd^2+5ae^2)\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^5(cd^2+ae^2)^{3/2}}\end{aligned}$$

output
```
-d*(-a*e^2+4*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^5-d^4*(5*
a*e^2+4*c*d^2)*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e
^5/(a*e^2+c*d^2)^(3/2)+1/3*(-2*a*e^2+13*c*d^2)*(c*x^2+a)^(1/2)/c^2/e^4+d^5
*(c*x^2+a)^(1/2)/e^4/(a*e^2+c*d^2)/(e*x+d)-5/3*d*(e*x+d)*(c*x^2+a)^(1/2)/c
/e^4+1/3*(e*x+d)^2*(c*x^2+a)^(1/2)/c/e^4
```

3.342.2 Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.03

$$\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx$$
$$= \frac{\frac{e\sqrt{a+cx^2}\left(-2a^2e^4(d+ex)+ace^2\left(7d^3+4d^2ex-2de^2x^2+e^3x^3\right)+c^2d^2\left(12d^3+6d^2ex-2de^2x^2+e^3x^3\right)\right)}{c^2(cd^2+ae^2)(d+ex)}+\frac{6d^4\left(4cd^2+5ae^2\right)\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+}}{\sqrt{-cd^2-ae^2}}\right.}{\left(-cd^2-ae^2\right)^{3/2}}}{3e^5}$$

input
```
Integrate[x^5/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

output
```
((e*Sqrt[a + c*x^2]*(-2*a^2*e^4*(d + e*x) + a*c*e^2*(7*d^3 + 4*d^2*e*x - 2
*d*e^2*x^2 + e^3*x^3) + c^2*d^2*(12*d^3 + 6*d^2*e*x - 2*d*e^2*x^2 + e^3*x^
3)))/(c^2*(c*d^2 + a*e^2)*(d + e*x)) + (6*d^4*(4*c*d^2 + 5*a*e^2)*ArcTan[(
Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2)
- a*e^2)^(3/2) + (3*(4*c*d^3 - a*d*e^2)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]
])/c^(3/2))/(3*e^5)
```

3.342.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.30, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {603, 25, 2185, 2185, 27, 2185, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{a+cx^2}(d+ex)^2}\,dx$$
$$\Big\downarrow \ 603$$
$$\frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)\left(ae^2+cd^2\right)}-\frac{\int -\frac{\frac{ad^4}{e^3}-\frac{\left(cd^2+ae^2\right)xd^3}{e^4}+\frac{\left(cd^2+ae^2\right)x^2d^2}{e^3}-\left(\frac{cd^2}{e^2}+a\right)x^3d+\frac{\left(cd^2+ae^2\right)x^4}{e}}{(d+ex)\sqrt{cx^2+a}}dx}{ae^2+cd^2}$$
$$\Big\downarrow \ 25$$
$$\frac{\int \frac{\frac{ad^4}{e^3}-\frac{\left(cd^2+ae^2\right)xd^3}{e^4}+\frac{\left(cd^2+ae^2\right)x^2d^2}{e^3}-\left(\frac{cd^2}{e^2}+a\right)x^3d+\frac{\left(cd^2+ae^2\right)x^4}{e}}{(d+ex)\sqrt{cx^2+a}}dx}{ae^2+cd^2}+\frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 2185$$

$$\frac{\frac{\int \frac{-10cde^2\left(cd^2+ae^2\right)x^3-2e\left(cd^2+ae^2\right)^2x^2-4d\left(cd^2+ae^2\right)^2x+ad^2e\left(cd^2-2ae^2\right)}{(d+ex)\sqrt{cx^2+a}}dx}{3ce^4}+\frac{\sqrt{a+cx^2}(d+ex)^2\left(ae^2+cd^2\right)}{3ce^4}}{ae^2+cd^2}+\frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 2185$$

$$\frac{\frac{\frac{\int \frac{2\left(c\left(13cd^2-2ae^2\right)\left(cd^2+ae^2\right)x^2e^4+3acd^2\left(2cd^2+ae^2\right)e^4+cd\left(cd^2+ae^2\right)^2xe^3\right)}{(d+ex)\sqrt{cx^2+a}}dx}{2ce^3}-5d\sqrt{a+cx^2}(d+ex)\left(ae^2+cd^2\right)}{3ce^4}+\frac{\sqrt{a+cx^2}(d+ex)^2\left(ae^2+cd^2\right)}{3ce^4}}{ae^2+cd^2}+\frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{c\left(13cd^2-2ae^2\right)\left(cd^2+ae^2\right)x^2e^4+3acd^2\left(2cd^2+ae^2\right)e^4+cd\left(cd^2+ae^2\right)^2xe^3}{(d+ex)\sqrt{cx^2+a}}dx}{ce^3}-5d\sqrt{a+cx^2}(d+ex)\left(ae^2+cd^2\right)}{3ce^4}+\frac{\sqrt{a+cx^2}(d+ex)^2\left(ae^2+cd^2\right)}{3ce^4}}{ae^2+cd^2}+\frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 2185$$

$$\frac{\frac{\frac{\frac{\int \frac{3c^2de^5\left(ade\left(2cd^2+ae^2\right)-\left(4cd^2-ae^2\right)\left(cd^2+ae^2\right)x\right)}{(d+ex)\sqrt{cx^2+a}}dx}{ce^2}+e^3\sqrt{a+cx^2}\left(13cd^2-2ae^2\right)\left(ae^2+cd^2\right)}{ce^3}-5d\sqrt{a+cx^2}(d+ex)\left(ae^2+cd^2\right)}{3ce^4}+\frac{\sqrt{a+cx^2}(d+ex)^2\left(ae^2+cd^2\right)}{3ce^4}}{ae^2+cd^2}+\frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3cde^3\int \frac{ade\left(2cd^2+ae^2\right)-\left(4cd^2-ae^2\right)\left(cd^2+ae^2\right)x}{(d+ex)\sqrt{cx^2+a}}dx+e^3\sqrt{a+cx^2}\left(13cd^2-2ae^2\right)\left(ae^2+cd^2\right)}{ce^3}-5d\sqrt{a+cx^2}(d+ex)\left(ae^2+cd^2\right)}{3ce^4}+\frac{\sqrt{a+cx^2}(d+ex)^2\left(ae^2+cd^2\right)}{3ce^4}}{ae^2+cd^2}+\frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 719$$

3.342. $\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

$$\frac{\frac{\frac{3cde^3\left(\frac{cd^3\left(5ae^2+4cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{\left(4cd^2-ae^2\right)\left(ae^2+cd^2\right)\int \frac{1}{\sqrt{cx^2+a}}dx}{e}\right)+e^3\sqrt{a+cx^2}\left(13cd^2-2ae^2\right)\left(ae^2+cd^2\right)}{ce^3}-5d\sqrt{a+cx^2}(d+ex)\left(ae^2+cd^2\right.}{3ce^4}}{ae^2+cd^2}$$
$$\frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\frac{3cde^3\left(\frac{cd^3\left(5ae^2+4cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{\left(4cd^2-ae^2\right)\left(ae^2+cd^2\right)\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{e}\right)+e^3\sqrt{a+cx^2}\left(13cd^2-2ae^2\right)\left(ae^2+cd^2\right)}{ce^3}-5d\sqrt{a+cx^2}(d+ex)\left(\right.}{3ce^4}}{ae^2+cd^2}$$
$$\frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{3cde^3\left(\frac{cd^3\left(5ae^2+4cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(4cd^2-ae^2\right)\left(ae^2+cd^2\right)}{\sqrt{c}e}\right)+e^3\sqrt{a+cx^2}\left(13cd^2-2ae^2\right)\left(ae^2+cd^2\right)}{ce^3}-5d\sqrt{a+cx^2}(d+ex)\left(\right.}{3ce^4}}{ae^2+cd^2}$$
$$\frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 488$$

$$\frac{\frac{\frac{3cde^3\left(-\frac{cd^3\left(5ae^2+4cd^2\right)\int \frac{1}{cd^2+ae^2-\frac{(ae-cdx)^2}{cx^2+a}}d\frac{ae-cdx}{\sqrt{cx^2+a}}}{e}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(4cd^2-ae^2\right)\left(ae^2+cd^2\right)}{\sqrt{c}e}\right)+e^3\sqrt{a+cx^2}\left(13cd^2-2ae^2\right)\left(ae^2+cd^2\right)}{ce^3}-5d\sqrt{}}{3ce^4}}{ae^2+cd^2}$$
$$\frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{3cde^3\left(-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(4cd^2-ae^2\right)\left(ae^2+cd^2\right)}{\sqrt{c}e}-\frac{cd^3\left(5ae^2+4cd^2\right)\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\sqrt{ae^2+cd^2}}\right)+e^3\sqrt{a+cx^2}\left(13cd^2-2ae^2\right)\left(ae^2+cd^2\right)}{ce^3}-5d\sqrt{a}}{3ce^4}}{ae^2+cd^2}$$
$$\frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)\left(ae^2+cd^2\right)}$$

3.342. $\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

input
```
Int[x^5/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

output
```
(d^5*Sqrt[a + c*x^2])/(e^4*(c*d^2 + a*e^2)*(d + e*x)) + (((c*d^2 + a*e^2)*
(d + e*x)^2*Sqrt[a + c*x^2])/(3*c*e^4) + (-5*d*(c*d^2 + a*e^2)*(d + e*x)*S
qrt[a + c*x^2] + (e^3*(13*c*d^2 - 2*a*e^2)*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]
 + 3*c*d*e^3*(-(((4*c*d^2 - a*e^2)*(c*d^2 + a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqr
t[a + c*x^2]])/(Sqrt[c]*e)) - (c*d^3*(4*c*d^2 + 5*a*e^2)*ArcTanh[(a*e - c*
d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e*Sqrt[c*d^2 + a*e^2])))/(c*
e^3))/(3*c*e^4))/(c*d^2 + a*e^2)
```

3.342.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 603
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[x^m, c + d*x, x], R = PolynomialRemainde
r[x^m, c + d*x, x]}, Simp[d*R*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n +
1)*(b*c^2 + a*d^2))), x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)
^(n + 1)*(a + b*x^2)^p*ExpandToSum[(n + 1)*(b*c^2 + a*d^2)*Qx + b*c*R*(n +
1) - b*d*R*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IGt
Q[m, 1] && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.342.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(218) = 436$.

Time = 0.48 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.80

method	result
risch	$-\frac{(-c e^2x^2+3cdex+2e^2a-9cd^2)\sqrt{cx^2+a}}{3c^2e^4}+\frac{d\left(\frac{(e^2a-4cd^2)\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{e\sqrt{c}}-\frac{5cd^3\ln\left(\frac{\frac{2e^2a+2cd^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+cd^2}{e^2}}\sqrt{(x}}{x+\frac{d}{e}}\right)}{e^2\sqrt{\frac{e^2a+cd^2}{e^2}}}\right.}{}$
default	$\frac{\frac{x^2\sqrt{cx^2+a}}{3c}-\frac{2a\sqrt{cx^2+a}}{3c^2}}{e^2}-\frac{4d^3\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{e^5\sqrt{c}}-\frac{2d\left(\frac{x\sqrt{cx^2+a}}{2c}-\frac{a\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{2c^{\frac{3}{2}}}\right)}{e^3}+\frac{3d^2\sqrt{cx^2+a}}{e^4c}-\frac{d^5\left(-\frac{e^2\sqrt{\left(x+\frac{d}{e}\right.}}{}\right.}{}$

3.342. $\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

input
```
int(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-c*e^2*x^2+3*c*d*e*x+2*a*e^2-9*c*d^2)*(c*x^2+a)^(1/2)/c^2/e^4+d/e^4/
c*((a*e^2-4*c*d^2)/e*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-5*c*d^3/e^2/((a
*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c
*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x
+d/e))-c*d^4/e^3*(-1/(a*e^2+c*d^2)*e^2/(x+d/e)*((x+d/e)^2*c-2/e*c*d*(x+d/e
)+(a*e^2+c*d^2)/e^2)^(1/2)-e*c*d/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*l
n((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e
)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))))
```

3.342.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(219) = 438$.

Time = 25.37 (sec) , antiderivative size = 2025, normalized size of antiderivative = 8.30

$$\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(4*c^3*d^8 + 7*a*c^2*d^6*e^2 + 2*a^2*c*d^4*e^4 - a^3*d^2*e^6 + (4*
c^3*d^7*e + 7*a*c^2*d^5*e^3 + 2*a^2*c*d^3*e^5 - a^3*d*e^7)*x)*sqrt(c)*log(
-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 3*(4*c^3*d^7 + 5*a*c^2*d^5*e
^2 + (4*c^3*d^6*e + 5*a*c^2*d^4*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e
*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^
2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(12*c^3*d
^7*e + 19*a*c^2*d^5*e^3 + 5*a^2*c*d^3*e^5 - 2*a^3*d*e^7 + (c^3*d^4*e^4 + 2
*a*c^2*d^2*e^6 + a^2*c*e^8)*x^3 - 2*(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 + a^2*c
*d*e^7)*x^2 + 2*(3*c^3*d^6*e^2 + 5*a*c^2*d^4*e^4 + a^2*c*d^2*e^6 - a^3*e^8
)*x)*sqrt(c*x^2 + a))/(c^4*d^5*e^5 + 2*a*c^3*d^3*e^7 + a^2*c^2*d*e^9 + (c^
4*d^4*e^6 + 2*a*c^3*d^2*e^8 + a^2*c^2*e^10)*x), -1/6*(6*(4*c^3*d^7 + 5*a*c
^2*d^5*e^2 + (4*c^3*d^6*e + 5*a*c^2*d^4*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arcta
n(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 +
(c^2*d^2 + a*c*e^2)*x^2)) - 3*(4*c^3*d^8 + 7*a*c^2*d^6*e^2 + 2*a^2*c*d^4*e
^4 - a^3*d^2*e^6 + (4*c^3*d^7*e + 7*a*c^2*d^5*e^3 + 2*a^2*c*d^3*e^5 - a^3*
d*e^7)*x)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(12*
c^3*d^7*e + 19*a*c^2*d^5*e^3 + 5*a^2*c*d^3*e^5 - 2*a^3*d*e^7 + (c^3*d^4*e^
4 + 2*a*c^2*d^2*e^6 + a^2*c*e^8)*x^3 - 2*(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 +
a^2*c*d*e^7)*x^2 + 2*(3*c^3*d^6*e^2 + 5*a*c^2*d^4*e^4 + a^2*c*d^2*e^6 - a^
3*e^8)*x)*sqrt(c*x^2 + a))/(c^4*d^5*e^5 + 2*a*c^3*d^3*e^7 + a^2*c^2*d*e...
```

3.342.6 Sympy [F]

$$\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x^5}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

input
```
integrate(x**5/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

output
```
Integral(x**5/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.342.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.342.8 Giac [F]

$$\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x^5}{\sqrt{cx^2+a}(ex+d)^2}\,dx$$

input
```
integrate(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.342.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x^5}{\sqrt{c\,x^2+a}\,(d+e\,x)^2}\,dx$$

input
```
int(x^5/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

output
```
int(x^5/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.343 $\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

3.343.1 Optimal result

Integrand size = 22, antiderivative size = 204

$$\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{5d\sqrt{a+cx^2}}{2ce^3}-\frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)}+\frac{(d+ex)\sqrt{a+cx^2}}{2ce^3}+\frac{\left(6cd^2-ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4}+\frac{d^3\left(3cd^2+4ae^2\right)\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^4\left(cd^2+ae^2\right)^{3/2}}$$

output
```
1/2*(-a*e^2+6*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^4+d^3*(4
*a*e^2+3*c*d^2)*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/
e^4/(a*e^2+c*d^2)^(3/2)-5/2*d*(c*x^2+a)^(1/2)/c/e^3-d^4*(c*x^2+a)^(1/2)/e^
3/(a*e^2+c*d^2)/(e*x+d)+1/2*(e*x+d)*(c*x^2+a)^(1/2)/c/e^3
```

3.343.2 Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.09

$$\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{\frac{e\sqrt{a+cx^2}\left(cd^2\left(-6d^2-3dex+e^2x^2\right)+ae^2\left(-4d^2-3dex+e^2x^2\right)\right)}{c(cd^2+ae^2)(d+ex)}-\frac{4d^3\left(3cd^2+4ae^2\right)\arctan\left(\frac{\sqrt{-cd^2-ae^2}x}{\sqrt{a}(d+ex)-d\sqrt{a+cx^2}}\right)}{\left(-cd^2-ae^2\right)^{3/2}}+\frac{2\left(6cd^2-ae^2\right)\operatorname{arctanh}\left(\ldots\right.}{c^{3/2}}}{2e^4}$$

input
```
Integrate[x^4/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

output
```
((e*Sqrt[a + c*x^2]*(c*d^2*(-6*d^2 - 3*d*e*x + e^2*x^2) + a*e^2*(-4*d^2 -
3*d*e*x + e^2*x^2)))/(c*(c*d^2 + a*e^2)*(d + e*x)) - (4*d^3*(3*c*d^2 + 4*a
*e^2)*ArcTan[(Sqrt[-(c*d^2) - a*e^2]*x)/(Sqrt[a]*(d + e*x) - d*Sqrt[a + c*
x^2])])/(-(c*d^2) - a*e^2)^(3/2) + (2*(6*c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x
)/(-Sqrt[a] + Sqrt[a + c*x^2])])/c^(3/2))/(2*e^4)
```

3.343.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {603, 2185, 2185, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+cx^2}(d+ex)^2} \, dx$$

$$\Big\downarrow \ 603$$

$$-\frac{\int \frac{\frac{ad^3}{e^2}-\frac{\left(cd^2+ae^2\right)xd^2}{e^3}+\left(\frac{cd^2}{e^2}+a\right)x^2d-\frac{\left(cd^2+ae^2\right)x^3}{e}}{(d+ex)\sqrt{cx^2+a}}dx}{ae^2+cd^2}-\frac{d^4\sqrt{a+cx^2}}{e^3(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \ 2185$$

$$-\frac{\frac{\int \frac{5cde\left(cd^2+ae^2\right)x^2-\left(c^2d^4-a^2e^4\right)x+ade\left(3cd^2+ae^2\right)}{(d+ex)\sqrt{cx^2+a}}dx}{2ce^3}-\frac{\sqrt{a+cx^2}(d+ex)\left(ae^2+cd^2\right)}{2ce^3}}{ae^2+cd^2}-\frac{d^4\sqrt{a+cx^2}}{e^3(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \ 2185$$

$$-\frac{\frac{\frac{\int \frac{ce^2\left(ade\left(3cd^2+ae^2\right)-\left(6cd^2-ae^2\right)\left(cd^2+ae^2\right)x\right)}{(d+ex)\sqrt{cx^2+a}}dx}{ce^2}+5d\sqrt{a+cx^2}\left(ae^2+cd^2\right)}{2ce^3}-\frac{\sqrt{a+cx^2}(d+ex)\left(ae^2+cd^2\right)}{2ce^3}}{ae^2+cd^2}-\frac{d^4\sqrt{a+cx^2}}{e^3(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\int \frac{ade\left(3cd^2+ae^2\right)-\left(6cd^2-ae^2\right)\left(cd^2+ae^2\right)x}{(d+ex)\sqrt{cx^2+a}}dx+5d\sqrt{a+cx^2}\left(ae^2+cd^2\right)}{2ce^3}-\frac{\sqrt{a+cx^2}(d+ex)\left(ae^2+cd^2\right)}{2ce^3}}{ae^2+cd^2}-\frac{d^4\sqrt{a+cx^2}}{e^3(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 719$$

$$-\frac{\frac{-\frac{\left(6cd^2-ae^2\right)\left(ae^2+cd^2\right)\int \frac{1}{\sqrt{cx^2+a}}dx}{e}+\frac{2cd^3\left(4ae^2+3cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}+5d\sqrt{a+cx^2}\left(ae^2+cd^2\right)}{2ce^3}-\frac{\sqrt{a+cx^2}(d+ex)\left(ae^2+cd^2\right)}{2ce^3}}{ae^2+cd^2}-\frac{d^4\sqrt{a+cx^2}}{e^3(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 224$$

$$-\frac{\frac{-\frac{\left(6cd^2-ae^2\right)\left(ae^2+cd^2\right)\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{e}+\frac{2cd^3\left(4ae^2+3cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}+5d\sqrt{a+cx^2}\left(ae^2+cd^2\right)}{2ce^3}-\frac{\sqrt{a+cx^2}(d+ex)\left(ae^2+cd^2\right)}{2ce^3}}{ae^2+cd^2}-\frac{d^4\sqrt{a+cx^2}}{e^3(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{\frac{2cd^3\left(4ae^2+3cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(6cd^2-ae^2\right)\left(ae^2+cd^2\right)}{\sqrt{c}e}+5d\sqrt{a+cx^2}\left(ae^2+cd^2\right)}{2ce^3}-\frac{\sqrt{a+cx^2}(d+ex)\left(ae^2+cd^2\right)}{2ce^3}}{ae^2+cd^2}-\frac{d^4\sqrt{a+cx^2}}{e^3(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 488$$

$$-\frac{\frac{-\frac{2cd^3\left(4ae^2+3cd^2\right)\int \frac{1}{cd^2+ae^2-\frac{(ae-cdx)^2}{cx^2+a}}d\frac{ae-cdx}{\sqrt{cx^2+a}}}{e}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(6cd^2-ae^2\right)\left(ae^2+cd^2\right)}{\sqrt{c}e}+5d\sqrt{a+cx^2}\left(ae^2+cd^2\right)}{2ce^3}-\frac{\sqrt{a+cx^2}(d+ex)(ae}{2ce^3}}{ae^2+cd^2}-\frac{d^4\sqrt{a+cx^2}}{e^3(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 219$$

3.343. $\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

$$-\frac{\frac{-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(6cd^2-ae^2)(ae^2+cd^2)}{\sqrt{c}e}-\frac{2cd^3(4ae^2+3cd^2)\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\sqrt{ae^2+cd^2}}+5d\sqrt{a+cx^2}(ae^2+cd^2)}{2ce^3}-\frac{\sqrt{a+cx^2}(d+ex)(ae}{2ce^3}}{ae^2+cd^2}$$

$$\frac{d^4\sqrt{a+cx^2}}{e^3(d+ex)(ae^2+cd^2)}$$

input
```
Int[x^4/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

output
```
-((d^4*Sqrt[a + c*x^2])/(e^3*(c*d^2 + a*e^2)*(d + e*x))) - (-1/2*((c*d^2 +
 a*e^2)*(d + e*x)*Sqrt[a + c*x^2])/(c*e^3) + (5*d*(c*d^2 + a*e^2)*Sqrt[a +
 c*x^2] - ((6*c*d^2 - a*e^2)*(c*d^2 + a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a +
c*x^2]])/(Sqrt[c]*e) - (2*c*d^3*(3*c*d^2 + 4*a*e^2)*ArcTanh[(a*e - c*d*x)/
(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e*Sqrt[c*d^2 + a*e^2]))/(2*c*e^3)
)/(c*d^2 + a*e^2)
```

3.343.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 603
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[x^m, c + d*x, x], R = PolynomialRemainde
r[x^m, c + d*x, x]}, Simp[d*R*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n +
1)*(b*c^2 + a*d^2))), x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)
^(n + 1)*(a + b*x^2)^p*ExpandToSum[(n + 1)*(b*c^2 + a*d^2)*Qx + b*c*R*(n +
1) - b*d*R*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IGt
Q[m, 1] && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.343.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(180) = 360$.

Time = 0.45 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.06

method	result
risch	$-\frac{(-ex+4d)\sqrt{c\,x^2+a}}{2c\,e^3}-\frac{\frac{\left(e^2a-6c\,d^2\right)\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e\sqrt{c}}-\frac{8c\,d^3\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+e^2a}}{x+\frac{d}{e}}\right.}{e^2\sqrt{\frac{e^2a+c\,d^2}{e^2}}}}{}$
default	$\frac{\frac{x\sqrt{c\,x^2+a}}{2c}-\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}}}{e^2}+\frac{3d^2\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e^4\sqrt{c}}-\frac{2d\sqrt{c\,x^2+a}}{c\,e^3}+\frac{d^4\left(-\frac{e^2\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{\left(e^2a+c\,d^2\right)\left(x+\frac{d}{e}\right)}-\frac{ecd\ln\left(\right.}{}\right.}{}$

input
```
int(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e*x+4*d)*(c*x^2+a)^(1/2)/c/e^3-1/2/c/e^3*((a*e^2-6*c*d^2)/e*ln(x*c^
(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-8*c*d^3/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2
*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*
c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))-2*c*d^4/e^3*(-1/(a*e^
2+c*d^2)*e^2/(x+d/e)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)
-e*c*d/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e
*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e
^2+c*d^2)/e^2)^(1/2))/(x+d/e))))
```

3.343.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(181) = 362$.

Time = 46.20 (sec) , antiderivative size = 1786, normalized size of antiderivative = 8.75

$$\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*((6*c^3*d^7 + 11*a*c^2*d^5*e^2 + 4*a^2*c*d^3*e^4 - a^3*d*e^6 + (6*c^
3*d^6*e + 11*a*c^2*d^4*e^3 + 4*a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(c)*log(-2*
c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(3*c^3*d^6 + 4*a*c^2*d^4*e^2
+ (3*c^3*d^5*e + 4*a*c^2*d^3*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x
- a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*
(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(6*c^3*d^6*e
 + 10*a*c^2*d^4*e^3 + 4*a^2*c*d^2*e^5 - (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a
^2*c*e^7)*x^2 + 3*(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x)*sqrt(c*
x^2 + a))/(c^4*d^5*e^4 + 2*a*c^3*d^3*e^6 + a^2*c^2*d*e^8 + (c^4*d^4*e^5 +
2*a*c^3*d^2*e^7 + a^2*c^2*e^9)*x), 1/4*(4*(3*c^3*d^6 + 4*a*c^2*d^4*e^2 + (
3*c^3*d^5*e + 4*a*c^2*d^3*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2
- a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c
*e^2)*x^2)) - (6*c^3*d^7 + 11*a*c^2*d^5*e^2 + 4*a^2*c*d^3*e^4 - a^3*d*e^6
+ (6*c^3*d^6*e + 11*a*c^2*d^4*e^3 + 4*a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(c)*
log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(6*c^3*d^6*e + 10*a*c^
2*d^4*e^3 + 4*a^2*c*d^2*e^5 - (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*
x^2 + 3*(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x)*sqrt(c*x^2 + a))/
(c^4*d^5*e^4 + 2*a*c^3*d^3*e^6 + a^2*c^2*d*e^8 + (c^4*d^4*e^5 + 2*a*c^3*d^
2*e^7 + a^2*c^2*e^9)*x), -1/2*((6*c^3*d^7 + 11*a*c^2*d^5*e^2 + 4*a^2*c*d^3
*e^4 - a^3*d*e^6 + (6*c^3*d^6*e + 11*a*c^2*d^4*e^3 + 4*a^2*c*d^2*e^5 - ...
```

3.343.6 Sympy [F]

$$\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x^4}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

input
```
integrate(x**4/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

output
```
Integral(x**4/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.343.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.343.8 Giac [F]

$$\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x^4}{\sqrt{cx^2+a}(ex+d)^2}\,dx$$

input
```
integrate(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.343.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x^4}{\sqrt{c\,x^2+a}\,(d+e\,x)^2}\,dx$$

input
```
int(x^4/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

output
```
int(x^4/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.344 $\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

3.344.1 Optimal result

Integrand size = 22, antiderivative size = 160

$$\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{\sqrt{a+cx^2}}{ce^2}+\frac{d^3\sqrt{a+cx^2}}{e^2(cd^2+ae^2)(d+ex)}-\frac{2d\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^3}-\frac{d^2(2cd^2+3ae^2)\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^3(cd^2+ae^2)^{3/2}}$$

output
```
-d^2*(3*a*e^2+2*c*d^2)*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^
(1/2))/e^3/(a*e^2+c*d^2)^(3/2)-2*d*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/e^3/
c^(1/2)+(c*x^2+a)^(1/2)/c/e^2+d^3*(c*x^2+a)^(1/2)/e^2/(a*e^2+c*d^2)/(e*x+d
)
```

3.344.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{\frac{e\sqrt{a+cx^2}(ae^2(d+ex)+cd^2(2d+ex))}{c(cd^2+ae^2)(d+ex)}+\frac{2d^2(2cd^2+3ae^2)\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}}+\frac{2d\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{\sqrt{c}}}{e^3}$$

input
```
Integrate[x^3/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

output
```
((e*Sqrt[a + c*x^2]*(a*e^2*(d + e*x) + c*d^2*(2*d + e*x)))/(c*(c*d^2 + a*e
^2)*(d + e*x)) + (2*d^2*(2*c*d^2 + 3*a*e^2)*ArcTan[(Sqrt[c]*(d + e*x) - e*
Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^2)^(3/2) + (2*d*
Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/e^3
```

3.344.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {603, 25, 2185, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+cx^2}(d+ex)^2}dx$$

$$\Big\downarrow \text{603}$$

$$\frac{d^3\sqrt{a+cx^2}}{e^2(d+ex)\left(ae^2+cd^2\right)}-\frac{\int -\frac{\frac{ad^2}{e}-\left(\frac{cd^2}{e^2}+a\right)xd+\frac{\left(cd^2+ae^2\right)x^2}{e}}{(d+ex)\sqrt{cx^2+a}}dx}{ae^2+cd^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{ad^2}{e}-\left(\frac{cd^2}{e^2}+a\right)xd+\frac{\left(cd^2+ae^2\right)x^2}{e}}{(d+ex)\sqrt{cx^2+a}}dx}{ae^2+cd^2}+\frac{d^3\sqrt{a+cx^2}}{e^2(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\frac{\int \frac{cd\left(ade-2\left(cd^2+ae^2\right)x\right)}{(d+ex)\sqrt{cx^2+a}}dx}{ce^2}+\sqrt{a+cx^2}\left(\frac{a}{c}+\frac{d^2}{e^2}\right)}{ae^2+cd^2}+\frac{d^3\sqrt{a+cx^2}}{e^2(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{d\int \frac{ade-2\left(cd^2+ae^2\right)x}{(d+ex)\sqrt{cx^2+a}}dx}{e^2}+\sqrt{a+cx^2}\left(\frac{a}{c}+\frac{d^2}{e^2}\right)}{ae^2+cd^2}+\frac{d^3\sqrt{a+cx^2}}{e^2(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{719}$$

$$\frac{\frac{d\left(\frac{d(3ae^2+2cd^2)\int\frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{2(ae^2+cd^2)\int\frac{1}{\sqrt{cx^2+a}}dx}{e}\right)}{e^2}+\sqrt{a+cx^2}\left(\frac{a}{c}+\frac{d^2}{e^2}\right)}{ae^2+cd^2}+\frac{d^3\sqrt{a+cx^2}}{e^2(d+ex)\,(ae^2+cd^2)}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{d\left(\frac{d(3ae^2+2cd^2)\int\frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{2(ae^2+cd^2)\int\frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{e}\right)}{e^2}+\sqrt{a+cx^2}\left(\frac{a}{c}+\frac{d^2}{e^2}\right)}{ae^2+cd^2}+\frac{d^3\sqrt{a+cx^2}}{e^2(d+ex)\,(ae^2+cd^2)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{d\left(\frac{d(3ae^2+2cd^2)\int\frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(ae^2+cd^2)}{\sqrt{c}e}\right)}{e^2}+\sqrt{a+cx^2}\left(\frac{a}{c}+\frac{d^2}{e^2}\right)}{ae^2+cd^2}+\frac{d^3\sqrt{a+cx^2}}{e^2(d+ex)\,(ae^2+cd^2)}$$

$$\Big\downarrow \; 488$$

$$\frac{\frac{d\left(-\frac{d(3ae^2+2cd^2)\int\frac{1}{cd^2+ae^2-\frac{(ae-cdx)^2}{cx^2+a}}d\frac{ae-cdx}{\sqrt{cx^2+a}}}{e}-\frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(ae^2+cd^2)}{\sqrt{c}e}\right)}{e^2}+\sqrt{a+cx^2}\left(\frac{a}{c}+\frac{d^2}{e^2}\right)}{ae^2+cd^2}+\frac{d^3\sqrt{a+cx^2}}{e^2(d+ex)\,(ae^2+cd^2)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{d\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(ae^2+cd^2)}{\sqrt{c}e}-\frac{d(3ae^2+2cd^2)\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\sqrt{ae^2+cd^2}}\right)}{e^2}+\sqrt{a+cx^2}\left(\frac{a}{c}+\frac{d^2}{e^2}\right)}{ae^2+cd^2}+\frac{d^3\sqrt{a+cx^2}}{e^2(d+ex)\,(ae^2+cd^2)}$$

input
```
Int[x^3/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

3.344. $\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

output
```
(d^3*Sqrt[a + c*x^2])/(e^2*(c*d^2 + a*e^2)*(d + e*x)) + ((a/c + d^2/e^2)*S
qrt[a + c*x^2] + (d*((-2*(c*d^2 + a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^
2]])/(Sqrt[c]*e) - (d*(2*c*d^2 + 3*a*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^
2 + a*e^2]*Sqrt[a + c*x^2])])/(e*Sqrt[c*d^2 + a*e^2])))/e^2)/(c*d^2 + a*e^
2)
```

3.344.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 603
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[x^m, c + d*x, x], R = PolynomialRemainde
r[x^m, c + d*x, x]}, Simp[d*R*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n +
1)*(b*c^2 + a*d^2))), x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)
^(n + 1)*(a + b*x^2)^p*ExpandToSum[(n + 1)*(b*c^2 + a*d^2)*Qx + b*c*R*(n +
1) - b*d*R*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IGt
Q[m, 1] && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

3.344. $\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.344.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(144) = 288.

Time = 0.44 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.41

method	result
risch	$\frac{\sqrt{c\,x^{2}+a}}{c\,e^{2}}-\frac{2d\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{e^{3}\sqrt{c}}+\frac{d^{3}\sqrt{\left(x+\frac{d}{e}\right)^{2}c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^{2}a+c\,d^{2}}{e^{2}}}}{e^{3}\left(e^{2}a+c\,d^{2}\right)\left(x+\frac{d}{e}\right)}+\frac{d^{4}c\ln\left(\frac{\frac{2e^{2}a+2c\,d^{2}}{e^{2}}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^{2}a+c\,d^{2}}{e^{2}}}\sqrt{(\ldots}}{x+\frac{d}{e}}\right.}{e^{4}\left(e^{2}a+c\,d^{2}\right)\sqrt{\frac{e^{2}a+\ldots}{e^{2}}}}$
default	$\frac{\sqrt{c\,x^{2}+a}}{c\,e^{2}}-\frac{2d\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{e^{3}\sqrt{c}}-\frac{d^{3}\left(-\frac{e^{2}\sqrt{\left(x+\frac{d}{e}\right)^{2}c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^{2}a+c\,d^{2}}{e^{2}}}}{\left(e^{2}a+c\,d^{2}\right)\left(x+\frac{d}{e}\right)}-\frac{ecd\ln\left(\frac{\frac{2e^{2}a+2c\,d^{2}}{e^{2}}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^{2}a+c\,d^{2}}{e^{2}}}\sqrt{(\ldots}}{x+\frac{d}{e}}\right.}{\left(e^{2}a+c\,d^{2}\right)\sqrt{\frac{e^{2}a+\ldots}{e^{2}}}}\right.}{e^{5}}$

input
```
int(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+a)^(1/2)/c/e^2-2*d/e^3*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)+d^3/e^
3/(a*e^2+c*d^2)/(x+d/e)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1
/2)+d^4/e^4*c/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/
e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/
e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))-3/e^4*d^2/((a*e^2+c*d^2)/e^2)^(1/2)*
ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/
e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

3.344.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(145) = 290$.

Time = 5.64 (sec) , antiderivative size = 1449, normalized size of antiderivative = 9.06

$$\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^
3 + a^2*d*e^5)*x)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a)
+ (2*c^2*d^5 + 3*a*c*d^3*e^2 + (2*c^2*d^4*e + 3*a*c*d^2*e^3)*x)*sqrt(c*d^2
 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x
^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e
*x + d^2)) + 2*(2*c^2*d^5*e + 3*a*c*d^3*e^3 + a^2*d*e^5 + (c^2*d^4*e^2 + 2
*a*c*d^2*e^4 + a^2*e^6)*x)*sqrt(c*x^2 + a))/(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5
 + a^2*c*d*e^7 + (c^3*d^4*e^4 + 2*a*c^2*d^2*e^6 + a^2*c*e^8)*x), -((2*c^2*
d^5 + 3*a*c*d^3*e^2 + (2*c^2*d^4*e + 3*a*c*d^2*e^3)*x)*sqrt(-c*d^2 - a*e^2
)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2
*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4
+ (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt(c)*log(-2*c*x^2 + 2*sqrt
(c*x^2 + a)*sqrt(c)*x - a) - (2*c^2*d^5*e + 3*a*c*d^3*e^3 + a^2*d*e^5 + (c
^2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x)*sqrt(c*x^2 + a))/(c^3*d^5*e^3 + 2
*a*c^2*d^3*e^5 + a^2*c*d*e^7 + (c^3*d^4*e^4 + 2*a*c^2*d^2*e^6 + a^2*c*e^8)
*x), 1/2*(4*(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^
3*e^3 + a^2*d*e^5)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (2*c^2
*d^5 + 3*a*c*d^3*e^2 + (2*c^2*d^4*e + 3*a*c*d^2*e^3)*x)*sqrt(c*d^2 + a*e^2
)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*s
qrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + ...
```

3.344.6 Sympy [F]

$$\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

input
```
integrate(x**3/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

output
```
Integral(x**3/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.344.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.344.8 Giac [F]

$$\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x^3}{\sqrt{cx^2+a}(ex+d)^2}\,dx$$

input
```
integrate(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.344.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx=\int \frac{x^3}{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2}\,dx$$

input
```
int(x^3/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

output
```
int(x^3/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.345 $\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

3.345.1 Optimal result

Integrand size = 22, antiderivative size = 137

$$\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{d^2\sqrt{a+cx^2}}{e\,(cd^2+ae^2)\,(d+ex)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^2}+\frac{d(cd^2+2ae^2)\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^2\,(cd^2+ae^2)^{3/2}}$$

output
```
d*(2*a*e^2+c*d^2)*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2)
)/e^2/(a*e^2+c*d^2)^(3/2)+arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/e^2/c^(1/2)-d
^2*(c*x^2+a)^(1/2)/e/(a*e^2+c*d^2)/(e*x+d)
```

3.345.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.15

$$\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{d\left(-\frac{de\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)}-\frac{2(cd^2+2ae^2)\arctan\left(\frac{\sqrt{-cd^2-ae^2}x}{\sqrt{a}(d+ex)-d\sqrt{a+cx^2}}\right)}{(-cd^2-ae^2)^{3/2}}\right)+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+cx^2}}\right)}{\sqrt{c}}}{e^2}$$

input
```
Integrate[x^2/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

output
```
(d*(-((d*e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) - (2*(c*d^2 + 2*a
*e^2)*ArcTan[(Sqrt[-(c*d^2) - a*e^2]*x)/(Sqrt[a]*(d + e*x) - d*Sqrt[a + c*
x^2])])/(-(c*d^2) - a*e^2)^(3/2)) + (2*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqr
t[a + c*x^2])])/Sqrt[c])/e^2
```

3.345.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {603, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+cx^2}(d+ex)^2}dx$$

$$\Big\downarrow \text{603}$$

$$-\frac{\int \frac{ade-(cd^2+ae^2)x}{e(d+ex)\sqrt{cx^2+a}}dx}{ae^2+cd^2}-\frac{d^2\sqrt{a+cx^2}}{e(d+ex)(ae^2+cd^2)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{ade-(cd^2+ae^2)x}{(d+ex)\sqrt{cx^2+a}}dx}{e(ae^2+cd^2)}-\frac{d^2\sqrt{a+cx^2}}{e(d+ex)(ae^2+cd^2)}$$

$$\Big\downarrow \text{719}$$

$$-\frac{\frac{d(2ae^2+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{(ae^2+cd^2)\int \frac{1}{\sqrt{cx^2+a}}dx}{e}}{e(ae^2+cd^2)}-\frac{d^2\sqrt{a+cx^2}}{e(d+ex)(ae^2+cd^2)}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\frac{d(2ae^2+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{(ae^2+cd^2)\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{e}}{e(ae^2+cd^2)}-\frac{d^2\sqrt{a+cx^2}}{e(d+ex)(ae^2+cd^2)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{d(2ae^2+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{e}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(ae^2+cd^2)}{\sqrt{c}e}}{e(ae^2+cd^2)}-\frac{d^2\sqrt{a+cx^2}}{e(d+ex)(ae^2+cd^2)}$$

$$\Big\downarrow \text{488}$$

$$-\frac{-\frac{d(2ae^2+cd^2)\int \frac{1}{cd^2+ae^2-\frac{(ae-cdx)^2}{cx^2+a}}d\frac{ae-cdx}{\sqrt{cx^2+a}}}{e}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(ae^2+cd^2)}{\sqrt{c}e}}{e\,(ae^2+cd^2)}-\frac{d^2\sqrt{a+cx^2}}{e(d+ex)\,(ae^2+cd^2)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(ae^2+cd^2)}{\sqrt{c}e}-\frac{d(2ae^2+cd^2)\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\sqrt{ae^2+cd^2}}}{e\,(ae^2+cd^2)}-\frac{d^2\sqrt{a+cx^2}}{e(d+ex)\,(ae^2+cd^2)}$$

input
```
Int[x^2/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

output
```
-((d^2*Sqrt[a + c*x^2])/(e*(c*d^2 + a*e^2)*(d + e*x))) - (-(((c*d^2 + a*e^
2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*e)) - (d*(c*d^2 + 2*a*e^
2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e*Sqrt[c
*d^2 + a*e^2]))/(e*(c*d^2 + a*e^2))
```

3.345.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 603
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[x^m, c + d*x, x], R = PolynomialRemainde
r[x^m, c + d*x, x]}, Simp[d*R*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n +
1)*(b*c^2 + a*d^2))), x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)
^(n + 1)*(a + b*x^2)^p*ExpandToSum[(n + 1)*(b*c^2 + a*d^2)*Qx + b*c*R*(n +
1) - b*d*R*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IGt
Q[m, 1] && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.345.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(123) = 246$.

Time = 0.42 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.69

method	result
default	$\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e^2\sqrt{c}}+\frac{d^2\left(-\frac{e^2\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{\left(e^2a+c\,d^2\right)\left(x+\frac{d}{e}\right)}-\frac{ecd\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\cdots\right.}{e}\cdots}}{x+\frac{d}{e}}\right)}{\left(e^2a+c\,d^2\right)\sqrt{\frac{e^2a+c\,d^2}{e^2}}}\cdots\right)}{e^4}$

input
```
int(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/e^2*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)+d^2/e^4*(-1/(a*e^2+c*d^2)*e^2/
(x+d/e)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-e*c*d/(a*e^2
+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+
2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2
)^(1/2))/(x+d/e)))+2*d/e^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e
^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e
)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

3.345.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(124) = 248.

Time = 5.49 (sec) , antiderivative size = 1260, normalized size of antiderivative = 9.20

$$\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*((c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 +
a^2*e^5)*x)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + (c^2
*d^4 + 2*a*c*d^2*e^2 + (c^2*d^3*e + 2*a*c*d*e^3)*x)*sqrt(c*d^2 + a*e^2)*lo
g((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(
c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) -
 2*(c^2*d^4*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^3*d^5*e^2 + 2*a*c^2*d^3*e
^4 + a^2*c*d*e^6 + (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x), 1/2*(2*
(c^2*d^4 + 2*a*c*d^2*e^2 + (c^2*d^3*e + 2*a*c*d*e^3)*x)*sqrt(-c*d^2 - a*e^
2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^
2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 +
 (c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*x)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*
x^2 + a)*sqrt(c)*x - a) - 2*(c^2*d^4*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^
3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*d*e^6 + (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5
 + a^2*c*e^7)*x), -1/2*(2*(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*
e + 2*a*c*d^2*e^3 + a^2*e^5)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)
) - (c^2*d^4 + 2*a*c*d^2*e^2 + (c^2*d^3*e + 2*a*c*d*e^3)*x)*sqrt(c*d^2 + a
*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 +
 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x +
 d^2)) + 2*(c^2*d^4*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^3*d^5*e^2 + 2*a*c
^2*d^3*e^4 + a^2*c*d*e^6 + (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*...
```

3.345.6 Sympy [F]

$$\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

input
```
integrate(x**2/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.345.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.345.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.345.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x^2}{\sqrt{c\,x^2+a}\,(d+e\,x)^2}\,dx$$

input
```
int(x^2/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

output
```
int(x^2/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.346 $\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

3.346.1 Optimal result

Integrand size = 20, antiderivative size = 90

$$\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx=\frac{d\sqrt{a+cx^2}}{(cd^2+ae^2)\,(d+ex)}-\frac{ae\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{(cd^2+ae^2)^{3/2}}$$

output
```
-a*e*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^
2)^(3/2)+d*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(e*x+d)
```

3.346.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx=\frac{d\sqrt{a+cx^2}}{(cd^2+ae^2)\,(d+ex)}+\frac{2ae\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}}$$

input
```
Integrate[x/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

output
```
(d*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)) + (2*a*e*ArcTan[(Sqrt[c]*(
d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^2)^
(3/2)
```

3.346.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {588, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+cx^2}(d+ex)^2}dx$$
$$\Big\downarrow \text{ 588}$$
$$\frac{ae\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{ae^2+cd^2}+\frac{d\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}$$
$$\Big\downarrow \text{ 488}$$
$$\frac{d\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}-\frac{ae\int \frac{1}{cd^2+ae^2-\frac{(ae-cdx)^2}{cx^2+a}}d\frac{ae-cdx}{\sqrt{cx^2+a}}}{ae^2+cd^2}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{d\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}-\frac{a e\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

input
```
Int[x/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

output
```
(d*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)) - (a*e*ArcTanh[(a*e - c*d*
x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

3.346.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 588
```
Int[(x_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 + a*d^2)))
, x] + Simp[a*(d/(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x]
, x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[Simplify[n + 2*p + 3], 0] && Ne
Q[b*c^2 + a*d^2, 0]
```

3.346.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(82) = 164$.

Time = 0.42 (sec) , antiderivative size = 344, normalized size of antiderivative = 3.82

method	result
default	$-\frac{\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{e^2\sqrt{\frac{e^2a+c\,d^2}{e^2}}}-\frac{d\left(-\frac{e^2\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{\left(e^2a+c\,d^2\right)\left(x+\frac{d}{e}\right)}-\frac{ecd\ln}{}\right.}{}$

input
```
int(x/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2
*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)
^(1/2))/(x+d/e))-d/e^3*(-1/(a*e^2+c*d^2)*e^2/(x+d/e)*((x+d/e)^2*c-2/e*c*d*
(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-e*c*d/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(
1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(
(x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

3.346.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(83) = 166$.

Time = 0.45 (sec) , antiderivative size = 382, normalized size of antiderivative = 4.24

$$\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx$$
$$= \left[\frac{(ae^2x+ade)\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+2\,(cd^3+ade^2)\sqrt{}}{2\,(c^2d^5+2\,acd^3e^2+a^2de^4+(c^2d^4e+2\,acd^2e^3+a^2e^5)x)}\right.$$
$$\left.-\frac{(ae^2x+ade)\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-(cd^3+ade^2)\sqrt{cx^2+a}}{c^2d^5+2\,acd^3e^2+a^2de^4+(c^2d^4e+2\,acd^2e^3+a^2e^5)x}\right]$$

input
```
integrate(x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*((a*e^2*x + a*d*e)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2
*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)
*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(c*d^3 + a*d*e^2)*sqrt(c*
x^2 + a))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^
3 + a^2*e^5)*x), -((a*e^2*x + a*d*e)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d
^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 +
a*c*e^2)*x^2)) - (c*d^3 + a*d*e^2)*sqrt(c*x^2 + a))/(c^2*d^5 + 2*a*c*d^3*e
^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*x)]
```

3.346.6 Sympy [F]

$$\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

input
```
integrate(x/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

output
```
Integral(x/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.346.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.346.8 Giac [F]

$$\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x}{\sqrt{cx^2+a}(ex+d)^2}\,dx$$

input
```
integrate(x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.346.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x}{\sqrt{c\,x^2+a}\,(d+e\,x)^2}\,dx$$

input
```
int(x/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

output
```
int(x/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.347 $\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

3.347.1 Optimal result

Integrand size = 19, antiderivative size = 91

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{e\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{cd\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{(cd^2+ae^2)^{3/2}}$$

output
```
-c*d*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^
2)^(3/2)-e*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(e*x+d)
```

3.347.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{e\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} + \frac{2cd\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}}$$

input
```
Integrate[1/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

output
```
-((e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) + (2*c*d*ArcTan[(Sqrt[c
]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^
2)^(3/2)
```

3.347.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {491, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^2}(d+ex)^2}\,dx$$

$$\Big\downarrow \text{491}$$

$$\frac{cd\int \frac{1}{(d+ex)\sqrt{cx^2+a}}dx}{ae^2+cd^2}-\frac{e\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{488}$$

$$-\frac{cd\int \frac{1}{cd^2+ae^2-\frac{(ae-cdx)^2}{cx^2+a}}d\frac{ae-cdx}{\sqrt{cx^2+a}}}{ae^2+cd^2}-\frac{e\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{cd\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}-\frac{e\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}$$

input
```
Int[1/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

output
```
-((e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) - (c*d*ArcTanh[(a*e - c
*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

3.347.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 491
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b*(c/(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 3, 0]
```

3.347.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(83) = 166$.

Time = 0.40 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.36

method	result	size
default	$\frac{-\frac{e^{2}\sqrt{\left(x+\frac{d}{e}\right)^{2}c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^{2}a+c\,d^{2}}{e^{2}}}}{\left(e^{2}a+c\,d^{2}\right)\left(x+\frac{d}{e}\right)}-\frac{ecd\ln\left(\frac{\frac{2e^{2}a+2c\,d^{2}}{e^{2}}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^{2}a+c\,d^{2}}{e^{2}}}\,\sqrt{\left(x+\frac{d}{e}\right)^{2}c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^{2}a+c\,d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\left(e^{2}a+c\,d^{2}\right)\sqrt{\frac{e^{2}a+c\,d^{2}}{e^{2}}}}}{e^{2}}$	215

input
```
int(1/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/e^2*(-1/(a*e^2+c*d^2)*e^2/(x+d/e)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*
d^2)/e^2)^(1/2)-e*c*d/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2
+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c
*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

3.347.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(84) = 168$.

Time = 0.45 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.19

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx$$

$$= \left[\frac{(cdex+cd^2)\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-2\,(cd^2e+ae^3)\sqrt{c}}{2\,(c^2d^5+2\,acd^3e^2+a^2de^4+(c^2d^4e+2\,acd^2e^3+a^2e^5)x)}\right.$$

$$\left.-\frac{(cdex+cd^2)\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)+(cd^2e+ae^3)\sqrt{cx^2+a}}{c^2d^5+2\,acd^3e^2+a^2de^4+(c^2d^4e+2\,acd^2e^3+a^2e^5)x}\right]$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*((c*d*e*x + c*d^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2
*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)
*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c*d^2*e + a*e^3)*sqrt(c*
x^2 + a))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^
3 + a^2*e^5)*x), -((c*d*e*x + c*d^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d
^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 +
a*c*e^2)*x^2)) + (c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2*d^5 + 2*a*c*d^3*e
^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*x)]
```

3.347.6 Sympy [F]

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.347.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.347.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.347.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2}\,dx$$

input
```
int(1/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

output
```
int(1/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.348 $\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx$

3.348.1 Optimal result

Integrand size = 22, antiderivative size = 179

$$\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{e^2\sqrt{a+cx^2}}{d\left(cd^2+ae^2\right)(d+ex)} + \frac{c e \operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}} + \frac{e \operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}$$

output
```
c*e*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2
)^(3/2)-arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^2/a^(1/2)+e*arctanh((-c*d*x+a*e
)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^2/(a*e^2+c*d^2)^(1/2)+e^2*(c*x^2+
a)^(1/2)/d/(a*e^2+c*d^2)/(e*x+d)
```

3.348.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{e\left(\frac{de\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{2(2cd^2+ae^2)\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}}\right) + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{d^2}$$

input
```
Integrate[1/(x*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

output
```
(e*((d*e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)) - (2*(2*c*d^2 + a*e^
2)*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])
/(-(c*d^2) - a*e^2)^(3/2)) + (2*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt
[a]])/Sqrt[a])/d^2
```

3.348.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+cx^2}(d+ex)^2}\,dx$$

$$\Big\downarrow \text{617}$$

$$\int \left(-\frac{e}{d^2\sqrt{a+cx^2}(d+ex)}+\frac{1}{d^2x\sqrt{a+cx^2}}-\frac{e}{d\sqrt{a+cx^2}(d+ex)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e\,\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\sqrt{ae^2+cd^2}}+\frac{ce\,\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}+\frac{e^2\sqrt{a+cx^2}}{d(d+ex)(ae^2+cd^2)}$$

input
```
Int[1/(x*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

output
```
(e^2*Sqrt[a + c*x^2])/(d*(c*d^2 + a*e^2)*(d + e*x)) + (c*e*ArcTanh[(a*e -
c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2) + (e*
ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^2*Sqrt[c*
d^2 + a*e^2]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d^2)
```

3.348.3.1 Defintions of rubi rules used

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.348.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(159) = 318$.

Time = 0.38 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.10

method	result
default	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c x^2+a}}{x}\right)}{d^2\sqrt{a}}-\frac{-\frac{e^2\sqrt{\left(x+\frac{d}{e}\right)^2 c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{\left(e^2a+c\,d^2\right)\left(x+\frac{d}{e}\right)}-\frac{ecd\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2 c-\frac{2cd(x-}{e}}}{x+\frac{d}{e}}\right)}{\left(e^2a+c\,d^2\right)\sqrt{\frac{e^2a+c\,d^2}{e^2}}}}{ed}\ldots$

input
```
int(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d^2/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-1/e/d*(-1/(a*e^2+c*d^
2)*e^2/(x+d/e)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-e*c*d
/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(
x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d
^2)/e^2)^(1/2))/(x+d/e)))+1/d^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d
^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(
x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

3.348.5 Fricas [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 1261, normalized size of antiderivative = 7.04

$$\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*((2*a*c*d^3*e + a^2*d*e^3 + (2*a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(c*d^2 +
 a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2
 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x
 + d^2)) + (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e
^3 + a^2*e^5)*x)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^
2) + 2*(a*c*d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 + a))/(a*c^2*d^7 + 2*a^2*c*d^5
*e^2 + a^3*d^3*e^4 + (a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5)*x), 1/2
*(2*(2*a*c*d^3*e + a^2*d*e^3 + (2*a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(-c*d^2 -
a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2
+ a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e
^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*x)*sqrt(a)*log(-(c*x^2 - 2*sqrt
(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(a*c*d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 +
 a))/(a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3*d^3*e^4 + (a*c^2*d^6*e + 2*a^2*c*d
^4*e^3 + a^3*d^2*e^5)*x), 1/2*(2*(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c
^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2
 + a)) + (2*a*c*d^3*e + a^2*d*e^3 + (2*a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(c*d^
2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*
x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*
e*x + d^2)) + 2*(a*c*d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 + a))/(a*c^2*d^7 + 2*
a^2*c*d^5*e^2 + a^3*d^3*e^4 + (a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*d^2*...
```

3.348.6 Sympy [F]

$$\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{x\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

input
```
integrate(1/x/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.348.7 Maxima [F]

$$\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^2x}\,dx$$

input
```
integrate(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*x), x)
```

3.348.8 Giac [F]

$$\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^2x}\,dx$$

input
```
integrate(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.348.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{x\sqrt{c\,x^2+a}\,(d+e\,x)^2}\,dx$$

input
```
int(1/(x*(a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

output
```
int(1/(x*(a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.349 $\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx$

3.349.1 Optimal result

Integrand size = 22, antiderivative size = 212

$$\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{\sqrt{a+cx^2}}{ad^2x} - \frac{e^3\sqrt{a+cx^2}}{d^2(cd^2+ae^2)(d+ex)} - \frac{ce^2\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d(cd^2+ae^2)^{3/2}} - \frac{2e^2\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}} + \frac{2e\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3}$$

output
```
-c*e^2*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d/(a*e^2+
c*d^2)^(3/2)+2*e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^3/a^(1/2)-2*e^2*arctan
h((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^3/(a*e^2+c*d^2)^(1/2
)-(c*x^2+a)^(1/2)/a/d^2/x-e^3*(c*x^2+a)^(1/2)/d^2/(a*e^2+c*d^2)/(e*x+d)
```

3.349.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{-\frac{d\sqrt{a+cx^2}(cd^2(d+ex)+ae^2(d+2ex))}{a(cd^2+ae^2)x(d+ex)} + \frac{2e^2(3cd^2+2ae^2)\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}} - \frac{4e\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{d^3}$$

input
```
Integrate[1/(x^2*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

output
```
(-((d*Sqrt[a + c*x^2]*(c*d^2*(d + e*x) + a*e^2*(d + 2*e*x)))/(a*(c*d^2 + a
*e^2)*x*(d + e*x))) + (2*e^2*(3*c*d^2 + 2*a*e^2)*ArcTan[(Sqrt[c]*(d + e*x)
 - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^2)^(3/2) -
(4*e*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/Sqrt[a])/d^3
```

3.349.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+cx^2}(d+ex)^2}dx$$

$$\Big\downarrow \text{ 617}$$

$$\int \left(\frac{2e^2}{d^3\sqrt{a+cx^2}(d+ex)}-\frac{2e}{d^3x\sqrt{a+cx^2}}+\frac{e^2}{d^2\sqrt{a+cx^2}(d+ex)^2}+\frac{1}{d^2x^2\sqrt{a+cx^2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2e\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3}-\frac{ce^2\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\left(ae^2+cd^2\right)^{3/2}}-\frac{2e^2\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}}-\frac{e^3\sqrt{a+cx^2}}{d^2(d+ex)\left(ae^2+cd^2\right)}-\frac{\sqrt{a+cx^2}}{ad^2x}$$

input
```
Int[1/(x^2*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

output
```
-(Sqrt[a + c*x^2]/(a*d^2*x)) - (e^3*Sqrt[a + c*x^2])/(d^2*(c*d^2 + a*e^2)*
(d + e*x)) - (c*e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*
x^2])])/(d*(c*d^2 + a*e^2)^(3/2)) - (2*e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d
^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^3*Sqrt[c*d^2 + a*e^2]) + (2*e*ArcTanh[Sq
rt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)
```

3.349.3.1 Defintions of rubi rules used

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.349.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(190) = 380$.

Time = 0.39 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.86

method	result
default	$-\frac{\sqrt{c x^2+a}}{a d^2 x}+\frac{2e\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c x^2+a}}{x}\right)}{d^3\sqrt{a}}+\frac{-\frac{e^2\sqrt{\left(x+\frac{d}{e}\right)^2 c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c d^2}{e^2}}}{\left(e^2a+c d^2\right)\left(x+\frac{d}{e}\right)}-\frac{ecd\ln\left(\frac{\frac{2e^2a+2c d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c d^2}{e^2}}\sqrt{(}}{x+\frac{d}{e}}\right)}{\left(e^2a+c d^2\right)\sqrt{\frac{e^2a+}{e^2}}}}{d^2}$
risch	$-\frac{\sqrt{c x^2+a}}{a d^2 x}+\frac{2e\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c x^2+a}}{x}\right)}{d^3\sqrt{a}}-\frac{e^2\sqrt{\left(x+\frac{d}{e}\right)^2 c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c d^2}{e^2}}}{d^2(e^2a+c d^2)\left(x+\frac{d}{e}\right)}-\frac{ec\ln\left(\frac{\frac{2e^2a+2c d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{d(e^2a+c d^2)\sqrt{e}}$

input
```
int(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2+a)^(1/2)/a/d^2/x+2/d^3*e/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2)
)/x)+1/d^2*(-1/(a*e^2+c*d^2)*e^2/(x+d/e)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e
^2+c*d^2)/e^2)^(1/2)-e*c*d/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(
a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-
2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))-2*e/d^3/((a*e^2+c*d^2)
/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^
(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

3.349.5 Fricas [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 1512, normalized size of antiderivative = 7.13

$$\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(c*d^2 + a*e^2)*((3*a*c*d^2*e^3 + 2*a^2*e^5)*x^2 + (3*a*c*d^3*e^
2 + 2*a^2*d*e^4)*x)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 +
a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x
^2 + 2*d*e*x + d^2)) + 2*((c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x^2 + (c
^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt(a)*log(-(c*x^2 + 2*sqrt(c*x^
2 + a)*sqrt(a) + 2*a)/x^2) - 2*(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c
^2*d^5*e + 3*a*c*d^3*e^3 + 2*a^2*d*e^5)*x)*sqrt(c*x^2 + a))/((a*c^2*d^7*e
+ 2*a^2*c*d^5*e^3 + a^3*d^3*e^5)*x^2 + (a*c^2*d^8 + 2*a^2*c*d^6*e^2 + a^3*
d^4*e^4)*x), -(sqrt(-c*d^2 - a*e^2)*((3*a*c*d^2*e^3 + 2*a^2*e^5)*x^2 + (3*
a*c*d^3*e^2 + 2*a^2*d*e^4)*x)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sq
rt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - ((c^2*d^4*e
^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x^2 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5
)*x)*sqrt(a)*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + (c^2*d^
6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 3*a*c*d^3*e^3 + 2*a^2*d*e^5
)*x)*sqrt(c*x^2 + a))/((a*c^2*d^7*e + 2*a^2*c*d^5*e^3 + a^3*d^3*e^5)*x^2 +
 (a*c^2*d^8 + 2*a^2*c*d^6*e^2 + a^3*d^4*e^4)*x), -1/2*(4*((c^2*d^4*e^2 + 2
*a*c*d^2*e^4 + a^2*e^6)*x^2 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*s
qrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - sqrt(c*d^2 + a*e^2)*((3*a*c*d^2
*e^3 + 2*a^2*e^5)*x^2 + (3*a*c*d^3*e^2 + 2*a^2*d*e^4)*x)*log((2*a*c*d*e*x
- a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^...
```

3.349.6 Sympy [F]

$$\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

input
```
integrate(1/x**2/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.349.7 Maxima [F]

$$\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^2x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*x^2), x)
```

3.349.8 Giac [F]

$$\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^2x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.349.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{x^2\sqrt{c\,x^2+a}\,(d+e\,x)^2}\,dx$$

input
```
int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

output
```
int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.350 $\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx$

3.350.1 Optimal result

Integrand size = 22, antiderivative size = 268

$$\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{\sqrt{a+cx^2}}{2ad^2x^2}+\frac{2e\sqrt{a+cx^2}}{ad^3x}+\frac{e^4\sqrt{a+cx^2}}{d^3\left(cd^2+ae^2\right)(d+ex)} + \frac{ce^3\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\left(cd^2+ae^2\right)^{3/2}}+\frac{3e^3\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^4\sqrt{cd^2+ae^2}} + \frac{c\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d^2}-\frac{3e^2\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^4}$$

output
```
c*e^3*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^2/(a*e^2
+c*d^2)^(3/2)+1/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d^2-3*e^2*arc
tanh((c*x^2+a)^(1/2)/a^(1/2))/d^4/a^(1/2)+3*e^3*arctanh((-c*d*x+a*e)/(a*e^
2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^4/(a*e^2+c*d^2)^(1/2)-1/2*(c*x^2+a)^(1/2
)/a/d^2/x^2+2*e*(c*x^2+a)^(1/2)/a/d^3/x+e^4*(c*x^2+a)^(1/2)/d^3/(a*e^2+c*d
^2)/(e*x+d)
```

3.350.2 Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx$$
$$=\frac{\frac{d\sqrt{a+cx^2}\left(cd^2(-d^2+3dex+4e^2x^2)+ae^2(-d^2+3dex+6e^2x^2)\right)}{a(cd^2+ae^2)x^2(d+ex)}-\frac{4e^3(4cd^2+3ae^2)\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}}+\frac{2(-cd^2+6ae^2)\text{arctanh}}{a^{3/2}}}{2d^4}$$

input
```
Integrate[1/(x^3*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

output
```
((d*Sqrt[a + c*x^2]*(c*d^2*(-d^2 + 3*d*e*x + 4*e^2*x^2) + a*e^2*(-d^2 + 3*
d*e*x + 6*e^2*x^2)))/(a*(c*d^2 + a*e^2)*x^2*(d + e*x)) - (4*e^3*(4*c*d^2 +
 3*a*e^2)*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a
*e^2]])/(-(c*d^2) - a*e^2)^(3/2) + (2*(-(c*d^2) + 6*a*e^2)*ArcTanh[(Sqrt[c
]*x - Sqrt[a + c*x^2])/Sqrt[a]])/a^(3/2))/(2*d^4)
```

3.350.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+cx^2}(d+ex)^2}\,dx$$
$$\Big\downarrow \ 617$$
$$\int \left(-\frac{3e^3}{d^4\sqrt{a+cx^2}(d+ex)}+\frac{3e^2}{d^4x\sqrt{a+cx^2}}-\frac{e^3}{d^3\sqrt{a+cx^2}(d+ex)^2}-\frac{2e}{d^3x^2\sqrt{a+cx^2}}+\frac{1}{d^2x^3\sqrt{a+cx^2}}\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d^2}-\frac{3e^2\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^4}+\frac{ce^3\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\left(ae^2+cd^2\right)^{3/2}}+\frac{3e^3\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^4\sqrt{ae^2+cd^2}}+\frac{2e\sqrt{a+cx^2}}{ad^3x}-\frac{\sqrt{a+cx^2}}{2ad^2x^2}+\frac{e^4\sqrt{a+cx^2}}{d^3(d+ex)\left(ae^2+cd^2\right)}$$

input
```
Int[1/(x^3*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

output
```
-1/2*Sqrt[a + c*x^2]/(a*d^2*x^2) + (2*e*Sqrt[a + c*x^2])/(a*d^3*x) + (e^4*
Sqrt[a + c*x^2])/(d^3*(c*d^2 + a*e^2)*(d + e*x)) + (c*e^3*ArcTanh[(a*e - c
*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^2*(c*d^2 + a*e^2)^(3/2))
+ (3*e^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^
4*Sqrt[c*d^2 + a*e^2]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(3/2)*d
^2) - (3*e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^4)
```

3.350.3.1 Defintions of rubi rules used

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.350.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.58

method	result
risch	$-\frac{\sqrt{c\,x^{2}+a}\,(-4ex+d)}{2a\,d^{3}x^{2}}-\frac{2ae\left(-\frac{e^{2}\sqrt{\left(x+\frac{d}{e}\right)^{2}c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^{2}a+c\,d^{2}}{e^{2}}}}{\left(e^{2}a+c\,d^{2}\right)\left(x+\frac{d}{e}\right)}-\frac{ecd\ln\left(\frac{\frac{2e^{2}a+2c\,d^{2}}{e^{2}}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^{2}a+c\,d^{2}}{e^{2}}}\sqrt{\left(x+\frac{d}{e}\right)^{2}c-\frac{2cd}{}\cdots}}{x+\frac{d}{e}}\cdots\right)}{\left(e^{2}a+c\,d^{2}\right)\sqrt{\frac{e^{2}a+c\,d^{2}}{e^{2}}}}\cdots\right)}{\cdots}$
default	$\frac{-\frac{\sqrt{c\,x^{2}+a}}{2a\,x^{2}}+\frac{c\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}}{d^{2}}-\frac{3e^{2}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^{2}+a}}{x}\right)}{d^{4}\sqrt{a}}+\frac{2e\sqrt{c\,x^{2}+a}}{a\,d^{3}x}-\frac{e\left(-\frac{e^{2}\sqrt{\left(x+\frac{d}{e}\right)^{2}c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^{2}a+c\,d^{2}}{e^{2}}}}{\left(e^{2}a+c\,d^{2}\right)\left(x+\frac{d}{e}\right)}-\frac{ec\cdots}{\cdots}\cdots\right)}{\cdots}$

input
```
int(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*x^2+a)^(1/2)*(-4*e*x+d)/a/d^3/x^2-1/2/d^3/a*(2*a*e*(-1/(a*e^2+c*d^
2)*e^2/(x+d/e)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-e*c*d
/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(
x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d
^2)/e^2)^(1/2))/(x+d/e)))-6*e^2*a/d/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2
+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c
*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))-(-6*a*e^2+c*d^2)/d/a^(1/2)*l
n((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x))
```

3.350.5 Fricas [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 1867, normalized size of antiderivative = 6.97

$$\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*((4*a^2*c*d^2*e^4 + 3*a^3*e^6)*x^3 + (4*a^2*c*d^3*e^3 + 3*a^3*d*e^
5)*x^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^
2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)
)/(e^2*x^2 + 2*d*e*x + d^2)) - ((c^3*d^6*e - 4*a*c^2*d^4*e^3 - 11*a^2*c*d^
2*e^5 - 6*a^3*e^7)*x^3 + (c^3*d^7 - 4*a*c^2*d^5*e^2 - 11*a^2*c*d^3*e^4 - 6
*a^3*d*e^6)*x^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^
2) - 2*(a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3*d^3*e^4 - 2*(2*a*c^2*d^5*e^2 + 5
*a^2*c*d^3*e^4 + 3*a^3*d*e^6)*x^2 - 3*(a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3
*d^2*e^5)*x)*sqrt(c*x^2 + a))/((a^2*c^2*d^8*e + 2*a^3*c*d^6*e^3 + a^4*d^4*
e^5)*x^3 + (a^2*c^2*d^9 + 2*a^3*c*d^7*e^2 + a^4*d^5*e^4)*x^2), 1/4*(4*((4*
a^2*c*d^2*e^4 + 3*a^3*e^6)*x^3 + (4*a^2*c*d^3*e^3 + 3*a^3*d*e^5)*x^2)*sqrt
(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)
/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - ((c^3*d^6*e - 4*a*c^2*d^
4*e^3 - 11*a^2*c*d^2*e^5 - 6*a^3*e^7)*x^3 + (c^3*d^7 - 4*a*c^2*d^5*e^2 - 1
1*a^2*c*d^3*e^4 - 6*a^3*d*e^6)*x^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a
)*sqrt(a) + 2*a)/x^2) - 2*(a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3*d^3*e^4 - 2*(
2*a*c^2*d^5*e^2 + 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)*x^2 - 3*(a*c^2*d^6*e + 2*
a^2*c*d^4*e^3 + a^3*d^2*e^5)*x)*sqrt(c*x^2 + a))/((a^2*c^2*d^8*e + 2*a^3*c
*d^6*e^3 + a^4*d^4*e^5)*x^3 + (a^2*c^2*d^9 + 2*a^3*c*d^7*e^2 + a^4*d^5*e^4
)*x^2), -1/2*(((c^3*d^6*e - 4*a*c^2*d^4*e^3 - 11*a^2*c*d^2*e^5 - 6*a^3*...
```

3.350.6 Sympy [F]

$$\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

input
```
integrate(1/x**3/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.350.7 Maxima [F]

$$\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^2x^3}\,dx$$

input
```
integrate(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*x^3), x)
```

3.350.8 Giac [F]

$$\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^2x^3}\,dx$$

input
```
integrate(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.350.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{x^3\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)^2}\,dx$$

input
```
int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

output
```
int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.351 $\int x^2(a+bx)^n(c+dx^2)\,dx$

3.351.1 Optimal result

Integrand size = 18, antiderivative size = 135

$$\int x^2(a+bx)^n(c+dx^2)\,dx=\frac{a^2(b^2c+a^2d)(a+bx)^{1+n}}{b^5(1+n)}-\frac{2a(b^2c+2a^2d)(a+bx)^{2+n}}{b^5(2+n)}+\frac{(b^2c+6a^2d)(a+bx)^{3+n}}{b^5(3+n)}-\frac{4ad(a+bx)^{4+n}}{b^5(4+n)}+\frac{d(a+bx)^{5+n}}{b^5(5+n)}$$

output
```
a^2*(a^2*d+b^2*c)*(b*x+a)^(1+n)/b^5/(1+n)-2*a*(2*a^2*d+b^2*c)*(b*x+a)^(2+n
)/b^5/(2+n)+(6*a^2*d+b^2*c)*(b*x+a)^(3+n)/b^5/(3+n)-4*a*d*(b*x+a)^(4+n)/b^
5/(4+n)+d*(b*x+a)^(5+n)/b^5/(5+n)
```

3.351.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.84

$$\int x^2(a+bx)^n(c+dx^2)\,dx=\frac{(a+bx)^{1+n}\left(\frac{a^2b^2c+a^4d}{1+n}-\frac{2a(b^2c+2a^2d)(a+bx)}{2+n}+\frac{(b^2c+6a^2d)(a+bx)^2}{3+n}-\frac{4ad(a+bx)^3}{4+n}+\frac{d(a+bx)^4}{5+n}\right)}{b^5}$$

input
```
Integrate[x^2*(a + b*x)^n*(c + d*x^2),x]
```

output
```
((a + b*x)^(1 + n)*((a^2*b^2*c + a^4*d)/(1 + n) - (2*a*(b^2*c + 2*a^2*d)*(
a + b*x))/(2 + n) + ((b^2*c + 6*a^2*d)*(a + b*x)^2)/(3 + n) - (4*a*d*(a +
b*x)^3)/(4 + n) + (d*(a + b*x)^4)/(5 + n)))/b^5
```

3.351.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(c+dx^2\right) (a+bx)^n \, dx$$

$$\Big\downarrow \text{522}$$

$$\int \left(-\frac{2\left(2a^3d+ab^2c\right)(a+bx)^{n+1}}{b^4}+\frac{\left(6a^2d+b^2c\right)(a+bx)^{n+2}}{b^4}+\frac{\left(a^4d+a^2b^2c\right)(a+bx)^n}{b^4}-\frac{4ad(a+bx)^{n+3}}{b^4}+\frac{d(\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\left(a^2d+b^2c\right)(a+bx)^{n+1}}{b^5(n+1)}-\frac{2a\left(2a^2d+b^2c\right)(a+bx)^{n+2}}{b^5(n+2)}+\frac{\left(6a^2d+b^2c\right)(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad(a+bx)^{n+4}}{b^5(n+4)}+\frac{d(a+bx)^{n+5}}{b^5(n+5)}$$

input
```
Int[x^2*(a + b*x)^n*(c + d*x^2),x]
```

output
```
(a^2*(b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^5*(1 + n)) - (2*a*(b^2*c + 2*a^
2*d)*(a + b*x)^(2 + n))/(b^5*(2 + n)) + ((b^2*c + 6*a^2*d)*(a + b*x)^(3 +
n))/(b^5*(3 + n)) - (4*a*d*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d*(a + b*x)
^(5 + n))/(b^5*(5 + n))
```

3.351.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.351.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(135) = 270$.

Time = 0.37 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.37

method	result
norman	$\frac{d\,x^{5} e^{n \ln(bx+a)}}{5+n}+\frac{n a d\,x^{4} e^{n \ln(bx+a)}}{b(n^{2}+9n+20)}+\frac{(b^{2}c\,n^{2}+9b^{2}cn+12a^{2}d+20b^{2}c)\,a n\,x^{2} e^{n \ln(bx+a)}}{b^{3}(n^{4}+14n^{3}+71n^{2}+154n+120)}+\frac{2a^{3}(b^{2}c\,n^{2}+9b^{2}cn+12a^{2}d+20b}{b^{5}(n^{5}+15n^{4}+85n^{3}+225n^{2}+}$
gosper	$\frac{(bx+a)^{1+n}(b^{4}d\,n^{4}x^{4}+10b^{4}d\,n^{3}x^{4}-4a\,b^{3}d\,n^{3}x^{3}+b^{4}c\,n^{4}x^{2}+35b^{4}d\,n^{2}x^{4}-24a\,b^{3}d\,n^{2}x^{3}+12b^{4}c\,n^{3}x^{2}+50b^{4}dn\,x^{4}+12a^{2}b^{2}d\,n^{2}x^{2}}{}$
risch	$\frac{(b^{5}d\,n^{4}x^{5}+a\,b^{4}d\,n^{4}x^{4}+10b^{5}d\,n^{3}x^{5}+6a\,b^{4}d\,n^{3}x^{4}+b^{5}c\,n^{4}x^{3}+35b^{5}d\,n^{2}x^{5}-4a^{2}b^{3}d\,n^{3}x^{3}+a\,b^{4}c\,n^{4}x^{2}+11a\,b^{4}d\,n^{2}x^{4}+12b^{5}c\,n^{3}x^{3}}{}$
parallelrisch	$\frac{x^{3}(bx+a)^{n}b^{5}c\,n^{4}+50x^{5}(bx+a)^{n}b^{5}dn+12x^{3}(bx+a)^{n}b^{5}c\,n^{3}+49x^{3}(bx+a)^{n}b^{5}c\,n^{2}+78x^{3}(bx+a)^{n}b^{5}cn+2(bx+a)^{n}a^{3}b^{2}c\,n^{2}+18(}{}$

input
```
int(x^2*(b*x+a)^n*(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
d/(5+n)*x^5*exp(n*ln(b*x+a))+n*a/b*d/(n^2+9*n+20)*x^4*exp(n*ln(b*x+a))+(b^
2*c*n^2+9*b^2*c*n+12*a^2*d+20*b^2*c)*a/b^3*n/(n^4+14*n^3+71*n^2+154*n+120)
*x^2*exp(n*ln(b*x+a))+2*a^3*(b^2*c*n^2+9*b^2*c*n+12*a^2*d+20*b^2*c)/b^5/(n
^5+15*n^4+85*n^3+225*n^2+274*n+120)*exp(n*ln(b*x+a))-(-b^2*c*n^2+4*a^2*d*n
-9*b^2*c*n-20*b^2*c)/b^2/(n^3+12*n^2+47*n+60)*x^3*exp(n*ln(b*x+a))-2/b^4*n
*a^2*(b^2*c*n^2+9*b^2*c*n+12*a^2*d+20*b^2*c)/(n^5+15*n^4+85*n^3+225*n^2+27
4*n+120)*x*exp(n*ln(b*x+a))
```

3.351.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(135) = 270$.

Time = 0.30 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.73

$$\int x^2(a+bx)^n\left(c+dx^2\right)\,dx$$
$$=\frac{(2\,a^3b^2cn^2+18\,a^3b^2cn+40\,a^3b^2c+24\,a^5d+(b^5dn^4+10\,b^5dn^3+35\,b^5dn^2+50\,b^5dn+24\,b^5d)x^5+(ab^4dn}{}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^2+c),x, algorithm="fricas")
```

output
```
(2*a^3*b^2*c*n^2 + 18*a^3*b^2*c*n + 40*a^3*b^2*c + 24*a^5*d + (b^5*d*n^4 +
 10*b^5*d*n^3 + 35*b^5*d*n^2 + 50*b^5*d*n + 24*b^5*d)*x^5 + (a*b^4*d*n^4 +
 6*a*b^4*d*n^3 + 11*a*b^4*d*n^2 + 6*a*b^4*d*n)*x^4 + (b^5*c*n^4 + 40*b^5*c
 + 4*(3*b^5*c - a^2*b^3*d)*n^3 + (49*b^5*c - 12*a^2*b^3*d)*n^2 + 2*(39*b^5
*c - 4*a^2*b^3*d)*n)*x^3 + (a*b^4*c*n^4 + 10*a*b^4*c*n^3 + (29*a*b^4*c + 1
2*a^3*b^2*d)*n^2 + 4*(5*a*b^4*c + 3*a^3*b^2*d)*n)*x^2 - 2*(a^2*b^3*c*n^3 +
 9*a^2*b^3*c*n^2 + 4*(5*a^2*b^3*c + 3*a^4*b*d)*n)*x)*(b*x + a)^n/(b^5*n^5
+ 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

3.351.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4134 vs. $2(122) = 244$.

Time = 1.26 (sec) , antiderivative size = 4134, normalized size of antiderivative = 30.62

$$\int x^2(a+bx)^n\left(c+dx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(b*x+a)**n*(d*x**2+c),x)
```

output
```
Piecewise((a**n*(c*x**3/3 + d*x**5/5), Eq(b, 0)), (12*a**4*d*log(a/b + x)/
(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b
**9*x**4) + 25*a**4*d/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 +
 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a**3*b*d*x*log(a/b + x)/(12*a**4*b**5
 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 8
8*a**3*b*d*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**
8*x**3 + 12*b**9*x**4) - a**2*b**2*c/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a
**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 72*a**2*b**2*d*x**2*log(a
/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**
3 + 12*b**9*x**4) + 108*a**2*b**2*d*x**2/(12*a**4*b**5 + 48*a**3*b**6*x +
72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 4*a*b**3*c*x/(12*a**4
*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4
) + 48*a*b**3*d*x**3*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2
*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**3*d*x**3/(12*a**4*b*
*5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) -
 6*b**4*c*x**2/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b
**8*x**3 + 12*b**9*x**4) + 12*b**4*d*x**4*log(a/b + x)/(12*a**4*b**5 + 48*
a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4), Eq(n, -5
)), (-12*a**4*d*log(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2
+ 3*b**8*x**3) - 22*a**4*d/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2...
```

3.351.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.56

$$\int x^2(a+bx)^n\left(c+dx^2\right)\,dx$$
$$=\frac{((n^2+3n+2)b^3x^3+(n^2+n)ab^2x^2-2a^2bnx+2a^3)(bx+a)^nc}{(n^3+6n^2+11n+6)b^3}$$
$$+\frac{((n^4+10n^3+35n^2+50n+24)b^5x^5+(n^4+6n^3+11n^2+6n)ab^4x^4-4(n^3+3n^2+2n)a^2b^3x^3+1}{(n^5+15n^4+85n^3+225n^2+274n+120)b^5}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^2+c),x, algorithm="maxima")
```

output
```
((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
 + a)^n*c/((n^3 + 6*n^2 + 11*n + 6)*b^3) + ((n^4 + 10*n^3 + 35*n^2 + 50*n
+ 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 +
2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x
+ a)^n*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5)
```

3.351.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(135) = 270$.

Time = 0.29 (sec) , antiderivative size = 624, normalized size of antiderivative = 4.62

$$\int x^2(a+bx)^n\left(c+dx^2\right)\,dx$$
$$=\frac{(bx+a)^nb^5dn^4x^5+(bx+a)^nab^4dn^4x^4+10(bx+a)^nb^5dn^3x^5+(bx+a)^nb^5cn^4x^3+6(bx+a)^nab^4dn^3x^4}{}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^2+c),x, algorithm="giac")
```

output
```
((b*x + a)^n*b^5*d*n^4*x^5 + (b*x + a)^n*a*b^4*d*n^4*x^4 + 10*(b*x + a)^n*
b^5*d*n^3*x^5 + (b*x + a)^n*b^5*c*n^4*x^3 + 6*(b*x + a)^n*a*b^4*d*n^3*x^4
+ 35*(b*x + a)^n*b^5*d*n^2*x^5 + (b*x + a)^n*a*b^4*c*n^4*x^2 + 12*(b*x + a
)^n*b^5*c*n^3*x^3 - 4*(b*x + a)^n*a^2*b^3*d*n^3*x^3 + 11*(b*x + a)^n*a*b^4
*d*n^2*x^4 + 50*(b*x + a)^n*b^5*d*n*x^5 + 10*(b*x + a)^n*a*b^4*c*n^3*x^2 +
 49*(b*x + a)^n*b^5*c*n^2*x^3 - 12*(b*x + a)^n*a^2*b^3*d*n^2*x^3 + 6*(b*x
+ a)^n*a*b^4*d*n*x^4 + 24*(b*x + a)^n*b^5*d*x^5 - 2*(b*x + a)^n*a^2*b^3*c*
n^3*x + 29*(b*x + a)^n*a*b^4*c*n^2*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n^2*x^2
+ 78*(b*x + a)^n*b^5*c*n*x^3 - 8*(b*x + a)^n*a^2*b^3*d*n*x^3 - 18*(b*x + a
)^n*a^2*b^3*c*n^2*x + 20*(b*x + a)^n*a*b^4*c*n*x^2 + 12*(b*x + a)^n*a^3*b^
2*d*n*x^2 + 40*(b*x + a)^n*b^5*c*x^3 + 2*(b*x + a)^n*a^3*b^2*c*n^2 - 40*(b
*x + a)^n*a^2*b^3*c*n*x - 24*(b*x + a)^n*a^4*b*d*n*x + 18*(b*x + a)^n*a^3*
b^2*c*n + 40*(b*x + a)^n*a^3*b^2*c + 24*(b*x + a)^n*a^5*d)/(b^5*n^5 + 15*b
^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

3.351.9 Mupad [B] (verification not implemented)

Time = 11.63 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.69

$$\int x^2(a+bx)^n\left(c+dx^2\right)\,dx=(a+b\,x)^n\left(\frac{2\,a^3\,\left(12\,d\,a^2+c\,b^2\,n^2+9\,c\,b^2\,n+20\,c\,b^2\right)}{b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{d\,x^5\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120}+\frac{x^3\,\left(n^2+3\,n+2\right)\,\left(-4\,d\,a^2\,n+c\,b^2\,n^2+9\,c\,b^2\,n+20\,c\,b^2\right)}{b^2\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}-\frac{2\,a^2\,n\,x\,\left(12\,d\,a^2+c\,b^2\,n^2+9\,c\,b^2\,n+20\,c\,b^2\right)}{b^4\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{a\,n\,x^2\,\left(n+1\right)\,\left(12\,d\,a^2+c\,b^2\,n^2+9\,c\,b^2\,n+20\,c\,b^2\right)}{b^3\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{a\,d\,n\,x^4\,\left(n^3+6\,n^2+11\,n+6\right)}{b\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\right)$$

input
```
int(x^2*(c + d*x^2)*(a + b*x)^n,x)
```

output
```
(a + b*x)^n*((2*a^3*(12*a^2*d + 20*b^2*c + b^2*c*n^2 + 9*b^2*c*n))/(b^5*(2
74*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (d*x^5*(50*n + 35*n^2 + 1
0*n^3 + n^4 + 24))/(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) + (x^3*
(3*n + n^2 + 2)*(20*b^2*c + b^2*c*n^2 - 4*a^2*d*n + 9*b^2*c*n))/(b^2*(274*
n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (2*a^2*n*x*(12*a^2*d + 20*b^
2*c + b^2*c*n^2 + 9*b^2*c*n))/(b^4*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^
5 + 120)) + (a*n*x^2*(n + 1)*(12*a^2*d + 20*b^2*c + b^2*c*n^2 + 9*b^2*c*n)
)/(b^3*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*d*n*x^4*(11*n
 + 6*n^2 + n^3 + 6))/(b*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)))
```

3.351. $\int x^2(a+bx)^n\left(c+dx^2\right)\,dx$

3.352 $\int x(a+bx)^n\left(c+dx^2\right)\,dx$

3.352.1 Optimal result

Integrand size = 16, antiderivative size = 102

$$\int x(a+bx)^n\left(c+dx^2\right)\,dx=-\frac{a(b^2c+a^2d)\,(a+bx)^{1+n}}{b^4(1+n)}+\frac{(b^2c+3a^2d)\,(a+bx)^{2+n}}{b^4(2+n)}-\frac{3ad(a+bx)^{3+n}}{b^4(3+n)}+\frac{d(a+bx)^{4+n}}{b^4(4+n)}$$

output
```
-a*(a^2*d+b^2*c)*(b*x+a)^(1+n)/b^4/(1+n)+(3*a^2*d+b^2*c)*(b*x+a)^(2+n)/b^4
/(2+n)-3*a*d*(b*x+a)^(3+n)/b^4/(3+n)+d*(b*x+a)^(4+n)/b^4/(4+n)
```

3.352.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.07

$$\int x(a+bx)^n\left(c+dx^2\right)\,dx$$
$$=\frac{(a+bx)^{1+n}\left(-6a^3d+6a^2bd(1+n)x+b^3(3+4n+n^2)\,x(c(4+n)+d(2+n)x^2)-ab^2(c(12+7n+n^2)\right.}{b^4(1+n)(2+n)(3+n)(4+n)}$$

input
```
Integrate[x*(a + b*x)^n*(c + d*x^2),x]
```

output
```
((a + b*x)^(1 + n)*(-6*a^3*d + 6*a^2*b*d*(1 + n)*x + b^3*(3 + 4*n + n^2)*x
*(c*(4 + n) + d*(2 + n)*x^2) - a*b^2*(c*(12 + 7*n + n^2) + 3*d*(2 + 3*n +
n^2)*x^2)))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n))
```

3.352.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c+dx^2\right)(a+bx)^n\,dx$$

$$\Big\downarrow \ 522$$

$$\int \left(\frac{a\left(a^2(-d)-b^2c\right)(a+bx)^n}{b^3}+\frac{\left(3a^2d+b^2c\right)(a+bx)^{n+1}}{b^3}-\frac{3ad(a+bx)^{n+2}}{b^3}+\frac{d(a+bx)^{n+3}}{b^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a\left(a^2d+b^2c\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{\left(3a^2d+b^2c\right)(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

input
```
Int[x*(a + b*x)^n*(c + d*x^2),x]
```

output
```
-((a*(b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^4*(1 + n))) + ((b^2*c + 3*a^2*d
)*(a + b*x)^(2 + n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n
)) + (d*(a + b*x)^(4 + n))/(b^4*(4 + n))
```

3.352.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.352.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.91

method	result
gosper	$-\frac{(bx+a)^{1+n}(-b^3dn^3x^3-6b^3dn^2x^3+3ab^2dn^2x^2-b^3cn^3x-11b^3dnx^3+9ab^2dnx^2-8b^3cn^2x-6x^3b^3d-6a^2bdnx+ab^2cn^2+}{b^4(n^4+10n^3+35n^2+50n+24)}$
norman	$\frac{dx^4e^{n\ln(bx+a)}}{4+n}+\frac{na(b^2cn^2+7b^2cn+6a^2d+12b^2c)xe^{n\ln(bx+a)}}{b^3(n^4+10n^3+35n^2+50n+24)}+\frac{nadx^3e^{n\ln(bx+a)}}{b(n^2+7n+12)}-\frac{a^2(b^2cn^2+7b^2cn+6a^2d+12b^2c)e}{b^4(n^4+10n^3+35n^2+50n+}$
risch	$-\frac{(-b^4dn^3x^4-ab^3dn^3x^3-6b^4dn^2x^4-3ab^3dn^2x^3-b^4cn^3x^2-11b^4dnx^4+3a^2b^2dn^2x^2-ab^3cn^3x-2ab^3dnx^3-8b^4cn^2x^2-}{(3+n)(4+n)(}$
parallelrisch	$\frac{-(bx+a)^na^3b^2cn^2-7(bx+a)^na^3b^2cn-12(bx+a)^na^3b^2c+12x(bx+a)^na^2b^3cn+11x^4(bx+a)^nab^4dn+3x^3(bx+a)^na^2b^3dn^2+x}{}$

input
```
int(x*(b*x+a)^n*(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/b^4*(b*x+a)^(1+n)/(n^4+10*n^3+35*n^2+50*n+24)*(-b^3*d*n^3*x^3-6*b^3*d*n
^2*x^3+3*a*b^2*d*n^2*x^2-b^3*c*n^3*x-11*b^3*d*n*x^3+9*a*b^2*d*n*x^2-8*b^3*
c*n^2*x-6*b^3*d*x^3-6*a^2*b*d*n*x+a*b^2*c*n^2+6*a*b^2*d*x^2-19*b^3*c*n*x-6
*a^2*b*d*x+7*a*b^2*c*n-12*b^3*c*x+6*a^3*d+12*a*b^2*c)
```

3.352.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(102) = 204$.

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.45

$$\int x(a+bx)^n\left(c+dx^2\right)\,dx = -\frac{(a^2b^2cn^2+7\,a^2b^2cn+12\,a^2b^2c+6\,a^4d-(b^4dn^3+6\,b^4dn^2+11\,b^4dn+6\,b^4d)x^4-(ab^3dn^3+3\,ab^3dn^2+}{b^4n^4+}$$

input
```
integrate(x*(b*x+a)^n*(d*x^2+c),x, algorithm="fricas")
```

output
```
-(a^2*b^2*c*n^2 + 7*a^2*b^2*c*n + 12*a^2*b^2*c + 6*a^4*d - (b^4*d*n^3 + 6*
b^4*d*n^2 + 11*b^4*d*n + 6*b^4*d)*x^4 - (a*b^3*d*n^3 + 3*a*b^3*d*n^2 + 2*a
*b^3*d*n)*x^3 - (b^4*c*n^3 + 12*b^4*c + (8*b^4*c - 3*a^2*b^2*d)*n^2 + (19*
b^4*c - 3*a^2*b^2*d)*n)*x^2 - (a*b^3*c*n^3 + 7*a*b^3*c*n^2 + 6*(2*a*b^3*c
+ a^3*b*d)*n)*x)*(b*x + a)^n/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n
 + 24*b^4)
```

3.352.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2181 vs. $2(90) = 180$.

Time = 0.78 (sec) , antiderivative size = 2181, normalized size of antiderivative = 21.38

$$\int x(a+bx)^n\left(c+dx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(x*(b*x+a)**n*(d*x**2+c),x)
```

output
```
Piecewise((a**n*(c*x**2/2 + d*x**4/4), Eq(b, 0)), (6*a**3*d*log(a/b + x)/(
6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3*d/(
6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*d
*x*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x*
*3) + 27*a**2*b*d*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**
7*x**3) - a*b**2*c/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7
*x**3) + 18*a*b**2*d*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*
a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*d*x**2/(6*a**3*b**4 + 18*a**2*b**5*
x + 18*a*b**6*x**2 + 6*b**7*x**3) - 3*b**3*c*x/(6*a**3*b**4 + 18*a**2*b**5
*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 6*b**3*d*x**3*log(a/b + x)/(6*a**3*b*
*4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3), Eq(n, -4)), (-6*a**3*
d*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 9*a**3*d/(2*a**2
*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*d*x*log(a/b + x)/(2*a**2*b**
4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*d*x/(2*a**2*b**4 + 4*a*b**5*x +
2*b**6*x**2) - a*b**2*c/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**
2*d*x**2*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 2*b**3*c*
x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*x**3/(2*a**2*b**4 +
4*a*b**5*x + 2*b**6*x**2), Eq(n, -3)), (6*a**3*d*log(a/b + x)/(2*a*b**4 +
2*b**5*x) + 6*a**3*d/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d*x*log(a/b + x)/(2*
a*b**4 + 2*b**5*x) + 2*a*b**2*c*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*...
```

3.352.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.43

$$\int x(a+bx)^n\left(c+dx^2\right)\,dx = \frac{(b^2(n+1)x^2+abnx-a^2)(bx+a)^nc}{(n^2+3\,n+2)b^2} + \frac{((n^3+6\,n^2+11\,n+6)b^4x^4+(n^3+3\,n^2+2\,n)ab^3x^3-3\,(n^2+n)a^2b^2x^2+6\,a^3bnx-6\,a^4)(bx+a)^nd}{(n^4+10\,n^3+35\,n^2+50\,n+24)b^4}$$

input
```
integrate(x*(b*x+a)^n*(d*x^2+c),x, algorithm="maxima")
```

output
```
(b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c/((n^2 + 3*n + 2)*b^2) + ((
n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 +
 n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n*d/((n^4 + 10*n^3 + 35*n
^2 + 50*n + 24)*b^4)
```

3.352.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(102) = 204$.

Time = 0.28 (sec) , antiderivative size = 410, normalized size of antiderivative = 4.02

$$\int x(a+bx)^n\left(c+dx^2\right)\,dx$$
$$=\frac{(bx+a)^n b^4dn^3x^4+(bx+a)^n ab^3dn^3x^3+6\,(bx+a)^n b^4dn^2x^4+(bx+a)^n b^4cn^3x^2+3\,(bx+a)^n ab^3dn^2x^3 -}{}$$

input
```
integrate(x*(b*x+a)^n*(d*x^2+c),x, algorithm="giac")
```

output
```
((b*x + a)^n*b^4*d*n^3*x^4 + (b*x + a)^n*a*b^3*d*n^3*x^3 + 6*(b*x + a)^n*b
^4*d*n^2*x^4 + (b*x + a)^n*b^4*c*n^3*x^2 + 3*(b*x + a)^n*a*b^3*d*n^2*x^3 +
 11*(b*x + a)^n*b^4*d*n*x^4 + (b*x + a)^n*a*b^3*c*n^3*x + 8*(b*x + a)^n*b^
4*c*n^2*x^2 - 3*(b*x + a)^n*a^2*b^2*d*n^2*x^2 + 2*(b*x + a)^n*a*b^3*d*n*x^
3 + 6*(b*x + a)^n*b^4*d*x^4 + 7*(b*x + a)^n*a*b^3*c*n^2*x + 19*(b*x + a)^n
*b^4*c*n*x^2 - 3*(b*x + a)^n*a^2*b^2*d*n*x^2 - (b*x + a)^n*a^2*b^2*c*n^2 +
 12*(b*x + a)^n*a*b^3*c*n*x + 6*(b*x + a)^n*a^3*b*d*n*x + 12*(b*x + a)^n*b
^4*c*x^2 - 7*(b*x + a)^n*a^2*b^2*c*n - 12*(b*x + a)^n*a^2*b^2*c - 6*(b*x +
 a)^n*a^4*d)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

3.352.9 Mupad [B] (verification not implemented)

Time = 11.56 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.50

$$\int x(a+bx)^n\left(c+dx^2\right)\,dx=(a+b\,x)^n\left(\frac{d\,x^4\,(n^3+6\,n^2+11\,n+6)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{a^2\,(6\,d\,a^2+c\,b^2\,n^2+7\,c\,b^2\,n+12\,c\,b^2)}{b^4\,(n^4+10\,n^3+35\,n^2+50\,n+24)}+\frac{x^2\,(n+1)\,(-3\,d\,a^2\,n+c\,b^2\,n^2+7\,c\,b^2\,n+12\,c\,b^2)}{b^2\,(n^4+10\,n^3+35\,n^2+50\,n+24)}+\frac{a\,n\,x\,(6\,d\,a^2+c\,b^2\,n^2+7\,c\,b^2\,n+12\,c\,b^2)}{b^3\,(n^4+10\,n^3+35\,n^2+50\,n+24)}+\frac{a\,d\,n\,x^3\,(n^2+3\,n+2)}{b\,(n^4+10\,n^3+35\,n^2+50\,n+24)}\right)$$

input
```
int(x*(c + d*x^2)*(a + b*x)^n,x)
```

output
```
(a + b*x)^n*((d*x^4*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^
4 + 24) - (a^2*(6*a^2*d + 12*b^2*c + b^2*c*n^2 + 7*b^2*c*n))/(b^4*(50*n +
35*n^2 + 10*n^3 + n^4 + 24)) + (x^2*(n + 1)*(12*b^2*c + b^2*c*n^2 - 3*a^2*
d*n + 7*b^2*c*n))/(b^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*n*x*(6*a^
2*d + 12*b^2*c + b^2*c*n^2 + 7*b^2*c*n))/(b^3*(50*n + 35*n^2 + 10*n^3 + n^
4 + 24)) + (a*d*n*x^3*(3*n + n^2 + 2))/(b*(50*n + 35*n^2 + 10*n^3 + n^4 +
24)))
```

3.353 $\int (a+bx)^n (c+dx^2)\, dx$

3.353.1 Optimal result

Integrand size = 15, antiderivative size = 70

$$\int (a+bx)^n (c+dx^2)\, dx=\frac{(b^2c+a^2d)\,(a+bx)^{1+n}}{b^3(1+n)}-\frac{2ad(a+bx)^{2+n}}{b^3(2+n)}+\frac{d(a+bx)^{3+n}}{b^3(3+n)}$$

output
```
(a^2*d+b^2*c)*(b*x+a)^(1+n)/b^3/(1+n)-2*a*d*(b*x+a)^(2+n)/b^3/(2+n)+d*(b*x
+a)^(3+n)/b^3/(3+n)
```

3.353.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int (a+bx)^n (c+dx^2)\, dx = \frac{(a+bx)^{1+n}\,(2a^2d-2abd(1+n)x+b^2(2+n)\,(c(3+n)+d(1+n)x^2))}{b^3(1+n)(2+n)(3+n)}$$

input
```
Integrate[(a + b*x)^n*(c + d*x^2),x]
```

output
```
((a + b*x)^(1 + n)*(2*a^2*d - 2*a*b*d*(1 + n)*x + b^2*(2 + n)*(c*(3 + n) +
 d*(1 + n)*x^2)))/(b^3*(1 + n)*(2 + n)*(3 + n))
```

3.353.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^2)(a+bx)^n\,dx$$

$$\Big\downarrow \ 476$$

$$\int \left(\frac{(a^2d+b^2c)(a+bx)^n}{b^2}-\frac{2ad(a+bx)^{n+1}}{b^2}+\frac{d(a+bx)^{n+2}}{b^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{(a^2d+b^2c)(a+bx)^{n+1}}{b^3(n+1)}-\frac{2ad(a+bx)^{n+2}}{b^3(n+2)}+\frac{d(a+bx)^{n+3}}{b^3(n+3)}$$

input
```
Int[(a + b*x)^n*(c + d*x^2),x]
```

output
```
((b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^3*(1 + n)) - (2*a*d*(a + b*x)^(2 +
n))/(b^3*(2 + n)) + (d*(a + b*x)^(3 + n))/(b^3*(3 + n))
```

3.353.3.1 Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.353.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.43

method	result
gosper	$\frac{(bx+a)^{1+n}\left(b^{2}d\,n^{2}x^{2}+3b^{2}dn\,x^{2}-2abdnx+b^{2}c\,n^{2}+2d\,x^{2}b^{2}-2abdx+5b^{2}cn+2a^{2}d+6b^{2}c\right)}{b^{3}\left(n^{3}+6n^{2}+11n+6\right)}$
risch	$\frac{\left(b^{3}d\,n^{2}x^{3}+a\,b^{2}d\,n^{2}x^{2}+3b^{3}dn\,x^{3}+a\,b^{2}dn\,x^{2}+b^{3}c\,n^{2}x+2x^{3}b^{3}d-2a^{2}bdnx+a\,b^{2}c\,n^{2}+5b^{3}cnx+5a\,b^{2}cn+6b^{3}cx+2d\,a^{3}+6a\,b^{2}c\right)(b}{(2+n)(3+n)(1+n)b^{3}}$
norman	$\frac{d\,x^{3}{\mathrm e}^{n\ln(bx+a)}}{3+n}+\frac{a\left(b^{2}c\,n^{2}+5b^{2}cn+2a^{2}d+6b^{2}c\right){\mathrm e}^{n\ln(bx+a)}}{b^{3}\left(n^{3}+6n^{2}+11n+6\right)}+\frac{adn\,x^{2}{\mathrm e}^{n\ln(bx+a)}}{b\left(n^{2}+5n+6\right)}-\frac{\left(-b^{2}c\,n^{2}+2a^{2}dn-5b^{2}cn-6b^{2}c\right)x\,{\mathrm e}^{n\ln}}{b^{2}\left(n^{3}+6n^{2}+11n+6\right)}$
parallelrisch	$\frac{x^{3}(bx+a)^{n}b^{3}d\,n^{2}+3x^{3}(bx+a)^{n}b^{3}dn+x^{2}(bx+a)^{n}a\,b^{2}d\,n^{2}+2x^{3}(bx+a)^{n}b^{3}d+x^{2}(bx+a)^{n}a\,b^{2}dn+x(bx+a)^{n}b^{3}c\,n^{2}-2x(bx+a)^{n}}{b^{3}\left(n^{3}+6n^{2}+11n+6\right.}$

input
```
int((b*x+a)^n*(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(b*x+a)^(1+n)/(n^3+6*n^2+11*n+6)*(b^2*d*n^2*x^2+3*b^2*d*n*x^2-2*a*b*
d*n*x+b^2*c*n^2+2*b^2*d*x^2-2*a*b*d*x+5*b^2*c*n+2*a^2*d+6*b^2*c)
```

3.353.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(70) = 140.

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.11

$$\int (a+bx)^n\left(c+dx^2\right)\,dx$$
$$=\frac{(ab^2cn^2+5\,ab^2cn+6\,ab^2c+2\,a^3d+(b^3dn^2+3\,b^3dn+2\,b^3d)x^3+(ab^2dn^2+ab^2dn)x^2+(b^3cn^2+6\,b^3c+}{b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3}$$

input
```
integrate((b*x+a)^n*(d*x^2+c),x, algorithm="fricas")
```

output
```
(a*b^2*c*n^2 + 5*a*b^2*c*n + 6*a*b^2*c + 2*a^3*d + (b^3*d*n^2 + 3*b^3*d*n
+ 2*b^3*d)*x^3 + (a*b^2*d*n^2 + a*b^2*d*n)*x^2 + (b^3*c*n^2 + 6*b^3*c + (5
*b^3*c - 2*a^2*b*d)*n)*x)*(b*x + a)^n/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*
b^3)
```

3.353.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(61) = 122$.

Time = 0.50 (sec) , antiderivative size = 952, normalized size of antiderivative = 13.60

$$\int (a+bx)^n (c+dx^2)\,dx$$

$$= \begin{cases} a^n\left(cx+\frac{dx^3}{3}\right) \\ \frac{2a^2d\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{3a^2d}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{4abdx\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{4abdx}{2a^2b^3+4ab^4x+2b^5x^2}-\frac{b^2c}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{2b^2dx^2\log}{2a^2b^3+4ab^4} \\ -\frac{2a^2d\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x}-\frac{2a^2d}{ab^3+b^4x}-\frac{2abdx\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x}-\frac{b^2c}{ab^3+b^4x}+\frac{b^2dx^2}{ab^3+b^4x} \\ \frac{a^2d\log\left(\frac{a}{b}+x\right)}{b^3}-\frac{adx}{b^2}+\frac{c\log\left(\frac{a}{b}+x\right)}{b}+\frac{dx^2}{2b} \\ \frac{2a^3d(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}-\frac{2a^2bdnx(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{ab^2cn^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{5ab^2cn(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{6ab^2c(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+} \end{cases}$$

input
```
integrate((b*x+a)**n*(d*x**2+c),x)
```

output
```
Piecewise((a**n*(c*x + d*x**3/3), Eq(b, 0)), (2*a**2*d*log(a/b + x)/(2*a**
2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2*d/(2*a**2*b**3 + 4*a*b**4*x +
2*b**5*x**2) + 4*a*b*d*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x
**2) + 4*a*b*d*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) - b**2*c/(2*a**2
*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*d*x**2*log(a/b + x)/(2*a**2*b**
3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*d*log(a/b + x)/(a*b**3
 + b**4*x) - 2*a**2*d/(a*b**3 + b**4*x) - 2*a*b*d*x*log(a/b + x)/(a*b**3 +
 b**4*x) - b**2*c/(a*b**3 + b**4*x) + b**2*d*x**2/(a*b**3 + b**4*x), Eq(n,
 -2)), (a**2*d*log(a/b + x)/b**3 - a*d*x/b**2 + c*log(a/b + x)/b + d*x**2/
(2*b), Eq(n, -1)), (2*a**3*d*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b*
*3*n + 6*b**3) - 2*a**2*b*d*n*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11
*b**3*n + 6*b**3) + a*b**2*c*n**2*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 +
11*b**3*n + 6*b**3) + 5*a*b**2*c*n*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 +
 11*b**3*n + 6*b**3) + 6*a*b**2*c*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 +
11*b**3*n + 6*b**3) + a*b**2*d*n**2*x**2*(a + b*x)**n/(b**3*n**3 + 6*b**3*
n**2 + 11*b**3*n + 6*b**3) + a*b**2*d*n*x**2*(a + b*x)**n/(b**3*n**3 + 6*b
**3*n**2 + 11*b**3*n + 6*b**3) + b**3*c*n**2*x*(a + b*x)**n/(b**3*n**3 + 6
*b**3*n**2 + 11*b**3*n + 6*b**3) + 5*b**3*c*n*x*(a + b*x)**n/(b**3*n**3 +
6*b**3*n**2 + 11*b**3*n + 6*b**3) + 6*b**3*c*x*(a + b*x)**n/(b**3*n**3 + 6
*b**3*n**2 + 11*b**3*n + 6*b**3) + b**3*d*n**2*x**3*(a + b*x)**n/(b**3*...
```

3.353.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.27

$$\int (a+bx)^n \left(c+dx^2\right) dx = \frac{(bx+a)^{n+1}c}{b(n+1)} + \frac{((n^2+3\,n+2)b^3x^3+(n^2+n)ab^2x^2-2\,a^2bnx+2\,a^3)(bx+a)^n d}{(n^3+6\,n^2+11\,n+6)b^3}$$

input
```
integrate((b*x+a)^n*(d*x^2+c),x, algorithm="maxima")
```

output
```
(b*x + a)^(n + 1)*c/(b*(n + 1)) + ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b
^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x + a)^n*d/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

3.353.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(70) = 140.

Time = 0.27 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.39

$$\int (a+bx)^n \left(c+dx^2\right) dx = \frac{(bx+a)^n b^3 dn^2x^3 + (bx+a)^n ab^2dn^2x^2 + 3\,(bx+a)^n b^3dnx^3 + (bx+a)^n b^3cn^2x + (bx+a)^n ab^2dnx^2 + 2\,(b}{}$$

input
```
integrate((b*x+a)^n*(d*x^2+c),x, algorithm="giac")
```

output
```
((b*x + a)^n*b^3*d*n^2*x^3 + (b*x + a)^n*a*b^2*d*n^2*x^2 + 3*(b*x + a)^n*b
^3*d*n*x^3 + (b*x + a)^n*b^3*c*n^2*x + (b*x + a)^n*a*b^2*d*n*x^2 + 2*(b*x
+ a)^n*b^3*d*x^3 + (b*x + a)^n*a*b^2*c*n^2 + 5*(b*x + a)^n*b^3*c*n*x - 2*(
b*x + a)^n*a^2*b*d*n*x + 5*(b*x + a)^n*a*b^2*c*n + 6*(b*x + a)^n*b^3*c*x +
 6*(b*x + a)^n*a*b^2*c + 2*(b*x + a)^n*a^3*d)/(b^3*n^3 + 6*b^3*n^2 + 11*b^
3*n + 6*b^3)
```

3.353.9 Mupad [B] (verification not implemented)

Time = 11.52 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.33

$$\int (a+bx)^n\left(c+dx^2\right)\,dx = (a+b\,x)^n\left(\frac{d\,x^3\,(n^2+3\,n+2)}{n^3+6\,n^2+11\,n+6} + \frac{x\,(-2\,d\,a^2\,b\,n+c\,b^3\,n^2+5\,c\,b^3\,n+6\,c\,b^3)}{b^3\,\left(n^3+6\,n^2+11\,n+6\right)} + \frac{a\,(2\,d\,a^2+c\,b^2\,n^2+5\,c\,b^2\,n+6\,c\,b^2)}{b^3\,\left(n^3+6\,n^2+11\,n+6\right)} + \frac{a\,d\,n\,x^2\,(n+1)}{b\,\left(n^3+6\,n^2+11\,n+6\right)}\right)$$

input
```
int((c + d*x^2)*(a + b*x)^n,x)
```

output
```
(a + b*x)^n*((d*x^3*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) + (x*(6*b^3*
c + b^3*c*n^2 + 5*b^3*c*n - 2*a^2*b*d*n))/(b^3*(11*n + 6*n^2 + n^3 + 6)) +
 (a*(2*a^2*d + 6*b^2*c + b^2*c*n^2 + 5*b^2*c*n))/(b^3*(11*n + 6*n^2 + n^3
+ 6)) + (a*d*n*x^2*(n + 1))/(b*(11*n + 6*n^2 + n^3 + 6)))
```

3.354 $\int \frac{(a+bx)^n(c+dx^2)}{x}\,dx$

3.354.1 Optimal result

Integrand size = 18, antiderivative size = 77

$$\int \frac{(a+bx)^n(c+dx^2)}{x}\,dx = -\frac{ad(a+bx)^{1+n}}{b^2(1+n)}+\frac{d(a+bx)^{2+n}}{b^2(2+n)} - \frac{c(a+bx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{bx}{a}\right)}{a(1+n)}$$

output
```
-a*d*(b*x+a)^(1+n)/b^2/(1+n)+d*(b*x+a)^(2+n)/b^2/(2+n)-c*(b*x+a)^(1+n)*hyp
ergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

3.354.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^n(c+dx^2)}{x}\,dx = -\frac{(a+bx)^{1+n}\left(ad(a-b(1+n)x)+b^2c(2+n)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{bx}{a}\right)\right)}{ab^2(1+n)(2+n)}$$

input
```
Integrate[((a + b*x)^n*(c + d*x^2))/x,x]
```

output
```
-(((a + b*x)^(1 + n)*(a*d*(a - b*(1 + n)*x) + b^2*c*(2 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, 1 + (b*x)/a]))/(a*b^2*(1 + n)*(2 + n)))
```

3.354.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)(a+bx)^n}{x} \, dx$$

$$\Big\downarrow \text{522}$$

$$\int \left(\frac{c(a+bx)^n}{x} - \frac{ad(a+bx)^n}{b} + \frac{d(a+bx)^{n+1}}{b}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ad(a+bx)^{n+1}}{b^2(n+1)} + \frac{d(a+bx)^{n+2}}{b^2(n+2)} - \frac{c(a+bx)^{n+1} \text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{bx}{a}+1\right)}{a(n+1)}$$

input
```
Int[((a + b*x)^n*(c + d*x^2))/x,x]
```

output
```
-((a*d*(a + b*x)^(1 + n))/(b^2*(1 + n))) + (d*(a + b*x)^(2 + n))/(b^2*(2 +
 n)) - (c*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a
])/(a*(1 + n))
```

3.354.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.354.4 Maple [F]

$$\int \frac{(bx+a)^n (d\,x^2+c)}{x}dx$$

input
```
int((b*x+a)^n*(d*x^2+c)/x,x)
```

output
```
int((b*x+a)^n*(d*x^2+c)/x,x)
```

3.354.5 Fricas [F]

$$\int \frac{(a+bx)^n (c+dx^2)}{x}\,dx = \int \frac{(dx^2+c)(bx+a)^n}{x}\,dx$$

input
```
integrate((b*x+a)^n*(d*x^2+c)/x,x, algorithm="fricas")
```

output
```
integral((d*x^2 + c)*(b*x + a)^n/x, x)
```

3.354.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(61) = 122$.

Time = 2.29 (sec) , antiderivative size = 279, normalized size of antiderivative = 3.62

$$\int \frac{(a+bx)^n (c+dx^2)}{x}\,dx = d \left(\begin{cases} \frac{a^n x^2}{2} & \text{for } b = 0 \\ \frac{a \log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} + \frac{a}{ab^2+b^3x} + \frac{bx \log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for } n = -2 \\ -\frac{a \log\left(\frac{a}{b}+x\right)}{b^2} + \frac{x}{b} & \text{for } n = -1 \\ -\frac{a^2(a+bx)^n}{b^2n^2+3b^2n+2b^2} + \frac{abnx(a+bx)^n}{b^2n^2+3b^2n+2b^2} + \frac{b^2nx^2(a+bx)^n}{b^2n^2+3b^2n+2b^2} + \frac{b^2x^2(a+bx)^n}{b^2n^2+3b^2n+2b^2} & \text{otherwise} \end{cases} \right) - \frac{b^{n+1}cn\left(\frac{a}{b}+x\right)^{n+1} \Phi\left(1+\frac{bx}{a}, 1, n+1\right)\Gamma(n+1)}{a\Gamma(n+2)} - \frac{b^{n+1}c\left(\frac{a}{b}+x\right)^{n+1} \Phi\left(1+\frac{bx}{a}, 1, n+1\right)\Gamma(n+1)}{a\Gamma(n+2)}$$

input
```
integrate((b*x+a)**n*(d*x**2+c)/x,x)
```

output
```
d*Piecewise((a**n*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x) + a
/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(n, -2)), (-a*l
og(a/b + x)/b**2 + x/b, Eq(n, -1)), (-a**2*(a + b*x)**n/(b**2*n**2 + 3*b**
2*n + 2*b**2) + a*b*n*x*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**
2*n*x**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2*x**2*(a + b*x
)**n/(b**2*n**2 + 3*b**2*n + 2*b**2), True)) - b**(n + 1)*c*n*(a/b + x)**(
n + 1)*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2)) - b**(n
 + 1)*c*(a/b + x)**(n + 1)*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*g
amma(n + 2))
```

3.354.7 Maxima [F]

$$\int \frac{(a+bx)^n(c+dx^2)}{x}\,dx=\int \frac{(dx^2+c)(bx+a)^n}{x}\,dx$$

input
```
integrate((b*x+a)^n*(d*x^2+c)/x,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*(b*x + a)^n/x, x)
```

3.354.8 Giac [F]

$$\int \frac{(a+bx)^n(c+dx^2)}{x}\,dx=\int \frac{(dx^2+c)(bx+a)^n}{x}\,dx$$

input
```
integrate((b*x+a)^n*(d*x^2+c)/x,x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*(b*x + a)^n/x, x)
```

3.354.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^n\,(c+dx^2)}{x}\,dx = \int \frac{(d\,x^2+c)\,(a+b\,x)^n}{x}\,dx$$

input
```
int(((c + d*x^2)*(a + b*x)^n)/x,x)
```

output
```
int(((c + d*x^2)*(a + b*x)^n)/x, x)
```

3.355 $\int x^2(a+bx)^n\left(c+dx^2\right)^2\,dx$

3.355.1 Optimal result

Integrand size = 20, antiderivative size = 232

$$\begin{aligned}\int x^2(a+bx)^n\left(c+dx^2\right)^2\,dx &= \frac{a^2\left(b^2c+a^2d\right)^2(a+bx)^{1+n}}{b^7(1+n)}\\&\quad-\frac{2a\left(b^2c+a^2d\right)\left(b^2c+3a^2d\right)(a+bx)^{2+n}}{b^7(2+n)}\\&\quad+\frac{\left(b^4c^2+12a^2b^2cd+15a^4d^2\right)(a+bx)^{3+n}}{b^7(3+n)}\\&\quad-\frac{4ad\left(2b^2c+5a^2d\right)(a+bx)^{4+n}}{b^7(4+n)}\\&\quad+\frac{d\left(2b^2c+15a^2d\right)(a+bx)^{5+n}}{b^7(5+n)}\\&\quad-\frac{6ad^2(a+bx)^{6+n}}{b^7(6+n)}+\frac{d^2(a+bx)^{7+n}}{b^7(7+n)}\end{aligned}$$

output
```
a^2*(a^2*d+b^2*c)^2*(b*x+a)^(1+n)/b^7/(1+n)-2*a*(a^2*d+b^2*c)*(3*a^2*d+b^2
*c)*(b*x+a)^(2+n)/b^7/(2+n)+(15*a^4*d^2+12*a^2*b^2*c*d+b^4*c^2)*(b*x+a)^(3
+n)/b^7/(3+n)-4*a*d*(5*a^2*d+2*b^2*c)*(b*x+a)^(4+n)/b^7/(4+n)+d*(15*a^2*d+
2*b^2*c)*(b*x+a)^(5+n)/b^7/(5+n)-6*a*d^2*(b*x+a)^(6+n)/b^7/(6+n)+d^2*(b*x+
a)^(7+n)/b^7/(7+n)
```

3.355.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.86

$$\int x^2(a+bx)^n \left(c+dx^2\right)^2 dx$$
$$= \frac{(a+bx)^{1+n}\left(\frac{\left(ab^2c+a^3d\right)^2}{1+n} - \frac{2a(b^2c+a^2d)(b^2c+3a^2d)(a+bx)}{2+n} + \frac{(b^4c^2+12a^2b^2cd+15a^4d^2)(a+bx)^2}{3+n} - \frac{4ad(2b^2c+5a^2d)(a+bx)^3}{4+n} + \frac{d}{}\right.}{b^7}$$

input
```
Integrate[x^2*(a + b*x)^n*(c + d*x^2)^2,x]
```

output
```
((a + b*x)^(1 + n)*((a*b^2*c + a^3*d)^2/(1 + n) - (2*a*(b^2*c + a^2*d)*(b^
2*c + 3*a^2*d)*(a + b*x))/(2 + n) + ((b^4*c^2 + 12*a^2*b^2*c*d + 15*a^4*d^
2)*(a + b*x)^2)/(3 + n) - (4*a*d*(2*b^2*c + 5*a^2*d)*(a + b*x)^3)/(4 + n)
+ (d*(2*b^2*c + 15*a^2*d)*(a + b*x)^4)/(5 + n) - (6*a*d^2*(a + b*x)^5)/(6
+ n) + (d^2*(a + b*x)^6)/(7 + n)))/b^7
```

3.355.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(c+dx^2\right)^2(a+bx)^n\,dx$$

$$\Big\downarrow \text{522}$$

$$\int \left(\frac{\left(a^3d+ab^2c\right)^2(a+bx)^n}{b^6} + \frac{2a\left(-3a^2d-b^2c\right)\left(a^2d+b^2c\right)(a+bx)^{n+1}}{b^6} - \frac{4ad\left(5a^2d+2b^2c\right)(a+bx)^{n+3}}{b^6} + \frac{d(15}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\left(a^2d+b^2c\right)^2(a+bx)^{n+1}}{b^7(n+1)} - \frac{2a\left(a^2d+b^2c\right)\left(3a^2d+b^2c\right)(a+bx)^{n+2}}{b^7(n+2)} - \frac{4ad\left(5a^2d+2b^2c\right)(a+bx)^{n+4}}{b^7(n+4)} + \frac{d\left(15a^2d+2b^2c\right)(a+bx)^{n+5}}{b^7(n+5)} + \frac{\left(15a^4d^2+12a^2b^2cd+b^4c^2\right)(a+bx)^{n+3}}{b^7(n+3)} - \frac{6ad^2(a+bx)^{n+6}}{b^7(n+6)} + \frac{d^2(a+bx)^{n+7}}{b^7(n+7)}$$

input
```
Int[x^2*(a + b*x)^n*(c + d*x^2)^2,x]
```

output
```
(a^2*(b^2*c + a^2*d)^2*(a + b*x)^(1 + n))/(b^7*(1 + n)) - (2*a*(b^2*c + a^
2*d)*(b^2*c + 3*a^2*d)*(a + b*x)^(2 + n))/(b^7*(2 + n)) + ((b^4*c^2 + 12*a
^2*b^2*c*d + 15*a^4*d^2)*(a + b*x)^(3 + n))/(b^7*(3 + n)) - (4*a*d*(2*b^2*
c + 5*a^2*d)*(a + b*x)^(4 + n))/(b^7*(4 + n)) + (d*(2*b^2*c + 15*a^2*d)*(a
 + b*x)^(5 + n))/(b^7*(5 + n)) - (6*a*d^2*(a + b*x)^(6 + n))/(b^7*(6 + n))
 + (d^2*(a + b*x)^(7 + n))/(b^7*(7 + n))
```

3.355.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.355.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 753 vs. $2(232) = 464$.

Time = 0.48 (sec) , antiderivative size = 754, normalized size of antiderivative = 3.25

method	result
norman	$\frac{d^2x^7 e^{n\ln(bx+a)}}{7+n}+\frac{n\,d^2a\,x^6 e^{n\ln(bx+a)}}{b(n^2+13n+42)}+\frac{(b^4c^2n^4+22b^4c^2n^3+24a^2b^2cd\,n^2+179b^4c^2n^2+312a^2b^2cdn+638b^4c^2n+360d^2a^4+}{b^5(n^6+27n^5+295n^4+1665n^3+5104n^2+8028n+}$
gosper	$\frac{(bx+a)^{1+n}(b^6d^2n^6x^6+21b^6d^2n^5x^6-6a\,b^5d^2n^5x^5+2b^6cd\,n^6x^4+175b^6d^2n^4x^6-90a\,b^5d^2n^4x^5+46b^6cd\,n^5x^4+735b^6d^2n^3x^6+30}{}$
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int(x^2*(b*x+a)^n*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
d^2/(7+n)*x^7*exp(n*ln(b*x+a))+n*d^2/b*a/(n^2+13*n+42)*x^6*exp(n*ln(b*x+a)
)+(b^4*c^2*n^4+22*b^4*c^2*n^3+24*a^2*b^2*c*d*n^2+179*b^4*c^2*n^2+312*a^2*b
^2*c*d*n+638*b^4*c^2*n+360*a^4*d^2+1008*a^2*b^2*c*d+840*b^4*c^2)*a/b^5*n/(
n^6+27*n^5+295*n^4+1665*n^3+5104*n^2+8028*n+5040)*x^2*exp(n*ln(b*x+a))+2*a
^3*(b^4*c^2*n^4+22*b^4*c^2*n^3+24*a^2*b^2*c*d*n^2+179*b^4*c^2*n^2+312*a^2*
b^2*c*d*n+638*b^4*c^2*n+360*a^4*d^2+1008*a^2*b^2*c*d+840*b^4*c^2)/b^7/(n^7
+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*n+5040)*exp(n*ln(b*x+a))
-(-b^4*c^2*n^4+8*a^2*b^2*c*d*n^3-22*b^4*c^2*n^3+104*a^2*b^2*c*d*n^2-179*b^
4*c^2*n^2+120*a^4*d^2*n+336*a^2*b^2*c*d*n-638*b^4*c^2*n-840*b^4*c^2)/b^4/(
n^5+25*n^4+245*n^3+1175*n^2+2754*n+2520)*x^3*exp(n*ln(b*x+a))-2*d*(-b^2*c*
n^2+3*a^2*d*n-13*b^2*c*n-42*b^2*c)/b^2/(n^3+18*n^2+107*n+210)*x^5*exp(n*ln
(b*x+a))-2/b^6*n*a^2*(b^4*c^2*n^4+22*b^4*c^2*n^3+24*a^2*b^2*c*d*n^2+179*b^
4*c^2*n^2+312*a^2*b^2*c*d*n+638*b^4*c^2*n+360*a^4*d^2+1008*a^2*b^2*c*d+840
*b^4*c^2)/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*n+5040)*x*
exp(n*ln(b*x+a))+2*(b^2*c*n^2+13*b^2*c*n+15*a^2*d+42*b^2*c)*a/b^3*d*n/(n^4
+22*n^3+179*n^2+638*n+840)*x^4*exp(n*ln(b*x+a))
```

3.355.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(232) = 464$.

Time = 0.32 (sec) , antiderivative size = 1027, normalized size of antiderivative = 4.43

$$\int x^2(a+bx)^n\left(c+dx^2\right)^2\,dx$$
$$= \frac{(2\,a^3b^4c^2n^4 + 44\,a^3b^4c^2n^3 + 1680\,a^3b^4c^2 + 2016\,a^5b^2cd + 720\,a^7d^2 + (b^7d^2n^6 + 21\,b^7d^2n^5 + 175\,b^7d^2n^4 + 7}{}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
(2*a^3*b^4*c^2*n^4 + 44*a^3*b^4*c^2*n^3 + 1680*a^3*b^4*c^2 + 2016*a^5*b^2*
c*d + 720*a^7*d^2 + (b^7*d^2*n^6 + 21*b^7*d^2*n^5 + 175*b^7*d^2*n^4 + 735*
b^7*d^2*n^3 + 1624*b^7*d^2*n^2 + 1764*b^7*d^2*n + 720*b^7*d^2)*x^7 + (a*b^
6*d^2*n^6 + 15*a*b^6*d^2*n^5 + 85*a*b^6*d^2*n^4 + 225*a*b^6*d^2*n^3 + 274*
a*b^6*d^2*n^2 + 120*a*b^6*d^2*n)*x^6 + 2*(b^7*c*d*n^6 + 1008*b^7*c*d + (23
*b^7*c*d - 3*a^2*b^5*d^2)*n^5 + 3*(69*b^7*c*d - 10*a^2*b^5*d^2)*n^4 + 5*(1
85*b^7*c*d - 21*a^2*b^5*d^2)*n^3 + 2*(1072*b^7*c*d - 75*a^2*b^5*d^2)*n^2 +
 36*(67*b^7*c*d - 2*a^2*b^5*d^2)*n)*x^5 + 2*(a*b^6*c*d*n^6 + 19*a*b^6*c*d*
n^5 + (131*a*b^6*c*d + 15*a^3*b^4*d^2)*n^4 + (401*a*b^6*c*d + 90*a^3*b^4*d
^2)*n^3 + 15*(36*a*b^6*c*d + 11*a^3*b^4*d^2)*n^2 + 18*(14*a*b^6*c*d + 5*a^
3*b^4*d^2)*n)*x^4 + (b^7*c^2*n^6 + 1680*b^7*c^2 + (25*b^7*c^2 - 8*a^2*b^5*
c*d)*n^5 + (247*b^7*c^2 - 128*a^2*b^5*c*d)*n^4 + (1219*b^7*c^2 - 664*a^2*b
^5*c*d - 120*a^4*b^3*d^2)*n^3 + 8*(389*b^7*c^2 - 152*a^2*b^5*c*d - 45*a^4*
b^3*d^2)*n^2 + 4*(949*b^7*c^2 - 168*a^2*b^5*c*d - 60*a^4*b^3*d^2)*n)*x^3 +
 2*(179*a^3*b^4*c^2 + 24*a^5*b^2*c*d)*n^2 + (a*b^6*c^2*n^6 + 23*a*b^6*c^2*
n^5 + 3*(67*a*b^6*c^2 + 8*a^3*b^4*c*d)*n^4 + (817*a*b^6*c^2 + 336*a^3*b^4*
c*d)*n^3 + 2*(739*a*b^6*c^2 + 660*a^3*b^4*c*d + 180*a^5*b^2*d^2)*n^2 + 24*
(35*a*b^6*c^2 + 42*a^3*b^4*c*d + 15*a^5*b^2*d^2)*n)*x^2 + 4*(319*a^3*b^4*c
^2 + 156*a^5*b^2*c*d)*n - 2*(a^2*b^5*c^2*n^5 + 22*a^2*b^5*c^2*n^4 + (179*a
^2*b^5*c^2 + 24*a^4*b^3*c*d)*n^3 + 2*(319*a^2*b^5*c^2 + 156*a^4*b^3*c*d...
```

3.355.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14317 vs. $2(218) = 436$.

Time = 4.19 (sec) , antiderivative size = 14317, normalized size of antiderivative = 61.71

$$\int x^2(a+bx)^n\left(c+dx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(b*x+a)**n*(d*x**2+c)**2,x)
```

output
```
Piecewise((a**n*(c**2*x**3/3 + 2*c*d*x**5/5 + d**2*x**7/7), Eq(b, 0)), (60
*a**6*d**2*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x*
*2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b*
*13*x**6) + 147*a**6*d**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*
x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*
b**13*x**6) + 360*a**5*b*d**2*x*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8
*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360
*a*b**12*x**5 + 60*b**13*x**6) + 822*a**5*b*d**2*x/(60*a**6*b**7 + 360*a**
5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4
 + 360*a*b**12*x**5 + 60*b**13*x**6) - 4*a**4*b**2*c*d/(60*a**6*b**7 + 360
*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*
x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 900*a**4*b**2*d**2*x**2*log(a/b
 + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**
10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1875*a
**4*b**2*d**2*x**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 +
1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x
**6) - 24*a**3*b**3*c*d*x/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*
x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*
b**13*x**6) + 1200*a**3*b**3*d**2*x**3*log(a/b + x)/(60*a**6*b**7 + 360*a*
*5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*...
```

3.355.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.93

$$\int x^2(a+bx)^n\left(c+dx^2\right)^2\,dx$$
$$= \frac{((n^2+3\,n+2)b^3x^3+(n^2+n)ab^2x^2-2\,a^2bnx+2\,a^3)(bx+a)^nc^2}{(n^3+6\,n^2+11\,n+6)b^3}$$
$$+ \frac{2\,((n^4+10\,n^3+35\,n^2+50\,n+24)b^5x^5+(n^4+6\,n^3+11\,n^2+6\,n)ab^4x^4-4\,(n^3+3\,n^2+2\,n)a^2b^3x^3+}{(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)b^5}$$
$$+ \frac{((n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)b^7x^7+(n^6+15\,n^5+85\,n^4+225\,n^3+274\,n^2}{}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
 + a)^n*c^2/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 2*((n^4 + 10*n^3 + 35*n^2 + 5
0*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^
2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(
b*x + a)^n*c*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5) + ((n
^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6
+ 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4
 + 35*n^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a
^3*b^4*x^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x
^2 - 720*a^6*b*n*x + 720*a^7)*(b*x + a)^n*d^2/((n^7 + 28*n^6 + 322*n^5 + 1
960*n^4 + 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^7)
```

3.355.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1750 vs. $2(232) = 464$.

Time = 0.30 (sec) , antiderivative size = 1750, normalized size of antiderivative = 7.54

$$\int x^2(a+bx)^n \left(c+dx^2\right)^2 dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
((b*x + a)^n*b^7*d^2*n^6*x^7 + (b*x + a)^n*a*b^6*d^2*n^6*x^6 + 21*(b*x + a
)^n*b^7*d^2*n^5*x^7 + 2*(b*x + a)^n*b^7*c*d*n^6*x^5 + 15*(b*x + a)^n*a*b^6
*d^2*n^5*x^6 + 175*(b*x + a)^n*b^7*d^2*n^4*x^7 + 2*(b*x + a)^n*a*b^6*c*d*n
^6*x^4 + 46*(b*x + a)^n*b^7*c*d*n^5*x^5 - 6*(b*x + a)^n*a^2*b^5*d^2*n^5*x^
5 + 85*(b*x + a)^n*a*b^6*d^2*n^4*x^6 + 735*(b*x + a)^n*b^7*d^2*n^3*x^7 + (
b*x + a)^n*b^7*c^2*n^6*x^3 + 38*(b*x + a)^n*a*b^6*c*d*n^5*x^4 + 414*(b*x +
 a)^n*b^7*c*d*n^4*x^5 - 60*(b*x + a)^n*a^2*b^5*d^2*n^4*x^5 + 225*(b*x + a)
^n*a*b^6*d^2*n^3*x^6 + 1624*(b*x + a)^n*b^7*d^2*n^2*x^7 + (b*x + a)^n*a*b^
6*c^2*n^6*x^2 + 25*(b*x + a)^n*b^7*c^2*n^5*x^3 - 8*(b*x + a)^n*a^2*b^5*c*d
*n^5*x^3 + 262*(b*x + a)^n*a*b^6*c*d*n^4*x^4 + 30*(b*x + a)^n*a^3*b^4*d^2*
n^4*x^4 + 1850*(b*x + a)^n*b^7*c*d*n^3*x^5 - 210*(b*x + a)^n*a^2*b^5*d^2*n
^3*x^5 + 274*(b*x + a)^n*a*b^6*d^2*n^2*x^6 + 1764*(b*x + a)^n*b^7*d^2*n*x^
7 + 23*(b*x + a)^n*a*b^6*c^2*n^5*x^2 + 247*(b*x + a)^n*b^7*c^2*n^4*x^3 - 1
28*(b*x + a)^n*a^2*b^5*c*d*n^4*x^3 + 802*(b*x + a)^n*a*b^6*c*d*n^3*x^4 + 1
80*(b*x + a)^n*a^3*b^4*d^2*n^3*x^4 + 4288*(b*x + a)^n*b^7*c*d*n^2*x^5 - 30
0*(b*x + a)^n*a^2*b^5*d^2*n^2*x^5 + 120*(b*x + a)^n*a*b^6*d^2*n*x^6 + 720*
(b*x + a)^n*b^7*d^2*x^7 - 2*(b*x + a)^n*a^2*b^5*c^2*n^5*x + 201*(b*x + a)^
n*a*b^6*c^2*n^4*x^2 + 24*(b*x + a)^n*a^3*b^4*c*d*n^4*x^2 + 1219*(b*x + a)^
n*b^7*c^2*n^3*x^3 - 664*(b*x + a)^n*a^2*b^5*c*d*n^3*x^3 - 120*(b*x + a)^n*
a^4*b^3*d^2*n^3*x^3 + 1080*(b*x + a)^n*a*b^6*c*d*n^2*x^4 + 330*(b*x + a...
```

3.355. $\int x^2(a+bx)^n\left(c+dx^2\right)^2\,dx$

3.355.9 Mupad [B] (verification not implemented)

Time = 11.96 (sec) , antiderivative size = 932, normalized size of antiderivative = 4.02

$$\int x^2(a+bx)^n\left(c+dx^2\right)^2\,dx$$
$$= \frac{2a^3(a+bx)^n(360a^4d^2+24a^2b^2cdn^2+312a^2b^2cdn+1008a^2b^2cd+b^4c^2n^4+22b^4c^2n^3+179b^4c}{b^7(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040)}$$
$$+\frac{d^2x^7(a+bx)^n(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)}{n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040}$$
$$+\frac{x^3(a+bx)^n(n^2+3n+2)(-120a^4d^2n-8a^2b^2cdn^3-104a^2b^2cdn^2-336a^2b^2cdn+b^4c^2n^4+}{b^4(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068}$$
$$-\frac{2a^2nx(a+bx)^n(360a^4d^2+24a^2b^2cdn^2+312a^2b^2cdn+1008a^2b^2cd+b^4c^2n^4+22b^4c^2n^3+1}{b^6(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+50}$$
$$+\frac{2dx^5(a+bx)^n(-3da^2n+cb^2n^2+13cb^2n+42cb^2)(n^4+10n^3+35n^2+50n+24)}{b^2(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040)}$$
$$+\frac{ad^2nx^6(a+bx)^n(n^5+15n^4+85n^3+225n^2+274n+120)}{b(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040)}$$
$$+\frac{anx^2(n+1)(a+bx)^n(360a^4d^2+24a^2b^2cdn^2+312a^2b^2cdn+1008a^2b^2cd+b^4c^2n^4+22b^4c^2}{b^5(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+}$$
$$+\frac{2adnx^4(a+bx)^n(n^3+6n^2+11n+6)(15da^2+cb^2n^2+13cb^2n+42cb^2)}{b^3(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040)}$$

input
```
int(x^2*(c + d*x^2)^2*(a + b*x)^n,x)
```

output
```
(2*a^3*(a + b*x)^n*(360*a^4*d^2 + 840*b^4*c^2 + 638*b^4*c^2*n + 179*b^4*c^
2*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n^4 + 1008*a^2*b^2*c*d + 312*a^2*b^2*c*d*
n + 24*a^2*b^2*c*d*n^2))/(b^7*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 +
 322*n^5 + 28*n^6 + n^7 + 5040)) + (d^2*x^7*(a + b*x)^n*(1764*n + 1624*n^2
 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))/(13068*n + 13132*n^2 + 6769*n^
3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040) + (x^3*(a + b*x)^n*(3*n + n^
2 + 2)*(840*b^4*c^2 - 120*a^4*d^2*n + 638*b^4*c^2*n + 179*b^4*c^2*n^2 + 22
*b^4*c^2*n^3 + b^4*c^2*n^4 - 336*a^2*b^2*c*d*n - 104*a^2*b^2*c*d*n^2 - 8*a
^2*b^2*c*d*n^3))/(b^4*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5
 + 28*n^6 + n^7 + 5040)) - (2*a^2*n*x*(a + b*x)^n*(360*a^4*d^2 + 840*b^4*c
^2 + 638*b^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n^4 + 1008
*a^2*b^2*c*d + 312*a^2*b^2*c*d*n + 24*a^2*b^2*c*d*n^2))/(b^6*(13068*n + 13
132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (2*d*x^5
*(a + b*x)^n*(42*b^2*c + b^2*c*n^2 - 3*a^2*d*n + 13*b^2*c*n)*(50*n + 35*n^
2 + 10*n^3 + n^4 + 24))/(b^2*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 +
322*n^5 + 28*n^6 + n^7 + 5040)) + (a*d^2*n*x^6*(a + b*x)^n*(274*n + 225*n^
2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b*(13068*n + 13132*n^2 + 6769*n^3 + 196
0*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (a*n*x^2*(n + 1)*(a + b*x)^n*(36
0*a^4*d^2 + 840*b^4*c^2 + 638*b^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^2*n^3
 + b^4*c^2*n^4 + 1008*a^2*b^2*c*d + 312*a^2*b^2*c*d*n + 24*a^2*b^2*c*d*...
```

3.355. $\int x^2(a+bx)^n \left(c+dx^2\right)^2 \, dx$

3.356 $\int x(a+bx)^n \left(c+dx^2\right)^2 \, dx$

3.356.1 Optimal result

Integrand size = 18, antiderivative size = 185

$$\int x(a+bx)^n \left(c+dx^2\right)^2 \, dx = -\frac{a(b^2c+a^2d)^2 (a+bx)^{1+n}}{b^6(1+n)} + \frac{(b^2c+a^2d)(b^2c+5a^2d)(a+bx)^{2+n}}{b^6(2+n)} - \frac{2ad(3b^2c+5a^2d)(a+bx)^{3+n}}{b^6(3+n)} + \frac{2d(b^2c+5a^2d)(a+bx)^{4+n}}{b^6(4+n)} - \frac{5ad^2(a+bx)^{5+n}}{b^6(5+n)} + \frac{d^2(a+bx)^{6+n}}{b^6(6+n)}$$

output
```
-a*(a^2*d+b^2*c)^2*(b*x+a)^(1+n)/b^6/(1+n)+(a^2*d+b^2*c)*(5*a^2*d+b^2*c)*(
b*x+a)^(2+n)/b^6/(2+n)-2*a*d*(5*a^2*d+3*b^2*c)*(b*x+a)^(3+n)/b^6/(3+n)+2*d
*(5*a^2*d+b^2*c)*(b*x+a)^(4+n)/b^6/(4+n)-5*a*d^2*(b*x+a)^(5+n)/b^6/(5+n)+d
^2*(b*x+a)^(6+n)/b^6/(6+n)
```

3.356.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.75

$$\int x(a+bx)^n \left(c+dx^2\right)^2 dx$$
$$= \frac{(a+bx)^{1+n} \left(b^4(1+n)(2+n)(3+n)(4+n)(5+n)(a+bx) \left(c+dx^2\right)^2 - a(6+n) \left(b^4(1+n)(2+n)(3+\right.\right.}{}$$

input
```
Integrate[x*(a + b*x)^n*(c + d*x^2)^2,x]
```

output
```
((a + b*x)^(1 + n)*(b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(a + b*x)*
(c + d*x^2)^2 - a*(6 + n)*(b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(c + d*x^2)
^2 + 4*(b^2*c + a^2*d)*(4 + n)*(2*a^2*d - 2*a*b*d*(1 + n)*x + b^2*(2 + n)*
(c*(3 + n) + d*(1 + n)*x^2)) - 4*a*d*(1 + n)*(a + b*x)*(2*a^2*d - 2*a*b*d*
(2 + n)*x + b^2*(3 + n)*(c*(4 + n) + d*(2 + n)*x^2))) + 4*(1 + n)*(a + b*x
)*((b^2*c + a^2*d)*(5 + n)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(3 + n)*(c*(
4 + n) + d*(2 + n)*x^2)) - a*d*(2 + n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(3 + n
)*x + b^2*(4 + n)*(c*(5 + n) + d*(3 + n)*x^2)))))/(b^6*(1 + n)*(2 + n)*(3
+ n)*(4 + n)*(5 + n)*(6 + n))
```

3.356.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c+dx^2\right)^2 (a+bx)^n dx$$
$$\Big\downarrow \text{522}$$
$$\int \left(-\frac{a\left(a^2d+b^2c\right)^2 (a+bx)^n}{b^5} + \frac{\left(a^2d+b^2c\right)\left(5a^2d+b^2c\right)(a+bx)^{n+1}}{b^5} - \frac{2ad\left(5a^2d+3b^2c\right)(a+bx)^{n+2}}{b^5} + \frac{2d(5a^2\right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{a(a^2d+b^2c)^2(a+bx)^{n+1}}{b^6(n+1)}+\frac{(a^2d+b^2c)(5a^2d+b^2c)(a+bx)^{n+2}}{b^6(n+2)}-\frac{2ad(5a^2d+3b^2c)(a+bx)^{n+3}}{b^6(n+3)}+\frac{2d(5a^2d+b^2c)(a+bx)^{n+4}}{b^6(n+4)}-\frac{5ad^2(a+bx)^{n+5}}{b^6(n+5)}+\frac{d^2(a+bx)^{n+6}}{b^6(n+6)}$$

input
```
Int[x*(a + b*x)^n*(c + d*x^2)^2,x]
```

output
```
-((a*(b^2*c + a^2*d)^2*(a + b*x)^(1 + n))/(b^6*(1 + n))) + ((b^2*c + a^2*d
)*(b^2*c + 5*a^2*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) - (2*a*d*(3*b^2*c + 5
*a^2*d)*(a + b*x)^(3 + n))/(b^6*(3 + n)) + (2*d*(b^2*c + 5*a^2*d)*(a + b*x
)^(4 + n))/(b^6*(4 + n)) - (5*a*d^2*(a + b*x)^(5 + n))/(b^6*(5 + n)) + (d^
2*(a + b*x)^(6 + n))/(b^6*(6 + n))
```

3.356.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.356.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(185) = 370.

Time = 0.43 (sec) , antiderivative size = 601, normalized size of antiderivative = 3.25

method	result
norman	$\frac{d^2x^6e^{n\ln(bx+a)}}{6+n}+\frac{na(b^4c^2n^4+18b^4c^2n^3+12a^2b^2cd\,n^2+119b^4c^2n^2+132a^2b^2cdn+342b^4c^2n+120d^2a^4+360a^2b^2cd+360b^4c^2)}{b^5(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)}$
gosper	$-\frac{(bx+a)^{1+n}(-b^5d^2n^5x^5-15b^5d^2n^4x^5+5a\,b^4d^2n^4x^4-2b^5cd\,n^5x^3-85b^5d^2n^3x^5+50a\,b^4d^2n^3x^4-34b^5cd\,n^4x^3-225b^5d^2n^2x^5}{}$
risch	$-\frac{(-b^6d^2n^5x^6-a\,b^5d^2n^5x^5-15b^6d^2n^4x^6-10a\,b^5d^2n^4x^5-2b^6cd\,n^5x^4-85b^6d^2n^3x^6+5a^2b^4d^2n^4x^4-2a\,b^5cd\,n^5x^3-35a\,b^5d^2n}{}$
parallelrisch	Expression too large to display

input
```
int(x*(b*x+a)^n*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
d^2/(6+n)*x^6*exp(n*ln(b*x+a))+1/b^5*n*a*(b^4*c^2*n^4+18*b^4*c^2*n^3+12*a^
2*b^2*c*d*n^2+119*b^4*c^2*n^2+132*a^2*b^2*c*d*n+342*b^4*c^2*n+120*a^4*d^2+
360*a^2*b^2*c*d+360*b^4*c^2)/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+7
20)*x*exp(n*ln(b*x+a))+n*d^2/b*a/(n^2+11*n+30)*x^5*exp(n*ln(b*x+a))-a^2*(b
^4*c^2*n^4+18*b^4*c^2*n^3+12*a^2*b^2*c*d*n^2+119*b^4*c^2*n^2+132*a^2*b^2*c
*d*n+342*b^4*c^2*n+120*a^4*d^2+360*a^2*b^2*c*d+360*b^4*c^2)/b^6/(n^6+21*n^
5+175*n^4+735*n^3+1624*n^2+1764*n+720)*exp(n*ln(b*x+a))-(-b^4*c^2*n^4+6*a^
2*b^2*c*d*n^3-18*b^4*c^2*n^3+66*a^2*b^2*c*d*n^2-119*b^4*c^2*n^2+60*a^4*d^2
*n+180*a^2*b^2*c*d*n-342*b^4*c^2*n-360*b^4*c^2)/b^4/(n^5+20*n^4+155*n^3+58
0*n^2+1044*n+720)*x^2*exp(n*ln(b*x+a))-d*(-2*b^2*c*n^2+5*a^2*d*n-22*b^2*c*
n-60*b^2*c)/b^2/(n^3+15*n^2+74*n+120)*x^4*exp(n*ln(b*x+a))+2*(b^2*c*n^2+11
*b^2*c*n+10*a^2*d+30*b^2*c)*a/b^3*d*n/(n^4+18*n^3+119*n^2+342*n+360)*x^3*e
xp(n*ln(b*x+a))
```

3.356.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 757 vs. $2(185) = 370$.

Time = 0.30 (sec) , antiderivative size = 757, normalized size of antiderivative = 4.09

$$\int x(a+bx)^n \left(c+dx^2\right)^2 \, dx = -\frac{(a^2b^4c^2n^4 + 18\,a^2b^4c^2n^3 + 360\,a^2b^4c^2 + 360\,a^4b^2cd + 120\,a^6d^2 - (b^6d^2n^5 + 15\,b^6d^2n^4 + 85\,b^6d^2n^3 + 225}{}$$

input
```
integrate(x*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
-(a^2*b^4*c^2*n^4 + 18*a^2*b^4*c^2*n^3 + 360*a^2*b^4*c^2 + 360*a^4*b^2*c*d
 + 120*a^6*d^2 - (b^6*d^2*n^5 + 15*b^6*d^2*n^4 + 85*b^6*d^2*n^3 + 225*b^6*
d^2*n^2 + 274*b^6*d^2*n + 120*b^6*d^2)*x^6 - (a*b^5*d^2*n^5 + 10*a*b^5*d^2
*n^4 + 35*a*b^5*d^2*n^3 + 50*a*b^5*d^2*n^2 + 24*a*b^5*d^2*n)*x^5 - (2*b^6*
c*d*n^5 + 360*b^6*c*d + (34*b^6*c*d - 5*a^2*b^4*d^2)*n^4 + 2*(107*b^6*c*d
- 15*a^2*b^4*d^2)*n^3 + (614*b^6*c*d - 55*a^2*b^4*d^2)*n^2 + 6*(132*b^6*c*
d - 5*a^2*b^4*d^2)*n)*x^4 - 2*(a*b^5*c*d*n^5 + 14*a*b^5*c*d*n^4 + 5*(13*a*
b^5*c*d + 2*a^3*b^3*d^2)*n^3 + 2*(56*a*b^5*c*d + 15*a^3*b^3*d^2)*n^2 + 20*
(3*a*b^5*c*d + a^3*b^3*d^2)*n)*x^3 + (119*a^2*b^4*c^2 + 12*a^4*b^2*c*d)*n^
2 - (b^6*c^2*n^5 + 360*b^6*c^2 + (19*b^6*c^2 - 6*a^2*b^4*c*d)*n^4 + (137*b
^6*c^2 - 72*a^2*b^4*c*d)*n^3 + (461*b^6*c^2 - 246*a^2*b^4*c*d - 60*a^4*b^2
*d^2)*n^2 + 6*(117*b^6*c^2 - 30*a^2*b^4*c*d - 10*a^4*b^2*d^2)*n)*x^2 + 6*(
57*a^2*b^4*c^2 + 22*a^4*b^2*c*d)*n - (a*b^5*c^2*n^5 + 18*a*b^5*c^2*n^4 + (
119*a*b^5*c^2 + 12*a^3*b^3*c*d)*n^3 + 6*(57*a*b^5*c^2 + 22*a^3*b^3*c*d)*n^
2 + 120*(3*a*b^5*c^2 + 3*a^3*b^3*c*d + a^5*b*d^2)*n)*x)*(b*x + a)^n/(b^6*n
^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6*n +
720*b^6)
```

3.356.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8940 vs. $2(170) = 340$.

Time = 2.45 (sec) , antiderivative size = 8940, normalized size of antiderivative = 48.32

$$\int x(a+bx)^n \left(c+dx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate(x*(b*x+a)**n*(d*x**2+c)**2,x)
```

output
```
Piecewise((a**n*(c**2*x**2/2 + c*d*x**4/2 + d**2*x**6/6), Eq(b, 0)), (60*a
**5*d**2*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2
 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 137*a**5*d**2/
(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3
+ 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a**4*b*d**2*x*log(a/b + x)/(60*a
**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300
*a*b**10*x**4 + 60*b**11*x**5) + 625*a**4*b*d**2*x/(60*a**5*b**6 + 300*a**
4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60
*b**11*x**5) - 6*a**3*b**2*c*d/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*
b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a
**3*b**2*d**2*x**2*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3
*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 1100
*a**3*b**2*d**2*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2
+ 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 30*a**2*b**3*c*
d*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x
**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a**2*b**3*d**2*x**3*log(a/b
+ x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*
x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 900*a**2*b**3*d**2*x**3/(60*a**
5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a
*b**10*x**4 + 60*b**11*x**5) - 3*a*b**4*c**2/(60*a**5*b**6 + 300*a**4*b...
```

3.356.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.81

$$\int x(a+bx)^n \left(c+dx^2\right)^2 \, dx = \frac{(b^2(n+1)x^2 + abnx - a^2)(bx+a)^n c^2}{(n^2+3n+2)b^2} + \frac{2\left((n^3+6n^2+11n+6)b^4x^4 + (n^3+3n^2+2n)ab^3x^3 - 3(n^2+n)a^2b^2x^2 + 6a^3bnx - 6a^4\right)(bx+a)^n c}{(n^4+10n^3+35n^2+50n+24)b^4} + \frac{((n^5+15n^4+85n^3+225n^2+274n+120)b^6x^6 + (n^5+10n^4+35n^3+50n^2+24n)ab^5x^5 - 5(n^4+}{(n^6+21n^5+175n^4+73}$$

input
```
integrate(x*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
(b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^2/((n^2 + 3*n + 2)*b^2) +
2*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n
^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n*c*d/((n^4 + 10*n^3
+ 35*n^2 + 50*n + 24)*b^4) + ((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 1
20)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*x^5 - 5*(n^4 +
 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*x^3 -
60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*d^2/((n^6
+ 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6)
```

3.356.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1266 vs. $2(185) = 370$.

Time = 0.30 (sec) , antiderivative size = 1266, normalized size of antiderivative = 6.84

$$\int x(a+bx)^n \left(c+dx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate(x*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
((b*x + a)^n*b^6*d^2*n^5*x^6 + (b*x + a)^n*a*b^5*d^2*n^5*x^5 + 15*(b*x + a
)^n*b^6*d^2*n^4*x^6 + 2*(b*x + a)^n*b^6*c*d*n^5*x^4 + 10*(b*x + a)^n*a*b^5
*d^2*n^4*x^5 + 85*(b*x + a)^n*b^6*d^2*n^3*x^6 + 2*(b*x + a)^n*a*b^5*c*d*n^
5*x^3 + 34*(b*x + a)^n*b^6*c*d*n^4*x^4 - 5*(b*x + a)^n*a^2*b^4*d^2*n^4*x^4
 + 35*(b*x + a)^n*a*b^5*d^2*n^3*x^5 + 225*(b*x + a)^n*b^6*d^2*n^2*x^6 + (b
*x + a)^n*b^6*c^2*n^5*x^2 + 28*(b*x + a)^n*a*b^5*c*d*n^4*x^3 + 214*(b*x +
a)^n*b^6*c*d*n^3*x^4 - 30*(b*x + a)^n*a^2*b^4*d^2*n^3*x^4 + 50*(b*x + a)^n
*a*b^5*d^2*n^2*x^5 + 274*(b*x + a)^n*b^6*d^2*n*x^6 + (b*x + a)^n*a*b^5*c^2
*n^5*x + 19*(b*x + a)^n*b^6*c^2*n^4*x^2 - 6*(b*x + a)^n*a^2*b^4*c*d*n^4*x^
2 + 130*(b*x + a)^n*a*b^5*c*d*n^3*x^3 + 20*(b*x + a)^n*a^3*b^3*d^2*n^3*x^3
 + 614*(b*x + a)^n*b^6*c*d*n^2*x^4 - 55*(b*x + a)^n*a^2*b^4*d^2*n^2*x^4 +
24*(b*x + a)^n*a*b^5*d^2*n*x^5 + 120*(b*x + a)^n*b^6*d^2*x^6 + 18*(b*x + a
)^n*a*b^5*c^2*n^4*x + 137*(b*x + a)^n*b^6*c^2*n^3*x^2 - 72*(b*x + a)^n*a^2
*b^4*c*d*n^3*x^2 + 224*(b*x + a)^n*a*b^5*c*d*n^2*x^3 + 60*(b*x + a)^n*a^3*
b^3*d^2*n^2*x^3 + 792*(b*x + a)^n*b^6*c*d*n*x^4 - 30*(b*x + a)^n*a^2*b^4*d
^2*n*x^4 - (b*x + a)^n*a^2*b^4*c^2*n^4 + 119*(b*x + a)^n*a*b^5*c^2*n^3*x +
 12*(b*x + a)^n*a^3*b^3*c*d*n^3*x + 461*(b*x + a)^n*b^6*c^2*n^2*x^2 - 246*
(b*x + a)^n*a^2*b^4*c*d*n^2*x^2 - 60*(b*x + a)^n*a^4*b^2*d^2*n^2*x^2 + 120
*(b*x + a)^n*a*b^5*c*d*n*x^3 + 40*(b*x + a)^n*a^3*b^3*d^2*n*x^3 + 360*(b*x
 + a)^n*b^6*c*d*x^4 - 18*(b*x + a)^n*a^2*b^4*c^2*n^3 + 342*(b*x + a)^n*...
```

3.356.9 Mupad [B] (verification not implemented)

Time = 11.74 (sec) , antiderivative size = 723, normalized size of antiderivative = 3.91

$$\int x(a+bx)^n\left(c+dx^2\right)^2\,dx=\frac{d^2\,x^6\,(a+b\,x)^n\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}{n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720}$$
$$-\frac{a^2\,(a+b\,x)^n\,(120\,a^4\,d^2+12\,a^2\,b^2\,c\,d\,n^2+132\,a^2\,b^2\,c\,d\,n+360\,a^2\,b^2\,c\,d+b^4\,c^2\,n^4+18\,b^4\,c^2\,n^3+119\,b^4\,c^2}{b^6\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)}$$
$$+\frac{x^2\,(n+1)\,(a+b\,x)^n\,(-60\,a^4\,d^2\,n-6\,a^2\,b^2\,c\,d\,n^3-66\,a^2\,b^2\,c\,d\,n^2-180\,a^2\,b^2\,c\,d\,n+b^4\,c^2\,n^4+18\,b^4\,c^2\,n^3}{b^4\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)}$$
$$+\frac{d\,x^4\,(a+b\,x)^n\,(-5\,d\,a^2\,n+2\,c\,b^2\,n^2+22\,c\,b^2\,n+60\,c\,b^2)\,(n^3+6\,n^2+11\,n+6)}{b^2\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)}$$
$$+\frac{a\,n\,x\,(a+b\,x)^n\,(120\,a^4\,d^2+12\,a^2\,b^2\,c\,d\,n^2+132\,a^2\,b^2\,c\,d\,n+360\,a^2\,b^2\,c\,d+b^4\,c^2\,n^4+18\,b^4\,c^2\,n^3+119\,b}{b^5\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)}$$
$$+\frac{a\,d^2\,n\,x^5\,(a+b\,x)^n\,(n^4+10\,n^3+35\,n^2+50\,n+24)}{b\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)}$$
$$+\frac{2\,a\,d\,n\,x^3\,(a+b\,x)^n\,(n^2+3\,n+2)\,(10\,d\,a^2+c\,b^2\,n^2+11\,c\,b^2\,n+30\,c\,b^2)}{b^3\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)}$$

input
```
int(x*(c + d*x^2)^2*(a + b*x)^n,x)
```

output
```
(d^2*x^6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(176
4*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720) - (a^2*(a + b*x)^
n*(120*a^4*d^2 + 360*b^4*c^2 + 342*b^4*c^2*n + 119*b^4*c^2*n^2 + 18*b^4*c^
2*n^3 + b^4*c^2*n^4 + 360*a^2*b^2*c*d + 132*a^2*b^2*c*d*n + 12*a^2*b^2*c*d
*n^2))/(b^6*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))
 + (x^2*(n + 1)*(a + b*x)^n*(360*b^4*c^2 - 60*a^4*d^2*n + 342*b^4*c^2*n + 1
19*b^4*c^2*n^2 + 18*b^4*c^2*n^3 + b^4*c^2*n^4 - 180*a^2*b^2*c*d*n - 66*a^2
*b^2*c*d*n^2 - 6*a^2*b^2*c*d*n^3))/(b^4*(1764*n + 1624*n^2 + 735*n^3 + 175
*n^4 + 21*n^5 + n^6 + 720)) + (d*x^4*(a + b*x)^n*(60*b^2*c + 2*b^2*c*n^2 -
 5*a^2*d*n + 22*b^2*c*n)*(11*n + 6*n^2 + n^3 + 6))/(b^2*(1764*n + 1624*n^2
 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (a*n*x*(a + b*x)^n*(120*a^4*
d^2 + 360*b^4*c^2 + 342*b^4*c^2*n + 119*b^4*c^2*n^2 + 18*b^4*c^2*n^3 + b^4
*c^2*n^4 + 360*a^2*b^2*c*d + 132*a^2*b^2*c*d*n + 12*a^2*b^2*c*d*n^2))/(b^5
*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (a*d^2*n*
x^5*(a + b*x)^n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(b*(1764*n + 1624*n^2
 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (2*a*d*n*x^3*(a + b*x)^n*(3*
n + n^2 + 2)*(10*a^2*d + 30*b^2*c + b^2*c*n^2 + 11*b^2*c*n))/(b^3*(1764*n
+ 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))
```

3.357 $\int (a+bx)^n \left(c+dx^2\right)^2 \, dx$

3.357.1 Optimal result

Integrand size = 17, antiderivative size = 140

$$\int (a+bx)^n \left(c+dx^2\right)^2 \, dx = \frac{\left(b^2c+a^2d\right)^2 (a+bx)^{1+n}}{b^5(1+n)} - \frac{4ad\left(b^2c+a^2d\right) (a+bx)^{2+n}}{b^5(2+n)} + \frac{2d\left(b^2c+3a^2d\right) (a+bx)^{3+n}}{b^5(3+n)} - \frac{4ad^2(a+bx)^{4+n}}{b^5(4+n)} + \frac{d^2(a+bx)^{5+n}}{b^5(5+n)}$$

output
```
(a^2*d+b^2*c)^2*(b*x+a)^(1+n)/b^5/(1+n)-4*a*d*(a^2*d+b^2*c)*(b*x+a)^(2+n)/
b^5/(2+n)+2*d*(3*a^2*d+b^2*c)*(b*x+a)^(3+n)/b^5/(3+n)-4*a*d^2*(b*x+a)^(4+n
)/b^5/(4+n)+d^2*(b*x+a)^(5+n)/b^5/(5+n)
```

3.357.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.14

$$\int (a+bx)^n \left(c+dx^2\right)^2 \, dx = \frac{(a+bx)^{1+n} \left(\left(c+dx^2\right)^2 + \frac{4\left(b^2c+a^2d\right)\left(2a^2d-2abd(1+n)x+b^2(2+n)\left(c(3+n)+d(1+n)x^2\right)\right)}{b^4(1+n)(2+n)(3+n)} - \frac{4ad(a+bx)\left(2a^2d-2abd(2+n)x+b^2(3+\cdots\right.}{b^4(2+n)(3+n)(4\cdots}\right.}{b(5+n)}$$

input
```
Integrate[(a + b*x)^n*(c + d*x^2)^2,x]
```

output
```
((a + b*x)^(1 + n)*((c + d*x^2)^2 + (4*(b^2*c + a^2*d)*(2*a^2*d - 2*a*b*d*
(1 + n)*x + b^2*(2 + n)*(c*(3 + n) + d*(1 + n)*x^2)))/(b^4*(1 + n)*(2 + n)
*(3 + n)) - (4*a*d*(a + b*x)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(3 + n)*(c
*(4 + n) + d*(2 + n)*x^2)))/(b^4*(2 + n)*(3 + n)*(4 + n))))/(b*(5 + n))
```

3.357.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^2\right)^2 (a+bx)^n\,dx$$

$$\Big\downarrow \; 476$$

$$\int \left(\frac{\left(a^2d+b^2c\right)^2 (a+bx)^n}{b^4}-\frac{4ad\left(a^2d+b^2c\right) (a+bx)^{n+1}}{b^4}+\frac{2d\left(3a^2d+b^2c\right) (a+bx)^{n+2}}{b^4}-\frac{4ad^2(a+bx)^{n+3}}{b^4}+\frac{d}{}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{\left(a^2d+b^2c\right)^2 (a+bx)^{n+1}}{b^5(n+1)}-\frac{4ad\left(a^2d+b^2c\right) (a+bx)^{n+2}}{b^5(n+2)}+\frac{2d\left(3a^2d+b^2c\right) (a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad^2(a+bx)^{n+4}}{b^5(n+4)}+\frac{d^2(a+bx)^{n+5}}{b^5(n+5)}$$

input
```
Int[(a + b*x)^n*(c + d*x^2)^2,x]
```

output
```
((b^2*c + a^2*d)^2*(a + b*x)^(1 + n))/(b^5*(1 + n)) - (4*a*d*(b^2*c + a^2*
d)*(a + b*x)^(2 + n))/(b^5*(2 + n)) + (2*d*(b^2*c + 3*a^2*d)*(a + b*x)^(3
+ n))/(b^5*(3 + n)) - (4*a*d^2*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d^2*(a
+ b*x)^(5 + n))/(b^5*(5 + n))
```

3.357.3.1 Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.357.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(140) = 280$.

Time = 0.42 (sec) , antiderivative size = 420, normalized size of antiderivative = 3.00

method	result
gosper	$(bx+a)^{1+n} (b^4 d^2 n^4 x^4+10 b^4 d^2 n^3 x^4-4 a b^3 d^2 n^3 x^3+2 b^4 c d n^4 x^2+35 b^4 d^2 n^2 x^4-24 a b^3 d^2 n^2 x^3+24 b^4 c d n^3 x^2+50 b^4 d^2 n x^4+12 a^2$
norman	$\frac{d^2 x^5 e^{n \ln(bx+a)}}{5+n} + \frac{a(b^4 c^2 n^4+14 b^4 c^2 n^3+4 a^2 b^2 c d n^2+71 b^4 c^2 n^2+36 a^2 b^2 c d n+154 b^4 c^2 n+24 d^2 a^4+80 a^2 b^2 c d+120 b^4 c^2) e^{n \ln(b}}{b^5 (n^5+15 n^4+85 n^3+225 n^2+274 n+120)}$
risch	$(b^5 d^2 n^4 x^5+a b^4 d^2 n^4 x^4+10 b^5 d^2 n^3 x^5+6 a b^4 d^2 n^3 x^4+2 b^5 c d n^4 x^3+35 b^5 d^2 n^2 x^5-4 a^2 b^3 d^2 n^3 x^3+2 a b^4 c d n^4 x^2+11 a b^4 d^2 n^2 x^4+$
parallelrisch	$-36x(bx+a)^n a^3 b^3 c d n^2-80x(bx+a)^n a^3 b^3 c d n-24x(bx+a)^n a^5 b d^2 n+154x(bx+a)^n a b^5 c^2 n+4(bx+a)^n a^4 b^2 c d n^2+36(bx+a)^n$

input
```
int((b*x+a)^n*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^5*(b*x+a)^(1+n)/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)*(b^4*d^2*n^4*x^4
+10*b^4*d^2*n^3*x^4-4*a*b^3*d^2*n^3*x^3+2*b^4*c*d*n^4*x^2+35*b^4*d^2*n^2*x
^4-24*a*b^3*d^2*n^2*x^3+24*b^4*c*d*n^3*x^2+50*b^4*d^2*n*x^4+12*a^2*b^2*d^2
*n^2*x^2-4*a*b^3*c*d*n^3*x-44*a*b^3*d^2*n*x^3+b^4*c^2*n^4+98*b^4*c*d*n^2*x
^2+24*b^4*d^2*x^4+36*a^2*b^2*d^2*n*x^2-40*a*b^3*c*d*n^2*x-24*a*b^3*d^2*x^3
+14*b^4*c^2*n^3+156*b^4*c*d*n*x^2-24*a^3*b*d^2*n*x+4*a^2*b^2*c*d*n^2+24*a^
2*b^2*d^2*x^2-116*a*b^3*c*d*n*x+71*b^4*c^2*n^2+80*b^4*c*d*x^2-24*a^3*b*d^2
*x+36*a^2*b^2*c*d*n-80*a*b^3*c*d*x+154*b^4*c^2*n+24*a^4*d^2+80*a^2*b^2*c*d
+120*b^4*c^2)
```

3.357.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(140) = 280$.

Time = 0.31 (sec) , antiderivative size = 519, normalized size of antiderivative = 3.71

$$\int (a+bx)^n \left(c+dx^2\right)^2 \, dx = \frac{(ab^4c^2n^4 + 14\,ab^4c^2n^3 + 120\,ab^4c^2 + 80\,a^3b^2cd + 24\,a^5d^2 + (b^5d^2n^4 + 10\,b^5d^2n^3 + 35\,b^5d^2n^2 + 50\,b^5d^2n + 2}{}$$

input
```
integrate((b*x+a)^n*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
(a*b^4*c^2*n^4 + 14*a*b^4*c^2*n^3 + 120*a*b^4*c^2 + 80*a^3*b^2*c*d + 24*a^
5*d^2 + (b^5*d^2*n^4 + 10*b^5*d^2*n^3 + 35*b^5*d^2*n^2 + 50*b^5*d^2*n + 24
*b^5*d^2)*x^5 + (a*b^4*d^2*n^4 + 6*a*b^4*d^2*n^3 + 11*a*b^4*d^2*n^2 + 6*a*
b^4*d^2*n)*x^4 + 2*(b^5*c*d*n^4 + 40*b^5*c*d + 2*(6*b^5*c*d - a^2*b^3*d^2)
*n^3 + (49*b^5*c*d - 6*a^2*b^3*d^2)*n^2 + 2*(39*b^5*c*d - 2*a^2*b^3*d^2)*n
)*x^3 + (71*a*b^4*c^2 + 4*a^3*b^2*c*d)*n^2 + 2*(a*b^4*c*d*n^4 + 10*a*b^4*c
*d*n^3 + (29*a*b^4*c*d + 6*a^3*b^2*d^2)*n^2 + 2*(10*a*b^4*c*d + 3*a^3*b^2*
d^2)*n)*x^2 + 2*(77*a*b^4*c^2 + 18*a^3*b^2*c*d)*n + (b^5*c^2*n^4 + 120*b^5
*c^2 + 2*(7*b^5*c^2 - 2*a^2*b^3*c*d)*n^3 + (71*b^5*c^2 - 36*a^2*b^3*c*d)*n
^2 + 2*(77*b^5*c^2 - 40*a^2*b^3*c*d - 12*a^4*b*d^2)*n)*x)*(b*x + a)^n/(b^5
*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

3.357.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5097 vs. $2(128) = 256$.

Time = 1.44 (sec) , antiderivative size = 5097, normalized size of antiderivative = 36.41

$$\int (a+bx)^n \left(c+dx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**n*(d*x**2+c)**2,x)
```

output
```
Piecewise((a**n*(c**2*x + 2*c*d*x**3/3 + d**2*x**5/5), Eq(b, 0)), (12*a**4
*d**2*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48
*a*b**8*x**3 + 12*b**9*x**4) + 25*a**4*d**2/(12*a**4*b**5 + 48*a**3*b**6*x
 + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a**3*b*d**2*x*l
og(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8
*x**3 + 12*b**9*x**4) + 88*a**3*b*d**2*x/(12*a**4*b**5 + 48*a**3*b**6*x +
72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 2*a**2*b**2*c*d/(12*a
**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x
**4) + 72*a**2*b**2*d**2*x**2*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x
+ 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 108*a**2*b**2*d**2*
x**2/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 +
 12*b**9*x**4) - 8*a*b**3*c*d*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b
**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**3*d**2*x**3*log(a/b +
x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 1
2*b**9*x**4) + 48*a*b**3*d**2*x**3/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**
2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 3*b**4*c**2/(12*a**4*b**5 +
 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 12*
b**4*c*d*x**2/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b*
*8*x**3 + 12*b**9*x**4) + 12*b**4*d**2*x**4*log(a/b + x)/(12*a**4*b**5 + 4
8*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4), Eq(...
```

3.357.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.68

$$\int (a+bx)^n\left(c+dx^2\right)^2\,dx$$

$$= \frac{(bx+a)^{n+1}c^2}{b(n+1)} + \frac{2\left((n^2+3n+2)b^3x^3+(n^2+n)ab^2x^2-2a^2bnx+2a^3\right)(bx+a)^n cd}{(n^3+6n^2+11n+6)b^3}$$

$$+ \frac{((n^4+10n^3+35n^2+50n+24)b^5x^5+(n^4+6n^3+11n^2+6n)ab^4x^4-4(n^3+3n^2+2n)a^2b^3x^3+1}{(n^5+15n^4+85n^3+225n^2+274n+120)b^5}$$

input
```
integrate((b*x+a)^n*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
(b*x + a)^(n + 1)*c^2/(b*(n + 1)) + 2*((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)
*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x + a)^n*c*d/((n^3 + 6*n^2 + 11*n + 6
)*b^3) + ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*
n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^
3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*d^2/((n^5 + 15*n^4 + 85*n^3
 + 225*n^2 + 274*n + 120)*b^5)
```

3.357.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(140) = 280$.

Time = 0.28 (sec) , antiderivative size = 851, normalized size of antiderivative = 6.08

$$\int (a+bx)^n \left(c+dx^2\right)^2 \, dx$$
$$= \frac{(bx+a)^n b^5 d^2 n^4 x^5 + (bx+a)^n a b^4 d^2 n^4 x^4 + 10\,(bx+a)^n b^5 d^2 n^3 x^5 + 2\,(bx+a)^n b^5 c d n^4 x^3 + 6\,(bx+a)^n a b^4 d}{}$$

input
```
integrate((b*x+a)^n*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
((b*x + a)^n*b^5*d^2*n^4*x^5 + (b*x + a)^n*a*b^4*d^2*n^4*x^4 + 10*(b*x + a
)^n*b^5*d^2*n^3*x^5 + 2*(b*x + a)^n*b^5*c*d*n^4*x^3 + 6*(b*x + a)^n*a*b^4*
d^2*n^3*x^4 + 35*(b*x + a)^n*b^5*d^2*n^2*x^5 + 2*(b*x + a)^n*a*b^4*c*d*n^4
*x^2 + 24*(b*x + a)^n*b^5*c*d*n^3*x^3 - 4*(b*x + a)^n*a^2*b^3*d^2*n^3*x^3
+ 11*(b*x + a)^n*a*b^4*d^2*n^2*x^4 + 50*(b*x + a)^n*b^5*d^2*n*x^5 + (b*x +
 a)^n*b^5*c^2*n^4*x + 20*(b*x + a)^n*a*b^4*c*d*n^3*x^2 + 98*(b*x + a)^n*b^
5*c*d*n^2*x^3 - 12*(b*x + a)^n*a^2*b^3*d^2*n^2*x^3 + 6*(b*x + a)^n*a*b^4*d
^2*n*x^4 + 24*(b*x + a)^n*b^5*d^2*x^5 + (b*x + a)^n*a*b^4*c^2*n^4 + 14*(b*
x + a)^n*b^5*c^2*n^3*x - 4*(b*x + a)^n*a^2*b^3*c*d*n^3*x + 58*(b*x + a)^n*
a*b^4*c*d*n^2*x^2 + 12*(b*x + a)^n*a^3*b^2*d^2*n^2*x^2 + 156*(b*x + a)^n*b
^5*c*d*n*x^3 - 8*(b*x + a)^n*a^2*b^3*d^2*n*x^3 + 14*(b*x + a)^n*a*b^4*c^2*
n^3 + 71*(b*x + a)^n*b^5*c^2*n^2*x - 36*(b*x + a)^n*a^2*b^3*c*d*n^2*x + 40
*(b*x + a)^n*a*b^4*c*d*n*x^2 + 12*(b*x + a)^n*a^3*b^2*d^2*n*x^2 + 80*(b*x
+ a)^n*b^5*c*d*x^3 + 71*(b*x + a)^n*a*b^4*c^2*n^2 + 4*(b*x + a)^n*a^3*b^2*
c*d*n^2 + 154*(b*x + a)^n*b^5*c^2*n*x - 80*(b*x + a)^n*a^2*b^3*c*d*n*x - 2
4*(b*x + a)^n*a^4*b*d^2*n*x + 154*(b*x + a)^n*a*b^4*c^2*n + 36*(b*x + a)^n
*a^3*b^2*c*d*n + 120*(b*x + a)^n*b^5*c^2*x + 120*(b*x + a)^n*a*b^4*c^2 + 8
0*(b*x + a)^n*a^3*b^2*c*d + 24*(b*x + a)^n*a^5*d^2)/(b^5*n^5 + 15*b^5*n^4
+ 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

3.357.9 Mupad [B] (verification not implemented)

Time = 11.62 (sec) , antiderivative size = 496, normalized size of antiderivative = 3.54

$$\int (a+bx)^n\left(c+dx^2\right)^2\,dx$$
$$=(a+b\,x)^n\left(\frac{a\,(24\,a^4\,d^2+4\,a^2\,b^2\,c\,d\,n^2+36\,a^2\,b^2\,c\,d\,n+80\,a^2\,b^2\,c\,d+b^4\,c^2\,n^4+14\,b^4\,c^2\,n^3+71\,b^4\,c^2\,n^2+1}{b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\right.$$
$$+\frac{d^2\,x^5\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120}$$
$$+\frac{x\,(-24\,a^4\,b\,d^2\,n-4\,a^2\,b^3\,c\,d\,n^3-36\,a^2\,b^3\,c\,d\,n^2-80\,a^2\,b^3\,c\,d\,n+b^5\,c^2\,n^4+14\,b^5\,c^2\,n^3+71\,b^5\,c^2\,n^2+15}{b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}$$
$$+\frac{2\,d\,x^3\,\left(n^2+3\,n+2\right)\,\left(-2\,d\,a^2\,n+c\,b^2\,n^2+9\,c\,b^2\,n+20\,c\,b^2\right)}{b^2\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}$$
$$+\frac{a\,d^2\,n\,x^4\,\left(n^3+6\,n^2+11\,n+6\right)}{b\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}$$
$$\left.+\frac{2\,a\,d\,n\,x^2\,(n+1)\,\left(6\,d\,a^2+c\,b^2\,n^2+9\,c\,b^2\,n+20\,c\,b^2\right)}{b^3\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\right)$$

input
```
int((c + d*x^2)^2*(a + b*x)^n,x)
```

output
```
(a + b*x)^n*((a*(24*a^4*d^2 + 120*b^4*c^2 + 154*b^4*c^2*n + 71*b^4*c^2*n^2
 + 14*b^4*c^2*n^3 + b^4*c^2*n^4 + 80*a^2*b^2*c*d + 36*a^2*b^2*c*d*n + 4*a^
2*b^2*c*d*n^2))/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (d
^2*x^5*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(274*n + 225*n^2 + 85*n^3 + 15
*n^4 + n^5 + 120) + (x*(120*b^5*c^2 + 154*b^5*c^2*n + 71*b^5*c^2*n^2 + 14*
b^5*c^2*n^3 + b^5*c^2*n^4 - 24*a^4*b*d^2*n - 80*a^2*b^3*c*d*n - 36*a^2*b^3
*c*d*n^2 - 4*a^2*b^3*c*d*n^3))/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n
^5 + 120)) + (2*d*x^3*(3*n + n^2 + 2)*(20*b^2*c + b^2*c*n^2 - 2*a^2*d*n +
9*b^2*c*n))/(b^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*d^2
*n*x^4*(11*n + 6*n^2 + n^3 + 6))/(b*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n
^5 + 120)) + (2*a*d*n*x^2*(n + 1)*(6*a^2*d + 20*b^2*c + b^2*c*n^2 + 9*b^2*
c*n))/(b^3*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)))
```

3.358 $\int \frac{(a+bx)^n (c+dx^2)^2}{x} \, dx$

3.358.1 Optimal result

Integrand size = 20, antiderivative size = 148

$$\int \frac{(a+bx)^n (c+dx^2)^2}{x} \, dx = -\frac{ad(2b^2c+a^2d)(a+bx)^{1+n}}{b^4(1+n)} + \frac{d(2b^2c+3a^2d)(a+bx)^{2+n}}{b^4(2+n)} - \frac{3ad^2(a+bx)^{3+n}}{b^4(3+n)} + \frac{d^2(a+bx)^{4+n}}{b^4(4+n)} - \frac{c^2(a+bx)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{bx}{a}\right)}{a(1+n)}$$

output
```
-a*d*(a^2*d+2*b^2*c)*(b*x+a)^(1+n)/b^4/(1+n)+d*(3*a^2*d+2*b^2*c)*(b*x+a)^(
2+n)/b^4/(2+n)-3*a*d^2*(b*x+a)^(3+n)/b^4/(3+n)+d^2*(b*x+a)^(4+n)/b^4/(4+n)
-c^2*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

3.358.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^n (c+dx^2)^2}{x} \, dx = (a+bx)^{1+n} \left(-\frac{ad(2b^2c+a^2d)}{b^4(1+n)} + \frac{d(2b^2c+3a^2d)(a+bx)}{b^4(2+n)} - \frac{3ad^2(a+bx)^2}{b^4(3+n)} + \frac{d^2(a+bx)^3}{b^4(4+n)} - \frac{c^2 \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+bx}{a}\right)}{a+an}\right)$$

input
```
Integrate[((a + b*x)^n*(c + d*x^2)^2)/x,x]
```

output
```
(a + b*x)^(1 + n)*(-((a*d*(2*b^2*c + a^2*d))/(b^4*(1 + n))) + (d*(2*b^2*c
+ 3*a^2*d)*(a + b*x))/(b^4*(2 + n)) - (3*a*d^2*(a + b*x)^2)/(b^4*(3 + n))
+ (d^2*(a + b*x)^3)/(b^4*(4 + n)) - (c^2*Hypergeometric2F1[1, 1 + n, 2 + n
, (a + b*x)/a])/(a + a*n))
```

3.358.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2 (a+bx)^n}{x} \, dx$$

$$\Big\downarrow \; 522$$

$$\int \left(-\frac{ad(a^2d+2b^2c)(a+bx)^n}{b^3}+\frac{d(3a^2d+2b^2c)(a+bx)^{n+1}}{b^3}-\frac{3ad^2(a+bx)^{n+2}}{b^3}+\frac{d^2(a+bx)^{n+3}}{b^3}+\frac{c^2(a+bx)}{x}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{ad(a^2d+2b^2c)(a+bx)^{n+1}}{b^4(n+1)}+\frac{d(3a^2d+2b^2c)(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad^2(a+bx)^{n+3}}{b^4(n+3)}+\frac{d^2(a+bx)^{n+4}}{b^4(n+4)}-\frac{c^2(a+bx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{bx}{a}+1\right)}{a(n+1)}$$

input
```
Int[((a + b*x)^n*(c + d*x^2)^2)/x,x]
```

output
```
-((a*d*(2*b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^4*(1 + n))) + (d*(2*b^2*c +
 3*a^2*d)*(a + b*x)^(2 + n))/(b^4*(2 + n)) - (3*a*d^2*(a + b*x)^(3 + n))/(
b^4*(3 + n)) + (d^2*(a + b*x)^(4 + n))/(b^4*(4 + n)) - (c^2*(a + b*x)^(1 +
 n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

3.358.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.358.4 Maple [F]

$$\int \frac{(bx+a)^n \left(d\,x^2+c\right)^2}{x}dx$$

input
```
int((b*x+a)^n*(d*x^2+c)^2/x,x)
```

output
```
int((b*x+a)^n*(d*x^2+c)^2/x,x)
```

3.358.5 Fricas [F]

$$\int \frac{(a+bx)^n (c+dx^2)^2}{x} \, dx = \int \frac{(dx^2+c)^2 (bx+a)^n}{x} \, dx$$

input
```
integrate((b*x+a)^n*(d*x^2+c)^2/x,x, algorithm="fricas")
```

output
```
integral((d^2*x^4 + 2*c*d*x^2 + c^2)*(b*x + a)^n/x, x)
```

3.358.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1321 vs. 2(129) = 258.

Time = 3.32 (sec) , antiderivative size = 1608, normalized size of antiderivative = 10.86

$$\int \frac{(a+bx)^n (c+dx^2)^2}{x} \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**n*(d*x**2+c)**2/x,x)
```

output
```
2*c*d*Piecewise((a**n*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x)
 + a/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(n, -2)), (
-a*log(a/b + x)/b**2 + x/b, Eq(n, -1)), (-a**2*(a + b*x)**n/(b**2*n**2 + 3
*b**2*n + 2*b**2) + a*b*n*x*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) +
 b**2*n*x**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2*x**2*(a +
 b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2), True)) + d**2*Piecewise((a**n*x*
*4/4, Eq(b, 0)), (6*a**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a
*b**6*x**2 + 6*b**7*x**3) + 11*a**3/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b
**6*x**2 + 6*b**7*x**3) + 18*a**2*b*x*log(a/b + x)/(6*a**3*b**4 + 18*a**2*
b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*x/(6*a**3*b**4 + 18*a**
2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*x**2*log(a/b + x)/(6*
a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*x**
2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 6*b**3*x
**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x
**3), Eq(n, -4)), (-6*a**3*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6
*x**2) - 9*a**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*x*log
(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*x/(2*a**2*b
**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*x**2*log(a/b + x)/(2*a**2*b**4
+ 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*x**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b*
*6*x**2), Eq(n, -3)), (6*a**3*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a*...
```

3.358.7 Maxima [F]

$$\int \frac{(a+bx)^n \left(c+dx^2\right)^2}{x}\,dx = \int \frac{\left(dx^2+c\right)^2 (bx+a)^n}{x}\,dx$$

input
```
integrate((b*x+a)^n*(d*x^2+c)^2/x,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^2*(b*x + a)^n/x, x)
```

3.358.8 Giac [F]

$$\int \frac{(a+bx)^n (c+dx^2)^2}{x} \, dx = \int \frac{(dx^2+c)^2 (bx+a)^n}{x} \, dx$$

input
```
integrate((b*x+a)^n*(d*x^2+c)^2/x,x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^2*(b*x + a)^n/x, x)
```

3.358.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^n (c+dx^2)^2}{x} \, dx = \int \frac{(d\,x^2+c)^2 (a+b\,x)^n}{x} \, dx$$

input
```
int(((c + d*x^2)^2*(a + b*x)^n)/x,x)
```

output
```
int(((c + d*x^2)^2*(a + b*x)^n)/x, x)
```

3.359 $\int x^2(a+bx)^n \left(c+dx^2\right)^3 \, dx$

3.359.1 Optimal result

Integrand size = 20, antiderivative size = 343

$$\begin{aligned}\int x^2(a+bx)^n \left(c+dx^2\right)^3 \, dx = {} & \frac{a^2(b^2c+a^2d)^3 (a+bx)^{1+n}}{b^9(1+n)} \\ & - \frac{2a(b^2c+a^2d)^2 (b^2c+4a^2d) (a+bx)^{2+n}}{b^9(2+n)} \\ & + \frac{(b^2c+a^2d) (b^4c^2+17a^2b^2cd+28a^4d^2) (a+bx)^{3+n}}{b^9(3+n)} \\ & - \frac{4ad(3b^4c^2+15a^2b^2cd+14a^4d^2) (a+bx)^{4+n}}{b^9(4+n)} \\ & + \frac{d(3b^4c^2+45a^2b^2cd+70a^4d^2) (a+bx)^{5+n}}{b^9(5+n)} \\ & - \frac{2ad^2(9b^2c+28a^2d) (a+bx)^{6+n}}{b^9(6+n)} \\ & + \frac{d^2(3b^2c+28a^2d) (a+bx)^{7+n}}{b^9(7+n)} \\ & - \frac{8ad^3(a+bx)^{8+n}}{b^9(8+n)} + \frac{d^3(a+bx)^{9+n}}{b^9(9+n)}\end{aligned}$$

output
```
a^2*(a^2*d+b^2*c)^3*(b*x+a)^(1+n)/b^9/(1+n)-2*a*(a^2*d+b^2*c)^2*(4*a^2*d+b
^2*c)*(b*x+a)^(2+n)/b^9/(2+n)+(a^2*d+b^2*c)*(28*a^4*d^2+17*a^2*b^2*c*d+b^4
*c^2)*(b*x+a)^(3+n)/b^9/(3+n)-4*a*d*(14*a^4*d^2+15*a^2*b^2*c*d+3*b^4*c^2)*
(b*x+a)^(4+n)/b^9/(4+n)+d*(70*a^4*d^2+45*a^2*b^2*c*d+3*b^4*c^2)*(b*x+a)^(5
+n)/b^9/(5+n)-2*a*d^2*(28*a^2*d+9*b^2*c)*(b*x+a)^(6+n)/b^9/(6+n)+d^2*(28*a
^2*d+3*b^2*c)*(b*x+a)^(7+n)/b^9/(7+n)-8*a*d^3*(b*x+a)^(8+n)/b^9/(8+n)+d^3*
(b*x+a)^(9+n)/b^9/(9+n)
```

3.359.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.88

$$\int x^2(a+bx)^n\left(c+dx^2\right)^3\,dx$$
$$= \frac{(a+bx)^{1+n}\left(\frac{a^2(b^2c+a^2d)^3}{1+n}-\frac{2a(b^2c+a^2d)^2(b^2c+4a^2d)(a+bx)}{2+n}+\frac{(b^2c+a^2d)(b^4c^2+17a^2b^2cd+28a^4d^2)(a+bx)^2}{3+n}-\frac{4ad(3b^4c^2+15a^2b^2 \cdots}{4 \cdots}\cdots\right.}{\cdots}$$

input
```
Integrate[x^2*(a + b*x)^n*(c + d*x^2)^3,x]
```

output
```
((a + b*x)^(1 + n)*((a^2*(b^2*c + a^2*d)^3)/(1 + n) - (2*a*(b^2*c + a^2*d)
^2*(b^2*c + 4*a^2*d)*(a + b*x))/(2 + n) + ((b^2*c + a^2*d)*(b^4*c^2 + 17*a
^2*b^2*c*d + 28*a^4*d^2)*(a + b*x)^2)/(3 + n) - (4*a*d*(3*b^4*c^2 + 15*a^2
*b^2*c*d + 14*a^4*d^2)*(a + b*x)^3)/(4 + n) + (d*(3*b^4*c^2 + 45*a^2*b^2*c
*d + 70*a^4*d^2)*(a + b*x)^4)/(5 + n) - (2*a*d^2*(9*b^2*c + 28*a^2*d)*(a +
 b*x)^5)/(6 + n) + (d^2*(3*b^2*c + 28*a^2*d)*(a + b*x)^6)/(7 + n) - (8*a*d
^3*(a + b*x)^7)/(8 + n) + (d^3*(a + b*x)^8)/(9 + n)))/b^9
```

3.359.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(c+dx^2\right)^3(a+bx)^n\,dx$$

$$\Big\downarrow 522$$

$$\int \left(-\frac{2ad^2\left(28a^2d+9b^2c\right)(a+bx)^{n+5}}{b^8}+\frac{d^2\left(28a^2d+3b^2c\right)(a+bx)^{n+6}}{b^8}+\frac{a^2\left(a^2d+b^2c\right)^3(a+bx)^n}{b^8}-\frac{2a(a^2d+b\right.$$

$$\Big\downarrow 2009$$

$$-\frac{2ad^2\left(28a^2d+9b^2c\right)(a+bx)^{n+6}}{b^9(n+6)}+\frac{d^2\left(28a^2d+3b^2c\right)(a+bx)^{n+7}}{b^9(n+7)}+\frac{a^2\left(a^2d+b^2c\right)^3(a+bx)^{n+1}}{b^9(n+1)}-\frac{2a\left(a^2d+b^2c\right)^2\left(4a^2d+b^2c\right)(a+bx)^{n+2}}{b^9(n+2)}+\frac{\left(a^2d+b^2c\right)\left(28a^4d^2+17a^2b^2cd+b^4c^2\right)(a+bx)^{n+3}}{b^9(n+3)}-\frac{4ad\left(14a^4d^2+15a^2b^2cd+3b^4c^2\right)(a+bx)^{n+4}}{b^9(n+4)}+\frac{d\left(70a^4d^2+45a^2b^2cd+3b^4c^2\right)(a+bx)^{n+5}}{b^9(n+5)}-\frac{8ad^3(a+bx)^{n+8}}{b^9(n+8)}+\frac{d^3(a+bx)^{n+9}}{b^9(n+9)}$$

input
```
Int[x^2*(a + b*x)^n*(c + d*x^2)^3,x]
```

output
```
(a^2*(b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^9*(1 + n)) - (2*a*(b^2*c + a^
2*d)^2*(b^2*c + 4*a^2*d)*(a + b*x)^(2 + n))/(b^9*(2 + n)) + ((b^2*c + a^2*
d)*(b^4*c^2 + 17*a^2*b^2*c*d + 28*a^4*d^2)*(a + b*x)^(3 + n))/(b^9*(3 + n)
) - (4*a*d*(3*b^4*c^2 + 15*a^2*b^2*c*d + 14*a^4*d^2)*(a + b*x)^(4 + n))/(b
^9*(4 + n)) + (d*(3*b^4*c^2 + 45*a^2*b^2*c*d + 70*a^4*d^2)*(a + b*x)^(5 +
n))/(b^9*(5 + n)) - (2*a*d^2*(9*b^2*c + 28*a^2*d)*(a + b*x)^(6 + n))/(b^9*
(6 + n)) + (d^2*(3*b^2*c + 28*a^2*d)*(a + b*x)^(7 + n))/(b^9*(7 + n)) - (8
*a*d^3*(a + b*x)^(8 + n))/(b^9*(8 + n)) + (d^3*(a + b*x)^(9 + n))/(b^9*(9
+ n))
```

3.359.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.359.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2231 vs. $2(343) = 686$.

Time = 0.45 (sec) , antiderivative size = 2232, normalized size of antiderivative = 6.51

method	result	size
gosper	Expression too large to display	2232
risch	Expression too large to display	2558
parallelrisch	Expression too large to display	3685

input
```
int(x^2*(b*x+a)^n*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^9*(b*x+a)^(1+n)/(n^9+45*n^8+870*n^7+9450*n^6+63273*n^5+269325*n^4+7236
80*n^3+1172700*n^2+1026576*n+362880)*(b^8*d^3*n^8*x^8+36*b^8*d^3*n^7*x^8-8
*a*b^7*d^3*n^7*x^7+3*b^8*c*d^2*n^8*x^6+546*b^8*d^3*n^6*x^8-224*a*b^7*d^3*n
^6*x^7+114*b^8*c*d^2*n^7*x^6+4536*b^8*d^3*n^5*x^8+56*a^2*b^6*d^3*n^6*x^6-1
8*a*b^7*c*d^2*n^7*x^5-2576*a*b^7*d^3*n^5*x^7+3*b^8*c^2*d*n^8*x^4+1812*b^8*
c*d^2*n^6*x^6+22449*b^8*d^3*n^4*x^8+1176*a^2*b^6*d^3*n^5*x^6-576*a*b^7*c*d
^2*n^6*x^5-15680*a*b^7*d^3*n^4*x^7+120*b^8*c^2*d*n^7*x^4+15666*b^8*c*d^2*n
^5*x^6+67284*b^8*d^3*n^3*x^8-336*a^3*b^5*d^3*n^5*x^5+90*a^2*b^6*c*d^2*n^6*
x^4+9800*a^2*b^6*d^3*n^4*x^6-12*a*b^7*c^2*d*n^7*x^3-7416*a*b^7*c*d^2*n^5*x
^5-54152*a*b^7*d^3*n^3*x^7+b^8*c^3*n^8*x^2+2010*b^8*c^2*d*n^6*x^4+80157*b^
8*c*d^2*n^4*x^6+118124*b^8*d^3*n^2*x^8-5040*a^3*b^5*d^3*n^4*x^5+2430*a^2*b
^6*c*d^2*n^5*x^4+41160*a^2*b^6*d^3*n^3*x^6-432*a*b^7*c^2*d*n^6*x^3-49500*a
*b^7*c*d^2*n^4*x^5-105056*a*b^7*d^3*n^2*x^7+42*b^8*c^3*n^7*x^2+18300*b^8*c
^2*d*n^5*x^4+246876*b^8*c*d^2*n^3*x^6+109584*b^8*d^3*n*x^8+1680*a^4*b^4*d^
3*n^4*x^4-360*a^3*b^5*c*d^2*n^5*x^3-28560*a^3*b^5*d^3*n^3*x^5+36*a^2*b^6*c
^2*d*n^6*x^2+24930*a^2*b^6*c*d^2*n^4*x^4+90944*a^2*b^6*d^3*n^2*x^6-2*a*b^7
*c^3*n^7*x-6312*a*b^7*c^2*d*n^5*x^3-183942*a*b^7*c*d^2*n^3*x^5-104544*a*b^
7*d^3*n*x^7+744*b^8*c^3*n^6*x^2+98319*b^8*c^2*d*n^4*x^4+442908*b^8*c*d^2*n
^2*x^6+40320*b^8*d^3*x^8+16800*a^4*b^4*d^3*n^3*x^4-8280*a^3*b^5*c*d^2*n^4*
x^3-75600*a^3*b^5*d^3*n^2*x^5+1188*a^2*b^6*c^2*d*n^5*x^2+122850*a^2*b^6...
```

3.359.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2165 vs. $2(343) = 686$.

Time = 0.31 (sec) , antiderivative size = 2165, normalized size of antiderivative = 6.31

$$\int x^2(a+bx)^n \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
(2*a^3*b^6*c^3*n^6 + 78*a^3*b^6*c^3*n^5 + 120960*a^3*b^6*c^3 + 217728*a^5*
b^4*c^2*d + 155520*a^7*b^2*c*d^2 + 40320*a^9*d^3 + (b^9*d^3*n^8 + 36*b^9*d
^3*n^7 + 546*b^9*d^3*n^6 + 4536*b^9*d^3*n^5 + 22449*b^9*d^3*n^4 + 67284*b^
9*d^3*n^3 + 118124*b^9*d^3*n^2 + 109584*b^9*d^3*n + 40320*b^9*d^3)*x^9 + (
a*b^8*d^3*n^8 + 28*a*b^8*d^3*n^7 + 322*a*b^8*d^3*n^6 + 1960*a*b^8*d^3*n^5
+ 6769*a*b^8*d^3*n^4 + 13132*a*b^8*d^3*n^3 + 13068*a*b^8*d^3*n^2 + 5040*a*
b^8*d^3*n)*x^8 + (3*b^9*c*d^2*n^8 + 155520*b^9*c*d^2 + 2*(57*b^9*c*d^2 - 4
*a^2*b^7*d^3)*n^7 + 12*(151*b^9*c*d^2 - 14*a^2*b^7*d^3)*n^6 + 14*(1119*b^9
*c*d^2 - 100*a^2*b^7*d^3)*n^5 + 21*(3817*b^9*c*d^2 - 280*a^2*b^7*d^3)*n^4
+ 28*(8817*b^9*c*d^2 - 464*a^2*b^7*d^3)*n^3 + 36*(12303*b^9*c*d^2 - 392*a^
2*b^7*d^3)*n^2 + 144*(2901*b^9*c*d^2 - 40*a^2*b^7*d^3)*n)*x^7 + (3*a*b^8*c
*d^2*n^8 + 96*a*b^8*c*d^2*n^7 + 4*(309*a*b^8*c*d^2 + 14*a^3*b^6*d^3)*n^6 +
 30*(275*a*b^8*c*d^2 + 28*a^3*b^6*d^3)*n^5 + (30657*a*b^8*c*d^2 + 4760*a^3
*b^6*d^3)*n^4 + 6*(10489*a*b^8*c*d^2 + 2100*a^3*b^6*d^3)*n^3 + 8*(8163*a*b
^8*c*d^2 + 1918*a^3*b^6*d^3)*n^2 + 960*(27*a*b^8*c*d^2 + 7*a^3*b^6*d^3)*n)
*x^6 + 3*(b^9*c^2*d*n^8 + 72576*b^9*c^2*d + 2*(20*b^9*c^2*d - 3*a^2*b^7*c*
d^2)*n^7 + 2*(335*b^9*c^2*d - 81*a^2*b^7*c*d^2)*n^6 + 2*(3050*b^9*c^2*d -
831*a^2*b^7*c*d^2 - 56*a^4*b^5*d^3)*n^5 + (32773*b^9*c^2*d - 8190*a^2*b^7*
c*d^2 - 1120*a^4*b^5*d^3)*n^4 + 4*(26365*b^9*c^2*d - 5091*a^2*b^7*c*d^2 -
980*a^4*b^5*d^3)*n^3 + 4*(49095*b^9*c^2*d - 6012*a^2*b^7*c*d^2 - 1400*a...
```

3.359.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35984 vs. $2(328) = 656$.

Time = 11.30 (sec) , antiderivative size = 35984, normalized size of antiderivative = 104.91

$$\int x^2(a+bx)^n \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(x**2*(b*x+a)**n*(d*x**2+c)**3,x)
```

output
```
Piecewise((a**n*(c**3*x**3/3 + 3*c**2*d*x**5/5 + 3*c*d**2*x**7/7 + d**3*x*
*9/9), Eq(b, 0)), (840*a**8*d**3*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b
**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*
x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 +
 840*b**17*x**8) + 2283*a**8*d**3/(840*a**8*b**9 + 6720*a**7*b**10*x + 235
20*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040
*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x
**8) + 6720*a**7*b*d**3*x*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x
+ 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 +
47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b*
*17*x**8) + 17424*a**7*b*d**3*x/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520
*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a
**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**
8) - 45*a**6*b**2*c*d**2/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b
**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**
14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 23
520*a**6*b**2*d**3*x**2*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x +
23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47
040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**1
7*x**8) + 57624*a**6*b**2*d**3*x**2/(840*a**8*b**9 + 6720*a**7*b**10*x ...
```

3.359.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(343) = 686$.

Time = 0.24 (sec) , antiderivative size = 795, normalized size of antiderivative = 2.32

$$\int x^2(a+bx)^n\left(c+dx^2\right)^3\,dx$$

$$= \frac{((n^2+3n+2)b^3x^3+(n^2+n)ab^2x^2-2a^2bnx+2a^3)(bx+a)^nc^3}{(n^3+6n^2+11n+6)b^3}$$

$$+ \frac{3((n^4+10n^3+35n^2+50n+24)b^5x^5+(n^4+6n^3+11n^2+6n)ab^4x^4-4(n^3+3n^2+2n)a^2b^3x^3+}{(n^5+15n^4+85n^3+225n^2+274n+120)b^5}$$

$$+ \frac{3((n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)b^7x^7+(n^6+15n^5+85n^4+225n^3+274n}{}$$

$$+ \frac{((n^8+36n^7+546n^6+4536n^5+22449n^4+67284n^3+118124n^2+109584n+40320)b^9x^9+(n^8+}{}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
 + a)^n*c^3/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 3*((n^4 + 10*n^3 + 35*n^2 + 5
0*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^
2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(
b*x + a)^n*c^2*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5) + 3
*((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (
n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10
*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*
n)*a^3*b^4*x^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b
^2*x^2 - 720*a^6*b*n*x + 720*a^7)*(b*x + a)^n*c*d^2/((n^7 + 28*n^6 + 322*n
^5 + 1960*n^4 + 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^7) + ((n^8 + 36*n
^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n^2 + 109584*n +
40320)*b^9*x^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3
 + 13068*n^2 + 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1
624*n^3 + 1764*n^2 + 720*n)*a^2*b^7*x^7 + 56*(n^6 + 15*n^5 + 85*n^4 + 225*
n^3 + 274*n^2 + 120*n)*a^3*b^6*x^6 - 336*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 +
 24*n)*a^4*b^5*x^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^5*b^4*x^4 - 6720*
(n^3 + 3*n^2 + 2*n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a^7*b^2*x^2 - 40320*a^8*
b*n*x + 40320*a^9)*(b*x + a)^n*d^3/((n^9 + 45*n^8 + 870*n^7 + 9450*n^6 + 6
3273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2 + 1026576*n + 362880)*...
```

3.359.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3713 vs. $2(343) = 686$.

Time = 0.30 (sec) , antiderivative size = 3713, normalized size of antiderivative = 10.83

$$\int x^2(a+bx)^n\left(c+dx^2\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
((b*x + a)^n*b^9*d^3*n^8*x^9 + (b*x + a)^n*a*b^8*d^3*n^8*x^8 + 36*(b*x + a
)^n*b^9*d^3*n^7*x^9 + 3*(b*x + a)^n*b^9*c*d^2*n^8*x^7 + 28*(b*x + a)^n*a*b
^8*d^3*n^7*x^8 + 546*(b*x + a)^n*b^9*d^3*n^6*x^9 + 3*(b*x + a)^n*a*b^8*c*d
^2*n^8*x^6 + 114*(b*x + a)^n*b^9*c*d^2*n^7*x^7 - 8*(b*x + a)^n*a^2*b^7*d^3
*n^7*x^7 + 322*(b*x + a)^n*a*b^8*d^3*n^6*x^8 + 4536*(b*x + a)^n*b^9*d^3*n^
5*x^9 + 3*(b*x + a)^n*b^9*c^2*d*n^8*x^5 + 96*(b*x + a)^n*a*b^8*c*d^2*n^7*x
^6 + 1812*(b*x + a)^n*b^9*c*d^2*n^6*x^7 - 168*(b*x + a)^n*a^2*b^7*d^3*n^6*
x^7 + 1960*(b*x + a)^n*a*b^8*d^3*n^5*x^8 + 22449*(b*x + a)^n*b^9*d^3*n^4*x
^9 + 3*(b*x + a)^n*a*b^8*c^2*d*n^8*x^4 + 120*(b*x + a)^n*b^9*c^2*d*n^7*x^5
 - 18*(b*x + a)^n*a^2*b^7*c*d^2*n^7*x^5 + 1236*(b*x + a)^n*a*b^8*c*d^2*n^6
*x^6 + 56*(b*x + a)^n*a^3*b^6*d^3*n^6*x^6 + 15666*(b*x + a)^n*b^9*c*d^2*n^
5*x^7 - 1400*(b*x + a)^n*a^2*b^7*d^3*n^5*x^7 + 6769*(b*x + a)^n*a*b^8*d^3*
n^4*x^8 + 67284*(b*x + a)^n*b^9*d^3*n^3*x^9 + (b*x + a)^n*b^9*c^3*n^8*x^3
+ 108*(b*x + a)^n*a*b^8*c^2*d*n^7*x^4 + 2010*(b*x + a)^n*b^9*c^2*d*n^6*x^5
 - 486*(b*x + a)^n*a^2*b^7*c*d^2*n^6*x^5 + 8250*(b*x + a)^n*a*b^8*c*d^2*n^
5*x^6 + 840*(b*x + a)^n*a^3*b^6*d^3*n^5*x^6 + 80157*(b*x + a)^n*b^9*c*d^2*
n^4*x^7 - 5880*(b*x + a)^n*a^2*b^7*d^3*n^4*x^7 + 13132*(b*x + a)^n*a*b^8*d
^3*n^3*x^8 + 118124*(b*x + a)^n*b^9*d^3*n^2*x^9 + (b*x + a)^n*a*b^8*c^3*n^
8*x^2 + 42*(b*x + a)^n*b^9*c^3*n^7*x^3 - 12*(b*x + a)^n*a^2*b^7*c^2*d*n^7*
x^3 + 1578*(b*x + a)^n*a*b^8*c^2*d*n^6*x^4 + 90*(b*x + a)^n*a^3*b^6*c*d...
```

3.359.9 Mupad [B] (verification not implemented)

Time = 12.66 (sec) , antiderivative size = 1796, normalized size of antiderivative = 5.24

$$\int x^2(a+bx)^n\left(c+dx^2\right)^3\,dx=\text{Too large to display}$$

input
```
int(x^2*(c + d*x^2)^3*(a + b*x)^n,x)
```

output
```
(d^3*x^9*(a + b*x)^n*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536
*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))/(1026576*n + 1172700*n^2 + 723680*
n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)
 + (2*a^3*(a + b*x)^n*(20160*a^6*d^3 + 60480*b^6*c^3 + 60216*b^6*c^3*n + 2
4574*b^6*c^3*n^2 + 5265*b^6*c^3*n^3 + 625*b^6*c^3*n^4 + 39*b^6*c^3*n^5 + b
^6*c^3*n^6 + 108864*a^2*b^4*c^2*d + 77760*a^4*b^2*c*d^2 + 59400*a^2*b^4*c^
2*d*n + 18360*a^4*b^2*c*d^2*n + 12060*a^2*b^4*c^2*d*n^2 + 1080*a^4*b^2*c*d
^2*n^2 + 1080*a^2*b^4*c^2*d*n^3 + 36*a^2*b^4*c^2*d*n^4))/(b^9*(1026576*n +
 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 +
45*n^8 + n^9 + 362880)) - (x^3*(a + b*x)^n*(3*n + n^2 + 2)*(6720*a^6*d^3*n
 - 60480*b^6*c^3 - 60216*b^6*c^3*n - 24574*b^6*c^3*n^2 - 5265*b^6*c^3*n^3
- 625*b^6*c^3*n^4 - 39*b^6*c^3*n^5 - b^6*c^3*n^6 + 36288*a^2*b^4*c^2*d*n +
 25920*a^4*b^2*c*d^2*n + 19800*a^2*b^4*c^2*d*n^2 + 6120*a^4*b^2*c*d^2*n^2
+ 4020*a^2*b^4*c^2*d*n^3 + 360*a^4*b^2*c*d^2*n^3 + 360*a^2*b^4*c^2*d*n^4 +
 12*a^2*b^4*c^2*d*n^5))/(b^6*(1026576*n + 1172700*n^2 + 723680*n^3 + 26932
5*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (3*d*x^
5*(a + b*x)^n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)*(3024*b^4*c^2 - 112*a^4*
d^2*n + 1650*b^4*c^2*n + 335*b^4*c^2*n^2 + 30*b^4*c^2*n^3 + b^4*c^2*n^4 -
432*a^2*b^2*c*d*n - 102*a^2*b^2*c*d*n^2 - 6*a^2*b^2*c*d*n^3))/(b^4*(102657
6*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 87...
```

3.359. $\int x^2(a+bx)^n \left(c+dx^2\right)^3 \, dx$

3.360 $\int x(a+bx)^n (c+dx^2)^3 \, dx$

3.360.1 Optimal result

Integrand size = 18, antiderivative size = 282

$$\begin{aligned}\int x(a+bx)^n (c+dx^2)^3 \, dx = &-\frac{a(b^2c+a^2d)^3 (a+bx)^{1+n}}{b^8(1+n)} \\ &+ \frac{(b^2c+a^2d)^2 (b^2c+7a^2d) (a+bx)^{2+n}}{b^8(2+n)} \\ &- \frac{3ad(b^2c+a^2d) (3b^2c+7a^2d) (a+bx)^{3+n}}{b^8(3+n)} \\ &+ \frac{d(3b^4c^2+30a^2b^2cd+35a^4d^2) (a+bx)^{4+n}}{b^8(4+n)} \\ &- \frac{5ad^2(3b^2c+7a^2d) (a+bx)^{5+n}}{b^8(5+n)} \\ &+ \frac{3d^2(b^2c+7a^2d) (a+bx)^{6+n}}{b^8(6+n)} \\ &- \frac{7ad^3(a+bx)^{7+n}}{b^8(7+n)} + \frac{d^3(a+bx)^{8+n}}{b^8(8+n)}\end{aligned}$$

output
```
-a*(a^2*d+b^2*c)^3*(b*x+a)^(1+n)/b^8/(1+n)+(a^2*d+b^2*c)^2*(7*a^2*d+b^2*c)
*(b*x+a)^(2+n)/b^8/(2+n)-3*a*d*(a^2*d+b^2*c)*(7*a^2*d+3*b^2*c)*(b*x+a)^(3+
n)/b^8/(3+n)+d*(35*a^4*d^2+30*a^2*b^2*c*d+3*b^4*c^2)*(b*x+a)^(4+n)/b^8/(4+
n)-5*a*d^2*(7*a^2*d+3*b^2*c)*(b*x+a)^(5+n)/b^8/(5+n)+3*d^2*(7*a^2*d+b^2*c)
*(b*x+a)^(6+n)/b^8/(6+n)-7*a*d^3*(b*x+a)^(7+n)/b^8/(7+n)+d^3*(b*x+a)^(8+n)
/b^8/(8+n)
```

3.360.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 709 vs. $2(282) = 564$.

Time = 0.87 (sec) , antiderivative size = 709, normalized size of antiderivative = 2.51

$$\int x(a+bx)^n \left(c+dx^2\right)^3 dx$$
$$= \frac{(a+bx)^{1+n} \left(b^6(1+n)(2+n)(3+n)(4+n)(5+n)(6+n)(7+n)(a+bx)\left(c+dx^2\right)^3 - a(8+n)\left(b^6(1+\right.\right.}{}$$

input
```
Integrate[x*(a + b*x)^n*(c + d*x^2)^3,x]
```

output
```
((a + b*x)^(1 + n)*(b^6*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(7
 + n)*(a + b*x)*(c + d*x^2)^3 - a*(8 + n)*(b^6*(1 + n)*(2 + n)*(3 + n)*(4
+ n)*(5 + n)*(6 + n)*(c + d*x^2)^3 + 6*(b^2*c + a^2*d)*(6 + n)*(b^4*(1 + n
)*(2 + n)*(3 + n)*(4 + n)*(c + d*x^2)^2 + 4*(b^2*c + a^2*d)*(4 + n)*(2*a^2
*d - 2*a*b*d*(1 + n)*x + b^2*(2 + n)*(c*(3 + n) + d*(1 + n)*x^2)) - 4*a*d*
(1 + n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(3 + n)*(c*(4 + n) +
d*(2 + n)*x^2))) - 6*a*d*(1 + n)*(a + b*x)*(b^4*(2 + n)*(3 + n)*(4 + n)*(5
 + n)*(c + d*x^2)^2 + 4*(b^2*c + a^2*d)*(5 + n)*(2*a^2*d - 2*a*b*d*(2 + n)
*x + b^2*(3 + n)*(c*(4 + n) + d*(2 + n)*x^2)) - 4*a*d*(2 + n)*(a + b*x)*(2
*a^2*d - 2*a*b*d*(3 + n)*x + b^2*(4 + n)*(c*(5 + n) + d*(3 + n)*x^2))) +
6*(1 + n)*(a + b*x)*((b^2*c + a^2*d)*(7 + n)*(b^4*(2 + n)*(3 + n)*(4 + n)*
(5 + n)*(c + d*x^2)^2 + 4*(b^2*c + a^2*d)*(5 + n)*(2*a^2*d - 2*a*b*d*(2 +
n)*x + b^2*(3 + n)*(c*(4 + n) + d*(2 + n)*x^2)) - 4*a*d*(2 + n)*(a + b*x)*
(2*a^2*d - 2*a*b*d*(3 + n)*x + b^2*(4 + n)*(c*(5 + n) + d*(3 + n)*x^2))) -
 a*d*(2 + n)*(a + b*x)*(b^4*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(c + d*x^2)^2
+ 4*(b^2*c + a^2*d)*(6 + n)*(2*a^2*d - 2*a*b*d*(3 + n)*x + b^2*(4 + n)*(c*
(5 + n) + d*(3 + n)*x^2)) - 4*a*d*(3 + n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(4
+ n)*x + b^2*(5 + n)*(c*(6 + n) + d*(4 + n)*x^2))))))/(b^8*(1 + n)*(2 + n)
*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(7 + n)*(8 + n))
```

3.360.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c+dx^2\right)^3(a+bx)^n\,dx$$

$$\Big\downarrow \ 522$$

$$\int \left(-\frac{5ad^2\left(7a^2d+3b^2c\right)(a+bx)^{n+4}}{b^7}+\frac{3d^2\left(7a^2d+b^2c\right)(a+bx)^{n+5}}{b^7}-\frac{a\left(a^2d+b^2c\right)^3(a+bx)^n}{b^7}+\frac{\left(a^2d+b^2c\right)^2\left(\right.}{}$$

$$\Big\downarrow \ 2009$$

$$-\frac{5ad^2\left(7a^2d+3b^2c\right)(a+bx)^{n+5}}{b^8(n+5)}+\frac{3d^2\left(7a^2d+b^2c\right)(a+bx)^{n+6}}{b^8(n+6)}-\frac{a\left(a^2d+b^2c\right)^3(a+bx)^{n+1}}{b^8(n+1)}+\frac{\left(a^2d+b^2c\right)^2\left(7a^2d+b^2c\right)(a+bx)^{n+2}}{b^8(n+2)}-\frac{3ad\left(a^2d+b^2c\right)\left(7a^2d+3b^2c\right)(a+bx)^{n+3}}{b^8(n+3)}+\frac{d\left(35a^4d^2+30a^2b^2cd+3b^4c^2\right)(a+bx)^{n+4}}{b^8(n+4)}-\frac{7ad^3(a+bx)^{n+7}}{b^8(n+7)}+\frac{d^3(a+bx)^{n+8}}{b^8(n+8)}$$

input
```
Int[x*(a + b*x)^n*(c + d*x^2)^3,x]
```

output
```
-((a*(b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^8*(1 + n))) + ((b^2*c + a^2*d
)^2*(b^2*c + 7*a^2*d)*(a + b*x)^(2 + n))/(b^8*(2 + n)) - (3*a*d*(b^2*c + a
^2*d)*(3*b^2*c + 7*a^2*d)*(a + b*x)^(3 + n))/(b^8*(3 + n)) + (d*(3*b^4*c^2
 + 30*a^2*b^2*c*d + 35*a^4*d^2)*(a + b*x)^(4 + n))/(b^8*(4 + n)) - (5*a*d^
2*(3*b^2*c + 7*a^2*d)*(a + b*x)^(5 + n))/(b^8*(5 + n)) + (3*d^2*(b^2*c + 7
*a^2*d)*(a + b*x)^(6 + n))/(b^8*(6 + n)) - (7*a*d^3*(a + b*x)^(7 + n))/(b^
8*(7 + n)) + (d^3*(a + b*x)^(8 + n))/(b^8*(8 + n))
```

3.360.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.360.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1213 vs. $2(282) = 564$.

Time = 0.48 (sec) , antiderivative size = 1214, normalized size of antiderivative = 4.30

method	result	size
norman	Expression too large to display	1214
gosper	Expression too large to display	1639
risch	Expression too large to display	1955
parallelrisch	Expression too large to display	2917

input
```
int(x*(b*x+a)^n*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
d^3/(8+n)*x^8*exp(n*ln(b*x+a))+1/b^7*n*a*(b^6*c^3*n^6+33*b^6*c^3*n^5+18*a^
2*b^4*c^2*d*n^4+445*b^6*c^3*n^4+468*a^2*b^4*c^2*d*n^3+3135*b^6*c^3*n^3+360
*a^4*b^2*c*d^2*n^2+4518*a^2*b^4*c^2*d*n^2+12154*b^6*c^3*n^2+5400*a^4*b^2*c
*d^2*n+19188*a^2*b^4*c^2*d*n+24552*b^6*c^3*n+5040*a^6*d^3+20160*a^4*b^2*c*
d^2+30240*a^2*b^4*c^2*d+20160*b^6*c^3)/(n^8+36*n^7+546*n^6+4536*n^5+22449*
n^4+67284*n^3+118124*n^2+109584*n+40320)*x*exp(n*ln(b*x+a))+n/b*d^3*a/(n^2
+15*n+56)*x^7*exp(n*ln(b*x+a))-a^2*(b^6*c^3*n^6+33*b^6*c^3*n^5+18*a^2*b^4*
c^2*d*n^4+445*b^6*c^3*n^4+468*a^2*b^4*c^2*d*n^3+3135*b^6*c^3*n^3+360*a^4*b
^2*c*d^2*n^2+4518*a^2*b^4*c^2*d*n^2+12154*b^6*c^3*n^2+5400*a^4*b^2*c*d^2*n
+19188*a^2*b^4*c^2*d*n+24552*b^6*c^3*n+5040*a^6*d^3+20160*a^4*b^2*c*d^2+30
240*a^2*b^4*c^2*d+20160*b^6*c^3)/b^8/(n^8+36*n^7+546*n^6+4536*n^5+22449*n^
4+67284*n^3+118124*n^2+109584*n+40320)*exp(n*ln(b*x+a))-(-b^6*c^3*n^6+9*a^
2*b^4*c^2*d*n^5-33*b^6*c^3*n^5+234*a^2*b^4*c^2*d*n^4-445*b^6*c^3*n^4+180*a
^4*b^2*c*d^2*n^3+2259*a^2*b^4*c^2*d*n^3-3135*b^6*c^3*n^3+2700*a^4*b^2*c*d^
2*n^2+9594*a^2*b^4*c^2*d*n^2-12154*b^6*c^3*n^2+2520*a^6*d^3*n+10080*a^4*b^
2*c*d^2*n+15120*a^2*b^4*c^2*d*n-24552*b^6*c^3*n-20160*b^6*c^3)/b^6/(n^7+35
*n^6+511*n^5+4025*n^4+18424*n^3+48860*n^2+69264*n+40320)*x^2*exp(n*ln(b*x+
a))-(-3*b^2*c*n^2+7*a^2*d*n-45*b^2*c*n-168*b^2*c)/b^2*d^2/(n^3+21*n^2+146*
n+336)*x^6*exp(n*ln(b*x+a))-3*(-b^4*c^2*n^4+5*a^2*b^2*c*d*n^3-26*b^4*c^2*n
^3+75*a^2*b^2*c*d*n^2-251*b^4*c^2*n^2+70*a^4*d^2*n+280*a^2*b^2*c*d*n-10...
```

3.360.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1675 vs. $2(282) = 564$.

Time = 0.31 (sec) , antiderivative size = 1675, normalized size of antiderivative = 5.94

$$\int x(a+bx)^n \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(x*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
-(a^2*b^6*c^3*n^6 + 33*a^2*b^6*c^3*n^5 + 20160*a^2*b^6*c^3 + 30240*a^4*b^4
*c^2*d + 20160*a^6*b^2*c*d^2 + 5040*a^8*d^3 - (b^8*d^3*n^7 + 28*b^8*d^3*n^
6 + 322*b^8*d^3*n^5 + 1960*b^8*d^3*n^4 + 6769*b^8*d^3*n^3 + 13132*b^8*d^3*
n^2 + 13068*b^8*d^3*n + 5040*b^8*d^3)*x^8 - (a*b^7*d^3*n^7 + 21*a*b^7*d^3*
n^6 + 175*a*b^7*d^3*n^5 + 735*a*b^7*d^3*n^4 + 1624*a*b^7*d^3*n^3 + 1764*a*
b^7*d^3*n^2 + 720*a*b^7*d^3*n)*x^7 - (3*b^8*c*d^2*n^7 + 20160*b^8*c*d^2 +
(90*b^8*c*d^2 - 7*a^2*b^6*d^3)*n^6 + 3*(366*b^8*c*d^2 - 35*a^2*b^6*d^3)*n^
5 + 5*(1404*b^8*c*d^2 - 119*a^2*b^6*d^3)*n^4 + 9*(2803*b^8*c*d^2 - 175*a^2
*b^6*d^3)*n^3 + 2*(25245*b^8*c*d^2 - 959*a^2*b^6*d^3)*n^2 + 24*(2143*b^8*c
*d^2 - 35*a^2*b^6*d^3)*n)*x^6 - 3*(a*b^7*c*d^2*n^7 + 25*a*b^7*c*d^2*n^6 +
(241*a*b^7*c*d^2 + 14*a^3*b^5*d^3)*n^5 + 5*(227*a*b^7*c*d^2 + 28*a^3*b^5*d
^3)*n^4 + 2*(1367*a*b^7*c*d^2 + 245*a^3*b^5*d^3)*n^3 + 20*(158*a*b^7*c*d^2
 + 35*a^3*b^5*d^3)*n^2 + 336*(4*a*b^7*c*d^2 + a^3*b^5*d^3)*n)*x^5 + (445*a
^2*b^6*c^3 + 18*a^4*b^4*c^2*d)*n^4 - 3*(b^8*c^2*d*n^7 + 10080*b^8*c^2*d +
(32*b^8*c^2*d - 5*a^2*b^6*c*d^2)*n^6 + (418*b^8*c^2*d - 105*a^2*b^6*c*d^2)
*n^5 + (2864*b^8*c^2*d - 785*a^2*b^6*c*d^2 - 70*a^4*b^4*d^3)*n^4 + (10993*
b^8*c^2*d - 2535*a^2*b^6*c*d^2 - 420*a^4*b^4*d^3)*n^3 + 2*(11656*b^8*c^2*d
 - 1765*a^2*b^6*c*d^2 - 385*a^4*b^4*d^3)*n^2 + 12*(2073*b^8*c^2*d - 140*a^
2*b^6*c*d^2 - 35*a^4*b^4*d^3)*n)*x^4 + 3*(1045*a^2*b^6*c^3 + 156*a^4*b^4*c
^2*d)*n^3 - 3*(a*b^7*c^2*d*n^7 + 29*a*b^7*c^2*d*n^6 + (331*a*b^7*c^2*d ...
```

3.360.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24687 vs. $2(265) = 530$.

Time = 6.42 (sec) , antiderivative size = 24687, normalized size of antiderivative = 87.54

$$\int x(a+bx)^n \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(x*(b*x+a)**n*(d*x**2+c)**3,x)
```

output
```
Piecewise((a**n*(c**3*x**2/2 + 3*c**2*d*x**4/4 + c*d**2*x**6/2 + d**3*x**8
/8), Eq(b, 0)), (420*a**7*d**3*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**
9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4
 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 1089*a**7*
d**3/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4
*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*
x**6 + 420*b**15*x**7) + 2940*a**6*b*d**3*x*log(a/b + x)/(420*a**7*b**8 +
2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a*
*3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7)
 + 7203*a**6*b*d**3*x/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*
x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**
5 + 2940*a*b**14*x**6 + 420*b**15*x**7) - 30*a**5*b**2*c*d**2/(420*a**7*b*
*8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 147
00*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*
x**7) + 8820*a**5*b**2*d**3*x**2*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b
**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x*
*4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 20139*a*
*5*b**2*d**3*x**2/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2
 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 +
2940*a*b**14*x**6 + 420*b**15*x**7) - 210*a**4*b**3*c*d**2*x/(420*a**7*...
```

3.360.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(282) = 564$.

Time = 0.23 (sec) , antiderivative size = 625, normalized size of antiderivative = 2.22

$$\int x(a+bx)^n\left(c+dx^2\right)^3\,dx=\frac{(b^2(n+1)x^2+abnx-a^2)(bx+a)^nc^3}{(n^2+3\,n+2)b^2}$$
$$+\frac{3\,((n^3+6\,n^2+11\,n+6)b^4x^4+(n^3+3\,n^2+2\,n)ab^3x^3-3\,(n^2+n)a^2b^2x^2+6\,a^3bnx-6\,a^4)(bx+a)^nc^2}{(n^4+10\,n^3+35\,n^2+50\,n+24)b^4}$$
$$+\frac{3\,((n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)b^6x^6+(n^5+10\,n^4+35\,n^3+50\,n^2+24\,n)ab^5x^5-5\,(n^4}{(n^6+21\,n^5+175\,n^4+7}$$
$$+\frac{((n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040)b^8x^8+(n^7+21\,n^6+175\,n^5+}{}$$

input
```
integrate(x*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
(b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^3/((n^2 + 3*n + 2)*b^2) +
3*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n
^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n*c^2*d/((n^4 + 10*n^
3 + 35*n^2 + 50*n + 24)*b^4) + 3*((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n
 + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*x^5 - 5*(n
^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*x^
3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*c*d^2/
((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6) + ((n^7
 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^
8*x^8 + (n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a
*b^7*x^7 - 7*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^2*b^6*x
^6 + 42*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^3*b^5*x^5 - 210*(n^4 + 6
*n^3 + 11*n^2 + 6*n)*a^4*b^4*x^4 + 840*(n^3 + 3*n^2 + 2*n)*a^5*b^3*x^3 - 2
520*(n^2 + n)*a^6*b^2*x^2 + 5040*a^7*b*n*x - 5040*a^8)*(b*x + a)^n*d^3/((n
^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n^2 + 10
9584*n + 40320)*b^8)
```

3.360.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2851 vs. $2(282) = 564$.

Time = 0.31 (sec) , antiderivative size = 2851, normalized size of antiderivative = 10.11

$$\int x(a+bx)^n \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(x*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
((b*x + a)^n*b^8*d^3*n^7*x^8 + (b*x + a)^n*a*b^7*d^3*n^7*x^7 + 28*(b*x + a
)^n*b^8*d^3*n^6*x^8 + 3*(b*x + a)^n*b^8*c*d^2*n^7*x^6 + 21*(b*x + a)^n*a*b
^7*d^3*n^6*x^7 + 322*(b*x + a)^n*b^8*d^3*n^5*x^8 + 3*(b*x + a)^n*a*b^7*c*d
^2*n^7*x^5 + 90*(b*x + a)^n*b^8*c*d^2*n^6*x^6 - 7*(b*x + a)^n*a^2*b^6*d^3*
n^6*x^6 + 175*(b*x + a)^n*a*b^7*d^3*n^5*x^7 + 1960*(b*x + a)^n*b^8*d^3*n^4
*x^8 + 3*(b*x + a)^n*b^8*c^2*d*n^7*x^4 + 75*(b*x + a)^n*a*b^7*c*d^2*n^6*x^
5 + 1098*(b*x + a)^n*b^8*c*d^2*n^5*x^6 - 105*(b*x + a)^n*a^2*b^6*d^3*n^5*x
^6 + 735*(b*x + a)^n*a*b^7*d^3*n^4*x^7 + 6769*(b*x + a)^n*b^8*d^3*n^3*x^8
+ 3*(b*x + a)^n*a*b^7*c^2*d*n^7*x^3 + 96*(b*x + a)^n*b^8*c^2*d*n^6*x^4 - 1
5*(b*x + a)^n*a^2*b^6*c*d^2*n^6*x^4 + 723*(b*x + a)^n*a*b^7*c*d^2*n^5*x^5
+ 42*(b*x + a)^n*a^3*b^5*d^3*n^5*x^5 + 7020*(b*x + a)^n*b^8*c*d^2*n^4*x^6
- 595*(b*x + a)^n*a^2*b^6*d^3*n^4*x^6 + 1624*(b*x + a)^n*a*b^7*d^3*n^3*x^7
 + 13132*(b*x + a)^n*b^8*d^3*n^2*x^8 + (b*x + a)^n*b^8*c^3*n^7*x^2 + 87*(b
*x + a)^n*a*b^7*c^2*d*n^6*x^3 + 1254*(b*x + a)^n*b^8*c^2*d*n^5*x^4 - 315*(
b*x + a)^n*a^2*b^6*c*d^2*n^5*x^4 + 3405*(b*x + a)^n*a*b^7*c*d^2*n^4*x^5 +
420*(b*x + a)^n*a^3*b^5*d^3*n^4*x^5 + 25227*(b*x + a)^n*b^8*c*d^2*n^3*x^6
- 1575*(b*x + a)^n*a^2*b^6*d^3*n^3*x^6 + 1764*(b*x + a)^n*a*b^7*d^3*n^2*x^
7 + 13068*(b*x + a)^n*b^8*d^3*n*x^8 + (b*x + a)^n*a*b^7*c^3*n^7*x + 34*(b*
x + a)^n*b^8*c^3*n^6*x^2 - 9*(b*x + a)^n*a^2*b^6*c^2*d*n^6*x^2 + 993*(b*x
+ a)^n*a*b^7*c^2*d*n^5*x^3 + 60*(b*x + a)^n*a^3*b^5*c*d^2*n^5*x^3 + 859...
```

3.360.9 Mupad [B] (verification not implemented)

Time = 12.26 (sec) , antiderivative size = 1459, normalized size of antiderivative = 5.17

$$\int x(a+bx)^n \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
int(x*(c + d*x^2)^3*(a + b*x)^n,x)
```

output
```
(d^3*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5
+ 28*n^6 + n^7 + 5040))/(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4
536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320) - (a^2*(a + b*x)^n*(5040*a^6*d^3
 + 20160*b^6*c^3 + 24552*b^6*c^3*n + 12154*b^6*c^3*n^2 + 3135*b^6*c^3*n^3
+ 445*b^6*c^3*n^4 + 33*b^6*c^3*n^5 + b^6*c^3*n^6 + 30240*a^2*b^4*c^2*d + 2
0160*a^4*b^2*c*d^2 + 19188*a^2*b^4*c^2*d*n + 5400*a^4*b^2*c*d^2*n + 4518*a
^2*b^4*c^2*d*n^2 + 360*a^4*b^2*c*d^2*n^2 + 468*a^2*b^4*c^2*d*n^3 + 18*a^2*
b^4*c^2*d*n^4))/(b^8*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536
*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (x^2*(n + 1)*(a + b*x)^n*(2520*a
^6*d^3*n - 20160*b^6*c^3 - 24552*b^6*c^3*n - 12154*b^6*c^3*n^2 - 3135*b^6*
c^3*n^3 - 445*b^6*c^3*n^4 - 33*b^6*c^3*n^5 - b^6*c^3*n^6 + 15120*a^2*b^4*c
^2*d*n + 10080*a^4*b^2*c*d^2*n + 9594*a^2*b^4*c^2*d*n^2 + 2700*a^4*b^2*c*d
^2*n^2 + 2259*a^2*b^4*c^2*d*n^3 + 180*a^4*b^2*c*d^2*n^3 + 234*a^2*b^4*c^2*
d*n^4 + 9*a^2*b^4*c^2*d*n^5))/(b^6*(109584*n + 118124*n^2 + 67284*n^3 + 22
449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (d^2*x^6*(a + b*x)
^n*(168*b^2*c + 3*b^2*c*n^2 - 7*a^2*d*n + 45*b^2*c*n)*(274*n + 225*n^2 + 8
5*n^3 + 15*n^4 + n^5 + 120))/(b^2*(109584*n + 118124*n^2 + 67284*n^3 + 224
49*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (3*d*x^4*(a + b*x)^
n*(11*n + 6*n^2 + n^3 + 6)*(1680*b^4*c^2 - 70*a^4*d^2*n + 1066*b^4*c^2*n +
 251*b^4*c^2*n^2 + 26*b^4*c^2*n^3 + b^4*c^2*n^4 - 280*a^2*b^2*c*d*n - 7...
```

3.360. $\int x(a+bx)^n \left(c+dx^2\right)^3 \, dx$

3.361 $\int (a+bx)^n (c+dx^2)^3 \, dx$

3.361.1 Optimal result

Integrand size = 17, antiderivative size = 223

$$\int (a+bx)^n (c+dx^2)^3 \, dx = \frac{(b^2c+a^2d)^3 (a+bx)^{1+n}}{b^7(1+n)} - \frac{6ad(b^2c+a^2d)^2 (a+bx)^{2+n}}{b^7(2+n)} + \frac{3d(b^2c+a^2d)(b^2c+5a^2d)(a+bx)^{3+n}}{b^7(3+n)} - \frac{4ad^2(3b^2c+5a^2d)(a+bx)^{4+n}}{b^7(4+n)} + \frac{3d^2(b^2c+5a^2d)(a+bx)^{5+n}}{b^7(5+n)} - \frac{6ad^3(a+bx)^{6+n}}{b^7(6+n)} + \frac{d^3(a+bx)^{7+n}}{b^7(7+n)}$$

output
```
(a^2*d+b^2*c)^3*(b*x+a)^(1+n)/b^7/(1+n)-6*a*d*(a^2*d+b^2*c)^2*(b*x+a)^(2+n
)/b^7/(2+n)+3*d*(a^2*d+b^2*c)*(5*a^2*d+b^2*c)*(b*x+a)^(3+n)/b^7/(3+n)-4*a*
d^2*(5*a^2*d+3*b^2*c)*(b*x+a)^(4+n)/b^7/(4+n)+3*d^2*(5*a^2*d+b^2*c)*(b*x+a
)^(5+n)/b^7/(5+n)-6*a*d^3*(b*x+a)^(6+n)/b^7/(6+n)+d^3*(b*x+a)^(7+n)/b^7/(7
+n)
```

3.361.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.56

$$\int (a+bx)^n \left(c+dx^2\right)^3 \, dx$$
$$= \frac{(a+bx)^{1+n} \left(\left(c+dx^2\right)^3 + \frac{6\left(\left(b^2c+a^2d\right)(6+n)\left(b^4(1+n)(2+n)(3+n)(4+n)\left(c+dx^2\right)^2+4\left(b^2c+a^2d\right)(4+n)\left(2a^2d-2abd(1+n)x+b^2(2+n\right.\right.\right.}{}}{}$$

input
```
Integrate[(a + b*x)^n*(c + d*x^2)^3,x]
```

output
```
((a + b*x)^(1 + n)*((c + d*x^2)^3 + (6*((b^2*c + a^2*d)*(6 + n)*(b^4*(1 +
n)*(2 + n)*(3 + n)*(4 + n)*(c + d*x^2)^2 + 4*(b^2*c + a^2*d)*(4 + n)*(2*a^
2*d - 2*a*b*d*(1 + n)*x + b^2*(2 + n)*(c*(3 + n) + d*(1 + n)*x^2)) - 4*a*d
*(1 + n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(3 + n)*(c*(4 + n) +
 d*(2 + n)*x^2))) - a*d*(1 + n)*(a + b*x)*(b^4*(2 + n)*(3 + n)*(4 + n)*(5
+ n)*(c + d*x^2)^2 + 4*(b^2*c + a^2*d)*(5 + n)*(2*a^2*d - 2*a*b*d*(2 + n)*
x + b^2*(3 + n)*(c*(4 + n) + d*(2 + n)*x^2)) - 4*a*d*(2 + n)*(a + b*x)*(2*
a^2*d - 2*a*b*d*(3 + n)*x + b^2*(4 + n)*(c*(5 + n) + d*(3 + n)*x^2)))))/(b
^6*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n))))/(b*(7 + n))
```

3.361.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^2\right)^3 (a+bx)^n \, dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(-\frac{4ad^2\left(5a^2d+3b^2c\right)(a+bx)^{n+3}}{b^6}+\frac{3d^2\left(5a^2d+b^2c\right)(a+bx)^{n+4}}{b^6}+\frac{\left(a^2d+b^2c\right)^3(a+bx)^n}{b^6}-\frac{6ad\left(a^2d+b^2c\right)}{b^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4ad^2(5a^2d+3b^2c)(a+bx)^{n+4}}{b^7(n+4)}+\frac{3d^2(5a^2d+b^2c)(a+bx)^{n+5}}{b^7(n+5)}+\frac{(a^2d+b^2c)^3(a+bx)^{n+1}}{b^7(n+1)}-\frac{6ad(a^2d+b^2c)^2(a+bx)^{n+2}}{b^7(n+2)}+\frac{3d(a^2d+b^2c)(5a^2d+b^2c)(a+bx)^{n+3}}{b^7(n+3)}-\frac{6ad^3(a+bx)^{n+6}}{b^7(n+6)}+\frac{d^3(a+bx)^{n+7}}{b^7(n+7)}$$

input
```
Int[(a + b*x)^n*(c + d*x^2)^3,x]
```

output
```
((b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^7*(1 + n)) - (6*a*d*(b^2*c + a^2*
d)^2*(a + b*x)^(2 + n))/(b^7*(2 + n)) + (3*d*(b^2*c + a^2*d)*(b^2*c + 5*a^
2*d)*(a + b*x)^(3 + n))/(b^7*(3 + n)) - (4*a*d^2*(3*b^2*c + 5*a^2*d)*(a +
b*x)^(4 + n))/(b^7*(4 + n)) + (3*d^2*(b^2*c + 5*a^2*d)*(a + b*x)^(5 + n))/
(b^7*(5 + n)) - (6*a*d^3*(a + b*x)^(6 + n))/(b^7*(6 + n)) + (d^3*(a + b*x)
^(7 + n))/(b^7*(7 + n))
```

3.361.3.1 Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.361.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 957 vs. $2(223) = 446$.

Time = 0.44 (sec) , antiderivative size = 958, normalized size of antiderivative = 4.30

method	result
norman	$\frac{d^3x^7\mathrm{e}^{n\ln(bx+a)}}{7+n}+\frac{a\left(b^6c^3n^6+27b^6c^3n^5+6a^2b^4c^2d\,n^4+295b^6c^3n^4+132a^2b^4c^2d\,n^3+1665b^6c^3n^3+72a^4b^2c\,d^2n^2+1074a^2b^4c^2 \ldots}{b^7(n^7+28n^6+322n^5+19 \ldots}$
gosper	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((b*x+a)^n*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
d^3/(7+n)*x^7*exp(n*ln(b*x+a))+a*(b^6*c^3*n^6+27*b^6*c^3*n^5+6*a^2*b^4*c^2
*d*n^4+295*b^6*c^3*n^4+132*a^2*b^4*c^2*d*n^3+1665*b^6*c^3*n^3+72*a^4*b^2*c
*d^2*n^2+1074*a^2*b^4*c^2*d*n^2+5104*b^6*c^3*n^2+936*a^4*b^2*c*d^2*n+3828*
a^2*b^4*c^2*d*n+8028*b^6*c^3*n+720*a^6*d^3+3024*a^4*b^2*c*d^2+5040*a^2*b^4
*c^2*d+5040*b^6*c^3)/b^7/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+1
3068*n+5040)*exp(n*ln(b*x+a))+a*d^3*n/b/(n^2+13*n+42)*x^6*exp(n*ln(b*x+a))
-(-b^6*c^3*n^6+6*a^2*b^4*c^2*d*n^5-27*b^6*c^3*n^5+132*a^2*b^4*c^2*d*n^4-29
5*b^6*c^3*n^4+72*a^4*b^2*c*d^2*n^3+1074*a^2*b^4*c^2*d*n^3-1665*b^6*c^3*n^3
+936*a^4*b^2*c*d^2*n^2+3828*a^2*b^4*c^2*d*n^2-5104*b^6*c^3*n^2+720*a^6*d^3
*n+3024*a^4*b^2*c*d^2*n+5040*a^2*b^4*c^2*d*n-8028*b^6*c^3*n-5040*b^6*c^3)/
b^6/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*n+5040)*x*exp(n*
ln(b*x+a))-3*(-b^2*c*n^2+2*a^2*d*n-13*b^2*c*n-42*b^2*c)/b^2*d^2/(n^3+18*n^
2+107*n+210)*x^5*exp(n*ln(b*x+a))-3*(-b^4*c^2*n^4+4*a^2*b^2*c*d*n^3-22*b^4
*c^2*n^3+52*a^2*b^2*c*d*n^2-179*b^4*c^2*n^2+40*a^4*d^2*n+168*a^2*b^2*c*d*n
-638*b^4*c^2*n-840*b^4*c^2)/b^4*d/(n^5+25*n^4+245*n^3+1175*n^2+2754*n+2520
)*x^3*exp(n*ln(b*x+a))+3*(b^2*c*n^2+13*b^2*c*n+10*a^2*d+42*b^2*c)*d^2*a/b^
3*n/(n^4+22*n^3+179*n^2+638*n+840)*x^4*exp(n*ln(b*x+a))+3*(b^4*c^2*n^4+22*
b^4*c^2*n^3+12*a^2*b^2*c*d*n^2+179*b^4*c^2*n^2+156*a^2*b^2*c*d*n+638*b^4*c
^2*n+120*a^4*d^2+504*a^2*b^2*c*d+840*b^4*c^2)*d*a/b^5*n/(n^6+27*n^5+295*n^
4+1665*n^3+5104*n^2+8028*n+5040)*x^2*exp(n*ln(b*x+a))
```

3.361.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1244 vs. $2(223) = 446$.

Time = 0.27 (sec) , antiderivative size = 1244, normalized size of antiderivative = 5.58

$$\int (a+bx)^n \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^n*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
(a*b^6*c^3*n^6 + 27*a*b^6*c^3*n^5 + 5040*a*b^6*c^3 + 5040*a^3*b^4*c^2*d +
3024*a^5*b^2*c*d^2 + 720*a^7*d^3 + (b^7*d^3*n^6 + 21*b^7*d^3*n^5 + 175*b^7
*d^3*n^4 + 735*b^7*d^3*n^3 + 1624*b^7*d^3*n^2 + 1764*b^7*d^3*n + 720*b^7*d
^3)*x^7 + (a*b^6*d^3*n^6 + 15*a*b^6*d^3*n^5 + 85*a*b^6*d^3*n^4 + 225*a*b^6
*d^3*n^3 + 274*a*b^6*d^3*n^2 + 120*a*b^6*d^3*n)*x^6 + 3*(b^7*c*d^2*n^6 + 1
008*b^7*c*d^2 + (23*b^7*c*d^2 - 2*a^2*b^5*d^3)*n^5 + (207*b^7*c*d^2 - 20*a
^2*b^5*d^3)*n^4 + 5*(185*b^7*c*d^2 - 14*a^2*b^5*d^3)*n^3 + 4*(536*b^7*c*d^
2 - 25*a^2*b^5*d^3)*n^2 + 12*(201*b^7*c*d^2 - 4*a^2*b^5*d^3)*n)*x^5 + (295
*a*b^6*c^3 + 6*a^3*b^4*c^2*d)*n^4 + 3*(a*b^6*c*d^2*n^6 + 19*a*b^6*c*d^2*n^
5 + (131*a*b^6*c*d^2 + 10*a^3*b^4*d^3)*n^4 + (401*a*b^6*c*d^2 + 60*a^3*b^4
*d^3)*n^3 + 10*(54*a*b^6*c*d^2 + 11*a^3*b^4*d^3)*n^2 + 12*(21*a*b^6*c*d^2
+ 5*a^3*b^4*d^3)*n)*x^4 + 3*(555*a*b^6*c^3 + 44*a^3*b^4*c^2*d)*n^3 + 3*(b^
7*c^2*d*n^6 + 1680*b^7*c^2*d + (25*b^7*c^2*d - 4*a^2*b^5*c*d^2)*n^5 + (247
*b^7*c^2*d - 64*a^2*b^5*c*d^2)*n^4 + (1219*b^7*c^2*d - 332*a^2*b^5*c*d^2 -
 40*a^4*b^3*d^3)*n^3 + 8*(389*b^7*c^2*d - 76*a^2*b^5*c*d^2 - 15*a^4*b^3*d^
3)*n^2 + 4*(949*b^7*c^2*d - 84*a^2*b^5*c*d^2 - 20*a^4*b^3*d^3)*n)*x^3 + 2*
(2552*a*b^6*c^3 + 537*a^3*b^4*c^2*d + 36*a^5*b^2*c*d^2)*n^2 + 3*(a*b^6*c^2
*d*n^6 + 23*a*b^6*c^2*d*n^5 + 3*(67*a*b^6*c^2*d + 4*a^3*b^4*c*d^2)*n^4 + (
817*a*b^6*c^2*d + 168*a^3*b^4*c*d^2)*n^3 + 2*(739*a*b^6*c^2*d + 330*a^3*b^
4*c*d^2 + 60*a^5*b^2*d^3)*n^2 + 24*(35*a*b^6*c^2*d + 21*a^3*b^4*c*d^2 +...
```

3.361.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15990 vs. 2(207) = 414.

Time = 4.08 (sec) , antiderivative size = 15990, normalized size of antiderivative = 71.70

$$\int (a+bx)^n \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**n*(d*x**2+c)**3,x)
```

output
```
Piecewise((a**n*(c**3*x + c**2*d*x**3 + 3*c*d**2*x**5/5 + d**3*x**7/7), Eq
(b, 0)), (60*a**6*d**3*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*
a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*
x**5 + 60*b**13*x**6) + 147*a**6*d**3/(60*a**6*b**7 + 360*a**5*b**8*x + 90
0*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**1
2*x**5 + 60*b**13*x**6) + 360*a**5*b*d**3*x*log(a/b + x)/(60*a**6*b**7 + 3
60*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**1
1*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 822*a**5*b*d**3*x/(60*a**6*b*
*7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**
2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 6*a**4*b**2*c*d**2/(60*
a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 +
900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 900*a**4*b**2*d*
*3*x**2*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2
+ 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13
*x**6) + 1875*a**4*b**2*d**3*x**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a*
*4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x*
*5 + 60*b**13*x**6) - 36*a**3*b**3*c*d**2*x/(60*a**6*b**7 + 360*a**5*b**8*
x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*
a*b**12*x**5 + 60*b**13*x**6) + 1200*a**3*b**3*d**3*x**3*log(a/b + x)/(60*
a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3...
```

3.361.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(223) = 446$.

Time = 0.21 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.12

$$\int (a+bx)^n \left(c+dx^2\right)^3 \,dx$$

$$= \frac{(bx+a)^{n+1}c^3}{b(n+1)} + \frac{3\left((n^2+3n+2)b^3x^3+(n^2+n)ab^2x^2-2a^2bnx+2a^3\right)(bx+a)^n c^2 d}{(n^3+6n^2+11n+6)b^3} + \frac{3\left((n^4+10n^3+35n^2+50n+24)b^5x^5+(n^4+6n^3+11n^2+6n)ab^4x^4-4(n^3+3n^2+2n)a^2b^3x^3+\dots\right.}{(n^5+15n^4+85n^3+225n^2+274n+120)b^5} + \frac{\left((n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)b^7x^7+(n^6+15n^5+85n^4+225n^3+274n^2\dots\right.}{\dots}$$

input
```
integrate((b*x+a)^n*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
(b*x + a)^(n + 1)*c^3/(b*(n + 1)) + 3*((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)
*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x + a)^n*c^2*d/((n^3 + 6*n^2 + 11*n +
 6)*b^3) + 3*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 +
 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n
)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*c*d^2/((n^5 + 15*n^4 +
85*n^3 + 225*n^2 + 274*n + 120)*b^5) + ((n^6 + 21*n^5 + 175*n^4 + 735*n^3
+ 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 27
4*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^2*b
^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b^4*x^4 - 120*(n^3 + 3*n^2 +
2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 - 720*a^6*b*n*x + 720*a^7)*(b
*x + a)^n*d^3/((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 + 13132*n^2 +
 13068*n + 5040)*b^7)
```

3.361.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2085 vs. $2(223) = 446$.

Time = 0.28 (sec) , antiderivative size = 2085, normalized size of antiderivative = 9.35

$$\int (a+bx)^n \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^n*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
((b*x + a)^n*b^7*d^3*n^6*x^7 + (b*x + a)^n*a*b^6*d^3*n^6*x^6 + 21*(b*x + a
)^n*b^7*d^3*n^5*x^7 + 3*(b*x + a)^n*b^7*c*d^2*n^6*x^5 + 15*(b*x + a)^n*a*b
^6*d^3*n^5*x^6 + 175*(b*x + a)^n*b^7*d^3*n^4*x^7 + 3*(b*x + a)^n*a*b^6*c*d
^2*n^6*x^4 + 69*(b*x + a)^n*b^7*c*d^2*n^5*x^5 - 6*(b*x + a)^n*a^2*b^5*d^3*
n^5*x^5 + 85*(b*x + a)^n*a*b^6*d^3*n^4*x^6 + 735*(b*x + a)^n*b^7*d^3*n^3*x
^7 + 3*(b*x + a)^n*b^7*c^2*d*n^6*x^3 + 57*(b*x + a)^n*a*b^6*c*d^2*n^5*x^4
+ 621*(b*x + a)^n*b^7*c*d^2*n^4*x^5 - 60*(b*x + a)^n*a^2*b^5*d^3*n^4*x^5 +
 225*(b*x + a)^n*a*b^6*d^3*n^3*x^6 + 1624*(b*x + a)^n*b^7*d^3*n^2*x^7 + 3*
(b*x + a)^n*a*b^6*c^2*d*n^6*x^2 + 75*(b*x + a)^n*b^7*c^2*d*n^5*x^3 - 12*(b
*x + a)^n*a^2*b^5*c*d^2*n^5*x^3 + 393*(b*x + a)^n*a*b^6*c*d^2*n^4*x^4 + 30
*(b*x + a)^n*a^3*b^4*d^3*n^4*x^4 + 2775*(b*x + a)^n*b^7*c*d^2*n^3*x^5 - 21
0*(b*x + a)^n*a^2*b^5*d^3*n^3*x^5 + 274*(b*x + a)^n*a*b^6*d^3*n^2*x^6 + 17
64*(b*x + a)^n*b^7*d^3*n*x^7 + (b*x + a)^n*b^7*c^3*n^6*x + 69*(b*x + a)^n*
a*b^6*c^2*d*n^5*x^2 + 741*(b*x + a)^n*b^7*c^2*d*n^4*x^3 - 192*(b*x + a)^n*
a^2*b^5*c*d^2*n^4*x^3 + 1203*(b*x + a)^n*a*b^6*c*d^2*n^3*x^4 + 180*(b*x +
a)^n*a^3*b^4*d^3*n^3*x^4 + 6432*(b*x + a)^n*b^7*c*d^2*n^2*x^5 - 300*(b*x +
 a)^n*a^2*b^5*d^3*n^2*x^5 + 120*(b*x + a)^n*a*b^6*d^3*n*x^6 + 720*(b*x + a
)^n*b^7*d^3*x^7 + (b*x + a)^n*a*b^6*c^3*n^6 + 27*(b*x + a)^n*b^7*c^3*n^5*x
 - 6*(b*x + a)^n*a^2*b^5*c^2*d*n^5*x + 603*(b*x + a)^n*a*b^6*c^2*d*n^4*x^2
 + 36*(b*x + a)^n*a^3*b^4*c*d^2*n^4*x^2 + 3657*(b*x + a)^n*b^7*c^2*d*n^...
```

3.361. $\int (a+bx)^n \left(c+dx^2\right)^3 \, dx$

3.361.9 Mupad [B] (verification not implemented)

Time = 12.11 (sec) , antiderivative size = 1144, normalized size of antiderivative = 5.13

$$\int (a+bx)^n \left(c+dx^2\right)^3 \, dx$$
$$= \frac{(a+b\,x)^n\,(720\,a^7\,d^3+72\,a^5\,b^2\,c\,d^2\,n^2+936\,a^5\,b^2\,c\,d^2\,n+3024\,a^5\,b^2\,c\,d^2+6\,a^3\,b^4\,c^2\,d\,n^4+132\,a^3\,b^4\,c^2\,d\,n^3}{b^7\,(n^7+2}$$
$$-\frac{x\,(a+b\,x)^n\,(720\,a^6\,b\,d^3\,n+72\,a^4\,b^3\,c\,d^2\,n^3+936\,a^4\,b^3\,c\,d^2\,n^2+3024\,a^4\,b^3\,c\,d^2\,n+6\,a^2\,b^5\,c^2\,d\,n^5+132\,a^2}{b^7\,(n^7+}$$
$$+\frac{d^3\,x^7\,(a+b\,x)^n\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)}{n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040}$$
$$+\frac{3\,d^2\,x^5\,(a+b\,x)^n\,(-2\,d\,a^2\,n+c\,b^2\,n^2+13\,c\,b^2\,n+42\,c\,b^2)\,(n^4+10\,n^3+35\,n^2+50\,n+24)}{b^2\,(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040)}$$
$$+\frac{3\,d\,x^3\,(a+b\,x)^n\,(n^2+3\,n+2)\,(-40\,a^4\,d^2\,n-4\,a^2\,b^2\,c\,d\,n^3-52\,a^2\,b^2\,c\,d\,n^2-168\,a^2\,b^2\,c\,d\,n+b^4\,c^2\,n^4+}{b^4\,(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+1306}$$
$$+\frac{a\,d^3\,n\,x^6\,(a+b\,x)^n\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}{b\,(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040)}$$
$$+\frac{3\,a\,d^2\,n\,x^4\,(a+b\,x)^n\,(n^3+6\,n^2+11\,n+6)\,(10\,d\,a^2+c\,b^2\,n^2+13\,c\,b^2\,n+42\,c\,b^2)}{b^3\,(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040)}$$
$$+\frac{3\,a\,d\,n\,x^2\,(n+1)\,(a+b\,x)^n\,(120\,a^4\,d^2+12\,a^2\,b^2\,c\,d\,n^2+156\,a^2\,b^2\,c\,d\,n+504\,a^2\,b^2\,c\,d+b^4\,c^2\,n^4+22\,b^4\,c}{b^5\,(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n\,+}$$

input
```
int((c + d*x^2)^3*(a + b*x)^n,x)
```

output
```
((a + b*x)^n*(720*a^7*d^3 + 5040*a*b^6*c^3 + 5040*a^3*b^4*c^2*d + 3024*a^5
*b^2*c*d^2 + 5104*a*b^6*c^3*n^2 + 1665*a*b^6*c^3*n^3 + 295*a*b^6*c^3*n^4 +
 27*a*b^6*c^3*n^5 + a*b^6*c^3*n^6 + 8028*a*b^6*c^3*n + 3828*a^3*b^4*c^2*d*
n + 936*a^5*b^2*c*d^2*n + 1074*a^3*b^4*c^2*d*n^2 + 72*a^5*b^2*c*d^2*n^2 +
132*a^3*b^4*c^2*d*n^3 + 6*a^3*b^4*c^2*d*n^4))/(b^7*(13068*n + 13132*n^2 +
6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) - (x*(a + b*x)^n*(72
0*a^6*b*d^3*n - 8028*b^7*c^3*n - 5104*b^7*c^3*n^2 - 1665*b^7*c^3*n^3 - 295
*b^7*c^3*n^4 - 27*b^7*c^3*n^5 - b^7*c^3*n^6 - 5040*b^7*c^3 + 5040*a^2*b^5*
c^2*d*n + 3024*a^4*b^3*c*d^2*n + 3828*a^2*b^5*c^2*d*n^2 + 936*a^4*b^3*c*d^
2*n^2 + 1074*a^2*b^5*c^2*d*n^3 + 72*a^4*b^3*c*d^2*n^3 + 132*a^2*b^5*c^2*d*
n^4 + 6*a^2*b^5*c^2*d*n^5))/(b^7*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^
4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (d^3*x^7*(a + b*x)^n*(1764*n + 1624*
n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))/(13068*n + 13132*n^2 + 6769
*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040) + (3*d^2*x^5*(a + b*x)^n*
(42*b^2*c + b^2*c*n^2 - 2*a^2*d*n + 13*b^2*c*n)*(50*n + 35*n^2 + 10*n^3 +
n^4 + 24))/(b^2*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*
n^6 + n^7 + 5040)) + (3*d*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(840*b^4*c^2 - 4
0*a^4*d^2*n + 638*b^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n
^4 - 168*a^2*b^2*c*d*n - 52*a^2*b^2*c*d*n^2 - 4*a^2*b^2*c*d*n^3))/(b^4*(13
068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040...
```

3.361. $\int (a+bx)^n \left(c+dx^2\right)^3 \, dx$

3.362 $\int \frac{(a+bx)^n (c+dx^2)^3}{x} \, dx$

3.362.1 Optimal result

Integrand size = 20, antiderivative size = 246

$$\begin{aligned}\int \frac{(a+bx)^n (c+dx^2)^3}{x} \, dx = & -\frac{ad(3b^4c^2+3a^2b^2cd+a^4d^2)(a+bx)^{1+n}}{b^6(1+n)} \\ & + \frac{d(3b^4c^2+9a^2b^2cd+5a^4d^2)(a+bx)^{2+n}}{b^6(2+n)} \\ & - \frac{ad^2(9b^2c+10a^2d)(a+bx)^{3+n}}{b^6(3+n)} \\ & + \frac{d^2(3b^2c+10a^2d)(a+bx)^{4+n}}{b^6(4+n)} \\ & - \frac{5ad^3(a+bx)^{5+n}}{b^6(5+n)} + \frac{d^3(a+bx)^{6+n}}{b^6(6+n)} \\ & - \frac{c^3(a+bx)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{bx}{a}\right)}{a(1+n)}\end{aligned}$$

output
```
-a*d*(a^4*d^2+3*a^2*b^2*c*d+3*b^4*c^2)*(b*x+a)^(1+n)/b^6/(1+n)+d*(5*a^4*d^
2+9*a^2*b^2*c*d+3*b^4*c^2)*(b*x+a)^(2+n)/b^6/(2+n)-a*d^2*(10*a^2*d+9*b^2*c
)*(b*x+a)^(3+n)/b^6/(3+n)+d^2*(10*a^2*d+3*b^2*c)*(b*x+a)^(4+n)/b^6/(4+n)-5
*a*d^3*(b*x+a)^(5+n)/b^6/(5+n)+d^3*(b*x+a)^(6+n)/b^6/(6+n)-c^3*(b*x+a)^(1+
n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

3.362.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^n (c+dx^2)^3}{x} dx = (a+bx)^{1+n} \left(-\frac{ad(3b^4c^2+3a^2b^2cd+a^4d^2)}{b^6(1+n)} + \frac{d(3b^4c^2+9a^2b^2cd+5a^4d^2)(a+bx)}{b^6(2+n)} - \frac{ad^2(9b^2c+10a^2d)(a+bx)^2}{b^6(3+n)} + \frac{d^2(3b^2c+10a^2d)(a+bx)^3}{b^6(4+n)} - \frac{5ad^3(a+bx)^4}{b^6(5+n)} + \frac{d^3(a+bx)^5}{b^6(6+n)} - \frac{c^3 \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+bx}{a}\right)}{a+an} \right)$$

input
```
Integrate[((a + b*x)^n*(c + d*x^2)^3)/x,x]
```

output
```
(a + b*x)^(1 + n)*(-((a*d*(3*b^4*c^2 + 3*a^2*b^2*c*d + a^4*d^2))/(b^6*(1 +
 n))) + (d*(3*b^4*c^2 + 9*a^2*b^2*c*d + 5*a^4*d^2)*(a + b*x))/(b^6*(2 + n)
) - (a*d^2*(9*b^2*c + 10*a^2*d)*(a + b*x)^2)/(b^6*(3 + n)) + (d^2*(3*b^2*c
 + 10*a^2*d)*(a + b*x)^3)/(b^6*(4 + n)) - (5*a*d^3*(a + b*x)^4)/(b^6*(5 +
n)) + (d^3*(a + b*x)^5)/(b^6*(6 + n)) - (c^3*Hypergeometric2F1[1, 1 + n, 2
 + n, (a + b*x)/a])/(a + a*n))
```

3.362.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3 (a+bx)^n}{x} dx$$

$$\Big\downarrow \; 522$$

$$\int \left(-\frac{ad^2(10a^2d+9b^2c)(a+bx)^{n+2}}{b^5}+\frac{d^2(10a^2d+3b^2c)(a+bx)^{n+3}}{b^5}-\frac{ad(a^4d^2+3a^2b^2cd+3b^4c^2)(a+bx)^n}{b^5}+\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{ad^2(10a^2d+9b^2c)(a+bx)^{n+3}}{b^6(n+3)}+\frac{d^2(10a^2d+3b^2c)(a+bx)^{n+4}}{b^6(n+4)}-\frac{ad(a^4d^2+3a^2b^2cd+3b^4c^2)(a+bx)^{n+1}}{b^6(n+1)}+\frac{d(5a^4d^2+9a^2b^2cd+3b^4c^2)(a+bx)^{n+2}}{b^6(n+2)}-\frac{5ad^3(a+bx)^{n+5}}{b^6(n+5)}+\frac{d^3(a+bx)^{n+6}}{b^6(n+6)}-\frac{c^3(a+bx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{bx}{a}+1\right)}{a(n+1)}$$

input
```
Int[((a + b*x)^n*(c + d*x^2)^3)/x,x]
```

output
```
-((a*d*(3*b^4*c^2 + 3*a^2*b^2*c*d + a^4*d^2)*(a + b*x)^(1 + n))/(b^6*(1 +
n))) + (d*(3*b^4*c^2 + 9*a^2*b^2*c*d + 5*a^4*d^2)*(a + b*x)^(2 + n))/(b^6*
(2 + n)) - (a*d^2*(9*b^2*c + 10*a^2*d)*(a + b*x)^(3 + n))/(b^6*(3 + n)) +
(d^2*(3*b^2*c + 10*a^2*d)*(a + b*x)^(4 + n))/(b^6*(4 + n)) - (5*a*d^3*(a +
 b*x)^(5 + n))/(b^6*(5 + n)) + (d^3*(a + b*x)^(6 + n))/(b^6*(6 + n)) - (c^
3*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1
 + n))
```

3.362.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.362.4 Maple [F]

$$\int \frac{(bx+a)^n (d\,x^2+c)^3}{x}dx$$

input
```
int((b*x+a)^n*(d*x^2+c)^3/x,x)
```

output
```
int((b*x+a)^n*(d*x^2+c)^3/x,x)
```

3.362.5 Fricas [F]

$$\int \frac{(a+bx)^n (c+dx^2)^3}{x}\,dx = \int \frac{(dx^2+c)^3(bx+a)^n}{x}\,dx$$

input
```
integrate((b*x+a)^n*(d*x^2+c)^3/x,x, algorithm="fricas")
```

output
```
integral((d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3)*(b*x + a)^n/x, x)
```

3.362.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4007 vs. 2(226) = 452.

Time = 4.96 (sec) , antiderivative size = 5622, normalized size of antiderivative = 22.85

$$\int \frac{(a+bx)^n (c+dx^2)^3}{x}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**n*(d*x**2+c)**3/x,x)
```

output
```
3*c**2*d*Piecewise((a**n*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3
*x) + a/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(n, -2))
, (-a*log(a/b + x)/b**2 + x/b, Eq(n, -1)), (-a**2*(a + b*x)**n/(b**2*n**2
+ 3*b**2*n + 2*b**2) + a*b*n*x*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2
) + b**2*n*x**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2*x**2*(
a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2), True)) + 3*c*d**2*Piecewise((
a**n*x**4/4, Eq(b, 0)), (6*a**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x
 + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3/(6*a**3*b**4 + 18*a**2*b**5*x +
 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*x*log(a/b + x)/(6*a**3*b**4 + 1
8*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*x/(6*a**3*b**4 +
 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*x**2*log(a/b +
 x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b
**2*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 6
*b**3*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6
*b**7*x**3), Eq(n, -4)), (-6*a**3*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x +
 2*b**6*x**2) - 9*a**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*
b*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*x/(2
*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*x**2*log(a/b + x)/(2*a**
2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*x**3/(2*a**2*b**4 + 4*a*b**5*x
 + 2*b**6*x**2), Eq(n, -3)), (6*a**3*log(a/b + x)/(2*a*b**4 + 2*b**5*x)...
```

3.362.7 Maxima [F]

$$\int \frac{(a+bx)^n (c+dx^2)^3}{x}\,dx = \int \frac{(dx^2+c)^3(bx+a)^n}{x}\,dx$$

input
```
integrate((b*x+a)^n*(d*x^2+c)^3/x,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^3*(b*x + a)^n/x, x)
```

3.362.8 Giac [F]

$$\int \frac{(a+bx)^n (c+dx^2)^3}{x} \, dx = \int \frac{(dx^2+c)^3 (bx+a)^n}{x} \, dx$$

input
```
integrate((b*x+a)^n*(d*x^2+c)^3/x,x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^3*(b*x + a)^n/x, x)
```

3.362.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^n (c+dx^2)^3}{x} \, dx = \int \frac{(d\,x^2+c)^3\,(a+b\,x)^n}{x} \, dx$$

input
```
int(((c + d*x^2)^3*(a + b*x)^n)/x,x)
```

output
```
int(((c + d*x^2)^3*(a + b*x)^n)/x, x)
```

3.363 $\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx$

3.363.1 Optimal result

Integrand size = 20, antiderivative size = 250

$$\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx = \frac{(cd^2-ae^2)(d+ex)^{1+n}}{c^2e^3(1+n)} - \frac{2d(d+ex)^{2+n}}{ce^3(2+n)} + \frac{(d+ex)^{3+n}}{ce^3(3+n)} + \frac{(-a)^{3/2}(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c^2\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} - \frac{(-a)^{3/2}(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2c^2\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}$$

output
```
(-a*e^2+c*d^2)*(e*x+d)^(1+n)/c^2/e^3/(1+n)-2*d*(e*x+d)^(2+n)/c/e^3/(2+n)+(
e*x+d)^(3+n)/c/e^3/(3+n)+1/2*(-a)^(3/2)*(e*x+d)^(1+n)*hypergeom([1, 1+n],[
2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))/c^2/(1+n)/(-e*(-a)^(1/2)+d
*c^(1/2))-1/2*(-a)^(3/2)*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^
(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))/c^2/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))
```

3.363.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.87

$$\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx = \frac{(d+ex)^{1+n}\left(\frac{2(cd^2-ae^2)}{e^3(1+n)}-\frac{4cd(d+ex)}{e^3(2+n)}+\frac{2c(d+ex)^2}{e^3(3+n)}+\frac{(-a)^{3/2}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{(\sqrt{c}d-\sqrt{-a}e)(1+n)}+\frac{\sqrt{-a}a\,\text{Hypergeom}}{(\sqrt{}}\right.}{2c^2}$$

input
```
Integrate[(x^4*(d + e*x)^n)/(a + c*x^2),x]
```

output
```
((d + e*x)^(1 + n)*((2*(c*d^2 - a*e^2))/(e^3*(1 + n)) - (4*c*d*(d + e*x))/
(e^3*(2 + n)) + (2*c*(d + e*x)^2)/(e^3*(3 + n)) + ((-a)^(3/2)*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/((Sq
rt[c]*d - Sqrt[-a]*e)*(1 + n)) + (Sqrt[-a]*a*Hypergeometric2F1[1, 1 + n, 2
 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/((Sqrt[c]*d + Sqrt[-a
]*e)*(1 + n))))/(2*c^2)
```

3.363.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {604, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx$$

$$\Big\downarrow \; 604$$

$$\frac{\int -\frac{(d+ex)^n\left(2cd(n+3)x^3e^3+2ad(n+3)xe^3+(cd^2+ae^2)(n+3)x^2e^2+ad^2(n+3)e^2\right)}{cx^2+a}dx}{ce^4(n+3)}+\frac{(d+ex)^{n+3}}{ce^3(n+3)}$$

$$\Big\downarrow \; 25$$

$$\frac{(d+ex)^{n+3}}{ce^3(n+3)}-\frac{\int \frac{(d+ex)^n\left(2cd(n+3)x^3e^3+2ad(n+3)xe^3+(cd^2+ae^2)(n+3)x^2e^2+ad^2(n+3)e^2\right)}{cx^2+a}dx}{ce^4(n+3)}$$

$$\Big\downarrow \; 2160$$

$$\frac{(d+ex)^{n+3}}{ce^3(n+3)}-\frac{\int \left(-\frac{e^2(cd^2-ae^2)(n+3)(d+ex)^n}{c}+\frac{\left(-\frac{a^2ne^4}{c}-\frac{3a^2e^4}{c}\right)(d+ex)^n}{cx^2+a}+2de^2(n+3)(d+ex)^{n+1}\right)dx}{ce^4(n+3)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(d+ex)^{n+3}}{ce^3(n+3)}-\frac{-\frac{e(n+3)(cd^2-ae^2)(d+ex)^{n+1}}{c(n+1)}-\frac{(-a)^{3/2}e^4(n+3)(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}+\frac{(-a)^{3/2}e^4(n+3)(d+ex)^{n+1}\text{Hy}}{2c(n}}{ce^4(n+3)}$$

input
```
Int[(x^4*(d + e*x)^n)/(a + c*x^2),x]
```

output
```
(d + e*x)^(3 + n)/(c*e^3*(3 + n)) - (-((e*(c*d^2 - a*e^2)*(3 + n)*(d + e*x
)^(1 + n))/(c*(1 + n))) + (2*d*e*(3 + n)*(d + e*x)^(2 + n))/(2 + n) - ((-a
)^(3/2)*e^4*(3 + n)*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (
Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*c*(Sqrt[c]*d - Sqrt[-a]*e
)*(1 + n)) + ((-a)^(3/2)*e^4*(3 + n)*(d + e*x)^(1 + n)*Hypergeometric2F1[1
, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*c*(Sqrt[
c]*d + Sqrt[-a]*e)*(1 + n)))/(c*e^4*(3 + n))
```

3.363.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 604
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(b*d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*
x^2)^p*ExpandToSum[b*d^m*(m + n + 2*p + 1)*x^m - b*(m + n + 2*p + 1)*(c + d
*x)^m - (c + d*x)^(m - 2)*(a*d^2*(m + n - 1) - b*c^2*(m + n + 2*p + 1) - 2*
b*c*d*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && IGtQ[m,
 1] && NeQ[m + n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.363. $\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx$

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.363.4 Maple [F]

$$\int \frac{x^4 (ex+d)^n}{c\,x^2+a}dx$$

input
```
int(x^4*(e*x+d)^n/(c*x^2+a),x)
```

output
```
int(x^4*(e*x+d)^n/(c*x^2+a),x)
```

3.363.5 Fricas [F]

$$\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n x^4}{cx^2+a}\,dx$$

input
```
integrate(x^4*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
```

output
```
integral((e*x + d)^n*x^4/(c*x^2 + a), x)
```

3.363.6 Sympy [F]

$$\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx = \int \frac{x^4(d+ex)^n}{a+cx^2}\,dx$$

input
```
integrate(x**4*(e*x+d)**n/(c*x**2+a),x)
```

output
```
Integral(x**4*(d + e*x)**n/(a + c*x**2), x)
```

3.363.7 Maxima [F]

$$\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n x^4}{cx^2+a}\,dx$$

input
```
integrate(x^4*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^n*x^4/(c*x^2 + a), x)
```

3.363.8 Giac [F]

$$\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n x^4}{cx^2+a}\,dx$$

input
```
integrate(x^4*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
```

output
```
integrate((e*x + d)^n*x^4/(c*x^2 + a), x)
```

3.363.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx = \int \frac{x^4\,(d+e\,x)^n}{c\,x^2+a}\,dx$$

input
```
int((x^4*(d + e*x)^n)/(a + c*x^2),x)
```

output
```
int((x^4*(d + e*x)^n)/(a + c*x^2), x)
```

3.364 $\int \frac{x^3(d+ex)^n}{a+cx^2}\,dx$

3.364.1 Optimal result

Integrand size = 20, antiderivative size = 209

$$\int \frac{x^3(d+ex)^n}{a+cx^2}\,dx = -\frac{d(d+ex)^{1+n}}{ce^2(1+n)}+\frac{(d+ex)^{2+n}}{ce^2(2+n)} + \frac{a(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c^{3/2}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} + \frac{a(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2c^{3/2}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}$$

output
```
-d*(e*x+d)^(1+n)/c/e^2/(1+n)+(e*x+d)^(2+n)/c/e^2/(2+n)+1/2*a*(e*x+d)^(1+n)
*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))/c^(3/
2)/(1+n)/(-e*(-a)^(1/2)+d*c^(1/2))+1/2*a*(e*x+d)^(1+n)*hypergeom([1, 1+n],
[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))/c^(3/2)/(1+n)/(e*(-a)^(1/2
)+d*c^(1/2))
```

3.364.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.80

$$\int \frac{x^3(d+ex)^n}{a+cx^2}\,dx$$
$$= \frac{(d+ex)^{1+n}\left(-\frac{2\sqrt{c}(d-e(1+n)x)}{e^2(2+n)} + \frac{a\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{c}d-\sqrt{-a}e} + \frac{a\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{c}d+\sqrt{-a}e}\right.}{2c^{3/2}(1+n)}$$

input
```
Integrate[(x^3*(d + e*x)^n)/(a + c*x^2),x]
```

output
```
((d + e*x)^(1 + n)*((-2*Sqrt[c]*(d - e*(1 + n)*x))/(e^2*(2 + n)) + (a*Hype
rgeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e
)])/(Sqrt[c]*d - Sqrt[-a]*e) + (a*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt
[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[c]*d + Sqrt[-a]*e)))/(2*c^
(3/2)*(1 + n))
```

3.364.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {604, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex)^n}{a+cx^2}\,dx$$

$$\Big\downarrow \; 604$$

$$\frac{\int -\frac{(d+ex)^n\left(a(n+2)xe^3+cd(n+2)x^2e^2+ad(n+2)e^2\right)}{cx^2+a}dx}{ce^3(n+2)} + \frac{(d+ex)^{n+2}}{ce^2(n+2)}$$

$$\Big\downarrow \; 25$$

$$\frac{(d+ex)^{n+2}}{ce^2(n+2)} - \frac{\int \frac{(d+ex)^n\left(a(n+2)xe^3+cd(n+2)x^2e^2+ad(n+2)e^2\right)}{cx^2+a}dx}{ce^3(n+2)}$$

$$\Big\downarrow \; 2160$$

$$\frac{(d+ex)^{n+2}}{ce^2(n+2)} - \frac{\int \left(de^2(n+2)(d+ex)^n + \frac{(2ae^3+ane^3)x(d+ex)^n}{cx^2+a}\right) dx}{ce^3(n+2)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(d+ex)^{n+2}}{ce^2(n+2)} - \frac{-\frac{ae^3(n+2)(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{c}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)} - \frac{ae^3(n+2)(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{c}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)} +}{ce^3(n+2)}$$

input
```
Int[(x^3*(d + e*x)^n)/(a + c*x^2),x]
```

output
```
(d + e*x)^(2 + n)/(c*e^2*(2 + n)) - ((d*e*(2 + n)*(d + e*x)^(1 + n))/(1 +
n) - (a*e^3*(2 + n)*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (
Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*Sqrt[c]*(Sqrt[c]*d - Sqrt
[-a]*e)*(1 + n)) - (a*e^3*(2 + n)*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1
 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*Sqrt[c]*(Sq
rt[c]*d + Sqrt[-a]*e)*(1 + n)))/(c*e^3*(2 + n))
```

3.364.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 604
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(b*d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*
x^2)^p*ExpandToSum[b*d^m*(m + n + 2*p + 1)*x^m - b*(m + n + 2*p + 1)*(c + d
*x)^m - (c + d*x)^(m - 2)*(a*d^2*(m + n - 1) - b*c^2*(m + n + 2*p + 1) - 2*
b*c*d*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && IGtQ[m,
 1] && NeQ[m + n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.364. $\int \frac{x^3(d+ex)^n}{a+cx^2}\,dx$

3.364.4 Maple [F]

$$\int \frac{x^3(ex+d)^n}{c\,x^2+a}dx$$

input
```
int(x^3*(e*x+d)^n/(c*x^2+a),x)
```

output
```
int(x^3*(e*x+d)^n/(c*x^2+a),x)
```

3.364.5 Fricas [F]

$$\int \frac{x^3(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n x^3}{cx^2+a}\,dx$$

input
```
integrate(x^3*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
```

output
```
integral((e*x + d)^n*x^3/(c*x^2 + a), x)
```

3.364.6 Sympy [F]

$$\int \frac{x^3(d+ex)^n}{a+cx^2}\,dx = \int \frac{x^3(d+ex)^n}{a+cx^2}\,dx$$

input
```
integrate(x**3*(e*x+d)**n/(c*x**2+a),x)
```

output
```
Integral(x**3*(d + e*x)**n/(a + c*x**2), x)
```

3.364.7 Maxima [F]

$$\int \frac{x^3(d+ex)^n}{a+cx^2}\,dx=\int \frac{(ex+d)^n x^3}{cx^2+a}\,dx$$

input
```
integrate(x^3*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^n*x^3/(c*x^2 + a), x)
```

3.364.8 Giac [F]

$$\int \frac{x^3(d+ex)^n}{a+cx^2}\,dx=\int \frac{(ex+d)^n x^3}{cx^2+a}\,dx$$

input
```
integrate(x^3*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
```

output
```
integrate((e*x + d)^n*x^3/(c*x^2 + a), x)
```

3.364.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex)^n}{a+cx^2}\,dx=\int \frac{x^3\,(d+e\,x)^n}{c\,x^2+a}\,dx$$

input
```
int((x^3*(d + e*x)^n)/(a + c*x^2),x)
```

output
```
int((x^3*(d + e*x)^n)/(a + c*x^2), x)
```

3.365 $\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx$

3.365.1 Optimal result

Integrand size = 20, antiderivative size = 194

$$\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx=\frac{(d+ex)^{1+n}}{ce(1+n)}+\frac{\sqrt{-a}(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)}-\frac{\sqrt{-a}(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2c\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}$$

output
```
(e*x+d)^(1+n)/c/e/(1+n)+1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)
*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(-a)^(1/2)/c/(1+n)/(-e*(-a)^(1/2)+d*c^
(1/2))-1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^
(1/2)+d*c^(1/2)))*(-a)^(1/2)/c/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))
```

3.365.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.88

$$\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx=\frac{(d+ex)^{1+n}\left(2(cd^2+ae^2)+e\left(\sqrt{-a}\sqrt{c}d-ae\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)-e\left(\sqrt{-c}\right.\right.}{2ce\left(cd^2+ae^2\right)(1+n)}$$

input
```
Integrate[(x^2*(d + e*x)^n)/(a + c*x^2),x]
```

output
```
((d + e*x)^(1 + n)*(2*(c*d^2 + a*e^2) + e*(Sqrt[-a]*Sqrt[c]*d - a*e)*Hyper
geometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)
] - e*(Sqrt[-a]*Sqrt[c]*d + a*e)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[
c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]))/(2*c*e*(c*d^2 + a*e^2)*(1 + n))
```

3.365.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {604, 25, 27, 485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)^n}{a+cx^2}dx$$

$$\Big\downarrow \text{ 604}$$

$$\frac{\int -\frac{ae^2(n+1)(d+ex)^n}{cx^2+a}dx}{ce^2(n+1)}+\frac{(d+ex)^{n+1}}{ce(n+1)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{(d+ex)^{n+1}}{ce(n+1)}-\frac{\int \frac{ae^2(n+1)(d+ex)^n}{cx^2+a}dx}{ce^2(n+1)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(d+ex)^{n+1}}{ce(n+1)}-\frac{a\int \frac{(d+ex)^n}{cx^2+a}dx}{c}$$

$$\Big\downarrow \text{ 485}$$

$$\frac{(d+ex)^{n+1}}{ce(n+1)}-\frac{a\int \left(\frac{\sqrt{-a}(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)}+\frac{\sqrt{-a}(d+ex)^n}{2a\left(\sqrt{c}x+\sqrt{-a}\right)}\right)dx}{c}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(d+ex)^{n+1}}{ce(n+1)} - \frac{a\left(\frac{(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{-a}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)} - \frac{(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{-a}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}\right)}{c}$$

input
```
Int[(x^2*(d + e*x)^n)/(a + c*x^2),x]
```

output
```
(d + e*x)^(1 + n)/(c*e*(1 + n)) - (a*(((d + e*x)^(1 + n)*Hypergeometric2F1
[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*Sqrt[-
a]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - ((d + e*x)^(1 + n)*Hypergeometric2F
1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*Sqrt[
-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))))/c
```

3.365.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 485
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n, 1/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, n}, x] &
&  !IntegerQ[2*n]
```

rule 604
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(b*d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*
x^2)^p*ExpandToSum[b*d^m*(m + n + 2*p + 1)*x^m - b*(m + n + 2*p + 1)*(c + d
*x)^m - (c + d*x)^(m - 2)*(a*d^2*(m + n - 1) - b*c^2*(m + n + 2*p + 1) - 2*
b*c*d*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && IGtQ[m,
 1] && NeQ[m + n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.365.4 Maple [F]

$$\int \frac{x^2 (ex+d)^n}{c\,x^2+a}dx$$

input
```
int(x^2*(e*x+d)^n/(c*x^2+a),x)
```

output
```
int(x^2*(e*x+d)^n/(c*x^2+a),x)
```

3.365.5 Fricas [F]

$$\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n x^2}{cx^2+a}\,dx$$

input
```
integrate(x^2*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
```

output
```
integral((e*x + d)^n*x^2/(c*x^2 + a), x)
```

3.365.6 Sympy [F]

$$\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx = \int \frac{x^2(d+ex)^n}{a+cx^2}\,dx$$

input
```
integrate(x**2*(e*x+d)**n/(c*x**2+a),x)
```

output
```
Integral(x**2*(d + e*x)**n/(a + c*x**2), x)
```

3.365.7 Maxima [F]

$$\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n x^2}{cx^2+a}\,dx$$

input
```
integrate(x^2*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^n*x^2/(c*x^2 + a), x)
```

3.365.8 Giac [F]

$$\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n x^2}{cx^2+a}\,dx$$

input
```
integrate(x^2*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
```

output
```
integrate((e*x + d)^n*x^2/(c*x^2 + a), x)
```

3.365.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx = \int \frac{x^2\,(d+e\,x)^n}{c\,x^2+a}\,dx$$

input
```
int((x^2*(d + e*x)^n)/(a + c*x^2),x)
```

output
```
int((x^2*(d + e*x)^n)/(a + c*x^2), x)
```

3.366 $\int \frac{x(d+ex)^n}{a+cx^2} \, dx$

3.366.1 Optimal result

Integrand size = 18, antiderivative size = 163

$$\int \frac{x(d+ex)^n}{a+cx^2} \, dx = -\frac{(d+ex)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{c}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} - \frac{(d+ex)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{c}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}$$

output
```
-1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)
+d*c^(1/2)))/(1+n)/c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))-1/2*(e*x+d)^(1+n)*hyp
ergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))/(1+n)/c^(1
/2)/(e*(-a)^(1/2)+d*c^(1/2))
```

3.366.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.93

$$\int \frac{x(d+ex)^n}{a+cx^2} \, dx = -\frac{(d+ex)^{1+n}\left(\left(\sqrt{c}d+\sqrt{-a}e\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)+\left(\sqrt{c}d-\sqrt{-a}e\right)\text{Hyper}\right.}{2\sqrt{c}\left(cd^2+ae^2\right)(1+n)}$$

input
```
Integrate[(x*(d + e*x)^n)/(a + c*x^2),x]
```

output
```
-1/2*((d + e*x)^(1 + n)*((Sqrt[c]*d + Sqrt[-a]*e)*Hypergeometric2F1[1, 1 +
 n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)] + (Sqrt[c]*d - Sq
rt[-a]*e)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*
d + Sqrt[-a]*e)]))/(Sqrt[c]*(c*d^2 + a*e^2)*(1 + n))
```

3.366.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex)^n}{a+cx^2} \, dx$$

$$\Big\downarrow \text{ 615}$$

$$\int \left(\frac{(d+ex)^n}{2\sqrt{c}\left(\sqrt{-a}+\sqrt{c}x\right)}-\frac{(d+ex)^n}{2\sqrt{c}\left(\sqrt{-a}-\sqrt{c}x\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{c}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{c}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

input
```
Int[(x*(d + e*x)^n)/(a + c*x^2),x]
```

output
```
-1/2*((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e
*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[c]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n))
 - ((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x
))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*Sqrt[c]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))
```

3.366.3.1 Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.366.4 Maple [F]

$$\int \frac{x(ex+d)^n}{c\,x^2+a}dx$$

input
```
int(x*(e*x+d)^n/(c*x^2+a),x)
```

output
```
int(x*(e*x+d)^n/(c*x^2+a),x)
```

3.366.5 Fricas [F]

$$\int \frac{x(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n x}{cx^2+a}\,dx$$

input
```
integrate(x*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
```

output
```
integral((e*x + d)^n*x/(c*x^2 + a), x)
```

3.366.6 Sympy [F]

$$\int \frac{x(d+ex)^n}{a+cx^2}\,dx = \int \frac{x(d+ex)^n}{a+cx^2}\,dx$$

input
```
integrate(x*(e*x+d)**n/(c*x**2+a),x)
```

output
```
Integral(x*(d + e*x)**n/(a + c*x**2), x)
```

3.366.7 Maxima [F]

$$\int \frac{x(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n x}{cx^2+a}\,dx$$

input
```
integrate(x*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^n*x/(c*x^2 + a), x)
```

3.366.8 Giac [F]

$$\int \frac{x(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n x}{cx^2+a}\,dx$$

input
```
integrate(x*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
```

output
```
integrate((e*x + d)^n*x/(c*x^2 + a), x)
```

3.366.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(d+ex)^n}{a+cx^2}\,dx = \int \frac{x\,(d+e\,x)^n}{c\,x^2+a}\,dx$$

input
```
int((x*(d + e*x)^n)/(a + c*x^2),x)
```

output
```
int((x*(d + e*x)^n)/(a + c*x^2), x)
```

3.367 $\int \frac{(d+ex)^n}{a+cx^2} dx$

3.367.1 Optimal result

Integrand size = 17, antiderivative size = 167

$$\int \frac{(d+ex)^n}{a+cx^2} dx = \frac{(d+ex)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} - \frac{(d+ex)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}$$

output
```
1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+
d*c^(1/2)))/(1+n)/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))-1/2*(e*x+d)^(1+n)*h
ypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))/(1+n)/(-
a)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))
```

3.367.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^n}{a+cx^2} dx = \frac{(d+ex)^{1+n}\left(\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{c}d-\sqrt{-a}e}-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{-a}(1+n)}$$

input
```
Integrate[(d + e*x)^n/(a + c*x^2),x]
```

output
```
((d + e*x)^(1 + n)*(Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))
/(Sqrt[c]*d - Sqrt[-a]*e)]/(Sqrt[c]*d - Sqrt[-a]*e) - Hypergeometric2F1[1,
 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]/(Sqrt[c]*d +
Sqrt[-a]*e)))/(2*Sqrt[-a]*(1 + n))
```

3.367.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^n}{a+cx^2} \, dx$$
$$\Big\downarrow \text{485}$$
$$\int \left(\frac{\sqrt{-a}(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)}+\frac{\sqrt{-a}(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}x\right)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{-a}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{-a}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

input
```
Int[(d + e*x)^n/(a + c*x^2),x]
```

output
```
((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/
(Sqrt[c]*d - Sqrt[-a]*e)])/(2*Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) -
 ((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))
/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))
```

3.367.3.1 Defintions of rubi rules used

rule 485
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n, 1/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, n}, x] &
&  !IntegerQ[2*n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.367.4 Maple [F]

$$\int \frac{(ex+d)^n}{c\,x^2+a}dx$$

input
```
int((e*x+d)^n/(c*x^2+a),x)
```

output
```
int((e*x+d)^n/(c*x^2+a),x)
```

3.367.5 Fricas [F]

$$\int \frac{(d+ex)^n}{a+cx^2}\,dx=\int \frac{(ex+d)^n}{cx^2+a}\,dx$$

input
```
integrate((e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
```

output
```
integral((e*x + d)^n/(c*x^2 + a), x)
```

3.367.6 Sympy [F]

$$\int \frac{(d+ex)^n}{a+cx^2}\,dx=\int \frac{(d+ex)^n}{a+cx^2}\,dx$$

input
```
integrate((e*x+d)**n/(c*x**2+a),x)
```

output
```
Integral((d + e*x)**n/(a + c*x**2), x)
```

3.367.7 Maxima [F]

$$\int \frac{(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n}{cx^2+a}\,dx$$

input
```
integrate((e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^n/(c*x^2 + a), x)
```

3.367.8 Giac [F]

$$\int \frac{(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n}{cx^2+a}\,dx$$

input
```
integrate((e*x+d)^n/(c*x^2+a),x, algorithm="giac")
```

output
```
integrate((e*x + d)^n/(c*x^2 + a), x)
```

3.367.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^n}{a+cx^2}\,dx = \int \frac{(d+e\,x)^n}{c\,x^2+a}\,dx$$

input
```
int((d + e*x)^n/(a + c*x^2),x)
```

output
```
int((d + e*x)^n/(a + c*x^2), x)
```

3.368 $\int \frac{(d+ex)^n}{x(a+cx^2)} dx$

3.368.1 Optimal result

Integrand size = 20, antiderivative size = 207

$$\int \frac{(d+ex)^n}{x(a+cx^2)} dx = \frac{\sqrt{c}(d+ex)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} + \frac{\sqrt{c}(d+ex)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2a\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)} - \frac{(d+ex)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{ex}{d}\right)}{ad(1+n)}$$

output
```
-(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],1+e*x/d)/a/d/(1+n)+1/2*(e*x+d)^(1+
n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*c^(
1/2)/a/(1+n)/(-e*(-a)^(1/2)+d*c^(1/2))+1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n
],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))*c^(1/2)/a/(1+n)/(e*(-a)^
(1/2)+d*c^(1/2))
```

3.368.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^n}{x\left(a+cx^2\right)}\,dx$$
$$= \frac{(d+ex)^{1+n}\left(\left(cd^2+\sqrt{-a}\sqrt{c}de\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)+\left(cd^2-\sqrt{-a}\sqrt{c}de\right)\text{Hy}\ldots}{2ad\left(cd^2+\ldots\right.}$$

input
```
Integrate[(d + e*x)^n/(x*(a + c*x^2)),x]
```

output
```
((d + e*x)^(1 + n)*((c*d^2 + Sqrt[-a]*Sqrt[c]*d*e)*Hypergeometric2F1[1, 1
+ n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)] + (c*d^2 - Sqrt[
-a]*Sqrt[c]*d*e)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(S
qrt[c]*d + Sqrt[-a]*e)] - 2*(c*d^2 + a*e^2)*Hypergeometric2F1[1, 1 + n, 2
+ n, 1 + (e*x)/d]))/(2*a*d*(c*d^2 + a*e^2)*(1 + n))
```

3.368.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^n}{x\left(a+cx^2\right)}\,dx$$
$$\Big\downarrow \text{615}$$
$$\int \left(\frac{(d+ex)^n}{ax}-\frac{cx(d+ex)^n}{a\left(a+cx^2\right)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{c}(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}+\frac{\sqrt{c}(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2a(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}-\frac{(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{ex}{d}+1\right)}{ad(n+1)}$$

input
```
Int[(d + e*x)^n/(x*(a + c*x^2)),x]
```

output
```
(Sqrt[c]*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d
+ e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*a*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n))
+ (Sqrt[c]*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(
d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*a*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)
) - ((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (e*x)/d])/(a
*d*(1 + n))
```

3.368.3.1 Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.368.4 Maple [F]

$$\int \frac{(ex+d)^n}{x\,(c\,x^2+a)}dx$$

input
```
int((e*x+d)^n/x/(c*x^2+a),x)
```

output
```
int((e*x+d)^n/x/(c*x^2+a),x)
```

3.368.5 Fricas [F]

$$\int \frac{(d+ex)^n}{x\,(a+cx^2)}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)x}\,dx$$

input
```
integrate((e*x+d)^n/x/(c*x^2+a),x, algorithm="fricas")
```

output
```
integral((e*x + d)^n/(c*x^3 + a*x), x)
```

3.368.6 Sympy [F]

$$\int \frac{(d+ex)^n}{x\,(a+cx^2)}\,dx = \int \frac{(d+ex)^n}{x\,(a+cx^2)}\,dx$$

input
```
integrate((e*x+d)**n/x/(c*x**2+a),x)
```

output
```
Integral((d + e*x)**n/(x*(a + c*x**2)), x)
```

3.368.7 Maxima [F]

$$\int \frac{(d+ex)^n}{x\,(a+cx^2)}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)x}\,dx$$

input
```
integrate((e*x+d)^n/x/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^n/((c*x^2 + a)*x), x)
```

3.368.8 Giac [F]

$$\int \frac{(d+ex)^n}{x\,(a+cx^2)}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)x}\,dx$$

input
```
integrate((e*x+d)^n/x/(c*x^2+a),x, algorithm="giac")
```

output
```
integrate((e*x + d)^n/((c*x^2 + a)*x), x)
```

3.368.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^n}{x\,(a+cx^2)}\,dx = \int \frac{(d+e\,x)^n}{x\,\left(c\,x^2+a\right)}\,dx$$

input
```
int((d + e*x)^n/(x*(a + c*x^2)),x)
```

output
```
int((d + e*x)^n/(x*(a + c*x^2)), x)
```

3.369 $\int \frac{(d+ex)^n}{x^2(a+cx^2)}\,dx$

3.369.1 Optimal result

Integrand size = 20, antiderivative size = 207

$$\begin{aligned}\int \frac{(d+ex)^n}{x^2(a+cx^2)}\,dx &= \frac{c(d+ex)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2(-a)^{3/2}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)}\\ &\quad-\frac{c(d+ex)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2(-a)^{3/2}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}\\ &\quad+\frac{e(d+ex)^{1+n}\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{ex}{d}\right)}{ad^2(1+n)}\end{aligned}$$

output
```
e*(e*x+d)^(1+n)*hypergeom([2, 1+n],[2+n],1+e*x/d)/a/d^2/(1+n)+1/2*c*(e*x+d
)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))
)/(-a)^(3/2)/(1+n)/(-e*(-a)^(1/2)+d*c^(1/2))-1/2*c*(e*x+d)^(1+n)*hypergeom
([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))/(-a)^(3/2)/(1+n)
/(e*(-a)^(1/2)+d*c^(1/2))
```

3.369.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)^n}{x^2(a+cx^2)}\,dx = \frac{(d+ex)^{1+n}\left(-\frac{c\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{-a}\sqrt{c}d+ae}+\frac{c\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{-a}\sqrt{c}d-ae}+\frac{2e\,\text{Hypergeometri}}{}\right.}{2a(1+n)}$$

input
```
Integrate[(d + e*x)^n/(x^2*(a + c*x^2)),x]
```

output
```
((d + e*x)^(1 + n)*(-((c*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d +
e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[c]*d + a*e)) + (c*Hypergeo
metric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/
(Sqrt[-a]*Sqrt[c]*d - a*e) + (2*e*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (
e*x)/d])/d^2))/(2*a*(1 + n))
```

3.369.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^n}{x^2(a+cx^2)}\,dx$$

$$\Big\downarrow \text{615}$$

$$\int \left(\frac{(d+ex)^n}{ax^2}-\frac{c(d+ex)^n}{a(a+cx^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2(-a)^{3/2}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{c(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2(-a)^{3/2}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}+\frac{e(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{ex}{d}+1\right)}{ad^2(n+1)}$$

input
```
Int[(d + e*x)^n/(x^2*(a + c*x^2)),x]
```

output
```
(c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x)
)/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*(-a)^(3/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n
)) - (c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d +
 e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*(-a)^(3/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(
1 + n)) + (e*(d + e*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (e*x
)/d])/(a*d^2*(1 + n))
```

3.369.3.1 Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.369.4 Maple [F]

$$\int \frac{(ex+d)^n}{x^2\,(c\,x^2+a)}dx$$

input
```
int((e*x+d)^n/x^2/(c*x^2+a),x)
```

output
```
int((e*x+d)^n/x^2/(c*x^2+a),x)
```

3.369.5 Fricas [F]

$$\int \frac{(d+ex)^n}{x^2(a+cx^2)}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)x^2}\,dx$$

input
```
integrate((e*x+d)^n/x^2/(c*x^2+a),x, algorithm="fricas")
```

output
```
integral((e*x + d)^n/(c*x^4 + a*x^2), x)
```

3.369.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^n}{x^2(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**n/x**2/(c*x**2+a),x)
```

output
```
Timed out
```

3.369.7 Maxima [F]

$$\int \frac{(d+ex)^n}{x^2(a+cx^2)}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)x^2}\,dx$$

input
```
integrate((e*x+d)^n/x^2/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^n/((c*x^2 + a)*x^2), x)
```

3.369.8 Giac [F]

$$\int \frac{(d+ex)^n}{x^2(a+cx^2)}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)x^2}\,dx$$

input
```
integrate((e*x+d)^n/x^2/(c*x^2+a),x, algorithm="giac")
```

output
```
integrate((e*x + d)^n/((c*x^2 + a)*x^2), x)
```

3.369.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^n}{x^2(a+cx^2)}\,dx = \int \frac{(d+e\,x)^n}{x^2\,(c\,x^2+a)}\,dx$$

input
```
int((d + e*x)^n/(x^2*(a + c*x^2)),x)
```

output
```
int((d + e*x)^n/(x^2*(a + c*x^2)), x)
```

3.370 $\int \frac{x^4(d+ex)^n}{(a+cx^2)^2} dx$

3.370.1 Optimal result

Integrand size = 20, antiderivative size = 332

$$\int \frac{x^4(d+ex)^n}{(a+cx^2)^2} dx = \frac{(d+ex)^{1+n}}{c^2e(1+n)} + \frac{a(ae+cdx)(d+ex)^{1+n}}{2c^2(cd^2+ae^2)(a+cx^2)} + \frac{\left(3\sqrt{-a}cd^2+a\sqrt{c}den+\sqrt{-a}ae^2(3+n)\right)(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4c^2\left(\sqrt{c}d-\sqrt{-a}e\right)(cd^2+ae^2)(1+n)} - \frac{\left(3\sqrt{-a}cd^2-a\sqrt{c}den+\sqrt{-a}ae^2(3+n)\right)(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4c^2\left(\sqrt{c}d+\sqrt{-a}e\right)(cd^2+ae^2)(1+n)}$$

output
```
(e*x+d)^(1+n)/c^2/e/(1+n)+1/2*a*(c*d*x+a*e)*(e*x+d)^(1+n)/c^2/(a*e^2+c*d^2
)/(c*x^2+a)-1/4*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*
(-a)^(1/2)+d*c^(1/2)))*(3*c*d^2*(-a)^(1/2)+a*e^2*(3+n)*(-a)^(1/2)-a*d*e*n*
c^(1/2))/c^2/(a*e^2+c*d^2)/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))+1/4*(e*x+d)^(1+n
)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(3*c
*d^2*(-a)^(1/2)+a*e^2*(3+n)*(-a)^(1/2)+a*d*e*n*c^(1/2))/c^2/(a*e^2+c*d^2)/
(1+n)/(-e*(-a)^(1/2)+d*c^(1/2))
```

3.370.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.24

$$\int \frac{x^4(d+ex)^n}{(a+cx^2)^2}\,dx$$
$$= \frac{(d+ex)^{1+n}\left(\frac{4}{e+en}+\frac{2a(ae+cdx)}{(cd^2+ae^2)(a+cx^2)}+\frac{4\sqrt{-a}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{(\sqrt{c}d-\sqrt{-a}e)(1+n)}-\frac{4\sqrt{-a}\,\text{Hypergeometric2F1}\left(1,1+\right.}{(\sqrt{c}d+\sqrt{-a}e)(1+}\right.}{}$$

input
```
Integrate[(x^4*(d + e*x)^n)/(a + c*x^2)^2,x]
```

output
```
((d + e*x)^(1 + n)*(4/(e + e*n) + (2*a*(a*e + c*d*x))/((c*d^2 + a*e^2)*(a
+ c*x^2)) + (4*Sqrt[-a]*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e
*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/((Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (4*Sq
rt[-a]*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d +
 Sqrt[-a]*e)])/((Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) + (a*(((c*d^2 - a*e^2*(-
1 + n) + Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[
c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[c]*d - Sqrt[-a]*e) - ((c*d^
2 - a*e^2*(-1 + n) - Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2
 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[c]*d + Sqrt[-a]
*e)))/(Sqrt[-a]*(c*d^2 + a*e^2)*(1 + n))))/(4*c^2)
```

3.370.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {602, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d+ex)^n}{(a+cx^2)^2}\,dx$$
$$\Big\downarrow \; 602$$

$$\frac{a(d+ex)^{n+1}(ae+cdx)}{2c^2\left(a+cx^2\right)\left(ae^2+cd^2\right)}-\frac{\int \frac{(d+ex)^n\left(\frac{\left(cd^2+ae^2(n+1)\right)a^2}{c^2}+\frac{denxa^2}{c}-2\left(d^2+\frac{ae^2}{c}\right)x^2a\right)}{cx^2+a}dx}{2a\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{ 2160}$$

$$\frac{a(d+ex)^{n+1}(ae+cdx)}{2c^2\left(a+cx^2\right)\left(ae^2+cd^2\right)}-\frac{\int \left(-\frac{2a\left(cd^2+ae^2\right)(d+ex)^n}{c^2}+\frac{\left(\sqrt{-a}\left(\frac{3e^2a^3}{c^2}+\frac{e^2na^3}{c^2}+\frac{3d^2a^2}{c}\right)-\frac{a^3den}{c^{3/2}}\right)(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)}+\frac{\left(\frac{dena^3}{c^{3/2}}+\sqrt{-a}\left(\frac{3e^2a^3}{c^2}+\frac{e^2na^3}{c^2}+\frac{3d^2a^2}{c}\right)\right)(d+ex)^n}{2a\left(\sqrt{c}x+\sqrt{-a}\right)}\right)dx}{2a\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a(d+ex)^{n+1}(ae+cdx)}{2c^2\left(a+cx^2\right)\left(ae^2+cd^2\right)}-\frac{-\frac{a(d+ex)^{n+1}\left(3\sqrt{-a}cd^2+a\sqrt{c}den+\sqrt{-a}ae^2(n+3)\right)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c^2(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}+\frac{a(d+ex)^{n+1}\left(3\sqrt{-a}cd^2-a\sqrt{c}den+\sqrt{-a}ae\right.}{2c^2(n}}{2a\left(ae^2+cd^2\right)}$$

input
```
Int[(x^4*(d + e*x)^n)/(a + c*x^2)^2,x]
```

output
```
(a*(a*e + c*d*x)*(d + e*x)^(1 + n))/(2*c^2*(c*d^2 + a*e^2)*(a + c*x^2)) -
((-2*a*(c*d^2 + a*e^2)*(d + e*x)^(1 + n))/(c^2*e*(1 + n)) - (a*(3*Sqrt[-a]
*c*d^2 + a*Sqrt[c]*d*e*n + Sqrt[-a]*a*e^2*(3 + n))*(d + e*x)^(1 + n)*Hyper
geometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)
])/(2*c^2*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) + (a*(3*Sqrt[-a]*c*d^2 - a*Sqr
t[c]*d*e*n + Sqrt[-a]*a*e^2*(3 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[1
, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*c^2*(Sqr
t[c]*d + Sqrt[-a]*e)*(1 + n)))/(2*a*(c*d^2 + a*e^2))
```

3.370. $\int \frac{x^4(d+ex)^n}{(a+cx^2)^2}\,dx$

3.370.3.1 Defintions of rubi rules used

rule 602
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a + b*x^2, x], e = Coeff[Polynomia
lRemainder[x^m, a + b*x^2, x], x, 0], f = Coeff[PolynomialRemainder[x^m, a
+ b*x^2, x], x, 1]}, Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^(p + 1)*((a*(d*e
 - c*f) + (b*c*e + a*d*f)*x)/(2*a*(p + 1)*(b*c^2 + a*d^2))), x] + Simp[1/(2
*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*x^2)^(p + 1)*ExpandToS
um[2*a*(p + 1)*(b*c^2 + a*d^2)*Qx + e*(b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3
)) - a*c*d*f*n + d*(b*c*e + a*d*f)*(n + 2*p + 4)*x, x], x], x]] /; FreeQ[{a
, b, c, d, n}, x] && IGtQ[m, 1] && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.370.4 Maple [F]

$$\int \frac{x^4 (ex+d)^n}{(c\,x^2+a)^2}dx$$

input
```
int(x^4*(e*x+d)^n/(c*x^2+a)^2,x)
```

output
```
int(x^4*(e*x+d)^n/(c*x^2+a)^2,x)
```

3.370.5 Fricas [F]

$$\int \frac{x^4(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x^4}{(cx^2+a)^2}\,dx$$

input
```
integrate(x^4*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((e*x + d)^n*x^4/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

3.370.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex)^n}{(a+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(e*x+d)**n/(c*x**2+a)**2,x)
```

output
```
Timed out
```

3.370.7 Maxima [F]

$$\int \frac{x^4(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x^4}{(cx^2+a)^2}\,dx$$

input
```
integrate(x^4*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^n*x^4/(c*x^2 + a)^2, x)
```

3.370.8 Giac [F]

$$\int \frac{x^4(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x^4}{(cx^2+a)^2}\,dx$$

input
```
integrate(x^4*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((e*x + d)^n*x^4/(c*x^2 + a)^2, x)
```

3.370.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex)^n}{\left(a+cx^2\right)^2}\,dx=\int \frac{x^4\,\left(d+e\,x\right)^n}{\left(c\,x^2+a\right)^2}\,dx$$

input
```
int((x^4*(d + e*x)^n)/(a + c*x^2)^2,x)
```

output
```
int((x^4*(d + e*x)^n)/(a + c*x^2)^2, x)
```

3.371 $\int \frac{x^3(d+ex)^n}{(a+cx^2)^2} dx$

3.371.1 Optimal result

Integrand size = 20, antiderivative size = 297

$$\int \frac{x^3(d+ex)^n}{(a+cx^2)^2} dx = \frac{a(d-ex)(d+ex)^{1+n}}{2c(cd^2+ae^2)(a+cx^2)} + \frac{\left(\sqrt{-a}den - \frac{2cd^2+ae^2(2+n)}{\sqrt{c}}\right)(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4c\left(\sqrt{c}d-\sqrt{-a}e\right)(cd^2+ae^2)(1+n)} - \frac{\left(2cd^2+\sqrt{-a}\sqrt{c}den+ae^2(2+n)\right)(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4c^{3/2}\left(\sqrt{c}d+\sqrt{-a}e\right)(cd^2+ae^2)(1+n)}$$

output
```
1/2*a*(-e*x+d)*(e*x+d)^(1+n)/c/(a*e^2+c*d^2)/(c*x^2+a)+1/4*(e*x+d)^(1+n)*h
ypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(d*e*n*
(-a)^(1/2)+(-2*c*d^2-a*e^2*(2+n))/c^(1/2))/c/(a*e^2+c*d^2)/(1+n)/(-e*(-a)^
(1/2)+d*c^(1/2))-1/4*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2
)/(e*(-a)^(1/2)+d*c^(1/2)))*(2*c*d^2+a*e^2*(2+n)+d*e*n*(-a)^(1/2)*c^(1/2))
/c^(3/2)/(a*e^2+c*d^2)/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))
```

3.371.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.83

$$\int \frac{x^3(d+ex)^n}{(a+cx^2)^2}\,dx =$$
$$-\frac{(d+ex)^{1+n}\left(\frac{2a\sqrt{c}(-d+ex)}{a+cx^2}+\frac{(2cd^2-\sqrt{-a}\sqrt{c}den+ae^2(2+n))\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{(\sqrt{c}d-\sqrt{-a}e)(1+n)}+\frac{(2cd^2+\sqrt{-a}\sqrt{c}den}{}\right.}{4c^{3/2}\left(cd^2+ae^2\right)}$$

input
```
Integrate[(x^3*(d + e*x)^n)/(a + c*x^2)^2,x]
```

output
```
-1/4*((d + e*x)^(1 + n)*((2*a*Sqrt[c]*(-d + e*x))/(a + c*x^2) + ((2*c*d^2
- Sqrt[-a]*Sqrt[c]*d*e*n + a*e^2*(2 + n))*Hypergeometric2F1[1, 1 + n, 2 +
n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/((Sqrt[c]*d - Sqrt[-a]*e
)*(1 + n)) + ((2*c*d^2 + Sqrt[-a]*Sqrt[c]*d*e*n + a*e^2*(2 + n))*Hypergeom
etric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(
(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))))/(c^(3/2)*(c*d^2 + a*e^2))
```

3.371.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {602, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex)^n}{(a+cx^2)^2}\,dx$$
$$\Big\downarrow \; 602$$
$$\frac{a(d-ex)(d+ex)^{n+1}}{2c\,(a+cx^2)\,(ae^2+cd^2)}-\frac{\int \frac{a(d+ex)^n\left(aden-\left(2cd^2+ae^2(n+2)\right)x\right)}{c(cx^2+a)}dx}{2a\,(ae^2+cd^2)}$$
$$\Big\downarrow \; 27$$
$$\frac{a(d-ex)(d+ex)^{n+1}}{2c\,(a+cx^2)\,(ae^2+cd^2)}-\frac{\int \frac{(d+ex)^n\left(aden-\left(2cd^2+ae^2(n+2)\right)x\right)}{cx^2+a}dx}{2c\,(ae^2+cd^2)}$$
$$\Big\downarrow \; 657$$

$$\frac{\frac{a(d-ex)(d+ex)^{n+1}}{2c(a+cx^2)(ae^2+cd^2)}-\int\left(\frac{\left(\sqrt{-a}aden-\frac{a\left(-2cd^2-ae^2(n+2)\right)}{\sqrt{c}}\right)(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)}+\frac{\left(\sqrt{-a}aden+\frac{a\left(-2cd^2-ae^2(n+2)\right)}{\sqrt{c}}\right)(d+ex)^n}{2a\left(\sqrt{c}x+\sqrt{-a}\right)}\right)dx}{2c(ae^2+cd^2)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{a(d-ex)(d+ex)^{n+1}}{2c(a+cx^2)(ae^2+cd^2)}-\frac{(d+ex)^{n+1}\left(-\sqrt{-a}\sqrt{c}den+ae^2(n+2)+2cd^2\right)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{c}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}+\frac{(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}den+ae^2(n+2)+2cd^2\right)\text{Hyper}}{2\sqrt{c}(n+1)\left(\sqrt{-ae}\right.}}{2c(ae^2+cd^2)}$$

input
```
Int[(x^3*(d + e*x)^n)/(a + c*x^2)^2,x]
```

output
```
(a*(d - e*x)*(d + e*x)^(1 + n))/(2*c*(c*d^2 + a*e^2)*(a + c*x^2)) - (((2*c
*d^2 - Sqrt[-a]*Sqrt[c]*d*e*n + a*e^2*(2 + n))*(d + e*x)^(1 + n)*Hypergeom
etric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(
2*Sqrt[c]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) + ((2*c*d^2 + Sqrt[-a]*Sqrt[c]
*d*e*n + a*e^2*(2 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 +
n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*Sqrt[c]*(Sqrt[c]*d +
Sqrt[-a]*e)*(1 + n)))/(2*c*(c*d^2 + a*e^2))
```

3.371.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 602
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a + b*x^2, x], e = Coeff[Polynomia
lRemainder[x^m, a + b*x^2, x], x, 0], f = Coeff[PolynomialRemainder[x^m, a
+ b*x^2, x], x, 1]}, Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^(p + 1)*((a*(d*e
 - c*f) + (b*c*e + a*d*f)*x)/(2*a*(p + 1)*(b*c^2 + a*d^2))), x] + Simp[1/(2
*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*x^2)^(p + 1)*ExpandToS
um[2*a*(p + 1)*(b*c^2 + a*d^2)*Qx + e*(b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3
)) - a*c*d*f*n + d*(b*c*e + a*d*f)*(n + 2*p + 4)*x, x], x], x]] /; FreeQ[{a
, b, c, d, n}, x] && IGtQ[m, 1] && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

3.371. $\int \frac{x^3(d+ex)^n}{(a+cx^2)^2}\,dx$

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.371.4 Maple [F]

$$\int \frac{x^3(ex+d)^n}{(c\,x^2+a)^2}dx$$

input
```
int(x^3*(e*x+d)^n/(c*x^2+a)^2,x)
```

output
```
int(x^3*(e*x+d)^n/(c*x^2+a)^2,x)
```

3.371.5 Fricas [F]

$$\int \frac{x^3(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x^3}{(cx^2+a)^2}\,dx$$

input
```
integrate(x^3*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((e*x + d)^n*x^3/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

3.371.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex)^n}{(a+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(e*x+d)**n/(c*x**2+a)**2,x)
```

output
```
Timed out
```

3.371.7 Maxima [F]

$$\int \frac{x^3(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x^3}{(cx^2+a)^2}\,dx$$

input
```
integrate(x^3*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^n*x^3/(c*x^2 + a)^2, x)
```

3.371.8 Giac [F]

$$\int \frac{x^3(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x^3}{(cx^2+a)^2}\,dx$$

input
```
integrate(x^3*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((e*x + d)^n*x^3/(c*x^2 + a)^2, x)
```

3.371.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{x^3\,(d+e\,x)^n}{(c\,x^2+a)^2}\,dx$$

input
```
int((x^3*(d + e*x)^n)/(a + c*x^2)^2,x)
```

output
```
int((x^3*(d + e*x)^n)/(a + c*x^2)^2, x)
```

3.372 $\int \frac{x^2(d+ex)^n}{(a+cx^2)^2} dx$

3.372.1 Optimal result

Integrand size = 20, antiderivative size = 306

$$\int \frac{x^2(d+ex)^n}{(a+cx^2)^2} dx = -\frac{(ae+cdx)(d+ex)^{1+n}}{2c(cd^2+ae^2)(a+cx^2)} + \frac{\left(cd^2-\sqrt{-a}\sqrt{c}den+ae^2(1+n)\right)(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4\sqrt{-a}c\left(\sqrt{c}d-\sqrt{-a}e\right)(cd^2+ae^2)(1+n)} - \frac{\left(cd^2+\sqrt{-a}\sqrt{c}den+ae^2(1+n)\right)(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4\sqrt{-a}c\left(\sqrt{c}d+\sqrt{-a}e\right)(cd^2+ae^2)(1+n)}$$

output
```
-1/2*(c*d*x+a*e)*(e*x+d)^(1+n)/c/(a*e^2+c*d^2)/(c*x^2+a)+1/4*(e*x+d)^(1+n)
*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(c*d^
2+a*e^2*(1+n)-d*e*n*(-a)^(1/2)*c^(1/2))/c/(a*e^2+c*d^2)/(1+n)/(-a)^(1/2)/(
-e*(-a)^(1/2)+d*c^(1/2))-1/4*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d
)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))*(c*d^2+a*e^2*(1+n)+d*e*n*(-a)^(1/2)*c^
(1/2))/c/(a*e^2+c*d^2)/(1+n)/(-a)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))
```

3.372.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.32

$$\int \frac{x^2(d+ex)^n}{(a+cx^2)^2}\,dx$$
$$= \frac{(d+ex)^{1+n}\left(-\frac{2(ae+cdx)}{(cd^2+ae^2)(a+cx^2)}+\frac{2\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)}-\frac{2\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}\right.}{}$$

input
```
Integrate[(x^2*(d + e*x)^n)/(a + c*x^2)^2,x]
```

output
```
((d + e*x)^(1 + n)*((-2*(a*e + c*d*x))/((c*d^2 + a*e^2)*(a + c*x^2)) + (2*
Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-
a]*e)])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (2*Hypergeometric2F1
[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[-a]
*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) + (a*(((c*d^2 - a*e^2*(-1 + n) + Sqrt[-
a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(
Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[c]*d - Sqrt[-a]*e) - ((c*d^2 - a*e^2*(-1 +
 n) - Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*
(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[c]*d + Sqrt[-a]*e)))/((-a)^(3/
2)*(c*d^2 + a*e^2)*(1 + n))))/(4*c)
```

3.372.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {602, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)^n}{(a+cx^2)^2}\,dx$$
$$\Big\downarrow \text{602}$$
$$-\frac{\int -\frac{a(d+ex)^n\left(cd^2+cenxd+ae^2(n+1)\right)}{c(cx^2+a)}dx}{2a\left(ae^2+cd^2\right)}-\frac{(d+ex)^{n+1}(ae+cdx)}{2c\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a(d+ex)^n(cd^2+cenxd+ae^2(n+1))}{c(cx^2+a)}dx}{2a\left(ae^2+cd^2\right)}-\frac{(d+ex)^{n+1}(ae+cdx)}{2c\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(d+ex)^n(cd^2+cenxd+ae^2(n+1))}{cx^2+a}dx}{2c\left(ae^2+cd^2\right)}-\frac{(d+ex)^{n+1}(ae+cdx)}{2c\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{657}$$

$$\frac{\int \left(\frac{(\sqrt{-a}(cd^2+ae^2(n+1))-a\sqrt{c}den)(d+ex)^n}{2a(\sqrt{-a}-\sqrt{c}x)}+\frac{(a\sqrt{c}den+\sqrt{-a}(cd^2+ae^2(n+1)))(d+ex)^n}{2a(\sqrt{c}x+\sqrt{-a})}\right)dx}{2c\left(ae^2+cd^2\right)}-\frac{(d+ex)^{n+1}(ae+cdx)}{2c\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(d+ex)^{n+1}(-\sqrt{-a}\sqrt{c}den+ae^2(n+1)+cd^2)\text{ Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{-a}(n+1)(\sqrt{c}d-\sqrt{-a}e)}-\frac{(d+ex)^{n+1}(\sqrt{-a}\sqrt{c}den+ae^2(n+1)+cd^2)\text{ Hyperge}}{2\sqrt{-a}(n+1)(\sqrt{-a}e}}{2c\left(ae^2+cd^2\right)}$$

$$\frac{(d+ex)^{n+1}(ae+cdx)}{2c\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

input
```
Int[(x^2*(d + e*x)^n)/(a + c*x^2)^2,x]
```

output
```
-1/2*((a*e + c*d*x)*(d + e*x)^(1 + n))/(c*(c*d^2 + a*e^2)*(a + c*x^2)) + (
((c*d^2 - Sqrt[-a]*Sqrt[c]*d*e*n + a*e^2*(1 + n))*(d + e*x)^(1 + n)*Hyperg
eometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)]
)/(2*Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - ((c*d^2 + Sqrt[-a]*Sqrt[
c]*d*e*n + a*e^2*(1 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
+ n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*Sqrt[-a]*(Sqrt[c]*d
 + Sqrt[-a]*e)*(1 + n)))/(2*c*(c*d^2 + a*e^2))
```

3.372. $\int \frac{x^2(d+ex)^n}{(a+cx^2)^2}\,dx$

3.372.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 602
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a + b*x^2, x], e = Coeff[Polynomia
lRemainder[x^m, a + b*x^2, x], x, 0], f = Coeff[PolynomialRemainder[x^m, a
+ b*x^2, x], x, 1]}, Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^(p + 1)*((a*(d*e
 - c*f) + (b*c*e + a*d*f)*x)/(2*a*(p + 1)*(b*c^2 + a*d^2))), x] + Simp[1/(2
*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*x^2)^(p + 1)*ExpandToS
um[2*a*(p + 1)*(b*c^2 + a*d^2)*Qx + e*(b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3
)) - a*c*d*f*n + d*(b*c*e + a*d*f)*(n + 2*p + 4)*x, x], x], x]] /; FreeQ[{a
, b, c, d, n}, x] && IGtQ[m, 1] && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.372.4 Maple [F]

$$\int \frac{x^2 (ex+d)^n}{(c\,x^2+a)^2} dx$$

input
```
int(x^2*(e*x+d)^n/(c*x^2+a)^2,x)
```

output
```
int(x^2*(e*x+d)^n/(c*x^2+a)^2,x)
```

3.372.5 Fricas [F]

$$\int \frac{x^2(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x^2}{(cx^2+a)^2}\,dx$$

input
```
integrate(x^2*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((e*x + d)^n*x^2/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

3.372.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex)^n}{(a+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(e*x+d)**n/(c*x**2+a)**2,x)
```

output
```
Timed out
```

3.372.7 Maxima [F]

$$\int \frac{x^2(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x^2}{(cx^2+a)^2}\,dx$$

input
```
integrate(x^2*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^n*x^2/(c*x^2 + a)^2, x)
```

3.372.8 Giac [F]

$$\int \frac{x^2(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x^2}{(cx^2+a)^2}\,dx$$

input
```
integrate(x^2*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((e*x + d)^n*x^2/(c*x^2 + a)^2, x)
```

3.372.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{x^2\,(d+e\,x)^n}{(c\,x^2+a)^2}\,dx$$

input
```
int((x^2*(d + e*x)^n)/(a + c*x^2)^2,x)
```

output
```
int((x^2*(d + e*x)^n)/(a + c*x^2)^2, x)
```

3.373 $\int \frac{x(d+ex)^n}{(a+cx^2)^2} dx$

3.373.1 Optimal result

Integrand size = 18, antiderivative size = 279

$$\begin{aligned}\int \frac{x(d+ex)^n}{(a+cx^2)^2} dx &= -\frac{(d-ex)(d+ex)^{1+n}}{2(cd^2+ae^2)(a+cx^2)} \\ &+ \frac{e(\sqrt{c}d+\sqrt{-a}e)n(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{c}(\sqrt{c}d-\sqrt{-a}e)(cd^2+ae^2)(1+n)} \\ &+ \frac{e(\sqrt{-a}\sqrt{c}d+ae)n(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4a\sqrt{c}(\sqrt{c}d+\sqrt{-a}e)(cd^2+ae^2)(1+n)}\end{aligned}$$

output
```
-1/2*(-e*x+d)*(e*x+d)^(1+n)/(a*e^2+c*d^2)/(c*x^2+a)+1/4*e*n*(e*x+d)^(1+n)*
hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(e*(-a
)^(1/2)+d*c^(1/2))/(a*e^2+c*d^2)/(1+n)/(-a)^(1/2)/c^(1/2)/(-e*(-a)^(1/2)+d
*c^(1/2))+1/4*e*n*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(
e*(-a)^(1/2)+d*c^(1/2)))*(a*e+d*(-a)^(1/2)*c^(1/2))/a/(a*e^2+c*d^2)/(1+n)/
c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))
```

3.373.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.82

$$\int \frac{x(d+ex)^n}{(a+cx^2)^2}\,dx = \frac{(d+ex)^{1+n}\left(-\frac{2ac(d-ex)}{a+cx^2}-\frac{\left(\sqrt{-a}cden-a\sqrt{c}e^2n\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)}+\frac{\left(\sqrt{-a}cden+a\sqrt{c}e^2n\right)\text{Hypergeome}}{\left(\sqrt{c}d+\sqrt{-}\right.}\right.}{4ac\left(cd^2+ae^2\right)}$$

input
```
Integrate[(x*(d + e*x)^n)/(a + c*x^2)^2,x]
```

output
```
((d + e*x)^(1 + n)*((-2*a*c*(d - e*x))/(a + c*x^2) - ((Sqrt[-a]*c*d*e*n -
a*Sqrt[c]*e^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(S
qrt[c]*d - Sqrt[-a]*e)])/((Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) + ((Sqrt[-a]*c
*d*e*n + a*Sqrt[c]*e^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d +
 e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/((Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))))/(4*
a*c*(c*d^2 + a*e^2))
```

3.373.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {593, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex)^n}{(a+cx^2)^2}\,dx$$

$$\Big\downarrow \; 593$$

$$\frac{e\int \frac{n(d-ex)(d+ex)^n}{cx^2+a}dx}{2\left(ae^2+cd^2\right)}-\frac{(d-ex)(d+ex)^{n+1}}{2\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{en\int \frac{(d-ex)(d+ex)^n}{cx^2+a}dx}{2\left(ae^2+cd^2\right)}-\frac{(d-ex)(d+ex)^{n+1}}{2\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 657$$

$$\frac{en\int \left(\frac{\left(\sqrt{-a}d+\frac{ae}{\sqrt{c}}\right)(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)}+\frac{\left(\sqrt{-a}d-\frac{ae}{\sqrt{c}}\right)(d+ex)^n}{2a\left(\sqrt{c}x+\sqrt{-a}\right)}\right)dx}{2\left(ae^2+cd^2\right)}-\frac{(d-ex)(d+ex)^{n+1}}{2\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

$$\downarrow \text{2009}$$

$$\frac{en\left(\frac{\left(\frac{\sqrt{-a}e}{\sqrt{c}}+d\right)(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{-a}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}+\frac{\left(\sqrt{-a}\sqrt{c}d+ae\right)(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d}{\sqrt{c}d+}\right.}{2a\sqrt{c}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}\right.}{2\left(ae^2+cd^2\right)}$$

$$\frac{(d-ex)(d+ex)^{n+1}}{2\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

input
```
Int[(x*(d + e*x)^n)/(a + c*x^2)^2,x]
```

output
```
-1/2*((d - e*x)*(d + e*x)^(1 + n))/((c*d^2 + a*e^2)*(a + c*x^2)) + (e*n*((
(d + (Sqrt[-a]*e)/Sqrt[c])*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*Sqrt[-a]*(Sqrt[c]*
d - Sqrt[-a]*e)*(1 + n)) + ((Sqrt[-a]*Sqrt[c]*d + a*e)*(d + e*x)^(1 + n)*H
ypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a
]*e)])/(2*a*Sqrt[c]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))))/(2*(c*d^2 + a*e^2)
)
```

3.373.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

3.373. $\int \frac{x(d+ex)^n}{(a+cx^2)^2}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.373.4 Maple [F]

$$\int \frac{x(ex+d)^n}{(c\,x^2+a)^2}dx$$

input
```
int(x*(e*x+d)^n/(c*x^2+a)^2,x)
```

output
```
int(x*(e*x+d)^n/(c*x^2+a)^2,x)
```

3.373.5 Fricas [F]

$$\int \frac{x(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x}{(cx^2+a)^2}\,dx$$

input
```
integrate(x*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((e*x + d)^n*x/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

3.373.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(d+ex)^n}{(a+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(e*x+d)**n/(c*x**2+a)**2,x)
```

output
```
Timed out
```

3.373.7 Maxima [F]

$$\int \frac{x(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x}{(cx^2+a)^2}\,dx$$

input
```
integrate(x*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^n*x/(c*x^2 + a)^2, x)
```

3.373.8 Giac [F]

$$\int \frac{x(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x}{(cx^2+a)^2}\,dx$$

input
```
integrate(x*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((e*x + d)^n*x/(c*x^2 + a)^2, x)
```

3.373.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{x\,(d+e\,x)^n}{(c\,x^2+a)^2}\,dx$$

input
```
int((x*(d + e*x)^n)/(a + c*x^2)^2,x)
```

output
```
int((x*(d + e*x)^n)/(a + c*x^2)^2, x)
```

3.374 $\int \frac{(d+ex)^n}{(a+cx^2)^2}\,dx$

3.374.1 Optimal result

Integrand size = 17, antiderivative size = 304

$$\int \frac{(d+ex)^n}{(a+cx^2)^2}\,dx = \frac{(ae+cdx)(d+ex)^{1+n}}{2a(cd^2+ae^2)(a+cx^2)} - \frac{\left(cd^2+ae^2(1-n)+\sqrt{-a}\sqrt{c}den\right)(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4(-a)^{3/2}\left(\sqrt{c}d-\sqrt{-a}e\right)(cd^2+ae^2)(1+n)} + \frac{\left(cd^2+ae^2(1-n)-\sqrt{-a}\sqrt{c}den\right)(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4(-a)^{3/2}\left(\sqrt{c}d+\sqrt{-a}e\right)(cd^2+ae^2)(1+n)}$$

output
```
1/2*(c*d*x+a*e)*(e*x+d)^(1+n)/a/(a*e^2+c*d^2)/(c*x^2+a)+1/4*(e*x+d)^(1+n)*
hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))*(c*d^2+
a*e^2*(1-n)-d*e*n*(-a)^(1/2)*c^(1/2))/(-a)^(3/2)/(a*e^2+c*d^2)/(1+n)/(e*(-
a)^(1/2)+d*c^(1/2))-1/4*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(
1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(c*d^2+a*e^2*(1-n)+d*e*n*(-a)^(1/2)*c^(1/2
))/(-a)^(3/2)/(a*e^2+c*d^2)/(1+n)/(-e*(-a)^(1/2)+d*c^(1/2))
```

3.374.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^n}{(a+cx^2)^2}\,dx = \frac{(d+ex)^{1+n}\left(\frac{2(ae+cdx)}{a+cx^2}+\frac{(cd^2-ae^2(-1+n)+\sqrt{-a}\sqrt{c}den)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)}+\frac{(-cd^2+ae^2(-1+n)+\sqrt{-c}\cdots}{\cdots}\right.}{4a\left(cd^2+ae^2\right)}$$

input
```
Integrate[(d + e*x)^n/(a + c*x^2)^2,x]
```

output
```
((d + e*x)^(1 + n)*((2*(a*e + c*d*x))/(a + c*x^2) + ((c*d^2 - a*e^2*(-1 +
n) + Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(
d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(1
 + n)) + ((-(c*d^2) + a*e^2*(-1 + n) + Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sq
rt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))))/(4*a*(c*d^2 + a*e^2))
```

3.374.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {496, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^n}{(a+cx^2)^2}\,dx$$

$$\Big\downarrow \; 496$$

$$\frac{(d+ex)^{n+1}(ae+cdx)}{2a(a+cx^2)(ae^2+cd^2)}-\frac{\int -\frac{(d+ex)^n(cd^2-cenxd+ae^2(1-n))}{cx^2+a}dx}{2a(ae^2+cd^2)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(d+ex)^n(cd^2-cenxd+ae^2(1-n))}{cx^2+a}dx}{2a(ae^2+cd^2)}+\frac{(d+ex)^{n+1}(ae+cdx)}{2a(a+cx^2)(ae^2+cd^2)}$$

$$\Big\downarrow \; 657$$

$$\frac{\int \left(\frac{\left(\sqrt{-a}\left(cd^2+ae^2(1-n)\right)+a\sqrt{c}den\right)(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)}+\frac{\left(\sqrt{-a}\left(cd^2+ae^2(1-n)\right)-a\sqrt{c}den\right)(d+ex)^n}{2a\left(\sqrt{c}x+\sqrt{-a}\right)}\right)dx}{2a\left(ae^2+cd^2\right)}+\frac{(d+ex)^{n+1}(ae+cdx)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}den+ae^2(1-n)+cd^2\right)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{-a}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{(d+ex)^{n+1}\left(-\sqrt{-a}\sqrt{c}den+ae^2(1-n)+cd^2\right)\text{Hyperge}}{2\sqrt{-a}(n+1)\left(\sqrt{-a}e+\right.}}{2a\left(ae^2+cd^2\right)}$$

$$\frac{(d+ex)^{n+1}(ae+cdx)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

input
```
Int[(d + e*x)^n/(a + c*x^2)^2,x]
```

output
```
((a*e + c*d*x)*(d + e*x)^(1 + n))/(2*a*(c*d^2 + a*e^2)*(a + c*x^2)) + (((c
*d^2 + a*e^2*(1 - n) + Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hypergeom
etric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(
2*Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - ((c*d^2 + a*e^2*(1 - n) - S
qrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n
, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*Sqrt[-a]*(Sqrt[c]*d +
Sqrt[-a]*e)*(1 + n)))/(2*a*(c*d^2 + a*e^2))
```

3.374.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.374.4 Maple [F]

$$\int \frac{(ex+d)^n}{(c\,x^2+a)^2}dx$$

input
```
int((e*x+d)^n/(c*x^2+a)^2,x)
```

output
```
int((e*x+d)^n/(c*x^2+a)^2,x)
```

3.374.5 Fricas [F]

$$\int \frac{(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)^2}\,dx$$

input
```
integrate((e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((e*x + d)^n/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

3.374.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^n}{(a+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**n/(c*x**2+a)**2,x)
```

output
```
Timed out
```

3.374.7 Maxima [F]

$$\int \frac{(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)^2}\,dx$$

input
```
integrate((e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^n/(c*x^2 + a)^2, x)
```

3.374.8 Giac [F]

$$\int \frac{(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)^2}\,dx$$

input
```
integrate((e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((e*x + d)^n/(c*x^2 + a)^2, x)
```

3.374.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(d+e\,x)^n}{(c\,x^2+a)^2}\,dx$$

input
```
int((d + e*x)^n/(a + c*x^2)^2,x)
```

output
```
int((d + e*x)^n/(a + c*x^2)^2, x)
```

3.375 $\int \frac{(d+ex)^n}{x(a+cx^2)^2}\,dx$

3.375.1 Optimal result

Integrand size = 20, antiderivative size = 489

$$\begin{aligned}
&\int \frac{(d+ex)^n}{x(a+cx^2)^2}\,dx \\
&= \frac{c(d-ex)(d+ex)^{1+n}}{2a(cd^2+ae^2)(a+cx^2)} \\
&\quad + \frac{\sqrt{c}(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a^2\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} \\
&\quad + \frac{\sqrt{c}e\left(\sqrt{c}d+\sqrt{-a}e\right)n(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4(-a)^{3/2}\left(\sqrt{c}d-\sqrt{-a}e\right)(cd^2+ae^2)(1+n)} \\
&\quad + \frac{\sqrt{c}(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2a^2\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)} \\
&\quad - \frac{\sqrt{c}e\left(\sqrt{-a}\sqrt{c}d+ae\right)n(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4a^2\left(\sqrt{c}d+\sqrt{-a}e\right)(cd^2+ae^2)(1+n)} \\
&\quad - \frac{(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{ex}{d}\right)}{a^2d(1+n)}
\end{aligned}$$

output
```
1/2*c*(-e*x+d)*(e*x+d)^(1+n)/a/(a*e^2+c*d^2)/(c*x^2+a)-(e*x+d)^(1+n)*hyper
geom([1, 1+n],[2+n],1+e*x/d)/a^2/d/(1+n)+1/2*(e*x+d)^(1+n)*hypergeom([1, 1
+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*c^(1/2)/a^2/(1+n)/(-e
*(-a)^(1/2)+d*c^(1/2))+1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*
c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))*c^(1/2)/a^2/(1+n)/(e*(-a)^(1/2)+d*c^(1/2
))+1/4*e*n*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)
^(1/2)+d*c^(1/2)))*c^(1/2)*(e*(-a)^(1/2)+d*c^(1/2))/(-a)^(3/2)/(a*e^2+c*d^
2)/(1+n)/(-e*(-a)^(1/2)+d*c^(1/2))-1/4*e*n*(e*x+d)^(1+n)*hypergeom([1, 1+n
],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))*c^(1/2)*(a*e+d*(-a)^(1/2
)*c^(1/2))/a^2/(a*e^2+c*d^2)/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))
```

3.375.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex)^n}{x\left(a+cx^2\right)^2}\,dx = \frac{(d+ex)^{1+n}\left(\frac{2ac(d-ex)}{(cd^2+ae^2)(a+cx^2)}-\frac{4\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{d+ex}{d}\right)}{d+dn}+\frac{2\sqrt{c}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)}+\right.}{}$$

input
```
Integrate[(d + e*x)^n/(x*(a + c*x^2)^2),x]
```

output
```
((d + e*x)^(1 + n)*((2*a*c*(d - e*x))/((c*d^2 + a*e^2)*(a + c*x^2)) - (4*H
ypergeometric2F1[1, 1 + n, 2 + n, (d + e*x)/d])/(d + d*n) + (2*Sqrt[c]*Hyp
ergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*
e)])/((Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) + (2*Sqrt[c]*Hypergeometric2F1[1,
1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/((Sqrt[c]*d +
 Sqrt[-a]*e)*(1 + n)) + (Sqrt[c]*e*n*((Sqrt[-a]*c*d^2 - 2*a*Sqrt[c]*d*e +
(-a)^(3/2)*e^2)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sq
rt[c]*d - Sqrt[-a]*e)] + (-(Sqrt[-a]*c*d^2) - 2*a*Sqrt[c]*d*e + Sqrt[-a]*a
*e^2)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d +
Sqrt[-a]*e)]))/((c*d^2 + a*e^2)^2*(1 + n))))/(4*a^2)
```

3.375.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 484, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^n}{x\left(a+cx^2\right)^2}dx$$

$$\Big\downarrow \text{ 615}$$

$$\int \left(-\frac{cx(d+ex)^n}{a^2\left(a+cx^2\right)}+\frac{(d+ex)^n}{a^2x}-\frac{cx(d+ex)^n}{a\left(a+cx^2\right)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{c}en\left(\sqrt{-a}\sqrt{c}d+ae\right)(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4a^2(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)}+\frac{\sqrt{c}(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a^2(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}+\frac{\sqrt{c}(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2a^2(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}-\frac{(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{ex}{d}+1\right)}{a^2d(n+1)}+\frac{cen\left(\frac{\sqrt{-a}e}{\sqrt{c}}+d\right)(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4(-a)^{3/2}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)\left(ae^2+cd^2\right)}+\frac{c(d-ex)(d+ex)^{n+1}}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

input
```
Int[(d + e*x)^n/(x*(a + c*x^2)^2),x]
```

output
```
(c*(d - e*x)*(d + e*x)^(1 + n))/(2*a*(c*d^2 + a*e^2)*(a + c*x^2)) + (Sqrt[
c]*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x)
)/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*a^2*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) + (c
*e*(d + (Sqrt[-a]*e)/Sqrt[c])*n*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 +
 n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*(-a)^(3/2)*(S
qrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (Sqrt[c]*(d + e*x)^(1 +
n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqr
t[-a]*e)])/(2*a^2*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) - (Sqrt[c]*e*(Sqrt[-a]
*Sqrt[c]*d + a*e)*n*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (
Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*a^2*(Sqrt[c]*d + Sqrt[-a]
*e)*(c*d^2 + a*e^2)*(1 + n)) - ((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 +
 n, 2 + n, 1 + (e*x)/d])/(a^2*d*(1 + n))
```

3.375.3.1 Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.375.4 Maple [F]

$$\int \frac{(ex+d)^n}{x\,(c\,x^2+a)^2}dx$$

input
```
int((e*x+d)^n/x/(c*x^2+a)^2,x)
```

output
```
int((e*x+d)^n/x/(c*x^2+a)^2,x)
```

3.375.5 Fricas [F]

$$\int \frac{(d+ex)^n}{x\left(a+cx^2\right)^2}\,dx=\int \frac{(ex+d)^n}{\left(cx^2+a\right)^2x}\,dx$$

input
```
integrate((e*x+d)^n/x/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((e*x + d)^n/(c^2*x^5 + 2*a*c*x^3 + a^2*x), x)
```

3.375.6 Sympy [F]

$$\int \frac{(d+ex)^n}{x\left(a+cx^2\right)^2}\,dx=\int \frac{(d+ex)^n}{x\left(a+cx^2\right)^2}\,dx$$

input
```
integrate((e*x+d)**n/x/(c*x**2+a)**2,x)
```

output
```
Integral((d + e*x)**n/(x*(a + c*x**2)**2), x)
```

3.375.7 Maxima [F]

$$\int \frac{(d+ex)^n}{x\left(a+cx^2\right)^2}\,dx=\int \frac{(ex+d)^n}{\left(cx^2+a\right)^2x}\,dx$$

input
```
integrate((e*x+d)^n/x/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^n/((c*x^2 + a)^2*x), x)
```

3.375.8 Giac [F]

$$\int \frac{(d+ex)^n}{x\left(a+cx^2\right)^2}\,dx = \int \frac{(ex+d)^n}{\left(cx^2+a\right)^2 x}\,dx$$

input
```
integrate((e*x+d)^n/x/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((e*x + d)^n/((c*x^2 + a)^2*x), x)
```

3.375.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^n}{x\left(a+cx^2\right)^2}\,dx = \int \frac{(d+e\,x)^n}{x\left(c\,x^2+a\right)^2}\,dx$$

input
```
int((d + e*x)^n/(x*(a + c*x^2)^2),x)
```

output
```
int((d + e*x)^n/(x*(a + c*x^2)^2), x)
```

3.376 $\int \frac{(d+ex)^n}{x^2(a+cx^2)^2}\,dx$

3.376.1 Optimal result

Integrand size = 20, antiderivative size = 513

$$\begin{aligned}\int \frac{(d+ex)^n}{x^2(a+cx^2)^2}\,dx &= -\frac{c(ae+cdx)(d+ex)^{1+n}}{2a^2(cd^2+ae^2)(a+cx^2)} \\ &- \frac{c(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2(-a)^{5/2}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} \\ &- \frac{c\left(cd^2+ae^2(1-n)+\sqrt{-a}\sqrt{c}den\right)(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4(-a)^{5/2}\left(\sqrt{c}d-\sqrt{-a}e\right)(cd^2+ae^2)(1+n)} \\ &+ \frac{c(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2(-a)^{5/2}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)} \\ &+ \frac{c\left(cd^2+ae^2(1-n)-\sqrt{-a}\sqrt{c}den\right)(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4(-a)^{5/2}\left(\sqrt{c}d+\sqrt{-a}e\right)(cd^2+ae^2)(1+n)} \\ &+ \frac{e(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{ex}{d}\right)}{a^2d^2(1+n)}\end{aligned}$$

output
```
-1/2*c*(c*d*x+a*e)*(e*x+d)^(1+n)/a^2/(a*e^2+c*d^2)/(c*x^2+a)+e*(e*x+d)^(1+
n)*hypergeom([2, 1+n],[2+n],1+e*x/d)/a^2/d^2/(1+n)-1/2*c*(e*x+d)^(1+n)*hyp
ergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))/(-a)^(5/2
)/(1+n)/(-e*(-a)^(1/2)+d*c^(1/2))+1/2*c*(e*x+d)^(1+n)*hypergeom([1, 1+n],[
2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))/(-a)^(5/2)/(1+n)/(e*(-a)^(1
/2)+d*c^(1/2))+1/4*c*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2
)/(e*(-a)^(1/2)+d*c^(1/2)))*(c*d^2+a*e^2*(1-n)-d*e*n*(-a)^(1/2)*c^(1/2))/(
-a)^(5/2)/(a*e^2+c*d^2)/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))-1/4*c*(e*x+d)^(1+n)
*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(c*d^
2+a*e^2*(1-n)+d*e*n*(-a)^(1/2)*c^(1/2))/(-a)^(5/2)/(a*e^2+c*d^2)/(1+n)/(-e
*(-a)^(1/2)+d*c^(1/2))
```

3.376.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^n}{x^2\left(a+cx^2\right)^2}\,dx = \frac{1}{4}(d+ex)^{1+n}\left(-\frac{2c(ae+cdx)}{a^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{2c\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{(-a)^{5/2}\left(-\sqrt{c}d+\sqrt{-a}e\right)(1+n)} + \frac{2c\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{(-a)^{5/2}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)} + \frac{ac\left(\frac{\left(cd^2-ae^2(-1+n)+\sqrt{-a}\sqrt{c}den\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{c}d-\sqrt{-a}e} - \frac{\left(cd^2-ae^2(-1+n)-\sqrt{-a}\sqrt{c}den\right)\text{Hypergeometric2F}\ldots}{\sqrt{c}d+\sqrt{-a}e}\ldots}{(-a)^{7/2}\left(cd^2+ae^2\right)(1+n)} + \frac{4e\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{ex}{d}\right)}{a^2d^2(1+n)}\right)$$

input
```
Integrate[(d + e*x)^n/(x^2*(a + c*x^2)^2),x]
```

output
```
((d + e*x)^(1 + n)*((-2*c*(a*e + c*d*x))/(a^2*(c*d^2 + a*e^2)*(a + c*x^2))
 + (2*c*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d
- Sqrt[-a]*e)])/((-a)^(5/2)*(-(Sqrt[c]*d) + Sqrt[-a]*e)*(1 + n)) + (2*c*Hy
pergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]
*e)])/((-a)^(5/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) + (a*c*(((c*d^2 - a*e^
2*(-1 + n) + Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (S
qrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[c]*d - Sqrt[-a]*e) - ((
c*d^2 - a*e^2*(-1 + n) - Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 +
n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[c]*d + Sqrt
[-a]*e)))/((-a)^(7/2)*(c*d^2 + a*e^2)*(1 + n)) + (4*e*Hypergeometric2F1[2,
 1 + n, 2 + n, 1 + (e*x)/d])/(a^2*d^2*(1 + n))))/4
```

3.376.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^n}{x^2(a+cx^2)^2}dx$$

$$\Big\downarrow \text{615}$$

$$\int \left(-\frac{c(d+ex)^n}{a^2(a+cx^2)}+\frac{(d+ex)^n}{a^2x^2}-\frac{c(d+ex)^n}{a(a+cx^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c(d+ex)^{n+1}(ae+cdx)}{2a^2(a+cx^2)(ae^2+cd^2)}+\frac{e(d+ex)^{n+1}\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{ex}{d}+1\right)}{a^2d^2(n+1)}-\frac{c(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}den+ae^2(1-n)+cd^2\right)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4(-a)^{5/2}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)(ae^2+cd^2)}+\frac{c(d+ex)^{n+1}\left(-\sqrt{-a}\sqrt{c}den+ae^2(1-n)+cd^2\right)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4(-a)^{5/2}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)(ae^2+cd^2)}-\frac{c(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2(-a)^{5/2}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}+\frac{c(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2(-a)^{5/2}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

input
```
Int[(d + e*x)^n/(x^2*(a + c*x^2)^2),x]
```

output
```
-1/2*(c*(a*e + c*d*x)*(d + e*x)^(1 + n))/(a^2*(c*d^2 + a*e^2)*(a + c*x^2))
 - (c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e
*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*(-a)^(5/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(1
+ n)) - (c*(c*d^2 + a*e^2*(1 - n) + Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 +
 n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sq
rt[-a]*e)])/(4*(-a)^(5/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)
) + (c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d +
e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*(-a)^(5/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(1
 + n)) + (c*(c*d^2 + a*e^2*(1 - n) - Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1
+ n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + S
qrt[-a]*e)])/(4*(-a)^(5/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n
)) + (e*(d + e*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (e*x)/d])
/(a^2*d^2*(1 + n))
```

3.376.3.1 Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.376.4 Maple [F]

$$\int \frac{(ex+d)^n}{x^2\,(c\,x^2+a)^2}dx$$

input
```
int((e*x+d)^n/x^2/(c*x^2+a)^2,x)
```

output
```
int((e*x+d)^n/x^2/(c*x^2+a)^2,x)
```

3.376.5 Fricas [F]

$$\int \frac{(d+ex)^n}{x^2(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)^2x^2}\,dx$$

input
```
integrate((e*x+d)^n/x^2/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((e*x + d)^n/(c^2*x^6 + 2*a*c*x^4 + a^2*x^2), x)
```

3.376.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^n}{x^2(a+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**n/x**2/(c*x**2+a)**2,x)
```

output
```
Timed out
```

3.376.7 Maxima [F]

$$\int \frac{(d+ex)^n}{x^2(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)^2x^2}\,dx$$

input
```
integrate((e*x+d)^n/x^2/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^n/((c*x^2 + a)^2*x^2), x)
```

3.376.8 Giac [F]

$$\int \frac{(d+ex)^n}{x^2\left(a+cx^2\right)^2}\,dx=\int \frac{(ex+d)^n}{\left(cx^2+a\right)^2x^2}\,dx$$

input
```
integrate((e*x+d)^n/x^2/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((e*x + d)^n/((c*x^2 + a)^2*x^2), x)
```

3.376.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^n}{x^2\left(a+cx^2\right)^2}\,dx=\int \frac{(d+e\,x)^n}{x^2\,\left(c\,x^2+a\right)^2}\,dx$$

input
```
int((d + e*x)^n/(x^2*(a + c*x^2)^2),x)
```

output
```
int((d + e*x)^n/(x^2*(a + c*x^2)^2), x)
```

3.377 $\int (gx)^m(d+ex)^n \left(a+cx^2\right)^2 \, dx$

3.377.1 Optimal result

Integrand size = 22, antiderivative size = 399

$$\int (gx)^m(d+ex)^n \left(a+cx^2\right)^2 \, dx = -\frac{cd(2+m)\left(cd^2(12+7m+m^2)+2ae^2(20+m^2+9n+n^2+m(9+2n))\right)(gx)^{1+m}(d+ex)^{1+n}}{e^4g(2+m+n)(3+m+n)(4+m+n)(5+m+n)} + \frac{c\left(cd^2(12+7m+m^2)+2ae^2(20+m^2+9n+n^2+m(9+2n))\right)(gx)^{2+m}(d+ex)^{1+n}}{e^3g^2(3+m+n)(4+m+n)(5+m+n)} - \frac{c^2d(4+m)(gx)^{3+m}(d+ex)^{1+n}}{e^2g^3(4+m+n)(5+m+n)} + \frac{c^2(gx)^{4+m}(d+ex)^{1+n}}{eg^4(5+m+n)} + \frac{\left(a^2e^4(2+m+n)(3+m+n)(4+m+n)(5+m+n)+cd^2(1+m)(2+m)\left(cd^2(12+7m+m^2)+2a\right.\right.}{e^4g(1+m)(2+m+n)(}$$

output
```
-c*d*(2+m)*(c*d^2*(m^2+7*m+12)+2*a*e^2*(20+m^2+9*n+n^2+m*(9+2*n)))*(g*x)^(
1+m)*(e*x+d)^(1+n)/e^4/g/(2+m+n)/(3+m+n)/(4+m+n)/(5+m+n)+c*(c*d^2*(m^2+7*m
+12)+2*a*e^2*(20+m^2+9*n+n^2+m*(9+2*n)))*(g*x)^(2+m)*(e*x+d)^(1+n)/e^3/g^2
/(3+m+n)/(4+m+n)/(5+m+n)-c^2*d*(4+m)*(g*x)^(3+m)*(e*x+d)^(1+n)/e^2/g^3/(4+
m+n)/(5+m+n)+c^2*(g*x)^(4+m)*(e*x+d)^(1+n)/e/g^4/(5+m+n)+(a^2*e^4*(2+m+n)*
(3+m+n)*(4+m+n)*(5+m+n)+c*d^2*(1+m)*(2+m)*(c*d^2*(m^2+7*m+12)+2*a*e^2*(20+
m^2+9*n+n^2+m*(9+2*n))))*(g*x)^(1+m)*(e*x+d)^n*hypergeom([-n, 1+m],[2+m],-
e*x/d)/e^4/g/(1+m)/(2+m+n)/(3+m+n)/(4+m+n)/(5+m+n)/((1+e*x/d)^n)
```

3.377.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.69

$$\int (gx)^m(d+ex)^n\left(a+cx^2\right)^2\,dx$$
$$= \frac{x(gx)^m(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\left(c^2d^4\,\text{Hypergeometric2F1}\left(1+m,-4-n,2+m,-\frac{ex}{d}\right)-4c^2d^4\,\text{Hypergeome}\right.}{}$$

input
```
Integrate[(g*x)^m*(d + e*x)^n*(a + c*x^2)^2,x]
```

output
```
(x*(g*x)^m*(d + e*x)^n*(c^2*d^4*Hypergeometric2F1[1 + m, -4 - n, 2 + m, -(
(e*x)/d)] - 4*c^2*d^4*Hypergeometric2F1[1 + m, -3 - n, 2 + m, -((e*x)/d)]
+ 6*c^2*d^4*Hypergeometric2F1[1 + m, -2 - n, 2 + m, -((e*x)/d)] + 2*a*c*d^
2*e^2*Hypergeometric2F1[1 + m, -2 - n, 2 + m, -((e*x)/d)] - 4*c^2*d^4*Hype
rgeometric2F1[1 + m, -1 - n, 2 + m, -((e*x)/d)] - 4*a*c*d^2*e^2*Hypergeome
tric2F1[1 + m, -1 - n, 2 + m, -((e*x)/d)] + c^2*d^4*Hypergeometric2F1[1 +
m, -n, 2 + m, -((e*x)/d)] + 2*a*c*d^2*e^2*Hypergeometric2F1[1 + m, -n, 2 +
 m, -((e*x)/d)] + a^2*e^4*Hypergeometric2F1[1 + m, -n, 2 + m, -((e*x)/d)])
)/(e^4*(1 + m)*(1 + (e*x)/d)^n)
```

3.377.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {521, 2125, 27, 521, 27, 90, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^2(gx)^m(d+ex)^n\,dx$$

$$\Big\downarrow \text{ 521}$$

$$\frac{\int (gx)^m(d+ex)^n\left(-c^2d(m+4)x^3g^4+2ace(m+n+5)x^2g^4+a^2e(m+n+5)g^4\right)dx}{eg^4(m+n+5)}+\frac{c^2(gx)^{m+4}(d+ex)^{n+1}}{eg^4(m+n+5)}$$

$$\Big\downarrow \text{ 2125}$$

$$\frac{\frac{\int g^7(gx)^m(d+ex)^n\left(a^2(m+n+4)(m+n+5)e^2+c\left(c\left(m^2+7m+12\right)d^2+2ae^2\left(m^2+(2n+9)m+n^2+9n+20\right)\right)x^2\right)dx}{eg^3(m+n+4)}-\frac{c^2dg(m+4)(gx)^{m+3}(d+ex)^{n+}}{e(m+n+4)}}{eg^4(m+n+5)}$$

$$\frac{c^2(gx)^{m+4}(d+ex)^{n+1}}{eg^4(m+n+5)}$$

$\big\downarrow$ 27

$$\frac{\frac{g^4\int(gx)^m(d+ex)^n\left(a^2(m+n+4)(m+n+5)e^2+c\left(c\left(m^2+7m+12\right)d^2+2ae^2\left(m^2+(2n+9)m+n^2+9n+20\right)\right)x^2\right)dx}{e(m+n+4)}-\frac{c^2dg(m+4)(gx)^{m+3}(d+ex)^{n+}}{e(m+n+4)}}{eg^4(m+n+5)}$$

$$\frac{c^2(gx)^{m+4}(d+ex)^{n+1}}{eg^4(m+n+5)}$$

$\big\downarrow$ 521

$$\frac{\frac{g^4\left(\frac{\int g^2(gx)^m(d+ex)^n\left(a^2e^3(m+n+3)(m+n+4)(m+n+5)-cd(m+2)\left(c\left(m^2+7m+12\right)d^2+2ae^2\left(m^2+(2n+9)m+n^2+9n+20\right)\right)x\right)dx}{eg^2(m+n+3)}+\frac{c(gx)^{m+2}(d+ex)^{n+1}\left(2ae^{2}\right.}{}\right.}{e(m+n+4)}}{eg^4(m+n+5)}$$

$$\frac{c^2(gx)^{m+4}(d+ex)^{n+1}}{eg^4(m+n+5)}$$

$\big\downarrow$ 27

$$\frac{\frac{g^4\left(\frac{\int(gx)^m(d+ex)^n\left(a^2e^3(m+n+3)(m+n+4)(m+n+5)-cd(m+2)\left(c\left(m^2+7m+12\right)d^2+2ae^2\left(m^2+(2n+9)m+n^2+9n+20\right)\right)x\right)dx}{e(m+n+3)}+\frac{c(gx)^{m+2}(d+ex)^{n+1}\left(2ae^2\left(\right.\right.}{}\right.}{e(m+n+4)}}{eg^4(m+n+5)}$$

$$\frac{c^2(gx)^{m+4}(d+ex)^{n+1}}{eg^4(m+n+5)}$$

$\big\downarrow$ 90

$$\frac{g^4\left(\frac{\frac{\left(a^2e^4(m+n+3)(m+n+4)(m+n+5)+\frac{cd^2(m+1)(m+2)\left(2ae^2\left(m^2+m(2n+9)+n^2+9n+20\right)+cd^2\left(m^2+7m+12\right)\right)}{m+n+2}\right)\int(gx)^m(d+ex)^ndx}{e}-\frac{cd(m+2)(gx)^{m+1}(d}{}}{e(m+n+3)}\right.}{e(m+n+4)}$$

$$\frac{c^2(gx)^{m+4}(d+ex)^{n+1}}{eg^4(m+n+5)}$$

$\big\downarrow$ 76

3.377. $\int(gx)^m(d+ex)^n\left(a+cx^2\right)^2\,dx$

$$g^4 \left(\frac{\frac{(d+ex)^n \left(\frac{ex}{d}+1\right)^{-n} \left(a^2 e^4 (m+n+3)(m+n+4)(m+n+5)+\frac{cd^2(m+1)(m+2)\left(2ae^2\left(m^2+m(2n+9)+n^2+9n+20\right)+cd^2\left(m^2+7m+12\right)\right)}{m+n+2}\right) \int (gx)^m \left(\frac{ex}{d}+1\right)^n dx}{e}}{e(m+n+3)} \right.$$

$$\overline{\qquad e}$$

$$\frac{c^2(gx)^{m+4}(d+ex)^{n+1}}{eg^4(m+n+5)}$$

$$\Big\downarrow \; 74$$

$$g^4 \left(\frac{\frac{(gx)^{m+1}(d+ex)^n \left(\frac{ex}{d}+1\right)^{-n} \left(a^2 e^4 (m+n+3)(m+n+4)(m+n+5)+\frac{cd^2(m+1)(m+2)\left(2ae^2\left(m^2+m(2n+9)+n^2+9n+20\right)+cd^2\left(m^2+7m+12\right)\right)}{m+n+2}\right) \text{Hypergeome}}{eg(m+1)}}{e(m+n+3)} \right.$$

$$\frac{c^2(gx)^{m+4}(d+ex)^{n+1}}{eg^4(m+n+5)}$$

input
```
Int[(g*x)^m*(d + e*x)^n*(a + c*x^2)^2,x]
```

output
```
(c^2*(g*x)^(4 + m)*(d + e*x)^(1 + n))/(e*g^4*(5 + m + n)) + (-((c^2*d*g*(4
 + m)*(g*x)^(3 + m)*(d + e*x)^(1 + n))/(e*(4 + m + n))) + (g^4*((c*(c*d^2*
(12 + 7*m + m^2) + 2*a*e^2*(20 + m^2 + 9*n + n^2 + m*(9 + 2*n)))*(g*x)^(2
+ m)*(d + e*x)^(1 + n))/(e*g^2*(3 + m + n)) + (-((c*d*(2 + m)*(c*d^2*(12 +
 7*m + m^2) + 2*a*e^2*(20 + m^2 + 9*n + n^2 + m*(9 + 2*n)))*(g*x)^(1 + m)*
(d + e*x)^(1 + n))/(e*g*(2 + m + n))) + ((a^2*e^4*(3 + m + n)*(4 + m + n)*
(5 + m + n) + (c*d^2*(1 + m)*(2 + m)*(c*d^2*(12 + 7*m + m^2) + 2*a*e^2*(20
 + m^2 + 9*n + n^2 + m*(9 + 2*n))))/(2 + m + n))*(g*x)^(1 + m)*(d + e*x)^n
*Hypergeometric2F1[1 + m, -n, 2 + m, -((e*x)/d)])/(e*g*(1 + m)*(1 + (e*x)/
d)^n))/(e*(3 + m + n))))/(e*(4 + m + n)))/(e*g^4*(5 + m + n))
```

3.377. $\int (gx)^m(d+ex)^n \left(a+cx^2\right)^2 \, dx$

3.377.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 521
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> Simp[b^p*(e*x)^(m + 2*p)*((c + d*x)^(n + 1)/(d*e^(2*p)*(m + n
 + 2*p + 1))), x] + Simp[1/(d*e^(2*p)*(m + n + 2*p + 1))   Int[(e*x)^m*(c +
 d*x)^n*ExpandToSum[d*(m + n + 2*p + 1)*(e^(2*p)*(a + b*x^2)^p - b^p*(e*x)^
(2*p)) - b^p*(e*c)*(m + 2*p)*(e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c,
 d, e}, x] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] &&  !IntegerQ[m] &&  !I
ntegerQ[n]
```

rule 2125
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[k*(a + b*x
)^(m + q)*((c + d*x)^(n + 1)/(d*b^q*(m + n + q + 1))), x] + Simp[1/(d*b^q*(
m + n + q + 1))   Int[(a + b*x)^m*(c + d*x)^n*ExpandToSum[d*b^q*(m + n + q
+ 1)*Px - d*k*(m + n + q + 1)*(a + b*x)^q - k*(b*c - a*d)*(m + q)*(a + b*x)
^(q - 1), x], x], x] /; NeQ[m + n + q + 1, 0]] /; FreeQ[{a, b, c, d, m, n},
 x] && PolyQ[Px, x]
```

3.377. $\int (gx)^m(d+ex)^n\left(a+cx^2\right)^2\,dx$

3.377.4 Maple [F]

$$\int (gx)^m (ex+d)^n \left(c\,x^2+a\right)^2 dx$$

input
```
int((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x)
```

output
```
int((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x)
```

3.377.5 Fricas [F]

$$\int (gx)^m (d+ex)^n \left(a+cx^2\right)^2 \, dx = \int \left(cx^2+a\right)^2 (ex+d)^n (gx)^m \, dx$$

input
```
integrate((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((c^2*x^4 + 2*a*c*x^2 + a^2)*(e*x + d)^n*(g*x)^m, x)
```

3.377.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 16.57 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.32

$$\int (gx)^m (d+ex)^n \left(a+cx^2\right)^2 \, dx = \frac{a^2 d^n g^m x^{m+1} \Gamma(m+1)\, {}_2F_1\left(\begin{matrix} -n, m+1 \\ m+2 \end{matrix}\middle| \frac{exe^{i\pi}}{d}\right)}{\Gamma(m+2)} + \frac{2acd^n g^m x^{m+3} \Gamma(m+3)\, {}_2F_1\left(\begin{matrix} -n, m+3 \\ m+4 \end{matrix}\middle| \frac{exe^{i\pi}}{d}\right)}{\Gamma(m+4)} + \frac{c^2 d^n g^m x^{m+5} \Gamma(m+5)\, {}_2F_1\left(\begin{matrix} -n, m+5 \\ m+6 \end{matrix}\middle| \frac{exe^{i\pi}}{d}\right)}{\Gamma(m+6)}$$

input
```
integrate((g*x)**m*(e*x+d)**n*(c*x**2+a)**2,x)
```

output
```
a**2*d**n*g**m*x**(m + 1)*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), e*x*ex
p_polar(I*pi)/d)/gamma(m + 2) + 2*a*c*d**n*g**m*x**(m + 3)*gamma(m + 3)*hy
per((-n, m + 3), (m + 4,), e*x*exp_polar(I*pi)/d)/gamma(m + 4) + c**2*d**n
*g**m*x**(m + 5)*gamma(m + 5)*hyper((-n, m + 5), (m + 6,), e*x*exp_polar(I
*pi)/d)/gamma(m + 6)
```

3.377.7 Maxima [F]

$$\int (gx)^m (d+ex)^n \left(a+cx^2\right)^2 dx = \int (cx^2+a)^2 (ex+d)^n (gx)^m dx$$

input
```
integrate((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)^2*(e*x + d)^n*(g*x)^m, x)
```

3.377.8 Giac [F]

$$\int (gx)^m (d+ex)^n \left(a+cx^2\right)^2 dx = \int (cx^2+a)^2 (ex+d)^n (gx)^m dx$$

input
```
integrate((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)^2*(e*x + d)^n*(g*x)^m, x)
```

3.377.9 Mupad [F(-1)]

Timed out.

$$\int (gx)^m (d+ex)^n \left(a+cx^2\right)^2 dx = \int (g\,x)^m \left(c\,x^2+a\right)^2 (d+e\,x)^n\, dx$$

input
```
int((g*x)^m*(a + c*x^2)^2*(d + e*x)^n,x)
```

output
```
int((g*x)^m*(a + c*x^2)^2*(d + e*x)^n, x)
```

3.378 $\int (gx)^m(d+ex)^n\left(a+cx^2\right)\,dx$

3.378.1 Optimal result

Integrand size = 20, antiderivative size = 164

$$\int (gx)^m(d+ex)^n\left(a+cx^2\right)\,dx=-\frac{cd(2+m)(gx)^{1+m}(d+ex)^{1+n}}{e^2g(2+m+n)(3+m+n)}+\frac{c(gx)^{2+m}(d+ex)^{1+n}}{eg^2(3+m+n)}+\frac{\left(cd^2(1+m)(2+m)+ae^2(2+m+n)(3+m+n)\right)(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\text{Hypergeometric2F1}}{e^2g(1+m)(2+m+n)(3+m+n)}$$

output
```
-c*d*(2+m)*(g*x)^(1+m)*(e*x+d)^(1+n)/e^2/g/(2+m+n)/(3+m+n)+c*(g*x)^(2+m)*(
e*x+d)^(1+n)/e/g^2/(3+m+n)+(c*d^2*(1+m)*(2+m)+a*e^2*(2+m+n)*(3+m+n))*(g*x)
^(1+m)*(e*x+d)^n*hypergeom([-n, 1+m],[2+m],-e*x/d)/e^2/g/(1+m)/(2+m+n)/(3+
m+n)/((1+e*x/d)^n)
```

3.378.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.69

$$\int (gx)^m(d+ex)^n\left(a+cx^2\right)\,dx$$
$$=\frac{x(gx)^m(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\left(cd^2\,\text{Hypergeometric2F1}\left(1+m,-2-n,2+m,-\frac{ex}{d}\right)-2cd^2\,\text{Hypergeometr}\right.}{e^2(1+m}$$

input
```
Integrate[(g*x)^m*(d + e*x)^n*(a + c*x^2),x]
```

output
```
(x*(g*x)^m*(d + e*x)^n*(c*d^2*Hypergeometric2F1[1 + m, -2 - n, 2 + m, -((e
*x)/d)] - 2*c*d^2*Hypergeometric2F1[1 + m, -1 - n, 2 + m, -((e*x)/d)] + (c
*d^2 + a*e^2)*Hypergeometric2F1[1 + m, -n, 2 + m, -((e*x)/d)]))/(e^2*(1 +
m)*(1 + (e*x)/d)^n)
```

3.378.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {521, 27, 90, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right) (gx)^m (d+ex)^n \, dx$$

$$\Big\downarrow \text{ 521}$$

$$\frac{\int g^2 (gx)^m (d+ex)^n (ae(m+n+3)-cd(m+2)x)dx}{eg^2(m+n+3)}+\frac{c(gx)^{m+2}(d+ex)^{n+1}}{eg^2(m+n+3)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int (gx)^m (d+ex)^n (ae(m+n+3)-cd(m+2)x)dx}{e(m+n+3)}+\frac{c(gx)^{m+2}(d+ex)^{n+1}}{eg^2(m+n+3)}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{\left(ae(m+n+3)+\frac{cd^2(m+1)(m+2)}{e(m+n+2)}\right)\int (gx)^m (d+ex)^n dx-\frac{cd(m+2)(gx)^{m+1}(d+ex)^{n+1}}{eg(m+n+2)}}{e(m+n+3)}+\frac{c(gx)^{m+2}(d+ex)^{n+1}}{eg^2(m+n+3)}$$

$$\Big\downarrow \text{ 76}$$

$$\frac{(d+ex)^n \left(\frac{ex}{d}+1\right)^{-n} \left(ae(m+n+3)+\frac{cd^2(m+1)(m+2)}{e(m+n+2)}\right)\int (gx)^m \left(\frac{ex}{d}+1\right)^n dx-\frac{cd(m+2)(gx)^{m+1}(d+ex)^{n+1}}{eg(m+n+2)}}{e(m+n+3)}+\frac{c(gx)^{m+2}(d+ex)^{n+1}}{eg^2(m+n+3)}$$

$$\Big\downarrow \text{ 74}$$

$$\frac{\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\left(ae(m+n+3)+\frac{cd^2(m+1)(m+2)}{e(m+n+2)}\right)\text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{ex}{d}\right)}{g(m+1)}-\frac{cd(m+2)(gx)^{m+1}(d+ex)^{n+1}}{eg(m+n+2)}}{e(m+n+3)}+\frac{c(gx)^{m+2}(d+ex)^{n+1}}{eg^2(m+n+3)}$$

input
```
Int[(g*x)^m*(d + e*x)^n*(a + c*x^2),x]
```

output
```
(c*(g*x)^(2 + m)*(d + e*x)^(1 + n))/(e*g^2*(3 + m + n)) + (-((c*d*(2 + m)*
(g*x)^(1 + m)*(d + e*x)^(1 + n))/(e*g*(2 + m + n))) + (((c*d^2*(1 + m)*(2
+ m))/(e*(2 + m + n)) + a*e*(3 + m + n))*(g*x)^(1 + m)*(d + e*x)^n*Hyperge
ometric2F1[1 + m, -n, 2 + m, -((e*x)/d)])/(g*(1 + m)*(1 + (e*x)/d)^n))/(e*
(3 + m + n))
```

3.378.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 521
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> Simp[b^p*(e*x)^(m + 2*p)*((c + d*x)^(n + 1)/(d*e^(2*p)*(m + n
 + 2*p + 1))), x] + Simp[1/(d*e^(2*p)*(m + n + 2*p + 1))   Int[(e*x)^m*(c +
 d*x)^n*ExpandToSum[d*(m + n + 2*p + 1)*(e^(2*p)*(a + b*x^2)^p - b^p*(e*x)^
(2*p)) - b^p*(e*c)*(m + 2*p)*(e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c,
 d, e}, x] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] &&  !IntegerQ[m] &&  !I
ntegerQ[n]
```

3.378.4 Maple [F]

$$\int (gx)^m (ex+d)^n (c\,x^2+a)\,dx$$

input
```
int((g*x)^m*(e*x+d)^n*(c*x^2+a),x)
```

output
```
int((g*x)^m*(e*x+d)^n*(c*x^2+a),x)
```

3.378.5 Fricas [F]

$$\int (gx)^m(d+ex)^n\left(a+cx^2\right)\,dx=\int (cx^2+a)(ex+d)^n(gx)^m\,dx$$

input
```
integrate((g*x)^m*(e*x+d)^n*(c*x^2+a),x, algorithm="fricas")
```

output
```
integral((c*x^2 + a)*(e*x + d)^n*(g*x)^m, x)
```

3.378.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.49

$$\int (gx)^m(d+ex)^n\left(a+cx^2\right)\,dx=\frac{ad^ng^mx^{m+1}\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}-n,m+1\\m+2\end{matrix}\middle|\frac{exe^{i\pi}}{d}\right)}{\Gamma(m+2)}+\frac{cd^ng^mx^{m+3}\Gamma(m+3)\,{}_2F_1\left(\begin{matrix}-n,m+3\\m+4\end{matrix}\middle|\frac{exe^{i\pi}}{d}\right)}{\Gamma(m+4)}$$

input
```
integrate((g*x)**m*(e*x+d)**n*(c*x**2+a),x)
```

output
```
a*d**n*g**m*x**(m + 1)*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), e*x*exp_p
olar(I*pi)/d)/gamma(m + 2) + c*d**n*g**m*x**(m + 3)*gamma(m + 3)*hyper((-n
, m + 3), (m + 4,), e*x*exp_polar(I*pi)/d)/gamma(m + 4)
```

3.378.7 Maxima [F]

$$\int (gx)^m (d+ex)^n \left(a+cx^2\right) dx = \int (cx^2+a)(ex+d)^n (gx)^m dx$$

input
```
integrate((g*x)^m*(e*x+d)^n*(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)*(e*x + d)^n*(g*x)^m, x)
```

3.378.8 Giac [F]

$$\int (gx)^m (d+ex)^n \left(a+cx^2\right) dx = \int (cx^2+a)(ex+d)^n (gx)^m dx$$

input
```
integrate((g*x)^m*(e*x+d)^n*(c*x^2+a),x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)*(e*x + d)^n*(g*x)^m, x)
```

3.378.9 Mupad [F(-1)]

Timed out.

$$\int (gx)^m (d+ex)^n \left(a+cx^2\right) dx = \int (g\,x)^m \left(c\,x^2+a\right) (d+e\,x)^n dx$$

input
```
int((g*x)^m*(a + c*x^2)*(d + e*x)^n,x)
```

output
```
int((g*x)^m*(a + c*x^2)*(d + e*x)^n, x)
```

3.379 $\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx$

3.379.1 Optimal result

Integrand size = 22, antiderivative size = 148

$$\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx = \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\text{AppellF1}\left(1+m,-n,1,2+m,-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{2ag(1+m)} + \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\text{AppellF1}\left(1+m,-n,1,2+m,-\frac{ex}{d},\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{2ag(1+m)}$$

output
```
1/2*(g*x)^(1+m)*(e*x+d)^n*AppellF1(1+m,-n,1,2+m,-e*x/d,-x*c^(1/2)/(-a)^(1/
2))/a/g/(1+m)/((1+e*x/d)^n)+1/2*(g*x)^(1+m)*(e*x+d)^n*AppellF1(1+m,1,-n,2+
m,x*c^(1/2)/(-a)^(1/2),-e*x/d)/a/g/(1+m)/((1+e*x/d)^n)
```

3.379.2 Mathematica [F]

$$\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx = \int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx$$

input
```
Integrate[((g*x)^m*(d + e*x)^n)/(a + c*x^2),x]
```

output
```
Integrate[((g*x)^m*(d + e*x)^n)/(a + c*x^2), x]
```

3.379.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx$$

$$\Big\downarrow \text{615}$$

$$\int \left(\frac{\sqrt{-a}(gx)^m(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)}+\frac{\sqrt{-a}(gx)^m(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{2ag(m+1)}+\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{ex}{d},\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{2ag(m+1)}$$

input
```
Int[((g*x)^m*(d + e*x)^n)/(a + c*x^2),x]
```

output
```
((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), -((Sq
rt[c]*x)/Sqrt[-a])])/(2*a*g*(1 + m)*(1 + (e*x)/d)^n) + ((g*x)^(1 + m)*(d +
 e*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), (Sqrt[c]*x)/Sqrt[-a]])/(
2*a*g*(1 + m)*(1 + (e*x)/d)^n)
```

3.379.3.1 Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.379.4 Maple [F]

$$\int \frac{(gx)^{m} (ex+d)^{n}}{c\,x^{2}+a}dx$$

input
```
int((g*x)^m*(e*x+d)^n/(c*x^2+a),x)
```

output
```
int((g*x)^m*(e*x+d)^n/(c*x^2+a),x)
```

3.379.5 Fricas [F]

$$\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n(gx)^m}{cx^2+a}\,dx$$

input
```
integrate((g*x)^m*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
```

output
```
integral((e*x + d)^n*(g*x)^m/(c*x^2 + a), x)
```

3.379.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx = \text{Timed out}$$

input
```
integrate((g*x)**m*(e*x+d)**n/(c*x**2+a),x)
```

output
```
Timed out
```

3.379.7 Maxima [F]

$$\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n(gx)^m}{cx^2+a}\,dx$$

input
```
integrate((g*x)^m*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a), x)
```

3.379.8 Giac [F]

$$\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n(gx)^m}{cx^2+a}\,dx$$

input
```
integrate((g*x)^m*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
```

output
```
integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a), x)
```

3.379.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx = \int \frac{(g\,x)^m\,(d+e\,x)^n}{c\,x^2+a}\,dx$$

input
```
int(((g*x)^m*(d + e*x)^n)/(a + c*x^2),x)
```

output
```
int(((g*x)^m*(d + e*x)^n)/(a + c*x^2), x)
```

3.380 $\int \frac{(gx)^m(d+ex)^n}{(a+cx^2)^2}\,dx$

3.380.1 Optimal result

Integrand size = 22, antiderivative size = 295

$$\begin{aligned}\int \frac{(gx)^m(d+ex)^n}{(a+cx^2)^2}\,dx &= \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\text{AppellF1}\left(1+m,-n,1,2+m,-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(1+m)}\\ &+\frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\text{AppellF1}\left(1+m,-n,1,2+m,-\frac{ex}{d},\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(1+m)}\\ &+\frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\text{AppellF1}\left(1+m,-n,2,2+m,-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(1+m)}\\ &+\frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\text{AppellF1}\left(1+m,-n,2,2+m,-\frac{ex}{d},\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(1+m)}\end{aligned}$$

output
```
1/4*(g*x)^(1+m)*(e*x+d)^n*AppellF1(1+m,-n,1,2+m,-e*x/d,-x*c^(1/2)/(-a)^(1/
2))/a^2/g/(1+m)/((1+e*x/d)^n)+1/4*(g*x)^(1+m)*(e*x+d)^n*AppellF1(1+m,1,-n,
2+m,x*c^(1/2)/(-a)^(1/2),-e*x/d)/a^2/g/(1+m)/((1+e*x/d)^n)+1/4*(g*x)^(1+m)
*(e*x+d)^n*AppellF1(1+m,-n,2,2+m,-e*x/d,-x*c^(1/2)/(-a)^(1/2))/a^2/g/(1+m)
/((1+e*x/d)^n)+1/4*(g*x)^(1+m)*(e*x+d)^n*AppellF1(1+m,2,-n,2+m,x*c^(1/2)/(
-a)^(1/2),-e*x/d)/a^2/g/(1+m)/((1+e*x/d)^n)
```

3.380.2 Mathematica [F]

$$\int \frac{(gx)^m(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(gx)^m(d+ex)^n}{(a+cx^2)^2}\,dx$$

input
```
Integrate[((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2,x]
```

output
```
Integrate[((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2, x]
```

3.380.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(gx)^m(d+ex)^n}{(a+cx^2)^2}\,dx$$

$$\Big\downarrow \text{615}$$

$$\int \left(-\frac{c(gx)^m(d+ex)^n}{2a(-ac-c^2x^2)}-\frac{c(gx)^m(d+ex)^n}{4a\left(\sqrt{-a}\sqrt{c}-cx\right)^2}-\frac{c(gx)^m(d+ex)^n}{4a\left(\sqrt{-a}\sqrt{c}+cx\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(m+1)}+\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{ex}{d},\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(m+1)}+\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,2,m+2,-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(m+1)}+\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,2,m+2,-\frac{ex}{d},\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(m+1)}$$

input
```
Int[((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2,x]
```

output
```
((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), -((Sq
rt[c]*x)/Sqrt[-a])])/(4*a^2*g*(1 + m)*(1 + (e*x)/d)^n) + ((g*x)^(1 + m)*(d
 + e*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), (Sqrt[c]*x)/Sqrt[-a]])
/(4*a^2*g*(1 + m)*(1 + (e*x)/d)^n) + ((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1
 + m, -n, 2, 2 + m, -((e*x)/d), -((Sqrt[c]*x)/Sqrt[-a])])/(4*a^2*g*(1 + m)
*(1 + (e*x)/d)^n) + ((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m, -n, 2, 2 +
m, -((e*x)/d), (Sqrt[c]*x)/Sqrt[-a]])/(4*a^2*g*(1 + m)*(1 + (e*x)/d)^n)
```

3.380.3.1 Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.380.4 Maple [F]

$$\int \frac{(gx)^m (ex+d)^n}{(c\,x^2+a)^2}dx$$

input
```
int((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x)
```

output
```
int((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x)
```

3.380.5 Fricas [F]

$$\int \frac{(gx)^m(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n(gx)^m}{(cx^2+a)^2}\,dx$$

input
```
integrate((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((e*x + d)^n*(g*x)^m/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

3.380.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(gx)^m(d+ex)^n}{(a+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((g*x)**m*(e*x+d)**n/(c*x**2+a)**2,x)
```

output
```
Timed out
```

3.380.7 Maxima [F]

$$\int \frac{(gx)^m(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n(gx)^m}{(cx^2+a)^2}\,dx$$

input
```
integrate((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a)^2, x)
```

3.380.8 Giac [F]

$$\int \frac{(gx)^m(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n(gx)^m}{(cx^2+a)^2}\,dx$$

input
```
integrate((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a)^2, x)
```

3.380.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(g\,x)^m\,(d+e\,x)^n}{(c\,x^2+a)^2}\,dx$$

input
```
int(((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2,x)
```

output
```
int(((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2, x)
```

3.381 $\int x^5(d+ex)\left(a+bx^2\right)^p\,dx$

3.381.1 Optimal result

Integrand size = 18, antiderivative size = 125

$$\int x^5(d+ex)\left(a+bx^2\right)^p\,dx=\frac{a^2d\left(a+bx^2\right)^{1+p}}{2b^3(1+p)}-\frac{ad\left(a+bx^2\right)^{2+p}}{b^3(2+p)}+\frac{d\left(a+bx^2\right)^{3+p}}{2b^3(3+p)}+\frac{1}{7}ex^7\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)$$

output
```
1/2*a^2*d*(b*x^2+a)^(p+1)/b^3/(p+1)-a*d*(b*x^2+a)^(2+p)/b^3/(2+p)+1/2*d*(b
*x^2+a)^(3+p)/b^3/(3+p)+1/7*e*x^7*(b*x^2+a)^p*hypergeom([7/2, -p],[9/2],-b
*x^2/a)/((1+b*x^2/a)^p)
```

3.381.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int x^5(d+ex)\left(a+bx^2\right)^p\,dx=\frac{1}{14}\left(a+bx^2\right)^p\left(\frac{7d\left(a+bx^2\right)\left(2a^2-2ab(1+p)x^2+b^2\left(2+3p+p^2\right)x^4\right)}{b^3(1+p)(2+p)(3+p)}+2ex^7\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)\right)$$

input
```
Integrate[x^5*(d + e*x)*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*((7*d*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p
 + p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (2*e*x^7*Hypergeometric2F1[7
/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/14
```

3.381.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {542, 243, 53, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(d+ex)\left(a+bx^2\right)^p\,dx$$

$$\Big\downarrow \text{542}$$

$$d\int x^5\left(bx^2+a\right)^p dx+e\int x^6\left(bx^2+a\right)^p dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}d\int x^4\left(bx^2+a\right)^p dx^2+e\int x^6\left(bx^2+a\right)^p dx$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}d\int \left(\frac{a^2\left(bx^2+a\right)^p}{b^2}-\frac{2a\left(bx^2+a\right)^{p+1}}{b^2}+\frac{\left(bx^2+a\right)^{p+2}}{b^2}\right)dx^2+e\int x^6\left(bx^2+a\right)^p dx$$

$$\Big\downarrow \text{279}$$

$$\frac{1}{2}d\int \left(\frac{a^2\left(bx^2+a\right)^p}{b^2}-\frac{2a\left(bx^2+a\right)^{p+1}}{b^2}+\frac{\left(bx^2+a\right)^{p+2}}{b^2}\right)dx^2+ e\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^6\left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{278}$$

$$\frac{1}{2}d\int \left(\frac{a^2\left(bx^2+a\right)^p}{b^2}-\frac{2a\left(bx^2+a\right)^{p+1}}{b^2}+\frac{\left(bx^2+a\right)^{p+2}}{b^2}\right)dx^2+ \frac{1}{7}ex^7\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}d\left(\frac{a^2(a+bx^2)^{p+1}}{b^3(p+1)}-\frac{2a(a+bx^2)^{p+2}}{b^3(p+2)}+\frac{(a+bx^2)^{p+3}}{b^3(p+3)}\right)+$$
$$\frac{1}{7}ex^7(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)$$

input
```
Int[x^5*(d + e*x)*(a + b*x^2)^p,x]
```

output
```
(d*((a^2*(a + b*x^2)^(1 + p))/(b^3*(1 + p)) - (2*a*(a + b*x^2)^(2 + p))/(b
^3*(2 + p)) + (a + b*x^2)^(3 + p)/(b^3*(3 + p))))/2 + (e*x^7*(a + b*x^2)^p
*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(7*(1 + (b*x^2)/a)^p)
```

3.381.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.381.4 Maple [F]

$$\int x^5(ex+d)\left(b\,x^2+a\right)^p dx$$

input
```
int(x^5*(e*x+d)*(b*x^2+a)^p,x)
```

output
```
int(x^5*(e*x+d)*(b*x^2+a)^p,x)
```

3.381.5 Fricas [F]

$$\int x^5(d+ex)\left(a+bx^2\right)^p\,dx=\int(ex+d)\left(bx^2+a\right)^px^5\,dx$$

input
```
integrate(x^5*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e*x^6 + d*x^5)*(b*x^2 + a)^p, x)
```

3.381.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(104) = 208$.

Time = 12.80 (sec) , antiderivative size = 950, normalized size of antiderivative = 7.60

$$\int x^5(d+ex)\left(a+bx^2\right)^p\,dx=\frac{a^pex^7{}_2F_1\left(\begin{matrix}\frac{7}{2},-p\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7}+d\begin{cases}\frac{a^px^6}{6}\\ \frac{2a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{2a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{3a^2}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{4abx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{4abx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\\ -\frac{2a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2}-\frac{2a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2}-\frac{2a^2}{2ab^3+2b^4x^2}-\frac{2abx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2}-\frac{2abx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2}+\frac{b^2x^4}{2ab^3+2b^4x^2}\\ \frac{a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^3}+\frac{a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^3}-\frac{ax^2}{2b^2}+\frac{x^4}{4b}\\ \frac{2a^3(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3}-\frac{2a^2bpx^2(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3}+\frac{ab^2p^2x^4(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3}+\frac{ab^2px^4(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3}+\frac{}{2b}\end{cases}$$

input
```
integrate(x**5*(e*x+d)*(b*x**2+a)**p,x)
```

output
```
a**p*e*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + d*Piece
wise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8
*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*
a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*
x**4) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b*
*5*x**4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4
*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b
**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) +
 2*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**
4), Eq(p, -3)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*
a**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b
**4*x**2) - 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a*
b*x**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b**3
+ 2*b**4*x**2), Eq(p, -2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a**2*log(
x + sqrt(-a/b))/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a*
*3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*
a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12
*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22
*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*
p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p*...
```

3.381. $\int x^5(d+ex)\left(a+bx^2\right)^p\,dx$

3.381.7 Maxima [F]

$$\int x^5(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p x^5\,dx$$

input
```
integrate(x^5*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
e*integrate((b*x^2 + a)^p*x^6, x) + 1/2*((p^2 + 3*p + 2)*b^3*x^6 + (p^2 +
p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + 2*a^3)*(b*x^2 + a)^p*d/((p^3 + 6*p^2 + 11*p
 + 6)*b^3)
```

3.381.8 Giac [F]

$$\int x^5(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p x^5\,dx$$

input
```
integrate(x^5*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(b*x^2 + a)^p*x^5, x)
```

3.381.9 Mupad [F(-1)]

Timed out.

$$\int x^5(d+ex)\left(a+bx^2\right)^p\,dx=\int x^5\,\left(b\,x^2+a\right)^p\,(d+e\,x)\,dx$$

input
```
int(x^5*(a + b*x^2)^p*(d + e*x),x)
```

output
```
int(x^5*(a + b*x^2)^p*(d + e*x), x)
```

3.382 $\int x^4(d+ex)\left(a+bx^2\right)^p\,dx$

3.382.1 Optimal result

Integrand size = 18, antiderivative size = 125

$$\int x^4(d+ex)\left(a+bx^2\right)^p\,dx=\frac{a^2e\left(a+bx^2\right)^{1+p}}{2b^3(1+p)}-\frac{ae\left(a+bx^2\right)^{2+p}}{b^3(2+p)}+\frac{e\left(a+bx^2\right)^{3+p}}{2b^3(3+p)}+\frac{1}{5}dx^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

output
```
1/2*a^2*e*(b*x^2+a)^(p+1)/b^3/(p+1)-a*e*(b*x^2+a)^(2+p)/b^3/(2+p)+1/2*e*(b
*x^2+a)^(3+p)/b^3/(3+p)+1/5*d*x^5*(b*x^2+a)^p*hypergeom([5/2, -p],[7/2],-b
*x^2/a)/((1+b*x^2/a)^p)
```

3.382.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int x^4(d+ex)\left(a+bx^2\right)^p\,dx=\frac{1}{10}\left(a+bx^2\right)^p\left(\frac{5e\left(a+bx^2\right)\left(2a^2-2ab(1+p)x^2+b^2\left(2+3p+p^2\right)x^4\right)}{b^3(1+p)(2+p)(3+p)}+2dx^5\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)\right)$$

input
```
Integrate[x^4*(d + e*x)*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*((5*e*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p
 + p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (2*d*x^5*Hypergeometric2F1[5
/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/10
```

3.382.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {542, 243, 53, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(d+ex)\left(a+bx^2\right)^p\,dx$$

$$\Big\downarrow \text{542}$$

$$d\int x^4\left(bx^2+a\right)^p dx+e\int x^5\left(bx^2+a\right)^p dx$$

$$\Big\downarrow \text{243}$$

$$d\int x^4\left(bx^2+a\right)^p dx+\frac{1}{2}e\int x^4\left(bx^2+a\right)^p dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}e\int\left(\frac{a^2\left(bx^2+a\right)^p}{b^2}-\frac{2a\left(bx^2+a\right)^{p+1}}{b^2}+\frac{\left(bx^2+a\right)^{p+2}}{b^2}\right)dx^2+d\int x^4\left(bx^2+a\right)^p dx$$

$$\Big\downarrow \text{279}$$

$$\frac{1}{2}e\int\left(\frac{a^2\left(bx^2+a\right)^p}{b^2}-\frac{2a\left(bx^2+a\right)^{p+1}}{b^2}+\frac{\left(bx^2+a\right)^{p+2}}{b^2}\right)dx^2+d\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^4\left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{278}$$

$$\frac{1}{2}e\int\left(\frac{a^2\left(bx^2+a\right)^p}{b^2}-\frac{2a\left(bx^2+a\right)^{p+1}}{b^2}+\frac{\left(bx^2+a\right)^{p+2}}{b^2}\right)dx^2+\frac{1}{5}dx^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}e\left(\frac{a^2(a+bx^2)^{p+1}}{b^3(p+1)}-\frac{2a(a+bx^2)^{p+2}}{b^3(p+2)}+\frac{(a+bx^2)^{p+3}}{b^3(p+3)}\right)+\frac{1}{5}dx^5(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

input
```
Int[x^4*(d + e*x)*(a + b*x^2)^p,x]
```

output
```
(e*((a^2*(a + b*x^2)^(1 + p))/(b^3*(1 + p)) - (2*a*(a + b*x^2)^(2 + p))/(b
^3*(2 + p)) + (a + b*x^2)^(3 + p)/(b^3*(3 + p))))/2 + (d*x^5*(a + b*x^2)^p
*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

3.382.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.382.4 Maple [F]

$$\int x^4(ex+d)\left(b\,x^2+a\right)^p dx$$

input
```
int(x^4*(e*x+d)*(b*x^2+a)^p,x)
```

output
```
int(x^4*(e*x+d)*(b*x^2+a)^p,x)
```

3.382.5 Fricas [F]

$$\int x^4(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p x^4\,dx$$

input
```
integrate(x^4*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e*x^5 + d*x^4)*(b*x^2 + a)^p, x)
```

3.382.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(104) = 208$.

Time = 8.51 (sec) , antiderivative size = 950, normalized size of antiderivative = 7.60

$$\int x^4(d+ex)\left(a+bx^2\right)^p\,dx=\frac{a^pdx^5{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\\frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5}+e\left(\begin{cases}\frac{a^px^6}{6}\\ \frac{2a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{2a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{3a^2}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{4abx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{4abx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\\ -\frac{2a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2}-\frac{2a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2}-\frac{2a^2}{2ab^3+2b^4x^2}-\frac{2abx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2}-\frac{2abx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2}+\frac{b^2x^4}{2ab^3+2b^4x^2}\\ \frac{a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^3}+\frac{a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^3}-\frac{ax^2}{2b^2}+\frac{x^4}{4b}\\ \frac{2a^3\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3}-\frac{2a^2bpx^2\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3}+\frac{ab^2p^2x^4\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3}+\frac{ab^2px^4\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3}+\frac{}{2b^3}\end{cases}\right.$$

input
```
integrate(x**4*(e*x+d)*(b*x**2+a)**p,x)
```

output
```
a**p*d*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + e*Piece
wise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8
*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*
a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*
x**4) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b*
*5*x**4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4
*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b
**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) +
 2*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**
4), Eq(p, -3)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*
a**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b
**4*x**2) - 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a*
b*x**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b**3
+ 2*b**4*x**2), Eq(p, -2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a**2*log(
x + sqrt(-a/b))/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a*
*3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*
a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12
*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22
*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*
p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p*...
```

3.382. $\int x^4(d+ex)\left(a+bx^2\right)^p\,dx$

3.382.7 Maxima [F]

$$\int x^4(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p x^4\,dx$$

input
```
integrate(x^4*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)*(b*x^2 + a)^p*x^4, x)
```

3.382.8 Giac [F]

$$\int x^4(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p x^4\,dx$$

input
```
integrate(x^4*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(b*x^2 + a)^p*x^4, x)
```

3.382.9 Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex)\left(a+bx^2\right)^p\,dx=\int x^4\,\left(b\,x^2+a\right)^p\,\left(d+e\,x\right)\,dx$$

input
```
int(x^4*(a + b*x^2)^p*(d + e*x),x)
```

output
```
int(x^4*(a + b*x^2)^p*(d + e*x), x)
```

3.383 $\int x^3(d+ex)\left(a+bx^2\right)^p\,dx$

3.383.1 Optimal result

Integrand size = 18, antiderivative size = 100

$$\int x^3(d+ex)\left(a+bx^2\right)^p\,dx=-\frac{ad\left(a+bx^2\right)^{1+p}}{2b^2(1+p)}+\frac{d\left(a+bx^2\right)^{2+p}}{2b^2(2+p)}+\frac{1}{5}ex^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

output
```
-1/2*a*d*(b*x^2+a)^(p+1)/b^2/(p+1)+1/2*d*(b*x^2+a)^(2+p)/b^2/(2+p)+1/5*e*x
^5*(b*x^2+a)^p*hypergeom([5/2, -p],[7/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

3.383.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.87

$$\int x^3(d+ex)\left(a+bx^2\right)^p\,dx=\frac{1}{10}\left(a+bx^2\right)^p\left(-\frac{5d\left(a+bx^2\right)\left(a-b(1+p)x^2\right)}{b^2(1+p)(2+p)}+2ex^5\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)\right)$$

input
```
Integrate[x^3*(d + e*x)*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*((-5*d*(a + b*x^2)*(a - b*(1 + p)*x^2))/(b^2*(1 + p)*(2 + p
)) + (2*e*x^5*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/
a)^p))/10
```

3.383.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {542, 243, 53, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+ex)\left(a+bx^2\right)^p\,dx$$

$$\Big\downarrow \text{542}$$

$$d\int x^3\left(bx^2+a\right)^p dx+e\int x^4\left(bx^2+a\right)^p dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}d\int x^2\left(bx^2+a\right)^p dx^2+e\int x^4\left(bx^2+a\right)^p dx$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}d\int \left(\frac{\left(bx^2+a\right)^{p+1}}{b}-\frac{a\left(bx^2+a\right)^p}{b}\right)dx^2+e\int x^4\left(bx^2+a\right)^p dx$$

$$\Big\downarrow \text{279}$$

$$\frac{1}{2}d\int \left(\frac{\left(bx^2+a\right)^{p+1}}{b}-\frac{a\left(bx^2+a\right)^p}{b}\right)dx^2+e\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^4\left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{278}$$

$$\frac{1}{2}d\int \left(\frac{\left(bx^2+a\right)^{p+1}}{b}-\frac{a\left(bx^2+a\right)^p}{b}\right)dx^2+\frac{1}{5}ex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}d\left(\frac{\left(a+bx^2\right)^{p+2}}{b^2(p+2)}-\frac{a\left(a+bx^2\right)^{p+1}}{b^2(p+1)}\right)+\frac{1}{5}ex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

input
```
Int[x^3*(d + e*x)*(a + b*x^2)^p,x]
```

output
```
(d*(-((a*(a + b*x^2)^(1 + p))/(b^2*(1 + p))) + (a + b*x^2)^(2 + p)/(b^2*(2
 + p))))/2 + (e*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2
)/a)])/(5*(1 + (b*x^2)/a)^p)
```

3.383.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.383. $\int x^3(d+ex)\left(a+bx^2\right)^p\,dx$

3.383.4 Maple [F]

$$\int x^{3}(ex+d)\left(b\,x^{2}+a\right)^{p}dx$$

input
```
int(x^3*(e*x+d)*(b*x^2+a)^p,x)
```

output
```
int(x^3*(e*x+d)*(b*x^2+a)^p,x)
```

3.383.5 Fricas [F]

$$\int x^{3}(d+ex)\left(a+bx^{2}\right)^{p}\,dx=\int(ex+d)\left(bx^{2}+a\right)^{p}x^{3}\,dx$$

input
```
integrate(x^3*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e*x^4 + d*x^3)*(b*x^2 + a)^p, x)
```

3.383.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(82) = 164$.

Time = 6.90 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.64

$$\int x^{3}(d+ex)\left(a+bx^{2}\right)^{p}\,dx=\frac{a^{p}ex^{5}{}_{2}F_{1}\left(\begin{matrix}\frac{5}{2},-p\\\frac{7}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{5}+d\left(\begin{cases}\frac{a^{p}x^{4}}{4}&\text{for }b=0\\\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^{2}+2b^{3}x^{2}}+\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^{2}+2b^{3}x^{2}}+\frac{a}{2ab^{2}+2b^{3}x^{2}}+\frac{bx^{2}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^{2}+2b^{3}x^{2}}+\frac{bx^{2}\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^{2}+2b^{3}x^{2}}&\text{for }p=-2\\-\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^{2}}-\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^{2}}+\frac{x^{2}}{2b}&\text{for }p=-1\\-\frac{a^{2}(a+bx^{2})^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}}+\frac{abpx^{2}(a+bx^{2})^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}}+\frac{b^{2}px^{4}(a+bx^{2})^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}}+\frac{b^{2}x^{4}(a+bx^{2})^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}}&\text{otherwise}\end{cases}\right)$$

input
```
integrate(x**3*(e*x+d)*(b*x**2+a)**p,x)
```

output
```
a**p*e*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + d*Piece
wise((a**p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x*
*2) + a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**
3*x**2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log
(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(-a
/b))/(2*b**2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (
-a**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a +
b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p
/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**
2 + 6*b**2*p + 4*b**2), True))
```

3.383.7 Maxima [F]

$$\int x^3(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
e*integrate((b*x^2 + a)^p*x^4, x) + 1/2*(b^2*(p + 1)*x^4 + a*b*p*x^2 - a^2
)*(b*x^2 + a)^p*d/((p^2 + 3*p + 2)*b^2)
```

3.383.8 Giac [F]

$$\int x^3(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(b*x^2 + a)^p*x^3, x)
```

3.383.9 Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex)\left(a+bx^2\right)^p\,dx=\int x^3\left(b\,x^2+a\right)^p(d+e\,x)\,dx$$

input
```
int(x^3*(a + b*x^2)^p*(d + e*x),x)
```

output
```
int(x^3*(a + b*x^2)^p*(d + e*x), x)
```

3.384 $\int x^2(d+ex)\left(a+bx^2\right)^p\,dx$

3.384.1 Optimal result

Integrand size = 18, antiderivative size = 100

$$\int x^2(d+ex)\left(a+bx^2\right)^p\,dx=-\frac{ae\left(a+bx^2\right)^{1+p}}{2b^2(1+p)}+\frac{e\left(a+bx^2\right)^{2+p}}{2b^2(2+p)}+\frac{1}{3}dx^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)$$

output
```
-1/2*a*e*(b*x^2+a)^(p+1)/b^2/(p+1)+1/2*e*(b*x^2+a)^(2+p)/b^2/(2+p)+1/3*d*x
^3*(b*x^2+a)^p*hypergeom([3/2, -p],[5/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

3.384.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.87

$$\int x^2(d+ex)\left(a+bx^2\right)^p\,dx=\frac{1}{6}\left(a+bx^2\right)^p\left(-\frac{3e\left(a+bx^2\right)\left(a-b(1+p)x^2\right)}{b^2(1+p)(2+p)}+2dx^3\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)\right)$$

input
```
Integrate[x^2*(d + e*x)*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*((-3*e*(a + b*x^2)*(a - b*(1 + p)*x^2))/(b^2*(1 + p)*(2 + p
)) + (2*d*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(1 + (b*x^2)/
a)^p))/6
```

3.384.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {542, 243, 53, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+ex)\left(a+bx^2\right)^p\,dx$$

$$\Big\downarrow \text{ 542}$$

$$d\int x^2\left(bx^2+a\right)^pdx+e\int x^3\left(bx^2+a\right)^pdx$$

$$\Big\downarrow \text{ 243}$$

$$d\int x^2\left(bx^2+a\right)^pdx+\frac{1}{2}e\int x^2\left(bx^2+a\right)^pdx^2$$

$$\Big\downarrow \text{ 53}$$

$$d\int x^2\left(bx^2+a\right)^pdx+\frac{1}{2}e\int\left(\frac{\left(bx^2+a\right)^{p+1}}{b}-\frac{a\left(bx^2+a\right)^p}{b}\right)dx^2$$

$$\Big\downarrow \text{ 279}$$

$$d\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^2\left(\frac{bx^2}{a}+1\right)^pdx+\frac{1}{2}e\int\left(\frac{\left(bx^2+a\right)^{p+1}}{b}-\frac{a\left(bx^2+a\right)^p}{b}\right)dx^2$$

$$\Big\downarrow \text{ 278}$$

$$\frac{1}{2}e\int\left(\frac{\left(bx^2+a\right)^{p+1}}{b}-\frac{a\left(bx^2+a\right)^p}{b}\right)dx^2+$$
$$\frac{1}{3}dx^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}e\left(\frac{\left(a+bx^2\right)^{p+2}}{b^2(p+2)}-\frac{a\left(a+bx^2\right)^{p+1}}{b^2(p+1)}\right)+$$
$$\frac{1}{3}dx^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)$$

input
```
Int[x^2*(d + e*x)*(a + b*x^2)^p,x]
```

output
```
(e*(-((a*(a + b*x^2)^(1 + p))/(b^2*(1 + p))) + (a + b*x^2)^(2 + p)/(b^2*(2
 + p))))/2 + (d*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2
)/a)])/(3*(1 + (b*x^2)/a)^p)
```

3.384.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.384. $\int x^2(d+ex)\left(a+bx^2\right)^p\,dx$

3.384.4 Maple [F]

$$\int x^2(ex+d)\left(b\,x^2+a\right)^p dx$$

input
```
int(x^2*(e*x+d)*(b*x^2+a)^p,x)
```

output
```
int(x^2*(e*x+d)*(b*x^2+a)^p,x)
```

3.384.5 Fricas [F]

$$\int x^2(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e*x^3 + d*x^2)*(b*x^2 + a)^p, x)
```

3.384.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(82)=164$.

Time = 4.56 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.64

$$\int x^2(d+ex)\left(a+bx^2\right)^p\,dx=\frac{a^p dx^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3}+e\left(\begin{cases}\frac{a^px^4}{4} & \text{for}\ b=0\\ \frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{a}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} & \text{for}\ p=-2\\ -\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^2}-\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^2}+\frac{x^2}{2b} & \text{for}\ p=-1\\ -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**2*(e*x+d)*(b*x**2+a)**p,x)
```

output
```
a**p*d*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + e*Piece
wise((a**p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x*
*2) + a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**
3*x**2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log
(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(-a
/b))/(2*b**2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (
-a**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a +
b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p
/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**
2 + 6*b**2*p + 4*b**2), True))
```

3.384.7 Maxima [F]

$$\int x^2(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)*(b*x^2 + a)^p*x^2, x)
```

3.384.8 Giac [F]

$$\int x^2(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(b*x^2 + a)^p*x^2, x)
```

3.384.9 Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex)\left(a+bx^2\right)^p\,dx=\int x^2\,\left(b\,x^2+a\right)^p\,(d+e\,x)\,dx$$

input
```
int(x^2*(a + b*x^2)^p*(d + e*x),x)
```

output
```
int(x^2*(a + b*x^2)^p*(d + e*x), x)
```

3.385 $\int x(d+ex)\left(a+bx^2\right)^p\,dx$

3.385.1 Optimal result

Integrand size = 16, antiderivative size = 75

$$\int x(d+ex)\left(a+bx^2\right)^p\,dx=\frac{d\left(a+bx^2\right)^{1+p}}{2b(1+p)}+\frac{1}{3}ex^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)$$

output
```
1/2*d*(b*x^2+a)^(p+1)/b/(p+1)+1/3*e*x^3*(b*x^2+a)^p*hypergeom([3/2, -p],[5
/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

3.385.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int x(d+ex)\left(a+bx^2\right)^p\,dx=\frac{1}{6}\left(a+bx^2\right)^p\left(\frac{3d\left(a+bx^2\right)}{b(1+p)}+2ex^3\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)\right)$$

input
```
Integrate[x*(d + e*x)*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*((3*d*(a + b*x^2))/(b*(1 + p)) + (2*e*x^3*Hypergeometric2F1
[3/2, -p, 5/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/6
```

3.385.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {542, 241, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex)\left(a+bx^2\right)^p\,dx$$

$$\Big\downarrow \text{542}$$

$$d\int x\left(bx^2+a\right)^p dx+e\int x^2\left(bx^2+a\right)^p dx$$

$$\Big\downarrow \text{241}$$

$$e\int x^2\left(bx^2+a\right)^p dx+\frac{d\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

$$\Big\downarrow \text{279}$$

$$e\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^2\left(\frac{bx^2}{a}+1\right)^p dx+\frac{d\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

$$\Big\downarrow \text{278}$$

$$\frac{d\left(a+bx^2\right)^{p+1}}{2b(p+1)}+\frac{1}{3}ex^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)$$

input
```
Int[x*(d + e*x)*(a + b*x^2)^p,x]
```

output
```
(d*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (e*x^3*(a + b*x^2)^p*Hypergeometri
c2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(3*(1 + (b*x^2)/a)^p)
```

3.385.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

3.385.4 Maple [F]

$$\int x(ex+d)\left(b\,x^{2}+a\right)^{p}dx$$

input
```
int(x*(e*x+d)*(b*x^2+a)^p,x)
```

output
```
int(x*(e*x+d)*(b*x^2+a)^p,x)
```

3.385.5 Fricas [F]

$$\int x(d+ex)\left(a+bx^2\right)^p\,dx=\int(ex+d)(bx^2+a)^p x\,dx$$

input
```
integrate(x*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e*x^2 + d*x)*(b*x^2 + a)^p, x)
```

3.385.6 Sympy [A] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int x(d+ex)\left(a+bx^2\right)^p\,dx=\frac{a^pex^3{}_2F_1\left(\left.\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{3}+d\left(\begin{cases}\frac{a^px^2}{2} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(a+bx^2\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x*(e*x+d)*(b*x**2+a)**p,x)
```

output
```
a**p*e*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + d*Piece
wise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), N
e(p, -1)), (log(a + b*x**2), True))/(2*b), True))
```

3.385.7 Maxima [F]

$$\int x(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^px\,dx$$

input
```
integrate(x*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
e*integrate((b*x^2 + a)^p*x^2, x) + 1/2*(b*x^2 + a)^(p + 1)*d/(b*(p + 1))
```

3.385.8 Giac [F]

$$\int x(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p x\,dx$$

input
```
integrate(x*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(b*x^2 + a)^p*x, x)
```

3.385.9 Mupad [F(-1)]

Timed out.

$$\int x(d+ex)\left(a+bx^2\right)^p\,dx=\int x\,\left(b\,x^2+a\right)^p\,(d+e\,x)\,dx$$

input
```
int(x*(a + b*x^2)^p*(d + e*x),x)
```

output
```
int(x*(a + b*x^2)^p*(d + e*x), x)
```

3.386 $\int (d+ex)\left(a+bx^2\right)^p\,dx$

3.386.1 Optimal result

Integrand size = 15, antiderivative size = 70

$$\int (d+ex)\left(a+bx^2\right)^p\,dx=\frac{e\left(a+bx^2\right)^{1+p}}{2b(1+p)}+dx\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

output
```
1/2*e*(b*x^2+a)^(p+1)/b/(p+1)+d*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b
*x^2/a)/((1+b*x^2/a)^p)
```

3.386.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.40

$$\int (d+ex)\left(a+bx^2\right)^p\,dx$$
$$=\frac{\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(bex^2\left(1+\frac{bx^2}{a}\right)^p+ae\left(-1+\left(1+\frac{bx^2}{a}\right)^p\right)+2bd(1+p)x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\right.\right.}{2b(1+p)}$$

input
```
Integrate[(d + e*x)*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*(b*e*x^2*(1 + (b*x^2)/a)^p + a*e*(-1 + (1 + (b*x^2)/a)^p) +
 2*b*d*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)]))/(2*b*(1 +
 p)*(1 + (b*x^2)/a)^p)
```

3.386.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {455, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(a+bx^2\right)^p dx$$

$$\Big\downarrow \text{ 455}$$

$$d\int \left(bx^2+a\right)^p dx+\frac{e\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

$$\Big\downarrow \text{ 238}$$

$$d\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \left(\frac{bx^2}{a}+1\right)^p dx+\frac{e\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

$$\Big\downarrow \text{ 237}$$

$$dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)+\frac{e\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

input
```
Int[(d + e*x)*(a + b*x^2)^p,x]
```

output
```
(e*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (d*x*(a + b*x^2)^p*Hypergeometric2
F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

3.386.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

3.386.4 Maple [F]

$$\int (ex+d)\left(b\,x^{2}+a\right)^{p}dx$$

input
```
int((e*x+d)*(b*x^2+a)^p,x)
```

output
```
int((e*x+d)*(b*x^2+a)^p,x)
```

3.386.5 Fricas [F]

$$\int (d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)(bx^2+a)^p\,dx$$

input
```
integrate((e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e*x + d)*(b*x^2 + a)^p, x)
```

3.386.6 Sympy [A] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int (d+ex)\left(a+bx^2\right)^p\,dx=a^pdx\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},-p\\\frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)+e\left(\begin{cases}\frac{a^px^2}{2} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(a+bx^2\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((e*x+d)*(b*x**2+a)**p,x)
```

output
```
a**p*d*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + e*Piecewise(
(a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(p,
-1)), (log(a + b*x**2), True))/(2*b), True))
```

3.386.7 Maxima [F]

$$\int (d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p\,dx$$

input
```
integrate((e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)*(b*x^2 + a)^p, x)
```

3.386.8 Giac [F]

$$\int (d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p\,dx$$

input
```
integrate((e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(b*x^2 + a)^p, x)
```

3.386.9 Mupad [B] (verification not implemented)

Time = 12.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int (d+ex)\left(a+bx^2\right)^p\,dx=\frac{e\left(b\,x^2+a\right)^{p+1}}{2\,b\,(p+1)}+\frac{d\,x\left(b\,x^2+a\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^p}$$

input
```
int((a + b*x^2)^p*(d + e*x),x)
```

output
```
(e*(a + b*x^2)^(p + 1))/(2*b*(p + 1)) + (d*x*(a + b*x^2)^p*hypergeom([1/2,
 -p], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^p
```

3.387 $\int \frac{(d+ex)(a+bx^2)^p}{x} dx$

3.387.1 Optimal result

Integrand size = 18, antiderivative size = 88

$$\int \frac{(d+ex)(a+bx^2)^p}{x} dx = ex(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) - \frac{d(a+bx^2)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

output
```
e*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/((1+b*x^2/a)^p)-1/2*d*
(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*x^2/a)/a/(p+1)
```

3.387.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)(a+bx^2)^p}{x} dx = ex(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) - \frac{d(a+bx^2)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

input
```
Integrate[((d + e*x)*(a + b*x^2)^p)/x,x]
```

output
```
(e*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*
x^2)/a)^p - (d*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 +
(b*x^2)/a])/(2*a*(1 + p))
```

3.387.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {542, 238, 237, 243, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x}dx$$

$$\Big\downarrow \text{542}$$

$$d\int \frac{\left(bx^2+a\right)^p}{x}dx+e\int \left(bx^2+a\right)^pdx$$

$$\Big\downarrow \text{238}$$

$$d\int \frac{\left(bx^2+a\right)^p}{x}dx+e\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \left(\frac{bx^2}{a}+1\right)^pdx$$

$$\Big\downarrow \text{237}$$

$$d\int \frac{\left(bx^2+a\right)^p}{x}dx+ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}d\int \frac{\left(bx^2+a\right)^p}{x^2}dx^2+ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

$$\Big\downarrow \text{75}$$

$$ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)-\frac{d\left(a+bx^2\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

input
```
Int[((d + e*x)*(a + b*x^2)^p)/x,x]
```

output
```
(e*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*
x^2)/a)^p - (d*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 +
(b*x^2)/a])/(2*a*(1 + p))
```

3.387.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

3.387.4 Maple [F]

$$\int \frac{(ex+d)\left(b\,x^{2}+a\right)^{p}}{x}dx$$

input
```
int((e*x+d)*(b*x^2+a)^p/x,x)
```

output
```
int((e*x+d)*(b*x^2+a)^p/x,x)
```

3.387.5 Fricas [F]

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x}\,dx = \int \frac{(ex+d)(bx^2+a)^p}{x}\,dx$$

input
```
integrate((e*x+d)*(b*x^2+a)^p/x,x, algorithm="fricas")
```

output
```
integral((e*x + d)*(b*x^2 + a)^p/x, x)
```

3.387.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.70 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x}\,dx = a^p e x {}_2F_1\left(\begin{matrix}\frac{1}{2}, -p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right) - \frac{b^p d x^{2p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p, -p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(1-p)}$$

input
```
integrate((e*x+d)*(b*x**2+a)**p/x,x)
```

output
```
a**p*e*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - b**p*d*x**(2
*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*gam
ma(1 - p))
```

3.387.7 Maxima [F]

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x}\,dx = \int \frac{(ex+d)(bx^2+a)^p}{x}\,dx$$

input
```
integrate((e*x+d)*(b*x^2+a)^p/x,x, algorithm="maxima")
```

output
```
integrate((e*x + d)*(b*x^2 + a)^p/x, x)
```

3.387.8 Giac [F]

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x}\,dx = \int \frac{(ex+d)(bx^2+a)^p}{x}\,dx$$

input
```
integrate((e*x+d)*(b*x^2+a)^p/x,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(b*x^2 + a)^p/x, x)
```

3.387.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x}\,dx = \int \frac{\left(b\,x^2+a\right)^p\,\left(d+e\,x\right)}{x}\,dx$$

input
```
int(((a + b*x^2)^p*(d + e*x))/x,x)
```

output
```
int(((a + b*x^2)^p*(d + e*x))/x, x)
```

3.388 $\int \frac{(d+ex)(a+bx^2)^p}{x^2}\,dx$

3.388.1 Optimal result

Integrand size = 18, antiderivative size = 91

$$\int \frac{(d+ex)(a+bx^2)^p}{x^2}\,dx = -\frac{d(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x} - \frac{e(a+bx^2)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

output
```
-d*(b*x^2+a)^p*hypergeom([-1/2, -p],[1/2],-b*x^2/a)/x/((1+b*x^2/a)^p)-1/2*
e*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*x^2/a)/a/(p+1)
```

3.388.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)(a+bx^2)^p}{x^2}\,dx = -\frac{d(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x} - \frac{e(a+bx^2)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

input
```
Integrate[((d + e*x)*(a + b*x^2)^p)/x^2,x]
```

output
```
-((d*(a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 +
 (b*x^2)/a)^p)) - (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p
, 1 + (b*x^2)/a])/(2*a*(1 + p))
```

3.388.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {542, 243, 75, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x^2}dx$$

$$\Big\downarrow \text{542}$$

$$d\int \frac{\left(bx^2+a\right)^p}{x^2}dx+e\int \frac{\left(bx^2+a\right)^p}{x}dx$$

$$\Big\downarrow \text{243}$$

$$d\int \frac{\left(bx^2+a\right)^p}{x^2}dx+\frac{1}{2}e\int \frac{\left(bx^2+a\right)^p}{x^2}dx^2$$

$$\Big\downarrow \text{75}$$

$$d\int \frac{\left(bx^2+a\right)^p}{x^2}dx-\frac{e\left(a+bx^2\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

$$\Big\downarrow \text{279}$$

$$d\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^2}{a}+1\right)^p}{x^2}dx-\frac{e\left(a+bx^2\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

$$\Big\downarrow \text{278}$$

$$-\frac{d(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x}-\frac{e(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

input
```
Int[((d + e*x)*(a + b*x^2)^p)/x^2,x]
```

output
```
-((d*(a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 +
 (b*x^2)/a)^p)) - (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p
, 1 + (b*x^2)/a])/(2*a*(1 + p))
```

3.388.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

3.388.4 Maple [F]

$$\int \frac{(ex+d)\left(b\,x^{2}+a\right)^{p}}{x^{2}}dx$$

input
```
int((e*x+d)*(b*x^2+a)^p/x^2,x)
```

output
```
int((e*x+d)*(b*x^2+a)^p/x^2,x)
```

3.388.5 Fricas [F]

$$\int \frac{(d+ex)\left(a+bx^{2}\right)^{p}}{x^{2}}\,dx = \int \frac{(ex+d)(bx^{2}+a)^{p}}{x^{2}}\,dx$$

input
```
integrate((e*x+d)*(b*x^2+a)^p/x^2,x, algorithm="fricas")
```

output
```
integral((e*x + d)*(b*x^2 + a)^p/x^2, x)
```

3.388.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.95 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)\left(a+bx^{2}\right)^{p}}{x^{2}}\,dx = -\frac{a^{p}d\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-p\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{x} - \frac{b^{p}ex^{2p}\Gamma(-p)\,{}_{2}F_{1}\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^{2}}\right)}{2\Gamma\left(1-p\right)}$$

input
```
integrate((e*x+d)*(b*x**2+a)**p/x**2,x)
```

output
```
-a**p*d*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x - b**p*e*x**
(2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*g
amma(1 - p))
```

3.388.7 Maxima [F]

$$\int \frac{(d+ex)(a+bx^2)^p}{x^2}\,dx = \int \frac{(ex+d)(bx^2+a)^p}{x^2}\,dx$$

input
```
integrate((e*x+d)*(b*x^2+a)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((e*x + d)*(b*x^2 + a)^p/x^2, x)
```

3.388.8 Giac [F]

$$\int \frac{(d+ex)(a+bx^2)^p}{x^2}\,dx = \int \frac{(ex+d)(bx^2+a)^p}{x^2}\,dx$$

input
```
integrate((e*x+d)*(b*x^2+a)^p/x^2,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(b*x^2 + a)^p/x^2, x)
```

3.388.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)(a+bx^2)^p}{x^2}\,dx = \int \frac{(b\,x^2+a)^p\,(d+e\,x)}{x^2}\,dx$$

input
```
int(((a + b*x^2)^p*(d + e*x))/x^2,x)
```

output
```
int(((a + b*x^2)^p*(d + e*x))/x^2, x)
```

3.389 $\int \frac{(d+ex)(a+bx^2)^p}{x^3} dx$

3.389.1 Optimal result

Integrand size = 18, antiderivative size = 92

$$\int \frac{(d+ex)(a+bx^2)^p}{x^3} dx = -\frac{e(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x} + \frac{bd(a+bx^2)^{1+p} \text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a^2(1+p)}$$

output
```
-e*(b*x^2+a)^p*hypergeom([-1/2, -p],[1/2],-b*x^2/a)/x/((1+b*x^2/a)^p)+1/2*
b*d*(b*x^2+a)^(p+1)*hypergeom([2, p+1],[2+p],1+b*x^2/a)/a^2/(p+1)
```

3.389.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)(a+bx^2)^p}{x^3} dx = \frac{1}{2}(a+bx^2)^p \left(-\frac{2e\left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x} + \frac{bd(a+bx^2) \text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{bx^2}{a}\right)}{a^2(1+p)}\right)$$

input
```
Integrate[((d + e*x)*(a + b*x^2)^p)/x^3,x]
```

output
```
((a + b*x^2)^p*((-2*e*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(
1 + (b*x^2)/a)^p) + (b*d*(a + b*x^2)*Hypergeometric2F1[2, 1 + p, 2 + p, 1
+ (b*x^2)/a])/(a^2*(1 + p))))/2
```

3.389.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {542, 243, 75, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x^3}dx$$

$$\Big\downarrow \text{ 542}$$

$$d\int \frac{\left(bx^2+a\right)^p}{x^3}dx+e\int \frac{\left(bx^2+a\right)^p}{x^2}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}d\int \frac{\left(bx^2+a\right)^p}{x^4}dx^2+e\int \frac{\left(bx^2+a\right)^p}{x^2}dx$$

$$\Big\downarrow \text{ 75}$$

$$e\int \frac{\left(bx^2+a\right)^p}{x^2}dx+\frac{bd\left(a+bx^2\right)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{bx^2}{a}+1\right)}{2a^2(p+1)}$$

$$\Big\downarrow \text{ 279}$$

$$e\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^2}{a}+1\right)^p}{x^2}dx+\frac{bd\left(a+bx^2\right)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{bx^2}{a}+1\right)}{2a^2(p+1)}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{bd(a+bx^2)^{p+1}\,\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{bx^2}{a}+1\right)}{2a^2(p+1)}-\frac{e(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x}$$

input
```
Int[((d + e*x)*(a + b*x^2)^p)/x^3,x]
```

output
```
-((e*(a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 +
 (b*x^2)/a)^p)) + (b*d*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 +
 p, 1 + (b*x^2)/a])/(2*a^2*(1 + p))
```

3.389.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

3.389. $\int \frac{(d+ex)(a+bx^2)^p}{x^3}\,dx$

3.389.4 Maple [F]

$$\int \frac{(ex+d)\,(b\,x^2+a)^p}{x^3}dx$$

input
```
int((e*x+d)*(b*x^2+a)^p/x^3,x)
```

output
```
int((e*x+d)*(b*x^2+a)^p/x^3,x)
```

3.389.5 Fricas [F]

$$\int \frac{(d+ex)\,(a+bx^2)^p}{x^3}\,dx = \int \frac{(ex+d)(bx^2+a)^p}{x^3}\,dx$$

input
```
integrate((e*x+d)*(b*x^2+a)^p/x^3,x, algorithm="fricas")
```

output
```
integral((e*x + d)*(b*x^2 + a)^p/x^3, x)
```

3.389.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.50 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)\,(a+bx^2)^p}{x^3}\,dx = -\frac{a^p e\, {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{x} - \frac{b^p d x^{2p-2}\Gamma(1-p)\, {}_2F_1\left(\begin{matrix} -p, 1-p \\ 2-p \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\,(2-p)}$$

input
```
integrate((e*x+d)*(b*x**2+a)**p/x**3,x)
```

output
```
-a**p*e*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x - b**p*d*x**
(2*p - 2)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), a*exp_polar(I*pi)/(b*x
**2))/(2*gamma(2 - p))
```

3.389.7 Maxima [F]

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x^3}\,dx=\int \frac{(ex+d)(bx^2+a)^p}{x^3}\,dx$$

input
```
integrate((e*x+d)*(b*x^2+a)^p/x^3,x, algorithm="maxima")
```

output
```
integrate((e*x + d)*(b*x^2 + a)^p/x^3, x)
```

3.389.8 Giac [F]

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x^3}\,dx=\int \frac{(ex+d)(bx^2+a)^p}{x^3}\,dx$$

input
```
integrate((e*x+d)*(b*x^2+a)^p/x^3,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(b*x^2 + a)^p/x^3, x)
```

3.389.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x^3}\,dx=\int \frac{\left(b\,x^2+a\right)^p\,\left(d+e\,x\right)}{x^3}\,dx$$

input
```
int(((a + b*x^2)^p*(d + e*x))/x^3,x)
```

output
```
int(((a + b*x^2)^p*(d + e*x))/x^3, x)
```

3.390 $\int x^5(d+ex)^2\left(a+bx^2\right)^p\,dx$

3.390.1 Optimal result

Integrand size = 20, antiderivative size = 188

$$\int x^5(d+ex)^2\left(a+bx^2\right)^p\,dx=\frac{a^2\left(bd^2-ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^4(1+p)}-\frac{a\left(2bd^2-3ae^2\right)\left(a+bx^2\right)^{2+p}}{2b^4(2+p)}+\frac{\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{3+p}}{2b^4(3+p)}+\frac{e^2\left(a+bx^2\right)^{4+p}}{2b^4(4+p)}+\frac{2}{7}dex^7\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)$$

output
```
1/2*a^2*(-a*e^2+b*d^2)*(b*x^2+a)^(p+1)/b^4/(p+1)-1/2*a*(-3*a*e^2+2*b*d^2)*
(b*x^2+a)^(2+p)/b^4/(2+p)+1/2*(-3*a*e^2+b*d^2)*(b*x^2+a)^(3+p)/b^4/(3+p)+1
/2*e^2*(b*x^2+a)^(4+p)/b^4/(4+p)+2/7*d*e*x^7*(b*x^2+a)^p*hypergeom([7/2, -
p],[9/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

3.390.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.09

$$\int x^5(d+ex)^2\left(a+bx^2\right)^p\,dx$$
$$=\frac{1}{14}\left(a+bx^2\right)^p\left(\frac{7d^2(a+bx^2)\left(2a^2-2ab(1+p)x^2+b^2\left(2+3p+p^2\right)x^4\right)}{b^3(1+p)(2+p)(3+p)}+\frac{7e^2(a+bx^2)\left(-6a^3+6a^2b(1+p)x^2-3ab^2\left(2+3p+p^2\right)x^4+b^3\left(6+11p+6p^2+p^3\right)x^6\right)}{b^4(1+p)(2+p)(3+p)(4+p)}+4dex^7\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)\right)$$

input
```
Integrate[x^5*(d + e*x)^2*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*((7*d^2*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3
*p + p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (7*e^2*(a + b*x^2)*(-6*a^3
 + 6*a^2*b*(1 + p)*x^2 - 3*a*b^2*(2 + 3*p + p^2)*x^4 + b^3*(6 + 11*p + 6*p
^2 + p^3)*x^6))/(b^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)) + (4*d*e*x^7*Hyperge
ometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/14
```

3.390.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {543, 27, 279, 278, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(d+ex)^2\left(a+bx^2\right)^p\,dx$$
$$\Big\downarrow \text{543}$$
$$\int x^5\left(bx^2+a\right)^p\left(d^2+e^2x^2\right)dx+\int 2dex^6\left(bx^2+a\right)^p\,dx$$
$$\Big\downarrow \text{27}$$
$$\int x^5\left(bx^2+a\right)^p\left(d^2+e^2x^2\right)dx+2de\int x^6\left(bx^2+a\right)^p\,dx$$

$$\Big\downarrow \text{279}$$

$$\int x^5 \left(bx^2+a\right)^p \left(d^2+e^2x^2\right)dx+2de\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^6\left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{278}$$

$$\int x^5 \left(bx^2+a\right)^p \left(d^2+e^2x^2\right)dx+$$
$$\frac{2}{7}dex^7\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int x^4 \left(bx^2+a\right)^p \left(d^2+e^2x^2\right)dx^2+$$
$$\frac{2}{7}dex^7\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int\left(-\frac{a^2\left(ae^2-bd^2\right)\left(bx^2+a\right)^p}{b^3}+\frac{a\left(3ae^2-2bd^2\right)\left(bx^2+a\right)^{p+1}}{b^3}+\frac{\left(bd^2-3ae^2\right)\left(bx^2+a\right)^{p+2}}{b^3}+\frac{e^2\left(bx^2+a\right)^{p+3}}{b^3}\right.$$
$$\frac{2}{7}dex^7\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^2\left(bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{b^4(p+1)}-\frac{a\left(2bd^2-3ae^2\right)\left(a+bx^2\right)^{p+2}}{b^4(p+2)}+\frac{\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{p+3}}{b^4(p+3)}+\frac{e^2\left(a+bx^2\right)^{p+4}}{b^4(p+4)}\right)$$
$$\frac{2}{7}dex^7\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)$$

input
```
Int[x^5*(d + e*x)^2*(a + b*x^2)^p,x]
```

output
```
((a^2*(b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(b^4*(1 + p)) - (a*(2*b*d^2 - 3
*a*e^2)*(a + b*x^2)^(2 + p))/(b^4*(2 + p)) + ((b*d^2 - 3*a*e^2)*(a + b*x^2
)^(3 + p))/(b^4*(3 + p)) + (e^2*(a + b*x^2)^(4 + p))/(b^4*(4 + p)))/2 + (2
*d*e*x^7*(a + b*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(7*(
1 + (b*x^2)/a)^p)
```

3.390. $\int x^5(d+ex)^2\left(a+bx^2\right)^p\,dx$

3.390.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.390. $\int x^5(d+ex)^2\left(a+bx^2\right)^p\,dx$

3.390.4 Maple [F]

$$\int x^{5}(ex+d)^{2}\left(b\,x^{2}+a\right)^{p}dx$$

input
```
int(x^5*(e*x+d)^2*(b*x^2+a)^p,x)
```

output
```
int(x^5*(e*x+d)^2*(b*x^2+a)^p,x)
```

3.390.5 Fricas [F]

$$\int x^5(d+ex)^2\left(a+bx^2\right)^p\,dx=\int (ex+d)^2\left(bx^2+a\right)^p x^5\,dx$$

input
```
integrate(x^5*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e^2*x^7 + 2*d*e*x^6 + d^2*x^5)*(b*x^2 + a)^p, x)
```

3.390.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 923 vs. $2(163) = 326$.

Time = 17.85 (sec) , antiderivative size = 2883, normalized size of antiderivative = 15.34

$$\int x^5(d+ex)^2\left(a+bx^2\right)^p\,dx=\text{Too large to display}$$

input
```
integrate(x**5*(e*x+d)**2*(b*x**2+a)**p,x)
```

output
```
2*a**p*d*e*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + d**
2*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b
**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b*
*3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 +
4*b**5*x**4) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2
 + 4*b**5*x**4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x
**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4
) + 2*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*
x**4) + 2*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b
**5*x**4), Eq(p, -3)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**
2) - 2*a**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**
3 + 2*b**4*x**2) - 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2)
 - 2*a*b*x**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*
a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a*
*2*log(x + sqrt(-a/b))/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1))
, (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**
3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3
*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p*
*2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 1
2*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a + b*x**2)**p/(2*...
```

3.390.7 Maxima [F]

$$\int x^5(d+ex)^2\left(a+bx^2\right)^p\,dx=\int (ex+d)^2\left(bx^2+a\right)^p x^5\,dx$$

input
```
integrate(x^5*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
1/2*((p^2 + 3*p + 2)*b^3*x^6 + (p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + 2*a^3
)*(b*x^2 + a)^p*d^2/((p^3 + 6*p^2 + 11*p + 6)*b^3) + integrate((e^2*x^7 +
2*d*e*x^6)*(b*x^2 + a)^p, x)
```

3.390.8 Giac [F]

$$\int x^5(d+ex)^2\left(a+bx^2\right)^p\,dx=\int (ex+d)^2\left(bx^2+a\right)^p x^5\,dx$$

input
```
integrate(x^5*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(b*x^2 + a)^p*x^5, x)
```

3.390.9 Mupad [F(-1)]

Timed out.

$$\int x^5(d+ex)^2\left(a+bx^2\right)^p\,dx=\int x^5\,\left(b\,x^2+a\right)^p\,\left(d+e\,x\right)^2\,dx$$

input
```
int(x^5*(a + b*x^2)^p*(d + e*x)^2,x)
```

output
```
int(x^5*(a + b*x^2)^p*(d + e*x)^2, x)
```

3.391 $\int x^4(d+ex)^2 \left(a+bx^2\right)^p \, dx$

3.391.1 Optimal result

Integrand size = 20, antiderivative size = 177

$$\int x^4(d+ex)^2 \left(a+bx^2\right)^p \, dx = \frac{a^2de\left(a+bx^2\right)^{1+p}}{b^3(1+p)}+\frac{e^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}-\frac{2ade\left(a+bx^2\right)^{2+p}}{b^3(2+p)}+\frac{de\left(a+bx^2\right)^{3+p}}{b^3(3+p)}-\frac{\left(5ae^2-bd^2(7+2p)\right)x^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)}{5b(7+2p)}$$

output
```
a^2*d*e*(b*x^2+a)^(p+1)/b^3/(p+1)+e^2*x^5*(b*x^2+a)^(p+1)/b/(7+2*p)-2*a*d*
e*(b*x^2+a)^(2+p)/b^3/(2+p)+d*e*(b*x^2+a)^(3+p)/b^3/(3+p)-1/5*(5*a*e^2-b*d
^2*(7+2*p))*x^5*(b*x^2+a)^p*hypergeom([5/2, -p],[7/2],-b*x^2/a)/b/(7+2*p)/
((1+b*x^2/a)^p)
```

3.391.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.88

$$\int x^4(d+ex)^2\left(a+bx^2\right)^p\,dx$$
$$=\frac{1}{35}\left(a+bx^2\right)^p\left(\frac{35de(a+bx^2)\left(2a^2-2ab(1+p)x^2+b^2(2+3p+p^2)\,x^4\right)}{b^3(1+p)(2+p)(3+p)}+7d^2x^5\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)+5e^2x^7\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)\right)$$

input
```
Integrate[x^4*(d + e*x)^2*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*((35*d*e*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 +
3*p + p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (7*d^2*x^5*Hypergeometric
2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (5*e^2*x^7*Hypergeome
tric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/35
```

3.391.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {543, 27, 243, 53, 363, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(d+ex)^2\left(a+bx^2\right)^p\,dx$$
$$\Big\downarrow \text{ 543}$$
$$\int x^4\left(bx^2+a\right)^p\left(d^2+e^2x^2\right)dx+\int 2dex^5\left(bx^2+a\right)^p\,dx$$
$$\Big\downarrow \text{ 27}$$
$$\int x^4\left(bx^2+a\right)^p\left(d^2+e^2x^2\right)dx+2de\int x^5\left(bx^2+a\right)^p\,dx$$
$$\Big\downarrow \text{ 243}$$

$$\int x^4 \left(bx^2+a\right)^p \left(d^2+e^2x^2\right)dx+de\int x^4 \left(bx^2+a\right)^p dx^2$$

$$\Big\downarrow \; 53$$

$$de\int \left(\frac{a^2 \left(bx^2+a\right)^p}{b^2}-\frac{2a \left(bx^2+a\right)^{p+1}}{b^2}+\frac{\left(bx^2+a\right)^{p+2}}{b^2}\right)dx^2+\int x^4 \left(bx^2+a\right)^p \left(d^2+e^2x^2\right)dx$$

$$\Big\downarrow \; 363$$

$$de\int \left(\frac{a^2 \left(bx^2+a\right)^p}{b^2}-\frac{2a \left(bx^2+a\right)^{p+1}}{b^2}+\frac{\left(bx^2+a\right)^{p+2}}{b^2}\right)dx^2+ \left(d^2-\frac{5ae^2}{2bp+7b}\right)\int x^4 \left(bx^2+a\right)^p dx+\frac{e^2x^5 \left(a+bx^2\right)^{p+1}}{b(2p+7)}$$

$$\Big\downarrow \; 279$$

$$de\int \left(\frac{a^2 \left(bx^2+a\right)^p}{b^2}-\frac{2a \left(bx^2+a\right)^{p+1}}{b^2}+\frac{\left(bx^2+a\right)^{p+2}}{b^2}\right)dx^2+ \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{5ae^2}{2bp+7b}\right)\int x^4 \left(\frac{bx^2}{a}+1\right)^p dx+\frac{e^2x^5 \left(a+bx^2\right)^{p+1}}{b(2p+7)}$$

$$\Big\downarrow \; 278$$

$$de\int \left(\frac{a^2 \left(bx^2+a\right)^p}{b^2}-\frac{2a \left(bx^2+a\right)^{p+1}}{b^2}+\frac{\left(bx^2+a\right)^{p+2}}{b^2}\right)dx^2+ \frac{1}{5}x^5 \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{5ae^2}{2bp+7b}\right)\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)+ \frac{e^2x^5 \left(a+bx^2\right)^{p+1}}{b(2p+7)}$$

$$\Big\downarrow \; 2009$$

$$de\left(\frac{a^2 \left(a+bx^2\right)^{p+1}}{b^3(p+1)}-\frac{2a \left(a+bx^2\right)^{p+2}}{b^3(p+2)}+\frac{\left(a+bx^2\right)^{p+3}}{b^3(p+3)}\right)+ \frac{1}{5}x^5 \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{5ae^2}{2bp+7b}\right)\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)+ \frac{e^2x^5 \left(a+bx^2\right)^{p+1}}{b(2p+7)}$$

input
```
Int[x^4*(d + e*x)^2*(a + b*x^2)^p,x]
```

3.391. $\int x^4(d+ex)^2 \left(a+bx^2\right)^p \, dx$

output
```
(e^2*x^5*(a + b*x^2)^(1 + p))/(b*(7 + 2*p)) + d*e*((a^2*(a + b*x^2)^(1 + p
))/(b^3*(1 + p)) - (2*a*(a + b*x^2)^(2 + p))/(b^3*(2 + p)) + (a + b*x^2)^(
3 + p)/(b^3*(3 + p))) + ((d^2 - (5*a*e^2)/(7*b + 2*b*p))*x^5*(a + b*x^2)^p
*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

3.391.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.391. $\int x^4(d+ex)^2\left(a+bx^2\right)^p\,dx$

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.391.4 Maple [F]

$$\int x^4(ex+d)^2\left(b\,x^2+a\right)^p dx$$

input
```
int(x^4*(e*x+d)^2*(b*x^2+a)^p,x)
```

output
```
int(x^4*(e*x+d)^2*(b*x^2+a)^p,x)
```

3.391.5 Fricas [F]

$$\int x^4(d+ex)^2\left(a+bx^2\right)^p\,dx=\int (ex+d)^2\left(bx^2+a\right)^p x^4\,dx$$

input
```
integrate(x^4*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e^2*x^6 + 2*d*e*x^5 + d^2*x^4)*(b*x^2 + a)^p, x)
```

3.391.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(151) = 302$.

Time = 16.94 (sec) , antiderivative size = 986, normalized size of antiderivative = 5.57

$$\int x^4(d+ex)^2\left(a+bx^2\right)^p\,dx=\frac{a^pd^2x^5{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5}+\frac{a^pe^2x^7{}_2F_1\left(\begin{matrix}\frac{7}{2},-p\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7}$$
$$+2de\begin{cases}\frac{a^px^6}{6}\\ \frac{2a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{2a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{3a^2}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{4abx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{4abx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}\\ -\frac{2a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2}-\frac{2a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2}-\frac{2a^2}{2ab^3+2b^4x^2}-\frac{2abx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2}-\frac{2abx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2}+\frac{b^2x^4}{2ab^3+2b^4x^2}\\ \frac{a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^3}+\frac{a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^3}-\frac{ax^2}{2b^2}+\frac{x^4}{4b}\\ \frac{2a^3(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3}-\frac{2a^2bpx^2(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3}+\frac{ab^2p^2x^4(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3}+\frac{ab^2px^4(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3}+\end{cases}$$

input
```
integrate(x**4*(e*x+d)**2*(b*x**2+a)**p,x)
```

output
```
a**p*d**2*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + a**p
*e**2*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + 2*d*e*Pi
ecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b**3
+ 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b**3 +
 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b*
*5*x**4) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4
*b**5*x**4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2
+ 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) +
2*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4
) + 2*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*
x**4), Eq(p, -3)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) -
 2*a**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 +
2*b**4*x**2) - 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2
*a*b*x**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b*
*3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a**2*l
og(x + sqrt(-a/b))/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2
*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) -
 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p +
 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 +
 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12...
```

3.391. $\int x^4(d+ex)^2\left(a+bx^2\right)^p\,dx$

3.391.7 Maxima [F]

$$\int x^4(d+ex)^2\left(a+bx^2\right)^p\,dx=\int (ex+d)^2\left(bx^2+a\right)^p x^4\,dx$$

input
```
integrate(x^4*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2*(b*x^2 + a)^p*x^4, x)
```

3.391.8 Giac [F]

$$\int x^4(d+ex)^2\left(a+bx^2\right)^p\,dx=\int (ex+d)^2\left(bx^2+a\right)^p x^4\,dx$$

input
```
integrate(x^4*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(b*x^2 + a)^p*x^4, x)
```

3.391.9 Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex)^2\left(a+bx^2\right)^p\,dx=\int x^4\left(b\,x^2+a\right)^p\,(d+e\,x)^2\,dx$$

input
```
int(x^4*(a + b*x^2)^p*(d + e*x)^2,x)
```

output
```
int(x^4*(a + b*x^2)^p*(d + e*x)^2, x)
```

3.392 $\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx$

3.392.1 Optimal result

Integrand size = 20, antiderivative size = 149

$$\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx=-\frac{a\left(bd^2-ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^3(1+p)}+\frac{\left(bd^2-2ae^2\right)\left(a+bx^2\right)^{2+p}}{2b^3(2+p)}+\frac{e^2\left(a+bx^2\right)^{3+p}}{2b^3(3+p)}+\frac{2}{5}dex^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

output
```
-1/2*a*(-a*e^2+b*d^2)*(b*x^2+a)^(p+1)/b^3/(p+1)+1/2*(-2*a*e^2+b*d^2)*(b*x^
2+a)^(2+p)/b^3/(2+p)+1/2*e^2*(b*x^2+a)^(3+p)/b^3/(3+p)+2/5*d*e*x^5*(b*x^2+
a)^p*hypergeom([5/2, -p],[7/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

3.392.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.02

$$\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx=\frac{1}{10}\left(a+bx^2\right)^p\left(\frac{5d^2\left(a+bx^2\right)\left(-a+b(1+p)x^2\right)}{b^2(1+p)(2+p)}+\frac{5e^2\left(a+bx^2\right)\left(2a^2-2ab(1+p)x^2+b^2\left(2+3p+p^2\right)x^4\right)}{b^3(1+p)(2+p)(3+p)}+4dex^5\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)\right)$$

input
```
Integrate[x^3*(d + e*x)^2*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*((5*d^2*(a + b*x^2)*(-a + b*(1 + p)*x^2))/(b^2*(1 + p)*(2 +
 p)) + (5*e^2*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p + p^2)
*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (4*d*e*x^5*Hypergeometric2F1[5/2, -
p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/10
```

3.392.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {543, 27, 279, 278, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx$$

$$\Big\downarrow \text{ 543}$$

$$\int x^3\left(bx^2+a\right)^p\left(d^2+e^2x^2\right)dx+\int 2dex^4\left(bx^2+a\right)^p dx$$

$$\Big\downarrow \text{ 27}$$

$$\int x^3\left(bx^2+a\right)^p\left(d^2+e^2x^2\right)dx+2de\int x^4\left(bx^2+a\right)^p dx$$

$$\Big\downarrow \text{ 279}$$

$$\int x^3\left(bx^2+a\right)^p\left(d^2+e^2x^2\right)dx+2de\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^4\left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{ 278}$$

$$\int x^3\left(bx^2+a\right)^p\left(d^2+e^2x^2\right)dx+\frac{2}{5}dex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int x^2\left(bx^2+a\right)^p\left(d^2+e^2x^2\right)dx^2+\frac{2}{5}dex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{a(ae^2-bd^2)(bx^2+a)^p}{b^2}+\frac{(bd^2-2ae^2)(bx^2+a)^{p+1}}{b^2}+\frac{e^2(bx^2+a)^{p+2}}{b^2}\right)dx^2+ \frac{2}{5}dex^5(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a(bd^2-ae^2)(a+bx^2)^{p+1}}{b^3(p+1)}+\frac{(bd^2-2ae^2)(a+bx^2)^{p+2}}{b^3(p+2)}+\frac{e^2(a+bx^2)^{p+3}}{b^3(p+3)}\right)+ \frac{2}{5}dex^5(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

input
```
Int[x^3*(d + e*x)^2*(a + b*x^2)^p,x]
```

output
```
(-((a*(b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(b^3*(1 + p))) + ((b*d^2 - 2*a*
e^2)*(a + b*x^2)^(2 + p))/(b^3*(2 + p)) + (e^2*(a + b*x^2)^(3 + p))/(b^3*(
3 + p)))/2 + (2*d*e*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b
*x^2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

3.392.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

3.392. $\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx$

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.392.4 Maple [F]

$$\int x^{3}(ex+d)^{2} \left(b\,x^{2}+a\right)^{p} dx$$

input
```
int(x^3*(e*x+d)^2*(b*x^2+a)^p,x)
```

output
```
int(x^3*(e*x+d)^2*(b*x^2+a)^p,x)
```

3.392.5 Fricas [F]

$$\int x^3(d+ex)^2 \left(a+bx^2\right)^p \, dx = \int (ex+d)^2 \left(bx^2+a\right)^p x^3 \, dx$$

input
```
integrate(x^3*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e^2*x^5 + 2*d*e*x^4 + d^2*x^3)*(b*x^2 + a)^p, x)
```

3.392.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(124) = 248.

Time = 9.55 (sec) , antiderivative size = 1294, normalized size of antiderivative = 8.68

$$\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx = \text{Too large to display}$$

input
```
integrate(x**3*(e*x+d)**2*(b*x**2+a)**p,x)
```

output
```
2*a**p*d*e*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + d**
2*Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*
b**3*x**2) + a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2
+ 2*b**3*x**2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x
**2*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x -
sqrt(-a/b))/(2*b**2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p,
-1)), (-a**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**
2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x
**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b
**2*p**2 + 6*b**2*p + 4*b**2), True)) + e**2*Piecewise((a**p*x**6/6, Eq(b,
 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x*
*4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**
4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(x
 - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*lo
g(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2
/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(x - sqrt(-a
/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(x + sqr
t(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a**2
*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(x + sqrt(-a/b))
/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**...
```

3.392.7 Maxima [F]

$$\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx=\int (ex+d)^2\left(bx^2+a\right)^px^3\,dx$$

input
```
integrate(x^3*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
1/2*(b^2*(p + 1)*x^4 + a*b*p*x^2 - a^2)*(b*x^2 + a)^p*d^2/((p^2 + 3*p + 2)
*b^2) + integrate((e^2*x^5 + 2*d*e*x^4)*(b*x^2 + a)^p, x)
```

3.392.8 Giac [F]

$$\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx=\int (ex+d)^2\left(bx^2+a\right)^px^3\,dx$$

input
```
integrate(x^3*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(b*x^2 + a)^p*x^3, x)
```

3.392.9 Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx=\int x^3\,\left(b\,x^2+a\right)^p\,\left(d+e\,x\right)^2\,dx$$

input
```
int(x^3*(a + b*x^2)^p*(d + e*x)^2,x)
```

output
```
int(x^3*(a + b*x^2)^p*(d + e*x)^2, x)
```

3.393 $\int x^2(d+ex)^2\left(a+bx^2\right)^p\,dx$

3.393.1 Optimal result

Integrand size = 20, antiderivative size = 152

$$\int x^2(d+ex)^2\left(a+bx^2\right)^p\,dx = -\frac{ade\left(a+bx^2\right)^{1+p}}{b^2(1+p)}+\frac{e^2x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{de\left(a+bx^2\right)^{2+p}}{b^2(2+p)} - \frac{\left(3ae^2-bd^2(5+2p)\right)x^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)}{3b(5+2p)}$$

output
```
-a*d*e*(b*x^2+a)^(p+1)/b^2/(p+1)+e^2*x^3*(b*x^2+a)^(p+1)/b/(5+2*p)+d*e*(b*
x^2+a)^(2+p)/b^2/(2+p)-1/3*(3*a*e^2-b*d^2*(5+2*p))*x^3*(b*x^2+a)^p*hyperge
om([3/2, -p],[5/2],-b*x^2/a)/b/(5+2*p)/((1+b*x^2/a)^p)
```

3.393.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.91

$$\int x^2(d+ex)^2\left(a+bx^2\right)^p\,dx$$
$$=\frac{1}{15}\left(a+bx^2\right)^p\left(5d^2x^3\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)\right.$$
$$\left.+\frac{3e\left(-\frac{5d(a+bx^2)(a-b(1+p)x^2)}{b^2}+e(2+3p+p^2)\,x^5\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)\right)}{(1+p)(2+p)}\right)$$

input
```
Integrate[x^2*(d + e*x)^2*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*((5*d^2*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])/
(1 + (b*x^2)/a)^p + (3*e*((-5*d*(a + b*x^2)*(a - b*(1 + p)*x^2))/b^2 + (e*
(2 + 3*p + p^2)*x^5*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b
*x^2)/a)^p))/((1 + p)*(2 + p))))/15
```

3.393.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {543, 27, 243, 53, 363, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+ex)^2\left(a+bx^2\right)^p\,dx$$
$$\Big\downarrow \text{ 543}$$
$$\int x^2\left(bx^2+a\right)^p\left(d^2+e^2x^2\right)dx+\int 2dex^3\left(bx^2+a\right)^p dx$$
$$\Big\downarrow \text{ 27}$$
$$\int x^2\left(bx^2+a\right)^p\left(d^2+e^2x^2\right)dx+2de\int x^3\left(bx^2+a\right)^p dx$$

$$\Big\downarrow \text{243}$$

$$\int x^2 (bx^2+a)^p (d^2+e^2x^2)\, dx + de \int x^2 (bx^2+a)^p\, dx^2$$

$$\Big\downarrow \text{53}$$

$$\int x^2 (bx^2+a)^p (d^2+e^2x^2)\, dx + de \int \left(\frac{(bx^2+a)^{p+1}}{b} - \frac{a(bx^2+a)^p}{b}\right) dx^2$$

$$\Big\downarrow \text{363}$$

$$\left(d^2 - \frac{3ae^2}{2bp+5b}\right) \int x^2 (bx^2+a)^p\, dx + de \int \left(\frac{(bx^2+a)^{p+1}}{b} - \frac{a(bx^2+a)^p}{b}\right) dx^2 + \frac{e^2x^3(a+bx^2)^{p+1}}{b(2p+5)}$$

$$\Big\downarrow \text{279}$$

$$(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(d^2 - \frac{3ae^2}{2bp+5b}\right) \int x^2 \left(\frac{bx^2}{a}+1\right)^p dx + de \int \left(\frac{(bx^2+a)^{p+1}}{b} - \frac{a(bx^2+a)^p}{b}\right) dx^2 + \frac{e^2x^3(a+bx^2)^{p+1}}{b(2p+5)}$$

$$\Big\downarrow \text{278}$$

$$de \int \left(\frac{(bx^2+a)^{p+1}}{b} - \frac{a(bx^2+a)^p}{b}\right) dx^2 + \frac{1}{3}x^3(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(d^2 - \frac{3ae^2}{2bp+5b}\right) \text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right) + \frac{e^2x^3(a+bx^2)^{p+1}}{b(2p+5)}$$

$$\Big\downarrow \text{2009}$$

$$de\left(\frac{(a+bx^2)^{p+2}}{b^2(p+2)} - \frac{a(a+bx^2)^{p+1}}{b^2(p+1)}\right) + \frac{1}{3}x^3(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(d^2 - \frac{3ae^2}{2bp+5b}\right) \text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right) + \frac{e^2x^3(a+bx^2)^{p+1}}{b(2p+5)}$$

input
```
Int[x^2*(d + e*x)^2*(a + b*x^2)^p,x]
```

3.393. $\int x^2(d+ex)^2\,(a+bx^2)^p\, dx$

output
```
(e^2*x^3*(a + b*x^2)^(1 + p))/(b*(5 + 2*p)) + d*e*(-((a*(a + b*x^2)^(1 + p
))/(b^2*(1 + p))) + (a + b*x^2)^(2 + p)/(b^2*(2 + p))) + ((d^2 - (3*a*e^2)
/(5*b + 2*b*p))*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2
)/a)])/(3*(1 + (b*x^2)/a)^p)
```

3.393.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.393. $\int x^2(d+ex)^2\left(a+bx^2\right)^p\,dx$

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.393.4 Maple [F]

$$\int x^{2}(ex+d)^{2} \left(b\,x^{2}+a\right)^{p}dx$$

input
```
int(x^2*(e*x+d)^2*(b*x^2+a)^p,x)
```

output
```
int(x^2*(e*x+d)^2*(b*x^2+a)^p,x)
```

3.393.5 Fricas [F]

$$\int x^2(d+ex)^2 \left(a+bx^2\right)^p \, dx=\int (ex+d)^2 \left(bx^2+a\right)^p x^2 \, dx$$

input
```
integrate(x^2*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e^2*x^4 + 2*d*e*x^3 + d^2*x^2)*(b*x^2 + a)^p, x)
```

3.393.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(126) = 252$.

Time = 8.84 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.63

$$\int x^2(d+ex)^2\left(a+bx^2\right)^p\,dx=\frac{a^pd^2x^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3}+\frac{a^pe^2x^5{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5}+2de\left(\begin{cases}\frac{a^px^4}{4} & \text{for } b=0\\ \frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{a}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} & \text{for } p=-2\\ -\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^2}-\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^2}+\frac{x^2}{2b} & \text{for } p=-1\\ -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**2*(e*x+d)**2*(b*x**2+a)**p,x)
```

output
```
a**p*d**2*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + a**p
*e**2*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + 2*d*e*Pi
ecewise((a**p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3
*x**2) + a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*
b**3*x**2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*
log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt
(-a/b))/(2*b**2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1))
, (-a**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a
 + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)
**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*
p**2 + 6*b**2*p + 4*b**2), True))
```

3.393.7 Maxima [F]

$$\int x^2(d+ex)^2\left(a+bx^2\right)^p\,dx=\int(ex+d)^2\left(bx^2+a\right)^px^2\,dx$$

input
```
integrate(x^2*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2*(b*x^2 + a)^p*x^2, x)
```

3.393.8 Giac [F]

$$\int x^2(d+ex)^2\left(a+bx^2\right)^p\,dx=\int (ex+d)^2\left(bx^2+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(b*x^2 + a)^p*x^2, x)
```

3.393.9 Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex)^2\left(a+bx^2\right)^p\,dx=\int x^2\left(b\,x^2+a\right)^p(d+e\,x)^2\,dx$$

input
```
int(x^2*(a + b*x^2)^p*(d + e*x)^2,x)
```

output
```
int(x^2*(a + b*x^2)^p*(d + e*x)^2, x)
```

3.394 $\int x(d+ex)^2 (a+bx^2)^p \, dx$

3.394.1 Optimal result

Integrand size = 18, antiderivative size = 113

$$\int x(d+ex)^2 (a+bx^2)^p \, dx = \frac{(bd^2-ae^2)(a+bx^2)^{1+p}}{2b^2(1+p)} + \frac{e^2(a+bx^2)^{2+p}}{2b^2(2+p)} + \frac{2}{3}dex^3(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)$$

output
```
1/2*(-a*e^2+b*d^2)*(b*x^2+a)^(p+1)/b^2/(p+1)+1/2*e^2*(b*x^2+a)^(2+p)/b^2/(
2+p)+2/3*d*e*x^3*(b*x^2+a)^p*hypergeom([3/2, -p],[5/2],-b*x^2/a)/((1+b*x^2
/a)^p)
```

3.394.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.63

$$\int x(d+ex)^2 (a+bx^2)^p \, dx = \frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(3b^2x^2\left(1+\frac{bx^2}{a}\right)^p (d^2(2+p)+e^2(1+p)x^2) - 3a^2e^2\left(-1+\left(1+\frac{bx^2}{a}\right)^p\right) + 3ab\left(e^2 \cdots\right.\right.}{6b^2(1 \cdots}$$

input
```
Integrate[x*(d + e*x)^2*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*(3*b^2*x^2*(1 + (b*x^2)/a)^p*(d^2*(2 + p) + e^2*(1 + p)*x^2
) - 3*a^2*e^2*(-1 + (1 + (b*x^2)/a)^p) + 3*a*b*(e^2*p*x^2*(1 + (b*x^2)/a)^
p + d^2*(2 + p)*(-1 + (1 + (b*x^2)/a)^p)) + 4*b^2*d*e*(2 + 3*p + p^2)*x^3*
Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)]))/(6*b^2*(1 + p)*(2 + p)*(1
+ (b*x^2)/a)^p)
```

3.394.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {543, 27, 279, 278, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex)^2\left(a+bx^2\right)^p\,dx$$

$$\Big\downarrow \text{ 543}$$

$$\int x\left(bx^2+a\right)^p\left(d^2+e^2x^2\right)dx+\int 2dex^2\left(bx^2+a\right)^p dx$$

$$\Big\downarrow \text{ 27}$$

$$\int x\left(bx^2+a\right)^p\left(d^2+e^2x^2\right)dx+2de\int x^2\left(bx^2+a\right)^p dx$$

$$\Big\downarrow \text{ 279}$$

$$\int x\left(bx^2+a\right)^p\left(d^2+e^2x^2\right)dx+2de\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^2\left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{ 278}$$

$$\int x\left(bx^2+a\right)^p\left(d^2+e^2x^2\right)dx+\frac{2}{3}dex^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)$$

$$\Big\downarrow \text{ 353}$$

$$\frac{1}{2}\int\left(bx^2+a\right)^p\left(d^2+e^2x^2\right)dx^2+\frac{2}{3}dex^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2}\int\left(\frac{(bd^2-ae^2)\,(bx^2+a)^p}{b}+\frac{e^2(bx^2+a)^{p+1}}{b}\right)dx^2+$$
$$\frac{2}{3}dex^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(\frac{(bd^2-ae^2)\,(a+bx^2)^{p+1}}{b^2(p+1)}+\frac{e^2(a+bx^2)^{p+2}}{b^2(p+2)}\right)+$$
$$\frac{2}{3}dex^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)$$

input
```
Int[x*(d + e*x)^2*(a + b*x^2)^p,x]
```

output
```
(((b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(b^2*(1 + p)) + (e^2*(a + b*x^2)^(2
 + p))/(b^2*(2 + p)))/2 + (2*d*e*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2,
-p, 5/2, -((b*x^2)/a)])/(3*(1 + (b*x^2)/a)^p)
```

3.394.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

3.394. $\int x(d+ex)^2\,(a+bx^2)^p\,dx$

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.394.4 Maple [F]

$$\int x(ex+d)^{2} \left(b\,x^{2}+a\right)^{p}dx$$

input
```
int(x*(e*x+d)^2*(b*x^2+a)^p,x)
```

output
```
int(x*(e*x+d)^2*(b*x^2+a)^p,x)
```

3.394.5 Fricas [F]

$$\int x(d+ex)^2 \left(a+bx^2\right)^p \, dx = \int (ex+d)^2 \left(bx^2+a\right)^p x \, dx$$

input
```
integrate(x*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e^2*x^3 + 2*d*e*x^2 + d^2*x)*(b*x^2 + a)^p, x)
```

3.394.6 Sympy [A] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 408, normalized size of antiderivative = 3.61

$$\int x(d+ex)^2\left(a+bx^2\right)^p\,dx$$

$$= \frac{2a^p dex^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3} + d^2\left(\begin{cases}\frac{a^px^2}{2} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log(a+bx^2) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

$$+e^2\left(\begin{cases}\frac{a^px^4}{4} & \text{for } b=0\\ \frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{a}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} & \text{for } p=-2\\ -\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^2}-\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^2}+\frac{x^2}{2b} & \text{for } p=-1\\ -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x*(e*x+d)**2*(b*x**2+a)**p,x)
```

output
```
2*a**p*d*e*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + d**
2*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p
+ 1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b), True)) + e**2*Piecewise(
(a**p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) +
 a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x**
2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(x +
sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(-a/b))/
(2*b**2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2
*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**
2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b
**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6
*b**2*p + 4*b**2), True))
```

3.394.7 Maxima [F]

$$\int x(d+ex)^2 \left(a+bx^2\right)^p dx = \int (ex+d)^2 \left(bx^2+a\right)^p x \, dx$$

input
```
integrate(x*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
1/2*(b*x^2 + a)^(p + 1)*d^2/(b*(p + 1)) + integrate((e^2*x^3 + 2*d*e*x^2)*
(b*x^2 + a)^p, x)
```

3.394.8 Giac [F]

$$\int x(d+ex)^2 \left(a+bx^2\right)^p dx = \int (ex+d)^2 \left(bx^2+a\right)^p x \, dx$$

input
```
integrate(x*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(b*x^2 + a)^p*x, x)
```

3.394.9 Mupad [F(-1)]

Timed out.

$$\int x(d+ex)^2 \left(a+bx^2\right)^p dx = \int x \left(b\,x^2+a\right)^p (d+e\,x)^2 \, dx$$

input
```
int(x*(a + b*x^2)^p*(d + e*x)^2,x)
```

output
```
int(x*(a + b*x^2)^p*(d + e*x)^2, x)
```

3.395 $\int (d+ex)^2 (a+bx^2)^p \, dx$

3.395.1 Optimal result

Integrand size = 17, antiderivative size = 133

$$\int (d+ex)^2 (a+bx^2)^p \, dx = \frac{de(2+p)(a+bx^2)^{1+p}}{b(1+p)(3+2p)} + \frac{e(d+ex)(a+bx^2)^{1+p}}{b(3+2p)} - \frac{(ae^2-bd^2(3+2p))\,x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{b(3+2p)}$$

output
```
d*e*(2+p)*(b*x^2+a)^(p+1)/b/(2*p^2+5*p+3)+e*(e*x+d)*(b*x^2+a)^(p+1)/b/(3+2
*p)-(a*e^2-b*d^2*(3+2*p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a
)/b/(3+2*p)/((1+b*x^2/a)^p)
```

3.395.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00

$$\int (d+ex)^2 (a+bx^2)^p \, dx = \frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(3bd^2(1+p)x \,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) + e\left(3d\left(bx^2\left(1+\frac{bx^2}{a}\right)^p + a\left(\right.\right.\right.\right.}{3b(1+p)}$$

input
```
Integrate[(d + e*x)^2*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*(3*b*d^2*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2
)/a)] + e*(3*d*(b*x^2*(1 + (b*x^2)/a)^p + a*(-1 + (1 + (b*x^2)/a)^p)) + b*
e*(1 + p)*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)]))/(3*b*(1 + p
)*(1 + (b*x^2)/a)^p)
```

3.395.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {497, 25, 455, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \left(a+bx^2\right)^p \, dx$$

$$\Big\downarrow \ 497$$

$$\frac{\int -\left(\left(-b(2p+3)d^2-2be(p+2)xd+ae^2\right)\left(bx^2+a\right)^p\right)dx}{b(2p+3)}+\frac{e(d+ex)\left(a+bx^2\right)^{p+1}}{b(2p+3)}$$

$$\Big\downarrow \ 25$$

$$\frac{e(d+ex)\left(a+bx^2\right)^{p+1}}{b(2p+3)}-\frac{\int \left(-b(2p+3)d^2-2be(p+2)xd+ae^2\right)\left(bx^2+a\right)^p dx}{b(2p+3)}$$

$$\Big\downarrow \ 455$$

$$\frac{e(d+ex)\left(a+bx^2\right)^{p+1}}{b(2p+3)}-\frac{\left(ae^2-bd^2(2p+3)\right)\int \left(bx^2+a\right)^p dx-\frac{de(p+2)\left(a+bx^2\right)^{p+1}}{p+1}}{b(2p+3)}$$

$$\Big\downarrow \ 238$$

$$\frac{e(d+ex)\left(a+bx^2\right)^{p+1}}{b(2p+3)}-\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2-bd^2(2p+3)\right)\int \left(\frac{bx^2}{a}+1\right)^p dx-\frac{de(p+2)\left(a+bx^2\right)^{p+1}}{p+1}}{b(2p+3)}$$

$$\Big\downarrow \ 237$$

$$\frac{e(d+ex)\left(a+bx^2\right)^{p+1}}{b(2p+3)} - \frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2-bd^2(2p+3)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)-\frac{de(p+2)\left(a+bx^2\right)^{p+1}}{p+1}}{b(2p+3)}$$

input
```
Int[(d + e*x)^2*(a + b*x^2)^p,x]
```

output
```
(e*(d + e*x)*(a + b*x^2)^(1 + p))/(b*(3 + 2*p)) - (-((d*e*(2 + p)*(a + b*x
^2)^(1 + p))/(1 + p)) + ((a*e^2 - b*d^2*(3 + 2*p))*x*(a + b*x^2)^p*Hyperge
ometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p)/(b*(3 + 2*p))
```

3.395.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

3.395.4 Maple [F]

$$\int (ex+d)^2 \left(b\,x^2+a\right)^p dx$$

input
```
int((e*x+d)^2*(b*x^2+a)^p,x)
```

output
```
int((e*x+d)^2*(b*x^2+a)^p,x)
```

3.395.5 Fricas [F]

$$\int (d+ex)^2 \left(a+bx^2\right)^p \, dx = \int (ex+d)^2 \left(bx^2+a\right)^p dx$$

input
```
integrate((e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e^2*x^2 + 2*d*e*x + d^2)*(b*x^2 + a)^p, x)
```

3.395.6 Sympy [A] (verification not implemented)

Time = 4.57 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.73

$$\int (d+ex)^2 \left(a+bx^2\right)^p \, dx = a^p d^2 x\, {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, -p\\ \frac{3}{2}\end{matrix}\right| \frac{bx^2 e^{i\pi}}{a}\right) + \frac{a^p e^2 x^3\, {}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, -p\\ \frac{5}{2}\end{matrix}\right| \frac{bx^2 e^{i\pi}}{a}\right)}{3} + 2de \left(\begin{cases} \frac{a^p x^2}{2} & \text{for}\ b=0 \\ \frac{\begin{cases} \frac{(a+bx^2)^{p+1}}{p+1} & \text{for}\ p \neq -1 \\ \log\left(a+bx^2\right) & \text{otherwise} \end{cases}}{2b} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x+d)**2*(b*x**2+a)**p,x)
```

output
```
a**p*d**2*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*e**2
*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + 2*d*e*Piecewi
se((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(
p, -1)), (log(a + b*x**2), True))/(2*b), True))
```

3.395.7 Maxima [F]

$$\int (d+ex)^2 \left(a+bx^2\right)^p \, dx = \int (ex+d)^2 (bx^2+a)^p \, dx$$

input
```
integrate((e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2*(b*x^2 + a)^p, x)
```

3.395.8 Giac [F]

$$\int (d+ex)^2 \left(a+bx^2\right)^p \, dx = \int (ex+d)^2 (bx^2+a)^p \, dx$$

input
```
integrate((e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(b*x^2 + a)^p, x)
```

3.395.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(a+bx^2\right)^p \, dx = \int (b\,x^2+a)^p \,(d+e\,x)^2 \, dx$$

input
```
int((a + b*x^2)^p*(d + e*x)^2,x)
```

output
```
int((a + b*x^2)^p*(d + e*x)^2, x)
```

3.396 $\int \frac{(d+ex)^2(a+bx^2)^p}{x} dx$

3.396.1 Optimal result

Integrand size = 20, antiderivative size = 118

$$\int \frac{(d+ex)^2(a+bx^2)^p}{x} dx = \frac{e^2(a+bx^2)^{1+p}}{2b(1+p)} + 2dex(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) - \frac{d^2(a+bx^2)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

output
```
1/2*e^2*(b*x^2+a)^(p+1)/b/(p+1)+2*d*e*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3
/2],-b*x^2/a)/((1+b*x^2/a)^p)-1/2*d^2*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[
2+p],1+b*x^2/a)/a/(p+1)
```

3.396.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)^2(a+bx^2)^p}{x} dx = \frac{1}{2}(a+bx^2)^p \left(4dex\left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) + \frac{(a+bx^2)\left(ae^2-bd^2 \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)\right)}{ab(1+p)}\right)$$

input
```
Integrate[((d + e*x)^2*(a + b*x^2)^p)/x,x]
```

output
```
((a + b*x^2)^p*((4*d*e*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1
 + (b*x^2)/a)^p + ((a + b*x^2)*(a*e^2 - b*d^2*Hypergeometric2F1[1, 1 + p,
2 + p, 1 + (b*x^2)/a]))/(a*b*(1 + p))))/2
```

3.396.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {543, 27, 238, 237, 354, 90, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x} \, dx$$

$$\Big\downarrow \text{543}$$

$$\int \frac{(bx^2+a)^p (d^2+e^2x^2)}{x}dx+\int 2de(bx^2+a)^p dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(bx^2+a)^p (d^2+e^2x^2)}{x}dx+2de\int (bx^2+a)^p dx$$

$$\Big\downarrow \text{238}$$

$$\int \frac{(bx^2+a)^p (d^2+e^2x^2)}{x}dx+2de(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int \left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{237}$$

$$\int \frac{(bx^2+a)^p (d^2+e^2x^2)}{x}dx+ 2dex(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^p (d^2+e^2x^2)}{x^2}dx^2+ 2dex(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

$$\Big\downarrow \ 90$$

$$\frac{1}{2}\left(d^2\int \frac{(bx^2+a)^p}{x^2}dx^2+\frac{e^2(a+bx^2)^{p+1}}{b(p+1)}\right)+ 2dex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

$$\Big\downarrow \ 75$$

$$\frac{1}{2}\left(\frac{e^2(a+bx^2)^{p+1}}{b(p+1)}-\frac{d^2(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{a(p+1)}\right)+ 2dex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

input
```
Int[((d + e*x)^2*(a + b*x^2)^p)/x,x]
```

output
```
(2*d*e*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^p + ((e^2*(a + b*x^2)^(1 + p))/(b*(1 + p)) - (d^2*(a + b*x^2)^
(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(a*(1 + p)))/2
```

3.396.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

3.396. $\int \frac{(d+ex)^2(a+bx^2)^p}{x}\,dx$

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

3.396.4 Maple [F]

$$\int \frac{(ex+d)^2 \left(b\,x^2+a\right)^p}{x}dx$$

input
```
int((e*x+d)^2*(b*x^2+a)^p/x,x)
```

output
```
int((e*x+d)^2*(b*x^2+a)^p/x,x)
```

3.396.5 Fricas [F]

$$\int \frac{(d+ex)^2\,(a+bx^2)^p}{x}\,dx = \int \frac{(ex+d)^2(bx^2+a)^p}{x}\,dx$$

input
```
integrate((e*x+d)^2*(b*x^2+a)^p/x,x, algorithm="fricas")
```

output
```
integral((e^2*x^2 + 2*d*e*x + d^2)*(b*x^2 + a)^p/x, x)
```

3.396.6 Sympy [A] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^2\,(a+bx^2)^p}{x}\,dx = 2a^p dex\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right) - \frac{b^p d^2 x^{2p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(1-p)} + e^2\left(\begin{cases}\frac{a^p x^2}{2} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log(a+bx^2) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((e*x+d)**2*(b*x**2+a)**p/x,x)
```

output
```
2*a**p*d*e*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - b**p*d**
2*x**(2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))
/(2*gamma(1 - p)) + e**2*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a
 + b*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b), T
rue))
```

3.396.7 Maxima [F]

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x} \, dx = \int \frac{(ex+d)^2 (bx^2+a)^p}{x} \, dx$$

input
```
integrate((e*x+d)^2*(b*x^2+a)^p/x,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2*(b*x^2 + a)^p/x, x)
```

3.396.8 Giac [F]

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x} \, dx = \int \frac{(ex+d)^2 (bx^2+a)^p}{x} \, dx$$

input
```
integrate((e*x+d)^2*(b*x^2+a)^p/x,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(b*x^2 + a)^p/x, x)
```

3.396.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x} \, dx = \int \frac{(b\,x^2+a)^p\,(d+e\,x)^2}{x} \, dx$$

input
```
int(((a + b*x^2)^p*(d + e*x)^2)/x,x)
```

output
```
int(((a + b*x^2)^p*(d + e*x)^2)/x, x)
```

3.397 $\int \frac{(d+ex)^2(a+bx^2)^p}{x^2}\,dx$

3.397.1 Optimal result

Integrand size = 20, antiderivative size = 127

$$\int \frac{(d+ex)^2(a+bx^2)^p}{x^2}\,dx = -\frac{d^2(a+bx^2)^{1+p}}{ax} + \frac{(ae^2+bd^2(1+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{a} - \frac{de(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{a(1+p)}$$

output
```
-d^2*(b*x^2+a)^(p+1)/a/x+(a*e^2+b*d^2*(1+2*p))*x*(b*x^2+a)^p*hypergeom([1/
2, -p],[3/2],-b*x^2/a)/a/((1+b*x^2/a)^p)-d*e*(b*x^2+a)^(p+1)*hypergeom([1,
 p+1],[2+p],1+b*x^2/a)/a/(p+1)
```

3.397.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^2} dx = -\frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(ad^2(1+p) \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)+ex\left(-ae(1+p)x \text{Hyperge}\right.\right.}{a(1-}$$

input
```
Integrate[((d + e*x)^2*(a + b*x^2)^p)/x^2,x]
```

output
```
-(((a + b*x^2)^p*(a*d^2*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)
/a)] + e*x*(-(a*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])
 + d*(a + b*x^2)*(1 + (b*x^2)/a)^p*Hypergeometric2F1[1, 1 + p, 2 + p, 1 +
(b*x^2)/a])))/(a*(1 + p)*x*(1 + (b*x^2)/a)^p))
```

3.397.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {543, 27, 243, 75, 359, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^2} dx$$

$$\Big\downarrow \text{ 543}$$

$$\int \frac{(bx^2+a)^p (d^2+e^2x^2)}{x^2} dx + \int \frac{2de(bx^2+a)^p}{x} dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(bx^2+a)^p (d^2+e^2x^2)}{x^2} dx + 2de \int \frac{(bx^2+a)^p}{x} dx$$

$$\Big\downarrow \text{ 243}$$

$$\int \frac{(bx^2+a)^p (d^2+e^2x^2)}{x^2} dx + de \int \frac{(bx^2+a)^p}{x^2} dx^2$$

$$\Big\downarrow \text{ 75}$$

$$\int \frac{(bx^2+a)^p(d^2+e^2x^2)}{x^2}dx - \frac{de(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{a(p+1)}$$

$$\Big\downarrow \text{359}$$

$$\frac{(ae^2+bd^2(2p+1))\int (bx^2+a)^p dx}{a} - \frac{d^2(a+bx^2)^{p+1}}{ax} - \frac{de(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{a(p+1)}$$

$$\Big\downarrow \text{238}$$

$$\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ae^2+bd^2(2p+1))\int \left(\frac{bx^2}{a}+1\right)^p dx}{a} - \frac{d^2(a+bx^2)^{p+1}}{ax} - \frac{de(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{a(p+1)}$$

$$\Big\downarrow \text{237}$$

$$\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ae^2+bd^2(2p+1))\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{a} - \frac{d^2(a+bx^2)^{p+1}}{ax} - \frac{de(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{a(p+1)}$$

input
```
Int[((d + e*x)^2*(a + b*x^2)^p)/x^2,x]
```

output
```
-((d^2*(a + b*x^2)^(1 + p))/(a*x)) + ((a*e^2 + b*d^2*(1 + 2*p))*x*(a + b*x
^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(a*(1 + (b*x^2)/a)^p)
 - (d*e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)
/a])/(a*(1 + p))
```

3.397. $\int \frac{(d+ex)^2(a+bx^2)^p}{x^2}\,dx$

3.397.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

3.397. $\int \frac{(d+ex)^2(a+bx^2)^p}{x^2}\,dx$

3.397.4 Maple [F]

$$\int \frac{(ex+d)^2 (b\,x^2+a)^p}{x^2}dx$$

input
```
int((e*x+d)^2*(b*x^2+a)^p/x^2,x)
```

output
```
int((e*x+d)^2*(b*x^2+a)^p/x^2,x)
```

3.397.5 Fricas [F]

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^2}\,dx = \int \frac{(ex+d)^2(bx^2+a)^p}{x^2}\,dx$$

input
```
integrate((e*x+d)^2*(b*x^2+a)^p/x^2,x, algorithm="fricas")
```

output
```
integral((e^2*x^2 + 2*d*e*x + d^2)*(b*x^2 + a)^p/x^2, x)
```

3.397.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.99 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^2}\,dx = -\frac{a^p d^2 {}_2F_1\left(\begin{matrix}-\frac{1}{2}, -p\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{x} + a^p e^2 x {}_2F_1\left(\begin{matrix}\frac{1}{2}, -p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right) - \frac{b^p dex^{2p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p, -p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{\Gamma(1-p)}$$

input
```
integrate((e*x+d)**2*(b*x**2+a)**p/x**2,x)
```

output
```
-a**p*d**2*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x + a**p*e*
*2*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - b**p*d*e*x**(2*p
)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/gamma(1
- p)
```

3.397.7 Maxima [F]

$$\int \frac{(d+ex)^2\,(a+bx^2)^p}{x^2}\,dx = \int \frac{(ex+d)^2(bx^2+a)^p}{x^2}\,dx$$

input
```
integrate((e*x+d)^2*(b*x^2+a)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2*(b*x^2 + a)^p/x^2, x)
```

3.397.8 Giac [F]

$$\int \frac{(d+ex)^2\,(a+bx^2)^p}{x^2}\,dx = \int \frac{(ex+d)^2(bx^2+a)^p}{x^2}\,dx$$

input
```
integrate((e*x+d)^2*(b*x^2+a)^p/x^2,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(b*x^2 + a)^p/x^2, x)
```

3.397.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2\,(a+bx^2)^p}{x^2}\,dx = \int \frac{(b\,x^2+a)^p\,(d+e\,x)^2}{x^2}\,dx$$

input
```
int(((a + b*x^2)^p*(d + e*x)^2)/x^2,x)
```

output
```
int(((a + b*x^2)^p*(d + e*x)^2)/x^2, x)
```

3.398 $\int \frac{(d+ex)^2(a+bx^2)^p}{x^3}\,dx$

3.398.1 Optimal result

Integrand size = 20, antiderivative size = 127

$$\int \frac{(d+ex)^2(a+bx^2)^p}{x^3}\,dx = -\frac{d^2(a+bx^2)^{1+p}}{2ax^2} - \frac{2de(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x} - \frac{(ae^2+bd^2p)(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a^2(1+p)}$$

output
```
-1/2*d^2*(b*x^2+a)^(p+1)/a/x^2-2*d*e*(b*x^2+a)^p*hypergeom([-1/2, -p],[1/2
],-b*x^2/a)/x/((1+b*x^2/a)^p)-1/2*(b*d^2*p+a*e^2)*(b*x^2+a)^(p+1)*hypergeo
m([1, p+1],[2+p],1+b*x^2/a)/a^2/(p+1)
```

3.398.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^3} dx$$
$$= \frac{1}{2}(a+bx^2)^p \left(-\frac{4de\left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x} \right.$$
$$- \frac{(a+bx^2)\left(ae^2 \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right) - bd^2 \text{Hypergeometric2F1}\left(2,1+p,2+p\right.\right.}{a^2(1+p)}$$

input
```
Integrate[((d + e*x)^2*(a + b*x^2)^p)/x^3,x]
```

output
```
((a + b*x^2)^p*((-4*d*e*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x
*(1 + (b*x^2)/a)^p) - ((a + b*x^2)*(a*e^2*Hypergeometric2F1[1, 1 + p, 2 +
p, 1 + (b*x^2)/a] - b*d^2*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x^2)/a
]))/(a^2*(1 + p))))/2
```

3.398.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {543, 27, 279, 278, 354, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^3} dx$$
$$\Big\downarrow \; 543$$
$$\int \frac{(bx^2+a)^p (d^2+e^2x^2)}{x^3} dx + \int \frac{2de(bx^2+a)^p}{x^2} dx$$
$$\Big\downarrow \; 27$$
$$\int \frac{(bx^2+a)^p (d^2+e^2x^2)}{x^3} dx + 2de \int \frac{(bx^2+a)^p}{x^2} dx$$

$$\Big\downarrow \text{279}$$

$$\int \frac{(bx^2+a)^p (d^2+e^2x^2)}{x^3}dx + 2de(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int \frac{\left(\frac{bx^2}{a}+1\right)^p}{x^2}dx$$

$$\Big\downarrow \text{278}$$

$$\int \frac{(bx^2+a)^p (d^2+e^2x^2)}{x^3}dx - \frac{2de(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x}$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^p (d^2+e^2x^2)}{x^4}dx^2 - \frac{2de(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x}$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{(ae^2+bd^2p)\int \frac{(bx^2+a)^p}{x^2}dx^2}{a} - \frac{d^2(a+bx^2)^{p+1}}{ax^2}\right) - \frac{2de(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x}$$

$$\Big\downarrow \text{75}$$

$$\frac{1}{2}\left(-\frac{(a+bx^2)^{p+1}(ae^2+bd^2p)\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{a^2(p+1)} - \frac{d^2(a+bx^2)^{p+1}}{ax^2}\right) - \frac{2de(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x}$$

input
```
Int[((d + e*x)^2*(a + b*x^2)^p)/x^3,x]
```

output
```
(-2*d*e*(a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(
1 + (b*x^2)/a)^p) + (-((d^2*(a + b*x^2)^(1 + p))/(a*x^2)) - ((a*e^2 + b*d^
2*p)*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a]
)/(a^2*(1 + p)))/2
```

3.398. $\int \frac{(d+ex)^2(a+bx^2)^p}{x^3}\,dx$

3.398.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.398. $\int \frac{(d+ex)^2(a+bx^2)^p}{x^3}\,dx$

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

3.398.4 Maple [F]

$$\int \frac{(ex+d)^2 (b\,x^2+a)^p}{x^3}dx$$

input
```
int((e*x+d)^2*(b*x^2+a)^p/x^3,x)
```

output
```
int((e*x+d)^2*(b*x^2+a)^p/x^3,x)
```

3.398.5 Fricas [F]

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^3}\,dx = \int \frac{(ex+d)^2(bx^2+a)^p}{x^3}\,dx$$

input
```
integrate((e*x+d)^2*(b*x^2+a)^p/x^3,x, algorithm="fricas")
```

output
```
integral((e^2*x^2 + 2*d*e*x + d^2)*(b*x^2 + a)^p/x^3, x)
```

3.398.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.65 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^3} dx = -\frac{2a^p d e {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{x} - \frac{b^p d^2 x^{2p-2} \Gamma(1-p) {}_2F_1\left(\begin{matrix} -p, 1-p \\ 2-p \end{matrix} \middle| \frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(2-p)} - \frac{b^p e^2 x^{2p} \Gamma(-p) {}_2F_1\left(\begin{matrix} -p, -p \\ 1-p \end{matrix} \middle| \frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(1-p)}$$

input
```
integrate((e*x+d)**2*(b*x**2+a)**p/x**3,x)
```

output
```
-2*a**p*d*e*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x - b**p*d
**2*x**(2*p - 2)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), a*exp_polar(I*p
i)/(b*x**2))/(2*gamma(2 - p)) - b**p*e**2*x**(2*p)*gamma(-p)*hyper((-p, -p
), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*gamma(1 - p))
```

3.398.7 Maxima [F]

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^3} dx = \int \frac{(ex+d)^2 (bx^2+a)^p}{x^3} dx$$

input
```
integrate((e*x+d)^2*(b*x^2+a)^p/x^3,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2*(b*x^2 + a)^p/x^3, x)
```

3.398.8 Giac [F]

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^3} \, dx = \int \frac{(ex+d)^2 (bx^2+a)^p}{x^3} \, dx$$

input
```
integrate((e*x+d)^2*(b*x^2+a)^p/x^3,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(b*x^2 + a)^p/x^3, x)
```

3.398.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^3} \, dx = \int \frac{(b\,x^2+a)^p\,(d+e\,x)^2}{x^3} \, dx$$

input
```
int(((a + b*x^2)^p*(d + e*x)^2)/x^3,x)
```

output
```
int(((a + b*x^2)^p*(d + e*x)^2)/x^3, x)
```

3.399 $\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx$

3.399.1 Optimal result

Integrand size = 20, antiderivative size = 247

$$\begin{aligned}\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx &= \frac{a^2d(bd^2-3ae^2)\left(a+bx^2\right)^{1+p}}{2b^4(1+p)}+\frac{e^3x^7\left(a+bx^2\right)^{1+p}}{b(9+2p)}\\ &\quad-\frac{ad(2bd^2-9ae^2)\left(a+bx^2\right)^{2+p}}{2b^4(2+p)}+\frac{d(bd^2-9ae^2)\left(a+bx^2\right)^{3+p}}{2b^4(3+p)}+\frac{3de^2\left(a+bx^2\right)^{4+p}}{2b^4(4+p)}\\ &\quad-\frac{e(7ae^2-3bd^2(9+2p))\,x^7\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)}{7b(9+2p)}\end{aligned}$$

output
```
1/2*a^2*d*(-3*a*e^2+b*d^2)*(b*x^2+a)^(p+1)/b^4/(p+1)+e^3*x^7*(b*x^2+a)^(p+
1)/b/(9+2*p)-1/2*a*d*(-9*a*e^2+2*b*d^2)*(b*x^2+a)^(2+p)/b^4/(2+p)+1/2*d*(-
9*a*e^2+b*d^2)*(b*x^2+a)^(3+p)/b^4/(3+p)+3/2*d*e^2*(b*x^2+a)^(4+p)/b^4/(4+
p)-1/7*e*(7*a*e^2-3*b*d^2*(9+2*p))*x^7*(b*x^2+a)^p*hypergeom([7/2, -p],[9/
2],-b*x^2/a)/b/(9+2*p)/((1+b*x^2/a)^p)
```

3.399.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.01

$$\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx$$
$$=\frac{1}{126}\left(a+bx^2\right)^p\left(\frac{63d^3(a+bx^2)\left(2a^2-2ab(1+p)x^2+b^2(2+3p+p^2)x^4\right)}{b^3(1+p)(2+p)(3+p)}\right.$$
$$+\frac{189de^2(a+bx^2)\left(-6a^3+6a^2b(1+p)x^2-3ab^2(2+3p+p^2)x^4+b^3(6+11p+6p^2+p^3)x^6\right)}{b^4(1+p)(2+p)(3+p)(4+p)}$$
$$+54d^2ex^7\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)$$
$$\left.+14e^3x^9\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{9}{2},-p,\frac{11}{2},-\frac{bx^2}{a}\right)\right)$$

input
```
Integrate[x^5*(d + e*x)^3*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*((63*d^3*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 +
3*p + p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (189*d*e^2*(a + b*x^2)*(-
6*a^3 + 6*a^2*b*(1 + p)*x^2 - 3*a*b^2*(2 + 3*p + p^2)*x^4 + b^3*(6 + 11*p
+ 6*p^2 + p^3)*x^6))/(b^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)) + (54*d^2*e*x^7
*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (14*e^
3*x^9*Hypergeometric2F1[9/2, -p, 11/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/
126
```

3.399.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {543, 354, 27, 86, 363, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx$$
$$\Big\downarrow \text{543}$$
$$\int x^5\left(bx^2+a\right)^p\left(d^3+3e^2x^2d\right)dx+\int x^6\left(bx^2+a\right)^p\left(x^2e^3+3d^2e\right)dx$$

$$\Big\downarrow \text{354}$$

$$\int x^6 \left(bx^2+a\right)^p \left(x^2e^3+3d^2e\right)dx+\frac{1}{2}\int dx^4 \left(bx^2+a\right)^p \left(d^2+3e^2x^2\right)dx^2$$

$$\Big\downarrow \text{27}$$

$$\int x^6 \left(bx^2+a\right)^p \left(x^2e^3+3d^2e\right)dx+\frac{1}{2}d\int x^4 \left(bx^2+a\right)^p \left(d^2+3e^2x^2\right)dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}d\int \left(-\frac{a^2\left(3ae^2-bd^2\right)\left(bx^2+a\right)^p}{b^3}+\frac{a\left(9ae^2-2bd^2\right)\left(bx^2+a\right)^{p+1}}{b^3}+\frac{\left(bd^2-9ae^2\right)\left(bx^2+a\right)^{p+2}}{b^3}+\frac{3e^2\left(bx^2+a\right)}{b^3}\right.$$
$$\int x^6 \left(bx^2+a\right)^p \left(x^2e^3+3d^2e\right)dx$$

$$\Big\downarrow \text{363}$$

$$\frac{1}{2}d\int \left(-\frac{a^2\left(3ae^2-bd^2\right)\left(bx^2+a\right)^p}{b^3}+\frac{a\left(9ae^2-2bd^2\right)\left(bx^2+a\right)^{p+1}}{b^3}+\frac{\left(bd^2-9ae^2\right)\left(bx^2+a\right)^{p+2}}{b^3}+\frac{3e^2\left(bx^2+a\right)}{b^3}\right.$$
$$e\left(3d^2-\frac{7ae^2}{2bp+9b}\right)\int x^6 \left(bx^2+a\right)^p dx+\frac{e^3x^7\left(a+bx^2\right)^{p+1}}{b(2p+9)}$$

$$\Big\downarrow \text{279}$$

$$\frac{1}{2}d\int \left(-\frac{a^2\left(3ae^2-bd^2\right)\left(bx^2+a\right)^p}{b^3}+\frac{a\left(9ae^2-2bd^2\right)\left(bx^2+a\right)^{p+1}}{b^3}+\frac{\left(bd^2-9ae^2\right)\left(bx^2+a\right)^{p+2}}{b^3}+\frac{3e^2\left(bx^2+a\right)}{b^3}\right.$$
$$e\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(3d^2-\frac{7ae^2}{2bp+9b}\right)\int x^6 \left(\frac{bx^2}{a}+1\right)^p dx+\frac{e^3x^7\left(a+bx^2\right)^{p+1}}{b(2p+9)}$$

$$\Big\downarrow \text{278}$$

$$\frac{1}{2}d\int \left(-\frac{a^2\left(3ae^2-bd^2\right)\left(bx^2+a\right)^p}{b^3}+\frac{a\left(9ae^2-2bd^2\right)\left(bx^2+a\right)^{p+1}}{b^3}+\frac{\left(bd^2-9ae^2\right)\left(bx^2+a\right)^{p+2}}{b^3}+\frac{3e^2\left(bx^2+a\right)}{b^3}\right.$$
$$\frac{1}{7}ex^7\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(3d^2-\frac{7ae^2}{2bp+9b}\right)\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)+$$
$$\frac{e^3x^7\left(a+bx^2\right)^{p+1}}{b(2p+9)}$$

$$\Big\downarrow \text{2009}$$

3.399. $\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx$

$$\frac{1}{2}d\left(\frac{a^2(bd^2-3ae^2)(a+bx^2)^{p+1}}{b^4(p+1)}-\frac{a(2bd^2-9ae^2)(a+bx^2)^{p+2}}{b^4(p+2)}+\frac{(bd^2-9ae^2)(a+bx^2)^{p+3}}{b^4(p+3)}+\frac{3e^2(a+bx^2)^{p+4}}{b^4(p+4)}\right)+\frac{1}{7}ex^7(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3d^2-\frac{7ae^2}{2bp+9b}\right)\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)+\frac{e^3x^7(a+bx^2)^{p+1}}{b(2p+9)}$$

input
```
Int[x^5*(d + e*x)^3*(a + b*x^2)^p,x]
```

output
```
(e^3*x^7*(a + b*x^2)^(1 + p))/(b*(9 + 2*p)) + (d*((a^2*(b*d^2 - 3*a*e^2)*(
a + b*x^2)^(1 + p))/(b^4*(1 + p)) - (a*(2*b*d^2 - 9*a*e^2)*(a + b*x^2)^(2
+ p))/(b^4*(2 + p)) + ((b*d^2 - 9*a*e^2)*(a + b*x^2)^(3 + p))/(b^4*(3 + p)
) + (3*e^2*(a + b*x^2)^(4 + p))/(b^4*(4 + p))))/2 + (e*(3*d^2 - (7*a*e^2)/
(9*b + 2*b*p))*x^7*(a + b*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)
/a)])/(7*(1 + (b*x^2)/a)^p)
```

3.399.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.399.4 Maple [F]

$$\int x^5(ex+d)^3 \left(b\,x^2+a\right)^p dx$$

input
```
int(x^5*(e*x+d)^3*(b*x^2+a)^p,x)
```

output
```
int(x^5*(e*x+d)^3*(b*x^2+a)^p,x)
```

3.399.5 Fricas [F]

$$\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3\left(bx^2+a\right)^p x^5\,dx$$

input
```
integrate(x^5*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e^3*x^8 + 3*d*e^2*x^7 + 3*d^2*e*x^6 + d^3*x^5)*(b*x^2 + a)^p, x)
```

3.399.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 923 vs. $2(218) = 436$.

Time = 31.28 (sec) , antiderivative size = 2919, normalized size of antiderivative = 11.82

$$\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx=\text{Too large to display}$$

input
```
integrate(x**5*(e*x+d)**3*(b*x**2+a)**p,x)
```

output
```
3*a**p*d**2*e*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 +
a**p*e**3*x**9*hyper((9/2, -p), (11/2,), b*x**2*exp_polar(I*pi)/a)/9 + d**
3*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b
**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b*
*3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 +
4*b**5*x**4) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2
 + 4*b**5*x**4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x
**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4
) + 2*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*
x**4) + 2*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b
**5*x**4), Eq(p, -3)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**
2) - 2*a**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**
3 + 2*b**4*x**2) - 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2)
 - 2*a*b*x**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*
a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a*
*2*log(x + sqrt(-a/b))/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1))
, (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**
3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3
*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p*
*2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 ...
```

3.399.7 Maxima [F]

$$\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3\left(bx^2+a\right)^p x^5\,dx$$

input
```
integrate(x^5*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
1/2*((p^2 + 3*p + 2)*b^3*x^6 + (p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + 2*a^3
)*(b*x^2 + a)^p*d^3/((p^3 + 6*p^2 + 11*p + 6)*b^3) + integrate((e^3*x^8 +
3*d*e^2*x^7 + 3*d^2*e*x^6)*(b*x^2 + a)^p, x)
```

3.399.8 Giac [F]

$$\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3\left(bx^2+a\right)^p x^5\,dx$$

input
```
integrate(x^5*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(b*x^2 + a)^p*x^5, x)
```

3.399.9 Mupad [F(-1)]

Timed out.

$$\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx=\int x^5\left(b\,x^2+a\right)^p(d+e\,x)^3\,dx$$

input
```
int(x^5*(a + b*x^2)^p*(d + e*x)^3,x)
```

output
```
int(x^5*(a + b*x^2)^p*(d + e*x)^3, x)
```

3.400 $\int x^4(d+ex)^3 \left(a+bx^2\right)^p \,dx$

3.400.1 Optimal result

Integrand size = 20, antiderivative size = 249

$$\begin{aligned}\int x^4(d+ex)^3 \left(a+bx^2\right)^p \,dx &= \frac{a^2e(3bd^2-ae^2)\left(a+bx^2\right)^{1+p}}{2b^4(1+p)}+\frac{3de^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}\\ &\quad-\frac{3ae(2bd^2-ae^2)\left(a+bx^2\right)^{2+p}}{2b^4(2+p)}+\frac{3e(bd^2-ae^2)\left(a+bx^2\right)^{3+p}}{2b^4(3+p)}+\frac{e^3\left(a+bx^2\right)^{4+p}}{2b^4(4+p)}\\ &\quad-\frac{d(15ae^2-bd^2(7+2p))\,x^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)}{5b(7+2p)}\end{aligned}$$

output
```
1/2*a^2*e*(-a*e^2+3*b*d^2)*(b*x^2+a)^(p+1)/b^4/(p+1)+3*d*e^2*x^5*(b*x^2+a)
^(p+1)/b/(7+2*p)-3/2*a*e*(-a*e^2+2*b*d^2)*(b*x^2+a)^(2+p)/b^4/(2+p)+3/2*e*
(-a*e^2+b*d^2)*(b*x^2+a)^(3+p)/b^4/(3+p)+1/2*e^3*(b*x^2+a)^(4+p)/b^4/(4+p)
-1/5*d*(15*a*e^2-b*d^2*(7+2*p))*x^5*(b*x^2+a)^p*hypergeom([5/2, -p],[7/2],
-b*x^2/a)/b/(7+2*p)/((1+b*x^2/a)^p)
```

3.400.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00

$$\int x^4(d+ex)^3\left(a+bx^2\right)^p\,dx$$
$$=\frac{1}{70}\left(a+bx^2\right)^p\left(\frac{105d^2e(a+bx^2)\left(2a^2-2ab(1+p)x^2+b^2\left(2+3p+p^2\right)x^4\right)}{b^3(1+p)(2+p)(3+p)}\right.$$
$$+\frac{35e^3(a+bx^2)\left(-6a^3+6a^2b(1+p)x^2-3ab^2\left(2+3p+p^2\right)x^4+b^3\left(6+11p+6p^2+p^3\right)x^6\right)}{b^4(1+p)(2+p)(3+p)(4+p)}$$
$$+14d^3x^5\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)$$
$$\left.+30de^2x^7\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)\right)$$

input
```
Integrate[x^4*(d + e*x)^3*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*((105*d^2*e*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2
 + 3*p + p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (35*e^3*(a + b*x^2)*(-
6*a^3 + 6*a^2*b*(1 + p)*x^2 - 3*a*b^2*(2 + 3*p + p^2)*x^4 + b^3*(6 + 11*p
+ 6*p^2 + p^3)*x^6))/(b^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)) + (14*d^3*x^5*H
ypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (30*d*e^
2*x^7*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/7
0
```

3.400.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {543, 354, 27, 86, 363, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(d+ex)^3\left(a+bx^2\right)^p\,dx$$
$$\Big\downarrow \ 543$$
$$\int x^4\left(bx^2+a\right)^p\left(d^3+3e^2x^2d\right)dx+\int x^5\left(bx^2+a\right)^p\left(x^2e^3+3d^2e\right)dx$$

$$\Big\downarrow \text{354}$$

$$\int x^4 \left(bx^2+a\right)^p \left(d^3+3e^2x^2d\right) dx + \frac{1}{2}\int ex^4 \left(bx^2+a\right)^p \left(3d^2+e^2x^2\right) dx^2$$

$$\Big\downarrow \text{27}$$

$$\int x^4 \left(bx^2+a\right)^p \left(d^3+3e^2x^2d\right) dx + \frac{1}{2}e\int x^4 \left(bx^2+a\right)^p \left(3d^2+e^2x^2\right) dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}e\int \left(-\frac{a^2\left(ae^2-3bd^2\right)\left(bx^2+a\right)^p}{b^3}+\frac{3a\left(ae^2-2bd^2\right)\left(bx^2+a\right)^{p+1}}{b^3}+\frac{3\left(bd^2-ae^2\right)\left(bx^2+a\right)^{p+2}}{b^3}+\frac{e^2\left(bx^2+a\right)^p}{b^3}\right.$$
$$\int x^4 \left(bx^2+a\right)^p \left(d^3+3e^2x^2d\right) dx$$

$$\Big\downarrow \text{363}$$

$$\frac{1}{2}e\int \left(-\frac{a^2\left(ae^2-3bd^2\right)\left(bx^2+a\right)^p}{b^3}+\frac{3a\left(ae^2-2bd^2\right)\left(bx^2+a\right)^{p+1}}{b^3}+\frac{3\left(bd^2-ae^2\right)\left(bx^2+a\right)^{p+2}}{b^3}+\frac{e^2\left(bx^2+a\right)^p}{b^3}\right.$$
$$d\left(d^2-\frac{15ae^2}{2bp+7b}\right)\int x^4 \left(bx^2+a\right)^p dx + \frac{3de^2x^5\left(a+bx^2\right)^{p+1}}{b(2p+7)}$$

$$\Big\downarrow \text{279}$$

$$\frac{1}{2}e\int \left(-\frac{a^2\left(ae^2-3bd^2\right)\left(bx^2+a\right)^p}{b^3}+\frac{3a\left(ae^2-2bd^2\right)\left(bx^2+a\right)^{p+1}}{b^3}+\frac{3\left(bd^2-ae^2\right)\left(bx^2+a\right)^{p+2}}{b^3}+\frac{e^2\left(bx^2+a\right)^p}{b^3}\right.$$
$$d\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{15ae^2}{2bp+7b}\right)\int x^4 \left(\frac{bx^2}{a}+1\right)^p dx + \frac{3de^2x^5\left(a+bx^2\right)^{p+1}}{b(2p+7)}$$

$$\Big\downarrow \text{278}$$

$$\frac{1}{2}e\int \left(-\frac{a^2\left(ae^2-3bd^2\right)\left(bx^2+a\right)^p}{b^3}+\frac{3a\left(ae^2-2bd^2\right)\left(bx^2+a\right)^{p+1}}{b^3}+\frac{3\left(bd^2-ae^2\right)\left(bx^2+a\right)^{p+2}}{b^3}+\frac{e^2\left(bx^2+a\right)^p}{b^3}\right.$$
$$\frac{1}{5}dx^5\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{15ae^2}{2bp+7b}\right)\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)+$$
$$\frac{3de^2x^5\left(a+bx^2\right)^{p+1}}{b(2p+7)}$$

$$\Big\downarrow \text{2009}$$

3.400. $\int x^4(d+ex)^3\left(a+bx^2\right)^p\,dx$

$$\frac{1}{2}e\left(\frac{a^2(3bd^2-ae^2)\,(a+bx^2)^{p+1}}{b^4(p+1)}-\frac{3a(2bd^2-ae^2)\,(a+bx^2)^{p+2}}{b^4(p+2)}+\frac{3(bd^2-ae^2)\,(a+bx^2)^{p+3}}{b^4(p+3)}+\frac{e^2(a+bx^2)^{p+4}}{b^4(p+4)}\right.$$
$$\frac{1}{5}dx^5(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{15ae^2}{2bp+7b}\right)\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)+$$
$$\frac{3de^2x^5(a+bx^2)^{p+1}}{b(2p+7)}$$

input
```
Int[x^4*(d + e*x)^3*(a + b*x^2)^p,x]
```

output
```
(3*d*e^2*x^5*(a + b*x^2)^(1 + p))/(b*(7 + 2*p)) + (e*((a^2*(3*b*d^2 - a*e^
2)*(a + b*x^2)^(1 + p))/(b^4*(1 + p)) - (3*a*(2*b*d^2 - a*e^2)*(a + b*x^2)
^(2 + p))/(b^4*(2 + p)) + (3*(b*d^2 - a*e^2)*(a + b*x^2)^(3 + p))/(b^4*(3
+ p)) + (e^2*(a + b*x^2)^(4 + p))/(b^4*(4 + p))))/2 + (d*(d^2 - (15*a*e^2)
/(7*b + 2*b*p))*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2
)/a)])/(5*(1 + (b*x^2)/a)^p)
```

3.400.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.400.4 Maple [F]

$$\int x^4(ex+d)^3\left(b\,x^2+a\right)^p dx$$

input
```
int(x^4*(e*x+d)^3*(b*x^2+a)^p,x)
```

output
```
int(x^4*(e*x+d)^3*(b*x^2+a)^p,x)
```

3.400.5 Fricas [F]

$$\int x^4(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3\left(bx^2+a\right)^p x^4\,dx$$

input
```
integrate(x^4*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e^3*x^7 + 3*d*e^2*x^6 + 3*d^2*e*x^5 + d^3*x^4)*(b*x^2 + a)^p, x)
```

3.400.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1926 vs. 2(216) = 432.

Time = 21.45 (sec) , antiderivative size = 2919, normalized size of antiderivative = 11.72

$$\int x^4(d+ex)^3\left(a+bx^2\right)^p\,dx=\text{Too large to display}$$

input
```
integrate(x**4*(e*x+d)**3*(b*x**2+a)**p,x)
```

output
```
a**p*d**3*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + 3*a*
*p*d*e**2*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + 3*d*
*2*e*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**
2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2
*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2
 + 4*b**5*x**4) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x
**2 + 4*b**5*x**4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**
4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x
**4) + 2*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b*
*5*x**4) + 2*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 +
4*b**5*x**4), Eq(p, -3)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*
x**2) - 2*a**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*
b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x*
*2) - 2*a*b*x**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/
(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) +
 a**2*log(x + sqrt(-a/b))/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -
1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*
b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b
**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3
*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p*...
```

3.400.7 Maxima [F]

$$\int x^4(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3\left(bx^2+a\right)^p x^4\,dx$$

input
```
integrate(x^4*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3*(b*x^2 + a)^p*x^4, x)
```

3.400.8 Giac [F]

$$\int x^4(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3\left(bx^2+a\right)^p x^4\,dx$$

input
```
integrate(x^4*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(b*x^2 + a)^p*x^4, x)
```

3.400.9 Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex)^3\left(a+bx^2\right)^p\,dx=\int x^4\left(b\,x^2+a\right)^p\,(d+e\,x)^3\,dx$$

input
```
int(x^4*(a + b*x^2)^p*(d + e*x)^3,x)
```

output
```
int(x^4*(a + b*x^2)^p*(d + e*x)^3, x)
```

3.401 $\int x^3(d+ex)^3\left(a+bx^2\right)^p\,dx$

3.401.1 Optimal result

Integrand size = 20, antiderivative size = 207

$$\int x^3(d+ex)^3\left(a+bx^2\right)^p\,dx = -\frac{ad(bd^2-3ae^2)\left(a+bx^2\right)^{1+p}}{2b^3(1+p)}+\frac{e^3x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}+\frac{d(bd^2-6ae^2)\left(a+bx^2\right)^{2+p}}{2b^3(2+p)}+\frac{3de^2\left(a+bx^2\right)^{3+p}}{2b^3(3+p)}-\frac{e(5ae^2-3bd^2(7+2p))\,x^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)}{5b(7+2p)}$$

output
```
-1/2*a*d*(-3*a*e^2+b*d^2)*(b*x^2+a)^(p+1)/b^3/(p+1)+e^3*x^5*(b*x^2+a)^(p+1
)/b/(7+2*p)+1/2*d*(-6*a*e^2+b*d^2)*(b*x^2+a)^(2+p)/b^3/(2+p)+3/2*d*e^2*(b*
x^2+a)^(3+p)/b^3/(3+p)-1/5*e*(5*a*e^2-3*b*d^2*(7+2*p))*x^5*(b*x^2+a)^p*hyp
ergeom([5/2, -p],[7/2],-b*x^2/a)/b/(7+2*p)/((1+b*x^2/a)^p)
```

3.401.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.95

$$\int x^3(d+ex)^3 \left(a+bx^2\right)^p \, dx = \frac{1}{70}\left(a+bx^2\right)^p \left(\frac{35d^3(a+bx^2)\left(-a+b(1+p)x^2\right)}{b^2(1+p)(2+p)} + \frac{105de^2(a+bx^2)\left(2a^2-2ab(1+p)x^2+b^2(2+3p+p^2)\,x^4\right)}{b^3(1+p)(2+p)(3+p)} + 42d^2ex^5\left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right) + 10e^3x^7\left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)\right)$$

input
```
Integrate[x^3*(d + e*x)^3*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*((35*d^3*(a + b*x^2)*(-a + b*(1 + p)*x^2))/(b^2*(1 + p)*(2
+ p)) + (105*d*e^2*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p +
 p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (42*d^2*e*x^5*Hypergeometric2F
1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (10*e^3*x^7*Hypergeomet
ric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/70
```

3.401.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {543, 354, 27, 86, 363, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+ex)^3 \left(a+bx^2\right)^p \, dx$$

$$\Big\downarrow \text{543}$$

$$\int x^3\left(bx^2+a\right)^p \left(d^3+3e^2x^2d\right)dx+\int x^4\left(bx^2+a\right)^p \left(x^2e^3+3d^2e\right)dx$$

$$\Big\downarrow \text{354}$$

$$\int x^4\left(bx^2+a\right)^p\left(x^2e^3+3d^2e\right)dx+\frac{1}{2}\int dx^2\left(bx^2+a\right)^p\left(d^2+3e^2x^2\right)dx^2$$

$$\Big\downarrow \text{27}$$

$$\int x^4\left(bx^2+a\right)^p\left(x^2e^3+3d^2e\right)dx+\frac{1}{2}d\int x^2\left(bx^2+a\right)^p\left(d^2+3e^2x^2\right)dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}d\int\left(\frac{a\left(3ae^2-bd^2\right)\left(bx^2+a\right)^p}{b^2}+\frac{\left(bd^2-6ae^2\right)\left(bx^2+a\right)^{p+1}}{b^2}+\frac{3e^2\left(bx^2+a\right)^{p+2}}{b^2}\right)dx^2+\int x^4\left(bx^2+a\right)^p\left(x^2e^3+3d^2e\right)dx$$

$$\Big\downarrow \text{363}$$

$$\frac{1}{2}d\int\left(\frac{a\left(3ae^2-bd^2\right)\left(bx^2+a\right)^p}{b^2}+\frac{\left(bd^2-6ae^2\right)\left(bx^2+a\right)^{p+1}}{b^2}+\frac{3e^2\left(bx^2+a\right)^{p+2}}{b^2}\right)dx^2+e\left(3d^2-\frac{5ae^2}{2bp+7b}\right)\int x^4\left(bx^2+a\right)^p dx+\frac{e^3x^5\left(a+bx^2\right)^{p+1}}{b(2p+7)}$$

$$\Big\downarrow \text{279}$$

$$\frac{1}{2}d\int\left(\frac{a\left(3ae^2-bd^2\right)\left(bx^2+a\right)^p}{b^2}+\frac{\left(bd^2-6ae^2\right)\left(bx^2+a\right)^{p+1}}{b^2}+\frac{3e^2\left(bx^2+a\right)^{p+2}}{b^2}\right)dx^2+e\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3d^2-\frac{5ae^2}{2bp+7b}\right)\int x^4\left(\frac{bx^2}{a}+1\right)^p dx+\frac{e^3x^5\left(a+bx^2\right)^{p+1}}{b(2p+7)}$$

$$\Big\downarrow \text{278}$$

$$\frac{1}{2}d\int\left(\frac{a\left(3ae^2-bd^2\right)\left(bx^2+a\right)^p}{b^2}+\frac{\left(bd^2-6ae^2\right)\left(bx^2+a\right)^{p+1}}{b^2}+\frac{3e^2\left(bx^2+a\right)^{p+2}}{b^2}\right)dx^2+\frac{1}{5}ex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3d^2-\frac{5ae^2}{2bp+7b}\right)\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)+\frac{e^3x^5\left(a+bx^2\right)^{p+1}}{b(2p+7)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}d\left(-\frac{a\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{p+1}}{b^3(p+1)}+\frac{\left(bd^2-6ae^2\right)\left(a+bx^2\right)^{p+2}}{b^3(p+2)}+\frac{3e^2\left(a+bx^2\right)^{p+3}}{b^3(p+3)}\right)+\frac{1}{5}ex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3d^2-\frac{5ae^2}{2bp+7b}\right)\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)+\frac{e^3x^5\left(a+bx^2\right)^{p+1}}{b(2p+7)}$$

3.401. $\int x^3(d+ex)^3\left(a+bx^2\right)^p\,dx$

input
```
Int[x^3*(d + e*x)^3*(a + b*x^2)^p,x]
```

output
```
(e^3*x^5*(a + b*x^2)^(1 + p))/(b*(7 + 2*p)) + (d*(-((a*(b*d^2 - 3*a*e^2)*(
a + b*x^2)^(1 + p))/(b^3*(1 + p))) + ((b*d^2 - 6*a*e^2)*(a + b*x^2)^(2 + p
))/(b^3*(2 + p)) + (3*e^2*(a + b*x^2)^(3 + p))/(b^3*(3 + p))))/2 + (e*(3*d
^2 - (5*a*e^2)/(7*b + 2*b*p))*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p,
 7/2, -((b*x^2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

3.401.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.401.4 Maple [F]

$$\int x^{3}(ex+d)^{3}\left(b\,x^{2}+a\right)^{p}dx$$

input
```
int(x^3*(e*x+d)^3*(b*x^2+a)^p,x)
```

output
```
int(x^3*(e*x+d)^3*(b*x^2+a)^p,x)
```

3.401.5 Fricas [F]

$$\int x^3(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3\left(bx^2+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e^3*x^6 + 3*d*e^2*x^5 + 3*d^2*e*x^4 + d^3*x^3)*(b*x^2 + a)^p, x)
```

3.401.6 Sympy [A] (verification not implemented)

Time = 17.32 (sec) , antiderivative size = 1329, normalized size of antiderivative = 6.42

$$\int x^3(d+ex)^3\left(a+bx^2\right)^p\,dx = \text{Too large to display}$$

input
```
integrate(x**3*(e*x+d)**3*(b*x**2+a)**p,x)
```

output
```
3*a**p*d**2*e*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 +
a**p*e**3*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + d**3
*Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b
**3*x**2) + a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 +
 2*b**3*x**2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x*
*2*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - s
qrt(-a/b))/(2*b**2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -
1)), (-a**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2
*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x*
*2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b*
*2*p**2 + 6*b**2*p + 4*b**2), True)) + 3*d*e**2*Piecewise((a**p*x**6/6, Eq
(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5
*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*
x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*lo
g(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2
*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x
**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(x - sqrt
(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(x +
sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a
**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(x + sqrt(...
```

3.401.7 Maxima [F]

$$\int x^3(d+ex)^3\left(a+bx^2\right)^p\,dx = \int (ex+d)^3\left(bx^2+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
1/2*(b^2*(p + 1)*x^4 + a*b*p*x^2 - a^2)*(b*x^2 + a)^p*d^3/((p^2 + 3*p + 2)
*b^2) + integrate((e^3*x^6 + 3*d*e^2*x^5 + 3*d^2*e*x^4)*(b*x^2 + a)^p, x)
```

3.401.8 Giac [F]

$$\int x^3(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3\left(bx^2+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(b*x^2 + a)^p*x^3, x)
```

3.401.9 Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex)^3\left(a+bx^2\right)^p\,dx=\int x^3\left(b\,x^2+a\right)^p\,(d+e\,x)^3\,dx$$

input
```
int(x^3*(a + b*x^2)^p*(d + e*x)^3,x)
```

output
```
int(x^3*(a + b*x^2)^p*(d + e*x)^3, x)
```

3.402 $\int x^2(d+ex)^3\left(a+bx^2\right)^p\,dx$

3.402.1 Optimal result

Integrand size = 20, antiderivative size = 210

$$\begin{aligned}\int x^2(d+ex)^3\left(a+bx^2\right)^p\,dx\\ =-\frac{ae(3bd^2-ae^2)\left(a+bx^2\right)^{1+p}}{2b^3(1+p)}+\frac{3de^2x^3(a+bx^2)^{1+p}}{b(5+2p)}\\ +\frac{e(3bd^2-2ae^2)\left(a+bx^2\right)^{2+p}}{2b^3(2+p)}+\frac{e^3(a+bx^2)^{3+p}}{2b^3(3+p)}\\ -\frac{d(9ae^2-bd^2(5+2p))\,x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)}{3b(5+2p)}\end{aligned}$$

output
```
-1/2*a*e*(-a*e^2+3*b*d^2)*(b*x^2+a)^(p+1)/b^3/(p+1)+3*d*e^2*x^3*(b*x^2+a)^
(p+1)/b/(5+2*p)+1/2*e*(-2*a*e^2+3*b*d^2)*(b*x^2+a)^(2+p)/b^3/(2+p)+1/2*e^3
*(b*x^2+a)^(3+p)/b^3/(3+p)-1/3*d*(9*a*e^2-b*d^2*(5+2*p))*x^3*(b*x^2+a)^p*h
ypergeom([3/2, -p],[5/2],-b*x^2/a)/b/(5+2*p)/((1+b*x^2/a)^p)
```

3.402.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.93

$$\int x^2(d+ex)^3\left(a+bx^2\right)^p\,dx=\frac{1}{30}\left(a+bx^2\right)^p\left(\frac{45d^2e(a+bx^2)\left(-a+b(1+p)x^2\right)}{b^2(1+p)(2+p)}+\frac{15e^3(a+bx^2)\left(2a^2-2ab(1+p)x^2+b^2\left(2+3p+p^2\right)x^4\right)}{b^3(1+p)(2+p)(3+p)}+10d^3x^3\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)+18de^2x^5\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)\right)$$

input
```
Integrate[x^2*(d + e*x)^3*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*((45*d^2*e*(a + b*x^2)*(-a + b*(1 + p)*x^2))/(b^2*(1 + p)*(
2 + p)) + (15*e^3*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p +
p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (10*d^3*x^3*Hypergeometric2F1[3
/2, -p, 5/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (18*d*e^2*x^5*Hypergeometr
ic2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/30
```

3.402.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {543, 354, 27, 86, 363, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+ex)^3\left(a+bx^2\right)^p\,dx$$

$$\Big\downarrow \text{ 543}$$

$$\int x^2\left(bx^2+a\right)^p\left(d^3+3e^2x^2d\right)dx+\int x^3\left(bx^2+a\right)^p\left(x^2e^3+3d^2e\right)dx$$

$$\Big\downarrow \text{ 354}$$

$$\int x^2\left(bx^2+a\right)^p\left(d^3+3e^2x^2d\right)dx+\frac{1}{2}\int ex^2\left(bx^2+a\right)^p\left(3d^2+e^2x^2\right)dx^2$$

$$\Big\downarrow \; 27$$

$$\int x^2\left(bx^2+a\right)^p\left(d^3+3e^2x^2d\right)dx+\frac{1}{2}e\int x^2\left(bx^2+a\right)^p\left(3d^2+e^2x^2\right)dx^2$$

$$\Big\downarrow \; 86$$

$$\frac{1}{2}e\int\left(\frac{a\left(ae^2-3bd^2\right)\left(bx^2+a\right)^p}{b^2}+\frac{\left(3bd^2-2ae^2\right)\left(bx^2+a\right)^{p+1}}{b^2}+\frac{e^2\left(bx^2+a\right)^{p+2}}{b^2}\right)dx^2+\int x^2\left(bx^2+a\right)^p\left(d^3+3e^2x^2d\right)dx$$

$$\Big\downarrow \; 363$$

$$\frac{1}{2}e\int\left(\frac{a\left(ae^2-3bd^2\right)\left(bx^2+a\right)^p}{b^2}+\frac{\left(3bd^2-2ae^2\right)\left(bx^2+a\right)^{p+1}}{b^2}+\frac{e^2\left(bx^2+a\right)^{p+2}}{b^2}\right)dx^2+d\left(d^2-\frac{9ae^2}{2bp+5b}\right)\int x^2\left(bx^2+a\right)^p dx+\frac{3de^2x^3\left(a+bx^2\right)^{p+1}}{b(2p+5)}$$

$$\Big\downarrow \; 279$$

$$\frac{1}{2}e\int\left(\frac{a\left(ae^2-3bd^2\right)\left(bx^2+a\right)^p}{b^2}+\frac{\left(3bd^2-2ae^2\right)\left(bx^2+a\right)^{p+1}}{b^2}+\frac{e^2\left(bx^2+a\right)^{p+2}}{b^2}\right)dx^2+d\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{9ae^2}{2bp+5b}\right)\int x^2\left(\frac{bx^2}{a}+1\right)^p dx+\frac{3de^2x^3\left(a+bx^2\right)^{p+1}}{b(2p+5)}$$

$$\Big\downarrow \; 278$$

$$\frac{1}{2}e\int\left(\frac{a\left(ae^2-3bd^2\right)\left(bx^2+a\right)^p}{b^2}+\frac{\left(3bd^2-2ae^2\right)\left(bx^2+a\right)^{p+1}}{b^2}+\frac{e^2\left(bx^2+a\right)^{p+2}}{b^2}\right)dx^2+\frac{1}{3}dx^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{9ae^2}{2bp+5b}\right)\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)+\frac{3de^2x^3\left(a+bx^2\right)^{p+1}}{b(2p+5)}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}e\left(-\frac{a\left(3bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{b^3(p+1)}+\frac{\left(3bd^2-2ae^2\right)\left(a+bx^2\right)^{p+2}}{b^3(p+2)}+\frac{e^2\left(a+bx^2\right)^{p+3}}{b^3(p+3)}\right)+\frac{1}{3}dx^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{9ae^2}{2bp+5b}\right)\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)+\frac{3de^2x^3\left(a+bx^2\right)^{p+1}}{b(2p+5)}$$

3.402. $\int x^2(d+ex)^3\left(a+bx^2\right)^p\,dx$

input
```
Int[x^2*(d + e*x)^3*(a + b*x^2)^p,x]
```

output
```
(3*d*e^2*x^3*(a + b*x^2)^(1 + p))/(b*(5 + 2*p)) + (e*(-((a*(3*b*d^2 - a*e^
2)*(a + b*x^2)^(1 + p))/(b^3*(1 + p))) + ((3*b*d^2 - 2*a*e^2)*(a + b*x^2)^
(2 + p))/(b^3*(2 + p)) + (e^2*(a + b*x^2)^(3 + p))/(b^3*(3 + p))))/2 + (d*
(d^2 - (9*a*e^2)/(5*b + 2*b*p))*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -
p, 5/2, -((b*x^2)/a)])/(3*(1 + (b*x^2)/a)^p)
```

3.402.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.402.4 Maple [F]

$$\int x^{2}(ex+d)^{3}\left(b\,x^{2}+a\right)^{p}dx$$

input
```
int(x^2*(e*x+d)^3*(b*x^2+a)^p,x)
```

output
```
int(x^2*(e*x+d)^3*(b*x^2+a)^p,x)
```

3.402.5 Fricas [F]

$$\int x^2(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3\left(bx^2+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*e*x^3 + d^3*x^2)*(b*x^2 + a)^p, x)
```

3.402.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 923 vs. $2(180) = 360$.

Time = 11.37 (sec) , antiderivative size = 1329, normalized size of antiderivative = 6.33

$$\int x^2(d+ex)^3\left(a+bx^2\right)^p\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(e*x+d)**3*(b*x**2+a)**p,x)
```

output
```
a**p*d**3*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + 3*a*
*p*d*e**2*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + 3*d*
*2*e*Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 +
 2*b**3*x**2) + a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b*
*2 + 2*b**3*x**2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) +
b*x**2*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x
 - sqrt(-a/b))/(2*b**2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(
p, -1)), (-a**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*
x**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a +
b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(
2*b**2*p**2 + 6*b**2*p + 4*b**2), True)) + e**3*Piecewise((a**p*x**6/6, Eq
(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5
*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*
x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*lo
g(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2
*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x
**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(x - sqrt
(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(x +
sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a
**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(x + sqrt(...
```

3.402.7 Maxima [F]

$$\int x^2(d+ex)^3\left(a+bx^2\right)^p\,dx = \int (ex+d)^3\left(bx^2+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3*(b*x^2 + a)^p*x^2, x)
```

3.402.8 Giac [F]

$$\int x^2(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3\left(bx^2+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(b*x^2 + a)^p*x^2, x)
```

3.402.9 Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex)^3\left(a+bx^2\right)^p\,dx=\int x^2\left(b\,x^2+a\right)^p\left(d+e\,x\right)^3\,dx$$

input
```
int(x^2*(a + b*x^2)^p*(d + e*x)^3,x)
```

output
```
int(x^2*(a + b*x^2)^p*(d + e*x)^3, x)
```

3.403 $\int x(d+ex)^3 \left(a+bx^2\right)^p \, dx$

3.403.1 Optimal result

Integrand size = 18, antiderivative size = 167

$$\int x(d+ex)^3 \left(a+bx^2\right)^p \, dx = \frac{d(bd^2-3ae^2)\left(a+bx^2\right)^{1+p}}{2b^2(1+p)} + \frac{e^3x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)} + \frac{3de^2\left(a+bx^2\right)^{2+p}}{2b^2(2+p)} - \frac{e(ae^2-bd^2(5+2p))\,x^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)}{b(5+2p)}$$

output
```
1/2*d*(-3*a*e^2+b*d^2)*(b*x^2+a)^(p+1)/b^2/(p+1)+e^3*x^3*(b*x^2+a)^(p+1)/b
/(5+2*p)+3/2*d*e^2*(b*x^2+a)^(2+p)/b^2/(2+p)-e*(a*e^2-b*d^2*(5+2*p))*x^3*(
b*x^2+a)^p*hypergeom([3/2, -p],[5/2],-b*x^2/a)/b/(5+2*p)/((1+b*x^2/a)^p)
```

3.403.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.37

$$\int x(d+ex)^3 \left(a+bx^2\right)^p \, dx = \frac{\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(5d\left(b^2x^2\left(1+\frac{bx^2}{a}\right)^p\left(d^2(2+p)+3e^2(1+p)x^2\right)-3a^2e^2\left(-1+\left(1+\frac{bx^2}{a}\right)^p\right)+ab\left(\right.\right.\right.}{}$$

input
```
Integrate[x*(d + e*x)^3*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*(5*d*(b^2*x^2*(1 + (b*x^2)/a)^p*(d^2*(2 + p) + 3*e^2*(1 + p
)*x^2) - 3*a^2*e^2*(-1 + (1 + (b*x^2)/a)^p) + a*b*(3*e^2*p*x^2*(1 + (b*x^2
)/a)^p + d^2*(2 + p)*(-1 + (1 + (b*x^2)/a)^p))) + 10*b^2*d^2*e*(2 + 3*p +
p^2)*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)] + 2*b^2*e^3*(2 + 3*
p + p^2)*x^5*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)]))/(10*b^2*(1 +
p)*(2 + p)*(1 + (b*x^2)/a)^p)
```

3.403.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {543, 353, 27, 53, 363, 279, 278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex)^3 \left(a+bx^2\right)^p dx$$

$$\Big\downarrow \text{ 543}$$

$$\int x\left(bx^2+a\right)^p \left(d^3+3e^2x^2d\right)dx+\int x^2\left(bx^2+a\right)^p \left(x^2e^3+3d^2e\right)dx$$

$$\Big\downarrow \text{ 353}$$

$$\int x^2\left(bx^2+a\right)^p \left(x^2e^3+3d^2e\right)dx+\frac{1}{2}\int d\left(bx^2+a\right)^p \left(d^2+3e^2x^2\right)dx^2$$

$$\Big\downarrow \text{ 27}$$

$$\int x^2\left(bx^2+a\right)^p \left(x^2e^3+3d^2e\right)dx+\frac{1}{2}d\int \left(bx^2+a\right)^p \left(d^2+3e^2x^2\right)dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\int x^2\left(bx^2+a\right)^p \left(x^2e^3+3d^2e\right)dx+\frac{1}{2}d\int \left(\frac{\left(bd^2-3ae^2\right)\left(bx^2+a\right)^p}{b}+\frac{3e^2\left(bx^2+a\right)^{p+1}}{b}\right)dx^2$$

$$\Big\downarrow \text{ 363}$$

$$3e\left(d^2-\frac{ae^2}{2bp+5b}\right)\int x^2\left(bx^2+a\right)^p dx+$$
$$\frac{1}{2}d\int\left(\frac{\left(bd^2-3ae^2\right)\left(bx^2+a\right)^p}{b}+\frac{3e^2\left(bx^2+a\right)^{p+1}}{b}\right)dx^2+\frac{e^3x^3\left(a+bx^2\right)^{p+1}}{b(2p+5)}$$

$$\Big\downarrow \text{ 279}$$

$$3e\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{ae^2}{2bp+5b}\right)\int x^2\left(\frac{bx^2}{a}+1\right)^p dx+$$
$$\frac{1}{2}d\int\left(\frac{\left(bd^2-3ae^2\right)\left(bx^2+a\right)^p}{b}+\frac{3e^2\left(bx^2+a\right)^{p+1}}{b}\right)dx^2+\frac{e^3x^3\left(a+bx^2\right)^{p+1}}{b(2p+5)}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{1}{2}d\int\left(\frac{\left(bd^2-3ae^2\right)\left(bx^2+a\right)^p}{b}+\frac{3e^2\left(bx^2+a\right)^{p+1}}{b}\right)dx^2+$$
$$ex^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{ae^2}{2bp+5b}\right)\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)+$$
$$\frac{e^3x^3\left(a+bx^2\right)^{p+1}}{b(2p+5)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}d\left(\frac{\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{p+1}}{b^2(p+1)}+\frac{3e^2\left(a+bx^2\right)^{p+2}}{b^2(p+2)}\right)+$$
$$ex^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{ae^2}{2bp+5b}\right)\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)+$$
$$\frac{e^3x^3\left(a+bx^2\right)^{p+1}}{b(2p+5)}$$

input
```
Int[x*(d + e*x)^3*(a + b*x^2)^p,x]
```

output
```
(e^3*x^3*(a + b*x^2)^(1 + p))/(b*(5 + 2*p)) + (d*(((b*d^2 - 3*a*e^2)*(a +
b*x^2)^(1 + p))/(b^2*(1 + p)) + (3*e^2*(a + b*x^2)^(2 + p))/(b^2*(2 + p)))
)/2 + (e*(d^2 - (a*e^2)/(5*b + 2*b*p))*x^3*(a + b*x^2)^p*Hypergeometric2F1
[3/2, -p, 5/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

3.403. $\int x(d+ex)^3\left(a+bx^2\right)^p\,dx$

3.403.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

3.403. $\int x(d+ex)^3\left(a+bx^2\right)^p\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.403.4 Maple [F]

$$\int x(ex+d)^3 \left(b\,x^2+a\right)^p dx$$

input
```
int(x*(e*x+d)^3*(b*x^2+a)^p,x)
```

output
```
int(x*(e*x+d)^3*(b*x^2+a)^p,x)
```

3.403.5 Fricas [F]

$$\int x(d+ex)^3 \left(a+bx^2\right)^p \, dx = \int (ex+d)^3 \left(bx^2+a\right)^p x \, dx$$

input
```
integrate(x*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e*x^2 + d^3*x)*(b*x^2 + a)^p, x)
```

3.403.6 Sympy [A] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.63

$$\int x(d+ex)^3\left(a+bx^2\right)^p\,dx = a^pd^2ex^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right) + \frac{a^pe^3x^5{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5} + d^3\left(\begin{cases}\frac{a^px^2}{2} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log(a+bx^2) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right) + 3de^2\left(\begin{cases}\frac{a^px^4}{4} & \text{for } b=0\\ \frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{a}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} & \text{for } p=-2\\ -\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^2}-\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^2}+\frac{x^2}{2b} & \text{for } p=-1\\ -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x*(e*x+d)**3*(b*x**2+a)**p,x)
```

output
```
a**p*d**2*e*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a) + a**p
*e**3*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + d**3*Pie
cewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1),
 Ne(p, -1)), (log(a + b*x**2), True))/(2*b), True)) + 3*d*e**2*Piecewise((
a**p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) +
a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x**2
) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(x + s
qrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(-a/b))/(
2*b**2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*
(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2
)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b*
*2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*
b**2*p + 4*b**2), True))
```

3.403.7 Maxima [F]

$$\int x(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3\left(bx^2+a\right)^p x\,dx$$

input
```
integrate(x*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
1/2*(b*x^2 + a)^(p + 1)*d^3/(b*(p + 1)) + integrate((e^3*x^4 + 3*d*e^2*x^3
 + 3*d^2*e*x^2)*(b*x^2 + a)^p, x)
```

3.403.8 Giac [F]

$$\int x(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3\left(bx^2+a\right)^p x\,dx$$

input
```
integrate(x*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(b*x^2 + a)^p*x, x)
```

3.403.9 Mupad [F(-1)]

Timed out.

$$\int x(d+ex)^3\left(a+bx^2\right)^p\,dx=\int x\left(b\,x^2+a\right)^p(d+e\,x)^3\,dx$$

input
```
int(x*(a + b*x^2)^p*(d + e*x)^3,x)
```

output
```
int(x*(a + b*x^2)^p*(d + e*x)^3, x)
```

3.404 $\int (d+ex)^3 (a+bx^2)^p \, dx$

3.404.1 Optimal result

Integrand size = 17, antiderivative size = 176

$$\int (d+ex)^3 (a+bx^2)^p \, dx = -\frac{e(ae^2-3bd^2(2+p))(a+bx^2)^{1+p}}{2b^2(1+p)(2+p)} + \frac{3de^2x(a+bx^2)^{1+p}}{b(3+2p)} + \frac{e^3x^2(a+bx^2)^{1+p}}{2b(2+p)} - \frac{d(3ae^2-bd^2(3+2p))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{b(3+2p)}$$

output
```
-1/2*e*(a*e^2-3*b*d^2*(2+p))*(b*x^2+a)^(p+1)/b^2/(p+1)/(2+p)+3*d*e^2*x*(b*
x^2+a)^(p+1)/b/(3+2*p)+1/2*e^3*x^2*(b*x^2+a)^(p+1)/b/(2+p)-d*(3*a*e^2-b*d^
2*(3+2*p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/b/(3+2*p)/((1
+b*x^2/a)^p)
```

3.404.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.27

$$\int (d+ex)^3 (a+bx^2)^p \, dx = \frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(2b^2d^3(2+3p+p^2)x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)+e\left(b^2x^2\left(1+\frac{bx^2}{a}\right)^p\right.\right.}{}$$

input
```
Integrate[(d + e*x)^3*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*(2*b^2*d^3*(2 + 3*p + p^2)*x*Hypergeometric2F1[1/2, -p, 3/2
, -((b*x^2)/a)] + e*(b^2*x^2*(1 + (b*x^2)/a)^p*(3*d^2*(2 + p) + e^2*(1 + p
)*x^2) - a^2*e^2*(-1 + (1 + (b*x^2)/a)^p) + a*b*(e^2*p*x^2*(1 + (b*x^2)/a)
^p + 3*d^2*(2 + p)*(-1 + (1 + (b*x^2)/a)^p)) + 2*b^2*d*e*(2 + 3*p + p^2)*x
^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])))/(2*b^2*(1 + p)*(2 + p)
*(1 + (b*x^2)/a)^p)
```

3.404.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {497, 27, 676, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \left(a+bx^2\right)^p \, dx$$

$$\Big\downarrow \ 497$$

$$\frac{\int -2(d+ex)\left(-b(p+2)d^2-be(p+3)xd+ae^2\right)\left(bx^2+a\right)^p dx}{2b(p+2)}+\frac{e(d+ex)^2\left(a+bx^2\right)^{p+1}}{2b(p+2)}$$

$$\Big\downarrow \ 27$$

$$\frac{e(d+ex)^2\left(a+bx^2\right)^{p+1}}{2b(p+2)}-\frac{\int (d+ex)\left(-b(p+2)d^2-be(p+3)xd+ae^2\right)\left(bx^2+a\right)^p dx}{b(p+2)}$$

$$\Big\downarrow \ 676$$

$$\frac{e(d+ex)^2\left(a+bx^2\right)^{p+1}}{2b(p+2)}-\frac{\frac{d(p+2)\left(3ae^2-bd^2(2p+3)\right)\int \left(bx^2+a\right)^p dx}{2p+3}+\frac{e\left(a+bx^2\right)^{p+1}\left(ae^2-bd^2(2p+5)\right)}{2b(p+1)}-\frac{de^2(p+3)x\left(a+bx^2\right)^{p+1}}{2p+3}}{b(p+2)}$$

$$\Big\downarrow \ 238$$

$$\frac{e(d+ex)^2\left(a+bx^2\right)^{p+1}}{2b(p+2)}-\frac{\frac{d(p+2)\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3ae^2-bd^2(2p+3)\right)\int \left(\frac{bx^2}{a}+1\right)^p dx}{2p+3}+\frac{e\left(a+bx^2\right)^{p+1}\left(ae^2-bd^2(2p+5)\right)}{2b(p+1)}-\frac{de^2(p+3)x\left(a+bx^2\right)^{p+1}}{2p+3}}{b(p+2)}$$

$$\Big\downarrow \text{237}$$

$$\frac{e(d+ex)^2\left(a+bx^2\right)^{p+1}}{2b(p+2)} - \frac{\frac{d(p+2)x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(3ae^2-bd^2(2p+3))\text{ Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{2p+3} + \frac{e(a+bx^2)^{p+1}(ae^2-bd^2(2p+5))}{2b(p+1)} - \frac{de^2(p+3)x(a+bx^2}{2p+3}}{b(p+2)}$$

input
```
Int[(d + e*x)^3*(a + b*x^2)^p,x]
```

output
```
(e*(d + e*x)^2*(a + b*x^2)^(1 + p))/(2*b*(2 + p)) - ((e*(a*e^2 - b*d^2*(5
+ 2*p))*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) - (d*e^2*(3 + p)*x*(a + b*x^2)^
(1 + p))/(3 + 2*p) + (d*(2 + p)*(3*a*e^2 - b*d^2*(3 + 2*p))*x*(a + b*x^2)^
p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/((3 + 2*p)*(1 + (b*x^2)/a
)^p))/(b*(2 + p))
```

3.404.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] && !LeQ[p, -1]
```

3.404.4 Maple [F]

$$\int (ex+d)^3 \left(b\,x^2+a\right)^p dx$$

input
```
int((e*x+d)^3*(b*x^2+a)^p,x)
```

output
```
int((e*x+d)^3*(b*x^2+a)^p,x)
```

3.404.5 Fricas [F]

$$\int (d+ex)^3 \left(a+bx^2\right)^p \, dx = \int (ex+d)^3 \left(bx^2+a\right)^p dx$$

input
```
integrate((e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(b*x^2 + a)^p, x)
```

3.404.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(148) = 296$.

Time = 5.71 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.48

$$\int (d+ex)^3 \left(a+bx^2\right)^p \, dx = a^p d^3 x \, {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right) + a^p d e^2 x^3 \, {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right) + 3d^2 e \left(\begin{cases} \frac{a^p x^2}{2} & \text{for } b = 0 \\ \frac{\begin{cases} \frac{(a+bx^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(a+bx^2) & \text{otherwise} \end{cases}}{2b} & \text{otherwise} \end{cases}\right) + e^3 \left(\begin{cases} \frac{a^p x^4}{4} & \text{for } b = 0 \\ \frac{a \log\left(x - \sqrt{-\frac{a}{b}}\right)}{2ab^2 + 2b^3x^2} + \frac{a \log\left(x + \sqrt{-\frac{a}{b}}\right)}{2ab^2 + 2b^3x^2} + \frac{a}{2ab^2 + 2b^3x^2} + \frac{bx^2 \log\left(x - \sqrt{-\frac{a}{b}}\right)}{2ab^2 + 2b^3x^2} + \frac{bx^2 \log\left(x + \sqrt{-\frac{a}{b}}\right)}{2ab^2 + 2b^3x^2} & \text{for } p = -2 \\ -\frac{a \log\left(x - \sqrt{-\frac{a}{b}}\right)}{2b^2} - \frac{a \log\left(x + \sqrt{-\frac{a}{b}}\right)}{2b^2} + \frac{x^2}{2b} & \text{for } p = -1 \\ -\frac{a^2 (a+bx^2)^p}{2b^2p^2 + 6b^2p + 4b^2} + \frac{abpx^2 (a+bx^2)^p}{2b^2p^2 + 6b^2p + 4b^2} + \frac{b^2px^4 (a+bx^2)^p}{2b^2p^2 + 6b^2p + 4b^2} + \frac{b^2x^4 (a+bx^2)^p}{2b^2p^2 + 6b^2p + 4b^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x+d)**3*(b*x**2+a)**p,x)
```

output
```
a**p*d**3*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*d*e*
*2*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a) + 3*d**2*e*Piec
ewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1),
Ne(p, -1)), (log(a + b*x**2), True))/(2*b), True)) + e**3*Piecewise((a**p*
x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a*log
(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x**2) + b
*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(x + sqrt(-
a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(-a/b))/(2*b**
2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a +
b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/
(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p*
*2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*
p + 4*b**2), True))
```

3.404.7 Maxima [F]

$$\int (d+ex)^3 \left(a+bx^2\right)^p \, dx = \int (ex+d)^3 (bx^2+a)^p \, dx$$

input
```
integrate((e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3*(b*x^2 + a)^p, x)
```

3.404.8 Giac [F]

$$\int (d+ex)^3 \left(a+bx^2\right)^p \, dx = \int (ex+d)^3 (bx^2+a)^p \, dx$$

input
```
integrate((e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(b*x^2 + a)^p, x)
```

3.404.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3 \left(a+bx^2\right)^p \, dx = \int \left(b\,x^2+a\right)^p \, (d+e\,x)^3 \, dx$$

input
```
int((a + b*x^2)^p*(d + e*x)^3,x)
```

output
```
int((a + b*x^2)^p*(d + e*x)^3, x)
```

3.405 $\int \frac{(d+ex)^3(a+bx^2)^p}{x}\,dx$

3.405.1 Optimal result

Integrand size = 20, antiderivative size = 171

$$\begin{aligned}\int \frac{(d+ex)^3(a+bx^2)^p}{x}\,dx &= \frac{3de^2(a+bx^2)^{1+p}}{2b(1+p)}+\frac{e^3x(a+bx^2)^{1+p}}{b(3+2p)}\\ &\quad-\frac{e(ae^2-3bd^2(3+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{b(3+2p)}\\ &\quad-\frac{d^3(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a(1+p)}\end{aligned}$$

output
```
3/2*d*e^2*(b*x^2+a)^(p+1)/b/(p+1)+e^3*x*(b*x^2+a)^(p+1)/b/(3+2*p)-e*(a*e^2
-3*b*d^2*(3+2*p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/b/(3+2
*p)/((1+b*x^2/a)^p)-1/2*d^3*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*x
^2/a)/a/(p+1)
```

3.405.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x} dx$$
$$= \frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(18abd^2e(1+p)x \operatorname{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) - 3bd^3(a+bx^2)\left(1+\frac{bx^2}{a}\right)\right.}{}$$

input
```
Integrate[((d + e*x)^3*(a + b*x^2)^p)/x,x]
```

output
```
((a + b*x^2)^p*(18*a*b*d^2*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((
b*x^2)/a)] - 3*b*d^3*(a + b*x^2)*(1 + (b*x^2)/a)^p*Hypergeometric2F1[1, 1
+ p, 2 + p, 1 + (b*x^2)/a] + a*e^2*(9*d*(a + b*x^2)*(1 + (b*x^2)/a)^p + 2*
b*e*(1 + p)*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])))/(6*a*b*(1
 + p)*(1 + (b*x^2)/a)^p)
```

3.405.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {543, 299, 238, 237, 354, 27, 90, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x} dx$$
$$\Big\downarrow \text{ 543}$$
$$\int \frac{(bx^2+a)^p (d^3+3e^2x^2d)}{x} dx + \int (bx^2+a)^p (x^2e^3+3d^2e) dx$$
$$\Big\downarrow \text{ 299}$$
$$\int \frac{(bx^2+a)^p (d^3+3e^2x^2d)}{x} dx + e\left(3d^2 - \frac{ae^2}{2bp+3b}\right)\int (bx^2+a)^p dx + \frac{e^3x(a+bx^2)^{p+1}}{b(2p+3)}$$
$$\Big\downarrow \text{ 238}$$

$$\int \frac{(bx^2+a)^p (d^3+3e^2x^2d)}{x}dx + e(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(3d^2-\frac{ae^2}{2bp+3b}\right) \int \left(\frac{bx^2}{a}+1\right)^p dx + \frac{e^3x(a+bx^2)^{p+1}}{b(2p+3)}$$

$\Big\downarrow$ 237

$$\int \frac{(bx^2+a)^p (d^3+3e^2x^2d)}{x}dx + ex(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(3d^2-\frac{ae^2}{2bp+3b}\right) \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) + \frac{e^3x(a+bx^2)^{p+1}}{b(2p+3)}$$

$\Big\downarrow$ 354

$$\frac{1}{2}\int \frac{d(bx^2+a)^p (d^2+3e^2x^2)}{x^2}dx^2 + ex(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(3d^2-\frac{ae^2}{2bp+3b}\right) \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) + \frac{e^3x(a+bx^2)^{p+1}}{b(2p+3)}$$

$\Big\downarrow$ 27

$$\frac{1}{2}d\int \frac{(bx^2+a)^p (d^2+3e^2x^2)}{x^2}dx^2 + ex(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(3d^2-\frac{ae^2}{2bp+3b}\right) \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) + \frac{e^3x(a+bx^2)^{p+1}}{b(2p+3)}$$

$\Big\downarrow$ 90

$$\frac{1}{2}d\left(d^2\int \frac{(bx^2+a)^p}{x^2}dx^2 + \frac{3e^2(a+bx^2)^{p+1}}{b(p+1)}\right) + ex(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(3d^2-\frac{ae^2}{2bp+3b}\right) \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) + \frac{e^3x(a+bx^2)^{p+1}}{b(2p+3)}$$

$\Big\downarrow$ 75

3.405. $\int \frac{(d+ex)^3(a+bx^2)^p}{x}\,dx$

$$ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3d^2-\frac{ae^2}{2bp+3b}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)+\frac{1}{2}d\left(\frac{3e^2\left(a+bx^2\right)^{p+1}}{b(p+1)}-\frac{d^2\left(a+bx^2\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{a(p+1)}\right)+\frac{e^3x\left(a+bx^2\right)^{p+1}}{b(2p+3)}$$

input
```
Int[((d + e*x)^3*(a + b*x^2)^p)/x,x]
```

output
```
(e^3*x*(a + b*x^2)^(1 + p))/(b*(3 + 2*p)) + (e*(3*d^2 - (a*e^2)/(3*b + 2*b
*p))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (
b*x^2)/a)^p + (d*((3*e^2*(a + b*x^2)^(1 + p))/(b*(1 + p)) - (d^2*(a + b*x^
2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(a*(1 + p)))
)/2
```

3.405.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

3.405. $\int \frac{(d+ex)^3(a+bx^2)^p}{x}\,dx$

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

3.405.4 Maple [F]

$$\int \frac{(ex+d)^3 \left(b\,x^2+a\right)^p}{x}dx$$

input
```
int((e*x+d)^3*(b*x^2+a)^p/x,x)
```

output
```
int((e*x+d)^3*(b*x^2+a)^p/x,x)
```

3.405.5 Fricas [F]

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x}\,dx = \int \frac{(ex+d)^3(bx^2+a)^p}{x}\,dx$$

input
```
integrate((e*x+d)^3*(b*x^2+a)^p/x,x, algorithm="fricas")
```

output
```
integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(b*x^2 + a)^p/x, x)
```

3.405.6 Sympy [A] (verification not implemented)

Time = 6.36 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x}\,dx = 3a^p d^2 e x\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, -p \\ \frac{3}{2}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right) + \frac{a^p e^3 x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2}, -p \\ \frac{5}{2}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{3} - \frac{b^p d^3 x^{2p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p, -p \\ 1-p\end{matrix}\middle|\,\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(1-p)} + 3de^2\left(\begin{cases}\frac{a^p x^2}{2} & \text{for}\ b=0 \\ \frac{\begin{cases}\frac{(a+bx^2)^{p+1}}{p+1} & \text{for}\ p\neq -1 \\ \log(a+bx^2) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((e*x+d)**3*(b*x**2+a)**p/x,x)
```

output
```
3*a**p*d**2*e*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*
e**3*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 - b**p*d**3
*x**(2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/
(2*gamma(1 - p)) + 3*d*e**2*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(
((a + b*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b)
, True))
```

3.405.7 Maxima [F]

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x}\,dx = \int \frac{(ex+d)^3 (bx^2+a)^p}{x}\,dx$$

input
```
integrate((e*x+d)^3*(b*x^2+a)^p/x,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3*(b*x^2 + a)^p/x, x)
```

3.405.8 Giac [F]

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x}\,dx = \int \frac{(ex+d)^3 (bx^2+a)^p}{x}\,dx$$

input
```
integrate((e*x+d)^3*(b*x^2+a)^p/x,x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(b*x^2 + a)^p/x, x)
```

3.405.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x}\,dx = \int \frac{(b\,x^2+a)^p\,(d+e\,x)^3}{x}\,dx$$

input
```
int(((a + b*x^2)^p*(d + e*x)^3)/x,x)
```

output
```
int(((a + b*x^2)^p*(d + e*x)^3)/x, x)
```

3.406 $\int \frac{(d+ex)^3(a+bx^2)^p}{x^2}\,dx$

3.406.1 Optimal result

Integrand size = 20, antiderivative size = 159

$$\int \frac{(d+ex)^3\,(a+bx^2)^p}{x^2}\,dx = \frac{e^3(a+bx^2)^{1+p}}{2b(1+p)} - \frac{d^3(a+bx^2)^{1+p}}{ax} + \frac{d(3ae^2+bd^2(1+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{a} - \frac{3d^2e(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

output
```
1/2*e^3*(b*x^2+a)^(p+1)/b/(p+1)-d^3*(b*x^2+a)^(p+1)/a/x+d*(3*a*e^2+b*d^2*(
1+2*p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/a/((1+b*x^2/a)^p
)-3/2*d^2*e*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*x^2/a)/a/(p+1)
```

3.406.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x^2} \, dx$$
$$= \frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(-2abd^3(1+p) \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)+ex\Big(6abde(1+p)x \text{ Hyp}\right.}{}$$

input
```
Integrate[((d + e*x)^3*(a + b*x^2)^p)/x^2,x]
```

output
```
((a + b*x^2)^p*(-2*a*b*d^3*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x
^2)/a)] + e*x*(6*a*b*d*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^
2)/a)] + (a + b*x^2)*(1 + (b*x^2)/a)^p*(a*e^2 - 3*b*d^2*Hypergeometric2F1[
1, 1 + p, 2 + p, 1 + (b*x^2)/a]))))/(2*a*b*(1 + p)*x*(1 + (b*x^2)/a)^p)
```

3.406.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {543, 354, 27, 90, 75, 359, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x^2} \, dx$$
$$\Big\downarrow \; 543$$
$$\int \frac{(bx^2+a)^p (d^3+3e^2x^2d)}{x^2}dx+\int \frac{(bx^2+a)^p (x^2e^3+3d^2e)}{x}dx$$
$$\Big\downarrow \; 354$$
$$\int \frac{(bx^2+a)^p (d^3+3e^2x^2d)}{x^2}dx+\frac{1}{2}\int \frac{e(bx^2+a)^p (3d^2+e^2x^2)}{x^2}dx^2$$
$$\Big\downarrow \; 27$$
$$\int \frac{(bx^2+a)^p (d^3+3e^2x^2d)}{x^2}dx+\frac{1}{2}e\int \frac{(bx^2+a)^p (3d^2+e^2x^2)}{x^2}dx^2$$
$$\Big\downarrow \; 90$$

$$\int \frac{(bx^2+a)^p\left(d^3+3e^2x^2d\right)}{x^2}dx+\frac{1}{2}e\left(3d^2\int \frac{(bx^2+a)^p}{x^2}dx^2+\frac{e^2(a+bx^2)^{p+1}}{b(p+1)}\right)$$

$$\Big\downarrow \text{75}$$

$$\int \frac{(bx^2+a)^p\left(d^3+3e^2x^2d\right)}{x^2}dx+$$
$$\frac{1}{2}e\left(\frac{e^2(a+bx^2)^{p+1}}{b(p+1)}-\frac{3d^2(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{a(p+1)}\right)$$

$$\Big\downarrow \text{359}$$

$$\frac{d(3ae^2+bd^2(2p+1))\int (bx^2+a)^p\,dx}{a}-\frac{d^3(a+bx^2)^{p+1}}{ax}+$$
$$\frac{1}{2}e\left(\frac{e^2(a+bx^2)^{p+1}}{b(p+1)}-\frac{3d^2(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{a(p+1)}\right)$$

$$\Big\downarrow \text{238}$$

$$\frac{d(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(3ae^2+bd^2(2p+1))\int \left(\frac{bx^2}{a}+1\right)^p dx}{a}-\frac{d^3(a+bx^2)^{p+1}}{ax}+$$
$$\frac{1}{2}e\left(\frac{e^2(a+bx^2)^{p+1}}{b(p+1)}-\frac{3d^2(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{a(p+1)}\right)$$

$$\Big\downarrow \text{237}$$

$$-\frac{d^3(a+bx^2)^{p+1}}{ax}+$$
$$\frac{dx(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(3ae^2+bd^2(2p+1))\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{a}+$$
$$\frac{1}{2}e\left(\frac{e^2(a+bx^2)^{p+1}}{b(p+1)}-\frac{3d^2(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{a(p+1)}\right)$$

input
```
Int[((d + e*x)^3*(a + b*x^2)^p)/x^2,x]
```

output
```
-((d^3*(a + b*x^2)^(1 + p))/(a*x)) + (d*(3*a*e^2 + b*d^2*(1 + 2*p))*x*(a +
 b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(a*(1 + (b*x^2)/a
)^p) + (e*((e^2*(a + b*x^2)^(1 + p))/(b*(1 + p)) - (3*d^2*(a + b*x^2)^(1 +
 p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(a*(1 + p))))/2
```

3.406. $\int \frac{(d+ex)^3(a+bx^2)^p}{x^2}\,dx$

3.406.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.406. $\int \frac{(d+ex)^3(a+bx^2)^p}{x^2}\,dx$

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

3.406.4 Maple [F]

$$\int \frac{(ex+d)^3 (b\,x^2+a)^p}{x^2}dx$$

input
```
int((e*x+d)^3*(b*x^2+a)^p/x^2,x)
```

output
```
int((e*x+d)^3*(b*x^2+a)^p/x^2,x)
```

3.406.5 Fricas [F]

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x^2}\,dx = \int \frac{(ex+d)^3(bx^2+a)^p}{x^2}\,dx$$

input
```
integrate((e*x+d)^3*(b*x^2+a)^p/x^2,x, algorithm="fricas")
```

output
```
integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(b*x^2 + a)^p/x^2, x)
```

3.406.6 Sympy [A] (verification not implemented)

Time = 5.47 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^3(a+bx^2)^p}{x^2}\,dx = -\frac{a^p d^3 {}_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{x} + 3a^p de^2 x {}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right) - \frac{3b^p d^2 e x^{2p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(1-p)} + e^3\left(\begin{cases}\frac{a^px^2}{2} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log(a+bx^2) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((e*x+d)**3*(b*x**2+a)**p/x**2,x)
```

output
```
-a**p*d**3*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x + 3*a**p*
d*e**2*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - 3*b**p*d**2*
e*x**(2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))
/(2*gamma(1 - p)) + e**3*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a
 + b*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b), T
rue))
```

3.406.7 Maxima [F]

$$\int \frac{(d+ex)^3(a+bx^2)^p}{x^2}\,dx = \int \frac{(ex+d)^3(bx^2+a)^p}{x^2}\,dx$$

input
```
integrate((e*x+d)^3*(b*x^2+a)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3*(b*x^2 + a)^p/x^2, x)
```

3.406.8 Giac [F]

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x^2} \, dx = \int \frac{(ex+d)^3 (bx^2+a)^p}{x^2} \, dx$$

input
```
integrate((e*x+d)^3*(b*x^2+a)^p/x^2,x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(b*x^2 + a)^p/x^2, x)
```

3.406.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x^2} \, dx = \int \frac{(b\,x^2+a)^p\,(d+e\,x)^3}{x^2} \, dx$$

input
```
int(((a + b*x^2)^p*(d + e*x)^3)/x^2,x)
```

output
```
int(((a + b*x^2)^p*(d + e*x)^3)/x^2, x)
```

3.407 $\int \frac{(d+ex)^3(a+bx^2)^p}{x^3}\,dx$

3.407.1 Optimal result

Integrand size = 20, antiderivative size = 168

$$\begin{aligned}\int \frac{(d+ex)^3(a+bx^2)^p}{x^3}\,dx &= -\frac{d^3(a+bx^2)^{1+p}}{2ax^2}-\frac{3d^2e(a+bx^2)^{1+p}}{ax} \\ &\quad+\frac{e(ae^2+3bd^2(1+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{a} \\ &\quad-\frac{d(3ae^2+bd^2p)\,(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a^2(1+p)}\end{aligned}$$

output
```
-1/2*d^3*(b*x^2+a)^(p+1)/a/x^2-3*d^2*e*(b*x^2+a)^(p+1)/a/x+e*(a*e^2+3*b*d^
2*(1+2*p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/a/((1+b*x^2/a
)^p)-1/2*d*(b*d^2*p+3*a*e^2)*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*
x^2/a)/a^2/(p+1)
```

3.407.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x^3} dx =$$
$$-\frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(6a^2d^2e(1+p)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)+x\left(-2a^2e^3(1+p)x\,\text{Hy}\right.\right.}{}$$

input
```
Integrate[((d + e*x)^3*(a + b*x^2)^p)/x^3,x]
```

output
```
-1/2*((a + b*x^2)^p*(6*a^2*d^2*e*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2,
-((b*x^2)/a)] + x*(-2*a^2*e^3*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -(
(b*x^2)/a)] + d*(a + b*x^2)*(1 + (b*x^2)/a)^p*(3*a*e^2*Hypergeometric2F1[1
, 1 + p, 2 + p, 1 + (b*x^2)/a] - b*d^2*Hypergeometric2F1[2, 1 + p, 2 + p,
1 + (b*x^2)/a]))))/(a^2*(1 + p)*x*(1 + (b*x^2)/a)^p)
```

3.407.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {543, 354, 27, 87, 75, 359, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x^3} dx$$
$$\Big\downarrow \; 543$$
$$\int \frac{(bx^2+a)^p (d^3+3e^2x^2d)}{x^3} dx + \int \frac{(bx^2+a)^p (x^2e^3+3d^2e)}{x^2} dx$$
$$\Big\downarrow \; 354$$
$$\int \frac{(bx^2+a)^p (x^2e^3+3d^2e)}{x^2} dx + \frac{1}{2}\int \frac{d(bx^2+a)^p (d^2+3e^2x^2)}{x^4} dx^2$$
$$\Big\downarrow \; 27$$
$$\int \frac{(bx^2+a)^p (x^2e^3+3d^2e)}{x^2} dx + \frac{1}{2}d\int \frac{(bx^2+a)^p (d^2+3e^2x^2)}{x^4} dx^2$$

$$\Big\downarrow \text{87}$$

$$\int \frac{(bx^2+a)^p\,(x^2e^3+3d^2e)}{x^2}dx+\frac{1}{2}d\left(\frac{(3ae^2+bd^2p)\int \frac{(bx^2+a)^p}{x^2}dx^2}{a}-\frac{d^2(a+bx^2)^{p+1}}{ax^2}\right)$$

$$\Big\downarrow \text{75}$$

$$\int \frac{(bx^2+a)^p\,(x^2e^3+3d^2e)}{x^2}dx+\frac{1}{2}d\left(-\frac{(a+bx^2)^{p+1}\,(3ae^2+bd^2p)\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{a^2(p+1)}-\frac{d^2(a+bx^2)^{p+1}}{ax^2}\right)$$

$$\Big\downarrow \text{359}$$

$$\frac{e(ae^2+3bd^2(2p+1))\int (bx^2+a)^p\,dx}{a}+\frac{1}{2}d\left(-\frac{(a+bx^2)^{p+1}\,(3ae^2+bd^2p)\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{a^2(p+1)}-\frac{d^2(a+bx^2)^{p+1}}{ax^2}\right)-\frac{3d^2e(a+bx^2)^{p+1}}{ax}$$

$$\Big\downarrow \text{238}$$

$$\frac{e(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ae^2+3bd^2(2p+1))\int \left(\frac{bx^2}{a}+1\right)^p dx}{a}+\frac{1}{2}d\left(-\frac{(a+bx^2)^{p+1}\,(3ae^2+bd^2p)\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{a^2(p+1)}-\frac{d^2(a+bx^2)^{p+1}}{ax^2}\right)-\frac{3d^2e(a+bx^2)^{p+1}}{ax}$$

$$\Big\downarrow \text{237}$$

$$\frac{1}{2}d\left(-\frac{(a+bx^2)^{p+1}\,(3ae^2+bd^2p)\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{a^2(p+1)}-\frac{d^2(a+bx^2)^{p+1}}{ax^2}\right)+\frac{ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ae^2+3bd^2(2p+1))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{a}-\frac{3d^2e(a+bx^2)^{p+1}}{ax}$$

input
```
Int[((d + e*x)^3*(a + b*x^2)^p)/x^3,x]
```

3.407. $\int \frac{(d+ex)^3(a+bx^2)^p}{x^3}\,dx$

output
```
(-3*d^2*e*(a + b*x^2)^(1 + p))/(a*x) + (e*(a*e^2 + 3*b*d^2*(1 + 2*p))*x*(a
 + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(a*(1 + (b*x^2)
/a)^p) + (d*(-((d^2*(a + b*x^2)^(1 + p))/(a*x^2)) - ((3*a*e^2 + b*d^2*p)*(
a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(a^2
*(1 + p))))/2
```

3.407.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.407. $\int \frac{(d+ex)^3(a+bx^2)^p}{x^3}\,dx$

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

3.407.4 Maple [F]

$$\int \frac{(ex+d)^3 \left(b\,x^2+a\right)^p}{x^3}dx$$

input
```
int((e*x+d)^3*(b*x^2+a)^p/x^3,x)
```

output
```
int((e*x+d)^3*(b*x^2+a)^p/x^3,x)
```

3.407.5 Fricas [F]

$$\int \frac{(d+ex)^3 \left(a+bx^2\right)^p}{x^3}\,dx = \int \frac{(ex+d)^3(bx^2+a)^p}{x^3}\,dx$$

input
```
integrate((e*x+d)^3*(b*x^2+a)^p/x^3,x, algorithm="fricas")
```

output
```
integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(b*x^2 + a)^p/x^3, x)
```

3.407.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.17 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x^3}\,dx = -\frac{3a^p d^2 e\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{x} + a^p e^3 x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right) - \frac{b^p d^3 x^{2p-2}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}-p,1-p\\ 2-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(2-p)} - \frac{3b^p d e^2 x^{2p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(1-p)}$$

input
```
integrate((e*x+d)**3*(b*x**2+a)**p/x**3,x)
```

output
```
-3*a**p*d**2*e*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x + a**
p*e**3*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - b**p*d**3*x*
*(2*p - 2)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), a*exp_polar(I*pi)/(b*
x**2))/(2*gamma(2 - p)) - 3*b**p*d*e**2*x**(2*p)*gamma(-p)*hyper((-p, -p),
 (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*gamma(1 - p))
```

3.407.7 Maxima [F]

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x^3}\,dx = \int \frac{(ex+d)^3(bx^2+a)^p}{x^3}\,dx$$

input
```
integrate((e*x+d)^3*(b*x^2+a)^p/x^3,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3*(b*x^2 + a)^p/x^3, x)
```

3.407.8 Giac [F]

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x^3} dx = \int \frac{(ex+d)^3 (bx^2+a)^p}{x^3} dx$$

input
```
integrate((e*x+d)^3*(b*x^2+a)^p/x^3,x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(b*x^2 + a)^p/x^3, x)
```

3.407.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x^3} dx = \int \frac{(b x^2+a)^p (d+e x)^3}{x^3} dx$$

input
```
int(((a + b*x^2)^p*(d + e*x)^3)/x^3,x)
```

output
```
int(((a + b*x^2)^p*(d + e*x)^3)/x^3, x)
```

3.408 $\int \frac{x^4(a+bx^2)^p}{d+ex}\,dx$

3.408.1 Optimal result

Integrand size = 20, antiderivative size = 199

$$\int \frac{x^4(a+bx^2)^p}{d+ex}\,dx = \frac{(bd^2-ae^2)\,(a+bx^2)^{1+p}}{2b^2e^3(1+p)} + \frac{(a+bx^2)^{2+p}}{2b^2e(2+p)} + \frac{x^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{5}{2},-p,1,\frac{7}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d} - \frac{d^4(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2e^3\,(bd^2+ae^2)\,(1+p)}$$

output
```
1/2*(-a*e^2+b*d^2)*(b*x^2+a)^(p+1)/b^2/e^3/(p+1)+1/2*(b*x^2+a)^(2+p)/b^2/e
/(2+p)+1/5*x^5*(b*x^2+a)^p*AppellF1(5/2,1,-p,7/2,e^2*x^2/d^2,-b*x^2/a)/d/(
(1+b*x^2/a)^p)-1/2*d^4*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],e^2*(b*x^2
+a)/(a*e^2+b*d^2))/e^3/(a*e^2+b*d^2)/(p+1)
```

3.408.2 Mathematica [F]

$$\int \frac{x^4(a+bx^2)^p}{d+ex}\,dx = \int \frac{x^4(a+bx^2)^p}{d+ex}\,dx$$

input
```
Integrate[(x^4*(a + b*x^2)^p)/(d + e*x),x]
```

output
```
Integrate[(x^4*(a + b*x^2)^p)/(d + e*x), x]
```

3.408.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {621, 354, 99, 395, 394, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+bx^2)^p}{d+ex}dx$$

$$\Big\downarrow \text{621}$$

$$d\int \frac{x^4(bx^2+a)^p}{d^2-e^2x^2}dx-e\int \frac{x^5(bx^2+a)^p}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{354}$$

$$d\int \frac{x^4(bx^2+a)^p}{d^2-e^2x^2}dx-\frac{1}{2}e\int \frac{x^4(bx^2+a)^p}{d^2-e^2x^2}dx^2$$

$$\Big\downarrow \text{99}$$

$$d\int \frac{x^4(bx^2+a)^p}{d^2-e^2x^2}dx-\frac{1}{2}e\int \left(\frac{(ae^2-bd^2)(bx^2+a)^p}{be^4}+\frac{d^4(bx^2+a)^p}{e^4(d^2-e^2x^2)}-\frac{(bx^2+a)^{p+1}}{be^2}\right)dx^2$$

$$\Big\downarrow \text{395}$$

$$d(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \frac{x^4\left(\frac{bx^2}{a}+1\right)^p}{d^2-e^2x^2}dx-\frac{1}{2}e\int \left(\frac{(ae^2-bd^2)(bx^2+a)^p}{be^4}+\frac{d^4(bx^2+a)^p}{e^4(d^2-e^2x^2)}-\frac{(bx^2+a)^{p+1}}{be^2}\right)dx^2$$

$$\Big\downarrow \text{394}$$

$$\frac{x^5(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{5}{2},-p,1,\frac{7}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d}-\frac{1}{2}e\int \left(\frac{(ae^2-bd^2)(bx^2+a)^p}{be^4}+\frac{d^4(bx^2+a)^p}{e^4(d^2-e^2x^2)}-\frac{(bx^2+a)^{p+1}}{be^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^5(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{5}{2},-p,1,\frac{7}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d}-\frac{1}{2}e\left(-\frac{(bd^2-ae^2)(a+bx^2)^{p+1}}{b^2e^4(p+1)}-\frac{(a+bx^2)^{p+2}}{b^2e^2(p+2)}+\frac{d^4(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{e^4(p+1)(ae^2+bd^2)}\right)$$

input
```
Int[(x^4*(a + b*x^2)^p)/(d + e*x),x]
```

output
```
(x^5*(a + b*x^2)^p*AppellF1[5/2, -p, 1, 7/2, -((b*x^2)/a), (e^2*x^2)/d^2])
/(5*d*(1 + (b*x^2)/a)^p) - (e*(-(((b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(b^
2*e^4*(1 + p))) - (a + b*x^2)^(2 + p)/(b^2*e^2*(2 + p)) + (d^4*(a + b*x^2)
^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e
^2)])/(e^4*(b*d^2 + a*e^2)*(1 + p))))/2
```

3.408.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 621
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[c   Int[x^m*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] - Simp[d   Int[
x^(m + 1)*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, m,
p}, x]
```

3.408. $\int \frac{x^4(a+bx^2)^p}{d+ex}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.408.4 Maple [F]

$$\int \frac{x^4(b\,x^2+a)^p}{ex+d}dx$$

input
```
int(x^4*(b*x^2+a)^p/(e*x+d),x)
```

output
```
int(x^4*(b*x^2+a)^p/(e*x+d),x)
```

3.408.5 Fricas [F]

$$\int \frac{x^4(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x^4}{ex+d}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^4/(e*x + d), x)
```

3.408.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)^p}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(b*x**2+a)**p/(e*x+d),x)
```

output
```
Timed out
```

3.408.7 Maxima [F]

$$\int \frac{x^4(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x^4}{ex+d}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^4/(e*x + d), x)
```

3.408.8 Giac [F]

$$\int \frac{x^4(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x^4}{ex+d}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^p/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^4/(e*x + d), x)
```

3.408.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)^p}{d+ex}\,dx = \int \frac{x^4\,(b\,x^2+a)^p}{d+e\,x}\,dx$$

input
```
int((x^4*(a + b*x^2)^p)/(d + e*x),x)
```

output
```
int((x^4*(a + b*x^2)^p)/(d + e*x), x)
```

3.409 $\int \frac{x^3(a+bx^2)^p}{d+ex}\,dx$

3.409.1 Optimal result

Integrand size = 20, antiderivative size = 163

$$\int \frac{x^3(a+bx^2)^p}{d+ex}\,dx = -\frac{d(a+bx^2)^{1+p}}{2be^2(1+p)} - \frac{ex^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{5}{2},-p,1,\frac{7}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d^2} + \frac{d^3(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2e^2(bd^2+ae^2)(1+p)}$$

output
```
-1/2*d*(b*x^2+a)^(p+1)/b/e^2/(p+1)-1/5*e*x^5*(b*x^2+a)^p*AppellF1(5/2,1,-p
,7/2,e^2*x^2/d^2,-b*x^2/a)/d^2/((1+b*x^2/a)^p)+1/2*d^3*(b*x^2+a)^(p+1)*hyp
ergeom([1, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e^2/(a*e^2+b*d^2)/(p+1)
```

3.409.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.60

$$\int \frac{x^3(a+bx^2)^p}{d+ex}\,dx = \frac{(a+bx^2)^p\left(-\frac{3d^3\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{p} + \frac{e\left(1+\frac{bx^2}{a}\right)^{-p}\left(6bd^2(1+p)\cdots\right.}{\cdots}\right.}{6e\cdots}$$

input
```
Integrate[(x^3*(a + b*x^2)^p)/(d + e*x),x]
```

output
```
((a + b*x^2)^p*((-3*d^3*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*
e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))
/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p) + (e*(6*b*d^2*(1 + p)*
x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)] + e*(-3*d*(b*x^2*(1 + (b*x
^2)/a)^p + a*(-1 + (1 + (b*x^2)/a)^p)) + 2*b*e*(1 + p)*x^3*Hypergeometric2
F1[3/2, -p, 5/2, -((b*x^2)/a)])))/(b*(1 + p)*(1 + (b*x^2)/a)^p)))/(6*e^4)
```

3.409.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {621, 354, 90, 78, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (a+bx^2)^p}{d+ex} \, dx$$

$$\Big\downarrow \text{621}$$

$$d\int \frac{x^3 (bx^2+a)^p}{d^2-e^2x^2}dx - e\int \frac{x^4 (bx^2+a)^p}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}d\int \frac{x^2 (bx^2+a)^p}{d^2-e^2x^2}dx^2 - e\int \frac{x^4 (bx^2+a)^p}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}d\left(\frac{d^2\int \frac{(bx^2+a)^p}{d^2-e^2x^2}dx^2}{e^2} - \frac{(a+bx^2)^{p+1}}{be^2(p+1)}\right) - e\int \frac{x^4 (bx^2+a)^p}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{78}$$

$$\frac{1}{2}d\left(\frac{d^2 (a+bx^2)^{p+1} \text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{e^2(p+1)(ae^2+bd^2)} - \frac{(a+bx^2)^{p+1}}{be^2(p+1)}\right) - e\int \frac{x^4 (bx^2+a)^p}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{395}$$

$$\frac{1}{2}d\left(\frac{d^2(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{e^2(p+1)(ae^2+bd^2)}-\frac{(a+bx^2)^{p+1}}{be^2(p+1)}\right)-e(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int\frac{x^4\left(\frac{bx^2}{a}+1\right)^p}{d^2-e^2x^2}dx$$

$$\Big\downarrow \; 394$$

$$\frac{1}{2}d\left(\frac{d^2(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{e^2(p+1)(ae^2+bd^2)}-\frac{(a+bx^2)^{p+1}}{be^2(p+1)}\right)-\frac{ex^5(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{5}{2},-p,1,\frac{7}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d^2}$$

input
```
Int[(x^3*(a + b*x^2)^p)/(d + e*x),x]
```

output
```
-1/5*(e*x^5*(a + b*x^2)^p*AppellF1[5/2, -p, 1, 7/2, -((b*x^2)/a), (e^2*x^2
)/d^2])/(d^2*(1 + (b*x^2)/a)^p) + (d*(-((a + b*x^2)^(1 + p)/(b*e^2*(1 + p)
)) + (d^2*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a +
 b*x^2))/(b*d^2 + a*e^2)])/(e^2*(b*d^2 + a*e^2)*(1 + p))))/2
```

3.409.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.409. $\int \frac{x^3(a+bx^2)^p}{d+ex}\,dx$

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 621
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[c   Int[x^m*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] - Simp[d   Int[
x^(m + 1)*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, m,
p}, x]
```

3.409.4 Maple [F]

$$\int \frac{x^3(b\,x^2+a)^p}{ex+d}dx$$

input
```
int(x^3*(b*x^2+a)^p/(e*x+d),x)
```

output
```
int(x^3*(b*x^2+a)^p/(e*x+d),x)
```

3.409.5 Fricas [F]

$$\int \frac{x^3(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x^3}{ex+d}\,dx$$

input
```
integrate(x^3*(b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^3/(e*x + d), x)
```

3.409.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^2)^p}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(b*x**2+a)**p/(e*x+d),x)
```

output
```
Timed out
```

3.409.7 Maxima [F]

$$\int \frac{x^3(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x^3}{ex+d}\,dx$$

input
```
integrate(x^3*(b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^3/(e*x + d), x)
```

3.409.8 Giac [F]

$$\int \frac{x^3(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x^3}{ex+d}\,dx$$

input
```
integrate(x^3*(b*x^2+a)^p/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^3/(e*x + d), x)
```

3.409.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^2)^p}{d+ex}\,dx=\int \frac{x^3\,\left(b\,x^2+a\right)^p}{d+e\,x}\,dx$$

input
```
int((x^3*(a + b*x^2)^p)/(d + e*x),x)
```

output
```
int((x^3*(a + b*x^2)^p)/(d + e*x), x)
```

3.410 $\int \frac{x^2(a+bx^2)^p}{d+ex}\,dx$

3.410.1 Optimal result

Integrand size = 20, antiderivative size = 161

$$\int \frac{x^2(a+bx^2)^p}{d+ex}\,dx=\frac{(a+bx^2)^{1+p}}{2be(1+p)}+\frac{x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},-p,1,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d}-\frac{d^2(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2e\left(bd^2+ae^2\right)(1+p)}$$

output
```
1/2*(b*x^2+a)^(p+1)/b/e/(p+1)+1/3*x^3*(b*x^2+a)^p*AppellF1(3/2,1,-p,5/2,e^
2*x^2/d^2,-b*x^2/a)/d/((1+b*x^2/a)^p)-1/2*d^2*(b*x^2+a)^(p+1)*hypergeom([1
, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e/(a*e^2+b*d^2)/(p+1)
```

3.410.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.41

$$\int \frac{x^2(a+bx^2)^p}{d+ex}\,dx = \frac{(a+bx^2)^p\left(ae^2p+be^2px^2-ae^2p\left(1+\frac{bx^2}{a}\right)^{-p}+bd^2(1+p)\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\operatorname{AppellF1}\left(\right.\right.}{2be^3}$$

input
```
Integrate[(x^2*(a + b*x^2)^p)/(d + e*x),x]
```

output
```
((a + b*x^2)^p*(a*e^2*p + b*e^2*p*x^2 - (a*e^2*p)/(1 + (b*x^2)/a)^p + (b*d
^2*(1 + p)*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x),
 (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*(
(e*(Sqrt[-(a/b)] + x))/(d + e*x))^p) - (2*b*d*e*p*(1 + p)*x*Hypergeometric
2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/(2*b*e^3*p*(1 + p))
```

3.410.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {621, 354, 90, 78, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(a+b x^2\right)^p}{d+e x} \, dx$$

$$\Big\downarrow \text{621}$$

$$d \int \frac{x^2 \left(b x^2+a\right)^p}{d^2-e^2 x^2}dx-e \int \frac{x^3 \left(b x^2+a\right)^p}{d^2-e^2 x^2}dx$$

$$\Big\downarrow \text{354}$$

$$d \int \frac{x^2 \left(b x^2+a\right)^p}{d^2-e^2 x^2}dx-\frac{1}{2} e \int \frac{x^2 \left(b x^2+a\right)^p}{d^2-e^2 x^2}dx^2$$

$$\Big\downarrow \text{90}$$

$$d \int \frac{x^2 \left(b x^2+a\right)^p}{d^2-e^2 x^2}dx-\frac{1}{2} e \left(\frac{d^2 \int \frac{\left(b x^2+a\right)^p}{d^2-e^2 x^2}dx^2}{e^2}-\frac{\left(a+b x^2\right)^{p+1}}{b e^2 (p+1)}\right)$$

$$\Big\downarrow \text{78}$$

$$d \int \frac{x^2 \left(b x^2+a\right)^p}{d^2-e^2 x^2}dx-\frac{1}{2} e \left(\frac{d^2 \left(a+b x^2\right)^{p+1} \text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2 \left(b x^2+a\right)}{b d^2+a e^2}\right)}{e^2 (p+1) \left(a e^2+b d^2\right)}-\frac{\left(a+b x^2\right)^{p+1}}{b e^2 (p+1)}\right)$$

$$\Big\downarrow \text{395}$$

$$d(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int\frac{x^2\left(\frac{bx^2}{a}+1\right)^p}{d^2-e^2x^2}dx-$$
$$\frac{1}{2}e\left(\frac{d^2(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{e^2(p+1)(ae^2+bd^2)}-\frac{(a+bx^2)^{p+1}}{be^2(p+1)}\right)$$

$$\Big\downarrow \text{394}$$

$$\frac{x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,1,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d}-$$
$$\frac{1}{2}e\left(\frac{d^2(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{e^2(p+1)(ae^2+bd^2)}-\frac{(a+bx^2)^{p+1}}{be^2(p+1)}\right)$$

input
```
Int[(x^2*(a + b*x^2)^p)/(d + e*x),x]
```

output
```
(x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 1, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])
/(3*d*(1 + (b*x^2)/a)^p) - (e*(-((a + b*x^2)^(1 + p)/(b*e^2*(1 + p))) + (d
^2*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2)
)/(b*d^2 + a*e^2)])/(e^2*(b*d^2 + a*e^2)*(1 + p))))/2
```

3.410.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.410. $\int \frac{x^2(a+bx^2)^p}{d+ex}\,dx$

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 621
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[c   Int[x^m*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] - Simp[d   Int[
x^(m + 1)*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, m,
p}, x]
```

3.410.4 Maple [F]

$$\int \frac{x^2(b\,x^2+a)^p}{ex+d}dx$$

input
```
int(x^2*(b*x^2+a)^p/(e*x+d),x)
```

output
```
int(x^2*(b*x^2+a)^p/(e*x+d),x)
```

3.410.5 Fricas [F]

$$\int \frac{x^2(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x^2}{ex+d}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^2/(e*x + d), x)
```

3.410.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)^p}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(b*x**2+a)**p/(e*x+d),x)
```

output
```
Timed out
```

3.410.7 Maxima [F]

$$\int \frac{x^2(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x^2}{ex+d}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^2/(e*x + d), x)
```

3.410.8 Giac [F]

$$\int \frac{x^2(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x^2}{ex+d}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^p/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^2/(e*x + d), x)
```

3.410.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)^p}{d+ex}\,dx=\int \frac{x^2\,{\left(b\,x^2+a\right)}^p}{d+e\,x} \,dx$$

input
```
int((x^2*(a + b*x^2)^p)/(d + e*x),x)
```

output
```
int((x^2*(a + b*x^2)^p)/(d + e*x), x)
```

3.411 $\int \frac{x(a+bx^2)^p}{d+ex}\,dx$

3.411.1 Optimal result

Integrand size = 18, antiderivative size = 173

$$\int \frac{x(a+bx^2)^p}{d+ex}\,dx = -\frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e} + \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e} + \frac{d(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2(bd^2+ae^2)(1+p)}$$

output
```
-x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/e/((1+b*x^2/a)^
p)+x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e/((1+b*x^2/a)^p)+1/2
*d*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(
a*e^2+b*d^2)/(p+1)
```

3.411.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.99

$$\int \frac{x(a+bx^2)^p}{d+ex}\,dx = \frac{(a+bx^2)^p\left(-\frac{d\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{p} + 2ex\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hyp}\right.}{2e^2}$$

input
```
Integrate[(x*(a + b*x^2)^p)/(d + e*x),x]
```

output
```
((a + b*x^2)^p*(-((d*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/
(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d
 + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p)) + (2*e*x*Hypergeometric2
F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/(2*e^2)
```

3.411.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {621, 353, 78, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^2)^p}{d+ex}dx$$

$$\Big\downarrow \text{621}$$

$$d\int \frac{x(bx^2+a)^p}{d^2-e^2x^2}dx-e\int \frac{x^2(bx^2+a)^p}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}d\int \frac{(bx^2+a)^p}{d^2-e^2x^2}dx^2-e\int \frac{x^2(bx^2+a)^p}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{78}$$

$$\frac{d(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)}-e\int \frac{x^2(bx^2+a)^p}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{395}$$

$$\frac{d(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)}-e(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \frac{x^2\left(\frac{bx^2}{a}+1\right)^p}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{394}$$

$$\frac{d(a+bx^2)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)\,(ae^2+bd^2)} - \frac{ex^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,1,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^2}$$

input
```
Int[(x*(a + b*x^2)^p)/(d + e*x),x]
```

output
```
-1/3*(e*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 1, 5/2, -((b*x^2)/a), (e^2*x^2
)/d^2])/(d^2*(1 + (b*x^2)/a)^p) + (d*(a + b*x^2)^(1 + p)*Hypergeometric2F1
[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)*(
1 + p))
```

3.411.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && !IntegerQ[m] && IntegerQ[n]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 621
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[c   Int[x^m*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] - Simp[d   Int[
x^(m + 1)*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, m,
p}, x]
```

3.411.4 Maple [F]

$$\int \frac{x(b\,x^2+a)^p}{ex+d}dx$$

input
```
int(x*(b*x^2+a)^p/(e*x+d),x)
```

output
```
int(x*(b*x^2+a)^p/(e*x+d),x)
```

3.411.5 Fricas [F]

$$\int \frac{x(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x}{ex+d}\,dx$$

input
```
integrate(x*(b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x/(e*x + d), x)
```

3.411.6 Sympy [F]

$$\int \frac{x(a+bx^2)^p}{d+ex}\,dx = \int \frac{x(a+bx^2)^p}{d+ex}\,dx$$

input
```
integrate(x*(b*x**2+a)**p/(e*x+d),x)
```

output
```
Integral(x*(a + b*x**2)**p/(d + e*x), x)
```

3.411.7 Maxima [F]

$$\int \frac{x(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x}{ex+d}\,dx$$

input
```
integrate(x*(b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x/(e*x + d), x)
```

3.411.8 Giac [F]

$$\int \frac{x(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x}{ex+d}\,dx$$

input
```
integrate(x*(b*x^2+a)^p/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x/(e*x + d), x)
```

3.411.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^2)^p}{d+ex}\,dx = \int \frac{x\,(b\,x^2+a)^p}{d+e\,x}\,dx$$

input
```
int((x*(a + b*x^2)^p)/(d + e*x),x)
```

output
```
int((x*(a + b*x^2)^p)/(d + e*x), x)
```

3.412 $\int \frac{(a+bx^2)^p}{d+ex} \, dx$

3.412.1 Optimal result

Integrand size = 17, antiderivative size = 125

$$\int \frac{(a+bx^2)^p}{d+ex} \, dx = \frac{x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d} - \frac{e(a+bx^2)^{1+p} \operatorname{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2(bd^2+ae^2)(1+p)}$$

output
```
x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d/((1+b*x^2/a)^p
)-1/2*e*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^
2))/(a*e^2+b*d^2)/(p+1)
```

3.412.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^p}{d+ex} \, dx = \frac{\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p} \left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p} (a+bx^2)^p \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{2ep}$$

input
```
Integrate[(a + b*x^2)^p/(d + e*x),x]
```

output
```
((a + b*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e
*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(2*e*p*((e*(-Sqrt[-(a/b)] + x))/(d +
 e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p)
```

3.412.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {504, 334, 333, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{d+ex}dx$$

$$\Big\downarrow \text{504}$$

$$d\int \frac{(bx^2+a)^p}{d^2-e^2x^2}dx-e\int \frac{x(bx^2+a)^p}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{334}$$

$$d(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^2}{a}+1\right)^p}{d^2-e^2x^2}dx-e\int \frac{x(bx^2+a)^p}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{333}$$

$$\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-e\int \frac{x(bx^2+a)^p}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{1}{2}e\int \frac{(bx^2+a)^p}{d^2-e^2x^2}dx^2$$

$$\Big\downarrow \text{78}$$

$$\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{e(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)}$$

input
```
Int[(a + b*x^2)^p/(d + e*x),x]
```

output
```
(x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(
d*(1 + (b*x^2)/a)^p) - (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p,
2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)*(1 + p))
```

3.412.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

3.412.4 Maple [F]

$$\int \frac{(b\,x^{2}+a)^{p}}{ex+d}dx$$

input
```
int((b*x^2+a)^p/(e*x+d),x)
```

output
```
int((b*x^2+a)^p/(e*x+d),x)
```

3.412.5 Fricas [F]

$$\int \frac{(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p}{ex+d}\,dx$$

input
```
integrate((b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/(e*x + d), x)
```

3.412.6 Sympy [F]

$$\int \frac{(a+bx^2)^p}{d+ex}\,dx = \int \frac{(a+bx^2)^p}{d+ex}\,dx$$

input
```
integrate((b*x**2+a)**p/(e*x+d),x)
```

output
```
Integral((a + b*x**2)**p/(d + e*x), x)
```

3.412.7 Maxima [F]

$$\int \frac{(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p}{ex+d}\,dx$$

input
```
integrate((b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/(e*x + d), x)
```

3.412.8 Giac [F]

$$\int \frac{(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p}{ex+d}\,dx$$

input
```
integrate((b*x^2+a)^p/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/(e*x + d), x)
```

3.412.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{d+ex}\,dx = \int \frac{(b\,x^2+a)^p}{d+e\,x}\,dx$$

input
```
int((a + b*x^2)^p/(d + e*x),x)
```

output
```
int((a + b*x^2)^p/(d + e*x), x)
```

3.413 $\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx$

3.413.1 Optimal result

Integrand size = 20, antiderivative size = 176

$$\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx = -\frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2} + \frac{e^2(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2d\left(bd^2+ae^2\right)(1+p)} - \frac{(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2ad(1+p)}$$

output
```
-e*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^2/((1+b*x^2
/a)^p)+1/2*e^2*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],e^2*(b*x^2+a)/(a*e
^2+b*d^2))/d/(a*e^2+b*d^2)/(p+1)-1/2*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2
+p],1+b*x^2/a)/a/d/(p+1)
```

3.413.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx$$
$$= \frac{(a+bx^2)^p\left(-\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)+(1}{2dp}$$

input
```
Integrate[(a + b*x^2)^p/(x*(d + e*x)),x]
```

output
```
((a + b*x^2)^p*(-(AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d
+ e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)]/(((e*(-Sqrt[-(a/b)] + x))/(d + e*x
))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p)) + Hypergeometric2F1[-p, -p, 1
- p, -(a/(b*x^2))]/(1 + a/(b*x^2))^p))/(2*d*p)
```

3.413.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {621, 334, 333, 354, 97, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx$$
$$\Big\downarrow \text{ 621}$$
$$d\int \frac{(bx^2+a)^p}{x\,(d^2-e^2x^2)}dx-e\int \frac{(bx^2+a)^p}{d^2-e^2x^2}dx$$
$$\Big\downarrow \text{ 334}$$
$$d\int \frac{(bx^2+a)^p}{x\,(d^2-e^2x^2)}dx-e(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^2}{a}+1\right)^p}{d^2-e^2x^2}dx$$
$$\Big\downarrow \text{ 333}$$

$$d\int \frac{(bx^2+a)^p}{x(d^2-e^2x^2)}dx - \frac{ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}$$

$\Big\downarrow$ 354

$$\frac{1}{2}d\int \frac{(bx^2+a)^p}{x^2(d^2-e^2x^2)}dx^2 - \frac{ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}$$

$\Big\downarrow$ 97

$$\frac{1}{2}d\left(\frac{e^2\int \frac{(bx^2+a)^p}{d^2-e^2x^2}dx^2}{d^2}+\frac{\int \frac{(bx^2+a)^p}{x^2}dx^2}{d^2}\right) - \frac{ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}$$

$\Big\downarrow$ 75

$$\frac{1}{2}d\left(\frac{e^2\int \frac{(bx^2+a)^p}{d^2-e^2x^2}dx^2}{d^2}-\frac{(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{ad^2(p+1)}\right) - \frac{ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}$$

$\Big\downarrow$ 78

$$\frac{1}{2}d\left(\frac{e^2(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{d^2(p+1)(ae^2+bd^2)}-\frac{(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1\right.}{ad^2(p+1)}\right. \frac{ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}$$

input
```
Int[(a + b*x^2)^p/(x*(d + e*x)),x]
```

output
```
-((e*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2
])/(d^2*(1 + (b*x^2)/a)^p)) + (d*((e^2*(a + b*x^2)^(1 + p)*Hypergeometric2
F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(d^2*(b*d^2 + a*e^
2)*(1 + p)) - ((a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 +
(b*x^2)/a])/(a*d^2*(1 + p))))/2
```

3.413. $\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx$

3.413.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 621
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[c   Int[x^m*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] - Simp[d   Int[
x^(m + 1)*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, m,
p}, x]
```

3.413. $\int \frac{(a+bx^2)^p}{x(d+ex)} dx$

3.413.4 Maple [F]

$$\int \frac{(b\,x^2+a)^p}{x\,(ex+d)}dx$$

input
```
int((b*x^2+a)^p/x/(e*x+d),x)
```

output
```
int((b*x^2+a)^p/x/(e*x+d),x)
```

3.413.5 Fricas [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)x}\,dx$$

input
```
integrate((b*x^2+a)^p/x/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/(e*x^2 + d*x), x)
```

3.413.6 Sympy [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx = \int \frac{(a+bx^2)^p}{x\,(d+ex)}\,dx$$

input
```
integrate((b*x**2+a)**p/x/(e*x+d),x)
```

output
```
Integral((a + b*x**2)**p/(x*(d + e*x)), x)
```

3.413.7 Maxima [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)x}\,dx$$

input
```
integrate((b*x^2+a)^p/x/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/((e*x + d)*x), x)
```

3.413.8 Giac [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)x}\,dx$$

input
```
integrate((b*x^2+a)^p/x/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/((e*x + d)*x), x)
```

3.413.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx = \int \frac{(b\,x^2+a)^p}{x\,(d+e\,x)}\,dx$$

input
```
int((a + b*x^2)^p/(x*(d + e*x)),x)
```

output
```
int((a + b*x^2)^p/(x*(d + e*x)), x)
```

3.414 $\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx$

3.414.1 Optimal result

Integrand size = 20, antiderivative size = 178

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx = -\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(-\frac{1}{2},-p,1,\frac{1}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{dx} - \frac{e^3(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2d^2\left(bd^2+ae^2\right)(1+p)} + \frac{e(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2ad^2(1+p)}$$

output
```
-(b*x^2+a)^p*AppellF1(-1/2,1,-p,1/2,e^2*x^2/d^2,-b*x^2/a)/d/x/((1+b*x^2/a)
^p)-1/2*e^3*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+
b*d^2))/d^2/(a*e^2+b*d^2)/(p+1)+1/2*e*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[
2+p],1+b*x^2/a)/a/d^2/(p+1)
```

3.414.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx = \frac{(a+bx^2)^p\left(\frac{e\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{p}-\frac{2d\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric}}{x}\right.}{2d^2}$$

input
```
Integrate[(a + b*x^2)^p/(x^2*(d + e*x)),x]
```

output
```
((a + b*x^2)^p*((e*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d
 + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d +
 e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p) - (2*d*Hypergeometric2F1[-1
/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 + (b*x^2)/a)^p) - (e*Hypergeometric2F1[-
p, -p, 1 - p, -(a/(b*x^2))])/(p*(1 + a/(b*x^2))^p)))/(2*d^2)
```

3.414.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {621, 354, 97, 75, 78, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx$$

$$\Big\downarrow \; 621$$

$$d\int \frac{(bx^2+a)^p}{x^2(d^2-e^2x^2)}dx - e\int \frac{(bx^2+a)^p}{x(d^2-e^2x^2)}dx$$

$$\Big\downarrow \; 354$$

$$d\int \frac{(bx^2+a)^p}{x^2(d^2-e^2x^2)}dx - \frac{1}{2}e\int \frac{(bx^2+a)^p}{x^2(d^2-e^2x^2)}dx^2$$

$$\Big\downarrow \; 97$$

$$d\int \frac{(bx^2+a)^p}{x^2(d^2-e^2x^2)}dx-\frac{1}{2}e\left(\frac{e^2\int \frac{(bx^2+a)^p}{d^2-e^2x^2}dx^2}{d^2}+\frac{\int \frac{(bx^2+a)^p}{x^2}dx^2}{d^2}\right)$$

$$\Big\downarrow \text{75}$$

$$d\int \frac{(bx^2+a)^p}{x^2(d^2-e^2x^2)}dx-\frac{1}{2}e\left(\frac{e^2\int \frac{(bx^2+a)^p}{d^2-e^2x^2}dx^2}{d^2}-\frac{(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{ad^2(p+1)}\right)$$

$$\Big\downarrow \text{78}$$

$$d\int \frac{(bx^2+a)^p}{x^2(d^2-e^2x^2)}dx-\frac{1}{2}e\left(\frac{e^2(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{d^2(p+1)(ae^2+bd^2)}-\frac{(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1\right.}{ad^2(p+1)}\right.$$

$$\Big\downarrow \text{395}$$

$$d(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^2}{a}+1\right)^p}{x^2(d^2-e^2x^2)}dx-\frac{1}{2}e\left(\frac{e^2(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{d^2(p+1)(ae^2+bd^2)}-\frac{(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1\right.}{ad^2(p+1)}\right.$$

$$\Big\downarrow \text{394}$$

$$-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(-\frac{1}{2},-p,1,\frac{1}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{dx}-\frac{1}{2}e\left(\frac{e^2(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{d^2(p+1)(ae^2+bd^2)}-\frac{(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1\right.}{ad^2(p+1)}\right.$$

input
```
Int[(a + b*x^2)^p/(x^2*(d + e*x)),x]
```

output
```
-(((a + b*x^2)^p*AppellF1[-1/2, -p, 1, 1/2, -((b*x^2)/a), (e^2*x^2)/d^2])/
(d*x*(1 + (b*x^2)/a)^p)) - (e*((e^2*(a + b*x^2)^(1 + p)*Hypergeometric2F1[
1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(d^2*(b*d^2 + a*e^2)*
(1 + p)) - ((a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*
x^2)/a])/(a*d^2*(1 + p))))/2
```

3.414. $\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx$

3.414.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.414. $\int \frac{(a+bx^2)^p}{x^2(d+ex)} dx$

rule 621
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[c   Int[x^m*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] - Simp[d   Int[
x^(m + 1)*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, m,
p}, x]
```

3.414.4 Maple [F]

$$\int \frac{(b\,x^2+a)^p}{x^2\,(ex+d)}dx$$

input
```
int((b*x^2+a)^p/x^2/(e*x+d),x)
```

output
```
int((b*x^2+a)^p/x^2/(e*x+d),x)
```

3.414.5 Fricas [F]

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)x^2}\,dx$$

input
```
integrate((b*x^2+a)^p/x^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/(e*x^3 + d*x^2), x)
```

3.414.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p/x**2/(e*x+d),x)
```

output
```
Timed out
```

3.414.7 Maxima [F]

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)x^2}\,dx$$

input
```
integrate((b*x^2+a)^p/x^2/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/((e*x + d)*x^2), x)
```

3.414.8 Giac [F]

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)x^2}\,dx$$

input
```
integrate((b*x^2+a)^p/x^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/((e*x + d)*x^2), x)
```

3.414.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx = \int \frac{(b\,x^2+a)^p}{x^2\,(d+e\,x)}\,dx$$

input
```
int((a + b*x^2)^p/(x^2*(d + e*x)),x)
```

output
```
int((a + b*x^2)^p/(x^2*(d + e*x)), x)
```

3.415 $\int \frac{(a+bx^2)^p}{x^3(d+ex)}\,dx$

3.415.1 Optimal result

Integrand size = 20, antiderivative size = 213

$$\int \frac{(a+bx^2)^p}{x^3(d+ex)}\,dx = -\frac{(a+bx^2)^{1+p}}{2adx^2} + \frac{e(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(-\frac{1}{2},-p,1,\frac{1}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x} + \frac{e^4(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2d^3(bd^2+ae^2)(1+p)} - \frac{(ae^2+bd^2p)(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a^2d^3(1+p)}$$

output
```
-1/2*(b*x^2+a)^(p+1)/a/d/x^2+e*(b*x^2+a)^p*AppellF1(-1/2,1,-p,1/2,e^2*x^2/
d^2,-b*x^2/a)/d^2/x/((1+b*x^2/a)^p)+1/2*e^4*(b*x^2+a)^(p+1)*hypergeom([1,
p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/d^3/(a*e^2+b*d^2)/(p+1)-1/2*(b*d^2
*p+a*e^2)*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*x^2/a)/a^2/d^3/(p+1
)
```

3.415.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^2)^p}{x^3(d+ex)}\,dx$$
$$= \frac{(a+bx^2)^p \left(-\frac{e^2 \left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p} \left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p} \text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{p} + \frac{2de\left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeon}}{}\right.}{}$$

input
```
Integrate[(a + b*x^2)^p/(x^3*(d + e*x)),x]
```

output
```
((a + b*x^2)^p*(-((e^2*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e
)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/
(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p)) + (2*d*e*Hypergeometri
c2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 + (b*x^2)/a)^p) + ((d^2*Hypergeom
etric2F1[1 - p, -p, 2 - p, -(a/(b*x^2))])/((-1 + p)*x^2) + (e^2*Hypergeome
tric2F1[-p, -p, 1 - p, -(a/(b*x^2))])/p)/(1 + a/(b*x^2))^p))/(2*d^3)
```

3.415.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {621, 354, 114, 25, 174, 75, 78, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{x^3(d+ex)}\,dx$$

$$\Big\downarrow \; 621$$

$$d\int \frac{(bx^2+a)^p}{x^3(d^2-e^2x^2)}dx - e\int \frac{(bx^2+a)^p}{x^2(d^2-e^2x^2)}dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}d\int \frac{(bx^2+a)^p}{x^4(d^2-e^2x^2)}dx^2 - e\int \frac{(bx^2+a)^p}{x^2(d^2-e^2x^2)}dx$$

$$\Big\downarrow \; 114$$

$$\frac{1}{2}d\left(-\frac{\int -\frac{(bx^2+a)^p(bpd^2+ae^2-be^2px^2)}{x^2(d^2-e^2x^2)}dx^2}{ad^2}-\frac{(a+bx^2)^{p+1}}{ad^2x^2}\right)-e\int\frac{(bx^2+a)^p}{x^2(d^2-e^2x^2)}dx$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}d\left(\frac{\int \frac{(bx^2+a)^p(bpd^2+ae^2-be^2px^2)}{x^2(d^2-e^2x^2)}dx^2}{ad^2}-\frac{(a+bx^2)^{p+1}}{ad^2x^2}\right)-e\int\frac{(bx^2+a)^p}{x^2(d^2-e^2x^2)}dx$$

$$\Big\downarrow \; 174$$

$$\frac{1}{2}d\left(\frac{\left(\frac{ae^2}{d^2}+bp\right)\int\frac{(bx^2+a)^p}{x^2}dx^2+\frac{ae^4\int\frac{(bx^2+a)^p}{d^2-e^2x^2}dx^2}{d^2}}{ad^2}-\frac{(a+bx^2)^{p+1}}{ad^2x^2}\right)-e\int\frac{(bx^2+a)^p}{x^2(d^2-e^2x^2)}dx$$

$$\Big\downarrow \; 75$$

$$\frac{1}{2}d\left(\frac{\frac{ae^4\int\frac{(bx^2+a)^p}{d^2-e^2x^2}dx^2}{d^2}-\frac{(a+bx^2)^{p+1}\left(\frac{ae^2}{d^2}+bp\right)\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{a(p+1)}}{ad^2}-\frac{(a+bx^2)^{p+1}}{ad^2x^2}\right)-$$

$$e\int\frac{(bx^2+a)^p}{x^2(d^2-e^2x^2)}dx$$

$$\Big\downarrow \; 78$$

$$\frac{1}{2}d\left(\frac{\frac{ae^4(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{d^2(p+1)(ae^2+bd^2)}-\frac{(a+bx^2)^{p+1}\left(\frac{ae^2}{d^2}+bp\right)\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{a(p+1)}}{ad^2}-\frac{(a\right.$$

$$e\int\frac{(bx^2+a)^p}{x^2(d^2-e^2x^2)}dx$$

$$\Big\downarrow \; 395$$

$$\frac{1}{2}d\left(\frac{\frac{ae^4(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{d^2(p+1)(ae^2+bd^2)}-\frac{(a+bx^2)^{p+1}\left(\frac{ae^2}{d^2}+bp\right)\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{a(p+1)}}{ad^2}-\frac{(a\right.$$

$$e(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int\frac{\left(\frac{bx^2}{a}+1\right)^p}{x^2(d^2-e^2x^2)}dx$$

$$\Big\downarrow \; 394$$

3.415. $\int\frac{(a+bx^2)^p}{x^3(d+ex)}dx$

$$\frac{e(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{AppellF1}\left(-\frac{1}{2},-p,1,\frac{1}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x} +$$

$$\frac{1}{2}d\left(\frac{\frac{ae^4(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{d^2(p+1)(ae^2+bd^2)} - \frac{(a+bx^2)^{p+1}\left(\frac{ae^2}{d^2}+bp\right)\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{a(p+1)}}{ad^2} - \frac{(a}{}\right.$$

input
```
Int[(a + b*x^2)^p/(x^3*(d + e*x)),x]
```

output
```
(e*(a + b*x^2)^p*AppellF1[-1/2, -p, 1, 1/2, -((b*x^2)/a), (e^2*x^2)/d^2])/
(d^2*x*(1 + (b*x^2)/a)^p) + (d*(-((a + b*x^2)^(1 + p)/(a*d^2*x^2)) + ((a*e
^4*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2)
)/(b*d^2 + a*e^2)])/(d^2*(b*d^2 + a*e^2)*(1 + p)) - (((a*e^2)/d^2 + b*p)*(
a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(a*(
1 + p)))/(a*d^2)))/2
```

3.415.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 621
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[c   Int[x^m*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] - Simp[d   Int[
x^(m + 1)*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, m,
p}, x]
```

3.415. $\int \frac{(a+bx^2)^p}{x^3(d+ex)} \, dx$

3.415.4 Maple [F]

$$\int \frac{(b\,x^2+a)^p}{x^3\,(ex+d)}dx$$

input
```
int((b*x^2+a)^p/x^3/(e*x+d),x)
```

output
```
int((b*x^2+a)^p/x^3/(e*x+d),x)
```

3.415.5 Fricas [F]

$$\int \frac{(a+bx^2)^p}{x^3(d+ex)}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)x^3}\,dx$$

input
```
integrate((b*x^2+a)^p/x^3/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/(e*x^4 + d*x^3), x)
```

3.415.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x^3(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p/x**3/(e*x+d),x)
```

output
```
Timed out
```

3.415.7 Maxima [F]

$$\int \frac{(a+bx^2)^p}{x^3(d+ex)}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)x^3}\,dx$$

input
```
integrate((b*x^2+a)^p/x^3/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/((e*x + d)*x^3), x)
```

3.415.8 Giac [F]

$$\int \frac{(a+bx^2)^p}{x^3(d+ex)}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)x^3}\,dx$$

input
```
integrate((b*x^2+a)^p/x^3/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/((e*x + d)*x^3), x)
```

3.415.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x^3(d+ex)}\,dx = \int \frac{(b\,x^2+a)^p}{x^3\,(d+e\,x)}\,dx$$

input
```
int((a + b*x^2)^p/(x^3*(d + e*x)),x)
```

output
```
int((a + b*x^2)^p/(x^3*(d + e*x)), x)
```

3.416 $\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx$

3.416.1 Optimal result

Integrand size = 20, antiderivative size = 392

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx = -\frac{d(4+3p)\left(a+bx^2\right)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\left(a+bx^2\right)^{1+p}}{be^3(3+2p)}$$
$$-\frac{2d^2\left(2ae^2+bd^2(2+p)\right)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^4\left(bd^2+ae^2\right)}$$
$$-\frac{\left(a^2e^4-2abd^2e^2(4+3p)-2b^2d^4\left(6+7p+2p^2\right)\right)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2}\right.}{be^4\left(bd^2+ae^2\right)(3+2p)}$$
$$+\frac{d^3\left(2ae^2+bd^2(2+p)\right)\left(a+bx^2\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{e^3\left(bd^2+ae^2\right)^2(1+p)}$$

output
```
-d*(4+3*p)*(b*x^2+a)^(p+1)/b/e^3/(p+1)/(3+2*p)-d^4*(b*x^2+a)^(p+1)/e^3/(a*
e^2+b*d^2)/(e*x+d)+(e*x+d)*(b*x^2+a)^(p+1)/b/e^3/(3+2*p)-2*d^2*(2*a*e^2+b*
d^2*(2+p))*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/e^4/(
a*e^2+b*d^2)/((1+b*x^2/a)^p)-(a^2*e^4-2*a*b*d^2*e^2*(4+3*p)-2*b^2*d^4*(2*p
^2+7*p+6))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/b/e^4/(a*e^2+
b*d^2)/(3+2*p)/((1+b*x^2/a)^p)+d^3*(2*a*e^2+b*d^2*(2+p))*(b*x^2+a)^(p+1)*h
ypergeom([1, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e^3/(a*e^2+b*d^2)^2/(
p+1)
```

3.416.2 Mathematica [F]

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx$$

input
```
Integrate[(x^4*(a + b*x^2)^p)/(d + e*x)^2,x]
```

output
```
Integrate[(x^4*(a + b*x^2)^p)/(d + e*x)^2, x]
```

3.416.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {603, 2185, 2185, 27, 719, 238, 237, 504, 334, 333, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(a+bx^2\right)^p}{(d+ex)^2}dx$$

$$\Big\downarrow \text{603}$$

$$-\frac{\int \frac{(bx^2+a)^p\left(\frac{ad^3}{e^2}-\frac{\left(2b(p+1)d^2+ae^2\right)xd^2}{e^3}+\left(\frac{bd^2}{e^2}+a\right)x^2d-\frac{\left(bd^2+ae^2\right)x^3}{e}\right)}{d+ex}dx}{ae^2+bd^2}-\frac{d^4\left(a+bx^2\right)^{p+1}}{e^3(d+ex)\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \text{2185}$$

$$-\frac{\frac{\int \frac{\left(bx^2+a\right)^p\left(2bde\left(bd^2+ae^2\right)(3p+4)x^2+\left(a^2e^4-4b^2d^4(p+1)^2\right)x+ade\left(2b(p+2)d^2+ae^2\right)\right)}{d+ex}dx}{be^3(2p+3)}-\frac{(d+ex)\left(ae^2+bd^2\right)\left(a+bx^2\right)^{p+1}}{be^3(2p+3)}}{ae^2+bd^2}-\frac{d^4\left(a+bx^2\right)^{p+1}}{e^3(d+ex)\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \text{2185}$$

$$-\frac{\frac{\frac{\int \frac{2be^2(p+1)\left(ade\left(2b(p+2)d^2+ae^2\right)+\left(-2b^2\left(2p^2+7p+6\right)d^4-2abe^2(3p+4)d^2+a^2e^4\right)x\right)\left(bx^2+a\right)^p}{d+ex}dx}{2be^2(p+1)}+\frac{d(3p+4)\left(ae^2+bd^2\right)\left(a+bx^2\right)^{p+1}}{p+1}}{be^3(2p+3)}-\frac{(d+ex)\left(ae^2+bd^2\right.}{be^3(2p}}{ae^2+bd^2}$$
$$\frac{d^4\left(a+bx^2\right)^{p+1}}{e^3(d+ex)\left(ae^2+bd^2\right)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\int \frac{\left(ade\left(2b(p+2)d^2+ae^2\right)+\left(-2b^2\left(2p^2+7p+6\right)d^4-2abe^2(3p+4)d^2+a^2e^4\right)x\right)\left(bx^2+a\right)^p}{d+ex}dx+\frac{d(3p+4)\left(ae^2+bd^2\right)\left(a+bx^2\right)^{p+1}}{p+1}}{be^3(2p+3)}-\frac{(d+ex)(ae^2+bd^2)(a+bx^2}{be^3(2p+3)}}{ae^2+bd^2}$$

$$\frac{d^4\left(a+bx^2\right)^{p+1}}{e^3(d+ex)\left(ae^2+bd^2\right)}$$

$\Big\downarrow$ 719

$$-\frac{\frac{\frac{\left(a^2e^4-2abd^2e^2(3p+4)-2b^2d^4\left(2p^2+7p+6\right)\right)\int\left(bx^2+a\right)^pdx}{e}+\frac{2bd^3(2p+3)\left(2ae^2+bd^2(p+2)\right)\int\frac{\left(bx^2+a\right)^p}{d+ex}dx}{e}+\frac{d(3p+4)\left(ae^2+bd^2\right)\left(a+bx^2\right)^{p+1}}{p+1}}{be^3(2p+3)}-\frac{(d+ex)(}{}}{ae^2+bd^2}$$

$$\frac{d^4\left(a+bx^2\right)^{p+1}}{e^3(d+ex)\left(ae^2+bd^2\right)}$$

$\Big\downarrow$ 238

$$-\frac{\frac{\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2e^4-2abd^2e^2(3p+4)-2b^2d^4\left(2p^2+7p+6\right)\right)\int\left(\frac{bx^2}{a}+1\right)^pdx}{e}+\frac{2bd^3(2p+3)\left(2ae^2+bd^2(p+2)\right)\int\frac{\left(bx^2+a\right)^p}{d+ex}dx}{e}+\frac{d(3p+4)\left(ae^2+bd^2\right)\left(a\right.}{p+1}}{be^3(2p+3)}}{ae^2+bd^2}$$

$$\frac{d^4\left(a+bx^2\right)^{p+1}}{e^3(d+ex)\left(ae^2+bd^2\right)}$$

$\Big\downarrow$ 237

$$-\frac{\frac{\frac{2bd^3(2p+3)\left(2ae^2+bd^2(p+2)\right)\int\frac{\left(bx^2+a\right)^p}{d+ex}dx}{e}+\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2e^4-2abd^2e^2(3p+4)-2b^2d^4\left(2p^2+7p+6\right)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e}+}{be^3(2p+3)}}{ae^2+bd^2}$$

$$\frac{d^4\left(a+bx^2\right)^{p+1}}{e^3(d+ex)\left(ae^2+bd^2\right)}$$

$\Big\downarrow$ 504

$$-\frac{\frac{\frac{2bd^3(2p+3)\left(2ae^2+bd^2(p+2)\right)\left(d\int\frac{\left(bx^2+a\right)^p}{d^2-e^2x^2}dx-e\int\frac{x\left(bx^2+a\right)^p}{d^2-e^2x^2}dx\right)}{e}+\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2e^4-2abd^2e^2(3p+4)-2b^2d^4\left(2p^2+7p+6\right)\right)\text{Hypergeometric}}{e}}{be^3(2p+3)}}{ae^2+bd^2}$$

$$\frac{d^4\left(a+bx^2\right)^{p+1}}{e^3(d+ex)\left(ae^2+bd^2\right)}$$

$\Big\downarrow$ 334

3.416. $\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx$

$$-\frac{\frac{\frac{2bd^3(2p+3)\left(2ae^2+bd^2(p+2)\right)\left(d\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int\frac{\left(\frac{bx^2}{a}+1\right)^p}{d^2-e^2x^2}dx-e\int\frac{x\left(bx^2+a\right)^p}{d^2-e^2x^2}dx\right)}{e}+\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2e^4-2abd^2e^2(3p+4)-2b^2d^4\left(2p^2\cdots\right.\right.}{e}}{be^3(2p+3)}}{ae^2+bd^2}\cdots$$

$$\frac{d^4\left(a+bx^2\right)^{p+1}}{e^3(d+ex)\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \; 333$$

$$-\frac{\frac{\frac{2bd^3(2p+3)\left(2ae^2+bd^2(p+2)\right)\left(\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-e\int\frac{x\left(bx^2+a\right)^p}{d^2-e^2x^2}dx\right)}{e}+\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2e^4-2abd^2e^2\cdots\right.}{\cdots}}{be^3(2p+3)}}{ae^2+bd^2}\cdots$$

$$\frac{d^4\left(a+bx^2\right)^{p+1}}{e^3(d+ex)\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \; 353$$

$$-\frac{\frac{\frac{2bd^3(2p+3)\left(2ae^2+bd^2(p+2)\right)\left(\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{1}{2}e\int\frac{\left(bx^2+a\right)^p}{d^2-e^2x^2}dx^2\right)}{e}+\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2e^4-2abd^2\cdots\right.}{\cdots}}{be^3(2p+3)}}{ae^2+bd^2}\cdots$$

$$\frac{d^4\left(a+bx^2\right)^{p+1}}{e^3(d+ex)\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \; 78$$

$$-\frac{\frac{\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2e^4-2abd^2e^2(3p+4)-2b^2d^4\left(2p^2+7p+6\right)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e}+\frac{2bd^3(2p+3)\left(2ae^2+bd^2(p+2)\right)\left(\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}\cdots\right.}{\cdots}\right.}{\cdots}}{be^3(2p+3)}}{\cdots}$$

$$\frac{d^4\left(a+bx^2\right)^{p+1}}{e^3(d+ex)\left(ae^2+bd^2\right)}$$

input

```
Int[(x^4*(a + b*x^2)^p)/(d + e*x)^2,x]
```

3.416. $\int \frac{x^4\left(a+bx^2\right)^p}{(d+ex)^2}\,dx$

output
```
-((d^4*(a + b*x^2)^(1 + p))/(e^3*(b*d^2 + a*e^2)*(d + e*x))) - (-(((b*d^2
+ a*e^2)*(d + e*x)*(a + b*x^2)^(1 + p))/(b*e^3*(3 + 2*p))) + ((d*(b*d^2 +
a*e^2)*(4 + 3*p)*(a + b*x^2)^(1 + p))/(1 + p) + ((a^2*e^4 - 2*a*b*d^2*e^2*
(4 + 3*p) - 2*b^2*d^4*(6 + 7*p + 2*p^2))*x*(a + b*x^2)^p*Hypergeometric2F1
[1/2, -p, 3/2, -((b*x^2)/a)])/(e*(1 + (b*x^2)/a)^p) + (2*b*d^3*(3 + 2*p)*(
2*a*e^2 + b*d^2*(2 + p))*((x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*
x^2)/a), (e^2*x^2)/d^2])/(d*(1 + (b*x^2)/a)^p) - (e*(a + b*x^2)^(1 + p)*Hy
pergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b
*d^2 + a*e^2)*(1 + p))))/e)/(b*e^3*(3 + 2*p)))/(b*d^2 + a*e^2)
```

3.416.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

3.416. $\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx$

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 603
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[x^m, c + d*x, x], R = PolynomialRemainde
r[x^m, c + d*x, x]}, Simp[d*R*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n +
1)*(b*c^2 + a*d^2))), x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)
^(n + 1)*(a + b*x^2)^p*ExpandToSum[(n + 1)*(b*c^2 + a*d^2)*Qx + b*c*R*(n +
1) - b*d*R*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IGt
Q[m, 1] && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.416. $\int \frac{x^4(a+bx^2)^p}{(d+ex)^2} dx$

3.416.4 Maple [F]

$$\int \frac{x^4(b\,x^2+a)^p}{(ex+d)^2}dx$$

input
```
int(x^4*(b*x^2+a)^p/(e*x+d)^2,x)
```

output
```
int(x^4*(b*x^2+a)^p/(e*x+d)^2,x)
```

3.416.5 Fricas [F]

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x^4}{(ex+d)^2}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^4/(e^2*x^2 + 2*d*e*x + d^2), x)
```

3.416.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(b*x**2+a)**p/(e*x+d)**2,x)
```

output
```
Timed out
```

3.416.7 Maxima [F]

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x^4}{(ex+d)^2}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^4/(e*x + d)^2, x)
```

3.416.8 Giac [F]

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x^4}{(ex+d)^2}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^4/(e*x + d)^2, x)
```

3.416.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{x^4\,(b\,x^2+a)^p}{(d+e\,x)^2}\,dx$$

input
```
int((x^4*(a + b*x^2)^p)/(d + e*x)^2,x)
```

output
```
int((x^4*(a + b*x^2)^p)/(d + e*x)^2, x)
```

3.417 $\int \frac{x^3(a+bx^2)^p}{(d+ex)^2}\,dx$

3.417.1 Optimal result

Integrand size = 20, antiderivative size = 321

$$\begin{aligned}\int \frac{x^3(a+bx^2)^p}{(d+ex)^2}\,dx &= \frac{(a+bx^2)^{1+p}}{2be^2(1+p)} + \frac{d^3(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)(d+ex)} \\ &+ \frac{d(3ae^2+bd^2(3+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^3(bd^2+ae^2)} \\ &- \frac{d(2ae^2+bd^2(3+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e^3(bd^2+ae^2)} \\ &- \frac{d^2(3ae^2+bd^2(3+2p))\,(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2e^2(bd^2+ae^2)^2(1+p)}\end{aligned}$$

output
```
1/2*(b*x^2+a)^(p+1)/b/e^2/(p+1)+d^3*(b*x^2+a)^(p+1)/e^2/(a*e^2+b*d^2)/(e*x
+d)+d*(3*a*e^2+b*d^2*(3+2*p))*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/
d^2,-b*x^2/a)/e^3/(a*e^2+b*d^2)/((1+b*x^2/a)^p)-d*(2*a*e^2+b*d^2*(3+2*p))*
x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e^3/(a*e^2+b*d^2)/((1+b*
x^2/a)^p)-1/2*d^2*(3*a*e^2+b*d^2*(3+2*p))*(b*x^2+a)^(p+1)*hypergeom([1, p+
1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e^2/(a*e^2+b*d^2)^2/(p+1)
```

3.417.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.07

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^2}\,dx$$
$$= \frac{(a+bx^2)^p \left(-\frac{2d^3\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+2p)(d+ex)} + \frac{3d^2\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(x^3*(a + b*x^2)^p)/(d + e*x)^2,x]
```

output
```
((a + b*x^2)^p*((-2*d^3*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b
)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-
(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x))
+ (3*d^2*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (
d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*(
(e*(Sqrt[-(a/b)] + x))/(d + e*x))^p) + e*((e*(a + b*x^2 - a/(1 + (b*x^2)/a
)^p))/(b + b*p) - (4*d*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1
 + (b*x^2)/a)^p)))/(2*e^4)
```

3.417.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {603, 25, 2185, 27, 719, 238, 237, 504, 334, 333, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^2}\,dx$$
$$\Big\downarrow \ 603$$
$$\frac{d^3(a+bx^2)^{p+1}}{e^2(d+ex)(ae^2+bd^2)} - \frac{\int -\frac{(bx^2+a)^p\left(\frac{ad^2}{e}-\left(\frac{2b(p+1)d^2}{e^2}+a\right)xd+\frac{(bd^2+ae^2)x^2}{e}\right)}{d+ex}dx}{ae^2+bd^2}$$
$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(bx^2+a)^p\left(\frac{ad^2}{e}-\left(\frac{2b(p+1)d^2}{e^2}+a\right)xd+\frac{(bd^2+ae^2)x^2}{e}\right)}{d+ex}dx}{ae^2+bd^2}+\frac{d^3(a+bx^2)^{p+1}}{e^2(d+ex)(ae^2+bd^2)}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\frac{\int \frac{2bd(p+1)\left(ade-\left(b(2p+3)d^2+2ae^2\right)x\right)(bx^2+a)^p}{d+ex}dx}{2be^2(p+1)}+\frac{(ae^2+bd^2)(a+bx^2)^{p+1}}{2be^2(p+1)}}{ae^2+bd^2}+\frac{d^3(a+bx^2)^{p+1}}{e^2(d+ex)(ae^2+bd^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{d\int \frac{\left(ade-\left(b(2p+3)d^2+2ae^2\right)x\right)(bx^2+a)^p}{d+ex}dx}{e^2}+\frac{(ae^2+bd^2)(a+bx^2)^{p+1}}{2be^2(p+1)}}{ae^2+bd^2}+\frac{d^3(a+bx^2)^{p+1}}{e^2(d+ex)(ae^2+bd^2)}$$

$$\Big\downarrow \text{719}$$

$$\frac{\frac{d\left(\frac{d(3ae^2+bd^2(2p+3))\int \frac{(bx^2+a)^p}{d+ex}dx}{e}-\frac{(2ae^2+bd^2(2p+3))\int (bx^2+a)^pdx}{e}\right)}{e^2}+\frac{(ae^2+bd^2)(a+bx^2)^{p+1}}{2be^2(p+1)}}{ae^2+bd^2}+\frac{d^3(a+bx^2)^{p+1}}{e^2(d+ex)(ae^2+bd^2)}$$

$$\Big\downarrow \text{238}$$

$$\frac{\frac{d\left(\frac{d(3ae^2+bd^2(2p+3))\int \frac{(bx^2+a)^p}{d+ex}dx}{e}-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(2ae^2+bd^2(2p+3))\int \left(\frac{bx^2}{a}+1\right)^pdx}{e}\right)}{e^2}+\frac{(ae^2+bd^2)(a+bx^2)^{p+1}}{2be^2(p+1)}}{ae^2+bd^2}+\frac{d^3(a+bx^2)^{p+1}}{e^2(d+ex)(ae^2+bd^2)}$$

$$\Big\downarrow \text{237}$$

$$\frac{\frac{d\left(\frac{d(3ae^2+bd^2(2p+3))\int \frac{(bx^2+a)^p}{d+ex}dx}{e}-\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(2ae^2+bd^2(2p+3))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e}\right)}{e^2}+\frac{(ae^2+bd^2)(a+bx^2)^{p+1}}{2be^2(p+1)}}{ae^2+bd^2}+\frac{d^3(a+bx^2)^{p+1}}{e^2(d+ex)(ae^2+bd^2)}$$

$$\Big\downarrow \text{504}$$

3.417. $\int \frac{x^3(a+bx^2)^p}{(d+ex)^2}\,dx$

$$\frac{\frac{d\left(\frac{d\left(3ae^2+bd^2(2p+3)\right)\left(d\int\frac{\left(bx^2+a\right)^p}{d^2-e^2x^2}dx-e\int\frac{x\left(bx^2+a\right)^p}{d^2-e^2x^2}dx\right)}{e}-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(2ae^2+bd^2(2p+3)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e}\right)}{e^2}+\frac{(ae}{}}{ae^2+bd^2}$$

$$\frac{d^3\left(a+bx^2\right)^{p+1}}{e^2(d+ex)\left(ae^2+bd^2\right)}$$

$\downarrow$ 334

$$\frac{\frac{d\left(\frac{d\left(3ae^2+bd^2(2p+3)\right)\left(d\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int\frac{\left(\frac{bx^2}{a}+1\right)^p}{d^2-e^2x^2}dx-e\int\frac{x\left(bx^2+a\right)^p}{d^2-e^2x^2}dx\right)}{e}-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(2ae^2+bd^2(2p+3)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\right.}{e}}{e^2}}{ae^2+bd^2}$$

$$\frac{d^3\left(a+bx^2\right)^{p+1}}{e^2(d+ex)\left(ae^2+bd^2\right)}$$

$\downarrow$ 333

$$\frac{\frac{d\left(\frac{d\left(3ae^2+bd^2(2p+3)\right)\left(\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-e\int\frac{x\left(bx^2+a\right)^p}{d^2-e^2x^2}dx\right)}{e}-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(2ae^2+bd^2(2p+3)\right)\text{Hyp}}{e}}{e^2}}{ae^2+bd^2}$$

$$\frac{d^3\left(a+bx^2\right)^{p+1}}{e^2(d+ex)\left(ae^2+bd^2\right)}$$

$\downarrow$ 353

$$\frac{\frac{d\left(\frac{d\left(3ae^2+bd^2(2p+3)\right)\left(\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{1}{2}e\int\frac{\left(bx^2+a\right)^p}{d^2-e^2x^2}dx^2\right)}{e}-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(2ae^2+bd^2(2p+3)\right)\text{Hy}}{e}}{e^2}}{ae^2+bd^2}$$

$$\frac{d^3\left(a+bx^2\right)^{p+1}}{e^2(d+ex)\left(ae^2+bd^2\right)}$$

$\downarrow$ 78

3.417. $\int\frac{x^3\left(a+bx^2\right)^p}{(d+ex)^2}dx$

$$\frac{\frac{d\left(\frac{d\left(3ae^2+bd^2(2p+3)\right)\left(\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{e\left(a+bx^2\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2\left(bx^2+a\right)}{bd^2+ae^2}\right)}{2(p+1)\left(ae^2+bd^2\right)}\right)}{e}-\frac{x\left(a+\cdots\right.}{}\right)}{e^2}}{ae^2+bd^2}$$

$$\frac{d^3\left(a+bx^2\right)^{p+1}}{e^2(d+ex)\left(ae^2+bd^2\right)}$$

input
```
Int[(x^3*(a + b*x^2)^p)/(d + e*x)^2,x]
```

output
```
(d^3*(a + b*x^2)^(1 + p))/(e^2*(b*d^2 + a*e^2)*(d + e*x)) + (((b*d^2 + a*e
^2)*(a + b*x^2)^(1 + p))/(2*b*e^2*(1 + p)) + (d*(-(((2*a*e^2 + b*d^2*(3 +
2*p))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e*(1
 + (b*x^2)/a)^p)) + (d*(3*a*e^2 + b*d^2*(3 + 2*p))*((x*(a + b*x^2)^p*Appel
lF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d*(1 + (b*x^2)/a)^p) -
 (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2
))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)*(1 + p))))/e))/e^2)/(b*d^2 + a*e^2
)
```

3.417.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

3.417. $\int \frac{x^3\left(a+bx^2\right)^p}{(d+ex)^2}\,dx$

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 603
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[x^m, c + d*x, x], R = PolynomialRemainde
r[x^m, c + d*x, x]}, Simp[d*R*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n +
1)*(b*c^2 + a*d^2))), x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)
^(n + 1)*(a + b*x^2)^p*ExpandToSum[(n + 1)*(b*c^2 + a*d^2)*Qx + b*c*R*(n +
1) - b*d*R*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IGt
Q[m, 1] && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.417. $\int \frac{x^3(a+bx^2)^p}{(d+ex)^2}\,dx$

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.417.4 Maple [F]

$$\int \frac{x^3(b\,x^2+a)^p}{(ex+d)^2}dx$$

input
```
int(x^3*(b*x^2+a)^p/(e*x+d)^2,x)
```

output
```
int(x^3*(b*x^2+a)^p/(e*x+d)^2,x)
```

3.417.5 Fricas [F]

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x^3}{(ex+d)^2}\,dx$$

input
```
integrate(x^3*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^3/(e^2*x^2 + 2*d*e*x + d^2), x)
```

3.417.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(b*x**2+a)**p/(e*x+d)**2,x)
```

output
```
Timed out
```

3.417.7 Maxima [F]

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x^3}{(ex+d)^2}\,dx$$

input
```
integrate(x^3*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^3/(e*x + d)^2, x)
```

3.417.8 Giac [F]

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x^3}{(ex+d)^2}\,dx$$

input
```
integrate(x^3*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^3/(e*x + d)^2, x)
```

3.417.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{x^3\,(b\,x^2+a)^p}{(d+e\,x)^2}\,dx$$

input
```
int((x^3*(a + b*x^2)^p)/(d + e*x)^2,x)
```

output
```
int((x^3*(a + b*x^2)^p)/(d + e*x)^2, x)
```

3.418 $\int \frac{x^2(a+bx^2)^p}{(d+ex)^2} dx$

3.418.1 Optimal result

Integrand size = 20, antiderivative size = 281

$$\begin{aligned}\int \frac{x^2(a+bx^2)^p}{(d+ex)^2}\,dx &= -\frac{d^2(a+bx^2)^{1+p}}{e(bd^2+ae^2)(d+ex)} \\ &\quad - \frac{2(ae^2+bd^2(1+p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^2(bd^2+ae^2)} \\ &\quad + \frac{(ae^2+2bd^2(1+p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e^2(bd^2+ae^2)} \\ &\quad + \frac{d(ae^2+bd^2(1+p))\,(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{e(bd^2+ae^2)^2(1+p)}\end{aligned}$$

output
```
-d^2*(b*x^2+a)^(p+1)/e/(a*e^2+b*d^2)/(e*x+d)-2*(a*e^2+b*d^2*(p+1))*x*(b*x^
2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/e^2/(a*e^2+b*d^2)/((1+b
*x^2/a)^p)+(a*e^2+2*b*d^2*(p+1))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-
b*x^2/a)/e^2/(a*e^2+b*d^2)/((1+b*x^2/a)^p)+d*(a*e^2+b*d^2*(p+1))*(b*x^2+a)
^(p+1)*hypergeom([1, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e/(a*e^2+b*d^
2)^2/(p+1)
```

3.418.2 Mathematica [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.07

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^2}\,dx$$
$$= \frac{(a+bx^2)^p\left(\frac{d^2\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+2p)(d+ex)}-\frac{d\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\ldots}\right.}{d-\ldots}\right.}{\ldots}\right.}{e^3}$$

input
```
Integrate[(x^2*(a + b*x^2)^p)/(d + e*x)^2,x]
```

output
```
((a + b*x^2)^p*((d^2*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*
e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/
b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) - (
d*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqr
t[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqr
t[-(a/b)] + x))/(d + e*x))^p) + (e*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*
x^2)/a)])/(1 + (b*x^2)/a)^p))/e^3
```

3.418.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {603, 27, 719, 238, 237, 504, 334, 333, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^2}\,dx$$
$$\Big\downarrow \text{603}$$
$$-\frac{\int \frac{(ade-(2b(p+1)d^2+ae^2)x)(bx^2+a)^p}{e(d+ex)}dx}{ae^2+bd^2}-\frac{d^2(a+bx^2)^{p+1}}{e(d+ex)(ae^2+bd^2)}$$
$$\Big\downarrow \text{27}$$
$$-\frac{\int \frac{(ade-(2b(p+1)d^2+ae^2)x)(bx^2+a)^p}{d+ex}dx}{e(ae^2+bd^2)}-\frac{d^2(a+bx^2)^{p+1}}{e(d+ex)(ae^2+bd^2)}$$

$$\Big\downarrow \; 719$$

$$-\frac{\frac{2d(ae^2+bd^2(p+1))\int \frac{(bx^2+a)^p}{d+ex}dx}{e}-\frac{(ae^2+2bd^2(p+1))\int (bx^2+a)^p dx}{e}}{e\left(ae^2+bd^2\right)}-\frac{d^2\left(a+bx^2\right)^{p+1}}{e(d+ex)\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \; 238$$

$$-\frac{\frac{2d(ae^2+bd^2(p+1))\int \frac{(bx^2+a)^p}{d+ex}dx}{e}-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ae^2+2bd^2(p+1))\int \left(\frac{bx^2}{a}+1\right)^p dx}{e}}{e\left(ae^2+bd^2\right)}-\frac{d^2\left(a+bx^2\right)^{p+1}}{e(d+ex)\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \; 237$$

$$-\frac{\frac{2d(ae^2+bd^2(p+1))\int \frac{(bx^2+a)^p}{d+ex}dx}{e}-\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ae^2+2bd^2(p+1))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e}}{e\left(ae^2+bd^2\right)}-\frac{d^2\left(a+bx^2\right)^{p+1}}{e(d+ex)\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \; 504$$

$$-\frac{\frac{2d(ae^2+bd^2(p+1))\left(d\int \frac{(bx^2+a)^p}{d^2-e^2x^2}dx-e\int \frac{x(bx^2+a)^p}{d^2-e^2x^2}dx\right)}{e}-\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ae^2+2bd^2(p+1))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e}}{e\left(ae^2+bd^2\right)}-\frac{d^2\left(a+bx^2\right)^{p+1}}{e(d+ex)\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \; 334$$

$$-\frac{\frac{2d(ae^2+bd^2(p+1))\left(d(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^2}{a}+1\right)^p}{d^2-e^2x^2}dx-e\int \frac{x(bx^2+a)^p}{d^2-e^2x^2}dx\right)}{e}-\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ae^2+2bd^2(p+1))\,\text{Hypergeometr}}{e}}{e\left(ae^2+bd^2\right)}$$

$$\frac{d^2\left(a+bx^2\right)^{p+1}}{e(d+ex)\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \; 333$$

$$-\frac{\frac{2d(ae^2+bd^2(p+1))\left(\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-e\int \frac{x(bx^2+a)^p}{d^2-e^2x^2}dx\right)}{e}-\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ae^2+2bd^2(p+1))}{e}}{e\left(ae^2+bd^2\right)}$$

$$\frac{d^2\left(a+bx^2\right)^{p+1}}{e(d+ex)\left(ae^2+bd^2\right)}$$

3.418. $\displaystyle\int \frac{x^2(a+bx^2)^p}{(d+ex)^2}\,dx$

$$\Big\downarrow \text{353}$$

$$-\frac{\frac{2d\left(ae^2+bd^2(p+1)\right)\left(\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{1}{2}e\int\frac{\left(bx^2+a\right)^p}{d^2-e^2x^2}dx^2\right)}{e}-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+2bd^2(p+1\right.}{e}}{e\left(ae^2+bd^2\right)}$$

$$\frac{d^2\left(a+bx^2\right)^{p+1}}{e(d+ex)\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \text{78}$$

$$-\frac{\frac{2d\left(ae^2+bd^2(p+1)\right)\left(\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{e\left(a+bx^2\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2\left(bx^2+a\right)}{bd^2+ae^2}\right)}{2(p+1)\left(ae^2+bd^2\right)}\right)}{e}-\frac{x(}{}}{e\left(ae^2+bd^2\right)}$$

$$\frac{d^2\left(a+bx^2\right)^{p+1}}{e(d+ex)\left(ae^2+bd^2\right)}$$

input
```
Int[(x^2*(a + b*x^2)^p)/(d + e*x)^2,x]
```

output
```
-((d^2*(a + b*x^2)^(1 + p))/(e*(b*d^2 + a*e^2)*(d + e*x))) - (-(((a*e^2 +
2*b*d^2*(1 + p))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)
/a)])/(e*(1 + (b*x^2)/a)^p)) + (2*d*(a*e^2 + b*d^2*(1 + p))*((x*(a + b*x^2
)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d*(1 + (b*x^2
)/a)^p) - (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(
a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)*(1 + p))))/e)/(e*(b*d^2 +
 a*e^2))
```

3.418.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

3.418. $\int \frac{x^2(a+bx^2)^p}{(d+ex)^2}\,dx$

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 603
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[x^m, c + d*x, x], R = PolynomialRemainde
r[x^m, c + d*x, x]}, Simp[d*R*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n +
1)*(b*c^2 + a*d^2))), x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)
^(n + 1)*(a + b*x^2)^p*ExpandToSum[(n + 1)*(b*c^2 + a*d^2)*Qx + b*c*R*(n +
1) - b*d*R*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IGt
Q[m, 1] && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

3.418. $\int \frac{x^2(a+bx^2)^p}{(d+ex)^2} dx$

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

3.418.4 Maple [F]

$$\int \frac{x^{2} \left(b \,x^{2}+a \right)^{p}}{\left(e x +d \right)^{2}}d x$$

input
```
int(x^2*(b*x^2+a)^p/(e*x+d)^2,x)
```

output
```
int(x^2*(b*x^2+a)^p/(e*x+d)^2,x)
```

3.418.5 Fricas [F]

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x^2}{(ex+d)^2}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^2/(e^2*x^2 + 2*d*e*x + d^2), x)
```

3.418.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(b*x**2+a)**p/(e*x+d)**2,x)
```

output
```
Timed out
```

3.418.7 Maxima [F]

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x^2}{(ex+d)^2}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^2/(e*x + d)^2, x)
```

3.418.8 Giac [F]

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x^2}{(ex+d)^2}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^2/(e*x + d)^2, x)
```

3.418.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{x^2\,(b\,x^2+a)^p}{(d+e\,x)^2}\,dx$$

input
```
int((x^2*(a + b*x^2)^p)/(d + e*x)^2,x)
```

output
```
int((x^2*(a + b*x^2)^p)/(d + e*x)^2, x)
```

3.419 $\int \frac{x(a+bx^2)^p}{(d+ex)^2}\,dx$

3.419.1 Optimal result

Integrand size = 18, antiderivative size = 273

$$\begin{aligned}
&\int \frac{x(a+bx^2)^p}{(d+ex)^2}\,dx \\
&= \frac{d(a+bx^2)^{1+p}}{(bd^2+ae^2)(d+ex)} \\
&+ \frac{(ae^2+bd^2(1+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{de\,(bd^2+ae^2)} \\
&- \frac{bd(1+2p)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e\,(bd^2+ae^2)} \\
&- \frac{(ae^2+bd^2(1+2p))\,(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2\,(bd^2+ae^2)^2\,(1+p)}
\end{aligned}$$

output
```
d*(b*x^2+a)^(p+1)/(a*e^2+b*d^2)/(e*x+d)+(a*e^2+b*d^2*(1+2*p))*x*(b*x^2+a)^
p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d/e/(a*e^2+b*d^2)/((1+b*x^2/
a)^p)-b*d*(1+2*p)*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e/(a*e
^2+b*d^2)/((1+b*x^2/a)^p)-1/2*(a*e^2+b*d^2*(1+2*p))*(b*x^2+a)^(p+1)*hyperg
eom([1, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^2/(p+1)
```

3.419.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.82

$$\int \frac{x(a+bx^2)^p}{(d+ex)^2} dx$$
$$= \frac{\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}(a+bx^2)^p\left(-2dp\,\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)\right.}{2e^2p(-1+2p)(d+ex)}$$

input
```
Integrate[(x*(a + b*x^2)^p)/(d + e*x)^2,x]
```

output
```
((a + b*x^2)^p*(-2*d*p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)
]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)] + (-1 + 2*p)*(d + e*x)*App
ellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a
/b)]*e)/(d + e*x)]))/(2*e^2*p*(-1 + 2*p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x
))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x))
```

3.419.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {594, 25, 719, 238, 237, 504, 334, 333, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^2)^p}{(d+ex)^2} dx$$

$$\Big\downarrow \text{594}$$

$$\frac{d(a+bx^2)^{p+1}}{(d+ex)(ae^2+bd^2)} - \frac{\int -\frac{(ae-bd(2p+1)x)(bx^2+a)^p}{d+ex}dx}{ae^2+bd^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(ae-bd(2p+1)x)(bx^2+a)^p}{d+ex}dx}{ae^2+bd^2} + \frac{d(a+bx^2)^{p+1}}{(d+ex)(ae^2+bd^2)}$$

$$\Big\downarrow \text{719}$$

$$\frac{\frac{(ae^2+bd^2(2p+1))\int \frac{(bx^2+a)^p}{d+ex}dx}{e}-\frac{bd(2p+1)\int (bx^2+a)^p dx}{e}}{ae^2+bd^2}+\frac{d(a+bx^2)^{p+1}}{(d+ex)(ae^2+bd^2)}$$

$\downarrow$ 238

$$\frac{\frac{(ae^2+bd^2(2p+1))\int \frac{(bx^2+a)^p}{d+ex}dx}{e}-\frac{bd(2p+1)(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \left(\frac{bx^2}{a}+1\right)^p dx}{e}}{ae^2+bd^2}+\frac{d(a+bx^2)^{p+1}}{(d+ex)(ae^2+bd^2)}$$

$\downarrow$ 237

$$\frac{\frac{(ae^2+bd^2(2p+1))\int \frac{(bx^2+a)^p}{d+ex}dx}{e}-\frac{bd(2p+1)x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e}}{ae^2+bd^2}+\frac{d(a+bx^2)^{p+1}}{(d+ex)(ae^2+bd^2)}$$

$\downarrow$ 504

$$\frac{\frac{(ae^2+bd^2(2p+1))\left(d\int \frac{(bx^2+a)^p}{d^2-e^2x^2}dx-e\int \frac{x(bx^2+a)^p}{d^2-e^2x^2}dx\right)}{e}-\frac{bd(2p+1)x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e}}{ae^2+bd^2}+\frac{d(a+bx^2)^{p+1}}{(d+ex)(ae^2+bd^2)}$$

$\downarrow$ 334

$$\frac{\frac{(ae^2+bd^2(2p+1))\left(d(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^2}{a}+1\right)^p}{d^2-e^2x^2}dx-e\int \frac{x(bx^2+a)^p}{d^2-e^2x^2}dx\right)}{e}-\frac{bd(2p+1)x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p\right.}{e}}{ae^2+bd^2}$$
$$\frac{d(a+bx^2)^{p+1}}{(d+ex)(ae^2+bd^2)}$$

$\downarrow$ 333

$$\frac{\frac{(ae^2+bd^2(2p+1))\left(\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-e\int \frac{x(bx^2+a)^p}{d^2-e^2x^2}dx\right)}{e}-\frac{bd(2p+1)x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeom}}{e}}{ae^2+bd^2}$$
$$\frac{d(a+bx^2)^{p+1}}{(d+ex)(ae^2+bd^2)}$$

$\downarrow$ 353

3.419. $\int \frac{x(a+bx^2)^p}{(d+ex)^2}\,dx$

$$\frac{\frac{(ae^2+bd^2(2p+1))\left(\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{1}{2}e\int\frac{(bx^2+a)^p}{d^2-e^2x^2}dx^2\right)}{e}-\frac{bd(2p+1)x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeo}}{e}}{ae^2+bd^2}$$

$$\frac{d(a+bx^2)^{p+1}}{(d+ex)(ae^2+bd^2)}$$

$\Big\downarrow$ 78

$$\frac{\frac{(ae^2+bd^2(2p+1))\left(\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{e(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)}\right)}{e}-\frac{bd(2p\cdots}{}}{ae^2+bd^2}$$

$$\frac{d(a+bx^2)^{p+1}}{(d+ex)(ae^2+bd^2)}$$

input
```
Int[(x*(a + b*x^2)^p)/(d + e*x)^2,x]
```

output
```
(d*(a + b*x^2)^(1 + p))/((b*d^2 + a*e^2)*(d + e*x)) + (-((b*d*(1 + 2*p)*x*
(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e*(1 + (b*x^
2)/a)^p)) + ((a*e^2 + b*d^2*(1 + 2*p))*((x*(a + b*x^2)^p*AppellF1[1/2, -p,
 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d*(1 + (b*x^2)/a)^p) - (e*(a + b*x
^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 +
a*e^2)])/(2*(b*d^2 + a*e^2)*(1 + p))))/e)/(b*d^2 + a*e^2)
```

3.419.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

3.419. $\int \frac{x(a+bx^2)^p}{(d+ex)^2}\,dx$

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.419. $\int \frac{x(a+bx^2)^p}{(d+ex)^2} \, dx$

3.419.4 Maple [F]

$$\int \frac{x(b\,x^2+a)^p}{(ex+d)^2}dx$$

input
```
int(x*(b*x^2+a)^p/(e*x+d)^2,x)
```

output
```
int(x*(b*x^2+a)^p/(e*x+d)^2,x)
```

3.419.5 Fricas [F]

$$\int \frac{x(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x}{(ex+d)^2}\,dx$$

input
```
integrate(x*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x/(e^2*x^2 + 2*d*e*x + d^2), x)
```

3.419.6 Sympy [F]

$$\int \frac{x(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{x(a+bx^2)^p}{(d+ex)^2}\,dx$$

input
```
integrate(x*(b*x**2+a)**p/(e*x+d)**2,x)
```

output
```
Integral(x*(a + b*x**2)**p/(d + e*x)**2, x)
```

3.419.7 Maxima [F]

$$\int \frac{x(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x}{(ex+d)^2}\,dx$$

input
```
integrate(x*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x/(e*x + d)^2, x)
```

3.419.8 Giac [F]

$$\int \frac{x(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x}{(ex+d)^2}\,dx$$

input
```
integrate(x*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x/(e*x + d)^2, x)
```

3.419.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{x\,(b\,x^2+a)^p}{(d+e\,x)^2}\,dx$$

input
```
int((x*(a + b*x^2)^p)/(d + e*x)^2,x)
```

output
```
int((x*(a + b*x^2)^p)/(d + e*x)^2, x)
```

3.420 $\int \frac{(a+bx^2)^p}{(d+ex)^2} dx$

3.420.1 Optimal result

Integrand size = 17, antiderivative size = 244

$$\int \frac{(a+bx^2)^p}{(d+ex)^2} dx = \frac{e^2x(a+bx^2)^{1+p}}{(bd^2+ae^2)(d^2-e^2x^2)} - \frac{2bpx(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{bd^2+ae^2} + \frac{b(1+2p)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{bd^2+ae^2} - \frac{bde(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(2,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{(bd^2+ae^2)^2(1+p)}$$

output
```
e^2*x*(b*x^2+a)^(p+1)/(a*e^2+b*d^2)/(-e^2*x^2+d^2)-2*b*p*x*(b*x^2+a)^p*App
ellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/(a*e^2+b*d^2)/((1+b*x^2/a)^p)+b*(
1+2*p)*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/(a*e^2+b*d^2)/((1
+b*x^2/a)^p)-b*d*e*(b*x^2+a)^(p+1)*hypergeom([2, p+1],[2+p],e^2*(b*x^2+a)/
(a*e^2+b*d^2))/(a*e^2+b*d^2)^2/(p+1)
```

3.420.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx$$
$$= \frac{\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}(a+bx^2)^p\,\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{e(-1+2p)(d+ex)}$$

input
```
Integrate[(a + b*x^2)^p/(d + e*x)^2,x]
```

output
```
((a + b*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*e)/(d
+ e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(e*(-1 + 2*p)*((e*(-Sqrt[-(a/b)]
+ x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x))
```

3.420.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.78, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {505, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx$$
$$\Big\downarrow \text{505}$$
$$\int \left(\frac{d^2(a+bx^2)^p}{(d^2-e^2x^2)^2}-\frac{2dex(a+bx^2)^p}{(d^2-e^2x^2)^2}+\frac{e^2x^2(a+bx^2)^p}{(e^2x^2-d^2)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}+\frac{e^2x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,2,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^4}-\frac{bde(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{(p+1)(ae^2+bd^2)^2}$$

input
```
Int[(a + b*x^2)^p/(d + e*x)^2,x]
```

output
```
(x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(
d^2*(1 + (b*x^2)/a)^p) + (e^2*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 2, 5/2,
-((b*x^2)/a), (e^2*x^2)/d^2])/(3*d^4*(1 + (b*x^2)/a)^p) - (b*d*e*(a + b*x^
2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a
*e^2)])/((b*d^2 + a*e^2)^2*(1 + p))
```

3.420.3.1 Defintions of rubi rules used

rule 505
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2 - d^2*x^2)))^(
-n), x], x] /; FreeQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.420.4 Maple [F]

$$\int \frac{(b\,x^2 + a)^p}{(ex + d)^2}dx$$

input
```
int((b*x^2+a)^p/(e*x+d)^2,x)
```

output
```
int((b*x^2+a)^p/(e*x+d)^2,x)
```

3.420.5 Fricas [F]

$$\int \frac{(a + bx^2)^p}{(d + ex)^2}\,dx = \int \frac{(bx^2 + a)^p}{(ex + d)^2}\,dx$$

input
```
integrate((b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/(e^2*x^2 + 2*d*e*x + d^2), x)
```

3.420.6 Sympy [F]

$$\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx$$

input
```
integrate((b*x**2+a)**p/(e*x+d)**2,x)
```

output
```
Integral((a + b*x**2)**p/(d + e*x)**2, x)
```

3.420.7 Maxima [F]

$$\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^2}\,dx$$

input
```
integrate((b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/(e*x + d)^2, x)
```

3.420.8 Giac [F]

$$\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^2}\,dx$$

input
```
integrate((b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/(e*x + d)^2, x)
```

3.420.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx=\int \frac{(b\,x^2+a)^p}{(d+e\,x)^2}\,dx$$

input
```
int((a + b*x^2)^p/(d + e*x)^2,x)
```

output
```
int((a + b*x^2)^p/(d + e*x)^2, x)
```

3.421 $\int \frac{(a+bx^2)^p}{x(d+ex)^2}\,dx$

3.421.1 Optimal result

Integrand size = 20, antiderivative size = 368

$$\begin{aligned}\int \frac{(a+bx^2)^p}{x(d+ex)^2}\,dx = & -\frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}\\ & -\frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}\\ & -\frac{e^3x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},-p,2,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^5}\\ & +\frac{e^2(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2d^2(bd^2+ae^2)(1+p)}\\ & -\frac{(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2ad^2(1+p)}\\ & +\frac{be^2(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(2,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{(bd^2+ae^2)^2(1+p)}\end{aligned}$$

output
```
-e*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^3/((1+b*x^2
/a)^p)-e*x*(b*x^2+a)^p*AppellF1(1/2,2,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^3/((1
+b*x^2/a)^p)-1/3*e^3*x^3*(b*x^2+a)^p*AppellF1(3/2,2,-p,5/2,e^2*x^2/d^2,-b*
x^2/a)/d^5/((1+b*x^2/a)^p)+1/2*e^2*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p
],e^2*(b*x^2+a)/(a*e^2+b*d^2))/d^2/(a*e^2+b*d^2)/(p+1)-1/2*(b*x^2+a)^(p+1)
*hypergeom([1, p+1],[2+p],1+b*x^2/a)/a/d^2/(p+1)+b*e^2*(b*x^2+a)^(p+1)*hyp
ergeom([2, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^2/(p+1)
```

3.421.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^p}{x(d+ex)^2}\,dx = \frac{(a+bx^2)^p\left(-\frac{2d\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+2p)(d+ex)}+\frac{-\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e(\cdots}{}\right.}{}\right.}{2d^2}$$

input
```
Integrate[(a + b*x^2)^p/(x*(d + e*x)^2),x]
```

output
```
((a + b*x^2)^p*((-2*d*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]
*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a
/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) +
(-(AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sq
rt[-(a/b)]*e)/(d + e*x)]/(((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[
-(a/b)] + x))/(d + e*x))^p)) + Hypergeometric2F1[-p, -p, 1 - p, -(a/(b*x^2
))]/(1 + a/(b*x^2))^p)/p))/(2*d^2)
```

3.421.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.83, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {622, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{x(d+ex)^2}\,dx$$

$$\Big\downarrow \text{622}$$

$$\int \left(-\frac{2de(a+bx^2)^p}{(d^2-e^2x^2)^2}+\frac{d^2(a+bx^2)^p}{x(d^2-e^2x^2)^2}+\frac{e^2x(a+bx^2)^p}{(e^2x^2-d^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}+\frac{e^2(a+bx^2)^{p+1}(ae^2+bd^2(1-p))\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^2(p+1)(ae^2+bd^2)^2}+\frac{be^2(a+bx^2)^{p+1}\,\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)^2}+\frac{e^2(a+bx^2)^{p+1}}{2(d^2-e^2x^2)(ae^2+bd^2)}-\frac{(a+bx^2)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{2ad^2(p+1)}$$

input
```
Int[(a + b*x^2)^p/(x*(d + e*x)^2),x]
```

output
```
(e^2*(a + b*x^2)^(1 + p))/(2*(b*d^2 + a*e^2)*(d^2 - e^2*x^2)) - (2*e*x*(a
+ b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^3*(1
 + (b*x^2)/a)^p) + (e^2*(a*e^2 + b*d^2*(1 - p))*(a + b*x^2)^(1 + p)*Hyperg
eometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*d^2*(b
*d^2 + a*e^2)^2*(1 + p)) - ((a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p
, 2 + p, 1 + (b*x^2)/a])/(2*a*d^2*(1 + p)) + (b*e^2*(a + b*x^2)^(1 + p)*Hy
pergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b
*d^2 + a*e^2)^2*(1 + p))
```

3.421.3.1 Defintions of rubi rules used

rule 622
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[x^m*(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2
 - d^2*x^2)))^(-n), x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[n, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.421. $\int \frac{(a+bx^2)^p}{x(d+ex)^2}\,dx$

3.421.4 Maple [F]

$$\int \frac{(b\,x^2+a)^p}{x\,(ex+d)^2}dx$$

input
```
int((b*x^2+a)^p/x/(e*x+d)^2,x)
```

output
```
int((b*x^2+a)^p/x/(e*x+d)^2,x)
```

3.421.5 Fricas [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^2x}\,dx$$

input
```
integrate((b*x^2+a)^p/x/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/(e^2*x^3 + 2*d*e*x^2 + d^2*x), x)
```

3.421.6 Sympy [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)^2}\,dx = \int \frac{(a+bx^2)^p}{x\,(d+ex)^2}\,dx$$

input
```
integrate((b*x**2+a)**p/x/(e*x+d)**2,x)
```

output
```
Integral((a + b*x**2)**p/(x*(d + e*x)**2), x)
```

3.421.7 Maxima [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^2x}\,dx$$

input
```
integrate((b*x^2+a)^p/x/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/((e*x + d)^2*x), x)
```

3.421.8 Giac [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^2x}\,dx$$

input
```
integrate((b*x^2+a)^p/x/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/((e*x + d)^2*x), x)
```

3.421.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x(d+ex)^2}\,dx = \int \frac{(b\,x^2+a)^p}{x\,(d+e\,x)^2}\,dx$$

input
```
int((a + b*x^2)^p/(x*(d + e*x)^2),x)
```

output
```
int((a + b*x^2)^p/(x*(d + e*x)^2), x)
```

3.422 $\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx$

3.422.1 Optimal result

Integrand size = 20, antiderivative size = 421

$$\begin{aligned}\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx &= \frac{2e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}\\
&\quad+\frac{e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}\\
&\quad+\frac{e^4x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},-p,2,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^6}\\
&\quad-\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{d^2x}\\
&\quad-\frac{e^3(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{d^3(bd^2+ae^2)(1+p)}\\
&\quad+\frac{e(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{ad^3(1+p)}\\
&\quad-\frac{be^3(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(2,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{d(bd^2+ae^2)^2(1+p)}\end{aligned}$$

output
```
2*e^2*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^4/((1+b*
x^2/a)^p)+e^2*x*(b*x^2+a)^p*AppellF1(1/2,2,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^
4/((1+b*x^2/a)^p)+1/3*e^4*x^3*(b*x^2+a)^p*AppellF1(3/2,2,-p,5/2,e^2*x^2/d^
2,-b*x^2/a)/d^6/((1+b*x^2/a)^p)-(b*x^2+a)^p*hypergeom([-1/2, -p],[1/2],-b*
x^2/a)/d^2/x/((1+b*x^2/a)^p)-e^3*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],
e^2*(b*x^2+a)/(a*e^2+b*d^2))/d^3/(a*e^2+b*d^2)/(p+1)+e*(b*x^2+a)^(p+1)*hyp
ergeom([1, p+1],[2+p],1+b*x^2/a)/a/d^3/(p+1)-b*e^3*(b*x^2+a)^(p+1)*hyperge
om([2, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/d/(a*e^2+b*d^2)^2/(p+1)
```

3.422.2 Mathematica [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx$$

$$= \frac{(a+bx^2)^p \left(\frac{de\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+2p)(d+ex)} + \frac{e\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\right.}\right.}{d+}\right.}{}}{}$$

input
```
Integrate[(a + b*x^2)^p/(x^2*(d + e*x)^2),x]
```

output
```
((a + b*x^2)^p*((d*e*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*
e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/
b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) + (
e*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqr
t[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqr
t[-(a/b)] + x))/(d + e*x))^p) - (d*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x
^2)/a)])/(x*(1 + (b*x^2)/a)^p) - (e*Hypergeometric2F1[-p, -p, 1 - p, -(a/(
b*x^2))])/(p*(1 + a/(b*x^2))^p)))/d^3
```

3.422.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.70, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {622, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^2} \, dx$$

$$\Big\downarrow \text{622}$$

$$\int \left(-\frac{2de(a+bx^2)^p}{x(d^2-e^2x^2)^2}+\frac{d^2(a+bx^2)^p}{x^2(d^2-e^2x^2)^2}+\frac{e^2(a+bx^2)^p}{(e^2x^2-d^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(-\frac{1}{2},-p,2,\frac{1}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x}+\frac{e^2x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}+\frac{e(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{ad^3(p+1)}-\frac{e^3(a+bx^2)^{p+1}}{d(d^2-e^2x^2)(ae^2+bd^2)}-\frac{e^3(a+bx^2)^{p+1}(ae^2+bd^2(1-p))\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{d^3(p+1)(ae^2+bd^2)^2}$$

input
```
Int[(a + b*x^2)^p/(x^2*(d + e*x)^2),x]
```

output
```
-((e^3*(a + b*x^2)^(1 + p))/(d*(b*d^2 + a*e^2)*(d^2 - e^2*x^2))) - ((a + b
*x^2)^p*AppellF1[-1/2, -p, 2, 1/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^2*x*(1
 + (b*x^2)/a)^p) + (e^2*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2
)/a), (e^2*x^2)/d^2])/(d^4*(1 + (b*x^2)/a)^p) - (e^3*(a*e^2 + b*d^2*(1 - p
))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2)
)/(b*d^2 + a*e^2)])/(d^3*(b*d^2 + a*e^2)^2*(1 + p)) + (e*(a + b*x^2)^(1 +
p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(a*d^3*(1 + p))
```

3.422.3.1 Defintions of rubi rules used

rule 622
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[x^m*(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2
 - d^2*x^2)))^(-n), x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[n, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.422.4 Maple [F]

$$\int \frac{(b\,x^{2}+a)^{p}}{x^{2}\left(ex+d\right)^{2}}dx$$

input
```
int((b*x^2+a)^p/x^2/(e*x+d)^2,x)
```

output
```
int((b*x^2+a)^p/x^2/(e*x+d)^2,x)
```

3.422.5 Fricas [F]

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^2x^2}\,dx$$

input
```
integrate((b*x^2+a)^p/x^2/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/(e^2*x^4 + 2*d*e*x^3 + d^2*x^2), x)
```

3.422.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p/x**2/(e*x+d)**2,x)
```

output
```
Timed out
```

3.422.7 Maxima [F]

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^2x^2}\,dx$$

input
```
integrate((b*x^2+a)^p/x^2/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/((e*x + d)^2*x^2), x)
```

3.422.8 Giac [F]

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^2x^2}\,dx$$

input
```
integrate((b*x^2+a)^p/x^2/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/((e*x + d)^2*x^2), x)
```

3.422.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx = \int \frac{(b\,x^2+a)^p}{x^2\,(d+e\,x)^2}\,dx$$

input
```
int((a + b*x^2)^p/(x^2*(d + e*x)^2),x)
```

output
```
int((a + b*x^2)^p/(x^2*(d + e*x)^2), x)
```

3.423 $\int \frac{x^4(a+bx^2)^p}{(d+ex)^3}\,dx$

3.423.1 Optimal result

Integrand size = 20, antiderivative size = 449

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^3}\,dx$$
$$= \frac{(a+bx^2)^{1+p}}{2be^3(1+p)} - \frac{d^4(a+bx^2)^{1+p}}{2e^3(bd^2+ae^2)(d+ex)^2} + \frac{d^3(4ae^2+bd^2(3+p))(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)^2(d+ex)}$$
$$+ \frac{d(6a^2e^4+3abd^2e^2(4+3p)+b^2d^4(6+7p+2p^2))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\right.}{e^4(bd^2+ae^2)^2}$$
$$- \frac{d(3a^2e^4+2abd^2e^2(5+4p)+b^2d^4(6+7p+2p^2))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\right.}{e^4(bd^2+ae^2)^2}$$
$$- \frac{d^2(6a^2e^4+3abd^2e^2(4+3p)+b^2d^4(6+7p+2p^2))(a+bx^2)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(}{bd}\right.}{2e^3(bd^2+ae^2)^3(1+p)}$$

output
```
1/2*(b*x^2+a)^(p+1)/b/e^3/(p+1)-1/2*d^4*(b*x^2+a)^(p+1)/e^3/(a*e^2+b*d^2)/
(e*x+d)^2+d^3*(4*a*e^2+b*d^2*(3+p))*(b*x^2+a)^(p+1)/e^3/(a*e^2+b*d^2)^2/(e
*x+d)+d*(6*a^2*e^4+3*a*b*d^2*e^2*(4+3*p)+b^2*d^4*(2*p^2+7*p+6))*x*(b*x^2+a
)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/e^4/(a*e^2+b*d^2)^2/((1+b*
x^2/a)^p)-d*(3*a^2*e^4+2*a*b*d^2*e^2*(5+4*p)+b^2*d^4*(2*p^2+7*p+6))*x*(b*x
^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e^4/(a*e^2+b*d^2)^2/((1+b*x^2/
a)^p)-1/2*d^2*(6*a^2*e^4+3*a*b*d^2*e^2*(4+3*p)+b^2*d^4*(2*p^2+7*p+6))*(b*x
^2+a)^(p+1)*hypergeom([1, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e^3/(a*e
^2+b*d^2)^3/(p+1)
```

3.423.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.03

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^3}\,dx$$
$$= \frac{(a+bx^2)^p\left(\frac{ae^2}{b+bp}+\frac{e^2x^2}{1+p}-\frac{8d^3\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+2p)(d+ex)}+\frac{d^4\left(\frac{e(-\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(x^4*(a + b*x^2)^p)/(d + e*x)^3,x]
```

output
```
((a + b*x^2)^p*((a*e^2)/(b + b*p) + (e^2*x^2)/(1 + p) - (8*d^3*AppellF1[1
- 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*
e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt
[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) + (d^4*AppellF1[2 - 2*p, -p, -p, 3
- 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((
-1 + p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d +
 e*x))^p*(d + e*x)^2) + (6*d^2*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-
(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)
] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p) - (6*d*e*x*Hype
rgeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/(2*e^5)
```

3.423.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {603, 27, 2182, 2185, 27, 719, 238, 237, 504, 334, 333, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^3}\,dx$$
$$\Big\downarrow \text{603}$$
$$-\frac{\int \frac{2(bx^2+a)^p\left(\frac{ad^3}{e^2}-\frac{(b(p+1)d^2+ae^2)xd^2}{e^3}+\left(\frac{bd^2}{e^2}+a\right)x^2d-\left(\frac{bd^2}{e}+ae\right)x^3\right)}{(d+ex)^2}dx}{2(ae^2+bd^2)}-\frac{d^4(a+bx^2)^{p+1}}{2e^3(d+ex)^2(ae^2+bd^2)}$$

$\downarrow$ 27

$$-\frac{\int \frac{(bx^2+a)^p\left(\frac{ad^3}{e^2}-\frac{\left(b(p+1)d^2+ae^2\right)xd^2}{e^3}+\left(\frac{bd^2}{e^2}+a\right)x^2d-\frac{\left(bd^2+ae^2\right)x^3}{e}\right)}{(d+ex)^2}dx}{ae^2+bd^2}-\frac{d^4\left(a+bx^2\right)^{p+1}}{2e^3(d+ex)^2\left(ae^2+bd^2\right)}$$

$\downarrow$ 2182

$$-\frac{-\frac{\int \frac{\left(bx^2+a\right)^p\left(a\left(\frac{b(p+2)d^2}{e^2}+3a\right)d^2-\frac{\left(b^2\left(2p^2+7p+5\right)d^4+8abe^2(p+1)d^2+2a^2e^4\right)xd}{e^3}+\frac{\left(bd^2+ae^2\right)^2x^2}{e^2}\right)}{d+ex}dx}{ae^2+bd^2}-\frac{d^3\left(a+bx^2\right)^{p+1}\left(4ae^2+bd^2(p+3)\right)}{e^3(d+ex)(ae^2+bd^2)}}{ae^2+bd^2}-\frac{d^4\left(a+bx^2\right)^{p+1}}{2e^3(d+ex)^2\left(ae^2+bd^2\right)}$$

$\downarrow$ 2185

$$-\frac{-\frac{\frac{\int \frac{2bd(p+1)\left(ade\left(b(p+2)d^2+3ae^2\right)-\left(b^2\left(2p^2+7p+6\right)d^4+2abe^2(4p+5)d^2+3a^2e^4\right)x\right)\left(bx^2+a\right)^p}{e(d+ex)}dx}{2be^2(p+1)}+\frac{\left(ae^2+bd^2\right)^2\left(a+bx^2\right)^{p+1}}{2be^3(p+1)}}{ae^2+bd^2}-\frac{d^3\left(a+bx^2\right)^{p+1}\left(4ae^2+bd^2(p+3)\right)}{e^3(d+ex)(ae^2+bd^2)}}{ae^2+bd^2}-\frac{d^4\left(a+bx^2\right)^{p+1}}{2e^3(d+ex)^2\left(ae^2+bd^2\right)}$$

$\downarrow$ 27

$$-\frac{-\frac{\frac{d\int \frac{\left(ade\left(b(p+2)d^2+3ae^2\right)-\left(b^2\left(2p^2+7p+6\right)d^4+2abe^2(4p+5)d^2+3a^2e^4\right)x\right)\left(bx^2+a\right)^p}{d+ex}dx}{e^3}+\frac{\left(ae^2+bd^2\right)^2\left(a+bx^2\right)^{p+1}}{2be^3(p+1)}}{ae^2+bd^2}-\frac{d^3\left(a+bx^2\right)^{p+1}\left(4ae^2+bd^2(p+3)\right)}{e^3(d+ex)(ae^2+bd^2)}}{ae^2+bd^2}-\frac{d^4\left(a+bx^2\right)^{p+1}}{2e^3(d+ex)^2\left(ae^2+bd^2\right)}$$

$\downarrow$ 719

$$-\frac{-\frac{\frac{d\left(\frac{d\left(6a^2e^4+3abd^2e^2(3p+4)+b^2d^4\left(2p^2+7p+6\right)\right)\int \frac{\left(bx^2+a\right)^p}{d+ex}dx}{e}-\frac{\left(3a^2e^4+2abd^2e^2(4p+5)+b^2d^4\left(2p^2+7p+6\right)\right)\int \left(bx^2+a\right)^p dx}{e}\right)}{e^3}+\frac{\left(ae^2+bd^2\right)^2\left(a+bx^2\right)^{p+1}}{2be^3(p+1)}}{ae^2+bd^2}-\frac{d^3\left(a+bx^2\right)^{p+1}\left(4ae^2+bd^2(p+3)\right)}{e^3(d+ex)(ae^2+bd^2)}}{ae^2+bd^2}-\frac{d^4\left(a+bx^2\right)^{p+1}}{2e^3(d+ex)^2\left(ae^2+bd^2\right)}$$

$\downarrow$ 238

3.423. $\int \frac{x^4\left(a+bx^2\right)^p}{(d+ex)^3}\,dx$

$$-\frac{-\frac{\frac{d\left(\frac{d\left(6a^2e^4+3abd^2e^2(3p+4)+b^2d^4\left(2p^2+7p+6\right)\right)\int\frac{\left(bx^2+a\right)^p}{d+ex}dx}{e}-\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3a^2e^4+2abd^2e^2(4p+5)+b^2d^4\left(2p^2+7p+6\right)\right)\int\left(\frac{bx^2}{a}+1\right)^pdx}{e}\right)}{e^3}+}{ae^2+bd^2}}{ae^2+bd^2}$$

$$\frac{d^4\left(a+bx^2\right)^{p+1}}{2e^3(d+ex)^2\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \text{237}$$

$$-\frac{-\frac{\frac{d\left(\frac{d\left(6a^2e^4+3abd^2e^2(3p+4)+b^2d^4\left(2p^2+7p+6\right)\right)\int\frac{\left(bx^2+a\right)^p}{d+ex}dx}{e}-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3a^2e^4+2abd^2e^2(4p+5)+b^2d^4\left(2p^2+7p+6\right)\right)\text{Hypergeometric2F}}{e}}{e^3}}{ae^2+bd^2}}{ae^2+bd^2}$$

$$\frac{d^4\left(a+bx^2\right)^{p+1}}{2e^3(d+ex)^2\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \text{504}$$

$$-\frac{-\frac{\frac{d\left(\frac{d\left(6a^2e^4+3abd^2e^2(3p+4)+b^2d^4\left(2p^2+7p+6\right)\right)\left(d\int\frac{\left(bx^2+a\right)^p}{d^2-e^2x^2}dx-e\int\frac{x\left(bx^2+a\right)^p}{d^2-e^2x^2}dx\right)}{e}-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3a^2e^4+2abd^2e^2(4p+5)+b^2d^4\left(2p^2+7\right.\right.}{e}}{e^3}}{ae^2+bd^2}}{ae^2+bd^2}$$

$$\frac{d^4\left(a+bx^2\right)^{p+1}}{2e^3(d+ex)^2\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \text{334}$$

$$-\frac{-\frac{\frac{d\left(\frac{d\left(6a^2e^4+3abd^2e^2(3p+4)+b^2d^4\left(2p^2+7p+6\right)\right)\left(d\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int\frac{\left(\frac{bx^2}{a}+1\right)^p}{d^2-e^2x^2}dx-e\int\frac{x\left(bx^2+a\right)^p}{d^2-e^2x^2}dx\right)}{e}-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3a^2e^4+2abd\right.}{}}{e^3}}{ae^2+bd^2}}{ae^2+bd^2}$$

$$\frac{d^4\left(a+bx^2\right)^{p+1}}{2e^3(d+ex)^2\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \text{333}$$

3.423. $\int\frac{x^4\left(a+bx^2\right)^p}{(d+ex)^3}\,dx$

$$-\frac{-\frac{d\left(\frac{d\left(6a^2e^4+3abd^2e^2(3p+4)+b^2d^4\left(2p^2+7p+6\right)\right)\left(\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-e\int\frac{x\left(bx^2+a\right)^p}{d^2-e^2x^2}dx\right)}{e}-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+\right.}{}\right)}{e^3}}{ae^2+bd^2}}{ae^2}$$

$$\frac{d^4\left(a+bx^2\right)^{p+1}}{2e^3(d+ex)^2\left(ae^2+bd^2\right)}$$

$\downarrow$ 353

$$-\frac{-\frac{d\left(\frac{d\left(6a^2e^4+3abd^2e^2(3p+4)+b^2d^4\left(2p^2+7p+6\right)\right)\left(\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{1}{2}e\int\frac{\left(bx^2+a\right)^p}{d^2-e^2x^2}dx^2\right)}{e}-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}\right.}{}\right)}{e^3}}{ae^2+bd^2}}{ae^2}$$

$$\frac{d^4\left(a+bx^2\right)^{p+1}}{2e^3(d+ex)^2\left(ae^2+bd^2\right)}$$

$\downarrow$ 78

$$-\frac{-\frac{d\left(\frac{d\left(6a^2e^4+3abd^2e^2(3p+4)+b^2d^4\left(2p^2+7p+6\right)\right)\left(\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{e\left(a+bx^2\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+\right.}{2(p+1)\left(ae^2+bd^2\right)}\right.}{e}\right.}{e^3}}{a}$$

$$\frac{d^4\left(a+bx^2\right)^{p+1}}{2e^3(d+ex)^2\left(ae^2+bd^2\right)}$$

input
```
Int[(x^4*(a + b*x^2)^p)/(d + e*x)^3,x]
```

3.423. $\int\frac{x^4\left(a+bx^2\right)^p}{(d+ex)^3}\,dx$

output
```
-1/2*(d^4*(a + b*x^2)^(1 + p))/(e^3*(b*d^2 + a*e^2)*(d + e*x)^2) - (-((d^3
*(4*a*e^2 + b*d^2*(3 + p))*(a + b*x^2)^(1 + p))/(e^3*(b*d^2 + a*e^2)*(d +
e*x))) - (((b*d^2 + a*e^2)^2*(a + b*x^2)^(1 + p))/(2*b*e^3*(1 + p)) + (d*(
-(((3*a^2*e^4 + 2*a*b*d^2*e^2*(5 + 4*p) + b^2*d^4*(6 + 7*p + 2*p^2))*x*(a
+ b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e*(1 + (b*x^2)/
a)^p)) + (d*(6*a^2*e^4 + 3*a*b*d^2*e^2*(4 + 3*p) + b^2*d^4*(6 + 7*p + 2*p^
2))*((x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^
2])/(d*(1 + (b*x^2)/a)^p) - (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1
+ p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)*(1 + p)
)))/e))/e^3)/(b*d^2 + a*e^2))/(b*d^2 + a*e^2)
```

3.423.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

3.423. $\int \frac{x^4(a+bx^2)^p}{(d+ex)^3}\,dx$

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 603
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[x^m, c + d*x, x], R = PolynomialRemainde
r[x^m, c + d*x, x]}, Simp[d*R*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n +
1)*(b*c^2 + a*d^2))), x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)
^(n + 1)*(a + b*x^2)^p*ExpandToSum[(n + 1)*(b*c^2 + a*d^2)*Qx + b*c*R*(n +
1) - b*d*R*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IGt
Q[m, 1] && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.423. $\int \frac{x^4(a+bx^2)^p}{(d+ex)^3}\,dx$

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.423.4 Maple [F]

$$\int \frac{x^4(b\,x^2+a)^p}{(ex+d)^3}dx$$

input
```
int(x^4*(b*x^2+a)^p/(e*x+d)^3,x)
```

output
```
int(x^4*(b*x^2+a)^p/(e*x+d)^3,x)
```

3.423.5 Fricas [F]

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p x^4}{(ex+d)^3}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^4/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

3.423.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(b*x**2+a)**p/(e*x+d)**3,x)
```

output
```
Timed out
```

3.423.7 Maxima [F]

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p x^4}{(ex+d)^3}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^4/(e*x + d)^3, x)
```

3.423.8 Giac [F]

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p x^4}{(ex+d)^3}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^4/(e*x + d)^3, x)
```

3.423.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{x^4\,(b\,x^2+a)^p}{(d+e\,x)^3}\,dx$$

input
```
int((x^4*(a + b*x^2)^p)/(d + e*x)^3,x)
```

output
```
int((x^4*(a + b*x^2)^p)/(d + e*x)^3, x)
```

3.424 $\int \frac{x^3(a+bx^2)^p}{(d+ex)^3}\,dx$

3.424.1 Optimal result

Integrand size = 20, antiderivative size = 416

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^3}\,dx = \frac{d^3(a+bx^2)^{1+p}}{2e^2(bd^2+ae^2)(d+ex)^2} - \frac{d^2(3ae^2+bd^2(2+p))(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)^2(d+ex)} - \frac{(3a^2e^4+abd^2e^2(6+7p)+b^2d^4(3+5p+2p^2))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^3(bd^2+ae^2)^2} + \frac{(a^2e^4+abd^2e^2(5+6p)+b^2d^4(3+5p+2p^2))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e^3(bd^2+ae^2)^2} + \frac{d(3a^2e^4+abd^2e^2(6+7p)+b^2d^4(3+5p+2p^2))(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2e^2(bd^2+ae^2)^3(1+p)}$$

output
```
1/2*d^3*(b*x^2+a)^(p+1)/e^2/(a*e^2+b*d^2)/(e*x+d)^2-d^2*(3*a*e^2+b*d^2*(2+
p))*(b*x^2+a)^(p+1)/e^2/(a*e^2+b*d^2)^2/(e*x+d)-(3*a^2*e^4+a*b*d^2*e^2*(6+
7*p)+b^2*d^4*(2*p^2+5*p+3))*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^
2,-b*x^2/a)/e^3/(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)+(a^2*e^4+a*b*d^2*e^2*(5+6*
p)+b^2*d^4*(2*p^2+5*p+3))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a
)/e^3/(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)+1/2*d*(3*a^2*e^4+a*b*d^2*e^2*(6+7*p)
+b^2*d^4*(2*p^2+5*p+3))*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],e^2*(b*x^
2+a)/(a*e^2+b*d^2))/e^2/(a*e^2+b*d^2)^3/(p+1)
```

3.424.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.05

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^3}\,dx$$
$$= \frac{(a+bx^2)^p\left(\frac{6d^2\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+2p)(d+ex)} - \frac{d^3\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e(\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(x^3*(a + b*x^2)^p)/(d + e*x)^3,x]
```

output
```
((a + b*x^2)^p*((6*d^2*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)
]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(
a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) -
 (d^3*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (
d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*
x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)^2) - (3*d*AppellF1[-2
*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/
(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x
))/(d + e*x))^p) + (2*e*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(
1 + (b*x^2)/a)^p))/(2*e^4)
```

3.424.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {603, 27, 2182, 27, 719, 238, 237, 504, 334, 333, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^3}\,dx$$
$$\Big\downarrow \text{603}$$
$$\frac{d^3(a+bx^2)^{p+1}}{2e^2(d+ex)^2(ae^2+bd^2)} - \frac{\int -\frac{2(bx^2+a)^p\left(\frac{ad^2}{e}-\left(\frac{b(p+1)d^2}{e^2}+a\right)xd+\left(\frac{bd^2}{e}+ae\right)x^2\right)}{(d+ex)^2}dx}{2(ae^2+bd^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(bx^2+a)^p\left(\frac{ad^2}{e}-\left(\frac{b(p+1)d^2}{e^2}+a\right)xd+\left(\frac{bd^2}{e}+ae\right)x^2\right)}{(d+ex)^2}dx}{ae^2+bd^2}+\frac{d^3\left(a+bx^2\right)^{p+1}}{2e^2(d+ex)^2\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{-\frac{\int \frac{\left(ade^2\left(\frac{b(p+1)d^2}{e}+2ae\right)-\left(b^2\left(2p^2+5p+3\right)d^4+abe^2(6p+5)d^2+a^2e^4\right)x\right)\left(bx^2+a\right)^p}{e^2(d+ex)}dx}{ae^2+bd^2}-\frac{d^2\left(a+bx^2\right)^{p+1}\left(3ae^2+bd^2(p+2)\right)}{e^2(d+ex)(ae^2+bd^2)}}{ae^2+bd^2}+\frac{d^3\left(a+bx^2\right)^{p+1}}{2e^2(d+ex)^2\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{\left(ade\left(b(p+1)d^2+2ae^2\right)-\left(b^2\left(2p^2+5p+3\right)d^4+abe^2(6p+5)d^2+a^2e^4\right)x\right)\left(bx^2+a\right)^p}{d+ex}dx}{e^2(ae^2+bd^2)}-\frac{d^2\left(a+bx^2\right)^{p+1}\left(3ae^2+bd^2(p+2)\right)}{e^2(d+ex)(ae^2+bd^2)}}{ae^2+bd^2}+\frac{d^3\left(a+bx^2\right)^{p+1}}{2e^2(d+ex)^2\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \; 719$$

$$\frac{-\frac{\frac{d\left(3a^2e^4+abd^2e^2(7p+6)+b^2d^4\left(2p^2+5p+3\right)\right)\int \frac{\left(bx^2+a\right)^p}{d+ex}dx}{e}-\frac{\left(a^2e^4+abd^2e^2(6p+5)+b^2d^4\left(2p^2+5p+3\right)\right)\int \left(bx^2+a\right)^pdx}{e}}{e^2(ae^2+bd^2)}-\frac{d^2\left(a+bx^2\right)^{p+1}\left(3ae^2+bd^2(p+\right.}{e^2(d+ex)(ae^2+bd^2)}}{ae^2+bd^2}$$

$$\frac{d^3\left(a+bx^2\right)^{p+1}}{2e^2(d+ex)^2\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \; 238$$

$$\frac{-\frac{\frac{d\left(3a^2e^4+abd^2e^2(7p+6)+b^2d^4\left(2p^2+5p+3\right)\right)\int \frac{\left(bx^2+a\right)^p}{d+ex}dx}{e}-\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2e^4+abd^2e^2(6p+5)+b^2d^4\left(2p^2+5p+3\right)\right)\int \left(\frac{bx^2}{a}+1\right)^pdx}{e}}{e^2(ae^2+bd^2)}-\frac{d^2(a+}{}}{ae^2+bd^2}$$

$$\frac{d^3\left(a+bx^2\right)^{p+1}}{2e^2(d+ex)^2\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \; 237$$

$$\frac{-\frac{\frac{d\left(3a^2e^4+abd^2e^2(7p+6)+b^2d^4\left(2p^2+5p+3\right)\right)\int \frac{\left(bx^2+a\right)^p}{d+ex}dx}{e}-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2e^4+abd^2e^2(6p+5)+b^2d^4\left(2p^2+5p+3\right)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\right.}{e}}{e^2(ae^2+bd^2)}}{ae^2+bd^2}$$

$$\frac{d^3\left(a+bx^2\right)^{p+1}}{2e^2(d+ex)^2\left(ae^2+bd^2\right)}$$

3.424. $\int \frac{x^3\left(a+bx^2\right)^p}{(d+ex)^3}\,dx$

$$\Big\downarrow \; 504$$

$$-\frac{\frac{\frac{d\left(3a^2e^4+abd^2e^2(7p+6)+b^2d^4\left(2p^2+5p+3\right)\right)\left(d\int \frac{\left(bx^2+a\right)^p}{d^2-e^2x^2}dx-e\int \frac{x\left(bx^2+a\right)^p}{d^2-e^2x^2}dx\right)}{e}-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2e^4+abd^2e^2(6p+5)+b^2d^4\left(2p^2+5p+3\right)\right)\text{Hy}}{e}}{e^2(ae^2+bd^2)}}{ae^2+bd^2}$$

$$\frac{d^3\left(a+bx^2\right)^{p+1}}{2e^2(d+ex)^2\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \; 334$$

$$-\frac{\frac{\frac{d\left(3a^2e^4+abd^2e^2(7p+6)+b^2d^4\left(2p^2+5p+3\right)\right)\left(d\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^2}{a}+1\right)^p}{d^2-e^2x^2}dx-e\int \frac{x\left(bx^2+a\right)^p}{d^2-e^2x^2}dx\right)}{e}-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2e^4+abd^2e^2(6p+\right.}{}}{e^2(ae^2+bd^2)}}{ae^2+bd^2}$$

$$\frac{d^3\left(a+bx^2\right)^{p+1}}{2e^2(d+ex)^2\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \; 333$$

$$-\frac{\frac{\frac{d\left(3a^2e^4+abd^2e^2(7p+6)+b^2d^4\left(2p^2+5p+3\right)\right)\left(\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-e\int \frac{x\left(bx^2+a\right)^p}{d^2-e^2x^2}dx\right)}{e}-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(\right.}{}}{e^2(ae^2+bd^2)}}{ae^2+bd^2}$$

$$\frac{d^3\left(a+bx^2\right)^{p+1}}{2e^2(d+ex)^2\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \; 353$$

$$-\frac{\frac{\frac{d\left(3a^2e^4+abd^2e^2(7p+6)+b^2d^4\left(2p^2+5p+3\right)\right)\left(\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{1}{2}e\int \frac{\left(bx^2+a\right)^p}{d^2-e^2x^2}dx^2\right)}{e}-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-}}{}}{e^2(ae^2+bd^2)}}{ae^2+bd^2}$$

$$\frac{d^3\left(a+bx^2\right)^{p+1}}{2e^2(d+ex)^2\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \; 78$$

3.424. $\int \frac{x^3\left(a+bx^2\right)^p}{(d+ex)^3}\,dx$

$$-\frac{\frac{d\left(3a^2e^4+abd^2e^2(7p+6)+b^2d^4\left(2p^2+5p+3\right)\right)\left(\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{e\left(a+bx^2\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\ldots\right.}{2(p+1)\left(ae^2+bd^2\right)}\right.}{e}}{e^2(ae^2+bd^2)}}{ae^2 \ldots}$$

$$\frac{d^3\left(a+bx^2\right)^{p+1}}{2e^2(d+ex)^2\left(ae^2+bd^2\right)}$$

input
```
Int[(x^3*(a + b*x^2)^p)/(d + e*x)^3,x]
```

output
```
(d^3*(a + b*x^2)^(1 + p))/(2*e^2*(b*d^2 + a*e^2)*(d + e*x)^2) + (-((d^2*(3
*a*e^2 + b*d^2*(2 + p))*(a + b*x^2)^(1 + p))/(e^2*(b*d^2 + a*e^2)*(d + e*x
))) - (-(((a^2*e^4 + a*b*d^2*e^2*(5 + 6*p) + b^2*d^4*(3 + 5*p + 2*p^2))*x*
(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e*(1 + (b*x^
2)/a)^p)) + (d*(3*a^2*e^4 + a*b*d^2*e^2*(6 + 7*p) + b^2*d^4*(3 + 5*p + 2*p
^2))*((x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d
^2])/(d*(1 + (b*x^2)/a)^p) - (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1
 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)*(1 + p
))))/e)/(e^2*(b*d^2 + a*e^2)))/(b*d^2 + a*e^2)
```

3.424.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

3.424. $\int \frac{x^3(a+bx^2)^p}{(d+ex)^3}\,dx$

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 603
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[x^m, c + d*x, x], R = PolynomialRemainde
r[x^m, c + d*x, x]}, Simp[d*R*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n +
1)*(b*c^2 + a*d^2))), x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)
^(n + 1)*(a + b*x^2)^p*ExpandToSum[(n + 1)*(b*c^2 + a*d^2)*Qx + b*c*R*(n +
1) - b*d*R*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IGt
Q[m, 1] && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

3.424. $\int \frac{x^3(a+bx^2)^p}{(d+ex)^3}\,dx$

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.424.4 Maple [F]

$$\int \frac{x^3 (b\,x^2+a)^p}{(ex+d)^3}dx$$

input
```
int(x^3*(b*x^2+a)^p/(e*x+d)^3,x)
```

output
```
int(x^3*(b*x^2+a)^p/(e*x+d)^3,x)
```

3.424.5 Fricas [F]

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^3}\,dx=\int \frac{(bx^2+a)^p x^3}{(ex+d)^3}\,dx$$

input
```
integrate(x^3*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^3/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

3.424.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^3}\,dx=\text{Timed out}$$

input
```
integrate(x**3*(b*x**2+a)**p/(e*x+d)**3,x)
```

output
```
Timed out
```

3.424.7 Maxima [F]

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p x^3}{(ex+d)^3}\,dx$$

input
```
integrate(x^3*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^3/(e*x + d)^3, x)
```

3.424.8 Giac [F]

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p x^3}{(ex+d)^3}\,dx$$

input
```
integrate(x^3*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^3/(e*x + d)^3, x)
```

3.424.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{x^3\,(b\,x^2+a)^p}{(d+e\,x)^3}\,dx$$

input
```
int((x^3*(a + b*x^2)^p)/(d + e*x)^3,x)
```

output
```
int((x^3*(a + b*x^2)^p)/(d + e*x)^3, x)
```

3.425 $\int \frac{x^2(a+bx^2)^p}{(d+ex)^3}\,dx$

3.425.1 Optimal result

Integrand size = 20, antiderivative size = 396

$$\begin{aligned}\int \frac{x^2(a+bx^2)^p}{(d+ex)^3}\,dx = & -\frac{d^2(a+bx^2)^{1+p}}{2e(bd^2+ae^2)(d+ex)^2}+\frac{d(2ae^2+bd^2(1+p))(a+bx^2)^{1+p}}{e(bd^2+ae^2)^2(d+ex)} \\ & +\frac{(a^2e^4+abd^2e^2(2+5p)+b^2d^4(1+3p+2p^2))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right.}{de^2(bd^2+ae^2)^2} \\ & -\frac{bd(1+2p)(2ae^2+bd^2(1+p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e^2(bd^2+ae^2)^2} \\ & -\frac{(a^2e^4+abd^2e^2(2+5p)+b^2d^4(1+3p+2p^2))(a+bx^2)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2}{bd^2+ae^2}\right.}{2e(bd^2+ae^2)^3(1+p)}\end{aligned}$$

output
```
-1/2*d^2*(b*x^2+a)^(p+1)/e/(a*e^2+b*d^2)/(e*x+d)^2+d*(2*a*e^2+b*d^2*(p+1))
*(b*x^2+a)^(p+1)/e/(a*e^2+b*d^2)^2/(e*x+d)+(a^2*e^4+a*b*d^2*e^2*(2+5*p)+b^
2*d^4*(2*p^2+3*p+1))*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^
2/a)/d/e^2/(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)-b*d*(1+2*p)*(2*a*e^2+b*d^2*(p+1
))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e^2/(a*e^2+b*d^2)^2/(
(1+b*x^2/a)^p)-1/2*(a^2*e^4+a*b*d^2*e^2*(2+5*p)+b^2*d^4*(2*p^2+3*p+1))*(b*
x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e/(a*e^
2+b*d^2)^3/(p+1)
```

3.425.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.73

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^3}\,dx$$
$$= \frac{\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}(a+bx^2)^p\left(-\frac{4d\,\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+2p)(d+ex)}+\frac{d^2\,\text{AppellF1}\left(2-2\right.}{}\right.}{2e^3}$$

input
```
Integrate[(x^2*(a + b*x^2)^p)/(d + e*x)^3,x]
```

output
```
((a + b*x^2)^p*((-4*d*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]
*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*(d + e*x)) + (
d^2*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d
+ Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + p)*(d + e*x)^2) + AppellF1[-2*p, -p,
-p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x
)]/p))/(2*e^3*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x)
)/(d + e*x))^p)
```

3.425.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {603, 27, 688, 719, 238, 237, 504, 334, 333, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^3}\,dx$$
$$\Big\downarrow \text{603}$$
$$-\frac{\int \frac{2(ade-(b(p+1)d^2+ae^2)x)(bx^2+a)^p}{e(d+ex)^2}dx}{2(ae^2+bd^2)}-\frac{d^2(a+bx^2)^{p+1}}{2e(d+ex)^2(ae^2+bd^2)}$$
$$\Big\downarrow \text{27}$$
$$-\frac{\int \frac{(ade-(b(p+1)d^2+ae^2)x)(bx^2+a)^p}{(d+ex)^2}dx}{e(ae^2+bd^2)}-\frac{d^2(a+bx^2)^{p+1}}{2e(d+ex)^2(ae^2+bd^2)}$$

$$\Big\downarrow \text{688}$$

$$-\frac{-\frac{\int \frac{\left(ae\left(bpd^2+ae^2\right)-bd(2p+1)\left(b(p+1)d^2+2ae^2\right)x\right)\left(bx^2+a\right)^p}{d+ex}dx}{ae^2+bd^2}-\frac{d(a+bx^2)^{p+1}(2ae^2+bd^2(p+1))}{(d+ex)(ae^2+bd^2)}}{e\left(ae^2+bd^2\right)}-\frac{d^2\left(a+bx^2\right)^{p+1}}{2e(d+ex)^2\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \text{719}$$

$$-\frac{-\frac{\frac{\left(a^2e^4+abd^2e^2(5p+2)+b^2d^4\left(2p^2+3p+1\right)\right)\int \frac{\left(bx^2+a\right)^p}{d+ex}dx}{e}-\frac{bd(2p+1)\left(2ae^2+bd^2(p+1)\right)\int \left(bx^2+a\right)^p dx}{e}}{ae^2+bd^2}-\frac{d(a+bx^2)^{p+1}(2ae^2+bd^2(p+1))}{(d+ex)(ae^2+bd^2)}}{e\left(ae^2+bd^2\right)}-\frac{d^2\left(a+bx^2\right)^{p+1}}{2e(d+ex)^2\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \text{238}$$

$$-\frac{-\frac{\frac{\left(a^2e^4+abd^2e^2(5p+2)+b^2d^4\left(2p^2+3p+1\right)\right)\int \frac{\left(bx^2+a\right)^p}{d+ex}dx}{e}-\frac{bd(2p+1)\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(2ae^2+bd^2(p+1)\right)\int \left(\frac{bx^2}{a}+1\right)^p dx}{e}}{ae^2+bd^2}-\frac{d(a+bx^2)^{p+1}(2ae^2+bd^2(p+1))}{(d+ex)(ae^2+bd^2)}}{e\left(ae^2+bd^2\right)}-\frac{d^2\left(a+bx^2\right)^{p+1}}{2e(d+ex)^2\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \text{237}$$

$$-\frac{-\frac{\frac{\left(a^2e^4+abd^2e^2(5p+2)+b^2d^4\left(2p^2+3p+1\right)\right)\int \frac{\left(bx^2+a\right)^p}{d+ex}dx}{e}-\frac{bd(2p+1)x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(2ae^2+bd^2(p+1)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e}}{ae^2+bd^2}-\frac{d(a+bx^2)^{p+1}(2ae^2+bd^2(p+1))}{(d+ex)(ae^2+bd^2)}}{e\left(ae^2+bd^2\right)}-\frac{d^2\left(a+bx^2\right)^{p+1}}{2e(d+ex)^2\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \text{504}$$

$$-\frac{-\frac{\frac{\left(a^2e^4+abd^2e^2(5p+2)+b^2d^4\left(2p^2+3p+1\right)\right)\left(d\int \frac{\left(bx^2+a\right)^p}{d^2-e^2x^2}dx-e\int \frac{x\left(bx^2+a\right)^p}{d^2-e^2x^2}dx\right)}{e}-\frac{bd(2p+1)x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(2ae^2+bd^2(p+1)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e}}{ae^2+bd^2}-\frac{d(a+bx^2)^{p+1}(2ae^2+bd^2(p+1))}{(d+ex)(ae^2+bd^2)}}{e\left(ae^2+bd^2\right)}-\frac{d^2\left(a+bx^2\right)^{p+1}}{2e(d+ex)^2\left(ae^2+bd^2\right)}$$

$$\Big\downarrow \text{334}$$

3.425. $\int \frac{x^2(a+bx^2)^p}{(d+ex)^3}\,dx$

$$-\frac{-\frac{\frac{\left(a^2e^4+abd^2e^2(5p+2)+b^2d^4\left(2p^2+3p+1\right)\right)\left(d\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int\frac{\left(\frac{bx^2}{a}+1\right)^p}{d^2-e^2x^2}dx-e\int\frac{x\left(bx^2+a\right)^p}{d^2-e^2x^2}dx\right)}{e}-\frac{bd(2p+1)x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(2ae^2+bd^2\right.}{}}{ae^2+bd^2}}{e\left(ae^2+bd^2\right)}$$

$$\frac{d^2\left(a+bx^2\right)^{p+1}}{2e(d+ex)^2\left(ae^2+bd^2\right)}$$

$\Big\downarrow$ 333

$$-\frac{-\frac{\frac{\left(a^2e^4+abd^2e^2(5p+2)+b^2d^4\left(2p^2+3p+1\right)\right)\left(\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-e\int\frac{x\left(bx^2+a\right)^p}{d^2-e^2x^2}dx\right)}{e}-\frac{bd(2p+1)x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}\right.}{}}{ae^2+bd^2}}{e\left(ae^2+bd^2\right)}$$

$$\frac{d^2\left(a+bx^2\right)^{p+1}}{2e(d+ex)^2\left(ae^2+bd^2\right)}$$

$\Big\downarrow$ 353

$$-\frac{-\frac{\frac{\left(a^2e^4+abd^2e^2(5p+2)+b^2d^4\left(2p^2+3p+1\right)\right)\left(\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{1}{2}e\int\frac{\left(bx^2+a\right)^p}{d^2-e^2x^2}dx^2\right)}{e}-\frac{bd(2p+1)x\left(a+bx^2\right)^p\left(b\right.}{}}{ae^2+bd^2}}{e\left(ae^2+bd^2\right)}$$

$$\frac{d^2\left(a+bx^2\right)^{p+1}}{2e(d+ex)^2\left(ae^2+bd^2\right)}$$

$\Big\downarrow$ 78

$$-\frac{-\frac{\frac{\left(a^2e^4+abd^2e^2(5p+2)+b^2d^4\left(2p^2+3p+1\right)\right)\left(\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{e\left(a+bx^2\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\right.}{2(p+1)\left(ae^2+bd^2\right)}\right.}{e}}{ae^2+bd^2}}{e\left(ae^2+b\right.}$$

$$\frac{d^2\left(a+bx^2\right)^{p+1}}{2e(d+ex)^2\left(ae^2+bd^2\right)}$$

input
```
Int[(x^2*(a + b*x^2)^p)/(d + e*x)^3,x]
```

3.425. $\int\frac{x^2\left(a+bx^2\right)^p}{(d+ex)^3}dx$

output
```
-1/2*(d^2*(a + b*x^2)^(1 + p))/(e*(b*d^2 + a*e^2)*(d + e*x)^2) - (-((d*(2*
a*e^2 + b*d^2*(1 + p))*(a + b*x^2)^(1 + p))/((b*d^2 + a*e^2)*(d + e*x))) -
 (-((b*d*(1 + 2*p)*(2*a*e^2 + b*d^2*(1 + p))*x*(a + b*x^2)^p*Hypergeometri
c2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e*(1 + (b*x^2)/a)^p)) + ((a^2*e^4 + a*b
*d^2*e^2*(2 + 5*p) + b^2*d^4*(1 + 3*p + 2*p^2))*((x*(a + b*x^2)^p*AppellF1
[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d*(1 + (b*x^2)/a)^p) - (e
*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/
(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)*(1 + p))))/e)/(b*d^2 + a*e^2))/(e*(b*
d^2 + a*e^2))
```

3.425.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

3.425. $\int \frac{x^2(a+bx^2)^p}{(d+ex)^3} dx$

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 603
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[x^m, c + d*x, x], R = PolynomialRemainde
r[x^m, c + d*x, x]}, Simp[d*R*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n +
1)*(b*c^2 + a*d^2))), x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)
^(n + 1)*(a + b*x^2)^p*ExpandToSum[(n + 1)*(b*c^2 + a*d^2)*Qx + b*c*R*(n +
1) - b*d*R*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IGt
Q[m, 1] && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.425. $\int \frac{x^2(a+bx^2)^p}{(d+ex)^3}\,dx$

3.425.4 Maple [F]

$$\int \frac{x^2(b\,x^2+a)^p}{(ex+d)^3}dx$$

input
```
int(x^2*(b*x^2+a)^p/(e*x+d)^3,x)
```

output
```
int(x^2*(b*x^2+a)^p/(e*x+d)^3,x)
```

3.425.5 Fricas [F]

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p x^2}{(ex+d)^3}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^2/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

3.425.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(b*x**2+a)**p/(e*x+d)**3,x)
```

output
```
Timed out
```

3.425.7 Maxima [F]

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p x^2}{(ex+d)^3}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^2/(e*x + d)^3, x)
```

3.425.8 Giac [F]

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p x^2}{(ex+d)^3}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^2/(e*x + d)^3, x)
```

3.425.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{x^2\,(b\,x^2+a)^p}{(d+e\,x)^3}\,dx$$

input
```
int((x^2*(a + b*x^2)^p)/(d + e*x)^3,x)
```

output
```
int((x^2*(a + b*x^2)^p)/(d + e*x)^3, x)
```

3.426 $\int \frac{x(a+bx^2)^p}{(d+ex)^3} dx$

3.426.1 Optimal result

Integrand size = 18, antiderivative size = 336

$$\begin{aligned}
&\int \frac{x(a+bx^2)^p}{(d+ex)^3} dx \\
&= \frac{d(a+bx^2)^{1+p}}{2(bd^2+ae^2)(d+ex)^2} - \frac{(ae^2+bd^2p)(a+bx^2)^{1+p}}{(bd^2+ae^2)^2(d+ex)} \\
&- \frac{bp(3ae^2+bd^2(1+2p))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e(bd^2+ae^2)^2} \\
&+ \frac{b(1+2p)(ae^2+bd^2p)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e(bd^2+ae^2)^2} \\
&+ \frac{bdp(3ae^2+bd^2(1+2p))(a+bx^2)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2(bd^2+ae^2)^3(1+p)}
\end{aligned}$$

output
```
1/2*d*(b*x^2+a)^(p+1)/(a*e^2+b*d^2)/(e*x+d)^2-(b*d^2*p+a*e^2)*(b*x^2+a)^(p
+1)/(a*e^2+b*d^2)^2/(e*x+d)-b*p*(3*a*e^2+b*d^2*(1+2*p))*x*(b*x^2+a)^p*Appe
llF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/e/(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)+
b*(1+2*p)*(b*d^2*p+a*e^2)*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a
)/e/(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)+1/2*b*d*p*(3*a*e^2+b*d^2*(1+2*p))*(b*x
^2+a)^(p+1)*hypergeom([1, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b
*d^2)^3/(p+1)
```

3.426.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.68

$$\int \frac{x(a+bx^2)^p}{(d+ex)^3}\,dx$$
$$= \frac{\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}(a+bx^2)^p\left(2(-1+p)(d+ex)\operatorname{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-}}{d+e}\right.\right.}{2e^2(-1+p)(-1+2p)(d+}$$

input
```
Integrate[(x*(a + b*x^2)^p)/(d + e*x)^3,x]
```

output
```
((a + b*x^2)^p*(2*(-1 + p)*(d + e*x)*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d
 - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)] + d*(1 - 2*p
)*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d +
Sqrt[-(a/b)]*e)/(d + e*x)]))/(2*e^2*(-1 + p)*(-1 + 2*p)*((e*(-Sqrt[-(a/b)]
 + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)^2)
```

3.426.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {594, 27, 688, 25, 27, 719, 238, 237, 504, 334, 333, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^2)^p}{(d+ex)^3}\,dx$$

$$\Big\downarrow \; 594$$

$$\frac{d(a+bx^2)^{p+1}}{2(d+ex)^2(ae^2+bd^2)}-\frac{\int -\frac{2(ae-bdpx)(bx^2+a)^p}{(d+ex)^2}dx}{2(ae^2+bd^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(ae-bdpx)(bx^2+a)^p}{(d+ex)^2}dx}{ae^2+bd^2}+\frac{d(a+bx^2)^{p+1}}{2(d+ex)^2(ae^2+bd^2)}$$

$$\Big\downarrow \; 688$$

$$\frac{-\frac{\int -\frac{b\left(ade(1-p)+(2p+1)\left(bpd^2+ae^2\right)x\right)\left(bx^2+a\right)^p}{d+ex}dx}{ae^2+bd^2}-\frac{(a+bx^2)^{p+1}(ae^2+bd^2p)}{(d+ex)(ae^2+bd^2)}}{ae^2+bd^2}+\frac{d(a+bx^2)^{p+1}}{2(d+ex)^2(ae^2+bd^2)}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{b\left(ade(1-p)+(2p+1)\left(bpd^2+ae^2\right)x\right)\left(bx^2+a\right)^p}{d+ex}dx}{ae^2+bd^2}-\frac{(a+bx^2)^{p+1}(ae^2+bd^2p)}{(d+ex)(ae^2+bd^2)}}{ae^2+bd^2}+\frac{d(a+bx^2)^{p+1}}{2(d+ex)^2(ae^2+bd^2)}$$

$\downarrow$ 27

$$\frac{\frac{b\int \frac{\left(ade(1-p)+(2p+1)\left(bpd^2+ae^2\right)x\right)\left(bx^2+a\right)^p}{d+ex}dx}{ae^2+bd^2}-\frac{(a+bx^2)^{p+1}(ae^2+bd^2p)}{(d+ex)(ae^2+bd^2)}}{ae^2+bd^2}+\frac{d(a+bx^2)^{p+1}}{2(d+ex)^2(ae^2+bd^2)}$$

$\downarrow$ 719

$$\frac{\frac{b\left(\frac{(2p+1)\left(ae^2+bd^2p\right)\int\left(bx^2+a\right)^pdx}{e}-\frac{dp\left(3ae^2+bd^2(2p+1)\right)\int\frac{\left(bx^2+a\right)^p}{d+ex}dx}{e}\right)}{ae^2+bd^2}-\frac{(a+bx^2)^{p+1}(ae^2+bd^2p)}{(d+ex)(ae^2+bd^2)}}{ae^2+bd^2}+\frac{d(a+bx^2)^{p+1}}{2(d+ex)^2(ae^2+bd^2)}$$

$\downarrow$ 238

$$\frac{\frac{b\left(\frac{(2p+1)\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+bd^2p\right)\int\left(\frac{bx^2}{a}+1\right)^pdx}{e}-\frac{dp\left(3ae^2+bd^2(2p+1)\right)\int\frac{\left(bx^2+a\right)^p}{d+ex}dx}{e}\right)}{ae^2+bd^2}-\frac{(a+bx^2)^{p+1}(ae^2+bd^2p)}{(d+ex)(ae^2+bd^2)}}{ae^2+bd^2}+\frac{d(a+bx^2)^{p+1}}{2(d+ex)^2(ae^2+bd^2)}$$

$\downarrow$ 237

$$\frac{\frac{b\left(\frac{(2p+1)x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+bd^2p\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e}-\frac{dp\left(3ae^2+bd^2(2p+1)\right)\int\frac{\left(bx^2+a\right)^p}{d+ex}dx}{e}\right)}{ae^2+bd^2}-\frac{(a+bx^2)^{p+1}(ae^2+bd^2p)}{(d+ex)(ae^2+bd^2)}}{ae^2+bd^2}+\frac{d(a+bx^2)^{p+1}}{2(d+ex)^2(ae^2+bd^2)}$$

$\downarrow$ 504

3.426. $\int \frac{x(a+bx^2)^p}{(d+ex)^3}dx$

$$\frac{b\left(\frac{(2p+1)x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+bd^2p\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e}-\frac{dp\left(3ae^2+bd^2(2p+1)\right)\left(d\int\frac{\left(bx^2+a\right)^p}{d^2-e^2x^2}dx-e\int\frac{x\left(bx^2+a\right)^p}{d^2-e^2x^2}dx\right)}{e}\right)}{ae^2+bd^2} - \frac{(a}{}$$

$$ae^2+bd^2$$

$$\frac{d\left(a+bx^2\right)^{p+1}}{2(d+ex)^2\left(ae^2+bd^2\right)}$$

$\downarrow$ 334

$$\frac{b\left(\frac{(2p+1)x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+bd^2p\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e}-\frac{dp\left(3ae^2+bd^2(2p+1)\right)\left(d\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int\frac{\left(\frac{bx^2}{a}+1\right)^p}{d^2-e^2x^2}dx-e\int\frac{x(}{d}\right.}{e}\right.}{ae^2+bd^2}$$

$$ae^2+bd^2$$

$$\frac{d\left(a+bx^2\right)^{p+1}}{2(d+ex)^2\left(ae^2+bd^2\right)}$$

$\downarrow$ 333

$$\frac{b\left(\frac{(2p+1)x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+bd^2p\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e}-\frac{dp\left(3ae^2+bd^2(2p+1)\right)\left(\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\right.}{d}\right.}{e}\right.}{ae^2+bd^2}$$

$$ae^2+bd^2$$

$$\frac{d\left(a+bx^2\right)^{p+1}}{2(d+ex)^2\left(ae^2+bd^2\right)}$$

$\downarrow$ 353

$$\frac{b\left(\frac{(2p+1)x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+bd^2p\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e}-\frac{dp\left(3ae^2+bd^2(2p+1)\right)\left(\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\right.}{d}\right.}{e}\right.}{ae^2+bd^2}$$

$$ae^2+bd^2$$

$$\frac{d\left(a+bx^2\right)^{p+1}}{2(d+ex)^2\left(ae^2+bd^2\right)}$$

$\downarrow$ 78

3.426. $\int\frac{x\left(a+bx^2\right)^p}{(d+ex)^3}\,dx$

$$b\left(\frac{(2p+1)x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+bd^2p\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e}-\frac{dp\left(3ae^2+bd^2(2p+1)\right)\left(\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\right.}{d}\right.}{}\right.$$

$$ae^2+bd^2$$

$$ae^2+bd^2$$

$$\frac{d\left(a+bx^2\right)^{p+1}}{2(d+ex)^2\left(ae^2+bd^2\right)}$$

input
```
Int[(x*(a + b*x^2)^p)/(d + e*x)^3,x]
```

output
```
(d*(a + b*x^2)^(1 + p))/(2*(b*d^2 + a*e^2)*(d + e*x)^2) + (-(((a*e^2 + b*d
^2*p)*(a + b*x^2)^(1 + p))/((b*d^2 + a*e^2)*(d + e*x))) + (b*(((1 + 2*p)*(
a*e^2 + b*d^2*p)*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)
/a)])/(e*(1 + (b*x^2)/a)^p) - (d*p*(3*a*e^2 + b*d^2*(1 + 2*p))*((x*(a + b*
x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d*(1 + (b*
x^2)/a)^p) - (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^
2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)*(1 + p))))/e))/(b*d^2
+ a*e^2))/(b*d^2 + a*e^2)
```

3.426.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.426. $\int \frac{x(a+bx^2)^p}{(d+ex)^3} \, dx$

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

3.426.4 Maple [F]

$$\int \frac{x(b\,x^2+a)^p}{(ex+d)^3}dx$$

input
```
int(x*(b*x^2+a)^p/(e*x+d)^3,x)
```

output
```
int(x*(b*x^2+a)^p/(e*x+d)^3,x)
```

3.426.5 Fricas [F]

$$\int \frac{x(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p x}{(ex+d)^3}\,dx$$

input
```
integrate(x*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

3.426.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+bx^2)^p}{(d+ex)^3}\,dx = \text{Timed out}$$

input
```
integrate(x*(b*x**2+a)**p/(e*x+d)**3,x)
```

output
```
Timed out
```

3.426.7 Maxima [F]

$$\int \frac{x(a+bx^2)^p}{(d+ex)^3} dx = \int \frac{(bx^2+a)^p x}{(ex+d)^3} dx$$

input
```
integrate(x*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x/(e*x + d)^3, x)
```

3.426.8 Giac [F]

$$\int \frac{x(a+bx^2)^p}{(d+ex)^3} dx = \int \frac{(bx^2+a)^p x}{(ex+d)^3} dx$$

input
```
integrate(x*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x/(e*x + d)^3, x)
```

3.426.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^2)^p}{(d+ex)^3} dx = \int \frac{x\,(b\,x^2+a)^p}{(d+e\,x)^3} dx$$

input
```
int((x*(a + b*x^2)^p)/(d + e*x)^3,x)
```

output
```
int((x*(a + b*x^2)^p)/(d + e*x)^3, x)
```

3.427 $\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx$

3.427.1 Optimal result

Integrand size = 17, antiderivative size = 322

$$\begin{aligned}\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx &= -\frac{d^2e(a+bx^2)^{1+p}}{4(bd^2+ae^2)(d^2-e^2x^2)^2}+\frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,3,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}\\&\quad+\frac{e^2x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,3,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}\\&\quad+\frac{be(2ae^2+bd^2(1+p))(a+bx^2)^{1+p}\text{Hypergeometric2F1}\left(2,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{4(bd^2+ae^2)^3(1+p)}\\&\quad-\frac{3b^2d^2e(a+bx^2)^{1+p}\text{Hypergeometric2F1}\left(3,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2(bd^2+ae^2)^3(1+p)}\end{aligned}$$

output
```
-1/4*d^2*e*(b*x^2+a)^(p+1)/(a*e^2+b*d^2)/(-e^2*x^2+d^2)^2+x*(b*x^2+a)^p*Ap
pellF1(1/2,3,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^3/((1+b*x^2/a)^p)+e^2*x^3*(b*x
^2+a)^p*AppellF1(3/2,3,-p,5/2,e^2*x^2/d^2,-b*x^2/a)/d^5/((1+b*x^2/a)^p)+1/
4*b*e*(2*a*e^2+b*d^2*(p+1))*(b*x^2+a)^(p+1)*hypergeom([2, p+1],[2+p],e^2*(
b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^3/(p+1)-3/2*b^2*d^2*e*(b*x^2+a)^(p+1
)*hypergeom([3, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^3/(p
+1)
```

3.427.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.44

$$\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx$$
$$=\frac{\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}(a+bx^2)^p\operatorname{AppellF1}\left(2-2p,-p,-p,3-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{2e(-1+p)(d+ex)^2}$$

input
```
Integrate[(a + b*x^2)^p/(d + e*x)^3,x]
```

output
```
((a + b*x^2)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d
+ e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(2*e*(-1 + p)*((e*(-Sqrt[-(a/b)]
+ x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)^2)
```

3.427.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {505, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx$$
$$\Big\downarrow \text{505}$$
$$\int \left(\frac{3de^2x^2(a+bx^2)^p}{(d^2-e^2x^2)^3}-\frac{3d^2ex(a+bx^2)^p}{(d^2-e^2x^2)^3}+\frac{e^3x^3(a+bx^2)^p}{(e^2x^2-d^2)^3}+\frac{d^3(a+bx^2)^p}{(d^2-e^2x^2)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{e^2x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,3,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}+\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,3,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}-\frac{3b^2d^2e(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(3,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)^3}+\frac{be(a+bx^2)^{p+1}(2ae^2+bd^2(p+1))\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{4(p+1)(ae^2+bd^2)^3}-\frac{d^2e(a+bx^2)^{p+1}}{4(d^2-e^2x^2)^2(ae^2+bd^2)}$$

input
```
Int[(a + b*x^2)^p/(d + e*x)^3,x]
```

output
```
-1/4*(d^2*e*(a + b*x^2)^(1 + p))/((b*d^2 + a*e^2)*(d^2 - e^2*x^2)^2) + (x*
(a + b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^3
*(1 + (b*x^2)/a)^p) + (e^2*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 3, 5/2, -((
b*x^2)/a), (e^2*x^2)/d^2])/(d^5*(1 + (b*x^2)/a)^p) + (b*e*(2*a*e^2 + b*d^2
*(1 + p))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a +
 b*x^2))/(b*d^2 + a*e^2)])/(4*(b*d^2 + a*e^2)^3*(1 + p)) - (3*b^2*d^2*e*(a
 + b*x^2)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*
d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)^3*(1 + p))
```

3.427.3.1 Defintions of rubi rules used

rule 505
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2 - d^2*x^2)))^(
-n), x], x] /; FreeQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.427.4 Maple [F]

$$\int \frac{(b\,x^2+a)^p}{(ex+d)^3}dx$$

input
```
int((b*x^2+a)^p/(e*x+d)^3,x)
```

output
```
int((b*x^2+a)^p/(e*x+d)^3,x)
```

3.427.5 Fricas [F]

$$\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^3}\,dx$$

input
```
integrate((b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

3.427.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p/(e*x+d)**3,x)
```

output
```
Timed out
```

3.427.7 Maxima [F]

$$\int \frac{(a+bx^2)^p}{(d+ex)^3} \, dx = \int \frac{(bx^2+a)^p}{(ex+d)^3} \, dx$$

input
```
integrate((b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/(e*x + d)^3, x)
```

3.427.8 Giac [F]

$$\int \frac{(a+bx^2)^p}{(d+ex)^3} \, dx = \int \frac{(bx^2+a)^p}{(ex+d)^3} \, dx$$

input
```
integrate((b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/(e*x + d)^3, x)
```

3.427.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{(d+ex)^3} \, dx = \int \frac{(b\,x^2+a)^p}{(d+e\,x)^3} \, dx$$

input
```
int((a + b*x^2)^p/(d + e*x)^3,x)
```

output
```
int((a + b*x^2)^p/(d + e*x)^3, x)
```

3.428 $\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx$

3.428.1 Optimal result

Integrand size = 20, antiderivative size = 700

$$\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx$$
$$= \frac{de^2(a+bx^2)^{1+p}}{4\,(bd^2+ae^2)\,(d^2-e^2x^2)^2} - \frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}$$
$$- \frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}$$
$$- \frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,3,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}$$
$$- \frac{e^3x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,2,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^6}$$
$$- \frac{e^3x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,3,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^6}$$
$$+ \frac{e^2(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2d^3\,(bd^2+ae^2)\,(1+p)}$$
$$- \frac{(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2ad^3(1+p)}$$
$$+ \frac{be^2(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(2,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{d\,(bd^2+ae^2)^2\,(1+p)}$$
$$- \frac{be^2(2ae^2+bd^2(1+p))\,(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(2,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{4d\,(bd^2+ae^2)^3\,(1+p)}$$
$$+ \frac{3b^2de^2(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(3,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2\,(bd^2+ae^2)^3\,(1+p)}$$

output
```
1/4*d*e^2*(b*x^2+a)^(p+1)/(a*e^2+b*d^2)/(-e^2*x^2+d^2)^2-e*x*(b*x^2+a)^p*A
ppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^4/((1+b*x^2/a)^p)-e*x*(b*x^2+
a)^p*AppellF1(1/2,2,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^4/((1+b*x^2/a)^p)-e*x*(
b*x^2+a)^p*AppellF1(1/2,3,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^4/((1+b*x^2/a)^p)
-1/3*e^3*x^3*(b*x^2+a)^p*AppellF1(3/2,2,-p,5/2,e^2*x^2/d^2,-b*x^2/a)/d^6/(
(1+b*x^2/a)^p)-e^3*x^3*(b*x^2+a)^p*AppellF1(3/2,3,-p,5/2,e^2*x^2/d^2,-b*x^
2/a)/d^6/((1+b*x^2/a)^p)+1/2*e^2*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],
e^2*(b*x^2+a)/(a*e^2+b*d^2))/d^3/(a*e^2+b*d^2)/(p+1)-1/2*(b*x^2+a)^(p+1)*h
ypergeom([1, p+1],[2+p],1+b*x^2/a)/a/d^3/(p+1)+b*e^2*(b*x^2+a)^(p+1)*hyper
geom([2, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/d/(a*e^2+b*d^2)^2/(p+1)-1
/4*b*e^2*(2*a*e^2+b*d^2*(p+1))*(b*x^2+a)^(p+1)*hypergeom([2, p+1],[2+p],e^
2*(b*x^2+a)/(a*e^2+b*d^2))/d/(a*e^2+b*d^2)^3/(p+1)+3/2*b^2*d*e^2*(b*x^2+a)
^(p+1)*hypergeom([3, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)
^3/(p+1)
```

3.428.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx$$
$$= \frac{(a+bx^2)^p \left(-\frac{2d\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+2p)(d+ex)} - \frac{d^2\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*x^2)^p/(x*(d + e*x)^3),x]
```

output
```
((a + b*x^2)^p*((-2*d*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]
*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a
/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) -
(d^2*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d
 + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x
))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)^2) + (-(AppellF1[-2*p,
 -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d
+ e*x)]/(((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d
+ e*x))^p)) + Hypergeometric2F1[-p, -p, 1 - p, -(a/(b*x^2))]/(1 + a/(b*x^2
))^p)/p))/(2*d^3)
```

3.428.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 464, normalized size of antiderivative = 0.66, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {622, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx$$

$$\Big\downarrow \; 622$$

$$\int \left(-\frac{3d^2e(a+bx^2)^p}{(d^2-e^2x^2)^3}+\frac{3de^2x(a+bx^2)^p}{(d^2-e^2x^2)^3}+\frac{e^3x^2(a+bx^2)^p}{(e^2x^2-d^2)^3}+\frac{d^3(a+bx^2)^p}{x(d^2-e^2x^2)^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{e^2(a+bx^2)^{p+1}\left(2a^2e^4+2abd^2e^2(2-p)+b^2d^4(p^2-3p+2)\right)\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{4d^3(p+1)(ae^2+bd^2)^3}-\frac{e^3x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,3,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^6}-\frac{3ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,3,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}+\frac{3b^2de^2(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(3,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)^3}-\frac{(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{2ad^3(p+1)}+\frac{e^2(a+bx^2)^{p+1}(2ae^2+bd^2(3-p))}{4d(d^2-e^2x^2)(ae^2+bd^2)^2}+\frac{de^2(a+bx^2)^{p+1}}{4(d^2-e^2x^2)^2(ae^2+bd^2)}$$

input
```
Int[(a + b*x^2)^p/(x*(d + e*x)^3),x]
```

output
```
(d*e^2*(a + b*x^2)^(1 + p))/(4*(b*d^2 + a*e^2)*(d^2 - e^2*x^2)^2) + (e^2*(
2*a*e^2 + b*d^2*(3 - p))*(a + b*x^2)^(1 + p))/(4*d*(b*d^2 + a*e^2)^2*(d^2
- e^2*x^2)) - (3*e*x*(a + b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a),
 (e^2*x^2)/d^2])/(d^4*(1 + (b*x^2)/a)^p) - (e^3*x^3*(a + b*x^2)^p*AppellF1
[3/2, -p, 3, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(3*d^6*(1 + (b*x^2)/a)^p)
+ (e^2*(2*a^2*e^4 + 2*a*b*d^2*e^2*(2 - p) + b^2*d^4*(2 - 3*p + p^2))*(a +
b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2
 + a*e^2)])/(4*d^3*(b*d^2 + a*e^2)^3*(1 + p)) - ((a + b*x^2)^(1 + p)*Hyper
geometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*d^3*(1 + p)) + (3*b^2*d
*e^2*(a + b*x^2)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, (e^2*(a + b*x^
2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)^3*(1 + p))
```

3.428.3.1 Defintions of rubi rules used

rule 622
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[x^m*(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2
 - d^2*x^2)))^(-n), x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[n, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.428.4 Maple [F]

$$\int \frac{(b\,x^2+a)^p}{x\,(ex+d)^3}dx$$

input
```
int((b*x^2+a)^p/x/(e*x+d)^3,x)
```

output
```
int((b*x^2+a)^p/x/(e*x+d)^3,x)
```

3.428.5 Fricas [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^3x}\,dx$$

input
```
integrate((b*x^2+a)^p/x/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/(e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e*x^2 + d^3*x), x)
```

3.428.6 Sympy [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx = \int \frac{(a+bx^2)^p}{x\,(d+ex)^3}\,dx$$

input
```
integrate((b*x**2+a)**p/x/(e*x+d)**3,x)
```

output
```
Integral((a + b*x**2)**p/(x*(d + e*x)**3), x)
```

3.428.7 Maxima [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^3x}\,dx$$

input
```
integrate((b*x^2+a)^p/x/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/((e*x + d)^3*x), x)
```

3.428.8 Giac [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^3x}\,dx$$

input
```
integrate((b*x^2+a)^p/x/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/((e*x + d)^3*x), x)
```

3.428.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx = \int \frac{(b\,x^2+a)^p}{x\,(d+e\,x)^3}\,dx$$

input
```
int((a + b*x^2)^p/(x*(d + e*x)^3),x)
```

output
```
int((a + b*x^2)^p/(x*(d + e*x)^3), x)
```

3.429 $\int \frac{(a+bx^2)^p}{x^2(d+ex)^3}\,dx$

3.429.1 Optimal result

Integrand size = 20, antiderivative size = 754

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^3}\,dx$$
$$= -\frac{e^3(a+bx^2)^{1+p}}{4(bd^2+ae^2)(d^2-e^2x^2)^2}$$
$$+\frac{3e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}$$
$$+\frac{2e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}$$
$$+\frac{e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,3,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}$$
$$+\frac{2e^4x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},-p,2,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^7}$$
$$+\frac{e^4x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},-p,3,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^7}$$
$$-\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{d^3x}$$
$$-\frac{3e^3(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2d^4(bd^2+ae^2)(1+p)}$$
$$+\frac{3e(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2ad^4(1+p)}$$
$$-\frac{2be^3(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(2,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{d^2(bd^2+ae^2)^2(1+p)}$$
$$+\frac{be^3(2ae^2+bd^2(1+p))(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(2,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{4d^2(bd^2+ae^2)^3(1+p)}$$
$$-\frac{3b^2e^3(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(3,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2(bd^2+ae^2)^3(1+p)}$$

output
```
-1/4*e^3*(b*x^2+a)^(p+1)/(a*e^2+b*d^2)/(-e^2*x^2+d^2)^2+3*e^2*x*(b*x^2+a)^
p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^5/((1+b*x^2/a)^p)+2*e^2*x*
(b*x^2+a)^p*AppellF1(1/2,2,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^5/((1+b*x^2/a)^p
)+e^2*x*(b*x^2+a)^p*AppellF1(1/2,3,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^5/((1+b*
x^2/a)^p)+2/3*e^4*x^3*(b*x^2+a)^p*AppellF1(3/2,2,-p,5/2,e^2*x^2/d^2,-b*x^2
/a)/d^7/((1+b*x^2/a)^p)+e^4*x^3*(b*x^2+a)^p*AppellF1(3/2,3,-p,5/2,e^2*x^2/
d^2,-b*x^2/a)/d^7/((1+b*x^2/a)^p)-(b*x^2+a)^p*hypergeom([-1/2, -p],[1/2],-
b*x^2/a)/d^3/x/((1+b*x^2/a)^p)-3/2*e^3*(b*x^2+a)^(p+1)*hypergeom([1, p+1],
[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/d^4/(a*e^2+b*d^2)/(p+1)+3/2*e*(b*x^2+a)
^(p+1)*hypergeom([1, p+1],[2+p],1+b*x^2/a)/a/d^4/(p+1)-2*b*e^3*(b*x^2+a)^(
p+1)*hypergeom([2, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/d^2/(a*e^2+b*d^
2)^2/(p+1)+1/4*b*e^3*(2*a*e^2+b*d^2*(p+1))*(b*x^2+a)^(p+1)*hypergeom([2, p
+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/d^2/(a*e^2+b*d^2)^3/(p+1)-3/2*b^2*e
^3*(b*x^2+a)^(p+1)*hypergeom([3, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(
a*e^2+b*d^2)^3/(p+1)
```

3.429.2 Mathematica [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 478, normalized size of antiderivative = 0.63

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^3}\,dx = \frac{(a+bx^2)^p\left(\frac{4de\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+2p)(d+ex)}+\frac{d^2e\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e(\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*x^2)^p/(x^2*(d + e*x)^3),x]
```

output
```
((a + b*x^2)^p*((4*d*e*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)
]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(
a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) +
 (d^2*e*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x),
 (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + p)*((e*(-Sqrt[-(a/b)] + x))/(d +
e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)^2) + (3*e*AppellF1[
-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e
)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] +
 x))/(d + e*x))^p) - (2*d*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/
(x*(1 + (b*x^2)/a)^p) - (3*e*Hypergeometric2F1[-p, -p, 1 - p, -(a/(b*x^2))
])/(p*(1 + a/(b*x^2))^p)))/(2*d^4)
```

3.429. $\int \frac{(a+bx^2)^p}{x^2(d+ex)^3}\,dx$

3.429.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.61, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {622, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^3} \, dx$$

$$\Big\downarrow \ 622$$

$$\int \left(\frac{3de^2(a+bx^2)^p}{(d^2-e^2x^2)^3}-\frac{3d^2e(a+bx^2)^p}{x\,(d^2-e^2x^2)^3}+\frac{e^3x(a+bx^2)^p}{(e^2x^2-d^2)^3}+\frac{d^3(a+bx^2)^p}{x^2\,(d^2-e^2x^2)^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{3e^3(a+bx^2)^{p+1}\left(2a^2e^4+2abd^2e^2(2-p)+b^2d^4(p^2-3p+2)\right)\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{4d^4(p+1)\,(ae^2+bd^2)^3}-\frac{3e^2x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,3,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(-\frac{1}{2},-p,3,\frac{1}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3x}-\frac{b^2e^3(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(3,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)\,(ae^2+bd^2)^3}+\frac{3e(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{2ad^4(p+1)}-\frac{3e^3(a+bx^2)^{p+1}\left(2ae^2+bd^2(3-p)\right)}{4d^2\,(d^2-e^2x^2)\,(ae^2+bd^2)^2}-\frac{3e^3(a+bx^2)^{p+1}}{4\,(d^2-e^2x^2)^2\,(ae^2+bd^2)}$$

input
```
Int[(a + b*x^2)^p/(x^2*(d + e*x)^3),x]
```

output
```
(-3*e^3*(a + b*x^2)^(1 + p))/(4*(b*d^2 + a*e^2)*(d^2 - e^2*x^2)^2) - (3*e^
3*(2*a*e^2 + b*d^2*(3 - p))*(a + b*x^2)^(1 + p))/(4*d^2*(b*d^2 + a*e^2)^2*
(d^2 - e^2*x^2)) - ((a + b*x^2)^p*AppellF1[-1/2, -p, 3, 1/2, -((b*x^2)/a),
 (e^2*x^2)/d^2])/(d^3*x*(1 + (b*x^2)/a)^p) + (3*e^2*x*(a + b*x^2)^p*Appell
F1[1/2, -p, 3, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^5*(1 + (b*x^2)/a)^p)
- (3*e^3*(2*a^2*e^4 + 2*a*b*d^2*e^2*(2 - p) + b^2*d^4*(2 - 3*p + p^2))*(a
+ b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d
^2 + a*e^2)])/(4*d^4*(b*d^2 + a*e^2)^3*(1 + p)) + (3*e*(a + b*x^2)^(1 + p)
*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*d^4*(1 + p)) - (b
^2*e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, (e^2*(a + b*
x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)^3*(1 + p))
```

3.429.3.1 Defintions of rubi rules used

rule 622
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[x^m*(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2
 - d^2*x^2)))^(-n), x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[n, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.429.4 Maple [F]

$$\int \frac{(b\,x^2+a)^p}{x^2\,(ex+d)^3}dx$$

input
```
int((b*x^2+a)^p/x^2/(e*x+d)^3,x)
```

output
```
int((b*x^2+a)^p/x^2/(e*x+d)^3,x)
```

3.429.5 Fricas [F]

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^3} \, dx = \int \frac{(bx^2+a)^p}{(ex+d)^3 x^2} \, dx$$

input
```
integrate((b*x^2+a)^p/x^2/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/(e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*e*x^3 + d^3*x^2), x)
```

3.429.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^3} \, dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p/x**2/(e*x+d)**3,x)
```

output
```
Timed out
```

3.429.7 Maxima [F]

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^3} \, dx = \int \frac{(bx^2+a)^p}{(ex+d)^3 x^2} \, dx$$

input
```
integrate((b*x^2+a)^p/x^2/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/((e*x + d)^3*x^2), x)
```

3.429.8 Giac [F]

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^3x^2}\,dx$$

input
```
integrate((b*x^2+a)^p/x^2/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/((e*x + d)^3*x^2), x)
```

3.429.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^3}\,dx = \int \frac{(b\,x^2+a)^p}{x^2\,(d+e\,x)^3}\,dx$$

input
```
int((a + b*x^2)^p/(x^2*(d + e*x)^3),x)
```

output
```
int((a + b*x^2)^p/(x^2*(d + e*x)^3), x)
```

3.430 $\int (gx)^m (d+ex)^3 \left(a+cx^2\right)^p \, dx$

3.430.1 Optimal result

Integrand size = 22, antiderivative size = 276

$$\int (gx)^m (d+ex)^3 \left(a+cx^2\right)^p \, dx = \frac{3de^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)} + \frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)}$$
$$- \frac{d\left(3ae^2(1+m)-cd^2(3+m+2p)\right)(gx)^{1+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2}\right.}{cg(1+m)(3+m+2p)}$$
$$- \frac{e\left(ae^2(2+m)-3cd^2(4+m+2p)\right)(gx)^{2+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2+m}{2},-p,\frac{4+m}{2}\right.}{cg^2(2+m)(4+m+2p)}$$

output
```
3*d*e^2*(g*x)^(1+m)*(c*x^2+a)^(p+1)/c/g/(3+m+2*p)+e^3*(g*x)^(2+m)*(c*x^2+a
)^(p+1)/c/g^2/(4+m+2*p)-d*(3*a*e^2*(1+m)-c*d^2*(3+m+2*p))*(g*x)^(1+m)*(c*x
^2+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-c*x^2/a)/c/g/(1+m)/(3+m+2*p
)/((1+c*x^2/a)^p)-e*(a*e^2*(2+m)-3*c*d^2*(4+m+2*p))*(g*x)^(2+m)*(c*x^2+a)^
p*hypergeom([-p, 1+1/2*m],[2+1/2*m],-c*x^2/a)/c/g^2/(2+m)/(4+m+2*p)/((1+c*
x^2/a)^p)
```

3.430.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.66

$$\int (gx)^m(d+ex)^3\left(a+cx^2\right)^p\,dx$$
$$= x(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(\frac{d^3\operatorname{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},-\frac{cx^2}{a}\right)}{1+m}+ex\left(\frac{3d^2\operatorname{Hypergeometric2F1}\left(\frac{2+m}{2},-p,\frac{4+m}{2},-\frac{cx^2}{a}\right)}{2+m}+ex\left(\frac{3d\operatorname{Hypergeometric2F1}\left(\frac{3+m}{2},-p,\frac{5+m}{2},-\frac{cx^2}{a}\right)}{3+m}+\frac{ex\operatorname{Hypergeometric2F1}\left(\frac{4+m}{2},-p,\frac{6+m}{2},-\frac{cx^2}{a}\right)}{4+m}\right)\right)\right)$$

input
```
Integrate[(g*x)^m*(d + e*x)^3*(a + c*x^2)^p,x]
```

output
```
(x*(g*x)^m*(a + c*x^2)^p*((d^3*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2,
 -((c*x^2)/a)])/(1 + m) + e*x*((3*d^2*Hypergeometric2F1[(2 + m)/2, -p, (4
+ m)/2, -((c*x^2)/a)])/(2 + m) + e*x*((3*d*Hypergeometric2F1[(3 + m)/2, -p
, (5 + m)/2, -((c*x^2)/a)])/(3 + m) + (e*x*Hypergeometric2F1[(4 + m)/2, -p
, (6 + m)/2, -((c*x^2)/a)])/(4 + m)))))/(1 + (c*x^2)/a)^p
```

3.430.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {559, 2340, 25, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3(gx)^m\left(a+cx^2\right)^p\,dx$$
$$\Big\downarrow \; 559$$

$$\frac{\int (gx)^m \left(cx^2+a\right)^p \left(c(m+2p+4)d^3+3ce^2(m+2p+4)x^2d-e\left(ae^2(m+2)-3cd^2(m+2p+4)\right)x\right)dx}{c(m+2p+4)}+\frac{e^3(gx)^{m+2}\left(a+cx^2\right)^{p+1}}{cg^2(m+2p+4)}$$

$\Big\downarrow$ 2340

$$\frac{\frac{\int -c(gx)^m \left(d(m+2p+4)\left(3ae^2(m+1)-cd^2(m+2p+3)\right)+e(m+2p+3)\left(ae^2(m+2)-3cd^2(m+2p+4)\right)x\right)\left(cx^2+a\right)^p dx}{c(m+2p+3)}+\frac{3de^2(m+2p+4)(gx)^{m+1}(a\cdots}{g(m+2p+3)}}{c(m+2p+4)}+\frac{e^3(gx)^{m+2}\left(a+cx^2\right)^{p+1}}{cg^2(m+2p+4)}$$

$\Big\downarrow$ 25

$$\frac{\frac{3de^2(m+2p+4)(gx)^{m+1}\left(a+cx^2\right)^{p+1}}{g(m+2p+3)}-\frac{\int c(gx)^m \left(d(m+2p+4)\left(3ae^2(m+1)-cd^2(m+2p+3)\right)+e(m+2p+3)\left(ae^2(m+2)-3cd^2(m+2p+4)\right)x\right)\left(cx^2\cdots\right.}{c(m+2p+3)}}{c(m+2p+4)}+\frac{e^3(gx)^{m+2}\left(a+cx^2\right)^{p+1}}{cg^2(m+2p+4)}$$

$\Big\downarrow$ 27

$$\frac{\frac{3de^2(m+2p+4)(gx)^{m+1}\left(a+cx^2\right)^{p+1}}{g(m+2p+3)}-\frac{\int (gx)^m \left(d(m+2p+4)\left(3ae^2(m+1)-cd^2(m+2p+3)\right)+e(m+2p+3)\left(ae^2(m+2)-3cd^2(m+2p+4)\right)x\right)\left(cx^2+\cdots\right.}{m+2p+3}}{c(m+2p+4)}+\frac{e^3(gx)^{m+2}\left(a+cx^2\right)^{p+1}}{cg^2(m+2p+4)}$$

$\Big\downarrow$ 557

$$\frac{\frac{3de^2(m+2p+4)(gx)^{m+1}\left(a+cx^2\right)^{p+1}}{g(m+2p+3)}-\frac{d(m+2p+4)\left(3ae^2(m+1)-cd^2(m+2p+3)\right)\int (gx)^m\left(cx^2+a\right)^p dx+\frac{e(m+2p+3)\left(ae^2(m+2)-3cd^2(m+2p+4)\right)\int\cdots}{g}}{m+2p+3}}{c(m+2p+4)}+\frac{e^3(gx)^{m+2}\left(a+cx^2\right)^{p+1}}{cg^2(m+2p+4)}$$

$\Big\downarrow$ 279

$$\frac{\frac{3de^2(m+2p+4)(gx)^{m+1}\left(a+cx^2\right)^{p+1}}{g(m+2p+3)}-\frac{d(m+2p+4)\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(3ae^2(m+1)-cd^2(m+2p+3)\right)\int (gx)^m\left(\frac{cx^2}{a}+1\right)^p dx+\frac{e(m+2p+3)\left(a+c\cdots\right.}{\cdots}}{m+2p+3}}{c(m+2p+4)}+\frac{e^3(gx)^{m+2}\left(a+cx^2\right)^{p+1}}{cg^2(m+2p+4)}$$

3.430. $\int (gx)^m(d+ex)^3\left(a+cx^2\right)^p\,dx$

$$\Big\downarrow \text{278}$$

$$\frac{\frac{3de^2(m+2p+4)(gx)^{m+1}\left(a+cx^2\right)^{p+1}}{g(m+2p+3)}-\frac{\frac{e(m+2p+3)(gx)^{m+2}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(ae^2(m+2)-3cd^2(m+2p+4)\right)\text{Hypergeometric2F1}\left(\frac{m+2}{2},-p,\frac{m+4}{2},-\text{c}\right.}{g^2(m+2)}}{}}{c(m+2p+4)}$$

$$\frac{e^3(gx)^{m+2}\left(a+cx^2\right)^{p+1}}{cg^2(m+2p+4)}$$

input
```
Int[(g*x)^m*(d + e*x)^3*(a + c*x^2)^p,x]
```

output
```
(e^3*(g*x)^(2 + m)*(a + c*x^2)^(1 + p))/(c*g^2*(4 + m + 2*p)) + ((3*d*e^2*
(4 + m + 2*p)*(g*x)^(1 + m)*(a + c*x^2)^(1 + p))/(g*(3 + m + 2*p)) - ((d*(
4 + m + 2*p)*(3*a*e^2*(1 + m) - c*d^2*(3 + m + 2*p))*(g*x)^(1 + m)*(a + c*
x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -((c*x^2)/a)])/(g*(1 +
m)*(1 + (c*x^2)/a)^p) + (e*(3 + m + 2*p)*(a*e^2*(2 + m) - 3*c*d^2*(4 + m +
 2*p))*(g*x)^(2 + m)*(a + c*x^2)^p*Hypergeometric2F1[(2 + m)/2, -p, (4 + m
)/2, -((c*x^2)/a)])/(g^2*(2 + m)*(1 + (c*x^2)/a)^p))/(3 + m + 2*p))/(c*(4
+ m + 2*p))
```

3.430.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

3.430. $\int (gx)^m(d+ex)^3\left(a+cx^2\right)^p\,dx$

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 559
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[d^n*(e*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*e^(n - 1)*(
m + n + 2*p + 1))), x] + Simp[1/(b*(m + n + 2*p + 1))   Int[(e*x)^m*(a + b*
x^2)^p*ExpandToSum[b*(m + n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1
)*x^n - a*d^n*(m + n - 1)*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && IGtQ[n, 1] &&  !IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

3.430.4 Maple [F]

$$\int (gx)^m (ex+d)^3 (c\,x^2+a)^p\,dx$$

input
```
int((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x)
```

output
```
int((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x)
```

3.430.5 Fricas [F]

$$\int (gx)^m(d+ex)^3\left(a+cx^2\right)^p\,dx=\int (ex+d)^3(cx^2+a)^p(gx)^m\,dx$$

input
```
integrate((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(c*x^2 + a)^p*(g*x)^m,
x)
```

3.430.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 113.06 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.83

$$\int (gx)^m(d+ex)^3\left(a+cx^2\right)^p\,dx = \frac{a^p d^3 g^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{cx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{3a^p d^2 e g^m x^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{cx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+2\right)} + \frac{3a^p d e^2 g^m x^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|\frac{cx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{a^p e^3 g^m x^{m+4}\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\middle|\frac{cx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+3\right)}$$

input
```
integrate((g*x)**m*(e*x+d)**3*(c*x**2+a)**p,x)
```

output
```
a**p*d**3*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3
/2,), c*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2)) + 3*a**p*d**2*e*g**m*
x**(m + 2)*gamma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), c*x**2*exp_pola
r(I*pi)/a)/(2*gamma(m/2 + 2)) + 3*a**p*d*e**2*g**m*x**(m + 3)*gamma(m/2 +
3/2)*hyper((-p, m/2 + 3/2), (m/2 + 5/2,), c*x**2*exp_polar(I*pi)/a)/(2*gam
ma(m/2 + 5/2)) + a**p*e**3*g**m*x**(m + 4)*gamma(m/2 + 2)*hyper((-p, m/2 +
 2), (m/2 + 3,), c*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3))
```

3.430.7 Maxima [F]

$$\int (gx)^m (d+ex)^3 \left(a+cx^2\right)^p \, dx = \int (ex+d)^3 (cx^2+a)^p (gx)^m \, dx$$

input
```
integrate((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3*(c*x^2 + a)^p*(g*x)^m, x)
```

3.430.8 Giac [F]

$$\int (gx)^m (d+ex)^3 \left(a+cx^2\right)^p \, dx = \int (ex+d)^3 (cx^2+a)^p (gx)^m \, dx$$

input
```
integrate((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(c*x^2 + a)^p*(g*x)^m, x)
```

3.430.9 Mupad [F(-1)]

Timed out.

$$\int (gx)^m (d+ex)^3 \left(a+cx^2\right)^p \, dx = \int (g\,x)^m \left(c\,x^2+a\right)^p (d+e\,x)^3 \, dx$$

input
```
int((g*x)^m*(a + c*x^2)^p*(d + e*x)^3,x)
```

output
```
int((g*x)^m*(a + c*x^2)^p*(d + e*x)^3, x)
```

3.431 $\int (gx)^m(d+ex)^2\left(a+cx^2\right)^p\,dx$

3.431.1 Optimal result

Integrand size = 22, antiderivative size = 205

$$\int (gx)^m(d+ex)^2\left(a+cx^2\right)^p\,dx=\frac{e^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)}-\frac{\left(ae^2(1+m)-cd^2(3+m+2p)\right)(gx)^{1+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},-\right.}{cg(1+m)(3+m+2p)}+\frac{2de(gx)^{2+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2+m}{2},-p,\frac{4+m}{2},-\frac{cx^2}{a}\right)}{g^2(2+m)}$$

output
```
e^2*(g*x)^(1+m)*(c*x^2+a)^(p+1)/c/g/(3+m+2*p)-(a*e^2*(1+m)-c*d^2*(3+m+2*p)
)*(g*x)^(1+m)*(c*x^2+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-c*x^2/a)/
c/g/(1+m)/(3+m+2*p)/((1+c*x^2/a)^p)+2*d*e*(g*x)^(2+m)*(c*x^2+a)^p*hypergeo
m([-p, 1+1/2*m],[2+1/2*m],-c*x^2/a)/g^2/(2+m)/((1+c*x^2/a)^p)
```

3.431.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.77

$$\int (gx)^m(d+ex)^2\left(a+cx^2\right)^p\,dx=\frac{x(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d^2(6+5m+m^2)\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},-\frac{cx^2}{a}\right)+e(1+m)\right.}{(1+m}$$

input
```
Integrate[(g*x)^m*(d + e*x)^2*(a + c*x^2)^p,x]
```

output
```
(x*(g*x)^m*(a + c*x^2)^p*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[(1 + m)/2,
 -p, (3 + m)/2, -((c*x^2)/a)] + e*(1 + m)*x*(2*d*(3 + m)*Hypergeometric2F1
[(2 + m)/2, -p, (4 + m)/2, -((c*x^2)/a)] + e*(2 + m)*x*Hypergeometric2F1[(
3 + m)/2, -p, (5 + m)/2, -((c*x^2)/a)])))/((1 + m)*(2 + m)*(3 + m)*(1 + (c
*x^2)/a)^p)
```

3.431.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {559, 25, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2(gx)^m \left(a+cx^2\right)^p \, dx$$

$$\Big\downarrow \; 559$$

$$\frac{\int -(gx)^m \left(-c(m+2p+3)d^2-2ce(m+2p+3)xd+ae^2(m+1)\right) \left(cx^2+a\right)^p dx}{c(m+2p+3)}+\frac{e^2(gx)^{m+1} \left(a+cx^2\right)^{p+1}}{cg(m+2p+3)}$$

$$\Big\downarrow \; 25$$

$$\frac{e^2(gx)^{m+1} \left(a+cx^2\right)^{p+1}}{cg(m+2p+3)}-\frac{\int (gx)^m \left(-c(m+2p+3)d^2-2ce(m+2p+3)xd+ae^2(m+1)\right) \left(cx^2+a\right)^p dx}{c(m+2p+3)}$$

$$\Big\downarrow \; 557$$

$$\frac{e^2(gx)^{m+1} \left(a+cx^2\right)^{p+1}}{cg(m+2p+3)}-\frac{\left(ae^2(m+1)-cd^2(m+2p+3)\right) \int (gx)^m \left(cx^2+a\right)^p dx-\frac{2cde(m+2p+3) \int (gx)^{m+1} \left(cx^2+a\right)^p dx}{g}}{c(m+2p+3)}$$

$$\Big\downarrow \; 279$$

$$\frac{e^2(gx)^{m+1}\left(a+cx^2\right)^{p+1}}{cg(m+2p+3)}-$$

$$\frac{\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(ae^2(m+1)-cd^2(m+2p+3)\right)\int (gx)^m\left(\frac{cx^2}{a}+1\right)^p dx-\frac{2cde(m+2p+3)\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\int}{g}}{c(m+2p+3)}$$

$$\Big\downarrow \text{278}$$

$$\frac{e^2(gx)^{m+1}\left(a+cx^2\right)^{p+1}}{cg(m+2p+3)}-$$

$$\frac{\frac{(gx)^{m+1}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(ae^2(m+1)-cd^2(m+2p+3)\right)\text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},-\frac{cx^2}{a}\right)}{g(m+1)}-\frac{2cde(m+2p+3)(gx)^{m+2}\left(a+cx^2\right)^p\left(\right.}{}}{c(m+2p+3)}$$

input
```
Int[(g*x)^m*(d + e*x)^2*(a + c*x^2)^p,x]
```

output
```
(e^2*(g*x)^(1 + m)*(a + c*x^2)^(1 + p))/(c*g*(3 + m + 2*p)) - (((a*e^2*(1
+ m) - c*d^2*(3 + m + 2*p))*(g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1[
(1 + m)/2, -p, (3 + m)/2, -((c*x^2)/a)])/(g*(1 + m)*(1 + (c*x^2)/a)^p) - (
2*c*d*e*(3 + m + 2*p)*(g*x)^(2 + m)*(a + c*x^2)^p*Hypergeometric2F1[(2 + m
)/2, -p, (4 + m)/2, -((c*x^2)/a)])/(g^2*(2 + m)*(1 + (c*x^2)/a)^p))/(c*(3
+ m + 2*p))
```

3.431.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 559
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[d^n*(e*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*e^(n - 1)*(
m + n + 2*p + 1))), x] + Simp[1/(b*(m + n + 2*p + 1))   Int[(e*x)^m*(a + b*
x^2)^p*ExpandToSum[b*(m + n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1
)*x^n - a*d^n*(m + n - 1)*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && IGtQ[n, 1] &&  !IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

3.431.4 Maple [F]

$$\int (gx)^m (ex+d)^2 (c\,x^2+a)^p\,dx$$

input
```
int((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x)
```

output
```
int((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x)
```

3.431.5 Fricas [F]

$$\int (gx)^m (d+ex)^2 (a+cx^2)^p\,dx = \int (ex+d)^2 (cx^2+a)^p (gx)^m\,dx$$

input
```
integrate((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e^2*x^2 + 2*d*e*x + d^2)*(c*x^2 + a)^p*(g*x)^m, x)
```

3.431.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 70.15 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.82

$$\int (gx)^m (d+ex)^2 \left(a+cx^2\right)^p \, dx = \frac{a^p d^2 g^m x^{m+1} \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{cx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{a^p d e g^m x^{m+2} \Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+1 \\ \frac{m}{2}+2 \end{matrix}\middle| \frac{cx^2 e^{i\pi}}{a}\right)}{\Gamma\left(\frac{m}{2}+2\right)} + \frac{a^p e^2 g^m x^{m+3} \Gamma\left(\frac{m}{2}+\frac{3}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{3}{2} \\ \frac{m}{2}+\frac{5}{2} \end{matrix}\middle| \frac{cx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

input
```
integrate((g*x)**m*(e*x+d)**2*(c*x**2+a)**p,x)
```

output
```
a**p*d**2*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3
/2,), c*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2)) + a**p*d*e*g**m*x**(m
 + 2)*gamma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), c*x**2*exp_polar(I*p
i)/a)/gamma(m/2 + 2) + a**p*e**2*g**m*x**(m + 3)*gamma(m/2 + 3/2)*hyper((-
p, m/2 + 3/2), (m/2 + 5/2,), c*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 5/2)
)
```

3.431.7 Maxima [F]

$$\int (gx)^m (d+ex)^2 \left(a+cx^2\right)^p \, dx = \int (ex+d)^2 (cx^2+a)^p (gx)^m \, dx$$

input
```
integrate((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2*(c*x^2 + a)^p*(g*x)^m, x)
```

3.431.8 Giac [F]

$$\int (gx)^m (d+ex)^2 \left(a+cx^2\right)^p \,dx = \int (ex+d)^2 (cx^2+a)^p (gx)^m \,dx$$

input
```
integrate((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(c*x^2 + a)^p*(g*x)^m, x)
```

3.431.9 Mupad [F(-1)]

Timed out.

$$\int (gx)^m (d+ex)^2 \left(a+cx^2\right)^p \,dx = \int (g\,x)^m \left(c\,x^2+a\right)^p \left(d+e\,x\right)^2 dx$$

input
```
int((g*x)^m*(a + c*x^2)^p*(d + e*x)^2,x)
```

output
```
int((g*x)^m*(a + c*x^2)^p*(d + e*x)^2, x)
```

3.432 $\int (gx)^m(d+ex)\left(a+cx^2\right)^p\,dx$

3.432.1 Optimal result

Integrand size = 20, antiderivative size = 135

$$\int (gx)^m(d+ex)\left(a+cx^2\right)^p\,dx = \frac{d(gx)^{1+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},-\frac{cx^2}{a}\right)}{g(1+m)} + \frac{e(gx)^{2+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2+m}{2},-p,\frac{4+m}{2},-\frac{cx^2}{a}\right)}{g^2(2+m)}$$

output
```
d*(g*x)^(1+m)*(c*x^2+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-c*x^2/a)/
g/(1+m)/((1+c*x^2/a)^p)+e*(g*x)^(2+m)*(c*x^2+a)^p*hypergeom([-p, 1+1/2*m],
[2+1/2*m],-c*x^2/a)/g^2/(2+m)/((1+c*x^2/a)^p)
```

3.432.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.79

$$\int (gx)^m(d+ex)\left(a+cx^2\right)^p\,dx = \frac{x(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d(2+m)\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},-\frac{cx^2}{a}\right)+e(1+m)x\,\text{Hyper}\right.}{(1+m)(2+m)}$$

input
```
Integrate[(g*x)^m*(d + e*x)*(a + c*x^2)^p,x]
```

output
```
(x*(g*x)^m*(a + c*x^2)^p*(d*(2 + m)*Hypergeometric2F1[(1 + m)/2, -p, (3 +
m)/2, -((c*x^2)/a)] + e*(1 + m)*x*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)
/2, -((c*x^2)/a)]))/((1 + m)*(2 + m)*(1 + (c*x^2)/a)^p)
```

3.432.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)(gx)^m \left(a+cx^2\right)^p dx$$

$$\Big\downarrow \text{557}$$

$$d\int (gx)^m \left(cx^2+a\right)^p dx+\frac{e\int (gx)^{m+1} \left(cx^2+a\right)^p dx}{g}$$

$$\Big\downarrow \text{279}$$

$$d\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p}\int (gx)^m \left(\frac{cx^2}{a}+1\right)^p dx+\frac{e\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p}\int (gx)^{m+1} \left(\frac{cx^2}{a}+1\right)^p dx}{g}$$

$$\Big\downarrow \text{278}$$

$$\frac{d(gx)^{m+1} \left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},-\frac{cx^2}{a}\right)}{g(m+1)}+\frac{e(gx)^{m+2} \left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+2}{2},-p,\frac{m+4}{2},-\frac{cx^2}{a}\right)}{g^2(m+2)}$$

input
```
Int[(g*x)^m*(d + e*x)*(a + c*x^2)^p,x]
```

output
```
(d*(g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2,
 -((c*x^2)/a)])/(g*(1 + m)*(1 + (c*x^2)/a)^p) + (e*(g*x)^(2 + m)*(a + c*x^
2)^p*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2, -((c*x^2)/a)])/(g^2*(2 +
m)*(1 + (c*x^2)/a)^p)
```

3.432.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

3.432.4 Maple [F]

$$\int (gx)^m (ex+d) \left(c\,x^2+a\right)^p dx$$

input
```
int((g*x)^m*(e*x+d)*(c*x^2+a)^p,x)
```

output
```
int((g*x)^m*(e*x+d)*(c*x^2+a)^p,x)
```

3.432.5 Fricas [F]

$$\int (gx)^m(d+ex)\left(a+cx^2\right)^p\,dx=\int (ex+d)(cx^2+a)^p(gx)^m\,dx$$

input
```
integrate((g*x)^m*(e*x+d)*(c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e*x + d)*(c*x^2 + a)^p*(g*x)^m, x)
```

3.432.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 37.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.79

$$\int (gx)^m(d+ex)\left(a+cx^2\right)^p\,dx=\frac{a^p d g^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{cx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{a^p e g^m x^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{cx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}$$

input
```
integrate((g*x)**m*(e*x+d)*(c*x**2+a)**p,x)
```

output
```
a**p*d*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,
), c*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2)) + a**p*e*g**m*x**(m + 2)
*gamma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), c*x**2*exp_polar(I*pi)/a)
/(2*gamma(m/2 + 2))
```

3.432.7 Maxima [F]

$$\int (gx)^m(d+ex)\left(a+cx^2\right)^p\,dx = \int (ex+d)(cx^2+a)^p(gx)^m\,dx$$

input
```
integrate((g*x)^m*(e*x+d)*(c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)*(c*x^2 + a)^p*(g*x)^m, x)
```

3.432.8 Giac [F]

$$\int (gx)^m(d+ex)\left(a+cx^2\right)^p\,dx = \int (ex+d)(cx^2+a)^p(gx)^m\,dx$$

input
```
integrate((g*x)^m*(e*x+d)*(c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(c*x^2 + a)^p*(g*x)^m, x)
```

3.432.9 Mupad [F(-1)]

Timed out.

$$\int (gx)^m(d+ex)\left(a+cx^2\right)^p\,dx = \int (g\,x)^m\,\left(c\,x^2+a\right)^p\,(d+e\,x)\,dx$$

input
```
int((g*x)^m*(a + c*x^2)^p*(d + e*x),x)
```

output
```
int((g*x)^m*(a + c*x^2)^p*(d + e*x), x)
```

3.433 $\int (gx)^m (a+cx^2)^p \, dx$

3.433.1 Optimal result

Integrand size = 15, antiderivative size = 66

$$\int (gx)^m (a+cx^2)^p \, dx = \frac{(gx)^{1+m} (a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},-\frac{cx^2}{a}\right)}{g(1+m)}$$

output
```
(g*x)^(1+m)*(c*x^2+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-c*x^2/a)/g/
(1+m)/((1+c*x^2/a)^p)
```

3.433.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int (gx)^m (a+cx^2)^p \, dx = \frac{x(gx)^m (a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,1+\frac{1+m}{2},-\frac{cx^2}{a}\right)}{1+m}$$

input
```
Integrate[(g*x)^m*(a + c*x^2)^p,x]
```

output
```
(x*(g*x)^m*(a + c*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, 1 + (1 + m)/2, -
((c*x^2)/a)])/((1 + m)*(1 + (c*x^2)/a)^p)
```

3.433.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (gx)^m \left(a+cx^2\right)^p \, dx$$

$$\Big\downarrow \text{ 279}$$

$$\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} \int (gx)^m \left(\frac{cx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{ 278}$$

$$\frac{(gx)^{m+1} \left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},-\frac{cx^2}{a}\right)}{g(m+1)}$$

input
```
Int[(g*x)^m*(a + c*x^2)^p,x]
```

output
```
((g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -
((c*x^2)/a)])/(g*(1 + m)*(1 + (c*x^2)/a)^p)
```

3.433.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

3.433.4 Maple [F]

$$\int (gx)^m \left(c\,x^2+a\right)^p dx$$

input
```
int((g*x)^m*(c*x^2+a)^p,x)
```

output
```
int((g*x)^m*(c*x^2+a)^p,x)
```

3.433.5 Fricas [F]

$$\int (gx)^m \left(a+cx^2\right)^p \,dx = \int \left(cx^2+a\right)^p (gx)^m \,dx$$

input
```
integrate((g*x)^m*(c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + a)^p*(g*x)^m, x)
```

3.433.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.52 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int (gx)^m \left(a+cx^2\right)^p \,dx = \frac{a^p g^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{cx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

input
```
integrate((g*x)**m*(c*x**2+a)**p,x)
```

output
```
a**p*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,),
 c*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))
```

3.433.7 Maxima [F]

$$\int (gx)^m (a+cx^2)^p \, dx = \int (cx^2+a)^p (gx)^m \, dx$$

input
```
integrate((g*x)^m*(c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)^p*(g*x)^m, x)
```

3.433.8 Giac [F]

$$\int (gx)^m (a+cx^2)^p \, dx = \int (cx^2+a)^p (gx)^m \, dx$$

input
```
integrate((g*x)^m*(c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)^p*(g*x)^m, x)
```

3.433.9 Mupad [F(-1)]

Timed out.

$$\int (gx)^m (a+cx^2)^p \, dx = \int (g\,x)^m \left(c\,x^2+a\right)^p dx$$

input
```
int((g*x)^m*(a + c*x^2)^p,x)
```

output
```
int((g*x)^m*(a + c*x^2)^p, x)
```

3.434 $\int \frac{(gx)^m(a+cx^2)^p}{d+ex} dx$

3.434.1 Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \frac{(gx)^m (a+cx^2)^p}{d+ex} dx = \frac{x(gx)^m (a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p} \text{AppellF1}\left(\frac{1+m}{2},-p,1,\frac{3+m}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d(1+m)} - \frac{ex^2(gx)^m (a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p} \text{AppellF1}\left(\frac{2+m}{2},-p,1,\frac{4+m}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2(2+m)}$$

output
```
x*(g*x)^m*(c*x^2+a)^p*AppellF1(1/2+1/2*m,1,-p,3/2+1/2*m,e^2*x^2/d^2,-c*x^2
/a)/d/(1+m)/((1+c*x^2/a)^p)-e*x^2*(g*x)^m*(c*x^2+a)^p*AppellF1(1+1/2*m,1,-
p,2+1/2*m,e^2*x^2/d^2,-c*x^2/a)/d^2/(2+m)/((1+c*x^2/a)^p)
```

3.434.2 Mathematica [F]

$$\int \frac{(gx)^m (a+cx^2)^p}{d+ex} dx = \int \frac{(gx)^m (a+cx^2)^p}{d+ex} dx$$

input
```
Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x),x]
```

output
```
Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x), x]
```

3.434.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {623, 621, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(gx)^m (a+cx^2)^p}{d+ex} \, dx$$

$$\Big\downarrow \ 623$$

$$x^{-m}(gx)^m \int \frac{x^m (cx^2+a)^p}{d+ex}dx$$

$$\Big\downarrow \ 621$$

$$x^{-m}(gx)^m \left(d \int \frac{x^m (cx^2+a)^p}{d^2-e^2x^2}dx - e \int \frac{x^{m+1} (cx^2+a)^p}{d^2-e^2x^2}dx\right)$$

$$\Big\downarrow \ 395$$

$$x^{-m}(gx)^m \left(d(a+cx^2)^p \left(\frac{cx^2}{a}+1\right)^{-p} \int \frac{x^m \left(\frac{cx^2}{a}+1\right)^p}{d^2-e^2x^2}dx - e(a+cx^2)^p \left(\frac{cx^2}{a}+1\right)^{-p} \int \frac{x^{m+1} \left(\frac{cx^2}{a}+1\right)^p}{d^2-e^2x^2}dx\right)$$

$$\Big\downarrow \ 394$$

$$x^{-m}(gx)^m \left(\frac{x^{m+1}(a+cx^2)^p \left(\frac{cx^2}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+1}{2},-p,1,\frac{m+3}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d(m+1)} - \frac{ex^{m+2}(a+cx^2)^p \left(\frac{cx^2}{a}+1\right)}{}\right.$$

input
```
Int[((g*x)^m*(a + c*x^2)^p)/(d + e*x),x]
```

output
```
((g*x)^m*((x^(1 + m)*(a + c*x^2)^p*AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -
((c*x^2)/a), (e^2*x^2)/d^2])/(d*(1 + m)*(1 + (c*x^2)/a)^p) - (e*x^(2 + m)*
(a + c*x^2)^p*AppellF1[(2 + m)/2, -p, 1, (4 + m)/2, -((c*x^2)/a), (e^2*x^2
)/d^2])/(d^2*(2 + m)*(1 + (c*x^2)/a)^p)))/x^m
```

3.434.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 621
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[c   Int[x^m*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] - Simp[d   Int[
x^(m + 1)*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, m,
p}, x]
```

rule 623
```
Int[((e_)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(e*x)^m/x^m   Int[x^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /
; FreeQ[{a, b, c, d, e, m, p}, x] && ILtQ[n, 0]
```

3.434.4 Maple [F]

$$\int \frac{(gx)^m (c\,x^2+a)^p}{ex+d}dx$$

input
```
int((g*x)^m*(c*x^2+a)^p/(e*x+d),x)
```

output
```
int((g*x)^m*(c*x^2+a)^p/(e*x+d),x)
```

3.434.5 Fricas [F]

$$\int \frac{(gx)^m (a + cx^2)^p}{d + ex} dx = \int \frac{(cx^2 + a)^p (gx)^m}{ex + d} dx$$

input
```
integrate((g*x)^m*(c*x^2+a)^p/(e*x+d),x, algorithm="fricas")
```

output
```
integral((c*x^2 + a)^p*(g*x)^m/(e*x + d), x)
```

3.434.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(gx)^m (a + cx^2)^p}{d + ex} dx = \text{Timed out}$$

input
```
integrate((g*x)**m*(c*x**2+a)**p/(e*x+d),x)
```

output
```
Timed out
```

3.434.7 Maxima [F]

$$\int \frac{(gx)^m (a + cx^2)^p}{d + ex} dx = \int \frac{(cx^2 + a)^p (gx)^m}{ex + d} dx$$

input
```
integrate((g*x)^m*(c*x^2+a)^p/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d), x)
```

3.434.8 Giac [F]

$$\int \frac{(gx)^m (a+cx^2)^p}{d+ex}\,dx = \int \frac{(cx^2+a)^p (gx)^m}{ex+d}\,dx$$

input
```
integrate((g*x)^m*(c*x^2+a)^p/(e*x+d),x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d), x)
```

3.434.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m (a+cx^2)^p}{d+ex}\,dx = \int \frac{(g\,x)^m\,(c\,x^2+a)^p}{d+e\,x}\,dx$$

input
```
int(((g*x)^m*(a + c*x^2)^p)/(d + e*x),x)
```

output
```
int(((g*x)^m*(a + c*x^2)^p)/(d + e*x), x)
```

3.435 $\int \frac{(gx)^m(a+cx^2)^p}{(d+ex)^2}\,dx$

3.435.1 Optimal result

Integrand size = 22, antiderivative size = 238

$$\begin{aligned}\int \frac{(gx)^m\,(a+cx^2)^p}{(d+ex)^2}\,dx &= \frac{x(gx)^m\,(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1+m}{2},-p,2,\frac{3+m}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2(1+m)}\\&\quad-\frac{2ex^2(gx)^m\,(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{2+m}{2},-p,2,\frac{4+m}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3(2+m)}\\&\quad+\frac{e^2x^3(gx)^m\,(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{3+m}{2},-p,2,\frac{5+m}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4(3+m)}\end{aligned}$$

output
```
x*(g*x)^m*(c*x^2+a)^p*AppellF1(1/2+1/2*m,2,-p,3/2+1/2*m,e^2*x^2/d^2,-c*x^2
/a)/d^2/(1+m)/((1+c*x^2/a)^p)-2*e*x^2*(g*x)^m*(c*x^2+a)^p*AppellF1(1+1/2*m
,2,-p,2+1/2*m,e^2*x^2/d^2,-c*x^2/a)/d^3/(2+m)/((1+c*x^2/a)^p)+e^2*x^3*(g*x
)^m*(c*x^2+a)^p*AppellF1(3/2+1/2*m,2,-p,5/2+1/2*m,e^2*x^2/d^2,-c*x^2/a)/d^
4/(3+m)/((1+c*x^2/a)^p)
```

3.435.2 Mathematica [F]

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^2}\,dx = \int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^2}\,dx$$

input
```
Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2,x]
```

output
```
Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2, x]
```

3.435.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {623, 622, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^2}\,dx$$

$$\Big\downarrow \text{623}$$

$$x^{-m}(gx)^m \int \frac{x^m (cx^2+a)^p}{(d+ex)^2}dx$$

$$\Big\downarrow \text{622}$$

$$x^{-m}(gx)^m \int \left(\frac{d^2 (cx^2+a)^p x^m}{(d^2-e^2x^2)^2} - \frac{2de (cx^2+a)^p x^{m+1}}{(d^2-e^2x^2)^2} + \frac{e^2 (cx^2+a)^p x^{m+2}}{(e^2x^2-d^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x^{-m}(gx)^m \left(\frac{x^{m+1}(a+cx^2)^p \left(\frac{cx^2}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+1}{2},-p,2,\frac{m+3}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2(m+1)} + \frac{e^2x^{m+3}(a+cx^2)^p \left(\frac{cx^2}{a}+1\right.}{}\right.$$

input
```
Int[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2,x]
```

output
```
((g*x)^m*((x^(1 + m)*(a + c*x^2)^p*AppellF1[(1 + m)/2, -p, 2, (3 + m)/2, -
((c*x^2)/a), (e^2*x^2)/d^2])/(d^2*(1 + m)*(1 + (c*x^2)/a)^p) - (2*e*x^(2 +
 m)*(a + c*x^2)^p*AppellF1[(2 + m)/2, -p, 2, (4 + m)/2, -((c*x^2)/a), (e^2
*x^2)/d^2])/(d^3*(2 + m)*(1 + (c*x^2)/a)^p) + (e^2*x^(3 + m)*(a + c*x^2)^p
*AppellF1[(3 + m)/2, -p, 2, (5 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^4*
(3 + m)*(1 + (c*x^2)/a)^p)))/x^m
```

3.435.3.1 Defintions of rubi rules used

rule 622
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[x^m*(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2
 - d^2*x^2)))^(-n), x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[n, -1]
```

rule 623
```
Int[((e_)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(e*x)^m/x^m   Int[x^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /
; FreeQ[{a, b, c, d, e, m, p}, x] && ILtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.435.4 Maple [F]

$$\int \frac{(gx)^m (c\,x^2+a)^p}{(ex+d)^2}dx$$

input
```
int((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x)
```

output
```
int((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x)
```

3.435.5 Fricas [F]

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^2} \, dx = \int \frac{(cx^2+a)^p (gx)^m}{(ex+d)^2} \, dx$$

input
```
integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((c*x^2 + a)^p*(g*x)^m/(e^2*x^2 + 2*d*e*x + d^2), x)
```

3.435.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^2} \, dx = \text{Timed out}$$

input
```
integrate((g*x)**m*(c*x**2+a)**p/(e*x+d)**2,x)
```

output
```
Timed out
```

3.435.7 Maxima [F]

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^2} \, dx = \int \frac{(cx^2+a)^p (gx)^m}{(ex+d)^2} \, dx$$

input
```
integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d)^2, x)
```

3.435.8 Giac [F]

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^2}\,dx = \int \frac{(cx^2+a)^p (gx)^m}{(ex+d)^2}\,dx$$

input
```
integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d)^2, x)
```

3.435.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^2}\,dx = \int \frac{(g\,x)^m\,(c\,x^2+a)^p}{(d+e\,x)^2}\,dx$$

input
```
int(((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2,x)
```

output
```
int(((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2, x)
```

3.436 $\int \frac{(gx)^m(a+cx^2)^p}{(d+ex)^3}\,dx$

3.436.1 Optimal result

Integrand size = 22, antiderivative size = 321

$$\begin{aligned}\int \frac{(gx)^m(a+cx^2)^p}{(d+ex)^3}\,dx &= \frac{x(gx)^m(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1+m}{2},-p,3,\frac{3+m}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3(1+m)}\\&\quad-\frac{3ex^2(gx)^m(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{2+m}{2},-p,3,\frac{4+m}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4(2+m)}\\&\quad+\frac{3e^2x^3(gx)^m(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{3+m}{2},-p,3,\frac{5+m}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5(3+m)}\\&\quad-\frac{e^3x^4(gx)^m(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{4+m}{2},-p,3,\frac{6+m}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^6(4+m)}\end{aligned}$$

output
```
x*(g*x)^m*(c*x^2+a)^p*AppellF1(1/2+1/2*m,3,-p,3/2+1/2*m,e^2*x^2/d^2,-c*x^2
/a)/d^3/(1+m)/((1+c*x^2/a)^p)-3*e*x^2*(g*x)^m*(c*x^2+a)^p*AppellF1(1+1/2*m
,3,-p,2+1/2*m,e^2*x^2/d^2,-c*x^2/a)/d^4/(2+m)/((1+c*x^2/a)^p)+3*e^2*x^3*(g
*x)^m*(c*x^2+a)^p*AppellF1(3/2+1/2*m,3,-p,5/2+1/2*m,e^2*x^2/d^2,-c*x^2/a)/
d^5/(3+m)/((1+c*x^2/a)^p)-e^3*x^4*(g*x)^m*(c*x^2+a)^p*AppellF1(2+1/2*m,3,-
p,3+1/2*m,e^2*x^2/d^2,-c*x^2/a)/d^6/(4+m)/((1+c*x^2/a)^p)
```

3.436.2 Mathematica [F]

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^3} dx = \int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^3} dx$$

input
```
Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3,x]
```

output
```
Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3, x]
```

3.436.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {623, 622, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^3} dx$$

$$\Big\downarrow \text{623}$$

$$x^{-m}(gx)^m \int \frac{x^m (cx^2+a)^p}{(d+ex)^3} dx$$

$$\Big\downarrow \text{622}$$

$$x^{-m}(gx)^m \int \left(\frac{d^3 (cx^2+a)^p x^m}{(d^2-e^2x^2)^3} - \frac{3d^2e (cx^2+a)^p x^{m+1}}{(d^2-e^2x^2)^3} + \frac{3de^2 (cx^2+a)^p x^{m+2}}{(d^2-e^2x^2)^3} + \frac{e^3 (cx^2+a)^p x^{m+3}}{(e^2x^2-d^2)^3}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$x^{-m}(gx)^m \left(-\frac{e^3 x^{m+4} (a+cx^2)^p \left(\frac{cx^2}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+4}{2}, -p, 3, \frac{m+6}{2}, -\frac{cx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d^6(m+4)} + \frac{3e^2 x^{m+3} (a+cx^2)^p \left(\frac{cx}{a}\right.}{}\right.$$

input
```
Int[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3,x]
```

output
```
((g*x)^m*((x^(1 + m)*(a + c*x^2)^p*AppellF1[(1 + m)/2, -p, 3, (3 + m)/2, -
((c*x^2)/a), (e^2*x^2)/d^2])/(d^3*(1 + m)*(1 + (c*x^2)/a)^p) - (3*e*x^(2 +
 m)*(a + c*x^2)^p*AppellF1[(2 + m)/2, -p, 3, (4 + m)/2, -((c*x^2)/a), (e^2
*x^2)/d^2])/(d^4*(2 + m)*(1 + (c*x^2)/a)^p) + (3*e^2*x^(3 + m)*(a + c*x^2)
^p*AppellF1[(3 + m)/2, -p, 3, (5 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^
5*(3 + m)*(1 + (c*x^2)/a)^p) - (e^3*x^(4 + m)*(a + c*x^2)^p*AppellF1[(4 +
m)/2, -p, 3, (6 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^6*(4 + m)*(1 + (c
*x^2)/a)^p)))/x^m
```

3.436.3.1 Defintions of rubi rules used

rule 622
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[x^m*(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2
 - d^2*x^2)))^(-n), x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[n, -1]
```

rule 623
```
Int[((e_)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(e*x)^m/x^m   Int[x^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /
; FreeQ[{a, b, c, d, e, m, p}, x] && ILtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.436.4 Maple [F]

$$\int \frac{(gx)^m (c\,x^2+a)^p}{(ex+d)^3}dx$$

input
```
int((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x)
```

output
```
int((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x)
```

3.436.5 Fricas [F]

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^3} dx = \int \frac{(cx^2+a)^p (gx)^m}{(ex+d)^3} dx$$

input
```
integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((c*x^2 + a)^p*(g*x)^m/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3),
x)
```

3.436.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^3} dx = \text{Timed out}$$

input
```
integrate((g*x)**m*(c*x**2+a)**p/(e*x+d)**3,x)
```

output
```
Timed out
```

3.436.7 Maxima [F]

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^3} dx = \int \frac{(cx^2+a)^p (gx)^m}{(ex+d)^3} dx$$

input
```
integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d)^3, x)
```

3.436.8 Giac [F]

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^3} \, dx = \int \frac{(cx^2+a)^p (gx)^m}{(ex+d)^3} \, dx$$

input
```
integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d)^3, x)
```

3.436.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^3} \, dx = \int \frac{(g\,x)^m\,(c\,x^2+a)^p}{(d+e\,x)^3} \, dx$$

input
```
int(((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3,x)
```

output
```
int(((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3, x)
```

3.437 $\int \frac{x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

3.437.1 Optimal result

Integrand size = 40, antiderivative size = 345

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$= \frac{1}{24}\left(\frac{a}{cd}-\frac{7d}{e^2}\right)x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}+\frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4e}$$
$$-\frac{(105c^3d^6-25ac^2d^4e^2-17a^2cd^2e^4-15a^3e^6-2cde(35c^2d^4-6acd^2e^2-5a^2e^4)\,x)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{192c^3d^3e^4}$$
$$+\frac{(cd^2-ae^2)\,(35c^3d^6+15ac^2d^4e^2+9a^2cd^2e^4+5a^3e^6)\,\mathrm{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128c^{7/2}d^{7/2}e^{9/2}}$$

output
```
1/128*(-a*e^2+c*d^2)*(5*a^3*e^6+9*a^2*c*d^2*e^4+15*a*c^2*d^4*e^2+35*c^3*d^
6)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(7/2)/d^(7/2)/e^(9/2)+1/24*(a/c/d-7*d/e^2)
*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/4*x^3*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)/e-1/192*(105*c^3*d^6-25*a*c^2*d^4*e^2-17*a^2*c*d^2*e^4-
15*a^3*e^6-2*c*d*e*(-5*a^2*e^4-6*a*c*d^2*e^2+35*c^2*d^4)*x)*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^4
```

3.437.2 Mathematica [A] (verified)

Time = 11.13 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.88

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$= \frac{\sqrt{(ae+cdx)(d+ex)}\left(-\sqrt{c}\sqrt{d}\sqrt{e}(-15a^3e^6+a^2cde^4(-17d+10ex)+ac^2d^2e^2(-25d^2+12dex-8e^2x^2)\right.}{192}$$

input
```
Integrate[(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-15*a^3*e^6 + a
^2*c*d*e^4*(-17*d + 10*e*x) + a*c^2*d^2*e^2*(-25*d^2 + 12*d*e*x - 8*e^2*x^
2) + c^3*d^3*(105*d^3 - 70*d^2*e*x + 56*d*e^2*x^2 - 48*e^3*x^3))) + (3*Sqr
t[c*d]*Sqrt[c*d^2 - a*e^2]*(35*c^3*d^6 + 15*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^
4 + 5*a^3*e^6)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c
*d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2
- a*e^2)])))/(192*c^(7/2)*d^(7/2)*e^(9/2))
```

3.437.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1215, 1236, 27, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{d+ex}\,dx$$
$$\Big\downarrow \text{1215}$$
$$\int \frac{x^3(ae+cdx)}{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}dx$$
$$\Big\downarrow \text{1236}$$
$$\frac{\int -\frac{cdx^2(6ade+(7cd^2-ae^2)x)}{2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4cde}+\frac{x^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{4e}$$

$$\Big\downarrow \; 27$$

$$\frac{x^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e}-\frac{\int \frac{x^2\left(6ade+\left(7cd^2-ae^2\right)x\right)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8e}$$

$$\Big\downarrow \; 1236$$

$$\frac{x^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e}-\frac{\frac{\int -\frac{x\left(4ade\left(7cd^2-ae^2\right)+\left(35c^2d^4-6ace^2d^2-5a^2e^4\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3cde}+\frac{1}{3}x^2\left(\frac{7d}{e}-\frac{ae}{cd}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8e}$$

$$\Big\downarrow \; 27$$

$$\frac{x^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e}-\frac{\frac{1}{3}x^2\left(\frac{7d}{e}-\frac{ae}{cd}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-\frac{\int \frac{x\left(4ade\left(7cd^2-ae^2\right)+\left(35c^2d^4-6ace^2d^2-5a^2e^4\right)x\right)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6cde}}{8e}$$

$$\Big\downarrow \; 1225$$

$$\frac{x^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e}-\frac{\frac{1}{3}x^2\left(\frac{7d}{e}-\frac{ae}{cd}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-\frac{\frac{3\left(cd^2-ae^2\right)\left(5a^3e^6+9a^2cd^2e^4+15ac^2d^4e^2+35c^3d^6\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8c^2d^2e^2}-\frac{\left(-15\cdots\right.}{\cdots}}{\cdots}}{8e}$$

$$\Big\downarrow \; 1092$$

$$\frac{x^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e}-\frac{\frac{1}{3}x^2\left(\frac{7d}{e}-\frac{ae}{cd}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-\frac{\frac{3\left(cd^2-ae^2\right)\left(5a^3e^6+9a^2cd^2e^4+15ac^2d^4e^2+35c^3d^6\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{\cdots}{\sqrt{c\cdots}}}{4c^2d^2e^2}\cdots}{\cdots}}{8e}$$

$$\Big\downarrow \; 219$$

$$\frac{x^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e}-\frac{\frac{1}{3}x^2\left(\frac{7d}{e}-\frac{ae}{cd}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-\frac{\frac{3\left(cd^2-ae^2\right)\left(5a^3e^6+9a^2cd^2e^4+15ac^2d^4e^2+35c^3d^6\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+\cdots}}\cdots\right.}{8c^{5/2}d^{5/2}e^{5/2}}\cdots}{\cdots}}{8e}$$

3.437. $\int \frac{x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx$

input
```
Int[(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

output
```
(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e) - ((((7*d)/e - (a*
e)/(c*d))*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/3 - (-1/4*((105
*c^3*d^6 - 25*a*c^2*d^4*e^2 - 17*a^2*c*d^2*e^4 - 15*a^3*e^6 - 2*c*d*e*(35*
c^2*d^4 - 6*a*c*d^2*e^2 - 5*a^2*e^4)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2])/(c^2*d^2*e^2) + (3*(c*d^2 - a*e^2)*(35*c^3*d^6 + 15*a*c^2*d^4*e
^2 + 9*a^2*c*d^2*e^4 + 5*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*S
qrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c
^(5/2)*d^(5/2)*e^(5/2)))/(6*c*d*e))/(8*e)
```

3.437.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

3.437.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 954 vs. $2(315) = 630$.

Time = 0.68 (sec) , antiderivative size = 955, normalized size of antiderivative = 2.77

method	result
default	$\frac{x\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}{4cde}-\frac{5\left(e^2a+c\,d^2\right)\left(\frac{\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}{3cde}-\frac{\left(e^2a+c\,d^2\right)\left(\frac{\left(2cdex+e^2a+c\,d^2\right)\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}{4cde}\right.}{}\right.}{8cde}$

input
```
int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(1/4*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/e-5/8*(a*e^2+c*d^2)
/c/d/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/e-1/2*(a*e^2+c*d^2
)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2
+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(
1/2)))-1/4*a/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/
2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c
*d*e)^(1/2)))+d^2/e^3*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e
^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2))/(c*d*e)^(1/2))-d/e^2*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d
/e-1/2*(a*e^2+c*d^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((
1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2))/(c*d*e)^(1/2)))-d^3/e^4*((c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))
^(1/2)+1/2*(a*e^2-c*d^2)*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1
/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))
```

3.437.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.97

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$= \left[-\frac{3\,(35\,c^4d^8-20\,ac^3d^6e^2-6\,a^2c^2d^4e^4-4\,a^3cd^2e^6-5\,a^4e^8)\sqrt{cde}\log\Big(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e\right.$$
$$-\frac{3\,(35\,c^4d^8-20\,ac^3d^6e^2-6\,a^2c^2d^4e^4-4\,a^3cd^2e^6-5\,a^4e^8)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2}{2\,(c^2d^2e^2x^2+acd^2e^2+(c^2d^3e+ac}\right.}{}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/768*(3*(35*c^4*d^8 - 20*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^
2*e^6 - 5*a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2
*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x
+ c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(48*c^4*d^
4*e^4*x^3 - 105*c^4*d^7*e + 25*a*c^3*d^5*e^3 + 17*a^2*c^2*d^3*e^5 + 15*a^3
*c*d*e^7 - 8*(7*c^4*d^5*e^3 - a*c^3*d^3*e^5)*x^2 + 2*(35*c^4*d^6*e^2 - 6*a
*c^3*d^4*e^4 - 5*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x))/(c^4*d^4*e^5), -1/384*(3*(35*c^4*d^8 - 20*a*c^3*d^6*e^2 - 6*a^2*c^
2*d^4*e^4 - 4*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e
)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(48*c^4
*d^4*e^4*x^3 - 105*c^4*d^7*e + 25*a*c^3*d^5*e^3 + 17*a^2*c^2*d^3*e^5 + 15*
a^3*c*d*e^7 - 8*(7*c^4*d^5*e^3 - a*c^3*d^3*e^5)*x^2 + 2*(35*c^4*d^6*e^2 -
6*a*c^3*d^4*e^4 - 5*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x))/(c^4*d^4*e^5)]
```

3.437.6 Sympy [F]

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\int \frac{x^3\sqrt{(d+ex)\,(ae+cdx)}}{d+ex}\,dx$$

input
```
integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

output
```
Integral(x**3*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

3.437.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.437.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.88

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$$
$$=\frac{1}{192}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\,x\left(\frac{6\,x}{e}-\frac{7\,c^3d^4e^2-ac^2d^2e^4}{c^3d^3e^4}\right)+\frac{35\,c^3d^5e-6\,ac^2d^3e^3-5\,a^2cde^5}{c^3d^3e^4}\right.\right.$$
$$-\frac{(35\,c^4d^8-20\,ac^3d^6e^2-6\,a^2c^2d^4e^4-4\,a^3cd^2e^6-5\,a^4e^8)\log\left(\left|-cd^2-ae^2-2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2}\right.\right.\right.}{128\,\sqrt{cde}c^3d^3e^4}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm
="giac")
```

output
```
1/192*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*x*(6*x/e - (7*c^3*
d^4*e^2 - a*c^2*d^2*e^4)/(c^3*d^3*e^4)) + (35*c^3*d^5*e - 6*a*c^2*d^3*e^3
- 5*a^2*c*d*e^5)/(c^3*d^3*e^4))*x - (105*c^3*d^6 - 25*a*c^2*d^4*e^2 - 17*a
^2*c*d^2*e^4 - 15*a^3*e^6)/(c^3*d^3*e^4)) - 1/128*(35*c^4*d^8 - 20*a*c^3*d
^6*e^2 - 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 - 5*a^4*e^8)*log(abs(-c*d^2 -
 a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))))/(sqrt(c*d*e)*c^3*d^3*e^4)
```

3.437.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx=\int \frac{x^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{d+e\,x}\,dx$$

input
```
int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),x)
```

output
```
int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x), x)
```

3.438 $\int \frac{x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

3.438.1 Optimal result

Integrand size = 40, antiderivative size = 251

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx = \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3e} + \frac{((5cd^2-3ae^2)\,(3cd^2+ae^2)-2cde(5cd^2-ae^2)\,x)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24c^2d^2e^3} - \frac{(cd^2-ae^2)\,(5c^2d^4+2acd^2e^2+a^2e^4)\,\text{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{16c^{5/2}d^{5/2}e^{7/2}}$$

output
```
-1/16*(-a*e^2+c*d^2)*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)*arctanh(1/2*(2*c*d*
e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2))/c^(5/2)/d^(5/2)/e^(7/2)+1/3*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)/e+1/24*((-3*a*e^2+5*c*d^2)*(a*e^2+3*c*d^2)-2*c*d*e*(-a*e^2+5*c*d^2)*
x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e^3
```

3.438.2 Mathematica [A] (verified)

Time = 10.56 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.98

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$$
$$= \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(-3a^2e^4+2acde^2(-2d+ex)+c^2d^2(15d^2-10dex+8e^2x^2))-\frac{3\sqrt{cd}\sqrt{cd^2\cdots}}{\cdots}\right.}{24c^{5/2}d^{5/2}e^{7/2}}$$

input
```
Integrate[(x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-3*a^2*e^4 + 2*a*
c*d*e^2*(-2*d + e*x) + c^2*d^2*(15*d^2 - 10*d*e*x + 8*e^2*x^2)) - (3*Sqrt[
c*d]*Sqrt[c*d^2 - a*e^2]*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*ArcSinh[(Sq
rt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])
/(Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])))/(24*c^(5/2)*d
^(5/2)*e^(7/2))
```

3.438.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1215, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d+ex}\,dx$$
$$\Big\downarrow \text{ 1215}$$
$$\int \frac{x^2(ae+cdx)}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}dx$$
$$\Big\downarrow \text{ 1236}$$
$$\frac{\int -\frac{cdx(4ade+(5cd^2-ae^2)x)}{2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{3cde}+\frac{x^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3e}$$
$$\Big\downarrow \text{ 27}$$

$$\frac{x^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3e}-\frac{\int \frac{x\left(4ade+\left(5cd^2-ae^2\right)x\right)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6e}$$

$\downarrow$ 1225

$$\frac{x^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3e}-\frac{\frac{3\left(cd^2-ae^2\right)\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8c^2d^2e^2}-\frac{\left(\left(5cd^2-3ae^2\right)\left(ae^2+3cd^2\right)-2cdex\left(5cd^2-ae^2\right)\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4c^2d^2e^2}}{6e}$$

$\downarrow$ 1092

$$\frac{x^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3e}-\frac{\frac{3\left(cd^2-ae^2\right)\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4c^2d^2e^2}-\frac{\left(\left(5cd^2-3ae^2\right)\left(ae^2+3cd^2\right)-2cdex\left(5cd^2-ae^2\right)\right)}{4c^2d^2e^2}}{6e}$$

$\downarrow$ 219

$$\frac{x^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3e}-\frac{\frac{3\left(cd^2-ae^2\right)\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{5/2}d^{5/2}e^{5/2}}-\frac{\left(\left(5cd^2-3ae^2\right)\left(ae^2+3cd^2\right)-2cdex\left(5cd^2-ae^2\right)\right)\sqrt{x(ae^2+}}{4c^2d^2e^2}}{6e}$$

input
```
Int[(x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

output
```
(x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*e) - (-1/4*(((5*c*d^2
 - 3*a*e^2)*(3*c*d^2 + a*e^2) - 2*c*d*e*(5*c*d^2 - a*e^2)*x)*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^2*d^2*e^2) + (3*(c*d^2 - a*e^2)*(5*c^2*
d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt
[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(8*c^(5
/2)*d^(5/2)*e^(5/2)))/(6*e)
```

3.438. $\int \frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}dx$

3.438.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

3.438. $\int \frac{x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

3.438.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(225) = 450$.

Time = 0.66 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.04

method	result
default	$\frac{\frac{\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}{3cde}-\frac{\left(e^2a+c\,d^2\right)\left(\frac{\left(2cdex+e^2a+c\,d^2\right)\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}{4cde}+\frac{\left(4ac\,d^2e^2-\left(e^2a+c\,d^2\right)^2\right)\ln\left(\frac{\frac{1}{2}e^2a+\frac{1}{2}c\,d^2+cde}{\sqrt{cde}}\right.}{8cde\sqrt{cde}}}{2cde}}{e}$

input
```
int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/e-1/2*(a*e^2+c*d^2)/c
/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*
d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2
)))-d/e^2*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d
^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)
^(1/2))+d^2/e^3*((c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-
c*d^2)*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)
^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))
```

3.438.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.14

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$= \left[-\frac{3\left(5\,c^3d^6-3\,ac^2d^4e^2-a^2cd^2e^4-a^3e^6\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+4\,\sqrt{cdex^2+a\cdots}\right.}{\cdots}\right.$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/96*(3*(5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d
*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e
) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^3*x^2 + 15*c^3*d^5*e - 4
*a*c^2*d^3*e^3 - 3*a^2*c*d*e^5 - 2*(5*c^3*d^4*e^2 - a*c^2*d^2*e^4)*x)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4), 1/48*(3*(5*c^3*d^6
 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1/2*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-
c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(8
*c^3*d^3*e^3*x^2 + 15*c^3*d^5*e - 4*a*c^2*d^3*e^3 - 3*a^2*c*d*e^5 - 2*(5*c
^3*d^4*e^2 - a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
)/(c^3*d^3*e^4)]
```

3.438.6 Sympy [F]

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\int \frac{x^2\sqrt{(d+ex)\,(ae+cdx)}}{d+ex}\,dx$$

input
```
integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

output
```
Integral(x**2*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

3.438.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.438.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.89

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$=\frac{1}{24}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\,x\left(\frac{4\,x}{e}-\frac{5\,c^2d^3e-acde^3}{c^2d^2e^3}\right)+\frac{15\,c^2d^4-4\,acd^2e^2-3\,a^2e^4}{c^2d^2e^3}\right)$$
$$+\frac{(5\,c^3d^6-3\,ac^2d^4e^2-a^2cd^2e^4-a^3e^6)\log\left(\left|-cd^2-ae^2-2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ac}\right.\right.\right.}{16\,\sqrt{cde}c^2d^2e^3}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm
="giac")
```

output
```
1/24*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*x*(4*x/e - (5*c^2*d^3*
e - a*c*d*e^3)/(c^2*d^2*e^3)) + (15*c^2*d^4 - 4*a*c*d^2*e^2 - 3*a^2*e^4)/(
c^2*d^2*e^3)) + 1/16*(5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^
6)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
+ c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^2*d^2*e^3)
```

3.438.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\int \frac{x^2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{d+e\,x}\,dx$$

input
```
int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),x)
```

output
```
int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x), x)
```

3.439 $\int \frac{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

3.439.1 Optimal result

Integrand size = 38, antiderivative size = 207

$$\begin{aligned}\int \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx &= -\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}+\frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{2cde(d+ex)}\\ &\quad+\frac{(cd^2-ae^2)\,(3cd^2+ae^2)\,\mathrm{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{5/2}}\end{aligned}$$

output
```
1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/e/(e*x+d)+1/8*(-a*e^2+c*d^
2)*(a*e^2+3*c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(
1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/e^(5/2)-1/4*
(a/c/d+3*d/e^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

3.439.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.87

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$

$$= \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(ae^2+cd(-3d+2ex))+\frac{(-6c^2d^4+4acd^2e^2+2a^2e^4)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\left(\sqrt{d-\frac{ae^2}{cd}}-\sqrt{d+ex}\right.}\right.}{\sqrt{ae+cdx}\sqrt{d+ex}}\right.}{4c^{3/2}d^{3/2}e^{5/2}}$$

input
```
Integrate[(x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e^2 + c*d*(-3*d
 + 2*e*x)) + ((-6*c^2*d^4 + 4*a*c*d^2*e^2 + 2*a^2*e^4)*ArcTanh[(Sqrt[e]*Sq
rt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*(Sqrt[d - (a*e^2)/(c*d)] - Sqrt[d + e*x]
))])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(4*c^(3/2)*d^(3/2)*e^(5/2))
```

3.439.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1215, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{d+ex}\,dx$$

$$\Big\downarrow \ 1215$$

$$\int \frac{x(ae+cdx)}{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \ 1225$$

$$-\frac{\left(\frac{a^2e^2}{c}+2ad^2-\frac{3cd^4}{e^2}\right)\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8d}-\frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\,(-ae^2+3cd^2-2cdex)}{4cde^2}$$

$$\Big\downarrow \text{1092}$$

$$-\frac{\left(\frac{a^2e^2}{c}+2ad^2-\frac{3cd^4}{e^2}\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{4d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(-ae^2+3cd^2-2cdex\right)}{4cde^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\left(\frac{a^2e^2}{c}+2ad^2-\frac{3cd^4}{e^2}\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8\sqrt{c}d^{3/2}\sqrt{e}}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(-ae^2+3cd^2-2cdex\right)}{4cde^2}$$

input
```
Int[(x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

output
```
-1/4*((3*c*d^2 - a*e^2 - 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(c*d*e^2) - ((2*a*d^2 - (3*c*d^4)/e^2 + (a^2*e^2)/c)*ArcTanh[(c*d^2
 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])])/(8*Sqrt[c]*d^(3/2)*Sqrt[e])
```

3.439.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

3.439. $\int \frac{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] && !LeQ[p, -1]
```

3.439.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.41

method	result
default	$\frac{\frac{\left(2cdex+e^2a+c\,d^2\right)\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}{4cde}+\frac{\left(4ac\,d^2e^2-\left(e^2a+c\,d^2\right)^2\right)\ln\left(\frac{\frac{1}{2}e^2a+\frac{1}{2}c\,d^2+cdex}{\sqrt{cde}}+\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}\right)}{8cde\sqrt{cde}}}{e}-\frac{d\left(\sqrt{c\right.}}{}$

input
```
int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x,method=_RETURNVERB
OSE)
```

output
```
1/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d
*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)
)-d/e^2*((c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*l
n((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^
2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))
```

3.439.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.02

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$=\left[-\frac{(3\,c^2d^4-2\,acd^2e^2-a^2e^4)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4-4\,\sqrt{cdex^2+ade+(cd^2+a\right.}}{}\right.$$
$$-\frac{(3\,c^2d^4-2\,acd^2e^2-a^2e^4)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+ae^2)\sqrt{-cde}}{2\,(c^2d^2e^2x^2+acd^2e^2+(c^2d^3e+acde^3)x)}\right)-2\,(2\,c^2d^2e^2x-3\,c}{8\,c^2d^2e^3}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="
fricas")
```

output
```
[-1/16*((3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^
2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a
*c*d*e^3)*x) - 4*(2*c^2*d^2*e^2*x - 3*c^2*d^3*e + a*c*d*e^3)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3), -1/8*((3*c^2*d^4 - 2*a*c*d^
2*e^2 - a^2*e^4)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c
*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(2*c^2*d^2*e^2*x - 3*c^2*d^3*e
+ a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3)]
```

3.439.6 Sympy [F]

$$\int \frac{x\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{d + ex}\,dx = \int \frac{x\sqrt{(d + ex)\,(ae + cdx)}}{d + ex}\,dx$$

input
```
integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

output
```
Integral(x*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

3.439.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{d + ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.439.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.79

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx = \frac{1}{4}\,\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(\frac{2\,x}{e}-\frac{3\,cd^2-ae^2}{cde^2}\right) - \frac{(3\,c^2d^4-2\,acd^2e^2-a^2e^4)\log\left(\left|-cd^2-ae^2-2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{8\,\sqrt{cde}cde^2}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="
giac")
```

output
```
1/4*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*x/e - (3*c*d^2 - a*e^2)
/(c*d*e^2)) - 1/8*(3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*log(abs(-c*d^2 - a
*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
 a*d*e))))/(sqrt(c*d*e)*c*d*e^2)
```

3.439.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx = \int \frac{x\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{d+e\,x}\,dx$$

input
```
int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),x)
```

output
```
int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x), x)
```

3.440 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

3.440.1 Optimal result

Integrand size = 37, antiderivative size = 131

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx = \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e} - \frac{(cd^2-ae^2)\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}e^{3/2}}$$

output
```
-1/2*(-a*e^2+c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^
(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/e^(3/2)/c^(1/2)/d^(1/2)+(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e
```

3.440.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{e}+\frac{(-cd^2+ae^2)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{e^{3/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[e] + ((-(c*d^2) + a*e^2)*ArcTanh[(Sqr
t[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[c]*Sqrt[d]
*Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/e^(3/2)
```

3.440.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {1131, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d+ex}dx$$

$$\Big\downarrow \text{1131}$$

$$\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e}-\frac{(cd^2-ae^2)\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2e}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e}-\frac{(cd^2-ae^2)\int \frac{1}{4cde-\frac{(cd^2+2cexd+ae^2)^2}{cdex^2+(cd^2+ae^2)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e}-\frac{(cd^2-ae^2)\,\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}e^{3/2}}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x),x]
```

output
```
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/e - ((c*d^2 - a*e^2)*ArcTanh[(
c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*e^(3/2))
```

3.440.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

3.440.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\sqrt{cde\left(x+\frac{d}{e}\right)^{2}+\left(e^{2}a-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}+\frac{\left(e^{2}a-c\,d^{2}\right)\ln\left(\frac{\frac{e^{2}a}{2}-\frac{c\,d^{2}}{2}+cde\left(x+\frac{d}{e}\right)}{\sqrt{cde}}+\sqrt{cde\left(x+\frac{d}{e}\right)^{2}+\left(e^{2}a-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}\right)}{2\sqrt{cde}}}{e}$	131

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOS
E)
```

output
```
1/e*((c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1
/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*
d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))
```

3.440.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.57

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{d + ex} dx$$
$$= \left[\frac{4\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}cde - (cd^2 - ae^2)\sqrt{cde}\log\left(8c^2d^2e^2x^2 + c^2d^4 + 6acd^2e^2 + a^2e^4 + 4\sqrt{cd\right.}{4cde^2} \right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="fr
icas")
```

output
```
[1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e - (c*d^2 - a*e^2
)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 +
4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*
sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x))/(c*d*e^2), 1/2*(2*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e + (c*d^2 - a*e^2)*sqrt(-c*d*e)*arcta
n(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e
^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*
x)))/(c*d*e^2)]
```

3.440.6 Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{d + ex} dx = \int \frac{\sqrt{(d + ex)(ae + cdx)}}{d + ex} dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

3.440.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{d + ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

3.440.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{d + ex}\,dx = \frac{(cd^2 - ae^2)\log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)\right|\right)}{2\sqrt{cde}e} + \frac{\sqrt{cdex^2 + cd^2x + ae^2x + ade}}{e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="gi
ac")
```

output
```
1/2*(c*d^2 - a*e^2)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*e) + sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)/e
```

3.440.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{d + ex}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}{d + e\,x}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x), x)
```

3.441 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x(d+ex)}\,dx$

3.441.1 Optimal result

Integrand size = 40, antiderivative size = 168

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x(d+ex)}\,dx = \frac{\sqrt{c}\sqrt{d}\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{e}} - \frac{\sqrt{a}\sqrt{e}\operatorname{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{d}}$$

output

```
arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2))*c^(1/2)*d^(1/2)/e^(1/2)-arctanh(1/2*(2*a*d*e+(a
*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2))*a^(1/2)*e^(1/2)/d^(1/2)
```

3.441.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 476 vs. 2(168) = 336.

Time = 1.41 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.83

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x(d+ex)}\,dx =$$

$$-\frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(-\left(\left(-\sqrt{c}d+\sqrt{cd^2-ae^2}\right)\sqrt{-2cd^2+ae^2-2\sqrt{c}d\sqrt{cd^2-ae^2}}\arctan\left(\frac{\sqrt{-2cd^2+ae\cdots}}{\sqrt{a}\sqrt{c}\sqrt{\cdots}}\right.\right.\right.}{\cdots}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x*(d + e*x)),x]
```

output
```
(-2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-((-(Sqrt[c]*d) + Sqrt[c*d^2 - a*e^2]
)*Sqrt[-2*c*d^2 + a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 - a*e^2]]*ArcTan[(Sqrt[-2
*c*d^2 + a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 - a*e^2]]*Sqrt[a*e + c*d*x])/(Sqrt
[a]*Sqrt[c]*Sqrt[d]*Sqrt[e]*(Sqrt[d - (a*e^2)/(c*d)] - Sqrt[d + e*x]))]) +
 (Sqrt[c]*d + Sqrt[c*d^2 - a*e^2])*Sqrt[-2*c*d^2 + a*e^2 + 2*Sqrt[c]*d*Sqr
t[c*d^2 - a*e^2]]*ArcTan[(Sqrt[-2*c*d^2 + a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d^2 -
 a*e^2]]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[c]*Sqrt[d]*Sqrt[e]*(Sqrt[d - (a*
e^2)/(c*d)] - Sqrt[d + e*x]))] + 2*Sqrt[a]*Sqrt[c]*d*e*ArcTanh[(Sqrt[e]*Sq
rt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*(Sqrt[d - (a*e^2)/(c*d)] - Sqrt[d + e*x]
))]))/(Sqrt[a]*Sqrt[d]*e^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.441.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1215, 1268, 140, 27, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x(d+ex)}dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{ae+cdx}{x\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1268}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{\sqrt{ae+cdx}}{x\sqrt{d+ex}}dx}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{140}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(cd\int \frac{1}{\sqrt{ae+cdx}\sqrt{d+ex}}dx+\int \frac{ae}{x\sqrt{ae+cdx}\sqrt{d+ex}}dx\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(cd\int \frac{1}{\sqrt{ae+cdx}\sqrt{d+ex}}dx+ae\int \frac{1}{x\sqrt{ae+cdx}\sqrt{d+ex}}dx\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 66}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(ae\int \frac{1}{x\sqrt{ae+cdx}\sqrt{d+ex}}dx+2cd\int \frac{1}{cd-\frac{e(ae+cdx)}{d+ex}}d\frac{\sqrt{ae+cdx}}{\sqrt{d+ex}}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 104}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(2cd\int \frac{1}{cd-\frac{e(ae+cdx)}{d+ex}}d\frac{\sqrt{ae+cdx}}{\sqrt{d+ex}}+2ae\int \frac{1}{\frac{ae(d+ex)}{ae+cdx}-d}d\frac{\sqrt{d+ex}}{\sqrt{ae+cdx}}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{2\sqrt{c}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{e}}-\frac{2\sqrt{a}\sqrt{e}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}\sqrt{d+ex}}{\sqrt{d}\sqrt{ae+cdx}}\right)}{\sqrt{d}}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x*(d + e*x)),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*((2*Sqrt[c]*Sqrt[d]*ArcTanh[(Sqrt[e]*Sqrt
[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/Sqrt[e] - (2*Sqrt[a]*Sqrt
[e]*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/
Sqrt[d]))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
```

3.441.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.441.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(136) = 272$.

Time = 0.53 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.83

method	result
default	$\frac{\sqrt{ade+(e^2a+c\,d^2)x+cde\,x^2}+\frac{\left(e^2a+c\,d^2\right)\ln\left(\frac{\frac{1}{2}e^2a+\frac{1}{2}c\,d^2+cdex}{\sqrt{cde}}+\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}\right)}{2\sqrt{cde}}-\frac{ade\ln\left(\frac{2ade+\left(e^2a+c\,d^2\right)x+2\sqrt{ade}\,\sqrt{ade+}}{x}\right.}{\sqrt{ade}}}{d}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x,method=_RETURNVERB
OSE)
```

output
```
1/d*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2
*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)
^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))-1/d*((c*d*e*(x+d/e)^2+
(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1/2*e^2*a-1/2*c*d^2+c*d
*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(
c*d*e)^(1/2))
```

3.441.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 947, normalized size of antiderivative = 5.64

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x(d+ex)}\,dx = \left[\frac{1}{2}\sqrt{\frac{cd}{e}}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4\right.\right.$$

$$\left.+4\left(2\,cde^2x+cd^2e+ae^3\right)\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{\frac{cd}{e}}+8\left(c^2d^3e+acde^3\right)x\right)$$

$$+\frac{1}{2}\sqrt{\frac{ae}{d}}\log\left(\frac{8\,a^2d^2e^2+(c^2d^4+6\,acd^2e^2+a^2e^4)x^2-4\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,ad^2e+(cd^3+a}{x^2}\right.$$

$$-\sqrt{-\frac{cd}{e}}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,cdex+cd^2+ae^2)\sqrt{-\frac{cd}{e}}}{2\left(c^2d^2ex^2+acd^2e+(c^2d^3+acde^2)x\right)}\right)$$

$$+\frac{1}{2}\sqrt{\frac{ae}{d}}\log\left(\frac{8\,a^2d^2e^2+(c^2d^4+6\,acd^2e^2+a^2e^4)x^2-4\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,ad^2e+(cd^3+a}{x^2}\right.$$

$$+\frac{1}{2}\sqrt{\frac{cd}{e}}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4\right.$$

$$\left.+4\left(2\,cde^2x+cd^2e+ae^3\right)\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{\frac{cd}{e}}+8\left(c^2d^3e+acde^3\right)x\right),$$

$$-\sqrt{-\frac{cd}{e}}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,cdex+cd^2+ae^2)\sqrt{-\frac{cd}{e}}}{2\left(c^2d^2ex^2+acd^2e+(c^2d^3+acde^2)x\right)}\right)$$

$$\left.+\sqrt{-\frac{ae}{d}}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,ade+(cd^2+ae^2)x)\sqrt{-\frac{ae}{d}}}{2\left(acde^2x^2+a^2de^2+(acd^2e+a^2e^3)x\right)}\right)\right]$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x, algorithm="
fricas")
```

output
```
[1/2*sqrt(c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4
 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 1/2*sqrt(a*e/d)*log((8
*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/d)
 + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2), -sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/
(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) + 1/2*sqrt(a*e/d)*l
og((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqrt(a
*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2), sqrt(-a*e/d)*arctan(1/2*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a
*e/d)/(a*c*d*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) + 1/2*sqrt(c*
d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*
e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x), -sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)
/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) + sqrt(-a*e/d)*arc
tan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(-a*e/d)/(a*c*d*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)...
```

3.441.6 Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x(d + ex)}\,dx = \int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{x\,(d + ex)}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x/(e*x+d),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x*(d + e*x)), x)
```

3.441.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x(d + ex)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.441.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x(d + ex)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

3.441.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x(d + ex)}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}{x\,(d + e\,x)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x*(d + e*x)), x)
```

3.442 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^2(d+ex)}\,dx$

3.442.1 Optimal result

Integrand size = 40, antiderivative size = 137

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^2(d+ex)}\,dx=-\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{dx}-\frac{(cd^2-ae^2)\,\mathrm{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2\sqrt{a}d^{3/2}\sqrt{e}}$$

output

```
-1/2*(-a*e^2+c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/
e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/d^(3/2)/a^(1/2)/e^(1/2)-(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/x
```

3.442.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 841 vs. 2(137) = 274.

Time = 9.25 (sec) , antiderivative size = 841, normalized size of antiderivative = 6.14

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^2(d+ex)}\,dx$$

$$=\frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(-\frac{\sqrt{ae+cdx}\left(a^2e^4\sqrt{d+ex}+2acde^2\left(2d\sqrt{d-\frac{ae^2}{cd}}+2e\sqrt{d-\frac{ae^2}{cd}}x-4d\sqrt{d+ex}-3ex\sqrt{d+ex}\right)+c^2\left(d^2e^2x^2\left(-4\sqrt{d-\frac{ae^2}{cd}}\right.\right.\right.}{a^2de^4x-2acd^2e^2x\left(4d+3ex-2\sqrt{d-\frac{ae^2}{cd}}\sqrt{d+ex}\right)+c^2d^3x\left(8d^2+8dex+e^2x^2\right.}}{}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^2*(d + e*x)),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-((Sqrt[a*e + c*d*x]*(a^2*e^4*Sqrt[d + e
*x] + 2*a*c*d*e^2*(2*d*Sqrt[d - (a*e^2)/(c*d)] + 2*e*Sqrt[d - (a*e^2)/(c*d
)]*x - 4*d*Sqrt[d + e*x] - 3*e*x*Sqrt[d + e*x]) + c^2*(d^2*e^2*x^2*(-4*Sqr
t[d - (a*e^2)/(c*d)] + Sqrt[d + e*x]) + d^3*e*x*(-12*Sqrt[d - (a*e^2)/(c*d
)] + 8*Sqrt[d + e*x]) + d^4*(-8*Sqrt[d - (a*e^2)/(c*d)] + 8*Sqrt[d + e*x])
)))/(a^2*d*e^4*x - 2*a*c*d^2*e^2*x*(4*d + 3*e*x - 2*Sqrt[d - (a*e^2)/(c*d)
]*Sqrt[d + e*x]) + c^2*d^3*x*(8*d^2 + 8*d*e*x + e^2*x^2 - 8*d*Sqrt[d - (a*
e^2)/(c*d)]*Sqrt[d + e*x] - 4*e*Sqrt[d - (a*e^2)/(c*d)]*x*Sqrt[d + e*x])))
 + (Sqrt[c*d^2 - a*e^2]*(c*d^2 - a*e^2 + Sqrt[c]*d*Sqrt[c*d^2 - a*e^2])*Ar
cTan[(Sqrt[-2*c*d^2 + a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 - a*e^2]]*Sqrt[a*e +
c*d*x])/(Sqrt[a]*Sqrt[c]*Sqrt[d]*Sqrt[e]*(Sqrt[d - (a*e^2)/(c*d)] - Sqrt[d
 + e*x]))])/(Sqrt[a]*d^(3/2)*Sqrt[e]*Sqrt[-2*c*d^2 + a*e^2 - 2*Sqrt[c]*d*S
qrt[c*d^2 - a*e^2]]) + (Sqrt[c*d^2 - a*e^2]*(-(c*d^2) + a*e^2 + Sqrt[c]*d*
Sqrt[c*d^2 - a*e^2])*ArcTan[(Sqrt[-2*c*d^2 + a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d^
2 - a*e^2]]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[c]*Sqrt[d]*Sqrt[e]*(Sqrt[d -
(a*e^2)/(c*d)] - Sqrt[d + e*x]))])/(Sqrt[a]*d^(3/2)*Sqrt[e]*Sqrt[-2*c*d^2
+ a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d^2 - a*e^2]])))/Sqrt[(a*e + c*d*x)*(d + e*x)
]
```

3.442.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1215, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x^2(d+ex)}dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{ae+cdx}{x^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1228}$$

$$\frac{\left(cd^2-ae^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{\left(cd^2-ae^2\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\left(cd^2-ae^2\right)\text{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{a}d^{3/2}\sqrt{e}}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^2*(d + e*x)),x]
```

output
```
-(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x)) - ((c*d^2 - a*e^2)*Ar
cTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[a]*d^(3/2)*Sqrt[e])
```

3.442.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

3.442. $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^2(d+ex)}\,dx$

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

3.442.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(117) = 234$.

Time = 0.72 (sec) , antiderivative size = 708, normalized size of antiderivative = 5.17

method	result
default	$-\frac{\left(ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}\right)^{\frac{3}{2}}}{adex}+\frac{\left(e^{2}a+c\,d^{2}\right)\left(\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}}+\frac{\left(e^{2}a+c\,d^{2}\right)\ln\left(\frac{\frac{1}{2}e^{2}a+\frac{1}{2}c\,d^{2}+cdex}{\sqrt{cde}}+\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}}\right.}{2\sqrt{cde}}\right.}{2ade}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/d*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)/
a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e
^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*
e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+2*c/a*(1/4*(2*c*d*e*
x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^
2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2
)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))-e/d^2*((a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+
c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1
/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+e/d^2*((c*d*e*(x+d/e)^2+(a*e^2-c*d^
2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))
/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2
))
```

3.442.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.59

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{x^2(d + ex)}\,dx$$

$$= \left[-\frac{4\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\,ade + (cd^2 - ae^2)\sqrt{ade}\,x\log\left(\frac{8a^2d^2e^2 + (c^2d^4 + 6acd^2e^2 + a^2e^4)x^2 + 4\sqrt{cdex^2 + ade + }}{}\right)}{4ad^2ex} \right.$$

$$\left. -\frac{2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\,ade - (cd^2 - ae^2)\sqrt{-ade}\,x\arctan\left(\frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\,(2ade + (cd^2 + ae^2)x)}{2(acd^2e^2x^2 + a^2d^2e^2 + (acd^3e + a^2de^3)x)}\right)}{2ad^2ex} \right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e + (c*d^2 - a*e^
2)*sqrt(a*d*e)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*
x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2))/(a*d^2*e*x), -1/2
*(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e - (c*d^2 - a*e^2)*sq
rt(-a*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c
*d^3*e + a^2*d*e^3)*x)))/(a*d^2*e*x)]
```

3.442.6 Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{x^2(d + ex)}\,dx = \int \frac{\sqrt{(d + ex)(ae + cdx)}}{x^2(d + ex)}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**2/(e*x+d),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x**2*(d + e*x)), x)
```

3.442.7 Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^2(d + ex)}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)x^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^2), x)
```

3.442.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^2(d + ex)}\,dx = \frac{(cd^2 - ae^2)\arctan\left(-\frac{\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}}{\sqrt{-ade}}\right)}{\sqrt{-ade}d} - \frac{\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)cd^2 + \left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)ae^2 + 2\sqrt{cde}}{\left(ade - \left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)^2\right)d}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x, algorithm
="giac")
```

output
```
(c*d^2 - a*e^2)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*
x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*d) - ((sqrt(c*d*e)*x - sqrt(c*d*e*
x^2 + c*d^2*x + a*e^2*x + a*d*e))*c*d^2 + (sqrt(c*d*e)*x - sqrt(c*d*e*x^2
+ c*d^2*x + a*e^2*x + a*d*e))*a*e^2 + 2*sqrt(c*d*e)*a*d*e)/((a*d*e - (sqrt
(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)*d)
```

3.442.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^2(d+ex)}\,dx=\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^2\,\left(d+e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^2*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^2*(d + e*x)), x)
```

3.443 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^3(d+ex)}\,dx$

3.443.1 Optimal result

Integrand size = 40, antiderivative size = 202

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^3(d+ex)}\,dx = -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2dx^2}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4x}+\frac{(cd^2-ae^2)\,(cd^2+3ae^2)\,\text{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8a^{3/2}d^{5/2}e^{3/2}}$$

output
```
1/8*(-a*e^2+c*d^2)*(3*a*e^2+c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a
^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/d^
(5/2)/e^(3/2)-1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/x^2-1/4*(c/a/e
-3*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x
```

3.443.2 Mathematica [A] (verified)

Time = 10.12 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^3(d+ex)}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(-cd^2x+ae(-2d+3ex))}{x^2}+\frac{(c^2d^4+2acd^2e^2-3a^2e^4)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{4a^{3/2}d^{5/2}e^{3/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^3*(d + e*x)),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-(c*d^2*x) + a*e
*(-2*d + 3*e*x)))/x^2 + ((c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*ArcTanh[(Sq
rt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d
*x]*Sqrt[d + e*x])))/(4*a^(3/2)*d^(5/2)*e^(3/2))
```

3.443.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1215, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x \left(a e^2+c d^2\right)+a d e+c d e x^2}}{x^3 (d+e x)} \, dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{a e+c d x}{x^3 \sqrt{x \left(a e^2+c d^2\right)+a d e+c d e x^2}}dx$$

$$\Big\downarrow \text{1237}$$

$$-\frac{\int -\frac{a e \left(c d^2-2 c e x d-3 a e^2\right)}{2 x^2 \sqrt{c d e x^2+\left(c d^2+a e^2\right) x+a d e}}dx}{2 a d e}-\frac{\sqrt{x \left(a e^2+c d^2\right)+a d e+c d e x^2}}{2 d x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{c d^2-2 c e x d-3 a e^2}{x^2 \sqrt{c d e x^2+\left(c d^2+a e^2\right) x+a d e}}dx}{4 d}-\frac{\sqrt{x \left(a e^2+c d^2\right)+a d e+c d e x^2}}{2 d x^2}$$

$$\Big\downarrow \text{1228}$$

$$\frac{-\frac{\left(\frac{c^2 d^4}{a}-3 a e^4+2 c d^2 e^2\right) \int \frac{1}{x \sqrt{c d e x^2+\left(c d^2+a e^2\right) x+a d e}}dx}{2 d e}-\frac{\left(\frac{c d}{a e}-\frac{3 e}{d}\right) \sqrt{x \left(a e^2+c d^2\right)+a d e+c d e x^2}}{x}}{4 d}-\frac{\sqrt{x \left(a e^2+c d^2\right)+a d e+c d e x^2}}{2 d x^2}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\frac{\left(\frac{c^2d^4}{a}-3ae^4+2cd^2e^2\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{de}-\frac{\left(\frac{cd}{ae}-\frac{3e}{d}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}}{4d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2dx^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\left(\frac{c^2d^4}{a}-3ae^4+2cd^2e^2\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}d^{3/2}e^{3/2}}-\frac{\left(\frac{cd}{ae}-\frac{3e}{d}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}}{4d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2dx^2}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^3*(d + e*x)),x]
```

output
```
-1/2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x^2) + (-((((c*d)/(a*e
) - (3*e)/d)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x) + (((c^2*d^4)
/a + 2*c*d^2*e^2 - 3*a*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[
a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[
a]*d^(3/2)*e^(3/2)))/(4*d)
```

3.443.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.443. $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^3(d+ex)}\,dx$

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.443.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1352 vs. $2(176) = 352$.

Time = 0.76 (sec) , antiderivative size = 1353, normalized size of antiderivative = 6.70

method	result	size
default	Expression too large to display	1353

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/d*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/4*(a*e^2+c*d
^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c
*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln(
(1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2
*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+2*c/a*(1/4*(2*
c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*
a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e
)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+1/2*c/a*(
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/
2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c
*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2
)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+e^2/d^3*((a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/
(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e
/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2))/x))-e/d^2*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*...
```

3.443.5 Fricas [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.19

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{x^3(d + ex)}\, dx$$
$$= \Bigg[-\frac{(c^2d^4 + 2\,acd^2e^2 - 3\,a^2e^4)\sqrt{ade}x^2 \log\left(\frac{8\,a^2d^2e^2+(c^2d^4+6\,acd^2e^2+a^2e^4)x^2-4\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,ade+(cd^2+ae^2)}{x^2}\right.}{16\,a^2d^3e^2x^2}$$
$$-\frac{(c^2d^4 + 2\,acd^2e^2 - 3\,a^2e^4)\sqrt{-ade}x^2 \arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,ade+(cd^2+ae^2)x)\sqrt{-ade}}{2\,(acd^2e^2x^2+a^2d^2e^2+(acd^3e+a^2de^3)x)}\right) + 2\,(2\,a^2d^2e^2 +}{8\,a^2d^3e^2x^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/16*((c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(a*d*e)*x^2*log((8*a^2*d
^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*
d^3*e + a^2*d*e^3)*x)/x^2) + 4*(2*a^2*d^2*e^2 + (a*c*d^3*e - 3*a^2*d*e^3)*
x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^2), -1/8*((
c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(-a*d*e)*x^2*arctan(1/2*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d
*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(2*a^
2*d^2*e^2 + (a*c*d^3*e - 3*a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x))/(a^2*d^3*e^2*x^2)]
```

3.443.6 Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^3(d + ex)}\,dx = \int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{x^3\,(d + ex)}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**3/(e*x+d),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x**3*(d + e*x)), x)
```

3.443.7 Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^3(d + ex)}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)x^3}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^3), x)
```

3.443.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(176) = 352$.

Time = 0.31 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.50

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{x^3(d + ex)}\,dx$$
$$= -\frac{(c^2d^4 + 2\,acd^2e^2 - 3\,a^2e^4)\arctan\left(-\frac{\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}}{\sqrt{-ade}}\right)}{4\sqrt{-ade}ad^2e}$$
$$+ \frac{\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)ac^2d^5e + 10\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^2cd^3e^3}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x, algorithm
="giac")
```

output
```
-1/4*(c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*arctan(-(sqrt(c*d*e)*x - sqrt(c
*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a*d^2*e
) + 1/4*((sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a*c
^2*d^5*e + 10*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)
)*a^2*c*d^3*e^3 + 5*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
a*d*e))*a^3*d*e^5 + 8*sqrt(c*d*e)*a^3*d^2*e^4 + (sqrt(c*d*e)*x - sqrt(c*d*
e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*c^2*d^4 + 2*(sqrt(c*d*e)*x - sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a*c*d^2*e^2 - 3*(sqrt(c*d*e)*x - s
qrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^2*e^4 + 8*sqrt(c*d*e)*(sqr
t(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a*c*d^3*e)/((a
*d*e - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)^2*
a*d^2*e)
```

3.443.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{x^3(d + ex)}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}{x^3\,(d + e\,x)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^3*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^3*(d + e*x)), x)
```

3.444 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^4(d+ex)}\,dx$

3.444.1 Optimal result

Integrand size = 40, antiderivative size = 286

$$\begin{aligned}\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^4(d+ex)}\,dx &= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3dx^3} - \frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12x^2} \\ &+ \frac{(3cd^2-5ae^2)\,(cd^2+3ae^2)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24a^2d^3e^2x} \\ &- \frac{(cd^2-ae^2)\,(c^2d^4+2acd^2e^2+5a^2e^4)\,\mathrm{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{16a^{5/2}d^{7/2}e^{5/2}}\end{aligned}$$

output
```
-1/16*(-a*e^2+c*d^2)*(5*a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)*arctanh(1/2*(2*a*d*
e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2))/a^(5/2)/d^(7/2)/e^(5/2)-1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/d/x^3-1/12*(c/a/e-5*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2
+1/24*(-5*a*e^2+3*c*d^2)*(3*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)/a^2/d^3/e^2/x
```

3.444.2 Mathematica [A] (verified)

Time = 10.17 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^4(d+ex)}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(3c^2d^4x^2-2acd^2ex(d-2ex)+a^2e^2(-8d^2+10dex-15e^2x^2))}{x^3}-\frac{3(c^3d^6+ac^2d^4e^2+3a^2cd^2e^4-5a^3e^6)\mathrm{ar}}{\sqrt{ae+cdx}\sqrt{d+e}}\right.}{24a^{5/2}d^{7/2}e^{5/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^4*(d + e*x)),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(3*c^2*d^4*x^2 -
2*a*c*d^2*e*x*(d - 2*e*x) + a^2*e^2*(-8*d^2 + 10*d*e*x - 15*e^2*x^2)))/x^3
 - (3*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*ArcTanh[(Sqr
t[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d*
x]*Sqrt[d + e*x])))/(24*a^(5/2)*d^(7/2)*e^(5/2))
```

3.444.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1215, 1237, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{x^4(d+ex)}\,dx$$

$$\Big\downarrow \ 1215$$

$$\int \frac{ae+cdx}{x^4\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \ 1237$$

$$-\frac{\int -\frac{ae(cd^2-4cexd-5ae^2)}{2x^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{3ade}-\frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{3dx^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{cd^2-4cexd-5ae^2}{x^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{6d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3dx^3}$$

$$\Big\downarrow \ 1237$$

$$\frac{-\frac{\int \frac{\left(3cd^2-5ae^2\right)\left(cd^2+3ae^2\right)+2cde\left(cd^2-5ae^2\right)x}{2x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(\frac{cd}{ae}-\frac{5e}{d}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2x^2}}{6d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3dx^3}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{\int \frac{\left(3cd^2-5ae^2\right)\left(cd^2+3ae^2\right)+2cde\left(cd^2-5ae^2\right)x}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4ade}-\frac{\left(\frac{cd}{ae}-\frac{5e}{d}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2x^2}}{6d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3dx^3}$$

$$\Big\downarrow \ 1228$$

$$\frac{-\frac{-\frac{3\left(cd^2-ae^2\right)\left(5a^2e^4+2acd^2e^2+c^2d^4\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(3cd^2-5ae^2\right)\left(3ae^2+cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{adex}}{4ade}-\frac{\left(\frac{cd}{ae}-\frac{5e}{d}\right)\sqrt{x(ae^2+c}}{2x^2}}{6d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3dx^3}$$

$$\Big\downarrow \ 1154$$

$$\frac{-\frac{\frac{3\left(cd^2-ae^2\right)\left(5a^2e^4+2acd^2e^2+c^2d^4\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade}-\frac{\left(3cd^2-5ae^2\right)\left(3ae^2+cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{adex}}{4ade}}{6d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3dx^3}$$

$$\Big\downarrow \ 219$$

$$\frac{-\frac{\frac{3\left(cd^2-ae^2\right)\left(5a^2e^4+2acd^2e^2+c^2d^4\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2a^{3/2}d^{3/2}e^{3/2}}-\frac{\left(3cd^2-5ae^2\right)\left(3ae^2+cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{adex}}{4ade}-\left(\frac{cd}{ae}-\right.}{6d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3dx^3}$$

3.444. $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^4(d+ex)}\,dx$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^4*(d + e*x)),x]
```

output
```
-1/3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x^3) + (-1/2*(((c*d)/(
a*e) - (5*e)/d)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x^2 - (-(((3*
c*d^2 - 5*a*e^2)*(c*d^2 + 3*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(a*d*e*x)) + (3*(c*d^2 - a*e^2)*(c^2*d^4 + 2*a*c*d^2*e^2 + 5*a^2*e^4
)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*a^(3/2)*d^(3/2)*e^(3/2)))/(4*a*
d*e))/(6*d)
```

3.444.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

3.444. $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^4(d+ex)}\,dx$

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.444.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2058 vs. $2(256) = 512$.

Time = 1.16 (sec) , antiderivative size = 2059, normalized size of antiderivative = 7.20

method	result	size
default	Expression too large to display	2059

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/d*(-1/3/a/d/e/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/2*(a*e^2+c*d
^2)/a/d/e*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/4*(a*e
^2+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a
*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^
2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^
2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+2*c/a*(1
/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1
/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/
(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+1/2
*c/a*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^
2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e
)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+e^2/d^3*(-1/a/d/e/x
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)/a/d/e*((a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+
c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1
/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+2*c/a*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/
c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2...
```

3.444.5 Fricas [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{x^4(d + ex)}\,dx$$
$$= \left[-\frac{3\,(c^3d^6 + ac^2d^4e^2 + 3\,a^2cd^2e^4 - 5\,a^3e^6)\sqrt{ade}x^3\log\left(\frac{8\,a^2d^2e^2+(c^2d^4+6\,acd^2e^2+a^2e^4)x^2+4\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(}{x^2}\right.}{} \right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/96*(3*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*sqrt(a*d
*e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(8*a^3*d^3*e^3 - (3*a*c
^2*d^5*e + 4*a^2*c*d^3*e^3 - 15*a^3*d*e^5)*x^2 + 2*(a^2*c*d^4*e^2 - 5*a^3*
d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^3*d^4*e^3*x^3)
, 1/48*(3*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*sqrt(-a*
d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e +
 (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3
*e + a^2*d*e^3)*x)) - 2*(8*a^3*d^3*e^3 - (3*a*c^2*d^5*e + 4*a^2*c*d^3*e^3
- 15*a^3*d*e^5)*x^2 + 2*(a^2*c*d^4*e^2 - 5*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x))/(a^3*d^4*e^3*x^3)]
```

3.444.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{x^4(d + ex)}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**4/(e*x+d),x)
```

output
```
Timed out
```

3.444.7 Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^4(d+ex)}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex+d)x^4}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^4), x)
```

3.444.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(256) = 512$.

Time = 0.33 (sec) , antiderivative size = 912, normalized size of antiderivative = 3.19

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^4(d+ex)}\,dx$$
$$= \frac{(c^3d^6 + ac^2d^4e^2 + 3\,a^2cd^2e^4 - 5\,a^3e^6)\arctan\left(-\frac{\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}}{\sqrt{-ade}}\right)}{8\,\sqrt{-ade}a^2d^3e^2}$$
$$- \frac{3\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^2c^3d^8e^2 + 51\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^3c^2}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x, algorithm
="giac")
```

output
```
1/8*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*arctan(-(sqrt(
c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqr
t(-a*d*e)*a^2*d^3*e^2) - 1/24*(3*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x
 + a*e^2*x + a*d*e))*a^2*c^3*d^8*e^2 + 51*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
+ c*d^2*x + a*e^2*x + a*d*e))*a^3*c^2*d^6*e^4 + 105*(sqrt(c*d*e)*x - sqrt(
c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*c*d^4*e^6 + 33*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^5*d^2*e^8 + 16*sqrt(c*d*e
)*a^4*c*d^5*e^5 + 48*sqrt(c*d*e)*a^5*d^3*e^7 + 8*(sqrt(c*d*e)*x - sqrt(c*d
*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a*c^3*d^7*e + 72*(sqrt(c*d*e)*x - s
qrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^2*c^2*d^5*e^3 + 24*(sqrt(c
*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*c*d^3*e^5 - 4
0*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^4*d*e^
7 + 48*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a
*d*e))^2*a^2*c^2*d^6*e^2 + 144*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
 + c*d^2*x + a*e^2*x + a*d*e))^2*a^3*c*d^4*e^4 - 3*(sqrt(c*d*e)*x - sqrt(c
*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*c^3*d^6 - 3*(sqrt(c*d*e)*x - sqrt
(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a*c^2*d^4*e^2 - 9*(sqrt(c*d*e)*
x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^2*c*d^2*e^4 + 15*(sqr
t(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^3*e^6)/((a*d
*e - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)^3...
```

3.444.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^4(d+ex)}\,dx=\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^4\,\left(d+e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^4*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^4*(d + e*x)), x)
```

3.445 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^5(d+ex)} dx$

3.445.1 Optimal result

Integrand size = 40, antiderivative size = 389

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^5(d+ex)}\,dx = -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24x^3} + \frac{(5c^2d^4+6acd^2e^2-35a^2e^4)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{96a^2d^3e^2x^2} - \frac{(15c^3d^6+17ac^2d^4e^2+25a^2cd^2e^4-105a^3e^6)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{192a^3d^4e^3x} + \frac{(cd^2-ae^2)(5c^3d^6+9ac^2d^4e^2+15a^2cd^2e^4+35a^3e^6)\,\text{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128a^{7/2}d^{9/2}e^{7/2}}$$

output
```
1/128*(-a*e^2+c*d^2)*(35*a^3*e^6+15*a^2*c*d^2*e^4+9*a*c^2*d^4*e^2+5*c^3*d^
6)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(7/2)/d^(9/2)/e^(7/2)-1/4*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/d/x^4-1/24*(c/a/e-7*e/d^2)*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)/x^3+1/96*(-35*a^2*e^4+6*a*c*d^2*e^2+5*c^2*d^4)*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/d^3/e^2/x^2-1/192*(-105*a^3*e^6+25*a^2
*c*d^2*e^4+17*a*c^2*d^4*e^2+15*c^3*d^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)/a^3/d^4/e^3/x
```

3.445.2 Mathematica [A] (verified)

Time = 10.24 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^5(d+ex)}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(-15c^3d^6x^3+ac^2d^4ex^2(10d-17ex)+a^2cd^2e^2x(-8d^2+12dex-25e^2x^2)+a^3e^3(-48d^3+56d^2ex-70de^2x}{x^4}\right.}{192a^{7/2}d^{9/2}e^{7/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^5*(d + e*x)),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-15*c^3*d^6*x^3
+ a*c^2*d^4*e*x^2*(10*d - 17*e*x) + a^2*c*d^2*e^2*x*(-8*d^2 + 12*d*e*x - 2
5*e^2*x^2) + a^3*e^3*(-48*d^3 + 56*d^2*e*x - 70*d*e^2*x^2 + 105*e^3*x^3)))
/x^4 + (3*(5*c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*
e^6 - 35*a^4*e^8)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqr
t[d + e*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(192*a^(7/2)*d^(9/2)*e^(
7/2))
```

3.445.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1215, 1237, 27, 1237, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{x^5(d+ex)}\,dx$$

$$\Big\downarrow \; 1215$$

$$\int \frac{ae+cdx}{x^5\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \; 1237$$

$$-\frac{\int -\frac{ae(cd^2-6cexd-7ae^2)}{2x^4\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4ade}-\frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{4dx^4}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{cd^2-6cexd-7ae^2}{x^4\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^4}$$

$$\Big\downarrow \; 1237$$

$$\frac{-\frac{\int \frac{5c^2d^4+6ace^2d^2+4ce\left(cd^2-7ae^2\right)xd-35a^2e^4}{2x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3ade}-\frac{\left(\frac{cd}{ae}-\frac{7e}{d}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3x^3}}{8d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^4}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{5c^2d^4+6ace^2d^2+4ce\left(cd^2-7ae^2\right)xd-35a^2e^4}{x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6ade}-\frac{\left(\frac{cd}{ae}-\frac{7e}{d}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3x^3}}{8d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^4}$$

$$\Big\downarrow \; 1237$$

$$\frac{-\frac{-\frac{\int \frac{15c^3d^6+17ac^2e^2d^4+25a^2ce^4d^2+2ce\left(5c^2d^4+6ace^2d^2-35a^2e^4\right)xd-105a^3e^6}{2x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(\frac{5c^2d^4}{a}-35ae^4+6cd^2e^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2dex^2}}{6ade}-\frac{\left(\frac{cd}{ae}-\frac{7e}{d}\right)\sqrt{\cdots}}{\cdots}}{8d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^4}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{-\frac{\int \frac{15c^3d^6+17ac^2e^2d^4+25a^2ce^4d^2+2ce\left(5c^2d^4+6ace^2d^2-35a^2e^4\right)xd-105a^3e^6}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4ade}-\frac{\left(\frac{5c^2d^4}{a}-35ae^4+6cd^2e^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2dex^2}}{6ade}-\frac{\left(\frac{cd}{ae}-\frac{7e}{d}\right)\sqrt{\cdots}}{\cdots}}{8d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^4}$$

$$\Big\downarrow \; 1228$$

$$\frac{-\frac{-\frac{-\frac{3\left(cd^2-ae^2\right)\left(35a^3e^6+15a^2cd^2e^4+9ac^2d^4e^2+5c^3d^6\right)\int\frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(-105a^3e^6+25a^2cd^2e^4+17ac^2d^4e^2+15c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+a\cdots}}{adex}}{4ade}-\cdots}{6ade}-\cdots}{8d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^4}$$

$$\Big\downarrow \; 1154$$

3.445. $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^5(d+ex)}\,dx$

$$-\frac{-\frac{-\frac{\frac{3\left(cd^2-ae^2\right)\left(35a^3e^6+15a^2cd^2e^4+9ac^2d^4e^2+5c^3d^6\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade}}{4ade}-\frac{\left(-105a^3e^6+25a^2cd^2e^4+17ac^2d^4e^2\cdots\right.}{a\cdots}}{6ade}\cdots}{8d} - \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^4}$$

$\Big\downarrow$ 219

$$-\frac{-\frac{-\frac{\frac{3\left(cd^2-ae^2\right)\left(35a^3e^6+15a^2cd^2e^4+9ac^2d^4e^2+5c^3d^6\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2a^{3/2}d^{3/2}e^{3/2}}}{4ade}-\frac{\left(-105a^3e^6+25a^2cd^2e^4+17ac^2d^4e^2+15c^3d^6\right)\cdots}{adex}}{6ade}\cdots}{8d} - \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^4}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^5*(d + e*x)),x]
```

output
```
-1/4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x^4) + (-1/3*(((c*d)/(
a*e) - (7*e)/d)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x^3 - (-1/2*(
((5*c^2*d^4)/a + 6*c*d^2*e^2 - 35*a*e^4)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(d*e*x^2) - (-(((15*c^3*d^6 + 17*a*c^2*d^4*e^2 + 25*a^2*c*d^2*
e^4 - 105*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x))
 + (3*(c*d^2 - a*e^2)*(5*c^3*d^6 + 9*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 + 35
*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*a^(3/2)*d^(3/2)*e^(3/2)
))/(4*a*d*e))/(6*a*d*e))/(8*d)
```

3.445.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.445. $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^5(d+ex)}\,dx$

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.445.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3470 vs. $2(355) = 710$.

Time = 1.12 (sec) , antiderivative size = 3471, normalized size of antiderivative = 8.92

method	result	size
default	Expression too large to display	3471

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/d*(-1/4/a/d/e/x^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-5/8*(a*e^2+c*d
^2)/a/d/e*(-1/3/a/d/e/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/2*(a*e
^2+c*d^2)/a/d/e*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/
4*(a*e^2+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+
1/2*(a*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^
2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^
2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+2*
c/a*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d
*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)
))+1/2*c/a*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((
1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*
(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))))-1/4*c/a*(-1/2
/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/4*(a*e^2+c*d^2)/a/d/e
*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)/a/d
/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*
a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
)/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*...
```

3.445.5 Fricas [A] (verification not implemented)

Time = 8.55 (sec) , antiderivative size = 702, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^5(d+ex)}\,dx$$
$$= \left[-\frac{3\left(5\,c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+20\,a^3cd^2e^6-35\,a^4e^8\right)\sqrt{ade}x^4\log\left(\frac{8\,a^2d^2e^2+\left(c^2d^4+6\,acd^2e^2+a^2e^4\right)x^2-4\,\sqrt{c}}{}\right.}{}\right.$$
$$-\frac{3\left(5\,c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+20\,a^3cd^2e^6-35\,a^4e^8\right)\sqrt{-ade}x^4\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,ade+(}{2\left(acd^2e^2x^2+a^2d^2e^2+(acd^3e\right.}\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/768*(3*(5*c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2
*e^6 - 35*a^4*e^8)*sqrt(a*d*e)*x^4*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d
^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2)
+ 4*(48*a^4*d^4*e^4 + (15*a*c^3*d^7*e + 17*a^2*c^2*d^5*e^3 + 25*a^3*c*d^3*
e^5 - 105*a^4*d*e^7)*x^3 - 2*(5*a^2*c^2*d^6*e^2 + 6*a^3*c*d^4*e^4 - 35*a^4
*d^2*e^6)*x^2 + 8*(a^3*c*d^5*e^3 - 7*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x))/(a^4*d^5*e^4*x^4), -1/384*(3*(5*c^4*d^8 + 4*a*c^3*
d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 35*a^4*e^8)*sqrt(-a*d*e)*
x^4*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e +
a^2*d*e^3)*x)) + 2*(48*a^4*d^4*e^4 + (15*a*c^3*d^7*e + 17*a^2*c^2*d^5*e^3
+ 25*a^3*c*d^3*e^5 - 105*a^4*d*e^7)*x^3 - 2*(5*a^2*c^2*d^6*e^2 + 6*a^3*c*d
^4*e^4 - 35*a^4*d^2*e^6)*x^2 + 8*(a^3*c*d^5*e^3 - 7*a^4*d^3*e^5)*x)*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^4*d^5*e^4*x^4)]
```

3.445.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^5(d + ex)}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**5/(e*x+d),x)
```

output
```
Timed out
```

3.445.7 Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^5(d + ex)}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)x^5}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^5), x)
```

3.445.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1483 vs. 2(355) = 710.

Time = 0.34 (sec) , antiderivative size = 1483, normalized size of antiderivative = 3.81

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^5(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x, algorithm
="giac")
```

output
```
-1/64*(5*c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6
- 35*a^4*e^8)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
+ a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a^3*d^4*e^3) + 1/192*(15*(sqrt(c*d*e
)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^3*c^4*d^11*e^3 + 396*
(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*c^3*d^9*
e^5 + 1170*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a
^5*c^2*d^7*e^7 + 1212*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
+ a*d*e))*a^6*c*d^5*e^9 + 279*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
a*e^2*x + a*d*e))*a^7*d^3*e^11 + 128*sqrt(c*d*e)*a^5*c^2*d^8*e^6 + 256*sqr
t(c*d*e)*a^6*c*d^6*e^8 + 384*sqrt(c*d*e)*a^7*d^4*e^10 + 73*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^2*c^4*d^10*e^2 + 980*(s
qrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*c^3*d^8*
e^4 + 2238*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3
*a^4*c^2*d^6*e^6 + 292*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))^3*a^5*c*d^4*e^8 - 511*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x
 + a*e^2*x + a*d*e))^3*a^6*d^2*e^10 + 384*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqr
t(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^3*c^3*d^9*e^3 + 1792*sqrt(c*
d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^4*c
^2*d^7*e^5 + 2432*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
a*e^2*x + a*d*e))^2*a^5*c*d^5*e^7 - 55*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 ...
```

3.445.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^5(d+ex)}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^5\,\left(d+e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^5*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^5*(d + e*x)), x)
```

3.445. $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^5(d+ex)}\,dx$

3.446 $\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

3.446.1 Optimal result

Integrand size = 40, antiderivative size = 449

$$\int \frac{x^3(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{d+ex}\,dx = \frac{(21c^4d^8-6a^2c^2d^4e^4-8a^3cd^2e^6-7a^4e^8)\,(cd^2+ae^2+2cdex)\,\sqrt{}}{512c^4d^4e^5}$$
$$+\frac{1}{20}\left(\frac{a}{cd}-\frac{3d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}+\frac{x^3(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{6e}$$
$$-\frac{(105c^3d^6-21ac^2d^4e^2-33a^2cd^2e^4-35a^3e^6-6cde(21c^2d^4-6acd^2e^2-7a^2e^4)\,x)\,(ade+(cd^2+ae^2)\,x+}{960c^3d^3e^4}$$
$$-\frac{(cd^2-ae^2)^3\,(21c^3d^6+21ac^2d^4e^2+15a^2cd^2e^4+7a^3e^6)\,\text{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{1024c^{9/2}d^{9/2}e^{11/2}}$$

output
```
1/20*(a/c/d-3*d/e^2)*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/6*x^3*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e-1/960*(105*c^3*d^6-21*a*c^2*d^4*e
^2-33*a^2*c*d^2*e^4-35*a^3*e^6-6*c*d*e*(-7*a^2*e^4-6*a*c*d^2*e^2+21*c^2*d^
4)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/e^4-1/1024*(-a*e^2+c
*d^2)^3*(7*a^3*e^6+15*a^2*c*d^2*e^4+21*a*c^2*d^4*e^2+21*c^3*d^6)*arctanh(1
/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2))/c^(9/2)/d^(9/2)/e^(11/2)+1/512*(-7*a^4*e^8-8*a^3*c*d^2*e
^6-6*a^2*c^2*d^4*e^4+21*c^4*d^8)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e^5
```

3.446.2 Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.86

$$\int \frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\Big(\sqrt{c}\sqrt{d}\sqrt{e}(-105a^5e^{10}+5a^4cde^8(11d+14}{}$$

input
```
Integrate[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-105*a^5*e^10 + 5
*a^4*c*d*e^8*(11*d + 14*e*x) + 2*a^3*c^2*d^2*e^6*(27*d^2 - 16*d*e*x - 28*e
^2*x^2) + 6*a^2*c^3*d^3*e^4*(13*d^3 - 6*d^2*e*x + 4*d*e^2*x^2 + 8*e^3*x^3)
 + a*c^4*d^4*e^2*(-525*d^4 + 336*d^3*e*x - 264*d^2*e^2*x^2 + 224*d*e^3*x^3
 + 1664*e^4*x^4) + c^5*d^5*(315*d^5 - 210*d^4*e*x + 168*d^3*e^2*x^2 - 144*
d^2*e^3*x^3 + 128*d*e^4*x^4 + 1280*e^5*x^5)) - (15*(c*d^2 - a*e^2)^3*(21*c
^3*d^6 + 21*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 + 7*a^3*e^6)*ArcTanh[(Sqrt[c]
*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*S
qrt[d + e*x])))/(7680*c^(9/2)*d^(9/2)*e^(11/2))
```

3.446.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {1215, 1236, 27, 1236, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{d+ex}\,dx$$

$$\Big\downarrow \text{1215}$$

$$\int x^3(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{\int -\frac{3}{2}cdx^2\left(2ade+\left(3cd^2-ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{6cde}+\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}$$

$$\Big\downarrow \; 27$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}-\frac{\int x^2\left(2ade+\left(3cd^2-ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{4e}$$

$$\Big\downarrow \; 1236$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}-\frac{\frac{\int -\frac{1}{2}x\left(4ade\left(3cd^2-ae^2\right)+\left(21c^2d^4-6ace^2d^2-7a^2e^4\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{5cde}+\frac{1}{5}x^2\left(\frac{3d}{e}-\frac{ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}{4e}$$

$$\Big\downarrow \; 27$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}-\frac{\frac{1}{5}x^2\left(\frac{3d}{e}-\frac{ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}-\frac{\int x\left(4ade\left(3cd^2-ae^2\right)+\left(21c^2d^4-6ace^2d^2-7a^2e^4\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{10cde}}{4e}$$

$$\Big\downarrow \; 1225$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}-\frac{\frac{1}{5}x^2\left(\frac{3d}{e}-\frac{ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}-\frac{\frac{5\left(-7a^4e^8-8a^3cd^2e^6-6a^2c^2d^4e^4+21c^4d^8\right)\int\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{16c^2d^2e^2}-\frac{\left(-35a^3e^6\ldots\right.}{}}{}}{4e}$$

$$\Big\downarrow \; 1087$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}-\frac{\frac{1}{5}x^2\left(\frac{3d}{e}-\frac{ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}-\frac{\frac{5\left(-7a^4e^8-8a^3cd^2e^6-6a^2c^2d^4e^4+21c^4d^8\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cde\ldots}}{4cde}\ldots\right)}{16c^2d^2e^2}}{}}{}$$

$$\Big\downarrow \; 1092$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}-\frac{\frac{1}{5}x^2\left(\frac{3d}{e}-\frac{ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}-\frac{\frac{5\left(-7a^4e^8-8a^3cd^2e^6-6a^2c^2d^4e^4+21c^4d^8\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cde\ldots}}{4cde}\ldots\right)}{16c^2d^2\ldots}}{}}{}$$

3.446. $\displaystyle\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}dx$

$$\Big\downarrow 219$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}-$$

$$\frac{1}{5}x^2\left(\frac{3d}{e}-\frac{ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}-\frac{5\left(-7a^4e^8-8a^3cd^2e^6-6a^2c^2d^4e^4+21c^4d^8\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cde}}{4cde}\right.}{16c^2d^2e^2}$$

input
```
Int[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

output
```
(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(6*e) - ((((3*d)/e - (
a*e)/(c*d))*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/5 - (-1/24*
((105*c^3*d^6 - 21*a*c^2*d^4*e^2 - 33*a^2*c*d^2*e^4 - 35*a^3*e^6 - 6*c*d*e
*(21*c^2*d^4 - 6*a*c*d^2*e^2 - 7*a^2*e^4)*x)*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2))/(c^2*d^2*e^2) + (5*(21*c^4*d^8 - 6*a^2*c^2*d^4*e^4 - 8*a
^3*c*d^2*e^6 - 7*a^4*e^8)*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2]])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 +
 a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(16*c^2*d^2*e^2))/(10*c*
d*e))/(4*e)
```

3.446.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.446. $\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

3.446.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1425 vs. 2(415) = 830.

Time = 0.64 (sec) , antiderivative size = 1426, normalized size of antiderivative = 3.18

method	result	size
default	Expression too large to display	1426

input
```
int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(1/6*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/e-7/12*(a*e^2+c*d^2
)/c/d/e*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/e-1/2*(a*e^2+c*d^
2)/c/d/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2
+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(
a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))))-1/6*a/c*(1/8*(2*c*d*e
*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*
d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*
ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2))/(c*d*e)^(1/2))))+d^2/e^3*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^
2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^
2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^
(1/2)))-d/e^2*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/e-1/2*(a*e^
2+c*d^2)/c/d/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x
+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*...
```

3.446.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 1044, normalized size of antiderivative = 2.33

$$\int \frac{x^3(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{d+ex}\,dx = \left[-\frac{15\,(21\,c^6d^{12}-42\,ac^5d^{10}e^2+15\,a^2c^4d^8e^4+4\,a^3c^3d^6e^6+3\,a^4c\right.$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/30720*(15*(21*c^6*d^12 - 42*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 + 4*a^
3*c^3*d^6*e^6 + 3*a^4*c^2*d^4*e^8 + 6*a^5*c*d^2*e^10 - 7*a^6*e^12)*sqrt(c*
d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*
e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(1280*c^6*d^6*e^6*x^5 + 315*c^6*d^11
*e - 525*a*c^5*d^9*e^3 + 78*a^2*c^4*d^7*e^5 + 54*a^3*c^3*d^5*e^7 + 55*a^4*
c^2*d^3*e^9 - 105*a^5*c*d*e^11 + 128*(c^6*d^7*e^5 + 13*a*c^5*d^5*e^7)*x^4
- 16*(9*c^6*d^8*e^4 - 14*a*c^5*d^6*e^6 - 3*a^2*c^4*d^4*e^8)*x^3 + 8*(21*c^
6*d^9*e^3 - 33*a*c^5*d^7*e^5 + 3*a^2*c^4*d^5*e^7 - 7*a^3*c^3*d^3*e^9)*x^2
- 2*(105*c^6*d^10*e^2 - 168*a*c^5*d^8*e^4 + 18*a^2*c^4*d^6*e^6 + 16*a^3*c^
3*d^4*e^8 - 35*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x))/(c^5*d^5*e^6), 1/15360*(15*(21*c^6*d^12 - 42*a*c^5*d^10*e^2 + 15*a^
2*c^4*d^8*e^4 + 4*a^3*c^3*d^6*e^6 + 3*a^4*c^2*d^4*e^8 + 6*a^5*c*d^2*e^10 -
 7*a^6*e^12)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2
*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(1280*c^6*d^6*e^6*x^5 + 315*c^6*d^1
1*e - 525*a*c^5*d^9*e^3 + 78*a^2*c^4*d^7*e^5 + 54*a^3*c^3*d^5*e^7 + 55*a^4
*c^2*d^3*e^9 - 105*a^5*c*d*e^11 + 128*(c^6*d^7*e^5 + 13*a*c^5*d^5*e^7)*x^4
 - 16*(9*c^6*d^8*e^4 - 14*a*c^5*d^6*e^6 - 3*a^2*c^4*d^4*e^8)*x^3 + 8*(21*c
^6*d^9*e^3 - 33*a*c^5*d^7*e^5 + 3*a^2*c^4*d^5*e^7 - 7*a^3*c^3*d^3*e^9)*...
```

3.446.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 (ade + (cd^2 + ae^2)\, x + cdex^2)^{3/2}}{d + ex}\, dx = \text{Timed out}$$

input
```
integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

output
```
Timed out
```

3.446.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{d+ex}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.446.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.14

$$\int \frac{x^3(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{d+ex}\,dx=\frac{1}{7680}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(2\left(8\left(10\,cdx+\frac{c^6d^7}{}\right.\right.\right.\right.\right.$$
$$+\frac{(21\,c^6d^{12}-42\,ac^5d^{10}e^2+15\,a^2c^4d^8e^4+4\,a^3c^3d^6e^6+3\,a^4c^2d^4e^8+6\,a^5cd^2e^{10}-7\,a^6e^{12})\log\left(\left|-cd^2-ae^2-\right.\right.}{1024\sqrt{cde}c^4d^4e^5}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm
="giac")
```

output
```
1/7680*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*(10*c*d*x +
 (c^6*d^7*e^4 + 13*a*c^5*d^5*e^6)/(c^5*d^5*e^5))*x - (9*c^6*d^8*e^3 - 14*a
*c^5*d^6*e^5 - 3*a^2*c^4*d^4*e^7)/(c^5*d^5*e^5))*x + (21*c^6*d^9*e^2 - 33*
a*c^5*d^7*e^4 + 3*a^2*c^4*d^5*e^6 - 7*a^3*c^3*d^3*e^8)/(c^5*d^5*e^5))*x -
(105*c^6*d^10*e - 168*a*c^5*d^8*e^3 + 18*a^2*c^4*d^6*e^5 + 16*a^3*c^3*d^4*
e^7 - 35*a^4*c^2*d^2*e^9)/(c^5*d^5*e^5))*x + (315*c^6*d^11 - 525*a*c^5*d^9
*e^2 + 78*a^2*c^4*d^7*e^4 + 54*a^3*c^3*d^5*e^6 + 55*a^4*c^2*d^3*e^8 - 105*
a^5*c*d*e^10)/(c^5*d^5*e^5)) + 1/1024*(21*c^6*d^12 - 42*a*c^5*d^10*e^2 + 1
5*a^2*c^4*d^8*e^4 + 4*a^3*c^3*d^6*e^6 + 3*a^4*c^2*d^4*e^8 + 6*a^5*c*d^2*e^
10 - 7*a^6*e^12)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - s
qrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^4*d^4*e^5)
```

3.446.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{d + ex}\,dx = \int \frac{x^3\,\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{d + e\,x}\,dx$$

input
```
int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),x)
```

output
```
int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x), x)
```

3.447 $\int \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

3.447.1 Optimal result

Integrand size = 40, antiderivative size = 352

$$\int \frac{x^2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{d+ex}\,dx = -\frac{(cd^2-ae^2)\,(7c^2d^4+6acd^2e^2+3a^2e^4)\,(cd^2+ae^2+2cdex)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{128c^3d^3e^4} + \frac{x^2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{5e} + \frac{(35c^2d^4-12acd^2e^2-15a^2e^4-6cde(7cd^2-3ae^2)\,x)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{240c^2d^2e^3} + \frac{(cd^2-ae^2)^3\,(7c^2d^4+6acd^2e^2+3a^2e^4)\,\text{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{256c^{7/2}d^{7/2}e^{9/2}}$$

output
```
1/5*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e+1/240*(35*c^2*d^4-12*a*c
*d^2*e^2-15*a^2*e^4-6*c*d*e*(-3*a*e^2+7*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(3/2)/c^2/d^2/e^3+1/256*(-a*e^2+c*d^2)^3*(3*a^2*e^4+6*a*c*d^2*e^
2+7*c^2*d^4)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(7/2)/d^(7/2)/e^(9/2)-1/128*(-a*
e^2+c*d^2)*(3*a^2*e^4+6*a*c*d^2*e^2+7*c^2*d^4)*(2*c*d*e*x+a*e^2+c*d^2)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^4
```

3.447.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.86

$$\int \frac{x^2(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex} dx = \frac{\sqrt{(ae + cdx)(d + ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(45a^4e^8 - 30a^3cde^6(d + ex) - \right.}{}$$

input
```
Integrate[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(45*a^4*e^8 - 30*a
^3*c*d*e^6*(d + e*x) - 6*a^2*c^2*d^2*e^4*(6*d^2 - 3*d*e*x - 4*e^2*x^2) + 2
*a*c^3*d^3*e^2*(95*d^3 - 61*d^2*e*x + 48*d*e^2*x^2 + 264*e^3*x^3) + c^4*d^
4*(-105*d^4 + 70*d^3*e*x - 56*d^2*e^2*x^2 + 48*d*e^3*x^3 + 384*e^4*x^4)) +
 (15*(c*d^2 - a*e^2)^3*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(Sq
rt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d
*x]*Sqrt[d + e*x])))/(1920*c^(7/2)*d^(7/2)*e^(9/2))
```

3.447.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1215, 1236, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{d+ex}dx$$

$$\Big\downarrow \text{ 1215}$$

$$\int x^2(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}dx$$

$$\Big\downarrow \text{ 1236}$$

$$\frac{\int -\frac{1}{2}cdx\left(4ade+\left(7cd^2-3ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{5cde}+\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}-\frac{\int x\left(4ade+\left(7cd^2-3ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{10e}$$

$\Big\downarrow$ 1225

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}-\frac{\frac{5\left(cd^2-ae^2\right)\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\int\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{16c^2d^2e^2}-\frac{\left(-15a^2e^4-6cdex\left(7cd^2-3ae^2\right)-12acd^2e^2+35c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+\cdots\right.}{24c^2d^2e^2}}{10e}$$

$\Big\downarrow$ 1087

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}-\frac{\frac{5\left(cd^2-ae^2\right)\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cde}\right)}{16c^2d^2e^2}-\frac{\left(-15a^2e^4-6c\cdots\right.}{\cdots}}{10e}$$

$\Big\downarrow$ 1092

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}-\frac{\frac{5\left(cd^2-ae^2\right)\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+\cdots}{\sqrt{cdex^2+\left(cd^2+ae^2\right.\cdots}}}{4cde}\cdots\right.}{16c^2d^2e^2}\cdots}{10e}$$

$\Big\downarrow$ 219

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}-\frac{\frac{5\left(cd^2-ae^2\right)\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{3/2}}\right)}{16c^2d^2e^2}\cdots}{10e}$$

input

```
Int[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

3.447. $\int\frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}dx$

output
```
(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*e) - (-1/24*((35*c^
2*d^4 - 12*a*c*d^2*e^2 - 15*a^2*e^4 - 6*c*d*e*(7*c*d^2 - 3*a*e^2)*x)*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(c^2*d^2*e^2) + (5*(c*d^2 - a*e^
2)*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*(((c*d^2 + a*e^2 + 2*c*d*e*x)*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2
*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2]])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(16*c^2
*d^2*e^2))/(10*e)
```

3.447.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

3.447. $\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx$

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

3.447.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 796 vs. $2(322) = 644$.

Time = 0.64 (sec) , antiderivative size = 797, normalized size of antiderivative = 2.26

method	result
default	$\frac{\left(ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}\right)^{\frac{5}{2}}}{5cde}-\frac{\left(e^{2}a+c\,d^{2}\right)\left(\frac{\left(2cdex+e^{2}a+c\,d^{2}\right)\left(ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}\right)^{\frac{3}{2}}}{8cde}+\frac{3\left(4ac\,d^{2}e^{2}-\left(e^{2}a+c\,d^{2}\right)^{2}\right)\left(\frac{(2cdex+e^{2}a+c\cdots}{\cdots}\right.}{\cdots}\right.}{e}$

input
```
int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/e-1/2*(a*e^2+c*d^2)/c
/d/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d
^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^
2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))))-d/e^2*(1/8*(2*c*d*e*x+a*e
^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^
2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/
2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2))/(c*d*e)^(1/2)))+d^2/e^3*(1/3*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e
))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/c/d/e*(c*d*e
*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/e*ln((1/2*
e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2
)*(x+d/e))^(1/2))/(c*d*e)^(1/2)))
```

3.447.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 846, normalized size of antiderivative = 2.40

$$\int \frac{x^2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{d+ex}\,dx = \left[-\frac{15\,(7\,c^5d^{10}-15\,ac^4d^8e^2+6\,a^2c^3d^6e^4+2\,a^3c^2d^4e^6+3\,a^4cd^2e\cdots}{\cdots}\right.$$

$$-\frac{15\,(7\,c^5d^{10}-15\,ac^4d^8e^2+6\,a^2c^3d^6e^4+2\,a^3c^2d^4e^6+3\,a^4cd^2e^8-3\,a^5e^{10})\sqrt{-cde}\,\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae\cdots}}{2\,(c^2d^2e^2x^2+acd\cdots}\right.}{\cdots}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/7680*(15*(7*c^5*d^10 - 15*a*c^4*d^8*e^2 + 6*a^2*c^3*d^6*e^4 + 2*a^3*c^
2*d^4*e^6 + 3*a^4*c*d^2*e^8 - 3*a^5*e^10)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^
2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d
*e^3)*x) - 4*(384*c^5*d^5*e^5*x^4 - 105*c^5*d^9*e + 190*a*c^4*d^7*e^3 - 36
*a^2*c^3*d^5*e^5 - 30*a^3*c^2*d^3*e^7 + 45*a^4*c*d*e^9 + 48*(c^5*d^6*e^4 +
 11*a*c^4*d^4*e^6)*x^3 - 8*(7*c^5*d^7*e^3 - 12*a*c^4*d^5*e^5 - 3*a^2*c^3*d
^3*e^7)*x^2 + 2*(35*c^5*d^8*e^2 - 61*a*c^4*d^6*e^4 + 9*a^2*c^3*d^4*e^6 - 1
5*a^3*c^2*d^2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^
4*e^5), -1/3840*(15*(7*c^5*d^10 - 15*a*c^4*d^8*e^2 + 6*a^2*c^3*d^6*e^4 + 2
*a^3*c^2*d^4*e^6 + 3*a^4*c*d^2*e^8 - 3*a^5*e^10)*sqrt(-c*d*e)*arctan(1/2*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqr
t(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2
*(384*c^5*d^5*e^5*x^4 - 105*c^5*d^9*e + 190*a*c^4*d^7*e^3 - 36*a^2*c^3*d^5
*e^5 - 30*a^3*c^2*d^3*e^7 + 45*a^4*c*d*e^9 + 48*(c^5*d^6*e^4 + 11*a*c^4*d^
4*e^6)*x^3 - 8*(7*c^5*d^7*e^3 - 12*a*c^4*d^5*e^5 - 3*a^2*c^3*d^3*e^7)*x^2
+ 2*(35*c^5*d^8*e^2 - 61*a*c^4*d^6*e^4 + 9*a^2*c^3*d^4*e^6 - 15*a^3*c^2*d^
2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e^5)]
```

3.447.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{d+ex}\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

output
```
Timed out
```

3.447.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{d+ex}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.447.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.15

$$\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \frac{1}{1920}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(6\left(8\,cdx+\frac{c^5d^6e^3+}{c^4}\right.\right.\right.\right.$$
$$-\frac{(7\,c^5d^{10}-15\,ac^4d^8e^2+6\,a^2c^3d^6e^4+2\,a^3c^2d^4e^6+3\,a^4cd^2e^8-3\,a^5e^{10})\log\left(\left|-cd^2-ae^2-2\,\sqrt{cde}\left(\sqrt{cde}x\right.\right.\right.}{256\,\sqrt{cde}c^3d^3e^4}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm
="giac")
```

output
```
1/1920*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(6*(8*c*d*x + (c^
5*d^6*e^3 + 11*a*c^4*d^4*e^5)/(c^4*d^4*e^4))*x - (7*c^5*d^7*e^2 - 12*a*c^4
*d^5*e^4 - 3*a^2*c^3*d^3*e^6)/(c^4*d^4*e^4))*x + (35*c^5*d^8*e - 61*a*c^4*
d^6*e^3 + 9*a^2*c^3*d^4*e^5 - 15*a^3*c^2*d^2*e^7)/(c^4*d^4*e^4))*x - (105*
c^5*d^9 - 190*a*c^4*d^7*e^2 + 36*a^2*c^3*d^5*e^4 + 30*a^3*c^2*d^3*e^6 - 45
*a^4*c*d*e^8)/(c^4*d^4*e^4)) - 1/256*(7*c^5*d^10 - 15*a*c^4*d^8*e^2 + 6*a^
2*c^3*d^6*e^4 + 2*a^3*c^2*d^4*e^6 + 3*a^4*c*d^2*e^8 - 3*a^5*e^10)*log(abs(
-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
 a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^3*d^3*e^4)
```

3.447.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \int \frac{x^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{d+e\,x}\,dx$$

input
```
int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),x)
```

output
```
int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x), x)
```

3.448 $\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

3.448.1 Optimal result

Integrand size = 38, antiderivative size = 295

$$\int \frac{x(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{d+ex}\,dx = \frac{(cd^2-ae^2)\,(5cd^2+3ae^2)\,(cd^2+ae^2+2cdex)\,\sqrt{ade+(cd^2+ae^2}}{64c^2d^2e^3}$$
$$-\frac{1}{24}\left(\frac{3a}{cd}+\frac{5d}{e^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}+\frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{4cde(d+ex)}$$
$$-\frac{(cd^2-ae^2)^3\,(5cd^2+3ae^2)\,\text{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128c^{5/2}d^{5/2}e^{7/2}}$$

output
```
-1/24*(3*a/c/d+5*d/e^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/4*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/e/(e*x+d)-1/128*(-a*e^2+c*d^2)^3*(3*
a*e^2+5*c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)
/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(5/2)/e^(7/2)+1/64*(-a
*e^2+c*d^2)*(3*a*e^2+5*c*d^2)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)/c^2/d^2/e^3
```

3.448.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.80

$$\int \frac{x(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(-9a^3e^6+3a^2cde^4(3d+2ex)+\right.}{}$$

input
```
Integrate[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-9*a^3*e^6 + 3*a^
2*c*d*e^4*(3*d + 2*e*x) + a*c^2*d^2*e^2*(-31*d^2 + 20*d*e*x + 72*e^2*x^2)
+ c^3*d^3*(15*d^3 - 10*d^2*e*x + 8*d*e^2*x^2 + 48*e^3*x^3)) - (3*(c*d^2 -
a*e^2)^3*(5*c*d^2 + 3*a*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt
[e]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(192*c^(5/2)*
d^(5/2)*e^(7/2))
```

3.448.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {1215, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{d+ex}dx$$

$$\Big\downarrow \text{1215}$$

$$\int x(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}dx$$

$$\Big\downarrow \text{1225}$$

$$-\frac{\left(\frac{3a^2e^2}{c}+2ad^2-\frac{5cd^4}{e^2}\right)\int \sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{16d}-\frac{\left(-3ae^2+5cd^2-6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24cde^2}$$

$$\Big\downarrow \text{1087}$$

$$-\frac{\left(\frac{3a^2e^2}{c}+2ad^2-\frac{5cd^4}{e^2}\right)\left(\frac{(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{(cd^2-ae^2)^2\int\frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8cde}\right)}{16d}-\frac{(-3ae^2+5cd^2-6cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{24cde^2}$$

$\Big\downarrow$ 1092

$$-\frac{\left(\frac{3a^2e^2}{c}+2ad^2-\frac{5cd^4}{e^2}\right)\left(\frac{(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{(cd^2-ae^2)^2\int\frac{1}{4cde-\frac{(cd^2+2cexd+ae^2)^2}{cdex^2+(cd^2+ae^2)x+ade}}d\frac{cd^2+2cexd-}{\sqrt{cdex^2+(cd^2+a}}}{4cde}\right.}{16d}$$

$$\frac{(-3ae^2+5cd^2-6cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{24cde^2}$$

$\Big\downarrow$ 219

$$-\frac{\left(\frac{3a^2e^2}{c}+2ad^2-\frac{5cd^4}{e^2}\right)\left(\frac{(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{(cd^2-ae^2)^2\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{3/2}}\right.}{16d}$$

$$\frac{(-3ae^2+5cd^2-6cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{24cde^2}$$

input
```
Int[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

output
```
-1/24*((5*c*d^2 - 3*a*e^2 - 6*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2))/(c*d*e^2) - ((2*a*d^2 - (5*c*d^4)/e^2 + (3*a^2*e^2)/c)*(((c*d^
2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c*d
*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sq
rt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^
(3/2)*e^(3/2))))/(16*d)
```

3.448. $\int\frac{x\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{d+ex}dx$

3.448.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

3.448. $\int \frac{x(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx$

3.448.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.64

method	result
default	$\frac{\frac{\left(2cdex+e^{2}a+c\,d^{2}\right)\left(ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}\right)^{\frac{3}{2}}}{8cde}+\frac{3\left(4ac\,d^{2}e^{2}-\left(e^{2}a+c\,d^{2}\right)^{2}\right)\left(\frac{\left(2cdex+e^{2}a+c\,d^{2}\right)\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}}}{4cde}+\frac{\left(4ac\,d^{2}e^{2}-\left(\right.\right.}{}\right.}{16cde}}{e}$

input
```
int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x,method=_RETURNVERB
OSE)
```

output
```
1/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^
2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2
+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))-d/e^2*(1/3*(c*d*e*(x+d/e)^2
+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*c*d*e*(x+d/e)+e^2*
a-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*
d^2)^2/c/d/e*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(
x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)))
```

3.448.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 676, normalized size of antiderivative = 2.29

$$\int \frac{x(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{d+ex}\,dx = \left[-\frac{3\left(5\,c^4d^8-12\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6-3\,a^4e^8\right)\sqrt{cde}}{}\right.$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="
fricas")
```

output
```
[-1/768*(3*(5*c^4*d^8 - 12*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2
*e^6 - 3*a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*
e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x +
 c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(48*c^4*d^4
*e^4*x^3 + 15*c^4*d^7*e - 31*a*c^3*d^5*e^3 + 9*a^2*c^2*d^3*e^5 - 9*a^3*c*d
*e^7 + 8*(c^4*d^5*e^3 + 9*a*c^3*d^3*e^5)*x^2 - 2*(5*c^4*d^6*e^2 - 10*a*c^3
*d^4*e^4 - 3*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x))/(c^3*d^3*e^4), 1/384*(3*(5*c^4*d^8 - 12*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*
e^4 + 4*a^3*c*d^2*e^6 - 3*a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2
*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(48*c^4*d^4*e
^4*x^3 + 15*c^4*d^7*e - 31*a*c^3*d^5*e^3 + 9*a^2*c^2*d^3*e^5 - 9*a^3*c*d*e
^7 + 8*(c^4*d^5*e^3 + 9*a*c^3*d^3*e^5)*x^2 - 2*(5*c^4*d^6*e^2 - 10*a*c^3*d
^4*e^4 - 3*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
)/(c^3*d^3*e^4)]
```

3.448.6 Sympy [A] (verification not implemented)

Time = 10.98 (sec) , antiderivative size = 1093, normalized size of antiderivative = 3.71

$$\int \frac{x(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex} dx = \text{Too large to display}$$

input
```
integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

output
```
a*e*Piecewise(((-a*(a*e**2/6 + c*d**2/6)/(2*c) - (a*e**2 + c*d**2)*(a*d*e/
3 - (a*e**2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(2*c*d*e))*
Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)*sqrt(a*d*e + c*
d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e - (a*e**2 + c*d**2)
**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*log(x - (-a*e**2 -
 c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*c*d*e))**2), Tru
e)) + (x**2/3 + x*(a*e**2/6 + c*d**2/6)/(2*c*d*e) + (a*d*e/3 - (a*e**2/6 +
 c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(c*d*e))*sqrt(a*d*e + c*d*
e*x**2 + x*(a*e**2 + c*d**2)), Ne(c*d*e, 0)), (2*(-a*d*e*(a*d*e + x*(a*e**
2 + c*d**2))**(3/2)/3 + (a*d*e + x*(a*e**2 + c*d**2))**(5/2)/5)/(a*e**2 +
c*d**2)**2, Ne(a*e**2 + c*d**2, 0)), (x**2*sqrt(a*d*e)/2, True)) + c*d*Pie
cewise(((-a*(a*d*e/4 - (a*e**2/8 + c*d**2/8)*(5*a*e**2/2 + 5*c*d**2/2)/(3*
c*d*e))/(2*c) - (a*e**2 + c*d**2)*(-2*a*(a*e**2/8 + c*d**2/8)/(3*c) - (3*a
*e**2/2 + 3*c*d**2/2)*(a*d*e/4 - (a*e**2/8 + c*d**2/8)*(5*a*e**2/2 + 5*c*d
**2/2)/(3*c*d*e))/(2*c*d*e))/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2
*c*d*e*x + 2*sqrt(c*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/s
qrt(c*d*e), Ne(a*d*e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2
 - c*d**2)/(2*c*d*e))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x
- (-a*e**2 - c*d**2)/(2*c*d*e))**2), True)) + sqrt(a*d*e + c*d*e*x**2 + x*
(a*e**2 + c*d**2))*(x**3/4 + x**2*(a*e**2/8 + c*d**2/8)/(3*c*d*e) + x*(...
```

3.448.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.448.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.05

$$\int \frac{x(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \frac{1}{192}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(6\,cdx+\frac{c^4d^5e^2+9\,ac^3d\right.\right.\right.$$
$$+\frac{(5\,c^4d^8-12\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6-3\,a^4e^8)\log\left(\left|-cd^2-ae^2-2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2}\right.\right.\right.}{128\,\sqrt{cde}c^2d^2e^3}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="
giac")
```

output
```
1/192*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(6*c*d*x + (c^4*d^
5*e^2 + 9*a*c^3*d^3*e^4)/(c^3*d^3*e^3))*x - (5*c^4*d^6*e - 10*a*c^3*d^4*e^
3 - 3*a^2*c^2*d^2*e^5)/(c^3*d^3*e^3))*x + (15*c^4*d^7 - 31*a*c^3*d^5*e^2 +
 9*a^2*c^2*d^3*e^4 - 9*a^3*c*d*e^6)/(c^3*d^3*e^3)) + 1/128*(5*c^4*d^8 - 12
*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 - 3*a^4*e^8)*log(abs(
-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
 a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^2*d^2*e^3)
```

3.448.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \int \frac{x\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{d+e\,x}\,dx$$

input
```
int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),x)
```

output
```
int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x), x)
```

3.449 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx$

3.449.1 Optimal result

Integrand size = 37, antiderivative size = 201

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}+\frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3e}+\frac{(cd^2-ae^2)^3\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{16c^{3/2}d^{3/2}e^{5/2}}$$

output
```
1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e+1/16*(-a*e^2+c*d^2)^3*arctan
h(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/e^(5/2)+1/8*(a/c/d-d/e^2)*(2*c*d*e*x+
a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

3.449.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.90

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(3a^2e^4+2acde^2(4d+7ex)+c^2d^2\right.}{24c^{3/2}d^{3/2}e}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(3*a^2*e^4 + 2*a*c
*d*e^2*(4*d + 7*e*x) + c^2*d^2*(-3*d^2 + 2*d*e*x + 8*e^2*x^2)) + (3*(c*d^2
 - a*e^2)^3*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*
d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(24*c^(3/2)*d^(3/2)*e^(5/2))
```

3.449.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {1131, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{d+ex}dx$$

$$\Big\downarrow \text{ 1131}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\int \sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{2e}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cde}\right)}{2e}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4cde}\right)}{2e}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{3e}-\frac{(cd^2-ae^2)\left(\frac{(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{(cd^2-ae^2)^2\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{3/2}}\right)}{2e}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x),x]
```

output
```
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(3*e) - ((c*d^2 - a*e^2)*(((
c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4
*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c
]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2
)*d^(3/2)*e^(3/2))))/(2*e)
```

3.449.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

3.449.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.14

method	result
default	$\frac{\frac{\left(cde\left(x+\frac{d}{e}\right)^{2}+\left(e^{2}a-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3}+\frac{\left(e^{2}a-c\,d^{2}\right)\left(\frac{\left(2cde\left(x+\frac{d}{e}\right)+e^{2}a-c\,d^{2}\right)\sqrt{cde\left(x+\frac{d}{e}\right)^{2}+\left(e^{2}a-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}{4cde}-\frac{\left(e^{2}a-c\,d^{2}\right)^{2}\ln\left(\frac{\frac{e^{2}a}{2}-\frac{c\,d^{2}}{2}}{\sqrt{}}\right.}{}\right.}{2}}{e}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x,method=_RETURNVERBOS
E)
```

output
```
1/e*(1/3*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(
1/4*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+
d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/e*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e
))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1
/2)))
```

3.449.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.65

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \left[-\frac{3\left(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+\right.}{}\right.$$

$$-\frac{3\left(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6\right)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+ae^2)\sqrt{-cde}}{2\,(c^2d^2e^2x^2+acd^2e^2+(c^2d^3e+acde^3)x)}\right)-2\left(8\,c^3d\right.}{48\,c^2d^2e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="fr
icas")
```

output
```
[-1/96*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d
*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e
) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^3*x^2 - 3*c^3*d^5*e + 8*
a*c^2*d^3*e^3 + 3*a^2*c*d*e^5 + 2*(c^3*d^4*e^2 + 7*a*c^2*d^2*e^4)*x)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3), -1/48*(3*(c^3*d^6 -
 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1/2*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-
c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(8
*c^3*d^3*e^3*x^2 - 3*c^3*d^5*e + 8*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5 + 2*(c^3*
d^4*e^2 + 7*a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))
/(c^2*d^2*e^3)]
```

3.449.6 Sympy [A] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 751, normalized size of antiderivative = 3.74

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{d+ex}\,dx = ae\left(\begin{cases}\left(\frac{x}{2}+\frac{\frac{ae^2}{4}+\frac{cd^2}{4}}{cde}\right)\sqrt{ade+cdex^2+x\,(ae^2+cd^2)}+\left(\frac{ade}{2}-\frac{\left(\frac{ae^2}{4}\right.}{\cdots}\right.\\ \frac{2\left(ade+x\left(ae^2+cd^2\right)\right)^{\frac{3}{2}}}{3(ae^2+cd^2)}\\ x\sqrt{ade}\end{cases}\right.$$

$$+cd\left(\begin{cases}\left(-\frac{a\left(\frac{ae^2}{6}+\frac{cd^2}{6}\right)}{2c}-\frac{(ae^2+cd^2)\left(\frac{ade}{3}-\frac{\left(\frac{ae^2}{6}+\frac{cd^2}{6}\right)\left(\frac{3ae^2}{2}+\frac{3cd^2}{2}\right)}{2cde}\right)}{2cde}\right)\left(\begin{cases}\frac{\log\left(ae^2+cd^2+2cdex+2\sqrt{cde}\sqrt{ade+cdex^2+x(ae^2+cd^2}\right.}{\sqrt{cde}}\\ \frac{\left(x-\frac{-ae^2-cd^2}{2cde}\right)\log\left(x-\frac{-ae^2-cd^2}{2cde}\right)}{\sqrt{cde\left(x-\frac{-ae^2-cd^2}{2cde}\right)^2}}\end{cases}\right.\\ \frac{2\left(-\frac{ade\left(ade+x\left(ae^2+cd^2\right)\right)^{\frac{3}{2}}}{3}+\frac{\left(ade+x\left(ae^2+cd^2\right)\right)^{\frac{5}{2}}}{5}\right)}{(ae^2+cd^2)^2}\\ \frac{x^2\sqrt{ade}}{2}\end{cases}\right.$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

output
```
a*e*Piecewise(((x/2 + (a*e**2/4 + c*d**2/4)/(c*d*e))*sqrt(a*d*e + c*d*e*x*
*2 + x*(a*e**2 + c*d**2)) + (a*d*e/2 - (a*e**2/4 + c*d**2/4)*(a*e**2 + c*d
**2)/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)
*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e - (
a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*lo
g(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*
c*d*e))**2), True)), Ne(c*d*e, 0)), (2*(a*d*e + x*(a*e**2 + c*d**2))**(3/2
)/(3*(a*e**2 + c*d**2)), Ne(a*e**2 + c*d**2, 0)), (x*sqrt(a*d*e), True)) +
 c*d*Piecewise(((-a*(a*e**2/6 + c*d**2/6)/(2*c) - (a*e**2 + c*d**2)*(a*d*e
/3 - (a*e**2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(2*c*d*e))
*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)*sqrt(a*d*e + c
*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e - (a*e**2 + c*d**2
)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*log(x - (-a*e**2
- c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*c*d*e))**2), Tr
ue)) + (x**2/3 + x*(a*e**2/6 + c*d**2/6)/(2*c*d*e) + (a*d*e/3 - (a*e**2/6
+ c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(c*d*e))*sqrt(a*d*e + c*d
*e*x**2 + x*(a*e**2 + c*d**2)), Ne(c*d*e, 0)), (2*(-a*d*e*(a*d*e + x*(a*e*
*2 + c*d**2))**(3/2)/3 + (a*d*e + x*(a*e**2 + c*d**2))**(5/2)/5)/(a*e**2 +
 c*d**2)**2, Ne(a*e**2 + c*d**2, 0)), (x**2*sqrt(a*d*e)/2, True))
```

3.449.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.449.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.14

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \frac{1}{24}\sqrt{cdex^2+cd^2x+ae^2x+ade}\Bigg(2\left(4\,cdx+\frac{c^3d^4e+7\,ac^2d^2e^3}{c^2d^2e^2}\right)x$$
$$-\frac{\left(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6\right)\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right.\right.}{16\sqrt{cde}cde^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="gi
ac")
```

output
```
1/24*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*c*d*x + (c^3*d^4*e
+ 7*a*c^2*d^2*e^3)/(c^2*d^2*e^2))*x - (3*c^3*d^5 - 8*a*c^2*d^3*e^2 - 3*a^2
*c*d*e^4)/(c^2*d^2*e^2)) - 1/16*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e
^4 - a^3*e^6)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt
(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c*d*e^2)
```

3.449.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{d+e\,x}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x), x)
```

3.450 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x(d+ex)}\,dx$

3.450.1 Optimal result

Integrand size = 40, antiderivative size = 251

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x(d+ex)}\,dx=\frac{(cd^2+5ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4e} -\frac{(c^2d^4-6acd^2e^2-3a^2e^4)\,\text{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8\sqrt{c}\sqrt{d}e^{3/2}} - a^{3/2}\sqrt{d}e^{3/2}\text{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)$$

output
```
-1/8*(-3*a^2*e^4-6*a*c*d^2*e^2+c^2*d^4)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2
)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/e^(3/2)
/c^(1/2)/d^(1/2)-a^(3/2)*e^(3/2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(
1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*d^(1/2)+1/4*
(2*c*d*e*x+5*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e
```

3.450.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.92

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x(d+ex)}\,dx=\frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}\sqrt{d+ex}(5ae^2+cd(d+2\right.}{4\sqrt{}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x*(d + e*x)),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x
]*Sqrt[d + e*x]*(5*a*e^2 + c*d*(d + 2*e*x)) + (-(c^2*d^4) + 6*a*c*d^2*e^2
+ 3*a^2*e^4)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c
*d*x])] - 8*a^(3/2)*Sqrt[c]*d*e^3*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/
(Sqrt[d]*Sqrt[a*e + c*d*x])]))/(4*Sqrt[c]*Sqrt[d]*e^(3/2)*Sqrt[(a*e + c*d*
x)*(d + e*x)])
```

3.450.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1215, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x(d+ex)}\,dx$$

$$\Big\downarrow \ 1215$$

$$\int \frac{(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}dx$$

$$\Big\downarrow \ 1231$$

$$\frac{\left(5ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e}-\frac{\int -\frac{cd\left(8a^2de^3-\left(c^2d^4-6ace^2d^2-3a^2e^4\right)x\right)}{2x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4cde}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{8a^2de^3-\left(c^2d^4-6ace^2d^2-3a^2e^4\right)x}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8e}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(5ae^2+cd^2+2cdex\right)}{4e}$$

$\Big\downarrow$ 1269

$$\frac{8a^2de^3\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\left(-3a^2e^4-6acd^2e^2+c^2d^4\right)\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8e}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(5ae^2+cd^2+2cdex\right)}{4e}$$

$\Big\downarrow$ 1092

$$\frac{8a^2de^3\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-2\left(-3a^2e^4-6acd^2e^2+c^2d^4\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ad}}}{8e}$$
$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(5ae^2+cd^2+2cdex\right)}{4e}$$

$\Big\downarrow$ 219

$$\frac{8a^2de^3\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\frac{\left(-3a^2e^4-6acd^2e^2+c^2d^4\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{e}}}{8e}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(5ae^2+cd^2+2cdex\right)}{4e}$$

$\Big\downarrow$ 1154

$$\frac{-16a^2de^3\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+(cd^2+ae^2)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-\frac{\left(-3a^2e^4-6acd^2e^2+c^2d^4\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade-}}\right.}{\sqrt{c}\sqrt{d}\sqrt{e}}}{8e}$$
$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(5ae^2+cd^2+2cdex\right)}{4e}$$

$\Big\downarrow$ 219

$$\frac{-8a^{3/2}\sqrt{d}e^{5/2}\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)-\frac{\left(-3a^2e^4-6acd^2e^2+c^2d^4\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}\right.}{\sqrt{c}\sqrt{d}\sqrt{e}}}{8e}$$
$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(5ae^2+cd^2+2cdex\right)}{4e}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x*(d + e*x)),x]
```

3.450. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x(d+ex)}dx$

output
```
((c*d^2 + 5*a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(4*e) + (-(((c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2
 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])])/(Sqrt[c]*Sqrt[d]*Sqrt[e])) - 8*a^(3/2)*Sqrt[d]*e^(5/2)*ArcTa
nh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*e)
```

3.450.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

3.450. $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x(d+ex)}\,dx$

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.450.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(213) = 426$.

Time = 0.64 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.42

method	result
default	$\frac{\frac{\left(ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}\right)^{\frac{3}{2}}}{3}+\frac{\left(e^{2}a+c\,d^{2}\right)\left(\frac{\left(2cdex+e^{2}a+c\,d^{2}\right)\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}}}{4cde}+\frac{\left(4ac\,d^{2}e^{2}-\left(e^{2}a+c\,d^{2}\right)^{2}\right)\ln\left(\frac{\frac{1}{2}e^{2}a+\frac{1}{2}c\,d^{2}+cde}{\sqrt{cde}}\right.}{8cde\sqrt{cde}}\right.}{2}}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x,method=_RETURNVERB
OSE)
```

output
```
1/d*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2
*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4
*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*
e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*
c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d
*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))-1/d*(1/3*(c*d*e*(x+d/e)^2+(a
*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*c*d*e*(x+d/e)+e^2*a-c
*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2
)^2/c/d/e*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d
/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)))
```

3.450.5 Fricas [A] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 1327, normalized size of antiderivative = 5.29

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x, algorithm="
fricas")
```

output
```
[1/16*(8*sqrt(a*d*e)*a*c*d*e^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e
^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e
 + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - (c
^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^
2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*
x) + 4*(2*c^2*d^2*e^2*x + c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x))/(c*d*e^2), 1/8*(4*sqrt(a*d*e)*a*c*d*e^3*log((8*a^2*d
^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*
d^3*e + a^2*d*e^3)*x)/x^2) + (c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(-c
*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x +
c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e +
a*c*d*e^3)*x)) + 2*(2*c^2*d^2*e^2*x + c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c*d*e^2), 1/16*(16*sqrt(-a*d*e)*a*c*d*e
^3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^
2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a
^2*d*e^3)*x)) - (c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(c*d*e)*log(8*c^
2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^...
```

3.450.6 Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^{3/2}}{x(d + ex)}\, dx = \int \frac{((d + ex)\,(ae + cdx))^{\frac{3}{2}}}{x\,(d + ex)}\, dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x/(e*x+d),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(x*(d + e*x)), x)
```

3.450.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x(d + ex)} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.450.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x(d + ex)} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

3.450.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x(d + ex)} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\ x + a\,d\,e)^{3/2}}{x\ (d + e\,x)} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x*(d + e*x)), x)
```

3.451 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^2(d+ex)}\,dx$

3.451.1 Optimal result

Integrand size = 40, antiderivative size = 240

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^2(d+ex)}\,dx = -\frac{(ae-cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x} + \frac{\sqrt{c}\sqrt{d}\left(cd^2+3ae^2\right)\text{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2\sqrt{e}} - \frac{\sqrt{a}\sqrt{e}\left(3cd^2+ae^2\right)\text{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2\sqrt{d}}$$

output
```
1/2*(3*a*e^2+c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^
(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*c^(1/2)*d^(1/2)/e^(1/2)-1/2
*(a*e^2+3*c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(
1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*a^(1/2)*e^(1/2)/d^(1/2)-(-c*
d*x+a*e)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x
```

3.451.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.89

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^2(d + ex)}\,dx =$$
$$-\frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{d}\sqrt{e}(ae-cdx)\sqrt{ae+cdx}\sqrt{d+ex} - \sqrt{c}d(cd^2+3ae^2)\,x\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right) + \sqrt{c}\right.}{\sqrt{d}\sqrt{e}x\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)),x]
```

output
```
-((Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[d]*Sqrt[e]*(a*e - c*d*x)*Sqrt[a*e
 + c*d*x]*Sqrt[d + e*x] - Sqrt[c]*d*(c*d^2 + 3*a*e^2)*x*ArcTanh[(Sqrt[c]*S
qrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])] + Sqrt[a]*e*(3*c*d^2 +
a*e^2)*x*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x
])]))/(Sqrt[d]*Sqrt[e]*x*Sqrt[(a*e + c*d*x)*(d + e*x)]))
```

3.451.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1215, 1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^2(d+ex)}dx$$
$$\Big\downarrow \text{1215}$$
$$\int \frac{(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x^2}dx$$
$$\Big\downarrow \text{1230}$$
$$-\frac{1}{2}\int -\frac{ae\left(3cd^2+ae^2\right)+cd\left(cd^2+3ae^2\right)x}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx - \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(ae-cdx)}{x}$$
$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{ae\left(3cd^2+ae^2\right)+cd\left(cd^2+3ae^2\right)x}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{(ae-cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}$$

$\downarrow$ 1269

$$\frac{1}{2}\left(cd\left(3ae^2+cd^2\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+ae\left(ae^2+3cd^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx\right)-\frac{(ae-cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}$$

$\downarrow$ 1092

$$\frac{1}{2}\left(ae\left(ae^2+3cd^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+2cd\left(3ae^2+cd^2\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}\right)-\frac{(ae-cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}$$

$\downarrow$ 219

$$\frac{1}{2}\left(ae\left(ae^2+3cd^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{\sqrt{c}\sqrt{d}\left(3ae^2+cd^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{e}}\right)-\frac{(ae-cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}$$

$\downarrow$ 1154

$$\frac{1}{2}\left(\frac{\sqrt{c}\sqrt{d}\left(3ae^2+cd^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{e}}-2ae\left(ae^2+3cd^2\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}\right)-\frac{(ae-cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{\sqrt{c}\sqrt{d}\left(3ae^2+cd^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{e}}-\frac{\sqrt{a}\sqrt{e}\left(ae^2+3cd^2\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{d}}\right)-\frac{(ae-cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)),x]
```

3.451. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^2(d+ex)}\,dx$

output
```
-(((a*e - c*d*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x) + ((Sqrt[
c]*Sqrt[d]*(c*d^2 + 3*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c
]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/Sqrt[e] -
 (Sqrt[a]*Sqrt[e]*(3*c*d^2 + a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/
(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/
Sqrt[d])/2
```

3.451.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

3.451. $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^2(d+ex)}\,dx$

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.451.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1299 vs. $2(202) = 404$.

Time = 0.72 (sec) , antiderivative size = 1300, normalized size of antiderivative = 5.42

method	result	size
default	Expression too large to display	1300

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/d*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+3/2*(a*e^2+c*d^2)/
a/d/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*
(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*
(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*
d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*(
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/
2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c
*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2
)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+4*c/a*(1/8*(2*c*d*e*x+a*e^
2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2
-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2
*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2))/(c*d*e)^(1/2))))-e/d^2*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*
a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
)/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2
+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*...
```

3.451.5 Fricas [A] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 1221, normalized size of antiderivative = 5.09

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^2(d+ex)}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x, algorithm
="fricas")
```

output
```
[1/4*((c*d^2 + 3*a*e^2)*sqrt(c*d/e)*x*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*
a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + (
3*c*d^2 + a*e^2)*sqrt(a*e/d)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e
^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2
*e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2)
+ 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x - a*e))/x, -1/4*(2*
(c*d^2 + 3*a*e^2)*sqrt(-c*d/e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*
c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) - (3*c*d^2 + a*e^2)*sqrt(a*e/d)*x*log(
(8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/
d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*(c*d*x - a*e))/x, 1/4*(2*(3*c*d^2 + a*e^2)*sqrt(-a*e/d)*x*arc
tan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(-a*e/d)/(a*c*d*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)
) + (c*d^2 + 3*a*e^2)*sqrt(c*d/e)*x*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*
c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x - a*e))/x, -1/2*((c*d...
```

3.451.6 Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^2(d + ex)}\,dx = \int \frac{((d + ex)\,(ae + cdx))^{\frac{3}{2}}}{x^2\,(d + ex)}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**2/(e*x+d),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(x**2*(d + e*x)), x)
```

3.451.7 Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^2(d + ex)}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)x^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^2), x
)
```

3.451.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.41

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^2(d + ex)}\,dx = \sqrt{cdex^2 + cd^2x + ae^2x + ade}cd + \frac{(3\,acd^2e + a^2e^3)\arctan\left(-\frac{\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}}{\sqrt{-ade}}\right)}{\sqrt{-ade}} - \frac{(c^2d^3 + 3\,acde^2)\log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)\right|\right)}{2\sqrt{cde}} - \frac{\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)acd^2e + \left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^2e^3 + 2\sqrt{cde}}{ade - \left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x, algorithm
="giac")
```

output
```
sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*c*d + (3*a*c*d^2*e + a^2*e^3)*
arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt
(-a*d*e))/sqrt(-a*d*e) - 1/2*(c^2*d^3 + 3*a*c*d*e^2)*log(abs(-c*d^2 - a*e^
2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*
d*e))))/sqrt(c*d*e) - ((sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))*a*c*d^2*e + (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))*a^2*e^3 + 2*sqrt(c*d*e)*a^2*d*e^2)/(a*d*e - (sqrt(c*d*e)*x - sq
rt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)
```

3.451.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^2(d + ex)}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{x^2\,\left(d + e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)), x)
```

3.452 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^3(d+ex)}\,dx$

3.452.1 Optimal result

Integrand size = 40, antiderivative size = 256

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^3(d+ex)}\,dx = -\frac{(2ade+(5cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4dx^2} + c^{3/2}d^{3/2}\sqrt{e}\,\mathrm{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right) - \frac{(3c^2d^4+6acd^2e^2-a^2e^4)\,\mathrm{arctanh}\left(\frac{}{2\sqrt{a}\sqrt{}}\right.}{8\sqrt{a}d^{3/2}\sqrt{e}}$$

output

```
-1/8*(-a^2*e^4+6*a*c*d^2*e^2+3*c^2*d^4)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)
*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/d^(3/
2)/a^(1/2)/e^(1/2)+c^(3/2)*d^(3/2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(
1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*e^(1/2)-1/4*
(2*a*d*e+(a*e^2+5*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/x^2
```

3.452.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.95

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^3(d+ex)}\,dx =$$
$$-\frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}\sqrt{d+ex}(5cd^2x+ae(2d+ex))-8\sqrt{a}c^{3/2}d^3ex^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d}}{\sqrt{e}\sqrt{ae+}}\right.\right.}{4\sqrt{a}d^{3/2}\sqrt{e}x^2\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)),x]
```

output
```
-1/4*(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e +
c*d*x]*Sqrt[d + e*x]*(5*c*d^2*x + a*e*(2*d + e*x)) - 8*Sqrt[a]*c^(3/2)*d^3
*e*x^2*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])
] + (3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*x^2*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqr
t[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])]))/(Sqrt[a]*d^(3/2)*Sqrt[e]*x^2*Sq
rt[(a*e + c*d*x)*(d + e*x)])
```

3.452.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1215, 1229, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{x^3(d+ex)}\,dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{(ae+cdx)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x^3}dx$$

$$\Big\downarrow \text{1229}$$

$$-\frac{\int -\frac{ae(3c^2d^4+8c^2exd^3+6ace^2d^2-a^2e^4)}{2x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4ade}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(x(ae^2+5cd^2)+2ade)}{4dx^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3c^2d^4+8c^2exd^3+6ace^2d^2-a^2e^4}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8d} - \frac{(x(ae^2+5cd^2)+2ade)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4dx^2}$$

$\big\downarrow$ 1269

$$\frac{(-a^2e^4+6acd^2e^2+3c^2d^4)\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx + 8c^2d^3e\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8d} - \frac{(x(ae^2+5cd^2)+2ade)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4dx^2}$$

$\big\downarrow$ 1092

$$\frac{(-a^2e^4+6acd^2e^2+3c^2d^4)\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx + 16c^2d^3e\int \frac{1}{4cde-\frac{(cd^2+2cexd+ae^2)^2}{cdex^2+(cd^2+ae^2)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ad}}}{8d} \\ \frac{(x(ae^2+5cd^2)+2ade)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4dx^2}$$

$\big\downarrow$ 219

$$\frac{(-a^2e^4+6acd^2e^2+3c^2d^4)\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx + 8c^{3/2}d^{5/2}\sqrt{e}\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8d} - \\ \frac{(x(ae^2+5cd^2)+2ade)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4dx^2}$$

$\big\downarrow$ 1154

$$\frac{8c^{3/2}d^{5/2}\sqrt{e}\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right) - 2(-a^2e^4+6acd^2e^2+3c^2d^4)\int \frac{1}{4ade-\frac{(2ade+(cd^2+ae^2)x)^2}{cdex^2+(cd^2+ae^2)x+ade}}d\frac{\ }{\sqrt{\ }}}{8d} \\ \frac{(x(ae^2+5cd^2)+2ade)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4dx^2}$$

$\big\downarrow$ 219

$$\frac{8c^{3/2}d^{5/2}\sqrt{e}\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right) - \frac{(-a^2e^4+6acd^2e^2+3c^2d^4)\text{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{a}\sqrt{d}\sqrt{e}}}{8d} \\ \frac{(x(ae^2+5cd^2)+2ade)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4dx^2}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)),x]
```

3.452. $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^3(d+ex)}dx$

output
```
-1/4*((2*a*d*e + (5*c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(d*x^2) + (8*c^(3/2)*d^(5/2)*Sqrt[e]*ArcTanh[(c*d^2 + a*e^2 + 2*c
*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])] - ((3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2
+ a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])])/(Sqrt[a]*Sqrt[d]*Sqrt[e]))/(8*d)
```

3.452.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

3.452. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3(d+ex)}\,dx$

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.452.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2437 vs. 2(218) = 436.

Time = 0.77 (sec) , antiderivative size = 2438, normalized size of antiderivative = 9.52

method	result	size
default	Expression too large to display	2438

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/d*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/4*(a*e^2+c*d
^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+3/2*(a*e^2+c
*d^2)/a/d/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)
*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*
x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a
*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^
2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e
)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+4*c/a*(1/8*(2*c*d*e*
x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d
^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*l
n((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2))/(c*d*e)^(1/2)))+3/2*c/a*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((
1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/
2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(...
```

3.452.5 Fricas [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 1375, normalized size of antiderivative = 5.37

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x, algorithm
="fricas")
```

output
```
[1/16*(8*sqrt(c*d*e)*a*c*d^3*e*x^2*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c
*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - (3*c^2*d
^4 + 6*a*c*d^2*e^2 - a^2*e^4)*sqrt(a*d*e)*x^2*log((8*a^2*d^2*e^2 + (c^2*d^
4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^
3)*x)/x^2) - 4*(2*a^2*d^2*e^2 + (5*a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^2), -1/16*(16*sqrt(-c*d*e)*a*c*
d^3*e*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*
x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*
e + a*c*d*e^3)*x)) + (3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*sqrt(a*d*e)*x^2
*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d
*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(2*a^2*d^2*e^2 + (5*a*c*d^3*e
+ a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^2)
, 1/8*(4*sqrt(c*d*e)*a*c*d^3*e*x^2*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c
*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + (3*c^2*d
^4 + 6*a*c*d^2*e^2 - a^2*e^4)*sqrt(-a*d*e)*x^2*arctan(1/2*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/...
```

3.452.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3(d+ex)}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**3/(e*x+d),x)
```

output
```
Timed out
```

3.452.7 Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^3(d + ex)} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)x^3} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^3), x
)
```

3.452.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(218) = 436$.

Time = 0.44 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.45

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^3(d + ex)} dx =$$
$$-\frac{c^2d^2e\log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)\right|\right)}{\sqrt{cde}}$$
$$+\frac{(3c^2d^4 + 6acd^2e^2 - a^2e^4)\arctan\left(-\frac{\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}}{\sqrt{-ade}}\right)}{4\sqrt{-ade}d}$$
$$-\frac{3\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)ac^2d^5e - 2\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^2cd^3e^3 -}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x, algorithm
="giac")
```

output
```
-c^2*d^2*e*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/sqrt(c*d*e) + 1/4*(3*c^2*d^4 + 6*a
*c*d^2*e^2 - a^2*e^4)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*d) - 1/4*(3*(sqrt(c*d*e)*x -
 sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a*c^2*d^5*e - 2*(sqrt(c*d*e)
*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^2*c*d^3*e^3 - (sqrt(c*
d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^3*d*e^5 + 8*sqrt(c
*d*e)*a^2*c*d^4*e^2 - 5*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*
x + a*d*e))^3*c^2*d^4 - 10*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e
^2*x + a*d*e))^3*a*c*d^2*e^2 - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
 a*e^2*x + a*d*e))^3*a^2*e^4 - 16*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*
x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a*c*d^3*e - 8*sqrt(c*d*e)*(sqrt(c*d*e)
*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^2*d*e^3)/((a*d*e - (
sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)^2*d)
```

3.452.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^3(d+ex)}\,dx=\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{x^3\,\left(d+e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)), x)
```

3.453 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)}\,dx$

3.453.1 Optimal result

Integrand size = 40, antiderivative size = 211

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)}\,dx = -\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8x^2} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3dx^3} + \frac{\left(cd^2-ae^2\right)^3\operatorname{arctanh}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{16a^{3/2}d^{5/2}e^{3/2}}$$

output
```
-1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^3+1/16*(-a*e^2+c*d^2)^3*a
rctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/d^(5/2)/e^(3/2)-1/8*(c/a/e-e/d^2)*(2*a
*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2
```

3.453.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.95

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^4(d + ex)} dx = \frac{(-cd^2 + ae^2)^3 \sqrt{(ae + cdx)(d + ex)} \left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(3c^2d^4x^2+2acd^2ex(7d+4ex)}{(cd^2-ae^2)^3x} \right.}{24a^{3/2}d^{5/2}e^{3/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)),x]
```

output
```
((-(c*d^2) + a*e^2)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt
[e]*(3*c^2*d^4*x^2 + 2*a*c*d^2*e*x*(7*d + 4*e*x) + a^2*e^2*(8*d^2 + 2*d*e*
x - 3*e^2*x^2)))/((c*d^2 - a*e^2)^3*x^3) - (3*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqr
t[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]
)))/(24*a^(3/2)*d^(5/2)*e^(3/2))
```

3.453.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1215, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{3/2}}{x^4(d + ex)} dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{(ae + cdx)\sqrt{x\left(ae^2 + cd^2\right) + ade + cdex^2}}{x^4} dx$$

$$\Big\downarrow \text{1228}$$

$$\frac{\left(cd^2 - ae^2\right) \int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{x^3} dx}{2d} - \frac{\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{3/2}}{3dx^3}$$

$$\Big\downarrow \text{1152}$$

$$\frac{\left(cd^2-ae^2\right)\left(-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4adex^2}\right)}{2d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3dx^3}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\left(cd^2-ae^2\right)\left(\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4adex^2}\right)}{2d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3dx^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(cd^2-ae^2\right)\left(\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{3/2}d^{3/2}e^{3/2}}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4adex^2}\right)}{2d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3dx^3}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)),x]
```

output
```
-1/3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d*x^3) + ((c*d^2 - a*e
^2)*(-1/4*((2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(a*d*e*x^2) + ((c*d^2 - a*e^2)^2*ArcTanh[(2*a*d*e + (c*d^2 + a
*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])])/(8*a^(3/2)*d^(3/2)*e^(3/2))))/(2*d)
```

3.453. $\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)}\,dx$

3.453.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

3.453. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)}\,dx$

3.453.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4329 vs. 2(185) = 370.

Time = 1.01 (sec) , antiderivative size = 4330, normalized size of antiderivative = 20.52

method	result	size
default	Expression too large to display	4330

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/d*(-1/3/a/d/e/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)-1/6*(a*e^2+c*d
^2)/a/d/e*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/4*(a*e
^2+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+3/2*(a
*e^2+c*d^2)/a/d/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+
c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+
c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1
/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((
1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*
(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+4*c/a*(1/8*(2*
c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4
*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c
/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))))+3/2*c/a*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/
e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)...
```

3.453.5 Fricas [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.64

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^4(d+ex)}\,dx = \left[-\frac{3\,(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)\sqrt{ade}x^3\log\left(\frac{8\,a^2d^2e^2+(c^2d\cdots}{\cdots}\right)\cdots}{\cdots}\right.$$

$$-\frac{3\,(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)\sqrt{-ade}x^3\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,ade+(cd^2+ae^2)x)\sqrt{-ade}}{2\,(acd^2e^2x^2+a^2d^2e^2+(acd^3e+a^2de^3)x)}\right)+2\,(8\cdots}{48\,a^2d^3e\cdots}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/96*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(a*d
*e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(8*a^3*d^3*e^3 + (3*a*c
^2*d^5*e + 8*a^2*c*d^3*e^3 - 3*a^3*d*e^5)*x^2 + 2*(7*a^2*c*d^4*e^2 + a^3*d
^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^3),
 -1/48*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-a*
d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e +
 (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3
*e + a^2*d*e^3)*x)) + 2*(8*a^3*d^3*e^3 + (3*a*c^2*d^5*e + 8*a^2*c*d^3*e^3
- 3*a^3*d*e^5)*x^2 + 2*(7*a^2*c*d^4*e^2 + a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^3)]
```

3.453.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^4(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**4/(e*x+d),x)
```

output
```
Timed out
```

3.453.7 Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^4(d + ex)} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)x^4} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^4), x
)
```

3.453.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1005 vs. 2(185) = 370.

Time = 0.34 (sec) , antiderivative size = 1005, normalized size of antiderivative = 4.76

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^4(d + ex)} dx =$$
$$-\frac{(c^3d^6 - 3ac^2d^4e^2 + 3a^2cd^2e^4 - a^3e^6)\arctan\left(-\frac{\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}}{\sqrt{-ade}}\right)}{8\sqrt{-ade}ad^2e}$$
$$+\frac{3\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^2c^3d^8e^2 - 9\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^3c^2d^6e^4}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x, algorithm
="giac")
```

output
```
-1/8*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*arctan(-(sqrt
(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sq
rt(-a*d*e)*a*d^2*e) + 1/24*(3*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
a*e^2*x + a*d*e))*a^2*c^3*d^8*e^2 - 9*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*
d^2*x + a*e^2*x + a*d*e))*a^3*c^2*d^6*e^4 - 39*(sqrt(c*d*e)*x - sqrt(c*d*e
*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*c*d^4*e^6 - 3*(sqrt(c*d*e)*x - sqrt
(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^5*d^2*e^8 - 16*sqrt(c*d*e)*a^4*
c*d^5*e^5 - 8*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)
)^3*a*c^3*d^7*e - 72*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
 a*d*e))^3*a^2*c^2*d^5*e^3 - 72*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x
+ a*e^2*x + a*d*e))^3*a^3*c*d^3*e^5 - 8*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
c*d^2*x + a*e^2*x + a*d*e))^3*a^4*d*e^7 - 48*sqrt(c*d*e)*(sqrt(c*d*e)*x -
sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^3*c*d^4*e^4 - 48*sqrt(c*d
*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^4*d^
2*e^6 - 3*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*
c^3*d^6 - 39*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))
^5*a*c^2*d^4*e^2 - 9*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
 a*d*e))^5*a^2*c*d^2*e^4 + 3*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a
*e^2*x + a*d*e))^5*a^3*e^6 - 48*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^
2 + c*d^2*x + a*e^2*x + a*d*e))^4*a*c^2*d^5*e - 96*sqrt(c*d*e)*(sqrt(c*...
```

3.453.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^4(d + ex)}\,dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{3/2}}{x^4\,(d + e\,x)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)), x)
```

3.454 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^5(d+ex)}\,dx$

3.454.1 Optimal result

Integrand size = 40, antiderivative size = 295

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^5(d+ex)}\,dx = \frac{(cd^2-ae^2)(3cd^2+5ae^2)(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+a}}{64a^2d^3e^2x^2}$$

$$-\frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{4dx^4}-\frac{\left(\frac{3c}{ae}-\frac{5e}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{24x^3}$$

$$-\frac{(cd^2-ae^2)^3(3cd^2+5ae^2)\operatorname{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128a^{5/2}d^{7/2}e^{5/2}}$$

output
```
-1/4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^4-1/24*(3*c/a/e-5*e/d^2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3-1/128*(-a*e^2+c*d^2)^3*(5*a*e^
2+3*c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/d^(7/2)/e^(5/2)+1/64*(-a*e
^2+c*d^2)*(5*a*e^2+3*c*d^2)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)/a^2/d^3/e^2/x^2
```

3.454.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.84

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5(d+ex)}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(-9c^3d^6x^3+3ac^2d^4ex^2(2d+3ex)+a^2cd^2e^2x(7\right.}{}\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-9*c^3*d^6*x^3
 + 3*a*c^2*d^4*e*x^2*(2*d + 3*e*x) + a^2*c*d^2*e^2*x*(72*d^2 + 20*d*e*x -
31*e^2*x^2) + a^3*e^3*(48*d^3 + 8*d^2*e*x - 10*d*e^2*x^2 + 15*e^3*x^3)))/x
^4) - (3*(c*d^2 - a*e^2)^3*(3*c*d^2 + 5*a*e^2)*ArcTanh[(Sqrt[a]*Sqrt[e]*Sq
rt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x
])))/(192*a^(5/2)*d^(7/2)*e^(5/2))
```

3.454.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1215, 1237, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^5(d+ex)}dx$$

$$\Big\downarrow \; 1215$$

$$\int \frac{(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x^5}dx$$

$$\Big\downarrow \; 1237$$

$$-\frac{\int -\frac{ae\left(3cd^2-2cexd-5ae^2\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{2x^4}dx}{4ade}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4dx^4}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(3cd^2-2cexd-5ae^2\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^4}dx}{8d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4dx^4}$$

$$\Big\downarrow 1228$$

$$-\frac{-\frac{\left(\frac{3c^2d^4}{a}-5ae^4+2cd^2e^2\right)\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^3}dx}{2de}-\frac{\left(\frac{3cd}{ae}-\frac{5e}{d}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x^3}}{8d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4dx^4}$$

$$\Big\downarrow 1152$$

$$-\frac{-\frac{\left(\frac{3c^2d^4}{a}-5ae^4+2cd^2e^2\right)\left(-\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4adex^2}\right)}{2de}-\frac{\left(\frac{3cd}{ae}-\frac{5e}{d}\right)\left(x\left(ae^2+\right.\right.}{3}}{8d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4dx^4}$$

$$\Big\downarrow 1154$$

$$-\frac{-\frac{\left(\frac{3c^2d^4}{a}-5ae^4+2cd^2e^2\right)\left(\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4adex^2}\right.}{2de}}{8d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4dx^4}$$

$$\Big\downarrow 219$$

$$-\frac{-\frac{\left(\frac{3c^2d^4}{a}-5ae^4+2cd^2e^2\right)\left(\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{3/2}d^{3/2}e^{3/2}}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4adex^2}\right)}{2de}-\frac{\left(\frac{3c}{ae}\right.}{}}{8d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4dx^4}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)),x]
```

3.454. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5(d+ex)}\,dx$

output
```
-1/4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d*x^4) + (-1/3*(((3*c*
d)/(a*e) - (5*e)/d)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/x^3 - (
((3*c^2*d^4)/a + 2*c*d^2*e^2 - 5*a*e^4)*(-1/4*((2*a*d*e + (c*d^2 + a*e^2)*
x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x^2) + ((c*d^2 - a*
e^2)^2*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*a^(3/2)*d^(3/2)*e^(3/2))))
/(2*d*e))/(8*d)
```

3.454.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

3.454. $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^5(d+ex)}\,dx$

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.454.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7420 vs. 2(265) = 530.

Time = 1.06 (sec) , antiderivative size = 7421, normalized size of antiderivative = 25.16

method	result	size
default	Expression too large to display	7421

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

3.454.5 Fricas [A] (verification not implemented)

Time = 4.94 (sec) , antiderivative size = 704, normalized size of antiderivative = 2.39

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^5(d+ex)}\,dx = \left[-\frac{3\left(3\,c^4d^8-4\,ac^3d^6e^2-6\,a^2c^2d^4e^4+12\,a^3cd^2e^6-5\,a^4e^8\right)\sqrt{adex}}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/768*(3*(3*c^4*d^8 - 4*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2
*e^6 - 5*a^4*e^8)*sqrt(a*d*e)*x^4*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^
2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) +
 4*(48*a^4*d^4*e^4 - (9*a*c^3*d^7*e - 9*a^2*c^2*d^5*e^3 + 31*a^3*c*d^3*e^5
 - 15*a^4*d*e^7)*x^3 + 2*(3*a^2*c^2*d^6*e^2 + 10*a^3*c*d^4*e^4 - 5*a^4*d^2
*e^6)*x^2 + 8*(9*a^3*c*d^5*e^3 + a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x))/(a^3*d^4*e^3*x^4), 1/384*(3*(3*c^4*d^8 - 4*a*c^3*d^6*e
^2 - 6*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(-a*d*e)*x^4*ar
ctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*
e^3)*x)) - 2*(48*a^4*d^4*e^4 - (9*a*c^3*d^7*e - 9*a^2*c^2*d^5*e^3 + 31*a^3
*c*d^3*e^5 - 15*a^4*d*e^7)*x^3 + 2*(3*a^2*c^2*d^6*e^2 + 10*a^3*c*d^4*e^4 -
 5*a^4*d^2*e^6)*x^2 + 8*(9*a^3*c*d^5*e^3 + a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x))/(a^3*d^4*e^3*x^4)]
```

3.454.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^5(d + ex)}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**5/(e*x+d),x)
```

output
```
Timed out
```

3.454.7 Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^5(d + ex)}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)x^5}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^5), x
)
```

3.454.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1618 vs. 2(265) = 530.

Time = 0.37 (sec) , antiderivative size = 1618, normalized size of antiderivative = 5.48

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^5(d + ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x, algorithm
="giac")
```

output
```
1/64*(3*c^4*d^8 - 4*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 -
 5*a^4*e^8)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a^2*d^3*e^2) - 1/192*(9*(sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^3*c^4*d^11*e^3 - 12*(sqr
t(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*c^3*d^9*e^5
- 402*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^5*c^
2*d^7*e^7 - 348*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*
e))*a^6*c*d^5*e^9 - 15*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))*a^7*d^3*e^11 - 128*sqrt(c*d*e)*a^6*c*d^6*e^8 - 33*(sqrt(c*d*e)*
x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^2*c^4*d^10*e^2 - 724*
(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*c^3*d^
8*e^4 - 1854*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))
^3*a^4*c^2*d^6*e^6 - 900*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2
*x + a*d*e))^3*a^5*c*d^4*e^8 - 73*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*
x + a*e^2*x + a*d*e))^3*a^6*d^2*e^10 - 768*sqrt(c*d*e)*(sqrt(c*d*e)*x - sq
rt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^4*c^2*d^7*e^5 - 1024*sqrt(c
*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^5*
c*d^5*e^7 - 384*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*
e^2*x + a*d*e))^2*a^6*d^3*e^9 - 33*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e))^5*a*c^4*d^9*e - 596*(sqrt(c*d*e)*x - sqrt(c*d*e*...
```

3.454.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5(d+ex)}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{x^5\,\left(d+e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)), x)
```

3.455 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^6(d+ex)}\,dx$

3.455.1 Optimal result

Integrand size = 40, antiderivative size = 395

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^6(d+ex)}\,dx = -\frac{(cd^2-ae^2)(3c^2d^4+6acd^2e^2+7a^2e^4)(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128a^3d^4e^3x^2} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5dx^5} - \frac{\left(\frac{3c}{ae}-\frac{7e}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{40x^4} + \frac{(15c^2d^4+12acd^2e^2-35a^2e^4)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{240a^2d^3e^2x^3} + \frac{(cd^2-ae^2)^3(3c^2d^4+6acd^2e^2+7a^2e^4)\operatorname{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{256a^{7/2}d^{9/2}e^{7/2}}$$

output
```
-1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^5-1/40*(3*c/a/e-7*e/d^2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4+1/240*(-35*a^2*e^4+12*a*c*d^2*
e^2+15*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^2/d^3/e^2/x^3+1/
256*(-a*e^2+c*d^2)^3*(7*a^2*e^4+6*a*c*d^2*e^2+3*c^2*d^4)*arctanh(1/2*(2*a*
d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2))/a^(7/2)/d^(9/2)/e^(7/2)-1/128*(-a*e^2+c*d^2)*(7*a^2*e^4+6*a*c*
d^2*e^2+3*c^2*d^4)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/a^3/d^4/e^3/x^2
```

3.455.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.80

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^6(d+ex)}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}(45c^4d^8x^4-30ac^3d^6ex^3(d+ex)+6a^2c^2d^4e^2x^2}{}\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(45*c^4*d^8*x^4
 - 30*a*c^3*d^6*e*x^3*(d + e*x) + 6*a^2*c^2*d^4*e^2*x^2*(4*d^2 + 3*d*e*x -
 6*e^2*x^2) + 2*a^3*c*d^2*e^3*x*(264*d^3 + 48*d^2*e*x - 61*d*e^2*x^2 + 95*
e^3*x^3) + a^4*e^4*(384*d^4 + 48*d^3*e*x - 56*d^2*e^2*x^2 + 70*d*e^3*x^3 -
 105*e^4*x^4)))/x^5) + (15*(c*d^2 - a*e^2)^3*(3*c^2*d^4 + 6*a*c*d^2*e^2 +
7*a^2*e^4)*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d
*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(1920*a^(7/2)*d^(9/2)*e^(7/2))
```

3.455.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {1215, 1237, 27, 1237, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{x^6(d+ex)}\,dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{(ae+cdx)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x^6}dx$$

$$\Big\downarrow \text{1237}$$

$$-\frac{\int -\frac{ae(3cd^2-4cexd-7ae^2)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{2x^5}dx}{5ade}-\frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{5dx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(3cd^2-4cexd-7ae^2)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{x^5}dx}{10d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5dx^5}$$

$\downarrow$ 1237

$$\frac{-\frac{\int \frac{\left(15c^2d^4+12ace^2d^2+2ce\left(3cd^2-7ae^2\right)xd-35a^2e^4\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{2x^4}dx}{4ade}-\frac{\left(\frac{3cd}{ae}-\frac{7e}{d}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4x^4}}{10d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5dx^5}$$

$\downarrow$ 27

$$\frac{-\frac{\int \frac{\left(15c^2d^4+12ace^2d^2+2ce\left(3cd^2-7ae^2\right)xd-35a^2e^4\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^4}dx}{8ade}-\frac{\left(\frac{3cd}{ae}-\frac{7e}{d}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4x^4}}{10d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5dx^5}$$

$\downarrow$ 1228

$$\frac{-\frac{-\frac{5\left(cd^2-ae^2\right)\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^3}dx}{2ade}-\frac{\left(\frac{15c^2d^4}{a}-35ae^4+12cd^2e^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3dex^3}}{8ade}-\frac{\left(\frac{3cd}{ae}-\frac{7e}{d}\right)\left(x\left(a\right.\right.}{}}{10d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5dx^5}$$

$\downarrow$ 1152

$$\frac{-\frac{-\frac{5\left(cd^2-ae^2\right)\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(-\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4adex^2}\right)}{2ade}-\frac{\left(\frac{15c^2d^4}{a}-\right.}{}}{8ade}}{10d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5dx^5}$$

$\downarrow$ 1154

3.455. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^6(d+ex)}\,dx$

$$-\frac{-\frac{-\frac{5\left(cd^2-ae^2\right)\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right.}}{4adex^2}\right.}{2ade}}{8ade}}{10d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5dx^5}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-\frac{-\frac{5\left(cd^2-ae^2\right)\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{3/2}d^{3/2}e^{3/2}}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{4adex^2}\right.}{2ade}}{8ade}}{10d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5dx^5}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)),x]
```

output
```
-1/5*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d*x^5) + (-1/4*(((3*c*
d)/(a*e) - (7*e)/d)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/x^4 - (
-1/3*(((15*c^2*d^4)/a + 12*c*d^2*e^2 - 35*a*e^4)*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(3/2))/(d*e*x^3) - (5*(c*d^2 - a*e^2)*(3*c^2*d^4 + 6*a*c*d^
2*e^2 + 7*a^2*e^4)*(-1/4*((2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x^2) + ((c*d^2 - a*e^2)^2*ArcTanh[(2*a*d
*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])])/(8*a^(3/2)*d^(3/2)*e^(3/2))))/(2*a*d*e))/(8*a*d*e)
)/(10*d)
```

3.455. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^6(d+ex)}\,dx$

3.455.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

3.455. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^6(d+ex)}\,dx$

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.455.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 10574 vs. $2(361) = 722$.

Time = 1.65 (sec) , antiderivative size = 10575, normalized size of antiderivative = 26.77

method	result	size
default	Expression too large to display	10575

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

3.455.5 Fricas [A] (verification not implemented)

Time = 14.93 (sec) , antiderivative size = 872, normalized size of antiderivative = 2.21

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^6(d+ex)}\,dx = \left[-\frac{15\,(3\,c^5d^{10}-3\,ac^4d^8e^2-2\,a^2c^3d^6e^4-6\,a^3c^2d^4e^6+15\,a^4cd^2e^8 -\cdots}{\cdots}\right.$$

$$-\frac{15\,(3\,c^5d^{10}-3\,ac^4d^8e^2-2\,a^2c^3d^6e^4-6\,a^3c^2d^4e^6+15\,a^4cd^2e^8-7\,a^5e^{10})\sqrt{-ade}\,x^5\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2\cdots}}{2\,(acd^2e^2x^2\cdots}\right.}{\cdots}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/7680*(15*(3*c^5*d^10 - 3*a*c^4*d^8*e^2 - 2*a^2*c^3*d^6*e^4 - 6*a^3*c^2
*d^4*e^6 + 15*a^4*c*d^2*e^8 - 7*a^5*e^10)*sqrt(a*d*e)*x^5*log((8*a^2*d^2*e
^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*
e + a^2*d*e^3)*x)/x^2) + 4*(384*a^5*d^5*e^5 + (45*a*c^4*d^9*e - 30*a^2*c^3
*d^7*e^3 - 36*a^3*c^2*d^5*e^5 + 190*a^4*c*d^3*e^7 - 105*a^5*d*e^9)*x^4 - 2
*(15*a^2*c^3*d^8*e^2 - 9*a^3*c^2*d^6*e^4 + 61*a^4*c*d^4*e^6 - 35*a^5*d^2*e
^8)*x^3 + 8*(3*a^3*c^2*d^7*e^3 + 12*a^4*c*d^5*e^5 - 7*a^5*d^3*e^7)*x^2 + 4
8*(11*a^4*c*d^6*e^4 + a^5*d^4*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x))/(a^4*d^5*e^4*x^5), -1/3840*(15*(3*c^5*d^10 - 3*a*c^4*d^8*e^2 - 2*
a^2*c^3*d^6*e^4 - 6*a^3*c^2*d^4*e^6 + 15*a^4*c*d^2*e^8 - 7*a^5*e^10)*sqrt(
-a*d*e)*x^5*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*
e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*
d^3*e + a^2*d*e^3)*x)) + 2*(384*a^5*d^5*e^5 + (45*a*c^4*d^9*e - 30*a^2*c^3
*d^7*e^3 - 36*a^3*c^2*d^5*e^5 + 190*a^4*c*d^3*e^7 - 105*a^5*d*e^9)*x^4 - 2
*(15*a^2*c^3*d^8*e^2 - 9*a^3*c^2*d^6*e^4 + 61*a^4*c*d^4*e^6 - 35*a^5*d^2*e
^8)*x^3 + 8*(3*a^3*c^2*d^7*e^3 + 12*a^4*c*d^5*e^5 - 7*a^5*d^3*e^7)*x^2 + 4
8*(11*a^4*c*d^6*e^4 + a^5*d^4*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x))/(a^4*d^5*e^4*x^5)]
```

3.455.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^6(d+ex)}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**6/(e*x+d),x)
```

output
```
Timed out
```

3.455.7 Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^6(d+ex)}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)x^6}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^6), x
)
```

3.455.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2352 vs. 2(361) = 722.

Time = 0.40 (sec) , antiderivative size = 2352, normalized size of antiderivative = 5.95

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^{3/2}}{x^6(d + ex)}\, dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x, algorithm
="giac")
```

output
```
-1/128*(3*c^5*d^10 - 3*a*c^4*d^8*e^2 - 2*a^2*c^3*d^6*e^4 - 6*a^3*c^2*d^4*e
^6 + 15*a^4*c*d^2*e^8 - 7*a^5*e^10)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^
2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a^3*d^4*e^3) +
 1/1920*(45*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*
a^4*c^5*d^14*e^4 - 45*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
+ a*d*e))*a^5*c^4*d^12*e^6 - 3870*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*
x + a*e^2*x + a*d*e))*a^6*c^3*d^10*e^8 - 7770*(sqrt(c*d*e)*x - sqrt(c*d*e*
x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^7*c^2*d^8*e^10 - 3615*(sqrt(c*d*e)*x -
 sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^8*c*d^6*e^12 - 105*(sqrt(c
*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^9*d^4*e^14 - 768*
sqrt(c*d*e)*a^7*c^2*d^9*e^9 - 1280*sqrt(c*d*e)*a^8*c*d^7*e^11 - 210*(sqrt(
c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*c^5*d^13*e^3
 - 7470*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^
4*c^4*d^11*e^5 - 34420*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))^3*a^5*c^3*d^9*e^7 - 41820*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d
^2*x + a*e^2*x + a*d*e))^3*a^6*c^2*d^7*e^9 - 12570*(sqrt(c*d*e)*x - sqrt(c
*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^7*c*d^5*e^11 - 790*(sqrt(c*d*e)
*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^8*d^3*e^13 - 7680*sq
rt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*
a^5*c^3*d^10*e^6 - 23040*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + ...
```

3.455.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{x^6(d+ex)}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{x^6\,\left(d+e\,x\right)} \,d x$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)), x)
```

3.456 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^7(d+ex)}\,dx$

3.456.1 Optimal result

Integrand size = 40, antiderivative size = 498

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^7(d+ex)}\,dx = \frac{\left(7c^4d^8+8ac^3d^6e^2+6a^2c^2d^4e^4-21a^4e^8\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{}}{512a^4d^5e^4x^2}$$
$$-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6dx^6}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{20x^5}$$
$$+\frac{\left(7c^2d^4+6acd^2e^2-21a^2e^4\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{160a^2d^3e^2x^4}$$
$$-\frac{\left(35c^3d^6+33ac^2d^4e^2+21a^2cd^2e^4-105a^3e^6\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{960a^3d^4e^3x^3}$$
$$-\frac{\left(cd^2-ae^2\right)^3\left(7c^3d^6+15ac^2d^4e^2+21a^2cd^2e^4+21a^3e^6\right)\operatorname{arctanh}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{1024a^{9/2}d^{11/2}e^{9/2}}$$

output
```
-1/6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^6-1/20*(c/a/e-3*e/d^2)*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5+1/160*(-21*a^2*e^4+6*a*c*d^2*e^2
+7*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^2/d^3/e^2/x^4-1/960*
(-105*a^3*e^6+21*a^2*c*d^2*e^4+33*a*c^2*d^4*e^2+35*c^3*d^6)*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(3/2)/a^3/d^4/e^3/x^3-1/1024*(-a*e^2+c*d^2)^3*(21*a^3*
e^6+21*a^2*c*d^2*e^4+15*a*c^2*d^4*e^2+7*c^3*d^6)*arctanh(1/2*(2*a*d*e+(a*e
^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2))/a^(9/2)/d^(11/2)/e^(9/2)+1/512*(-21*a^4*e^8+6*a^2*c^2*d^4*e^4+8*a*c^3*
d^6*e^2+7*c^4*d^8)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/a^4/d^5/e^4/x^2
```

3.456.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.81

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^7(d+ex)}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}(-105c^5d^{10}x^5+5ac^4d^8ex^4(14d+11ex)-2a^2c^3}{}\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-105*c^5*d^10*
x^5 + 5*a*c^4*d^8*e*x^4*(14*d + 11*e*x) - 2*a^2*c^3*d^6*e^2*x^3*(28*d^2 +
16*d*e*x - 27*e^2*x^2) + 6*a^3*c^2*d^4*e^3*x^2*(8*d^3 + 4*d^2*e*x - 6*d*e^
2*x^2 + 13*e^3*x^3) + a^4*c*d^2*e^4*x*(1664*d^4 + 224*d^3*e*x - 264*d^2*e^
2*x^2 + 336*d*e^3*x^3 - 525*e^4*x^4) + a^5*e^5*(1280*d^5 + 128*d^4*e*x - 1
44*d^3*e^2*x^2 + 168*d^2*e^3*x^3 - 210*d*e^4*x^4 + 315*e^5*x^5)))/x^6) - (
15*(c*d^2 - a*e^2)^3*(7*c^3*d^6 + 15*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4 + 21
*a^3*e^6)*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*
x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(7680*a^(9/2)*d^(11/2)*e^(9/2))
```

3.456.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.275, Rules used = {1215, 1237, 27, 1237, 27, 1237, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^7(d+ex)}\,dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x^7}dx$$

$$\Big\downarrow \text{1237}$$

$$-\frac{\int -\frac{3ae\left(cd^2-2cexd-3ae^2\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{2x^6}dx}{6ade}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6dx^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(cd^2-2cexd-3ae^2)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{x^6}dx}{4d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6dx^6}$$

$\big\downarrow$ 1237

$$\frac{-\frac{\int \frac{\left(7c^2d^4+6ace^2d^2+4ce\left(cd^2-3ae^2\right)xd-21a^2e^4\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{2x^5}dx}{5ade}-\frac{\left(\frac{cd}{ae}-\frac{3e}{d}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5x^5}}{4d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6dx^6}$$

$\big\downarrow$ 27

$$\frac{-\frac{\int \frac{\left(7c^2d^4+6ace^2d^2+4ce\left(cd^2-3ae^2\right)xd-21a^2e^4\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^5}dx}{10ade}-\frac{\left(\frac{cd}{ae}-\frac{3e}{d}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5x^5}}{4d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6dx^6}$$

$\big\downarrow$ 1237

$$-\frac{-\frac{\frac{\int \frac{\left(35c^3d^6+33ac^2e^2d^4+21a^2ce^4d^2+2ce\left(7c^2d^4+6ace^2d^2-21a^2e^4\right)xd-105a^3e^6\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{2x^4}dx}{4ade}-\frac{\left(\frac{7c^2d^4}{a}-21ae^4+6cd^2e^2\right)\left(x\left(ae^2+cd^2\right)+c\cdots\right.}{4dex^4}}{10ade}\cdots}{4d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6dx^6}$$

$\big\downarrow$ 27

$$-\frac{-\frac{\frac{\int \frac{\left(35c^3d^6+33ac^2e^2d^4+21a^2ce^4d^2+2ce\left(7c^2d^4+6ace^2d^2-21a^2e^4\right)xd-105a^3e^6\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^4}dx}{8ade}-\frac{\left(\frac{7c^2d^4}{a}-21ae^4+6cd^2e^2\right)\left(x\left(ae^2+cd^2\right)+c\cdots\right.}{4dex^4}}{10ade}\cdots}{4d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6dx^6}$$

$\big\downarrow$ 1228

$$-\frac{-\frac{-\frac{\frac{5\left(-21a^4e^8+6a^2c^2d^4e^4+8ac^3d^6e^2+7c^4d^8\right)\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^3}dx}{2ade}-\frac{\left(-105a^3e^6+21a^2cd^2e^4+33ac^2d^4e^2+35c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3\cdots}}{3adex^3}}{8ade}\cdots}{10ade}\cdots}{4d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6dx^6}$$

$\big\downarrow$ 1152

3.456. $\displaystyle\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^7(d+ex)}\,dx$

$$-\frac{-\frac{-\frac{5\left(-21a^4e^8+6a^2c^2d^4e^4+8ac^3d^6e^2+7c^4d^8\right)\left(-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4adex^2}\right)}{2ade}-\frac{\left(-\cdots\right.}{\cdots}}{8ade}}{10ade}\cdots$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6dx^6}$$

$\Big\downarrow$ 1154

$$-\frac{-\frac{-\frac{5\left(-21a^4e^8+6a^2c^2d^4e^4+8ac^3d^6e^2+7c^4d^8\right)\left(\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2\cdots\right.}}{4adex^2}\cdots\right)}{2ade}\cdots}{8ade}\cdots}{\cdots}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6dx^6}$$

$\Big\downarrow$ 219

$$-\frac{-\frac{-\frac{\left(-105a^3e^6+21a^2cd^2e^4+33ac^2d^4e^2+35c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3adex^3}-\frac{5\left(-21a^4e^8+6a^2c^2d^4e^4+8ac^3d^6e^2+7c^4d^8\right)\left(\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{\cdots}{2\sqrt{\cdots}}\right)}{8a\cdots}\cdots\right)}{\cdots}}{8ade}\cdots}{10ade\cdots}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6dx^6}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)),x]
```

3.456. $\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^7(d+ex)}\,dx$

output
```
-1/6*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d*x^6) + (-1/5*(((c*d)
/(a*e) - (3*e)/d)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/x^5 - (-1
/4*(((7*c^2*d^4)/a + 6*c*d^2*e^2 - 21*a*e^4)*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2))/(d*e*x^4) - (-1/3*((35*c^3*d^6 + 33*a*c^2*d^4*e^2 + 21*a
^2*c*d^2*e^4 - 105*a^3*e^6)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))
/(a*d*e*x^3) - (5*(7*c^4*d^8 + 8*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 21*a^
4*e^8)*(-1/4*((2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(a*d*e*x^2) + ((c*d^2 - a*e^2)^2*ArcTanh[(2*a*d*e + (c*d^2
+ a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])])/(8*a^(3/2)*d^(3/2)*e^(3/2))))/(2*a*d*e))/(8*a*d*e))/(10*a*d*e)
)/(4*d)
```

3.456.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

3.456. $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^7(d+ex)}\,dx$

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.456.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 16882 vs. $2(460) = 920$.

Time = 1.93 (sec) , antiderivative size = 16883, normalized size of antiderivative = 33.90

method	result	size
default	Expression too large to display	16883

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

3.456.5 Fricas [A] (verification not implemented)

Time = 37.47 (sec) , antiderivative size = 1072, normalized size of antiderivative = 2.15

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^7(d + ex)}\,dx = \left[-\frac{15\,(7\,c^6d^{12} - 6\,ac^5d^{10}e^2 - 3\,a^2c^4d^8e^4 - 4\,a^3c^3d^6e^6 - 15\,a^4c^2d^4e^8}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/30720*(15*(7*c^6*d^12 - 6*a*c^5*d^10*e^2 - 3*a^2*c^4*d^8*e^4 - 4*a^3*c
^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 42*a^5*c*d^2*e^10 - 21*a^6*e^12)*sqrt(a*
d*e)*x^6*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(1280*a^6*d^6*e^6 - (1
05*a*c^5*d^11*e - 55*a^2*c^4*d^9*e^3 - 54*a^3*c^3*d^7*e^5 - 78*a^4*c^2*d^5
*e^7 + 525*a^5*c*d^3*e^9 - 315*a^6*d*e^11)*x^5 + 2*(35*a^2*c^4*d^10*e^2 -
16*a^3*c^3*d^8*e^4 - 18*a^4*c^2*d^6*e^6 + 168*a^5*c*d^4*e^8 - 105*a^6*d^2*
e^10)*x^4 - 8*(7*a^3*c^3*d^9*e^3 - 3*a^4*c^2*d^7*e^5 + 33*a^5*c*d^5*e^7 -
21*a^6*d^3*e^9)*x^3 + 16*(3*a^4*c^2*d^8*e^4 + 14*a^5*c*d^6*e^6 - 9*a^6*d^4
*e^8)*x^2 + 128*(13*a^5*c*d^7*e^5 + a^6*d^5*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x))/(a^5*d^6*e^5*x^6), 1/15360*(15*(7*c^6*d^12 - 6*a*c^
5*d^10*e^2 - 3*a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 +
42*a^5*c*d^2*e^10 - 21*a^6*e^12)*sqrt(-a*d*e)*x^6*arctan(1/2*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/
(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 2*(1280*a^6
*d^6*e^6 - (105*a*c^5*d^11*e - 55*a^2*c^4*d^9*e^3 - 54*a^3*c^3*d^7*e^5 - 7
8*a^4*c^2*d^5*e^7 + 525*a^5*c*d^3*e^9 - 315*a^6*d*e^11)*x^5 + 2*(35*a^2*c^
4*d^10*e^2 - 16*a^3*c^3*d^8*e^4 - 18*a^4*c^2*d^6*e^6 + 168*a^5*c*d^4*e^8 -
 105*a^6*d^2*e^10)*x^4 - 8*(7*a^3*c^3*d^9*e^3 - 3*a^4*c^2*d^7*e^5 + 33*...
```

3.456.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^7(d + ex)}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**7/(e*x+d),x)
```

output
```
Timed out
```

3.456.7 Maxima [F]

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{3/2}}{x^7(d + ex)}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)x^7}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^7), x
)
```

3.456.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3251 vs. 2(460) = 920.

Time = 0.48 (sec) , antiderivative size = 3251, normalized size of antiderivative = 6.53

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{3/2}}{x^7(d + ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x, algorithm
="giac")
```

output
```
1/512*(7*c^6*d^12 - 6*a*c^5*d^10*e^2 - 3*a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e
^6 - 15*a^4*c^2*d^4*e^8 + 42*a^5*c*d^2*e^10 - 21*a^6*e^12)*arctan(-(sqrt(c
*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt
(-a*d*e)*a^4*d^5*e^4) - 1/7680*(105*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^
2*x + a*e^2*x + a*d*e))*a^5*c^6*d^17*e^5 - 90*(sqrt(c*d*e)*x - sqrt(c*d*e*
x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^6*c^5*d^15*e^7 - 15405*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^7*c^4*d^13*e^9 - 46140*(s
qrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^8*c^3*d^11*e
^11 - 46305*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*
a^9*c^2*d^9*e^13 - 14730*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2
*x + a*d*e))*a^10*c*d^7*e^15 - 315*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e))*a^11*d^5*e^17 - 3072*sqrt(c*d*e)*a^8*c^3*d^12*e^10
- 6144*sqrt(c*d*e)*a^9*c^2*d^10*e^12 - 5120*sqrt(c*d*e)*a^10*c*d^8*e^14 -
595*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^4*c^
6*d^16*e^4 - 30210*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a
*d*e))^3*a^5*c^5*d^14*e^6 - 199425*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e))^3*a^6*c^4*d^12*e^8 - 419500*(sqrt(c*d*e)*x - sqrt(c
*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^7*c^3*d^10*e^10 - 305925*(sqrt(
c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^8*c^2*d^8*e^12
 - 65010*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^...
```

3.456.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{3/2}}{x^7(d + ex)}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{x^7\,\left(d + e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)), x)
```

3.457 $\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

3.457.1 Optimal result

Integrand size = 40, antiderivative size = 574

$$\int \frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx =$$

$$-\frac{3(cd^2-ae^2)^3\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+(cd^2+ae^2)x+cde}}{16384c^5d^5e^6}$$

$$+\frac{(cd^2-ae^2)\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\left(cd^2+ae^2+2cdex\right)\left(ade+(cd^2+ae^2)x+cdex^2\right)}{2048c^4d^4e^5}$$

$$+\frac{1}{112}\left(\frac{5a}{cd}-\frac{11d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}+\frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{8e}$$

$$-\frac{\left(231c^3d^6-15ac^2d^4e^2-95a^2cd^2e^4-105a^3e^6-10cde(33c^2d^4-10acd^2e^2-15a^2e^4)x\right)\left(ade+(cd^2+ae^2)\right.}{4480c^3d^3e^4}$$

$$+\frac{3(cd^2-ae^2)^5\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{32768c^{11/2}d^{11/2}e^{13/2}}$$

output
```
1/2048*(-a*e^2+c*d^2)*(15*a^3*e^6+35*a^2*c*d^2*e^4+45*a*c^2*d^4*e^2+33*c^3
*d^6)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^4/
d^4/e^5+1/112*(5*a/c/d-11*d/e^2)*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2)+1/8*x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e-1/4480*(231*c^3*d^6-1
5*a*c^2*d^4*e^2-95*a^2*c*d^2*e^4-105*a^3*e^6-10*c*d*e*(-15*a^2*e^4-10*a*c*
d^2*e^2+33*c^2*d^4)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^3/d^3/e^4
+3/32768*(-a*e^2+c*d^2)^5*(15*a^3*e^6+35*a^2*c*d^2*e^4+45*a*c^2*d^4*e^2+33
*c^3*d^6)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(11/2)/d^(11/2)/e^(13/2)-3/16384*(-
a*e^2+c*d^2)^3*(15*a^3*e^6+35*a^2*c*d^2*e^4+45*a*c^2*d^4*e^2+33*c^3*d^6)*(
2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^5/d^5/e^6
```

3.457.2 Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 549, normalized size of antiderivative = 0.96

$$\int \frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(1575a^7e^{14}-525a^6cde^{12}(7d+\right.}{}$$

input
```
Integrate[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(1575*a^7*e^14 - 5
25*a^6*c*d*e^12*(7*d + 2*e*x) + 35*a^5*c^2*d^2*e^10*(29*d^2 + 68*d*e*x + 2
4*e^2*x^2) + 5*a^4*c^3*d^3*e^8*(185*d^3 - 110*d^2*e*x - 376*d*e^2*x^2 - 14
4*e^3*x^3) + 5*a^3*c^4*d^4*e^6*(265*d^4 - 120*d^3*e*x + 80*d^2*e^2*x^2 + 3
20*d*e^3*x^3 + 128*e^4*x^4) + a^2*c^5*d^5*e^4*(-11193*d^5 + 7034*d^4*e*x -
 5488*d^3*e^2*x^2 + 4640*d^2*e^3*x^3 + 137600*d*e^4*x^4 + 103680*e^5*x^5)
+ a*c^6*d^6*e^2*(11445*d^6 - 7476*d^5*e*x + 5928*d^4*e^2*x^2 - 5056*d^3*e^
3*x^3 + 4480*d^2*e^4*x^4 + 212480*d*e^5*x^5 + 168960*e^6*x^6) + c^7*d^7*(-
3465*d^7 + 2310*d^6*e*x - 1848*d^5*e^2*x^2 + 1584*d^4*e^3*x^3 - 1408*d^3*e
^4*x^4 + 1280*d^2*e^5*x^5 + 87040*d*e^6*x^6 + 71680*e^7*x^7)) + (105*(c*d^
2 - a*e^2)^5*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 + 15*a^3*e^
6)*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(
Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(573440*c^(11/2)*d^(11/2)*e^(13/2))
```

3.457.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 543, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1215, 1236, 27, 1236, 27, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(x \left(a e^2+c d^2\right)+a d e+c d e x^2\right)^{5/2}}{d+e x} \, dx$$

$$\Big\downarrow \text{1215}$$

$$\int x^3 (a e+c d x) \left(x \left(a e^2+c d^2\right)+a d e+c d e x^2\right)^{3/2} dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{\int -\frac{1}{2} c d x^2 \left(6 a d e+\left(11 c d^2-5 a e^2\right) x\right) \left(c d e x^2+\left(c d^2+a e^2\right) x+a d e\right)^{3/2} dx}{8 c d e}+\frac{x^3 \left(x \left(a e^2+c d^2\right)+a d e+c d e x^2\right)^{5/2}}{8 e}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3 \left(x \left(a e^2+c d^2\right)+a d e+c d e x^2\right)^{5/2}}{8 e}-\frac{\int x^2 \left(6 a d e+\left(11 c d^2-5 a e^2\right) x\right) \left(c d e x^2+\left(c d^2+a e^2\right) x+a d e\right)^{3/2} dx}{16 e}$$

$$\Big\downarrow \text{1236}$$

$$\frac{x^3 \left(x \left(a e^2+c d^2\right)+a d e+c d e x^2\right)^{5/2}}{8 e}-\frac{\frac{\int -\frac{1}{2} x \left(4 a d e \left(11 c d^2-5 a e^2\right)+3 \left(33 c^2 d^4-10 a c e^2 d^2-15 a^2 e^4\right) x\right) \left(c d e x^2+\left(c d^2+a e^2\right) x+a d e\right)^{3/2} dx}{7 c d e}+\frac{1}{7} x^2 \left(\frac{11 d}{e}-\frac{5 a e}{c d}\right) \left(x \left(a e^2+c d^2\right)+a d e\right.}{16 e}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3 \left(x \left(a e^2+c d^2\right)+a d e+c d e x^2\right)^{5/2}}{8 e}-\frac{\frac{1}{7} x^2 \left(\frac{11 d}{e}-\frac{5 a e}{c d}\right) \left(x \left(a e^2+c d^2\right)+a d e+c d e x^2\right)^{5/2}-\frac{\int x \left(4 a d e \left(11 c d^2-5 a e^2\right)+3 \left(33 c^2 d^4-10 a c e^2 d^2-15 a^2 e^4\right) x\right) \left(c d e x^2+\left(c d^2+a e^2\right) x\right.}{14 c d e}}{16 e}$$

$$\Big\downarrow \text{1225}$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8e}-$$

$$\frac{\frac{1}{7}x^2\left(\frac{11d}{e}-\frac{5ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}-\frac{\frac{7\left(cd^2-ae^2\right)\left(15a^3e^6+35a^2cd^2e^4+45ac^2d^4e^2+33c^3d^6\right)\int\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/}}{8c^2d^2e^2}}{}}{16e}$$

$\downarrow$ 1087

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8e}-$$

$$\frac{1}{7}x^2\left(\frac{11d}{e}-\frac{5ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}-\frac{\frac{7\left(cd^2-ae^2\right)\left(15a^3e^6+35a^2cd^2e^4+45ac^2d^4e^2+33c^3d^6\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+c\right.\right.}{8cde}\right.}{8c^2d^2e^2}}{}$$

$\downarrow$ 1087

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8e}-$$

$$\frac{1}{7}x^2\left(\frac{11d}{e}-\frac{5ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}-\frac{\frac{7\left(cd^2-ae^2\right)\left(15a^3e^6+35a^2cd^2e^4+45ac^2d^4e^2+33c^3d^6\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+c\right.\right.}{8cde}\right.}{}}{}$$

$\downarrow$ 1092

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8e}-$$

$$\frac{1}{7}x^2\left(\frac{11d}{e}-\frac{5ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}-\frac{\frac{7\left(cd^2-ae^2\right)\left(15a^3e^6+35a^2cd^2e^4+45ac^2d^4e^2+33c^3d^6\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+c\right.\right.}{8cde}\right.}{}}{}$$

$\downarrow$ 219

3.457. $\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8e}-$$

$$\frac{1}{7}x^2\left(\frac{11d}{e}-\frac{5ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}-\frac{7\left(cd^2-ae^2\right)\left(15a^3e^6+35a^2cd^2e^4+45ac^2d^4e^2+33c^3d^6\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+c\cdots\right.\right.}{8cde}\right.}{\cdots}$$

input
```
Int[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

output
```
(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(8*e) - ((((11*d)/e -
(5*a*e)/(c*d))*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/7 - (-1/
20*((231*c^3*d^6 - 15*a*c^2*d^4*e^2 - 95*a^2*c*d^2*e^4 - 105*a^3*e^6 - 10*
c*d*e*(33*c^2*d^4 - 10*a*c*d^2*e^2 - 15*a^2*e^4)*x)*(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^(5/2))/(c^2*d^2*e^2) + (7*(c*d^2 - a*e^2)*(33*c^3*d^6 +
45*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 + 15*a^3*e^6)*(((c*d^2 + a*e^2 + 2*c*d
*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*c*d*e) - (3*(c*d^2
 - a*e^2)^2*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d
*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(16*c*d*e)))/(8*c^2*d^2*e^2))/(14*c*d*
e))/(16*e)
```

3.457.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

3.457.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1894 vs. $2(536) = 1072$.

Time = 0.66 (sec) , antiderivative size = 1895, normalized size of antiderivative = 3.30

method	result	size
default	Expression too large to display	1895

3.457. $\int \frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx$

input
```
int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(1/8*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/e-9/16*(a*e^2+c*d^2
)/c/d/e*(1/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/e-1/2*(a*e^2+c*d^
2)/c/d/e*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/8*(2*c*d*e*x+a*e^
2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2
-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2
*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2))/(c*d*e)^(1/2)))))-1/8*a/c*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/
d/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^
2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2
+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))))+d^2/e^3*(1/12*(2*c*d*e*x+
a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+5/24*(4*a*c*d^2
*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1
/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1
/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*...
```

3.457.5 Fricas [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 1524, normalized size of antiderivative = 2.66

$$\int \frac{x^3(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{d + ex}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm
="fricas")
```

output
```
[1/2293760*(105*(33*c^8*d^16 - 120*a*c^7*d^14*e^2 + 140*a^2*c^6*d^12*e^4 -
 40*a^3*c^5*d^10*e^6 - 10*a^4*c^4*d^8*e^8 - 8*a^5*c^3*d^6*e^10 - 20*a^6*c^
2*d^4*e^12 + 40*a^7*c*d^2*e^14 - 15*a^8*e^16)*sqrt(c*d*e)*log(8*c^2*d^2*e^
2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a
*c*d*e^3)*x) + 4*(71680*c^8*d^8*e^8*x^7 - 3465*c^8*d^15*e + 11445*a*c^7*d^
13*e^3 - 11193*a^2*c^6*d^11*e^5 + 1325*a^3*c^5*d^9*e^7 + 925*a^4*c^4*d^7*e
^9 + 1015*a^5*c^3*d^5*e^11 - 3675*a^6*c^2*d^3*e^13 + 1575*a^7*c*d*e^15 + 5
120*(17*c^8*d^9*e^7 + 33*a*c^7*d^7*e^9)*x^6 + 1280*(c^8*d^10*e^6 + 166*a*c
^7*d^8*e^8 + 81*a^2*c^6*d^6*e^10)*x^5 - 128*(11*c^8*d^11*e^5 - 35*a*c^7*d^
9*e^7 - 1075*a^2*c^6*d^7*e^9 - 5*a^3*c^5*d^5*e^11)*x^4 + 16*(99*c^8*d^12*e
^4 - 316*a*c^7*d^10*e^6 + 290*a^2*c^6*d^8*e^8 + 100*a^3*c^5*d^6*e^10 - 45*
a^4*c^4*d^4*e^12)*x^3 - 8*(231*c^8*d^13*e^3 - 741*a*c^7*d^11*e^5 + 686*a^2
*c^6*d^9*e^7 - 50*a^3*c^5*d^7*e^9 + 235*a^4*c^4*d^5*e^11 - 105*a^5*c^3*d^3
*e^13)*x^2 + 2*(1155*c^8*d^14*e^2 - 3738*a*c^7*d^12*e^4 + 3517*a^2*c^6*d^1
0*e^6 - 300*a^3*c^5*d^8*e^8 - 275*a^4*c^4*d^6*e^10 + 1190*a^5*c^3*d^4*e^12
 - 525*a^6*c^2*d^2*e^14)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(
c^6*d^6*e^7), -1/1146880*(105*(33*c^8*d^16 - 120*a*c^7*d^14*e^2 + 140*a^2*
c^6*d^12*e^4 - 40*a^3*c^5*d^10*e^6 - 10*a^4*c^4*d^8*e^8 - 8*a^5*c^3*d^6*e^
10 - 20*a^6*c^2*d^4*e^12 + 40*a^7*c*d^2*e^14 - 15*a^8*e^16)*sqrt(-c*d*e...
```

3.457.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{d + ex}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

output
```
Timed out
```

3.457.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.457.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.37

$$\int \frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx = \frac{1}{573440}\sqrt{cdex^2+cd^2x+ae^2x+ade}\Bigg(2\Bigg(4\Bigg(2\Bigg(8\Bigg(10\Bigg(4\Bigg(14\,c$$
$$-\frac{3\,(33\,c^8d^{16}-120\,ac^7d^{14}e^2+140\,a^2c^6d^{12}e^4-40\,a^3c^5d^{10}e^6-10\,a^4c^4d^8e^8-8\,a^5c^3d^6e^{10}-20\,a^6c^2d^4e^{12}+40}{32768\sqrt{cde}c^5d^5}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm
="giac")
```

output
```
1/573440*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*(10*(4*(1
4*c^2*d^2*e*x + (17*c^9*d^10*e^7 + 33*a*c^8*d^8*e^9)/(c^7*d^7*e^7))*x + (c
^9*d^11*e^6 + 166*a*c^8*d^9*e^8 + 81*a^2*c^7*d^7*e^10)/(c^7*d^7*e^7))*x -
(11*c^9*d^12*e^5 - 35*a*c^8*d^10*e^7 - 1075*a^2*c^7*d^8*e^9 - 5*a^3*c^6*d^
6*e^11)/(c^7*d^7*e^7))*x + (99*c^9*d^13*e^4 - 316*a*c^8*d^11*e^6 + 290*a^2
*c^7*d^9*e^8 + 100*a^3*c^6*d^7*e^10 - 45*a^4*c^5*d^5*e^12)/(c^7*d^7*e^7))*
x - (231*c^9*d^14*e^3 - 741*a*c^8*d^12*e^5 + 686*a^2*c^7*d^10*e^7 - 50*a^3
*c^6*d^8*e^9 + 235*a^4*c^5*d^6*e^11 - 105*a^5*c^4*d^4*e^13)/(c^7*d^7*e^7))
*x + (1155*c^9*d^15*e^2 - 3738*a*c^8*d^13*e^4 + 3517*a^2*c^7*d^11*e^6 - 30
0*a^3*c^6*d^9*e^8 - 275*a^4*c^5*d^7*e^10 + 1190*a^5*c^4*d^5*e^12 - 525*a^6
*c^3*d^3*e^14)/(c^7*d^7*e^7))*x - (3465*c^9*d^16*e - 11445*a*c^8*d^14*e^3
+ 11193*a^2*c^7*d^12*e^5 - 1325*a^3*c^6*d^10*e^7 - 925*a^4*c^5*d^8*e^9 - 1
015*a^5*c^4*d^6*e^11 + 3675*a^6*c^3*d^4*e^13 - 1575*a^7*c^2*d^2*e^15)/(c^7
*d^7*e^7)) - 3/32768*(33*c^8*d^16 - 120*a*c^7*d^14*e^2 + 140*a^2*c^6*d^12*
e^4 - 40*a^3*c^5*d^10*e^6 - 10*a^4*c^4*d^8*e^8 - 8*a^5*c^3*d^6*e^10 - 20*a
^6*c^2*d^4*e^12 + 40*a^7*c*d^2*e^14 - 15*a^8*e^16)*log(abs(-c*d^2 - a*e^2
- 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*
e))))/(sqrt(c*d*e)*c^5*d^5*e^6)
```

3.457.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx=\int \frac{x^3\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{d+e\,x}\,dx$$

input
```
int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x),x)
```

output
```
int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x), x)
```

3.458 $\int \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

3.458.1 Optimal result

Integrand size = 40, antiderivative size = 452

$$\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx=\frac{(cd^2-ae^2)^3(9c^2d^4+10acd^2e^2+5a^2e^4)(cd^2+ae^2+2cdex)\sqrt{c}}{1024c^4d^4e^5}$$
$$-\frac{(cd^2-ae^2)(9c^2d^4+10acd^2e^2+5a^2e^4)(cd^2+ae^2+2cdex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{384c^3d^3e^4}$$
$$+\frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{7e}$$
$$+\frac{(63c^2d^4-20acd^2e^2-35a^2e^4-10cde(9cd^2-5ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{840c^2d^2e^3}$$
$$-\frac{(cd^2-ae^2)^5(9c^2d^4+10acd^2e^2+5a^2e^4)\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2048c^{9/2}d^{9/2}e^{11/2}}$$

output
```
-1/384*(-a*e^2+c*d^2)*(5*a^2*e^4+10*a*c*d^2*e^2+9*c^2*d^4)*(2*c*d*e*x+a*e^
2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/e^4+1/7*x^2*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e+1/840*(63*c^2*d^4-20*a*c*d^2*e^2-35*a
^2*e^4-10*c*d*e*(-5*a*e^2+9*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2)/c^2/d^2/e^3-1/2048*(-a*e^2+c*d^2)^5*(5*a^2*e^4+10*a*c*d^2*e^2+9*c^2*d^
4)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(9/2)/d^(9/2)/e^(11/2)+1/1024*(-a*e^2+c*d^
2)^3*(5*a^2*e^4+10*a*c*d^2*e^2+9*c^2*d^4)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e^5
```

3.458.2 Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.06

$$\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx = \frac{(cd^2-ae^2)^5((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(-525a^6e^{12}+350a^5cde^{10}}{}\right.}{}$$

input
```
Integrate[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

output
```
((c*d^2 - a*e^2)^5*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[
e]*(-525*a^6*e^12 + 350*a^5*c*d*e^10*(4*d + e*x) - 35*a^4*c^2*d^2*e^8*(15*
d^2 + 26*d*e*x + 8*e^2*x^2) - 60*a^3*c^3*d^3*e^6*(10*d^3 - 5*d^2*e*x - 12*
d*e^2*x^2 - 4*e^3*x^3) + a^2*c^4*d^4*e^4*(3689*d^4 - 2332*d^3*e*x + 1824*d
^2*e^2*x^2 + 33520*d*e^3*x^3 + 23680*e^4*x^4) + 2*a*c^5*d^5*e^2*(-1680*d^5
 + 1099*d^4*e*x - 872*d^3*e^2*x^2 + 744*d^2*e^3*x^3 + 24320*d*e^4*x^4 + 18
560*e^5*x^5) + 3*c^6*d^6*(315*d^6 - 210*d^5*e*x + 168*d^4*e^2*x^2 - 144*d^
3*e^3*x^3 + 128*d^2*e^4*x^4 + 6400*d*e^5*x^5 + 5120*e^6*x^6)))/((c*d^2 - a
*e^2)^5*(a*e + c*d*x)*(d + e*x)) - (105*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^
2*e^4)*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])
])/((a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(107520*c^(9/2)*d^(9/2)*e^(11/2
))
```

3.458.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1215, 1236, 27, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{d+ex}dx$$

$$\Big\downarrow \text{ 1215}$$

$$\int x^2(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 1236}$$

$$\frac{\int -\frac{1}{2}cdx\left(4ade+\left(9cd^2-5ae^2\right)x\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{7cde}+\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7e}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7e}-\frac{\int x\left(4ade+\left(9cd^2-5ae^2\right)x\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{14e}$$

$$\Big\downarrow \text{1225}$$

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7e}-\frac{\frac{7\left(cd^2-ae^2\right)\left(5a^2e^4+10acd^2e^2+9c^2d^4\right)\int\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{24c^2d^2e^2}-\frac{\left(-35a^2e^4-10cdex\left(9cd^2-5ae^2\right)-20acd^2e^2+63c^2d^4\right)\left(x\left(ae^2+cd^2\right)+\right.}{60c^2d^2e^2}}{14e}$$

$$\Big\downarrow \text{1087}$$

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7e}-\frac{\frac{7\left(cd^2-ae^2\right)\left(5a^2e^4+10acd^2e^2+9c^2d^4\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\int\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{16cde}\right)}{24c^2d^2e^2}-\frac{\left(-35a^2e\right.}{}}{14e}$$

$$\Big\downarrow \text{1087}$$

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7e}-\frac{\frac{7\left(cd^2-ae^2\right)\left(5a^2e^4+10acd^2e^2+9c^2d^4\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}\right.}{16cde}\right.}{24c^2d^2e^2}}{14e}$$

$$\Big\downarrow \text{1092}$$

3.458. $\int \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7e}-$$

$$\frac{7\left(cd^2-ae^2\right)\left(5a^2e^4+10acd^2e^2+9c^2d^4\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}\right.}{}\right.}{24c^2d^2e^2}$$

$\downarrow$ 219

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7e}-$$

$$\frac{7\left(cd^2-ae^2\right)\left(5a^2e^4+10acd^2e^2+9c^2d^4\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}\right.}{}\right.}{24c^2d^2e^2}$$

input
```
Int[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

output
```
(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*e) - (-1/60*((63*c^
2*d^4 - 20*a*c*d^2*e^2 - 35*a^2*e^4 - 10*c*d*e*(9*c*d^2 - 5*a*e^2)*x)*(a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(c^2*d^2*e^2) + (7*(c*d^2 - a*e
^2)*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(((c*d^2 + a*e^2 + 2*c*d*e*x)
*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*c*d*e) - (3*(c*d^2 - a*
e^2)^2*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)
/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])
/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(16*c*d*e)))/(24*c^2*d^2*e^2))/(14*e)
```

3.458. $\int \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

3.458.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

3.458. $\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx$

3.458.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. 2(418) = 836.

Time = 0.66 (sec) , antiderivative size = 1080, normalized size of antiderivative = 2.39

method	result	size
default	Expression too large to display	1080

input
```
int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(1/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/e-1/2*(a*e^2+c*d^2)/c
/d/e*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/8*(2*c*d*e*x+a*e^2+c*
d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*
e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2
*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
))/(c*d*e)^(1/2)))))-d/e^2*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1
/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3
/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d
/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2
)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))))+d^2/e^3*(1/5*(c*d*e*(x+d/e)^2+(a
*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e^2-c*d^2)*(1/8*(2*c*d*e*(x+d/e)+e^2*a-c
*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^
2)^2/c/d/e*(1/4*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^
2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/e*ln((1/2*e^2*a-1/2*c*d^2+
c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
)/(c*d*e)^(1/2))))
```

3.458.5 Fricas [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 1272, normalized size of antiderivative = 2.81

$$\int \frac{x^2(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{d + ex}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/430080*(105*(9*c^7*d^14 - 35*a*c^6*d^12*e^2 + 45*a^2*c^5*d^10*e^4 - 15
*a^3*c^4*d^8*e^6 - 5*a^4*c^3*d^6*e^8 - 9*a^5*c^2*d^4*e^10 + 15*a^6*c*d^2*e
^12 - 5*a^7*e^14)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*
e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x +
 c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(15360*c^7*
d^7*e^7*x^6 + 945*c^7*d^13*e - 3360*a*c^6*d^11*e^3 + 3689*a^2*c^5*d^9*e^5
- 600*a^3*c^4*d^7*e^7 - 525*a^4*c^3*d^5*e^9 + 1400*a^5*c^2*d^3*e^11 - 525*
a^6*c*d*e^13 + 1280*(15*c^7*d^8*e^6 + 29*a*c^6*d^6*e^8)*x^5 + 128*(3*c^7*d
^9*e^5 + 380*a*c^6*d^7*e^7 + 185*a^2*c^5*d^5*e^9)*x^4 - 16*(27*c^7*d^10*e^
4 - 93*a*c^6*d^8*e^6 - 2095*a^2*c^5*d^6*e^8 - 15*a^3*c^4*d^4*e^10)*x^3 + 8
*(63*c^7*d^11*e^3 - 218*a*c^6*d^9*e^5 + 228*a^2*c^5*d^7*e^7 + 90*a^3*c^4*d
^5*e^9 - 35*a^4*c^3*d^3*e^11)*x^2 - 2*(315*c^7*d^12*e^2 - 1099*a*c^6*d^10*
e^4 + 1166*a^2*c^5*d^8*e^6 - 150*a^3*c^4*d^6*e^8 + 455*a^4*c^3*d^4*e^10 -
175*a^5*c^2*d^2*e^12)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5
*d^5*e^6), 1/215040*(105*(9*c^7*d^14 - 35*a*c^6*d^12*e^2 + 45*a^2*c^5*d^10
*e^4 - 15*a^3*c^4*d^8*e^6 - 5*a^4*c^3*d^6*e^8 - 9*a^5*c^2*d^4*e^10 + 15*a^
6*c*d^2*e^12 - 5*a^7*e^14)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2
*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(15360*c^7*d^7*e^7*x^
6 + 945*c^7*d^13*e - 3360*a*c^6*d^11*e^3 + 3689*a^2*c^5*d^9*e^5 - 600*a...
```

3.458.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 (ade + (cd^2 + ae^2)\, x + cdex^2)^{5/2}}{d + ex}\, dx = \text{Timed out}$$

input
```
integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

output
```
Timed out
```

3.458.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.458.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.43

$$\int \frac{x^2(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex}\,dx = \frac{1}{107520}\sqrt{cdex^2 + cd^2x + ae^2x + ade}\left(2\left(4\left(2\left(8\left(10\left(12\,c^2d^2\right.\right.\right.\right.\right.\right.$$
$$+ \frac{(9\,c^7d^{14} - 35\,ac^6d^{12}e^2 + 45\,a^2c^5d^{10}e^4 - 15\,a^3c^4d^8e^6 - 5\,a^4c^3d^6e^8 - 9\,a^5c^2d^4e^{10} + 15\,a^6cd^2e^{12} - 5\,a^7e^{14})\log}{2048\,\sqrt{cde}c^4d^4e^5}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm
="giac")
```

output
```
1/107520*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*(10*(12*c
^2*d^2*e*x + (15*c^8*d^9*e^6 + 29*a*c^7*d^7*e^8)/(c^6*d^6*e^6))*x + (3*c^8
*d^10*e^5 + 380*a*c^7*d^8*e^7 + 185*a^2*c^6*d^6*e^9)/(c^6*d^6*e^6))*x - (2
7*c^8*d^11*e^4 - 93*a*c^7*d^9*e^6 - 2095*a^2*c^6*d^7*e^8 - 15*a^3*c^5*d^5*
e^10)/(c^6*d^6*e^6))*x + (63*c^8*d^12*e^3 - 218*a*c^7*d^10*e^5 + 228*a^2*c
^6*d^8*e^7 + 90*a^3*c^5*d^6*e^9 - 35*a^4*c^4*d^4*e^11)/(c^6*d^6*e^6))*x -
(315*c^8*d^13*e^2 - 1099*a*c^7*d^11*e^4 + 1166*a^2*c^6*d^9*e^6 - 150*a^3*c
^5*d^7*e^8 + 455*a^4*c^4*d^5*e^10 - 175*a^5*c^3*d^3*e^12)/(c^6*d^6*e^6))*x
 + (945*c^8*d^14*e - 3360*a*c^7*d^12*e^3 + 3689*a^2*c^6*d^10*e^5 - 600*a^3
*c^5*d^8*e^7 - 525*a^4*c^4*d^6*e^9 + 1400*a^5*c^3*d^4*e^11 - 525*a^6*c^2*d
^2*e^13)/(c^6*d^6*e^6)) + 1/2048*(9*c^7*d^14 - 35*a*c^6*d^12*e^2 + 45*a^2*
c^5*d^10*e^4 - 15*a^3*c^4*d^8*e^6 - 5*a^4*c^3*d^6*e^8 - 9*a^5*c^2*d^4*e^10
 + 15*a^6*c*d^2*e^12 - 5*a^7*e^14)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*
(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*
e)*c^4*d^4*e^5)
```

3.458.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{d+ex}\,dx=\int \frac{x^2\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{d+e\,x}\,dx$$

input
```
int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x),x)
```

output
```
int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x), x)
```

3.459 $\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

3.459.1 Optimal result

Integrand size = 38, antiderivative size = 381

$$\int \frac{x(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{d+ex}\,dx = -\frac{(cd^2-ae^2)^3\,(7cd^2+5ae^2)\,(cd^2+ae^2+2cdex)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{512c^3d^3e^4} + \frac{(cd^2-ae^2)\,(7cd^2+5ae^2)\,(cd^2+ae^2+2cdex)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{192c^2d^2e^3} - \frac{1}{60}\left(\frac{5a}{cd}+\frac{7d}{e^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} + \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}{6cde(d+ex)} + \frac{(cd^2-ae^2)^5\,(7cd^2+5ae^2)\,\text{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{1024c^{7/2}d^{7/2}e^{9/2}}$$

output
```
1/192*(-a*e^2+c*d^2)*(5*a*e^2+7*c*d^2)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/e^3-1/60*(5*a/c/d+7*d/e^2)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/
c/d/e/(e*x+d)+1/1024*(-a*e^2+c*d^2)^5*(5*a*e^2+7*c*d^2)*arctanh(1/2*(2*c*d
*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2))/c^(7/2)/d^(7/2)/e^(9/2)-1/512*(-a*e^2+c*d^2)^3*(5*a*e^2+7*c*d^2)*
(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^
4
```

3.459.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.02

$$\int \frac{x(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx = \frac{(cd^2-ae^2)^5\left((ae+cdx)(d+ex)\right)^{3/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(75a^5e^{10}-5a^4cde^8(49d+1\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

output
```
((c*d^2 - a*e^2)^5*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[
e]*(75*a^5*e^10 - 5*a^4*c*d*e^8*(49*d + 10*e*x) + 10*a^3*c^2*d^2*e^6*(15*d
^2 + 16*d*e*x + 4*e^2*x^2) - 6*a^2*c^3*d^3*e^4*(91*d^3 - 58*d^2*e*x - 564*
d*e^2*x^2 - 360*e^3*x^3) + a*c^4*d^4*e^2*(415*d^4 - 272*d^3*e*x + 216*d^2*
e^2*x^2 + 4448*d*e^3*x^3 + 3200*e^4*x^4) + c^5*d^5*(-105*d^5 + 70*d^4*e*x
- 56*d^3*e^2*x^2 + 48*d^2*e^3*x^3 + 1664*d*e^4*x^4 + 1280*e^5*x^5)))/((c*d
^2 - a*e^2)^5*(a*e + c*d*x)*(d + e*x)) + (15*(7*c*d^2 + 5*a*e^2)*ArcTanh[(
Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/((a*e + c*d*x
)^(3/2)*(d + e*x)^(3/2))))/(7680*c^(7/2)*d^(7/2)*e^(9/2))
```

3.459.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1215, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{d+ex}dx$$

$$\Big\downarrow \text{1215}$$

$$\int x(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}dx$$

$$\Big\downarrow \text{1225}$$

$$-\frac{\left(\frac{5a^2e^2}{c}+2ad^2-\frac{7cd^4}{e^2}\right)\int\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{24d}-\frac{\left(-5ae^2+7cd^2-10cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{60cde^2}$$

$$\Big\downarrow \text{1087}$$

$$-\frac{\left(\frac{5a^2e^2}{c}+2ad^2-\frac{7cd^4}{e^2}\right)\left(\frac{(ae^2+cd^2+2cdex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\int\sqrt{cdex^2+(cd^2+ae^2)x+ade}dx}{16cde}\right)}{24d}-\frac{\left(-5ae^2+7cd^2-10cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{60cde^2}$$

$$\Big\downarrow \text{1087}$$

$$-\frac{\left(\frac{5a^2e^2}{c}+2ad^2-\frac{7cd^4}{e^2}\right)\left(\frac{(ae^2+cd^2+2cdex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}\right.}{16cde}\right.}{24d}$$

$$\frac{\left(-5ae^2+7cd^2-10cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{60cde^2}$$

$$\Big\downarrow \text{1092}$$

$$-\frac{\left(\frac{5a^2e^2}{c}+2ad^2-\frac{7cd^4}{e^2}\right)\left(\frac{(ae^2+cd^2+2cdex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}\right.}{}\right.}{24d}$$

$$\frac{\left(-5ae^2+7cd^2-10cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{60cde^2}$$

$$\Big\downarrow \text{219}$$

3.459. $\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

$$-\frac{\left(\frac{5a^2e^2}{c}+2ad^2-\frac{7cd^4}{e^2}\right)\left(\frac{(ae^2+cd^2+2cdex)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{8cde}-\frac{3(cd^2-ae^2)^2\left(\frac{(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}\right.}{}\right.}{24d}$$

$$\frac{(-5ae^2+7cd^2-10cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{60cde^2}$$

input
```
Int[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

output
```
-1/60*((7*c*d^2 - 5*a*e^2 - 10*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(5/2))/(c*d*e^2) - ((2*a*d^2 - (7*c*d^4)/e^2 + (5*a^2*e^2)/c)*(((c*d
^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*
c*d*e) - (3*(c*d^2 - a*e^2)^2*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d
^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(16*c*d*e)))/(24*d)
```

3.459.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

3.459. $\int \frac{x(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx$

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

3.459.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.77

method	result
default	$\frac{\left(2cdex+e^2a+c\,d^2\right)\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{5}{2}}}{12cde}+\frac{5\left(4ac\,d^2e^2-\left(e^2a+c\,d^2\right)^2\right)\left(\frac{\left(2cdex+e^2a+c\,d^2\right)\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}{8cde}+\frac{3\left(4ac\,d^2e^2 \cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x,method=_RETURNVERB
OSE)
```

output
```
1/e*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/8*(2*c*d*e*x+a*e^2+c*d
^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e
^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*
a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
)/(c*d*e)^(1/2))))-d/e^2*(1/5*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2
)+1/2*(a*e^2-c*d^2)*(1/8*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/c/d/e*(c*d*e*(x+d/e
)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/c/d/e*(1/4*(2*c*d*e*
(x+d/e)+e^2*a-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1
/8*(a*e^2-c*d^2)^2/c/d/e*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1
/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))))
```

3.459.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 1046, normalized size of antiderivative = 2.75

$$\int \frac{x(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{d + ex}\,dx = \left[-\frac{15\,(7\,c^6d^{12} - 30\,ac^5d^{10}e^2 + 45\,a^2c^4d^8e^4 - 20\,a^3c^3d^6e^6 - 15\,a^4c \cdots}{\cdots}\right.$$

$$-\frac{15\,(7\,c^6d^{12} - 30\,ac^5d^{10}e^2 + 45\,a^2c^4d^8e^4 - 20\,a^3c^3d^6e^6 - 15\,a^4c^2d^4e^8 + 18\,a^5cd^2e^{10} - 5\,a^6e^{12})\sqrt{-cde}\,\arctan \cdots}{\cdots}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="
fricas")
```

output
```
[-1/30720*(15*(7*c^6*d^12 - 30*a*c^5*d^10*e^2 + 45*a^2*c^4*d^8*e^4 - 20*a^
3*c^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 - 5*a^6*e^12)*sqrt(
c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*
d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(1280*c^6*d^6*e^6*x^5 - 105*c^6*d^
11*e + 415*a*c^5*d^9*e^3 - 546*a^2*c^4*d^7*e^5 + 150*a^3*c^3*d^5*e^7 - 245
*a^4*c^2*d^3*e^9 + 75*a^5*c*d*e^11 + 128*(13*c^6*d^7*e^5 + 25*a*c^5*d^5*e^
7)*x^4 + 16*(3*c^6*d^8*e^4 + 278*a*c^5*d^6*e^6 + 135*a^2*c^4*d^4*e^8)*x^3
- 8*(7*c^6*d^9*e^3 - 27*a*c^5*d^7*e^5 - 423*a^2*c^4*d^5*e^7 - 5*a^3*c^3*d^
3*e^9)*x^2 + 2*(35*c^6*d^10*e^2 - 136*a*c^5*d^8*e^4 + 174*a^2*c^4*d^6*e^6
+ 80*a^3*c^3*d^4*e^8 - 25*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x))/(c^4*d^4*e^5), -1/15360*(15*(7*c^6*d^12 - 30*a*c^5*d^10*
e^2 + 45*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 18*a^
5*c*d^2*e^10 - 5*a^6*e^12)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2
*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(1280*c^6*d^6*e^6*x^5
 - 105*c^6*d^11*e + 415*a*c^5*d^9*e^3 - 546*a^2*c^4*d^7*e^5 + 150*a^3*c^3*
d^5*e^7 - 245*a^4*c^2*d^3*e^9 + 75*a^5*c*d*e^11 + 128*(13*c^6*d^7*e^5 + 25
*a*c^5*d^5*e^7)*x^4 + 16*(3*c^6*d^8*e^4 + 278*a*c^5*d^6*e^6 + 135*a^2*c^4*
d^4*e^8)*x^3 - 8*(7*c^6*d^9*e^3 - 27*a*c^5*d^7*e^5 - 423*a^2*c^4*d^5*e^...
```

3.459.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(ade + (cd^2 + ae^2)\, x + cdex^2)^{5/2}}{d + ex}\, dx = \text{Timed out}$$

input
```
integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

output
```
Timed out
```

3.459.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.459.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.38

$$\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx = \frac{1}{7680}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(2\left(8\left(10\,c^2d^2ex+\frac{13}{}\right.\right.\right.\right.\right.$$
$$-\frac{\left(7\,c^6d^{12}-30\,ac^5d^{10}e^2+45\,a^2c^4d^8e^4-20\,a^3c^3d^6e^6-15\,a^4c^2d^4e^8+18\,a^5cd^2e^{10}-5\,a^6e^{12}\right)\log\left(\left|-cd^2-ae^2\right.\right.}{1024\sqrt{cde}c^3d^3e^4}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="
giac")
```

output
```
1/7680*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*(10*c^2*d^2
*e*x + (13*c^7*d^8*e^5 + 25*a*c^6*d^6*e^7)/(c^5*d^5*e^5))*x + (3*c^7*d^9*e
^4 + 278*a*c^6*d^7*e^6 + 135*a^2*c^5*d^5*e^8)/(c^5*d^5*e^5))*x - (7*c^7*d^
10*e^3 - 27*a*c^6*d^8*e^5 - 423*a^2*c^5*d^6*e^7 - 5*a^3*c^4*d^4*e^9)/(c^5*
d^5*e^5))*x + (35*c^7*d^11*e^2 - 136*a*c^6*d^9*e^4 + 174*a^2*c^5*d^7*e^6 +
 80*a^3*c^4*d^5*e^8 - 25*a^4*c^3*d^3*e^10)/(c^5*d^5*e^5))*x - (105*c^7*d^1
2*e - 415*a*c^6*d^10*e^3 + 546*a^2*c^5*d^8*e^5 - 150*a^3*c^4*d^6*e^7 + 245
*a^4*c^3*d^4*e^9 - 75*a^5*c^2*d^2*e^11)/(c^5*d^5*e^5)) - 1/1024*(7*c^6*d^1
2 - 30*a*c^5*d^10*e^2 + 45*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 - 15*a^4*c
^2*d^4*e^8 + 18*a^5*c*d^2*e^10 - 5*a^6*e^12)*log(abs(-c*d^2 - a*e^2 - 2*sq
rt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/
(sqrt(c*d*e)*c^3*d^3*e^4)
```

3.459.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex} dx = \int \frac{x\,(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\;x + a\,d\,e)^{5/2}}{d + e\,x} dx$$

input
```
int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x),x)
```

output
```
int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x), x)
```

3.460 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx$

3.460.1 Optimal result

Integrand size = 37, antiderivative size = 274

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx = \frac{3(cd^2-ae^2)^3(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128c^2d^2e^3}$$
$$+\frac{1}{16}\left(\frac{a}{cd}-\frac{d}{e^2}\right)(cd^2+ae^2+2cdex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}$$
$$+\frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{5e}-\frac{3(cd^2-ae^2)^5\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{256c^{5/2}d^{5/2}e^{7/2}}$$

output
```
1/16*(a/c/d-d/e^2)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(3/2)+1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e-3/256*(-a*e^2+c*d^2
)^5*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(5/2)/e^(7/2)+3/128*(-a*e^2+c*d^2
)^3*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^
2/e^3
```

3.460.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.08

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx = \frac{((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(-15a^4e^8+10a^3cde^6(7d+ex)+2a^2c^2d^2e^4(64c\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[e]*(-15*a^4*e^8 +
10*a^3*c*d*e^6*(7*d + e*x) + 2*a^2*c^2*d^2*e^4*(64*d^2 + 233*d*e*x + 124*e
^2*x^2) + 2*a*c^3*d^3*e^2*(-35*d^3 + 23*d^2*e*x + 256*d*e^2*x^2 + 168*e^3*
x^3) + c^4*d^4*(15*d^4 - 10*d^3*e*x + 8*d^2*e^2*x^2 + 176*d*e^3*x^3 + 128*
e^4*x^4)))/((a*e + c*d*x)*(d + e*x)) - (15*(c*d^2 - a*e^2)^5*ArcTanh[(Sqrt
[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/((a*e + c*d*x)^(3
/2)*(d + e*x)^(3/2))))/(640*c^(5/2)*d^(5/2)*e^(7/2))
```

3.460.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1131, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{d+ex}dx$$

$$\Big\downarrow \text{ 1131}$$

$$\frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{5e}-\frac{(cd^2-ae^2)\int (cdex^2+(cd^2+ae^2)x+ade)^{3/2}dx}{2e}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{5e}-\frac{(cd^2-ae^2)\left(\frac{(ae^2+cd^2+2cdex)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{8cde}-\frac{3(cd^2-ae^2)^2\int \sqrt{cdex^2+(cd^2+ae^2)x+ade}dx}{16cde}\right)}{2e}$$

$\big\downarrow$ 1087

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e}-$$

$$\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int \cdots}{\cdots}\right.}{16cde}\right.}{2e}$$

$\big\downarrow$ 1092

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e}-$$

$$\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int \cdots}{\cdots}\right.}{16cde}\right.}{2e}$$

$\big\downarrow$ 219

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e}-$$

$$\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\text{ar}\cdots}{\cdots}\right.}{16cde}\right.}{2e}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x),x]
```

3.460. $\displaystyle\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

output
```
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*e) - ((c*d^2 - a*e^2)*(((
c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/
(8*c*d*e) - (3*(c*d^2 - a*e^2)^2*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(
c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(16*c*d*e)))/(2*e
)
```

3.460.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

3.460.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.19

method	result
default	$\frac{\frac{\left(cde\left(x+\frac{d}{e}\right)^{2}+\left(e^{2}a-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+\frac{\left(e^{2}a-c\,d^{2}\right)\left(\frac{\left(2cde\left(x+\frac{d}{e}\right)+e^{2}a-c\,d^{2}\right)\left(cde\left(x+\frac{d}{e}\right)^{2}+\left(e^{2}a-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8cde}-\frac{3\left(e^{2}a-c\,d^{2}\right)^{2}\left(\frac{\left(2cde\left(x\right.\right.}{}\right.}{}\right.}{}}{e}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x,method=_RETURNVERBOS
E)
```

output
```
1/e*(1/5*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e^2-c*d^2)*(
1/8*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+
d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/c/d/e*(1/4*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/
c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/
d/e*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+
(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))))
```

3.460.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 844, normalized size of antiderivative = 3.08

$$\int \frac{\left(ade+\left(cd^{2}+ae^{2}\right)x+cdex^{2}\right)^{5/2}}{d+ex}\,dx=\left[\frac{15\left(c^{5}d^{10}-5\,ac^{4}d^{8}e^{2}+10\,a^{2}c^{3}d^{6}e^{4}-10\,a^{3}c^{2}d^{4}e^{6}+5\,a^{4}cd^{2}e^{8}-\right.}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="fr
icas")
```

output
```
[1/2560*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*
d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 +
c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3
)*x) + 4*(128*c^5*d^5*e^5*x^4 + 15*c^5*d^9*e - 70*a*c^4*d^7*e^3 + 128*a^2*
c^3*d^5*e^5 + 70*a^3*c^2*d^3*e^7 - 15*a^4*c*d*e^9 + 16*(11*c^5*d^6*e^4 + 2
1*a*c^4*d^4*e^6)*x^3 + 8*(c^5*d^7*e^3 + 64*a*c^4*d^5*e^5 + 31*a^2*c^3*d^3*
e^7)*x^2 - 2*(5*c^5*d^8*e^2 - 23*a*c^4*d^6*e^4 - 233*a^2*c^3*d^4*e^6 - 5*a
^3*c^2*d^2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e
^4), 1/1280*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*
c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*
e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(128*c
^5*d^5*e^5*x^4 + 15*c^5*d^9*e - 70*a*c^4*d^7*e^3 + 128*a^2*c^3*d^5*e^5 + 7
0*a^3*c^2*d^3*e^7 - 15*a^4*c*d*e^9 + 16*(11*c^5*d^6*e^4 + 21*a*c^4*d^4*e^6
)*x^3 + 8*(c^5*d^7*e^3 + 64*a*c^4*d^5*e^5 + 31*a^2*c^3*d^3*e^7)*x^2 - 2*(5
*c^5*d^8*e^2 - 23*a*c^4*d^6*e^4 - 233*a^2*c^3*d^4*e^6 - 5*a^3*c^2*d^2*e^8)
*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4)]
```

3.460.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{d + ex}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

output
```
Timed out
```

3.460.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{d + ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.460.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.51

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex}\,dx = \frac{1}{640}\sqrt{cdex^2 + cd^2x + ae^2x + ade}\left(2\left(4\left(2\left(8c^2d^2ex + \frac{11c^6d^7e^4}{c^4\cdots}\right.\right.\right.\right.$$
$$+ \frac{3(c^5d^{10} - 5ac^4d^8e^2 + 10a^2c^3d^6e^4 - 10a^3c^2d^4e^6 + 5a^4cd^2e^8 - a^5e^{10})\log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x - \cdots\right.\right.\right.}{256\sqrt{cde}c^2d^2e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="gi
ac")
```

output
```
1/640*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*c^2*d^2*e*x
+ (11*c^6*d^7*e^4 + 21*a*c^5*d^5*e^6)/(c^4*d^4*e^4))*x + (c^6*d^8*e^3 + 64
*a*c^5*d^6*e^5 + 31*a^2*c^4*d^4*e^7)/(c^4*d^4*e^4))*x - (5*c^6*d^9*e^2 - 2
3*a*c^5*d^7*e^4 - 233*a^2*c^4*d^5*e^6 - 5*a^3*c^3*d^3*e^8)/(c^4*d^4*e^4))*
x + (15*c^6*d^10*e - 70*a*c^5*d^8*e^3 + 128*a^2*c^4*d^6*e^5 + 70*a^3*c^3*d
^4*e^7 - 15*a^4*c^2*d^2*e^9)/(c^4*d^4*e^4)) + 3/256*(c^5*d^10 - 5*a*c^4*d^
8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^
10)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^2*d^2*e^3)
```

3.460.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex}\,dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{5/2}}{d + e\,x}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x), x)
```

3.461 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x(d+ex)}\,dx$

3.461.1 Optimal result

Integrand size = 40, antiderivative size = 394

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x(d+ex)}\,dx =$$
$$-\frac{(3c^3d^6-11ac^2d^4e^2-83a^2cd^2e^4-5a^3e^6+2cde(cd^2-5ae^2)(3cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex}}{64cde^2}$$
$$+\frac{(3cd^2+11ae^2+6cdex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{24e}$$
$$+\frac{(3c^4d^8-20ac^3d^6e^2+90a^2c^2d^4e^4+60a^3cd^2e^6-5a^4e^8)\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128c^{3/2}d^{3/2}e^{5/2}}$$
$$-a^{5/2}d^{3/2}e^{5/2}\operatorname{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)$$

output
```
1/24*(6*c*d*e*x+11*a*e^2+3*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/
e-a^(5/2)*d^(3/2)*e^(5/2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^
(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))+1/128*(-5*a^4*e^8+6
0*a^3*c*d^2*e^6+90*a^2*c^2*d^4*e^4-20*a*c^3*d^6*e^2+3*c^4*d^8)*arctanh(1/2
*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/e^(5/2)-1/64*(3*c^3*d^6-11*a*c^2*d^4*e^2-8
3*a^2*c*d^2*e^4-5*a^3*e^6+2*c*d*e*(-5*a*e^2+c*d^2)*(a*e^2+3*c*d^2)*x)*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e^2
```

3.461.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.87

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x(d + ex)} dx = \frac{\sqrt{ae + cdx}\sqrt{d + ex}\left(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae + cdx}\sqrt{d + ex}(15a^3e^6 + a^2cde\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x*(d + e*x)),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x
]*Sqrt[d + e*x]*(15*a^3*e^6 + a^2*c*d*e^4*(337*d + 118*e*x) + a*c^2*d^2*e^
2*(57*d^2 + 244*d*e*x + 136*e^2*x^2) + c^3*(-9*d^6 + 6*d^5*e*x + 72*d^4*e^
2*x^2 + 48*d^3*e^3*x^3)) - 384*a^(5/2)*c^(3/2)*d^3*e^5*ArcTanh[(Sqrt[d]*Sq
rt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])] + 3*(3*c^4*d^8 - 20*a*c^
3*d^6*e^2 + 90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*ArcTanh[(Sq
rt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]))/(192*c^(3/2)*d
^(3/2)*e^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.461.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1215, 1231, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{5/2}}{x(d + ex)} dx$$

$$\Big\downarrow \text{ 1215}$$

$$\int \frac{(ae + cdx)\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{3/2}}{x} dx$$

$$\Big\downarrow \text{ 1231}$$

$$\frac{\left(11ae^2 + 3cd^2 + 6cdex\right)\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{3/2}}{24e} - \frac{\int -\frac{cd\left(16a^2de^3 - \left(cd^2 - 5ae^2\right)\left(3cd^2 + ae^2\right)x\right)\sqrt{cdex^2 + \left(cd^2 + ae^2\right)x + ade}}{2x} dx}{8cde}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(16a^2de^3-\left(cd^2-5ae^2\right)\left(3cd^2+ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x}dx}{16e}+\frac{\left(11ae^2+3cd^2+6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24e}$$

$$\Big\downarrow \; 1231$$

$$\frac{-\frac{\int -\frac{128a^3cd^3e^5+\left(3c^4d^8-20ac^3e^2d^6+90a^2c^2e^4d^4+60a^3ce^6d^2-5a^4e^8\right)x}{2x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4cde}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(-5a^3e^6-83a^2cd^2e^4-11ac^2d^4e^2+2cdex\left(\right.\right.}{4cde}}{16e}+\frac{\left(11ae^2+3cd^2+6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24e}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{128a^3cd^3e^5+\left(3c^4d^8-20ac^3e^2d^6+90a^2c^2e^4d^4+60a^3ce^6d^2-5a^4e^8\right)x}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cde}-\frac{\left(-5a^3e^6-83a^2cd^2e^4-11ac^2d^4e^2+2cdex\left(cd^2-5ae^2\right)\left(ae^2+3cd^2\right)+3c^3d^6\right)}{4cde}}{16e}+\frac{\left(11ae^2+3cd^2+6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24e}$$

$$\Big\downarrow \; 1269$$

$$\frac{\frac{128a^3cd^3e^5\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\left(-5a^4e^8+60a^3cd^2e^6+90a^2c^2d^4e^4-20ac^3d^6e^2+3c^4d^8\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cde}-\frac{\left(-5a^3e^6-\right.}{}}{16e}+\frac{\left(11ae^2+3cd^2+6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24e}$$

$$\Big\downarrow \; 1092$$

$$\frac{\frac{128a^3cd^3e^5\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+2\left(-5a^4e^8+60a^3cd^2e^6+90a^2c^2d^4e^4-20ac^3d^6e^2+3c^4d^8\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2}{\sqrt{cdex^2+(c}}}{8cde}}{16e}+\frac{\left(11ae^2+3cd^2+6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24e}$$

$$\Big\downarrow \; 219$$

3.461. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x(d+ex)}\,dx$

$$\frac{\frac{128a^3cd^3e^5\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{\left(-5a^4e^8+60a^3cd^2e^6+90a^2c^2d^4e^4-20ac^3d^6e^2+3c^4d^8\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{e}}}{8cde}}{16e}$$

$$\frac{\left(11ae^2+3cd^2+6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24e}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{\frac{\left(-5a^4e^8+60a^3cd^2e^6+90a^2c^2d^4e^4-20ac^3d^6e^2+3c^4d^8\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{e}}-256a^3cd^3e^5\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d}{8cde}}{16e}$$

$$\frac{\left(11ae^2+3cd^2+6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24e}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{\left(-5a^4e^8+60a^3cd^2e^6+90a^2c^2d^4e^4-20ac^3d^6e^2+3c^4d^8\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{e}}-128a^{5/2}cd^{5/2}e^{9/2}\text{arctanh}\left(\frac{x\left(ae^2\right.}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(a\right.}}\right.}{8cde}}{16e}$$

$$\frac{\left(11ae^2+3cd^2+6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24e}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x*(d + e*x)),x]
```

output
```
((3*c*d^2 + 11*a*e^2 + 6*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2))/(24*e) + (-1/4*((3*c^3*d^6 - 11*a*c^2*d^4*e^2 - 83*a^2*c*d^2*e^4 -
5*a^3*e^6 + 2*c*d*e*(c*d^2 - 5*a*e^2)*(3*c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*e) + (((3*c^4*d^8 - 20*a*c^3*d^6*e^2 +
90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*ArcTanh[(c*d^2 + a*e^2
+ 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2])])/(Sqrt[c]*Sqrt[d]*Sqrt[e]) - 128*a^(5/2)*c*d^(5/2)*e^(9/2)*Arc
Tanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c*d*e))/(16*e)
```

3.461. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x(d+ex)}\,dx$

3.461.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.461. $\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{x(d+ex)}\,dx$

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

3.461.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 996 vs. $2(352) = 704$.

Time = 0.60 (sec) , antiderivative size = 997, normalized size of antiderivative = 2.53

method	result
default	$\frac{\frac{\left(ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}\right)^{\frac{5}{2}}}{5}+\left(e^{2}a+c\,d^{2}\right)\left(\frac{\left(2cdex+e^{2}a+c\,d^{2}\right)\left(ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}\right)^{\frac{3}{2}}}{8cde}+\frac{3\left(4ac\,d^{2}e^{2}-\left(e^{2}a+c\,d^{2}\right)^{2}\right)\left(\frac{\left(2cdex+e^{2}a+c\right.}{}\right.}{}\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x,method=_RETURNVERB
OSE)
```

output
```
1/d*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/2*(a*e^2+c*d^2)*(1/8*(2
*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(
4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/
c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+a*d*e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*
ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*
a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2))/x))))-1/d*(1/5*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e
^2-c*d^2)*(1/8*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2
-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/c/d/e*(1/4*(2*c*d*e*(x+d/e)+e^
2*a-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-
c*d^2)^2/c/d/e*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e
*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))))
```

3.461.5 Fricas [A] (verification not implemented)

Time = 30.62 (sec) , antiderivative size = 1873, normalized size of antiderivative = 4.75

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x(d + ex)} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x, algorithm="
fricas")
```

output
```
[1/768*(384*sqrt(a*d*e)*a^2*c^2*d^3*e^5*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*
a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/
x^2) - 3*(3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2
*e^6 - 5*a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*
e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x +
 c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(48*c^4*d^4
*e^4*x^3 - 9*c^4*d^7*e + 57*a*c^3*d^5*e^3 + 337*a^2*c^2*d^3*e^5 + 15*a^3*c
*d*e^7 + 8*(9*c^4*d^5*e^3 + 17*a*c^3*d^3*e^5)*x^2 + 2*(3*c^4*d^6*e^2 + 122
*a*c^3*d^4*e^4 + 59*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x))/(c^2*d^2*e^3), 1/384*(192*sqrt(a*d*e)*a^2*c^2*d^3*e^5*log((8*a^
2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a
*c*d^3*e + a^2*d*e^3)*x)/x^2) - 3*(3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*a^2*c
^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d
*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(48*c
^4*d^4*e^4*x^3 - 9*c^4*d^7*e + 57*a*c^3*d^5*e^3 + 337*a^2*c^2*d^3*e^5 + 15
*a^3*c*d*e^7 + 8*(9*c^4*d^5*e^3 + 17*a*c^3*d^3*e^5)*x^2 + 2*(3*c^4*d^6*e^2
 + 122*a*c^3*d^4*e^4 + 59*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + ...
```

3.461.6 Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x(d + ex)}\,dx = \int \frac{((d + ex)(ae + cdx))^{\frac{5}{2}}}{x(d + ex)}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x/(e*x+d),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(x*(d + e*x)), x)
```

3.461.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{5/2}}{x(d + ex)} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.461.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{5/2}}{x(d + ex)} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

3.461.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{5/2}}{x(d + ex)} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\;x + a\,d\,e)^{5/2}}{x\;(d + e\,x)} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x*(d + e*x)), x)
```

3.462 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^2(d+ex)}\,dx$

3.462.1 Optimal result

Integrand size = 40, antiderivative size = 352

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^2(d+ex)}\,dx = \frac{(c^2d^4+28acd^2e^2+19a^2e^4+2cde(cd^2+7ae^2)x)\sqrt{ade+(cd^2+a}}{8e}$$

$$-\frac{(3ae-cdx)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3x}$$

$$-\frac{(c^3d^6-15ac^2d^4e^2-45a^2cd^2e^4-5a^3e^6)\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{16\sqrt{c}\sqrt{d}e^{3/2}}$$

$$-\frac{1}{2}a^{3/2}\sqrt{d}e^{3/2}(5cd^2+3ae^2)\operatorname{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)$$

output
```
-1/3*(-c*d*x+3*a*e)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x-1/16*(-5*a^3
*e^6-45*a^2*c*d^2*e^4-15*a*c^2*d^4*e^2+c^3*d^6)*arctanh(1/2*(2*c*d*e*x+a*e
^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))
/e^(3/2)/c^(1/2)/d^(1/2)-1/2*a^(3/2)*e^(3/2)*(3*a*e^2+5*c*d^2)*arctanh(1/2
*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2))*d^(1/2)+1/8*(c^2*d^4+28*a*c*d^2*e^2+19*a^2*e^4+2*c*d*e*(
7*a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e
```

3.462.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.88

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^2(d+ex)}\,dx = \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}\sqrt{d+ex}(3a^2e^3(-8d+11\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x
]*Sqrt[d + e*x]*(3*a^2*e^3*(-8*d + 11*e*x) + 2*a*c*d*e^2*x*(34*d + 13*e*x)
 + c^2*d^2*x*(3*d^2 + 14*d*e*x + 8*e^2*x^2)) - 24*a^(3/2)*Sqrt[c]*d*e^3*(5
*c*d^2 + 3*a*e^2)*x*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*S
qrt[d + e*x])] - 3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*e^4 - 5*a^3*
e^6)*x*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])
]))/(24*Sqrt[c]*Sqrt[d]*e^(3/2)*x*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.462.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1215, 1230, 25, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{x^2(d+ex)}\,dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{1230}$$

$$-\frac{1}{2}\int -\frac{\left(ae\left(5cd^2+3ae^2\right)+cd\left(cd^2+7ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x}dx-\frac{(3ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{\left(ae\left(5cd^2+3ae^2\right)+cd\left(cd^2+7ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x}dx-$$
$$\frac{(3ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x}$$

$\downarrow$ 1231

$$\frac{1}{2}\left(\frac{\left(19a^2e^4+2cdex\left(7ae^2+cd^2\right)+28acd^2e^2+c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e}-\frac{\int -\frac{cd\left(8a^2de^3\left(5cd^2+3ae^2\right)-\left(c^3\right.\right.}{2x\sqrt{cdex^2}}\right.$$
$$\frac{(3ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x}$$

$\downarrow$ 27

$$\frac{1}{2}\left(\frac{\int \frac{8a^2de^3\left(5cd^2+3ae^2\right)-\left(c^3d^6-15ac^2e^2d^4-45a^2ce^4d^2-5a^3e^6\right)x}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8e}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(19a^2e^4+2cdex\left(7ae^2\right.\right.}{4e}\right.$$
$$\frac{(3ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x}$$

$\downarrow$ 1269

$$\frac{1}{2}\left(\frac{8a^2de^3\left(3ae^2+5cd^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\left(-5a^3e^6-45a^2cd^2e^4-15ac^2d^4e^2+c^3d^6\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+a\right.}}}{8e}\right.$$
$$\frac{(3ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x}$$

$\downarrow$ 1092

$$\frac{1}{2}\left(\frac{8a^2de^3\left(3ae^2+5cd^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-2\left(-5a^3e^6-45a^2cd^2e^4-15ac^2d^4e^2+c^3d^6\right)\int \frac{1}{4cde-\frac{\left(cd^2+2\right.}{cdex^2+\left(c\right.}}}{8e}\right.$$
$$\frac{(3ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x}$$

$\downarrow$ 219

3.462. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^2(d+ex)}dx$

$$\frac{1}{2}\left(\frac{8a^2de^3\left(3ae^2+5cd^2\right)\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\frac{\left(-5a^3e^6-45a^2cd^2e^4-15ac^2d^4e^2+c^3d^6\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cd}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)}}\right.}{\sqrt{c}\sqrt{d}\sqrt{e}}}{8e}\right.$$

$$\frac{(3ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x}$$

$$\Big\downarrow \ 1154$$

$$\frac{1}{2}\left(\frac{-16a^2de^3\left(3ae^2+5cd^2\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-\frac{\left(-5a^3e^6-45a^2cd^2e^4-15ac^2d^4e^2+c^3d^6\right)\text{ar}}{\sqrt{c}}}{8e}\right.$$

$$\frac{(3ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x}$$

$$\Big\downarrow \ 219$$

$$\frac{1}{2}\left(\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(19a^2e^4+2cdex\left(7ae^2+cd^2\right)+28acd^2e^2+c^2d^4\right)}{4e}+\frac{-8a^{3/2}\sqrt{d}e^{5/2}\left(3ae^2+5cd\right.}{}\right.$$

$$\frac{(3ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

3.462. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^2(d+ex)}\,dx$

output
```
-1/3*((3*a*e - c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/x + (
((c^2*d^4 + 28*a*c*d^2*e^2 + 19*a^2*e^4 + 2*c*d*e*(c*d^2 + 7*a*e^2)*x)*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e) + (-(((c^3*d^6 - 15*a*c^2*
d^4*e^2 - 45*a^2*c*d^2*e^4 - 5*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x
)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]
)/(Sqrt[c]*Sqrt[d]*Sqrt[e])) - 8*a^(3/2)*Sqrt[d]*e^(5/2)*(5*c*d^2 + 3*a*e^
2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*e))/2
```

3.462.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

3.462. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^2(d+ex)}\,dx$

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.462.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2075 vs. $2(308) = 616$.

Time = 0.75 (sec) , antiderivative size = 2076, normalized size of antiderivative = 5.90

method	result	size
default	Expression too large to display	2076

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/d*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)+5/2*(a*e^2+c*d^2)/
a/d/e*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/2*(a*e^2+c*d^2)*(1/8*
(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16
*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2
)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+a*d*e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/
e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((
2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2))/x))))+6*c/a*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/8*(2*c*
d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a
*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d
/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*...
```

3.462.5 Fricas [A] (verification not implemented)

Time = 10.89 (sec) , antiderivative size = 1717, normalized size of antiderivative = 4.88

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^2(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/96*(3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*e^4 - 5*a^3*e^6)*sqrt
(c*d*e)*x*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sq
rt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt
(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 24*(5*a*c^2*d^3*e^3 + 3*a^2*c*d*e
^5)*sqrt(a*d*e)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)
*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(8*c^3*d^3*e^
3*x^3 - 24*a^2*c*d^2*e^4 + 2*(7*c^3*d^4*e^2 + 13*a*c^2*d^2*e^4)*x^2 + (3*c
^3*d^5*e + 68*a*c^2*d^3*e^3 + 33*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x))/(c*d*e^2*x), 1/48*(3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*
a^2*c*d^2*e^4 - 5*a^3*e^6)*sqrt(-c*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e
^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 12*(5*a*c^2*d^3*e^3 +
 3*a^2*c*d*e^5)*sqrt(a*d*e)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^
2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 2*(
8*c^3*d^3*e^3*x^3 - 24*a^2*c*d^2*e^4 + 2*(7*c^3*d^4*e^2 + 13*a*c^2*d^2*e^4
)*x^2 + (3*c^3*d^5*e + 68*a*c^2*d^3*e^3 + 33*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/(c*d*e^2*x), 1/96*(48*(5*a*c^2*d^3*e^3 + 3
*a^2*c*d*e^5)*sqrt(-a*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2...
```

3.462.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{5/2}}{x^2(d + ex)}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**2/(e*x+d),x)
```

output
```
Timed out
```

3.462.7 Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^2(d + ex)}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)x^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^2), x
)
```

3.462.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.34

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^2(d + ex)}\,dx = \frac{1}{24}\sqrt{cdex^2 + cd^2x + ae^2x + ade}\left(2\left(4\,c^2d^2ex + \frac{7\,c^4d^5e^2 + 13\,ac^3d}{c^2d^2e^2}\right.\right.$$
$$+ \frac{(5\,a^2cd^3e^2 + 3\,a^3de^4)\arctan\left(-\frac{\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}}{\sqrt{-ade}}\right)}{\sqrt{-ade}}$$
$$+ \frac{(c^3d^6 - 15\,ac^2d^4e^2 - 45\,a^2cd^2e^4 - 5\,a^3e^6)\log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + a\right.\right.\right.}{16\sqrt{cde}e}$$
$$- \frac{\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^2cd^3e^2 + \left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^3de^4 + 2\sqrt{}}{ade - \left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x, algorithm
="giac")
```

output
```
1/24*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*c^2*d^2*e*x + (7*c^
4*d^5*e^2 + 13*a*c^3*d^3*e^4)/(c^2*d^2*e^2))*x + (3*c^4*d^6*e + 68*a*c^3*d
^4*e^3 + 33*a^2*c^2*d^2*e^5)/(c^2*d^2*e^2)) + (5*a^2*c*d^3*e^2 + 3*a^3*d*e
^4)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/
sqrt(-a*d*e))/sqrt(-a*d*e) + 1/16*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d
^2*e^4 - 5*a^3*e^6)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*e) - ((sqrt(
c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^2*c*d^3*e^2 + (s
qrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^3*d*e^4 + 2*
sqrt(c*d*e)*a^3*d^2*e^3)/(a*d*e - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*
x + a*e^2*x + a*d*e))^2)
```

3.462.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^2(d + ex)}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^2\,\left(d + e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)), x)
```

3.463 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^3(d+ex)}\,dx$

3.463.1 Optimal result

Integrand size = 40, antiderivative size = 339

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^3(d+ex)}\,dx = -\frac{3(ae(3cd^2+ae^2)-cd(cd^2+3ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4x} - \frac{(ae-cdx)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{2x^2} + \frac{3\sqrt{c}\sqrt{d}(c^2d^4+10acd^2e^2+5a^2e^4)\,\text{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8\sqrt{e}} - \frac{3\sqrt{a}\sqrt{e}(5c^2d^4+10acd^2e^2+a^2e^4)\,\text{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8\sqrt{d}}$$

output
```
-1/2*(-c*d*x+a*e)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2+3/8*(5*a^2*e
^4+10*a*c*d^2*e^2+c^2*d^4)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(
1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*c^(1/2)*d^(1/2)/e^(1
/2)-3/8*(a^2*e^4+10*a*c*d^2*e^2+5*c^2*d^4)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d
^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*a^
(1/2)*e^(1/2)/d^(1/2)-3/4*(a*e*(a*e^2+3*c*d^2)-c*d*(3*a*e^2+c*d^2)*x)*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x
```

3.463.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.85

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^3(d+ex)}\,dx = \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{d}\sqrt{e}\sqrt{ae+cdx}\sqrt{d+ex}(-9acdex(d-ex)\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d
 + e*x]*(-9*a*c*d*e*x*(d - e*x) + c^2*d^2*x^2*(5*d + 2*e*x) - a^2*e^2*(2*d
 + 5*e*x)) - 3*Sqrt[a]*e*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*x^2*ArcTan
h[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])] + 3*Sqrt[c]
*d*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*x^2*ArcTanh[(Sqrt[e]*Sqrt[a*e +
c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]))/(4*Sqrt[d]*Sqrt[e]*x^2*Sqrt[(a*
e + c*d*x)*(d + e*x)])
```

3.463.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1215, 1230, 27, 1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{x^3(d+ex)}\,dx$$

$$\Big\downarrow \text{ 1215}$$

$$\int \frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{ 1230}$$

$$-\frac{3}{8}\int -\frac{2\left(ae\left(3cd^2+ae^2\right)+cd\left(cd^2+3ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^2}dx-\frac{(ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{4}\int \frac{\left(ae\left(3cd^2+ae^2\right)+cd\left(cd^2+3ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^2}dx-$$
$$\frac{(ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2x^2}$$

$\downarrow$ 1230

$$\frac{3}{4}\left(-\frac{1}{2}\int -\frac{ae\left(5c^2d^4+10ace^2d^2+a^2e^4\right)+cd\left(c^2d^4+10ace^2d^2+5a^2e^4\right)x}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(a\right.$$
$$\frac{(ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2x^2}$$

$\downarrow$ 25

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{ae\left(5c^2d^4+10ace^2d^2+a^2e^4\right)+cd\left(c^2d^4+10ace^2d^2+5a^2e^4\right)x}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{\left(ae\left(ae^2+3cd^2\right)-cdx\left(3ae^2+cd^2\right)\right)}{x}\right.$$
$$\frac{(ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2x^2}$$

$\downarrow$ 1269

$$\frac{3}{4}\left(\frac{1}{2}\left(cd\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+ae\left(a^2e^4+10acd^2e^2+5c^2d^4\right)\int \frac{}{x\sqrt{cd}}\right.\right.$$
$$\frac{(ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2x^2}$$

$\downarrow$ 1092

$$\frac{3}{4}\left(\frac{1}{2}\left(ae\left(a^2e^4+10acd^2e^2+5c^2d^4\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+2cd\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\int \frac{}{4cd}\right.\right.$$
$$\frac{(ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2x^2}$$

$\downarrow$ 219

$$\frac{3}{4}\left(\frac{1}{2}\left(ae\left(a^2e^4+10acd^2e^2+5c^2d^4\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{\sqrt{c}\sqrt{d}\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\text{arc}}{}\right.\right.$$
$$\frac{(ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2x^2}$$

$\downarrow$ 1154

3.463. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^3(d+ex)}\,dx$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\sqrt{c}\sqrt{d}(5a^2e^4+10acd^2e^2+c^2d^4)\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{e}} - 2ae(a^2e^4+10acd^2e^2+5c^2d^4\right.\right.$$

$$\frac{(ae-cdx)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow 219$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\sqrt{c}\sqrt{d}(5a^2e^4+10acd^2e^2+c^2d^4)\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{e}} - \frac{\sqrt{a}\sqrt{e}(a^2e^4+10acd^2e^2+5c\right.\right.$$

$$\frac{(ae-cdx)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{2x^2}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

output
```
-1/2*((a*e - c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/x^2 + (
3*(-(((a*e*(3*c*d^2 + a*e^2) - c*d*(c*d^2 + 3*a*e^2)*x)*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])/x) + ((Sqrt[c]*Sqrt[d]*(c^2*d^4 + 10*a*c*d^2*e^
2 + 5*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt
[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/Sqrt[e] - (Sqrt[a]*Sqrt
[e]*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e
^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])])/Sqrt[d])/2))/4
```

3.463.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.463. $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^3(d+ex)}\,dx$

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.463.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3892 vs. $2(295) = 590$.

Time = 0.81 (sec) , antiderivative size = 3893, normalized size of antiderivative = 11.48

method	result	size
default	Expression too large to display	3893

3.463. $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^3(d+ex)}\,dx$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/d*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)+3/4*(a*e^2+c*d
^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)+5/2*(a*e^2+c
*d^2)/a/d/e*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/2*(a*e^2+c*d^2)
*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/
c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*
d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+a*d*e*(1/3*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2
)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)
+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2
)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2))/x))))+6*c/a*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/8
*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/1
6*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^...
```

3.463.5 Fricas [A] (verification not implemented)

Time = 4.42 (sec) , antiderivative size = 1569, normalized size of antiderivative = 4.63

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^3(d + ex)} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x, algorithm
="fricas")
```

output
```
[1/16*(3*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*sqrt(c*d/e)*x^2*log(8*c^2*
d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e
 + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2
*d^3*e + a*c*d*e^3)*x) + 3*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*e
/d)*x^2*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*
x)*sqrt(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(2*c^2*d^2*e*x^3 -
2*a^2*d*e^2 + (5*c^2*d^3 + 9*a*c*d*e^2)*x^2 - (9*a*c*d^2*e + 5*a^2*e^3)*x)
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/x^2, -1/16*(6*(c^2*d^4 + 10*
a*c*d^2*e^2 + 5*a^2*e^4)*sqrt(-c*d/e)*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e
*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) - 3*(5*c^2*d^4 + 10*a*c*d^2*e
^2 + a^2*e^4)*sqrt(a*e/d)*x^2*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^
2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*
e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) -
 4*(2*c^2*d^2*e*x^3 - 2*a^2*d*e^2 + (5*c^2*d^3 + 9*a*c*d*e^2)*x^2 - (9*a*c
*d^2*e + 5*a^2*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/x^2, 1
/16*(6*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*sqrt(-a*e/d)*x^2*arctan(1/2*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(-a*e/d)/(a*c*d*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) + 3...
```

3.463.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^3(d + ex)} dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**3/(e*x+d),x)
```

output
```
Timed out
```

3.463.7 Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^3(d + ex)} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)x^3} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^3), x
)
```

3.463.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 746 vs. $2(295) = 590$.

Time = 0.46 (sec) , antiderivative size = 746, normalized size of antiderivative = 2.20

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^3(d + ex)} dx = \frac{1}{4}\left(2c^2d^2ex + \frac{5c^3d^4e + 9ac^2d^2e^3}{cde}\right)\sqrt{cdex^2 + cd^2x + ae^2x + ade}$$
$$+ \frac{3(5ac^2d^4e + 10a^2cd^2e^3 + a^3e^5)\arctan\left(-\frac{\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}}{\sqrt{-ade}}\right)}{4\sqrt{-ade}}$$
$$- \frac{3(c^3d^5 + 10ac^2d^3e^2 + 5a^2cde^4)\log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)\right|\right)}{8\sqrt{cde}}$$
$$- \frac{7\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^2c^2d^5e^2 + 6\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^3cd^3e^4}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x, algorithm
="giac")
```

output
```
1/4*(2*c^2*d^2*e*x + (5*c^3*d^4*e + 9*a*c^2*d^2*e^3)/(c*d*e))*sqrt(c*d*e*x
^2 + c*d^2*x + a*e^2*x + a*d*e) + 3/4*(5*a*c^2*d^4*e + 10*a^2*c*d^2*e^3 +
a^3*e^5)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d
*e))/sqrt(-a*d*e))/sqrt(-a*d*e) - 3/8*(c^3*d^5 + 10*a*c^2*d^3*e^2 + 5*a^2*
c*d*e^4)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*
e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/sqrt(c*d*e) - 1/4*(7*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^2*c^2*d^5*e^2 + 6*(sqrt(c
*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^3*c*d^3*e^4 + 3*(
sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*d*e^6 + 1
6*sqrt(c*d*e)*a^3*c*d^4*e^3 + 8*sqrt(c*d*e)*a^4*d^2*e^5 - 9*(sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a*c^2*d^4*e - 18*(sqrt(c
*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^2*c*d^2*e^3 - 5
*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*e^5 -
 24*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*
e))^2*a^2*c*d^3*e^2 - 16*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d
^2*x + a*e^2*x + a*d*e))^2*a^3*d*e^4)/(a*d*e - (sqrt(c*d*e)*x - sqrt(c*d*e
*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)^2
```

3.463.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^3(d+ex)}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^3\,\left(d + e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)), x)
```

3.464 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^4(d+ex)}\,dx$

3.464.1 Optimal result

Integrand size = 40, antiderivative size = 371

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^4(d+ex)}\,dx = -\frac{(5c^2d^4+12acd^2e^2-a^2e^4-2cde(7cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8dx} - \frac{(4ade+3(3cd^2+ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{12dx^3} + \frac{1}{2}c^{3/2}d^{3/2}\sqrt{e}(3cd^2+5ae^2)\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right) - \frac{(5c^3d^6+45ac^2d^4e^2+15a}{}$$

output
```
-1/12*(4*a*d*e+3*(a*e^2+3*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)/d/x^3-1/16*(-a^3*e^6+15*a^2*c*d^2*e^4+45*a*c^2*d^4*e^2+5*c^3*d^6)*arctan
h(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2))/d^(3/2)/a^(1/2)/e^(1/2)+1/2*c^(3/2)*d^(3/2)*(5*a*e^
2+3*c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*e^(1/2)-1/8*(5*c^2*d^4+12*a*c*d^2*e^
2-a^2*e^4-2*c*d*e*(a*e^2+7*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)/d/x
```

3.464.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.85

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^4(d+ex)}\,dx =$$
$$-\frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}\sqrt{d+ex}(3c^2d^3x^2(11d-8ex)+2acd^2ex(13d+34ex)+a^2e^2(8d^2\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

output
```
-1/24*(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e +
 c*d*x]*Sqrt[d + e*x]*(3*c^2*d^3*x^2*(11*d - 8*e*x) + 2*a*c*d^2*e*x*(13*d
+ 34*e*x) + a^2*e^2*(8*d^2 + 14*d*e*x + 3*e^2*x^2)) + 3*(5*c^3*d^6 + 45*a*
c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*x^3*ArcTanh[(Sqrt[d]*Sqrt[a*e +
c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])] - 24*Sqrt[a]*c^(3/2)*d^3*e*(3*c*d
^2 + 5*a*e^2)*x^3*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqr
t[d + e*x])]))/(Sqrt[a]*d^(3/2)*Sqrt[e]*x^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.464.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1215, 1229, 27, 1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{x^4(d+ex)}\,dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{1229}$$

$$-\frac{\int -\frac{ae\left(5c^2d^4+12ace^2d^2+2ce\left(7cd^2+ae^2\right)xd-a^2e^4\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{2x^2}dx}{4ade}-\frac{\left(3x\left(ae^2+3cd^2\right)+4ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12dx^3}$$

$\downarrow$ 27

$$\frac{\int \frac{\left(5c^2d^4+12ace^2d^2+2ce\left(7cd^2+ae^2\right)xd-a^2e^4\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^2}dx}{8d}-\frac{\left(3x\left(ae^2+3cd^2\right)+4ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12dx^3}$$

$\downarrow$ 1230

$$\frac{-\frac{1}{2}\int -\frac{5c^3d^6+45ac^2e^2d^4+8c^2e\left(3cd^2+5ae^2\right)xd^3+15a^2ce^4d^2-a^3e^6}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(-a^2e^4-2cdex\left(ae^2+7cd^2\right)+12acd^2e^2+5\right.}{x}}{8d}-\frac{\left(3x\left(ae^2+3cd^2\right)+4ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12dx^3}$$

$\downarrow$ 25

$$\frac{\frac{1}{2}\int \frac{5c^3d^6+45ac^2e^2d^4+8c^2e\left(3cd^2+5ae^2\right)xd^3+15a^2ce^4d^2-a^3e^6}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{\left(-a^2e^4-2cdex\left(ae^2+7cd^2\right)+12acd^2e^2+5c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}}{8d}-\frac{\left(3x\left(ae^2+3cd^2\right)+4ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12dx^3}$$

$\downarrow$ 1269

$$\frac{\frac{1}{2}\left(\left(-a^3e^6+15a^2cd^2e^4+45ac^2d^4e^2+5c^3d^6\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+8c^2d^3e\left(5ae^2+3cd^2\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae\right.}}\right.}{8d}-\frac{\left(3x\left(ae^2+3cd^2\right)+4ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12dx^3}$$

$\downarrow$ 1092

$$\frac{\frac{1}{2}\left(\left(-a^3e^6+15a^2cd^2e^4+45ac^2d^4e^2+5c^3d^6\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+16c^2d^3e\left(5ae^2+3cd^2\right)\int \frac{1}{4cde-\frac{\left(cd^2+2\right.}{cdex^2+\left(c\right.}}\right.}{8d}-\frac{\left(3x\left(ae^2+3cd^2\right)+4ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12dx^3}$$

$\downarrow$ 219

$$\frac{\frac{1}{2}\left(\left(-a^3e^6+15a^2cd^2e^4+45ac^2d^4e^2+5c^3d^6\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+8c^{3/2}d^{5/2}\sqrt{e}\left(5ae^2+3cd^2\right)\operatorname{arctanh}\left(\frac{}{2\sqrt{}}\right.\right.}{8d}-\frac{\left(3x\left(ae^2+3cd^2\right)+4ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12dx^3}$$

3.464. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^4(d+ex)}dx$

$\big\downarrow$ 1154

$$\frac{\frac{1}{2}\left(8c^{3/2}d^{5/2}\sqrt{e}\left(5ae^2+3cd^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)-2\left(-a^3e^6+15a^2cd^2e^4+45ac^2d^4e^2+5c^3\right.\right.}{}$$

$$\frac{\left(3x\left(ae^2+3cd^2\right)+4ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12dx^3}$$

$\big\downarrow$ 219

$$\frac{\frac{1}{2}\left(8c^{3/2}d^{5/2}\sqrt{e}\left(5ae^2+3cd^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)-\frac{\left(-a^3e^6+15a^2cd^2e^4+45ac^2d^4e^2+5c^3d^6\right)\text{arctanh}}{\sqrt{a}\sqrt{d}\sqrt{e}}\right.}{8d}$$

$$\frac{\left(3x\left(ae^2+3cd^2\right)+4ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12dx^3}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

output
```
-1/12*((4*a*d*e + 3*(3*c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2))/(d*x^3) + (-(((5*c^2*d^4 + 12*a*c*d^2*e^2 - a^2*e^4 - 2*c*d*
e*(7*c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x) + (
8*c^(3/2)*d^(5/2)*Sqrt[e]*(3*c*d^2 + 5*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c
*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])] - ((5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*ArcT
anh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])])/(Sqrt[a]*Sqrt[d]*Sqrt[e]))/2)/(8*d)
```

3.464.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

3.464. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^4(d+ex)}\,dx$

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.464.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 6849 vs. 2(327) = 654.

Time = 1.06 (sec) , antiderivative size = 6850, normalized size of antiderivative = 18.46

method	result	size
default	Expression too large to display	6850

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

3.464.5 Fricas [A] (verification not implemented)

Time = 5.78 (sec) , antiderivative size = 1741, normalized size of antiderivative = 4.69

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{5/2}}{x^4(d + ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x, algorithm
="fricas")
```

output
```
[1/96*(24*(3*a*c^2*d^5*e + 5*a^2*c*d^3*e^3)*sqrt(c*d*e)*x^3*log(8*c^2*d^2*
e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e +
 a*c*d*e^3)*x) - 3*(5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*
e^6)*sqrt(a*d*e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e
^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(24*a*c^2*
d^4*e^2*x^3 - 8*a^3*d^3*e^3 - (33*a*c^2*d^5*e + 68*a^2*c*d^3*e^3 + 3*a^3*d
*e^5)*x^2 - 2*(13*a^2*c*d^4*e^2 + 7*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^3), -1/96*(48*(3*a*c^2*d^5*e + 5*a^2*c*d
^3*e^3)*sqrt(-c*d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*
e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 3*(5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a
^2*c*d^2*e^4 - a^3*e^6)*sqrt(a*d*e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*
a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/
x^2) - 4*(24*a*c^2*d^4*e^2*x^3 - 8*a^3*d^3*e^3 - (33*a*c^2*d^5*e + 68*a^2*
c*d^3*e^3 + 3*a^3*d*e^5)*x^2 - 2*(13*a^2*c*d^4*e^2 + 7*a^3*d^2*e^4)*x)*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^3), 1/48*(3*(5*c^3*d^
6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-a*d*e)*x^3*arc...
```

3.464.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^4(d + ex)}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**4/(e*x+d),x)
```

output
```
Timed out
```

3.464.7 Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^4(d + ex)}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)x^4}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^4), x
)
```

3.464.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1195 vs. 2(327) = 654.

Time = 0.50 (sec) , antiderivative size = 1195, normalized size of antiderivative = 3.22

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^4(d + ex)}\,dx = \sqrt{cdex^2 + cd^2x + ae^2x + ade}\,c^2d^2e - \frac{(3\,c^3d^4e + 5\,ac^2d^2e^3)\log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)\right|\right)}{2\sqrt{cde}} + \frac{(5\,c^3d^6 + 45\,ac^2d^4e^2 + 15\,a^2cd^2e^4 - a^3e^6)\arctan\left(-\frac{\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}}{\sqrt{-ade}}\right)}{8\sqrt{-ade}d} - \frac{15\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^2c^3d^8e^2 + 39\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^3c^2d}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x, algorithm
="giac")
```

output
```
sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*c^2*d^2*e - 1/2*(3*c^3*d^4*e +
 5*a*c^2*d^2*e^3)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x -
sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/sqrt(c*d*e) + 1/8*(5*c^3*d^
6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*arctan(-(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)
*d) - 1/24*(15*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e
))*a^2*c^3*d^8*e^2 + 39*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*
x + a*d*e))*a^3*c^2*d^6*e^4 + 45*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x
 + a*e^2*x + a*d*e))*a^4*c*d^4*e^6 - 3*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c
*d^2*x + a*e^2*x + a*d*e))*a^5*d^2*e^8 + 48*sqrt(c*d*e)*a^3*c^2*d^7*e^3 +
112*sqrt(c*d*e)*a^4*c*d^5*e^5 - 40*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e))^3*a*c^3*d^7*e - 72*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
+ c*d^2*x + a*e^2*x + a*d*e))^3*a^2*c^2*d^5*e^3 - 24*(sqrt(c*d*e)*x - sqrt
(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*c*d^3*e^5 + 8*(sqrt(c*d*e)*
x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^4*d*e^7 - 144*sqrt(c*
d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^2*c
^2*d^6*e^2 - 240*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a
*e^2*x + a*d*e))^2*a^3*c*d^4*e^4 + 33*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*
d^2*x + a*e^2*x + a*d*e))^5*c^3*d^6 + 153*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
+ c*d^2*x + a*e^2*x + a*d*e))^5*a*c^2*d^4*e^2 + 99*(sqrt(c*d*e)*x - sqr...
```

3.464.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^4(d+ex)}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^4\,\left(d + e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)), x)
```

3.465 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^5(d+ex)}\,dx$

3.465.1 Optimal result

Integrand size = 40, antiderivative size = 404

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^5(d+ex)}\,dx =$$
$$-\frac{(2ade(5cd^2-ae^2)(cd^2+3ae^2)+(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e^4-3a^3e^6)x)\sqrt{ade+(cd^2+ae^2)x+cde}}{64ad^2ex^2}$$
$$-\frac{(6ade+(11cd^2+3ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{24dx^4}$$
$$+c^{5/2}d^{5/2}e^{3/2}\text{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)+\frac{(5c^4d^8-60ac^3d^6e^2-90a^2c^2d^4e^4+20a}{}$$

output
```
-1/24*(6*a*d*e+(3*a*e^2+11*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2)/d/x^4+1/128*(-3*a^4*e^8+20*a^3*c*d^2*e^6-90*a^2*c^2*d^4*e^4-60*a*c^3*d^
6*e^2+5*c^4*d^8)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(
1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/d^(5/2)/e^(3/2)+c^(5
/2)*d^(5/2)*e^(3/2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^
(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))-1/64*(2*a*d*e*(-a*e^2+5*c*d
^2)*(3*a*e^2+c*d^2)+(-3*a^3*e^6+11*a^2*c*d^2*e^4+83*a*c^2*d^4*e^2+5*c^3*d^
6)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/d^2/e/x^2
```

3.465.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.89

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^5(d+ex)}\,dx = \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(-\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}\sqrt{d+ex}(15c^3d^6x^3+a\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d
*x]*Sqrt[d + e*x]*(15*c^3*d^6*x^3 + a*c^2*d^4*e*x^2*(118*d + 337*e*x) + a^
2*c*d^2*e^2*x*(136*d^2 + 244*d*e*x + 57*e^2*x^2) + 3*a^3*e^3*(16*d^3 + 24*
d^2*e*x + 2*d*e^2*x^2 - 3*e^3*x^3))) + 3*(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 9
0*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*x^4*ArcTanh[(Sqrt[d]*Sqr
t[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])] + 384*a^(3/2)*c^(5/2)*d^5
*e^3*x^4*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x
])]))/(192*a^(3/2)*d^(5/2)*e^(3/2)*x^4*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.465.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1215, 1229, 27, 1229, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{x^5(d+ex)}\,dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{1229}$$

$$-\frac{\int -\frac{ae\left(16c^2exd^3+\left(5cd^2-ae^2\right)\left(cd^2+3ae^2\right)\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{2x^3}dx}{8ade}-\frac{\left(x\left(3ae^2+11cd^2\right)+6ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24dx^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(16c^2exd^3+(5cd^2-ae^2)(cd^2+3ae^2))\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{x^3}dx}{16d}-\frac{(x(3ae^2+11cd^2)+6ade)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{24dx^4}$$

$$\Big\downarrow \text{1229}$$

$$\frac{-\frac{\int \frac{5c^4d^8-60ac^3e^2d^6-128ac^3e^3xd^5-90a^2c^2e^4d^4+20a^3ce^6d^2-3a^4e^8}{2x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4ade}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(x\left(-3a^3e^6+11a^2cd^2e^4+83ac^2d^4e^2+5c^3d^6\right)+\cdots}{4adex^2}}{16d}-\frac{(x(3ae^2+11cd^2)+6ade)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{24dx^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{5c^4d^8-60ac^3e^2d^6-128ac^3e^3xd^5-90a^2c^2e^4d^4+20a^3ce^6d^2-3a^4e^8}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8ade}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(x\left(-3a^3e^6+11a^2cd^2e^4+83ac^2d^4e^2+5c^3d^6\right)+\cdots}{4adex^2}}{16d}-\frac{(x(3ae^2+11cd^2)+6ade)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{24dx^4}$$

$$\Big\downarrow \text{1269}$$

$$\frac{-\frac{(-3a^4e^8+20a^3cd^2e^6-90a^2c^2d^4e^4-60ac^3d^6e^2+5c^4d^8)\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-128ac^3d^5e^3\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8ade}-\frac{\sqrt{x(ae^2\cdots}}{\cdots}}{16d}-\frac{(x(3ae^2+11cd^2)+6ade)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{24dx^4}$$

$$\Big\downarrow \text{1092}$$

$$\frac{-\frac{(-3a^4e^8+20a^3cd^2e^6-90a^2c^2d^4e^4-60ac^3d^6e^2+5c^4d^8)\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-256ac^3d^5e^3\int \frac{1}{4cde-\frac{(cd^2+2cexd+ae^2)^2}{cdex^2+(cd^2+ae^2)x+ade}}d\frac{cd^2+\cdots}{\sqrt{cdex^2+(\cdots}}}{8ade}\cdots}{16d}-\frac{(x(3ae^2+11cd^2)+6ade)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{24dx^4}$$

$$\Big\downarrow \text{219}$$

3.465. $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^5(d+ex)}\,dx$

$$-\frac{\frac{\left(-3a^4e^8+20a^3cd^2e^6-90a^2c^2d^4e^4-60ac^3d^6e^2+5c^4d^8\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-128ac^{5/2}d^{9/2}e^{5/2}\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+a}}\right.}{8ade}}{16d}$$

$$\frac{\left(x\left(3ae^2+11cd^2\right)+6ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24dx^4}$$

$\downarrow$ 1154

$$-\frac{-2\left(-3a^4e^8+20a^3cd^2e^6-90a^2c^2d^4e^4-60ac^3d^6e^2+5c^4d^8\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}-128ac^{5/2}d^{9/2}e^{5/2}\text{arcta}}{8ade}$$

$$\frac{\left(x\left(3ae^2+11cd^2\right)+6ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24dx^4}$$

$\downarrow$ 219

$$-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(x\left(-3a^3e^6+11a^2cd^2e^4+83ac^2d^4e^2+5c^3d^6\right)+2ade\left(5cd^2-ae^2\right)\left(3ae^2+cd^2\right)\right)}{4adex^2}-\frac{\left(-3a^4e^8+20a^3cd^2e^6-90a^2c^2d^4e^4-\right.}{16d}$$

$$\frac{\left(x\left(3ae^2+11cd^2\right)+6ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24dx^4}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

output
```
-1/24*((6*a*d*e + (11*c*d^2 + 3*a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2))/(d*x^4) + (-1/4*((2*a*d*e*(5*c*d^2 - a*e^2)*(c*d^2 + 3*a*e^
2) + (5*c^3*d^6 + 83*a*c^2*d^4*e^2 + 11*a^2*c*d^2*e^4 - 3*a^3*e^6)*x)*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x^2) - (-128*a*c^(5/2)*d^(
9/2)*e^(5/2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e
]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])] - ((5*c^4*d^8 - 60*a*c^3*d
^6*e^2 - 90*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*ArcTanh[(2*a*d
*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])])/(Sqrt[a]*Sqrt[d]*Sqrt[e]))/(8*a*d*e))/(16*d)
```

3.465. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^5(d+ex)}\,dx$

3.465.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

3.465. $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^5(d+ex)}\,dx$

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.465.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 11684 vs. $2(362) = 724$.

Time = 1.20 (sec) , antiderivative size = 11685, normalized size of antiderivative = 28.92

method	result	size
default	Expression too large to display	11685

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

3.465.5 Fricas [A] (verification not implemented)

Time = 16.14 (sec) , antiderivative size = 1917, normalized size of antiderivative = 4.75

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^5(d + ex)} \, dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x, algorithm
="fricas")
```

output
```
[1/768*(384*sqrt(c*d*e)*a^2*c^2*d^5*e^3*x^4*log(8*c^2*d^2*e^2*x^2 + c^2*d^
4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) -
 3*(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 -
 3*a^4*e^8)*sqrt(a*d*e)*x^4*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2
+ a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e +
(c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(48
*a^4*d^4*e^4 + (15*a*c^3*d^7*e + 337*a^2*c^2*d^5*e^3 + 57*a^3*c*d^3*e^5 -
9*a^4*d*e^7)*x^3 + 2*(59*a^2*c^2*d^6*e^2 + 122*a^3*c*d^4*e^4 + 3*a^4*d^2*e
^6)*x^2 + 8*(17*a^3*c*d^5*e^3 + 9*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^4), -1/768*(768*sqrt(-c*d*e)*a^2*c^2*d
^5*e^3*x^4*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e
*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3
*e + a*c*d*e^3)*x)) + 3*(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^2*c^2*d^4*e^4
 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*sqrt(a*d*e)*x^4*log((8*a^2*d^2*e^2 + (c^2
*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d
*e^3)*x)/x^2) + 4*(48*a^4*d^4*e^4 + (15*a*c^3*d^7*e + 337*a^2*c^2*d^5*e^3
+ 57*a^3*c*d^3*e^5 - 9*a^4*d*e^7)*x^3 + 2*(59*a^2*c^2*d^6*e^2 + 122*a^3*c*
d^4*e^4 + 3*a^4*d^2*e^6)*x^2 + 8*(17*a^3*c*d^5*e^3 + 9*a^4*d^3*e^5)*x)*...
```

3.465.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^5(d + ex)}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**5/(e*x+d),x)
```

output
```
Timed out
```

3.465.7 Maxima [F]

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{x^5(d+ex)}\,dx=\int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}{(ex+d)x^5}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^5), x
)
```

3.465.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1750 vs. 2(362) = 724.

Time = 0.68 (sec) , antiderivative size = 1750, normalized size of antiderivative = 4.33

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{x^5(d+ex)}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x, algorithm
="giac")
```

output
```
-c^3*d^3*e^2*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(
c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/sqrt(c*d*e) - 1/64*(5*c^4*d^8 -
60*a*c^3*d^6*e^2 - 90*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*arct
an(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*
d*e))/(sqrt(-a*d*e)*a*d^2*e) + 1/192*(15*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
 c*d^2*x + a*e^2*x + a*d*e))*a^3*c^4*d^11*e^3 - 180*(sqrt(c*d*e)*x - sqrt(
c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*c^3*d^9*e^5 + 114*(sqrt(c*d*e)
*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^5*c^2*d^7*e^7 + 60*(sq
rt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^6*c*d^5*e^9 -
 9*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^7*d^3*e
^11 - 512*sqrt(c*d*e)*a^5*c^2*d^8*e^6 - 55*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
 + c*d^2*x + a*e^2*x + a*d*e))^3*a^2*c^4*d^10*e^2 + 660*(sqrt(c*d*e)*x - s
qrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*c^3*d^8*e^4 + 1374*(sqrt
(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^4*c^2*d^6*e^6
 + 548*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^5
*c*d^4*e^8 + 33*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*
e))^3*a^6*d^2*e^10 + 2048*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*
d^2*x + a*e^2*x + a*d*e))^2*a^4*c^2*d^7*e^5 + 768*sqrt(c*d*e)*(sqrt(c*d*e)
*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^5*c*d^5*e^7 + 73*(sq
rt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a*c^4*d^9*...
```

3.465.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^5(d + ex)} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{5/2}}{x^5\,(d + e\,x)} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)), x)
```

3.466 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^6(d+ex)}\,dx$

3.466.1 Optimal result

Integrand size = 40, antiderivative size = 289

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^6(d+ex)}\,dx=\frac{3(cd^2-ae^2)^3(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex}}{128a^2d^3e^2x^2}$$
$$-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)(2ade+(cd^2+ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{16x^4}$$
$$-\frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{5dx^5}-\frac{3(cd^2-ae^2)^5\,\mathrm{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{256a^{5/2}d^{7/2}e^{5/2}}$$

output
```
-1/16*(c/a/e-e/d^2)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(3/2)/x^4-1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/d/x^5-3/256*(-
a*e^2+c*d^2)^5*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/
2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/d^(7/2)/e^(5/2)+3/128*
(-a*e^2+c*d^2)^3*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)/a^2/d^3/e^2/x^2
```

3.466.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.94

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^6(d+ex)}\,dx=\frac{\left(-cd^2+ae^2\right)^5\left((ae+cdx)(d+ex)\right)^{3/2}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}(d+ex)^3\left(15a^4e^4-\frac{15d^4}{\cdots}\right.}{\cdots}\right.}{64\cdots}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

output
```
((-(c*d^2) + a*e^2)^5*((a*e + c*d*x)*(d + e*x))^(3/2)*(-((Sqrt[a]*Sqrt[d]*
Sqrt[e]*(d + e*x)^3*(15*a^4*e^4 - (15*d^4*(a*e + c*d*x)^4)/(d + e*x)^4 + (
70*a*d^3*e*(a*e + c*d*x)^3)/(d + e*x)^3 + (128*a^2*d^2*e^2*(a*e + c*d*x)^2
)/(d + e*x)^2 - (70*a^3*d*e^3*(a*e + c*d*x))/(d + e*x)))/((-(c*d^2) + a*e^
2)^5*x^5*(a*e + c*d*x))) + (15*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a
]*Sqrt[e]*Sqrt[d + e*x])])/((a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(640*a^
(5/2)*d^(7/2)*e^(5/2))
```

3.466.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1215, 1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{x^6(d+ex)}dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^6}dx$$

$$\Big\downarrow \text{1228}$$

$$\frac{\left(cd^2-ae^2\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{x^5}dx}{2d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5dx^5}$$

$$\Big\downarrow \text{1152}$$

$$\frac{\left(cd^2-ae^2\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\int\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^3}dx}{16ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8adex^4}\right)}{2d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5dx^5}$$

$$\Big\downarrow \text{1152}$$

$$\frac{\left(cd^2-ae^2\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\left(-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4adex^2}\right)}{16ade}-\frac{(x(ae^2+cd^2}{}\right)}{2d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5dx^5}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\left(cd^2-ae^2\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade}}{4adex^2}\right)}{16ade}\right)}{2d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5dx^5}$$

$$\Big\downarrow \text{219}$$

3.466. $\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^6(d+ex)}dx$

$$\frac{\left(cd^2-ae^2\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(cd^2-ae^2\right)^2\operatorname{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{3/2}d^{3/2}e^{3/2}}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4adex^2}\right)}{16ade}\right.}{2d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5dx^5}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

output
```
-1/5*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d*x^5) + ((c*d^2 - a*e
^2)*(-1/8*((2*a*d*e + (c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2))/(a*d*e*x^4) - (3*(c*d^2 - a*e^2)^2*(-1/4*((2*a*d*e + (c*d^2
+ a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x^2) + ((c
*d^2 - a*e^2)^2*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*a^(3/2)*d^(3/2)*e
^(3/2))))/(16*a*d*e)))/(2*d)
```

3.466.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

3.466.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 19538 vs. 2(259) = 518.

Time = 1.57 (sec) , antiderivative size = 19539, normalized size of antiderivative = 67.61

method	result	size
default	Expression too large to display	19539

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

3.466.5 Fricas [A] (verification not implemented)

Time = 13.96 (sec) , antiderivative size = 872, normalized size of antiderivative = 3.02

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^6(d+ex)}\,dx = \left[\frac{15\,(c^5d^{10}-5\,ac^4d^8e^2+10\,a^2c^3d^6e^4-10\,a^3c^2d^4e^6+5\,a^4cd^2e^8-c\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x, algorithm
="fricas")
```

output
```
[1/2560*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*
d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(a*d*e)*x^5*log((8*a^2*d^2*e^2 +
 (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e +
a^2*d*e^3)*x)/x^2) - 4*(128*a^5*d^5*e^5 - (15*a*c^4*d^9*e - 70*a^2*c^3*d^7
*e^3 - 128*a^3*c^2*d^5*e^5 + 70*a^4*c*d^3*e^7 - 15*a^5*d*e^9)*x^4 + 2*(5*a
^2*c^3*d^8*e^2 + 233*a^3*c^2*d^6*e^4 + 23*a^4*c*d^4*e^6 - 5*a^5*d^2*e^8)*x
^3 + 8*(31*a^3*c^2*d^7*e^3 + 64*a^4*c*d^5*e^5 + a^5*d^3*e^7)*x^2 + 16*(21*
a^4*c*d^6*e^4 + 11*a^5*d^4*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x))/(a^3*d^4*e^3*x^5), 1/1280*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c
^3*d^6*e^4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(-a*d*e)
*x^5*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e +
 a^2*d*e^3)*x)) - 2*(128*a^5*d^5*e^5 - (15*a*c^4*d^9*e - 70*a^2*c^3*d^7*e^
3 - 128*a^3*c^2*d^5*e^5 + 70*a^4*c*d^3*e^7 - 15*a^5*d*e^9)*x^4 + 2*(5*a^2*
c^3*d^8*e^2 + 233*a^3*c^2*d^6*e^4 + 23*a^4*c*d^4*e^6 - 5*a^5*d^2*e^8)*x^3
+ 8*(31*a^3*c^2*d^7*e^3 + 64*a^4*c*d^5*e^5 + a^5*d^3*e^7)*x^2 + 16*(21*a^4
*c*d^6*e^4 + 11*a^5*d^4*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
))/(a^3*d^4*e^3*x^5)]
```

3.466.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^6(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**6/(e*x+d),x)
```

output
```
Timed out
```

3.466.7 Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^6(d+ex)}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)x^6}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^6), x
)
```

3.466.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2449 vs. 2(259) = 518.

Time = 0.40 (sec) , antiderivative size = 2449, normalized size of antiderivative = 8.47

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^6(d+ex)}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x, algorithm
="giac")
```

output
```
3/128*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^
6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a^2*d^3*e^2) - 1/6
40*(15*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*c
^5*d^14*e^4 - 75*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d
*e))*a^5*c^4*d^12*e^6 + 150*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*
e^2*x + a*d*e))*a^6*c^3*d^10*e^8 + 1130*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
c*d^2*x + a*e^2*x + a*d*e))*a^7*c^2*d^8*e^10 + 75*(sqrt(c*d*e)*x - sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^8*c*d^6*e^12 - 15*(sqrt(c*d*e)*x -
 sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^9*d^4*e^14 + 256*sqrt(c*d*
e)*a^7*c^2*d^9*e^9 - 70*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*
x + a*d*e))^3*a^3*c^5*d^13*e^3 + 350*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d
^2*x + a*e^2*x + a*d*e))^3*a^4*c^4*d^11*e^5 + 5700*(sqrt(c*d*e)*x - sqrt(c
*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^5*c^3*d^9*e^7 + 7100*(sqrt(c*d*
e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^6*c^2*d^7*e^9 + 22
10*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^7*c*d
^5*e^11 + 70*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))
^3*a^8*d^3*e^13 + 2560*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e))^2*a^6*c^2*d^8*e^8 + 2560*sqrt(c*d*e)*(sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^7*c*d^6*e^10 + 128*...
```

3.466.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^6(d + ex)}\,dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{5/2}}{x^6\,(d + e\,x)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)), x)
```

3.467 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^7(d+ex)}\,dx$

3.467.1 Optimal result

Integrand size = 40, antiderivative size = 386

$$\begin{aligned}\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^7(d+ex)}\,dx &= \\ &-\frac{(cd^2-ae^2)^3(5cd^2+7ae^2)(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{512a^3d^4e^3x^2} \\ &+\frac{(cd^2-ae^2)(5cd^2+7ae^2)(2ade+(cd^2+ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{192a^2d^3e^2x^4} \\ &-\frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{6dx^6}-\frac{\left(\frac{5c}{ae}-\frac{7e}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{60x^5} \\ &+\frac{(cd^2-ae^2)^5(5cd^2+7ae^2)\operatorname{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{1024a^{7/2}d^{9/2}e^{7/2}}\end{aligned}$$

output
```
1/192*(-a*e^2+c*d^2)*(7*a*e^2+5*c*d^2)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^2/d^3/e^2/x^4-1/6*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(5/2)/d/x^6-1/60*(5*c/a/e-7*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(5/2)/x^5+1/1024*(-a*e^2+c*d^2)^5*(7*a*e^2+5*c*d^2)*arctanh(1/2*(2*a
*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2))/a^(7/2)/d^(9/2)/e^(7/2)-1/512*(-a*e^2+c*d^2)^3*(7*a*e^2+5*c*d
^2)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^3/
d^4/e^3/x^2
```

3.467.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.05

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^7(d+ex)}\,dx = \frac{(-cd^2+ae^2)^5\,((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(75c^5d^{10}x^5-5ac^4d^8ex^4(1\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

output
```
((-(c*d^2) + a*e^2)^5*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[a]*Sqrt[d]*Sq
rt[e]*(75*c^5*d^10*x^5 - 5*a*c^4*d^8*e*x^4*(10*d + 49*e*x) + 10*a^2*c^3*d^
6*e^2*x^3*(4*d^2 + 16*d*e*x + 15*e^2*x^2) + 6*a^3*c^2*d^4*e^3*x^2*(360*d^3
 + 564*d^2*e*x + 58*d*e^2*x^2 - 91*e^3*x^3) + a^4*c*d^2*e^4*x*(3200*d^4 +
4448*d^3*e*x + 216*d^2*e^2*x^2 - 272*d*e^3*x^3 + 415*e^4*x^4) + a^5*e^5*(1
280*d^5 + 1664*d^4*e*x + 48*d^3*e^2*x^2 - 56*d^2*e^3*x^3 + 70*d*e^4*x^4 -
105*e^5*x^5)))/((c*d^2 - a*e^2)^5*x^6*(a*e + c*d*x)*(d + e*x)) - (15*(5*c*
d^2 + 7*a*e^2)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d
 + e*x])])/((a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(7680*a^(7/2)*d^(9/2)*e
^(7/2))
```

3.467.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1215, 1237, 27, 1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{x^7(d+ex)}\,dx$$

$$\Big\downarrow \; 1215$$

$$\int \frac{(ae+cdx)\,(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{x^7}\,dx$$

$$\Big\downarrow \; 1237$$

$$-\frac{\int -\frac{ae(5cd^2-2cexd-7ae^2)(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}{2x^6}dx}{6ade} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{6dx^6}$$

$$\downarrow \textcolor{blue}{27}$$

$$\frac{\int \frac{\left(5cd^2-2cexd-7ae^2\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{x^6}dx}{12d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6dx^6}$$

$$\downarrow \textcolor{blue}{1228}$$

$$\frac{-\frac{\left(\frac{5c^2d^4}{a}-7ae^4+2cd^2e^2\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{x^5}dx}{2de}-\frac{\left(\frac{5cd}{ae}-\frac{7e}{d}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5x^5}}{12d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6dx^6}$$

$$\downarrow \textcolor{blue}{1152}$$

$$\frac{-\frac{\left(\frac{5c^2d^4}{a}-7ae^4+2cd^2e^2\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^3}dx}{16ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8adex^4}\right)}{2de}-\frac{\left(\frac{5cd}{ae}-\frac{7e}{d}\right)\left(x\left(a\cdots\right.\right.}{\cdots}}{12d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6dx^6}$$

$$\downarrow \textcolor{blue}{1152}$$

$$\frac{-\frac{\left(\frac{5c^2d^4}{a}-7ae^4+2cd^2e^2\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\left(-\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4adex^2}\right)}{16ade}-\frac{\left(x\left(ae\cdots\right.\right.}{\cdots}\cdots\right)}{2de}\cdots}{12d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6dx^6}$$

$$\downarrow \textcolor{blue}{1154}$$

3.467. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^7(d+ex)}\,dx$

$$-\frac{\left(\frac{5c^2d^4}{a}-7ae^4+2cd^2e^2\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+\right.}}{4adex^2}\right.}{16ade}\right.}{2de}}{12d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6dx^6}$$

$$\Big\downarrow \; 219$$

$$-\frac{\left(\frac{5c^2d^4}{a}-7ae^4+2cd^2e^2\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{3/2}d^{3/2}e^{3/2}}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+}}{4adex^2}\right.}{16ade}\right.}{2de}}{12d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6dx^6}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

output
```
-1/6*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d*x^6) + (-1/5*(((5*c*
d)/(a*e) - (7*e)/d)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/x^5 - (
((5*c^2*d^4)/a + 2*c*d^2*e^2 - 7*a*e^4)*(-1/8*((2*a*d*e + (c*d^2 + a*e^2)*
x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(a*d*e*x^4) - (3*(c*d^2
- a*e^2)^2*(-1/4*((2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(a*d*e*x^2) + ((c*d^2 - a*e^2)^2*ArcTanh[(2*a*d*e + (c*
d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])])/(8*a^(3/2)*d^(3/2)*e^(3/2))))/(16*a*d*e)))/(2*d*e))/(12*d)
```

3.467. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^7(d+ex)}\,dx$

3.467.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

3.467. $\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{x^7(d+ex)}\,dx$

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.467.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 32290 vs. $2(352) = 704$.

Time = 1.97 (sec) , antiderivative size = 32291, normalized size of antiderivative = 83.66

method	result	size
default	Expression too large to display	32291

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

3.467.5 Fricas [A] (verification not implemented)

Time = 41.44 (sec) , antiderivative size = 1072, normalized size of antiderivative = 2.78

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^7(d + ex)}\,dx = \left[-\frac{15\,(5\,c^6d^{12} - 18\,ac^5d^{10}e^2 + 15\,a^2c^4d^8e^4 + 20\,a^3c^3d^6e^6 - 45\,a^4c^2\cdots}{\cdots}\right.$$

$$-\frac{15\,(5\,c^6d^{12} - 18\,ac^5d^{10}e^2 + 15\,a^2c^4d^8e^4 + 20\,a^3c^3d^6e^6 - 45\,a^4c^2d^4e^8 + 30\,a^5cd^2e^{10} - 7\,a^6e^{12})\sqrt{-ade}\,x^6\,\mathrm{arct}\cdots}{\cdots}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/30720*(15*(5*c^6*d^12 - 18*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 + 20*a^
3*c^3*d^6*e^6 - 45*a^4*c^2*d^4*e^8 + 30*a^5*c*d^2*e^10 - 7*a^6*e^12)*sqrt(
a*d*e)*x^6*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 -
4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x
)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(1280*a^6*d^6*e^6 +
(75*a*c^5*d^11*e - 245*a^2*c^4*d^9*e^3 + 150*a^3*c^3*d^7*e^5 - 546*a^4*c^2
*d^5*e^7 + 415*a^5*c*d^3*e^9 - 105*a^6*d*e^11)*x^5 - 2*(25*a^2*c^4*d^10*e^
2 - 80*a^3*c^3*d^8*e^4 - 174*a^4*c^2*d^6*e^6 + 136*a^5*c*d^4*e^8 - 35*a^6*
d^2*e^10)*x^4 + 8*(5*a^3*c^3*d^9*e^3 + 423*a^4*c^2*d^7*e^5 + 27*a^5*c*d^5*
e^7 - 7*a^6*d^3*e^9)*x^3 + 16*(135*a^4*c^2*d^8*e^4 + 278*a^5*c*d^6*e^6 + 3
*a^6*d^4*e^8)*x^2 + 128*(25*a^5*c*d^7*e^5 + 13*a^6*d^5*e^7)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^4*d^5*e^4*x^6), -1/15360*(15*(5*c^6*d
^12 - 18*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 + 20*a^3*c^3*d^6*e^6 - 45*a^4
*c^2*d^4*e^8 + 30*a^5*c*d^2*e^10 - 7*a^6*e^12)*sqrt(-a*d*e)*x^6*arctan(1/2
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x))
 + 2*(1280*a^6*d^6*e^6 + (75*a*c^5*d^11*e - 245*a^2*c^4*d^9*e^3 + 150*a^3*
c^3*d^7*e^5 - 546*a^4*c^2*d^5*e^7 + 415*a^5*c*d^3*e^9 - 105*a^6*d*e^11)*x^
5 - 2*(25*a^2*c^4*d^10*e^2 - 80*a^3*c^3*d^8*e^4 - 174*a^4*c^2*d^6*e^6 + 13
6*a^5*c*d^4*e^8 - 35*a^6*d^2*e^10)*x^4 + 8*(5*a^3*c^3*d^9*e^3 + 423*a^4...
```

3.467.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^7(d+ex)}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**7/(e*x+d),x)
```

output
```
Timed out
```

3.467.7 Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^7(d + ex)}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)x^7}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^7), x
)
```

3.467.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3388 vs. 2(352) = 704.

Time = 0.51 (sec) , antiderivative size = 3388, normalized size of antiderivative = 8.78

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^7(d + ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x, algorithm
="giac")
```

output
```
-1/512*(5*c^6*d^12 - 18*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 + 20*a^3*c^3*d
^6*e^6 - 45*a^4*c^2*d^4*e^8 + 30*a^5*c*d^2*e^10 - 7*a^6*e^12)*arctan(-(sqr
t(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(s
qrt(-a*d*e)*a^3*d^4*e^3) + 1/7680*(75*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*
d^2*x + a*e^2*x + a*d*e))*a^5*c^6*d^17*e^5 - 270*(sqrt(c*d*e)*x - sqrt(c*d
*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^6*c^5*d^15*e^7 + 225*(sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^7*c^4*d^13*e^9 + 15660*(
sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^8*c^3*d^11*
e^11 + 14685*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))
*a^9*c^2*d^9*e^13 + 450*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*
x + a*d*e))*a^10*c*d^7*e^15 - 105*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*
x + a*e^2*x + a*d*e))*a^11*d^5*e^17 + 3072*sqrt(c*d*e)*a^9*c^2*d^10*e^12 -
 425*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^4*c
^6*d^16*e^4 + 1530*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a
*d*e))^3*a^5*c^5*d^14*e^6 + 75525*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*
x + a*e^2*x + a*d*e))^3*a^6*c^4*d^12*e^8 + 203100*(sqrt(c*d*e)*x - sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^7*c^3*d^10*e^10 + 142065*(sqrt(c
*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^8*c^2*d^8*e^12
+ 28170*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^
9*c*d^6*e^14 + 595*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x ...
```

3.467.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^7(d + ex)} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{5/2}}{x^7\,(d + e\,x)} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)), x)
```

3.468 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^8(d+ex)}\,dx$

3.468.1 Optimal result

Integrand size = 40, antiderivative size = 500

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^8(d+ex)}\,dx = \frac{(cd^2-ae^2)^3(5c^2d^4+10acd^2e^2+9a^2e^4)(2ade+(cd^2+ae^2)x)\sqrt{\ }}{1024a^4d^5e^4x^2}$$
$$-\frac{(cd^2-ae^2)(5c^2d^4+10acd^2e^2+9a^2e^4)(2ade+(cd^2+ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{384a^3d^4e^3x^4}$$
$$-\frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{7dx^7}-\frac{\left(\frac{5c}{ae}-\frac{9e}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{84x^6}$$
$$+\frac{(35c^2d^4+20acd^2e^2-63a^2e^4)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{840a^2d^3e^2x^5}$$
$$-\frac{(cd^2-ae^2)^5(5c^2d^4+10acd^2e^2+9a^2e^4)\operatorname{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2048a^{9/2}d^{11/2}e^{9/2}}$$

output
```
-1/384*(-a*e^2+c*d^2)*(9*a^2*e^4+10*a*c*d^2*e^2+5*c^2*d^4)*(2*a*d*e+(a*e^2
+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^3/d^4/e^3/x^4-1/7*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/d/x^7-1/84*(5*c/a/e-9*e/d^2)*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6+1/840*(-63*a^2*e^4+20*a*c*d^2*e^2+35*c
^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/a^2/d^3/e^2/x^5-1/2048*(-a
*e^2+c*d^2)^5*(9*a^2*e^4+10*a*c*d^2*e^2+5*c^2*d^4)*arctanh(1/2*(2*a*d*e+(a
*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2))/a^(9/2)/d^(11/2)/e^(9/2)+1/1024*(-a*e^2+c*d^2)^3*(9*a^2*e^4+10*a*c*d
^2*e^2+5*c^2*d^4)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)/a^4/d^5/e^4/x^2
```

3.468.2 Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 497, normalized size of antiderivative = 0.99

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^8(d + ex)}\,dx = \frac{(-cd^2 + ae^2)^5\,((ae + cdx)(d + ex))^{3/2}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(-525c^6d^{12}x^6 + 350ac^5d^1}{}\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

output
```
((-(c*d^2) + a*e^2)^5*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[a]*Sqrt[d]*Sq
rt[e]*(-525*c^6*d^12*x^6 + 350*a*c^5*d^10*e*x^5*(d + 4*e*x) - 35*a^2*c^4*d
^8*e^2*x^4*(8*d^2 + 26*d*e*x + 15*e^2*x^2) + 60*a^3*c^3*d^6*e^3*x^3*(4*d^3
 + 12*d^2*e*x + 5*d*e^2*x^2 - 10*e^3*x^3) + a^4*c^2*d^4*e^4*x^2*(23680*d^4
 + 33520*d^3*e*x + 1824*d^2*e^2*x^2 - 2332*d*e^3*x^3 + 3689*e^4*x^4) + 2*a
^5*c*d^2*e^5*x*(18560*d^5 + 24320*d^4*e*x + 744*d^3*e^2*x^2 - 872*d^2*e^3*
x^3 + 1099*d*e^4*x^4 - 1680*e^5*x^5) + 3*a^6*e^6*(5120*d^6 + 6400*d^5*e*x
+ 128*d^4*e^2*x^2 - 144*d^3*e^3*x^3 + 168*d^2*e^4*x^4 - 210*d*e^5*x^5 + 31
5*e^6*x^6)))/((c*d^2 - a*e^2)^5*x^7*(a*e + c*d*x)*(d + e*x)) + (105*(5*c^2
*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sq
rt[a]*Sqrt[e]*Sqrt[d + e*x])])/((a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(10
7520*a^(9/2)*d^(11/2)*e^(9/2))
```

3.468.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 494, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1215, 1237, 27, 1237, 27, 1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{x^8(d+ex)}dx$$

$$\Big\downarrow \; 1215$$

$$\int \frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^8}dx$$

$$\Big\downarrow \; 1237$$

$$-\frac{\int -\frac{ae(5cd^2-4cexd-9ae^2)(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}{2x^7}dx}{7ade}-\frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{7dx^7}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(5cd^2-4cexd-9ae^2)(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}{x^7}dx}{14d}-\frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{7dx^7}$$

$\Big\downarrow$ 1237

$$\frac{-\frac{\int \frac{(35c^2d^4+20ace^2d^2+2ce(5cd^2-9ae^2)xd-63a^2e^4)(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}{2x^6}dx}{6ade}-\frac{\left(\frac{5cd}{ae}-\frac{9e}{d}\right)(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{6x^6}}{14d}-\frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{7dx^7}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\int \frac{(35c^2d^4+20ace^2d^2+2ce(5cd^2-9ae^2)xd-63a^2e^4)(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}{x^6}dx}{12ade}-\frac{\left(\frac{5cd}{ae}-\frac{9e}{d}\right)(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{6x^6}}{14d}-\frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{7dx^7}$$

$\Big\downarrow$ 1228

$$\frac{-\frac{-\frac{7(cd^2-ae^2)(9a^2e^4+10acd^2e^2+5c^2d^4)\int \frac{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}{x^5}dx}{2ade}-\frac{\left(\frac{35c^2d^4}{a}-63ae^4+20cd^2e^2\right)(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{5dex^5}}{12ade}-\frac{\left(\frac{5cd}{ae}-\frac{9e}{d}\right)}{}}{14d}$$

$$-\frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{7dx^7}$$

$\Big\downarrow$ 1152

$$\frac{-\frac{-\frac{7(cd^2-ae^2)(9a^2e^4+10acd^2e^2+5c^2d^4)\left(-\frac{3(cd^2-ae^2)^2\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{x^3}dx}{16ade}-\frac{(x(ae^2+cd^2)+2ade)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{8adex^4}\right)}{2ade}-\frac{\left(35c\right.}{}}{12ade}}{14d}$$

$$-\frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{7dx^7}$$

$\Big\downarrow$ 1152

3.468. $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^8(d+ex)}dx$

$$-\frac{-\frac{7\left(cd^2-ae^2\right)\left(9a^2e^4+10acd^2e^2+5c^2d^4\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\left(-\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cd}}{4adex^2}\right.}{16ade}\right.}{2ade}}{12ade}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7dx^7}$$

$\big\downarrow$ 1154

$$-\frac{-\frac{7\left(cd^2-ae^2\right)\left(9a^2e^4+10acd^2e^2+5c^2d^4\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4ade}-\frac{\left(x\left(ae^2+cd^2\right)+2\right.}{}\right.}{16ade}\right.}{2ade}}{12ad}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7dx^7}$$

$\big\downarrow$ 219

$$-\frac{-\frac{7\left(cd^2-ae^2\right)\left(9a^2e^4+10acd^2e^2+5c^2d^4\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{3/2}d^{3/2}e^{3/2}}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae\right.}}{4adex^2}\right.}{16ade}\right.}{2ade}}{12ade}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7dx^7}$$

3.468. $\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{x^8(d+ex)}\,dx$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

output
```
-1/7*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d*x^7) + (-1/6*(((5*c*
d)/(a*e) - (9*e)/d)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/x^6 - (
-1/5*(((35*c^2*d^4)/a + 20*c*d^2*e^2 - 63*a*e^4)*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(5/2))/(d*e*x^5) - (7*(c*d^2 - a*e^2)*(5*c^2*d^4 + 10*a*c*d
^2*e^2 + 9*a^2*e^4)*(-1/8*((2*a*d*e + (c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2))/(a*d*e*x^4) - (3*(c*d^2 - a*e^2)^2*(-1/4*((2
*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
a*d*e*x^2) + ((c*d^2 - a*e^2)^2*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*S
qrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*a
^(3/2)*d^(3/2)*e^(3/2))))/(16*a*d*e)))/(2*a*d*e))/(12*a*d*e))/(14*d)
```

3.468.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.468. $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^8(d+ex)}\,dx$

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.468.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 45105 vs. $2(462) = 924$.

Time = 3.19 (sec) , antiderivative size = 45106, normalized size of antiderivative = 90.21

method	result	size
default	Expression too large to display	45106

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

3.468.5 Fricas [A] (verification not implemented)

Time = 78.30 (sec) , antiderivative size = 1300, normalized size of antiderivative = 2.60

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^8(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/430080*(105*(5*c^7*d^14 - 15*a*c^6*d^12*e^2 + 9*a^2*c^5*d^10*e^4 + 5*a
^3*c^4*d^8*e^6 + 15*a^4*c^3*d^6*e^8 - 45*a^5*c^2*d^4*e^10 + 35*a^6*c*d^2*e
^12 - 9*a^7*e^14)*sqrt(a*d*e)*x^7*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^
2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) +
 4*(15360*a^7*d^7*e^7 - (525*a*c^6*d^13*e - 1400*a^2*c^5*d^11*e^3 + 525*a^
3*c^4*d^9*e^5 + 600*a^4*c^3*d^7*e^7 - 3689*a^5*c^2*d^5*e^9 + 3360*a^6*c*d^
3*e^11 - 945*a^7*d*e^13)*x^6 + 2*(175*a^2*c^5*d^12*e^2 - 455*a^3*c^4*d^10*
e^4 + 150*a^4*c^3*d^8*e^6 - 1166*a^5*c^2*d^6*e^8 + 1099*a^6*c*d^4*e^10 - 3
15*a^7*d^2*e^12)*x^5 - 8*(35*a^3*c^4*d^11*e^3 - 90*a^4*c^3*d^9*e^5 - 228*a
^5*c^2*d^7*e^7 + 218*a^6*c*d^5*e^9 - 63*a^7*d^3*e^11)*x^4 + 16*(15*a^4*c^3
*d^10*e^4 + 2095*a^5*c^2*d^8*e^6 + 93*a^6*c*d^6*e^8 - 27*a^7*d^4*e^10)*x^3
 + 128*(185*a^5*c^2*d^9*e^5 + 380*a^6*c*d^7*e^7 + 3*a^7*d^5*e^9)*x^2 + 128
0*(29*a^6*c*d^8*e^6 + 15*a^7*d^6*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x))/(a^5*d^6*e^5*x^7), 1/215040*(105*(5*c^7*d^14 - 15*a*c^6*d^12*e
^2 + 9*a^2*c^5*d^10*e^4 + 5*a^3*c^4*d^8*e^6 + 15*a^4*c^3*d^6*e^8 - 45*a^5*
c^2*d^4*e^10 + 35*a^6*c*d^2*e^12 - 9*a^7*e^14)*sqrt(-a*d*e)*x^7*arctan(1/2
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x))
 - 2*(15360*a^7*d^7*e^7 - (525*a*c^6*d^13*e - 1400*a^2*c^5*d^11*e^3 + 5...
```

3.468.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^8(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**8/(e*x+d),x)
```

output
```
Timed out
```

3.468.7 Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^8(d+ex)}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)x^8}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^8), x
)
```

3.468.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4452 vs. 2(462) = 924.

Time = 0.78 (sec) , antiderivative size = 4452, normalized size of antiderivative = 8.90

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^8(d+ex)}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x, algorithm
="giac")
```

output
```
1/1024*(5*c^7*d^14 - 15*a*c^6*d^12*e^2 + 9*a^2*c^5*d^10*e^4 + 5*a^3*c^4*d^
8*e^6 + 15*a^4*c^3*d^6*e^8 - 45*a^5*c^2*d^4*e^10 + 35*a^6*c*d^2*e^12 - 9*a
^7*e^14)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d
*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a^4*d^5*e^4) - 1/107520*(525*(sqrt(c*d*e)
*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^6*c^7*d^20*e^6 - 1575*
(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^7*c^6*d^18
*e^8 + 945*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a
^8*c^5*d^16*e^10 + 215565*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^
2*x + a*d*e))*a^9*c^4*d^14*e^12 + 431655*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
 c*d^2*x + a*e^2*x + a*d*e))*a^10*c^3*d^12*e^14 + 210315*(sqrt(c*d*e)*x -
sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^11*c^2*d^10*e^16 + 3675*(sq
rt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^12*c*d^8*e^18
 - 945*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^13*
d^6*e^20 + 30720*sqrt(c*d*e)*a^10*c^3*d^13*e^13 + 43008*sqrt(c*d*e)*a^11*c
^2*d^11*e^15 - 3500*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
a*d*e))^3*a^5*c^7*d^19*e^5 + 10500*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e))^3*a^6*c^6*d^17*e^7 + 1068900*(sqrt(c*d*e)*x - sqrt(
c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^7*c^5*d^15*e^9 + 4655700*(sqrt
(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^8*c^4*d^13*e^
11 + 6225660*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*...
```

3.468.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^8(d + ex)}\,dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{5/2}}{x^8\,(d + e\,x)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)), x)
```

3.469 $\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{x^9(d+ex)}\,dx$

3.469.1 Optimal result

Integrand size = 40, antiderivative size = 628

$$\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{x^9(d+ex)}\,dx =$$

$$-\frac{3(cd^2-ae^2)^3\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\left(2ade+(cd^2+ae^2)x\right)\sqrt{ade+(cd^2+ae^2)x+ \cdots}}{16384a^5d^6e^5x^2}$$

$$+\frac{(cd^2-ae^2)\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\left(2ade+(cd^2+ae^2)x\right)\left(ade+(cd^2+ae^2)x+cdex \cdots\right.}{2048a^4d^5e^4x^4}$$

$$-\frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{8dx^8}-\frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{112x^7}$$

$$+\frac{\left(15c^2d^4+10acd^2e^2-33a^2e^4\right)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{448a^2d^3e^2x^6}$$

$$-\frac{\left(105c^3d^6+95ac^2d^4e^2+15a^2cd^2e^4-231a^3e^6\right)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{4480a^3d^4e^3x^5}$$

$$+\frac{3(cd^2-ae^2)^5\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\operatorname{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{32768a^{11/2}d^{13/2}e^{11/2}}$$

output
```
1/2048*(-a*e^2+c*d^2)*(33*a^3*e^6+45*a^2*c*d^2*e^4+35*a*c^2*d^4*e^2+15*c^3
*d^6)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^
4/d^5/e^4/x^4-1/8*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/d/x^8-1/112*(5*c
/a/e-11*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7+1/448*(-33*a^2*
e^4+10*a*c*d^2*e^2+15*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/a^2
/d^3/e^2/x^6-1/4480*(-231*a^3*e^6+15*a^2*c*d^2*e^4+95*a*c^2*d^4*e^2+105*c^
3*d^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/a^3/d^4/e^3/x^5+3/32768*(-a
*e^2+c*d^2)^5*(33*a^3*e^6+45*a^2*c*d^2*e^4+35*a*c^2*d^4*e^2+15*c^3*d^6)*ar
ctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2))/a^(11/2)/d^(13/2)/e^(11/2)-3/16384*(-a*e^2+c*d^
2)^3*(33*a^3*e^6+45*a^2*c*d^2*e^4+35*a*c^2*d^4*e^2+15*c^3*d^6)*(2*a*d*e+(a
*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^5/d^6/e^5/x^2
```

3.469.2 Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 572, normalized size of antiderivative = 0.91

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^9(d+ex)}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}(1575c^7d^{14}x^7-525ac^6d^{12}ex^6(2d+7ex)+35a^2c\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(1575*c^7*d^14*
x^7 - 525*a*c^6*d^12*e*x^6*(2*d + 7*e*x) + 35*a^2*c^5*d^10*e^2*x^5*(24*d^2
 + 68*d*e*x + 29*e^2*x^2) - 5*a^3*c^4*d^8*e^3*x^4*(144*d^3 + 376*d^2*e*x +
 110*d*e^2*x^2 - 185*e^3*x^3) + 5*a^4*c^3*d^6*e^4*x^3*(128*d^4 + 320*d^3*e
*x + 80*d^2*e^2*x^2 - 120*d*e^3*x^3 + 265*e^4*x^4) + a^5*c^2*d^4*e^5*x^2*(
103680*d^5 + 137600*d^4*e*x + 4640*d^3*e^2*x^2 - 5488*d^2*e^3*x^3 + 7034*d
*e^4*x^4 - 11193*e^5*x^5) + a^6*c*d^2*e^6*x*(168960*d^6 + 212480*d^5*e*x +
 4480*d^4*e^2*x^2 - 5056*d^3*e^3*x^3 + 5928*d^2*e^4*x^4 - 7476*d*e^5*x^5 +
 11445*e^6*x^6) + a^7*e^7*(71680*d^7 + 87040*d^6*e*x + 1280*d^5*e^2*x^2 -
1408*d^4*e^3*x^3 + 1584*d^3*e^4*x^4 - 1848*d^2*e^5*x^5 + 2310*d*e^6*x^6 -
3465*e^7*x^7)))/x^8) + (105*(c*d^2 - a*e^2)^5*(15*c^3*d^6 + 35*a*c^2*d^4*e
^2 + 45*a^2*c*d^2*e^4 + 33*a^3*e^6)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(S
qrt[a]*Sqrt[e]*Sqrt[d + e*x])]))/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(57344
0*a^(11/2)*d^(13/2)*e^(11/2))
```

3.469.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 608, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1215, 1237, 27, 1237, 27, 1237, 27, 1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{x^9(d+ex)}dx$$

$$\Big\downarrow \text{ 1215}$$

$$\int \frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^9}dx$$

$$\Big\downarrow \text{ 1237}$$

$$-\frac{\int -\frac{ae\left(5cd^2-6cexd-11ae^2\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{2x^8}dx}{8ade}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\left(5cd^2-6cexd-11ae^2\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{x^8}dx}{16d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \text{ 1237}$$

$$\frac{-\frac{\int \frac{\left(3\left(15c^2d^4+10ace^2d^2-33a^2e^4\right)+4cde\left(5cd^2-11ae^2\right)x\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{2x^7}dx}{7ade}-\frac{\left(\frac{5cd}{ae}-\frac{11e}{d}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7x^7}}{16d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{\int \frac{\left(3\left(15c^2d^4+10ace^2d^2-33a^2e^4\right)+4cde\left(5cd^2-11ae^2\right)x\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{x^7}dx}{14ade}-\frac{\left(\frac{5cd}{ae}-\frac{11e}{d}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7x^7}}{16d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \text{ 1237}$$

$$-\frac{-\frac{-\frac{\int \frac{3\left(105c^3d^6+95ac^2e^2d^4+15a^2ce^4d^2+2ce\left(15c^2d^4+10ace^2d^2-33a^2e^4\right)xd-231a^3e^6\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{2x^6}dx}{6ade}-\frac{\left(\frac{15c^2d^4}{a}-33ae^4+10cd^2e^2\right)\left(x\left(a\ldots\right.\right.}{2dex^6}\ldots}{14ade}\ldots}{16d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8dx^8}$$

$\big\downarrow$ 27

$$-\frac{-\frac{-\frac{\int \frac{\left(105c^3d^6+95ac^2e^2d^4+15a^2ce^4d^2+2ce\left(15c^2d^4+10ace^2d^2-33a^2e^4\right)xd-231a^3e^6\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{x^6}dx}{4ade}-\frac{\left(\frac{15c^2d^4}{a}-33ae^4+10cd^2e^2\right)\left(x\left(ae\ldots\right.\right.}{2dex^6}\ldots}{14ade}\ldots}{16d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8dx^8}$$

$\big\downarrow$ 1228

$$-\frac{-\frac{-\frac{-\frac{7\left(cd^2-ae^2\right)\left(33a^3e^6+45a^2cd^2e^4+35ac^2d^4e^2+15c^3d^6\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{x^5}dx}{2ade}-\frac{\left(-231a^3e^6+15a^2cd^2e^4+95ac^2d^4e^2+105c^3d^6\right)\left(x\left(ae^2+cd^2\ldots\right.\right.}{5adex^5}\ldots}{4ade}\ldots}{14ade}\ldots}{16d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8dx^8}$$

$\big\downarrow$ 1152

$$-\frac{-\frac{-\frac{7\left(cd^2-ae^2\right)\left(33a^3e^6+45a^2cd^2e^4+35ac^2d^4e^2+15c^3d^6\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^3}dx}{16ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\left(x\left(ae^2+cd^2\right)+ade+cde\ldots\right.}{8adex^4}\ldots\right.}{2ade}\ldots}{4ade}\ldots}{14ad\ldots}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8dx^8}$$

$\big\downarrow$ 1152

3.469. $\displaystyle\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^9(d+ex)}\,dx$

$$-\frac{-\frac{-\frac{7\left(cd^2-ae^2\right)\left(33a^3e^6+45a^2cd^2e^4+35ac^2d^4e^2+15c^3d^6\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\left(-\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(a\ldots\right.}}{4adex^2\ldots}\right)}{16ade}\ldots\right)}{2ade}\ldots}{\ldots}}{\ldots}}{8dx^8}\ldots\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}$$

$\downarrow$ 1154

$$-\frac{-\frac{-\frac{7\left(cd^2-ae^2\right)\left(33a^3e^6+45a^2cd^2e^4+35ac^2d^4e^2+15c^3d^6\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4ade}-\ldots\right)}{16ade}\ldots\right)}{2ade}\ldots}{\ldots}}{\ldots}}{8dx^8}\ldots\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}$$

$\downarrow$ 219

$$-\frac{-\frac{-\frac{\left(-231a^3e^6+15a^2cd^2e^4+95ac^2d^4e^2+105c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5adex^5}-\frac{7\left(cd^2-ae^2\right)\left(33a^3e^6+45a^2cd^2e^4+35ac^2d^4e^2+15c^3d^6\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\ldots}{\ldots}\ldots\right)}{\ldots}}{\ldots}}{\ldots}}{8dx^8}\ldots\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}$$

3.469. $\displaystyle\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^9(d+ex)}\,dx$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

output
```
-1/8*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d*x^8) + (-1/7*(((5*c*
d)/(a*e) - (11*e)/d)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/x^7 -
(-1/2*(((15*c^2*d^4)/a + 10*c*d^2*e^2 - 33*a*e^4)*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(5/2))/(d*e*x^6) - (-1/5*((105*c^3*d^6 + 95*a*c^2*d^4*e^2
+ 15*a^2*c*d^2*e^4 - 231*a^3*e^6)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(5/2))/(a*d*e*x^5) - (7*(c*d^2 - a*e^2)*(15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 4
5*a^2*c*d^2*e^4 + 33*a^3*e^6)*(-1/8*((2*a*d*e + (c*d^2 + a*e^2)*x)*(a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(a*d*e*x^4) - (3*(c*d^2 - a*e^2)^2
*(-1/4*((2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(a*d*e*x^2) + ((c*d^2 - a*e^2)^2*ArcTanh[(2*a*d*e + (c*d^2 + a*e^
2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])])/(8*a^(3/2)*d^(3/2)*e^(3/2))))/(16*a*d*e)))/(2*a*d*e))/(4*a*d*e))/(14
*a*d*e))/(16*d)
```

3.469.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.469. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^9(d+ex)}\,dx$

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.469.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 70735 vs. $2(586) = 1172$.

Time = 3.29 (sec) , antiderivative size = 70736, normalized size of antiderivative = 112.64

method	result	size
default	Expression too large to display	70736

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

3.469.5 Fricas [A] (verification not implemented)

Time = 176.45 (sec) , antiderivative size = 1550, normalized size of antiderivative = 2.47

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^{5/2}}{x^9(d + ex)}\, dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x, algorithm
="fricas")
```

output
```
[1/2293760*(105*(15*c^8*d^16 - 40*a*c^7*d^14*e^2 + 20*a^2*c^6*d^12*e^4 + 8
*a^3*c^5*d^10*e^6 + 10*a^4*c^4*d^8*e^8 + 40*a^5*c^3*d^6*e^10 - 140*a^6*c^2
*d^4*e^12 + 120*a^7*c*d^2*e^14 - 33*a^8*e^16)*sqrt(a*d*e)*x^8*log((8*a^2*d
^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*
d^3*e + a^2*d*e^3)*x)/x^2) - 4*(71680*a^8*d^8*e^8 + (1575*a*c^7*d^15*e - 3
675*a^2*c^6*d^13*e^3 + 1015*a^3*c^5*d^11*e^5 + 925*a^4*c^4*d^9*e^7 + 1325*
a^5*c^3*d^7*e^9 - 11193*a^6*c^2*d^5*e^11 + 11445*a^7*c*d^3*e^13 - 3465*a^8
*d*e^15)*x^7 - 2*(525*a^2*c^6*d^14*e^2 - 1190*a^3*c^5*d^12*e^4 + 275*a^4*c
^4*d^10*e^6 + 300*a^5*c^3*d^8*e^8 - 3517*a^6*c^2*d^6*e^10 + 3738*a^7*c*d^4
*e^12 - 1155*a^8*d^2*e^14)*x^6 + 8*(105*a^3*c^5*d^13*e^3 - 235*a^4*c^4*d^1
1*e^5 + 50*a^5*c^3*d^9*e^7 - 686*a^6*c^2*d^7*e^9 + 741*a^7*c*d^5*e^11 - 23
1*a^8*d^3*e^13)*x^5 - 16*(45*a^4*c^4*d^12*e^4 - 100*a^5*c^3*d^10*e^6 - 290
*a^6*c^2*d^8*e^8 + 316*a^7*c*d^6*e^10 - 99*a^8*d^4*e^12)*x^4 + 128*(5*a^5*
c^3*d^11*e^5 + 1075*a^6*c^2*d^9*e^7 + 35*a^7*c*d^7*e^9 - 11*a^8*d^5*e^11)*
x^3 + 1280*(81*a^6*c^2*d^10*e^6 + 166*a^7*c*d^8*e^8 + a^8*d^6*e^10)*x^2 +
5120*(33*a^7*c*d^9*e^7 + 17*a^8*d^7*e^9)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x))/(a^6*d^7*e^6*x^8), -1/1146880*(105*(15*c^8*d^16 - 40*a*c^7*
d^14*e^2 + 20*a^2*c^6*d^12*e^4 + 8*a^3*c^5*d^10*e^6 + 10*a^4*c^4*d^8*e^8 +
 40*a^5*c^3*d^6*e^10 - 140*a^6*c^2*d^4*e^12 + 120*a^7*c*d^2*e^14 - 33*a...
```

3.469.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^{5/2}}{x^9(d + ex)}\, dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**9/(e*x+d),x)
```

output
```
Timed out
```

3.469.7 Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^9(d + ex)}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)x^9}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^9), x
)
```

3.469.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5681 vs. 2(586) = 1172.

Time = 1.30 (sec) , antiderivative size = 5681, normalized size of antiderivative = 9.05

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^9(d + ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x, algorithm
="giac")
```

output
```
-3/16384*(15*c^8*d^16 - 40*a*c^7*d^14*e^2 + 20*a^2*c^6*d^12*e^4 + 8*a^3*c^
5*d^10*e^6 + 10*a^4*c^4*d^8*e^8 + 40*a^5*c^3*d^6*e^10 - 140*a^6*c^2*d^4*e^
12 + 120*a^7*c*d^2*e^14 - 33*a^8*e^16)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e
*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a^5*d^6*e^5
) + 1/573440*(1575*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a
*d*e))*a^7*c^8*d^23*e^7 - 4200*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
 a*e^2*x + a*d*e))*a^8*c^7*d^21*e^9 + 2100*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
 + c*d^2*x + a*e^2*x + a*d*e))*a^9*c^6*d^19*e^11 + 1147720*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^10*c^5*d^17*e^13 + 344169
0*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^11*c^4*d
^15*e^15 + 3444840*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a
*d*e))*a^12*c^3*d^13*e^17 + 1132180*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^
2*x + a*e^2*x + a*d*e))*a^13*c^2*d^11*e^19 + 12600*(sqrt(c*d*e)*x - sqrt(c
*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^14*c*d^9*e^21 - 3465*(sqrt(c*d*e)
*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^15*d^7*e^23 + 163840*s
qrt(c*d*e)*a^11*c^4*d^16*e^14 + 327680*sqrt(c*d*e)*a^12*c^3*d^14*e^16 + 22
9376*sqrt(c*d*e)*a^13*c^2*d^12*e^18 - 12075*(sqrt(c*d*e)*x - sqrt(c*d*e*x^
2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^6*c^8*d^22*e^6 + 32200*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^7*c^7*d^20*e^8 + 571830
0*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^8*c...
```

3.469.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^9(d + ex)} \, dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{5/2}}{x^9\,(d + e\,x)} \, dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)), x)
```

3.470 $\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.470.1 Optimal result

Integrand size = 40, antiderivative size = 271

$$\begin{aligned}\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx &= -\frac{3(3cd^2+ae^2)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^2d^2e^3} \\ &\quad -\frac{2d^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{e^3\,(cd^2-ae^2)\,(d+ex)}+\frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2cde^3} \\ &\quad +\frac{3(5c^2d^4+2acd^2e^2+a^2e^4)\,\text{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8c^{5/2}d^{5/2}e^{7/2}}\end{aligned}$$

output
```
3/8*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/
c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d
^(5/2)/e^(7/2)-3/4*(a*e^2+3*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
/c^2/d^2/e^3-2*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^3/(-a*e^2+c*d
^2)/(e*x+d)+1/2*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e^3
```

3.470.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$$
$$=\frac{\sqrt{c}\sqrt{d}\sqrt{e}(3a^3e^5(d+ex)+a^2cde^3(4d^2+5dex+e^2x^2)+c^3d^4x(-15d^2-5dex+2e^2x^2)-ac^2d^2e(15d^3+c}{4c^{5/2}d^{5/2}e^{7/2}\left(cd^2-ae^2\right.}$$

input
```
Integrate[x^3/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(Sqrt[c]*Sqrt[d]*Sqrt[e]*(3*a^3*e^5*(d + e*x) + a^2*c*d*e^3*(4*d^2 + 5*d*e
*x + e^2*x^2) + c^3*d^4*x*(-15*d^2 - 5*d*e*x + 2*e^2*x^2) - a*c^2*d^2*e*(1
5*d^3 + d^2*e*x - 4*d*e^2*x^2 + 2*e^3*x^3)) + 3*(5*c^3*d^6 - 3*a*c^2*d^4*e
^2 - a^2*c*d^2*e^4 - a^3*e^6)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqr
t[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(4*c^(5/2)*d^(5/
2)*e^(7/2)*(c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.470.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1213, 25, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\,dx$$
$$\Big\downarrow \ 1213$$
$$-\frac{\int -\frac{d^2-exd+e^2x^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^3}-\frac{2d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)(cd^2-ae^2)}$$
$$\Big\downarrow \ 25$$
$$\frac{\int \frac{d^2-exd+e^2x^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^3}-\frac{2d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)(cd^2-ae^2)}$$
$$\Big\downarrow \ 2192$$

$$\frac{\frac{\int \frac{e\left(2d\left(2cd^2-ae^2\right)-e\left(7cd^2+3ae^2\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}+\frac{ex\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd}}{e^3}-\frac{2d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3(d+ex)\left(cd^2-ae^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{2d\left(2cd^2-ae^2\right)-e\left(7cd^2+3ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4cd}+\frac{ex\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd}}{e^3}-\frac{2d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3(d+ex)\left(cd^2-ae^2\right)}$$

$\Big\downarrow$ 1160

$$\frac{\frac{\frac{3\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}-\left(\frac{3ae^2}{cd}+7d\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cd}+\frac{ex\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd}}{e^3}-\frac{2d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3(d+ex)\left(cd^2-ae^2\right)}$$

$\Big\downarrow$ 1092

$$\frac{\frac{\frac{3\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{cd}-\left(\frac{3ae^2}{cd}+7d\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cd}+\frac{ex\sqrt{x(ae^2+cd^2)+}}{2cd}}{e^3}$$
$$\frac{2d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3(d+ex)\left(cd^2-ae^2\right)}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{3\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2c^{3/2}d^{3/2}\sqrt{e}}-\left(\frac{3ae^2}{cd}+7d\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cd}+\frac{ex\sqrt{x(ae^2+cd^2)+ade+cdex}}{2cd}}{e^3}$$
$$\frac{2d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3(d+ex)\left(cd^2-ae^2\right)}$$

input
```
Int[x^3/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

3.470. $\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

output
```
(-2*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^3*(c*d^2 - a*e^2)*
(d + e*x)) + ((e*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*c*d) +
(-((7*d + (3*a*e^2)/(c*d))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) +
(3*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*
x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
])/(2*c^(3/2)*d^(3/2)*Sqrt[e]))/(4*c*d))/e^3
```

3.470.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1213
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m - n + 2)   Int[Expan
dToSum[((-d)^n*(-2*c*d + b*e)^(-m - 1) - e^n*x^n*((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IGtQ[n, 0] && EqQ[m
+ p, -3/2]
```

3.470. $\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.470.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(243) = 486$.

Time = 0.69 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.90

method	result
default	$\frac{\frac{x\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}{2cde}-\frac{3\left(e^2a+c\,d^2\right)\left(\frac{\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}{cde}-\frac{\left(e^2a+c\,d^2\right)\ln\left(\frac{\frac{1}{2}e^2a+\frac{1}{2}c\,d^2+cdex}{\sqrt{cde}}+\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}\right)}{2cde\sqrt{cde}}\right.}{4cde}}{e}$

input
```
int(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(1/2*x/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/4*(a*e^2+c*d^2)
/c/d/e*(1/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e^2+c*d^2)/
c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))-1/2*a/c*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x
)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+d^
2/e^3*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-d/e^2*(1/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)-1/2*(a*e^2+c*d^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c
*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+2*d^3/
e^4/(a*e^2-c*d^2)/(x+d/e)*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

3.470.5 Fricas [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 758, normalized size of antiderivative = 2.80

$$\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$

$$= \left[\frac{3\,(5\,c^3d^7-3\,ac^2d^5e^2-a^2cd^3e^4-a^3de^6+(5\,c^3d^6e-3\,ac^2d^4e^3-a^2cd^2e^5-a^3e^7)x)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2\right.}{}\right.$$

$$-\frac{3\,(5\,c^3d^7-3\,ac^2d^5e^2-a^2cd^3e^4-a^3de^6+(5\,c^3d^6e-3\,ac^2d^4e^3-a^2cd^2e^5-a^3e^7)x)\sqrt{-cde}\arctan\left(\frac{\sqrt{cd}}{}\right.}{}$$

input
```
integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="fricas")
```

output
```
[1/16*(3*(5*c^3*d^7 - 3*a*c^2*d^5*e^2 - a^2*c*d^3*e^4 - a^3*d*e^6 + (5*c^3
*d^6*e - 3*a*c^2*d^4*e^3 - a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(c*d*e)*log(8*c
^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*
d^3*e + a*c*d*e^3)*x) - 4*(15*c^3*d^6*e - 4*a*c^2*d^4*e^3 - 3*a^2*c*d^2*e^
5 - 2*(c^3*d^4*e^3 - a*c^2*d^2*e^5)*x^2 + (5*c^3*d^5*e^2 - 2*a*c^2*d^3*e^4
 - 3*a^2*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^6
*e^4 - a*c^3*d^4*e^6 + (c^4*d^5*e^5 - a*c^3*d^3*e^7)*x), -1/8*(3*(5*c^3*d^
7 - 3*a*c^2*d^5*e^2 - a^2*c*d^3*e^4 - a^3*d*e^6 + (5*c^3*d^6*e - 3*a*c^2*d
^4*e^3 - a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c
^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(15*c^3*d^6
*e - 4*a*c^2*d^4*e^3 - 3*a^2*c*d^2*e^5 - 2*(c^3*d^4*e^3 - a*c^2*d^2*e^5)*x
^2 + (5*c^3*d^5*e^2 - 2*a*c^2*d^3*e^4 - 3*a^2*c*d*e^6)*x)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^6*e^4 - a*c^3*d^4*e^6 + (c^4*d^5*e^5 -
 a*c^3*d^3*e^7)*x)]
```

3.470.6 Sympy [F]

$$\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{x^3}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

input
```
integrate(x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(x**3/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

3.470.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.470.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{1}{4}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(\frac{2\,x}{cde^2}-\frac{7\,cd^2e^5+3\,ae^7}{c^2d^2e^8}\right)$$
$$-\frac{2\,d^3}{\left(\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)e+\sqrt{cde}d\right)e^3}$$
$$-\frac{3\,(5\,c^2d^4+2\,acd^2e^2+a^2e^4)\log\left(\left|cd^2+ae^2+2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{8\,\sqrt{cde}c^2d^2e^3}$$

input
```
integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="giac")
```

output
```
1/4*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*x/(c*d*e^2) - (7*c*d^2*
e^5 + 3*a*e^7)/(c^2*d^2*e^8)) - 2*d^3/(((sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
c*d^2*x + a*e^2*x + a*d*e))*e + sqrt(c*d*e)*d)*e^3) - 3/8*(5*c^2*d^4 + 2*a
*c*d^2*e^2 + a^2*e^4)*log(abs(c*d^2 + a*e^2 + 2*sqrt(c*d*e)*(sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^2*d^2*e^3
)
```

3.470.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{x^3}{(d+e\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

3.471 $\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.471.1 Optimal result

Integrand size = 40, antiderivative size = 195

$$\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cde^2} + \frac{2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e^2(cd^2-ae^2)(d+ex)} - \frac{(3cd^2+ae^2)\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2c^{3/2}d^{3/2}e^{5/2}}$$

output
```
-1/2*(a*e^2+3*c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e
^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/e^(5/2)+(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e^2+2*d^2*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/e^2/(-a*e^2+c*d^2)/(e*x+d)
```

3.471.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.03

$$\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{\sqrt{c}\sqrt{d}\sqrt{e}(-a^2e^3(d+ex)+c^2d^3x(3d+ex)+acde(3d^2-e^2x^2))-(3c^2d^4-2acd^2e^2-a^2e^4)\sqrt{ae+cdx}\sqrt{}}{c^{3/2}d^{3/2}e^{5/2}(cd^2-ae^2)\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[x^2/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-(a^2*e^3*(d + e*x)) + c^2*d^3*x*(3*d + e*x) + a
*c*d*e*(3*d^2 - e^2*x^2)) - (3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*Sqrt[a*e
 + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[
d]*Sqrt[d + e*x])])/(c^(3/2)*d^(3/2)*e^(5/2)*(c*d^2 - a*e^2)*Sqrt[(a*e + c
*d*x)*(d + e*x)])
```

3.471.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1213, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{ 1213}$$

$$\frac{2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^2(d+ex)\left(cd^2-ae^2\right)}-\frac{\int \frac{d-ex}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^2}$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^2(d+ex)\left(cd^2-ae^2\right)}-\frac{\frac{1}{2}\left(\frac{ae^2}{cd}+3d\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd}}{e^2}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^2(d+ex)\left(cd^2-ae^2\right)}-\frac{\left(\frac{ae^2}{cd}+3d\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd}}{e^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^2(d+ex)\left(cd^2-ae^2\right)}-\frac{\frac{\left(\frac{ae^2}{cd}+3d\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\sqrt{e}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd}}{e^2}}{}$$

input
```
Int[x^2/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^2*(c*d^2 - a*e^2)*(
d + e*x)) - (-(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(c*d)) + ((3*d
+ (a*e^2)/(c*d))*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sq
rt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*Sq
rt[e]))/e^2
```

3.471.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1213
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m - n + 2)   Int[Expan
dToSum[((-d)^n*(-2*c*d + b*e)^(-m - 1) - e^n*x^n*((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IGtQ[n, 0] && EqQ[m
+ p, -3/2]
```

3.471.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.33

method	result
default	$\frac{\frac{\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}}}{cde}-\frac{\left(e^{2}a+c\,d^{2}\right)\ln\left(\frac{\frac{1}{2}e^{2}a+\frac{1}{2}c\,d^{2}+cdex}{\sqrt{cde}}+\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}}\right)}{2cde\sqrt{cde}}}{e}-\frac{d\ln\left(\frac{\frac{1}{2}e^{2}a+\frac{1}{2}c\,d^{2}+cdex}{\sqrt{cde}}+\sqrt{ade+(e^{2}a\right.}{e^{2}\sqrt{cde}}$

input
```
int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(1/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e^2+c*d^2)/c/d
/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2))/(c*d*e)^(1/2))-d/e^2*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*
d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-2*d^2/e^
3/(a*e^2-c*d^2)/(x+d/e)*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

3.471.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 586, normalized size of antiderivative = 3.01

$$\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$$
$$=\left[\frac{(3\,c^2d^5-2\,acd^3e^2-a^2de^4+(3\,c^2d^4e-2\,acd^2e^3-a^2e^5)x)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4\right.}{}\right.$$

input
```
integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="fricas")
```

output
```
[1/4*((3*c^2*d^5 - 2*a*c*d^3*e^2 - a^2*d*e^4 + (3*c^2*d^4*e - 2*a*c*d^2*e^
3 - a^2*e^5)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^
2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c
*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(3*c^2*d^4*e
- a*c*d^2*e^3 + (c^2*d^3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x))/(c^3*d^5*e^3 - a*c^2*d^3*e^5 + (c^3*d^4*e^4 - a*c^2*d^2*e^
6)*x), 1/2*((3*c^2*d^5 - 2*a*c*d^3*e^2 - a^2*d*e^4 + (3*c^2*d^4*e - 2*a*c*
d^2*e^3 - a^2*e^5)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2
+ a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(3*c^2*d^4*e - a*c*d^2*e^3
 + (c^2*d^3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
))/(c^3*d^5*e^3 - a*c^2*d^3*e^5 + (c^3*d^4*e^4 - a*c^2*d^2*e^6)*x)]
```

3.471.6 Sympy [F]

$$\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{x^2}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

input
```
integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(x**2/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

3.471.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.471.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{2\,d^2}{\left(\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)e+\sqrt{cde}d\right)e^2} + \frac{(3\,cd^2+ae^2)\log\left(\left|cd^2+ae^2+2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{2\,\sqrt{cde}cde^2} + \frac{\sqrt{cdex^2+cd^2x+ae^2x+ade}}{cde^2}$$

input
```
integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="giac")
```

output
```
2*d^2/(((sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*e +
sqrt(c*d*e)*d)*e^2) + 1/2*(3*c*d^2 + a*e^2)*log(abs(c*d^2 + a*e^2 + 2*sqrt
(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(s
qrt(c*d*e)*c*d*e^2) + sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)/(c*d*e^2
)
```

3.471.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{x^2}{(d+e\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

3.472 $\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.472.1 Optimal result

Integrand size = 38, antiderivative size = 139

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{2d\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e(cd^2-ae^2)(d+ex)} + \frac{\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{c}\sqrt{d}e^{3/2}}$$

output
```
arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2))/e^(3/2)/c^(1/2)/d^(1/2)-2*d*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)/e/(-a*e^2+c*d^2)/(e*x+d)
```

3.472.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\left(-\frac{d^{3/2}\sqrt{e}(ae+cdx)}{cd^2-ae^2}+\frac{\sqrt{ae+cdx}\sqrt{d+ex}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{c}}\right)}{\sqrt{d}e^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[x/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(2*(-((d^(3/2)*Sqrt[e]*(a*e + c*d*x))/(c*d^2 - a*e^2)) + (Sqrt[a*e + c*d*x
]*Sqrt[d + e*x]*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[
d + e*x])])/Sqrt[c]))/(Sqrt[d]*e^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.472.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1213, 25, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \ 1213$$

$$-\frac{\int -\frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e}-\frac{2d\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e}-\frac{2d\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 1092$$

$$\frac{2\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{e}-\frac{2d\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 219$$

$$\frac{\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}e^{3/2}}-\frac{2d\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)\left(cd^2-ae^2\right)}$$

input
```
Int[x/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(-2*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*(c*d^2 - a*e^2)*(d +
 e*x)) + ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(Sqrt[c]*Sqrt[d]*e^(3/2))
```

3.472.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1213
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m - n + 2)   Int[Expan
dToSum[((-d)^n*(-2*c*d + b*e)^(-m - 1) - e^n*x^n*((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IGtQ[n, 0] && EqQ[m
+ p, -3/2]
```

3.472.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\ln\left(\frac{\frac{1}{2}e^2a+\frac{1}{2}c\,d^2+cdex}{\sqrt{cde}}+\sqrt{ade+(e^2a+c\,d^2)x+cde\,x^2}\right)}{e\sqrt{cde}}+\frac{2d\sqrt{cde\left(x+\frac{d}{e}\right)^2+(e^2a-c\,d^2)\left(x+\frac{d}{e}\right)}}{e^2(e^2a-c\,d^2)\left(x+\frac{d}{e}\right)}$	131

input
```
int(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
1/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)+2*d/e^2/(a*e^2-c*d^2)/(x+d/e)*(c*d*e*(x+d/
e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

3.472.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.19

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \left[-\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}cd^2e-(cd^3-ade^2+(cd^2e-ae^3)x)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,a\right.}{2\,(c^2d^4e^2-acd^2e^4+(c}\right.$$
$$-\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}cd^2e+(cd^3-ade^2+(cd^2e-ae^3)x)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2}}{2\,(c^2d^2e^2x^2+acd^2}\right.}{c^2d^4e^2-acd^2e^4+(c^2d^3e^3-acde^5)x}$$

input
```
integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
fricas")
```

output
```
[-1/2*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d^2*e - (c*d^3 - a*
d*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 +
 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(
2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x))/(c^
2*d^4*e^2 - a*c*d^2*e^4 + (c^2*d^3*e^3 - a*c*d*e^5)*x), -(2*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*c*d^2*e + (c*d^3 - a*d*e^2 + (c*d^2*e - a*e^
3)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 +
(c^2*d^3*e + a*c*d*e^3)*x)))/(c^2*d^4*e^2 - a*c*d^2*e^4 + (c^2*d^3*e^3 - a
*c*d*e^5)*x)]
```

3.472.6 Sympy [F]

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{x}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

input
```
integrate(x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(x/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

3.472.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.472.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{1,[0,0,5]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[2,2]
%%%}+%%%{
```

3.472.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \int \frac{x}{(d+e\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int(x/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
int(x/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

3.473 $\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.473.1 Optimal result

Integrand size = 37, antiderivative size = 52

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(cd^2-ae^2)\,(d+ex)}$$

output
```
2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)/(e*x+d)
```

3.473.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2(ae+cdx)}{(cd^2-ae^2)\,\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[1/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(2*(a*e + c*d*x))/((c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.473.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1123}$$

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)\left(cd^2-ae^2\right)}$$

input
```
Int[1/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d^2 - a*e^2)*(d + e*x)
)
```

3.473.3.1 Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

3.473.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

method	result	size
trager	$-\frac{2\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{\left(e^2a-c\,d^2\right)(ex+d)}$	50
gosper	$-\frac{2(cdx+ae)}{\left(e^2a-c\,d^2\right)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	51
default	$-\frac{2\sqrt{cde\left(x+\frac{d}{e}\right)^2+\left(e^2a-c\,d^2\right)\left(x+\frac{d}{e}\right)}}{e\left(e^2a-c\,d^2\right)\left(x+\frac{d}{e}\right)}$	65

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
-2/(a*e^2-c*d^2)/(e*x+d)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

3.473.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{cd^3-ade^2+(cd^2e-ae^3)x}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
fricas")
```

output
```
2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c*d^3 - a*d*e^2 + (c*d^2*e
- a*e^3)*x)
```

3.473.6 Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(1/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

3.473.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

3.473.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,0,1]%%%},[2]%%%}+%%%{%%{[%%%{-2,[0,1,0]%%%},0]:[1
,0,%%%{-1
```

3.473.9 Mupad [B] (verification not implemented)

Time = 11.71 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{2\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{(a\,e^2-c\,d^2)\,(d+e\,x)}$$

input
```
int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
-(2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((a*e^2 - c*d^2)*(d + e
*x))
```

3.473. $\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.474 $\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.474.1 Optimal result

Integrand size = 40, antiderivative size = 143

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{2e(ae+cdx)}{d(cd^2-ae^2)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{\operatorname{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{a}d^{3/2}\sqrt{e}}$$

output
```
-arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2))/d^(3/2)/a^(1/2)/e^(1/2)-2*e*(c*d*x+a*e)/d/(-
a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

3.474.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\left(-\frac{\sqrt{d}e^{3/2}(ae+cdx)}{cd^2-ae^2} - \frac{\sqrt{ae+cdx}\sqrt{d+ex}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{a}}\right)}{d^{3/2}\sqrt{e}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[1/(x*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(2*(-((Sqrt[d]*e^(3/2)*(a*e + c*d*x))/(c*d^2 - a*e^2)) - (Sqrt[a*e + c*d*x
]*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[
d + e*x])])/Sqrt[a]))/(d^(3/2)*Sqrt[e]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.474.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1214, 25, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{ 1214}$$

$$-\int -\frac{1}{dx\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{1}{dx\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{d}-\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 1154}$$

$$-\frac{2\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{d}-\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{a}d^{3/2}\sqrt{e}}-\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d(d+ex)\left(cd^2-ae^2\right)}$$

input
```
Int[1/(x*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(-2*e*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d*(c*d^2 - a*e^2)*(d +
 e*x)) - ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(Sqrt[a]*d^(3/2)*Sqrt[e])
```

3.474.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1214
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[ExpandToS
um[(((-d)^n*(-2*c*d + b*e)^(-m - 1))/(e^n*x^n) - ((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/(Sqrt[a + b*x + c*x^2]/x^n), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && ILtQ[n, 0] &&
EqQ[m + p, -3/2]
```

3.474.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{\ln\left(\frac{2ade+\left(e^{2}a+c\,d^{2}\right)x+2\sqrt{ade}\,\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}}}{x}\right)}{d\sqrt{ade}}+\frac{2\sqrt{cde\left(x+\frac{d}{e}\right)^{2}+\left(e^{2}a-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}{d\left(e^{2}a-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}$	136

input
```
int(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
-1/d/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)+2/d/(a*e^2-c*d^2)/(x+d/e)*(c*d*e*(x+d/e)^
2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

3.474.5 Fricas [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 454, normalized size of antiderivative = 3.17

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\left[-\frac{4\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}ade^2-(cd^3-ade^2+(cd^2e-ae^3)x)\sqrt{ade}\log\left(\frac{8\,a^2d^2e^2+(c^2d^4+6\,acd^2e^2+a^2\cdots}{\cdots}\right.}{2\,(acd^5e-a^2d^3e^3+(acd^4e^2-a^2d^2e^4)x\cdots}\right.$$
$$-\frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}ade^2-(cd^3-ade^2+(cd^2e-ae^3)x)\sqrt{-ade}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2\cdots}}{2\,(acd^2e^2x^2+a^2\cdots}\right.}{acd^5e-a^2d^3e^3+(acd^4e^2-a^2d^2e^4)x}$$

input
```
integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="fricas")
```

output
```
[-1/2*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e^2 - (c*d^3 - a*
d*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6
*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)
/x^2))/(a*c*d^5*e - a^2*d^3*e^3 + (a*c*d^4*e^2 - a^2*d^2*e^4)*x), -(2*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e^2 - (c*d^3 - a*d*e^2 + (c*d^
2*e - a*e^3)*x)*sqrt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^
2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)))/(a*c*d^5*e - a^2*d^3*e^3 + (a*c*d
^4*e^2 - a^2*d^2*e^4)*x)]
```

3.474.6 Sympy [F]

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{x\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

input
```
integrate(1/x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

3.474.7 Maxima [F]

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(ex+d)x}\,dx$$

input
```
integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)*x), x)
```

3.474.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,1,5]%%%},[2,2]%%%}+%%%{%%%{-2,[1,3,3]%%%},[2,1]%%
%}+%%%{%%
```

3.474.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{x\,(d+e\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

3.475 $\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.475.1 Optimal result

Integrand size = 40, antiderivative size = 229

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{2e(ae+cdx)}{d\,(cd^2-ae^2)\,x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} - \frac{(cd^2-3ae^2)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{ad^2e\,(cd^2-ae^2)\,x} + \frac{(cd^2+3ae^2)\,\text{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2a^{3/2}d^{5/2}e^{3/2}}$$

output
```
1/2*(3*a*e^2+c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/
e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/d^(5/2)/e^(3/2)-2
*e*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-
(-3*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/d^2/e/(-a*e^2+c
*d^2)/x
```

3.475.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{\sqrt{a}\sqrt{d}\sqrt{e}(-c^2d^3x(d+ex)+a^2e^3(d+3ex)-acde(d^2-3e^2x^2))+(c^2d^4+2acd^2e^2-3a^2e^4)\,x\sqrt{ae+cdx}\sqrt{}}{a^{3/2}d^{5/2}e^{3/2}\left(cd^2-ae^2\right)x\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[1/(x^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(Sqrt[a]*Sqrt[d]*Sqrt[e]*(-(c^2*d^3*x*(d + e*x)) + a^2*e^3*(d + 3*e*x) - a
*c*d*e*(d^2 - 3*e^2*x^2)) + (c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*x*Sqrt[a
*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqr
t[e]*Sqrt[d + e*x])]])/(a^(3/2)*d^(5/2)*e^(3/2)*(c*d^2 - a*e^2)*x*Sqrt[(a*e
 + c*d*x)*(d + e*x)])
```

3.475.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1214, 25, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$
$$\Big\downarrow \text{1214}$$
$$\frac{2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2(d+ex)\left(cd^2-ae^2\right)}-\int -\frac{d-ex}{d^2x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx$$
$$\Big\downarrow \text{25}$$
$$\int \frac{d-ex}{d^2x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2(d+ex)\left(cd^2-ae^2\right)}$$
$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{d-ex}{x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{d^2}+\frac{2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{-\frac{1}{2}\left(\frac{cd^2}{ae}+3e\right)\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{aex}}{d^2}+\frac{2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{\left(\frac{cd^2}{ae}+3e\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{aex}}{d^2}+\frac{2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\left(\frac{cd^2}{ae}+3e\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}\sqrt{d}\sqrt{e}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{aex}}{d^2}+\frac{2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2(d+ex)\left(cd^2-ae^2\right)}$$

input
```
Int[1/(x^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(2*e^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^2*(c*d^2 - a*e^2)*(
d + e*x)) + (-(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(a*e*x)) + (((c
*d^2)/(a*e) + 3*e)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d
]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[a]*Sqrt[d
]*Sqrt[e]))/d^2
```

3.475. $\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.475.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1214
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[ExpandToS
um[(((-d)^n*(-2*c*d + b*e)^(-m - 1))/(e^n*x^n) - ((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/(Sqrt[a + b*x + c*x^2]/x^n), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && ILtQ[n, 0] &&
EqQ[m + p, -3/2]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

3.475. $\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.475.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.18

method	result
default	$\frac{-\frac{\sqrt{ade+\left(e^{2} a+c\,d^{2}\right) x+cde\,x^{2}}}{adex}+\frac{\left(e^{2} a+c\,d^{2}\right) \ln\left(\frac{2ade+\left(e^{2} a+c\,d^{2}\right) x+2\sqrt{ade}\,\sqrt{ade+\left(e^{2} a+c\,d^{2}\right) x+cde\,x^{2}}}{x}\right)}{2ade\sqrt{ade}}}{d}+\frac{e\ln\left(\frac{2ade+\left(e^{2} a+c\,d^{2}\right) x+2\sqrt{a}}{}\right.}{d^{2}\sqrt{}}$

input
```
int(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
1/d*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)/
a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+e/d^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+
c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)-2*e/d
^2/(a*e^2-c*d^2)/(x+d/e)*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

3.475.5 Fricas [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.66

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \left[\frac{\sqrt{ade}((c^2d^4e+2\,acd^2e^3-3\,a^2e^5)x^2+(c^2d^5+2\,acd^3e^2-3\,a^2de^4)x)\log\left(\frac{8\,a^2d^2e^2+(c^2d^4+6\,acd^2e^2+a^2e^4)x^2+4\,\sqrt{}}{}\right.}{4\,((a^2cd^5e^3}\right.$$
$$-\frac{\sqrt{-ade}((c^2d^4e+2\,acd^2e^3-3\,a^2e^5)x^2+(c^2d^5+2\,acd^3e^2-3\,a^2de^4)x)\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,ad}{2\,(acd^2e^2x^2+a^2d^2e^2+(a}\right.}{2\,((a^2cd^5e^3-a^3d^3e^5)x^2+(a}$$

input
```
integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="fricas")
```

output
```
[1/4*(sqrt(a*d*e)*((c^2*d^4*e + 2*a*c*d^2*e^3 - 3*a^2*e^5)*x^2 + (c^2*d^5
+ 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^
2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) -
 4*(a*c*d^4*e - a^2*d^2*e^3 + (a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^2*c*d^5*e^3 - a^3*d^3*e^5)*x^2 + (a^2*
c*d^6*e^2 - a^3*d^4*e^4)*x), -1/2*(sqrt(-a*d*e)*((c^2*d^4*e + 2*a*c*d^2*e^
3 - 3*a^2*e^5)*x^2 + (c^2*d^5 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*arctan(1/2
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x))
 + 2*(a*c*d^4*e - a^2*d^2*e^3 + (a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^2*c*d^5*e^3 - a^3*d^3*e^5)*x^2 + (a^
2*c*d^6*e^2 - a^3*d^4*e^4)*x)]
```

3.475.6 Sympy [F]

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{x^2\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

input
```
integrate(1/x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

3.475.7 Maxima [F]

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(ex+d)x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)*x^2), x
)
```

3.475.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,0,1]%%%},[6,0]%%%}+%%%{%%{[%%%{-2,[0,1,0]%%%},0]:
[1,0,%%%{
```

3.475.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{x^2\,\left(d+e\,x\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

input
```
int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

3.476 $\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.476.1 Optimal result

Integrand size = 40, antiderivative size = 329

$$\begin{aligned}&\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx\\&=-\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\\&-\frac{\left(cd^2-5ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2ad^2e\left(cd^2-ae^2\right)x^2}\\&+\frac{\left(3cd^2-5ae^2\right)\left(cd^2+3ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4a^2d^3e^2\left(cd^2-ae^2\right)x}\\&-\frac{3(c^2d^4+2acd^2e^2+5a^2e^4)\,\mathrm{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8a^{5/2}d^{7/2}e^{5/2}}\end{aligned}$$

output
```
-3/8*(5*a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*
x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2
)/d^(7/2)/e^(5/2)-2*e*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/x^2/(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)-1/2*(-5*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)/a/d^2/e/(-a*e^2+c*d^2)/x^2+1/4*(-5*a*e^2+3*c*d^2)*(3*a*e^2+c*d^2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/d^3/e^2/(-a*e^2+c*d^2)/x
```

3.476.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$$
$$= \frac{\sqrt{a}\sqrt{d}\sqrt{e}(3c^3d^5x^2(d+ex)+a^3e^4(2d^2-5dex-15e^2x^2)+ac^2d^3ex(d^2+5dex+4e^2x^2)-a^2cde^2(2d^3-4}{4a^{5/2}d^{7/2}e^{5/2}\left(cd^2-ae\right.}$$

input
```
Integrate[1/(x^3*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(Sqrt[a]*Sqrt[d]*Sqrt[e]*(3*c^3*d^5*x^2*(d + e*x) + a^3*e^4*(2*d^2 - 5*d*e
*x - 15*e^2*x^2) + a*c^2*d^3*e*x*(d^2 + 5*d*e*x + 4*e^2*x^2) - a^2*c*d*e^2
*(2*d^3 - 4*d^2*e*x + d*e^2*x^2 + 15*e^3*x^3)) - 3*(c^3*d^6 + a*c^2*d^4*e^
2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*x^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTa
nh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(4*a^(5/2
)*d^(7/2)*e^(5/2)*(c*d^2 - a*e^2)*x^2*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.476.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.85, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1214, 25, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$
$$\Big\downarrow \text{1214}$$
$$-\int -\frac{\frac{e^2x^2}{d^3}-\frac{ex}{d^2}+\frac{1}{d}}{x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{2e^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)\left(cd^2-ae^2\right)}$$
$$\Big\downarrow \text{25}$$
$$\int \frac{\frac{e^2x^2}{d^3}-\frac{ex}{d^2}+\frac{1}{d}}{x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{2e^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)\left(cd^2-ae^2\right)}$$
$$\Big\downarrow \text{2181}$$

$$-\frac{\int \frac{\frac{7ae^2}{d}+2\left(c-\frac{2ae^2}{d^2}\right)xe+3cd}{2x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2ade}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2ad^2ex^2}-\frac{2e^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d^3(d+ex)(cd^2-ae^2)}$$

$\downarrow$ 27

$$-\frac{\int \frac{\frac{7ae^2}{d}+2\left(c-\frac{2ae^2}{d^2}\right)xe+3cd}{x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4ade}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2ad^2ex^2}-\frac{2e^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d^3(d+ex)(cd^2-ae^2)}$$

$\downarrow$ 1228

$$-\frac{-\frac{3\left(\frac{c^2d^2}{a}+\frac{5ae^4}{d^2}+2ce^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2e}-\frac{\left(\frac{3c}{ae}+\frac{7e}{d^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}}{4ade}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2ad^2ex^2}-\frac{2e^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d^3(d+ex)(cd^2-ae^2)}$$

$\downarrow$ 1154

$$-\frac{\frac{3\left(\frac{c^2d^2}{a}+\frac{5ae^4}{d^2}+2ce^2\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{e}-\frac{\left(\frac{3c}{ae}+\frac{7e}{d^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}}{4ade}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2ad^2ex^2}-\frac{2e^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d^3(d+ex)(cd^2-ae^2)}$$

$\downarrow$ 219

$$-\frac{\frac{3\left(\frac{c^2d^2}{a}+\frac{5ae^4}{d^2}+2ce^2\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}\sqrt{d}e^{3/2}}-\frac{\left(\frac{3c}{ae}+\frac{7e}{d^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}}{4ade}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2ad^2ex^2}-\frac{2e^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d^3(d+ex)(cd^2-ae^2)}$$

input
```
Int[1/(x^3*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

3.476. $\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

output
```
-1/2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(a*d^2*e*x^2) - (2*e^3*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^3*(c*d^2 - a*e^2)*(d + e*x))
 - (-((((3*c)/(a*e) + (7*e)/d^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/x) + (3*((c^2*d^2)/a + 2*c*e^2 + (5*a*e^4)/d^2)*ArcTanh[(2*a*d*e + (c*
d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])])/(2*Sqrt[a]*Sqrt[d]*e^(3/2)))/(4*a*d*e)
```

3.476.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1214
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[ExpandToS
um[(((-d)^n*(-2*c*d + b*e)^(-m - 1))/(e^n*x^n) - ((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/(Sqrt[a + b*x + c*x^2]/x^n), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && ILtQ[n, 0] &&
EqQ[m + p, -3/2]
```

3.476. $\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.476.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.66

method	result
default	$\frac{-\frac{\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}}}{2ade\,x^{2}}-\frac{3\left(e^{2}a+c\,d^{2}\right)\left(-\frac{\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}}}{adex}+\frac{\left(e^{2}a+c\,d^{2}\right)\ln\left(\frac{2ade+\left(e^{2}a+c\,d^{2}\right)x+2\sqrt{ade}\,\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x\cdots}}{x}\right)}{2ade\sqrt{ade}}\cdots\right)}{4ade}\cdots}{d}$

input
```
int(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
1/d*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/4*(a*e^2+c*d
^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c
*d^2)/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+1/2*c/a/(a*d*e)^(1/2)*ln((2*a*d*e
+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/
x))-e^2/d^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)-e/d^2*(-1/a/d/e/x*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+
(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x
))+2*e^2/d^3/(a*e^2-c*d^2)/(x+d/e)*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))
^(1/2)
```

3.476.5 Fricas [A] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 792, normalized size of antiderivative = 2.41

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$

$$= \left[\frac{3\left((c^3d^6e+ac^2d^4e^3+3\,a^2cd^2e^5-5\,a^3e^7)x^3+(c^3d^7+ac^2d^5e^2+3\,a^2cd^3e^4-5\,a^3de^6)x^2\right)\sqrt{ade}\log\left(\frac{8\,a^2d^2e}{}\right.}{}\right.$$

input
```
integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="fricas")
```

output
```
[1/16*(3*((c^3*d^6*e + a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - 5*a^3*e^7)*x^3 +
(c^3*d^7 + a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 - 5*a^3*d*e^6)*x^2)*sqrt(a*d*e)
*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d
*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(2*a^2*c*d^5*e^2 - 2*a^3*d^3*e
^4 - (3*a*c^2*d^5*e^2 + 4*a^2*c*d^3*e^4 - 15*a^3*d*e^6)*x^2 - (3*a*c^2*d^6
*e + 2*a^2*c*d^4*e^3 - 5*a^3*d^2*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x))/((a^3*c*d^6*e^4 - a^4*d^4*e^6)*x^3 + (a^3*c*d^7*e^3 - a^4*d^5*
e^5)*x^2), 1/8*(3*((c^3*d^6*e + a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - 5*a^3*e^
7)*x^3 + (c^3*d^7 + a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 - 5*a^3*d*e^6)*x^2)*sq
rt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e
 + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d
^3*e + a^2*d*e^3)*x)) - 2*(2*a^2*c*d^5*e^2 - 2*a^3*d^3*e^4 - (3*a*c^2*d^5*
e^2 + 4*a^2*c*d^3*e^4 - 15*a^3*d*e^6)*x^2 - (3*a*c^2*d^6*e + 2*a^2*c*d^4*e
^3 - 5*a^3*d^2*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^3*
c*d^6*e^4 - a^4*d^4*e^6)*x^3 + (a^3*c*d^7*e^3 - a^4*d^5*e^5)*x^2)]
```

3.476.6 Sympy [F]

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{x^3\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

input
```
integrate(1/x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

3.476.7 Maxima [F]

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(ex+d)x^3}\,dx$$

input
```
integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)*x^3), x
)
```

3.476.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,3,9]%%%},[2,4]%%%}+%%%{%%%{-4,[1,5,7]%%%},[2,3]%%
%}+%%%{%%
```

3.476.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{x^3\,\left(d+e\,x\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

input
```
int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

3.477 $\int \frac{x^5}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

3.477.1 Optimal result

Integrand size = 40, antiderivative size = 515

$$\int \frac{x^5}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2dx^4\left(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$
$$-\frac{2x^2\left(ade(cd^2-ae^2)\left(7c^2d^4-12acd^2e^2-3a^2e^4\right)+\left(cd^2-ae^2\right)\left(7c^3d^6-11ac^2d^4e^2-a^2cd^2e^4-3a^3e^6\right)x\right)}{3cde^2\left(cd^2-ae^2\right)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$-\frac{\left(105c^4d^8-190ac^3d^6e^2+36a^2c^2d^4e^4+30a^3cd^2e^6-45a^4e^8-2cde(35c^3d^6-61ac^2d^4e^2+9a^2cd^2e^4-15a^3\right.}{12c^3d^3e^4\left(cd^2-ae^2\right)^3}$$
$$+\frac{5(7c^2d^4+6acd^2e^2+3a^2e^4)\,\text{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8c^{7/2}d^{7/2}e^{9/2}}$$

output
```
-2/3*d*x^4*(a*e*(-a*e^2+c*d^2)+c*d*(-a*e^2+c*d^2)*x)/e/(-a*e^2+c*d^2)^2/(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+5/8*(3*a^2*e^4+6*a*c*d^2*e^2+7*c^2*d
^4)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(7/2)/d^(7/2)/e^(9/2)-2/3*x^2*(a*d*e*(-a*
e^2+c*d^2)*(-3*a^2*e^4-12*a*c*d^2*e^2+7*c^2*d^4)+(-a*e^2+c*d^2)*(-3*a^3*e^
6-a^2*c*d^2*e^4-11*a*c^2*d^4*e^2+7*c^3*d^6)*x)/c/d/e^2/(-a*e^2+c*d^2)^4/(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/12*(105*c^4*d^8-190*a*c^3*d^6*e^2+
36*a^2*c^2*d^4*e^4+30*a^3*c*d^2*e^6-45*a^4*e^8-2*c*d*e*(-15*a^3*e^6+9*a^2*
c*d^2*e^4-61*a*c^2*d^4*e^2+35*c^3*d^6)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)/c^3/d^3/e^4/(-a*e^2+c*d^2)^3
```

3.477.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{-\frac{\sqrt{c}\sqrt{d}\sqrt{e}(ae+cdx)\left(-45a^5e^9(d+ex)^2+15a^4cde^7(2d-ex)(d+ex)^2+6a^3c^2d^2e \cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[x^5/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-((Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-45*a^5*e^9*(d + e*x)^2 + 15*a^
4*c*d*e^7*(2*d - e*x)*(d + e*x)^2 + 6*a^3*c^2*d^2*e^5*(d + e*x)^2*(6*d^2 +
 2*d*e*x + e^2*x^2) + c^5*d^8*x*(105*d^3 + 140*d^2*e*x + 21*d*e^2*x^2 - 6*
e^3*x^3) - 2*a^2*c^3*d^4*e^3*(95*d^4 + 111*d^3*e*x - 6*d^2*e^2*x^2 - 9*d*e
^3*x^3 + 9*e^4*x^4) + a*c^4*d^6*e*(105*d^4 - 50*d^3*e*x - 237*d^2*e^2*x^2
- 48*d*e^3*x^3 + 18*e^4*x^4)))/(c*d^2 - a*e^2)^3) + 15*(7*c^2*d^4 + 6*a*c*
d^2*e^2 + 3*a^2*e^4)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*ArcTanh[(Sqrt[c]*
Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(12*c^(7/2)*d^(7/2)*e
^(9/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

3.477.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 511, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1244, 27, 1233, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(d+ex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 1244$$

$$-\frac{2\int -\frac{e^2x^3(8ade+(7cd^2-3ae^2)x)}{2(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3e^3(cd^2-ae^2)} - \frac{2dx^4}{3e(d+ex)(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{x^3(8ade+(7cd^2-3ae^2)x)}{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3e(cd^2-ae^2)} - \frac{2dx^4}{3e(d+ex)(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\downarrow$ 1233

$$\frac{\frac{2\int \frac{x\left(4ade\left(7c^2d^4-12ace^2d^2-3a^2e^4\right)+\left(35c^3d^6-61ac^2e^2d^4+9a^2ce^4d^2-15a^3e^6\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cde(cd^2-ae^2)^2}-\frac{2x^2\left(ade\left(-3a^2e^4-12acd^2e^2+7c^2d^4\right)+x\left(-3a^3e^6-a^2cd^2e^4-1\right.\right.}{cde(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3e\left(cd^2-ae^2\right)}$$

$$\frac{2dx^4}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{x\left(4ade\left(7c^2d^4-12ace^2d^2-3a^2e^4\right)+\left(35c^3d^6-61ac^2e^2d^4+9a^2ce^4d^2-15a^3e^6\right)x\right)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cde(cd^2-ae^2)^2}-\frac{2x^2\left(ade\left(-3a^2e^4-12acd^2e^2+7c^2d^4\right)+x\left(-3a^3e^6-a^2cd^2e^4-11c\right.\right.}{cde(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3e\left(cd^2-ae^2\right)}$$

$$\frac{2dx^4}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 1225

$$\frac{\frac{\frac{15\left(cd^2-ae^2\right)^3\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8c^2d^2e^2}-\frac{\left(-45a^4e^8+30a^3cd^2e^6+36a^2c^2d^4e^4-2cdex\left(-15a^3e^6+9a^2cd^2e^4-61ac^2d^4e^2+35c^3d\right.\right.}{4c^2d^2e^2}}{cde(cd^2-ae^2)^2}}{3e\left(cd^2-a\right.}$$

$$\frac{2dx^4}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 1092

$$\frac{\frac{\frac{15\left(cd^2-ae^2\right)^3\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4c^2d^2e^2}-\frac{\left(-45a^4e^8+30a^3cd^2e^6+36a^2c^2d^4e^4-2cdex\left(-15a^3e\right.\right.}{}}{cde(cd^2-ae^2)^2}}{}$$

$$\frac{2dx^4}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 219

3.477. $\int \frac{x^5}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

$$\frac{\frac{15\left(cd^2-ae^2\right)^3\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{5/2}d^{5/2}e^{5/2}}-\frac{\left(-45a^4e^8+30a^3cd^2e^6+36a^2c^2d^4e^4-2cdex\left(-15a^3e^6+9a^2cd^2e^4\cdots\right.\right.}{cde(cd^2-ae^2)^2}}{3e\,(c\cdots}$$

$$\frac{2dx^4}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[x^5/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*d*x^4)/(3*e*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2]) + ((-2*x^2*(a*d*e*(7*c^2*d^4 - 12*a*c*d^2*e^2 - 3*a^2*e^4) +
(7*c^3*d^6 - 11*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(c*d*e*(c*d
^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (-1/4*((105*c
^4*d^8 - 190*a*c^3*d^6*e^2 + 36*a^2*c^2*d^4*e^4 + 30*a^3*c*d^2*e^6 - 45*a^
4*e^8 - 2*c*d*e*(35*c^3*d^6 - 61*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 15*a^3*
e^6)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^2*d^2*e^2) + (15*(
c*d^2 - a*e^2)^3*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(c*d^2 +
a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])])/(8*c^(5/2)*d^(5/2)*e^(5/2)))/(c*d*e*(c*d^2 - a*e^2)^2))
/(3*e*(c*d^2 - a*e^2))
```

3.477.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

3.477. $\int \frac{x^5}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1244
```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d
_) + (e_.)*(x_)), x_Symbol] :> Simp[(-(e*f - d*g))*(f + g*x)^(n - 1)*((a +
b*x + c*x^2)^(p + 1)/(p*(2*c*d - b*e)*(d + e*x))), x] + Simp[1/(p*e^2*(2*c*
d - b*e))   Int[(f + g*x)^(n - 2)*(a + b*x + c*x^2)^p*Simp[b*e*g*((-e)*f +
d*g + e*f*n - d*g*n - e*f*p) + c*(d^2*g^2*(n - 1) - d*e*f*g*n + e^2*f^2*(2*
p + 1)) - e*g*(b*e*g*p - c*(e*f*n - d*g*n + 2*e*f*p))*x, x], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1] && LtQ[p, -1] && EqQ[c*d^2 - b*d*
e + a*e^2, 0]
```

3.477.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1922 vs. $2(485) = 970$.

Time = 0.87 (sec) , antiderivative size = 1923, normalized size of antiderivative = 3.73

method	result	size
default	Expression too large to display	1923

input
```
int(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(1/2*x^3/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-5/4*(a*e^2+c*d^
2)/c/d/e*(x^2/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/2*(a*e^2+c*d
^2)/c/d/e*(-x/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e^2+c*d
^2)/c/d/e*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/
c/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))+1/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d
*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))-2*a/c*(-
1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/e*(2*c*d
*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)))-3/2*a/c*(-x/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)-1/2*(a*e^2+c*d^2)/c/d/e*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)-(a*e^2+c*d^2)/c/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)
^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))+1/c/d/e*ln((1/2*e^2*a+1/2*c*d
^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)
^(1/2)))+2*d^4/e^5*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)
/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+d^2/e^3*(-x/c/d/e/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e^2+c*d^2)/c/d/e*(-1/c/d/e/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*
c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))+1/c/d/
e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x...
```

3.477.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1053 vs. $2(485) = 970$.

Time = 4.28 (sec) , antiderivative size = 2120, normalized size of antiderivative = 4.12

$$\int \frac{x^5}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
[1/48*(15*(7*a*c^5*d^12*e - 15*a^2*c^4*d^10*e^3 + 6*a^3*c^3*d^8*e^5 + 2*a^
4*c^2*d^6*e^7 + 3*a^5*c*d^4*e^9 - 3*a^6*d^2*e^11 + (7*c^6*d^11*e^2 - 15*a*
c^5*d^9*e^4 + 6*a^2*c^4*d^7*e^6 + 2*a^3*c^3*d^5*e^8 + 3*a^4*c^2*d^3*e^10 -
 3*a^5*c*d*e^12)*x^3 + (14*c^6*d^12*e - 23*a*c^5*d^10*e^3 - 3*a^2*c^4*d^8*
e^5 + 10*a^3*c^3*d^6*e^7 + 8*a^4*c^2*d^4*e^9 - 3*a^5*c*d^2*e^11 - 3*a^6*e^
13)*x^2 + (7*c^6*d^13 - a*c^5*d^11*e^2 - 24*a^2*c^4*d^9*e^4 + 14*a^3*c^3*d
^7*e^6 + 7*a^4*c^2*d^5*e^8 + 3*a^5*c*d^3*e^10 - 6*a^6*d*e^12)*x)*sqrt(c*d*
e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e)
 + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(105*a*c^5*d^11*e^2 - 190*a^2*c^4*d^9*
e^4 + 36*a^3*c^3*d^7*e^6 + 30*a^4*c^2*d^5*e^8 - 45*a^5*c*d^3*e^10 - 6*(c^6
*d^9*e^4 - 3*a*c^5*d^7*e^6 + 3*a^2*c^4*d^5*e^8 - a^3*c^3*d^3*e^10)*x^4 + 3
*(7*c^6*d^10*e^3 - 16*a*c^5*d^8*e^5 + 6*a^2*c^4*d^6*e^7 + 8*a^3*c^3*d^4*e^
9 - 5*a^4*c^2*d^2*e^11)*x^3 + (140*c^6*d^11*e^2 - 237*a*c^5*d^9*e^4 + 12*a
^2*c^4*d^7*e^6 + 66*a^3*c^3*d^5*e^8 - 45*a^5*c*d*e^12)*x^2 + (105*c^6*d^12
*e - 50*a*c^5*d^10*e^3 - 222*a^2*c^4*d^8*e^5 + 84*a^3*c^3*d^6*e^7 + 45*a^4
*c^2*d^4*e^9 - 90*a^5*c*d^2*e^11)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x))/(a*c^7*d^12*e^6 - 3*a^2*c^6*d^10*e^8 + 3*a^3*c^5*d^8*e^10 - a^4*c^
4*d^6*e^12 + (c^8*d^11*e^7 - 3*a*c^7*d^9*e^9 + 3*a^2*c^6*d^7*e^11 - a^3*c^
5*d^5*e^13)*x^3 + (2*c^8*d^12*e^6 - 5*a*c^7*d^10*e^8 + 3*a^2*c^6*d^8*e^...
```

3.477.6 Sympy [F]

$$\int \frac{x^5}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^5}{\left((d+ex)(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(x**5/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x**5/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.477.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.477.8 Giac [F]

$$\int \frac{x^5}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^5}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
integrate(x^5/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)), x
)
```

3.477.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^5}{(d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int(x^5/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(x^5/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

3.478 $\int \frac{x^4}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

3.478.1 Optimal result

Integrand size = 40, antiderivative size = 438

$$\int \frac{x^4}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2dx^3\left(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2x\left(ade(cd^2-ae^2)\left(5c^2d^4-10acd^2e^2-3a^2e^4\right)+\left(cd^2-ae^2\right)\left(5c^3d^6-9ac^2d^4e^2-a^2cd^2e^4-3a^3e^6\right)x\right)}{3cde^2\left(cd^2-ae^2\right)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{\left(15c^3d^6-31ac^2d^4e^2+9a^2cd^2e^4-9a^3e^6\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^2d^2e^3\left(cd^2-ae^2\right)^3} - \frac{\left(5cd^2+3ae^2\right)\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2c^{5/2}d^{5/2}e^{7/2}}$$

output
```
-2/3*d*x^3*(a*e*(-a*e^2+c*d^2)+c*d*(-a*e^2+c*d^2)*x)/e/(-a*e^2+c*d^2)^2/(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/2*(3*a*e^2+5*c*d^2)*arctanh(1/2*(2
*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2))/c^(5/2)/d^(5/2)/e^(7/2)-2/3*x*(a*d*e*(-a*e^2+c*d^2)*(-3*a^2*e
^4-10*a*c*d^2*e^2+5*c^2*d^4)+(-a*e^2+c*d^2)*(-3*a^3*e^6-a^2*c*d^2*e^4-9*a*
c^2*d^4*e^2+5*c^3*d^6)*x)/c/d/e^2/(-a*e^2+c*d^2)^4/(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)+1/3*(-9*a^3*e^6+9*a^2*c*d^2*e^4-31*a*c^2*d^4*e^2+15*c^3*d
^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e^3/(-a*e^2+c*d^2)^3
```

3.478.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.68

$$\int \frac{x^4}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{\sqrt{c}\sqrt{d}\sqrt{e}(ae+cdx)\left(-9a^4e^7(d+ex)^2+3a^3cde^5(3d-ex)(d+ex)^2+c^4d^7x\left(15d^2+\right.\right.}{}$$

input
```
Integrate[x^4/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
((Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-9*a^4*e^7*(d + e*x)^2 + 3*a^3*c*
d*e^5*(3*d - e*x)*(d + e*x)^2 + c^4*d^7*x*(15*d^2 + 20*d*e*x + 3*e^2*x^2)
+ a*c^3*d^5*e*(15*d^3 - 11*d^2*e*x - 39*d*e^2*x^2 - 9*e^3*x^3) + a^2*c^2*d
^3*e^3*(-31*d^3 - 33*d^2*e*x + 9*d*e^2*x^2 + 9*e^3*x^3)))/(c*d^2 - a*e^2)^
3 - 3*(5*c*d^2 + 3*a*e^2)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*ArcTanh[(Sqr
t[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(3*c^(5/2)*d^(5/
2)*e^(7/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

3.478.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 431, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1244, 27, 1233, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1244}$$

$$-\frac{2\int -\frac{e^2x^2\left(6ade+\left(5cd^2-3ae^2\right)x\right)}{2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3e^3\left(cd^2-ae^2\right)}-\frac{2dx^3}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{x^2\left(6ade+\left(5cd^2-3ae^2\right)x\right)}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3e\left(cd^2-ae^2\right)}-\frac{2dx^3}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 1233}$$

$$\frac{\frac{2\int \frac{2ade\left(5c^2d^4-10ace^2d^2-3a^2e^4\right)+\left(15c^3d^6-31ac^2e^2d^4+9a^2ce^4d^2-9a^3e^6\right)x}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cde(cd^2-ae^2)^2}-\frac{2x(ade(-3a^2e^4-10acd^2e^2+5c^2d^4)+x(-3a^3e^6-a^2cd^2e^4-9ac^2d^4}{cde(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3e\left(cd^2-ae^2\right)}$$

$$\frac{2dx^3}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{2ade\left(5c^2d^4-10ace^2d^2-3a^2e^4\right)+\left(15c^3d^6-31ac^2e^2d^4+9a^2ce^4d^2-9a^3e^6\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cde(cd^2-ae^2)^2}-\frac{2x(ade(-3a^2e^4-10acd^2e^2+5c^2d^4)+x(-3a^3e^6-a^2cd^2e^4-9ac^2d^4e^2}{cde(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3e\left(cd^2-ae^2\right)}$$

$$\frac{2dx^3}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 1160

$$\frac{\frac{\frac{\left(-9a^3e^6+9a^2cd^2e^4-31ac^2d^4e^2+15c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cde}-\frac{3\left(cd^2-ae^2\right)^3\left(3ae^2+5cd^2\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}}{cde(cd^2-ae^2)^2}-\frac{2x(ade(-3a^2e^4-10}{cd}}{3e\left(cd^2-ae^2\right)}$$

$$\frac{2dx^3}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 1092

$$\frac{\frac{\frac{\left(-9a^3e^6+9a^2cd^2e^4-31ac^2d^4e^2+15c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cde}-\frac{3\left(cd^2-ae^2\right)^3\left(3ae^2+5cd^2\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+}}}{cde}}{cde(cd^2-ae^2)^2}}{3e\left(cd^2-ae^2\right)}$$

$$\frac{2dx^3}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 219

$$\frac{\frac{\frac{\left(-9a^3e^6+9a^2cd^2e^4-31ac^2d^4e^2+15c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cde}-\frac{3\left(cd^2-ae^2\right)^3\left(3ae^2+5cd^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2c^{3/2}d^{3/2}e^{3/2}}}{cde(cd^2-ae^2)^2}-2x}{3e\left(cd^2-ae^2\right)}$$

$$\frac{2dx^3}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

3.478. $\int \frac{x^4}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

input
```
Int[x^4/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*d*x^3)/(3*e*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2]) + ((-2*x*(a*d*e*(5*c^2*d^4 - 10*a*c*d^2*e^2 - 3*a^2*e^4) + (5
*c^3*d^6 - 9*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(c*d*e*(c*d^2
- a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (((15*c^3*d^6 -
31*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 9*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(c*d*e) - (3*(c*d^2 - a*e^2)^3*(5*c*d^2 + 3*a*e^2)*ArcT
anh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(3/2)*d^(3/2)*e^(3/2)))/(c*d*e*(c*d^2
 - a*e^2)^2))/(3*e*(c*d^2 - a*e^2))
```

3.478.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1244
```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d
_) + (e_.)*(x_)), x_Symbol] :> Simp[(-(e*f - d*g))*(f + g*x)^(n - 1)*((a +
b*x + c*x^2)^(p + 1)/(p*(2*c*d - b*e)*(d + e*x))), x] + Simp[1/(p*e^2*(2*c*
d - b*e))   Int[(f + g*x)^(n - 2)*(a + b*x + c*x^2)^p*Simp[b*e*g*((-e)*f +
d*g + e*f*n - d*g*n - e*f*p) + c*(d^2*g^2*(n - 1) - d*e*f*g*n + e^2*f^2*(2*
p + 1)) - e*g*(b*e*g*p - c*(e*f*n - d*g*n + 2*e*f*p))*x, x], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1] && LtQ[p, -1] && EqQ[c*d^2 - b*d*
e + a*e^2, 0]
```

3.478.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1111 vs. $2(408) = 816$.

Time = 0.86 (sec) , antiderivative size = 1112, normalized size of antiderivative = 2.54

method	result	size
default	Expression too large to display	1112

input
```
int(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(x^2/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/2*(a*e^2+c*d^2)/c
/d/e*(-x/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e^2+c*d^2)/c
/d/e*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/e
*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2))+1/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(
1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))-2*a/c*(-1/c/d
/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/e*(2*c*d*e*x+
a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)))+d^2/e^3*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*
e^2+c*d^2)/c/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))-2*d^3/e^4*(2*c*d*e*x+a*e^2+c*d^2)/
(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-d/
e^2*(-x/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e^2+c*d^2)/c/
d/e*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/e*
(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2))+1/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1
/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+d^4/e^5*(-2/3/
(a*e^2-c*d^2)/(x+d/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*c*
d*e/(a*e^2-c*d^2)^3*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/(c*d*e*(x+d/e)^2+(a*e^2-
c*d^2)*(x+d/e))^(1/2))
```

3.478.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 884 vs. $2(408) = 816$.

Time = 1.67 (sec) , antiderivative size = 1782, normalized size of antiderivative = 4.07

$$\int \frac{x^4}{(d+ex)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
[1/12*(3*(5*a*c^4*d^10*e - 12*a^2*c^3*d^8*e^3 + 6*a^3*c^2*d^6*e^5 + 4*a^4*
c*d^4*e^7 - 3*a^5*d^2*e^9 + (5*c^5*d^9*e^2 - 12*a*c^4*d^7*e^4 + 6*a^2*c^3*
d^5*e^6 + 4*a^3*c^2*d^3*e^8 - 3*a^4*c*d*e^10)*x^3 + (10*c^5*d^10*e - 19*a*
c^4*d^8*e^3 + 14*a^3*c^2*d^4*e^7 - 2*a^4*c*d^2*e^9 - 3*a^5*e^11)*x^2 + (5*
c^5*d^11 - 2*a*c^4*d^9*e^2 - 18*a^2*c^3*d^7*e^4 + 16*a^3*c^2*d^5*e^6 + 5*a
^4*c*d^3*e^8 - 6*a^5*d*e^10)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^
4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) +
 4*(15*a*c^4*d^9*e^2 - 31*a^2*c^3*d^7*e^4 + 9*a^3*c^2*d^5*e^6 - 9*a^4*c*d^
3*e^8 + 3*(c^5*d^8*e^3 - 3*a*c^4*d^6*e^5 + 3*a^2*c^3*d^4*e^7 - a^3*c^2*d^2
*e^9)*x^3 + (20*c^5*d^9*e^2 - 39*a*c^4*d^7*e^4 + 9*a^2*c^3*d^5*e^6 + 3*a^3
*c^2*d^3*e^8 - 9*a^4*c*d*e^10)*x^2 + (15*c^5*d^10*e - 11*a*c^4*d^8*e^3 - 3
3*a^2*c^3*d^6*e^5 + 15*a^3*c^2*d^4*e^7 - 18*a^4*c*d^2*e^9)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*c^6*d^11*e^5 - 3*a^2*c^5*d^9*e^7 + 3*a
^3*c^4*d^7*e^9 - a^4*c^3*d^5*e^11 + (c^7*d^10*e^6 - 3*a*c^6*d^8*e^8 + 3*a^
2*c^5*d^6*e^10 - a^3*c^4*d^4*e^12)*x^3 + (2*c^7*d^11*e^5 - 5*a*c^6*d^9*e^7
 + 3*a^2*c^5*d^7*e^9 + a^3*c^4*d^5*e^11 - a^4*c^3*d^3*e^13)*x^2 + (c^7*d^1
2*e^4 - a*c^6*d^10*e^6 - 3*a^2*c^5*d^8*e^8 + 5*a^3*c^4*d^6*e^10 - 2*a^4*c^
3*d^4*e^12)*x), 1/6*(3*(5*a*c^4*d^10*e - 12*a^2*c^3*d^8*e^3 + 6*a^3*c^2*d^
6*e^5 + 4*a^4*c*d^4*e^7 - 3*a^5*d^2*e^9 + (5*c^5*d^9*e^2 - 12*a*c^4*d^7...
```

3.478.6 Sympy [F]

$$\int \frac{x^4}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^4}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(x**4/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x**4/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.478.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.478.8 Giac [F]

$$\int \frac{x^4}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^4}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
integrate(x^4/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)), x
)
```

3.478.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^4}{(d+e\,x)\,\left(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int(x^4/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(x^4/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

3.479 $\int \frac{x^3}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

3.479.1 Optimal result

Integrand size = 40, antiderivative size = 297

$$\int \frac{x^3}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = -\frac{2dx^2(ae(cd^2-ae^2)+cd(cd^2-ae^2)x)}{3e\left(cd^2-ae^2\right)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}} - \frac{2(ade(cd^2-3ae^2)\left(3cd^2+ae^2\right)+\left(3c^3d^6-7ac^2d^4e^2-a^2cd^2e^4-3a^3e^6\right)x)}{3cde^2\left(cd^2-ae^2\right)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{3/2}d^{3/2}e^{5/2}}$$

output
```
-2/3*d*x^2*(a*e*(-a*e^2+c*d^2)+c*d*(-a*e^2+c*d^2)*x)/e/(-a*e^2+c*d^2)^2/(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/
c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3/2)/d
^(3/2)/e^(5/2)-2/3*(a*d*e*(-3*a*e^2+c*d^2)*(a*e^2+3*c*d^2)+(-3*a^3*e^6-a^2
*c*d^2*e^4-7*a*c^2*d^4*e^2+3*c^3*d^6)*x)/c/d/e^2/(-a*e^2+c*d^2)^3/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

3.479.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{2\left(-\frac{\sqrt{c}\sqrt{d}\sqrt{e}(ae+cdx)\left(-3a^3e^5(d+ex)^2+c^3d^6x(3d+4ex)-a^2cd^3e^3(8d+9ex\right.}{(cd^2-ae^2)^3}\right.}{3c^{3/2}d^{3/2}e}$$

input
```
Integrate[x^3/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*(-((Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-3*a^3*e^5*(d + e*x)^2 + c^3
*d^6*x*(3*d + 4*e*x) - a^2*c*d^3*e^3*(8*d + 9*e*x) + a*c^2*d^4*e*(3*d^2 -
4*d*e*x - 9*e^2*x^2)))/(c*d^2 - a*e^2)^3) + 3*(a*e + c*d*x)^(3/2)*(d + e*x
)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x]
)]))/(3*c^(3/2)*d^(3/2)*e^(5/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

3.479.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1244, 27, 1224, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1244}$$

$$-\frac{2\int -\frac{e^2x\left(4ade+3\left(cd^2-ae^2\right)x\right)}{2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3e^3\left(cd^2-ae^2\right)}-\frac{2dx^2}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x\left(4ade+3\left(cd^2-ae^2\right)x\right)}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3e\left(cd^2-ae^2\right)}-\frac{2dx^2}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1224}$$

$$\frac{3\left(\frac{d}{e}-\frac{ae}{cd}\right)\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\frac{2(x(-3a^3e^6-a^2cd^2e^4-7ac^2d^4e^2+3c^3d^6)+ade(cd^2-3ae^2)(ae^2+3cd^2))}{cde(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3e(cd^2-ae^2)}-\frac{2dx^2}{3e(d+ex)(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\downarrow$ 1092

$$\frac{6\left(\frac{d}{e}-\frac{ae}{cd}\right)\int \frac{1}{4cde-\frac{(cd^2+2cexd+ae^2)^2}{cdex^2+(cd^2+ae^2)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-\frac{2(x(-3a^3e^6-a^2cd^2e^4-7ac^2d^4e^2+3c^3d^6)+ade(cd^2-3ae^2)(ae^2+3}{cde(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3e(cd^2-ae^2)}$$
$$\frac{2dx^2}{3e(d+ex)(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\downarrow$ 219

$$\frac{\frac{3\left(\frac{d}{e}-\frac{ae}{cd}\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{e}}-\frac{2(x(-3a^3e^6-a^2cd^2e^4-7ac^2d^4e^2+3c^3d^6)+ade(cd^2-3ae^2)(ae^2+3cd^2))}{cde(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3e(cd^2-ae^2)}-\frac{2dx^2}{3e(d+ex)(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

input
```
Int[x^3/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*d*x^2)/(3*e*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2]) + ((-2*(a*d*e*(c*d^2 - 3*a*e^2)*(3*c*d^2 + a*e^2) + (3*c^3*d^
6 - 7*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(c*d*e*(c*d^2 - a*e^2
)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (3*(d/e - (a*e)/(c*d))*
ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(Sqrt[c]*Sqrt[d]*Sqrt[e]))/(3*e*(c*d^2
 - a*e^2))
```

3.479. $\int \frac{x^3}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

3.479.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1224
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c))   Int[(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -
1] &&  !(IntegerQ[p] && NeQ[a, 0] && NiceSqrtQ[b^2 - 4*a*c])
```

rule 1244
```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d
_) + (e_.)*(x_)), x_Symbol] :> Simp[(-(e*f - d*g))*(f + g*x)^(n - 1)*((a +
b*x + c*x^2)^(p + 1)/(p*(2*c*d - b*e)*(d + e*x))), x] + Simp[1/(p*e^2*(2*c*
d - b*e))   Int[(f + g*x)^(n - 2)*(a + b*x + c*x^2)^p*Simp[b*e*g*((-e)*f +
d*g + e*f*n - d*g*n - e*f*p) + c*(d^2*g^2*(n - 1) - d*e*f*g*n + e^2*f^2*(2*
p + 1)) - e*g*(b*e*g*p - c*(e*f*n - d*g*n + 2*e*f*p))*x, x], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1] && LtQ[p, -1] && EqQ[c*d^2 - b*d*
e + a*e^2, 0]
```

3.479. $\int \frac{x^3}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

3.479.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(273) = 546$.

Time = 0.69 (sec) , antiderivative size = 639, normalized size of antiderivative = 2.15

method	result
default	$\frac{-\frac{x}{cde\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}}}-\frac{\left(e^{2}a+c\,d^{2}\right)\left(-\frac{1}{cde\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}}}-\frac{\left(e^{2}a+c\,d^{2}\right)\left(2cdex+e^{2}a+c\,d^{2}\right)}{cde\left(4ac\,d^{2}e^{2}-\left(e^{2}a+c\,d^{2}\right)^{2}\right)\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}}}\right.}{2cde}}{e}$

input
```
int(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(-x/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e^2+c*d^2)/c/
d/e*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/e*
(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2))+1/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1
/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+2*d^2/e^3*(2*c
*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)-d/e^2*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-
(a*e^2+c*d^2)/c/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2
)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))-d^3/e^4*(-2/3/(a*e^2-c*d^2)/(x+
d/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*c*d*e/(a*e^2-c*d^2)
^3*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(
1/2))
```

3.479.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(273) = 546$.

Time = 2.06 (sec) , antiderivative size = 1466, normalized size of antiderivative = 4.94

$$\int \frac{x^3}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
[1/6*(3*(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 +
 (c^4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (
2*c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^
9)*x^2 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 -
2*a^4*d*e^8)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^
2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c
*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(3*a*c^3*d^7*
e^2 - 8*a^2*c^2*d^5*e^4 - 3*a^3*c*d^3*e^6 + (4*c^4*d^7*e^2 - 9*a*c^3*d^5*e
^4 - 3*a^3*c*d*e^8)*x^2 + (3*c^4*d^8*e - 4*a*c^3*d^6*e^3 - 9*a^2*c^2*d^4*e
^5 - 6*a^3*c*d^2*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*c
^5*d^10*e^4 - 3*a^2*c^4*d^8*e^6 + 3*a^3*c^3*d^6*e^8 - a^4*c^2*d^4*e^10 + (
c^6*d^9*e^5 - 3*a*c^5*d^7*e^7 + 3*a^2*c^4*d^5*e^9 - a^3*c^3*d^3*e^11)*x^3
+ (2*c^6*d^10*e^4 - 5*a*c^5*d^8*e^6 + 3*a^2*c^4*d^6*e^8 + a^3*c^3*d^4*e^10
 - a^4*c^2*d^2*e^12)*x^2 + (c^6*d^11*e^3 - a*c^5*d^9*e^5 - 3*a^2*c^4*d^7*e
^7 + 5*a^3*c^3*d^5*e^9 - 2*a^4*c^2*d^3*e^11)*x), -1/3*(3*(a*c^3*d^8*e - 3*
a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3*a*c^3*d
^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a*c^3*d^6
*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9 - a*c^3
*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)*sqrt(-c*d
*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x +...
```

3.479.6 Sympy [F]

$$\int \frac{x^3}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx=\int \frac{x^3}{\left((d+ex)(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x**3/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.479.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.479.8 Giac [F]

$$\int \frac{x^3}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^3}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
integrate(x^3/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)), x
)
```

3.479.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^3}{(d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

3.480 $\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

3.480.1 Optimal result

Integrand size = 40, antiderivative size = 126

$$\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2x^2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{8ae\left(2ade+\left(cd^2+ae^2\right)x\right)}{3\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
2/3*x^2/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-8/3
*a*e*(2*a*d*e+(a*e^2+c*d^2)*x)/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)
```

3.480.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2(ae+cdx)^3\left(d^2-\frac{6ade(d+ex)}{ae+cdx}-\frac{3a^2e^2(d+ex)^2}{(ae+cdx)^2}\right)}{3\left(cd^2-ae^2\right)^3((ae+cdx)(d+ex))^{3/2}}$$

input
```
Integrate[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*(a*e + c*d*x)^3*(d^2 - (6*a*d*e*(d + e*x))/(a*e + c*d*x) - (3*a^2*e^2*(
d + e*x)^2)/(a*e + c*d*x)^2))/(3*(c*d^2 - a*e^2)^3*((a*e + c*d*x)*(d + e*x
))^(3/2))
```

3.480.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1243, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1243}$$

$$\frac{2x^2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{4ae\int \frac{x}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1158}$$

$$\frac{2x^2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8ae\left(x\left(ae^2+cd^2\right)+2ade\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*x^2)/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]) - (8*a*e*(2*a*d*e + (c*d^2 + a*e^2)*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

3.480.3.1 Defintions of rubi rules used

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1243
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-(2*c*d - b*e))*(f + g*x)^n*((a + b*
x + c*x^2)^(p + 1)/(e*p*(b^2 - 4*a*c)*(d + e*x))), x] + Simp[n*((a*g*(2*c*d
 - b*e) - c*f*(b*d - 2*a*e))/(d*e*p*(b^2 - 4*a*c)))   Int[(f + g*x)^(n - 1)
*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[n, 1] && LtQ[p, -1] && EqQ[n + 2*p + 1, 0]
```

3.480.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.15

method	result
gosper	$\frac{2(cdx+ae)(3a^2e^4x^2+6ac\,d^2e^2x^2-c^2d^4x^2+12a^2d\,e^3x+4ac\,d^3ex+8a^2d^2e^2)}{3(a^3e^6-3a^2c\,d^2e^4+3a\,c^2d^4e^2-c^3d^6)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$
trager	$\frac{2(3a^2e^4x^2+6ac\,d^2e^2x^2-c^2d^4x^2+12a^2d\,e^3x+4ac\,d^3ex+8a^2d^2e^2)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{3(a^2e^4-2ac\,d^2e^2+c^2d^4)(ex+d)^2(e^2a-c\,d^2)(cdx+ae)}$
default	$\frac{-\frac{1}{cde\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}-\frac{\left(e^2a+c\,d^2\right)\left(2cdex+e^2a+c\,d^2\right)}{cde\left(4ac\,d^2e^2-\left(e^2a+c\,d^2\right)^2\right)\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}}{e}-\frac{2d(2cdex+e^2a+c\,d^2)}{e^2\left(4ac\,d^2e^2-(e^2a+c\,d^2)^2\right)\sqrt{ade+(e^2a+}$

input
```
int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
2/3*(c*d*x+a*e)*(3*a^2*e^4*x^2+6*a*c*d^2*e^2*x^2-c^2*d^4*x^2+12*a^2*d*e^3*
x+4*a*c*d^3*e*x+8*a^2*d^2*e^2)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^
3*d^6)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

3.480.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(118) = 236$.

Time = 1.68 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.44

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\left(8a^2d^2e^2-(c^2d^4-6acd^2e^2-3a^2e^4)x^2+4(acd^3\right.}{3\left(ac^3d^8e-3a^2c^2d^6e^3+3a^3cd^4e^5-a^4d^2e^7+(c^4d^7e^2-3ac^3d^5e^4+3a^2c^2d^3e^6-a^3cde^8)x^3+(2c^4d^8e-5\right.}$$

input
```
integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
-2/3*(8*a^2*d^2*e^2 - (c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*x^2 + 4*(a*c*d
^3*e + 3*a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c^3*
d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 -
 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5
*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d
^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)
```

3.480.6 Sympy [F]

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^2}{\left((d+ex)(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x**2/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.480.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

3.480.8 Giac [F]

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^2}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
integrate(x^2/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)), x
)
```

3.480.9 Mupad [B] (verification not implemented)

Time = 12.58 (sec) , antiderivative size = 1071, normalized size of antiderivative = 8.50

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{4\,c\,d^3\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\ldots}}{3\,\left(a^3\,d\,e^7+x\,a^3\,e^8-3\,a^2\,c\,d^3\,e^5-3\,x\,a^2\,c\,d^2\,e^6+3\,a\,c^2\,d^5\ldots\right.}$$

$$-\frac{2\,d^2\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}}{3\,a^2\,d^2\,e^5+6\,a^2\,d\,e^6\,x+3\,a^2\,e^7\,x^2-6\,a\,c\,d^4\,e^3-12\,a\,c\,d^3\,e^4\,x-6\,a\,c\,d^2\,e^5\,x^2+3\,c^2\,d^6\,e+6\,c^2\,d^5\,e^2\,x+3\,c^2\ldots}$$

$$-\frac{4\,a\,d\,e^2\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}}{3\,\left(a^3\,d\,e^7+x\,a^3\,e^8-3\,a^2\,c\,d^3\,e^5-3\,x\,a^2\,c\,d^2\,e^6+3\,a\,c^2\,d^5\,e^3+3\,x\,a\,c^2\,d^4\,e^4-c^3\,d^7\,e-x\,c^3\,d^6\,e^2\right)}$$

$$+\frac{2\,c^4\,d^7\,x}{\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}\,\left(a^4\,c\,d\,e^9-4\,a^3\,c^2\,d^3\,e^7+6\,a^2\,c^3\,d^5\,e^5-4\,a\,c^4\,d^7\,e^3+c^5\,d^9\,e\right)}$$

$$+\frac{22\,a^3\,c\,d^2\,e^5}{3\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}\,\left(a^4\,c\,d\,e^9-4\,a^3\,c^2\,d^3\,e^7+6\,a^2\,c^3\,d^5\,e^5-4\,a\,c^4\,d^7\,e^3+c^5\,d^9\,e\right)}$$

$$-\frac{28\,a^2\,c^2\,d^4\,e^3}{3\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}\,\left(a^4\,c\,d\,e^9-4\,a^3\,c^2\,d^3\,e^7+6\,a^2\,c^3\,d^5\,e^5-4\,a\,c^4\,d^7\,e^3+c^5\,d^9\,e\right)}$$

$$+\frac{2\,a\,c^3\,d^6\,e}{\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}\,\left(a^4\,c\,d\,e^9-4\,a^3\,c^2\,d^3\,e^7+6\,a^2\,c^3\,d^5\,e^5-4\,a\,c^4\,d^7\,e^3+c^5\,d^9\,e\right)}$$

$$+\frac{10\,a^2\,c^2\,d^3\,e^4\,x}{3\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}\,\left(a^4\,c\,d\,e^9-4\,a^3\,c^2\,d^3\,e^7+6\,a^2\,c^3\,d^5\,e^5-4\,a\,c^4\,d^7\,e^3+c^5\,d^9\,e\right)}$$

$$+\frac{2\,a^3\,c\,d\,e^6\,x}{\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}\,\left(a^4\,c\,d\,e^9-4\,a^3\,c^2\,d^3\,e^7+6\,a^2\,c^3\,d^5\,e^5-4\,a\,c^4\,d^7\,e^3+c^5\,d^9\,e\right)}$$

$$-\frac{22\,a\,c^3\,d^5\,e^2\,x}{3\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}\,\left(a^4\,c\,d\,e^9-4\,a^3\,c^2\,d^3\,e^7+6\,a^2\,c^3\,d^5\,e^5-4\,a\,c^4\,d^7\,e^3+c^5\,d^9\,e\right)}$$

input
```
int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
(4*c*d^3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(3*(a^3*d*e^7 - c^
3*d^7*e + a^3*e^8*x + 3*a*c^2*d^5*e^3 - 3*a^2*c*d^3*e^5 - c^3*d^6*e^2*x +
3*a*c^2*d^4*e^4*x - 3*a^2*c*d^2*e^6*x)) - (2*d^2*(a*d*e + a*e^2*x + c*d^2*
x + c*d*e*x^2)^(1/2))/(3*c^2*d^6*e + 3*a^2*d^2*e^5 + 3*a^2*e^7*x^2 + 6*c^2
*d^5*e^2*x + 3*c^2*d^4*e^3*x^2 - 6*a*c*d^4*e^3 + 6*a^2*d*e^6*x - 12*a*c*d^
3*e^4*x - 6*a*c*d^2*e^5*x^2) - (4*a*d*e^2*(a*d*e + a*e^2*x + c*d^2*x + c*d
*e*x^2)^(1/2))/(3*(a^3*d*e^7 - c^3*d^7*e + a^3*e^8*x + 3*a*c^2*d^5*e^3 - 3
*a^2*c*d^3*e^5 - c^3*d^6*e^2*x + 3*a*c^2*d^4*e^4*x - 3*a^2*c*d^2*e^6*x)) +
 (2*c^4*d^7*x)/((a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*d^9*e -
 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9)) +
 (22*a^3*c*d^2*e^5)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*
d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*
e^9)) - (28*a^2*c^2*d^4*e^3)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1
/2)*(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 +
 a^4*c*d*e^9)) + (2*a*c^3*d^6*e)/((a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^
(1/2)*(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7
 + a^4*c*d*e^9)) + (10*a^2*c^2*d^3*e^4*x)/(3*(a*d*e + a*e^2*x + c*d^2*x +
c*d*e*x^2)^(1/2)*(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*
c^2*d^3*e^7 + a^4*c*d*e^9)) + (2*a^3*c*d*e^6*x)/((a*d*e + a*e^2*x + c*d^2*
x + c*d*e*x^2)^(1/2)*(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 -...
```

3.480. $\int \frac{x^2}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

3.481 $\int \frac{x}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

3.481.1 Optimal result

Integrand size = 38, antiderivative size = 138

$$\int \frac{x}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2d}{3e\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{2(cd^2+3ae^2)\left(cd^2+ae^2+2cdex\right)}{3e\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
-2/3*d/e/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2/
3*(3*a*e^2+c*d^2)*(2*c*d*e*x+a*e^2+c*d^2)/e/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)
```

3.481.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.72

$$\int \frac{x}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2(c^2d^3x(3d+2ex)+a^2e^3(2d+3ex)+2acde(3d^2+5dex}{3\left(cd^2-ae^2\right)^3(d+ex)\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[x/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*(c^2*d^3*x*(3*d + 2*e*x) + a^2*e^3*(2*d + 3*e*x) + 2*a*c*d*e*(3*d^2 + 5
*d*e*x + 3*e^2*x^2)))/(3*(c*d^2 - a*e^2)^3*(d + e*x)*Sqrt[(a*e + c*d*x)*(d
 + e*x)])
```

3.481.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1220, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1220}$$

$$-\frac{\left(3ae^2+cd^2\right)\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3e\left(cd^2-ae^2\right)}-\frac{2d}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{2\left(3ae^2+cd^2\right)\left(ae^2+cd^2+2cdex\right)}{3e\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2d}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[x/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*d)/(3*e*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]) + (2*(c*d^2 + 3*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*e*(c*d^2 -
 a*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

3.481.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

3.481.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.08

method	result
gosper	$-\frac{2(cdx+ae)\left(6acd\,e^{3}x^{2}+2c^{2}d^{3}e\,x^{2}+3a^{2}e^{4}x+10ac\,d^{2}e^{2}x+3c^{2}d^{4}x+2d\,e^{3}a^{2}+6ac\,d^{3}e\right)}{3(a^{3}e^{6}-3a^{2}c\,d^{2}e^{4}+3a\,c^{2}d^{4}e^{2}-c^{3}d^{6})(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{3}{2}}}$
trager	$-\frac{2\left(6acd\,e^{3}x^{2}+2c^{2}d^{3}e\,x^{2}+3a^{2}e^{4}x+10ac\,d^{2}e^{2}x+3c^{2}d^{4}x+2d\,e^{3}a^{2}+6ac\,d^{3}e\right)\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}}{3(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4})(ex+d)^{2}(e^{2}a-c\,d^{2})(cdx+ae)}$
default	$\frac{4cdex+2e^{2}a+2c\,d^{2}}{e\left(4ac\,d^{2}e^{2}-(e^{2}a+c\,d^{2})^{2}\right)\sqrt{ade+(e^{2}a+c\,d^{2})x+cde\,x^{2}}}-\frac{d\left(-\frac{2}{3\left(e^{2}a-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\sqrt{cde\left(x+\frac{d}{e}\right)^{2}+\left(e^{2}a-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}+\frac{8cde(2}{3\left(e^{2}a-c\,d^{2}\right)^{3}\sqrt{}}\right)}{e^{2}}$

input
```
int(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
-2/3*(c*d*x+a*e)*(6*a*c*d*e^3*x^2+2*c^2*d^3*e*x^2+3*a^2*e^4*x+10*a*c*d^2*e
^2*x+3*c^2*d^4*x+2*a^2*d*e^3+6*a*c*d^3*e)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2
*d^4*e^2-c^3*d^6)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

3.481.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(130) = 260.

Time = 1.87 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.28

$$\int \frac{x}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{2\,(6\,acd^3e}{3\,(ac^3d^8e-3\,a^2c^2d^6e^3+3\,a^3cd^4e^5-a^4d^2e^7+(c^4d^7e^2-3}$$

input
```
integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
fricas")
```

output
```
2/3*(6*a*c*d^3*e + 2*a^2*d*e^3 + 2*(c^2*d^3*e + 3*a*c*d*e^3)*x^2 + (3*c^2*
d^4 + 10*a*c*d^2*e^2 + 3*a^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)/(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 +
(c^4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2
*c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9
)*x^2 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2
*a^4*d*e^8)*x)
```

3.481.6 Sympy [F]

$$\int \frac{x}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x}{\left((d+ex)(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.481.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

3.481.8 Giac [F]

$$\int \frac{x}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{x}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
giac")
```

output
```
integrate(x/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)), x)
```

3.481.9 Mupad [B] (verification not implemented)

Time = 12.48 (sec) , antiderivative size = 499, normalized size of antiderivative = 3.62

$$\int \frac{x}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{4\,a^2\,d\,e^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}+6\,a^2\,e^4\,x\,\sqrt{c}}{-3\,a^4\,d^2\,e^7-6\,a^4\,d\,e^8\,x}$$

input
```
int(x/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
(4*a^2*d*e^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + 6*a^2*e^4*x*(
x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + 6*c^2*d^4*x*(x*(a*e^2 + c*d
^2) + a*d*e + c*d*e*x^2)^(1/2) + 4*c^2*d^3*e*x^2*(x*(a*e^2 + c*d^2) + a*d*
e + c*d*e*x^2)^(1/2) + 12*a*c*d^3*e*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2) + 20*a*c*d^2*e^2*x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) +
 12*a*c*d*e^3*x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(3*c^4*d^
9*x - 3*a^4*d^2*e^7 - 3*a^4*e^9*x^2 + 9*a^3*c*d^4*e^5 + 6*c^4*d^8*e*x^2 -
9*a^2*c^2*d^6*e^3 + 3*c^4*d^7*e^2*x^3 + 3*a*c^3*d^8*e - 6*a^4*d*e^8*x + 9*
a^2*c^2*d^4*e^5*x^2 + 9*a^2*c^2*d^3*e^6*x^3 - 3*a*c^3*d^7*e^2*x + 15*a^3*c
*d^3*e^6*x - 3*a^3*c*d*e^8*x^3 - 9*a^2*c^2*d^5*e^4*x - 15*a*c^3*d^6*e^3*x^
2 + 3*a^3*c*d^2*e^7*x^2 - 9*a*c^3*d^5*e^4*x^3)
```

3.481. $\int \frac{x}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

3.482 $\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

3.482.1 Optimal result

Integrand size = 37, antiderivative size = 121

$$\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{8cd(cd^2+ae^2+2cdex)}{3\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
2/3/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-8/3*c*d
*(2*c*d*e*x+a*e^2+c*d^2)/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)
```

3.482.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2a^2e^4-4acde^2(3d+2ex)-2c^2d^2(3d^2+12dex+8e^2x^2)}{3\left(cd^2-ae^2\right)^3(d+ex)\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*a^2*e^4 - 4*a*c*d*e^2*(3*d + 2*e*x) - 2*c^2*d^2*(3*d^2 + 12*d*e*x + 8*e
^2*x^2))/(3*(c*d^2 - a*e^2)^3*(d + e*x)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.482.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1129}$$

$$\frac{4cd\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3\left(cd^2-ae^2\right)}+\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8cd\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
2/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
) - (8*c*d*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])
```

3.482.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

3.482.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.14

method	result	size
gosper	$-\frac{2(cdx+ae)\left(-8c^2d^2e^2x^2-4acd\,e^3x-12c^2d^3ex+a^2e^4-6ac\,d^2e^2-3c^2d^4\right)}{3(a^3e^6-3a^2c\,d^2e^4+3a\,c^2d^4e^2-c^3d^6)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$	138
default	$\frac{-\frac{2}{3\left(e^2a-c\,d^2\right)\left(x+\frac{d}{e}\right)\sqrt{cde\left(x+\frac{d}{e}\right)^2+\left(e^2a-c\,d^2\right)\left(x+\frac{d}{e}\right)}}+\frac{8cde\left(2cde\left(x+\frac{d}{e}\right)+e^2a-c\,d^2\right)}{3\left(e^2a-c\,d^2\right)^3\sqrt{cde\left(x+\frac{d}{e}\right)^2+\left(e^2a-c\,d^2\right)\left(x+\frac{d}{e}\right)}}}{e}$	146
trager	$-\frac{2\left(-8c^2d^2e^2x^2-4acd\,e^3x-12c^2d^3ex+a^2e^4-6ac\,d^2e^2-3c^2d^4\right)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{3(a^2e^4-2ac\,d^2e^2+c^2d^4)(ex+d)^2(e^2a-c\,d^2)(cdx+ae)}$	146

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
-2/3*(c*d*x+a*e)*(-8*c^2*d^2*e^2*x^2-4*a*c*d*e^3*x-12*c^2*d^3*e*x+a^2*e^4-
6*a*c*d^2*e^2-3*c^2*d^4)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*d^6)
/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

3.482.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(113) = 226$.

Time = 2.12 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.53

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx =$$

$$-\frac{2\left(8\,c^2d^2e^2x^2+3\,c^2d^4+6\,acd^2e^2-a^2e^4+4\left(3\,c^2d^3\right.\right.}{3\left(ac^3d^8e-3\,a^2c^2d^6e^3+3\,a^3cd^4e^5-a^4d^2e^7+(c^4d^7e^2-3\,ac^3d^5e^4+3\,a^2c^2d^3e^6-a^3cde^8)x^3+(2\,c^4d^8e-5\right.}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
fricas")
```

output
```
-2/3*(8*c^2*d^2*e^2*x^2 + 3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4 + 4*(3*c^2*d
^3*e + a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c^3*d^
8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3
*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a
*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9
 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)
```

3.482.6 Sympy [F]

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.482.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

3.482.8 Giac [F]

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)), x)
```

3.482.9 Mupad [B] (verification not implemented)

Time = 12.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(-a^2\,e^4+6\,a\,c\,d^2\,e^2+\right.}{3\,\left(a\,e+c\,d\,x\right)\,\left(a\,e^2-c\right.}$$

input
```
int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
(2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(3*c^2*d^4 - a^2*e^4 + 8*
c^2*d^2*e^2*x^2 + 6*a*c*d^2*e^2 + 12*c^2*d^3*e*x + 4*a*c*d*e^3*x))/(3*(a*e
 + c*d*x)*(a*e^2 - c*d^2)^3*(d + e*x)^2)
```

3.483 $\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

3.483.1 Optimal result

Integrand size = 40, antiderivative size = 271

$$\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2(3c^3d^6+ac^2d^4e^2+7a^2cd^2e^4-3a^3e^6+cde(3cd^2-ae^2)\left(cd^2+3ae^2\right)x)}{3ad^2e\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\operatorname{arctanh}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{a^{3/2}d^{5/2}e^{3/2}}$$

output
```
-2/3*e*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)-arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/d^(5/2)/e^(3/2)+2/3*(3*c^3*d^6+a*c^
2*d^4*e^2+7*a^2*c*d^2*e^4-3*a^3*e^6+c*d*e*(-a*e^2+3*c*d^2)*(3*a*e^2+c*d^2)
*x)/a/d^2/e/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

3.483.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.80

$$\int \frac{1}{x(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(ae+cdx)(-3c^3d^5(d+ex)^2+a^3e^6(4d+3ex)-ac^2d^3e^3x(9d+8ex)}{(-cd^2+ae^2)^3}\right.}{3a^{3/2}d^{5/2}}$$

input
```
Integrate[1/(x*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-3*c^3*d^5*(d + e*x)^2 + a^3*e
^6*(4*d + 3*e*x) - a*c^2*d^3*e^3*x*(9*d + 8*e*x) + a^2*c*d*e^4*(-9*d^2 - 4
*d*e*x + 3*e^2*x^2)))/(-(c*d^2) + a*e^2)^3 - 3*(a*e + c*d*x)^(3/2)*(d + e*
x)^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x
])]))/(3*a^(3/2)*d^(5/2)*e^(3/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

3.483.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1246, 27, 1235, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 1246$$

$$\frac{2\int \frac{e\left(3\left(cd^2-ae^2\right)-4cdex\right)}{2x\left(cdex^2+(cd^2+ae^2)x+ade\right)^{3/2}}dx}{3de\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{3\left(cd^2-ae^2\right)-4cdex}{x\left(cdex^2+(cd^2+ae^2)x+ade\right)^{3/2}}dx}{3d\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 1235$$

$$\frac{\frac{2\left(-3a^3e^6+7a^2cd^2e^4+ac^2d^4e^2+cdex\left(3cd^2-ae^2\right)\left(3ae^2+cd^2\right)+3c^3d^6\right)}{ade(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\int -\frac{3\left(cd^2-ae^2\right)^3}{2x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{ade(cd^2-ae^2)^2}}{3d\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\big\downarrow$ 27

$$\frac{\frac{3\left(cd^2-ae^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{ade}+\frac{2\left(-3a^3e^6+7a^2cd^2e^4+ac^2d^4e^2+cdex\left(3cd^2-ae^2\right)\left(3ae^2+cd^2\right)+3c^3d^6\right)}{ade(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3d\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\big\downarrow$ 1154

$$\frac{\frac{2\left(-3a^3e^6+7a^2cd^2e^4+ac^2d^4e^2+cdex\left(3cd^2-ae^2\right)\left(3ae^2+cd^2\right)+3c^3d^6\right)}{ade(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{6\left(cd^2-ae^2\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade}}{3d\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\big\downarrow$ 219

$$\frac{\frac{2\left(-3a^3e^6+7a^2cd^2e^4+ac^2d^4e^2+cdex\left(3cd^2-ae^2\right)\left(3ae^2+cd^2\right)+3c^3d^6\right)}{ade(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{3\left(cd^2-ae^2\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{a^{3/2}d^{3/2}e^{3/2}}}{3d\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[1/(x*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2)) + ((2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 7*a^2*c*d^2*e^4 - 3*a^3*
e^6 + c*d*e*(3*c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2)*x))/(a*d*e*(c*d^2 - a*e^2)
^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (3*(c*d^2 - a*e^2)*ArcTa
nh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])])/(a^(3/2)*d^(3/2)*e^(3/2)))/(3*d*(c*d^2 -
a*e^2))
```

3.483. $\int \frac{1}{x(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

3.483.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1246
```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> Simp[(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p*
((c*d - b*e - c*e*x)/(p*(2*c*d - b*e)*(e*f - d*g))), x] + Simp[1/(p*(2*c*d
- b*e)*(e*f - d*g))   Int[(f + g*x)^n*(a + b*x + c*x^2)^p*(b*e*g*(n + p + 1
) + c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x), x], x]
/; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ
[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n, 0]
```

3.483. $\int \frac{1}{x(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

3.483.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.33

method	result
default	$\frac{\frac{1}{ade\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}-\frac{\left(e^2a+c\,d^2\right)\left(2cdex+e^2a+c\,d^2\right)}{ade\left(4ac\,d^2e^2-\left(e^2a+c\,d^2\right)^2\right)\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}-\frac{\ln\left(\frac{2ade+\left(e^2a+c\,d^2\right)x+2\sqrt{ade}\sqrt{ade+\left(e^2a+c\,d^2\right.}}{x}\right.}{ade\sqrt{ade}}}{d}$

input
```
int(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/d*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/a/d/e*(
2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(
a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))-1/d*(-2/3/(a*e^2
-c*d^2)/(x+d/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*c*d*e/(a
*e^2-c*d^2)^3*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)
*(x+d/e))^(1/2))
```

3.483.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(247) = 494$.

Time = 5.46 (sec) , antiderivative size = 1476, normalized size of antiderivative = 5.45

$$\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
[1/6*(3*(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 +
 (c^4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (
2*c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^
9)*x^2 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 -
2*a^4*d*e^8)*x)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2
+ a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e +
(c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(3*
a*c^3*d^8*e + 9*a^3*c*d^4*e^5 - 4*a^4*d^2*e^7 + (3*a*c^3*d^6*e^3 + 8*a^2*c
^2*d^4*e^5 - 3*a^3*c*d^2*e^7)*x^2 + (6*a*c^3*d^7*e^2 + 9*a^2*c^2*d^5*e^4 +
 4*a^3*c*d^3*e^6 - 3*a^4*d*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x))/(a^3*c^3*d^11*e^3 - 3*a^4*c^2*d^9*e^5 + 3*a^5*c*d^7*e^7 - a^6*d^5*e^
9 + (a^2*c^4*d^10*e^4 - 3*a^3*c^3*d^8*e^6 + 3*a^4*c^2*d^6*e^8 - a^5*c*d^4*
e^10)*x^3 + (2*a^2*c^4*d^11*e^3 - 5*a^3*c^3*d^9*e^5 + 3*a^4*c^2*d^7*e^7 +
a^5*c*d^5*e^9 - a^6*d^3*e^11)*x^2 + (a^2*c^4*d^12*e^2 - a^3*c^3*d^10*e^4 -
 3*a^4*c^2*d^8*e^6 + 5*a^5*c*d^6*e^8 - 2*a^6*d^4*e^10)*x), 1/3*(3*(a*c^3*d
^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 -
3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*
a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^
9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)*
sqrt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*...
```

3.483.6 Sympy [F]

$$\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{x\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate(1/x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(1/(x*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.483.7 Maxima [F]

$$\int \frac{1}{x(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)x}\,dx$$

input
```
integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x), x
)
```

3.483.8 Giac [F]

$$\int \frac{1}{x(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)x}\,dx$$

input
```
integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x), x
)
```

3.483.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{x\,\left(d+e\,x\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

input
```
int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

3.484 $\int \frac{1}{x^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

3.484.1 Optimal result

Integrand size = 40, antiderivative size = 394

$$\int \frac{1}{x^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+ac^2d^4e^2+9a^2cd^2e^4-5a^3e^6+cde\left(3c^2d^4+10acd^2e^2-5a^2e^4\right)x\right)}{3ad^2e\left(cd^2-ae^2\right)^3x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(9c^3d^6-9ac^2d^4e^2+31a^2cd^2e^4-15a^3e^6\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3a^2d^3e^2\left(cd^2-ae^2\right)^3x} + \frac{\left(3cd^2+5ae^2\right)\operatorname{arctanh}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{2a^{5/2}d^{7/2}e^{5/2}}$$

output
```
-2/3*e*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2)+1/2*(5*a*e^2+3*c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^
(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/d^(7/2)/e^(
5/2)+2/3*(3*c^3*d^6+a*c^2*d^4*e^2+9*a^2*c*d^2*e^4-5*a^3*e^6+c*d*e*(-5*a^2*
e^4+10*a*c*d^2*e^2+3*c^2*d^4)*x)/a/d^2/e/(-a*e^2+c*d^2)^3/x/(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)-1/3*(-15*a^3*e^6+31*a^2*c*d^2*e^4-9*a*c^2*d^4*e^
2+9*c^3*d^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/d^3/e^2/(-a*e^2+c
*d^2)^3/x
```

3.484.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{-\frac{\sqrt{a}\sqrt{d}\sqrt{e}(ae+cdx)\left(-9c^4d^7x(d+ex)^2-3ac^3d^5e(d-3ex)(d+ex)^2+a^4e^7(3\right.}{}}{}$$

input
```
Integrate[1/(x^2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-9*c^4*d^7*x*(d + e*x)^2 - 3*a*
c^3*d^5*e*(d - 3*e*x)*(d + e*x)^2 + a^4*e^7*(3*d^2 + 20*d*e*x + 15*e^2*x^2
) + a^2*c^2*d^3*e^3*(9*d^3 + 9*d^2*e*x - 33*d*e^2*x^2 - 31*e^3*x^3) - a^3*
c*d*e^5*(9*d^3 + 39*d^2*e*x + 11*d*e^2*x^2 - 15*e^3*x^3)))/((-(c*d^2) + a*
e^2)^3*x)) + 3*(3*c*d^2 + 5*a*e^2)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*Arc
Tanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(3*a^(5
/2)*d^(7/2)*e^(5/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

3.484.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1246, 27, 1235, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 1246$$

$$\frac{2\int \frac{e\left(3cd^2-6cexd-5ae^2\right)}{2x^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3de\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3dx\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{3cd^2-6cexd-5ae^2}{x^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3d\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3dx\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 1235$$

$$\frac{\frac{2(-5a^3e^6+cdex(-5a^2e^4+10acd^2e^2+3c^2d^4)+9a^2cd^2e^4+ac^2d^4e^2+3c^3d^6)}{adex(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\int -\frac{9c^3d^6-9ac^2e^2d^4+31a^2ce^4d^2+2ce\left(3c^2d^4+10ace^2d^2-5a^2e^4\right)xd-15}{2x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade(cd^2-ae^2)^2}}{3d\left(cd^2-ae^2\right)}$$

$$\frac{2e(ae+cdx)}{3dx\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{9c^3d^6-9ac^2e^2d^4+31a^2ce^4d^2+2ce\left(3c^2d^4+10ace^2d^2-5a^2e^4\right)xd-15a^3e^6}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{ade(cd^2-ae^2)^2}+\frac{2(-5a^3e^6+cdex(-5a^2e^4+10acd^2e^2+3c^2d^4)+9a^2cd^2e^4+ac^2d^4e^2+3c^3}{adex(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3d\left(cd^2-ae^2\right)}$$

$$\frac{2e(ae+cdx)}{3dx\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\downarrow$ 1228

$$\frac{\frac{-\frac{3\left(5ae^2+3cd^2\right)\left(cd^2-ae^2\right)^3\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(-15a^3e^6+31a^2cd^2e^4-9ac^2d^4e^2+9c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{adex}}{ade(cd^2-ae^2)^2}+\frac{2(-5a^3e^6+cdex(}{ad}}{3d\left(cd^2-ae^2\right)}$$

$$\frac{2e(ae+cdx)}{3dx\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\downarrow$ 1154

$$\frac{\frac{\frac{3\left(cd^2-ae^2\right)^3\left(5ae^2+3cd^2\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade}-\frac{\left(-15a^3e^6+31a^2cd^2e^4-9ac^2d^4e^2+9c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+c}}{adex}}{ade(cd^2-ae^2)^2}}{3d\left(cd^2-ae^2\right)}$$

$$\frac{2e(ae+cdx)}{3dx\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\downarrow$ 219

$$\frac{\frac{2(-5a^3e^6+cdex(-5a^2e^4+10acd^2e^2+3c^2d^4)+9a^2cd^2e^4+ac^2d^4e^2+3c^3d^6)}{adex(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{\frac{3\left(cd^2-ae^2\right)^3\left(5ae^2+3cd^2\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+c}}\right.}{2a^{3/2}d^{3/2}e^{3/2}}}{ade(}}{3d\left(cd^2-ae^2\right)}$$

$$\frac{2e(ae+cdx)}{3dx\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

3.484. $\int \frac{1}{x^2(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

input
```
Int[1/(x^2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x*(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(3/2)) + ((2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 5*a^
3*e^6 + c*d*e*(3*c^2*d^4 + 10*a*c*d^2*e^2 - 5*a^2*e^4)*x))/(a*d*e*(c*d^2 -
 a*e^2)^2*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (-(((9*c^3*d^6
- 9*a*c^2*d^4*e^2 + 31*a^2*c*d^2*e^4 - 15*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(a*d*e*x)) + (3*(c*d^2 - a*e^2)^3*(3*c*d^2 + 5*a*e^2
)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*a^(3/2)*d^(3/2)*e^(3/2)))/(a*d*
e*(c*d^2 - a*e^2)^2))/(3*d*(c*d^2 - a*e^2))
```

3.484.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1246
```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> Simp[(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p*
((c*d - b*e - c*e*x)/(p*(2*c*d - b*e)*(e*f - d*g))), x] + Simp[1/(p*(2*c*d
- b*e)*(e*f - d*g))   Int[(f + g*x)^n*(a + b*x + c*x^2)^p*(b*e*g*(n + p + 1
) + c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x), x], x]
/; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ
[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n, 0]
```

3.484.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.82

method	result
default	$\frac{-\frac{1}{adex\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}-\frac{3\left(e^2a+c\,d^2\right)\left(\frac{1}{ade\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}-\frac{\left(e^2a+c\,d^2\right)\left(2cdex+e^2a+c\,d^2\right)}{ade\left(4ac\,d^2e^2-\left(e^2a+c\,d^2\right)^2\right)\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}\right.}{2ade}}{d}$

input
```
int(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
1/d*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/2*(a*e^2+c*d^2)/
a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/a/d/e
*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2
*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))-4*c/a*(2*c*d*e
*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2))-e/d^2*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e
^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a
*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))
+e/d^2*(-2/3/(a*e^2-c*d^2)/(x+d/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))
^(1/2)+8/3*c*d*e/(a*e^2-c*d^2)^3*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/(c*d*e*(x+d
/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))
```

3.484.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(364) = 728$.

Time = 11.47 (sec) , antiderivative size = 1812, normalized size of antiderivative = 4.60

$$\int \frac{1}{x^2(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
[1/12*(3*((3*c^5*d^9*e^2 - 4*a*c^4*d^7*e^4 - 6*a^2*c^3*d^5*e^6 + 12*a^3*c^
2*d^3*e^8 - 5*a^4*c*d*e^10)*x^4 + (6*c^5*d^10*e - 5*a*c^4*d^8*e^3 - 16*a^2
*c^3*d^6*e^5 + 18*a^3*c^2*d^4*e^7 + 2*a^4*c*d^2*e^9 - 5*a^5*e^11)*x^3 + (3
*c^5*d^11 + 2*a*c^4*d^9*e^2 - 14*a^2*c^3*d^7*e^4 + 19*a^4*c*d^3*e^8 - 10*a
^5*d*e^10)*x^2 + (3*a*c^4*d^10*e - 4*a^2*c^3*d^8*e^3 - 6*a^3*c^2*d^6*e^5 +
 12*a^4*c*d^4*e^7 - 5*a^5*d^2*e^9)*x)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^
2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*
d*e^3)*x)/x^2) - 4*(3*a^2*c^3*d^9*e^2 - 9*a^3*c^2*d^7*e^4 + 9*a^4*c*d^5*e^
6 - 3*a^5*d^3*e^8 + (9*a*c^4*d^8*e^3 - 9*a^2*c^3*d^6*e^5 + 31*a^3*c^2*d^4*
e^7 - 15*a^4*c*d^2*e^9)*x^3 + (18*a*c^4*d^9*e^2 - 15*a^2*c^3*d^7*e^4 + 33*
a^3*c^2*d^5*e^6 + 11*a^4*c*d^3*e^8 - 15*a^5*d*e^10)*x^2 + (9*a*c^4*d^10*e
- 3*a^2*c^3*d^8*e^3 - 9*a^3*c^2*d^6*e^5 + 39*a^4*c*d^4*e^7 - 20*a^5*d^2*e^
9)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^3*c^4*d^11*e^5 - 3*
a^4*c^3*d^9*e^7 + 3*a^5*c^2*d^7*e^9 - a^6*c*d^5*e^11)*x^4 + (2*a^3*c^4*d^1
2*e^4 - 5*a^4*c^3*d^10*e^6 + 3*a^5*c^2*d^8*e^8 + a^6*c*d^6*e^10 - a^7*d^4*
e^12)*x^3 + (a^3*c^4*d^13*e^3 - a^4*c^3*d^11*e^5 - 3*a^5*c^2*d^9*e^7 + 5*a
^6*c*d^7*e^9 - 2*a^7*d^5*e^11)*x^2 + (a^4*c^3*d^12*e^4 - 3*a^5*c^2*d^10*e^
6 + 3*a^6*c*d^8*e^8 - a^7*d^6*e^10)*x), -1/6*(3*((3*c^5*d^9*e^2 - 4*a*c^4*
d^7*e^4 - 6*a^2*c^3*d^5*e^6 + 12*a^3*c^2*d^3*e^8 - 5*a^4*c*d*e^10)*x^4 ...
```

3.484.6 Sympy [F]

$$\int \frac{1}{x^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{x^2\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate(1/x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(1/(x**2*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.484.7 Maxima [F]

$$\int \frac{1}{x^2(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^2),
 x)
```

3.484.8 Giac [F]

$$\int \frac{1}{x^2(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^2),
 x)
```

3.484.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{x^2\,(d+e\,x)\,\left(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

3.485 $\int \frac{1}{x^3(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

3.485.1 Optimal result

Integrand size = 40, antiderivative size = 522

$$\int \frac{1}{x^3(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+ac^2d^4e^2+11a^2cd^2e^4-7a^3e^6+cde\left(3c^2d^4+12acd^2e^2-7a^2e^4\right)x\right)}{3ad^2e\left(cd^2-ae^2\right)^3x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(15c^3d^6-9ac^2d^4e^2+61a^2cd^2e^4-35a^3e^6\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{6a^2d^3e^2\left(cd^2-ae^2\right)^3x^2} + \frac{\left(45c^4d^8-30ac^3d^6e^2-36a^2c^2d^4e^4+190a^3cd^2e^6-105a^4e^8\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12a^3d^4e^3\left(cd^2-ae^2\right)^3x} - \frac{5\left(3c^2d^4+6acd^2e^2+7a^2e^4\right)\operatorname{arctanh}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{8a^{7/2}d^{9/2}e^{7/2}}$$

output
```
-2/3*e*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(3/2)-5/8*(7*a^2*e^4+6*a*c*d^2*e^2+3*c^2*d^4)*arctanh(1/2*(2*a*d*e+(a*e^2+
c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))
/a^(7/2)/d^(9/2)/e^(7/2)+2/3*(3*c^3*d^6+a*c^2*d^4*e^2+11*a^2*c*d^2*e^4-7*a
^3*e^6+c*d*e*(-7*a^2*e^4+12*a*c*d^2*e^2+3*c^2*d^4)*x)/a/d^2/e/(-a*e^2+c*d^
2)^3/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/6*(-35*a^3*e^6+61*a^2*c
*d^2*e^4-9*a*c^2*d^4*e^2+15*c^3*d^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)/a^2/d^3/e^2/(-a*e^2+c*d^2)^3/x^2+1/12*(-105*a^4*e^8+190*a^3*c*d^2*e^6-3
6*a^2*c^2*d^4*e^4-30*a*c^3*d^6*e^2+45*c^4*d^8)*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)/a^3/d^4/e^3/(-a*e^2+c*d^2)^3/x
```

3.485.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^3(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{\sqrt{a}\sqrt{d}\sqrt{e}(ae+cdx)\left(-45c^5d^9x^2(d+ex)^2-15ac^4d^7ex(d-2ex)(d+ex)^2+6a^2c\right.}{}$$

input
```
Integrate[1/(x^3*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
((Sqrt[a]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-45*c^5*d^9*x^2*(d + e*x)^2 - 15*
a*c^4*d^7*e*x*(d - 2*e*x)*(d + e*x)^2 + 6*a^2*c^3*d^5*e^2*(d + e*x)^2*(d^2
 + 2*d*e*x + 6*e^2*x^2) + a^5*e^8*(-6*d^3 + 21*d^2*e*x + 140*d*e^2*x^2 + 1
05*e^3*x^3) - 2*a^3*c^2*d^3*e^4*(9*d^4 - 9*d^3*e*x - 6*d^2*e^2*x^2 + 111*d
*e^3*x^3 + 95*e^4*x^4) + a^4*c*d*e^6*(18*d^4 - 48*d^3*e*x - 237*d^2*e^2*x^
2 - 50*d*e^3*x^3 + 105*e^4*x^4)))/((-(c*d^2) + a*e^2)^3*x^2) - 15*(3*c^2*d
^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*ArcTan
h[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(12*a^(7/2
)*d^(9/2)*e^(7/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

3.485.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {1246, 27, 1235, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \ 1246$$

$$\frac{2\int \frac{e(3cd^2-8cexd-7ae^2)}{2x^3(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3de\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3dx^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{3cd^2-8cexd-7ae^2}{x^3(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3d\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3dx^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \ 1235$$

$$\frac{\frac{2\left(-7a^3e^6+cdex\left(-7a^2e^4+12acd^2e^2+3c^2d^4\right)+11a^2cd^2e^4+ac^2d^4e^2+3c^3d^6\right)}{adex^2(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\int -\frac{15c^3d^6-9ac^2e^2d^4+61a^2ce^4d^2+4ce\left(3c^2d^4+12ace^2d^2-7a^2e^4\right)xd-}{2x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade(cd^2-ae^2)^2}}{3d\left(cd^2-ae^2\right)}$$

$$\frac{2e(ae+cdx)}{3dx^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{15c^3d^6-9ac^2e^2d^4+61a^2ce^4d^2+4ce\left(3c^2d^4+12ace^2d^2-7a^2e^4\right)xd-35a^3e^6}{x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{ade(cd^2-ae^2)^2}+\frac{2\left(-7a^3e^6+cdex\left(-7a^2e^4+12acd^2e^2+3c^2d^4\right)+11a^2cd^2e^4+ac^2d^4e^2+3\right.}{adex^2(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3d\left(cd^2-ae^2\right)}$$

$$\frac{2e(ae+cdx)}{3dx^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \ 1237$$

$$\frac{-\frac{-\frac{\int \frac{45c^4d^8-30ac^3e^2d^6-36a^2c^2e^4d^4+190a^3ce^6d^2+2ce\left(15c^3d^6-9ac^2e^2d^4+61a^2ce^4d^2-35a^3e^6\right)xd-105a^4e^8}{2x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(-35a^3e^6+61a^2cd^2e^4-9ac^2d^4e^2+15c^3d^6\right)\sqrt{\cdots}}{2adex^2}}{ade(cd^2-ae^2)^2}\cdots}{3d\left(cd^2-ae^2\right)\cdots}$$

$$\frac{2e(ae+cdx)}{3dx^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\big\downarrow$ 27

$$\frac{-\frac{-\frac{\int \frac{45c^4d^8-30ac^3e^2d^6-36a^2c^2e^4d^4+190a^3ce^6d^2+2ce\left(15c^3d^6-9ac^2e^2d^4+61a^2ce^4d^2-35a^3e^6\right)xd-105a^4e^8}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4ade}-\frac{\left(-35a^3e^6+61a^2cd^2e^4-9ac^2d^4e^2+15c^3d^6\right)\sqrt{\cdots}}{2adex^2}}{ade(cd^2-ae^2)^2}\cdots}{3d\left(cd^2-ae^2\right)\cdots}$$

$$\frac{2e(ae+cdx)}{3dx^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\big\downarrow$ 1228

$$\frac{-\frac{-\frac{-\frac{15\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(cd^2-ae^2\right)^3\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(-105a^4e^8+190a^3cd^2e^6-36a^2c^2d^4e^4-30ac^3d^6e^2+45c^4d^8\right)\sqrt{x\left(ae^2+cd^2\right)+a\cdots}}{adex}}{4ade}\cdots}{ade(cd^2-ae^2)^2}\cdots}{3d\cdots}$$

$$\frac{2e(ae+cdx)}{3dx^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\big\downarrow$ 1154

$$-\frac{\frac{\frac{15\left(cd^2-ae^2\right)^3\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade}-\frac{\left(-105a^4e^8+190a^3cd^2e^6-36a^2c^2d^4e^4-30ac^3d^6e^2\cdots\right)}{adex}}{4ade}\cdots}{ade(cd^2-ae^2)^2}\cdots$$

$$\frac{2e(ae+cdx)}{3dx^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\big\downarrow$ 219

3.485. $\int \frac{1}{x^3(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

$$\frac{\frac{2\left(-7a^3e^6+cdex\left(-7a^2e^4+12acd^2e^2+3c^2d^4\right)+11a^2cd^2e^4+ac^2d^4e^2+3c^3d^6\right)}{adex^2(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{-\frac{\left(-35a^3e^6+61a^2cd^2e^4-9ac^2d^4e^2+15c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2adex^2}}{}}{}$$

$$\frac{2e(ae+cdx)}{3dx^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[1/(x^3*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(3/2)) + ((2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 11*a^2*c*d^2*e^4 - 7
*a^3*e^6 + c*d*e*(3*c^2*d^4 + 12*a*c*d^2*e^2 - 7*a^2*e^4)*x))/(a*d*e*(c*d^
2 - a*e^2)^2*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (-1/2*((15
*c^3*d^6 - 9*a*c^2*d^4*e^2 + 61*a^2*c*d^2*e^4 - 35*a^3*e^6)*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x^2) - (-(((45*c^4*d^8 - 30*a*c^3*d^
6*e^2 - 36*a^2*c^2*d^4*e^4 + 190*a^3*c*d^2*e^6 - 105*a^4*e^8)*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x)) + (15*(c*d^2 - a*e^2)^3*(3*c^2
*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2
*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2
*a^(3/2)*d^(3/2)*e^(3/2)))/(4*a*d*e))/(a*d*e*(c*d^2 - a*e^2)^2))/(3*d*(c*d
^2 - a*e^2))
```

3.485.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.485. $\int \frac{1}{x^3(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1246
```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> Simp[(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p*
((c*d - b*e - c*e*x)/(p*(2*c*d - b*e)*(e*f - d*g))), x] + Simp[1/(p*(2*c*d
- b*e)*(e*f - d*g))   Int[(f + g*x)^n*(a + b*x + c*x^2)^p*(b*e*g*(n + p + 1
) + c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x), x], x]
/; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ
[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n, 0]
```

3.485. $\int \frac{1}{x^3(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

3.485.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1353 vs. $2(488) = 976$.

Time = 0.77 (sec) , antiderivative size = 1354, normalized size of antiderivative = 2.59

method	result	size
default	Expression too large to display	1354

input
```
int(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
1/d*(-1/2/a/d/e/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-5/4*(a*e^2+c*d
^2)/a/d/e*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/2*(a*e^2+c
*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)
/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^
2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))-4*c/a*(2
*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2))-3/2*c/a*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^
2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*
a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2))/x)))+e^2/d^3*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+
c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^
2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))-e/
d^2*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/2*(a*e^2+c*d^2)/
a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/a/d/e
*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2
*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))-4*c/a*(2*c*...
```

3.485.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(488) = 976$.

Time = 28.52 (sec) , antiderivative size = 2162, normalized size of antiderivative = 4.14

$$\int \frac{1}{x^3(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
[1/48*(15*((3*c^6*d^11*e^2 - 3*a*c^5*d^9*e^4 - 2*a^2*c^4*d^7*e^6 - 6*a^3*c
^3*d^5*e^8 + 15*a^4*c^2*d^3*e^10 - 7*a^5*c*d*e^12)*x^5 + (6*c^6*d^12*e - 3
*a*c^5*d^10*e^3 - 7*a^2*c^4*d^8*e^5 - 14*a^3*c^3*d^6*e^7 + 24*a^4*c^2*d^4*
e^9 + a^5*c*d^2*e^11 - 7*a^6*e^13)*x^4 + (3*c^6*d^13 + 3*a*c^5*d^11*e^2 -
8*a^2*c^4*d^9*e^4 - 10*a^3*c^3*d^7*e^6 + 3*a^4*c^2*d^5*e^8 + 23*a^5*c*d^3*
e^10 - 14*a^6*d*e^12)*x^3 + (3*a*c^5*d^12*e - 3*a^2*c^4*d^10*e^3 - 2*a^3*c
^3*d^8*e^5 - 6*a^4*c^2*d^6*e^7 + 15*a^5*c*d^4*e^9 - 7*a^6*d^2*e^11)*x^2)*s
qrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 -
4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x
)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(6*a^3*c^3*d^10*e^3
- 18*a^4*c^2*d^8*e^5 + 18*a^5*c*d^6*e^7 - 6*a^6*d^4*e^9 - (45*a*c^5*d^10*e
^3 - 30*a^2*c^4*d^8*e^5 - 36*a^3*c^3*d^6*e^7 + 190*a^4*c^2*d^4*e^9 - 105*a
^5*c*d^2*e^11)*x^4 - (90*a*c^5*d^11*e^2 - 45*a^2*c^4*d^9*e^4 - 84*a^3*c^3*
d^7*e^6 + 222*a^4*c^2*d^5*e^8 + 50*a^5*c*d^3*e^10 - 105*a^6*d*e^12)*x^3 -
(45*a*c^5*d^12*e - 66*a^3*c^3*d^8*e^5 - 12*a^4*c^2*d^6*e^7 + 237*a^5*c*d^4
*e^9 - 140*a^6*d^2*e^11)*x^2 - 3*(5*a^2*c^4*d^11*e^2 - 8*a^3*c^3*d^9*e^4 -
 6*a^4*c^2*d^7*e^6 + 16*a^5*c*d^5*e^8 - 7*a^6*d^3*e^10)*x)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x))/((a^4*c^4*d^12*e^6 - 3*a^5*c^3*d^10*e^8 + 3*
a^6*c^2*d^8*e^10 - a^7*c*d^6*e^12)*x^5 + (2*a^4*c^4*d^13*e^5 - 5*a^5*c^3*d
^11*e^7 + 3*a^6*c^2*d^9*e^9 + a^7*c*d^7*e^11 - a^8*d^5*e^13)*x^4 + (a^4...
```

3.485.6 Sympy [F]

$$\int \frac{1}{x^3(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{x^3\left((d+ex)(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(1/x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(1/(x**3*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.485.7 Maxima [F]

$$\int \frac{1}{x^3(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)x^3}\,dx$$

input
```
integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^3),
 x)
```

3.485.8 Giac [F]

$$\int \frac{1}{x^3(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)x^3}\,dx$$

input
```
integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^3),
 x)
```

3.485.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{x^3\,\left(d+e\,x\right)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

3.486 $\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

3.486.1 Optimal result

Integrand size = 40, antiderivative size = 664

$$\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx =$$

$$-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

$$+\frac{2(3c^3d^6+ac^2d^4e^2+13a^2cd^2e^4-9a^3e^6+cde(3c^2d^4+14acd^2e^2-9a^2e^4)\,x)}{3ad^2e\left(cd^2-ae^2\right)^3x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$-\frac{\left(7c^3d^6-3ac^2d^4e^2+33a^2cd^2e^4-21a^3e^6\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3a^2d^3e^2\left(cd^2-ae^2\right)^3x^3}$$

$$+\frac{\left(35c^4d^8-16ac^3d^6e^2-18a^2c^2d^4e^4+168a^3cd^2e^6-105a^4e^8\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12a^3d^4e^3\left(cd^2-ae^2\right)^3x^2}$$

$$-\frac{\left(105c^5d^{10}-55ac^4d^8e^2-54a^2c^3d^6e^4-78a^3c^2d^4e^6+525a^4cd^2e^8-315a^5e^{10}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{24a^4d^5e^4\left(cd^2-ae^2\right)^3x}$$

$$+\frac{5(7c^3d^6+15ac^2d^4e^2+21a^2cd^2e^4+21a^3e^6)\,\text{arctanh}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{16a^{9/2}d^{11/2}e^{9/2}}$$

output
```
-2/3*e*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(3/2)+5/16*(21*a^3*e^6+21*a^2*c*d^2*e^4+15*a*c^2*d^4*e^2+7*c^3*d^6)*arctan
h(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2))/a^(9/2)/d^(11/2)/e^(9/2)+2/3*(3*c^3*d^6+a*c^2*d^4*e
^2+13*a^2*c*d^2*e^4-9*a^3*e^6+c*d*e*(-9*a^2*e^4+14*a*c*d^2*e^2+3*c^2*d^4)*
x)/a/d^2/e/(-a*e^2+c*d^2)^3/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/
3*(-21*a^3*e^6+33*a^2*c*d^2*e^4-3*a*c^2*d^4*e^2+7*c^3*d^6)*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)/a^2/d^3/e^2/(-a*e^2+c*d^2)^3/x^3+1/12*(-105*a^4*e
^8+168*a^3*c*d^2*e^6-18*a^2*c^2*d^4*e^4-16*a*c^3*d^6*e^2+35*c^4*d^8)*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^3/d^4/e^3/(-a*e^2+c*d^2)^3/x^2-1/24*(
-315*a^5*e^10+525*a^4*c*d^2*e^8-78*a^3*c^2*d^4*e^6-54*a^2*c^3*d^6*e^4-55*a
*c^4*d^8*e^2+105*c^5*d^10)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^4/d^5
/e^4/(-a*e^2+c*d^2)^3/x
```

3.486.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 493, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(-105c^6d^{11}x^3(d+ex)^2-5ac^5d^9ex^2(7d-11ex)(d+ex)^2+a^2c^4d^7e^2\right.}{}$$

input
```
Integrate[1/(x^4*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-105*c^6*d^11*x^3*(d + e*x)^2 - 5*a*c^5*d^9*e*x
^2*(7*d - 11*e*x)*(d + e*x)^2 + a^2*c^4*d^7*e^2*x*(d + e*x)^2*(14*d^2 + 23
*d*e*x + 54*e^2*x^2) - 2*a^3*c^3*d^5*e^3*(d + e*x)^2*(4*d^3 + 4*d^2*e*x -
9*d*e^2*x^2 - 39*e^3*x^3) + a^6*e^9*(8*d^4 - 18*d^3*e*x + 63*d^2*e^2*x^2 +
 420*d*e^3*x^3 + 315*e^4*x^4) + a^4*c^2*d^3*e^5*(24*d^5 - 12*d^4*e*x + 62*
d^3*e^2*x^2 + 3*d^2*e^3*x^3 - 636*d*e^4*x^4 - 525*e^5*x^5) - a^5*c*d*e^7*(
24*d^5 - 40*d^4*e*x + 135*d^3*e^2*x^2 + 651*d^2*e^3*x^3 + 105*d*e^4*x^4 -
315*e^5*x^5)))/((c*d^2 - a*e^2)^3*x^3*(d + e*x)) + 15*(7*c^3*d^6 + 15*a*c^
2*d^4*e^2 + 21*a^2*c*d^2*e^4 + 21*a^3*e^6)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]
*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(24
*a^(9/2)*d^(11/2)*e^(9/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.486.3 Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.275, Rules used = {1246, 27, 1235, 27, 1237, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1246}$$

$$\frac{2\int \frac{e\left(3\left(cd^2-3ae^2\right)-10cdex\right)}{2x^4\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3de\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3\left(cd^2-3ae^2\right)-10cdex}{x^4\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3d\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1235}$$

$$\frac{\frac{2\left(-9a^3e^6+cdex\left(-9a^2e^4+14acd^2e^2+3c^2d^4\right)+13a^2cd^2e^4+ac^2d^4e^2+3c^3d^6\right)}{adex^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\int -\frac{3\left(7c^3d^6-3ac^2e^2d^4+33a^2ce^4d^2+2ce\left(3c^2d^4+14ace^2d^2-9a^2e^4\right)xd\right.}{2x^4\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade\left(cd^2-ae^2\right)^2}}{3d\left(cd^2-ae^2\right)}$$
$$\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{7c^3d^6-3ac^2e^2d^4+33a^2ce^4d^2+2ce\left(3c^2d^4+14ace^2d^2-9a^2e^4\right)xd-21a^3e^6}{x^4\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{ade\left(cd^2-ae^2\right)^2}+\frac{2\left(-9a^3e^6+cdex\left(-9a^2e^4+14acd^2e^2+3c^2d^4\right)+13a^2cd^2e^4+ac^2d^4e^2+\right.}{adex^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}}{3d\left(cd^2-ae^2\right)}$$
$$\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1237}$$

$$3\left(-\frac{\int \frac{35c^4d^8-16ac^3e^2d^6-18a^2c^2e^4d^4+168a^3ce^6d^2+4ce\left(7c^3d^6-3ac^2e^2d^4+33a^2ce^4d^2-21a^3e^6\right)xd-105a^4e^8}{2x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3ade}-\frac{\left(-21a^3e^6+33a^2cd^2e^4-3ac^2d^4e^2+7c^3d^6\right)}{3adex^3}\right.$$
$$\overline{\qquad ade(cd^2-ae^2)^2 \qquad}$$
$$\overline{\qquad 3d\left(cd^2-ae^2\right) \qquad}$$

$$\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$3\left(-\frac{\int \frac{35c^4d^8-16ac^3e^2d^6-18a^2c^2e^4d^4+168a^3ce^6d^2+4ce\left(7c^3d^6-3ac^2e^2d^4+33a^2ce^4d^2-21a^3e^6\right)xd-105a^4e^8}{x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6ade}-\frac{\left(-21a^3e^6+33a^2cd^2e^4-3ac^2d^4e^2+7c^3d^6\right)}{3adex^3}\right.$$
$$\overline{\qquad ade(cd^2-ae^2)^2 \qquad}$$
$$\overline{\qquad 3d\left(cd^2-ae^2\right) \qquad}$$

$$\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \ 1237$$

$$3\left(-\frac{-\frac{\int \frac{105c^5d^{10}-55ac^4e^2d^8-54a^2c^3e^4d^6-78a^3c^2e^6d^4+525a^4ce^8d^2+2ce\left(35c^4d^8-16ac^3e^2d^6-18a^2c^2e^4d^4+168a^3ce^6d^2-105a^4e^8\right)xd-315a^5e^{10}}{2x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(-105\right.}{}}{6ade}\right.$$
$$\overline{\qquad ade(cd^2- \qquad}$$

$$\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$3\left(-\frac{-\frac{\int \frac{105c^5d^{10}-55ac^4e^2d^8-54a^2c^3e^4d^6-78a^3c^2e^6d^4+525a^4ce^8d^2+2ce\left(35c^4d^8-16ac^3e^2d^6-18a^2c^2e^4d^4+168a^3ce^6d^2-105a^4e^8\right)xd-315a^5e^{10}}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4ade}-\frac{\left(-105\right.}{}}{6ade}\right.$$
$$\overline{\qquad ade(cd^2- \qquad}$$

$$\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \ 1228$$

3.486. $\int \frac{1}{x^4(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx$

$$3\left(-\frac{-\frac{-\frac{15\left(21a^3e^6+21a^2cd^2e^4+15ac^2d^4e^2+7c^3d^6\right)\left(cd^2-ae^2\right)^3\int\frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(-315a^5e^{10}+525a^4cd^2e^8-78a^3c^2d^4e^6-54a^2c^3d^6e^4-55 \cdots}{adex}}{4ade}}{6ade}\cdots\right.$$

$$\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\downarrow$ 1154

$$3\left(-\frac{-\frac{\frac{15\left(cd^2-ae^2\right)^3\left(21a^3e^6+21a^2cd^2e^4+15ac^2d^4e^2+7c^3d^6\right)\int\frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade}-\frac{\left(-315a^5e^{10}+525a^4cd^2e^8-7 \cdots}{\cdots}}{4ade}}{\cdots}\cdots\right.$$

$$\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\downarrow$ 219

$$\frac{2\left(-9a^3e^6+cdex\left(-9a^2e^4+14acd^2e^2+3c^2d^4\right)+13a^2cd^2e^4+ac^2d^4e^2+3c^3d^6\right)}{adex^3(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{3\left(-\frac{\left(-21a^3e^6+33a^2cd^2e^4-3ac^2d^4e^2+7c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cde \cdots}}{3adex^3}\cdots\right.}{\cdots}$$

$$\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input

```
Int[1/(x^4*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

3.486. $\int \frac{1}{x^4(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

output
```
(-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x^3*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(3/2)) + ((2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 13*a^2*c*d^2*e^4 - 9
*a^3*e^6 + c*d*e*(3*c^2*d^4 + 14*a*c*d^2*e^2 - 9*a^2*e^4)*x))/(a*d*e*(c*d^
2 - a*e^2)^2*x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (3*(-1/3*(
(7*c^3*d^6 - 3*a*c^2*d^4*e^2 + 33*a^2*c*d^2*e^4 - 21*a^3*e^6)*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x^3) - (-1/2*((35*c^4*d^8 - 16*a*c
^3*d^6*e^2 - 18*a^2*c^2*d^4*e^4 + 168*a^3*c*d^2*e^6 - 105*a^4*e^8)*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x^2) - (-(((105*c^5*d^10 - 55
*a*c^4*d^8*e^2 - 54*a^2*c^3*d^6*e^4 - 78*a^3*c^2*d^4*e^6 + 525*a^4*c*d^2*e
^8 - 315*a^5*e^10)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x))
 + (15*(c*d^2 - a*e^2)^3*(7*c^3*d^6 + 15*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4
+ 21*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqr
t[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*a^(3/2)*d^(3/2)*e^(
3/2)))/(4*a*d*e))/(6*a*d*e)))/(a*d*e*(c*d^2 - a*e^2)^2))/(3*d*(c*d^2 - a*e
^2))
```

3.486.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

3.486. $\int \frac{1}{x^4(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1246
```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> Simp[(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p*
((c*d - b*e - c*e*x)/(p*(2*c*d - b*e)*(e*f - d*g))), x] + Simp[1/(p*(2*c*d
- b*e)*(e*f - d*g))   Int[(f + g*x)^n*(a + b*x + c*x^2)^p*(b*e*g*(n + p + 1
) + c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x), x], x]
/; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ
[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n, 0]
```

3.486.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2408 vs. 2(626) = 1252.

Time = 1.14 (sec) , antiderivative size = 2409, normalized size of antiderivative = 3.63

method	result	size
default	Expression too large to display	2409

input
```
int(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
1/d*(-1/3/a/d/e/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-7/6*(a*e^2+c*d
^2)/a/d/e*(-1/2/a/d/e/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-5/4*(a*e
^2+c*d^2)/a/d/e*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/2*(a
*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+
c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^
2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))-4*
c/a*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2))-3/2*c/a*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^
2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*
ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2))/x)))-4/3*c/a*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)-3/2*(a*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^
2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d
*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
)/x))-4*c/a*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)))+e^2/d^3*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2)-3/2*(a*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*...
```

3.486.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1253 vs. 2(626) = 1252.

Time = 64.20 (sec) , antiderivative size = 2526, normalized size of antiderivative = 3.80

$$\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
[1/96*(15*((7*c^7*d^13*e^2 - 6*a*c^6*d^11*e^4 - 3*a^2*c^5*d^9*e^6 - 4*a^3*
c^4*d^7*e^8 - 15*a^4*c^3*d^5*e^10 + 42*a^5*c^2*d^3*e^12 - 21*a^6*c*d*e^14)
*x^6 + (14*c^7*d^14*e - 5*a*c^6*d^12*e^3 - 12*a^2*c^5*d^10*e^5 - 11*a^3*c^
4*d^8*e^7 - 34*a^4*c^3*d^6*e^9 + 69*a^5*c^2*d^4*e^11 - 21*a^7*e^15)*x^5 +
(7*c^7*d^15 + 8*a*c^6*d^13*e^2 - 15*a^2*c^5*d^11*e^4 - 10*a^3*c^4*d^9*e^6
- 23*a^4*c^3*d^7*e^8 + 12*a^5*c^2*d^5*e^10 + 63*a^6*c*d^3*e^12 - 42*a^7*d*
e^14)*x^4 + (7*a*c^6*d^14*e - 6*a^2*c^5*d^12*e^3 - 3*a^3*c^4*d^10*e^5 - 4*
a^4*c^3*d^8*e^7 - 15*a^5*c^2*d^6*e^9 + 42*a^6*c*d^4*e^11 - 21*a^7*d^2*e^13
)*x^3)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4
)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(8*a^4*c^3*d
^11*e^4 - 24*a^5*c^2*d^9*e^6 + 24*a^6*c*d^7*e^8 - 8*a^7*d^5*e^10 + (105*a*
c^6*d^12*e^3 - 55*a^2*c^5*d^10*e^5 - 54*a^3*c^4*d^8*e^7 - 78*a^4*c^3*d^6*e
^9 + 525*a^5*c^2*d^4*e^11 - 315*a^6*c*d^2*e^13)*x^5 + (210*a*c^6*d^13*e^2
- 75*a^2*c^5*d^11*e^4 - 131*a^3*c^4*d^9*e^6 - 174*a^4*c^3*d^7*e^8 + 636*a^
5*c^2*d^5*e^10 + 105*a^6*c*d^3*e^12 - 315*a^7*d*e^14)*x^4 + (105*a*c^6*d^1
4*e + 15*a^2*c^5*d^12*e^3 - 114*a^3*c^4*d^10*e^5 - 106*a^4*c^3*d^8*e^7 - 3
*a^5*c^2*d^6*e^9 + 651*a^6*c*d^4*e^11 - 420*a^7*d^2*e^13)*x^3 + (35*a^2*c^
5*d^13*e^2 - 51*a^3*c^4*d^11*e^4 + 6*a^4*c^3*d^9*e^6 - 62*a^5*c^2*d^7*e^8
+ 135*a^6*c*d^5*e^10 - 63*a^7*d^3*e^12)*x^2 - 2*(7*a^3*c^4*d^12*e^3 - 1...
```

3.486.6 Sympy [F]

$$\int \frac{1}{x^4(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{x^4\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(1/x**4/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(1/(x**4*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.486.7 Maxima [F]

$$\int \frac{1}{x^4(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)x^4}\,dx$$

input
```
integrate(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^4),
 x)
```

3.486.8 Giac [F]

$$\int \frac{1}{x^4(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)x^4}\,dx$$

input
```
integrate(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^4),
 x)
```

3.486.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{x^4\,\left(d+e\,x\right)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int(1/(x^4*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(1/(x^4*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

3.487 $\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

3.487.1 Optimal result

Integrand size = 40, antiderivative size = 259

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \frac{2x^2}{5(cd^2-ae^2)(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}} - \frac{8(ade(cd^2-ae^2)(cd^2+3ae^2)+(c^3d^6+a^2cd^2e^4-2a^3e^6)x)}{15e(cd^2-ae^2)^4\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}} + \frac{8(c^2d^4+10acd^2e^2+5a^2e^4)(cd^2+ae^2+2cdex)}{15e(cd^2-ae^2)^5\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

output
```
2/5*x^2/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-8/1
5*(a*d*e*(-a*e^2+c*d^2)*(3*a*e^2+c*d^2)+(-2*a^3*e^6+a^2*c*d^2*e^4+c^3*d^6)
*x)/e/(-a*e^2+c*d^2)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+8/15*(5*a^2
*e^4+10*a*c*d^2*e^2+c^2*d^4)*(2*c*d*e*x+a*e^2+c*d^2)/e/(-a*e^2+c*d^2)^5/(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

3.487.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \frac{2(c^4d^6x^2(15d^2+20dex+8e^2x^2)+a^4e^6(8d^2+20dex+15\cdots}{}$$

input
```
Integrate[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

output
```
(2*(c^4*d^6*x^2*(15*d^2 + 20*d*e*x + 8*e^2*x^2) + a^4*e^6*(8*d^2 + 20*d*e*
x + 15*e^2*x^2) + 4*a^3*c*d*e^4*(20*d^3 + 53*d^2*e*x + 45*d*e^2*x^2 + 15*e
^3*x^3) + 4*a*c^3*d^4*e*x*(15*d^3 + 45*d^2*e*x + 53*d*e^2*x^2 + 20*e^3*x^3
) + 2*a^2*c^2*d^2*e^2*(20*d^4 + 110*d^3*e*x + 189*d^2*e^2*x^2 + 110*d*e^3*
x^3 + 20*e^4*x^4)))/(15*(c*d^2 - a*e^2)^5*(d + e*x)*((a*e + c*d*x)*(d + e*
x))^(3/2))
```

3.487.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1244, 27, 1159, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1244}$$

$$-\frac{2\int -\frac{e^2(2ade-(cd^2+5ae^2)x)}{2(cdex^2+(cd^2+ae^2)x+ade)^{5/2}}dx}{5e^3\left(cd^2-ae^2\right)}-\frac{2dx}{5e(d+ex)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{2ade-(cd^2+5ae^2)x}{(cdex^2+(cd^2+ae^2)x+ade)^{5/2}}dx}{5e\left(cd^2-ae^2\right)}-\frac{2dx}{5e(d+ex)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1159}$$

$$\frac{-\frac{4(5a^2e^4+10acd^2e^2+c^2d^4)\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3(cd^2-ae^2)^2}-\frac{2\left(x\left(5a^2e^4+10acd^2e^2+c^2d^4\right)+4ade\left(3ae^2+cd^2\right)\right)}{3(cd^2-ae^2)^2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}}{5e\left(cd^2-ae^2\right)}-\frac{2dx}{5e(d+ex)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1088}$$

$$\frac{\frac{8(5a^2e^4+10acd^2e^2+c^2d^4)(ae^2+cd^2+2cdex)}{3(cd^2-ae^2)^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2(x(5a^2e^4+10acd^2e^2+c^2d^4)+4ade(3ae^2+cd^2))}{3(cd^2-ae^2)^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}}{5e\left(cd^2-ae^2\right)}-\frac{2dx}{5e(d+ex)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

output
```
(-2*d*x)/(5*e*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)) + ((-2*(4*a*d*e*(c*d^2 + 3*a*e^2) + (c^2*d^4 + 10*a*c*d^2*e^2
 + 5*a^2*e^4)*x))/(3*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)) + (8*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*e^2 + 2
*c*d*e*x))/(3*(c*d^2 - a*e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]))/(5*e*(c*d^2 - a*e^2))
```

3.487.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1244
```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d
_) + (e_.)*(x_)), x_Symbol] :> Simp[(-(e*f - d*g))*(f + g*x)^(n - 1)*((a +
b*x + c*x^2)^(p + 1)/(p*(2*c*d - b*e)*(d + e*x))), x] + Simp[1/(p*e^2*(2*c*
d - b*e))   Int[(f + g*x)^(n - 2)*(a + b*x + c*x^2)^p*Simp[b*e*g*((-e)*f +
d*g + e*f*n - d*g*n - e*f*p) + c*(d^2*g^2*(n - 1) - d*e*f*g*n + e^2*f^2*(2*
p + 1)) - e*g*(b*e*g*p - c*(e*f*n - d*g*n + 2*e*f*p))*x, x], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1] && LtQ[p, -1] && EqQ[c*d^2 - b*d*
e + a*e^2, 0]
```

3.487.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.41

method	result
gosper	$-\frac{2(cdx+ae)\left(40a^2c^2d^2e^6x^4+80a\,c^3d^4e^4x^4+8c^4d^6e^2x^4+60a^3cd\,e^7x^3+220a^2c^2d^3e^5x^3+212a\,c^3d^5e^3x^3+20c^4d^7e\,x^3+15a^4e^8x^2+180\right.}{15(a^5e^{10}-5a^4c\,d^2e^8+10a^3c^2d^4e^6-10}$
trager	$-\frac{2\left(40a^2c^2d^2e^6x^4+80a\,c^3d^4e^4x^4+8c^4d^6e^2x^4+60a^3cd\,e^7x^3+220a^2c^2d^3e^5x^3+212a\,c^3d^5e^3x^3+20c^4d^7e\,x^3+15a^4e^8x^2+180a^3c\,d^2e^6\right.}{15(a^4e^8-4a^3c\,d^2e^6+}$
default	$\frac{-\frac{1}{3cde\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}-\frac{\left(e^2a+c\,d^2\right)\left(\frac{\frac{4}{3}cdex+\frac{2}{3}e^2a+\frac{2}{3}c\,d^2}{\left(4ac\,d^2e^2-\left(e^2a+c\,d^2\right)^2\right)\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}+\frac{16cde\left(2cdex+e\right.}{3\left(4ac\,d^2e^2-\left(e^2a+c\,d^2\right)^2\right)^2\sqrt{a}}\right.}{2cde}}{e}$

input
```
int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,method=_RETURNVE
RBOSE)
```

output
```
-2/15*(c*d*x+a*e)*(40*a^2*c^2*d^2*e^6*x^4+80*a*c^3*d^4*e^4*x^4+8*c^4*d^6*e
^2*x^4+60*a^3*c*d*e^7*x^3+220*a^2*c^2*d^3*e^5*x^3+212*a*c^3*d^5*e^3*x^3+20
*c^4*d^7*e*x^3+15*a^4*e^8*x^2+180*a^3*c*d^2*e^6*x^2+378*a^2*c^2*d^4*e^4*x^
2+180*a*c^3*d^6*e^2*x^2+15*c^4*d^8*x^2+20*a^4*d*e^7*x+212*a^3*c*d^3*e^5*x+
220*a^2*c^2*d^5*e^3*x+60*a*c^3*d^7*e*x+8*a^4*d^2*e^6+80*a^3*c*d^4*e^4+40*a
^2*c^2*d^6*e^2)/(a^5*e^10-5*a^4*c*d^2*e^8+10*a^3*c^2*d^4*e^6-10*a^2*c^3*d^
6*e^4+5*a*c^4*d^8*e^2-c^5*d^10)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

3.487.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 820 vs. $2(247) = 494$.

Time = 16.88 (sec) , antiderivative size = 820, normalized size of antiderivative = 3.17

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \frac{}{15\left(a^2c^5d^{13}e^2-5\,a^3c^4d^{11}e^4+10\,a^4c^3d^9e^6-10\,a^5c^2d^7e^8+\right.}$$

input
```
integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="fricas")
```

output
```
2/15*(40*a^2*c^2*d^6*e^2 + 80*a^3*c*d^4*e^4 + 8*a^4*d^2*e^6 + 8*(c^4*d^6*e
^2 + 10*a*c^3*d^4*e^4 + 5*a^2*c^2*d^2*e^6)*x^4 + 4*(5*c^4*d^7*e + 53*a*c^3
*d^5*e^3 + 55*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7)*x^3 + 3*(5*c^4*d^8 + 60*a*
c^3*d^6*e^2 + 126*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 + 5*a^4*e^8)*x^2 + 4*
(15*a*c^3*d^7*e + 55*a^2*c^2*d^5*e^3 + 53*a^3*c*d^3*e^5 + 5*a^4*d*e^7)*x)*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a^2*c^5*d^13*e^2 - 5*a^3*c^4*
d^11*e^4 + 10*a^4*c^3*d^9*e^6 - 10*a^5*c^2*d^7*e^8 + 5*a^6*c*d^5*e^10 - a^
7*d^3*e^12 + (c^7*d^12*e^3 - 5*a*c^6*d^10*e^5 + 10*a^2*c^5*d^8*e^7 - 10*a^
3*c^4*d^6*e^9 + 5*a^4*c^3*d^4*e^11 - a^5*c^2*d^2*e^13)*x^5 + (3*c^7*d^13*e
^2 - 13*a*c^6*d^11*e^4 + 20*a^2*c^5*d^9*e^6 - 10*a^3*c^4*d^7*e^8 - 5*a^4*c
^3*d^5*e^10 + 7*a^5*c^2*d^3*e^12 - 2*a^6*c*d*e^14)*x^4 + (3*c^7*d^14*e - 9
*a*c^6*d^12*e^3 + a^2*c^5*d^10*e^5 + 25*a^3*c^4*d^8*e^7 - 35*a^4*c^3*d^6*e
^9 + 17*a^5*c^2*d^4*e^11 - a^6*c*d^2*e^13 - a^7*e^15)*x^3 + (c^7*d^15 + a*
c^6*d^13*e^2 - 17*a^2*c^5*d^11*e^4 + 35*a^3*c^4*d^9*e^6 - 25*a^4*c^3*d^7*e
^8 - a^5*c^2*d^5*e^10 + 9*a^6*c*d^3*e^12 - 3*a^7*d*e^14)*x^2 + (2*a*c^6*d^
14*e - 7*a^2*c^5*d^12*e^3 + 5*a^3*c^4*d^10*e^5 + 10*a^4*c^3*d^8*e^7 - 20*a
^5*c^2*d^6*e^9 + 13*a^6*c*d^4*e^11 - 3*a^7*d^2*e^13)*x)
```

3.487.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Timed out
```

3.487.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

3.487.8 Giac [F]

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \int \frac{x^2}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(ex+d)}\,dx$$

input
```
integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="giac")
```

output
```
integrate(x^2/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(e*x + d)), x
)
```

3.487.9 Mupad [B] (verification not implemented)

Time = 13.60 (sec) , antiderivative size = 3099, normalized size of antiderivative = 11.97

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)),x)
```

output
```
(((6*a*e^2 - 10*c*d^2)/(15*(a*e^2 - c*d^2)^4) - (4*c*d^2)/(5*(a*e^2 - c*d^
2)^4))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x) - (((d*((e
*(2*a*e^3 - 2*c*d^2*e))/(5*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e)) - (4*c
*d^2*e^2)/(5*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e))))/e + (e*(2*c*d^3 +
2*a*d*e^2))/(5*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*d^2
) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 + ((x*(a*e^2 + c*d^2) + a*d*e +
c*d*e*x^2)^(1/2)*(x*((((12*c^3*d^3*e^2)/(5*(a*e^2 - c*d^2)^2*(c^3*d^5*e -
2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^3*d^3*e^2*(a*e^2 + c*d^2))/(5*(a*e^
2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)))*(a*e^2 + c*d^2)
)/(c*d*e) - (6*c^2*d^2*e*(a*e^2 + c*d^2))/(5*(a*e^2 - c*d^2)^2*(c^3*d^5*e
- 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*a*c^3*d^4*e^3)/(5*(a*e^2 - c*d^2)^3
*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (2*c^2*d^2*e*(46*a^2*e^4 +
 4*c^2*d^4 + 66*a*c*d^2*e^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d
^3*e^3 + a^2*c*d*e^5))) + (a*((12*c^3*d^3*e^2)/(5*(a*e^2 - c*d^2)^2*(c^3*d
^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^3*d^3*e^2*(a*e^2 + c*d^2))/(
5*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - (c*
d*(a*e^2 + c*d^2)*(46*a^2*e^4 + 4*c^2*d^4 + 66*a*c*d^2*e^2))/(15*(a*e^2 -
c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/((a*e + c*d*x)*(d
+ e*x)) + ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(x*((a*(((a*e^2 +
 c*d^2)*((4*c^4*d^4*e^3*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5...
```

3.487. $\int \frac{x^2}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

3.488 $\int \frac{x^2}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}\,dx$

3.488.1 Optimal result

Integrand size = 40, antiderivative size = 341

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}\,dx = \frac{2x^2}{7(cd^2-ae^2)(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}} - \frac{8(2ade(cd^2+2ae^2)+(2c^2d^4+acd^2e^2+3a^2e^4)x)}{35e(cd^2-ae^2)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}} + \frac{16(3c^2d^4+14acd^2e^2+7a^2e^4)(cd^2+ae^2+2cdex)}{105e(cd^2-ae^2)^5\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}} - \frac{128cd(3c^2d^4+14acd^2e^2+7a^2e^4)(cd^2+ae^2+2cdex)}{105(cd^2-ae^2)^7\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

output
```
2/7*x^2/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)-8/3
5*(2*a*d*e*(2*a*e^2+c*d^2)+(3*a^2*e^4+a*c*d^2*e^2+2*c^2*d^4)*x)/e/(-a*e^2+
c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+16/105*(7*a^2*e^4+14*a*c*
d^2*e^2+3*c^2*d^4)*(2*c*d*e*x+a*e^2+c*d^2)/e/(-a*e^2+c*d^2)^5/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(3/2)-128/105*c*d*(7*a^2*e^4+14*a*c*d^2*e^2+3*c^2*d^
4)*(2*c*d*e*x+a*e^2+c*d^2)/(-a*e^2+c*d^2)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)
```

3.488.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.29

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}\,dx =$$
$$-\frac{2\sqrt{(ae+cdx)(d+ex)}\left(-15d^2e^4(ae+cdx)^6+84cd^3e^3(ae+cdx)^5(d+ex)+42ade^5(ae+cdx)^5(d+ex)\right.}{}$$

input
```
Integrate[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)),x]
```

output
```
(-2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-15*d^2*e^4*(a*e + c*d*x)^6 + 84*c*d^3*
e^3*(a*e + c*d*x)^5*(d + e*x) + 42*a*d*e^5*(a*e + c*d*x)^5*(d + e*x) - 210
*c^2*d^4*e^2*(a*e + c*d*x)^4*(d + e*x)^2 - 280*a*c*d^2*e^4*(a*e + c*d*x)^4
*(d + e*x)^2 - 35*a^2*e^6*(a*e + c*d*x)^4*(d + e*x)^2 + 420*c^3*d^5*e*(a*e
 + c*d*x)^3*(d + e*x)^3 + 1260*a*c^2*d^3*e^3*(a*e + c*d*x)^3*(d + e*x)^3 +
 420*a^2*c*d*e^5*(a*e + c*d*x)^3*(d + e*x)^3 + 105*c^4*d^6*(a*e + c*d*x)^2
*(d + e*x)^4 + 840*a*c^3*d^4*e^2*(a*e + c*d*x)^2*(d + e*x)^4 + 630*a^2*c^2
*d^2*e^4*(a*e + c*d*x)^2*(d + e*x)^4 - 70*a*c^4*d^5*e*(a*e + c*d*x)*(d + e
*x)^5 - 140*a^2*c^3*d^3*e^3*(a*e + c*d*x)*(d + e*x)^5 + 21*a^2*c^4*d^4*e^2
*(d + e*x)^6))/(105*(c*d^2 - a*e^2)^7*(a*e + c*d*x)^3*(d + e*x)^4)
```

3.488.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1244, 27, 1159, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}dx$$

$$\Big\downarrow \; 1244$$

$$-\frac{2\int -\frac{e^2\left(2ade-\left(3cd^2+7ae^2\right)x\right)}{2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{7/2}}dx}{7e^3\left(cd^2-ae^2\right)}-\frac{2dx}{7e(d+ex)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{2ade-(3cd^2+7ae^2)x}{(cdex^2+(cd^2+ae^2)x+ade)^{7/2}}dx}{7e(cd^2-ae^2)}-\frac{2dx}{7e(d+ex)(cd^2-ae^2)(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}$$

$\Big\downarrow$ 1159

$$\frac{-\frac{8(7a^2e^4+14acd^2e^2+3c^2d^4)\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}dx}{5(cd^2-ae^2)^2}-\frac{2(x(7a^2e^4+14acd^2e^2+3c^2d^4)+8ade(2ae^2+cd^2))}{5(cd^2-ae^2)^2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}}{7e(cd^2-ae^2)}-\frac{2dx}{7e(d+ex)(cd^2-ae^2)(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}$$

$\Big\downarrow$ 1089

$$\frac{-\frac{8(7a^2e^4+14acd^2e^2+3c^2d^4)\left(-\frac{8cde\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3\left(cd^2-ae^2\right)^2}-\frac{2\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}\right)}{5(cd^2-ae^2)^2}-\frac{2(x(7a^2e^4+14acd^2e^2+3}{5(cd^2-ae^2)^2(x(ae^2-}}{7e(cd^2-ae^2)}-\frac{2dx}{7e(d+ex)(cd^2-ae^2)(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}$$

$\Big\downarrow$ 1088

$$\frac{-\frac{2(x(7a^2e^4+14acd^2e^2+3c^2d^4)+8ade(2ae^2+cd^2))}{5(cd^2-ae^2)^2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}-\frac{8(7a^2e^4+14acd^2e^2+3c^2d^4)\left(\frac{16cde\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\left(ae^2+c\right.}{3\left(cd^2-ae^2\right)^2\left(x\left(ae^2\right.\right.}\right.}{5(cd^2-ae^2)^2}}{7e(cd^2-ae^2)}-\frac{2dx}{7e(d+ex)(cd^2-ae^2)(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}$$

input
```
Int[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)),x]
```

output
```
(-2*d*x)/(7*e*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(5/2)) + ((-2*(8*a*d*e*(c*d^2 + 2*a*e^2) + (3*c^2*d^4 + 14*a*c*d^2*e
^2 + 7*a^2*e^4)*x))/(5*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(5/2)) - (8*(3*c^2*d^4 + 14*a*c*d^2*e^2 + 7*a^2*e^4)*((-2*(c*d^2 +
a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2)) + (16*c*d*e*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^
4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(5*(c*d^2 - a*e^2)^2))/(7
*e*(c*d^2 - a*e^2))
```

3.488. $\int \frac{x^2}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}\,dx$

3.488.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1244
```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d
_) + (e_.)*(x_)), x_Symbol] :> Simp[(-(e*f - d*g))*(f + g*x)^(n - 1)*((a +
b*x + c*x^2)^(p + 1)/(p*(2*c*d - b*e)*(d + e*x))), x] + Simp[1/(p*e^2*(2*c*
d - b*e))   Int[(f + g*x)^(n - 2)*(a + b*x + c*x^2)^p*Simp[b*e*g*((-e)*f +
d*g + e*f*n - d*g*n - e*f*p) + c*(d^2*g^2*(n - 1) - d*e*f*g*n + e^2*f^2*(2*
p + 1)) - e*g*(b*e*g*p - c*(e*f*n - d*g*n + 2*e*f*p))*x, x], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1] && LtQ[p, -1] && EqQ[c*d^2 - b*d*
e + a*e^2, 0]
```

3.488.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(325) = 650$.

Time = 0.66 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.94

method	result
gosper	$-\frac{2(cdx+ae)\left(-896a^2c^4d^4e^8x^6-1792a\,c^5d^6e^6x^6-384c^6d^8e^4x^6-2240a^3c^3d^3e^9x^5-7616a^2c^4d^5e^7x^5-7232a\,c^5d^7e^5x^5-1344c^6d^9e^3x\right.}{}$
trager	$-\frac{2\left(-896a^2c^4d^4e^8x^6-1792a\,c^5d^6e^6x^6-384c^6d^8e^4x^6-2240a^3c^3d^3e^9x^5-7616a^2c^4d^5e^7x^5-7232a\,c^5d^7e^5x^5-1344c^6d^9e^3x^5-1680a\right.}{}$
default	$\frac{-\frac{1}{5cde\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{5}{2}}}-\frac{\left(e^2a+c\,d^2\right)\left(\frac{\frac{4}{5}cdex+\frac{2}{5}e^2a+\frac{2}{5}c\,d^2}{\left(4ac\,d^2e^2-\left(e^2a+c\,d^2\right)^2\right)\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{5}{2}}}+\frac{16cde\left(\frac{\frac{4}{3}cdex}{\left(4ac\,d^2e^2-\left(e^2a+c\,d^2\right)^2\right.}\right.}{}\right.}{2cde}}{e}$

input

```
int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x,method=_RETURNVE
RBOSE)
```

output

```
-2/105*(c*d*x+a*e)*(-896*a^2*c^4*d^4*e^8*x^6-1792*a*c^5*d^6*e^6*x^6-384*c^
6*d^8*e^4*x^6-2240*a^3*c^3*d^3*e^9*x^5-7616*a^2*c^4*d^5*e^7*x^5-7232*a*c^5
*d^7*e^5*x^5-1344*c^6*d^9*e^3*x^5-1680*a^4*c^2*d^2*e^10*x^4-11200*a^3*c^3*
d^4*e^8*x^4-20320*a^2*c^4*d^6*e^6*x^4-11200*a*c^5*d^8*e^4*x^4-1680*c^6*d^1
0*e^2*x^4-280*a^5*c*d*e^11*x^3-6440*a^4*c^2*d^3*e^9*x^3-21680*a^3*c^3*d^5*
e^7*x^3-24080*a^2*c^4*d^7*e^5*x^3-8120*a*c^5*d^9*e^3*x^3-840*c^6*d^11*e*x^
3+35*a^6*e^12*x^2-910*a^5*c*d^2*e^10*x^2-9295*a^4*c^2*d^4*e^8*x^2-20020*a^
3*c^3*d^6*e^6*x^2-13195*a^2*c^4*d^8*e^4*x^2-2590*a*c^5*d^10*e^2*x^2-105*c^
6*d^12*x^2+28*a^6*d*e^11*x-764*a^5*c*d^3*e^9*x-6440*a^4*c^2*d^5*e^7*x-8120
*a^3*c^3*d^7*e^5*x-2996*a^2*c^4*d^9*e^3*x-140*a*c^5*d^11*e*x+8*a^6*d^2*e^1
0-224*a^5*c*d^4*e^8-1680*a^4*c^2*d^6*e^6-1120*a^3*c^3*d^8*e^4-56*a^2*c^4*d
^10*e^2)/(a^7*e^14-7*a^6*c*d^2*e^12+21*a^5*c^2*d^4*e^10-35*a^4*c^3*d^6*e^8
+35*a^3*c^4*d^8*e^6-21*a^2*c^5*d^10*e^4+7*a*c^6*d^12*e^2-c^7*d^14)/(c*d*e*
x^2+a*e^2*x+c*d^2*x+a*d*e)^(7/2)
```

3.488. $\int \frac{x^2}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}\,dx$

3.488.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1540 vs. $2(325) = 650$.

Time = 115.68 (sec) , antiderivative size = 1540, normalized size of antiderivative = 4.52

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm
="fricas")
```

output
```
-2/105*(56*a^2*c^4*d^10*e^2 + 1120*a^3*c^3*d^8*e^4 + 1680*a^4*c^2*d^6*e^6
+ 224*a^5*c*d^4*e^8 - 8*a^6*d^2*e^10 + 128*(3*c^6*d^8*e^4 + 14*a*c^5*d^6*e
^6 + 7*a^2*c^4*d^4*e^8)*x^6 + 64*(21*c^6*d^9*e^3 + 113*a*c^5*d^7*e^5 + 119
*a^2*c^4*d^5*e^7 + 35*a^3*c^3*d^3*e^9)*x^5 + 80*(21*c^6*d^10*e^2 + 140*a*c
^5*d^8*e^4 + 254*a^2*c^4*d^6*e^6 + 140*a^3*c^3*d^4*e^8 + 21*a^4*c^2*d^2*e^
10)*x^4 + 40*(21*c^6*d^11*e + 203*a*c^5*d^9*e^3 + 602*a^2*c^4*d^7*e^5 + 54
2*a^3*c^3*d^5*e^7 + 161*a^4*c^2*d^3*e^9 + 7*a^5*c*d*e^11)*x^3 + 5*(21*c^6*
d^12 + 518*a*c^5*d^10*e^2 + 2639*a^2*c^4*d^8*e^4 + 4004*a^3*c^3*d^6*e^6 +
1859*a^4*c^2*d^4*e^8 + 182*a^5*c*d^2*e^10 - 7*a^6*e^12)*x^2 + 4*(35*a*c^5*
d^11*e + 749*a^2*c^4*d^9*e^3 + 2030*a^3*c^3*d^7*e^5 + 1610*a^4*c^2*d^5*e^7
 + 191*a^5*c*d^3*e^9 - 7*a^6*d*e^11)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)/(a^3*c^7*d^18*e^3 - 7*a^4*c^6*d^16*e^5 + 21*a^5*c^5*d^14*e^7 - 3
5*a^6*c^4*d^12*e^9 + 35*a^7*c^3*d^10*e^11 - 21*a^8*c^2*d^8*e^13 + 7*a^9*c*
d^6*e^15 - a^10*d^4*e^17 + (c^10*d^17*e^4 - 7*a*c^9*d^15*e^6 + 21*a^2*c^8*
d^13*e^8 - 35*a^3*c^7*d^11*e^10 + 35*a^4*c^6*d^9*e^12 - 21*a^5*c^5*d^7*e^1
4 + 7*a^6*c^4*d^5*e^16 - a^7*c^3*d^3*e^18)*x^7 + (4*c^10*d^18*e^3 - 25*a*c
^9*d^16*e^5 + 63*a^2*c^8*d^14*e^7 - 77*a^3*c^7*d^12*e^9 + 35*a^4*c^6*d^10*
e^11 + 21*a^5*c^5*d^8*e^13 - 35*a^6*c^4*d^6*e^15 + 17*a^7*c^3*d^4*e^17 - 3
*a^8*c^2*d^2*e^19)*x^6 + 3*(2*c^10*d^19*e^2 - 10*a*c^9*d^17*e^4 + 15*a^2*c
^8*d^15*e^6 + 7*a^3*c^7*d^13*e^8 - 49*a^4*c^6*d^11*e^10 + 63*a^5*c^5*d^...
```

3.488.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(7/2),x)
```

output
```
Timed out
```

3.488.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

3.488.8 Giac [F]

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{7/2}}\,dx = \int \frac{x^2}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{7}{2}}(ex+d)}\,dx$$

input
```
integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm
="giac")
```

output
```
integrate(x^2/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(7/2)*(e*x + d)), x
)
```

3.488.9 Mupad [B] (verification not implemented)

Time = 17.00 (sec) , antiderivative size = 11469, normalized size of antiderivative = 33.63

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{7/2}}\,dx = \text{Too large to display}$$

input
```
int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)),x)
```

output
```
((6*c^3*d^5 + 36*a*c^2*d^3*e^2 - 10*a^2*c*d*e^4)/(105*(a*e^2 - c*d^2)^6) -
 x*((16*c^2*d^2*e)/(105*(a*e^2 - c*d^2)^5) - (8*c^2*d^2*e*(a*e^2 + c*d^2))
/(105*(a*e^2 - c*d^2)^6)) + (8*a*c^2*d^3*e^2)/(105*(a*e^2 - c*d^2)^6))/(x*
(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + (x*((a*((64*c^5*d^5*e^4*(a*e^
2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*
e^5)) - (64*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d
^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c
*d^2)*((64*c^5*d^5*e^4*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e
- 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (64*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(
105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*
e) - (32*c^4*d^4*e^3*(7*c^2*d^4 - 9*a^2*e^4 + 18*a*c*d^2*e^2))/(105*(a*e^2
 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (128*a*c^5*d^6*
e^5)/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) +
 (32*c^4*d^4*e^3*(a*e^2 + c*d^2)*(5*a*e^2 - 3*c*d^2))/(105*(a*e^2 - c*d^2)
^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2
*(60*c^4*d^7 - 204*a*c^3*d^5*e^2 - 156*a^2*c^2*d^3*e^4 + 44*a^3*c*d*e^6))/
(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*
c^3*d^3*e^2*(a*e^2 + c*d^2)*(7*c^2*d^4 - 9*a^2*e^4 + 18*a*c*d^2*e^2))/(105
*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))) - (a*(((a
*e^2 + c*d^2)*((64*c^5*d^5*e^4*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*...
```

3.489 $\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx$

3.489.1 Optimal result

Integrand size = 23, antiderivative size = 170

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\frac{6}{55}x\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{11}x^4\sqrt{1+x}\sqrt{1-x+x^2}-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{55\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,(1+x^3)}$$

output
```
6/55*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/11*x^4*(1+x)^(1/2)*(x^2-x+1)^(1/2)-4/
55*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I
)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1
/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.489.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.67 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.30

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx$$

$$=\frac{2\left(x\sqrt{1+x}(3-3x+3x^2+5x^3-5x^4+5x^5)+\sqrt{-\frac{6i}{3i+\sqrt{3}}}\left(3i+\sqrt{3}\right)(1+x)\sqrt{\frac{3i+\sqrt{3}+\left(-3i+\sqrt{3}\right)x}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{\frac{-3i+\sqrt{3}+\cdots}{\left(3i+\sqrt{\cdots}\right.}}\cdots\right.}{55\sqrt{1-x+x^2}}$$

input
```
Integrate[x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

output
```
(2*(x*Sqrt[1 + x]*(3 - 3*x + 3*x^2 + 5*x^3 - 5*x^4 + 5*x^5) + Sqrt[(-6*I)/
(3*I + Sqrt[3])]*(3*I + Sqrt[3])*(1 + x)*Sqrt[(3*I + Sqrt[3] + (-3*I + Sqr
t[3])*x)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[(-3*I + Sqrt[3] + (3*I + Sqrt[3]
)*x)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqr
t[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/(55*Sqrt[1 - x + x
^2])
```

3.489.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1210, 811, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{x+1}\sqrt{x^2-x+1}\,dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int x^3\sqrt{x^3+1}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{811}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{11}\int \frac{x^3}{\sqrt{x^3+1}}dx+\frac{2}{11}\sqrt{x^3+1}x^4\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{843}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{11}\left(\frac{2}{5}x\sqrt{x^3+1}-\frac{2}{5}\int \frac{1}{\sqrt{x^3+1}}dx\right)+\frac{2}{11}\sqrt{x^3+1}x^4\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{11}\left(\frac{2}{5}x\sqrt{x^3+1}-\frac{4\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)+\frac{2}{11}\sqrt{x^3+1}x^4\right)}{\sqrt{x^3+1}}$$

input
```
Int[x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*((2*x^4*Sqrt[1 + x^3])/11 + (3*((2*x*Sqrt[1
 + x^3])/5 - (4*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3]
+ x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt
[3]])/(5*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/11))/S
qrt[1 + x^3]
```

3.489.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

3.489. $\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx$

3.489.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x^4\sqrt{x^3+1}}{11}+\frac{6x\sqrt{x^3+1}}{55}-\frac{12\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{55\sqrt{x^3+1}}\right)}{\sqrt{1+x}\sqrt{x^2-x+1}}$
risch	$\frac{2x(5x^3+3)\sqrt{1+x}\sqrt{x^2-x+1}}{55}-\frac{12\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)(x^2-x+1)}}{55\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$\frac{2\sqrt{1+x}\sqrt{x^2-x+1}\left(5x^7+3i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)-9\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\right.}{55(x^3+1)}$

input
```
int(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(2/11*x^4*(x^3+1)^(1/2
)+6/55*x*(x^3+1)^(1/2)-12/55*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)
))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*
3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1
/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

3.489.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.19

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{2}{55}\left(5\,x^4+3\,x\right)\sqrt{x^2-x+1}\sqrt{x+1} - \frac{12}{55}\,\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
2/55*(5*x^4 + 3*x)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 12/55*weierstrassPInver
se(0, -4, x)
```

3.489.6 Sympy [F]

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int x^3\sqrt{x+1}\sqrt{x^2-x+1}\,dx$$

input
```
integrate(x**3*(1+x)**(1/2)*(x**2-x+1)**(1/2),x)
```

output
```
Integral(x**3*sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

3.489.7 Maxima [F]

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int \sqrt{x^2-x+1}\sqrt{x+1}x^3\,dx$$

input
```
integrate(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3, x)
```

3.489.8 Giac [F]

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int \sqrt{x^2-x+1}\sqrt{x+1}x^3\,dx$$

input
```
integrate(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3, x)
```

3.489.9 Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\int x^3\,\sqrt{x+1}\,\sqrt{x^2-x+1}\,dx$$

input
```
int(x^3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2),x)
```

output
```
int(x^3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2), x)
```

3.490 $\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx$

3.490.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{2}{9}(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}$$

output
```
2/9*(1+x)^(3/2)*(x^2-x+1)^(3/2)
```

3.490.2 Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{2}{9}(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}$$

input
```
Integrate[x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

output
```
(2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))/9
```

3.490.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{x+1}\sqrt{x^2-x+1}\,dx$$
$$\Big\downarrow \ 1208$$
$$\frac{2}{9}(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}$$

input
```
Int[x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

output
```
(2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))/9
```

3.490.3.1 Defintions of rubi rules used

rule 1208
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^2*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g^2*(d + e*x)^(m + 1)*((a + b*x + c
*x^2)^(p + 1)/(c*e*(m + 2*p + 3))), x] /; FreeQ[{a, b, c, d, e, f, g, m, p}
, x] && EqQ[b*e*g*(m + p + 2) + 2*c*(d*g*(p + 1) - e*f*(m + 2*p + 3)), 0] &
& EqQ[e*(c*f^2 - b*f*g + a*g^2)*(m + 1) + (2*c*f - b*g)*(e*f - d*g)*(p + 1)
, 0] && NeQ[m + 2*p + 3, 0]
```

3.490.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{2(1+x)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}{9}$	18
default	$\frac{2\left(x^3+1\right)\sqrt{1+x}\,\sqrt{x^2-x+1}}{9}$	23
risch	$\frac{2\left(x^3+1\right)\sqrt{1+x}\,\sqrt{x^2-x+1}}{9}$	23
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x^3\sqrt{x^3+1}}{9}+\frac{2\sqrt{x^3+1}}{9}\right)}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$	53

input
```
int(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(1+x)^(3/2)*(x^2-x+1)^(3/2)
```

3.490.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{2}{9}\left(x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}$$

input
```
integrate(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

3.490.6 Sympy [F]

$$\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int x^2\sqrt{x+1}\sqrt{x^2-x+1}\,dx$$

input
```
integrate(x**2*(1+x)**(1/2)*(x**2-x+1)**(1/2),x)
```

output
```
Integral(x**2*sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

3.490.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{2}{9}\left(x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}$$

input
```
integrate(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
2/9*(x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

3.490.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(17) = 34$.

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.91

$$\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx$$
$$= \frac{2}{315}\left((5\,(7\,x-23)(x+1)+258)(x+1)-213\right)\sqrt{(x+1)^2-3\,x}\sqrt{x+1}$$
$$+ \frac{2}{105}\,(3\,(5\,x-12)(x+1)+71)\sqrt{(x+1)^2-3\,x}\sqrt{x+1}$$

input
```
integrate(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
2/315*((5*(7*x - 23)*(x + 1) + 258)*(x + 1) - 213)*sqrt((x + 1)^2 - 3*x)*s
qrt(x + 1) + 2/105*(3*(5*x - 12)*(x + 1) + 71)*sqrt((x + 1)^2 - 3*x)*sqrt(
x + 1)
```

3.490.9 Mupad [B] (verification not implemented)

Time = 11.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{2\,(x^3+1)\,\sqrt{x+1}\,\sqrt{x^2-x+1}}{9}$$

input
```
int(x^2*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2),x)
```

output
```
(2*(x^3 + 1)*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/9
```

3.491 $\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx$

3.491.1 Optimal result

Integrand size = 21, antiderivative size = 294

$$\begin{aligned}\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx &= \frac{2}{7}x^2\sqrt{1+x}\sqrt{1-x+x^2}+\frac{6\sqrt{1+x}\sqrt{1-x+x^2}}{7\left(1+\sqrt{3}+x\right)} \\ &\quad -\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{7\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)} \\ &\quad +\frac{2\sqrt{2}3^{3/4}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{7\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}\end{aligned}$$

output
```
2/7*x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2)+6/7*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(1+x+3
^(1/2))+2/7*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^
(1/2)+2*I)*2^(1/2)*(x^2-x+1)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+
1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-3/7*3^(1/4)*(1+x)^(3/2)*EllipticE((1+x-3^
(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/
2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2
)
```

3.491.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.40 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.18

$$\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{\sqrt{1+x}\left(4x^2\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}(1-x+x^2)-3\sqrt{2}(-3i+\sqrt{3})\sqrt{\frac{i+\sqrt{3}-2ix}{3i+\sqrt{3}}}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right)\right.\right.}{14\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-}}$$

input
```
Integrate[x*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

output
```
(Sqrt[1 + x]*(4*x^2*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]*(1 - x + x^2) - 3
*Sqrt[2]*(-3*I + Sqrt[3])*Sqrt[(I + Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])]*Sq
rt[(-I + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[2]*
Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])] +
3*Sqrt[2]*(-I + Sqrt[3])*Sqrt[(I + Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])]*Sqr
t[(-I + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*EllipticF[I*ArcSinh[Sqrt[2]*S
qrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/(
14*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])
```

3.491.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1210, 811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{x+1}\sqrt{x^2-x+1}\,dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int x\sqrt{x^3+1}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{811}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{7}\int \frac{x}{\sqrt{x^3+1}}dx+\frac{2}{7}\sqrt{x^3+1}x^2\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{832}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{7}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)+\frac{2}{7}\sqrt{x^3+1}x^2\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{7}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)+\frac{2}{7}\sqrt{x^3 ...}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{2416}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{7}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2- ...}{(x+\sqrt{ ...}}}}{\sqrt{(x- ...}}\right.\right.}{\sqrt{x^3+1}}$$

input
```
Int[x*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*((2*x^2*Sqrt[1 + x^3])/7 + (3*((2*Sqrt[1 +
x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x +
 x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3]
 + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
 - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt
[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*
Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/7))/
Sqrt[1 + x^3]
```

3.491. $\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx$

3.491.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.491. $\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx$

3.491.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.73

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x^2\sqrt{x^3+1}}{7}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right.\right.}{7\sqrt{x^3+1}}\right.}{\sqrt{1+x}\sqrt{x^2-x+1}}$
risch	$\frac{2x^2\sqrt{1+x}\sqrt{x^2-x+1}}{7}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)F\right.}{7\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$\frac{\sqrt{1+x}\sqrt{x^2-x+1}\left(3i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)+2x^5+9\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\right.}{7x^3+7}$

input
```
int(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(2/7*x^2*(x^3+1)^(1/2)
+6/7*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3
^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(
1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2
*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2
+1/2*I*3^(1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3
^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))))
```

3.491.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.10

$$\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\frac{2}{7}\sqrt{x^2-x+1}\sqrt{x+1}x^2-\frac{6}{7}\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

input
```
integrate(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
2/7*sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2 - 6/7*weierstrassZeta(0, -4, weierst
rassPInverse(0, -4, x))
```

3.491.6 Sympy [F]

$$\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int x\sqrt{x+1}\sqrt{x^2-x+1}\,dx$$

input
```
integrate(x*(1+x)**(1/2)*(x**2-x+1)**(1/2),x)
```

output
```
Integral(x*sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

3.491.7 Maxima [F]

$$\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int \sqrt{x^2-x+1}\sqrt{x+1}x\,dx$$

input
```
integrate(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x, x)
```

3.491.8 Giac [F]

$$\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int \sqrt{x^2-x+1}\sqrt{x+1}x\,dx$$

input
```
integrate(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x, x)
```

3.491.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\int x\,\sqrt{x+1}\,\sqrt{x^2-x+1}\,dx$$

input
```
int(x*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2),x)
```

output
```
int(x*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2), x)
```

3.492 $\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx$

3.492.1 Optimal result

Integrand size = 20, antiderivative size = 144

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{2}{5}x\sqrt{1+x}\sqrt{1-x+x^2} + \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{5\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,(1+x^3)}$$

output
```
2/5*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/5*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3
^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)+1/2*2^(1
/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/
2)
```

3.492.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.35 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.17

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{2x\sqrt{1+x}(1-x+x^2) + \frac{i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right),\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{5\sqrt{1-x+x^2}}$$

input
```
Integrate[Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

output
```
(2*x*Sqrt[1 + x]*(1 - x + x^2) + (I*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqrt[3
])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSin
h[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3
])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(5*Sqrt[1 - x + x^2])
```

3.492.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1151, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x+1}\sqrt{x^2-x+1}\,dx$$

$$\Big\downarrow \text{1151}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int \sqrt{x^3+1}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{748}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{5}\int \frac{1}{\sqrt{x^3+1}}dx+\frac{2}{5}\sqrt{x^3+1}x\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{5\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2}{5}\sqrt{x^3+1}x\right)}{\sqrt{x^3+1}}$$

input
```
Int[Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*((2*x*Sqrt[1 + x^3])/5 + (2*3^(3/4)*Sqrt[2
+ Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSi
n[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(5*Sqrt[(1 + x)/(
1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/Sqrt[1 + x^3]
```

3.492.3.1 Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 1151
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^FracPart[p]/(a*d + c
*e*x^3)^FracPart[p])   Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; F
reeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
 IGtQ[m - p + 1, 0] && !IntegerQ[p]
```

3.492.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x\sqrt{x^3+1}}{5}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{5\sqrt{x^3+1}}\right)}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$
risch	$\frac{2x\sqrt{1+x}\,\sqrt{x^2-x+1}}{5}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)(x^2-x+1)}}{5\sqrt{x^3+1}\,\sqrt{1+x}\,\sqrt{x^2-x+1}}$
default	$-\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\left(3i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)-9\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\right.}{5(x^3+1)}$

input
```
int((1+x)^(1/2)*(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(2/5*x*(x^3+1)^(1/2)+6
/5*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/
2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

3.492.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.17

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{2}{5}\sqrt{x^2-x+1}\sqrt{x+1}x + \frac{6}{5}\,\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
2/5*sqrt(x^2 - x + 1)*sqrt(x + 1)*x + 6/5*weierstrassPInverse(0, -4, x)
```

3.492.6 Sympy [F]

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int \sqrt{x+1}\sqrt{x^2-x+1}\,dx$$

input
```
integrate((1+x)**(1/2)*(x**2-x+1)**(1/2),x)
```

output
```
Integral(sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

3.492.7 Maxima [F]

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int \sqrt{x^2-x+1}\sqrt{x+1}\,dx$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

3.492.8 Giac [F]

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int \sqrt{x^2-x+1}\sqrt{x+1}\,dx$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

3.492.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int \sqrt{x+1}\,\sqrt{x^2-x+1}\,dx$$

input
```
int((x + 1)^(1/2)*(x^2 - x + 1)^(1/2),x)
```

output
```
int((x + 1)^(1/2)*(x^2 - x + 1)^(1/2), x)
```

3.493 $\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} \, dx$

3.493.1 Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} \, dx = \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2} - \frac{2\sqrt{1+x}\sqrt{1-x+x^2}\operatorname{arctanh}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x^3}}$$

output
```
2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)-2/3*arctanh((x^3+1)^(1/2))*(1+x)^(1/2)*(x^
2-x+1)^(1/2)/(x^3+1)^(1/2)
```

3.493.2 Mathematica [A] (verified)

Time = 15.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} \, dx = \frac{2}{3}\left(\sqrt{1+x}\sqrt{1-x+x^2} - \operatorname{arctanh}\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\right)$$

input
```
Integrate[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x,x]
```

output
```
(2*(Sqrt[1 + x]*Sqrt[1 - x + x^2] - ArcTanh[Sqrt[1 + x]*Sqrt[1 - x + x^2]]
))/3
```

3.493.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1210, 798, 60, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x}\,dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int \frac{\sqrt{x^3+1}}{x}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int \frac{\sqrt{x^3+1}}{x^3}dx^3}{3\sqrt{x^3+1}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\int \frac{1}{x^3\sqrt{x^3+1}}dx^3+2\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(2\int \frac{1}{x^6-1}d\sqrt{x^3+1}+2\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

$$\Big\downarrow \text{220}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(2\sqrt{x^3+1}-2\text{arctanh}\left(\sqrt{x^3+1}\right)\right)}{3\sqrt{x^3+1}}$$

input
```
Int[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x,x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*(2*Sqrt[1 + x^3] - 2*ArcTanh[Sqrt[1 + x^3]]
))/(3*Sqrt[1 + x^3])
```

3.493.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

3.493. $\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx$

3.493.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.65

method	result	size
default	$-\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}\,\left(-\sqrt{x^3+1}+\operatorname{arctanh}\left(\sqrt{x^3+1}\right)\right)}{3\sqrt{x^3+1}}$	43
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\,\left(\frac{2\sqrt{x^3+1}}{3}-\frac{2\,\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{3}\right)}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$	51

input
```
int((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(-(x^3+1)^(1/2)+arctanh((x^3+1)^(1/2)))/(
x^3+1)^(1/2)
```

3.493.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx = \frac{2}{3}\sqrt{x^2-x+1}\sqrt{x+1}-\frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}+1\right) + \frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}-1\right)$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x, algorithm="fricas")
```

output
```
2/3*sqrt(x^2 - x + 1)*sqrt(x + 1) - 1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1)
+ 1) + 1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) - 1)
```

3.493.6 Sympy [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx = \int \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x}\,dx$$

input
```
integrate((1+x)**(1/2)*(x**2-x+1)**(1/2)/x,x)
```

output
```
Integral(sqrt(x + 1)*sqrt(x**2 - x + 1)/x, x)
```

3.493.7 Maxima [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx = \int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x}\,dx$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x, x)
```

3.493.8 Giac [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx = \int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x}\,dx$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x, x)
```

3.493.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx = \int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x}\,dx$$

input
```
int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x,x)
```

output
```
int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x, x)
```

3.494 $\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx$

3.494.1 Optimal result

Integrand size = 23, antiderivative size = 287

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}+\frac{3\sqrt{1+x}\sqrt{1-x+x^2}}{1+\sqrt{3}+x} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}(1+x^3)} + \frac{\sqrt{2}3^{3/4}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}(1+x^3)}$$

output
```
-(1+x)^(1/2)*(x^2-x+1)^(1/2)/x+3*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(1+x+3^(1/2))
+3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*
2^(1/2)*(x^2-x+1)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(
1+x+3^(1/2))^2)^(1/2)-3/2*3^(1/4)*(1+x)^(3/2)*EllipticE((1+x-3^(1/2))/(1+x
+3^(1/2)),I*3^(1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x
+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.494.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.30 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} + \frac{3\sqrt{1+\frac{2i(1+x)}{-3i+\sqrt{3}}}\sqrt{1-\frac{2i(1+x)}{3i+\sqrt{3}}}\left(-\frac{\left(-3i+\sqrt{3}\right)\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1+x}E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}} + \frac{\left(-i+\sqrt{3}\right)\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-}}{}\right.}{2\sqrt{2}\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{3-3(1+x)+(1+x)^2}}$$

input
```
Integrate[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^2,x]
```

output
```
-((Sqrt[1 + x]*Sqrt[1 - x + x^2])/x) + (3*Sqrt[1 + ((2*I)*(1 + x))/(-3*I +
 Sqrt[3])]*Sqrt[1 - ((2*I)*(1 + x))/(3*I + Sqrt[3])]*(-(((-3*I + Sqrt[3])*
Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 + x]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[((
-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-
I)*(1 + x))/(3*I + Sqrt[3])]) + ((-I + Sqrt[3])*Sqrt[(-I)/(3*I + Sqrt[3])]
*Sqrt[1 + x]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3
])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3]
)]))/(2*Sqrt[2]*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[3 - 3*(1 + x) + (1 + x)^2]
)
```

3.494.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1210, 809, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x^2}dx$$

$$\Big\downarrow \text{ 1210}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int \frac{\sqrt{x^3+1}}{x^2}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \; 809$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{2}\int \frac{x}{\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{x}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \; 832$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)-\frac{\sqrt{x^3+1}}{x}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \; 759$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)-\frac{\sqrt{x^3+1}}{x}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \; 2416$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{2}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-\ldots}{\left(x+\sqrt{\ldots}\right.}}}{\sqrt{\frac{\ldots}{\left(x-\ldots\right.}}}\right.\right.}{\sqrt{x^3+1}}$$

input
```
Int[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^2,x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*(-(Sqrt[1 + x^3]/x) + (3*((2*Sqrt[1 + x^3])
/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)
/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)
], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (2
*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] +
 x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[
3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/2))/Sqrt[
1 + x^3]
```

3.494. $\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx$

3.494.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.494. $\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx$

3.494.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.75

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{x}+\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\cdots\right)}{\sqrt{x^3+1}}\right)}{\sqrt{1+x}\sqrt{x^2-x+1}}$
risch	$-\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}}{x}+\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)F\left(\cdots\right)\right)}{\sqrt{x^3+1}\,\sqrt{1+x}\,\sqrt{x^2-x+1}}$
default	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\left(3i\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\,x+9\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\cdots\right)}{2x(x^3+1\cdots)}$

input
```
int((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(-1/x*(x^3+1)^(1/2)+3*
(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2
))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^
(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*
I*3^(1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2
))/(-3/2-1/2*I*3^(1/2)))^(1/2))))
```

3.494.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.11

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = -\frac{3\,x\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))+\sqrt{x^2-x+1}\sqrt{x+1}}{x}$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-(3*x*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)) + sqrt(x^2 - x
 + 1)*sqrt(x + 1))/x
```

3.494.6 Sympy [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = \int \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x^2}\,dx$$

input
```
integrate((1+x)**(1/2)*(x**2-x+1)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(x + 1)*sqrt(x**2 - x + 1)/x**2, x)
```

3.494.7 Maxima [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = \int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^2}\,dx$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2, x)
```

3.494.8 Giac [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = \int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^2}\,dx$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2, x)
```

3.494.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = \int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^2}\,dx$$

input
```
int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x^2,x)
```

output
```
int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x^2, x)
```

3.495 $\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3} \, dx$

3.495.1 Optimal result

Integrand size = 23, antiderivative size = 146

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3} \, dx = -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{2x^2} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,(1+x^3)}$$

output
```
-1/2*(1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2+1/2*3^(3/4)*(1+x)^(3/2)*EllipticF((1+
x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)+1/2*2
^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^
(1/2)
```

3.495.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.31 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3} \, dx = \frac{\sqrt{1+x}\left(-\frac{2(1-x+x^2)}{x^2} - \frac{3i\sqrt{2}\sqrt{\frac{i+\sqrt{3}-2ix}{3i+\sqrt{3}}}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right),\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}}\right)}{4\sqrt{1-x+x^2}}$$

input
```
Integrate[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^3,x]
```

output
```
(Sqrt[1 + x]*((-2*(1 - x + x^2))/x^2 - ((3*I)*Sqrt[2]*Sqrt[(I + Sqrt[3] -
(2*I)*x)/(3*I + Sqrt[3])]*Sqrt[(-I + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*
EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I +
Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]))/(4*Sqrt[
1 - x + x^2])
```

3.495.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1210, 809, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x^3}\,dx$$

$$\Big\downarrow \text{ 1210}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int \frac{\sqrt{x^3+1}}{x^3}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{ 809}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{4}\int \frac{1}{\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{2x^2}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3^{3/4}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{2\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt{x^3+1}}{2x^2}\right)}{\sqrt{x^3+1}}$$

input
```
Int[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^3,x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*(-1/2*Sqrt[1 + x^3]/x^2 + (3^(3/4)*Sqrt[2 +
 Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin
[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1 + x)/(1
 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/Sqrt[1 + x^3]
```

3.495.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

3.495. $\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx$

3.495.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{2x^2}+\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^3+1}}\right)}{\sqrt{1+x}\sqrt{x^2-x+1}}$
risch	$-\frac{\sqrt{1+x}\sqrt{x^2-x+1}}{2x^2}+\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)(x^2-x+1)}}{2\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$-\frac{\sqrt{1+x}\sqrt{x^2-x+1}\left(3i\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\,x^2-9\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\cdots}\right.}{4(x^3+1)x^2}$

input
```
int((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(-1/2/x^2*(x^3+1)^(1/2
)+3/2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*
3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^
(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((
-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

3.495.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx = \frac{3\,x^2\text{weierstrassPInverse}(0,-4,x)-\sqrt{x^2-x+1}\sqrt{x+1}}{2\,x^2}$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(3*x^2*weierstrassPInverse(0, -4, x) - sqrt(x^2 - x + 1)*sqrt(x + 1))/
x^2
```

3.495.6 Sympy [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx = \int \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x^3}\,dx$$

input
```
integrate((1+x)**(1/2)*(x**2-x+1)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(x + 1)*sqrt(x**2 - x + 1)/x**3, x)
```

3.495.7 Maxima [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx = \int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^3}\,dx$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^3, x)
```

3.495.8 Giac [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx = \int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^3}\,dx$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^3, x)
```

3.495.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx = \int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^3}\,dx$$

input
```
int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x^3,x)
```

output
```
int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x^3, x)
```

3.496 $\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx$

3.496.1 Optimal result

Integrand size = 23, antiderivative size = 201

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{54}{935}x\sqrt{1+x}\sqrt{1-x+x^2}+\frac{18}{187}x^4\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{17}x^4\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)-\frac{36\ 3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{935\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}$$

output
```
54/935*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)+18/187*x^4*(1+x)^(1/2)*(x^2-x+1)^(1/2
)+2/17*x^4*(x^3+1)*(1+x)^(1/2)*(x^2-x+1)^(1/2)-36/935*3^(3/4)*(1+x)^(3/2)*
EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*
6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x
+3^(1/2))^2)^(1/2)
```

3.496.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.56 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.17

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx = \frac{2\left(x\sqrt{1+x}(27-27x+27x^2+100x^3-100x^4+100x^5+55x^6-55x^7+55x^8)-\frac{9i\sqrt{6}(1+x)\sqrt{\frac{3\cdots}{\cdots}}}{\cdots}\right)}{935\sqrt{1-x+x^2}}$$

input
```
Integrate[x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

output
```
(2*(x*Sqrt[1 + x]*(27 - 27*x + 27*x^2 + 100*x^3 - 100*x^4 + 100*x^5 + 55*x
^6 - 55*x^7 + 55*x^8) - ((9*I)*Sqrt[6]*(1 + x)*Sqrt[(3*I + Sqrt[3] + (-3*I
 + Sqrt[3])*x)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[(-3*I + Sqrt[3] + (3*I + S
qrt[3])*x)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I
 + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*
I + Sqrt[3])]))/(935*Sqrt[1 - x + x^2])
```

3.496.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1210, 811, 811, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}dx$$

$$\Big\downarrow \text{ 1210}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int x^3\left(x^3+1\right)^{3/2}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{ 811}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{17}\int x^3\sqrt{x^3+1}dx+\frac{2}{17}\left(x^3+1\right)^{3/2}x^4\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{ 811}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{17}\left(\frac{3}{11}\int\frac{x^3}{\sqrt{x^3+1}}dx+\frac{2}{11}\sqrt{x^3+1}x^4\right)+\frac{2}{17}\left(x^3+1\right)^{3/2}x^4\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{17}\left(\frac{3}{11}\left(\frac{2}{5}x\sqrt{x^3+1}-\frac{2}{5}\int\frac{1}{\sqrt{x^3+1}}dx\right)+\frac{2}{11}\sqrt{x^3+1}x^4\right)+\frac{2}{17}\left(x^3+1\right)^{3/2}x^4\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{17}\left(\frac{3}{11}\left(\frac{2}{5}x\sqrt{x^3+1}-\frac{4\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)+\frac{2}{11}\sqrt{x^3+\right.\right.}{\sqrt{x^3+1}}$$

input
```
Int[x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*((2*x^4*(1 + x^3)^(3/2))/17 + (9*((2*x^4*Sq
rt[1 + x^3])/11 + (3*((2*x*Sqrt[1 + x^3])/5 - (4*Sqrt[2 + Sqrt[3]]*(1 + x)
*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x
)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3
] + x)^2]*Sqrt[1 + x^3])))/11))/17))/Sqrt[1 + x^3]
```

3.496.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

3.496. $\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx$

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

3.496.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.88

method	result
risch	$\frac{2x\left(55x^{6}+100x^{3}+27\right)\sqrt{1+x}\,\sqrt{x^{2}-x+1}}{935}-\frac{108\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)}}{935\sqrt{x^{3}+1}\,\sqrt{1+x}\,\sqrt{x^{2}-x+1}}$
elliptic	$\frac{\sqrt{1+x}\,\sqrt{x^{2}-x+1}\,\sqrt{(1+x)(x^{2}-x+1)}\left(\frac{2x^{7}\sqrt{x^{3}+1}}{17}+\frac{40x^{4}\sqrt{x^{3}+1}}{187}+\frac{54x\sqrt{x^{3}+1}}{935}-\frac{108\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\right.}{935\sqrt{x^{3}+1}}\right.}{x^{3}+1}$
default	$\frac{2\sqrt{1+x}\,\sqrt{x^{2}-x+1}\left(55x^{10}+155x^{7}+27i\sqrt{3}\,\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)-81\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\right.}{935(x^{3}+1)}$

input
```
int(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/935*x*(55*x^6+100*x^3+27)*(1+x)^(1/2)*(x^2-x+1)^(1/2)-108/935*(3/2-1/2*I
*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1
/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x
^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-
x+1)^(1/2)
```

3.496.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.19

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2}{935}\left(55\,x^7+100\,x^4+27\,x\right)\sqrt{x^2-x+1}\sqrt{x+1}-\frac{108}{935}\,\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
```

output
```
2/935*(55*x^7 + 100*x^4 + 27*x)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 108/935*we
ierstrassPInverse(0, -4, x)
```

3.496.6 Sympy [F]

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int x^3(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**3*(1+x)**(3/2)*(x**2-x+1)**(3/2),x)
```

output
```
Integral(x**3*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

3.496.7 Maxima [F]

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int \left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^3\,dx$$

input
```
integrate(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3, x)
```

3.496.8 Giac [F]

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int \left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^3\,dx$$

input
```
integrate(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3, x)
```

3.496.9 Mupad [F(-1)]

Timed out.

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int x^3\,(x+1)^{3/2}\,\left(x^2-x+1\right)^{3/2}\,dx$$

input
```
int(x^3*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2),x)
```

output
```
int(x^3*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2), x)
```

3.497 $\int x^2(1+x)^{3/2} \left(1-x+x^2\right)^{3/2} dx$

3.497.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2(1+x)^{3/2} \left(1-x+x^2\right)^{3/2} dx = \frac{2}{15}(1+x)^{5/2} \left(1-x+x^2\right)^{5/2}$$

output
```
2/15*(1+x)^(5/2)*(x^2-x+1)^(5/2)
```

3.497.2 Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2(1+x)^{3/2} \left(1-x+x^2\right)^{3/2} dx = \frac{2}{15}(1+x)^{5/2} \left(1-x+x^2\right)^{5/2}$$

input
```
Integrate[x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

output
```
(2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2))/15
```

3.497.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}dx$$

$$\Big\downarrow \text{1208}$$

$$\frac{2}{15}(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}$$

input
```
Int[x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

output
```
(2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2))/15
```

3.497.3.1 Defintions of rubi rules used

rule 1208
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^2*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g^2*(d + e*x)^(m + 1)*((a + b*x + c
*x^2)^(p + 1)/(c*e*(m + 2*p + 3))), x] /; FreeQ[{a, b, c, d, e, f, g, m, p}
, x] && EqQ[b*e*g*(m + p + 2) + 2*c*(d*g*(p + 1) - e*f*(m + 2*p + 3)), 0] &
& EqQ[e*(c*f^2 - b*f*g + a*g^2)*(m + 1) + (2*c*f - b*g)*(e*f - d*g)*(p + 1)
, 0] && NeQ[m + 2*p + 3, 0]
```

3.497.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{2(1+x)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}{15}$	18
default	$\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}\,\left(x^6+2x^3+1\right)}{15}$	28
risch	$\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}\,\left(x^6+2x^3+1\right)}{15}$	28
elliptic	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\,\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x^6\sqrt{x^3+1}}{15}+\frac{4x^3\sqrt{x^3+1}}{15}+\frac{2\sqrt{x^3+1}}{15}\right)}{x^3+1}$	72

input
```
int(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(1+x)^(5/2)*(x^2-x+1)^(5/2)
```

3.497.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2}{15}\left(x^6+2\,x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}$$

input
```
integrate(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
```

output
```
2/15*(x^6 + 2*x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

3.497.6 Sympy [F]

$$\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int x^2(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(1+x)**(3/2)*(x**2-x+1)**(3/2),x)
```

output
```
Integral(x**2*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

3.497.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2}{15}\left(x^6+2\,x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}$$

input
```
integrate(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
```

output
```
2/15*(x^6 + 2*x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

3.497.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(17) = 34$.

Time = 0.31 (sec) , antiderivative size = 173, normalized size of antiderivative = 7.52

$$\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx = \frac{2}{45045}\left(\left(\left(7\left(3\left(11\left(13x-80\right)(x+1)+3165\right)(x+1)-16442\right)(x+1)+121227\right)(x+1)-8018\right.\right.$$
$$+\frac{2}{45045}\left(\left(5\left(7\left(9\left(11x-57\right)(x+1)+1601\right)(x+1)-15837\right)(x+1)+65172\right)(x+1)-34077\right)\sqrt{(x+1)^2-}$$
$$+\frac{2}{315}\left(\left(5\left(7x-23\right)(x+1)+258\right)(x+1)-213\right)\sqrt{(x+1)^2-3x}\sqrt{x+1}$$
$$+\frac{2}{105}\left(3\left(5x-12\right)(x+1)+71\right)\sqrt{(x+1)^2-3x}\sqrt{x+1}$$

input
```
integrate(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
```

output
```
2/45045*(((7*(3*(11*(13*x - 80)*(x + 1) + 3165)*(x + 1) - 16442)*(x + 1) +
 121227)*(x + 1) - 80187)*(x + 1) + 34077)*sqrt((x + 1)^2 - 3*x)*sqrt(x +
1) + 2/45045*((5*(7*(9*(11*x - 57)*(x + 1) + 1601)*(x + 1) - 15837)*(x + 1
) + 65172)*(x + 1) - 34077)*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1) + 2/315*((5*
(7*x - 23)*(x + 1) + 258)*(x + 1) - 213)*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1)
 + 2/105*(3*(5*x - 12)*(x + 1) + 71)*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1)
```

3.497.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx = \frac{2\sqrt{x+1}\,\left(x^2-x+1\right)^{5/2}\left(x^2+2x+1\right)}{15}$$

input
```
int(x^2*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2),x)
```

output
```
(2*(x + 1)^(1/2)*(x^2 - x + 1)^(5/2)*(2*x + x^2 + 1))/15
```

3.498 $\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx$

3.498.1 Optimal result

Integrand size = 21, antiderivative size = 325

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{18}{91}x^2\sqrt{1+x}\sqrt{1-x+x^2}+\frac{54\sqrt{1+x}\sqrt{1-x+x^2}}{91\left(1+\sqrt{3}+x\right)}+\frac{2}{13}x^2\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{91\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}+\frac{18\sqrt{2}3^{3/4}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{91\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}$$

output
```
18/91*x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/13*x^2*(x^3+1)*(1+x)^(1/2)*(x^2-x+
1)^(1/2)+54/91*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(1+x+3^(1/2))+18/91*3^(3/4)*(1+
x)^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(x^2
-x+1)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))
^2)^(1/2)-27/91*3^(1/4)*(1+x)^(3/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),
I*3^(1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3
^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.498.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.33 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.75

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{\sqrt{1+x}\left(4x^2(1-x+x^2)\left(16+7x^3\right)-\frac{27\sqrt{2}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\left(\left(-3i+\sqrt{3}\right)E\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)-\right.}{\sqrt{-\frac{i(1+x)}{i+\sqrt{3}-2ix}}}}{182\sqrt{1-x+x^2}}$$

input
```
Integrate[x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

output
```
(Sqrt[1 + x]*(4*x^2*(1 - x + x^2)*(16 + 7*x^3) - (27*Sqrt[2]*Sqrt[(-I + Sq
rt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*((-3*I + Sqrt[3])*EllipticE[I*ArcSinh[S
qrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[
3])] - (-I + Sqrt[3])*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I
 + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/Sqrt[((-I)*(1 + x))/(I +
 Sqrt[3] - (2*I)*x)]))/(182*Sqrt[1 - x + x^2])
```

3.498.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1210, 811, 811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}\,dx$$

$$\Big\downarrow \ 1210$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int x\left(x^3+1\right)^{3/2}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \ 811$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{13}\int x\sqrt{x^3+1}dx+\frac{2}{13}\left(x^3+1\right)^{3/2}x^2\right)}{\sqrt{x^3+1}}$$

$\downarrow$ 811

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{13}\left(\frac{3}{7}\int \frac{x}{\sqrt{x^3+1}}dx+\frac{2}{7}\sqrt{x^3+1}x^2\right)+\frac{2}{13}\left(x^3+1\right)^{3/2}x^2\right)}{\sqrt{x^3+1}}$$

$\downarrow$ 832

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{13}\left(\frac{3}{7}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)+\frac{2}{7}\sqrt{x^3+1}x^2\right)+\frac{2}{13}\left(x^3+1\right)^{3/2}x^2\right)}{\sqrt{x^3+1}}$$

$\downarrow$ 759

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{13}\left(\frac{3}{7}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)+\frac{2}{7}\cdots\right.\right.}{\sqrt{x^3+1}}$$

$\downarrow$ 2416

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{13}\left(\frac{3}{7}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\cdots}}{\cdots}\right.\right.\right.}{\sqrt{x^3+1}}$$

input
```
Int[x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*((2*x^2*(1 + x^3)^(3/2))/13 + (9*((2*x^2*Sq
rt[1 + x^3])/7 + (3*((2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2
 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcS
in[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1
 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1
+ x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3]
 + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt
[3] + x)^2]*Sqrt[1 + x^3])))/7))/13))/Sqrt[1 + x^3]
```

3.498. $\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx$

3.498.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.498. $\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx$

3.498.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.71

method	result
risch	$\frac{2x^2(7x^3+16)\sqrt{1+x}\,\sqrt{x^2-x+1}}{91}+\frac{54\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\right.\right.}{91\sqrt{x^3+1}\,\sqrt{1+x}\,\sqrt{x^2-x+1}}$
elliptic	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\,\sqrt{(1+x)(x^2-x+1)}\left(\frac{32x^2\sqrt{x^3+1}}{91}+\frac{54\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\right.\right.}{91\sqrt{x^3+1}}\right.}{x^3+1}$
default	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\left(14x^8+27i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)+46x^5-162\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\right.}{91}$

input
```
int(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/91*x^2*(7*x^3+16)*(1+x)^(1/2)*(x^2-x+1)^(1/2)+54/91*(3/2-1/2*I*3^(1/2))*
((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/
2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2
)*((-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF
(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/
2)))^(1/2)))*((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

3.498.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.12

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2}{91}\left(7\,x^5+16\,x^2\right)\sqrt{x^2-x+1}\sqrt{x+1}-\frac{54}{91}\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

input
```
integrate(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
```

output
```
2/91*(7*x^5 + 16*x^2)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 54/91*weierstrassZet
a(0, -4, weierstrassPInverse(0, -4, x))
```

3.498.6 Sympy [F]

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx = \int x(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(1+x)**(3/2)*(x**2-x+1)**(3/2),x)
```

output
```
Integral(x*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

3.498.7 Maxima [F]

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx = \int \left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x, x)
```

3.498.8 Giac [F]

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx = \int \left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x, x)
```

3.498.9 Mupad [F(-1)]

Timed out.

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int x\,(x+1)^{3/2}\,\left(x^2-x+1\right)^{3/2}\,dx$$

input
```
int(x*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2),x)
```

output
```
int(x*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2), x)
```

3.499 $\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} \, dx$

3.499.1 Optimal result

Integrand size = 20, antiderivative size = 173

$$\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} \, dx = \frac{18}{55} x\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{11} x\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right) + \frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\, \text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{55\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}$$

output
```
18/55*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/11*x*(x^3+1)*(1+x)^(1/2)*(x^2-x+1)^(
1/2)+18/55*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(
1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2
))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.499.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.48 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.02

$$\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} \, dx = \frac{2x\sqrt{1+x}\left(1-x+x^2\right)\left(14+5x^3\right)+\frac{9i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-3}}{\sqrt{1}}\right.\right.}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{55\sqrt{1-x+x^2}}$$

input
```
Integrate[(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

output
```
(2*x*Sqrt[1 + x]*(1 - x + x^2)*(14 + 5*x^3) + ((9*I)*(1 + x)*Sqrt[1 + (6*I
)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*E
llipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[
3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(55*Sqrt[1 - x + x^2])
```

3.499.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1151, 748, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+1)^{3/2} \left(x^2-x+1\right)^{3/2} \, dx$$

$$\Big\downarrow \text{1151}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int \left(x^3+1\right)^{3/2}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{748}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{11}\int \sqrt{x^3+1}dx+\frac{2}{11}x\left(x^3+1\right)^{3/2}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{748}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{11}\left(\frac{3}{5}\int \frac{1}{\sqrt{x^3+1}}dx+\frac{2}{5}\sqrt{x^3+1}x\right)+\frac{2}{11}x\left(x^3+1\right)^{3/2}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{11}\left(\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{5\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2}{5}\sqrt{x^3+1}x\right)+\frac{2}{11}x\left(x^3+\right.}{\sqrt{x^3+1}}$$

input
```
Int[(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*((2*x*(1 + x^3)^(3/2))/11 + (9*((2*x*Sqrt[1
 + x^3])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
- 4*Sqrt[3]])/(5*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/11))/S
qrt[1 + x^3]
```

3.499.3.1 Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 1151
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^FracPart[p]/(a*d + c
*e*x^3)^FracPart[p])   Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; F
reeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
 IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

3.499.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((1+x)^(3/2)*(x^2-x+1)^(3/2),x)
```

output
```
int((1+x)^(3/2)*(x^2-x+1)^(3/2),x)
```

3.499.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.19

$$\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} \, dx = \frac{2}{55} \left(5 x^4 + 14 x\right) \sqrt{x^2-x+1} \sqrt{x+1} + \frac{54}{55} \operatorname{weierstrassPInverse}(0, -4, x)$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
```

output
```
2/55*(5*x^4 + 14*x)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 54/55*weierstrassPInve
rse(0, -4, x)
```

3.499.6 Sympy [F]

$$\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} \, dx = \int (x+1)^{\frac{3}{2}} \left(x^2-x+1\right)^{\frac{3}{2}} \, dx$$

input
```
integrate((1+x)**(3/2)*(x**2-x+1)**(3/2),x)
```

output
```
Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

3.499.7 Maxima [F]

$$\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} \, dx = \int \left(x^2-x+1\right)^{\frac{3}{2}} (x+1)^{\frac{3}{2}} \, dx$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2), x)
```

3.499.8 Giac [F]

$$\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} \, dx = \int \left(x^2-x+1\right)^{\frac{3}{2}} (x+1)^{\frac{3}{2}} \, dx$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2), x)
```

3.499.9 Mupad [F(-1)]

Timed out.

$$\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} \, dx = \int (x+1)^{3/2} \left(x^2-x+1\right)^{3/2} \, dx$$

input
```
int((x + 1)^(3/2)*(x^2 - x + 1)^(3/2),x)
```

output
```
int((x + 1)^(3/2)*(x^2 - x + 1)^(3/2), x)
```

3.500 $\int \frac{(1+x)^{3/2}(1-x+x^2)^{3/2}}{x} \, dx$

3.500.1 Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \frac{(1+x)^{3/2} (1-x+x^2)^{3/2}}{x} \, dx = \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2} + \frac{2}{9}\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3) - \frac{2\sqrt{1+x}\sqrt{1-x+x^2}\operatorname{arctanh}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x^3}}$$

output
```
2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/9*(x^3+1)*(1+x)^(1/2)*(x^2-x+1)^(1/2)-2/
3*arctanh((x^3+1)^(1/2))*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(x^3+1)^(1/2)
```

3.500.2 Mathematica [A] (verified)

Time = 10.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.56

$$\int \frac{(1+x)^{3/2} (1-x+x^2)^{3/2}}{x} \, dx = \frac{2}{9}\left(\sqrt{1+x}\sqrt{1-x+x^2}(4+x^3) - 3\operatorname{arctanh}\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\right)$$

input
```
Integrate[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x,x]
```

output
```
(2*(Sqrt[1 + x]*Sqrt[1 - x + x^2]*(4 + x^3) - 3*ArcTanh[Sqrt[1 + x]*Sqrt[1
 - x + x^2]]))/9
```

3.500.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1210, 798, 60, 60, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{3/2} \left(x^2-x+1\right)^{3/2}}{x} \, dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int \frac{\left(x^3+1\right)^{3/2}}{x}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int \frac{\left(x^3+1\right)^{3/2}}{x^3}dx^3}{3\sqrt{x^3+1}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\int \frac{\sqrt{x^3+1}}{x^3}dx^3+\frac{2}{3}\left(x^3+1\right)^{3/2}\right)}{3\sqrt{x^3+1}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\int \frac{1}{x^3\sqrt{x^3+1}}dx^3+\frac{2}{3}\left(x^3+1\right)^{3/2}+2\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(2\int \frac{1}{x^6-1}d\sqrt{x^3+1}+\frac{2}{3}\left(x^3+1\right)^{3/2}+2\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

$$\Big\downarrow \text{220}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(-2\text{arctanh}\left(\sqrt{x^3+1}\right)+\frac{2}{3}\left(x^3+1\right)^{3/2}+2\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

input
```
Int[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x,x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*(2*Sqrt[1 + x^3] + (2*(1 + x^3)^(3/2))/3 -
2*ArcTanh[Sqrt[1 + x^3]]))/(3*Sqrt[1 + x^3])
```

3.500.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

3.500. $\int \frac{(1+x)^{3/2}(1-x+x^2)^{3/2}}{x} \, dx$

3.500.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

method	result	size
default	$-\frac{2\sqrt{1+x}\,\sqrt{x^{2}-x+1}\,\left(-x^{3}\sqrt{x^{3}+1}+3\,\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)-4\sqrt{x^{3}+1}\right)}{9\sqrt{x^{3}+1}}$	57
elliptic	$\frac{\sqrt{1+x}\,\sqrt{x^{2}-x+1}\,\sqrt{(1+x)(x^{2}-x+1)}\,\left(\frac{2x^{3}\sqrt{x^{3}+1}}{9}+\frac{8\sqrt{x^{3}+1}}{9}-\frac{2\,\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)}{3}\right)}{x^{3}+1}$	70

input
```
int((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2/9*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(-x^3*(x^3+1)^(1/2)+3*arctanh((x^3+1)^(1/
2))-4*(x^3+1)^(1/2))/(x^3+1)^(1/2)
```

3.500.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.69

$$\int \frac{(1+x)^{3/2}\left(1-x+x^{2}\right)^{3/2}}{x}\,dx=\frac{2}{9}\left(x^{3}+4\right)\sqrt{x^{2}-x+1}\sqrt{x+1}-\frac{1}{3}\log\left(\sqrt{x^{2}-x+1}\sqrt{x+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^{2}-x+1}\sqrt{x+1}-1\right)$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x, algorithm="fricas")
```

output
```
2/9*(x^3 + 4)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 1/3*log(sqrt(x^2 - x + 1)*sq
rt(x + 1) + 1) + 1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) - 1)
```

3.500.6 Sympy [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^{2}\right)^{3/2}}{x}\,dx=\int \frac{(x+1)^{\frac{3}{2}}\left(x^{2}-x+1\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x,x)
```

output
```
Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)/x, x)
```

3.500.7 Maxima [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx=\int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x, x)
```

3.500.8 Giac [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx=\int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x, x)
```

3.500.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx=\int \frac{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}{x}\,dx$$

input
```
int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x,x)
```

output
```
int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x, x)
```

3.501 $\int \frac{(1+x)^{3/2}(1-x+x^2)^{3/2}}{x^2} dx$

3.501.1 Optimal result

Integrand size = 23, antiderivative size = 323

$$\begin{aligned}
\int \frac{(1+x)^{3/2}(1-x+x^2)^{3/2}}{x^2} dx &= \frac{9}{7}x^2\sqrt{1+x}\sqrt{1-x+x^2} \\
&+ \frac{27\sqrt{1+x}\sqrt{1-x+x^2}}{7(1+\sqrt{3}+x)} - \frac{\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3)}{x} \\
&- \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{14\sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}(1+x^3)} \\
&+ \frac{9\sqrt{2}3^{3/4}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{7\sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}(1+x^3)}
\end{aligned}$$

output
```
9/7*x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2)-(x^3+1)*(1+x)^(1/2)*(x^2-x+1)^(1/2)/x+
27/7*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(1+x+3^(1/2))+9/7*3^(3/4)*(1+x)^(3/2)*Ell
ipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(x^2-x+1)^(1/2)*
((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-27
/14*3^(1/4)*(1+x)^(3/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*
I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(
1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.501.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.37 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.76

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^2}\,dx = \frac{\sqrt{1+x}\left(\frac{4(1-x+x^2)(-7+2x^3)}{x} - \frac{27\sqrt{2}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\left(\left(-3i+\sqrt{3}\right)E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{-\frac{i(1+}{3i+}}\right.\right.\right.}{}\right.}{28\sqrt{1-x+x^2}}$$

input
```
Integrate[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^2,x]
```

output
```
(Sqrt[1 + x]*((4*(1 - x + x^2)*(-7 + 2*x^3))/x - (27*Sqrt[2]*Sqrt[(-I + Sq
rt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*((-3*I + Sqrt[3])*EllipticE[I*ArcSinh[S
qrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[
3])] - (-I + Sqrt[3])*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I
 + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/Sqrt[((-I)*(1 + x))/(I +
 Sqrt[3] - (2*I)*x)]))/(28*Sqrt[1 - x + x^2])
```

3.501.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1210, 809, 811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int \frac{\left(x^3+1\right)^{3/2}}{x^2}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{809}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{2}\int x\sqrt{x^3+1}dx-\frac{\left(x^3+1\right)^{3/2}}{x}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{811}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{2}\left(\frac{3}{7}\int\frac{x}{\sqrt{x^3+1}}dx+\frac{2}{7}\sqrt{x^3+1}x^2\right)-\frac{\left(x^3+1\right)^{3/2}}{x}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{832}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{2}\left(\frac{3}{7}\left(\int\frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int\frac{1}{\sqrt{x^3+1}}dx\right)+\frac{2}{7}\sqrt{x^3+1}x^2\right)-\frac{\left(x^3+1\right)^{3/2}}{x}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{2}\left(\frac{3}{7}\left(\int\frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)+\frac{2}{7}\sqrt{\ldots}\right.\right.}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{2416}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{2}\left(\frac{3}{7}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\ldots}}{\ldots}\right.\right.\right.}{\sqrt{x^3+1}}$$

input
```
Int[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^2,x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*(-((1 + x^3)^(3/2)/x) + (9*((2*x^2*Sqrt[1 +
 x^3])/7 + (3*((2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqr
t[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1
- Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqr
t[3] + x)^2]*Sqrt[1 + x^3]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] +
x)^2]*Sqrt[1 + x^3])))/7))/2))/Sqrt[1 + x^3]
```

3.501. $\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^2}\,dx$

3.501.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

3.501. $\int \frac{(1+x)^{3/2}(1-x+x^2)^{3/2}}{x^2}\,dx$

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.501.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.71

method	result
risch	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\,(2x^3-7)}{7x}+\frac{27\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\cdots\right.\right.}{7\sqrt{x^3+1}\,\sqrt{1+x}\,\sqrt{x^2-x+1}}$
elliptic	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\,\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x^2\sqrt{x^3+1}}{7}+\frac{27\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\cdots}\right.\right.}{7\sqrt{x^3+1}}\right.}{x^3+1}$
default	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\left(27i\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\,x+4x^6-162\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}-2x+\cdots}{i\sqrt{3}+3}}\cdots\right.}{14x\cdots}$

input
```
int((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(2*x^3-7)/x+27/7*(3/2-1/2*I*3^(1/2))*((1+x
)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-
3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*
I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((1+
x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2)))*((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

3.501.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.12

$$\int \frac{(1+x)^{3/2}(1-x+x^2)^{3/2}}{x^2}\,dx = \frac{(2\,x^3-7)\sqrt{x^2-x+1}\sqrt{x+1}-27\,x\text{weierstrassZeta}(0,-4,\text{weierstrassP}$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
1/7*((2*x^3 - 7)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 27*x*weierstrassZeta(0, -
4, weierstrassPInverse(0, -4, x)))/x
```

3.501.6 Sympy [F]

$$\int \frac{(1+x)^{3/2}(1-x+x^2)^{3/2}}{x^2}\,dx = \int \frac{(x+1)^{\frac{3}{2}}(x^2-x+1)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x**2,x)
```

output
```
Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)/x**2, x)
```

3.501.7 Maxima [F]

$$\int \frac{(1+x)^{3/2}(1-x+x^2)^{3/2}}{x^2}\,dx = \int \frac{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^2, x)
```

3.501.8 Giac [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^2}\,dx = \int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^2, x)
```

3.501.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^2}\,dx = \int \frac{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}{x^2}\,dx$$

input
```
int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x^2,x)
```

output
```
int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x^2, x)
```

3.502 $\int \frac{(1+x)^{3/2}(1-x+x^2)^{3/2}}{x^3}\,dx$

3.502.1 Optimal result

Integrand size = 23, antiderivative size = 175

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx=\frac{9}{10}x\sqrt{1+x}\sqrt{1-x+x^2}-\frac{\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3)}{2x^2}+\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{10\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,(1+x^3)}$$

output
```
9/10*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)-1/2*(x^3+1)*(1+x)^(1/2)*(x^2-x+1)^(1/2)
/x^2+9/10*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1
/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2)
)^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.502.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.29 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.10

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx=\frac{\sqrt{1+x}\left(\frac{2(1-x+x^2)(-5+4x^3)}{x^2}-\frac{27i\sqrt{2}\sqrt{\frac{i+\sqrt{3}-2ix}{3i+\sqrt{3}}}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\,\text{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{2\cdots}\right.\right.}{\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}}\right.}{20\sqrt{1-x+x^2}}$$

input
```
Integrate[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^3,x]
```

output
```
(Sqrt[1 + x]*((2*(1 - x + x^2)*(-5 + 4*x^3))/x^2 - ((27*I)*Sqrt[2]*Sqrt[(I
 + Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])]*Sqrt[(-I + Sqrt[3] + (2*I)*x)/(-3*I
 + Sqrt[3])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3
])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3]
)]))/(20*Sqrt[1 - x + x^2])
```

3.502.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1210, 809, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{3/2} \left(x^2-x+1\right)^{3/2}}{x^3} \, dx$$

$$\Big\downarrow \text{ 1210}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int \frac{\left(x^3+1\right)^{3/2}}{x^3}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{ 809}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{4}\int \sqrt{x^3+1}dx-\frac{\left(x^3+1\right)^{3/2}}{2x^2}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{ 748}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{4}\left(\frac{3}{5}\int \frac{1}{\sqrt{x^3+1}}dx+\frac{2}{5}\sqrt{x^3+1}x\right)-\frac{\left(x^3+1\right)^{3/2}}{2x^2}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{4}\left(\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{5\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2}{5}\sqrt{x^3+1}x\right)-\frac{\left(x^3+1\right)^{3/2}}{2x^2}\right)}{\sqrt{x^3+1}}$$

input
```
Int[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^3,x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*(-1/2*(1 + x^3)^(3/2)/x^2 + (9*((2*x*Sqrt[1
 + x^3])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
- 4*Sqrt[3]])/(5*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/4))/Sq
rt[1 + x^3]
```

3.502.3.1 Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

3.502.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.99

method	result
risch	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\,(4x^3-5)}{10x^2}+\frac{27\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)(x^2-x+1)}}{10\sqrt{x^3+1}\,\sqrt{1+x}\,\sqrt{x^2-x+1}}$
elliptic	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\,\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x\sqrt{x^3+1}}{5}+\frac{27\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{10\sqrt{x^3+1}}-\frac{\sqrt{x}}{2}\right.}{x^3+1}$
default	$-\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\left(27i\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\,x^2-81\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\right.}{20(x^3+1)x^2}$

input
```
int((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(4*x^3-5)/x^2+27/10*(3/2-1/2*I*3^(1/2))*(
(1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2
)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)
*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1
/2*I*3^(1/2)))^(1/2))*((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

3.502.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.22

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx=\frac{27\,x^2\mathrm{weierstrassPInverse}(0,-4,x)+(4\,x^3-5)\sqrt{x^2-x+1}\sqrt{x+1}}{10\,x^2}$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
1/10*(27*x^2*weierstrassPInverse(0, -4, x) + (4*x^3 - 5)*sqrt(x^2 - x + 1)
*sqrt(x + 1))/x^2
```

3.502.6 Sympy [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx = \int \frac{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x**3,x)
```

output
```
Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)/x**3, x)
```

3.502.7 Maxima [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx = \int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^3, x)
```

3.502.8 Giac [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx = \int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^3, x)
```

3.502.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^{3/2}\,(1-x+x^2)^{3/2}}{x^3}\,dx = \int \frac{(x+1)^{3/2}\,(x^2-x+1)^{3/2}}{x^3}\,dx$$

input
```
int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x^3,x)
```

output
```
int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x^3, x)
```

3.503 $\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

3.503.1 Optimal result

Integrand size = 23, antiderivative size = 142

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2x(1+x^3)}{5\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{4\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

output
```
2/5*x*(x^3+1)/(1+x)^(1/2)/(x^2-x+1)^(1/2)-4/15*EllipticF((1+x-3^(1/2))/(1+
x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)
/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1
/2)
```

3.503.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.43 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.19

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{6x\sqrt{1+x}(1-x+x^2) - \frac{2i(1+x)\sqrt{1+\frac{6i}{(-3i+\sqrt{3})(1+x)}}\sqrt{6-\frac{36i}{(3i+\sqrt{3})(1+x)}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right),\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{15\sqrt{1-x+x^2}}$$

input
```
Integrate[x^3/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
(6*x*Sqrt[1 + x]*(1 - x + x^2) - ((2*I)*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sq
rt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*Ar
cSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sq
rt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(15*Sqrt[1 - x + x^2])
```

3.503.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1210, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{x^3}{\sqrt{x^3+1}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{843}$$

$$\frac{\sqrt{x^3+1}\left(\frac{2}{5}x\sqrt{x^3+1}-\frac{2}{5}\int \frac{1}{\sqrt{x^3+1}}dx\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x^3+1}\left(\frac{2}{5}x\sqrt{x^3+1}-\frac{4\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[x^3/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
(Sqrt[1 + x^3]*((2*x*Sqrt[1 + x^3])/5 - (4*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[
(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 +
 Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)
^2]*Sqrt[1 + x^3])))/(Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

3.503.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

3.503.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.11

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x\sqrt{x^3+1}}{5}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{5\sqrt{x^3+1}}\right)}{\sqrt{1+x}\sqrt{x^2-x+1}}$
risch	$\frac{2x\sqrt{1+x}\sqrt{x^2-x+1}}{5}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)(x^2-x+1)}}{5\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$\frac{2\sqrt{1+x}\sqrt{x^2-x+1}\left(i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)-3\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+\cdots}{-3+\cdots}}\cdots\right.}{5(x^3+1)}$

input
```
int(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(2/5*x*(x^3+1)^(1/2)-4
/5*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/
2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

3.503.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.18

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2}{5}\sqrt{x^2-x+1}\sqrt{x+1}x - \frac{4}{5}\,\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
2/5*sqrt(x^2 - x + 1)*sqrt(x + 1)*x - 4/5*weierstrassPInverse(0, -4, x)
```

3.503.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x^3}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

input
```
integrate(x**3/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

output
```
Integral(x**3/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

3.503.7 Maxima [F]

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x^3}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

input
```
integrate(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

3.503.8 Giac [F]

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x^3}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

input
```
integrate(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

3.503.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x^3}{\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

input
```
int(x^3/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

output
```
int(x^3/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

3.504 $\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}} \, dx$

3.504.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}} \, dx = \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}$$

output
```
2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)
```

3.504.2 Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}} \, dx = \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}$$

input
```
Integrate[x^2/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
(2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3
```

3.504.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$
$$\Big\downarrow \; 1208$$
$$\frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1}$$

input
```
Int[x^2/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
(2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3
```

3.504.3.1 Defintions of rubi rules used

rule 1208
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^2*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g^2*(d + e*x)^(m + 1)*((a + b*x + c
*x^2)^(p + 1)/(c*e*(m + 2*p + 3))), x] /; FreeQ[{a, b, c, d, e, f, g, m, p}
, x] && EqQ[b*e*g*(m + p + 2) + 2*c*(d*g*(p + 1) - e*f*(m + 2*p + 3)), 0] &
& EqQ[e*(c*f^2 - b*f*g + a*g^2)*(m + 1) + (2*c*f - b*g)*(e*f - d*g)*(p + 1)
, 0] && NeQ[m + 2*p + 3, 0]
```

3.504.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}}{3}$	18
default	$\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}}{3}$	18
risch	$\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}}{3}$	18
elliptic	$\frac{2\sqrt{(1+x)(x^2-x+1)}\,\sqrt{x^3+1}}{3\sqrt{1+x}\,\sqrt{x^2-x+1}}$	39

input
```
int(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)
```

3.504.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2}{3}\sqrt{x^2-x+1}\sqrt{x+1}$$

input
```
integrate(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

3.504.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x^2}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

input
```
integrate(x**2/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

3.504.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2\,(x^3+1)}{3\,\sqrt{x^2-x+1}\sqrt{x+1}}$$

input
```
integrate(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(x^3 + 1)/(sqrt(x^2 - x + 1)*sqrt(x + 1))
```

3.504.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2}{3}\sqrt{(x+1)^2-3x}\sqrt{x+1}$$

input
```
integrate(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1)
```

3.504.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.39

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2\sqrt{x^3+1}}{3}$$

input
```
int(x^2/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

output
```
(2*(x^3 + 1)^(1/2))/3
```

3.505 $\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}} \, dx$

3.505.1 Optimal result

Integrand size = 21, antiderivative size = 253

$$\begin{aligned}\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}} \, dx &= \frac{2(1+x^3)}{\sqrt{1+x}\,(1+\sqrt{3}+x)\,\sqrt{1-x+x^2}} \\ &- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}} \\ &+ \frac{2\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}\end{aligned}$$

output
```
2*(x^3+1)/(1+x+3^(1/2))/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/3*EllipticF((1+x-3^(
1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(
1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-3^(1/
4)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6
^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)/((1+
x)/(1+x+3^(1/2))^2)^(1/2)
```

3.505.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.74 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.48

$$\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}(1-x+x^2)}{(1+x)^2}+\frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}E\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}}+\frac{i\sqrt{2}\left(3i+\right.}{}\right.}{6\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

input
```
Integrate[x/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
((1 + x)^(3/2)*((12*Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 +
(3*Sqrt[2]*(1 - I*Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqr
t[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I
*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I -
 Sqrt[3])])/Sqrt[1 + x] + (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] -
 (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3
*I + Sqrt[3])]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x
]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x]))/(6*Sqrt[(-I)/(3*I + Sq
rt[3])]*Sqrt[1 - x + x^2])
```

3.505.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1210, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x+1}\sqrt{x^2-x+1}}dx$$

$$\Big\downarrow \ 1210$$

$$\frac{\sqrt{x^3+1}\int \frac{x}{\sqrt{x^3+1}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \color{blue}{832}$$

$$\frac{\sqrt{x^3+1}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \color{blue}{759}$$

$$\frac{\sqrt{x^3+1}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \color{blue}{2416}$$

$$\frac{\sqrt{x^3+1}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x-}{x+}\right.\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right.}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[x/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
(Sqrt[1 + x^3]*((2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sq
rt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1
 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sq
rt[3] + x)^2]*Sqrt[1 + x^3]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)
/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] +
 x)^2]*Sqrt[1 + x^3])))/(Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

3.505.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.505.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.80

method	result
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}} \cdots\right.\right.}{\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$\frac{\sqrt{1+x}\sqrt{x^2-x+1}\left(-3+i\sqrt{3}\right)\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\left(iE\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}-iF\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{- \cdots}\right.\right.}{2x^3+2}$

input
```
int(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*
3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/
2*I*3^(1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))*((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^
2-x+1)^(1/2)
```

3.505.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.04

$$\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = -2\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

input
```
integrate(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x))
```

3.505.6 Sympy [F]

$$\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

input
```
integrate(x/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

output
```
Integral(x/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

3.505.7 Maxima [F]

$$\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

input
```
integrate(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

3.505.8 Giac [F]

$$\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

input
```
integrate(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

3.505.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x}{\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

input
```
int(x/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

output
```
int(x/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

3.506 $\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

3.506.1 Optimal result

Integrand size = 20, antiderivative size = 110

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

output
```
2/3*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*
6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(
1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.506.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.35

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{\frac{2}{3}-\frac{4i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right),\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

input
```
Integrate[1/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
(I*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[2/3 - (4*I)/((3
*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/S
qrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/(Sqrt[(-I)/(3*I + Sqrt[3])]
*Sqrt[1 - x + x^2])
```

3.506.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1151, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

$$\Big\downarrow \text{ 1151}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{\sqrt{x^3+1}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{2\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$

input
```
Int[1/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
(2*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*E
llipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^
(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

3.506.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 1151
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^FracPart[p]/(a*d + c
*e*x^3)^FracPart[p])   Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; F
reeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
 IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

3.506.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.25

method	result	size
default	$\frac{\left(3-i\sqrt{3}\right)\sqrt{1+x}\,\sqrt{x^2-x+1}\,\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)}{x^3+1}$	137
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)(x^2-x+1)}}{\sqrt{x^3+1}\,\sqrt{1+x}\,\sqrt{x^2-x+1}}$	145

input
```
int(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(3-I*3^(1/2))*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*
((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))
^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/
2)+3))^(1/2))/(x^3+1)
```

3.506.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.05

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = 2\,\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
2*weierstrassPInverse(0, -4, x)
```

3.506.6 Sympy [F]

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

input
```
integrate(1/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

3.506.7 Maxima [F]

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

input
```
integrate(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

3.506.8 Giac [F]

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

input
```
integrate(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

3.506.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

input
```
int(1/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

output
```
int(1/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

3.507 $\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

3.507.1 Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = -\frac{2\sqrt{1+x^3}\operatorname{arctanh}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

output
```
-2/3*arctanh((x^3+1)^(1/2))*(x^3+1)^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

3.507.2 Mathematica [A] (verified)

Time = 4.92 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = -\frac{2}{3}\operatorname{arctanh}\left(\sqrt{1+x}\sqrt{3-3(1+x)+(1+x)^2}\right)$$

input
```
Integrate[1/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
(-2*ArcTanh[Sqrt[1 + x]*Sqrt[3 - 3*(1 + x) + (1 + x)^2]])/3
```

3.507.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1210, 798, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{x+1}\sqrt{x^2-x+1}}dx$$

$$\Big\downarrow \text{ 1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x\sqrt{x^3+1}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x^3\sqrt{x^3+1}}dx^3}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2\sqrt{x^3+1}\int \frac{1}{x^6-1}d\sqrt{x^3+1}}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 220}$$

$$-\frac{2\sqrt{x^3+1}\text{arctanh}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[1/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
(-2*Sqrt[1 + x^3]*ArcTanh[Sqrt[1 + x^3]])/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]
)
```

3.507.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

3.507.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{2 \operatorname{arctanh}\left(\sqrt{x^{3}+1}\right) \sqrt{1+x}\, \sqrt{x^{2}-x+1}}{3 \sqrt{x^{3}+1}}$	33
elliptic	$-\frac{2 \sqrt{\left(1+x\right) \left(x^{2}-x+1\right)}\, \operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)}{3 \sqrt{1+x}\, \sqrt{x^{2}-x+1}}$	40

input
```
int(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*arctanh((x^3+1)^(1/2))*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(x^3+1)^(1/2)
```

3.507.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = -\frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}+1\right) + \frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}-1\right)$$

input
```
integrate(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) + 1) + 1/3*log(sqrt(x^2 - x + 1)*sq
rt(x + 1) - 1)
```

3.507.6 Sympy [F]

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{x\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

input
```
integrate(1/x/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

3.507.7 Maxima [F]

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x}\,dx$$

input
```
integrate(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x), x)
```

3.507.8 Giac [F]

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x}\,dx$$

input
```
integrate(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x), x)
```

3.507.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{x\,\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

input
```
int(1/(x*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

output
```
int(1/(x*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

3.508 $\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

3.508.1 Optimal result

Integrand size = 23, antiderivative size = 282

$$\begin{aligned}\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx &= -\frac{1+x^3}{x\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{1+x^3}{\sqrt{1+x}\,(1+\sqrt{3}+x)\,\sqrt{1-x+x^2}}\\ &\quad-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}\\ &\quad+\frac{\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}\end{aligned}$$

output
```
(-x^3-1)/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+(x^3+1)/(1+x+3^(1/2))/(1+x)^(1/2)/(
x^2-x+1)^(1/2)+1/3*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^
(1/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2
)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-1/2*3^(1/4)*EllipticE((1+x-3^(1/2))/(1+x+3
^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1
+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.508.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.42 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} + \frac{(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}(1-x+x^2)}{(1+x)^2} + \frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}E\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}} + \frac{i\sqrt{2}(\cdots}{\cdots}\right)}{12\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

input
```
Integrate[1/(x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
-((Sqrt[1 + x]*Sqrt[1 - x + x^2])/x) + ((1 + x)^(3/2)*((12*Sqrt[(-I)/(3*I
+ Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1 - I*Sqrt[3])*Sqrt[(3*
I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)
/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3]
)]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x] + (I*Sqrt[2
]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sq
rt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticF[I*ArcSinh[
Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])
])/Sqrt[1 + x]))/(12*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])
```

3.508.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1210, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x^2\sqrt{x^3+1}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$\Big\downarrow$ 847

$$\frac{\sqrt{x^3+1}\left(\frac{1}{2}\int \frac{x}{\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{x}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$\Big\downarrow$ 832

$$\frac{\sqrt{x^3+1}\left(\frac{1}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)-\frac{\sqrt{x^3+1}}{x}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$\Big\downarrow$ 759

$$\frac{\sqrt{x^3+1}\left(\frac{1}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)-\frac{\sqrt{x^3+1}}{x}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$\Big\downarrow$ 2416

$$\frac{\sqrt{x^3+1}\left(\frac{1}{2}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+}}\right.\right.}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input

```
Int[1/(x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output

```
(Sqrt[1 + x^3]*(-(Sqrt[1 + x^3]/x) + ((2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x)
- (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^
2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])
/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (2*(1 - Sqrt[3])*Sqrt
[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[Ar
cSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[
(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]))/2))/(Sqrt[1 + x]*Sqrt[1 - x +
 x^2])
```

3.508. $\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

3.508.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.508. $\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

3.508.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.76

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{x}+\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\cdots\right.}{\sqrt{x^3+1}}\right.}{\sqrt{1+x}\sqrt{x^2-x+1}\cdots}$
risch	$-\frac{\sqrt{1+x}\sqrt{x^2-x+1}}{x}+\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)F\left(\cdots\right.\right.}{\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$\frac{\sqrt{1+x}\sqrt{x^2-x+1}\left(i\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}x-6\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+\cdots}{-3+\cdots}}\cdots\right.}{2x(x^3+1)\cdots}$

input
```
int(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(-1/x*(x^3+1)^(1/2)+(3
/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))
/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^
(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^(1
/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*
3^(1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))
/(-3/2-1/2*I*3^(1/2)))^(1/2))))
```

3.508.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.11

$$\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = -\frac{x\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))+\sqrt{x^2-x+1}\sqrt{x+1}}{x}$$

input
```
integrate(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
-(x*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)) + sqrt(x^2 - x +
 1)*sqrt(x + 1))/x
```

3.508.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{x^2\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

input
```
integrate(1/x**2/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

3.508.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x^2}\,dx$$

input
```
integrate(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2), x)
```

3.508.8 Giac [F]

$$\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x^2}\,dx$$

input
```
integrate(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2), x)
```

3.508.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{x^2\,\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

input
```
int(1/(x^2*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

output
```
int(1/(x^2*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

3.509 $\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

3.509.1 Optimal result

Integrand size = 23, antiderivative size = 146

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{-1-x^3}{2x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

output
```
1/2*(-x^3-1)/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)-1/6*EllipticF((1+x-3^(1/2))/(
1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+
1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^
(1/2)
```

3.509.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.41 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{-\frac{6\sqrt{1+x}(1-x+x^2)}{x^2}-\frac{i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right),\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{12\sqrt{1-x+x^2}}$$

input
```
Integrate[1/(x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
((-6*Sqrt[1 + x]*(1 - x + x^2))/x^2 - (I*(1 + x)*Sqrt[1 + (6*I)/((-3*I + S
qrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*A
rcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - S
qrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(12*Sqrt[1 - x + x^2])
```

3.509.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1210, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x^3\sqrt{x^3+1}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{847}$$

$$\frac{\sqrt{x^3+1}\left(-\frac{1}{4}\int \frac{1}{\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{2x^2}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x^3+1}\left(-\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt{x^3+1}}{2x^2}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[1/(x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
(Sqrt[1 + x^3]*(-1/2*Sqrt[1 + x^3]/x^2 - (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(
1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 +
Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^
2]*Sqrt[1 + x^3])))/(Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

3.509.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

3.509.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{2x^2}-\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^3+1}}\right)}{\sqrt{1+x}\sqrt{x^2-x+1}}$
risch	$-\frac{\sqrt{1+x}\sqrt{x^2-x+1}}{2x^2}-\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)(x^2-x+1)}}{2\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$\frac{\sqrt{1+x}\sqrt{x^2-x+1}\left(i\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\,x^2-3\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}-}{-3}}\right.}{4(x^3+1)x^2}$

input
```
int(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(-1/2/x^2*(x^3+1)^(1/2
)-1/2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*
3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^
(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((
-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

3.509.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = -\frac{x^2\text{weierstrassPInverse}(0,-4,x)+\sqrt{x^2-x+1}\sqrt{x+1}}{2\,x^2}$$

input
```
integrate(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(x^2*weierstrassPInverse(0, -4, x) + sqrt(x^2 - x + 1)*sqrt(x + 1))/x
^2
```

3.509.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{x^3\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

input
```
integrate(1/x**3/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

3.509.7 Maxima [F]

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x^3}\,dx$$

input
```
integrate(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3), x)
```

3.509.8 Giac [F]

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x^3}\,dx$$

input
```
integrate(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3), x)
```

3.509.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{x^3\,\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

input
```
int(1/(x^3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

output
```
int(1/(x^3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

3.510 $\int \frac{x^3}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

3.510.1 Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{x^3}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=-\frac{2x}{3\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{4\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

output
```
-2/3*x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+4/9*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2
)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^
(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.510.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.38 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\frac{-\frac{6x}{\sqrt{1+x}}+\frac{2i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)\right.}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{9\sqrt{1-x+x^2}}$$

input
```
Integrate[x^3/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
((-6*x)/Sqrt[1 + x] + ((2*I)*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqrt[3])*(1 +
 x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[
(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sq
rt[(-I)/(3*I + Sqrt[3])])/(9*Sqrt[1 - x + x^2])
```

3.510.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1210, 817, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{x^3}{\left(x^3+1\right)^{3/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 817}$$

$$\frac{\sqrt{x^3+1}\left(\frac{2}{3}\int \frac{1}{\sqrt{x^3+1}}dx-\frac{2x}{3\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{\sqrt{x^3+1}\left(\frac{4\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2x}{3\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[x^3/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + x^3]*((-2*x)/(3*Sqrt[1 + x^3]) + (4*Sqrt[2 + Sqrt[3]]*(1 + x)*Sq
rt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] +
 x)^2]*Sqrt[1 + x^3])))/(Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

3.510.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

3.510. $\int \frac{x^3}{(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

3.510.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{2x}{3\sqrt{x^3+1}}+\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3+1}}\right)}{\sqrt{1+x}\sqrt{x^2-x+1}}$
risch	$-\frac{2x}{3\sqrt{1+x}\sqrt{x^2-x+1}}+\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)(x^2-x+1)}}{3\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$-\frac{2\sqrt{1+x}\sqrt{x^2-x+1}\left(i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)-3\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\cdots}\right.}{3(x^3+1)}$

input
```
int(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(-2/3*x/(x^3+1)^(1/2)+
4/3*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3
/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

3.510.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.28

$$\int \frac{x^3}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = -\frac{2\left(\sqrt{x^2-x+1}\sqrt{x+1}x-2\left(x^3+1\right)\text{weierstrassPInverse}(0,-4,x)\right)}{3\left(x^3+1\right)}$$

input
```
integrate(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(x^2 - x + 1)*sqrt(x + 1)*x - 2*(x^3 + 1)*weierstrassPInverse(0,
 -4, x))/(x^3 + 1)
```

3.510.6 Sympy [F]

$$\int \frac{x^3}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{x^3}{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

output
```
Integral(x**3/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

3.510.7 Maxima [F]

$$\int \frac{x^3}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{x^3}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

3.510.8 Giac [F]

$$\int \frac{x^3}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{x^3}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

3.510.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{x^3}{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}\,dx$$

input
```
int(x^3/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

output
```
int(x^3/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

3.511 $\int \frac{x^2}{(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

3.511.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=-\frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

output
```
-2/3/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

3.511.2 Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=-\frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

input
```
Integrate[x^2/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
-2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

3.511.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}\,dx$$
$$\Big\downarrow \ 1208$$
$$-\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[x^2/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
-2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

3.511.3.1 Defintions of rubi rules used

rule 1208
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^2*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g^2*(d + e*x)^(m + 1)*((a + b*x + c
*x^2)^(p + 1)/(c*e*(m + 2*p + 3))), x] /; FreeQ[{a, b, c, d, e, f, g, m, p}
, x] && EqQ[b*e*g*(m + p + 2) + 2*c*(d*g*(p + 1) - e*f*(m + 2*p + 3)), 0] &
& EqQ[e*(c*f^2 - b*f*g + a*g^2)*(m + 1) + (2*c*f - b*g)*(e*f - d*g)*(p + 1)
, 0] && NeQ[m + 2*p + 3, 0]
```

3.511.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
gosper	$-\frac{2}{3\sqrt{1+x}\,\sqrt{x^{2}-x+1}}$	18
risch	$-\frac{2}{3\sqrt{1+x}\,\sqrt{x^{2}-x+1}}$	18
default	$-\frac{2\sqrt{1+x}\,\sqrt{x^{2}-x+1}}{3\left(x^{3}+1\right)}$	25
elliptic	$-\frac{2\sqrt{\left(1+x\right)\left(x^{2}-x+1\right)}}{3\sqrt{1+x}\,\sqrt{x^{2}-x+1}\,\sqrt{x^{3}+1}}$	39

input
```
int(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

3.511.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = -\frac{2\sqrt{x^2-x+1}\sqrt{x+1}}{3\left(x^3+1\right)}$$

input
```
integrate(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^3 + 1)
```

3.511.6 Sympy [F]

$$\int \frac{x^2}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{x^2}{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

output
```
Integral(x**2/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

3.511.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = -\frac{2}{3\sqrt{x^2-x+1}\sqrt{x+1}}$$

input
```
integrate(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

output
```
-2/3/(sqrt(x^2 - x + 1)*sqrt(x + 1))
```

3.511.8 Giac [F]

$$\int \frac{x^2}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{x^2}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

3.511.9 Mupad [B] (verification not implemented)

Time = 12.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=-\frac{2}{3\sqrt{x+1}\,\sqrt{x^2-x+1}}$$

input
```
int(x^2/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

output
```
-2/(3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2))
```

3.512 $\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

3.512.1 Optimal result

Integrand size = 21, antiderivative size = 282

$$\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{2x^2}{3\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{2(1+x^3)}{3\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} + \frac{\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}} - \frac{2\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

output
```
2/3*x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)-2/3*(x^3+1)/(1+x+3^(1/2))/(1+x)^(1/2)/
(x^2-x+1)^(1/2)-2/9*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2
^(1/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/
2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+1/3*3^(1/4)*EllipticE((1+x-3^(1/2))/(1+x+
3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(
1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.512.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.42 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.43

$$\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{2x^2}{3\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}(1-x+x^2)}{(1+x)^2} + \frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}} + \frac{i\sqrt{2}(3i+}{}\right.}{18\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

input
```
Integrate[x/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
(2*x^2)/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - ((1 + x)^(3/2)*((12*Sqrt[(-I)/
(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1 - I*Sqrt[3])*Sqr
t[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] +
(6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sq
rt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x] + (I*S
qrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3]
)]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticF[I*Arc
Sinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqr
t[3])])/Sqrt[1 + x]))/(18*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])
```

3.512.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1210, 819, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{x}{\left(x^3+1\right)^{3/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \; 819$$

$$\frac{\sqrt{x^3+1}\left(\frac{2x^2}{3\sqrt{x^3+1}}-\frac{1}{3}\int \frac{x}{\sqrt{x^3+1}}dx\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \; 832$$

$$\frac{\sqrt{x^3+1}\left(\frac{1}{3}\left(\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx-\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx\right)+\frac{2x^2}{3\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \; 759$$

$$\frac{\sqrt{x^3+1}\left(\frac{1}{3}\left(\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx\right)+\frac{2x^2}{3\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \; 2416$$

$$\frac{\sqrt{x^3+1}\left(\frac{1}{3}\left(\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\right.\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right.\right.}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[x/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + x^3]*((2*x^2)/(3*Sqrt[1 + x^3]) + ((-2*Sqrt[1 + x^3])/(1 + Sqrt[
3] + x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[
3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*S
qrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*(1 - Sqrt[
3])*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Elli
pticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/
4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]))/3))/(Sqrt[1 + x]*Sqrt
[1 - x + x^2])
```

3.512. $\int \frac{x}{(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

3.512.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.512. $\int \frac{x}{(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

3.512.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.77

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x^2}{3\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\cdots\right.}{3\sqrt{x^3+1}}\right.}{\sqrt{1+x}\sqrt{x^2-x+1}}$
risch	$\frac{2x^2}{3\sqrt{1+x}\sqrt{x^2-x+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)F\left(\cdots\right.\right.}{3\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$-\frac{\sqrt{1+x}\sqrt{x^2-x+1}\left(i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)+3\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}\cdots}{-3\cdots}}\right.}{3(x^3+1)}$

input
```
int(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(2/3/(x^3+1)^(1/2)*x^2
-2/3*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3
^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(
1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2
*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2
+1/2*I*3^(1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3
^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))))
```

3.512.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.15

$$\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\frac{2\left(\sqrt{x^2-x+1}\sqrt{x+1}x^2+(x^3+1)\text{weierstrassZeta}(0,-4,\text{weierstrassPI}\cdots\right.}{3\left(x^3+1\right)}$$

input
```
integrate(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2 + (x^3 + 1)*weierstrassZeta(0, -4,
weierstrassPInverse(0, -4, x)))/(x^3 + 1)
```

3.512.6 Sympy [F]

$$\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{x}{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

output
```
Integral(x/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

3.512.7 Maxima [F]

$$\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{x}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

3.512.8 Giac [F]

$$\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{x}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

3.512.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{x}{{\left(x+1\right)}^{3/2}\,{\left(x^2-x+1\right)}^{3/2}}\,dx$$

input
```
int(x/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

output
```
int(x/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

3.513 $\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

3.513.1 Optimal result

Integrand size = 20, antiderivative size = 137

$$\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\frac{2x}{3\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{2\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

output
```
2/3*x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)
),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(
1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.513.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.27 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.58

$$\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\sqrt{3-3(1+x)+(1+x)^2}\left(-\frac{2}{9\sqrt{1+x}}+\frac{2(1+x)^{3/2}}{9\left(3-3(1+x)+(1+x)^2\right)}\right)+$$

input
```
Integrate[1/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
Sqrt[3 - 3*(1 + x) + (1 + x)^2]*(-2/(9*Sqrt[1 + x]) + (2*(1 + x)^(3/2))/(9
*(3 - 3*(1 + x) + (1 + x)^2))) + ((I/3)*Sqrt[2/3]*(1 + x)*Sqrt[1 - 6/((3 -
 I*Sqrt[3])*(1 + x))]*Sqrt[1 - 6/((3 + I*Sqrt[3])*(1 + x))]*EllipticF[I*Ar
cSinh[Sqrt[-6/(3 - I*Sqrt[3])]/Sqrt[1 + x]], (3 - I*Sqrt[3])/(3 + I*Sqrt[3
])])/(Sqrt[-(3 - I*Sqrt[3])^(-1)]*Sqrt[3 - 3*(1 + x) + (1 + x)^2])
```

3.513.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1151, 749, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1151}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{\left(x^3+1\right)^{3/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 749}$$

$$\frac{\sqrt{x^3+1}\left(\frac{1}{3}\int \frac{1}{\sqrt{x^3+1}}dx+\frac{2x}{3\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{\sqrt{x^3+1}\left(\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2x}{3\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[1/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + x^3]*((2*x)/(3*Sqrt[1 + x^3]) + (2*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqr
t[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1
 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] +
x)^2]*Sqrt[1 + x^3])))/(Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

3.513.3.1 Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 1151
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^FracPart[p]/(a*d + c
*e*x^3)^FracPart[p])   Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; F
reeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
 IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

3.513.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x}{3\sqrt{x^3+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3+1}}\right)}{\sqrt{1+x}\sqrt{x^2-x+1}}$
risch	$\frac{2x}{3\sqrt{1+x}\sqrt{x^2-x+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)(x^2-x+1)}}{3\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$-\frac{\sqrt{1+x}\sqrt{x^2-x+1}\left(i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)-3\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}-}{-3}}\right.}{3(x^3+1)}$

input
```
int(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(2/3*x/(x^3+1)^(1/2)+2
/3*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/
2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

3.513.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.27

$$\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\frac{2\left(\sqrt{x^2-x+1}\sqrt{x+1}x+(x^3+1)\text{weierstrassPInverse}(0,-4,x)\right)}{3\left(x^3+1\right)}$$

input
```
integrate(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(x^2 - x + 1)*sqrt(x + 1)*x + (x^3 + 1)*weierstrassPInverse(0, -4
, x))/(x^3 + 1)
```

3.513.6 Sympy [F]

$$\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{1}{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

output
```
Integral(1/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

3.513.7 Maxima [F]

$$\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

3.513.8 Giac [F]

$$\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

3.513.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(x+1\right)^{3/2}\left(x^2-x+1\right)^{3/2}}\,dx$$

input
```
int(1/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

output
```
int(1/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

3.514 $\int \frac{1}{x(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

3.514.1 Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \frac{1}{x(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx=\frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}}-\frac{2\sqrt{1+x^3}\,\text{arctanh}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

output
```
2/3/(1+x)^(1/2)/(x^2-x+1)^(1/2)-2/3*arctanh((x^3+1)^(1/2))*(x^3+1)^(1/2)/(
1+x)^(1/2)/(x^2-x+1)^(1/2)
```

3.514.2 Mathematica [A] (verified)

Time = 11.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

$$\int \frac{1}{x(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx=\frac{2\left(\sqrt{1+x}-(1+x)^2\sqrt{\frac{1-x+x^2}{(1+x)^2}}\,\text{arctanh}\left(\frac{1}{(1+x)^{3/2}\sqrt{\frac{1-x+x^2}{(1+x)^2}}}\right)\right)}{3(1+x)\sqrt{1-x+x^2}}$$

input
```
Integrate[1/(x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
(2*(Sqrt[1 + x] - (1 + x)^2*Sqrt[(1 - x + x^2)/(1 + x)^2]*ArcTanh[1/((1 +
x)^(3/2)*Sqrt[(1 - x + x^2)/(1 + x)^2]])]))/(3*(1 + x)*Sqrt[1 - x + x^2])
```

3.514.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1210, 798, 61, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x\left(x^3+1\right)^{3/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x^3\left(x^3+1\right)^{3/2}}dx^3}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt{x^3+1}\left(\int \frac{1}{x^3\sqrt{x^3+1}}dx^3+\frac{2}{\sqrt{x^3+1}}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{x^3+1}\left(2\int \frac{1}{x^6-1}d\sqrt{x^3+1}+\frac{2}{\sqrt{x^3+1}}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{220}$$

$$\frac{\sqrt{x^3+1}\left(\frac{2}{\sqrt{x^3+1}}-2\text{arctanh}\left(\sqrt{x^3+1}\right)\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[1/(x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + x^3]*(2/Sqrt[1 + x^3] - 2*ArcTanh[Sqrt[1 + x^3]]))/(3*Sqrt[1 + x
]*Sqrt[1 - x + x^2])
```

3.514.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

3.514.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.65

method	result	size
default	$-\frac{2\sqrt{1+x}\,\sqrt{x^{2}-x+1}\,\left(\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)\sqrt{x^{3}+1}-1\right)}{3\left(x^{3}+1\right)}$	43
elliptic	$\frac{\sqrt{(1+x)\left(x^{2}-x+1\right)}\,\left(\frac{2}{3\sqrt{x^{3}+1}}-\frac{2\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)}{3}\right)}{\sqrt{1+x}\,\sqrt{x^{2}-x+1}}$	51
risch	$\frac{2}{3\sqrt{1+x}\,\sqrt{x^{2}-x+1}}-\frac{2\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)\sqrt{(1+x)\left(x^{2}-x+1\right)}}{3\sqrt{1+x}\,\sqrt{x^{2}-x+1}}$	58

input
```
int(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(arctanh((x^3+1)^(1/2))*(x^3+1)^(1/2)-1)/
(x^3+1)
```

3.514.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.18

$$\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^{2}\right)^{3/2}}\,dx = -\frac{\left(x^{3}+1\right)\log\left(\sqrt{x^{2}-x+1}\sqrt{x+1}+1\right)-\left(x^{3}+1\right)\log\left(\sqrt{x^{2}-x+1}\sqrt{x+1}-1\right)-2\sqrt{x^{2}-x+1}\sqrt{x+1}}{3\left(x^{3}+1\right)}$$

input
```
integrate(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*((x^3 + 1)*log(sqrt(x^2 - x + 1)*sqrt(x + 1) + 1) - (x^3 + 1)*log(sqr
t(x^2 - x + 1)*sqrt(x + 1) - 1) - 2*sqrt(x^2 - x + 1)*sqrt(x + 1))/(x^3 +
1)
```

3.514.6 Sympy [F]

$$\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{1}{x\,(x+1)^{\frac{3}{2}}\,(x^2-x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

output
```
Integral(1/(x*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

3.514.7 Maxima [F]

$$\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{1}{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x), x)
```

3.514.8 Giac [F]

$$\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{1}{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x), x)
```

3.514.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{1}{x\,(x+1)^{3/2}\,\left(x^2-x+1\right)^{3/2}}\,dx$$

input
```
int(1/(x*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

output
```
int(1/(x*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

3.515 $\int \frac{1}{x^2(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

3.515.1 Optimal result

Integrand size = 23, antiderivative size = 316

$$\begin{aligned}\int \frac{1}{x^2(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx &= \frac{2}{3x\sqrt{1+x}\sqrt{1-x+x^2}} \\ &- \frac{5(1+x^3)}{3x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{5(1+x^3)}{3\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} \\ &- \frac{5\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\,3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}} \\ &+ \frac{5\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}\end{aligned}$$

output
```
2/3/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)-5/3*(x^3+1)/x/(1+x)^(1/2)/(x^2-x+1)^(1/2
)+5/3*(x^3+1)/(1+x+3^(1/2))/(1+x)^(1/2)/(x^2-x+1)^(1/2)+5/9*EllipticF((1+x
-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x
+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-5
/6*3^(1/4)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2
)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1
/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.515.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.49 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = -\frac{3+5x^3}{3x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{5(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}(1-x+x^2)}{(1+x)^2} + \frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}} + \frac{i\sqrt{2}(3i+\ldots}{\ldots}\right)}{36\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

input
```
Integrate[1/(x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
-1/3*(3 + 5*x^3)/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (5*(1 + x)^(3/2)*((12
*Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1 - I*S
qrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I +
 Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(-6*I
)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1
+ x] + (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*
I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*Elli
pticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])
/(3*I - Sqrt[3])])/Sqrt[1 + x]))/(36*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x
 + x^2])
```

3.515.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1210, 819, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}dx$$

$$\Big\downarrow \; 1210$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x^2\left(x^3+1\right)^{3/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$\Big\downarrow$ 819

$$\frac{\sqrt{x^3+1}\left(\frac{5}{3}\int \frac{1}{x^2\sqrt{x^3+1}}dx+\frac{2}{3x\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$\Big\downarrow$ 847

$$\frac{\sqrt{x^3+1}\left(\frac{5}{3}\left(\frac{1}{2}\int \frac{x}{\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{x}\right)+\frac{2}{3x\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$\Big\downarrow$ 832

$$\frac{\sqrt{x^3+1}\left(\frac{5}{3}\left(\frac{1}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)-\frac{\sqrt{x^3+1}}{x}\right)+\frac{2}{3x\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$\Big\downarrow$ 759

$$\frac{\sqrt{x^3+1}\left(\frac{5}{3}\left(\frac{1}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)-\frac{\sqrt{x^3+1}}{x}\right)+\frac{}{3x\sqrt{}}}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$\Big\downarrow$ 2416

$$\frac{\sqrt{x^3+1}\left(\frac{5}{3}\left(\frac{1}{2}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(a\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{}}\right.\right.\right.}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[1/(x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + x^3]*(2/(3*x*Sqrt[1 + x^3]) + (5*(-(Sqrt[1 + x^3]/x) + ((2*Sqrt[
1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 -
 x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqr
t[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x
^3]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
- 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]))/2
))/3))/(Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

3.515. $\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

3.515.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

3.515. $\int \frac{1}{x^2(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.515.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.72

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{x}-\frac{2x^2}{3\sqrt{x^3+1}}+\frac{5\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\right)}{3\sqrt{x^3+1}}\right)}{\sqrt{1+x}\sqrt{x^2-x+1}}$
risch	$-\frac{5x^3+3}{3x\sqrt{1+x}\sqrt{x^2-x+1}}+\frac{5\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\right)}{3\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$\frac{\sqrt{1+x}\sqrt{x^2-x+1}\left(5i\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\,x+15\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\right)}{6(x^3+1}$

input
```
int(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(-1/x*(x^3+1)^(1/2)-2/
3/(x^3+1)^(1/2)*x^2+5/3*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1
/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/
2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*Ellipt
icE(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^
(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(
1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))))
```

3.515.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = -\frac{(5\,x^3+3)\sqrt{x^2-x+1}\sqrt{x+1}+5\,(x^4+x)\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))}{3\,(x^4+x)}$$

input
```
integrate(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*((5*x^3 + 3)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 5*(x^4 + x)*weierstrassZ
eta(0, -4, weierstrassPInverse(0, -4, x)))/(x^4 + x)
```

3.515.6 Sympy [F]

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{x^2\,(x+1)^{\frac{3}{2}}\,(x^2-x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

output
```
Integral(1/(x**2*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

3.515.7 Maxima [F]

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^2), x)
```

3.515.8 Giac [F]

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^2), x)
```

3.515.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{x^2\,(x+1)^{3/2}\,(x^2-x+1)^{3/2}}\,dx$$

input
```
int(1/(x^2*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

output
```
int(1/(x^2*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

3.516 $\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

3.516.1 Optimal result

Integrand size = 23, antiderivative size = 170

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\frac{2}{3x^2\sqrt{1+x}\sqrt{1-x+x^2}}-\frac{7(1+x^3)}{6x^2\sqrt{1+x}\sqrt{1-x+x^2}}-\frac{7\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{6\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

output
```
2/3/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)-7/6*(x^3+1)/x^2/(1+x)^(1/2)/(x^2-x+1)^
(1/2)-7/18*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2
)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2
-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.516.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.26 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\frac{-\frac{6(3+7x^3)}{x^2\sqrt{1+x}}-\frac{7i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-\frac{\cdots}{3i\cdots}}}{\sqrt{1-\cdots}}\right.\right.}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{36\sqrt{1-x+x^2}}$$

input
```
Integrate[1/(x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
((-6*(3 + 7*x^3))/(x^2*Sqrt[1 + x]) - ((7*I)*(1 + x)*Sqrt[1 + (6*I)/((-3*I
 + Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF
[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I
 - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(36*Sqrt[1 - x + x^2])
```

3.516.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1210, 819, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x^3\left(x^3+1\right)^{3/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{819}$$

$$\frac{\sqrt{x^3+1}\left(\frac{7}{3}\int \frac{1}{x^3\sqrt{x^3+1}}dx+\frac{2}{3x^2\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{847}$$

$$\frac{\sqrt{x^3+1}\left(\frac{7}{3}\left(-\frac{1}{4}\int \frac{1}{\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{2x^2}\right)+\frac{2}{3x^2\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x^3+1}\left(\frac{7}{3}\left(-\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt{x^3+1}}{2x^2}\right)+\frac{2}{3x^2\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[1/(x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + x^3]*(2/(3*x^2*Sqrt[1 + x^3]) + (7*(-1/2*Sqrt[1 + x^3]/x^2 - (Sq
rt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[
ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*S
qrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/3))/(Sqrt[1 + x]*Sqrt[1
- x + x^2])
```

3.516.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

3.516. $\int \frac{1}{x^3(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

3.516.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{2x^2}-\frac{2x}{3\sqrt{x^3+1}}-\frac{7\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{6\sqrt{x^3+1}}\right)}{\sqrt{1+x}\sqrt{x^2-x+1}}$
risch	$-\frac{7x^3+3}{6x^2\sqrt{1+x}\sqrt{x^2-x+1}}-\frac{7\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)(x^2-x+1)}}{6\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$\frac{\sqrt{1+x}\sqrt{x^2-x+1}\left(7i\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\,x^2-21\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{}}{}}\right.}{12(x^3+1)x^2}$

input
```
int(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(-1/2/x^2*(x^3+1)^(1/2
)-2/3*x/(x^3+1)^(1/2)-7/6*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^
(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(
1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*
I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

3.516.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.28

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = -\frac{(7\,x^3+3)\sqrt{x^2-x+1}\sqrt{x+1}+7\,(x^5+x^2)\text{weierstrassPInverse}(0,-4,x)}{6\,(x^5+x^2)}$$

input
```
integrate(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*((7*x^3 + 3)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 7*(x^5 + x^2)*weierstras
sPInverse(0, -4, x))/(x^5 + x^2)
```

3.516.6 Sympy [F]

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{x^3\left(x+1\right)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

output
```
Integral(1/(x**3*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

3.516.7 Maxima [F]

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}\left(x+1\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3), x)
```

3.516.8 Giac [F]

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}\left(x+1\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3), x)
```

3.516.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{1}{x^3\,\left(x+1\right)^{3/2}\,\left(x^2-x+1\right)^{3/2}} \,dx$$

input
```
int(1/(x^3*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

output
```
int(1/(x^3*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

3.517 $\int \frac{x^3}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

3.517.1 Optimal result

Integrand size = 23, antiderivative size = 168

$$\int \frac{x^3}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \frac{4x}{27\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} + \frac{4\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

output
```
4/27*x/(1+x)^(1/2)/(x^2-x+1)^(1/2)-2/9*x/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)^(1/
2)+4/81*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(
1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+
1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.517.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.35 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{(1+x)^{5/2} (1-x+x^2)^{5/2}} \, dx = \frac{\frac{6x(-1+2x^3)}{(1+x)^{3/2}(1-x+x^2)} + \frac{2i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\right.\right.}}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{81\sqrt{1-x+x^2}}$$

input
```
Integrate[x^3/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
((6*x*(-1 + 2*x^3))/((1 + x)^(3/2)*(1 - x + x^2)) + ((2*I)*(1 + x)*Sqrt[1
+ (6*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 +
x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I +
 Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(81*Sqrt[1 - x + x
^2])
```

3.517.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1210, 817, 749, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(x+1)^{5/2} (x^2-x+1)^{5/2}} \, dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{x^3}{(x^3+1)^{5/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{817}$$

$$\frac{\sqrt{x^3+1}\left(\frac{2}{9}\int \frac{1}{(x^3+1)^{3/2}}dx-\frac{2x}{9(x^3+1)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{749}$$

$$\frac{\sqrt{x^3+1}\left(\frac{2}{9}\left(\frac{1}{3}\int\frac{1}{\sqrt{x^3+1}}dx+\frac{2x}{3\sqrt{x^3+1}}\right)-\frac{2x}{9(x^3+1)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x^3+1}\left(\frac{2}{9}\left(\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2x}{3\sqrt{x^3+1}}\right)-\frac{2x}{9(x^3+1)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[x^3/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
(Sqrt[1 + x^3]*((-2*x)/(9*(1 + x^3)^(3/2)) + (2*((2*x)/(3*Sqrt[1 + x^3]) +
 (2*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Elli
pticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(
1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/9))/(Sqrt[1 + x]*S
qrt[1 - x + x^2])
```

3.517.3.1 Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

3.517. $\int \frac{x^3}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

3.517.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{2x}{9(x^3+1)^{\frac{3}{2}}}+\frac{4x}{27\sqrt{x^3+1}}+\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{27\sqrt{x^3+1}}\right)}{\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$-\frac{2\left(i\sqrt{3}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x^3\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}-3F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x^3\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\right.}{}$

input
```
int(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(-2/9*x/(x^3+1)^(3/2)+
4/27*x/(x^3+1)^(1/2)+4/27*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^
(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(
1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*
I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

3.517.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.33

$$\int \frac{x^3}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \frac{2\left((2x^4-x)\sqrt{x^2-x+1}\sqrt{x+1}+2(x^6+2x^3+1)\text{weierstrassPInverse}(\right.}{27(x^6+2x^3+1)}$$

input
```
integrate(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

output
```
2/27*((2*x^4 - x)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 2*(x^6 + 2*x^3 + 1)*weie
rstrassPInverse(0, -4, x))/(x^6 + 2*x^3 + 1)
```

3.517.6 Sympy [F]

$$\int \frac{x^3}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=\int \frac{x^3}{(x+1)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

output
```
Integral(x**3/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

3.517.7 Maxima [F]

$$\int \frac{x^3}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=\int \frac{x^3}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

3.517.8 Giac [F]

$$\int \frac{x^3}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=\int \frac{x^3}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^3/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

3.517.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{x^3}{(x+1)^{5/2}\,(x^2-x+1)^{5/2}}\,dx$$

input
```
int(x^3/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

output
```
int(x^3/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

3.518 $\int \frac{x^2}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

3.518.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = -\frac{2}{9(1+x)^{3/2}(1-x+x^2)^{3/2}}$$

output
```
-2/9/(1+x)^(3/2)/(x^2-x+1)^(3/2)
```

3.518.2 Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = -\frac{2}{9(1+x)^{3/2}(1-x+x^2)^{3/2}}$$

input
```
Integrate[x^2/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
-2/(9*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))
```

3.518.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x+1)^{5/2}(x^2-x+1)^{5/2}}\,dx$$

$$\Big\downarrow \text{1208}$$

$$-\frac{2}{9(x+1)^{3/2}(x^2-x+1)^{3/2}}$$

input
```
Int[x^2/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
-2/(9*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))
```

3.518.3.1 Defintions of rubi rules used

rule 1208
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^2*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g^2*(d + e*x)^(m + 1)*((a + b*x + c
*x^2)^(p + 1)/(c*e*(m + 2*p + 3))), x] /; FreeQ[{a, b, c, d, e, f, g, m, p}
, x] && EqQ[b*e*g*(m + p + 2) + 2*c*(d*g*(p + 1) - e*f*(m + 2*p + 3)), 0] &
& EqQ[e*(c*f^2 - b*f*g + a*g^2)*(m + 1) + (2*c*f - b*g)*(e*f - d*g)*(p + 1)
, 0] && NeQ[m + 2*p + 3, 0]
```

3.518.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
gosper	$-\frac{2}{9(1+x)^{\frac{3}{2}}(x^2-x+1)^{\frac{3}{2}}}$	18
default	$-\frac{2}{9(x^3+1)\sqrt{1+x}\,\sqrt{x^2-x+1}}$	25
elliptic	$-\frac{2\sqrt{(1+x)(x^2-x+1)}}{9\sqrt{1+x}\,\sqrt{x^2-x+1}\,(x^3+1)^{\frac{3}{2}}}$	39

input
```
int(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9/(1+x)^(3/2)/(x^2-x+1)^(3/2)
```

3.518.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{x^2}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = -\frac{2\sqrt{x^2-x+1}\sqrt{x+1}}{9\left(x^6+2x^3+1\right)}$$

input
```
integrate(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

output
```
-2/9*sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^6 + 2*x^3 + 1)
```

3.518.6 Sympy [F]

$$\int \frac{x^2}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{x^2}{(x+1)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

output
```
Integral(x**2/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

3.518.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = -\frac{2}{9\left(x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}}$$

input
```
integrate(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

output
```
-2/9/((x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1))
```

3.518.8 Giac [F]

$$\int \frac{x^2}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{x^2}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^2/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

3.518.9 Mupad [B] (verification not implemented)

Time = 12.44 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.57

$$\int \frac{x^2}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{18\sqrt{x+1}\left(x^2-x+1\right)^{5/2} - 18\,x\sqrt{x+1}\left(x^2-x+1\right)^{5/2}}{(x+1)\left(81\,x\left(x^2-x+1\right)^4 - 162\left(x^2-x+1\right)^4 + 81\left(x^2-x+1\right)^5\right)}$$

input
```
int(x^2/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

output
```
(18*(x + 1)^(1/2)*(x^2 - x + 1)^(5/2) - 18*x*(x + 1)^(1/2)*(x^2 - x + 1)^(
5/2))/((x + 1)*(81*x*(x^2 - x + 1)^4 - 162*(x^2 - x + 1)^4 + 81*(x^2 - x +
 1)^5))
```

3.519 $\int \frac{x}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

3.519.1 Optimal result

Integrand size = 21, antiderivative size = 318

$$\int \frac{x}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \frac{10x^2}{27\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2x^2}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} - \frac{10(1+x^3)}{27\sqrt{1+x}\,(1+\sqrt{3}+x)\sqrt{1-x+x^2}} + \frac{5\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}\,E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{9\,3^{3/4}\sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}\sqrt{1-x+x^2}} - \frac{10\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}\sqrt{1-x+x^2}}$$

output
```
10/27*x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9*x^2/(x^3+1)/(1+x)^(1/2)/(x^2-x+1
)^(1/2)-10/27*(x^3+1)/(1+x+3^(1/2))/(1+x)^(1/2)/(x^2-x+1)^(1/2)-10/81*Elli
pticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(1+x)^(1/2)*((x^2
-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/(((1+x)/(1+x+3^(1/2))^
2)^(1/2)+5/27*3^(1/4)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)
*(1+x)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(
x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.519.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.48 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.29

$$\int \frac{x}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{2x^2(8+5x^3)}{27(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}} - \frac{5(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}(1-x+x^2)}{(1+x)^2} + \frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}} + \frac{i\sqrt{2}\left(3i\cdots\right.}{}\right.}{162\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

input
```
Integrate[x/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
(2*x^2*(8 + 5*x^3))/(27*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)) - (5*(1 + x)^(3
/2)*((12*Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*
(1 - I*Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt
[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sq
rt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])
/Sqrt[1 + x] + (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 +
 x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3
])]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I +
Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x]))/(162*Sqrt[(-I)/(3*I + Sqrt[3])]*S
qrt[1 - x + x^2])
```

3.519.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1210, 819, 819, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 1210$$

$$\frac{\sqrt{x^3+1}\int \frac{x}{(x^3+1)^{5/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$\Big\downarrow$ 819

$$\frac{\sqrt{x^3+1}\left(\frac{5}{9}\int \frac{x}{(x^3+1)^{3/2}}dx+\frac{2x^2}{9(x^3+1)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$\Big\downarrow$ 819

$$\frac{\sqrt{x^3+1}\left(\frac{5}{9}\left(\frac{2x^2}{3\sqrt{x^3+1}}-\frac{1}{3}\int \frac{x}{\sqrt{x^3+1}}dx\right)+\frac{2x^2}{9(x^3+1)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$\Big\downarrow$ 832

$$\frac{\sqrt{x^3+1}\left(\frac{5}{9}\left(\frac{1}{3}\left(\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx-\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx\right)+\frac{2x^2}{3\sqrt{x^3+1}}\right)+\frac{2x^2}{9(x^3+1)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$\Big\downarrow$ 759

$$\frac{\sqrt{x^3+1}\left(\frac{5}{9}\left(\frac{1}{3}\left(\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx\right)+\frac{2x^2}{3\sqrt{x^3+1}}\right)+\frac{2x^2}{9(}\right.}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$\Big\downarrow$ 2416

$$\frac{\sqrt{x^3+1}\left(\frac{5}{9}\left(\frac{1}{3}\left(\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\text{arcs}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3}}\right.\right.\right.}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[x/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

3.519. $\int \frac{x}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

output
```
(Sqrt[1 + x^3]*((2*x^2)/(9*(1 + x^3)^(3/2)) + (5*((2*x^2)/(3*Sqrt[1 + x^3]
) + ((-2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1
+ x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3]
 + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)
^2]*Sqrt[1 + x^3]) + (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 -
x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt
[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqr
t[1 + x^3]))/3))/9))/(Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

3.519.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

3.519. $\int \frac{x}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.519.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.72

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x^2}{9\left(x^3+1\right)^{\frac{3}{2}}}+\frac{10x^2}{27\sqrt{x^3+1}}-\frac{10\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right.\right.}{27\sqrt{x^3+1}}\right.}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$
default	$-\frac{5i\sqrt{3}\,F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x^3\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}+15F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x^3\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}}{}$

input
```
int(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(2/9*x^2/(x^3+1)^(3/2)
+10/27/(x^3+1)^(1/2)*x^2-10/27*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/
2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*
I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))
*EllipticE(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1
/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1
/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))))
```

3.519.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.19

$$\int \frac{x}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \frac{2\left((5x^5+8x^2)\sqrt{x^2-x+1}\sqrt{x+1}+5(x^6+2x^3+1)\text{weierstrassZeta}(0\right.}{27(x^6+2x^3+1)}$$

input
```
integrate(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

output
```
2/27*((5*x^5 + 8*x^2)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 5*(x^6 + 2*x^3 + 1)*
weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)))/(x^6 + 2*x^3 + 1)
```

3.519.6 Sympy [F]

$$\int \frac{x}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \int \frac{x}{(x+1)^{\frac{5}{2}}(x^2-x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

output
```
Integral(x/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

3.519.7 Maxima [F]

$$\int \frac{x}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \int \frac{x}{(x^2-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

3.519.8 Giac [F]

$$\int \frac{x}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{x}{(x^2-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

output
```
integrate(x/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

3.519.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{x}{(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}}\,dx$$

input
```
int(x/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

output
```
int(x/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

3.520 $\int \frac{1}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

3.520.1 Optimal result

Integrand size = 20, antiderivative size = 168

$$\int \frac{1}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \frac{14x}{27\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} + \frac{14\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

output
```
14/27*x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9*x/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)^(1
/2)+14/81*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)
*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-
x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.520.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.33 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.06

$$\int \frac{1}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\frac{6x(10+7x^3)}{(1+x)^{3/2}(1-x+x^2)} + \frac{7i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\right.\right.}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{81\sqrt{1-x+x^2}}$$

input
```
Integrate[1/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
((6*x*(10 + 7*x^3))/((1 + x)^(3/2)*(1 - x + x^2)) + ((7*I)*(1 + x)*Sqrt[1
+ (6*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 +
x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I +
 Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(81*Sqrt[1 - x + x
^2])
```

3.520.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1151, 749, 749, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1151}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{\left(x^3+1\right)^{5/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 749}$$

$$\frac{\sqrt{x^3+1}\left(\frac{7}{9}\int \frac{1}{\left(x^3+1\right)^{3/2}}dx+\frac{2x}{9\left(x^3+1\right)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 749}$$

$$\frac{\sqrt{x^3+1}\left(\frac{7}{9}\left(\frac{1}{3}\int\frac{1}{\sqrt{x^3+1}}dx+\frac{2x}{3\sqrt{x^3+1}}\right)+\frac{2x}{9(x^3+1)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x^3+1}\left(\frac{7}{9}\left(\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2x}{3\sqrt{x^3+1}}\right)+\frac{2x}{9(x^3+1)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[1/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
(Sqrt[1 + x^3]*((2*x)/(9*(1 + x^3)^(3/2)) + (7*((2*x)/(3*Sqrt[1 + x^3]) +
(2*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1
/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/9))/(Sqrt[1 + x]*Sq
rt[1 - x + x^2])
```

3.520.3.1 Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 1151
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^FracPart[p]/(a*d + c
*e*x^3)^FracPart[p])   Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; F
reeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
 IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

3.520. $\int \frac{1}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

3.520.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x}{9(x^3+1)^{\frac{3}{2}}}+\frac{14x}{27\sqrt{x^3+1}}+\frac{14\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{27\sqrt{x^3+1}}\right)}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$
default	$-\frac{7i\sqrt{3}\,F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x^3\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}-21F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x^3\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}}{}$

input
```
int(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(2/9*x/(x^3+1)^(3/2)+1
4/27*x/(x^3+1)^(1/2)+14/27*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))
^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^
(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2
*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

3.520.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.33

$$\int \frac{1}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{2\left((7\,x^4+10\,x)\sqrt{x^2-x+1}\sqrt{x+1}+7\,(x^6+2\,x^3+1)\text{weierstrassPInver}\right.}{27\,(x^6+2\,x^3+1)}$$

input
```
integrate(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

output
```
2/27*((7*x^4 + 10*x)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 7*(x^6 + 2*x^3 + 1)*w
eierstrassPInverse(0, -4, x))/(x^6 + 2*x^3 + 1)
```

3.520.6 Sympy [F]

$$\int \frac{1}{(1+x)^{5/2} (1-x+x^2)^{5/2}} \, dx = \int \frac{1}{(x+1)^{\frac{5}{2}} (x^2-x+1)^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

output
```
Integral(1/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

3.520.7 Maxima [F]

$$\int \frac{1}{(1+x)^{5/2} (1-x+x^2)^{5/2}} \, dx = \int \frac{1}{(x^2-x+1)^{\frac{5}{2}} (x+1)^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

3.520.8 Giac [F]

$$\int \frac{1}{(1+x)^{5/2} (1-x+x^2)^{5/2}} \, dx = \int \frac{1}{(x^2-x+1)^{\frac{5}{2}} (x+1)^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

3.520.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=\int \frac{1}{\left(x+1\right)^{5/2}\left(x^2-x+1\right)^{5/2}}\,dx$$

input
```
int(1/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

output
```
int(1/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

3.521 $\int \frac{1}{x(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

3.521.1 Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=\frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{2}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)}-\frac{2\sqrt{1+x^3}\operatorname{arctanh}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

output
```
2/3/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)^(1/2)-2/
3*arctanh((x^3+1)^(1/2))*(x^3+1)^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

3.521.2 Mathematica [A] (verified)

Time = 10.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.99

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=\frac{\frac{2(4+3x^3)}{3(1+x)^{3/2}(1-x+x^2)}-2(1+x)\sqrt{\frac{1-x+x^2}{(1+x)^2}}\operatorname{arctanh}\left(\frac{1}{(1+x)^{3/2}\sqrt{\frac{1-x+x^2}{(1+x)^2}}}\right)}{3\sqrt{1-x+x^2}}$$

input
```
Integrate[1/(x*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
((2*(4 + 3*x^3))/(3*(1 + x)^(3/2)*(1 - x + x^2)) - 2*(1 + x)*Sqrt[(1 - x +
 x^2)/(1 + x)^2]*ArcTanh[1/((1 + x)^(3/2)*Sqrt[(1 - x + x^2)/(1 + x)^2])])
/(3*Sqrt[1 - x + x^2])
```

3.521.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.72, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1210, 798, 61, 61, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}}dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x\left(x^3+1\right)^{5/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x^3\left(x^3+1\right)^{5/2}}dx^3}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt{x^3+1}\left(\int \frac{1}{x^3\left(x^3+1\right)^{3/2}}dx^3+\frac{2}{3\left(x^3+1\right)^{3/2}}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt{x^3+1}\left(\int \frac{1}{x^3\sqrt{x^3+1}}dx^3+\frac{2}{\sqrt{x^3+1}}+\frac{2}{3\left(x^3+1\right)^{3/2}}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{x^3+1}\left(2\int \frac{1}{x^6-1}d\sqrt{x^3+1}+\frac{2}{\sqrt{x^3+1}}+\frac{2}{3\left(x^3+1\right)^{3/2}}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{220}$$

$$\frac{\sqrt{x^3+1}\left(-2\text{arctanh}\left(\sqrt{x^3+1}\right)+\frac{2}{\sqrt{x^3+1}}+\frac{2}{3\left(x^3+1\right)^{3/2}}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[1/(x*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
(Sqrt[1 + x^3]*(2/(3*(1 + x^3)^(3/2)) + 2/Sqrt[1 + x^3] - 2*ArcTanh[Sqrt[1
 + x^3]]))/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

3.521.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

3.521. $\int \frac{1}{x(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

3.521.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.62

method	result	size
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2}{9(x^3+1)^{\frac{3}{2}}}+\frac{2}{3\sqrt{x^3+1}}-\frac{2\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)}{3}\right)}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$	60
default	$-\frac{2\left(3\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)\sqrt{x^3+1}\,x^3-3x^3+3\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)\sqrt{x^3+1}-4\right)}{9(x^3+1)\sqrt{x^2-x+1}\,\sqrt{1+x}}$	69

input
```
int(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(2/9/(x^3+1)^(3/2)+2/3
/(x^3+1)^(1/2)-2/3*arctanh((x^3+1)^(1/2)))
```

3.521.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.05

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{2\left(3\,x^3+4\right)\sqrt{x^2-x+1}\sqrt{x+1}-3\left(x^6+2\,x^3+1\right)\log\left(\sqrt{x^2-x+1}\sqrt{x\right.}{9\left(x^6+2\,x^3+\right.}$$

input
```
integrate(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

output
```
1/9*(2*(3*x^3 + 4)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 3*(x^6 + 2*x^3 + 1)*log
(sqrt(x^2 - x + 1)*sqrt(x + 1) + 1) + 3*(x^6 + 2*x^3 + 1)*log(sqrt(x^2 - x
 + 1)*sqrt(x + 1) - 1))/(x^6 + 2*x^3 + 1)
```

3.521.6 Sympy [F]

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{x\left(x+1\right)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

output
```
Integral(1/(x*(x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

3.521.7 Maxima [F]

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x), x)
```

3.521.8 Giac [F]

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x), x)
```

3.521.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{x\,(x+1)^{5/2}\,\left(x^2-x+1\right)^{5/2}}\,dx$$

input
```
int(1/(x*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

output
```
int(1/(x*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

3.522 $\int \frac{1}{x^2(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

3.522.1 Optimal result

Integrand size = 23, antiderivative size = 349

$$\begin{aligned}\int \frac{1}{x^2(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx &= \frac{22}{27x\sqrt{1+x}\sqrt{1-x+x^2}} \\ &+ \frac{2}{9x\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} - \frac{55(1+x^3)}{27x\sqrt{1+x}\sqrt{1-x+x^2}} \\ &+ \frac{55(1+x^3)}{27\sqrt{1+x}\,(1+\sqrt{3}+x)\,\sqrt{1-x+x^2}} \\ &- \frac{55\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{18\ 3^{3/4}\sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}\sqrt{1-x+x^2}} \\ &+ \frac{55\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}\sqrt{1-x+x^2}}\end{aligned}$$

output
```
22/27/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9/x/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)^(1
/2)-55/27*(x^3+1)/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+55/27*(x^3+1)/(1+x+3^(1/2)
)/(1+x)^(1/2)/(x^2-x+1)^(1/2)+55/81*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),
I*3^(1/2)+2*I)*2^(1/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/
4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-55/54*3^(1/4)*EllipticE((
1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)-1/2*2^(
1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2
))^2)^(1/2)
```

3.522.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = -\frac{27+88x^3+55x^6}{27x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}} + \frac{55(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}(1-x+x^2)}{(1+x)^2} + \frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}} + \frac{i\sqrt{2}(3\ldots}{\ldots}\right)}{324\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

input
```
Integrate[1/(x^2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
-1/27*(27 + 88*x^3 + 55*x^6)/(x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)) + (55*(
1 + x)^(3/2)*((12*Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3
*Sqrt[2]*(1 - I*Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[
3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*A
rcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - S
qrt[3])])/Sqrt[1 + x] + (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (
6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I
 + Sqrt[3])]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]]
, (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x]))/(324*Sqrt[(-I)/(3*I + Sq
rt[3])]*Sqrt[1 - x + x^2])
```

3.522.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1210, 819, 819, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x^2\left(x^3+1\right)^{5/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 819}$$

$$\frac{\sqrt{x^3+1}\left(\frac{11}{9}\int \frac{1}{x^2\left(x^3+1\right)^{3/2}}dx+\frac{2}{9x\left(x^3+1\right)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 819}$$

$$\frac{\sqrt{x^3+1}\left(\frac{11}{9}\left(\frac{5}{3}\int \frac{1}{x^2\sqrt{x^3+1}}dx+\frac{2}{3x\sqrt{x^3+1}}\right)+\frac{2}{9x\left(x^3+1\right)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{\sqrt{x^3+1}\left(\frac{11}{9}\left(\frac{5}{3}\left(\frac{1}{2}\int \frac{x}{\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{x}\right)+\frac{2}{3x\sqrt{x^3+1}}\right)+\frac{2}{9x\left(x^3+1\right)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 832}$$

$$\frac{\sqrt{x^3+1}\left(\frac{11}{9}\left(\frac{5}{3}\left(\frac{1}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)-\frac{\sqrt{x^3+1}}{x}\right)+\frac{2}{3x\sqrt{x^3+1}}\right)+\frac{2}{9x\left(x^3+1\right)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{\sqrt{x^3+1}\left(\frac{11}{9}\left(\frac{5}{3}\left(\frac{1}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)-\frac{\sqrt{x^3+1}}{x}\right)+\right.\right.}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 2416}$$

$$\frac{\sqrt{x^3+1}\left(\frac{11}{9}\left(\frac{5}{3}\left(\frac{1}{2}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)}}}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+\right.}}}\right.\right.\right.\right.\right.}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[1/(x^2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
(Sqrt[1 + x^3]*(2/(9*x*(1 + x^3)^(3/2)) + (11*(2/(3*x*Sqrt[1 + x^3]) + (5*
(-(Sqrt[1 + x^3]/x) + ((2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt
[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[Ar
cSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/
(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(
1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[
3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sq
rt[3] + x)^2]*Sqrt[1 + x^3]))/2))/3))/9))/(Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

3.522.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.522.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.69

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{x}-\frac{2x^2}{9\left(x^3+1\right)^{\frac{3}{2}}}-\frac{28x^2}{27\sqrt{x^3+1}}+\frac{55\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right.\right.}{27\sqrt{x^3+1}}\right.}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$
default	$\frac{55i\sqrt{3}\,F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x^4\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}+165F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x^4\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{}}{}$

input
```
int(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

3.522. $\int \frac{1}{x^2(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

output
```
((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(-1/x*(x^3+1)^(1/2)-2/
9*x^2/(x^3+1)^(3/2)-28/27/(x^3+1)^(1/2)*x^2+55/27*(3/2-1/2*I*3^(1/2))*((1+
x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((
-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2
*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((1
+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2))))
```

3.522.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = -\frac{(55\,x^6+88\,x^3+27)\sqrt{x^2-x+1}\sqrt{x+1}+55\,(x^7+2\,x^4+x)\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))}{27\,(x^7+2\,x^4+x)}$$

input
```
integrate(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

output
```
-1/27*((55*x^6 + 88*x^3 + 27)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 55*(x^7 + 2*
x^4 + x)*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)))/(x^7 + 2*x
^4 + x)
```

3.522.6 Sympy [F]

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{x^2\,(x+1)^{\frac{5}{2}}\,(x^2-x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x**2/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

output
```
Integral(1/(x**2*(x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

3.522.7 Maxima [F]

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^2), x)
```

3.522.8 Giac [F]

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^2), x)
```

3.522.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{x^2\,(x+1)^{5/2}\,\left(x^2-x+1\right)^{5/2}}\,dx$$

input
```
int(1/(x^2*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

output
```
int(1/(x^2*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

3.523 $\int \frac{1}{x^3(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

3.523.1 Optimal result

Integrand size = 23, antiderivative size = 203

$$\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{26}{27x^2\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x^2\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} - \frac{91(1+x^3)}{54x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{91\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{54\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

output
```
26/27/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9/x^2/(x^3+1)/(1+x)^(1/2)/(x^2-x+1
)^(1/2)-91/54*(x^3+1)/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)-91/162*EllipticF((1+
x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/
2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+
3^(1/2))^2)^(1/2)
```

3.523.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.43 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{-\frac{6(27+130x^3+91x^6)}{x^2(1+x)^{3/2}} - \frac{91i(1+x)(1-x+x^2)\sqrt{6+\frac{36i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{1-\frac{6i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{Elliptic}}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{324\left(1-x+x^2\right)^{3/2}}$$

input
```
Integrate[1/(x^3*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
((-6*(27 + 130*x^3 + 91*x^6))/(x^2*(1 + x)^(3/2)) - ((91*I)*(1 + x)*(1 - x
 + x^2)*Sqrt[6 + (36*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[1 - (6*I)/((3*I +
 Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[
1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(32
4*(1 - x + x^2)^(3/2))
```

3.523.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1210, 819, 819, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}}dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x^3\left(x^3+1\right)^{5/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{819}$$

$$\frac{\sqrt{x^3+1}\left(\frac{13}{9}\int \frac{1}{x^3\left(x^3+1\right)^{3/2}}dx+\frac{2}{9x^2\left(x^3+1\right)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{819}$$

$$\frac{\sqrt{x^3+1}\left(\frac{13}{9}\left(\frac{7}{3}\int\frac{1}{x^3\sqrt{x^3+1}}dx+\frac{2}{3x^2\sqrt{x^3+1}}\right)+\frac{2}{9x^2(x^3+1)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{847}$$

$$\frac{\sqrt{x^3+1}\left(\frac{13}{9}\left(\frac{7}{3}\left(-\frac{1}{4}\int\frac{1}{\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{2x^2}\right)+\frac{2}{3x^2\sqrt{x^3+1}}\right)+\frac{2}{9x^2(x^3+1)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x^3+1}\left(\frac{13}{9}\left(\frac{7}{3}\left(-\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^3+1}}-\frac{\sqrt{x^3+1}}{2x^2}\right)+\frac{2}{3x^2\sqrt{x^3+1}}\right)+\frac{2}{9x^2(x^3+1)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[1/(x^3*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
(Sqrt[1 + x^3]*(2/(9*x^2*(1 + x^3)^(3/2)) + (13*(2/(3*x^2*Sqrt[1 + x^3]) +
 (7*(-1/2*Sqrt[1 + x^3]/x^2 - (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2
)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x
)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 +
 x^3])))/3))/9))/(Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

3.523.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

3.523. $\int \frac{1}{x^3(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

3.523.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.88

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{2x^2}-\frac{2x}{9\left(x^3+1\right)^{\frac{3}{2}}}-\frac{32x}{27\sqrt{x^3+1}}-\frac{91\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{54\sqrt{x^3+1}}\right.}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$
default	$\frac{91i\sqrt{3}\,F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x^5\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}-273F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x^5\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\,\sqrt{}}{}$

input
```
int(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(-1/2/x^2*(x^3+1)^(1/2
)-2/9*x/(x^3+1)^(3/2)-32/27*x/(x^3+1)^(1/2)-91/54*(3/2-1/2*I*3^(1/2))*((1+
x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*El
lipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*
I*3^(1/2)))^(1/2)))
```

3.523.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.31

$$\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = -\frac{\left(91\,x^6+130\,x^3+27\right)\sqrt{x^2-x+1}\sqrt{x+1}+91\left(x^8+2\,x^5+x^2\right)\text{weierstrassPInverse}(0,-4,x)}{54\left(x^8+2\,x^5+x^2\right)}$$

input
```
integrate(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

output
```
-1/54*((91*x^6 + 130*x^3 + 27)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 91*(x^8 + 2
*x^5 + x^2)*weierstrassPInverse(0, -4, x))/(x^8 + 2*x^5 + x^2)
```

3.523.6 Sympy [F]

$$\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{x^3\left(x+1\right)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x**3/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

output
```
Integral(1/(x**3*(x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

3.523.7 Maxima [F]

$$\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}\left(x+1\right)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^3), x)
```

3.523.8 Giac [F]

$$\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{(x^2-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^3), x)
```

3.523.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{x^3\,(x+1)^{5/2}\,(x^2-x+1)^{5/2}}\,dx$$

input
```
int(1/(x^3*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

output
```
int(1/(x^3*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

3.524 $\int \frac{x}{(-1+x)^3\left(3+5x+4x^2\right)^2}\,dx$

3.524.1 Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{x}{(-1+x)^3\left(3+5x+4x^2\right)^2}\,dx = -\frac{21}{736(1-x)^2}-\frac{97}{4416(1-x)} + \frac{39+44x}{276(1-x)^2\left(3+5x+4x^2\right)}+\frac{6023\arctan\left(\frac{5+8x}{\sqrt{23}}\right)}{52992\sqrt{23}} + \frac{11\log(1-x)}{2304}-\frac{11\log\left(3+5x+4x^2\right)}{4608}$$

output
```
-21/736/(1-x)^2-97/4416/(1-x)+1/276*(39+44*x)/(1-x)^2/(4*x^2+5*x+3)+11/230
4*ln(1-x)-11/4608*ln(4*x^2+5*x+3)+6023/1218816*arctan(1/23*(5+8*x)*23^(1/2
))*23^(1/2)
```

3.524.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int \frac{x}{(-1+x)^3\left(3+5x+4x^2\right)^2}\,dx = \frac{-\frac{25392}{(-1+x)^2}+\frac{59248}{-1+x}+\frac{184(975+2204x)}{3+5x+4x^2}+36138\sqrt{23}\arctan\left(\frac{5+8x}{\sqrt{23}}\right)+34914\log(1-x)-17457\log(3+5x+4x}{7312896}$$

input
```
Integrate[x/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
```

output
```
(-25392/(-1 + x)^2 + 59248/(-1 + x) + (184*(975 + 2204*x))/(3 + 5*x + 4*x^
2) + 36138*Sqrt[23]*ArcTan[(5 + 8*x)/Sqrt[23]] + 34914*Log[1 - x] - 17457*
Log[3 + 5*x + 4*x^2])/7312896
```

3.524.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x-1)^3 (4x^2+5x+3)^2} dx$$

$$\Big\downarrow \text{1235}$$

$$\frac{1}{276} \int -\frac{3(44x+19)}{(1-x)^3 (4x^2+5x+3)} dx + \frac{44x+39}{276(1-x)^2 (4x^2+5x+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{44x+39}{276(1-x)^2 (4x^2+5x+3)} - \frac{1}{92} \int \frac{44x+19}{(1-x)^3 (4x^2+5x+3)} dx$$

$$\Big\downarrow \text{1200}$$

$$\frac{44x+39}{276(1-x)^2 (4x^2+5x+3)} - \frac{1}{92} \int \left(\frac{1012x-2379}{576 (4x^2+5x+3)} - \frac{253}{576(x-1)} + \frac{97}{48(x-1)^2} - \frac{21}{4(x-1)^3}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{92} \left(\frac{6023 \arctan\left(\frac{8x+5}{\sqrt{23}}\right)}{576\sqrt{23}} - \frac{253 \log\left(4x^2+5x+3\right)}{1152} - \frac{97}{48(1-x)} - \frac{21}{8(1-x)^2} + \frac{253}{576} \log(1-x)\right) + \frac{44x+39}{276(1-x)^2 (4x^2+5x+3)}$$

input
```
Int[x/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
```

output
```
(39 + 44*x)/(276*(1 - x)^2*(3 + 5*x + 4*x^2)) + (-21/(8*(1 - x)^2) - 97/(4
8*(1 - x)) + (6023*ArcTan[(5 + 8*x)/Sqrt[23]])/(576*Sqrt[23]) + (253*Log[1
 - x])/576 - (253*Log[3 + 5*x + 4*x^2])/1152)/92
```

3.524.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.524. $\int \frac{x}{(-1+x)^3(3+5x+4x^2)^2} \, dx$

3.524.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.70

method	result
default	$-\frac{1}{288(-1+x)^{2}}+\frac{7}{864(-1+x)}+\frac{11\ln(-1+x)}{2304}-\frac{-\frac{2204x}{23}-\frac{975}{23}}{6912\left(x^{2}+\frac{5}{4}x+\frac{3}{4}\right)}-\frac{11\ln\left(4x^{2}+5x+3\right)}{4608}+\frac{6023\arctan\left(\frac{(5+8x)\sqrt{23}}{23}\right)\sqrt{23}}{1218816}$
risch	$\frac{\frac{97}{1104}x^{3}-\frac{407}{4416}x^{2}-\frac{5}{184}x-\frac{15}{1472}}{(-1+x)^{2}(4x^{2}+5x+3)}-\frac{11\ln\left(64x^{2}+80x+48\right)}{4608}+\frac{6023\arctan\left(\frac{(5+8x)\sqrt{23}}{23}\right)\sqrt{23}}{1218816}+\frac{11\ln(-1+x)}{2304}$

input
```
int(x/(-1+x)^3/(4*x^2+5*x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/288/(-1+x)^2+7/864/(-1+x)+11/2304*ln(-1+x)-1/6912*(-2204/23*x-975/23)/(
x^2+5/4*x+3/4)-11/4608*ln(4*x^2+5*x+3)+6023/1218816*arctan(1/23*(5+8*x)*23
^(1/2))*23^(1/2)
```

3.524.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.38

$$\int \frac{x}{(-1+x)^3(3+5x+4x^2)^2}\,dx$$
$$= \frac{214176\,x^{3}+12046\,\sqrt{23}(4\,x^{4}-3\,x^{3}-3\,x^{2}-x+3)\arctan\left(\frac{1}{23}\,\sqrt{23}(8\,x+5)\right)-224664\,x^{2}-5819\,(4\,x^{4}-}{2437632\,(4\,x^{4}-3}$$

input
```
integrate(x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="fricas")
```

output
```
1/2437632*(214176*x^3 + 12046*sqrt(23)*(4*x^4 - 3*x^3 - 3*x^2 - x + 3)*arc
tan(1/23*sqrt(23)*(8*x + 5)) - 224664*x^2 - 5819*(4*x^4 - 3*x^3 - 3*x^2 -
x + 3)*log(4*x^2 + 5*x + 3) + 11638*(4*x^4 - 3*x^3 - 3*x^2 - x + 3)*log(x
- 1) - 66240*x - 24840)/(4*x^4 - 3*x^3 - 3*x^2 - x + 3)
```

3.524.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\int \frac{x}{(-1+x)^3 (3+5x+4x^2)^2} \, dx = \frac{388x^3 - 407x^2 - 120x - 45}{17664x^4 - 13248x^3 - 13248x^2 - 4416x + 13248} + \frac{11 \log(x-1)}{2304} - \frac{11 \log\left(x^2 + \frac{5x}{4} + \frac{3}{4}\right)}{4608} + \frac{6023\sqrt{23} \operatorname{atan}\left(\frac{8\sqrt{23}x}{23} + \frac{5\sqrt{23}}{23}\right)}{1218816}$$

input
```
integrate(x/(-1+x)**3/(4*x**2+5*x+3)**2,x)
```

output
```
(388*x**3 - 407*x**2 - 120*x - 45)/(17664*x**4 - 13248*x**3 - 13248*x**2 -
 4416*x + 13248) + 11*log(x - 1)/2304 - 11*log(x**2 + 5*x/4 + 3/4)/4608 +
6023*sqrt(23)*atan(8*sqrt(23)*x/23 + 5*sqrt(23)/23)/1218816
```

3.524.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \frac{x}{(-1+x)^3 (3+5x+4x^2)^2} \, dx = \frac{6023}{1218816} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(8x+5)\right) + \frac{388x^3 - 407x^2 - 120x - 45}{4416(4x^4 - 3x^3 - 3x^2 - x + 3)} - \frac{11}{4608} \log(4x^2 + 5x + 3) + \frac{11}{2304} \log(x-1)$$

input
```
integrate(x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="maxima")
```

output
```
6023/1218816*sqrt(23)*arctan(1/23*sqrt(23)*(8*x + 5)) + 1/4416*(388*x^3 -
407*x^2 - 120*x - 45)/(4*x^4 - 3*x^3 - 3*x^2 - x + 3) - 11/4608*log(4*x^2
+ 5*x + 3) + 11/2304*log(x - 1)
```

3.524.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.73

$$\int \frac{x}{(-1+x)^3 (3+5x+4x^2)^2} \, dx = \frac{6023}{1218816} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(8\,x+5)\right) + \frac{388\,x^3 - 407\,x^2 - 120\,x - 45}{4416\,(4\,x^2+5\,x+3)(x-1)^2} - \frac{11}{4608} \log\left(4\,x^2+5\,x+3\right) + \frac{11}{2304} \log\left(|x-1|\right)$$

input
```
integrate(x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="giac")
```

output
```
6023/1218816*sqrt(23)*arctan(1/23*sqrt(23)*(8*x + 5)) + 1/4416*(388*x^3 -
407*x^2 - 120*x - 45)/((4*x^2 + 5*x + 3)*(x - 1)^2) - 11/4608*log(4*x^2 +
5*x + 3) + 11/2304*log(abs(x - 1))
```

3.524.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.87

$$\int \frac{x}{(-1+x)^3 (3+5x+4x^2)^2} \, dx = \frac{11\ln(x-1)}{2304} + \frac{-\frac{97\,x^3}{4416} + \frac{407\,x^2}{17664} + \frac{5\,x}{736} + \frac{15}{5888}}{-x^4 + \frac{3\,x^3}{4} + \frac{3\,x^2}{4} + \frac{x}{4} - \frac{3}{4}} - \ln\left(x + \frac{5}{8} - \frac{\sqrt{23}\,1\mathrm{i}}{8}\right)\left(\frac{11}{4608} + \frac{\sqrt{23}\,6023\mathrm{i}}{2437632}\right) + \ln\left(x + \frac{5}{8} + \frac{\sqrt{23}\,1\mathrm{i}}{8}\right)\left(-\frac{11}{4608} + \frac{\sqrt{23}\,6023\mathrm{i}}{2437632}\right)$$

input
```
int(x/((x - 1)^3*(5*x + 4*x^2 + 3)^2),x)
```

output
```
(11*log(x - 1))/2304 + ((5*x)/736 + (407*x^2)/17664 - (97*x^3)/4416 + 15/5
888)/(x/4 + (3*x^2)/4 + (3*x^3)/4 - x^4 - 3/4) - log(x - (23^(1/2)*1i)/8 +
 5/8)*((23^(1/2)*6023i)/2437632 + 11/4608) + log(x + (23^(1/2)*1i)/8 + 5/8
)*((23^(1/2)*6023i)/2437632 - 11/4608)
```

3.525 $\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx$

3.525.1 Optimal result

Integrand size = 25, antiderivative size = 490

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx = -\frac{2b(b^2-2ac)\sqrt{d+ex}}{c^4} + \frac{2(c^2d^2+b^2e^2+ce(bd-ae))(d+ex)^{3/2}}{3c^3e^3} - \frac{2(2cd+be)(d+ex)^{5/2}}{5c^2e^3} + \frac{2(d+ex)^{7/2}}{7ce^3}$$
$$+ \frac{\sqrt{2}\left(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e-\frac{b^4cd-4ab^2c^2d+2a^2c^3d-b^5e+5ab^3ce-5a^2bc^2e}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+}}{\sqrt{2cd-\left(b-\sqrt{b^2}\right.}}\right.}{c^{9/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$
$$+ \frac{\sqrt{2}\left(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e+\frac{b^4cd-4ab^2c^2d+2a^2c^3d-b^5e+5ab^3ce-5a^2bc^2e}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+}}{\sqrt{2cd-\left(b+\sqrt{b^2}\right.}}\right.}{c^{9/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
2/3*(c^2*d^2+b^2*e^2+c*e*(-a*e+b*d))*(e*x+d)^(3/2)/c^3/e^3-2/5*(b*e+2*c*d)
*(e*x+d)^(5/2)/c^2/e^3+2/7*(e*x+d)^(7/2)/c/e^3-2*b*(-2*a*c+b^2)*(e*x+d)^(1
/2)/c^4+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/
2)))^(1/2))*2^(1/2)*(b^3*c*d-2*a*b*c^2*d-b^4*e+3*a*b^2*c*e-a^2*c^2*e+(5*a^
2*b*c^2*e-2*a^2*c^3*d-5*a*b^3*c*e+4*a*b^2*c^2*d+b^5*e-b^4*c*d)/(-4*a*c+b^2
)^(1/2))/c^(9/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^
(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b^3*c
*d-2*a*b*c^2*d-b^4*e+3*a*b^2*c*e-a^2*c^2*e+(-5*a^2*b*c^2*e+2*a^2*c^3*d+5*a
*b^3*c*e-4*a*b^2*c^2*d-b^5*e+b^4*c*d)/(-4*a*c+b^2)^(1/2))/c^(9/2)/(2*c*d-e
*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.525.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.96 (sec) , antiderivative size = 627, normalized size of antiderivative = 1.28

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx$$
$$= \frac{2\sqrt{d+ex}(-105b^3e^3 - 7c^2e(d+ex)(-2bd+5ae+3bex) + c^3(8d^3 - 4d^2ex + 3de^2x^2 + 15e^3x^3) + 35bce^2(}{105c^4e^3}$$
$$+ \frac{(ib^5e - b^3c(\sqrt{-b^2+4ac}d + 5iae) + abc^2(2\sqrt{-b^2+4ac}d + 5iae) + ab^2c(4icd - 3\sqrt{-b^2+4ac}e) + b^4(-}{c^{9/2}\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+(b-i}}$$
$$+ \frac{(-ib^5e + abc^2(2\sqrt{-b^2+4ac}d - 5iae) + b^3c(-\sqrt{-b^2+4ac}d + 5iae) + ab^2c(-4icd - 3\sqrt{-b^2+4ac}e) -}{c^{9/2}\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+(b+}}$$

input
```
Integrate[(x^4*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

output
```
(2*Sqrt[d + e*x]*(-105*b^3*e^3 - 7*c^2*e*(d + e*x)*(-2*b*d + 5*a*e + 3*b*e
*x) + c^3*(8*d^3 - 4*d^2*e*x + 3*d*e^2*x^2 + 15*e^3*x^3) + 35*b*c*e^2*(6*a
*e + b*(d + e*x))))/(105*c^4*e^3) + ((I*b^5*e - b^3*c*(Sqrt[-b^2 + 4*a*c]*
d + (5*I)*a*e) + a*b*c^2*(2*Sqrt[-b^2 + 4*a*c]*d + (5*I)*a*e) + a*b^2*c*((
4*I)*c*d - 3*Sqrt[-b^2 + 4*a*c]*e) + b^4*((-I)*c*d + Sqrt[-b^2 + 4*a*c]*e)
 + a^2*c^2*((-2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sq
rt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(c^(9/2)*Sqrt[-
1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) + (((-I)*b^5
*e + a*b*c^2*(2*Sqrt[-b^2 + 4*a*c]*d - (5*I)*a*e) + b^3*c*(-(Sqrt[-b^2 + 4
*a*c]*d) + (5*I)*a*e) + a*b^2*c*((-4*I)*c*d - 3*Sqrt[-b^2 + 4*a*c]*e) + b^
4*(I*c*d + Sqrt[-b^2 + 4*a*c]*e) + a^2*c^2*((2*I)*c*d + Sqrt[-b^2 + 4*a*c]
*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^
2 + 4*a*c]*e]])/(c^(9/2)*Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[
-b^2 + 4*a*c])*e])
```

3.525.3 Rubi [A] (verified)

Time = 7.95 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1199}$$

$$\frac{2\int \left(\frac{(d+ex)^3}{ce^2}-\frac{(2cd+be)(d+ex)^2}{c^2e^2}+\frac{\left(c^2d^2+b^2e^2+ce(bd-ae)\right)(d+ex)}{c^3e^2}-\frac{b\left(b^2-2ac\right)e}{c^4}+\frac{b\left(b^2-2ac\right)\left(cd^2-bed+ae^2\right)-\left(-eb^4+cdb^3+3aceb^2-\right.}{c^4e\left(\frac{c(d+ex)^2}{e^2}-\frac{(2cd-be)(d+ex)}{e^2}+a+\frac{d}{}\right.}\right.}{e}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{e\left(-\frac{-5a^2bc^2e+2a^2c^3d+5ab^3ce-4ab^2c^2d+b^5(-e)+b^4cd}{\sqrt{b^2-4ac}}-a^2c^2e+3ab^2ce-2abc^2d+b^4(-e)+b^3cd\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{9/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{e\left(\frac{-5a^2}{}\right.}{}\right.$$

input
```
Int[(x^4*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

output
```
(2*(-((b*(b^2 - 2*a*c)*e*Sqrt[d + e*x])/c^4) + ((c^2*d^2 + b^2*e^2 + c*e*(
b*d - a*e))*(d + e*x)^(3/2))/(3*c^3*e^2) - ((2*c*d + b*e)*(d + e*x)^(5/2))
/(5*c^2*e^2) + (d + e*x)^(7/2)/(7*c*e^2) + (e*(b^3*c*d - 2*a*b*c^2*d - b^4
*e + 3*a*b^2*c*e - a^2*c^2*e - (b^4*c*d - 4*a*b^2*c^2*d + 2*a^2*c^3*d - b^
5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqr
t[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(
9/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (e*(b^3*c*d - 2*a*b*c^2*d
- b^4*e + 3*a*b^2*c*e - a^2*c^2*e + (b^4*c*d - 4*a*b^2*c^2*d + 2*a^2*c^3*d
 - b^5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2
]*c^(9/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])))/e
```

3.525.3.1 Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.525.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.21

method	result
pseudoelliptic	$-\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,e^3\left(\left(\left(a^2c^2-3a\,b^2c+b^4\right)e+2ab\,c^2d-b^3cd\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}-5e\left(b\left(a^2c^2-a\,b^2c+\right.\right.\right.$
derivativedivides	$\frac{2\left(\frac{(ex+d)^{\frac{7}{2}}c^3}{7}-\frac{b\,c^2e(ex+d)^{\frac{5}{2}}}{5}-\frac{2c^3d(ex+d)^{\frac{5}{2}}}{5}-\frac{a\,c^2e^2(ex+d)^{\frac{3}{2}}}{3}+\frac{b^2c\,e^2(ex+d)^{\frac{3}{2}}}{3}+\frac{b\,c^2de(ex+d)^{\frac{3}{2}}}{3}+\frac{c^3d^2(ex+d)^{\frac{3}{2}}}{3}+2abc\,e^3\sqrt{ex+\right.}{c^4}$
default	$\frac{2\left(\frac{(ex+d)^{\frac{7}{2}}c^3}{7}-\frac{b\,c^2e(ex+d)^{\frac{5}{2}}}{5}-\frac{2c^3d(ex+d)^{\frac{5}{2}}}{5}-\frac{a\,c^2e^2(ex+d)^{\frac{3}{2}}}{3}+\frac{b^2c\,e^2(ex+d)^{\frac{3}{2}}}{3}+\frac{b\,c^2de(ex+d)^{\frac{3}{2}}}{3}+\frac{c^3d^2(ex+d)^{\frac{3}{2}}}{3}+2abc\,e^3\sqrt{ex+\right.}{c^4}$
risch	$\frac{2(15c^3e^3x^3-21c^2e^3b\,x^2+3c^3d\,e^2x^2-35a\,c^2e^3x+35b^2c\,e^3x-7b\,c^2d\,e^2x-4c^3d^2ex+210c\,e^3ba-35a\,c^2d\,e^2-105b^3e^3+35b^2}{105e^3c^4}$

input
```
int(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/(-4*e^2*(a*c-1/4*b^2))^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c
)^(1/2)*(-2^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*e^3*(
((a^2*c^2-3*a*b^2*c+b^4)*e+2*a*b*c^2*d-b^3*c*d)*(-4*e^2*(a*c-1/4*b^2))^(1/
2)-5*e*(b*(a^2*c^2-a*b^2*c+1/5*b^4)*e-2/5*d*(a^2*c^2-2*a*b^2*c+1/2*b^4)*c)
)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2
))*c)^(1/2))+((((a^2*c^2-3*a*b^2*c+b^4)*e+2*a*b*c^2*d-b^3*c*d)*(-4*e^2*(a*
c-1/4*b^2))^(1/2)+5*e*(b*(a^2*c^2-a*b^2*c+1/5*b^4)*e-2/5*d*(a^2*c^2-2*a*b^
2*c+1/2*b^4)*c))*2^(1/2)*e^3*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-
4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+4*(-4*e^2*(a*c-1/4*b^2))^(1/2)*((b*e
-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*((1/14*c^3*x^3-1/6*(3/5*b*x+
a)*x*c^2+b*(1/6*b*x+a)*c-1/2*b^3)*e^3-1/6*d*(-3/35*c^2*x^2+(1/5*b*x+a)*c-b
^2)*c*e^2+1/15*d^2*(-2/7*c*x+b)*c^2*e+4/105*c^3*d^3)*(e*x+d)^(1/2))*((-b*e
+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))/((-b*e+2*c*d+(-4*e^2*(a*c-1
/4*b^2))^(1/2))*c)^(1/2)/e^3/c^4
```

3.525. $\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx$

3.525.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5507 vs. $2(436) = 872$.

Time = 0.99 (sec) , antiderivative size = 5507, normalized size of antiderivative = 11.24

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.525.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

3.525.7 Maxima [F]

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx = \int \frac{\sqrt{ex+d}x^4}{cx^2+bx+a}\,dx$$

input
```
integrate(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*x^4/(c*x^2 + b*x + a), x)
```

3.525.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1200 vs. $2(436) = 872$.

Time = 0.38 (sec) , antiderivative size = 1200, normalized size of antiderivative = 2.45

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^5*c - 6*a*b^3*c
^2 + 8*a^2*b*c^3)*d - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e)*c^
2*e^2 - 2*((b^3*c^3 - 2*a*b*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^4*c^2 - 2*a*b^
2*c^3)*sqrt(b^2 - 4*a*c)*d*e + (a*b^3*c^2 - 2*a^2*b*c^3)*sqrt(b^2 - 4*a*c)
*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) + (2*
(b^4*c^4 - 4*a*b^2*c^5 + 2*a^2*c^6)*d^2*e - (3*b^5*c^3 - 14*a*b^3*c^4 + 12
*a^2*b*c^5)*d*e^2 + (b^6*c^2 - 5*a*b^4*c^3 + 5*a^2*b^2*c^4)*e^3)*sqrt(-4*c
^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/
sqrt(-(2*c^8*d*e^24 - b*c^7*e^25 + sqrt(-4*(c^8*d^2*e^24 - b*c^7*d*e^25 +
a*c^7*e^26)*c^8*e^24 + (2*c^8*d*e^24 - b*c^7*e^25)^2))/(c^8*e^24)))/((sqrt
(b^2 - 4*a*c)*c^7*d^2 - sqrt(b^2 - 4*a*c)*b*c^6*d*e + sqrt(b^2 - 4*a*c)*a*
c^6*e^2)*c^2*abs(e)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*
e)*((b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d - (b^6 - 7*a*b^4*c + 13*a^2*b^2*
c^2 - 4*a^3*c^3)*e)*c^2*e^2 + 2*((b^3*c^3 - 2*a*b*c^4)*sqrt(b^2 - 4*a*c)*d
^2 - (b^4*c^2 - 2*a*b^2*c^3)*sqrt(b^2 - 4*a*c)*d*e + (a*b^3*c^2 - 2*a^2*b*
c^3)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*
e)*abs(c)*abs(e) + (2*(b^4*c^4 - 4*a*b^2*c^5 + 2*a^2*c^6)*d^2*e - (3*b^5*c
^3 - 14*a*b^3*c^4 + 12*a^2*b*c^5)*d*e^2 + (b^6*c^2 - 5*a*b^4*c^3 + 5*a^2*b
^2*c^4)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sq
rt(1/2)*sqrt(e*x + d)/sqrt(-(2*c^8*d*e^24 - b*c^7*e^25 - sqrt(-4*(c^8*d...
```

3.525.9 Mupad [B] (verification not implemented)

Time = 14.52 (sec) , antiderivative size = 13879, normalized size of antiderivative = 28.32

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((x^4*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

output

```
(d + e*x)^(3/2)*((4*d^2)/(c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(3*c^
2*e^6) + (((8*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c
*d*e^3))/(3*c*e^3)) - atan(((((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^
5*d*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^
3*c^7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 - (8*(d +
e*x)^(1/2)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10
*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7
*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)
^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*
a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d
*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^
2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)
^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*
c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e
^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)
^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^
5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e
*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e
 - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2)
 + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^...
```

3.525. $\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx$

3.526 $\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx$

3.526.1 Optimal result

Integrand size = 25, antiderivative size = 326

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx=\frac{2(b^2-ac)\sqrt{d+ex}}{c^3}-\frac{2(cd+be)(d+ex)^{3/2}}{3c^2e^2}+\frac{2(d+ex)^{5/2}}{5ce^2}+\frac{\left(b^3-3abc-\sqrt{b^2-4ac}(b^2-ac)\right)\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}}-\frac{\left(b^3-3abc+\sqrt{b^2-4ac}(b^2-ac)\right)\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}}$$

output
```
-2/3*(b*e+c*d)*(e*x+d)^(3/2)/c^2/e^2+2/5*(e*x+d)^(5/2)/c/e^2+2*(-a*c+b^2)*
(e*x+d)^(1/2)/c^3+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-
4*a*c+b^2)^(1/2)))^(1/2))*(b^3-3*a*b*c-(-a*c+b^2)*(-4*a*c+b^2)^(1/2))*(2*c
*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/c^(7/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)-1/2*
arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/
2))*(b^3-3*a*b*c+(-a*c+b^2)*(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)^(
1/2)))^(1/2)/c^(7/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)
```

3.526.2 Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.43

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

$$= \frac{\frac{2\sqrt{c}\sqrt{d+ex}\left(15b^2e^2+c^2\left(-2d^2+dex+3e^2x^2\right)-5ce(3ae+b(d+ex))\right)}{e^2} - \frac{15\sqrt{2}\left(-b^4e+ac^2\left(\sqrt{b^2-4ac}d-2ae\right)+b^2c\left(-\sqrt{b^2-4ac}d+4ae\right)+b^3\left(cd+\sqrt{b\right.\right.}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(\right.}}}{}$$

input
```
Integrate[(x^3*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

output
```
((2*Sqrt[c]*Sqrt[d + e*x]*(15*b^2*e^2 + c^2*(-2*d^2 + d*e*x + 3*e^2*x^2) -
 5*c*e*(3*a*e + b*(d + e*x))))/e^2 - (15*Sqrt[2]*(-(b^4*e) + a*c^2*(Sqrt[b
^2 - 4*a*c]*d - 2*a*e) + b^2*c*(-(Sqrt[b^2 - 4*a*c]*d) + 4*a*e) + b^3*(c*d
 + Sqrt[b^2 - 4*a*c]*e) - a*b*c*(3*c*d + 2*Sqrt[b^2 - 4*a*c]*e))*ArcTan[(S
qrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) - (15*Sqrt[2]*
(b^4*e + a*c^2*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - b^2*c*(Sqrt[b^2 - 4*a*c]*d
+ 4*a*e) + a*b*c*(3*c*d - 2*Sqrt[b^2 - 4*a*c]*e) + b^3*(-(c*d) + Sqrt[b^2
- 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + (b + Sqr
t[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*
c])*e]))/(15*c^(7/2))
```

3.526.3 Rubi [A] (verified)

Time = 4.16 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.24, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \; 1199$$

$$\frac{2\int\left(\frac{(d+ex)^2}{ce}-\frac{(cd+be)(d+ex)}{c^2e}+\frac{(b^2-ac)e}{c^3}-\frac{(b^2-ac)(cd^2-bed+ae^2)-(-eb^3+cdb^2+2aceb-ac^2d)(d+ex)}{c^3e\left(\frac{c(d+ex)^2}{e^2}-\frac{(2cd-be)(d+ex)}{e^2}+a+\frac{d(cd-be)}{e^2}\right)}\right)d\sqrt{d+ex}}{e}$$

$\downarrow$ 2009

$$\frac{2\left(-\frac{e\left(-\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^3(-e)+b^2cd\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{e\left(\frac{-2a^2c^2e+4ab^2ce-3abc^2d}{\sqrt{b^2-4ac}}\right.}{}\right.}{e}$$

input
```
Int[(x^3*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

output
```
(2*(((b^2 - a*c)*e*Sqrt[d + e*x])/c^3 - ((c*d + b*e)*(d + e*x)^(3/2))/(3*c
^2*e) + (d + e*x)^(5/2)/(5*c*e) - (e*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*
e - (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 -
 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])
 - (e*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e + (b^3*c*d - 3*a*b*c^2*d - b^
4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[
c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/
2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])))/e
```

3.526.3.1 Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.526. $\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx$

3.526.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.55

method	result
pseudoelliptic	$-\frac{2\left(\left(\left(\left(-\frac{1}{2}b^3+abc\right)e-\frac{cd\left(ac-b^2\right)}{2}\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+e\left(\left(a^2c^2-2a\,b^2c+\frac{1}{2}b^4\right)e+\frac{3db\left(ac-\frac{b^2}{3}\right)c}{2}\right)\right)\sqrt{2}\sqrt{\left(be-2cd+\cdots\right.}\cdots\right.}{\cdots}$
risch	$-\frac{2\left(-3c^2x^2e^2+5bc\,e^2x-c^2dex+15ac\,e^2-15b^2e^2+5bcde+2c^2d^2\right)\sqrt{ex+d}}{15e^2c^3}+\frac{-\frac{\left(2a^2c^2e^2-4a\,b^2c\,e^2+3ab\,c^2de+b^4e^2-b^3cde\cdots\right.}{\cdots}}{\cdots}$
derivativedivides	$\frac{-\frac{2\left(-\frac{(ex+d)^{\frac{5}{2}}c^2}{5}+\frac{bce(ex+d)^{\frac{3}{2}}}{3}+\frac{c^2d(ex+d)^{\frac{3}{2}}}{3}+ac\,e^2\sqrt{ex+d}-b^2e^2\sqrt{ex+d}\right)}{c^3}+\frac{8e^2\left(-\frac{\left(2a^2c^2e^2-4a\,b^2c\,e^2+3ab\,c^2de+b^4e^2-b^3cd\cdots\right.}{\cdots}\cdots\right.}{\cdots}}{\cdots}$
default	$\frac{-\frac{2\left(-\frac{(ex+d)^{\frac{5}{2}}c^2}{5}+\frac{bce(ex+d)^{\frac{3}{2}}}{3}+\frac{c^2d(ex+d)^{\frac{3}{2}}}{3}+ac\,e^2\sqrt{ex+d}-b^2e^2\sqrt{ex+d}\right)}{c^3}+\frac{8e^2\left(-\frac{\left(2a^2c^2e^2-4a\,b^2c\,e^2+3ab\,c^2de+b^4e^2-b^3cd\cdots\right.}{\cdots}\cdots\right.}{\cdots}}{\cdots}$

input
```
int(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/(-4*e^2*(a*c-1/4*b^2))^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*
c)^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*((((-1/2*b^3+
a*b*c)*e-1/2*c*d*(a*c-b^2))*(-4*e^2*(a*c-1/4*b^2))^(1/2)+e*((a^2*c^2-2*a*b
^2*c+1/2*b^4)*e+3/2*d*b*(a*c-1/3*b^2)*c))*2^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c
-1/4*b^2))^(1/2))*c)^(1/2)*e^2*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*
d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^
2))^(1/2))*c)^(1/2)*((((1/2*b^3-a*b*c)*e+1/2*c*d*(a*c-b^2))*(-4*e^2*(a*c-1
/4*b^2))^(1/2)+e*((a^2*c^2-2*a*b^2*c+1/2*b^4)*e+3/2*d*b*(a*c-1/3*b^2)*c))*
2^(1/2)*e^2*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^
2))^(1/2))*c)^(1/2))+((-1/5*c^2*x^2+(1/3*b*x+a)*c-b^2)*e^2+1/3*d*(-1/5*c*x
+b)*c*e+2/15*c^2*d^2)*(-4*e^2*(a*c-1/4*b^2))^(1/2)*((b*e-2*c*d+(-4*e^2*(a*
c-1/4*b^2))^(1/2))*c)^(1/2)*(e*x+d)^(1/2)))/c^3/e^2
```

3.526.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4245 vs. $2(273) = 546$.

Time = 0.95 (sec) , antiderivative size = 4245, normalized size of antiderivative = 13.02

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
1/30*(15*sqrt(2)*c^3*e^2*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^
3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e + (b^2*c^7 -
 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5
+ 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b
^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*
b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)
/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(sqrt(2)*((b^9*c - 9*a*b^
7*c^2 + 27*a^2*b^5*c^3 - 31*a^3*b^3*c^4 + 12*a^4*b*c^5)*d - (b^10 - 10*a*b
^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*e - (
b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a
^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2
 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (
b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12
*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^6*c - 6*a*b^
4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 -
 7*a^3*b*c^3)*e + (b^2*c^7 - 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a
^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2
 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (
b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12
*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8...
```

3.526.6 Sympy [F]

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx = \int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

input
```
integrate(x**3*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

output
```
Integral(x**3*sqrt(d + e*x)/(a + b*x + c*x**2), x)
```

3.526.7 Maxima [F]

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx = \int \frac{\sqrt{ex+d}x^3}{cx^2+bx+a}\,dx$$

input
```
integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*x^3/(c*x^2 + b*x + a), x)
```

3.526.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1074 vs. 2(273) = 546.

Time = 0.38 (sec) , antiderivative size = 1074, normalized size of antiderivative = 3.29

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^4*c - 5*a*b^2*c^
2 + 4*a^2*c^3)*d - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e)*c^2*e^2 - 2*((b^2*c^
3 - a*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^3*c^2 - a*b*c^3)*sqrt(b^2 - 4*a*c)*d
*e + (a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c -
 sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) + (2*(b^3*c^4 - 3*a*b*c^5)*d^2*e -
(3*b^4*c^3 - 11*a*b^2*c^4 + 4*a^2*c^5)*d*e^2 + (b^5*c^2 - 4*a*b^3*c^3 + 2*
a^2*b*c^4)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2
*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c^6*d*e^12 - b*c^5*e^13 + sqrt(-4*(c^6*d
^2*e^12 - b*c^5*d*e^13 + a*c^5*e^14)*c^6*e^12 + (2*c^6*d*e^12 - b*c^5*e^13
)^2))/(c^6*e^12)))/((sqrt(b^2 - 4*a*c)*c^6*d^2 - sqrt(b^2 - 4*a*c)*b*c^5*d
*e + sqrt(b^2 - 4*a*c)*a*c^5*e^2)*c^2*abs(e)) - 1/4*(sqrt(-4*c^2*d + 2*(b*
c + sqrt(b^2 - 4*a*c)*c)*e)*((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d - (b^5 -
6*a*b^3*c + 8*a^2*b*c^2)*e)*c^2*e^2 + 2*((b^2*c^3 - a*c^4)*sqrt(b^2 - 4*a*
c)*d^2 - (b^3*c^2 - a*b*c^3)*sqrt(b^2 - 4*a*c)*d*e + (a*b^2*c^2 - a^2*c^3)
*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*a
bs(c)*abs(e) + (2*(b^3*c^4 - 3*a*b*c^5)*d^2*e - (3*b^4*c^3 - 11*a*b^2*c^4
+ 4*a^2*c^5)*d*e^2 + (b^5*c^2 - 4*a*b^3*c^3 + 2*a^2*b*c^4)*e^3)*sqrt(-4*c^
2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/s
qrt(-(2*c^6*d*e^12 - b*c^5*e^13 - sqrt(-4*(c^6*d^2*e^12 - b*c^5*d*e^13 + a
*c^5*e^14)*c^6*e^12 + (2*c^6*d*e^12 - b*c^5*e^13)^2))/(c^6*e^12)))/((sq...
```

3.526.9 Mupad [B] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 11143, normalized size of antiderivative = 34.18

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((x^3*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

output
```
atan(((((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*
e^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^
6*d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 - (8*(d + e*x)^(1/2)*(-(b^9*e - 8*a^4*
c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a
^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c -
b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(
-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c
^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6
*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^
2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*
e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c
*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c
^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d
+ 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c
 - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*
(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(1
6*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^4 +
 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e
^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e
^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28...
```

3.527 $\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx$

3.527.1 Optimal result

Integrand size = 25, antiderivative size = 316

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx = -\frac{2b\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3ce} + \frac{\sqrt{2}\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{c^{5/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{2}\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{c^{5/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output

```
2/3*(e*x+d)^(3/2)/c/e-2*b*(e*x+d)^(1/2)/c^2+arctanh(2^(1/2)*c^(1/2)*(e*x+d
)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b*c*d-b^2*e+a*c*e
+(-3*a*b*c*e+2*a*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(2*c*d-e
*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*
d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b*c*d-b^2*e+a*c*e+(3*a*b*c*e-2
*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(2*c*d-e*(b+(-4*a*c+b^
2)^(1/2)))^(1/2)
```

3.527.2 Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.19

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

$$= \frac{\frac{2\sqrt{c}\sqrt{d+ex}(-3be+c(d+ex))}{e} + \frac{3\sqrt{2}\left(-b^3e+bc\left(-\sqrt{b^2-4ac}d+3ae\right)+b^2\left(cd+\sqrt{b^2-4ac}e\right)-ac\left(2cd+\sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}e}}\right.}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}}{3c^{5/2}}$$

input
```
Integrate[(x^2*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

output
```
((2*Sqrt[c]*Sqrt[d + e*x]*(-3*b*e + c*(d + e*x)))/e + (3*Sqrt[2]*(-(b^3*e)
 + b*c*(-(Sqrt[b^2 - 4*a*c]*d) + 3*a*e) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e)
- a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x]
)/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*
d + (b - Sqrt[b^2 - 4*a*c])*e]) + (3*Sqrt[2]*(b^3*e - b*c*(Sqrt[b^2 - 4*a*
c]*d + 3*a*e) + a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b^2*(-(c*d) + Sqrt[b^2
 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + (b + Sq
rt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a
*c])*e]))/(3*c^(5/2))
```

3.527.3 Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \; 1199$$

$$\frac{2\int \left(-\frac{be}{c^2}+\frac{d+ex}{c}+\frac{b\left(cd^2-bed+ae^2\right)-\left(-eb^2+cdb+ace\right)(d+ex)}{c^2\left(\frac{c(d+ex)^2}{e^2}-\frac{(2cd-be)(d+ex)}{e^2}+a+\frac{d(cd-be)}{e^2}\right)e}\right)d\sqrt{d+ex}}{e}$$

$$\Big\downarrow \; 2009$$

$$\frac{2\left(\frac{e\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{e\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)a}{\sqrt{2}c^{5/2}\sqrt{2cd-e\left(\sqrt{b^2-\right.}}}\right.}{e}$$

input
```
Int[(x^2*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

output
```
(2*(-((b*e*Sqrt[d + e*x])/c^2) + (d + e*x)^(3/2)/(3*c) + (e*(b*c*d - b^2*e
 + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*Ar
cTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e]])/(Sqrt[2]*c^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (e*(b*c*
d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4
*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2
- 4*a*c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])))
/e
```

3.527.3.1 Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.527.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.34

method	result
risch	$-\frac{2(-xce+3be-cd)\sqrt{ex+d}}{3e\,c^{2}}-\frac{8\left(-\frac{\left(-3c\,e^{2}ba+2a\,c^{2}de+b^{3}e^{2}-b^{2}cde+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,ace-\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,b^{2}e+\sqrt{-e^{2}\left(4ac-\right.}\right.}{8c\sqrt{-e^{2}\left(4ac-b^{2}\right)}\sqrt{\left(-be+2cd+\sqrt{-e^{2}\left(4\right.}\right.}}\right.}{}$
pseudoelliptic	$-\frac{-\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}\right)ce}\left(\left((ac-b^{2})e+bcd\right)\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}+(-3abc+b^{3})e^{2}+d(2a\,c^{2}-b^{2}c)e\right)\operatorname{arctan}}{}$
derivativedivides	$\frac{-\frac{2\left(-\frac{c(ex+d)^{\frac{3}{2}}}{3}+be\sqrt{ex+d}\right)}{c^{2}}+\frac{8e\left(\frac{\left(-3c\,e^{2}ba+2a\,c^{2}de+b^{3}e^{2}-b^{2}cde-\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,ace+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,b^{2}e-\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right.}{8c\sqrt{-e^{2}\left(4ac-b^{2}\right)}\sqrt{\left(be-2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right.}\right.}}\right.}{}}{}$
default	$\frac{-\frac{2\left(-\frac{c(ex+d)^{\frac{3}{2}}}{3}+be\sqrt{ex+d}\right)}{c^{2}}+\frac{8e\left(\frac{\left(-3c\,e^{2}ba+2a\,c^{2}de+b^{3}e^{2}-b^{2}cde-\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,ace+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,b^{2}e-\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right.}{8c\sqrt{-e^{2}\left(4ac-b^{2}\right)}\sqrt{\left(be-2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right.}\right.}}\right.}{}}{}$

input
```
int(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-c*e*x+3*b*e-c*d)*(e*x+d)^(1/2)/e/c^2-8/c*(-1/8*(-3*c*e^2*b*a+2*a*c^
2*d*e+b^3*e^2-b^2*c*d*e+(-e^2*(4*a*c-b^2))^(1/2)*a*c*e-(-e^2*(4*a*c-b^2))^
(1/2)*b^2*e+(-e^2*(4*a*c-b^2))^(1/2)*b*c*d)/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(
1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/
2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+1/8*(3*c*e^2*b
*a-2*a*c^2*d*e-b^3*e^2+b^2*c*d*e+(-e^2*(4*a*c-b^2))^(1/2)*a*c*e-(-e^2*(4*a
*c-b^2))^(1/2)*b^2*e+(-e^2*(4*a*c-b^2))^(1/2)*b*c*d)/c/(-e^2*(4*a*c-b^2))^
(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x
+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))
```

3.527.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2966 vs. $2(270) = 540$.

Time = 0.50 (sec) , antiderivative size = 2966, normalized size of antiderivative = 9.39

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
1/6*(3*sqrt(2)*c^2*e*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*
a*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^
3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*
c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^
2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(sqrt(2)*((b^6*c - 6*a*
b^4*c^2 + 8*a^2*b^2*c^3)*d - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c
^3)*e - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 +
 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4
)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/
(b^2*c^10 - 4*a*c^11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 -
 5*a*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4
*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3
*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)
*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*((a^2*b^3*c - 2*a^3
*b*c^2)*d - (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e)*sqrt(e*x + d)) - 3*sqrt(2
)*c^2*e*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a
^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2
*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10
- 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-sqrt(2)*((b^6*c - 6*a*b^4*c^2 +...
```

3.527.6 Sympy [F]

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx = \int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

input
```
integrate(x**2*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

output
```
Integral(x**2*sqrt(d + e*x)/(a + b*x + c*x**2), x)
```

3.527.7 Maxima [F]

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx=\int \frac{\sqrt{ex+d}x^2}{cx^2+bx+a}\,dx$$

input
```
integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*x^2/(c*x^2 + b*x + a), x)
```

3.527.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 897 vs. 2(270) = 540.

Time = 0.37 (sec) , antiderivative size = 897, normalized size of antiderivative = 2.84

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx=$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}c\right)e}((b^3c-4\,abc^2)d-(b^4-5\,ab^2c+4\,a^2c^2)e)c^2e^2-2\left(\sqrt{b^2-4\,ac}bc^3d^2\right.\right.}{}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}c\right)e}((b^3c-4\,abc^2)d-(b^4-5\,ab^2c+4\,a^2c^2)e)c^2e^2+2\left(\sqrt{b^2-4\,ac}bc^3d^2\right.\right.}{}$$

$$+\frac{2\left((ex+d)^{\frac{3}{2}}c^2e^2-3\sqrt{ex+d}bce^3\right)}{3\,c^3e^3}$$

input
```
integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^3*c - 4*a*b*c^2
)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*c^2*e^2 - 2*(sqrt(b^2 - 4*a*c)*b*c^
3*d^2 - sqrt(b^2 - 4*a*c)*b^2*c^2*d*e + sqrt(b^2 - 4*a*c)*a*b*c^2*e^2)*sqr
t(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) + (2*(b^2*c^4
- 2*a*c^5)*d^2*e - (3*b^3*c^3 - 8*a*b*c^4)*d*e^2 + (b^4*c^2 - 3*a*b^2*c^3)
*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)
*sqrt(e*x + d)/sqrt(-(2*c^4*d*e^4 - b*c^3*e^5 + sqrt(-4*(c^4*d^2*e^4 - b*c
^3*d*e^5 + a*c^3*e^6)*c^4*e^4 + (2*c^4*d*e^4 - b*c^3*e^5)^2))/(c^4*e^4)))/
((sqrt(b^2 - 4*a*c)*c^5*d^2 - sqrt(b^2 - 4*a*c)*b*c^4*d*e + sqrt(b^2 - 4*a
*c)*a*c^4*e^2)*c^2*abs(e)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*
c)*c)*e)*((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*c^2*e^2
 + 2*(sqrt(b^2 - 4*a*c)*b*c^3*d^2 - sqrt(b^2 - 4*a*c)*b^2*c^2*d*e + sqrt(b
^2 - 4*a*c)*a*b*c^2*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*
abs(c)*abs(e) + (2*(b^2*c^4 - 2*a*c^5)*d^2*e - (3*b^3*c^3 - 8*a*b*c^4)*d*e
^2 + (b^4*c^2 - 3*a*b^2*c^3)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*
c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c^4*d*e^4 - b*c^3*e^5
- sqrt(-4*(c^4*d^2*e^4 - b*c^3*d*e^5 + a*c^3*e^6)*c^4*e^4 + (2*c^4*d*e^4 -
 b*c^3*e^5)^2))/(c^4*e^4)))/((sqrt(b^2 - 4*a*c)*c^5*d^2 - sqrt(b^2 - 4*a*c
)*b*c^4*d*e + sqrt(b^2 - 4*a*c)*a*c^4*e^2)*c^2*abs(e)) + 2/3*((e*x + d)^(3
/2)*c^2*e^2 - 3*sqrt(e*x + d)*b*c*e^3)/(c^3*e^3)
```

3.527.9 Mupad [B] (verification not implemented)

Time = 13.25 (sec) , antiderivative size = 8171, normalized size of antiderivative = 25.86

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((x^2*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

output
```
(2*(d + e*x)^(3/2))/(3*c*e) - atan(((((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4
- b^4*c^3*d*e^3 + b^3*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3)
)/c^3 - (8*(d + e*x)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^
3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*
a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c
*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b
^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))
^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^
3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*
a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*
a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1
/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3
)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1
/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4
*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2
*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(
-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e +
a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*
b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3
)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^...
```

3.527. $\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx$

3.528 $\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx$

3.528.1 Optimal result

Integrand size = 23, antiderivative size = 287

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{2\sqrt{d+ex}}{c}+\frac{\sqrt{2}\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}(cd-be)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\sqrt{2}\left(bcd-b^2e+2ace+\sqrt{b^2-4ac}(cd-be)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
2*(e*x+d)^(1/2)/c+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*
c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b*c*d-b^2*e+2*a*c*e-(-b*e+c*d)*(-4*a*c+b^2)
^(1/2))/c^(3/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-
arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/
2))*2^(1/2)*(b*c*d-b^2*e+2*a*c*e+(-b*e+c*d)*(-4*a*c+b^2)^(1/2))/c^(3/2)/(-
4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.528.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.19

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{2\sqrt{c}\sqrt{d+ex} - \frac{\left(-ibcd-c\sqrt{-b^2+4ac}d+ib^2e-2iace+b\sqrt{-b^2+4ace}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ace}}}\right)}{\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}} - \frac{\left(ibcd-c\sqrt{-b^2+4ac}d-ib^2e+2ia\right.}{\sqrt{-\frac{b^2}{2}+}}}{c^{3/2}}$$

input
```
Integrate[(x*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

output
```
(2*Sqrt[c]*Sqrt[d + e*x] - (((-I)*b*c*d - c*Sqrt[-b^2 + 4*a*c]*d + I*b^2*e
 - (2*I)*a*c*e + b*Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d +
e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c
]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) - ((I*b*c*d - c*Sqrt[-b^2 +
 4*a*c]*d - I*b^2*e + (2*I)*a*c*e + b*Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(Sq
rt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e]))/c^(3/2)
```

3.528.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1196, 25, 1197, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{ 1196}$$

$$\frac{\int -\frac{ae-(cd-be)x}{\sqrt{d+ex}(cx^2+bx+a)}dx}{c} + \frac{2\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\sqrt{d+ex}}{c}-\frac{\int \frac{ae-(cd-be)x}{\sqrt{d+ex}(cx^2+bx+a)}dx}{c}$$

$$\Big\downarrow \; 1197$$

$$\frac{2\sqrt{d+ex}}{c}-\frac{2\int \frac{cd^2-bed+ae^2-(cd-be)(d+ex)}{cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex)}d\sqrt{d+ex}}{c}$$

$$\Big\downarrow \; 1480$$

$$\frac{2\sqrt{d+ex}}{c}-$$

$$\frac{2\left(\frac{\left(-\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\int \frac{1}{\frac{1}{2}\left(\left(b-\sqrt{b^2-4ac}\right)e-2cd\right)+c(d+ex)}d\sqrt{d+ex}}{2\sqrt{b^2-4ac}}-\frac{\left(\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\int \frac{}{\frac{1}{2}\left(\left(b+\sqrt{\right.\right.}}}{2\sqrt{b^2-4ac}}\right.}{c}$$

$$\Big\downarrow \; 221$$

$$\frac{2\sqrt{d+ex}}{c}-$$

$$\frac{2\left(\frac{\left(\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\left(-\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\operatorname{arctanh}\left(\frac{\sqrt{}}{\sqrt{2cd-}}\right.}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right.}{c}$$

input

```
Int[(x*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

output

```
(2*Sqrt[d + e*x])/c - (2*(-(((b*c*d - b^2*e + 2*a*c*e - Sqrt[b^2 - 4*a*c]*
(c*d - b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b -
Sqrt[b^2 - 4*a*c])*e])) + ((b*c*d - b^2*e + 2*a*c*e + Sqrt[b^2 - 4*a*c]*(c
*d - b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sq
rt[b^2 - 4*a*c])*e])))/c
```

3.528. $\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx$

3.528.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1196
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int
[(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] &
& GtQ[m, 0]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.528.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$\frac{2\sqrt{ex+d}+\frac{\left(2ac\,e^2-b^2e^2+bcde+\sqrt{-e^2\left(4ac-b^2\right)}\,be-\sqrt{-e^2\left(4ac-b^2\right)}\,cd\right)\sqrt{2}\,\text{arctanh}\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}-\frac{\left(-2a\right.}{}}{c}$
derivativedivides	$\frac{2\sqrt{ex+d}}{c}-\frac{\left(-2ac\,e^2+b^2e^2-bcde-\sqrt{-e^2(4ac-b^2)}\,be+\sqrt{-e^2(4ac-b^2)}\,cd\right)\sqrt{2}\,\text{arctanh}\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right.}}\right.}{c\sqrt{-e^2(4ac-b^2)}\,\sqrt{\left(-be+2cd+\sqrt{-e^2(4ac-b^2)}\right)c}}$
default	$\frac{2\sqrt{ex+d}}{c}-\frac{\left(-2ac\,e^2+b^2e^2-bcde-\sqrt{-e^2(4ac-b^2)}\,be+\sqrt{-e^2(4ac-b^2)}\,cd\right)\sqrt{2}\,\text{arctanh}\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right.}}\right.}{c\sqrt{-e^2(4ac-b^2)}\,\sqrt{\left(-be+2cd+\sqrt{-e^2(4ac-b^2)}\right)c}}$
risch	$\frac{2\sqrt{ex+d}}{c}+\frac{\left(2ac\,e^2-b^2e^2+bcde+\sqrt{-e^2(4ac-b^2)}\,be-\sqrt{-e^2(4ac-b^2)}\,cd\right)\sqrt{2}\,\text{arctanh}\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right.}{c\sqrt{-e^2(4ac-b^2)}\,\sqrt{\left(-be+2cd+\sqrt{-e^2(4ac-b^2)}\right)c}}$

input
```
int(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/c*(2*(e*x+d)^(1/2)+(2*a*c*e^2-b^2*e^2+b*c*d*e+(-e^2*(4*a*c-b^2))^(1/2)*b
*e-(-e^2*(4*a*c-b^2))^(1/2)*c*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2
*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((
-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-(-2*a*c*e^2+b^2*e^2-b*c*d*e
+(-e^2*(4*a*c-b^2))^(1/2)*b*e-(-e^2*(4*a*c-b^2))^(1/2)*c*d)/(-e^2*(4*a*c-b
^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c
*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))
```

3.528.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1721 vs. $2(241) = 482$.

Time = 0.40 (sec) , antiderivative size = 1721, normalized size of antiderivative = 6.00

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 -
4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c +
a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(sqrt(2)*((b^3
*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e - (b^3*c^3 - 4*a*b*c^4
)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2
)*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e
 + (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4
- 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + 4
*(a*b*c*d - (a*b^2 - a^2*c)*e)*sqrt(e*x + d)) - sqrt(2)*c*sqrt(((b^2*c - 2
*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*
(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^
7)))/(b^2*c^3 - 4*a*c^4))*log(-sqrt(2)*((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a
*b^2*c + 4*a^2*c^2)*e - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c
 - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))*s
qrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*a*c^4)*sqrt((b
^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b
^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + 4*(a*b*c*d - (a*b^2 - a^2*c)*e)
*sqrt(e*x + d)) + sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e
- (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 -
 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*l...
```

3.528.6 Sympy [F]

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx = \int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

input
```
integrate(x*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

output
```
Integral(x*sqrt(d + e*x)/(a + b*x + c*x**2), x)
```

3.528.7 Maxima [F]

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx = \int \frac{\sqrt{ex+d}x}{cx^2+bx+a}\,dx$$

input
```
integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*x/(c*x^2 + b*x + a), x)
```

3.528.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 758 vs. $2(241) = 482$.

Time = 0.34 (sec) , antiderivative size = 758, normalized size of antiderivative = 2.64

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{2\sqrt{ex+d}}{c}$$

$$+ \frac{\left(\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}c\right)e}((b^2c-4\,ac^2)d-(b^3-4\,abc)e)c^2e^2-2\left(\sqrt{b^2-4\,ac}c^3d^2-\sqrt{b^2-4\right.\right.}{}$$

$$- \frac{\left(\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}c\right)e}((b^2c-4\,ac^2)d-(b^3-4\,abc)e)c^2e^2+2\left(\sqrt{b^2-4\,ac}c^3d^2-\sqrt{b^2-4\right.\right.}{}$$

input
```
integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
2*sqrt(e*x + d)/c + 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*
((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*c^2*e^2 - 2*(sqrt(b^2 - 4*a*c)*c
^3*d^2 - sqrt(b^2 - 4*a*c)*b*c^2*d*e + sqrt(b^2 - 4*a*c)*a*c^2*e^2)*sqrt(-
4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) + (2*b*c^4*d^2*e
- (3*b^2*c^3 - 4*a*c^4)*d*e^2 + (b^3*c^2 - 2*a*b*c^3)*e^3)*sqrt(-4*c^2*d +
 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-
(2*c^2*d - b*c*e + sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d -
b*c*e)^2))/c^2))/((sqrt(b^2 - 4*a*c)*c^4*d^2 - sqrt(b^2 - 4*a*c)*b*c^3*d*e
 + sqrt(b^2 - 4*a*c)*a*c^3*e^2)*c^2*abs(e)) - 1/4*(sqrt(-4*c^2*d + 2*(b*c
+ sqrt(b^2 - 4*a*c)*c)*e)*((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*c^2*e^
2 + 2*(sqrt(b^2 - 4*a*c)*c^3*d^2 - sqrt(b^2 - 4*a*c)*b*c^2*d*e + sqrt(b^2
- 4*a*c)*a*c^2*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c
)*abs(e) + (2*b*c^4*d^2*e - (3*b^2*c^3 - 4*a*c^4)*d*e^2 + (b^3*c^2 - 2*a*b
*c^3)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt
(1/2)*sqrt(e*x + d)/sqrt(-(2*c^2*d - b*c*e - sqrt(-4*(c^2*d^2 - b*c*d*e +
a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/((sqrt(b^2 - 4*a*c)*c^4*d^2 - s
qrt(b^2 - 4*a*c)*b*c^3*d*e + sqrt(b^2 - 4*a*c)*a*c^3*e^2)*c^2*abs(e))
```

3.528.9 Mupad [B] (verification not implemented)

Time = 13.18 (sec) , antiderivative size = 5664, normalized size of antiderivative = 19.74

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2} \, dx = \text{Too large to display}$$

input
```
int((x*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

output
```
(2*(d + e*x)^(1/2))/c - atan(((((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^
4*d^2*e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c - (8*(d
+ e*x)^(1/2)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*
c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)
^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a
*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5
*d*e^2))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c
*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^
3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2) - (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d
^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3
))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7
*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/
2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^
4)))^(1/2)*1i - (((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^
3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c + (8*(d + e*x)^(1/2)*(
(8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*
c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*
a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1
/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)...
```

3.528. $\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx$

3.529 $\int \frac{\sqrt{d+ex}}{a+bx+cx^2} dx$

3.529.1 Optimal result

Integrand size = 22, antiderivative size = 198

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2} dx = -\frac{\sqrt{2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{c}\sqrt{b^2-4ac}} + \frac{\sqrt{2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}$$

output

```
-arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1
/2))*2^(1/2)*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/c^(1/2)/(-4*a*c+b^2)^(
1/2)+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))
)^(1/2))*2^(1/2)*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/c^(1/2)/(-4*a*c+b^
2)^(1/2)
```

3.529.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{\sqrt{2}\left(\frac{\left(-2icd+\left(ib+\sqrt{-b^2+4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}e}}\right)}{\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}+\frac{\left(2icd+\left(-ib+\sqrt{-b^2+4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be+i\sqrt{-b^2+4ac}e}}\right)}{\sqrt{-2cd+\left(b+i\sqrt{-b^2+4ac}\right)e}}\right)}{\sqrt{c}\sqrt{-b^2+4ac}}$$

input

```
Integrate[Sqrt[d + e*x]/(a + b*x + c*x^2),x]
```

output

```
(Sqrt[2]*((((-2*I)*c*d + (I*b + Sqrt[-b^2 + 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqr
t[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/Sqrt[-2*
c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e] + (((2*I)*c*d + ((-I)*b + Sqrt[-b^2 +
4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sq
rt[-b^2 + 4*a*c]*e]])/Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e]))/(Sqrt[
c]*Sqrt[-b^2 + 4*a*c])
```

3.529.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1148, 1450, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \; 1148$$

$$2e\int \frac{d+ex}{cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex)}d\sqrt{d+ex}$$

$$\Big\downarrow \; 1450$$

$$2e\left(\frac{1}{2}\left(\frac{2cd-be}{e\sqrt{b^2-4ac}}+1\right)\int\frac{1}{\frac{1}{2}\left(\left(b-\sqrt{b^2-4ac}\right)e-2cd\right)+c(d+ex)}d\sqrt{d+ex}+\frac{1}{2}\left(1-\frac{2cd-be}{e\sqrt{b^2-4ac}}\right)\int\frac{}{\frac{1}{2}\left(\left(b\right.\right.}\right.$$

$$\Big\downarrow 221$$

$$2e\left(-\frac{\left(\frac{2cd-be}{e\sqrt{b^2-4ac}}+1\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(1-\frac{2cd-be}{e\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)$$

input
```
Int[Sqrt[d + e*x]/(a + b*x + c*x^2),x]
```

output
```
2*e*(-(((1 + (2*c*d - b*e)/(Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]
*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]
*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])) - ((1 - (2*c*d - b*e)/(Sqrt[b^2
 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sq
rt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]))
```

3.529.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1148
```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[2*e   Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c
*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

3.529. $\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx$

3.529.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$\frac{e\sqrt{2}\left(\frac{\left(be-2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)\arctan\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)c}}\right)}{\sqrt{\left(be-2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)c}}-\frac{\left(-be+2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)\operatorname{arctanh}\left(\frac{}{\sqrt{\left(-be+2\right.}}\right.}{\sqrt{\left(-be+2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right.}\right.}}\right.}{\sqrt{-e^{2}(4ac-b^{2})}}$
derivativedivides	$8ec\left(\frac{\left(be-2cd+\sqrt{-e^{2}(4ac-b^{2})}\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)c}}\right)}{8c\sqrt{-e^{2}(4ac-b^{2})}\sqrt{\left(be-2cd+\sqrt{-e^{2}(4ac-b^{2})}\right)c}}-\frac{\left(-be+2cd+\sqrt{-e^{2}(4ac-b^{2})}\right)\sqrt{2}\,\text{ar}}{8c\sqrt{-e^{2}(4ac-b^{2})}\sqrt{\left(-\right.}}\right.$
default	$8ec\left(\frac{\left(be-2cd+\sqrt{-e^{2}(4ac-b^{2})}\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)c}}\right)}{8c\sqrt{-e^{2}(4ac-b^{2})}\sqrt{\left(be-2cd+\sqrt{-e^{2}(4ac-b^{2})}\right)c}}-\frac{\left(-be+2cd+\sqrt{-e^{2}(4ac-b^{2})}\right)\sqrt{2}\,\text{ar}}{8c\sqrt{-e^{2}(4ac-b^{2})}\sqrt{\left(-\right.}}\right.$

input
```
int((e*x+d)^(1/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
e*2^(1/2)/(-e^2*(4*a*c-b^2))^(1/2)*((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))/(
(b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/
2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-(-b*e+2*c*d+(-e^2*(4*a*
c-b^2))^(1/2))/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(
e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))
```

3.529.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 715 vs. $2(158) = 316$.

Time = 0.52 (sec) , antiderivative size = 715, normalized size of antiderivative = 3.61

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx =$$

$$-\frac{1}{2}\sqrt{2}\sqrt{\frac{2\,cd-be+(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\sqrt{2}(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}\sqrt{\frac{2\,cd-be+(b^2c-4}{b^2c-4}}\right.$$

$$\left.+2\sqrt{ex+d}e\right)$$

$$+\frac{1}{2}\sqrt{2}\sqrt{\frac{2\,cd-be+(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(-\sqrt{2}(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}\sqrt{\frac{2\,cd-be+(b^2c-}{b^2c-}}\right.$$

$$\left.+2\sqrt{ex+d}e\right)$$

$$+\frac{1}{2}\sqrt{2}\sqrt{\frac{2\,cd-be-(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\sqrt{2}(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}\sqrt{\frac{2\,cd-be-(b^2c-4}{b^2c-4}}\right.$$

$$\left.+2\sqrt{ex+d}e\right)$$

$$-\frac{1}{2}\sqrt{2}\sqrt{\frac{2\,cd-be-(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(-\sqrt{2}(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}\sqrt{\frac{2\,cd-be-(b^2c-}{b^2c-}}\right.$$

$$\left.+2\sqrt{ex+d}e\right)$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a
*c^3)))/(b^2*c - 4*a*c^2))*log(sqrt(2)*(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2
 - 4*a*c^3))*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a
*c^3)))/(b^2*c - 4*a*c^2)) + 2*sqrt(e*x + d)*e) + 1/2*sqrt(2)*sqrt((2*c*d
- b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)
)*log(-sqrt(2)*(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3))*sqrt((2*c*d
 - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2
)) + 2*sqrt(e*x + d)*e) + 1/2*sqrt(2)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2
)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log(sqrt(2)*(b^2*c - 4
*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3))*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2
)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) + 2*sqrt(e*x + d)*e) -
 1/2*sqrt(2)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a
*c^3)))/(b^2*c - 4*a*c^2))*log(-sqrt(2)*(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^
2 - 4*a*c^3))*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*
a*c^3)))/(b^2*c - 4*a*c^2)) + 2*sqrt(e*x + d)*e)
```

3.529.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx=\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

output
```
Integral(sqrt(d + e*x)/(a + b*x + c*x**2), x)
```

3.529.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx=\int \frac{\sqrt{ex+d}}{cx^2+bx+a}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(c*x^2 + b*x + a), x)
```

3.529.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(158) = 316$.

Time = 0.32 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

$$= \frac{\left(\sqrt{-4\,c^2d+2\,\left(bc-\sqrt{b^2-4\,ac}c\right)e}(b^2-4\,ac)e^3-\left(4\,c^2d^2e-4\,bcde^2+b^2e^3\right)\sqrt{-4\,c^2d+2\,\left(bc-\sqrt{b^2-4\,ac}\right.}\right.}{4\left(\sqrt{b^2-4\,ac}c^2d^2-\sqrt{b^2-4\,ac}bcde+\sqrt{b^2-4\,ac}ace^2\right)|c|}$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\,\left(bc+\sqrt{b^2-4\,ac}c\right)e}(b^2-4\,ac)e^3-\left(4\,c^2d^2e-4\,bcde^2+b^2e^3\right)\sqrt{-4\,c^2d+2\,\left(bc+\sqrt{b^2-\right.}\right.}{4\left(\sqrt{b^2-4\,ac}c^2d^2-\sqrt{b^2-4\,ac}bcde+\sqrt{b^2-4\,ac}ace^2\right)}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*e^3 -
(4*c^2*d^2*e - 4*b*c*d*e^2 + b^2*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 -
4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c*d - b*e + sqrt((
2*c*d - b*e)^2 - 4*(c*d^2 - b*d*e + a*e^2)*c))/c))/((sqrt(b^2 - 4*a*c)*c^2
*d^2 - sqrt(b^2 - 4*a*c)*b*c*d*e + sqrt(b^2 - 4*a*c)*a*c*e^2)*abs(c)*abs(e
)) - 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*e
^3 - (4*c^2*d^2*e - 4*b*c*d*e^2 + b^2*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b
^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c*d - b*e - s
qrt((2*c*d - b*e)^2 - 4*(c*d^2 - b*d*e + a*e^2)*c))/c))/((sqrt(b^2 - 4*a*c
)*c^2*d^2 - sqrt(b^2 - 4*a*c)*b*c*d*e + sqrt(b^2 - 4*a*c)*a*c*e^2)*abs(c)*
abs(e))
```

3.529.9 Mupad [B] (verification not implemented)

Time = 12.29 (sec) , antiderivative size = 709, normalized size of antiderivative = 3.58

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx =$$

$$-2\,\mathrm{atanh}\left(\frac{2\left(\sqrt{d+e\,x}\,\left(-8\,b^2\,c\,e^4+16\,b\,c^2\,d\,e^3-16\,c^3\,d^2\,e^2+16\,a\,c^2\,e^4\right)+\frac{\sqrt{d+e\,x}\,\left(8\,b^3\,c^2\,e^3-16\,d\,b^2\,c^3\,e^2-32\,a\,b\right.}{}\right.}{16\,c^2\,d^2\,e^3-16}\right.$$

$$-2\,\mathrm{atanh}\left(\frac{2\left(\sqrt{d+e\,x}\,\left(-8\,b^2\,c\,e^4+16\,b\,c^2\,d\,e^3-16\,c^3\,d^2\,e^2+16\,a\,c^2\,e^4\right)-\frac{\sqrt{d+e\,x}\,\left(8\,b^3\,c^2\,e^3-16\,d\,b^2\,c^3\,e^2-32\,a\,b\right.}{}\right.}{16\,c^2\,d^2\,e^3-16\,b}\right.$$

input
```
int((d + e*x)^(1/2)/(a + b*x + c*x^2),x)
```

output
```
- 2*atanh((2*((d + e*x)^(1/2)*(16*a*c^2*e^4 - 8*b^2*c*e^4 - 16*c^3*d^2*e^2
 + 16*b*c^2*d*e^3) + ((d + e*x)^(1/2)*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 -
32*a*b*c^3*e^3 + 64*a*c^4*d*e^2)*(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a
*c^2*d - 2*b^2*c*d - 4*a*b*c*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(
-(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d - 2*b^2*c*d - 4*a*b*c*e)/
(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(16*c^2*d^2*e^3 + 16*a*c*e^
5 - 16*b*c*d*e^4))*(-(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d - 2*b
^2*c*d - 4*a*b*c*e)/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atan
h((2*((d + e*x)^(1/2)*(16*a*c^2*e^4 - 8*b^2*c*e^4 - 16*c^3*d^2*e^2 + 16*b*
c^2*d*e^3) - ((d + e*x)^(1/2)*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 - 32*a*b*c
^3*e^3 + 64*a*c^4*d*e^2)*(e*(-(4*a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d +
 2*b^2*c*d + 4*a*b*c*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((e*(-(4*
a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b*c*e)/(2*(b^4*c
 + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(16*c^2*d^2*e^3 + 16*a*c*e^5 - 16*b*
c*d*e^4))*((e*(-(4*a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4
*a*b*c*e)/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2)
```

3.530 $\int \frac{\sqrt{d+ex}}{x(a+bx+cx^2)}\,dx$

3.530.1 Optimal result

Integrand size = 25, antiderivative size = 275

$$\int \frac{\sqrt{d+ex}}{x\,(a+bx+cx^2)}\,dx = -\frac{2\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} + \frac{\sqrt{2}\sqrt{c}\left(bd+\sqrt{b^2-4ac}d-2ae\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\sqrt{2}\sqrt{c}\left(bd-\sqrt{b^2-4ac}d-2ae\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-2*arctanh((e*x+d)^(1/2)/d^(1/2))*d^(1/2)/a+arctanh(2^(1/2)*c^(1/2)*(e*x+d
)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b*d-2*a*e
+d*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)
))^(1/2)-arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1
/2)))^(1/2))*2^(1/2)*c^(1/2)*(b*d-2*a*e-d*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^
2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.530.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{d+ex}}{x(a+bx+cx^2)}\,dx = -\frac{\frac{\sqrt{2}\sqrt{c}\left(bd+\sqrt{b^2-4ac}d-2ae\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\sqrt{2}\sqrt{c}\left(-bd+\sqrt{b^2-4ac}d+2ae\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}+2\sqrt{d}}{a}$$

input
```
Integrate[Sqrt[d + e*x]/(x*(a + b*x + c*x^2)),x]
```

output
```
-(((Sqrt[2]*Sqrt[c]*(b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcTan[(Sqrt[2]*Sq
rt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2
- 4*a*c]*Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(-(b
*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/
Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d
+ (b + Sqrt[b^2 - 4*a*c])*e]) + 2*Sqrt[d]*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/
a)
```

3.530.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{x(a+bx+cx^2)}\,dx$$

$$\Big\downarrow \; 1199$$

$$\frac{2\int \left(\frac{d}{ax}+\frac{e(cd^2-bed-c(d+ex)d+ae^2)}{a(cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex))}\right)d\sqrt{d+ex}}{e}$$

$$\Big\downarrow \; 2009$$

$$\frac{2\left(\frac{\sqrt{c}e\left(d\sqrt{b^2-4ac}-2ae+bd\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{c}e\left(-d\sqrt{b^2-4ac}-2ae+bd\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d}ea}{}\right.}{e}$$

input
```
Int[Sqrt[d + e*x]/(x*(a + b*x + c*x^2)),x]
```

output
```
(2*(-((Sqrt[d]*e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a) + (Sqrt[c]*e*(b*d + Sq
rt[b^2 - 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*
c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[c]*e*(b*d - Sqrt[b^2 - 4*a*c]*d - 2
*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e])))/e
```

3.530.3.1 Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.530.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.06

method	result
derivativedivides	$2e^2\left(\frac{4c\left(\frac{\left(-2e^2a+bde-\sqrt{-e^2\left(4ac-b^2\right)}\,d\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{8\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}-\frac{\left(2e^2a-bde-\sqrt{-e^2\left(4ac-b^2\right)}\,d\right)\sqrt{2}}{8\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{(}}\right.}{a\,e^2}\right.$
default	$2e^2\left(\frac{4c\left(\frac{\left(-2e^2a+bde-\sqrt{-e^2\left(4ac-b^2\right)}\,d\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{8\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}-\frac{\left(2e^2a-bde-\sqrt{-e^2\left(4ac-b^2\right)}\,d\right)\sqrt{2}}{8\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{(}}\right.}{a\,e^2}\right.$
pseudoelliptic	$-\frac{2\left(\sqrt{2}\,\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,c\left(e^2a-\frac{bde}{2}-\frac{\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\,d}{2}\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}}\right.\right.}{\sqrt{}}$

input
```
int((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*e^2*(4/a/e^2*c*(1/8*(-2*e^2*a+b*d*e-(-e^2*(4*a*c-b^2))^(1/2)*d)/(-e^2*(4
*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*ar
ctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2
))-1/8*(2*e^2*a-b*d*e-(-e^2*(4*a*c-b^2))^(1/2)*d)/(-e^2*(4*a*c-b^2))^(1/2)
*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)
^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))-d^(1/2)/a
/e^2*arctanh((e*x+d)^(1/2)/d^(1/2)))
```

3.530.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1220 vs. $2(225) = 450$.

Time = 0.54 (sec) , antiderivative size = 2446, normalized size of antiderivative = 8.89

$$\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt(
(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))
*log(sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b
*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*
e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*
e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e
*x + d)) - sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*
sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^
3*c))*log(-sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4
*a^3*b*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(
-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e
+ a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*
sqrt(e*x + d)) + sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a
^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2
- 4*a^3*c))*log(sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^
3 - 4*a^3*b*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*
sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b
*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*
c*e)*sqrt(e*x + d)) - sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2
- 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(...
```

3.530.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx = \int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx$$

input
```
integrate((e*x+d)**(1/2)/x/(c*x**2+b*x+a),x)
```

output
```
Integral(sqrt(d + e*x)/(x*(a + b*x + c*x**2)), x)
```

3.530.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx=\int \frac{\sqrt{ex+d}}{(cx^2+bx+a)x}\,dx$$

input
```
integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*x), x)
```

3.530.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 719 vs. 2(225) = 450.

Time = 0.32 (sec) , antiderivative size = 719, normalized size of antiderivative = 2.61

$$\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx=\frac{2\,d\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{a\sqrt{-d}}$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}c\right)e}(b^2-4\,ac)a^2de^2-2\left(\sqrt{b^2-4\,ac}acd^2-\sqrt{b^2-4\,ac}abde+\sqrt{b^2-4\,a\right.\right.}{}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}c\right)e}(b^2-4\,ac)a^2de^2+2\left(\sqrt{b^2-4\,ac}acd^2-\sqrt{b^2-4\,ac}abde+\sqrt{b^2-4\,a\right.\right.}{}$$

input
```
integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
2*d*arctan(sqrt(e*x + d)/sqrt(-d))/(a*sqrt(-d)) - 1/4*(sqrt(-4*c^2*d + 2*(
b*c - sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*a^2*d*e^2 - 2*(sqrt(b^2 - 4*a*
c)*a*c*d^2 - sqrt(b^2 - 4*a*c)*a*b*d*e + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-
4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(a)*abs(e) - (2*a^2*b*c*d^2*
e + 2*a^3*b*e^3 - (a^2*b^2 + 4*a^3*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt
(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a*c*d - a*b
*e + sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*
c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^
2 - 4*a*c)*a^3*e^2)*abs(a)*abs(c)*abs(e)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c +
sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*a^2*d*e^2 + 2*(sqrt(b^2 - 4*a*c)*a*c
*d^2 - sqrt(b^2 - 4*a*c)*a*b*d*e + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-4*c^2*
d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(a)*abs(e) - (2*a^2*b*c*d^2*e + 2*
a^3*b*e^3 - (a^2*b^2 + 4*a^3*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 -
 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a*c*d - a*b*e - s
qrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/(
(sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*
a*c)*a^3*e^2)*abs(a)*abs(c)*abs(e))
```

3.530.9 Mupad [B] (verification not implemented)

Time = 16.58 (sec) , antiderivative size = 10894, normalized size of antiderivative = 39.61

$$\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(1/2)/(x*(a + b*x + c*x^2)),x)
```

output
```
- atan(((((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) -
b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16
*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-
(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^
2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*
((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*
a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2
- 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b
^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c
^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*
d*e^9) - 384*a^4*c^4*d*e^10 - 384*a^3*c^5*d^3*e^8 + 96*a^2*b^2*c^4*d^3*e^8
 - 96*a^2*b^3*c^3*d^2*e^9 + 384*a^3*b*c^4*d^2*e^9 + 96*a^3*b^2*c^3*d*e^10)
 - (d + e*x)^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c
^2*e^11 + 576*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 +
64*a*b^4*c^2*d*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*
a^2*b*c^4*d^2*e^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3
*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2
*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 96*a
*c^5*d^4*e^8 + 96*a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e
^10 + 64*a*b*c^4*d^3*e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 -...
```

3.530. $\int \frac{\sqrt{d+ex}}{x(a+bx+cx^2)}\,dx$

3.531 $\int \frac{\sqrt{d+ex}}{x^2(a+bx+cx^2)}\,dx$

3.531.1 Optimal result

Integrand size = 25, antiderivative size = 368

$$\int \frac{\sqrt{d+ex}}{x^2(a+bx+cx^2)}\,dx = -\frac{\sqrt{d+ex}}{ax}+\frac{e\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a\sqrt{d}}+\frac{2(bd-ae)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}}-\frac{\sqrt{2}\sqrt{c}\left(b^2d-2acd-abe+\sqrt{b^2-4ac}(bd-ae)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\sqrt{2}\sqrt{c}\left(b^2d-b\left(\sqrt{b^2-4ac}d+ae\right)-a\left(2cd-\sqrt{b^2-4ac}e\right)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
e*arctanh((e*x+d)^(1/2)/d^(1/2))/a/d^(1/2)+2*(-a*e+b*d)*arctanh((e*x+d)^(1
/2)/d^(1/2))/a^2/d^(1/2)-(e*x+d)^(1/2)/a/x-arctanh(2^(1/2)*c^(1/2)*(e*x+d)
^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b^2*d-2*a*
c*d-a*b*e+(-a*e+b*d)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(
b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-
e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b^2*d-2*a*c*d-a*b*e-(-a*
e+b*d)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)
^(1/2)))^(1/2)
```

3.531.2 Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx$$

$$= \frac{-\frac{a\sqrt{d+ex}}{x}+\frac{\sqrt{2}\sqrt{c}\left(b^2d-2acd+b\sqrt{b^2-4ac}d-abe-a\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\sqrt{2}\sqrt{c}\left(-b^2d+2acd+b\sqrt{b^2-4ac}d+abe-a\right.}{\sqrt{b^2-4ac}\sqrt{-2}}}{a^2}$$

input
```
Integrate[Sqrt[d + e*x]/(x^2*(a + b*x + c*x^2)),x]
```

output
```
(-((a*Sqrt[d + e*x])/x) + (Sqrt[2]*Sqrt[c]*(b^2*d - 2*a*c*d + b*Sqrt[b^2 -
 4*a*c]*d - a*b*e - a*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d
+ e*x])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt
[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(-(b^2*d) + 2*a*c
*d + b*Sqrt[b^2 - 4*a*c]*d + a*b*e - a*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[
b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]) + ((2*b*d - a*e)*Ar
cTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d])/a^2
```

3.531.3 Rubi [A] (verified)

Time = 2.40 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx$$

$$\Big\downarrow \text{ 1199}$$

$$\frac{2\int \left(\frac{d}{ax^2}-\frac{bd-ae}{a^2x}-\frac{e\left(b\left(cd^2-bed+ae^2\right)-c(bd-ae)(d+ex)\right)}{a^2\left(cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex)\right)}\right)d\sqrt{d+ex}}{e}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2\left(-\frac{\sqrt{c}e\left(\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}e\left(-\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right)\operatorname{arctanh}\left(\ldots\right.}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\ldots\right.}}\ldots\right.}{e}$$

input
```
Int[Sqrt[d + e*x]/(x^2*(a + b*x + c*x^2)),x]
```

output
```
(2*(-1/2*(e*Sqrt[d + e*x])/(a*x) + (e^2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(2
*a*Sqrt[d]) + (e*(b*d - a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^2*Sqrt[d])
 - (Sqrt[c]*e*(b^2*d - 2*a*c*d - a*b*e + Sqrt[b^2 - 4*a*c]*(b*d - a*e))*Ar
cTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e]) + (Sqrt[c]*e*(b^2*d - 2*a*c*d - a*b*e - Sqrt[b^2 - 4*a*c]*(b*d - a*e))
*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e])))/e
```

3.531.3.1 Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.531.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.01

method	result
derivativedivides	$2e^{3}\left(\frac{-\frac{a\sqrt{ex+d}}{2x}-\frac{(ae-2bd)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{2\sqrt{d}}}{a^{2}e^{3}}+\frac{4c\left(-\frac{\left(-ab\,e^{2}-2acde+b^{2}de-\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,ae+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,bd\right)\sqrt{2}\,a\cdots}{8\sqrt{-e^{2}\left(4ac-b^{2}\right)}\sqrt{\left(-be+2cd+\sqrt{-e^{2}\cdots}\right.}}\cdots\right.}{\cdots}\cdots\right.$
default	$2e^{3}\left(\frac{-\frac{a\sqrt{ex+d}}{2x}-\frac{(ae-2bd)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{2\sqrt{d}}}{a^{2}e^{3}}+\frac{4c\left(-\frac{\left(-ab\,e^{2}-2acde+b^{2}de-\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,ae+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,bd\right)\sqrt{2}\,a\cdots}{8\sqrt{-e^{2}\left(4ac-b^{2}\right)}\sqrt{\left(-be+2cd+\sqrt{-e^{2}\cdots}\right.}}\cdots\right.}{\cdots}\cdots\right.$
risch	$-\frac{\sqrt{ex+d}}{ax}-\frac{e\left(-\frac{(-ae+2bd)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{ae\sqrt{d}}+\frac{8c\left(-\frac{\left(ab\,e^{2}+2acde-b^{2}de+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,ae-\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,bd\right)\sqrt{2}\,\mathrm{arct}\cdots}{8\sqrt{-e^{2}\left(4ac-b^{2}\right)}\sqrt{\left(-be+2cd+\sqrt{-e^{2}\left(4\cdots\right.}\right.}}\cdots\right.}{\cdots}\cdots\right)}{\cdots}$
pseudoelliptic	$\frac{2\sqrt{\left(be-2cd+\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}\right)c}\,\sqrt{2}\left(\frac{\left(\sqrt{d}\,ea-b\,d^{\frac{3}{2}}\right)\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}}{2}+\left(\left(ac-\frac{b^{2}}{2}\right)d^{\frac{3}{2}}+\frac{b\sqrt{d}\,ea}{2}\right)e\right)xc\operatorname{arctanh}\left(\frac{\cdots}{\sqrt{\left(-\cdots\right.}}\right.\cdots}{\cdots}$

input
```
int((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*e^3*(1/a^2/e^3*(-1/2*a*(e*x+d)^(1/2)/x-1/2*(a*e-2*b*d)/d^(1/2)*arctanh((
e*x+d)^(1/2)/d^(1/2)))+4/a^2/e^3*c*(-1/8*(-a*b*e^2-2*a*c*d*e+b^2*d*e-(-e^2
*(4*a*c-b^2))^(1/2)*a*e+(-e^2*(4*a*c-b^2))^(1/2)*b*d)/(-e^2*(4*a*c-b^2))^(
1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*
x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+1/8*(a
*b*e^2+2*a*c*d*e-b^2*d*e-(-e^2*(4*a*c-b^2))^(1/2)*a*e+(-e^2*(4*a*c-b^2))^(
1/2)*b*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^
(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^
2))^(1/2))*c)^(1/2))))
```

3.531.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2427 vs. $2(298) = 596$.

Time = 6.03 (sec) , antiderivative size = 4860, normalized size of antiderivative = 13.21

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a
^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^
2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a
^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(sqrt(2)*((b^6
- 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e - (
a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)
*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c
+ a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*
d - (a*b^3 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4
*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 -
 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) +
4*((b^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) - sqr
t(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e
 + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*
b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e
^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-sqrt(2)*((b^6 - 6*a*b^
4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e - (a^4*b^4
- 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2
*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^
2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (...
```

3.531.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx=\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx$$

input
```
integrate((e*x+d)**(1/2)/x**2/(c*x**2+b*x+a),x)
```

output
```
Integral(sqrt(d + e*x)/(x**2*(a + b*x + c*x**2)), x)
```

3.531.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx=\int \frac{\sqrt{ex+d}}{(cx^2+bx+a)x^2}\,dx$$

input
```
integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*x^2), x)
```

3.531.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 776 vs. 2(298) = 596.

Time = 0.34 (sec) , antiderivative size = 776, normalized size of antiderivative = 2.11

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx=-\frac{(2\,bd-ae)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{a^2\sqrt{-d}}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\,\left(bc-\sqrt{b^2-4\,ac}c\right)e}((b^3-4\,abc)d-(ab^2-4\,a^2c)e)e^2-2\,\left(\sqrt{b^2-4\,ac}bcd^2-\sqrt{b^2-4\,ac}\right.\right.}{}$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\,\left(bc+\sqrt{b^2-4\,ac}c\right)e}((b^3-4\,abc)d-(ab^2-4\,a^2c)e)e^2+2\,\left(\sqrt{b^2-4\,ac}bcd^2-\sqrt{b^2-4\,ac}\right.\right.}{}$$

$$-\frac{\sqrt{ex+d}}{ax}$$

input
```
integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-(2*b*d - a*e)*arctan(sqrt(e*x + d)/sqrt(-d))/(a^2*sqrt(-d)) + 1/4*(sqrt(-
4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4
*a^2*c)*e)*e^2 - 2*(sqrt(b^2 - 4*a*c)*b*c*d^2 - sqrt(b^2 - 4*a*c)*b^2*d*e
+ sqrt(b^2 - 4*a*c)*a*b*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)
*e)*abs(e) + (b^3*d*e^2 - a*b^2*e^3 - 2*(b^2*c - 2*a*c^2)*d^2*e)*sqrt(-4*c
^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/
sqrt(-(2*a^2*c*d - a^2*b*e + sqrt(-4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*a^2
*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - s
qrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(c)*abs(e)) - 1
/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^3 - 4*a*b*c)*d -
(a*b^2 - 4*a^2*c)*e)*e^2 + 2*(sqrt(b^2 - 4*a*c)*b*c*d^2 - sqrt(b^2 - 4*a*c
)*b^2*d*e + sqrt(b^2 - 4*a*c)*a*b*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 -
 4*a*c)*c)*e)*abs(e) + (b^3*d*e^2 - a*b^2*e^3 - 2*(b^2*c - 2*a*c^2)*d^2*e)
*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt
(e*x + d)/sqrt(-(2*a^2*c*d - a^2*b*e - sqrt(-4*(a^2*c*d^2 - a^2*b*d*e + a^
3*e^2)*a^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/((sqrt(b^2 - 4*a*c)*a^2
*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(c)*a
bs(e)) - sqrt(e*x + d)/(a*x)
```

3.531.9 Mupad [B] (verification not implemented)

Time = 16.14 (sec) , antiderivative size = 19887, normalized size of antiderivative = 54.04

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(1/2)/(x^2*(a + b*x + c*x^2)),x)
```

output
```
(atan((((a*e - 2*b*d)*((8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*
e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 1
8*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4 - ((
a*e - 2*b*d)*((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12
 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a
^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84
*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^
7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*
c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 - ((a*e -
 2*b*d)*((8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4
*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d
^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d
^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2
*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d
*e^10))/a^4 - (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*
e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9
+ 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32
*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 - (4*(a*e - 2*b*d)*(d + e
*x)^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96
*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112...
```

3.531. $\int \frac{\sqrt{d+ex}}{x^2(a+bx+cx^2)}\,dx$

3.532 $\int \frac{\sqrt{d+ex}}{x^3(a+bx+cx^2)}\,dx$

3.532.1 Optimal result

Integrand size = 25, antiderivative size = 531

$$\int \frac{\sqrt{d+ex}}{x^3(a+bx+cx^2)}\,dx = -\frac{\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4adx} + \frac{(bd-ae)\sqrt{d+ex}}{a^2dx}-\frac{3e^2\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4ad^{3/2}} - \frac{e(bd-ae)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2d^{3/2}}-\frac{2(b^2d-acd-abe)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^3\sqrt{d}} + \frac{\sqrt{2}\sqrt{c}\left(b^3d-ac\left(\sqrt{b^2-4ac}\,d-2ae\right)+b^2\left(\sqrt{b^2-4ac}\,d-ae\right)-ab\left(3cd+\sqrt{b^2-4ac}\,e\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}}{\sqrt{2cd-(}}\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\sqrt{2}\sqrt{c}\left(b^3d-b^2\left(\sqrt{b^2-4ac}\,d+ae\right)+ac\left(\sqrt{b^2-4ac}\,d+2ae\right)-ab\left(3cd-\sqrt{b^2-4ac}\,e\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}}{\sqrt{2cd-(}}\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-3/4*e^2*arctanh((e*x+d)^(1/2)/d^(1/2))/a/d^(3/2)-e*(-a*e+b*d)*arctanh((e*
x+d)^(1/2)/d^(1/2))/a^2/d^(3/2)-2*(-a*b*e-a*c*d+b^2*d)*arctanh((e*x+d)^(1/
2)/d^(1/2))/a^3/d^(1/2)-1/2*(e*x+d)^(1/2)/a/x^2+3/4*e*(e*x+d)^(1/2)/a/d/x+
(-a*e+b*d)*(e*x+d)^(1/2)/a^2/d/x+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*
c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b^3*d-a*c*(-2*a*e+d*
(-4*a*c+b^2)^(1/2))+b^2*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*b*(3*c*d+e*(-4*a*c+b
^2)^(1/2)))/a^3/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-
arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/
2))*2^(1/2)*c^(1/2)*(b^3*d-b^2*(a*e+d*(-4*a*c+b^2)^(1/2))+a*c*(2*a*e+d*(-4
*a*c+b^2)^(1/2))-a*b*(3*c*d-e*(-4*a*c+b^2)^(1/2)))/a^3/(-4*a*c+b^2)^(1/2)/
(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.532.2 Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx$$
$$= \frac{\frac{a\sqrt{d+ex}(4bdx-a(2d+ex))}{dx^2} + \frac{4\sqrt{2}\sqrt{c}\left(-b^3d+ac\left(\sqrt{b^2-4ac}d-2ae\right)+b^2\left(-\sqrt{b^2-4ac}d+ae\right)+ab\left(3cd+\sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}}}\right.}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}}{}$$

input
```
Integrate[Sqrt[d + e*x]/(x^3*(a + b*x + c*x^2)),x]
```

output
```
((a*Sqrt[d + e*x]*(4*b*d*x - a*(2*d + e*x)))/(d*x^2) + (4*Sqrt[2]*Sqrt[c]*
(-(b^3*d) + a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(-(Sqrt[b^2 - 4*a*c]*d
) + a*e) + a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt
[d + e*x])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*S
qrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (4*Sqrt[2]*Sqrt[c]*(b^3*d - b^2
*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) + a*b*(-3
*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-
2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b +
 Sqrt[b^2 - 4*a*c])*e]) + ((-8*b^2*d^2 + 4*a*b*d*e + a*(8*c*d^2 + a*e^2))*
ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/d^(3/2))/(4*a^3)
```

3.532.3 Rubi [A] (verified)

Time = 2.07 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx$$

$$\Big\downarrow \; 1199$$

$$\frac{2\int \left(\frac{d}{ax^3}+\frac{db^2-aeb-acd}{a^3x}+\frac{e\left(\left(b^2-ac\right)\left(cd^2-bed+ae^2\right)-c\left(db^2-aeb-acd\right)(d+ex)\right)}{a^3\left(cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex)\right)}-\frac{bd-ae}{a^2x^2}\right)d\sqrt{d+ex}}{e}$$

$$\Big\downarrow \; 2009$$

$$2\left(-\frac{e\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(-abe-acd+b^2d\right)}{a^3\sqrt{d}}+\frac{\sqrt{c}e\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)-ac\left(d\sqrt{b^2-4ac}-2ae\right)+b^3d\right)\text{arctanh}\left(\frac{\sqrt{}}{\sqrt{2cd-}}\right.}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right.$$

input
```
Int[Sqrt[d + e*x]/(x^3*(a + b*x + c*x^2)),x]
```

output
```
(2*(-1/4*(e*Sqrt[d + e*x])/(a*x^2) + (3*e^2*Sqrt[d + e*x])/(8*a*d*x) + (e*
(b*d - a*e)*Sqrt[d + e*x])/(2*a^2*d*x) - (3*e^3*ArcTanh[Sqrt[d + e*x]/Sqrt
[d]])/(8*a*d^(3/2)) - (e^2*(b*d - a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(2*
a^2*d^(3/2)) - (e*(b^2*d - a*c*d - a*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/
(a^3*Sqrt[d]) + (Sqrt[c]*e*(b^3*d - a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^
2*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh
[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])
/(Sqrt[2]*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) -
 (Sqrt[c]*e*(b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a
*c]*d + 2*a*e) - a*b*(3*c*d - Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[
c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^3*S
qrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])))/e
```

3.532.3.1 Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.532.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 489, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{\sqrt{ex+d}\,(aex-4bdx+2ad)}{4d\,a^2x^2}-\frac{e\left(-\frac{\left(e^2a^2+4abde+8c\,d^2a-8b^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{ea\sqrt{d}}-\frac{32dc\left(-\frac{\left(-2a^2c\,e^2+a\,b^2e^2+3abcde-\right.}{}\right.}{}\right.}{}$
derivativedivides	$2e^4\left(-\frac{\frac{\frac{ae(ae-4bd)(ex+d)^{\frac{3}{2}}}{8d}+\left(\frac{1}{2}abde+\frac{1}{8}e^2a^2\right)\sqrt{ex+d}}{e^2x^2}-\frac{\left(e^2a^2+4abde+8c\,d^2a-8b^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{8d^{\frac{3}{2}}}}{a^3e^4}+\frac{4c\left(-\frac{\left(-2a^2c\right.}{}\right.}{}\right.$
default	$2e^4\left(-\frac{\frac{\frac{ae(ae-4bd)(ex+d)^{\frac{3}{2}}}{8d}+\left(\frac{1}{2}abde+\frac{1}{8}e^2a^2\right)\sqrt{ex+d}}{e^2x^2}-\frac{\left(e^2a^2+4abde+8c\,d^2a-8b^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{8d^{\frac{3}{2}}}}{a^3e^4}+\frac{4c\left(-\frac{\left(-2a^2c\right.}{}\right.}{}\right.$
pseudoelliptic	$-\frac{-8\left(\frac{\left(-a\,d^{\frac{3}{2}}be-d^{\frac{5}{2}}\left(ac-b^2\right)\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}}{2}+\left(ae\left(ac-\frac{b^2}{2}\right)d^{\frac{3}{2}}-\frac{3b\,d^{\frac{5}{2}}\left(ac-\frac{b^2}{3}\right)}{2}\right)e\right)\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\right.}\right.}}{}$

input
```
int((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

3.532. $\int \frac{\sqrt{d+ex}}{x^3(a+bx+cx^2)}\,dx$

output
```
-1/4*(e*x+d)^(1/2)*(a*e*x-4*b*d*x+2*a*d)/d/a^2/x^2-1/4/a^2/d*e*(-1/e*(a^2*
e^2+4*a*b*d*e+8*a*c*d^2-8*b^2*d^2)/a/d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2)
)-32*d/a/e*c*(-1/8*(-2*a^2*c*e^2+a*b^2*e^2+3*a*b*c*d*e-b^3*d*e+(-e^2*(4*a*
c-b^2))^(1/2)*a*b*e+(-e^2*(4*a*c-b^2))^(1/2)*a*c*d-(-e^2*(4*a*c-b^2))^(1/2
)*b^2*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^
(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2))+1/8*(2*a^2*c*e^2-a*b^2*e^2-3*a*b*c*d*e+b^3*d*e+(-e^
2*(4*a*c-b^2))^(1/2)*a*b*e+(-e^2*(4*a*c-b^2))^(1/2)*a*c*d-(-e^2*(4*a*c-b^2
))^(1/2)*b^2*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2))))
```

3.532.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3710 vs. $2(450) = 900$.

Time = 104.00 (sec) , antiderivative size = 7425, normalized size of antiderivative = 13.98

$$\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.532.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx=\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx$$

input
```
integrate((e*x+d)**(1/2)/x**3/(c*x**2+b*x+a),x)
```

output
```
Integral(sqrt(d + e*x)/(x**3*(a + b*x + c*x**2)), x)
```

3.532.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx = \int \frac{\sqrt{ex+d}}{(cx^2+bx+a)x^3}\,dx$$

input
```
integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*x^3), x)
```

3.532.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1043 vs. $2(450) = 900$.

Time = 0.36 (sec) , antiderivative size = 1043, normalized size of antiderivative = 1.96

$$\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx =$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}\right)e}\left((b^4-5\,ab^2c+4\,a^2c^2)d-(ab^3-4\,a^2bc)e\right)a^2e^2-2\left((ab^2c-a^2c^2)\sqrt{\right.\right.}{}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}\right)e}\left((b^4-5\,ab^2c+4\,a^2c^2)d-(ab^3-4\,a^2bc)e\right)a^2e^2+2\left((ab^2c-a^2c^2)\sqrt{\right.\right.}{}$$

$$+\frac{(8\,b^2d^2-8\,acd^2-4\,abde-a^2e^2)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{4\,a^3\sqrt{-d}d}$$

$$+\frac{4\,(ex+d)^{\frac{3}{2}}bde-4\,\sqrt{ex+d}bd^2e-(ex+d)^{\frac{3}{2}}ae^2-\sqrt{ex+d}ade^2}{4\,a^2de^2x^2}$$

input
```
integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*b^2*c +
 4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e)*a^2*e^2 - 2*((a*b^2*c - a^2*c^2)*sq
rt(b^2 - 4*a*c)*d^2 - (a*b^3 - a^2*b*c)*sqrt(b^2 - 4*a*c)*d*e + (a^2*b^2 -
 a^3*c)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*
c)*e)*abs(a)*abs(e) - (2*(a^2*b^3*c - 3*a^3*b*c^2)*d^2*e - (a^2*b^4 - a^3*
b^2*c - 4*a^4*c^2)*d*e^2 + (a^3*b^3 - 2*a^4*b*c)*e^3)*sqrt(-4*c^2*d + 2*(b
*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a^
3*c*d - a^3*b*e + sqrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*a^3*c + (2*a^3
*c*d - a^3*b*e)^2))/(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c*d^2 - sqrt(b^2 - 4
*a*c)*a^4*b*d*e + sqrt(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(c)*abs(e)) + 1/4*(
sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*b^2*c + 4*a^2
*c^2)*d - (a*b^3 - 4*a^2*b*c)*e)*a^2*e^2 + 2*((a*b^2*c - a^2*c^2)*sqrt(b^2
 - 4*a*c)*d^2 - (a*b^3 - a^2*b*c)*sqrt(b^2 - 4*a*c)*d*e + (a^2*b^2 - a^3*c
)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*
abs(a)*abs(e) - (2*(a^2*b^3*c - 3*a^3*b*c^2)*d^2*e - (a^2*b^4 - a^3*b^2*c
- 4*a^4*c^2)*d*e^2 + (a^3*b^3 - 2*a^4*b*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c + s
qrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a^3*c*d
- a^3*b*e - sqrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*a^3*c + (2*a^3*c*d -
 a^3*b*e)^2))/(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c*d^2 - sqrt(b^2 - 4*a*c)*
a^4*b*d*e + sqrt(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(c)*abs(e)) + 1/4*(8*b...
```

3.532.9 Mupad [B] (verification not implemented)

Time = 17.85 (sec) , antiderivative size = 33838, normalized size of antiderivative = 63.73

$$\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(1/2)/(x^3*(a + b*x + c*x^2)),x)
```

output
```
atan(((((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e
^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^
3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c
^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a
^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 38
4*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) - ((d + e*x)^(
1/2)*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33
*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c
- b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*
b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*
c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(
2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^8 + 10
24*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9
 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d
^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*d^2
))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a
^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c -
b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^
5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^
2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/...
```

3.532. $\int \frac{\sqrt{d+ex}}{x^3(a+bx+cx^2)}\,dx$

3.533 $\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2} \, dx$

3.533.1 Optimal result

Integrand size = 25, antiderivative size = 650

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2} \, dx = -\frac{2(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e)\sqrt{d+ex}}{c^5}$$

$$-\frac{2b(b^2-2ac)(d+ex)^{3/2}}{3c^4}+\frac{2(c^2d^2+b^2e^2+ce(bd-ae))(d+ex)^{5/2}}{5c^3e^3}$$

$$-\frac{2(2cd+be)(d+ex)^{7/2}}{7c^2e^3}+\frac{2(d+ex)^{9/2}}{9ce^3}$$

$$+\frac{\sqrt{2}\Big((bcd-b^2e+ace)(b^2cd-2ac^2d-b^3e+3abce)+\frac{2b^5cde-10ab^3c^2de+10a^2bc^3de-b^6e^2+ab^2c^2(4cd^2-9ae^2)-b^4c(cd^2-6\cdots}{\sqrt{b^2-4ac}}\cdots}{c^{11/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\cdots}$$

$$+\frac{\sqrt{2}\Big((bcd-b^2e+ace)(b^2cd-2ac^2d-b^3e+3abce)-\frac{2b^5cde-10ab^3c^2de+10a^2bc^3de-b^6e^2+ab^2c^2(4cd^2-9ae^2)-b^4c(cd^2-6\cdots}{\sqrt{b^2-4ac}}\cdots}{c^{11/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\cdots}$$

output
```
-2/3*b*(-2*a*c+b^2)*(e*x+d)^(3/2)/c^4+2/5*(c^2*d^2+b^2*e^2+c*e*(-a*e+b*d))
*(e*x+d)^(5/2)/c^3/e^3-2/7*(b*e+2*c*d)*(e*x+d)^(7/2)/c^2/e^3+2/9*(e*x+d)^(
9/2)/c/e^3-2*(-a^2*c^2*e+3*a*b^2*c*e-2*a*b*c^2*d-b^4*e+b^3*c*d)*(e*x+d)^(1
/2)/c^5+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/
2)))^(1/2))*2^(1/2)*((a*c*e-b^2*e+b*c*d)*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*
d)+(2*b^5*c*d*e-10*a*b^3*c^2*d*e+10*a^2*b*c^3*d*e-b^6*e^2+a*b^2*c^2*(-9*a*
e^2+4*c*d^2)-b^4*c*(-6*a*e^2+c*d^2)-2*a^2*c^3*(-a*e^2+c*d^2))/(-4*a*c+b^2)
^(1/2))/c^(11/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^
(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*((a*c*
e-b^2*e+b*c*d)*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)+(-2*b^5*c*d*e+10*a*b^3*
c^2*d*e-10*a^2*b*c^3*d*e+b^6*e^2-a*b^2*c^2*(-9*a*e^2+4*c*d^2)+b^4*c*(-6*a*
e^2+c*d^2)+2*a^2*c^3*(-a*e^2+c*d^2))/(-4*a*c+b^2)^(1/2))/c^(11/2)/(2*c*d-e
*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.533.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.09 (sec) , antiderivative size = 901, normalized size of antiderivative = 1.39

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\sqrt{d+ex}\big(315b^4e^4-105b^2ce^3(4bd+9ae+bex)-9c^3e(d+ex)^2(-2bd+7ae+5bex)}{3}$$

$$-\frac{\big(ib^6e^2+b^5e\big(-2icd+\sqrt{-b^2+4ac}e\big)+ib^4c\big(cd^2+2i\sqrt{-b^2+4ac}de-6ae^2\big)+ab^2c^2\big(-4icd^2+6\sqrt{-b^2+4}}{}$$

$$-\frac{\big(-ib^6e^2+b^5e\big(2icd+\sqrt{-b^2+4ac}e\big)+b^4c\big(-icd^2-2\sqrt{-b^2+4ac}de+6iae^2\big)+ab^2c^2\big(4icd^2+6\sqrt{-b^2+4}}{}$$

input
```
Integrate[(x^4*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

output
```
(2*Sqrt[d + e*x]*(315*b^4*e^4 - 105*b^2*c*e^3*(4*b*d + 9*a*e + b*e*x) - 9*
c^3*e*(d + e*x)^2*(-2*b*d + 7*a*e + 5*b*e*x) + c^4*(d + e*x)^2*(8*d^2 - 20
*d*e*x + 35*e^2*x^2) + 21*c^2*e^2*(15*a^2*e^2 + 3*b^2*(d + e*x)^2 + 10*a*b
*e*(4*d + e*x))))/(315*c^5*e^3) - ((I*b^6*e^2 + b^5*e*((-2*I)*c*d + Sqrt[-
b^2 + 4*a*c]*e) + I*b^4*c*(c*d^2 + (2*I)*Sqrt[-b^2 + 4*a*c]*d*e - 6*a*e^2)
 + a*b^2*c^2*((-4*I)*c*d^2 + 6*Sqrt[-b^2 + 4*a*c]*d*e + (9*I)*a*e^2) + a*b
*c^2*(3*a*Sqrt[-b^2 + 4*a*c]*e^2 - 2*c*d*(Sqrt[-b^2 + 4*a*c]*d + (5*I)*a*e
)) + b^3*c*(-4*a*Sqrt[-b^2 + 4*a*c]*e^2 + c*d*(Sqrt[-b^2 + 4*a*c]*d + (10*
I)*a*e)) - (2*I)*a^2*c^3*(-(c*d^2) + e*((-I)*Sqrt[-b^2 + 4*a*c]*d + a*e)))
*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 +
4*a*c]*e]])/(c^(11/2)*Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^
2 + 4*a*c])*e]) - (((-I)*b^6*e^2 + b^5*e*((2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e
) + b^4*c*((-I)*c*d^2 - 2*Sqrt[-b^2 + 4*a*c]*d*e + (6*I)*a*e^2) + a*b^2*c^
2*((4*I)*c*d^2 + 6*Sqrt[-b^2 + 4*a*c]*d*e - (9*I)*a*e^2) + a*b*c^2*(3*a*Sq
rt[-b^2 + 4*a*c]*e^2 - 2*c*d*(Sqrt[-b^2 + 4*a*c]*d - (5*I)*a*e)) + b^3*c*(
-4*a*Sqrt[-b^2 + 4*a*c]*e^2 + c*d*(Sqrt[-b^2 + 4*a*c]*d - (10*I)*a*e)) + (
2*I)*a^2*c^3*(-(c*d^2) + e*(I*Sqrt[-b^2 + 4*a*c]*d + a*e)))*ArcTan[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(c^
(11/2)*Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e])
```

3.533.3 Rubi [A] (verified)

Time = 2.19 (sec) , antiderivative size = 659, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1199}$$

$$\frac{2\int \left(\frac{(d+ex)^4}{ce^2}-\frac{(2cd+be)(d+ex)^3}{c^2e^2}+\frac{(c^2d^2+b^2e^2+ce(bd-ae))(d+ex)^2}{c^3e^2}-\frac{b(b^2-2ac)e(d+ex)}{c^4}-\frac{e(-eb^4+cdb^3+3aceb^2-2ac^2db-a^2c^2e)}{c^5}+\right.}{e}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{e\left(\frac{10a^2bc^3de-2a^2c^3\left(cd^2-ae^2\right)-b^4c\left(cd^2-6ae^2\right)-10ab^3c^2de+ab^2c^2\left(4cd^2-9ae^2\right)+b^6\left(-e^2\right)+2b^5cde}{\sqrt{b^2-4ac}}+\left(ace+b^2(-e)+bcd\right)\left(3abce-2ac^2d+b^3(-e)+b^2\right.\right.}{\sqrt{2}c^{11/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right.$$

input
```
Int[(x^4*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

output
```
(2*(-((e*(b^3*c*d - 2*a*b*c^2*d - b^4*e + 3*a*b^2*c*e - a^2*c^2*e)*Sqrt[d
+ e*x])/c^5) - (b*(b^2 - 2*a*c)*e*(d + e*x)^(3/2))/(3*c^4) + ((c^2*d^2 + b
^2*e^2 + c*e*(b*d - a*e))*(d + e*x)^(5/2))/(5*c^3*e^2) - ((2*c*d + b*e)*(d
 + e*x)^(7/2))/(7*c^2*e^2) + (d + e*x)^(9/2)/(9*c*e^2) + (e*((b*c*d - b^2*
e + a*c*e)*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e) + (2*b^5*c*d*e - 10*a
*b^3*c^2*d*e + 10*a^2*b*c^3*d*e - b^6*e^2 + a*b^2*c^2*(4*c*d^2 - 9*a*e^2)
- b^4*c*(c*d^2 - 6*a*e^2) - 2*a^2*c^3*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])*
ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c
])*e]])/(Sqrt[2]*c^(11/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (e*((
b*c*d - b^2*e + a*c*e)*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e) - (2*b^5*
c*d*e - 10*a*b^3*c^2*d*e + 10*a^2*b*c^3*d*e - b^6*e^2 + a*b^2*c^2*(4*c*d^2
 - 9*a*e^2) - b^4*c*(c*d^2 - 6*a*e^2) - 2*a^2*c^3*(c*d^2 - a*e^2))/Sqrt[b^
2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(11/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e])))/e
```

3.533.3.1 Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.533.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 762, normalized size of antiderivative = 1.17

method	result
pseudoelliptic	$2\left(\frac{3\left(\left(ac-b^2\right)e+bcd\right)\left(\left(abc-\frac{1}{3}b^3\right)e-\frac{2d\left(ac-\frac{b^2}{2}\right)c}{3}\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}}{2}+e\left(\left(-\frac{9}{2}a^2b^2c^2+3a\,b^4c+a^3c^3-\frac{1}{2}b^6\right)e^2+5db\left(a^2c^2-a\right.\right.\right.$
derivativedivides	Expression too large to display
default	Expression too large to display
risch	Expression too large to display

input
```
int(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*((3/2*((a*c-b^2)*e+b*c*d)*((a*b*c-1/3*b^3)*e-2/3*d*(a*c-1/2*b^2)*c)*(-4*
e^2*(a*c-1/4*b^2))^(1/2)+e*((-9/2*a^2*b^2*c^2+3*a*b^4*c+a^3*c^3-1/2*b^6)*e
^2+5*d*b*(a^2*c^2-a*b^2*c+1/5*b^4)*c*e-d^2*(a^2*c^2-2*a*b^2*c+1/2*b^4)*c^2
))*2^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*e^3*arctanh(
c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2
))+(2^(1/2)*e^3*(-3/2*((a*c-b^2)*e+b*c*d)*((a*b*c-1/3*b^3)*e-2/3*d*(a*c-1/
2*b^2)*c)*(-4*e^2*(a*c-1/4*b^2))^(1/2)+e*((-9/2*a^2*b^2*c^2+3*a*b^4*c+a^3*
c^3-1/2*b^6)*e^2+5*d*b*(a^2*c^2-a*b^2*c+1/5*b^4)*c*e-d^2*(a^2*c^2-2*a*b^2*
c+1/2*b^4)*c^2))*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1
/4*b^2))^(1/2))*c)^(1/2))+(-4*e^2*(a*c-1/4*b^2))^(1/2)*((b*e-2*c*d+(-4*e^2
*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*((1/9*c^4*x^4-1/5*(5/7*b*x+a)*x^2*c^3+(2/3
*a*b*x+1/5*b^2*x^2+a^2)*c^2+(-1/3*b^3*x-3*b^2*a)*c+b^4)*e^4+8/3*d*(5/84*c^
3*x^3-3/20*(4/7*b*x+a)*x*c^2+b*(3/20*b*x+a)*c-1/2*b^3)*c*e^3-1/5*d^2*(-1/2
1*c^2*x^2+(1/7*b*x+a)*c-b^2)*c^2*e^2+2/35*d^3*(-2/9*c*x+b)*c^3*e+8/315*c^4
*d^4)*(e*x+d)^(1/2))*((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))/
(-4*e^2*(a*c-1/4*b^2))^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^
(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/e^3/c^5
```

3.533.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14340 vs. $2(592) = 1184$.

Time = 17.37 (sec) , antiderivative size = 14340, normalized size of antiderivative = 22.06

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.533.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

3.533.7 Maxima [F]

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}x^4}{cx^2+bx+a}\,dx$$

input
```
integrate(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)*x^4/(c*x^2 + b*x + a), x)
```

3.533.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1612 vs. $2(592) = 1184$.

Time = 0.40 (sec) , antiderivative size = 1612, normalized size of antiderivative = 2.48

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^5*c^2 - 6*a*b^3
*c^3 + 8*a^2*b*c^4)*d^2 - 2*(b^6*c - 7*a*b^4*c^2 + 13*a^2*b^2*c^3 - 4*a^3*
c^4)*d*e + (b^7 - 8*a*b^5*c + 19*a^2*b^3*c^2 - 12*a^3*b*c^3)*e^2)*c^2*e^2
- 2*((b^3*c^4 - 2*a*b*c^5)*sqrt(b^2 - 4*a*c)*d^3 - (2*b^4*c^3 - 5*a*b^2*c^
4 + a^2*c^5)*sqrt(b^2 - 4*a*c)*d^2*e + (b^5*c^2 - 2*a*b^3*c^3 - a^2*b*c^4)
*sqrt(b^2 - 4*a*c)*d*e^2 - (a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*sqrt(b^2
- 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(
e) + (2*(b^4*c^5 - 4*a*b^2*c^6 + 2*a^2*c^7)*d^3*e - (5*b^5*c^4 - 24*a*b^3*
c^5 + 22*a^2*b*c^6)*d^2*e^2 + 2*(2*b^6*c^3 - 11*a*b^4*c^4 + 14*a^2*b^2*c^5
 - 2*a^3*c^6)*d*e^3 - (b^7*c^2 - 6*a*b^5*c^3 + 9*a^2*b^3*c^4 - 2*a^3*b*c^5
)*e^4)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2
)*sqrt(e*x + d)/sqrt(-(2*c^10*d*e^30 - b*c^9*e^31 + sqrt(-4*(c^10*d^2*e^30
 - b*c^9*d*e^31 + a*c^9*e^32)*c^10*e^30 + (2*c^10*d*e^30 - b*c^9*e^31)^2))
/(c^10*e^30)))/((sqrt(b^2 - 4*a*c)*c^8*d^2 - sqrt(b^2 - 4*a*c)*b*c^7*d*e +
 sqrt(b^2 - 4*a*c)*a*c^7*e^2)*c^2*abs(e)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c +
sqrt(b^2 - 4*a*c)*c)*e)*((b^5*c^2 - 6*a*b^3*c^3 + 8*a^2*b*c^4)*d^2 - 2*(b^
6*c - 7*a*b^4*c^2 + 13*a^2*b^2*c^3 - 4*a^3*c^4)*d*e + (b^7 - 8*a*b^5*c + 1
9*a^2*b^3*c^2 - 12*a^3*b*c^3)*e^2)*c^2*e^2 + 2*((b^3*c^4 - 2*a*b*c^5)*sqrt
(b^2 - 4*a*c)*d^3 - (2*b^4*c^3 - 5*a*b^2*c^4 + a^2*c^5)*sqrt(b^2 - 4*a*c)*
d^2*e + (b^5*c^2 - 2*a*b^3*c^3 - a^2*b*c^4)*sqrt(b^2 - 4*a*c)*d*e^2 - (...
```

3.533.9 Mupad [B] (verification not implemented)

Time = 16.93 (sec) , antiderivative size = 31485, normalized size of antiderivative = 48.44

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((x^4*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)
```

output
```
(d + e*x)^(1/2)*((2*d^4)/(c*e^3) - ((a*e^5 + c*d^2*e^3 - b*d*e^4)*((12*d^2
)/(c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(c^2*e^6) + (((8*d)/(c*e^3)
+ (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c*d*e^3))/(c*e^3)))/(c*e^3
) + ((b*e^4 - 2*c*d*e^3)*((8*d^3)/(c*e^3) - (((8*d)/(c*e^3) + (2*(b*e^4 -
2*c*d*e^3))/(c^2*e^6))*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(c*e^3) + ((b*e^4 -
2*c*d*e^3)*((12*d^2)/(c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(c^2*e^6)
 + (((8*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c*d*e^3
))/(c*e^3)))/(c*e^3)))/(c*e^3)) - atan(((((8*(4*a^4*c^9*e^5 - a*b^6*c^6*e^
5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*d^
2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*a
*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^
2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 - (8*(d + e*x)^(1
/2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^
3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11
*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3
+ 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5
*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c
 - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(
-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35
*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c...
```

3.533. $\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

3.534 $\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2} dx$

3.534.1 Optimal result

Integrand size = 25, antiderivative size = 581

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2} dx = \frac{2(b^2cd-ac^2d-b^3e+2abce)\sqrt{d+ex}}{c^4}$$

$$+\frac{2(b^2-ac)(d+ex)^{3/2}}{3c^3}-\frac{2(cd+be)(d+ex)^{5/2}}{5c^2e^2}+\frac{2(d+ex)^{7/2}}{7ce^2}$$

$$+\frac{\sqrt{2}\Big(2b^3cde-4abc^2de-b^4e^2-b^2c(cd^2-3ae^2)+ac^2(cd^2-ae^2)-\frac{2b^4cde-8ab^2c^2de+4a^2c^3de-b^5e^2-b^3c(cd^2-5ae^2)+}{\sqrt{b^2-4ac}}}{c^{9/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

$$+\frac{\sqrt{2}\Big(2b^3cde-4abc^2de-b^4e^2-b^2c(cd^2-3ae^2)+ac^2(cd^2-ae^2)+\frac{2b^4cde-8ab^2c^2de+4a^2c^3de-b^5e^2-b^3c(cd^2-5ae^2)+}{\sqrt{b^2-4ac}}}{c^{9/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
2/3*(-a*c+b^2)*(e*x+d)^(3/2)/c^3-2/5*(b*e+c*d)*(e*x+d)^(5/2)/c^2/e^2+2/7*(
e*x+d)^(7/2)/c/e^2+2*(2*a*b*c*e-a*c^2*d-b^3*e+b^2*c*d)*(e*x+d)^(1/2)/c^4+a
rctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2
))*2^(1/2)*(2*b^3*c*d*e-4*a*b*c^2*d*e-b^4*e^2-b^2*c*(-3*a*e^2+c*d^2)+a*c^2
*(-a*e^2+c*d^2)+(-2*b^4*c*d*e+8*a*b^2*c^2*d*e-4*a^2*c^3*d*e+b^5*e^2+b^3*c*
(-5*a*e^2+c*d^2)-a*b*c^2*(-5*a*e^2+3*c*d^2))/(-4*a*c+b^2)^(1/2))/c^(9/2)/(
2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2
)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(2*b^3*c*d*e-4*a*b*c^2*d
*e-b^4*e^2-b^2*c*(-3*a*e^2+c*d^2)+a*c^2*(-a*e^2+c*d^2)+(2*b^4*c*d*e-8*a*b^
2*c^2*d*e+4*a^2*c^3*d*e-b^5*e^2-b^3*c*(-5*a*e^2+c*d^2)+a*b*c^2*(-5*a*e^2+3
*c*d^2))/(-4*a*c+b^2)^(1/2))/c^(9/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2
)
```

3.534.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.51 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.30

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx =$$
$$-\frac{2\sqrt{d+ex}(105b^3e^3+3c^3(2d-5ex)(d+ex)^2-35bce^2(4bd+6ae+bex)+7c^2e(3b(d+ex)^2+5ae(4d+e}{105c^4e^2}$$
$$+\frac{\left(ib^5e^2+b^4e\left(-2icd+\sqrt{-b^2+4ac}e\right)+ib^3c\left(cd^2+e\left(2i\sqrt{-b^2+4ac}d-5ae\right)\right)+ac^2\left(a\sqrt{-b^2+4ac}e^2-cd\right(}{}$$
$$+\frac{\left(-ib^5e^2+b^4e\left(2icd+\sqrt{-b^2+4ac}e\right)+ac^2\left(a\sqrt{-b^2+4ac}e^2+cd\left(-\sqrt{-b^2+4ac}d+4iae\right)\right)+abc^2\left(3icd^2+\right.}{}$$

input
```
Integrate[(x^3*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

output
```
(-2*Sqrt[d + e*x]*(105*b^3*e^3 + 3*c^3*(2*d - 5*e*x)*(d + e*x)^2 - 35*b*c*
e^2*(4*b*d + 6*a*e + b*e*x) + 7*c^2*e*(3*b*(d + e*x)^2 + 5*a*e*(4*d + e*x)
)))/(105*c^4*e^2) + ((I*b^5*e^2 + b^4*e*((-2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e
) + I*b^3*c*(c*d^2 + e*((2*I)*Sqrt[-b^2 + 4*a*c]*d - 5*a*e)) + a*c^2*(a*Sq
rt[-b^2 + 4*a*c]*e^2 - c*d*(Sqrt[-b^2 + 4*a*c]*d + (4*I)*a*e)) + a*b*c^2*(
(-3*I)*c*d^2 + e*(4*Sqrt[-b^2 + 4*a*c]*d + (5*I)*a*e)) + b^2*c*(-3*a*Sqrt[
-b^2 + 4*a*c]*e^2 + c*d*(Sqrt[-b^2 + 4*a*c]*d + (8*I)*a*e)))*ArcTan[(Sqrt[
2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(c
^(9/2)*Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e])
 + (((-I)*b^5*e^2 + b^4*e*((2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e) + a*c^2*(a*Sq
rt[-b^2 + 4*a*c]*e^2 + c*d*(-(Sqrt[-b^2 + 4*a*c]*d) + (4*I)*a*e)) + a*b*c^
2*((3*I)*c*d^2 + e*(4*Sqrt[-b^2 + 4*a*c]*d - (5*I)*a*e)) + b^3*c*((-I)*c*d
^2 + e*(-2*Sqrt[-b^2 + 4*a*c]*d + (5*I)*a*e)) + b^2*c*(-3*a*Sqrt[-b^2 + 4*
a*c]*e^2 + c*d*(Sqrt[-b^2 + 4*a*c]*d - (8*I)*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c
]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(c^(9/2)*Sq
rt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e])
```

3.534.3 Rubi [A] (verified)

Time = 8.09 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}dx$$

$$\Big\downarrow \text{1199}$$

$$\frac{2\int \left(\frac{(d+ex)^3}{ce}-\frac{(cd+be)(d+ex)^2}{c^2e}+\frac{(b^2-ac)e(d+ex)}{c^3}+\frac{e(-eb^3+cdb^2+2aceb-ac^2d)}{c^4}-\frac{(-eb^3+cdb^2+2aceb-ac^2d)(cd^2-bed+ae^2)+(-e^2b\cdots}{c^4e\left(\frac{c(d+ex)^2}{e^2}-\frac{(2c\cdots}{}\right)}\cdots\right.}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{e\left(-\frac{4a^2c^3de-b^3c\left(cd^2-5ae^2\right)-8ab^2c^2de+abc^2\left(3cd^2-5ae^2\right)+b^5\left(-e^2\right)+2b^4cde}{\sqrt{b^2-4ac}}-b^2c(cd^2-3ae^2)-4abc^2de+ac^2(cd^2-ae^2)+b^4(-e^2)+2b^3cde\right)\text{arc}\cdots}{\sqrt{2}c^{9/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\cdots\right.}{\cdots}$$

input
```
Int[(x^3*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

output
```
(2*((e*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*Sqrt[d + e*x])/c^4 + ((b^2
- a*c)*e*(d + e*x)^(3/2))/(3*c^3) - ((c*d + b*e)*(d + e*x)^(5/2))/(5*c^2*e
) + (d + e*x)^(7/2)/(7*c*e) + (e*(2*b^3*c*d*e - 4*a*b*c^2*d*e - b^4*e^2 -
b^2*c*(c*d^2 - 3*a*e^2) + a*c^2*(c*d^2 - a*e^2) - (2*b^4*c*d*e - 8*a*b^2*c
^2*d*e + 4*a^2*c^3*d*e - b^5*e^2 - b^3*c*(c*d^2 - 5*a*e^2) + a*b*c^2*(3*c*
d^2 - 5*a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])
/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(9/2)*Sqrt[2*c*d - (
b - Sqrt[b^2 - 4*a*c])*e]) + (e*(2*b^3*c*d*e - 4*a*b*c^2*d*e - b^4*e^2 - b
^2*c*(c*d^2 - 3*a*e^2) + a*c^2*(c*d^2 - a*e^2) + (2*b^4*c*d*e - 8*a*b^2*c^
2*d*e + 4*a^2*c^3*d*e - b^5*e^2 - b^3*c*(c*d^2 - 5*a*e^2) + a*b*c^2*(3*c*d
^2 - 5*a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/
Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(9/2)*Sqrt[2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e])))/e
```

3.534.3.1 Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.534. $\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

3.534.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.17

method	result
pseudoelliptic	$\frac{-\left(\left(\left(\left(a^{2}c^{2}-3a\,b^{2}c+b^{4}\right)e^{2}+2d\left(2ba\,c^{2}-b^{3}c\right)e-c^{2}d^{2}\left(ac-b^{2}\right)\right)\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}-5\left(b\left(a^{2}c^{2}-a\,b^{2}c+\frac{1}{5}b^{4}\right)e^{2}-\frac{4d\left(a^{2}c^{2}-2a\right.}{5}\right.\right.\right.}{}$
risch	$\frac{2\left(15c^{3}e^{3}x^{3}-21c^{2}e^{3}b\,x^{2}+24c^{3}d\,e^{2}x^{2}-35a\,c^{2}e^{3}x+35b^{2}c\,e^{3}x-42b\,c^{2}d\,e^{2}x+3c^{3}d^{2}ex+210c\,e^{3}ba-140a\,c^{2}d\,e^{2}-105b^{3}e^{3}+\right.}{105e^{2}c^{4}}$
derivativedivides	$\frac{2\left(\frac{(ex+d)^{\frac{7}{2}}c^{3}}{7}-\frac{b\,c^{2}e(ex+d)^{\frac{5}{2}}}{5}-\frac{c^{3}d(ex+d)^{\frac{5}{2}}}{5}-\frac{a\,c^{2}e^{2}(ex+d)^{\frac{3}{2}}}{3}+\frac{b^{2}c\,e^{2}(ex+d)^{\frac{3}{2}}}{3}+2abc\,e^{3}\sqrt{ex+d}-a\,c^{2}d\,e^{2}\sqrt{ex+d}-b^{3}e^{3}\sqrt{ex+d}+b\right.}{c^{4}}$
default	$\frac{2\left(\frac{(ex+d)^{\frac{7}{2}}c^{3}}{7}-\frac{b\,c^{2}e(ex+d)^{\frac{5}{2}}}{5}-\frac{c^{3}d(ex+d)^{\frac{5}{2}}}{5}-\frac{a\,c^{2}e^{2}(ex+d)^{\frac{3}{2}}}{3}+\frac{b^{2}c\,e^{2}(ex+d)^{\frac{3}{2}}}{3}+2abc\,e^{3}\sqrt{ex+d}-a\,c^{2}d\,e^{2}\sqrt{ex+d}-b^{3}e^{3}\sqrt{ex+d}+b\right.}{c^{4}}$

input
```
int(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/(-4*e^2*(a*c-1/4*b^2))^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c
)^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(-(((a^2*c^2-3
*a*b^2*c+b^4)*e^2+2*d*(2*a*b*c^2-b^3*c)*e-c^2*d^2*(a*c-b^2))*(-4*e^2*(a*c-
1/4*b^2))^(1/2)-5*(b*(a^2*c^2-a*b^2*c+1/5*b^4)*e^2-4/5*d*(a^2*c^2-2*a*b^2*
c+1/2*b^4)*c*e-3/5*d^2*b*(a*c-1/3*b^2)*c^2)*e)*2^(1/2)*((b*e-2*c*d+(-4*e^2
*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*e^2*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e
+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+((((a^2*c^2-3*a*b^2*c+b^4)*
e^2+2*d*(2*a*b*c^2-b^3*c)*e-c^2*d^2*(a*c-b^2))*(-4*e^2*(a*c-1/4*b^2))^(1/2
)+5*(b*(a^2*c^2-a*b^2*c+1/5*b^4)*e^2-4/5*d*(a^2*c^2-2*a*b^2*c+1/2*b^4)*c*e
-3/5*d^2*b*(a*c-1/3*b^2)*c^2)*e)*2^(1/2)*e^2*arctan(c*(e*x+d)^(1/2)*2^(1/2
)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+4*(-4*e^2*(a*c-1/4*b
^2))^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(e*x+d)^(1/2
)*((1/14*c^3*x^3-1/6*(3/5*b*x+a)*x*c^2+b*(1/6*b*x+a)*c-1/2*b^3)*e^3-2/3*d*
(-6/35*c^2*x^2+(3/10*b*x+a)*c-b^2)*c*e^2-1/10*d^2*(-1/7*c*x+b)*c^2*e-1/35*
c^3*d^3))*((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))/e^2/c^4
```

3.534.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11459 vs. $2(527) = 1054$.

Time = 7.34 (sec) , antiderivative size = 11459, normalized size of antiderivative = 19.72

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.534.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

3.534.7 Maxima [F]

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}x^3}{cx^2+bx+a}\,dx$$

input
```
integrate(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)*x^3/(c*x^2 + b*x + a), x)
```

3.534.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1397 vs. 2(527) = 1054.

Time = 0.40 (sec) , antiderivative size = 1397, normalized size of antiderivative = 2.40

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^4*c^2 - 5*a*b^2*
c^3 + 4*a^2*c^4)*d^2 - 2*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d*e + (b^6 -
7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^2)*c^2*e^2 - 2*((b^2*c^4 - a*c^5
)*sqrt(b^2 - 4*a*c)*d^3 - (2*b^3*c^3 - 3*a*b*c^4)*sqrt(b^2 - 4*a*c)*d^2*e
+ (b^4*c^2 - a*b^2*c^3 - a^2*c^4)*sqrt(b^2 - 4*a*c)*d*e^2 - (a*b^3*c^2 - 2
*a^2*b*c^3)*sqrt(b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a
*c)*c)*e)*abs(c)*abs(e) + (2*(b^3*c^5 - 3*a*b*c^6)*d^3*e - (5*b^4*c^4 - 19
*a*b^2*c^5 + 8*a^2*c^6)*d^2*e^2 + 2*(2*b^5*c^3 - 9*a*b^3*c^4 + 7*a^2*b*c^5
)*d*e^3 - (b^6*c^2 - 5*a*b^4*c^3 + 5*a^2*b^2*c^4)*e^4)*sqrt(-4*c^2*d + 2*(
b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c
^8*d*e^16 - b*c^7*e^17 + sqrt(-4*(c^8*d^2*e^16 - b*c^7*d*e^17 + a*c^7*e^18
)*c^8*e^16 + (2*c^8*d*e^16 - b*c^7*e^17)^2))/(c^8*e^16)))/((sqrt(b^2 - 4*a
*c)*c^7*d^2 - sqrt(b^2 - 4*a*c)*b*c^6*d*e + sqrt(b^2 - 4*a*c)*a*c^6*e^2)*c
^2*abs(e)) - 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^4*c
^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^2 - 2*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*
d*e + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^2)*c^2*e^2 + 2*((b^
2*c^4 - a*c^5)*sqrt(b^2 - 4*a*c)*d^3 - (2*b^3*c^3 - 3*a*b*c^4)*sqrt(b^2 -
4*a*c)*d^2*e + (b^4*c^2 - a*b^2*c^3 - a^2*c^4)*sqrt(b^2 - 4*a*c)*d*e^2 - (
a*b^3*c^2 - 2*a^2*b*c^3)*sqrt(b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c + s
qrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) + (2*(b^3*c^5 - 3*a*b*c^6)*d^3*e -...
```

3.534.9 Mupad [B] (verification not implemented)

Time = 16.51 (sec) , antiderivative size = 25497, normalized size of antiderivative = 43.88

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((x^3*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)
```

output
```
atan(((((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d*
e^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^5*c^6*
d^2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*e^3 -
 12*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 - (8*(d + e*x)^(1/2)*(-(
b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2)
+ 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e
 - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*
b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) -
 b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 +
15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c
- b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^
2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b
^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*
(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 -
 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3
*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) +
 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4
*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)...
```

3.535 $\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

3.535.1 Optimal result

Integrand size = 25, antiderivative size = 441

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = -\frac{2(bcd-b^2e+ace)\sqrt{d+ex}}{c^3} - \frac{2b(d+ex)^{3/2}}{3c^2} + \frac{2(d+ex)^{5/2}}{5ce}$$
$$+ \frac{\sqrt{2}\left((cd-be)\left(bcd-b^2e+2ace\right)+\frac{2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)}}\right.}{c^{7/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$
$$+ \frac{\sqrt{2}\left((cd-be)\left(bcd-b^2e+2ace\right)-\frac{2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)}}\right.}{c^{7/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-2/3*b*(e*x+d)^(3/2)/c^2+2/5*(e*x+d)^(5/2)/c/e-2*(a*c*e-b^2*e+b*c*d)*(e*x+
d)^(1/2)/c^3+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2
)^(1/2)))^(1/2))*2^(1/2)*((-b*e+c*d)*(2*a*c*e-b^2*e+b*c*d)+(2*b^3*c*d*e-6*
a*b*c^2*d*e-b^4*e^2-b^2*c*(-4*a*e^2+c*d^2)+2*a*c^2*(-a*e^2+c*d^2))/(-4*a*c
+b^2)^(1/2))/c^(7/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2
)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*((
-b*e+c*d)*(2*a*c*e-b^2*e+b*c*d)+(-2*b^3*c*d*e+6*a*b*c^2*d*e+b^4*e^2+b^2*c*
(-4*a*e^2+c*d^2)-2*a*c^2*(-a*e^2+c*d^2))/(-4*a*c+b^2)^(1/2))/c^(7/2)/(2*c*
d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.535.2 Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.22

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{\frac{2\sqrt{c}\sqrt{d+ex}(15b^2e^2+3c^2(d+ex)^2-5ce(4bd+3ae+bex))}{e} - \frac{15\sqrt{2}\left(-b^4e^2+b^3e\left(2cd+\sqrt{b^2-4ace}\right)+bc\left(-2a\sqrt{b^2-4ace}\right.\right.}{}}{}$$

input
```
Integrate[(x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

output
```
((2*Sqrt[c]*Sqrt[d + e*x]*(15*b^2*e^2 + 3*c^2*(d + e*x)^2 - 5*c*e*(4*b*d +
 3*a*e + b*e*x)))/e - (15*Sqrt[2]*(-(b^4*e^2) + b^3*e*(2*c*d + Sqrt[b^2 -
4*a*c]*e) + b*c*(-2*a*Sqrt[b^2 - 4*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d - 6
*a*e)) - b^2*c*(c*d^2 + 2*e*(Sqrt[b^2 - 4*a*c]*d - 2*a*e)) + 2*a*c^2*(c*d^
2 + e*(Sqrt[b^2 - 4*a*c]*d - a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])
/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d
 + (b - Sqrt[b^2 - 4*a*c])*e]) - (15*Sqrt[2]*(b^4*e^2 + b^3*e*(-2*c*d + Sq
rt[b^2 - 4*a*c]*e) + 2*a*c^2*(-(c*d^2) + e*(Sqrt[b^2 - 4*a*c]*d + a*e)) +
b^2*c*(c*d^2 - 2*e*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)) + b*c*(-2*a*Sqrt[b^2 - 4
*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d + 6*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sq
rt[d + e*x])/Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]
*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]))/(15*c^(7/2))
```

3.535.3 Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \; 1199$$

$$\frac{2\int \left(\frac{(d+ex)^2}{c}-\frac{be(d+ex)}{c^2}-\frac{e(-eb^2+cdb+ace)}{c^3}+\frac{(-eb^2+cdb+ace)(cd^2-bed+ae^2)-(cd-be)(-eb^2+cdb+2ace)(d+ex)}{c^3e\left(\frac{c(d+ex)^2}{e^2}-\frac{(2cd-be)(d+ex)}{e^2}+a+\frac{d(cd-be)}{e^2}\right)}\right)d\sqrt{d+ex}}{e}$$

$$\downarrow 2009$$

$$2\left(\frac{e\left((cd-be)\left(2ace+b^2(-e)+bcd\right)+\frac{-b^2c\left(cd^2-4ae^2\right)-6abc^2de+2ac^2\left(cd^2-ae^2\right)+b^4\left(-e^2\right)+2b^3cde}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{e\left((cd-\right.}{}\right.$$

input
```
Int[(x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

output
```
(2*(-((e*(b*c*d - b^2*e + a*c*e)*Sqrt[d + e*x])/c^3) - (b*e*(d + e*x)^(3/2
))/(3*c^2) + (d + e*x)^(5/2)/(5*c) + (e*((c*d - b*e)*(b*c*d - b^2*e + 2*a*
c*e) + (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) +
2*a*c^2*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[
d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/2)*Sqrt[
2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (e*((c*d - b*e)*(b*c*d - b^2*e + 2*a
*c*e) - (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) +
 2*a*c^2*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt
[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/2)*Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])))/e
```

3.535.3.1 Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.535. $\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

3.535.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.23

method	result
pseudoelliptic	$-\frac{2\left(\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,ce\left(\left(\left(\left(ac-\frac{b^2}{2}\right)e+\frac{bcd}{2}\right)(be-cd)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+e\left(\left(a^2c^2-2ab^2c+\frac{1}{2}b^4\right)e^2+\cdots\right.\right.\right.\right.}{\cdots}$
risch	$-\frac{2(-3c^2x^2e^2+5bce^2x-6c^2dex+15ace^2-15b^2e^2+20bcde-3c^2d^2)\sqrt{ex+d}}{15ec^3}+\frac{-\frac{\left(2a^2c^2e^3-4ab^2ce^3+6abc^2de^2-2ac^3d^2\cdots\right.}{\cdots}}{\cdots}$
derivativedivides	$-\frac{2\left(-\frac{(ex+d)^{\frac{5}{2}}c^2}{5}+\frac{bce(ex+d)^{\frac{3}{2}}}{3}+ace^2\sqrt{ex+d}-b^2e^2\sqrt{ex+d}+bcde\sqrt{ex+d}\right)}{c^3}+\frac{8e\left(-\frac{\left(2a^2c^2e^3-4ab^2ce^3+6abc^2de^2-2ac^3d^2e+b^4\cdots\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{2\left(-\frac{(ex+d)^{\frac{5}{2}}c^2}{5}+\frac{bce(ex+d)^{\frac{3}{2}}}{3}+ace^2\sqrt{ex+d}-b^2e^2\sqrt{ex+d}+bcde\sqrt{ex+d}\right)}{c^3}+\frac{8e\left(-\frac{\left(2a^2c^2e^3-4ab^2ce^3+6abc^2de^2-2ac^3d^2e+b^4\cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/(-4*e^2*(a*c-1/4*b^2))^(1/2)*(2^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2)
)^(1/2))*c)^(1/2)*e*(((a*c-1/2*b^2)*e+1/2*b*c*d)*(b*e-c*d)*(-4*e^2*(a*c-1/
4*b^2))^(1/2)+e*((a^2*c^2-2*a*b^2*c+1/2*b^4)*e^2+d*(3*a*b*c^2-b^3*c)*e-d^2
*(a*c-1/2*b^2)*c^2))*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-4*e^2*
(a*c-1/4*b^2))^(1/2))*c)^(1/2))+((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))
*c)^(1/2)*(2^(1/2)*e*(-((a*c-1/2*b^2)*e+1/2*b*c*d)*(b*e-c*d)*(-4*e^2*(a*c-
1/4*b^2))^(1/2)+e*((a^2*c^2-2*a*b^2*c+1/2*b^4)*e^2+d*(3*a*b*c^2-b^3*c)*e-d
^2*(a*c-1/2*b^2)*c^2))*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-4*e^2*
(a*c-1/4*b^2))^(1/2))*c)^(1/2))+(-4*e^2*(a*c-1/4*b^2))^(1/2)*((-1/5*c^2*x^
2+(1/3*b*x+a)*c-b^2)*e^2+4/3*d*(-3/10*c*x+b)*c*e-1/5*c^2*d^2)*((b*e-2*c*d+
(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(e*x+d)^(1/2)))/((b*e-2*c*d+(-4*e^2
*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))
*c)^(1/2)/e/c^3
```

3.535.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8530 vs. $2(391) = 782$.

Time = 3.55 (sec) , antiderivative size = 8530, normalized size of antiderivative = 19.34

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.535.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

3.535.7 Maxima [F]

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}x^2}{cx^2+bx+a}\,dx$$

input
```
integrate(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)*x^2/(c*x^2 + b*x + a), x)
```

3.535.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1195 vs. 2(391) = 782.

Time = 0.37 (sec) , antiderivative size = 1195, normalized size of antiderivative = 2.71

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^3*c^2 - 4*a*b*c
^3)*d^2 - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e + (b^5 - 6*a*b^3*c + 8*a
^2*b*c^2)*e^2)*c^2*e^2 - 2*(sqrt(b^2 - 4*a*c)*b*c^4*d^3 + sqrt(b^2 - 4*a*c
)*b^3*c^2*d*e^2 - (2*b^2*c^3 - a*c^4)*sqrt(b^2 - 4*a*c)*d^2*e - (a*b^2*c^2
 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a
*c)*c)*e)*abs(c)*abs(e) + (2*(b^2*c^5 - 2*a*c^6)*d^3*e - (5*b^3*c^4 - 14*a
*b*c^5)*d^2*e^2 + 2*(2*b^4*c^3 - 7*a*b^2*c^4 + 2*a^2*c^5)*d*e^3 - (b^5*c^2
 - 4*a*b^3*c^3 + 2*a^2*b*c^4)*e^4)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a
*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c^6*d*e^6 - b*c^5*e^7
 + sqrt(-4*(c^6*d^2*e^6 - b*c^5*d*e^7 + a*c^5*e^8)*c^6*e^6 + (2*c^6*d*e^6
- b*c^5*e^7)^2))/(c^6*e^6)))/((sqrt(b^2 - 4*a*c)*c^6*d^2 - sqrt(b^2 - 4*a*
c)*b*c^5*d*e + sqrt(b^2 - 4*a*c)*a*c^5*e^2)*c^2*abs(e)) + 1/4*(sqrt(-4*c^2
*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^3*c^2 - 4*a*b*c^3)*d^2 - 2*(b^4*
c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^2)*c^
2*e^2 + 2*(sqrt(b^2 - 4*a*c)*b*c^4*d^3 + sqrt(b^2 - 4*a*c)*b^3*c^2*d*e^2 -
 (2*b^2*c^3 - a*c^4)*sqrt(b^2 - 4*a*c)*d^2*e - (a*b^2*c^2 - a^2*c^3)*sqrt(
b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*
abs(e) + (2*(b^2*c^5 - 2*a*c^6)*d^3*e - (5*b^3*c^4 - 14*a*b*c^5)*d^2*e^2 +
 2*(2*b^4*c^3 - 7*a*b^2*c^4 + 2*a^2*c^5)*d*e^3 - (b^5*c^2 - 4*a*b^3*c^3 +
2*a^2*b*c^4)*e^4)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arc...
```

3.535.9 Mupad [B] (verification not implemented)

Time = 14.90 (sec) , antiderivative size = 19465, normalized size of antiderivative = 44.14

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)
```

output
```
atan(((((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*e^4 - 5*a^2*b^2*c^5*
e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d^2*e^3 + 4*a*b*c^7*
d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5 - (8*(d + e*x)^(1/
2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d
^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*
c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) -
11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2
) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)
^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e +
 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^
4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(
1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a
*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b
*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^
8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2)
)/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^
3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c
^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 +
a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(...
```

3.536 $\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

3.536.1 Optimal result

Integrand size = 23, antiderivative size = 453

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2(cd-be)\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3c}$$

$$+ \frac{\sqrt{2}\left(b^3e^2 - b^2e\left(2cd+\sqrt{b^2-4ac}e\right) + c\left(a\sqrt{b^2-4ac}e^2 - cd\left(\sqrt{b^2-4ac}d-4ae\right)\right) + bc\left(cd^2+e\left(2\sqrt{b^2-4ac}\right.\right.\right.}{c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

$$- \frac{\sqrt{2}\left(b^3e^2 - b^2e\left(2cd-\sqrt{b^2-4ac}e\right) + bc\left(cd^2-e\left(2\sqrt{b^2-4ac}d+3ae\right)\right) - c\left(a\sqrt{b^2-4ac}e^2 - cd\left(\sqrt{b^2-4ac}\right.\right.\right.}{c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
2/3*(e*x+d)^(3/2)/c+2*(-b*e+c*d)*(e*x+d)^(1/2)/c^2+arctanh(2^(1/2)*c^(1/2)
*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b^3*e^2-b^
2*e*(2*c*d+e*(-4*a*c+b^2)^(1/2))+c*(a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*e+d
*(-4*a*c+b^2)^(1/2)))+b*c*(c*d^2+e*(-3*a*e+2*d*(-4*a*c+b^2)^(1/2))))/c^(5/
2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-arctanh(2^(1/
2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(
b^3*e^2-b^2*e*(2*c*d-e*(-4*a*c+b^2)^(1/2))-c*(a*e^2*(-4*a*c+b^2)^(1/2)-c*d
*(4*a*e+d*(-4*a*c+b^2)^(1/2)))+b*c*(c*d^2-e*(3*a*e+2*d*(-4*a*c+b^2)^(1/2))
))/c^(5/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.536.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.75 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.09

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\sqrt{c}\sqrt{d+ex}(4cd-3be+cex)+\frac{3\left(ib^3e^2+b^2e\left(-2icd+\sqrt{-b^2+4ac}e\right)+ibc\left(cd^2+e\left(2i\sqrt{-b^2+4ac}d-3ae\right)\right)}{\sqrt{-\frac{b^2}{2}+2ac}\sqrt{}}}{}$$

input
```
Integrate[(x*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

output
```
(2*Sqrt[c]*Sqrt[d + e*x]*(4*c*d - 3*b*e + c*e*x) + (3*(I*b^3*e^2 + b^2*e*(
(-2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e) + I*b*c*(c*d^2 + e*((2*I)*Sqrt[-b^2 + 4
*a*c]*d - 3*a*e)) + c*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(Sqrt[-b^2 + 4*a*
c]*d + (4*I)*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b
*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b -
I*Sqrt[-b^2 + 4*a*c])*e]) + (3*((-I)*b^3*e^2 + b^2*e*((2*I)*c*d + Sqrt[-b^
2 + 4*a*c]*e) + b*c*((-I)*c*d^2 + e*(-2*Sqrt[-b^2 + 4*a*c]*d + (3*I)*a*e))
 + c*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(Sqrt[-b^2 + 4*a*c]*d - (4*I)*a*e)
))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2
+ 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a
*c])*e]))/(3*c^(5/2))
```

3.536.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1196, 25, 1196, 1197, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \ 1196$$

$$\frac{\int -\frac{\sqrt{d+ex}(ae-(cd-be)x)}{cx^2+bx+a}dx}{c}+\frac{2(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \ 25$$

$$\frac{2(d+ex)^{3/2}}{3c}-\frac{\int \frac{\sqrt{d+ex}(ae-(cd-be)x)}{cx^2+bx+a}dx}{c}$$

$\Big\downarrow$ 1196

$$\frac{2(d+ex)^{3/2}}{3c}-\frac{\frac{\int \frac{ae(2cd-be)-\left(c^2d^2+b^2e^2-ce(2bd+ae)\right)x}{\sqrt{d+ex}\left(cx^2+bx+a\right)}dx}{c}-\frac{2\sqrt{d+ex}(cd-be)}{c}}{c}$$

$\Big\downarrow$ 1197

$$\frac{2(d+ex)^{3/2}}{3c}-\frac{\frac{2\int \frac{(cd-be)\left(cd^2-bed+ae^2\right)-\left(c^2d^2+b^2e^2-ce(2bd+ae)\right)(d+ex)}{cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex)}d\sqrt{d+ex}}{c}-\frac{2\sqrt{d+ex}(cd-be)}{c}}{c}$$

$\Big\downarrow$ 1480

$$\frac{2(d+ex)^{3/2}}{3c}-\frac{\frac{2\left(-\frac{1}{2}\left(\frac{-bc\left(cd^2-3ae^2\right)-4ac^2de+b^3\left(-e^2\right)+2b^2cde}{\sqrt{b^2-4ac}}-ce(ae+2bd)+b^2e^2+c^2d^2\right)\int \frac{1}{\frac{1}{2}\left(\left(b-\sqrt{b^2-4ac}\right)e-2cd\right)+c(d+ex)}d\sqrt{d+ex}-\frac{1}{2}\left(-\frac{-bc\left(cd^2-3ae^2\right)-4a\ldots}{\sqrt{b\ldots}}\right.\right.}{c}}{c}$$

$\Big\downarrow$ 221

$$\frac{2(d+ex)^{3/2}}{3c}-\frac{\frac{2\left(\frac{\left(\frac{-bc\left(cd^2-3ae^2\right)-4ac^2de+b^3\left(-e^2\right)+2b^2cde}{\sqrt{b^2-4ac}}-ce(ae+2bd)+b^2e^2+c^2d^2\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(-\frac{-bc\left(cd^2-3ae^2\right)-4ac^2de+b^3\left(-e^2\right)+\ldots}{\sqrt{b^2-4ac}}\right.}{\ldots}\right.}{c}}{c}$$

input
```
Int[(x*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

output
```
(2*(d + e*x)^(3/2))/(3*c) - ((-2*(c*d - b*e)*Sqrt[d + e*x])/c + (2*(((c^2*
d^2 + b^2*e^2 - c*e*(2*b*d + a*e) + (2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 -
 b*c*(c*d^2 - 3*a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
 + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((c^2*d^2 + b^2*e^2 - c*e*(2*b*d + a*
e) - (2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c*d^2 - 3*a*e^2))/Sqrt[b^
2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e])))/c)/c
```

3.536. $\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

3.536.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1196
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int
[(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] &
& GtQ[m, 0]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.536.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{-\sqrt{2}\,\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,\left(\left(\left(ac-b^2\right)e^2+2bcde-c^2d^2\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+\left(-3abc+b^3\right)e^3+2d\left(2a\,c^2-b^2c\right.\right.}{}$
risch	$-\frac{2(-xce+3be-4cd)\sqrt{ex+d}}{3c^2}-\frac{8\left(-\frac{\left(-3c\,e^3ba+4a\,c^2d\,e^2+b^3e^3-2b^2cd\,e^2+b\,c^2d^2e+\sqrt{-e^2\left(4ac-b^2\right)}\,ac\,e^2-\sqrt{-e^2\left(4ac-b^2\right)}\right.}{8c\sqrt{-e^2\left(4ac-b^2\right)}}\right.}{}$
derivativedivides	$-\frac{2\left(-\frac{c(ex+d)^{\frac{3}{2}}}{3}+be\sqrt{ex+d}-cd\sqrt{ex+d}\right)}{c^2}+\frac{\left(-3c\,e^3ba+4a\,c^2d\,e^2+b^3e^3-2b^2cd\,e^2+b\,c^2d^2e-\sqrt{-e^2\left(4ac-b^2\right)}\,ac\,e^2+\sqrt{-e^2\left(4\right.}\right.}{c\sqrt{-e^2\left(4a\right.}}$
default	$-\frac{2\left(-\frac{c(ex+d)^{\frac{3}{2}}}{3}+be\sqrt{ex+d}-cd\sqrt{ex+d}\right)}{c^2}+\frac{\left(-3c\,e^3ba+4a\,c^2d\,e^2+b^3e^3-2b^2cd\,e^2+b\,c^2d^2e-\sqrt{-e^2\left(4ac-b^2\right)}\,ac\,e^2+\sqrt{-e^2\left(4\right.}\right.}{c\sqrt{-e^2\left(4a\right.}}$

input
```
int(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-(-2^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(((a*c-b^2)*
e^2+2*b*c*d*e-c^2*d^2)*(-4*e^2*(a*c-1/4*b^2))^(1/2)+(-3*a*b*c+b^3)*e^3+2*d
*(2*a*c^2-b^2*c)*e^2+b*c^2*d^2*e)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2
*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+((((a*c-b^2)*e^2+2*b*c*d*e-c^
2*d^2)*(-4*e^2*(a*c-1/4*b^2))^(1/2)+(3*a*b*c-b^3)*e^3+2*d*(-2*a*c^2+b^2*c)
*e^2-b*c^2*d^2*e)*2^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-4*e
^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+2*(-4*e^2*(a*c-1/4*b^2))^(1/2)*((b*e-2*
c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(e*x+d)^(1/2)*((-1/3*c*x+b)*e-4
/3*c*d))*((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))/(-4*e^2*(a*c
-1/4*b^2))^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/((-b*e
+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/c^2
```

3.536. $\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

3.536.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5572 vs. $2(392) = 784$.

Time = 1.65 (sec) , antiderivative size = 5572, normalized size of antiderivative = 12.30

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.536.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

3.536.7 Maxima [F]

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}x}{cx^2+bx+a}\,dx$$

input
```
integrate(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)*x/(c*x^2 + b*x + a), x)
```

3.536.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 986 vs. $2(392) = 784$.

Time = 0.36 (sec) , antiderivative size = 986, normalized size of antiderivative = 2.18

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\left((ex+d)^{\frac{3}{2}}c^2 + 3\sqrt{ex+d}c^2d - 3\sqrt{ex+d}bce\right)}{3\,c^3}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\,(bc-\sqrt{b^2-4\,ac}c)e}((b^2c^2-4\,ac^3)d^2-2\,(b^3c-4\,abc^2)de+(b^4-5\,ab^2c+4\,a^2c^2)e^2)c^2e^2-\right.}{}$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\,(bc+\sqrt{b^2-4\,ac}c)e}((b^2c^2-4\,ac^3)d^2-2\,(b^3c-4\,abc^2)de+(b^4-5\,ab^2c+4\,a^2c^2)e^2)c^2e^2+\right.}{}$$

input
```
integrate(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
2/3*((e*x + d)^(3/2)*c^2 + 3*sqrt(e*x + d)*c^2*d - 3*sqrt(e*x + d)*b*c*e)/
c^3 + 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^2*c^2 - 4*
a*c^3)*d^2 - 2*(b^3*c - 4*a*b*c^2)*d*e + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^2
)*c^2*e^2 - 2*(sqrt(b^2 - 4*a*c)*c^4*d^3 - 2*sqrt(b^2 - 4*a*c)*b*c^3*d^2*e
 - sqrt(b^2 - 4*a*c)*a*b*c^2*e^3 + (b^2*c^2 + a*c^3)*sqrt(b^2 - 4*a*c)*d*e
^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) + (2*b*
c^5*d^3*e - (5*b^2*c^4 - 8*a*c^5)*d^2*e^2 + 2*(2*b^3*c^3 - 5*a*b*c^4)*d*e^
3 - (b^4*c^2 - 3*a*b^2*c^3)*e^4)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c
)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c^4*d - b*c^3*e + sqrt(
-4*(c^4*d^2 - b*c^3*d*e + a*c^3*e^2)*c^4 + (2*c^4*d - b*c^3*e)^2))/c^4))/(
(sqrt(b^2 - 4*a*c)*c^5*d^2 - sqrt(b^2 - 4*a*c)*b*c^4*d*e + sqrt(b^2 - 4*a*
c)*a*c^4*e^2)*c^2*abs(e)) - 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c
)*c)*e)*((b^2*c^2 - 4*a*c^3)*d^2 - 2*(b^3*c - 4*a*b*c^2)*d*e + (b^4 - 5*a*
b^2*c + 4*a^2*c^2)*e^2)*c^2*e^2 + 2*(sqrt(b^2 - 4*a*c)*c^4*d^3 - 2*sqrt(b^
2 - 4*a*c)*b*c^3*d^2*e - sqrt(b^2 - 4*a*c)*a*b*c^2*e^3 + (b^2*c^2 + a*c^3)
*sqrt(b^2 - 4*a*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)
*abs(c)*abs(e) + (2*b*c^5*d^3*e - (5*b^2*c^4 - 8*a*c^5)*d^2*e^2 + 2*(2*b^3
*c^3 - 5*a*b*c^4)*d*e^3 - (b^4*c^2 - 3*a*b^2*c^3)*e^4)*sqrt(-4*c^2*d + 2*(
b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c
^4*d - b*c^3*e - sqrt(-4*(c^4*d^2 - b*c^3*d*e + a*c^3*e^2)*c^4 + (2*c^4...
```

3.536.9 Mupad [B] (verification not implemented)

Time = 13.80 (sec) , antiderivative size = 13841, normalized size of antiderivative = 30.55

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((x*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)
```

output
```
(2*(d + e*x)^(3/2))/(3*c) - ((2*d)/c + (2*(b*e - 2*c*d))/c^2)*(d + e*x)^(1
/2) - atan(((((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^
2*c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*
c^5*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 - (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*
a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4
*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*
d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)
^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e -
3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b
^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d
*e^2))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c -
b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c -
b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 +
a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a
*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*
e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*
c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2...
```

3.537 $\int \frac{(d+ex)^{3/2}}{a+bx+cx^2} dx$

3.537.1 Optimal result

Integrand size = 22, antiderivative size = 322

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2e\sqrt{d+ex}}{c} - \frac{\sqrt{2}\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{2}\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}\,d+ae\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
2*e*(e*x+d)^(1/2)/c-arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*
a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(2*c^2*d^2+b*e^2*(b-(-4*a*c+b^2)^(1/2))-2*
c*e*(b*d+a*e-d*(-4*a*c+b^2)^(1/2)))/c^(3/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b
-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e
*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(2*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(
1/2))-2*c*e*(b*d+a*e+d*(-4*a*c+b^2)^(1/2)))/c^(3/2)/(-4*a*c+b^2)^(1/2)/(2*
c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.537.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\sqrt{c}e\sqrt{d+ex}+\frac{\left(-2ic^2d^2-b\left(ib+\sqrt{-b^2+4ac}\right)e^2+2ce\left(ibd+\sqrt{-b^2+4ac}d+iae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-i\sqrt{-b^2+4a}}}\right.}{\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}}{c^{3/2}}$$

input
```
Integrate[(d + e*x)^(3/2)/(a + b*x + c*x^2),x]
```

output
```
(2*Sqrt[c]*e*Sqrt[d + e*x] + (((-2*I)*c^2*d^2 - b*(I*b + Sqrt[-b^2 + 4*a*c
])*e^2 + 2*c*e*(I*b*d + Sqrt[-b^2 + 4*a*c]*d + I*a*e))*ArcTan[(Sqrt[2]*Sqr
t[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1
/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) + (((2*I)*c^2
*d^2 - b*((-I)*b + Sqrt[-b^2 + 4*a*c])*e^2 + 2*c*e*((-I)*b*d + Sqrt[-b^2 +
 4*a*c]*d - I*a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b
*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b +
I*Sqrt[-b^2 + 4*a*c])*e]))/c^(3/2)
```

3.537.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1146, 1197, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}dx$$

$$\Big\downarrow \text{1146}$$

$$\frac{\int \frac{cd^2-ae^2+e(2cd-be)x}{\sqrt{d+ex}(cx^2+bx+a)}dx}{c}+\frac{2e\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{1197}$$

$$\frac{2\int -\frac{e(cd^2-bed+ae^2-(2cd-be)(d+ex))}{cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex)}d\sqrt{d+ex}}{c}+\frac{2e\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{25}$$

$$\frac{2e\sqrt{d+ex}}{c}-\frac{2\int \frac{e(cd^2-bed+ae^2-(2cd-be)(d+ex))}{cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex)}d\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{27}$$

$$\frac{2e\sqrt{d+ex}}{c}-\frac{2e\int \frac{cd^2-bed+ae^2-(2cd-be)(d+ex)}{cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex)}d\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{1480}$$

$$\frac{2e\sqrt{d+ex}}{c}-$$

$$\frac{2e\left(-\frac{1}{2}\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{e\sqrt{b^2-4ac}}-be+2cd\right)\int \frac{1}{\frac{1}{2}\left(\left(b-\sqrt{b^2-4ac}\right)e-2cd\right)+c(d+ex)}d\sqrt{d+ex}-\frac{1}{2}\left(-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{e\sqrt{b^2-4ac}}\right.\right.}{c}$$

$$\Big\downarrow \text{221}$$

$$\frac{2e\sqrt{d+ex}}{c}-$$

$$\frac{2e\left(\frac{\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{e\sqrt{b^2-4ac}}-be+2cd\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{e\sqrt{b^2-4ac}}-be+2cd\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\cdots}{\sqrt{2cd-e\left(\sqrt{\cdots}\right.}}\right.}{\sqrt{2}\sqrt{c}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right.}{c}$$

input
```
Int[(d + e*x)^(3/2)/(a + b*x + c*x^2),x]
```

output
```
(2*e*Sqrt[d + e*x])/c - (2*e*(((2*c*d - b*e + (2*c^2*d^2 + b^2*e^2 - 2*c*e
*(b*d + a*e))/(Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x
])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]) + ((2*c*d - b*e - (2*c^2*d^2 + b^2*e^2 - 2*c*
e*(b*d + a*e))/(Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*
x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e])))/c
```

3.537. $\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

3.537.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1146
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Simp[1/c   Int[(d + e*x)^
(m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
 x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.537.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{e\left(2\sqrt{ex+d}+\frac{\left(2ac\,e^{2}-b^{2}e^{2}+2bcde-2c^{2}d^{2}+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,be-2\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right.}}\right.}{\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,\sqrt{\left(-be+2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)c}}\right.}{c}$
derivativedivides	$2e\left(\frac{\sqrt{ex+d}}{c}+\frac{\left(2ac\,e^{2}-b^{2}e^{2}+2bcde-2c^{2}d^{2}-\sqrt{-e^{2}(4ac-b^{2})}\,be+2\sqrt{-e^{2}(4ac-b^{2})}\,cd\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(be-2cd+\sqrt{-e^{2}\right.}}}\right.}{2c\sqrt{-e^{2}(4ac-b^{2})}\,\sqrt{\left(be-2cd+\sqrt{-e^{2}(4ac-b^{2})}\right)c}}\right.$
default	$2e\left(\frac{\sqrt{ex+d}}{c}+\frac{\left(2ac\,e^{2}-b^{2}e^{2}+2bcde-2c^{2}d^{2}-\sqrt{-e^{2}(4ac-b^{2})}\,be+2\sqrt{-e^{2}(4ac-b^{2})}\,cd\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(be-2cd+\sqrt{-e^{2}\right.}}}\right.}{2c\sqrt{-e^{2}(4ac-b^{2})}\,\sqrt{\left(be-2cd+\sqrt{-e^{2}(4ac-b^{2})}\right)c}}\right.$
risch	$\frac{2e\sqrt{ex+d}}{c}-8e\left(\frac{\left(-2ac\,e^{2}+b^{2}e^{2}-2bcde+2c^{2}d^{2}+\sqrt{-e^{2}(4ac-b^{2})}\,be-2\sqrt{-e^{2}(4ac-b^{2})}\,cd\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{e}}{\sqrt{\left(be-2cd+\sqrt{\right.}}}\right.}{8c\sqrt{-e^{2}(4ac-b^{2})}\,\sqrt{\left(be-2cd+\sqrt{-e^{2}(4ac-b^{2})}\right)c}}\right.$

input
```
int((e*x+d)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
e/c*(2*(e*x+d)^(1/2)+(2*a*c*e^2-b^2*e^2+2*b*c*d*e-2*c^2*d^2+(-e^2*(4*a*c-b
^2))^(1/2)*b*e-2*(-e^2*(4*a*c-b^2))^(1/2)*c*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^
(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1
/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-(-2*a*c*e^2+b
^2*e^2-2*b*c*d*e+2*c^2*d^2+(-e^2*(4*a*c-b^2))^(1/2)*b*e-2*(-e^2*(4*a*c-b^2
))^(1/2)*c*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^
2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*
c-b^2))^(1/2))*c)^(1/2)))
```

3.537.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2770 vs. $2(272) = 544$.

Time = 1.11 (sec) , antiderivative size = 2770, normalized size of antiderivative = 8.60

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^
2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c
^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 +
 (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4
))*log(sqrt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^
3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 - (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^
3 - 4*a*b*c^4)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 -
2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)
*e^6)/(b^2*c^6 - 4*a*c^7)))*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2
*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*
e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*
c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c
^3 - 4*a*c^4)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^
2*e^3 - (b^3 + 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*sqrt(e*x + d)) - sqrt
(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 -
 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3
 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*
a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-sq
rt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^3 + (b^4
- 5*a*b^2*c + 4*a^2*c^2)*e^4 - (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*...
```

3.537.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{a+bx+cx^2}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

output
```
Integral((d + e*x)**(3/2)/(a + b*x + c*x**2), x)
```

3.537.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}}{cx^2+bx+a}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/(c*x^2 + b*x + a), x)
```

3.537.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 797 vs. $2(272) = 544$.

Time = 0.33 (sec) , antiderivative size = 797, normalized size of antiderivative = 2.48

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx=\frac{2\sqrt{ex+d}e}{c}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}c\right)e}(2\left(b^2c-4\,ac^2\right)de-\left(b^3-4\,abc\right)e^2)c^2e^2-2\left(\sqrt{b^2-4\,ac}c^3d^2e-\sqrt{b^2-\right.\right.}{}$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}c\right)e}(2\left(b^2c-4\,ac^2\right)de-\left(b^3-4\,abc\right)e^2)c^2e^2+2\left(\sqrt{b^2-4\,ac}c^3d^2e-\sqrt{b^2-\right.\right.}{}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
2*sqrt(e*x + d)*e/c + 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e
)*(2*(b^2*c - 4*a*c^2)*d*e - (b^3 - 4*a*b*c)*e^2)*c^2*e^2 - 2*(sqrt(b^2 -
4*a*c)*c^3*d^2*e - sqrt(b^2 - 4*a*c)*b*c^2*d*e^2 + sqrt(b^2 - 4*a*c)*a*c^2
*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) - (4*
c^5*d^3*e - 6*b*c^4*d^2*e^2 + 4*(b^2*c^3 - a*c^4)*d*e^3 - (b^3*c^2 - 2*a*b
*c^3)*e^4)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt
(1/2)*sqrt(e*x + d)/sqrt(-(2*c^2*d - b*c*e + sqrt(-4*(c^2*d^2 - b*c*d*e +
a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/((sqrt(b^2 - 4*a*c)*c^4*d^2 - s
qrt(b^2 - 4*a*c)*b*c^3*d*e + sqrt(b^2 - 4*a*c)*a*c^3*e^2)*c^2*abs(e)) - 1/
4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(2*(b^2*c - 4*a*c^2)*d
*e - (b^3 - 4*a*b*c)*e^2)*c^2*e^2 + 2*(sqrt(b^2 - 4*a*c)*c^3*d^2*e - sqrt(
b^2 - 4*a*c)*b*c^2*d*e^2 + sqrt(b^2 - 4*a*c)*a*c^2*e^3)*sqrt(-4*c^2*d + 2*
(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) - (4*c^5*d^3*e - 6*b*c^4*d^2*
e^2 + 4*(b^2*c^3 - a*c^4)*d*e^3 - (b^3*c^2 - 2*a*b*c^3)*e^4)*sqrt(-4*c^2*d
 + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt
(-(2*c^2*d - b*c*e - sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d
- b*c*e)^2))/c^2))/((sqrt(b^2 - 4*a*c)*c^4*d^2 - sqrt(b^2 - 4*a*c)*b*c^3*d
*e + sqrt(b^2 - 4*a*c)*a*c^3*e^2)*c^2*abs(e))
```

3.537.9 Mupad [B] (verification not implemented)

Time = 13.56 (sec) , antiderivative size = 8334, normalized size of antiderivative = 25.88

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(3/2)/(a + b*x + c*x^2),x)
```

output
```
(2*e*(d + e*x)^(1/2))/c - atan(((((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*a*
c^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c - (8*(
d + e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e -
 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c -
 b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c -
 b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^
4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2)
)/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2
*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(
1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(
1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)
 - (8*(d + e*x)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*
d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*
e^5 + 12*a*b*c^2*d*e^5))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2
*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^
3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e
^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e
^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*...
```

3.537. $\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

3.538 $\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx$

3.538.1 Optimal result

Integrand size = 25, antiderivative size = 340

$$\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx = -\frac{2d^{3/2}\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} - \frac{\sqrt{2}\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d-4ae\right)-b(cd^2+ae^2)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\sqrt{2}\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d+4ae\right)+b(cd^2+ae^2)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-2*d^(3/2)*arctanh((e*x+d)^(1/2)/d^(1/2))/a-arctanh(2^(1/2)*c^(1/2)*(e*x+d
)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(-b*(a*e^2+c*d^2)+
a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*e+d*(-4*a*c+b^2)^(1/2)))/a/c^(1/2)/(-4*
a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-arctanh(2^(1/2)*c^(1
/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b*(a*e^
2+c*d^2)+a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(4*a*e+d*(-4*a*c+b^2)^(1/2)))/a/c^(1
/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.538.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx = -\frac{\frac{\sqrt{2}\left(-a\sqrt{-b^2+4ac}e^2+cd\left(\sqrt{-b^2+4ac}d+4iae\right)-ib(cd^2+ae^2)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ace}}}\right)}{\sqrt{c}\sqrt{-b^2+4ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}+\frac{\sqrt{2}\left(-a\sqrt{-b^2+4ac}e^2+cd\left(\sqrt{-b^2+4ac}d-4i\right.\right.}{\sqrt{c}\sqrt{-b^2+4ac}\sqrt{}}}{a}$$

input
```
Integrate[(d + e*x)^(3/2)/(x*(a + b*x + c*x^2)),x]
```

output
```
-(((Sqrt[2]*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(Sqrt[-b^2 + 4*a*c]*d + (4*
I)*a*e) - I*b*(c*d^2 + a*e^2))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt
[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[c]*Sqrt[-b^2 + 4*a*c]*Sqrt
[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) + (Sqrt[2]*(-(a*Sqrt[-b^2 + 4*a*c
]*e^2) + c*d*(Sqrt[-b^2 + 4*a*c]*d - (4*I)*a*e) + I*b*(c*d^2 + a*e^2))*Arc
Tan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*
c]*e]])/(Sqrt[c]*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c
])*e]) + 2*d^(3/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a)
```

3.538.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx$$

$$\Big\downarrow \; 1199$$

$$\frac{2\int \left(\frac{d^2}{ax}+\frac{e(d(cd^2-bed+ae^2)-(cd^2-ae^2)(d+ex))}{a(cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex))}\right)d\sqrt{d+ex}}{e}$$

$$\Big\downarrow \; 2009$$

$$\frac{2\left(-\frac{e\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b(ae^2+cd^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}} - \frac{e\left(-cd\left(d\sqrt{b^2-4ac}+4ae\right)+ae^2\sqrt{b^2-4ac}+b(\right.}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{}}\right.}{e}$$

input
```
Int[(d + e*x)^(3/2)/(x*(a + b*x + c*x^2)),x]
```

output
```
(2*(-((d^(3/2)*e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a) - (e*(a*Sqrt[b^2 - 4*a
*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) - b*(c*d^2 + a*e^2))*ArcTanh[(
Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(
Sqrt[2]*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
]) - (e*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(
c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e])))/e
```

3.538.3.1 Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.538.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.09

method	result
derivativedivides	$2e^2\left(\frac{4c\left(\frac{\left(a b e^3-4acd e^2+bc d^2 e+\sqrt{-e^2(4ac-b^2)}\,a e^2-\sqrt{-e^2(4ac-b^2)}\,c d^2\right)\sqrt{2}\arctan\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2(4ac-b^2)}\right)c}}\right)}{8c\sqrt{-e^2(4ac-b^2)}\sqrt{\left(be-2cd+\sqrt{-e^2(4ac-b^2)}\right)c}}-\cdots\right)}{a e \cdots}\cdots\right.$
default	$2e^2\left(\frac{4c\left(\frac{\left(a b e^3-4acd e^2+bc d^2 e+\sqrt{-e^2(4ac-b^2)}\,a e^2-\sqrt{-e^2(4ac-b^2)}\,c d^2\right)\sqrt{2}\arctan\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2(4ac-b^2)}\right)c}}\right)}{8c\sqrt{-e^2(4ac-b^2)}\sqrt{\left(be-2cd+\sqrt{-e^2(4ac-b^2)}\right)c}}-\cdots\right)}{a e \cdots}\cdots\right.$
pseudoelliptic	$\frac{-\sqrt{2}\left((e^2a-cd^2)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}-abe^3+4acde^2-bcd^2e\right)\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,\operatorname{arctanh}\left(\frac{c\sqrt{e\cdots}}{\sqrt{\left(-be+2cd+\cdots\right.}}\right.\cdots}{\cdots}$

input
```
int((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*e^2*(4/a/e^2*c*(1/8*(a*b*e^3-4*a*c*d*e^2+b*c*d^2*e+(-e^2*(4*a*c-b^2))^(1
/2)*a*e^2-(-e^2*(4*a*c-b^2))^(1/2)*c*d^2)/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/
2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2
^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-1/8*(-a*b*e^3+4*a*c
*d*e^2-b*c*d^2*e+(-e^2*(4*a*c-b^2))^(1/2)*a*e^2-(-e^2*(4*a*c-b^2))^(1/2)*c
*d^2)/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(
1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b
^2))^(1/2))*c)^(1/2)))-d^(3/2)/a/e^2*arctanh((e*x+d)^(1/2)/d^(1/2)))
```

3.538.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2581 vs. $2(286) = 572$.

Time = 4.77 (sec) , antiderivative size = 5167, normalized size of antiderivative = 15.20

$$\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.538.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{x(a+bx+cx^2)}\,dx$$

input
```
integrate((e*x+d)**(3/2)/x/(c*x**2+b*x+a),x)
```

output
```
Integral((d + e*x)**(3/2)/(x*(a + b*x + c*x**2)), x)
```

3.538.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+bx+a)x}\,dx$$

input
```
integrate((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*x), x)
```

3.538.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 833 vs. $2(286) = 572$.

Time = 0.33 (sec) , antiderivative size = 833, normalized size of antiderivative = 2.45

$$\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx = \frac{2\,d^2\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{a\sqrt{-d}}$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}c\right)e}((b^2c-4\,ac^2)d^2-(ab^2-4\,a^2c)e^2)a^2e^2-2\left(\sqrt{b^2-4\,ac}ac^2d^3-\sqrt{b^2-\right.}{}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}c\right)e}((b^2c-4\,ac^2)d^2-(ab^2-4\,a^2c)e^2)a^2e^2+2\left(\sqrt{b^2-4\,ac}ac^2d^3-\sqrt{b^2-\right.}{}$$

input
```
integrate((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
2*d^2*arctan(sqrt(e*x + d)/sqrt(-d))/(a*sqrt(-d)) - 1/4*(sqrt(-4*c^2*d + 2
*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^2*c - 4*a*c^2)*d^2 - (a*b^2 - 4*a^2*c)
*e^2)*a^2*e^2 - 2*(sqrt(b^2 - 4*a*c)*a*c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d
^2*e + sqrt(b^2 - 4*a*c)*a^2*c*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 -
4*a*c)*c)*e)*abs(a)*abs(e) - (2*a^2*b*c^2*d^3*e + 6*a^3*b*c*d*e^3 - a^3*b^
2*e^4 - (a^2*b^2*c + 8*a^3*c^2)*d^2*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2
 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a*c*d - a*b*e +
 sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))
/((sqrt(b^2 - 4*a*c)*a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*c*d*e + sqrt(b^
2 - 4*a*c)*a^3*c*e^2)*abs(a)*abs(c)*abs(e)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c
+ sqrt(b^2 - 4*a*c)*c)*e)*((b^2*c - 4*a*c^2)*d^2 - (a*b^2 - 4*a^2*c)*e^2)*
a^2*e^2 + 2*(sqrt(b^2 - 4*a*c)*a*c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d^2*e +
 sqrt(b^2 - 4*a*c)*a^2*c*d*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)
*c)*e)*abs(a)*abs(e) - (2*a^2*b*c^2*d^3*e + 6*a^3*b*c*d*e^3 - a^3*b^2*e^4
- (a^2*b^2*c + 8*a^3*c^2)*d^2*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a
*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a*c*d - a*b*e - sqrt(
-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/((sqr
t(b^2 - 4*a*c)*a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*c*d*e + sqrt(b^2 - 4*
a*c)*a^3*c*e^2)*abs(a)*abs(c)*abs(e))
```

3.538.9 Mupad [B] (verification not implemented)

Time = 18.02 (sec) , antiderivative size = 20897, normalized size of antiderivative = 61.46

$$\int \frac{(d+ex)^{3/2}}{x\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(3/2)/(x*(a + b*x + c*x^2)),x)
```

output
```
atan(((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)
^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(
-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*
c - 8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a
^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a
^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^
2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*
(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)*((b^4*c*
d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a
*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^
3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2
*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c
^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10
 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8
- 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 38
4*a^3*c^5*d^4*e^8 - 384*a^4*c^4*d^2*e^10 + 96*a^2*b^2*c^4*d^4*e^8 - 128*a^
2*b^3*c^3*d^3*e^9 + 32*a^2*b^4*c^2*d^2*e^10 - 32*a^3*b^2*c^3*d^2*e^10 + 12
8*a^4*b*c^3*d*e^11 + 512*a^3*b*c^4*d^3*e^9 - 32*a^3*b^3*c^2*d*e^11) + (d +
 e*x)^(1/2)*(32*a^3*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^1...
```

3.539 $\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\,dx$

3.539.1 Optimal result

Integrand size = 25, antiderivative size = 403

$$\begin{aligned}\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\,dx &= -\frac{d\sqrt{d+ex}}{ax} \\ &+ \frac{\sqrt{d}e\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} + \frac{2\sqrt{d}(bd-2ae)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2} \\ &- \frac{\sqrt{2}\sqrt{c}\left(b^2d^2+bd\left(\sqrt{b^2-4ac}d-2ae\right)-2a\left(cd^2+e\left(\sqrt{b^2-4ac}d-ae\right)\right)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} \\ &+ \frac{\sqrt{2}\sqrt{c}\left(b^2d^2-bd\left(\sqrt{b^2-4ac}d+2ae\right)-2a\left(cd^2-e\left(\sqrt{b^2-4ac}d+ae\right)\right)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\end{aligned}$$

output
```
e*arctanh((e*x+d)^(1/2)/d^(1/2))*d^(1/2)/a+2*(-2*a*e+b*d)*arctanh((e*x+d)^
(1/2)/d^(1/2))*d^(1/2)/a^2-d*(e*x+d)^(1/2)/a/x-arctanh(2^(1/2)*c^(1/2)*(e*
x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b^2*d^
2+b*d*(-2*a*e+d*(-4*a*c+b^2)^(1/2))-2*a*(c*d^2+e*(-a*e+d*(-4*a*c+b^2)^(1/2
))))/a^2/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh
(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(
1/2)*c^(1/2)*(b^2*d^2-b*d*(2*a*e+d*(-4*a*c+b^2)^(1/2))-2*a*(c*d^2-e*(a*e+d
*(-4*a*c+b^2)^(1/2))))/a^2/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/
2)))^(1/2)
```

3.539.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.59 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\,dx = \frac{-\frac{ad\sqrt{d+ex}}{x}+\frac{\sqrt{2}\sqrt{c}\left(-ib^2d^2+bd\left(\sqrt{-b^2+4ac}d+2iae\right)-2ia\left(-cd^2+e\left(-i\sqrt{-b^2+4ac}d+ae\right)\right)\right)\arctan\left(\frac{\cdots}{\sqrt{-2c\cdots}}\right.}{\sqrt{-b^2+4ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}\cdots}{\cdots}$$

input
```
Integrate[(d + e*x)^(3/2)/(x^2*(a + b*x + c*x^2)),x]
```

output
```
(-((a*d*Sqrt[d + e*x])/x) + (Sqrt[2]*Sqrt[c]*((-I)*b^2*d^2 + b*d*(Sqrt[-b^
2 + 4*a*c]*d + (2*I)*a*e) - (2*I)*a*(-(c*d^2) + e*((-I)*Sqrt[-b^2 + 4*a*c]
*d + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*S
qrt[-b^2 + 4*a*c]*e]])/(Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2
+ 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(I*b^2*d^2 + b*d*(Sqrt[-b^2 + 4*a*c]*d -
(2*I)*a*e) + (2*I)*a*(-(c*d^2) + e*(I*Sqrt[-b^2 + 4*a*c]*d + a*e)))*ArcTan
[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*
e]])/(Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e]) + Sq
rt[d]*(2*b*d - 3*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a^2
```

3.539.3 Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{x^2\left(a+bx+cx^2\right)}dx$$

$$\Big\downarrow \ 1199$$

$$\frac{2\int \left(\frac{d^2}{ax^2}-\frac{(bd-2ae)d}{a^2x}-\frac{e\left((bd-ae)\left(cd^2-bed+ae^2\right)-cd(bd-2ae)(d+ex)\right)}{a^2\left(cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex)\right)}\right)d\sqrt{d+ex}}{e}$$

$$\Big\downarrow \ 2009$$

$$2\left(-\frac{\sqrt{c}e\left(-2a\left(e\left(d\sqrt{b^2-4ac}-ae\right)+cd^2\right)+bd\left(d\sqrt{b^2-4ac}-2ae\right)+b^2d^2\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}e\left(-bd\left(d\sqrt{b^2-4ac}+2ae\right)+2a\right.}{}\right.$$

input
```
Int[(d + e*x)^(3/2)/(x^2*(a + b*x + c*x^2)),x]
```

output
```
(2*(-1/2*(d*e*Sqrt[d + e*x])/(a*x) + (Sqrt[d]*e^2*ArcTanh[Sqrt[d + e*x]/Sq
rt[d]])/(2*a) + (Sqrt[d]*e*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a
^2 - (Sqrt[c]*e*(b^2*d^2 + b*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) - 2*a*(c*d^2
+ e*(Sqrt[b^2 - 4*a*c]*d - a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/
Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*S
qrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[c]*e*(b^2*d^2 - 2*a*c*d^2
+ 2*a*e*(Sqrt[b^2 - 4*a*c]*d + a*e) - b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e))*A
rcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e]))/e
```

3.539.3.1 Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.539.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.01

method	result
derivativedivides	$2e^3 \frac{4c\left(-\frac{\left(2a^2e^3-2abd\,e^2-2ac\,d^2e+b^2d^2e-2\sqrt{-e^2\left(4ac-b^2\right)}\,ade+\sqrt{-e^2\left(4ac-b^2\right)}\,b\,d^2\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4\right.}\right.}}\right.}{8\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right.}{}$
default	$2e^3 \frac{4c\left(-\frac{\left(2a^2e^3-2abd\,e^2-2ac\,d^2e+b^2d^2e-2\sqrt{-e^2\left(4ac-b^2\right)}\,ade+\sqrt{-e^2\left(4ac-b^2\right)}\,b\,d^2\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4\right.}\right.}}\right.}{8\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right.}{}$
risch	$-\frac{d\sqrt{ex+d}}{ax}-\frac{e\left(\frac{\sqrt{d}\,(3ae-2bd)\,\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{ea}+\frac{8c\left(-\frac{\left(-2a^2e^3+2abd\,e^2+2ac\,d^2e-b^2d^2e+2\sqrt{-e^2\left(4ac-b^2\right)}\,ade-\sqrt{-e^2\left(4\right.}\right.}{8\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(-be+2c\right.}}\right.}{}\right.}{}$
pseudoelliptic	$-\frac{2\left(\left(\left(-d^{\frac{3}{2}}ea+\frac{b\,d^{\frac{5}{2}}}{2}\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+e\left(\left(-ac+\frac{b^2}{2}\right)d^{\frac{5}{2}}+ae\left(\sqrt{d}\,ea-b\,d^{\frac{3}{2}}\right)\right)\right)\sqrt{2}\,\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right.}\right.}{}$

input
```
int((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

3.539. $\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\,dx$

output
```
2*e^3*(4/a^2/e^3*c*(-1/8*(2*a^2*e^3-2*a*b*d*e^2-2*a*c*d^2*e+b^2*d^2*e-2*(-
e^2*(4*a*c-b^2))^(1/2)*a*d*e+(-e^2*(4*a*c-b^2))^(1/2)*b*d^2)/(-e^2*(4*a*c-
b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan
h(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))
+1/8*(-2*a^2*e^3+2*a*b*d*e^2+2*a*c*d^2*e-b^2*d^2*e-2*(-e^2*(4*a*c-b^2))^(1
/2)*a*d*e+(-e^2*(4*a*c-b^2))^(1/2)*b*d^2)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)
/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(
1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))-d/a^2/e^3*(1/2*a*(e*
x+d)^(1/2)/x+1/2*(3*a*e-2*b*d)/d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))))
```

3.539.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4324 vs. 2(344) = 688.

Time = 24.31 (sec) , antiderivative size = 8653, normalized size of antiderivative = 21.47

$$\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.539.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)/x**2/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

3.539.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{x^2\,(a+bx+cx^2)}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+bx+a)x^2}\,dx$$

input
```
integrate((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*x^2), x)
```

3.539.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 890 vs. 2(344) = 688.

Time = 0.36 (sec) , antiderivative size = 890, normalized size of antiderivative = 2.21

$$\int \frac{(d+ex)^{3/2}}{x^2\,(a+bx+cx^2)}\,dx = -\frac{\sqrt{ex+d}d}{ax} - \frac{(2\,bd^2-3\,ade)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{a^2\sqrt{-d}}$$

$$+ \frac{\left(\sqrt{-4\,c^2d+2\,(bc-\sqrt{b^2-4\,ac}c)e}((b^3-4\,abc)d^2-2\,(ab^2-4\,a^2c)de)e^2-2\,(\sqrt{b^2-4\,ac}bcd^3+2\,\sqrt{b^2-4\right.}{}$$

$$- \frac{\left(\sqrt{-4\,c^2d+2\,(bc+\sqrt{b^2-4\,ac}c)e}((b^3-4\,abc)d^2-2\,(ab^2-4\,a^2c)de)e^2+2\,(\sqrt{b^2-4\,ac}bcd^3+2\,\sqrt{b^2-4\right.}{}$$

input
```
integrate((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-sqrt(e*x + d)*d/(a*x) - (2*b*d^2 - 3*a*d*e)*arctan(sqrt(e*x + d)/sqrt(-d)
)/(a^2*sqrt(-d)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*(
(b^3 - 4*a*b*c)*d^2 - 2*(a*b^2 - 4*a^2*c)*d*e)*e^2 - 2*(sqrt(b^2 - 4*a*c)*
b*c*d^3 + 2*sqrt(b^2 - 4*a*c)*a*b*d*e^2 - sqrt(b^2 - 4*a*c)*a^2*e^3 - (b^2
 + a*c)*sqrt(b^2 - 4*a*c)*d^2*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c
)*c)*e)*abs(e) + (2*a^2*b*e^4 - 2*(b^2*c - 2*a*c^2)*d^3*e + (b^3 + 2*a*b*c
)*d^2*e^2 - 2*(a*b^2 + 2*a^2*c)*d*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 -
 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a^2*c*d - a^2*b*e
 + sqrt(-4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*a^2*c + (2*a^2*c*d - a^2*b*e)
^2))/(a^2*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e
 + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(c)*abs(e)) - 1/4*(sqrt(-4*c^2*d + 2*(b*c
 + sqrt(b^2 - 4*a*c)*c)*e)*((b^3 - 4*a*b*c)*d^2 - 2*(a*b^2 - 4*a^2*c)*d*e)
*e^2 + 2*(sqrt(b^2 - 4*a*c)*b*c*d^3 + 2*sqrt(b^2 - 4*a*c)*a*b*d*e^2 - sqrt
(b^2 - 4*a*c)*a^2*e^3 - (b^2 + a*c)*sqrt(b^2 - 4*a*c)*d^2*e)*sqrt(-4*c^2*d
 + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(e) + (2*a^2*b*e^4 - 2*(b^2*c - 2*a
*c^2)*d^3*e + (b^3 + 2*a*b*c)*d^2*e^2 - 2*(a*b^2 + 2*a^2*c)*d*e^3)*sqrt(-4
*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d
)/sqrt(-(2*a^2*c*d - a^2*b*e - sqrt(-4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*a
^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 -
 sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(c)*abs(e)...
```

3.539.9 Mupad [B] (verification not implemented)

Time = 17.20 (sec) , antiderivative size = 29890, normalized size of antiderivative = 74.17

$$\int \frac{(d+ex)^{3/2}}{x^2 (a+bx+cx^2)}\, dx = \text{Too large to display}$$

input
```
int((d + e*x)^(3/2)/(x^2*(a + b*x + c*x^2)),x)
```

output
```
(d^(1/2)*atan(((d^(1/2)*((8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^
8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4
*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*
a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 +
 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*
a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*
b*c^4*d^3*e^13))/a^4 - (d^(1/2)*((8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e
^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e
^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d
^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2
*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a
^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*
c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d
^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4 + (d^(1/2)*(
(8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*
e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 -
 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 +
 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^
9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d
^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4...
```

3.539. $\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\,dx$

3.540 $\int \frac{(d+ex)^{3/2}}{x^3(a+bx+cx^2)}\,dx$

3.540.1 Optimal result

Integrand size = 25, antiderivative size = 607

$$\int \frac{(d+ex)^{3/2}}{x^3(a+bx+cx^2)}\,dx = -\frac{d\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4ax}+\frac{(bd-2ae)\sqrt{d+ex}}{a^2x}$$
$$-\frac{3e^2\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4a\sqrt{d}}-\frac{e(bd-2ae)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}}$$
$$-\frac{2(b^2d^2-2abde-a(cd^2-ae^2))\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^3\sqrt{d}}$$
$$+\frac{\sqrt{2}\sqrt{c}\left(b^3d^2+b^2d\left(\sqrt{b^2-4ac}d-2ae\right)+a\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d-4ae\right)\right)-ab\left(3cd^2+e\left(2\sqrt{b^2-\cdots}\right.\right.\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$
$$-\frac{\sqrt{2}\sqrt{c}\left(b^3d^2-b^2d\left(\sqrt{b^2-4ac}d+2ae\right)-ab\left(3cd^2-e\left(2\sqrt{b^2-4ac}d+ae\right)\right)-a\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-\cdots}\right.\right.\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-3/4*e^2*arctanh((e*x+d)^(1/2)/d^(1/2))/a/d^(1/2)-e*(-2*a*e+b*d)*arctanh((
e*x+d)^(1/2)/d^(1/2))/a^2/d^(1/2)-2*(b^2*d^2-2*a*b*d*e-a*(-a*e^2+c*d^2))*a
rctanh((e*x+d)^(1/2)/d^(1/2))/a^3/d^(1/2)-1/2*d*(e*x+d)^(1/2)/a/x^2+3/4*e*
(e*x+d)^(1/2)/a/x+(-2*a*e+b*d)*(e*x+d)^(1/2)/a^2/x+arctanh(2^(1/2)*c^(1/2)
*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b^
3*d^2+b^2*d*(-2*a*e+d*(-4*a*c+b^2)^(1/2))+a*(a*e^2*(-4*a*c+b^2)^(1/2)-c*d*
(-4*a*e+d*(-4*a*c+b^2)^(1/2)))-a*b*(3*c*d^2+e*(-a*e+2*d*(-4*a*c+b^2)^(1/2)
)))/a^3/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-arctanh(
2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1
/2)*c^(1/2)*(b^3*d^2-b^2*d*(2*a*e+d*(-4*a*c+b^2)^(1/2))-a*(a*e^2*(-4*a*c+b
^2)^(1/2)-c*d*(4*a*e+d*(-4*a*c+b^2)^(1/2)))-a*b*(3*c*d^2-e*(a*e+2*d*(-4*a*
c+b^2)^(1/2))))/a^3/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1
/2)
```

3.540.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.74 (sec) , antiderivative size = 560, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^{3/2}}{x^3(a+bx+cx^2)}\,dx = \frac{\frac{a\sqrt{d+ex}(-2ad+4bdx-5aex)}{x^2} + \frac{4\sqrt{2}\sqrt{c}\left(ib^3d^2-b^2d\left(\sqrt{-b^2+4ac}d+2iae\right)+ab\left(-3icd^2+e\left(2\sqrt{-b^2+4ac}d+iae\right)\right.\right.}{\sqrt{-b^2+4ac}\sqrt{-}}}{}$$

input
```
Integrate[(d + e*x)^(3/2)/(x^3*(a + b*x + c*x^2)),x]
```

output
```
((a*Sqrt[d + e*x]*(-2*a*d + 4*b*d*x - 5*a*e*x))/x^2 + (4*Sqrt[2]*Sqrt[c]*(
I*b^3*d^2 - b^2*d*(Sqrt[-b^2 + 4*a*c]*d + (2*I)*a*e) + a*b*((-3*I)*c*d^2 +
 e*(2*Sqrt[-b^2 + 4*a*c]*d + I*a*e)) + a*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*
d*(Sqrt[-b^2 + 4*a*c]*d + (4*I)*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*
x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-b^2 + 4*a*c]*Sqrt
[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) - (4*Sqrt[2]*Sqrt[c]*(I*b^3*d^2 +
 b^2*d*(Sqrt[-b^2 + 4*a*c]*d - (2*I)*a*e) + a*(a*Sqrt[-b^2 + 4*a*c]*e^2 +
c*d*(-(Sqrt[-b^2 + 4*a*c]*d) + (4*I)*a*e)) + I*a*b*(-3*c*d^2 + e*((2*I)*Sq
rt[-b^2 + 4*a*c]*d + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2
*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (
b + I*Sqrt[-b^2 + 4*a*c])*e]) + ((-8*b^2*d^2 + 12*a*b*d*e + a*(8*c*d^2 - 3
*a*e^2))*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d])/(4*a^3)
```

3.540.3 Rubi [A] (verified)

Time = 2.46 (sec) , antiderivative size = 626, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{x^3\left(a+bx+cx^2\right)}\,dx$$

$$\Big\downarrow \ 1199$$

$$\frac{2\int \left(\frac{d^2}{ax^3}-\frac{(bd-2ae)d}{a^2x^2}+\frac{b^2d^2-2abed-a\left(cd^2-ae^2\right)}{a^3x}+\frac{e\left(\left(db^2-aeb-acd\right)\left(cd^2-bed+ae^2\right)-c\left(b^2d^2-2abed-a\left(cd^2-ae^2\right)\right)(d+ex)\right)}{a^3\left(cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex)\right)}\right)d\sqrt{d+e}}{e}$$

$$\Big\downarrow \ 2009$$

$$\frac{2\left(-\frac{e\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(-2abde-a\left(cd^2-ae^2\right)+b^2d^2\right)}{a^3\sqrt{d}}+\frac{\sqrt{c}e\left(-ab\left(e\left(2d\sqrt{b^2-4ac}-ae\right)+3cd^2\right)+a\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)+b^2}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{\right.}\right.}}\right.}{}$$

input
```
Int[(d + e*x)^(3/2)/(x^3*(a + b*x + c*x^2)),x]
```

output
```
(2*(-1/4*(d*e*Sqrt[d + e*x])/(a*x^2) + (3*e^2*Sqrt[d + e*x])/(8*a*x) + (e*
(b*d - 2*a*e)*Sqrt[d + e*x])/(2*a^2*x) - (3*e^3*ArcTanh[Sqrt[d + e*x]/Sqrt
[d]])/(8*a*Sqrt[d]) - (e^2*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(
2*a^2*Sqrt[d]) - (e*(b^2*d^2 - 2*a*b*d*e - a*(c*d^2 - a*e^2))*ArcTanh[Sqrt
[d + e*x]/Sqrt[d]])/(a^3*Sqrt[d]) + (Sqrt[c]*e*(b^3*d^2 + b^2*d*(Sqrt[b^2
- 4*a*c]*d - 2*a*e) + a*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*
d - 4*a*e)) - a*b*(3*c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - a*e)))*ArcTanh[(Sq
rt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sq
rt[2]*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sq
rt[c]*e*(b^3*d^2 - b^2*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - a*b*(3*c*d^2 - e*
(2*Sqrt[b^2 - 4*a*c]*d + a*e)) - a*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^
2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e])))/e
```

3.540.3.1 Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.540.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 560, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$-\frac{-8\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,x^2c\left(\frac{\left(-a\,d^{\frac{3}{2}}be+\frac{(-ac+b^2)d^{\frac{5}{2}}}{2}+\frac{e^2a^2\sqrt{d}}{2}\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}}{2}+e\left(ae\left(ac-\frac{b^2}{2}\right)d^{\frac{3}{2}}\cdots\right.\right.}{\cdots}$
risch	$-\frac{\sqrt{ex+d}\,(5aex-4bdx+2ad)}{4a^2x^2}-\frac{e\left(-\frac{\left(-3e^2a^2+12abde+8c\,d^2a-8b^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{ae\sqrt{d}}+\frac{32c\left(-\frac{\left(a^2b\,e^3+4a^2cd\,e^2-2a\,b^2\cdots\right.}{\cdots}\right.}{\cdots}\cdots\right.}{\cdots}$
derivativedivides	$2e^4\left(-\frac{\frac{\frac{ae(5ae-4bd)(ex+d)^{\frac{3}{2}}}{8}+\left(\frac{1}{2}ab\,d^2e-\frac{3}{8}d\,a^2e^2\right)\sqrt{ex+d}}{e^2x^2}+\frac{\left(3e^2a^2-12abde-8c\,d^2a+8b^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{8\sqrt{d}}}{a^3e^4}+\frac{4c\left(-\cdots\right.}{\cdots}\cdots\right.$
default	$2e^4\left(-\frac{\frac{\frac{ae(5ae-4bd)(ex+d)^{\frac{3}{2}}}{8}+\left(\frac{1}{2}ab\,d^2e-\frac{3}{8}d\,a^2e^2\right)\sqrt{ex+d}}{e^2x^2}+\frac{\left(3e^2a^2-12abde-8c\,d^2a+8b^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{8\sqrt{d}}}{a^3e^4}+\frac{4c\left(-\cdots\right.}{\cdots}\cdots\right.$

input
```
int((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

3.540. $\int \frac{(d+ex)^{3/2}}{x^3(a+bx+cx^2)}\,dx$

output
```
-1/2/(-4*e^2*(a*c-1/4*b^2))^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2)
)*c)^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/d^(1/2)*(-8
*2^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*x^2*c*(1/2*(-a
*d^(3/2)*b*e+1/2*(-a*c+b^2)*d^(5/2)+1/2*e^2*a^2*d^(1/2))*(-4*e^2*(a*c-1/4*
b^2))^(1/2)+e*(a*e*(a*c-1/2*b^2)*d^(3/2)+1/4*b*((-3*a*c+b^2)*d^(5/2)+e^2*a
^2*d^(1/2))))*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/
4*b^2))^(1/2))*c)^(1/2))+(-8*2^(1/2)*(1/2*(a*d^(3/2)*b*e+1/2*d^(5/2)*(a*c-
b^2)-1/2*e^2*a^2*d^(1/2))*(-4*e^2*(a*c-1/4*b^2))^(1/2)+e*(a*e*(a*c-1/2*b^2
)*d^(3/2)+1/4*b*((-3*a*c+b^2)*d^(5/2)+e^2*a^2*d^(1/2))))*x^2*c*arctan(c*(e
*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+(-
4*e^2*(a*c-1/4*b^2))^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1
/2)*(3/2*(e^2*a^2+4*(-b*d*e-2/3*c*d^2)*a+8/3*b^2*d^2)*x^2*arctanh((e*x+d)^
(1/2)/d^(1/2))+((-2*b*x+a)*d^(3/2)+5/2*a*d^(1/2)*e*x)*a*(e*x+d)^(1/2)))*((
-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))/x^2/a^3
```

3.540.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7205 vs. $2(523) = 1046$.

Time = 213.71 (sec) , antiderivative size = 14417, normalized size of antiderivative = 23.75

$$\int \frac{(d+ex)^{3/2}}{x^3(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.540.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{x^3(a+bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)/x**3/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

3.540.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{x^3(a+bx+cx^2)}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+bx+a)x^3}\,dx$$

input
```
integrate((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*x^3), x)
```

3.540.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1129 vs. 2(523) = 1046.

Time = 0.38 (sec) , antiderivative size = 1129, normalized size of antiderivative = 1.86

$$\int \frac{(d+ex)^{3/2}}{x^3(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/4*(8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e + 3*a^2*e^2)*arctan(sqrt(e*x + d)/
sqrt(-d))/(a^3*sqrt(-d)) - 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)
*c)*e)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e + (a
^2*b^2 - 4*a^3*c)*e^2)*a^2*e^2 + 2*(sqrt(b^2 - 4*a*c)*a*b^3*d^2*e + sqrt(b
^2 - 4*a*c)*a^3*b*e^3 - (a*b^2*c - a^2*c^2)*sqrt(b^2 - 4*a*c)*d^3 - (2*a^2
*b^2 - a^3*c)*sqrt(b^2 - 4*a*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 -
 4*a*c)*c)*e)*abs(a)*abs(e) + (a^4*b^2*e^4 - 2*(a^2*b^3*c - 3*a^3*b*c^2)*d
^3*e + (a^2*b^4 + a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - a^4*b*c)*d
*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)
*sqrt(e*x + d)/sqrt(-(2*a^3*c*d - a^3*b*e + sqrt(-4*(a^3*c*d^2 - a^3*b*d*e
 + a^4*e^2)*a^3*c + (2*a^3*c*d - a^3*b*e)^2))/(a^3*c)))/((sqrt(b^2 - 4*a*c
)*a^4*c*d^2 - sqrt(b^2 - 4*a*c)*a^4*b*d*e + sqrt(b^2 - 4*a*c)*a^5*e^2)*abs
(a)*abs(c)*abs(e)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)
*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2
 - 4*a^3*c)*e^2)*a^2*e^2 - 2*(sqrt(b^2 - 4*a*c)*a*b^3*d^2*e + sqrt(b^2 - 4
*a*c)*a^3*b*e^3 - (a*b^2*c - a^2*c^2)*sqrt(b^2 - 4*a*c)*d^3 - (2*a^2*b^2 -
 a^3*c)*sqrt(b^2 - 4*a*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c
)*c)*e)*abs(a)*abs(e) + (a^4*b^2*e^4 - 2*(a^2*b^3*c - 3*a^3*b*c^2)*d^3*e +
 (a^2*b^4 + a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - a^4*b*c)*d*e^3)*
sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sq...
```

3.540.9 Mupad [B] (verification not implemented)

Time = 18.13 (sec) , antiderivative size = 44649, normalized size of antiderivative = 73.56

$$\int \frac{(d+ex)^{3/2}}{x^3(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(3/2)/(x^3*(a + b*x + c*x^2)),x)
```

output
```
(((3*a*d*e^2 - 4*b*d^2*e)*(d + e*x)^(1/2))/(4*a^2) - ((5*a*e^2 - 4*b*d*e)*
(d + e*x)^(3/2))/(4*a^2))/((d + e*x)^2 - 2*d*(d + e*x) + d^2) + atan((((((
(192*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^
10*c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*
b^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*
a^9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^1
0 + 1408*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^
9 - 576*a^10*b^3*c^3*d*e^11)/(2*a^8) - ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^
5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3
 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2
 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^
3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a
^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3
*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*
a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)
^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(
-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(
1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a
^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 -...
```

3.541 $\int \frac{x^m(e+fx)^n}{a+bx+cx^2} \, dx$

3.541.1 Optimal result

Integrand size = 23, antiderivative size = 201

$$\int \frac{x^m(e+fx)^n}{a+bx+cx^2} \, dx = \frac{2cx^{1+m}(e+fx)^n \left(1+\frac{fx}{e}\right)^{-n} \text{AppellF1}\left(1+m,-n,1,2+m,-\frac{fx}{e},-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)(1+m)} - \frac{2cx^{1+m}(e+fx)^n \left(1+\frac{fx}{e}\right)^{-n} \text{AppellF1}\left(1+m,-n,1,2+m,-\frac{fx}{e},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)(1+m)}$$

output
```
2*c*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,-2*c*x/(b-(-4*a*c+b^2)^(1/2)),
-f*x/e)/(1+m)/((1+f*x/e)^n)/(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)-2*c*
x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,-2*c*x/(b+(-4*a*c+b^2)^(1/2)),-f*x
/e)/(1+m)/((1+f*x/e)^n)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

3.541.2 Mathematica [F]

$$\int \frac{x^m(e+fx)^n}{a+bx+cx^2} \, dx = \int \frac{x^m(e+fx)^n}{a+bx+cx^2} \, dx$$

input
```
Integrate[(x^m*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

output
```
Integrate[(x^m*(e + f*x)^n)/(a + b*x + c*x^2), x]
```

3.541.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(e+fx)^n}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \ 1205$$

$$\int \left(\frac{2cx^m(e+fx)^n}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}+b+2cx\right)}-\frac{2cx^m(e+fx)^n}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b+2cx\right)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2cx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{fx}{e},-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{fx}{e},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[(x^m*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

output
```
(2*c*x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), (-2*c
*x)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*(
1 + m)*(1 + (f*x)/e)^n) - (2*c*x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1
, 2 + m, -((f*x)/e), (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]
*(b + Sqrt[b^2 - 4*a*c])*(1 + m)*(1 + (f*x)/e)^n)
```

3.541.3.1 Defintions of rubi rules used

rule 1205
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
 !IntegerQ[m] &&  !IntegerQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.541.4 Maple [F]

$$\int \frac{x^m (fx+e)^n}{c\,x^2+bx+a}dx$$

input
```
int(x^m*(f*x+e)^n/(c*x^2+b*x+a),x)
```

output
```
int(x^m*(f*x+e)^n/(c*x^2+b*x+a),x)
```

3.541.5 Fricas [F]

$$\int \frac{x^m (e+fx)^n}{a+bx+cx^2}\,dx = \int \frac{(fx+e)^n x^m}{cx^2+bx+a}\,dx$$

input
```
integrate(x^m*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n*x^m/(c*x^2 + b*x + a), x)
```

3.541.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m (e+fx)^n}{a+bx+cx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(f*x+e)**n/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

3.541.7 Maxima [F]

$$\int \frac{x^m(e+fx)^n}{a+bx+cx^2} \, dx = \int \frac{(fx+e)^n x^m}{cx^2+bx+a} \, dx$$

input
```
integrate(x^m*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n*x^m/(c*x^2 + b*x + a), x)
```

3.541.8 Giac [F]

$$\int \frac{x^m(e+fx)^n}{a+bx+cx^2} \, dx = \int \frac{(fx+e)^n x^m}{cx^2+bx+a} \, dx$$

input
```
integrate(x^m*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n*x^m/(c*x^2 + b*x + a), x)
```

3.541.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(e+fx)^n}{a+bx+cx^2} \, dx = \int \frac{x^m \, (e+f\,x)^n}{c\,x^2+b\,x+a} \, dx$$

input
```
int((x^m*(e + f*x)^n)/(a + b*x + c*x^2),x)
```

output
```
int((x^m*(e + f*x)^n)/(a + b*x + c*x^2), x)
```

3.542 $\int \frac{x^3(e+fx)^n}{a+bx+cx^2} dx$

3.542.1 Optimal result

Integrand size = 23, antiderivative size = 290

$$\int \frac{x^3(e+fx)^n}{a+bx+cx^2} dx = -\frac{(ce+bf)(e+fx)^{1+n}}{c^2f^2(1+n)} + \frac{(e+fx)^{2+n}}{cf^2(2+n)} + \frac{\left(a-\frac{b^2}{c}+\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} + \frac{\left(a-\frac{b^2}{c}-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{c\left(2ce-\left(b+\sqrt{b^2-4ac}\right)f\right)(1+n)}$$

output
```
-(b*f+c*e)*(f*x+e)^(1+n)/c^2/f^2/(1+n)+(f*x+e)^(2+n)/c/f^2/(2+n)+(f*x+e)^(
1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2)))
)*(a-b^2/c+b*(-3*a*c+b^2)/c/(-4*a*c+b^2)^(1/2))/c/(1+n)/(2*c*e-f*(b-(-4*a*
c+b^2)^(1/2)))+(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f
*(b+(-4*a*c+b^2)^(1/2))))*(a-b^2/c-b*(-3*a*c+b^2)/c/(-4*a*c+b^2)^(1/2))/c/
(1+n)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)))
```

3.542.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.10

$$\int \frac{x^3(e+fx)^n}{a+bx+cx^2}\,dx$$
$$= \frac{(e+fx)^{1+n}\left(\frac{\left(b^3-3abc-b^2\sqrt{b^2-4ac}+ac\sqrt{b^2-4ac}\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce+\left(-b+\sqrt{b^2-4ac}\right)f}\right)}{2ce+\left(-b+\sqrt{b^2-4ac}\right)f}+\frac{-\left(\left(b\left(b+\sqrt{b^2-4ac}\right)f-\right.\right.}{}\right.}{c^2\sqrt{b}}$$

input
```
Integrate[(x^3*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

output
```
((e + f*x)^(1 + n)*(((b^3 - 3*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + a*c*Sqrt[b^2
 - 4*a*c])*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e + (-b
 + Sqrt[b^2 - 4*a*c])*f)])/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f) + (-((b*(b
 + Sqrt[b^2 - 4*a*c])*f - 2*c*(Sqrt[b^2 - 4*a*c]*e + 2*a*f))*(b*f*(2 + n)
+ c*(e - f*(1 + n)*x))) + (b^3 - 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqr
t[b^2 - 4*a*c])*f^2*(2 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f
*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/(f^2*(-2*c*e + (b + Sqrt[b^2 -
4*a*c])*f)*(2 + n))))/(c^2*Sqrt[b^2 - 4*a*c]*(1 + n))
```

3.542.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(e+fx)^n}{a+bx+cx^2}\,dx$$
$$\Big\downarrow \text{ 1200}$$
$$\int \left(\frac{\left(-\frac{b(b^2-3ac)}{c^2\sqrt{b^2-4ac}}-\frac{a}{c}+\frac{b^2}{c^2}\right)(e+fx)^n}{-\sqrt{b^2-4ac}+b+2cx}+\frac{\left(\frac{b(b^2-3ac)}{c^2\sqrt{b^2-4ac}}-\frac{a}{c}+\frac{b^2}{c^2}\right)(e+fx)^n}{\sqrt{b^2-4ac}+b+2cx}+\frac{(-bf-ce)(e+fx)^n}{c^2f}+\frac{(e+fx)^{n+1}}{cf}\right.$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{c(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{(bf+ce)(e+fx)^{n+1}}{c^2f^2(n+1)}+\frac{(e+fx)^{n+2}}{cf^2(n+2)}$$

input
```
Int[(x^3*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

output
```
-(((c*e + b*f)*(e + f*x)^(1 + n))/(c^2*f^2*(1 + n))) + (e + f*x)^(2 + n)/(
c*f^2*(2 + n)) + ((a - b^2/c + (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*(e
 + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e
- (b - Sqrt[b^2 - 4*a*c])*f)])/(c*(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)*(1 +
 n)) + ((a - b^2/c - (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*(e + f*x)^(1
 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqr
t[b^2 - 4*a*c])*f)])/(c*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

3.542.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.542.4 Maple [F]

$$\int \frac{x^3(fx+e)^n}{c\,x^2+bx+a}dx$$

input
```
int(x^3*(f*x+e)^n/(c*x^2+b*x+a),x)
```

output
```
int(x^3*(f*x+e)^n/(c*x^2+b*x+a),x)
```

3.542.5 Fricas [F]

$$\int \frac{x^3(e+fx)^n}{a+bx+cx^2}\,dx = \int \frac{(fx+e)^n x^3}{cx^2+bx+a}\,dx$$

input
```
integrate(x^3*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n*x^3/(c*x^2 + b*x + a), x)
```

3.542.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{x^3(e+fx)^n}{a+bx+cx^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**3*(f*x+e)**n/(c*x**2+b*x+a),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.542.7 Maxima [F]

$$\int \frac{x^3(e+fx)^n}{a+bx+cx^2}\,dx=\int \frac{(fx+e)^nx^3}{cx^2+bx+a}\,dx$$

input
```
integrate(x^3*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n*x^3/(c*x^2 + b*x + a), x)
```

3.542.8 Giac [F]

$$\int \frac{x^3(e+fx)^n}{a+bx+cx^2}\,dx=\int \frac{(fx+e)^nx^3}{cx^2+bx+a}\,dx$$

input
```
integrate(x^3*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n*x^3/(c*x^2 + b*x + a), x)
```

3.542.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(e+fx)^n}{a+bx+cx^2}\,dx=\int \frac{x^3\,(e+f\,x)^n}{c\,x^2+b\,x+a}\,dx$$

input
```
int((x^3*(e + f*x)^n)/(a + b*x + c*x^2),x)
```

output
```
int((x^3*(e + f*x)^n)/(a + b*x + c*x^2), x)
```

3.543 $\int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx$

3.543.1 Optimal result

Integrand size = 23, antiderivative size = 237

$$\begin{aligned}\int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx &= \frac{(e+fx)^{1+n}}{cf(1+n)} \\ &+ \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} \\ &+ \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{c\left(2ce-\left(b+\sqrt{b^2-4ac}\right)f\right)(1+n)}\end{aligned}$$

output
```
(f*x+e)^(1+n)/c/f/(1+n)+(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)
/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2))))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/c/(1
+n)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2)))+(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n
],2*c*(f*x+e)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2))))*(b+(-2*a*c+b^2)/(-4*a*c+b^
2)^(1/2))/c/(1+n)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)))
```

3.543.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.22

$$\int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx =$$
$$-\frac{2(e+fx)^{1+n}\left(2\sqrt{b^2-4ac}(ce^2+f(-be+af))+f\left(-b^2e+2ace+b\sqrt{b^2-4ac}e+abf-a\sqrt{b^2-4ac}f\right.\right.}{\sqrt{b^2}}$$

input
```
Integrate[(x^2*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

output
```
(-2*(e + f*x)^(1 + n)*(2*Sqrt[b^2 - 4*a*c]*(c*e^2 + f*(-(b*e) + a*f)) + f*
(-(b^2*e) + 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e + a*b*f - a*Sqrt[b^2 - 4*a*c]*
f)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e + (-b + Sqrt[
b^2 - 4*a*c])*f)] + f*(b^2*e - 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e - a*b*f - a
*Sqrt[b^2 - 4*a*c]*f)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(
2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)]))/(Sqrt[b^2 - 4*a*c]*f*(2*c*e + (-b +
Sqrt[b^2 - 4*a*c])*f)*(-2*c*e + (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

3.543.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx$$
$$\Big\downarrow \; 1200$$
$$\int \left(\frac{\left(\frac{b^2-2ac}{c\sqrt{b^2-4ac}}-\frac{b}{c}\right)(e+fx)^n}{-\sqrt{b^2-4ac}+b+2cx}+\frac{\left(-\frac{b^2-2ac}{c\sqrt{b^2-4ac}}-\frac{b}{c}\right)(e+fx)^n}{\sqrt{b^2-4ac}+b+2cx}+\frac{(e+fx)^n}{c}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{c(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}+\frac{(e+fx)^{n+1}}{cf(n+1)}$$

input
```
Int[(x^2*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

output
```
(e + f*x)^(1 + n)/(c*f*(1 + n)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(
e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e
 - (b - Sqrt[b^2 - 4*a*c])*f)])/(c*(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)*(1
+ n)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeom
etric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])
*f)])/(c*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

3.543.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.543.4 Maple [F]

$$\int \frac{x^2(fx+e)^n}{c\,x^2+bx+a}dx$$

input
```
int(x^2*(f*x+e)^n/(c*x^2+b*x+a),x)
```

output
```
int(x^2*(f*x+e)^n/(c*x^2+b*x+a),x)
```

3.543.5 Fricas [F]

$$\int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx = \int \frac{(fx+e)^n x^2}{cx^2+bx+a}\,dx$$

input
```
integrate(x^2*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n*x^2/(c*x^2 + b*x + a), x)
```

3.543.6 Sympy [F]

$$\int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx = \int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx$$

input
```
integrate(x**2*(f*x+e)**n/(c*x**2+b*x+a),x)
```

output
```
Integral(x**2*(e + f*x)**n/(a + b*x + c*x**2), x)
```

3.543.7 Maxima [F]

$$\int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx = \int \frac{(fx+e)^n x^2}{cx^2+bx+a}\,dx$$

input
```
integrate(x^2*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n*x^2/(c*x^2 + b*x + a), x)
```

3.543.8 Giac [F]

$$\int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx = \int \frac{(fx+e)^n x^2}{cx^2+bx+a}\,dx$$

input
```
integrate(x^2*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n*x^2/(c*x^2 + b*x + a), x)
```

3.543.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx = \int \frac{x^2\,(e+f\,x)^n}{c\,x^2+b\,x+a}\,dx$$

input
```
int((x^2*(e + f*x)^n)/(a + b*x + c*x^2),x)
```

output
```
int((x^2*(e + f*x)^n)/(a + b*x + c*x^2), x)
```

3.544 $\int \frac{x(e+fx)^n}{a+bx+cx^2} dx$

3.544.1 Optimal result

Integrand size = 21, antiderivative size = 198

$$\int \frac{x(e+fx)^n}{a+bx+cx^2} dx = -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} - \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{\left(2ce-\left(b+\sqrt{b^2-4ac}\right)f\right)(1+n)}$$

output
```
-(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b-(-4*a*c+b^
2)^(1/2))))*(1-b/(-4*a*c+b^2)^(1/2))/(1+n)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2))
)-(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b+(-4*a*c+b
^2)^(1/2))))*(1+b/(-4*a*c+b^2)^(1/2))/(1+n)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)
))
```

3.544.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.92

$$\int \frac{x(e+fx)^n}{a+bx+cx^2}\,dx = \frac{(e+fx)^{1+n}\left(-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce+\left(-b+\sqrt{b^2-4ac}\right)f}\right)}{2ce+\left(-b+\sqrt{b^2-4ac}\right)f}-\frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+n\right.}{2ce-\left(b+\sqrt{b^2-4ac}\right.}\right.}{1+n}$$

input
```
Integrate[(x*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

output
```
((e + f*x)^(1 + n)*(-(((1 - b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 +
n, 2 + n, (2*c*(e + f*x))/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)])/(2*c*e +
(-b + Sqrt[b^2 - 4*a*c])*f)) - ((1 + b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F
1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/(
2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)))/(1 + n)
```

3.544.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(e+fx)^n}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^n}{-\sqrt{b^2-4ac}+b+2cx}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(e+fx)^n}{\sqrt{b^2-4ac}+b+2cx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}$$

input
```
Int[(x*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

output
```
-(((1 - b/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
 2 + n, (2*c*(e + f*x))/(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/((2*c*e - (b
 - Sqrt[b^2 - 4*a*c])*f)*(1 + n))) - ((1 + b/Sqrt[b^2 - 4*a*c])*(e + f*x)^
(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + S
qrt[b^2 - 4*a*c])*f)])/((2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

3.544.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.544.4 Maple [F]

$$\int \frac{x(fx+e)^n}{c\,x^2+bx+a}dx$$

input
```
int(x*(f*x+e)^n/(c*x^2+b*x+a),x)
```

output
```
int(x*(f*x+e)^n/(c*x^2+b*x+a),x)
```

3.544.5 Fricas [F]

$$\int \frac{x(e+fx)^n}{a+bx+cx^2} \, dx = \int \frac{(fx+e)^n x}{cx^2+bx+a} \, dx$$

input
```
integrate(x*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n*x/(c*x^2 + b*x + a), x)
```

3.544.6 Sympy [F]

$$\int \frac{x(e+fx)^n}{a+bx+cx^2} \, dx = \int \frac{x(e+fx)^n}{a+bx+cx^2} \, dx$$

input
```
integrate(x*(f*x+e)**n/(c*x**2+b*x+a),x)
```

output
```
Integral(x*(e + f*x)**n/(a + b*x + c*x**2), x)
```

3.544.7 Maxima [F]

$$\int \frac{x(e+fx)^n}{a+bx+cx^2} \, dx = \int \frac{(fx+e)^n x}{cx^2+bx+a} \, dx$$

input
```
integrate(x*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n*x/(c*x^2 + b*x + a), x)
```

3.544.8 Giac [F]

$$\int \frac{x(e+fx)^n}{a+bx+cx^2}\,dx=\int \frac{(fx+e)^n x}{cx^2+bx+a}\,dx$$

input
```
integrate(x*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n*x/(c*x^2 + b*x + a), x)
```

3.544.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(e+fx)^n}{a+bx+cx^2}\,dx=\int \frac{x\,(e+f\,x)^n}{c\,x^2+b\,x+a}\,dx$$

input
```
int((x*(e + f*x)^n)/(a + b*x + c*x^2),x)
```

output
```
int((x*(e + f*x)^n)/(a + b*x + c*x^2), x)
```

3.545 $\int \frac{(e+fx)^n}{a+bx+cx^2}\,dx$

3.545.1 Optimal result

Integrand size = 20, antiderivative size = 191

$$\int \frac{(e+fx)^n}{a+bx+cx^2}\,dx = -\frac{2c(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{\sqrt{b^2-4ac}\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} + \frac{2c(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{\sqrt{b^2-4ac}\left(2ce-\left(b+\sqrt{b^2-4ac}\right)f\right)(1+n)}$$

output
```
-2*c*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b-(-4*a*
c+b^2)^(1/2))))/(1+n)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)+
2*c*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b+(-4*a*c
+b^2)^(1/2))))/(1+n)/(-4*a*c+b^2)^(1/2)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)))
```

3.545.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.85

$$\int \frac{(e+fx)^n}{a+bx+cx^2}\,dx = \frac{2c(e+fx)^{1+n}\left(-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce+\left(-b+\sqrt{b^2-4ac}\right)f}\right)}{2ce+\left(-b+\sqrt{b^2-4ac}\right)f}+\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{\sqrt{b^2-4ac}(1+n)}$$

input
```
Integrate[(e + f*x)^n/(a + b*x + c*x^2),x]
```

output
```
(2*c*(e + f*x)^(1 + n)*(-(Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x
))/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)]/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])
*f)) + Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sq
rt[b^2 - 4*a*c])*f)]/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)))/(Sqrt[b^2 - 4*a
*c]*(1 + n))
```

3.545.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1150, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^n}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1150}$$

$$\int \left(\frac{2c(e+fx)^n}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}+b+2cx\right)}-\frac{2c(e+fx)^n}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b+2cx\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2c(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\sqrt{b^2-4ac}\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{2c(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\sqrt{b^2-4ac}\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}$$

input
```
Int[(e + f*x)^n/(a + b*x + c*x^2),x]
```

output
```
(-2*c*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))
/(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/(Sqrt[b^2 - 4*a*c]*(2*c*e - (b - Sq
rt[b^2 - 4*a*c])*f)*(1 + n)) + (2*c*(e + f*x)^(1 + n)*Hypergeometric2F1[1,
 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/(Sqrt
[b^2 - 4*a*c]*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

3.545.3.1 Defintions of rubi rules used

rule 1150
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[ExpandIntegrand[(d + e*x)^m, 1/(a + b*x + c*x^2), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && !IntegerQ[2*m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.545.4 Maple [F]

$$\int \frac{(fx+e)^n}{c\,x^2+bx+a}dx$$

input
```
int((f*x+e)^n/(c*x^2+b*x+a),x)
```

output
```
int((f*x+e)^n/(c*x^2+b*x+a),x)
```

3.545.5 Fricas [F]

$$\int \frac{(e+fx)^n}{a+bx+cx^2}\,dx=\int \frac{(fx+e)^n}{cx^2+bx+a}\,dx$$

input
```
integrate((f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n/(c*x^2 + b*x + a), x)
```

3.545.6 Sympy [F]

$$\int \frac{(e+fx)^n}{a+bx+cx^2}\,dx=\int \frac{(e+fx)^n}{a+bx+cx^2}\,dx$$

input
```
integrate((f*x+e)**n/(c*x**2+b*x+a),x)
```

output
```
Integral((e + f*x)**n/(a + b*x + c*x**2), x)
```

3.545.7 Maxima [F]

$$\int \frac{(e+fx)^n}{a+bx+cx^2}\,dx=\int \frac{(fx+e)^n}{cx^2+bx+a}\,dx$$

input
```
integrate((f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n/(c*x^2 + b*x + a), x)
```

3.545.8 Giac [F]

$$\int \frac{(e+fx)^n}{a+bx+cx^2} \, dx = \int \frac{(fx+e)^n}{cx^2+bx+a} \, dx$$

input
```
integrate((f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n/(c*x^2 + b*x + a), x)
```

3.545.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{a+bx+cx^2} \, dx = \int \frac{(e+f\,x)^n}{c\,x^2+b\,x+a} \, dx$$

input
```
int((e + f*x)^n/(a + b*x + c*x^2),x)
```

output
```
int((e + f*x)^n/(a + b*x + c*x^2), x)
```

3.546 $\int \frac{(e+fx)^n}{x(a+bx+cx^2)} dx$

3.546.1 Optimal result

Integrand size = 23, antiderivative size = 242

$$\int \frac{(e+fx)^n}{x(a+bx+cx^2)}\,dx = \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{a\left(2ce-\left(b+\sqrt{b^2-4ac}\right)f\right)(1+n)} - \frac{(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{fx}{e}\right)}{ae(1+n)}$$

output
```
-(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],1+f*x/e)/a/e/(1+n)+c*(f*x+e)^(1+n)
*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2))))*(1
+b/(-4*a*c+b^2)^(1/2))/a/(1+n)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2)))+c*(f*x+e)^
(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2))
))*(1-b/(-4*a*c+b^2)^(1/2))/a/(1+n)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)))
```

3.546.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.86

$$\int \frac{(e+fx)^n}{x\left(a+bx+cx^2\right)}\,dx$$

$$= \frac{(e+fx)^{1+n}\left(\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce+\left(-b+\sqrt{b^2-4ac}\right)f}\right)}{2ce+\left(-b+\sqrt{b^2-4ac}\right)f}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+n\right.}{2ce-\left(b+\sqrt{b^2-4ac}\right)}\right.}{a(1+n)}$$

input
```
Integrate[(e + f*x)^n/(x*(a + b*x + c*x^2)),x]
```

output
```
((e + f*x)^(1 + n)*((c*(1 + b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 +
n, 2 + n, (2*c*(e + f*x))/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)])/(2*c*e +
(-b + Sqrt[b^2 - 4*a*c])*f) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*Hypergeometric2
F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/
(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f) - Hypergeometric2F1[1, 1 + n, 2 + n, 1
 + (f*x)/e]/e))/(a*(1 + n))
```

3.546.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^n}{x\left(a+bx+cx^2\right)}\,dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(\frac{(-b-cx)(e+fx)^n}{a\left(a+bx+cx^2\right)}+\frac{(e+fx)^n}{ax}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{a(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{fx}{e}+1\right)}{ae(n+1)}$$

input
```
Int[(e + f*x)^n/(x*(a + b*x + c*x^2)),x]
```

output
```
(c*(1 + b/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
 2 + n, (2*c*(e + f*x))/(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/(a*(2*c*e -
(b - Sqrt[b^2 - 4*a*c])*f)*(1 + n)) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*(e + f*
x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b
+ Sqrt[b^2 - 4*a*c])*f)])/(a*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
- ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e
*(1 + n))
```

3.546.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.546.4 Maple [F]

$$\int \frac{(fx+e)^n}{x\,(c\,x^2+bx+a)}dx$$

input
```
int((f*x+e)^n/x/(c*x^2+b*x+a),x)
```

output
```
int((f*x+e)^n/x/(c*x^2+b*x+a),x)
```

3.546.5 Fricas [F]

$$\int \frac{(e+fx)^n}{x\,(a+bx+cx^2)}\,dx = \int \frac{(fx+e)^n}{(cx^2+bx+a)x}\,dx$$

input
```
integrate((f*x+e)^n/x/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n/(c*x^3 + b*x^2 + a*x), x)
```

3.546.6 Sympy [F]

$$\int \frac{(e+fx)^n}{x\,(a+bx+cx^2)}\,dx = \int \frac{(e+fx)^n}{x\,(a+bx+cx^2)}\,dx$$

input
```
integrate((f*x+e)**n/x/(c*x**2+b*x+a),x)
```

output
```
Integral((e + f*x)**n/(x*(a + b*x + c*x**2)), x)
```

3.546.7 Maxima [F]

$$\int \frac{(e+fx)^n}{x(a+bx+cx^2)}\,dx = \int \frac{(fx+e)^n}{(cx^2+bx+a)x}\,dx$$

input
```
integrate((f*x+e)^n/x/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n/((c*x^2 + b*x + a)*x), x)
```

3.546.8 Giac [F]

$$\int \frac{(e+fx)^n}{x(a+bx+cx^2)}\,dx = \int \frac{(fx+e)^n}{(cx^2+bx+a)x}\,dx$$

input
```
integrate((f*x+e)^n/x/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n/((c*x^2 + b*x + a)*x), x)
```

3.546.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{x(a+bx+cx^2)}\,dx = \int \frac{(e+f\,x)^n}{x\,(c\,x^2+b\,x+a)}\,dx$$

input
```
int((e + f*x)^n/(x*(a + b*x + c*x^2)),x)
```

output
```
int((e + f*x)^n/(x*(a + b*x + c*x^2)), x)
```

3.547 $\int \frac{(e+fx)^n}{x^2(a+bx+cx^2)}\,dx$

3.547.1 Optimal result

Integrand size = 23, antiderivative size = 296

$$\begin{aligned}\int \frac{(e+fx)^n}{x^2(a+bx+cx^2)}\,dx &= -\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a^2\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} \\ &\quad -\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{a^2\left(2ce-\left(b+\sqrt{b^2-4ac}\right)f\right)(1+n)} \\ &\quad +\frac{b(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{fx}{e}\right)}{a^2e(1+n)} \\ &\quad +\frac{f(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{fx}{e}\right)}{ae^2(1+n)}\end{aligned}$$

output
```
b*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],1+f*x/e)/a^2/e/(1+n)+f*(f*x+e)^(1
+n)*hypergeom([2, 1+n],[2+n],1+f*x/e)/a/e^2/(1+n)-c*(f*x+e)^(1+n)*hypergeo
m([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2))))*(b+(-2*a*c+
b^2)/(-4*a*c+b^2)^(1/2))/a^2/(1+n)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2)))-c*(f*x
+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b+(-4*a*c+b^2)^(1
/2))))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2/(1+n)/(2*c*e-f*(b+(-4*a*c+b^
2)^(1/2)))
```

3.547.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.83

$$\int \frac{(e+fx)^n}{x^2\left(a+bx+cx^2\right)}\,dx$$
$$= \frac{(e+fx)^{1+n}\left(-\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce+\left(-b+\sqrt{b^2-4ac}\right)f}\right)}{2ce+\left(-b+\sqrt{b^2-4ac}\right)f}-\frac{c\left(b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+\right.}{2ce-\left(b+\sqrt{b^2-4a\right.}}\right.}{a^2(1+n)}$$

input
```
Integrate[(e + f*x)^n/(x^2*(a + b*x + c*x^2)),x]
```

output
```
((e + f*x)^(1 + n)*(-((c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Hypergeomet
ric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*
f)])/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)) - (c*(b + (-b^2 + 2*a*c)/Sqrt[b
^2 - 4*a*c])*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (
b + Sqrt[b^2 - 4*a*c])*f)])/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f) + (b*Hyper
geometric2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])/e + (a*f*Hypergeometric2F1[2,
1 + n, 2 + n, 1 + (f*x)/e])/e^2))/(a^2*(1 + n))
```

3.547.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^n}{x^2\left(a+bx+cx^2\right)}\,dx$$
$$\Big\downarrow \ 1200$$
$$\int \left(\frac{\left(-ac+b^2+bcx\right)(e+fx)^n}{a^2\left(a+bx+cx^2\right)}-\frac{b(e+fx)^n}{a^2x}+\frac{(e+fx)^n}{ax^2}\right)dx$$
$$\Big\downarrow \ 2009$$

$$-\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a^2(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{a^2(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}+\frac{b(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{fx}{e}+1\right)}{a^2e(n+1)}+\frac{f(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{fx}{e}+1\right)}{ae^2(n+1)}$$

input
```
Int[(e + f*x)^n/(x^2*(a + b*x + c*x^2)),x]
```

output
```
-((c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometri
c2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)]
)/(a^2*(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)*(1 + n))) - (c*(b - (b^2 - 2*a*
c)/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n,
 (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/(a^2*(2*c*e - (b +
Sqrt[b^2 - 4*a*c])*f)*(1 + n)) + (b*(e + f*x)^(1 + n)*Hypergeometric2F1[1,
 1 + n, 2 + n, 1 + (f*x)/e])/(a^2*e*(1 + n)) + (f*(e + f*x)^(1 + n)*Hyperg
eometric2F1[2, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e^2*(1 + n))
```

3.547.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.547. $\int \frac{(e+fx)^n}{x^2(a+bx+cx^2)}\,dx$

3.547.4 Maple [F]

$$\int \frac{(fx+e)^n}{x^2\,(c\,x^2+bx+a)}dx$$

input
```
int((f*x+e)^n/x^2/(c*x^2+b*x+a),x)
```

output
```
int((f*x+e)^n/x^2/(c*x^2+b*x+a),x)
```

3.547.5 Fricas [F]

$$\int \frac{(e+fx)^n}{x^2\,(a+bx+cx^2)}\,dx = \int \frac{(fx+e)^n}{(cx^2+bx+a)x^2}\,dx$$

input
```
integrate((f*x+e)^n/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n/(c*x^4 + b*x^3 + a*x^2), x)
```

3.547.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{x^2\,(a+bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**n/x**2/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

3.547.7 Maxima [F]

$$\int \frac{(e+fx)^n}{x^2(a+bx+cx^2)}\,dx = \int \frac{(fx+e)^n}{(cx^2+bx+a)x^2}\,dx$$

input
```
integrate((f*x+e)^n/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n/((c*x^2 + b*x + a)*x^2), x)
```

3.547.8 Giac [F]

$$\int \frac{(e+fx)^n}{x^2(a+bx+cx^2)}\,dx = \int \frac{(fx+e)^n}{(cx^2+bx+a)x^2}\,dx$$

input
```
integrate((f*x+e)^n/x^2/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n/((c*x^2 + b*x + a)*x^2), x)
```

3.547.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{x^2(a+bx+cx^2)}\,dx = \int \frac{(e+f\,x)^n}{x^2\,(c\,x^2+b\,x+a)}\,dx$$

input
```
int((e + f*x)^n/(x^2*(a + b*x + c*x^2)),x)
```

output
```
int((e + f*x)^n/(x^2*(a + b*x + c*x^2)), x)
```

3.548 $\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx$

3.548.1 Optimal result

Integrand size = 29, antiderivative size = 141

$$\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{d^2(7e^2f^2+16defg+8d^2g^2)\,x}{e^2}-\frac{d(2e^2f^2+7defg+4d^2g^2)\,x^2}{e} -\frac{1}{3}(ef+dg)(ef+7dg)x^3-\frac{1}{2}eg(ef+2dg)x^4 -\frac{1}{5}e^2g^2x^5-\frac{8d^3(ef+dg)^2\log(d-ex)}{e^3}$$

output
```
-d^2*(8*d^2*g^2+16*d*e*f*g+7*e^2*f^2)*x/e^2-d*(4*d^2*g^2+7*d*e*f*g+2*e^2*f
^2)*x^2/e-1/3*(d*g+e*f)*(7*d*g+e*f)*x^3-1/2*e*g*(2*d*g+e*f)*x^4-1/5*e^2*g^
2*x^5-8*d^3*(d*g+e*f)^2*ln(-e*x+d)/e^3
```

3.548.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{x(240d^4g^2+120d^3eg(4f+gx)+70d^2e^2(3f^2+3fgx+g^2x^2)+10de^3x(6f^2+8fgx+3g^2x^2)+e^4x^2(1}{30e^2} -\frac{8d^3(ef+dg)^2\log(d-ex)}{e^3}$$

input
```
Integrate[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

output
```
-1/30*(x*(240*d^4*g^2 + 120*d^3*e*g*(4*f + g*x) + 70*d^2*e^2*(3*f^2 + 3*f*
g*x + g^2*x^2) + 10*d*e^3*x*(6*f^2 + 8*f*g*x + 3*g^2*x^2) + e^4*x^2*(10*f^
2 + 15*f*g*x + 6*g^2*x^2)))/e^2 - (8*d^3*(e*f + d*g)^2*Log[d - e*x])/e^3
```

3.548.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {639, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx$$

$$\Big\downarrow \text{639}$$

$$\int \frac{(d+ex)^3(f+gx)^2}{d-ex}dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(-\frac{8d^3(dg+ef)^2}{e^2(ex-d)}-\frac{2dx\left(4d^2g^2+7defg+2e^2f^2\right)}{e}-\frac{d^2\left(8d^2g^2+16defg+7e^2f^2\right)}{e^2}-2egx^3(2dg+ef)+x^2(-$$

$$\Big\downarrow \text{2009}$$

$$-\frac{8d^3(dg+ef)^2\log(d-ex)}{e^3}-\frac{dx^2\left(4d^2g^2+7defg+2e^2f^2\right)}{e}-\frac{d^2x\left(8d^2g^2+16defg+7e^2f^2\right)}{e^2}-\frac{1}{2}egx^4(2dg+ef)-\frac{1}{3}x^3(dg+ef)(7dg+ef)-\frac{1}{5}e^2g^2x^5$$

input
```
Int[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

output
```
-((d^2*(7*e^2*f^2 + 16*d*e*f*g + 8*d^2*g^2)*x)/e^2) - (d*(2*e^2*f^2 + 7*d*
e*f*g + 4*d^2*g^2)*x^2)/e - ((e*f + d*g)*(e*f + 7*d*g)*x^3)/3 - (e*g*(e*f
+ 2*d*g)*x^4)/2 - (e^2*g^2*x^5)/5 - (8*d^3*(e*f + d*g)^2*Log[d - e*x])/e^3
```

3.548.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.548.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.10

method	result
norman	$\left(-\frac{7}{3}d^2g^2-\frac{8}{3}defg-\frac{1}{3}e^2f^2\right)x^3-\frac{e^2g^2x^5}{5}-\frac{d\left(4d^2g^2+7defg+2e^2f^2\right)x^2}{e}-\frac{d^2\left(8d^2g^2+16defg+7e^2f^2\right)x}{e^2}-eg$
default	$-\frac{\frac{1}{5}g^2e^4x^5+d\,e^3g^2x^4+\frac{1}{2}e^4fg\,x^4+\frac{7}{3}d^2e^2g^2x^3+\frac{8}{3}d\,e^3fg\,x^3+\frac{1}{3}e^4f^2x^3+4d^3e\,g^2x^2+7d^2e^2fg\,x^2+2d\,e^3f^2x^2+8d^4g^2x+16d^3efg}{e^2}$
risch	$-\frac{e^2g^2x^5}{5}-ed\,g^2x^4-\frac{e^2fg\,x^4}{2}-\frac{7d^2g^2x^3}{3}-\frac{8edfg\,x^3}{3}-\frac{e^2f^2x^3}{3}-\frac{4d^3g^2x^2}{e}-7d^2fg\,x^2-2ed\,f^2x^2-8$
parallelrisch	$-\frac{6g^2e^5x^5+30x^4d\,e^4g^2+15x^4e^5fg+70x^3d^2e^3g^2+80x^3d\,e^4fg+10x^3e^5f^2+120x^2d^3e^2g^2+210x^2d^2e^3fg+60x^2d\,e^4f^2+240\ln(e}{30e^3}$

input
```
int((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

output
```
(-7/3*d^2*g^2-8/3*d*e*f*g-1/3*e^2*f^2)*x^3-1/5*e^2*g^2*x^5-d*(4*d^2*g^2+7*
d*e*f*g+2*e^2*f^2)*x^2/e-d^2*(8*d^2*g^2+16*d*e*f*g+7*e^2*f^2)*x/e^2-1/2*e*
g*(2*d*g+e*f)*x^4-8*d^3*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3*ln(-e*x+d)
```

3.548.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{6\,e^5g^2x^5+15\,(e^5fg+2\,de^4g^2)x^4+10\,(e^5f^2+8\,de^4fg+7\,d^2e^3g^2)x^3+30\,(2\,de^4f^2+7\,d^2e^3fg+4\,d^3e^2g}{30\,e^3}$$

input
```
integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

output
```
-1/30*(6*e^5*g^2*x^5 + 15*(e^5*f*g + 2*d*e^4*g^2)*x^4 + 10*(e^5*f^2 + 8*d*
e^4*f*g + 7*d^2*e^3*g^2)*x^3 + 30*(2*d*e^4*f^2 + 7*d^2*e^3*f*g + 4*d^3*e^2
*g^2)*x^2 + 30*(7*d^2*e^3*f^2 + 16*d^3*e^2*f*g + 8*d^4*e*g^2)*x + 240*(d^3
*e^2*f^2 + 2*d^4*e*f*g + d^5*g^2)*log(e*x - d))/e^3
```

3.548.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{8d^3(dg+ef)^2\log(-d+ex)}{e^3} - \frac{e^2g^2x^5}{5} - x^4\left(deg^2+\frac{e^2fg}{2}\right) - x^3\cdot\left(\frac{7d^2g^2}{3}+\frac{8defg}{3}+\frac{e^2f^2}{3}\right) - x^2\cdot\left(\frac{4d^3g^2}{e}+7d^2fg+2def^2\right) - x\left(\frac{8d^4g^2}{e^2}+\frac{16d^3fg}{e}+7d^2f^2\right)$$

input
```
integrate((e*x+d)**4*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

output
```
-8*d**3*(d*g + e*f)**2*log(-d + e*x)/e**3 - e**2*g**2*x**5/5 - x**4*(d*e*g
**2 + e**2*f*g/2) - x**3*(7*d**2*g**2/3 + 8*d*e*f*g/3 + e**2*f**2/3) - x**
2*(4*d**3*g**2/e + 7*d**2*f*g + 2*d*e*f**2) - x*(8*d**4*g**2/e**2 + 16*d**
3*f*g/e + 7*d**2*f**2)
```

3.548.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx =$$
$$-\frac{6\,e^4g^2x^5+15\,(e^4fg+2\,de^3g^2)x^4+10\,(e^4f^2+8\,de^3fg+7\,d^2e^2g^2)x^3+30\,(2\,de^3f^2+7\,d^2e^2fg+4\,d^3eg^2}{30\,e^2}$$
$$-\frac{8\,(d^3e^2f^2+2\,d^4efg+d^5g^2)\log{(ex-d)}}{e^3}$$

input
```
integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

output
```
-1/30*(6*e^4*g^2*x^5 + 15*(e^4*f*g + 2*d*e^3*g^2)*x^4 + 10*(e^4*f^2 + 8*d*
e^3*f*g + 7*d^2*e^2*g^2)*x^3 + 30*(2*d*e^3*f^2 + 7*d^2*e^2*f*g + 4*d^3*e*g
^2)*x^2 + 30*(7*d^2*e^2*f^2 + 16*d^3*e*f*g + 8*d^4*g^2)*x)/e^2 - 8*(d^3*e^
2*f^2 + 2*d^4*e*f*g + d^5*g^2)*log(e*x - d)/e^3
```

3.548.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{8\,(d^3e^2f^2+2\,d^4efg+d^5g^2)\log{(|ex-d|)}}{e^3}$$
$$-\frac{6\,e^7g^2x^5+15\,e^7fgx^4+30\,de^6g^2x^4+10\,e^7f^2x^3+80\,de^6fgx^3+70\,d^2e^5g^2x^3+60\,de^6f^2x^2+210\,d^2e^5fg}{30\,e^5}$$

input
```
integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

output
```
-8*(d^3*e^2*f^2 + 2*d^4*e*f*g + d^5*g^2)*log(abs(e*x - d))/e^3 - 1/30*(6*e
^7*g^2*x^5 + 15*e^7*f*g*x^4 + 30*d*e^6*g^2*x^4 + 10*e^7*f^2*x^3 + 80*d*e^6
*f*g*x^3 + 70*d^2*e^5*g^2*x^3 + 60*d*e^6*f^2*x^2 + 210*d^2*e^5*f*g*x^2 + 1
20*d^3*e^4*g^2*x^2 + 210*d^2*e^5*f^2*x + 480*d^3*e^4*f*g*x + 240*d^4*e^3*g
^2*x)/e^5
```

3.548.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.49

$$\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx$$
$$= -x^2\left(\frac{d^3\,g^2+6\,d^2\,e\,f\,g+3\,d\,e^2\,f^2}{2\,e}+\frac{d\left(\frac{3\,d^2\,e\,g^2+6\,d\,e^2\,f\,g+e^3\,f^2}{e}+\frac{d\,(e\,g\,(3\,d\,g+2\,e\,f)+d\,e\,g^2)}{e}\right)}{2\,e}\right)$$
$$-x^3\left(\frac{3\,d^2\,e\,g^2+6\,d\,e^2\,f\,g+e^3\,f^2}{3\,e}+\frac{d\,(e\,g\,(3\,d\,g+2\,e\,f)+d\,e\,g^2)}{3\,e}\right)$$
$$-x^4\left(\frac{e\,g\,(3\,d\,g+2\,e\,f)}{4}+\frac{d\,e\,g^2}{4}\right)$$
$$-x\left(\frac{d\left(\frac{d^3\,g^2+6\,d^2\,e\,f\,g+3\,d\,e^2\,f^2}{e}+\frac{d\left(\frac{3\,d^2\,e\,g^2+6\,d\,e^2\,f\,g+e^3\,f^2}{e}+\frac{d\,(e\,g\,(3\,d\,g+2\,e\,f)+d\,e\,g^2)}{e}\right)}{e}\right)}{e}+\frac{d^2\,f\,(2\,d\,g+3\,e\,f)}{e}\right)-\frac{\ln(e\,x-d)\,(8\,d^5\,g^2+16\,d^4\,e\,f\,g+8\,d^3\,e^2\,f^2)}{e^3}-\frac{e^2\,g^2\,x^5}{5}$$

input
```
int(((f + g*x)^2*(d + e*x)^4)/(d^2 - e^2*x^2),x)
```

output
```
- x^2*((d^3*g^2 + 3*d*e^2*f^2 + 6*d^2*e*f*g)/(2*e) + (d*((e^3*f^2 + 3*d^2*
e*g^2 + 6*d*e^2*f*g)/e + (d*(e*g*(3*d*g + 2*e*f) + d*e*g^2))/e))/(2*e)) -
x^3*((e^3*f^2 + 3*d^2*e*g^2 + 6*d*e^2*f*g)/(3*e) + (d*(e*g*(3*d*g + 2*e*f)
 + d*e*g^2))/(3*e)) - x^4*((e*g*(3*d*g + 2*e*f))/4 + (d*e*g^2)/4) - x*((d*
((d^3*g^2 + 3*d*e^2*f^2 + 6*d^2*e*f*g)/e + (d*((e^3*f^2 + 3*d^2*e*g^2 + 6*
d*e^2*f*g)/e + (d*(e*g*(3*d*g + 2*e*f) + d*e*g^2))/e))/e))/e + (d^2*f*(2*d
*g + 3*e*f))/e) - (log(e*x - d)*(8*d^5*g^2 + 8*d^3*e^2*f^2 + 16*d^4*e*f*g)
)/e^3 - (e^2*g^2*x^5)/5
```

3.549 $\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx$

3.549.1 Optimal result

Integrand size = 29, antiderivative size = 109

$$\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{d(ef+2dg)(3ef+2dg)x}{e^2} - \frac{(e^2f^2+6defg+4d^2g^2)\,x^2}{2e} - \frac{1}{3}g(2ef+3dg)x^3 - \frac{1}{4}eg^2x^4 - \frac{4d^2(ef+dg)^2\log(d-ex)}{e^3}$$

output
```
-d*(2*d*g+e*f)*(2*d*g+3*e*f)*x/e^2-1/2*(4*d^2*g^2+6*d*e*f*g+e^2*f^2)*x^2/e
-1/3*g*(3*d*g+2*e*f)*x^3-1/4*e*g^2*x^4-4*d^2*(d*g+e*f)^2*ln(-e*x+d)/e^3
```

3.549.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{ex(48d^3g^2+24d^2eg(4f+gx)+12de^2(3f^2+3fgx+g^2x^2)+e^3x(6f^2+8fgx+3g^2x^2))+48d^2(ef+}{12e^3}$$

input
```
Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

output
```
-1/12*(e*x*(48*d^3*g^2 + 24*d^2*e*g*(4*f + g*x) + 12*d*e^2*(3*f^2 + 3*f*g*
x + g^2*x^2) + e^3*x*(6*f^2 + 8*f*g*x + 3*g^2*x^2)) + 48*d^2*(e*f + d*g)^2
*Log[d - e*x])/e^3
```

3.549.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {639, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{639}$$

$$\int \frac{(d+ex)^2(f+gx)^2}{d-ex}dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(-\frac{x\left(4d^2g^2+6defg+e^2f^2\right)}{e}-\frac{4d^2(dg+ef)^2}{e^2(ex-d)}+\frac{d(-2dg-3ef)(2dg+ef)}{e^2}-gx^2(3dg+2ef)-eg^2x^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4d^2(dg+ef)^2\log(d-ex)}{e^3}-\frac{x^2\left(4d^2g^2+6defg+e^2f^2\right)}{2e}-\frac{dx(2dg+ef)(2dg+3ef)}{e^2}-\frac{1}{3}gx^3(3dg+2ef)-\frac{1}{4}eg^2x^4$$

input
```
Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

output
```
-((d*(e*f + 2*d*g)*(3*e*f + 2*d*g)*x)/e^2) - ((e^2*f^2 + 6*d*e*f*g + 4*d^2
*g^2)*x^2)/(2*e) - (g*(2*e*f + 3*d*g)*x^3)/3 - (e*g^2*x^4)/4 - (4*d^2*(e*f
 + d*g)^2*Log[d - e*x])/e^3
```

3.549.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.549.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.12

method	result
norman	$-\frac{e\,g^{2}x^{4}}{4}-\frac{g(3dg+2ef)x^{3}}{3}-\frac{(4d^{2}g^{2}+6defg+e^{2}f^{2})x^{2}}{2e}-\frac{d(4d^{2}g^{2}+8defg+3e^{2}f^{2})x}{e^{2}}-\frac{4d^{2}(d^{2}g^{2}+2defg+e^{2}f^{2})\ln(-ex}{e^{3}}$
risch	$-\frac{e\,g^{2}x^{4}}{4}-x^{3}d\,g^{2}-\frac{2e\,x^{3}fg}{3}-\frac{2x^{2}g^{2}d^{2}}{e}-3x^{2}fgd-\frac{e\,x^{2}f^{2}}{2}-\frac{4d^{3}g^{2}x}{e^{2}}-\frac{8d^{2}fgx}{e}-3d\,f^{2}x-\frac{4d^{4}\ln(-ex}{e^{3}}$
default	$-\frac{\frac{g^{2}e^{3}x^{4}}{4}+\frac{((2dg+ef)e^{2}g+eg(deg+e^{2}f))x^{3}}{3}+\frac{((2dg+ef)(deg+e^{2}f)+eg(2d^{2}g+3def))x^{2}}{2}+(2dg+ef)(2d^{2}g+3def)x}{e^{2}}-\frac{4d^{2}(d^{2}g^{2}}{}$
parallelrisch	$-\frac{3g^{2}e^{4}x^{4}+12x^{3}d\,e^{3}g^{2}+8x^{3}e^{4}fg+24x^{2}d^{2}e^{2}g^{2}+36x^{2}d\,e^{3}fg+6x^{2}e^{4}f^{2}+48\ln(ex-d)d^{4}g^{2}+96\ln(ex-d)d^{3}efg+48\ln(ex-d)d^{2}}{12e^{3}}$

input
```
int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*e*g^2*x^4-1/3*g*(3*d*g+2*e*f)*x^3-1/2*(4*d^2*g^2+6*d*e*f*g+e^2*f^2)*x
^2/e-d*(4*d^2*g^2+8*d*e*f*g+3*e^2*f^2)/e^2*x-4*d^2*(d^2*g^2+2*d*e*f*g+e^2*
f^2)/e^3*ln(-e*x+d)
```

3.549.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx =$$
$$-\frac{3\,e^{4}g^{2}x^{4}+4\,(2\,e^{4}fg+3\,de^{3}g^{2})x^{3}+6\,(e^{4}f^{2}+6\,de^{3}fg+4\,d^{2}e^{2}g^{2})x^{2}+12\,(3\,de^{3}f^{2}+8\,d^{2}e^{2}fg+4\,d^{3}eg^{2}}{12\,e^{3}}$$

input
```
integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

output
```
-1/12*(3*e^4*g^2*x^4 + 4*(2*e^4*f*g + 3*d*e^3*g^2)*x^3 + 6*(e^4*f^2 + 6*d*
e^3*f*g + 4*d^2*e^2*g^2)*x^2 + 12*(3*d*e^3*f^2 + 8*d^2*e^2*f*g + 4*d^3*e*g
^2)*x + 48*(d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2)*log(e*x - d))/e^3
```

3.549.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{4d^2(dg+ef)^2\log(-d+ex)}{e^3} - \frac{eg^2x^4}{4} - x^3\left(dg^2+\frac{2efg}{3}\right) - x^2\cdot\left(\frac{2d^2g^2}{e}+3dfg+\frac{ef^2}{2}\right) - x\left(\frac{4d^3g^2}{e^2}+\frac{8d^2fg}{e}+3df^2\right)$$

input
```
integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

output
```
-4*d**2*(d*g + e*f)**2*log(-d + e*x)/e**3 - e*g**2*x**4/4 - x**3*(d*g**2 +
 2*e*f*g/3) - x**2*(2*d**2*g**2/e + 3*d*f*g + e*f**2/2) - x*(4*d**3*g**2/e
**2 + 8*d**2*f*g/e + 3*d*f**2)
```

3.549.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{3e^3g^2x^4+4(2e^3fg+3de^2g^2)x^3+6(e^3f^2+6de^2fg+4d^2eg^2)x^2+12(3de^2f^2+8d^2efg+4d^3g^2)x}{12e^2} - \frac{4(d^2e^2f^2+2d^3efg+d^4g^2)\log(ex-d)}{e^3}$$

input
```
integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

output
```
-1/12*(3*e^3*g^2*x^4 + 4*(2*e^3*f*g + 3*d*e^2*g^2)*x^3 + 6*(e^3*f^2 + 6*d*
e^2*f*g + 4*d^2*e*g^2)*x^2 + 12*(3*d*e^2*f^2 + 8*d^2*e*f*g + 4*d^3*g^2)*x)
/e^2 - 4*(d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2)*log(e*x - d)/e^3
```

3.549.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.37

$$\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{4\,(d^2e^2f^2+2\,d^3efg+d^4g^2)\log(|ex-d|)}{e^3} - \frac{3\,e^5g^2x^4+8\,e^5fgx^3+12\,de^4g^2x^3+6\,e^5f^2x^2+36\,de^4fgx^2+24\,d^2e^3g^2x^2+36\,de^4f^2x+96\,d^2e^3fgx+4}{12\,e^4}$$

input
```
integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

output
```
-4*(d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2)*log(abs(e*x - d))/e^3 - 1/12*(3*e
^5*g^2*x^4 + 8*e^5*f*g*x^3 + 12*d*e^4*g^2*x^3 + 6*e^5*f^2*x^2 + 36*d*e^4*f
*g*x^2 + 24*d^2*e^3*g^2*x^2 + 36*d*e^4*f^2*x + 96*d^2*e^3*f*g*x + 48*d^3*e
^2*g^2*x)/e^4
```

3.549.9 Mupad [B] (verification not implemented)

Time = 12.02 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.81

$$\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx = -x^3\left(\frac{2\,g\,(d\,g+e\,f)}{3}+\frac{d\,g^2}{3}\right) - x^2\left(\frac{d^2\,g^2+4\,d\,e\,f\,g+e^2\,f^2}{2\,e}+\frac{d\,(2\,g\,(d\,g+e\,f)+d\,g^2)}{2\,e}\right) - x\left(\frac{d\left(\frac{d^2\,g^2+4\,d\,e\,f\,g+e^2\,f^2}{e}+\frac{d\,(2\,g\,(d\,g+e\,f)+d\,g^2)}{e}\right)}{e}+\frac{2\,d\,f\,(d\,g+e\,f)}{e}\right) - \frac{\ln(e\,x-d)\,(4\,d^4\,g^2+8\,d^3\,e\,f\,g+4\,d^2\,e^2\,f^2)}{e^3} - \frac{e\,g^2\,x^4}{4}$$

input
```
int(((f + g*x)^2*(d + e*x)^3)/(d^2 - e^2*x^2),x)
```

output
```
- x^3*((2*g*(d*g + e*f))/3 + (d*g^2)/3) - x^2*((d^2*g^2 + e^2*f^2 + 4*d*e*
f*g)/(2*e) + (d*(2*g*(d*g + e*f) + d*g^2))/(2*e)) - x*((d*((d^2*g^2 + e^2*
f^2 + 4*d*e*f*g)/e + (d*(2*g*(d*g + e*f) + d*g^2))/e))/e + (2*d*f*(d*g + e
*f))/e) - (log(e*x - d)*(4*d^4*g^2 + 4*d^2*e^2*f^2 + 8*d^3*e*f*g))/e^3 - (
e*g^2*x^4)/4
```

3.550 $\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx$

3.550.1 Optimal result

Integrand size = 29, antiderivative size = 65

$$\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{2dg(ef+dg)x}{e^2}-\frac{d(f+gx)^2}{e}-\frac{(f+gx)^3}{3g}-\frac{2d(ef+dg)^2\log(d-ex)}{e^3}$$

output
```
-2*d*g*(d*g+e*f)*x/e^2-d*(g*x+f)^2/e-1/3*(g*x+f)^3/g-2*d*(d*g+e*f)^2*ln(-e
*x+d)/e^3
```

3.550.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{ex(6d^2g^2+3deg(4f+gx)+e^2(3f^2+3fgx+g^2x^2))+6d(ef+dg)^2\log(d-ex)}{3e^3}$$

input
```
Integrate[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

output
```
-1/3*(e*x*(6*d^2*g^2 + 3*d*e*g*(4*f + g*x) + e^2*(3*f^2 + 3*f*g*x + g^2*x^
2)) + 6*d*(e*f + d*g)^2*Log[d - e*x])/e^3
```

3.550.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {639, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{ 639}$$

$$\int \frac{(d+ex)(f+gx)^2}{d-ex}dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(-\frac{2d(dg+ef)^2}{e^2(ex-d)}-\frac{2dg(dg+ef)}{e^2}-\frac{2dg(f+gx)}{e}-(f+gx)^2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2d(dg+ef)^2\log(d-ex)}{e^3}-\frac{2dgx(dg+ef)}{e^2}-\frac{d(f+gx)^2}{e}-\frac{(f+gx)^3}{3g}$$

input
```
Int[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

output
```
(-2*d*g*(e*f + d*g)*x)/e^2 - (d*(f + g*x)^2)/e - (f + g*x)^3/(3*g) - (2*d*
(e*f + d*g)^2*Log[d - e*x])/e^3
```

3.550.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.550.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.35

method	result
norman	$-\frac{g^{2} x^{3}}{3}-\frac{\left(2 d^{2} g^{2}+4 d e f g+e^{2} f^{2}\right) x}{e^{2}}-\frac{g \left(d g+e f\right) x^{2}}{e}-\frac{2 d \left(d^{2} g^{2}+2 d e f g+e^{2} f^{2}\right) \ln \left(-e x+d\right)}{e^{3}}$
default	$-\frac{\frac{1}{3} g^{2} x^{3} e^{2}+d e\,g^{2} x^{2}+e^{2} f g\,x^{2}+2 d^{2} g^{2} x+4 d e f g x+e^{2} f^{2} x}{e^{2}}-\frac{2 d \left(d^{2} g^{2}+2 d e f g+e^{2} f^{2}\right) \ln \left(-e x+d\right)}{e^{3}}$
risch	$-\frac{g^{2} x^{3}}{3}-\frac{d\,g^{2} x^{2}}{e}-f g\,x^{2}-\frac{2 d^{2} g^{2} x}{e^{2}}-\frac{4 d f g x}{e}-f^{2} x-\frac{2 d^{3} \ln \left(-e x+d\right) g^{2}}{e^{3}}-\frac{4 d^{2} \ln \left(-e x+d\right) f g}{e^{2}}-\frac{2 d \ln \left(-e x+d\right)}{e}$
parallelrisch	$-\frac{g^{2} x^{3} e^{3}+3 x^{2} d\,e^{2} g^{2}+3 x^{2} e^{3} f g+6 \ln \left(e x-d\right) d^{3} g^{2}+12 \ln \left(e x-d\right) d^{2} e f g+6 \ln \left(e x-d\right) d\,e^{2} f^{2}+6 x\,d^{2} e\,g^{2}+12 x d\,e^{2} f g+3 x\,e^{3} f^{2}}{3 e^{3}}$

input
```
int((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*g^2*x^3-(2*d^2*g^2+4*d*e*f*g+e^2*f^2)/e^2*x-1/e*g*(d*g+e*f)*x^2-2*d*(
d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3*ln(-e*x+d)
```

3.550.5 Fricas [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{e^3g^2x^3+3\,(e^3fg+de^2g^2)x^2+3\,(e^3f^2+4\,de^2fg+2\,d^2eg^2)x+6\,(de^2f^2+2\,d^2efg+d^3g^2)\log{(ex-d)}}{3\,e^3}$$

input
```
integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

output
```
-1/3*(e^3*g^2*x^3 + 3*(e^3*f*g + d*e^2*g^2)*x^2 + 3*(e^3*f^2 + 4*d*e^2*f*g
 + 2*d^2*e*g^2)*x + 6*(d*e^2*f^2 + 2*d^2*e*f*g + d^3*g^2)*log(e*x - d))/e^
3
```

3.550.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{2d(dg+ef)^2\log(-d+ex)}{e^3} - \frac{g^2x^3}{3} - x^2\left(\frac{dg^2}{e}+fg\right) - x\left(\frac{2d^2g^2}{e^2}+\frac{4dfg}{e}+f^2\right)$$

input
```
integrate((e*x+d)**2*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

output
```
-2*d*(d*g + e*f)**2*log(-d + e*x)/e**3 - g**2*x**3/3 - x**2*(d*g**2/e + f*
g) - x*(2*d**2*g**2/e**2 + 4*d*f*g/e + f**2)
```

3.550.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.49

$$\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{e^2g^2x^3+3\,(e^2fg+deg^2)x^2+3\,(e^2f^2+4\,defg+2\,d^2g^2)x}{3\,e^2} - \frac{2\,(de^2f^2+2\,d^2efg+d^3g^2)\log(ex-d)}{e^3}$$

input
```
integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

output
```
-1/3*(e^2*g^2*x^3 + 3*(e^2*f*g + d*e*g^2)*x^2 + 3*(e^2*f^2 + 4*d*e*f*g + 2
*d^2*g^2)*x)/e^2 - 2*(d*e^2*f^2 + 2*d^2*e*f*g + d^3*g^2)*log(e*x - d)/e^3
```

3.550.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.62

$$\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{2\left(de^2f^2+2\,d^2efg+d^3g^2\right)\log\left(|ex-d|\right)}{e^3} - \frac{e^3g^2x^3+3\,e^3fgx^2+3\,de^2g^2x^2+3\,e^3f^2x+12\,de^2fgx+6\,d^2eg^2x}{3\,e^3}$$

input
```
integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

output
```
-2*(d*e^2*f^2 + 2*d^2*e*f*g + d^3*g^2)*log(abs(e*x - d))/e^3 - 1/3*(e^3*g^
2*x^3 + 3*e^3*f*g*x^2 + 3*d*e^2*g^2*x^2 + 3*e^3*f^2*x + 12*d*e^2*f*g*x + 6
*d^2*e*g^2*x)/e^3
```

3.550.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.95

$$\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx = -x^2\left(\frac{d\,g^2+2\,e\,f\,g}{2\,e}+\frac{d\,g^2}{2\,e}\right) - x\left(\frac{e\,f^2+2\,d\,g\,f}{e}+\frac{d\left(\frac{d\,g^2+2\,e\,f\,g}{e}+\frac{d\,g^2}{e}\right)}{e}\right) - \frac{g^2\,x^3}{3} - \frac{\ln\left(e\,x-d\right)\,\left(2\,d^3\,g^2+4\,d^2\,e\,f\,g+2\,d\,e^2\,f^2\right)}{e^3}$$

input
```
int(((f + g*x)^2*(d + e*x)^2)/(d^2 - e^2*x^2),x)
```

output
```
- x^2*((d*g^2 + 2*e*f*g)/(2*e) + (d*g^2)/(2*e)) - x*((e*f^2 + 2*d*f*g)/e +
 (d*((d*g^2 + 2*e*f*g)/e + (d*g^2)/e))/e) - (g^2*x^3)/3 - (log(e*x - d)*(2
*d^3*g^2 + 2*d*e^2*f^2 + 4*d^2*e*f*g))/e^3
```

3.551 $\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2} dx$

3.551.1 Optimal result

Integrand size = 27, antiderivative size = 50

$$\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2} dx = -\frac{g(ef+dg)x}{e^2} - \frac{(f+gx)^2}{2e} - \frac{(ef+dg)^2 \log(d-ex)}{e^3}$$

output
```
-g*(d*g+e*f)*x/e^2-1/2*(g*x+f)^2/e-(d*g+e*f)^2*ln(-e*x+d)/e^3
```

3.551.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2} dx = -\frac{egx(4ef+2dg+egx)+2(ef+dg)^2 \log(d-ex)}{2e^3}$$

input
```
Integrate[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

output
```
-1/2*(e*g*x*(4*e*f + 2*d*g + e*g*x) + 2*(e*f + d*g)^2*Log[d - e*x])/e^3
```

3.551.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {639, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2} \, dx$$

$$\Big\downarrow \text{639}$$

$$\int \frac{(f+gx)^2}{d-ex}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{(dg+ef)^2}{e^2(d-ex)}-\frac{g(dg+ef)}{e^2}-\frac{g(f+gx)}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(dg+ef)^2\log(d-ex)}{e^3}-\frac{gx(dg+ef)}{e^2}-\frac{(f+gx)^2}{2e}$$

input
```
Int[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

output
```
-((g*(e*f + d*g)*x)/e^2) - (f + g*x)^2/(2*e) - ((e*f + d*g)^2*Log[d - e*x]
)/e^3
```

3.551.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.551.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

method	result	size
default	$-\frac{g\left(\frac{1}{2}eg\,x^{2}+dgx+2efx\right)}{e^{2}}+\frac{\left(-d^{2}g^{2}-2defg-e^{2}f^{2}\right)\ln\left(-ex+d\right)}{e^{3}}$	59
norman	$-\frac{g^{2}x^{2}}{2e}-\frac{g(dg+2ef)x}{e^{2}}-\frac{\left(d^{2}g^{2}+2defg+e^{2}f^{2}\right)\ln\left(-ex+d\right)}{e^{3}}$	61
risch	$-\frac{g^{2}x^{2}}{2e}-\frac{g^{2}dx}{e^{2}}-\frac{2gfx}{e}-\frac{\ln\left(-ex+d\right)d^{2}g^{2}}{e^{3}}-\frac{2\ln\left(-ex+d\right)dfg}{e^{2}}-\frac{\ln\left(-ex+d\right)f^{2}}{e}$	79
parallelrisch	$-\frac{g^{2}x^{2}e^{2}+2\ln\left(ex-d\right)d^{2}g^{2}+4\ln\left(ex-d\right)defg+2\ln\left(ex-d\right)e^{2}f^{2}+2xde\,g^{2}+4x\,e^{2}fg}{2e^{3}}$	79

input
```
int((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

output
```
-g/e^2*(1/2*e*g*x^2+d*g*x+2*e*f*x)+(-d^2*g^2-2*d*e*f*g-e^2*f^2)/e^3*ln(-e*
x+d)
```

3.551.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2}\,dx=-\frac{e^2g^2x^2+2\left(2\,e^2fg+deg^2\right)x+2\left(e^2f^2+2\,defg+d^2g^2\right)\log\left(ex-d\right)}{2\,e^3}$$

input
```
integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

output
```
-1/2*(e^2*g^2*x^2 + 2*(2*e^2*f*g + d*e*g^2)*x + 2*(e^2*f^2 + 2*d*e*f*g + d
^2*g^2)*log(e*x - d))/e^3
```

3.551.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2}\,dx = -x\left(\frac{dg^2}{e^2}+\frac{2fg}{e}\right)-\frac{g^2x^2}{2e}-\frac{(dg+ef)^2\log(-d+ex)}{e^3}$$

input
```
integrate((e*x+d)*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

output
```
-x*(d*g**2/e**2 + 2*f*g/e) - g**2*x**2/(2*e) - (d*g + e*f)**2*log(-d + e*x
)/e**3
```

3.551.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{eg^2x^2+2\,(2\,efg+dg^2)x}{2\,e^2}-\frac{(e^2f^2+2\,defg+d^2g^2)\log(ex-d)}{e^3}$$

input
```
integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

output
```
-1/2*(e*g^2*x^2 + 2*(2*e*f*g + d*g^2)*x)/e^2 - (e^2*f^2 + 2*d*e*f*g + d^2*
g^2)*log(e*x - d)/e^3
```

3.551.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{eg^2x^2+4\,efgx+2\,dg^2x}{2\,e^2}-\frac{(e^2f^2+2\,defg+d^2g^2)\log(|ex-d|)}{e^3}$$

input
```
integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

output
```
-1/2*(e*g^2*x^2 + 4*e*f*g*x + 2*d*g^2*x)/e^2 - (e^2*f^2 + 2*d*e*f*g + d^2*
g^2)*log(abs(e*x - d))/e^3
```

3.551.9 Mupad [B] (verification not implemented)

Time = 11.93 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2}\,dx = -x\left(\frac{d\,g^2}{e^2}+\frac{2\,f\,g}{e}\right)-\frac{\ln\left(e\,x-d\right)\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{e^3}-\frac{g^2\,x^2}{2\,e}$$

input
```
int(((f + g*x)^2*(d + e*x))/(d^2 - e^2*x^2),x)
```

output
```
- x*((d*g^2)/e^2 + (2*f*g)/e) - (log(e*x - d)*(d^2*g^2 + e^2*f^2 + 2*d*e*f
*g))/e^3 - (g^2*x^2)/(2*e)
```

3.552 $\int \frac{(f+gx)^2}{d^2-e^2x^2}\,dx$

3.552.1 Optimal result

Integrand size = 22, antiderivative size = 62

$$\int \frac{(f+gx)^2}{d^2-e^2x^2}\,dx=-\frac{g^2x}{e^2}-\frac{(ef+dg)^2\log(d-ex)}{2de^3}+\frac{(ef-dg)^2\log(d+ex)}{2de^3}$$

output
```
-g^2*x/e^2-1/2*(d*g+e*f)^2*ln(-e*x+d)/d/e^3+1/2*(-d*g+e*f)^2*ln(e*x+d)/d/e
^3
```

3.552.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{(f+gx)^2}{d^2-e^2x^2}\,dx=\frac{(e^2f^2+d^2g^2)\,\text{arctanh}\left(\frac{ex}{d}\right)-deg(gx+f\log(d^2-e^2x^2))}{de^3}$$

input
```
Integrate[(f + g*x)^2/(d^2 - e^2*x^2),x]
```

output
```
((e^2*f^2 + d^2*g^2)*ArcTanh[(e*x)/d] - d*e*g*(g*x + f*Log[d^2 - e^2*x^2])
)/(d*e^3)
```

3.552.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{d^2-e^2x^2} \, dx$$

$$\Big\downarrow \text{477}$$

$$\frac{\int \left(-\frac{d^2g^2}{e^2}+\frac{d(ef+dg)^2}{2e^2(d-ex)}+\frac{d(ef-dg)^2}{2e^2(d+ex)}\right)dx}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{d^2g^2x}{e^2}-\frac{d(dg+ef)^2\log(d-ex)}{2e^3}+\frac{d(ef-dg)^2\log(d+ex)}{2e^3}}{d^2}$$

input
```
Int[(f + g*x)^2/(d^2 - e^2*x^2),x]
```

output
```
(-((d^2*g^2*x)/e^2) - (d*(e*f + d*g)^2*Log[d - e*x])/(2*e^3) + (d*(e*f - d
*g)^2*Log[d + e*x])/(2*e^3))/d^2
```

3.552.3.1 Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.552.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

method	result	size
norman	$-\frac{g^2 x}{e^2}+\frac{(d^2g^2-2defg+e^2f^2)\ln(ex+d)}{2e^3d}-\frac{(d^2g^2+2defg+e^2f^2)\ln(-ex+d)}{2d\,e^3}$	82
default	$-\frac{g^2 x}{e^2}+\frac{(-d^2g^2-2defg-e^2f^2)\ln(-ex+d)}{2d\,e^3}+\frac{(d^2g^2-2defg+e^2f^2)\ln(ex+d)}{2e^3d}$	84
parallelrisch	$-\frac{\ln(ex-d)d^2g^2+2\ln(ex-d)defg+\ln(ex-d)e^2f^2-\ln(ex+d)d^2g^2+2\ln(ex+d)defg-\ln(ex+d)e^2f^2+2xde\,g^2}{2d\,e^3}$	102
risch	$-\frac{g^2x}{e^2}+\frac{d\ln(-ex-d)g^2}{2e^3}-\frac{\ln(-ex-d)fg}{e^2}+\frac{\ln(-ex-d)f^2}{2ed}-\frac{d\ln(ex-d)g^2}{2e^3}-\frac{\ln(ex-d)fg}{e^2}-\frac{\ln(ex-d)f^2}{2ed}$	116

input
```
int((g*x+f)^2/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

output
```
-g^2*x/e^2+1/2/e^3*(d^2*g^2-2*d*e*f*g+e^2*f^2)/d*ln(e*x+d)-1/2*(d^2*g^2+2*
d*e*f*g+e^2*f^2)/d/e^3*ln(-e*x+d)
```

3.552.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.23

$$\int \frac{(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{2\,deg^2x-(e^2f^2-2\,defg+d^2g^2)\log(ex+d)+(e^2f^2+2\,defg+d^2g^2)\log(ex-d)}{2\,de^3}$$

input
```
integrate((g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

output
```
-1/2*(2*d*e*g^2*x - (e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(e*x + d) + (e^2*f^
2 + 2*d*e*f*g + d^2*g^2)*log(e*x - d))/(d*e^3)
```

3.552.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(51) = 102$.

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.81

$$\int \frac{(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{g^2x}{e^2} + \frac{(dg-ef)^2\log\left(x+\frac{2d^2fg+\frac{d(dg-ef)^2}{e}}{d^2g^2+e^2f^2}\right)}{2de^3} - \frac{(dg+ef)^2\log\left(x+\frac{2d^2fg-\frac{d(dg+ef)^2}{e}}{d^2g^2+e^2f^2}\right)}{2de^3}$$

input
```
integrate((g*x+f)**2/(-e**2*x**2+d**2),x)
```

output
```
-g**2*x/e**2 + (d*g - e*f)**2*log(x + (2*d**2*f*g + d*(d*g - e*f)**2/e)/(d
**2*g**2 + e**2*f**2))/(2*d*e**3) - (d*g + e*f)**2*log(x + (2*d**2*f*g - d
*(d*g + e*f)**2/e)/(d**2*g**2 + e**2*f**2))/(2*d*e**3)
```

3.552.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int \frac{(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{g^2x}{e^2} + \frac{(e^2f^2-2\,defg+d^2g^2)\log(ex+d)}{2\,de^3} - \frac{(e^2f^2+2\,defg+d^2g^2)\log(ex-d)}{2\,de^3}$$

input
```
integrate((g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

output
```
-g^2*x/e^2 + 1/2*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(e*x + d)/(d*e^3) - 1/
2*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(e*x - d)/(d*e^3)
```

3.552.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.35

$$\int \frac{(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{g^2x}{e^2} + \frac{(e^2f^2-2\,defg+d^2g^2)\log(|ex+d|)}{2\,de^3} - \frac{(e^2f^2+2\,defg+d^2g^2)\log(|ex-d|)}{2\,de^3}$$

input
```
integrate((g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

output
```
-g^2*x/e^2 + 1/2*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(abs(e*x + d))/(d*e^3)
 - 1/2*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(abs(e*x - d))/(d*e^3)
```

3.552.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.31

$$\int \frac{(f+gx)^2}{d^2-e^2x^2}\,dx = \frac{\ln(d+e\,x)\,(d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2)}{2\,d\,e^3} - \frac{g^2\,x}{e^2} - \frac{\ln(d-e\,x)\,(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2)}{2\,d\,e^3}$$

input
```
int((f + g*x)^2/(d^2 - e^2*x^2),x)
```

output
```
(log(d + e*x)*(d^2*g^2 + e^2*f^2 - 2*d*e*f*g))/(2*d*e^3) - (g^2*x)/e^2 - (
log(d - e*x)*(d^2*g^2 + e^2*f^2 + 2*d*e*f*g))/(2*d*e^3)
```

3.553 $\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)}\,dx$

3.553.1 Optimal result

Integrand size = 29, antiderivative size = 86

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)}\,dx=-\frac{(ef-dg)^2}{2de^3(d+ex)}-\frac{(ef+dg)^2\log(d-ex)}{4d^2e^3}+\frac{(ef-dg)(ef+3dg)\log(d+ex)}{4d^2e^3}$$

output
```
-1/2*(-d*g+e*f)^2/d/e^3/(e*x+d)-1/4*(d*g+e*f)^2*ln(-e*x+d)/d^2/e^3+1/4*(-d
*g+e*f)*(3*d*g+e*f)*ln(e*x+d)/d^2/e^3
```

3.553.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)}\,dx=\frac{-(ef+dg)^2(d+ex)\log(d-ex)+(ef-dg)(2d(-ef+dg)+(ef+3dg)(d+ex)\log(d+ex))}{4d^2e^3(d+ex)}$$

input
```
Integrate[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)),x]
```

output
```
(-((e*f + d*g)^2*(d + e*x)*Log[d - e*x]) + (e*f - d*g)*(2*d*(-(e*f) + d*g)
 + (e*f + 3*d*g)*(d + e*x)*Log[d + e*x]))/(4*d^2*e^3*(d + e*x))
```

3.553.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {639, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)}dx$$

$$\Big\downarrow \text{639}$$

$$\int \frac{(f+gx)^2}{(d-ex)(d+ex)^2}dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(\frac{(dg+ef)^2}{4d^2e^2(d-ex)}+\frac{(ef-dg)(3dg+ef)}{4d^2e^2(d+ex)}+\frac{(dg-ef)^2}{2de^2(d+ex)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(3dg+ef)(ef-dg)\log(d+ex)}{4d^2e^3}-\frac{(dg+ef)^2\log(d-ex)}{4d^2e^3}-\frac{(ef-dg)^2}{2de^3(d+ex)}$$

input
```
Int[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)),x]
```

output
```
-1/2*(e*f - d*g)^2/(d*e^3*(d + e*x)) - ((e*f + d*g)^2*Log[d - e*x])/(4*d^2
*e^3) + ((e*f - d*g)*(e*f + 3*d*g)*Log[d + e*x])/(4*d^2*e^3)
```

3.553.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.553.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

method	result
default	$\frac{(-d^2g^2-2defg-e^2f^2)\ln(-ex+d)}{4d^2e^3}+\frac{(-3d^2g^2+2defg+e^2f^2)\ln(ex+d)}{4d^2e^3}-\frac{d^2g^2-2defg+e^2f^2}{2e^3d(ex+d)}$
norman	$\frac{-d^2g^2+2defg-e^2f^2}{2d\,e^3(ex+d)}-\frac{(d^2g^2+2defg+e^2f^2)\ln(-ex+d)}{4d^2e^3}-\frac{(3d^2g^2-2defg-e^2f^2)\ln(ex+d)}{4d^2e^3}$
risch	$-\frac{d\,g^2}{2e^3(ex+d)}+\frac{fg}{e^2(ex+d)}-\frac{f^2}{2ed(ex+d)}-\frac{\ln(ex-d)g^2}{4e^3}-\frac{\ln(ex-d)fg}{2d\,e^2}-\frac{\ln(ex-d)f^2}{4d^2e}-\frac{3\ln(-ex-d)g^2}{4e^3}+\frac{\ln(-ex-}{2d\,e^2}$
parallelrisch	$-\frac{\ln(ex-d)x\,d^2e\,g^2+2\ln(ex-d)xd\,e^2fg+\ln(ex-d)x\,e^3f^2+3\ln(ex+d)x\,d^2e\,g^2-2\ln(ex+d)xd\,e^2fg-\ln(ex+d)x\,e^3f^2+\ln(ex-}{4d^2e^3(}$

input
```
int((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-d^2*g^2-2*d*e*f*g-e^2*f^2)/d^2/e^3*ln(-e*x+d)+1/4*(-3*d^2*g^2+2*d*e*
f*g+e^2*f^2)/d^2/e^3*ln(e*x+d)-1/2/e^3*(d^2*g^2-2*d*e*f*g+e^2*f^2)/d/(e*x+
d)
```

3.553.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(80) = 160$.

Time = 0.30 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.92

$$\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)}\,dx = -\frac{2\,de^2f^2-4\,d^2efg+2\,d^3g^2-(de^2f^2+2\,d^2efg-3\,d^3g^2+(e^3f^2+2\,de^2fg-3\,d^2eg^2)x)\log(ex+d)+}{4\,(d^2e^4x+d^3e^3)}$$

input
```
integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x, algorithm="fricas")
```

output
```
-1/4*(2*d*e^2*f^2 - 4*d^2*e*f*g + 2*d^3*g^2 - (d*e^2*f^2 + 2*d^2*e*f*g - 3
*d^3*g^2 + (e^3*f^2 + 2*d*e^2*f*g - 3*d^2*e*g^2)*x)*log(e*x + d) + (d*e^2*
f^2 + 2*d^2*e*f*g + d^3*g^2 + (e^3*f^2 + 2*d*e^2*f*g + d^2*e*g^2)*x)*log(e
*x - d))/(d^2*e^4*x + d^3*e^3)
```

3.553.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(75) = 150$.

Time = 0.47 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.12

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)}\,dx = -\frac{d^2g^2-2defg+e^2f^2}{2d^2e^3+2de^4x} - \frac{(dg-ef)\,(3dg+ef)\log\left(x+\frac{-2d^3g^2+d(dg-ef)(3dg+ef)}{d^2eg^2-2de^2fg-e^3f^2}\right)}{4d^2e^3} - \frac{(dg+ef)^2\log\left(x+\frac{-2d^3g^2+d(dg+ef)^2}{d^2eg^2-2de^2fg-e^3f^2}\right)}{4d^2e^3}$$

input
```
integrate((g*x+f)**2/(e*x+d)/(-e**2*x**2+d**2),x)
```

output
```
-(d**2*g**2 - 2*d*e*f*g + e**2*f**2)/(2*d**2*e**3 + 2*d*e**4*x) - (d*g - e
*f)*(3*d*g + e*f)*log(x + (-2*d**3*g**2 + d*(d*g - e*f)*(3*d*g + e*f))/(d*
*2*e*g**2 - 2*d*e**2*f*g - e**3*f**2))/(4*d**2*e**3) - (d*g + e*f)**2*log(
x + (-2*d**3*g**2 + d*(d*g + e*f)**2)/(d**2*e*g**2 - 2*d*e**2*f*g - e**3*f
**2))/(4*d**2*e**3)
```

3.553.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.31

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)}\,dx = -\frac{e^2f^2-2\,defg+d^2g^2}{2\,(de^4x+d^2e^3)} + \frac{\left(e^2f^2+2\,defg-3\,d^2g^2\right)\log\left(ex+d\right)}{4\,d^2e^3} - \frac{\left(e^2f^2+2\,defg+d^2g^2\right)\log\left(ex-d\right)}{4\,d^2e^3}$$

input
```
integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x, algorithm="maxima")
```

output
```
-1/2*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(d*e^4*x + d^2*e^3) + 1/4*(e^2*f^2 +
2*d*e*f*g - 3*d^2*g^2)*log(e*x + d)/(d^2*e^3) - 1/4*(e^2*f^2 + 2*d*e*f*g +
 d^2*g^2)*log(e*x - d)/(d^2*e^3)
```

3.553.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.34

$$\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)}\,dx = \frac{(e^2f^2+2\,defg-3\,d^2g^2)\log(|ex+d|)}{4\,d^2e^3} - \frac{(e^2f^2+2\,defg+d^2g^2)\log(|ex-d|)}{4\,d^2e^3} - \frac{de^2f^2-2\,d^2efg+d^3g^2}{2\,(ex+d)d^2e^3}$$

input
```
integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x, algorithm="giac")
```

output
```
1/4*(e^2*f^2 + 2*d*e*f*g - 3*d^2*g^2)*log(abs(e*x + d))/(d^2*e^3) - 1/4*(e
^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(abs(e*x - d))/(d^2*e^3) - 1/2*(d*e^2*f^2
 - 2*d^2*e*f*g + d^3*g^2)/((e*x + d)*d^2*e^3)
```

3.553.9 Mupad [B] (verification not implemented)

Time = 12.02 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.27

$$\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)}\,dx = \frac{\ln(d+e\,x)\,\left(-3\,d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^2\,e^3} - \frac{\ln(d-e\,x)\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^2\,e^3} - \frac{d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2}{2\,d\,e^3\,(d+e\,x)}$$

input
```
int((f + g*x)^2/((d^2 - e^2*x^2)*(d + e*x)),x)
```

output
```
(log(d + e*x)*(e^2*f^2 - 3*d^2*g^2 + 2*d*e*f*g))/(4*d^2*e^3) - (log(d - e*
x)*(d^2*g^2 + e^2*f^2 + 2*d*e*f*g))/(4*d^2*e^3) - (d^2*g^2 + e^2*f^2 - 2*d
*e*f*g)/(2*d*e^3*(d + e*x))
```

3.554 $\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)}\,dx$

3.554.1 Optimal result

Integrand size = 29, antiderivative size = 87

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)}\,dx = -\frac{(ef-dg)^2}{4de^3(d+ex)^2}-\frac{(ef-dg)(ef+3dg)}{4d^2e^3(d+ex)}+\frac{(ef+dg)^2\text{arctanh}\left(\frac{ex}{d}\right)}{4d^3e^3}$$

output
```
-1/4*(-d*g+e*f)^2/d/e^3/(e*x+d)^2-1/4*(-d*g+e*f)*(3*d*g+e*f)/d^2/e^3/(e*x+
d)+1/4*(d*g+e*f)^2*arctanh(e*x/d)/d^3/e^3
```

3.554.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)}\,dx = \frac{\frac{2d(-ef+dg)(2d^2g+e^2fx+de(2f+3gx))}{(d+ex)^2}-(ef+dg)^2\log(d-ex)+(ef+dg)^2\log(d+ex)}{8d^3e^3}$$

input
```
Integrate[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)),x]
```

output
```
((2*d*(-(e*f) + d*g)*(2*d^2*g + e^2*f*x + d*e*(2*f + 3*g*x)))/(d + e*x)^2
- (e*f + d*g)^2*Log[d - e*x] + (e*f + d*g)^2*Log[d + e*x])/(8*d^3*e^3)
```

3.554.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {639, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{(d+ex)^2 (d^2-e^2x^2)} \, dx$$

$$\Big\downarrow \text{ 639}$$

$$\int \frac{(f+gx)^2}{(d-ex)(d+ex)^3}dx$$

$$\Big\downarrow \text{ 99}$$

$$\int \left(\frac{(dg+ef)^2}{4d^2e^2 (d^2-e^2x^2)}+\frac{(ef-dg)(3dg+ef)}{4d^2e^2(d+ex)^2}+\frac{(dg-ef)^2}{2de^2(d+ex)^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\text{arctanh}\left(\frac{ex}{d}\right) (dg+ef)^2}{4d^3e^3}-\frac{(3dg+ef)(ef-dg)}{4d^2e^3(d+ex)}-\frac{(ef-dg)^2}{4de^3(d+ex)^2}$$

input
```
Int[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)),x]
```

output
```
-1/4*(e*f - d*g)^2/(d*e^3*(d + e*x)^2) - ((e*f - d*g)*(e*f + 3*d*g))/(4*d^
2*e^3*(d + e*x)) + ((e*f + d*g)^2*ArcTanh[(e*x)/d])/(4*d^3*e^3)
```

3.554.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.554.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.59

method	result
norman	$\frac{\frac{d^2g^2-e^2f^2}{2d\,e^3}+\frac{\left(3d^2g^2-2defg-e^2f^2\right)x}{4d^2e^2}}{(ex+d)^2}-\frac{\left(d^2g^2+2defg+e^2f^2\right)\ln(-ex+d)}{8e^3d^3}+\frac{\left(d^2g^2+2defg+e^2f^2\right)\ln(ex+d)}{8e^3d^3}$
default	$\frac{\left(-d^2g^2-2defg-e^2f^2\right)\ln(-ex+d)}{8e^3d^3}-\frac{-3d^2g^2+2defg+e^2f^2}{4d^2e^3(ex+d)}-\frac{d^2g^2-2defg+e^2f^2}{4e^3d(ex+d)^2}+\frac{\left(d^2g^2+2defg+e^2f^2\right)\ln(ex+d)}{8e^3d^3}$
risch	$\frac{\frac{d^2g^2-e^2f^2}{2d\,e^3}+\frac{\left(3d^2g^2-2defg-e^2f^2\right)x}{4d^2e^2}}{(ex+d)^2}-\frac{\ln(-ex+d)g^2}{8e^3d}-\frac{\ln(-ex+d)fg}{4e^2d^2}-\frac{\ln(-ex+d)f^2}{8e\,d^3}+\frac{\ln(ex+d)g^2}{8e^3d}+\frac{\ln(ex+d)fg}{4e^2d^2}-$
parallelrisch	$-\frac{4d^2e^2f^2-4d^4g^2-\ln(ex+d)d^2e^2f^2+\ln(ex-d)x^2e^4f^2-\ln(ex+d)x^2e^4f^2+\ln(ex-d)d^2e^2f^2-6x\,d^3e\,g^2+2xd\,e^3f^2+2\ln(ex-d)}{}$

input
```
int((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

output
```
(1/2*(d^2*g^2-e^2*f^2)/d/e^3+1/4*(3*d^2*g^2-2*d*e*f*g-e^2*f^2)/d^2/e^2*x)/
(e*x+d)^2-1/8*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/d^3*ln(-e*x+d)+1/8*(d^2*g^2+
2*d*e*f*g+e^2*f^2)/e^3/d^3*ln(e*x+d)
```

3.554.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(81) = 162.

Time = 0.31 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.11

$$\int \frac{(f+gx)^2}{(d+ex)^2\,(d^2-e^2x^2)}\,dx =$$
$$-\frac{4\,d^2e^2f^2-4\,d^4g^2+2\,(de^3f^2+2\,d^2e^2fg-3\,d^3eg^2)x-(d^2e^2f^2+2\,d^3efg+d^4g^2+(e^4f^2+2\,de^3fg+d}{}$$

input
```
integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

output
```
-1/8*(4*d^2*e^2*f^2 - 4*d^4*g^2 + 2*(d*e^3*f^2 + 2*d^2*e^2*f*g - 3*d^3*e*g
^2)*x - (d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2 + (e^4*f^2 + 2*d*e^3*f*g + d^
2*e^2*g^2)*x^2 + 2*(d*e^3*f^2 + 2*d^2*e^2*f*g + d^3*e*g^2)*x)*log(e*x + d)
 + (d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2 + (e^4*f^2 + 2*d*e^3*f*g + d^2*e^2
*g^2)*x^2 + 2*(d*e^3*f^2 + 2*d^2*e^2*f*g + d^3*e*g^2)*x)*log(e*x - d))/(d^
3*e^5*x^2 + 2*d^4*e^4*x + d^5*e^3)
```

3.554.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(75) = 150$.

Time = 0.45 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.13

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)}\,dx = -\frac{-2d^3g^2+2de^2f^2+x(-3d^2eg^2+2de^2fg+e^3f^2)}{4d^4e^3+8d^3e^4x+4d^2e^5x^2} - \frac{(dg+ef)^2\log\left(-\frac{d(dg+ef)^2}{e(d^2g^2+2defg+e^2f^2)}+x\right)}{8d^3e^3} + \frac{(dg+ef)^2\log\left(\frac{d(dg+ef)^2}{e(d^2g^2+2defg+e^2f^2)}+x\right)}{8d^3e^3}$$

input
```
integrate((g*x+f)**2/(e*x+d)**2/(-e**2*x**2+d**2),x)
```

output
```
-(-2*d**3*g**2 + 2*d*e**2*f**2 + x*(-3*d**2*e*g**2 + 2*d*e**2*f*g + e**3*f
**2))/(4*d**4*e**3 + 8*d**3*e**4*x + 4*d**2*e**5*x**2) - (d*g + e*f)**2*lo
g(-d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g + e**2*f**2)) + x)/(8*d**3*e
**3) + (d*g + e*f)**2*log(d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g + e**
2*f**2)) + x)/(8*d**3*e**3)
```

3.554.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.71

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)}\,dx = -\frac{2\,de^2f^2-2\,d^3g^2+(e^3f^2+2\,de^2fg-3\,d^2eg^2)x}{4\,(d^2e^5x^2+2\,d^3e^4x+d^4e^3)} + \frac{(e^2f^2+2\,defg+d^2g^2)\log(ex+d)}{8\,d^3e^3} - \frac{(e^2f^2+2\,defg+d^2g^2)\log(ex-d)}{8\,d^3e^3}$$

input
```
integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

output
```
-1/4*(2*d*e^2*f^2 - 2*d^3*g^2 + (e^3*f^2 + 2*d*e^2*f*g - 3*d^2*e*g^2)*x)/(
d^2*e^5*x^2 + 2*d^3*e^4*x + d^4*e^3) + 1/8*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)
*log(e*x + d)/(d^3*e^3) - 1/8*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(e*x - d)
/(d^3*e^3)
```

3.554.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.71

$$\int \frac{(f+gx)^2}{(d+ex)^2\,(d^2-e^2x^2)}\,dx = -\frac{(e^2f^2+2\,defg+d^2g^2)\log\left(\left|-\frac{2\,d}{ex+d}+1\right|\right)}{8\,d^3e^3} - \frac{\frac{e^5f^2}{ex+d}+\frac{de^5f^2}{(ex+d)^2}+\frac{2\,de^4fg}{ex+d}-\frac{2\,d^2e^4fg}{(ex+d)^2}-\frac{3\,d^2e^3g^2}{ex+d}+\frac{d^3e^3g^2}{(ex+d)^2}}{4\,d^2e^6}$$

input
```
integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

output
```
-1/8*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(abs(-2*d/(e*x + d) + 1))/(d^3*e^3
) - 1/4*(e^5*f^2/(e*x + d) + d*e^5*f^2/(e*x + d)^2 + 2*d*e^4*f*g/(e*x + d)
 - 2*d^2*e^4*f*g/(e*x + d)^2 - 3*d^2*e^3*g^2/(e*x + d) + d^3*e^3*g^2/(e*x
+ d)^2)/(d^2*e^6)
```

3.554.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\int \frac{(f+gx)^2}{(d+ex)^2\,(d^2-e^2x^2)}\,dx = \frac{\frac{d^2\,g^2-e^2\,f^2}{2\,d\,e^3}-\frac{x\,\left(-3\,d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^2\,e^2}}{d^2+2\,d\,e\,x+e^2\,x^2}+\frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,{\left(d\,g+e\,f\right)}^2}{4\,d^3\,e^3}$$

input
```
int((f + g*x)^2/((d^2 - e^2*x^2)*(d + e*x)^2),x)
```

output
```
((d^2*g^2 - e^2*f^2)/(2*d*e^3) - (x*(e^2*f^2 - 3*d^2*g^2 + 2*d*e*f*g))/(4*
d^2*e^2))/(d^2 + e^2*x^2 + 2*d*e*x) + (atanh((e*x)/d)*(d*g + e*f)^2)/(4*d^
3*e^3)
```

3.555 $\int \frac{(f+gx)^2}{(d+ex)^3(d^2-e^2x^2)}\,dx$

3.555.1 Optimal result

Integrand size = 29, antiderivative size = 113

$$\int \frac{(f+gx)^2}{(d+ex)^3(d^2-e^2x^2)}\,dx = -\frac{(ef-dg)^2}{6de^3(d+ex)^3}-\frac{(ef-dg)(ef+3dg)}{8d^2e^3(d+ex)^2}-\frac{(ef+dg)^2}{8d^3e^3(d+ex)}+\frac{(ef+dg)^2\operatorname{arctanh}\left(\frac{ex}{d}\right)}{8d^4e^3}$$

output
```
-1/6*(-d*g+e*f)^2/d/e^3/(e*x+d)^3-1/8*(-d*g+e*f)*(3*d*g+e*f)/d^2/e^3/(e*x+
d)^2-1/8*(d*g+e*f)^2/d^3/e^3/(e*x+d)+1/8*(d*g+e*f)^2*arctanh(e*x/d)/d^4/e^
3
```

3.555.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^2}{(d+ex)^3(d^2-e^2x^2)}\,dx = \frac{-\frac{8d^3(ef-dg)^2}{(d+ex)^3}+\frac{6d^2(-e^2f^2-2defg+3d^2g^2)}{(d+ex)^2}-\frac{6d(ef+dg)^2}{d+ex}-3(ef+dg)^2\log(d-ex)+3(ef+dg)^2\log(d+ex)}{48d^4e^3}$$

input
```
Integrate[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)),x]
```

output
```
((-8*d^3*(e*f - d*g)^2)/(d + e*x)^3 + (6*d^2*(-(e^2*f^2) - 2*d*e*f*g + 3*d
^2*g^2))/(d + e*x)^2 - (6*d*(e*f + d*g)^2)/(d + e*x) - 3*(e*f + d*g)^2*Log
[d - e*x] + 3*(e*f + d*g)^2*Log[d + e*x])/(48*d^4*e^3)
```

3.555.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {639, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{(d+ex)^3 (d^2-e^2x^2)} \, dx$$

$$\Big\downarrow \text{639}$$

$$\int \frac{(f+gx)^2}{(d-ex)(d+ex)^4}dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(\frac{(dg+ef)^2}{8d^3e^2(d+ex)^2}+\frac{(ef-dg)(3dg+ef)}{4d^2e^2(d+ex)^3}+\frac{(dg+ef)^2}{8d^3e^2 (d^2-e^2x^2)}+\frac{(dg-ef)^2}{2de^2(d+ex)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{arctanh}\left(\frac{ex}{d}\right)(dg+ef)^2}{8d^4e^3}-\frac{(dg+ef)^2}{8d^3e^3(d+ex)}-\frac{(3dg+ef)(ef-dg)}{8d^2e^3(d+ex)^2}-\frac{(ef-dg)^2}{6de^3(d+ex)^3}$$

input
```
Int[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)),x]
```

output
```
-1/6*(e*f - d*g)^2/(d*e^3*(d + e*x)^3) - ((e*f - d*g)*(e*f + 3*d*g))/(8*d^
2*e^3*(d + e*x)^2) - (e*f + d*g)^2/(8*d^3*e^3*(d + e*x)) + ((e*f + d*g)^2*
ArcTanh[(e*x)/d])/(8*d^4*e^3)
```

3.555.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.555.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.57

method	result
norman	$\frac{-\frac{\left(d^2g^2-2defg-5e^2f^2\right)x^3}{12d^4}-\frac{\left(d^2g^2+2defg-7e^2f^2\right)x}{8d^2e^2}-\frac{\left(3d^2g^2-2defg-9e^2f^2\right)x^2}{8d^3e}}{(ex+d)^3}-\frac{\left(d^2g^2+2defg+e^2f^2\right)\ln(-ex+d)}{16e^3d^4}+\frac{(d^2g^2+$
default	$\frac{\left(-d^2g^2-2defg-e^2f^2\right)\ln(-ex+d)}{16e^3d^4}-\frac{-3d^2g^2+2defg+e^2f^2}{8d^2e^3(ex+d)^2}-\frac{d^2g^2-2defg+e^2f^2}{6e^3d(ex+d)^3}+\frac{\left(d^2g^2+2defg+e^2f^2\right)\ln(ex+d)}{16e^3d^4}-$
risch	$\frac{-\frac{\left(d^2g^2+2defg+e^2f^2\right)x^2}{8d^3e}+\frac{\left(d^2g^2-6defg-3e^2f^2\right)x}{8d^2e^2}+\frac{d^2g^2-2defg-5e^2f^2}{12d\,e^3}}{(ex+d)^3}-\frac{\ln(-ex+d)g^2}{16e^3d^2}-\frac{\ln(-ex+d)fg}{8e^2d^3}-\frac{\ln(-ex+d)f^2}{16e\,d^4}$
parallelrisch	$-\frac{-6\ln(ex+d)d^4efg+3\ln(ex-d)x^3d^2e^3g^2-18\ln(ex+d)x\,d^3e^2fg+18\ln(ex-d)x\,d^3e^2fg+18\ln(ex-d)x^2d^2e^3fg-18\ln(ex+d)$

input
```
int((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

output
```
(-1/12*(d^2*g^2-2*d*e*f*g-5*e^2*f^2)/d^4*x^3-1/8*(d^2*g^2+2*d*e*f*g-7*e^2*
f^2)/d^2/e^2*x-1/8*(3*d^2*g^2-2*d*e*f*g-9*e^2*f^2)/d^3/e*x^2)/(e*x+d)^3-1/
16*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/d^4*ln(-e*x+d)+1/16*(d^2*g^2+2*d*e*f*g+
e^2*f^2)/e^3/d^4*ln(e*x+d)
```

3.555.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(105) = 210$.

Time = 0.28 (sec) , antiderivative size = 400, normalized size of antiderivative = 3.54

$$\int \frac{(f+gx)^2}{(d+ex)^3\,(d^2-e^2x^2)}\,dx = -\frac{20\,d^3e^2f^2+8\,d^4efg-4\,d^5g^2+6\,(de^4f^2+2\,d^2e^3fg+d^3e^2g^2)x^2+6\,(3\,d^2e^3f^2+6\,d^3e^2fg-d^4eg^2)x-3}{}$$

input
```
integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x, algorithm="fricas")
```

output
```
-1/48*(20*d^3*e^2*f^2 + 8*d^4*e*f*g - 4*d^5*g^2 + 6*(d*e^4*f^2 + 2*d^2*e^3
*f*g + d^3*e^2*g^2)*x^2 + 6*(3*d^2*e^3*f^2 + 6*d^3*e^2*f*g - d^4*e*g^2)*x
- 3*(d^3*e^2*f^2 + 2*d^4*e*f*g + d^5*g^2 + (e^5*f^2 + 2*d*e^4*f*g + d^2*e^
3*g^2)*x^3 + 3*(d*e^4*f^2 + 2*d^2*e^3*f*g + d^3*e^2*g^2)*x^2 + 3*(d^2*e^3*
f^2 + 2*d^3*e^2*f*g + d^4*e*g^2)*x)*log(e*x + d) + 3*(d^3*e^2*f^2 + 2*d^4*
e*f*g + d^5*g^2 + (e^5*f^2 + 2*d*e^4*f*g + d^2*e^3*g^2)*x^3 + 3*(d*e^4*f^2
 + 2*d^2*e^3*f*g + d^3*e^2*g^2)*x^2 + 3*(d^2*e^3*f^2 + 2*d^3*e^2*f*g + d^4
*e*g^2)*x)*log(e*x - d))/(d^4*e^6*x^3 + 3*d^5*e^5*x^2 + 3*d^6*e^4*x + d^7*
e^3)
```

3.555.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(99) = 198$.

Time = 0.59 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.19

$$\int \frac{(f+gx)^2}{(d+ex)^3\,(d^2-e^2x^2)}\,dx = -\frac{-2d^4g^2+4d^3efg+10d^2e^2f^2+x^2\cdot(3d^2e^2g^2+6de^3fg+3e^4f^2)+x(-3d^3eg^2+18d^2e^2fg+9de^3f^2)}{24d^6e^3+72d^5e^4x+72d^4e^5x^2+24d^3e^6x^3} - \frac{(dg+ef)^2\log\left(-\frac{d(dg+ef)^2}{e(d^2g^2+2defg+e^2f^2)}+x\right)}{16d^4e^3} + \frac{(dg+ef)^2\log\left(\frac{d(dg+ef)^2}{e(d^2g^2+2defg+e^2f^2)}+x\right)}{16d^4e^3}$$

input
```
integrate((g*x+f)**2/(e*x+d)**3/(-e**2*x**2+d**2),x)
```

output
```
-(-2*d**4*g**2 + 4*d**3*e*f*g + 10*d**2*e**2*f**2 + x**2*(3*d**2*e**2*g**2
 + 6*d*e**3*f*g + 3*e**4*f**2) + x*(-3*d**3*e*g**2 + 18*d**2*e**2*f*g + 9*
d*e**3*f**2))/(24*d**6*e**3 + 72*d**5*e**4*x + 72*d**4*e**5*x**2 + 24*d**3
*e**6*x**3) - (d*g + e*f)**2*log(-d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f
*g + e**2*f**2)) + x)/(16*d**4*e**3) + (d*g + e*f)**2*log(d*(d*g + e*f)**2
/(e*(d**2*g**2 + 2*d*e*f*g + e**2*f**2)) + x)/(16*d**4*e**3)
```

3.555.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.82

$$\int \frac{(f+gx)^2}{(d+ex)^3(d^2-e^2x^2)}\,dx = -\frac{10\,d^2e^2f^2+4\,d^3efg-2\,d^4g^2+3\,(e^4f^2+2\,de^3fg+d^2e^2g^2)x^2+3\,(3\,de^3f^2+6\,d^2e^2fg-d^3eg^2)x}{24\,(d^3e^6x^3+3\,d^4e^5x^2+3\,d^5e^4x+d^6e^3)} + \frac{(e^2f^2+2\,defg+d^2g^2)\log(ex+d)}{16\,d^4e^3} - \frac{(e^2f^2+2\,defg+d^2g^2)\log(ex-d)}{16\,d^4e^3}$$

input
```
integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x, algorithm="maxima")
```

output
```
-1/24*(10*d^2*e^2*f^2 + 4*d^3*e*f*g - 2*d^4*g^2 + 3*(e^4*f^2 + 2*d*e^3*f*g
 + d^2*e^2*g^2)*x^2 + 3*(3*d*e^3*f^2 + 6*d^2*e^2*f*g - d^3*e*g^2)*x)/(d^3*
e^6*x^3 + 3*d^4*e^5*x^2 + 3*d^5*e^4*x + d^6*e^3) + 1/16*(e^2*f^2 + 2*d*e*f
*g + d^2*g^2)*log(e*x + d)/(d^4*e^3) - 1/16*(e^2*f^2 + 2*d*e*f*g + d^2*g^2
)*log(e*x - d)/(d^4*e^3)
```

3.555.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.65

$$\int \frac{(f+gx)^2}{(d+ex)^3(d^2-e^2x^2)}\,dx = \frac{(e^2f^2+2\,defg+d^2g^2)\log(|ex+d|)}{16\,d^4e^3} - \frac{(e^2f^2+2\,defg+d^2g^2)\log(|ex-d|)}{16\,d^4e^3} - \frac{10\,d^3e^2f^2+4\,d^4efg-2\,d^5g^2+3\,(de^4f^2+2\,d^2e^3fg+d^3e^2g^2)x^2+3\,(3\,d^2e^3f^2+6\,d^3e^2fg-d^4eg^2)x}{24\,(ex+d)^3d^4e^3}$$

input
```
integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x, algorithm="giac")
```

output
```
1/16*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(abs(e*x + d))/(d^4*e^3) - 1/16*(e
^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(abs(e*x - d))/(d^4*e^3) - 1/24*(10*d^3*e
^2*f^2 + 4*d^4*e*f*g - 2*d^5*g^2 + 3*(d*e^4*f^2 + 2*d^2*e^3*f*g + d^3*e^2*
g^2)*x^2 + 3*(3*d^2*e^3*f^2 + 6*d^3*e^2*f*g - d^4*e*g^2)*x)/((e*x + d)^3*d
^4*e^3)
```

3.555.9 Mupad [B] (verification not implemented)

Time = 12.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.35

$$\int \frac{(f+gx)^2}{(d+ex)^3(d^2-e^2x^2)}\,dx = \frac{\operatorname{atanh}\left(\frac{e\,x}{d}\right)\,\left(d\,g+e\,f\right)^2}{8\,d^4\,e^3} - \frac{\frac{-d^2\,g^2+2\,d\,e\,f\,g+5\,e^2\,f^2}{12\,d\,e^3}+\frac{x\,\left(-d^2\,g^2+6\,d\,e\,f\,g+3\,e^2\,f^2\right)}{8\,d^2\,e^2}+\frac{x^2\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{8\,d^3\,e}}{d^3+3\,d^2\,e\,x+3\,d\,e^2\,x^2+e^3\,x^3}$$

input
```
int((f + g*x)^2/((d^2 - e^2*x^2)*(d + e*x)^3),x)
```

output
```
(atanh((e*x)/d)*(d*g + e*f)^2)/(8*d^4*e^3) - ((5*e^2*f^2 - d^2*g^2 + 2*d*e
*f*g)/(12*d*e^3) + (x*(3*e^2*f^2 - d^2*g^2 + 6*d*e*f*g))/(8*d^2*e^2) + (x^
2*(d^2*g^2 + e^2*f^2 + 2*d*e*f*g))/(8*d^3*e))/(d^3 + e^3*x^3 + 3*d*e^2*x^2
 + 3*d^2*e*x)
```

3.556 $\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)}\,dx$

3.556.1 Optimal result

Integrand size = 29, antiderivative size = 139

$$\int \frac{(f+gx)^2}{(d+ex)^4\,(d^2-e^2x^2)}\,dx = -\frac{(ef-dg)^2}{8de^3(d+ex)^4}-\frac{(ef-dg)(ef+3dg)}{12d^2e^3(d+ex)^3}-\frac{(ef+dg)^2}{16d^3e^3(d+ex)^2}-\frac{(ef+dg)^2}{16d^4e^3(d+ex)}+\frac{(ef+dg)^2\operatorname{arctanh}\left(\frac{ex}{d}\right)}{16d^5e^3}$$

output
```
-1/8*(-d*g+e*f)^2/d/e^3/(e*x+d)^4-1/12*(-d*g+e*f)*(3*d*g+e*f)/d^2/e^3/(e*x
+d)^3-1/16*(d*g+e*f)^2/d^3/e^3/(e*x+d)^2-1/16*(d*g+e*f)^2/d^4/e^3/(e*x+d)+
1/16*(d*g+e*f)^2*arctanh(e*x/d)/d^5/e^3
```

3.556.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.02

$$\int \frac{(f+gx)^2}{(d+ex)^4\,(d^2-e^2x^2)}\,dx = -\frac{\frac{12d^4(ef-dg)^2}{(d+ex)^4}+\frac{8d^3(e^2f^2+2defg-3d^2g^2)}{(d+ex)^3}+\frac{6d^2(ef+dg)^2}{(d+ex)^2}+\frac{6d(ef+dg)^2}{d+ex}+3(ef+dg)^2\log(d-ex)-3(ef+dg)^2\log}{96d^5e^3}$$

input
```
Integrate[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)),x]
```

output
```
-1/96*((12*d^4*(e*f - d*g)^2)/(d + e*x)^4 + (8*d^3*(e^2*f^2 + 2*d*e*f*g -
3*d^2*g^2))/(d + e*x)^3 + (6*d^2*(e*f + d*g)^2)/(d + e*x)^2 + (6*d*(e*f +
d*g)^2)/(d + e*x) + 3*(e*f + d*g)^2*Log[d - e*x] - 3*(e*f + d*g)^2*Log[d +
 e*x])/(d^5*e^3)
```

3.556.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {639, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{(d+ex)^4 (d^2-e^2x^2)} \, dx$$

$$\Big\downarrow \text{ 639}$$

$$\int \frac{(f+gx)^2}{(d-ex)(d+ex)^5}dx$$

$$\Big\downarrow \text{ 99}$$

$$\int \left(\frac{(dg+ef)^2}{16d^4e^2(d+ex)^2}+\frac{(dg+ef)^2}{8d^3e^2(d+ex)^3}+\frac{(ef-dg)(3dg+ef)}{4d^2e^2(d+ex)^4}+\frac{(dg+ef)^2}{16d^4e^2(d^2-e^2x^2)}+\frac{(dg-ef)^2}{2de^2(d+ex)^5}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\text{arctanh}\left(\frac{ex}{d}\right)(dg+ef)^2}{16d^5e^3}-\frac{(dg+ef)^2}{16d^4e^3(d+ex)}-\frac{(dg+ef)^2}{16d^3e^3(d+ex)^2}-\frac{(3dg+ef)(ef-dg)}{12d^2e^3(d+ex)^3}-\frac{(ef-dg)^2}{8de^3(d+ex)^4}$$

input
```
Int[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)),x]
```

output
```
-1/8*(e*f - d*g)^2/(d*e^3*(d + e*x)^4) - ((e*f - d*g)*(e*f + 3*d*g))/(12*d
^2*e^3*(d + e*x)^3) - (e*f + d*g)^2/(16*d^3*e^3*(d + e*x)^2) - (e*f + d*g)
^2/(16*d^4*e^3*(d + e*x)) + ((e*f + d*g)^2*ArcTanh[(e*x)/d])/(16*d^5*e^3)
```

3.556.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.556.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.43

method	result
norman	$\frac{-\frac{\left(3d^2g^2-26defg-61e^2f^2\right)x^3}{48d^4}-\frac{\left(d^2g^2-2defg-7e^2f^2\right)x^2}{4e\,d^3}+\frac{e^2\left(dfg+2e\,f^2\right)x^4}{6d^5}-\frac{\left(d^2g^2+2defg-15e^2f^2\right)x}{16d^2e^2}}{\left(ex+d\right)^4}-\frac{\left(d^2g^2+2defg+e^2f^2\right)\ln}{32e^3d^5}$
default	$\frac{\left(-d^2g^2-2defg-e^2f^2\right)\ln\left(-ex+d\right)}{32e^3d^5}-\frac{-3d^2g^2+2defg+e^2f^2}{12d^2e^3\left(ex+d\right)^3}-\frac{d^2g^2-2defg+e^2f^2}{8e^3d\left(ex+d\right)^4}+\frac{\left(d^2g^2+2defg+e^2f^2\right)\ln\left(ex+d\right)}{32e^3d^5}-$
risch	$\frac{-\frac{\left(d^2g^2+2defg+e^2f^2\right)x^3}{16d^4}-\frac{\left(d^2g^2+2defg+e^2f^2\right)x^2}{4d^3e}-\frac{\left(3d^2g^2+38defg+19e^2f^2\right)x}{48d^2e^2}-\frac{f\left(dg+2ef\right)}{6e^2d}}{\left(ex+d\right)^4}-\frac{\ln\left(-ex+d\right)g^2}{32e^3d^3}-\frac{\ln\left(-ex+d\right)fg}{16e^2d^4}$
parallelrisch	$-\frac{-12\ln\left(ex+d\right)x\,d^3e^3f^2+6\ln\left(ex-d\right)d^5efg-6\ln\left(ex+d\right)d^5efg+3\ln\left(ex-d\right)x^4d^2e^4g^2-3\ln\left(ex+d\right)x^4d^2e^4g^2+12\ln\left(ex-d\right)x^3d^3}{}$

input
```
int((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

output
```
(-1/48*(3*d^2*g^2-26*d*e*f*g-61*e^2*f^2)/d^4*x^3-1/4/e*(d^2*g^2-2*d*e*f*g-
7*e^2*f^2)/d^3*x^2+1/6*e^2*(d*f*g+2*e*f^2)/d^5*x^4-1/16*(d^2*g^2+2*d*e*f*g
-15*e^2*f^2)/d^2/e^2*x)/(e*x+d)^4-1/32*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/d^5
*ln(-e*x+d)+1/32*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/d^5*ln(e*x+d)
```

3.556.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(129) = 258$.

Time = 0.30 (sec) , antiderivative size = 511, normalized size of antiderivative = 3.68

$$\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)}\,dx = -\frac{32\,d^4e^2f^2+16\,d^5efg+6\,(de^5f^2+2\,d^2e^4fg+d^3e^3g^2)x^3+24\,(d^2e^4f^2+2\,d^3e^3fg+d^4e^2g^2)x^2+2\,(19\,d}{}$$

input
```
integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x, algorithm="fricas")
```

output
```
-1/96*(32*d^4*e^2*f^2 + 16*d^5*e*f*g + 6*(d*e^5*f^2 + 2*d^2*e^4*f*g + d^3*
e^3*g^2)*x^3 + 24*(d^2*e^4*f^2 + 2*d^3*e^3*f*g + d^4*e^2*g^2)*x^2 + 2*(19*
d^3*e^3*f^2 + 38*d^4*e^2*f*g + 3*d^5*e*g^2)*x - 3*(d^4*e^2*f^2 + 2*d^5*e*f
*g + d^6*g^2 + (e^6*f^2 + 2*d*e^5*f*g + d^2*e^4*g^2)*x^4 + 4*(d*e^5*f^2 +
2*d^2*e^4*f*g + d^3*e^3*g^2)*x^3 + 6*(d^2*e^4*f^2 + 2*d^3*e^3*f*g + d^4*e^
2*g^2)*x^2 + 4*(d^3*e^3*f^2 + 2*d^4*e^2*f*g + d^5*e*g^2)*x)*log(e*x + d) +
 3*(d^4*e^2*f^2 + 2*d^5*e*f*g + d^6*g^2 + (e^6*f^2 + 2*d*e^5*f*g + d^2*e^4
*g^2)*x^4 + 4*(d*e^5*f^2 + 2*d^2*e^4*f*g + d^3*e^3*g^2)*x^3 + 6*(d^2*e^4*f
^2 + 2*d^3*e^3*f*g + d^4*e^2*g^2)*x^2 + 4*(d^3*e^3*f^2 + 2*d^4*e^2*f*g + d
^5*e*g^2)*x)*log(e*x - d))/(d^5*e^7*x^4 + 4*d^6*e^6*x^3 + 6*d^7*e^5*x^2 +
4*d^8*e^4*x + d^9*e^3)
```

3.556.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(122) = 244$.

Time = 0.68 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.03

$$\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)}\,dx = -\frac{8d^4fg+16d^3ef^2+x^3\cdot(3d^2e^2g^2+6de^3fg+3e^4f^2)+x^2\cdot(12d^3eg^2+24d^2e^2fg+12de^3f^2)+x(3d^4g^2}{48d^8e^2+192d^7e^3x+288d^6e^4x^2+192d^5e^5x^3+48d^4e^6x^4} - \frac{(dg+ef)^2\log\left(-\frac{d(dg+ef)^2}{e(d^2g^2+2defg+e^2f^2)}+x\right)}{32d^5e^3} + \frac{(dg+ef)^2\log\left(\frac{d(dg+ef)^2}{e(d^2g^2+2defg+e^2f^2)}+x\right)}{32d^5e^3}$$

input
```
integrate((g*x+f)**2/(e*x+d)**4/(-e**2*x**2+d**2),x)
```

output
```
-(8*d**4*f*g + 16*d**3*e*f**2 + x**3*(3*d**2*e**2*g**2 + 6*d*e**3*f*g + 3*
e**4*f**2) + x**2*(12*d**3*e*g**2 + 24*d**2*e**2*f*g + 12*d*e**3*f**2) + x
*(3*d**4*g**2 + 38*d**3*e*f*g + 19*d**2*e**2*f**2))/(48*d**8*e**2 + 192*d*
*7*e**3*x + 288*d**6*e**4*x**2 + 192*d**5*e**5*x**3 + 48*d**4*e**6*x**4) -
 (d*g + e*f)**2*log(-d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g + e**2*f**
2)) + x)/(32*d**5*e**3) + (d*g + e*f)**2*log(d*(d*g + e*f)**2/(e*(d**2*g**
2 + 2*d*e*f*g + e**2*f**2)) + x)/(32*d**5*e**3)
```

3.556.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.70

$$\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)}\,dx = -\frac{16\,d^3ef^2+8\,d^4fg+3\,(e^4f^2+2\,de^3fg+d^2e^2g^2)x^3+12\,(de^3f^2+2\,d^2e^2fg+d^3eg^2)x^2+(19\,d^2e^2f^2+3}{48\,(d^4e^6x^4+4\,d^5e^5x^3+6\,d^6e^4x^2+4\,d^7e^3x+d^8e^2)} + \frac{(e^2f^2+2\,defg+d^2g^2)\log(ex+d)}{32\,d^5e^3} - \frac{(e^2f^2+2\,defg+d^2g^2)\log(ex-d)}{32\,d^5e^3}$$

input
```
integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x, algorithm="maxima")
```

output
```
-1/48*(16*d^3*e*f^2 + 8*d^4*f*g + 3*(e^4*f^2 + 2*d*e^3*f*g + d^2*e^2*g^2)*
x^3 + 12*(d*e^3*f^2 + 2*d^2*e^2*f*g + d^3*e*g^2)*x^2 + (19*d^2*e^2*f^2 + 3
8*d^3*e*f*g + 3*d^4*g^2)*x)/(d^4*e^6*x^4 + 4*d^5*e^5*x^3 + 6*d^6*e^4*x^2 +
 4*d^7*e^3*x + d^8*e^2) + 1/32*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(e*x + d
)/(d^5*e^3) - 1/32*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(e*x - d)/(d^5*e^3)
```

3.556.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.53

$$\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)}\,dx = \frac{(e^2f^2+2\,defg+d^2g^2)\log(|ex+d|)}{32\,d^5e^3} - \frac{(e^2f^2+2\,defg+d^2g^2)\log(|ex-d|)}{32\,d^5e^3} - \frac{16\,d^4e^2f^2+8\,d^5efg+3\,(de^5f^2+2\,d^2e^4fg+d^3e^3g^2)x^3+12\,(d^2e^4f^2+2\,d^3e^3fg+d^4e^2g^2)x^2+(19\,d^3e^3}{48\,(ex+d)^4d^5e^3}$$

input
```
integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x, algorithm="giac")
```

output
```
1/32*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(abs(e*x + d))/(d^5*e^3) - 1/32*(e
^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(abs(e*x - d))/(d^5*e^3) - 1/48*(16*d^4*e
^2*f^2 + 8*d^5*e*f*g + 3*(d*e^5*f^2 + 2*d^2*e^4*f*g + d^3*e^3*g^2)*x^3 + 1
2*(d^2*e^4*f^2 + 2*d^3*e^3*f*g + d^4*e^2*g^2)*x^2 + (19*d^3*e^3*f^2 + 38*d
^4*e^2*f*g + 3*d^5*e*g^2)*x)/((e*x + d)^4*d^5*e^3)
```

3.556.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.29

$$\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)}\,dx$$
$$=\frac{\operatorname{atanh}\left(\frac{e\,x}{d}\right)\,\left(d\,g+e\,f\right)^2}{16\,d^5\,e^3}-\frac{\frac{x^3\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{16\,d^4}+\frac{2\,e\,f^2+d\,g\,f}{6\,d\,e^2}+\frac{x\,\left(3\,d^2\,g^2+38\,d\,e\,f\,g+19\,e^2\,f^2\right)}{48\,d^2\,e^2}+\frac{x^2\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^3\,e}}{d^4+4\,d^3\,e\,x+6\,d^2\,e^2\,x^2+4\,d\,e^3\,x^3+e^4\,x^4}$$

input
```
int((f + g*x)^2/((d^2 - e^2*x^2)*(d + e*x)^4),x)
```

output
```
(atanh((e*x)/d)*(d*g + e*f)^2)/(16*d^5*e^3) - ((x^3*(d^2*g^2 + e^2*f^2 + 2
*d*e*f*g))/(16*d^4) + (2*e*f^2 + d*f*g)/(6*d*e^2) + (x*(3*d^2*g^2 + 19*e^2
*f^2 + 38*d*e*f*g))/(48*d^2*e^2) + (x^2*(d^2*g^2 + e^2*f^2 + 2*d*e*f*g))/(
4*d^3*e))/(d^4 + e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x)
```

3.557 $\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$

3.557.1 Optimal result

Integrand size = 29, antiderivative size = 218

$$\begin{aligned}\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^2}\,dx &= \frac{d^3(49e^2f^2+160defg+112d^2g^2)\,x}{e^2} \\ &+ \frac{d^2(23e^2f^2+98defg+80d^2g^2)\,x^2}{2e} \\ &+ \frac{1}{3}d\left(7e^2f^2+46defg+49d^2g^2\right)x^3 \\ &+ \frac{1}{4}e\left(e^2f^2+14defg+23d^2g^2\right)x^4+\frac{1}{5}e^2g(2ef+7dg)x^5+\frac{1}{6}e^3g^2x^6 \\ &+ \frac{32d^5(ef+dg)^2}{e^3(d-ex)}+\frac{16d^4(ef+dg)(5ef+9dg)\log(d-ex)}{e^3}\end{aligned}$$

output
```
d^3*(112*d^2*g^2+160*d*e*f*g+49*e^2*f^2)*x/e^2+1/2*d^2*(80*d^2*g^2+98*d*e*
f*g+23*e^2*f^2)*x^2/e+1/3*d*(49*d^2*g^2+46*d*e*f*g+7*e^2*f^2)*x^3+1/4*e*(2
3*d^2*g^2+14*d*e*f*g+e^2*f^2)*x^4+1/5*e^2*g*(7*d*g+2*e*f)*x^5+1/6*e^3*g^2*
x^6+32*d^5*(d*g+e*f)^2/e^3/(-e*x+d)+16*d^4*(d*g+e*f)*(9*d*g+5*e*f)*ln(-e*x
+d)/e^3
```

3.557.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{d^3(49e^2f^2+160defg+112d^2g^2)\,x}{e^2} + \frac{d^2(23e^2f^2+98defg+80d^2g^2)\,x^2}{2e} + \frac{1}{3}d\left(7e^2f^2+46defg+49d^2g^2\right)x^3 + \frac{1}{4}e\left(e^2f^2+14defg+23d^2g^2\right)x^4 + \frac{1}{5}e^2g(2ef+7dg)x^5 + \frac{1}{6}e^3g^2x^6 - \frac{32d^5(ef+dg)^2}{e^3(-d+ex)} + \frac{16d^4(5e^2f^2+14defg+9d^2g^2)\log(d-ex)}{e^3}$$

input
```
Integrate[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

output
```
(d^3*(49*e^2*f^2 + 160*d*e*f*g + 112*d^2*g^2)*x)/e^2 + (d^2*(23*e^2*f^2 +
98*d*e*f*g + 80*d^2*g^2)*x^2)/(2*e) + (d*(7*e^2*f^2 + 46*d*e*f*g + 49*d^2*
g^2)*x^3)/3 + (e*(e^2*f^2 + 14*d*e*f*g + 23*d^2*g^2)*x^4)/4 + (e^2*g*(2*e*
f + 7*d*g)*x^5)/5 + (e^3*g^2*x^6)/6 - (32*d^5*(e*f + d*g)^2)/(e^3*(-d + e*
x)) + (16*d^4*(5*e^2*f^2 + 14*d*e*f*g + 9*d^2*g^2)*Log[d - e*x])/e^3
```

3.557.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {639, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$$

$$\Big\downarrow \; 639$$

$$\int \frac{(d+ex)^5(f+gx)^2}{(d-ex)^2}dx$$

$$\Big\downarrow \; 99$$

$$\int \left(\frac{32d^5(dg+ef)^2}{e^2(ex-d)^2}+\frac{16d^4(-9dg-5ef)(dg+ef)}{e^2(d-ex)}+ex^3(23d^2g^2+14defg+e^2f^2)+dx^2(49d^2g^2+46defg+7e\right.$$

$\big\downarrow$ 2009

$$\frac{32d^5(dg+ef)^2}{e^3(d-ex)}+\frac{16d^4(dg+ef)(9dg+5ef)\log(d-ex)}{e^3}+\frac{1}{4}ex^4(23d^2g^2+14defg+e^2f^2)+\frac{1}{3}dx^3(49d^2g^2+46defg+7e^2f^2)+\frac{d^2x^2(80d^2g^2+98defg+23e^2f^2)}{2e}+\frac{d^3x(112d^2g^2+160defg+49e^2f^2)}{e^2}+\frac{1}{5}e^2gx^5(7dg+2ef)+\frac{1}{6}e^3g^2x^6$$

input
```
Int[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

output
```
(d^3*(49*e^2*f^2 + 160*d*e*f*g + 112*d^2*g^2)*x)/e^2 + (d^2*(23*e^2*f^2 +
98*d*e*f*g + 80*d^2*g^2)*x^2)/(2*e) + (d*(7*e^2*f^2 + 46*d*e*f*g + 49*d^2*
g^2)*x^3)/3 + (e*(e^2*f^2 + 14*d*e*f*g + 23*d^2*g^2)*x^4)/4 + (e^2*g*(2*e*
f + 7*d*g)*x^5)/5 + (e^3*g^2*x^6)/6 + (32*d^5*(e*f + d*g)^2)/(e^3*(d - e*x
)) + (16*d^4*(e*f + d*g)*(5*e*f + 9*d*g)*Log[d - e*x])/e^3
```

3.557.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.557.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.19

method	result
default	$\frac{\frac{1}{6}g^2e^5x^6+\frac{7}{5}x^5d\,e^4g^2+\frac{2}{5}x^5e^5fg+\frac{23}{4}x^4d^2e^3g^2+\frac{7}{2}x^4d\,e^4fg+\frac{1}{4}x^4e^5f^2+\frac{49}{3}x^3d^3e^2g^2+\frac{46}{3}x^3d^2e^3fg+\frac{7}{3}x^3d\,e^4f^2+40x^2d^4e\,g^2+4}{e^2}$
risch	$\frac{e^3g^2x^6}{6}+\frac{7e^2x^5d\,g^2}{5}+\frac{2e^3x^5fg}{5}+\frac{23e\,x^4d^2g^2}{4}+\frac{7e^2x^4dfg}{2}+\frac{e^3x^4f^2}{4}+\frac{49x^3d^3g^2}{3}+\frac{46e\,x^3d^2fg}{3}+\frac{7e^2x^3d\,f^2}{3}+$
norman	$\frac{(-\frac{287}{3}g^2d^5-\frac{434}{3}fg\,d^4e-\frac{140}{3}f^2d^3e^2)x^3+(-\frac{67}{12}g^2d^2e^3-\frac{7}{2}fgd\,e^4-\frac{1}{4}f^2e^5)x^6+(-\frac{224}{15}g^2d^3e^2-\frac{224}{15}fg\,d^2e^3-\frac{7}{3}f^2d\,e^4)x^5+(-\frac{137}{4}}{-e^2}$
parallelrisch	$\frac{-4860d^5e^2f^2+8640\ln(ex-d)x\,d^6e\,g^2+4800\ln(ex-d)x\,d^4e^3f^2-13440\ln(ex-d)d^6efg-8640d^7g^2+2250x^2d^3e^4f^2+1420x^3d^4e}{}$

input
```
int((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e^2*(1/6*g^2*e^5*x^6+7/5*x^5*d*e^4*g^2+2/5*x^5*e^5*f*g+23/4*x^4*d^2*e^3*
g^2+7/2*x^4*d*e^4*f*g+1/4*x^4*e^5*f^2+49/3*x^3*d^3*e^2*g^2+46/3*x^3*d^2*e^
3*f*g+7/3*x^3*d*e^4*f^2+40*x^2*d^4*e*g^2+49*x^2*d^3*e^2*f*g+23/2*x^2*d^2*e
^3*f^2+112*g^2*d^5*x+160*f*g*d^4*e*x+49*f^2*d^3*e^2*x)+32*d^5*(d^2*g^2+2*d
*e*f*g+e^2*f^2)/e^3/(-e*x+d)+16*d^4/e^3*(9*d^2*g^2+14*d*e*f*g+5*e^2*f^2)*l
n(-e*x+d)
```

3.557.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.50

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$$
$$= \frac{10\,e^7g^2x^7-1920\,d^5e^2f^2-3840\,d^6efg-1920\,d^7g^2+2\,(12\,e^7fg+37\,de^6g^2)x^6+3\,(5\,e^7f^2+62\,de^6fg+8}{}$$

input
```
integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

output
```
1/60*(10*e^7*g^2*x^7 - 1920*d^5*e^2*f^2 - 3840*d^6*e*f*g - 1920*d^7*g^2 +
2*(12*e^7*f*g + 37*d*e^6*g^2)*x^6 + 3*(5*e^7*f^2 + 62*d*e^6*f*g + 87*d^2*e
^5*g^2)*x^5 + 5*(25*d*e^6*f^2 + 142*d^2*e^5*f*g + 127*d^3*e^4*g^2)*x^4 + 1
0*(55*d^2*e^5*f^2 + 202*d^3*e^4*f*g + 142*d^4*e^3*g^2)*x^3 + 90*(25*d^3*e^
4*f^2 + 74*d^4*e^3*f*g + 48*d^5*e^2*g^2)*x^2 - 60*(49*d^4*e^3*f^2 + 160*d^
5*e^2*f*g + 112*d^6*e*g^2)*x - 960*(5*d^5*e^2*f^2 + 14*d^6*e*f*g + 9*d^7*g
^2 - (5*d^4*e^3*f^2 + 14*d^5*e^2*f*g + 9*d^6*e*g^2)*x)*log(e*x - d))/(e^4*
x - d*e^3)
```

3.557.6 Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{16d^4(dg+ef)\,(9dg+5ef)\log{(-d+ex)}}{e^3} + \frac{e^3g^2x^6}{6} + x^5 \cdot \left(\frac{7de^2g^2}{5} + \frac{2e^3fg}{5}\right) + x^4 \cdot \left(\frac{23d^2eg^2}{4} + \frac{7de^2fg}{2} + \frac{e^3f^2}{4}\right) + x^3 \cdot \left(\frac{49d^3g^2}{3} + \frac{46d^2efg}{3} + \frac{7de^2f^2}{3}\right) + x^2 \cdot \left(\frac{40d^4g^2}{e} + 49d^3fg + \frac{23d^2ef^2}{2}\right) + x\left(\frac{112d^5g^2}{e^2} + \frac{160d^4fg}{e} + 49d^3f^2\right) + \frac{-32d^7g^2 - 64d^6efg - 32d^5e^2f^2}{-de^3 + e^4x}$$

input
```
integrate((e*x+d)**7*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

output
```
16*d**4*(d*g + e*f)*(9*d*g + 5*e*f)*log(-d + e*x)/e**3 + e**3*g**2*x**6/6
+ x**5*(7*d*e**2*g**2/5 + 2*e**3*f*g/5) + x**4*(23*d**2*e*g**2/4 + 7*d*e**
2*f*g/2 + e**3*f**2/4) + x**3*(49*d**3*g**2/3 + 46*d**2*e*f*g/3 + 7*d*e**2
*f**2/3) + x**2*(40*d**4*g**2/e + 49*d**3*f*g + 23*d**2*e*f**2/2) + x*(112
*d**5*g**2/e**2 + 160*d**4*f*g/e + 49*d**3*f**2) + (-32*d**7*g**2 - 64*d**
6*e*f*g - 32*d**5*e**2*f**2)/(-d*e**3 + e**4*x)
```

3.557.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = -\frac{32\,(d^5e^2f^2+2\,d^6efg+d^7g^2)}{e^4x-de^3} + \frac{10\,e^5g^2x^6+12\,(2\,e^5fg+7\,de^4g^2)x^5+15\,(e^5f^2+14\,de^4fg+23\,d^2e^3g^2)x^4+20\,(7\,de^4f^2+46\,d^2e^3fg+\cdots}{60\,e^2\cdots} + \frac{16\,(5\,d^4e^2f^2+14\,d^5efg+9\,d^6g^2)\log(ex-d)}{e^3}$$

input
```
integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

output
```
-32*(d^5*e^2*f^2 + 2*d^6*e*f*g + d^7*g^2)/(e^4*x - d*e^3) + 1/60*(10*e^5*g
^2*x^6 + 12*(2*e^5*f*g + 7*d*e^4*g^2)*x^5 + 15*(e^5*f^2 + 14*d*e^4*f*g + 2
3*d^2*e^3*g^2)*x^4 + 20*(7*d*e^4*f^2 + 46*d^2*e^3*f*g + 49*d^3*e^2*g^2)*x^
3 + 30*(23*d^2*e^3*f^2 + 98*d^3*e^2*f*g + 80*d^4*e*g^2)*x^2 + 60*(49*d^3*e
^2*f^2 + 160*d^4*e*f*g + 112*d^5*g^2)*x)/e^2 + 16*(5*d^4*e^2*f^2 + 14*d^5*
e*f*g + 9*d^6*g^2)*log(e*x - d)/e^3
```

3.557.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{16\,(5\,d^4e^2f^2+14\,d^5efg+9\,d^6g^2)\log(|ex-d|)}{e^3} - \frac{32\,(d^5e^2f^2+2\,d^6efg+d^7g^2)}{(ex-d)e^3} + \frac{10\,e^{15}g^2x^6+24\,e^{15}fgx^5+84\,de^{14}g^2x^5+15\,e^{15}f^2x^4+210\,de^{14}fgx^4+345\,d^2e^{13}g^2x^4+140\,de^{14}f^2x^3+\cdots}{\cdots}$$

input
```
integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

output
```
16*(5*d^4*e^2*f^2 + 14*d^5*e*f*g + 9*d^6*g^2)*log(abs(e*x - d))/e^3 - 32*(
d^5*e^2*f^2 + 2*d^6*e*f*g + d^7*g^2)/((e*x - d)*e^3) + 1/60*(10*e^15*g^2*x
^6 + 24*e^15*f*g*x^5 + 84*d*e^14*g^2*x^5 + 15*e^15*f^2*x^4 + 210*d*e^14*f*
g*x^4 + 345*d^2*e^13*g^2*x^4 + 140*d*e^14*f^2*x^3 + 920*d^2*e^13*f*g*x^3 +
 980*d^3*e^12*g^2*x^3 + 690*d^2*e^13*f^2*x^2 + 2940*d^3*e^12*f*g*x^2 + 240
0*d^4*e^11*g^2*x^2 + 2940*d^3*e^12*f^2*x + 9600*d^4*e^11*f*g*x + 6720*d^5*
e^10*g^2*x)/e^12
```

3.557.9 Mupad [B] (verification not implemented)

Time = 11.99 (sec) , antiderivative size = 1029, normalized size of antiderivative = 4.72

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$$

$$= x^5\left(\frac{e^2\,g\,(5\,d\,g+2\,e\,f)}{5}+\frac{2\,d\,e^2\,g^2}{5}\right)+x^3\left(\frac{5\,d\,(2\,d^2\,g^2+4\,d\,e\,f\,g+e^2\,f^2)}{3}+\frac{2\,d\left(\frac{10\,d^2\,e^3\,g^2+10\,d\,e^4\,f\,g+e^5\,f^2}{e^2}-d^2\,e\,g^2+\frac{2\,d\,(e^2\,g\,(5\,d\,g+2\,e\,f)+2\,d\,e^2\,g^2)}{e}\right)}{3\,e}-\frac{d^2\,(e^2\,g\,(5\,d\,g+2\,e\,f)+2\,d\,e^2\,g^2)}{3\,e^2}\right)$$

$$+x^4\left(\frac{10\,d^2\,e^3\,g^2+10\,d\,e^4\,f\,g+e^5\,f^2}{4\,e^2}-\frac{d^2\,e\,g^2}{4}+\frac{d\,(e^2\,g\,(5\,d\,g+2\,e\,f)+2\,d\,e^2\,g^2)}{2\,e}\right)$$

$$+x^2\left(\frac{5\,d^2\,(d^2\,g^2+4\,d\,e\,f\,g+2\,e^2\,f^2)}{2\,e}-\frac{d^2\left(\frac{10\,d^2\,e^3\,g^2+10\,d\,e^4\,f\,g+e^5\,f^2}{e^2}-d^2\,e\,g^2+\frac{2\,d\,(e^2\,g\,(5\,d\,g+2\,e\,f)+2\,d\,e^2\,g^2)}{e}\right)}{2\,e^2}\right.$$

$$+\frac{d\left(5\,d\,(2\,d^2\,g^2+4\,d\,e\,f\,g+e^2\,f^2)+\frac{2\,d\left(\frac{10\,d^2\,e^3\,g^2+10\,d\,e^4\,f\,g+e^5\,f^2}{e^2}-d^2\,e\,g^2+\frac{2\,d\,\left(e^2\,g\,(5\,d\,g+2\,e\,f)+2\,d\,e^2\,g^2\right)}{e}\right)}{e}-\frac{d^2\,(e^2\,g\,(5\,d}{}\right.}{e}$$

$$+x\left(\frac{d^5\,g^2+10\,d^4\,e\,f\,g+10\,d^3\,e^2\,f^2}{e^2}\right.$$

3.557. $\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$

$$d^2\left(5\,d\,(2\,d^2\,g^2+4\,d\,e\,f\,g+e^2\,f^2)+\frac{2\,d\left(\frac{10\,d^2\,e^3\,g^2+10\,d\,e^4\,f\,g+e^5\,f^2}{e^2}-d^2\,e\,g^2+\frac{2\,d\,\left(e^2\,g\,(5\,d\,g+2\,e\,f)+2\,d\,e^2\,g^2\right)}{e}\right)}{e}-\frac{d^2\,(e^2\,g\,(5}{}\right.$$

input
```
int(((f + g*x)^2*(d + e*x)^7)/(d^2 - e^2*x^2)^2,x)
```

output
```
x^5*((e^2*g*(5*d*g + 2*e*f))/5 + (2*d*e^2*g^2)/5) + x^3*((5*d*(2*d^2*g^2 +
 e^2*f^2 + 4*d*e*f*g))/3 + (2*d*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)
/e^2 - d^2*e*g^2 + (2*d*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/(3*e) -
 (d^2*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/(3*e^2)) + x^4*((e^5*f^2 + 10
*d^2*e^3*g^2 + 10*d*e^4*f*g)/(4*e^2) - (d^2*e*g^2)/4 + (d*(e^2*g*(5*d*g +
2*e*f) + 2*d*e^2*g^2))/(2*e)) + x^2*((5*d^2*(d^2*g^2 + 2*e^2*f^2 + 4*d*e*f
*g))/(2*e) - (d^2*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^2 - d^2*e*g
^2 + (2*d*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/(2*e^2) + (d*(5*d*(2*
d^2*g^2 + e^2*f^2 + 4*d*e*f*g) + (2*d*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^
4*f*g)/e^2 - d^2*e*g^2 + (2*d*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/e
 - (d^2*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e^2))/e) + x*((d^5*g^2 + 10
*d^3*e^2*f^2 + 10*d^4*e*f*g)/e^2 - (d^2*(5*d*(2*d^2*g^2 + e^2*f^2 + 4*d*e*
f*g) + (2*d*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^2 - d^2*e*g^2 + (
2*d*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/e - (d^2*(e^2*g*(5*d*g + 2*
e*f) + 2*d*e^2*g^2))/e^2))/e^2 + (2*d*((5*d^2*(d^2*g^2 + 2*e^2*f^2 + 4*d*e
*f*g))/e - (d^2*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^2 - d^2*e*g^2
 + (2*d*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/e^2 + (2*d*(5*d*(2*d^2*
g^2 + e^2*f^2 + 4*d*e*f*g) + (2*d*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*
g)/e^2 - d^2*e*g^2 + (2*d*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/e - (
d^2*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e^2))/e))/e) + (log(e*x - d)...
```

3.557. $\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$

3.558 $\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$

3.558.1 Optimal result

Integrand size = 29, antiderivative size = 177

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{d^2(17e^2f^2+64defg+48d^2g^2)\,x}{e^2} + \frac{d(3e^2f^2+17defg+16d^2g^2)\,x^2}{e} + \frac{1}{3}\left(e^2f^2+12defg+17d^2g^2\right)x^3 + \frac{1}{2}eg(ef+3dg)x^4 + \frac{1}{5}e^2g^2x^5 + \frac{16d^4(ef+dg)^2}{e^3(d-ex)} + \frac{32d^3(ef+dg)(ef+2dg)\log(d-ex)}{e^3}$$

output
```
d^2*(48*d^2*g^2+64*d*e*f*g+17*e^2*f^2)*x/e^2+d*(16*d^2*g^2+17*d*e*f*g+3*e^
2*f^2)*x^2/e+1/3*(17*d^2*g^2+12*d*e*f*g+e^2*f^2)*x^3+1/2*e*g*(3*d*g+e*f)*x
^4+1/5*e^2*g^2*x^5+16*d^4*(d*g+e*f)^2/e^3/(-e*x+d)+32*d^3*(d*g+e*f)*(2*d*g
+e*f)*ln(-e*x+d)/e^3
```

3.558.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{d^2(17e^2f^2+64defg+48d^2g^2)\,x}{e^2} + \frac{d(3e^2f^2+17defg+16d^2g^2)\,x^2}{e} + \frac{1}{3}\left(e^2f^2+12defg+17d^2g^2\right)x^3 + \frac{1}{2}eg(ef+3dg)x^4 + \frac{1}{5}e^2g^2x^5 - \frac{16d^4(ef+dg)^2}{e^3(-d+ex)} + \frac{32d^3(e^2f^2+3defg+2d^2g^2)\log(d-ex)}{e^3}$$

input
```
Integrate[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

output
```
(d^2*(17*e^2*f^2 + 64*d*e*f*g + 48*d^2*g^2)*x)/e^2 + (d*(3*e^2*f^2 + 17*d*
e*f*g + 16*d^2*g^2)*x^2)/e + ((e^2*f^2 + 12*d*e*f*g + 17*d^2*g^2)*x^3)/3 +
 (e*g*(e*f + 3*d*g)*x^4)/2 + (e^2*g^2*x^5)/5 - (16*d^4*(e*f + d*g)^2)/(e^3
*(-d + e*x)) + (32*d^3*(e^2*f^2 + 3*d*e*f*g + 2*d^2*g^2)*Log[d - e*x])/e^3
```

3.558.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {639, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$$

$$\Big\downarrow \text{639}$$

$$\int \frac{(d+ex)^4(f+gx)^2}{(d-ex)^2}dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(\frac{16d^4(dg+ef)^2}{e^2(ex-d)^2} + \frac{32d^3(-2dg-ef)(dg+ef)}{e^2(d-ex)} + x^2\left(17d^2g^2+12defg+e^2f^2\right) + \frac{2dx\left(16d^2g^2+17defg+3e^2\right.}{e}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{16d^4(dg+ef)^2}{e^3(d-ex)}+\frac{32d^3(dg+ef)(2dg+ef)\log(d-ex)}{e^3}+\frac{1}{3}x^3\left(17d^2g^2+12defg+e^2f^2\right)+\frac{dx^2\left(16d^2g^2+17defg+3e^2f^2\right)}{e}+\frac{d^2x\left(48d^2g^2+64defg+17e^2f^2\right)}{e^2}+\frac{1}{2}egx^4(3dg+ef)+\frac{1}{5}e^2g^2x^5$$

input
```
Int[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

output
```
(d^2*(17*e^2*f^2 + 64*d*e*f*g + 48*d^2*g^2)*x)/e^2 + (d*(3*e^2*f^2 + 17*d*
e*f*g + 16*d^2*g^2)*x^2)/e + ((e^2*f^2 + 12*d*e*f*g + 17*d^2*g^2)*x^3)/3 +
 (e*g*(e*f + 3*d*g)*x^4)/2 + (e^2*g^2*x^5)/5 + (16*d^4*(e*f + d*g)^2)/(e^3
*(d - e*x)) + (32*d^3*(e*f + d*g)*(e*f + 2*d*g)*Log[d - e*x])/e^3
```

3.558.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.558. $\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$

3.558.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.23

method	result
default	$\frac{\frac{1}{5}g^2e^4x^5+\frac{3}{2}de^3g^2x^4+\frac{1}{2}e^4fgx^4+\frac{17}{3}d^2e^2g^2x^3+4de^3fgx^3+\frac{1}{3}e^4f^2x^3+16d^3eg^2x^2+17d^2e^2fgx^2+3de^3f^2x^2+48d^4g^2x+64d^3e}{e^2}$
risch	$\frac{e^2g^2x^5}{5}+\frac{3edg^2x^4}{2}+\frac{e^2fgx^4}{2}+\frac{17d^2g^2x^3}{3}+4edfgx^3+\frac{e^2f^2x^3}{3}+\frac{16d^3g^2x^2}{e}+17d^2fgx^2+3edf^2x^2+$
norman	$\frac{\left(-\frac{127}{3}d^4g^2-60fged^3-\frac{50}{3}d^2e^2f^2\right)x^3+\left(-\frac{82}{15}d^2g^2e^2-4dfge^3-\frac{1}{3}f^2e^4\right)x^5+\left(-\frac{29}{2}g^2ed^3-\frac{33}{2}e^2fgd^2-3e^3f^2d\right)x^4+\frac{d^2\left(32g^2d^5+4\right.}{}}{-e^2x^2+d^2}$
parallelrisch	$\frac{6g^2e^6x^6+39x^5de^5g^2+15x^5e^6fg+125x^4d^2e^4g^2+105de^5fgx^4+10e^6f^2x^4+310d^3e^3g^2x^3+390d^2e^4fgx^3+80de^5f^2x^3+1920\ln}{}$

input
```
int((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e^2*(1/5*g^2*e^4*x^5+3/2*d*e^3*g^2*x^4+1/2*e^4*f*g*x^4+17/3*d^2*e^2*g^2*
x^3+4*d*e^3*f*g*x^3+1/3*e^4*f^2*x^3+16*d^3*e*g^2*x^2+17*d^2*e^2*f*g*x^2+3*
d*e^3*f^2*x^2+48*d^4*g^2*x+64*d^3*e*f*g*x+17*d^2*e^2*f^2*x)+32*d^3/e^3*(2*
d^2*g^2+3*d*e*f*g+e^2*f^2)*ln(-e*x+d)+16*d^4*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e
^3/(-e*x+d)
```

3.558.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.63

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$$
$$=\frac{6e^6g^2x^6-480d^4e^2f^2-960d^5efg-480d^6g^2+3(5e^6fg+13de^5g^2)x^5+5(2e^6f^2+21de^5fg+25d^2e^4g}{}$$

input
```
integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

output
```
1/30*(6*e^6*g^2*x^6 - 480*d^4*e^2*f^2 - 960*d^5*e*f*g - 480*d^6*g^2 + 3*(5
*e^6*f*g + 13*d*e^5*g^2)*x^5 + 5*(2*e^6*f^2 + 21*d*e^5*f*g + 25*d^2*e^4*g^
2)*x^4 + 10*(8*d*e^5*f^2 + 39*d^2*e^4*f*g + 31*d^3*e^3*g^2)*x^3 + 30*(14*d
^2*e^4*f^2 + 47*d^3*e^3*f*g + 32*d^4*e^2*g^2)*x^2 - 30*(17*d^3*e^3*f^2 + 6
4*d^4*e^2*f*g + 48*d^5*e*g^2)*x - 960*(d^4*e^2*f^2 + 3*d^5*e*f*g + 2*d^6*g
^2 - (d^3*e^3*f^2 + 3*d^4*e^2*f*g + 2*d^5*e*g^2)*x)*log(e*x - d))/(e^4*x -
 d*e^3)
```

3.558.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{32d^3(dg+ef)\,(2dg+ef)\log(-d+ex)}{e^3} + \frac{e^2g^2x^5}{5} + x^4 \cdot \left(\frac{3deg^2}{2} + \frac{e^2fg}{2}\right) + x^3 \cdot \left(\frac{17d^2g^2}{3} + 4defg + \frac{e^2f^2}{3}\right) + x^2 \cdot \left(\frac{16d^3g^2}{e} + 17d^2fg + 3def^2\right) + x\left(\frac{48d^4g^2}{e^2} + \frac{64d^3fg}{e} + 17d^2f^2\right) + \frac{-16d^6g^2 - 32d^5efg - 16d^4e^2f^2}{-de^3 + e^4x}$$

input
```
integrate((e*x+d)**6*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

output
```
32*d**3*(d*g + e*f)*(2*d*g + e*f)*log(-d + e*x)/e**3 + e**2*g**2*x**5/5 +
x**4*(3*d*e*g**2/2 + e**2*f*g/2) + x**3*(17*d**2*g**2/3 + 4*d*e*f*g + e**2
*f**2/3) + x**2*(16*d**3*g**2/e + 17*d**2*f*g + 3*d*e*f**2) + x*(48*d**4*g
**2/e**2 + 64*d**3*f*g/e + 17*d**2*f**2) + (-16*d**6*g**2 - 32*d**5*e*f*g
- 16*d**4*e**2*f**2)/(-d*e**3 + e**4*x)
```

3.558.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = -\frac{16\,(d^4e^2f^2 + 2\,d^5efg + d^6g^2)}{e^4x - de^3} + \frac{6\,e^4g^2x^5 + 15\,(e^4fg + 3\,de^3g^2)x^4 + 10\,(e^4f^2 + 12\,de^3fg + 17\,d^2e^2g^2)x^3 + 30\,(3\,de^3f^2 + 17\,d^2e^2fg + 16\,d}{30\,e^2} + \frac{32\,(d^3e^2f^2 + 3\,d^4efg + 2\,d^5g^2)\log(ex-d)}{e^3}$$

input
```
integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

output
```
-16*(d^4*e^2*f^2 + 2*d^5*e*f*g + d^6*g^2)/(e^4*x - d*e^3) + 1/30*(6*e^4*g^
2*x^5 + 15*(e^4*f*g + 3*d*e^3*g^2)*x^4 + 10*(e^4*f^2 + 12*d*e^3*f*g + 17*d
^2*e^2*g^2)*x^3 + 30*(3*d*e^3*f^2 + 17*d^2*e^2*f*g + 16*d^3*e*g^2)*x^2 + 3
0*(17*d^2*e^2*f^2 + 64*d^3*e*f*g + 48*d^4*g^2)*x)/e^2 + 32*(d^3*e^2*f^2 +
3*d^4*e*f*g + 2*d^5*g^2)*log(e*x - d)/e^3
```

3.558.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)^6(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$$
$$= \frac{32\left(d^3e^2f^2+3\,d^4efg+2\,d^5g^2\right)\log\left(|ex-d|\right)}{e^3}-\frac{16\left(d^4e^2f^2+2\,d^5efg+d^6g^2\right)}{(ex-d)e^3}$$
$$+\frac{6\,e^{12}g^2x^5+15\,e^{12}fgx^4+45\,de^{11}g^2x^4+10\,e^{12}f^2x^3+120\,de^{11}fgx^3+170\,d^2e^{10}g^2x^3+90\,de^{11}f^2x^2+510}{30\,e^{10}}$$

input
```
integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

output
```
32*(d^3*e^2*f^2 + 3*d^4*e*f*g + 2*d^5*g^2)*log(abs(e*x - d))/e^3 - 16*(d^4
*e^2*f^2 + 2*d^5*e*f*g + d^6*g^2)/((e*x - d)*e^3) + 1/30*(6*e^12*g^2*x^5 +
 15*e^12*f*g*x^4 + 45*d*e^11*g^2*x^4 + 10*e^12*f^2*x^3 + 120*d*e^11*f*g*x^
3 + 170*d^2*e^10*g^2*x^3 + 90*d*e^11*f^2*x^2 + 510*d^2*e^10*f*g*x^2 + 480*
d^3*e^9*g^2*x^2 + 510*d^2*e^10*f^2*x + 1920*d^3*e^9*f*g*x + 1440*d^4*e^8*g
^2*x)/e^10
```

3.558.9 Mupad [B] (verification not implemented)

Time = 11.85 (sec) , antiderivative size = 565, normalized size of antiderivative = 3.19

$$\int \frac{(d+ex)^6(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$$

$$= x^2\left(\frac{2\,d\,\left(d^2\,g^2+3\,d\,e\,f\,g+e^2\,f^2\right)}{e}-\frac{d^2\,\left(2\,e\,g\,\left(2\,d\,g+e\,f\right)+2\,d\,e\,g^2\right)}{2\,e^2}+\frac{d\,\left(\frac{6\,d^2\,e^2\,g^2+8\,d\,e^3\,f\,g+e^4\,f^2}{e^2}-d^2\,g^2+\frac{2\,d\,\left(2\,e\,g\,\left(2\,d\,g+e\,f\right)+2\,d\,e\,g^2\right)}{e}\right)}{e}\right)$$
$$+x^4\left(\frac{e\,g\,\left(2\,d\,g+e\,f\right)}{2}+\frac{d\,e\,g^2}{2}\right)+x\left(\frac{d^4\,g^2+8\,d^3\,e\,f\,g+6\,d^2\,e^2\,f^2}{e^2}-\frac{d^2\,\left(\frac{6\,d^2\,e^2\,g^2+8\,d\,e^3\,f\,g+e^4\,f^2}{e^2}-d^2\,g^2+\frac{2\,d\,\left(2\,e\,g\,\left(2\,d\,g+e\,f\right)+2\,d\,e\,g^2\right)}{e}\right)}{e^2}\right.$$
$$+\frac{2\,d\,\left(\frac{4\,d\,\left(d^2\,g^2+3\,d\,e\,f\,g+e^2\,f^2\right)}{e}-\frac{d^2\,\left(2\,e\,g\,\left(2\,d\,g+e\,f\right)+2\,d\,e\,g^2\right)}{e^2}+\frac{2\,d\,\left(\frac{6\,d^2\,e^2\,g^2+8\,d\,e^3\,f\,g+e^4\,f^2}{e^2}-d^2\,g^2+\frac{2\,d\,\left(2\,e\,g\,\left(2\,d\,g+e\,f\right)+2\,d\,e\,g^2\right)}{e}\right)}{e}\right.}{e}$$
$$+x^3\left(\frac{6\,d^2\,e^2\,g^2+8\,d\,e^3\,f\,g+e^4\,f^2}{3\,e^2}-\frac{d^2\,g^2}{3}+\frac{2\,d\,\left(2\,e\,g\,\left(2\,d\,g+e\,f\right)+2\,d\,e\,g^2\right)}{3\,e}\right)$$
$$+\frac{\ln\left(e\,x-d\right)\,\left(64\,d^5\,g^2+96\,d^4\,e\,f\,g+32\,d^3\,e^2\,f^2\right)}{e^3}$$
$$+\frac{16\,\left(d^6\,g^2+2\,d^5\,e\,f\,g+d^4\,e^2\,f^2\right)}{e\,\left(d\,e^2-e^3\,x\right)}+\frac{e^2\,g^2\,x^5}{5}$$

input

```
int(((f + g*x)^2*(d + e*x)^6)/(d^2 - e^2*x^2)^2,x)
```

output
```
x^2*((2*d*(d^2*g^2 + e^2*f^2 + 3*d*e*f*g))/e - (d^2*(2*e*g*(2*d*g + e*f) +
 2*d*e*g^2))/(2*e^2) + (d*((e^4*f^2 + 6*d^2*e^2*g^2 + 8*d*e^3*f*g)/e^2 - d
^2*g^2 + (2*d*(2*e*g*(2*d*g + e*f) + 2*d*e*g^2))/e))/e) + x^4*((e*g*(2*d*g
 + e*f))/2 + (d*e*g^2)/2) + x*((d^4*g^2 + 6*d^2*e^2*f^2 + 8*d^3*e*f*g)/e^2
 - (d^2*((e^4*f^2 + 6*d^2*e^2*g^2 + 8*d*e^3*f*g)/e^2 - d^2*g^2 + (2*d*(2*e
*g*(2*d*g + e*f) + 2*d*e*g^2))/e))/e^2 + (2*d*((4*d*(d^2*g^2 + e^2*f^2 + 3
*d*e*f*g))/e - (d^2*(2*e*g*(2*d*g + e*f) + 2*d*e*g^2))/e^2 + (2*d*((e^4*f^
2 + 6*d^2*e^2*g^2 + 8*d*e^3*f*g)/e^2 - d^2*g^2 + (2*d*(2*e*g*(2*d*g + e*f)
 + 2*d*e*g^2))/e))/e))/e) + x^3*((e^4*f^2 + 6*d^2*e^2*g^2 + 8*d*e^3*f*g)/(
3*e^2) - (d^2*g^2)/3 + (2*d*(2*e*g*(2*d*g + e*f) + 2*d*e*g^2))/(3*e)) + (l
og(e*x - d)*(64*d^5*g^2 + 32*d^3*e^2*f^2 + 96*d^4*e*f*g))/e^3 + (16*(d^6*g
^2 + d^4*e^2*f^2 + 2*d^5*e*f*g))/(e*(d*e^2 - e^3*x)) + (e^2*g^2*x^5)/5
```

3.558. $\int \frac{(d+ex)^6(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

3.559 $\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$

3.559.1 Optimal result

Integrand size = 29, antiderivative size = 146

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{d(5e^2f^2+24defg+20d^2g^2)\,x}{e^2}+\frac{(e^2f^2+10defg+12d^2g^2)\,x^2}{2e}+\frac{1}{3}g(2ef+5dg)x^3+\frac{1}{4}eg^2x^4+\frac{8d^3(ef+dg)^2}{e^3(d-ex)}+\frac{4d^2(ef+dg)(3ef+7dg)\log(d-ex)}{e^3}$$

output
```
d*(20*d^2*g^2+24*d*e*f*g+5*e^2*f^2)*x/e^2+1/2*(12*d^2*g^2+10*d*e*f*g+e^2*f
^2)*x^2/e+1/3*g*(5*d*g+2*e*f)*x^3+1/4*e*g^2*x^4+8*d^3*(d*g+e*f)^2/e^3/(-e*
x+d)+4*d^2*(d*g+e*f)*(7*d*g+3*e*f)*ln(-e*x+d)/e^3
```

3.559.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{d(5e^2f^2+24defg+20d^2g^2)\,x}{e^2}+\frac{(e^2f^2+10defg+12d^2g^2)\,x^2}{2e}+\frac{1}{3}g(2ef+5dg)x^3+\frac{1}{4}eg^2x^4-\frac{8d^3(ef+dg)^2}{e^3(-d+ex)}+\frac{4d^2(3e^2f^2+10defg+7d^2g^2)\log(d-ex)}{e^3}$$

input
```
Integrate[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

output
```
(d*(5*e^2*f^2 + 24*d*e*f*g + 20*d^2*g^2)*x)/e^2 + ((e^2*f^2 + 10*d*e*f*g +
 12*d^2*g^2)*x^2)/(2*e) + (g*(2*e*f + 5*d*g)*x^3)/3 + (e*g^2*x^4)/4 - (8*d
^3*(e*f + d*g)^2)/(e^3*(-d + e*x)) + (4*d^2*(3*e^2*f^2 + 10*d*e*f*g + 7*d^
2*g^2)*Log[d - e*x])/e^3
```

3.559.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {639, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$$
$$\Big\downarrow \; 639$$
$$\int \frac{(d+ex)^3(f+gx)^2}{(d-ex)^2}dx$$
$$\Big\downarrow \; 99$$
$$\int \left(\frac{8d^3(dg+ef)^2}{e^2(ex-d)^2}+\frac{x\left(12d^2g^2+10defg+e^2f^2\right)}{e}+\frac{d\left(20d^2g^2+24defg+5e^2f^2\right)}{e^2}+\frac{4d^2(-7dg-3ef)(dg+ef)}{e^2(d-ex)}\right.$$
$$\Big\downarrow \; 2009$$
$$\frac{8d^3(dg+ef)^2}{e^3(d-ex)}+\frac{4d^2(dg+ef)(7dg+3ef)\log(d-ex)}{e^3}+\frac{x^2\left(12d^2g^2+10defg+e^2f^2\right)}{2e}+\frac{dx\left(20d^2g^2+24defg+5e^2f^2\right)}{e^2}+\frac{1}{3}gx^3(5dg+2ef)+\frac{1}{4}eg^2x^4$$

input
```
Int[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

output
```
(d*(5*e^2*f^2 + 24*d*e*f*g + 20*d^2*g^2)*x)/e^2 + ((e^2*f^2 + 10*d*e*f*g +
 12*d^2*g^2)*x^2)/(2*e) + (g*(2*e*f + 5*d*g)*x^3)/3 + (e*g^2*x^4)/4 + (8*d
^3*(e*f + d*g)^2)/(e^3*(d - e*x)) + (4*d^2*(e*f + d*g)*(3*e*f + 7*d*g)*Log
[d - e*x])/e^3
```

3.559.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.559.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.21

method	result
default	$\frac{\frac{1}{4}g^2e^3x^4+\frac{5}{3}x^3de^2g^2+\frac{2}{3}x^3e^3fg+6x^2d^2eg^2+5x^2de^2fg+\frac{1}{2}x^2e^3f^2+20d^3g^2x+24d^2efgx+5de^2f^2x}{e^2}+\frac{4d^2(7d^2g^2+10defg+3}{e^3}$
risch	$\frac{eg^2x^4}{4}+\frac{5x^3dg^2}{3}+\frac{2ex^3fg}{3}+\frac{6x^2g^2d^2}{e}+5x^2fgd+\frac{ex^2f^2}{2}+\frac{20d^3g^2x}{e^2}+\frac{24d^2fgx}{e}+5df^2x+\frac{28d^4\ln(-ex}{e^3}$
norman	$\frac{(-\frac{55}{3}d^3g^2-\frac{70}{3}d^2efg-5de^2f^2)x^3+(-\frac{23}{4}d^2g^2e-5dfge^2-\frac{1}{2}f^2e^3)x^4+\frac{d^3(28d^2g^2+40defg+13e^2f^2)x}{e^2}+\frac{d^2(28d^4g^2+42fged^3+17d^2}{2e^3}}{-e^2x^2+d^2}$
parallelrisch	$\frac{3g^2e^5x^5+17x^4de^4g^2+8x^4e^5fg+52x^3d^2e^3g^2+52x^3de^4fg+6x^3e^5f^2+336\ln(ex-d)xd^4eg^2+480\ln(ex-d)xd^3e^2fg+144\ln(ex}{}$

input
```
int((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e^2*(1/4*g^2*e^3*x^4+5/3*x^3*d*e^2*g^2+2/3*x^3*e^3*f*g+6*x^2*d^2*e*g^2+5
*x^2*d*e^2*f*g+1/2*x^2*e^3*f^2+20*d^3*g^2*x+24*d^2*e*f*g*x+5*d*e^2*f^2*x)+
4*d^2/e^3*(7*d^2*g^2+10*d*e*f*g+3*e^2*f^2)*ln(-e*x+d)+8*d^3*(d^2*g^2+2*d*e
*f*g+e^2*f^2)/e^3/(-e*x+d)
```

3.559.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.72

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$$
$$= \frac{3\,e^5g^2x^5 - 96\,d^3e^2f^2 - 192\,d^4efg - 96\,d^5g^2 + (8\,e^5fg + 17\,de^4g^2)x^4 + 2\,(3\,e^5f^2 + 26\,de^4fg + 26\,d^2e^3g^2)x$$

input
```
integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

output
```
1/12*(3*e^5*g^2*x^5 - 96*d^3*e^2*f^2 - 192*d^4*e*f*g - 96*d^5*g^2 + (8*e^5
*f*g + 17*d*e^4*g^2)*x^4 + 2*(3*e^5*f^2 + 26*d*e^4*f*g + 26*d^2*e^3*g^2)*x
^3 + 6*(9*d*e^4*f^2 + 38*d^2*e^3*f*g + 28*d^3*e^2*g^2)*x^2 - 12*(5*d^2*e^3
*f^2 + 24*d^3*e^2*f*g + 20*d^4*e*g^2)*x - 48*(3*d^3*e^2*f^2 + 10*d^4*e*f*g
 + 7*d^5*g^2 - (3*d^2*e^3*f^2 + 10*d^3*e^2*f*g + 7*d^4*e*g^2)*x)*log(e*x -
 d))/(e^4*x - d*e^3)
```

3.559.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{4d^2(dg+ef)\,(7dg+3ef)\log{(-d+ex)}}{e^3} + \frac{eg^2x^4}{4} + x^3 \cdot \left(\frac{5dg^2}{3} + \frac{2efg}{3}\right) + x^2 \cdot \left(\frac{6d^2g^2}{e} + 5dfg + \frac{ef^2}{2}\right) + x\left(\frac{20d^3g^2}{e^2} + \frac{24d^2fg}{e} + 5df^2\right) + \frac{-8d^5g^2 - 16d^4efg - 8d^3e^2f^2}{-de^3 + e^4x}$$

input
```
integrate((e*x+d)**5*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

output
```
4*d**2*(d*g + e*f)*(7*d*g + 3*e*f)*log(-d + e*x)/e**3 + e*g**2*x**4/4 + x*
*3*(5*d*g**2/3 + 2*e*f*g/3) + x**2*(6*d**2*g**2/e + 5*d*f*g + e*f**2/2) +
x*(20*d**3*g**2/e**2 + 24*d**2*f*g/e + 5*d*f**2) + (-8*d**5*g**2 - 16*d**4
*e*f*g - 8*d**3*e**2*f**2)/(-d*e**3 + e**4*x)
```

3.559.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx = -\frac{8\,(d^3e^2f^2+2\,d^4efg+d^5g^2)}{e^4x-de^3} + \frac{3\,e^3g^2x^4+4\,(2\,e^3fg+5\,de^2g^2)x^3+6\,(e^3f^2+10\,de^2fg+12\,d^2eg^2)x^2+12\,(5\,de^2f^2+24\,d^2efg+20\,d^3g}{12\,e^2} + \frac{4\,(3\,d^2e^2f^2+10\,d^3efg+7\,d^4g^2)\log{(ex-d)}}{e^3}$$

input
```
integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

output
```
-8*(d^3*e^2*f^2 + 2*d^4*e*f*g + d^5*g^2)/(e^4*x - d*e^3) + 1/12*(3*e^3*g^2
*x^4 + 4*(2*e^3*f*g + 5*d*e^2*g^2)*x^3 + 6*(e^3*f^2 + 10*d*e^2*f*g + 12*d^
2*e*g^2)*x^2 + 12*(5*d*e^2*f^2 + 24*d^2*e*f*g + 20*d^3*g^2)*x)/e^2 + 4*(3*
d^2*e^2*f^2 + 10*d^3*e*f*g + 7*d^4*g^2)*log(e*x - d)/e^3
```

3.559.8 Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx = \frac{4\,(3\,d^2e^2f^2+10\,d^3efg+7\,d^4g^2)\log{(|ex-d|)}}{e^3} - \frac{8\,(d^3e^2f^2+2\,d^4efg+d^5g^2)}{(ex-d)e^3} + \frac{3\,e^9g^2x^4+8\,e^9fgx^3+20\,de^8g^2x^3+6\,e^9f^2x^2+60\,de^8fgx^2+72\,d^2e^7g^2x^2+60\,de^8f^2x+288\,d^2e^7fgx+}{12\,e^8}$$

input
```
integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

output
```
4*(3*d^2*e^2*f^2 + 10*d^3*e*f*g + 7*d^4*g^2)*log(abs(e*x - d))/e^3 - 8*(d^
3*e^2*f^2 + 2*d^4*e*f*g + d^5*g^2)/((e*x - d)*e^3) + 1/12*(3*e^9*g^2*x^4 +
 8*e^9*f*g*x^3 + 20*d*e^8*g^2*x^3 + 6*e^9*f^2*x^2 + 60*d*e^8*f*g*x^2 + 72*
d^2*e^7*g^2*x^2 + 60*d*e^8*f^2*x + 288*d^2*e^7*f*g*x + 240*d^3*e^6*g^2*x)/
e^8
```

3.559.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.16

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = x\left(\frac{d^3\,g^2+6\,d^2\,e\,f\,g+3\,d\,e^2\,f^2}{e^2}-\frac{d^2\,(g\,(3\,d\,g+2\,e\,f)+2\,d\,g^2)}{e^2}+\frac{2\,d\left(\frac{3\,d^2\,e\,g^2+6\,d\,e^2\,f\,g+e^3\,f^2}{e^2}-\frac{d^2\,g^2}{e}+\frac{2\,d\,(g\,(3\,d\,g+2\,e\,f)+2\,d\,g^2)}{e}\right)}{e}\right)+x^2\left(\frac{3\,d^2\,e\,g^2+6\,d\,e^2\,f\,g+e^3\,f^2}{2\,e^2}-\frac{d^2\,g^2}{2\,e}+\frac{d\,(g\,(3\,d\,g+2\,e\,f)+2\,d\,g^2)}{e}\right)+x^3\left(\frac{g\,(3\,d\,g+2\,e\,f)}{3}+\frac{2\,d\,g^2}{3}\right)+\frac{\ln(e\,x-d)\,(28\,d^4\,g^2+40\,d^3\,e\,f\,g+12\,d^2\,e^2\,f^2)}{e^3}+\frac{8\,(d^5\,g^2+2\,d^4\,e\,f\,g+d^3\,e^2\,f^2)}{e\,(d\,e^2-e^3\,x)}+\frac{e\,g^2\,x^4}{4}$$

input
```
int(((f + g*x)^2*(d + e*x)^5)/(d^2 - e^2*x^2)^2,x)
```

output
```
x*((d^3*g^2 + 3*d*e^2*f^2 + 6*d^2*e*f*g)/e^2 - (d^2*(g*(3*d*g + 2*e*f) + 2
*d*g^2))/e^2 + (2*d*((e^3*f^2 + 3*d^2*e*g^2 + 6*d*e^2*f*g)/e^2 - (d^2*g^2)
/e + (2*d*(g*(3*d*g + 2*e*f) + 2*d*g^2))/e))/e) + x^2*((e^3*f^2 + 3*d^2*e*
g^2 + 6*d*e^2*f*g)/(2*e^2) - (d^2*g^2)/(2*e) + (d*(g*(3*d*g + 2*e*f) + 2*d
*g^2))/e) + x^3*((g*(3*d*g + 2*e*f))/3 + (2*d*g^2)/3) + (log(e*x - d)*(28*
d^4*g^2 + 12*d^2*e^2*f^2 + 40*d^3*e*f*g))/e^3 + (8*(d^5*g^2 + d^3*e^2*f^2
+ 2*d^4*e*f*g))/(e*(d*e^2 - e^3*x)) + (e*g^2*x^4)/4
```

3.560 $\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$

3.560.1 Optimal result

Integrand size = 29, antiderivative size = 107

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{(e^2f^2+8defg+8d^2g^2)\,x}{e^2}+\frac{g(ef+2dg)x^2}{e}+\frac{g^2x^3}{3} + \frac{4d^2(ef+dg)^2}{e^3(d-ex)}+\frac{4d(ef+dg)(ef+3dg)\log(d-ex)}{e^3}$$

output
```
(8*d^2*g^2+8*d*e*f*g+e^2*f^2)*x/e^2+g*(2*d*g+e*f)*x^2/e+1/3*g^2*x^3+4*d^2*
(d*g+e*f)^2/e^3/(-e*x+d)+4*d*(d*g+e*f)*(3*d*g+e*f)*ln(-e*x+d)/e^3
```

3.560.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{(e^2f^2+8defg+8d^2g^2)\,x}{e^2}+\frac{g(ef+2dg)x^2}{e}+\frac{g^2x^3}{3} - \frac{4d^2(ef+dg)^2}{e^3(-d+ex)}+\frac{4d(e^2f^2+4defg+3d^2g^2)\log(d-ex)}{e^3}$$

input
```
Integrate[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

output
```
((e^2*f^2 + 8*d*e*f*g + 8*d^2*g^2)*x)/e^2 + (g*(e*f + 2*d*g)*x^2)/e + (g^2
*x^3)/3 - (4*d^2*(e*f + d*g)^2)/(e^3*(-d + e*x)) + (4*d*(e^2*f^2 + 4*d*e*f
*g + 3*d^2*g^2)*Log[d - e*x])/e^3
```

3.560.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {639, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^2} \, dx$$

$$\Big\downarrow \text{639}$$

$$\int \frac{(d+ex)^2(f+gx)^2}{(d-ex)^2}dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(\frac{8d^2g^2+8defg+e^2f^2}{e^2}+\frac{4d^2(dg+ef)^2}{e^2(ex-d)^2}+\frac{4d(-3dg-ef)(dg+ef)}{e^2(d-ex)}+\frac{2gx(2dg+ef)}{e}+g^2x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4d^2(dg+ef)^2}{e^3(d-ex)}+\frac{x(8d^2g^2+8defg+e^2f^2)}{e^2}+\frac{4d(dg+ef)(3dg+ef)\log(d-ex)}{e^3}+\frac{gx^2(2dg+ef)}{e}+\frac{g^2x^3}{3}$$

input
```
Int[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

output
```
((e^2*f^2 + 8*d*e*f*g + 8*d^2*g^2)*x)/e^2 + (g*(e*f + 2*d*g)*x^2)/e + (g^2
*x^3)/3 + (4*d^2*(e*f + d*g)^2)/(e^3*(d - e*x)) + (4*d*(e*f + d*g)*(e*f +
3*d*g)*Log[d - e*x])/e^3
```

3.560.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.560.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.24

method	result
default	$\frac{\frac{1}{3}g^2x^3e^2+2de\,g^2x^2+e^2fg\,x^2+8d^2g^2x+8defgx+e^2f^2x}{e^2}+\frac{4d(3d^2g^2+4defg+e^2f^2)\ln(-ex+d)}{e^3}+\frac{4d^2(d^2g^2+2defg+e^2f^2)}{e^3(-ex+d)}$
risch	$\frac{g^2x^3}{3}+\frac{2d\,g^2x^2}{e}+fg\,x^2+\frac{8d^2g^2x}{e^2}+\frac{8dfgx}{e}+f^2x+\frac{12d^3\ln(-ex+d)g^2}{e^3}+\frac{16d^2\ln(-ex+d)fg}{e^2}+\frac{4d\ln(-ex+d)}{e}$
norman	$\frac{(-\frac{23}{3}d^2g^2-8defg-e^2f^2)x^3+\frac{d^2(6d^3g^2+9d^2efg+4d\,e^2f^2)}{e^3}+\frac{d^2(12d^2g^2+16defg+5e^2f^2)x}{e^2}-\frac{e^2g^2x^5}{3}-eg(2dg+ef)x^4}{-e^2x^2+d^2}+\frac{4d(3d^2g}{}$
parallelrisch	$\frac{g^2e^4x^4+5x^3d\,e^3g^2+3x^3e^4fg+36\ln(ex-d)x\,d^3e\,g^2+48\ln(ex-d)x\,d^2e^2fg+12\ln(ex-d)xd\,e^3f^2+18x^2d^2e^2g^2+21x^2d\,e^3fg+3x}{3e^3(ex-d)}$

input
```
int((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e^2*(1/3*g^2*x^3*e^2+2*d*e*g^2*x^2+e^2*f*g*x^2+8*d^2*g^2*x+8*d*e*f*g*x+e
^2*f^2*x)+4*d/e^3*(3*d^2*g^2+4*d*e*f*g+e^2*f^2)*ln(-e*x+d)+4*d^2*(d^2*g^2+
2*d*e*f*g+e^2*f^2)/e^3/(-e*x+d)
```

3.560.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.93

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$$
$$=\frac{e^4g^2x^4-12\,d^2e^2f^2-24\,d^3efg-12\,d^4g^2+(3\,e^4fg+5\,de^3g^2)x^3+3\,(e^4f^2+7\,de^3fg+6\,d^2e^2g^2)x^2-3\,(d}{3\,(e^4x}$$

input
```
integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

output
```
1/3*(e^4*g^2*x^4 - 12*d^2*e^2*f^2 - 24*d^3*e*f*g - 12*d^4*g^2 + (3*e^4*f*g
 + 5*d*e^3*g^2)*x^3 + 3*(e^4*f^2 + 7*d*e^3*f*g + 6*d^2*e^2*g^2)*x^2 - 3*(d
*e^3*f^2 + 8*d^2*e^2*f*g + 8*d^3*e*g^2)*x - 12*(d^2*e^2*f^2 + 4*d^3*e*f*g
+ 3*d^4*g^2 - (d*e^3*f^2 + 4*d^2*e^2*f*g + 3*d^3*e*g^2)*x)*log(e*x - d))/(
e^4*x - d*e^3)
```

3.560.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{4d(dg+ef)\,(3dg+ef)\log{(-d+ex)}}{e^3} + \frac{g^2x^3}{3} + x^2\cdot\left(\frac{2dg^2}{e} + fg\right) + x\left(\frac{8d^2g^2}{e^2} + \frac{8dfg}{e} + f^2\right) + \frac{-4d^4g^2 - 8d^3efg - 4d^2e^2f^2}{-de^3 + e^4x}$$

input
```
integrate((e*x+d)**4*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

output
```
4*d*(d*g + e*f)*(3*d*g + e*f)*log(-d + e*x)/e**3 + g**2*x**3/3 + x**2*(2*d
*g**2/e + f*g) + x*(8*d**2*g**2/e**2 + 8*d*f*g/e + f**2) + (-4*d**4*g**2 -
 8*d**3*e*f*g - 4*d**2*e**2*f**2)/(-d*e**3 + e**4*x)
```

3.560.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.32

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = -\frac{4\,(d^2e^2f^2 + 2\,d^3efg + d^4g^2)}{e^4x - de^3} + \frac{e^2g^2x^3 + 3\,(e^2fg + 2\,deg^2)x^2 + 3\,(e^2f^2 + 8\,defg + 8\,d^2g^2)x}{3\,e^2} + \frac{4\,(de^2f^2 + 4\,d^2efg + 3\,d^3g^2)\log{(ex-d)}}{e^3}$$

input
```
integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

output
```
-4*(d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2)/(e^4*x - d*e^3) + 1/3*(e^2*g^2*x^
3 + 3*(e^2*f*g + 2*d*e*g^2)*x^2 + 3*(e^2*f^2 + 8*d*e*f*g + 8*d^2*g^2)*x)/e
^2 + 4*(d*e^2*f^2 + 4*d^2*e*f*g + 3*d^3*g^2)*log(e*x - d)/e^3
```

3.560.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{4\,(de^2f^2+4\,d^2efg+3\,d^3g^2)\log{(|ex-d|)}}{e^3} - \frac{4\,(d^2e^2f^2+2\,d^3efg+d^4g^2)}{(ex-d)e^3} + \frac{e^6g^2x^3+3\,e^6fgx^2+6\,de^5g^2x^2+3\,e^6f^2x+24\,de^5fgx+24\,d^2e^4g^2x}{3\,e^6}$$

input
```
integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

output
```
4*(d*e^2*f^2 + 4*d^2*e*f*g + 3*d^3*g^2)*log(abs(e*x - d))/e^3 - 4*(d^2*e^2
*f^2 + 2*d^3*e*f*g + d^4*g^2)/((e*x - d)*e^3) + 1/3*(e^6*g^2*x^3 + 3*e^6*f
*g*x^2 + 6*d*e^5*g^2*x^2 + 3*e^6*f^2*x + 24*d*e^5*f*g*x + 24*d^2*e^4*g^2*x
)/e^6
```

3.560.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.73

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = x^2\left(\frac{g\,(d\,g+e\,f)}{e}+\frac{d\,g^2}{e}\right)+x\left(\frac{d^2\,g^2+4\,d\,e\,f\,g+e^2\,f^2}{e^2}+\frac{2\,d\,\left(\frac{2\,g\,(d\,g+e\,f)}{e}+\frac{2\,d\,g^2}{e}\right)}{e}-\frac{d^2\,g^2}{e^2}\right)+\frac{g^2\,x^3}{3}+\frac{4\,(d^4\,g^2+2\,d^3\,e\,f\,g+d^2\,e^2\,f^2)}{e\,(d\,e^2-e^3\,x)}+\frac{\ln{(e\,x-d)}\,\left(12\,d^3\,g^2+16\,d^2\,e\,f\,g+4\,d\,e^2\,f^2\right)}{e^3}$$

input
```
int(((f + g*x)^2*(d + e*x)^4)/(d^2 - e^2*x^2)^2,x)
```

output
```
x^2*((g*(d*g + e*f))/e + (d*g^2)/e) + x*((d^2*g^2 + e^2*f^2 + 4*d*e*f*g)/e
^2 + (2*d*((2*g*(d*g + e*f))/e + (2*d*g^2)/e))/e - (d^2*g^2)/e^2) + (g^2*x
^3)/3 + (4*(d^4*g^2 + d^2*e^2*f^2 + 2*d^3*e*f*g))/(e*(d*e^2 - e^3*x)) + (l
og(e*x - d)*(12*d^3*g^2 + 4*d*e^2*f^2 + 16*d^2*e*f*g))/e^3
```

3.561 $\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

3.561.1 Optimal result

Integrand size = 29, antiderivative size = 78

$$\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx = \frac{g(2ef+3dg)x}{e^2}+\frac{g^2x^2}{2e}+\frac{2d(ef+dg)^2}{e^3(d-ex)}+\frac{(ef+dg)(ef+5dg)\log(d-ex)}{e^3}$$

output
```
g*(3*d*g+2*e*f)*x/e^2+1/2*g^2*x^2/e+2*d*(d*g+e*f)^2/e^3/(-e*x+d)+(d*g+e*f)
*(5*d*g+e*f)*ln(-e*x+d)/e^3
```

3.561.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx = \frac{2eg(2ef+3dg)x+e^2g^2x^2+\frac{4d(ef+dg)^2}{d-ex}+2(e^2f^2+6defg+5d^2g^2)\log(d-ex)}{2e^3}$$

input
```
Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

output
```
(2*e*g*(2*e*f + 3*d*g)*x + e^2*g^2*x^2 + (4*d*(e*f + d*g)^2)/(d - e*x) + 2
*(e^2*f^2 + 6*d*e*f*g + 5*d^2*g^2)*Log[d - e*x])/(2*e^3)
```

3.561.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {639, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^2}dx$$

$$\Big\downarrow \text{639}$$

$$\int \frac{(d+ex)(f+gx)^2}{(d-ex)^2}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{(-5dg-ef)(dg+ef)}{e^2(d-ex)}+\frac{2d(dg+ef)^2}{e^2(ex-d)^2}+\frac{g(3dg+2ef)}{e^2}+\frac{g^2x}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2d(dg+ef)^2}{e^3(d-ex)}+\frac{(5dg+ef)(dg+ef)\log(d-ex)}{e^3}+\frac{gx(3dg+2ef)}{e^2}+\frac{g^2x^2}{2e}$$

input
```
Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

output
```
(g*(2*e*f + 3*d*g)*x)/e^2 + (g^2*x^2)/(2*e) + (2*d*(e*f + d*g)^2)/(e^3*(d
- e*x)) + ((e*f + d*g)*(e*f + 5*d*g)*Log[d - e*x])/e^3
```

3.561.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.561.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19

method	result
default	$\frac{g\left(\frac{1}{2}eg\,x^{2}+3dgx+2efx\right)}{e^{2}}+\frac{\left(5d^{2}g^{2}+6defg+e^{2}f^{2}\right)\ln\left(-ex+d\right)}{e^{3}}+\frac{2d\left(d^{2}g^{2}+2defg+e^{2}f^{2}\right)}{e^{3}\left(-ex+d\right)}$
risch	$\frac{g^{2}x^{2}}{2e}+\frac{3g^{2}dx}{e^{2}}+\frac{2gfx}{e}+\frac{5\ln\left(-ex+d\right)d^{2}g^{2}}{e^{3}}+\frac{6\ln\left(-ex+d\right)dfg}{e^{2}}+\frac{\ln\left(-ex+d\right)f^{2}}{e}+\frac{2d^{3}g^{2}}{e^{3}\left(-ex+d\right)}+\frac{4d^{2}fg}{e^{2}\left(-ex+d\right)}+\frac{}{e(}$
norman	$\frac{\frac{d\left(5d^{2}g^{2}+6defg+2e^{2}f^{2}\right)x}{e^{2}}+\frac{d^{2}\left(5d^{2}g^{2}+8defg+4e^{2}f^{2}\right)}{2e^{3}}-\frac{e\,g^{2}x^{4}}{2}-g\left(3dg+2ef\right)x^{3}}{-e^{2}x^{2}+d^{2}}+\frac{\left(5d^{2}g^{2}+6defg+e^{2}f^{2}\right)\ln\left(-ex+d\right)}{e^{3}}$
parallelrisch	$\frac{g^{2}x^{3}e^{3}+10\ln\left(ex-d\right)x\,d^{2}e\,g^{2}+12\ln\left(ex-d\right)xd\,e^{2}fg+2\ln\left(ex-d\right)x\,e^{3}f^{2}+5x^{2}d\,e^{2}g^{2}+4x^{2}e^{3}fg-10\ln\left(ex-d\right)d^{3}g^{2}-12\ln\left(ex-d\right)d^{2}}{2e^{3}\left(ex-d\right)}$

input
```
int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

output
```
g/e^2*(1/2*e*g*x^2+3*d*g*x+2*e*f*x)+1/e^3*(5*d^2*g^2+6*d*e*f*g+e^2*f^2)*ln
(-e*x+d)+2*d*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/(-e*x+d)
```

3.561.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(77) = 154.

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.01

$$\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$$
$$=\frac{e^3g^2x^3-4\,de^2f^2-8\,d^2efg-4\,d^3g^2+(4\,e^3fg+5\,de^2g^2)x^2-2\,(2\,de^2fg+3\,d^2eg^2)x-2\,(de^2f^2+6\,d^2efg}{2\,(e^4x-de^3)}$$

input
```
integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

output
```
1/2*(e^3*g^2*x^3 - 4*d*e^2*f^2 - 8*d^2*e*f*g - 4*d^3*g^2 + (4*e^3*f*g + 5*
d*e^2*g^2)*x^2 - 2*(2*d*e^2*f*g + 3*d^2*e*g^2)*x - 2*(d*e^2*f^2 + 6*d^2*e*
f*g + 5*d^3*g^2 - (e^3*f^2 + 6*d*e^2*f*g + 5*d^2*e*g^2)*x)*log(e*x - d))/(
e^4*x - d*e^3)
```

3.561.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = x\left(\frac{3dg^2}{e^2}+\frac{2fg}{e}\right)+\frac{-2d^3g^2-4d^2efg-2de^2f^2}{-de^3+e^4x} + \frac{g^2x^2}{2e}+\frac{(dg+ef)\,(5dg+ef)\log{(-d+ex)}}{e^3}$$

input
```
integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

output
```
x*(3*d*g**2/e**2 + 2*f*g/e) + (-2*d**3*g**2 - 4*d**2*e*f*g - 2*d*e**2*f**2
)/(-d*e**3 + e**4*x) + g**2*x**2/(2*e) + (d*g + e*f)*(5*d*g + e*f)*log(-d
+ e*x)/e**3
```

3.561.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.33

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = -\frac{2\,(de^2f^2+2\,d^2efg+d^3g^2)}{e^4x-de^3}+\frac{eg^2x^2+2\,(2\,efg+3\,dg^2)x}{2\,e^2} + \frac{(e^2f^2+6\,defg+5\,d^2g^2)\log{(ex-d)}}{e^3}$$

input
```
integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

output
```
-2*(d*e^2*f^2 + 2*d^2*e*f*g + d^3*g^2)/(e^4*x - d*e^3) + 1/2*(e*g^2*x^2 +
2*(2*e*f*g + 3*d*g^2)*x)/e^2 + (e^2*f^2 + 6*d*e*f*g + 5*d^2*g^2)*log(e*x -
 d)/e^3
```

3.561.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{(e^2f^2+6\,defg+5\,d^2g^2)\log(|ex-d|)}{e^3} + \frac{e^3g^2x^2+4\,e^3fgx+6\,de^2g^2x}{2\,e^4} - \frac{2\,(de^2f^2+2\,d^2efg+d^3g^2)}{(ex-d)e^3}$$

input
```
integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

output
```
(e^2*f^2 + 6*d*e*f*g + 5*d^2*g^2)*log(abs(e*x - d))/e^3 + 1/2*(e^3*g^2*x^2
 + 4*e^3*f*g*x + 6*d*e^2*g^2*x)/e^4 - 2*(d*e^2*f^2 + 2*d^2*e*f*g + d^3*g^2
)/((e*x - d)*e^3)
```

3.561.9 Mupad [B] (verification not implemented)

Time = 12.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = x\left(\frac{d\,g^2+2\,e\,f\,g}{e^2}+\frac{2\,d\,g^2}{e^2}\right) + \frac{\ln(e\,x-d)\,\left(5\,d^2\,g^2+6\,d\,e\,f\,g+e^2\,f^2\right)}{e^3} + \frac{g^2\,x^2}{2\,e} + \frac{2\,(d^3\,g^2+2\,d^2\,e\,f\,g+d\,e^2\,f^2)}{e\,\left(d\,e^2-e^3\,x\right)}$$

input
```
int(((f + g*x)^2*(d + e*x)^3)/(d^2 - e^2*x^2)^2,x)
```

output
```
x*((d*g^2 + 2*e*f*g)/e^2 + (2*d*g^2)/e^2) + (log(e*x - d)*(5*d^2*g^2 + e^2
*f^2 + 6*d*e*f*g))/e^3 + (g^2*x^2)/(2*e) + (2*(d^3*g^2 + d*e^2*f^2 + 2*d^2
*e*f*g))/(e*(d*e^2 - e^3*x))
```

3.562 $\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^2} dx$

3.562.1 Optimal result

Integrand size = 29, antiderivative size = 50

$$\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^2} dx = \frac{g^2x}{e^2} + \frac{(ef+dg)^2}{e^3(d-ex)} + \frac{2g(ef+dg)\log(d-ex)}{e^3}$$

output
```
g^2*x/e^2+(d*g+e*f)^2/e^3/(-e*x+d)+2*g*(d*g+e*f)*ln(-e*x+d)/e^3
```

3.562.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^2} dx = \frac{eg^2x + \frac{(ef+dg)^2}{d-ex} + 2g(ef+dg)\log(d-ex)}{e^3}$$

input
```
Integrate[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

output
```
(e*g^2*x + (e*f + d*g)^2/(d - e*x) + 2*g*(e*f + d*g)*Log[d - e*x])/e^3
```

3.562.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {639, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^2}dx$$

$$\Big\downarrow \text{639}$$

$$\int \frac{(f+gx)^2}{(d-ex)^2}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{2g(dg+ef)}{e^2(ex-d)}+\frac{(dg+ef)^2}{e^2(ex-d)^2}+\frac{g^2}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(dg+ef)^2}{e^3(d-ex)}+\frac{2g(dg+ef)\log(d-ex)}{e^3}+\frac{g^2x}{e^2}$$

input
```
Int[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

output
```
(g^2*x)/e^2 + (e*f + d*g)^2/(e^3*(d - e*x)) + (2*g*(e*f + d*g)*Log[d - e*x
])/e^3
```

3.562.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.562.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{g^{2} x}{e^{2}}+\frac{2 g \left(d g +e f \right) \ln \left(-e x +d \right)}{e^{3}}+\frac{d^{2} g^{2}+2 d e f g +e^{2} f^{2}}{e^{3} \left(-e x +d \right)}$	63
risch	$\frac{g^{2} x}{e^{2}}+\frac{2 g^{2} \ln \left(-e x +d \right) d}{e^{3}}+\frac{2 g \ln \left(-e x +d \right) f}{e^{2}}+\frac{d^{2} g^{2}}{e^{3} \left(-e x +d \right)}+\frac{2 d f g}{e^{2} \left(-e x +d \right)}+\frac{f^{2}}{e \left(-e x +d \right)}$	89
norman	$\frac{\frac{d \left(d^{2} g^{2}+2 d e f g +e^{2} f^{2}\right)}{e^{3}}+\frac{\left(2 d^{2} g^{2}+2 d e f g +e^{2} f^{2}\right) x}{e^{2}}-g^{2} x^{3}}{-e^{2} x^{2}+d^{2}}+\frac{2 g \left(d g +e f \right) \ln \left(-e x +d \right)}{e^{3}}$	99
parallelrisch	$\frac{2 \ln \left(e x -d \right) x d e \,g^{2}+2 \ln \left(e x -d \right) x \,e^{2} f g +g^{2} x^{2} e^{2}-2 \ln \left(e x -d \right) d^{2} g^{2}-2 \ln \left(e x -d \right) d e f g -2 d^{2} g^{2}-2 d e f g -e^{2} f^{2}}{e^{3} \left(e x -d \right)}$	109

input
```
int((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

output
```
g^2*x/e^2+2*g*(d*g+e*f)*ln(-e*x+d)/e^3+(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/(-e
*x+d)
```

3.562.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.90

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{e^2g^2x^2-deg^2x-e^2f^2-2\,defg-d^2g^2-2\,(defg+d^2g^2-(e^2fg+deg^2)x)\log{(ex-d)}}{e^4x-de^3}$$

input
```
integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

output
```
(e^2*g^2*x^2 - d*e*g^2*x - e^2*f^2 - 2*d*e*f*g - d^2*g^2 - 2*(d*e*f*g + d^
2*g^2 - (e^2*f*g + d*e*g^2)*x)*log(e*x - d))/(e^4*x - d*e^3)
```

3.562.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{-d^2g^2-2defg-e^2f^2}{-de^3+e^4x}+\frac{g^2x}{e^2}+\frac{2g(dg+ef)\log(-d+ex)}{e^3}$$

input
```
integrate((e*x+d)**2*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

output
```
(-d**2*g**2 - 2*d*e*f*g - e**2*f**2)/(-d*e**3 + e**4*x) + g**2*x/e**2 + 2*
g*(d*g + e*f)*log(-d + e*x)/e**3
```

3.562.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{g^2x}{e^2}-\frac{e^2f^2+2\,defg+d^2g^2}{e^4x-de^3}+\frac{2\,(efg+dg^2)\log(ex-d)}{e^3}$$

input
```
integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

output
```
g^2*x/e^2 - (e^2*f^2 + 2*d*e*f*g + d^2*g^2)/(e^4*x - d*e^3) + 2*(e*f*g + d
*g^2)*log(e*x - d)/e^3
```

3.562.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{g^2x}{e^2}+\frac{2\,(efg+dg^2)\log(|ex-d|)}{e^3}-\frac{e^2f^2+2\,defg+d^2g^2}{(ex-d)e^3}$$

input
```
integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

output
```
g^2*x/e^2 + 2*(e*f*g + d*g^2)*log(abs(e*x - d))/e^3 - (e^2*f^2 + 2*d*e*f*g
 + d^2*g^2)/((e*x - d)*e^3)
```

3.562.9 Mupad [B] (verification not implemented)

Time = 11.90 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2}{e\,\left(d\,e^2-e^3\,x\right)}+\frac{g^2\,x}{e^2}+\frac{\ln\left(e\,x-d\right)\,\left(2\,d\,g^2+2\,e\,f\,g\right)}{e^3}$$

input
```
int(((f + g*x)^2*(d + e*x)^2)/(d^2 - e^2*x^2)^2,x)
```

output
```
(d^2*g^2 + e^2*f^2 + 2*d*e*f*g)/(e*(d*e^2 - e^3*x)) + (g^2*x)/e^2 + (log(e
*x - d)*(2*d*g^2 + 2*e*f*g))/e^3
```

3.563 $\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^2} dx$

3.563.1 Optimal result

Integrand size = 27, antiderivative size = 86

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^2} dx = \frac{(ef+dg)^2}{2de^3(d-ex)} - \frac{(ef-3dg)(ef+dg)\log(d-ex)}{4d^2e^3} + \frac{(ef-dg)^2\log(d+ex)}{4d^2e^3}$$

output
```
1/2*(d*g+e*f)^2/d/e^3/(-e*x+d)-1/4*(-3*d*g+e*f)*(d*g+e*f)*ln(-e*x+d)/d^2/e
^3+1/4*(-d*g+e*f)^2*ln(e*x+d)/d^2/e^3
```

3.563.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^2} dx = \frac{2d(ef+dg)^2 + (-e^2f^2+2defg+3d^2g^2)(d-ex)\log(d-ex) + (ef-dg)^2(d-ex)\log(d+ex)}{4d^2e^3(d-ex)}$$

input
```
Integrate[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

output
```
(2*d*(e*f + d*g)^2 + (-(e^2*f^2) + 2*d*e*f*g + 3*d^2*g^2)*(d - e*x)*Log[d
- e*x] + (e*f - d*g)^2*(d - e*x)*Log[d + e*x])/(4*d^2*e^3*(d - e*x))
```

3.563.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {639, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^2} dx$$

$$\Big\downarrow \text{639}$$

$$\int \frac{(f+gx)^2}{(d-ex)^2(d+ex)}dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(\frac{(dg-ef)^2}{4d^2e^2(d+ex)}+\frac{(ef-3dg)(dg+ef)}{4d^2e^2(d-ex)}+\frac{(dg+ef)^2}{2de^2(d-ex)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ef-dg)^2\log(d+ex)}{4d^2e^3}-\frac{(ef-3dg)(dg+ef)\log(d-ex)}{4d^2e^3}+\frac{(dg+ef)^2}{2de^3(d-ex)}$$

input
```
Int[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

output
```
(e*f + d*g)^2/(2*d*e^3*(d - e*x)) - ((e*f - 3*d*g)*(e*f + d*g)*Log[d - e*x
])/(4*d^2*e^3) + ((e*f - d*g)^2*Log[d + e*x])/(4*d^2*e^3)
```

3.563.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.563.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

method	result
default	$\frac{d^2g^2+2defg+e^2f^2}{2d\,e^3(-ex+d)}+\frac{(3d^2g^2+2defg-e^2f^2)\ln(-ex+d)}{4d^2e^3}+\frac{(d^2g^2-2defg+e^2f^2)\ln(ex+d)}{4d^2e^3}$
norman	$\frac{-\frac{-d^2g^2-2defg-e^2f^2}{2e^3}+\frac{(d^2g^2+2defg+e^2f^2)x}{2d\,e^2}}{-e^2x^2+d^2}+\frac{(d^2g^2-2defg+e^2f^2)\ln(ex+d)}{4d^2e^3}+\frac{(3d^2g^2+2defg-e^2f^2)\ln(-ex+d)}{4d^2e^3}$
risch	$\frac{d\,g^2}{2e^3(-ex+d)}+\frac{fg}{e^2(-ex+d)}+\frac{f^2}{2de(-ex+d)}+\frac{\ln(-ex-d)g^2}{4e^3}-\frac{\ln(-ex-d)fg}{2d\,e^2}+\frac{\ln(-ex-d)f^2}{4d^2e}+\frac{3\ln(ex-d)g^2}{4e^3}+\ln($
parallelrisch	$\frac{3\ln(ex-d)x\,d^2e\,g^2+2\ln(ex-d)xd\,e^2fg-\ln(ex-d)x\,e^3f^2+\ln(ex+d)x\,d^2e\,g^2-2\ln(ex+d)xd\,e^2fg+\ln(ex+d)x\,e^3f^2-3\ln(ex-d}{4d^2e^3(ex}$

input
```
int((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(d^2*g^2+2*d*e*f*g+e^2*f^2)/d/e^3/(-e*x+d)+1/4*(3*d^2*g^2+2*d*e*f*g-e^
2*f^2)/d^2/e^3*ln(-e*x+d)+1/4*(d^2*g^2-2*d*e*f*g+e^2*f^2)/d^2/e^3*ln(e*x+d
)
```

3.563.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(81) = 162.

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.95

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = -\frac{2\,de^2f^2+4\,d^2efg+2\,d^3g^2+(de^2f^2-2\,d^2efg+d^3g^2-(e^3f^2-2\,de^2fg+d^2eg^2)x)\log(ex+d)-(de}{4\,(d^2e^4x-d^3e^3)}$$

input
```
integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*d*e^2*f^2 + 4*d^2*e*f*g + 2*d^3*g^2 + (d*e^2*f^2 - 2*d^2*e*f*g + d
^3*g^2 - (e^3*f^2 - 2*d*e^2*f*g + d^2*e*g^2)*x)*log(e*x + d) - (d*e^2*f^2
- 2*d^2*e*f*g - 3*d^3*g^2 - (e^3*f^2 - 2*d*e^2*f*g - 3*d^2*e*g^2)*x)*log(e
*x - d))/(d^2*e^4*x - d^3*e^3)
```

3.563.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(75) = 150$.

Time = 0.47 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.12

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{-d^2g^2-2defg-e^2f^2}{-2d^2e^3+2de^4x} + \frac{(dg-ef)^2\log\left(x+\frac{2d^3g^2-d(dg-ef)^2}{d^2eg^2+2de^2fg-e^3f^2}\right)}{4d^2e^3} + \frac{(dg+ef)\,(3dg-ef)\log\left(x+\frac{2d^3g^2-d(dg+ef)(3dg-ef)}{d^2eg^2+2de^2fg-e^3f^2}\right)}{4d^2e^3}$$

input
```
integrate((e*x+d)*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

output
```
(-d**2*g**2 - 2*d*e*f*g - e**2*f**2)/(-2*d**2*e**3 + 2*d*e**4*x) + (d*g -
e*f)**2*log(x + (2*d**3*g**2 - d*(d*g - e*f)**2)/(d**2*e*g**2 + 2*d*e**2*f
*g - e**3*f**2))/(4*d**2*e**3) + (d*g + e*f)*(3*d*g - e*f)*log(x + (2*d**3
*g**2 - d*(d*g + e*f)*(3*d*g - e*f))/(d**2*e*g**2 + 2*d*e**2*f*g - e**3*f*
*2))/(4*d**2*e**3)
```

3.563.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.33

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = -\frac{e^2f^2+2\,defg+d^2g^2}{2\,(de^4x-d^2e^3)} + \frac{(e^2f^2-2\,defg+d^2g^2)\log{(ex+d)}}{4\,d^2e^3} - \frac{(e^2f^2-2\,defg-3\,d^2g^2)\log{(ex-d)}}{4\,d^2e^3}$$

input
```
integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)/(d*e^4*x - d^2*e^3) + 1/4*(e^2*f^2 -
2*d*e*f*g + d^2*g^2)*log(e*x + d)/(d^2*e^3) - 1/4*(e^2*f^2 - 2*d*e*f*g - 3
*d^2*g^2)*log(e*x - d)/(d^2*e^3)
```

3.563.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{(e^2f^2-2\,defg+d^2g^2)\log(|ex+d|)}{4\,d^2e^3} - \frac{(e^2f^2-2\,defg-3\,d^2g^2)\log(|ex-d|)}{4\,d^2e^3} - \frac{de^2f^2+2\,d^2efg+d^3g^2}{2\,(ex-d)d^2e^3}$$

input
```
integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

output
```
1/4*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(abs(e*x + d))/(d^2*e^3) - 1/4*(e^2
*f^2 - 2*d*e*f*g - 3*d^2*g^2)*log(abs(e*x - d))/(d^2*e^3) - 1/2*(d*e^2*f^2
 + 2*d^2*e*f*g + d^3*g^2)/((e*x - d)*d^2*e^3)
```

3.563.9 Mupad [B] (verification not implemented)

Time = 11.88 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2}{2\,d\,e^3\,(d-e\,x)} + \frac{\ln(d+e\,x)\,(d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2)}{4\,d^2\,e^3} + \frac{\ln(d-e\,x)\,(3\,d^2\,g^2+2\,d\,e\,f\,g-e^2\,f^2)}{4\,d^2\,e^3}$$

input
```
int(((f + g*x)^2*(d + e*x))/(d^2 - e^2*x^2)^2,x)
```

output
```
(d^2*g^2 + e^2*f^2 + 2*d*e*f*g)/(2*d*e^3*(d - e*x)) + (log(d + e*x)*(d^2*g
^2 + e^2*f^2 - 2*d*e*f*g))/(4*d^2*e^3) + (log(d - e*x)*(3*d^2*g^2 - e^2*f^
2 + 2*d*e*f*g))/(4*d^2*e^3)
```

3.564 $\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

3.564.1 Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx=\frac{\left(d^2g+e^2fx\right)(f+gx)}{2d^2e^2\left(d^2-e^2x^2\right)}+\frac{(ef-dg)(ef+dg)\text{arctanh}\left(\frac{ex}{d}\right)}{2d^3e^3}$$

output
```
1/2*(e^2*f*x+d^2*g)*(g*x+f)/d^2/e^2/(-e^2*x^2+d^2)+1/2*(-d*g+e*f)*(d*g+e*f
)*arctanh(e*x/d)/d^3/e^3
```

3.564.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx=\frac{-2d^2fg-e^2f^2x-d^2g^2x}{2d^2e^2\left(-d^2+e^2x^2\right)}-\frac{\left(-e^2f^2+d^2g^2\right)\text{arctanh}\left(\frac{ex}{d}\right)}{2d^3e^3}$$

input
```
Integrate[(f + g*x)^2/(d^2 - e^2*x^2)^2,x]
```

output
```
(-2*d^2*f*g - e^2*f^2*x - d^2*g^2*x)/(2*d^2*e^2*(-d^2 + e^2*x^2)) - ((-(e^
2*f^2) + d^2*g^2)*ArcTanh[(e*x)/d])/(2*d^3*e^3)
```

3.564.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^2} \, dx$$

$$\Big\downarrow \text{477}$$

$$\frac{\int \left(\frac{\left(f^2-\frac{d^2g^2}{e^2}\right)d^2}{2(d^2-e^2x^2)}+\frac{(ef+dg)^2d^2}{4e^2(d-ex)^2}+\frac{(ef-dg)^2d^2}{4e^2(d+ex)^2}\right)dx}{d^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{d\text{arctanh}\left(\frac{ex}{d}\right)\left(f^2-\frac{d^2g^2}{e^2}\right)}{2e}-\frac{d^2(ef-dg)^2}{4e^3(d+ex)}+\frac{d^2(dg+ef)^2}{4e^3(d-ex)}}{d^4}$$

input
```
Int[(f + g*x)^2/(d^2 - e^2*x^2)^2,x]
```

output
```
((d^2*(e*f + d*g)^2)/(4*e^3*(d - e*x)) - (d^2*(e*f - d*g)^2)/(4*e^3*(d + e
*x)) + (d*(f^2 - (d^2*g^2)/e^2)*ArcTanh[(e*x)/d])/(2*e))/d^4
```

3.564.3.1 Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.564.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.47

method	result
norman	$\frac{\frac{fg}{e^2}+\frac{\left(d^2g^2+e^2f^2\right)x}{2d^2e^2}}{-e^2x^2+d^2}+\frac{\left(d^2g^2-e^2f^2\right)\ln\left(-ex+d\right)}{4e^3d^3}-\frac{\left(d^2g^2-e^2f^2\right)\ln\left(ex+d\right)}{4e^3d^3}$
risch	$\frac{\frac{fg}{e^2}+\frac{\left(d^2g^2+e^2f^2\right)x}{2d^2e^2}}{-e^2x^2+d^2}+\frac{\ln\left(ex-d\right)g^2}{4e^3d}-\frac{\ln\left(ex-d\right)f^2}{4e\,d^3}-\frac{\ln\left(-ex-d\right)g^2}{4e^3d}+\frac{\ln\left(-ex-d\right)f^2}{4e\,d^3}$
default	$\frac{\left(d^2g^2-e^2f^2\right)\ln\left(-ex+d\right)}{4e^3d^3}+\frac{d^2g^2+2defg+e^2f^2}{4d^2e^3\left(-ex+d\right)}+\frac{\left(-d^2g^2+e^2f^2\right)\ln\left(ex+d\right)}{4e^3d^3}-\frac{d^2g^2-2defg+e^2f^2}{4d^2e^3\left(ex+d\right)}$
parallelrisch	$\frac{\ln\left(ex-d\right)x^2d^2e^2g^2-\ln\left(ex-d\right)x^2e^4f^2-\ln\left(ex+d\right)x^2d^2e^2g^2+\ln\left(ex+d\right)x^2e^4f^2-\ln\left(ex-d\right)d^4g^2+\ln\left(ex-d\right)d^2e^2f^2+\ln\left(ex+d\right)d^4g^2}{4d^3e^3\left(e^2x^2-d^2\right)}$

input
```
int((g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(f*g/e^2+1/2*(d^2*g^2+e^2*f^2)/d^2/e^2*x)/(-e^2*x^2+d^2)+1/4/e^3*(d^2*g^2-
e^2*f^2)/d^3*ln(-e*x+d)-1/4/e^3*(d^2*g^2-e^2*f^2)/d^3*ln(e*x+d)
```

3.564.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(71) = 142.

Time = 0.26 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.09

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^2}\,dx =$$
$$-\frac{4\,d^3efg+2\,(de^3f^2+d^3eg^2)x+(d^2e^2f^2-d^4g^2-(e^4f^2-d^2e^2g^2)x^2)\log\left(ex+d\right)-(d^2e^2f^2-d^4g^2-(}{4\,(d^3e^5x^2-d^5e^3)}$$

input
```
integrate((g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*d^3*e*f*g + 2*(d*e^3*f^2 + d^3*e*g^2)*x + (d^2*e^2*f^2 - d^4*g^2 -
 (e^4*f^2 - d^2*e^2*g^2)*x^2)*log(e*x + d) - (d^2*e^2*f^2 - d^4*g^2 - (e^4
*f^2 - d^2*e^2*g^2)*x^2)*log(e*x - d))/(d^3*e^5*x^2 - d^5*e^3)
```

3.564.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(66) = 132$.

Time = 0.33 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.11

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{-2d^2fg + x(-d^2g^2-e^2f^2)}{-2d^4e^2+2d^2e^4x^2} + \frac{(dg-ef)\,(dg+ef)\log\left(-\frac{d(dg-ef)(dg+ef)}{e(d^2g^2-e^2f^2)}+x\right)}{4d^3e^3} - \frac{(dg-ef)\,(dg+ef)\log\left(\frac{d(dg-ef)(dg+ef)}{e(d^2g^2-e^2f^2)}+x\right)}{4d^3e^3}$$

input
```
integrate((g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

output
```
(-2*d**2*f*g + x*(-d**2*g**2 - e**2*f**2))/(-2*d**4*e**2 + 2*d**2*e**4*x**
2) + (d*g - e*f)*(d*g + e*f)*log(-d*(d*g - e*f)*(d*g + e*f)/(e*(d**2*g**2
- e**2*f**2)) + x)/(4*d**3*e**3) - (d*g - e*f)*(d*g + e*f)*log(d*(d*g - e*
f)*(d*g + e*f)/(e*(d**2*g**2 - e**2*f**2)) + x)/(4*d**3*e**3)
```

3.564.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.50

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = -\frac{2\,d^2fg + (e^2f^2+d^2g^2)x}{2\,(d^2e^4x^2-d^4e^2)} + \frac{(e^2f^2-d^2g^2)\log(ex+d)}{4\,d^3e^3} - \frac{(e^2f^2-d^2g^2)\log(ex-d)}{4\,d^3e^3}$$

input
```
integrate((g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*d^2*f*g + (e^2*f^2 + d^2*g^2)*x)/(d^2*e^4*x^2 - d^4*e^2) + 1/4*(e^
2*f^2 - d^2*g^2)*log(e*x + d)/(d^3*e^3) - 1/4*(e^2*f^2 - d^2*g^2)*log(e*x
- d)/(d^3*e^3)
```

3.564.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.54

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = -\frac{e^2f^2x+d^2g^2x+2\,d^2fg}{2\,(e^2x^2-d^2)d^2e^2} + \frac{(e^3f^2-d^2eg^2)\log\left(|ex+d|\right)}{4\,d^3e^4} - \frac{(e^3f^2-d^2eg^2)\log\left(|ex-d|\right)}{4\,d^3e^4}$$

input
```
integrate((g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

output
```
-1/2*(e^2*f^2*x + d^2*g^2*x + 2*d^2*f*g)/((e^2*x^2 - d^2)*d^2*e^2) + 1/4*(
e^3*f^2 - d^2*e*g^2)*log(abs(e*x + d))/(d^3*e^4) - 1/4*(e^3*f^2 - d^2*e*g^
2)*log(abs(e*x - d))/(d^3*e^4)
```

3.564.9 Mupad [B] (verification not implemented)

Time = 12.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.55

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{\frac{f\,g}{e^2}+\frac{x\,(d^2\,g^2+e^2\,f^2)}{2\,d^2\,e^2}}{d^2-e^2\,x^2} - \frac{2\,\mathrm{atanh}\left(\frac{4\,e\,x\,\left(\frac{d^2\,g^2}{4}-\frac{e^2\,f^2}{4}\right)}{d\,(d^2\,g^2-e^2\,f^2)}\right)\,\left(\frac{d^2\,g^2}{4}-\frac{e^2\,f^2}{4}\right)}{d^3\,e^3}$$

input
```
int((f + g*x)^2/(d^2 - e^2*x^2)^2,x)
```

output
```
((f*g)/e^2 + (x*(d^2*g^2 + e^2*f^2))/(2*d^2*e^2))/(d^2 - e^2*x^2) - (2*ata
nh((4*e*x*((d^2*g^2)/4 - (e^2*f^2)/4))/(d*(d^2*g^2 - e^2*f^2)))*((d^2*g^2)
/4 - (e^2*f^2)/4))/(d^3*e^3)
```

3.565 $\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2}\,dx$

3.565.1 Optimal result

Integrand size = 29, antiderivative size = 121

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2}\,dx = \frac{(ef+dg)^2}{8d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^2e^3(d+ex)^2}-\frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)}+\frac{(3ef-dg)(ef+dg)\text{arctanh}\left(\frac{ex}{d}\right)}{8d^4e^3}$$

output
```
1/8*(d*g+e*f)^2/d^3/e^3/(-e*x+d)-1/8*(-d*g+e*f)^2/d^2/e^3/(e*x+d)^2+1/4*(d
^2*g^2-e^2*f^2)/d^3/e^3/(e*x+d)+1/8*(-d*g+3*e*f)*(d*g+e*f)*arctanh(e*x/d)/
d^4/e^3
```

3.565.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.15

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2}\,dx = \frac{\frac{2d(ef+dg)^2}{d-ex}-\frac{2d^2(ef-dg)^2}{(d+ex)^2}+\frac{4d\left(-e^2f^2+d^2g^2\right)}{d+ex}+\left(-3e^2f^2-2defg+d^2g^2\right)\log(d-ex)+\left(3e^2f^2+2defg-d^2g^2\right.}{16d^4e^3}$$

input
```
Integrate[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^2),x]
```

output
```
((2*d*(e*f + d*g)^2)/(d - e*x) - (2*d^2*(e*f - d*g)^2)/(d + e*x)^2 + (4*d*
(-(e^2*f^2) + d^2*g^2))/(d + e*x) + (-3*e^2*f^2 - 2*d*e*f*g + d^2*g^2)*Log
[d - e*x] + (3*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*Log[d + e*x])/(16*d^4*e^3)
```

3.565.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {639, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)^2}dx$$

$$\Big\downarrow \text{639}$$

$$\int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^3}dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(\frac{(dg+ef)^2}{8d^3e^2(d-ex)^2}+\frac{(dg-ef)^2}{4d^2e^2(d+ex)^3}+\frac{e^2f^2-d^2g^2}{4d^3e^2(d+ex)^2}+\frac{(3ef-dg)(dg+ef)}{8d^3e^2(d^2-e^2x^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{arctanh}\left(\frac{ex}{d}\right)(3ef-dg)(dg+ef)}{8d^4e^3}+\frac{(dg+ef)^2}{8d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^2e^3(d+ex)^2}-\frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)}$$

input
```
Int[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^2),x]
```

output
```
(e*f + d*g)^2/(8*d^3*e^3*(d - e*x)) - (e*f - d*g)^2/(8*d^2*e^3*(d + e*x)^2
) - (e^2*f^2 - d^2*g^2)/(4*d^3*e^3*(d + e*x)) + ((3*e*f - d*g)*(e*f + d*g)
*ArcTanh[(e*x)/d])/(8*d^4*e^3)
```

3.565.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.565.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.49

method	result
default	$\frac{\left(d^2g^2-2defg-3e^2f^2\right)\ln(-ex+d)}{16e^3d^4}+\frac{d^2g^2+2defg+e^2f^2}{8e^3d^3(-ex+d)}-\frac{-d^2g^2+e^2f^2}{4e^3d^3(ex+d)}+\frac{\left(-d^2g^2+2defg+3e^2f^2\right)\ln(ex+d)}{16e^3d^4}-\frac{d^2g^2-}{8d^2}$
norman	$\frac{-\frac{-d^2g^2-2defg+e^2f^2}{4d\,e^3}-\frac{\left(-3d^2g^2-2defg-3e^2f^2\right)x}{8e^2d^2}+\frac{\left(-d^2g^2+2defg+3e^2f^2\right)x^2}{8d^3e}}{(-ex+d)(ex+d)^2}+\frac{\left(d^2g^2-2defg-3e^2f^2\right)\ln(-ex+d)}{16e^3d^4}-\frac{(d^2g^2-}{}$
risch	$\frac{-\frac{\left(d^2g^2-2defg-3e^2f^2\right)x^2}{8e\,d^3}+\frac{\left(3d^2g^2+2defg+3e^2f^2\right)x}{8d^2e^2}+\frac{d^2g^2+2defg-e^2f^2}{4d\,e^3}}{(ex+d)(-e^2x^2+d^2)}+\frac{\ln(ex-d)g^2}{16e^3d^2}-\frac{\ln(ex-d)fg}{8e^2d^3}-\frac{3\ln(ex-d)f^2}{16e\,d^4}-$
parallelrisch	$\frac{-2\ln(ex+d)d^4efg+\ln(ex-d)x^3d^2e^3g^2-2\ln(ex+d)x\,d^3e^2fg+2\ln(ex-d)x\,d^3e^2fg-2\ln(ex-d)x^2d^2e^3fg+2\ln(ex+d)x^2d^2e^3f}{}$

input
```
int((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16/e^3*(d^2*g^2-2*d*e*f*g-3*e^2*f^2)/d^4*ln(-e*x+d)+1/8*(d^2*g^2+2*d*e*f
*g+e^2*f^2)/e^3/d^3/(-e*x+d)-1/4*(-d^2*g^2+e^2*f^2)/e^3/d^3/(e*x+d)+1/16*(
-d^2*g^2+2*d*e*f*g+3*e^2*f^2)/e^3/d^4*ln(e*x+d)-1/8*(d^2*g^2-2*d*e*f*g+e^2
*f^2)/d^2/e^3/(e*x+d)^2
```

3.565.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(114) = 228$.

Time = 0.35 (sec) , antiderivative size = 417, normalized size of antiderivative = 3.45

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2}\,dx = \frac{4\,d^3e^2f^2 - 8\,d^4efg - 4\,d^5g^2 - 2\,(3\,de^4f^2 + 2\,d^2e^3fg - d^3e^2g^2)x^2 - 2\,(3\,d^2e^3f^2 + 2\,d^3e^2fg + 3\,d^4eg^2)x - (}{}$$

input
```
integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

output
```
1/16*(4*d^3*e^2*f^2 - 8*d^4*e*f*g - 4*d^5*g^2 - 2*(3*d*e^4*f^2 + 2*d^2*e^3
*f*g - d^3*e^2*g^2)*x^2 - 2*(3*d^2*e^3*f^2 + 2*d^3*e^2*f*g + 3*d^4*e*g^2)*
x - (3*d^3*e^2*f^2 + 2*d^4*e*f*g - d^5*g^2 - (3*e^5*f^2 + 2*d*e^4*f*g - d^
2*e^3*g^2)*x^3 - (3*d*e^4*f^2 + 2*d^2*e^3*f*g - d^3*e^2*g^2)*x^2 + (3*d^2*
e^3*f^2 + 2*d^3*e^2*f*g - d^4*e*g^2)*x)*log(e*x + d) + (3*d^3*e^2*f^2 + 2*
d^4*e*f*g - d^5*g^2 - (3*e^5*f^2 + 2*d*e^4*f*g - d^2*e^3*g^2)*x^3 - (3*d*e
^4*f^2 + 2*d^2*e^3*f*g - d^3*e^2*g^2)*x^2 + (3*d^2*e^3*f^2 + 2*d^3*e^2*f*g
 - d^4*e*g^2)*x)*log(e*x - d))/(d^4*e^6*x^3 + d^5*e^5*x^2 - d^6*e^4*x - d^
7*e^3)
```

3.565.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(105) = 210$.

Time = 0.58 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.31

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2}\,dx = \frac{-2d^4g^2 - 4d^3efg + 2d^2e^2f^2 + x^2(d^2e^2g^2 - 2de^3fg - 3e^4f^2) + x(-3d^3eg^2 - 2d^2e^2fg - 3de^3f^2)}{-8d^6e^3 - 8d^5e^4x + 8d^4e^5x^2 + 8d^3e^6x^3} + \frac{(dg-3ef)(dg+ef)\log\left(-\frac{d(dg-3ef)(dg+ef)}{e(d^2g^2-2defg-3e^2f^2)} + x\right)}{16d^4e^3} - \frac{(dg-3ef)(dg+ef)\log\left(\frac{d(dg-3ef)(dg+ef)}{e(d^2g^2-2defg-3e^2f^2)} + x\right)}{16d^4e^3}$$

input
```
integrate((g*x+f)**2/(e*x+d)/(-e**2*x**2+d**2)**2,x)
```

output
```
(-2*d**4*g**2 - 4*d**3*e*f*g + 2*d**2*e**2*f**2 + x**2*(d**2*e**2*g**2 - 2
*d*e**3*f*g - 3*e**4*f**2) + x*(-3*d**3*e*g**2 - 2*d**2*e**2*f*g - 3*d*e**
3*f**2))/(-8*d**6*e**3 - 8*d**5*e**4*x + 8*d**4*e**5*x**2 + 8*d**3*e**6*x*
*3) + (d*g - 3*e*f)*(d*g + e*f)*log(-d*(d*g - 3*e*f)*(d*g + e*f)/(e*(d**2*
g**2 - 2*d*e*f*g - 3*e**2*f**2)) + x)/(16*d**4*e**3) - (d*g - 3*e*f)*(d*g
+ e*f)*log(d*(d*g - 3*e*f)*(d*g + e*f)/(e*(d**2*g**2 - 2*d*e*f*g - 3*e**2*
f**2)) + x)/(16*d**4*e**3)
```

3.565.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.75

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2}\,dx = \frac{2\,d^2e^2f^2-4\,d^3efg-2\,d^4g^2-(3\,e^4f^2+2\,de^3fg-d^2e^2g^2)x^2-(3\,de^3f^2+2\,d^2e^2fg+3\,d^3eg^2)x}{8\left(d^3e^6x^3+d^4e^5x^2-d^5e^4x-d^6e^3\right)} + \frac{(3\,e^2f^2+2\,defg-d^2g^2)\log(ex+d)}{16\,d^4e^3} - \frac{(3\,e^2f^2+2\,defg-d^2g^2)\log(ex-d)}{16\,d^4e^3}$$

input
```
integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

output
```
1/8*(2*d^2*e^2*f^2 - 4*d^3*e*f*g - 2*d^4*g^2 - (3*e^4*f^2 + 2*d*e^3*f*g -
d^2*e^2*g^2)*x^2 - (3*d*e^3*f^2 + 2*d^2*e^2*f*g + 3*d^3*e*g^2)*x)/(d^3*e^6
*x^3 + d^4*e^5*x^2 - d^5*e^4*x - d^6*e^3) + 1/16*(3*e^2*f^2 + 2*d*e*f*g -
d^2*g^2)*log(e*x + d)/(d^4*e^3) - 1/16*(3*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*l
og(e*x - d)/(d^4*e^3)
```

3.565.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.66

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2}\,dx = \frac{(3\,e^2f^2+2\,defg-d^2g^2)\log(|ex+d|)}{16\,d^4e^3} - \frac{(3\,e^2f^2+2\,defg-d^2g^2)\log(|ex-d|)}{16\,d^4e^3} + \frac{2\,d^3e^2f^2-4\,d^4efg-2\,d^5g^2-(3\,de^4f^2+2\,d^2e^3fg-d^3e^2g^2)x^2-(3\,d^2e^3f^2+2\,d^3e^2fg+3\,d^4eg^2)x}{8\,(ex+d)^2(ex-d)d^4e^3}$$

input
```
integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

output
```
1/16*(3*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*log(abs(e*x + d))/(d^4*e^3) - 1/16*
(3*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*log(abs(e*x - d))/(d^4*e^3) + 1/8*(2*d^3
*e^2*f^2 - 4*d^4*e*f*g - 2*d^5*g^2 - (3*d*e^4*f^2 + 2*d^2*e^3*f*g - d^3*e^
2*g^2)*x^2 - (3*d^2*e^3*f^2 + 2*d^3*e^2*f*g + 3*d^4*e*g^2)*x)/((e*x + d)^2
*(e*x - d)*d^4*e^3)
```

3.565.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.64

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2}\,dx = \frac{\frac{d^2\,g^2+2\,d\,e\,f\,g-e^2\,f^2}{4\,d\,e^3}+\frac{x\,\left(3\,d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2\right)}{8\,d^2\,e^2}+\frac{x^2\,\left(-d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2\right)}{8\,d^3\,e}}{d^3+d^2\,e\,x-d\,e^2\,x^2-e^3\,x^3} + \frac{\mathrm{atanh}\left(\frac{e\,x\,(d\,g+e\,f)\,(d\,g-3\,e\,f)}{d\,\left(-d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2\right)}\right)\,\left(d\,g+e\,f\right)\,\left(d\,g-3\,e\,f\right)}{8\,d^4\,e^3}$$

input
```
int((f + g*x)^2/((d^2 - e^2*x^2)^2*(d + e*x)),x)
```

output
```
((d^2*g^2 - e^2*f^2 + 2*d*e*f*g)/(4*d*e^3) + (x*(3*d^2*g^2 + 3*e^2*f^2 + 2
*d*e*f*g))/(8*d^2*e^2) + (x^2*(3*e^2*f^2 - d^2*g^2 + 2*d*e*f*g))/(8*d^3*e)
)/(d^3 - e^3*x^3 - d*e^2*x^2 + d^2*e*x) + (atanh((e*x*(d*g + e*f)*(d*g - 3
*e*f))/(d*(3*e^2*f^2 - d^2*g^2 + 2*d*e*f*g)))*(d*g + e*f)*(d*g - 3*e*f))/(
8*d^4*e^3)
```

3.566 $\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^2}\,dx$

3.566.1 Optimal result

Integrand size = 29, antiderivative size = 146

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^2}\,dx = \frac{(ef+dg)^2}{16d^4e^3(d-ex)} - \frac{(ef-dg)^2}{12d^2e^3(d+ex)^3} - \frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2} - \frac{(3ef-dg)(ef+dg)}{16d^4e^3(d+ex)} + \frac{f(ef+dg)\operatorname{arctanh}\left(\frac{ex}{d}\right)}{4d^5e^2}$$

output
```
1/16*(d*g+e*f)^2/d^4/e^3/(-e*x+d)-1/12*(-d*g+e*f)^2/d^2/e^3/(e*x+d)^3+1/8*
(d^2*g^2-e^2*f^2)/d^3/e^3/(e*x+d)^2-1/16*(-d*g+3*e*f)*(d*g+e*f)/d^4/e^3/(e
*x+d)+1/4*f*(d*g+e*f)*arctanh(e*x/d)/d^5/e^2
```

3.566.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.17

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^2}\,dx = \frac{2d(2d^5g^2+3e^5f^2x^3+d^3e^2f(-4f+gx)+3de^4fx^2(2f+gx)+2d^4eg(f+2gx)+d^2e^3fx(f+6gx))+3e}{24d^5e^3(d-ex)(d+}$$

input
```
Integrate[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^2),x]
```

output
```
(2*d*(2*d^5*g^2 + 3*e^5*f^2*x^3 + d^3*e^2*f*(-4*f + g*x) + 3*d*e^4*f*x^2*(
2*f + g*x) + 2*d^4*e*g*(f + 2*g*x) + d^2*e^3*f*x*(f + 6*g*x)) + 3*e*f*(e*f
 + d*g)*(-d + e*x)*(d + e*x)^3*Log[d - e*x] + 3*e*f*(e*f + d*g)*(d - e*x)*
(d + e*x)^3*Log[d + e*x])/(24*d^5*e^3*(d - e*x)*(d + e*x)^3)
```

3.566.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {639, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{(d+ex)^2 \left(d^2-e^2x^2\right)^2} \, dx$$
$$\Big\downarrow \text{ 639}$$
$$\int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^4}dx$$
$$\Big\downarrow \text{ 99}$$
$$\int \left(\frac{(dg+ef)^2}{16d^4e^2(d-ex)^2}+\frac{(3ef-dg)(dg+ef)}{16d^4e^2(d+ex)^2}+\frac{(dg-ef)^2}{4d^2e^2(d+ex)^4}+\frac{f(dg+ef)}{4d^4e\left(d^2-e^2x^2\right)}+\frac{e^2f^2-d^2g^2}{4d^3e^2(d+ex)^3}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{f\text{arctanh}\left(\frac{ex}{d}\right)(dg+ef)}{4d^5e^2}+\frac{(dg+ef)^2}{16d^4e^3(d-ex)}-\frac{(3ef-dg)(dg+ef)}{16d^4e^3(d+ex)}-\frac{(ef-dg)^2}{12d^2e^3(d+ex)^3}-\frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2}$$

input
```
Int[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^2),x]
```

output
```
(e*f + d*g)^2/(16*d^4*e^3*(d - e*x)) - (e*f - d*g)^2/(12*d^2*e^3*(d + e*x)
^3) - (e^2*f^2 - d^2*g^2)/(8*d^3*e^3*(d + e*x)^2) - ((3*e*f - d*g)*(e*f +
d*g))/(16*d^4*e^3*(d + e*x)) + (f*(e*f + d*g)*ArcTanh[(e*x)/d])/(4*d^5*e^2
)
```

3.566.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.566.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.11

method	result
norman	$\frac{\frac{dfg-3ef^2}{8de^2}-\frac{\left(-d^2g^2-defg-e^2f^2\right)x^3}{3d^4}+\frac{f(dg+ef)x^2}{2d^3}-\frac{e\left(-4d^2g^2-defg-e^2f^2\right)x^4}{24d^5}}{(ex+d)^3(-ex+d)}-\frac{f(dg+ef)\ln(-ex+d)}{8d^5e^2}+\frac{f(dg+ef)\ln(ex+d)}{8d^5e^2}$
risch	$\frac{\frac{ef(dg+ef)x^3}{4d^4}+\frac{f(dg+ef)x^2}{2d^3}+\frac{\left(4d^2g^2+defg+e^2f^2\right)x}{12d^2e^2}+\frac{d^2g^2+defg-2e^2f^2}{6de^3}}{(ex+d)^2(-e^2x^2+d^2)}-\frac{f\ln(-ex+d)g}{8d^4e^2}-\frac{\ln(-ex+d)f^2}{8ed^5}+\frac{f\ln(ex+d)g}{8d^4e^2}-$
default	$\frac{d^2g^2+2defg+e^2f^2}{16e^3d^4(-ex+d)}-\frac{f(dg+ef)\ln(-ex+d)}{8d^5e^2}-\frac{-d^2g^2+e^2f^2}{8e^3d^3(ex+d)^2}-\frac{-d^2g^2+2defg+3e^2f^2}{16e^3d^4(ex+d)}-\frac{d^2g^2-2defg+e^2f^2}{12d^2e^3(ex+d)^3}+$
parallelrisch	$-\frac{3\ln(ex-d)x^4e^5f^2-3\ln(ex+d)x^4e^5f^2-3\ln(ex-d)d^5fg-3\ln(ex-d)d^4ef^2+3\ln(ex+d)d^5fg+3\ln(ex+d)d^4ef^2+18f^2d^3e^2}{}$

input
```
int((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(d*f*g-3*e*f^2)/d/e^2-1/3*(-d^2*g^2-d*e*f*g-e^2*f^2)/d^4*x^3+1/2/d^3*
f*(d*g+e*f)*x^2-1/24*e*(-4*d^2*g^2-d*e*f*g-e^2*f^2)/d^5*x^4)/(e*x+d)^3/(-e
*x+d)-1/8*f*(d*g+e*f)/d^5/e^2*ln(-e*x+d)+1/8*f*(d*g+e*f)/d^5/e^2*ln(e*x+d)
```

3.566.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(137) = 274$.

Time = 0.37 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.31

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^2}\,dx = \frac{8\,d^4e^2f^2-4\,d^5efg-4\,d^6g^2-6\,(de^5f^2+d^2e^4fg)x^3-12\,(d^2e^4f^2+d^3e^3fg)x^2-2\,(d^3e^3f^2+d^4e^2fg+4\,d}{}$$

input
```
integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

output
```
1/24*(8*d^4*e^2*f^2 - 4*d^5*e*f*g - 4*d^6*g^2 - 6*(d*e^5*f^2 + d^2*e^4*f*g
)*x^3 - 12*(d^2*e^4*f^2 + d^3*e^3*f*g)*x^2 - 2*(d^3*e^3*f^2 + d^4*e^2*f*g
+ 4*d^5*e*g^2)*x - 3*(d^4*e^2*f^2 + d^5*e*f*g - (e^6*f^2 + d*e^5*f*g)*x^4
- 2*(d*e^5*f^2 + d^2*e^4*f*g)*x^3 + 2*(d^3*e^3*f^2 + d^4*e^2*f*g)*x)*log(e
*x + d) + 3*(d^4*e^2*f^2 + d^5*e*f*g - (e^6*f^2 + d*e^5*f*g)*x^4 - 2*(d*e^
5*f^2 + d^2*e^4*f*g)*x^3 + 2*(d^3*e^3*f^2 + d^4*e^2*f*g)*x)*log(e*x - d))/
(d^5*e^7*x^4 + 2*d^6*e^6*x^3 - 2*d^8*e^4*x - d^9*e^3)
```

3.566.6 Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.65

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^2}\,dx = \frac{-2d^5g^2-2d^4efg+4d^3e^2f^2+x^3(-3de^4fg-3e^5f^2)+x^2(-6d^2e^3fg-6de^4f^2)+x(-4d^4eg^2-d^3e^2fg-}{-12d^8e^3-24d^7e^4x+24d^5e^6x^3+12d^4e^7x^4} - \frac{f(dg+ef)\log\left(-\frac{df(dg+ef)}{e(dfg+ef^2)}+x\right)}{8d^5e^2} + \frac{f(dg+ef)\log\left(\frac{df(dg+ef)}{e(dfg+ef^2)}+x\right)}{8d^5e^2}$$

input
```
integrate((g*x+f)**2/(e*x+d)**2/(-e**2*x**2+d**2)**2,x)
```

output
```
(-2*d**5*g**2 - 2*d**4*e*f*g + 4*d**3*e**2*f**2 + x**3*(-3*d*e**4*f*g - 3*
e**5*f**2) + x**2*(-6*d**2*e**3*f*g - 6*d*e**4*f**2) + x*(-4*d**4*e*g**2 -
 d**3*e**2*f*g - d**2*e**3*f**2))/(-12*d**8*e**3 - 24*d**7*e**4*x + 24*d**
5*e**6*x**3 + 12*d**4*e**7*x**4) - f*(d*g + e*f)*log(-d*f*(d*g + e*f)/(e*(
d*f*g + e*f**2)) + x)/(8*d**5*e**2) + f*(d*g + e*f)*log(d*f*(d*g + e*f)/(e
*(d*f*g + e*f**2)) + x)/(8*d**5*e**2)
```

3.566.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.35

$$\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)^2}\,dx$$
$$= \frac{4\,d^3e^2f^2-2\,d^4efg-2\,d^5g^2-3\,(e^5f^2+de^4fg)x^3-6\,(de^4f^2+d^2e^3fg)x^2-(d^2e^3f^2+d^3e^2fg+4\,d^4eg^2)}{12\,(d^4e^7x^4+2\,d^5e^6x^3-2\,d^7e^4x-d^8e^3)} + \frac{(ef^2+dfg)\log(ex+d)}{8\,d^5e^2} - \frac{(ef^2+dfg)\log(ex-d)}{8\,d^5e^2}$$

input
```
integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

output
```
1/12*(4*d^3*e^2*f^2 - 2*d^4*e*f*g - 2*d^5*g^2 - 3*(e^5*f^2 + d*e^4*f*g)*x^
3 - 6*(d*e^4*f^2 + d^2*e^3*f*g)*x^2 - (d^2*e^3*f^2 + d^3*e^2*f*g + 4*d^4*e
*g^2)*x)/(d^4*e^7*x^4 + 2*d^5*e^6*x^3 - 2*d^7*e^4*x - d^8*e^3) + 1/8*(e*f^
2 + d*f*g)*log(e*x + d)/(d^5*e^2) - 1/8*(e*f^2 + d*f*g)*log(e*x - d)/(d^5*
e^2)
```

3.566.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.55

$$\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)^2}\,dx$$
$$= -\frac{(ef^2+dfg)\log\left(\left|-\frac{2\,d}{ex+d}+1\right|\right)}{8\,d^5e^2} + \frac{e^2f^2+2\,defg+d^2g^2}{32\,d^5e^3\left(\frac{2\,d}{ex+d}-1\right)} - \frac{\frac{9\,d^2e^5f^2}{ex+d}+\frac{6\,d^3e^5f^2}{(ex+d)^2}+\frac{4\,d^4e^5f^2}{(ex+d)^3}+\frac{6\,d^3e^4fg}{ex+d}-\frac{8\,d^5e^4fg}{(ex+d)^3}-\frac{3\,d^4e^3g^2}{ex+d}-\frac{6\,d^5e^3g^2}{(ex+d)^2}+\frac{4\,d^6e^3g^2}{(ex+d)^3}}{48\,d^6e^6}$$

input
```
integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

output
```
-1/8*(e*f^2 + d*f*g)*log(abs(-2*d/(e*x + d) + 1))/(d^5*e^2) + 1/32*(e^2*f^
2 + 2*d*e*f*g + d^2*g^2)/(d^5*e^3*(2*d/(e*x + d) - 1)) - 1/48*(9*d^2*e^5*f
^2/(e*x + d) + 6*d^3*e^5*f^2/(e*x + d)^2 + 4*d^4*e^5*f^2/(e*x + d)^3 + 6*d
^3*e^4*f*g/(e*x + d) - 8*d^5*e^4*f*g/(e*x + d)^3 - 3*d^4*e^3*g^2/(e*x + d)
 - 6*d^5*e^3*g^2/(e*x + d)^2 + 4*d^6*e^3*g^2/(e*x + d)^3)/(d^6*e^6)
```

3.566.9 Mupad [B] (verification not implemented)

Time = 12.06 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^2}\,dx$$
$$=\frac{\frac{d^2\,g^2+d\,e\,f\,g-2\,e^2\,f^2}{6\,d\,e^3}+\frac{f\,x^2\,(d\,g+e\,f)}{2\,d^3}+\frac{x\,(4\,d^2\,g^2+d\,e\,f\,g+e^2\,f^2)}{12\,d^2\,e^2}+\frac{e\,f\,x^3\,(d\,g+e\,f)}{4\,d^4}}{d^4+2\,d^3\,e\,x-2\,d\,e^3\,x^3-e^4\,x^4}$$
$$+\frac{f\,\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,(d\,g+e\,f)}{4\,d^5\,e^2}$$

input
```
int((f + g*x)^2/((d^2 - e^2*x^2)^2*(d + e*x)^2),x)
```

output
```
((d^2*g^2 - 2*e^2*f^2 + d*e*f*g)/(6*d*e^3) + (f*x^2*(d*g + e*f))/(2*d^3) +
 (x*(4*d^2*g^2 + e^2*f^2 + d*e*f*g))/(12*d^2*e^2) + (e*f*x^3*(d*g + e*f))/
(4*d^4))/(d^4 - e^4*x^4 - 2*d*e^3*x^3 + 2*d^3*e*x) + (f*atanh((e*x)/d)*(d*
g + e*f))/(4*d^5*e^2)
```

3.567 $\int \frac{(f+gx)^2}{(d+ex)^3(d^2-e^2x^2)^2}\,dx$

3.567.1 Optimal result

Integrand size = 29, antiderivative size = 178

$$\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)^2}\,dx = \frac{(ef+dg)^2}{32d^5e^3(d-ex)} - \frac{(ef-dg)^2}{16d^2e^3(d+ex)^4} - \frac{e^2f^2-d^2g^2}{12d^3e^3(d+ex)^3} - \frac{(3ef-dg)(ef+dg)}{32d^4e^3(d+ex)^2} - \frac{f(ef+dg)}{8d^5e^2(d+ex)} + \frac{(ef+dg)(5ef+dg)\operatorname{arctanh}\left(\frac{ex}{d}\right)}{32d^6e^3}$$

output

```
1/32*(d*g+e*f)^2/d^5/e^3/(-e*x+d)-1/16*(-d*g+e*f)^2/d^2/e^3/(e*x+d)^4+1/12
*(d^2*g^2-e^2*f^2)/d^3/e^3/(e*x+d)^3-1/32*(-d*g+3*e*f)*(d*g+e*f)/d^4/e^3/(
e*x+d)^2-1/8*f*(d*g+e*f)/d^5/e^2/(e*x+d)+1/32*(d*g+e*f)*(d*g+5*e*f)*arctan
h(e*x/d)/d^6/e^3
```

3.567.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.10

$$\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)^2}\,dx = \frac{\frac{6d(ef+dg)^2}{d-ex} - \frac{12d^4(ef-dg)^2}{(d+ex)^4} + \frac{16d^3\left(-e^2f^2+d^2g^2\right)}{(d+ex)^3} + \frac{6d^2\left(-3e^2f^2-2defg+d^2g^2\right)}{(d+ex)^2} - \frac{24def(ef+dg)}{d+ex} - 3(5e^2f^2+6defg+d^2g^2}{192d^6e^3}$$

input
```
Integrate[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)^2),x]
```

output
```
((6*d*(e*f + d*g)^2)/(d - e*x) - (12*d^4*(e*f - d*g)^2)/(d + e*x)^4 + (16*
d^3*(-(e^2*f^2) + d^2*g^2))/(d + e*x)^3 + (6*d^2*(-3*e^2*f^2 - 2*d*e*f*g +
 d^2*g^2))/(d + e*x)^2 - (24*d*e*f*(e*f + d*g))/(d + e*x) - 3*(5*e^2*f^2 +
 6*d*e*f*g + d^2*g^2)*Log[d - e*x] + 3*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*L
og[d + e*x])/(192*d^6*e^3)
```

3.567.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {639, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{(d+ex)^3 \left(d^2-e^2x^2\right)^2}dx$$

$$\Big\downarrow \text{ 639}$$

$$\int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^5}dx$$

$$\Big\downarrow \text{ 99}$$

$$\int \left(\frac{(dg+ef)^2}{32d^5e^2(d-ex)^2}+\frac{f(dg+ef)}{8d^5e(d+ex)^2}+\frac{(3ef-dg)(dg+ef)}{16d^4e^2(d+ex)^3}+\frac{(dg-ef)^2}{4d^2e^2(d+ex)^5}+\frac{(dg+ef)(dg+5ef)}{32d^5e^2\left(d^2-e^2x^2\right)}+\frac{e^2f^2}{4d^3e^2(\cdots}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\text{arctanh}\left(\frac{ex}{d}\right)(dg+ef)(dg+5ef)}{32d^6e^3}+\frac{(dg+ef)^2}{32d^5e^3(d-ex)}-\frac{f(dg+ef)}{8d^5e^2(d+ex)}-\frac{(3ef-dg)(dg+ef)}{32d^4e^3(d+ex)^2}-\frac{(ef-dg)^2}{16d^2e^3(d+ex)^4}-\frac{e^2f^2-d^2g^2}{12d^3e^3(d+ex)^3}$$

input
```
Int[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)^2),x]
```

output
```
(e*f + d*g)^2/(32*d^5*e^3*(d - e*x)) - (e*f - d*g)^2/(16*d^2*e^3*(d + e*x)
^4) - (e^2*f^2 - d^2*g^2)/(12*d^3*e^3*(d + e*x)^3) - ((3*e*f - d*g)*(e*f +
 d*g))/(32*d^4*e^3*(d + e*x)^2) - (f*(e*f + d*g))/(8*d^5*e^2*(d + e*x)) +
((e*f + d*g)*(5*e*f + d*g)*ArcTanh[(e*x)/d])/(32*d^6*e^3)
```

3.567.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.567.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.35

method	result
default	$\frac{\left(-d^2g^2-6defg-5e^2f^2\right)\ln(-ex+d)}{64e^3d^6}+\frac{d^2g^2+2defg+e^2f^2}{32e^3d^5(-ex+d)}+\frac{\left(d^2g^2+6defg+5e^2f^2\right)\ln(ex+d)}{64e^3d^6}-\frac{-d^2g^2+e^2f^2}{12e^3d^3(ex+d)^3}-$ …
norman	$\frac{\frac{\left(25d^2g^2+54defg-19e^2f^2\right)x^3}{96d^4}-\frac{\left(3d^2g^2-14defg-33e^2f^2\right)x^2}{32e\,d^3}+\frac{3e\left(3d^2g^2+2defg-9e^2f^2\right)x^4}{32d^5}+\frac{e^2\left(d^2g^2-4e^2f^2\right)x^5}{12d^6}-\ldots}{(ex+d)^4(-ex+d)}$
risch	$\frac{\frac{e\left(d^2g^2+6defg+5e^2f^2\right)x^4}{32d^5}+\frac{3\left(d^2g^2+6defg+5e^2f^2\right)x^3}{32d^4}+\frac{7\left(d^2g^2+6defg+5e^2f^2\right)x^2}{96d^3e}+\frac{\left(7d^2g^2-6defg-5e^2f^2\right)x}{32d^2e^2}+\frac{d^2g^2-4e^2f^2}{12d\,e^3}}{(ex+d)^3(-e^2x^2+d^2)}-$ …
parallelrisch	$-\frac{-15\ln(ex+d)x^5e^7f^2+15\ln(ex-d)x^5e^7f^2+162x\,d^4e^3f^2+54\ln(ex+d)x\,d^5e^2fg+3\ln(ex-d)x^5d^2e^5g^2-3\ln(ex+d)x^5d^2e^5g^2 \ldots}{\ldots}$

input
```
int((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/64*(-d^2*g^2-6*d*e*f*g-5*e^2*f^2)/e^3/d^6*ln(-e*x+d)+1/32*(d^2*g^2+2*d*e
*f*g+e^2*f^2)/e^3/d^5/(-e*x+d)+1/64/e^3*(d^2*g^2+6*d*e*f*g+5*e^2*f^2)/d^6*
ln(e*x+d)-1/12*(-d^2*g^2+e^2*f^2)/e^3/d^3/(e*x+d)^3-1/32*(-d^2*g^2+2*d*e*f
*g+3*e^2*f^2)/e^3/d^4/(e*x+d)^2-1/16*(d^2*g^2-2*d*e*f*g+e^2*f^2)/d^2/e^3/(
e*x+d)^4-1/8*f*(d*g+e*f)/d^5/e^2/(e*x+d)
```

3.567.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(167) = 334$.

Time = 0.35 (sec) , antiderivative size = 648, normalized size of antiderivative = 3.64

$$\int \frac{(f+gx)^2}{(d+ex)^3(d^2-e^2x^2)^2}\,dx$$
$$= \frac{64\,d^5e^2f^2 - 16\,d^7g^2 - 6\,(5\,de^6f^2 + 6\,d^2e^5fg + d^3e^4g^2)x^4 - 18\,(5\,d^2e^5f^2 + 6\,d^3e^4fg + d^4e^3g^2)x^3 - 14\,(5\,d^3}{}$$

input
```
integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

output
```
1/192*(64*d^5*e^2*f^2 - 16*d^7*g^2 - 6*(5*d*e^6*f^2 + 6*d^2*e^5*f*g + d^3*
e^4*g^2)*x^4 - 18*(5*d^2*e^5*f^2 + 6*d^3*e^4*f*g + d^4*e^3*g^2)*x^3 - 14*(
5*d^3*e^4*f^2 + 6*d^4*e^3*f*g + d^5*e^2*g^2)*x^2 + 6*(5*d^4*e^3*f^2 + 6*d^
5*e^2*f*g - 7*d^6*e*g^2)*x - 3*(5*d^5*e^2*f^2 + 6*d^6*e*f*g + d^7*g^2 - (5
*e^7*f^2 + 6*d*e^6*f*g + d^2*e^5*g^2)*x^5 - 3*(5*d*e^6*f^2 + 6*d^2*e^5*f*g
 + d^3*e^4*g^2)*x^4 - 2*(5*d^2*e^5*f^2 + 6*d^3*e^4*f*g + d^4*e^3*g^2)*x^3
+ 2*(5*d^3*e^4*f^2 + 6*d^4*e^3*f*g + d^5*e^2*g^2)*x^2 + 3*(5*d^4*e^3*f^2 +
 6*d^5*e^2*f*g + d^6*e*g^2)*x)*log(e*x + d) + 3*(5*d^5*e^2*f^2 + 6*d^6*e*f
*g + d^7*g^2 - (5*e^7*f^2 + 6*d*e^6*f*g + d^2*e^5*g^2)*x^5 - 3*(5*d*e^6*f^
2 + 6*d^2*e^5*f*g + d^3*e^4*g^2)*x^4 - 2*(5*d^2*e^5*f^2 + 6*d^3*e^4*f*g +
d^4*e^3*g^2)*x^3 + 2*(5*d^3*e^4*f^2 + 6*d^4*e^3*f*g + d^5*e^2*g^2)*x^2 + 3
*(5*d^4*e^3*f^2 + 6*d^5*e^2*f*g + d^6*e*g^2)*x)*log(e*x - d))/(d^6*e^8*x^5
 + 3*d^7*e^7*x^4 + 2*d^8*e^6*x^3 - 2*d^9*e^5*x^2 - 3*d^10*e^4*x - d^11*e^3
)
```

3.567.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(162) = 324$.

Time = 0.80 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.11

$$\int \frac{(f+gx)^2}{(d+ex)^3(d^2-e^2x^2)^2}\,dx = \frac{-8d^6g^2+32d^4e^2f^2+x^4(-3d^2e^4g^2-18de^5fg-15e^6f^2)+x^3(-9d^3e^3g^2-54d^2e^4fg-45de^5f^2)+x^2(-7\cdots}{-96d^{10}e^3-288d^9e^4x-192d^8e^5x^2+192d^7e^6x^3+28\cdots} - \frac{(dg+ef)(dg+5ef)\log\left(-\frac{d(dg+ef)(dg+5ef)}{e(d^2g^2+6defg+5e^2f^2)}+x\right)}{64d^6e^3} + \frac{(dg+ef)(dg+5ef)\log\left(\frac{d(dg+ef)(dg+5ef)}{e(d^2g^2+6defg+5e^2f^2)}+x\right)}{64d^6e^3}$$

input
```
integrate((g*x+f)**2/(e*x+d)**3/(-e**2*x**2+d**2)**2,x)
```

output
```
(-8*d**6*g**2 + 32*d**4*e**2*f**2 + x**4*(-3*d**2*e**4*g**2 - 18*d*e**5*f*
g - 15*e**6*f**2) + x**3*(-9*d**3*e**3*g**2 - 54*d**2*e**4*f*g - 45*d*e**5
*f**2) + x**2*(-7*d**4*e**2*g**2 - 42*d**3*e**3*f*g - 35*d**2*e**4*f**2) +
 x*(-21*d**5*e*g**2 + 18*d**4*e**2*f*g + 15*d**3*e**3*f**2))/(-96*d**10*e*
*3 - 288*d**9*e**4*x - 192*d**8*e**5*x**2 + 192*d**7*e**6*x**3 + 288*d**6*
e**7*x**4 + 96*d**5*e**8*x**5) - (d*g + e*f)*(d*g + 5*e*f)*log(-d*(d*g + e
*f)*(d*g + 5*e*f)/(e*(d**2*g**2 + 6*d*e*f*g + 5*e**2*f**2)) + x)/(64*d**6*
e**3) + (d*g + e*f)*(d*g + 5*e*f)*log(d*(d*g + e*f)*(d*g + 5*e*f)/(e*(d**2
*g**2 + 6*d*e*f*g + 5*e**2*f**2)) + x)/(64*d**6*e**3)
```

3.567.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.67

$$\int \frac{(f+gx)^2}{(d+ex)^3(d^2-e^2x^2)^2}\,dx = \frac{32\,d^4e^2f^2-8\,d^6g^2-3\,(5\,e^6f^2+6\,de^5fg+d^2e^4g^2)x^4-9\,(5\,de^5f^2+6\,d^2e^4fg+d^3e^3g^2)x^3-7\,(5\,d^2e^4f^2+\cdots}{96\,(d^5e^8x^5+3\,d^6e^7x^4+2\,d^7e^6x^3-2\,d^8e^5x^2-3\,d^9e^4x\cdots} + \frac{(5\,e^2f^2+6\,defg+d^2g^2)\log(ex+d)}{64\,d^6e^3} - \frac{(5\,e^2f^2+6\,defg+d^2g^2)\log(ex-d)}{64\,d^6e^3}$$

input
```
integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

output
```
1/96*(32*d^4*e^2*f^2 - 8*d^6*g^2 - 3*(5*e^6*f^2 + 6*d*e^5*f*g + d^2*e^4*g^
2)*x^4 - 9*(5*d*e^5*f^2 + 6*d^2*e^4*f*g + d^3*e^3*g^2)*x^3 - 7*(5*d^2*e^4*
f^2 + 6*d^3*e^3*f*g + d^4*e^2*g^2)*x^2 + 3*(5*d^3*e^3*f^2 + 6*d^4*e^2*f*g
- 7*d^5*e*g^2)*x)/(d^5*e^8*x^5 + 3*d^6*e^7*x^4 + 2*d^7*e^6*x^3 - 2*d^8*e^5
*x^2 - 3*d^9*e^4*x - d^10*e^3) + 1/64*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*lo
g(e*x + d)/(d^6*e^3) - 1/64*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*log(e*x - d)
/(d^6*e^3)
```

3.567.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.48

$$\int \frac{(f+gx)^2}{(d+ex)^3(d^2-e^2x^2)^2}\,dx$$
$$= \frac{(5e^2f^2+6defg+d^2g^2)\log(|ex+d|)}{64d^6e^3} - \frac{(5e^2f^2+6defg+d^2g^2)\log(|ex-d|)}{64d^6e^3}$$
$$+ \frac{32d^5e^2f^2-8d^7g^2-3(5de^6f^2+6d^2e^5fg+d^3e^4g^2)x^4-9(5d^2e^5f^2+6d^3e^4fg+d^4e^3g^2)x^3-7(5d^3e}{96(ex+d)^4(ex-d)d^6e^3}$$

input
```
integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

output
```
1/64*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*log(abs(e*x + d))/(d^6*e^3) - 1/64*
(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*log(abs(e*x - d))/(d^6*e^3) + 1/96*(32*d
^5*e^2*f^2 - 8*d^7*g^2 - 3*(5*d*e^6*f^2 + 6*d^2*e^5*f*g + d^3*e^4*g^2)*x^4
 - 9*(5*d^2*e^5*f^2 + 6*d^3*e^4*f*g + d^4*e^3*g^2)*x^3 - 7*(5*d^3*e^4*f^2
+ 6*d^4*e^3*f*g + d^5*e^2*g^2)*x^2 + 3*(5*d^4*e^3*f^2 + 6*d^5*e^2*f*g - 7*
d^6*e*g^2)*x)/((e*x + d)^4*(e*x - d)*d^6*e^3)
```

3.567.9 Mupad [B] (verification not implemented)

Time = 11.80 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.54

$$\int \frac{(f+gx)^2}{(d+ex)^3(d^2-e^2x^2)^2}\,dx$$
$$= \frac{\frac{d^2g^2-4e^2f^2}{12de^3}+\frac{3x^3(d^2g^2+6defg+5e^2f^2)}{32d^4}+\frac{ex^4(d^2g^2+6defg+5e^2f^2)}{32d^5}-\frac{x(-7d^2g^2+6defg+5e^2f^2)}{32d^2e^2}+\frac{7x^2(d^2g^2+6defg+}{96d^3e}}{d^5+3d^4ex+2d^3e^2x^2-2d^2e^3x^3-3de^4x^4-e^5x^5}$$
$$+ \frac{\operatorname{atanh}\left(\frac{ex(dg+ef)(dg+5ef)}{d(d^2g^2+6defg+5e^2f^2)}\right)(dg+ef)(dg+5ef)}{32d^6e^3}$$

input
```
int((f + g*x)^2/((d^2 - e^2*x^2)^2*(d + e*x)^3),x)
```

output
```
((d^2*g^2 - 4*e^2*f^2)/(12*d*e^3) + (3*x^3*(d^2*g^2 + 5*e^2*f^2 + 6*d*e*f*
g))/(32*d^4) + (e*x^4*(d^2*g^2 + 5*e^2*f^2 + 6*d*e*f*g))/(32*d^5) - (x*(5*
e^2*f^2 - 7*d^2*g^2 + 6*d*e*f*g))/(32*d^2*e^2) + (7*x^2*(d^2*g^2 + 5*e^2*f
^2 + 6*d*e*f*g))/(96*d^3*e))/(d^5 - e^5*x^5 - 3*d*e^4*x^4 + 2*d^3*e^2*x^2
- 2*d^2*e^3*x^3 + 3*d^4*e*x) + (atanh((e*x*(d*g + e*f)*(d*g + 5*e*f))/(d*(
d^2*g^2 + 5*e^2*f^2 + 6*d*e*f*g)))*(d*g + e*f)*(d*g + 5*e*f))/(32*d^6*e^3)
```

3.567. $\int \frac{(f+gx)^2}{(d+ex)^3(d^2-e^2x^2)^2}\,dx$

3.568 $\int \frac{(f+gx)^2}{(d+ex)^4 \left(d^2-e^2x^2\right)^2} \, dx$

3.568.1 Optimal result

Integrand size = 29, antiderivative size = 210

$$\int \frac{(f+gx)^2}{(d+ex)^4 \left(d^2-e^2x^2\right)^2} \, dx = \frac{(ef+dg)^2}{64d^6e^3(d-ex)} - \frac{(ef-dg)^2}{20d^2e^3(d+ex)^5} - \frac{e^2f^2-d^2g^2}{16d^3e^3(d+ex)^4} - \frac{(3ef-dg)(ef+dg)}{48d^4e^3(d+ex)^3} - \frac{f(ef+dg)}{16d^5e^2(d+ex)^2} - \frac{(ef+dg)(5ef+dg)}{64d^6e^3(d+ex)} + \frac{(ef+dg)(3ef+dg)\text{arctanh}\left(\frac{ex}{d}\right)}{32d^7e^3}$$

output
```
1/64*(d*g+e*f)^2/d^6/e^3/(-e*x+d)-1/20*(-d*g+e*f)^2/d^2/e^3/(e*x+d)^5+1/16
*(d^2*g^2-e^2*f^2)/d^3/e^3/(e*x+d)^4-1/48*(-d*g+3*e*f)*(d*g+e*f)/d^4/e^3/(
e*x+d)^3-1/16*f*(d*g+e*f)/d^5/e^2/(e*x+d)^2-1/64*(d*g+e*f)*(d*g+5*e*f)/d^6
/e^3/(e*x+d)+1/32*(d*g+e*f)*(d*g+3*e*f)*arctanh(e*x/d)/d^7/e^3
```

3.568.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.09

$$\int \frac{(f+gx)^2}{(d+ex)^4 \left(d^2-e^2x^2\right)^2} \, dx = \frac{\frac{15d(ef+dg)^2}{d-ex} - \frac{48d^5(ef-dg)^2}{(d+ex)^5} + \frac{60d^4\left(-e^2f^2+d^2g^2\right)}{(d+ex)^4} + \frac{20d^3\left(-3e^2f^2-2defg+d^2g^2\right)}{(d+ex)^3} - \frac{60d^2ef(ef+dg)}{(d+ex)^2} - \frac{15d\left(5e^2f^2+6defg+d^2g^2\right)}{d+ex} - \cdots}{960d^7e^3}$$

input
```
Integrate[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)^2),x]
```

output
```
((15*d*(e*f + d*g)^2)/(d - e*x) - (48*d^5*(e*f - d*g)^2)/(d + e*x)^5 + (60
*d^4*(-(e^2*f^2) + d^2*g^2))/(d + e*x)^4 + (20*d^3*(-3*e^2*f^2 - 2*d*e*f*g
 + d^2*g^2))/(d + e*x)^3 - (60*d^2*e*f*(e*f + d*g))/(d + e*x)^2 - (15*d*(5
*e^2*f^2 + 6*d*e*f*g + d^2*g^2))/(d + e*x) - 15*(3*e^2*f^2 + 4*d*e*f*g + d
^2*g^2)*Log[d - e*x] + 15*(3*e^2*f^2 + 4*d*e*f*g + d^2*g^2)*Log[d + e*x])/
(960*d^7*e^3)
```

3.568.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {639, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{(d+ex)^4 (d^2-e^2x^2)^2} \, dx$$

$$\Big\downarrow \text{639}$$

$$\int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^6}dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(\frac{(dg+ef)^2}{64d^6e^2(d-ex)^2}+\frac{(dg+ef)(dg+5ef)}{64d^6e^2(d+ex)^2}+\frac{f(dg+ef)}{8d^5e(d+ex)^3}+\frac{(3ef-dg)(dg+ef)}{16d^4e^2(d+ex)^4}+\frac{(dg-ef)^2}{4d^2e^2(d+ex)^6}+\frac{(dg+$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{arctanh}\left(\frac{ex}{d}\right)(dg+ef)(dg+3ef)}{32d^7e^3}+\frac{(dg+ef)^2}{64d^6e^3(d-ex)}-\frac{(dg+ef)(dg+5ef)}{64d^6e^3(d+ex)}-\frac{f(dg+ef)}{16d^5e^2(d+ex)^2}-\frac{(3ef-dg)(dg+ef)}{48d^4e^3(d+ex)^3}-\frac{(ef-dg)^2}{20d^2e^3(d+ex)^5}-\frac{e^2f^2-d^2g^2}{16d^3e^3(d+ex)^4}$$

input
```
Int[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)^2),x]
```

output
```
(e*f + d*g)^2/(64*d^6*e^3*(d - e*x)) - (e*f - d*g)^2/(20*d^2*e^3*(d + e*x)
^5) - (e^2*f^2 - d^2*g^2)/(16*d^3*e^3*(d + e*x)^4) - ((3*e*f - d*g)*(e*f +
 d*g))/(48*d^4*e^3*(d + e*x)^3) - (f*(e*f + d*g))/(16*d^5*e^2*(d + e*x)^2)
 - ((e*f + d*g)*(5*e*f + d*g))/(64*d^6*e^3*(d + e*x)) + ((e*f + d*g)*(3*e*
f + d*g)*ArcTanh[(e*x)/d])/(32*d^7*e^3)
```

3.568.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.568.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.32

method	result
default	$\frac{\left(-d^2g^2-4defg-3e^2f^2\right)\ln(-ex+d)}{64e^3d^7}+\frac{d^2g^2+2defg+e^2f^2}{64e^3d^6(-ex+d)}+\frac{\left(d^2g^2+4defg+3e^2f^2\right)\ln(ex+d)}{64e^3d^7}-\frac{d^2g^2+6defg+5e^2f^2}{64e^3d^6(ex+d)}-$
norman	$\frac{\frac{\left(d^2g^2+4defg+3e^2f^2\right)x^3}{6d^4}-\frac{\left(d^2g^2-4defg-13e^2f^2\right)x^2}{8e\,d^3}+\frac{e\left(7d^2g^2+4defg-27e^2f^2\right)x^4}{24d^5}+\frac{e^2\left(79d^2g^2-68defg-531e^2f^2\right)x^5}{480d^6}+\frac{e^3\left(d^2g^2-2d\right.}{3}}{(ex+d)^5(-ex+d)}$
risch	$\frac{\frac{e^2\left(d^2g^2+4defg+3e^2f^2\right)x^5}{32d^6}+\frac{\left(d^2g^2+4defg+3e^2f^2\right)e\,x^4}{8d^5}+\frac{\left(d^2g^2+4defg+3e^2f^2\right)x^3}{6d^4}+\frac{\left(d^2g^2+4defg+3e^2f^2\right)x^2}{24d^3e}+\frac{\left(49d^2g^2-188defg-14\right.}{480e^2d^2}}{(ex+d)^4(-e^2x^2+d^2)}$
parallelrisch	$-\frac{-300\ln(ex-d)x^2d^5e^3fg-64x^6d\,e^7fg-136x^5d^2e^6fg+160x^4d^3e^5fg+640x^3d^4e^4fg-180\ln(ex-d)x\,d^5e^3f^2+60\ln(ex+d)x\,d}{}$

input
```
int((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/64*(-d^2*g^2-4*d*e*f*g-3*e^2*f^2)/e^3/d^7*ln(-e*x+d)+1/64*(d^2*g^2+2*d*e
*f*g+e^2*f^2)/e^3/d^6/(-e*x+d)+1/64/e^3*(d^2*g^2+4*d*e*f*g+3*e^2*f^2)/d^7*
ln(e*x+d)-1/64/e^3*(d^2*g^2+6*d*e*f*g+5*e^2*f^2)/d^6/(e*x+d)-1/16*(-d^2*g^
2+e^2*f^2)/e^3/d^3/(e*x+d)^4-1/48*(-d^2*g^2+2*d*e*f*g+3*e^2*f^2)/e^3/d^4/(
e*x+d)^3-1/20*(d^2*g^2-2*d*e*f*g+e^2*f^2)/d^2/e^3/(e*x+d)^5-1/16*f*(d*g+e*
f)/d^5/e^2/(e*x+d)^2
```

3.568.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 693 vs. $2(197) = 394$.

Time = 0.34 (sec) , antiderivative size = 693, normalized size of antiderivative = 3.30

$$\int \frac{(f+gx)^2}{(d+ex)^4 (d^2-e^2x^2)^2} \, dx$$
$$= \frac{288\, d^6e^2f^2 + 64\, d^7efg - 32\, d^8g^2 - 30\, (3\, de^7f^2 + 4\, d^2e^6fg + d^3e^5g^2)x^5 - 120\, (3\, d^2e^6f^2 + 4\, d^3e^5fg + d^4e^4g}{}$$

input
```
integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

output
```
1/960*(288*d^6*e^2*f^2 + 64*d^7*e*f*g - 32*d^8*g^2 - 30*(3*d*e^7*f^2 + 4*d
^2*e^6*f*g + d^3*e^5*g^2)*x^5 - 120*(3*d^2*e^6*f^2 + 4*d^3*e^5*f*g + d^4*e
^4*g^2)*x^4 - 160*(3*d^3*e^5*f^2 + 4*d^4*e^4*f*g + d^5*e^3*g^2)*x^3 - 40*(
3*d^4*e^4*f^2 + 4*d^5*e^3*f*g + d^6*e^2*g^2)*x^2 + 2*(141*d^5*e^3*f^2 + 18
8*d^6*e^2*f*g - 49*d^7*e*g^2)*x - 15*(3*d^6*e^2*f^2 + 4*d^7*e*f*g + d^8*g^
2 - (3*e^8*f^2 + 4*d*e^7*f*g + d^2*e^6*g^2)*x^6 - 4*(3*d*e^7*f^2 + 4*d^2*e
^6*f*g + d^3*e^5*g^2)*x^5 - 5*(3*d^2*e^6*f^2 + 4*d^3*e^5*f*g + d^4*e^4*g^2
)*x^4 + 5*(3*d^4*e^4*f^2 + 4*d^5*e^3*f*g + d^6*e^2*g^2)*x^2 + 4*(3*d^5*e^3
*f^2 + 4*d^6*e^2*f*g + d^7*e*g^2)*x)*log(e*x + d) + 15*(3*d^6*e^2*f^2 + 4*
d^7*e*f*g + d^8*g^2 - (3*e^8*f^2 + 4*d*e^7*f*g + d^2*e^6*g^2)*x^6 - 4*(3*d
*e^7*f^2 + 4*d^2*e^6*f*g + d^3*e^5*g^2)*x^5 - 5*(3*d^2*e^6*f^2 + 4*d^3*e^5
*f*g + d^4*e^4*g^2)*x^4 + 5*(3*d^4*e^4*f^2 + 4*d^5*e^3*f*g + d^6*e^2*g^2)*
x^2 + 4*(3*d^5*e^3*f^2 + 4*d^6*e^2*f*g + d^7*e*g^2)*x)*log(e*x - d))/(d^7*
e^9*x^6 + 4*d^8*e^8*x^5 + 5*d^9*e^7*x^4 - 5*d^11*e^5*x^2 - 4*d^12*e^4*x -
d^13*e^3)
```

3.568.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(192) = 384$.

Time = 0.93 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.03

$$\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)^2}\,dx = \frac{-16d^7g^2+32d^6efg+144d^5e^2f^2+x^5(-15d^2e^5g^2-60de^6fg-45e^7f^2)+x^4(-60d^3e^4g^2-240d^2e^5fg-}{-480d^{12}e^3-1920d^{11}e^4x} - \frac{(dg+ef)(dg+3ef)\log\left(-\frac{d(dg+ef)(dg+3ef)}{e(d^2g^2+4defg+3e^2f^2)}+x\right)}{64d^7e^3} + \frac{(dg+ef)(dg+3ef)\log\left(\frac{d(dg+ef)(dg+3ef)}{e(d^2g^2+4defg+3e^2f^2)}+x\right)}{64d^7e^3}$$

input
```
integrate((g*x+f)**2/(e*x+d)**4/(-e**2*x**2+d**2)**2,x)
```

output
```
(-16*d**7*g**2 + 32*d**6*e*f*g + 144*d**5*e**2*f**2 + x**5*(-15*d**2*e**5*
g**2 - 60*d*e**6*f*g - 45*e**7*f**2) + x**4*(-60*d**3*e**4*g**2 - 240*d**2
*e**5*f*g - 180*d*e**6*f**2) + x**3*(-80*d**4*e**3*g**2 - 320*d**3*e**4*f*
g - 240*d**2*e**5*f**2) + x**2*(-20*d**5*e**2*g**2 - 80*d**4*e**3*f*g - 60
*d**3*e**4*f**2) + x*(-49*d**6*e*g**2 + 188*d**5*e**2*f*g + 141*d**4*e**3*
f**2))/(-480*d**12*e**3 - 1920*d**11*e**4*x - 2400*d**10*e**5*x**2 + 2400*
d**8*e**7*x**4 + 1920*d**7*e**8*x**5 + 480*d**6*e**9*x**6) - (d*g + e*f)*(
d*g + 3*e*f)*log(-d*(d*g + e*f)*(d*g + 3*e*f)/(e*(d**2*g**2 + 4*d*e*f*g +
3*e**2*f**2)) + x)/(64*d**7*e**3) + (d*g + e*f)*(d*g + 3*e*f)*log(d*(d*g +
 e*f)*(d*g + 3*e*f)/(e*(d**2*g**2 + 4*d*e*f*g + 3*e**2*f**2)) + x)/(64*d**
7*e**3)
```

3.568.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.63

$$\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)^2}\,dx = \frac{144\,d^5e^2f^2+32\,d^6efg-16\,d^7g^2-15\,(3\,e^7f^2+4\,de^6fg+d^2e^5g^2)x^5-60\,(3\,de^6f^2+4\,d^2e^5fg+d^3e^4g^2)x^4}{480\,(d^6e^9x^6+4\,d^7e^8x^5+} + \frac{(3\,e^2f^2+4\,defg+d^2g^2)\log(ex+d)}{64\,d^7e^3} - \frac{(3\,e^2f^2+4\,defg+d^2g^2)\log(ex-d)}{64\,d^7e^3}$$

input
```
integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

output
```
1/480*(144*d^5*e^2*f^2 + 32*d^6*e*f*g - 16*d^7*g^2 - 15*(3*e^7*f^2 + 4*d*e
^6*f*g + d^2*e^5*g^2)*x^5 - 60*(3*d*e^6*f^2 + 4*d^2*e^5*f*g + d^3*e^4*g^2)
*x^4 - 80*(3*d^2*e^5*f^2 + 4*d^3*e^4*f*g + d^4*e^3*g^2)*x^3 - 20*(3*d^3*e^
4*f^2 + 4*d^4*e^3*f*g + d^5*e^2*g^2)*x^2 + (141*d^4*e^3*f^2 + 188*d^5*e^2*
f*g - 49*d^6*e*g^2)*x)/(d^6*e^9*x^6 + 4*d^7*e^8*x^5 + 5*d^8*e^7*x^4 - 5*d^
10*e^5*x^2 - 4*d^11*e^4*x - d^12*e^3) + 1/64*(3*e^2*f^2 + 4*d*e*f*g + d^2*
g^2)*log(e*x + d)/(d^7*e^3) - 1/64*(3*e^2*f^2 + 4*d*e*f*g + d^2*g^2)*log(e
*x - d)/(d^7*e^3)
```

3.568.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.47

$$\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)^2}\,dx$$
$$= \frac{(3e^2f^2+4defg+d^2g^2)\log(|ex+d|)}{64\,d^7e^3} - \frac{(3e^2f^2+4defg+d^2g^2)\log(|ex-d|)}{64\,d^7e^3}$$
$$+ \frac{144\,d^6e^2f^2+32\,d^7efg-16\,d^8g^2-15\,(3\,de^7f^2+4\,d^2e^6fg+d^3e^5g^2)x^5-60\,(3\,d^2e^6f^2+4\,d^3e^5fg+d^4e^4 \cdots}{48\cdots}$$

input
```
integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

output
```
1/64*(3*e^2*f^2 + 4*d*e*f*g + d^2*g^2)*log(abs(e*x + d))/(d^7*e^3) - 1/64*
(3*e^2*f^2 + 4*d*e*f*g + d^2*g^2)*log(abs(e*x - d))/(d^7*e^3) + 1/480*(144
*d^6*e^2*f^2 + 32*d^7*e*f*g - 16*d^8*g^2 - 15*(3*d*e^7*f^2 + 4*d^2*e^6*f*g
 + d^3*e^5*g^2)*x^5 - 60*(3*d^2*e^6*f^2 + 4*d^3*e^5*f*g + d^4*e^4*g^2)*x^4
 - 80*(3*d^3*e^5*f^2 + 4*d^4*e^4*f*g + d^5*e^3*g^2)*x^3 - 20*(3*d^4*e^4*f^
2 + 4*d^5*e^3*f*g + d^6*e^2*g^2)*x^2 + (141*d^5*e^3*f^2 + 188*d^6*e^2*f*g
- 49*d^7*e*g^2)*x)/((e*x + d)^5*(e*x - d)*d^7*e^3)
```

3.568.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.50

$$\int \frac{(f+gx)^2}{(d+ex)^4\left(d^2-e^2x^2\right)^2}\,dx$$
$$=\frac{\frac{x^3\,\left(d^2\,g^2+4\,d\,e\,f\,g+3\,e^2\,f^2\right)}{6\,d^4}-\frac{-d^2\,g^2+2\,d\,e\,f\,g+9\,e^2\,f^2}{30\,d\,e^3}+\frac{e\,x^4\,\left(d^2\,g^2+4\,d\,e\,f\,g+3\,e^2\,f^2\right)}{8\,d^5}-\frac{x\,\left(-49\,d^2\,g^2+188\,d\,e\,f\,g+141\,e^2\,f^2\right)}{480\,d^2\,e^2}+\frac{x^2\,\left(d^2\right.}{}}{d^6+4\,d^5\,e\,x+5\,d^4\,e^2\,x^2-5\,d^2\,e^4\,x^4-4\,d\,e^5\,x^5-e^6\,x^6}$$
$$+\frac{\mathrm{atanh}\left(\frac{e\,x\,(d\,g+e\,f)\,(d\,g+3\,e\,f)}{d\,\left(d^2\,g^2+4\,d\,e\,f\,g+3\,e^2\,f^2\right)}\right)\,(d\,g+e\,f)\,(d\,g+3\,e\,f)}{32\,d^7\,e^3}$$

input
```
int((f + g*x)^2/((d^2 - e^2*x^2)^2*(d + e*x)^4),x)
```

output
```
((x^3*(d^2*g^2 + 3*e^2*f^2 + 4*d*e*f*g))/(6*d^4) - (9*e^2*f^2 - d^2*g^2 +
2*d*e*f*g)/(30*d*e^3) + (e*x^4*(d^2*g^2 + 3*e^2*f^2 + 4*d*e*f*g))/(8*d^5)
- (x*(141*e^2*f^2 - 49*d^2*g^2 + 188*d*e*f*g))/(480*d^2*e^2) + (x^2*(d^2*g
^2 + 3*e^2*f^2 + 4*d*e*f*g))/(24*d^3*e) + (e^2*x^5*(d^2*g^2 + 3*e^2*f^2 +
4*d*e*f*g))/(32*d^6))/(d^6 - e^6*x^6 - 4*d*e^5*x^5 + 5*d^4*e^2*x^2 - 5*d^2
*e^4*x^4 + 4*d^5*e*x) + (atanh((e*x*(d*g + e*f)*(d*g + 3*e*f))/(d*(d^2*g^2
 + 3*e^2*f^2 + 4*d*e*f*g)))*(d*g + e*f)*(d*g + 3*e*f))/(32*d^7*e^3)
```

3.569 $\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^3} dx$

3.569.1 Optimal result

Integrand size = 29, antiderivative size = 179

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^3} dx = -\frac{d(7e^2f^2+48defg+56d^2g^2)\,x}{e^2} - \frac{(ef+2dg)(ef+12dg)x^2}{2e} - \frac{1}{3}g(2ef+7dg)x^3 - \frac{1}{4}eg^2x^4 + \frac{8d^4(ef+dg)^2}{e^3(d-ex)^2} - \frac{32d^3(ef+dg)(ef+2dg)}{e^3(d-ex)} - \frac{8d^2(3e^2f^2+14defg+13d^2g^2)\log(d-ex)}{e^3}$$

output
```
-d*(56*d^2*g^2+48*d*e*f*g+7*e^2*f^2)*x/e^2-1/2*(2*d*g+e*f)*(12*d*g+e*f)*x^
2/e-1/3*g*(7*d*g+2*e*f)*x^3-1/4*e*g^2*x^4+8*d^4*(d*g+e*f)^2/e^3/(-e*x+d)^2
-32*d^3*(d*g+e*f)*(2*d*g+e*f)/e^3/(-e*x+d)-8*d^2*(13*d^2*g^2+14*d*e*f*g+3*
e^2*f^2)*ln(-e*x+d)/e^3
```

3.569.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{d(7e^2f^2+48defg+56d^2g^2)\,x}{e^2} - \frac{(e^2f^2+14defg+24d^2g^2)\,x^2}{2e} - \frac{1}{3}g(2ef+7dg)x^3 - \frac{1}{4}eg^2x^4 + \frac{8d^4(ef+dg)^2}{e^3(d-ex)^2} + \frac{32d^3(e^2f^2+3defg+2d^2g^2)}{e^3(-d+ex)} - \frac{8d^2(3e^2f^2+14defg+13d^2g^2)\log(d-ex)}{e^3}$$

input
```
Integrate[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

output
```
-((d*(7*e^2*f^2 + 48*d*e*f*g + 56*d^2*g^2)*x)/e^2) - ((e^2*f^2 + 14*d*e*f*
g + 24*d^2*g^2)*x^2)/(2*e) - (g*(2*e*f + 7*d*g)*x^3)/3 - (e*g^2*x^4)/4 + (
8*d^4*(e*f + d*g)^2)/(e^3*(d - e*x)^2) + (32*d^3*(e^2*f^2 + 3*d*e*f*g + 2*
d^2*g^2))/(e^3*(-d + e*x)) - (8*d^2*(3*e^2*f^2 + 14*d*e*f*g + 13*d^2*g^2)*
Log[d - e*x])/e^3
```

3.569.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {639, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$$

$$\Big\downarrow \text{639}$$

$$\int \frac{(d+ex)^4(f+gx)^2}{(d-ex)^3}dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(-\frac{16d^4(dg+ef)^2}{e^2(ex-d)^3} + \frac{32d^3(-2dg-ef)(dg+ef)}{e^2(d-ex)^2} - \frac{8d^2\left(13d^2g^2+14defg+3e^2f^2\right)}{e^2(ex-d)} - \frac{d\left(56d^2g^2+48defg+7\right.}{e^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{8d^4(dg+ef)^2}{e^3(d-ex)^2}-\frac{32d^3(dg+ef)(2dg+ef)}{e^3(d-ex)}-\frac{dx(56d^2g^2+48defg+7e^2f^2)}{e^2}-\frac{8d^2(13d^2g^2+14defg+3e^2f^2)\log(d-ex)}{e^3}-\frac{1}{3}gx^3(7dg+2ef)-\frac{x^2(2dg+ef)(12dg+ef)}{2e}-\frac{1}{4}eg^2x^4$$

input
```
Int[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

output
```
-((d*(7*e^2*f^2 + 48*d*e*f*g + 56*d^2*g^2)*x)/e^2) - ((e*f + 2*d*g)*(e*f +
 12*d*g)*x^2)/(2*e) - (g*(2*e*f + 7*d*g)*x^3)/3 - (e*g^2*x^4)/4 + (8*d^4*(
e*f + d*g)^2)/(e^3*(d - e*x)^2) - (32*d^3*(e*f + d*g)*(e*f + 2*d*g))/(e^3*
(d - e*x)) - (8*d^2*(3*e^2*f^2 + 14*d*e*f*g + 13*d^2*g^2)*Log[d - e*x])/e^
3
```

3.569.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.569. $\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$

3.569.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.21

method	result
default	$-\frac{\frac{1}{4}g^2e^3x^4+\frac{7}{3}x^3d\,e^2g^2+\frac{2}{3}x^3e^3fg+12x^2d^2e\,g^2+7x^2d\,e^2fg+\frac{1}{2}x^2e^3f^2+56d^3g^2x+48d^2efgx+7d\,e^2f^2x}{e^2}-\frac{8d^2\left(13d^2g^2+14de\right.}{}$
risch	$-\frac{e\,g^2x^4}{4}-\frac{7x^3d\,g^2}{3}-\frac{2e\,x^3fg}{3}-\frac{12x^2g^2d^2}{e}-7x^2fgd-\frac{e\,x^2f^2}{2}-\frac{56d^3g^2x}{e^2}-\frac{48d^2fgx}{e}-7d\,f^2x+\frac{(64g^2d^5}{}$
norman	$\frac{\left(\frac{521}{3}g^2d^5+\frac{574}{3}fg\,d^4e+46f^2d^3e^2\right)x^3+\left(-\frac{23}{2}g^2d^2e^3-7fgd\,e^4-\frac{1}{2}f^2e^5\right)x^6+\left(-\frac{154}{3}g^2d^3e^2-\frac{140}{3}fg\,d^2e^3-7f^2d\,e^4\right)x^5-\frac{d^4(319g^2d}{}}{(-e^2x^2}$
parallelrisch	$-\frac{1344\ln(ex-d)d^5efg+1248\ln(ex-d)x^2d^4e^2g^2+288\ln(ex-d)x^2d^2e^4f^2-2496\ln(ex-d)x\,d^5e\,g^2-576\ln(ex-d)x\,d^3e^3f^2+6e}{}$

input
```
int((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/e^2*(1/4*g^2*e^3*x^4+7/3*x^3*d*e^2*g^2+2/3*x^3*e^3*f*g+12*x^2*d^2*e*g^2
+7*x^2*d*e^2*f*g+1/2*x^2*e^3*f^2+56*d^3*g^2*x+48*d^2*e*f*g*x+7*d*e^2*f^2*x
)-8*d^2*(13*d^2*g^2+14*d*e*f*g+3*e^2*f^2)*ln(-e*x+d)/e^3-32*d^3/e^3*(2*d^2
*g^2+3*d*e*f*g+e^2*f^2)/(-e*x+d)+8*d^4*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/(-e
*x+d)^2
```

3.569.5 Fricas [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.88

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^3}\,dx =$$
$$-\frac{3\,e^6g^2x^6+288\,d^4e^2f^2+960\,d^5efg+672\,d^6g^2+2\,(4\,e^6fg+11\,de^5g^2)x^5+(6\,e^6f^2+68\,de^5fg+91\,d^2e^4}{}$$

input
```
integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

output
```
-1/12*(3*e^6*g^2*x^6 + 288*d^4*e^2*f^2 + 960*d^5*e*f*g + 672*d^6*g^2 + 2*(
4*e^6*f*g + 11*d*e^5*g^2)*x^5 + (6*e^6*f^2 + 68*d*e^5*f*g + 91*d^2*e^4*g^2
)*x^4 + 4*(18*d*e^5*f^2 + 104*d^2*e^4*f*g + 103*d^3*e^3*g^2)*x^3 - 6*(27*d
^2*e^4*f^2 + 178*d^3*e^3*f*g + 200*d^4*e^2*g^2)*x^2 - 12*(25*d^3*e^3*f^2 +
 48*d^4*e^2*f*g + 8*d^5*e*g^2)*x + 96*(3*d^4*e^2*f^2 + 14*d^5*e*f*g + 13*d
^6*g^2 + (3*d^2*e^4*f^2 + 14*d^3*e^3*f*g + 13*d^4*e^2*g^2)*x^2 - 2*(3*d^3*
e^3*f^2 + 14*d^4*e^2*f*g + 13*d^5*e*g^2)*x)*log(e*x - d))/(e^5*x^2 - 2*d*e
^4*x + d^2*e^3)
```

3.569.6 Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$$
$$= -\frac{8d^2\cdot(13d^2g^2+14defg+3e^2f^2)\log(-d+ex)}{e^3} - \frac{eg^2x^4}{4} - x^3\cdot\left(\frac{7dg^2}{3}+\frac{2efg}{3}\right) - x^2\cdot\left(\frac{12d^2g^2}{e}+7dfg+\frac{ef^2}{2}\right) - x\left(\frac{56d^3g^2}{e^2}+\frac{48d^2fg}{e}+7df^2\right) - \frac{56d^6g^2+80d^5efg+24d^4e^2f^2+x(-64d^5eg^2-96d^4e^2fg-32d^3e^3f^2)}{d^2e^3-2de^4x+e^5x^2}$$

input
```
integrate((e*x+d)**7*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

output
```
-8*d**2*(13*d**2*g**2 + 14*d*e*f*g + 3*e**2*f**2)*log(-d + e*x)/e**3 - e*g
**2*x**4/4 - x**3*(7*d*g**2/3 + 2*e*f*g/3) - x**2*(12*d**2*g**2/e + 7*d*f*
g + e*f**2/2) - x*(56*d**3*g**2/e**2 + 48*d**2*f*g/e + 7*d*f**2) - (56*d**
6*g**2 + 80*d**5*e*f*g + 24*d**4*e**2*f**2 + x*(-64*d**5*e*g**2 - 96*d**4*
e**2*f*g - 32*d**3*e**3*f**2))/(d**2*e**3 - 2*d*e**4*x + e**5*x**2)
```

3.569.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$$
$$= -\frac{8\,(3\,d^4e^2f^2+10\,d^5efg+7\,d^6g^2-4\,(d^3e^3f^2+3\,d^4e^2fg+2\,d^5eg^2)x)}{e^5x^2-2\,de^4x+d^2e^3}$$
$$-\frac{3\,e^3g^2x^4+4\,(2\,e^3fg+7\,de^2g^2)x^3+6\,(e^3f^2+14\,de^2fg+24\,d^2eg^2)x^2+12\,(7\,de^2f^2+48\,d^2efg+56\,d^3g}{12\,e^2}$$
$$-\frac{8\,(3\,d^2e^2f^2+14\,d^3efg+13\,d^4g^2)\log{(ex-d)}}{e^3}$$

input
```
integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

output
```
-8*(3*d^4*e^2*f^2 + 10*d^5*e*f*g + 7*d^6*g^2 - 4*(d^3*e^3*f^2 + 3*d^4*e^2*
f*g + 2*d^5*e*g^2)*x)/(e^5*x^2 - 2*d*e^4*x + d^2*e^3) - 1/12*(3*e^3*g^2*x^
4 + 4*(2*e^3*f*g + 7*d*e^2*g^2)*x^3 + 6*(e^3*f^2 + 14*d*e^2*f*g + 24*d^2*e
*g^2)*x^2 + 12*(7*d*e^2*f^2 + 48*d^2*e*f*g + 56*d^3*g^2)*x)/e^2 - 8*(3*d^2
*e^2*f^2 + 14*d^3*e*f*g + 13*d^4*g^2)*log(e*x - d)/e^3
```

3.569.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{8\,(3\,d^2e^2f^2+14\,d^3efg+13\,d^4g^2)\log{(|ex-d|)}}{e^3}$$
$$-\frac{8\,(3\,d^4e^2f^2+10\,d^5efg+7\,d^6g^2-4\,(d^3e^3f^2+3\,d^4e^2fg+2\,d^5eg^2)x)}{(ex-d)^2e^3}$$
$$-\frac{3\,e^{13}g^2x^4+8\,e^{13}fgx^3+28\,de^{12}g^2x^3+6\,e^{13}f^2x^2+84\,de^{12}fgx^2+144\,d^2e^{11}g^2x^2+84\,de^{12}f^2x+576\,d^2e}{12\,e^{12}}$$

input
```
integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

output
```
-8*(3*d^2*e^2*f^2 + 14*d^3*e*f*g + 13*d^4*g^2)*log(abs(e*x - d))/e^3 - 8*(
3*d^4*e^2*f^2 + 10*d^5*e*f*g + 7*d^6*g^2 - 4*(d^3*e^3*f^2 + 3*d^4*e^2*f*g
+ 2*d^5*e*g^2)*x)/((e*x - d)^2*e^3) - 1/12*(3*e^13*g^2*x^4 + 8*e^13*f*g*x^
3 + 28*d*e^12*g^2*x^3 + 6*e^13*f^2*x^2 + 84*d*e^12*f*g*x^2 + 144*d^2*e^11*
g^2*x^2 + 84*d*e^12*f^2*x + 576*d^2*e^11*f*g*x + 672*d^3*e^10*g^2*x)/e^12
```

3.569.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.09

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{x\,(64\,d^5\,g^2+96\,d^4\,e\,f\,g+32\,d^3\,e^2\,f^2)-\frac{8\,(7\,d^6\,g^2+10\,d^5\,e\,f\,g+3\,d^4\,e^2\,f^2)}{e}}{d^2\,e^2-2\,d\,e^3\,x+e^4\,x^2} - x^2\left(\frac{6\,d^2\,e^2\,g^2+8\,d\,e^3\,f\,g+e^4\,f^2}{2\,e^3}-\frac{3\,d^2\,g^2}{2\,e}+\frac{3\,d\,(2\,g\,(2\,d\,g+e\,f)+3\,d\,g^2)}{2\,e}\right) - x\left(\frac{d^3\,g^2}{e^2}-\frac{3\,d^2\,(2\,g\,(2\,d\,g+e\,f)+3\,d\,g^2)}{e^2}+\frac{4\,d\,(d^2\,g^2+3\,d\,e\,f\,g+e^2\,f^2)}{e^2}+\frac{3\,d\left(\frac{6\,d^2\,e^2\,g^2+8\,d\,e^3\,f\,g+e^4\,f^2}{e^3}-\frac{3\,d^2\,g^2}{e}+\frac{3\,d\,(2\,g\,(2\,d\,g+e\,f)+3\,d\,g^2)}{e}\right)}{e}\right) - x^3\left(\frac{2\,g\,(2\,d\,g+e\,f)}{3}+d\,g^2\right) - \frac{\ln(e\,x-d)\,(104\,d^4\,g^2+112\,d^3\,e\,f\,g+24\,d^2\,e^2\,f^2)}{e^3} - \frac{e\,g^2\,x^4}{4}$$

input
```
int(((f + g*x)^2*(d + e*x)^7)/(d^2 - e^2*x^2)^3,x)
```

output
```
(x*(64*d^5*g^2 + 32*d^3*e^2*f^2 + 96*d^4*e*f*g) - (8*(7*d^6*g^2 + 3*d^4*e^
2*f^2 + 10*d^5*e*f*g))/e)/(d^2*e^2 + e^4*x^2 - 2*d*e^3*x) - x^2*((e^4*f^2
+ 6*d^2*e^2*g^2 + 8*d*e^3*f*g)/(2*e^3) - (3*d^2*g^2)/(2*e) + (3*d*(2*g*(2*
d*g + e*f) + 3*d*g^2))/(2*e)) - x*((d^3*g^2)/e^2 - (3*d^2*(2*g*(2*d*g + e*
f) + 3*d*g^2))/e^2 + (4*d*(d^2*g^2 + e^2*f^2 + 3*d*e*f*g))/e^2 + (3*d*((e^
4*f^2 + 6*d^2*e^2*g^2 + 8*d*e^3*f*g)/e^3 - (3*d^2*g^2)/e + (3*d*(2*g*(2*d*
g + e*f) + 3*d*g^2))/e))/e) - x^3*((2*g*(2*d*g + e*f))/3 + d*g^2) - (log(e
*x - d)*(104*d^4*g^2 + 24*d^2*e^2*f^2 + 112*d^3*e*f*g))/e^3 - (e*g^2*x^4)/
4
```

3.570 $\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$

3.570.1 Optimal result

Integrand size = 29, antiderivative size = 149

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{(e^2f^2+12defg+18d^2g^2)\,x}{e^2} - \frac{g(ef+3dg)x^2}{e} - \frac{g^2x^3}{3} + \frac{4d^3(ef+dg)^2}{e^3(d-ex)^2} - \frac{4d^2(ef+dg)(3ef+7dg)}{e^3(d-ex)} - \frac{2d(3e^2f^2+18defg+19d^2g^2)\log(d-ex)}{e^3}$$

output
```
-(18*d^2*g^2+12*d*e*f*g+e^2*f^2)*x/e^2-g*(3*d*g+e*f)*x^2/e-1/3*g^2*x^3+4*d
^3*(d*g+e*f)^2/e^3/(-e*x+d)^2-4*d^2*(d*g+e*f)*(7*d*g+3*e*f)/e^3/(-e*x+d)-2
*d*(19*d^2*g^2+18*d*e*f*g+3*e^2*f^2)*ln(-e*x+d)/e^3
```

3.570.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{(e^2f^2+12defg+18d^2g^2)\,x}{e^2} - \frac{g(ef+3dg)x^2}{e} - \frac{g^2x^3}{3} + \frac{4d^3(ef+dg)^2}{e^3(d-ex)^2} + \frac{4d^2(3e^2f^2+10defg+7d^2g^2)}{e^3(-d+ex)} - \frac{2d(3e^2f^2+18defg+19d^2g^2)\log(d-ex)}{e^3}$$

input
```
Integrate[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

output
```
-(((e^2*f^2 + 12*d*e*f*g + 18*d^2*g^2)*x)/e^2) - (g*(e*f + 3*d*g)*x^2)/e -
 (g^2*x^3)/3 + (4*d^3*(e*f + d*g)^2)/(e^3*(d - e*x)^2) + (4*d^2*(3*e^2*f^2
 + 10*d*e*f*g + 7*d^2*g^2))/(e^3*(-d + e*x)) - (2*d*(3*e^2*f^2 + 18*d*e*f*
g + 19*d^2*g^2)*Log[d - e*x])/e^3
```

3.570.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {639, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^3} \, dx$$

$$\Big\downarrow \text{639}$$

$$\int \frac{(d+ex)^3(f+gx)^2}{(d-ex)^3}dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(-\frac{8d^3(dg+ef)^2}{e^2(ex-d)^3}-\frac{2d\left(19d^2g^2+18defg+3e^2f^2\right)}{e^2(ex-d)}+\frac{-18d^2g^2-12defg-e^2f^2}{e^2}+\frac{4d^2(-7dg-3ef)(dg+ef)}{e^2(d-ex)^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{4d^3(dg+ef)^2}{e^3(d-ex)^2}-\frac{4d^2(dg+ef)(7dg+3ef)}{e^3(d-ex)}-\frac{x\left(18d^2g^2+12defg+e^2f^2\right)}{e^2}-\frac{2d\left(19d^2g^2+18defg+3e^2f^2\right)\log(d-ex)}{e^3}-\frac{gx^2(3dg+ef)}{e}-\frac{g^2x^3}{3}$$

input
```
Int[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

output
```
-(((e^2*f^2 + 12*d*e*f*g + 18*d^2*g^2)*x)/e^2) - (g*(e*f + 3*d*g)*x^2)/e -
 (g^2*x^3)/3 + (4*d^3*(e*f + d*g)^2)/(e^3*(d - e*x)^2) - (4*d^2*(e*f + d*g
)*(3*e*f + 7*d*g))/(e^3*(d - e*x)) - (2*d*(3*e^2*f^2 + 18*d*e*f*g + 19*d^2
*g^2)*Log[d - e*x])/e^3
```

3.570.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.570.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.17

method	result
default	$-\frac{\frac{1}{3} g^{2} x^{3} e^{2}+3 d e g^{2} x^{2}+e^{2} f g x^{2}+18 d^{2} g^{2} x+12 d e f g x+e^{2} f^{2} x}{e^{2}}-\frac{2 d \left(19 d^{2} g^{2}+18 d e f g+3 e^{2} f^{2}\right) \ln \left(-e x+d\right)}{e^{3}}-\frac{4 d^{2} \left(7 d^{2} g^{2}+10 d \ldots\right.}{e^{3} \left(-e x \ldots\right.}$
risch	$-\frac{g^{2} x^{3}}{3}-\frac{3 d g^{2} x^{2}}{e}-f g x^{2}-\frac{18 d^{2} g^{2} x}{e^{2}}-\frac{12 d f g x}{e}-f^{2} x-\frac{\left(-28 d^{4} g^{2}-40 f g e d^{3}-12 d^{2} e^{2} f^{2}\right) x+\frac{8 d^{3} \left(3 d^{2} g^{2}+4 d e f g+e \ldots\right.}{e}}{e^{2} \left(-e x+d\right)^{2}}$
norman	$\frac{\left(\frac{191}{3} d^{4} g^{2}+64 f g e d^{3}+14 d^{2} e^{2} f^{2}\right) x^{3}+\left(-\frac{52}{3} d^{2} g^{2} e^{2}-12 d f g e^{3}-f^{2} e^{4}\right) x^{5}+\frac{d^{2} \left(41 g^{2} e d^{3}+51 e^{2} f g d^{2}+16 e^{3} f^{2} d\right) x^{2}}{e^{2}}-\frac{d^{4} \left(30 g^{2} e d^{3}+34 e^{2} \ldots\right.}{e^{4}}}{\left(-e^{2} x^{2}+d^{2}\right)^{2}}$
parallelrisch	$-\frac{g^{2} e^{5} x^{5}+7 x^{4} d e^{4} g^{2}+3 x^{4} e^{5} f g+114 \ln \left(e x-d\right) x^{2} d^{3} e^{2} g^{2}+108 \ln \left(e x-d\right) x^{2} d^{2} e^{3} f g+18 \ln \left(e x-d\right) x^{2} d e^{4} f^{2}+37 x^{3} d^{2} e^{3} g^{2}+30 x^{3} d \ldots}{\ldots}$

input
```
int((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/e^2*(1/3*g^2*x^3*e^2+3*d*e*g^2*x^2+e^2*f*g*x^2+18*d^2*g^2*x+12*d*e*f*g*
x+e^2*f^2*x)-2*d*(19*d^2*g^2+18*d*e*f*g+3*e^2*f^2)*ln(-e*x+d)/e^3-4*d^2/e^
3*(7*d^2*g^2+10*d*e*f*g+3*e^2*f^2)/(-e*x+d)+4*d^3*(d^2*g^2+2*d*e*f*g+e^2*f
^2)/e^3/(-e*x+d)^2
```

3.570.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.97

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{e^5g^2x^5+24\,d^3e^2f^2+96\,d^4efg+72\,d^5g^2+(3\,e^5fg+7\,de^4g^2)x^4+(3\,e^5f^2+30\,de^4fg+37\,d^2e^3g^2)x^3-$$

input
```
integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

output
```
-1/3*(e^5*g^2*x^5 + 24*d^3*e^2*f^2 + 96*d^4*e*f*g + 72*d^5*g^2 + (3*e^5*f*
g + 7*d*e^4*g^2)*x^4 + (3*e^5*f^2 + 30*d*e^4*f*g + 37*d^2*e^3*g^2)*x^3 - 3
*(2*d*e^4*f^2 + 23*d^2*e^3*f*g + 33*d^3*e^2*g^2)*x^2 - 3*(11*d^2*e^3*f^2 +
 28*d^3*e^2*f*g + 10*d^4*e*g^2)*x + 6*(3*d^3*e^2*f^2 + 18*d^4*e*f*g + 19*d
^5*g^2 + (3*d*e^4*f^2 + 18*d^2*e^3*f*g + 19*d^3*e^2*g^2)*x^2 - 2*(3*d^2*e^
3*f^2 + 18*d^3*e^2*f*g + 19*d^4*e*g^2)*x)*log(e*x - d))/(e^5*x^2 - 2*d*e^4
*x + d^2*e^3)
```

3.570.6 Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^3}\,dx &= -\frac{2d(19d^2g^2+18defg+3e^2f^2)\log(-d+ex)}{e^3}-\frac{g^2x^3}{3}\\ &\quad -x^2\cdot\left(\frac{3dg^2}{e}+fg\right)-x\left(\frac{18d^2g^2}{e^2}+\frac{12dfg}{e}+f^2\right)\\ &\quad -\frac{24d^5g^2+32d^4efg+8d^3e^2f^2+x(-28d^4eg^2-40d^3e^2fg-12d^2e^3f^2)}{d^2e^3-2de^4x+e^5x^2}\end{aligned}$$

input
```
integrate((e*x+d)**6*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

output
```
-2*d*(19*d**2*g**2 + 18*d*e*f*g + 3*e**2*f**2)*log(-d + e*x)/e**3 - g**2*x
**3/3 - x**2*(3*d*g**2/e + f*g) - x*(18*d**2*g**2/e**2 + 12*d*f*g/e + f**2
) - (24*d**5*g**2 + 32*d**4*e*f*g + 8*d**3*e**2*f**2 + x*(-28*d**4*e*g**2
- 40*d**3*e**2*f*g - 12*d**2*e**3*f**2))/(d**2*e**3 - 2*d*e**4*x + e**5*x*
*2)
```

3.570.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{4\,(2\,d^3e^2f^2+8\,d^4efg+6\,d^5g^2-(3\,d^2e^3f^2+10\,d^3e^2fg+7\,d^4eg^2)x)}{e^5x^2-2\,de^4x+d^2e^3} - \frac{e^2g^2x^3+3\,(e^2fg+3\,deg^2)x^2+3\,(e^2f^2+12\,defg+18\,d^2g^2)x}{3\,e^2} - \frac{2\,(3\,de^2f^2+18\,d^2efg+19\,d^3g^2)\log(ex-d)}{e^3}$$

input
```
integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

output
```
-4*(2*d^3*e^2*f^2 + 8*d^4*e*f*g + 6*d^5*g^2 - (3*d^2*e^3*f^2 + 10*d^3*e^2*
f*g + 7*d^4*e*g^2)*x)/(e^5*x^2 - 2*d*e^4*x + d^2*e^3) - 1/3*(e^2*g^2*x^3 +
 3*(e^2*f*g + 3*d*e*g^2)*x^2 + 3*(e^2*f^2 + 12*d*e*f*g + 18*d^2*g^2)*x)/e^
2 - 2*(3*d*e^2*f^2 + 18*d^2*e*f*g + 19*d^3*g^2)*log(e*x - d)/e^3
```

3.570.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{2\,(3\,de^2f^2+18\,d^2efg+19\,d^3g^2)\log(|ex-d|)}{e^3} - \frac{4\,(2\,d^3e^2f^2+8\,d^4efg+6\,d^5g^2-(3\,d^2e^3f^2+10\,d^3e^2fg+7\,d^4eg^2)x)}{(ex-d)^2e^3} - \frac{e^9g^2x^3+3\,e^9fgx^2+9\,de^8g^2x^2+3\,e^9f^2x+36\,de^8fgx+54\,d^2e^7g^2x}{3\,e^9}$$

input
```
integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

output
```
-2*(3*d*e^2*f^2 + 18*d^2*e*f*g + 19*d^3*g^2)*log(abs(e*x - d))/e^3 - 4*(2*
d^3*e^2*f^2 + 8*d^4*e*f*g + 6*d^5*g^2 - (3*d^2*e^3*f^2 + 10*d^3*e^2*f*g +
7*d^4*e*g^2)*x)/((e*x - d)^2*e^3) - 1/3*(e^9*g^2*x^3 + 3*e^9*f*g*x^2 + 9*d
*e^8*g^2*x^2 + 3*e^9*f^2*x + 36*d*e^8*f*g*x + 54*d^2*e^7*g^2*x)/e^9
```

3.570.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.61

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{x\,(28\,d^4\,g^2+40\,d^3\,e\,f\,g+12\,d^2\,e^2\,f^2)-\frac{8\,(3\,d^5\,g^2+4\,d^4\,e\,f\,g+d^3\,e^2\,f^2)}{e}}{d^2\,e^2-2\,d\,e^3\,x+e^4\,x^2} - x\left(\frac{3\,d^2\,e\,g^2+6\,d\,e^2\,f\,g+e^3\,f^2}{e^3}+\frac{3\,d\left(\frac{g\,(3\,d\,g+2\,e\,f)}{e}+\frac{3\,d\,g^2}{e}\right)}{e}-\frac{3\,d^2\,g^2}{e^2}\right) - x^2\left(\frac{g\,(3\,d\,g+2\,e\,f)}{2\,e}+\frac{3\,d\,g^2}{2\,e}\right) - \frac{g^2\,x^3}{3} - \frac{\ln(e\,x-d)\,(38\,d^3\,g^2+36\,d^2\,e\,f\,g+6\,d\,e^2\,f^2)}{e^3}$$

input
```
int(((f + g*x)^2*(d + e*x)^6)/(d^2 - e^2*x^2)^3,x)
```

output
```
(x*(28*d^4*g^2 + 12*d^2*e^2*f^2 + 40*d^3*e*f*g) - (8*(3*d^5*g^2 + d^3*e^2*
f^2 + 4*d^4*e*f*g))/e)/(d^2*e^2 + e^4*x^2 - 2*d*e^3*x) - x*((e^3*f^2 + 3*d
^2*e*g^2 + 6*d*e^2*f*g)/e^3 + (3*d*((g*(3*d*g + 2*e*f))/e + (3*d*g^2)/e))/
e - (3*d^2*g^2)/e^2) - x^2*((g*(3*d*g + 2*e*f))/(2*e) + (3*d*g^2)/(2*e)) -
 (g^2*x^3)/3 - (log(e*x - d)*(38*d^3*g^2 + 6*d*e^2*f^2 + 36*d^2*e*f*g))/e^
3
```

3.571 $\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$

3.571.1 Optimal result

Integrand size = 29, antiderivative size = 118

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{g(2ef+5dg)x}{e^2}-\frac{g^2x^2}{2e}+\frac{2d^2(ef+dg)^2}{e^3(d-ex)^2} - \frac{4d(ef+dg)(ef+3dg)}{e^3(d-ex)} - \frac{(e^2f^2+10defg+13d^2g^2)\log(d-ex)}{e^3}$$

output
```
-g*(5*d*g+2*e*f)*x/e^2-1/2*g^2*x^2/e+2*d^2*(d*g+e*f)^2/e^3/(-e*x+d)^2-4*d*
(d*g+e*f)*(3*d*g+e*f)/e^3/(-e*x+d)-(13*d^2*g^2+10*d*e*f*g+e^2*f^2)*ln(-e*x
+d)/e^3
```

3.571.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{2eg(2ef+5dg)x+e^2g^2x^2-\frac{4d^2(ef+dg)^2}{(d-ex)^2}+\frac{8d(e^2f^2+4defg+3d^2g^2)}{d-ex}+2(e^2f^2+10defg+13d^2g^2)\log(d-ex)}{2e^3}$$

input
```
Integrate[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

output
```
-1/2*(2*e*g*(2*e*f + 5*d*g)*x + e^2*g^2*x^2 - (4*d^2*(e*f + d*g)^2)/(d - e
*x)^2 + (8*d*(e^2*f^2 + 4*d*e*f*g + 3*d^2*g^2))/(d - e*x) + 2*(e^2*f^2 + 1
0*d*e*f*g + 13*d^2*g^2)*Log[d - e*x])/e^3
```

3.571.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {639, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$$

$$\Big\downarrow \text{639}$$

$$\int \frac{(d+ex)^2(f+gx)^2}{(d-ex)^3}dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(\frac{-13d^2g^2-10defg-e^2f^2}{e^2(ex-d)}-\frac{4d^2(dg+ef)^2}{e^2(ex-d)^3}+\frac{4d(-3dg-ef)(dg+ef)}{e^2(d-ex)^2}-\frac{g(5dg+2ef)}{e^2}-\frac{g^2x}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2d^2(dg+ef)^2}{e^3(d-ex)^2}-\frac{\left(13d^2g^2+10defg+e^2f^2\right)\log(d-ex)}{e^3}-\frac{4d(3dg+ef)(dg+ef)}{e^3(d-ex)}-\frac{gx(5dg+2ef)}{e^2}-\frac{g^2x^2}{2e}$$

input
```
Int[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

output
```
-((g*(2*e*f + 5*d*g)*x)/e^2) - (g^2*x^2)/(2*e) + (2*d^2*(e*f + d*g)^2)/(e^
3*(d - e*x)^2) - (4*d*(e*f + d*g)*(e*f + 3*d*g))/(e^3*(d - e*x)) - ((e^2*f
^2 + 10*d*e*f*g + 13*d^2*g^2)*Log[d - e*x])/e^3
```

3.571.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.571.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.13

method	result
default	$-\frac{g\left(\frac{1}{2}eg\,x^{2}+5dgx+2efx\right)}{e^{2}}+\frac{\left(-13d^{2}g^{2}-10defg-e^{2}f^{2}\right)\ln\left(-ex+d\right)}{e^{3}}-\frac{4d\left(3d^{2}g^{2}+4defg+e^{2}f^{2}\right)}{e^{3}\left(-ex+d\right)}+\frac{2d^{2}\left(d^{2}g^{2}+2defg+e^{2}}{e^{3}\left(-ex+d\right)^{2}}$
risch	$-\frac{g^{2}x^{2}}{2e}-\frac{5g^{2}dx}{e^{2}}-\frac{2gfx}{e}+\frac{\left(12d^{3}g^{2}+16d^{2}efg+4d\,e^{2}f^{2}\right)x-\frac{2d^{2}\left(5d^{2}g^{2}+6defg+e^{2}f^{2}\right)}{e}}{e^{2}\left(-ex+d\right)^{2}}-\frac{13\ln\left(-ex+d\right)d^{2}g^{2}}{e^{3}}-\frac{10\ln(-e}{e}$
norman	$\frac{\left(22d^{3}g^{2}+20d^{2}efg+4d\,e^{2}f^{2}\right)x^{3}-\frac{d^{4}\left(11d^{2}g^{2}e+12dfg\,e^{2}+2f^{2}e^{3}\right)}{e^{4}}-\frac{e^{3}g^{2}x^{6}}{2}+\frac{d^{2}\left(31d^{2}g^{2}e+40dfg\,e^{2}+12f^{2}e^{3}\right)x^{2}}{2e^{2}}-e^{2}g\left(5dg+2ef\right)x^{5}-}{\left(-e^{2}x^{2}+d^{2}\right)^{2}}$
parallelrisch	$-\frac{g^{2}e^{4}x^{4}+26\ln\left(ex-d\right)x^{2}d^{2}e^{2}g^{2}+20\ln\left(ex-d\right)x^{2}d\,e^{3}fg+2\ln\left(ex-d\right)x^{2}e^{4}f^{2}+8x^{3}d\,e^{3}g^{2}+4x^{3}e^{4}fg-52\ln\left(ex-d\right)x\,d^{3}e\,g^{2}-40\ln}{}$

input
```
int((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-g/e^2*(1/2*e*g*x^2+5*d*g*x+2*e*f*x)+(-13*d^2*g^2-10*d*e*f*g-e^2*f^2)/e^3*
ln(-e*x+d)-4*d/e^3*(3*d^2*g^2+4*d*e*f*g+e^2*f^2)/(-e*x+d)+2*d^2*(d^2*g^2+2
*d*e*f*g+e^2*f^2)/e^3/(-e*x+d)^2
```

3.571.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(118) = 236.

Time = 0.36 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.04

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{e^4g^2x^4+4\,d^2e^2f^2+24\,d^3efg+20\,d^4g^2+4\,(e^4fg+2\,de^3g^2)x^3-(8\,de^3fg+19\,d^2e^2g^2)x^2-2\,(4\,de^3f^2}{}$$

input
```
integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

output
```
-1/2*(e^4*g^2*x^4 + 4*d^2*e^2*f^2 + 24*d^3*e*f*g + 20*d^4*g^2 + 4*(e^4*f*g
 + 2*d*e^3*g^2)*x^3 - (8*d*e^3*f*g + 19*d^2*e^2*g^2)*x^2 - 2*(4*d*e^3*f^2
+ 14*d^2*e^2*f*g + 7*d^3*e*g^2)*x + 2*(d^2*e^2*f^2 + 10*d^3*e*f*g + 13*d^4
*g^2 + (e^4*f^2 + 10*d*e^3*f*g + 13*d^2*e^2*g^2)*x^2 - 2*(d*e^3*f^2 + 10*d
^2*e^2*f*g + 13*d^3*e*g^2)*x)*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x + d^2*e^3
)
```

3.571.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -x\left(\frac{5dg^2}{e^2}+\frac{2fg}{e}\right)-\frac{10d^4g^2+12d^3efg+2d^2e^2f^2+x(-12d^3eg^2-16d^2e^2fg-4de^3f^2)}{d^2e^3-2de^4x+e^5x^2} - \frac{g^2x^2}{2e}-\frac{(13d^2g^2+10defg+e^2f^2)\log(-d+ex)}{e^3}$$

input
```
integrate((e*x+d)**5*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

output
```
-x*(5*d*g**2/e**2 + 2*f*g/e) - (10*d**4*g**2 + 12*d**3*e*f*g + 2*d**2*e**2
*f**2 + x*(-12*d**3*e*g**2 - 16*d**2*e**2*f*g - 4*d*e**3*f**2))/(d**2*e**3
 - 2*d*e**4*x + e**5*x**2) - g**2*x**2/(2*e) - (13*d**2*g**2 + 10*d*e*f*g
+ e**2*f**2)*log(-d + e*x)/e**3
```

3.571.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{2\,(d^2e^2f^2+6\,d^3efg+5\,d^4g^2-2\,(de^3f^2+4\,d^2e^2fg+3\,d^3eg^2)x)}{e^5x^2-2\,de^4x+d^2e^3} - \frac{eg^2x^2+2\,(2\,efg+5\,dg^2)x}{2\,e^2} - \frac{(e^2f^2+10\,defg+13\,d^2g^2)\log(ex-d)}{e^3}$$

input
```
integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

output
```
-2*(d^2*e^2*f^2 + 6*d^3*e*f*g + 5*d^4*g^2 - 2*(d*e^3*f^2 + 4*d^2*e^2*f*g +
 3*d^3*e*g^2)*x)/(e^5*x^2 - 2*d*e^4*x + d^2*e^3) - 1/2*(e*g^2*x^2 + 2*(2*e
*f*g + 5*d*g^2)*x)/e^2 - (e^2*f^2 + 10*d*e*f*g + 13*d^2*g^2)*log(e*x - d)/
e^3
```

3.571.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{(e^2f^2+10\,defg+13\,d^2g^2)\log(|ex-d|)}{e^3} - \frac{2\,(d^2e^2f^2+6\,d^3efg+5\,d^4g^2-2\,(de^3f^2+4\,d^2e^2fg+3\,d^3eg^2)x)}{(ex-d)^2e^3} - \frac{e^5g^2x^2+4\,e^5fgx+10\,de^4g^2x}{2\,e^6}$$

input
```
integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

output
```
-(e^2*f^2 + 10*d*e*f*g + 13*d^2*g^2)*log(abs(e*x - d))/e^3 - 2*(d^2*e^2*f^
2 + 6*d^3*e*f*g + 5*d^4*g^2 - 2*(d*e^3*f^2 + 4*d^2*e^2*f*g + 3*d^3*e*g^2)*
x)/((e*x - d)^2*e^3) - 1/2*(e^5*g^2*x^2 + 4*e^5*f*g*x + 10*d*e^4*g^2*x)/e^
6
```

3.571.9 Mupad [B] (verification not implemented)

Time = 11.94 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{\frac{2\,(5\,d^4\,g^2+6\,d^3\,e\,f\,g+d^2\,e^2\,f^2)}{e}-x\,(12\,d^3\,g^2+16\,d^2\,e\,f\,g+4\,d\,e^2\,f^2)}{d^2\,e^2-2\,d\,e^3\,x+e^4\,x^2} - x\left(\frac{2\,g\,(d\,g+e\,f)}{e^2}+\frac{3\,d\,g^2}{e^2}\right) - \frac{\ln(e\,x-d)\,(13\,d^2\,g^2+10\,d\,e\,f\,g+e^2\,f^2)}{e^3} - \frac{g^2\,x^2}{2\,e}$$

input
```
int(((f + g*x)^2*(d + e*x)^5)/(d^2 - e^2*x^2)^3,x)
```

output
```
- ((2*(5*d^4*g^2 + d^2*e^2*f^2 + 6*d^3*e*f*g))/e - x*(12*d^3*g^2 + 4*d*e^2
*f^2 + 16*d^2*e*f*g))/(d^2*e^2 + e^4*x^2 - 2*d*e^3*x) - x*((2*g*(d*g + e*f
))/e^2 + (3*d*g^2)/e^2) - (log(e*x - d)*(13*d^2*g^2 + e^2*f^2 + 10*d*e*f*g
))/e^3 - (g^2*x^2)/(2*e)
```

3.572 $\int \frac{(d+ex)^4(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

3.572.1 Optimal result

Integrand size = 29, antiderivative size = 81

$$\int \frac{(d+ex)^4(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx=-\frac{g^2x}{e^2}+\frac{d(ef+dg)^2}{e^3(d-ex)^2}-\frac{(ef+dg)(ef+5dg)}{e^3(d-ex)}-\frac{2g(ef+2dg)\log(d-ex)}{e^3}$$

output
```
-g^2*x/e^2+d*(d*g+e*f)^2/e^3/(-e*x+d)^2-(d*g+e*f)*(5*d*g+e*f)/e^3/(-e*x+d)
-2*g*(2*d*g+e*f)*ln(-e*x+d)/e^3
```

3.572.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)^4(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx=\frac{-4d^3g^2+4d^2eg(-f+gx)+2de^2gx(3f+gx)+e^3x\left(f^2-g^2x^2\right)-2g(ef+2dg)(d-ex)^2\log(d-ex)}{e^3(d-ex)^2}$$

input
```
Integrate[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

output
```
(-4*d^3*g^2 + 4*d^2*e*g*(-f + g*x) + 2*d*e^2*g*x*(3*f + g*x) + e^3*x*(f^2
- g^2*x^2) - 2*g*(e*f + 2*d*g)*(d - e*x)^2*Log[d - e*x])/(e^3*(d - e*x)^2)
```

3.572.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {639, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^3}dx$$

$$\Big\downarrow \text{639}$$

$$\int \frac{(d+ex)(f+gx)^2}{(d-ex)^3}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{2g(2dg+ef)}{e^2(ex-d)}+\frac{(-5dg-ef)(dg+ef)}{e^2(d-ex)^2}-\frac{2d(dg+ef)^2}{e^2(ex-d)^3}-\frac{g^2}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(dg+ef)(5dg+ef)}{e^3(d-ex)}+\frac{d(dg+ef)^2}{e^3(d-ex)^2}-\frac{2g(2dg+ef)\log(d-ex)}{e^3}-\frac{g^2x}{e^2}$$

input
```
Int[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

output
```
-((g^2*x)/e^2) + (d*(e*f + d*g)^2)/(e^3*(d - e*x)^2) - ((e*f + d*g)*(e*f +
 5*d*g))/(e^3*(d - e*x)) - (2*g*(e*f + 2*d*g)*Log[d - e*x])/e^3
```

3.572.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.572.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.19

method	result
risch	$-\frac{g^2 x}{e^2}-\frac{(-5d^2g^2-6defg-e^2f^2)x+\frac{4d^2g(dg+ef)}{e}}{e^2(-ex+d)^2}-\frac{4g^2\ln(-ex+d)d}{e^3}-\frac{2g\ln(-ex+d)f}{e^2}$
default	$-\frac{g^2 x}{e^2}-\frac{2g(2dg+ef)\ln(-ex+d)}{e^3}+\frac{-5d^2g^2-6defg-e^2f^2}{e^3(-ex+d)}+\frac{d(d^2g^2+2defg+e^2f^2)}{e^3(-ex+d)^2}$
norman	$\frac{(7d^2g^2+6defg+e^2f^2)x^3-\frac{d^4(4deg^2+4e^2fg)}{e^4}-e^2g^2x^5+\frac{2d(3d^2g^2e+4dfge^2+f^2e^3)x^2}{e^2}-\frac{d^2(4d^2g^2+2defg-e^2f^2)x}{e^2}}{(-e^2x^2+d^2)^2}-\frac{2g(2dg+ef}{}$
parallelrisch	$-\frac{4\ln(ex-d)x^2de^2g^2+2\ln(ex-d)x^2e^3fg+g^2x^3e^3-8\ln(ex-d)xd^2eg^2-4\ln(ex-d)xde^2fg+4\ln(ex-d)d^3g^2+2\ln(ex-d)d^2e}{e^3(ex-d)^2}$

input
```
int((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-g^2*x/e^2-((-5*d^2*g^2-6*d*e*f*g-e^2*f^2)*x+4*d^2*g*(d*g+e*f)/e)/e^2/(-e*
x+d)^2-4*g^2*ln(-e*x+d)/e^3*d-2*g*ln(-e*x+d)/e^2*f
```

3.572.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.96

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{e^3g^2x^3-2\,de^2g^2x^2+4\,d^2efg+4\,d^3g^2-(e^3f^2+6\,de^2fg+4\,d^2eg^2)x+2\,(d^2efg+2\,d^3g^2+(e^3fg+2\,d}{e^5x^2-2\,de^4x+d^2e^3}$$

input
```
integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

output
```
-(e^3*g^2*x^3 - 2*d*e^2*g^2*x^2 + 4*d^2*e*f*g + 4*d^3*g^2 - (e^3*f^2 + 6*d
*e^2*f*g + 4*d^2*e*g^2)*x + 2*(d^2*e*f*g + 2*d^3*g^2 + (e^3*f*g + 2*d*e^2*
g^2)*x^2 - 2*(d*e^2*f*g + 2*d^2*e*g^2)*x)*log(e*x - d))/(e^5*x^2 - 2*d*e^4
*x + d^2*e^3)
```

3.572.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{4d^3g^2+4d^2efg+x(-5d^2eg^2-6de^2fg-e^3f^2)}{d^2e^3-2de^4x+e^5x^2} - \frac{g^2x}{e^2} - \frac{2g(2dg+ef)\log(-d+ex)}{e^3}$$

input
```
integrate((e*x+d)**4*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

output
```
-(4*d**3*g**2 + 4*d**2*e*f*g + x*(-5*d**2*e*g**2 - 6*d*e**2*f*g - e**3*f**
2))/(d**2*e**3 - 2*d*e**4*x + e**5*x**2) - g**2*x/e**2 - 2*g*(2*d*g + e*f)
*log(-d + e*x)/e**3
```

3.572.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{g^2x}{e^2} - \frac{4\,d^2efg+4\,d^3g^2-(e^3f^2+6\,de^2fg+5\,d^2eg^2)x}{e^5x^2-2\,de^4x+d^2e^3} - \frac{2\,(efg+2\,dg^2)\log(ex-d)}{e^3}$$

input
```
integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

output
```
-g^2*x/e^2 - (4*d^2*e*f*g + 4*d^3*g^2 - (e^3*f^2 + 6*d*e^2*f*g + 5*d^2*e*g
^2)*x)/(e^5*x^2 - 2*d*e^4*x + d^2*e^3) - 2*(e*f*g + 2*d*g^2)*log(e*x - d)/
e^3
```

3.572.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{g^2x}{e^2} - \frac{2\,(efg+2\,dg^2)\log\left(|ex-d|\right)}{e^3} - \frac{4\,d^2efg+4\,d^3g^2-(e^3f^2+6\,de^2fg+5\,d^2eg^2)x}{(ex-d)^2e^3}$$

input
```
integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

output
```
-g^2*x/e^2 - 2*(e*f*g + 2*d*g^2)*log(abs(e*x - d))/e^3 - (4*d^2*e*f*g + 4*
d^3*g^2 - (e^3*f^2 + 6*d*e^2*f*g + 5*d^2*e*g^2)*x)/((e*x - d)^2*e^3)
```

3.572.9 Mupad [B] (verification not implemented)

Time = 11.74 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.32

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{\frac{4\,(d^3\,g^2+e\,f\,d^2\,g)}{e}-x\,(5\,d^2\,g^2+6\,d\,e\,f\,g+e^2\,f^2)}{d^2\,e^2-2\,d\,e^3\,x+e^4\,x^2} - \frac{g^2\,x}{e^2} - \frac{\ln\left(e\,x-d\right)\,\left(4\,d\,g^2+2\,e\,f\,g\right)}{e^3}$$

input
```
int(((f + g*x)^2*(d + e*x)^4)/(d^2 - e^2*x^2)^3,x)
```

output
```
- ((4*(d^3*g^2 + d^2*e*f*g))/e - x*(5*d^2*g^2 + e^2*f^2 + 6*d*e*f*g))/(d^2
*e^2 + e^4*x^2 - 2*d*e^3*x) - (g^2*x)/e^2 - (log(e*x - d)*(4*d*g^2 + 2*e*f
*g))/e^3
```

3.573 $\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$

3.573.1 Optimal result

Integrand size = 29, antiderivative size = 61

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{(ef+dg)^2}{2e^3(d-ex)^2} - \frac{2g(ef+dg)}{e^3(d-ex)} - \frac{g^2\log(d-ex)}{e^3}$$

output
```
1/2*(d*g+e*f)^2/e^3/(-e*x+d)^2-2*g*(d*g+e*f)/e^3/(-e*x+d)-g^2*ln(-e*x+d)/e
^3
```

3.573.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{\frac{(ef+dg)(-3dg+e(f+4gx))}{(d-ex)^2} - 2g^2\log(d-ex)}{2e^3}$$

input
```
Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

output
```
(((e*f + d*g)*(-3*d*g + e*(f + 4*g*x)))/(d - e*x)^2 - 2*g^2*Log[d - e*x])/
(2*e^3)
```

3.573.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {639, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^3} \, dx$$

$$\Big\downarrow \text{639}$$

$$\int \frac{(f+gx)^2}{(d-ex)^3}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{2g(dg+ef)}{e^2(d-ex)^2}+\frac{(dg+ef)^2}{e^2(d-ex)^3}+\frac{g^2}{e^2(d-ex)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2g(dg+ef)}{e^3(d-ex)}+\frac{(dg+ef)^2}{2e^3(d-ex)^2}-\frac{g^2\log(d-ex)}{e^3}$$

input
```
Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

output
```
(e*f + d*g)^2/(2*e^3*(d - e*x)^2) - (2*g*(e*f + d*g))/(e^3*(d - e*x)) - (g
^2*Log[d - e*x])/e^3
```

3.573.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.573.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

method	result	size
risch	$\frac{\frac{2g(dg+ef)x}{e^2}-\frac{3d^2g^2+2defg-e^2f^2}{2e^3}}{(-ex+d)^2}-\frac{g^2\ln(-ex+d)}{e^3}$	69
default	$-\frac{g^2\ln(-ex+d)}{e^3}-\frac{2g(dg+ef)}{e^3(-ex+d)}-\frac{-d^2g^2-2defg-e^2f^2}{2e^3(-ex+d)^2}$	74
parallelrisch	$-\frac{2\ln(ex-d)x^2e^2g^2-4\ln(ex-d)xde\,g^2+2\ln(ex-d)d^2g^2-4xde\,g^2-4x\,e^2fg+3d^2g^2+2defg-e^2f^2}{2e^3(ex-d)^2}$	105
norman	$\frac{(2d\,g^2+2fge)x^3-\frac{d^2(3d^2g^2e+2dfg\,e^2-f^2e^3)}{2e^4}+\frac{(5d^2g^2e+6dfg\,e^2+f^2e^3)x^2}{2e^2}-\frac{d(d^2g^2-e^2f^2)x}{e^2}}{(-e^2x^2+d^2)^2}-\frac{g^2\ln(-ex+d)}{e^3}$	139

input
```
int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(2*g*(d*g+e*f)*x/e^2-1/2*(3*d^2*g^2+2*d*e*f*g-e^2*f^2)/e^3)/(-e*x+d)^2-g^2
*ln(-e*x+d)/e^3
```

3.573.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{e^2f^2-2\,defg-3\,d^2g^2+4\,(e^2fg+deg^2)x-2\,(e^2g^2x^2-2\,deg^2x+d^2g^2)\log(ex-d)}{2\,(e^5x^2-2\,de^4x+d^2e^3)}$$

input
```
integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

output
```
1/2*(e^2*f^2 - 2*d*e*f*g - 3*d^2*g^2 + 4*(e^2*f*g + d*e*g^2)*x - 2*(e^2*g^
2*x^2 - 2*d*e*g^2*x + d^2*g^2)*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x + d^2*e^
3)
```

3.573.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{3d^2g^2+2defg-e^2f^2+x(-4deg^2-4e^2fg)}{2d^2e^3-4de^4x+2e^5x^2} - \frac{g^2\log(-d+ex)}{e^3}$$

input
```
integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

output
```
-(3*d**2*g**2 + 2*d*e*f*g - e**2*f**2 + x*(-4*d*e*g**2 - 4*e**2*f*g))/(2*d
**2*e**3 - 4*d*e**4*x + 2*e**5*x**2) - g**2*log(-d + e*x)/e**3
```

3.573.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.33

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{e^2f^2-2\,defg-3\,d^2g^2+4\,(e^2fg+deg^2)x}{2\,(e^5x^2-2\,de^4x+d^2e^3)} - \frac{g^2\log(ex-d)}{e^3}$$

input
```
integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

output
```
1/2*(e^2*f^2 - 2*d*e*f*g - 3*d^2*g^2 + 4*(e^2*f*g + d*e*g^2)*x)/(e^5*x^2 -
 2*d*e^4*x + d^2*e^3) - g^2*log(e*x - d)/e^3
```

3.573.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{g^2\log(|ex-d|)}{e^3} + \frac{4\,(efg+dg^2)x+\frac{e^2f^2-2\,defg-3\,d^2g^2}{e}}{2\,(ex-d)^2e^2}$$

input
```
integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

output
```
-g^2*log(abs(e*x - d))/e^3 + 1/2*(4*(e*f*g + d*g^2)*x + (e^2*f^2 - 2*d*e*f
*g - 3*d^2*g^2)/e)/((e*x - d)^2*e^2)
```

3.573.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{\frac{3\,d^2\,g^2+2\,d\,e\,f\,g-e^2\,f^2}{2\,e^3}-\frac{2\,g\,x\,(d\,g+e\,f)}{e^2}}{d^2-2\,d\,e\,x+e^2\,x^2}-\frac{g^2\,\ln(e\,x-d)}{e^3}$$

input
```
int(((f + g*x)^2*(d + e*x)^3)/(d^2 - e^2*x^2)^3,x)
```

output
```
- ((3*d^2*g^2 - e^2*f^2 + 2*d*e*f*g)/(2*e^3) - (2*g*x*(d*g + e*f))/e^2)/(d
^2 + e^2*x^2 - 2*d*e*x) - (g^2*log(e*x - d))/e^3
```

3.574 $\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$

3.574.1 Optimal result

Integrand size = 29, antiderivative size = 88

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{(ef+dg)^2}{4de^3(d-ex)^2} + \frac{(ef-3dg)(ef+dg)}{4d^2e^3(d-ex)} + \frac{(ef-dg)^2\text{arctanh}\left(\frac{ex}{d}\right)}{4d^3e^3}$$

output
```
1/4*(d*g+e*f)^2/d/e^3/(-e*x+d)^2+1/4*(-3*d*g+e*f)*(d*g+e*f)/d^2/e^3/(-e*x+
d)+1/4*(-d*g+e*f)^2*arctanh(e*x/d)/d^3/e^3
```

3.574.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{-\frac{2d(ef+dg)(2d^2g+e^2fx-de(2f+3gx))}{(d-ex)^2} - (ef-dg)^2\log(d-ex) + (ef-dg)^2\log(d+ex)}{8d^3e^3}$$

input
```
Integrate[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

output
```
((-2*d*(e*f + d*g)*(2*d^2*g + e^2*f*x - d*e*(2*f + 3*g*x)))/(d - e*x)^2 -
(e*f - d*g)^2*Log[d - e*x] + (e*f - d*g)^2*Log[d + e*x])/(8*d^3*e^3)
```

3.574.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {639, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^3}dx$$

$$\Big\downarrow \text{639}$$

$$\int \frac{(f+gx)^2}{(d-ex)^3(d+ex)}dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(\frac{(dg-ef)^2}{4d^2e^2(d^2-e^2x^2)}+\frac{(ef-3dg)(dg+ef)}{4d^2e^2(d-ex)^2}+\frac{(dg+ef)^2}{2de^2(d-ex)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{arctanh}\left(\frac{ex}{d}\right)(ef-dg)^2}{4d^3e^3}+\frac{(ef-3dg)(dg+ef)}{4d^2e^3(d-ex)}+\frac{(dg+ef)^2}{4de^3(d-ex)^2}$$

input
```
Int[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

output
```
(e*f + d*g)^2/(4*d*e^3*(d - e*x)^2) + ((e*f - 3*d*g)*(e*f + d*g))/(4*d^2*e
^3*(d - e*x)) + ((e*f - d*g)^2*ArcTanh[(e*x)/d])/(4*d^3*e^3)
```

3.574.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.574.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.73

method	result
default	$\frac{-3d^2g^2-2defg+e^2f^2}{4e^3d^2(-ex+d)}-\frac{-d^2g^2-2defg-e^2f^2}{4d\,e^3(-ex+d)^2}+\frac{(-d^2g^2+2defg-e^2f^2)\ln(-ex+d)}{8e^3d^3}+\frac{(d^2g^2-2defg+e^2f^2)\ln(ex+d)}{8e^3d^3}$
risch	$\frac{\frac{(3d^2g^2+2defg-e^2f^2)x}{4d^2e^2}-\frac{d^2g^2-e^2f^2}{2d\,e^3}}{(-ex+d)^2}-\frac{\ln(-ex+d)g^2}{8e^3d}+\frac{\ln(-ex+d)fg}{4e^2d^2}-\frac{\ln(-ex+d)f^2}{8e\,d^3}+\frac{\ln(ex+d)g^2}{8e^3d}-\frac{\ln(ex+d)fg}{4e^2d^2}$
norman	$\frac{\frac{d(-d^2g^2e+f^2e^3)}{2e^4}-\frac{(d^2g^2-2defg-3e^2f^2)x}{4e^2}+\frac{(3d^2g^2+2defg-e^2f^2)x^3}{4d^2}-\frac{(-de\,g^2-e^2fg)x^2}{e^2}}{(-e^2x^2+d^2)^2}-\frac{(d^2g^2-2defg+e^2f^2)\ln(-ex+d)}{8e^3d^3}$
parallelrisch	$-\frac{-4d^2e^2f^2+4d^4g^2-\ln(ex+d)d^2e^2f^2+\ln(ex-d)x^2e^4f^2-\ln(ex+d)x^2e^4f^2+\ln(ex-d)d^2e^2f^2-6x\,d^3e\,g^2+2xd\,e^3f^2-2\ln(ex-}{}$

input
```
int((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-3*d^2*g^2-2*d*e*f*g+e^2*f^2)/e^3/d^2/(-e*x+d)-1/4*(-d^2*g^2-2*d*e*f*
g-e^2*f^2)/d/e^3/(-e*x+d)^2+1/8*(-d^2*g^2+2*d*e*f*g-e^2*f^2)/e^3/d^3*ln(-e
*x+d)+1/8*(d^2*g^2-2*d*e*f*g+e^2*f^2)/e^3/d^3*ln(e*x+d)
```

3.574.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(84) = 168.

Time = 0.27 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.08

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$$
$$= \frac{4\,d^2e^2f^2-4\,d^4g^2-2\,(de^3f^2-2\,d^2e^2fg-3\,d^3eg^2)x+(d^2e^2f^2-2\,d^3efg+d^4g^2+(e^4f^2-2\,de^3fg+d^2e}{}$$

input
```
integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

output
```
1/8*(4*d^2*e^2*f^2 - 4*d^4*g^2 - 2*(d*e^3*f^2 - 2*d^2*e^2*f*g - 3*d^3*e*g^
2)*x + (d^2*e^2*f^2 - 2*d^3*e*f*g + d^4*g^2 + (e^4*f^2 - 2*d*e^3*f*g + d^2
*e^2*g^2)*x^2 - 2*(d*e^3*f^2 - 2*d^2*e^2*f*g + d^3*e*g^2)*x)*log(e*x + d)
- (d^2*e^2*f^2 - 2*d^3*e*f*g + d^4*g^2 + (e^4*f^2 - 2*d*e^3*f*g + d^2*e^2*
g^2)*x^2 - 2*(d*e^3*f^2 - 2*d^2*e^2*f*g + d^3*e*g^2)*x)*log(e*x - d))/(d^3
*e^5*x^2 - 2*d^4*e^4*x + d^5*e^3)
```

3.574.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(75) = 150$.

Time = 0.46 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.10

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{2d^3g^2-2de^2f^2+x(-3d^2eg^2-2de^2fg+e^3f^2)}{4d^4e^3-8d^3e^4x+4d^2e^5x^2} - \frac{(dg-ef)^2\log\left(-\frac{d(dg-ef)^2}{e(d^2g^2-2defg+e^2f^2)}+x\right)}{8d^3e^3} + \frac{(dg-ef)^2\log\left(\frac{d(dg-ef)^2}{e(d^2g^2-2defg+e^2f^2)}+x\right)}{8d^3e^3}$$

input
```
integrate((e*x+d)**2*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

output
```
-(2*d**3*g**2 - 2*d*e**2*f**2 + x*(-3*d**2*e*g**2 - 2*d*e**2*f*g + e**3*f*
*2))/(4*d**4*e**3 - 8*d**3*e**4*x + 4*d**2*e**5*x**2) - (d*g - e*f)**2*log
(-d*(d*g - e*f)**2/(e*(d**2*g**2 - 2*d*e*f*g + e**2*f**2)) + x)/(8*d**3*e*
*3) + (d*g - e*f)**2*log(d*(d*g - e*f)**2/(e*(d**2*g**2 - 2*d*e*f*g + e**2
*f**2)) + x)/(8*d**3*e**3)
```

3.574.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.70

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{2\,de^2f^2-2\,d^3g^2-(e^3f^2-2\,de^2fg-3\,d^2eg^2)x}{4\,(d^2e^5x^2-2\,d^3e^4x+d^4e^3)} + \frac{(e^2f^2-2\,defg+d^2g^2)\log(ex+d)}{8\,d^3e^3} - \frac{(e^2f^2-2\,defg+d^2g^2)\log(ex-d)}{8\,d^3e^3}$$

input
```
integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

output
```
1/4*(2*d*e^2*f^2 - 2*d^3*g^2 - (e^3*f^2 - 2*d*e^2*f*g - 3*d^2*e*g^2)*x)/(d
^2*e^5*x^2 - 2*d^3*e^4*x + d^4*e^3) + 1/8*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*
log(e*x + d)/(d^3*e^3) - 1/8*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(e*x - d)/
(d^3*e^3)
```

3.574.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.62

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{(e^2f^2-2\,defg+d^2g^2)\log(|ex+d|)}{8\,d^3e^3} - \frac{(e^2f^2-2\,defg+d^2g^2)\log(|ex-d|)}{8\,d^3e^3} + \frac{2\,d^2e^2f^2-2\,d^4g^2-(de^3f^2-2\,d^2e^2fg-3\,d^3eg^2)x}{4\,(ex-d)^2d^3e^3}$$

input
```
integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

output
```
1/8*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(abs(e*x + d))/(d^3*e^3) - 1/8*(e^2
*f^2 - 2*d*e*f*g + d^2*g^2)*log(abs(e*x - d))/(d^3*e^3) + 1/4*(2*d^2*e^2*f
^2 - 2*d^4*g^2 - (d*e^3*f^2 - 2*d^2*e^2*f*g - 3*d^3*e*g^2)*x)/((e*x - d)^2
*d^3*e^3)
```

3.574.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{\operatorname{atanh}\left(\frac{e\,x}{d}\right)\,(d\,g-e\,f)^2}{4\,d^3\,e^3} - \frac{\frac{d^2\,g^2-e^2\,f^2}{2\,d\,e^3}-\frac{x\,(3\,d^2\,g^2+2\,d\,e\,f\,g-e^2\,f^2)}{4\,d^2\,e^2}}{d^2-2\,d\,e\,x+e^2\,x^2}$$

input
```
int(((f + g*x)^2*(d + e*x)^2)/(d^2 - e^2*x^2)^3,x)
```

output
```
(atanh((e*x)/d)*(d*g - e*f)^2)/(4*d^3*e^3) - ((d^2*g^2 - e^2*f^2)/(2*d*e^3
) - (x*(3*d^2*g^2 - e^2*f^2 + 2*d*e*f*g))/(4*d^2*e^2))/(d^2 + e^2*x^2 - 2*
d*e*x)
```

3.575 $\int \frac{(d+ex)(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

3.575.1 Optimal result

Integrand size = 27, antiderivative size = 122

$$\int \frac{(d+ex)(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx=\frac{(ef+dg)^2}{8d^2e^3(d-ex)^2}+\frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^3e^3(d+ex)}+\frac{(ef-dg)(3ef+dg)\text{arctanh}\left(\frac{ex}{d}\right)}{8d^4e^3}$$

output
```
1/8*(d*g+e*f)^2/d^2/e^3/(-e*x+d)^2+1/4*(-d^2*g^2+e^2*f^2)/d^3/e^3/(-e*x+d)
-1/8*(-d*g+e*f)^2/d^3/e^3/(e*x+d)+1/8*(-d*g+e*f)*(d*g+3*e*f)*arctanh(e*x/d
)/d^4/e^3
```

3.575.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx=\frac{\frac{2d^2(ef+dg)^2}{(d-ex)^2}+\frac{4de^2f^2-4d^3g^2}{d-ex}-\frac{2d(ef-dg)^2}{d+ex}+\left(-3e^2f^2+2defg+d^2g^2\right)\log(d-ex)+\left(3e^2f^2-2defg-d^2g^2\right)}{16d^4e^3}$$

input
```
Integrate[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

output
```
((2*d^2*(e*f + d*g)^2)/(d - e*x)^2 + (4*d*e^2*f^2 - 4*d^3*g^2)/(d - e*x) -
 (2*d*(e*f - d*g)^2)/(d + e*x) + (-3*e^2*f^2 + 2*d*e*f*g + d^2*g^2)*Log[d
- e*x] + (3*e^2*f^2 - 2*d*e*f*g - d^2*g^2)*Log[d + e*x])/(16*d^4*e^3)
```

3.575.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {639, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$$

$$\Big\downarrow \text{639}$$

$$\int \frac{(f+gx)^2}{(d-ex)^3(d+ex)^2}dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(\frac{(dg-ef)^2}{8d^3e^2(d+ex)^2}+\frac{(dg+ef)^2}{4d^2e^2(d-ex)^3}+\frac{e^2f^2-d^2g^2}{4d^3e^2(d-ex)^2}+\frac{(ef-dg)(dg+3ef)}{8d^3e^2\,(d^2-e^2x^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{arctanh}\left(\frac{ex}{d}\right)(dg+3ef)(ef-dg)}{8d^4e^3}-\frac{(ef-dg)^2}{8d^3e^3(d+ex)}+\frac{(dg+ef)^2}{8d^2e^3(d-ex)^2}+\frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)}$$

input
```
Int[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

output
```
(e*f + d*g)^2/(8*d^2*e^3*(d - e*x)^2) + (e^2*f^2 - d^2*g^2)/(4*d^3*e^3*(d
- e*x)) - (e*f - d*g)^2/(8*d^3*e^3*(d + e*x)) + ((e*f - d*g)*(3*e*f + d*g)
*ArcTanh[(e*x)/d])/(8*d^4*e^3)
```

3.575.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.575.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.50

method	result
default	$\frac{-d^2g^2+e^2f^2}{4d^3e^3(-ex+d)}-\frac{-d^2g^2-2defg-e^2f^2}{8e^3d^2(-ex+d)^2}+\frac{(d^2g^2+2defg-3e^2f^2)\ln(-ex+d)}{16e^3d^4}+\frac{(-d^2g^2-2defg+3e^2f^2)\ln(ex+d)}{16e^3d^4}-\frac{d^2}{}$
norman	$\frac{\frac{-d^2g^2e+2dfg\,e^2+f^2e^3}{4e^4}+\frac{(d^2g^2+2defg-3e^2f^2)x^3}{8d^3}+\frac{g^2x^2}{2e}+\frac{(d^2g^2+2defg+5e^2f^2)x}{8d\,e^2}}{(-e^2x^2+d^2)^2}+\frac{(d^2g^2+2defg-3e^2f^2)\ln(-ex+d)}{16e^3d^4}-\frac{(d^2}{}$
risch	$\frac{\frac{(d^2g^2+2defg-3e^2f^2)x^2}{8d^3e}+\frac{(3d^2g^2-2defg+3e^2f^2)x}{8e^2d^2}-\frac{d^2g^2-2defg-e^2f^2}{4d\,e^3}}{(-ex+d)(-e^2x^2+d^2)}-\frac{\ln(-ex-d)g^2}{16e^3d^2}-\frac{\ln(-ex-d)fg}{8e^2d^3}+\frac{3\ln(-ex-d)f^2}{16e\,d^4}$
parallelrisch	$\frac{-2\ln(ex+d)d^4efg+\ln(ex-d)x^3d^2e^3g^2+2\ln(ex+d)x\,d^3e^2fg-2\ln(ex-d)x\,d^3e^2fg-2\ln(ex-d)x^2d^2e^3fg+2\ln(ex+d)x^2d^2e^3f}{}$

input
```
int((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-d^2*g^2+e^2*f^2)/d^3/e^3/(-e*x+d)-1/8*(-d^2*g^2-2*d*e*f*g-e^2*f^2)/e
^3/d^2/(-e*x+d)^2+1/16/e^3*(d^2*g^2+2*d*e*f*g-3*e^2*f^2)/d^4*ln(-e*x+d)+1/
16*(-d^2*g^2-2*d*e*f*g+3*e^2*f^2)/e^3/d^4*ln(e*x+d)-1/8*(d^2*g^2-2*d*e*f*g
+e^2*f^2)/e^3/d^3/(e*x+d)
```

3.575.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(116) = 232$.

Time = 0.29 (sec) , antiderivative size = 417, normalized size of antiderivative = 3.42

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$$
$$= \frac{4\,d^3e^2f^2+8\,d^4efg-4\,d^5g^2-2\,(3\,de^4f^2-2\,d^2e^3fg-d^3e^2g^2)x^2+2\,(3\,d^2e^3f^2-2\,d^3e^2fg+3\,d^4eg^2)x+(}{}$$

input
```
integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

output
```
1/16*(4*d^3*e^2*f^2 + 8*d^4*e*f*g - 4*d^5*g^2 - 2*(3*d*e^4*f^2 - 2*d^2*e^3
*f*g - d^3*e^2*g^2)*x^2 + 2*(3*d^2*e^3*f^2 - 2*d^3*e^2*f*g + 3*d^4*e*g^2)*
x + (3*d^3*e^2*f^2 - 2*d^4*e*f*g - d^5*g^2 + (3*e^5*f^2 - 2*d*e^4*f*g - d^
2*e^3*g^2)*x^3 - (3*d*e^4*f^2 - 2*d^2*e^3*f*g - d^3*e^2*g^2)*x^2 - (3*d^2*
e^3*f^2 - 2*d^3*e^2*f*g - d^4*e*g^2)*x)*log(e*x + d) - (3*d^3*e^2*f^2 - 2*
d^4*e*f*g - d^5*g^2 + (3*e^5*f^2 - 2*d*e^4*f*g - d^2*e^3*g^2)*x^3 - (3*d*e
^4*f^2 - 2*d^2*e^3*f*g - d^3*e^2*g^2)*x^2 - (3*d^2*e^3*f^2 - 2*d^3*e^2*f*g
 - d^4*e*g^2)*x)*log(e*x - d))/(d^4*e^6*x^3 - d^5*e^5*x^2 - d^6*e^4*x + d^
7*e^3)
```

3.575.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(105) = 210$.

Time = 0.58 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.27

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{2d^4g^2-4d^3efg-2d^2e^2f^2+x^2(-d^2e^2g^2-2de^3fg+3e^4f^2)+x(-3d^3eg^2+2d^2e^2fg-3de^3f^2)}{8d^6e^3-8d^5e^4x-8d^4e^5x^2+8d^3e^6x^3} + \frac{(dg-ef)\,(dg+3ef)\log\left(-\frac{d(dg-ef)(dg+3ef)}{e(d^2g^2+2defg-3e^2f^2)}+x\right)}{16d^4e^3} - \frac{(dg-ef)\,(dg+3ef)\log\left(\frac{d(dg-ef)(dg+3ef)}{e(d^2g^2+2defg-3e^2f^2)}+x\right)}{16d^4e^3}$$

input
```
integrate((e*x+d)*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

output
```
-(2*d**4*g**2 - 4*d**3*e*f*g - 2*d**2*e**2*f**2 + x**2*(-d**2*e**2*g**2 -
2*d*e**3*f*g + 3*e**4*f**2) + x*(-3*d**3*e*g**2 + 2*d**2*e**2*f*g - 3*d*e*
*3*f**2))/(8*d**6*e**3 - 8*d**5*e**4*x - 8*d**4*e**5*x**2 + 8*d**3*e**6*x*
*3) + (d*g - e*f)*(d*g + 3*e*f)*log(-d*(d*g - e*f)*(d*g + 3*e*f)/(e*(d**2*
g**2 + 2*d*e*f*g - 3*e**2*f**2)) + x)/(16*d**4*e**3) - (d*g - e*f)*(d*g +
3*e*f)*log(d*(d*g - e*f)*(d*g + 3*e*f)/(e*(d**2*g**2 + 2*d*e*f*g - 3*e**2*
f**2)) + x)/(16*d**4*e**3)
```

3.575.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.73

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$$
$$= \frac{2\,d^2e^2f^2+4\,d^3efg-2\,d^4g^2-(3\,e^4f^2-2\,de^3fg-d^2e^2g^2)x^2+(3\,de^3f^2-2\,d^2e^2fg+3\,d^3eg^2)x}{8\,(d^3e^6x^3-d^4e^5x^2-d^5e^4x+d^6e^3)} + \frac{(3\,e^2f^2-2\,defg-d^2g^2)\log(ex+d)}{16\,d^4e^3} - \frac{(3\,e^2f^2-2\,defg-d^2g^2)\log(ex-d)}{16\,d^4e^3}$$

input
```
integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

output
```
1/8*(2*d^2*e^2*f^2 + 4*d^3*e*f*g - 2*d^4*g^2 - (3*e^4*f^2 - 2*d*e^3*f*g -
d^2*e^2*g^2)*x^2 + (3*d*e^3*f^2 - 2*d^2*e^2*f*g + 3*d^3*e*g^2)*x)/(d^3*e^6
*x^3 - d^4*e^5*x^2 - d^5*e^4*x + d^6*e^3) + 1/16*(3*e^2*f^2 - 2*d*e*f*g -
d^2*g^2)*log(e*x + d)/(d^4*e^3) - 1/16*(3*e^2*f^2 - 2*d*e*f*g - d^2*g^2)*l
og(e*x - d)/(d^4*e^3)
```

3.575.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$$
$$= \frac{(3\,e^2f^2-2\,defg-d^2g^2)\log(|ex+d|)}{16\,d^4e^3} - \frac{(3\,e^2f^2-2\,defg-d^2g^2)\log(|ex-d|)}{16\,d^4e^3} + \frac{2\,d^3e^2f^2+4\,d^4efg-2\,d^5g^2-(3\,de^4f^2-2\,d^2e^3fg-d^3e^2g^2)x^2+(3\,d^2e^3f^2-2\,d^3e^2fg+3\,d^4eg^2)x}{8\,(ex+d)(ex-d)^2d^4e^3}$$

input
```
integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

output
```
1/16*(3*e^2*f^2 - 2*d*e*f*g - d^2*g^2)*log(abs(e*x + d))/(d^4*e^3) - 1/16*
(3*e^2*f^2 - 2*d*e*f*g - d^2*g^2)*log(abs(e*x - d))/(d^4*e^3) + 1/8*(2*d^3
*e^2*f^2 + 4*d^4*e*f*g - 2*d^5*g^2 - (3*d*e^4*f^2 - 2*d^2*e^3*f*g - d^3*e^
2*g^2)*x^2 + (3*d^2*e^3*f^2 - 2*d^3*e^2*f*g + 3*d^4*e*g^2)*x)/((e*x + d)*(
e*x - d)^2*d^4*e^3)
```

3.575.9 Mupad [B] (verification not implemented)

Time = 11.81 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.62

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{\frac{-d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2}{4\,d\,e^3}+\frac{x\,\left(3\,d^2\,g^2-2\,d\,e\,f\,g+3\,e^2\,f^2\right)}{8\,d^2\,e^2}+\frac{x^2\,\left(d^2\,g^2+2\,d\,e\,f\,g-3\,e^2\,f^2\right)}{8\,d^3\,e}}{d^3-d^2\,e\,x-d\,e^2\,x^2+e^3\,x^3} - \frac{\mathrm{atanh}\left(\frac{e\,x\,\left(d\,g-e\,f\right)\,\left(d\,g+3\,e\,f\right)}{d\,\left(d^2\,g^2+2\,d\,e\,f\,g-3\,e^2\,f^2\right)}\right)\,\left(d\,g-e\,f\right)\,\left(d\,g+3\,e\,f\right)}{8\,d^4\,e^3}$$

input
```
int(((f + g*x)^2*(d + e*x))/(d^2 - e^2*x^2)^3,x)
```

output
```
((e^2*f^2 - d^2*g^2 + 2*d*e*f*g)/(4*d*e^3) + (x*(3*d^2*g^2 + 3*e^2*f^2 - 2
*d*e*f*g))/(8*d^2*e^2) + (x^2*(d^2*g^2 - 3*e^2*f^2 + 2*d*e*f*g))/(8*d^3*e)
)/(d^3 + e^3*x^3 - d*e^2*x^2 - d^2*e*x) - (atanh((e*x*(d*g - e*f)*(d*g + 3
*e*f))/(d*(d^2*g^2 - 3*e^2*f^2 + 2*d*e*f*g)))*(d*g - e*f)*(d*g + 3*e*f))/(
8*d^4*e^3)
```

3.576 $\int \frac{(f+gx)^2}{(d^2-e^2x^2)^3} dx$

3.576.1 Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^3} dx = \frac{(d^2g+e^2fx)(f+gx)}{4d^2e^2(d^2-e^2x^2)^2} + \frac{2d^2fg+(3e^2f^2-d^2g^2)x}{8d^4e^2(d^2-e^2x^2)} + \frac{(3e^2f^2-d^2g^2)\operatorname{arctanh}\left(\frac{ex}{d}\right)}{8d^5e^3}$$

output
```
1/4*(e^2*f*x+d^2*g)*(g*x+f)/d^2/e^2/(-e^2*x^2+d^2)^2+1/8*(2*d^2*f*g+(-d^2*
g^2+3*e^2*f^2)*x)/d^4/e^2/(-e^2*x^2+d^2)+1/8*(-d^2*g^2+3*e^2*f^2)*arctanh(
e*x/d)/d^5/e^3
```

3.576.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^3} dx = \frac{-3de^5f^2x^3+d^5eg(4f+gx)+d^3e^3x(5f^2+g^2x^2)+(3e^2f^2-d^2g^2)(d^2-e^2x^2)^2\operatorname{arctanh}\left(\frac{ex}{d}\right)}{8d^5e^3(d^2-e^2x^2)^2}$$

input
```
Integrate[(f + g*x)^2/(d^2 - e^2*x^2)^3,x]
```

output
```
(-3*d*e^5*f^2*x^3 + d^5*e*g*(4*f + g*x) + d^3*e^3*x*(5*f^2 + g^2*x^2) + (3
*e^2*f^2 - d^2*g^2)*(d^2 - e^2*x^2)^2*ArcTanh[(e*x)/d])/(8*d^5*e^3*(d^2 -
e^2*x^2)^2)
```

3.576.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$$

$$\Big\downarrow \; 477$$

$$\frac{\int \left(\frac{(ef+dg)^2d^3}{8e^2(d-ex)^3}+\frac{(ef-dg)^2d^3}{8e^2(d+ex)^3}+\frac{\left(3f^2-\frac{d^2g^2}{e^2}\right)d^2}{8(d^2-e^2x^2)}+\frac{(3ef-dg)(ef+dg)d^2}{16e^2(d-ex)^2}+\frac{(ef-dg)(3ef+dg)d^2}{16e^2(d+ex)^2}\right)dx}{d^6}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{d\,\text{arctanh}\left(\frac{ex}{d}\right)(3e^2f^2-d^2g^2)}{8e^3}+\frac{d^3(dg+ef)^2}{16e^3(d-ex)^2}-\frac{d^3(ef-dg)^2}{16e^3(d+ex)^2}+\frac{d^2(3ef-dg)(dg+ef)}{16e^3(d-ex)}-\frac{d^2(ef-dg)(dg+3ef)}{16e^3(d+ex)}}{d^6}$$

input
```
Int[(f + g*x)^2/(d^2 - e^2*x^2)^3,x]
```

output
```
((d^3*(e*f + d*g)^2)/(16*e^3*(d - e*x)^2) + (d^2*(3*e*f - d*g)*(e*f + d*g)
)/(16*e^3*(d - e*x)) - (d^3*(e*f - d*g)^2)/(16*e^3*(d + e*x)^2) - (d^2*(e*
f - d*g)*(3*e*f + d*g))/(16*e^3*(d + e*x)) + (d*(3*e^2*f^2 - d^2*g^2)*ArcT
anh[(e*x)/d])/(8*e^3))/d^6
```

3.576.3.1 Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.576.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.06

method	result
norman	$\frac{\frac{fg}{2e^2}+\frac{\left(d^2g^2-3e^2f^2\right)x^3}{8d^4}+\frac{\left(d^2g^2+5e^2f^2\right)x}{8d^2e^2}}{\left(-e^2x^2+d^2\right)^2}+\frac{\left(d^2g^2-3e^2f^2\right)\ln\left(-ex+d\right)}{16d^5e^3}-\frac{\left(d^2g^2-3e^2f^2\right)\ln\left(ex+d\right)}{16d^5e^3}$
risch	$\frac{\frac{fg}{2e^2}+\frac{\left(d^2g^2-3e^2f^2\right)x^3}{8d^4}+\frac{\left(d^2g^2+5e^2f^2\right)x}{8d^2e^2}}{\left(-e^2x^2+d^2\right)^2}-\frac{\ln\left(-ex-d\right)g^2}{16d^3e^3}+\frac{3\ln\left(-ex-d\right)f^2}{16d^5e}+\frac{\ln\left(ex-d\right)g^2}{16d^3e^3}-\frac{3\ln\left(ex-d\right)f^2}{16d^5e}$
default	$\frac{\left(d^2g^2-3e^2f^2\right)\ln\left(-ex+d\right)}{16d^5e^3}-\frac{-d^2g^2-2defg-e^2f^2}{16e^3d^3\left(-ex+d\right)^2}+\frac{-d^2g^2+2defg+3e^2f^2}{16e^3d^4\left(-ex+d\right)}+\frac{\left(-d^2g^2+3e^2f^2\right)\ln\left(ex+d\right)}{16e^3d^5}-\frac{-d^2g^2-2de}{16e^3d^4}$
parallelrisch	$\ln(ex-d)x^4d^2e^5g^2-3\ln(ex-d)x^4e^7f^2-\ln(ex+d)x^4d^2e^5g^2+3\ln(ex+d)x^4e^7f^2-2\ln(ex-d)x^2d^4e^3g^2+6\ln(ex-d)x^2d^2e^5f^2+$

input
```
int((g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/2*f*g/e^2+1/8/d^4*(d^2*g^2-3*e^2*f^2)*x^3+1/8*(d^2*g^2+5*e^2*f^2)/d^2/e
^2*x)/(-e^2*x^2+d^2)^2+1/16*(d^2*g^2-3*e^2*f^2)/d^5/e^3*ln(-e*x+d)-1/16*(d
^2*g^2-3*e^2*f^2)/d^5/e^3*ln(e*x+d)
```

3.576.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(123) = 246.

Time = 0.27 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.98

$$\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx = \frac{8\,d^5efg-2\,(3\,de^5f^2-d^3e^3g^2)x^3+2\,(5\,d^3e^3f^2+d^5eg^2)x+(3\,d^4e^2f^2-d^6g^2+(3\,e^6f^2-d^2e^4g^2)x^4-2\,(3}{16\,(d^5e^7x^4}$$

input
```
integrate((g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

output
```
1/16*(8*d^5*e*f*g - 2*(3*d*e^5*f^2 - d^3*e^3*g^2)*x^3 + 2*(5*d^3*e^3*f^2 +
 d^5*e*g^2)*x + (3*d^4*e^2*f^2 - d^6*g^2 + (3*e^6*f^2 - d^2*e^4*g^2)*x^4 -
 2*(3*d^2*e^4*f^2 - d^4*e^2*g^2)*x^2)*log(e*x + d) - (3*d^4*e^2*f^2 - d^6*
g^2 + (3*e^6*f^2 - d^2*e^4*g^2)*x^4 - 2*(3*d^2*e^4*f^2 - d^4*e^2*g^2)*x^2)
*log(e*x - d))/(d^5*e^7*x^4 - 2*d^7*e^5*x^2 + d^9*e^3)
```

3.576.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.13

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{-4d^4fg + x^3(-d^2e^2g^2+3e^4f^2) + x(-d^4g^2-5d^2e^2f^2)}{8d^8e^2-16d^6e^4x^2+8d^4e^6x^4} + \frac{(d^2g^2-3e^2f^2)\log\left(-\frac{d}{e}+x\right)}{16d^5e^3} - \frac{(d^2g^2-3e^2f^2)\log\left(\frac{d}{e}+x\right)}{16d^5e^3}$$

input
```
integrate((g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

output
```
-(-4*d**4*f*g + x**3*(-d**2*e**2*g**2 + 3*e**4*f**2) + x*(-d**4*g**2 - 5*d
**2*e**2*f**2))/(8*d**8*e**2 - 16*d**6*e**4*x**2 + 8*d**4*e**6*x**4) + (d*
*2*g**2 - 3*e**2*f**2)*log(-d/e + x)/(16*d**5*e**3) - (d**2*g**2 - 3*e**2*
f**2)*log(d/e + x)/(16*d**5*e**3)
```

3.576.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.20

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{4\,d^4fg - (3\,e^4f^2-d^2e^2g^2)x^3 + (5\,d^2e^2f^2+d^4g^2)x}{8\,(d^4e^6x^4-2\,d^6e^4x^2+d^8e^2)} + \frac{(3\,e^2f^2-d^2g^2)\log(ex+d)}{16\,d^5e^3} - \frac{(3\,e^2f^2-d^2g^2)\log(ex-d)}{16\,d^5e^3}$$

input
```
integrate((g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

output
```
1/8*(4*d^4*f*g - (3*e^4*f^2 - d^2*e^2*g^2)*x^3 + (5*d^2*e^2*f^2 + d^4*g^2)
*x)/(d^4*e^6*x^4 - 2*d^6*e^4*x^2 + d^8*e^2) + 1/16*(3*e^2*f^2 - d^2*g^2)*l
og(e*x + d)/(d^5*e^3) - 1/16*(3*e^2*f^2 - d^2*g^2)*log(e*x - d)/(d^5*e^3)
```

3.576.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.15

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{3\,e^4f^2x^3-d^2e^2g^2x^3-5\,d^2e^2f^2x-d^4g^2x-4\,d^4fg}{8\,(e^2x^2-d^2)^2d^4e^2} + \frac{(3\,e^3f^2-d^2eg^2)\log(|ex+d|)}{16\,d^5e^4} - \frac{(3\,e^3f^2-d^2eg^2)\log(|ex-d|)}{16\,d^5e^4}$$

input
```
integrate((g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

output
```
-1/8*(3*e^4*f^2*x^3 - d^2*e^2*g^2*x^3 - 5*d^2*e^2*f^2*x - d^4*g^2*x - 4*d^
4*f*g)/((e^2*x^2 - d^2)^2*d^4*e^2) + 1/16*(3*e^3*f^2 - d^2*e*g^2)*log(abs(
e*x + d))/(d^5*e^4) - 1/16*(3*e^3*f^2 - d^2*e*g^2)*log(abs(e*x - d))/(d^5*
e^4)
```

3.576.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{\frac{x^3\,(d^2\,g^2-3\,e^2\,f^2)}{8\,d^4}+\frac{f\,g}{2\,e^2}+\frac{x\,(d^2\,g^2+5\,e^2\,f^2)}{8\,d^2\,e^2}}{d^4-2\,d^2\,e^2\,x^2+e^4\,x^4} - \frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,(d^2\,g^2-3\,e^2\,f^2)}{8\,d^5\,e^3}$$

input
```
int((f + g*x)^2/(d^2 - e^2*x^2)^3,x)
```

output
```
((x^3*(d^2*g^2 - 3*e^2*f^2))/(8*d^4) + (f*g)/(2*e^2) + (x*(d^2*g^2 + 5*e^2
*f^2))/(8*d^2*e^2))/(d^4 + e^4*x^4 - 2*d^2*e^2*x^2) - (atanh((e*x)/d)*(d^2
*g^2 - 3*e^2*f^2))/(8*d^5*e^3)
```

3.577 $\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)^3}\,dx$

3.577.1 Optimal result

Integrand size = 29, antiderivative size = 188

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^3}\,dx = \frac{(ef+dg)^2}{32d^4e^3(d-ex)^2}+\frac{f(ef+dg)}{8d^5e^2(d-ex)}-\frac{(ef-dg)^2}{24d^3e^3(d+ex)^3}-\frac{(ef-dg)(3ef+dg)}{32d^4e^3(d+ex)^2}-\frac{3e^2f^2-d^2g^2}{16d^5e^3(d+ex)}+\frac{(5e^2f^2+2defg-d^2g^2)\operatorname{arctanh}\left(\frac{ex}{d}\right)}{16d^6e^3}$$

output
```
1/32*(d*g+e*f)^2/d^4/e^3/(-e*x+d)^2+1/8*f*(d*g+e*f)/d^5/e^2/(-e*x+d)-1/24*
(-d*g+e*f)^2/d^3/e^3/(e*x+d)^3-1/32*(-d*g+e*f)*(d*g+3*e*f)/d^4/e^3/(e*x+d)
^2+1/16*(d^2*g^2-3*e^2*f^2)/d^5/e^3/(e*x+d)+1/16*(-d^2*g^2+2*d*e*f*g+5*e^2
*f^2)*arctanh(e*x/d)/d^6/e^3
```

3.577.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.05

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^3}\,dx = \frac{\frac{3d^2(ef+dg)^2}{(d-ex)^2}+\frac{12def(ef+dg)}{d-ex}-\frac{4d^3(ef-dg)^2}{(d+ex)^3}+\frac{3d^2(-3e^2f^2+2defg+d^2g^2)}{(d+ex)^2}+\frac{6d(-3e^2f^2+d^2g^2)}{d+ex}+3(-5e^2f^2-2defg+d^2g}{96d^6e^3}$$

input
```
Integrate[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^3),x]
```

output
```
((3*d^2*(e*f + d*g)^2)/(d - e*x)^2 + (12*d*e*f*(e*f + d*g))/(d - e*x) - (4
*d^3*(e*f - d*g)^2)/(d + e*x)^3 + (3*d^2*(-3*e^2*f^2 + 2*d*e*f*g + d^2*g^2
))/(d + e*x)^2 + (6*d*(-3*e^2*f^2 + d^2*g^2))/(d + e*x) + 3*(-5*e^2*f^2 -
2*d*e*f*g + d^2*g^2)*Log[d - e*x] + 3*(5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*Lo
g[d + e*x])/(96*d^6*e^3)
```

3.577.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {639, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^3}dx$$

$$\Big\downarrow \text{639}$$

$$\int \frac{(f+gx)^2}{(d-ex)^3(d+ex)^4}dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(\frac{f(dg+ef)}{8d^5e(d-ex)^2}+\frac{(dg+ef)^2}{16d^4e^2(d-ex)^3}+\frac{(ef-dg)(dg+3ef)}{16d^4e^2(d+ex)^3}+\frac{(dg-ef)^2}{8d^3e^2(d+ex)^4}+\frac{d^2g^2-2defg-5e^2f^2}{16d^5e^2\left(e^2x^2-d^2\right)}+\frac{3e^2}{16d^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{arctanh}\left(\frac{ex}{d}\right)\left(-d^2g^2+2defg+5e^2f^2\right)}{16d^6e^3}+\frac{f(dg+ef)}{8d^5e^2(d-ex)}-\frac{(dg+3ef)(ef-dg)}{32d^4e^3(d+ex)^2}+\frac{(dg+ef)^2}{32d^4e^3(d-ex)^2}-\frac{(ef-dg)^2}{24d^3e^3(d+ex)^3}-\frac{3e^2f^2-d^2g^2}{16d^5e^3(d+ex)}$$

input
```
Int[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^3),x]
```

output
```
(e*f + d*g)^2/(32*d^4*e^3*(d - e*x)^2) + (f*(e*f + d*g))/(8*d^5*e^2*(d - e
*x)) - (e*f - d*g)^2/(24*d^3*e^3*(d + e*x)^3) - ((e*f - d*g)*(3*e*f + d*g)
)/(32*d^4*e^3*(d + e*x)^2) - (3*e^2*f^2 - d^2*g^2)/(16*d^5*e^3*(d + e*x))
+ ((5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*ArcTanh[(e*x)/d])/(16*d^6*e^3)
```

3.577.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.577.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.30

method	result
default	$-\frac{-d^2g^2-2defg-e^2f^2}{32e^3d^4(-ex+d)^2}+\frac{(d^2g^2-2defg-5e^2f^2)\ln(-ex+d)}{32d^6e^3}+\frac{f(dg+ef)}{8d^5e^2(-ex+d)}-\frac{-d^2g^2+3e^2f^2}{16e^3d^5(ex+d)}-$ (cut off: $-d^2g^2-2defg+3e^2$ over $32e^3d^4(ex+d)^2$)
norman	$\frac{\frac{(11d^2g^2+26defg-31e^2f^2)x^3}{48d^4}+\frac{(d^2g^2+14defg+3e^2f^2)x^2}{16e\,d^3}-\frac{e(d^2g^2+22defg+7e^2f^2)x^4}{48d^5}-\frac{e^2(d^2g^2+4defg-2e^2f^2)x^5}{12d^6}+\dots}{(ex+d)^3(-ex+d)^2}$ (cut off: $+(d^2g^2-2defg+$ over $16d^2e$)
risch	$\frac{\frac{(d^2g^2-2defg-5e^2f^2)e\,x^4}{16d^5}+\frac{(d^2g^2-2defg-5e^2f^2)x^3}{16d^4}-\frac{5(d^2g^2-2defg-5e^2f^2)x^2}{48d^3e}+\frac{(7d^2g^2+10defg+25e^2f^2)x}{48d^2e^2}+\frac{d^2g^2+4defg-2e^2f^2}{12d\,e^3}\dots}{(ex+d)(-e^2x^2+d^2)^2}$
parallelrisch	$15\ln(ex+d)x^5e^7f^2-15\ln(ex-d)x^5e^7f^2+66x\,d^4e^3f^2+6\ln(ex+d)x\,d^5e^2fg+3\ln(ex-d)x^5d^2e^5g^2-3\ln(ex+d)x^5d^2e^5g^2+3\ln($ (cut off)

input
```
int((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/32*(-d^2*g^2-2*d*e*f*g-e^2*f^2)/e^3/d^4/(-e*x+d)^2+1/32*(d^2*g^2-2*d*e*
f*g-5*e^2*f^2)/d^6/e^3*ln(-e*x+d)+1/8*f*(d*g+e*f)/d^5/e^2/(-e*x+d)-1/16*(-
d^2*g^2+3*e^2*f^2)/e^3/d^5/(e*x+d)-1/32*(-d^2*g^2-2*d*e*f*g+3*e^2*f^2)/e^3
/d^4/(e*x+d)^2+1/32*(-d^2*g^2+2*d*e*f*g+5*e^2*f^2)/e^3/d^6*ln(e*x+d)-1/24*
(d^2*g^2-2*d*e*f*g+e^2*f^2)/e^3/d^3/(e*x+d)^3
```

3.577.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(178) = 356$.

Time = 0.31 (sec) , antiderivative size = 662, normalized size of antiderivative = 3.52

$$\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)^3}\,dx = -\frac{16\,d^5e^2f^2 - 32\,d^6efg - 8\,d^7g^2 + 6\,(5\,de^6f^2 + 2\,d^2e^5fg - d^3e^4g^2)x^4 + 6\,(5\,d^2e^5f^2 + 2\,d^3e^4fg - d^4e^3g^2)x}{}$$

input
```
integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

output
```
-1/96*(16*d^5*e^2*f^2 - 32*d^6*e*f*g - 8*d^7*g^2 + 6*(5*d*e^6*f^2 + 2*d^2*
e^5*f*g - d^3*e^4*g^2)*x^4 + 6*(5*d^2*e^5*f^2 + 2*d^3*e^4*f*g - d^4*e^3*g^
2)*x^3 - 10*(5*d^3*e^4*f^2 + 2*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 - 2*(25*d^4*
e^3*f^2 + 10*d^5*e^2*f*g + 7*d^6*e*g^2)*x - 3*(5*d^5*e^2*f^2 + 2*d^6*e*f*g
 - d^7*g^2 + (5*e^7*f^2 + 2*d*e^6*f*g - d^2*e^5*g^2)*x^5 + (5*d*e^6*f^2 +
2*d^2*e^5*f*g - d^3*e^4*g^2)*x^4 - 2*(5*d^2*e^5*f^2 + 2*d^3*e^4*f*g - d^4*
e^3*g^2)*x^3 - 2*(5*d^3*e^4*f^2 + 2*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 + (5*d^
4*e^3*f^2 + 2*d^5*e^2*f*g - d^6*e*g^2)*x)*log(e*x + d) + 3*(5*d^5*e^2*f^2
+ 2*d^6*e*f*g - d^7*g^2 + (5*e^7*f^2 + 2*d*e^6*f*g - d^2*e^5*g^2)*x^5 + (5
*d*e^6*f^2 + 2*d^2*e^5*f*g - d^3*e^4*g^2)*x^4 - 2*(5*d^2*e^5*f^2 + 2*d^3*e
^4*f*g - d^4*e^3*g^2)*x^3 - 2*(5*d^3*e^4*f^2 + 2*d^4*e^3*f*g - d^5*e^2*g^2
)*x^2 + (5*d^4*e^3*f^2 + 2*d^5*e^2*f*g - d^6*e*g^2)*x)*log(e*x - d))/(d^6*
e^8*x^5 + d^7*e^7*x^4 - 2*d^8*e^6*x^3 - 2*d^9*e^5*x^2 + d^10*e^4*x + d^11*
e^3)
```

3.577.6 Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.71

$$\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)^3}\,dx = -\frac{-4d^6g^2-16d^5efg+8d^4e^2f^2+x^4(-3d^2e^4g^2+6de^5fg+15e^6f^2)+x^3(-3d^3e^3g^2+6d^2e^4fg+15de^5f^2 \ldots}{48d^{10}e^3+48d^9e^4x-96d^8e^5x^2-96d^7e^6x^3+ \ldots} + \frac{(d^2g^2-2defg-5e^2f^2)\log\left(-\frac{d}{e}+x\right)}{32d^6e^3} - \frac{(d^2g^2-2defg-5e^2f^2)\log\left(\frac{d}{e}+x\right)}{32d^6e^3}$$

input
```
integrate((g*x+f)**2/(e*x+d)/(-e**2*x**2+d**2)**3,x)
```

output
```
-(-4*d**6*g**2 - 16*d**5*e*f*g + 8*d**4*e**2*f**2 + x**4*(-3*d**2*e**4*g**
2 + 6*d*e**5*f*g + 15*e**6*f**2) + x**3*(-3*d**3*e**3*g**2 + 6*d**2*e**4*f
*g + 15*d*e**5*f**2) + x**2*(5*d**4*e**2*g**2 - 10*d**3*e**3*f*g - 25*d**2
*e**4*f**2) + x*(-7*d**5*e*g**2 - 10*d**4*e**2*f*g - 25*d**3*e**3*f**2))/(
48*d**10*e**3 + 48*d**9*e**4*x - 96*d**8*e**5*x**2 - 96*d**7*e**6*x**3 + 4
8*d**6*e**7*x**4 + 48*d**5*e**8*x**5) + (d**2*g**2 - 2*d*e*f*g - 5*e**2*f*
*2)*log(-d/e + x)/(32*d**6*e**3) - (d**2*g**2 - 2*d*e*f*g - 5*e**2*f**2)*l
og(d/e + x)/(32*d**6*e**3)
```

3.577.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.64

$$\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)^3}\,dx = -\frac{8\,d^4e^2f^2-16\,d^5efg-4\,d^6g^2+3\,(5\,e^6f^2+2\,de^5fg-d^2e^4g^2)x^4+3\,(5\,de^5f^2+2\,d^2e^4fg-d^3e^3g^2)x^3- \ldots}{48\,(d^5e^8x^5+d^6e^7x^4-2\,d^7e^6x^3-2\,d^8e^5x^2+ \ldots} + \frac{(5\,e^2f^2+2\,defg-d^2g^2)\log(ex+d)}{32\,d^6e^3} - \frac{(5\,e^2f^2+2\,defg-d^2g^2)\log(ex-d)}{32\,d^6e^3}$$

input
```
integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

output
```
-1/48*(8*d^4*e^2*f^2 - 16*d^5*e*f*g - 4*d^6*g^2 + 3*(5*e^6*f^2 + 2*d*e^5*f
*g - d^2*e^4*g^2)*x^4 + 3*(5*d*e^5*f^2 + 2*d^2*e^4*f*g - d^3*e^3*g^2)*x^3
- 5*(5*d^2*e^4*f^2 + 2*d^3*e^3*f*g - d^4*e^2*g^2)*x^2 - (25*d^3*e^3*f^2 +
10*d^4*e^2*f*g + 7*d^5*e*g^2)*x)/(d^5*e^8*x^5 + d^6*e^7*x^4 - 2*d^7*e^6*x^
3 - 2*d^8*e^5*x^2 + d^9*e^4*x + d^10*e^3) + 1/32*(5*e^2*f^2 + 2*d*e*f*g -
d^2*g^2)*log(e*x + d)/(d^6*e^3) - 1/32*(5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*l
og(e*x - d)/(d^6*e^3)
```

3.577.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.47

$$\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)^3}\,dx = \frac{(5e^2f^2+2defg-d^2g^2)\log(|ex+d|)}{32d^6e^3} - \frac{(5e^2f^2+2defg-d^2g^2)\log(|ex-d|)}{32d^6e^3} - \frac{8d^5e^2f^2-16d^6efg-4d^7g^2+3(5de^6f^2+2d^2e^5fg-d^3e^4g^2)x^4+3(5d^2e^5f^2+2d^3e^4fg-d^4e^3g^2)x^3}{48(ex+d)^3(ex-d)^2d^6e^3}$$

input
```
integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

output
```
1/32*(5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*log(abs(e*x + d))/(d^6*e^3) - 1/32*
(5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*log(abs(e*x - d))/(d^6*e^3) - 1/48*(8*d^
5*e^2*f^2 - 16*d^6*e*f*g - 4*d^7*g^2 + 3*(5*d*e^6*f^2 + 2*d^2*e^5*f*g - d^
3*e^4*g^2)*x^4 + 3*(5*d^2*e^5*f^2 + 2*d^3*e^4*f*g - d^4*e^3*g^2)*x^3 - 5*(
5*d^3*e^4*f^2 + 2*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 - (25*d^4*e^3*f^2 + 10*d^
5*e^2*f*g + 7*d^6*e*g^2)*x)/((e*x + d)^3*(e*x - d)^2*d^6*e^3)
```

3.577.9 Mupad [B] (verification not implemented)

Time = 11.80 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.32

$$\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)^3}\,dx = \frac{\frac{d^2g^2+4defg-2e^2f^2}{12de^3} - \frac{x^3(-d^2g^2+2defg+5e^2f^2)}{16d^4} - \frac{ex^4(-d^2g^2+2defg+5e^2f^2)}{16d^5} + \frac{x(7d^2g^2+10defg+25e^2f^2)}{48d^2e^2} + \frac{5x^2(-d^2}{}}{d^5+d^4ex-2d^3e^2x^2-2d^2e^3x^3+de^4x^4+e^5x^5} + \frac{\operatorname{atanh}\left(\frac{ex}{d}\right)(-d^2g^2+2defg+5e^2f^2)}{16d^6e^3}$$

input
```
int((f + g*x)^2/((d^2 - e^2*x^2)^3*(d + e*x)),x)
```

output
```
((d^2*g^2 - 2*e^2*f^2 + 4*d*e*f*g)/(12*d*e^3) - (x^3*(5*e^2*f^2 - d^2*g^2
+ 2*d*e*f*g))/(16*d^4) - (e*x^4*(5*e^2*f^2 - d^2*g^2 + 2*d*e*f*g))/(16*d^5
) + (x*(7*d^2*g^2 + 25*e^2*f^2 + 10*d*e*f*g))/(48*d^2*e^2) + (5*x^2*(5*e^2
*f^2 - d^2*g^2 + 2*d*e*f*g))/(48*d^3*e))/(d^5 + e^5*x^5 + d*e^4*x^4 - 2*d^
3*e^2*x^2 - 2*d^2*e^3*x^3 + d^4*e*x) + (atanh((e*x)/d)*(5*e^2*f^2 - d^2*g^
2 + 2*d*e*f*g))/(16*d^6*e^3)
```

3.577. $\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)^3}\,dx$

3.578 $\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^3}\,dx$

3.578.1 Optimal result

Integrand size = 29, antiderivative size = 235

$$\begin{aligned}\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^3}\,dx &= \frac{(ef+dg)^2}{64d^5e^3(d-ex)^2}+\frac{(ef+dg)(5ef+dg)}{64d^6e^3(d-ex)} \\ &\quad -\frac{(ef-dg)^2}{32d^3e^3(d+ex)^4}-\frac{(ef-dg)(3ef+dg)}{48d^4e^3(d+ex)^3} \\ &\quad -\frac{3e^2f^2-d^2g^2}{32d^5e^3(d+ex)^2}-\frac{5e^2f^2+2defg-d^2g^2}{32d^6e^3(d+ex)} \\ &\quad +\frac{(15e^2f^2+10defg-d^2g^2)\operatorname{arctanh}\left(\frac{ex}{d}\right)}{64d^7e^3}\end{aligned}$$

output
```
1/64*(d*g+e*f)^2/d^5/e^3/(-e*x+d)^2+1/64*(d*g+e*f)*(d*g+5*e*f)/d^6/e^3/(-e
*x+d)-1/32*(-d*g+e*f)^2/d^3/e^3/(e*x+d)^4-1/48*(-d*g+e*f)*(d*g+3*e*f)/d^4/
e^3/(e*x+d)^3+1/32*(d^2*g^2-3*e^2*f^2)/d^5/e^3/(e*x+d)^2+1/32*(d^2*g^2-2*d
*e*f*g-5*e^2*f^2)/d^6/e^3/(e*x+d)+1/64*(-d^2*g^2+10*d*e*f*g+15*e^2*f^2)*ar
ctanh(e*x/d)/d^7/e^3
```

3.578.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.04

$$\int \frac{(f+gx)^2}{(d+ex)^2 (d^2-e^2x^2)^3} dx$$
$$= \frac{\frac{6d^2(ef+dg)^2}{(d-ex)^2} + \frac{6d(5e^2f^2+6defg+d^2g^2)}{d-ex} - \frac{12d^4(ef-dg)^2}{(d+ex)^4} + \frac{8d^3(-3e^2f^2+2defg+d^2g^2)}{(d+ex)^3} + \frac{12d^2(-3e^2f^2+d^2g^2)}{(d+ex)^2} + \frac{12d(-5e^2f^2-2def}{d+ex}}{384d^7e^3}$$

input
```
Integrate[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^3),x]
```

output
```
((6*d^2*(e*f + d*g)^2)/(d - e*x)^2 + (6*d*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2
))/(d - e*x) - (12*d^4*(e*f - d*g)^2)/(d + e*x)^4 + (8*d^3*(-3*e^2*f^2 + 2
*d*e*f*g + d^2*g^2))/(d + e*x)^3 + (12*d^2*(-3*e^2*f^2 + d^2*g^2))/(d + e*
x)^2 + (12*d*(-5*e^2*f^2 - 2*d*e*f*g + d^2*g^2))/(d + e*x) + 3*(-15*e^2*f^
2 - 10*d*e*f*g + d^2*g^2)*Log[d - e*x] + 3*(15*e^2*f^2 + 10*d*e*f*g - d^2*
g^2)*Log[d + e*x])/(384*d^7*e^3)
```

3.578.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {639, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{(d+ex)^2 (d^2-e^2x^2)^3} dx$$
$$\Big\downarrow \text{ 639}$$
$$\int \frac{(f+gx)^2}{(d-ex)^3(d+ex)^5}dx$$
$$\Big\downarrow \text{ 99}$$
$$\int \left(\frac{(dg+ef)(dg+5ef)}{64d^6e^2(d-ex)^2} + \frac{(dg+ef)^2}{32d^5e^2(d-ex)^3} + \frac{(ef-dg)(dg+3ef)}{16d^4e^2(d+ex)^4} + \frac{(dg-ef)^2}{8d^3e^2(d+ex)^5} + \frac{d^2g^2-10defg-15e^2f}{64d^6e^2(e^2x^2-d^2)}\right.$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{\operatorname{arctanh}\left(\frac{ex}{d}\right)\left(-d^2g^2+10defg+15e^2f^2\right)}{64d^7e^3}+\frac{(dg+ef)(dg+5ef)}{64d^6e^3(d-ex)}+\frac{(dg+ef)^2}{64d^5e^3(d-ex)^2}-\frac{(dg+3ef)(ef-dg)}{48d^4e^3(d+ex)^3}-\frac{(ef-dg)^2}{32d^3e^3(d+ex)^4}-\frac{-d^2g^2+2defg+5e^2f^2}{32d^6e^3(d+ex)}-\frac{3e^2f^2-d^2g^2}{32d^5e^3(d+ex)^2}$$

input
```
Int[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^3),x]
```

output
```
(e*f + d*g)^2/(64*d^5*e^3*(d - e*x)^2) + ((e*f + d*g)*(5*e*f + d*g))/(64*d
^6*e^3*(d - e*x)) - (e*f - d*g)^2/(32*d^3*e^3*(d + e*x)^4) - ((e*f - d*g)*
(3*e*f + d*g))/(48*d^4*e^3*(d + e*x)^3) - (3*e^2*f^2 - d^2*g^2)/(32*d^5*e^
3*(d + e*x)^2) - (5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)/(32*d^6*e^3*(d + e*x))
+ ((15*e^2*f^2 + 10*d*e*f*g - d^2*g^2)*ArcTanh[(e*x)/d])/(64*d^7*e^3)
```

3.578.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.578.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.22

method	result
norman	$\frac{\frac{\left(31d^2g^2+74defg-81e^2f^2\right)x^3}{96d^4}+\frac{\left(d^2g^2+22defg+17e^2f^2\right)x^2}{32e\,d^3}+\frac{e\left(11d^2g^2-14defg-69e^2f^2\right)x^4}{96d^5}-\frac{e^2\left(29d^2g^2+94defg-51e^2f^2\right)x^5}{192d^6}-\frac{e^3\left(d^2\right.}{}}{(ex+d)^4(-ex+d)^2}$
default	$\frac{d^2g^2+6defg+5e^2f^2}{64e^3d^6(-ex+d)}-\frac{-d^2g^2-2defg-e^2f^2}{64e^3d^5(-ex+d)^2}+\frac{(d^2g^2-10defg-15e^2f^2)\ln(-ex+d)}{128d^7e^3}-\frac{-d^2g^2+3e^2f^2}{32e^3d^5(ex+d)^2}-\frac{-d^2g^2-2def}{48e^3d^4(ex}$
risch	$\frac{\frac{\left(d^2g^2-10defg-15e^2f^2\right)e^2x^5}{64d^6}+\frac{\left(d^2g^2-10defg-15e^2f^2\right)e\,x^4}{32d^5}-\frac{\left(d^2g^2-10defg-15e^2f^2\right)x^3}{96d^4}-\frac{5\left(d^2g^2-10defg-15e^2f^2\right)x^2}{96d^3e}+\frac{\left(35d^2g^2+3\right.}{1}}{(ex+d)^2(-e^2x^2+d^2)^2}$
parallelrisch	Expression too large to display

input
```
int((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/96*(31*d^2*g^2+74*d*e*f*g-81*e^2*f^2)/d^4*x^3+1/32/e*(d^2*g^2+22*d*e*f*
g+17*e^2*f^2)/d^3*x^2+1/96*e*(11*d^2*g^2-14*d*e*f*g-69*e^2*f^2)/d^5*x^4-1/
192*e^2*(29*d^2*g^2+94*d*e*f*g-51*e^2*f^2)/d^6*x^5-1/12*e^3*(d^2*g^2+2*d*e
*f*g-3*e^2*f^2)/d^7*x^6+1/64*(d^2*g^2-10*d*e*f*g+49*e^2*f^2)/d^2/e^2*x)/(e
*x+d)^4/(-e*x+d)^2+1/128*(d^2*g^2-10*d*e*f*g-15*e^2*f^2)/d^7/e^3*ln(-e*x+d
)-1/128*(d^2*g^2-10*d*e*f*g-15*e^2*f^2)/d^7/e^3*ln(e*x+d)
```

3.578.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(223) = 446$.

Time = 0.35 (sec) , antiderivative size = 793, normalized size of antiderivative = 3.37

$$\int \frac{(f+gx)^2}{(d+ex)^2\,(d^2-e^2x^2)^3}\,dx =$$
$$-\frac{96\,d^6e^2f^2-64\,d^7efg-32\,d^8g^2+6\,(15\,de^7f^2+10\,d^2e^6fg-d^3e^5g^2)x^5+12\,(15\,d^2e^6f^2+10\,d^3e^5fg-d}{}$$

input
```
integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

output
```
-1/384*(96*d^6*e^2*f^2 - 64*d^7*e*f*g - 32*d^8*g^2 + 6*(15*d*e^7*f^2 + 10*
d^2*e^6*f*g - d^3*e^5*g^2)*x^5 + 12*(15*d^2*e^6*f^2 + 10*d^3*e^5*f*g - d^4
*e^4*g^2)*x^4 - 4*(15*d^3*e^5*f^2 + 10*d^4*e^4*f*g - d^5*e^3*g^2)*x^3 - 20
*(15*d^4*e^4*f^2 + 10*d^5*e^3*f*g - d^6*e^2*g^2)*x^2 - 2*(51*d^5*e^3*f^2 +
 34*d^6*e^2*f*g + 35*d^7*e*g^2)*x - 3*(15*d^6*e^2*f^2 + 10*d^7*e*f*g - d^8
*g^2 + (15*e^8*f^2 + 10*d*e^7*f*g - d^2*e^6*g^2)*x^6 + 2*(15*d*e^7*f^2 + 1
0*d^2*e^6*f*g - d^3*e^5*g^2)*x^5 - (15*d^2*e^6*f^2 + 10*d^3*e^5*f*g - d^4*
e^4*g^2)*x^4 - 4*(15*d^3*e^5*f^2 + 10*d^4*e^4*f*g - d^5*e^3*g^2)*x^3 - (15
*d^4*e^4*f^2 + 10*d^5*e^3*f*g - d^6*e^2*g^2)*x^2 + 2*(15*d^5*e^3*f^2 + 10*
d^6*e^2*f*g - d^7*e*g^2)*x)*log(e*x + d) + 3*(15*d^6*e^2*f^2 + 10*d^7*e*f*
g - d^8*g^2 + (15*e^8*f^2 + 10*d*e^7*f*g - d^2*e^6*g^2)*x^6 + 2*(15*d*e^7*
f^2 + 10*d^2*e^6*f*g - d^3*e^5*g^2)*x^5 - (15*d^2*e^6*f^2 + 10*d^3*e^5*f*g
 - d^4*e^4*g^2)*x^4 - 4*(15*d^3*e^5*f^2 + 10*d^4*e^4*f*g - d^5*e^3*g^2)*x^
3 - (15*d^4*e^4*f^2 + 10*d^5*e^3*f*g - d^6*e^2*g^2)*x^2 + 2*(15*d^5*e^3*f^
2 + 10*d^6*e^2*f*g - d^7*e*g^2)*x)*log(e*x - d))/(d^7*e^9*x^6 + 2*d^8*e^8*
x^5 - d^9*e^7*x^4 - 4*d^10*e^6*x^3 - d^11*e^5*x^2 + 2*d^12*e^4*x + d^13*e^
3)
```

3.578.6 Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.58

$$\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)^3}\,dx = -\frac{-16d^7g^2-32d^6efg+48d^5e^2f^2+x^5(-3d^2e^5g^2+30de^6fg+45e^7f^2)+x^4(-6d^3e^4g^2+60d^2e^5fg+90d}{192d^{12}e^3+384d^{11}e^4x-192d^1} + \frac{(d^2g^2-10defg-15e^2f^2)\log\left(-\frac{d}{e}+x\right)}{128d^7e^3} - \frac{(d^2g^2-10defg-15e^2f^2)\log\left(\frac{d}{e}+x\right)}{128d^7e^3}$$

input
```
integrate((g*x+f)**2/(e*x+d)**2/(-e**2*x**2+d**2)**3,x)
```

output
```
-(-16*d**7*g**2 - 32*d**6*e*f*g + 48*d**5*e**2*f**2 + x**5*(-3*d**2*e**5*g
**2 + 30*d*e**6*f*g + 45*e**7*f**2) + x**4*(-6*d**3*e**4*g**2 + 60*d**2*e*
*5*f*g + 90*d*e**6*f**2) + x**3*(2*d**4*e**3*g**2 - 20*d**3*e**4*f*g - 30*
d**2*e**5*f**2) + x**2*(10*d**5*e**2*g**2 - 100*d**4*e**3*f*g - 150*d**3*e
**4*f**2) + x*(-35*d**6*e*g**2 - 34*d**5*e**2*f*g - 51*d**4*e**3*f**2))/(1
92*d**12*e**3 + 384*d**11*e**4*x - 192*d**10*e**5*x**2 - 768*d**9*e**6*x**
3 - 192*d**8*e**7*x**4 + 384*d**7*e**8*x**5 + 192*d**6*e**9*x**6) + (d**2*
g**2 - 10*d*e*f*g - 15*e**2*f**2)*log(-d/e + x)/(128*d**7*e**3) - (d**2*g*
*2 - 10*d*e*f*g - 15*e**2*f**2)*log(d/e + x)/(128*d**7*e**3)
```

3.578.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.53

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^3}\,dx = -\frac{48\,d^5e^2f^2-32\,d^6efg-16\,d^7g^2+3\,(15\,e^7f^2+10\,de^6fg-d^2e^5g^2)x^5+6\,(15\,de^6f^2+10\,d^2e^5fg-d^3e^4g}{192\,(d^6e^9x^6+2\,d^7e^8x^5-}$$
$$+\frac{(15\,e^2f^2+10\,defg-d^2g^2)\log(ex+d)}{128\,d^7e^3}-\frac{(15\,e^2f^2+10\,defg-d^2g^2)\log(ex-d)}{128\,d^7e^3}$$

input
```
integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

output
```
-1/192*(48*d^5*e^2*f^2 - 32*d^6*e*f*g - 16*d^7*g^2 + 3*(15*e^7*f^2 + 10*d*
e^6*f*g - d^2*e^5*g^2)*x^5 + 6*(15*d*e^6*f^2 + 10*d^2*e^5*f*g - d^3*e^4*g^
2)*x^4 - 2*(15*d^2*e^5*f^2 + 10*d^3*e^4*f*g - d^4*e^3*g^2)*x^3 - 10*(15*d^
3*e^4*f^2 + 10*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 - (51*d^4*e^3*f^2 + 34*d^5*e
^2*f*g + 35*d^6*e*g^2)*x)/(d^6*e^9*x^6 + 2*d^7*e^8*x^5 - d^8*e^7*x^4 - 4*d
^9*e^6*x^3 - d^10*e^5*x^2 + 2*d^11*e^4*x + d^12*e^3) + 1/128*(15*e^2*f^2 +
 10*d*e*f*g - d^2*g^2)*log(e*x + d)/(d^7*e^3) - 1/128*(15*e^2*f^2 + 10*d*e
*f*g - d^2*g^2)*log(e*x - d)/(d^7*e^3)
```

3.578.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.43

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^3}\,dx = -\frac{(15\,e^2f^2+10\,defg-d^2g^2)\log\left(\left|-\frac{2\,d}{ex+d}+1\right|\right)}{128\,d^7e^3}$$
$$-\frac{11\,e^2f^2+14\,defg+3\,d^2g^2-\frac{8\,(3\,de^3f^2+4\,d^2e^2fg+d^3eg^2)}{(ex+d)e}}{256\,d^7e^3\left(\frac{2\,d}{ex+d}-1\right)^2}$$
$$-\frac{\frac{15\,d^6e^{11}f^2}{ex+d}+\frac{9\,d^7e^{11}f^2}{(ex+d)^2}+\frac{6\,d^8e^{11}f^2}{(ex+d)^3}+\frac{3\,d^9e^{11}f^2}{(ex+d)^4}+\frac{6\,d^7e^{10}fg}{ex+d}-\frac{4\,d^9e^{10}fg}{(ex+d)^3}-\frac{6\,d^{10}e^{10}fg}{(ex+d)^4}-\frac{3\,d^8e^9g^2}{ex+d}-\frac{3\,d^9e^9g^2}{(ex+d)^2}-\frac{2\,d^{10}e^9g^2}{(ex+d)^3}}{96\,d^{12}e^{12}}$$

input
```
integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

output
```
-1/128*(15*e^2*f^2 + 10*d*e*f*g - d^2*g^2)*log(abs(-2*d/(e*x + d) + 1))/(d
^7*e^3) - 1/256*(11*e^2*f^2 + 14*d*e*f*g + 3*d^2*g^2 - 8*(3*d*e^3*f^2 + 4*
d^2*e^2*f*g + d^3*e*g^2)/((e*x + d)*e))/(d^7*e^3*(2*d/(e*x + d) - 1)^2) -
1/96*(15*d^6*e^11*f^2/(e*x + d) + 9*d^7*e^11*f^2/(e*x + d)^2 + 6*d^8*e^11*
f^2/(e*x + d)^3 + 3*d^9*e^11*f^2/(e*x + d)^4 + 6*d^7*e^10*f*g/(e*x + d) -
4*d^9*e^10*f*g/(e*x + d)^3 - 6*d^10*e^10*f*g/(e*x + d)^4 - 3*d^8*e^9*g^2/(
e*x + d) - 3*d^9*e^9*g^2/(e*x + d)^2 - 2*d^10*e^9*g^2/(e*x + d)^3 + 3*d^11
*e^9*g^2/(e*x + d)^4)/(d^12*e^12)
```

3.578.9 Mupad [B] (verification not implemented)

Time = 11.94 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.26

$$\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)^3}\,dx$$
$$= \frac{\frac{d^2g^2+2\,d\,e\,f\,g-3\,e^2f^2}{12\,d\,e^3}+\frac{x^3\left(-d^2g^2+10\,d\,e\,f\,g+15\,e^2f^2\right)}{96\,d^4}-\frac{e\,x^4\left(-d^2g^2+10\,d\,e\,f\,g+15\,e^2f^2\right)}{32\,d^5}+\frac{x\left(35\,d^2g^2+34\,d\,e\,f\,g+51\,e^2f^2\right)}{192\,d^2e^2}+\frac{5\,x^2}{}}{d^6+2\,d^5\,e\,x-d^4\,e^2\,x^2-4\,d^3\,e^3\,x^3-d^2\,e^4\,x^4+2\,d\,e^5\,x^5+e}$$
$$+\frac{\operatorname{atanh}\left(\frac{e\,x}{d}\right)\left(-d^2g^2+10\,d\,e\,f\,g+15\,e^2f^2\right)}{64\,d^7e^3}$$

input
```
int((f + g*x)^2/((d^2 - e^2*x^2)^3*(d + e*x)^2),x)
```

output
```
((d^2*g^2 - 3*e^2*f^2 + 2*d*e*f*g)/(12*d*e^3) + (x^3*(15*e^2*f^2 - d^2*g^2
 + 10*d*e*f*g))/(96*d^4) - (e*x^4*(15*e^2*f^2 - d^2*g^2 + 10*d*e*f*g))/(32
*d^5) + (x*(35*d^2*g^2 + 51*e^2*f^2 + 34*d*e*f*g))/(192*d^2*e^2) + (5*x^2*
(15*e^2*f^2 - d^2*g^2 + 10*d*e*f*g))/(96*d^3*e) - (e^2*x^5*(15*e^2*f^2 - d
^2*g^2 + 10*d*e*f*g))/(64*d^6))/(d^6 + e^6*x^6 + 2*d*e^5*x^5 - d^4*e^2*x^2
 - 4*d^3*e^3*x^3 - d^2*e^4*x^4 + 2*d^5*e*x) + (atanh((e*x)/d)*(15*e^2*f^2
- d^2*g^2 + 10*d*e*f*g))/(64*d^7*e^3)
```

3.579 $\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx$

3.579.1 Optimal result

Integrand size = 31, antiderivative size = 269

$$\begin{aligned}\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx &= \frac{(ef+dg)^5(d+ex)^3}{5de^6(d^2-e^2x^2)^{5/2}}+\frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6(d^2-e^2x^2)^{3/2}}\\ &+\frac{(ef+dg)^3(2e^2f^2-21defg+127d^2g^2)(d+ex)}{15d^3e^6\sqrt{d^2-e^2x^2}}+\frac{g^4(5ef+3dg)\sqrt{d^2-e^2x^2}}{e^6}\\ &+\frac{g^5x\sqrt{d^2-e^2x^2}}{2e^5}-\frac{g^3(20e^2f^2+30defg+13d^2g^2)\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}\end{aligned}$$

output
```
1/5*(d*g+e*f)^5*(e*x+d)^3/d/e^6/(-e^2*x^2+d^2)^(5/2)+1/15*(-23*d*g+2*e*f)*
(d*g+e*f)^4*(e*x+d)^2/d^2/e^6/(-e^2*x^2+d^2)^(3/2)-1/2*g^3*(13*d^2*g^2+30*
d*e*f*g+20*e^2*f^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+1/15*(d*g+e*f)^3*
(127*d^2*g^2-21*d*e*f*g+2*e^2*f^2)*(e*x+d)/d^3/e^6/(-e^2*x^2+d^2)^(1/2)+g^
4*(3*d*g+5*e*f)*(-e^2*x^2+d^2)^(1/2)/e^6+1/2*g^5*x*(-e^2*x^2+d^2)^(1/2)/e^
5
```

3.579.2 Mathematica [A] (verified)

Time = 3.38 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}\left(304d^7g^5+4e^7f^5x^2+3d^6eg^4(240f-239gx)-6de^6f^4x(2f+5gx)+2d^2e^5f^3\left(7f^2+45fgx+70g^2x^2\right)+\right.}{}$$

input
```
Integrate[((d + e*x)^3*(f + g*x)^5)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(304*d^7*g^5 + 4*e^7*f^5*x^2 + 3*d^6*e*g^4*(240*f -
239*g*x) - 6*d*e^6*f^4*x*(2*f + 5*g*x) + 2*d^2*e^5*f^3*(7*f^2 + 45*f*g*x +
 70*g^2*x^2) + d^5*e^2*g^3*(440*f^2 - 1710*f*g*x + 479*g^2*x^2) + 5*d^4*e^
3*g^2*(8*f^3 - 204*f^2*g*x + 234*f*g^2*x^2 - 9*g^3*x^3) - 5*d^3*e^4*g*(6*f
^4 + 24*f^3*g*x - 128*f^2*g^2*x^2 + 30*f*g^3*x^3 + 3*g^4*x^4)))/(d^3*(d -
e*x)^3) + 30*g^3*(20*e^2*f^2 + 30*d*e*f*g + 13*d^2*g^2)*ArcTan[(e*x)/(Sqrt
[d^2] - Sqrt[d^2 - e^2*x^2])])/(30*e^6)
```

3.579.3 Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {691, 25, 2166, 25, 2166, 27, 2346, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{691}$$

$$\frac{(d+ex)^3(dg+ef)^5}{5de^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int -\frac{(d+ex)^2\left(-\frac{5dx^4g^5}{e}-\frac{5d(5ef+dg)x^3g^4}{e^2}-\frac{5d\left(10e^2f^2+5degf+d^2g^2\right)x^2g^3}{e^3}-\frac{5d\left(10e^3f^3+10de^2gf^2+5d^2eg^2f+d^3g^3\right)xg^2}{e^4}+\frac{2e^5f^5-15de^4gf^4-30d^2e^3g^2f^3\cdots}{e\cdots}\right.}{\left(d^2-e^2x^2\right)^{5/2}}}{5d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(d+ex)^2\left(-\frac{5dx^4g^5}{e}-\frac{5d(5ef+dg)x^3g^4}{e^2}-\frac{5d\left(10e^2f^2+5degf+d^2g^2\right)x^2g^3}{e^3}-\frac{5d\left(10e^3f^3+10de^2gf^2+5d^2eg^2f+d^3g^3\right)xg^2}{e^4}+\frac{2e^5f^5-15de^4gf^4-30d^2e^3g^2f^3-3}{e^5}\right.}{(d^2-e^2x^2)^{5/2}}}{5d}$$

$$\frac{(d+ex)^3(dg+ef)^5}{5de^6\left(d^2-e^2x^2\right)^{5/2}}$$

$\downarrow$ 2166

$$\frac{\frac{(d+ex)^2(2ef-23dg)(dg+ef)^4}{3de^6(d^2-e^2x^2)^{3/2}}-\frac{\int -\frac{(d+ex)\left(\frac{15d^2x^3g^5}{e^2}+\frac{15d^2(5ef+2dg)x^2g^4}{e^3}+\frac{15d^2\left(10e^2f^2+10degf+3d^2g^2\right)xg^3}{e^4}+\frac{2e^5f^5-15de^4gf^4+70d^2e^3g^2f^3+170d}{e^5}\right.}{\left(d^2-e^2x^2\right)^{3/2}}}{3d}}{5d}$$

$$\frac{(d+ex)^3(dg+ef)^5}{5de^6\left(d^2-e^2x^2\right)^{5/2}}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{(d+ex)\left(\frac{15d^2x^3g^5}{e^2}+\frac{15d^2(5ef+2dg)x^2g^4}{e^3}+\frac{15d^2\left(10e^2f^2+10degf+3d^2g^2\right)xg^3}{e^4}+\frac{2e^5f^5-15de^4gf^4+70d^2e^3g^2f^3+170d^3e^2g^3f^2+135d^4eg^4f+37d^5g^5}{e^5}\right)}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d}+}{5d}$$

$$\frac{(d+ex)^3(dg+ef)^5}{5de^6\left(d^2-e^2x^2\right)^{5/2}}$$

$\downarrow$ 2166

$$\frac{\frac{\frac{(d+ex)(dg+ef)^3\left(127d^2g^2-21defg+2e^2f^2\right)}{de^6\sqrt{d^2-e^2x^2}}-\frac{\int \frac{15\left(\frac{d^3x^2g^5}{e^3}+\frac{d^3(5ef+3dg)xg^4}{e^4}+\frac{d^3\left(10e^2f^2+15degf+6d^2g^2\right)g^3}{e^5}\right)}{\sqrt{d^2-e^2x^2}}dx}{d}}{3d}+\frac{(d+ex)^2(2ef-23dg)(dg+ef)^4}{3de^6(d^2-e^2x^2)^{3/2}}+}{5d}$$

$$\frac{(d+ex)^3(dg+ef)^5}{5de^6\left(d^2-e^2x^2\right)^{5/2}}$$

$\downarrow$ 27

$$\frac{\frac{\frac{(d+ex)(dg+ef)^3\left(127d^2g^2-21defg+2e^2f^2\right)}{de^6\sqrt{d^2-e^2x^2}}-\frac{15\int \frac{\frac{d^3x^2g^5}{e^3}+\frac{d^3(5ef+3dg)xg^4}{e^4}+\frac{d^3\left(10e^2f^2+15degf+6d^2g^2\right)g^3}{e^5}}{\sqrt{d^2-e^2x^2}}dx}{d}}{3d}+\frac{(d+ex)^2(2ef-23dg)(dg+ef)^4}{3de^6(d^2-e^2x^2)^{3/2}}+}{5d}$$

$$\frac{(d+ex)^3(dg+ef)^5}{5de^6\left(d^2-e^2x^2\right)^{5/2}}$$

3.579. $\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx$

$$\Big\downarrow \; 2346$$

$$\frac{\frac{\frac{(d+ex)(dg+ef)^3\left(127d^2g^2-21defg+2e^2f^2\right)}{de^6\sqrt{d^2-e^2x^2}}-\frac{15\left(-\frac{\int -\frac{d^3g^3\left(20e^2f^2+30degf+13d^2g^2+2eg(5ef+3dg)x\right)}{e^3\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{d^3g^5x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d}}{3d}+\frac{(d+ex)^2(2ef-23dg)(d\cdots}{3de^6(d^2-e^2x^2)^3\cdots}}{5d}$$
$$\frac{(d+ex)^3(dg+ef)^5}{5de^6\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{(d+ex)(dg+ef)^3\left(127d^2g^2-21defg+2e^2f^2\right)}{de^6\sqrt{d^2-e^2x^2}}-\frac{15\left(\frac{\int \frac{d^3g^3\left(20e^2f^2+30degf+13d^2g^2+2eg(5ef+3dg)x\right)}{e^3\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{d^3g^5x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d}}{3d}+\frac{(d+ex)^2(2ef-23dg)(dg+\cdots}{3de^6(d^2-e^2x^2)^{3/2}}}{5d}$$
$$\frac{(d+ex)^3(dg+ef)^5}{5de^6\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{(d+ex)(dg+ef)^3\left(127d^2g^2-21defg+2e^2f^2\right)}{de^6\sqrt{d^2-e^2x^2}}-\frac{15\left(\frac{d^3g^3\int \frac{20e^2f^2+30degf+13d^2g^2+2eg(5ef+3dg)x}{\sqrt{d^2-e^2x^2}}dx}{2e^5}-\frac{d^3g^5x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d}}{3d}+\frac{(d+ex)^2(2ef-23dg)(dg+e\cdots}{3de^6(d^2-e^2x^2)^{3/2}}}{5d}$$
$$\frac{(d+ex)^3(dg+ef)^5}{5de^6\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 455$$

$$\frac{\frac{\frac{(d+ex)(dg+ef)^3\left(127d^2g^2-21defg+2e^2f^2\right)}{de^6\sqrt{d^2-e^2x^2}}-\frac{15\left(\frac{d^3g^3\left(\left(13d^2g^2+30defg+20e^2f^2\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{2g\sqrt{d^2-e^2x^2}(3dg+5ef)}{e}\right)}{2e^5}-\frac{d^3g^5x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d}}{3d}+\frac{(d+\cdots}{\cdots}}{5d}$$
$$\frac{(d+ex)^3(dg+ef)^5}{5de^6\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 224$$

3.579. $\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx$

$$\frac{\frac{\frac{(d+ex)(dg+ef)^3\left(127d^2g^2-21defg+2e^2f^2\right)}{de^6\sqrt{d^2-e^2x^2}}-\frac{15\left(\frac{d^3g^3\left(\left(13d^2g^2+30defg+20e^2f^2\right)\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{2g\sqrt{d^2-e^2x^2}(3dg+5ef)}{e}\right)}{2e^5}-\frac{d^3g^5x\sqrt{d^2-e^2x^2}}{2e^5}\right.}{d}}{3d}}{5d}$$

$$\frac{(d+ex)^3(dg+ef)^5}{5de^6\left(d^2-e^2x^2\right)^{5/2}}$$

$$\downarrow \ 216$$

$$\frac{\frac{\frac{(d+ex)(dg+ef)^3\left(127d^2g^2-21defg+2e^2f^2\right)}{de^6\sqrt{d^2-e^2x^2}}-\frac{15\left(\frac{d^3g^3\left(\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(13d^2g^2+30defg+20e^2f^2\right)}{e}-\frac{2g\sqrt{d^2-e^2x^2}(3dg+5ef)}{e}\right)}{2e^5}-\frac{d^3g^5x\sqrt{d^2-e^2x^2}}{2e^5}\right)}{d}}{3d}+}{5d}$$

$$\frac{(d+ex)^3(dg+ef)^5}{5de^6\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[((d + e*x)^3*(f + g*x)^5)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
((e*f + d*g)^5*(d + e*x)^3)/(5*d*e^6*(d^2 - e^2*x^2)^(5/2)) + (((2*e*f - 2
3*d*g)*(e*f + d*g)^4*(d + e*x)^2)/(3*d*e^6*(d^2 - e^2*x^2)^(3/2)) + (((e*f
 + d*g)^3*(2*e^2*f^2 - 21*d*e*f*g + 127*d^2*g^2)*(d + e*x))/(d*e^6*Sqrt[d^
2 - e^2*x^2]) - (15*(-1/2*(d^3*g^5*x*Sqrt[d^2 - e^2*x^2])/e^5 + (d^3*g^3*(
(-2*g*(5*e*f + 3*d*g)*Sqrt[d^2 - e^2*x^2])/e + ((20*e^2*f^2 + 30*d*e*f*g +
 13*d^2*g^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/(2*e^5)))/d)/(3*d))/(5
*d)
```

3.579.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 691
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)
^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(f + g*x)^n, a*e + c*d*
x, x], R = PolynomialRemainder[(f + g*x)^n, a*e + c*d*x, x]}, Simp[(-d)*R*(
d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*(p + 1))), x] + Simp[d/(2*a*(p + 1))
   Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*e*(p + 1)*Q +
R*(m + 2*p + 2), x], x], x]] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[n, 1]
&& IGtQ[m, 0] && LtQ[p, -1] && EqQ[c*d^2 + a*e^2, 0]
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.579. $\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx$

3.579.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(247) = 494$.

Time = 1.14 (sec) , antiderivative size = 604, normalized size of antiderivative = 2.25

method	result
risch	$\frac{g^4(egx+6dg+10ef)\sqrt{-e^2x^2+d^2}}{2e^6} - \frac{\frac{13d^2g^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} + \frac{20e^2f^2g^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} + \frac{30defg^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} +$
default	Expression too large to display

input
```
int((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*g^4*(e*g*x+6*d*g+10*e*f)/e^6*(-e^2*x^2+d^2)^(1/2)-1/2/e^5*(13*d^2*g^5/
(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+20*e^2*f^2*g^3/(e^2
)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+30*d*e*f*g^4/(e^2)^(1/2
)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+20*g^2*(d^3*g^3+3*d^2*e*f*g^2
+3*d*e^2*f^2*g+e^3*f^3)/e^2/d/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
+10*g*(d^4*g^4+4*d^3*e*f*g^3+6*d^2*e^2*f^2*g^2+4*d*e^3*f^3*g+e^4*f^4)/e^2*
(1/3/d/e/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-1/3/d^2/(x-d/e)*(-
(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2))+(2*d^5*g^5+10*d^4*e*f*g^4+20*d^3*e^2*f
^2*g^3+20*d^2*e^3*f^3*g^2+10*d*e^4*f^4*g+2*e^5*f^5)/e^3*(1/5/d/e/(x-d/e)^3
*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-2/5*e/d*(1/3/d/e/(x-d/e)^2*(-(x-d/e)
^2*e^2-2*d*e*(x-d/e))^(1/2)-1/3/d^2/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))
^(1/2))))
```

3.579.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(247) = 494$.

Time = 0.33 (sec) , antiderivative size = 807, normalized size of antiderivative = 3.00

$$\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx =$$

$$-\frac{14\,d^3e^5f^5 - 30\,d^4e^4f^4g + 40\,d^5e^3f^3g^2 + 440\,d^6e^2f^2g^3 + 720\,d^7efg^4 + 304\,d^8g^5 - 2\,(7\,e^8f^5 - 15\,de^7f^4g + 2}{}$$

input
```
integrate((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/30*(14*d^3*e^5*f^5 - 30*d^4*e^4*f^4*g + 40*d^5*e^3*f^3*g^2 + 440*d^6*e^
2*f^2*g^3 + 720*d^7*e*f*g^4 + 304*d^8*g^5 - 2*(7*e^8*f^5 - 15*d*e^7*f^4*g
+ 20*d^2*e^6*f^3*g^2 + 220*d^3*e^5*f^2*g^3 + 360*d^4*e^4*f*g^4 + 152*d^5*e
^3*g^5)*x^3 + 6*(7*d*e^7*f^5 - 15*d^2*e^6*f^4*g + 20*d^3*e^5*f^3*g^2 + 220
*d^4*e^4*f^2*g^3 + 360*d^5*e^3*f*g^4 + 152*d^6*e^2*g^5)*x^2 - 6*(7*d^2*e^6
*f^5 - 15*d^3*e^5*f^4*g + 20*d^4*e^4*f^3*g^2 + 220*d^5*e^3*f^2*g^3 + 360*d
^6*e^2*f*g^4 + 152*d^7*e*g^5)*x + 30*(20*d^6*e^2*f^2*g^3 + 30*d^7*e*f*g^4
+ 13*d^8*g^5 - (20*d^3*e^5*f^2*g^3 + 30*d^4*e^4*f*g^4 + 13*d^5*e^3*g^5)*x^
3 + 3*(20*d^4*e^4*f^2*g^3 + 30*d^5*e^3*f*g^4 + 13*d^6*e^2*g^5)*x^2 - 3*(20
*d^5*e^3*f^2*g^3 + 30*d^6*e^2*f*g^4 + 13*d^7*e*g^5)*x)*arctan(-(d - sqrt(-
e^2*x^2 + d^2))/(e*x)) - (15*d^3*e^4*g^5*x^4 - 14*d^2*e^5*f^5 + 30*d^3*e^4
*f^4*g - 40*d^4*e^3*f^3*g^2 - 440*d^5*e^2*f^2*g^3 - 720*d^6*e*f*g^4 - 304*
d^7*g^5 + 15*(10*d^3*e^4*f*g^4 + 3*d^4*e^3*g^5)*x^3 - (4*e^7*f^5 - 30*d*e^
6*f^4*g + 140*d^2*e^5*f^3*g^2 + 640*d^3*e^4*f^2*g^3 + 1170*d^4*e^3*f*g^4 +
 479*d^5*e^2*g^5)*x^2 + 3*(4*d*e^6*f^5 - 30*d^2*e^5*f^4*g + 40*d^3*e^4*f^3
*g^2 + 340*d^4*e^3*f^2*g^3 + 570*d^5*e^2*f*g^4 + 239*d^6*e*g^5)*x)*sqrt(-e
^2*x^2 + d^2))/(d^3*e^9*x^3 - 3*d^4*e^8*x^2 + 3*d^5*e^7*x - d^6*e^6)
```

3.579.6 Sympy [F]

$$\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^3\,(f+gx)^5}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**3*(g*x+f)**5/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**3*(f + g*x)**5/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.579.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1603 vs. 2(247) = 494.

Time = 0.29 (sec) , antiderivative size = 1603, normalized size of antiderivative = 5.96

$$\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
-1/2*e*g^5*x^7/(-e^2*x^2 + d^2)^(5/2) + 7/30*d^2*e*g^5*x*(15*x^4/((-e^2*x^
2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e
^2*x^2 + d^2)^(5/2)*e^6)) - 7/6*d^2*g^5*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e
^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e + 1/5*d*f^5*x/(-e^2*x^2 + d^2)
^(5/2) + 3/5*d^2*f^5/((-e^2*x^2 + d^2)^(5/2)*e) + d^3*f^4*g/((-e^2*x^2 + d
^2)^(5/2)*e^2) + 4/15*f^5*x/((-e^2*x^2 + d^2)^(3/2)*d) + 14/15*d^4*g^5*x/(
(-e^2*x^2 + d^2)^(3/2)*e^5) + 1/15*(10*e^3*f^2*g^3 + 15*d*e^2*f*g^4 + 3*d^
2*e*g^5)*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 +
d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) + 8/15*f^5*x/(sqrt(-
e^2*x^2 + d^2)*d^3) - 49/30*d^2*g^5*x/(sqrt(-e^2*x^2 + d^2)*e^5) - (5*e^3*
f*g^4 + 3*d*e^2*g^5)*x^6/((-e^2*x^2 + d^2)^(5/2)*e^2) - 7/2*d^2*g^5*arcsin
(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^5) - 1/3*(10*e^3*f^2*g^3 + 15*d*e^2*f*g
^4 + 3*d^2*e*g^5)*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2
 + d^2)^(3/2)*e^4))/e^2 + 6*(5*e^3*f*g^4 + 3*d*e^2*g^5)*d^2*x^4/((-e^2*x^2
 + d^2)^(5/2)*e^4) + (10*e^3*f^3*g^2 + 30*d*e^2*f^2*g^3 + 15*d^2*e*f*g^4 +
 d^3*g^5)*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) + 5/2*(e^3*f^4*g + 6*d*e^2*f^3*
g^2 + 6*d^2*e*f^2*g^3 + d^3*f*g^4)*x^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 8*(5
*e^3*f*g^4 + 3*d*e^2*g^5)*d^4*x^2/((-e^2*x^2 + d^2)^(5/2)*e^6) - 4/3*(10*e
^3*f^3*g^2 + 30*d*e^2*f^2*g^3 + 15*d^2*e*f*g^4 + d^3*g^5)*d^2*x^2/((-e^2*x
^2 + d^2)^(5/2)*e^4) + 1/3*(e^3*f^5 + 15*d*e^2*f^4*g + 30*d^2*e*f^3*g^2...
```

3.579.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(247) = 494$.

Time = 0.32 (sec) , antiderivative size = 969, normalized size of antiderivative = 3.60

$$\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{1}{2}\sqrt{-e^2x^2+d^2}\left(\frac{g^5x}{e^5}+\frac{2\,(5\,e^{11}fg^4+3\,de^{10}g^5)}{e^{16}}\right) - \frac{(20\,e^2f^2g^3+30\,defg^4+13\,d^2g^5)\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,e^5|e|} + \frac{2\left(7\,e^5f^5-15\,de^4f^4g+20\,d^2e^3f^3g^2+220\,d^3e^2f^2g^3+285\,d^4efg^4+107\,d^5g^5-\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^3f^5}{x}+\cdots\right.}{}$$

input
```
integrate((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-e^2*x^2 + d^2)*(g^5*x/e^5 + 2*(5*e^11*f*g^4 + 3*d*e^10*g^5)/e^16
) - 1/2*(20*e^2*f^2*g^3 + 30*d*e*f*g^4 + 13*d^2*g^5)*arcsin(e*x/d)*sgn(d)*
sgn(e)/(e^5*abs(e)) + 2/15*(7*e^5*f^5 - 15*d*e^4*f^4*g + 20*d^2*e^3*f^3*g^
2 + 220*d^3*e^2*f^2*g^3 + 285*d^4*e*f*g^4 + 107*d^5*g^5 - 20*(d*e + sqrt(-
e^2*x^2 + d^2)*abs(e))*e^3*f^5/x + 75*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*
d*e^2*f^4*g/x - 100*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^2*e*f^3*g^2/x -
950*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^3*f^2*g^3/x - 1200*(d*e + sqrt(-
e^2*x^2 + d^2)*abs(e))*d^4*f*g^4/(e*x) - 445*(d*e + sqrt(-e^2*x^2 + d^2)*a
bs(e))*d^5*g^5/(e^2*x) + 40*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*e*f^5/x^
2 - 75*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d*f^4*g/x^2 + 200*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^2*d^2*f^3*g^2/(e*x^2) + 1450*(d*e + sqrt(-e^2*x^2
 + d^2)*abs(e))^2*d^3*f^2*g^3/(e^2*x^2) + 1800*(d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))^2*d^4*f*g^4/(e^3*x^2) + 665*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2
*d^5*g^5/(e^4*x^2) - 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*f^5/(e*x^3)
+ 75*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d*f^4*g/(e^2*x^3) - 750*(d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^3*f^2*g^3/(e^4*x^3) - 1050*(d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))^3*d^4*f*g^4/(e^5*x^3) - 405*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))^3*d^5*g^5/(e^6*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4
*f^5/(e^3*x^4) + 150*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^3*f^2*g^3/(e^
6*x^4) + 225*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^4*f*g^4/(e^7*x^4) ...
```

3.579.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx=\int \frac{\left(f+g\,x\right)^5\,\left(d+e\,x\right)^3}{\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

input
```
int(((f + g*x)^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
int(((f + g*x)^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

3.580 $\int \frac{(d+ex)^3(f+gx)^4}{(d^2-e^2x^2)^{7/2}}\,dx$

3.580.1 Optimal result

Integrand size = 31, antiderivative size = 215

$$\int \frac{(d+ex)^3(f+gx)^4}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{(ef+dg)^4(d+ex)^3}{5de^5(d^2-e^2x^2)^{5/2}} + \frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5(d^2-e^2x^2)^{3/2}} + \frac{2(ef+dg)^2(e^2f^2-8defg+36d^2g^2)(d+ex)}{15d^3e^5\sqrt{d^2-e^2x^2}} + \frac{g^4\sqrt{d^2-e^2x^2}}{e^5} - \frac{g^3(4ef+3dg)\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

output
```
1/5*(d*g+e*f)^4*(e*x+d)^3/d/e^5/(-e^2*x^2+d^2)^(5/2)+2/15*(-9*d*g+e*f)*(d*
g+e*f)^3*(e*x+d)^2/d^2/e^5/(-e^2*x^2+d^2)^(3/2)-g^3*(3*d*g+4*e*f)*arctan(e
*x/(-e^2*x^2+d^2)^(1/2))/e^5+2/15*(d*g+e*f)^2*(36*d^2*g^2-8*d*e*f*g+e^2*f^
2)*(e*x+d)/d^3/e^5/(-e^2*x^2+d^2)^(1/2)+g^4*(-e^2*x^2+d^2)^(1/2)/e^5
```

3.580.2 Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)^3(f+gx)^4}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(72d^6g^4+2e^6f^4x^2+d^5eg^3(88f-171gx)-6de^5f^3x(f+2gx)+3d}{} + \frac{g^3(4ef+3dg)\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{e^4\sqrt{-e^2}}$$

input
```
Integrate[((d + e*x)^3*(f + g*x)^4)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(72*d^6*g^4 + 2*e^6*f^4*x^2 + d^5*e*g^3*(88*f - 171*g
*x) - 6*d*e^5*f^3*x*(f + 2*g*x) + 3*d^4*e^2*g^2*(4*f^2 - 68*f*g*x + 39*g^2
*x^2) + d^2*e^4*f^2*(7*f^2 + 36*f*g*x + 42*g^2*x^2) - d^3*e^3*g*(12*f^3 +
36*f^2*g*x - 128*f*g^2*x^2 + 15*g^3*x^3)))/(15*d^3*e^5*(d - e*x)^3) + (g^3
*(4*e*f + 3*d*g)*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(e^4*Sqrt[-e^
2])
```

3.580.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {691, 25, 2166, 25, 2166, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3(f+gx)^4}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{691}$$

$$\frac{(d+ex)^3(dg+ef)^4}{5de^5(d^2-e^2x^2)^{5/2}}-\frac{\int -\frac{(d+ex)^2\left(-\frac{5dx^3g^4}{e}-\frac{5d(4ef+dg)x^2g^3}{e^2}-\frac{5d\left(6e^2f^2+4degf+d^2g^2\right)xg^2}{e^3}+\frac{2e^4f^4-12de^3gf^3-18d^2e^2g^2f^2-12d^3eg^3f-3d^4g^4}{e^4}\right)}{(d^2-e^2x^2)^{5/2}}dx}{5d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(d+ex)^2\left(-\frac{5dx^3g^4}{e}-\frac{5d(4ef+dg)x^2g^3}{e^2}-\frac{5d\left(6e^2f^2+4degf+d^2g^2\right)xg^2}{e^3}+\frac{2e^4f^4-12de^3gf^3-18d^2e^2g^2f^2-12d^3eg^3f-3d^4g^4}{e^4}\right)}{(d^2-e^2x^2)^{5/2}}dx}{5d}+\frac{(d+ex)^3(dg+ef)^4}{5de^5(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{2166}$$

$$\frac{\frac{2(d+ex)^2(ef-9dg)(dg+ef)^3}{3de^5(d^2-e^2x^2)^{3/2}}-\frac{\int -\frac{(d+ex)\left(\frac{15d^2x^2g^4}{e^2}+\frac{30d^2(2ef+dg)xg^3}{e^3}+\frac{2e^4f^4-12de^3gf^3+42d^2e^2g^2f^2+68d^3eg^3f+27d^4g^4}{e^4}\right)}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d}}{5d}+\frac{(d+ex)^3(dg+ef)^4}{5de^5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{(d+ex)\left(\frac{15d^2x^2g^4}{e^2}+\frac{30d^2(2ef+dg)xg^3}{e^3}+\frac{2e^4f^4-12de^3gf^3+42d^2e^2g^2f^2+68d^3eg^3f+27d^4g^4}{e^4}\right)}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d}+\frac{2(d+ex)^2(ef-9dg)(dg+ef)^3}{3de^5(d^2-e^2x^2)^{3/2}}}{5d}+\frac{(d+ex)^3(dg+ef)^4}{5de^5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 2166$$

$$\frac{\frac{\frac{2(d+ex)(dg+ef)^2\left(36d^2g^2-8defg+e^2f^2\right)}{de^5\sqrt{d^2-e^2x^2}}-\frac{\int \frac{15d^3g^3(4ef+3dg+egx)}{e^4\sqrt{d^2-e^2x^2}}dx}{d}}{3d}+\frac{2(d+ex)^2(ef-9dg)(dg+ef)^3}{3de^5(d^2-e^2x^2)^{3/2}}}{5d}+\frac{(d+ex)^3(dg+ef)^4}{5de^5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{2(d+ex)(dg+ef)^2\left(36d^2g^2-8defg+e^2f^2\right)}{de^5\sqrt{d^2-e^2x^2}}-\frac{15d^2g^3\int \frac{4ef+3dg+egx}{\sqrt{d^2-e^2x^2}}dx}{e^4}}{3d}+\frac{2(d+ex)^2(ef-9dg)(dg+ef)^3}{3de^5(d^2-e^2x^2)^{3/2}}}{5d}+\frac{(d+ex)^3(dg+ef)^4}{5de^5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 455$$

$$\frac{\frac{\frac{2(d+ex)(dg+ef)^2\left(36d^2g^2-8defg+e^2f^2\right)}{de^5\sqrt{d^2-e^2x^2}}-\frac{15d^2g^3\left((3dg+4ef)\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{g\sqrt{d^2-e^2x^2}}{e}\right)}{e^4}}{3d}+\frac{2(d+ex)^2(ef-9dg)(dg+ef)^3}{3de^5(d^2-e^2x^2)^{3/2}}}{5d}+\frac{(d+ex)^3(dg+ef)^4}{5de^5\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 224$$

3.580. $\int \frac{(d+ex)^3(f+gx)^4}{(d^2-e^2x^2)^{7/2}}\,dx$

$$\frac{\frac{\frac{2(d+ex)(dg+ef)^2\left(36d^2g^2-8defg+e^2f^2\right)}{de^5\sqrt{d^2-e^2x^2}}-\frac{15d^2g^3\left((3dg+4ef)\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{g\sqrt{d^2-e^2x^2}}{e}\right)}{e^4}}{3d}+\frac{2(d+ex)^2(ef-9dg)(dg+ef)^3}{3de^5(d^2-e^2x^2)^{3/2}}}{5d}+\frac{(d+ex)^3(dg+ef)^4}{5de^5\left(d^2-e^2x^2\right)^{5/2}}$$

$\big\downarrow$ 216

$$\frac{\frac{\frac{2(d+ex)(dg+ef)^2\left(36d^2g^2-8defg+e^2f^2\right)}{de^5\sqrt{d^2-e^2x^2}}-\frac{15d^2g^3\left(\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(3dg+4ef)}{e}-\frac{g\sqrt{d^2-e^2x^2}}{e}\right)}{e^4}}{3d}+\frac{2(d+ex)^2(ef-9dg)(dg+ef)^3}{3de^5(d^2-e^2x^2)^{3/2}}}{5d}+\frac{(d+ex)^3(dg+ef)^4}{5de^5\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[((d + e*x)^3*(f + g*x)^4)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
((e*f + d*g)^4*(d + e*x)^3)/(5*d*e^5*(d^2 - e^2*x^2)^(5/2)) + ((2*(e*f - 9
*d*g)*(e*f + d*g)^3*(d + e*x)^2)/(3*d*e^5*(d^2 - e^2*x^2)^(3/2)) + ((2*(e*
f + d*g)^2*(e^2*f^2 - 8*d*e*f*g + 36*d^2*g^2)*(d + e*x))/(d*e^5*Sqrt[d^2 -
 e^2*x^2]) - (15*d^2*g^3*(-((g*Sqrt[d^2 - e^2*x^2])/e) + ((4*e*f + 3*d*g)*
ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/e^4)/(3*d))/(5*d)
```

3.580.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 691
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)
^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(f + g*x)^n, a*e + c*d*
x, x], R = PolynomialRemainder[(f + g*x)^n, a*e + c*d*x, x]}, Simp[(-d)*R*(
d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*(p + 1))), x] + Simp[d/(2*a*(p + 1))
   Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*e*(p + 1)*Q +
R*(m + 2*p + 2), x], x], x]] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[n, 1]
&& IGtQ[m, 0] && LtQ[p, -1] && EqQ[c*d^2 + a*e^2, 0]
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

3.580.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(199) = 398.

Time = 0.90 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.24

method	result
risch	$\frac{g^4\sqrt{-e^2x^2+d^2}}{e^5} - \frac{\frac{(3dg+4ef)g^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e\sqrt{e^2}} + \frac{6g^2\left(d^2g^2+2defg+e^2f^2\right)\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{e^3d\left(x-\frac{d}{e}\right)} + \frac{4g\left(d^3g^3+3d^2efg^2+3de^2f^2\right.}{}}{}$
default	$d^3f^4\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right) + e^3g^4\left(-\frac{x^6}{e^2(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{6d^2\left(\frac{x^4}{e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}\right.}{}\right.$

input
```
int((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
g^4*(-e^2*x^2+d^2)^(1/2)/e^5-1/e^3*((3*d*g+4*e*f)*g^3/e/(e^2)^(1/2)*arctan
((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+6*g^2/e^3*(d^2*g^2+2*d*e*f*g+e^2*f^2)
/d/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)+4/e^3*g*(d^3*g^3+3*d^2*e*f
*g^2+3*d*e^2*f^2*g+e^3*f^3)*(1/3/d/e/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/
e))^(1/2)-1/3/d^2/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2))+1/e^4*(d^4
*g^4+4*d^3*e*f*g^3+6*d^2*e^2*f^2*g^2+4*d*e^3*f^3*g+e^4*f^4)*(1/5/d/e/(x-d/
e)^3*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-2/5*e/d*(1/3/d/e/(x-d/e)^2*(-(x-
d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-1/3/d^2/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d
/e))^(1/2))))
```

3.580.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(199) = 398$.

Time = 0.40 (sec) , antiderivative size = 624, normalized size of antiderivative = 2.90

$$\int \frac{(d+ex)^3(f+gx)^4}{\left(d^2-e^2x^2\right)^{7/2}}\,dx =$$

$$-\frac{7\,d^3e^4f^4 - 12\,d^4e^3f^3g + 12\,d^5e^2f^2g^2 + 88\,d^6efg^3 + 72\,d^7g^4 - (7\,e^7f^4 - 12\,de^6f^3g + 12\,d^2e^5f^2g^2 + 88\,d^3e^4}{}$$

input
```
integrate((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/15*(7*d^3*e^4*f^4 - 12*d^4*e^3*f^3*g + 12*d^5*e^2*f^2*g^2 + 88*d^6*e*f*
g^3 + 72*d^7*g^4 - (7*e^7*f^4 - 12*d*e^6*f^3*g + 12*d^2*e^5*f^2*g^2 + 88*d
^3*e^4*f*g^3 + 72*d^4*e^3*g^4)*x^3 + 3*(7*d*e^6*f^4 - 12*d^2*e^5*f^3*g + 1
2*d^3*e^4*f^2*g^2 + 88*d^4*e^3*f*g^3 + 72*d^5*e^2*g^4)*x^2 - 3*(7*d^2*e^5*
f^4 - 12*d^3*e^4*f^3*g + 12*d^4*e^3*f^2*g^2 + 88*d^5*e^2*f*g^3 + 72*d^6*e*
g^4)*x + 30*(4*d^6*e*f*g^3 + 3*d^7*g^4 - (4*d^3*e^4*f*g^3 + 3*d^4*e^3*g^4)
*x^3 + 3*(4*d^4*e^3*f*g^3 + 3*d^5*e^2*g^4)*x^2 - 3*(4*d^5*e^2*f*g^3 + 3*d^
6*e*g^4)*x)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (15*d^3*e^3*g^4*x^
3 - 7*d^2*e^4*f^4 + 12*d^3*e^3*f^3*g - 12*d^4*e^2*f^2*g^2 - 88*d^5*e*f*g^3
 - 72*d^6*g^4 - (2*e^6*f^4 - 12*d*e^5*f^3*g + 42*d^2*e^4*f^2*g^2 + 128*d^3
*e^3*f*g^3 + 117*d^4*e^2*g^4)*x^2 + 3*(2*d*e^5*f^4 - 12*d^2*e^4*f^3*g + 12
*d^3*e^3*f^2*g^2 + 68*d^4*e^2*f*g^3 + 57*d^5*e*g^4)*x)*sqrt(-e^2*x^2 + d^2
))/(d^3*e^8*x^3 - 3*d^4*e^7*x^2 + 3*d^5*e^6*x - d^6*e^5)
```

3.580.6 Sympy [F]

$$\int \frac{(d+ex)^3(f+gx)^4}{(d^2-e^2x^2)^{7/2}}\,dx=\int \frac{(d+ex)^3\,(f+gx)^4}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**3*(g*x+f)**4/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**3*(f + g*x)**4/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.580.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1190 vs. $2(199) = 398$.

Time = 0.29 (sec) , antiderivative size = 1190, normalized size of antiderivative = 5.53

$$\int \frac{(d+ex)^3(f+gx)^4}{(d^2-e^2x^2)^{7/2}}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
-e*g^4*x^6/(-e^2*x^2 + d^2)^(5/2) + 6*d^2*g^4*x^4/((-e^2*x^2 + d^2)^(5/2)*
e) - 8*d^4*g^4*x^2/((-e^2*x^2 + d^2)^(5/2)*e^3) + 1/5*d*f^4*x/(-e^2*x^2 +
d^2)^(5/2) + 1/15*(4*e^3*f*g^3 + 3*d*e^2*g^4)*x*(15*x^4/((-e^2*x^2 + d^2)^
(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 +
d^2)^(5/2)*e^6)) + 3/5*d^2*f^4/((-e^2*x^2 + d^2)^(5/2)*e) + 4/5*d^3*f^3*g/
((-e^2*x^2 + d^2)^(5/2)*e^2) + 16/5*d^6*g^4/((-e^2*x^2 + d^2)^(5/2)*e^5) +
 4/15*f^4*x/((-e^2*x^2 + d^2)^(3/2)*d) + 8/15*f^4*x/(sqrt(-e^2*x^2 + d^2)*
d^3) - 1/3*(4*e^3*f*g^3 + 3*d*e^2*g^4)*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^
2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e^2 + 3*(2*e^3*f^2*g^2 + 4*d*e^2*
f*g^3 + d^2*e*g^4)*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) + 1/2*(4*e^3*f^3*g + 1
8*d*e^2*f^2*g^2 + 12*d^2*e*f*g^3 + d^3*g^4)*x^3/((-e^2*x^2 + d^2)^(5/2)*e^
2) - 4*(2*e^3*f^2*g^2 + 4*d*e^2*f*g^3 + d^2*e*g^4)*d^2*x^2/((-e^2*x^2 + d^
2)^(5/2)*e^4) + 1/3*(e^3*f^4 + 12*d*e^2*f^3*g + 18*d^2*e*f^2*g^2 + 4*d^3*f
*g^3)*x^2/((-e^2*x^2 + d^2)^(5/2)*e^2) - 3/10*(4*e^3*f^3*g + 18*d*e^2*f^2*
g^2 + 12*d^2*e*f*g^3 + d^3*g^4)*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^4) + 3/5*(
d*e^2*f^4 + 4*d^2*e*f^3*g + 2*d^3*f^2*g^2)*x/((-e^2*x^2 + d^2)^(5/2)*e^2)
+ 8/5*(2*e^3*f^2*g^2 + 4*d*e^2*f*g^3 + d^2*e*g^4)*d^4/((-e^2*x^2 + d^2)^(5
/2)*e^6) - 2/15*(e^3*f^4 + 12*d*e^2*f^3*g + 18*d^2*e*f^2*g^2 + 4*d^3*f*g^3
)*d^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 4/15*(4*e^3*f*g^3 + 3*d*e^2*g^4)*d^2*
x/((-e^2*x^2 + d^2)^(3/2)*e^6) + 1/10*(4*e^3*f^3*g + 18*d*e^2*f^2*g^2 +...
```

3.580.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 757 vs. $2(199) = 398$.

Time = 0.31 (sec) , antiderivative size = 757, normalized size of antiderivative = 3.52

$$\int \frac{(d+ex)^3(f+gx)^4}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{-e^2x^2+d^2}g^4}{e^5} - \frac{(4\,efg^3+3\,dg^4)\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^4|e|} + \frac{2\left(7\,e^4f^4-12\,de^3f^3g+12\,d^2e^2f^2g^2+88\,d^3efg^3+57\,d^4g^4-\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^2f^4}{x}+\frac{60\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d}{x}\right.}{}$$

input
```
integrate((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
sqrt(-e^2*x^2 + d^2)*g^4/e^5 - (4*e*f*g^3 + 3*d*g^4)*arcsin(e*x/d)*sgn(d)*
sgn(e)/(e^4*abs(e)) + 2/15*(7*e^4*f^4 - 12*d*e^3*f^3*g + 12*d^2*e^2*f^2*g^
2 + 88*d^3*e*f*g^3 + 57*d^4*g^4 - 20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e
^2*f^4/x + 60*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d*e*f^3*g/x - 60*(d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))*d^2*f^2*g^2/x - 380*(d*e + sqrt(-e^2*x^2 + d^
2)*abs(e))*d^3*f*g^3/(e*x) - 240*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^4*g
^4/(e^2*x) + 40*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*f^4/x^2 - 60*(d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))^2*d*f^3*g/(e*x^2) + 120*(d*e + sqrt(-e^2*x^2
+ d^2)*abs(e))^2*d^2*f^2*g^2/(e^2*x^2) + 580*(d*e + sqrt(-e^2*x^2 + d^2)*a
bs(e))^2*d^3*f*g^3/(e^3*x^2) + 360*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d
^4*g^4/(e^4*x^2) - 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*f^4/(e^2*x^3)
+ 60*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d*f^3*g/(e^3*x^3) - 300*(d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^3*f*g^3/(e^5*x^3) - 210*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^3*d^4*g^4/(e^6*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))^4*f^4/(e^4*x^4) + 60*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^3*f*g^3
/(e^7*x^4) + 45*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^4*g^4/(e^8*x^4))/(
d^3*e^4*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^5*abs(e))
```

3.580.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3(f+gx)^4}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(f+g\,x)^4\,(d+e\,x)^3}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

input
```
int(((f + g*x)^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
int(((f + g*x)^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

3.581 $\int \frac{(d+ex)^3(f+gx)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

3.581.1 Optimal result

Integrand size = 31, antiderivative size = 183

$$\int \frac{(d+ex)^3(f+gx)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{(ef+dg)\left(2e^2f^2-11defg+32d^2g^2\right)(d+ex)}{15d^3e^4\sqrt{d^2-e^2x^2}}-\frac{g^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

output
```
1/5*(d*g+e*f)^3*(e*x+d)^3/d/e^4/(-e^2*x^2+d^2)^(5/2)+1/15*(-13*d*g+2*e*f)*
(d*g+e*f)^2*(e*x+d)^2/d^2/e^4/(-e^2*x^2+d^2)^(3/2)-g^3*arctan(e*x/(-e^2*x^
2+d^2)^(1/2))/e^4+1/15*(d*g+e*f)*(32*d^2*g^2-11*d*e*f*g+2*e^2*f^2)*(e*x+d)
/d^3/e^4/(-e^2*x^2+d^2)^(1/2)
```

3.581.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)^3(f+gx)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{\frac{(ef+dg)\sqrt{d^2-e^2x^2}\left(22d^4g^2+2e^4f^2x^2-de^3fx(6f+11gx)-d^3eg(16f+51gx)+d^2e^2\left(7f^2+33fgx+32g^2x^2\right)\right)}{d^3(d-ex)^3}+}{15e^4}$$

input
```
Integrate[((d + e*x)^3*(f + g*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(((e*f + d*g)*Sqrt[d^2 - e^2*x^2]*(22*d^4*g^2 + 2*e^4*f^2*x^2 - d*e^3*f*x*
(6*f + 11*g*x) - d^3*e*g*(16*f + 51*g*x) + d^2*e^2*(7*f^2 + 33*f*g*x + 32*
g^2*x^2)))/(d^3*(d - e*x)^3) + 30*g^3*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 -
 e^2*x^2])])/(15*e^4)
```

3.581.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {691, 25, 2166, 25, 27, 665, 27, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3(f+gx)^3}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{691}$$

$$\frac{(d+ex)^3(dg+ef)^3}{5de^4(d^2-e^2x^2)^{5/2}}-\frac{\int -\frac{(d+ex)^2\left(-\frac{5dx^2g^3}{e}-\frac{5d(3ef+dg)xg^2}{e^2}+\frac{2e^3f^3-9de^2gf^2-9d^2eg^2f-3d^3g^3}{e^3}\right)}{(d^2-e^2x^2)^{5/2}}dx}{5d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(d+ex)^2\left(-\frac{5dx^2g^3}{e}-\frac{5d(3ef+dg)xg^2}{e^2}+\frac{2e^3f^3-9de^2gf^2-9d^2eg^2f-3d^3g^3}{e^3}\right)}{(d^2-e^2x^2)^{5/2}}dx}{5d}+\frac{(d+ex)^3(dg+ef)^3}{5de^4(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{2166}$$

$$\frac{\frac{(d+ex)^2(2ef-13dg)(dg+ef)^2}{3de^4(d^2-e^2x^2)^{3/2}}-\frac{\int -\frac{(d+ex)\left(2e^3f^3-9de^2gf^2+21d^2eg^2f+17d^3g^3+15d^2eg^3x\right)}{e^3\left(d^2-e^2x^2\right)^{3/2}}dx}{3d}}{5d}+\frac{(d+ex)^3(dg+ef)^3}{5de^4(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{(d+ex)\left(2e^3f^3-9de^2gf^2+21d^2eg^2f+17d^3g^3+15d^2eg^3x\right)}{e^3\left(d^2-e^2x^2\right)^{3/2}}dx}{3d}+\frac{(d+ex)^2(2ef-13dg)(dg+ef)^2}{3de^4(d^2-e^2x^2)^{3/2}}}{5d}+\frac{(d+ex)^3(dg+ef)^3}{5de^4(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{(d+ex)\left(2e^3f^3-9de^2gf^2+21d^2eg^2f+17d^3g^3+15d^2eg^3x\right)}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3de^3}+\frac{(d+ex)^2(2ef-13dg)(dg+ef)^2}{3de^4(d^2-e^2x^2)^{3/2}}}{5d}+\frac{(d+ex)^3(dg+ef)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 665$$

$$\frac{\frac{\frac{(d+ex)(dg+ef)\left(32d^2g^2-11defg+2e^2f^2\right)}{de\sqrt{d^2-e^2x^2}}-\frac{\int \frac{15d^2eg^3}{\sqrt{d^2-e^2x^2}}dx}{e}}{3de^3}+\frac{(d+ex)^2(2ef-13dg)(dg+ef)^2}{3de^4(d^2-e^2x^2)^{3/2}}}{5d}+\frac{(d+ex)^3(dg+ef)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{(d+ex)(dg+ef)\left(32d^2g^2-11defg+2e^2f^2\right)}{de\sqrt{d^2-e^2x^2}}-15d^2g^3\int \frac{1}{\sqrt{d^2-e^2x^2}}dx}{3de^3}+\frac{(d+ex)^2(2ef-13dg)(dg+ef)^2}{3de^4(d^2-e^2x^2)^{3/2}}}{5d}+\frac{(d+ex)^3(dg+ef)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\frac{(d+ex)(dg+ef)\left(32d^2g^2-11defg+2e^2f^2\right)}{de\sqrt{d^2-e^2x^2}}-15d^2g^3\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}}{3de^3}+\frac{(d+ex)^2(2ef-13dg)(dg+ef)^2}{3de^4(d^2-e^2x^2)^{3/2}}}{5d}+\frac{(d+ex)^3(dg+ef)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\frac{(d+ex)(dg+ef)\left(32d^2g^2-11defg+2e^2f^2\right)}{de\sqrt{d^2-e^2x^2}}-\frac{15d^2g^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}}{3de^3}+\frac{(d+ex)^2(2ef-13dg)(dg+ef)^2}{3de^4(d^2-e^2x^2)^{3/2}}}{5d}+\frac{(d+ex)^3(dg+ef)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}}$$

input

```
Int[((d + e*x)^3*(f + g*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

output

```
((e*f + d*g)^3*(d + e*x)^3)/(5*d*e^4*(d^2 - e^2*x^2)^(5/2)) + (((2*e*f - 1
3*d*g)*(e*f + d*g)^2*(d + e*x)^2)/(3*d*e^4*(d^2 - e^2*x^2)^(3/2)) + (((e*f
 + d*g)*(2*e^2*f^2 - 11*d*e*f*g + 32*d^2*g^2)*(d + e*x))/(d*e*Sqrt[d^2 - e
^2*x^2]) - (15*d^2*g^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e)/(3*d*e^3))/(5
*d)
```

3.581. $\int \frac{(d+ex)^3(f+gx)^3}{(d^2-e^2x^2)^{7/2}}\,dx$

3.581.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 665
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2)^(3/2), x_Symbol] :> Simp[(-2^(m - 1))*d^(m - 2)*(e*f + d*g)^n*((d +
e*x)/(c*e^(n - 1)*Sqrt[a + c*x^2])), x] + Simp[1/(c*e^(n - 2))   Int[Expand
ToSum[(2^(m - 1)*d^(m - 1)*(e*f + d*g)^n - e^n*(d + e*x)^(m - 1)*(f + g*x)^
n)/(d - e*x), x]/Sqrt[a + c*x^2], x], x] /; FreeQ[{a, c, d, e, f, g}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 691
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)
^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(f + g*x)^n, a*e + c*d*
x, x], R = PolynomialRemainder[(f + g*x)^n, a*e + c*d*x, x]}, Simp[(-d)*R*(
d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*(p + 1))), x] + Simp[d/(2*a*(p + 1))
   Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*e*(p + 1)*Q +
R*(m + 2*p + 2), x], x], x]] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[n, 1]
&& IGtQ[m, 0] && LtQ[p, -1] && EqQ[c*d^2 + a*e^2, 0]
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

3.581. $\int \frac{(d+ex)^3(f+gx)^3}{(d^2-e^2x^2)^{7/2}}\,dx$

3.581.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 687 vs. $2(169) = 338$.

Time = 0.65 (sec) , antiderivative size = 688, normalized size of antiderivative = 3.76

method	result
default	$d^3f^3\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+e^3g^3\left(\frac{x^5}{5e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{\frac{x^3}{3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{e^2\sqrt{}}{}}{}\right.$

input
```
int((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
d^3*f^3*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^
(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))+e^3*g^3*(1/5*x^5/e^2/(-e^2*x^2+d^2)
^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)
^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))+(3*
d*e^2*g^3+3*e^3*f*g^2)*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*d^2/e^2*(1/3*x^2/e^
2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2)))+1/5*(3*d^3*f^2*
g+3*d^2*e*f^3)/e^2/(-e^2*x^2+d^2)^(5/2)+(3*d^2*e*g^3+9*d*e^2*f*g^2+3*e^3*f
^2*g)*(1/2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)-3/2*d^2/e^2*(1/4*x/e^2/(-e^2*x^2+d
^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(
-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))))+(3*d^3*f*g^2+9*d^2*
e*f^2*g+3*d*e^2*f^3)*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^
2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(
-e^2*x^2+d^2)^(1/2))))+(d^3*g^3+9*d^2*e*f*g^2+9*d*e^2*f^2*g+e^3*f^3)*(1/3*
x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2))
```

3.581.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(169) = 338$.

Time = 0.33 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.48

$$\int \frac{(d+ex)^3(f+gx)^3}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{7\,d^3e^3f^3 - 9\,d^4e^2f^2g + 6\,d^5efg^2 + 22\,d^6g^3 - (7\,e^6f^3 - 9\,de^5f^2g + 6\,d^2e^4fg^2 + 22\,d^3e^3g^3)x^3 + 3\,(7\,de^5f^3 -}{}$$

input
```
integrate((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/15*(7*d^3*e^3*f^3 - 9*d^4*e^2*f^2*g + 6*d^5*e*f*g^2 + 22*d^6*g^3 - (7*e
^6*f^3 - 9*d*e^5*f^2*g + 6*d^2*e^4*f*g^2 + 22*d^3*e^3*g^3)*x^3 + 3*(7*d*e^
5*f^3 - 9*d^2*e^4*f^2*g + 6*d^3*e^3*f*g^2 + 22*d^4*e^2*g^3)*x^2 - 3*(7*d^2
*e^4*f^3 - 9*d^3*e^3*f^2*g + 6*d^4*e^2*f*g^2 + 22*d^5*e*g^3)*x - 30*(d^3*e
^3*g^3*x^3 - 3*d^4*e^2*g^3*x^2 + 3*d^5*e*g^3*x - d^6*g^3)*arctan(-(d - sqr
t(-e^2*x^2 + d^2))/(e*x)) + (7*d^2*e^3*f^3 - 9*d^3*e^2*f^2*g + 6*d^4*e*f*g
^2 + 22*d^5*g^3 + (2*e^5*f^3 - 9*d*e^4*f^2*g + 21*d^2*e^3*f*g^2 + 32*d^3*e
^2*g^3)*x^2 - 3*(2*d*e^4*f^3 - 9*d^2*e^3*f^2*g + 6*d^3*e^2*f*g^2 + 17*d^4*
e*g^3)*x)*sqrt(-e^2*x^2 + d^2))/(d^3*e^7*x^3 - 3*d^4*e^6*x^2 + 3*d^5*e^5*x
 - d^6*e^4)
```

3.581.6 Sympy [F]

$$\int \frac{(d+ex)^3(f+gx)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^3\,(f+gx)^3}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**3*(g*x+f)**3/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**3*(f + g*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.581.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 903 vs. 2(169) = 338.

Time = 0.28 (sec) , antiderivative size = 903, normalized size of antiderivative = 4.93

$$\int \frac{(d+ex)^3(f+gx)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{1}{15}\,e^3g^3x\left(\frac{15\,x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right) - \frac{1}{3}\,eg^3x\left(\frac{3\,x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right) + \frac{df^3x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{3\,d^2f^3}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{3\,d^3f^2g}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} + \frac{4\,f^3x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d} + \frac{4\,d^2g^3x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}e^3} + \frac{8\,f^3x}{15\,\sqrt{-e^2x^2+d^2}d^3} - \frac{7\,g^3x}{15\,\sqrt{-e^2x^2+d^2}e^3} + \frac{3\,(e^3fg^2+de^2g^3)x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{g^3\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}e^3} + \frac{3\,(e^3f^2g+3\,de^2fg^2+d^2eg^3)x^3}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{4\,(e^3fg^2+de^2g^3)d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{(e^3f^3+9\,de^2f^2g+9\,d^2efg^2+d^3g^3)x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{9\,(e^3f^2g+3\,de^2fg^2+d^2eg^3)d^2x}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{3\,(de^2f^3+3\,d^2ef^2g+d^3fg^2)x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} + \frac{8\,(e^3fg^2+de^2g^3)d^4}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^6} - \frac{2\,(e^3f^3+9\,de^2f^2g+9\,d^2efg^2+d^3g^3)d^2}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{3\,(e^3f^2g+3\,de^2fg^2+d^2eg^3)x}{10\,(-e^2x^2+d^2)^{\frac{3}{2}}e^4} - \frac{(de^2f^3+3\,d^2ef^2g+d^3fg^2)x}{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2e^2} + \frac{3\,(e^3f^2g+3\,de^2fg^2+d^2eg^3)x}{5\,\sqrt{-e^2x^2+d^2}d^2e^4} - \frac{2\,(de^2f^3+3\,d^2ef^2g+d^3fg^2)x}{5\,\sqrt{-e^2x^2+d^2}d^4e^2}$$

input
```
integrate((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/15*e^3*g^3*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^
2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 1/3*e*g^3*x*(3
*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4)) +
1/5*d*f^3*x/(-e^2*x^2 + d^2)^(5/2) + 3/5*d^2*f^3/((-e^2*x^2 + d^2)^(5/2)*e
) + 3/5*d^3*f^2*g/((-e^2*x^2 + d^2)^(5/2)*e^2) + 4/15*f^3*x/((-e^2*x^2 + d
^2)^(3/2)*d) + 4/15*d^2*g^3*x/((-e^2*x^2 + d^2)^(3/2)*e^3) + 8/15*f^3*x/(s
qrt(-e^2*x^2 + d^2)*d^3) - 7/15*g^3*x/(sqrt(-e^2*x^2 + d^2)*e^3) + 3*(e^3*
f*g^2 + d*e^2*g^3)*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - g^3*arcsin(e^2*x/(d*
sqrt(e^2)))/(sqrt(e^2)*e^3) + 3/2*(e^3*f^2*g + 3*d*e^2*f*g^2 + d^2*e*g^3)*
x^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 4*(e^3*f*g^2 + d*e^2*g^3)*d^2*x^2/((-e^
2*x^2 + d^2)^(5/2)*e^4) + 1/3*(e^3*f^3 + 9*d*e^2*f^2*g + 9*d^2*e*f*g^2 + d
^3*g^3)*x^2/((-e^2*x^2 + d^2)^(5/2)*e^2) - 9/10*(e^3*f^2*g + 3*d*e^2*f*g^2
 + d^2*e*g^3)*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^4) + 3/5*(d*e^2*f^3 + 3*d^2*
e*f^2*g + d^3*f*g^2)*x/((-e^2*x^2 + d^2)^(5/2)*e^2) + 8/5*(e^3*f*g^2 + d*e
^2*g^3)*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6) - 2/15*(e^3*f^3 + 9*d*e^2*f^2*g +
 9*d^2*e*f*g^2 + d^3*g^3)*d^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 3/10*(e^3*f^2
*g + 3*d*e^2*f*g^2 + d^2*e*g^3)*x/((-e^2*x^2 + d^2)^(3/2)*e^4) - 1/5*(d*e^
2*f^3 + 3*d^2*e*f^2*g + d^3*f*g^2)*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e^2) + 3/
5*(e^3*f^2*g + 3*d*e^2*f*g^2 + d^2*e*g^3)*x/(sqrt(-e^2*x^2 + d^2)*d^2*e^4)
 - 2/5*(d*e^2*f^3 + 3*d^2*e*f^2*g + d^3*f*g^2)*x/(sqrt(-e^2*x^2 + d^2)*...
```

3.581.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 560 vs. $2(169) = 338$.

Time = 0.31 (sec) , antiderivative size = 560, normalized size of antiderivative = 3.06

$$\int \frac{(d+ex)^3(f+gx)^3}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{g^3\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^3|e|}$$

$$+\frac{2\left(7e^3f^3 - 9de^2f^2g + 6d^2efg^2 + 22d^3g^3 - \frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)ef^3}{x} + \frac{45\left(de+\sqrt{-e^2x^2+d^2}|e|\right)df^2g}{x} - \frac{30\left(de+\sqrt{-e^2x^2\cdots}\right.}{ex\cdots}\right.}{\cdots}$$

input
```
integrate((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
-g^3*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^3*abs(e)) + 2/15*(7*e^3*f^3 - 9*d*e^2*
f^2*g + 6*d^2*e*f*g^2 + 22*d^3*g^3 - 20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e)
)*e*f^3/x + 45*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d*f^2*g/x - 30*(d*e + s
qrt(-e^2*x^2 + d^2)*abs(e))*d^2*f*g^2/(e*x) - 95*(d*e + sqrt(-e^2*x^2 + d^
2)*abs(e))*d^3*g^3/(e^2*x) + 40*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*f^3/
(e*x^2) - 45*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d*f^2*g/(e^2*x^2) + 60*
(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^2*f*g^2/(e^3*x^2) + 145*(d*e + sqr
t(-e^2*x^2 + d^2)*abs(e))^2*d^3*g^3/(e^4*x^2) - 30*(d*e + sqrt(-e^2*x^2 +
d^2)*abs(e))^3*f^3/(e^3*x^3) + 45*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d*
f^2*g/(e^4*x^3) - 75*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^3*g^3/(e^6*x^
3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*f^3/(e^5*x^4) + 15*(d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))^4*d^3*g^3/(e^8*x^4))/(d^3*e^3*((d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))/(e^2*x) - 1)^5*abs(e))
```

3.581.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3(f+gx)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(f+g\,x)^3\,(d+e\,x)^3}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

input
```
int(((f + g*x)^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
int(((f + g*x)^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

3.582 $\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

3.582.1 Optimal result

Integrand size = 31, antiderivative size = 145

$$\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{(ef+dg)^2(d+ex)^3}{5de^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(ef-4dg)(ef+dg)(d+ex)^2}{15d^2e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{\left(2e^2f^2-6defg+7d^2g^2\right)(d+ex)}{15d^3e^3\sqrt{d^2-e^2x^2}}$$

output
```
1/5*(d*g+e*f)^2*(e*x+d)^3/d/e^3/(-e^2*x^2+d^2)^(5/2)+2/15*(-4*d*g+e*f)*(d*
g+e*f)*(e*x+d)^2/d^2/e^3/(-e^2*x^2+d^2)^(3/2)+1/15*(7*d^2*g^2-6*d*e*f*g+2*
e^2*f^2)*(e*x+d)/d^3/e^3/(-e^2*x^2+d^2)^(1/2)
```

3.582.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}\left(2d^4g^2+2e^4f^2x^2-6d^3eg(f+gx)-6de^3fx(f+gx)+d^2e^2(7f^2+\right.}{15d^3e^3(d-ex)^3}$$

input
```
Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(2*d^4*g^2 + 2*e^4*f^2*x^2 - 6*d^3*e*g*(f + g*x) - 6*
d*e^3*f*x*(f + g*x) + d^2*e^2*(7*f^2 + 18*f*g*x + 7*g^2*x^2)))/(15*d^3*e^3
*(d - e*x)^3)
```

3.582.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {691, 25, 27, 669, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{ 691}$$

$$\frac{(d+ex)^3(dg+ef)^2}{5de^3(d^2-e^2x^2)^{5/2}}-\frac{\int -\frac{(d+ex)^2\left(e\left(2f^2-\frac{6dgf}{e}-\frac{3d^2g^2}{e^2}\right)-5dg^2x\right)}{e(d^2-e^2x^2)^{5/2}}dx}{5d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(d+ex)^2\left(2ef^2-6dgf-\frac{3d^2g^2}{e}-5dg^2x\right)}{e(d^2-e^2x^2)^{5/2}}dx}{5d}+\frac{(d+ex)^3(dg+ef)^2}{5de^3(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(d+ex)^2\left(2ef^2-6dgf-\frac{3d^2g^2}{e}-5dg^2x\right)}{(d^2-e^2x^2)^{5/2}}dx}{5de}+\frac{(d+ex)^3(dg+ef)^2}{5de^3(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{ 669}$$

$$\frac{\frac{1}{3}\left(\frac{2ef^2}{d}+\frac{7dg^2}{e}-6fg\right)\int \frac{d+ex}{(d^2-e^2x^2)^{3/2}}dx+\frac{2(d+ex)^2(ef-4dg)(dg+ef)}{3de^2(d^2-e^2x^2)^{3/2}}}{5de}+\frac{(d+ex)^3(dg+ef)^2}{5de^3(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{ 453}$$

$$\frac{\frac{(d+ex)\left(\frac{2ef^2}{d}+\frac{7dg^2}{e}-6fg\right)}{3de\sqrt{d^2-e^2x^2}}+\frac{2(d+ex)^2(ef-4dg)(dg+ef)}{3de^2(d^2-e^2x^2)^{3/2}}}{5de}+\frac{(d+ex)^3(dg+ef)^2}{5de^3(d^2-e^2x^2)^{5/2}}$$

input
```
Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
((e*f + d*g)^2*(d + e*x)^3)/(5*d*e^3*(d^2 - e^2*x^2)^(5/2)) + ((2*(e*f - 4
*d*g)*(e*f + d*g)*(d + e*x)^2)/(3*d*e^2*(d^2 - e^2*x^2)^(3/2)) + (((2*e*f^
2)/d - 6*f*g + (7*d*g^2)/e)*(d + e*x))/(3*d*e*Sqrt[d^2 - e^2*x^2]))/(5*d*e
)
```

3.582.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 669
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*
(p + 1))), x] - Simp[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1)))
Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

rule 691
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)
^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(f + g*x)^n, a*e + c*d*
x, x], R = PolynomialRemainder[(f + g*x)^n, a*e + c*d*x, x]}, Simp[(-d)*R*(
d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*(p + 1))), x] + Simp[d/(2*a*(p + 1))
   Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*e*(p + 1)*Q +
R*(m + 2*p + 2), x], x], x]] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[n, 1]
&& IGtQ[m, 0] && LtQ[p, -1] && EqQ[c*d^2 + a*e^2, 0]
```

3.582. $\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^{7/2}}\,dx$

3.582.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.87

method	result
trager	$\frac{\left(7x^2d^2e^2g^2-6x^2d\,e^3fg+2x^2e^4f^2-6x\,d^3e\,g^2+18x\,d^2e^2fg-6xd\,e^3f^2+2d^4g^2-6fge\,d^3+7d^2e^2f^2\right)\sqrt{-e^2x^2+d^2}}{15d^3e^3\left(-ex+d\right)^3}$
gosper	$\frac{\left(-ex+d\right)\left(ex+d\right)^4\left(7x^2d^2e^2g^2-6x^2d\,e^3fg+2x^2e^4f^2-6x\,d^3e\,g^2+18x\,d^2e^2fg-6xd\,e^3f^2+2d^4g^2-6fge\,d^3+7d^2e^2f^2\right)}{15d^3e^3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}$
default	$d^3f^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+g^2e^3\left(\frac{x^4}{e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{4d^2\left(\frac{x^2}{3e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\right.}{e^2}\right.$

input
```
int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(7*d^2*e^2*g^2*x^2-6*d*e^3*f*g*x^2+2*e^4*f^2*x^2-6*d^3*e*g^2*x+18*d^2
*e^2*f*g*x-6*d*e^3*f^2*x+2*d^4*g^2-6*d^3*e*f*g+7*d^2*e^2*f^2)/d^3/e^3/(-e*
x+d)^3*(-e^2*x^2+d^2)^(1/2)
```

3.582.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(133) = 266.

Time = 0.31 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.92

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^{7/2}}\,dx =$$
$$-\frac{7\,d^3e^2f^2-6\,d^4efg+2\,d^5g^2-(7\,e^5f^2-6\,de^4fg+2\,d^2e^3g^2)x^3+3\,(7\,de^4f^2-6\,d^2e^3fg+2\,d^3e^2g^2)x^2-3\,(}{}$$

input
```
integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/15*(7*d^3*e^2*f^2 - 6*d^4*e*f*g + 2*d^5*g^2 - (7*e^5*f^2 - 6*d*e^4*f*g
+ 2*d^2*e^3*g^2)*x^3 + 3*(7*d*e^4*f^2 - 6*d^2*e^3*f*g + 2*d^3*e^2*g^2)*x^2
 - 3*(7*d^2*e^3*f^2 - 6*d^3*e^2*f*g + 2*d^4*e*g^2)*x + (7*d^2*e^2*f^2 - 6*
d^3*e*f*g + 2*d^4*g^2 + (2*e^4*f^2 - 6*d*e^3*f*g + 7*d^2*e^2*g^2)*x^2 - 6*
(d*e^3*f^2 - 3*d^2*e^2*f*g + d^3*e*g^2)*x)*sqrt(-e^2*x^2 + d^2))/(d^3*e^6*
x^3 - 3*d^4*e^5*x^2 + 3*d^5*e^4*x - d^6*e^3)
```

3.582.6 Sympy [F]

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^3\,(f+gx)^2}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**3*(f + g*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.582.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(133) = 266$.

Time = 0.19 (sec) , antiderivative size = 583, normalized size of antiderivative = 4.02

$$\begin{aligned}\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^{7/2}}\,dx &= \frac{eg^2x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{4\,d^2g^2x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e} \\ &+ \frac{df^2x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{3\,d^2f^2}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{2\,d^3fg}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} \\ &+ \frac{8\,d^4g^2}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^3} + \frac{4\,f^2x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d} + \frac{8\,f^2x}{15\,\sqrt{-e^2x^2+d^2}d^3} \\ &+ \frac{(2\,e^3fg+3\,de^2g^2)x^3}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} + \frac{(e^3f^2+6\,de^2fg+3\,d^2eg^2)x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{3\,(2\,e^3fg+3\,de^2g^2)d^2x}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} \\ &+ \frac{(3\,de^2f^2+6\,d^2efg+d^3g^2)x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{2\,(e^3f^2+6\,de^2fg+3\,d^2eg^2)d^2}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} \\ &+ \frac{(2\,e^3fg+3\,de^2g^2)x}{10\,(-e^2x^2+d^2)^{\frac{3}{2}}e^4} - \frac{(3\,de^2f^2+6\,d^2efg+d^3g^2)x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2e^2} \\ &+ \frac{(2\,e^3fg+3\,de^2g^2)x}{5\,\sqrt{-e^2x^2+d^2}d^2e^4} - \frac{2\,(3\,de^2f^2+6\,d^2efg+d^3g^2)x}{15\,\sqrt{-e^2x^2+d^2}d^4e^2}\end{aligned}$$

input
```
integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
e*g^2*x^4/(-e^2*x^2 + d^2)^(5/2) - 4/3*d^2*g^2*x^2/((-e^2*x^2 + d^2)^(5/2)
*e) + 1/5*d*f^2*x/(-e^2*x^2 + d^2)^(5/2) + 3/5*d^2*f^2/((-e^2*x^2 + d^2)^(
5/2)*e) + 2/5*d^3*f*g/((-e^2*x^2 + d^2)^(5/2)*e^2) + 8/15*d^4*g^2/((-e^2*x
^2 + d^2)^(5/2)*e^3) + 4/15*f^2*x/((-e^2*x^2 + d^2)^(3/2)*d) + 8/15*f^2*x/
(sqrt(-e^2*x^2 + d^2)*d^3) + 1/2*(2*e^3*f*g + 3*d*e^2*g^2)*x^3/((-e^2*x^2
+ d^2)^(5/2)*e^2) + 1/3*(e^3*f^2 + 6*d*e^2*f*g + 3*d^2*e*g^2)*x^2/((-e^2*x
^2 + d^2)^(5/2)*e^2) - 3/10*(2*e^3*f*g + 3*d*e^2*g^2)*d^2*x/((-e^2*x^2 + d
^2)^(5/2)*e^4) + 1/5*(3*d*e^2*f^2 + 6*d^2*e*f*g + d^3*g^2)*x/((-e^2*x^2 +
d^2)^(5/2)*e^2) - 2/15*(e^3*f^2 + 6*d*e^2*f*g + 3*d^2*e*g^2)*d^2/((-e^2*x^
2 + d^2)^(5/2)*e^4) + 1/10*(2*e^3*f*g + 3*d*e^2*g^2)*x/((-e^2*x^2 + d^2)^(
3/2)*e^4) - 1/15*(3*d*e^2*f^2 + 6*d^2*e*f*g + d^3*g^2)*x/((-e^2*x^2 + d^2)
^(3/2)*d^2*e^2) + 1/5*(2*e^3*f*g + 3*d*e^2*g^2)*x/(sqrt(-e^2*x^2 + d^2)*d^
2*e^4) - 2/15*(3*d*e^2*f^2 + 6*d^2*e*f*g + d^3*g^2)*x/(sqrt(-e^2*x^2 + d^2
)*d^4*e^2)
```

3.582.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(133) = 266$.

Time = 0.30 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.55

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2\left(7e^2f^2-6defg+2d^2g^2-\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)f^2}{x}+\frac{30\left(de+\sqrt{-e^2x^2+d^2}|e|\right)dfg}{ex}-\frac{10}{}\right.}{}$$

input
```
integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
2/15*(7*e^2*f^2 - 6*d*e*f*g + 2*d^2*g^2 - 20*(d*e + sqrt(-e^2*x^2 + d^2)*a
bs(e))*f^2/x + 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d*f*g/(e*x) - 10*(d*
e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^2*g^2/(e^2*x) + 40*(d*e + sqrt(-e^2*x^2
 + d^2)*abs(e))^2*f^2/(e^2*x^2) - 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2
*d*f*g/(e^3*x^2) + 20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^2*g^2/(e^4*x
^2) - 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*f^2/(e^4*x^3) + 30*(d*e + s
qrt(-e^2*x^2 + d^2)*abs(e))^3*d*f*g/(e^5*x^3) + 15*(d*e + sqrt(-e^2*x^2 +
d^2)*abs(e))^4*f^2/(e^6*x^4))/(d^3*e^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e)
)/(e^2*x) - 1)^5*abs(e))
```

3.582.9 Mupad [B] (verification not implemented)

Time = 12.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^4\,g^2-6\,d^3\,e\,f\,g-6\,d^3\,e\,g^2\,x+7\,d^2\,e^2\,f^2+18\,d^2\,e^2\,f\,g\,x+7\,d^2\,e\right.}{15\,d^3\,e^3\,\left(d-e\,x\right)^3}$$

input
```
int(((f + g*x)^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(2*d^4*g^2 + 7*d^2*e^2*f^2 + 2*e^4*f^2*x^2 - 6*d^3*
e*f*g + 7*d^2*e^2*g^2*x^2 - 6*d*e^3*f^2*x - 6*d^3*e*g^2*x + 18*d^2*e^2*f*g
*x - 6*d*e^3*f*g*x^2))/(15*d^3*e^3*(d - e*x)^3)
```

3.583 $\int \frac{(d+ex)^3(f+gx)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

3.583.1 Optimal result

Integrand size = 29, antiderivative size = 117

$$\int \frac{(d+ex)^3(f+gx)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{(ef+dg)(d+ex)^3}{5de^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(2ef-3dg)(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{(2ef-3dg)x}{15d^3e\sqrt{d^2-e^2x^2}}$$

output
```
1/5*(d*g+e*f)*(e*x+d)^3/d/e^2/(-e^2*x^2+d^2)^(5/2)+2/15*(-3*d*g+2*e*f)*(e*
x+d)/d/e^2/(-e^2*x^2+d^2)^(3/2)+1/15*(-3*d*g+2*e*f)*x/d^3/e/(-e^2*x^2+d^2)
^(1/2)
```

3.583.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex)^3(f+gx)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}\left(-3d^3g+2e^3fx^2-3de^2x(2f+gx)+d^2e(7f+9gx)\right)}{15d^3e^2(d-ex)^3}$$

input
```
Integrate[((d + e*x)^3*(f + g*x))/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-3*d^3*g + 2*e^3*f*x^2 - 3*d*e^2*x*(2*f + g*x) + d^2
*e*(7*f + 9*g*x)))/(15*d^3*e^2*(d - e*x)^3)
```

3.583.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {669, 457, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3(f+gx)}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{669}$$

$$\frac{(2ef-3dg)\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{5/2}}dx}{5de}+\frac{(d+ex)^3(dg+ef)}{5de^2(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{457}$$

$$\frac{(2ef-3dg)\left(\frac{1}{3}\int \frac{1}{(d^2-e^2x^2)^{3/2}}dx+\frac{2(d+ex)}{3e(d^2-e^2x^2)^{3/2}}\right)}{5de}+\frac{(d+ex)^3(dg+ef)}{5de^2(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{(d+ex)^3(dg+ef)}{5de^2(d^2-e^2x^2)^{5/2}}+\frac{\left(\frac{x}{3d^2\sqrt{d^2-e^2x^2}}+\frac{2(d+ex)}{3e(d^2-e^2x^2)^{3/2}}\right)(2ef-3dg)}{5de}$$

input
```
Int[((d + e*x)^3*(f + g*x))/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
((e*f + d*g)*(d + e*x)^3)/(5*d*e^2*(d^2 - e^2*x^2)^(5/2)) + ((2*e*f - 3*d*
g)*((2*(d + e*x))/(3*e*(d^2 - e^2*x^2)^(3/2)) + x/(3*d^2*Sqrt[d^2 - e^2*x^
2])))/(5*d*e)
```

3.583.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 669
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*
(p + 1))), x] - Simp[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1)))
Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

3.583.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.68

method	result
trager	$-\frac{\left(3d\,e^{2}g\,x^{2}-2e^{3}f\,x^{2}-9d^{2}egx+6d\,e^{2}fx+3d^{3}g-7d^{2}ef\right)\sqrt{-e^{2}x^{2}+d^{2}}}{15d^{3}\left(-ex+d\right)^{3}e^{2}}$
gosper	$-\frac{\left(-ex+d\right)\left(ex+d\right)^{4}\left(3d\,e^{2}g\,x^{2}-2e^{3}f\,x^{2}-9d^{2}egx+6d\,e^{2}fx+3d^{3}g-7d^{2}ef\right)}{15d^{3}e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}$
default	$d^{3}f\left(\frac{x}{5d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{8x}{15d^{4}\sqrt{-e^{2}x^{2}+d^{2}}}}{d^{2}}\right)+e^{3}g\left(\frac{x^{3}}{2e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{3d^{2}\left(\frac{x}{4e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\cdots\right.}{\cdots}\right.$

input
```
int((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(3*d*e^2*g*x^2-2*e^3*f*x^2-9*d^2*e*g*x+6*d*e^2*f*x+3*d^3*g-7*d^2*e*f
)/d^3/(-e*x+d)^3/e^2*(-e^2*x^2+d^2)^(1/2)
```

3.583.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.56

$$\int \frac{(d+ex)^3(f+gx)}{(d^2-e^2x^2)^{7/2}}\,dx =$$
$$-\frac{7\,d^3ef-3\,d^4g-(7\,e^4f-3\,de^3g)x^3+3\,(7\,de^3f-3\,d^2e^2g)x^2-3\,(7\,d^2e^2f-3\,d^3eg)x+(7\,d^2ef-3\,d^3g+}{15\,(d^3e^5x^3-3\,d^4e^4x^2+3\,d^5e^3x-d^6e^2)}$$

input
```
integrate((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/15*(7*d^3*e*f - 3*d^4*g - (7*e^4*f - 3*d*e^3*g)*x^3 + 3*(7*d*e^3*f - 3*
d^2*e^2*g)*x^2 - 3*(7*d^2*e^2*f - 3*d^3*e*g)*x + (7*d^2*e*f - 3*d^3*g + (2
*e^3*f - 3*d*e^2*g)*x^2 - 3*(2*d*e^2*f - 3*d^2*e*g)*x)*sqrt(-e^2*x^2 + d^2
))/(d^3*e^5*x^3 - 3*d^4*e^4*x^2 + 3*d^5*e^3*x - d^6*e^2)
```

3.583.6 Sympy [F]

$$\int \frac{(d+ex)^3(f+gx)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^3\,(f+gx)}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**3*(g*x+f)/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**3*(f + g*x)/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.583.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(105) = 210.

Time = 0.20 (sec) , antiderivative size = 373, normalized size of antiderivative = 3.19

$$\begin{aligned}\int \frac{(d+ex)^3(f+gx)}{(d^2-e^2x^2)^{7/2}}\,dx &= \frac{egx^3}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{dfx}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{3\,d^2gx}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}e}\\
&+\frac{3\,d^2f}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e}+\frac{d^3g}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2}+\frac{4\,fx}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d}+\frac{gx}{10\,(-e^2x^2+d^2)^{\frac{3}{2}}e}\\
&+\frac{8\,fx}{15\,\sqrt{-e^2x^2+d^2}d^3}+\frac{gx}{5\,\sqrt{-e^2x^2+d^2}d^2e}+\frac{(e^3f+3\,de^2g)x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2}+\frac{3\,(de^2f+d^2eg)x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2}\\
&-\frac{2\,(e^3f+3\,de^2g)d^2}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4}-\frac{(de^2f+d^2eg)x}{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2e^2}-\frac{2\,(de^2f+d^2eg)x}{5\,\sqrt{-e^2x^2+d^2}d^4e^2}\end{aligned}$$

input
```
integrate((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/2*e*g*x^3/(-e^2*x^2 + d^2)^(5/2) + 1/5*d*f*x/(-e^2*x^2 + d^2)^(5/2) - 3/
10*d^2*g*x/((-e^2*x^2 + d^2)^(5/2)*e) + 3/5*d^2*f/((-e^2*x^2 + d^2)^(5/2)*
e) + 1/5*d^3*g/((-e^2*x^2 + d^2)^(5/2)*e^2) + 4/15*f*x/((-e^2*x^2 + d^2)^(
3/2)*d) + 1/10*g*x/((-e^2*x^2 + d^2)^(3/2)*e) + 8/15*f*x/(sqrt(-e^2*x^2 +
d^2)*d^3) + 1/5*g*x/(sqrt(-e^2*x^2 + d^2)*d^2*e) + 1/3*(e^3*f + 3*d*e^2*g)
*x^2/((-e^2*x^2 + d^2)^(5/2)*e^2) + 3/5*(d*e^2*f + d^2*e*g)*x/((-e^2*x^2 +
 d^2)^(5/2)*e^2) - 2/15*(e^3*f + 3*d*e^2*g)*d^2/((-e^2*x^2 + d^2)^(5/2)*e^
4) - 1/5*(d*e^2*f + d^2*e*g)*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e^2) - 2/5*(d*e
^2*f + d^2*e*g)*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^2)
```

3.583.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(105) = 210$.

Time = 0.31 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.36

$$\int \frac{(d+ex)^3(f+gx)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2\left(7\,ef - 3\,dg - \frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)f}{ex} + \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)dg}{e^2x} + \frac{40\left(de+\sqrt{-e^2x^2+d^2}|e|\cdots}{e^3x^2}\cdots\right)}{15\,d^3e\cdots}$$

input
```
integrate((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
2/15*(7*e*f - 3*d*g - 20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*f/(e*x) + 15*
(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d*g/(e^2*x) + 40*(d*e + sqrt(-e^2*x^2
+ d^2)*abs(e))^2*f/(e^3*x^2) - 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d*
g/(e^4*x^2) - 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*f/(e^5*x^3) + 15*(d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d*g/(e^6*x^3) + 15*(d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))^4*f/(e^7*x^4))/(d^3*e*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e)
)/(e^2*x) - 1)^5*abs(e))
```

3.583.9 Mupad [B] (verification not implemented)

Time = 12.72 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex)^3(f+gx)}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}\,\left(3\,g\,d^3-9\,g\,d^2\,e\,x-7\,f\,d^2\,e+3\,g\,d\,e^2\,x^2+6\,f\,d\,e^2\,x-2\,f\,e^3\,x^2\right)}{15\,d^3\,e^2\,{\left(d-e\,x\right)}^3}$$

input
```
int(((f + g*x)*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
-((d^2 - e^2*x^2)^(1/2)*(3*d^3*g - 2*e^3*f*x^2 - 7*d^2*e*f + 6*d*e^2*f*x -
 9*d^2*e*g*x + 3*d*e^2*g*x^2))/(15*d^3*e^2*(d - e*x)^3)
```

3.584 $\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}} \, dx$

3.584.1 Optimal result

Integrand size = 24, antiderivative size = 103

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}} \, dx = \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2} + \frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

output
```
1/5*(-e^2*x^2+d^2)^(1/2)/d/e/(-e*x+d)^3+2/15*(-e^2*x^2+d^2)^(1/2)/d^2/e/(-
e*x+d)^2+2/15*(-e^2*x^2+d^2)^(1/2)/d^3/e/(-e*x+d)
```

3.584.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.51

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}} \, dx = \frac{\sqrt{d^2-e^2x^2}(7d^2-6dex+2e^2x^2)}{15d^3e(d-ex)^3}$$

input
```
Integrate[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(7*d^2 - 6*d*e*x + 2*e^2*x^2))/(15*d^3*e*(d - e*x)^3)
```

3.584.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {464, 461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \; 464$$

$$\int \frac{1}{(d-ex)^3\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \; 461$$

$$\frac{2\int \frac{1}{(d-ex)^2\sqrt{d^2-e^2x^2}}dx}{5d}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}$$

$$\Big\downarrow \; 461$$

$$\frac{2\left(\frac{\int \frac{1}{(d-ex)\sqrt{d^2-e^2x^2}}dx}{3d}+\frac{\sqrt{d^2-e^2x^2}}{3de(d-ex)^2}\right)}{5d}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}$$

$$\Big\downarrow \; 460$$

$$\frac{2\left(\frac{\sqrt{d^2-e^2x^2}}{3d^2e(d-ex)}+\frac{\sqrt{d^2-e^2x^2}}{3de(d-ex)^2}\right)}{5d}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}$$

input
```
Int[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
Sqrt[d^2 - e^2*x^2]/(5*d*e*(d - e*x)^3) + (2*(Sqrt[d^2 - e^2*x^2]/(3*d*e*(
d - e*x)^2) + Sqrt[d^2 - e^2*x^2]/(3*d^2*e*(d - e*x))))/(5*d)
```

3.584.3.1 Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 464
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
a + b*x^2)^(n + p)/(a/c + b*(x/d))^n, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c^2 + a*d^2, 0] && IntegerQ[n] && RationalQ[p] && (LtQ[0, -n, p] || LtQ[p,
 -n, 0]) && NeQ[n, 2] && NeQ[n, -1]
```

3.584.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.49

method	result
trager	$\frac{\left(2 e^{2} x^{2}-6 d e x+7 d^{2}\right) \sqrt{-e^{2} x^{2}+d^{2}}}{15 d^{3} \left(-e x+d\right)^{3} e}$
gosper	$\frac{\left(-e x+d\right) \left(e x+d\right)^{4} \left(2 e^{2} x^{2}-6 d e x+7 d^{2}\right)}{15 d^{3} e \left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}}}$
default	$d^{3} \left(\frac{x}{5 d^{2} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{\frac{4 x}{15 d^{2} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{8 x}{15 d^{4} \sqrt{-e^{2} x^{2}+d^{2}}}}{d^{2}}\right)+e^{3} \left(\frac{x^{2}}{3 e^{2} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{2 d^{2}}{15 e^{4} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}\right)+3 d\, e$

input
```
int((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(2*e^2*x^2-6*d*e*x+7*d^2)/d^3/(-e*x+d)^3/e*(-e^2*x^2+d^2)^(1/2)
```

3.584.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{7\,e^3x^3-21\,de^2x^2+21\,d^2ex-7\,d^3-(2\,e^2x^2-6\,dex+7\,d^2)\sqrt{-e^2x^2+d^2}}{15\,(d^3e^4x^3-3\,d^4e^3x^2+3\,d^5e^2x-d^6e)}$$

input
```
integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(7*e^3*x^3 - 21*d*e^2*x^2 + 21*d^2*e*x - 7*d^3 - (2*e^2*x^2 - 6*d*e*x
 + 7*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^4*x^3 - 3*d^4*e^3*x^2 + 3*d^5*e^2*x
 - d^6*e)
```

3.584.6 Sympy [F]

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^3}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.584.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{ex^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{4\,dx}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{7\,d^2}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d} + \frac{2\,x}{15\,\sqrt{-e^2x^2+d^2}d^3}$$

input
```
integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/3*e*x^2/(-e^2*x^2 + d^2)^(5/2) + 4/5*d*x/(-e^2*x^2 + d^2)^(5/2) + 7/15*d
^2/((-e^2*x^2 + d^2)^(5/2)*e) + 1/15*x/((-e^2*x^2 + d^2)^(3/2)*d) + 2/15*x
/(sqrt(-e^2*x^2 + d^2)*d^3)
```

3.584.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{2\left(\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x}-\frac{40\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2}+\frac{30\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3}-\frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4}-7\right)}{15\,d^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^5|e|}$$

input
```
integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
-2/15*(20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 40*(d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^2/(e^4*x^2) + 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^
3/(e^6*x^3) - 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) - 7)/(d^3
*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^5*abs(e))
```

3.584.9 Mupad [B] (verification not implemented)

Time = 12.66 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.48

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(7\,d^2-6\,d\,e\,x+2\,e^2\,x^2\right)}{15\,d^3\,e\,{\left(d-e\,x\right)}^3}$$

input
```
int((d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(7*d^2 + 2*e^2*x^2 - 6*d*e*x))/(15*d^3*e*(d - e*x)^
3)
```

3.585 $\int \frac{(d+ex)^3}{(f+gx)\left(d^2-e^2x^2\right)^{7/2}}\,dx$

3.585.1 Optimal result

Integrand size = 31, antiderivative size = 242

$$\int \frac{(d+ex)^3}{(f+gx)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} - \frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d^3g^2+e\left(2e^2f^2+9defg+22d^2g^2\right)x}{15d^3(ef+dg)^3\sqrt{d^2-e^2x^2}} + \frac{g^3\arctan\left(\frac{d^2g+e^2fx}{\sqrt{e^2f^2-d^2g^2}\sqrt{d^2-e^2x^2}}\right)}{(ef+dg)^3\sqrt{e^2f^2-d^2g^2}}$$

output
```
4/5*d*(e*x+d)/(d*g+e*f)/(-e^2*x^2+d^2)^(5/2)+1/15*(-5*d*(-d*g+e*f)+e*(11*d
*g+e*f)*x)/d/(d*g+e*f)^2/(-e^2*x^2+d^2)^(3/2)+g^3*arctan((e^2*f*x+d^2*g)/(
-d^2*g^2+e^2*f^2)^(1/2)/(-e^2*x^2+d^2)^(1/2))/(d*g+e*f)^3/(-d^2*g^2+e^2*f^
2)^(1/2)+1/15*(15*d^3*g^2+e*(22*d^2*g^2+9*d*e*f*g+2*e^2*f^2)*x)/d^3/(d*g+e
*f)^3/(-e^2*x^2+d^2)^(1/2)
```

3.585.2 Mathematica [A] (verified)

Time = 10.30 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^3}{(f+gx)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\frac{(-e^2f^2+d^2g^2)(d+ex)(32d^4g^2+2e^4f^2x^2+3d^3eg(8f-17gx)+3de^3fx(-2f+3gx)+d^2e^2(7f^2-27fgx+2}{d^3(d-ex)^2\sqrt{d^2-e^2x^2}}}{15(-ef+dg)(ef+dg}$$

input
```
Integrate[(d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(((-(e^2*f^2) + d^2*g^2)*(d + e*x)*(32*d^4*g^2 + 2*e^4*f^2*x^2 + 3*d^3*e*g
*(8*f - 17*g*x) + 3*d*e^3*f*x*(-2*f + 3*g*x) + d^2*e^2*(7*f^2 - 27*f*g*x +
 22*g^2*x^2)))/(d^3*(d - e*x)^2*Sqrt[d^2 - e^2*x^2]) - 15*g^3*Sqrt[e^2*f^2
 - d^2*g^2]*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2*f^2 - d^2*g^2]*Sqrt[d^2 - e
^2*x^2])])/(15*(-(e*f) + d*g)*(e*f + d*g)^4)
```

3.585.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.34, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {713, 27, 686, 25, 27, 686, 27, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}(f+gx)}dx$$

$$\Big\downarrow \text{713}$$

$$\frac{\int \frac{d^2e^2(d(ef+5dg)-e(5ef-11dg)x)}{(ef+dg)(f+gx)(d^2-e^2x^2)^{5/2}}dx}{5d^2e^2}+\frac{4d(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{d(ef+5dg)-e(5ef-11dg)x}{(f+gx)(d^2-e^2x^2)^{5/2}}dx}{5(dg+ef)}+\frac{4d(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)}$$

$$\Big\downarrow \text{686}$$

$$\frac{-\frac{\int -\frac{de^2\left((ef-dg)\left(2e^2f^2+7degf+15d^2g^2\right)+2eg(ef-dg)(ef+11dg)x\right)}{(f+gx)\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2e^2(e^2f^2-d^2g^2)}-\frac{5d(ef-dg)^2-ex(ef-dg)(11dg+ef)}{3d(d^2-e^2x^2)^{3/2}(e^2f^2-d^2g^2)}}{5(dg+ef)}+\frac{4d(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{de^2(ef-dg)\left(2e^2f^2+7degf+15d^2g^2+2eg(ef+11dg)x\right)}{(f+gx)\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2e^2(e^2f^2-d^2g^2)}-\frac{5d(ef-dg)^2-ex(ef-dg)(11dg+ef)}{3d(d^2-e^2x^2)^{3/2}(e^2f^2-d^2g^2)}}{5(dg+ef)}+\frac{4d(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{(ef-dg)\int \frac{2e^2f^2+7degf+15d^2g^2+2eg(ef+11dg)x}{(f+gx)\left(d^2-e^2x^2\right)^{3/2}}dx}{3d(e^2f^2-d^2g^2)}-\frac{5d(ef-dg)^2-ex(ef-dg)(11dg+ef)}{3d(d^2-e^2x^2)^{3/2}(e^2f^2-d^2g^2)}}{5(dg+ef)}+\frac{4d(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)}$$

$$\Big\downarrow \; 686$$

$$\frac{\frac{(ef-dg)\left(\frac{(ef-dg)\left(15d^3g^2+ex\left(22d^2g^2+9defg+2e^2f^2\right)\right)}{d^2\sqrt{d^2-e^2x^2}\left(e^2f^2-d^2g^2\right)}-\frac{\int -\frac{15d^3e^2g^3(ef-dg)}{(f+gx)\sqrt{d^2-e^2x^2}}dx}{d^2e^2\left(e^2f^2-d^2g^2\right)}\right)}{3d(e^2f^2-d^2g^2)}-\frac{5d(ef-dg)^2-ex(ef-dg)(11dg+ef)}{3d(d^2-e^2x^2)^{3/2}(e^2f^2-d^2g^2)}}{5(dg+ef)}+\frac{4d(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{(ef-dg)\left(\frac{15dg^3(ef-dg)\int \frac{1}{(f+gx)\sqrt{d^2-e^2x^2}}dx}{e^2f^2-d^2g^2}+\frac{(ef-dg)\left(15d^3g^2+ex\left(22d^2g^2+9defg+2e^2f^2\right)\right)}{d^2\sqrt{d^2-e^2x^2}\left(e^2f^2-d^2g^2\right)}\right)}{3d(e^2f^2-d^2g^2)}-\frac{5d(ef-dg)^2-ex(ef-dg)(11dg+ef)}{3d(d^2-e^2x^2)^{3/2}(e^2f^2-d^2g^2)}}{5(dg+ef)}+\frac{4d(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)}$$

$$\Big\downarrow \; 488$$

3.585. $\int \frac{(d+ex)^3}{(f+gx)(d^2-e^2x^2)^{7/2}}\,dx$

$$\frac{\frac{(ef-dg)\left(\frac{(ef-dg)\left(15d^3g^2+ex\left(22d^2g^2+9defg+2e^2f^2\right)\right)}{d^2\sqrt{d^2-e^2x^2}\left(e^2f^2-d^2g^2\right)}-\frac{15dg^3(ef-dg)\int\frac{1}{-e^2f^2+d^2g^2-\frac{\left(gd^2+e^2fx\right)^2}{d^2-e^2x^2}}d\frac{gd^2+e^2fx}{\sqrt{d^2-e^2x^2}}}{e^2f^2-d^2g^2}\right)}{3d(e^2f^2-d^2g^2)}-\frac{5d(ef-dg)^2-ex(ef-dg)(11d}{3d(d^2-e^2x^2)^{3/2}(e^2f^2-d}}{5(dg+ef)}$$
$$\frac{4d(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\frac{(ef-dg)\left(\frac{15dg^3(ef-dg)\arctan\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{\left(e^2f^2-d^2g^2\right)^{3/2}}+\frac{(ef-dg)\left(15d^3g^2+ex\left(22d^2g^2+9defg+2e^2f^2\right)\right)}{d^2\sqrt{d^2-e^2x^2}\left(e^2f^2-d^2g^2\right)}\right)}{3d(e^2f^2-d^2g^2)}-\frac{5d(ef-dg)^2-ex(ef-dg)(11dg+ef)}{3d(d^2-e^2x^2)^{3/2}(e^2f^2-d^2g^2)}}{5(dg+ef)}+$$
$$\frac{4d(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)}$$

input
```
Int[(d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(4*d*(d + e*x))/(5*(e*f + d*g)*(d^2 - e^2*x^2)^(5/2)) + (-1/3*(5*d*(e*f -
d*g)^2 - e*(e*f - d*g)*(e*f + 11*d*g)*x)/(d*(e^2*f^2 - d^2*g^2)*(d^2 - e^2
*x^2)^(3/2)) + ((e*f - d*g)*(((e*f - d*g)*(15*d^3*g^2 + e*(2*e^2*f^2 + 9*d
*e*f*g + 22*d^2*g^2)*x))/(d^2*(e^2*f^2 - d^2*g^2)*Sqrt[d^2 - e^2*x^2]) + (
15*d*g^3*(e*f - d*g)*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2*f^2 - d^2*g^2]*Sqr
t[d^2 - e^2*x^2])])/(e^2*f^2 - d^2*g^2)^(3/2)))/(3*d*(e^2*f^2 - d^2*g^2)))
/(5*(e*f + d*g))
```

3.585.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 713
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*(f + g*x)^n,
 a + c*x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*(f + g*x)^n, a +
c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*(f + g*x)^n, a
+ c*x^2, x], x, 1]}, Simp[(a*S - c*R*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)
)), x] + Simp[1/(2*a*c*(p + 1))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Expan
dToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*R*(2*p + 3))/(d + e*x)^m, x], x],
 x]] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[n, 1] && LtQ[p, -1] && ILtQ[m,
 0] && NeQ[c*d^2 + a*e^2, 0]
```

3.585.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1666 vs. $2(223) = 446$.

Time = 0.52 (sec) , antiderivative size = 1667, normalized size of antiderivative = 6.89

method	result	size
default	Expression too large to display	1667

input
```
int((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(d^3*g^3-3*d^2*e*f*g^2+3*d*e^2*f^2*g-e^3*f^3)/g^4*(1/5/(d^2*g^2-e^2*f^2)*g
^2/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2)-e^2*f*g/
(d^2*g^2-e^2*f^2)*(2/5*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e^2*(d^2*g^2-e^2*f^2
)/g^2-4*e^4*f^2/g^2)/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g
^2)^(5/2)-16/5*e^2/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)*(2/3*(-2*e
^2*(x+f/g)+2*e^2*f/g)/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-e^2*(
x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)-16/3*e^2/(-4*e^2*(
d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)^2*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-e^2*(x+
f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)))+1/(d^2*g^2-e^2*f^2
)*g^2*(1/3/(d^2*g^2-e^2*f^2)*g^2/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^
2-e^2*f^2)/g^2)^(3/2)-e^2*f*g/(d^2*g^2-e^2*f^2)*(2/3*(-2*e^2*(x+f/g)+2*e^2
*f/g)/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-e^2*(x+f/g)^2+2*e^2*f
/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)-16/3*e^2/(-4*e^2*(d^2*g^2-e^2*f^2)
/g^2-4*e^4*f^2/g^2)^2*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-e^2*(x+f/g)^2+2*e^2*f/g
*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2))+1/(d^2*g^2-e^2*f^2)*g^2*(1/(d^2*g^2
-e^2*f^2)*g^2/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/
2)-2*e^2*f*g/(d^2*g^2-e^2*f^2)*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e^2*(d^2*g^2
-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^
2*f^2)/g^2)^(1/2)-1/(d^2*g^2-e^2*f^2)*g^2/((d^2*g^2-e^2*f^2)/g^2)^(1/2)*ln
((2*(d^2*g^2-e^2*f^2)/g^2+2*e^2*f/g*(x+f/g)+2*((d^2*g^2-e^2*f^2)/g^2)^(...
```

3.585.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 854 vs. $2(223) = 446$.

Time = 0.37 (sec) , antiderivative size = 1767, normalized size of antiderivative = 7.30

$$\int \frac{(d+ex)^3}{(f+gx)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
[1/15*(7*d^3*e^4*f^4 + 24*d^4*e^3*f^3*g + 25*d^5*e^2*f^2*g^2 - 24*d^6*e*f*
g^3 - 32*d^7*g^4 - (7*e^7*f^4 + 24*d*e^6*f^3*g + 25*d^2*e^5*f^2*g^2 - 24*d
^3*e^4*f*g^3 - 32*d^4*e^3*g^4)*x^3 + 3*(7*d*e^6*f^4 + 24*d^2*e^5*f^3*g + 2
5*d^3*e^4*f^2*g^2 - 24*d^4*e^3*f*g^3 - 32*d^5*e^2*g^4)*x^2 + 15*(d^3*e^3*g
^3*x^3 - 3*d^4*e^2*g^3*x^2 + 3*d^5*e*g^3*x - d^6*g^3)*sqrt(-e^2*f^2 + d^2*
g^2)*log((d*e^2*f*g*x + d^3*g^2 - sqrt(-e^2*f^2 + d^2*g^2)*(e^2*f*x + d^2*
g + sqrt(-e^2*x^2 + d^2)*d*g) - (e^2*f^2 - d^2*g^2)*sqrt(-e^2*x^2 + d^2))/
(g*x + f)) - 3*(7*d^2*e^5*f^4 + 24*d^3*e^4*f^3*g + 25*d^4*e^3*f^2*g^2 - 24
*d^5*e^2*f*g^3 - 32*d^6*e*g^4)*x + (7*d^2*e^4*f^4 + 24*d^3*e^3*f^3*g + 25*
d^4*e^2*f^2*g^2 - 24*d^5*e*f*g^3 - 32*d^6*g^4 + (2*e^6*f^4 + 9*d*e^5*f^3*g
 + 20*d^2*e^4*f^2*g^2 - 9*d^3*e^3*f*g^3 - 22*d^4*e^2*g^4)*x^2 - 3*(2*d*e^5
*f^4 + 9*d^2*e^4*f^3*g + 15*d^3*e^3*f^2*g^2 - 9*d^4*e^2*f*g^3 - 17*d^5*e*g
^4)*x)*sqrt(-e^2*x^2 + d^2))/(d^6*e^5*f^5 + 3*d^7*e^4*f^4*g + 2*d^8*e^3*f^
3*g^2 - 2*d^9*e^2*f^2*g^3 - 3*d^10*e*f*g^4 - d^11*g^5 - (d^3*e^8*f^5 + 3*d
^4*e^7*f^4*g + 2*d^5*e^6*f^3*g^2 - 2*d^6*e^5*f^2*g^3 - 3*d^7*e^4*f*g^4 - d
^8*e^3*g^5)*x^3 + 3*(d^4*e^7*f^5 + 3*d^5*e^6*f^4*g + 2*d^6*e^5*f^3*g^2 - 2
*d^7*e^4*f^2*g^3 - 3*d^8*e^3*f*g^4 - d^9*e^2*g^5)*x^2 - 3*(d^5*e^6*f^5 + 3
*d^6*e^5*f^4*g + 2*d^7*e^4*f^3*g^2 - 2*d^8*e^3*f^2*g^3 - 3*d^9*e^2*f*g^4 -
 d^10*e*g^5)*x), 1/15*(7*d^3*e^4*f^4 + 24*d^4*e^3*f^3*g + 25*d^5*e^2*f^2*g
^2 - 24*d^6*e*f*g^3 - 32*d^7*g^4 - (7*e^7*f^4 + 24*d*e^6*f^3*g + 25*d^2...
```

3.585.6 Sympy [F]

$$\int \frac{(d+ex)^3}{(f+gx)(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}}(f+gx)}\,dx$$

input
```
integrate((e*x+d)**3/(g*x+f)/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**3/((-(-d + e*x)*(d + e*x))**(7/2)*(f + g*x)), x)
```

3.585.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{(f+gx)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.585.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(223) = 446$.

Time = 0.31 (sec) , antiderivative size = 642, normalized size of antiderivative = 2.65

$$\int \frac{(d+ex)^3}{(f+gx)\left(d^2-e^2x^2\right)^{7/2}}\,dx = -\frac{2\,eg^3\arctan\left(\frac{dg+\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)f}{ex}}{\sqrt{e^2f^2-d^2g^2}}\right)}{\left(e^3f^3|e|+3\,de^2f^2g|e|+3\,d^2efg^2|e|+d^3g^3|e|\right)\sqrt{e^2f^2-d^2g^2}} + \frac{2\left(7\,e^3f^2+24\,de^2fg+32\,d^2eg^2-\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)ef^2}{x}-\frac{75\left(de+\sqrt{-e^2x^2+d^2}|e|\right)dfg}{x}-\frac{115\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2g^2}{ex}\right.}{}$$

input
```
integrate((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
-2*e*g^3*arctan((d*g + (d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*f/(e*x))/sqrt(e
^2*f^2 - d^2*g^2))/((e^3*f^3*abs(e) + 3*d*e^2*f^2*g*abs(e) + 3*d^2*e*f*g^2
*abs(e) + d^3*g^3*abs(e))*sqrt(e^2*f^2 - d^2*g^2)) + 2/15*(7*e^3*f^2 + 24*
d*e^2*f*g + 32*d^2*e*g^2 - 20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e*f^2/x
- 75*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d*f*g/x - 115*(d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))*d^2*g^2/(e*x) + 40*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*
f^2/(e*x^2) + 135*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d*f*g/(e^2*x^2) +
185*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^2*g^2/(e^3*x^2) - 30*(d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))^3*f^2/(e^3*x^3) - 105*(d*e + sqrt(-e^2*x^2 + d^
2)*abs(e))^3*d*f*g/(e^4*x^3) - 135*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d
^2*g^2/(e^5*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*f^2/(e^5*x^4)
+ 45*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d*f*g/(e^6*x^4) + 45*(d*e + sqr
t(-e^2*x^2 + d^2)*abs(e))^4*d^2*g^2/(e^7*x^4))/((d^3*e^3*f^3*abs(e) + 3*d^
4*e^2*f^2*g*abs(e) + 3*d^5*e*f*g^2*abs(e) + d^6*g^3*abs(e))*((d*e + sqrt(-
e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^5)
```

3.585.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{(f+gx)\,(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+e\,x)^3}{(f+g\,x)\,\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

input
```
int((d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)),x)
```

output
```
int((d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)), x)
```

3.586 $\int \frac{(d+ex)^3}{(f+gx)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$

3.586.1 Optimal result

Integrand size = 31, antiderivative size = 311

$$\int \frac{(d+ex)^3}{(f+gx)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e\left(45d^3g^2+e\left(2e^2f^2+14defg+57d^2g^2\right)x\right)}{15d^3(ef+dg)^4\sqrt{d^2-e^2x^2}} + \frac{g^4\sqrt{d^2-e^2x^2}}{(ef-dg)(ef+dg)^4(f+gx)} + \frac{eg^3(4ef-3dg)\arctan\left(\frac{d^2g+e^2fx}{\sqrt{e^2f^2-d^2g^2}\sqrt{d^2-e^2x^2}}\right)}{(ef-dg)(ef+dg)^4\sqrt{e^2f^2-d^2g^2}}$$

output
```
4/5*d*e*(e*x+d)/(d*g+e*f)^2/(-e^2*x^2+d^2)^(5/2)-1/15*e*(5*d*(-3*d*g+e*f)-
e*(21*d*g+e*f)*x)/d/(d*g+e*f)^3/(-e^2*x^2+d^2)^(3/2)+e*g^3*(-3*d*g+4*e*f)*
arctan((e^2*f*x+d^2*g)/(-d^2*g^2+e^2*f^2)^(1/2)/(-e^2*x^2+d^2)^(1/2))/(-d*
g+e*f)/(d*g+e*f)^4/(-d^2*g^2+e^2*f^2)^(1/2)+1/15*e*(45*d^3*g^2+e*(57*d^2*g
^2+14*d*e*f*g+2*e^2*f^2)*x)/d^3/(d*g+e*f)^4/(-e^2*x^2+d^2)^(1/2)+g^4*(-e^2
*x^2+d^2)^(1/2)/(-d*g+e*f)/(d*g+e*f)^4/(g*x+f)
```

3.586.2 Mathematica [A] (verified)

Time = 10.43 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)^3}{(f+gx)^2 (d^2-e^2x^2)^{7/2}}\,dx = \frac{(e^2f^2-d^2g^2)(d+ex)(15d^6g^4+2e^6f^3x^2(f+gx)-9d^5eg^3(8f+13gx)+6de^5f^2x(-f^2+fgx+2g^2x^2)+d^4e \cdots}{d^3(d-e \cdots}$$

input
```
Integrate[(d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(((e^2*f^2 - d^2*g^2)*(d + e*x)*(15*d^6*g^4 + 2*e^6*f^3*x^2*(f + g*x) - 9*
d^5*e*g^3*(8*f + 13*g*x) + 6*d*e^5*f^2*x*(-f^2 + f*g*x + 2*g^2*x^2) + d^4*
e^2*g^2*(38*f^2 + 164*f*g*x + 171*g^2*x^2) - 3*d^3*e^3*g*(-9*f^3 + 19*f^2*
g*x + 47*f*g^2*x^2 + 24*g^3*x^3) + d^2*e^4*f*(7*f^3 - 29*f^2*g*x + 7*f*g^2
*x^2 + 43*g^3*x^3)))/(d^3*(d - e*x)^2*(f + g*x)*Sqrt[d^2 - e^2*x^2]) + 15*
e*g^3*(4*e*f - 3*d*g)*Sqrt[e^2*f^2 - d^2*g^2]*ArcTan[(d^2*g + e^2*f*x)/(Sq
rt[e^2*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x^2])])/(15*(e*f - d*g)^2*(e*f + d*g)
^5)
```

3.586.3 Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {713, 2178, 2178, 27, 679, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}(f+gx)^2}dx$$

$$\Big\downarrow \text{ 713}$$

$$\frac{\int \frac{\frac{16d^3g^2x^2e^4}{(ef+dg)^2}-\frac{d^2(ef-5dg)(5ef+3dg)xe^3}{(ef+dg)^2}+\frac{d^3\left(e^2f^2+10degf+5d^2g^2\right)e^2}{(ef+dg)^2}}{(f+gx)^2(d^2-e^2x^2)^{5/2}}dx}{5d^2e^2}+\frac{4de(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)^2}$$

$$\Big\downarrow \text{ 2178}$$

$$\frac{\frac{\int \frac{\frac{2d^3g^2(ef+21dg)x^2e^6}{(ef+dg)^3}+\frac{d^3g\left(4e^2f^2+69degf+45d^2g^2\right)xe^5}{(ef+dg)^3}+\frac{d^3\left(2e^3f^3+12de^2gf^2+45d^2eg^2f+15d^3g^3\right)e^4}{(ef+dg)^3}}{(f+gx)^2\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2e^2}-\frac{de^3(5d(ef-3dg)-ex(21dg+ef))}{3(d^2-e^2x^2)^{3/2}(dg+ef)^3}}{5d^2e^2}+\frac{4de(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)^2}$$

$$\Big\downarrow \text{2178}$$

$$\frac{\frac{\frac{\int \frac{15d^6e^6g^3(4ef+dg+3egx)}{(ef+dg)^4(f+gx)^2\sqrt{d^2-e^2x^2}}dx}{d^2e^2}+\frac{de^5\left(45d^3g^2+ex\left(57d^2g^2+14defg+2e^2f^2\right)\right)}{\sqrt{d^2-e^2x^2}(dg+ef)^4}}{3d^2e^2}-\frac{de^3(5d(ef-3dg)-ex(21dg+ef))}{3(d^2-e^2x^2)^{3/2}(dg+ef)^3}}{5d^2e^2}+\frac{4de(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{15d^4e^4g^3\int \frac{4ef+dg+3egx}{(f+gx)^2\sqrt{d^2-e^2x^2}}dx}{(dg+ef)^4}+\frac{de^5\left(45d^3g^2+ex\left(57d^2g^2+14defg+2e^2f^2\right)\right)}{\sqrt{d^2-e^2x^2}(dg+ef)^4}}{3d^2e^2}-\frac{de^3(5d(ef-3dg)-ex(21dg+ef))}{3(d^2-e^2x^2)^{3/2}(dg+ef)^3}}{5d^2e^2}+\frac{4de(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)^2}$$

$$\Big\downarrow \text{679}$$

$$\frac{\frac{\frac{15d^4e^4g^3\left(\frac{e(4ef-3dg)\int \frac{1}{(f+gx)\sqrt{d^2-e^2x^2}}dx}{ef-dg}+\frac{g\sqrt{d^2-e^2x^2}}{(f+gx)(ef-dg)}\right)}{(dg+ef)^4}+\frac{de^5\left(45d^3g^2+ex\left(57d^2g^2+14defg+2e^2f^2\right)\right)}{\sqrt{d^2-e^2x^2}(dg+ef)^4}}{3d^2e^2}-\frac{de^3(5d(ef-3dg)-ex(21dg+ef))}{3(d^2-e^2x^2)^{3/2}(dg+ef)^3}}{5d^2e^2}+\frac{4de(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)^2}$$

$$\Big\downarrow \text{488}$$

$$\frac{\frac{\frac{15d^4e^4g^3\left(\frac{g\sqrt{d^2-e^2x^2}}{(f+gx)(ef-dg)}-\frac{e(4ef-3dg)\int \frac{1}{-e^2f^2+d^2g^2-\frac{\left(gd^2+e^2fx\right)^2}{d^2-e^2x^2}}d\frac{gd^2+e^2fx}{\sqrt{d^2-e^2x^2}}}{ef-dg}\right)}{(dg+ef)^4}+\frac{de^5\left(45d^3g^2+ex\left(57d^2g^2+14defg+2e^2f^2\right)\right)}{\sqrt{d^2-e^2x^2}(dg+ef)^4}}{3d^2e^2}-\frac{de^3(5d(ef-3dg)-\ldots}{3(d^2-e^2x^2)^{3/\ldots}}}{5d^2e^2}$$

$$\frac{4de(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)^2}$$

$$\Big\downarrow \text{217}$$

3.586. $\displaystyle\int \frac{(d+ex)^3}{(f+gx)^2(d^2-e^2x^2)^{7/2}}\,dx$

$$\frac{\frac{\frac{15d^4e^4g^3\left(\frac{e(4ef-3dg)\arctan\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{(ef-dg)\sqrt{e^2f^2-d^2g^2}}+\frac{g\sqrt{d^2-e^2x^2}}{(f+gx)(ef-dg)}\right)}{(dg+ef)^4}+\frac{de^5\left(45d^3g^2+ex\left(57d^2g^2+14defg+2e^2f^2\right)\right)}{\sqrt{d^2-e^2x^2}(dg+ef)^4}}{3d^2e^2}-\frac{de^3(5d(ef-3dg)-ex(21dg-}{3(d^2-e^2x^2)^{3/2}(dg+ef)}}{5d^2e^2}$$

$$\frac{4de(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)^2}$$

input
```
Int[(d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(4*d*e*(d + e*x))/(5*(e*f + d*g)^2*(d^2 - e^2*x^2)^(5/2)) + (-1/3*(d*e^3*(
5*d*(e*f - 3*d*g) - e*(e*f + 21*d*g)*x))/((e*f + d*g)^3*(d^2 - e^2*x^2)^(3
/2)) + ((d*e^5*(45*d^3*g^2 + e*(2*e^2*f^2 + 14*d*e*f*g + 57*d^2*g^2)*x))/(
(e*f + d*g)^4*Sqrt[d^2 - e^2*x^2]) + (15*d^4*e^4*g^3*((g*Sqrt[d^2 - e^2*x^
2])/((e*f - d*g)*(f + g*x)) + (e*(4*e*f - 3*d*g)*ArcTan[(d^2*g + e^2*f*x)/
(Sqrt[e^2*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x^2])])/((e*f - d*g)*Sqrt[e^2*f^2
- d^2*g^2])))/(e*f + d*g)^4)/(3*d^2*e^2))/(5*d^2*e^2)
```

3.586.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

3.586. $\int \frac{(d+ex)^3}{(f+gx)^2(d^2-e^2x^2)^{7/2}}\,dx$

rule 713
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*(f + g*x)^n,
 a + c*x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*(f + g*x)^n, a +
c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*(f + g*x)^n, a
+ c*x^2, x], x, 1]}, Simp[(a*S - c*R*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)
)), x] + Simp[1/(2*a*c*(p + 1))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Expan
dToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*R*(2*p + 3))/(d + e*x)^m, x], x],
 x]] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[n, 1] && LtQ[p, -1] && ILtQ[m,
 0] && NeQ[c*d^2 + a*e^2, 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.586.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3288 vs. $2(291) = 582$.

Time = 0.52 (sec) , antiderivative size = 3289, normalized size of antiderivative = 10.58

method	result	size
default	Expression too large to display	3289

input
```
int((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
e^2/g^3*(1/5/e*g/(-e^2*x^2+d^2)^(5/2)+3*d*g*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2
)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))
-2*e*f*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(
3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))))+3*e/g^4*(d^2*g^2-2*d*e*f*g+e^2*f^2)
*(1/5/(d^2*g^2-e^2*f^2)*g^2/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2
*f^2)/g^2)^(5/2)-e^2*f*g/(d^2*g^2-e^2*f^2)*(2/5*(-2*e^2*(x+f/g)+2*e^2*f/g)
/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-e^2*(x+f/g)^2+2*e^2*f/g*(x
+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2)-16/5*e^2/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-
4*e^4*f^2/g^2)*(2/3*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e^2*(d^2*g^2-e^2*f^2)/g
^2-4*e^4*f^2/g^2)/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)
^(3/2)-16/3*e^2/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)^2*(-2*e^2*(x+
f/g)+2*e^2*f/g)/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(
1/2)))+1/(d^2*g^2-e^2*f^2)*g^2*(1/3/(d^2*g^2-e^2*f^2)*g^2/(-e^2*(x+f/g)^2+
2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)-e^2*f*g/(d^2*g^2-e^2*f^2)*(
2/3*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2
)/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)-16/3*e^2/
(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)^2*(-2*e^2*(x+f/g)+2*e^2*f/g)/
(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2))+1/(d^2*g^2
-e^2*f^2)*g^2*(1/(d^2*g^2-e^2*f^2)*g^2/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(
d^2*g^2-e^2*f^2)/g^2)^(1/2)-2*e^2*f*g/(d^2*g^2-e^2*f^2)*(-2*e^2*(x+f/g)...
```

3.586.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1623 vs. $2(290) = 580$.

Time = 0.75 (sec) , antiderivative size = 3305, normalized size of antiderivative = 10.63

$$\int \frac{(d+ex)^3}{(f+gx)^2 (d^2-e^2x^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
[1/15*(7*d^3*e^6*f^7 + 27*d^4*e^5*f^6*g + 31*d^5*e^4*f^5*g^2 - 99*d^6*e^3*
f^4*g^3 - 23*d^7*e^2*f^3*g^4 + 72*d^8*e*f^2*g^5 - 15*d^9*f*g^6 - (7*e^9*f^
6*g + 27*d*e^8*f^5*g^2 + 31*d^2*e^7*f^4*g^3 - 99*d^3*e^6*f^3*g^4 - 23*d^4*
e^5*f^2*g^5 + 72*d^5*e^4*f*g^6 - 15*d^6*e^3*g^7)*x^4 - (7*e^9*f^7 + 6*d*e^
8*f^6*g - 50*d^2*e^7*f^5*g^2 - 192*d^3*e^6*f^4*g^3 + 274*d^4*e^5*f^3*g^4 +
 141*d^5*e^4*f^2*g^5 - 231*d^6*e^3*f*g^6 + 45*d^7*e^2*g^7)*x^3 + 3*(7*d*e^
8*f^7 + 20*d^2*e^7*f^6*g + 4*d^3*e^6*f^5*g^2 - 130*d^4*e^5*f^4*g^3 + 76*d^
5*e^4*f^3*g^4 + 95*d^6*e^3*f^2*g^5 - 87*d^7*e^2*f*g^6 + 15*d^8*e*g^7)*x^2
 - 15*(4*d^6*e^2*f^3*g^3 - 3*d^7*e*f^2*g^4 - (4*d^3*e^5*f^2*g^4 - 3*d^4*e^4
*f*g^5)*x^4 - (4*d^3*e^5*f^3*g^3 - 15*d^4*e^4*f^2*g^4 + 9*d^5*e^3*f*g^5)*x
^3 + 3*(4*d^4*e^4*f^3*g^3 - 7*d^5*e^3*f^2*g^4 + 3*d^6*e^2*f*g^5)*x^2 - (12
*d^5*e^3*f^3*g^3 - 13*d^6*e^2*f^2*g^4 + 3*d^7*e*f*g^5)*x)*sqrt(-e^2*f^2 +
d^2*g^2)*log((d*e^2*f*g*x + d^3*g^2 - sqrt(-e^2*f^2 + d^2*g^2)*(e^2*f*x +
d^2*g + sqrt(-e^2*x^2 + d^2)*d*g) - (e^2*f^2 - d^2*g^2)*sqrt(-e^2*x^2 + d^
2))/(g*x + f)) - (21*d^2*e^7*f^7 + 74*d^3*e^6*f^6*g + 66*d^4*e^5*f^5*g^2 -
 328*d^5*e^4*f^4*g^3 + 30*d^6*e^3*f^3*g^4 + 239*d^7*e^2*f^2*g^5 - 117*d^8*
e*f*g^6 + 15*d^9*g^7)*x + (7*d^2*e^6*f^7 + 27*d^3*e^5*f^6*g + 31*d^4*e^4*f
^5*g^2 - 99*d^5*e^3*f^4*g^3 - 23*d^6*e^2*f^3*g^4 + 72*d^7*e*f^2*g^5 - 15*d
^8*f*g^6 + (2*e^8*f^6*g + 12*d*e^7*f^5*g^2 + 41*d^2*e^6*f^4*g^3 - 84*d^3*e
^5*f^3*g^4 - 43*d^4*e^4*f^2*g^5 + 72*d^5*e^3*f*g^6)*x^3 + (2*e^8*f^7 + ...
```

3.586.6 Sympy [F]

$$\int \frac{(d+ex)^3}{(f+gx)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx=\int \frac{(d+ex)^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(f+gx\right)^2}\,dx$$

input
```
integrate((e*x+d)**3/(g*x+f)**2/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**3/((-(-d + e*x)*(d + e*x))**(7/2)*(f + g*x)**2), x)
```

3.586.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{(f+gx)^2 (d^2-e^2x^2)^{7/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.586.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{(f+gx)^2 (d^2-e^2x^2)^{7/2}} \, dx = \text{Timed out}$$

input
```
integrate((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.586.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{(f+gx)^2 (d^2-e^2x^2)^{7/2}} \, dx = \int \frac{(d+e\,x)^3}{(f+g\,x)^2\,(d^2-e^2\,x^2)^{7/2}} \,dx$$

input
```
int((d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)),x)
```

output
```
int((d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)), x)
```

3.587 $\int \frac{(d+ex)^3}{(f+gx)^3(d^2-e^2x^2)^{7/2}}\,dx$

3.587.1 Optimal result

Integrand size = 31, antiderivative size = 398

$$\begin{aligned}\int \frac{(d+ex)^3}{(f+gx)^3(d^2-e^2x^2)^{7/2}}\,dx = {} & \frac{4de^2(d+ex)}{5(ef+dg)^3(d^2-e^2x^2)^{5/2}} \\ & - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4(d^2-e^2x^2)^{3/2}} + \frac{e^2(90d^3g^2+e(2e^2f^2+19defg+107d^2g^2)x)}{15d^3(ef+dg)^5\sqrt{d^2-e^2x^2}} \\ & + \frac{g^4\sqrt{d^2-e^2x^2}}{2(ef-dg)(ef+dg)^4(f+gx)^2} + \frac{3eg^4(3ef-2dg)\sqrt{d^2-e^2x^2}}{2(ef-dg)^2(ef+dg)^5(f+gx)} \\ & + \frac{e^2g^3(20e^2f^2-30defg+13d^2g^2)\arctan\left(\frac{d^2g+e^2fx}{\sqrt{e^2f^2-d^2g^2}\sqrt{d^2-e^2x^2}}\right)}{2(ef-dg)^2(ef+dg)^5\sqrt{e^2f^2-d^2g^2}}\end{aligned}$$

output

```
4/5*d*e^2*(e*x+d)/(d*g+e*f)^3/(-e^2*x^2+d^2)^(5/2)-1/15*e^2*(5*d*(-5*d*g+e
*f)-e*(31*d*g+e*f)*x)/d/(d*g+e*f)^4/(-e^2*x^2+d^2)^(3/2)+1/2*e^2*g^3*(13*d
^2*g^2-30*d*e*f*g+20*e^2*f^2)*arctan((e^2*f*x+d^2*g)/(-d^2*g^2+e^2*f^2)^(1
/2)/(-e^2*x^2+d^2)^(1/2))/(-d*g+e*f)^2/(d*g+e*f)^5/(-d^2*g^2+e^2*f^2)^(1/2
)+1/15*e^2*(90*d^3*g^2+e*(107*d^2*g^2+19*d*e*f*g+2*e^2*f^2)*x)/d^3/(d*g+e*
f)^5/(-e^2*x^2+d^2)^(1/2)+1/2*g^4*(-e^2*x^2+d^2)^(1/2)/(-d*g+e*f)/(d*g+e*f
)^4/(g*x+f)^2+3/2*e*g^4*(-2*d*g+3*e*f)*(-e^2*x^2+d^2)^(1/2)/(-d*g+e*f)^2/(
d*g+e*f)^5/(g*x+f)
```

3.587.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.99 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^3}{(f+gx)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}\left(\frac{6e^2(ef+dg)^2}{d(d-ex)^3}+\frac{2e^2(ef+dg)(2ef+17dg)}{d^2(d-ex)^2}+\frac{2e^2\left(2e^2f^2+19defg+107d^2g^2\right)}{d^3(d-ex)}+\frac{1}{(e\ldots}\right.}{}$$

input
```
Integrate[(d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*((6*e^2*(e*f + d*g)^2)/(d*(d - e*x)^3) + (2*e^2*(e*f
+ d*g)*(2*e*f + 17*d*g))/(d^2*(d - e*x)^2) + (2*e^2*(2*e^2*f^2 + 19*d*e*f*
g + 107*d^2*g^2))/(d^3*(d - e*x)) + (15*g^4*(e*f + d*g))/((e*f - d*g)*(f +
 g*x)^2) + (45*e*g^4*(3*e*f - 2*d*g))/((e*f - d*g)^2*(f + g*x))) - ((15*I)
*e^2*g^3*(20*e^2*f^2 - 30*d*e*f*g + 13*d^2*g^2)*Log[(4*(e*f - d*g)^2*(e*f
+ d*g)^5*(I*d^2*g + I*e^2*f*x + Sqrt[e^2*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x^2
]))/(e^2*g^2*Sqrt[e^2*f^2 - d^2*g^2]*(20*e^2*f^2 - 30*d*e*f*g + 13*d^2*g^2
)*(f + g*x))])/((e*f - d*g)^2*Sqrt[e^2*f^2 - d^2*g^2]))/(30*(e*f + d*g)^5)
```

3.587.3 Rubi [A] (verified)

Time = 3.01 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {713, 2178, 2178, 27, 2182, 27, 679, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}(f+gx)^3}dx$$

$$\Big\downarrow \text{ 713}$$

$$\frac{\int \frac{\frac{16d^3g^3x^3e^5}{(ef+dg)^3}+\frac{4d^3g^2(12ef+5dg)x^2e^4}{(ef+dg)^3}-\frac{d^2\left(5e^3f^3-33de^2gf^2-45d^2eg^2f-15d^3g^3\right)xe^3}{(ef+dg)^3}+\frac{d^3\left(e^3f^3+15de^2gf^2+15d^2eg^2f+5d^3g^3\right)e^2}{(ef+dg)^3}}{(f+gx)^3\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2e^2}+\frac{4de^2(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)^3}$$

$$\Big\downarrow \; 2178$$

$$\frac{\frac{\int \frac{\frac{2d^3g^3(ef+31dg)x^3e^7}{(ef+dg)^4}+\frac{3d^3g^2\left(2e^2f^2+57degf+25d^2g^2\right)x^2e^6}{(ef+dg)^4}+\frac{3d^3g\left(2e^2f^2+45degf+15d^2g^2\right)xe^5}{(ef+dg)^3}+\frac{d^3\left(2e^4f^4+17de^3gf^3+90d^2e^2g^2f^2+60d^3eg^3f+15d^4g^4\right)e^{}}{(ef+dg)^4}}{(f+gx)^3\left(d^2-e^2x^2\right)^{3/2}}}{3d^2e^2}}{5d^2e^2}$$

$$\frac{4de^2(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)^3}$$

$$\Big\downarrow \; 2178$$

$$\frac{\frac{\frac{\int \frac{15\left(\frac{6d^6g^5x^2e^8}{(ef+dg)^5}+\frac{3d^6g^4(5ef+dg)xe^7}{(ef+dg)^5}+\frac{d^6g^3\left(10e^2f^2+5degf+d^2g^2\right)e^6}{(ef+dg)^5}\right)}{(f+gx)^3\sqrt{d^2-e^2x^2}}dx}{d^2e^2}+\frac{de^6\left(90d^3g^2+ex\left(107d^2g^2+19defg+2e^2f^2\right)\right)}{\sqrt{d^2-e^2x^2}(dg+ef)^5}}{3d^2e^2}-\frac{de^4(5d(ef-5dg)-ex(31dg+}{3(d^2-e^2x^2)^{3/2}(dg+ef)}}{5d^2e^2}$$

$$\frac{4de^2(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{15\int \frac{\frac{6d^6g^5x^2e^8}{(ef+dg)^5}+\frac{3d^6g^4(5ef+dg)xe^7}{(ef+dg)^5}+\frac{d^6g^3\left(10e^2f^2+5degf+d^2g^2\right)e^6}{(ef+dg)^5}}{(f+gx)^3\sqrt{d^2-e^2x^2}}dx}{d^2e^2}+\frac{de^6\left(90d^3g^2+ex\left(107d^2g^2+19defg+2e^2f^2\right)\right)}{\sqrt{d^2-e^2x^2}(dg+ef)^5}}{3d^2e^2}-\frac{de^4(5d(ef-5dg)-ex(31dg+ef}{3(d^2-e^2x^2)^{3/2}(dg+ef)^4}}{5d^2e^2}$$

$$\frac{4de^2(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)^3}$$

$$\Big\downarrow \; 2182$$

$$\frac{\frac{\frac{15\left(\frac{\int \frac{d^6e^7g^3\left(2\left(10e^2f^2-5degf-3d^2g^2\right)+eg(11ef-13dg)x\right)}{(ef+dg)^4(f+gx)^2\sqrt{d^2-e^2x^2}}dx}{2\left(e^2f^2-d^2g^2\right)}+\frac{d^6e^6g^4\sqrt{d^2-e^2x^2}}{2(f+gx)^2(ef-dg)(dg+ef)^4}\right)}{d^2e^2}+\frac{de^6\left(90d^3g^2+ex\left(107d^2g^2+19defg+2e^2f^2\right)\right)}{\sqrt{d^2-e^2x^2}(dg+ef)^5}}{3d^2e^2}-\frac{de^4(5d(ef-}{3(d^2-e}}{5d^2e^2}$$

$$\frac{4de^2(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)^3}$$

$$\Big\downarrow \; 27$$

3.587. $\int \frac{(d+ex)^3}{(f+gx)^3(d^2-e^2x^2)^{7/2}}\,dx$

$$\frac{\frac{\frac{15\left(\frac{d^6e^7g^3\int\frac{2\left(10e^2f^2-5degf-3d^2g^2\right)+eg(11ef-13dg)x}{(f+gx)^2\sqrt{d^2-e^2x^2}}dx}{2(dg+ef)^4\left(e^2f^2-d^2g^2\right)}+\frac{d^6e^6g^4\sqrt{d^2-e^2x^2}}{2(f+gx)^2(ef-dg)(dg+ef)^4}\right)}{d^2e^2}+\frac{de^6\left(90d^3g^2+ex\left(107d^2g^2+19defg+2e^2f^2\right)\right)}{\sqrt{d^2-e^2x^2}(dg+ef)^5}}{3d^2e^2}-\frac{de^4(5d(ef-5\cdots}{3(d^2-e^2x\cdots}}{5d^2e^2}$$

$$\frac{4de^2(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)^3}$$

$\Big\downarrow$ 679

$$\frac{\frac{\frac{15\left(\frac{d^6e^7g^3\left(\frac{e\left(13d^2g^2-30defg+20e^2f^2\right)\int\frac{1}{(f+gx)\sqrt{d^2-e^2x^2}}dx}{ef-dg}+\frac{3g\sqrt{d^2-e^2x^2}(3ef-2dg)}{(f+gx)(ef-dg)}\right)}{2(dg+ef)^4\left(e^2f^2-d^2g^2\right)}+\frac{d^6e^6g^4\sqrt{d^2-e^2x^2}}{2(f+gx)^2(ef-dg)(dg+ef)^4}\right)}{d^2e^2}+\frac{de^6\left(90d^3g^2+ex\left(107d^2g^2+19d\cdots\right.\right.}{\sqrt{d^2-e^2x^2}(dg+ef\cdots}}{3d^2e^2}\cdots}{5d^2e^2}$$

$$\frac{4de^2(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)^3}$$

$\Big\downarrow$ 488

$$\frac{\frac{\frac{15\left(\frac{d^6e^7g^3\left(\frac{3g\sqrt{d^2-e^2x^2}(3ef-2dg)}{(f+gx)(ef-dg)}-\frac{e\left(13d^2g^2-30defg+20e^2f^2\right)\int\frac{1}{-e^2f^2+d^2g^2-\frac{\left(gd^2+e^2fx\right)^2}{d^2-e^2x^2}}d\frac{gd^2+e^2fx}{\sqrt{d^2-e^2x^2}}}{ef-dg}\right)}{2(dg+ef)^4\left(e^2f^2-d^2g^2\right)}+\frac{d^6e^6g^4\sqrt{d^2-e^2x^2}}{2(f+gx)^2(ef-dg)(dg+ef)^4}\right)}{d^2e^2}+\frac{de^6\left(90d^3\cdots\right.}{\cdots}}{3d^2e^2}\cdots}{5d^2e^2}$$

$$\frac{4de^2(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)^3}$$

$\Big\downarrow$ 217

3.587. $\int\frac{(d+ex)^3}{(f+gx)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$

$$\frac{\frac{\frac{15\left(\frac{d^6e^7g^3\left(\frac{e\left(13d^2g^2-30defg+20e^2f^2\right)\arctan\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{(ef-dg)\sqrt{e^2f^2-d^2g^2}}+\frac{3g\sqrt{d^2-e^2x^2}(3ef-2dg)}{(f+gx)(ef-dg)}\right)}{2(dg+ef)^4\left(e^2f^2-d^2g^2\right)}+\frac{d^6e^6g^4\sqrt{d^2-e^2x^2}}{2(f+gx)^2(ef-dg)(dg+ef)^4}\right)}{d^2e^2}+\frac{de^6\left(90d^3g^2+ex\left(1\cdots\right.\right.}{\sqrt{d^2-e\cdots}}}{3d^2e^2}}{5d^2e^2}$$

$$\frac{4de^2(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)^3}$$

input
```
Int[(d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(4*d*e^2*(d + e*x))/(5*(e*f + d*g)^3*(d^2 - e^2*x^2)^(5/2)) + (-1/3*(d*e^4
*(5*d*(e*f - 5*d*g) - e*(e*f + 31*d*g)*x))/((e*f + d*g)^4*(d^2 - e^2*x^2)^
(3/2)) + ((d*e^6*(90*d^3*g^2 + e*(2*e^2*f^2 + 19*d*e*f*g + 107*d^2*g^2)*x)
)/((e*f + d*g)^5*Sqrt[d^2 - e^2*x^2]) + (15*((d^6*e^6*g^4*Sqrt[d^2 - e^2*x
^2])/(2*(e*f - d*g)*(e*f + d*g)^4*(f + g*x)^2) + (d^6*e^7*g^3*((3*g*(3*e*f
 - 2*d*g)*Sqrt[d^2 - e^2*x^2])/((e*f - d*g)*(f + g*x)) + (e*(20*e^2*f^2 -
30*d*e*f*g + 13*d^2*g^2)*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2*f^2 - d^2*g^2]
*Sqrt[d^2 - e^2*x^2])])/((e*f - d*g)*Sqrt[e^2*f^2 - d^2*g^2])))/(2*(e*f +
d*g)^4*(e^2*f^2 - d^2*g^2))))/(d^2*e^2))/(3*d^2*e^2))/(5*d^2*e^2)
```

3.587.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

3.587. $\int \frac{(d+ex)^3}{(f+gx)^3(d^2-e^2x^2)^{7/2}}\,dx$

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 713
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*(f + g*x)^n,
 a + c*x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*(f + g*x)^n, a +
c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*(f + g*x)^n, a
+ c*x^2, x], x, 1]}, Simp[(a*S - c*R*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)
)), x] + Simp[1/(2*a*c*(p + 1))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Expan
dToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*R*(2*p + 3))/(d + e*x)^m, x], x],
 x]] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[n, 1] && LtQ[p, -1] && ILtQ[m,
 0] && NeQ[c*d^2 + a*e^2, 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.587. $\int \frac{(d+ex)^3}{(f+gx)^3(d^2-e^2x^2)^{7/2}}\,dx$

3.587.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 6395 vs. $2(370) = 740$.

Time = 0.48 (sec) , antiderivative size = 6396, normalized size of antiderivative = 16.07

method	result	size
default	Expression too large to display	6396

input
```
int((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.587.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2651 vs. $2(369) = 738$.

Time = 2.54 (sec) , antiderivative size = 5361, normalized size of antiderivative = 13.47

$$\int \frac{(d+ex)^3}{(f+gx)^3 (d^2-e^2x^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.587.6 Sympy [F]

$$\int \frac{(d+ex)^3}{(f+gx)^3 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}}(f+gx)^3}\,dx$$

input
```
integrate((e*x+d)**3/(g*x+f)**3/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**3/((-(-d + e*x)*(d + e*x))**(7/2)*(f + g*x)**3), x)
```

3.587.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{(f+gx)^3(d^2-e^2x^2)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.587.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1401 vs. $2(369) = 738$.

Time = 0.43 (sec) , antiderivative size = 1401, normalized size of antiderivative = 3.52

$$\int \frac{(d+ex)^3}{(f+gx)^3(d^2-e^2x^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
-(20*e^5*f^2*g^3 - 30*d*e^4*f*g^4 + 13*d^2*e^3*g^5)*arctan((d*g + (d*e + s
qrt(-e^2*x^2 + d^2)*abs(e))*f/(e*x))/sqrt(e^2*f^2 - d^2*g^2))/((e^7*f^7*ab
s(e) + 3*d*e^6*f^6*g*abs(e) + d^2*e^5*f^5*g^2*abs(e) - 5*d^3*e^4*f^4*g^3*a
bs(e) - 5*d^4*e^3*f^3*g^4*abs(e) + d^5*e^2*f^2*g^5*abs(e) + 3*d^6*e*f*g^6*
abs(e) + d^7*g^7*abs(e))*sqrt(e^2*f^2 - d^2*g^2)) - (10*d*e^5*f^4*g^4 - 6*
d^2*e^4*f^3*g^5 - d^3*e^3*f^2*g^6 + 29*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
*d^2*e^2*f^3*g^5/x - 18*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^3*e*f^2*g^6/
x - 2*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^4*f*g^7/x + 10*(d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^2*d*e*f^4*g^4/x^2 - 6*(d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))^2*d^2*f^3*g^5/x^2 + 19*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^3*f^2*
g^6/(e*x^2) - 12*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^4*f*g^7/(e^2*x^2)
 - 2*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^5*g^8/(e^3*x^2) + 11*(d*e + s
qrt(-e^2*x^2 + d^2)*abs(e))^3*d^2*f^3*g^5/(e^2*x^3) - 6*(d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))^3*d^3*f^2*g^6/(e^3*x^3) - 2*(d*e + sqrt(-e^2*x^2 + d^2)*
abs(e))^3*d^4*f*g^7/(e^4*x^3))/((e^7*f^9*abs(e) + 3*d*e^6*f^8*g*abs(e) + d
^2*e^5*f^7*g^2*abs(e) - 5*d^3*e^4*f^6*g^3*abs(e) - 5*d^4*e^3*f^5*g^4*abs(e
) + d^5*e^2*f^4*g^5*abs(e) + 3*d^6*e*f^3*g^6*abs(e) + d^7*f^2*g^7*abs(e))*
(e*f + 2*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d*g/(e^2*x) + (d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^2*f/(e^3*x^2))^2) + 2/15*(7*e^5*f^2 + 44*d*e^4*f*g +
127*d^2*e^3*g^2 - 20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^3*f^2/x - 14...
```

3.587.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{(f+gx)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx=\int \frac{\left(d+e\,x\right)^3}{\left(f+g\,x\right)^3\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

input
```
int((d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)),x)
```

output
```
int((d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)), x)
```

3.588 $\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx$

3.588.1 Optimal result

Integrand size = 24, antiderivative size = 112

$$\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx=-\frac{2(cd^2+ae^2)}{e^2(ef-dg)\sqrt{d+ex}}+\frac{2c\sqrt{d+ex}}{e^2g}-\frac{2(cf^2+ag^2)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}}$$

output
```
-2*(a*g^2+c*f^2)*arctan(g^(1/2)*(e*x+d)^(1/2)/(-d*g+e*f)^(1/2))/g^(3/2)/(-
d*g+e*f)^(3/2)-2*(a*e^2+c*d^2)/e^2/(-d*g+e*f)/(e*x+d)^(1/2)+2*c*(e*x+d)^(1
/2)/e^2/g
```

3.588.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05

$$\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx=-\frac{2(cd^2g+ae^2g-cef(d+ex)+cdg(d+ex))}{e^2g(ef-dg)\sqrt{d+ex}}-\frac{2(cf^2+ag^2)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}}$$

input
```
Integrate[(a + c*x^2)/((d + e*x)^(3/2)*(f + g*x)),x]
```

output
```
(-2*(c*d^2*g + a*e^2*g - c*e*f*(d + e*x) + c*d*g*(d + e*x)))/(e^2*g*(e*f -
 d*g)*Sqrt[d + e*x]) - (2*(c*f^2 + a*g^2)*ArcTan[(Sqrt[g]*Sqrt[d + e*x])/S
qrt[e*f - d*g]])/(g^(3/2)*(e*f - d*g)^(3/2))
```

3.588.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {649, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx$$

$$\Big\downarrow \text{ 649}$$

$$\frac{2\int \frac{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}{(d+ex)(ef-dg+g(d+ex))}d\sqrt{d+ex}}{e^2}$$

$$\Big\downarrow \text{ 1584}$$

$$\frac{2\int \left(-\frac{(cf^2+ag^2)e^2}{g(dg-ef)(-ef+dg-g(d+ex))}+\frac{c}{g}-\frac{-cd^2-ae^2}{(ef-dg)(d+ex)}\right)d\sqrt{d+ex}}{e^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2\left(-\frac{e^2\left(ag^2+cf^2\right)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}}-\frac{ae^2+cd^2}{\sqrt{d+ex}(ef-dg)}+\frac{c\sqrt{d+ex}}{g}\right)}{e^2}$$

input
```
Int[(a + c*x^2)/((d + e*x)^(3/2)*(f + g*x)),x]
```

output
```
(2*(-((c*d^2 + a*e^2)/((e*f - d*g)*Sqrt[d + e*x])) + (c*Sqrt[d + e*x])/g -
 (e^2*(c*f^2 + a*g^2)*ArcTan[(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]])/(g^
(3/2)*(e*f - d*g)^(3/2))))/e^2
```

3.588.3.1 Defintions of rubi rules used

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.588.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.99

method	result	size
pseudoelliptic	$\frac{\frac{2c\sqrt{ex+d}}{g}-\frac{2e^{2}\left(a\,g^{2}+c\,f^{2}\right)\operatorname{arctanh}\left(\frac{g\sqrt{ex+d}}{\sqrt{(dg-ef)g}}\right)}{(dg-ef)g\sqrt{(dg-ef)g}}+\frac{2\left(e^{2}a+c\,d^{2}\right)}{(dg-ef)\sqrt{ex+d}}}{e^{2}}$	111
derivativedivides	$\frac{\frac{2c\sqrt{ex+d}}{g}-\frac{2e^{2}\left(a\,g^{2}+c\,f^{2}\right)\operatorname{arctanh}\left(\frac{g\sqrt{ex+d}}{\sqrt{(dg-ef)g}}\right)}{(dg-ef)g\sqrt{(dg-ef)g}}-\frac{2\left(-e^{2}a-c\,d^{2}\right)}{(dg-ef)\sqrt{ex+d}}}{e^{2}}$	114
default	$\frac{\frac{2c\sqrt{ex+d}}{g}-\frac{2e^{2}\left(a\,g^{2}+c\,f^{2}\right)\operatorname{arctanh}\left(\frac{g\sqrt{ex+d}}{\sqrt{(dg-ef)g}}\right)}{(dg-ef)g\sqrt{(dg-ef)g}}-\frac{2\left(-e^{2}a-c\,d^{2}\right)}{(dg-ef)\sqrt{ex+d}}}{e^{2}}$	114
risch	$\frac{2c\sqrt{ex+d}}{e^{2}g}-\frac{2\left(\frac{e^{2}\left(a\,g^{2}+c\,f^{2}\right)\operatorname{arctanh}\left(\frac{g\sqrt{ex+d}}{\sqrt{(dg-ef)g}}\right)}{(dg-ef)\sqrt{(dg-ef)g}}-\frac{\left(e^{2}a+c\,d^{2}\right)g}{(dg-ef)\sqrt{ex+d}}\right)}{g\,e^{2}}$	117

input
```
int((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
2/e^2*(c/g*(e*x+d)^(1/2)-e^2*(a*g^2+c*f^2)/(d*g-e*f)/g/((d*g-e*f)*g)^(1/2)
*arctanh(g*(e*x+d)^(1/2)/((d*g-e*f)*g)^(1/2))+(a*e^2+c*d^2)/(d*g-e*f)/(e*x
+d)^(1/2))
```

3.588.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(98) = 196.

Time = 0.30 (sec) , antiderivative size = 499, normalized size of antiderivative = 4.46

$$\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx = \left[\frac{(cde^2f^2+ade^2g^2+(ce^3f^2+ae^3g^2)x)\sqrt{-efg+dg^2}\log\left(\frac{egx-ef+2\,dg-2\,\sqrt{-efg+}}{gx+f}\right.}{de^4f^2g^2-2\,d^2e^3}\right.$$

input
```
integrate((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x, algorithm="fricas")
```

output
```
[((c*d*e^2*f^2 + a*d*e^2*g^2 + (c*e^3*f^2 + a*e^3*g^2)*x)*sqrt(-e*f*g + d*
g^2)*log((e*g*x - e*f + 2*d*g - 2*sqrt(-e*f*g + d*g^2)*sqrt(e*x + d))/(g*x
 + f)) + 2*(c*d*e^2*f^2*g - (3*c*d^2*e + a*e^3)*f*g^2 + (2*c*d^3 + a*d*e^2
)*g^3 + (c*e^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(e*x + d))/(d
*e^4*f^2*g^2 - 2*d^2*e^3*f*g^3 + d^3*e^2*g^4 + (e^5*f^2*g^2 - 2*d*e^4*f*g^
3 + d^2*e^3*g^4)*x), 2*((c*d*e^2*f^2 + a*d*e^2*g^2 + (c*e^3*f^2 + a*e^3*g^
2)*x)*sqrt(e*f*g - d*g^2)*arctan(sqrt(e*f*g - d*g^2)*sqrt(e*x + d)/(e*g*x
+ d*g)) + (c*d*e^2*f^2*g - (3*c*d^2*e + a*e^3)*f*g^2 + (2*c*d^3 + a*d*e^2)
*g^3 + (c*e^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(e*x + d))/(d*
e^4*f^2*g^2 - 2*d^2*e^3*f*g^3 + d^3*e^2*g^4 + (e^5*f^2*g^2 - 2*d*e^4*f*g^3
 + d^2*e^3*g^4)*x)]
```

3.588.6 Sympy [A] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.38

$$\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx = \begin{cases} \frac{2\left(\frac{c\sqrt{d+ex}}{eg}+\frac{e(ag^2+cf^2)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-\frac{dg-ef}{g}}}\right)}{g^2\sqrt{-\frac{dg-ef}{g}}(dg-ef)}+\frac{ae^2+cd^2}{e\sqrt{d+ex}(dg-ef)}\right)}{e} & \text{for } e\neq 0 \\ \frac{-\frac{cfx}{g^2}+\frac{cx^2}{2g}+\frac{(ag^2+cf^2)\left(\begin{cases}\frac{x}{f} & \text{for } g=0\\ \frac{\log(f+gx)}{g} & \text{otherwise}\end{cases}\right)}{g^2}}{d^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+a)/(e*x+d)**(3/2)/(g*x+f),x)
```

output
```
Piecewise((2*(c*sqrt(d + e*x)/(e*g) + e*(a*g**2 + c*f**2)*atan(sqrt(d + e*
x)/sqrt(-(d*g - e*f)/g))/(g**2*sqrt(-(d*g - e*f)/g)*(d*g - e*f)) + (a*e**2
 + c*d**2)/(e*sqrt(d + e*x)*(d*g - e*f)))/e, Ne(e, 0)), ((-c*f*x/g**2 + c*
x**2/(2*g) + (a*g**2 + c*f**2)*Piecewise((x/f, Eq(g, 0)), (log(f + g*x)/g,
 True))/g**2)/d**(3/2), True))
```

3.588.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for m
ore detail
```

3.588.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.90

$$\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx = -\frac{2\,(cf^2+ag^2)\arctan\left(\frac{\sqrt{ex+d}g}{\sqrt{efg-dg^2}}\right)}{(efg-dg^2)^{\frac{3}{2}}} - \frac{2\,(cd^2+ae^2)}{(e^3f-de^2g)\sqrt{ex+d}} + \frac{2\,\sqrt{ex+d}c}{e^2g}$$

input
```
integrate((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x, algorithm="giac")
```

output
```
-2*(c*f^2 + a*g^2)*arctan(sqrt(e*x + d)*g/sqrt(e*f*g - d*g^2))/(e*f*g - d*
g^2)^(3/2) - 2*(c*d^2 + a*e^2)/((e^3*f - d*e^2*g)*sqrt(e*x + d)) + 2*sqrt(
e*x + d)*c/(e^2*g)
```

3.588.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11

$$\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx = \frac{2\,c\,\sqrt{d+e\,x}}{e^2\,g}+\frac{2\,\left(c\,g\,d^2+a\,g\,e^2\right)}{e^2\,g\,\left(d\,g-e\,f\right)\,\sqrt{d+e\,x}} + \frac{\mathrm{atan}\left(\frac{d\,g^{3/2}\,\sqrt{d+e\,x}\,1\mathrm{i}-e\,f\,\sqrt{g}\,\sqrt{d+e\,x}\,1\mathrm{i}}{{\left(d\,g-e\,f\right)}^{3/2}}\right)\,\left(c\,f^2+a\,g^2\right)\,2\mathrm{i}}{g^{3/2}\,{\left(d\,g-e\,f\right)}^{3/2}}$$

input
```
int((a + c*x^2)/((f + g*x)*(d + e*x)^(3/2)),x)
```

output
```
(atan((d*g^(3/2)*(d + e*x)^(1/2)*1i - e*f*g^(1/2)*(d + e*x)^(1/2)*1i)/(d*g
 - e*f)^(3/2))*(a*g^2 + c*f^2)*2i)/(g^(3/2)*(d*g - e*f)^(3/2)) + (2*c*(d +
 e*x)^(1/2))/(e^2*g) + (2*(a*e^2*g + c*d^2*g))/(e^2*g*(d*g - e*f)*(d + e*x
)^(1/2))
```

3.589 $\int \frac{(d+ex)^3(a+cx^2)}{\sqrt{f+gx}}\,dx$

3.589.1 Optimal result

Integrand size = 24, antiderivative size = 240

$$\begin{aligned}\int \frac{(d+ex)^3(a+cx^2)}{\sqrt{f+gx}}\,dx = &-\frac{2(ef-dg)^3(cf^2+ag^2)\sqrt{f+gx}}{g^6}\\ &+\frac{2(ef-dg)^2(3aeg^2+cf(5ef-2dg))(f+gx)^{3/2}}{3g^6}\\ &-\frac{2(ef-dg)(3ae^2g^2+c(10e^2f^2-8defg+d^2g^2))(f+gx)^{5/2}}{5g^6}\\ &+\frac{2e(ae^2g^2+c(10e^2f^2-12defg+3d^2g^2))(f+gx)^{7/2}}{7g^6}\\ &-\frac{2ce^2(5ef-3dg)(f+gx)^{9/2}}{9g^6}+\frac{2ce^3(f+gx)^{11/2}}{11g^6}\end{aligned}$$

output
```
2/3*(-d*g+e*f)^2*(3*a*e*g^2+c*f*(-2*d*g+5*e*f))*(g*x+f)^(3/2)/g^6-2/5*(-d*
g+e*f)*(3*a*e^2*g^2+c*(d^2*g^2-8*d*e*f*g+10*e^2*f^2))*(g*x+f)^(5/2)/g^6+2/
7*e*(a*e^2*g^2+c*(3*d^2*g^2-12*d*e*f*g+10*e^2*f^2))*(g*x+f)^(7/2)/g^6-2/9*
c*e^2*(-3*d*g+5*e*f)*(g*x+f)^(9/2)/g^6+2/11*c*e^3*(g*x+f)^(11/2)/g^6-2*(-d
*g+e*f)^3*(a*g^2+c*f^2)*(g*x+f)^(1/2)/g^6
```

3.589.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^3\,(a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\sqrt{f+gx}(99ag^2(35d^3g^3+35d^2eg^2(-2f+gx)+7de^2g(8f^2-4fgx+3g^2x^2)+e^3(-16f^3+8f^2gx-6fg}{}$$

input
```
Integrate[((d + e*x)^3*(a + c*x^2))/Sqrt[f + g*x],x]
```

output
```
(2*Sqrt[f + g*x]*(99*a*g^2*(35*d^3*g^3 + 35*d^2*e*g^2*(-2*f + g*x) + 7*d*e
^2*g*(8*f^2 - 4*f*g*x + 3*g^2*x^2) + e^3*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^
2 + 5*g^3*x^3)) + c*(231*d^3*g^3*(8*f^2 - 4*f*g*x + 3*g^2*x^2) + 297*d^2*e
*g^2*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^3) + 33*d*e^2*g*(128*f^4
 - 64*f^3*g*x + 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + 35*g^4*x^4) - 5*e^3*(256*f
^5 - 128*f^4*g*x + 96*f^3*g^2*x^2 - 80*f^2*g^3*x^3 + 70*f*g^4*x^4 - 63*g^5
*x^5))))/(3465*g^6)
```

3.589.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)\,(d+ex)^3}{\sqrt{f+gx}}\,dx$$
$$\Big\downarrow \text{652}$$
$$\int \left(\frac{e(f+gx)^{5/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{g^5}+\frac{(f+gx)^{3/2}(ef-dg)\left(-3ae^2g^2-c\left(d^2g^2-8defg+1\right.\right.}{g^5}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{2e(f+gx)^{7/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{7g^6}-\frac{2(f+gx)^{5/2}(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10e^2f^2\right)\right)}{5g^6}-\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)^3}{g^6}+\frac{2(f+gx)^{3/2}(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)}{3g^6}-\frac{2ce^2(f+gx)^{9/2}(5ef-3dg)}{9g^6}+\frac{2ce^3(f+gx)^{11/2}}{11g^6}$$

input
```
Int[((d + e*x)^3*(a + c*x^2))/Sqrt[f + g*x],x]
```

output
```
(-2*(e*f - d*g)^3*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^6 + (2*(e*f - d*g)^2*(3
*a*e*g^2 + c*f*(5*e*f - 2*d*g))*(f + g*x)^(3/2))/(3*g^6) - (2*(e*f - d*g)*
(3*a*e^2*g^2 + c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g
^6) + (2*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)
^(7/2))/(7*g^6) - (2*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^(9/2))/(9*g^6) + (2*c
*e^3*(f + g*x)^(11/2))/(11*g^6)
```

3.589.3.1 Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.589.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{2ce^3(gx+f)^{\frac{11}{2}}}{11}+\frac{2(3(dg-ef)e^2c-2fce^3)(gx+f)^{\frac{9}{2}}}{9}+\frac{2(3(dg-ef)^2ec-6(dg-ef)e^2cf+e^3(ag^2+cf^2))(gx+f)^{\frac{7}{2}}}{7}+\frac{2((dg-ef)^3c-$
default	$\frac{2ce^3(gx+f)^{\frac{11}{2}}}{11}+\frac{2(3(dg-ef)e^2c-2fce^3)(gx+f)^{\frac{9}{2}}}{9}+\frac{2(3(dg-ef)^2ec-6(dg-ef)e^2cf+e^3(ag^2+cf^2))(gx+f)^{\frac{7}{2}}}{7}+\frac{2((dg-ef)^3c-6$
pseudoelliptic	$2\left(\left(\frac{x^3\left(\frac{7cx^2}{11}+a\right)e^3}{7}+\frac{3x^2\left(\frac{5cx^2}{9}+a\right)de^2}{5}+d^2x\left(\frac{3cx^2}{7}+a\right)e+d^3\left(\frac{cx^2}{5}+a\right)\right)g^5-2\left(\left(\frac{5}{99}cx^4+\frac{3}{35}ax^2\right)e^3+\frac{2\left(\frac{10cx^2}{21}+a\right)x\right.\right.\right.$
gosper	$2\sqrt{gx+f}\,(315ce^3x^5g^5+1155cde^2g^5x^4-350ce^3fg^4x^4+495ae^3g^5x^3+1485cd^2eg^5x^3-1320cde^2fg^4x^3+400ce^3f^2g^3x^3$
trager	$2\sqrt{gx+f}\,(315ce^3x^5g^5+1155cde^2g^5x^4-350ce^3fg^4x^4+495ae^3g^5x^3+1485cd^2eg^5x^3-1320cde^2fg^4x^3+400ce^3f^2g^3x^3$
risch	$2\sqrt{gx+f}\,(315ce^3x^5g^5+1155cde^2g^5x^4-350ce^3fg^4x^4+495ae^3g^5x^3+1485cd^2eg^5x^3-1320cde^2fg^4x^3+400ce^3f^2g^3x^3$

input
```
int((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/g^6*(1/11*c*e^3*(g*x+f)^(11/2)+1/9*(3*(d*g-e*f)*e^2*c-2*f*c*e^3)*(g*x+f)
^(9/2)+1/7*(3*(d*g-e*f)^2*e*c-6*(d*g-e*f)*e^2*c*f+e^3*(a*g^2+c*f^2))*(g*x+
f)^(7/2)+1/5*((d*g-e*f)^3*c-6*(d*g-e*f)^2*e*c*f+3*(d*g-e*f)*e^2*(a*g^2+c*f
^2))*(g*x+f)^(5/2)+1/3*(-2*(d*g-e*f)^3*c*f+3*(d*g-e*f)^2*e*(a*g^2+c*f^2))*
(g*x+f)^(3/2)+(d*g-e*f)^3*(a*g^2+c*f^2)*(g*x+f)^(1/2))
```

3.589.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^3\,(a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$=\frac{2\,(315\,ce^3g^5x^5-1280\,ce^3f^5+4224\,cde^2f^4g-6930\,ad^2efg^4+3465\,ad^3g^5-1584\,(3\,cd^2e+ae^3)f^3g^2+18}{}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
2/3465*(315*c*e^3*g^5*x^5 - 1280*c*e^3*f^5 + 4224*c*d*e^2*f^4*g - 6930*a*d
^2*e*f*g^4 + 3465*a*d^3*g^5 - 1584*(3*c*d^2*e + a*e^3)*f^3*g^2 + 1848*(c*d
^3 + 3*a*d*e^2)*f^2*g^3 - 35*(10*c*e^3*f*g^4 - 33*c*d*e^2*g^5)*x^4 + 5*(80
*c*e^3*f^2*g^3 - 264*c*d*e^2*f*g^4 + 99*(3*c*d^2*e + a*e^3)*g^5)*x^3 - 3*(
160*c*e^3*f^3*g^2 - 528*c*d*e^2*f^2*g^3 + 198*(3*c*d^2*e + a*e^3)*f*g^4 -
231*(c*d^3 + 3*a*d*e^2)*g^5)*x^2 + (640*c*e^3*f^4*g - 2112*c*d*e^2*f^3*g^2
 + 3465*a*d^2*e*g^5 + 792*(3*c*d^2*e + a*e^3)*f^2*g^3 - 924*(c*d^3 + 3*a*d
*e^2)*f*g^4)*x)*sqrt(g*x + f)/g^6
```

3.589.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(241) = 482$.

Time = 1.00 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.09

$$\int \frac{(d+ex)^3\,(a+cx^2)}{\sqrt{f+gx}}\,dx$$

$$= \begin{cases} \frac{2\left(\frac{ce^3(f+gx)^{\frac{11}{2}}}{11g^5}+\frac{(f+gx)^{\frac{9}{2}}\cdot\left(3cde^2g-5ce^3f\right)}{9g^5}+\frac{(f+gx)^{\frac{7}{2}}\left(ae^3g^2+3cd^2eg^2-12cde^2fg+10ce^3f^2\right)}{7g^5}+\frac{(f+gx)^{\frac{5}{2}}\cdot\left(3ade^2g^3-3ae^3fg^2+cd^3g^3-9cd^2efg^2+18cd\right.}{5g^5}}{} \\ \frac{ad^3x+\frac{3ad^2ex^2}{2}+\frac{3cde^2x^5}{5}+\frac{ce^3x^6}{6}+\frac{x^4\left(ae^3+3cd^2e\right)}{4}+\frac{x^3\cdot\left(3ade^2+cd^3\right)}{3}}{\sqrt{f}} \end{cases}$$

input
```
integrate((e*x+d)**3*(c*x**2+a)/(g*x+f)**(1/2),x)
```

output
```
Piecewise((2*(c*e**3*(f + g*x)**(11/2)/(11*g**5) + (f + g*x)**(9/2)*(3*c*d
*e**2*g - 5*c*e**3*f)/(9*g**5) + (f + g*x)**(7/2)*(a*e**3*g**2 + 3*c*d**2*
e*g**2 - 12*c*d*e**2*f*g + 10*c*e**3*f**2)/(7*g**5) + (f + g*x)**(5/2)*(3*
a*d*e**2*g**3 - 3*a*e**3*f*g**2 + c*d**3*g**3 - 9*c*d**2*e*f*g**2 + 18*c*d
*e**2*f**2*g - 10*c*e**3*f**3)/(5*g**5) + (f + g*x)**(3/2)*(3*a*d**2*e*g**
4 - 6*a*d*e**2*f*g**3 + 3*a*e**3*f**2*g**2 - 2*c*d**3*f*g**3 + 9*c*d**2*e*
f**2*g**2 - 12*c*d*e**2*f**3*g + 5*c*e**3*f**4)/(3*g**5) + sqrt(f + g*x)*(
a*d**3*g**5 - 3*a*d**2*e*f*g**4 + 3*a*d*e**2*f**2*g**3 - a*e**3*f**3*g**2
+ c*d**3*f**2*g**3 - 3*c*d**2*e*f**3*g**2 + 3*c*d*e**2*f**4*g - c*e**3*f**
5)/g**5)/g, Ne(g, 0)), ((a*d**3*x + 3*a*d**2*e*x**2/2 + 3*c*d*e**2*x**5/5
+ c*e**3*x**6/6 + x**4*(a*e**3 + 3*c*d**2*e)/4 + x**3*(3*a*d*e**2 + c*d**3
)/3)/sqrt(f), True))
```

3.589.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^3 (a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(315\,(gx+f)^{\frac{11}{2}}ce^3 - 385\,(5\,ce^3f - 3\,cde^2g)(gx+f)^{\frac{9}{2}} + 495\,(10\,ce^3f^2 - 12\,cde^2fg + (3\,cd^2e + ae^3)g^2)(\right.}{}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
2/3465*(315*(g*x + f)^(11/2)*c*e^3 - 385*(5*c*e^3*f - 3*c*d*e^2*g)*(g*x +
f)^(9/2) + 495*(10*c*e^3*f^2 - 12*c*d*e^2*f*g + (3*c*d^2*e + a*e^3)*g^2)*(
g*x + f)^(7/2) - 693*(10*c*e^3*f^3 - 18*c*d*e^2*f^2*g + 3*(3*c*d^2*e + a*e
^3)*f*g^2 - (c*d^3 + 3*a*d*e^2)*g^3)*(g*x + f)^(5/2) + 1155*(5*c*e^3*f^4 -
 12*c*d*e^2*f^3*g + 3*a*d^2*e*g^4 + 3*(3*c*d^2*e + a*e^3)*f^2*g^2 - 2*(c*d
^3 + 3*a*d*e^2)*f*g^3)*(g*x + f)^(3/2) - 3465*(c*e^3*f^5 - 3*c*d*e^2*f^4*g
 + 3*a*d^2*e*f*g^4 - a*d^3*g^5 + (3*c*d^2*e + a*e^3)*f^3*g^2 - (c*d^3 + 3*
a*d*e^2)*f^2*g^3)*sqrt(g*x + f))/g^6
```

3.589.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.58

$$\int \frac{(d+ex)^3 (a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(3465\,\sqrt{gx+f}ad^3 + \frac{3465\left((gx+f)^{\frac{3}{2}} - 3\,\sqrt{gx+f}f\right)ad^2e}{g} + \frac{231\left(3\,(gx+f)^{\frac{5}{2}} - 10\,(gx+f)^{\frac{3}{2}}f + 15\,\sqrt{gx+f}f^2\right)cd^3}{g^2} + \frac{693\left(3\,(gx+f)^{\frac{5}{2}}\right.}{}\right.}{}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
2/3465*(3465*sqrt(g*x + f)*a*d^3 + 3465*((g*x + f)^(3/2) - 3*sqrt(g*x + f)
*f)*a*d^2*e/g + 231*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*
x + f)*f^2)*c*d^3/g^2 + 693*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15
*sqrt(g*x + f)*f^2)*a*d*e^2/g^2 + 297*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5
/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d^2*e/g^3 + 99*(5
*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt
(g*x + f)*f^3)*a*e^3/g^3 + 33*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f
+ 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^
4)*c*d*e^2/g^4 + 5*(63*(g*x + f)^(11/2) - 385*(g*x + f)^(9/2)*f + 990*(g*x
 + f)^(7/2)*f^2 - 1386*(g*x + f)^(5/2)*f^3 + 1155*(g*x + f)^(3/2)*f^4 - 69
3*sqrt(g*x + f)*f^5)*c*e^3/g^5)/g
```

3.589.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^3 (a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{(f+g\,x)^{7/2}\,(6\,c\,d^2\,e\,g^2-24\,c\,d\,e^2\,f\,g+20\,c\,e^3\,f^2+2\,a\,e^3\,g^2)}{7\,g^6} + \frac{2\sqrt{f+g\,x}\,(c\,f^2+a\,g^2)\,(d\,g-e\,f)^3}{g^6} + \frac{2\,c\,e^3\,(f+g\,x)^{11/2}}{11\,g^6} + \frac{2\,(f+g\,x)^{3/2}\,(d\,g-e\,f)^2\,(5\,c\,e\,f^2-2\,c\,d\,f\,g+3\,a\,e\,g^2)}{3\,g^6} + \frac{2\,(f+g\,x)^{5/2}\,(d\,g-e\,f)\,(c\,d^2\,g^2-8\,c\,d\,e\,f\,g+10\,c\,e^2\,f^2+3\,a\,e^2\,g^2)}{5\,g^6} + \frac{2\,c\,e^2\,(f+g\,x)^{9/2}\,(3\,d\,g-5\,e\,f)}{9\,g^6}$$

input
```
int(((a + c*x^2)*(d + e*x)^3)/(f + g*x)^(1/2),x)
```

output
```
((f + g*x)^(7/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 + 6*c*d^2*e*g^2 - 24*c*d*e^2*
f*g))/(7*g^6) + (2*(f + g*x)^(1/2)*(a*g^2 + c*f^2)*(d*g - e*f)^3)/g^6 + (2
*c*e^3*(f + g*x)^(11/2))/(11*g^6) + (2*(f + g*x)^(3/2)*(d*g - e*f)^2*(3*a*
e*g^2 + 5*c*e*f^2 - 2*c*d*f*g))/(3*g^6) + (2*(f + g*x)^(5/2)*(d*g - e*f)*(
3*a*e^2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 - 8*c*d*e*f*g))/(5*g^6) + (2*c*e^2*
(f + g*x)^(9/2)*(3*d*g - 5*e*f))/(9*g^6)
```

3.590 $\int \frac{(d+ex)^2(a+cx^2)}{\sqrt{f+gx}}\,dx$

3.590.1 Optimal result

Integrand size = 24, antiderivative size = 175

$$\int \frac{(d+ex)^2(a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{2(ef-dg)^2(cf^2+ag^2)\sqrt{f+gx}}{g^5} - \frac{4(ef-dg)(aeg^2+cf(2ef-dg))(f+gx)^{3/2}}{3g^5} + \frac{2(ae^2g^2+c(6e^2f^2-6defg+d^2g^2))(f+gx)^{5/2}}{5g^5} - \frac{4ce(2ef-dg)(f+gx)^{7/2}}{7g^5} + \frac{2ce^2(f+gx)^{9/2}}{9g^5}$$

output
```
-4/3*(-d*g+e*f)*(a*e*g^2+c*f*(-d*g+2*e*f))*(g*x+f)^(3/2)/g^5+2/5*(a*e^2*g^
2+c*(d^2*g^2-6*d*e*f*g+6*e^2*f^2))*(g*x+f)^(5/2)/g^5-4/7*c*e*(-d*g+2*e*f)*
(g*x+f)^(7/2)/g^5+2/9*c*e^2*(g*x+f)^(9/2)/g^5+2*(-d*g+e*f)^2*(a*g^2+c*f^2)
*(g*x+f)^(1/2)/g^5
```

3.590.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^2 (a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}(21ag^2(15d^2g^2+10deg(-2f+gx)+e^2(8f^2-4fgx+3g^2x^2))+c(21d^2g^2(8f^2-4fgx+3g^2x^2)}{315}$$

input
```
Integrate[((d + e*x)^2*(a + c*x^2))/Sqrt[f + g*x],x]
```

output
```
(2*Sqrt[f + g*x]*(21*a*g^2*(15*d^2*g^2 + 10*d*e*g*(-2*f + g*x) + e^2*(8*f^
2 - 4*f*g*x + 3*g^2*x^2)) + c*(21*d^2*g^2*(8*f^2 - 4*f*g*x + 3*g^2*x^2) +
18*d*e*g*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^3) + e^2*(128*f^4 -
64*f^3*g*x + 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + 35*g^4*x^4))))/(315*g^5)
```

3.590.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(d+ex)^2}{\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \; 652$$

$$\int \left(\frac{(f+gx)^{3/2}(ae^2g^2+c(d^2g^2-6defg+6e^2f^2))}{g^4}+\frac{(ag^2+cf^2)(dg-ef)^2}{g^4\sqrt{f+gx}}+\frac{2\sqrt{f+gx}(ef-dg)(-aeg^2-c}{g^4}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{2(f+gx)^{5/2}(ae^2g^2+c(d^2g^2-6defg+6e^2f^2))}{5g^5}+\frac{2\sqrt{f+gx}(ag^2+cf^2)(ef-dg)^2}{g^5}-\frac{4(f+gx)^{3/2}(ef-dg)(aeg^2+cf(2ef-dg))}{3g^5}-\frac{4ce(f+gx)^{7/2}(2ef-dg)}{7g^5}+\frac{2ce^2(f+gx)^{9/2}}{9g^5}$$

input
```
Int[((d + e*x)^2*(a + c*x^2))/Sqrt[f + g*x],x]
```

output
```
(2*(e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^5 - (4*(e*f - d*g)*(a*e*
g^2 + c*f*(2*e*f - d*g))*(f + g*x)^(3/2))/(3*g^5) + (2*(a*e^2*g^2 + c*(6*e
^2*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g^5) - (4*c*e*(2*e*f -
d*g)*(f + g*x)^(7/2))/(7*g^5) + (2*c*e^2*(f + g*x)^(9/2))/(9*g^5)
```

3.590.3.1 Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.590.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$\frac{2\sqrt{gx+f}\left(\left(\left(\frac{x^{2}\left(\frac{5c\,x^{2}}{9}+a\right)e^{2}}{5}+\frac{2xd\left(\frac{3c\,x^{2}}{7}+a\right)e}{3}+d^{2}\left(\frac{c\,x^{2}}{5}+a\right)\right)g^{4}-\frac{4\left(\left(\frac{2}{21}c\,x^{3}+\frac{1}{5}ax\right)e^{2}+d\left(\frac{9c\,x^{2}}{35}+a\right)e+\frac{c\,d^{2}x}{5}\right)f\,g^{3}}{3}+\frac{8f^{2}\cdots}{\cdots}\right.\right.}{g^{5}}$
derivativedivides	$\frac{\frac{2c\,e^{2}(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(2(dg-ef)ec-2c\,e^{2}f\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left((dg-ef)^{2}c-4(dg-ef)ecf+e^{2}\left(a\,g^{2}+c\,f^{2}\right)\right)(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left(-2(dg-ef)^{2}cf+2(\cdots\right.}{\cdots}}{g^{5}}$
default	$\frac{\frac{2c\,e^{2}(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(2(dg-ef)ec-2c\,e^{2}f\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left((dg-ef)^{2}c-4(dg-ef)ecf+e^{2}\left(a\,g^{2}+c\,f^{2}\right)\right)(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left(-2(dg-ef)^{2}cf+2(\cdots\right.}{\cdots}}{g^{5}}$
gosper	$\frac{2\sqrt{gx+f}\,\left(35c\,e^{2}x^{4}g^{4}+90cde\,g^{4}x^{3}-40c\,e^{2}f\,g^{3}x^{3}+63a\,e^{2}g^{4}x^{2}+63c\,d^{2}g^{4}x^{2}-108cdef\,g^{3}x^{2}+48c\,e^{2}f^{2}g^{2}x^{2}+210ade\,g^{4}x-\cdots\right.}{\cdots}$
trager	$\frac{2\sqrt{gx+f}\,\left(35c\,e^{2}x^{4}g^{4}+90cde\,g^{4}x^{3}-40c\,e^{2}f\,g^{3}x^{3}+63a\,e^{2}g^{4}x^{2}+63c\,d^{2}g^{4}x^{2}-108cdef\,g^{3}x^{2}+48c\,e^{2}f^{2}g^{2}x^{2}+210ade\,g^{4}x-\cdots\right.}{\cdots}$
risch	$\frac{2\sqrt{gx+f}\,\left(35c\,e^{2}x^{4}g^{4}+90cde\,g^{4}x^{3}-40c\,e^{2}f\,g^{3}x^{3}+63a\,e^{2}g^{4}x^{2}+63c\,d^{2}g^{4}x^{2}-108cdef\,g^{3}x^{2}+48c\,e^{2}f^{2}g^{2}x^{2}+210ade\,g^{4}x-\cdots\right.}{\cdots}$

input
```
int((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(g*x+f)^(1/2)*((1/5*x^2*(5/9*c*x^2+a)*e^2+2/3*x*d*(3/7*c*x^2+a)*e+d^2*(1
/5*c*x^2+a))*g^4-4/3*((2/21*c*x^3+1/5*a*x)*e^2+d*(9/35*c*x^2+a)*e+1/5*c*d^
2*x)*f*g^3+8/15*f^2*((2/7*c*x^2+a)*e^2+6/7*c*d*e*x+c*d^2)*g^2-32/35*e*c*f^
3*(2/9*e*x+d)*g+128/315*c*e^2*f^4)/g^5
```

3.590.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^2\,(a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\,(35\,ce^2g^4x^4 + 128\,ce^2f^4 - 288\,cdef^3g - 420\,adefg^3 + 315\,ad^2g^4 + 168\,(cd^2+ae^2)f^2g^2 - 10\,(4\,ce^2fg^3 - \cdots}{\cdots}$$

input
```
integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*c*e^2*g^4*x^4 + 128*c*e^2*f^4 - 288*c*d*e*f^3*g - 420*a*d*e*f*g^
3 + 315*a*d^2*g^4 + 168*(c*d^2 + a*e^2)*f^2*g^2 - 10*(4*c*e^2*f*g^3 - 9*c*
d*e*g^4)*x^3 + 3*(16*c*e^2*f^2*g^2 - 36*c*d*e*f*g^3 + 21*(c*d^2 + a*e^2)*g
^4)*x^2 - 2*(32*c*e^2*f^3*g - 72*c*d*e*f^2*g^2 - 105*a*d*e*g^4 + 42*(c*d^2
 + a*e^2)*f*g^3)*x)*sqrt(g*x + f)/g^5
```

3.590.6 Sympy [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.78

$$\int \frac{(d+ex)^2\,(a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{ce^2(f+gx)^{\frac{9}{2}}}{9g^4} + \frac{(f+gx)^{\frac{7}{2}}\cdot\left(2cdeg-4ce^2f\right)}{7g^4} + \frac{(f+gx)^{\frac{5}{2}}\left(ae^2g^2+cd^2g^2-6cdefg+6ce^2f^2\right)}{5g^4} + \frac{(f+gx)^{\frac{3}{2}}\cdot\left(2adeg^3-2ae^2fg^2-2cd^2fg^2+6cdef^2g-4ce^2f^3\right)}{3g^4} + \cdots\right.}{g} \\ \frac{ad^2x+adex^2+\frac{cdex^4}{2}+\frac{ce^2x^5}{5}+\frac{x^3\left(ae^2+cd^2\right)}{3}}{\sqrt{f}} \end{cases}$$

input
```
integrate((e*x+d)**2*(c*x**2+a)/(g*x+f)**(1/2),x)
```

output
```
Piecewise((2*(c*e**2*(f + g*x)**(9/2)/(9*g**4) + (f + g*x)**(7/2)*(2*c*d*e
*g - 4*c*e**2*f)/(7*g**4) + (f + g*x)**(5/2)*(a*e**2*g**2 + c*d**2*g**2 -
6*c*d*e*f*g + 6*c*e**2*f**2)/(5*g**4) + (f + g*x)**(3/2)*(2*a*d*e*g**3 - 2
*a*e**2*f*g**2 - 2*c*d**2*f*g**2 + 6*c*d*e*f**2*g - 4*c*e**2*f**3)/(3*g**4
) + sqrt(f + g*x)*(a*d**2*g**4 - 2*a*d*e*f*g**3 + a*e**2*f**2*g**2 + c*d**
2*f**2*g**2 - 2*c*d*e*f**3*g + c*e**2*f**4)/g**4)/g, Ne(g, 0)), ((a*d**2*x
 + a*d*e*x**2 + c*d*e*x**4/2 + c*e**2*x**5/5 + x**3*(a*e**2 + c*d**2)/3)/s
qrt(f), True))
```

3.590.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^2 (a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(35\,(gx+f)^{\frac{9}{2}}ce^2 - 90\,(2\,ce^2f - cdeg)(gx+f)^{\frac{7}{2}} + 63\,(6\,ce^2f^2 - 6\,cdefg + (cd^2+ae^2)g^2)(gx+f)^{\frac{5}{2}} - 2\right.}{}$$

input
```
integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
2/315*(35*(g*x + f)^(9/2)*c*e^2 - 90*(2*c*e^2*f - c*d*e*g)*(g*x + f)^(7/2)
 + 63*(6*c*e^2*f^2 - 6*c*d*e*f*g + (c*d^2 + a*e^2)*g^2)*(g*x + f)^(5/2) -
210*(2*c*e^2*f^3 - 3*c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 + a*e^2)*f*g^2)*(g*x
 + f)^(3/2) + 315*(c*e^2*f^4 - 2*c*d*e*f^3*g - 2*a*d*e*f*g^3 + a*d^2*g^4 +
 (c*d^2 + a*e^2)*f^2*g^2)*sqrt(g*x + f))/g^5
```

3.590.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.39

$$\int \frac{(d+ex)^2 (a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(315\,\sqrt{gx+f}ad^2 + \frac{210\left((gx+f)^{\frac{3}{2}} - 3\,\sqrt{gx+f}f\right)ade}{g} + \frac{21\left(3\,(gx+f)^{\frac{5}{2}} - 10\,(gx+f)^{\frac{3}{2}}f + 15\,\sqrt{gx+f}f^2\right)cd^2}{g^2} + \frac{21\left(3\,(gx+f)^{\frac{5}{2}} - 10\,(g\right.}{}\right.}{}$$

input
```
integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
2/315*(315*sqrt(g*x + f)*a*d^2 + 210*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)
*a*d*e/g + 21*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)
*f^2)*c*d^2/g^2 + 21*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g
*x + f)*f^2)*a*e^2/g^2 + 18*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35
*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d*e/g^3 + (35*(g*x + f)^(9/
2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)
*f^3 + 315*sqrt(g*x + f)*f^4)*c*e^2/g^4)/g
```

3.590.9 Mupad [B] (verification not implemented)

Time = 12.19 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^2 (a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{(f+g\,x)^{5/2}\,(2\,c\,d^2\,g^2-12\,c\,d\,e\,f\,g+12\,c\,e^2\,f^2+2\,a\,e^2\,g^2)}{5\,g^5} + \frac{2\,\sqrt{f+g\,x}\,(c\,f^2+a\,g^2)\,\left(d\,g-e\,f\right)^2}{g^5} + \frac{4\,(f+g\,x)^{3/2}\,(d\,g-e\,f)\,\left(2\,c\,e\,f^2-c\,d\,f\,g+a\,e\,g^2\right)}{3\,g^5} + \frac{2\,c\,e^2\,(f+g\,x)^{9/2}}{9\,g^5} + \frac{4\,c\,e\,(f+g\,x)^{7/2}\,(d\,g-2\,e\,f)}{7\,g^5}$$

input
```
int(((a + c*x^2)*(d + e*x)^2)/(f + g*x)^(1/2),x)
```

output
```
((f + g*x)^(5/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 - 12*c*d*e*f*g)
)/(5*g^5) + (2*(f + g*x)^(1/2)*(a*g^2 + c*f^2)*(d*g - e*f)^2)/g^5 + (4*(f
+ g*x)^(3/2)*(d*g - e*f)*(a*e*g^2 + 2*c*e*f^2 - c*d*f*g))/(3*g^5) + (2*c*e
^2*(f + g*x)^(9/2))/(9*g^5) + (4*c*e*(f + g*x)^(7/2)*(d*g - 2*e*f))/(7*g^5
)
```

3.591 $\int \frac{(d+ex)(a+cx^2)}{\sqrt{f+gx}}\,dx$

3.591.1 Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \frac{(d+ex)(a+cx^2)}{\sqrt{f+gx}}\,dx = -\frac{2(ef-dg)(cf^2+ag^2)\sqrt{f+gx}}{g^4} + \frac{2(aeg^2+cf(3ef-2dg))(f+gx)^{3/2}}{3g^4} - \frac{2c(3ef-dg)(f+gx)^{5/2}}{5g^4} + \frac{2ce(f+gx)^{7/2}}{7g^4}$$

output
```
2/3*(a*e*g^2+c*f*(-2*d*g+3*e*f))*(g*x+f)^(3/2)/g^4-2/5*c*(-d*g+3*e*f)*(g*x
+f)^(5/2)/g^4+2/7*c*e*(g*x+f)^(7/2)/g^4-2*(-d*g+e*f)*(a*g^2+c*f^2)*(g*x+f)
^(1/2)/g^4
```

3.591.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)(a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}(35ag^2(-2ef+3dg+egx)+7cdg(8f^2-4fgx+3g^2x^2)-3ce(16f^3-8f^2gx+6fg^2x^2-5g^3x}{105g^4}$$

input
```
Integrate[((d + e*x)*(a + c*x^2))/Sqrt[f + g*x],x]
```

output
```
(2*Sqrt[f + g*x]*(35*a*g^2*(-2*e*f + 3*d*g + e*g*x) + 7*c*d*g*(8*f^2 - 4*f
*g*x + 3*g^2*x^2) - 3*c*e*(16*f^3 - 8*f^2*g*x + 6*f*g^2*x^2 - 5*g^3*x^3)))
/(105*g^4)
```

3.591.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(d+ex)}{\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{ 652}$$

$$\int \left(\frac{(ag^2+cf^2)(dg-ef)}{g^3\sqrt{f+gx}}+\frac{\sqrt{f+gx}(aeg^2+cf(3ef-2dg))}{g^3}+\frac{c(f+gx)^{3/2}(dg-3ef)}{g^3}+\frac{ce(f+gx)^{5/2}}{g^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2\sqrt{f+gx}(ag^2+cf^2)(ef-dg)}{g^4}+\frac{2(f+gx)^{3/2}(aeg^2+cf(3ef-2dg))}{3g^4}-\frac{2c(f+gx)^{5/2}(3ef-dg)}{5g^4}+\frac{2ce(f+gx)^{7/2}}{7g^4}$$

input
```
Int[((d + e*x)*(a + c*x^2))/Sqrt[f + g*x],x]
```

output
```
(-2*(e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^4 + (2*(a*e*g^2 + c*f*(3*
e*f - 2*d*g))*(f + g*x)^(3/2))/(3*g^4) - (2*c*(3*e*f - d*g)*(f + g*x)^(5/2
))/(5*g^4) + (2*c*e*(f + g*x)^(7/2))/(7*g^4)
```

3.591.3.1 Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.591.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{2\sqrt{gx+f}\left(\left(\frac{x^{2}\left(\frac{5ex}{7}+d\right)c}{5}+a\left(\frac{ex}{3}+d\right)\right)g^{3}-\frac{2f\left(\frac{2\left(\frac{9ex}{14}+d\right)xc}{5}+ae\right)g^{2}}{3}+\frac{8c\left(\frac{3ex}{7}+d\right)f^{2}g}{15}-\frac{16ce\,f^{3}}{35}\right)}{g^{4}}$
gosper	$\frac{2\sqrt{gx+f}\,\left(15ce\,x^{3}g^{3}+21cd\,g^{3}x^{2}-18cef\,g^{2}x^{2}+35ae\,g^{3}x-28cdf\,g^{2}x+24ce\,f^{2}gx+105ad\,g^{3}-70aef\,g^{2}+56cd\,f^{2}g-48ce\,f^{3}\right.}{105g^{4}}$
trager	$\frac{2\sqrt{gx+f}\,\left(15ce\,x^{3}g^{3}+21cd\,g^{3}x^{2}-18cef\,g^{2}x^{2}+35ae\,g^{3}x-28cdf\,g^{2}x+24ce\,f^{2}gx+105ad\,g^{3}-70aef\,g^{2}+56cd\,f^{2}g-48ce\,f^{3}\right.}{105g^{4}}$
risch	$\frac{2\sqrt{gx+f}\,\left(15ce\,x^{3}g^{3}+21cd\,g^{3}x^{2}-18cef\,g^{2}x^{2}+35ae\,g^{3}x-28cdf\,g^{2}x+24ce\,f^{2}gx+105ad\,g^{3}-70aef\,g^{2}+56cd\,f^{2}g-48ce\,f^{3}\right.}{105g^{4}}$
derivativedivides	$\frac{\frac{2ce(gx+f)^{\frac{7}{2}}}{7}+\frac{2((dg-ef)c-2cef)(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left(-2(dg-ef)cf+e\left(a\,g^{2}+c\,f^{2}\right)\right)(gx+f)^{\frac{3}{2}}}{3}+2(dg-ef)\left(a\,g^{2}+c\,f^{2}\right)\sqrt{gx+f}}{g^{4}}$
default	$\frac{\frac{2ce(gx+f)^{\frac{7}{2}}}{7}+\frac{2((dg-ef)c-2cef)(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left(-2(dg-ef)cf+e\left(a\,g^{2}+c\,f^{2}\right)\right)(gx+f)^{\frac{3}{2}}}{3}+2(dg-ef)\left(a\,g^{2}+c\,f^{2}\right)\sqrt{gx+f}}{g^{4}}$

input
```
int((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(g*x+f)^(1/2)*((1/5*x^2*(5/7*e*x+d)*c+a*(1/3*e*x+d))*g^3-2/3*f*(2/5*(9/1
4*e*x+d)*x*c+a*e)*g^2+8/15*c*(3/7*e*x+d)*f^2*g-16/35*c*e*f^3)/g^4
```

3.591.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)(a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{2\,(15\,ceg^3x^3 - 48\,cef^3 + 56\,cdf^2g - 70\,aefg^2 + 105\,adg^3 - 3\,(6\,cefg^2 - 7\,cdg^3)x^2 + (24\,cef^2g - 28\,cdfg^2}{105\,g^4}$$

input
```
integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*c*e*g^3*x^3 - 48*c*e*f^3 + 56*c*d*f^2*g - 70*a*e*f*g^2 + 105*a*d
*g^3 - 3*(6*c*e*f*g^2 - 7*c*d*g^3)*x^2 + (24*c*e*f^2*g - 28*c*d*f*g^2 + 35
*a*e*g^3)*x)*sqrt(g*x + f)/g^4
```

3.591.6 Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.46

$$\int \frac{(d+ex)(a+cx^2)}{\sqrt{f+gx}}\,dx = \begin{cases} \frac{2\left(\frac{ce(f+gx)^{\frac{7}{2}}}{7g^3} + \frac{(f+gx)^{\frac{5}{2}}(cdg-3cef)}{5g^3} + \frac{(f+gx)^{\frac{3}{2}}\left(aeg^2-2cdfg+3cef^2\right)}{3g^3} + \frac{\sqrt{f+gx}\left(adg^3-aefg^2+cdf^2g-cef^3\right)}{g^3}\right)}{g} & \text{for } g \neq 0 \\ \frac{adx+\frac{aex^2}{2}+\frac{cdx^3}{3}+\frac{cex^4}{4}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

input
```
integrate((e*x+d)*(c*x**2+a)/(g*x+f)**(1/2),x)
```

output
```
Piecewise((2*(c*e*(f + g*x)**(7/2)/(7*g**3) + (f + g*x)**(5/2)*(c*d*g - 3*
c*e*f)/(5*g**3) + (f + g*x)**(3/2)*(a*e*g**2 - 2*c*d*f*g + 3*c*e*f**2)/(3*
g**3) + sqrt(f + g*x)*(a*d*g**3 - a*e*f*g**2 + c*d*f**2*g - c*e*f**3)/g**3
)/g, Ne(g, 0)), ((a*d*x + a*e*x**2/2 + c*d*x**3/3 + c*e*x**4/4)/sqrt(f), T
rue))
```

3.591.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)\,(a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(15\,(gx+f)^{\frac{7}{2}}ce-21\,(3\,cef-cdg)(gx+f)^{\frac{5}{2}}+35\,(3\,cef^2-2\,cdfg+aeg^2)(gx+f)^{\frac{3}{2}}-105\,(cef^3-cdf\right.}{105\,g^4}$$

input
```
integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
2/105*(15*(g*x + f)^(7/2)*c*e - 21*(3*c*e*f - c*d*g)*(g*x + f)^(5/2) + 35*
(3*c*e*f^2 - 2*c*d*f*g + a*e*g^2)*(g*x + f)^(3/2) - 105*(c*e*f^3 - c*d*f^2
*g + a*e*f*g^2 - a*d*g^3)*sqrt(g*x + f))/g^4
```

3.591.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)\,(a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(105\,\sqrt{gx+f}ad+\frac{35\left((gx+f)^{\frac{3}{2}}-3\,\sqrt{gx+f}f\right)ae}{g}+\frac{7\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\,\sqrt{gx+f}f^2\right)cd}{g^2}+\frac{3\left(5\,(gx+f)^{\frac{7}{2}}-21\,(gx+f)^{\frac{5}{2}}\right.}{}\right.}{105\,g}$$

input
```
integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
2/105*(105*sqrt(g*x + f)*a*d + 35*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*
e/g + 7*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*
c*d/g^2 + 3*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)
*f^2 - 35*sqrt(g*x + f)*f^3)*c*e/g^3)/g
```

3.591.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)(a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{(f+g\,x)^{3/2}\,(6\,c\,e\,f^2-4\,c\,d\,f\,g+2\,a\,e\,g^2)}{3\,g^4} + \frac{2\,c\,e\,(f+g\,x)^{7/2}}{7\,g^4} + \frac{2\,c\,(f+g\,x)^{5/2}\,(d\,g-3\,e\,f)}{5\,g^4} + \frac{2\,\sqrt{f+g\,x}\,(c\,f^2+a\,g^2)\,(d\,g-e\,f)}{g^4}$$

input
```
int(((a + c*x^2)*(d + e*x))/(f + g*x)^(1/2),x)
```

output
```
((f + g*x)^(3/2)*(2*a*e*g^2 + 6*c*e*f^2 - 4*c*d*f*g))/(3*g^4) + (2*c*e*(f
+ g*x)^(7/2))/(7*g^4) + (2*c*(f + g*x)^(5/2)*(d*g - 3*e*f))/(5*g^4) + (2*(
f + g*x)^(1/2)*(a*g^2 + c*f^2)*(d*g - e*f))/g^4
```

3.592 $\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx$

3.592.1 Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx=\frac{2(cf^2+ag^2)\sqrt{f+gx}}{g^3}-\frac{4cf(f+gx)^{3/2}}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}$$

output
```
-4/3*c*f*(g*x+f)^(3/2)/g^3+2/5*c*(g*x+f)^(5/2)/g^3+2*(a*g^2+c*f^2)*(g*x+f)
^(1/2)/g^3
```

3.592.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx=\frac{2\sqrt{f+gx}(15ag^2+c(8f^2-4fgx+3g^2x^2))}{15g^3}$$

input
```
Integrate[(a + c*x^2)/Sqrt[f + g*x],x]
```

output
```
(2*Sqrt[f + g*x]*(15*a*g^2 + c*(8*f^2 - 4*f*g*x + 3*g^2*x^2)))/(15*g^3)
```

3.592.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{ag^2+cf^2}{g^2\sqrt{f+gx}}+\frac{c(f+gx)^{3/2}}{g^2}-\frac{2cf\sqrt{f+gx}}{g^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)}{g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}-\frac{4cf(f+gx)^{3/2}}{3g^3}$$

input
```
Int[(a + c*x^2)/Sqrt[f + g*x],x]
```

output
```
(2*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^3 - (4*c*f*(f + g*x)^(3/2))/(3*g^3) +
(2*c*(f + g*x)^(5/2))/(5*g^3)
```

3.592.3.1 Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.592.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

method	result	size
pseudoelliptic	$\frac{2(3(c x^2+5a) g^2-4c f x g+8c f^2) \sqrt{g x+f}}{15 g^3}$	40
gosper	$\frac{2\sqrt{g x+f}\, (3c x^2 g^2-4c f x g+15a g^2+8c f^2)}{15 g^3}$	41
trager	$\frac{2\sqrt{g x+f}\, (3c x^2 g^2-4c f x g+15a g^2+8c f^2)}{15 g^3}$	41
risch	$\frac{2\sqrt{g x+f}\, (3c x^2 g^2-4c f x g+15a g^2+8c f^2)}{15 g^3}$	41
derivativedivides	$\frac{\frac{2c(g x+f)^{\frac{5}{2}}}{5}-\frac{4c f(g x+f)^{\frac{3}{2}}}{3}+2a g^2\sqrt{g x+f}+2c f^2\sqrt{g x+f}}{g^3}$	52
default	$\frac{\frac{2c(g x+f)^{\frac{5}{2}}}{5}-\frac{4c f(g x+f)^{\frac{3}{2}}}{3}+2a g^2\sqrt{g x+f}+2c f^2\sqrt{g x+f}}{g^3}$	52

input
```
int((c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(3*(c*x^2+5*a)*g^2-4*c*f*x*g+8*c*f^2)*(g*x+f)^(1/2)/g^3
```

3.592.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\, dx = \frac{2\,(3\,cg^2x^2 - 4\,cfgx + 8\,cf^2 + 15\,ag^2)\sqrt{gx+f}}{15\,g^3}$$

input
```
integrate((c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*c*g^2*x^2 - 4*c*f*g*x + 8*c*f^2 + 15*a*g^2)*sqrt(g*x + f)/g^3
```

3.592.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx = \begin{cases} \frac{2a\sqrt{f+gx}+\frac{2c\left(f^2\sqrt{f+gx}-\frac{2f(f+gx)^{\frac{3}{2}}}{3}+\frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2}}{g} & \text{for } g\neq 0 \\ \frac{ax+\frac{cx^3}{3}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+a)/(g*x+f)**(1/2),x)
```

output
```
Piecewise(((2*a*sqrt(f + g*x) + 2*c*(f**2*sqrt(f + g*x) - 2*f*(f + g*x)**(
3/2)/3 + (f + g*x)**(5/2)/5)/g**2)/g, Ne(g, 0)), ((a*x + c*x**3/3)/sqrt(f)
, True))
```

3.592.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx = \frac{2\left(15\sqrt{gx+f}a+\frac{\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}f^2\right)c}{g^2}\right)}{15g}$$

input
```
integrate((c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(15*sqrt(g*x + f)*a + (3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*
sqrt(g*x + f)*f^2)*c/g^2)/g
```

3.592.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx = \frac{2\left(15\sqrt{gx+f}a+\frac{\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}f^2\right)c}{g^2}\right)}{15g}$$

input
```
integrate((c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
2/15*(15*sqrt(g*x + f)*a + (3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*
sqrt(g*x + f)*f^2)*c/g^2)/g
```

3.592.9 Mupad [B] (verification not implemented)

Time = 11.83 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+g\,x}\,\left(3\,c\,{\left(f+g\,x\right)}^2+15\,a\,g^2+15\,c\,f^2-10\,c\,f\,\left(f+g\,x\right)\right)}{15\,g^3}$$

input
```
int((a + c*x^2)/(f + g*x)^(1/2),x)
```

output
```
(2*(f + g*x)^(1/2)*(3*c*(f + g*x)^2 + 15*a*g^2 + 15*c*f^2 - 10*c*f*(f + g*
x)))/(15*g^3)
```

3.593 $\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}} \, dx$

3.593.1 Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}} \, dx = -\frac{2c(ef+dg)\sqrt{f+gx}}{e^2g^2} + \frac{2c(f+gx)^{3/2}}{3eg^2} - \frac{2(cd^2+ae^2)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}$$

output
```
2/3*c*(g*x+f)^(3/2)/e/g^2-2*(a*e^2+c*d^2)*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-
d*g+e*f)^(1/2))/e^(5/2)/(-d*g+e*f)^(1/2)-2*c*(d*g+e*f)*(g*x+f)^(1/2)/e^2/g
^2
```

3.593.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}} \, dx = \frac{2c\sqrt{f+gx}(-2ef-3dg+egx)}{3e^2g^2} + \frac{2(cd^2+ae^2)\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{5/2}\sqrt{-ef+dg}}$$

input
```
Integrate[(a + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

output
```
(2*c*Sqrt[f + g*x]*(-2*e*f - 3*d*g + e*g*x))/(3*e^2*g^2) + (2*(c*d^2 + a*e
^2)*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(5/2)*Sqrt[-(e*
f) + d*g])
```

3.593.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {649, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \; 649$$

$$\frac{2\int -\frac{cf^2-2c(f+gx)f+ag^2+c(f+gx)^2}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{2\int \frac{cf^2-2c(f+gx)f+ag^2+c(f+gx)^2}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \; 1467$$

$$-\frac{2\int \left(\frac{c(ef+dg)}{e^2}-\frac{c(f+gx)}{e}+\frac{cd^2g^2+ae^2g^2}{e^2(ef-dg-e(f+gx))}\right)d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{2\left(-\frac{g^2(ae^2+cd^2)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}-\frac{c\sqrt{f+gx}(dg+ef)}{e^2}+\frac{c(f+gx)^{3/2}}{3e}\right)}{g^2}$$

input
```
Int[(a + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

output
```
(2*(-((c*(e*f + d*g)*Sqrt[f + g*x])/e^2) + (c*(f + g*x)^(3/2))/(3*e) - ((c
*d^2 + a*e^2)*g^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(5/
2)*Sqrt[e*f - d*g])))/g^2
```

3.593.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.593.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{2c(-egx+3dg+2ef)\sqrt{gx+f}}{3g^{2}e^{2}}+\frac{2(e^{2}a+c\,d^{2})\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e^{2}\sqrt{(dg-ef)e}}$	82
pseudoelliptic	$\frac{-\frac{2c\sqrt{gx+f}\,(-egx+3dg+2ef)}{3}+\frac{2g^{2}\left(e^{2}a+c\,d^{2}\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{\sqrt{(dg-ef)e}}}{g^{2}e^{2}}$	83
derivativedivides	$\frac{-\frac{2c\left(-\frac{e(gx+f)^{\frac{3}{2}}}{3}+dg\sqrt{gx+f}+ef\sqrt{gx+f}\right)}{e^{2}}+\frac{2g^{2}\left(e^{2}a+c\,d^{2}\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e^{2}\sqrt{(dg-ef)e}}}{g^{2}}$	96
default	$\frac{-\frac{2c\left(-\frac{e(gx+f)^{\frac{3}{2}}}{3}+dg\sqrt{gx+f}+ef\sqrt{gx+f}\right)}{e^{2}}+\frac{2g^{2}\left(e^{2}a+c\,d^{2}\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e^{2}\sqrt{(dg-ef)e}}}{g^{2}}$	96

input
```
int((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*c*(-e*g*x+3*d*g+2*e*f)*(g*x+f)^(1/2)/g^2/e^2+2*(a*e^2+c*d^2)/e^2/((d*
g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))
```

3.593.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.86

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx$$
$$= \left[\frac{3\left(cd^2+ae^2\right)\sqrt{e^2f-deg}\,g^2\log\left(\frac{egx+2\,ef-dg-2\,\sqrt{e^2f-deg}\sqrt{gx+f}}{ex+d}\right)-2\left(2\,ce^3f^2+cde^2fg-3\,cd^2eg^2-\left(ce^3fg\right.\right.}{3\left(e^4fg^2-de^3g^3\right)}\right.$$

input
```
integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(c*d^2 + a*e^2)*sqrt(e^2*f - d*e*g)*g^2*log((e*g*x + 2*e*f - d*g -
 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*e^3*f^2 + c*d*e^
2*f*g - 3*c*d^2*e*g^2 - (c*e^3*f*g - c*d*e^2*g^2)*x)*sqrt(g*x + f))/(e^4*f
*g^2 - d*e^3*g^3), 2/3*(3*(c*d^2 + a*e^2)*sqrt(-e^2*f + d*e*g)*g^2*arctan(
sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) - (2*c*e^3*f^2 + c*d*e^2
*f*g - 3*c*d^2*e*g^2 - (c*e^3*f*g - c*d*e^2*g^2)*x)*sqrt(g*x + f))/(e^4*f*
g^2 - d*e^3*g^3)]
```

3.593.6 Sympy [A] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.38

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \begin{cases} \frac{2\left(\frac{c(f+gx)^{\frac{3}{2}}}{3eg}+\frac{\sqrt{f+gx}(-cdg-cef)}{e^2g}+\frac{g\left(ae^2+cd^2\right)\operatorname{atan}\left(\frac{\sqrt{f+gx}}{\sqrt{\frac{dg-ef}{e}}}\right)}{e^3\sqrt{\frac{dg-ef}{e}}}\right)}{g} & \text{for } g\neq 0 \\ \frac{-\frac{cdx}{e^2}+\frac{cx^2}{2e}+\frac{\left(ae^2+cd^2\right)\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{e^2}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+a)/(e*x+d)/(g*x+f)**(1/2),x)
```

output
```
Piecewise((2*(c*(f + g*x)**(3/2)/(3*e*g) + sqrt(f + g*x)*(-c*d*g - c*e*f)/
(e**2*g) + g*(a*e**2 + c*d**2)*atan(sqrt(f + g*x)/sqrt((d*g - e*f)/e))/(e*
*3*sqrt((d*g - e*f)/e)))/g, Ne(g, 0)), ((-c*d*x/e**2 + c*x**2/(2*e) + (a*e
**2 + c*d**2)*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/e**2)/sqr
t(f), True))
```

3.593.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.593.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.05

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \frac{2\,(cd^2+ae^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{\sqrt{-e^2f+deg}e^2} + \frac{2\left((gx+f)^{\frac{3}{2}}ce^2g^4-3\sqrt{gx+f}ce^2fg^4-3\sqrt{gx+f}cdeg^5\right)}{3\,e^3g^6}$$

input
```
integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
2*(c*d^2 + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/(sqrt(-e^2*
f + d*e*g)*e^2) + 2/3*((g*x + f)^(3/2)*c*e^2*g^4 - 3*sqrt(g*x + f)*c*e^2*f
*g^4 - 3*sqrt(g*x + f)*c*d*e*g^5)/(e^3*g^6)
```

3.593.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(c\,d^2+a\,e^2\right)}{e^{5/2}\,\sqrt{d\,g-e\,f}} - \sqrt{f+g\,x}\,\left(\frac{2\,c\,\left(d\,g^3-e\,f\,g^2\right)}{e^2\,g^4}+\frac{4\,c\,f}{e\,g^2}\right)+\frac{2\,c\,{\left(f+g\,x\right)}^{3/2}}{3\,e\,g^2}$$

input
```
int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)),x)
```

output
```
(2*atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2 + c*d^2))/(e^(
5/2)*(d*g - e*f)^(1/2)) - (f + g*x)^(1/2)*((2*c*(d*g^3 - e*f*g^2))/(e^2*g^
4) + (4*c*f)/(e*g^2)) + (2*c*(f + g*x)^(3/2))/(3*e*g^2)
```

3.594 $\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx$

3.594.1 Optimal result

Integrand size = 24, antiderivative size = 122

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx=\frac{2c\sqrt{f+gx}}{e^2g}-\frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)}+\frac{(ae^2g+cd(4ef-3dg))\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}}$$

output
```
(a*e^2*g+c*d*(-3*d*g+4*e*f))*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2
))/e^(5/2)/(-d*g+e*f)^(3/2)+2*c*(g*x+f)^(1/2)/e^2/g-(a+c*d^2/e^2)*(g*x+f)^
(1/2)/(-d*g+e*f)/(e*x+d)
```

3.594.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.09

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx=\frac{\sqrt{f+gx}(-ae^2g+c(-3d^2g+2e^2fx+2de(f-gx)))}{e^2g(ef-dg)(d+ex)}+\frac{(ae^2g+cd(4ef-3dg))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{5/2}(-ef+dg)^{3/2}}$$

input
```
Integrate[(a + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

output
```
(Sqrt[f + g*x]*(-(a*e^2*g) + c*(-3*d^2*g + 2*e^2*f*x + 2*d*e*(f - g*x))))/
(e^2*g*(e*f - d*g)*(d + e*x)) + ((a*e^2*g + c*d*(4*e*f - 3*d*g))*ArcTan[(S
qrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(5/2)*(-(e*f) + d*g)^(3/2))
```

3.594.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.34, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {649, 1471, 25, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{649}$$

$$\frac{2\int \frac{cf^2-2c(f+gx)f+ag^2+c(f+gx)^2}{(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{g}$$

$$\Big\downarrow \text{1471}$$

$$\frac{2\left(\frac{g^2\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{2(ef-dg)(-dg-e(f+gx)+ef)}-\frac{\int -\frac{2cf^2+ag^2-\frac{cd^2g^2}{e^2}-2c\left(f-\frac{dg}{e}\right)(f+gx)}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{2(ef-dg)}\right)}{g}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(\frac{\int \frac{2cf^2+ag^2-\frac{cd^2g^2}{e^2}-2c\left(f-\frac{dg}{e}\right)(f+gx)}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{2(ef-dg)}+\frac{g^2\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{2(ef-dg)(-dg-e(f+gx)+ef)}\right)}{g}$$

$$\Big\downarrow \text{299}$$

$$\frac{2\left(\frac{\frac{g\left(ae^2g+cd(4ef-3dg)\right)\int \frac{1}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{e^2}+\frac{2c\sqrt{f+gx}(ef-dg)}{e^2}}{2(ef-dg)}+\frac{g^2\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{2(ef-dg)(-dg-e(f+gx)+ef)}\right)}{g}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\left(\frac{\frac{g\left(ae^2g+cd(4ef-3dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}+\frac{2c\sqrt{f+gx}(ef-dg)}{e^2}}{2(ef-dg)}+\frac{g^2\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{2(ef-dg)(-dg-e(f+gx)+ef)}\right)}{g}$$

input
```
Int[(a + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

output
```
(2*(((a + (c*d^2)/e^2)*g^2*Sqrt[f + g*x])/(2*(e*f - d*g)*(e*f - d*g - e*(f
 + g*x))) + ((2*c*(e*f - d*g)*Sqrt[f + g*x])/e^2 + (g*(a*e^2*g + c*d*(4*e*
f - 3*d*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(5/2)*Sqr
t[e*f - d*g]))/(2*(e*f - d*g))))/g
```

3.594.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.594.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.13

method	result
risch	$\frac{2c\sqrt{gx+f}}{e^{2} g}-\frac{-\frac{g\left(e^{2} a+c\,d^{2}\right)\sqrt{gx+f}}{(dg-ef)(e(gx+f)+dg-ef)}-\frac{\left(a\,e^{2} g-3c\,d^{2} g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)\sqrt{(dg-ef)e}}}{e^{2}}$
derivativedivides	$\frac{\frac{2c\sqrt{gx+f}}{e^{2}}+\frac{2g\left(\frac{g\left(e^{2} a+c\,d^{2}\right)\sqrt{gx+f}}{2(dg-ef)(e(gx+f)+dg-ef)}+\frac{\left(a\,e^{2} g-3c\,d^{2} g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2(dg-ef)\sqrt{(dg-ef)e}}\right)}{e^{2}}}{g}$
default	$\frac{\frac{2c\sqrt{gx+f}}{e^{2}}+\frac{2g\left(\frac{g\left(e^{2} a+c\,d^{2}\right)\sqrt{gx+f}}{2(dg-ef)(e(gx+f)+dg-ef)}+\frac{\left(a\,e^{2} g-3c\,d^{2} g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2(dg-ef)\sqrt{(dg-ef)e}}\right)}{e^{2}}}{g}$
pseudoelliptic	$\frac{g(ex+d)\left(a\,e^{2} g-3c\,d^{2} g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)+\sqrt{(dg-ef)e}\,\sqrt{gx+f}\,\left((-2cfx+ag)e^{2}-2cd(-gx+f)e+3c\,d^{2} g\right)}{\sqrt{(dg-ef)e}\,g\,e^{2}(dg-ef)(ex+d)}$

input
```
int((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*c*(g*x+f)^(1/2)/e^2/g-1/e^2*(-g*(a*e^2+c*d^2)/(d*g-e*f)*(g*x+f)^(1/2)/(e
*(g*x+f)+d*g-e*f)-(a*e^2*g-3*c*d^2*g+4*c*d*e*f)/(d*g-e*f)/((d*g-e*f)*e)^(1
/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)))
```

3.594.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(108) = 216$.

Time = 0.30 (sec) , antiderivative size = 539, normalized size of antiderivative = 4.42

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx$$
$$= \left[-\frac{(4\,cd^2efg-(3\,cd^3-ade^2)g^2+(4\,cde^2fg-(3\,cd^2e-ae^3)g^2)x)\sqrt{e^2f-deg}\log\left(\frac{egx+2\,ef-dg-2\,\sqrt{e^2f-deg}}{ex+d}\right.}{2\,(de^5f^2g-2\,d^2e^4fg^2+d^3e^3g}\right.$$
$$-\frac{(4\,cd^2efg-(3\,cd^3-ade^2)g^2+(4\,cde^2fg-(3\,cd^2e-ae^3)g^2)x)\sqrt{-e^2f+deg}\arctan\left(\frac{\sqrt{-e^2f+deg}\sqrt{gx+f}}{egx+ef}\right)}{de^5f^2g-2\,d^2e^4fg^2+d^3e^3g^3+(}$$

input
```
integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((4*c*d^2*e*f*g - (3*c*d^3 - a*d*e^2)*g^2 + (4*c*d*e^2*f*g - (3*c*d^
2*e - a*e^3)*g^2)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*g - 2*sqrt
(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*d*e^3*f^2 - (5*c*d^2*e^
2 + a*e^4)*f*g + (3*c*d^3*e + a*d*e^3)*g^2 + 2*(c*e^4*f^2 - 2*c*d*e^3*f*g
+ c*d^2*e^2*g^2)*x)*sqrt(g*x + f))/(d*e^5*f^2*g - 2*d^2*e^4*f*g^2 + d^3*e^
3*g^3 + (e^6*f^2*g - 2*d*e^5*f*g^2 + d^2*e^4*g^3)*x), -((4*c*d^2*e*f*g - (
3*c*d^3 - a*d*e^2)*g^2 + (4*c*d*e^2*f*g - (3*c*d^2*e - a*e^3)*g^2)*x)*sqrt
(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f))
- (2*c*d*e^3*f^2 - (5*c*d^2*e^2 + a*e^4)*f*g + (3*c*d^3*e + a*d*e^3)*g^2 +
 2*(c*e^4*f^2 - 2*c*d*e^3*f*g + c*d^2*e^2*g^2)*x)*sqrt(g*x + f))/(d*e^5*f^
2*g - 2*d^2*e^4*f*g^2 + d^3*e^3*g^3 + (e^6*f^2*g - 2*d*e^5*f*g^2 + d^2*e^4
*g^3)*x)]
```

3.594.6 Sympy [F]

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \int \frac{a+cx^2}{(d+ex)^2\,\sqrt{f+gx}}\,dx$$

input
```
integrate((c*x**2+a)/(e*x+d)**2/(g*x+f)**(1/2),x)
```

output
```
Integral((a + c*x**2)/((d + e*x)**2*sqrt(f + g*x)), x)
```

3.594.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + cx^2}{(d + ex)^2 \sqrt{f + gx}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.594.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.24

$$\int \frac{a + cx^2}{(d + ex)^2 \sqrt{f + gx}} \, dx = -\frac{(4\,cdef - 3\,cd^2g + ae^2g)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{(e^3f - de^2g)\sqrt{-e^2f + deg}} - \frac{\sqrt{gx + f}cd^2g + \sqrt{gx + f}ae^2g}{(e^3f - de^2g)((gx + f)e - ef + dg)} + \frac{2\sqrt{gx + f}c}{e^2g}$$

input
```
integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
-(4*c*d*e*f - 3*c*d^2*g + a*e^2*g)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*
e*g))/((e^3*f - d*e^2*g)*sqrt(-e^2*f + d*e*g)) - (sqrt(g*x + f)*c*d^2*g +
sqrt(g*x + f)*a*e^2*g)/((e^3*f - d*e^2*g)*((g*x + f)*e - e*f + d*g)) + 2*s
qrt(g*x + f)*c/(e^2*g)
```

3.594.9 Mupad [B] (verification not implemented)

Time = 11.97 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.05

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(-3\,c\,g\,d^2+4\,c\,f\,d\,e+a\,g\,e^2\right)}{e^{5/2}\,{\left(d\,g-e\,f\right)}^{3/2}} + \frac{\sqrt{f+g\,x}\,\left(c\,g\,d^2+a\,g\,e^2\right)}{\left(d\,g-e\,f\right)\,\left(e^3\,\left(f+g\,x\right)-e^3\,f+d\,e^2\,g\right)} + \frac{2\,c\,\sqrt{f+g\,x}}{e^2\,g}$$

input
```
int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^2),x)
```

output
```
(atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2*g - 3*c*d^2*g +
4*c*d*e*f))/(e^(5/2)*(d*g - e*f)^(3/2)) + ((f + g*x)^(1/2)*(a*e^2*g + c*d^
2*g))/((d*g - e*f)*(e^3*(f + g*x) - e^3*f + d*e^2*g)) + (2*c*(f + g*x)^(1/
2))/(e^2*g)
```

3.595 $\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx$

3.595.1 Optimal result

Integrand size = 24, antiderivative size = 178

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx=-\frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2}+\frac{(3ae^2g+cd(8ef-5dg))\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)}-\frac{(3ae^2g^2+c(8e^2f^2-8defg+3d^2g^2))\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{5/2}(ef-dg)^{5/2}}$$

output
```
-1/4*(3*a*e^2*g^2+c*(3*d^2*g^2-8*d*e*f*g+8*e^2*f^2))*arctanh(e^(1/2)*(g*x+
f)^(1/2)/(-d*g+e*f)^(1/2))/e^(5/2)/(-d*g+e*f)^(5/2)-1/2*(a+c*d^2/e^2)*(g*x
+f)^(1/2)/(-d*g+e*f)/(e*x+d)^2+1/4*(3*a*e^2*g+c*d*(-5*d*g+8*e*f))*(g*x+f)^
(1/2)/e^2/(-d*g+e*f)^2/(e*x+d)
```

3.595.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.93

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx=\frac{\frac{\sqrt{e}\sqrt{f+gx}(ae^2(-2ef+5dg+3egx)+cd(-3d^2g+8e^2fx+de(6f-5gx)))}{(ef-dg)^2(d+ex)^2}+\frac{(3ae^2g^2+c(8e^2f^2-8defg+3d^2g^2))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{(-ef+dg)^{5/2}}}{4e^{5/2}}$$

input
```
Integrate[(a + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

output
```
((Sqrt[e]*Sqrt[f + g*x]*(a*e^2*(-2*e*f + 5*d*g + 3*e*g*x) + c*d*(-3*d^2*g
+ 8*e^2*f*x + d*e*(6*f - 5*g*x))))/((e*f - d*g)^2*(d + e*x)^2) + ((3*a*e^2
*g^2 + c*(8*e^2*f^2 - 8*d*e*f*g + 3*d^2*g^2))*ArcTan[(Sqrt[e]*Sqrt[f + g*x
])/Sqrt[-(e*f) + d*g]])/(-(e*f) + d*g)^(5/2))/(4*e^(5/2))
```

3.595.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {649, 25, 1471, 25, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{649}$$

$$2\int -\frac{cf^2-2c(f+gx)f+ag^2+c(f+gx)^2}{(ef-dg-e(f+gx))^3}d\sqrt{f+gx}$$

$$\Big\downarrow \text{25}$$

$$-2\int \frac{cf^2-2c(f+gx)f+ag^2+c(f+gx)^2}{(ef-dg-e(f+gx))^3}d\sqrt{f+gx}$$

$$\Big\downarrow \text{1471}$$

$$2\left(\frac{\int -\frac{4cf^2+3ag^2-\frac{cd^2g^2}{e^2}-4c\left(f-\frac{dg}{e}\right)(f+gx)}{(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{4(ef-dg)}-\frac{g^2\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{4(ef-dg)(-dg-e(f+gx)+ef)^2}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(-\frac{\int \frac{4cf^2+3ag^2-\frac{cd^2g^2}{e^2}-4c\left(f-\frac{dg}{e}\right)(f+gx)}{(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{4(ef-dg)}-\frac{g^2\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{4(ef-dg)(-dg-e(f+gx)+ef)^2}\right)$$

$$\Big\downarrow \text{298}$$

$$2\left(-\frac{\frac{\left(3ae^2g^2+c\left(3d^2g^2-8defg+8e^2f^2\right)\right)\int \frac{1}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{2e^2(ef-dg)}+\frac{g\sqrt{f+gx}\left(3ae^2g+cd(8ef-5dg)\right)}{2e^2(ef-dg)(-dg-e(f+gx)+ef)}}{4(ef-dg)}-\frac{g^2\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right.}{4(ef-dg)(-dg-e(f+gx}$$

$$\Big\downarrow \; 221$$

$$2\left(-\frac{\frac{\left(3ae^2g^2+c\left(3d^2g^2-8defg+8e^2f^2\right)\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{2e^{5/2}(ef-dg)^{3/2}}+\frac{g\sqrt{f+gx}\left(3ae^2g+cd(8ef-5dg)\right)}{2e^2(ef-dg)(-dg-e(f+gx)+ef)}}{4(ef-dg)}-\frac{g^2\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{4(ef-dg)(-dg-e(f+gx)+}$$

input
```
Int[(a + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

output
```
2*(-1/4*((a + (c*d^2)/e^2)*g^2*Sqrt[f + g*x])/((e*f - d*g)*(e*f - d*g - e*
(f + g*x))^2) - ((g*(3*a*e^2*g + c*d*(8*e*f - 5*d*g))*Sqrt[f + g*x])/(2*e^
2*(e*f - d*g)*(e*f - d*g - e*(f + g*x))) + ((3*a*e^2*g^2 + c*(8*e^2*f^2 -
8*d*e*f*g + 3*d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/
(2*e^(5/2)*(e*f - d*g)^(3/2)))/(4*(e*f - d*g)))
```

3.595.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.595.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$\frac{\frac{3\left(\left(a\,g^{2}+\frac{8c\,f^{2}}{3}\right)e^{2}-\frac{8cdefg}{3}+c\,d^{2}g^{2}\right)(ex+d)^{2}\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{4}+\frac{5\left(-\frac{2a\left(-\frac{3gx}{2}+f\right)e^{3}}{5}+d\left(\frac{8cfx}{5}+ag\right)e^{2}+\frac{6c\left(-\frac{5gx}{6}+f\right)d^{2}e}{5}\right.}{4}}{\sqrt{(dg-ef)e}\,(dg-ef)^{2}(ex+d)^{2}e^{2}}$
derivativedivides	$\frac{\frac{g\left(3a\,e^{2}g-5c\,d^{2}g+8cdef\right)(gx+f)^{\frac{3}{2}}}{4e\left(d^{2}g^{2}-2defg+e^{2}f^{2}\right)}+\frac{\left(5a\,e^{2}g-3c\,d^{2}g+8cdef\right)g\sqrt{gx+f}}{4e^{2}(dg-ef)}}{(e(gx+f)+dg-ef)^{2}}+\frac{(3a\,e^{2}g^{2}+3c\,d^{2}g^{2}-8cdefg+8c\,e^{2}f^{2})\arctan\left(\frac{e\sqrt{}}{\sqrt{(dg}}\right.}{4(d^{2}g^{2}-2defg+e^{2}f^{2})e^{2}\sqrt{(dg-ef)e}}$
default	$\frac{\frac{g\left(3a\,e^{2}g-5c\,d^{2}g+8cdef\right)(gx+f)^{\frac{3}{2}}}{4e\left(d^{2}g^{2}-2defg+e^{2}f^{2}\right)}+\frac{\left(5a\,e^{2}g-3c\,d^{2}g+8cdef\right)g\sqrt{gx+f}}{4e^{2}(dg-ef)}}{(e(gx+f)+dg-ef)^{2}}+\frac{(3a\,e^{2}g^{2}+3c\,d^{2}g^{2}-8cdefg+8c\,e^{2}f^{2})\arctan\left(\frac{e\sqrt{}}{\sqrt{(dg}}\right.}{4(d^{2}g^{2}-2defg+e^{2}f^{2})e^{2}\sqrt{(dg-ef)e}}$

input
```
int((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
5/4/((d*g-e*f)*e)^(1/2)/(d*g-e*f)^2*(3/5*((a*g^2+8/3*c*f^2)*e^2-8/3*c*d*e*
f*g+c*d^2*g^2)*(e*x+d)^2*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))+(-2/5
*a*(-3/2*g*x+f)*e^3+d*(8/5*c*f*x+a*g)*e^2+6/5*c*(-5/6*g*x+f)*d^2*e-3/5*c*d
^3*g)*((d*g-e*f)*e)^(1/2)*(g*x+f)^(1/2))/(e*x+d)^2/e^2
```

3.595.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(158) = 316.

Time = 0.30 (sec) , antiderivative size = 896, normalized size of antiderivative = 5.03

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx$$

$$= \left[\frac{(8\,cd^2e^2f^2-8\,cd^3efg+3\,(cd^4+ad^2e^2)g^2+(8\,ce^4f^2-8\,cde^3fg+3\,(cd^2e^2+ae^4)g^2)x^2+2\,(8\,cde^3f^2-}{8\,(d^2e^6f^3}\right.$$

input
```
integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*((8*c*d^2*e^2*f^2 - 8*c*d^3*e*f*g + 3*(c*d^4 + a*d^2*e^2)*g^2 + (8*c*
e^4*f^2 - 8*c*d*e^3*f*g + 3*(c*d^2*e^2 + a*e^4)*g^2)*x^2 + 2*(8*c*d*e^3*f^
2 - 8*c*d^2*e^2*f*g + 3*(c*d^3*e + a*d*e^3)*g^2)*x)*sqrt(e^2*f - d*e*g)*lo
g((e*g*x + 2*e*f - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) +
 2*(2*(3*c*d^2*e^3 - a*e^5)*f^2 - (9*c*d^3*e^2 - 7*a*d*e^4)*f*g + (3*c*d^4
*e - 5*a*d^2*e^3)*g^2 + (8*c*d*e^4*f^2 - (13*c*d^2*e^3 - 3*a*e^5)*f*g + (5
*c*d^3*e^2 - 3*a*d*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^3 - 3*d^3*e^5*f^
2*g + 3*d^4*e^4*f*g^2 - d^5*e^3*g^3 + (e^8*f^3 - 3*d*e^7*f^2*g + 3*d^2*e^6
*f*g^2 - d^3*e^5*g^3)*x^2 + 2*(d*e^7*f^3 - 3*d^2*e^6*f^2*g + 3*d^3*e^5*f*g
^2 - d^4*e^4*g^3)*x), 1/4*((8*c*d^2*e^2*f^2 - 8*c*d^3*e*f*g + 3*(c*d^4 + a
*d^2*e^2)*g^2 + (8*c*e^4*f^2 - 8*c*d*e^3*f*g + 3*(c*d^2*e^2 + a*e^4)*g^2)*
x^2 + 2*(8*c*d*e^3*f^2 - 8*c*d^2*e^2*f*g + 3*(c*d^3*e + a*d*e^3)*g^2)*x)*s
qrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f
)) + (2*(3*c*d^2*e^3 - a*e^5)*f^2 - (9*c*d^3*e^2 - 7*a*d*e^4)*f*g + (3*c*d
^4*e - 5*a*d^2*e^3)*g^2 + (8*c*d*e^4*f^2 - (13*c*d^2*e^3 - 3*a*e^5)*f*g +
(5*c*d^3*e^2 - 3*a*d*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^3 - 3*d^3*e^5*
f^2*g + 3*d^4*e^4*f*g^2 - d^5*e^3*g^3 + (e^8*f^3 - 3*d*e^7*f^2*g + 3*d^2*e
^6*f*g^2 - d^3*e^5*g^3)*x^2 + 2*(d*e^7*f^3 - 3*d^2*e^6*f^2*g + 3*d^3*e^5*f
*g^2 - d^4*e^4*g^3)*x)]
```

3.595.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + cx^2}{(d + ex)^3 \sqrt{f + gx}} \, dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)/(e*x+d)**3/(g*x+f)**(1/2),x)
```

output
```
Timed out
```

3.595.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.595.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.60

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \frac{(8\,ce^2f^2-8\,cdefg+3\,cd^2g^2+3\,ae^2g^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{4\,(e^4f^2-2\,de^3fg+d^2e^2g^2)\sqrt{-e^2f+deg}} + \frac{8\,(gx+f)^{\frac{3}{2}}cde^2fg-8\,\sqrt{gx+f}cde^2f^2g-5\,(gx+f)^{\frac{3}{2}}cd^2eg^2+3\,(gx+f)^{\frac{3}{2}}ae^3g^2+11\,\sqrt{gx+f}cd^2efg^2}{4\,(e^4f^2-2\,de^3fg+d^2e^2g^2)((gx+f)e-ef+}$$

input
```
integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
1/4*(8*c*e^2*f^2 - 8*c*d*e*f*g + 3*c*d^2*g^2 + 3*a*e^2*g^2)*arctan(sqrt(g*
x + f)*e/sqrt(-e^2*f + d*e*g))/((e^4*f^2 - 2*d*e^3*f*g + d^2*e^2*g^2)*sqrt
(-e^2*f + d*e*g)) + 1/4*(8*(g*x + f)^(3/2)*c*d*e^2*f*g - 8*sqrt(g*x + f)*c
*d*e^2*f^2*g - 5*(g*x + f)^(3/2)*c*d^2*e*g^2 + 3*(g*x + f)^(3/2)*a*e^3*g^2
 + 11*sqrt(g*x + f)*c*d^2*e*f*g^2 - 5*sqrt(g*x + f)*a*e^3*f*g^2 - 3*sqrt(g
*x + f)*c*d^3*g^3 + 5*sqrt(g*x + f)*a*d*e^2*g^3)/((e^4*f^2 - 2*d*e^3*f*g +
 d^2*e^2*g^2)*((g*x + f)*e - e*f + d*g)^2)
```

3.595.9 Mupad [B] (verification not implemented)

Time = 12.01 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.26

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \frac{\frac{\sqrt{f+g\,x}\,\left(-3\,c\,d^2\,g^2+8\,c\,f\,d\,e\,g+5\,a\,e^2\,g^2\right)}{4\,e^2\,\left(d\,g-e\,f\right)}+\frac{{\left(f+g\,x\right)}^{3/2}\,\left(-5\,c\,d^2\,g^2+8\,c\,f\,d\,e\,g+3\,a\,e^2\,g^2\right)}{4\,e\,{\left(d\,g-e\,f\right)}^2}}{e^2\,{\left(f+g\,x\right)}^2-\left(f+g\,x\right)\,\left(2\,e^2\,f-2\,d\,e\,g\right)+d^2\,g^2+e^2\,f^2-2\,d\,e\,f\,g}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(3\,c\,d^2\,g^2-8\,c\,d\,e\,f\,g+8\,c\,e^2\,f^2+3\,a\,e^2\,g^2\right)}{4\,e^{5/2}\,{\left(d\,g-e\,f\right)}^{5/2}}$$

input
```
int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^3),x)
```

output
```
(((f + g*x)^(1/2)*(5*a*e^2*g^2 - 3*c*d^2*g^2 + 8*c*d*e*f*g))/(4*e^2*(d*g -
 e*f)) + ((f + g*x)^(3/2)*(3*a*e^2*g^2 - 5*c*d^2*g^2 + 8*c*d*e*f*g))/(4*e*
(d*g - e*f)^2))/(e^2*(f + g*x)^2 - (f + g*x)*(2*e^2*f - 2*d*e*g) + d^2*g^2
 + e^2*f^2 - 2*d*e*f*g) + (atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2
))*(3*a*e^2*g^2 + 3*c*d^2*g^2 + 8*c*e^2*f^2 - 8*c*d*e*f*g))/(4*e^(5/2)*(d*
g - e*f)^(5/2))
```

3.596 $\int \frac{(d+ex)^3(a+cx^2)}{(f+gx)^{3/2}} dx$

3.596.1 Optimal result

Integrand size = 24, antiderivative size = 238

$$\begin{aligned}\int \frac{(d+ex)^3(a+cx^2)}{(f+gx)^{3/2}}\,dx &= \frac{2(ef-dg)^3(cf^2+ag^2)}{g^6\sqrt{f+gx}} \\ &+ \frac{2(ef-dg)^2(3aeg^2+cf(5ef-2dg))\sqrt{f+gx}}{g^6} \\ &- \frac{2(ef-dg)(3ae^2g^2+c(10e^2f^2-8defg+d^2g^2))(f+gx)^{3/2}}{3g^6} \\ &+ \frac{2e(ae^2g^2+c(10e^2f^2-12defg+3d^2g^2))(f+gx)^{5/2}}{5g^6} \\ &- \frac{2ce^2(5ef-3dg)(f+gx)^{7/2}}{7g^6} + \frac{2ce^3(f+gx)^{9/2}}{9g^6}\end{aligned}$$

output
```
-2/3*(-d*g+e*f)*(3*a*e^2*g^2+c*(d^2*g^2-8*d*e*f*g+10*e^2*f^2))*(g*x+f)^(3/
2)/g^6+2/5*e*(a*e^2*g^2+c*(3*d^2*g^2-12*d*e*f*g+10*e^2*f^2))*(g*x+f)^(5/2)
/g^6-2/7*c*e^2*(-3*d*g+5*e*f)*(g*x+f)^(7/2)/g^6+2/9*c*e^3*(g*x+f)^(9/2)/g^
6+2*(-d*g+e*f)^3*(a*g^2+c*f^2)/g^6/(g*x+f)^(1/2)+2*(-d*g+e*f)^2*(3*a*e*g^2
+c*f*(-2*d*g+5*e*f))*(g*x+f)^(1/2)/g^6
```

3.596.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^3 (a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2(63ag^2(-5d^3g^3+15d^2eg^2(2f+gx)+5de^2g(-8f^2-4fgx+g^2x^2)+e^3(16f^3}{}$$

input
```
Integrate[((d + e*x)^3*(a + c*x^2))/(f + g*x)^(3/2),x]
```

output
```
(2*(63*a*g^2*(-5*d^3*g^3 + 15*d^2*e*g^2*(2*f + g*x) + 5*d*e^2*g*(-8*f^2 -
4*f*g*x + g^2*x^2) + e^3*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3)) + c
*(105*d^3*g^3*(-8*f^2 - 4*f*g*x + g^2*x^2) + 189*d^2*e*g^2*(16*f^3 + 8*f^2
*g*x - 2*f*g^2*x^2 + g^3*x^3) + 27*d*e^2*g*(-128*f^4 - 64*f^3*g*x + 16*f^2
*g^2*x^2 - 8*f*g^3*x^3 + 5*g^4*x^4) + 5*e^3*(256*f^5 + 128*f^4*g*x - 32*f^
3*g^2*x^2 + 16*f^2*g^3*x^3 - 10*f*g^4*x^4 + 7*g^5*x^5))))/(315*g^6*Sqrt[f
+ g*x])
```

3.596.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(d+ex)^3}{(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{e(f+gx)^{3/2}\left(ae^2g^2+c(3d^2g^2-12defg+10e^2f^2)\right)}{g^5}+\frac{\sqrt{f+gx}(ef-dg)\left(-3ae^2g^2-c(d^2g^2-8defg+10e\right.}{g^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2e(f+gx)^{5/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{5g^6}-\frac{2(f+gx)^{3/2}(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10e^2f^2\right)\right)}{3g^6}+\frac{2\left(ag^2+cf^2\right)(ef-dg)^3}{g^6\sqrt{f+gx}}+\frac{2\sqrt{f+gx}(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)}{g^6}-\frac{2ce^2(f+gx)^{7/2}(5ef-3dg)}{7g^6}+\frac{2ce^3(f+gx)^{9/2}}{9g^6}$$

input
```
Int[((d + e*x)^3*(a + c*x^2))/(f + g*x)^(3/2),x]
```

output
```
(2*(e*f - d*g)^3*(c*f^2 + a*g^2))/(g^6*Sqrt[f + g*x]) + (2*(e*f - d*g)^2*(
3*a*e*g^2 + c*f*(5*e*f - 2*d*g))*Sqrt[f + g*x])/g^6 - (2*(e*f - d*g)*(3*a*
e^2*g^2 + c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^6) +
 (2*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(5/2
))/(5*g^6) - (2*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^(7/2))/(7*g^6) + (2*c*e^3*
(f + g*x)^(9/2))/(9*g^6)
```

3.596.3.1 Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.596.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{2\left(\left(\left(-\frac{x^{3}\left(\frac{5c\,x^{2}}{9}+a\right)e^{3}}{5}-x^{2}d\left(\frac{3c\,x^{2}}{7}+a\right)e^{2}-3\left(\frac{c\,x^{2}}{5}+a\right)x\,d^{2}e+d^{3}\left(-\frac{c\,x^{2}}{3}+a\right)\right)g^{5}-6f\left(-\frac{\left(\frac{25c\,x^{2}}{63}+a\right)x^{2}e^{3}}{15}-\frac{2x\left(\frac{6c\,x^{2}}{35}\right.}{3}\right.\right.}{}$
risch	$\frac{2\left(35c\,e^{3}x^{4}g^{4}+135cd\,e^{2}g^{4}x^{3}-85c\,e^{3}f\,g^{3}x^{3}+63a\,e^{3}g^{4}x^{2}+189c\,d^{2}e\,g^{4}x^{2}-351cd\,e^{2}f\,g^{3}x^{2}+165c\,e^{3}f^{2}g^{2}x^{2}+315ad\,e^{2}g^{4}x\right.}{}$
gosper	$-\frac{2\left(-35c\,e^{3}x^{5}g^{5}-135cd\,e^{2}g^{5}x^{4}+50c\,e^{3}f\,g^{4}x^{4}-63a\,e^{3}g^{5}x^{3}-189c\,d^{2}e\,g^{5}x^{3}+216cd\,e^{2}f\,g^{4}x^{3}-80c\,e^{3}f^{2}g^{3}x^{3}-315ad\,e^{2}g\right.}{}$
trager	$-\frac{2\left(-35c\,e^{3}x^{5}g^{5}-135cd\,e^{2}g^{5}x^{4}+50c\,e^{3}f\,g^{4}x^{4}-63a\,e^{3}g^{5}x^{3}-189c\,d^{2}e\,g^{5}x^{3}+216cd\,e^{2}f\,g^{4}x^{3}-80c\,e^{3}f^{2}g^{3}x^{3}-315ad\,e^{2}g\right.}{}$
derivativedivides	$\frac{\frac{2c\,e^{3}(gx+f)^{\frac{9}{2}}}{9}+\frac{6cd\,e^{2}g(gx+f)^{\frac{7}{2}}}{7}-\frac{10c\,e^{3}f(gx+f)^{\frac{7}{2}}}{7}+\frac{2a\,e^{3}g^{2}(gx+f)^{\frac{5}{2}}}{5}+\frac{6c\,d^{2}e\,g^{2}(gx+f)^{\frac{5}{2}}}{5}-\frac{24cd\,e^{2}fg(gx+f)^{\frac{5}{2}}}{5}+4c\,e^{3}f^{2}(gx+}{}$
default	$\frac{\frac{2c\,e^{3}(gx+f)^{\frac{9}{2}}}{9}+\frac{6cd\,e^{2}g(gx+f)^{\frac{7}{2}}}{7}-\frac{10c\,e^{3}f(gx+f)^{\frac{7}{2}}}{7}+\frac{2a\,e^{3}g^{2}(gx+f)^{\frac{5}{2}}}{5}+\frac{6c\,d^{2}e\,g^{2}(gx+f)^{\frac{5}{2}}}{5}-\frac{24cd\,e^{2}fg(gx+f)^{\frac{5}{2}}}{5}+4c\,e^{3}f^{2}(gx+}{}$

input
```
int((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((-1/5*x^3*(5/9*c*x^2+a)*e^3-x^2*d*(3/7*c*x^2+a)*e^2-3*(1/5*c*x^2+a)*x*
d^2*e+d^3*(-1/3*c*x^2+a))*g^5-6*f*(-1/15*(25/63*c*x^2+a)*x^2*e^3-2/3*x*(6/
35*c*x^2+a)*d*e^2+d^2*(-1/5*c*x^2+a)*e-2/9*c*d^3*x)*g^4+8*f^2*((-2/63*c*x^
3-1/5*a*x)*e^3+d*(-6/35*c*x^2+a)*e^2-3/5*c*d^2*e*x+1/3*c*d^3)*g^3-16/5*e*(
(-10/63*c*x^2+a)*e^2-12/7*c*d*e*x+3*c*d^2)*f^3*g^2+384/35*e^2*c*(-5/27*e*x
+d)*f^4*g-256/63*c*e^3*f^5)/(g*x+f)^(1/2)/g^6
```

3.596.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex)^3\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\,(35\,ce^3g^5x^5+1280\,ce^3f^5-3456\,cde^2f^4g+1890\,ad^2efg^4-315\,ad^3g^5+1008\,(\cdots}{\cdots}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
2/315*(35*c*e^3*g^5*x^5 + 1280*c*e^3*f^5 - 3456*c*d*e^2*f^4*g + 1890*a*d^2
*e*f*g^4 - 315*a*d^3*g^5 + 1008*(3*c*d^2*e + a*e^3)*f^3*g^2 - 840*(c*d^3 +
 3*a*d*e^2)*f^2*g^3 - 5*(10*c*e^3*f*g^4 - 27*c*d*e^2*g^5)*x^4 + (80*c*e^3*
f^2*g^3 - 216*c*d*e^2*f*g^4 + 63*(3*c*d^2*e + a*e^3)*g^5)*x^3 - (160*c*e^3
*f^3*g^2 - 432*c*d*e^2*f^2*g^3 + 126*(3*c*d^2*e + a*e^3)*f*g^4 - 105*(c*d^
3 + 3*a*d*e^2)*g^5)*x^2 + (640*c*e^3*f^4*g - 1728*c*d*e^2*f^3*g^2 + 945*a*
d^2*e*g^5 + 504*(3*c*d^2*e + a*e^3)*f^2*g^3 - 420*(c*d^3 + 3*a*d*e^2)*f*g^
4)*x)*sqrt(g*x + f)/(g^7*x + f*g^6)
```

3.596.6 Sympy [A] (verification not implemented)

Time = 11.71 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.76

$$\int \frac{(d+ex)^3\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{ce^3(f+gx)^{\frac{9}{2}}}{9g^5}+\frac{(f+gx)^{\frac{7}{2}}\cdot\left(3cde^2g-5ce^3f\right)}{7g^5}+\frac{(f+gx)^{\frac{5}{2}}\left(ae^3g^2+3cd^2eg^2-12cde^2fg+10ce^3f^2\right)}{5g^5}+\frac{(f+gx)^{\frac{3}{2}}\cdot\left(3ade^2\cdots\right.}{\cdots}\cdots\right.}{\cdots} \\ \frac{ad^3x+\frac{3ad^2ex^2}{2}+\frac{3cde^2x^5}{5}+\frac{ce^3x^6}{6}+\frac{x^4\left(ae^3+3cd^2e\right)}{4}+\frac{x^3\cdot\left(3ade^2+cd^3\right)}{3}}{f^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((e*x+d)**3*(c*x**2+a)/(g*x+f)**(3/2),x)
```

output
```
Piecewise((2*(c*e**3*(f + g*x)**(9/2)/(9*g**5) + (f + g*x)**(7/2)*(3*c*d*e
**2*g - 5*c*e**3*f)/(7*g**5) + (f + g*x)**(5/2)*(a*e**3*g**2 + 3*c*d**2*e*
g**2 - 12*c*d*e**2*f*g + 10*c*e**3*f**2)/(5*g**5) + (f + g*x)**(3/2)*(3*a*
d*e**2*g**3 - 3*a*e**3*f*g**2 + c*d**3*g**3 - 9*c*d**2*e*f*g**2 + 18*c*d*e
**2*f**2*g - 10*c*e**3*f**3)/(3*g**5) + sqrt(f + g*x)*(3*a*d**2*e*g**4 - 6
*a*d*e**2*f*g**3 + 3*a*e**3*f**2*g**2 - 2*c*d**3*f*g**3 + 9*c*d**2*e*f**2*
g**2 - 12*c*d*e**2*f**3*g + 5*c*e**3*f**4)/g**5 - (a*g**2 + c*f**2)*(d*g -
 e*f)**3/(g**5*sqrt(f + g*x)))/g, Ne(g, 0)), ((a*d**3*x + 3*a*d**2*e*x**2/
2 + 3*c*d*e**2*x**5/5 + c*e**3*x**6/6 + x**4*(a*e**3 + 3*c*d**2*e)/4 + x**
3*(3*a*d*e**2 + c*d**3)/3)/f**(3/2), True))
```

3.596.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex)^3\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\left(\frac{35\,(gx+f)^{\frac{9}{2}}ce^3-45\,(5\,ce^3f-3\,cde^2g)(gx+f)^{\frac{7}{2}}+63\,(10\,ce^3f^2-12\,cde^2fg+(3\,cd^2e+ae^3)g^2)(gx+f)^{\frac{5}{2}}-10\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
2/315*((35*(g*x + f)^(9/2)*c*e^3 - 45*(5*c*e^3*f - 3*c*d*e^2*g)*(g*x + f)^
(7/2) + 63*(10*c*e^3*f^2 - 12*c*d*e^2*f*g + (3*c*d^2*e + a*e^3)*g^2)*(g*x
+ f)^(5/2) - 105*(10*c*e^3*f^3 - 18*c*d*e^2*f^2*g + 3*(3*c*d^2*e + a*e^3)*
f*g^2 - (c*d^3 + 3*a*d*e^2)*g^3)*(g*x + f)^(3/2) + 315*(5*c*e^3*f^4 - 12*c
*d*e^2*f^3*g + 3*a*d^2*e*g^4 + 3*(3*c*d^2*e + a*e^3)*f^2*g^2 - 2*(c*d^3 +
3*a*d*e^2)*f*g^3)*sqrt(g*x + f))/g^5 + 315*(c*e^3*f^5 - 3*c*d*e^2*f^4*g +
3*a*d^2*e*f*g^4 - a*d^3*g^5 + (3*c*d^2*e + a*e^3)*f^3*g^2 - (c*d^3 + 3*a*d
*e^2)*f^2*g^3)/(sqrt(g*x + f)*g^5))/g
```

3.596.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(218) = 436.

Time = 0.28 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.95

$$\int \frac{(d+ex)^3\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\,(ce^3f^5-3\,cde^2f^4g+3\,cd^2ef^3g^2+ae^3f^3g^2-cd^3f^2g^3-3\,ade^2f^2g^3+3\,ad^2ef\cdots}{\sqrt{gx+f}g^6} + \frac{2\left(35\,(gx+f)^{\frac{9}{2}}ce^3g^{48}-225\,(gx+f)^{\frac{7}{2}}ce^3fg^{48}+630\,(gx+f)^{\frac{5}{2}}ce^3f^2g^{48}-1050\,(gx+f)^{\frac{3}{2}}ce^3f^3g^{48}+1575\cdots\right.}{\cdots}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
2*(c*e^3*f^5 - 3*c*d*e^2*f^4*g + 3*c*d^2*e*f^3*g^2 + a*e^3*f^3*g^2 - c*d^3
*f^2*g^3 - 3*a*d*e^2*f^2*g^3 + 3*a*d^2*e*f*g^4 - a*d^3*g^5)/(sqrt(g*x + f)
*g^6) + 2/315*(35*(g*x + f)^(9/2)*c*e^3*g^48 - 225*(g*x + f)^(7/2)*c*e^3*f
*g^48 + 630*(g*x + f)^(5/2)*c*e^3*f^2*g^48 - 1050*(g*x + f)^(3/2)*c*e^3*f^
3*g^48 + 1575*sqrt(g*x + f)*c*e^3*f^4*g^48 + 135*(g*x + f)^(7/2)*c*d*e^2*g
^49 - 756*(g*x + f)^(5/2)*c*d*e^2*f*g^49 + 1890*(g*x + f)^(3/2)*c*d*e^2*f^
2*g^49 - 3780*sqrt(g*x + f)*c*d*e^2*f^3*g^49 + 189*(g*x + f)^(5/2)*c*d^2*e
*g^50 + 63*(g*x + f)^(5/2)*a*e^3*g^50 - 945*(g*x + f)^(3/2)*c*d^2*e*f*g^50
 - 315*(g*x + f)^(3/2)*a*e^3*f*g^50 + 2835*sqrt(g*x + f)*c*d^2*e*f^2*g^50
+ 945*sqrt(g*x + f)*a*e^3*f^2*g^50 + 105*(g*x + f)^(3/2)*c*d^3*g^51 + 315*
(g*x + f)^(3/2)*a*d*e^2*g^51 - 630*sqrt(g*x + f)*c*d^3*f*g^51 - 1890*sqrt(
g*x + f)*a*d*e^2*f*g^51 + 945*sqrt(g*x + f)*a*d^2*e*g^52)/g^54
```

3.596.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^3\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{(f+g\,x)^{5/2}\,(6\,c\,d^2\,e\,g^2-24\,c\,d\,e^2\,f\,g+20\,c\,e^3\,f^2+2\,a\,e^3\,g^2)}{5\,g^6}$$
$$-\frac{2\,c\,d^3\,f^2\,g^3+2\,a\,d^3\,g^5-6\,c\,d^2\,e\,f^3\,g^2-6\,a\,d^2\,e\,f\,g^4+6\,c\,d\,e^2\,f^4\,g+6\,a\,d\,e^2\,f^2\,g^3-2\,c\,e^3\,f^5-2\,a\,e^3\,f^3\,g^2}{g^6\,\sqrt{f+g\,x}}$$
$$+\frac{2\,c\,e^3\,(f+g\,x)^{9/2}}{9\,g^6}+\frac{2\,\sqrt{f+g\,x}\,(d\,g-e\,f)^2\,(5\,c\,e\,f^2-2\,c\,d\,f\,g+3\,a\,e\,g^2)}{g^6}$$
$$+\frac{2\,(f+g\,x)^{3/2}\,(d\,g-e\,f)\,(c\,d^2\,g^2-8\,c\,d\,e\,f\,g+10\,c\,e^2\,f^2+3\,a\,e^2\,g^2)}{3\,g^6}$$
$$+\frac{2\,c\,e^2\,(f+g\,x)^{7/2}\,(3\,d\,g-5\,e\,f)}{7\,g^6}$$

input
```
int(((a + c*x^2)*(d + e*x)^3)/(f + g*x)^(3/2),x)
```

output
```
((f + g*x)^(5/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 + 6*c*d^2*e*g^2 - 24*c*d*e^2*
f*g))/(5*g^6) - (2*a*d^3*g^5 - 2*c*e^3*f^5 - 2*a*e^3*f^3*g^2 + 2*c*d^3*f^2
*g^3 - 6*a*d^2*e*f*g^4 + 6*c*d*e^2*f^4*g + 6*a*d*e^2*f^2*g^3 - 6*c*d^2*e*f
^3*g^2)/(g^6*(f + g*x)^(1/2)) + (2*c*e^3*(f + g*x)^(9/2))/(9*g^6) + (2*(f
+ g*x)^(1/2)*(d*g - e*f)^2*(3*a*e*g^2 + 5*c*e*f^2 - 2*c*d*f*g))/g^6 + (2*(
f + g*x)^(3/2)*(d*g - e*f)*(3*a*e^2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 - 8*c*d
*e*f*g))/(3*g^6) + (2*c*e^2*(f + g*x)^(7/2)*(3*d*g - 5*e*f))/(7*g^6)
```

3.597 $\int \frac{(d+ex)^2(a+cx^2)}{(f+gx)^{3/2}}\,dx$

3.597.1 Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{(d+ex)^2\left(a+cx^2\right)}{(f+gx)^{3/2}}\,dx = -\frac{2(ef-dg)^2\left(cf^2+ag^2\right)}{g^5\sqrt{f+gx}} - \frac{4(ef-dg)\left(aeg^2+cf(2ef-dg)\right)\sqrt{f+gx}}{g^5} + \frac{2\left(ae^2g^2+c(6e^2f^2-6defg+d^2g^2)\right)(f+gx)^{3/2}}{3g^5} - \frac{4ce(2ef-dg)(f+gx)^{5/2}}{5g^5} + \frac{2ce^2(f+gx)^{7/2}}{7g^5}$$

output
```
2/3*(a*e^2*g^2+c*(d^2*g^2-6*d*e*f*g+6*e^2*f^2))*(g*x+f)^(3/2)/g^5-4/5*c*e*
(-d*g+2*e*f)*(g*x+f)^(5/2)/g^5+2/7*c*e^2*(g*x+f)^(7/2)/g^5-2*(-d*g+e*f)^2*
(a*g^2+c*f^2)/g^5/(g*x+f)^(1/2)-4*(-d*g+e*f)*(a*e*g^2+c*f*(-d*g+2*e*f))*(g
*x+f)^(1/2)/g^5
```

3.597.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^2 (a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{-70ag^2(3d^2g^2-6deg(2f+gx)+e^2(8f^2+4fgx-g^2x^2))+2c(35d^2g^2(-8f^2-\cdots}{\cdots}$$

input
```
Integrate[((d + e*x)^2*(a + c*x^2))/(f + g*x)^(3/2),x]
```

output
```
(-70*a*g^2*(3*d^2*g^2 - 6*d*e*g*(2*f + g*x) + e^2*(8*f^2 + 4*f*g*x - g^2*x
^2)) + 2*c*(35*d^2*g^2*(-8*f^2 - 4*f*g*x + g^2*x^2) + 42*d*e*g*(16*f^3 + 8
*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3) - 3*e^2*(128*f^4 + 64*f^3*g*x - 16*f^2*g
^2*x^2 + 8*f*g^3*x^3 - 5*g^4*x^4)))/(105*g^5*Sqrt[f + g*x])
```

3.597.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(d+ex)^2}{(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{\sqrt{f+gx}(ae^2g^2+c(d^2g^2-6defg+6e^2f^2))}{g^4}+\frac{(ag^2+cf^2)(dg-ef)^2}{g^4(f+gx)^{3/2}}+\frac{2(ef-dg)(-aeg^2-cf(2ef-dg)\cdots}{g^4\sqrt{f+gx}}\cdots\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(f+gx)^{3/2}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{3g^5}-\frac{2\left(ag^2+cf^2\right)(ef-dg)^2}{g^5\sqrt{f+gx}}-\frac{4\sqrt{f+gx}(ef-dg)\left(aeg^2+cf(2ef-dg)\right)}{g^5}-\frac{4ce(f+gx)^{5/2}(2ef-dg)}{5g^5}+\frac{2ce^2(f+gx)^{7/2}}{7g^5}$$

input
```
Int[((d + e*x)^2*(a + c*x^2))/(f + g*x)^(3/2),x]
```

output
```
(-2*(e*f - d*g)^2*(c*f^2 + a*g^2))/(g^5*Sqrt[f + g*x]) - (4*(e*f - d*g)*(a
*e*g^2 + c*f*(2*e*f - d*g))*Sqrt[f + g*x])/g^5 + (2*(a*e^2*g^2 + c*(6*e^2*
f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^5) - (4*c*e*(2*e*f - d*g
)*(f + g*x)^(5/2))/(5*g^5) + (2*c*e^2*(f + g*x)^(7/2))/(7*g^5)
```

3.597.3.1 Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.597.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$-\frac{2\left(\left(\left(-\frac{x^{2}\left(\frac{3c\,x^{2}}{7}+a\right)e^{2}}{3}-2\left(\frac{c\,x^{2}}{5}+a\right)xde+d^{2}\left(-\frac{c\,x^{2}}{3}+a\right)\right)g^{4}-4\left(-\frac{x\left(\frac{6c\,x^{2}}{35}+a\right)e^{2}}{3}+d\left(-\frac{c\,x^{2}}{5}+a\right)e-\frac{c\,d^{2}x}{3}\right)f\,g^{3}+\frac{8f^{2}}{\cdots}\right.\right.}{\sqrt{gx+f}\,g^{5}}$
risch	$\frac{2(15c\,e^{2}x^{3}g^{3}+42cde\,g^{3}x^{2}-39c\,e^{2}f\,g^{2}x^{2}+35a\,e^{2}g^{3}x+35c\,d^{2}g^{3}x-126cdef\,g^{2}x+87c\,e^{2}f^{2}gx+210ade\,g^{3}-175a\,e^{2}f\,g^{2}-1\cdots}{105g^{5}}$
gosper	$-\frac{2(-15c\,e^{2}x^{4}g^{4}-42cde\,g^{4}x^{3}+24c\,e^{2}f\,g^{3}x^{3}-35a\,e^{2}g^{4}x^{2}-35c\,d^{2}g^{4}x^{2}+84cdef\,g^{3}x^{2}-48c\,e^{2}f^{2}g^{2}x^{2}-210ade\,g^{4}x+140a\cdots}{\cdots}$
trager	$-\frac{2(-15c\,e^{2}x^{4}g^{4}-42cde\,g^{4}x^{3}+24c\,e^{2}f\,g^{3}x^{3}-35a\,e^{2}g^{4}x^{2}-35c\,d^{2}g^{4}x^{2}+84cdef\,g^{3}x^{2}-48c\,e^{2}f^{2}g^{2}x^{2}-210ade\,g^{4}x+140a\cdots}{\cdots}$
derivativedivides	$\frac{\frac{2c\,e^{2}(gx+f)^{\frac{7}{2}}}{7}+\frac{4cdeg(gx+f)^{\frac{5}{2}}}{5}-\frac{8c\,e^{2}f(gx+f)^{\frac{5}{2}}}{5}+\frac{2a\,e^{2}g^{2}(gx+f)^{\frac{3}{2}}}{3}+\frac{2c\,d^{2}g^{2}(gx+f)^{\frac{3}{2}}}{3}-4cdefg(gx+f)^{\frac{3}{2}}+4c\,e^{2}f^{2}(gx+f)^{\frac{3}{2}}-\cdots}{\cdots}$
default	$\frac{\frac{2c\,e^{2}(gx+f)^{\frac{7}{2}}}{7}+\frac{4cdeg(gx+f)^{\frac{5}{2}}}{5}-\frac{8c\,e^{2}f(gx+f)^{\frac{5}{2}}}{5}+\frac{2a\,e^{2}g^{2}(gx+f)^{\frac{3}{2}}}{3}+\frac{2c\,d^{2}g^{2}(gx+f)^{\frac{3}{2}}}{3}-4cdefg(gx+f)^{\frac{3}{2}}+4c\,e^{2}f^{2}(gx+f)^{\frac{3}{2}}-\cdots}{\cdots}$

input
```
int((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(g*x+f)^(1/2)*((-1/3*x^2*(3/7*c*x^2+a)*e^2-2*(1/5*c*x^2+a)*x*d*e+d^2*(-
1/3*c*x^2+a))*g^4-4*(-1/3*x*(6/35*c*x^2+a)*e^2+d*(-1/5*c*x^2+a)*e-1/3*c*d^
2*x)*f*g^3+8/3*f^2*((-6/35*c*x^2+a)*e^2-6/5*c*d*e*x+c*d^2)*g^2-32/5*e*(-2/
7*e*x+d)*c*f^3*g+128/35*c*e^2*f^4)/g^5
```

3.597.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex)^2 (a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\left(15\,ce^2g^4x^4 - 384\,ce^2f^4 + 672\,cdef^3g + 420\,adefg^3 - 105\,ad^2g^4 - 280\,(cd^2 + \right.}{}$$

input
```
integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
2/105*(15*c*e^2*g^4*x^4 - 384*c*e^2*f^4 + 672*c*d*e*f^3*g + 420*a*d*e*f*g^
3 - 105*a*d^2*g^4 - 280*(c*d^2 + a*e^2)*f^2*g^2 - 6*(4*c*e^2*f*g^3 - 7*c*d
*e*g^4)*x^3 + (48*c*e^2*f^2*g^2 - 84*c*d*e*f*g^3 + 35*(c*d^2 + a*e^2)*g^4)
*x^2 - 2*(96*c*e^2*f^3*g - 168*c*d*e*f^2*g^2 - 105*a*d*e*g^4 + 70*(c*d^2 +
 a*e^2)*f*g^3)*x)*sqrt(g*x + f)/(g^6*x + f*g^5)
```

3.597.6 Sympy [A] (verification not implemented)

Time = 4.82 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)^2 (a+cx^2)}{(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{ce^2(f+gx)^{\frac{7}{2}}}{7g^4} + \frac{(f+gx)^{\frac{5}{2}}\cdot(2cdeg-4ce^2f)}{5g^4} + \frac{(f+gx)^{\frac{3}{2}}\left(ae^2g^2+cd^2g^2-6cdefg+6ce^2f^2\right)}{3g^4} + \frac{\sqrt{f+gx}\left(2adeg^3-2ae^2f\right.}{}\right.}{g} \\ \frac{ad^2x+adex^2+\frac{cdex^4}{2}+\frac{ce^2x^5}{5}+\frac{x^3\left(ae^2+cd^2\right)}{3}}{f^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((e*x+d)**2*(c*x**2+a)/(g*x+f)**(3/2),x)
```

output
```
Piecewise((2*(c*e**2*(f + g*x)**(7/2)/(7*g**4) + (f + g*x)**(5/2)*(2*c*d*e
*g - 4*c*e**2*f)/(5*g**4) + (f + g*x)**(3/2)*(a*e**2*g**2 + c*d**2*g**2 -
6*c*d*e*f*g + 6*c*e**2*f**2)/(3*g**4) + sqrt(f + g*x)*(2*a*d*e*g**3 - 2*a*
e**2*f*g**2 - 2*c*d**2*f*g**2 + 6*c*d*e*f**2*g - 4*c*e**2*f**3)/g**4 - (a*
g**2 + c*f**2)*(d*g - e*f)**2/(g**4*sqrt(f + g*x)))/g, Ne(g, 0)), ((a*d**2
*x + a*d*e*x**2 + c*d*e*x**4/2 + c*e**2*x**5/5 + x**3*(a*e**2 + c*d**2)/3)
/f**(3/2), True))
```

3.597.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^2\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\left(\frac{15\,(gx+f)^{\frac{7}{2}}ce^2-42\,(2\,ce^2f-cdeg)(gx+f)^{\frac{5}{2}}+35\,(6\,ce^2f^2-6\,cdefg+(cd^2+ae^2)g^2)(gx+f)^{\frac{3}{2}}-210\,(2\,ce^2 \cdots}{g^4}\right.}{10\cdots}$$

input
```
integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
2/105*((15*(g*x + f)^(7/2)*c*e^2 - 42*(2*c*e^2*f - c*d*e*g)*(g*x + f)^(5/2
) + 35*(6*c*e^2*f^2 - 6*c*d*e*f*g + (c*d^2 + a*e^2)*g^2)*(g*x + f)^(3/2) -
 210*(2*c*e^2*f^3 - 3*c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 + a*e^2)*f*g^2)*sqr
t(g*x + f))/g^4 - 105*(c*e^2*f^4 - 2*c*d*e*f^3*g - 2*a*d*e*f*g^3 + a*d^2*g
^4 + (c*d^2 + a*e^2)*f^2*g^2)/(sqrt(g*x + f)*g^4))/g
```

3.597.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)^2\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = -\frac{2\,(ce^2f^4-2\,cdef^3g+cd^2f^2g^2+ae^2f^2g^2-2\,adefg^3+ad^2g^4)}{\sqrt{gx+f}\,g^5} + \frac{2\left(15\,(gx+f)^{\frac{7}{2}}ce^2g^{30}-84\,(gx+f)^{\frac{5}{2}}ce^2fg^{30}+210\,(gx+f)^{\frac{3}{2}}ce^2f^2g^{30}-420\,\sqrt{gx+f}\,ce^2f^3g^{30}+42\,(gx+\cdots\right.}{\cdots}$$

input
```
integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
-2*(c*e^2*f^4 - 2*c*d*e*f^3*g + c*d^2*f^2*g^2 + a*e^2*f^2*g^2 - 2*a*d*e*f*
g^3 + a*d^2*g^4)/(sqrt(g*x + f)*g^5) + 2/105*(15*(g*x + f)^(7/2)*c*e^2*g^3
0 - 84*(g*x + f)^(5/2)*c*e^2*f*g^30 + 210*(g*x + f)^(3/2)*c*e^2*f^2*g^30 -
 420*sqrt(g*x + f)*c*e^2*f^3*g^30 + 42*(g*x + f)^(5/2)*c*d*e*g^31 - 210*(g
*x + f)^(3/2)*c*d*e*f*g^31 + 630*sqrt(g*x + f)*c*d*e*f^2*g^31 + 35*(g*x +
f)^(3/2)*c*d^2*g^32 + 35*(g*x + f)^(3/2)*a*e^2*g^32 - 210*sqrt(g*x + f)*c*
d^2*f*g^32 - 210*sqrt(g*x + f)*a*e^2*f*g^32 + 210*sqrt(g*x + f)*a*d*e*g^33
)/g^35
```

3.597.9 Mupad [B] (verification not implemented)

Time = 11.82 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)^2 (a+cx^2)}{(f+gx)^{3/2}} \, dx = \frac{(f+g\,x)^{3/2}\,(2\,c\,d^2\,g^2-12\,c\,d\,e\,f\,g+12\,c\,e^2\,f^2+2\,a\,e^2\,g^2)}{3\,g^5} - \frac{2\,c\,d^2\,f^2\,g^2+2\,a\,d^2\,g^4-4\,c\,d\,e\,f^3\,g-4\,a\,d\,e\,f\,g^3+2\,c\,e^2\,f^4+2\,a\,e^2\,f^2\,g^2}{g^5\,\sqrt{f+g\,x}} + \frac{4\,\sqrt{f+g\,x}\,(d\,g-e\,f)\,(2\,c\,e\,f^2-c\,d\,f\,g+a\,e\,g^2)}{g^5} + \frac{2\,c\,e^2\,(f+g\,x)^{7/2}}{7\,g^5} + \frac{4\,c\,e\,(f+g\,x)^{5/2}\,(d\,g-2\,e\,f)}{5\,g^5}$$

input
```
int(((a + c*x^2)*(d + e*x)^2)/(f + g*x)^(3/2),x)
```

output
```
((f + g*x)^(3/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 - 12*c*d*e*f*g)
)/(3*g^5) - (2*a*d^2*g^4 + 2*c*e^2*f^4 + 2*a*e^2*f^2*g^2 + 2*c*d^2*f^2*g^2
 - 4*a*d*e*f*g^3 - 4*c*d*e*f^3*g)/(g^5*(f + g*x)^(1/2)) + (4*(f + g*x)^(1/
2)*(d*g - e*f)*(a*e*g^2 + 2*c*e*f^2 - c*d*f*g))/g^5 + (2*c*e^2*(f + g*x)^(
7/2))/(7*g^5) + (4*c*e*(f + g*x)^(5/2)*(d*g - 2*e*f))/(5*g^5)
```

3.598 $\int \frac{(d+ex)(a+cx^2)}{(f+gx)^{3/2}}\,dx$

3.598.1 Optimal result

Integrand size = 22, antiderivative size = 111

$$\int \frac{(d+ex)\left(a+cx^2\right)}{(f+gx)^{3/2}}\,dx=\frac{2(ef-dg)\left(cf^2+ag^2\right)}{g^4\sqrt{f+gx}}+\frac{2\left(aeg^2+cf(3ef-2dg)\right)\sqrt{f+gx}}{g^4}-\frac{2c(3ef-dg)(f+gx)^{3/2}}{3g^4}+\frac{2ce(f+gx)^{5/2}}{5g^4}$$

output
```
-2/3*c*(-d*g+3*e*f)*(g*x+f)^(3/2)/g^4+2/5*c*e*(g*x+f)^(5/2)/g^4+2*(-d*g+e*
f)*(a*g^2+c*f^2)/g^4/(g*x+f)^(1/2)+2*(a*e*g^2+c*f*(-2*d*g+3*e*f))*(g*x+f)^
(1/2)/g^4
```

3.598.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)\left(a+cx^2\right)}{(f+gx)^{3/2}}\,dx=\frac{30ag^2(2ef-dg+egx)+10cdg\left(-8f^2-4fgx+g^2x^2\right)+6ce\left(16f^3+8f^2gx-2\right.}{15g^4\sqrt{f+gx}}$$

input
```
Integrate[((d + e*x)*(a + c*x^2))/(f + g*x)^(3/2),x]
```

output
```
(30*a*g^2*(2*e*f - d*g + e*g*x) + 10*c*d*g*(-8*f^2 - 4*f*g*x + g^2*x^2) +
6*c*e*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3))/(15*g^4*Sqrt[f + g*x])
```

3.598.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(d+ex)}{(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{(ag^2+cf^2)(dg-ef)}{g^3(f+gx)^{3/2}}+\frac{aeg^2+cf(3ef-2dg)}{g^3\sqrt{f+gx}}+\frac{c\sqrt{f+gx}(dg-3ef)}{g^3}+\frac{ce(f+gx)^{3/2}}{g^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(ag^2+cf^2)(ef-dg)}{g^4\sqrt{f+gx}}+\frac{2\sqrt{f+gx}(aeg^2+cf(3ef-2dg))}{g^4}-\frac{2c(f+gx)^{3/2}(3ef-dg)}{3g^4}+\frac{2ce(f+gx)^{5/2}}{5g^4}$$

input
```
Int[((d + e*x)*(a + c*x^2))/(f + g*x)^(3/2),x]
```

output
```
(2*(e*f - d*g)*(c*f^2 + a*g^2))/(g^4*Sqrt[f + g*x]) + (2*(a*e*g^2 + c*f*(3
*e*f - 2*d*g))*Sqrt[f + g*x])/g^4 - (2*c*(3*e*f - d*g)*(f + g*x)^(3/2))/(3
*g^4) + (2*c*e*(f + g*x)^(5/2))/(5*g^4)
```

3.598.3.1 Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.598.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$\frac{\left(\left(6 e \,x^{3}+10 d \,x^{2}\right) c -30 a \left(-e x +d \right)\right) g^{3}+60 \left(\left(-\frac{1}{5} e \,x^{2}-\frac{2}{3} d x \right) c +a e \right) f \,g^{2}-80 c \,f^{2} \left(-\frac{3 e x}{5}+d \right) g +96 c e \,f^{3}}{15 \sqrt{g x +f}\, g^{4}}$
gosper	$-\frac{2 \left(-3 c e \,x^{3} g^{3}-5 c d \,g^{3} x^{2}+6 c e f \,g^{2} x^{2}-15 a e \,g^{3} x +20 c d f \,g^{2} x -24 c e \,f^{2} g x +15 a d \,g^{3}-30 a e f \,g^{2}+40 c d \,f^{2} g -48 c e \,f^{3}\right)}{15 \sqrt{g x +f}\, g^{4}}$
trager	$-\frac{2 \left(-3 c e \,x^{3} g^{3}-5 c d \,g^{3} x^{2}+6 c e f \,g^{2} x^{2}-15 a e \,g^{3} x +20 c d f \,g^{2} x -24 c e \,f^{2} g x +15 a d \,g^{3}-30 a e f \,g^{2}+40 c d \,f^{2} g -48 c e \,f^{3}\right)}{15 \sqrt{g x +f}\, g^{4}}$
risch	$\frac{2 \left(3 c e \,x^{2} g^{2}+5 c d x \,g^{2}-9 c e f g x +15 a e \,g^{2}-25 c d f g +33 c e \,f^{2}\right) \sqrt{g x +f}}{15 g^{4}}-\frac{2 \left(a d \,g^{3}-a e f \,g^{2}+c d \,f^{2} g -c e \,f^{3}\right)}{g^{4} \sqrt{g x +f}}$
derivativedivides	$\frac{\frac{2 c e \left(g x +f \right)^{\frac{5}{2}}}{5}+\frac{2 c d g \left(g x +f \right)^{\frac{3}{2}}}{3}-2 c e f \left(g x +f \right)^{\frac{3}{2}}+2 a e \,g^{2} \sqrt{g x +f}-4 c d f g \sqrt{g x +f}+6 c e \,f^{2} \sqrt{g x +f}-\frac{2 \left(a d \,g^{3}-a e f \,g^{2}+c d \,f^{2} g -c e \,f^{3}\right)}{\sqrt{g x +f}}}{g^{4}}$
default	$\frac{\frac{2 c e \left(g x +f \right)^{\frac{5}{2}}}{5}+\frac{2 c d g \left(g x +f \right)^{\frac{3}{2}}}{3}-2 c e f \left(g x +f \right)^{\frac{3}{2}}+2 a e \,g^{2} \sqrt{g x +f}-4 c d f g \sqrt{g x +f}+6 c e \,f^{2} \sqrt{g x +f}-\frac{2 \left(a d \,g^{3}-a e f \,g^{2}+c d \,f^{2} g -c e \,f^{3}\right)}{\sqrt{g x +f}}}{g^{4}}$

input
```
int((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(((6*e*x^3+10*d*x^2)*c-30*a*(-e*x+d))*g^3+60*((-1/5*e*x^2-2/3*d*x)*c+
a*e)*f*g^2-80*c*f^2*(-3/5*e*x+d)*g+96*c*e*f^3)/(g*x+f)^(1/2)/g^4
```

3.598.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)\left(a+cx^2\right)}{(f+gx)^{3/2}}\,dx = \frac{2\left(3\,ceg^3x^3+48\,cef^3-40\,cdf^2g+30\,aefg^2-15\,adg^3-(6\,cefg^2-5\,cdg^3)x^2+\right.}{15\left(g^5x+fg^4\right)}$$

input
```
integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
2/15*(3*c*e*g^3*x^3 + 48*c*e*f^3 - 40*c*d*f^2*g + 30*a*e*f*g^2 - 15*a*d*g^
3 - (6*c*e*f*g^2 - 5*c*d*g^3)*x^2 + (24*c*e*f^2*g - 20*c*d*f*g^2 + 15*a*e*
g^3)*x)*sqrt(g*x + f)/(g^5*x + f*g^4)
```

3.598.6 Sympy [A] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{ce(f+gx)^{\frac{5}{2}}}{5g^3}+\frac{(f+gx)^{\frac{3}{2}}(cdg-3cef)}{3g^3}+\frac{\sqrt{f+gx}\left(aeg^2-2cdfg+3cef^2\right)}{g^3}-\frac{\left(ag^2+cf^2\right)(dg-ef)}{g^3\sqrt{f+gx}}\right)}{g} & \text{for } g\neq 0 \\ \frac{adx+\frac{aex^2}{2}+\frac{cdx^3}{3}+\frac{cex^4}{4}}{f^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((e*x+d)*(c*x**2+a)/(g*x+f)**(3/2),x)
```

output
```
Piecewise((2*(c*e*(f + g*x)**(5/2)/(5*g**3) + (f + g*x)**(3/2)*(c*d*g - 3*
c*e*f)/(3*g**3) + sqrt(f + g*x)*(a*e*g**2 - 2*c*d*f*g + 3*c*e*f**2)/g**3 -
 (a*g**2 + c*f**2)*(d*g - e*f)/(g**3*sqrt(f + g*x)))/g, Ne(g, 0)), ((a*d*x
 + a*e*x**2/2 + c*d*x**3/3 + c*e*x**4/4)/f**(3/2), True))
```

3.598.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\left(\frac{3\,(gx+f)^{\frac{5}{2}}ce-5\,(3\,cef-cdg)(gx+f)^{\frac{3}{2}}+15\,(3\,cef^2-2\,cdfg+aeg^2)\sqrt{gx+f}}{g^3}+\frac{15\,(cef^3-cdf^2g+aefg^2-ad}{\sqrt{gx+f}\,g^3}\right.}{15\,g}$$

input
```
integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
2/15*((3*(g*x + f)^(5/2)*c*e - 5*(3*c*e*f - c*d*g)*(g*x + f)^(3/2) + 15*(3
*c*e*f^2 - 2*c*d*f*g + a*e*g^2)*sqrt(g*x + f))/g^3 + 15*(c*e*f^3 - c*d*f^2
*g + a*e*f*g^2 - a*d*g^3)/(sqrt(g*x + f)*g^3))/g
```

3.598.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)\left(a+cx^2\right)}{(f+gx)^{3/2}}\,dx = \frac{2\left(cef^3-cdf^2g+aefg^2-adg^3\right)}{\sqrt{gx+f}g^4} + \frac{2\left(3\left(gx+f\right)^{\frac{5}{2}}ceg^{16}-15\left(gx+f\right)^{\frac{3}{2}}cefg^{16}+45\sqrt{gx+f}cef^2g^{16}+5\left(gx+f\right)^{\frac{3}{2}}cdg^{17}-30\sqrt{gx+f}cdfg^{17}\right.}{15\,g^{20}}$$

input
```
integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
2*(c*e*f^3 - c*d*f^2*g + a*e*f*g^2 - a*d*g^3)/(sqrt(g*x + f)*g^4) + 2/15*(
3*(g*x + f)^(5/2)*c*e*g^16 - 15*(g*x + f)^(3/2)*c*e*f*g^16 + 45*sqrt(g*x +
 f)*c*e*f^2*g^16 + 5*(g*x + f)^(3/2)*c*d*g^17 - 30*sqrt(g*x + f)*c*d*f*g^1
7 + 15*sqrt(g*x + f)*a*e*g^18)/g^20
```

3.598.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)\left(a+cx^2\right)}{(f+gx)^{3/2}}\,dx = \frac{\sqrt{f+g\,x}\left(6\,c\,e\,f^2-4\,c\,d\,f\,g+2\,a\,e\,g^2\right)}{g^4} - \frac{-2\,c\,e\,f^3+2\,c\,d\,f^2\,g-2\,a\,e\,f\,g^2+2\,a\,d\,g^3}{g^4\sqrt{f+g\,x}} + \frac{2\,c\,e\,{\left(f+g\,x\right)}^{5/2}}{5\,g^4} + \frac{2\,c\,{\left(f+g\,x\right)}^{3/2}\left(d\,g-3\,e\,f\right)}{3\,g^4}$$

input
```
int(((a + c*x^2)*(d + e*x))/(f + g*x)^(3/2),x)
```

output
```
((f + g*x)^(1/2)*(2*a*e*g^2 + 6*c*e*f^2 - 4*c*d*f*g))/g^4 - (2*a*d*g^3 - 2
*c*e*f^3 - 2*a*e*f*g^2 + 2*c*d*f^2*g)/(g^4*(f + g*x)^(1/2)) + (2*c*e*(f +
g*x)^(5/2))/(5*g^4) + (2*c*(f + g*x)^(3/2)*(d*g - 3*e*f))/(3*g^4)
```

3.599 $\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx$

3.599.1 Optimal result

Integrand size = 17, antiderivative size = 59

$$\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx=-\frac{2(cf^2+ag^2)}{g^3\sqrt{f+gx}}-\frac{4cf\sqrt{f+gx}}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}$$

output
```
2/3*c*(g*x+f)^(3/2)/g^3-2*(a*g^2+c*f^2)/g^3/(g*x+f)^(1/2)-4*c*f*(g*x+f)^(1
/2)/g^3
```

3.599.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx=\frac{2(-3ag^2+c(-8f^2-4fgx+g^2x^2))}{3g^3\sqrt{f+gx}}$$

input
```
Integrate[(a + c*x^2)/(f + g*x)^(3/2),x]
```

output
```
(2*(-3*a*g^2 + c*(-8*f^2 - 4*f*g*x + g^2*x^2)))/(3*g^3*Sqrt[f + g*x])
```

3.599.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{ag^2+cf^2}{g^2(f+gx)^{3/2}}+\frac{c\sqrt{f+gx}}{g^2}-\frac{2cf}{g^2\sqrt{f+gx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(ag^2+cf^2\right)}{g^3\sqrt{f+gx}}+\frac{2c(f+gx)^{3/2}}{3g^3}-\frac{4cf\sqrt{f+gx}}{g^3}$$

input
```
Int[(a + c*x^2)/(f + g*x)^(3/2),x]
```

output
```
(-2*(c*f^2 + a*g^2))/(g^3*Sqrt[f + g*x]) - (4*c*f*Sqrt[f + g*x])/g^3 + (2*
c*(f + g*x)^(3/2))/(3*g^3)
```

3.599.3.1 Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.599.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

method	result	size
pseudoelliptic	$\frac{\frac{2\left(c\,x^{2}-3 a\right) g^{2}}{3}-\frac{8 c f x g}{3}-\frac{16 c\,f^{2}}{3}}{\sqrt{g x +f}\, g^{3}}$	39
gosper	$-\frac{2\left(-c\,x^{2} g^{2}+4 c f x g +3 a\,g^{2}+8 c\,f^{2}\right)}{3 \sqrt{g x +f}\, g^{3}}$	41
trager	$-\frac{2\left(-c\,x^{2} g^{2}+4 c f x g +3 a\,g^{2}+8 c\,f^{2}\right)}{3 \sqrt{g x +f}\, g^{3}}$	41
risch	$-\frac{2 c \left(-g x +5 f \right) \sqrt{g x +f}}{3 g^{3}}-\frac{2\left(a\,g^{2}+c\,f^{2}\right)}{g^{3} \sqrt{g x +f}}$	46
derivativedivides	$\frac{\frac{2 c \left(g x +f \right)^{\frac{3}{2}}}{3}-4 c f \sqrt{g x +f}-\frac{2\left(a\,g^{2}+c\,f^{2}\right)}{\sqrt{g x +f}}}{g^{3}}$	48
default	$\frac{\frac{2 c \left(g x +f \right)^{\frac{3}{2}}}{3}-4 c f \sqrt{g x +f}-\frac{2\left(a\,g^{2}+c\,f^{2}\right)}{\sqrt{g x +f}}}{g^{3}}$	48

input
```
int((c*x^2+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*((c*x^2-3*a)*g^2-4*c*f*x*g-8*c*f^2)/(g*x+f)^(1/2)/g^3
```

3.599.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx = \frac{2\left(cg^2x^2-4\,cfgx-8\,cf^2-3\,ag^2\right)\sqrt{gx+f}}{3\left(g^4x+fg^3\right)}$$

input
```
integrate((c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(c*g^2*x^2 - 4*c*f*g*x - 8*c*f^2 - 3*a*g^2)*sqrt(g*x + f)/(g^4*x + f*g
^3)
```

3.599.6 Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int \frac{a + cx^2}{(f + gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(-\frac{2cf\sqrt{f+gx}}{g^2} + \frac{c(f+gx)^{\frac{3}{2}}}{3g^2} - \frac{ag^2+cf^2}{g^2\sqrt{f+gx}}\right)}{g} & \text{for } g \neq 0 \\ \frac{ax+\frac{cx^3}{3}}{f^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+a)/(g*x+f)**(3/2),x)
```

output
```
Piecewise((2*(-2*c*f*sqrt(f + g*x)/g**2 + c*(f + g*x)**(3/2)/(3*g**2) - (a
*g**2 + c*f**2)/(g**2*sqrt(f + g*x)))/g, Ne(g, 0)), ((a*x + c*x**3/3)/f**(
3/2), True))
```

3.599.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{a + cx^2}{(f + gx)^{3/2}}\,dx = \frac{2\left(\frac{(gx+f)^{\frac{3}{2}}c - 6\sqrt{gx+f}cf}{g^2} - \frac{3(cf^2+ag^2)}{\sqrt{gx+f}g^2}\right)}{3g}$$

input
```
integrate((c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
2/3*(((g*x + f)^(3/2)*c - 6*sqrt(g*x + f)*c*f)/g^2 - 3*(c*f^2 + a*g^2)/(sq
rt(g*x + f)*g^2))/g
```

3.599.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{a + cx^2}{(f + gx)^{3/2}}\,dx = -\frac{2(cf^2 + ag^2)}{\sqrt{gx+f}g^3} + \frac{2\left((gx+f)^{\frac{3}{2}}cg^6 - 6\sqrt{gx+f}cfg^6\right)}{3g^9}$$

input
```
integrate((c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
-2*(c*f^2 + a*g^2)/(sqrt(g*x + f)*g^3) + 2/3*((g*x + f)^(3/2)*c*g^6 - 6*sq
rt(g*x + f)*c*f*g^6)/g^9
```

3.599.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx = -\frac{6\,a\,g^2-2\,c\,(f+g\,x)^2+6\,c\,f^2+12\,c\,f\,(f+g\,x)}{3\,g^3\,\sqrt{f+g\,x}}$$

input
```
int((a + c*x^2)/(f + g*x)^(3/2),x)
```

output
```
-(6*a*g^2 - 2*c*(f + g*x)^2 + 6*c*f^2 + 12*c*f*(f + g*x))/(3*g^3*(f + g*x)
^(1/2))
```

3.600 $\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx$

3.600.1 Optimal result

Integrand size = 24, antiderivative size = 112

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2(cf^2+ag^2)}{g^2(ef-dg)\sqrt{f+gx}} + \frac{2c\sqrt{f+gx}}{eg^2} - \frac{2(cd^2+ae^2)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}$$

output
```
-2*(a*e^2+c*d^2)*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(3/2)/(
-d*g+e*f)^(3/2)+2*(a*g^2+c*f^2)/g^2/(-d*g+e*f)/(g*x+f)^(1/2)+2*c*(g*x+f)^(
1/2)/e/g^2
```

3.600.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = -\frac{2(aeg^2-cdg(f+gx)+cef(2f+gx))}{eg^2(-ef+dg)\sqrt{f+gx}} - \frac{2(cd^2+ae^2)\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{3/2}(-ef+dg)^{3/2}}$$

input
```
Integrate[(a + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

output
```
(-2*(a*e*g^2 - c*d*g*(f + g*x) + c*e*f*(2*f + g*x)))/(e*g^2*(-(e*f) + d*g)
*Sqrt[f + g*x]) - (2*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-
(e*f) + d*g]])/(e^(3/2)*(-(e*f) + d*g)^(3/2))
```

3.600.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {649, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \text{649}$$

$$\frac{2\int -\frac{cf^2-2c(f+gx)f+ag^2+c(f+gx)^2}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\int \frac{cf^2-2c(f+gx)f+ag^2+c(f+gx)^2}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \text{1584}$$

$$-\frac{2\int \left(\frac{(cd^2+ae^2)g^2}{e(ef-dg)(ef-dg-e(f+gx))}-\frac{c}{e}+\frac{cf^2+ag^2}{(ef-dg)(f+gx)}\right)d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\frac{g^2(ae^2+cd^2)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}+\frac{ag^2+cf^2}{\sqrt{f+gx}(ef-dg)}+\frac{c\sqrt{f+gx}}{e}\right)}{g^2}$$

input
```
Int[(a + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

output
```
(2*((c*f^2 + a*g^2)/((e*f - d*g)*Sqrt[f + g*x]) + (c*Sqrt[f + g*x])/e - ((
c*d^2 + a*e^2)*g^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(3
/2)*(e*f - d*g)^(3/2))))/g^2
```

3.600.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.600.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{2c\sqrt{gx+f}}{e}-\frac{2g^{2}\left(e^{2}a+c\,d^{2}\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)e\sqrt{(dg-ef)e}}-\frac{2\left(a\,g^{2}+c\,f^{2}\right)}{(dg-ef)\sqrt{gx+f}}}{g^{2}}$	112
default	$\frac{\frac{2c\sqrt{gx+f}}{e}-\frac{2g^{2}\left(e^{2}a+c\,d^{2}\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)e\sqrt{(dg-ef)e}}-\frac{2\left(a\,g^{2}+c\,f^{2}\right)}{(dg-ef)\sqrt{gx+f}}}{g^{2}}$	112
pseudoelliptic	$\frac{\frac{2c\sqrt{gx+f}}{e}-\frac{2g^{2}\left(e^{2}a+c\,d^{2}\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)e\sqrt{(dg-ef)e}}-\frac{2\left(a\,g^{2}+c\,f^{2}\right)}{(dg-ef)\sqrt{gx+f}}}{g^{2}}$	112
risch	$\frac{2c\sqrt{gx+f}}{e\,g^{2}}-\frac{2\left(\frac{g^{2}\left(e^{2}a+c\,d^{2}\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)\sqrt{(dg-ef)e}}+\frac{\left(a\,g^{2}+c\,f^{2}\right)e}{(dg-ef)\sqrt{gx+f}}\right)}{g^{2}e}$	116

input
```
int((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/g^2*(c/e*(g*x+f)^(1/2)-g^2*(a*e^2+c*d^2)/(d*g-e*f)/e/((d*g-e*f)*e)^(1/2)
*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))-(a*g^2+c*f^2)/(d*g-e*f)/(g*x+
f)^(1/2))
```

3.600.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(98) = 196$.

Time = 0.28 (sec) , antiderivative size = 492, normalized size of antiderivative = 4.39

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \left[-\frac{((cd^2+ae^2)g^3x+(cd^2+ae^2)fg^2)\sqrt{e^2f-deg}\log\left(\frac{egx+2\,ef-dg+2\,\sqrt{e^2f-deg}\sqrt{g\cdots}}{ex+d}\right.}{e^4f^3g^2-2\,de^3f^2g\cdots}\right.$$

input
```
integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
[-(((c*d^2 + a*e^2)*g^3*x + (c*d^2 + a*e^2)*f*g^2)*sqrt(e^2*f - d*e*g)*log
((e*g*x + 2*e*f - d*g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) -
2*(2*c*e^3*f^3 - 3*c*d*e^2*f^2*g - a*d*e^2*g^3 + (c*d^2*e + a*e^3)*f*g^2 +
 (c*e^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(g*x + f))/(e^4*f^3*
g^2 - 2*d*e^3*f^2*g^3 + d^2*e^2*f*g^4 + (e^4*f^2*g^3 - 2*d*e^3*f*g^4 + d^2
*e^2*g^5)*x), 2*(((c*d^2 + a*e^2)*g^3*x + (c*d^2 + a*e^2)*f*g^2)*sqrt(-e^2
*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*
c*e^3*f^3 - 3*c*d*e^2*f^2*g - a*d*e^2*g^3 + (c*d^2*e + a*e^3)*f*g^2 + (c*e
^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(g*x + f))/(e^4*f^3*g^2 -
 2*d*e^3*f^2*g^3 + d^2*e^2*f*g^4 + (e^4*f^2*g^3 - 2*d*e^3*f*g^4 + d^2*e^2*
g^5)*x)]
```

3.600.6 Sympy [A] (verification not implemented)

Time = 4.86 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.35

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{c\sqrt{f+gx}}{eg}-\frac{ag^2+cf^2}{g\sqrt{f+gx}(dg-ef)}-\frac{g\left(ae^2+cd^2\right)\operatorname{atan}\left(\frac{\sqrt{f+gx}}{\sqrt{\frac{dg-ef}{e}}}\right)}{e^2\sqrt{\frac{dg-ef}{e}}(dg-ef)}\right)}{g} & \text{for } g\neq 0 \\ \frac{-\frac{cdx}{e^2}+\frac{cx^2}{2e}+\frac{\left(ae^2+cd^2\right)\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{e^2}}{f^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+a)/(e*x+d)/(g*x+f)**(3/2),x)
```

output
```
Piecewise((2*(c*sqrt(f + g*x)/(e*g) - (a*g**2 + c*f**2)/(g*sqrt(f + g*x)*(
d*g - e*f)) - g*(a*e**2 + c*d**2)*atan(sqrt(f + g*x)/sqrt((d*g - e*f)/e))/
(e**2*sqrt((d*g - e*f)/e)*(d*g - e*f)))/g, Ne(g, 0)), ((-c*d*x/e**2 + c*x*
*2/(2*e) + (a*e**2 + c*d**2)*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, T
rue))/e**2)/f**(3/2), True))
```

3.600.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.600.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2\,(cd^2+ae^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{(e^2f-deg)\sqrt{-e^2f+deg}} + \frac{2\,(cf^2+ag^2)}{(efg^2-dg^3)\sqrt{gx+f}} + \frac{2\sqrt{gx+f}c}{eg^2}$$

input
```
integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
2*(c*d^2 + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/((e^2*f - d
*e*g)*sqrt(-e^2*f + d*e*g)) + 2*(c*f^2 + a*g^2)/((e*f*g^2 - d*g^3)*sqrt(g*
x + f)) + 2*sqrt(g*x + f)*c/(e*g^2)
```

3.600.9 Mupad [B] (verification not implemented)

Time = 12.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.26

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2\,\mathrm{atan}\left(\frac{2\,\sqrt{f+g\,x}\,\left(c\,d^2+a\,e^2\right)\,\left(e^2\,f-d\,e\,g\right)}{\sqrt{e}\,\left(2\,c\,d^2+2\,a\,e^2\right)\,{\left(d\,g-e\,f\right)}^{3/2}}\right)\,\left(c\,d^2+a\,e^2\right)}{e^{3/2}\,{\left(d\,g-e\,f\right)}^{3/2}} + \frac{2\,c\,\sqrt{f+g\,x}}{e\,g^2} - \frac{2\,\left(c\,e\,f^2+a\,e\,g^2\right)}{e\,g^2\,\sqrt{f+g\,x}\,\left(d\,g-e\,f\right)}$$

input
```
int((a + c*x^2)/((f + g*x)^(3/2)*(d + e*x)),x)
```

output
```
(2*atan((2*(f + g*x)^(1/2)*(a*e^2 + c*d^2)*(e^2*f - d*e*g))/(e^(1/2)*(2*a*
e^2 + 2*c*d^2)*(d*g - e*f)^(3/2)))*(a*e^2 + c*d^2))/(e^(3/2)*(d*g - e*f)^(
3/2)) + (2*c*(f + g*x)^(1/2))/(e*g^2) - (2*(a*e*g^2 + c*e*f^2))/(e*g^2*(f
+ g*x)^(1/2)*(d*g - e*f))
```

3.601 $\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx$

3.601.1 Optimal result

Integrand size = 24, antiderivative size = 144

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = -\frac{2(cf^2+ag^2)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{(cd^2+ae^2)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} + \frac{(3ae^2g+cd(4ef-dg))\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{5/2}}$$

output
```
(3*a*e^2*g+c*d*(-d*g+4*e*f))*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2
))/e^(3/2)/(-d*g+e*f)^(5/2)-2*(a*g^2+c*f^2)/g/(-d*g+e*f)^2/(g*x+f)^(1/2)-(
a*e^2+c*d^2)*(g*x+f)^(1/2)/e/(-d*g+e*f)^2/(e*x+d)
```

3.601.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \frac{-c(2def^2+2e^2f^2x+d^2g(f+gx))-aeg(2dg+e(f+3gx))}{eg(ef-dg)^2(d+ex)\sqrt{f+gx}} + \frac{(-3ae^2g+cd(-4ef+dg))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{3/2}(-ef+dg)^{5/2}}$$

input
```
Integrate[(a + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

output
```
(-(c*(2*d*e*f^2 + 2*e^2*f^2*x + d^2*g*(f + g*x))) - a*e*g*(2*d*g + e*(f +
3*g*x)))/(e*g*(e*f - d*g)^2*(d + e*x)*Sqrt[f + g*x]) + ((-3*a*e^2*g + c*d*
(-4*e*f + d*g))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(3/
2)*(-(e*f) + d*g)^(5/2))
```

3.601.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {649, 1582, 25, 27, 359, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{649}$$

$$\frac{2\int \frac{cf^2-2c(f+gx)f+ag^2+c(f+gx)^2}{(f+gx)(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{g}$$

$$\Big\downarrow \text{1582}$$

$$\frac{2\left(\frac{g^2\sqrt{f+gx}(ae^2+cd^2)}{2e(ef-dg)^2(-dg-e(f+gx)+ef)}-\frac{\int -\frac{e\left(2e(ef-dg)\left(cf^2+ag^2\right)+\left(ae^2g^2-c\left(2e^2f^2-4degf+d^2g^2\right)\right)(f+gx)\right)}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}}{2e^2(ef-dg)^2}\right)}{g}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(\frac{\int \frac{e\left(2e(ef-dg)\left(cf^2+ag^2\right)+\left(ae^2g^2-c\left(2e^2f^2-4degf+d^2g^2\right)\right)(f+gx)\right)}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}}{2e^2(ef-dg)^2}+\frac{g^2\sqrt{f+gx}(ae^2+cd^2)}{2e(ef-dg)^2(-dg-e(f+gx)+ef)}\right)}{g}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{\int \frac{2e(ef-dg)\left(cf^2+ag^2\right)+\left(ae^2g^2-c\left(2e^2f^2-4degf+d^2g^2\right)\right)(f+gx)}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}}{2e(ef-dg)^2}+\frac{g^2\sqrt{f+gx}(ae^2+cd^2)}{2e(ef-dg)^2(-dg-e(f+gx)+ef)}\right)}{g}$$

$$\Big\downarrow \text{359}$$

$$\frac{2\left(\frac{g(3ae^2g+cd(4ef-dg))\int\frac{1}{ef-dg-e(f+gx)}d\sqrt{f+gx}-\frac{2e(ag^2+cf^2)}{\sqrt{f+gx}}}{2e(ef-dg)^2}+\frac{g^2\sqrt{f+gx}(ae^2+cd^2)}{2e(ef-dg)^2(-dg-e(f+gx)+ef)}\right)}{g}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\left(\frac{\frac{g(3ae^2g+cd(4ef-dg))\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}\sqrt{ef-dg}}-\frac{2e(ag^2+cf^2)}{\sqrt{f+gx}}}{2e(ef-dg)^2}+\frac{g^2\sqrt{f+gx}(ae^2+cd^2)}{2e(ef-dg)^2(-dg-e(f+gx)+ef)}\right)}{g}$$

input
```
Int[(a + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

output
```
(2*(((c*d^2 + a*e^2)*g^2*Sqrt[f + g*x])/(2*e*(e*f - d*g)^2*(e*f - d*g - e*
(f + g*x))) + ((-2*e*(c*f^2 + a*g^2))/Sqrt[f + g*x] + (g*(3*a*e^2*g + c*d*
(4*e*f - d*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(Sqrt[e]*
Sqrt[e*f - d*g]))/(2*e*(e*f - d*g)^2)))/g
```

3.601.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

3.601.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{-\frac{2g\left(\frac{g\left(e^2a+c\,d^2\right)\sqrt{gx+f}}{2e(e(gx+f)+dg-ef)}+\frac{\left(3a\,e^2g-c\,d^2g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2e\sqrt{(dg-ef)e}}\right)}{(dg-ef)^2}-\frac{2\left(a\,g^2+c\,f^2\right)}{(dg-ef)^2\sqrt{gx+f}}}{g}$
default	$\frac{-\frac{2g\left(\frac{g\left(e^2a+c\,d^2\right)\sqrt{gx+f}}{2e(e(gx+f)+dg-ef)}+\frac{\left(3a\,e^2g-c\,d^2g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2e\sqrt{(dg-ef)e}}\right)}{(dg-ef)^2}-\frac{2\left(a\,g^2+c\,f^2\right)}{(dg-ef)^2\sqrt{gx+f}}}{g}$
pseudoelliptic	$-\frac{2\left(\frac{3\sqrt{gx+f}\,\left(a\,e^2g-\frac{1}{3}c\,d^2g+\frac{4}{3}cdef\right)(ex+d)g\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2}+\sqrt{(dg-ef)e}\,\left(\left(\frac{3}{2}a\,g^2x+c\,f^2x+\frac{1}{2}afg\right)e^2+d(a\,g^2+c\,f^2 \right.\right.}{\sqrt{(dg-ef)e}\,\sqrt{gx+f}\,g(ex+d)(dg-ef)^2e}$

input
```
int((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/g*(-g/(d*g-e*f)^2*(1/2*g*(a*e^2+c*d^2)/e*(g*x+f)^(1/2)/(e*(g*x+f)+d*g-e*
f)+1/2*(3*a*e^2*g-c*d^2*g+4*c*d*e*f)/e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f
)^(1/2)/((d*g-e*f)*e)^(1/2)))-(a*g^2+c*f^2)/(d*g-e*f)^2/(g*x+f)^(1/2))
```

3.601.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(130) = 260$.

Time = 0.32 (sec) , antiderivative size = 906, normalized size of antiderivative = 6.29

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \Bigg[\frac{(4\,cd^2ef^2g - (cd^3 - 3\,ade^2)fg^2 + (4\,cde^2fg^2 - (cd^2e - 3\,ae^3)g^3)x^2 + (4\,cde^2}{}$$

$$-\frac{(4\,cd^2ef^2g - (cd^3 - 3\,ade^2)fg^2 + (4\,cde^2fg^2 - (cd^2e - 3\,ae^3)g^3)x^2 + (4\,cde^2f^2g + 3\,(cd^2e + ae^3)fg^2 - (c}{de^5f^4g - 3\,d^2e^4f^3g^2 +}$$

input
```
integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((4*c*d^2*e*f^2*g - (c*d^3 - 3*a*d*e^2)*f*g^2 + (4*c*d*e^2*f*g^2 - (c
*d^2*e - 3*a*e^3)*g^3)*x^2 + (4*c*d*e^2*f^2*g + 3*(c*d^2*e + a*e^3)*f*g^2
- (c*d^3 - 3*a*d*e^2)*g^3)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*g
 + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*d*e^3*f^3 - 2*
a*d^2*e^2*g^3 - (c*d^2*e^2 - a*e^4)*f^2*g - (c*d^3*e - a*d*e^3)*f*g^2 + (2
*c*e^4*f^3 - 2*c*d*e^3*f^2*g + (c*d^2*e^2 + 3*a*e^4)*f*g^2 - (c*d^3*e + 3*
a*d*e^3)*g^3)*x)*sqrt(g*x + f))/(d*e^5*f^4*g - 3*d^2*e^4*f^3*g^2 + 3*d^3*e
^3*f^2*g^3 - d^4*e^2*f*g^4 + (e^6*f^3*g^2 - 3*d*e^5*f^2*g^3 + 3*d^2*e^4*f*
g^4 - d^3*e^3*g^5)*x^2 + (e^6*f^4*g - 2*d*e^5*f^3*g^2 + 2*d^3*e^3*f*g^4 -
d^4*e^2*g^5)*x), -((4*c*d^2*e*f^2*g - (c*d^3 - 3*a*d*e^2)*f*g^2 + (4*c*d*e
^2*f*g^2 - (c*d^2*e - 3*a*e^3)*g^3)*x^2 + (4*c*d*e^2*f^2*g + 3*(c*d^2*e +
a*e^3)*f*g^2 - (c*d^3 - 3*a*d*e^2)*g^3)*x)*sqrt(-e^2*f + d*e*g)*arctan(sqr
t(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*c*d*e^3*f^3 - 2*a*d^2*
e^2*g^3 - (c*d^2*e^2 - a*e^4)*f^2*g - (c*d^3*e - a*d*e^3)*f*g^2 + (2*c*e^4
*f^3 - 2*c*d*e^3*f^2*g + (c*d^2*e^2 + 3*a*e^4)*f*g^2 - (c*d^3*e + 3*a*d*e^
3)*g^3)*x)*sqrt(g*x + f))/(d*e^5*f^4*g - 3*d^2*e^4*f^3*g^2 + 3*d^3*e^3*f^2
*g^3 - d^4*e^2*f*g^4 + (e^6*f^3*g^2 - 3*d*e^5*f^2*g^3 + 3*d^2*e^4*f*g^4 -
d^3*e^3*g^5)*x^2 + (e^6*f^4*g - 2*d*e^5*f^3*g^2 + 2*d^3*e^3*f*g^4 - d^4*e^
2*g^5)*x)]
```

3.601.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)/(e*x+d)**2/(g*x+f)**(3/2),x)
```

output
```
Timed out
```

3.601.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.601.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.58

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = -\frac{(4\,cdef-cd^2g+3\,ae^2g)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{(e^3f^2-2\,de^2fg+d^2eg^2)\sqrt{-e^2f+deg}} - \frac{2\,(gx+f)ce^2f^2-2\,ce^2f^3+2\,cdef^2g+(gx+f)cd^2g^2+3\,(gx+f)ae^2g^2-2\,ae^2fg^2+2\,adeg^3}{(e^3f^2g-2\,de^2fg^2+d^2eg^3)\left((gx+f)^{\frac{3}{2}}e-\sqrt{gx+f}ef+\sqrt{gx+f}dg\right)}$$

input
```
integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
-(4*c*d*e*f - c*d^2*g + 3*a*e^2*g)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*
e*g))/((e^3*f^2 - 2*d*e^2*f*g + d^2*e*g^2)*sqrt(-e^2*f + d*e*g)) - (2*(g*x
 + f)*c*e^2*f^2 - 2*c*e^2*f^3 + 2*c*d*e*f^2*g + (g*x + f)*c*d^2*g^2 + 3*(g
*x + f)*a*e^2*g^2 - 2*a*e^2*f*g^2 + 2*a*d*e*g^3)/((e^3*f^2*g - 2*d*e^2*f*g
^2 + d^2*e*g^3)*((g*x + f)^(3/2)*e - sqrt(g*x + f)*e*f + sqrt(g*x + f)*d*g
))
```

3.601.9 Mupad [B] (verification not implemented)

Time = 12.67 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.30

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = -\frac{\frac{2\,(c\,f^2+a\,g^2)}{d\,g-e\,f}+\frac{(f+g\,x)\,(c\,d^2\,g^2+2\,c\,e^2\,f^2+3\,a\,e^2\,g^2)}{e\,(d\,g-e\,f)^2}}{\sqrt{f+g\,x}\,(d\,g^2-e\,f\,g)+e\,g\,(f+g\,x)^{3/2}} - \frac{\mathrm{atan}\left(\frac{\sqrt{f+g\,x}\,(d^2\,e\,g^2-2\,d\,e^2\,f\,g+e^3\,f^2)}{\sqrt{e}\,(d\,g-e\,f)^{5/2}}\right)\,(-c\,g\,d^2+4\,c\,f\,d\,e+3\,a\,g\,e^2)}{e^{3/2}\,(d\,g-e\,f)^{5/2}}$$

input
```
int((a + c*x^2)/((f + g*x)^(3/2)*(d + e*x)^2),x)
```

output
```
- ((2*(a*g^2 + c*f^2))/(d*g - e*f) + ((f + g*x)*(3*a*e^2*g^2 + c*d^2*g^2 +
 2*c*e^2*f^2))/(e*(d*g - e*f)^2))/((f + g*x)^(1/2)*(d*g^2 - e*f*g) + e*g*(
f + g*x)^(3/2)) - (atan(((f + g*x)^(1/2)*(e^3*f^2 + d^2*e*g^2 - 2*d*e^2*f*
g))/(e^(1/2)*(d*g - e*f)^(5/2)))*(3*a*e^2*g - c*d^2*g + 4*c*d*e*f))/(e^(3/
2)*(d*g - e*f)^(5/2))
```

3.602 $\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx$

3.602.1 Optimal result

Integrand size = 24, antiderivative size = 214

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx=\frac{2(cf^2+ag^2)}{(ef-dg)^3\sqrt{f+gx}}-\frac{(cd^2+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2}+\frac{(7ae^2g+cd(8ef-dg))\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)}-\frac{(15ae^2g^2+c(8e^2f^2+8defg-d^2g^2))\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{3/2}(ef-dg)^{7/2}}$$

output
```
-1/4*(15*a*e^2*g^2+c*(-d^2*g^2+8*d*e*f*g+8*e^2*f^2))*arctanh(e^(1/2)*(g*x+
f)^(1/2)/(-d*g+e*f)^(1/2))/e^(3/2)/(-d*g+e*f)^(7/2)+2*(a*g^2+c*f^2)/(-d*g+
e*f)^3/(g*x+f)^(1/2)-1/2*(a*e^2+c*d^2)*(g*x+f)^(1/2)/e/(-d*g+e*f)^2/(e*x+d
)^2+1/4*(7*a*e^2*g+c*d*(-d*g+8*e*f))*(g*x+f)^(1/2)/e/(-d*g+e*f)^3/(e*x+d)
```

3.602.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.07

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx=\frac{\frac{\sqrt{e}\left(c\left(8e^3f^2x^2+d^3g(f+gx)+8de^2fx(3f+gx)+d^2e\left(14f^2+5fgx-g^2x^2\right)\right)+ae\left(8d^2g^2+deg(9f+25gx)+e^2\left(-\right.\right.\right.}{(ef-dg)^3(d+ex)^2\sqrt{f+gx}}}{4e^{3/2}}$$

input
```
Integrate[(a + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

output
```
((Sqrt[e]*(c*(8*e^3*f^2*x^2 + d^3*g*(f + g*x) + 8*d*e^2*f*x*(3*f + g*x) +
d^2*e*(14*f^2 + 5*f*g*x - g^2*x^2)) + a*e*(8*d^2*g^2 + d*e*g*(9*f + 25*g*x
) + e^2*(-2*f^2 + 5*f*g*x + 15*g^2*x^2))))/((e*f - d*g)^3*(d + e*x)^2*Sqrt
[f + g*x]) - ((15*a*e^2*g^2 + c*(8*e^2*f^2 + 8*d*e*f*g - d^2*g^2))*ArcTan[
(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(-(e*f) + d*g)^(7/2))/(4*e^(3
/2))
```

3.602.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {649, 25, 1582, 25, 27, 361, 25, 359, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \text{649}$$

$$2\int -\frac{cf^2-2c(f+gx)f+ag^2+c(f+gx)^2}{(f+gx)(ef-dg-e(f+gx))^3}d\sqrt{f+gx}$$

$$\Big\downarrow \text{25}$$

$$-2\int \frac{cf^2-2c(f+gx)f+ag^2+c(f+gx)^2}{(f+gx)(ef-dg-e(f+gx))^3}d\sqrt{f+gx}$$

$$\Big\downarrow \text{1582}$$

$$2\left(\frac{\int -\frac{e(4e(ef-dg)(cf^2+ag^2)+(3ae^2g^2-c(4e^2f^2-8degf+d^2g^2))(f+gx))}{(f+gx)(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{4e^2(ef-dg)^2}-\frac{g^2\sqrt{f+gx}(ae^2+cd^2)}{4e(ef-dg)^2(-dg-e(f+gx)+ef)^2}\right.$$

$$\Big\downarrow \text{25}$$

$$2\left(-\frac{\int \frac{e(4e(ef-dg)(cf^2+ag^2)+(3ae^2g^2-c(4e^2f^2-8degf+d^2g^2))(f+gx))}{(f+gx)(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{4e^2(ef-dg)^2}-\frac{g^2\sqrt{f+gx}(ae^2+cd^2)}{4e(ef-dg)^2(-dg-e(f+gx)+ef)^2}\right.$$

$$\Big\downarrow \text{27}$$

$$2\left(-\frac{\int \frac{4e(ef-dg)(cf^2+ag^2)+(3ae^2g^2-c(4e^2f^2-8degf+d^2g^2))(f+gx)}{(f+gx)(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{4e(ef-dg)^2}-\frac{g^2\sqrt{f+gx}(ae^2+cd^2)}{4e(ef-dg)^2(-dg-e(f+gx)+ef)^2}\right)$$

$\Big\downarrow$ 361

$$2\left(-\frac{\frac{g\sqrt{f+gx}(7ae^2g+cd(8ef-dg))}{2(ef-dg)(-dg-e(f+gx)+ef)}-\frac{1}{2}\int -\frac{8e(cf^2+ag^2)+\frac{g\left(7age^2+cd(8ef-dg)\right)(f+gx)}{ef-dg}}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}}{4e(ef-dg)^2}-\frac{g^2\sqrt{f+gx}(ae^2+cd}{4e(ef-dg)^2(-dg-e(f+g}\right.$$

$\Big\downarrow$ 25

$$2\left(-\frac{\frac{1}{2}\int \frac{8e(cf^2+ag^2)+\frac{g\left(7age^2+cd(8ef-dg)\right)(f+gx)}{ef-dg}}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}+\frac{g\sqrt{f+gx}(7ae^2g+cd(8ef-dg))}{2(ef-dg)(-dg-e(f+gx)+ef)}}{4e(ef-dg)^2}-\frac{g^2\sqrt{f+gx}(ae^2+cd^2)}{4e(ef-dg)^2(-dg-e(f+gx}\right.$$

$\Big\downarrow$ 359

$$2\left(-\frac{\frac{1}{2}\left(\frac{(15ae^2g^2+c(-d^2g^2+8defg+8e^2f^2))\int \frac{1}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{ef-dg}-\frac{8e(ag^2+cf^2)}{\sqrt{f+gx}(ef-dg)}\right)+\frac{g\sqrt{f+gx}(7ae^2g+cd(8ef-dg))}{2(ef-dg)(-dg-e(f+gx)+ef)}}{4e(ef-dg)^2}-\frac{}{4e(ef}\right.$$

$\Big\downarrow$ 221

$$2\left(-\frac{\frac{1}{2}\left(\frac{(15ae^2g^2+c(-d^2g^2+8defg+8e^2f^2))\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}(ef-dg)^{3/2}}-\frac{8e(ag^2+cf^2)}{\sqrt{f+gx}(ef-dg)}\right)+\frac{g\sqrt{f+gx}(7ae^2g+cd(8ef-dg))}{2(ef-dg)(-dg-e(f+gx)+ef)}}{4e(ef-dg)^2}-\frac{g^2}{4e(ef-d}\right.$$

input
```
Int[(a + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

output
```
2*(-1/4*((c*d^2 + a*e^2)*g^2*Sqrt[f + g*x])/(e*(e*f - d*g)^2*(e*f - d*g -
e*(f + g*x))^2) - ((g*(7*a*e^2*g + c*d*(8*e*f - d*g))*Sqrt[f + g*x])/(2*(e
*f - d*g)*(e*f - d*g - e*(f + g*x))) + ((-8*e*(c*f^2 + a*g^2))/((e*f - d*g
)*Sqrt[f + g*x]) + ((15*a*e^2*g^2 + c*(8*e^2*f^2 + 8*d*e*f*g - d^2*g^2))*A
rcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(Sqrt[e]*(e*f - d*g)^(3/2
)))/2)/(4*e*(e*f - d*g)^2))
```

3.602. $\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx$

3.602.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
 + 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

3.602. $\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx$

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

3.602.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.07

method	result
derivativedivides	$-\frac{2(a g^2+c f^2)}{(dg-ef)^3\sqrt{gx+f}}-\frac{2\left(\frac{\left(\frac{7}{8}a e^2 g^2-\frac{1}{8}c d^2 g^2+cdefg\right)(gx+f)^{\frac{3}{2}}+\frac{g\left(9ad e^2 g^2-9a e^3 fg+c d^3 g^2+7c d^2 efg-8cd e^2 f^2\right)\sqrt{gx+f}}{8e}}{(e(gx+f)+dg-ef)^2}+\cdots\right.}{(dg-ef)^3}$
default	$-\frac{2(a g^2+c f^2)}{(dg-ef)^3\sqrt{gx+f}}-\frac{2\left(\frac{\left(\frac{7}{8}a e^2 g^2-\frac{1}{8}c d^2 g^2+cdefg\right)(gx+f)^{\frac{3}{2}}+\frac{g\left(9ad e^2 g^2-9a e^3 fg+c d^3 g^2+7c d^2 efg-8cd e^2 f^2\right)\sqrt{gx+f}}{8e}}{(e(gx+f)+dg-ef)^2}+\cdots\right.}{(dg-ef)^3}$
pseudoelliptic	$-\frac{2\left(\frac{15\sqrt{gx+f}\left(\left(a g^2+\frac{8c f^2}{15}\right)e^2+\frac{8cdefg}{15}-\frac{c d^2 g^2}{15}\right)(ex+d)^2\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{8}+\left(\left(\frac{15a g^2 x^2}{8}+\frac{5afgx}{8}-\frac{f^2\left(-4c x^2+a\right)}{4}\right)e\cdots\right.}{\sqrt{(dg-ef)e}\sqrt{gx+f}\,(dg\cdots}$

input
```
int((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(a*g^2+c*f^2)/(d*g-e*f)^3/(g*x+f)^(1/2)-2/(d*g-e*f)^3*(((7/8*a*e^2*g^2-
1/8*c*d^2*g^2+c*d*e*f*g)*(g*x+f)^(3/2)+1/8*g*(9*a*d*e^2*g^2-9*a*e^3*f*g+c*
d^3*g^2+7*c*d^2*e*f*g-8*c*d*e^2*f^2)/e*(g*x+f)^(1/2))/(e*(g*x+f)+d*g-e*f)^
2+1/8*(15*a*e^2*g^2-c*d^2*g^2+8*c*d*e*f*g+8*c*e^2*f^2)/e/((d*g-e*f)*e)^(1/
2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)))
```

3.602.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(192) = 384$.

Time = 0.34 (sec) , antiderivative size = 1539, normalized size of antiderivative = 7.19

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*((8*c*d^2*e^2*f^3 + 8*c*d^3*e*f^2*g - (c*d^4 - 15*a*d^2*e^2)*f*g^2 +
 (8*c*e^4*f^2*g + 8*c*d*e^3*f*g^2 - (c*d^2*e^2 - 15*a*e^4)*g^3)*x^3 + (8*c
*e^4*f^3 + 24*c*d*e^3*f^2*g + 15*(c*d^2*e^2 + a*e^4)*f*g^2 - 2*(c*d^3*e -
15*a*d*e^3)*g^3)*x^2 + (16*c*d*e^3*f^3 + 24*c*d^2*e^2*f^2*g + 6*(c*d^3*e +
 5*a*d*e^3)*f*g^2 - (c*d^4 - 15*a*d^2*e^2)*g^3)*x)*sqrt(e^2*f - d*e*g)*log
((e*g*x + 2*e*f - d*g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) +
2*(8*a*d^3*e^2*g^3 - 2*(7*c*d^2*e^3 - a*e^5)*f^3 + (13*c*d^3*e^2 - 11*a*d*
e^4)*f^2*g + (c*d^4*e + a*d^2*e^3)*f*g^2 - (8*c*e^5*f^3 - 3*(3*c*d^2*e^3 -
 5*a*e^5)*f*g^2 + (c*d^3*e^2 - 15*a*d*e^4)*g^3)*x^2 - (24*c*d*e^4*f^3 - (1
9*c*d^2*e^3 - 5*a*e^5)*f^2*g - 4*(c*d^3*e^2 - 5*a*d*e^4)*f*g^2 - (c*d^4*e
+ 25*a*d^2*e^3)*g^3)*x)*sqrt(g*x + f))/(d^2*e^6*f^5 - 4*d^3*e^5*f^4*g + 6*
d^4*e^4*f^3*g^2 - 4*d^5*e^3*f^2*g^3 + d^6*e^2*f*g^4 + (e^8*f^4*g - 4*d*e^7
*f^3*g^2 + 6*d^2*e^6*f^2*g^3 - 4*d^3*e^5*f*g^4 + d^4*e^4*g^5)*x^3 + (e^8*f
^5 - 2*d*e^7*f^4*g - 2*d^2*e^6*f^3*g^2 + 8*d^3*e^5*f^2*g^3 - 7*d^4*e^4*f*g
^4 + 2*d^5*e^3*g^5)*x^2 + (2*d*e^7*f^5 - 7*d^2*e^6*f^4*g + 8*d^3*e^5*f^3*g
^2 - 2*d^4*e^4*f^2*g^3 - 2*d^5*e^3*f*g^4 + d^6*e^2*g^5)*x), 1/4*((8*c*d^2*
e^2*f^3 + 8*c*d^3*e*f^2*g - (c*d^4 - 15*a*d^2*e^2)*f*g^2 + (8*c*e^4*f^2*g
+ 8*c*d*e^3*f*g^2 - (c*d^2*e^2 - 15*a*e^4)*g^3)*x^3 + (8*c*e^4*f^3 + 24*c*
d*e^3*f^2*g + 15*(c*d^2*e^2 + a*e^4)*f*g^2 - 2*(c*d^3*e - 15*a*d*e^3)*g^3)
*x^2 + (16*c*d*e^3*f^3 + 24*c*d^2*e^2*f^2*g + 6*(c*d^3*e + 5*a*d*e^3)*f...
```

3.602.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)/(e*x+d)**3/(g*x+f)**(3/2),x)
```

output
```
Timed out
```

3.602.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.602.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.72

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{(8\,ce^2f^2+8\,cdefg-cd^2g^2+15\,ae^2g^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{4\,(e^4f^3-3\,de^3f^2g+3\,d^2e^2fg^2-d^3eg^3)\sqrt{-e^2f+deg}}$$
$$+\frac{2\,(cf^2+ag^2)}{(e^3f^3-3\,de^2f^2g+3\,d^2efg^2-d^3g^3)\sqrt{gx+f}}$$
$$+\frac{8\,(gx+f)^{\frac{3}{2}}cde^2fg-8\,\sqrt{gx+f}cde^2f^2g-(gx+f)^{\frac{3}{2}}cd^2eg^2+7\,(gx+f)^{\frac{3}{2}}ae^3g^2+7\,\sqrt{gx+f}cd^2efg^2-9\,\sqrt{}}{4\,(e^4f^3-3\,de^3f^2g+3\,d^2e^2fg^2-d^3eg^3)((gx+f)e-ef+}$$

input
```
integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
1/4*(8*c*e^2*f^2 + 8*c*d*e*f*g - c*d^2*g^2 + 15*a*e^2*g^2)*arctan(sqrt(g*x
 + f)*e/sqrt(-e^2*f + d*e*g))/((e^4*f^3 - 3*d*e^3*f^2*g + 3*d^2*e^2*f*g^2
- d^3*e*g^3)*sqrt(-e^2*f + d*e*g)) + 2*(c*f^2 + a*g^2)/((e^3*f^3 - 3*d*e^2
*f^2*g + 3*d^2*e*f*g^2 - d^3*g^3)*sqrt(g*x + f)) + 1/4*(8*(g*x + f)^(3/2)*
c*d*e^2*f*g - 8*sqrt(g*x + f)*c*d*e^2*f^2*g - (g*x + f)^(3/2)*c*d^2*e*g^2
+ 7*(g*x + f)^(3/2)*a*e^3*g^2 + 7*sqrt(g*x + f)*c*d^2*e*f*g^2 - 9*sqrt(g*x
 + f)*a*e^3*f*g^2 + sqrt(g*x + f)*c*d^3*g^3 + 9*sqrt(g*x + f)*a*d*e^2*g^3)
/((e^4*f^3 - 3*d*e^3*f^2*g + 3*d^2*e^2*f*g^2 - d^3*e*g^3)*((g*x + f)*e - e
*f + d*g)^2)
```

3.602.9 Mupad [B] (verification not implemented)

Time = 12.68 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.45

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{f+g\,x}\,\left(-d^3\,e\,g^3+3\,d^2\,e^2\,f\,g^2-3\,d\,e^3\,f^2\,g+e^4\,f^3\right)}{\sqrt{e}\,{\left(d\,g-e\,f\right)}^{7/2}}\right)\,\left(-c\,d^2\,g^2+8\,c\,d\,e\,f\,g+8\,c\,e^2\,f^2\right.}{4\,e^{3/2}\,{\left(d\,g-e\,f\right)}^{7/2}}$$
$$-\frac{\frac{2\,\left(c\,f^2+a\,g^2\right)}{d\,g-e\,f}+\frac{{\left(f+g\,x\right)}^2\,\left(-c\,d^2\,g^2+8\,c\,d\,e\,f\,g+8\,c\,e^2\,f^2+15\,a\,e^2\,g^2\right)}{4\,{\left(d\,g-e\,f\right)}^3}+\frac{\left(f+g\,x\right)\,\left(c\,d^2\,g^2+8\,c\,d\,e\,f\,g+16\,c\,e^2\,f^2+25\,a\,e^2\,g^2\right)}{4\,e\,{\left(d\,g-e\,f\right)}^2}}{e^2\,{\left(f+g\,x\right)}^{5/2}-{\left(f+g\,x\right)}^{3/2}\,\left(2\,e^2\,f-2\,d\,e\,g\right)+\sqrt{f+g\,x}\,\left(d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2\right)}$$

input
```
int((a + c*x^2)/((f + g*x)^(3/2)*(d + e*x)^3),x)
```

output
```
(atan(((f + g*x)^(1/2)*(e^4*f^3 - d^3*e*g^3 + 3*d^2*e^2*f*g^2 - 3*d*e^3*f^
2*g))/(e^(1/2)*(d*g - e*f)^(7/2)))*(15*a*e^2*g^2 - c*d^2*g^2 + 8*c*e^2*f^2
 + 8*c*d*e*f*g))/(4*e^(3/2)*(d*g - e*f)^(7/2)) - ((2*(a*g^2 + c*f^2))/(d*g
 - e*f) + ((f + g*x)^2*(15*a*e^2*g^2 - c*d^2*g^2 + 8*c*e^2*f^2 + 8*c*d*e*f
*g))/(4*(d*g - e*f)^3) + ((f + g*x)*(25*a*e^2*g^2 + c*d^2*g^2 + 16*c*e^2*f
^2 + 8*c*d*e*f*g))/(4*e*(d*g - e*f)^2))/(e^2*(f + g*x)^(5/2) - (f + g*x)^(
3/2)*(2*e^2*f - 2*d*e*g) + (f + g*x)^(1/2)*(d^2*g^2 + e^2*f^2 - 2*d*e*f*g)
)
```

3.603 $\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}} \, dx$

3.603.1 Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}} \, dx = -\frac{c(3ef+5dg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{(8ae^2g^2+c(3e^2f^2+2defg+3d^2g^2))\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{4e^{5/2}g^{5/2}}$$

output
```
1/4*(8*a*e^2*g^2+c*(3*d^2*g^2+2*d*e*f*g+3*e^2*f^2))*arctanh(g^(1/2)*(e*x+d
)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(5/2)+1/2*c*(e*x+d)^(3/2)*(g*x+f)
^(1/2)/e^2/g-1/4*c*(5*d*g+3*e*f)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/e^2/g^2
```

3.603.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.84

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}} \, dx = \frac{c\sqrt{d+ex}\sqrt{f+gx}(-3ef-3dg+2egx)}{4e^2g^2} + \frac{(8ae^2g^2+c(3e^2f^2+2defg+3d^2g^2))\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{4e^{5/2}g^{5/2}}$$

input
```
Integrate[(a + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

output
```
(c*Sqrt[d + e*x]*Sqrt[f + g*x]*(-3*e*f - 3*d*g + 2*e*g*x))/(4*e^2*g^2) + (
(8*a*e^2*g^2 + c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2))*ArcTanh[(Sqrt[e]*Sqr
t[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/(4*e^(5/2)*g^(5/2))
```

3.603.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {651, 27, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{651}$$

$$\frac{\int \frac{4age^2-c(3ef+5dg)xe-cd(3ef+dg)}{2\sqrt{d+ex}\sqrt{f+gx}}dx}{2e^2g}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4age^2-c(3ef+5dg)xe-cd(3ef+dg)}{\sqrt{d+ex}\sqrt{f+gx}}dx}{4e^2g}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{(8ae^2g^2+c(3d^2g^2+2defg+3e^2f^2))\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}dx}{2g}-\frac{c\sqrt{d+ex}\sqrt{f+gx}(5dg+3ef)}{g}}{4e^2g}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

$$\Big\downarrow \text{66}$$

$$\frac{\frac{(8ae^2g^2+c(3d^2g^2+2defg+3e^2f^2))\int \frac{1}{e-\frac{g(d+ex)}{f+gx}}d\frac{\sqrt{d+ex}}{\sqrt{f+gx}}}{g}-\frac{c\sqrt{d+ex}\sqrt{f+gx}(5dg+3ef)}{g}}{4e^2g}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{(8ae^2g^2+c(3d^2g^2+2defg+3e^2f^2))\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{e}g^{3/2}}-\frac{c\sqrt{d+ex}\sqrt{f+gx}(5dg+3ef)}{g}}{4e^2g}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

input
```
Int[(a + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

output
```
(c*(d + e*x)^(3/2)*Sqrt[f + g*x])/(2*e^2*g) + (-((c*(3*e*f + 5*d*g)*Sqrt[d
 + e*x]*Sqrt[f + g*x])/g) + ((8*a*e^2*g^2 + c*(3*e^2*f^2 + 2*d*e*f*g + 3*d
^2*g^2))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt[e
]*g^(3/2)))/(4*e^2*g)
```

3.603.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 651
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x)^(n + 1)/(g*e
^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2*p + 1))   Int
[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^
2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1)
, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] &&  !IntegerQ[m]
 &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

3.603.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(121) = 242$.

Time = 0.42 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.08

method	result
default	$\frac{\left(8\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)a\,e^{2}g^{2}+3\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)c\,d^{2}g^{2}+2\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+\right.\right.}{}$

input
```
int((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(8*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g
)^(1/2))*a*e^2*g^2+3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)
+d*g+e*f)/(e*g)^(1/2))*c*d^2*g^2+2*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/
2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e*f*g+3*ln(1/2*(2*e*g*x+2*((g*x+f
)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*e^2*f^2+4*(e*g)^(1/2)
*((g*x+f)*(e*x+d))^(1/2)*c*e*g*x-6*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*d
*g-6*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*e*f)*(e*x+d)^(1/2)*(g*x+f)^(1/2
)/(e*g)^(1/2)/g^2/e^2/((g*x+f)*(e*x+d))^(1/2)
```

3.603.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.29

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$
$$= \left[\frac{(3\,ce^2f^2 + 2\,cdefg + (3\,cd^2 + 8\,ae^2)g^2)\sqrt{eg}\log\left(8\,e^2g^2x^2 + e^2f^2 + 6\,defg + d^2g^2 + 4\,(2\,egx + ef + dg)\right.}{16\,e^3g^3}\right.$$
$$- \frac{(3\,ce^2f^2 + 2\,cdefg + (3\,cd^2 + 8\,ae^2)g^2)\sqrt{-eg}\arctan\left(\frac{(2\,egx+ef+dg)\sqrt{-eg}\sqrt{ex+d}\sqrt{gx+f}}{2\,(e^2g^2x^2+defg+(e^2fg+deg^2)x)}\right) - 2\,(2\,ce^2g^2x - 3\,ce}{8\,e^3g^3}$$

input
```
integrate((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((3*c*e^2*f^2 + 2*c*d*e*f*g + (3*c*d^2 + 8*a*e^2)*g^2)*sqrt(e*g)*log
(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*x + e*f + d*g)*s
qrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2)*x) + 4*(2*c*e
^2*g^2*x - 3*c*e^2*f*g - 3*c*d*e*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^
3), -1/8*((3*c*e^2*f^2 + 2*c*d*e*f*g + (3*c*d^2 + 8*a*e^2)*g^2)*sqrt(-e*g)
*arctan(1/2*(2*e*g*x + e*f + d*g)*sqrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(
e^2*g^2*x^2 + d*e*f*g + (e^2*f*g + d*e*g^2)*x)) - 2*(2*c*e^2*g^2*x - 3*c*e
^2*f*g - 3*c*d*e*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^3)]
```

3.603.6 Sympy [F]

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx=\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

input
```
integrate((c*x**2+a)/(e*x+d)**(1/2)/(g*x+f)**(1/2),x)
```

output
```
Integral((a + c*x**2)/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

3.603.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for m
ore detail
```

3.603.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.12

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{\left(\sqrt{e^2f+(ex+d)eg-deg}\sqrt{ex+d}\left(\frac{2\,(ex+d)c}{e^3g}-\frac{3\,ce^6fg+5\,cde^5g^2}{e^8g^3}\right)-\frac{(3\,ce^2f^2+2\,cdefg+3\,cd^2g^2+8\,ae^2g^2)\log\left(\left|-\sqrt{eg}\sqrt{ex\right.\right.}{\sqrt{eg}e^2g^2}\right.}{4\,|e|}$$

input
```
integrate((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
1/4*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(e*x + d)*(2*(e*x + d)*c/(e^3
*g) - (3*c*e^6*f*g + 5*c*d*e^5*g^2)/(e^8*g^3)) - (3*c*e^2*f^2 + 2*c*d*e*f*
g + 3*c*d^2*g^2 + 8*a*e^2*g^2)*log(abs(-sqrt(e*g)*sqrt(e*x + d) + sqrt(e^2
*f + (e*x + d)*e*g - d*e*g)))/(sqrt(e*g)*e^2*g^2))*e/abs(e)
```

3.603.9 Mupad [B] (verification not implemented)

Time = 32.61 (sec) , antiderivative size = 569, normalized size of antiderivative = 3.87

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{c\,\mathrm{atanh}\left(\frac{\sqrt{g}\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{e}\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)}\right)\,\left(3\,d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2\right)}{2\,e^{5/2}\,g^{5/2}} - \frac{4\,a\,\mathrm{atan}\left(\frac{e\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)}{\sqrt{-e\,g}\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{-e\,g}} - \frac{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)\,\left(\frac{3\,c\,d^2\,e\,g^2}{2}+c\,d\,e^2\,f\,g+\frac{3\,c\,e^3\,f^2}{2}\right)}{g^6\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)} - \frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3\,\left(\frac{11\,c\,d^2\,g^2}{2}+25\,c\,d\,e\,f\,g+\frac{11\,c\,e^2\,f^2}{2}\right)}{g^5\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)^3} + \frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^7\,\left(\frac{3\,c\,d^2\,g^2}{2}+c\right.}{e^2\,g^3\,\left(\sqrt{f+g\,x}-\right.}}{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^8}{\left(\sqrt{f+g\,x}-\sqrt{f}\right)^8}+\frac{e^4}{g^4}-\frac{4\,e\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^6}{g\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)^6}-\frac{4\,e^3\,\left(\right.}{g^3\,\left(\right.}}$$

input
```
int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(1/2)),x)
```

output
```
(c*atanh((g^(1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x)^(1/2) -
 f^(1/2))))*(3*d^2*g^2 + 3*e^2*f^2 + 2*d*e*f*g))/(2*e^(5/2)*g^(5/2)) - (4*
a*atan((e*((f + g*x)^(1/2) - f^(1/2)))/((-e*g)^(1/2)*((d + e*x)^(1/2) - d^
(1/2)))))/(-e*g)^(1/2) - ((((d + e*x)^(1/2) - d^(1/2))*((3*c*e^3*f^2)/2 +
(3*c*d^2*e*g^2)/2 + c*d*e^2*f*g))/(g^6*((f + g*x)^(1/2) - f^(1/2))) - (((d
 + e*x)^(1/2) - d^(1/2))^3*((11*c*d^2*g^2)/2 + (11*c*e^2*f^2)/2 + 25*c*d*e
*f*g))/(g^5*((f + g*x)^(1/2) - f^(1/2))^3) + (((d + e*x)^(1/2) - d^(1/2))^
7*((3*c*d^2*g^2)/2 + (3*c*e^2*f^2)/2 + c*d*e*f*g))/(e^2*g^3*((f + g*x)^(1/
2) - f^(1/2))^7) - (((d + e*x)^(1/2) - d^(1/2))^5*((11*c*d^2*g^2)/2 + (11*
c*e^2*f^2)/2 + 25*c*d*e*f*g))/(e*g^4*((f + g*x)^(1/2) - f^(1/2))^5) + (d^(
1/2)*f^(1/2)*(32*c*d*g + 32*c*e*f)*((d + e*x)^(1/2) - d^(1/2))^4)/(g^4*((f
 + g*x)^(1/2) - f^(1/2))^4))/(((d + e*x)^(1/2) - d^(1/2))^8/((f + g*x)^(1/
2) - f^(1/2))^8 + e^4/g^4 - (4*e*((d + e*x)^(1/2) - d^(1/2))^6)/(g*((f + g
*x)^(1/2) - f^(1/2))^6) - (4*e^3*((d + e*x)^(1/2) - d^(1/2))^2)/(g^3*((f +
 g*x)^(1/2) - f^(1/2))^2) + (6*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(g^2*((f
 + g*x)^(1/2) - f^(1/2))^4))
```

3.603. $\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

3.604 $\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx$

3.604.1 Optimal result

Integrand size = 22, antiderivative size = 16

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx = \sqrt{-1+x}x\sqrt{1+x}$$

output
```
x*(-1+x)^(1/2)*(1+x)^(1/2)
```

3.604.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx = \sqrt{-1+x}x\sqrt{1+x}$$

input
```
Integrate[(-1 + 2*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]),x]
```

output
```
Sqrt[-1 + x]*x*Sqrt[1 + x]
```

3.604.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {644}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2-1}{\sqrt{x-1}\sqrt{x+1}}\,dx$$

$$\Big\downarrow \text{644}$$

$$\sqrt{x-1}x\sqrt{x+1}$$

input
```
Int[(-1 + 2*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]),x]
```

output
```
Sqrt[-1 + x]*x*Sqrt[1 + x]
```

3.604.3.1 Defintions of rubi rules used

rule 644
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2), x_Symbol] :> Simp[a*x*(c + d*x)^(m + 1)*((e + f*x)^(n + 1)/(c*e)), x]
/; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
EqQ[b*c*e - a*d*f*(2*m + 3), 0]
```

3.604.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$x\sqrt{-1+x}\,\sqrt{1+x}$	13
default	$x\sqrt{-1+x}\,\sqrt{1+x}$	13
risch	$x\sqrt{-1+x}\,\sqrt{1+x}$	13

input
```
int((2*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*(-1+x)^(1/2)*(1+x)^(1/2)
```

3.604.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx = \sqrt{x+1}\sqrt{x-1}x$$

input
```
integrate((2*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x + 1)*sqrt(x - 1)*x
```

3.604.6 Sympy [F(-1)]

Timed out.

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx = \text{Timed out}$$

input
```
integrate((2*x**2-1)/(-1+x)**(1/2)/(1+x)**(1/2),x)
```

output
```
Timed out
```

3.604.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx = \sqrt{x^2-1}x$$

input
```
integrate((2*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x^2 - 1)*x
```

3.604.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx = \sqrt{x+1}\sqrt{x-1}x$$

input
```
integrate((2*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
sqrt(x + 1)*sqrt(x - 1)*x
```

3.604.9 Mupad [B] (verification not implemented)

Time = 12.48 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx = \frac{(x^2+x)\sqrt{x-1}}{\sqrt{x+1}}$$

input
```
int((2*x^2 - 1)/((x - 1)^(1/2)*(x + 1)^(1/2)),x)
```

output
```
((x + x^2)*(x - 1)^(1/2))/(x + 1)^(1/2)
```

3.605 $\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx$

3.605.1 Optimal result

Integrand size = 28, antiderivative size = 411

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \frac{e\sqrt{d+ex}\sqrt{f+gx}}{c}+\frac{\sqrt{e}(ef+3dg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}}$$
$$+\frac{\left(\frac{a(ae^2g-cd(2ef+dg))}{\sqrt{c}}-\sqrt{-a}(cd^2f-ae(ef+2dg))\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{ac\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}$$
$$+\frac{\left(\frac{a(ae^2g-cd(2ef+dg))}{\sqrt{c}}+\sqrt{-a}(cd^2f-ae(ef+2dg))\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{ac\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

output
```
(3*d*g+e*f)*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))*e^(1/2)/c
/g^(1/2)+e*(e*x+d)^(1/2)*(g*x+f)^(1/2)/c+arctanh((e*x+d)^(1/2)*(-g*(-a)^(1
/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2))*(-(c*d
^2*f-a*e*(2*d*g+e*f))*(-a)^(1/2)+a*(a*e^2*g-c*d*(d*g+2*e*f))/c^(1/2))/a/c/
(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)+arctanh((e
*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c
^(1/2))^(1/2))*((c*d^2*f-a*e*(2*d*g+e*f))*(-a)^(1/2)+a*(a*e^2*g-c*d*(d*g+2
*e*f))/c^(1/2))/a/c/(e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)
)^(1/2)
```

3.605.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \frac{\sqrt{c}e\sqrt{d+ex}\sqrt{f+gx} + \frac{(i\sqrt{c}d+\sqrt{a}e)\sqrt{cd^2+ae^2}(\sqrt{c}f-i\sqrt{a}g)\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-((\sqrt{c}d+i\sqrt{a}e)(\sqrt{c}f-i\sqrt{a}g)}}\right.}{\sqrt{a}\sqrt{-((\sqrt{c}d+i\sqrt{a}e)(\sqrt{c}f-i\sqrt{a}g))}}}{}$$

input
```
Integrate[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a + c*x^2),x]
```

output
```
(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[f + g*x] + ((I*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[c
*d^2 + a*e^2]*(Sqrt[c]*f - I*Sqrt[a]*g)*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f
 + g*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*Sqr
t[d + e*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqr
t[a]*g))]) + (((-I)*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[c*d^2 + a*e^2]*(Sqrt[c]*f
+ I*Sqrt[a]*g)*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]
*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*Sq
rt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]) + (Sqrt[c]*Sqr
t[e]*(e*f + 3*d*g)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])
])/Sqrt[g])/c^(3/2)
```

3.605.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {658, 90, 66, 221, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx$$

$$\Big\downarrow \text{658}$$

$$\frac{\int \frac{cfd^2-ae(ef+2dg)-(ae^2g-cd(2ef+dg))x}{\sqrt{d+ex}\sqrt{f+gx}(cx^2+a)}dx}{c}+\frac{e\int \frac{ef+2dg+egx}{\sqrt{d+ex}\sqrt{f+gx}}dx}{c}$$

$$\Big\downarrow \text{90}$$

$$\frac{\int \frac{cfd^2-ae(ef+2dg)-(ae^2g-cd(2ef+dg))x}{\sqrt{d+ex}\sqrt{f+gx}(cx^2+a)}dx}{c}+\frac{e\left(\frac{1}{2}(3dg+ef)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}dx+\sqrt{d+ex}\sqrt{f+gx}\right)}{c}$$

$$\Big\downarrow \; 66$$

$$\frac{\int \frac{cfd^2-ae(ef+2dg)-(ae^2g-cd(2ef+dg))x}{\sqrt{d+ex}\sqrt{f+gx}(cx^2+a)}dx}{c}+\frac{e\left((3dg+ef)\int \frac{1}{e-\frac{g(d+ex)}{f+gx}}d\frac{\sqrt{d+ex}}{\sqrt{f+gx}}+\sqrt{d+ex}\sqrt{f+gx}\right)}{c}$$

$$\Big\downarrow \; 221$$

$$\frac{\int \frac{cfd^2-ae(ef+2dg)-(ae^2g-cd(2ef+dg))x}{\sqrt{d+ex}\sqrt{f+gx}(cx^2+a)}dx}{c}+\frac{e\left(\frac{(3dg+ef)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{e}\sqrt{g}}+\sqrt{d+ex}\sqrt{f+gx}\right)}{c}$$

$$\Big\downarrow \; 2348$$

$$\frac{\int \left(\frac{\sqrt{-a}(cd^2f-ae(ef+2dg))-\frac{a\left(cd(2ef+dg)-ae^2g\right)}{\sqrt{c}}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}+\frac{\frac{a\left(cd(2ef+dg)-ae^2g\right)}{\sqrt{c}}+\sqrt{-a}(cd^2f-ae(ef+2dg))}{2a\left(\sqrt{c}x+\sqrt{-a}\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx}{c}+\frac{e\left(\frac{(3dg+ef)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{e}\sqrt{g}}+\sqrt{d+ex}\sqrt{f+gx}\right)}{c}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\left(\frac{a\left(ae^2g-cd(dg+2ef)\right)}{\sqrt{c}}-\sqrt{-a}(cd^2f-ae(2dg+ef))\right)\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{a\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}+\frac{\left(\sqrt{-a}(cd^2f-ae(2dg+ef))+\frac{a\left(ae^2g-cd(dg+2ef)\right)}{\sqrt{c}}\right)\text{arc}}{a\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}}}}{c}+$$

$$\frac{e\left(\frac{(3dg+ef)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{e}\sqrt{g}}+\sqrt{d+ex}\sqrt{f+gx}\right)}{c}$$

input
```
Int[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a + c*x^2),x]
```

3.605. $\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx$

output
```
(e*(Sqrt[d + e*x]*Sqrt[f + g*x] + ((e*f + 3*d*g)*ArcTanh[(Sqrt[g]*Sqrt[d +
 e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt[e]*Sqrt[g])))/c + ((((a*(a*e^2*g -
c*d*(2*e*f + d*g)))/Sqrt[c] - Sqrt[-a]*(c*d^2*f - a*e*(e*f + 2*d*g)))*ArcT
anh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a
]*e]*Sqrt[f + g*x])])/(a*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqr
t[-a]*g]) + (((a*(a*e^2*g - c*d*(2*e*f + d*g)))/Sqrt[c] + Sqrt[-a]*(c*d^2*
f - a*e*(e*f + 2*d*g)))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x
])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(a*Sqrt[Sqrt[c]*d + Sqrt
[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]))/c
```

3.605.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 658
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Simp[g/c   Int[Simp[2*e*f + d*g + e*g*x, x]*(d + e*x)^(m
 - 1)*(f + g*x)^(n - 2), x], x] + Simp[1/c   Int[Simp[c*d*f^2 - 2*a*e*f*g -
 a*d*g^2 + (c*e*f^2 + 2*c*d*f*g - a*e*g^2)*x, x]*(d + e*x)^(m - 1)*((f + g*
x)^(n - 2)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}, x] &&  !Intege
rQ[m] &&  !IntegerQ[n] && GtQ[m, 0] && GtQ[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.605. $\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx$

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

3.605.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2384 vs. $2(331) = 662$.

Time = 0.45 (sec) , antiderivative size = 2385, normalized size of antiderivative = 5.80

method	result	size
default	Expression too large to display	2385

input
```
int((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(3*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e
*g+c*d*f)/c)^(1/2)*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d
*g+e*f)/(e*g)^(1/2))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^
(1/2)*(-a*c)^(1/2)*c*d*e*g+(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f
)/c)^(1/2)*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/
(e*g)^(1/2))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(-
a*c)^(1/2)*c*e^2*f-2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(
1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c
)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d
*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*a*c*(e*g)^(1/2)*d*e*g-(((
-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*ln((-2*(-a*c)^(1/2)
*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)
/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*
d*f)/(c*x+(-a*c)^(1/2)))*a*c*(e*g)^(1/2)*e^2*f+(((-a*c)^(1/2)*d*g+(-a*c)^(
1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2
*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+
d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))
*(-a*c)^(1/2)*(e*g)^(1/2)*a*e^2*g-(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*
g+c*d*f)/c)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1
/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*...
```

3.605.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.605.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx=\int \frac{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}}{a+cx^2}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)**(1/2)/(c*x**2+a),x)
```

output
```
Integral((d + e*x)**(3/2)*sqrt(f + g*x)/(a + c*x**2), x)
```

3.605.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}{cx^2+a}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)*sqrt(g*x + f)/(c*x^2 + a), x)
```

3.605.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.605.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \int \frac{\sqrt{f+g\,x}\,(d+e\,x)^{3/2}}{c\,x^2+a}\,dx$$

input
```
int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(a + c*x^2),x)
```

output
```
int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(a + c*x^2), x)
```

3.606 $\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2} dx$

3.606.1 Optimal result

Integrand size = 28, antiderivative size = 342

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2} dx = \frac{2\sqrt{e}\sqrt{g}\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c} + \frac{\left(cdf-aeg-\sqrt{-a}\sqrt{c}(ef+dg)\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\left(cdf-aeg+\sqrt{-a}\sqrt{c}(ef+dg)\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

output
```
2*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))*e^(1/2)*g^(1/2)/c+a
rctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a
)^(1/2)+d*c^(1/2))^(1/2))*(c*d*f-a*e*g-(d*g+e*f)*(-a)^(1/2)*c^(1/2))/c/(-a
)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-ar
ctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(
1/2)+d*c^(1/2))^(1/2))*(c*d*f-a*e*g+(d*g+e*f)*(-a)^(1/2)*c^(1/2))/c/(-a)^(
1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

3.606.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx$$
$$= \frac{\frac{\sqrt{cd^2+ae^2}(i\sqrt{c}f+\sqrt{a}g)\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-((\sqrt{c}d+i\sqrt{a}e)(\sqrt{c}f-i\sqrt{a}g))}\sqrt{d+ex}}\right)}{\sqrt{a}\sqrt{-((\sqrt{c}d+i\sqrt{a}e)(\sqrt{c}f-i\sqrt{a}g))}} + \frac{\sqrt{cd^2+ae^2}(-i\sqrt{c}f+\sqrt{a}g)\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-((\sqrt{c}d-i\sqrt{a}e)(\sqrt{c}f+i\sqrt{a}g))}\sqrt{d+e}}\right.}{\sqrt{a}\sqrt{-((\sqrt{c}d-i\sqrt{a}e)(\sqrt{c}f+i\sqrt{a}g))}}}{c}$$

input
```
Integrate[(Sqrt[d + e*x]*Sqrt[f + g*x])/(a + c*x^2),x]
```

output
```
((Sqrt[c*d^2 + a*e^2]*(I*Sqrt[c]*f + Sqrt[a]*g)*ArcTan[(Sqrt[c*d^2 + a*e^2
]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*
g))]*Sqrt[d + e*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f
 - I*Sqrt[a]*g))]) + (Sqrt[c*d^2 + a*e^2]*((-I)*Sqrt[c]*f + Sqrt[a]*g)*Arc
Tan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*
(Sqrt[c]*f + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d -
I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]) + 2*Sqrt[e]*Sqrt[g]*ArcTanh[(Sqr
t[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/c
```

3.606.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {659, 66, 221, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx$$
$$\Big\downarrow \text{659}$$
$$\frac{\int \frac{cdf-aeg+c(ef+dg)x}{\sqrt{d+ex}\sqrt{f+gx}(cx^2+a)}dx}{c}+\frac{eg\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}dx}{c}$$
$$\Big\downarrow \text{66}$$

$$\frac{\int \frac{cdf-aeg+c(ef+dg)x}{\sqrt{d+ex}\sqrt{f+gx}(cx^2+a)}dx}{c}+\frac{2eg\int \frac{1}{e-\frac{g(d+ex)}{f+gx}}d\frac{\sqrt{d+ex}}{\sqrt{f+gx}}}{c}$$

$$\Big\downarrow \ 221$$

$$\frac{\int \frac{cdf-aeg+c(ef+dg)x}{\sqrt{d+ex}\sqrt{f+gx}(cx^2+a)}dx}{c}+\frac{2\sqrt{e}\sqrt{g}\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c}$$

$$\Big\downarrow \ 2348$$

$$\frac{\int \left(\frac{\sqrt{-a}(cdf-aeg)-a\sqrt{c}(ef+dg)}{2a(\sqrt{-a}-\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}+\frac{a\sqrt{c}(ef+dg)+\sqrt{-a}(cdf-aeg)}{2a(\sqrt{c}x+\sqrt{-a})\sqrt{d+ex}\sqrt{f+gx}}\right)dx}{c}+\frac{2\sqrt{e}\sqrt{g}\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{(-\sqrt{-a}\sqrt{c}(dg+ef)-aeg+cdf)\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{(\sqrt{-a}\sqrt{c}(dg+ef)-aeg+cdf)\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}}{c}+\frac{2\sqrt{e}\sqrt{g}\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c}$$

input
```
Int[(Sqrt[d + e*x]*Sqrt[f + g*x])/(a + c*x^2),x]
```

output
```
(2*Sqrt[e]*Sqrt[g]*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])
])/c + (((c*d*f - a*e*g - Sqrt[-a]*Sqrt[c]*(e*f + d*g))*ArcTanh[(Sqrt[Sqrt
[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f +
g*x])])/(Sqrt[-a]*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g
]) - ((c*d*f - a*e*g + Sqrt[-a]*Sqrt[c]*(e*f + d*g))*ArcTanh[(Sqrt[Sqrt[c]
*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x
])])/(Sqrt[-a]*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]))
/c
```

3.606.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 659
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Simp[e*(g/c)   Int[(d + e*x)^(m - 1)*(f + g*x)^(n - 1),
x], x] + Simp[1/c   Int[Simp[c*d*f - a*e*g + (c*e*f + c*d*g)*x, x]*(d + e*x
)^(m - 1)*((f + g*x)^(n - 1)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f,
g}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[m, 0] && GtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

3.606.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1498 vs. 2(262) = 524.

Time = 0.45 (sec) , antiderivative size = 1499, normalized size of antiderivative = 4.38

method	result	size
default	Expression too large to display	1499

input
```
int((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(g*x+f)^(1/2)*(e*x+d)^(1/2)*((e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1
/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(
((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))
^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*
e*g-(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)
*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2
)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a
*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c*d*f-(e*g)^(1/2)*(-((-a*c)^(1/
2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d
*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(
(g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(
-a*c)^(1/2)))*(-a*c)^(1/2)*d*g-(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2
)*e*f+a*e*g-c*d*f)/c)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((
-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(
1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*(-a*
c)^(1/2)*e*f-(e*g)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-
a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1
/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*(((-a
*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*a*e*g+(e*g)^(1/2)*ln(
(-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/...
```

3.606.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.606.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx = \int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(g*x+f)**(1/2)/(c*x**2+a),x)
```

output
```
Integral(sqrt(d + e*x)*sqrt(f + g*x)/(a + c*x**2), x)
```

3.606.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx = \int \frac{\sqrt{ex+d}\sqrt{gx+f}}{cx^2+a}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*sqrt(g*x + f)/(c*x^2 + a), x)
```

3.606.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.606.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx = \text{Hanged}$$

input
```
int(((f + g*x)^(1/2)*(d + e*x)^(1/2))/(a + c*x^2),x)
```

output
```
\text{Hanged}
```

3.607 $\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx$

3.607.1 Optimal result

Integrand size = 28, antiderivative size = 240

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\,\mathrm{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}d-\sqrt{-a}e}} - \frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\,\mathrm{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}d+\sqrt{-a}e}}$$

output
```
arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-
a)^(1/2)+d*c^(1/2))^(1/2))*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(-a)^(1/2)/c^(1
/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*
c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*(g*(-a)^(1/2)
+f*c^(1/2))^(1/2)/(-a)^(1/2)/c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2)
```

3.607.2 Mathematica [A] (verified)

Time = 10.30 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \frac{\frac{\sqrt{-\sqrt{c}f+\sqrt{-a}g}\,\mathrm{arctanh}\left(\frac{\sqrt{-\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-\sqrt{c}d+\sqrt{-a}e}} - \frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\,\mathrm{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{\sqrt{c}d+\sqrt{-a}e}}}{\sqrt{-a}\sqrt{c}}$$

input
```
Integrate[Sqrt[f + g*x]/(Sqrt[d + e*x]*(a + c*x^2)),x]
```

output
```
((Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]
*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/Sqrt[-(S
qrt[c]*d) + Sqrt[-a]*e] - (Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*ArcTanh[(Sqrt[Sqrt
[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f +
g*x])]])/Sqrt[Sqrt[c]*d + Sqrt[-a]*e])/(Sqrt[-a]*Sqrt[c])
```

3.607.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {661, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}}{(a+cx^2)\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{ 661}$$

$$\int \left(\frac{\sqrt{-a}f-\frac{ag}{\sqrt{c}}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}+\frac{\frac{ag}{\sqrt{c}}+\sqrt{-a}f}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}d-\sqrt{-a}e}}-\frac{\sqrt{\sqrt{-a}g+\sqrt{c}f}\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{-a}e+\sqrt{c}d}}$$

input
```
Int[Sqrt[f + g*x]/(Sqrt[d + e*x]*(a + c*x^2)),x]
```

output
```
(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d
 + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[c]*
Sqrt[Sqrt[c]*d - Sqrt[-a]*e]) - (Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*ArcTanh[(Sqr
t[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqr
t[f + g*x])]])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*d + Sqrt[-a]*e])
```

3.607.3.1 Defintions of rubi rules used

rule 661
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGt
Q[m + 1/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.607.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1386 vs. $2(176) = 352$.

Time = 0.40 (sec) , antiderivative size = 1387, normalized size of antiderivative = 5.78

method	result	size
default	Expression too large to display	1387

input
```
int((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(g*x+f)^(1/2)*(e*x+d)^(1/2)*(ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x
+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*
x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)
))*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*a*c*e^2*f-ln(
(-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*
e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c
)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*(-a*c)^(1/2)*(((-a*c)^(1/2)*d*g+(
-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*a*e^2*g+ln((-2*(-a*c)^(1/2)*e*g*x+c*
d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)
*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x
+(-a*c)^(1/2)))*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*
c^2*d^2*f-ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+
(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1
/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*(-a*c)^(1/2)*(((-a*c
)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c*d^2*g-ln((2*(-a*c)^(1
/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*
f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*
c*d*f)/(c*x-(-a*c)^(1/2)))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*
f)/c)^(1/2)*a*c*e^2*f-ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^
(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2...
```

3.607.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1921 vs. $2(176) = 352$.

Time = 6.81 (sec) , antiderivative size = 1921, normalized size of antiderivative = 8.00

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*sqrt(-(c*d*f + a*e*g + (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e
*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 +
 a^2*c*e^2))*log(-(e^2*f^2 - d^2*g^2 + 2*(c*d*e*f - c*d^2*g - (a*c^2*d^2*e
 + a^2*c*e^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2
*d^2*e^2 + a^3*c*e^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g +
 (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4
+ 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2)) + 2*(e^2*f*g -
 d*e*g^2)*x + (2*(c^2*d^3 + a*c*d*e^2)*f + ((c^2*d^2*e + a*c*e^3)*f + (c^2
*d^3 + a*c*d*e^2)*g)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 +
 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/x) + 1/4*sqrt(-(c*d*f + a*e*g + (a*c^2*d
^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c
^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2))*log(-(e^2*f^2 - d^2*g^2
 - 2*(c*d*e*f - c*d^2*g - (a*c^2*d^2*e + a^2*c*e^3)*sqrt(-(e^2*f^2 - 2*d*e
*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*sqrt(e*x + d
)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g + (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*
f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(
a*c^2*d^2 + a^2*c*e^2)) + 2*(e^2*f*g - d*e*g^2)*x + (2*(c^2*d^3 + a*c*d*e^
2)*f + ((c^2*d^2*e + a*c*e^3)*f + (c^2*d^3 + a*c*d*e^2)*g)*x)*sqrt(-(e^2*f
^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/x)
 - 1/4*sqrt(-(c*d*f + a*e*g - (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - ...
```

3.607.6 Sympy [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \int \frac{\sqrt{f+gx}}{(a+cx^2)\sqrt{d+ex}}\,dx$$

input
```
integrate((g*x+f)**(1/2)/(e*x+d)**(1/2)/(c*x**2+a),x)
```

output
```
Integral(sqrt(f + g*x)/((a + c*x**2)*sqrt(d + e*x)), x)
```

3.607.7 Maxima [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \int \frac{\sqrt{gx+f}}{(cx^2+a)\sqrt{ex+d}}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(g*x + f)/((c*x^2 + a)*sqrt(e*x + d)), x)
```

3.607.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

3.607.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \text{Hanged}$$

input
```
int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

3.608 $\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}(a+cx^2)}\,dx$

3.608.1 Optimal result

Integrand size = 28, antiderivative size = 351

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}(a+cx^2)}\,dx = -\frac{2e\sqrt{f+gx}}{(cd^2+ae^2)\sqrt{d+ex}} + \frac{\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\,(cd^2+ae^2)\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d+\sqrt{-a}e}\,(cd^2+ae^2)\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

output
```
-2*e*(g*x+f)^(1/2)/(a*e^2+c*d^2)/(e*x+d)^(1/2)+arctanh((e*x+d)^(1/2)*(-g*(
-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2))*
(c*d*f+a*e*g+(-d*g+e*f)*(-a)^(1/2)*c^(1/2))/(a*e^2+c*d^2)/(-a)^(1/2)/(-e*(
-a)^(1/2)+d*c^(1/2))^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e*x+d)
^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2
))^(1/2))*(c*d*f+a*e*g-(-d*g+e*f)*(-a)^(1/2)*c^(1/2))/(a*e^2+c*d^2)/(-a)^(
1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

3.608.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}(a+cx^2)}\,dx = -\frac{2e\sqrt{f+gx}}{(cd^2+ae^2)\sqrt{d+ex}} - \frac{i\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\left(\sqrt{c}d-i\sqrt{a}e\right)\sqrt{cd^2+ae^2}} + \frac{i\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\left(\sqrt{c}d+i\sqrt{a}e\right)\sqrt{cd^2+ae^2}}$$

input
```
Integrate[Sqrt[f + g*x]/((d + e*x)^(3/2)*(a + c*x^2)),x]
```

output
```
(-2*e*Sqrt[f + g*x])/((c*d^2 + a*e^2)*Sqrt[d + e*x]) - (I*Sqrt[-((Sqrt[c]*
d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*ArcTan[(Sqrt[c*d^2 + a*e^2]*S
qrt[f + g*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))
]*Sqrt[d + e*x])])/(Sqrt[a]*(Sqrt[c]*d - I*Sqrt[a]*e)*Sqrt[c*d^2 + a*e^2])
 + (I*Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]*ArcTan[
(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqr
t[c]*f + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*(Sqrt[c]*d + I*Sqrt[a]*e
)*Sqrt[c*d^2 + a*e^2])
```

3.608.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {660, 48, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}}{(a+cx^2)(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 660}$$

$$\frac{\int \frac{cdf+aeg-c(ef-dg)x}{\sqrt{d+ex}\sqrt{f+gx}(cx^2+a)}dx}{ae^2+cd^2}+\frac{e(ef-dg)\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}}dx}{ae^2+cd^2}$$

$$\Big\downarrow \text{48}$$

$$\frac{\int \frac{cdf+aeg-c(ef-dg)x}{\sqrt{d+ex}\sqrt{f+gx}(cx^2+a)}dx}{ae^2+cd^2}-\frac{2e\sqrt{f+gx}}{\sqrt{d+ex}\,(ae^2+cd^2)}$$

$$\Big\downarrow \text{2348}$$

$$\frac{\int \left(\frac{\sqrt{-a}(cdf+aeg)-a\sqrt{c}(ef-dg)}{2a(\sqrt{c}x+\sqrt{-a})\sqrt{d+ex}\sqrt{f+gx}}+\frac{a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a(\sqrt{-a}-\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\right)dx}{ae^2+cd^2}-\frac{2e\sqrt{f+gx}}{\sqrt{d+ex}\,(ae^2+cd^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}}{ae^2+cd^2}-\frac{2e\sqrt{f+gx}}{\sqrt{d+ex}\,(ae^2+cd^2)}$$

input
```
Int[Sqrt[f + g*x]/((d + e*x)^(3/2)*(a + c*x^2)),x]
```

output
```
(-2*e*Sqrt[f + g*x])/((c*d^2 + a*e^2)*Sqrt[d + e*x]) + (((c*d*f + a*e*g +
Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d
 + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[Sqr
t[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - ((c*d*f + a*e*g - Sqr
t[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d +
e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[Sqrt[c
]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]))/(c*d^2 + a*e^2)
```

3.608. $\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}(a+cx^2)}\,dx$

3.608.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 660
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Simp[(-g)*((e*f - d*g)/(c*f^2 + a*g^2))   Int[(d + e*x)^
(m - 1)*(f + g*x)^n, x], x] + Simp[1/(c*f^2 + a*g^2)   Int[Simp[c*d*f + a*e
*g + c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*((f + g*x)^(n + 1)/(a + c*x^2)),
 x], x] /; FreeQ[{a, c, d, e, f, g}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &
& GtQ[m, 0] && LtQ[n, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

3.608.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5382 vs. $2(279) = 558$.

Time = 0.46 (sec) , antiderivative size = 5383, normalized size of antiderivative = 15.34

method	result	size
default	Expression too large to display	5383

input
```
int((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.608.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5816 vs. $2(279) = 558$.

Time = 35.16 (sec) , antiderivative size = 5816, normalized size of antiderivative = 16.57

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.608.6 Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\,(a+cx^2)}\,dx = \int \frac{\sqrt{f+gx}}{(a+cx^2)\,(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)**(1/2)/(e*x+d)**(3/2)/(c*x**2+a),x)
```

output
```
Integral(sqrt(f + g*x)/((a + c*x**2)*(d + e*x)**(3/2)), x)
```

3.608.7 Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\,(a+cx^2)}\,dx = \int \frac{\sqrt{gx+f}}{(cx^2+a)(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(g*x + f)/((c*x^2 + a)*(e*x + d)^(3/2)), x)
```

3.608.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

3.608.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}(a+cx^2)}\,dx = \int \frac{\sqrt{f+g\,x}}{(c\,x^2+a)\,(d+e\,x)^{3/2}}\,dx$$

input
```
int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(3/2)),x)
```

output
```
int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(3/2)), x)
```

3.609 $\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}(a+cx^2)}\,dx$

3.609.1 Optimal result

Integrand size = 28, antiderivative size = 613

$$\begin{aligned}\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}(a+cx^2)}\,dx &= -\frac{2e\sqrt{f+gx}}{3(cd^2+ae^2)(d+ex)^{3/2}} \\ &+ \frac{4eg\sqrt{f+gx}}{3(cd^2+ae^2)(ef-dg)\sqrt{d+ex}} \\ &+ \frac{e\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(cd^2+ae^2)(ef-dg)\sqrt{d+ex}} \\ &- \frac{e\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(cd^2+ae^2)(ef-dg)\sqrt{d+ex}} \\ &+ \frac{\sqrt{c}\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)^{3/2}(cd^2+ae^2)\sqrt{\sqrt{c}f-\sqrt{-a}g}} \\ &+ \frac{\sqrt{c}\left(\sqrt{-a}cdf+\sqrt{-a}aeg+a\sqrt{c}(ef-dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{a\left(\sqrt{c}d+\sqrt{-a}e\right)^{3/2}(cd^2+ae^2)\sqrt{\sqrt{c}f+\sqrt{-a}g}}\end{aligned}$$

output
```
-2/3*e*(g*x+f)^(1/2)/(a*e^2+c*d^2)/(e*x+d)^(3/2)+4/3*e*g*(g*x+f)^(1/2)/(a*
e^2+c*d^2)/(-d*g+e*f)/(e*x+d)^(1/2)+e*(c*d*f+a*e*g-(-d*g+e*f)*(-a)^(1/2)*c
^(1/2))*(g*x+f)^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(-a)^(1/2)/(e*(-a)^(1/2)+d*
c^(1/2))/(e*x+d)^(1/2)-e*(c*d*f+a*e*g+(-d*g+e*f)*(-a)^(1/2)*c^(1/2))*(g*x+
f)^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))/(e*
x+d)^(1/2)+arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(
1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2)*(c*d*f+a*e*g+(-d*g+e*f)*(-a)
^(1/2)*c^(1/2))/(a*e^2+c*d^2)/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(3/2)/(
-g*(-a)^(1/2)+f*c^(1/2))^(1/2)+arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/
2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2)*(c*d*f*(-a
)^(1/2)+a*e*g*(-a)^(1/2)+a*(-d*g+e*f)*c^(1/2))/a/(a*e^2+c*d^2)/(e*(-a)^(1/
2)+d*c^(1/2))^(3/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

3.609.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.65 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}(a+cx^2)}\,dx = \frac{2\sqrt{f+gx}(ae^4(f+gx)+cde(-6d^2g+6e^2fx+de(7f-5gx)))}{3(cd^2+ae^2)^2(-ef+dg)(d+ex)^{3/2}} - \frac{i\sqrt{c}\sqrt{-((\sqrt{c}d+i\sqrt{a}e)(\sqrt{c}f-i\sqrt{a}g))}\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-((\sqrt{c}d+i\sqrt{a}e)(\sqrt{c}f-i\sqrt{a}g))}\sqrt{d+ex}}\right)}{\sqrt{a}(\sqrt{c}d-i\sqrt{a}e)^2\sqrt{cd^2+ae^2}} + \frac{i\sqrt{c}\sqrt{-((\sqrt{c}d-i\sqrt{a}e)(\sqrt{c}f+i\sqrt{a}g))}\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-((\sqrt{c}d-i\sqrt{a}e)(\sqrt{c}f+i\sqrt{a}g))}\sqrt{d+ex}}\right)}{\sqrt{a}(\sqrt{c}d+i\sqrt{a}e)^2\sqrt{cd^2+ae^2}}$$

input
```
Integrate[Sqrt[f + g*x]/((d + e*x)^(5/2)*(a + c*x^2)),x]
```

output
```
(2*Sqrt[f + g*x]*(a*e^4*(f + g*x) + c*d*e*(-6*d^2*g + 6*e^2*f*x + d*e*(7*f
 - 5*g*x))))/(3*(c*d^2 + a*e^2)^2*(-(e*f) + d*g)*(d + e*x)^(3/2)) - (I*Sqr
t[c]*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*ArcTan[(
Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt
[c]*f - I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*(Sqrt[c]*d - I*Sqrt[a]*e)
^2*Sqrt[c*d^2 + a*e^2]) + (I*Sqrt[c]*Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqr
t[c]*f + I*Sqrt[a]*g))]*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-
((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(S
qrt[a]*(Sqrt[c]*d + I*Sqrt[a]*e)^2*Sqrt[c*d^2 + a*e^2])
```

3.609.3 Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 582, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {660, 55, 48, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}}{(a+cx^2)(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{ 660}$$

$$\frac{\int \frac{cdf+aeg-c(ef-dg)x}{(d+ex)^{3/2}\sqrt{f+gx}(cx^2+a)}dx}{ae^2+cd^2}+\frac{e(ef-dg)\int \frac{1}{(d+ex)^{5/2}\sqrt{f+gx}}dx}{ae^2+cd^2}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{\int \frac{cdf+aeg-c(ef-dg)x}{(d+ex)^{3/2}\sqrt{f+gx}(cx^2+a)}dx}{ae^2+cd^2}+\frac{e(ef-dg)\left(-\frac{2g\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}}dx}{3(ef-dg)}-\frac{2\sqrt{f+gx}}{3(d+ex)^{3/2}(ef-dg)}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{\int \frac{cdf+aeg-c(ef-dg)x}{(d+ex)^{3/2}\sqrt{f+gx}(cx^2+a)}dx}{ae^2+cd^2}+\frac{e(ef-dg)\left(\frac{4g\sqrt{f+gx}}{3\sqrt{d+ex}(ef-dg)^2}-\frac{2\sqrt{f+gx}}{3(d+ex)^{3/2}(ef-dg)}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \text{ 2348}$$

$$\frac{\int \left(\frac{\sqrt{-a}(cdf+aeg)-a\sqrt{c}(ef-dg)}{2a(\sqrt{c}x+\sqrt{-a})(d+ex)^{3/2}\sqrt{f+gx}}+\frac{a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a(\sqrt{-a}-\sqrt{c}x)(d+ex)^{3/2}\sqrt{f+gx}}\right)dx}{ae^2+cd^2}+\frac{e(ef-dg)\left(\frac{4g\sqrt{f+gx}}{3\sqrt{d+ex}(ef-dg)^2}-\frac{2\sqrt{f+gx}}{3(d+ex)^{3/2}(ef-dg)}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\sqrt{c}(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}(\sqrt{c}d-\sqrt{-a}e)^{3/2}\sqrt{\sqrt{c}f-\sqrt{-a}g}}+\frac{\sqrt{c}(a\sqrt{c}(ef-dg)+\sqrt{-a}cdf+\sqrt{-a}aeg)\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{a(\sqrt{-a}e+\sqrt{c}d)^{3/2}\sqrt{\sqrt{-a}g+\sqrt{c}f}}}{ae^2+cd^2}+\frac{e(ef-dg)\left(\frac{4g\sqrt{f+gx}}{3\sqrt{d+ex}(ef-dg)^2}-\frac{2\sqrt{f+gx}}{3(d+ex)^{3/2}(ef-dg)}\right)}{ae^2+cd^2}$$

input
```
Int[Sqrt[f + g*x]/((d + e*x)^(5/2)*(a + c*x^2)),x]
```

output
```
(e*(e*f - d*g)*((-2*Sqrt[f + g*x])/(3*(e*f - d*g)*(d + e*x)^(3/2)) + (4*g*
Sqrt[f + g*x])/(3*(e*f - d*g)^2*Sqrt[d + e*x])))/(c*d^2 + a*e^2) + ((e*(c*
d*f + a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*g))*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt
[c]*d + Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d + e*x]) - (e*(c*d*f + a*e*g + Sqrt[
-a]*Sqrt[c]*(e*f - d*g))*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)
*(e*f - d*g)*Sqrt[d + e*x]) + (Sqrt[c]*(c*d*f + a*e*g + Sqrt[-a]*Sqrt[c]*(
e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqr
t[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)^(
3/2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) + (Sqrt[c]*(Sqrt[-a]*c*d*f + Sqrt[-a]*a
*e*g + a*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d
 + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(a*(Sqrt[c]*d + Sq
rt[-a]*e)^(3/2)*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]))/(c*d^2 + a*e^2)
```

3.609.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 660
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Simp[(-g)*((e*f - d*g)/(c*f^2 + a*g^2))   Int[(d + e*x)^
(m - 1)*(f + g*x)^n, x], x] + Simp[1/(c*f^2 + a*g^2)   Int[Simp[c*d*f + a*e
*g + c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*((f + g*x)^(n + 1)/(a + c*x^2)),
 x], x] /; FreeQ[{a, c, d, e, f, g}, x] && !IntegerQ[m] &&  !IntegerQ[n] &
& GtQ[m, 0] && LtQ[n, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.609. $\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}(a+cx^2)}\,dx$

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

3.609.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 14860 vs. 2(501) = 1002.

Time = 0.46 (sec) , antiderivative size = 14861, normalized size of antiderivative = 24.24

method	result	size
default	Expression too large to display	14861

input
```
int((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.609.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10812 vs. 2(501) = 1002.

Time = 160.90 (sec) , antiderivative size = 10812, normalized size of antiderivative = 17.64

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.609.6 Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}\,(a+cx^2)}\,dx = \int \frac{\sqrt{f+gx}}{(a+cx^2)\,(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)**(1/2)/(e*x+d)**(5/2)/(c*x**2+a),x)
```

output
```
Integral(sqrt(f + g*x)/((a + c*x**2)*(d + e*x)**(5/2)), x)
```

3.609.7 Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}\,(a+cx^2)}\,dx = \int \frac{\sqrt{gx+f}}{(cx^2+a)(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(g*x + f)/((c*x^2 + a)*(e*x + d)^(5/2)), x)
```

3.609.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}\,(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

3.609.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}\left(a+cx^2\right)}\,dx=\int \frac{\sqrt{f+g\,x}}{\left(c\,x^2+a\right)\,{\left(d+e\,x\right)}^{5/2}}\,dx$$

input
```
int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(5/2)),x)
```

output
```
int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(5/2)), x)
```

3.610 $\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}(a+cx^2)} dx$

3.610.1 Optimal result

Integrand size = 28, antiderivative size = 337

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \frac{2e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}} + \frac{\left(cd^2-2\sqrt{-a}\sqrt{c}de-ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\left(cd^2+2\sqrt{-a}\sqrt{c}de-ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

output
```
2*e^(3/2)*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/c/g^(1/2)+a
rctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a
)^(1/2)+d*c^(1/2))^(1/2))*(c*d^2-a*e^2-2*d*e*(-a)^(1/2)*c^(1/2))/c/(-a)^(1
/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctan
h((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)
+d*c^(1/2))^(1/2))*(c*d^2-a*e^2+2*d*e*(-a)^(1/2)*c^(1/2))/c/(-a)^(1/2)/(e*
(-a)^(1/2)+d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

3.610.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \frac{\frac{\left(i\sqrt{c}d+\sqrt{a}e\right)\sqrt{cd^2+ae^2}\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}} + \frac{\left(-i\sqrt{c}d+\sqrt{a}e\right)\sqrt{cd^2+ae^2}\arctan\left(\cdots\right.}{\sqrt{a}\sqrt{-\left(\left(\sqrt{c}d-i\cdots\right.\right.}}}{c}$$

input
```
Integrate[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

output
```
(((I*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[c*d^2 + a*e^2]*ArcTan[(Sqrt[c*d^2 + a*e^2
]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*
g))]*Sqrt[d + e*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f
 - I*Sqrt[a]*g))]) + (((-I)*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[c*d^2 + a*e^2]*Arc
Tan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*
(Sqrt[c]*f + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d -
I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]) + (2*e^(3/2)*ArcTanh[(Sqrt[e]*Sq
rt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/Sqrt[g])/c
```

3.610.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {661, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)\sqrt{f+gx}}dx$$

$$\Big\downarrow \ 661$$

$$\int \left(\frac{-ae^2+cd^2+2cdex}{c(a+cx^2)\sqrt{d+ex}\sqrt{f+gx}}+\frac{e^2}{c\sqrt{d+ex}\sqrt{f+gx}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\left(-2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\left(2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}c\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}+\frac{2e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}}$$

input
```
Int[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

output
```
(2*e^(3/2)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(c*Sq
rt[g]) + ((c*d^2 - 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(Sqrt[Sqrt[c]*f
 - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])
])/(Sqrt[-a]*c*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g])
- ((c*d^2 + 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt
[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqr
t[-a]*c*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

3.610.3.1 Defintions of rubi rules used

rule 661
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGt
Q[m + 1/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.610.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2335 vs. 2(257) = 514.

Time = 0.46 (sec) , antiderivative size = 2336, normalized size of antiderivative = 6.93

method	result	size
default	Expression too large to display	2336

input
```
int((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+
2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+
d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))
*a^2*e^2*g^2*(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)
/c)^(1/2)-ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2
)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*c*d^2*g^2*(e*g)^(1/2
)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)+ln((2*(-a*c)^
(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*
d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+
2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*c*e^2*f^2*(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(
-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)-2*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c
*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f
)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^
(1/2)))*a*d*e*g^2*(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c
*d*f)/c)^(1/2)*(-a*c)^(1/2)-ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((
-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(
1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c^2*
d^2*f^2*(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(
1/2)-2*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-...
```

3.610.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.610.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{(a+cx^2)\,\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
```

output
```
Integral((d + e*x)**(3/2)/((a + c*x**2)*sqrt(f + g*x)), x)
```

3.610.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+a)\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*x^2 + a)*sqrt(g*x + f)), x)
```

3.610.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.610.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{(d+e\,x)^{3/2}}{\sqrt{f+g\,x}\,\left(c\,x^2+a\right)}\,dx$$

input
```
int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(a + c*x^2)),x)
```

output
```
int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(a + c*x^2)), x)
```

3.611 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}(a+cx^2)}\,dx$

3.611.1 Optimal result

Integrand size = 28, antiderivative size = 240

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\sqrt{\sqrt{c}d+\sqrt{-a}e}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

output
```
arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-
a)^(1/2)+d*c^(1/2))^(1/2))*(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(-a)^(1/2)/c^(1
/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*
c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*(e*(-a)^(1/2)
+d*c^(1/2))^(1/2)/(-a)^(1/2)/c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

3.611.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 6.75 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.40

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = -\frac{1}{4}(ef-dg)\text{RootSum}\Big[ce^4f^2+ae^4g^2+4ce^3f^2g\#1^2 - 8cde^2fg^2\#1^2-4ae^3g^3\#1^2+6ce^2f^2g^2\#1^4-16cdefg^3\#1^4+16cd^2g^4\#1^4+6ae^2g^4\#1^4 + 4cef^2g^3\#1^6-8cdfg^4\#1^6-4aeg^5\#1^6+cf^2g^4\#1^8 + ag^6\#1^8\&, \frac{-e^3\log(f+gx)+2e^3\log\left(\sqrt{d-\frac{ef}{g}}-\sqrt{d+ex}+\sqrt{f+gx}\#1\right)-e^2g\log(f+gx)\#1^2+2e}{ce^3f^2\#1-2cde^2fg\#1-}$$

input
```
Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

output
```
-1/4*((e*f - d*g)*RootSum[c*e^4*f^2 + a*e^4*g^2 + 4*c*e^3*f^2*g*#1^2 - 8*c
*d*e^2*f*g^2*#1^2 - 4*a*e^3*g^3*#1^2 + 6*c*e^2*f^2*g^2*#1^4 - 16*c*d*e*f*g
^3*#1^4 + 16*c*d^2*g^4*#1^4 + 6*a*e^2*g^4*#1^4 + 4*c*e*f^2*g^3*#1^6 - 8*c*
d*f*g^4*#1^6 - 4*a*e*g^5*#1^6 + c*f^2*g^4*#1^8 + a*g^6*#1^8 & , (-(e^3*Log
[f + g*x]) + 2*e^3*Log[Sqrt[d - (e*f)/g] - Sqrt[d + e*x] + Sqrt[f + g*x]*#
1] - e^2*g*Log[f + g*x]*#1^2 + 2*e^2*g*Log[Sqrt[d - (e*f)/g] - Sqrt[d + e*
x] + Sqrt[f + g*x]*#1]*#1^2 + e*g^2*Log[f + g*x]*#1^4 - 2*e*g^2*Log[Sqrt[d
 - (e*f)/g] - Sqrt[d + e*x] + Sqrt[f + g*x]*#1]*#1^4 + g^3*Log[f + g*x]*#1
^6 - 2*g^3*Log[Sqrt[d - (e*f)/g] - Sqrt[d + e*x] + Sqrt[f + g*x]*#1]*#1^6)
/(c*e^3*f^2*#1 - 2*c*d*e^2*f*g*#1 - a*e^3*g^2*#1 + 3*c*e^2*f^2*g*#1^3 - 8*
c*d*e*f*g^2*#1^3 + 8*c*d^2*g^3*#1^3 + 3*a*e^2*g^3*#1^3 + 3*c*e*f^2*g^2*#1^
5 - 6*c*d*f*g^3*#1^5 - 3*a*e*g^4*#1^5 + c*f^2*g^3*#1^7 + a*g^5*#1^7) & ])
```

3.611.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {661, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)\sqrt{f+gx}}\,dx$$

$$\downarrow 661$$

$$\int \left(\frac{\sqrt{-a}d - \frac{ae}{\sqrt{c}}}{2a\left(\sqrt{-a} - \sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}} + \frac{\frac{ae}{\sqrt{c}} + \sqrt{-a}d}{2a\left(\sqrt{-a} + \sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}} \right) dx$$

$$\downarrow 2009$$

$$\frac{\sqrt{\sqrt{c}d - \sqrt{-a}e}\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f - \sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d - \sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}f - \sqrt{-a}g}} - \frac{\sqrt{\sqrt{-a}e + \sqrt{c}d}\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g + \sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e + \sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{-a}g + \sqrt{c}f}}$$

input
```
Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

output
```
(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d
 + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[c]*
Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - (Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*ArcTanh[(Sqr
t[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqr
t[f + g*x])])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

3.611.3.1 Defintions of rubi rules used

rule 661
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
 + e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGt
Q[m + 1/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.611.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1382 vs. $2(176) = 352$.

Time = 0.46 (sec) , antiderivative size = 1383, normalized size of antiderivative = 5.76

method	result	size
default	Expression too large to display	1383

input
```
int((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*((-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e
*g-c*d*f)/c)^(1/2)*(-a*c)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2
*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d
))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*
a*e*g^2+(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(-a*c)^
(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)
^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*
g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c*e*f^2+(-((-a*c)^(1/2)*d*
g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+
c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+
f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)
^(1/2)))*a*c*d*g^2+(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1
/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(
1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+
(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c^2*d*f^2+(((-a*c)^(1/2)*d*g
+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(-a*c)^(1/2)*ln((-2*(-a*c)^(1/2)*e
*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c
)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*
f)/(c*x+(-a*c)^(1/2)))*a*e*g^2+(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c
*d*f)/c)^(1/2)*(-a*c)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2...
```

3.611.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1913 vs. $2(176) = 352$.

Time = 8.91 (sec) , antiderivative size = 1913, normalized size of antiderivative = 7.97

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(-(c*d*f + a*e*g + (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e
*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 +
 a^2*c*g^2))*log(-(e^2*f^2 - d^2*g^2 + 2*(c*e*f^2 - c*d*f*g + (a*c^2*f^2*g
 + a^2*c*g^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2
*f^2*g^2 + a^3*c*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g +
 (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4
+ 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2)) + 2*(e^2*f*g -
 d*e*g^2)*x - (2*c^2*d*f^3 + 2*a*c*d*f*g^2 + (c^2*e*f^3 + c^2*d*f^2*g + a*
c*e*f*g^2 + a*c*d*g^3)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4
 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/x) + 1/4*sqrt(-(c*d*f + a*e*g + (a*c^2
*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2
*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2))*log(-(e^2*f^2 - d^2*g
^2 - 2*(c*e*f^2 - c*d*f*g + (a*c^2*f^2*g + a^2*c*g^3)*sqrt(-(e^2*f^2 - 2*d
*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))*sqrt(e*x +
 d)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g + (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^
2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))
/(a*c^2*f^2 + a^2*c*g^2)) + 2*(e^2*f*g - d*e*g^2)*x - (2*c^2*d*f^3 + 2*a*c
*d*f*g^2 + (c^2*e*f^3 + c^2*d*f^2*g + a*c*e*f*g^2 + a*c*d*g^3)*x)*sqrt(-(e
^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4))
)/x) - 1/4*sqrt(-(c*d*f + a*e*g - (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^...
```

3.611.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{\sqrt{d+ex}}{(a+cx^2)\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/((a + c*x**2)*sqrt(f + g*x)), x)
```

3.611.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{\sqrt{ex+d}}{(cx^2+a)\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/((c*x^2 + a)*sqrt(g*x + f)), x)
```

3.611.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.611.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Hanged}$$

input
```
int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(a + c*x^2)),x)
```

output
```
\text{Hanged}
```

3.612 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}(a+cx^2)}\,dx$

3.612.1 Optimal result

Integrand size = 28, antiderivative size = 230

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

output
```
arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-
a)^(1/2)+d*c^(1/2))^(1/2))/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(-g*
(-a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))
^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))/(-a)^(1/2)/(e*(-a)^(1
/2)+d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

3.612.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.24

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx = \frac{\sqrt[4]{-1}\left(-\frac{\sqrt{-i\sqrt{c}d+\sqrt{a}e}\arctan\left(\frac{\sqrt[4]{-1}\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-i\sqrt{c}d+\sqrt{a}e}\sqrt{\sqrt{c}f-i\sqrt{a}g}\sqrt{d+ex}}\right)}{\sqrt{\sqrt{c}f-i\sqrt{a}g}}+\frac{\sqrt{i\sqrt{c}d+\sqrt{a}e}\operatorname{arctanh}\left(\frac{\sqrt[4]{-1}\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{i\sqrt{c}d+\sqrt{a}e}\sqrt{\sqrt{c}f+i\sqrt{a}g}\sqrt{d+ex}}\right)}{\sqrt{\sqrt{c}f+i\sqrt{a}g}}\right)}{\sqrt{a}\sqrt{cd^2+ae^2}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + c*x^2)),x]
```

output
```
((-1)^(1/4)*(-((Sqrt[(-I)*Sqrt[c]*d + Sqrt[a]*e]*ArcTan[((-1)^(1/4)*Sqrt[c
*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[(-I)*Sqrt[c]*d + Sqrt[a]*e]*Sqrt[Sqrt[c
]*f - I*Sqrt[a]*g]*Sqrt[d + e*x])])/Sqrt[Sqrt[c]*f - I*Sqrt[a]*g]) + (Sqrt
[I*Sqrt[c]*d + Sqrt[a]*e]*ArcTanh[((-1)^(1/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[f +
 g*x])/(Sqrt[I*Sqrt[c]*d + Sqrt[a]*e]*Sqrt[Sqrt[c]*f + I*Sqrt[a]*g]*Sqrt[d
 + e*x])])/Sqrt[Sqrt[c]*f + I*Sqrt[a]*g]))/(Sqrt[a]*Sqrt[c*d^2 + a*e^2])
```

3.612.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {662, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)\sqrt{d+ex}\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{ 662}$$

$$\int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}+\frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}$$

input
```
Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + c*x^2)),x]
```

output
```
ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqr
t[-a]*e]*Sqrt[f + g*x])]/(Sqrt[-a]*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[
c]*f - Sqrt[-a]*g]) - ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])
/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]/(Sqrt[-a]*Sqrt[Sqrt[c]*d +
Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

3.612.3.1 Defintions of rubi rules used

rule 662
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && !IntegerQ[m] && !Inte
gerQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.612.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1414 vs. $2(170) = 340$.

Time = 0.45 (sec) , antiderivative size = 1415, normalized size of antiderivative = 6.15

method	result	size
default	Expression too large to display	1415

input
```
int(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*c^2*(ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2
)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a^2*e^2*g^2*(-((-a*c)^
(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)+ln((2*(-a*c)^(1/2)*e*g*x+
c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2
)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*
x-(-a*c)^(1/2)))*a*c*d^2*g^2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*
d*f)/c)^(1/2)+ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*
g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^
(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*c*e^2*f^2*(-((-a
*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)+ln((2*(-a*c)^(1/2)*e*
g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^
(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)
/(c*x-(-a*c)^(1/2)))*c^2*d^2*f^2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*
g-c*d*f)/c)^(1/2)-ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1
/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(
-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*a^2*e^2*g^2*
(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)-ln((-2*(-a*c)^(1
/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d
*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*...
```

3.612.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4325 vs. $2(170) = 340$.

Time = 15.59 (sec) , antiderivative size = 4325, normalized size of antiderivative = 18.80

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(-(c*d*f - a*e*g + ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^
3*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^
2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^
4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^
2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2))*log((e^2*f^2 + 2*d*e*
f*g + d^2*g^2 + 2*(c*d*e*f^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*f*g - ((a*c^2*d
^2*e + a^2*c*e^3)*f^3 + (a*c^2*d^3 + a^2*c*d*e^2)*f^2*g + (a^2*c*d^2*e + a
^3*e^3)*f*g^2 + (a^2*c*d^3 + a^3*d*e^2)*g^3)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*
g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2
*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c
*d^2*e^2 + a^5*e^4)*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f - a*e*
g + ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqrt(-(c*e^2
*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*
e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*
c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)*f^2 +
 (a^2*c*d^2 + a^3*e^2)*g^2)) + 2*(e^2*f*g + d*e*g^2)*x + (2*(c^2*d^3 + a*c
*d*e^2)*f^3 + 2*(a*c*d^3 + a^2*d*e^2)*f*g^2 + ((c^2*d^2*e + a*c*e^3)*f^3 +
 (c^2*d^3 + a*c*d*e^2)*f^2*g + (a*c*d^2*e + a^2*e^3)*f*g^2 + (a*c*d^3 + a^
2*d*e^2)*g^3)*x)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 +
 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*...
```

3.612.6 Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{1}{(a+cx^2)\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
```

output
```
Integral(1/((a + c*x**2)*sqrt(d + e*x)*sqrt(f + g*x)), x)
```

3.612.7 Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{1}{(cx^2+a)\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

3.612.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.612.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Hanged}$$

input
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)*(d + e*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

3.613 $\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}(a+cx^2)}\,dx$

3.613.1 Optimal result

Integrand size = 28, antiderivative size = 354

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx = -\frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}} + \frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}} + \frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)^{3/2}\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)^{3/2}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

output
```
-e*(g*x+f)^(1/2)/(-d*g+e*f)/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))/(e*x+d)^(
1/2)+e*(g*x+f)^(1/2)/(-d*g+e*f)/(-a)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))/(e*x+d
)^(1/2)+arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2
)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2)/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(
1/2))^(3/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^
(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/
2)/(-a)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(3/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2
)
```

3.613.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx = \frac{2e^2\sqrt{f+gx}}{(cd^2+ae^2)\,(-ef+dg)\sqrt{d+ex}} + \frac{i\sqrt{c}\left(\sqrt{c}d+i\sqrt{a}e\right)^2 \arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\,(cd^2+ae^2)^{3/2}\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}} - \frac{i\sqrt{c}\left(\sqrt{c}d-i\sqrt{a}e\right)^2 \arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\,(cd^2+ae^2)^{3/2}\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}}$$

input
```
Integrate[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + c*x^2)),x]
```

output
```
(2*e^2*Sqrt[f + g*x])/((c*d^2 + a*e^2)*(-(e*f) + d*g)*Sqrt[d + e*x]) + (I*
Sqrt[c]*(Sqrt[c]*d + I*Sqrt[a]*e)^2*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g
*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*Sqrt[d
+ e*x])])/(Sqrt[a]*(c*d^2 + a*e^2)^(3/2)*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*
(Sqrt[c]*f - I*Sqrt[a]*g))]) - (I*Sqrt[c]*(Sqrt[c]*d - I*Sqrt[a]*e)^2*ArcT
an[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(
Sqrt[c]*f + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*(c*d^2 + a*e^2)^(3/2)
*Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))])
```

3.613.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {662, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)\,(d+ex)^{3/2}\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \text{ 662}$$

$$\int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)(d+ex)^{3/2}\sqrt{f+gx}}+\frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)(d+ex)^{3/2}\sqrt{f+gx}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)^{3/2}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\left(\sqrt{-a}e+\sqrt{c}d\right)^{3/2}\sqrt{\sqrt{-a}g+\sqrt{c}f}}-\frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)}+\frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{-a}e+\sqrt{c}d\right)(ef-dg)}$$

input
```
Int[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + c*x^2)),x]
```

output
```
-((e*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d
+ e*x])) + (e*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(e*f - d*g
)*Sqrt[d + e*x]) + (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d +
 e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*(Sqrt[c]*d
 - Sqrt[-a]*e)^(3/2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - (Sqrt[c]*ArcTanh[(Sqr
t[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqr
t[f + g*x])])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*Sqrt[Sqrt[c]*f + Sq
rt[-a]*g])
```

3.613.3.1 Defintions of rubi rules used

rule 662
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && !IntegerQ[m] && !Inte
gerQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.613.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 10976 vs. 2(270) = 540.

Time = 0.47 (sec) , antiderivative size = 10977, normalized size of antiderivative = 31.01

method	result	size
default	Expression too large to display	10977

input
```
int(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.613.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11846 vs. 2(270) = 540.

Time = 56.03 (sec) , antiderivative size = 11846, normalized size of antiderivative = 33.46

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.613.6 Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{1}{(a+cx^2)\,(d+ex)^{\frac{3}{2}}\,\sqrt{f+gx}}\,dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
```

output
```
Integral(1/((a + c*x**2)*(d + e*x)**(3/2)*sqrt(f + g*x)), x)
```

3.613.7 Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx=\int \frac{1}{(cx^2+a)(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

3.613.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.613.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx=\int \frac{1}{\sqrt{f+g\,x}\,\left(c\,x^2+a\right)\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)*(d + e*x)^(3/2)),x)
```

output
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)*(d + e*x)^(3/2)), x)
```

3.614 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}(a+cx^2)}\,dx$

3.614.1 Optimal result

Integrand size = 28, antiderivative size = 625

$$
\begin{aligned}
\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\,(a+cx^2)}\,dx &= \frac{2(ef-dg)\sqrt{d+ex}}{(cf^2+ag^2)\,\sqrt{f+gx}} \\
&- \frac{2\sqrt{e}(ef-dg)\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{g}\,(cf^2+ag^2)} \\
&- \frac{\sqrt{e}\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{g}\,(cf^2+ag^2)} \\
&+ \frac{\sqrt{e}\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{g}\,(cf^2+ag^2)} \\
&+ \frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}f-\sqrt{-a}g}\,(cf^2+ag^2)} \\
&- \frac{\sqrt{\sqrt{c}d+\sqrt{-a}e}\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}f+\sqrt{-a}g}\,(cf^2+ag^2)}
\end{aligned}
$$

output
```
-2*(-d*g+e*f)*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))*e^(1/2)
/(a*g^2+c*f^2)/g^(1/2)-arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2)
)*(c*d*f+a*e*g-(-d*g+e*f)*(-a)^(1/2)*c^(1/2))*e^(1/2)/(a*g^2+c*f^2)/(-a)^(
1/2)/c^(1/2)/g^(1/2)+arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))*
(c*d*f+a*e*g+(-d*g+e*f)*(-a)^(1/2)*c^(1/2))*e^(1/2)/(a*g^2+c*f^2)/(-a)^(1/
2)/c^(1/2)/g^(1/2)+2*(-d*g+e*f)*(e*x+d)^(1/2)/(a*g^2+c*f^2)/(g*x+f)^(1/2)+
arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-
a)^(1/2)+d*c^(1/2))^(1/2))*(c*d*f+a*e*g-(-d*g+e*f)*(-a)^(1/2)*c^(1/2))*(-e
*(-a)^(1/2)+d*c^(1/2))^(1/2)/(a*g^2+c*f^2)/(-a)^(1/2)/c^(1/2)/(-g*(-a)^(1/
2)+f*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(
g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*(c*d*f+a*e*g+(-d*g+e*f)*(-a)^
(1/2)*c^(1/2))*(e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(a*g^2+c*f^2)/(-a)^(1/2)/c^(
1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

3.614.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 8.15 (sec) , antiderivative size = 1049, normalized size of antiderivative = 1.68

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\,(a+cx^2)}\,dx = \frac{(ef-dg)\left(2\sqrt{d+ex}-\frac{1}{2}\sqrt{f+gx}\text{RootSum}\left[ce^4f^2+ae^4g^2+4ce^3f^2g\#1^2-\right.\right.}{}$$

input
```
Integrate[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

output
```
((e*f - d*g)*(2*Sqrt[d + e*x] - (Sqrt[f + g*x]*RootSum[c*e^4*f^2 + a*e^4*g
^2 + 4*c*e^3*f^2*g*#1^2 - 8*c*d*e^2*f*g^2*#1^2 - 4*a*e^3*g^3*#1^2 + 6*c*e^
2*f^2*g^2*#1^4 - 16*c*d*e*f*g^3*#1^4 + 16*c*d^2*g^4*#1^4 + 6*a*e^2*g^4*#1^
4 + 4*c*e*f^2*g^3*#1^6 - 8*c*d*f*g^4*#1^6 - 4*a*e*g^5*#1^6 + c*f^2*g^4*#1^
8 + a*g^6*#1^8 & , (-(c*d*e^3*f*Log[f + g*x]) - a*e^4*g*Log[f + g*x] + 2*c
*d*e^3*f*Log[Sqrt[d - (e*f)/g] - Sqrt[d + e*x] + Sqrt[f + g*x]*#1] + 2*a*e
^4*g*Log[Sqrt[d - (e*f)/g] - Sqrt[d + e*x] + Sqrt[f + g*x]*#1] - c*d*e^2*f
*g*Log[f + g*x]*#1^2 + 2*c*d^2*e*g^2*Log[f + g*x]*#1^2 + a*e^3*g^2*Log[f +
 g*x]*#1^2 + 2*c*d*e^2*f*g*Log[Sqrt[d - (e*f)/g] - Sqrt[d + e*x] + Sqrt[f
+ g*x]*#1]*#1^2 - 4*c*d^2*e*g^2*Log[Sqrt[d - (e*f)/g] - Sqrt[d + e*x] + Sq
rt[f + g*x]*#1]*#1^2 - 2*a*e^3*g^2*Log[Sqrt[d - (e*f)/g] - Sqrt[d + e*x] +
 Sqrt[f + g*x]*#1]*#1^2 + c*d*e*f*g^2*Log[f + g*x]*#1^4 - 2*c*d^2*g^3*Log[
f + g*x]*#1^4 - a*e^2*g^3*Log[f + g*x]*#1^4 - 2*c*d*e*f*g^2*Log[Sqrt[d - (
e*f)/g] - Sqrt[d + e*x] + Sqrt[f + g*x]*#1]*#1^4 + 4*c*d^2*g^3*Log[Sqrt[d
- (e*f)/g] - Sqrt[d + e*x] + Sqrt[f + g*x]*#1]*#1^4 + 2*a*e^2*g^3*Log[Sqrt
[d - (e*f)/g] - Sqrt[d + e*x] + Sqrt[f + g*x]*#1]*#1^4 + c*d*f*g^3*Log[f +
 g*x]*#1^6 + a*e*g^4*Log[f + g*x]*#1^6 - 2*c*d*f*g^3*Log[Sqrt[d - (e*f)/g]
 - Sqrt[d + e*x] + Sqrt[f + g*x]*#1]*#1^6 - 2*a*e*g^4*Log[Sqrt[d - (e*f)/g
] - Sqrt[d + e*x] + Sqrt[f + g*x]*#1]*#1^6)/(c*e^3*f^2*#1 - 2*c*d*e^2*f*g*
#1 - a*e^3*g^2*#1 + 3*c*e^2*f^2*g*#1^3 - 8*c*d*e*f*g^2*#1^3 + 8*c*d^2*g...
```

3.614.3 Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 574, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {660, 57, 66, 221, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{ 660}$$

$$\frac{\int \frac{\sqrt{d+ex}(cdf+aeg+c(ef-dg)x)}{\sqrt{f+gx}(cx^2+a)}dx}{ag^2+cf^2}-\frac{g(ef-dg)\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}}dx}{ag^2+cf^2}$$

$$\Big\downarrow \text{ 57}$$

$$\frac{\int \frac{\sqrt{d+ex}(cdf+aeg+c(ef-dg)x)}{\sqrt{f+gx}(cx^2+a)}dx}{ag^2+cf^2}-\frac{g(ef-dg)\left(\frac{e\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}dx}{g}-\frac{2\sqrt{d+ex}}{g\sqrt{f+gx}}\right)}{ag^2+cf^2}$$

$\Big\downarrow$ 66

$$\frac{\int \frac{\sqrt{d+ex}(cdf+aeg+c(ef-dg)x)}{\sqrt{f+gx}(cx^2+a)}dx}{ag^2+cf^2}-\frac{g(ef-dg)\left(\frac{2e\int \frac{1}{e-\frac{g(d+ex)}{f+gx}}d\frac{\sqrt{d+ex}}{\sqrt{f+gx}}}{g}-\frac{2\sqrt{d+ex}}{g\sqrt{f+gx}}\right)}{ag^2+cf^2}$$

$\Big\downarrow$ 221

$$\frac{\int \frac{\sqrt{d+ex}(cdf+aeg+c(ef-dg)x)}{\sqrt{f+gx}(cx^2+a)}dx}{ag^2+cf^2}-\frac{g(ef-dg)\left(\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{g^{3/2}}-\frac{2\sqrt{d+ex}}{g\sqrt{f+gx}}\right)}{ag^2+cf^2}$$

$\Big\downarrow$ 2348

$$\frac{\int \left(\frac{\sqrt{d+ex}\left(\sqrt{-a}(cdf+aeg)-a\sqrt{c}(ef-dg)\right)}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{f+gx}}+\frac{\left(a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)\right)\sqrt{d+ex}}{2a\left(\sqrt{c}x+\sqrt{-a}\right)\sqrt{f+gx}}\right)dx}{ag^2+cf^2}-\frac{g(ef-dg)\left(\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{g^{3/2}}-\frac{2\sqrt{d+ex}}{g\sqrt{f+gx}}\right)}{ag^2+cf^2}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{\sqrt{e}\left(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{g}}+\frac{\sqrt{e}\left(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{g}}+\frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(-\sqrt{-a}\cdots\right.}{\cdots}}{ag^2+c\cdots}-\frac{g(ef-dg)\left(\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{g^{3/2}}-\frac{2\sqrt{d+ex}}{g\sqrt{f+gx}}\right)}{ag^2+cf^2}$$

input
```
Int[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

3.614. $\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}(a+cx^2)}\,dx$

output
```
-((g*(e*f - d*g)*((-2*Sqrt[d + e*x])/(g*Sqrt[f + g*x]) + (2*Sqrt[e]*ArcTan
h[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/g^(3/2)))/(c*f^2 + a*g
^2)) + (-((Sqrt[e]*(c*d*f + a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[
(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[c]*Sqrt[g
])) + (Sqrt[e]*(c*d*f + a*e*g + Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqr
t[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[c]*Sqrt[g]) +
 (Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*(c*d*f + a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*
g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d -
 Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*f - Sqrt[-a]*
g]) - (Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*(c*d*f + a*e*g + Sqrt[-a]*Sqrt[c]*(e*f
 - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c
]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*f + Sqrt
[-a]*g]))/(c*f^2 + a*g^2)
```

3.614.3.1 Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 660
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Simp[(-g)*((e*f - d*g)/(c*f^2 + a*g^2))   Int[(d + e*x)^
(m - 1)*(f + g*x)^n, x], x] + Simp[1/(c*f^2 + a*g^2)   Int[Simp[c*d*f + a*e
*g + c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*((f + g*x)^(n + 1)/(a + c*x^2)),
 x], x] /; FreeQ[{a, c, d, e, f, g}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &
& GtQ[m, 0] && LtQ[n, -1]
```

3.614. $\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}(a+cx^2)}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

3.614.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8263 vs. $2(497) = 994$.

Time = 0.48 (sec) , antiderivative size = 8264, normalized size of antiderivative = 13.22

method	result	size
default	Expression too large to display	8264

input
```
int((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.614.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.614.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{(a+cx^2)\,(f+gx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
```

output
```
Integral((d + e*x)**(3/2)/((a + c*x**2)*(f + g*x)**(3/2)), x)
```

3.614.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+a)(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*x^2 + a)*(g*x + f)^(3/2)), x)
```

3.614.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\,(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

3.614.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \frac{(d+e\,x)^{3/2}}{(f+g\,x)^{3/2}\,(c\,x^2+a)}\,dx$$

input
```
int((d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)),x)
```

output
```
int((d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)), x)
```

3.615 $\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)}\,dx$

3.615.1 Optimal result

Integrand size = 28, antiderivative size = 351

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)}\,dx = -\frac{2g\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} + \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}\,(cf^2+ag^2)} - \frac{\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}\,(cf^2+ag^2)}$$

output
```
-2*g*(e*x+d)^(1/2)/(a*g^2+c*f^2)/(g*x+f)^(1/2)+arctanh((e*x+d)^(1/2)*(-g*(
-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2))*
(c*d*f+a*e*g-(-d*g+e*f)*(-a)^(1/2)*c^(1/2))/(a*g^2+c*f^2)/(-a)^(1/2)/(-e*(
-a)^(1/2)+d*c^(1/2))^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e*x+d)
^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2
))^(1/2))*(c*d*f+a*e*g+(-d*g+e*f)*(-a)^(1/2)*c^(1/2))/(a*g^2+c*f^2)/(-a)^(
1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

3.615.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)}\,dx = -\frac{2g\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} + \frac{i\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\arctan\left(\frac{\sqrt{cf^2+ag^2}\sqrt{d+ex}}{\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\sqrt{f+gx}}\right)}{\sqrt{a}\left(\sqrt{c}f+i\sqrt{a}g\right)\sqrt{cf^2+ag^2}} - \frac{i\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\arctan\left(\frac{\sqrt{cf^2+ag^2}\sqrt{d+ex}}{\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\sqrt{f+gx}}\right)}{\sqrt{a}\left(\sqrt{c}f-i\sqrt{a}g\right)\sqrt{cf^2+ag^2}}$$

input
```
Integrate[Sqrt[d + e*x]/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

output
```
(-2*g*Sqrt[d + e*x])/((c*f^2 + a*g^2)*Sqrt[f + g*x]) + (I*Sqrt[-((Sqrt[c]*
d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*ArcTan[(Sqrt[c*f^2 + a*g^2]*S
qrt[d + e*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))
]*Sqrt[f + g*x])])/(Sqrt[a]*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[c*f^2 + a*g^2])
 - (I*Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]*ArcTan[
(Sqrt[c*f^2 + a*g^2]*Sqrt[d + e*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqr
t[c]*f + I*Sqrt[a]*g))]*Sqrt[f + g*x])])/(Sqrt[a]*(Sqrt[c]*f - I*Sqrt[a]*g
)*Sqrt[c*f^2 + a*g^2])
```

3.615.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {660, 48, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 660}$$

$$\frac{\int \frac{cdf+aeg+c(ef-dg)x}{\sqrt{d+ex}\sqrt{f+gx}(cx^2+a)}dx}{ag^2+cf^2}-\frac{g(ef-dg)\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}}dx}{ag^2+cf^2}$$

$$\Big\downarrow \text{48}$$

$$\frac{\int \frac{cdf+aeg+c(ef-dg)x}{\sqrt{d+ex}\sqrt{f+gx}(cx^2+a)}dx}{ag^2+cf^2}-\frac{2g\sqrt{d+ex}}{\sqrt{f+gx}\,(ag^2+cf^2)}$$

$$\Big\downarrow \text{2348}$$

$$\frac{\int \left(\frac{\sqrt{-a}(cdf+aeg)-a\sqrt{c}(ef-dg)}{2a(\sqrt{-a}-\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}+\frac{a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a(\sqrt{c}x+\sqrt{-a})\sqrt{d+ex}\sqrt{f+gx}}\right)dx}{ag^2+cf^2}-\frac{2g\sqrt{d+ex}}{\sqrt{f+gx}\,(ag^2+cf^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}}{ag^2+cf^2}-\frac{2g\sqrt{d+ex}}{\sqrt{f+gx}\,(ag^2+cf^2)}$$

input
```
Int[Sqrt[d + e*x]/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

output
```
(-2*g*Sqrt[d + e*x])/((c*f^2 + a*g^2)*Sqrt[f + g*x]) + (((c*d*f + a*e*g -
Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d
 + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[Sqr
t[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - ((c*d*f + a*e*g + Sqr
t[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d +
e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[Sqrt[c
]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]))/(c*f^2 + a*g^2)
```

3.615. $\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)}\,dx$

3.615.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 660
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Simp[(-g)*((e*f - d*g)/(c*f^2 + a*g^2))   Int[(d + e*x)^
(m - 1)*(f + g*x)^n, x], x] + Simp[1/(c*f^2 + a*g^2)   Int[Simp[c*d*f + a*e
*g + c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*((f + g*x)^(n + 1)/(a + c*x^2)),
 x], x] /; FreeQ[{a, c, d, e, f, g}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &
& GtQ[m, 0] && LtQ[n, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

3.615.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5382 vs. $2(279) = 558$.

Time = 0.46 (sec) , antiderivative size = 5383, normalized size of antiderivative = 15.34

method	result	size
default	Expression too large to display	5383

input
```
int((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.615.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5844 vs. $2(279) = 558$.

Time = 46.42 (sec) , antiderivative size = 5844, normalized size of antiderivative = 16.65

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.615.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \frac{\sqrt{d+ex}}{(a+cx^2)\,(f+gx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
```

output
```
Integral(sqrt(d + e*x)/((a + c*x**2)*(f + g*x)**(3/2)), x)
```

3.615.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \frac{\sqrt{ex+d}}{(cx^2+a)(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/((c*x^2 + a)*(g*x + f)^(3/2)), x)
```

3.615.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

3.615.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)}\,dx = \int \frac{\sqrt{d+e\,x}}{(f+g\,x)^{3/2}\,(c\,x^2+a)}\,dx$$

input
```
int((d + e*x)^(1/2)/((f + g*x)^(3/2)*(a + c*x^2)),x)
```

output
```
int((d + e*x)^(1/2)/((f + g*x)^(3/2)*(a + c*x^2)), x)
```

3.616 $\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}(a+cx^2)} dx$

3.616.1 Optimal result

Integrand size = 28, antiderivative size = 354

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}(a+cx^2)} dx = \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)\sqrt{f+gx}} - \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}f+\sqrt{-a}g\right)(ef-dg)\sqrt{f+gx}} + \frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(\sqrt{c}f-\sqrt{-a}g\right)^{3/2}} - \frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d+\sqrt{-a}e}\left(\sqrt{c}f+\sqrt{-a}g\right)^{3/2}}$$

output
```
g*(e*x+d)^(1/2)/(-d*g+e*f)/(-a)^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))/(g*x+f)^(1
/2)-g*(e*x+d)^(1/2)/(-d*g+e*f)/(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))/(g*x+f)
^(1/2)+arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)
/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2)/(-a)^(1/2)/(-g*(-a)^(1/2)+f*c^(1
/2))^(3/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(
1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2
)/(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(3/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2)
```

3.616.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}(a+cx^2)}\,dx = \frac{2g^2\sqrt{d+ex}}{(ef-dg)(cf^2+ag^2)\sqrt{f+gx}} - \frac{i\sqrt{c}\left(\sqrt{c}f-i\sqrt{a}g\right)^2\arctan\left(\frac{\sqrt{cf^2+ag^2}\sqrt{d+ex}}{\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\sqrt{f+gx}}\right)}{\sqrt{a}\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}(cf^2+ag^2)^{3/2}} + \frac{i\sqrt{c}\left(\sqrt{c}f+i\sqrt{a}g\right)^2\arctan\left(\frac{\sqrt{cf^2+ag^2}\sqrt{d+ex}}{\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\sqrt{f+gx}}\right)}{\sqrt{a}\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}(cf^2+ag^2)^{3/2}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

output
```
(2*g^2*Sqrt[d + e*x])/((e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x]) - (I*Sqr
t[c]*(Sqrt[c]*f - I*Sqrt[a]*g)^2*ArcTan[(Sqrt[c*f^2 + a*g^2]*Sqrt[d + e*x]
)/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*Sqrt[f + g
*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g)
)]*(c*f^2 + a*g^2)^(3/2)) + (I*Sqrt[c]*(Sqrt[c]*f + I*Sqrt[a]*g)^2*ArcTan[
(Sqrt[c*f^2 + a*g^2]*Sqrt[d + e*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqr
t[c]*f + I*Sqrt[a]*g))]*Sqrt[f + g*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d - I*Sq
rt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]*(c*f^2 + a*g^2)^(3/2))
```

3.616.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {662, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)\sqrt{d+ex}(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \; 662$$

$$\int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}(f+gx)^{3/2}}+\frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}(f+gx)^{3/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(\sqrt{c}f-\sqrt{-a}g\right)^{3/2}}-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\left(\sqrt{-a}g+\sqrt{c}f\right)^{3/2}}+\frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)}-\frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}\left(\sqrt{-a}g+\sqrt{c}f\right)(ef-dg)}$$

input
```
Int[1/(Sqrt[d + e*x]*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

output
```
(g*Sqrt[d + e*x])/(Sqrt[-a]*(Sqrt[c]*f - Sqrt[-a]*g)*(e*f - d*g)*Sqrt[f +
g*x]) - (g*Sqrt[d + e*x])/(Sqrt[-a]*(Sqrt[c]*f + Sqrt[-a]*g)*(e*f - d*g)*S
qrt[f + g*x]) + (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*
x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[Sqrt[c]*
d - Sqrt[-a]*e]*(Sqrt[c]*f - Sqrt[-a]*g)^(3/2)) - (Sqrt[c]*ArcTanh[(Sqrt[S
qrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f
 + g*x])]])/(Sqrt[-a]*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*(Sqrt[c]*f + Sqrt[-a]*g)
^(3/2))
```

3.616.3.1 Defintions of rubi rules used

rule 662
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] &&  !IntegerQ[m] &&  !Inte
gerQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.616.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 10976 vs. 2(270) = 540.

Time = 0.46 (sec) , antiderivative size = 10977, normalized size of antiderivative = 31.01

method	result	size
default	Expression too large to display	10977

input
```
int(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.616.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12028 vs. 2(270) = 540.

Time = 90.51 (sec) , antiderivative size = 12028, normalized size of antiderivative = 33.98

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\left(a+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.616.6 Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\left(a+cx^2\right)}\,dx = \int \frac{1}{\left(a+cx^2\right)\sqrt{d+ex}\,(f+gx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
```

output
```
Integral(1/((a + c*x**2)*sqrt(d + e*x)*(f + g*x)**(3/2)), x)
```

3.616.7 Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\left(a+cx^2\right)}\,dx = \int \frac{1}{(cx^2+a)\sqrt{ex+d}(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)*sqrt(e*x + d)*(g*x + f)^(3/2)), x)
```

3.616.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\left(a+cx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

3.616.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\left(a+cx^2\right)}\,dx = \int \frac{1}{{\left(f+g\,x\right)}^{3/2}\,\left(c\,x^2+a\right)\,\sqrt{d+e\,x}}\,dx$$

input
```
int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(1/2)),x)
```

output
```
int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(1/2)), x)
```

3.617 $\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}(a+cx^2)}\,dx$

3.617.1 Optimal result

Integrand size = 28, antiderivative size = 549

$$\begin{aligned}
&\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\left(a+cx^2\right)}\,dx = \\
&-\frac{e}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} \\
&+\frac{e}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} \\
&+\frac{g\left(2\sqrt{-a}eg-\sqrt{c}(ef+dg)\right)\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)^2\sqrt{f+gx}} \\
&+\frac{g\left(2\sqrt{-a}eg+\sqrt{c}(ef+dg)\right)\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)\left(\sqrt{c}f+\sqrt{-a}g\right)(ef-dg)^2\sqrt{f+gx}} \\
&+\frac{c\,\mathrm{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)^{3/2}\left(\sqrt{c}f-\sqrt{-a}g\right)^{3/2}} \\
&-\frac{c\,\mathrm{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)^{3/2}\left(\sqrt{c}f+\sqrt{-a}g\right)^{3/2}}
\end{aligned}$$

output
```
c*arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*
(-a)^(1/2)+d*c^(1/2))^(1/2))/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(3/2)/(-
g*(-a)^(1/2)+f*c^(1/2))^(3/2)-c*arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1
/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))/(-a)^(1/2)/(e*(-a
)^(1/2)+d*c^(1/2))^(3/2)/(g*(-a)^(1/2)+f*c^(1/2))^(3/2)-e/(-d*g+e*f)/(-a)^
(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))/(e*x+d)^(1/2)/(g*x+f)^(1/2)+e/(-d*g+e*f)/(
-a)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))/(e*x+d)^(1/2)/(g*x+f)^(1/2)+g*(2*e*g*(-
a)^(1/2)-(d*g+e*f)*c^(1/2))*(e*x+d)^(1/2)/(-d*g+e*f)^2/(-a)^(1/2)/(-e*(-a)
^(1/2)+d*c^(1/2))/(-g*(-a)^(1/2)+f*c^(1/2))/(g*x+f)^(1/2)+g*(2*e*g*(-a)^(1
/2)+(d*g+e*f)*c^(1/2))*(e*x+d)^(1/2)/(-d*g+e*f)^2/(-a)^(1/2)/(e*(-a)^(1/2)
+d*c^(1/2))/(g*(-a)^(1/2)+f*c^(1/2))/(g*x+f)^(1/2)
```

3.617.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.81 (sec) , antiderivative size = 477, normalized size of antiderivative = 0.87

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}(a+cx^2)}\,dx = -\frac{2(c(d^3g^3+d^2eg^3x+e^3f^2(f+gx))+ae^2g^2(dg+e(f+2gx)))}{(cd^2+ae^2)(ef-dg)^2(cf^2+ag^2)\sqrt{d+ex}\sqrt{f+gx}} - \frac{ic\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\left(\sqrt{c}d-i\sqrt{a}e\right)\sqrt{cd^2+ae^2}\left(\sqrt{c}f-i\sqrt{a}g\right)^2} + \frac{ic\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\left(\sqrt{c}d+i\sqrt{a}e\right)\sqrt{cd^2+ae^2}\left(\sqrt{c}f+i\sqrt{a}g\right)^2}$$

input
```
Integrate[1/((d + e*x)^(3/2)*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

output
```
(-2*(c*(d^3*g^3 + d^2*e*g^3*x + e^3*f^2*(f + g*x)) + a*e^2*g^2*(d*g + e*(f
 + 2*g*x))))/((c*d^2 + a*e^2)*(e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[d + e*x]*
Sqrt[f + g*x]) - (I*c*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt
[a]*g))]*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d + I
*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*(Sqrt[c]
*d - I*Sqrt[a]*e)*Sqrt[c*d^2 + a*e^2]*(Sqrt[c]*f - I*Sqrt[a]*g)^2) + (I*c*
Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]*ArcTan[(Sqrt[
c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f
 + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*(Sqrt[c]*d + I*Sqrt[a]*e)*Sqrt
[c*d^2 + a*e^2]*(Sqrt[c]*f + I*Sqrt[a]*g)^2)
```

3.617.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 543, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {662, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)(d+ex)^{3/2}(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{662}$$

$$\int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)(d+ex)^{3/2}(f+gx)^{3/2}}+\frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)(d+ex)^{3/2}(f+gx)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)^{3/2}\left(\sqrt{c}f-\sqrt{-a}g\right)^{3/2}}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\left(\sqrt{-a}e+\sqrt{c}d\right)^{3/2}\left(\sqrt{-a}g+\sqrt{c}f\right)^{3/2}}-\frac{e}{\sqrt{-a}\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)}+\frac{e}{\sqrt{-a}\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{-a}e+\sqrt{c}d\right)(ef-dg)}+\frac{g\sqrt{d+ex}\left(2aeg-\sqrt{-a}\sqrt{c}(dg+ef)\right)}{a\sqrt{f+gx}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(\sqrt{-a}g+\sqrt{c}f\right)(ef-dg)^2}+\frac{g\sqrt{d+ex}\left(\sqrt{-a}\sqrt{c}(dg+ef)+2aeg\right)}{a\sqrt{f+gx}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)^2}$$

input
```
Int[1/((d + e*x)^(3/2)*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

3.617. $\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}(a+cx^2)}dx$

output
```
-(e/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d + e*x]*Sqrt[f +
g*x])) + e/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d + e*x]*Sq
rt[f + g*x]) + (g*(2*a*e*g - Sqrt[-a]*Sqrt[c]*(e*f + d*g))*Sqrt[d + e*x])/
(a*(Sqrt[c]*d + Sqrt[-a]*e)*(Sqrt[c]*f + Sqrt[-a]*g)*(e*f - d*g)^2*Sqrt[f
+ g*x]) + (g*(2*a*e*g + Sqrt[-a]*Sqrt[c]*(e*f + d*g))*Sqrt[d + e*x])/(a*(S
qrt[c]*d - Sqrt[-a]*e)*(Sqrt[c]*f - Sqrt[-a]*g)*(e*f - d*g)^2*Sqrt[f + g*x
]) + (c*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]
*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)^(3/2)
*(Sqrt[c]*f - Sqrt[-a]*g)^(3/2)) - (c*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g
]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*
(Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*(Sqrt[c]*f + Sqrt[-a]*g)^(3/2))
```

3.617.3.1 Defintions of rubi rules used

rule 662
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && !IntegerQ[m] && !Inte
gerQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.617.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 30647 vs. $2(433) = 866$.

Time = 0.46 (sec) , antiderivative size = 30648, normalized size of antiderivative = 55.83

method	result	size
default	Expression too large to display	30648

input
```
int(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.617.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\,(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.617.6 Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \frac{1}{(a+cx^2)\,(d+ex)^{\frac{3}{2}}\,(f+gx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
```

output
```
Integral(1/((a + c*x**2)*(d + e*x)**(3/2)*(f + g*x)**(3/2)), x)
```

3.617.7 Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \frac{1}{(cx^2+a)(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)*(e*x + d)^(3/2)*(g*x + f)^(3/2)), x)
```

3.617.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

3.617.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}(a+cx^2)}\,dx = \int \frac{1}{(f+g\,x)^{3/2}\,(c\,x^2+a)\,(d+e\,x)^{3/2}}\,dx$$

input
```
int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(3/2)),x)
```

output
```
int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(3/2)), x)
```

3.618 $\int \frac{\sqrt{x}}{\sqrt{1+x}(1+x^2)}\,dx$

3.618.1 Optimal result

Integrand size = 20, antiderivative size = 65

$$\int \frac{\sqrt{x}}{\sqrt{1+x}\,(1+x^2)}\,dx=-\frac{1}{2}(1-i)^{3/2}\text{arctanh}\left(\frac{\sqrt{1-i}\sqrt{x}}{\sqrt{1+x}}\right)-\frac{1}{2}(1+i)^{3/2}\text{arctanh}\left(\frac{\sqrt{1+i}\sqrt{x}}{\sqrt{1+x}}\right)$$

output
```
-1/2*(1-I)^(3/2)*arctanh((1-I)^(1/2)*x^(1/2)/(1+x)^(1/2))-1/2*(1+I)^(3/2)*
arctanh((1+I)^(1/2)*x^(1/2)/(1+x)^(1/2))
```

3.618.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{x}}{\sqrt{1+x}\,(1+x^2)}\,dx=-\text{RootSum}\left[16+32\#1+16\#1^2+\#1^4\&,\frac{\log\left(-2x+2\sqrt{x}\sqrt{1+x}+\#1\right)\#1^2}{8+8\#1+\#1^3}\&\right]$$

input
```
Integrate[Sqrt[x]/(Sqrt[1 + x]*(1 + x^2)),x]
```

output
```
-RootSum[16 + 32*#1 + 16*#1^2 + #1^4 & , (Log[-2*x + 2*Sqrt[x]*Sqrt[1 + x]
 + #1]*#1^2)/(8 + 8*#1 + #1^3) & ]
```

3.618.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {613, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{x+1}\,(x^2+1)}dx$$

$$\Big\downarrow \text{613}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x}(x+i)\sqrt{x+1}}dx-\frac{1}{2}\int \frac{1}{(i-x)\sqrt{x}\sqrt{x+1}}dx$$

$$\Big\downarrow \text{104}$$

$$\int \frac{1}{\frac{(1-i)x}{x+1}+i}d\frac{\sqrt{x}}{\sqrt{x+1}}-\int \frac{1}{i-\frac{(1+i)x}{x+1}}d\frac{\sqrt{x}}{\sqrt{x+1}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{1}{2}(1-i)^{3/2}\text{arctanh}\left(\frac{\sqrt{1-i}\sqrt{x}}{\sqrt{x+1}}\right)-\frac{1}{2}(1+i)^{3/2}\text{arctanh}\left(\frac{\sqrt{1+i}\sqrt{x}}{\sqrt{x+1}}\right)$$

input
```
Int[Sqrt[x]/(Sqrt[1 + x]*(1 + x^2)),x]
```

output
```
-1/2*((1 - I)^(3/2)*ArcTanh[(Sqrt[1 - I]*Sqrt[x])/Sqrt[1 + x]]) - ((1 + I)
^(3/2)*ArcTanh[(Sqrt[1 + I]*Sqrt[x])/Sqrt[1 + x]])/2
```

3.618.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 613
```
Int[Sqrt[(e_.)*(x_)]/(Sqrt[(c_) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^2)), x_Sym
bol] :> Simp[e/(2*b)   Int[1/(Sqrt[e*x]*Sqrt[c + d*x]*(Rt[-a/b, 2] + x)), x
], x] - Simp[e/(2*b)   Int[1/(Sqrt[e*x]*Sqrt[c + d*x]*(Rt[-a/b, 2] - x)), x
], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.618.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(45) = 90$.

Time = 0.43 (sec) , antiderivative size = 305, normalized size of antiderivative = 4.69

method	result
default	$\sqrt{\frac{(1+x)x}{\left(\sqrt{2}-1+x\right)^{2}}}\left(\sqrt{2}-1+x\right)\left(\sqrt{-2+2\sqrt{2}}\,\arctan\left(\frac{\sqrt{-2+2\sqrt{2}}\,\sqrt{\frac{\left(3\sqrt{2}-4\right)x\left(4+3\sqrt{2}\right)(1+x)}{\left(\sqrt{2}-1+x\right)^{2}}}\left(3+2\sqrt{2}\right)\left(\sqrt{2}+1-x\right)\left(3\sqrt{2}-4\right)\left(\sqrt{2}-1+x\right)}{4(1+x)x}\right)\sqrt{1+$

input
```
int(x^(1/2)/(x^2+1)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/x^(1/2)/(1+x)^(1/2)*((1+x)*x/(2^(1/2)-1+x)^2)^(1/2)*(2^(1/2)-1+x)*((-2
+2*2^(1/2))^(1/2)*arctan(1/4*(-2+2*2^(1/2))^(1/2)*((3*2^(1/2)-4)*x*(4+3*2^
(1/2))*(1+x)/(2^(1/2)-1+x)^2)^(1/2)*(3+2*2^(1/2))*(2^(1/2)+1-x)*(3*2^(1/2)
-4)*(2^(1/2)-1+x)/(1+x)/x)*(1+2^(1/2))^(1/2)*2^(1/2)-2*(-2+2*2^(1/2))^(1/2
)*arctan(1/4*(-2+2*2^(1/2))^(1/2)*((3*2^(1/2)-4)*x*(4+3*2^(1/2))*(1+x)/(2^
(1/2)-1+x)^2)^(1/2)*(3+2*2^(1/2))*(2^(1/2)+1-x)*(3*2^(1/2)-4)*(2^(1/2)-1+x
)/(1+x)/x)*(1+2^(1/2))^(1/2)+4*arctanh(2^(1/2)*((1+x)*x/(2^(1/2)-1+x)^2)^(
1/2)/(1+2^(1/2))^(1/2))*2^(1/2)-6*arctanh(2^(1/2)*((1+x)*x/(2^(1/2)-1+x)^2
)^(1/2)/(1+2^(1/2))^(1/2)))*2^(1/2)/(3*2^(1/2)-4)/(1+2^(1/2))^(1/2)
```

3.618.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(37) = 74.

Time = 0.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.98

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{1+x}\,(1+x^2)}\,dx &= \frac{1}{4}\,\sqrt{2}\sqrt{i-1}\log\left((i+1)\,\sqrt{2}\sqrt{i-1}+2\,\sqrt{x+1}\sqrt{x}-2\,x-2i\right)\\ &\quad-\frac{1}{4}\,\sqrt{2}\sqrt{i-1}\log\left(-(i+1)\,\sqrt{2}\sqrt{i-1}+2\,\sqrt{x+1}\sqrt{x}-2\,x-2i\right)\\ &\quad+\frac{1}{4}\,\sqrt{2}\sqrt{-i-1}\log\left(-(i-1)\,\sqrt{2}\sqrt{-i-1}+2\,\sqrt{x+1}\sqrt{x}-2\,x+2i\right)\\ &\quad-\frac{1}{4}\,\sqrt{2}\sqrt{-i-1}\log\left((i-1)\,\sqrt{2}\sqrt{-i-1}+2\,\sqrt{x+1}\sqrt{x}-2\,x+2i\right)\end{aligned}$$

input
```
integrate(x^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*sqrt(I - 1)*log((I + 1)*sqrt(2)*sqrt(I - 1) + 2*sqrt(x + 1)*sq
rt(x) - 2*x - 2*I) - 1/4*sqrt(2)*sqrt(I - 1)*log(-(I + 1)*sqrt(2)*sqrt(I -
 1) + 2*sqrt(x + 1)*sqrt(x) - 2*x - 2*I) + 1/4*sqrt(2)*sqrt(-I - 1)*log(-(
I - 1)*sqrt(2)*sqrt(-I - 1) + 2*sqrt(x + 1)*sqrt(x) - 2*x + 2*I) - 1/4*sqr
t(2)*sqrt(-I - 1)*log((I - 1)*sqrt(2)*sqrt(-I - 1) + 2*sqrt(x + 1)*sqrt(x)
 - 2*x + 2*I)
```

3.618.6 Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt{1+x}\,(1+x^2)}\,dx = \int \frac{\sqrt{x}}{\sqrt{x+1}\,(x^2+1)}\,dx$$

input
```
integrate(x**(1/2)/(x**2+1)/(1+x)**(1/2),x)
```

output
```
Integral(sqrt(x)/(sqrt(x + 1)*(x**2 + 1)), x)
```

3.618.7 Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{1+x}\,(1+x^2)}\,dx = \int \frac{\sqrt{x}}{(x^2+1)\sqrt{x+1}}\,dx$$

input
```
integrate(x^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/((x^2 + 1)*sqrt(x + 1)), x)
```

3.618.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(37) = 74.

Time = 0.78 (sec) , antiderivative size = 375, normalized size of antiderivative = 5.77

$$\begin{aligned}
&\int \frac{\sqrt{x}}{\sqrt{1+x}\,(1+x^2)}\,dx \\
&= \frac{1}{4}\left(\sqrt{2\sqrt{2}+2}+\sqrt{2\sqrt{2}-2}\right)\arctan\left(\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{-\frac{1}{x+1}+1}\right)}{\sqrt{-\sqrt{2}+2}}\right) \\
&+\frac{1}{4}\left(\sqrt{2\sqrt{2}+2}+\sqrt{2\sqrt{2}-2}\right)\arctan\left(-\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{-\frac{1}{x+1}+1}\right)}{\sqrt{-\sqrt{2}+2}}\right) \\
&-\frac{1}{8}\left(\sqrt{2\sqrt{2}+2}-\sqrt{2\sqrt{2}-2}\right)\log\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{-\frac{1}{x+1}+1}+\sqrt{\frac{1}{2}}-\frac{1}{x+1}\right. \\
&\left.+1\right)+\frac{1}{8}\left(\sqrt{2\sqrt{2}+2}-\sqrt{2\sqrt{2}-2}\right)\log\left(-\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{-\frac{1}{x+1}+1}+\sqrt{\frac{1}{2}}\right. \\
&\left.-\frac{1}{x+1}+1\right)-\frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\right)}{\sqrt{-\sqrt{2}+2}}\right) \\
&-\frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(-\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\right)}{\sqrt{-\sqrt{2}+2}}\right) \\
&-\frac{1}{8}\sqrt{2\sqrt{2}-2}\log\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+\sqrt{\frac{1}{2}}+1\right) \\
&+\frac{1}{8}\sqrt{2\sqrt{2}-2}\log\left(-\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+\sqrt{\frac{1}{2}}+1\right)
\end{aligned}$$

input
```
integrate(x^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
1/4*(sqrt(2*sqrt(2) + 2) + sqrt(2*sqrt(2) - 2))*arctan(2*(1/2)^(3/4)*((1/2
)^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(-1/(x + 1) + 1))/sqrt(-sqrt(2) + 2)) +
1/4*(sqrt(2*sqrt(2) + 2) + sqrt(2*sqrt(2) - 2))*arctan(-2*(1/2)^(3/4)*((1/
2)^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(-1/(x + 1) + 1))/sqrt(-sqrt(2) + 2)) -
 1/8*(sqrt(2*sqrt(2) + 2) - sqrt(2*sqrt(2) - 2))*log((1/2)^(1/4)*sqrt(sqrt
(2) + 2)*sqrt(-1/(x + 1) + 1) + sqrt(1/2) - 1/(x + 1) + 1) + 1/8*(sqrt(2*s
qrt(2) + 2) - sqrt(2*sqrt(2) - 2))*log(-(1/2)^(1/4)*sqrt(sqrt(2) + 2)*sqrt
(-1/(x + 1) + 1) + sqrt(1/2) - 1/(x + 1) + 1) - 1/4*sqrt(2*sqrt(2) + 2)*ar
ctan(2*(1/2)^(3/4)*((1/2)^(1/4)*sqrt(sqrt(2) + 2) + 2)/sqrt(-sqrt(2) + 2))
 - 1/4*sqrt(2*sqrt(2) + 2)*arctan(-2*(1/2)^(3/4)*((1/2)^(1/4)*sqrt(sqrt(2)
 + 2) - 2)/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(2*sqrt(2) - 2)*log((1/2)^(1/4)*s
qrt(sqrt(2) + 2) + sqrt(1/2) + 1) + 1/8*sqrt(2*sqrt(2) - 2)*log(-(1/2)^(1/
4)*sqrt(sqrt(2) + 2) + sqrt(1/2) + 1)
```

3.618.9 Mupad [B] (verification not implemented)

Time = 17.83 (sec) , antiderivative size = 1610, normalized size of antiderivative = 24.77

$$\int \frac{\sqrt{x}}{\sqrt{1+x}\,(1+x^2)}\,dx = \text{Too large to display}$$

input
```
int(x^(1/2)/((x^2 + 1)*(x + 1)^(1/2)),x)
```

output

```
- atan(((((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*(((2845
4158336*x^(1/2))/((x + 1)^(1/2) - 1) + ((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(
1/2)/16 - 1/16)^(1/2))*(((112742891520*x^(1/2))/((x + 1)^(1/2) - 1) - ((53
1502202880*x)/((x + 1)^(1/2) - 1)^2 - 241591910400)*((- 2^(1/2)/16 - 1/16)
^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/
2)/16 - 1/16)^(1/2)) - (12079595520*x)/((x + 1)^(1/2) - 1)^2 + 68451041280
))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)) + (1355599052
8*x)/((x + 1)^(1/2) - 1)^2 + 9529458688) + (3556769792*x^(1/2))/((x + 1)^(
1/2) - 1))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*1i -
(((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*((13555990528*x
)/((x + 1)^(1/2) - 1)^2 - ((28454158336*x^(1/2))/((x + 1)^(1/2) - 1) + ((-
 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*(((112742891520*x^(
1/2))/((x + 1)^(1/2) - 1) + ((531502202880*x)/((x + 1)^(1/2) - 1)^2 - 2415
91910400)*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)))*((- 2
^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)) + (12079595520*x)/((x
 + 1)^(1/2) - 1)^2 - 68451041280))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)
/16 - 1/16)^(1/2)) + 9529458688) - (3556769792*x^(1/2))/((x + 1)^(1/2) - 1
))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*1i)/((((- 2^(
1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*(((28454158336*x^(1/2))
/((x + 1)^(1/2) - 1) + ((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/1...
```

3.618. $\int \frac{\sqrt{x}}{\sqrt{1+x}(1+x^2)}\,dx$

3.619 $\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4}\,dx$

3.619.1 Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4}\,dx = \frac{(f+g)^2(1+x)^4}{5\left(1-x^2\right)^{5/2}} + \frac{(f-9g)(f+g)(1+x)^3}{15\left(1-x^2\right)^{3/2}} + \frac{2g^2(1+x)}{\sqrt{1-x^2}} - g^2\arcsin(x)$$

output
```
1/5*(f+g)^2*(1+x)^4/(-x^2+1)^(5/2)+1/15*(f-9*g)*(f+g)*(1+x)^3/(-x^2+1)^(3/
2)-g^2*arcsin(x)+2*g^2*(1+x)/(-x^2+1)^(1/2)
```

3.619.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4}\,dx = \frac{\sqrt{1-x^2}\left(f^2(-4-3x+x^2)-2fg(-1+3x+4x^2)-3g^2(8-19x+13x^2)\right)}{15(-1+x)^3} + 2g^2\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[((f + g*x)^2*Sqrt[1 - x^2])/(1 - x)^4,x]
```

output
```
(Sqrt[1 - x^2]*(f^2*(-4 - 3*x + x^2) - 2*f*g*(-1 + 3*x + 4*x^2) - 3*g^2*(8
 - 19*x + 13*x^2)))/(15*(-1 + x)^3) + 2*g^2*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

3.619.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {717, 100, 25, 87, 57, 39, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}(f+gx)^2}{(1-x)^4} \, dx$$

$$\Big\downarrow \text{717}$$

$$\int \frac{\sqrt{x+1}(f+gx)^2}{(1-x)^{7/2}}dx$$

$$\Big\downarrow \text{100}$$

$$\frac{(x+1)^{3/2}(f+g)^2}{5(1-x)^{5/2}}-\frac{1}{5}\int -\frac{\sqrt{x+1}\left(f^2-8gf-4g^2-5g^2x\right)}{(1-x)^{5/2}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\int \frac{\sqrt{x+1}\left(f^2-8gf-4g^2-5g^2x\right)}{(1-x)^{5/2}}dx+\frac{(x+1)^{3/2}(f+g)^2}{5(1-x)^{5/2}}$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{5}\left(5g^2\int \frac{\sqrt{x+1}}{(1-x)^{3/2}}dx+\frac{(x+1)^{3/2}(f-9g)(f+g)}{3(1-x)^{3/2}}\right)+\frac{(x+1)^{3/2}(f+g)^2}{5(1-x)^{5/2}}$$

$$\Big\downarrow \text{57}$$

$$\frac{1}{5}\left(5g^2\left(\frac{2\sqrt{x+1}}{\sqrt{1-x}}-\int \frac{1}{\sqrt{1-x}\sqrt{x+1}}dx\right)+\frac{(x+1)^{3/2}(f-9g)(f+g)}{3(1-x)^{3/2}}\right)+\frac{(x+1)^{3/2}(f+g)^2}{5(1-x)^{5/2}}$$

$$\Big\downarrow \text{39}$$

$$\frac{1}{5}\left(5g^2\left(\frac{2\sqrt{x+1}}{\sqrt{1-x}}-\int \frac{1}{\sqrt{1-x^2}}dx\right)+\frac{(x+1)^{3/2}(f-9g)(f+g)}{3(1-x)^{3/2}}\right)+\frac{(x+1)^{3/2}(f+g)^2}{5(1-x)^{5/2}}$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{5}\left(5g^2\left(\frac{2\sqrt{x+1}}{\sqrt{1-x}}-\arcsin(x)\right)+\frac{(x+1)^{3/2}(f-9g)(f+g)}{3(1-x)^{3/2}}\right)+\frac{(x+1)^{3/2}(f+g)^2}{5(1-x)^{5/2}}$$

input
```
Int[((f + g*x)^2*Sqrt[1 - x^2])/(1 - x)^4,x]
```

output
```
((f + g)^2*(1 + x)^(3/2))/(5*(1 - x)^(5/2)) + (((f - 9*g)*(f + g)*(1 + x)^
(3/2))/(3*(1 - x)^(3/2)) + 5*g^2*((2*Sqrt[1 + x])/Sqrt[1 - x] - ArcSin[x])
)/5
```

3.619.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 717
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)
^2)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a
, 0] && GtQ[d, 0]
```

3.619.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{(1+x)(f^2x^2-8x^2fg-39g^2x^2-3f^2x-6fgx+57g^2x-4f^2+2fg-24g^2)}{15(-1+x)^2\sqrt{-x^2+1}}-g^2\arcsin(x)$
trager	$\frac{(f^2x^2-8x^2fg-39g^2x^2-3f^2x-6fgx+57g^2x-4f^2+2fg-24g^2)\sqrt{-x^2+1}}{15(-1+x)^3}+g^2\operatorname{RootOf}(\_Z^2+1)\ln(-\operatorname{RootOf}(\_Z$
default	$g^2\left(\frac{\left(-(-1+x)^2+2-2x\right)^{\frac{3}{2}}}{(-1+x)^2}+\sqrt{-(-1+x)^2+2-2x}-\arcsin(x)\right)+\frac{2g(f+g)\left(-(-1+x)^2+2-2x\right)^{\frac{3}{2}}}{3(-1+x)^3}+(f^2$

input
```
int((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(1+x)*(f^2*x^2-8*f*g*x^2-39*g^2*x^2-3*f^2*x-6*f*g*x+57*g^2*x-4*f^2+2
*f*g-24*g^2)/(-1+x)^2/(-x^2+1)^(1/2)-g^2*arcsin(x)
```

3.619.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(70) = 140$.

Time = 0.32 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.41

$$\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4}\,dx$$
$$= \frac{2\,(2\,f^2 - fg + 12\,g^2)x^3 - 6\,(2\,f^2 - fg + 12\,g^2)x^2 - 4\,f^2 + 2\,fg - 24\,g^2 + 6\,(2\,f^2 - fg + 12\,g^2)x + 30\,(g^2}{}$$

input
```
integrate((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x, algorithm="fricas")
```

output
```
1/15*(2*(2*f^2 - f*g + 12*g^2)*x^3 - 6*(2*f^2 - f*g + 12*g^2)*x^2 - 4*f^2
+ 2*f*g - 24*g^2 + 6*(2*f^2 - f*g + 12*g^2)*x + 30*(g^2*x^3 - 3*g^2*x^2 +
3*g^2*x - g^2)*arctan((sqrt(-x^2 + 1) - 1)/x) + ((f^2 - 8*f*g - 39*g^2)*x^
2 - 4*f^2 + 2*f*g - 24*g^2 - 3*(f^2 + 2*f*g - 19*g^2)*x)*sqrt(-x^2 + 1))/(
x^3 - 3*x^2 + 3*x - 1)
```

3.619.6 Sympy [F]

$$\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4}\,dx = \int \frac{\sqrt{-(x-1)\,(x+1)}(f+gx)^2}{(x-1)^4}\,dx$$

input
```
integrate((g*x+f)**2*(-x**2+1)**(1/2)/(1-x)**4,x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1))*(f + g*x)**2/(x - 1)**4, x)
```

3.619.7 Maxima [F]

$$\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4}\,dx = \int \frac{(gx+f)^2\sqrt{-x^2+1}}{(x-1)^4}\,dx$$

input
```
integrate((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x, algorithm="maxima")
```

output
```
integrate((g*x + f)^2*sqrt(-x^2 + 1)/(x - 1)^4, x)
```

3.619.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(70) = 140$.

Time = 0.29 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.32

$$\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4}\,dx = -g^2\arcsin(x) + \frac{2\left(4f^2-2fg+24g^2+\frac{5f^2\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{10fg\left(\sqrt{-x^2+1}-1\right)}{x}+\frac{105g^2\left(\sqrt{-x^2+1}-1\right)}{x}+\frac{25f^2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{10fg\cdots}{}\right.}{}$$

input
```
integrate((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x, algorithm="giac")
```

output
```
-g^2*arcsin(x) + 2/15*(4*f^2 - 2*f*g + 24*g^2 + 5*f^2*(sqrt(-x^2 + 1) - 1)
/x - 10*f*g*(sqrt(-x^2 + 1) - 1)/x + 105*g^2*(sqrt(-x^2 + 1) - 1)/x + 25*f
^2*(sqrt(-x^2 + 1) - 1)^2/x^2 + 10*f*g*(sqrt(-x^2 + 1) - 1)^2/x^2 + 165*g^
2*(sqrt(-x^2 + 1) - 1)^2/x^2 + 15*f^2*(sqrt(-x^2 + 1) - 1)^3/x^3 - 30*f*g*
(sqrt(-x^2 + 1) - 1)^3/x^3 + 75*g^2*(sqrt(-x^2 + 1) - 1)^3/x^3 + 15*f^2*(s
qrt(-x^2 + 1) - 1)^4/x^4 + 15*g^2*(sqrt(-x^2 + 1) - 1)^4/x^4)/((sqrt(-x^2
+ 1) - 1)/x + 1)^5
```

3.619.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.05

$$\begin{aligned}\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4}\,dx &= \sqrt{1-x^2}\left(\frac{\frac{f^2}{3}+2\,f\,g+\frac{5\,g^2}{3}}{x-1}-\frac{\frac{f^2}{3}+2\,f\,g+\frac{5\,g^2}{3}}{(x-1)^2}\right) \\ &\quad -\sqrt{1-x^2}\left(\frac{\frac{2\,f^2}{5}+\frac{4\,f\,g}{5}+\frac{2\,g^2}{5}}{(x-1)^3}+\frac{\frac{4\,f^2}{15}+\frac{8\,f\,g}{15}+\frac{4\,g^2}{15}}{x-1}\right. \\ &\quad \left.-\frac{\frac{4\,f^2}{15}+\frac{8\,f\,g}{15}+\frac{4\,g^2}{15}}{(x-1)^2}\right)-g^2\,\mathrm{asin}(x)-\frac{\sqrt{1-x^2}\,(4\,g^2+2\,f\,g)}{x-1}\end{aligned}$$

input
```
int(((f + g*x)^2*(1 - x^2)^(1/2))/(x - 1)^4,x)
```

output
```
(1 - x^2)^(1/2)*((2*f*g + f^2/3 + (5*g^2)/3)/(x - 1) - (2*f*g + f^2/3 + (5
*g^2)/3)/(x - 1)^2) - (1 - x^2)^(1/2)*(((4*f*g)/5 + (2*f^2)/5 + (2*g^2)/5)
/(x - 1)^3 + ((8*f*g)/15 + (4*f^2)/15 + (4*g^2)/15)/(x - 1) - ((8*f*g)/15
+ (4*f^2)/15 + (4*g^2)/15)/(x - 1)^2) - g^2*asin(x) - ((1 - x^2)^(1/2)*(2*
f*g + 4*g^2))/(x - 1)
```

3.619. $\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4}\,dx$

3.620 $\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)}\,dx$

3.620.1 Optimal result

Integrand size = 30, antiderivative size = 107

$$\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)}\,dx = -\frac{\sqrt{1-a^2x^2}}{d} - \frac{(ac-2d)\arcsin(ax)}{d^2} + \frac{(ac-d)^2\arctan\left(\frac{d+a^2cx}{\sqrt{a^2c^2-d^2}\sqrt{1-a^2x^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}$$

output
```
-(a*c-2*d)*arcsin(a*x)/d^2+(a*c-d)^2*arctan((a^2*c*x+d)/(a^2*c^2-d^2)^(1/2
)/(-a^2*x^2+1)^(1/2))/d^2/(a^2*c^2-d^2)^(1/2)-(-a^2*x^2+1)^(1/2)/d
```

3.620.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.21

$$\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)}\,dx = \frac{-d\sqrt{1-a^2x^2} + (-2ac+4d)\arctan\left(\frac{ax}{-1+\sqrt{1-a^2x^2}}\right) - \frac{2(ac-d)\sqrt{a^2c^2-d^2}\arctan\left(\frac{\sqrt{a^2c^2}}{c+dx-c\sqrt{}}\right.}{ac+d}}{d^2}$$

input
```
Integrate[(1 - a^2*x^2)^(3/2)/((1 - a*x)^2*(c + d*x)),x]
```

output
```
(-(d*Sqrt[1 - a^2*x^2]) + (-2*a*c + 4*d)*ArcTan[(a*x)/(-1 + Sqrt[1 - a^2*x
^2])] - (2*(a*c - d)*Sqrt[a^2*c^2 - d^2]*ArcTan[(Sqrt[a^2*c^2 - d^2]*x)/(c
 + d*x - c*Sqrt[1 - a^2*x^2])])/(a*c + d))/d^2
```

3.620.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {708, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)}\,dx$$

$$\Big\downarrow \; 708$$

$$\int \left(-\frac{a(ac-2d)}{d^2\sqrt{1-a^2x^2}}+\frac{(d-ac)^2}{d^2\sqrt{1-a^2x^2}(c+dx)}+\frac{a^2x}{d\sqrt{1-a^2x^2}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{(ac-d)^2\arctan\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}-\frac{\sqrt{1-a^2x^2}}{d}-\frac{\arcsin(ax)(ac-2d)}{d^2}$$

input
```
Int[(1 - a^2*x^2)^(3/2)/((1 - a*x)^2*(c + d*x)),x]
```

output
```
-(Sqrt[1 - a^2*x^2]/d) - ((a*c - 2*d)*ArcSin[a*x])/d^2 + ((a*c - d)^2*ArcT
an[(d + a^2*c*x)/(Sqrt[a^2*c^2 - d^2]*Sqrt[1 - a^2*x^2])])/(d^2*Sqrt[a^2*c
^2 - d^2])
```

3.620.3.1 Defintions of rubi rules used

rule 708
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^2], (d + e*x)^m*(f
+ g*x)^n*(a + c*x^2)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x
] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p - 1/2] && ILtQ[m, 0] && ILtQ[n, 0]
 &&  !IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.620.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(99) = 198$.

Time = 0.56 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.18

method	result
risch	$\frac{a^2x^2-1}{d\sqrt{-a^2x^2+1}} - \frac{\frac{a(ac-2d)\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{d\sqrt{a^2}} - \frac{\left(-a^2c^2+2acd-d^2\right)\ln\left(\frac{-\frac{2\left(a^2c^2-d^2\right)}{d^2}+\frac{2a^2c\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{a^2c^2-d^2}{d^2}}\sqrt{-a^2\left(x+\frac{c}{d}\right)^2+\frac{2a^2 \cdots}{}}}{x+\frac{c}{d}}\right)}{d^2\sqrt{-\frac{a^2c^2-d^2}{d^2}}}}{d}$
default	$\frac{-\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{1}{a}\right)^2}-3a\left(\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{3}-a\left(-\frac{\left(-2a^2\left(x-\frac{1}{a}\right)-2a\right)\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{4a^2}+\frac{\arctan\left(\frac{\cdots}{\sqrt{-a^2\left(x \cdots\right.}}\right)}{2\sqrt{\cdots}}\cdots\right.\right.}{a(ac+d)}$

input
```
int((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*x^2-1)/(-a^2*x^2+1)^(1/2)-1/d*(a*(a*c-2*d)/d/(a^2)^(1/2)*arctan((
a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-(-a^2*c^2+2*a*c*d-d^2)/d^2/(-(a^2*c^2-d^2
)/d^2)^(1/2)*ln((-2*(a^2*c^2-d^2)/d^2+2*a^2*c/d*(x+c/d)+2*(-(a^2*c^2-d^2)/
d^2)^(1/2)*(-a^2*(x+c/d)^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(1/2))/(x+
c/d)))
```

3.620.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.97

$$\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)}\,dx = \left[-\frac{(ac-d)\sqrt{-\frac{ac-d}{ac+d}}\log\left(\frac{a^2cdx+d^2-(a^2c^2-d^2)\sqrt{-a^2x^2+1}-\left(acd+d^2+(a^3c^2+a^2cd)x+\sqrt{-a^2x^2+1}\cdots\right.}{dx+c}\right.}{d^2}\cdots\right.$$

input
```
integrate((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x, algorithm="fricas")
```

output
```
[-((a*c - d)*sqrt(-(a*c - d)/(a*c + d))*log((a^2*c*d*x + d^2 - (a^2*c^2 -
d^2)*sqrt(-a^2*x^2 + 1) - (a*c*d + d^2 + (a^3*c^2 + a^2*c*d)*x + sqrt(-a^2
*x^2 + 1)*(a*c*d + d^2))*sqrt(-(a*c - d)/(a*c + d)))/(d*x + c)) - 2*(a*c -
 2*d)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 + 1)*d)/d^2,
(2*(a*c - d)*sqrt((a*c - d)/(a*c + d))*arctan((d*x - sqrt(-a^2*x^2 + 1)*c
+ c)*sqrt((a*c - d)/(a*c + d))/((a*c - d)*x)) + 2*(a*c - 2*d)*arctan((sqrt
(-a^2*x^2 + 1) - 1)/(a*x)) - sqrt(-a^2*x^2 + 1)*d)/d^2]
```

3.620.6 Sympy [F]

$$\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)}\,dx = \int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}}{(c+dx)(ax-1)^2}\,dx$$

input
```
integrate((-a**2*x**2+1)**(3/2)/(-a*x+1)**2/(d*x+c),x)
```

output
```
Integral((-(a*x - 1)*(a*x + 1))**(3/2)/((c + d*x)*(a*x - 1)**2), x)
```

3.620.7 Maxima [F]

$$\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)}\,dx = \int \frac{(-a^2x^2+1)^{\frac{3}{2}}}{(ax-1)^2(dx+c)}\,dx$$

input
```
integrate((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((-a^2*x^2 + 1)^(3/2)/((a*x - 1)^2*(d*x + c)), x)
```

3.620.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(99) = 198.

Time = 0.30 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.94

$$\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)}\,dx =$$

$$-\left(\frac{(ax-1)\sqrt{-\frac{2}{ax-1}-1}\,\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}(a)}{ad} - \frac{2\left(ac\,\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}(a) - 2\,d\,\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}(a)\right)\arctan\left(\sqrt{-\frac{2}{ax}\right.}{ad^2}\right.$$

input
```
integrate((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x, algorithm="giac")
```

output
```
-((a*x - 1)*sqrt(-2/(a*x - 1) - 1)*sgn(1/(a*x - 1))*sgn(a)/(a*d) - 2*(a*c*
sgn(1/(a*x - 1))*sgn(a) - 2*d*sgn(1/(a*x - 1))*sgn(a))*arctan(sqrt(-2/(a*x
 - 1) - 1))/(a*d^2) + 2*(a^2*c^2*sgn(1/(a*x - 1))*sgn(a) - 2*a*c*d*sgn(1/(
a*x - 1))*sgn(a) + d^2*sgn(1/(a*x - 1))*sgn(a))*arctan((a*c*sqrt(-2/(a*x -
 1) - 1) + d*sqrt(-2/(a*x - 1) - 1))/sqrt(a^2*c^2 - d^2))/(sqrt(a^2*c^2 -
d^2)*a*d^2))*abs(a)
```

3.620.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.38

$$\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)}\,dx = -\frac{\sqrt{1-a^2\,x^2}}{d} - \frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)\,\left(2\,a\,\sqrt{-a^2}-\frac{a^2\,c\,\sqrt{-a^2}}{d}\right)}{a^2\,d} - \frac{\left(\ln\left(\sqrt{1-\frac{a^2\,c^2}{d^2}}\,\sqrt{1-a^2\,x^2}+\frac{a^2\,c\,x}{d}+1\right)-\ln\left(c+d\,x\right)\right)\,\left(a^2\,c^2-2\,a\,c\,d+d^2\right)}{d^3\,\sqrt{1-\frac{a^2\,c^2}{d^2}}}$$

input
```
int((1 - a^2*x^2)^(3/2)/((a*x - 1)^2*(c + d*x)),x)
```

output
```
- (1 - a^2*x^2)^(1/2)/d - (asinh(x*(-a^2)^(1/2))*(2*a*(-a^2)^(1/2) - (a^2*
c*(-a^2)^(1/2))/d))/(a^2*d) - ((log((1 - (a^2*c^2)/d^2)^(1/2)*(1 - a^2*x^2
)^(1/2) + (a^2*c*x)/d + 1) - log(c + d*x))*(d^2 + a^2*c^2 - 2*a*c*d))/(d^3
*(1 - (a^2*c^2)/d^2)^(1/2))
```

3.621 $\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx$

3.621.1 Optimal result

Integrand size = 29, antiderivative size = 107

$$\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\arcsin(ax)}{d^2}+\frac{(ac-d)^2\arctan\left(\frac{d+a^2cx}{\sqrt{a^2c^2-d^2}\sqrt{1-a^2x^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}$$

output
```
-(a*c-2*d)*arcsin(a*x)/d^2+(a*c-d)^2*arctan((a^2*c*x+d)/(a^2*c^2-d^2)^(1/2
)/(-a^2*x^2+1)^(1/2))/d^2/(a^2*c^2-d^2)^(1/2)-(-a^2*x^2+1)^(1/2)/d
```

3.621.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.21

$$\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx = \frac{-d\sqrt{1-a^2x^2}+(-2ac+4d)\arctan\left(\frac{ax}{-1+\sqrt{1-a^2x^2}}\right)-\frac{2(ac-d)\sqrt{a^2c^2-d^2}\arctan\left(\frac{\sqrt{a^2c^2-d^2}x}{c+dx-c\sqrt{1-a^2x^2}}\right)}{ac+d}}{d^2}$$

input
```
Integrate[(1 + a*x)^2/((c + d*x)*Sqrt[1 - a^2*x^2]),x]
```

output
```
(-(d*Sqrt[1 - a^2*x^2]) + (-2*a*c + 4*d)*ArcTan[(a*x)/(-1 + Sqrt[1 - a^2*x
^2])] - (2*(a*c - d)*Sqrt[a^2*c^2 - d^2]*ArcTan[(Sqrt[a^2*c^2 - d^2]*x)/(c
 + d*x - c*Sqrt[1 - a^2*x^2])])/(a*c + d))/d^2
```

3.621.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {716, 25, 27, 719, 223, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+1)^2}{\sqrt{1-a^2x^2}(c+dx)}dx$$

$$\Big\downarrow \text{716}$$

$$-\frac{\int -\frac{a^2d(d-a(ac-2d)x)}{(c+dx)\sqrt{1-a^2x^2}}dx}{a^2d^2}-\frac{\sqrt{1-a^2x^2}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a^2d(d-a(ac-2d)x)}{(c+dx)\sqrt{1-a^2x^2}}dx}{a^2d^2}-\frac{\sqrt{1-a^2x^2}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{d-a(ac-2d)x}{(c+dx)\sqrt{1-a^2x^2}}dx}{d}-\frac{\sqrt{1-a^2x^2}}{d}$$

$$\Big\downarrow \text{719}$$

$$\frac{\frac{(ac-d)^2\int \frac{1}{(c+dx)\sqrt{1-a^2x^2}}dx}{d}-\frac{a(ac-2d)\int \frac{1}{\sqrt{1-a^2x^2}}dx}{d}}{d}-\frac{\sqrt{1-a^2x^2}}{d}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{(ac-d)^2\int \frac{1}{(c+dx)\sqrt{1-a^2x^2}}dx}{d}-\frac{\arcsin(ax)(ac-2d)}{d}}{d}-\frac{\sqrt{1-a^2x^2}}{d}$$

$$\Big\downarrow \text{488}$$

$$\frac{-\frac{(ac-d)^2\int \frac{1}{-a^2c^2+d^2-\frac{\left(cxa^2+d\right)^2}{1-a^2x^2}}d\frac{cxa^2+d}{\sqrt{1-a^2x^2}}}{d}-\frac{\arcsin(ax)(ac-2d)}{d}}{d}-\frac{\sqrt{1-a^2x^2}}{d}$$

$$\downarrow 217$$

$$\frac{\frac{(ac-d)^2\arctan\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}}\right)}{d\sqrt{a^2c^2-d^2}}-\frac{\arcsin(ax)(ac-2d)}{d}}{d}-\frac{\sqrt{1-a^2x^2}}{d}$$

input
```
Int[(1 + a*x)^2/((c + d*x)*Sqrt[1 - a^2*x^2]),x]
```

output
```
-(Sqrt[1 - a^2*x^2]/d) + (-(((a*c - 2*d)*ArcSin[a*x])/d) + ((a*c - d)^2*Ar
cTan[(d + a^2*c*x)/(Sqrt[a^2*c^2 - d^2]*Sqrt[1 - a^2*x^2])])/(d*Sqrt[a^2*c
^2 - d^2]))/d
```

3.621.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 716
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + c*x^2)^(p + 1)/
(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m + n + 2*p + 1))   I
nt[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c*e^n*(m + n + 2*p + 1)*(f + g*x)^
n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d + e*x)^(n - 2)*(a*e^2*(m +
 n - 1) - c*d^2*(m + n + 2*p + 1) - 2*c*d*e*(m + n + p)*x), x], x], x] /; F
reeQ[{a, c, d, e, f, g, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n
 + 2*p + 1, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.621.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(99) = 198$.

Time = 0.47 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.18

method	result
risch	$\frac{a^2x^2-1}{d\sqrt{-a^2x^2+1}}-\frac{\frac{a(ac-2d)\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{d\sqrt{a^2}}-\frac{\left(-a^2c^2+2acd-d^2\right)\ln\left(\frac{-\frac{2\left(a^2c^2-d^2\right)}{d^2}+\frac{2a^2c\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{a^2c^2-d^2}{d^2}}\sqrt{-a^2\left(x+\frac{c}{d}\right)^2+2a^2\cdots}}{x+\frac{c}{d}}\right.\cdots}{d^2\sqrt{-\frac{a^2c^2-d^2}{d^2}}}\cdots}{d}$
default	$-\frac{a\left(-\frac{2d\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+\frac{ac\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+\frac{d\sqrt{-a^2x^2+1}}{a}\right)}{d^2}-\frac{\left(a^2c^2-2acd+d^2\right)\ln\left(\frac{-\frac{2\left(a^2c^2-d^2\right)}{d^2}+\frac{2a^2c\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\cdots}}{\cdots}\right.\cdots}{d^3\sqrt{-\frac{a^2c^2\cdots}{d^2}}}$

input
```
int((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*x^2-1)/(-a^2*x^2+1)^(1/2)-1/d*(a*(a*c-2*d)/d/(a^2)^(1/2)*arctan((
a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-(-a^2*c^2+2*a*c*d-d^2)/d^2/(-(a^2*c^2-d^2
)/d^2)^(1/2)*ln((-2*(a^2*c^2-d^2)/d^2+2*a^2*c/d*(x+c/d)+2*(-(a^2*c^2-d^2)/
d^2)^(1/2)*(-a^2*(x+c/d)^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(1/2))/(x+
c/d)))
```

3.621.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.97

$$\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx$$
$$= \left[-\frac{(ac-d)\sqrt{-\frac{ac-d}{ac+d}}\log\left(\frac{a^2cdx+d^2-(a^2c^2-d^2)\sqrt{-a^2x^2+1}-\left(acd+d^2+(a^3c^2+a^2cd)x+\sqrt{-a^2x^2+1}(acd+d^2)\right)\sqrt{-\frac{ac-d}{ac+d}}}{dx+c}\right)-2\,(ac}{d^2}\right.$$

input
```
integrate((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
[-((a*c - d)*sqrt(-(a*c - d)/(a*c + d))*log((a^2*c*d*x + d^2 - (a^2*c^2 -
d^2)*sqrt(-a^2*x^2 + 1) - (a*c*d + d^2 + (a^3*c^2 + a^2*c*d)*x + sqrt(-a^2
*x^2 + 1)*(a*c*d + d^2))*sqrt(-(a*c - d)/(a*c + d)))/(d*x + c)) - 2*(a*c -
 2*d)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 + 1)*d)/d^2,
(2*(a*c - d)*sqrt((a*c - d)/(a*c + d))*arctan((d*x - sqrt(-a^2*x^2 + 1)*c
+ c)*sqrt((a*c - d)/(a*c + d))/((a*c - d)*x)) + 2*(a*c - 2*d)*arctan((sqrt
(-a^2*x^2 + 1) - 1)/(a*x)) - sqrt(-a^2*x^2 + 1)*d)/d^2]
```

3.621.6 Sympy [F]

$$\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx = \int \frac{(ax+1)^2}{\sqrt{-(ax-1)(ax+1)}\,(c+dx)}\,dx$$

input
```
integrate((a*x+1)**2/(d*x+c)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral((a*x + 1)**2/(sqrt(-(a*x - 1)*(a*x + 1))*(c + d*x)), x)
```

3.621.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d-a*c>0)', see `assume?` for mor
e details)
```

3.621.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.22

$$\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx = -\frac{(a^2c-2\,ad)\arcsin{(ax)}\,\mathrm{sgn}(a)}{d^2|a|} - \frac{\sqrt{-a^2x^2+1}}{d} - \frac{2\,(a^3c^2-2\,a^2cd+ad^2)\arctan\left(\frac{d+\frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)c}{ax}}{\sqrt{a^2c^2-d^2}}\right)}{\sqrt{a^2c^2-d^2}d^2|a|}$$

input
```
integrate((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-(a^2*c - 2*a*d)*arcsin(a*x)*sgn(a)/(d^2*abs(a)) - sqrt(-a^2*x^2 + 1)/d -
2*(a^3*c^2 - 2*a^2*c*d + a*d^2)*arctan((d + (sqrt(-a^2*x^2 + 1)*abs(a) + a
)*c/(a*x))/sqrt(a^2*c^2 - d^2))/(sqrt(a^2*c^2 - d^2)*d^2*abs(a))
```

3.621.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.38

$$\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{\sqrt{1-a^2\,x^2}}{d} - \frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)\,\left(2\,a\,\sqrt{-a^2}-\frac{a^2\,c\,\sqrt{-a^2}}{d}\right)}{a^2\,d} - \frac{\left(\ln\left(\sqrt{1-\frac{a^2\,c^2}{d^2}}\,\sqrt{1-a^2\,x^2}+\frac{a^2\,c\,x}{d}+1\right)-\ln\left(c+d\,x\right)\right)\,\left(a^2\,c^2-2\,a\,c\,d+d^2\right)}{d^3\,\sqrt{1-\frac{a^2\,c^2}{d^2}}}$$

input

```
int((a*x + 1)^2/((1 - a^2*x^2)^(1/2)*(c + d*x)),x)
```

output

```
- (1 - a^2*x^2)^(1/2)/d - (asinh(x*(-a^2)^(1/2))*(2*a*(-a^2)^(1/2) - (a^2*
c*(-a^2)^(1/2))/d))/(a^2*d) - ((log((1 - (a^2*c^2)/d^2)^(1/2)*(1 - a^2*x^2
)^(1/2) + (a^2*c*x)/d + 1) - log(c + d*x))*(d^2 + a^2*c^2 - 2*a*c*d))/(d^3
*(1 - (a^2*c^2)/d^2)^(1/2))
```

3.622 $\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

3.622.1 Optimal result

Integrand size = 28, antiderivative size = 851

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx =$$
$$-\frac{2(150a^2e^4g^4-6ace^2g^2(2e^2f^2-33defg+165d^2g^2)+c^2(187e^4f^4-732de^3f^3g+1098d^2e^2f^2g^2-798d}{3465c^2eg^4}$$
$$+\frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+cx^2}}{11e}$$
$$-\frac{2(2ae^2g^2(74ef-231dg)-c(233e^3f^3-843de^2f^2g+1107d^2efg^2-567d^3g^3))(f+gx)^{3/2}\sqrt{a+cx^2}}{3465cg^4}$$
$$+\frac{2e(18ae^2g^2-c(29e^2f^2-96defg+81d^2g^2))(f+gx)^{5/2}\sqrt{a+cx^2}}{693cg^4}$$
$$+\frac{2e^2(ef-3dg)(f+gx)^{7/2}\sqrt{a+cx^2}}{99g^4}$$
$$+\frac{4\sqrt{-a}(3a^2e^2g^4(26ef+231dg)-c^2f^2(64e^3f^3-264de^2f^2g+396d^2efg^2-231d^3g^3)-9acg^2(6e^3f^3-3}{3465c^{3/2}g^5\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}}$$
$$-\frac{4\sqrt{-a}(cf^2+ag^2)(75a^2e^3g^4-3aceg^2(2e^2f^2-33defg+165d^2g^2)-c^2f(64e^3f^3-264de^2f^2g+396d^2e}{3465c^{5/2}g^5\sqrt{f+gx}\sqrt{a}}$$

output
```
-2/3465*(2*a*e^2*g^2*(-231*d*g+74*e*f)-c*(-567*d^3*g^3+1107*d^2*e*f*g^2-84
3*d*e^2*f^2*g+233*e^3*f^3))*(g*x+f)^(3/2)*(c*x^2+a)^(1/2)/c/g^4+2/693*e*(1
8*a*e^2*g^2-c*(81*d^2*g^2-96*d*e*f*g+29*e^2*f^2))*(g*x+f)^(5/2)*(c*x^2+a)^
(1/2)/c/g^4+2/99*e^2*(-3*d*g+e*f)*(g*x+f)^(7/2)*(c*x^2+a)^(1/2)/g^4-2/3465
*(150*a^2*e^4*g^4-6*a*c*e^2*g^2*(165*d^2*g^2-33*d*e*f*g+2*e^2*f^2)+c^2*(31
5*d^4*g^4-798*d^3*e*f*g^3+1098*d^2*e^2*f^2*g^2-732*d*e^3*f^3*g+187*e^4*f^4
))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c^2/e/g^4+2/11*(e*x+d)^4*(g*x+f)^(1/2)*(c
*x^2+a)^(1/2)/e+4/3465*(3*a^2*e^2*g^4*(231*d*g+26*e*f)-c^2*f^2*(-231*d^3*g
^3+396*d^2*e*f*g^2-264*d*e^2*f^2*g+64*e^3*f^3)-9*a*c*g^2*(77*d^3*g^3+88*d^
2*e*f*g^2-33*d*e^2*f^2*g+6*e^3*f^3))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2)
)^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*
x+f)^(1/2)*(1+c*x^2/a)^(1/2)/c^(3/2)/g^5/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/
(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-4/3465*(a*g^2+c*f^2)*(75*a^2*e^3*g^4-3*a*c
*e*g^2*(165*d^2*g^2-33*d*e*f*g+2*e^2*f^2)-c^2*f*(-231*d^3*g^3+396*d^2*e*f*
g^2-264*d*e^2*f^2*g+64*e^3*f^3))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1
/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^
2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(5/2)/g^5/(g
*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

3.622.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.02 (sec) , antiderivative size = 1045, normalized size of antiderivative = 1.23

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2(a+cx^2)\left(-150a^2e^3g^4+2aceg^2\left(495d^2g^2+33deg(4f+7gx)+e^2\left(-23f^2+16fgx+45g^2x^2\right)\right)+c^2\left(231d^3g^3(f+3gx)+99d^2eg^2\left(-4f^2\cdots\right.\right.\right.}{c^2g^4}\cdots\right.}{\cdots}$$

input
```
Integrate[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

output
```
(Sqrt[f + g*x]*((2*(a + c*x^2)*(-150*a^2*e^3*g^4 + 2*a*c*e*g^2*(495*d^2*g^
2 + 33*d*e*g*(4*f + 7*g*x) + e^2*(-23*f^2 + 16*f*g*x + 45*g^2*x^2)) + c^2*
(231*d^3*g^3*(f + 3*g*x) + 99*d^2*e*g^2*(-4*f^2 + 3*f*g*x + 15*g^2*x^2) +
33*d*e^2*g*(8*f^3 - 6*f^2*g*x + 5*f*g^2*x^2 + 35*g^3*x^3) + e^3*(-64*f^4 +
 48*f^3*g*x - 40*f^2*g^2*x^2 + 35*f*g^3*x^3 + 315*g^4*x^4))))/(c^2*g^4) -
(4*(f + g*x)*((g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(3*a^2*e^2*g^4*(26*e*f
 + 231*d*g) - 9*a*c*g^2*(6*e^3*f^3 - 33*d*e^2*f^2*g + 88*d^2*e*f*g^2 + 77*
d^3*g^3) + c^2*f^2*(-64*e^3*f^3 + 264*d*e^2*f^2*g - 396*d^2*e*f*g^2 + 231*
d^3*g^3))*(a + c*x^2))/(f + g*x)^2 + (Sqrt[c]*((-I)*Sqrt[c]*f + Sqrt[a]*g)
*(3*a^2*e^2*g^4*(26*e*f + 231*d*g) - 9*a*c*g^2*(6*e^3*f^3 - 33*d*e^2*f^2*g
 + 88*d^2*e*f*g^2 + 77*d^3*g^3) + c^2*f^2*(-64*e^3*f^3 + 264*d*e^2*f^2*g -
 396*d^2*e*f*g^2 + 231*d^3*g^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g
*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*EllipticE[I*ArcSinh[
Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)
/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + (Sqrt[a]*g*((-I)*Sqrt[c]*f +
Sqrt[a]*g)*((-75*I)*a^2*e^3*g^4 - 3*a^(3/2)*Sqrt[c]*e^2*g^3*(e*f + 231*d*g
) + (3*I)*a*c*e*g^2*(2*e^2*f^2 - 33*d*e*f*g + 165*d^2*g^2) + I*c^2*f*(64*e
^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3) + 3*Sqrt[a]*c^(3
/2)*g*(16*e^3*f^3 - 66*d*e^2*f^2*g + 99*d^2*e*f*g^2 + 231*d^3*g^3))*Sqrt[(
g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] -...
```

3.622.3 Rubi [A] (warning: unable to verify)

Time = 3.64 (sec) , antiderivative size = 1311, normalized size of antiderivative = 1.54, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {722, 2185, 27, 2185, 27, 2185, 27, 2185, 25, 27, 599, 25, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}\,dx$$

$$\Big\downarrow \text{722}$$

$$\frac{\int \frac{(d+ex)^3\left(c(ef-3dg)x^2-2(cdf-aeg)x+a(3ef-dg)\right)}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{11e}+\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$$\Big\downarrow \text{2185}$$

$$2\int -\frac{-ce^2g^4\left(18ae^2g^2-c\left(29e^2f^2-96degf+81d^2g^2\right)\right)x^4-ceg^3\left(2ae^2g^2(10ef+33dg)-3c\left(11e^3f^3-33de^2gf^2+9d^2eg^2f+27d^3g^3\right)\right)x^3+3cg^2\left(ae^2\left(7e^2f^2-48degf-\right.\right.}{}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$$\Big\downarrow \; 27$$

$$\frac{2e^3\sqrt{a+cx^2}(f+gx)^{7/2}(ef-3dg)}{9g^4}-\frac{\int \frac{-ce^2g^4\left(18ae^2g^2-c\left(29e^2f^2-96degf+81d^2g^2\right)\right)x^4-ceg^3\left(2ae^2g^2(10ef+33dg)-3c\left(11e^3f^3-33de^2gf^2+9d^2eg^2f+27d\right.\right.}{}}{}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$$\Big\downarrow \; 2185$$

$$\frac{2e^3\sqrt{a+cx^2}(f+gx)^{7/2}(ef-3dg)}{9g^4}-\frac{2\int \frac{c^2e\left(2ae^2g^2(74ef-231dg)-c\left(233e^3f^3-843de^2gf^2+1107d^2eg^2f-567d^3g^3\right)\right)x^3g^7+c\left(90a^2e^4g^4+2ace^2\left(100e^2f^2-26\right.\right.}{}}{}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$$\Big\downarrow \; 27$$

$$\frac{2e^3\sqrt{a+cx^2}(f+gx)^{7/2}(ef-3dg)}{9g^4}-\frac{\int \frac{c^2e\left(2ae^2g^2(74ef-231dg)-c\left(233e^3f^3-843de^2gf^2+1107d^2eg^2f-567d^3g^3\right)\right)x^3g^7+c\left(90a^2e^4g^4+2ace^2\left(100e^2f^2-264\right.\right.}{}}{}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$$\Big\downarrow \; 2185$$

$$\frac{2e^3\sqrt{a+cx^2}(f+gx)^{7/2}(ef-3dg)}{9g^4}-\frac{2\int \frac{3\left(c^2\left(150a^2e^4g^4-6ace^2\left(2e^2f^2-33degf+165d^2g^2\right)g^2+c^2\left(187e^4f^4-732de^3gf^3+1098d^2e^2g^2f^2-798d^3eg^3f+315d^4\right.\right.\right.}{}}{}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$$\Big\downarrow \; 27$$

3.622. $\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

$$\frac{2e^3\sqrt{a+cx^2}(f+gx)^{7/2}(ef-3dg)}{9g^4} - \frac{3\int \frac{c^2\left(150a^2e^4g^4-6ace^2\left(2e^2f^2-33degf+165d^2g^2\right)g^2+c^2\left(187e^4f^4-732de^3gf^3+1098d^2e^2g^2f^2-798d^3eg^3f+315d^4g^4 \cdots\right.\right.}{\cdots}}{\cdots}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$$\Big\downarrow \; 2185$$

$$\frac{2e^3\sqrt{a+cx^2}(f+gx)^{7/2}(ef-3dg)}{9g^4} - \frac{3\left(2\int -\frac{c^2eg^{10}\left(ag\left(75a^2e^3g^4-9ace\left(e^2f^2+66degf+55d^2g^2\right)g^2-c^2f\left(16e^3f^3-66de^2gf^2+99d^2eg^2f-924d^3g^3\right)\right)-c\left(3a \cdots\right.\right.}{\sqrt{\cdots}}\cdots\right.}{\cdots}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$$\Big\downarrow \; 25$$

$$\frac{2e^3\sqrt{a+cx^2}(f+gx)^{7/2}(ef-3dg)}{9g^4} - \frac{3\left(\frac{2}{3}cg^8\sqrt{a+cx^2}\sqrt{f+gx}\left(150a^2e^4g^4-6ace^2g^2\left(165d^2g^2-33defg+2e^2f^2\right)+c^2\left(315d^4g^4-798d^3efg^3+1098d^2e^2f^2g^2-73\cdots\right.\right.\right.}{\cdots}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$$\Big\downarrow \; 27$$

$$\frac{2e^3\sqrt{a+cx^2}(f+gx)^{7/2}(ef-3dg)}{9g^4} - \frac{3\left(\frac{2}{3}cg^8\sqrt{a+cx^2}\sqrt{f+gx}\left(150a^2e^4g^4-6ace^2g^2\left(165d^2g^2-33defg+2e^2f^2\right)+c^2\left(315d^4g^4-798d^3efg^3+1098d^2e^2f^2g^2-73\cdots\right.\right.\right.}{\cdots}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$$\Big\downarrow \; 599$$

3.622. $\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

$$\frac{2e^3\sqrt{a+cx^2}(f+gx)^{7/2}(ef-3dg)}{9g^4} - \frac{3\left(\frac{4}{3}ceg^6\int -\frac{\left(cf^2+ag^2\right)\left(75a^2e^3g^4-3ace\left(2e^2f^2-33degf+165d^2g^2\right)g^2-c^2f\left(64e^3f^3-264de^2gf^2+396d^2eg^2f-231d^3g\right.\right.}{\sqrt{}}\right.}{}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$$\Big\downarrow \; 25$$

$$\frac{2e^3\sqrt{a+cx^2}(f+gx)^{7/2}(ef-3dg)}{9g^4} - \frac{3\left(\frac{2}{3}cg^8\sqrt{a+cx^2}\sqrt{f+gx}\left(150a^2e^4g^4-6ace^2g^2\left(165d^2g^2-33defg+2e^2f^2\right)+c^2\left(315d^4g^4-798d^3efg^3+1098d^2e^2f^2g^2-73\right.\right.\right.}{}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$$\Big\downarrow \; 1511$$

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+a}(d+ex)^4}{11e} +$$

$$\frac{2e^3(ef-3dg)(f+gx)^{7/2}\sqrt{cx^2+a}}{9g^4} - \frac{\frac{2}{5}ce\left(2ae^2g^2(74ef-231dg)-c\left(233e^3f^3-843de^2gf^2+1107d^2eg^2f-567d^3g^3\right)\right)(f+gx)^{3/2}\sqrt{cx^2+a}g^5+\frac{3\left(\frac{2}{3}c\left(150a^2e^4g^4-\right.\right.}{}}{}$$

$$\Big\downarrow \; 1416$$

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+a}(d+ex)^4}{11e} +$$

$$\frac{2e^3(ef-3dg)(f+gx)^{7/2}\sqrt{cx^2+a}}{9g^4} - \frac{\frac{2}{5}ce\left(2ae^2g^2(74ef-231dg)-c\left(233e^3f^3-843de^2gf^2+1107d^2eg^2f-567d^3g^3\right)\right)(f+gx)^{3/2}\sqrt{cx^2+a}g^5+\frac{3\left(\frac{2}{3}c\left(150a^2e^4g^4-\right.\right.}{}}{}$$

$$\Big\downarrow \; 1509$$

3.622. $\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+a}(d+ex)^4}{11e}+$$

$$\frac{2e^3(ef-3dg)(f+gx)^{7/2}\sqrt{cx^2+a}}{9g^4}-\frac{\frac{2}{5}ce\left(2ae^2g^2(74ef-231dg)-c\left(233e^3f^3-843de^2gf^2+1107d^2eg^2f-567d^3g^3\right)\right)(f+gx)^{3/2}\sqrt{cx^2+a}g^5+\frac{3\left(\frac{2}{3}c\left(150a^2e^4g^4-\right.\right.}{}}{}$$

input
```
Int[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

output
```
(2*(d + e*x)^4*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(11*e) + ((2*e^3*(e*f - 3*d*
g)*(f + g*x)^(7/2)*Sqrt[a + c*x^2])/(9*g^4) - ((-2*e^2*g*(18*a*e^2*g^2 - c
*(29*e^2*f^2 - 96*d*e*f*g + 81*d^2*g^2))*(f + g*x)^(5/2)*Sqrt[a + c*x^2])/
7 + ((2*c*e*g^5*(2*a*e^2*g^2*(74*e*f - 231*d*g) - c*(233*e^3*f^3 - 843*d*e
^2*f^2*g + 1107*d^2*e*f*g^2 - 567*d^3*g^3))*(f + g*x)^(3/2)*Sqrt[a + c*x^2
])/5 + (3*((2*c*g^8*(150*a^2*e^4*g^4 - 6*a*c*e^2*g^2*(2*e^2*f^2 - 33*d*e*f
*g + 165*d^2*g^2) + c^2*(187*e^4*f^4 - 732*d*e^3*f^3*g + 1098*d^2*e^2*f^2*
g^2 - 798*d^3*e*f*g^3 + 315*d^4*g^4))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/3 + (
4*c*e*g^6*(-(Sqrt[c]*Sqrt[c*f^2 + a*g^2]*(3*a^2*e^2*g^4*(26*e*f + 231*d*g)
 - c^2*f^2*(64*e^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3)
 - 9*a*c*g^2*(6*e^3*f^3 - 33*d*e^2*f^2*g + 88*d^2*e*f*g^2 + 77*d^3*g^3))*(-
((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x
)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2]
))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])
*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a
+ (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticE
[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)
/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x)
)/g^2 + (c*(f + g*x)^2)/g^2]))) - ((c*f^2 + a*g^2)^(3/4)*(Sqrt[c*f^2 + a*g
^2]*(75*a^2*e^3*g^4 - 3*a*c*e*g^2*(2*e^2*f^2 - 33*d*e*f*g + 165*d^2*g^2...
```

3.622. $\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

3.622.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 722
```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2
]/(e*(2*m + 5))), x] + Simp[1/(e*(2*m + 5))   Int[((d + e*x)^m/(Sqrt[f + g*
x]*Sqrt[a + c*x^2]))*Simp[3*a*e*f - a*d*g - 2*(c*d*f - a*e*g)*x + (c*e*f -
3*c*d*g)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegerQ[2*m
] &&  !LtQ[m, -1]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.622. $\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.622.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1823 vs. $2(761) = 1522$.

Time = 2.82 (sec) , antiderivative size = 1824, normalized size of antiderivative = 2.14

method	result	size
elliptic	Expression too large to display	1824
risch	Expression too large to display	2571
default	Expression too large to display	6457

input
```
int((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/11*e^3*x^4*(c*g
*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/9*(3*c*d*e^2*g+1/11*f*c*e^3)/c/g*x^3*(c*g*
x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/7*(2/11*a*e^3*g+3*c*d^2*e*g+3*c*d*e^2*f-8/9
*(3*c*d*e^2*g+1/11*f*c*e^3)/g*f)/c/g*x^2*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)
+2/5*(3*a*e^2*g*d+3/11*a*e^3*f+c*d^3*g+3*c*d^2*e*f-7/9*(3*c*d*e^2*g+1/11*f
*c*e^3)/c*a-6/7*(2/11*a*e^3*g+3*c*d^2*e*g+3*c*d*e^2*f-8/9*(3*c*d*e^2*g+1/1
1*f*c*e^3)/g*f)/g*f)/c/g*x*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/3*(3*a*d^2*
e*g+3*a*d*e^2*f+c*d^3*f-2/3*(3*c*d*e^2*g+1/11*f*c*e^3)/c/g*f*a-5/7*(2/11*a
*e^3*g+3*c*d^2*e*g+3*c*d*e^2*f-8/9*(3*c*d*e^2*g+1/11*f*c*e^3)/g*f)/c*a-4/5
*(3*a*e^2*g*d+3/11*a*e^3*f+c*d^3*g+3*c*d^2*e*f-7/9*(3*c*d*e^2*g+1/11*f*c*e
^3)/c*a-6/7*(2/11*a*e^3*g+3*c*d^2*e*g+3*c*d*e^2*f-8/9*(3*c*d*e^2*g+1/11*f*
c*e^3)/g*f)/g*f)/g*f)/c/g*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2*(a*d^3*f-2/5
*(3*a*e^2*g*d+3/11*a*e^3*f+c*d^3*g+3*c*d^2*e*f-7/9*(3*c*d*e^2*g+1/11*f*c*e
^3)/c*a-6/7*(2/11*a*e^3*g+3*c*d^2*e*g+3*c*d*e^2*f-8/9*(3*c*d*e^2*g+1/11*f*
c*e^3)/g*f)/g*f)/c/g*f*a-1/3*(3*a*d^2*e*g+3*a*d*e^2*f+c*d^3*f-2/3*(3*c*d*e
^2*g+1/11*f*c*e^3)/c/g*f*a-5/7*(2/11*a*e^3*g+3*c*d^2*e*g+3*c*d*e^2*f-8/9*(
3*c*d*e^2*g+1/11*f*c*e^3)/g*f)/c*a-4/5*(3*a*e^2*g*d+3/11*a*e^3*f+c*d^3*g+3
*c*d^2*e*f-7/9*(3*c*d*e^2*g+1/11*f*c*e^3)/c*a-6/7*(2/11*a*e^3*g+3*c*d^2*e*
g+3*c*d*e^2*f-8/9*(3*c*d*e^2*g+1/11*f*c*e^3)/g*f)/g*f)/g*f)/c*a)*(f/g-(-a*
c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-...
```

3.622.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 765, normalized size of antiderivative = 0.90

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx =$$

$$-\frac{2\left(2\left(64\,c^3e^3f^6-264\,c^3de^2f^5g+6\left(66\,c^3d^2e+17\,ac^2e^3\right)f^4g^2-33\left(7\,c^3d^3+15\,ac^2de^2\right)f^3g^3+3\left(363\,ac^2\right.\right.\right.}{}$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/10395*(2*(64*c^3*e^3*f^6 - 264*c^3*d*e^2*f^5*g + 6*(66*c^3*d^2*e + 17*a
*c^2*e^3)*f^4*g^2 - 33*(7*c^3*d^3 + 15*a*c^2*d*e^2)*f^3*g^3 + 3*(363*a*c^2
*d^2*e - 17*a^2*c*e^3)*f^2*g^4 - 99*(21*a*c^2*d^3 - 11*a^2*c*d*e^2)*f*g^5
+ 45*(33*a^2*c*d^2*e - 5*a^3*e^3)*g^6)*sqrt(c*g)*weierstrassPInverse(4/3*(
c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x +
f)/g) + 6*(64*c^3*e^3*f^5*g - 264*c^3*d*e^2*f^4*g^2 + 18*(22*c^3*d^2*e + 3
*a*c^2*e^3)*f^3*g^3 - 33*(7*c^3*d^3 + 9*a*c^2*d*e^2)*f^2*g^4 + 6*(132*a*c^
2*d^2*e - 13*a^2*c*e^3)*f*g^5 + 693*(a*c^2*d^3 - a^2*c*d*e^2)*g^6)*sqrt(c*
g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2
)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3
 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) - 3*(315*c^3*e^3*g^6*x^4 - 64*c
^3*e^3*f^4*g^2 + 264*c^3*d*e^2*f^3*g^3 - 2*(198*c^3*d^2*e + 23*a*c^2*e^3)*
f^2*g^4 + 33*(7*c^3*d^3 + 8*a*c^2*d*e^2)*f*g^5 + 30*(33*a*c^2*d^2*e - 5*a^
2*c*e^3)*g^6 + 35*(c^3*e^3*f*g^5 + 33*c^3*d*e^2*g^6)*x^3 - 5*(8*c^3*e^3*f^
2*g^4 - 33*c^3*d*e^2*f*g^5 - 9*(33*c^3*d^2*e + 2*a*c^2*e^3)*g^6)*x^2 + (48
*c^3*e^3*f^3*g^3 - 198*c^3*d*e^2*f^2*g^4 + (297*c^3*d^2*e + 32*a*c^2*e^3)*
f*g^5 + 231*(3*c^3*d^3 + 2*a*c^2*d*e^2)*g^6)*x)*sqrt(c*x^2 + a)*sqrt(g*x +
 f))/(c^3*g^6)
```

3.622.6 Sympy [F]

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}\,dx$$

input
```
integrate((e*x+d)**3*(g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)*(d + e*x)**3*sqrt(f + g*x), x)
```

3.622.7 Maxima [F]

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{cx^2+a}(ex+d)^3\sqrt{gx+f}\,dx$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f), x)
```

3.622.8 Giac [F]

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{cx^2+a}(ex+d)^3\sqrt{gx+f}\,dx$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f), x)
```

3.622.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,(d+e\,x)^3\,dx$$

input
```
int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^3,x)
```

output
```
int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^3, x)
```

3.623 $\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

3.623.1 Optimal result

Integrand size = 28, antiderivative size = 635

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx$$
$$= -\frac{2(6ae^2g^2(ef-10dg)-c(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3))\sqrt{f+gx}\sqrt{a+cx^2}}{315ceg^3}$$
$$+\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9e}$$
$$+\frac{4(7ae^2g^2-c(8e^2f^2-24defg+21d^2g^2))(f+gx)^{3/2}\sqrt{a+cx^2}}{315cg^3}$$
$$+\frac{2e(ef-3dg)(f+gx)^{5/2}\sqrt{a+cx^2}}{63g^3}$$
$$+\frac{4\sqrt{-a}(21a^2e^2g^4+3acg^2(3e^2f^2-16defg-21d^2g^2)+c^2f^2(8e^2f^2-24defg+21d^2g^2))\sqrt{f+gx}\sqrt{1+}}{315c^{3/2}g^4\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$-\frac{4\sqrt{-a}(cf^2+ag^2)(3aeg^2(ef-10dg)+cf(8e^2f^2-24defg+21d^2g^2))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}}{315c^{3/2}g^4\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
4/315*(7*a*e^2*g^2-c*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2))*(g*x+f)^(3/2)*(c*x
^2+a)^(1/2)/c/g^3+2/63*e*(-3*d*g+e*f)*(g*x+f)^(5/2)*(c*x^2+a)^(1/2)/g^3-2/
315*(6*a*e^2*g^2*(-10*d*g+e*f)-c*(-35*d^3*g^3+63*d^2*e*f*g^2-57*d*e^2*f^2*
g+19*e^3*f^3))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/e/g^3+2/9*(e*x+d)^3*(g*x+f)
^(1/2)*(c*x^2+a)^(1/2)/e+4/315*(21*a^2*e^2*g^4+3*a*c*g^2*(-21*d^2*g^2-16*d
*e*f*g+3*e^2*f^2)+c^2*f^2*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2))*EllipticE(1/2
*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)
))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/c^(3/2)/g^4/(c*x^2+a)
^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-4/315*(a*g^2+c*f^2
)*(3*a*e*g^2*(-10*d*g+e*f)+c*f*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2))*Elliptic
F(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^
(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)
+f*c^(1/2)))^(1/2)/c^(3/2)/g^4/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

3.623.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.89 (sec) , antiderivative size = 809, normalized size of antiderivative = 1.27

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2(a+cx^2)(2aeg^2(4ef+30dg+7egx)+c(21d^2g^2(f+3gx)+6deg(-4f^2+3fgx+15g^2x^2)+e^2(8f^3-6f^2gx+5fg^2x^2+35g^3x^3)))}{cg^3} - \cdots\right.}{\cdots}$$

input
```
Integrate[(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

output
```
(Sqrt[f + g*x]*((2*(a + c*x^2)*(2*a*e*g^2*(4*e*f + 30*d*g + 7*e*g*x) + c*(
21*d^2*g^2*(f + 3*g*x) + 6*d*e*g*(-4*f^2 + 3*f*g*x + 15*g^2*x^2) + e^2*(8*
f^3 - 6*f^2*g*x + 5*f*g^2*x^2 + 35*g^3*x^3))))/(c*g^3) - (4*(g^2*Sqrt[-f -
 (I*Sqrt[a]*g)/Sqrt[c]]*(21*a^2*e^2*g^4 + c^2*f^2*(8*e^2*f^2 - 24*d*e*f*g
+ 21*d^2*g^2) - 3*a*c*g^2*(-3*e^2*f^2 + 16*d*e*f*g + 21*d^2*g^2))*(a + c*x
^2) - I*Sqrt[c]*(Sqrt[c]*f + I*Sqrt[a]*g)*(21*a^2*e^2*g^4 + c^2*f^2*(8*e^2
*f^2 - 24*d*e*f*g + 21*d^2*g^2) - 3*a*c*g^2*(-3*e^2*f^2 + 16*d*e*f*g + 21*
d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]
*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[
c]*f + I*Sqrt[a]*g)] + Sqrt[a]*Sqrt[c]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*((21*I)
*a^(3/2)*e^2*g^3 - 3*a*Sqrt[c]*e*g^2*(e*f - 10*d*g) + c^(3/2)*f*(-8*e^2*f^
2 + 24*d*e*f*g - 21*d^2*g^2) - (3*I)*Sqrt[a]*c*g*(-2*e^2*f^2 + 6*d*e*f*g +
 21*d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqr
t[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqr
t[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(S
qrt[c]*f + I*Sqrt[a]*g)]))/(c^2*g^5*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f +
g*x))))/(315*Sqrt[a + c*x^2])
```

3.623.3 Rubi [A] (warning: unable to verify)

Time = 2.63 (sec) , antiderivative size = 1041, normalized size of antiderivative = 1.64, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {722, 2185, 27, 2185, 27, 2185, 27, 599, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}\,dx$$

$$\Big\downarrow \ 722$$

$$\frac{\int \frac{(d+ex)^2\left(c(ef-3dg)x^2-2(cdf-aeg)x+a(3ef-dg)\right)}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{9e}+\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9e}$$

$$\Big\downarrow \ 2185$$

$$\frac{\frac{2\int -\frac{-2ceg^3\left(7ae^2g^2-c\left(8e^2f^2-24degf+21d^2g^2\right)\right)x^3-cg^2\left(4ae^2g^2(4ef+9dg)-c\left(11e^3f^3-33de^2gf^2+21d^2eg^2f+21d^3g^3\right)\right)x^2+2cfg\left(ae^2(5ef-36dg)g^2+c\left(e^3f^3 \cdots\right.\right.}{2\sqrt{f+gx}\sqrt{cx^2+a}}}{7cg^4}}{9e}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9e}$$

$\downarrow$ 27

$$\frac{\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(ef-3dg)}{7g^3}-\frac{\int\frac{-2ceg^3\left(7ae^2g^2-c\left(8e^2f^2-24degf+21d^2g^2\right)\right)x^3-cg^2\left(4ae^2g^2(4ef+9dg)-c\left(11e^3f^3-33de^2gf^2+21d^2eg^2f+21d^3g^3\right)\right)}{\sqrt{f+gx}\sqrt{cx^2+}}}{7cg^4}}{9e}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9e}$$

$\downarrow$ 2185

$$\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(ef-3dg)}{7g^3}-\frac{2\int\frac{3c^2\left(6ae^2g^2(ef-10dg)-c\left(19e^3f^3-57de^2gf^2+63d^2eg^2f-35d^3g^3\right)\right)x^2g^5+ac\left(42ae^3fg^2-c\left(23e^3f^3-69de^2gf^2+231d\right.\right.}{2\sqrt{f+}}}{}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9e}$$

$\downarrow$ 27

$$\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(ef-3dg)}{7g^3}-\frac{\int\frac{3c^2\left(6ae^2g^2(ef-10dg)-c\left(19e^3f^3-57de^2gf^2+63d^2eg^2f-35d^3g^3\right)\right)x^2g^5+ac\left(42ae^3fg^2-c\left(23e^3f^3-69de^2gf^2+231d^2\right.\right.}{\sqrt{f+gx}}}{5}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9e}$$

$\downarrow$ 2185

$$\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(ef-3dg)}{7g^3}-\frac{\frac{2\int\frac{3c^2eg^6\left(2ag\left(3aeg^2(3ef+5dg)-cf\left(e^2f^2-3degf+42d^2g^2\right)\right)+\left(21a^2e^2g^4+3ac\left(3e^2f^2-16degf-21d^2g^2\right)g^2+c^2f^2\left(8e^2\right.\right.\right.}{\sqrt{f+gx}\sqrt{cx^2+a}}}{3cg^2}}{5cg}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9e}$$

$\downarrow$ 27

$$\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(ef-3dg)}{7g^3}-\frac{2ceg^4\int\frac{2ag\left(3aeg^2(3ef+5dg)-cf\left(e^2f^2-3degf+42d^2g^2\right)\right)+\left(21a^2e^2g^4+3ac\left(3e^2f^2-16degf-21d^2g^2\right)g^2+c^2f^2\left(8e^2f^2-\right.\right.}{\sqrt{f+gx}\sqrt{cx^2+a}}}{5cg^3}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9e}$$

$\downarrow$ 599

3.623. $\int(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

$$2cg^4\sqrt{a+cx^2}\sqrt{f+gx}\left(6ae^2g^2(ef-10dg)-c\left(-35d^3g^3+63d^2efg^2-57de^2f^2g+19e^3f^3\right)\right)-4ceg^2\int\frac{\left(cf^2+ag^2\right)\left(3ae(ef-10\right.}{}$$

$$\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(ef-3dg)}{7g^3}-$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9e}$$

$\downarrow$ 1511

$$2cg^4\sqrt{a+cx^2}\sqrt{f+gx}\left(6ae^2g^2(ef-10dg)-c\left(-35d^3g^3+63d^2efg^2-57de^2f^2g+19e^3f^3\right)\right)-4ceg^2\left(\frac{\sqrt{ag^2+cf^2}\left(21a^2e^2g^4+\right.}{}\right.$$

$$\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(ef-3dg)}{7g^3}-$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9e}$$

$\downarrow$ 1416

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+a}(d+ex)^3}{9e}+$$

$$2cg^4\left(6ae^2g^2(ef-10dg)-c\left(19e^3f^3-57de^2gf^2+63d^2eg^2f-35d^3g^3\right)\right)\sqrt{f+gx}\sqrt{cx^2+a}-4ceg^2\left(\frac{\sqrt{cf^2+ag^2}\left(21a^2e^2g^4+3a\right.}{}\right.$$

$$\frac{2e^2(ef-3dg)(f+gx)^{5/2}\sqrt{cx^2+a}}{7g^3}-$$

$\downarrow$ 1509

3.623. $\int(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+a}(d+ex)^3}{9e}+$$

$$\frac{\frac{2e^2(ef-3dg)(f+gx)^{5/2}\sqrt{cx^2+a}}{7g^3}-\frac{2cg^4\left(6ae^2g^2(ef-10dg)-c\left(19e^3f^3-57de^2gf^2+63d^2eg^2f-35d^3g^3\right)\right)\sqrt{f+gx}\sqrt{cx^2+a}-4ceg^2\left(\frac{\sqrt{cf^2+ag^2}\left(21a^2e^2g^4+3a\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

input
```
Int[(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

output
```
(2*(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(9*e) + ((2*e^2*(e*f - 3*d*g
)*(f + g*x)^(5/2)*Sqrt[a + c*x^2])/(7*g^3) - ((-4*e*g*(7*a*e^2*g^2 - c*(8*
e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2))*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/5 + (
2*c*g^4*(6*a*e^2*g^2*(e*f - 10*d*g) - c*(19*e^3*f^3 - 57*d*e^2*f^2*g + 63*
d^2*e*f*g^2 - 35*d^3*g^3))*Sqrt[f + g*x]*Sqrt[a + c*x^2] - 4*c*e*g^2*((Sqr
t[c*f^2 + a*g^2]*(21*a^2*e^2*g^4 + 3*a*c*g^2*(3*e^2*f^2 - 16*d*e*f*g - 21*
d^2*g^2) + c^2*f^2*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2))*(-((Sqrt[f + g*x
]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a
 + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2
+ a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*
f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)
*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^(
1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 +
a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f
+ g*x)^2)/g^2]))))/Sqrt[c] - ((c*f^2 + a*g^2)^(3/4)*(21*a^2*e^2*g^4 + 3*a*c
*g^2*(3*e^2*f^2 - 16*d*e*f*g - 21*d^2*g^2) + c^2*f^2*(8*e^2*f^2 - 24*d*e*f
*g + 21*d^2*g^2) - Sqrt[c]*Sqrt[c*f^2 + a*g^2]*(3*a*e*g^2*(e*f - 10*d*g) +
 c*f*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2)))*(1 + (Sqrt[c]*(f + g*x))/Sqrt
[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g
*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*...
```

3.623. $\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

3.623.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 722
```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2
]/(e*(2*m + 5))), x] + Simp[1/(e*(2*m + 5))   Int[((d + e*x)^m/(Sqrt[f + g*
x]*Sqrt[a + c*x^2]))*Simp[3*a*e*f - a*d*g - 2*(c*d*f - a*e*g)*x + (c*e*f -
3*c*d*g)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegerQ[2*m
] &&  !LtQ[m, -1]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.623. $\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.623.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1141 vs. $2(551) = 1102$.

Time = 1.40 (sec) , antiderivative size = 1142, normalized size of antiderivative = 1.80

method	result	size
elliptic	Expression too large to display	1142
risch	Expression too large to display	1677
default	Expression too large to display	4352

input
```
int((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/9*e^2*x^3*(c*g*
x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/7*(2*c*d*e*g+1/9*c*e^2*f)/c/g*x^2*(c*g*x^3+
c*f*x^2+a*g*x+a*f)^(1/2)+2/5*(2/9*a*e^2*g+c*d^2*g+2*c*d*e*f-6/7*f/g*(2*c*d
*e*g+1/9*c*e^2*f))/c/g*x*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/3*(2*a*d*e*g+
1/3*a*e^2*f+c*d^2*f-4/5*f/g*(2/9*a*e^2*g+c*d^2*g+2*c*d*e*f-6/7*f/g*(2*c*d*
e*g+1/9*c*e^2*f))-5/7*a/c*(2*c*d*e*g+1/9*c*e^2*f))/c/g*(c*g*x^3+c*f*x^2+a*
g*x+a*f)^(1/2)+2*(a*d^2*f-2/5*f*a/c/g*(2/9*a*e^2*g+c*d^2*g+2*c*d*e*f-6/7*f
/g*(2*c*d*e*g+1/9*c*e^2*f))-1/3*a/c*(2*a*d*e*g+1/3*a*e^2*f+c*d^2*f-4/5*f/g
*(2/9*a*e^2*g+c*d^2*g+2*c*d*e*f-6/7*f/g*(2*c*d*e*g+1/9*c*e^2*f))-5/7*a/c*(
2*c*d*e*g+1/9*c*e^2*f)))*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c
))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)
/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*Ellipti
cF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c
)^(1/2)/c))^(1/2))+2*(a*d^2*g+2*a*d*e*f-4/7*f*a/c/g*(2*c*d*e*g+1/9*c*e^2*f
)-3/5*a/c*(2/9*a*e^2*g+c*d^2*g+2*c*d*e*f-6/7*f/g*(2*c*d*e*g+1/9*c*e^2*f))-
2/3*f/g*(2*a*d*e*g+1/3*a*e^2*f+c*d^2*f-4/5*f/g*(2/9*a*e^2*g+c*d^2*g+2*c*d*
e*f-6/7*f/g*(2*c*d*e*g+1/9*c*e^2*f))-5/7*a/c*(2*c*d*e*g+1/9*c*e^2*f)))*(f/
g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)
/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(
1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE...
```

3.623.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 510, normalized size of antiderivative = 0.80

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx$$

$$= \frac{2\left(2\left(8c^2e^2f^5 - 24c^2def^4g - 66acdef^2g^3 - 90a^2deg^5 + 3\left(7c^2d^2 + 5ace^2\right)f^3g^2 + 3\left(63acd^2 - 11a^2e^2\right)fg\right.\right.}{}$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/945*(2*(8*c^2*e^2*f^5 - 24*c^2*d*e*f^4*g - 66*a*c*d*e*f^2*g^3 - 90*a^2*d
*e*g^5 + 3*(7*c^2*d^2 + 5*a*c*e^2)*f^3*g^2 + 3*(63*a*c*d^2 - 11*a^2*e^2)*f
*g^4)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(
c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 6*(8*c^2*e^2*f^4*g - 24*c
^2*d*e*f^3*g^2 - 48*a*c*d*e*f*g^4 + 3*(7*c^2*d^2 + 3*a*c*e^2)*f^2*g^3 - 21
*(3*a*c*d^2 - a^2*e^2)*g^5)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2
)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f
^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/
g)) + 3*(35*c^2*e^2*g^5*x^3 + 8*c^2*e^2*f^3*g^2 - 24*c^2*d*e*f^2*g^3 + 60*
a*c*d*e*g^5 + (21*c^2*d^2 + 8*a*c*e^2)*f*g^4 + 5*(c^2*e^2*f*g^4 + 18*c^2*d
*e*g^5)*x^2 - (6*c^2*e^2*f^2*g^3 - 18*c^2*d*e*f*g^4 - 7*(9*c^2*d^2 + 2*a*c
*e^2)*g^5)*x)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c^2*g^5)
```

3.623.6 Sympy [F]

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx=\int \sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}\,dx$$

input
```
integrate((e*x+d)**2*(g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)*(d + e*x)**2*sqrt(f + g*x), x)
```

3.623.7 Maxima [F]

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx=\int \sqrt{cx^2+a}(ex+d)^2\sqrt{gx+f}\,dx$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f), x)
```

3.623.8 Giac [F]

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{cx^2+a}(ex+d)^2\sqrt{gx+f}\,dx$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f), x)
```

3.623.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,(d+e\,x)^2\,dx$$

input
```
int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^2,x)
```

output
```
int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^2, x)
```

3.624 $\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

3.624.1 Optimal result

Integrand size = 26, antiderivative size = 434

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx$$
$$= -\frac{2\sqrt{f+gx}(5aeg^2+cf(4ef-7dg)-3cg(ef+7dg)x)\sqrt{a+cx^2}}{105cg^2}$$
$$+\frac{2e\sqrt{f+gx}(a+cx^2)^{3/2}}{7c}$$
$$-\frac{4\sqrt{-a}(cf^2(4ef-7dg)+ag^2(8ef+21dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{105\sqrt{c}g^3\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$+\frac{4\sqrt{-a}(cf^2+ag^2)(5aeg^2+cf(4ef-7dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{}{\sqrt{-a}}\right.}{105c^{3/2}g^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
2/7*e*(c*x^2+a)^(3/2)*(g*x+f)^(1/2)/c-2/105*(5*a*e*g^2+c*f*(-7*d*g+4*e*f)-
3*c*g*(7*d*g+e*f)*x)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^2-4/105*(c*f^2*(-7*
d*g+4*e*f)+a*g^2*(21*d*g+8*e*f))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1
/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)
^(1/2)*(1+c*x^2/a)^(1/2)/g^3/c^(1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(
-a)^(1/2)+f*c^(1/2)))^(1/2)+4/105*(a*g^2+c*f^2)*(5*a*e*g^2+c*f*(-7*d*g+4*e
*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*
(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/
(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

3.624.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.39 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.41

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2(a+cx^2)(10aeg^2+7cdg(f+3gx)+ce(-4f^2+3fgx+15g^2x^2))}{cg^2}+\frac{4\left(g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(cf^2(4ef-7dg)+ag^2(8ef+21dg))(a+cx^2)+\right.}{}\right.}{}$$

input
```
Integrate[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

output
```
(Sqrt[f + g*x]*((2*(a + c*x^2)*(10*a*e*g^2 + 7*c*d*g*(f + 3*g*x) + c*e*(-4
*f^2 + 3*f*g*x + 15*g^2*x^2)))/(c*g^2) + (4*(g^2*Sqrt[-f - (I*Sqrt[a]*g)/S
qrt[c]]*(c*f^2*(4*e*f - 7*d*g) + a*g^2*(8*e*f + 21*d*g))*(a + c*x^2) + I*S
qrt[c]*(Sqrt[c]*f + I*Sqrt[a]*g)*(c*f^2*(-4*e*f + 7*d*g) - a*g^2*(8*e*f +
21*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*
g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f
- (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c
]*f + I*Sqrt[a]*g)] + Sqrt[a]*g*(I*Sqrt[c]*f - Sqrt[a]*g)*((5*I)*a*e*g^2 +
 I*c*f*(4*e*f - 7*d*g) + 3*Sqrt[a]*Sqrt[c]*g*(e*f + 7*d*g))*Sqrt[(g*((I*Sq
rt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f +
g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]
]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(
c*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/(105*Sqrt[a + c*x^2])
```

3.624.3 Rubi [A] (warning: unable to verify)

Time = 1.07 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.81, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {687, 27, 682, 27, 599, 25, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^2}(d+ex)\sqrt{f+gx}\,dx$$

$$\Big\downarrow \text{687}$$

$$\frac{2\int \frac{(7cdf-aeg+c(ef+7dg)x)\sqrt{cx^2+a}}{2\sqrt{f+gx}}dx}{7c}+\frac{2e\left(a+cx^2\right)^{3/2}\sqrt{f+gx}}{7c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(7cdf-aeg+c(ef+7dg)x)\sqrt{cx^2+a}}{\sqrt{f+gx}}dx}{7c}+\frac{2e\left(a+cx^2\right)^{3/2}\sqrt{f+gx}}{7c}$$

$$\Big\downarrow \text{682}$$

$$\frac{\frac{4\int -\frac{c\left(ag\left(5aeg^2+cf(ef-28dg)\right)-c\left(c(4ef-7dg)f^2+ag^2(8ef+21dg)\right)x\right)}{2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{15cg^2}-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(5aeg^2-3cgx(7dg+ef)+cf(4ef-7dg))}{15g^2}}{7c}+\frac{2e\left(a+cx^2\right)^{3/2}\sqrt{f+gx}}{7c}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{2\int \frac{ag\left(5aeg^2+cf(ef-28dg)\right)-c\left(c(4ef-7dg)f^2+ag^2(8ef+21dg)\right)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{15g^2}-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(5aeg^2-3cgx(7dg+ef)+cf(4ef-7dg))}{15g^2}}{7c}+\frac{2e\left(a+cx^2\right)^{3/2}\sqrt{f+gx}}{7c}$$

$$\Big\downarrow \text{599}$$

$$\frac{\frac{4\int -\frac{\left(cf^2+ag^2\right)\left(5aeg^2+cf(4ef-7dg)\right)-c\left(c(4ef-7dg)f^2+ag^2(8ef+21dg)\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{15g^4}-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(5aeg^2-3cgx(7dg+ef)+cf(4ef-7dg)}{15g^2}}{7c}$$
$$\frac{2e\left(a+cx^2\right)^{3/2}\sqrt{f+gx}}{7c}$$

$\downarrow$ 25

$$-\frac{\frac{4\int \frac{(cf^2+ag^2)(5aeg^2+cf(4ef-7dg))-c(c(4ef-7dg)f^2+ag^2(8ef+21dg))(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{15g^4}-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(5aeg^2-3cgx(7dg+ef)+cf(4ef-7dg}{15g^2}}{7c}$$

$$\frac{2e(a+cx^2)^{3/2}\sqrt{f+gx}}{7c}$$

$\downarrow$ 1511

$$\frac{4\left(-\sqrt{ag^2+cf^2}\left(\sqrt{ag^2+cf^2}(5aeg^2+cf(4ef-7dg))-\sqrt{c}(ag^2(21dg+8ef)+cf^2(4ef-7dg))\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\sqrt{c}\sqrt{ag^2}\right.}{15g^4}$$

$$\frac{2e(a+cx^2)^{3/2}\sqrt{f+gx}}{7c}$$

$\downarrow$ 1416

$$\frac{4\left(-\sqrt{c}\sqrt{ag^2+cf^2}(ag^2(21dg+8ef)+cf^2(4ef-7dg))\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{(ag^2+cf^2)^{3/4}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f-}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{}}{\sqrt{}}\right.}}}{}\right.}{15g}$$

$$\frac{2e(a+cx^2)^{3/2}\sqrt{f+gx}}{7c}$$

$\downarrow$ 1509

$$4\left(-\frac{(ag^2+cf^2)^{3/4}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(\sqrt{ag^2+cf^2}(5aeg^2+cf(4ef-7dg))-\sqrt{c}(ag^2(21dg+8ef)+cf^2(4ef-7dg))\right)\text{EllipticF}\left(\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right.$$

$$\frac{2e(a+cx^2)^{3/2}\sqrt{f+gx}}{7c}$$

3.624. $\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

input
```
Int[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

output
```
(2*e*Sqrt[f + g*x]*(a + c*x^2)^(3/2))/(7*c) + ((-2*Sqrt[f + g*x]*(5*a*e*g^
2 + c*f*(4*e*f - 7*d*g) - 3*c*g*(e*f + 7*d*g)*x)*Sqrt[a + c*x^2])/(15*g^2)
 + (4*(-(Sqrt[c]*Sqrt[c*f^2 + a*g^2]*(c*f^2*(4*e*f - 7*d*g) + a*g^2*(8*e*f
 + 21*d*g))*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2
 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[
c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c
*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x
)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])
^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1
 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2
*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2]))) - ((c*f^2 + a*g^2)^(3/4)*(Sq
rt[c*f^2 + a*g^2]*(5*a*e*g^2 + c*f*(4*e*f - 7*d*g)) - Sqrt[c]*(c*f^2*(4*e*
f - 7*d*g) + a*g^2*(8*e*f + 21*d*g)))*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2
+ a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/
g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*
EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (S
qrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(2*c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*
f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/(15*g^4))/(7*c)
```

3.624.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.624. $\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

3.624.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(362) = 724$.

Time = 1.44 (sec) , antiderivative size = 794, normalized size of antiderivative = 1.83

method	result
elliptic	$\sqrt{(gx+f)(cx^2+a)}\left(\frac{2e\,x^2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{7}+\frac{2\left(cdg+\frac{1}{7}cef\right)x\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{5cg}+\frac{2\left(\frac{2aeg}{7}+cdf-\frac{4f\left(cdg+\frac{1}{7}cef\right)}{5g}\right)\sqrt{cg\,x^3+cf\,x^2+}}{3cg}\right.$
risch	Expression too large to display
default	Expression too large to display

input
```
int((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/7*e*x^2*(c*g*x^
3+c*f*x^2+a*g*x+a*f)^(1/2)+2/5*(c*d*g+1/7*c*e*f)/c/g*x*(c*g*x^3+c*f*x^2+a*
g*x+a*f)^(1/2)+2/3*(2/7*a*e*g+c*d*f-4/5*f/g*(c*d*g+1/7*c*e*f))/c/g*(c*g*x^
3+c*f*x^2+a*g*x+a*f)^(1/2)+2*(a*d*f-2/5*a/c*f/g*(c*d*g+1/7*c*e*f)-1/3*a/c*
(2/7*a*e*g+c*d*f-4/5*f/g*(c*d*g+1/7*c*e*f)))*(f/g-(-a*c)^(1/2)/c)*((x+f/g)
/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1
/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*
x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^
(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(a*d*g+3/7*a*e*f-3/5*a/c*(c*d*g+1
/7*c*e*f)-2/3*f/g*(2/7*a*e*g+c*d*f-4/5*f/g*(c*d*g+1/7*c*e*f)))*(f/g-(-a*c)
^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(
-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*
g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(
f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(
1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g
+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))
```

3.624.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.79

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx =$$
$$-\frac{2\left(2\left(4c^2ef^4-7c^2df^3g+11acef^2g^2-63acdfg^3+15a^2eg^4\right)\sqrt{cg}\,\text{weierstrassPInverse}\left(\frac{4(cf^2-3ag^2)}{3cg^2},-\frac{8}{}\right.}{}$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/315*(2*(4*c^2*e*f^4 - 7*c^2*d*f^3*g + 11*a*c*e*f^2*g^2 - 63*a*c*d*f*g^3
 + 15*a^2*e*g^4)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^
2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 6*(4*c^2*e*f^3
*g - 7*c^2*d*f^2*g^2 + 8*a*c*e*f*g^3 + 21*a*c*d*g^4)*sqrt(c*g)*weierstrass
Zeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), wei
erstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/
(c*g^3), 1/3*(3*g*x + f)/g)) - 3*(15*c^2*e*g^4*x^2 - 4*c^2*e*f^2*g^2 + 7*c
^2*d*f*g^3 + 10*a*c*e*g^4 + 3*(c^2*e*f*g^3 + 7*c^2*d*g^4)*x)*sqrt(c*x^2 +
a)*sqrt(g*x + f))/(c^2*g^4)
```

3.624.6 Sympy [F]

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{a+cx^2}(d+ex)\sqrt{f+gx}\,dx$$

input
```
integrate((e*x+d)*(g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)*(d + e*x)*sqrt(f + g*x), x)
```

3.624.7 Maxima [F]

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{cx^2+a}(ex+d)\sqrt{gx+f}\,dx$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f), x)
```

3.624.8 Giac [F]

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{cx^2+a}(ex+d)\sqrt{gx+f}\,dx$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f), x)
```

3.624.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,(d+e\,x)\,dx$$

input
```
int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x),x)
```

output
```
int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x), x)
```

3.625 $\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx$

3.625.1 Optimal result

Integrand size = 21, antiderivative size = 362

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx = -\frac{4f\sqrt{f+gx}\sqrt{a+cx^2}}{15g}+\frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g} + \frac{4\sqrt{-a}(cf^2-3ag^2)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \frac{4\sqrt{-a}f(cf^2+ag^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
2/5*(g*x+f)^(3/2)*(c*x^2+a)^(1/2)/g-4/15*f*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g
+4/15*(-3*a*g^2+c*f^2)*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2
),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+
c*x^2/a)^(1/2)/g^2/c^(1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+
f*c^(1/2)))^(1/2)-4/15*f*(a*g^2+c*f^2)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/
2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(
1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/g^2/c^(1
/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

3.625.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.16 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.44

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx$$
$$= \frac{\sqrt{f+gx}\left(\frac{2(f+3gx)(a+cx^2)}{g} - \frac{4\left(g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(cf^2-3ag^2)(a+cx^2)+\sqrt{c}(-ic^{3/2}f^3+\sqrt{a}cf^2g+3ia\sqrt{c}fg^2-3a^{3/2}g^3)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

output
```
(Sqrt[f + g*x]*((2*(f + 3*g*x)*(a + c*x^2))/g - (4*(g^2*Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]*(c*f^2 - 3*a*g^2)*(a + c*x^2) + Sqrt[c]*((-I)*c^(3/2)*f^3 +
 Sqrt[a]*c*f^2*g + (3*I)*a*Sqrt[c]*f*g^2 - 3*a^(3/2)*g^3)*Sqrt[(g*((I*Sqrt
[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*
x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - Sqr
t[a]*Sqrt[c]*g*(c*f^2 + (4*I)*Sqrt[a]*Sqrt[c]*f*g - 3*a*g^2)*Sqrt[(g*((I*S
qrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f +
 g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c
]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/
(c*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/(15*Sqrt[a + c*x^2])
```

3.625.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {493, 687, 27, 599, 25, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^2}\sqrt{f+gx}\,dx$$

$$\Big\downarrow \text{493}$$

$$\frac{2\int \frac{(ag-cfx)\sqrt{f+gx}}{\sqrt{cx^2+a}}dx}{5g}+\frac{2\sqrt{a+cx^2}(f+gx)^{3/2}}{5g}$$

$$\Big\downarrow \; 687$$

$$\frac{2\left(\frac{2\int \frac{c\left(4afg-\left(cf^2-3ag^2\right)x\right)}{2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3c}-\frac{2}{3}f\sqrt{a+cx^2}\sqrt{f+gx}\right)}{5g}+\frac{2\sqrt{a+cx^2}(f+gx)^{3/2}}{5g}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(\frac{1}{3}\int \frac{4afg-(cf^2-3ag^2)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2}{3}f\sqrt{a+cx^2}\sqrt{f+gx}\right)}{5g}+\frac{2\sqrt{a+cx^2}(f+gx)^{3/2}}{5g}$$

$$\Big\downarrow \; 599$$

$$\frac{2\left(-\frac{2\int -\frac{f\left(cf^2+ag^2\right)-\left(cf^2-3ag^2\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3g^2}-\frac{2}{3}f\sqrt{a+cx^2}\sqrt{f+gx}\right)}{5g}+\frac{2\sqrt{a+cx^2}(f+gx)^{3/2}}{5g}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(\frac{2\int \frac{f\left(cf^2+ag^2\right)-\left(cf^2-3ag^2\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3g^2}-\frac{2}{3}f\sqrt{a+cx^2}\sqrt{f+gx}\right)}{5g}+\frac{2\sqrt{a+cx^2}(f+gx)^{3/2}}{5g}$$

$$\Big\downarrow \; 1511$$

$$\frac{2\left(-\frac{2\left(\frac{\sqrt{ag^2+cf^2}\left(-\sqrt{c}f\sqrt{ag^2+cf^2}-3ag^2+cf^2\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\frac{\left(cf^2-3ag^2\right)\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}}}}{\sqrt{c}}\right)}{3g^2}\right)}{5g}$$

$$\frac{2\sqrt{a+cx^2}(f+gx)^{3/2}}{5g}$$

3.625. $\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx$

$$\Big\downarrow \text{1416}$$

$$2\left(-\frac{2\left(\frac{\left(ag^2+cf^2\right)^{3/4}\left(-\sqrt{c}f\sqrt{ag^2+cf^2}-3ag^2+cf^2\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{}{\sqrt{c}}\right.\right.}{2c^{3/4}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right)}{3g^2}\right)$$

$$\frac{}{5g}$$

$$\frac{2\sqrt{a+cx^2}(f+gx)^{3/2}}{5g}$$

$$\Big\downarrow \text{1509}$$

3.625. $\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx$

$$2\left(-\frac{2\left(\frac{\left(ag^2+cf^2\right)^{3/4}\left(-\sqrt{c}f\sqrt{ag^2+cf^2}-3ag^2+cf^2\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\ldots}{\sqrt{c\ldots}}\right.\right.}{2c^{3/4}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right.}{\ldots}\right.$$

$$\frac{2\sqrt{a+cx^2}(f+gx)^{3/2}}{5g}$$

input
```
Int[Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

3.625. $\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx$

output
```
(2*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(5*g) + (2*((-2*f*Sqrt[f + g*x]*Sqrt[a
 + c*x^2])/3 - (2*(-(((c*f^2 - 3*a*g^2)*Sqrt[c*f^2 + a*g^2]*(-((Sqrt[f + g
*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/(
(a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^
2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (
c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^
2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c
^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2
+ a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(
f + g*x)^2)/g^2]))))/Sqrt[c]) + ((c*f^2 + a*g^2)^(3/4)*(c*f^2 - 3*a*g^2 - S
qrt[c]*f*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2]
)*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a
 + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*Elliptic
F[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f
)/Sqrt[c*f^2 + a*g^2])/2])/(2*c^(3/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g
*x))/g^2 + (c*(f + g*x)^2)/g^2])))/(3*g^2)))/(5*g)
```

3.625.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.625.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(290) = 580$.

Time = 0.93 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.66

method	result
risch	$\frac{2(3gx+f)\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{15g}+\frac{2\left(\frac{8afg\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}+\frac{2\left(3a\,g\cdots\right.}{\cdots}\right.}{\cdots}$
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(\frac{2x\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{5}+\frac{2f\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{15g}+\frac{16fa\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\cdots}{15\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(3*g*x+f)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+2/15/g*(8*a*f*g*(f/g-(-a*c)
^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(
-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*
g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1
/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(3*a*g^2-c*f^2)
*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2
)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c
))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*Elliptic
E(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)
^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^
(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))*((g*x+f)*(c*x
^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

3.625. $\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx$

3.625.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.63

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx$$
$$= \frac{2\left(2\,(cf^3+9\,afg^2)\sqrt{cg}\,\text{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)+6\,(cf^2g-3\,ag^3)\sqrt{cg}\,\text{wei}\right.}{}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(2*(c*f^3 + 9*a*f*g^2)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a
*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 6*(
c*f^2*g - 3*a*g^3)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2)
, -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*
g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) + 3*(
3*c*g^3*x + c*f*g^2)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c*g^3)
```

3.625.6 Sympy [F]

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{a+cx^2}\sqrt{f+gx}\,dx$$

input
```
integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)*sqrt(f + g*x), x)
```

3.625.7 Maxima [F]

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{cx^2+a}\sqrt{gx+f}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*sqrt(g*x + f), x)
```

3.625.8 Giac [F]

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx=\int \sqrt{cx^2+a}\sqrt{gx+f}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*sqrt(g*x + f), x)
```

3.625.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx=\int \sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,dx$$

input
```
int((f + g*x)^(1/2)*(a + c*x^2)^(1/2),x)
```

output
```
int((f + g*x)^(1/2)*(a + c*x^2)^(1/2), x)
```

3.626 $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx$

3.626.1 Optimal result

Integrand size = 28, antiderivative size = 683

$$\begin{aligned}\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx &= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} \\ &- \frac{2\sqrt{-a}\sqrt{c}(ef-3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3e^2g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} \\ &+ \frac{2\sqrt{-a}\sqrt{c}f(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3e^2g\sqrt{f+gx}\sqrt{a+cx^2}} \\ &- \frac{2\sqrt{-a}(2ae^2g-3cd(ef-dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}e^3\sqrt{f+gx}\sqrt{a+cx^2}} \\ &- \frac{2(cd^2+ae^2)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}\end{aligned}$$

output
```
2/3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e-2/3*(-3*d*g+e*f)*EllipticE(1/2*(1-x*c^
(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2)
)*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/e^2/g/(c*x^2+a)^(1/2)
/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-2/3*(2*a*e^2*g-3*c*d*(-d
*g+e*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*
g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1
/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^3/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1
/2)+2/3*f*(-3*d*g+e*f)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2
),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/
a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/g/(g*x+f)^(1
/2)/(c*x^2+a)^(1/2)-2*(a*e^2+c*d^2)*(-d*g+e*f)*EllipticPi(1/2*(1-x*c^(1/2)
/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1
/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g
*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^3/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(
c*x^2+a)^(1/2)
```

3.626.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.08 (sec) , antiderivative size = 1216, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e}$$

$$+ \frac{(f+gx)^{3/2}\left(2ce^2f\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}} - 6cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}} + \frac{2ce^2f^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{(f+gx)^2} - \frac{6cdef^2g\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{(f+gx)^2} + \frac{2ae^2fg^2\sqrt{-\cdots}}{(f+gx\cdots}\right.}{\cdots}$$

input
```
Integrate[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x),x]
```

output
```
(2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*e) + ((f + g*x)^(3/2)*(2*c*e^2*f*Sqrt
[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 6*c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]
+ (2*c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (6*c*d*e*f^
2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (2*a*e^2*f*g^2*Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (6*a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]])/(f + g*x)^2 - (4*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])
/(f + g*x) + (12*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) + (
2*Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(e*f - 3*d*g)*Sqrt[1 - f/(f + g*x
) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g
)/(Sqrt[c]*(f + g*x))]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]
]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sq
rt[f + g*x] + (2*e*(3*Sqrt[c]*d - I*Sqrt[a]*e)*g*((-I)*Sqrt[c]*f + Sqrt[a]
*g)*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(
f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticF[I*ArcSinh[Sqrt[-f
- (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c
]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + ((6*I)*c*d^2*g^2*Sqrt[1 - f/(f + g*x)
 - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)
/(Sqrt[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*S
qrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sq
rt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + ((6*...
```

3.626.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}\sqrt{f+gx}}{d+ex}\,dx$$

$$\Big\downarrow \; 722$$

$$\frac{\int \frac{c(ef-3dg)x^2-2(cdf-aeg)x+a(3ef-dg)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 2349$$

$$\frac{\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}+\int \frac{\frac{3cgd^2}{e^2}-\frac{3cfd}{e}+2ag+\left(cf-\frac{3cdg}{e}\right)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 599$$

$$\frac{\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \ 25$$

3.626. $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3\left(ae^2+cd^2\right)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \color{blue}{25}$$

$$\frac{\frac{3\left(ae^2+cd^2\right)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \color{blue}{25}$$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3\left(ae^2+cd^2\right)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \color{blue}{25}$$

$$\frac{\frac{3\left(ae^2+cd^2\right)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \color{blue}{25}$$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3\left(ae^2+cd^2\right)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \color{blue}{25}$$

3.626. $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx$

$$\frac{\frac{\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}}{}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \ 25$$

3.626. $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx$

$$\frac{\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}}{}$$

$\downarrow$ 25

$$\frac{\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$\downarrow$ 25

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$\downarrow$ 25

$$\frac{\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$\downarrow$ 25

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$\downarrow$ 25

3.626. $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx$

$$\frac{\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 25$$

3.626. $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx$

$$\frac{\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

input
```
Int[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x),x]
```

output
```
$Aborted
```

3.626.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

3.626. $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx$

rule 722
```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2
]/(e*(2*m + 5))), x] + Simp[1/(e*(2*m + 5))   Int[((d + e*x)^m/(Sqrt[f + g*
x]*Sqrt[a + c*x^2]))*Simp[3*a*e*f - a*d*g - 2*(c*d*f - a*e*g)*x + (c*e*f -
3*c*d*g)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegerQ[2*m
] &&  !LtQ[m, -1]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

3.626.4 Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 922, normalized size of antiderivative = 1.35

method	result
elliptic	$\frac{\sqrt{(gx+f)(c x^2+a)}\left(\frac{2\sqrt{cg x^3+cf x^2+agx+fa}}{3e}+\frac{2\left(\frac{a e^2 g+c d^2 g-cdef}{e^3}-\frac{ag}{3e}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}F\left(\sqrt{\frac{f}{g}\cdots}\right.}{\sqrt{cg x^3+cf x^2+agx+fa}}\cdots\right.}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/3/e*(c*g*x^3+c*
f*x^2+a*g*x+a*f)^(1/2)+2*((a*e^2*g+c*d^2*g-c*d*e*f)/e^3-1/3/e*a*g)*(f/g-(-
a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f
/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)
/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c)
)^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(-(d*g-e*f)
/e^2*c-2/3/e*c*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2
)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f
/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^
(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2
)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-
(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)
))-2*(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/e^4*(f/g-(-a*c)^(1/2)/c)*((x+f/
g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^
(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*
g*x+a*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),
(-f/g+(-a*c)^(1/2)/c)/(-f/g+d/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)
/c))^(1/2)))
```

3.626.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.626.6 Sympy [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{a+cx^2}\sqrt{f+gx}}{d+ex}\,dx$$

input
```
integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2)/(e*x+d),x)
```

output
```
Integral(sqrt(a + c*x**2)*sqrt(f + g*x)/(d + e*x), x)
```

3.626.7 Maxima [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{cx^2+a}\sqrt{gx+f}}{ex+d}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d), x)
```

3.626.8 Giac [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{cx^2+a}\sqrt{gx+f}}{ex+d}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d), x)
```

3.626.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

input
```
int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x),x)
```

output
```
int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x), x)
```

3.627 $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx$

3.627.1 Optimal result

Integrand size = 28, antiderivative size = 650

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)} - \frac{3\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$+\frac{3\sqrt{-a}\sqrt{c}f\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$-\frac{\sqrt{-a}\sqrt{c}(2ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^3\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$-\frac{(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
-(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e/(e*x+d)-3*EllipticE(1/2*(1-x*c^(1/2)/(-a)
^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/
2)*c^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/e^2/(c*x^2+a)^(1/2)/((g*x+f)*c^
(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+3*f*EllipticF(1/2*(1-x*c^(1/2)/(-a)^
(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2
)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/
2)/e^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-(-3*d*g+2*e*f)*EllipticF(1/2*(1-x*c^(
1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))
*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(
1/2)))^(1/2)/e^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-(a*e^2*g-c*d*(-3*d*g+2*e*f)
)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(
-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(1+c*x^2
/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^3/(e+d*c^(1/2
)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

3.627.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.77 (sec) , antiderivative size = 1331, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx$$

$$= \frac{\sqrt{f+gx}\left(-\frac{e^2(a+cx^2)}{d+ex} - \frac{-3ce^2f^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+3cdef^2g\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}-3ae^2fg^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+3adeg^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+6ce^2f^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^2,x]
```

output
```
(Sqrt[f + g*x]*(-((e^2*(a + c*x^2))/(d + e*x)) - (-3*c*e^2*f^3*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]] + 3*c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3
*a*e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 3*a*d*e*g^3*Sqrt[-f - (I*S
qrt[a]*g)/Sqrt[c]] + 6*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x
) - 6*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 3*c*e^2*f*Sqr
t[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + 3*c*d*e*g*Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]]*(f + g*x)^2 + 3*Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f
) + d*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*
g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f
- (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c
]*f + I*Sqrt[a]*g)] + e*(Sqrt[c]*f + I*Sqrt[a]*g)*(Sqrt[a]*e*g - I*Sqrt[c]
*(2*e*f - 3*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I
*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh
[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g
)/(Sqrt[c]*f + I*Sqrt[a]*g)] + (2*I)*c*d*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c
] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g
*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)),
I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*
Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (3*I)*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a
])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g...
```

3.627.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1547 vs. $2(650) = 1300$.

Time = 2.97 (sec) , antiderivative size = 1547, normalized size of antiderivative = 2.38, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {721, 2349, 599, 27, 729, 25, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)^2}\,dx$$

$$\Big\downarrow \text{721}$$

$$\frac{\int \frac{3cgx^2+2cfx+ag}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2e}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{e(d+ex)}$$

$$\Big\downarrow \text{2349}$$

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{\frac{2cf}{e}-\frac{3cdg}{e^2}+\frac{3cgx}{e}}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2e}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{e(d+ex)}$$

$$\Big\downarrow \; 599$$

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2\int \frac{cg(ef+3dg-3e(f+gx))}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2e}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{e(d+ex)}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2c\int \frac{ef+3dg-3e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e^2g}}{2e}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{e(d+ex)}$$

$$\Big\downarrow \; 729$$

$$\frac{2\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\int -\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\int \frac{ef+3dg-3e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e^2g}}{2e}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{e(d+ex)}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{2c\int \frac{ef+3dg-3e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e^2g}-2\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{2e}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{e(d+ex)}$$

$$\Big\downarrow \; 1511$$

3.627. $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx$

$$\frac{-2\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\left(\left(e\left(f-\frac{3\sqrt{ag^2+cf^2}}{\sqrt{c}}\right)+3dg\right)\int \frac{}{\sqrt{\frac{cf^2}{g^2}-\frac{2c}{}}}\right.}{2e}}{\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{e(d+ex)}}$$

$\Big\downarrow$ 1416

$$\frac{-\frac{2c\left(\frac{3e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}+\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(e\left(f-\frac{3\sqrt{ag^2+cf^2}}{\sqrt{c}}\right.\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}}}\right.}{e^2g}}{\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{e(d+ex)}}$$

$\Big\downarrow$ 1509

$$\frac{-2\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-2c\left(\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}}{\left(a+\right.}}}{}\right.}{\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{e(d+ex)}}$$

3.627. $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx$

$$\Big\downarrow \text{1540}$$

$$\frac{2\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\int \frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g(age^2+cd(2ef-dg))}-\frac{\sqrt{c}\left(cef^2+c\cdots\right.}{\cdots}\right.}{\cdots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{e(d+ex)}$$

$$\Big\downarrow \text{1416}$$

$$\frac{2\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\int \frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g(age^2+cd(2ef-dg))}-\frac{\sqrt[4]{c}\left(cef^2+\cdots\right.}{\cdots}\right.}{\cdots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{e(d+ex)}$$

$$\Big\downarrow \text{2222}$$

3.627. $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx$

$$\frac{2\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\left(\frac{\left(e+\frac{\sqrt{c}(ef-dg)}{\sqrt{cf^2+ag^2}}\right)\operatorname{arctanh}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{e}\sqrt{ef-dg}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}\right)}{2\sqrt{e}\sqrt{cd^2+ae^2}\sqrt{ef-dg}} - \cdots\right)}{\cdots}\right)}{\cdots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{e(d+ex)}$$

input
```
Int[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^2,x]
```

3.627. $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx$

output
```
-((Sqrt[f + g*x]*Sqrt[a + c*x^2])/(e*(d + e*x))) + ((-2*c*((3*e*Sqrt[c*f^2
 + a*g^2]*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 +
 (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*
f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f
^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^
2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2
)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 +
 (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c
*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2]))))/Sqrt[c] + ((c*f^2 + a*g^2)^(1/
4)*(3*d*g + e*(f - (3*Sqrt[c*f^2 + a*g^2])/Sqrt[c]))*(1 + (Sqrt[c]*(f + g*
x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (
c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2
 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^
(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(2*c^(1/4)*Sqrt[a + (c*f
^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/(e^2*g) + 2*(a*g
 - (c*d*(2*e*f - 3*d*g))/e^2)*(-1/2*(c^(1/4)*(c*e*f^2 + a*e*g^2 - Sqrt[c]*
(e*f - d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g
^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/
((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*Ellip
ticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqr...
```

3.627.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 721
```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2]/
(e*(m + 1))), x] - Simp[1/(2*e*(m + 1))   Int[((d + e*x)^(m + 1)/(Sqrt[f +
g*x]*Sqrt[a + c*x^2]))*Simp[a*g + 2*c*f*x + 3*c*g*x^2, x], x], x] /; FreeQ[
{a, c, d, e, f, g}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

3.627. $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx$

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.627. $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2} dx$

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

3.627.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 895, normalized size of antiderivative = 1.38

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(-\frac{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{e(ex+d)}+\frac{2\left(-\frac{c(2dg-ef)}{e^3}+\frac{cdg}{2e^3}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}F\left(\sqrt{\frac{x+\cdots}{\frac{f}{g}-\cdots}}\right.}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\right.\cdots}{\cdots}$
default	Expression too large to display

input
```
int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-1/e*(c*g*x^3+c*f
*x^2+a*g*x+a*f)^(1/2)/(e*x+d)+2*(-c*(2*d*g-e*f)/e^3+1/2*c*d/e^3*g)*(f/g-(-
a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f
/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)
/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c)
)^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+3*c/e^2*g*(f/
g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)
/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(
1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((
x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/
2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2
),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+1/e^4*(a*e^2*g+3*c
*d^2*g-2*c*d*e*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2
)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f
/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d/e)*Ell
ipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-a*c)^(1/2)/c)/(-f/g+d
/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))
```

3.627.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.627.6 Sympy [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)^2}\,dx$$

input
```
integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2)/(e*x+d)**2,x)
```

output
```
Integral(sqrt(a + c*x**2)*sqrt(f + g*x)/(d + e*x)**2, x)
```

3.627.7 Maxima [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{cx^2+a}\sqrt{gx+f}}{(ex+d)^2}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^2, x)
```

3.627.8 Giac [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{cx^2+a}\sqrt{gx+f}}{(ex+d)^2}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^2, x)
```

3.627.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}{(d+e\,x)^2}\,dx$$

input
```
int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x)^2,x)
```

output
```
int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x)^2, x)
```

3.628 $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx$

3.628.1 Optimal result

Integrand size = 28, antiderivative size = 1205

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4e\,(cd^2+ae^2)\,(ef-dg)(d+ex)}$$
$$-\frac{\sqrt{-a}\sqrt{c}(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e^2\,(cd^2+ae^2)\,(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$-\frac{3\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e^3\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$+\frac{\sqrt{-a}\sqrt{c}f(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e^2\,(cd^2+ae^2)\,(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$-\frac{\sqrt{-a}\sqrt{c}dg(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e^3\,(cd^2+ae^2)\,(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$-\frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$+\frac{(ae^2g-cd(2ef-3dg))^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{4e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)\,(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
-1/2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e/(e*x+d)^2-1/4*(a*e^2*g-c*d*(-3*d*g+2*
e*f))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e/(a*e^2+c*d^2)/(-d*g+e*f)/(e*x+d)-1/4
*(a*e^2*g-c*d*(-3*d*g+2*e*f))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)
*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g
*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/e^2/(a*e^2+c*d^2)/(-d*g+e*f)/(c*x^2+a)^(1/2)
/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-3/2*g*EllipticF(1/2*(1-x
*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1
/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f
*c^(1/2)))^(1/2)/e^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+1/4*f*(a*e^2*g-c*d*(-3*
d*g+2*e*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(
-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((
g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/(a*e^2+c*d^2)/(-d*g+e*f
)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-1/4*d*g*(a*e^2*g-c*d*(-3*d*g+2*e*f))*Ellip
ticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)
*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g
*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^3/(a*e^2+c*d^2)/(-d*g+e*f)/(g*x+f)^(1/2)/(
c*x^2+a)^(1/2)-c*(-3*d*g+e*f)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2
)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)
+f*c^(1/2)))^(1/2))*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(
1/2)))^(1/2)/e^3/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)...
```

3.628.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.36 (sec) , antiderivative size = 2526, normalized size of antiderivative = 2.10

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^3,x]
```

output
```
(Sqrt[f + g*x]*Sqrt[a + c*x^2]*(-2 + ((a*e^2*g + c*d*(-2*e*f + 3*d*g))*(d
+ e*x))/((c*d^2 + a*e^2)*(-(e*f) + d*g))))/(4*e*(d + e*x)^2) + (-2*c^2*d*e
^3*f^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 5*c^2*d^2*e^2*f^3*g*Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]] + a*c*e^4*f^3*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*
c^2*d^3*e*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*a*c*d*e^3*f^2*g^2*S
qrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 5*a*c*d^2*e^2*f*g^3*Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]] + a^2*e^4*f*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*a*c*d^3*
e*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - a^2*d*e^3*g^4*Sqrt[-f - (I*Sqrt[a
]*g)/Sqrt[c]] + 4*c^2*d*e^3*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)
 - 10*c^2*d^2*e^2*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 2*a*c
*e^4*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 6*c^2*d^3*e*f*g^2*
Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 2*a*c*d*e^3*f*g^2*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 2*c^2*d*e^3*f^2*Sqrt[-f - (I*Sqrt[a]*g)/
Sqrt[c]]*(f + g*x)^2 + 5*c^2*d^2*e^2*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*
(f + g*x)^2 + a*c*e^4*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 - 3
*c^2*d^3*e*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 - a*c*d*e^3*g^
2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + I*Sqrt[c]*e*(Sqrt[c]*f +
I*Sqrt[a]*g)*(-(e*f) + d*g)*(a*e^2*g + c*d*(-2*e*f + 3*d*g))*Sqrt[(g*((I*S
qrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f +
 g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqr...
```

3.628.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2763 vs. 2(1205) = 2410.

Time = 5.36 (sec) , antiderivative size = 2763, normalized size of antiderivative = 2.29, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {721, 2349, 734, 2349, 27, 510, 599, 25, 27, 729, 25, 1416, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{721}$$

$$\frac{\int \frac{3cgx^2+2cfx+ag}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4e}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$$\Big\downarrow \text{2349}$$

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{\frac{2cf}{e}-\frac{3cdg}{e^2}+\frac{3cgx}{e}}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4e}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$$\Big\downarrow \text{734}$$

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(-\frac{\int \frac{-cgx^2e^2+age^2-2cdgxe-2cd(ef-dg)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)+\int \frac{\frac{2cf}{e}-\frac{3cdg}{e^2}+\frac{3cgx}{e}}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4e}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$$\Big\downarrow \text{2349}$$

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)+\frac{2c(ef-3c}{\cdots}}{4e}$$
$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)+\frac{2c(ef-3c}{\cdots}}{4e}$$
$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$$\Big\downarrow \text{510}$$

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)+\frac{2c(ef-3c}{\cdots}}{4e}$$
$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$$\Big\downarrow \text{599}$$

3.628. $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx$

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2\int -\frac{cg(ef-dg-e(f+gx))}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{}}{(d+ex)(ae^2+cd^2}\right.}{4e}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$\downarrow$ 25

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\frac{2\int \frac{cg(ef-dg-e(f+gx))}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f}}{(d+ex)(ae^2+cd^2)(}\right.}{4e}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$\downarrow$ 27

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{}}{(d+ex)(ae^2+cd^2)}\right.}{4e}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$\downarrow$ 729

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(-\frac{2(ae^2g-cd(2ef-3dg))\int -\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}+\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+g}{g^2}}}}{g}}{2(ae^2+cd^2)(ef-dg)}\right.}{}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$\downarrow$ 25

3.628. $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx$

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(-\frac{\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}-2(ae^2g-cd(2ef-3dg))\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}}}}{2(ae^2+cd^2)(ef-dg)}\right.}{}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$\big\downarrow$ 1416

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(-\frac{\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}-2(ae^2g-cd(2ef-3dg))\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}}}}{2(ae^2+cd^2)(ef-dg)}\right.}{}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$\big\downarrow$ 1511

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(-\frac{\frac{2c\left(\frac{e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\left(dg-e\left(f-\frac{\sqrt{ag^2+cf^2}}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+}}\right.}{g}}{2(ae^2+cd^2)(ef-dg)}\right.}{}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$\big\downarrow$ 1416

3.628. $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx$

$$\frac{3c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-\frac{4c(ef-\cdots}{\cdots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2e(d+ex)^2}$$

$\downarrow$ 1509

3.628. $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx$

$$\frac{3c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}} - \frac{4c(ef-\ldots}{\ldots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2e(d+ex)^2}$$

$$\Big\downarrow \; 1540$$

3.628. $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx$

$$\frac{3c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}+\frac{4c(ef-\ldots}{\ldots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2e(d+ex)^2}$$

$$\Big\downarrow \ 1416$$

3.628. $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx$

$$\frac{3c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}+\frac{4c(ef-}{}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2e(d+ex)^2}$$

$$\Big\downarrow \text{2222}$$

3.628. $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx$

$4c(ef-$

$$\frac{3c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}+$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2e(d+ex)^2}$$

input
```
Int[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^3,x]
```

3.628. $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx$

output
```
-1/2*(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(e*(d + e*x)^2) + ((3*c^(3/4)*(c*f^2
+ a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*
f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)
*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(
1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 +
a*g^2])/2])/(e^2*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*
x)^2)/g^2]) + (4*c*(e*f - 3*d*g)*(-1/2*(c^(1/4)*(c*e*f^2 + a*e*g^2 - Sqrt[
c]*(e*f - d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 +
a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^
2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*El
lipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqr
t[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(g*(c*f^2 + a*g^2)^(1/4)*(a*e^2*g + c*d*(
2*e*f - d*g))*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^
2)/g^2]) + (e*Sqrt[c*f^2 + a*g^2]*(Sqrt[c]*(e*f - d*g) - e*Sqrt[c*f^2 + a*
g^2])*(((e + (Sqrt[c]*(e*f - d*g))/Sqrt[c*f^2 + a*g^2])*ArcTanh[(Sqrt[c*d^
2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[e]*Sqrt[e*f - d*g]*Sqrt[a + (c*f^2)/g^2 -
(2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])])/(2*Sqrt[e]*Sqrt[c*d^2 + a*
e^2]*Sqrt[e*f - d*g]) - ((Sqrt[c]/e - Sqrt[c*f^2 + a*g^2]/(e*f - d*g))*(1
+ (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*
(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*...
```

3.628.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 510
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2/d   Subst[Int[1/Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2
)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

3.628. $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx$

rule 721
```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2]/
(e*(m + 1))), x] - Simp[1/(2*e*(m + 1))   Int[((d + e*x)^(m + 1)/(Sqrt[f +
g*x]*Sqrt[a + c*x^2]))*Simp[a*g + 2*c*f*x + 3*c*g*x^2, x], x], x] /; FreeQ[
{a, c, d, e, f, g}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 734
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2))), x] + Simp[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2))   Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^
2]))*Simp[2*c*d*(e*f - d*g)*(m + 1) - a*e^2*g*(2*m + 3) + 2*c*e*(d*g*(m + 1
) - e*f*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.628. $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3} \, dx$

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

3.628.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 1161, normalized size of antiderivative = 0.96

method	result	size
elliptic	Expression too large to display	1161
default	Expression too large to display	19181

input
```
int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-1/2/e*(c*g*x^3+c
*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)^2+1/4*(a*e^2*g+3*c*d^2*g-2*c*d*e*f)/e/(a*d
*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d
)+2*(c*g/e^3-1/8*c*g*(3*a*d*e^2*g-2*a*e^3*f+5*c*d^3*g-4*c*d^2*e*f)/e^3/(a*
d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f))*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a
*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(
-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1
/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/
(-f/g-(-a*c)^(1/2)/c))^(1/2))-1/4*c/e^2*g*(a*e^2*g+3*c*d^2*g-2*c*d*e*f)/(a
*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a
*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(
-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1
/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),
((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*Ellipt
icF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*
c)^(1/2)/c))^(1/2)))+1/4*(a^2*e^4*g^2-6*a*c*d^2*e^2*g^2+12*a*c*d*e^3*f*g-4
*a*c*e^4*f^2-3*c^2*d^4*g^2+4*c^2*d^3*e*f*g)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d
^2*e*f)/e^4*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-
(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a
*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d/e)*Ellipt...
```

3.628.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.628.6 Sympy [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx = \int \frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)^3}\,dx$$

input
```
integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2)/(e*x+d)**3,x)
```

output
```
Integral(sqrt(a + c*x**2)*sqrt(f + g*x)/(d + e*x)**3, x)
```

3.628.7 Maxima [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx = \int \frac{\sqrt{cx^2+a}\sqrt{gx+f}}{(ex+d)^3}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^3, x)
```

3.628.8 Giac [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx = \int \frac{\sqrt{cx^2+a}\sqrt{gx+f}}{(ex+d)^3}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^3, x)
```

3.628.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx = \int \frac{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}{{\left(d+e\,x\right)}^3}\,dx$$

input
```
int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x)^3,x)
```

output
```
int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x)^3, x)
```

3.629 $\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

3.629.1 Optimal result

Integrand size = 28, antiderivative size = 666

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx =$$
$$-\frac{4(9ae^2g^2(2ef-5dg)+c(76e^3f^3-204de^2f^2g+168d^2efg^2-35d^3g^3))\sqrt{f+gx}\sqrt{a+cx^2}}{315cg^4}$$
$$+\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9g}$$
$$+\frac{4e(7ae^2g^2+c(64e^2f^2-111defg+42d^2g^2))(f+gx)^{3/2}\sqrt{a+cx^2}}{315cg^4}$$
$$-\frac{4e^2(4ef-3dg)(f+gx)^{5/2}\sqrt{a+cx^2}}{63g^4}$$
$$+\frac{4\sqrt{-a}(21a^2e^3g^4-3aceg^2(10e^2f^2-39defg+63d^2g^2)-c^2f(64e^3f^3-216de^2f^2g+252d^2efg^2-105d}{315c^{3/2}g^5\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$-\frac{4\sqrt{-a}(cf^2+ag^2)(9ae^2g^2(2ef-5dg)-c(64e^3f^3-216de^2f^2g+252d^2efg^2-105d^3g^3))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{}{315c^{3/2}g^5\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
4/315*e*(7*a*e^2*g^2+c*(42*d^2*g^2-111*d*e*f*g+64*e^2*f^2))*(g*x+f)^(3/2)*
(c*x^2+a)^(1/2)/c/g^4-4/63*e^2*(-3*d*g+4*e*f)*(g*x+f)^(5/2)*(c*x^2+a)^(1/2
)/g^4-4/315*(9*a*e^2*g^2*(-5*d*g+2*e*f)+c*(-35*d^3*g^3+168*d^2*e*f*g^2-204
*d*e^2*f^2*g+76*e^3*f^3))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^4+2/9*(e*x+d)^
3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+4/315*(21*a^2*e^3*g^4-3*a*c*e*g^2*(63*d^
2*g^2-39*d*e*f*g+10*e^2*f^2)-c^2*f*(-105*d^3*g^3+252*d^2*e*f*g^2-216*d*e^2
*f^2*g+64*e^3*f^3))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(
-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x
^2/a)^(1/2)/c^(3/2)/g^5/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c
^(1/2)))^(1/2)-4/315*(a*g^2+c*f^2)*(9*a*e^2*g^2*(-5*d*g+2*e*f)-c*(-105*d^3
*g^3+252*d^2*e*f*g^2-216*d*e^2*f^2*g+64*e^3*f^3))*EllipticF(1/2*(1-x*c^(1/
2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(
-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/
2)/c^(3/2)/g^5/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

3.629.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.66 (sec) , antiderivative size = 872, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2(a+cx^2)\left(2ae^2g^2(-11ef+45dg+7egx)+c\left(105d^3g^3+63d^2eg^2(-4f+3gx)+27de^2g\left(8f^2-6fgx+5g^2x^2\right)+e^3\left(-64f^3+48f^2gx-40f\right.\right.\right.}{cg^4}\right.}{}$$

input
```
Integrate[((d + e*x)^3*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

output
```
(Sqrt[f + g*x]*((2*(a + c*x^2)*(2*a*e^2*g^2*(-11*e*f + 45*d*g + 7*e*g*x) +
 c*(105*d^3*g^3 + 63*d^2*e*g^2*(-4*f + 3*g*x) + 27*d*e^2*g*(8*f^2 - 6*f*g*
x + 5*g^2*x^2) + e^3*(-64*f^3 + 48*f^2*g*x - 40*f*g^2*x^2 + 35*g^3*x^3))))
/(c*g^4) + (4*(f + g*x)*((g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(-21*a^2*e^
3*g^4 + 3*a*c*e*g^2*(10*e^2*f^2 - 39*d*e*f*g + 63*d^2*g^2) + c^2*f*(64*e^3
*f^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3))*(a + c*x^2))/(f +
 g*x)^2 + (I*Sqrt[c]*(Sqrt[c]*f + I*Sqrt[a]*g)*(21*a^2*e^3*g^4 - 3*a*c*e*g
^2*(10*e^2*f^2 - 39*d*e*f*g + 63*d^2*g^2) + c^2*f*(-64*e^3*f^3 + 216*d*e^2
*f^2*g - 252*d^2*e*f*g^2 + 105*d^3*g^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x)
)/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*EllipticE[I*
ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sq
rt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + (Sqrt[a]*Sqrt[c]*g*(S
qrt[c]*f + I*Sqrt[a]*g)*((-21*I)*a^(3/2)*e^3*g^3 + 9*a*Sqrt[c]*e^2*g^2*(2*
e*f - 5*d*g) + (3*I)*Sqrt[a]*c*e*g*(16*e^2*f^2 - 54*d*e*f*g + 63*d^2*g^2)
+ c^(3/2)*(-64*e^3*f^3 + 216*d*e^2*f^2*g - 252*d^2*e*f*g^2 + 105*d^3*g^3))
*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[
c] - g*x)/(f + g*x))]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]
/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqr
t[f + g*x]))/(c^2*g^6*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])))/(315*Sqrt[a + c*
x^2])
```

3.629.3 Rubi [A] (warning: unable to verify)

Time = 2.70 (sec) , antiderivative size = 1085, normalized size of antiderivative = 1.63, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {723, 27, 2185, 27, 2185, 27, 2185, 27, 599, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}(d+ex)^3}{\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{723}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g}-\frac{\int \frac{2(d+ex)^2(c(4ef-3dg)x^2+(cdf-aeg)x+a(3ef-4dg))}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{9g}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g}-\frac{2\int \frac{(d+ex)^2(c(4ef-3dg)x^2+(cdf-aeg)x+a(3ef-4dg))}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{9g}$$

$\Big\downarrow$ 2185

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g}-$$

$$2\left(\frac{2\int -\frac{ceg^3\left(7ae^2g^2+c\left(64e^2f^2-111degf+42d^2g^2\right)\right)x^3-cg^2\left(ae^2g^2(ef-27dg)-c\left(44e^3f^3-33de^2gf^2-42d^2eg^2f+21d^3g^3\right)\right)x^2+cg\left(ae\left(40e^2f^2-72degf+63d^2\right.\right.}{2\sqrt{f+gx}\sqrt{cx^2+a}}}{7cg^4}\right.$$

$$9g$$

$\Big\downarrow$ 27

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g}-$$

$$2\left(\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(4ef-3dg)}{7g^3}-\frac{\int \frac{ceg^3\left(7ae^2g^2+c\left(64e^2f^2-111degf+42d^2g^2\right)\right)x^3-cg^2\left(ae^2g^2(ef-27dg)-c\left(44e^3f^3-33de^2gf^2-42d^2eg^2f+21d^3\right.\right.}{\sqrt{}}}{}\right.$$

$$9g$$

$\Big\downarrow$ 2185

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g}-$$

$$2\left(\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(4ef-3dg)}{7g^3}-\frac{2\int -\frac{3c^2\left(9ae^2(2ef-5dg)g^2+c\left(76e^3f^3-204de^2gf^2+168d^2eg^2f-35d^3g^3\right)\right)x^2g^5+ac\left(21afg^2e^3+c\left(92e^3f^3-258de\right.\right.}{}}{}\right.$$

$\Big\downarrow$ 27

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g}-$$

$$2\left(\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(4ef-3dg)}{7g^3}-\frac{\frac{2}{5}eg\sqrt{a+cx^2}(f+gx)^{3/2}\left(7ae^2g^2+c\left(42d^2g^2-111defg+64e^2f^2\right)\right)-\frac{\int\frac{3c^2\left(9ae^2(2ef-5dg)g^2+c\left(76e^3f^3-204\right.\right.}{}}{}}{}\right.$$

$\Big\downarrow$ 2185

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g}-$$

$$2\left(\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(4ef-3dg)}{7g^3}-\frac{\frac{2}{5}eg\sqrt{a+cx^2}(f+gx)^{3/2}\left(7ae^2g^2+c\left(42d^2g^2-111defg+64e^2f^2\right)\right)-\frac{2\int\frac{3c^2g^6\left(ag\left(3ae^2(ef+15dg)g^2+c\left(16e^3\right.\right.\right.}{}}{}}{}\right.$$

3.629. $\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

$\downarrow$ 27

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g}-$$

$$2\left(\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(4ef-3dg)}{7g^3}-\frac{\frac{2}{5}eg\sqrt{a+cx^2}(f+gx)^{3/2}\left(7ae^2g^2+c\left(42d^2g^2-111defg+64e^2f^2\right)\right)-\frac{cg^4\int\frac{ag\left(3ae^2(ef+15dg)g^2+c\left(16e^3f^3-5\right.\right.}{}}{}}{}\right.$$

$\downarrow$ 599

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g}-$$

$$2\left(\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(4ef-3dg)}{7g^3}-\frac{\frac{2}{5}eg\sqrt{a+cx^2}(f+gx)^{3/2}\left(7ae^2g^2+c\left(42d^2g^2-111defg+64e^2f^2\right)\right)-\frac{2cg^4\sqrt{a+cx^2}\sqrt{f+gx}\left(9ae^2g^2(2ef-5dg)+\right.}{}}{}\right.$$

$\downarrow$ 1511

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g}-$$

$$2\left(\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(4ef-3dg)}{7g^3}-\frac{\frac{2}{5}eg\sqrt{a+cx^2}(f+gx)^{3/2}\left(7ae^2g^2+c\left(42d^2g^2-111defg+64e^2f^2\right)\right)-\frac{2cg^4\sqrt{a+cx^2}\sqrt{f+gx}\left(9ae^2g^2(2ef-5dg)+\right.}{}}{}\right.$$

$\downarrow$ 1416

3.629. $\int\frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{cx^2+a}}{9g}-$$

$$2\left(\frac{2e^2(4ef-3dg)(f+gx)^{5/2}\sqrt{cx^2+a}}{7g^3}-\frac{\frac{2}{5}eg\left(7ae^2g^2+c\left(64e^2f^2-111degf+42d^2g^2\right)\right)(f+gx)^{3/2}\sqrt{cx^2+a}-\frac{2cg^4\left(9ae^2(2ef-5dg)g^2+c\left(76e^3f^3-204de\right.\right.}{}}{}\right.$$

$\downarrow$ 1509

3.629. $\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{cx^2+a}}{9g} -$$

$$2\left(\frac{2e^2(4ef-3dg)(f+gx)^{5/2}\sqrt{cx^2+a}}{7g^3} - \frac{\frac{2}{5}eg\left(7ae^2g^2+c\left(64e^2f^2-111degf+42d^2g^2\right)\right)(f+gx)^{3/2}\sqrt{cx^2+a} - \frac{2cg^4\left(9ae^2(2ef-5dg)g^2+c\left(76e^3f^3-204de\right.\right.}{}}{}\right.$$

input
```
Int[((d + e*x)^3*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

3.629. $\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

output
```
(2*(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(9*g) - (2*((2*e^2*(4*e*f -
3*d*g)*(f + g*x)^(5/2)*Sqrt[a + c*x^2])/(7*g^3) - ((2*e*g*(7*a*e^2*g^2 + c
*(64*e^2*f^2 - 111*d*e*f*g + 42*d^2*g^2))*(f + g*x)^(3/2)*Sqrt[a + c*x^2])
/5 - (2*c*g^4*(9*a*e^2*g^2*(2*e*f - 5*d*g) + c*(76*e^3*f^3 - 204*d*e^2*f^2
*g + 168*d^2*e*f*g^2 - 35*d^3*g^3))*Sqrt[f + g*x]*Sqrt[a + c*x^2] - 2*c*g^
2*((Sqrt[c*f^2 + a*g^2]*(21*a^2*e^3*g^4 - 3*a*c*e*g^2*(10*e^2*f^2 - 39*d*e
*f*g + 63*d^2*g^2) - c^2*f*(64*e^3*f^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2
 - 105*d^3*g^3))*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x)
)/g^2 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/
Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/S
qrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f
+ g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*
g^2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)
], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/g^2
 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/Sqrt[c] - ((c*f^2 + a*g
^2)^(3/4)*(21*a^2*e^3*g^4 - 3*a*c*e*g^2*(10*e^2*f^2 - 39*d*e*f*g + 63*d^2*
g^2) - c^2*f*(64*e^3*f^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3
) - Sqrt[c]*Sqrt[c*f^2 + a*g^2]*(9*a*e^2*g^2*(2*e*f - 5*d*g) - c*(64*e^3*f
^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3)))*(1 + (Sqrt[c]*(f +
 g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g...
```

3.629.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 723
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[2*(d + e*x)^m*Sqrt[f + g*x]*(Sqrt[a + c*x^2]/(g
*(2*m + 3))), x] - Simp[1/(g*(2*m + 3))   Int[((d + e*x)^(m - 1)/(Sqrt[f +
g*x]*Sqrt[a + c*x^2]))*Simp[2*a*(e*f*m - d*g*(m + 1)) + (2*c*d*f - 2*a*e*g)
*x - (2*c*(d*g*m - e*f*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g
}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.629.4 Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 1156, normalized size of antiderivative = 1.74

method	result	size
elliptic	Expression too large to display	1156
risch	Expression too large to display	1937
default	Expression too large to display	5079

input
```
int((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/9*e^3/g*x^3*(c*
g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/7*(3*c*d*e^2-8/9*e^3/g*c*f)/c/g*x^2*(c*g*
x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/5*(2/9*a*e^3+3*c*d^2*e-6/7*(3*c*d*e^2-8/9*e
^3/g*c*f)/g*f)/c/g*x*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/3*(3*a*d*e^2+c*d^
3-2/3*e^3/g*f*a-5/7*(3*c*d*e^2-8/9*e^3/g*c*f)/c*a-4/5*(2/9*a*e^3+3*c*d^2*e
-6/7*(3*c*d*e^2-8/9*e^3/g*c*f)/g*f)/g*f)/c/g*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(
1/2)+2*(a*d^3-2/5*(2/9*a*e^3+3*c*d^2*e-6/7*(3*c*d*e^2-8/9*e^3/g*c*f)/g*f)/
c/g*f*a-1/3*(3*a*d*e^2+c*d^3-2/3*e^3/g*f*a-5/7*(3*c*d*e^2-8/9*e^3/g*c*f)/c
*a-4/5*(2/9*a*e^3+3*c*d^2*e-6/7*(3*c*d*e^2-8/9*e^3/g*c*f)/g*f)/g*f)/c*a)*(
f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/
c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))
^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1
/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(3*a*
d^2*e-4/7*(3*c*d*e^2-8/9*e^3/g*c*f)/c/g*f*a-3/5*(2/9*a*e^3+3*c*d^2*e-6/7*(
3*c*d*e^2-8/9*e^3/g*c*f)/g*f)/c*a-2/3*(3*a*d*e^2+c*d^3-2/3*e^3/g*f*a-5/7*(
3*c*d*e^2-8/9*e^3/g*c*f)/c*a-4/5*(2/9*a*e^3+3*c*d^2*e-6/7*(3*c*d*e^2-8/9*e
^3/g*c*f)/g*f)/g*f)/g*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c
))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)
/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-
(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(...
```

3.629.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 578, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = -\frac{2\left(2\left(64\,c^2e^3f^5-216\,c^2de^2f^4g+6\left(42\,c^2d^2e+13\,ace^3\right)f^3g^2-3\left(35\,c^2d^3+93\,acde^2\right)f^2g^3+6\left(63\,acd^2e\right.\right.\right.}{}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
-2/945*(2*(64*c^2*e^3*f^5 - 216*c^2*d*e^2*f^4*g + 6*(42*c^2*d^2*e + 13*a*c
*e^3)*f^3*g^2 - 3*(35*c^2*d^3 + 93*a*c*d*e^2)*f^2*g^3 + 6*(63*a*c*d^2*e -
2*a^2*e^3)*f*g^4 - 45*(7*a*c*d^3 - 3*a^2*d*e^2)*g^5)*sqrt(c*g)*weierstrass
PInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3),
 1/3*(3*g*x + f)/g) + 6*(64*c^2*e^3*f^4*g - 216*c^2*d*e^2*f^3*g^2 + 6*(42*
c^2*d^2*e + 5*a*c*e^3)*f^2*g^3 - 3*(35*c^2*d^3 + 39*a*c*d*e^2)*f*g^4 + 21*
(9*a*c*d^2*e - a^2*e^3)*g^5)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^
2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*
f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)
/g)) - 3*(35*c^2*e^3*g^5*x^3 - 64*c^2*e^3*f^3*g^2 + 216*c^2*d*e^2*f^2*g^3
- 2*(126*c^2*d^2*e + 11*a*c*e^3)*f*g^4 + 15*(7*c^2*d^3 + 6*a*c*d*e^2)*g^5
- 5*(8*c^2*e^3*f*g^4 - 27*c^2*d*e^2*g^5)*x^2 + (48*c^2*e^3*f^2*g^3 - 162*c
^2*d*e^2*f*g^4 + 7*(27*c^2*d^2*e + 2*a*c*e^3)*g^5)*x)*sqrt(c*x^2 + a)*sqrt
(g*x + f))/(c^2*g^6)
```

3.629.6 Sympy [F]

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+cx^2}(d+ex)^3}{\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)**3*(c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)*(d + e*x)**3/sqrt(f + g*x), x)
```

3.629.7 Maxima [F]

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}(ex+d)^3}{\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)^3/sqrt(g*x + f), x)
```

3.629.8 Giac [F]

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}(ex+d)^3}{\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)^3/sqrt(g*x + f), x)
```

3.629.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{c\,x^2+a}\,{(d+e\,x)}^3}{\sqrt{f+g\,x}}\,dx$$

input
```
int(((a + c*x^2)^(1/2)*(d + e*x)^3)/(f + g*x)^(1/2),x)
```

output
```
int(((a + c*x^2)^(1/2)*(d + e*x)^3)/(f + g*x)^(1/2), x)
```

3.630 $\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

3.630.1 Optimal result

Integrand size = 28, antiderivative size = 508

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \frac{4(5ae^2g^2+c(21e^2f^2-34defg+10d^2g^2))\sqrt{f+gx}\sqrt{a+cx^2}}{105cg^3}$$
$$+\frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7g}-\frac{4e(3ef-2dg)(f+gx)^{3/2}\sqrt{a+cx^2}}{35g^3}$$
$$+\frac{4\sqrt{-a}(aeg^2(13ef-42dg)+cf(24e^2f^2-56defg+35d^2g^2))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\right.}{105\sqrt{c}g^4\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$+\frac{4\sqrt{-a}(cf^2+ag^2)(5ae^2g^2-c(24e^2f^2-56defg+35d^2g^2))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\right.\right.}{105c^{3/2}g^4\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
-4/35*e*(-2*d*g+3*e*f)*(g*x+f)^(3/2)*(c*x^2+a)^(1/2)/g^3+4/105*(5*a*e^2*g^
2+c*(10*d^2*g^2-34*d*e*f*g+21*e^2*f^2))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^
3+2/7*(e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+4/105*(a*e*g^2*(-42*d*g+13
*e*f)+c*f*(35*d^2*g^2-56*d*e*f*g+24*e^2*f^2))*EllipticE(1/2*(1-x*c^(1/2)/(
-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^
(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/g^4/c^(1/2)/(c*x^2+a)^(1/2)/((g*x+f)
*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+4/105*(a*g^2+c*f^2)*(5*a*e^2*g^2-
c*(35*d^2*g^2-56*d*e*f*g+24*e^2*f^2))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2
))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1
+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g
^4/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

3.630.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.16 (sec) , antiderivative size = 665, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2(a+cx^2)\left(10ae^2g^2+c\left(35d^2g^2+14deg(-4f+3gx)+3e^2\left(8f^2-6fgx+5g^2x^2\right)\right)\right)}{cg^3} - \frac{4\left(g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\left(aeg^2(13ef-42dg)+cf\left(24e^2\right.\right.\right.}{}\right.}{}$$

input
```
Integrate[((d + e*x)^2*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

output
```
(Sqrt[f + g*x]*((2*(a + c*x^2)*(10*a*e^2*g^2 + c*(35*d^2*g^2 + 14*d*e*g*(-
4*f + 3*g*x) + 3*e^2*(8*f^2 - 6*f*g*x + 5*g^2*x^2))))/(c*g^3) - (4*(g^2*Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(a*e*g^2*(13*e*f - 42*d*g) + c*f*(24*e^2*f^
2 - 56*d*e*f*g + 35*d^2*g^2))*(a + c*x^2) - I*Sqrt[c]*(Sqrt[c]*f + I*Sqrt[
a]*g)*(a*e*g^2*(13*e*f - 42*d*g) + c*f*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g
^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/S
qrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f
+ I*Sqrt[a]*g)] + Sqrt[a]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*(5*a*e^2*g^2 + (6*I)
*Sqrt[a]*Sqrt[c]*e*g*(3*e*f - 7*d*g) + c*(-24*e^2*f^2 + 56*d*e*f*g - 35*d^
2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g
)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f -
 (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]
*f + I*Sqrt[a]*g)]))/(c*g^5*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/
(105*Sqrt[a + c*x^2])
```

3.630.3 Rubi [A] (warning: unable to verify)

Time = 1.62 (sec) , antiderivative size = 878, normalized size of antiderivative = 1.73, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {723, 27, 2185, 27, 2185, 27, 599, 25, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}(d+ex)^2}{\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{723}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g}-\frac{\int \frac{2(d+ex)\left(c(3ef-2dg)x^2+(cdf-aeg)x+a(2ef-3dg)\right)}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{7g}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g}-\frac{2\int \frac{(d+ex)\left(c(3ef-2dg)x^2+(cdf-aeg)x+a(2ef-3dg)\right)}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{7g}$$

$$\Big\downarrow \text{2185}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g}-\frac{2\left(\frac{2\int-\frac{c\left(5ae^2g^2+c\left(21e^2f^2-34degf+10d^2g^2\right)\right)x^2g^2+ac\left(9e^2f^2-16degf+15d^2g^2\right)g^2-c\left(aeg^2(ef-14dg)-cf\left(6e^2f^2-4degf-5d^2g^2\right)\right)xg}{2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{5cg^3}+\frac{2e\sqrt{a+cx^2}\cdots}{\cdots}\right)}{7g}$$

$\downarrow$ 27

$$\frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g}-\frac{2\left(\frac{2e\sqrt{a+cx^2}(f+gx)^{3/2}(3ef-2dg)}{5g^2}-\frac{\int\frac{c\left(5ae^2g^2+c\left(21e^2f^2-34degf+10d^2g^2\right)\right)x^2g^2+ac\left(9e^2f^2-16degf+15d^2g^2\right)g^2-c\left(aeg^2(ef-14dg)-cf\left(6e^2f^2-4d\cdots\right.\right.}{\sqrt{f+gx}\sqrt{cx^2+a}}\cdots}{5cg^3}\right)}{7g}$$

$\downarrow$ 2185

$$\frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g}-\frac{2\left(\frac{2e\sqrt{a+cx^2}(f+gx)^{3/2}(3ef-2dg)}{5g^2}-\frac{\frac{2\int-\frac{cg^3\left(ag\left(5ae^2g^2-c\left(6e^2f^2-14degf+35d^2g^2\right)\right)+c\left(ae(13ef-42dg)g^2+cf\left(24e^2f^2-56degf+35d^2g^2\right)\right)x\right)}{2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3cg^2}+\frac{2}{3}\cdots}{5cg^3}\right)}{7g}$$

$\downarrow$ 27

$$\frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g}-\frac{2\left(\frac{2e\sqrt{a+cx^2}(f+gx)^{3/2}(3ef-2dg)}{5g^2}-\frac{\frac{2}{3}g\sqrt{a+cx^2}\sqrt{f+gx}\left(5ae^2g^2+c\left(10d^2g^2-34defg+21e^2f^2\right)\right)-\frac{1}{3}g\int\frac{ag\left(5ae^2g^2-c\left(6e^2f^2-14degf+35d^2g^2\right)\right)+\cdots}{\sqrt{f\cdots}}}{5cg^3}\right)}{7g}$$

$\downarrow$ 599

$$\frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g}-\frac{2\left(\frac{2e\sqrt{a+cx^2}(f+gx)^{3/2}(3ef-2dg)}{5g^2}-\frac{\frac{2\int-\frac{\left(cf^2+ag^2\right)\left(5ae^2g^2-c\left(24e^2f^2-56degf+35d^2g^2\right)\right)+c\left(ae(13ef-42dg)g^2+cf\left(24e^2f^2-56degf+35d^2g^2\right)\right)(f+\cdots}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}}{3g}\cdots}{5cg^3}\right)}{7g}$$

$\downarrow$ 25

3.630. $\int\frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

$$\frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g} - \frac{2\left(\frac{2e\sqrt{a+cx^2}(f+gx)^{3/2}(3ef-2dg)}{5g^2} - \frac{\frac{2}{3}g\sqrt{a+cx^2}\sqrt{f+gx}\left(5ae^2g^2+c\left(10d^2g^2-34defg+21e^2f^2\right)\right) - \frac{2\int \frac{\left(cf^2+ag^2\right)\left(5ae^2g^2-c\left(24e^2f^2-56degf+35d\right.\right.}{\sqrt{\frac{cf^2}{g^2}}}}{}}{5cg^3}\right)}{7g}$$

$\downarrow$ 1511

$$\frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g} - \frac{2\left(\frac{2e\sqrt{a+cx^2}(f+gx)^{3/2}(3ef-2dg)}{5g^2} - \frac{2\left(\sqrt{c}\sqrt{ag^2+cf^2}\left(aeg^2(13ef-42dg)+cf\left(35d^2g^2-56defg+24e^2f^2\right)\right)\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\right.}{}\right)}{}$$

$\downarrow$ 1416

$$\frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g} - \frac{2\left(\frac{2e\sqrt{a+cx^2}(f+gx)^{3/2}(3ef-2dg)}{5g^2} - \frac{2\left(\sqrt{c}\sqrt{ag^2+cf^2}\left(aeg^2(13ef-42dg)+cf\left(35d^2g^2-56defg+24e^2f^2\right)\right)\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\right.}{}\right)}{}$$

$\downarrow$ 1509

3.630. $\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

$$\frac{2(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+a}}{7g} -$$

$$2\left(\frac{2e(3ef-2dg)(f+gx)^{3/2}\sqrt{cx^2+a}}{5g^2} - \frac{\frac{2}{3}g\sqrt{f+gx}\sqrt{cx^2+a}\left(5ae^2g^2+c\left(21e^2f^2-34degf+10d^2g^2\right)\right)+\frac{2\left(\sqrt{c}\sqrt{cf^2+ag^2}\left(ae(13ef-42dg)g^2+cf\left(24e^2f^2\right.\right.\right.}{}}{}\right.$$

input
```
Int[((d + e*x)^2*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

output
```
(2*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(7*g) - (2*((2*e*(3*e*f - 2*
d*g)*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(5*g^2) - ((2*g*(5*a*e^2*g^2 + c*(21
*e^2*f^2 - 34*d*e*f*g + 10*d^2*g^2))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/3 + (2
*(Sqrt[c]*Sqrt[c*f^2 + a*g^2]*(a*e*g^2*(13*e*f - 42*d*g) + c*f*(24*e^2*f^2
 - 56*d*e*f*g + 35*d^2*g^2))*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c
*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]
*(f + g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*
(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/
g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqr
t[c*f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 +
a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a +
 (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])) - ((c*f^2 +
a*g^2)^(3/4)*(Sqrt[c*f^2 + a*g^2]*(5*a*e^2*g^2 - c*(24*e^2*f^2 - 56*d*e*f*
g + 35*d^2*g^2)) + Sqrt[c]*(a*e*g^2*(13*e*f - 42*d*g) + c*f*(24*e^2*f^2 -
56*d*e*f*g + 35*d^2*g^2)))*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*S
qrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a +
(c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2
*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/S
qrt[c*f^2 + a*g^2])/2])/(2*c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x)
)/g^2 + (c*(f + g*x)^2)/g^2])))/(3*g))/(5*c*g^3)))/(7*g)
```

3.630. $\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

3.630.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 723
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[2*(d + e*x)^m*Sqrt[f + g*x]*(Sqrt[a + c*x^2]/(g
*(2*m + 3))), x] - Simp[1/(g*(2*m + 3))   Int[((d + e*x)^(m - 1)/(Sqrt[f +
g*x]*Sqrt[a + c*x^2]))*Simp[2*a*(e*f*m - d*g*(m + 1)) + (2*c*d*f - 2*a*e*g)
*x - (2*c*(d*g*m - e*f*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g
}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.630. $\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.630.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.67

method	result
elliptic	$\frac{\sqrt{(gx+f)(c x^2+a)}\left(\frac{2e^2x^2\sqrt{cg x^3+cf x^2+agx+fa}}{7g}+\frac{2\left(2cde-\frac{6e^2cf}{7g}\right)x\sqrt{cg x^3+cf x^2+agx+fa}}{5cg}+\frac{2\left(\frac{2e^2a}{7}+c d^2-\frac{4\left(2cde-\frac{6e^2cf}{7g}\right)f}{5g}\right)\sqrt{cg x^3 \cdots}}{3cg}\right.}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/7*e^2/g*x^2*(c*
g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/5*(2*c*d*e-6/7*e^2/g*c*f)/c/g*x*(c*g*x^3+
c*f*x^2+a*g*x+a*f)^(1/2)+2/3*(2/7*e^2*a+c*d^2-4/5*(2*c*d*e-6/7*e^2/g*c*f)/
g*f)/c/g*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2*(a*d^2-2/5*(2*c*d*e-6/7*e^2/g
*c*f)/c/g*f*a-1/3*(2/7*e^2*a+c*d^2-4/5*(2*c*d*e-6/7*e^2/g*c*f)/g*f)/c*a)*(
f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/
c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))
^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1
/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(2*a*
d*e-4/7*e^2/g*f*a-3/5*(2*c*d*e-6/7*e^2/g*c*f)/c*a-2/3*(2/7*e^2*a+c*d^2-4/5
*(2*c*d*e-6/7*e^2/g*c*f)/g*f)/g*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*
c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-
a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/
2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(
(-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*Ellipti
cF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c
)^(1/2)/c))^(1/2))))
```

3.630.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(2\left(24\,c^2e^2f^4-56\,c^2def^3g-84\,acdefg^3+\left(35\,c^2d^2+31\,ace^2\right)f^2g^2+15\left(7\,acd^2-a^2e^2\right)g^4\right)\sqrt{cg}\text{weierstra}\right.}{}$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(2*(24*c^2*e^2*f^4 - 56*c^2*d*e*f^3*g - 84*a*c*d*e*f*g^3 + (35*c^2*d
^2 + 31*a*c*e^2)*f^2*g^2 + 15*(7*a*c*d^2 - a^2*e^2)*g^4)*sqrt(c*g)*weierst
rassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g
^3), 1/3*(3*g*x + f)/g) + 6*(24*c^2*e^2*f^3*g - 56*c^2*d*e*f^2*g^2 - 42*a*
c*d*e*g^4 + (35*c^2*d^2 + 13*a*c*e^2)*f*g^3)*sqrt(c*g)*weierstrassZeta(4/3
*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrass
PInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3),
 1/3*(3*g*x + f)/g)) + 3*(15*c^2*e^2*g^4*x^2 + 24*c^2*e^2*f^2*g^2 - 56*c^2
*d*e*f*g^3 + 5*(7*c^2*d^2 + 2*a*c*e^2)*g^4 - 6*(3*c^2*e^2*f*g^3 - 7*c^2*d*
e*g^4)*x)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c^2*g^5)
```

3.630.6 Sympy [F]

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+cx^2}(d+ex)^2}{\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)**2*(c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)*(d + e*x)**2/sqrt(f + g*x), x)
```

3.630.7 Maxima [F]

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}(ex+d)^2}{\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)^2/sqrt(g*x + f), x)
```

3.630.8 Giac [F]

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}(ex+d)^2}{\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)^2/sqrt(g*x + f), x)
```

3.630.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{c\,x^2+a}\,(d+e\,x)^2}{\sqrt{f+g\,x}}\,dx$$

input
```
int(((a + c*x^2)^(1/2)*(d + e*x)^2)/(f + g*x)^(1/2),x)
```

output
```
int(((a + c*x^2)^(1/2)*(d + e*x)^2)/(f + g*x)^(1/2), x)
```

3.631 $\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}} \, dx$

3.631.1 Optimal result

Integrand size = 26, antiderivative size = 364

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}} \, dx = -\frac{2\sqrt{f+gx}(4ef-5dg-3egx)\sqrt{a+cx^2}}{15g^2} - \frac{4\sqrt{-a}(3aeg^2+cf(4ef-5dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^3\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{4\sqrt{-a}(4ef-5dg)(cf^2+ag^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
-2/15*(-3*e*g*x-5*d*g+4*e*f)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g^2-4/15*(3*a*e
*g^2+c*f*(-5*d*g+4*e*f))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1
/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(
1+c*x^2/a)^(1/2)/g^3/c^(1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2
)+f*c^(1/2)))^(1/2)+4/15*(-5*d*g+4*e*f)*(a*g^2+c*f^2)*EllipticF(1/2*(1-x*c
^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2
))*(-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))
^(1/2)/g^3/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

3.631.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.32 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.50

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2(-4ef+5dg+3egx)(a+cx^2)}{g^2} + \frac{4\left(g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(3aeg^2+cf(4ef-5dg))(a+cx^2)-\sqrt{c}(i\sqrt{c}f-\sqrt{a}g)(3aeg^2+cf(4ef-5dg))\sqrt{\frac{g}{-}}\right.}{}\right.}{}$$

input
```
Integrate[((d + e*x)*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

output
```
(Sqrt[f + g*x]*((2*(-4*e*f + 5*d*g + 3*e*g*x)*(a + c*x^2))/g^2 + (4*(g^2*S
qrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*(a + c*x
^2) - Sqrt[c]*(I*Sqrt[c]*f - Sqrt[a]*g)*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*
Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c
] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*S
qrt[a]*g)] + Sqrt[a]*Sqrt[c]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*((3*I)*Sqrt[a]*e*
g + Sqrt[c]*(-4*e*f + 5*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)
]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Ellipti
cF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f -
 I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(c*g^4*Sqrt[-f - (I*Sqrt[a]*g)/
Sqrt[c]]*(f + g*x))))/(15*Sqrt[a + c*x^2])
```

3.631.3 Rubi [A] (warning: unable to verify)

Time = 0.89 (sec) , antiderivative size = 698, normalized size of antiderivative = 1.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {682, 27, 599, 25, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}(d+ex)}{\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \ 682$$

$$\frac{4\int -\frac{c(ag(ef-5dg)-(3aeg^2+cf(4ef-5dg))x)}{2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{15cg^2}-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(-5dg+4ef-3egx)}{15g^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{2\int \frac{ag(ef-5dg)-(3aeg^2+cf(4ef-5dg))x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{15g^2}-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(-5dg+4ef-3egx)}{15g^2}$$

$$\Big\downarrow \ 599$$

$$\frac{4\int -\frac{(4ef-5dg)(cf^2+ag^2)-(3aeg^2+cf(4ef-5dg))(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{15g^4}-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(-5dg+4ef-3egx)}{15g^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{4\int \frac{(4ef-5dg)(cf^2+ag^2)-(3aeg^2+cf(4ef-5dg))(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{15g^4}-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(-5dg+4ef-3egx)}{15g^2}$$

$$\Big\downarrow \ 1511$$

$$\frac{4\left(\frac{\sqrt{ag^2+cf^2}\left(-\sqrt{c}\sqrt{ag^2+cf^2}(4ef-5dg)+3aeg^2+cf(4ef-5dg)\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\frac{\sqrt{ag^2+cf^2}(3aeg^2+cf(4ef-5\cdots}{\cdots}\right.}{15g^4}$$

$$\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(-5dg+4ef-3egx)}{15g^2}$$

$$\Big\downarrow \ 1416$$

3.631. $\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

$$4\left(\frac{\left(ag^2+cf^2\right)^{3/4}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(-\sqrt{c}\sqrt{ag^2+cf^2}(4ef-5dg)+3aeg^2+cf(4ef-5dg)\right)\text{EllipticF}\left(2\arctan\left(\frac{}{\sqrt[4]{}}\right.\right.}{2c^{3/4}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right.$$

$$\overline{\qquad\qquad\qquad 15g^4}$$

$$\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(-5dg+4ef-3egx)}{15g^2}$$

$\downarrow$ 1509

$$4\left(\frac{\left(ag^2+cf^2\right)^{3/4}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(-\sqrt{c}\sqrt{ag^2+cf^2}(4ef-5dg)+3aeg^2+cf(4ef-5dg)\right)\text{EllipticF}\left(2\arctan\left(\frac{}{\sqrt[4]{}}\right.\right.}{2c^{3/4}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right.$$

$$\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(-5dg+4ef-3egx)}{15g^2}$$

input
```
Int[((d + e*x)*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

3.631. $\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

output
```
(-2*Sqrt[f + g*x]*(4*e*f - 5*d*g - 3*e*g*x)*Sqrt[a + c*x^2])/(15*g^2) + (4
*(-((Sqrt[c*f^2 + a*g^2]*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*(-((Sqrt[f + g*
x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((
a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2
 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c
*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2
)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^
(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 +
 a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f
 + g*x)^2)/g^2]))))/Sqrt[c]) + ((c*f^2 + a*g^2)^(3/4)*(3*a*e*g^2 + c*f*(4*e
*f - 5*d*g) - Sqrt[c]*(4*e*f - 5*d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(
f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g
^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt
[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a
*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(2*c^(3/4)*Sqrt[a
+ (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/(15*g^4)
```

3.631.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.631. $\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.631.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(298) = 596$.

Time = 1.08 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.86

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(\frac{2ex\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{5g}+\frac{2\left(cd-\frac{4cfe}{5g}\right)\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{3cg}+\frac{2\left(ad-\frac{2fae}{5g}-\frac{a\left(cd-\frac{4cfe}{5g}\right)}{3c}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+}{\frac{f}{g}-\sqrt{}}}}{\sqrt{c}}\right.}{}$
risch	$\frac{2(3egx+5dg-4ef)\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{15g^2}+\frac{2\left(\frac{10ad\,g^2\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right.}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\right.}{}$
default	Expression too large to display

input
```
int((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/5*e/g*x*(c*g*x^
3+c*f*x^2+a*g*x+a*f)^(1/2)+2/3*(c*d-4/5*c*f/g*e)/c/g*(c*g*x^3+c*f*x^2+a*g*
x+a*f)^(1/2)+2*(a*d-2/5*f*a/g*e-1/3*a/c*(c*d-4/5*c*f/g*e))*(f/g-(-a*c)^(1/
2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c
)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^
3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),
((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(2/5*a*e-2/3*f/g*(c
*d-4/5*c*f/g*e))*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)
*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/
g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(
1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)
/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(
-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))
))
```

3.631. $\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

3.631.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.74

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx =$$
$$-\frac{2\left(2\left(4\,cef^3-5\,cdf^2g+6\,aefg^2-15\,adg^3\right)\sqrt{cg}\text{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)\right.}{}$$

input
```
integrate((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*(2*(4*c*e*f^3 - 5*c*d*f^2*g + 6*a*e*f*g^2 - 15*a*d*g^3)*sqrt(c*g)*we
ierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)
/(c*g^3), 1/3*(3*g*x + f)/g) + 6*(4*c*e*f^2*g - 5*c*d*f*g^2 + 3*a*e*g^3)*s
qrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a
*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*
(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) - 3*(3*c*e*g^3*x - 4*c*e*
f*g^2 + 5*c*d*g^3)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c*g^4)
```

3.631.6 Sympy [F]

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+cx^2}\,(d+ex)}{\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)*(c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)*(d + e*x)/sqrt(f + g*x), x)
```

3.631.7 Maxima [F]

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}(ex+d)}{\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)/sqrt(g*x + f), x)
```

3.631.8 Giac [F]

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}(ex+d)}{\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)/sqrt(g*x + f), x)
```

3.631.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{c\,x^2+a}\,(d+e\,x)}{\sqrt{f+g\,x}}\,dx$$

input
```
int(((a + c*x^2)^(1/2)*(d + e*x))/(f + g*x)^(1/2),x)
```

output
```
int(((a + c*x^2)^(1/2)*(d + e*x))/(f + g*x)^(1/2), x)
```

3.632 $\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

3.632.1 Optimal result

Integrand size = 21, antiderivative size = 322

$$\begin{aligned}\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx &= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3g} \\ &+ \frac{4\sqrt{-a}\sqrt{c}f\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3g^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} \\ &- \frac{4\sqrt{-a}(cf^2+ag^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}g^2\sqrt{f+gx}\sqrt{a+cx^2}}\end{aligned}$$

output
```
2/3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+4/3*f*EllipticE(1/2*(1-x*c^(1/2)/(-a)^
(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2
)*c^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/g^2/(c*x^2+a)^(1/2)/((g*x+f)*c^(
1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-4/3*(a*g^2+c*f^2)*EllipticF(1/2*(1-x*
c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/
2))*(-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))
)^(1/2)/g^2/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

3.632.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.50 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}\left(g^2(a+cx^2)-\frac{2\left(fg^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(a+cx^2)+\sqrt{c}f(-i\sqrt{c}f+\sqrt{a}g)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}(f+gx)^{3/2}E\left(i\,\mathrm{arcsinh}\left(\sqrt{\cdots}\right.\right.\cdots\right)}{\cdots}\right)}{3g^3\sqrt{a\cdots}}$$

input
```
Integrate[Sqrt[a + c*x^2]/Sqrt[f + g*x],x]
```

output
```
(2*Sqrt[f + g*x]*(g^2*(a + c*x^2) - (2*(f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]]*(a + c*x^2) + Sqrt[c]*f*((-I)*Sqrt[c]*f + Sqrt[a]*g)*Sqrt[(g*((I*Sqrt
[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*
x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - Sqr
t[a]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g
*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Elli
pticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*
f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]]*(f + g*x))))/(3*g^3*Sqrt[a + c*x^2])
```

3.632.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {493, 599, 25, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \; 493$$

$$\frac{2\int \frac{ag-cfx}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3g}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3g}$$

$\Big\downarrow$ 599

$$\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3g}-\frac{4\int -\frac{cf^2-c(f+gx)f+ag^2}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3g^3}$$

$\Big\downarrow$ 25

$$\frac{4\int \frac{cf^2-c(f+gx)f+ag^2}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3g^3}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3g}$$

$\Big\downarrow$ 1511

$$\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3g}-\frac{4\left(\sqrt{ag^2+cf^2}\left(\sqrt{c}f-\sqrt{ag^2+cf^2}\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\sqrt{c}f\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+g}{\sqrt{cf^2+c}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+}}\right.}{3g^3}$$

$\Big\downarrow$ 1416

$$\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3g}-\frac{4\left(\frac{(ag^2+cf^2)^{3/4}\left(\sqrt{c}f-\sqrt{ag^2+cf^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+\right.\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right.}{3g^3}$$

$\Big\downarrow$ 1509

$$\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3g}-\frac{4\left(\frac{(ag^2+cf^2)^{3/4}\left(\sqrt{c}f-\sqrt{ag^2+cf^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+\right.\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right.}{}$$

3.632. $\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

input
```
Int[Sqrt[a + c*x^2]/Sqrt[f + g*x],x]
```

output
```
(2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*g) - (4*(-(Sqrt[c]*f*Sqrt[c*f^2 + a*g
^2]*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f
 + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 +
a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a
*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2
)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*Ell
ipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt
[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f
+ g*x))/g^2 + (c*(f + g*x)^2)/g^2]))) + ((c*f^2 + a*g^2)^(3/4)*(Sqrt[c]*f
- Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[
(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f
^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*Arc
Tan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[
c*f^2 + a*g^2])/2])/(2*c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^
2 + (c*(f + g*x)^2)/g^2])))/(3*g^3)
```

3.632.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.632.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(256) = 512$.

Time = 0.98 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.81

method	result
risch	$\frac{2\sqrt{gx+f}\,\sqrt{cx^2+a}}{3g}+\frac{2\left(\frac{2ag\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}-\frac{2cf\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\cdots}{\cdots}\cdots\right)}{\cdots}$
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(\frac{2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{3g}+\frac{4a\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\cdots\right)}{3\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\cdots\right)}{\cdots}$
default	$-\frac{2\sqrt{c\,x^2+a}\,\sqrt{gx+f}\left(2\sqrt{-ac}\,\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}}\,\sqrt{\frac{(-cx+\sqrt{-ac})g}{g\sqrt{-ac}+cf}}\,\sqrt{\frac{(cx+\sqrt{-ac})g}{g\sqrt{-ac}-cf}}\,F\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)a\,g^3+2\sqrt{-ac}\,\sqrt{-\cdots}\right)}{\cdots}$

input
```
int((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+2/3/g*(2*a*g*(f/g-(-a*c)^(1/2)/c)*((x+
f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c)
)^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+
a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a
*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))-2*c*f*(f/g-(-a*c)^(1/2)/c)*((x+
f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c)
)^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+
a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/
2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(
1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/
c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))*((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)
/(c*x^2+a)^(1/2)
```

3.632. $\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

3.632.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2\left(6\sqrt{cg}cfg\text{weierstrassZeta}\left(\frac{4(cf^2-3ag^2)}{3cg^2},-\frac{8(cf^3+9afg^2)}{27cg^3},\text{weierstrassPInverse}\left(\frac{4(cf^2-3ag^2)}{3cg^2},-\frac{8(cf^3+9afg^2)}{27cg^3},\right.\right.\right.}{}$$

input
```
integrate((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(6*sqrt(c*g)*c*f*g*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/2
7*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(
c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) + 3*sqrt(c*
x^2 + a)*sqrt(g*x + f)*c*g^2 + 2*(c*f^2 + 3*a*g^2)*sqrt(c*g)*weierstrassPI
nverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1
/3*(3*g*x + f)/g))/(c*g^3)
```

3.632.6 Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$

input
```
integrate((c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)/sqrt(f + g*x), x)
```

3.632.7 Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}}{\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)/sqrt(g*x + f), x)
```

3.632.8 Giac [F]

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx=\int \frac{\sqrt{cx^2+a}}{\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)/sqrt(g*x + f), x)
```

3.632.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx=\int \frac{\sqrt{c\,x^2+a}}{\sqrt{f+g\,x}}\,dx$$

input
```
int((a + c*x^2)^(1/2)/(f + g*x)^(1/2),x)
```

output
```
int((a + c*x^2)^(1/2)/(f + g*x)^(1/2), x)
```

3.633 $\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$

3.633.1 Optimal result

Integrand size = 28, antiderivative size = 473

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = -\frac{2\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{eg\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{2\sqrt{-a}\sqrt{c}(ef+dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2g\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{2(cd^2+ae^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^2\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
-2*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-
a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/
2)/e/g/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+2*
(d*g+e*f)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a
*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*
x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/g/(g*x+f)^(1/2)/(c*x^2+a)
^(1/2)-2*(a*e^2+c*d^2)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/
2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1
/2)))^(1/2))*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^
(1/2)/e^2/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

3.633.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.77 (sec) , antiderivative size = 1096, normalized size of antiderivative = 2.32

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx =$$

$$-\frac{2\left(-ce^2f^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+cdef^2g\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}-ae^2fg^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+adeg^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+2ce^2f^2\sqrt{\right.}{}$$

input
```
Integrate[Sqrt[a + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]
```

output
```
(-2*(-(c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) + c*d*e*f^2*g*Sqrt[-f -
 (I*Sqrt[a]*g)/Sqrt[c]] - a*e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a
*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]*(f + g*x) - 2*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f
 + g*x) - c*e^2*f*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + c*d*e*g*S
qrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + Sqrt[c]*e*((-I)*Sqrt[c]*f +
Sqrt[a]*g)*(-(e*f) + d*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sq
rt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I
*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*S
qrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + e*(I*Sqrt[c]*d + Sqrt[a]*e)*g*(Sqrt
[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(
((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcS
inh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a
]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - I*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c]
 + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*
x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I
*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*S
qrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - I*a*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sq
rt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f
 + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a...
```

3.633.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1496 vs. $2(473) = 946$.

Time = 2.52 (sec) , antiderivative size = 1496, normalized size of antiderivative = 3.16, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = $\{724, 27, 599, 25, 729, 25, 1511, 1416, 1509, 1540, 1416, 2222\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{724}$$

$$\left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{\int \frac{c(d-ex)}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}$$

$$\Big\downarrow \text{27}$$

$$\left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{c\int \frac{d-ex}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}$$

$$\Big\downarrow \text{599}$$

$$\left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\frac{2c\int -\frac{ef+dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e^2g^2}$$

$$\Big\downarrow \text{25}$$

$$\left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2c\int \frac{ef+dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e^2g^2}$$

$$\Big\downarrow \text{729}$$

$$2\left(a+\frac{cd^2}{e^2}\right)\int -\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\int \frac{ef+dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e^2g^2}$$

$$\Big\downarrow \text{25}$$

$$-2\left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\int \frac{ef+dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e^2g^2}$$

$$\Big\downarrow \text{1511}$$

$$\frac{2c\left(-\left(\left(e\left(f-\frac{\sqrt{ag^2+cf^2}}{\sqrt{c}}\right)+dg\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}\right)-\frac{e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}}{\sqrt{c}}\right.}{e^2g^2}$$

$$2\left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}$$

$$\Big\downarrow \text{1416}$$

3.633. $\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$

$$\frac{2c\left(-\frac{e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(e\left(f-\frac{\sqrt{ag}}{}\right.\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cj}{}}}\right.}{e^2g^2}$$

$$2\left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}$$

$$\Big\downarrow \text{1509}$$

$$\frac{2c\left(-\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(e\left(f-\frac{\sqrt{ag^2+cf^2}}{\sqrt{c}}\right)+dg\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\right.\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right.}{}$$

$$2\left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}$$

$$\Big\downarrow \text{1540}$$

3.633. $\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$

$$2c\left(-\frac{e\sqrt{cf^2+ag^2}\left(\frac{\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-\frac{\sqrt{f+g\cdots}}{\cdots}\right.}{\sqrt{c}}\cdots\right.$$

$$2\left(\frac{cd^2}{e^2}+a\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\int\frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g\left(age^2+cd(2ef-dg)\right)}\cdots\right.$$

$$\Big\downarrow \text{1416}$$

$$2c\left(-\frac{e\sqrt{cf^2+ag^2}\left(\frac{\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-\frac{\sqrt{f+g\cdots}}{\cdots}\right.}{\sqrt{c}}\cdots\right.$$

$$2\left(\frac{cd^2}{e^2}+a\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\int\frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g\left(age^2+cd(2ef-dg)\right)}\cdots\right.$$

$$\Big\downarrow \text{2222}$$

3.633. $\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$

$$\frac{2c\left(-\frac{e\sqrt{cf^2+ag^2}\left(\frac{\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-\frac{\sqrt{f+g}}{}\right.}{\sqrt{c}}\right.}{2\left(\frac{cd^2}{e^2}+a\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\left(\frac{\left(e+\frac{\sqrt{c}(ef-dg)}{\sqrt{cf^2+ag^2}}\right)\text{arctanh}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{e}\sqrt{ef-dg}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+g}{g^2}}}\right.}{2\sqrt{e}\sqrt{cd^2+ae^2}\sqrt{ef-dg}}\right.}{}\right.}$$

input
```
Int[Sqrt[a + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]
```

3.633. $\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$

output
```
(2*c*(-((e*Sqrt[c*f^2 + a*g^2]*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2
*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[
c]*(f + g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c
]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x)
)/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/S
qrt[c*f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2
+ a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a
 + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2]))))/Sqrt[c])
- ((c*f^2 + a*g^2)^(1/4)*(d*g + e*(f - Sqrt[c*f^2 + a*g^2]/Sqrt[c]))*(1 +
(Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f
 + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f +
g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/
(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(2*c^(1/
4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/
(e^2*g^2) + 2*(a + (c*d^2)/e^2)*(-1/2*(c^(1/4)*(c*e*f^2 + a*e*g^2 - Sqrt[c
]*(e*f - d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a
*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2
)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*Ell
ipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt
[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(g*(c*f^2 + a*g^2)^(1/4)*(a*e^2*g + c*d...
```

3.633.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 724
```
Int[Sqrt[(a_) + (c_.)*(x_)^2]/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)
]), x_Symbol] :> Simp[(c*d^2 + a*e^2)/e^2   Int[1/((d + e*x)*Sqrt[f + g*x]*
Sqrt[a + c*x^2]), x], x] - Simp[1/e^2   Int[(c*d - c*e*x)/(Sqrt[f + g*x]*Sq
rt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

3.633. $\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.633. $\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

3.633.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 832 vs. $2(386) = 772$.

Time = 0.78 (sec) , antiderivative size = 833, normalized size of antiderivative = 1.76

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(-\frac{2cd\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{e^2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}+\frac{2c\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\cdots}}{\cdots}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-2*c*d/e^2*(f/g-(
-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-
f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2
)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c
))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*c/e*(f/g-(
-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-
f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2
)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f
/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/
c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(
(-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+2*(a*e^2+c*d^2)/e^3*(
f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/
c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))
^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f
/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-a*c)^(1/2)/c)/(-f/g+d/e),((-f/g+(-a*c)^(
1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))
```

3.633.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.633.6 Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+cx^2}}{(d+ex)\,\sqrt{f+gx}}\,dx$$

input
```
integrate((c*x**2+a)**(1/2)/(e*x+d)/(g*x+f)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)/((d + e*x)*sqrt(f + g*x)), x)
```

3.633.7 Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}}{(ex+d)\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)/((e*x + d)*sqrt(g*x + f)), x)
```

3.633.8 Giac [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}}{(ex+d)\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)/((e*x + d)*sqrt(g*x + f)), x)
```

3.633.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \int \frac{\sqrt{c\,x^2+a}}{\sqrt{f+g\,x}\,(d+e\,x)}\,dx$$

input
```
int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)),x)
```

output
```
int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)), x)
```

3.634 $\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$

3.634.1 Optimal result

Integrand size = 28, antiderivative size = 694

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$+ \frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$- \frac{\sqrt{-a}\sqrt{c}(2ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+ \frac{(ae^2g+cd(2ef-dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^2\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
-(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(-d*g+e*f)/(e*x+d)-EllipticE(1/2*(1-x*c^(1/
2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(
-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/e/(-d*g+e*f)/(c*x^2+a)^(
1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+f*EllipticF(1/2*(1-x
*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1
/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f
*c^(1/2)))^(1/2)/e/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-(-d*g+2*e*f)*E
llipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(
1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2
)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(
1/2)+(a*e^2*g+c*d*(-d*g+2*e*f))*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1
/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/
2)+f*c^(1/2)))^(1/2))*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c
^(1/2)))^(1/2)/e^2/(-d*g+e*f)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^
2+a)^(1/2)
```

3.634.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.29 (sec) , antiderivative size = 1336, normalized size of antiderivative = 1.93

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{a+cx^2}{d+ex} - \frac{ce^2f^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}} - cdef^2g\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}} + ae^2fg^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}} - adeg^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}} - 2ce^2f^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx) + 2cde\ldots}{\ldots}\right.}{\ldots}$$

input
```
Integrate[Sqrt[a + c*x^2]/((d + e*x)^2*Sqrt[f + g*x]),x]
```

output
```
(Sqrt[f + g*x]*((a + c*x^2)/(d + e*x) - (c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)
/Sqrt[c]] - c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*e^2*f*g^2*Sqr
t[-f - (I*Sqrt[a]*g)/Sqrt[c]] - a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]
 - 2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 2*c*d*e*f*g*Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + c*e^2*f*Sqrt[-f - (I*Sqrt[a]*g)
/Sqrt[c]]*(f + g*x)^2 - c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)
^2 + I*Sqrt[c]*e*(Sqrt[c]*f + I*Sqrt[a]*g)*(-(e*f) + d*g)*Sqrt[(g*((I*Sqrt
[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*
x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + e*(
Sqrt[c]*f + I*Sqrt[a]*g)*(Sqrt[a]*e*g + I*Sqrt[c]*(2*e*f - d*g))*Sqrt[(g*(
(I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/
(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)
] - (2*I)*c*d*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-((
(I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[
c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*S
qrt[a]*g)] + I*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqr
t[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticP...
```

3.634.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1559 vs. $2(694) = 1388$.

Time = 2.73 (sec) , antiderivative size = 1559, normalized size of antiderivative = 2.25, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {725, 25, 2349, 599, 27, 729, 25, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \; 725$$

$$\frac{\int -\frac{-cgx^2-2cfx+ag}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ef-dg)}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{-cgx^2-2cfx+ag}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ef-dg)}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)}$$

$$\Big\downarrow \; 2349$$

$$-\frac{\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-\frac{2cf}{e}+\frac{cdg}{e^2}-\frac{cgx}{e}}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ef-dg)}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)}$$

$$\Big\downarrow \; 599$$

$$-\frac{\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2\int \frac{cg(ef-dg+e(f+gx))}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2(ef-dg)}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2c\int \frac{ef-dg+e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e^2g}}{2(ef-dg)}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)}$$

$$\Big\downarrow \; 729$$

$$-\frac{2\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\int -\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\int \frac{ef-dg+e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e^2g}}{2(ef-dg)}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)}$$

$$\Big\downarrow \; 25$$

$$-\frac{-2\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\int \frac{ef-dg+e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e^2g}}{2(ef-dg)}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)}$$

$$\Big\downarrow \; 1511$$

3.634. $\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$

$$-\frac{-2\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\left(-\left(\left(dg-e\left(\frac{\sqrt{ag^2+cf^2}}{\sqrt{c}}+f\right)\right)\int \frac{}{\sqrt{\frac{cf^2}{g^2}-}}\right.\right.}{}}{2(ef-dg)}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)}$$

$$\Big\downarrow 1416$$

$$-\frac{-\frac{2c\left(-\frac{e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(dg-e\left(\frac{\sqrt{ag^2+}}{\sqrt{c}}\right.\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx}{g^2}}}\right.}{e^2g}}{}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)}$$

$$\Big\downarrow 1509$$

$$-\frac{-2\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\left(-\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+}{(}}}{}\right.}{}}{}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)}$$

3.634. $\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$

$\downarrow$ 1540

$$-\frac{2\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\int\frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g(age^2+cd(2ef-dg))}-\frac{\sqrt{c}\left(cef^2+\cdots\right.}{\cdots}\right.}{\cdots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{(ef-dg)(d+ex)}$$

$\downarrow$ 1416

$$-\frac{2\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\int\frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g(age^2+cd(2ef-dg))}-\frac{\sqrt[4]{c}\left(cef^2\cdots\right.}{\cdots}\right.}{\cdots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{(ef-dg)(d+ex)}$$

$\downarrow$ 2222

3.634. $\int\frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$

$$-\frac{2\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\left(\frac{\left(e+\frac{\sqrt{c}(ef-dg)}{\sqrt{cf^2+ag^2}}\right)\operatorname{arctanh}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{e}\sqrt{ef-dg}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}\right)}{2\sqrt{e}\sqrt{cd^2+ae^2}\sqrt{ef-dg}}\right.}{}\right.}{}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{(ef-dg)(d+ex)}$$

input
```
Int[Sqrt[a + c*x^2]/((d + e*x)^2*Sqrt[f + g*x]),x]
```

3.634. $\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$

output
```
-((Sqrt[f + g*x]*Sqrt[a + c*x^2])/((e*f - d*g)*(d + e*x))) - ((-2*c*(-((e*
Sqrt[c*f^2 + a*g^2]*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g
*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x
))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x)
)/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*
(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 +
 a*g^2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1
/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/
g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/Sqrt[c]) - ((c*f^2 +
 a*g^2)^(1/4)*(d*g - e*(f + Sqrt[c*f^2 + a*g^2]/Sqrt[c]))*(1 + (Sqrt[c]*(f
 + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^
2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[
c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*
g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(2*c^(1/4)*Sqrt[a +
 (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/(e^2*g) + 2
*(a*g + (c*d*(2*e*f - d*g))/e^2)*(-1/2*(c^(1/4)*(c*e*f^2 + a*e*g^2 - Sqrt[
c]*(e*f - d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 +
a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^
2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*El
lipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (...
```

3.634.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 725
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2
]/((m + 1)*(e*f - d*g))), x] - Simp[1/(2*(m + 1)*(e*f - d*g))   Int[((d + e
*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*g*(2*m + 3) + 2*(c*f)*x
 + c*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && Integer
Q[2*m] && LtQ[m, -1]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.634. $\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

3.634.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 921, normalized size of antiderivative = 1.33

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(\frac{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{(dg-ef)(ex+d)}+\frac{2\left(\frac{c}{e^2}-\frac{cdg}{2e^2(dg-ef)}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right.}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\right.}{}$
default	Expression too large to display

input
```
int((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(1/(d*g-e*f)*(c*g*
x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)+2*(c/e^2-1/2*c*d/e^2*g/(d*g-e*f))*(f/
g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)
/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(
1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2
)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))-c*g/(d*g-
e*f)/e*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c
)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(
1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*El
lipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-
(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2
)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+(a*e^2*g
-c*d^2*g+2*c*d*e*f)/e^3/(d*g-e*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c
)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a
*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2
)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-a*c)^
(1/2)/c)/(-f/g+d/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))
```

3.634.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.634.6 Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$$

input
```
integrate((c*x**2+a)**(1/2)/(e*x+d)**2/(g*x+f)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)/((d + e*x)**2*sqrt(f + g*x)), x)
```

3.634.7 Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}}{(ex+d)^2\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)/((e*x + d)^2*sqrt(g*x + f)), x)
```

3.634.8 Giac [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}}{(ex+d)^2\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)/((e*x + d)^2*sqrt(g*x + f)), x)
```

3.634.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \int \frac{\sqrt{c\,x^2+a}}{\sqrt{f+g\,x}\,(d+e\,x)^2}\,dx$$

input
```
int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^2),x)
```

output
```
int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^2), x)
```

3.635 $\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

3.635.1 Optimal result

Integrand size = 28, antiderivative size = 1241

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$$
$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(3ae^2g+cd(2ef+dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4(cd^2+ae^2)(ef-dg)^2(d+ex)}$$
$$+ \frac{\sqrt{-a}\sqrt{c}(3ae^2g+cd(2ef+dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e(cd^2+ae^2)(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$+ \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$- \frac{\sqrt{-a}\sqrt{c}f(3ae^2g+cd(2ef+dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e(cd^2+ae^2)(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$+ \frac{\sqrt{-a}\sqrt{c}dg(3ae^2g+cd(2ef+dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e^2(cd^2+ae^2)(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$- \frac{c(ef+dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^2\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$- \frac{(ae^2g-cd(2ef-3dg))(3ae^2g+cd(2ef+dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\dots}}{\sqrt{2}}\right.\right.}{4e^2\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
-1/2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(-d*g+e*f)/(e*x+d)^2+1/4*(3*a*e^2*g+c*d
*(d*g+2*e*f))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)^2/(e*
x+d)+1/4*(3*a*e^2*g+c*d*(d*g+2*e*f))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2)
)^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(
1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/e/(a*e^2+c*d^2)/(-d*g+e*f)^2/(c*x^2+a
)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+1/2*g*EllipticF(1
/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/
2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^
(1/2)+f*c^(1/2)))^(1/2)/e^2/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-1/4*f
*(3*a*e^2*g+c*d*(d*g+2*e*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*
2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+
c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/(a*e^2+c
*d^2)/(-d*g+e*f)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+1/4*d*g*(3*a*e^2*g+c*d*(d
*g+2*e*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-
a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g
*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/(a*e^2+c*d^2)/(-d*g+e*f)
^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-c*(d*g+e*f)*EllipticPi(1/2*(1-x*c^(1/2)/(
-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2
)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(
-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/(-d*g+e*f)/(e+d*c^(1/2)/(-a)^(1/2))/(g*...
```

3.635.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.72 (sec) , antiderivative size = 2197, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
```

output
```
(c^2*d^2*f^3 - 3*a*c*e^2*f^3 - (2*c^2*d*e*f^4)/g + (c^2*d^3*f^2*g)/e + a*c
*d*e*f^2*g + a*c*d^2*f*g^2 - 3*a^2*e^2*f*g^2 + (a*c*d^3*g^3)/e + 3*a^2*d*e
*g^3 - 2*c^2*d^2*f^2*(f + g*x) + 6*a*c*e^2*f^2*(f + g*x) + (4*c^2*d*e*f^3*
(f + g*x))/g - (2*c^2*d^3*f*g*(f + g*x))/e - 6*a*c*d*e*f*g*(f + g*x) + c^2
*d^2*f*(f + g*x)^2 - 3*a*c*e^2*f*(f + g*x)^2 - (2*c^2*d*e*f^2*(f + g*x)^2)
/g + (c^2*d^3*g*(f + g*x)^2)/e + 3*a*c*d*e*g*(f + g*x)^2 - ((e*f - d*g)*(f
 + g*x)*(a + c*x^2)*(a*e^2*(2*e*f - 5*d*g - 3*e*g*x) - c*d*(3*d^2*g + 2*e^
2*f*x + d*e*g*x)))/(d + e*x)^2 + (Sqrt[c]*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(
e*f) + d*g)*(3*a*e^2*g + c*d*(2*e*f + d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] +
 x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)
^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]]
, (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(e*g*Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]]) + ((I*Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g
)*(-3*a*e^2*g - (6*I)*Sqrt[a]*Sqrt[c]*e*(e*f - d*g) + c*d*(-4*e*f + d*g))*
Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c
] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*S
qrt[a]*g)])/(e*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) + ((4*I)*a*c*e^2*f^2*Sqrt
[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] -
g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(S...
```

3.635.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2774 vs. 2(1241) = 2482.

Time = 5.31 (sec) , antiderivative size = 2774, normalized size of antiderivative = 2.24, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.679$, Rules used = {725, 25, 2349, 734, 2349, 27, 510, 599, 25, 27, 729, 25, 1416, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{725}$$

$$\frac{\int -\frac{cgx^2-2cfx+3ag}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4(ef-dg)}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{cgx^2-2cfx+3ag}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4(ef-dg)}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$\Big\downarrow$ 2349

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-\frac{2cf}{e}-\frac{cdg}{e^2}+\frac{cgx}{e}}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4(ef-dg)}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$\Big\downarrow$ 734

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\left(-\frac{\int \frac{-cgx^2e^2+age^2-2cdgxe-2cd(ef-dg)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)+\int \frac{-\frac{2cf}{e}-\frac{cdg}{e^2}+\frac{cgx}{e}}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4(ef-dg)}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$\Big\downarrow$ 2349

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)-\frac{2c(dg+}{}}{4(ef-dg)}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$\Big\downarrow$ 27

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)-\frac{2c(dg+}{}}{4(ef-dg)}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$\Big\downarrow$ 510

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)-\frac{2c(dg+}{}}{4(ef-dg)}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$\Big\downarrow$ 599

3.635. $\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2\int-\frac{cg(ef-dg-e(f+gx))}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}}{(d+ex)(ae^2+c}\right.}{4(ef-dg)}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$\downarrow$ 25

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\frac{2\int\frac{cg(ef-dg-e(f+gx))}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{}}{(d+ex)(ae^2+cd}\right.}{4(ef-dg)}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$\downarrow$ 27

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\frac{2c\int\frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}}{(d+ex)(ae^2+c}\right.}{4(ef-dg)}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$\downarrow$ 729

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\left(-\frac{2(ae^2g-cd(2ef-3dg))\int-\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}+\frac{2c\int\frac{ef-dg-e(f+gx}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f}{}}}}{g}}{2(ae^2+cd^2)(ef-dg)}\right.}{}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$\downarrow$ 25

3.635. $\int\frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}dx$

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\left(-\frac{\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}-2\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+g}{g^2}}}}{2(ae^2+cd^2)(ef-dg)}\right.}{}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$\downarrow$ 1416

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\left(-\frac{\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}-2\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+g}{g^2}}}}{2(ae^2+cd^2)(ef-dg)}\right.}{}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$\downarrow$ 1511

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\left(-\frac{\frac{2c\left(\frac{e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\left(dg-e\left(f-\frac{\sqrt{ag^2+cf^2}}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)}{g^2}}}\right.}{g}}{2(ae^2+cd^2)(ef-}\right.}{}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$\downarrow$ 1416

3.635. $\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

$$\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}+\frac{4c(ef+}{}$$

$$-\frac{}{}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2(ef-dg)(d+ex)^2}$$

$$\Big\downarrow \; 1509$$

3.635. $\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

$$\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}+\frac{4c(ef+}{}$$

$$-\frac{}{}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2(ef-dg)(d+ex)^2}$$

$\downarrow$ 1540

3.635. $\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

$$\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}} - \frac{4c(ef+\cdots}{\cdots}$$

$$-\frac{\cdots}{\cdots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2(ef-dg)(d+ex)^2}$$

$\downarrow$ 1416

3.635. $\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

$$\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}} - \frac{4c(ef+\cdots}{\cdots}$$

$$-\frac{\cdots}{\cdots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2(ef-dg)(d+ex)^2}$$

$\downarrow$ 2222

3.635. $\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

$4c(ef+$

$$\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}} - \text{———}$$

$$-\text{————————————————}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2(ef-dg)(d+ex)^2}$$

input
```
Int[Sqrt[a + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
```

3.635. $\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

output
```
-1/2*(Sqrt[f + g*x]*Sqrt[a + c*x^2])/((e*f - d*g)*(d + e*x)^2) - ((c^(3/4)
*(c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[
(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f
^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*Arc
Tan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[
c*f^2 + a*g^2])/2])/(e^2*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c
*(f + g*x)^2)/g^2]) - (4*c*(e*f + d*g)*(-1/2*(c^(1/4)*(c*e*f^2 + a*e*g^2 -
 Sqrt[c]*(e*f - d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*
f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)
^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^
2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1
+ (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(g*(c*f^2 + a*g^2)^(1/4)*(a*e^2*g +
 c*d*(2*e*f - d*g))*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f +
 g*x)^2)/g^2]) + (e*Sqrt[c*f^2 + a*g^2]*(Sqrt[c]*(e*f - d*g) - e*Sqrt[c*f^
2 + a*g^2])*(((e + (Sqrt[c]*(e*f - d*g))/Sqrt[c*f^2 + a*g^2])*ArcTanh[(Sqr
t[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[e]*Sqrt[e*f - d*g]*Sqrt[a + (c*f^2)/
g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])])/(2*Sqrt[e]*Sqrt[c*d^
2 + a*e^2]*Sqrt[e*f - d*g]) - ((Sqrt[c]/e - Sqrt[c*f^2 + a*g^2]/(e*f - d*g
))*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (
2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sq...
```

3.635.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 510
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2/d   Subst[Int[1/Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2
)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

3.635. $\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

rule 725
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2
]/((m + 1)*(e*f - d*g))), x] - Simp[1/(2*(m + 1)*(e*f - d*g))   Int[((d + e
*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*g*(2*m + 3) + 2*(c*f)*x
 + c*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && Integer
Q[2*m] && LtQ[m, -1]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 734
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2))), x] + Simp[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2))   Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^
2]))*Simp[2*c*d*(e*f - d*g)*(m + 1) - a*e^2*g*(2*m + 3) + 2*c*e*(d*g*(m + 1
) - e*f*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.635. $\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

3.635.4 Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 1196, normalized size of antiderivative = 0.96

method	result	size
elliptic	Expression too large to display	1196
default	Expression too large to display	19170

input
```
int((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(1/2/(d*g-e*f)*(c*
g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)^2+1/4*(3*a*e^2*g+c*d^2*g+2*c*d*e*f)
/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/(d*g-e*f)*(c*g*x^3+c*f*x^2+a*g*x+a*
f)^(1/2)/(e*x+d)-1/4*c*g*(a*d*e^2*g+2*a*e^3*f-c*d^3*g+4*c*d^2*e*f)/(a*d*e^
2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/(d*g-e*f)/e^2*(f/g-(-a*c)^(1/2)/c)*((x+f/g)
/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1
/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*
x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^
(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))-1/4*c*g*(3*a*e^2*g+c*d^2*g+2*c*d*e*
f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/(d*g-e*f)/e*(f/g-(-a*c)^(1/2)/c)*
((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2
)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*
x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)
^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*
c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1
/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+1/4*(3*a^2*e^4*g^2+6*a*c*d^2*e^2*g^2
-4*a*c*d*e^3*f*g+4*a*c*e^4*f^2-c^2*d^4*g^2+4*c^2*d^3*e*f*g)/(a*d*e^2*g-a*e
^3*f+c*d^3*g-c*d^2*e*f)/e^3/(d*g-e*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(
-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x
+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*...
```

3.635.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.635.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)**(1/2)/(e*x+d)**3/(g*x+f)**(1/2),x)
```

output
```
Timed out
```

3.635.7 Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}}{(ex+d)^3\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)/((e*x + d)^3*sqrt(g*x + f)), x)
```

3.635.8 Giac [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}}{(ex+d)^3\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)/((e*x + d)^3*sqrt(g*x + f)), x)
```

3.635.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx=\int \frac{\sqrt{c\,x^2+a}}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^3}\,dx$$

input
```
int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^3),x)
```

output
```
int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^3), x)
```

3.636 $\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

3.636.1 Optimal result

Integrand size = 28, antiderivative size = 531

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = -\frac{2e(25ae^2g^2+c(7e^2f^2+12defg-90d^2g^2))\sqrt{f+gx}\sqrt{a+cx^2}}{105c^2g^2} + \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7c} + \frac{2e^2(ef+11dg)(f+gx)^{3/2}\sqrt{a+cx^2}}{35cg^2} + \frac{2\sqrt{-a}(ae^2g^2(19ef+189dg)-c(8e^3f^3-42de^2f^2g+105d^2efg^2+105d^3g^3))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\Big(\text{arc}}{105c^{3/2}g^3\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \frac{2\sqrt{-a}e(cf^2+ag^2)(25ae^2g^2-c(8e^2f^2-42defg+105d^2g^2))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\Big(\arcsin\Big(}{105c^{5/2}g^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
2/35*e^2*(11*d*g+e*f)*(g*x+f)^(3/2)*(c*x^2+a)^(1/2)/c/g^2-2/105*e*(25*a*e^
2*g^2+c*(-90*d^2*g^2+12*d*e*f*g+7*e^2*f^2))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/
c^2/g^2+2/7*e*(e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c+2/105*(a*e^2*g^2*(
189*d*g+19*e*f)-c*(105*d^3*g^3+105*d^2*e*f*g^2-42*d*e^2*f^2*g+8*e^3*f^3))*
EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^
(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/c^(3/2)/
g^3/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-2/105
*e*(a*g^2+c*f^2)*(25*a*e^2*g^2-c*(105*d^2*g^2-42*d*e*f*g+8*e^2*f^2))*Ellip
ticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)
*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1
/2)+f*c^(1/2)))^(1/2)/c^(5/2)/g^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

3.636.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.52 (sec) , antiderivative size = 777, normalized size of antiderivative = 1.46

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2(a+cx^2)\left(-25ae^3g^2+ce\left(105d^2g^2+21deg(f+3gx)+e^2\left(-4f^2+3fgx+15g^2x^2\right)\right)\right)}{c^2g^2} - \frac{2\left(g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\left(ae^2g^2(19ef+189dg)-c(8\right.\right.}{}\right.}{}$$

input
```
Integrate[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

output
```
(Sqrt[f + g*x]*((2*(a + c*x^2)*(-25*a*e^3*g^2 + c*e*(105*d^2*g^2 + 21*d*e*
g*(f + 3*g*x) + e^2*(-4*f^2 + 3*f*g*x + 15*g^2*x^2))))/(c^2*g^2) - (2*(g^2
*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(a*e^2*g^2*(19*e*f + 189*d*g) - c*(8*e^3
*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^2 + 105*d^3*g^3))*(a + c*x^2) - Sqrt
[c]*(I*a*Sqrt[c]*e^2*f*g^2*(19*e*f + 189*d*g) - a^(3/2)*e^2*g^3*(19*e*f +
189*d*g) - I*c^(3/2)*f*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^2 + 105
*d^3*g^3) + Sqrt[a]*c*g*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^2 + 10
5*d^3*g^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[
a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqr
t[c]*f + I*Sqrt[a]*g)] - g*(Sqrt[c]*f + I*Sqrt[a]*g)*((105*I)*c^(3/2)*d^3*
g^2 + 25*a^(3/2)*e^3*g^2 + (3*I)*a*Sqrt[c]*e^2*g*(2*e*f - 63*d*g) + Sqrt[a
]*c*e*(-8*e^2*f^2 + 42*d*e*f*g - 105*d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c
] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g
*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*
x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(c^2*g^4*Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/(105*Sqrt[a + c*x^2])
```

3.636.3 Rubi [A] (warning: unable to verify)

Time = 1.80 (sec) , antiderivative size = 916, normalized size of antiderivative = 1.73, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {735, 25, 2185, 27, 2185, 27, 599, 25, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

$$\Big\downarrow \text{735}$$

$$\frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}-\frac{\int -\frac{(d+ex)\left(7cfd^2+ce(ef+11dg)x^2-ae(4ef+dg)-\left(5ae^2g-cd(12ef+7dg)\right)x\right)}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{7c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(d+ex)\left(7cfd^2+ce(ef+11dg)x^2-ae(4ef+dg)-\left(5ae^2g-cd(12ef+7dg)\right)x\right)}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{7c}+\frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\frac{2\int \frac{-ce\left(25ae^2g^2+c\left(7e^2f^2+12degf-90d^2g^2\right)\right)x^2g^2+c\left(35cd^3fg-ae\left(3e^2f^2+53degf+5d^2g^2\right)\right)g^2-c\left(ae^2(23ef+63dg)g^2+c\left(2e^3f^3+22de^2gf^2-95d^2eg^2f-35d^3g\right.\right.}{2\sqrt{f+gx}\sqrt{cx^2+a}}}{5cg^3}}{7c}$$

$$\frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{-ce\left(25ae^2g^2+c\left(7e^2f^2+12degf-90d^2g^2\right)\right)x^2g^2+c\left(35cd^3fg-ae\left(3e^2f^2+53degf+5d^2g^2\right)\right)g^2-c\left(ae^2(23ef+63dg)g^2+c\left(2e^3f^3+22de^2gf^2-95d^2eg^2f-35d^3g^3\right.\right.}{\sqrt{f+gx}\sqrt{cx^2+a}}}{5cg^3}}{7c}$$

$$\frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}$$

$\downarrow$ 2185

$$\frac{\frac{\frac{2\int \frac{cg^3\left(g\left(105c^2fgd^3+25a^2e^3g^2-ace\left(2e^2f^2+147degf+105d^2g^2\right)\right)-c\left(ae^2g^2(19ef+189dg)-c\left(8e^3f^3-42de^2gf^2+105d^2eg^2f+105d^3g^3\right)\right)x\right)}{2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3cg^2}-\frac{2}{3}eg\sqrt{a+cx^2}\sqrt{}}{5cg^3}}{7c}$$

$$\frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}$$

$\downarrow$ 27

$$\frac{\frac{\frac{1}{3}g\int \frac{g\left(105c^2fgd^3+25a^2e^3g^2-ace\left(2e^2f^2+147degf+105d^2g^2\right)\right)-c\left(ae^2g^2(19ef+189dg)-c\left(8e^3f^3-42de^2gf^2+105d^2eg^2f+105d^3g^3\right)\right)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2}{3}eg\sqrt{a+cx^2}\sqrt{f+}}{5cg^3}}{7c}$$

$$\frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}$$

$\downarrow$ 599

$$\frac{\frac{-\frac{2\int -\frac{e\left(cf^2+ag^2\right)\left(25ae^2g^2-c\left(8e^2f^2-42degf+105d^2g^2\right)\right)-c\left(ae^2g^2(19ef+189dg)-c\left(8e^3f^3-42de^2gf^2+105d^2eg^2f+105d^3g^3\right)\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3g}-\frac{2}{3}eg\sqrt{a+c}}{5cg^3}}{7c}$$

$$\frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}$$

$\downarrow$ 25

3.636. $\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

$$\frac{\frac{\frac{2\int \frac{e\left(cf^2+ag^2\right)\left(25ae^2g^2-c\left(8e^2f^2-42degf+105d^2g^2\right)\right)-c\left(ae^2g^2(19ef+189dg)-c\left(8e^3f^3-42de^2gf^2+105d^2eg^2f+105d^3g^3\right)\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3g}-\frac{2}{3}eg\sqrt{a+cx^2}\cdots}{5cg^3}}{7c}$$

$$\frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}$$

$\downarrow$ 1511

$$-\frac{2\left(-\sqrt{ag^2+cf^2}\left(e\sqrt{ag^2+cf^2}\left(25ae^2g^2-c\left(105d^2g^2-42defg+8e^2f^2\right)\right)-\sqrt{c}\left(ae^2g^2(189dg+19ef)-c\left(105d^3g^3+105d^2efg^2-42de^2f^2g+8e^3f^3\right)\right)\right)\int \frac{\cdots}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(\cdots}{\cdots}}}\cdots\right.}{\cdots}$$

$$\frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}$$

$\downarrow$ 1416

$$-\frac{2\left(-\sqrt{c}\sqrt{ag^2+cf^2}\left(ae^2g^2(189dg+19ef)-c\left(105d^3g^3+105d^2efg^2-42de^2f^2g+8e^3f^3\right)\right)\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{\left(ag^2+cf^2\right)^{3/4}\left(\frac{\sqrt{c}(f+\cdots}{\sqrt{ag^2+\cdots}}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}$$

$\downarrow$ 1509

$$\frac{2e\sqrt{f+gx}\sqrt{cx^2+a}(d+ex)^2}{7c}+$$

$$\frac{\frac{2(ef+11dg)(f+gx)^{3/2}\sqrt{cx^2+a}e^2}{5g^2}+\frac{-\frac{2}{3}eg\sqrt{f+gx}\sqrt{cx^2+a}\left(25ae^2g^2+c\left(7e^2f^2+12degf-90d^2g^2\right)\right)-\frac{2\left(-\sqrt{c}\sqrt{cf^2+ag^2}\left(ae^2g^2(19ef+189dg)-c\left(8e^3f^{\cdots}\right.\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

input
```
Int[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

3.636. $\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

output
```
(2*e*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(7*c) + ((2*e^2*(e*f + 11*
d*g)*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(5*g^2) + ((-2*e*g*(25*a*e^2*g^2 + c
*(7*e^2*f^2 + 12*d*e*f*g - 90*d^2*g^2))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/3 -
 (2*(-(Sqrt[c]*Sqrt[c*f^2 + a*g^2]*(a*e^2*g^2*(19*e*f + 189*d*g) - c*(8*e^
3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^2 + 105*d^3*g^3))*(-((Sqrt[f + g*x]
*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a
+ (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 +
 a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f
^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*
(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^(1
/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a
*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f +
 g*x)^2)/g^2]))) - ((c*f^2 + a*g^2)^(3/4)*(e*Sqrt[c*f^2 + a*g^2]*(25*a*e^2
*g^2 - c*(8*e^2*f^2 - 42*d*e*f*g + 105*d^2*g^2)) - Sqrt[c]*(a*e^2*g^2*(19*
e*f + 189*d*g) - c*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^2 + 105*d^3
*g^3)))*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^
2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (
Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sq
rt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])
/2])/(2*c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + ...
```

3.636.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 735
```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/(c*(2*m + 1))), x] - Simp[1/(c*(2*m + 1))   Int[((d + e*x)^(m - 2)/(Sq
rt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e*(d*g + 2*e*f*(m - 1)) - c*d^2*f*(2*m
 + 1) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f*m + d*g*(2*m + 1)))*x - c*e*(e*f +
d*g*(4*m - 1))*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && IntegerQ[
2*m] && GtQ[m, 1]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.636. $\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

3.636.4 Maple [A] (verified)

Time = 3.37 (sec) , antiderivative size = 882, normalized size of antiderivative = 1.66

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(\frac{2e^3x^2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{7c}+\frac{2\left(3d\,e^2g+\frac{1}{7}e^3f\right)x\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{5cg}+\frac{2\left(3d^2eg+3d\,e^2f-\frac{4f\left(3d\,e^2g+\frac{1}{7}e^3f\right)}{5g}-\frac{5ga}{7c}\right.}{3cg}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/7*e^3/c*x^2*(c*
g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/5*(3*d*e^2*g+1/7*e^3*f)/c/g*x*(c*g*x^3+c*
f*x^2+a*g*x+a*f)^(1/2)+2/3*(3*d^2*e*g+3*d*e^2*f-4/5*f/g*(3*d*e^2*g+1/7*e^3
*f)-5/7/c*g*a*e^3)/c/g*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2*(d^3*f-2/5*a/c*
f/g*(3*d*e^2*g+1/7*e^3*f)-1/3*c*a*(3*d^2*e*g+3*d*e^2*f-4/5*f/g*(3*d*e^2*g+
1/7*e^3*f)-5/7/c*g*a*e^3))*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)
/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/
2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*Ellip
ticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a
*c)^(1/2)/c))^(1/2))+2*(d^3*g+3*d^2*e*f-4/7*a/c*f*e^3-3/5/c*a*(3*d*e^2*g+1
/7*e^3*f)-2/3*f/g*(3*d^2*e*g+3*d*e^2*f-4/5*f/g*(3*d*e^2*g+1/7*e^3*f)-5/7/c
*g*a*e^3))*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(
-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*
c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c
)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-
f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^
(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))
```

3.636.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 445, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx =$$
$$-\frac{2\left((8\,c^2e^3f^4-42\,c^2de^2f^3g+(105\,c^2d^2e-13\,ace^3)f^2g^2-42\,(5\,c^2d^3-6\,acde^2)fg^3+15\,(21\,acd^2e-5\,a\right.}{}$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/315*((8*c^2*e^3*f^4 - 42*c^2*d*e^2*f^3*g + (105*c^2*d^2*e - 13*a*c*e^3)
*f^2*g^2 - 42*(5*c^2*d^3 - 6*a*c*d*e^2)*f*g^3 + 15*(21*a*c*d^2*e - 5*a^2*e
^3)*g^4)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/2
7*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 3*(8*c^2*e^3*f^3*g - 4
2*c^2*d*e^2*f^2*g^2 + (105*c^2*d^2*e - 19*a*c*e^3)*f*g^3 + 21*(5*c^2*d^3 -
 9*a*c*d*e^2)*g^4)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2)
, -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*
g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) - 3*(
15*c^2*e^3*g^4*x^2 - 4*c^2*e^3*f^2*g^2 + 21*c^2*d*e^2*f*g^3 + 5*(21*c^2*d^
2*e - 5*a*c*e^3)*g^4 + 3*(c^2*e^3*f*g^3 + 21*c^2*d*e^2*g^4)*x)*sqrt(c*x^2
+ a)*sqrt(g*x + f))/(c^3*g^4)
```

3.636.6 Sympy [F]

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

input
```
integrate((e*x+d)**3*(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x)**3*sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

3.636.7 Maxima [F]

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^3\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

3.636.8 Giac [F]

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^3\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

3.636.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}\,(d+e\,x)^3}{\sqrt{c\,x^2+a}}\,dx$$

input
```
int(((f + g*x)^(1/2)*(d + e*x)^3)/(a + c*x^2)^(1/2),x)
```

output
```
int(((f + g*x)^(1/2)*(d + e*x)^3)/(a + c*x^2)^(1/2), x)
```

3.637 $\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

3.637.1 Optimal result

Integrand size = 28, antiderivative size = 410

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx=\frac{2e(ef+7dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg}+\frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5c}$$
$$+\frac{2\sqrt{-a}(9ae^2g^2+c(2e^2f^2-10defg-15d^2g^2))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15c^{3/2}g^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$-\frac{4\sqrt{-a}e(ef-5dg)(cf^2+ag^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15c^{3/2}g^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
2/15*e*(7*d*g+e*f)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g+2/5*e*(e*x+d)*(g*x+f)
^(1/2)*(c*x^2+a)^(1/2)/c+2/15*(9*a*e^2*g^2+c*(-15*d^2*g^2-10*d*e*f*g+2*e^2
*f^2))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+
f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/c
^(3/2)/g^2/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2
)-4/15*e*(-5*d*g+e*f)*(a*g^2+c*f^2)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))
^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c
*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^2
/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

3.637.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.09 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2e(a+cx^2)(10dg+e(f+3gx))}{cg}+\frac{(f+gx)\left(\frac{2g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\left(-9ae^2g^2+c\left(-2e^2f^2+10defg+15d^2g^2\right)\right)\left(a+cx^2\right)}{(f+gx)^2}+\frac{2\sqrt{c}\left(-i\sqrt{c}f+\sqrt{a}g\right)\left(-9a\right.}{}\right.}{}\right.}{}$$

input
```
Integrate[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

output
```
(Sqrt[f + g*x]*((2*e*(a + c*x^2)*(10*d*g + e*(f + 3*g*x)))/(c*g) + ((f + g
*x)*((2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(-9*a*e^2*g^2 + c*(-2*e^2*f^2
 + 10*d*e*f*g + 15*d^2*g^2))*(a + c*x^2))/(f + g*x)^2 + (2*Sqrt[c]*((-I)*S
qrt[c]*f + Sqrt[a]*g)*(-9*a*e^2*g^2 + c*(-2*e^2*f^2 + 10*d*e*f*g + 15*d^2*
g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/
Sqrt[c] - g*x)/(f + g*x))]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]
)/Sqrt[f + g*x] + (2*Sqrt[c]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*((15*I)*c*d^2*g -
 (9*I)*a*e^2*g + 2*Sqrt[a]*Sqrt[c]*e*(e*f - 5*d*g))*Sqrt[(g*((I*Sqrt[a])/S
qrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*E
llipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[
c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x]))/(c^2*g^3*S
qrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])))/(15*Sqrt[a + c*x^2])
```

3.637.3 Rubi [A] (warning: unable to verify)

Time = 1.16 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.86, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {735, 25, 2185, 27, 599, 25, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}dx$$

$$\Big\downarrow \text{735}$$

$$\frac{2e\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5c}-\frac{\int -\frac{5cfd^2+ce(ef+7dg)x^2-ae(2ef+dg)-\left(3ae^2g-cd(8ef+5dg)\right)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{5c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5cfd^2+ce(ef+7dg)x^2-ae(2ef+dg)-\left(3ae^2g-cd(8ef+5dg)\right)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{5c}+\frac{2e\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5c}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\frac{2\int \frac{cg\left(g\left(15cd^2f-ae(7ef+10dg)\right)-\left(9ae^2g^2+c\left(2e^2f^2-10degf-15d^2g^2\right)\right)x\right)}{2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3cg^2}+\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}(7dg+ef)}{3g}}{5c}+\frac{2e\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{g\left(15cd^2f-ae(7ef+10dg)\right)-\left(9ae^2g^2+c\left(2e^2f^2-10degf-15d^2g^2\right)\right)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3g}+\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}(7dg+ef)}{3g}}{5c}+\frac{2e\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5c}$$

$$\Big\downarrow \text{599}$$

$$\frac{\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}(7dg+ef)}{3g}-\frac{2\int -\frac{2e(ef-5dg)\left(cf^2+ag^2\right)-\left(9ae^2g^2+c\left(2e^2f^2-10degf-15d^2g^2\right)\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3g^3}}{5c}+\frac{2e\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2\int \frac{2e(ef-5dg)\left(cf^2+ag^2\right)-\left(9ae^2g^2+c\left(2e^2f^2-10degf-15d^2g^2\right)\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3g^3}+\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}(7dg+ef)}{3g}}{5c}+\frac{2e\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5c}$$

$\big\downarrow$ 1511

$$\frac{\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}(7dg+ef)}{3g}-\frac{2\left(\frac{\sqrt{ag^2+cf^2}\left(-2\sqrt{c}e\sqrt{ag^2+cf^2}(ef-5dg)+9ae^2g^2+c\left(-15d^2g^2-10defg+2e^2f^2\right)\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{\cdots}}{\sqrt{c}}\cdots\right.}{3g^3}}{5c}+\frac{2e\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5c}$$

$\big\downarrow$ 1416

$$\frac{\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}(7dg+ef)}{3g}-2\left(\frac{\left(ag^2+cf^2\right)^{3/4}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(-2\sqrt{c}e\sqrt{ag^2+cf^2}(ef-5dg)+9ae^2g^2+c\left(-15d^2g^2-\cdots\right.\right.}{2c^{3/4}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}\cdots}}\cdots\right.}{\cdots}+\frac{2e\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5c}$$

$\big\downarrow$ 1509

3.637. $\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

$$\frac{\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}(7dg+ef)}{3g} - \frac{2\left(\frac{\left(ag^2+cf^2\right)^{3/4}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(-2\sqrt{c}e\sqrt{ag^2+cf^2}(ef-5dg)+9ae^2g^2+c\left(-15d^2g^2-\right.\right.}{2c^{3/4}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right)}{}}{} + \frac{2e\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5c}$$

input
```
Int[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

output
```
(2*e*(d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(5*c) + ((2*e*(e*f + 7*d*g)*
Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*g) - (2*(-((Sqrt[c*f^2 + a*g^2]*(9*a*e^2
*g^2 + c*(2*e^2*f^2 - 10*d*e*f*g - 15*d^2*g^2))*(-((Sqrt[f + g*x]*Sqrt[a +
 (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)
/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(
1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 -
 (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqr
t[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[
f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2]
)/(c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/
g^2]))))/Sqrt[c]) + ((c*f^2 + a*g^2)^(3/4)*(9*a*e^2*g^2 - 2*Sqrt[c]*e*(e*f
 - 5*d*g)*Sqrt[c*f^2 + a*g^2] + c*(2*e^2*f^2 - 10*d*e*f*g - 15*d^2*g^2))*(1
 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f
*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f
 + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x
])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(2*c^
(3/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])
))/(3*g^3))/(5*c)
```

3.637. $\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

3.637.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 735
```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/(c*(2*m + 1))), x] - Simp[1/(c*(2*m + 1))   Int[((d + e*x)^(m - 2)/(Sq
rt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e*(d*g + 2*e*f*(m - 1)) - c*d^2*f*(2*m
 + 1) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f*m + d*g*(2*m + 1)))*x - c*e*(e*f +
d*g*(4*m - 1))*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && IntegerQ[
2*m] && GtQ[m, 1]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.637. $\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.637.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(338) = 676$.

Time = 2.09 (sec), antiderivative size = 700, normalized size of antiderivative = 1.71

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(\frac{2e^2x\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{5c}+\frac{2\left(2deg+\frac{1}{5}e^2f\right)\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{3cg}+\frac{2\left(d^2f-\frac{2e^2fa}{5c}-\frac{\left(2deg+\frac{1}{5}e^2f\right)a}{3c}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/5*e^2/c*x*(c*g*
x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/3*(2*d*e*g+1/5*e^2*f)/c/g*(c*g*x^3+c*f*x^2+
a*g*x+a*f)^(1/2)+2*(d^2*f-2/5*e^2/c*f*a-1/3*(2*d*e*g+1/5*e^2*f)/c*a)*(f/g-
(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(
-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/
2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/
c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(d^2*g+2*
d*e*f-3/5*e^2/c*a*g-2/3*(2*d*e*g+1/5*e^2*f)/g*f)*(f/g-(-a*c)^(1/2)/c)*((x+
f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c)
)^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+
a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/
2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(
1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/
c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))
```

3.637.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.73

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{2\left(2\left(ce^2f^3-5\,cdef^2g-15\,adeg^3+3\left(5\,cd^2-2\,ae^2\right)fg^2\right)\sqrt{cg}\text{weierstrassPInverse}\left(\frac{4\left(cf^2-3\,ag^2\right)}{3\,cg^2},-\frac{8\left(cf^3+9\,a\right.}{27\,cg^3}\right.}{}$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(2*(c*e^2*f^3 - 5*c*d*e*f^2*g - 15*a*d*e*g^3 + 3*(5*c*d^2 - 2*a*e^2)*
f*g^2)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*
(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 3*(2*c*e^2*f^2*g - 10*c*
d*e*f*g^2 - 3*(5*c*d^2 - 3*a*e^2)*g^3)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^
2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInver
se(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(
3*g*x + f)/g)) + 3*(3*c*e^2*g^3*x + c*e^2*f*g^2 + 10*c*d*e*g^3)*sqrt(c*x^2
 + a)*sqrt(g*x + f))/(c^2*g^3)
```

3.637.6 Sympy [F]

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

input
```
integrate((e*x+d)**2*(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x)**2*sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

3.637.7 Maxima [F]

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^2\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

3.637.8 Giac [F]

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^2\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

3.637.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx=\int \frac{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^2}{\sqrt{c\,x^2+a}}\,dx$$

input
```
int(((f + g*x)^(1/2)*(d + e*x)^2)/(a + c*x^2)^(1/2),x)
```

output
```
int(((f + g*x)^(1/2)*(d + e*x)^2)/(a + c*x^2)^(1/2), x)
```

3.638 $\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

3.638.1 Optimal result

Integrand size = 26, antiderivative size = 331

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{2e\sqrt{f+gx}\sqrt{a+cx^2}}{3c} - \frac{2\sqrt{-a}(ef+3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{2\sqrt{-a}e(cf^2+ag^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3c^{3/2}g\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
2/3*e*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c-2/3*(3*d*g+e*f)*EllipticE(1/2*(1-x*c
^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2
))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/g/c^(1/2)/(c*x^2+a)^(1/2)/((
g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+2/3*e*(a*g^2+c*f^2)*Ellipti
cF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c
^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2
)+f*c^(1/2)))^(1/2)/c^(3/2)/g/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

3.638.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.52 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{2\sqrt{f+gx}\left(e(a+cx^2)+\frac{(ef+3dg)(a+cx^2)}{f+gx}+\frac{ic\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(ef+3dg)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}\sqrt{f+gx}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-f-\frac{i\sqrt{}}{\sqrt{}}}}{\sqrt{f+gx}}\right.\right.}{g^2}\right.}{3c\sqrt{a-}}$$

input
```
Integrate[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

output
```
(2*Sqrt[f + g*x]*(e*(a + c*x^2) + ((e*f + 3*d*g)*(a + c*x^2))/(f + g*x) +
(I*c*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f + 3*d*g)*Sqrt[(g*((I*Sqrt[a])/S
qrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*S
qrt[f + g*x]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
 g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/g^2 + (I*(3*
Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sq
rt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sq
rt[f + g*x]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(g*Sqrt[-f -
(I*Sqrt[a]*g)/Sqrt[c]])))/(3*c*Sqrt[a + c*x^2])
```

3.638.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 667 vs. 2(331) = 662.

Time = 0.71 (sec) , antiderivative size = 667, normalized size of antiderivative = 2.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {687, 27, 599, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

$$\Big\downarrow \text{ 687}$$

$$\frac{2\int \frac{3cdf-aeg+c(ef+3dg)x}{2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3c}+\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}}{3c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3cdf-aeg+c(ef+3dg)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3c}+\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}}{3c}$$

$$\Big\downarrow \text{599}$$

$$\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}}{3c}-\frac{2\int \frac{e(cf^2+ag^2)-c(ef+3dg)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3cg^2}$$

$$\Big\downarrow \text{1511}$$

$$\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}}{3c}-$$

$$\frac{2\left(\left(-\sqrt{c}\sqrt{ag^2+cf^2}(3dg+ef)+aeg^2+cef^2\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}+\sqrt{c}\sqrt{ag^2+cf^2}(3dg+e\right.}{3cg^2}$$

$$\Big\downarrow \text{1416}$$

$$\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}}{3c}-$$

$$\frac{2\left(\sqrt{c}\sqrt{ag^2+cf^2}(3dg+ef)\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}+\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}\cdots}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+\cdots}{\sqrt{ag^2+\cdots}}\right.}}}{\cdots}\right.}{3cg^2}$$

$$\Big\downarrow \text{1509}$$

$$\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}}{3c}-$$

$$2\left(\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(-\sqrt{c}\sqrt{ag^2+cf^2}(3dg+ef)+aeg^2+cef^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+g\cdots}}{\sqrt[4]{cf^2+\cdots}}\right.\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right.$$

3.638. $\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

input
```
Int[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

output
```
(2*e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c) - (2*(Sqrt[c]*(e*f + 3*d*g)*Sqrt
[c*f^2 + a*g^2]*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))
/g^2 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/S
qrt[c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sq
rt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f +
 g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g
^2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)]
, (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/g^2
- (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])) + ((c*f^2 + a*g^2)^(1/4)*
(c*e*f^2 + a*e*g^2 - Sqrt[c]*(e*f + 3*d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt
[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*
x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))
/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^
2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(2*c^(1/4)*Sq
rt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/(3*c*
g^2)
```

3.638.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

3.638. $\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.638.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(265) = 530$.

Time = 1.29 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.82

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(\frac{2e\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{3c}+\frac{2\left(df-\frac{eag}{3c}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}\cdots}{-\frac{f}{g}\cdots}}\right.}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\right.}{}$
risch	$\frac{2e\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{3c}-\frac{\left(\frac{2aeg\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}-\frac{6cdf\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\cdots\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/3/c*e*(c*g*x^3+
c*f*x^2+a*g*x+a*f)^(1/2)+2*(d*f-1/3/c*e*a*g)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)
/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1
/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*
x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^
(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(d*g+1/3*e*f)*(f/g-(-a*c)^(1/2)/c
)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1
/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*
f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*
c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-
a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^
(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))
```

3.638. $\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

3.638.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$
$$= \frac{2\left(3\sqrt{cx^2+a}\sqrt{gx+f}ceg^2-(cef^2-6\,cdfg+3\,aeg^2)\sqrt{cg}\text{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3}\right.\right.}{}$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(3*sqrt(c*x^2 + a)*sqrt(g*x + f)*c*e*g^2 - (c*e*f^2 - 6*c*d*f*g + 3*a*
e*g^2)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*
(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) - 3*(c*e*f*g + 3*c*d*g^2)*
sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*
a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27
*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)))/(c^2*g^2)
```

3.638.6 Sympy [F]

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(d+ex)\,\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

input
```
integrate((e*x+d)*(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x)*sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

3.638.7 Maxima [F]

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

3.638.8 Giac [F]

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

3.638.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}\,(d+e\,x)}{\sqrt{c\,x^2+a}}\,dx$$

input
```
int(((f + g*x)^(1/2)*(d + e*x))/(a + c*x^2)^(1/2),x)
```

output
```
int(((f + g*x)^(1/2)*(d + e*x))/(a + c*x^2)^(1/2), x)
```

3.639 $\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

3.639.1 Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx=-\frac{2\sqrt{-a}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

output
```
-2*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-
a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/c^(1/
2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)
```

3.639.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.38 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.16

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{2i\left(\sqrt{c}f+i\sqrt{a}g\right)\sqrt{\frac{g(\sqrt{a}+i\sqrt{c}x)}{-i\sqrt{c}f+\sqrt{a}g}}\sqrt{f+gx}\left(E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-i\sqrt{a}g}}\right)|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)-\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\right.\right.\right.}{\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{g(i\sqrt{a}+\sqrt{c}x)}}\sqrt{a+cx^2}}$$

input
```
Integrate[Sqrt[f + g*x]/Sqrt[a + c*x^2],x]
```

output
```
((2*I)*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*(Sqrt[a] + I*Sqrt[c]*x))/((-I)*Sq
rt[c]*f + Sqrt[a]*g)]*Sqrt[f + g*x]*(EllipticE[I*ArcSinh[Sqrt[-((Sqrt[c]*(
f + g*x))/(Sqrt[c]*f - I*Sqrt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]
*f + I*Sqrt[a]*g)] - EllipticF[I*ArcSinh[Sqrt[-((Sqrt[c]*(f + g*x))/(Sqrt[
c]*f - I*Sqrt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g
)]))/(Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(g*(I*Sqrt[a] + Sqrt[c]*x))]*Sqrt
[a + c*x^2])
```

3.639.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 583 vs. 2(136) = 272.

Time = 0.59 (sec) , antiderivative size = 583, normalized size of antiderivative = 4.29, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {507, 1459, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}dx$$

$$\Big\downarrow \text{ 507}$$

$$\frac{2\int \frac{f+gx}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}$$

$$\Big\downarrow \text{ 1459}$$

$$\frac{2\left(\frac{\sqrt{ag^2+cf^2}\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\frac{\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}\right)}{g}$$

$$\Big\downarrow \text{ 1416}$$

$$2\left(\frac{\left(ag^2+cf^2\right)^{3/4}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{2c^{3/4}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}-\frac{\sqrt{ag^2+cf^2}\cdots}{\cdots}\right)$$
$$\overline{\qquad g \qquad}$$

$\downarrow$ 1509

$$2\left(\frac{\left(ag^2+cf^2\right)^{3/4}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{2c^{3/4}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}-\frac{\sqrt{ag^2+cf^2}\cdots}{\cdots}\right)$$

input
```
Int[Sqrt[f + g*x]/Sqrt[a + c*x^2],x]
```

output
```
(2*(-((Sqrt[c*f^2 + a*g^2]*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f
*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(
f + g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f
 + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^
2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[
c*f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*
g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a + (
c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/Sqrt[c]) + ((
c*f^2 + a*g^2)^(3/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a
 + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2
)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTa
n[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*
f^2 + a*g^2])/2])/(2*c^(3/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2
+ (c*(f + g*x)^2)/g^2])))/g
```

3.639. $\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

3.639.3.1 Defintions of rubi rules used

rule 507
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[2/
d   Subst[Int[x^2/Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]
, x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1459
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[1/q   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[1/q
 Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

3.639.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(108) = 216$.

Time = 0.44 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.91

method	result
default	$\frac{2\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,(cf-g\sqrt{-ac})\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}}\,\sqrt{\frac{(-cx+\sqrt{-ac})g}{g\sqrt{-ac}+cf}}\,\sqrt{\frac{(cx+\sqrt{-ac})g}{g\sqrt{-ac}-cf}}\,\left(\sqrt{-ac}\,F\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)g-\sqrt{-a}\right.}{g(cg\,x^3+cf\,x^2+agx}$
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(\frac{2f\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}+\frac{2g\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+}{\frac{f}{g}-\sqrt{}}}}{}\right.}{\sqrt{gx+f}\,\sqrt{c\,x}}$

input
```
int((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(c*f-g*(-a*c)^(1/2))*(-(g*x+f)*c/(g*(-a*c)
^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*
x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-a*c)^(1/2)*EllipticF((-(g
*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)
+c*f))^(1/2))*g-(-a*c)^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(
1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*g+f*EllipticF((-(
g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2
)+c*f))^(1/2))*c-EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-
a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*f)/g/(c*g*x^3+c*f*x^2+a*g*x
+a*f)/c^2
```

3.639.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{2\left(2\sqrt{cg}f\text{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)-3\sqrt{cg}g\text{weierstrassZeta}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},\right.\right.}{3\,cg}$$

input
```
integrate((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(2*sqrt(c*g)*f*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/2
7*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) - 3*sqrt(c*g)*g*weierstr
assZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3),
weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^
2)/(c*g^3), 1/3*(3*g*x + f)/g)))/(c*g)
```

3.639.6 Sympy [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

input
```
integrate((g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

3.639.7 Maxima [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

3.639.8 Giac [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

3.639.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx=\int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+a}}\,dx$$

input

```
int((f + g*x)^(1/2)/(a + c*x^2)^(1/2),x)
```

output

```
int((f + g*x)^(1/2)/(a + c*x^2)^(1/2), x)
```

3.640 $\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx$

3.640.1 Optimal result

Integrand size = 28, antiderivative size = 319

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx = -\frac{2\sqrt{-a}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{2(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
-2*g*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*
(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/
(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-2*
(-d*g+e*f)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*
c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))
*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/(e+d
*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

3.640.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.36 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx = -\frac{2i\sqrt{\frac{g(\sqrt{a}+i\sqrt{c}x)}{-i\sqrt{c}f+\sqrt{a}g}}\sqrt{f+gx}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-i\sqrt{a}g}}\right),\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)-\text{EllipticPi}\left(\frac{e\left(f-\frac{i\sqrt{a}g}{\sqrt{c}}\right)}{ef-dg},i\text{arcsi}\right.\right.}{e\sqrt{\frac{\sqrt{c}(f+gx)}{g(i\sqrt{a}+\sqrt{c}x)}}\sqrt{a+cx^2}}$$

input
```
Integrate[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
((-2*I)*Sqrt[(g*(Sqrt[a] + I*Sqrt[c]*x))/((-I)*Sqrt[c]*f + Sqrt[a]*g)]*Sqr
t[f + g*x]*(EllipticF[I*ArcSinh[Sqrt[-((Sqrt[c]*(f + g*x))/(Sqrt[c]*f - I*
Sqrt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - Elli
pticPi[(e*(f - (I*Sqrt[a]*g)/Sqrt[c]))/(e*f - d*g), I*ArcSinh[Sqrt[-((Sqrt
[c]*(f + g*x))/(Sqrt[c]*f - I*Sqrt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sq
rt[c]*f + I*Sqrt[a]*g)]))/(e*Sqrt[(Sqrt[c]*(f + g*x))/(g*(I*Sqrt[a] + Sqrt
[c]*x))]*Sqrt[a + c*x^2])
```

3.640.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1112 vs. 2(319) = 638.

Time = 1.89 (sec) , antiderivative size = 1112, normalized size of antiderivative = 3.49, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {736, 510, 729, 25, 1416, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}(d+ex)}dx$$

$$\Big\downarrow \; 736$$

$$\frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e}+\frac{g\int \frac{1}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e}$$

$$\Big\downarrow \; 510$$

$$\frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e}+\frac{2\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e}$$

$$\Big\downarrow \; 729$$

$$\frac{2(ef-dg)\int -\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e}+\frac{2\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e}$$

$$\Big\downarrow \; 25$$

$$\frac{2\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e}-\frac{2(ef-dg)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e}$$

$$\Big\downarrow \; 1416$$

$$\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}e\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}-\frac{2(ef-dg)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e}$$

$$\Big\downarrow \; 1540$$

$$\frac{2(ef-dg)\left(\frac{e\sqrt{ag^2+cf^2}\left(\sqrt{c}(ef-dg)-e\sqrt{ag^2+cf^2}\right)\int \frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g(ae^2g+cd(2ef-dg))}-\frac{\sqrt{c}\left(-\sqrt{c}\sqrt{ag^2+cf^2}(ef\cdots\right.}{\cdots}\right.}{e}$$

$$+\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}e\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}$$

$$\Big\downarrow \; 1416$$

3.640. $\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx$

$$2(ef-dg)\left(\frac{e\sqrt{ag^2+cf^2}\left(\sqrt{c}(ef-dg)-e\sqrt{ag^2+cf^2}\right)\int \frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g(ae^2g+cd(2ef-dg))}-\frac{\sqrt[4]{c}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\ldots}}{\ldots}\right.$$

$$\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}e\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}$$

e

$\Big\downarrow$ 2222

$$\frac{\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}e\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}$$

$$2(ef-dg)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\left(\frac{\left(e+\frac{\sqrt{c}(ef-dg)}{\sqrt{cf^2+ag^2}}\right)\text{arctanh}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{e}\sqrt{ef-dg}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}\right)}{2\sqrt{e}\sqrt{cd^2+ae^2}\sqrt{ef-dg}}-\frac{\left(\frac{\sqrt{c}}{e}-\frac{\sqrt{cf\ldots}}{ef\ldots}\right.}{\ldots}\right.}{g(\ldots}\right.$$

input
```
Int[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + c*x^2]),x]
```

3.640. $\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx$

output
```
((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[
(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f
^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*Arc
Tan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[
c*f^2 + a*g^2])/2])/(c^(1/4)*e*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^
2 + (c*(f + g*x)^2)/g^2]) + (2*(e*f - d*g)*(-1/2*(c^(1/4)*(c*e*f^2 + a*e*g
^2 - Sqrt[c]*(e*f - d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f + g*x))/Sqr
t[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f +
g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^
2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)],
 (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(g*(c*f^2 + a*g^2)^(1/4)*(a*e^2
*g + c*d*(2*e*f - d*g))*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*
(f + g*x)^2)/g^2]) + (e*Sqrt[c*f^2 + a*g^2]*(Sqrt[c]*(e*f - d*g) - e*Sqrt[
c*f^2 + a*g^2])*(((e + (Sqrt[c]*(e*f - d*g))/Sqrt[c*f^2 + a*g^2])*ArcTanh[
(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[e]*Sqrt[e*f - d*g]*Sqrt[a + (c*f
^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])])/(2*Sqrt[e]*Sqrt[
c*d^2 + a*e^2]*Sqrt[e*f - d*g]) - ((Sqrt[c]/e - Sqrt[c*f^2 + a*g^2]/(e*f -
 d*g))*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2
 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (S
qrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticPi[(Sqrt[c]*(e*f - d...
```

3.640.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 510
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2/d   Subst[Int[1/Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2
)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 736
```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Simp[g/e   Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
 Simp[(e*f - d*g)/e   Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x],
x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

3.640.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.38

method	result
default	$\frac{2\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}}\,\sqrt{\frac{(-cx+\sqrt{-ac})g}{g\sqrt{-ac}+cf}}\,\sqrt{\frac{(cx+\sqrt{-ac})g}{g\sqrt{-ac}-cf}}\left(fF\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)c-\sqrt{-ac}\,F\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cj}}\right.\right.}{ec(cg\,x^3+cj}$
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(\frac{2g\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{e\sqrt{cg\,x^3+cf\,x^2+agx+fa}}-\frac{2(dg-ef)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{}}{}\right.}{\sqrt{gx+f}\,\sqrt{c\,x^2+a}}$

input
```
int((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

3.640. $\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx$

output
```
2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((
-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*
(-a*c)^(1/2)-c*f))^(1/2)*(f*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1
/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c-(-a*c)^(1/2)*Ell
ipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(
-a*c)^(1/2)+c*f))^(1/2))*g-EllipticPi((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1
/2),(g*(-a*c)^(1/2)-c*f)*e/c/(d*g-e*f),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1
/2)+c*f))^(1/2))*c*f+EllipticPi((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g
*(-a*c)^(1/2)-c*f)*e/c/(d*g-e*f),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*
f))^(1/2))*(-a*c)^(1/2)*g)/e/c/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

3.640.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.640.6 Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx=\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

input
```
integrate((g*x+f)**(1/2)/(e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(f + g*x)/(sqrt(a + c*x**2)*(d + e*x)), x)
```

3.640.7 Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx=\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

3.640.8 Giac [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx=\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

3.640.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx=\int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+a}\,(d+e\,x)}\,dx$$

input
```
int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.641 $\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

3.641.1 Optimal result

Integrand size = 28, antiderivative size = 698

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(cd^2+ae^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$+\frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(cd^2+ae^2)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$-\frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e(cd^2+ae^2)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$-\frac{(ae^2g+cd(2ef-dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
-e*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(e*x+d)-EllipticE(1/2*(1-x*
c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/
2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/(a*e^2+c*d^2)/(c*x^
2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+f*EllipticF(1/
2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2
)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(
1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-d*g*Ell
ipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/
2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/
(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/(a*e^2+c*d^2)/(g*x+f)^(1/2)/(c*x^2+a)^(1
/2)-(a*e^2*g+c*d*(-d*g+2*e*f))*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/
2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2
)+f*c^(1/2)))^(1/2))*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^
(1/2)))^(1/2)/e/(a*e^2+c*d^2)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^
2+a)^(1/2)
```

3.641.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.59 (sec) , antiderivative size = 1330, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(-\frac{e^2(a+cx^2)}{d+ex} - \frac{-ce^2f^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+cdef^2g\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}-ae^2fg^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+adeg^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+2ce^2f^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+g\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

output
```
(Sqrt[f + g*x]*(-((e^2*(a + c*x^2))/(d + e*x)) - (-(c*e^2*f^3*Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]]) + c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - a*e
^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]] + 2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 2*
c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - c*e^2*f*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c
]]*(f + g*x)^2 + Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d*g)*Sqr
t[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] -
 g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt
[a]*g)] + e*(I*Sqrt[c]*d + Sqrt[a]*e)*g*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*
((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)
/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/S
qrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g
)] - (2*I)*c*d*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(
((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt
[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqr
t[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*
Sqrt[a]*g)] + I*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sq
rt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Elliptic...
```

3.641.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1567 vs. $2(698) = 1396$.

Time = 2.70 (sec) , antiderivative size = 1567, normalized size of antiderivative = 2.24, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {737, 25, 2349, 599, 27, 729, 25, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}(d+ex)^2}\,dx$$

$$\Big\downarrow \text{737}$$

$$-\frac{\int -\frac{cegx^2+2cdgx+2cdf+aeg}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)}-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{cegx^2+2cdgx+2cdf+aeg}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2\left(ae^2+cd^2\right)}-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{2349}$$

$$\frac{\left(aeg+\frac{cd(2ef-dg)}{e}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{cxg+\frac{cdg}{e}}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2\left(ae^2+cd^2\right)}-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{599}$$

$$\frac{\left(aeg+\frac{cd(2ef-dg)}{e}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2\int \frac{cg(ef-dg-e(f+gx))}{e\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2\left(ae^2+cd^2\right)}-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(aeg+\frac{cd(2ef-dg)}{e}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{eg}}{2\left(ae^2+cd^2\right)}-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{729}$$

$$\frac{2\left(aeg+\frac{cd(2ef-dg)}{e}\right)\int -\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{eg}}{2\left(ae^2+cd^2\right)}-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{-2\left(aeg+\frac{cd(2ef-dg)}{e}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{eg}}{2\left(ae^2+cd^2\right)}-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{1511}$$

3.641. $\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

$$\frac{-2\left(aeg+\frac{cd(2ef-dg)}{e}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\left(\frac{e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}\cdots}}}{\sqrt{c}}\cdots\right)}{\cdots}}{2\left(ae^2+cd^2\right)\cdots}$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 1416

$$\frac{-\frac{2c\left(\frac{e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(dg-e\left(f-\frac{\sqrt{ag^2+c\cdots}}{\sqrt{c}}\cdots\right.\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\cdots}}\cdots\right)}{eg}\cdots}{\cdots}$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 1509

$$\frac{-2\left(aeg+\frac{cd(2ef-dg)}{e}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\left(\frac{e\sqrt{ag^2+cf^2}\left(\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}\cdots\right.}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}}{\cdots}$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)}$$

3.641. $\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

$\downarrow$ 1540

$$\frac{2\left(aeg+\frac{cd(2ef-dg)}{e}\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\int \frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g(age^2+cd(2ef-dg))}-\frac{\sqrt{c}\left(cef^2+c\right.}{\cdots}\right.}{\cdots}$$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+a}}{(cd^2+ae^2)(d+ex)}$$

$\downarrow$ 1416

$$\frac{2\left(aeg+\frac{cd(2ef-dg)}{e}\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\int \frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g(age^2+cd(2ef-dg))}-\frac{\sqrt[4]{c}\left(cef^2+\right.}{\cdots}\right.}{\cdots}$$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+a}}{(cd^2+ae^2)(d+ex)}$$

$\downarrow$ 2222

3.641. $\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

$$\frac{2\left(aeg+\frac{cd(2ef-dg)}{e}\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\left(\frac{\left(e+\frac{\sqrt{c}(ef-dg)}{\sqrt{cf^2+ag^2}}\right)\operatorname{arctanh}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{e}\sqrt{ef-dg}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}\right)}{2\sqrt{e}\sqrt{cd^2+ae^2}\sqrt{ef-dg}}\right.}{}\right.}{}$$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+a}}{(cd^2+ae^2)\,(d+ex)}$$

input
```
Int[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

3.641. $\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

output
```
-((e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) + ((-2*c*
((e*Sqrt[c*f^2 + a*g^2]*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f
 + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f +
 g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f +
g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 +
 (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f
^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2
)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f
^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/Sqrt[c] - ((c*f^
2 + a*g^2)^(1/4)*(d*g - e*(f - Sqrt[c*f^2 + a*g^2]/Sqrt[c]))*(1 + (Sqrt[c]
*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))
/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sq
rt[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 +
 a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(2*c^(1/4)*Sqrt[
a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/(e*g) +
2*(a*e*g + (c*d*(2*e*f - d*g))/e)*(-1/2*(c^(1/4)*(c*e*f^2 + a*e*g^2 - Sqrt
[c]*(e*f - d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 +
 a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g
^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*E
llipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + ...
```

3.641.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

3.641. $\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

rule 737
```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^
2]/((m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/(2*(m + 1)*(c*d^2 + a*e^2))   In
t[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[2*c*d*f*(m + 1)
- e*(a*g) + 2*c*(d*g*(m + 1) - e*f*(m + 2))*x - c*e*g*(2*m + 5)*x^2, x], x]
, x] /; FreeQ[{a, c, d, e, f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.641. $\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

3.641.4 Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 922, normalized size of antiderivative = 1.32

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(-\frac{e\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{\left(e^2a+c\,d^2\right)(ex+d)}+\frac{cdg\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-a}}{c}}{-\frac{f}{g}-\frac{\sqrt{-a}}{c}}}\right.}{\left(e^2a+c\,d^2\right)e\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\right.}{}$
default	Expression too large to display

input
```
int((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-e/(a*e^2+c*d^2)*
(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)+c*d*g/(a*e^2+c*d^2)/e*(f/g-(-a*c
)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-
(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c
*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(
1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+c*g/(a*e^2+c*d^2
)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/
2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/
c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*Ellipti
cE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c
)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))
^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+(a*e^2*g-c*d^
2*g+2*c*d*e*f)/e^2/(a*e^2+c*d^2)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)
^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*
c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)
/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-a*c)^(
1/2)/c)/(-f/g+d/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))
```

3.641.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.641.6 Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

input
```
integrate((g*x+f)**(1/2)/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(f + g*x)/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.641.7 Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)^2}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^2), x)
```

3.641.8 Giac [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)^2}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^2), x)
```

3.641.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2}\,dx$$

input
```
int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

output
```
int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.642 $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx$

3.642.1 Optimal result

Integrand size = 28, antiderivative size = 1246

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx$$
$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2}-\frac{e(ae^2g+cd(6ef-5dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)}$$
$$-\frac{\sqrt{-a}\sqrt{c}(ae^2g+cd(6ef-5dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4\left(cd^2+ae^2\right)^2(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$+\frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e\left(cd^2+ae^2\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$+\frac{\sqrt{-a}\sqrt{c}f(ae^2g+cd(6ef-5dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4\left(cd^2+ae^2\right)^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$-\frac{\sqrt{-a}\sqrt{c}dg(ae^2g+cd(6ef-5dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e\left(cd^2+ae^2\right)^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$+\frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\left(cd^2+ae^2\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$+\frac{(ae^2g+cd(6ef-5dg))(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-}}{\sqrt{2}}\right.\right.}{4e\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\left(cd^2+ae^2\right)^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
-1/2*e*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(e*x+d)^2-1/4*e*(a*e^2*
g+c*d*(-5*d*g+6*e*f))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)^2/(-d*g+
e*f)/(e*x+d)-1/4*(a*e^2*g+c*d*(-5*d*g+6*e*f))*EllipticE(1/2*(1-x*c^(1/2)/(
-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^
(1/2)*c^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/(a*e^2+c*d^2)^2/(-d*g+e*f)/(
c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+1/2*g*Elli
pticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2
)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(
g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/(a*e^2+c*d^2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/
2)+1/4*f*(a*e^2*g+c*d*(-5*d*g+6*e*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2
))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^
(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(
a*e^2+c*d^2)^2/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-1/4*d*g*(a*e^2*g+c
*d*(-5*d*g+6*e*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-
2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(
1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/(a*e^2+c*d^2)^2/(-
d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+c*(-3*d*g+e*f)*EllipticPi(1/2*(1-x*
c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*
(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(
1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/(a*e^2+c*d^2)/(e+d*c^(1/2)/(-a)^...
```

3.642.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.04 (sec) , antiderivative size = 2450, normalized size of antiderivative = 1.97

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + c*x^2]),x]
```

output
```
(-11*c^2*d^2*e^2*f^3 + a*c*e^4*f^3 + (6*c^2*d*e^3*f^4)/g + 5*c^2*d^3*e*f^2
*g + 5*a*c*d*e^3*f^2*g - 11*a*c*d^2*e^2*f*g^2 + a^2*e^4*f*g^2 + 5*a*c*d^3*
e*g^3 - a^2*d*e^3*g^3 + 22*c^2*d^2*e^2*f^2*(f + g*x) - 2*a*c*e^4*f^2*(f +
g*x) - (12*c^2*d*e^3*f^3*(f + g*x))/g - 10*c^2*d^3*e*f*g*(f + g*x) + 2*a*c
*d*e^3*f*g*(f + g*x) - 11*c^2*d^2*e^2*f*(f + g*x)^2 + a*c*e^4*f*(f + g*x)^
2 + (6*c^2*d*e^3*f^2*(f + g*x)^2)/g + 5*c^2*d^3*e*g*(f + g*x)^2 - a*c*d*e^
3*g*(f + g*x)^2 - (e^2*(e*f - d*g)*(f + g*x)*(a + c*x^2)*(2*(c*d^2 + a*e^2
)*(e*f - d*g) + (a*e^2*g + c*d*(6*e*f - 5*d*g))*(d + e*x)))/(d + e*x)^2 +
(Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(e*f - d*g)*(a*e^2*g + c*d*(6*e*f
- 5*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]
*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[
c]*f + I*Sqrt[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) + (e*(I*Sqrt[c]
*d + Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g)*(a*e^2*g + (2*I)*Sqrt[a]*Sqrt[c]
*e*(e*f - d*g) + c*d*(-4*e*f + 5*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/
(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2
)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sq
rt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)
/Sqrt[c]] + ((8*I)*c^2*d^2*e^2*f^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f +
 g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)...
```

3.642.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2782 vs. 2(1246) = 2492.

Time = 5.15 (sec) , antiderivative size = 2782, normalized size of antiderivative = 2.23, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {737, 25, 2349, 734, 2349, 25, 27, 510, 599, 25, 27, 729, 25, 1416, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}(d+ex)^3}\,dx$$

$$\Big\downarrow \text{737}$$

$$-\frac{\int -\frac{-cegx^2-2c(ef-2dg)x+4cdf+aeg}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4\left(ae^2+cd^2\right)}-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-cegx^2-2c(ef-2dg)x+4cdf+aeg}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4\left(ae^2+cd^2\right)}-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{ 2349}$$

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-2cf+\frac{5cdg}{e}-cgx}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4\left(ae^2+cd^2\right)}-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{ 734}$$

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{\int \frac{-cgx^2e^2+age^2-2cdgxe-2cd(ef-dg)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)+\int \frac{-2cf+\frac{5cdg}{e}-cgx}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4\left(ae^2+cd^2\right)}-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{ 2349}$$

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)-2c\Big(f-}{4\left(ae^2+cd^2\right)}$$
$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)-2c\Big(f-}{4\left(ae^2+cd^2\right)}$$
$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)-2c\Big(f-}{4\left(ae^2+cd^2\right)}$$
$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{ 510}$$

3.642. $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx$

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int\frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)-2c\Big(f-}{4\left(ae^2+cd^2\right)}$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$\downarrow$ 599

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2\int-\frac{cg(ef-dg-e(f+gx))}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{}}{(d+ex)(ae^2+cd}\right.}{4\left(ae^2+cd^2\right)}$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$\downarrow$ 25

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\frac{2\int\frac{cg(ef-dg-e(f+gx))}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{}}{(d+ex)(ae^2+cd^2}\right.}{4\left(ae^2+cd^2\right)}$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$\downarrow$ 27

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\frac{2c\int\frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{}}{(d+ex)(ae^2+cd}\right.}{4\left(ae^2+cd^2\right)}$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$\downarrow$ 729

3.642. $\int\frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx$

$$\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{2\left(ae^2g-cd(2ef-3dg)\right)\int -\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}+\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+\cdots}{\cdots}}}\cdots}{g}}{2(ae^2+cd^2)(ef-dg)}\cdots\right.$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$\downarrow$ 25

$$\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}-2\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx\cdots}{g^2}}}\cdots}{2(ae^2+cd^2)(ef-dg)}\cdots\right.$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$\downarrow$ 1416

$$\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}-2\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx\cdots}{g^2}}}\cdots}{2(ae^2+cd^2)(ef-dg)}\cdots\right.$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$\downarrow$ 1511

3.642. $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx$

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{\frac{2c\left(\frac{e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\left(dg-e\left(f-\frac{\sqrt{ag^2+cf^2}}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right.}{g}}{2(ae^2+cd^2)(ef-dg}\right.}{}$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$\downarrow$ 1416

$$-\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}+4c\Big(f$$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+a}}{2\left(cd^2+ae^2\right)(d+ex)^2}$$

$\downarrow$ 1509

3.642. $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx$

$$-\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}+4c\Big(f$$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+a}}{2\left(cd^2+ae^2\right)(d+ex)^2}$$

$$\downarrow 1540$$

3.642. $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx$

$$-\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-4c\Big(f$$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+a}}{2\left(cd^2+ae^2\right)(d+ex)^2}$$

$\downarrow$ 1416

3.642. $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx$

$$-\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-4c\Big(f$$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+a}}{2\left(cd^2+ae^2\right)(d+ex)^2}$$

$$\Big\downarrow \text{ 2222}$$

3.642. $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx$

$$-\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-4c\Big(f$$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+a}}{2\left(cd^2+ae^2\right)(d+ex)^2}$$

input
```
Int[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + c*x^2]),x]
```

3.642. $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx$

output
```
-1/2*(e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)^2) + (-(
(c^(3/4)*(c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2
])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((
a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*Ellipti
cF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*
f)/Sqrt[c*f^2 + a*g^2])/2])/(e*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^
2 + (c*(f + g*x)^2)/g^2])) - 4*c*(f - (3*d*g)/e)*(-1/2*(c^(1/4)*(c*e*f^2 +
 a*e*g^2 - Sqrt[c]*(e*f - d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f + g*x
))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c
*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2
+ a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(
1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(g*(c*f^2 + a*g^2)^(1/4)*
(a*e^2*g + c*d*(2*e*f - d*g))*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2
 + (c*(f + g*x)^2)/g^2]) + (e*Sqrt[c*f^2 + a*g^2]*(Sqrt[c]*(e*f - d*g) - e
*Sqrt[c*f^2 + a*g^2])*(((e + (Sqrt[c]*(e*f - d*g))/Sqrt[c*f^2 + a*g^2])*Ar
cTanh[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[e]*Sqrt[e*f - d*g]*Sqrt[a
+ (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])])/(2*Sqrt[e]
*Sqrt[c*d^2 + a*e^2]*Sqrt[e*f - d*g]) - ((Sqrt[c]/e - Sqrt[c*f^2 + a*g^2]/
(e*f - d*g))*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^
2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2...
```

3.642.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 510
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2/d   Subst[Int[1/Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2
)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

3.642. $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx$

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 734
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2))), x] + Simp[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2))   Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^
2]))*Simp[2*c*d*(e*f - d*g)*(m + 1) - a*e^2*g*(2*m + 3) + 2*c*e*(d*g*(m + 1
) - e*f*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 737
```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^
2]/((m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/(2*(m + 1)*(c*d^2 + a*e^2))   In
t[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[2*c*d*f*(m + 1)
- e*(a*g) + 2*c*(d*g*(m + 1) - e*f*(m + 2))*x - c*e*g*(2*m + 5)*x^2, x], x]
, x] /; FreeQ[{a, c, d, e, f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.642. $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx$

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

3.642.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 1224, normalized size of antiderivative = 0.98

method	result	size
elliptic	Expression too large to display	1224
default	Expression too large to display	20359

input
```
int((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-1/2*e/(a*e^2+c*d
^2)*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)^2+1/4*e*(a*e^2*g-5*c*d^2*g+6
*c*d*e*f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/(a*e^2+c*d^2)*(c*g*x^3+c*f
*x^2+a*g*x+a*f)^(1/2)/(e*x+d)-1/4*c*g*(3*a*d*e^2*g-2*a*e^3*f-3*c*d^3*g+4*c
*d^2*e*f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/(a*e^2+c*d^2)/e*(f/g-(-a*c
)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-
(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c
*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(
1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))-1/4*c*g*(a*e^2*g
-5*c*d^2*g+6*c*d*e*f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/(a*e^2+c*d^2)*
(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)
/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c)
)^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE
(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^
(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(
1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+1/4*(a^2*e^4*g^
2+10*a*c*d^2*e^2*g^2-12*a*c*d*e^3*f*g+4*a*c*e^4*f^2-3*c^2*d^4*g^2+12*c^2*d
^3*e*f*g-8*c^2*d^2*e^2*f^2)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/(a*e^2+c
*d^2)/e^2*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-
a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-...
```

3.642.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.642.6 Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}\,(d+ex)^3}\,dx$$

input
```
integrate((g*x+f)**(1/2)/(e*x+d)**3/(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(f + g*x)/(sqrt(a + c*x**2)*(d + e*x)**3), x)
```

3.642.7 Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)^3}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^3), x)
```

3.642.8 Giac [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)^3}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^3), x)
```

3.642.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx=\int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^3}\,dx$$

input
```
int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)^3),x)
```

output
```
int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)^3), x)
```

3.643 $\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx$

3.643.1 Optimal result

Integrand size = 28, antiderivative size = 600

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx=\frac{2g^2\sqrt{f+gx}\sqrt{a+cx^2}}{3ce}$$
$$-\frac{2\sqrt{-a}g(7ef-3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1+\frac{a\sqrt{c}x}{(-a)^{3/2}}}}{\sqrt{2}}\right)|\frac{2ag}{-\sqrt{-a}\sqrt{c}f+ag}\right)}{3\sqrt{c}e^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$+\frac{2\sqrt{-a}g(ae^2g^2+c(-2e^2f^2+6defg-3d^2g^2))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{a\sqrt{c}x}{(-a)^{3/2}}}}{\sqrt{2}}\right),\frac{}{-\sqrt{-}}\right.}{3c^{3/2}e^3\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$-\frac{2(ef-dg)^2\sqrt{\frac{g(\sqrt{-a}-\sqrt{c}x)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{-\frac{g(\sqrt{-a}+\sqrt{c}x)}{\sqrt{c}f-\sqrt{-a}g}}\,\mathrm{EllipticPi}\left(\frac{e\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}{ef-dg},\arcsin\left(\sqrt{\frac{c}{cf+\sqrt{-a}\sqrt{c}g}}\sqrt{f+gx}\right),\frac{\sqrt{c}f+\sqrt{-a}}{\sqrt{c}f-\sqrt{-a}}\right.}{e^3\sqrt{\frac{c}{cf+\sqrt{-a}\sqrt{c}g}}\sqrt{a+cx^2}}$$

output
```
2/3*g^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/e-2/3*g*(-3*d*g+7*e*f)*EllipticE(1
/2*(1+a*x*c^(1/2)/(-a)^(3/2))^(1/2)*2^(1/2),2^(1/2)*(a*g/(a*g-f*(-a)^(1/2)
*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/e^2/c^(1/2)/(
c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+2/3*g*(a*e
^2*g^2+c*(-3*d^2*g^2+6*d*e*f*g-2*e^2*f^2))*EllipticF(1/2*(1+a*x*c^(1/2)/(-
a)^(3/2))^(1/2)*2^(1/2),2^(1/2)*(a*g/(a*g-f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-
a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2
)/c^(3/2)/e^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-2*(-d*g+e*f)^2*EllipticPi((g*x
+f)^(1/2)*(c/(c*f+g*(-a)^(1/2)*c^(1/2)))^(1/2),e*(f+g*(-a)^(1/2)/c^(1/2))/
(-d*g+e*f),((g*(-a)^(1/2)+f*c^(1/2))/(-g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(g*
((-a)^(1/2)-x*c^(1/2))/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)*(-g*((-a)^(1/2)+x*c
^(1/2))/(-g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^3/(c*x^2+a)^(1/2)/(c/(c*f+g*(-a
)^(1/2)*c^(1/2)))^(1/2)
```

3.643.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.46 (sec) , antiderivative size = 1440, normalized size of antiderivative = 2.40

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{2g^2\sqrt{f+gx}\sqrt{a+cx^2}}{3ce}$$

$$+\frac{2(f+gx)^{3/2}\left(7ce^2f\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}-3cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+\frac{7ce^2f^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{(f+gx)^2}-\frac{3cdef^2g\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{(f+gx)^2}+\frac{7ae^2fg^2\sqrt{-f-}}{(f+gx)^2}\right.}{}$$

input
```
Integrate[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
(2*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c*e) + (2*(f + g*x)^(3/2)*(7*c*e^
2*f*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/S
qrt[c]] + (7*c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (3*
c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (7*a*e^2*f*g^2
*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (3*a*d*e*g^3*Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (14*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/
Sqrt[c]])/(f + g*x) + (6*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f +
g*x) + (Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(7*e*f - 3*d*g)*Sqrt[1 - f/
(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*S
qrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)
/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]
*g)])/Sqrt[f + g*x] + (I*e*(Sqrt[c]*f + I*Sqrt[a]*g)*(I*Sqrt[a]*e*g + Sqrt
[c]*(6*e*f - 3*d*g))*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*
x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticF[I
*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*S
qrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + ((3*I)*c*e^2*f^2*Sqr
t[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x
) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e
*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sq
rt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqr...
```

3.643.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1848 vs. $2(600) = 1200$.

Time = 4.03 (sec) , antiderivative size = 1848, normalized size of antiderivative = 3.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {740, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^{5/2}}{\sqrt{a+cx^2}(d+ex)}\,dx$$

$$\Big\downarrow \text{740}$$

$$\int \left(\frac{(ef-dg)^3}{e^3\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}+\frac{g(ef-dg)^2}{e^3\sqrt{a+cx^2}\sqrt{f+gx}}+\frac{g\sqrt{f+gx}(ef-dg)}{e^2\sqrt{a+cx^2}}+\frac{g(f+gx)^{3/2}}{e\sqrt{a+cx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{c}\sqrt[4]{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\arctan\left(\frac{}{\sqrt[4]{}}\right.\right.}{e^3g\left(age^2+cd(2ef-dg)\right)\sqrt{cx^2+a}}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{e}\sqrt{ef-dg}\sqrt{cx^2+a}}\right)(ef-dg)^{5/2}}{e^{5/2}\sqrt{cd^2+ae^2}}+$$

$$\frac{\sqrt[4]{cf^2+ag^2}\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)(e}{\sqrt[4]{c}e^3\sqrt{cx^2+a}}$$

$$\frac{\sqrt[4]{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)^2\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticPi}\left(\frac{\left(\sqrt{cf^2+ag^2}e+\sqrt{}\right.}{4\sqrt{c}e(ef-dg)\sqrt{}}\right.}{2\sqrt[4]{c}e^3g\left(age^2+cd(2ef-dg)\right)\sqrt{cx^2+a}}$$

$$\frac{2\left(cf^2+ag^2\right)^{3/4}\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)(ef-}{c^{3/4}e^2\sqrt{cx^2+a}}$$

$$\frac{\left(cf^2+ag^2\right)^{3/4}\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right.}{c^{3/4}e^2\sqrt{cx^2+a}}$$

$$\frac{2g^2\sqrt{f+gx}\sqrt{cx^2+a}(ef-dg)}{\sqrt{c}e^2\sqrt{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)}-$$

$$\frac{8f\left(cf^2+ag^2\right)^{3/4}\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{3c^{3/4}e\sqrt{cx^2+a}}-$$

$$\frac{\sqrt[4]{cf^2+ag^2}\left(cf^2-4\sqrt{c}\sqrt{cf^2+ag^2}f+ag^2\right)\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{}}{\sqrt[4]{c}}\right.\right.}{3c^{5/4}e\sqrt{cx^2+a}}$$

$$\frac{2g^2\sqrt{f+gx}\sqrt{cx^2+a}}{3ce}+\frac{8fg^2\sqrt{f+gx}\sqrt{cx^2+a}}{3\sqrt{c}e\sqrt{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)}$$

input
```
Int[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

3.643. $\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx$

output
```
(2*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c*e) + (8*f*g^2*Sqrt[f + g*x]*Sqr
t[a + c*x^2])/(3*Sqrt[c]*e*Sqrt[c*f^2 + a*g^2]*(1 + (Sqrt[c]*(f + g*x))/Sq
rt[c*f^2 + a*g^2])) + (2*g^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(S
qrt[c]*e^2*Sqrt[c*f^2 + a*g^2]*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2
])) - ((e*f - d*g)^(5/2)*ArcTanh[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt
[e]*Sqrt[e*f - d*g]*Sqrt[a + c*x^2])]])/(e^(5/2)*Sqrt[c*d^2 + a*e^2]) - (8*
f*(c*f^2 + a*g^2)^(3/4)*Sqrt[(g^2*(a + c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt
[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^
2 + a*g^2])*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/
4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(3*c^(3/4)*e*Sqrt[a + c*x^2
]) - (2*(e*f - d*g)*(c*f^2 + a*g^2)^(3/4)*Sqrt[(g^2*(a + c*x^2))/((c*f^2 +
 a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*(1 + (Sqrt[c]*(f
 + g*x))/Sqrt[c*f^2 + a*g^2])*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(
c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(3/4)*
e^2*Sqrt[a + c*x^2]) + ((e*f - d*g)^2*(c*f^2 + a*g^2)^(1/4)*Sqrt[(g^2*(a +
 c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)
]*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*EllipticF[2*ArcTan[(c^(1/4
)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g
^2])/2])/(c^(1/4)*e^3*Sqrt[a + c*x^2]) + ((e*f - d*g)*(c*f^2 + a*g^2)^(3/4
)*Sqrt[(g^2*(a + c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt...
```

3.643.3.1 Defintions of rubi rules used

rule 740
```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && Inte
gerQ[n + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.643. $\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx$

3.643.4 Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 948, normalized size of antiderivative = 1.58

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(\frac{2g^2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{3ec}+\frac{2\left(\frac{g\left(d^2g^2-3defg+3e^2f^2\right)}{e^3}-\frac{g^3a}{3ec}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-a}}{c}}}}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/3/e*g^2/c*(c*g*
x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2*(g*(d^2*g^2-3*d*e*f*g+3*e^2*f^2)/e^3-1/3/e*
g^3/c*a)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a
*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)
^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g
-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2
))+2*(-g^2/e^2*(d*g-3*e*f)-2/3/e*g^2*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g
-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*(
(x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f
)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1
/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*El
lipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-
(-a*c)^(1/2)/c))^(1/2)))-2*(d^3*g^3-3*d^2*e*f*g^2+3*d*e^2*f^2*g-e^3*f^3)/e
^4*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1
/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)
/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g
)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-a*c)^(1/2)/c)/(-f/g+d/e),((-f/g+(-a*
c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))
```

3.643.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.643.6 Sympy [F]

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(f+gx)^{\frac{5}{2}}}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

input
```
integrate((g*x+f)**(5/2)/(e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Integral((f + g*x)**(5/2)/(sqrt(a + c*x**2)*(d + e*x)), x)
```

3.643.7 Maxima [F]

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(gx+f)^{\frac{5}{2}}}{\sqrt{cx^2+a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

3.643.8 Giac [F]

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(gx+f)^{\frac{5}{2}}}{\sqrt{cx^2+a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

3.643.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(f+g\,x)^{5/2}}{\sqrt{c\,x^2+a}\,(d+e\,x)}\,dx$$

input
```
int((f + g*x)^(5/2)/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int((f + g*x)^(5/2)/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.644 $\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx$

3.644.1 Optimal result

Integrand size = 28, antiderivative size = 469

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = -\frac{2\sqrt{-a}g\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$-\frac{2\sqrt{-a}g(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e^2\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$-\frac{2(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^2\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
-2*g*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*
(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/e/c
^(1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-2*
g*(-d*g+e*f)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/
(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*
c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a
)^(1/2)-2*(-d*g+e*f)^2*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/
2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1
/2)))^(1/2))*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^
(1/2)/e^2/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

3.644.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.87 (sec) , antiderivative size = 927, normalized size of antiderivative = 1.98

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-i\sqrt{a}g}}\left(\frac{2i\sqrt{a}fg\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}g}{i\sqrt{c}f+\sqrt{a}g}\right)}{\sqrt{c}e} - \frac{i\sqrt{a}dg^2\sqrt{1+\frac{cx^2}{a}}\,\text{Ellipti}}{}\right.}{}$$

input
```
Integrate[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
(2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f - I*Sqrt[a]*g)]*(((2*I)*Sqrt[a]*f*g
*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (I*Sqrt[c]*x)/Sqrt[a]]/Sqrt
[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f + Sqrt[a]*g)])/(Sqrt[c]*e) - (I*Sqrt[a]*d
*g^2*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (I*Sqrt[c]*x)/Sqrt[a]]/
Sqrt[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f + Sqrt[a]*g)])/(Sqrt[c]*e^2) + (g*Sqr
t[(g*(Sqrt[a] + I*Sqrt[c]*x))/((-I)*Sqrt[c]*f + Sqrt[a]*g)]*(I*Sqrt[a] + S
qrt[c]*x)*((Sqrt[c]*f + I*Sqrt[a]*g)*EllipticE[ArcSin[Sqrt[(Sqrt[c]*(f + g
*x))/(Sqrt[c]*f - I*Sqrt[a]*g)]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I
*Sqrt[a]*g)] - I*Sqrt[a]*g*EllipticF[ArcSin[Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt
[c]*f - I*Sqrt[a]*g)]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g
)]))/(c*e*Sqrt[(g*(Sqrt[a] - I*Sqrt[c]*x))/(I*Sqrt[c]*f + Sqrt[a]*g)]) - (
Sqrt[a]*f^2*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*Sqrt[a]*e)/(I*Sqrt[c]*d + Sq
rt[a]*e), ArcSin[Sqrt[1 - (I*Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*g)/(
I*Sqrt[c]*f + Sqrt[a]*g)])/(I*Sqrt[c]*d + Sqrt[a]*e) + (2*Sqrt[a]*d*f*g*Sq
rt[1 + (c*x^2)/a]*EllipticPi[(2*Sqrt[a]*e)/(I*Sqrt[c]*d + Sqrt[a]*e), ArcS
in[Sqrt[1 - (I*Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f +
Sqrt[a]*g)])/(I*Sqrt[c]*d*e + Sqrt[a]*e^2) - (Sqrt[a]*d^2*g^2*Sqrt[1 + (c*
x^2)/a]*EllipticPi[(2*Sqrt[a]*e)/(I*Sqrt[c]*d + Sqrt[a]*e), ArcSin[Sqrt[1
- (I*Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f + Sqrt[a]*g)
])/(e^2*(I*Sqrt[c]*d + Sqrt[a]*e))))/(Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

3.644.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1299 vs. $2(469) = 938$.

Time = 2.84 (sec) , antiderivative size = 1299, normalized size of antiderivative = 2.77, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {740, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^{3/2}}{\sqrt{a+cx^2}(d+ex)}\,dx$$

$$\Big\downarrow \text{740}$$

$$\int \left(\frac{(ef-dg)^2}{e^2\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}+\frac{g(ef-dg)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}}+\frac{g\sqrt{f+gx}}{e\sqrt{a+cx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+a}g^2}{\sqrt{c}e\sqrt{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)}-\frac{(ef-dg)^{3/2}\text{arctanh}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{e}\sqrt{ef-dg}\sqrt{cx^2+a}}\right)}{e^{3/2}\sqrt{cd^2+ae^2}}-$$

$$\frac{2\left(cf^2+ag^2\right)^{3/4}\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{c^{3/4}e\sqrt{cx^2+a}}+$$

$$\frac{\left(cf^2+ag^2\right)^{3/4}\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right.}{c^{3/4}e\sqrt{cx^2+a}}$$

$$\frac{(ef-dg)\sqrt[4]{cf^2+ag^2}\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}\right.\right.}{\sqrt[4]{c}e^2\sqrt{cx^2+a}}$$

$$\frac{\sqrt[4]{c}(ef-dg)^2\sqrt[4]{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\right.}{e^2\left(age^2+cd(2ef-dg)\right)\sqrt{cx^2+ag}}$$

$$\frac{(ef-dg)\sqrt[4]{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)^2\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticPi}\left(\frac{\left(\sqrt{c}\right.}{4\sqrt{}}\right.}{2\sqrt[4]{c}e^2\left(age^2+cd(2ef-dg)\right)\sqrt{cx^2+ag}}$$

input
```
Int[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

3.644. $\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx$

output
```
(2*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(Sqrt[c]*e*Sqrt[c*f^2 + a*g^2]*(1 +
(Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])) - ((e*f - d*g)^(3/2)*ArcTanh[(Sq
rt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[e]*Sqrt[e*f - d*g]*Sqrt[a + c*x^2])
])/(e^(3/2)*Sqrt[c*d^2 + a*e^2]) - (2*(c*f^2 + a*g^2)^(3/4)*Sqrt[(g^2*(a +
 c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)
]*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*EllipticE[2*ArcTan[(c^(1/4
)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g
^2])/2])/(c^(3/4)*e*Sqrt[a + c*x^2]) + ((e*f - d*g)*(c*f^2 + a*g^2)^(1/4)*
Sqrt[(g^2*(a + c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^
2 + a*g^2])^2)]*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*EllipticF[2*
ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sq
rt[c*f^2 + a*g^2])/2])/(c^(1/4)*e^2*Sqrt[a + c*x^2]) + ((c*f^2 + a*g^2)^(3
/4)*Sqrt[(g^2*(a + c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[
c*f^2 + a*g^2])^2)]*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Elliptic
F[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f
)/Sqrt[c*f^2 + a*g^2])/2])/(c^(3/4)*e*Sqrt[a + c*x^2]) + (c^(1/4)*(e*f - d
*g)^2*(c*f^2 + a*g^2)^(1/4)*(Sqrt[c]*(e*f - d*g) - e*Sqrt[c*f^2 + a*g^2])*
Sqrt[(g^2*(a + c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^
2 + a*g^2])^2)]*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*EllipticF[2*
ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)...
```

3.644.3.1 Defintions of rubi rules used

rule 740
```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && Inte
gerQ[n + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.644. $\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx$

3.644.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(382) = 764$.

Time = 1.37 (sec) , antiderivative size = 852, normalized size of antiderivative = 1.82

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(-\frac{2g(dg-2ef)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{e^2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}+\frac{2g^2\left(\frac{f}{g}-\frac{\sqrt{-\cdots}}{c}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
default	$\frac{2\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}}\,\sqrt{\frac{(-cx+\sqrt{-ac})g}{g\sqrt{-ac}+cf}}\,\sqrt{\frac{(cx+\sqrt{-ac})g}{g\sqrt{-ac}-cf}}\left(\sqrt{-ac}\,F\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)d\,g^2-\sqrt{-ac}\,F\left(\sqrt{-\cdots}\right.\cdots\right.}{\cdots}$

input
```
int((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-2*g*(d*g-2*e*f)/
e^2*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(
1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2
)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*
c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*
g^2/e*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)
^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1
/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*Ell
ipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(
-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)
/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+2*(d^2*g^
2-2*d*e*f*g+e^2*f^2)/e^3*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c
))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)
/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d
/e)*EllipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-a*c)^(1/2)/c)/
(-f/g+d/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))
```

3.644.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.644.6 Sympy [F]

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(f+gx)^{\frac{3}{2}}}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

input
```
integrate((g*x+f)**(3/2)/(e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Integral((f + g*x)**(3/2)/(sqrt(a + c*x**2)*(d + e*x)), x)
```

3.644.7 Maxima [F]

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(gx+f)^{\frac{3}{2}}}{\sqrt{cx^2+a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

3.644.8 Giac [F]

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(gx+f)^{\frac{3}{2}}}{\sqrt{cx^2+a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

3.644.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(f+g\,x)^{3/2}}{\sqrt{c\,x^2+a}\,(d+e\,x)}\,dx$$

input
```
int((f + g*x)^(3/2)/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int((f + g*x)^(3/2)/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.645 $\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

3.645.1 Optimal result

Integrand size = 28, antiderivative size = 457

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$

$$= -\frac{8e^2(ef-3dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg^2}+\frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5cg}$$

$$+\frac{2\sqrt{-a}e(9ae^2g^2-c(8e^2f^2-30defg+45d^2g^2))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right.}{15c^{3/2}g^3\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$-\frac{2\sqrt{-a}(ae^2g^2(7ef-15dg)-c(8e^3f^3-30de^2f^2g+45d^2efg^2-15d^3g^3))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{Elliptic}}{15c^{3/2}g^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
-8/15*e^2*(-3*d*g+e*f)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^2+2/5*e^2*(e*x+d)
*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g+2/15*e*(9*a*e^2*g^2-c*(45*d^2*g^2-30*d*
e*f*g+8*e^2*f^2))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2
*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2
/a)^(1/2)/c^(3/2)/g^3/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(
1/2)))^(1/2)-2/15*(a*e^2*g^2*(-15*d*g+7*e*f)-c*(-15*d^3*g^3+45*d^2*e*f*g^2
-30*d*e^2*f^2*g+8*e^3*f^3))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2
^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a)^
(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^3/(g*x+f)
^(1/2)/(c*x^2+a)^(1/2)
```

3.645.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.72 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2e^2(-4ef+15dg+3egx)(a+cx^2)}{cg^2} + \frac{2(f+gx)\left(\frac{eg^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\left(-9ae^2g^2+c\left(8e^2f^2-30defg+45d^2g^2\right)\right)\left(a+cx^2\right)}{(f+gx)^2} + \frac{\sqrt{c}e(-i\sqrt{c}f+\sqrt{a}g)\left(\cdots\right.}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

input
```
Integrate[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
(Sqrt[f + g*x]*((2*e^2*(-4*e*f + 15*d*g + 3*e*g*x)*(a + c*x^2))/(c*g^2) +
(2*(f + g*x)*((e*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(-9*a*e^2*g^2 + c*(8
*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*(a + c*x^2))/(f + g*x)^2 + (Sqrt[c]*e
*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-9*a*e^2*g^2 + c*(8*e^2*f^2 - 30*d*e*f*g +
45*d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt
[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt
[a]*g)])/Sqrt[f + g*x] + (Sqrt[c]*g*((15*I)*c^(3/2)*d^3*g^2 + 9*a^(3/2)*e^
3*g^2 - I*a*Sqrt[c]*e^2*g*(2*e*f + 15*d*g) + Sqrt[a]*c*e*(-8*e^2*f^2 + 30*
d*e*f*g - 45*d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[
-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*EllipticF[I*ArcSinh[Sqrt[-f -
(I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*
f + I*Sqrt[a]*g)])/Sqrt[f + g*x]))/(c^2*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c
]])))/(15*Sqrt[a + c*x^2])
```

3.645.3 Rubi [A] (warning: unable to verify)

Time = 1.31 (sec) , antiderivative size = 831, normalized size of antiderivative = 1.82, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {728, 25, 2185, 27, 599, 25, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^2}\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{728}$$

$$\frac{2e^2\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5cg}-\frac{\int -\frac{5cgd^3-4ce^2(ef-3dg)x^2-ae^2(2ef+dg)-e\left(3age^2+cd(2ef-15dg)\right)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{5cg}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5cgd^3-4ce^2(ef-3dg)x^2-ae^2(2ef+dg)-e\left(3age^2+cd(2ef-15dg)\right)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{5cg}+\frac{2e^2\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5cg}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\frac{2\int \frac{cg\left(g\left(15cd^3g-ae^2(2ef+15dg)\right)-e\left(9ae^2g^2-c\left(8e^2f^2-30degf+45d^2g^2\right)\right)x\right)}{2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3cg^2}-\frac{8e^2\sqrt{a+cx^2}\sqrt{f+gx}(ef-3dg)}{3g}}{5cg}+\frac{2e^2\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5cg}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{g\left(15cd^3g-ae^2(2ef+15dg)\right)-e\left(9ae^2g^2-c\left(8e^2f^2-30degf+45d^2g^2\right)\right)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3g}-\frac{8e^2\sqrt{a+cx^2}\sqrt{f+gx}(ef-3dg)}{3g}}{5cg}+\frac{2e^2\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5cg}$$

$$\Big\downarrow \; 599$$

$$\frac{-\frac{2\int -\frac{ae^2(7ef-15dg)g^2-c\left(8e^3f^3-30de^2gf^2+45d^2eg^2f-15d^3g^3\right)-e\left(9ae^2g^2-c\left(8e^2f^2-30degf+45d^2g^2\right)\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3g^3}-\frac{8e^2\sqrt{a+cx^2}\sqrt{f+gx}(ef-}{3g}}{5cg}$$
$$\frac{2e^2\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5cg}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2\int \frac{ae^2(7ef-15dg)g^2-c\left(8e^3f^3-30de^2gf^2+45d^2eg^2f-15d^3g^3\right)-e\left(9ae^2g^2-c\left(8e^2f^2-30degf+45d^2g^2\right)\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3g^3}-\frac{8e^2\sqrt{a+cx^2}\sqrt{f+gx}(ef-3dg)}{3g}}{5cg}$$
$$\frac{2e^2\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5cg}$$

$$\Big\downarrow \; 1511$$

$$\frac{-\frac{2\left(-\frac{\left(-e\sqrt{ag^2+cf^2}\left(9ae^2g^2-c\left(45d^2g^2-30defg+8e^2f^2\right)\right)+a\sqrt{c}e^2g^2(7ef-15dg)-c^{3/2}\left(-15d^3g^3+45d^2efg^2-30de^2f^2g+8e^3f^3\right)\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(}{}}}}{\sqrt{c}}\right.}{3g^3}}{5c}$$
$$\frac{2e^2\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5cg}$$

$$\Big\downarrow \; 1416$$

3.645. $\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

$$-\frac{2\left(-\frac{e\sqrt{ag^2+cf^2}\left(9ae^2g^2-c\left(45d^2g^2-30defg+8e^2f^2\right)\right)\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(\ldots}{g\ldots}}{\left(a+\frac{cf^2}{g^2}\right)\left(\ldots\right.}}}{}\right.}{}$$

$$\frac{2e^2\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5cg}$$

$\downarrow$ 1509

$$\frac{2(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}e^2}{5cg}+$$

$$\frac{-\frac{8(ef-3dg)\sqrt{f+gx}\sqrt{cx^2+a}e^2}{3g}-\frac{2\left(-\frac{e\sqrt{cf^2+ag^2}\left(9ae^2g^2-c\left(8e^2f^2-30degf+45d^2g^2\right)\right)\left(\frac{\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f\ldots}{\ldots}}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx\ldots}{\sqrt{cf^2+ag\ldots}}\right.}}}{\sqrt[4]{c}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f\ldots}{g\ldots}}}\right.}{\sqrt{c}\ldots}\right.}{}}{}$$

input
```
Int[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

3.645. $\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

output
```
(2*e^2*(d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(5*c*g) + ((-8*e^2*(e*f -
3*d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*g) - (2*(-((e*Sqrt[c*f^2 + a*g^2]
*(9*a*e^2*g^2 - c*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*(-((Sqrt[f + g*x]
*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a
+ (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 +
 a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f
^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*
(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^(1
/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a
*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f +
 g*x)^2)/g^2]))))/Sqrt[c]) - ((c*f^2 + a*g^2)^(1/4)*(a*Sqrt[c]*e^2*g^2*(7*e
*f - 15*d*g) - c^(3/2)*(8*e^3*f^3 - 30*d*e^2*f^2*g + 45*d^2*e*f*g^2 - 15*d
^3*g^3) - e*Sqrt[c*f^2 + a*g^2]*(9*a*e^2*g^2 - c*(8*e^2*f^2 - 30*d*e*f*g +
 45*d^2*g^2)))*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*
f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)
*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(
1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 +
a*g^2])/2])/(2*c^(3/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(
f + g*x)^2)/g^2])))/(3*g^3))/(5*c*g)
```

3.645.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

3.645. $\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

rule 728
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*(Sqrt[a +
c*x^2]/(c*g*(2*m - 1))), x] - Simp[1/(c*g*(2*m - 1))   Int[((d + e*x)^(m -
3)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^
3*g*(2*m - 1) + e*(e*(a*e*g*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x +
 2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g
}, x] && IntegerQ[2*m] && GeQ[m, 2]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.645. $\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

3.645.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 711, normalized size of antiderivative = 1.56

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(\frac{2e^3x\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{5cg}+\frac{2\left(3d\,e^2-\frac{4f\,e^3}{5g}\right)\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{3cg}+\frac{2\left(d^3-\frac{2fa\,e^3}{5cg}-\frac{a\left(3d\,e^2-\frac{4f\,e^3}{5g}\right)}{3c}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/5*e^3/c/g*x*(c*
g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/3*(3*d*e^2-4/5*f/g*e^3)/c/g*(c*g*x^3+c*f*
x^2+a*g*x+a*f)^(1/2)+2*(d^3-2/5*f*a/c/g*e^3-1/3*a/c*(3*d*e^2-4/5*f/g*e^3))
*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2
)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c
))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^
(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(3*
d^2*e-3/5*e^3/c*a-2/3*f/g*(3*d*e^2-4/5*f/g*e^3))*(f/g-(-a*c)^(1/2)/c)*((x+
f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c)
)^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+
a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/
2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(
1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/
c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))
```

3.645.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.70

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{2\left(\left(8\,ce^3f^3-30\,cde^2f^2g+3\,(15\,cd^2e-ae^3)fg^2-45\,(cd^3-ade^2)g^3\right)\sqrt{cg}\,\text{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,a}{3\,cg^2}\right.\right.}{}$$

input
```
integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*((8*c*e^3*f^3 - 30*c*d*e^2*f^2*g + 3*(15*c*d^2*e - a*e^3)*f*g^2 - 45
*(c*d^3 - a*d*e^2)*g^3)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2
)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 3*(8*c*
e^3*f^2*g - 30*c*d*e^2*f*g^2 + 9*(5*c*d^2*e - a*e^3)*g^3)*sqrt(c*g)*weiers
trassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3)
, weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*
g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) - 3*(3*c*e^3*g^3*x - 4*c*e^3*f*g^2 + 15*
c*d*e^2*g^3)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c^2*g^4)
```

3.645.6 Sympy [F]

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(d+ex)^3}{\sqrt{a+cx^2}\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x)**3/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

3.645.7 Maxima [F]

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^3}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

3.645.8 Giac [F]

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^3}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^3/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

3.645.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(d+e\,x)^3}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}\,dx$$

input
```
int((d + e*x)^3/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x)^3/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)), x)
```

3.646 $\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

3.646.1 Optimal result

Integrand size = 28, antiderivative size = 356

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \frac{2e^2\sqrt{f+gx}\sqrt{a+cx^2}}{3cg}$$
$$+\frac{4\sqrt{-a}e(ef-3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}g^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$-\frac{2\sqrt{-a}\left((3cd^2-ae^2)g^2+2cef(ef-3dg)\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2a}{\sqrt{-a}\sqrt{}}\right.}{3c^{3/2}g^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
2/3*e^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g+4/3*e*(-3*d*g+e*f)*EllipticE(1/2
*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)
))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/g^2/c^(1/2)/(c*x^2+a)
^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-2/3*((-a*e^2+3*c*d
^2)*g^2+2*c*e*f*(-3*d*g+e*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)
*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a
)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^2/(g*x+
f)^(1/2)/(c*x^2+a)^(1/2)
```

3.646.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.65 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.33

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$

$$= \frac{2\sqrt{f+gx}\left(e^2g^2(a+cx^2)-\frac{2eg^2(ef-3dg)(a+cx^2)}{f+gx}-2ice\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(ef-3dg)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}\sqrt{f+\cdots}\right.}{\cdots}$$

input
```
Integrate[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
(2*Sqrt[f + g*x]*(e^2*g^2*(a + c*x^2) - (2*e*g^2*(e*f - 3*d*g)*(a + c*x^2)
)/(f + g*x) - (2*I)*c*e*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - 3*d*g)*Sqr
t[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] -
 g*x)/(f + g*x))]*Sqrt[f + g*x]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g
)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a
]*g)] + (g*((3*I)*c*d^2*g - I*a*e^2*g + 2*Sqrt[a]*Sqrt[c]*e*(e*f - 3*d*g))
*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[
c] - g*x)/(f + g*x))]*Sqrt[f + g*x]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sq
rt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]))/(3*c*g^3*Sqrt[a + c*x^2])
```

3.646.3 Rubi [A] (warning: unable to verify)

Time = 0.89 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {728, 25, 2004, 599, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^2}\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \ 728$$

$$\frac{2e^2\sqrt{a+cx^2}\sqrt{f+gx}}{3cg}-\frac{\int -\frac{-2ce^2(ef-3dg)x^2-e\left(age^2+cd(2ef-9dg)\right)x+d\left(3cd^2-ae^2\right)g}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3cg}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-2ce^2(ef-3dg)x^2-e\left(age^2+cd(2ef-9dg)\right)x+d\left(3cd^2-ae^2\right)g}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3cg}+\frac{2e^2\sqrt{a+cx^2}\sqrt{f+gx}}{3cg}$$

$$\Big\downarrow \; 2004$$

$$\frac{\int \frac{\left(3cd^2-ae^2\right)g-2ce(ef-3dg)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3cg}+\frac{2e^2\sqrt{a+cx^2}\sqrt{f+gx}}{3cg}$$

$$\Big\downarrow \; 599$$

$$\frac{2e^2\sqrt{a+cx^2}\sqrt{f+gx}}{3cg}-\frac{2\int \frac{ae^2g^2-c\left(2e^2f^2-6degf+3d^2g^2\right)+2ce(ef-3dg)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3cg^3}$$

$$\Big\downarrow \; 1511$$

$$\frac{2e^2\sqrt{a+cx^2}\sqrt{f+gx}}{3cg}-$$

$$\frac{2\left(\left(2\sqrt{c}e\sqrt{ag^2+cf^2}(ef-3dg)+ae^2g^2-c\left(3d^2g^2-6defg+2e^2f^2\right)\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-$$

$$3cg^3$$

$$\Big\downarrow \; 1416$$

$$\frac{2e^2\sqrt{a+cx^2}\sqrt{f+gx}}{3cg}-$$

$$2\left(\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(2\sqrt{c}e\sqrt{ag^2+cf^2}(ef-3dg)+ae^2g^2-c\left(3d^2g^2-6defg+2e^2f^2\right)\right)\text{EllipticF}\left(2\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right.$$

$$3cg$$

$$\Big\downarrow \; 1509$$

3.646. $\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

$$\frac{2e^2\sqrt{a+cx^2}\sqrt{f+gx}}{3cg} -$$

$$2\left(\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(2\sqrt{c}e\sqrt{ag^2+cf^2}(ef-3dg)+ae^2g^2-c\left(3d^2g^2-6defg+2e^2f^2\right)\right)\text{EllipticF}\left(2\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right.$$

input
```
Int[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
(2*e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c*g) - (2*(-2*Sqrt[c]*e*(e*f - 3*
d*g)*Sqrt[c*f^2 + a*g^2]*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(
f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f
+ g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f +
 g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2
+ (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*
f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^
2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*
f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])) + ((c*f^2 + a*g^
2)^(1/4)*(a*e^2*g^2 + 2*Sqrt[c]*e*(e*f - 3*d*g)*Sqrt[c*f^2 + a*g^2] - c*(2
*e^2*f^2 - 6*d*e*f*g + 3*d^2*g^2))*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a
*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2
)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*Ell
ipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt
[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(2*c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(
f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/(3*c*g^3)
```

3.646.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 728
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*(Sqrt[a +
c*x^2]/(c*g*(2*m - 1))), x] - Simp[1/(c*g*(2*m - 1))   Int[((d + e*x)^(m -
3)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^
3*g*(2*m - 1) + e*(e*(a*e*g*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x +
 2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g
}, x] && IntegerQ[2*m] && GeQ[m, 2]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

3.646. $\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

3.646.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(290) = 580$.

Time = 2.54 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.72

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(\frac{2e^2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{3cg}+\frac{2\left(d^2-\frac{a\,e^2}{3c}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\cdots}\right.}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\cdots\right.}{\cdots}$
risch	$\frac{2e^2\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{3cg}-\frac{\left(\frac{2a\,e^2g\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}-\frac{6c\,d^2g\left(\frac{f}{g}-\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/3*e^2/c/g*(c*g*
x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2*(d^2-1/3*a*e^2/c)*(f/g-(-a*c)^(1/2)/c)*((x+
f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c)
)^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+
a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a
*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(2*d*e-2/3*e^2*f/g)*(f/g-(-a*
c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g
-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(
c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)
/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))
^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f
/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))
```

3.646.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \frac{2\left(3\sqrt{cx^2+a}\sqrt{gx+f}ce^2g^2 + (2ce^2f^2 - 6cdefg + 3(3cd^2 - ae^2)g^2)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4(cf^2-3ag^2)}{3cg^2}\right.\right.}{}$$

input
```
integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(3*sqrt(c*x^2 + a)*sqrt(g*x + f)*c*e^2*g^2 + (2*c*e^2*f^2 - 6*c*d*e*f*
g + 3*(3*c*d^2 - a*e^2)*g^2)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*
a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 6*
(c*e^2*f*g - 3*c*d*e*g^2)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/
(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2
 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)
))/(c^2*g^3)
```

3.646.6 Sympy [F]

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(d+ex)^2}{\sqrt{a+cx^2}\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x)**2/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

3.646.7 Maxima [F]

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^2}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

3.646.8 Giac [F]

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^2}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

3.646.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(d+e\,x)^2}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}\,dx$$

input
```
int((d + e*x)^2/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x)^2/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)), x)
```

3.647 $\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

3.647.1 Optimal result

Integrand size = 26, antiderivative size = 288

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{2\sqrt{-a}e\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{2\sqrt{-a}(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}g\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
-2*e*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*
(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/g/c
^(1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+2*
(-d*g+e*f)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-
a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^
(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/g/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1
/2)
```

3.647.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.29 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.52

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx =$$

$$-\frac{2\left(-eg^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(a+cx^2)+i\sqrt{c}e\left(\sqrt{c}f+i\sqrt{a}g\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}(f+gx)^{3/2}E\left(i\text{arcsinh}\left(\right.\right.\right.}{cg^2}$$

input
```
Integrate[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
(-2*(-(e*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(a + c*x^2)) + I*Sqrt[c]*e*(
Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqr
t[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*
ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sq
rt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + Sqrt[c]*((-I)*Sqrt[c]*d + Sqrt[a]*e)
*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqr
t[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*S
qrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f +
I*Sqrt[a]*g)]))/(c*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*Sqrt[f + g*x]*Sqrt
[a + c*x^2])
```

3.647.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 614 vs. 2(288) = 576.

Time = 0.65 (sec) , antiderivative size = 614, normalized size of antiderivative = 2.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {599, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{\sqrt{a+cx^2}\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \ 599$$

$$-\frac{2\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}$$

$\Big\downarrow$ 1511

$$-\frac{2\left(\frac{e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\left(dg-e\left(f-\frac{\sqrt{ag^2+cf^2}}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+g}\right.}{g^2}$$

$\Big\downarrow$ 1416

$$-\frac{2\left(\frac{e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(dg-e\left(f-\right.\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(}{}}}\right.}{g^2}$$

$\Big\downarrow$ 1509

$$-\frac{2\left(\frac{e\sqrt{ag^2+cf^2}\left(\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}-\frac{\sqrt{f+gx}}{}\right.\right.}{\sqrt{c}}}{}$$

input
```
Int[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

3.647. $\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

output
```
(-2*((e*Sqrt[c*f^2 + a*g^2]*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*
f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*
(f + g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(
f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g
^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt
[c*f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a
*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a +
(c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2]))))/Sqrt[c] - ((
c*f^2 + a*g^2)^(1/4)*(d*g - e*(f - Sqrt[c*f^2 + a*g^2]/Sqrt[c]))*(1 + (Sqr
t[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g
*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x)
)/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f
^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(2*c^(1/4)*S
qrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/g^2
```

3.647.3.1 Defintions of rubi rules used

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.647.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(232) = 464$.

Time = 1.22 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.81

method	result
default	$\frac{2\left(F\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)ae\,g^{2}+F\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)cdfg-\sqrt{-ac}\,F\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)d\ldots}{\ldots}$
elliptic	$\frac{\sqrt{(gx+f)(c\,x^{2}+a)}\left(\frac{2d\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{\sqrt{cg\,x^{3}+cf\,x^{2}+agx+fa}}+\frac{2e\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\ldots}{\frac{f}{g}-\sqrt{\ldots}}}\ldots}{\ldots}\right.}{\sqrt{gx+f}\,\sqrt{c\,x^{2}\ldots}}$

input
```
int((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f
)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*e*g^2+EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/
2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*d*f*g
-(-a*c)^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c
)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*d*g^2+(-a*c)^(1/2)*EllipticF((-(
g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2
)+c*f))^(1/2))*e*f*g-EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(
g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*e*g^2-EllipticE((-(g*x+
f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*
f))^(1/2))*c*e*f^2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c
*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)
-c*f))^(1/2)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^2/(c*g*x^3+c*f*x^2+a*g*x+a*
f)
```

3.647.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.62

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{2\left(3\sqrt{cg}eg\,\text{weierstrassZeta}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\text{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\right.\right.\right.}{3\,cg^2}$$

input
```
integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(3*sqrt(c*g)*e*g*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27
*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c
*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) + (e*f - 3*d
*g)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*
f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g))/(c*g^2)
```

3.647.6 Sympy [F]

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{d+ex}{\sqrt{a+cx^2}\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x)/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

3.647.7 Maxima [F]

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{ex+d}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

3.647.8 Giac [F]

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{ex+d}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

3.647.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{d+e\,x}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}\,dx$$

input
```
int((d + e*x)/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x)/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)), x)
```

3.648 $\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}} dx$

3.648.1 Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}} dx = -\frac{2\sqrt{-a}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
-2*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-
a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g
*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

3.648.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.20 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.37

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}} dx = \frac{2i\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}(f+gx)\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right),\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)}{g\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\sqrt{a+cx^2}}$$

input
```
Integrate[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
((2*I)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)
/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqr
t[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*
Sqrt[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*Sqrt[a + c*x^2])
```

3.648.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.71, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {510, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^2}\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{ 510}$$

$$\frac{2\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}g\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}$$

input
```
Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[
(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f
^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*Arc
Tan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[
c*f^2 + a*g^2])/2])/(c^(1/4)*g*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^
2 + (c*(f + g*x)^2)/g^2])
```

3.648.3.1 Defintions of rubi rules used

rule 510
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2/d   Subst[Int[1/Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2
)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

3.648.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.47

method	result	size
default	$\frac{2\left(cf-g\sqrt{-ac}\right)F\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)\sqrt{\frac{(cx+\sqrt{-ac})g}{g\sqrt{-ac}-cf}}\,\sqrt{\frac{(-cx+\sqrt{-ac})g}{g\sqrt{-ac}+cf}}\,\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}}\,\sqrt{c\,x^{2}+a}\,\sqrt{gx+f}}{cg\left(cg\,x^{3}+cf\,x^{2}+agx+fa\right)}$	200
elliptic	$\frac{2\sqrt{(gx+f)(c\,x^{2}+a)}\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{\sqrt{gx+f}\,\sqrt{c\,x^{2}+a}\,\sqrt{cg\,x^{3}+cf\,x^{2}+agx+fa}}$	242

input
```
int(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(c*f-g*(-a*c)^(1/2))*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(
-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*((c*x+(-a*c)^(1/2))*g/(
g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1
/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)/
c/g/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

3.648.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \frac{2\,\sqrt{cg}\text{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)}{cg}$$

input
```
integrate(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^
3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)/(c*g)
```

3.648.6 Sympy [F]

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{a+cx^2}\sqrt{f+gx}}\,dx$$

input
```
integrate(1/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

3.648.7 Maxima [F]

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

3.648.8 Giac [F]

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

3.648.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)),x)
```

output
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)), x)
```

3.649 $\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

3.649.1 Optimal result

Integrand size = 28, antiderivative size = 167

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

output

```
-2*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/
(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(1+c*x^
2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(e+d*c^(1/2)/(
-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

3.649.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.36 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.86

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{2i\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}(f+gx)\left(\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right),\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)-\operatorname{EllipticPi}\left(\frac{\sqrt{c}(e}{e(\sqrt{c}f}\right.\right.}{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(ef-dg)\sqrt{a+cx^2}}$$

input
```
Integrate[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
((-2*I)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g
)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)*(EllipticF[I*ArcSinh[Sqrt[-f - (I*S
qrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f +
I*Sqrt[a]*g)] - EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]
*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*
f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]]*(e*f - d*g)*Sqrt[a + c*x^2])
```

3.649.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 867 vs. 2(167) = 334.

Time = 1.56 (sec) , antiderivative size = 867, normalized size of antiderivative = 5.19, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {729, 25, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{729}$$

$$2\int -\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}$$

$$\Big\downarrow \text{25}$$

$$-2\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}$$

$$\Big\downarrow \text{1540}$$

$$2\left(\frac{e\sqrt{ag^2+cf^2}\left(\sqrt{c}(ef-dg)-e\sqrt{ag^2+cf^2}\right)\int \frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g\left(ae^2g+cd(2ef-dg)\right)}-\frac{\sqrt{c}\left(-\sqrt{c}\right.}{}\right.$$

$$\Big\downarrow \text{1416}$$

$$2\left(\frac{e\sqrt{ag^2+cf^2}\left(\sqrt{c}(ef-dg)-e\sqrt{ag^2+cf^2}\right)\int \frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g\left(ae^2g+cd(2ef-dg)\right)}-\frac{\sqrt[4]{c}\left(\frac{\sqrt{c}(f}{\sqrt{ag^2}}\right.}{}\right.$$

$\downarrow$ 2222

$$2\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\left(\frac{\left(e+\frac{\sqrt{c}(ef-dg)}{\sqrt{cf^2+ag^2}}\right)\operatorname{arctanh}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{e}\sqrt{ef-dg}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}\right)}{2\sqrt{e}\sqrt{cd^2+ae^2}\sqrt{ef-dg}}-\right.}{g\,(ag}\right.$$

input
```
Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

3.649. $\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

output
```
2*(-1/2*(c^(1/4)*(c*e*f^2 + a*e*g^2 - Sqrt[c]*(e*f - d*g)*Sqrt[c*f^2 + a*g
^2])*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 -
 (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqr
t[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[
f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2]
)/(g*(c*f^2 + a*g^2)^(1/4)*(a*e^2*g + c*d*(2*e*f - d*g))*Sqrt[a + (c*f^2)/
g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2]) + (e*Sqrt[c*f^2 + a*g^
2]*(Sqrt[c]*(e*f - d*g) - e*Sqrt[c*f^2 + a*g^2])*(((e + (Sqrt[c]*(e*f - d*
g))/Sqrt[c*f^2 + a*g^2])*ArcTanh[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt
[e]*Sqrt[e*f - d*g]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f +
 g*x)^2)/g^2])])/(2*Sqrt[e]*Sqrt[c*d^2 + a*e^2]*Sqrt[e*f - d*g]) - ((Sqrt[
c]/e - Sqrt[c*f^2 + a*g^2]/(e*f - d*g))*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^
2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2
)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)
]*EllipticPi[(Sqrt[c]*(e*f - d*g) + e*Sqrt[c*f^2 + a*g^2])^2/(4*Sqrt[c]*e*
(e*f - d*g)*Sqrt[c*f^2 + a*g^2]), 2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2
+ a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(4*c^(1/4)*(c*f
^2 + a*g^2)^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g
*x)^2)/g^2])))/(g*(a*e^2*g + c*d*(2*e*f - d*g))))
```

3.649.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

3.649.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.41

method	result
default	$\frac{2\left(cf-g\sqrt{-ac}\right)\Pi\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\frac{\left(g\sqrt{-ac}-cf\right)e}{c(dg-ef)},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)\sqrt{\frac{\left(cx+\sqrt{-ac}\right)g}{g\sqrt{-ac}-cf}}\,\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)g}{g\sqrt{-ac}+cf}}\,\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}}\,\sqrt{c\,x^{2}+a}\,\sqrt{gx+f}}{c(dg-ef)\left(cg\,x^{3}+cf\,x^{2}+agx+fa\right)}$
elliptic	$\frac{2\sqrt{(gx+f)\left(c\,x^{2}+a\right)}\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,\Pi\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{d}{e}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{\sqrt{gx+f}\,\sqrt{c\,x^{2}+a}\,e\sqrt{cg\,x^{3}+cf\,x^{2}+agx+fa}\left(-\frac{f}{g}+\frac{d}{e}\right)}$

input
```
int(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(c*f-g*(-a*c)^(1/2))*EllipticPi((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),
(g*(-a*c)^(1/2)-c*f)*e/c/(d*g-e*f),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+
c*f))^(1/2))*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*
c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^
(1/2)*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)/c/(d*g-e*f)/(c*g*x^3+c*f*x^2+a*g*x+a*f
)
```

3.649.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.649.6 Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx=\int \frac{1}{\sqrt{a+cx^2}\,(d+ex)\,\sqrt{f+gx}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**2)*(d + e*x)*sqrt(f + g*x)), x)
```

3.649.7 Maxima [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx=\int \frac{1}{\sqrt{cx^2+a}(ex+d)\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f)), x)
```

3.649.8 Giac [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f)), x)
```

3.649.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.650 $\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

3.650.1 Optimal result

Integrand size = 28, antiderivative size = 746

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{(cd^2+ae^2)(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}e\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(cd^2+ae^2)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{\sqrt{-a}\sqrt{c}ef\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
-e^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(e*x+d)-e*Elli
pticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2
)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/(a*e
^2+c*d^2)/(-d*g+e*f)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1
/2)))^(1/2)+e*f*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a
*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2
)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f
)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-d*g*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))
^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1
/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*
e^2+c*d^2)/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+(a*e^2*g-c*d*(-3*d*g+2
*e*f))*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1
/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(1+
c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d
^2)/(-d*g+e*f)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

3.650.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.97 (sec) , antiderivative size = 1349, normalized size of antiderivative = 1.81

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(-\frac{2e^2(a+cx^2)}{d+ex}+\frac{2\left(-ce^2f^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+cdef^2g\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}-ae^2fg^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+adeg^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+2ce^2f^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(\ldots}{\ldots}\right.}{\ldots}$$

input
```
Integrate[1/((d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
(Sqrt[f + g*x]*((-2*e^2*(a + c*x^2))/(d + e*x) + (2*(-(c*e^2*f^3*Sqrt[-f -
 (I*Sqrt[a]*g)/Sqrt[c]]) + c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] -
a*e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*d*e*g^3*Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]] + 2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) -
 2*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - c*e^2*f*Sqrt[-f
- (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]]*(f + g*x)^2 + Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d*g)*
Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c
] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*S
qrt[a]*g)] + (Sqrt[c]*d - I*Sqrt[a]*e)*g*(Sqrt[a]*e*g + I*Sqrt[c]*(e*f - 2
*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)
/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f -
(I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*
f + I*Sqrt[a]*g)] - (2*I)*c*d*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f
+ g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*E
llipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[S
qrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/
(Sqrt[c]*f + I*Sqrt[a]*g)] + (3*I)*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c]
+ x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + ...
```

3.650.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1586 vs. $2(746) = 1492$.

Time = 2.76 (sec) , antiderivative size = 1586, normalized size of antiderivative = 2.13, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {734, 2349, 599, 25, 27, 729, 25, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \text{734}$$

$$-\frac{\int \frac{-cgx^2e^2+age^2-2cdgxe-2cd(ef-dg)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2\left(ae^2+cd^2\right)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \text{2349}$$

$$-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int\frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2\left(ae^2+cd^2\right)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}$$

$\downarrow$ 599

$$-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2\int-\frac{cg(ef-dg-e(f+gx))}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2\left(ae^2+cd^2\right)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}$$

$\downarrow$ 25

$$-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\frac{2\int\frac{cg(ef-dg-e(f+gx))}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2\left(ae^2+cd^2\right)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}$$

$\downarrow$ 27

$$-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\frac{2c\int\frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}}{2\left(ae^2+cd^2\right)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}$$

$\downarrow$ 729

$$-\frac{2\left(ae^2g-cd(2ef-3dg)\right)\int-\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}+\frac{2c\int\frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}}{g}}{2\left(ae^2+cd^2\right)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}$$

$\downarrow$ 25

3.650. $\int\frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

$$-\frac{\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}-2\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f}}{2\left(ae^2+cd^2\right)(ef-dg)}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \text{ 1511}$$

$$-\frac{\frac{2c\left(\frac{e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\left(dg-e\left(f-\frac{\sqrt{ag^2+cf^2}}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}\right)}{g}-2\left(ae^2g\right.}{2\left(ae^2+cd^2\right)(ef-dg)}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \text{ 1416}$$

$$-\frac{\frac{2c\left(\frac{e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(dg-e\left(f-\frac{\sqrt{ag^2+c}}{\sqrt{c}}\right.\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+c}}\right)}{g}}{2}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \text{ 1509}$$

3.650. $\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

$$2c\left(\frac{e\sqrt{ag^2+cf^2}\left(\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}-\frac{\sqrt{f+gx}\sqrt{\cdots}}{(\cdots}\right.}{\sqrt{c}}\right.$$

$$-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}$$

$\downarrow$ 1540

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+a}e^2}{\left(cd^2+ae^2\right)(ef-dg)(d+ex)}-$$

$$2c\left(\frac{e\sqrt{cf^2+ag^2}\left(\frac{\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-\frac{\sqrt{f+gx}\sqrt{\frac{c\cdots}{g\cdots}}}{\left(\frac{cf^2}{g^2}\cdots\right.}\right.}{\sqrt{c}}\right.$$

$\downarrow$ 1416

3.650. $\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+a}e^2}{(cd^2+ae^2)(ef-dg)(d+ex)}-$$

$$2c\left(\frac{e\sqrt{cf^2+ag^2}\left(\frac{\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-\frac{\sqrt{f+gx}\sqrt{\ldots}}{\left(\frac{cf^2}{g^2}\ldots\right.}\right.}{\sqrt{c}}\right.$$

$\downarrow$ 2222

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+a}e^2}{(cd^2+ae^2)(ef-dg)(d+ex)}-$$

$$2c\left(\frac{e\sqrt{cf^2+ag^2}\left(\frac{\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-\frac{\sqrt{f+gx}\sqrt{\ldots}}{\left(\frac{cf^2}{g^2}\ldots\right.}\right.}{\sqrt{c}}\right.$$

3.650. $\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

input
```
Int[1/((d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
-((e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(e*f - d*g)*(d + e*
x))) - ((2*c*((e*Sqrt[c*f^2 + a*g^2]*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^
2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 +
(Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (
Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f
+ g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g
*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(
c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*
Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/Sqr
t[c] - ((c*f^2 + a*g^2)^(1/4)*(d*g - e*(f - Sqrt[c*f^2 + a*g^2]/Sqrt[c]))*
(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c
*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*
(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g
*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(2*
c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2
])))/g + 2*(a*e^2*g - c*d*(2*e*f - 3*d*g))*(-1/2*(c^(1/4)*(c*e*f^2 + a*e*g
^2 - Sqrt[c]*(e*f - d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f + g*x))/Sqr
t[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f +
g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^
2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4...
```

3.650.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 734
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2))), x] + Simp[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2))   Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^
2]))*Simp[2*c*d*(e*f - d*g)*(m + 1) - a*e^2*g*(2*m + 3) + 2*c*e*(d*g*(m + 1
) - e*f*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.650. $\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

3.650.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 995, normalized size of antiderivative = 1.33

method	result
elliptic	$\frac{\sqrt{(gx+f)(c x^2+a)}\left(\frac{e^2\sqrt{cg x^3+cf x^2+agx+fa}}{\left(a e^2 gd-a e^3 f+c d^3 g-c d^2 ef\right)(ex+d)}-\frac{cdg\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\right.}{\left(a e^2 gd-a e^3 f+c d^3 g-c d^2 ef\right)\sqrt{cg x^3+cf x^2+agx+fa}}\right.}{}$
default	Expression too large to display

input
```
int(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(e^2/(a*d*e^2*g-a*
e^3*f+c*d^3*g-c*d^2*e*f)*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)-c*d*g/(
a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-
a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+
(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(
1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)
/(-f/g-(-a*c)^(1/2)/c))^(1/2))-c*e*g/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)
*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2
)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c
))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*Elliptic
E(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)
^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^
(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+(a*e^2*g+3*c*d
^2*g-2*c*d*e*f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/e*(f/g-(-a*c)^(1/2)/
c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(
1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c
*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c
))^(1/2),(-f/g+(-a*c)^(1/2)/c)/(-f/g+d/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a
*c)^(1/2)/c))^(1/2)))
```

3.650.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.650.6 Sympy [F]

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{a+cx^2}\,(d+ex)^2\,\sqrt{f+gx}}\,dx$$

input
```
integrate(1/(e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**2)*(d + e*x)**2*sqrt(f + g*x)), x)
```

3.650.7 Maxima [F]

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^2\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f)), x)
```

3.650.8 Giac [F]

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^2\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f)), x)
```

3.650.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

output
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.651 $\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

3.651.1 Optimal result

Integrand size = 28, antiderivative size = 1257

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$
$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2(cd^2+ae^2)(ef-dg)(d+ex)^2} + \frac{3e^2(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4(cd^2+ae^2)^2(ef-dg)^2(d+ex)}$$
$$+ \frac{3\sqrt{-a}\sqrt{c}e(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4(cd^2+ae^2)^2(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$+ \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$- \frac{3\sqrt{-a}\sqrt{c}ef(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4(cd^2+ae^2)^2(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$+ \frac{3\sqrt{-a}\sqrt{c}dg(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4(cd^2+ae^2)^2(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$+ \frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$- \frac{3(ae^2g-cd(2ef-3dg))^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{4\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)^2(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
-1/2*e^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(e*x+d)^2+
3/4*e^2*(a*e^2*g-c*d*(-3*d*g+2*e*f))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+
c*d^2)^2/(-d*g+e*f)^2/(e*x+d)+3/4*e*(a*e^2*g-c*d*(-3*d*g+2*e*f))*EllipticE
(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(
1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/(a*e^2+c*
d^2)^2/(-d*g+e*f)^2/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/
2)))^(1/2)+1/2*g*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*
a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/
2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)/(-d*g+e*
f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-3/4*e*f*(a*e^2*g-c*d*(-3*d*g+2*e*f))*Elli
pticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2
)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(
g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)^2/(-d*g+e*f)^2/(g*x+f)^(1/2)/
(c*x^2+a)^(1/2)+3/4*d*g*(a*e^2*g-c*d*(-3*d*g+2*e*f))*EllipticF(1/2*(1-x*c^
(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2)
)*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^
(1/2)))^(1/2)/(a*e^2+c*d^2)^2/(-d*g+e*f)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+c
*(-3*d*g+e*f)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e
+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/
2))*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/...
```

3.651.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.75 (sec) , antiderivative size = 2491, normalized size of antiderivative = 1.98

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
(-15*c^2*d^2*e^2*f^3 - 3*a*c*e^4*f^3 + (6*c^2*d*e^3*f^4)/g + 9*c^2*d^3*e*f
^2*g + 9*a*c*d*e^3*f^2*g - 15*a*c*d^2*e^2*f*g^2 - 3*a^2*e^4*f*g^2 + 9*a*c*
d^3*e*g^3 + 3*a^2*d*e^3*g^3 + 30*c^2*d^2*e^2*f^2*(f + g*x) + 6*a*c*e^4*f^2
*(f + g*x) - (12*c^2*d*e^3*f^3*(f + g*x))/g - 18*c^2*d^3*e*f*g*(f + g*x) -
 6*a*c*d*e^3*f*g*(f + g*x) - 15*c^2*d^2*e^2*f*(f + g*x)^2 - 3*a*c*e^4*f*(f
 + g*x)^2 + (6*c^2*d*e^3*f^2*(f + g*x)^2)/g + 9*c^2*d^3*e*g*(f + g*x)^2 +
3*a*c*d*e^3*g*(f + g*x)^2 - (e^2*(e*f - d*g)*(f + g*x)*(a + c*x^2)*(2*(c*d
^2 + a*e^2)*(e*f - d*g) - 3*(a*e^2*g + c*d*(-2*e*f + 3*d*g))*(d + e*x)))/(
d + e*x)^2 + (3*Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d*g)*(a*e
^2*g + c*d*(-2*e*f + 3*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]
*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Elliptic
E[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f -
I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c
]]) + ((Sqrt[c]*d - I*Sqrt[a]*e)*(3*a^(3/2)*e^3*g^2 + (3*I)*a*Sqrt[c]*e^2*
g*(e*f - 2*d*g) - Sqrt[a]*c*e*(2*e^2*f^2 - 6*d*e*f*g + d^2*g^2) - I*c^(3/2
)*d*(4*e^2*f^2 - 9*d*e*f*g + 8*d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x)
)/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3
/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (
Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*
g)/Sqrt[c]] + ((8*I)*c^2*d^2*e^2*f^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))...
```

3.651.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2800 vs. $2(1257) = 2514$.

Time = 5.12 (sec) , antiderivative size = 2800, normalized size of antiderivative = 2.23, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {734, 2349, 734, 2349, 27, 510, 599, 25, 27, 729, 25, 1416, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{734}$$

$$-\frac{\int \frac{cgx^2e^2+3age^2+2c(ef-2dg)xe-4cd(ef-dg)}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4\left(ae^2+cd^2\right)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \text{2349}$$

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int\frac{2cef-5cdg+cegx}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4\left(ae^2+cd^2\right)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \text{ 734}$$

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\left(-\frac{\int\frac{-cgx^2e^2+age^2-2cdgxe-2cd(ef-dg)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)+\int\frac{2cef-5cdg+cegx}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4\left(ae^2+cd^2\right)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \text{ 2349}$$

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int\frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)+}{4\left(ae^2+cd^2\right)(ef-dg)}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int\frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)+}{4\left(ae^2+cd^2\right)(ef-dg)}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \text{ 510}$$

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int\frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)+}{4\left(ae^2+cd^2\right)(ef-dg)}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \text{ 599}$$

3.651. $\int\frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2\int-\frac{cg(ef-dg-e(f+gx))}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2}{(d+ex}\right.}{4\left(ae^2+cd^2\right)(ef}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$\Big\downarrow$ 25

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\frac{2\int\frac{cg(ef-dg-e(f+gx))}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{}}{(d+ex)(}\right.}{4\left(ae^2+cd^2\right)(ef}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$\Big\downarrow$ 27

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\frac{2c\int\frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{}}{(d+ex)}\right.}{4\left(ae^2+cd^2\right)(ef}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$\Big\downarrow$ 729

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\left(-\frac{2\left(ae^2g-cd(2ef-3dg)\right)\int-\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}+\frac{2c\int\frac{ef-dg-}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+g}{g^2}}}}{}}{2(ae^2+cd^2)(ef-dg)}\right.}{}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$\Big\downarrow$ 25

3.651. $\int\frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\left(-\frac{\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}-2\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}}}}{2(ae^2+cd^2)(ef-dg)}\right.}{}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$\Big\downarrow$ 1416

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\left(-\frac{\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}-2\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}}}}{2(ae^2+cd^2)(ef-dg)}\right.}{}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$\Big\downarrow$ 1511

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\left(-\frac{\frac{2c\left(\frac{e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\left(dg-e\left(f-\frac{\sqrt{ag^2+cf^2}}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+}}\right.}{g}}{2(ae^2+cd}\right.}{}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$\Big\downarrow$ 1416

3.651. $\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+a}e^2}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2}-$$

$$\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-4c(ef-$$

$\downarrow$ 1509

3.651. $\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+a}e^2}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2}-$$

$$\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-4c(ef-$$

$\downarrow$ 1540

3.651. $\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+a}e^2}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2}-$$

$$\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}+4c(ef-$$

$$\Big\downarrow \text{1416}$$

3.651. $\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+a}e^2}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2}-$$

$$\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}+4c(ef-$$

$\downarrow$ 2222

3.651. $\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+a}e^2}{2\left(cd^2+ae^2\right)\left(ef-dg\right)(d+ex)^2}-$$

$$\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}+4c(ef-$$

input
```
Int[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

3.651. $\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

output
```
-1/2*(e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(e*f - d*g)*(d +
 e*x)^2) - ((c^(3/4)*(c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c
*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x
)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])
^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1
 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f +
 g*x))/g^2 + (c*(f + g*x)^2)/g^2] + 4*c*(e*f - 3*d*g)*(-1/2*(c^(1/4)*(c*e*
f^2 + a*e*g^2 - Sqrt[c]*(e*f - d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f
+ g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2
 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c
*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g
^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(g*(c*f^2 + a*g^2)^(
1/4)*(a*e^2*g + c*d*(2*e*f - d*g))*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x)
)/g^2 + (c*(f + g*x)^2)/g^2]) + (e*Sqrt[c*f^2 + a*g^2]*(Sqrt[c]*(e*f - d*g
) - e*Sqrt[c*f^2 + a*g^2])*(((e + (Sqrt[c]*(e*f - d*g))/Sqrt[c*f^2 + a*g^2
])*ArcTanh[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[e]*Sqrt[e*f - d*g]*Sq
rt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])])/(2*Sq
rt[e]*Sqrt[c*d^2 + a*e^2]*Sqrt[e*f - d*g]) - ((Sqrt[c]/e - Sqrt[c*f^2 + a*
g^2]/(e*f - d*g))*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a +
(c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2...
```

3.651.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 510
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2/d   Subst[Int[1/Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2
)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 734
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2))), x] + Simp[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2))   Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^
2]))*Simp[2*c*d*(e*f - d*g)*(m + 1) - a*e^2*g*(2*m + 3) + 2*c*e*(d*g*(m + 1
) - e*f*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.651. $\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

3.651.4 Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 1192, normalized size of antiderivative = 0.95

method	result	size
elliptic	Expression too large to display	1192
default	Expression too large to display	20366

input
```
int(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(1/2*e^2/(a*d*e^2*
g-a*e^3*f+c*d^3*g-c*d^2*e*f)*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)^2+3
/4*e^2*(a*e^2*g+3*c*d^2*g-2*c*d*e*f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)
^2*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)-1/4*c*g*(a*d*e^2*g+2*a*e^3*f+
7*c*d^3*g-4*c*d^2*e*f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)^2*(f/g-(-a*c)
^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(
-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*
g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1
/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))-3/4*c*e*g*(a*e^2*
g+3*c*d^2*g-2*c*d*e*f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)^2*(f/g-(-a*c)
^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(
-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*
g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(
f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(
1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g
+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+1/4*(3*a^2*e^4*g^2+6*a*c*d
^2*e^2*g^2+4*a*c*d*e^3*f*g-4*a*c*e^4*f^2+15*c^2*d^4*g^2-20*c^2*d^3*e*f*g+8
*c^2*d^2*e^2*f^2)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)^2/e*(f/g-(-a*c)^(1
/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*
c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*...
```

3.651.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.651.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Timed out
```

3.651.7 Maxima [F]

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^3\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f)), x)
```

3.651.8 Giac [F]

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^3\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f)), x)
```

3.651.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^3}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^3),x)
```

output
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^3), x)
```

3.652 $\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx$

3.652.1 Optimal result

Integrand size = 28, antiderivative size = 387

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \frac{2g^2\sqrt{a+cx^2}}{(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}} + \frac{2\sqrt{-a}\sqrt{c}g\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(ef-dg)\left(cf^2+ag^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \frac{2e\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
2*g^2*(c*x^2+a)^(1/2)/(-d*g+e*f)/(a*g^2+c*f^2)/(g*x+f)^(1/2)+2*g*EllipticE
(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(
1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/(-d*g+e*f
)/(a*g^2+c*f^2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))
^(1/2)-2*e*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*
c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))
*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(-d*g+
e*f)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

3.652.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.30 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.21

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \frac{2i\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}(f+gx)\left(\sqrt{c}(ef-dg)E\left(i\text{arcsinh}\left(\frac{\sqrt{-f-\cdots}}{\sqrt{f+\cdots}}\right.\right.\right.}{\cdots}$$

input
```
Integrate[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + c*x^2]),x]
```

output
```
((2*I)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)
/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)*(Sqrt[c]*(e*f - d*g)*EllipticE[I*Arc
Sinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[
a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + (I*Sqrt[a]*e*g + Sqrt[c]*(-2*e*f + d*g)
)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sq
rt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + e*(Sqrt[c]*f - I*Sqrt[
a]*g)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*Ar
cSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt
[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/((Sqrt[c]*f - I*Sqrt[a]*g)*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)^2*Sqrt[a + c*x^2])
```

3.652.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1176 vs. 2(387) = 774.

Time = 2.93 (sec) , antiderivative size = 1176, normalized size of antiderivative = 3.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {740, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^2}(d+ex)(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{ 740}$$

$$\int \left(\frac{e}{\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}(ef-dg)}-\frac{g}{\sqrt{a+cx^2}(f+gx)^{3/2}(ef-dg)}\right)dx$$

↓ 2009

$$\frac{2\sqrt{cx^2+a}g^2}{(ef-dg)(cf^2+ag^2)\sqrt{f+gx}}-\frac{2\sqrt{c}\sqrt{f+gx}\sqrt{cx^2+a}g^2}{(ef-dg)(cf^2+ag^2)^{3/2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)}-\frac{e^{3/2}\text{arctanh}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{e}\sqrt{ef-dg}\sqrt{cx^2+a}}\right)}{\sqrt{cd^2+ae^2}(ef-dg)^{3/2}}+\frac{2\sqrt[4]{c}\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{(ef-dg)\sqrt[4]{cf^2+ag^2}\sqrt{cx^2+a}}-\frac{\sqrt[4]{c}\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{(ef-dg)\sqrt[4]{cf^2+ag^2}\sqrt{cx^2+a}}+\frac{\sqrt[4]{c}e\sqrt[4]{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\arctan\left(\right.\right.}{(ef-dg)(age^2+cd(2ef-dg))\sqrt{cx^2+a}g}$$

$$\frac{e\sqrt[4]{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)^2\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticPi}\left(\frac{\left(\sqrt{cf^2+ag^2}e+\right.}{4\sqrt{c}e(ef-dg)}\right.}{2\sqrt[4]{c}(ef-dg)^2(age^2+cd(2ef-dg))\sqrt{cx^2+a}g}$$

input
```
Int[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + c*x^2]),x]
```

3.652. $\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx$

output
```
(2*g^2*Sqrt[a + c*x^2])/((e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x]) - (2*S
qrt[c]*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((e*f - d*g)*(c*f^2 + a*g^2)^(3/
2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])) - (e^(3/2)*ArcTanh[(Sqrt
[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[e]*Sqrt[e*f - d*g]*Sqrt[a + c*x^2])])
/(Sqrt[c*d^2 + a*e^2]*(e*f - d*g)^(3/2)) + (2*c^(1/4)*Sqrt[(g^2*(a + c*x^2
))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*(1 +
 (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*EllipticE[2*ArcTan[(c^(1/4)*Sqrt
[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2
])/((e*f - d*g)*(c*f^2 + a*g^2)^(1/4)*Sqrt[a + c*x^2]) - (c^(1/4)*Sqrt[(g^
2*(a + c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^
2])^2)]*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*EllipticF[2*ArcTan[(
c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2
 + a*g^2])/2])/((e*f - d*g)*(c*f^2 + a*g^2)^(1/4)*Sqrt[a + c*x^2]) + (c^(1
/4)*e*(c*f^2 + a*g^2)^(1/4)*(Sqrt[c]*(e*f - d*g) - e*Sqrt[c*f^2 + a*g^2])*
Sqrt[(g^2*(a + c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^
2 + a*g^2])^2)]*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*EllipticF[2*
ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sq
rt[c*f^2 + a*g^2])/2])/(g*(e*f - d*g)*(a*e^2*g + c*d*(2*e*f - d*g))*Sqrt[a
 + c*x^2]) - (e*(c*f^2 + a*g^2)^(1/4)*(Sqrt[c]*(e*f - d*g) - e*Sqrt[c*f^2
+ a*g^2])^2*Sqrt[(g^2*(a + c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + ...
```

3.652.3.1 Defintions of rubi rules used

rule 740
```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && Inte
gerQ[n + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.652.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 928 vs. $2(324) = 648$.

Time = 2.53 (sec) , antiderivative size = 929, normalized size of antiderivative = 2.40

method	result
elliptic	$\sqrt{(gx+f)(c\,x^2+a)}\left(-\frac{2\left(cg\,x^2+ag\right)g}{\left(a\,g^2+c\,f^2\right)(dg-ef)\sqrt{\left(x+\frac{f}{g}\right)\left(cg\,x^2+ag\right)}}+\frac{2cgf\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+}{\frac{f}{g}-\sqrt{}}}\right.}{\left(a\,g^2+c\,f^2\right)(dg-ef)\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\right.$
default	Expression too large to display

input
```
int(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-2*(c*g*x^2+a*g)/
(a*g^2+c*f^2)*g/(d*g-e*f)/((x+f/g)*(c*g*x^2+a*g))^(1/2)+2/(a*g^2+c*f^2)*c*
g*f/(d*g-e*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((
x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(
-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)
/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))
^(1/2))+2/(a*g^2+c*f^2)*c*g^2/(d*g-e*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g
-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*(
(x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f
)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1
/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*El
lipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-
(-a*c)^(1/2)/c))^(1/2)))-2/(d*g-e*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-
a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+
(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(
1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-a*
c)^(1/2)/c)/(-f/g+d/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)
))
```

3.652.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.652.6 Sympy [F]

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{a+cx^2}\,(d+ex)\,(f+gx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)**(3/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**2)*(d + e*x)*(f + g*x)**(3/2)), x)
```

3.652.7 Maxima [F]

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*(g*x + f)^(3/2)), x)
```

3.652.8 Giac [F]

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*(g*x + f)^(3/2)), x)
```

3.652.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{{\left(f+g\,x\right)}^{3/2}\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

input
```
int(1/((f + g*x)^(3/2)*(a + c*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int(1/((f + g*x)^(3/2)*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.653 $\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx$

3.653.1 Optimal result

Integrand size = 28, antiderivative size = 818

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}} + \frac{8cfg^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)^2\sqrt{f+gx}} + \frac{2eg^2\sqrt{a+cx^2}}{(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{f+gx}} + \frac{8\sqrt{-a}c^{3/2}fg\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3(ef-dg)\left(cf^2+ag^2\right)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{2\sqrt{-a}\sqrt{c}eg\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \frac{2\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{2e^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

output
```
2/3*g^2*(c*x^2+a)^(1/2)/(-d*g+e*f)/(a*g^2+c*f^2)/(g*x+f)^(3/2)+8/3*c*f*g^2
*(c*x^2+a)^(1/2)/(-d*g+e*f)/(a*g^2+c*f^2)^2/(g*x+f)^(1/2)+2*e*g^2*(c*x^2+a
)^(1/2)/(-d*g+e*f)^2/(a*g^2+c*f^2)/(g*x+f)^(1/2)+8/3*c^(3/2)*f*g*EllipticE
(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(
1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/(-d*g+e*f)/(a*g^2
+c*f^2)^2/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)
+2*e*g*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+
f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)
^(1/2)/(-d*g+e*f)^2/(a*g^2+c*f^2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)
^(1/2)+f*c^(1/2)))^(1/2)-2/3*g*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2
)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(
1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(-d*g+e*
f)/(a*g^2+c*f^2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-2*e^2*EllipticPi(1/2*(1-x*c
^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(
-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1
/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(-d*g+e*f)^2/(e+d*c^(1/2)/(-a)^(1/2))/
(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

3.653.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.37 (sec) , antiderivative size = 1917, normalized size of antiderivative = 2.34

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((d + e*x)*(f + g*x)^(5/2)*Sqrt[a + c*x^2]),x]
```

output
```
(2*(g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)*(a + c*x^2)*(a*g^2*(4
*e*f - d*g + 3*e*g*x) + c*f*(-(d*g*(5*f + 4*g*x)) + e*f*(8*f + 7*g*x))) -
(f + g*x)*(7*c^2*e^2*f^5*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 11*c^2*d*e*f^4
*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 4*c^2*d^2*f^3*g^2*Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]] + 10*a*c*e^2*f^3*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 14*
a*c*d*e*f^2*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 4*a*c*d^2*f*g^4*Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]] + 3*a^2*e^2*f*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c
]] - 3*a^2*d*e*g^5*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 14*c^2*e^2*f^4*Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 22*c^2*d*e*f^3*g*Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]]*(f + g*x) - 8*c^2*d^2*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]]*(f + g*x) - 6*a*c*e^2*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g
*x) + 6*a*c*d*e*f*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 7*c^2*e
^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 - 11*c^2*d*e*f^2*g*Sqr
t[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + 4*c^2*d^2*f*g^2*Sqrt[-f - (I*S
qrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + 3*a*c*e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/S
qrt[c]]*(f + g*x)^2 - 3*a*c*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f +
g*x)^2 + Sqrt[c]*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(e*f - d*g)*(3*a*e*g^2 + c*f
*(7*e*f - 4*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I
*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh
[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a...
```

3.653.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1795 vs. 2(818) = 1636.

Time = 3.99 (sec) , antiderivative size = 1795, normalized size of antiderivative = 2.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {740, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^2}(d+ex)(f+gx)^{5/2}}\,dx$$

$$\Big\downarrow \text{740}$$

$$\int \left(\frac{e^2}{\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}(ef-dg)^2}-\frac{eg}{\sqrt{a+cx^2}(f+gx)^{3/2}(ef-dg)^2}-\frac{g}{\sqrt{a+cx^2}(f+gx)^{5/2}(ef-dg)}\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{e}\sqrt{ef-dg}\sqrt{cx^2+a}}\right)e^{5/2}}{\sqrt{cd^2+ae^2}(ef-dg)^{5/2}}+$$

$$\frac{\sqrt[4]{c}\sqrt[4]{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\arctan\left(\frac{\cdots}{\sqrt[4]{\cdots}}\right.\right.}{g(ef-dg)^2\left(age^2+cd(2ef-dg)\right)\sqrt{cx^2+a}}$$

$$\frac{\sqrt[4]{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)^2\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticPi}\left(\frac{\left(\sqrt{cf^2+ag^2}e+\cdots\right.}{4\sqrt{c}e(ef-dg)\cdots}\right.}{2\sqrt[4]{c}g(ef-dg)^3\left(age^2+cd(2ef-dg)\right)\sqrt{cx^2+a}}$$

$$\frac{2\sqrt[4]{c}\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)e}{(ef-dg)^2\sqrt[4]{cf^2+ag^2}\sqrt{cx^2+a}}-$$

$$\frac{\sqrt[4]{c}\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)e}{(ef-dg)^2\sqrt[4]{cf^2+ag^2}\sqrt{cx^2+a}}+$$

$$\frac{2g^2\sqrt{cx^2+a}e}{(ef-dg)^2(cf^2+ag^2)\sqrt{f+gx}}-\frac{2\sqrt{c}g^2\sqrt{f+gx}\sqrt{cx^2+a}e}{(ef-dg)^2(cf^2+ag^2)^{3/2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)}+$$

$$\frac{8c^{5/4}f\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{3(ef-dg)(cf^2+ag^2)^{5/4}\sqrt{cx^2+a}}+$$

$$\frac{c^{3/4}\left(cf^2-4\sqrt{c}\sqrt{cf^2+ag^2}f+ag^2\right)\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right.\right.}{3(ef-dg)(cf^2+ag^2)^{7/4}\sqrt{cx^2+a}}$$

$$\frac{8cfg^2\sqrt{cx^2+a}}{3(ef-dg)(cf^2+ag^2)^2\sqrt{f+gx}}+\frac{2g^2\sqrt{cx^2+a}}{3(ef-dg)(cf^2+ag^2)(f+gx)^{3/2}}-\frac{8c^{3/2}fg^2\sqrt{f+gx}\sqrt{cx^2+a}}{3(ef-dg)(cf^2+ag^2)^{5/2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)}$$

input
```
Int[1/((d + e*x)*(f + g*x)^(5/2)*Sqrt[a + c*x^2]),x]
```

3.653. $\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx$

output
```
(2*g^2*Sqrt[a + c*x^2])/(3*(e*f - d*g)*(c*f^2 + a*g^2)*(f + g*x)^(3/2)) +
(8*c*f*g^2*Sqrt[a + c*x^2])/(3*(e*f - d*g)*(c*f^2 + a*g^2)^2*Sqrt[f + g*x]
) + (2*e*g^2*Sqrt[a + c*x^2])/((e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[f + g*x]
) - (8*c^(3/2)*f*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*(e*f - d*g)*(c*f^2
+ a*g^2)^(5/2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])) - (2*Sqrt[c]
*e*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((e*f - d*g)^2*(c*f^2 + a*g^2)^(3/2)
*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])) - (e^(5/2)*ArcTanh[(Sqrt[c
*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[e]*Sqrt[e*f - d*g]*Sqrt[a + c*x^2])])/(
Sqrt[c*d^2 + a*e^2]*(e*f - d*g)^(5/2)) + (8*c^(5/4)*f*Sqrt[(g^2*(a + c*x^2
))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*(1 +
 (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*EllipticE[2*ArcTan[(c^(1/4)*Sqrt
[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2
])/(3*(e*f - d*g)*(c*f^2 + a*g^2)^(5/4)*Sqrt[a + c*x^2]) + (2*c^(1/4)*e*Sq
rt[(g^2*(a + c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2
+ a*g^2])^2)]*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*EllipticE[2*Ar
cTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt
[c*f^2 + a*g^2])/2])/((e*f - d*g)^2*(c*f^2 + a*g^2)^(1/4)*Sqrt[a + c*x^2])
 - (c^(1/4)*e*Sqrt[(g^2*(a + c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g
*x))/Sqrt[c*f^2 + a*g^2])^2)]*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2]
)*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1...
```

3.653.3.1 Defintions of rubi rules used

rule 740
```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && Inte
gerQ[n + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.653.4 Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 1079, normalized size of antiderivative = 1.32

method	result	size
elliptic	Expression too large to display	1079
default	Expression too large to display	9415

input
```
int(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-2/3/(a*g^2+c*f^2
)/(d*g-e*f)*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(x+f/g)^2+2/3*(c*g*x^2+a*g)/
(a*g^2+c*f^2)^2*g*(3*a*e*g^2-4*c*d*f*g+7*c*e*f^2)/(d*g-e*f)^2/((x+f/g)*(c*
g*x^2+a*g))^(1/2)+2*(-1/3*c*g/(a*g^2+c*f^2)/(d*g-e*f)-1/3*c*f*g*(3*a*e*g^2
-4*c*d*f*g+7*c*e*f^2)/(a*g^2+c*f^2)^2/(d*g-e*f)^2)*(f/g-(-a*c)^(1/2)/c)*((
x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/
c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^
2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(
-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))-2/3*g^2*c*(3*a*e*g^2-4*c*d*f*
g+7*c*e*f^2)/(a*g^2+c*f^2)^2/(d*g-e*f)^2*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/
g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*
((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*
f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(
1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*E
llipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g
-(-a*c)^(1/2)/c))^(1/2)))+2*e/(d*g-e*f)^2*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f
/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)
*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a
*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g
+(-a*c)^(1/2)/c)/(-f/g+d/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c...
```

3.653.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.653.6 Sympy [F]

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{a+cx^2}\,(d+ex)\,(f+gx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)**(5/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**2)*(d + e*x)*(f + g*x)**(5/2)), x)
```

3.653.7 Maxima [F]

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)(gx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*(g*x + f)^(5/2)), x)
```

3.653.8 Giac [F]

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)(gx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*(g*x + f)^(5/2)), x)
```

3.653.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{(f+g\,x)^{5/2}\,\sqrt{c\,x^2+a}\,(d+e\,x)}\,dx$$

input
```
int(1/((f + g*x)^(5/2)*(a + c*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int(1/((f + g*x)^(5/2)*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.654 $\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx$

3.654.1 Optimal result

Integrand size = 28, antiderivative size = 110

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = -\frac{2\sqrt{\frac{\sqrt{-c}(f+gx)}{\sqrt{-c}f+g}}\,\text{EllipticPi}\left(\frac{2e}{\sqrt{-c}d+e},\arcsin\left(\frac{\sqrt{1-\sqrt{-c}x}}{\sqrt{2}}\right),\frac{2g}{\sqrt{-c}f+g}\right)}{\left(\sqrt{-c}d+e\right)\sqrt{f+gx}}$$

output
```
-2*EllipticPi(1/2*(1-x*(-c)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*(-c)^(1/2)),2^(1
/2)*(g/(g+f*(-c)^(1/2)))^(1/2))*((g*x+f)*(-c)^(1/2)/(g+f*(-c)^(1/2)))^(1/2
)/(e+d*(-c)^(1/2))/(g*x+f)^(1/2)
```

3.654.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.00 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.37

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = -\frac{2i\sqrt{\frac{g\left(\frac{i}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{ig}{\sqrt{c}}-gx}{f+gx}}(f+gx)\left(\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{-f-\frac{ig}{\sqrt{c}}}}{\sqrt{f+gx}}\right),\frac{\sqrt{c}f-ig}{\sqrt{c}f+ig}\right)-\text{EllipticPi}\left(\frac{\sqrt{c}(ef-dg)}{e(\sqrt{c}f+ig)},i\text{arc}\right.\right.}{\sqrt{-f-\frac{ig}{\sqrt{c}}}(ef-dg)\sqrt{1+cx^2}}$$

input
```
Integrate[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 + c*x^2]),x]
```

output
```
((-2*I)*Sqrt[(g*(I/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*g)/Sqrt[c] - g*x)/(
f + g*x))]*(f + g*x)*(EllipticF[I*ArcSinh[Sqrt[-f - (I*g)/Sqrt[c]]/Sqrt[f
+ g*x]], (Sqrt[c]*f - I*g)/(Sqrt[c]*f + I*g)] - EllipticPi[(Sqrt[c]*(e*f -
 d*g))/(e*(Sqrt[c]*f + I*g)), I*ArcSinh[Sqrt[-f - (I*g)/Sqrt[c]]/Sqrt[f +
g*x]], (Sqrt[c]*f - I*g)/(Sqrt[c]*f + I*g)]))/(Sqrt[-f - (I*g)/Sqrt[c]]*(e
*f - d*g)*Sqrt[1 + c*x^2])
```

3.654.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 826 vs. 2(110) = 220.

Time = 1.64 (sec) , antiderivative size = 826, normalized size of antiderivative = 7.51, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {729, 25, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx^2+1}(d+ex)\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{729}$$

$$2\int -\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+1}}d\sqrt{f+gx}$$

$$\Big\downarrow \text{25}$$

$$-2\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+1}}d\sqrt{f+gx}$$

$$\Big\downarrow \text{1540}$$

$$2\left(\frac{e\sqrt{cf^2+g^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+g^2}\right)\int \frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+g^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+1}}d\sqrt{f+gx}}{g\left(cd(2ef-dg)+e^2g\right)}-\frac{\sqrt{c}\left(-\sqrt{c}\sqrt{\right.}}{}\right.$$

$$\Big\downarrow \text{1416}$$

$$2\left(\frac{e\sqrt{cf^2+g^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+g^2}\right)\int \frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+g^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+1}}d\sqrt{f+gx}}{g\left(cd(2ef-dg)+e^2g\right)}-\frac{\sqrt[4]{c}\left(\frac{\sqrt{c}(f+g}{\sqrt{cf^2+g}}\right.}{}\right.$$

$\downarrow$ 2222

$$2\left(\frac{e\sqrt{cf^2+g^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+g^2}\right)\left(\frac{\left(e+\frac{\sqrt{c}(ef-dg)}{\sqrt{cf^2+g^2}}\right)\text{arctanh}\left(\frac{\sqrt{cd^2+e^2}\sqrt{f+gx}}{\sqrt{e}\sqrt{ef-dg}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+1}}\right)}{2\sqrt{e}\sqrt{cd^2+e^2}\sqrt{ef-dg}}-\frac{\left(\frac{\sqrt{c}}{e}-\right.}{}\right.}{g\left(ge^2+\right.}\right.$$

input
```
Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 + c*x^2]),x]
```

3.654. $\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx$

output
```
2*(-1/2*(c^(1/4)*(c*e*f^2 + e*g^2 - Sqrt[c]*(e*f - d*g)*Sqrt[c*f^2 + g^2])
*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + g^2])*Sqrt[(1 + (c*f^2)/g^2 - (2*c*
f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((1 + (c*f^2)/g^2)*(1 + (Sqrt[c]*(
f + g*x))/Sqrt[c*f^2 + g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x]
)/(c*f^2 + g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + g^2])/2])/(g*(c*f^2
+ g^2)^(1/4)*(e^2*g + c*d*(2*e*f - d*g))*Sqrt[1 + (c*f^2)/g^2 - (2*c*f*(f
+ g*x))/g^2 + (c*(f + g*x)^2)/g^2]) + (e*Sqrt[c*f^2 + g^2]*(Sqrt[c]*(e*f -
 d*g) - e*Sqrt[c*f^2 + g^2])*(((e + (Sqrt[c]*(e*f - d*g))/Sqrt[c*f^2 + g^2
])*ArcTanh[(Sqrt[c*d^2 + e^2]*Sqrt[f + g*x])/(Sqrt[e]*Sqrt[e*f - d*g]*Sqrt
[1 + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])])/(2*Sqrt
[e]*Sqrt[c*d^2 + e^2]*Sqrt[e*f - d*g]) - ((Sqrt[c]/e - Sqrt[c*f^2 + g^2]/(
e*f - d*g))*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + g^2])*Sqrt[(1 + (c*f^2)/
g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((1 + (c*f^2)/g^2)*(1 +
 (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + g^2])^2)]*EllipticPi[(Sqrt[c]*(e*f - d*g
) + e*Sqrt[c*f^2 + g^2])^2/(4*Sqrt[c]*e*(e*f - d*g)*Sqrt[c*f^2 + g^2]), 2*
ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt
[c*f^2 + g^2])/2])/(4*c^(1/4)*(c*f^2 + g^2)^(1/4)*Sqrt[1 + (c*f^2)/g^2 - (
2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/(g*(e^2*g + c*d*(2*e*f - d*
g))))
```

3.654.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

3.654. $\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx$

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

3.654.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(95) = 190$.

Time = 1.81 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.95

method	result	size
default	$\frac{2\left(g+f\sqrt{-c}\right)\Pi\left(\sqrt{\frac{(gx+f)\sqrt{-c}}{g+f\sqrt{-c}}},-\frac{\left(g+f\sqrt{-c}\right)e}{\sqrt{-c}\,(dg-ef)},\sqrt{\frac{g+f\sqrt{-c}}{f\sqrt{-c}-g}}\right)\sqrt{-\frac{\left(x\sqrt{-c}-1\right)g}{g+f\sqrt{-c}}}\,\sqrt{-\frac{\left(x\sqrt{-c}+1\right)g}{f\sqrt{-c}-g}}\,\sqrt{\frac{(gx+f)\sqrt{-c}}{g+f\sqrt{-c}}}\,\sqrt{c\,x^{2}+1}\,\sqrt{gx+f}}{\sqrt{-c}\,(dg-ef)\left(cg\,x^{3}+cf\,x^{2}+gx+f\right)}$	215
elliptic	$\frac{2\sqrt{(gx+f)\left(c\,x^{2}+1\right)}\left(\frac{f}{g}+\frac{1}{\sqrt{-c}}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}+\frac{1}{\sqrt{-c}}}}\,\sqrt{\frac{x+\frac{1}{\sqrt{-c}}}{-\frac{f}{g}+\frac{1}{\sqrt{-c}}}}\,\sqrt{\frac{x-\frac{1}{\sqrt{-c}}}{-\frac{f}{g}-\frac{1}{\sqrt{-c}}}}\,\Pi\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}+\frac{1}{\sqrt{-c}}}},\frac{-\frac{f}{g}-\frac{1}{\sqrt{-c}}}{-\frac{f}{g}+\frac{d}{e}},\sqrt{\frac{-\frac{f}{g}-\frac{1}{\sqrt{-c}}}{-\frac{f}{g}+\frac{1}{\sqrt{-c}}}}\right)}{\sqrt{gx+f}\,\sqrt{c\,x^{2}+1}\,e\sqrt{cg\,x^{3}+cf\,x^{2}+gx+f}\,\left(-\frac{f}{g}+\frac{d}{e}\right)}$	240

input
```
int(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(g+f*(-c)^(1/2))/(-c)^(1/2)*EllipticPi(((g*x+f)*(-c)^(1/2)/(g+f*(-c)^(1/
2)))^(1/2),-(g+f*(-c)^(1/2))*e/(-c)^(1/2)/(d*g-e*f),((g+f*(-c)^(1/2))/(f*(
-c)^(1/2)-g))^(1/2))*(-(x*(-c)^(1/2)-1)*g/(g+f*(-c)^(1/2)))^(1/2)*(-(x*(-c
)^(1/2)+1)*g/(f*(-c)^(1/2)-g))^(1/2)*((g*x+f)*(-c)^(1/2)/(g+f*(-c)^(1/2)))
^(1/2)*(c*x^2+1)^(1/2)*(g*x+f)^(1/2)/(d*g-e*f)/(c*g*x^3+c*f*x^2+g*x+f)
```

3.654.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.654.6 Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = \int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+1}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)**(1/2)/(c*x**2+1)**(1/2),x)
```

output
```
Integral(1/((d + e*x)*sqrt(f + g*x)*sqrt(c*x**2 + 1)), x)
```

3.654.7 Maxima [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+1}(ex+d)\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + 1)*(e*x + d)*sqrt(g*x + f)), x)
```

3.654.8 Giac [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+1}(ex+d)\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + 1)*(e*x + d)*sqrt(g*x + f)), x)
```

3.654.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+1}\,(d+e\,x)}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(c*x^2 + 1)^(1/2)*(d + e*x)),x)
```

output
```
int(1/((f + g*x)^(1/2)*(c*x^2 + 1)^(1/2)*(d + e*x)), x)
```

3.655 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

3.655.1 Optimal result

Integrand size = 30, antiderivative size = 454

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{\sqrt[4]{cf^2+ag^2}(d+ex)\sqrt{\frac{(ef-dg)^2(a+cx^2)}{(cf^2+ag^2)(d+ex)^2}}\left(1+\frac{\sqrt{cd^2+ae^2}(f+gx)}{\sqrt{cf^2+ag^2}(d+ex)}\right)\sqrt{\frac{1-\frac{2(cdf+aeg)(f+gx)}{(cf^2+ag^2)(d+ex)}+\frac{(cd^2+ae^2)(f+gx)^2}{(cf^2+ag^2)(d+ex)^2}}{\left(1+\frac{\sqrt{cd^2+ae^2}(f+gx)}{\sqrt{cf^2+ag^2}(d+ex)}\right)^2}}\text{EllipticF}\left(\ldots\right.}{\sqrt[4]{cd^2+ae^2}(ef-dg)\sqrt{a+cx^2}\sqrt{1-\frac{2(cdf+aeg)(f+gx)}{(cf^2+ag^2)(d+ex)}+\frac{(c\ldots}{(c\ldots}}}$$

output
```
-(a*g^2+c*f^2)^(1/4)*(e*x+d)*(cos(2*arctan((a*e^2+c*d^2)^(1/4)*(g*x+f)^(1/
2)/(a*g^2+c*f^2)^(1/4)/(e*x+d)^(1/2)))^2)^(1/2)/cos(2*arctan((a*e^2+c*d^2)
^(1/4)*(g*x+f)^(1/2)/(a*g^2+c*f^2)^(1/4)/(e*x+d)^(1/2)))*EllipticF(sin(2*a
rctan((a*e^2+c*d^2)^(1/4)*(g*x+f)^(1/2)/(a*g^2+c*f^2)^(1/4)/(e*x+d)^(1/2))
),1/2*(2+2*(a*e*g+c*d*f)/(a*e^2+c*d^2)^(1/2)/(a*g^2+c*f^2)^(1/2))^(1/2))*(
1+(g*x+f)*(a*e^2+c*d^2)^(1/2)/(e*x+d)/(a*g^2+c*f^2)^(1/2))*((-d*g+e*f)^2*(
c*x^2+a)/(a*g^2+c*f^2)/(e*x+d)^2)^(1/2)*((1-2*(a*e*g+c*d*f)*(g*x+f)/(a*g^2
+c*f^2)/(e*x+d)+(a*e^2+c*d^2)*(g*x+f)^2/(a*g^2+c*f^2)/(e*x+d)^2)/(1+(g*x+f
)*(a*e^2+c*d^2)^(1/2)/(e*x+d)/(a*g^2+c*f^2)^(1/2))^2)^(1/2)/(a*e^2+c*d^2)^
(1/4)/(-d*g+e*f)/(c*x^2+a)^(1/2)/(1-2*(a*e*g+c*d*f)*(g*x+f)/(a*g^2+c*f^2)/
(e*x+d)+(a*e^2+c*d^2)*(g*x+f)^2/(a*g^2+c*f^2)/(e*x+d)^2)^(1/2)
```

3.655.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.18 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$
$$= \frac{\sqrt{2}\left(i\sqrt{a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{\frac{d-\frac{i\sqrt{a}e}{\sqrt{c}}+\frac{i\sqrt{c}dx}{\sqrt{a}}+ex}{d+ex}}\sqrt{\frac{(i\sqrt{c}d+\sqrt{a}e)(f+gx)}{(i\sqrt{c}f+\sqrt{a}g)(d+ex)}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(ef-dg)(i\sqrt{a}+\sqrt{c}x)}{(\sqrt{c}f-i\sqrt{a}g)(d+ex)}}\right), -\right.}{\left(\sqrt{c}d-i\sqrt{a}e\right)\sqrt{\frac{(ef-dg)(i\sqrt{a}+\sqrt{c}x)}{(\sqrt{c}f-i\sqrt{a}g)(d+ex)}}\sqrt{f+gx}\sqrt{a+cx^2}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
(Sqrt[2]*(I*Sqrt[a] + Sqrt[c]*x)*Sqrt[d + e*x]*Sqrt[(d - (I*Sqrt[a]*e)/Sqr
t[c] + (I*Sqrt[c]*d*x)/Sqrt[a] + e*x)/(d + e*x)]*Sqrt[((I*Sqrt[c]*d + Sqrt
[a]*e)*(f + g*x))/((I*Sqrt[c]*f + Sqrt[a]*g)*(d + e*x))]*EllipticF[ArcSin[
Sqrt[((e*f - d*g)*(I*Sqrt[a] + Sqrt[c]*x))/((Sqrt[c]*f - I*Sqrt[a]*g)*(d +
 e*x))]], -(((I*Sqrt[c]*d*f)/Sqrt[a] - e*f + d*g + (I*Sqrt[a]*e*g)/Sqrt[c]
)/(2*e*f - 2*d*g))])/((Sqrt[c]*d - I*Sqrt[a]*e)*Sqrt[((e*f - d*g)*(I*Sqrt[
a] + Sqrt[c]*x))/((Sqrt[c]*f - I*Sqrt[a]*g)*(d + e*x))]*Sqrt[f + g*x]*Sqrt
[a + c*x^2])
```

3.655.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {732, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^2}\sqrt{d+ex}\sqrt{f+gx}}\,dx$$
$$\Big\downarrow \text{732}$$
$$-\frac{2(d+ex)\sqrt{\frac{(a+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2+cf^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2+ae^2\right)(f+gx)^2}{\left(cf^2+ag^2\right)(d+ex)^2}-\frac{2(cdf+aeg)(f+gx)}{\left(cf^2+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a+cx^2}(ef-dg)}$$

$\big\downarrow$ 1416

$$-\frac{(d+ex)\sqrt[4]{ag^2+cf^2}\sqrt{\frac{(a+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2+cd^2}}{(d+ex)\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{\frac{(f+gx)^2(ae^2+cd^2)}{(d+ex)^2(ag^2+cf^2)}-\frac{2(f+gx)(aeg+cdf)}{(d+ex)(ag^2+cf^2)}+1}{\left(\frac{(f+gx)\sqrt{ae^2+cd^2}}{(d+ex)\sqrt{ag^2+cf^2}}+1\right)^2}}\text{EllipticF}\left(2\text{arc}\right.}{\sqrt{a+cx^2}\sqrt[4]{ae^2+cd^2}(ef-dg)\sqrt{\frac{(f+gx)^2(ae^2+cd^2)}{(d+ex)^2(ag^2+cf^2)}-\frac{2(f+gx)(aeg+}{(d+ex)(ag^2+c}}}$$

input
```
Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
-(((c*f^2 + a*g^2)^(1/4)*(d + e*x)*Sqrt[((e*f - d*g)^2*(a + c*x^2))/((c*f^
2 + a*g^2)*(d + e*x)^2)]*(1 + (Sqrt[c*d^2 + a*e^2]*(f + g*x))/(Sqrt[c*f^2
+ a*g^2]*(d + e*x)))*Sqrt[(1 - (2*(c*d*f + a*e*g)*(f + g*x))/((c*f^2 + a*g
^2)*(d + e*x)) + ((c*d^2 + a*e^2)*(f + g*x)^2)/((c*f^2 + a*g^2)*(d + e*x)^
2))/(1 + (Sqrt[c*d^2 + a*e^2]*(f + g*x))/(Sqrt[c*f^2 + a*g^2]*(d + e*x)))^
2]*EllipticF[2*ArcTan[((c*d^2 + a*e^2)^(1/4)*Sqrt[f + g*x])/((c*f^2 + a*g^
2)^(1/4)*Sqrt[d + e*x])], (1 + (c*d*f + a*e*g)/(Sqrt[c*d^2 + a*e^2]*Sqrt[c
*f^2 + a*g^2]))/2])/((c*d^2 + a*e^2)^(1/4)*(e*f - d*g)*Sqrt[a + c*x^2]*Sqr
t[1 - (2*(c*d*f + a*e*g)*(f + g*x))/((c*f^2 + a*g^2)*(d + e*x)) + ((c*d^2
+ a*e^2)*(f + g*x)^2)/((c*f^2 + a*g^2)*(d + e*x)^2)]))
```

3.655.3.1 Defintions of rubi rules used

rule 732
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(a_) + (b_.)*
(x_)^2]), x_Symbol] :> Simp[-2*(c + d*x)*(Sqrt[(d*e - c*f)^2*((a + b*x^2)/(
(b*e^2 + a*f^2)*(c + d*x)^2))]/((d*e - c*f)*Sqrt[a + b*x^2]))   Subst[Int[1
/Sqrt[Simp[1 - (2*b*c*e + 2*a*d*f)*(x^2/(b*e^2 + a*f^2)) + (b*c^2 + a*d^2)*
(x^4/(b*e^2 + a*f^2)), x]], x], x, Sqrt[e + f*x]/Sqrt[c + d*x]], x] /; Free
Q[{a, b, c, d, e, f}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

3.655.4 Maple [A] (verified)

Time = 5.08 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.88

method	result
default	$\frac{2\left(c e^{2} f x^{2}-\sqrt{-a c}\, e^{2} g x^{2}+2 c d e f x-2 \sqrt{-a c}\, d e g x+c d^{2} f-\sqrt{-a c}\, d^{2} g\right) F\left(\sqrt{\frac{\left(e \sqrt{-a c}-c d\right)\left(g x+f\right)}{\left(g \sqrt{-a c}-c f\right)\left(e x+d\right)}}, \sqrt{\frac{\left(e \sqrt{-a c}+c d\right)\left(g \sqrt{-a c}-c f\right)}{\left(g \sqrt{-a c}+c f\right)\left(e \sqrt{-a c}-c d\right)}}\right) \sqrt{\frac{\left(d g-e f\right)}{\left(g \sqrt{-a c}\right.}}}{\sqrt{-\frac{\left(g x+f\right)\left(e x+d\right)\left(-c x+\sqrt{-a c}\right)\left(c x+\sqrt{-a c}\right)}{c}}\, \left(-e \sqrt{-a c}+c d\right)\left(d g-e f\right) \sqrt{(g x}}$
elliptic	$\frac{2 \sqrt{\left(g x+f\right)\left(e x+d\right)\left(c x^{2}+a\right)}\, \left(-\frac{f}{g}+\frac{\sqrt{-a c}}{c}\right) \sqrt{\frac{\left(\frac{d}{e}-\frac{\sqrt{-a c}}{c}\right)\left(x+\frac{f}{g}\right)}{\left(\frac{f}{g}-\frac{\sqrt{-a c}}{c}\right)\left(x+\frac{d}{e}\right)}}\, \left(x+\frac{d}{e}\right)^{2} \sqrt{\frac{\left(-\frac{d}{e}+\frac{f}{g}\right)\left(x-\frac{\sqrt{-a c}}{c}\right)}{\left(\frac{f}{g}+\frac{\sqrt{-a c}}{c}\right)\left(x+\frac{d}{e}\right)}}\, \sqrt{\frac{\left(-\frac{d}{e}+\frac{f}{g}\right)\left(x+\frac{\sqrt{-a c}}{c}\right)}{\left(\frac{f}{g}-\frac{\sqrt{-a c}}{c}\right)\left(x+\frac{d}{e}\right)}}\, F\left(\sqrt{\ldots}\right.}{\sqrt{g x+f}\, \sqrt{e x+d}\, \sqrt{c x^{2}+a}\, \left(\frac{d}{e}-\frac{\sqrt{-a c}}{c}\right)\left(-\frac{d}{e}+\frac{f}{g}\right) \sqrt{c e g \left(x+\frac{f}{g}\right)\left(x+\frac{d}{e}\right)\left(x-\frac{\sqrt{-a c}}{c}\right)\left(x+\ldots\right.}}$

input
```
int(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(c*e^2*f*x^2-(-a*c)^(1/2)*e^2*g*x^2+2*c*d*e*f*x-2*(-a*c)^(1/2)*d*e*g*x+c
*d^2*f-(-a*c)^(1/2)*d^2*g)*EllipticF(((e*(-a*c)^(1/2)-c*d)*(g*x+f)/(g*(-a*
c)^(1/2)-c*f)/(e*x+d))^(1/2),((e*(-a*c)^(1/2)+c*d)*(g*(-a*c)^(1/2)-c*f)/(g
*(-a*c)^(1/2)+c*f)/(e*(-a*c)^(1/2)-c*d))^(1/2))*((d*g-e*f)*(c*x+(-a*c)^(1/
2))/(g*(-a*c)^(1/2)-c*f)/(e*x+d))^(1/2)*((d*g-e*f)*(-c*x+(-a*c)^(1/2))/(g*
(-a*c)^(1/2)+c*f)/(e*x+d))^(1/2)*((e*(-a*c)^(1/2)-c*d)*(g*x+f)/(g*(-a*c)^(
1/2)-c*f)/(e*x+d))^(1/2)*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)*(e*x+d)^(1/2)/(-1/c
*(g*x+f)*(e*x+d)*(-c*x+(-a*c)^(1/2))*(c*x+(-a*c)^(1/2)))^(1/2)/(-e*(-a*c)^
(1/2)+c*d)/(d*g-e*f)/((g*x+f)*(e*x+d)*(c*x^2+a))^(1/2)
```

3.655.5 Fricas [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\, dx = \int \frac{1}{\sqrt{cx^2+a}\sqrt{ex+d}\sqrt{gx+f}}\, dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)/(c*e*g*x^4 + (c*e*f +
 c*d*g)*x^3 + a*d*f + (c*d*f + a*e*g)*x^2 + (a*e*f + a*d*g)*x), x)
```

3.655.6 Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{a+cx^2}\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**2)*sqrt(d + e*x)*sqrt(f + g*x)), x)
```

3.655.7 Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

3.655.8 Giac [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

3.655.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx=\int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,\sqrt{d+e\,x}}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^(1/2)),x)
```

output
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^(1/2)), x)
```

3.656 $\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx$

3.656.1 Optimal result

Integrand size = 26, antiderivative size = 52

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = \frac{\sqrt{1-2x^2}\sqrt{1-x^2}\,\text{EllipticF}(\arcsin(x),2)}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}$$

output
```
EllipticF(x,2^(1/2))*(-2*x^2+1)^(1/2)*(-x^2+1)^(1/2)/(-1+x)^(1/2)/(1+x)^(1
/2)/(2*x^2-1)^(1/2)
```

3.656.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 107 vs. $2(52) = 104$.

Time = 34.62 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.06

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = -\frac{2(-1+x)^{3/2}\sqrt{\frac{1+x}{1-x}}\sqrt{\frac{1-2x^2}{(-1+x)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2+\sqrt{2}+\frac{1}{-1+x}}}{2^{3/4}}\right),4\left(-4+3\sqrt{2}\right)\right)}{\sqrt{3+2\sqrt{2}}\sqrt{1+x}\sqrt{-1+2x^2}}$$

input
```
Integrate[1/(Sqrt[-1 + x]*Sqrt[1 + x]*Sqrt[-1 + 2*x^2]),x]
```

output
```
(-2*(-1 + x)^(3/2)*Sqrt[(1 + x)/(1 - x)]*Sqrt[(1 - 2*x^2)/(-1 + x)^2]*Elli
pticF[ArcSin[Sqrt[2 + Sqrt[2] + (-1 + x)^(-1)]/2^(3/4)], 4*(-4 + 3*Sqrt[2]
)])/(Sqrt[3 + 2*Sqrt[2]]*Sqrt[1 + x]*Sqrt[-1 + 2*x^2])
```

3.656.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {648, 323, 323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x-1}\sqrt{x+1}\sqrt{2x^2-1}}dx$$

$$\Big\downarrow \text{ 648}$$

$$\frac{\sqrt{x^2-1}\int \frac{1}{\sqrt{x^2-1}\sqrt{2x^2-1}}dx}{\sqrt{x-1}\sqrt{x+1}}$$

$$\Big\downarrow \text{ 323}$$

$$\frac{\sqrt{1-2x^2}\sqrt{x^2-1}\int \frac{1}{\sqrt{1-2x^2}\sqrt{x^2-1}}dx}{\sqrt{x-1}\sqrt{x+1}\sqrt{2x^2-1}}$$

$$\Big\downarrow \text{ 323}$$

$$\frac{\sqrt{1-2x^2}\sqrt{1-x^2}\int \frac{1}{\sqrt{1-2x^2}\sqrt{1-x^2}}dx}{\sqrt{x-1}\sqrt{x+1}\sqrt{2x^2-1}}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{\sqrt{1-2x^2}\sqrt{1-x^2}\,\text{EllipticF}(\arcsin(x),2)}{\sqrt{x-1}\sqrt{x+1}\sqrt{2x^2-1}}$$

input
```
Int[1/(Sqrt[-1 + x]*Sqrt[1 + x]*Sqrt[-1 + 2*x^2]),x]
```

output
```
(Sqrt[1 - 2*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[x], 2])/(Sqrt[-1 + x]*Sqrt
[1 + x]*Sqrt[-1 + 2*x^2])
```

3.656.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

3.656.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\sqrt{-1+x}\,\sqrt{1+x}\,\sqrt{2x^{2}-1}\,\sqrt{-x^{2}+1}\,\sqrt{-2x^{2}+1}\,F\left(x,\sqrt{2}\right)}{2x^{4}-3x^{2}+1}$	58
elliptic	$\frac{\sqrt{\left(2x^{2}-1\right)\left(x^{2}-1\right)}\,\sqrt{-x^{2}+1}\,\sqrt{-2x^{2}+1}\,F\left(x,\sqrt{2}\right)}{\sqrt{-1+x}\,\sqrt{1+x}\,\sqrt{2x^{2}-1}\,\sqrt{2x^{4}-3x^{2}+1}}$	73

input
```
int(1/(-1+x)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-1+x)^(1/2)*(1+x)^(1/2)*(2*x^2-1)^(1/2)/(2*x^4-3*x^2+1)*(-x^2+1)^(1/2)*(-
2*x^2+1)^(1/2)*EllipticF(x,2^(1/2))
```

3.656.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.08

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = F(\arcsin{(x)}\,|2)$$

input
```
integrate(1/(-1+x)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="fricas"
)
```

output
```
elliptic_f(arcsin(x), 2)
```

3.656.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = \int \frac{1}{\sqrt{x-1}\sqrt{x+1}\sqrt{2x^2-1}}\,dx$$

input
```
integrate(1/(-1+x)**(1/2)/(1+x)**(1/2)/(2*x**2-1)**(1/2),x)
```

output
```
Integral(1/(sqrt(x - 1)*sqrt(x + 1)*sqrt(2*x**2 - 1)), x)
```

3.656.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2-1}\sqrt{x+1}\sqrt{x-1}}\,dx$$

input
```
integrate(1/(-1+x)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(2*x^2 - 1)*sqrt(x + 1)*sqrt(x - 1)), x)
```

3.656.8 Giac [F]

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2-1}\sqrt{x+1}\sqrt{x-1}}\,dx$$

input
```
integrate(1/(-1+x)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(2*x^2 - 1)*sqrt(x + 1)*sqrt(x - 1)), x)
```

3.656.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2-1}\,\sqrt{x-1}\,\sqrt{x+1}}\,dx$$

input
```
int(1/((2*x^2 - 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)),x)
```

output
```
int(1/((2*x^2 - 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)), x)
```

3.657 $\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.657.1 Optimal result

Integrand size = 46, antiderivative size = 269

$$\begin{aligned}
&\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx \\
&= -\frac{16(cdf-aeg)^2(2ae^2g-cd(3ef-dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35c^4d^4e\sqrt{d+ex}} \\
&\quad+\frac{16g(cdf-aeg)^2\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35c^3d^3e} \\
&\quad+\frac{12(cdf-aeg)(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35c^2d^2\sqrt{d+ex}} \\
&\quad+\frac{2(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{7cd\sqrt{d+ex}}
\end{aligned}$$

output
```
-16/35*(-a*e*g+c*d*f)^2*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)/c^4/d^4/e/(e*x+d)^(1/2)+12/35*(-a*e*g+c*d*f)*(g*x+f)^2*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)+2/7*(g*x+f)^
3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)+16/35*g*(-a*e*
g+c*d*f)^2*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e
```

3.657.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(-16a^3e^3g^3+8a^2cde^2g^2(7f+gx)-2ac^2d^2eg(35f^2+14fgx+3g^2x^2)+c^3d^3(35f^3+35f^2gx+21fg^2x^2+5g^3x^3))}{35c^4d^4\sqrt{d+ex}}$$

input
```
Integrate[(Sqrt[d + e*x]*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2],x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(7*f
 + g*x) - 2*a*c^2*d^2*e*g*(35*f^2 + 14*f*g*x + 3*g^2*x^2) + c^3*d^3*(35*f^
3 + 35*f^2*g*x + 21*f*g^2*x^2 + 5*g^3*x^3)))/(35*c^4*d^4*Sqrt[d + e*x])
```

3.657.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1253, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \ 1253$$

$$\frac{6(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{7cd}+\frac{2(f+gx)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{7cd\sqrt{d+ex}}$$

$$\Big\downarrow \ 1253$$

$$\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{5cd}+\frac{2(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5cd\sqrt{d+ex}}\right)}{7cd}+\frac{2(f+gx)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{7cd\sqrt{d+ex}}$$

$$\Big\downarrow \; 1221$$

$$\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\left(\frac{1}{3}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)\int \frac{\sqrt{d+ex}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{2g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}\right)}{5cd}+\frac{2(f+gx)^2\sqrt{x(ae^2+}}{5cd\sqrt{d}}\right.}{7cd}$$

$$\frac{2(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7cd\sqrt{d+ex}}$$

$$\Big\downarrow \; 1122$$

$$\frac{2(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7cd\sqrt{d+ex}}+$$

$$\frac{6(cdf-aeg)\left(\frac{2(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5cd\sqrt{d+ex}}+\frac{4(cdf-aeg)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)}{3cd\sqrt{d+ex}}+\frac{2g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ad}}{3cde}\right.}{5cd}\right.}{7cd}$$

input

```
Int[(Sqrt[d + e*x]*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
],x]
```

output

```
(2*(f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*c*d*Sqrt[d
+ e*x]) + (6*(c*d*f - a*e*g)*((2*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])/(5*c*d*Sqrt[d + e*x]) + (4*(c*d*f - a*e*g)*((2*(3*f - (d*g
)/e - (2*a*e*g)/(c*d))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d
*Sqrt[d + e*x]) + (2*g*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(3*c*d*e)))/(5*c*d)))/(7*c*d)
```

3.657. $\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.657.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

3.657.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.63

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-5g^3x^3c^3d^3+6a\,c^2d^2e\,g^3x^2-21c^3d^3f\,g^2x^2-8a^2cd\,e^2g^3x+28a\,c^2d^2ef\,g^2x-35c^3d^3f^2gx+16a^3e^3g^3-56a^2cd}{35\sqrt{ex+d}\,c^4d^4}$
gosper	$-\frac{2(cdx+ae)(-5g^3x^3c^3d^3+6a\,c^2d^2e\,g^3x^2-21c^3d^3f\,g^2x^2-8a^2cd\,e^2g^3x+28a\,c^2d^2ef\,g^2x-35c^3d^3f^2gx+16a^3e^3g^3-56a^2cd\,e^2f\,g^2+7}{35c^4d^4\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$

input
```
int((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
-2/35/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-5*c^3*d^3*g^3*x^3+6*a*c^
2*d^2*e*g^3*x^2-21*c^3*d^3*f*g^2*x^2-8*a^2*c*d*e^2*g^3*x+28*a*c^2*d^2*e*f*
g^2*x-35*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-56*a^2*c*d*e^2*f*g^2+70*a*c^2*d^2*
e*f^2*g-35*c^3*d^3*f^3)/c^4/d^4
```

3.657.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{2\,(5\,c^3d^3g^3x^3+35\,c^3d^3f^3-70\,ac^2d^2ef^2g+56\,a^2cde^2fg^2-16\,a^3e^3g^3+3\,(7\,c^3d^3fg^2-2\,ac^2d^2eg^3)x^2+(35}{35\,(c^4d^4ex+c^4d^5)}$$

input
```
integrate((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output
```
2/35*(5*c^3*d^3*g^3*x^3 + 35*c^3*d^3*f^3 - 70*a*c^2*d^2*e*f^2*g + 56*a^2*c
*d*e^2*f*g^2 - 16*a^3*e^3*g^3 + 3*(7*c^3*d^3*f*g^2 - 2*a*c^2*d^2*e*g^3)*x^
2 + (35*c^3*d^3*f^2*g - 28*a*c^2*d^2*e*f*g^2 + 8*a^2*c*d*e^2*g^3)*x)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^
5)
```

3.657.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}\,(f+gx)^3}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((g*x+f)**3*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output
```
Integral(sqrt(d + e*x)*(f + g*x)**3/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.657.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{2\sqrt{cdx+ae}f^3}{cd} + \frac{2\,(c^2d^2x^2-acdex-2\,a^2e^2)f^2g}{\sqrt{cdx+ae}c^2d^2} + \frac{2\,(3\,c^3d^3x^3-ac^2d^2ex^2+4\,a^2cde^2x+8\,a^3e^3)fg^2}{5\sqrt{cdx+ae}c^3d^3} + \frac{2\,(5\,c^4d^4x^4-ac^3d^3ex^3+2\,a^2c^2d^2e^2x^2-8\,a^3cde^3x-16\,a^4e^4)g^3}{35\sqrt{cdx+ae}c^4d^4}$$

input
```
integrate((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
2*sqrt(c*d*x + a*e)*f^3/(c*d) + 2*(c^2*d^2*x^2 - a*c*d*e*x - 2*a^2*e^2)*f^
2*g/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/5*(3*c^3*d^3*x^3 - a*c^2*d^2*e*x^2 + 4
*a^2*c*d*e^2*x + 8*a^3*e^3)*f*g^2/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/35*(5*c^
4*d^4*x^4 - a*c^3*d^3*e*x^3 + 2*a^2*c^2*d^2*e^2*x^2 - 8*a^3*c*d*e^3*x - 16
*a^4*e^4)*g^3/(sqrt(c*d*x + a*e)*c^4*d^4)
```

3.657.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(245) = 490$.

Time = 0.31 (sec) , antiderivative size = 613, normalized size of antiderivative = 2.28

$$\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{2\,e\left(\frac{35\,(c^3d^3f^3-3\,ac^2d^2ef^2g+3\,a^2cde^2fg^2-a^3e^3g^3)\sqrt{(ex+d)cde-cd^2e+ae^3}}{c^4d^4e} - \frac{35\sqrt{-cd^2e+ae^3}c^3d^3e^3f^3-35\sqrt{-cd^2e+ae^3}c^3d^4e^2f^2g-70\sqrt{}\ldots}{}\right.}{}$$

input
```
integrate((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output
```
2/35*e*(35*(c^3*d^3*f^3 - 3*a*c^2*d^2*e*f^2*g + 3*a^2*c*d*e^2*f*g^2 - a^3*
e^3*g^3)*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/(c^4*d^4*e) - (35*sqrt(-c
*d^2*e + a*e^3)*c^3*d^3*e^3*f^3 - 35*sqrt(-c*d^2*e + a*e^3)*c^3*d^4*e^2*f^
2*g - 70*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^2*e^4*f^2*g + 21*sqrt(-c*d^2*e + a
*e^3)*c^3*d^5*e*f*g^2 + 28*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^3*e^3*f*g^2 + 56
*sqrt(-c*d^2*e + a*e^3)*a^2*c*d*e^5*f*g^2 - 5*sqrt(-c*d^2*e + a*e^3)*c^3*d
^6*g^3 - 6*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^4*e^2*g^3 - 8*sqrt(-c*d^2*e + a*
e^3)*a^2*c*d^2*e^4*g^3 - 16*sqrt(-c*d^2*e + a*e^3)*a^3*e^6*g^3)/(c^4*d^4*e
^4) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*e^4*f^2*g - 70
*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c*d*e^5*f*g^2 + 35*((e*x + d)
*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6*g^3 + 21*((e*x + d)*c*d*e - c*d^2*
e + a*e^3)^(5/2)*c*d*e^2*f*g^2 - 21*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5
/2)*a*e^3*g^3 + 5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*g^3)/(c^4*d^4*
e^7))/abs(e)
```

3.657.9 Mupad [B] (verification not implemented)

Time = 12.61 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(32\,a^3\,e^3\,g^3-112\,a^2\,c\,d\,e^2\,f\,g^2+140\,a\,c^2\,d^2\,e\,f^2\,g-70\,c^3\,d^3\,f^3\right)}{35\,c^4\,d^4\,e}-\frac{2\,g^3\,x^3\,\sqrt{d+e\,x}}{7\,c\,d\,e}+\dots\right.}{x+\frac{d}{e}}$$

input
```
int(((f + g*x)^3*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(1/2),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(32*a^3*
e^3*g^3 - 70*c^3*d^3*f^3 + 140*a*c^2*d^2*e*f^2*g - 112*a^2*c*d*e^2*f*g^2))
/(35*c^4*d^4*e) - (2*g^3*x^3*(d + e*x)^(1/2))/(7*c*d*e) + (6*g^2*x^2*(2*a*
e*g - 7*c*d*f)*(d + e*x)^(1/2))/(35*c^2*d^2*e) - (2*g*x*(d + e*x)^(1/2)*(8
*a^2*e^2*g^2 + 35*c^2*d^2*f^2 - 28*a*c*d*e*f*g))/(35*c^3*d^3*e)))/(x + d/e
)
```

3.658 $\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.658.1 Optimal result

Integrand size = 46, antiderivative size = 200

$$\begin{aligned}\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx &= -\frac{8(cdf-aeg)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{15c^3d^3e\sqrt{d+ex}} \\ &\quad + \frac{8g(cdf-aeg)\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{15c^2d^2e} \\ &\quad + \frac{2(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{5cd\sqrt{d+ex}}\end{aligned}$$

output
```
-8/15*(-a*e*g+c*d*f)*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)/c^3/d^3/e/(e*x+d)^(1/2)+2/5*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)+8/15*g*(-a*e*g+c*d*f)*(e*x+d)^(1/2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e
```

3.658.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(8a^2e^2g^2-4acdeg(5f+gx)+c^2d^2(15f^2+10fgx+3g^2x^2))}{15c^3d^3\sqrt{d+ex}}$$

input
```
Integrate[(Sqrt[d + e*x]*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2],x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(5*f + g*x)
+ c^2*d^2*(15*f^2 + 10*f*g*x + 3*g^2*x^2)))/(15*c^3*d^3*Sqrt[d + e*x])
```

3.658.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \text{1253}$$

$$\frac{4(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{5cd}+\frac{2(f+gx)^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{5cd\sqrt{d+ex}}$$

$$\Big\downarrow \text{1221}$$

$$\frac{4(cdf-aeg)\left(\frac{1}{3}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)\int \frac{\sqrt{d+ex}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\frac{2g\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cde}\right)}{5cd}+\frac{2(f+gx)^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{5cd\sqrt{d+ex}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5cd\sqrt{d+ex}}+\frac{4(cdf-aeg)\left(\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)}{3cd\sqrt{d+ex}}+\frac{2g\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cde}\right)}{5cd}$$

input
```
Int[(Sqrt[d + e*x]*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
],x]
```

output
```
(2*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c*d*Sqrt[d
+ e*x]) + (4*(c*d*f - a*e*g)*((2*(3*f - (d*g)/e - (2*a*e*g)/(c*d))*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*Sqrt[d + e*x]) + (2*g*Sqrt[d
+ e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*e)))/(5*c*d)
```

3.658.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

3.658. $\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.658.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(3g^2x^2c^2d^2-4acde\,g^2x+10c^2d^2fgx+8a^2e^2g^2-20acdefg+15c^2d^2f^2)}{15\sqrt{ex+d}\,c^3d^3}$	98
gosper	$\frac{2(cdx+ae)(3g^2x^2c^2d^2-4acde\,g^2x+10c^2d^2fgx+8a^2e^2g^2-20acdefg+15c^2d^2f^2)\sqrt{ex+d}}{15c^3d^3\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	116

input
```
int((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/15/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*c^2*d^2*g^2*x^2-4*a*c*d*
e*g^2*x+10*c^2*d^2*f*g*x+8*a^2*e^2*g^2-20*a*c*d*e*f*g+15*c^2*d^2*f^2)/c^3/
d^3
```

3.658.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{2\,(3\,c^2d^2g^2x^2+15\,c^2d^2f^2-20\,acdefg+8\,a^2e^2g^2+2\,(5\,c^2d^2fg-2\,acdeg^2)x)\sqrt{cdex^2+ade+(cd^2+ae^2)}}{15\,(c^3d^3ex+c^3d^4)}$$

input
```
integrate((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output
```
2/15*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 20*a*c*d*e*f*g + 8*a^2*e^2*g^2
+ 2*(5*c^2*d^2*f*g - 2*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

3.658.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((g*x+f)**2*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output
```
Integral(sqrt(d + e*x)*(f + g*x)**2/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.658.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{cdx+ae}f^2}{cd} + \frac{4\,(c^2d^2x^2-acdex-2\,a^2e^2)fg}{3\sqrt{cdx+ae}c^2d^2} + \frac{2\,(3\,c^3d^3x^3-ac^2d^2ex^2+4\,a^2cde^2x+8\,a^3e^3)g^2}{15\sqrt{cdx+ae}c^3d^3}$$

input
```
integrate((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
2*sqrt(c*d*x + a*e)*f^2/(c*d) + 4/3*(c^2*d^2*x^2 - a*c*d*e*x - 2*a^2*e^2)*
f*g/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/15*(3*c^3*d^3*x^3 - a*c^2*d^2*e*x^2 +
4*a^2*c*d*e^2*x + 8*a^3*e^3)*g^2/(sqrt(c*d*x + a*e)*c^3*d^3)
```

3.658.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{2\,e\left(\frac{15\,(c^2d^2f^2-2\,acdefg+a^2e^2g^2)\sqrt{(ex+d)cde-cd^2e+ae^3}}{c^3d^3e} - \frac{15\sqrt{-cd^2e+ae^3}c^2d^2e^2f^2-10\sqrt{-cd^2e+ae^3}c^2d^3efg-20\sqrt{-cd^2e+ae^3}acde^3f}{c^3d^3}\right.}{}$$

input
```
integrate((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output
```
2/15*e*(15*(c^2*d^2*f^2 - 2*a*c*d*e*f*g + a^2*e^2*g^2)*sqrt((e*x + d)*c*d*
e - c*d^2*e + a*e^3)/(c^3*d^3*e) - (15*sqrt(-c*d^2*e + a*e^3)*c^2*d^2*e^2*
f^2 - 10*sqrt(-c*d^2*e + a*e^3)*c^2*d^3*e*f*g - 20*sqrt(-c*d^2*e + a*e^3)*
a*c*d*e^3*f*g + 3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4*g^2 + 4*sqrt(-c*d^2*e + a
*e^3)*a*c*d^2*e^2*g^2 + 8*sqrt(-c*d^2*e + a*e^3)*a^2*e^4*g^2)/(c^3*d^3*e^3
) + (10*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c*d*e^2*f*g - 10*((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3*g^2 + 3*((e*x + d)*c*d*e - c*d^2*
e + a*e^3)^(5/2)*g^2)/(c^3*d^3*e^5))/abs(e)
```

3.658.9 Mupad [B] (verification not implemented)

Time = 12.50 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(16\,a^2\,e^2\,g^2-40\,a\,c\,d\,e\,f\,g+30\,c^2\,d^2\,f^2\right)}{15\,c^3\,d^3\,e}+\frac{2\,g^2\,x^2\,\sqrt{d+e\,x}}{5\,c\,d\,e}-\frac{4\,g\,x\,\left(2\,a\,e\,g-5\,c\,d\,f\right)\,\sqrt{d}}{15\,c^2\,d^2\,e}\right.}{x+\frac{d}{e}}$$

input
```
int(((f + g*x)^2*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(1/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(16*a^2*e
^2*g^2 + 30*c^2*d^2*f^2 - 40*a*c*d*e*f*g))/(15*c^3*d^3*e) + (2*g^2*x^2*(d
+ e*x)^(1/2))/(5*c*d*e) - (4*g*x*(2*a*e*g - 5*c*d*f)*(d + e*x)^(1/2))/(15*
c^2*d^2*e)))/(x + d/e)
```

3.659 $\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.659.1 Optimal result

Integrand size = 44, antiderivative size = 125

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{2(2ae^2g-cd(3ef-dg))\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3c^2d^2e\sqrt{d+ex}} + \frac{2g\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3cde}$$

output
```
-2/3*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/
c^2/d^2/e/(e*x+d)^(1/2)+2/3*g*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)/c/d/e
```

3.659.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(-2aeg+cd(3f+gx))}{3c^2d^2\sqrt{d+ex}}$$

input
```
Integrate[(Sqrt[d + e*x]*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2],x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e*g + c*d*(3*f + g*x)))/(3*c^2*d^2*
Sqrt[d + e*x])
```

3.659.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{1}{3}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)\int \frac{\sqrt{d+ex}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\frac{2g\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cde}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)}{3cd\sqrt{d+ex}}+\frac{2g\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cde}$$

input
```
Int[(Sqrt[d + e*x]*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],
x]
```

output
```
(2*(3*f - (d*g)/e - (2*a*e*g)/(c*d))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(3*c*d*Sqrt[d + e*x]) + (2*g*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(3*c*d*e)
```

3.659.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

3.659.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.39

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-cdgx+2aeg-3cdf)}{3\sqrt{ex+d}\,c^2d^2}$	49
gosper	$-\frac{2(cdx+ae)(-cdgx+2aeg-3cdf)\sqrt{ex+d}}{3c^2d^2\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	67

input
```
int((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-c*d*g*x+2*a*e*g-3*c*d*f)/
c^2/d^2
```

3.659.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(cdgx+3\,cdf-2\,aeg)\sqrt{ex+d}}{3\,(c^2d^2ex+c^2d^3)}$$

input
```
integrate((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="fricas")
```

output
```
2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x + 3*c*d*f - 2*a*e
*g)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

3.659.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((g*x+f)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/
2),x)
```

output
```
Integral(sqrt(d + e*x)*(f + g*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.659.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{cdx+ae}f}{cd}+\frac{2\,(c^2d^2x^2-acdex-2\,a^2e^2)g}{3\sqrt{cdx+ae}c^2d^2}$$

input
```
integrate((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="maxima")
```

output
```
2*sqrt(c*d*x + a*e)*f/(c*d) + 2/3*(c^2*d^2*x^2 - a*c*d*e*x - 2*a^2*e^2)*g/
(sqrt(c*d*x + a*e)*c^2*d^2)
```

3.659.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2e\left(\frac{3\sqrt{(ex+d)cde-cd^2e+ae^3}(cdf-aeg)}{c^2d^2e}+\frac{((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}g}{c^2d^2e^3}-\frac{3\sqrt{-cd^2e+ae^3}cdef-\sqrt{-cd^2e+ae^3}cd^2g-2\sqrt{-cd^2e+ae^3}ae^2g}{c^2d^2e^2}\right)}{3|e|}$$

input
```
integrate((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="giac")
```

output
```
2/3*e*(3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(c*d*f - a*e*g)/(c^2*d^2*
e) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*g/(c^2*d^2*e^3) - (3*sqrt(-
c*d^2*e + a*e^3)*c*d*e*f - sqrt(-c*d^2*e + a*e^3)*c*d^2*g - 2*sqrt(-c*d^2*
e + a*e^3)*a*e^2*g)/(c^2*d^2*e^2))/abs(e)
```

3.659.9 Mupad [B] (verification not implemented)

Time = 12.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{\left(\frac{(4aeg-6cdf)\sqrt{d+ex}}{3c^2d^2e}-\frac{2gx\sqrt{d+ex}}{3cde}\right)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{x+\frac{d}{e}}$$

input
```
int(((f + g*x)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2),x)
```

output
```
-((((4*a*e*g - 6*c*d*f)*(d + e*x)^(1/2))/(3*c^2*d^2*e) - (2*g*x*(d + e*x)^
(1/2))/(3*c*d*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x + d/e)
```

3.660 $\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.660.1 Optimal result

Integrand size = 39, antiderivative size = 46

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cd\sqrt{d+ex}}$$

output
```
2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)
```

3.660.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{(ae+cdx)(d+ex)}}{cd\sqrt{d+ex}}$$

input
```
Integrate[Sqrt[d + e*x]/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)])/(c*d*Sqrt[d + e*x])
```

3.660.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \text{1122}$$

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

input
```
Int[Sqrt[d + e*x]/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x])
```

3.660.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

3.660.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}}{\sqrt{ex+d}\,cd}$	32
gosper	$\frac{2(cdx+ae)\sqrt{ex+d}}{cd\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	50

input
```
int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
2/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/c/d
```

3.660.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}}{cdex+cd^2}$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="fricas")
```

output
```
2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d
^2)
```

3.660.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.660.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\,\sqrt{cdx+ae}}{cd}$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="maxima")
```

output
```
2*sqrt(c*d*x + a*e)/(c*d)
```

3.660.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2e\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{cde}-\frac{\sqrt{-cd^2e+ae^3}}{cde}\right)}{|e|}$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="giac")
```

output
```
2*e*(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/(c*d*e) - sqrt(-c*d^2*e + a*e
^3)/(c*d*e))/abs(e)
```

3.660.9 Mupad [B] (verification not implemented)

Time = 12.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{d+e\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{c\,d\,e\,\left(x+\frac{d}{e}\right)}$$

input
```
int((d + e*x)^(1/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
(2*(d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(c*d*e*(
x + d/e))
```

3.661 $\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.661.1 Optimal result

Integrand size = 46, antiderivative size = 80

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{\sqrt{g}\sqrt{cdf-aeg}}$$

output
```
2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1
/2)/(e*x+d)^(1/2))/g^(1/2)/(-a*e*g+c*d*f)^(1/2)
```

3.661.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{ae+cdx}\sqrt{d+ex}\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{g}\sqrt{cdf-aeg}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[Sqrt[d + e*x]/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]),x]
```

output
```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt
[c*d*f - a*e*g]])/(Sqrt[g]*Sqrt[c*d*f - a*e*g]*Sqrt[(a*e + c*d*x)*(d + e*x
)])
```

3.661.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \ 1255$$

$$2e^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}$$

$$\Big\downarrow \ 218$$

$$\frac{2\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}\sqrt{cdf-aeg}}$$

input
```
Int[Sqrt[d + e*x]/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),
x]
```

output
```
(2*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*
f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*Sqrt[c*d*f - a*e*g])
```

3.661.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.661.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}}$	77

input
```
int((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
-2/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2)*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))
```

3.661.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.15

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \left[-\frac{\sqrt{-cdfg+aeg^2}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-(cdef-(cd^2+2\,ae^2)g)x-2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{-cdfg+aeg^2}\sqrt{ex+d}}{egx^2+df+(ef+dg)x}\right)}{cdfg-aeg^2}, -\frac{2\,\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{cdfg-aeg^2}\sqrt{ex+d}}{cdegx^2+adeg+(cd^2+ae^2)gx}\right)}{\sqrt{cdfg-aeg^2}}\right]$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="fricas")
```

output
```
[-sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*
e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)
)/(c*d*f*g - a*e*g^2), -2*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 +
 a*e^2)*g*x))/sqrt(c*d*f*g - a*e*g^2)]
```

3.661.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/
2),x)
```

output
```
Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)), x)
```

3.661.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
+ f)), x)
```

3.661.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\,e\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3g}}{\sqrt{cdfg-aeg^2}e}\right)}{\sqrt{cdfg-aeg^2}|e|}-\frac{2\,e\arctan\left(\frac{\sqrt{-cd^2e+ae^3g}}{\sqrt{cdfg-aeg^2}e}\right)}{\sqrt{cdfg-aeg^2}|e|}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="giac")
```

output
```
2*e*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g
^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*abs(e)) - 2*e*arctan(sqrt(-c*d^2*e + a*e^
3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*abs(e))
```

3.661.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \int \frac{\sqrt{d+e\,x}}{(f+g\,x)\ \sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\ x+a\,d\,e}}\,dx$$

input
```
int((d + e*x)^(1/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/
2)),x)
```

output
```
int((d + e*x)^(1/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/
2)), x)
```

3.662 $\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.662.1 Optimal result

Integrand size = 46, antiderivative size = 140

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}(f+gx)}+\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}$$

output
```
c*d*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^
(1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(3/2)/g^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)/(e*x+d)^(1/2)
```

3.662.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{d+ex}\left(\sqrt{g}\sqrt{cdf-aeg}(ae+cdx)+cd\sqrt{ae+cdx}(f+gx)\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{\sqrt{g}(cdf-aeg)^{3/2}\sqrt{(ae+cdx)(d+ex)}(f+gx)}$$

input
```
Integrate[Sqrt[d + e*x]/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]),x]
```

output
```
(Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*(a*e + c*d*x) + c*d*Sqrt[a*e +
 c*d*x]*(f + g*x)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])
)/(Sqrt[g]*(c*d*f - a*e*g)^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x))
```

3.662.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1254}$$

$$\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}$$

$$\Big\downarrow \text{1255}$$

$$\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}$$

$$\Big\downarrow \text{218}$$

$$\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}$$

input
```
Int[Sqrt[d + e*x]/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
),x]
```

output
```
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]
*(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2)
)
```

3.662.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.662.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cdgx+\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cdf-\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\right)}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,(aeg-cdf)(gx+f)\sqrt{(aeg-cdf)g}}$	158

input
```
int((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
((c*d*x+a*e)*(e*x+d))^(1/2)*(arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*c*d*g*x+arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*f
-(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2
)/(a*e*g-c*d*f)/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

3.662.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(124) = 248$.

Time = 0.31 (sec) , antiderivative size = 703, normalized size of antiderivative = 5.02

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \left[\frac{(cdegx^2+cd^2f+(cdef+cd^2g)x)\sqrt{-cdfg+aeg^2}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-(cdef-(cd^2+2\,ae^2)g)x+2\,\sqrt{cdex^2+ad}}{egx^2+df+(ef+dg)x}\right.}{2\,(c^2d^3f^3g-2\,acd^2ef^2g^2+a^2de^2fg^3+(c^2d^2ef^2g^2-2\,acde^2fg^3+a^2e^3g^4)x^2+(c^2d}\right.$$
$$-\frac{(cdegx^2+cd^2f+(cdef+cd^2g)x)\sqrt{cdfg-aeg^2}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{cdfg-aeg^2}\sqrt{ex+d}}{cdegx^2+adeg+(cd^2+ae^2)gx}\right)-\sqrt{cde}}{c^2d^3f^3g-2\,acd^2ef^2g^2+a^2de^2fg^3+(c^2d^2ef^2g^2-2\,acde^2fg^3+a^2e^3g^4)x^2+(c^2d^2ef^3g+a^2de^2g^4+}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output
```
[1/2*((c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(-c*d*f*g + a*e*
g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2
)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e
*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f*g - a*e*g^2)*sqrt(e*x + d))/(c^2*d^3*f^
3*g - 2*a*c*d^2*e*f^2*g^2 + a^2*d*e^2*f*g^3 + (c^2*d^2*e*f^2*g^2 - 2*a*c*d
*e^2*f*g^3 + a^2*e^3*g^4)*x^2 + (c^2*d^2*e*f^3*g + a^2*d*e^2*g^4 + (c^2*d^
3 - 2*a*c*d*e^2)*f^2*g^2 - (2*a*c*d^2*e - a^2*e^3)*f*g^3)*x), -((c*d*e*g*x
^2 + c*d^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x +
d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) - sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x)*(c*d*f*g - a*e*g^2)*sqrt(e*x + d))/(c^2*d^3*f^3*g -
2*a*c*d^2*e*f^2*g^2 + a^2*d*e^2*f*g^3 + (c^2*d^2*e*f^2*g^2 - 2*a*c*d*e^2*f
*g^3 + a^2*e^3*g^4)*x^2 + (c^2*d^2*e*f^3*g + a^2*d*e^2*g^4 + (c^2*d^3 - 2*
a*c*d*e^2)*f^2*g^2 - (2*a*c*d^2*e - a^2*e^3)*f*g^3)*x)]
```

3.662.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)(ae+cdx)}(f+gx)^2}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**2), x)
```

3.662.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^2}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
+ f)^2), x)
```

3.662.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(124) = 248$.

Time = 0.37 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.82

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = e^2\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}cd}{(cde^2f-ae^3g+((ex+d)cde-cd^2e+ae^3)g)(cdf|e|-aeg|e|)} + \frac{cd\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2e}}\right)}{\sqrt{cdfg-aeg^2}(cdf|e|-aeg|e|)e}\right) - \frac{cde^2f\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2e}}\right) - cd^2eg\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2e}}\right) + \sqrt{-cd^2e+ae^3}\sqrt{cdfg-aeg^2}e}{\sqrt{cdfg-aeg^2}cdef^2|e| - \sqrt{cdfg-aeg^2}cd^2fg|e| - \sqrt{cdfg-aeg^2}ae^2fg|e| + \sqrt{cdfg-aeg^2}adeg^2|e|}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output
```
e^2*(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d/((c*d*e^2*f - a*e^3*g + (
(e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)*(c*d*f*abs(e) - a*e*g*abs(e))) + c*d
*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)
*e))/(sqrt(c*d*f*g - a*e*g^2)*(c*d*f*abs(e) - a*e*g*abs(e))*e)) - (c*d*e^2
*f*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - c*d^2*e*
g*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + sqrt(-c*d
^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*e)/(sqrt(c*d*f*g - a*e*g^2)*c*d*e*f^
2*abs(e) - sqrt(c*d*f*g - a*e*g^2)*c*d^2*f*g*abs(e) - sqrt(c*d*f*g - a*e*g
^2)*a*e^2*f*g*abs(e) + sqrt(c*d*f*g - a*e*g^2)*a*d*e*g^2*abs(e))
```

3.662.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \int \frac{\sqrt{d+e\,x}}{{\left(f+g\,x\right)}^2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

input
```
int((d + e*x)^(1/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)),x)
```

output
```
int((d + e*x)^(1/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)), x)
```

3.663 $\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.663.1 Optimal result

Integrand size = 46, antiderivative size = 213

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+\frac{3cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4(cdf-aeg)^2\sqrt{d+ex}(f+gx)} + \frac{3c^2d^2\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4\sqrt{g}(cdf-aeg)^{5/2}}$$

output
```
3/4*c^2*d^2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g
+c*d*f)^(1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(5/2)/g^(1/2)+1/2*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^2/(e*x+d)^(1/2)+3/4*c*d
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)/(e*x+d)^
(1/2)
```

3.663.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\frac{\sqrt{d+ex}\Big(\sqrt{g}\sqrt{cdf-aeg}(ae+cdx)(-2aeg+cd(5f+3gx))+3c^2d^2\sqrt{ae+cdx}(f+gx)^2\arctan\Big(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\Big)\Big)}{4\sqrt{g}(cdf-aeg)^{5/2}\sqrt{(ae+cdx)(d+ex)}(f+gx)^2}$$

input
```
Integrate[Sqrt[d + e*x]/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]),x]
```

output
```
(Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*(a*e + c*d*x)*(-2*a*e*g + c*d*
(5*f + 3*g*x)) + 3*c^2*d^2*Sqrt[a*e + c*d*x]*(f + g*x)^2*ArcTan[(Sqrt[g]*S
qrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/(4*Sqrt[g]*(c*d*f - a*e*g)^(5/2)*
Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^2)
```

3.663.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1254, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$
$$\Big\downarrow \text{1254}$$
$$\frac{3cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4(cdf-aeg)}+\frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}$$
$$\Big\downarrow \text{1254}$$
$$\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}$$

$$\Big\downarrow \text{1255}$$

$$\frac{3cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}$$

$$\Big\downarrow \text{218}$$

$$\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}$$

input
```
Int[Sqrt[d + e*x]/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
),x]
```

output
```
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d*f - a*e*g)*Sqrt[d + e*
x]*(f + g*x)^2) + (3*c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*
d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqr
t[g]*(c*d*f - a*e*g)^(3/2))))/(4*(c*d*f - a*e*g))
```

3.663.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

3.663. $\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.663.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.29

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(3\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2g^2x^2+6\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2fgx+3\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2f^2-}{4\sqrt{ex+d}\,\sqrt{cdx+ae}\,(aeg-cdf)^2(gx+f)^2\sqrt{(a}}$

input
```
int((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
-1/4/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1
/2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^2*g^2*x^2+6*arctanh(g*(c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^2*f*g*x+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*
e*g-c*d*f)*g)^(1/2))*c^2*d^2*f^2-3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/
2)*c*d*g*x+2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*e*g-5*(c*d*x+a*e)
^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c*d*f)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)^2/(g
*x+f)^2/((a*e*g-c*d*f)*g)^(1/2)
```

3.663.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. 2(187) = 374.

Time = 0.35 (sec) , antiderivative size = 1283, normalized size of antiderivative = 6.02

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output
```
[-1/8*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2
)*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(
c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(
e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(5*c^2*d^2*f^2*g - 7*a*c*d*
e*f*g^2 + 2*a^2*e^2*g^3 + 3*(c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^5*g - 3*a*c^2*d^3
*e*f^4*g^2 + 3*a^2*c*d^2*e^2*f^3*g^3 - a^3*d*e^3*f^2*g^4 + (c^3*d^3*e*f^3*
g^3 - 3*a*c^2*d^2*e^2*f^2*g^4 + 3*a^2*c*d*e^3*f*g^5 - a^3*e^4*g^6)*x^3 + (
2*c^3*d^3*e*f^4*g^2 - a^3*d*e^3*g^6 + (c^3*d^4 - 6*a*c^2*d^2*e^2)*f^3*g^3
- 3*(a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^2*g^4 + (3*a^2*c*d^2*e^2 - 2*a^3*e^4)*
f*g^5)*x^2 + (c^3*d^3*e*f^5*g - 2*a^3*d*e^3*f*g^5 + (2*c^3*d^4 - 3*a*c^2*d
^2*e^2)*f^4*g^2 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^3 + (6*a^2*c*d^2*e
^2 - a^3*e^4)*f^2*g^4)*x), -1/4*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c
^2*d^2*e*f*g + c^2*d^3*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(
c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)
*g*x)) - (5*c^2*d^2*f^2*g - 7*a*c*d*e*f*g^2 + 2*a^2*e^2*g^3 + 3*(c^2*d^2*f
*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*
x + d))/(c^3*d^4*f^5*g - 3*a*c^2*d^3*e*f^4*g^2 + 3*a^2*c*d^2*e^2*f^3*g^...
```

3.663.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^3}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**3), x)
```

3.663.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^3}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
+ f)^3), x)
```

3.663.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 844 vs. $2(187) = 374$.

Time = 0.45 (sec) , antiderivative size = 844, normalized size of antiderivative = 3.96

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{1}{4}\left(\frac{3\,c^2d^2\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{(c^2d^2ef^2|e|-2\,acde^2fg|e|+a^2e^3g^2|e|)\sqrt{cdfg-aeg^2}e} + \frac{5\,\sqrt{(ex+d)cde-cd^2e+ae^3}c^3d^3e^2f-5\,\sqrt{}}{(c^2d^2ef^2|e|-2\,acde^2fg|e|+}\right.$$
$$-\frac{3\,c^2d^2e^3f^2\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)-6\,c^2d^3e^2fg\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)+3\,c^2d^4eg}{4\left(\sqrt{cdfg-aeg^2}c^2d^2e^2f^4|e|-2\,\sqrt{cdfg-aeg^2}c^2d^3ef^3g|e|-2\,\sqrt{cdfg-aeg^2}acde^3f^3g|e|+\sqrt{cdfg-aeg}\right.}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output
```
1/4*(3*c^2*d^2*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*
f*g - a*e*g^2)*e))/((c^2*d^2*e*f^2*abs(e) - 2*a*c*d*e^2*f*g*abs(e) + a^2*e
^3*g^2*abs(e))*sqrt(c*d*f*g - a*e*g^2)*e) + (5*sqrt((e*x + d)*c*d*e - c*d^
2*e + a*e^3)*c^3*d^3*e^2*f - 5*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c
^2*d^2*e^3*g + 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*g)/((c^
2*d^2*e*f^2*abs(e) - 2*a*c*d*e^2*f*g*abs(e) + a^2*e^3*g^2*abs(e))*(c*d*e^2
*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^2))*e^3 - 1/4*(3*c^2
*d^2*e^3*f^2*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))
- 6*c^2*d^3*e^2*f*g*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^
2)*e)) + 3*c^2*d^4*e*g^2*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a
*e*g^2)*e)) + 5*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c*d*e^2*f -
 3*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c*d^2*e*g - 2*sqrt(-c*d^
2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*e^3*g)/(sqrt(c*d*f*g - a*e*g^2)*c^2
*d^2*e^2*f^4*abs(e) - 2*sqrt(c*d*f*g - a*e*g^2)*c^2*d^3*e*f^3*g*abs(e) - 2
*sqrt(c*d*f*g - a*e*g^2)*a*c*d*e^3*f^3*g*abs(e) + sqrt(c*d*f*g - a*e*g^2)*
c^2*d^4*f^2*g^2*abs(e) + 4*sqrt(c*d*f*g - a*e*g^2)*a*c*d^2*e^2*f^2*g^2*abs
(e) + sqrt(c*d*f*g - a*e*g^2)*a^2*e^4*f^2*g^2*abs(e) - 2*sqrt(c*d*f*g - a*
e*g^2)*a*c*d^3*e*f*g^3*abs(e) - 2*sqrt(c*d*f*g - a*e*g^2)*a^2*d*e^3*f*g^3*
abs(e) + sqrt(c*d*f*g - a*e*g^2)*a^2*d^2*e^2*g^4*abs(e))
```

3.663.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+e\,x}}{{\left(f+g\,x\right)}^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

input
```
int((d + e*x)^(1/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)),x)
```

output
```
int((d + e*x)^(1/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)), x)
```

3.664 $\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.664.1 Optimal result

Integrand size = 46, antiderivative size = 280

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}(f+gx)^3}+\frac{5cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2}+\frac{5c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8(cdf-aeg)^3\sqrt{d+ex}(f+gx)}+\frac{5c^3d^3\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{8\sqrt{g}(cdf-aeg)^{7/2}}$$

output
```
5/8*c^3*d^3*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g
+c*d*f)^(1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(7/2)/g^(1/2)+1/3*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^3/(e*x+d)^(1/2)+5/12*c*
d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)^2/(e*x+
d)^(1/2)+5/8*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f
)^3/(g*x+f)/(e*x+d)^(1/2)
```

3.664.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\frac{c^3d^3\sqrt{d+ex}\left(\frac{(ae+cdx)(8a^2e^2g^2-2acdeg(13f+5gx)+c^2d^2(33f^2+40fgx+15g^2x^2))}{c^3d^3(cdf-aeg)^3(f+gx)^3}+\frac{15\sqrt{ae+cdx}\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{7/2}}\right)}{24\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[Sqrt[d + e*x]/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]),x]
```

output
```
(c^3*d^3*Sqrt[d + e*x]*(((a*e + c*d*x)*(8*a^2*e^2*g^2 - 2*a*c*d*e*g*(13*f
+ 5*g*x) + c^2*d^2*(33*f^2 + 40*f*g*x + 15*g^2*x^2)))/(c^3*d^3*(c*d*f - a*
e*g)^3*(f + g*x)^3) + (15*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d
*x])/Sqrt[c*d*f - a*e*g]])/(Sqrt[g]*(c*d*f - a*e*g)^(7/2))))/(24*Sqrt[(a*e
 + c*d*x)*(d + e*x)])
```

3.664.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {1254, 1254, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \text{1254}$$

$$\frac{5cd\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{6(cdf-aeg)}+\frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$\frac{5cd\left(\frac{3cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$\frac{5cd\left(\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

$$\Big\downarrow \text{1255}$$

$$\frac{5cd\left(\frac{3cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

$$\Big\downarrow \text{218}$$

$$\frac{5cd\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

3.664. $\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

input
```
Int[Sqrt[d + e*x]/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
),x]
```

output
```
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*(c*d*f - a*e*g)*Sqrt[d + e*
x]*(f + g*x)^3) + (5*c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(
c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c*d*(Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) + (c*d*Arc
Tan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*
e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2))))/(4*(c*d*f - a*e*g)
)))/(6*(c*d*f - a*e*g))
```

3.664.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.664.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.57

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(15 \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3g^3x^3+45 \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3fg^2x^2+45 \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3\right.}{}$

input
```
int((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/24*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^3*d^3*g^3*x^3+45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*c^3*d^3*f*g^2*x^2+45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*c^3*d^3*f^2*g*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^3*d^3*f^3-15*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d
^2*g^2*x^2+10*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*g^2*x-40*(
c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x-8*(c*d*x+a*e)^(1/2)
*((a*e*g-c*d*f)*g)^(1/2)*a^2*e^2*g^2+26*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g
)^(1/2)*a*c*d*e*f*g-33*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f
^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)^3/(g*x+f)^3/((a*e*g-c*d*
f)*g)^(1/2)
```

3.664.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 993 vs. 2(248) = 496.

Time = 0.67 (sec) , antiderivative size = 2027, normalized size of antiderivative = 7.24

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output
```
[1/48*(15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*
g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^
3*d^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a
*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f +
(e*f + d*g)*x)) + 2*(33*c^3*d^3*f^3*g - 59*a*c^2*d^2*e*f^2*g^2 + 34*a^2*c*
d*e^2*f*g^3 - 8*a^3*e^3*g^4 + 15*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 1
0*(4*c^3*d^3*f^2*g^2 - 5*a*c^2*d^2*e*f*g^3 + a^2*c*d*e^2*g^4)*x)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^7*g - 4*a*c^3
*d^4*e*f^6*g^2 + 6*a^2*c^2*d^3*e^2*f^5*g^3 - 4*a^3*c*d^2*e^3*f^4*g^4 + a^4
*d*e^4*f^3*g^5 + (c^4*d^4*e*f^4*g^4 - 4*a*c^3*d^3*e^2*f^3*g^5 + 6*a^2*c^2*
d^2*e^3*f^2*g^6 - 4*a^3*c*d*e^4*f*g^7 + a^4*e^5*g^8)*x^4 + (3*c^4*d^4*e*f^
5*g^3 + a^4*d*e^4*g^8 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*f^4*g^4 - 2*(2*a*c^3*
d^4*e - 9*a^2*c^2*d^2*e^3)*f^3*g^5 + 6*(a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f
^2*g^6 - (4*a^3*c*d^2*e^3 - 3*a^4*e^5)*f*g^7)*x^3 + 3*(c^4*d^4*e*f^6*g^2 +
 a^4*d*e^4*f*g^7 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f^5*g^3 - 2*(2*a*c^3*d^4*e
- 3*a^2*c^2*d^2*e^3)*f^4*g^4 + 2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^3*g
^5 - (4*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^6)*x^2 + (c^4*d^4*e*f^7*g + 3*a^4*d
*e^4*f^2*g^6 + (3*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^6*g^2 - 6*(2*a*c^3*d^4*e -
a^2*c^2*d^2*e^3)*f^5*g^3 + 2*(9*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*g^...
```

3.664.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^4}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**4), x)
```

3.664.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^4}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
+ f)^4), x)
```

3.664.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1460 vs. 2(248) = 496.

Time = 0.66 (sec) , antiderivative size = 1460, normalized size of antiderivative = 5.21

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output
```
1/24*(15*c^3*d^3*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*
d*f*g - a*e*g^2)*e))/((c^3*d^3*e^2*f^3*abs(e) - 3*a*c^2*d^2*e^3*f^2*g*abs(
e) + 3*a^2*c*d*e^4*f*g^2*abs(e) - a^3*e^5*g^3*abs(e))*sqrt(c*d*f*g - a*e*g
^2)*e) + (33*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^5*d^5*e^4*f^2 - 66*
sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^4*d^4*e^5*f*g + 33*sqrt((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^3*d^3*e^6*g^2 + 40*((e*x + d)*c*d*e - c
*d^2*e + a*e^3)^(3/2)*c^4*d^4*e^2*f*g - 40*((e*x + d)*c*d*e - c*d^2*e + a*
e^3)^(3/2)*a*c^3*d^3*e^3*g^2 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2
)*c^3*d^3*g^2)/((c^3*d^3*e^2*f^3*abs(e) - 3*a*c^2*d^2*e^3*f^2*g*abs(e) + 3
*a^2*c*d*e^4*f*g^2*abs(e) - a^3*e^5*g^3*abs(e))*(c*d*e^2*f - a*e^3*g + ((e
*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^3))*e^4 - 1/24*(15*c^3*d^3*e^4*f^3*arc
tan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 45*c^3*d^4*e^3
*f^2*g*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 45*c
^3*d^5*e^2*f*g^2*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*
e)) - 15*c^3*d^6*e*g^3*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e
*g^2)*e)) + 33*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d^2*e^3*
f^2 - 40*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d^3*e^2*f*g -
26*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c*d*e^4*f*g + 15*sqrt(
-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d^4*e*g^2 + 10*sqrt(-c*d^2*e
 + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c*d^2*e^3*g^2 + 8*sqrt(-c*d^2*e + a...
```

3.664.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+e\,x}}{{\left(f+g\,x\right)}^4\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

input
```
int((d + e*x)^(1/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)),x)
```

output
```
int((d + e*x)^(1/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)), x)
```

3.665 $\int \frac{(d+ex)^{3/2}(f+gx)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

3.665.1 Optimal result

Integrand size = 46, antiderivative size = 257

$$\begin{aligned}\int \frac{(d+ex)^{3/2}(f+gx)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx &= -\frac{2\sqrt{d+ex}(f+gx)^3}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\\ &- \frac{16g(cdf-aeg)\,(2ae^2g-cd(3ef-dg))\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5c^4d^4e\sqrt{d+ex}}\\ &+ \frac{16g^2(cdf-aeg)\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5c^3d^3e}\\ &+ \frac{12g(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5c^2d^2\sqrt{d+ex}}\end{aligned}$$

output
```
-2*(g*x+f)^3*(e*x+d)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-16/
5*g*(-a*e*g+c*d*f)*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)/c^4/d^4/e/(e*x+d)^(1/2)+12/5*g*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)+16/5*g^2*(-a*e*g+c*d*f)*(e*x+d)
^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e
```

3.665.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.52

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{2\sqrt{d+ex}(16a^3e^3g^3+8a^2cde^2g^2(-5f+gx)-2ac^2d^2eg(-15f^2+}{5c^4d^4\sqrt{(ae+cdx}}$$

input
```
Integrate[((d + e*x)^(3/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2),x]
```

output
```
(2*Sqrt[d + e*x]*(16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(-5*f + g*x) - 2*a*c^
2*d^2*e*g*(-15*f^2 + 10*f*g*x + g^2*x^2) + c^3*d^3*(-5*f^3 + 15*f^2*g*x +
5*f*g^2*x^2 + g^3*x^3)))/(5*c^4*d^4*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.665.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1251, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{(x\,(ae^2+cd^2)+ade+cdex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1251}$$

$$\frac{6g\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cd}-\frac{2\sqrt{d+ex}(f+gx)^3}{cd\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{1253}$$

$$\frac{6g\left(\frac{4(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{5cd}+\frac{2(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5cd\sqrt{d+ex}}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^3}{cd\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{1221}$$

$$\frac{6g\left(\frac{4(cdf-aeg)\left(\frac{1}{3}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)\int\frac{\sqrt{d+ex}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{2g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}\right)}{5cd}+\frac{2(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cd}}{5cd\sqrt{d+ex}}\right.}{cd}\frac{2\sqrt{d+ex}(f+gx)^3}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{6g\left(\frac{2(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5cd\sqrt{d+ex}}+\frac{4(cdf-aeg)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)}{3cd\sqrt{d+ex}}+\frac{2g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}\right)}{5cd}\right)}{cd}\frac{2\sqrt{d+ex}(f+gx)^3}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[((d + e*x)^(3/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(f + g*x)^3)/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]) + (6*g*((2*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
/(5*c*d*Sqrt[d + e*x]) + (4*(c*d*f - a*e*g)*((2*(3*f - (d*g)/e - (2*a*e*g)
/(c*d))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*Sqrt[d + e*x])
 + (2*g*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*
e)))/(5*c*d)))/(c*d)
```

3.665.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

3.665. $\int \frac{(d+ex)^{3/2}(f+gx)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1251
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[e*g*(n/(c*(p + 1)))   Int[(
d + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p,
 0] && LtQ[p, -1] && GtQ[n, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

3.665.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.70

method	result
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(g^3x^3c^3d^3-2a\,c^2d^2e\,g^3x^2+5c^3d^3f\,g^2x^2+8a^2cd\,e^2g^3x-20a\,c^2d^2ef\,g^2x+15c^3d^3f^2gx+16a^3e^3g^3-40a^2cd\,e^2f\,g^2}{5\sqrt{ex+d}\,(cdx+ae)c^4d^4}$
gosper	$\frac{2(cdx+ae)(g^3x^3c^3d^3-2a\,c^2d^2e\,g^3x^2+5c^3d^3f\,g^2x^2+8a^2cd\,e^2g^3x-20a\,c^2d^2ef\,g^2x+15c^3d^3f^2gx+16a^3e^3g^3-40a^2cd\,e^2f\,g^2+30a\,c^2d^2}{5c^4d^4(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$

input
```
int((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/5/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(c^3*d^3*g^3*x^3-2*a*c^2*d^2
*e*g^3*x^2+5*c^3*d^3*f*g^2*x^2+8*a^2*c*d*e^2*g^3*x-20*a*c^2*d^2*e*f*g^2*x+
15*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-40*a^2*c*d*e^2*f*g^2+30*a*c^2*d^2*e*f^2*
g-5*c^3*d^3*f^3)/(c*d*x+a*e)/c^4/d^4
```

3.665.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{2\,(c^3d^3g^3x^3-5\,c^3d^3f^3+30\,ac^2d^2ef^2g-40\,a^2cde^2fg^2+16\,a^3e^3g^3}{}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="fricas")
```

output
```
2/5*(c^3*d^3*g^3*x^3 - 5*c^3*d^3*f^3 + 30*a*c^2*d^2*e*f^2*g - 40*a^2*c*d*e
^2*f*g^2 + 16*a^3*e^3*g^3 + (5*c^3*d^3*f*g^2 - 2*a*c^2*d^2*e*g^3)*x^2 + (1
5*c^3*d^3*f^2*g - 20*a*c^2*d^2*e*f*g^2 + 8*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x^2 + a*c^4*d^5*
e + (c^5*d^6 + a*c^4*d^4*e^2)*x)
```

3.665.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(3/2),x)
```

output
```
Timed out
```

3.665.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.64

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = -\frac{2\,f^3}{\sqrt{cdx+ae}cd} + \frac{6\,(cdx+2\,ae)f^2g}{\sqrt{cdx+ae}c^2d^2} + \frac{2\,(c^2d^2x^2-4\,acdex-8\,a^2e^2)fg^2}{\sqrt{cdx+ae}c^3d^3} + \frac{2\,(c^3d^3x^3-2\,ac^2d^2ex^2+8\,a^2cde^2x+16\,a^3e^3)g^3}{5\,\sqrt{cdx+ae}c^4d^4}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="maxima")
```

output
```
-2*f^3/(sqrt(c*d*x + a*e)*c*d) + 6*(c*d*x + 2*a*e)*f^2*g/(sqrt(c*d*x + a*e
)*c^2*d^2) + 2*(c^2*d^2*x^2 - 4*a*c*d*e*x - 8*a^2*e^2)*f*g^2/(sqrt(c*d*x +
 a*e)*c^3*d^3) + 2/5*(c^3*d^3*x^3 - 2*a*c^2*d^2*e*x^2 + 8*a^2*c*d*e^2*x +
16*a^3*e^3)*g^3/(sqrt(c*d*x + a*e)*c^4*d^4)
```

3.665.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(235) = 470.

Time = 0.31 (sec) , antiderivative size = 513, normalized size of antiderivative = 2.00

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = -\frac{2\,(c^3d^3e^2f^3-3\,ac^2d^2e^3f^2g+3\,a^2cde^4fg^2-a^3e^5g^3)}{\sqrt{(ex+d)cde-cd^2e+ae^3}c^4d^4|e|} + \frac{2\,(5\,c^3d^3e^3f^3+15\,c^3d^4e^2f^2g-30\,ac^2d^2e^4f^2g-5\,c^3d^5efg^2-20\,ac^2d^3e^3fg^2+40\,a^2cde^5fg^2+c^3d^6g^3+2\,a}{5\,\sqrt{-cd^2e+ae^3}c^4d^4e|e|} + \frac{2\left(15\,\sqrt{(ex+d)cde-cd^2e+ae^3}c^{18}d^{18}e^{24}f^2g-30\,\sqrt{(ex+d)cde-cd^2e+ae^3}ac^{17}d^{17}e^{25}fg^2+15\,\sqrt{(ex+\right.}{}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="giac")
```

output
```
-2*(c^3*d^3*e^2*f^3 - 3*a*c^2*d^2*e^3*f^2*g + 3*a^2*c*d*e^4*f*g^2 - a^3*e^
5*g^3)/(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^4*d^4*abs(e)) + 2/5*(5*c
^3*d^3*e^3*f^3 + 15*c^3*d^4*e^2*f^2*g - 30*a*c^2*d^2*e^4*f^2*g - 5*c^3*d^5
*e*f*g^2 - 20*a*c^2*d^3*e^3*f*g^2 + 40*a^2*c*d*e^5*f*g^2 + c^3*d^6*g^3 + 2
*a*c^2*d^4*e^2*g^3 + 8*a^2*c*d^2*e^4*g^3 - 16*a^3*e^6*g^3)/(sqrt(-c*d^2*e
+ a*e^3)*c^4*d^4*e*abs(e)) + 2/5*(15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^
3)*c^18*d^18*e^24*f^2*g - 30*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^1
7*d^17*e^25*f*g^2 + 15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^16*d^
16*e^26*g^3 + 5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^17*d^17*e^22*f
*g^2 - 5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^16*d^16*e^23*g^3 +
((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^16*d^16*e^20*g^3)/(c^20*d^20*e
^24*abs(e))
```

3.665.9 Mupad [B] (verification not implemented)

Time = 12.73 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\left(\frac{\sqrt{d+e\,x}\,\left(32\,a^3\,e^3\,g^3-80\,a^2\,c\,d\,e^2\,f\,g^2+60}{5\,c^5\,d^5\,e}\right.}{}$$

input
```
int(((f + g*x)^3*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(3/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(32*a^3*e
^3*g^3 - 10*c^3*d^3*f^3 + 60*a*c^2*d^2*e*f^2*g - 80*a^2*c*d*e^2*f*g^2))/(5
*c^5*d^5*e) + (2*g^3*x^3*(d + e*x)^(1/2))/(5*c^2*d^2*e) - (2*g^2*x^2*(2*a*
e*g - 5*c*d*f)*(d + e*x)^(1/2))/(5*c^3*d^3*e) + (2*g*x*(d + e*x)^(1/2)*(8*
a^2*e^2*g^2 + 15*c^2*d^2*f^2 - 20*a*c*d*e*f*g))/(5*c^4*d^4*e)))/(a/c + x^2
 + (x*(5*c^5*d^6 + 5*a*c^4*d^4*e^2))/(5*c^5*d^5*e))
```

3.666 $\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

3.666.1 Optimal result

Integrand size = 46, antiderivative size = 181

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{8g\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^3d^3e\sqrt{d+ex}} + \frac{8g^2\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^2d^2e}$$

output
```
-2*(g*x+f)^2*(e*x+d)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-8/3
*g*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^
3/d^3/e/(e*x+d)^(1/2)+8/3*g^2*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)/c^2/d^2/e
```

3.666.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.49

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2\sqrt{d+ex}\left(-8a^2e^2g^2-4acdeg(-3f+gx)+c^2d^2(-3f^2+6fgx+\right.}{3c^3d^3\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[((d + e*x)^(3/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2),x]
```

output
```
(2*Sqrt[d + e*x]*(-8*a^2*e^2*g^2 - 4*a*c*d*e*g*(-3*f + g*x) + c^2*d^2*(-3*
f^2 + 6*f*g*x + g^2*x^2)))/(3*c^3*d^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.666.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1251, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1251}$$

$$\frac{4g\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cd}-\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1221}$$

$$\frac{4g\left(\frac{1}{3}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)\int \frac{\sqrt{d+ex}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\frac{2g\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cde}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{4g\left(\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)}{3cd\sqrt{d+ex}}+\frac{2g\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cde}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[((d + e*x)^(3/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(f + g*x)^2)/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]) + (4*g*((2*(3*f - (d*g)/e - (2*a*e*g)/(c*d))*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])/(3*c*d*Sqrt[d + e*x]) + (2*g*Sqrt[d + e*x]*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*e)))/(c*d)
```

3.666. $\int \frac{(d+ex)^{3/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}dx$

3.666.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1251
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[e*g*(n/(c*(p + 1)))   Int[(
d + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p,
 0] && LtQ[p, -1] && GtQ[n, 0]
```

3.666.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.60

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-g^2x^2c^2d^2+4acde\,g^2x-6c^2d^2fgx+8a^2e^2g^2-12acdefg+3c^2d^2f^2)}{3\sqrt{ex+d}\,(cdx+ae)c^3d^3}$	108
gosper	$-\frac{2(cdx+ae)(-g^2x^2c^2d^2+4acde\,g^2x-6c^2d^2fgx+8a^2e^2g^2-12acdefg+3c^2d^2f^2)(ex+d)^{\frac{3}{2}}}{3c^3d^3(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$	116

input
```
int((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,meth
od=_RETURNVERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-c^2*d^2*g^2*x^2+4*a*c*d*e
*g^2*x-6*c^2*d^2*f*g*x+8*a^2*e^2*g^2-12*a*c*d*e*f*g+3*c^2*d^2*f^2)/(c*d*x+
a*e)/c^3/d^3
```

3.666.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{2\left(c^2d^2g^2x^2-3\,c^2d^2f^2+12\,acdefg-8\,a^2e^2g^2+2\left(3\,c^2d^2fg-2\,a\right.\right.}{3\left(c^4d^4ex^2+ac^3d^4e+\left(c^4d^5+\right.\right.}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="fricas")
```

output
```
2/3*(c^2*d^2*g^2*x^2 - 3*c^2*d^2*f^2 + 12*a*c*d*e*f*g - 8*a^2*e^2*g^2 + 2*
(3*c^2*d^2*f*g - 2*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(e*x + d)/(c^4*d^4*e*x^2 + a*c^3*d^4*e + (c^4*d^5 + a*c^3*d^3*e^2
)*x)
```

3.666.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)^2}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(3/2),x)
```

output
```
Integral((d + e*x)**(3/2)*(f + g*x)**2/((d + e*x)*(a*e + c*d*x))**(3/2), x
)
```

3.666.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.54

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = -\frac{2\,f^2}{\sqrt{cdx+ae}cd} + \frac{4\,(cdx+2\,ae)fg}{\sqrt{cdx+ae}c^2d^2} + \frac{2\,(c^2d^2x^2-4\,acdex-8\,a^2e^2)g^2}{3\,\sqrt{cdx+ae}c^3d^3}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="maxima")
```

output
```
-2*f^2/(sqrt(c*d*x + a*e)*c*d) + 4*(c*d*x + 2*a*e)*f*g/(sqrt(c*d*x + a*e)*
c^2*d^2) + 2/3*(c^2*d^2*x^2 - 4*a*c*d*e*x - 8*a^2*e^2)*g^2/(sqrt(c*d*x + a
*e)*c^3*d^3)
```

3.666.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.65

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2\,(c^2d^2e^2f^2-2\,acde^3fg+a^2e^4g^2)}{\sqrt{(ex+d)cde-cd^2e+ae^3}c^3d^3|e|} + \frac{2\,(3\,c^2d^2e^2f^2+6\,c^2d^3efg-12\,acde^3fg-c^2d^4g^2-4\,acd^2e^2g^2+8\,a^2e^4g^2)}{3\,\sqrt{-cd^2e+ae^3}c^3d^3|e|} + \frac{2\left(6\,\sqrt{(ex+d)cde-cd^2e+ae^3}c^7d^7e^8fg-6\,\sqrt{(ex+d)cde-cd^2e+ae^3}ac^6d^6e^9g^2+((ex+d)cde-cd^2e\right.}{3\,c^9d^9e^8|e|}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="giac")
```

output
```
-2*(c^2*d^2*e^2*f^2 - 2*a*c*d*e^3*f*g + a^2*e^4*g^2)/(sqrt((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)*c^3*d^3*abs(e)) + 2/3*(3*c^2*d^2*e^2*f^2 + 6*c^2*d^3*e
*f*g - 12*a*c*d*e^3*f*g - c^2*d^4*g^2 - 4*a*c*d^2*e^2*g^2 + 8*a^2*e^4*g^2)
/(sqrt(-c*d^2*e + a*e^3)*c^3*d^3*abs(e)) + 2/3*(6*sqrt((e*x + d)*c*d*e - c
*d^2*e + a*e^3)*c^7*d^7*e^8*f*g - 6*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3
)*a*c^6*d^6*e^9*g^2 + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^6*d^6*e^
6*g^2)/(c^9*d^9*e^8*abs(e))
```

3.666.9 Mupad [B] (verification not implemented)

Time = 12.58 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx =$$
$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(16\,a^2\,e^2\,g^2-24\,a\,c\,d\,e\,f\,g+6\,c^2\,d^2\,f^2\right)}{3\,c^4\,d^4\,e}-\frac{2\,g^2\,x^2\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e}+\frac{4\,g\,x\,\left(2\,a\,e\,g-3\,c\,d\,f\right)\,\sqrt{d+}}{3\,c^3\,d^3\,e}\right.}{\frac{a}{c}+x^2+\frac{x\,\left(3\,c^4\,d^5+3\,a\,c^3\,d^3\,e^2\right)}{3\,c^4\,d^4\,e}}$$

input
```
int(((f + g*x)^2*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(3/2),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(16*a^2*
e^2*g^2 + 6*c^2*d^2*f^2 - 24*a*c*d*e*f*g))/(3*c^4*d^4*e) - (2*g^2*x^2*(d +
 e*x)^(1/2))/(3*c^2*d^2*e) + (4*g*x*(2*a*e*g - 3*c*d*f)*(d + e*x)^(1/2))/(
3*c^3*d^3*e)))/(a/c + x^2 + (x*(3*c^4*d^5 + 3*a*c^3*d^3*e^2))/(3*c^4*d^4*e
))
```

3.667 $\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

3.667.1 Optimal result

Integrand size = 44, antiderivative size = 150

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2(cdf-aeg)(d+ex)^{3/2}}{cd\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{2(2ae^2g-cd(ef+dg))\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^2d^2\left(cd^2-ae^2\right)\sqrt{d+ex}}$$

output
```
-2*(-a*e*g+c*d*f)*(e*x+d)^(3/2)/c/d/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)-2*(2*a*e^2*g-c*d*(d*g+e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/c^2/d^2/(-a*e^2+c*d^2)/(e*x+d)^(1/2)
```

3.667.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.34

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2\sqrt{d+ex}(2aeg+cd(-f+gx))}{c^2d^2\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[((d + e*x)^(3/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/2),x]
```

output
```
(2*Sqrt[d + e*x]*(2*a*e*g + c*d*(-f + g*x)))/(c^2*d^2*Sqrt[(a*e + c*d*x)*(
d + e*x)])
```

3.667.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1218, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$-\frac{\left(2ae^2g-cd(dg+ef)\right)\int \frac{\sqrt{d+ex}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cd\left(cd^2-ae^2\right)}-\frac{2(d+ex)^{3/2}(cdf-aeg)}{cd\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1122}$$

$$-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(2ae^2g-cd(dg+ef)\right)}{c^2d^2\sqrt{d+ex}\left(cd^2-ae^2\right)}-\frac{2(d+ex)^{3/2}(cdf-aeg)}{cd\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[((d + e*x)^(3/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/2),x]
```

output
```
(-2*(c*d*f - a*e*g)*(d + e*x)^(3/2))/(c*d*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2]) - (2*(2*a*e^2*g - c*d*(e*f + d*g))*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^2*d^2*(c*d^2 - a*e^2)*Sqrt[d + e*x])
```

3.667.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

3.667.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.39

method	result	size
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdgx+2aeg-cdf)}{\sqrt{ex+d}\,(cdx+ae)c^2d^2}$	58
gosper	$\frac{2(cdx+ae)(cdgx+2aeg-cdf)(ex+d)^{\frac{3}{2}}}{c^2d^2(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$	66

input
```
int((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method
=_RETURNVERBOSE)
```

output
```
2/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(c*d*g*x+2*a*e*g-c*d*f)/(c*d*x
+a*e)/c^2/d^2
```

3.667.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.64

$$\int \frac{(d+ex)^{3/2}(f+gx)}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}(cdgx-cdf+2\,aeg)\sqrt{ex+d}}{c^3d^3ex^2+ac^2d^3e+(c^3d^4+ac^2d^2e^2)x}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
 algorithm="fricas")
```

output
```
2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - c*d*f + 2*a*e*g)*
sqrt(e*x + d)/(c^3*d^3*e*x^2 + a*c^2*d^3*e + (c^3*d^4 + a*c^2*d^2*e^2)*x)
```

3.667.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}(f+gx)}{((d+ex)(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/
2),x)
```

output
```
Integral((d + e*x)**(3/2)*(f + g*x)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

3.667.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.32

$$\int \frac{(d+ex)^{3/2}(f+gx)}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = -\frac{2\,f}{\sqrt{cdx+ae}cd} + \frac{2\,(cdx+2\,ae)g}{\sqrt{cdx+ae}c^2d^2}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
 algorithm="maxima")
```

output
```
-2*f/(sqrt(c*d*x + a*e)*c*d) + 2*(c*d*x + 2*a*e)*g/(sqrt(c*d*x + a*e)*c^2*
d^2)
```

3.667.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{2\sqrt{(ex+d)cde-cd^2e+ae^3}g}{c^2d^2|e|} - \frac{2\left(cde^2f-ae^3g\right)}{\sqrt{(ex+d)cde-cd^2e+ae^3}c^2d^2|e|} + \frac{2\left(cde^2f+cd^2eg-2\,ae^3g\right)}{\sqrt{-cd^2e+ae^3}c^2d^2|e|}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
 algorithm="giac")
```

output
```
2*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(c^2*d^2*abs(e)) - 2*(c*d*e^2*
f - a*e^3*g)/(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^2*abs(e)) + 2*
(c*d*e^2*f + c*d^2*e*g - 2*a*e^3*g)/(sqrt(-c*d^2*e + a*e^3)*c^2*d^2*abs(e)
)
```

3.667.9 Mupad [B] (verification not implemented)

Time = 12.39 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{\left(\frac{(4\,a\,e\,g-2\,c\,d\,f)\,\sqrt{d+e\,x}}{c^3\,d^3\,e}+\frac{2\,g\,x\,\sqrt{d+e\,x}}{c^2\,d^2\,e}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d}}{\frac{a}{c}+x^2+\frac{x\,\left(c^3\,d^4+a\,c^2\,d^2\,e^2\right)}{c^3\,d^3\,e}}$$

input
```
int(((f + g*x)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3
/2),x)
```

output
```
((((4*a*e*g - 2*c*d*f)*(d + e*x)^(1/2))/(c^3*d^3*e) + (2*g*x*(d + e*x)^(1/
2))/(c^2*d^2*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(a/c + x^2
 + (x*(c^3*d^4 + a*c^2*d^2*e^2))/(c^3*d^3*e))
```

3.668 $\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

3.668.1 Optimal result

Integrand size = 39, antiderivative size = 46

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

output
```
-2*(e*x+d)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

3.668.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}}{cd\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x])/(c*d*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.668.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{1122}$$
$$-\frac{2\sqrt{d+ex}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

input
```
Int[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x])/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

3.668.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

3.668.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}}{\sqrt{ex+d}\,(cdx+ae)cd}$	42
gosper	$-\frac{2(cdx+ae)(ex+d)^{\frac{3}{2}}}{cd(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$	50

input
```
int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/(c*d*x+a*e)/c/d
```

3.668.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}}{c^2d^2ex^2+acd^2e+(c^2d^3+acde^2)x}$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
-2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*x^
2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)
```

3.668.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)**(3/2)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

3.668.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2}{\sqrt{cdx+ae}cd}$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
-2/(sqrt(c*d*x + a*e)*c*d)
```

3.668.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.52

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2\,e^2}{\sqrt{(ex+d)cde-cd^2e+ae^3}cd|e|}+\frac{2\,e^2}{\sqrt{-cd^2e+ae^3}cd|e|}$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
-2*e^2/(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*abs(e)) + 2*e^2/(sqrt(
-c*d^2*e + a*e^3)*c*d*abs(e))
```

3.668.9 Mupad [B] (verification not implemented)

Time = 12.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.78

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2\,\sqrt{d+e\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{c^2\,d^2\,e\,\left(\frac{a}{c}+x^2+\frac{x\,\left(c^2\,d^3+a\,c\,d\,e^2\right)}{c^2\,d^2\,e}\right)}$$

input
```
int((d + e*x)^(3/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
-(2*(d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(c^2*d^
2*e*(a/c + x^2 + (x*(c^2*d^3 + a*c*d*e^2))/(c^2*d^2*e)))
```

3.669 $\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

3.669.1 Optimal result

Integrand size = 46, antiderivative size = 133

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{2\sqrt{g}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{(cdf-aeg)^{3/2}}$$

output
```
-2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(
1/2)/(e*x+d)^(1/2))*g^(1/2)/(-a*e*g+c*d*f)^(3/2)-2*(e*x+d)^(1/2)/(-a*e*g+c
*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

3.669.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\left(\sqrt{cdf-aeg}+\sqrt{g}\sqrt{ae+cdx}\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{(cdf-aeg)^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)^(3/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2)),x]
```

output
```
(-2*Sqrt[d + e*x]*(Sqrt[c*d*f - a*e*g] + Sqrt[g]*Sqrt[a*e + c*d*x]*ArcTan[
(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/((c*d*f - a*e*g)^(3/2)*
Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.669.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1252, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1252}$$

$$-\frac{g\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cdf-aeg}-\frac{2\sqrt{d+ex}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \text{1255}$$

$$-\frac{2e^2g\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}-\frac{2\sqrt{d+ex}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \text{218}$$

$$-\frac{2\sqrt{g}\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{3/2}}-\frac{2\sqrt{d+ex}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

input
```
Int[(d + e*x)^(3/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2)),x]
```

output
```
(-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]) - (2*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a*e*g)^(3/2)
```

3.669.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1252
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.669.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\left(g \,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)\sqrt{cdx+ae}-\sqrt{(aeg-cdf)g}\right)}{\sqrt{ex+d}\,(cdx+ae)(aeg-cdf)\sqrt{(aeg-cdf)g}}$	118

input
```
int((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method
=_RETURNVERBOSE)
```

output
```
-2/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(g*arctanh(g*(c*d*x+a*e)^(1/2
)/((a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)-((a*e*g-c*d*f)*g)^(1/2))/(c*d
*x+a*e)/(a*e*g-c*d*f)/((a*e*g-c*d*f)*g)^(1/2)
```

3.669. $\int \frac{(d+ex)^{3/2}}{(f+gx)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

3.669.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(117) = 234$.

Time = 0.29 (sec) , antiderivative size = 553, normalized size of antiderivative = 4.16

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \left[-\frac{(cdex^2+ade+(cd^2+ae^2)x)\sqrt{-\frac{g}{cdf-aeg}}\log\left(-\frac{cdegx^2}{}\right.}{acd^2ef}\right.$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
 algorithm="fricas")
```

output
```
[-((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(
c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f
- (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c*d^2*e*f - a^2*d*e^2*g
 + (c^2*d^2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*d^3 + a*c*d*e^2)*f - (a*c*d^2*e
 + a^2*e^3)*g)*x), -2*((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g/(c*d
*f - a*e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f -
a*e*g)*sqrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a*d*e*g + (c*d
^2 + a*e^2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x +
 d))/(a*c*d^2*e*f - a^2*d*e^2*g + (c^2*d^2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*
d^3 + a*c*d*e^2)*f - (a*c*d^2*e + a^2*e^3)*g)*x)]
```

3.669.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/
2),x)
```

output
```
Timed out
```

3.669.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
 algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(
g*x + f)), x)
```

3.669.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(117) = 234$.

Time = 0.38 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.11

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = -2\,e^2\left(\frac{g\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{\sqrt{cdfg-aeg^2}(cdf|e|-aeg|e|)e}+\frac{1}{\sqrt{(ex+d)cde-cd^2e+ae^3}(cdf|e|-aeg|e|)}\right) + \frac{2\left(\sqrt{-cd^2e+ae^3}eg\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)+\sqrt{cdfg-aeg^2}e^2\right)}{\sqrt{-cd^2e+ae^3}\sqrt{cdfg-aeg^2}cdf|e|-\sqrt{-cd^2e+ae^3}\sqrt{cdfg-aeg^2}aeg|e|}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
 algorithm="giac")
```

output
```
-2*e^2*(g*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g -
 a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*(c*d*f*abs(e) - a*e*g*abs(e))*e) +
1/(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(c*d*f*abs(e) - a*e*g*abs(e))))
 + 2*(sqrt(-c*d^2*e + a*e^3)*e*g*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d
*f*g - a*e*g^2)*e)) + sqrt(c*d*f*g - a*e*g^2)*e^2)/(sqrt(-c*d^2*e + a*e^3)
*sqrt(c*d*f*g - a*e*g^2)*c*d*f*abs(e) - sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*
g - a*e*g^2)*a*e*g*abs(e))
```

3.669.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{{\left(d+e\,x\right)}^{3/2}}{\left(f+g\,x\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}} \,d x$$

input
```
int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/
2)),x)
```

output
```
int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/
2)), x)
```

3.670 $\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

3.670.1 Optimal result

Integrand size = 46, antiderivative size = 202

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{3g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(cdf-aeg)^2\sqrt{d+ex}(f+gx)} - \frac{3cd\sqrt{g}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{(cdf-aeg)^{5/2}}$$

output
```
-3*c*d*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*
f)^(1/2)/(e*x+d)^(1/2))*g^(1/2)/(-a*e*g+c*d*f)^(5/2)-2*(e*x+d)^(1/2)/(-a*e
*g+c*d*f)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3*g*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)/(e*x+d)^(1/2)
```

3.670.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.70

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{\sqrt{d+ex}\left(\sqrt{cdf-aeg}(aeg+cd(2f+3gx))+3cd\sqrt{g}\sqrt{ae+cdx}(f+gx)\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{(cdf-aeg)^{5/2}\sqrt{(ae+cdx)(d+ex)}(f+gx)}$$

input
```
Integrate[(d + e*x)^(3/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)),x]
```

output
```
-((Sqrt[d + e*x]*(Sqrt[c*d*f - a*e*g]*(a*e*g + c*d*(2*f + 3*g*x)) + 3*c*d*
Sqrt[g]*Sqrt[a*e + c*d*x]*(f + g*x)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqr
t[c*d*f - a*e*g]]))/((c*d*f - a*e*g)^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(
f + g*x)))
```

3.670.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1252, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1252}$$

$$-\frac{3g\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$-\frac{3g\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \text{1255}$$

$$-\frac{3g\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow 218$$

$$-\frac{3g\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

input
```
Int[(d + e*x)^(3/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2)),x]
```

output
```
(-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2]) - (3*g*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d
*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt
[g]*(c*d*f - a*e*g)^(3/2))))/(c*d*f - a*e*g)
```

3.670.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1252
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

3.670. $\int \frac{(d+ex)^{3/2}}{(f+gx)^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.670.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.06

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(3\sqrt{cdx+ae}\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cd\,g^{2}x+3\sqrt{cdx+ae}\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cdfg-3\sqrt{(aeg-cdf)g}\,cdgx-\sqrt{}\right.}{\sqrt{ex+d}\,(cdx+ae)(aeg-cdf)^{2}(gx+f)\sqrt{(aeg-cdf)g}}$

input
```
int((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*(c*d*x+a*e)^(1/2)*arctanh(g
*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*g^2*x+3*(c*d*x+a*e)^(1/2)*
arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*f*g-3*((a*e*g-c*d
*f)*g)^(1/2)*c*d*g*x-((a*e*g-c*d*f)*g)^(1/2)*a*e*g-2*((a*e*g-c*d*f)*g)^(1/
2)*c*d*f)/(c*d*x+a*e)/(a*e*g-c*d*f)^2/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

3.670.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(182) = 364$.

Time = 0.33 (sec) , antiderivative size = 1067, normalized size of antiderivative = 5.28

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2 \left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \left[\frac{3\left(c^2d^2egx^3+acd^2ef+(c^2d^2ef+(c^2d^3+acde^2)g)x^2\right.}{2\left(ac^2d^3ef^3-2\,a^2cd^2e^2f^2g+a^3de^3fg^2+(c^3d^3ef^2g-\right.}\right.$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="fricas")
```

output
```
[1/2*(3*(c^2*d^2*e*g*x^3 + a*c*d^2*e*f + (c^2*d^2*e*f + (c^2*d^3 + a*c*d*e
^2)*g)*x^2 + (a*c*d^2*e*g + (c^2*d^3 + a*c*d*e^2)*f)*x)*sqrt(-g/(c*d*f - a
*e*g))*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)
) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) -
2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*g*x + 2*c*d*f + a*e*g
)*sqrt(e*x + d))/(a*c^2*d^3*e*f^3 - 2*a^2*c*d^2*e^2*f^2*g + a^3*d*e^3*f*g^
2 + (c^3*d^3*e*f^2*g - 2*a*c^2*d^2*e^2*f*g^2 + a^2*c*d*e^3*g^3)*x^3 + (c^3
*d^3*e*f^3 + (c^3*d^4 - a*c^2*d^2*e^2)*f^2*g - (2*a*c^2*d^3*e + a^2*c*d*e^
3)*f*g^2 + (a^2*c*d^2*e^2 + a^3*e^4)*g^3)*x^2 + (a^3*d*e^3*g^3 + (c^3*d^4
+ a*c^2*d^2*e^2)*f^3 - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*f^2*g - (a^2*c*d^2*e^
2 - a^3*e^4)*f*g^2)*x), -(3*(c^2*d^2*e*g*x^3 + a*c*d^2*e*f + (c^2*d^2*e*f
+ (c^2*d^3 + a*c*d*e^2)*g)*x^2 + (a*c*d^2*e*g + (c^2*d^3 + a*c*d*e^2)*f)*x
)*sqrt(g/(c*d*f - a*e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a
*d*e*g + (c*d^2 + a*e^2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*(3*c*d*g*x + 2*c*d*f + a*e*g)*sqrt(e*x + d))/(a*c^2*d^3*e*f^3 - 2*a^2*c
*d^2*e^2*f^2*g + a^3*d*e^3*f*g^2 + (c^3*d^3*e*f^2*g - 2*a*c^2*d^2*e^2*f*g^
2 + a^2*c*d*e^3*g^3)*x^3 + (c^3*d^3*e*f^3 + (c^3*d^4 - a*c^2*d^2*e^2)*f^2*
g - (2*a*c^2*d^3*e + a^2*c*d*e^3)*f*g^2 + (a^2*c*d^2*e^2 + a^3*e^4)*g^3...
```

3.670.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2 \left(ade+(cd^2+ae^2) x+cdex^2\right)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(3/2),x)
```

output
```
Timed out
```

3.670.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2 \left(ade+(cd^2+ae^2) x+cdex^2\right)^{3/2}} \, dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^2} \, dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(
g*x + f)^2), x)
```

3.670.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 753 vs. $2(182) = 364$.

Time = 0.50 (sec) , antiderivative size = 753, normalized size of antiderivative = 3.73

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2 \left(ade+(cd^2+ae^2) x+cdex^2\right)^{3/2}} \, dx =$$

$$-\left(\frac{3\,cdg\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2e}}\right)}{(c^2d^2ef^2|e|-2\,acde^2fg|e|+a^2e^3g^2|e|)\sqrt{cdfg-aeg^2e}}+\frac{2\,c}{(c^2d^2ef^2|e|-2\,acde^2fg|e|+a^2e^3g^2|e|)\left(\sqrt{(ex}\right.}\right.$$

$$+\frac{3\sqrt{-cd^2e+ae^3}cde^2fg\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2e}}\right)-3\sqrt{-cd^2e+a}}{\sqrt{-cd^2e+ae^3}\sqrt{cdfg-aeg^2}c^2d^2ef^3|e|-\sqrt{-cd^2e+ae^3}\sqrt{cdfg-aeg^2}c^2d^3f^2g|e|-2\sqrt{-cd^2e+ae^3}\sqrt{cdfg}}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="giac")
```

output
```
-(3*c*d*g*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g -
 a*e*g^2)*e))/((c^2*d^2*e*f^2*abs(e) - 2*a*c*d*e^2*f*g*abs(e) + a^2*e^3*g^
2*abs(e))*sqrt(c*d*f*g - a*e*g^2)*e) + (2*c^2*d^2*e^2*f - 2*a*c*d*e^3*g +
3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)/((c^2*d^2*e*f^2*abs(e) - 2*a*
c*d*e^2*f*g*abs(e) + a^2*e^3*g^2*abs(e))*(sqrt((e*x + d)*c*d*e - c*d^2*e +
 a*e^3)*c*d*e^2*f - sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*e^3*g + ((e*
x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*g)))*e^3 + (3*sqrt(-c*d^2*e + a*e^3)
*c*d*e^2*f*g*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))
- 3*sqrt(-c*d^2*e + a*e^3)*c*d^2*e*g^2*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sq
rt(c*d*f*g - a*e*g^2)*e)) + 2*sqrt(c*d*f*g - a*e*g^2)*c*d*e^3*f - 3*sqrt(c
*d*f*g - a*e*g^2)*c*d^2*e^2*g + sqrt(c*d*f*g - a*e*g^2)*a*e^4*g)/(sqrt(-c*
d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d^2*e*f^3*abs(e) - sqrt(-c*d^2*
e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d^3*f^2*g*abs(e) - 2*sqrt(-c*d^2*e
+ a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c*d*e^2*f^2*g*abs(e) + 2*sqrt(-c*d^2*e
+ a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c*d^2*e*f*g^2*abs(e) + sqrt(-c*d^2*e +
a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*e^3*f*g^2*abs(e) - sqrt(-c*d^2*e + a*e^
3)*sqrt(c*d*f*g - a*e*g^2)*a^2*d*e^2*g^3*abs(e))
```

3.670.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{(d+e\,x)^{3/2}}{(f+g\,x)^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
3/2)),x)
```

output
```
int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
3/2)), x)
```

3.671 $\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

3.671.1 Optimal result

Integrand size = 46, antiderivative size = 274

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{5g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} - \frac{15cdg\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4(cdf-aeg)^3\sqrt{d+ex}(f+gx)} - \frac{15c^2d^2\sqrt{g}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4(cdf-aeg)^{7/2}}$$

output
```
-15/4*c^2*d^2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e
*g+c*d*f)^(1/2)/(e*x+d)^(1/2))*g^(1/2)/(-a*e*g+c*d*f)^(7/2)-2*(e*x+d)^(1/2
)/(-a*e*g+c*d*f)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-5/2*g*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)^2/(e*x+d)^
(1/2)-15/4*c*d*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^3/
(g*x+f)/(e*x+d)^(1/2)
```

3.671.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx =$$
$$-\frac{\sqrt{d+ex}\left(\sqrt{cdf-aeg}(-2a^2e^2g^2+acdeg(9f+5gx)+c^2d^2(8f^2+25fgx+15g^2x^2))+15c^2d^2\sqrt{g}\sqrt{ae+c}\right.}{4(cdf-aeg)^{7/2}\sqrt{(ae+cdx)(d+ex)}(f+gx)^2}$$

input
```
Integrate[(d + e*x)^(3/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)),x]
```

output
```
-1/4*(Sqrt[d + e*x]*(Sqrt[c*d*f - a*e*g]*(-2*a^2*e^2*g^2 + a*c*d*e*g*(9*f
+ 5*g*x) + c^2*d^2*(8*f^2 + 25*f*g*x + 15*g^2*x^2)) + 15*c^2*d^2*Sqrt[g]*S
qrt[a*e + c*d*x]*(f + g*x)^2*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f
 - a*e*g]]))/((c*d*f - a*e*g)^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x
)^2)
```

3.671.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {1252, 1254, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 1252$$

$$-\frac{5g\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \; 1254$$

$$-\frac{5g\left(\frac{3cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$\big\downarrow$ 1254

$$-\frac{5g\left(\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$\big\downarrow$ 1255

$$-\frac{5g\left(\frac{3cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+c}}{2\sqrt{d+ex}(f+gx)^2(cdf-}\right.}{cdf-aeg}$$

$$-\frac{2\sqrt{d+ex}}{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$\big\downarrow$ 218

3.671. $\int \frac{(d+ex)^{3/2}}{(f+gx)^3(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

$$-\frac{5g\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

input
```
Int[(d + e*x)^(3/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2)),x]
```

output
```
(-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2]) - (5*g*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*
(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c*d*(Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) + (c*d*Ar
cTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a
*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2))))/(4*(c*d*f - a*e*g
))))/(c*d*f - a*e*g)
```

3.671.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1252
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

3.671. $\int \frac{(d+ex)^{3/2}}{(f+gx)^3(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.671.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.35

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)\sqrt{cdx+ae}\,c^2d^2g^3x^2+30\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)\sqrt{cdx+ae}\,c^2d^2f\,g^2x-15\sqrt{(aeg-}\right.}{}$

input
```
int((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,meth
od=_RETURNVERBOSE)
```

output
```
-1/4/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(
1/2)/((a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*c^2*d^2*g^3*x^2+30*arctanh
(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*c^2*d^2*f*
g^2*x-15*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*g^2*x^2+15*arctanh(g*(c*d*x+a*e)^
(1/2)/((a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*c^2*d^2*f^2*g-5*((a*e*g-c
*d*f)*g)^(1/2)*a*c*d*e*g^2*x-25*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x+2*((
a*e*g-c*d*f)*g)^(1/2)*a^2*e^2*g^2-9*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*f*g-8*
((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f^2)/(c*d*x+a*e)/(a*e*g-c*d*f)^3/(g*x+f)^2
/((a*e*g-c*d*f)*g)^(1/2)
```

3.671.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(244) = 488$.

Time = 0.49 (sec) , antiderivative size = 1863, normalized size of antiderivative = 6.80

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="fricas")
```

output
```
[-1/8*(15*(c^3*d^3*e*g^2*x^4 + a*c^2*d^3*e*f^2 + (2*c^3*d^3*e*f*g + (c^3*d
^4 + a*c^2*d^2*e^2)*g^2)*x^3 + (c^3*d^3*e*f^2 + a*c^2*d^3*e*g^2 + 2*(c^3*d
^4 + a*c^2*d^2*e^2)*f*g)*x^2 + (2*a*c^2*d^3*e*f*g + (c^3*d^4 + a*c^2*d^2*e
^2)*f^2)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e
*g + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*
x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*
x^2 + d*f + (e*f + d*g)*x)) + 2*(15*c^2*d^2*g^2*x^2 + 8*c^2*d^2*f^2 + 9*a*
c*d*e*f*g - 2*a^2*e^2*g^2 + 5*(5*c^2*d^2*f*g + a*c*d*e*g^2)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c^3*d^4*e*f^5 - 3*a^2*c
^2*d^3*e^2*f^4*g + 3*a^3*c*d^2*e^3*f^3*g^2 - a^4*d*e^4*f^2*g^3 + (c^4*d^4*
e*f^3*g^2 - 3*a*c^3*d^3*e^2*f^2*g^3 + 3*a^2*c^2*d^2*e^3*f*g^4 - a^3*c*d*e^
4*g^5)*x^4 + (2*c^4*d^4*e*f^4*g + (c^4*d^5 - 5*a*c^3*d^3*e^2)*f^3*g^2 - 3*
(a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^2*g^3 + (3*a^2*c^2*d^3*e^2 + a^3*c*d*e^4
)*f*g^4 - (a^3*c*d^2*e^3 + a^4*e^5)*g^5)*x^3 + (c^4*d^4*e*f^5 - a^4*d*e^4*
g^5 + (2*c^4*d^5 - a*c^3*d^3*e^2)*f^4*g - (5*a*c^3*d^4*e + 3*a^2*c^2*d^2*e
^3)*f^3*g^2 + (3*a^2*c^2*d^3*e^2 + 5*a^3*c*d*e^4)*f^2*g^3 + (a^3*c*d^2*e^3
 - 2*a^4*e^5)*f*g^4)*x^2 - (2*a^4*d*e^4*f*g^4 - (c^4*d^5 + a*c^3*d^3*e^2)*
f^5 + (a*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*f^4*g + 3*(a^2*c^2*d^3*e^2 - a^3*c
*d*e^4)*f^3*g^2 - (5*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^3)*x), -1/4*(15*(c^3*d
^3*e*g^2*x^4 + a*c^2*d^3*e*f^2 + (2*c^3*d^3*e*f*g + (c^3*d^4 + a*c^2*d^...
```

3.671.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(3/2),x)
```

output
```
Timed out
```

3.671.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^3}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(
g*x + f)^3), x)
```

3.671.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1390 vs. $2(244) = 488$.

Time = 0.70 (sec) , antiderivative size = 1390, normalized size of antiderivative = 5.07

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="giac")
```

output
```
-1/4*(15*c^2*d^2*g*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(
c*d*f*g - a*e*g^2)*e))/((c^3*d^3*e^2*f^3*abs(e) - 3*a*c^2*d^2*e^3*f^2*g*ab
s(e) + 3*a^2*c*d*e^4*f*g^2*abs(e) - a^3*e^5*g^3*abs(e))*sqrt(c*d*f*g - a*e
*g^2)*e) + 8*c^2*d^2/((c^3*d^3*e^2*f^3*abs(e) - 3*a*c^2*d^2*e^3*f^2*g*abs(
e) + 3*a^2*c*d*e^4*f*g^2*abs(e) - a^3*e^5*g^3*abs(e))*sqrt((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)) + (9*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^3*d^3*
e^2*f*g - 9*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^2*d^2*e^3*g^2 + 7*
((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*g^2)/((c^3*d^3*e^2*f^3*a
bs(e) - 3*a*c^2*d^2*e^3*f^2*g*abs(e) + 3*a^2*c*d*e^4*f*g^2*abs(e) - a^3*e^
5*g^3*abs(e))*(c*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g
)^2))*e^4 + 1/4*(15*sqrt(-c*d^2*e + a*e^3)*c^2*d^2*e^3*f^2*g*arctan(sqrt(-
c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 30*sqrt(-c*d^2*e + a*e^3
)*c^2*d^3*e^2*f*g^2*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^
2)*e)) + 15*sqrt(-c*d^2*e + a*e^3)*c^2*d^4*e*g^3*arctan(sqrt(-c*d^2*e + a*
e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 8*sqrt(c*d*f*g - a*e*g^2)*c^2*d^2*e^
4*f^2 - 25*sqrt(c*d*f*g - a*e*g^2)*c^2*d^3*e^3*f*g + 9*sqrt(c*d*f*g - a*e*
g^2)*a*c*d*e^5*f*g + 15*sqrt(c*d*f*g - a*e*g^2)*c^2*d^4*e^2*g^2 - 5*sqrt(c
*d*f*g - a*e*g^2)*a*c*d^2*e^4*g^2 - 2*sqrt(c*d*f*g - a*e*g^2)*a^2*e^6*g^2)
/(sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^3*d^3*e^2*f^5*abs(e) -
2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^3*d^4*e*f^4*g*abs(e)...
```

3.671.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{(d+e\,x)^{3/2}}{(f+g\,x)^3\,(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{3/2}}\,dx$$

input
```
int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
3/2)),x)
```

output
```
int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
3/2)), x)
```

3.672 $\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

3.672.1 Optimal result

Integrand size = 46, antiderivative size = 239

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{4g\sqrt{d+ex}(f+gx)^2}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{16g^2(2ae^2g-cd(3ef-dg))\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^4d^4e\sqrt{d+ex}} + \frac{16g^3\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^3d^3e}$$

output
```
-2/3*(e*x+d)^(3/2)*(g*x+f)^3/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-4
*g*(g*x+f)^2*(e*x+d)^(1/2)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
-16/3*g^2*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)/c^4/d^4/e/(e*x+d)^(1/2)+16/3*g^3*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/c^3/d^3/e
```

3.672.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.55

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \frac{2(d+ex)^{3/2}\left(-16a^3e^3g^3+24a^2cde^2g^2(f-gx)-6ac^2d^2eg(f^2-6\right.}{3c^4d^4((ae+cdx)(d+}$$

input
```
Integrate[((d + e*x)^(5/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(5/2),x]
```

output
```
(2*(d + e*x)^(3/2)*(-16*a^3*e^3*g^3 + 24*a^2*c*d*e^2*g^2*(f - g*x) - 6*a*c
^2*d^2*e*g*(f^2 - 6*f*g*x + g^2*x^2) + c^3*d^3*(-f^3 - 9*f^2*g*x + 9*f*g^2
*x^2 + g^3*x^3)))/(3*c^4*d^4*((a*e + c*d*x)*(d + e*x))^(3/2))
```

3.672.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1251, 1251, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 1251$$

$$\frac{2g\int \frac{(d+ex)^{3/2}(f+gx)^2}{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{cd}-\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\,(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

$$\Big\downarrow \; 1251$$

$$\frac{2g\left(\frac{4g\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cd}-\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{cd}-\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\,(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

$$\Big\downarrow \; 1221$$

$$\frac{2g\left(\frac{4g\left(\frac{1}{3}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)\int\frac{\sqrt{d+ex}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{2g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{cd}-\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\downarrow$ 1122

$$\frac{2g\left(\frac{4g\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)}{3cd\sqrt{d+ex}}+\frac{2g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{cd}-\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[((d + e*x)^(5/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2)*(f + g*x)^3)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)) + (2*g*((-2*Sqrt[d + e*x]*(f + g*x)^2)/(c*d*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2]) + (4*g*((2*(3*f - (d*g)/e - (2*a*e*g)/(c*d))*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*Sqrt[d + e*x]) + (2*g*S
qrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*e)))/(c*d
)))/(c*d)
```

3.672.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

3.672. $\int \frac{(d+ex)^{5/2}(f+gx)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1251
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[e*g*(n/(c*(p + 1)))   Int[(
d + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p,
 0] && LtQ[p, -1] && GtQ[n, 0]
```

3.672.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.75

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-g^3x^3c^3d^3+6a\,c^2d^2e\,g^3x^2-9c^3d^3f\,g^2x^2+24a^2cd\,e^2g^3x-36a\,c^2d^2ef\,g^2x+9c^3d^3f^2gx+16a^3e^3g^3-24a^2cd\,e^{2}}{3\sqrt{ex+d}\,(cdx+ae)^2c^4d^4}$
gosper	$-\frac{2(cdx+ae)(-g^3x^3c^3d^3+6a\,c^2d^2e\,g^3x^2-9c^3d^3f\,g^2x^2+24a^2cd\,e^2g^3x-36a\,c^2d^2ef\,g^2x+9c^3d^3f^2gx+16a^3e^3g^3-24a^2cd\,e^2f\,g^2+6a}{3c^4d^4(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}$

input
```
int((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,meth
od=_RETURNVERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-c^3*d^3*g^3*x^3+6*a*c^2*d
^2*e*g^3*x^2-9*c^3*d^3*f*g^2*x^2+24*a^2*c*d*e^2*g^3*x-36*a*c^2*d^2*e*f*g^2
*x+9*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-24*a^2*c*d*e^2*f*g^2+6*a*c^2*d^2*e*f^2
*g+c^3*d^3*f^3)/(c*d*x+a*e)^2/c^4/d^4
```

3.672.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \frac{2\,(c^3d^3g^3x^3-c^3d^3f^3-6\,ac^2d^2ef^2g+24\,a^2cde^2fg^2-16\,a^3e^3g^3+}{3\,(c^6d^6ex^3-}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="fricas")
```

output
```
2/3*(c^3*d^3*g^3*x^3 - c^3*d^3*f^3 - 6*a*c^2*d^2*e*f^2*g + 24*a^2*c*d*e^2*
f*g^2 - 16*a^3*e^3*g^3 + 3*(3*c^3*d^3*f*g^2 - 2*a*c^2*d^2*e*g^3)*x^2 - 3*(
3*c^3*d^3*f^2*g - 12*a*c^2*d^2*e*f*g^2 + 8*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^6*d^6*e*x^3 + a^2*c^4*d^
5*e^2 + (c^6*d^7 + 2*a*c^5*d^5*e^2)*x^2 + (2*a*c^5*d^6*e + a^2*c^4*d^4*e^3
)*x)
```

3.672.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(5/2),x)
```

output
```
Timed out
```

3.672.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.92

$$\begin{aligned}\int \frac{(d+ex)^{5/2}(f+gx)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx &= \\ &-\frac{2\,(3\,cdx+2\,ae)f^2g}{(c^3d^3x+ac^2d^2e)\sqrt{cdx+ae}}+\frac{2\,(3\,c^2d^2x^2+12\,acdex+8\,a^2e^2)fg^2}{(c^4d^4x+ac^3d^3e)\sqrt{cdx+ae}} \\ &+\frac{2\,(c^3d^3x^3-6\,ac^2d^2ex^2-24\,a^2cde^2x-16\,a^3e^3)g^3}{3\,(c^5d^5x+ac^4d^4e)\sqrt{cdx+ae}}-\frac{2\,f^3}{3\,(c^2d^2x+acde)\sqrt{cdx+ae}}\end{aligned}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="maxima")
```

output
```
-2*(3*c*d*x + 2*a*e)*f^2*g/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x + a*e)) +
 2*(3*c^2*d^2*x^2 + 12*a*c*d*e*x + 8*a^2*e^2)*f*g^2/((c^4*d^4*x + a*c^3*d^
3*e)*sqrt(c*d*x + a*e)) + 2/3*(c^3*d^3*x^3 - 6*a*c^2*d^2*e*x^2 - 24*a^2*c*
d*e^2*x - 16*a^3*e^3)*g^3/((c^5*d^5*x + a*c^4*d^4*e)*sqrt(c*d*x + a*e)) -
2/3*f^3/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

3.672.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(217) = 434$.

Time = 0.34 (sec) , antiderivative size = 524, normalized size of antiderivative = 2.19

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx =$$

$$-\frac{2\left(c^3d^3e^3f^3-9\,c^3d^4e^2f^2g+6\,ac^2d^2e^4f^2g-9\,c^3d^5efg^2+36\,ac^2d^3e^3fg^2-24\,a^2cde^5fg^2+c^3d^6g^3+6\,ac^2d^4\right.}{3\left(\sqrt{-cd^2e+ae^3}c^5d^6|e|-\sqrt{-cd^2e+ae^3}ac^4d^4e^2|e|\right)}$$

$$-\frac{2\left(c^3d^3e^4f^3-3\,ac^2d^2e^5f^2g+3\,a^2cde^6fg^2-a^3e^7g^3+9\left((ex+d)cde-cd^2e+ae^3\right)c^2d^2e^2f^2g-18\left((ex+\right.\right.}{3\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}c^4d^4|e|}$$

$$+\frac{2\left(9\sqrt{(ex+d)cde-cd^2e+ae^3}c^9d^9e^8fg^2-9\sqrt{(ex+d)cde-cd^2e+ae^3}ac^8d^8e^9g^3+\left((ex+d)cde-cd^2\right.\right.}{3\,c^{12}d^{12}e^8|e|}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="giac")
```

output
```
-2/3*(c^3*d^3*e^3*f^3 - 9*c^3*d^4*e^2*f^2*g + 6*a*c^2*d^2*e^4*f^2*g - 9*c^
3*d^5*e*f*g^2 + 36*a*c^2*d^3*e^3*f*g^2 - 24*a^2*c*d*e^5*f*g^2 + c^3*d^6*g^
3 + 6*a*c^2*d^4*e^2*g^3 - 24*a^2*c*d^2*e^4*g^3 + 16*a^3*e^6*g^3)/(sqrt(-c*
d^2*e + a*e^3)*c^5*d^6*abs(e) - sqrt(-c*d^2*e + a*e^3)*a*c^4*d^4*e^2*abs(e
)) - 2/3*(c^3*d^3*e^4*f^3 - 3*a*c^2*d^2*e^5*f^2*g + 3*a^2*c*d*e^6*f*g^2 -
a^3*e^7*g^3 + 9*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^2*e^2*f^2*g - 18
*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c*d*e^3*f*g^2 + 9*((e*x + d)*c*d*e
- c*d^2*e + a*e^3)*a^2*e^4*g^3)/((((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)
*c^4*d^4*abs(e)) + 2/3*(9*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^9*d^9*
e^8*f*g^2 - 9*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^8*d^8*e^9*g^3 +
((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^8*d^8*e^6*g^3)/(c^12*d^12*e^8*
abs(e))
```

3.672.9 Mupad [B] (verification not implemented)

Time = 12.94 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx =$$

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(\frac{32\,a^3\,e^3\,g^3}{3}-16\,a^2\,c\,d\,e^2\,f\,g^2+4\,a\,c^2\,d^2\,e\,f^2\,g+\frac{2\,c^3\,d^3\,f^3}{3}\right)}{c^6\,d^6\,e}-\frac{2\,g^3\,x^3\,\sqrt{d+e\,x}}{3\,c^3\,d^3\,e}+\frac{g^2\,x^2}{}\right.}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}+\frac{x^2\,\left(c^6\,d^7+2\,a\,c^5\,d^5\,e^2\right)}{c^6\,d^6\,e}}$$

input
```
int(((f + g*x)^3*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(5/2),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*((32*a^3
*e^3*g^3)/3 + (2*c^3*d^3*f^3)/3 + 4*a*c^2*d^2*e*f^2*g - 16*a^2*c*d*e^2*f*g
^2))/(c^6*d^6*e) - (2*g^3*x^3*(d + e*x)^(1/2))/(3*c^3*d^3*e) + (g^2*x^2*(4
*a*e*g - 6*c*d*f)*(d + e*x)^(1/2))/(c^4*d^4*e) + (2*g*x*(d + e*x)^(1/2)*(8
*a^2*e^2*g^2 + 3*c^2*d^2*f^2 - 12*a*c*d*e*f*g))/(c^5*d^5*e)))/(x^3 + (a^2*
e)/(c^2*d) + (a*x*(a*e^2 + 2*c*d^2))/(c^2*d^2) + (x^2*(c^6*d^7 + 2*a*c^5*d
^5*e^2))/(c^6*d^6*e))
```

3.673 $\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

3.673.1 Optimal result

Integrand size = 46, antiderivative size = 211

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{8g(cdf-aeg)(d+ex)^{3/2}}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{8g\left(2ae^2g-cd(ef+dg)\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^3d^3\left(cd^2-ae^2\right)\sqrt{d+ex}}$$

output
```
-2/3*(e*x+d)^(3/2)*(g*x+f)^2/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-8
/3*g*(-a*e*g+c*d*f)*(e*x+d)^(3/2)/c^2/d^2/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)-8/3*g*(2*a*e^2*g-c*d*(d*g+e*f))*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)/c^3/d^3/(-a*e^2+c*d^2)/(e*x+d)^(1/2)
```

3.673.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.41

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2(d+ex)^{3/2}\left(8a^2e^2g^2-4acdeg(f-3gx)-c^2d^2(f^2+6fgx-3g^2\right.}{3c^3d^3((ae+cdx)(d+ex))^{3/2}}$$

input
```
Integrate[((d + e*x)^(5/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(5/2),x]
```

output
```
(2*(d + e*x)^(3/2)*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(f - 3*g*x) - c^2*d^2*(f^2
 + 6*f*g*x - 3*g^2*x^2)))/(3*c^3*d^3*((a*e + c*d*x)*(d + e*x))^(3/2))
```

3.673.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1251, 1218, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1251}$$

$$\frac{4g\int \frac{(d+ex)^{3/2}(f+gx)}{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3cd}-\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1218}$$

$$\frac{4g\left(-\frac{(2ae^2g-cd(dg+ef))\int \frac{\sqrt{d+ex}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cd(cd^2-ae^2)}-\frac{2(d+ex)^{3/2}(cdf-aeg)}{cd(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{4g\left(-\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(2ae^2g-cd(dg+ef))}{c^2d^2\sqrt{d+ex}(cd^2-ae^2)}-\frac{2(d+ex)^{3/2}(cdf-aeg)}{cd(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[((d + e*x)^(5/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2)*(f + g*x)^2)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)) + (4*g*((-2*(c*d*f - a*e*g)*(d + e*x)^(3/2))/(c*d*(c*d^2 - a*
e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (2*(2*a*e^2*g - c*d*(e
*f + d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^2*d^2*(c*d^2 -
a*e^2)*Sqrt[d + e*x])))/(3*c*d)
```

3.673.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

rule 1251
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[e*g*(n/(c*(p + 1)))   Int[(
d + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p,
 0] && LtQ[p, -1] && GtQ[n, 0]
```

3.673.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(3g^2x^2c^2d^2+12acde\,g^2x-6c^2d^2fgx+8a^2e^2g^2-4acdefg-c^2d^2f^2)}{3\sqrt{ex+d}\,(cdx+ae)^2c^3d^3}$	108
gosper	$\frac{2(cdx+ae)(3g^2x^2c^2d^2+12acde\,g^2x-6c^2d^2fgx+8a^2e^2g^2-4acdefg-c^2d^2f^2)(ex+d)^{\frac{5}{2}}}{3c^3d^3(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}$	116

3.673. $\int \frac{(d+ex)^{5/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

input
```
int((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/3/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*c^2*d^2*g^2*x^2+12*a*c*d*
e*g^2*x-6*c^2*d^2*f*g*x+8*a^2*e^2*g^2-4*a*c*d*e*f*g-c^2*d^2*f^2)/(c*d*x+a*
e)^2/c^3/d^3
```

3.673.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{2\,(3\,c^2d^2g^2x^2-c^2d^2f^2-4\,acdefg+8\,a^2e^2g^2-6\,(c^2d^2fg-2\,acde}{3\,(c^5d^5ex^3+a^2c^3d^4e^2+(c^5d^6+2\,ac^4d^4e^2)x^2+}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="fricas")
```

output
```
2/3*(3*c^2*d^2*g^2*x^2 - c^2*d^2*f^2 - 4*a*c*d*e*f*g + 8*a^2*e^2*g^2 - 6*(
c^2*d^2*f*g - 2*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*sqrt(e*x + d)/(c^5*d^5*e*x^3 + a^2*c^3*d^4*e^2 + (c^5*d^6 + 2*a*c^4*d^4*
e^2)*x^2 + (2*a*c^4*d^5*e + a^2*c^3*d^3*e^3)*x)
```

3.673.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(5/2),x)
```

output
```
Timed out
```

3.673.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.65

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{4\,(3\,cdx+2\,ae)fg}{3\,(c^3d^3x+ac^2d^2e)\sqrt{cdx+ae}} + \frac{2\,(3\,c^2d^2x^2+12\,acdex+8\,a^2e^2)g^2}{3\,(c^4d^4x+ac^3d^3e)\sqrt{cdx+ae}} - \frac{2\,f^2}{3\,(c^2d^2x+acde)\sqrt{cdx+ae}}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="maxima")
```

output
```
-4/3*(3*c*d*x + 2*a*e)*f*g/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x + a*e)) +
 2/3*(3*c^2*d^2*x^2 + 12*a*c*d*e*x + 8*a^2*e^2)*g^2/((c^4*d^4*x + a*c^3*d^
3*e)*sqrt(c*d*x + a*e)) - 2/3*f^2/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e)
)
```

3.673.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2\,(c^2d^2e^3f^2-6\,c^2d^3e^2fg+4\,acde^4fg-3\,c^2d^4eg^2+12\,acd^2e^3g^2-8\,a^2e^5g^2)}{3\,\left(\sqrt{-cd^2e+ae^3}c^4d^5|e|-\sqrt{-cd^2e+ae^3}ac^3d^3e^2|e|\right)} + \frac{2\,\sqrt{(ex+d)cde-cd^2e+ae^3}g^2}{c^3d^3|e|} - \frac{2\,(c^2d^2e^4f^2-2\,acde^5fg+a^2e^6g^2+6\,((ex+d)cde-cd^2e+ae^3)cde^2fg-6\,((ex+d)cde-cd^2e+ae^3)ae}{3\,((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}c^3d^3|e|}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="giac")
```

output
```
-2/3*(c^2*d^2*e^3*f^2 - 6*c^2*d^3*e^2*f*g + 4*a*c*d*e^4*f*g - 3*c^2*d^4*e*
g^2 + 12*a*c*d^2*e^3*g^2 - 8*a^2*e^5*g^2)/(sqrt(-c*d^2*e + a*e^3)*c^4*d^5*
abs(e) - sqrt(-c*d^2*e + a*e^3)*a*c^3*d^3*e^2*abs(e)) + 2*sqrt((e*x + d)*c
*d*e - c*d^2*e + a*e^3)*g^2/(c^3*d^3*abs(e)) - 2/3*(c^2*d^2*e^4*f^2 - 2*a*
c*d*e^5*f*g + a^2*e^6*g^2 + 6*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*e^2*
f*g - 6*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*e^3*g^2)/((((e*x + d)*c*d*e -
 c*d^2*e + a*e^3)^(3/2)*c^3*d^3*abs(e))
```

3.673.9 Mupad [B] (verification not implemented)

Time = 12.73 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\,\left(\frac{2\,g^2\,x^2\,\sqrt{d+e\,x}}{c^3\,d^3\,e}-\frac{\sqrt{d+e\,x}\,(-16\,a^2\,e^2\,g^2 \cdots}{3\,c \cdots}\right.}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,(2\,c\,d^2+a\,e^2)}{c^2\,d^2}+\frac{x^2\,(3\,c^5\,d^6 \cdots}{3 \cdots}}$$

input
```
int(((f + g*x)^2*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(5/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^2*(d + e*x)^(1/2)
)/(c^3*d^3*e) - ((d + e*x)^(1/2)*(2*c^2*d^2*f^2 - 16*a^2*e^2*g^2 + 8*a*c*d
*e*f*g))/(3*c^5*d^5*e) + (4*g*x*(2*a*e*g - c*d*f)*(d + e*x)^(1/2))/(c^4*d^
4*e)))/(x^3 + (a^2*e)/(c^2*d) + (a*x*(a*e^2 + 2*c*d^2))/(c^2*d^2) + (x^2*(
3*c^5*d^6 + 6*a*c^4*d^4*e^2))/(3*c^5*d^5*e))
```

3.674 $\int \frac{(d+ex)^{5/2}(f+gx)}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

3.674.1 Optimal result

Integrand size = 44, antiderivative size = 154

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2(cdf-aeg)(d+ex)^{5/2}}{3cd\,(cd^2-ae^2)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}} + \frac{2(2ae^2g+cd(ef-3dg))\,\sqrt{d+ex}}{3c^2d^2\,(cd^2-ae^2)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

output
```
-2/3*(-a*e*g+c*d*f)*(e*x+d)^(5/2)/c/d/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(3/2)+2/3*(2*a*e^2*g+c*d*(-3*d*g+e*f))*(e*x+d)^(1/2)/c^2/d^2/
(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

3.674.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.34

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(2aeg+cd(f+3gx))}{3c^2d^2((ae+cdx)(d+ex))^{3/2}}$$

input
```
Integrate[((d + e*x)^(5/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2)*(2*a*e*g + c*d*(f + 3*g*x)))/(3*c^2*d^2*((a*e + c*d*x)
*(d + e*x))^(3/2))
```

3.674.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1218, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$-\frac{(2ae^2g+cd(ef-3dg))\int \frac{(d+ex)^{3/2}}{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3cd(cd^2-ae^2)}-\frac{2(d+ex)^{5/2}(cdf-aeg)}{3cd(cd^2-ae^2)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2\sqrt{d+ex}(2ae^2g+cd(ef-3dg))}{3c^2d^2(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2(d+ex)^{5/2}(cdf-aeg)}{3cd(cd^2-ae^2)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

input
```
Int[((d + e*x)^(5/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5
/2),x]
```

output
```
(-2*(c*d*f - a*e*g)*(d + e*x)^(5/2))/(3*c*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (2*(2*a*e^2*g + c*d*(e*f - 3*d*g))*Sqrt
[d + e*x])/(3*c^2*d^2*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])
```

3.674.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

3.674.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.38

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(3cdgx+2aeg+cdf)}{3\sqrt{ex+d}\,(cdx+ae)^{2}c^{2}d^{2}}$	58
gosper	$-\frac{2(cdx+ae)(3cdgx+2aeg+cdf)(ex+d)^{\frac{5}{2}}}{3c^{2}d^{2}(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{5}{2}}}$	66

input
```
int((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,method
=_RETURNVERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*c*d*g*x+2*a*e*g+c*d*f)/(
c*d*x+a*e)^2/c^2/d^2
```

3.674.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(3\,cdgx+cdf+2\,aeg)\sqrt{ex+d}}{3\,(c^4d^4ex^3+a^2c^2d^3e^2+(c^4d^5+2\,ac^3d^3e^2)x^2+(2\,ac^3d^4e+a^2c^2d^2e^3)x)}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
 algorithm="fricas")
```

output
```
-2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*g*x + c*d*f + 2*a*
e*g)*sqrt(e*x + d)/(c^4*d^4*e*x^3 + a^2*c^2*d^3*e^2 + (c^4*d^5 + 2*a*c^3*d
^3*e^2)*x^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*x)
```

3.674.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/
2),x)
```

output
```
Timed out
```

3.674.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.47

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2\,(3\,cdx+2\,ae)g}{3\,(c^3d^3x+ac^2d^2e)\sqrt{cdx+ae}} - \frac{2\,f}{3\,(c^2d^2x+acde)\sqrt{cdx+ae}}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
 algorithm="maxima")
```

output
```
-2/3*(3*c*d*x + 2*a*e)*g/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x + a*e)) - 2
/3*f/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

3.674.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2\left(cde^3f-3\,cd^2e^2g+2\,ae^4g\right)}{3\left(\sqrt{-cd^2e+ae^3}c^3d^4|e|-\sqrt{-cd^2e+ae^3}ac^2d^2e^2|e|\right)} - \frac{2\left(cde^4f-ae^5g+3\left((ex+d)cde-cd^2e+ae^3\right)e^2g\right)}{3\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}c^2d^2|e|}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
 algorithm="giac")
```

output
```
-2/3*(c*d*e^3*f - 3*c*d^2*e^2*g + 2*a*e^4*g)/(sqrt(-c*d^2*e + a*e^3)*c^3*d
^4*abs(e) - sqrt(-c*d^2*e + a*e^3)*a*c^2*d^2*e^2*abs(e)) - 2/3*(c*d*e^4*f
- a*e^5*g + 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*e^2*g)/((((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*abs(e))
```

3.674.9 Mupad [B] (verification not implemented)

Time = 12.51 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{\left(\frac{\left(\frac{4\,a\,e\,g}{3}+\frac{2\,c\,d\,f}{3}\right)\sqrt{d+e\,x}}{c^4\,d^4\,e}+\frac{2\,g\,x\,\sqrt{d+e\,x}}{c^3\,d^3\,e}\right)\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}+\frac{x^2\,\left(c^4\,d^5+2\,a\,c^3\,d^3\,e^2\right)}{c^4\,d^4\,e}}$$

input
```
int(((f + g*x)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5
/2),x)
```

output
```
-(((((4*a*e*g)/3 + (2*c*d*f)/3)*(d + e*x)^(1/2))/(c^4*d^4*e) + (2*g*x*(d +
 e*x)^(1/2))/(c^3*d^3*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(
x^3 + (a^2*e)/(c^2*d) + (a*x*(a*e^2 + 2*c*d^2))/(c^2*d^2) + (x^2*(c^4*d^5
+ 2*a*c^3*d^3*e^2))/(c^4*d^4*e))
```

3.674. $\int \frac{(d+ex)^{5/2}(f+gx)}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

3.675 $\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

3.675.1 Optimal result

Integrand size = 39, antiderivative size = 48

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3cd\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}$$

output
```
-2/3*(e*x+d)^(3/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
```

3.675.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3cd((ae+cdx)(d+ex))^{3/2}}$$

input
```
Integrate[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2))/(3*c*d*((a*e + c*d*x)*(d + e*x))^(3/2))
```

3.675.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}dx$$
$$\Big\downarrow \text{1122}$$
$$-\frac{2(d+ex)^{3/2}}{3cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

input
```
Int[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))
```

3.675.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

3.675.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}}{3\sqrt{ex+d}\,(cdx+ae)^2 cd}$	42
gosper	$-\frac{2(cdx+ae)(ex+d)^{\frac{5}{2}}}{3cd(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}$	50

input
```
int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/(c*d*x+a*e)^2/c/d
```

3.675.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(42) = 84$.

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.23

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}}{3\,(c^3d^3ex^3+a^2cd^2e^2+(c^3d^4+2\,ac^2d^2e^2)x^2+(2\,ac^2d^3e+a^2cde^3)x)}$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="fricas")
```

output
```
-2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*
x^3 + a^2*c*d^2*e^2 + (c^3*d^4 + 2*a*c^2*d^2*e^2)*x^2 + (2*a*c^2*d^3*e + a
^2*c*d*e^3)*x)
```

3.675.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Timed out
```

3.675.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.58

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2}{3\,(c^2d^2x+acde)\sqrt{cdx+ae}}$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="maxima")
```

output
```
-2/3/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

3.675.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(42) = 84.

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.02

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2\,e^3}{3\,\left(\sqrt{-cd^2e+ae^3}c^2d^3|e|-\sqrt{-cd^2e+ae^3}acde^2|e|\right)} - \frac{2\,e^4}{3\,((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}cd|e|}$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="giac")
```

output
```
-2/3*e^3/(sqrt(-c*d^2*e + a*e^3)*c^2*d^3*abs(e) - sqrt(-c*d^2*e + a*e^3)*a
*c*d*e^2*abs(e)) - 2/3*e^4/(((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c*d*
abs(e))
```

3.675.9 Mupad [B] (verification not implemented)

Time = 12.47 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.29

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = -\frac{2\sqrt{d+e\,x}\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}}{3\,\left(a^2\,c\,d^2\,e^2+a^2\,c\,d\,e^3\,x+2\,a\,c^2\,d^3\,e\,x+2\,a\,c^2\,d^2\,e^2\,x^2+c^3\,d^4\,x^2+c^3\,d^3\,e\,x^3\right)}$$

input
```
int((d + e*x)^(5/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
-(2*(d + e*x)^(1/2)*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(3*(c^3
*d^4*x^2 + a^2*c*d^2*e^2 + c^3*d^3*e*x^3 + 2*a*c^2*d^3*e*x + a^2*c*d*e^3*x
 + 2*a*c^2*d^2*e^2*x^2))
```

3.676 $\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

3.676.1 Optimal result

Integrand size = 46, antiderivative size = 188

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2g\sqrt{d+ex}}{(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{2g^{3/2}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{(cdf-aeg)^{5/2}}$$

output
```
-2/3*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+
2*g^(3/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c
*d*f)^(1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(5/2)+2*g*(e*x+d)^(1/2)/(-a*e*g+
c*d*f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

3.676.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2(d+ex)^{3/2}\left(\sqrt{cdf-aeg}(4aeg-cd(f-3gx))+3g^{3/2}(\ldots\right.}{3(cdf-aeg)^{5/2}((ae+cdx)(d+\ldots}$$

input
```
Integrate[(d + e*x)^(5/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2)),x]
```

output
```
(2*(d + e*x)^(3/2)*(Sqrt[c*d*f - a*e*g]*(4*a*e*g - c*d*(f - 3*g*x)) + 3*g^
(3/2)*(a*e + c*d*x)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f -
a*e*g]]))/(3*(c*d*f - a*e*g)^(5/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

3.676.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1252, 1252, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1252}$$

$$-\frac{g\int \frac{(d+ex)^{3/2}}{(f+gx)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{cdf-aeg}-\frac{2(d+ex)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{ 1252}$$

$$-\frac{g\left(-\frac{g\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cdf-aeg}-\frac{2\sqrt{d+ex}}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2(d+ex)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{ 1255}$$

$$-\frac{g\left(-\frac{2e^2g\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}-\frac{2\sqrt{d+ex}}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2(d+ex)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{218}$$

$$-\frac{g\left(-\frac{2\sqrt{g}\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{3/2}}-\frac{2\sqrt{d+ex}}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2(d+ex)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

input
```
Int[(d + e*x)^(5/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/
2)),x]
```

output
```
(-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)) - (g*((-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2]) - (2*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a
*e*g)^(3/2)))/(c*d*f - a*e*g)
```

3.676.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1252
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.676. $\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

3.676.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.11

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\left(3\sqrt{cdx+ae}\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cd\,g^2x+3\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)ae\,g^2\sqrt{cdx+ae}-3\sqrt{(aeg-cdf)g}\,cdgx}{3\sqrt{ex+d}\,(cdx+ae)^2(aeg-cdf)^2\sqrt{(aeg-cdf)g}}$

input
```
int((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,method
=_RETURNVERBOSE)
```

output
```
-2/3*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*(c*d*x+a*e)^(1/2)*arctanh(g*(c*d*x+a*e
)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*g^2*x+3*arctanh(g*(c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2))*a*e*g^2*(c*d*x+a*e)^(1/2)-3*((a*e*g-c*d*f)*g)^(1/2
)*c*d*g*x-4*((a*e*g-c*d*f)*g)^(1/2)*a*e*g+((a*e*g-c*d*f)*g)^(1/2)*c*d*f)/(
e*x+d)^(1/2)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^2/((a*e*g-c*d*f)*g)^(1/2)
```

3.676.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. 2(166) = 332.

Time = 0.39 (sec) , antiderivative size = 1015, normalized size of antiderivative = 5.40

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \left[\frac{3\left(c^2d^2egx^3+a^2de^2g+(c^2d^3+2\,acde^2)gx^2+(2\right.}{3\left(a^2c^2d^3e^2f^2-2\,a^3cd^2e^3fg+a^4de^4g^2+(c^4d^4ef^2-2\,a\right.}\right.$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
 algorithm="fricas")
```

output
```
[1/3*(3*(c^2*d^2*e*g*x^3 + a^2*d*e^2*g + (c^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (
2*a*c*d^2*e + a^2*e^3)*g*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c
*d^2*f + 2*a*d*e*g + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f
- a*e*g)*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*
e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(3*c*d*g*x - c*d*f + 4*a*e*g)*sqrt(e*x + d))/(a^2*c^2*d^
3*e^2*f^2 - 2*a^3*c*d^2*e^3*f*g + a^4*d*e^4*g^2 + (c^4*d^4*e*f^2 - 2*a*c^3
*d^3*e^2*f*g + a^2*c^2*d^2*e^3*g^2)*x^3 + ((c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2
 - 2*(a*c^3*d^4*e + 2*a^2*c^2*d^2*e^3)*f*g + (a^2*c^2*d^3*e^2 + 2*a^3*c*d*
e^4)*g^2)*x^2 + ((2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*f^2 - 2*(2*a^2*c^2*d^3*
e^2 + a^3*c*d*e^4)*f*g + (2*a^3*c*d^2*e^3 + a^4*e^5)*g^2)*x), 2/3*(3*(c^2*
d^2*e*g*x^3 + a^2*d*e^2*g + (c^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (2*a*c*d^2*e +
 a^2*e^3)*g*x)*sqrt(g/(c*d*f - a*e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*
d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(3*c*d*g*x - c*d*f + 4*a*e*g)*sqrt(e*x + d))/(a^2*c^2*d^3*
e^2*f^2 - 2*a^3*c*d^2*e^3*f*g + a^4*d*e^4*g^2 + (c^4*d^4*e*f^2 - 2*a*c^3*d
^3*e^2*f*g + a^2*c^2*d^2*e^3*g^2)*x^3 + ((c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2 -
 2*(a*c^3*d^4*e + 2*a^2*c^2*d^2*e^3)*f*g + (a^2*c^2*d^3*e^2 + 2*a^3*c*d*e^
4)*g^2)*x^2 + ((2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*f^2 - 2*(2*a^2*c^2*d^3...
```

3.676.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/
2),x)
```

output
```
Timed out
```

3.676.7 Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\int \frac{(ex+d)^{\frac{5}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(gx+f)}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
 algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(
g*x + f)), x)
```

3.676.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(166) = 332$.

Time = 0.44 (sec) , antiderivative size = 683, normalized size of antiderivative = 3.63

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\frac{2}{3}e^3\left(\frac{3\,g^2\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{(c^2d^2ef^2|e|-2\,acde^2fg|e|+a^2e^3g^2|e|)\sqrt{cdfg-ae}}\right.$$

$$-\frac{2\left(3\sqrt{-cd^2e+ae^3}cd^2eg^2\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)-3\sqrt{-cd^2e}\right.}{3\left(\sqrt{-cd^2e+ae^3}\sqrt{cdfg-aeg^2}c^3d^4f^2|e|-\sqrt{-cd^2e+ae^3}\sqrt{cdfg-aeg^2}ac^2d^2e^2f^2|e|-2\sqrt{-cd^2e+ae^3}\sqrt{c}\right.}$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
 algorithm="giac")
```

output
```
2/3*e^3*(3*g^2*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*
f*g - a*e*g^2)*e))/((c^2*d^2*e*f^2*abs(e) - 2*a*c*d*e^2*f*g*abs(e) + a^2*e
^3*g^2*abs(e))*sqrt(c*d*f*g - a*e*g^2)*e) - (c*d*e^2*f - a*e^3*g - 3*((e*x
 + d)*c*d*e - c*d^2*e + a*e^3)*g)/((c^2*d^2*e*f^2*abs(e) - 2*a*c*d*e^2*f*g
*abs(e) + a^2*e^3*g^2*abs(e))*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)))
- 2/3*(3*sqrt(-c*d^2*e + a*e^3)*c*d^2*e*g^2*arctan(sqrt(-c*d^2*e + a*e^3)*
g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 3*sqrt(-c*d^2*e + a*e^3)*a*e^3*g^2*arctan
(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + sqrt(c*d*f*g - a*
e*g^2)*c*d*e^3*f + 3*sqrt(c*d*f*g - a*e*g^2)*c*d^2*e^2*g - 4*sqrt(c*d*f*g
- a*e*g^2)*a*e^4*g)/(sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^3*d^
4*f^2*abs(e) - sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^2*e^
2*f^2*abs(e) - 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^3*
e*f*g*abs(e) + 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d*e^
3*f*g*abs(e) + sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^2*e^
2*g^2*abs(e) - sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^3*e^4*g^2*
abs(e))
```

3.676.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\int \frac{(d+e\,x)^{5/2}}{(f+g\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}\,dx$$

input
```
int((d + e*x)^(5/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/
2)),x)
```

output
```
int((d + e*x)^(5/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/
2)), x)
```

3.677 $\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

3.677.1 Optimal result

Integrand size = 46, antiderivative size = 268

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{10g\sqrt{d+ex}}{3(cdf-aeg)^2(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{5g^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(cdf-aeg)^3\sqrt{d+ex}(f+gx)} + \frac{5cdg^{3/2}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{(cdf-aeg)^{7/2}}$$

output
```
-2/3*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(3/2)+5*c*d*g^(3/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(7/2)+10/3*g*(e*x+d)
^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5*
g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^3/(g*x+f)/(e*x+
d)^(1/2)
```

3.677.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.67

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \frac{(d+ex)^{3/2}\left(\sqrt{cdf-aeg}(3a^2e^2g^2+2acdeg(7f+10gx)\right.}{3(cdf-}$$

input
```
Integrate[(d + e*x)^(5/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2)),x]
```

output
```
((d + e*x)^(3/2)*(Sqrt[c*d*f - a*e*g]*(3*a^2*e^2*g^2 + 2*a*c*d*e*g*(7*f +
10*g*x) + c^2*d^2*(-2*f^2 + 10*f*g*x + 15*g^2*x^2)) + 15*c*d*g^(3/2)*(a*e
+ c*d*x)^(3/2)*(f + g*x)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a
*e*g]]))/(3*(c*d*f - a*e*g)^(7/2)*((a*e + c*d*x)*(d + e*x))^(3/2)*(f + g*x
))
```

3.677.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {1252, 1252, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \ 1252$$

$$-\frac{5g\int \frac{(d+ex)^{3/2}}{(f+gx)^2(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3(cdf-aeg)}-\frac{2(d+ex)^{3/2}}{3(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \ 1252$$

$$-\frac{5g\left(-\frac{3g\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{3(cdf-aeg)}-\frac{2(d+ex)^{3/2}}{3(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$-\frac{5g\left(-\frac{3g\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{3(cdf-aeg)}-\frac{2(d+ex)^{3/2}}{3(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1255}$$

$$-\frac{5g\left(-\frac{3g\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{\ }}{(f+gx)\sqrt{x(ae^2+cd^2}}\right)}{3(cdf-aeg)}$$

$$\frac{2(d+ex)^{3/2}}{3(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{218}$$

$$-\frac{5g\left(-\frac{3g\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{3(cdf-aeg)}-\frac{2(d+ex)^{3/2}}{3(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

3.677. $\int \frac{(d+ex)^{5/2}}{(f+gx)^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

input
```
Int[(d + e*x)^(5/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2)),x]
```

output
```
(-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(3/2)) - (5*g*((-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*
x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (3*g*(Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) + (c*d
*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f
- a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2))))/(c*d*f - a*e*g
)))/(3*(c*d*f - a*e*g))
```

3.677.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1252
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.677. $\int \frac{(d+ex)^{5/2}}{(f+gx)^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

3.677.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.54

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)\sqrt{cdx+ae}\,c^2d^2g^3x^2+15\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)acde\,g^3x\sqrt{cdx+ae}+15\,\operatorname{arctanh}\left(\ldots\right.\right.}{}$

input
```
int((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/3*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*c^2*d^2*g^3*x^2+15*arctanh(g*(c*d*x+a*e)^
(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c*d*e*g^3*x*(c*d*x+a*e)^(1/2)+15*arctanh(
g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g
^2*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c*d*e*f*g^2
*(c*d*x+a*e)^(1/2)-15*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*g^2*x^2-20*((a*e*g-c
*d*f)*g)^(1/2)*a*c*d*e*g^2*x-10*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x-3*((
a*e*g-c*d*f)*g)^(1/2)*a^2*e^2*g^2-14*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*f*g+2
*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^2/(a*e*g-c
*d*f)^3/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

3.677.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(240) = 480$.

Time = 0.63 (sec) , antiderivative size = 1907, normalized size of antiderivative = 7.12

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="fricas")
```

output
```
[-1/6*(15*(c^3*d^3*e*g^2*x^4 + a^2*c*d^2*e^2*f*g + (c^3*d^3*e*f*g + (c^3*d
^4 + 2*a*c^2*d^2*e^2)*g^2)*x^3 + ((c^3*d^4 + 2*a*c^2*d^2*e^2)*f*g + (2*a*c
^2*d^3*e + a^2*c*d*e^3)*g^2)*x^2 + (a^2*c*d^2*e^2*g^2 + (2*a*c^2*d^3*e + a
^2*c*d*e^3)*f*g)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c*d^2*f +
 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)
*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*
x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(15*c^2*d^2*g^2*x^2 - 2*c^2*d^2*f^
2 + 14*a*c*d*e*f*g + 3*a^2*e^2*g^2 + 10*(c^2*d^2*f*g + 2*a*c*d*e*g^2)*x)*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a^2*c^3*d^4*e^2
*f^4 - 3*a^3*c^2*d^3*e^3*f^3*g + 3*a^4*c*d^2*e^4*f^2*g^2 - a^5*d*e^5*f*g^3
 + (c^5*d^5*e*f^3*g - 3*a*c^4*d^4*e^2*f^2*g^2 + 3*a^2*c^3*d^3*e^3*f*g^3 -
a^3*c^2*d^2*e^4*g^4)*x^4 + (c^5*d^5*e*f^4 + (c^5*d^6 - a*c^4*d^4*e^2)*f^3*
g - 3*(a*c^4*d^5*e + a^2*c^3*d^3*e^3)*f^2*g^2 + (3*a^2*c^3*d^4*e^2 + 5*a^3
*c^2*d^2*e^4)*f*g^3 - (a^3*c^2*d^3*e^3 + 2*a^4*c*d*e^5)*g^4)*x^3 + ((c^5*d
^6 + 2*a*c^4*d^4*e^2)*f^4 - (a*c^4*d^5*e + 5*a^2*c^3*d^3*e^3)*f^3*g - 3*(a
^2*c^3*d^4*e^2 - a^3*c^2*d^2*e^4)*f^2*g^2 + (5*a^3*c^2*d^3*e^3 + a^4*c*d*e
^5)*f*g^3 - (2*a^4*c*d^2*e^4 + a^5*e^6)*g^4)*x^2 - (a^5*d*e^5*g^4 - (2*a*c
^4*d^5*e + a^2*c^3*d^3*e^3)*f^4 + (5*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4)*
f^3*g - 3*(a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f^2*g^2 - (a^4*c*d^2*e^4 - a^5*e
^6)*f*g^3)*x), 1/3*(15*(c^3*d^3*e*g^2*x^4 + a^2*c*d^2*e^2*f*g + (c^3*d^...
```

3.677.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(5/2),x)
```

output
```
Timed out
```

3.677.7 Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(gx+f)^2}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(
g*x + f)^2), x)
```

3.677.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1404 vs. $2(240) = 480$.

Time = 0.64 (sec) , antiderivative size = 1404, normalized size of antiderivative = 5.24

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="giac")
```

output
```
1/3*(3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g^2/((c^3*d^3*e^2*f^3*a
bs(e) - 3*a*c^2*d^2*e^3*f^2*g*abs(e) + 3*a^2*c*d*e^4*f*g^2*abs(e) - a^3*e^
5*g^3*abs(e))*(c*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g
)) + 15*c*d*g^2*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d
*f*g - a*e*g^2)*e))/((c^3*d^3*e^2*f^3*abs(e) - 3*a*c^2*d^2*e^3*f^2*g*abs(e
) + 3*a^2*c*d*e^4*f*g^2*abs(e) - a^3*e^5*g^3*abs(e))*sqrt(c*d*f*g - a*e*g^
2)*e) - 2*(c^2*d^2*e^2*f - a*c*d*e^3*g - 6*((e*x + d)*c*d*e - c*d^2*e + a*
e^3)*c*d*g)/((c^3*d^3*e^2*f^3*abs(e) - 3*a*c^2*d^2*e^3*f^2*g*abs(e) + 3*a^
2*c*d*e^4*f*g^2*abs(e) - a^3*e^5*g^3*abs(e))*((e*x + d)*c*d*e - c*d^2*e +
a*e^3)^(3/2)))*e^4 - 1/3*(15*sqrt(-c*d^2*e + a*e^3)*c^2*d^3*e^2*f*g^2*arct
an(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 15*sqrt(-c*d^2*
e + a*e^3)*a*c*d*e^4*f*g^2*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g -
 a*e*g^2)*e)) - 15*sqrt(-c*d^2*e + a*e^3)*c^2*d^4*e*g^3*arctan(sqrt(-c*d^2
*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 15*sqrt(-c*d^2*e + a*e^3)*a*c
*d^2*e^3*g^3*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))
+ 2*sqrt(c*d*f*g - a*e*g^2)*c^2*d^2*e^4*f^2 + 10*sqrt(c*d*f*g - a*e*g^2)*c
^2*d^3*e^3*f*g - 14*sqrt(c*d*f*g - a*e*g^2)*a*c*d*e^5*f*g - 15*sqrt(c*d*f*
g - a*e*g^2)*c^2*d^4*e^2*g^2 + 20*sqrt(c*d*f*g - a*e*g^2)*a*c*d^2*e^4*g^2
- 3*sqrt(c*d*f*g - a*e*g^2)*a^2*e^6*g^2)/(sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*
f*g - a*e*g^2)*c^4*d^5*e*f^4*abs(e) - sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f...
```

3.677.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx=\int \frac{(d+e\,x)^{5/2}}{(f+g\,x)^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}\,dx$$

input
```
int((d + e*x)^(5/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
5/2)),x)
```

output
```
int((d + e*x)^(5/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
5/2)), x)
```

3.678 $\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

3.678.1 Optimal result

Integrand size = 46, antiderivative size = 342

$$\begin{aligned}
&\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx= \\
&-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}} \\
&+\frac{14g\sqrt{d+ex}}{3(cdf-aeg)^2(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} \\
&+\frac{35g^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{6(cdf-aeg)^3\sqrt{d+ex}(f+gx)^2}+\frac{35cdg^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4(cdf-aeg)^4\sqrt{d+ex}(f+gx)} \\
&+\frac{35c^2d^2g^{3/2}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4(cdf-aeg)^{9/2}}
\end{aligned}$$

output

```
-2/3*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(3/2)+35/4*c^2*d^2*g^(3/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(9/2)+14/3*
g*(e*x+d)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)+35/6*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^3
/(g*x+f)^2/(e*x+d)^(1/2)+35/4*c*d*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/(-a*e*g+c*d*f)^4/(g*x+f)/(e*x+d)^(1/2)
```

3.678.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.70

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \frac{c^2d^2\sqrt{d+ex}\left(\frac{-6a^3e^3g^3+3a^2cde^2g^2(13f+7gx)+2ac^2d^2eg(40f^2+119f}{c^2d^2(cdf-aeg)^4}\right.}{12}$$

input
```
Integrate[(d + e*x)^(5/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2)),x]
```

output
```
(c^2*d^2*Sqrt[d + e*x]*((-6*a^3*e^3*g^3 + 3*a^2*c*d*e^2*g^2*(13*f + 7*g*x)
 + 2*a*c^2*d^2*e*g*(40*f^2 + 119*f*g*x + 70*g^2*x^2) + c^3*d^3*(-8*f^3 + 5
6*f^2*g*x + 175*f*g^2*x^2 + 105*g^3*x^3))/(c^2*d^2*(c*d*f - a*e*g)^4*(a*e
+ c*d*x)*(f + g*x)^2) + (105*g^(3/2)*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]*Sqr
t[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(c*d*f - a*e*g)^(9/2)))/(12*Sqrt[(a*
e + c*d*x)*(d + e*x)])
```

3.678.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1252, 1252, 1254, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1252}$$

$$-\frac{7g\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(cdex^2+(cd^2+ae^2)x+ade\right)^{3/2}}dx}{3(cdf-aeg)}-\frac{2(d+ex)^{3/2}}{3(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{ 1252}$$

$$-\frac{7g\left(-\frac{\frac{5g\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{3(cdf-aeg)}-\frac{2(d+ex)^{3/2}}{3(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \; 1254$$

$$-\frac{7g\left(-\frac{5g\left(\frac{3cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{3(cdf-aeg)}-\frac{2(d+ex)^{3/2}}{3(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \; 1254$$

$$-\frac{7g\left(-\frac{5g\left(\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d\cdots}}{(f+gx)^2\sqrt{x(ae^2+cd^2)+\cdots}}\right)}{3(cdf-aeg)}$$

$$-\frac{2(d+ex)^{3/2}}{3(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \; 1255$$

3.678. $\int \frac{(d+ex)^{5/2}}{(f+gx)^3(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

$$-\frac{7g\left(-\frac{5g\left(\frac{3cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade-}}{2\sqrt{d+ex}(f+gx)^2(cdf}\right)}{cdf-aeg}}{3(cdf-aeg)}$$

$$\frac{2(d+ex)^{3/2}}{3(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$\downarrow$ 218

$$-\frac{7g\left(-\frac{5g\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d}}{(f+gx)^2\sqrt{x(ae^2+cd^2)}}\right)}{3(cdf-aeg)}$$

$$\frac{2(d+ex)^{3/2}}{3(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

3.678. $\displaystyle\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

input
```
Int[(d + e*x)^(5/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2)),x]
```

output
```
(-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^(3/2)) - (7*g*((-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f +
g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (5*g*(Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2
) + (3*c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*S
qrt[d + e*x]*(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f -
a*e*g)^(3/2))))/(4*(c*d*f - a*e*g))))/(c*d*f - a*e*g)))/(3*(c*d*f - a*e*g)
)
```

3.678.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1252
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

3.678. $\int \frac{(d+ex)^{5/2}}{(f+gx)^3(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.678.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 659 vs. $2(304) = 608$.

Time = 0.54 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.93

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(105\ \mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3g^4x^3\sqrt{cdx+ae}+105\ \mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a\,c^2d^2e\,g^4x^2\sqrt{cdx+ae}+210\ \mathrm{ar}\right.}{}$

input
```
int((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,meth
od=_RETURNVERBOSE)
```

output
```
-1/12*((c*d*x+a*e)*(e*x+d))^(1/2)*(105*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2))*c^3*d^3*g^4*x^3*(c*d*x+a*e)^(1/2)+105*arctanh(g*(c*d*x+a
*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c^2*d^2*e*g^4*x^2*(c*d*x+a*e)^(1/2)+2
10*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^3*f*g^3*x^2*
(c*d*x+a*e)^(1/2)+210*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))
*a*c^2*d^2*e*f*g^3*x*(c*d*x+a*e)^(1/2)+105*arctanh(g*(c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2))*c^3*d^3*f^2*g^2*x*(c*d*x+a*e)^(1/2)-105*((a*e*g-c*d*
f)*g)^(1/2)*c^3*d^3*g^3*x^3+105*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*a*c^2*d^2*e*f^2*g^2*(c*d*x+a*e)^(1/2)-140*((a*e*g-c*d*f)*g)^(1/
2)*a*c^2*d^2*e*g^3*x^2-175*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f*g^2*x^2-21*((
a*e*g-c*d*f)*g)^(1/2)*a^2*c*d*e^2*g^3*x-238*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*
d^2*e*f*g^2*x-56*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f^2*g*x+6*((a*e*g-c*d*f)*
g)^(1/2)*a^3*e^3*g^3-39*((a*e*g-c*d*f)*g)^(1/2)*a^2*c*d*e^2*f*g^2-80*((a*e
*g-c*d*f)*g)^(1/2)*a*c^2*d^2*e*f^2*g+8*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f^3
)/(e*x+d)^(1/2)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^4/(g*x+f)^2/((a*e*g-c*d*f)*g)^
(1/2)
```

3.678.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1446 vs. $2(304) = 608$.

Time = 1.31 (sec) , antiderivative size = 2935, normalized size of antiderivative = 8.58

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="fricas")
```

output
```
[1/24*(105*(c^4*d^4*e*g^3*x^5 + a^2*c^2*d^3*e^2*f^2*g + (2*c^4*d^4*e*f*g^2
 + (c^4*d^5 + 2*a*c^3*d^3*e^2)*g^3)*x^4 + (c^4*d^4*e*f^2*g + 2*(c^4*d^5 +
2*a*c^3*d^3*e^2)*f*g^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*g^3)*x^3 + (a^2
*c^2*d^3*e^2*g^3 + (c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2*g + 2*(2*a*c^3*d^4*e +
a^2*c^2*d^2*e^3)*f*g^2)*x^2 + (2*a^2*c^2*d^3*e^2*f*g^2 + (2*a*c^3*d^4*e +
a^2*c^2*d^2*e^3)*f^2*g)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c*
d^2*f + 2*a*d*e*g + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f -
 a*e*g)*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*e
^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(105*c^3*d^3*g^3*x^3 - 8*c^
3*d^3*f^3 + 80*a*c^2*d^2*e*f^2*g + 39*a^2*c*d*e^2*f*g^2 - 6*a^3*e^3*g^3 +
35*(5*c^3*d^3*f*g^2 + 4*a*c^2*d^2*e*g^3)*x^2 + 7*(8*c^3*d^3*f^2*g + 34*a*c
^2*d^2*e*f*g^2 + 3*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(e*x + d))/(a^2*c^4*d^5*e^2*f^6 - 4*a^3*c^3*d^4*e^3*f^5*g + 6
*a^4*c^2*d^3*e^4*f^4*g^2 - 4*a^5*c*d^2*e^5*f^3*g^3 + a^6*d*e^6*f^2*g^4 + (
c^6*d^6*e*f^4*g^2 - 4*a*c^5*d^5*e^2*f^3*g^3 + 6*a^2*c^4*d^4*e^3*f^2*g^4 -
4*a^3*c^3*d^3*e^4*f*g^5 + a^4*c^2*d^2*e^5*g^6)*x^5 + (2*c^6*d^6*e*f^5*g +
(c^6*d^7 - 6*a*c^5*d^5*e^2)*f^4*g^2 - 4*(a*c^5*d^6*e - a^2*c^4*d^4*e^3)*f^
3*g^3 + 2*(3*a^2*c^4*d^5*e^2 + 2*a^3*c^3*d^3*e^4)*f^2*g^4 - 2*(2*a^3*c^3*d
^4*e^3 + 3*a^4*c^2*d^2*e^5)*f*g^5 + (a^4*c^2*d^3*e^4 + 2*a^5*c*d*e^6)*g^6)
*x^4 + (c^6*d^6*e*f^6 + 2*c^6*d^7*f^5*g - 6*a^4*c^2*d^3*e^4*f*g^5 - 3*(...
```

3.678.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(5/2),x)
```

output
```
Timed out
```

3.678.7 Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cdex^2+ade+(cd^2+ae^2)x\right)^{\frac{5}{2}}(gx+f)^3}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(
g*x + f)^3), x)
```

3.678.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2347 vs. 2(304) = 608.

Time = 1.14 (sec) , antiderivative size = 2347, normalized size of antiderivative = 6.86

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="giac")
```

output
```
1/12*(105*c^2*d^2*g^2*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sq
rt(c*d*f*g - a*e*g^2)*e))/((c^4*d^4*e^3*f^4*abs(e) - 4*a*c^3*d^3*e^4*f^3*g
*abs(e) + 6*a^2*c^2*d^2*e^5*f^2*g^2*abs(e) - 4*a^3*c*d*e^6*f*g^3*abs(e) +
a^4*e^7*g^4*abs(e))*sqrt(c*d*f*g - a*e*g^2)*e) - 8*(c^3*d^3*e^2*f - a*c^2*
d^2*e^3*g - 9*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^2*g)/((c^4*d^4*e^3
*f^4*abs(e) - 4*a*c^3*d^3*e^4*f^3*g*abs(e) + 6*a^2*c^2*d^2*e^5*f^2*g^2*abs
(e) - 4*a^3*c*d*e^6*f*g^3*abs(e) + a^4*e^7*g^4*abs(e))*((e*x + d)*c*d*e -
c*d^2*e + a*e^3)^(3/2)) + 3*(13*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^
3*d^3*e^2*f*g^2 - 13*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^2*d^2*e^3
*g^3 + 11*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*g^3)/((c^4*d^4
*e^3*f^4*abs(e) - 4*a*c^3*d^3*e^4*f^3*g*abs(e) + 6*a^2*c^2*d^2*e^5*f^2*g^2
*abs(e) - 4*a^3*c*d*e^6*f*g^3*abs(e) + a^4*e^7*g^4*abs(e))*(c*d*e^2*f - a*
e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^2))*e^5 - 1/12*(105*sqrt(-c
*d^2*e + a*e^3)*c^3*d^4*e^3*f^2*g^2*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(
c*d*f*g - a*e*g^2)*e)) - 105*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^2*e^5*f^2*g^2*
arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 210*sqrt(-c
*d^2*e + a*e^3)*c^3*d^5*e^2*f*g^3*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*
d*f*g - a*e*g^2)*e)) + 210*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^3*e^4*f*g^3*arct
an(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 105*sqrt(-c*d^2
*e + a*e^3)*c^3*d^6*e*g^4*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g...
```

3.678.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx=\int \frac{(d+e\,x)^{5/2}}{(f+g\,x)^3\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}\,dx$$

input
```
int((d + e*x)^(5/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
5/2)),x)
```

output
```
int((d + e*x)^(5/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
5/2)), x)
```

3.679 $\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

3.679.1 Optimal result

Integrand size = 46, antiderivative size = 336

$$\begin{aligned}\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx &= -\frac{128(cdf-aeg)^3(2ae^2g-cd(5ef-3dg))\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{3465c^5d^5e(d+ex)^{3/2}}\\ &\quad+\frac{128g(cdf-aeg)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{1155c^4d^4e\sqrt{d+ex}}\\ &\quad+\frac{32(cdf-aeg)^2(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{231c^3d^3(d+ex)^{3/2}}\\ &\quad+\frac{16(cdf-aeg)(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{99c^2d^2(d+ex)^{3/2}}\\ &\quad+\frac{2(f+gx)^4\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}}\end{aligned}$$

output
```
-128/3465*(-a*e*g+c*d*f)^3*(2*a*e^2*g-c*d*(-3*d*g+5*e*f))*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(3/2)/c^5/d^5/e/(e*x+d)^(3/2)+32/231*(-a*e*g+c*d*f)^2*(g
*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/(e*x+d)^(3/2)+16/9
9*(-a*e*g+c*d*f)*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2
/(e*x+d)^(3/2)+2/11*(g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/
(e*x+d)^(3/2)+128/1155*g*(-a*e*g+c*d*f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(3/2)/c^4/d^4/e/(e*x+d)^(1/2)
```

3.679.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.58

$$\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{2((ae+cdx)(d+ex))^{3/2}\,(128a^4e^4g^4-64a^3cde^3g^3(11f+3gx)+48a^2c^2d^2e^2g^2(33f^2+22fgx+5g^2x^2)-}{346}$$

input
```
Integrate[((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
 + e*x],x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2)*(128*a^4*e^4*g^4 - 64*a^3*c*d*e^3*g^3*(
11*f + 3*g*x) + 48*a^2*c^2*d^2*e^2*g^2*(33*f^2 + 22*f*g*x + 5*g^2*x^2) - 8
*a*c^3*d^3*e*g*(231*f^3 + 297*f^2*g*x + 165*f*g^2*x^2 + 35*g^3*x^3) + c^4*
d^4*(1155*f^4 + 2772*f^3*g*x + 2970*f^2*g^2*x^2 + 1540*f*g^3*x^3 + 315*g^4
*x^4)))/(3465*c^5*d^5*(d + e*x)^(3/2))
```

3.679.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {1253, 1253, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \; 1253$$

$$\frac{8(cdf-aeg)\int \frac{(f+gx)^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}dx}{11cd}+\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}}$$

$$\Big\downarrow \; 1253$$

$$\frac{8(cdf-aeg)\left(\frac{2(cdf-aeg)\int \frac{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}dx}{3cd}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}}\right)}{11cd}+\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}}$$

$\big\downarrow$ 1253

$$\frac{8(cdf-aeg)\left(\frac{2(cdf-aeg)\left(\frac{4(cdf-aeg)\int \frac{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}dx}{7cd}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}}\right)}{3cd}+\frac{2(f+gx)^3(x(ae^2+c}{9cd(d}\right.}{11cd}+\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}}$$

$\big\downarrow$ 1221

$$\frac{8(cdf-aeg)\left(\frac{2(cdf-aeg)\left(\frac{4(cdf-aeg)\left(\frac{1}{5}\left(-\frac{2aeg}{cd}-\frac{3dg}{e}+5f\right)\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}\right)}{7cd}+\frac{2(f+gx)^2\left(x\left(ae\right.\right.}{7c}\right.}{3cd}\right.}{11cd}+\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}}$$

$\big\downarrow$ 1122

3.679. $\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

$$\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}}+\frac{8(cdf-aeg)\left(\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}}+\frac{2(cdf-aeg)\left(\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}}+\frac{4(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)+\cdots\right.}{\cdots}\right.}{\cdots}\right)}{3cd}\right)}{11cd}$$

input
```
Int[((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x
],x]
```

output
```
(2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(11*c*d*(d +
 e*x)^(3/2)) + (8*(c*d*f - a*e*g)*((2*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(3/2))/(9*c*d*(d + e*x)^(3/2)) + (2*(c*d*f - a*e*g)*((2*(f
 + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*c*d*(d + e*x)^
(3/2)) + (4*(c*d*f - a*e*g)*((2*(5*f - (3*d*g)/e - (2*a*e*g)/(c*d))*(a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(15*c*d*(d + e*x)^(3/2)) + (2*g*(
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*c*d*e*Sqrt[d + e*x])))/(7
*c*d)))/(3*c*d)))/(11*c*d)
```

3.679.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

3.679. $\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

3.679.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.81

method	result
default	$\frac{2(cdx+ae)\left(315g^4x^4c^4d^4-280a\,c^3d^3e\,g^4x^3+1540c^4d^4f\,g^3x^3+240a^2c^2d^2e^2g^4x^2-1320a\,c^3d^3ef\,g^3x^2+2970c^4d^4f^2g^2x^2-192a^3cd\,e^3g\right.}{}$
gosper	$\frac{2(cdx+ae)\left(315g^4x^4c^4d^4-280a\,c^3d^3e\,g^4x^3+1540c^4d^4f\,g^3x^3+240a^2c^2d^2e^2g^4x^2-1320a\,c^3d^3ef\,g^3x^2+2970c^4d^4f^2g^2x^2-192a^3cd\,e^3g\right.}{}$

input
```
int((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/3465*(c*d*x+a*e)*(315*c^4*d^4*g^4*x^4-280*a*c^3*d^3*e*g^4*x^3+1540*c^4*d
^4*f*g^3*x^3+240*a^2*c^2*d^2*e^2*g^4*x^2-1320*a*c^3*d^3*e*f*g^3*x^2+2970*c
^4*d^4*f^2*g^2*x^2-192*a^3*c*d*e^3*g^4*x+1056*a^2*c^2*d^2*e^2*f*g^3*x-2376
*a*c^3*d^3*e*f^2*g^2*x+2772*c^4*d^4*f^3*g*x+128*a^4*e^4*g^4-704*a^3*c*d*e^
3*f*g^3+1584*a^2*c^2*d^2*e^2*f^2*g^2-1848*a*c^3*d^3*e*f^3*g+1155*c^4*d^4*f
^4)*((c*d*x+a*e)*(e*x+d))^(1/2)/c^5/d^5/(e*x+d)^(1/2)
```

3.679.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.12

$$\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\left(315\,c^5d^5g^4x^5+1155\,ac^4d^4ef^4-1848\,a^2c^3d^3e^2f^3g+1584\,a^3c^2d^2e^3f^2g^2-704\,a^4cde^4fg^3+128\,a^5e^5g^4+\right.}{}$$

input
```
integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="fricas")
```

output
```
2/3465*(315*c^5*d^5*g^4*x^5 + 1155*a*c^4*d^4*e*f^4 - 1848*a^2*c^3*d^3*e^2*
f^3*g + 1584*a^3*c^2*d^2*e^3*f^2*g^2 - 704*a^4*c*d*e^4*f*g^3 + 128*a^5*e^5
*g^4 + 35*(44*c^5*d^5*f*g^3 + a*c^4*d^4*e*g^4)*x^4 + 10*(297*c^5*d^5*f^2*g
^2 + 22*a*c^4*d^4*e*f*g^3 - 4*a^2*c^3*d^3*e^2*g^4)*x^3 + 6*(462*c^5*d^5*f^
3*g + 99*a*c^4*d^4*e*f^2*g^2 - 44*a^2*c^3*d^3*e^2*f*g^3 + 8*a^3*c^2*d^2*e^
3*g^4)*x^2 + (1155*c^5*d^5*f^4 + 924*a*c^4*d^4*e*f^3*g - 792*a^2*c^3*d^3*e
^2*f^2*g^2 + 352*a^3*c^2*d^2*e^3*f*g^3 - 64*a^4*c*d*e^4*g^4)*x)*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x + c^5*d^6)
```

3.679.6 Sympy [F]

$$\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}(f+gx)^4}{\sqrt{d+ex}}\,dx$$

input
```
integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**
(1/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**4/sqrt(d + e*x), x)
```

3.679.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.95

$$\begin{aligned}&\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx\\&=\frac{2\,(cdx+ae)^{\frac{3}{2}}f^4}{3\,cd}+\frac{8\,(3\,c^2d^2x^2+acdex-2\,a^2e^2)\sqrt{cdx+ae}\,f^3g}{15\,c^2d^2}\\&\quad+\frac{4\,(15\,c^3d^3x^3+3\,ac^2d^2ex^2-4\,a^2cde^2x+8\,a^3e^3)\sqrt{cdx+ae}\,f^2g^2}{35\,c^3d^3}\\&\quad+\frac{8\,(35\,c^4d^4x^4+5\,ac^3d^3ex^3-6\,a^2c^2d^2e^2x^2+8\,a^3cde^3x-16\,a^4e^4)\sqrt{cdx+ae}\,fg^3}{315\,c^4d^4}\\&\quad+\frac{2\,(315\,c^5d^5x^5+35\,ac^4d^4ex^4-40\,a^2c^3d^3e^2x^3+48\,a^3c^2d^2e^3x^2-64\,a^4cde^4x+128\,a^5e^5)\sqrt{cdx+ae}\,g^4}{3465\,c^5d^5}\end{aligned}$$

input
```
integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="maxima")
```

output
```
2/3*(c*d*x + a*e)^(3/2)*f^4/(c*d) + 8/15*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^
2*e^2)*sqrt(c*d*x + a*e)*f^3*g/(c^2*d^2) + 4/35*(15*c^3*d^3*x^3 + 3*a*c^2*
d^2*e*x^2 - 4*a^2*c*d*e^2*x + 8*a^3*e^3)*sqrt(c*d*x + a*e)*f^2*g^2/(c^3*d^
3) + 8/315*(35*c^4*d^4*x^4 + 5*a*c^3*d^3*e*x^3 - 6*a^2*c^2*d^2*e^2*x^2 + 8
*a^3*c*d*e^3*x - 16*a^4*e^4)*sqrt(c*d*x + a*e)*f*g^3/(c^4*d^4) + 2/3465*(3
15*c^5*d^5*x^5 + 35*a*c^4*d^4*e*x^4 - 40*a^2*c^3*d^3*e^2*x^3 + 48*a^3*c^2*
d^2*e^3*x^2 - 64*a^4*c*d*e^4*x + 128*a^5*e^5)*sqrt(c*d*x + a*e)*g^4/(c^5*d
^5)
```

3.679.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1107 vs. $2(306) = 612$.

Time = 0.32 (sec) , antiderivative size = 1107, normalized size of antiderivative = 3.29

$$\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="giac")
```

output
```
2/3465*(1155*f^4*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a
*e^2)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e)/e^
2 + 198*f^2*g^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*
e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^
2*e + a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e
^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 1
5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e^2 - 4
4*f*g^3*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c
^3*d^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e +
a*e^3)*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)/(c^4*d^4*e^3) +
(105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x + d)*c*
d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^2*e + a*
e^3)^(7/2)*a*e^3 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))/(c^4*d^4*
e^7))*abs(e)/e^2 + g^4*((315*sqrt(-c*d^2*e + a*e^3)*c^5*d^10 - 35*sqrt(-c*
d^2*e + a*e^3)*a*c^4*d^8*e^2 - 40*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^6*e^4 -
 48*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^4*e^6 - 64*sqrt(-c*d^2*e + a*e^3)*a^4
*c*d^2*e^8 - 128*sqrt(-c*d^2*e + a*e^3)*a^5*e^10)/(c^5*d^5*e^4) + (1155*((
e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^4*e^12 - 2772*((e*x + d)*c*d*e -
 c*d^2*e + a*e^3)^(5/2)*a^3*e^9 + 2970*((e*x + d)*c*d*e - c*d^2*e + a*e^3)
^(7/2)*a^2*e^6 - 1540*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a*e^3 +...
```

3.679.9 Mupad [B] (verification not implemented)

Time = 12.36 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.03

$$\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\left(\frac{2\,g^4\,x^5}{11}+\frac{256\,a^5\,e^5\,g^4-1408\,a^4\,c\,d\,e^4\,f\,g^3+3168\,a^3\,c^2\,d^2\,e^3\,f^2\,g^2-3696\,a^2\,c^3\,d^3\,e^2\,f^3\,g+2310\,a}{3465\,c^5\,d^5}\right.}{}$$

input
```
int(((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^
(1/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^4*x^5)/11 + (256*a^5*
e^5*g^4 + 2310*a*c^4*d^4*e*f^4 - 3696*a^2*c^3*d^3*e^2*f^3*g - 1408*a^4*c*d
*e^4*f*g^3 + 3168*a^3*c^2*d^2*e^3*f^2*g^2)/(3465*c^5*d^5) + (x*(2310*c^5*d
^5*f^4 - 128*a^4*c*d*e^4*g^4 + 704*a^3*c^2*d^2*e^3*f*g^3 + 1848*a*c^4*d^4*
e*f^3*g - 1584*a^2*c^3*d^3*e^2*f^2*g^2))/(3465*c^5*d^5) + (4*g*x^2*(8*a^3*
e^3*g^3 + 462*c^3*d^3*f^3 + 99*a*c^2*d^2*e*f^2*g - 44*a^2*c*d*e^2*f*g^2))/
(1155*c^3*d^3) + (4*g^2*x^3*(297*c^2*d^2*f^2 - 4*a^2*e^2*g^2 + 22*a*c*d*e*
f*g))/(693*c^2*d^2) + (2*g^3*x^4*(a*e*g + 44*c*d*f))/(99*c*d)))/(d + e*x)^
(1/2)
```

3.680 $\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

3.680.1 Optimal result

Integrand size = 46, antiderivative size = 269

$$\begin{aligned}\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx &= -\frac{16(cdf-aeg)^2(2ae^2g-cd(5ef-3dg))\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{315c^4d^4e(d+ex)^{3/2}}\\ &+\frac{16g(cdf-aeg)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{105c^3d^3e\sqrt{d+ex}}\\ &+\frac{4(cdf-aeg)(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{21c^2d^2(d+ex)^{3/2}}\\ &+\frac{2(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}}\end{aligned}$$

output
```
-16/315*(-a*e*g+c*d*f)^2*(2*a*e^2*g-c*d*(-3*d*g+5*e*f))*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(3/2)/c^4/d^4/e/(e*x+d)^(3/2)+4/21*(-a*e*g+c*d*f)*(g*x+f)^
2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/(e*x+d)^(3/2)+2/9*(g*x+f
)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/(e*x+d)^(3/2)+16/105*g*(-a
*e*g+c*d*f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/e/(e*x+d)^(1
/2)
```

3.680.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.51

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{2((ae+cdx)(d+ex))^{3/2}\,(-16a^3e^3g^3+24a^2cde^2g^2(3f+gx)-6ac^2d^2eg(21f^2+18fgx+5g^2x^2)+c^3d^3(}{315c^4d^4(d+ex)^{3/2}}$$

input
```
Integrate[((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
 + e*x],x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-16*a^3*e^3*g^3 + 24*a^2*c*d*e^2*g^2*(
3*f + g*x) - 6*a*c^2*d^2*e*g*(21*f^2 + 18*f*g*x + 5*g^2*x^2) + c^3*d^3*(10
5*f^3 + 189*f^2*g*x + 135*f*g^2*x^2 + 35*g^3*x^3)))/(315*c^4*d^4*(d + e*x)
^(3/2))
```

3.680.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1253, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$\Big\downarrow \text{1253}$$
$$\frac{2(cdf-aeg)\int \frac{(f+gx)^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}dx}{3cd}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}}$$
$$\Big\downarrow \text{1253}$$
$$\frac{2(cdf-aeg)\left(\frac{4(cdf-aeg)\int \frac{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}dx}{7cd}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}}\right)}{3cd}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}}$$

$$\Big\downarrow \text{1221}$$

$$\frac{2(cdf-aeg)\left(\frac{4(cdf-aeg)\left(\frac{1}{5}\left(-\frac{2aeg}{cd}-\frac{3dg}{e}+5f\right)\int\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}\right)}{7cd}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right.\right.}{7cd(d+e}\right.}{3cd}$$

$$\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}}+$$

$$\frac{2(cdf-aeg)\left(\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}}+\frac{4(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}\left(-\frac{2aeg}{cd}-\frac{3dg}{e}+5f\right)}{15cd(d+ex)^{3/2}}+\frac{2g\left(x\left(ae^2+cd^2\right)+a\right.}{5cde\sqrt{d-}}\right.}{7cd}\right.}{3cd}$$

input
```
Int[((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x
],x]
```

output
```
(2*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(9*c*d*(d +
e*x)^(3/2)) + (2*(c*d*f - a*e*g)*((2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(3/2))/(7*c*d*(d + e*x)^(3/2)) + (4*(c*d*f - a*e*g)*((2*(5*
f - (3*d*g)/e - (2*a*e*g)/(c*d))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2))/(15*c*d*(d + e*x)^(3/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/2))/(5*c*d*e*Sqrt[d + e*x])))/(7*c*d)))/(3*c*d)
```

3.680. $\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

3.680.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

3.680.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.66

method	result
default	$-\frac{2(cdx+ae)\left(-35g^3x^3c^3d^3+30a\,c^2d^2e\,g^3x^2-135c^3d^3f\,g^2x^2-24a^2cd\,e^2g^3x+108a\,c^2d^2ef\,g^2x-189c^3d^3f^2gx+16a^3e^3g^3-72a^2cd\,e^2\right.}{315c^4d^4\sqrt{ex+d}}$
gosper	$-\frac{2(cdx+ae)\left(-35g^3x^3c^3d^3+30a\,c^2d^2e\,g^3x^2-135c^3d^3f\,g^2x^2-24a^2cd\,e^2g^3x+108a\,c^2d^2ef\,g^2x-189c^3d^3f^2gx+16a^3e^3g^3-72a^2cd\,e^2\right.}{315c^4d^4\sqrt{ex+d}}$

input
```
int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
-2/315*(c*d*x+a*e)*(-35*c^3*d^3*g^3*x^3+30*a*c^2*d^2*e*g^3*x^2-135*c^3*d^3
*f*g^2*x^2-24*a^2*c*d*e^2*g^3*x+108*a*c^2*d^2*e*f*g^2*x-189*c^3*d^3*f^2*g*
x+16*a^3*e^3*g^3-72*a^2*c*d*e^2*f*g^2+126*a*c^2*d^2*e*f^2*g-105*c^3*d^3*f^
3)*((c*d*x+a*e)*(e*x+d))^(1/2)/c^4/d^4/(e*x+d)^(1/2)
```

3.680.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.98

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\,(35\,c^4d^4g^3x^4+105\,ac^3d^3ef^3-126\,a^2c^2d^2e^2f^2g+72\,a^3cde^3fg^2-16\,a^4e^4g^3+5\,(27\,c^4d^4fg^2+ac^3d^3eg^3)x}{}$$

input
```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="fricas")
```

output
```
2/315*(35*c^4*d^4*g^3*x^4 + 105*a*c^3*d^3*e*f^3 - 126*a^2*c^2*d^2*e^2*f^2*
g + 72*a^3*c*d*e^3*f*g^2 - 16*a^4*e^4*g^3 + 5*(27*c^4*d^4*f*g^2 + a*c^3*d^
3*e*g^3)*x^3 + 3*(63*c^4*d^4*f^2*g + 9*a*c^3*d^3*e*f*g^2 - 2*a^2*c^2*d^2*e
^2*g^3)*x^2 + (105*c^4*d^4*f^3 + 63*a*c^3*d^3*e*f^2*g - 36*a^2*c^2*d^2*e^2
*f*g^2 + 8*a^3*c*d*e^3*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

3.680.6 Sympy [F]

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}(f+gx)^3}{\sqrt{d+ex}}\,dx$$

input
```
integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**
(1/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**3/sqrt(d + e*x), x)
```

3.680.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.81

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\,(cdx+ae)^{\frac{3}{2}}f^3}{3\,cd} + \frac{2\,(3\,c^2d^2x^2+acdex-2\,a^2e^2)\sqrt{cdx+ae}f^2g}{5\,c^2d^2} + \frac{2\,(15\,c^3d^3x^3+3\,ac^2d^2ex^2-4\,a^2cde^2x+8\,a^3e^3)\sqrt{cdx+ae}fg^2}{35\,c^3d^3} + \frac{2\,(35\,c^4d^4x^4+5\,ac^3d^3ex^3-6\,a^2c^2d^2e^2x^2+8\,a^3cde^3x-16\,a^4e^4)\sqrt{cdx+ae}g^3}{315\,c^4d^4}$$

input
```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="maxima")
```

output
```
2/3*(c*d*x + a*e)^(3/2)*f^3/(c*d) + 2/5*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^2
*e^2)*sqrt(c*d*x + a*e)*f^2*g/(c^2*d^2) + 2/35*(15*c^3*d^3*x^3 + 3*a*c^2*d
^2*e*x^2 - 4*a^2*c*d*e^2*x + 8*a^3*e^3)*sqrt(c*d*x + a*e)*f*g^2/(c^3*d^3)
+ 2/315*(35*c^4*d^4*x^4 + 5*a*c^3*d^3*e*x^3 - 6*a^2*c^2*d^2*e^2*x^2 + 8*a^
3*c*d*e^3*x - 16*a^4*e^4)*sqrt(c*d*x + a*e)*g^3/(c^4*d^4)
```

3.680.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. 2(245) = 490.

Time = 0.30 (sec) , antiderivative size = 762, normalized size of antiderivative = 2.83

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\left(\frac{105\,f^3\left(\frac{\sqrt{-cd^2e+ae^3}cd^2-\sqrt{-cd^2e+ae^3}ae^2}{cd}+\frac{\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}}{cde}\right)|e|}{e^2}+\frac{9\,fg^2\left(\frac{15\,\sqrt{-cd^2e+ae^3}c^3d^6-3\,\sqrt{-cd^2e+ae^3}ac^2d^4e^2-4\,\sqrt{-cd^2e+\cdots}}{c^3d^3e^2}\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="giac")
```

output
```
2/315*(105*f^3*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e
^2)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e)/e^2
+ 9*f*g^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a
*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e +
a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3
/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*
x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e^2 - g^3*((3
5*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2
- 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*
c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)/(c^4*d^4*e^3) + (105*((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x + d)*c*d*e - c*d^2
*e + a*e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*
a*e^3 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))/(c^4*d^4*e^7))*abs(e
)/e^2 - 63*f^2*g*((3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^
3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x +
d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a
*e^3)^(5/2))/(c^2*d^2*e^2))*abs(e)/e^3)/e
```

3.680.9 Mupad [B] (verification not implemented)

Time = 12.29 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.90

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\left(\frac{2\,g^3\,x^4}{9}-\frac{32\,a^4\,e^4\,g^3-144\,a^3\,c\,d\,e^3\,f\,g^2+252\,a^2\,c^2\,d^2\,e^2\,f^2\,g-210\,a\,c^3\,d^3\,e\,f^3}{315\,c^4\,d^4}+\frac{x\,(16\,a^3\,c\,d\,e^3}{}\right.}{\sqrt{d+e\,x}}$$

input
```
int(((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^
(1/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^3*x^4)/9 - (32*a^4*e^
4*g^3 - 210*a*c^3*d^3*e*f^3 + 252*a^2*c^2*d^2*e^2*f^2*g - 144*a^3*c*d*e^3*
f*g^2)/(315*c^4*d^4) + (x*(210*c^4*d^4*f^3 + 16*a^3*c*d*e^3*g^3 - 72*a^2*c
^2*d^2*e^2*f*g^2 + 126*a*c^3*d^3*e*f^2*g))/(315*c^4*d^4) + (2*g*x^2*(63*c^
2*d^2*f^2 - 2*a^2*e^2*g^2 + 9*a*c*d*e*f*g))/(105*c^2*d^2) + (2*g^2*x^3*(a*
e*g + 27*c*d*f))/(63*c*d)))/(d + e*x)^(1/2)
```

3.681 $\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

3.681.1 Optimal result

Integrand size = 46, antiderivative size = 200

$$\begin{aligned}\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx &= -\frac{8(cdf-aeg)\left(2ae^2g-cd(5ef-3dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{105c^3d^3e(d+ex)^{3/2}}\\ &\quad+\frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{35c^2d^2e\sqrt{d+ex}}\\ &\quad+\frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}}\end{aligned}$$

output
```
-8/105*(-a*e*g+c*d*f)*(2*a*e^2*g-c*d*(-3*d*g+5*e*f))*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(3/2)/c^3/d^3/e/(e*x+d)^(3/2)+2/7*(g*x+f)^2*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(3/2)/c/d/(e*x+d)^(3/2)+8/35*g*(-a*e*g+c*d*f)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/e/(e*x+d)^(1/2)
```

3.681.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.45

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{2((ae+cdx)(d+ex))^{3/2}\,(8a^2e^2g^2-4acdeg(7f+3gx)+c^2d^2(35f^2+42fgx+15g^2x^2))}{105c^3d^3(d+ex)^{3/2}}$$

input
```
Integrate[((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
 + e*x],x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2)*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(7*f + 3*g
*x) + c^2*d^2*(35*f^2 + 42*f*g*x + 15*g^2*x^2)))/(105*c^3*d^3*(d + e*x)^(3
/2))
```

3.681.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$\Big\downarrow \text{1253}$$
$$\frac{4(cdf-aeg)\int \frac{(f+gx)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}dx}{7cd}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}}$$
$$\Big\downarrow \text{1221}$$
$$\frac{4(cdf-aeg)\left(\frac{1}{5}\left(-\frac{2aeg}{cd}-\frac{3dg}{e}+5f\right)\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}\right)}{7cd}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}}$$
$$\Big\downarrow \text{1122}$$

$$\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}}+\frac{4(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}\left(-\frac{2aeg}{cd}-\frac{3dg}{e}+5f\right)}{15cd(d+ex)^{3/2}}+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}\right)}{7cd}$$

input
```
Int[((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x
],x]
```

output
```
(2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*c*d*(d +
e*x)^(3/2)) + (4*(c*d*f - a*e*g)*((2*(5*f - (3*d*g)/e - (2*a*e*g)/(c*d))*(
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(15*c*d*(d + e*x)^(3/2)) + (
2*g*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*c*d*e*Sqrt[d + e*x])
))/(7*c*d)
```

3.681.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

3.681. $\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

3.681.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.53

method	result	size
default	$\frac{2(cdx+ae)\left(15g^{2}x^{2}c^{2}d^{2}-12acde\,g^{2}x+42c^{2}d^{2}fgx+8a^{2}e^{2}g^{2}-28acdefg+35c^{2}d^{2}f^{2}\right)\sqrt{(cdx+ae)(ex+d)}}{105c^{3}d^{3}\sqrt{ex+d}}$	106
gosper	$\frac{2(cdx+ae)\left(15g^{2}x^{2}c^{2}d^{2}-12acde\,g^{2}x+42c^{2}d^{2}fgx+8a^{2}e^{2}g^{2}-28acdefg+35c^{2}d^{2}f^{2}\right)\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}}{105c^{3}d^{3}\sqrt{ex+d}}$	116

input
```
int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/105*(c*d*x+a*e)*(15*c^2*d^2*g^2*x^2-12*a*c*d*e*g^2*x+42*c^2*d^2*f*g*x+8*
a^2*e^2*g^2-28*a*c*d*e*f*g+35*c^2*d^2*f^2)*((c*d*x+a*e)*(e*x+d))^(1/2)/c^3
/d^3/(e*x+d)^(1/2)
```

3.681.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.86

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\frac{2\left(15\,c^3d^3g^2x^3+35\,ac^2d^2ef^2-28\,a^2cde^2fg+8\,a^3e^3g^2+3\,(14\,c^3d^3fg+ac^2d^2eg^2)x^2+(35\,c^3d^3f^2+14\,ac\right.}{105\,(c^3d^3ex+c^3d^4)}$$

input
```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="fricas")
```

output
```
2/105*(15*c^3*d^3*g^2*x^3 + 35*a*c^2*d^2*e*f^2 - 28*a^2*c*d*e^2*f*g + 8*a^
3*e^3*g^2 + 3*(14*c^3*d^3*f*g + a*c^2*d^2*e*g^2)*x^2 + (35*c^3*d^3*f^2 + 1
4*a*c^2*d^2*e*f*g - 4*a^2*c*d*e^2*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

3.681.6 Sympy [F]

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}(f+gx)^2}{\sqrt{d+ex}}\,dx$$

input
```
integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**
(1/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**2/sqrt(d + e*x), x)
```

3.681.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.66

$$\begin{aligned}\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx &= \frac{2\,(cdx+ae)^{\frac{3}{2}}f^2}{3\,cd}+\frac{4\,(3\,c^2d^2x^2+acdex-2\,a^2e^2)\sqrt{cdx+ae}fg}{15\,c^2d^2}\\&\quad+\frac{2\,(15\,c^3d^3x^3+3\,ac^2d^2ex^2-4\,a^2cde^2x+8\,a^3e^3)\sqrt{cdx+ae}g^2}{105\,c^3d^3}\end{aligned}$$

input
```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="maxima")
```

output
```
2/3*(c*d*x + a*e)^(3/2)*f^2/(c*d) + 4/15*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^
2*e^2)*sqrt(c*d*x + a*e)*f*g/(c^2*d^2) + 2/105*(15*c^3*d^3*x^3 + 3*a*c^2*d
^2*e*x^2 - 4*a^2*c*d*e^2*x + 8*a^3*e^3)*sqrt(c*d*x + a*e)*g^2/(c^3*d^3)
```

3.681.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. 2(182) = 364.

Time = 0.30 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.38

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\left(\frac{35\,f^2\left(\frac{\sqrt{-cd^2e+ae^3}cd^2-\sqrt{-cd^2e+ae^3}ae^2}{cd}+\frac{\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}}{cde}\right)|e|}{e^2}+\frac{g^2\left(\frac{15\,\sqrt{-cd^2e+ae^3}c^3d^6-3\,\sqrt{-cd^2e+ae^3}ac^2d^4e^2-4\,\sqrt{-cd^2e+ae^3}a}{c^3d^3e^2}\right.}{}\right.}{}$$

input
```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="giac")
```

output
```
2/105*(35*f^2*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^
2)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e)/e^2 +
 g^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*
d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3
)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a
^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d
)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e^2 - 14*f*g*((3*s
qrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqr
t(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x + d)*c*d*e - c*d^2*e + a
*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))/(c^2*d^2*
e^2))*abs(e)/e^3)/e
```

3.681.9 Mupad [B] (verification not implemented)

Time = 12.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.78

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\left(\frac{2\,g^2\,x^3}{7}+\frac{16\,a^3\,e^3\,g^2-56\,a^2\,c\,d\,e^2\,f\,g+70\,a\,c^2\,d^2\,e\,f^2}{105\,c^3\,d^3}+\frac{x\left(-8\,a^2\,c\,d\,e^2\,g^2+28\,a\,c^2\,d^2\,e\,f\,g+70\right.}{105\,c^3\,d^3}\right.}{\sqrt{d+e\,x}}$$

input
```
int(((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^
(1/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^3)/7 + (16*a^3*e^
3*g^2 + 70*a*c^2*d^2*e*f^2 - 56*a^2*c*d*e^2*f*g)/(105*c^3*d^3) + (x*(70*c^
3*d^3*f^2 - 8*a^2*c*d*e^2*g^2 + 28*a*c^2*d^2*e*f*g))/(105*c^3*d^3) + (2*g*
x^2*(a*e*g + 14*c*d*f))/(35*c*d)))/(d + e*x)^(1/2)
```

3.681. $\int \frac{(f+gx)^2 \sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

3.682 $\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

3.682.1 Optimal result

Integrand size = 44, antiderivative size = 125

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = -\frac{2(2ae^2g-cd(5ef-3dg))\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{15c^2d^2e(d+ex)^{3/2}} + \frac{2g\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}$$

output
```
-2/15*(2*a*e^2*g-c*d*(-3*d*g+5*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2)/c^2/d^2/e/(e*x+d)^(3/2)+2/5*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c
/d/e/(e*x+d)^(1/2)
```

3.682.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.43

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2((ae+cdx)(d+ex))^{3/2}(-2aeg+cd(5f+3gx))}{15c^2d^2(d+ex)^{3/2}}$$

input
```
Integrate[((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +
 e*x],x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-2*a*e*g + c*d*(5*f + 3*g*x)))/(15*c^2
*d^2*(d + e*x)^(3/2))
```

3.682.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{1}{5}\left(-\frac{2aeg}{cd}-\frac{3dg}{e}+5f\right)\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}\left(-\frac{2aeg}{cd}-\frac{3dg}{e}+5f\right)}{15cd(d+ex)^{3/2}}+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}$$

input
```
Int[((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x],
x]
```

output
```
(2*(5*f - (3*d*g)/e - (2*a*e*g)/(c*d))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2))/(15*c*d*(d + e*x)^(3/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(3/2))/(5*c*d*e*Sqrt[d + e*x])
```

3.682.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

3.682.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.46

method	result	size
default	$-\frac{2(cdx+ae)(-3cdgx+2aeg-5cdf)\sqrt{(cdx+ae)(ex+d)}}{15c^{2}d^{2}\sqrt{ex+d}}$	57
gosper	$-\frac{2(cdx+ae)(-3cdgx+2aeg-5cdf)\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}}{15c^{2}d^{2}\sqrt{ex+d}}$	67

input
```
int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
-2/15*(c*d*x+a*e)*(-3*c*d*g*x+2*a*e*g-5*c*d*f)*((c*d*x+a*e)*(e*x+d))^(1/2)
/c^2/d^2/(e*x+d)^(1/2)
```

3.682.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\frac{2\,(3\,c^2d^2gx^2+5\,acdef-2\,a^2e^2g+(5\,c^2d^2f+acdeg)x)\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}}{15\,(c^2d^2ex+c^2d^3)}$$

input
```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
 algorithm="fricas")
```

output
```
2/15*(3*c^2*d^2*g*x^2 + 5*a*c*d*e*f - 2*a^2*e^2*g + (5*c^2*d^2*f + a*c*d*e
*g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*
e*x + c^2*d^3)
```

3.682.6 Sympy [F]

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}(f+gx)}{\sqrt{d+ex}}\,dx$$

input
```
integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/
2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)/sqrt(d + e*x), x)
```

3.682.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.52

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2\,(cdx+ae)^{\frac{3}{2}}f}{3\,cd}+\frac{2\,(3\,c^2d^2x^2+acdex-2\,a^2e^2)\sqrt{cdx+ae}g}{15\,c^2d^2}$$

input
```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
 algorithm="maxima")
```

output
```
2/3*(c*d*x + a*e)^(3/2)*f/(c*d) + 2/15*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^2*
e^2)*sqrt(c*d*x + a*e)*g/(c^2*d^2)
```

3.682.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(113) = 226$.

Time = 0.28 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.02

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\left(\frac{5f\left(\frac{\sqrt{-cd^2e+ae^3}cd^2-\sqrt{-cd^2e+ae^3}ae^2}{cd}+\frac{\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}}{cde}\right)|e|}{e^2}-\frac{g\left(\frac{3\sqrt{-cd^2e+ae^3}c^2d^4-\sqrt{-cd^2e+ae^3}acd^2e^2-2\sqrt{-cd^2e+ae^3}a^2e^4}{c^2d^2}+\cdots\right.}{e^3}\right.}{15e}$$

input
```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
 algorithm="giac")
```

output
```
2/15*(5*f*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^2)/(
c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e)/e^2 - g*(
(3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2
*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))/(c^2*
d^2*e^2))*abs(e)/e^3)/e
```

3.682.9 Mupad [B] (verification not implemented)

Time = 11.98 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.74

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{\left(\frac{2\,g\,x^2}{5}-\frac{4\,a^2\,e^2\,g-10\,a\,c\,d\,e\,f}{15\,c^2\,d^2}+\frac{x\,\left(10\,f\,c^2\,d^2+2\,a\,e\,g\,c\,d\right)}{15\,c^2\,d^2}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}$$

input
```
int(((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(1
/2),x)
```

output
```
(((2*g*x^2)/5 - (4*a^2*e^2*g - 10*a*c*d*e*f)/(15*c^2*d^2) + (x*(10*c^2*d^2
*f + 2*a*c*d*e*g))/(15*c^2*d^2))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2))/(d + e*x)^(1/2)
```

3.683 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

3.683.1 Optimal result

Integrand size = 39, antiderivative size = 48

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{3cd(d+ex)^{3/2}}$$

output
```
2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/(e*x+d)^(3/2)
```

3.683.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2((ae+cdx)(d+ex))^{3/2}}{3cd(d+ex)^{3/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/Sqrt[d + e*x],x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2))/(3*c*d*(d + e*x)^(3/2))
```

3.683.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \ 1122$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3cd(d+ex)^{3/2}}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/Sqrt[d + e*x],x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*c*d*(d + e*x)^(3/2))
```

3.683.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

3.683.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2(cdx+ae)\sqrt{(cdx+ae)(ex+d)}}{3cd\sqrt{ex+d}}$	40
gosper	$\frac{2(cdx+ae)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{3cd\sqrt{ex+d}}$	50

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
2/3*(c*d*x+a*e)*((c*d*x+a*e)*(e*x+d))^(1/2)/c/d/(e*x+d)^(1/2)
```

3.683.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(cdx+ae)\sqrt{ex+d}}{3\,(cdex+cd^2)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorit
hm="fricas")
```

output
```
2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x + a*e)*sqrt(e*x + d
)/(c*d*e*x + c*d^2)
```

3.683.6 Sympy [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{\sqrt{d+ex}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/sqrt(d + e*x), x)
```

3.683.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2\,(cdx+ae)^{\frac{3}{2}}}{3\,cd}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorit
hm="maxima")
```

output
```
2/3*(c*d*x + a*e)^(3/2)/(c*d)
```

3.683.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(42) = 84.

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.90

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}}\,dx = \frac{2\left(\frac{\sqrt{-cd^2e+ae^3}cd^2-\sqrt{-cd^2e+ae^3}ae^2}{cd} + \frac{((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}}{cde}\right)|e|}{3\,e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorit
hm="giac")
```

output
```
2/3*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^2)/(c*d) +
 ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e)/e^3
```

3.683.9 Mupad [B] (verification not implemented)

Time = 11.98 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}}\,dx = \frac{\left(\frac{2\,x}{3} + \frac{2\,a\,e}{3\,c\,d}\right)\,\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}{\sqrt{d + e\,x}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^(1/2),x)
```

output
```
(((2*x)/3 + (2*a*e)/(3*c*d))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)
)/(d + e*x)^(1/2)
```

3.684 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx$

3.684.1 Optimal result

Integrand size = 46, antiderivative size = 124

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx=\frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{3/2}}$$

output
```
-2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(
1/2)/(e*x+d)^(1/2))*(-a*e*g+c*d*f)^(1/2)/g^(3/2)+2*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)
```

3.684.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx=\frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}-\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{g^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f +
g*x)),x]
```

output
```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x] - Sqrt[c*d*f
 - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/(g^(3/
2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.684.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1250, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)}dx$$

$$\Big\downarrow \text{1250}$$

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{g}$$

$$\Big\downarrow \text{1255}$$

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2e^2(cdf-aeg)\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}}{g}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)),
x]
```

output
```
(2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]) - (2*Sqr
t[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(3/2)
```

3.684.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.684.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.15

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\left(\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)aeg-\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cdf-\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\right)}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,g\sqrt{(aeg-cdf)g}}$	143

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
-2*((c*d*x+a*e)*(e*x+d))^(1/2)*(arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*a*e*g-arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*
f-(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/
2)/g/((a*e*g-c*d*f)*g)^(1/2)
```

3.684.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.56

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx$$
$$= \left[\frac{(ex+d)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}g\sqrt{-\frac{cdf-aeg}{g}}-(cdef-(cd^2+2\,ae^2)g)x}{egx^2+df+(ef+dg)x}\right)+}{egx+dg}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x,
 algorithm="fricas")
```

output
```
[((e*x + d)*sqrt(-(c*d*f - a*e*g)/g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e
*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(
c*d*f - a*e*g)/g) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e
*f + d*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d
))/(e*g*x + d*g), 2*((e*x + d)*sqrt((c*d*f - a*e*g)/g)*arctan(sqrt(e*x + d
)*sqrt((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) + s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g*x + d*g)]
```

3.684.6 Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx = \int \frac{\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}\,(f+gx)}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)/(e*x+d)**(1/
2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)), x)
```

3.684.7 Maxima [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{\sqrt{ex+d}(gx+f)}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x,
 algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x
+ f)), x)
```

3.684.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(108) = 216$.

Time = 0.36 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.06

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx$$
$$=\frac{2\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{g}+\frac{cdef\arctan\left(\frac{\sqrt{-cd^2e+ae^3}\,g}{\sqrt{cdfg-aeg^2}\,e}\right)-ae^2g\arctan\left(\frac{\sqrt{-cd^2e+ae^3}\,g}{\sqrt{cdfg-aeg^2}\,e}\right)-\sqrt{-cd^2e+ae^3}\sqrt{cdfg-aeg^2}}{\sqrt{cdfg-aeg^2}\,g}-\frac{(cde^2f-ae^3g)\,a}{}\right.}{e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x,
 algorithm="giac")
```

output
```
2*(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/g + (c*d*e*f*arctan(sqrt(-c*d^2
*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - a*e^2*g*arctan(sqrt(-c*d^2*e
+ a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*
f*g - a*e*g^2))/(sqrt(c*d*f*g - a*e*g^2)*g) - (c*d*e^2*f - a*e^3*g)*arctan
(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/(s
qrt(c*d*f*g - a*e*g^2)*e*g))*abs(e)/e^2
```

3.684.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\;x + a\,d\,e}}{(f + g\,x)\;\sqrt{d + e\,x}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)*(d + e*x)^(1/
2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)*(d + e*x)^(1/
2)), x)
```

3.685 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^2}\,dx$

3.685.1 Optimal result

Integrand size = 46, antiderivative size = 132

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^2}\,dx=-\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{g\sqrt{d+ex}(f+gx)}+\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

output
```
c*d*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^
(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(1/2)-(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)/g/(g*x+f)/(e*x+d)^(1/2)
```

3.685.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^2}\,dx=\frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{g}}{f+gx}+\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{cdf-aeg}\sqrt{ae+cdx}}\right)}{g^{3/2}\sqrt{d+ex}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f +
g*x)^2),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[g]/(f + g*x)) + (c*d*ArcTan[(Sqrt[g
]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(Sqrt[c*d*f - a*e*g]*Sqrt[a*e +
 c*d*x])))/(g^(3/2)*Sqrt[d + e*x])
```

3.685.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1249, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x \left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)^2} \, dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{cd \int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2g}-\frac{\sqrt{x \left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}$$

$$\Big\downarrow \text{1255}$$

$$\frac{cde^2 \int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}}{g}-\frac{\sqrt{x \left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}$$

$$\Big\downarrow \text{218}$$

$$\frac{cd \arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}-\frac{\sqrt{x \left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^2
),x]
```

output
```
-(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x)))
 + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt
[c*d*f - a*e*g]*Sqrt[d + e*x])])/(g^(3/2)*Sqrt[c*d*f - a*e*g])
```

3.685.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.685.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.14

method	result	size
default	$\frac{\left(-\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cdgx-\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cdf-\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\right)\sqrt{(cdx+ae)(ex+d)}}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,g(gx+f)\sqrt{(aeg-cdf)g}}$	151

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
(-arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*g*x-arctanh(g*(
c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*f-(c*d*x+a*e)^(1/2)*((a*e*g-
c*d*f)*g)^(1/2))*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/
2)/g/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

3.685.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(116) = 232$.

Time = 0.44 (sec) , antiderivative size = 562, normalized size of antiderivative = 4.26

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}(f + gx)^2}\,dx$$
$$= \left[-\frac{(cdegx^2 + cd^2f + (cdef + cd^2g)x)\sqrt{-cdfg + aeg^2}\log\left(-\frac{cdegx^2 - cd^2f + 2\,adeg - (cdef - (cd^2 + 2\,ae^2)g)x - 2\sqrt{cdex^2 +}}{egx^2 + df + (ef + dg)x}\right.}{2\,(cd^2f^2g^2 - adefg^3 + (cdefg^3 - ae^2g^4)x^2 + (cd}\right.$$
$$-\frac{(cdegx^2 + cd^2f + (cdef + cd^2g)x)\sqrt{cdfg - aeg^2}\arctan\left(\frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{cdfg - aeg^2}\sqrt{ex + d}}{cdegx^2 + adeg + (cd^2 + ae^2)gx}\right) + \sqrt{cde}}{cd^2f^2g^2 - adefg^3 + (cdefg^3 - ae^2g^4)x^2 + (cdef^2g^2 - adeg^4 + (cd^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),
x, algorithm="fricas")
```

output
```
[-1/2*((c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(-c*d*f*g + a*e
*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^
2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*
e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f*g - a*e*g^2)*sqrt(e*x + d))/(c*d^2*f^2
*g^2 - a*d*e*f*g^3 + (c*d*e*f*g^3 - a*e^2*g^4)*x^2 + (c*d*e*f^2*g^2 - a*d*
e*g^4 + (c*d^2 - a*e^2)*f*g^3)*x), -((c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c
*d^2*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g +
 (c*d^2 + a*e^2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*
f*g - a*e*g^2)*sqrt(e*x + d))/(c*d^2*f^2*g^2 - a*d*e*f*g^3 + (c*d*e*f*g^3
- a*e^2*g^4)*x^2 + (c*d*e*f^2*g^2 - a*d*e*g^4 + (c*d^2 - a*e^2)*f*g^3)*x)]
```

3.685.6 Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}(f + gx)^2}\,dx = \int \frac{\sqrt{(d + ex)(ae + cdx)}}{\sqrt{d + ex}\,(f + gx)^2}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**2/(e*x+d)**
(1/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**2), x)
```

3.685.7 Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^2}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),
x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x
+ f)^2), x)
```

3.685.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(116) = 232$.

Time = 0.36 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.39

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^2}\,dx =$$
$$-\frac{\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}\,cde^2}{(cde^2f-ae^3g+((ex+d)cde-cd^2e+ae^3)g)g} - \frac{cde\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}\,g}{\sqrt{cdfg-aeg^2}\,e}\right)}{\sqrt{cdfg-aeg^2}\,g} + \frac{cde^2f\arctan\left(\frac{\sqrt{-cd^2e+ae^3}\,g}{\sqrt{cdfg-aeg^2}\,e}\right)-cd^2eg\arctan\left(\cdots\right.}{\sqrt{cdfg-aeg^2}\,efg-\cdots}\right.}{e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),
x, algorithm="giac")
```

output
```
-(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*e^2/((c*d*e^2*f - a*e^3*g +
((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)*g) - c*d*e*arctan(sqrt((e*x + d)*c*
d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*
g^2)*g) + (c*d*e^2*f*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g
^2)*e)) - c*d^2*e*g*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^
2)*e)) - sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*e)/(sqrt(c*d*f*g -
 a*e*g^2)*e*f*g - sqrt(c*d*f*g - a*e*g^2)*d*g^2))*abs(e)/e^2
```

3.685.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^2}\,dx=\int \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{(f+g\,x)^2\,\sqrt{d+e\,x}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^2*(d + e*x)^(
1/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^2*(d + e*x)^(
1/2)), x)
```

3.686 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^3}\,dx$

3.686.1 Optimal result

Integrand size = 46, antiderivative size = 207

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^3}\,dx = -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2} + \frac{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4g(cdf-aeg)\sqrt{d+ex}(f+gx)} + \frac{c^2d^2\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4g^{3/2}(cdf-aeg)^{3/2}}$$

output
```
1/4*c^2*d^2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g
+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(3/2)-1/2*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(g*x+f)^2/(e*x+d)^(1/2)+1/4*c*d*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)/(g*x+f)/(e*x+d)^(1/2)
```

3.686.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^3}\,dx = \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{cdf-aeg}\sqrt{ae+cdx}(2aeg+cd(-f+gx))+c^2d^2(f+gx)^2\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right.\right.}{4g^{3/2}(cdf-aeg)^{3/2}\sqrt{(ae+cdx)(d+ex)}(f+gx)^2}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f +
g*x)^3),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c
*d*x]*(2*a*e*g + c*d*(-f + g*x)) + c^2*d^2*(f + g*x)^2*ArcTan[(Sqrt[g]*Sqr
t[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/(4*g^(3/2)*(c*d*f - a*e*g)^(3/2)*Sq
rt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^2)
```

3.686.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1249, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)^3}dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}$$

$$\Big\downarrow \text{1254}$$

$$\frac{cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}$$

$$\Big\downarrow \text{1255}$$

$$\frac{cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^3
),x]
```

output
```
-1/2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x
)^2) + (c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*
Sqrt[d + e*x]*(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f -
 a*e*g)^(3/2))))/(4*g)
```

3.686.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.686.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.33

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2g^2x^2+2\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2fgx+\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2f^2-\sqrt{cdx}\right.}{4\sqrt{ex+d}\sqrt{cdx+ae}\,(aeg-cdf)g(gx+f)^2\sqrt{(aeg-c}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/4*((c*d*x+a*e)*(e*x+d))^(1/2)*(arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^2*d^2*g^2*x^2+2*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g
)^(1/2))*c^2*d^2*f*g*x+arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)
)*c^2*d^2*f^2-(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c*d*g*x-2*(c*d*x+a
*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*e*g+(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g
)^(1/2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)/g/(g*x+f)^2/(
(a*e*g-c*d*f)*g)^(1/2)
```

3.686.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(181) = 362$.

Time = 0.40 (sec) , antiderivative size = 1056, normalized size of antiderivative = 5.10

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^3}\,dx$$
$$=\left[\frac{(c^2d^2eg^2x^3+c^2d^3f^2+(2\,c^2d^2efg+c^2d^3g^2)x^2+(c^2d^2ef^2+2\,c^2d^3fg)x)\sqrt{-cdfg+aeg^2}\log\left(-\frac{cdegx^2-cd^2}{}\right.}{8\,(c^2d^3f^4g^2-2\,acd^2ef^3g^3+a^2de^2f^2g^4+(c^2d^2ef^2g^4-2\,acde^2fg^5+a^2e^3g^6)x^3+(2\,c}\right.$$
$$-\frac{(c^2d^2eg^2x^3+c^2d^3f^2+(2\,c^2d^2efg+c^2d^3g^2)x^2+(c^2d^2ef^2+2\,c^2d^3fg)x)\sqrt{cdfg-aeg^2}\arctan\left(\frac{\sqrt{cdex^2+}}{}\right.}{4\,(c^2d^3f^4g^2-2\,acd^2ef^3g^3+a^2de^2f^2g^4+(c^2d^2ef^2g^4-2\,acde^2fg^5+a^2e^3g^6)x^3+(2\,c^2d^2ef^3g^3+a^2de^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),
x, algorithm="fricas")
```

output
```
[1/8*((c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)*x
^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d
*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x
 + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(c^2*d^2*f^2*g - 3*a*c*d*e*f*g
^2 + 2*a^2*e^2*g^3 - (c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^4*g^2 - 2*a*c*d^2*e*f^3*
g^3 + a^2*d*e^2*f^2*g^4 + (c^2*d^2*e*f^2*g^4 - 2*a*c*d*e^2*f*g^5 + a^2*e^3
*g^6)*x^3 + (2*c^2*d^2*e*f^3*g^3 + a^2*d*e^2*g^6 + (c^2*d^3 - 4*a*c*d*e^2)
*f^2*g^4 - 2*(a*c*d^2*e - a^2*e^3)*f*g^5)*x^2 + (c^2*d^2*e*f^4*g^2 + 2*a^2
*d*e^2*f*g^5 + 2*(c^2*d^3 - a*c*d*e^2)*f^3*g^3 - (4*a*c*d^2*e - a^2*e^3)*f
^2*g^4)*x), -1/4*((c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^
2*d^3*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(c*d*f*g - a*e*g^2
)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^
2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (c^2*d^2
*f^2*g - 3*a*c*d*e*f*g^2 + 2*a^2*e^2*g^3 - (c^2*d^2*f*g^2 - a*c*d*e*g^3)*x
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^4*
g^2 - 2*a*c*d^2*e*f^3*g^3 + a^2*d*e^2*f^2*g^4 + (c^2*d^2*e*f^2*g^4 - 2*a*c
*d*e^2*f*g^5 + a^2*e^3*g^6)*x^3 + (2*c^2*d^2*e*f^3*g^3 + a^2*d*e^2*g^6 + (
c^2*d^3 - 4*a*c*d*e^2)*f^2*g^4 - 2*(a*c*d^2*e - a^2*e^3)*f*g^5)*x^2 + (...
```

3.686.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^3}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**3/(e*x+d)**
(1/2),x)
```

output
```
Timed out
```

3.686.7 Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^3}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^3}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),
x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x
+ f)^3), x)
```

3.686.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. 2(181) = 362.

Time = 0.49 (sec) , antiderivative size = 662, normalized size of antiderivative = 3.20

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^3}\,dx$$
$$= \frac{\left(\frac{c^2d^2e\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3g}}{\sqrt{cdfg-aeg^2e}}\right)}{(cdfg-aeg^2)^{\frac{3}{2}}} - \frac{c^2d^2e^3f^2\arctan\left(\frac{\sqrt{-cd^2e+ae^3g}}{\sqrt{cdfg-aeg^2e}}\right) - 2c^2d^3e^2fg\arctan\left(\frac{\sqrt{-cd^2e+ae^3g}}{\sqrt{cdfg-aeg^2e}}\right) + c^2d^4eg^2\arctan\left(\frac{\sqrt{-cd^2}}{\sqrt{cdfg}}\right.}{\sqrt{cdfg-aeg^2}cde^2f^3g - 2\sqrt{cdfg-aeg^2}cd^2ef^2g^2 - \sqrt{cdfg-aeg^2}}\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),
x, algorithm="giac")
```

output
```
1/4*(c^2*d^2*e*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*
f*g - a*e*g^2)*e))/(c*d*f*g - a*e*g^2)^(3/2) - (c^2*d^2*e^3*f^2*arctan(sqr
t(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 2*c^2*d^3*e^2*f*g*arc
tan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + c^2*d^4*e*g^2*
arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - sqrt(-c*d^2
*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c*d*e^2*f - sqrt(-c*d^2*e + a*e^3)*sqr
t(c*d*f*g - a*e*g^2)*c*d^2*e*g + 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a
*e*g^2)*a*e^3*g)/(sqrt(c*d*f*g - a*e*g^2)*c*d*e^2*f^3*g - 2*sqrt(c*d*f*g -
 a*e*g^2)*c*d^2*e*f^2*g^2 - sqrt(c*d*f*g - a*e*g^2)*a*e^3*f^2*g^2 + sqrt(c
*d*f*g - a*e*g^2)*c*d^3*f*g^3 + 2*sqrt(c*d*f*g - a*e*g^2)*a*d*e^2*f*g^3 -
sqrt(c*d*f*g - a*e*g^2)*a*d^2*e*g^4) - (sqrt((e*x + d)*c*d*e - c*d^2*e + a
*e^3)*c^3*d^3*e^4*f - sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^2*d^2*e^
5*g - ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*e^2*g)/((c*d*e^2*f
 - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^2*(c*d*f*g - a*e*g^2))
)*abs(e)/e^2
```

3.686.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}(f + gx)^3}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}{(f + g\,x)^3\,\sqrt{d + e\,x}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^3*(d + e*x)^(
1/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^3*(d + e*x)^(
1/2)), x)
```

3.687 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx$

3.687.1 Optimal result

Integrand size = 46, antiderivative size = 277

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx = -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3} + \frac{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12g(cdf-aeg)\sqrt{d+ex}(f+gx)^2} + \frac{c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8g(cdf-aeg)^2\sqrt{d+ex}(f+gx)} + \frac{c^3d^3\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{8g^{3/2}(cdf-aeg)^{5/2}}$$

output
```
1/8*c^3*d^3*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g
+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(5/2)-1/3*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(g*x+f)^3/(e*x+d)^(1/2)+1/12*c*d*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)/(g*x+f)^2/(e*x+d)^(1/2)+1/8*
c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)^2/(g*x+f)
/(e*x+d)^(1/2)
```

3.687.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx$$
$$= \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{cdf-aeg}\sqrt{ae+cdx}(-8a^2e^2g^2-2acdeg(-7f+gx)+c^2d^2(-3f^2+8fgx+3g\right.}{24g^{3/2}(cdf-aeg)^{5/2}\sqrt{(ae+cdx)(d+ex)}(f+gx)^3}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f +
g*x)^4),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c
*d*x]*(-8*a^2*e^2*g^2 - 2*a*c*d*e*g*(-7*f + g*x) + c^2*d^2*(-3*f^2 + 8*f*g
*x + 3*g^2*x^2)) + 3*c^3*d^3*(f + g*x)^3*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x]
)/Sqrt[c*d*f - a*e*g]]))/(24*g^(3/2)*(c*d*f - a*e*g)^(5/2)*Sqrt[(a*e + c*d
*x)*(d + e*x)]*(f + g*x)^3)
```

3.687.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {1249, 1254, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx$$
$$\Big\downarrow \text{1249}$$
$$\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{6g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}$$
$$\Big\downarrow \text{1254}$$
$$\frac{cd\left(\frac{3cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}$$

$\Big\downarrow$ 1254

$$\frac{cd\left(\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}$$

$\Big\downarrow$ 1255

$$\frac{cd\left(\frac{3cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cde}}{2\sqrt{d+ex}(f+gx)^2(cdf-a}\right.}{6g}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}$$

$\Big\downarrow$ 218

$$\frac{cd\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^4
),x]
```

3.687. $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx$

output
```
-1/3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x
)^3) + (c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d*f - a*e*g
)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d +
 e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2))))/(4*(c*d*f - a*e*g))))/(6*g)
```

3.687.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.687. $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx$

3.687.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.60

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(3\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3g^3x^3+9\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3f\,g^2x^2+9\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3f \cdots\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
-1/24*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^3*d^3*g^3*x^3+9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^3*d^3*f*g^2*x^2+9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^3*d^3*f^2*g*x+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g
)^(1/2))*c^3*d^3*f^3-3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*g
^2*x^2+2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*g^2*x-8*(c*d*x+
a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x+8*(c*d*x+a*e)^(1/2)*((a*e
*g-c*d*f)*g)^(1/2)*a^2*e^2*g^2-14*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2
)*a*c*d*e*f*g+3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f^2)/(e*
x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^3/g/(a*e*g-c*d*f)^2/(c*d*x+a*e)
^(1/2)
```

3.687.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 845 vs. 2(245) = 490.

Time = 0.80 (sec) , antiderivative size = 1732, normalized size of antiderivative = 6.25

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),
x, algorithm="fricas")
```

output
```
[-1/48*(3*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*
g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^
3*d^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a
*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f +
(e*f + d*g)*x)) + 2*(3*c^3*d^3*f^3*g - 17*a*c^2*d^2*e*f^2*g^2 + 22*a^2*c*d
*e^2*f*g^3 - 8*a^3*e^3*g^4 - 3*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 - 2*(
4*c^3*d^3*f^2*g^2 - 5*a*c^2*d^2*e*f*g^3 + a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^6*g^2 - 3*a*c^2*
d^3*e*f^5*g^3 + 3*a^2*c*d^2*e^2*f^4*g^4 - a^3*d*e^3*f^3*g^5 + (c^3*d^3*e*f
^3*g^5 - 3*a*c^2*d^2*e^2*f^2*g^6 + 3*a^2*c*d*e^3*f*g^7 - a^3*e^4*g^8)*x^4
+ (3*c^3*d^3*e*f^4*g^4 - a^3*d*e^3*g^8 + (c^3*d^4 - 9*a*c^2*d^2*e^2)*f^3*g
^5 - 3*(a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^2*g^6 + 3*(a^2*c*d^2*e^2 - a^3*e^4)
*f*g^7)*x^3 + 3*(c^3*d^3*e*f^5*g^3 - a^3*d*e^3*f*g^7 + (c^3*d^4 - 3*a*c^2*
d^2*e^2)*f^4*g^4 - 3*(a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^5 + (3*a^2*c*d^2*e^
2 - a^3*e^4)*f^2*g^6)*x^2 + (c^3*d^3*e*f^6*g^2 - 3*a^3*d*e^3*f^2*g^6 + 3*(
c^3*d^4 - a*c^2*d^2*e^2)*f^5*g^3 - 3*(3*a*c^2*d^3*e - a^2*c*d*e^3)*f^4*g^4
 + (9*a^2*c*d^2*e^2 - a^3*e^4)*f^3*g^5)*x), -1/24*(3*(c^3*d^3*e*g^3*x^4 +
c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g^3)*x^3 + 3*(c^3*d^3*e*f^2*g +
 c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3*d^4*f^2*g)*x)*sqrt(c*d*f*g...
```

3.687.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^4}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**4/(e*x+d)**
(1/2),x)
```

output
```
Timed out
```

3.687.7 Maxima [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{\sqrt{ex+d}(gx+f)^4}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),
x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x
+ f)^4), x)
```

3.687.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1234 vs. 2(245) = 490.

Time = 0.66 (sec) , antiderivative size = 1234, normalized size of antiderivative = 4.45

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx$$
$$=\frac{\left(\frac{3\,c^3d^3e\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3g}}{\sqrt{cdfg-aeg^2}e}\right)}{(c^2d^2f^2g-2\,acdefg^2+a^2e^2g^3)\sqrt{cdfg-aeg^2}}-\frac{3\,c^3d^3e^4f^3\arctan\left(\frac{\sqrt{-cd^2e+ae^3g}}{\sqrt{cdfg-aeg^2}e}\right)-9\,c^3d^4e^3f^2g\arctan\left(\frac{\sqrt{-cd^2e+ae^3g}}{\sqrt{cdfg-aeg^2}e}\right)+9\,c^3d^5e^2fg^2\,\text{arct}\cdots}{\sqrt{cdfg-aeg^2}c^2d^2e^3f^5g-3\sqrt{cdfg-aeg^2}c^2d^3e^2f^4g^2-2\sqrt{cd\cdots}}\right.}{\cdots}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),
x, algorithm="giac")
```

output
```
1/24*(3*c^3*d^3*e*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c
*d*f*g - a*e*g^2)*e))/((c^2*d^2*f^2*g - 2*a*c*d*e*f*g^2 + a^2*e^2*g^3)*sqr
t(c*d*f*g - a*e*g^2)) - (3*c^3*d^3*e^4*f^3*arctan(sqrt(-c*d^2*e + a*e^3)*g
/(sqrt(c*d*f*g - a*e*g^2)*e)) - 9*c^3*d^4*e^3*f^2*g*arctan(sqrt(-c*d^2*e +
 a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 9*c^3*d^5*e^2*f*g^2*arctan(sqrt(-
c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 3*c^3*d^6*e*g^3*arctan(s
qrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 3*sqrt(-c*d^2*e + a
*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d^2*e^3*f^2 - 8*sqrt(-c*d^2*e + a*e^3)*s
qrt(c*d*f*g - a*e*g^2)*c^2*d^3*e^2*f*g + 14*sqrt(-c*d^2*e + a*e^3)*sqrt(c*
d*f*g - a*e*g^2)*a*c*d*e^4*f*g + 3*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a
*e*g^2)*c^2*d^4*e*g^2 + 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a
*c*d^2*e^3*g^2 - 8*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*e^5*
g^2)/(sqrt(c*d*f*g - a*e*g^2)*c^2*d^2*e^3*f^5*g - 3*sqrt(c*d*f*g - a*e*g^2
)*c^2*d^3*e^2*f^4*g^2 - 2*sqrt(c*d*f*g - a*e*g^2)*a*c*d*e^4*f^4*g^2 + 3*sq
rt(c*d*f*g - a*e*g^2)*c^2*d^4*e*f^3*g^3 + 6*sqrt(c*d*f*g - a*e*g^2)*a*c*d^
2*e^3*f^3*g^3 + sqrt(c*d*f*g - a*e*g^2)*a^2*e^5*f^3*g^3 - sqrt(c*d*f*g - a
*e*g^2)*c^2*d^5*f^2*g^4 - 6*sqrt(c*d*f*g - a*e*g^2)*a*c*d^3*e^2*f^2*g^4 -
3*sqrt(c*d*f*g - a*e*g^2)*a^2*d*e^4*f^2*g^4 + 2*sqrt(c*d*f*g - a*e*g^2)*a*
c*d^4*e*f*g^5 + 3*sqrt(c*d*f*g - a*e*g^2)*a^2*d^2*e^3*f*g^5 - sqrt(c*d*f*g
 - a*e*g^2)*a^2*d^3*e^2*g^6) - (3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^...
```

3.687.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^4}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}{(f + g\,x)^4\,\sqrt{d + e\,x}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^4*(d + e*x)^(
1/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^4*(d + e*x)^(
1/2)), x)
```

3.688 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx$

3.688.1 Optimal result

Integrand size = 46, antiderivative size = 347

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx = -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4} + \frac{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{5c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{96g(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} + \frac{5c^3d^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{64g(cdf-aeg)^3\sqrt{d+ex}(f+gx)} + \frac{5c^4d^4\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{64g^{3/2}(cdf-aeg)^{7/2}}$$

output
```
5/64*c^4*d^4*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*
g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(7/2)-1/4*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(g*x+f)^4/(e*x+d)^(1/2)+1/24*c*d*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)/(g*x+f)^3/(e*x+d)^(1/2)+5/9
6*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)^2/(g*x+
f)^2/(e*x+d)^(1/2)+5/64*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/
(-a*e*g+c*d*f)^3/(g*x+f)/(e*x+d)^(1/2)
```

3.688.2 Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx$$
$$= \frac{c^4d^4\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{g}(48a^3e^3g^3+8a^2cde^2g^2(-17f+gx)-2ac^2d^2eg(-59f^2+18fgx+5g^2x^2)+c^3d^3(-15f^3+73f^2gx+55fg^2x^2}{c^4d^4(cdf-aeg)^3(f+gx)^4}\right.}{192g^{3/2}\sqrt{d+ex}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f +
g*x)^5),x]
```

output
```
(c^4*d^4*Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[g]*(48*a^3*e^3*g^3 + 8*a^2*c
*d*e^2*g^2*(-17*f + g*x) - 2*a*c^2*d^2*e*g*(-59*f^2 + 18*f*g*x + 5*g^2*x^2
) + c^3*d^3*(-15*f^3 + 73*f^2*g*x + 55*f*g^2*x^2 + 15*g^3*x^3)))/(c^4*d^4*
(c*d*f - a*e*g)^3*(f + g*x)^4) + (15*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sq
rt[c*d*f - a*e*g]])/((c*d*f - a*e*g)^(7/2)*Sqrt[a*e + c*d*x])))/(192*g^(3/
2)*Sqrt[d + e*x])
```

3.688.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1249, 1254, 1254, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx$$
$$\Big\downarrow \text{1249}$$
$$\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}$$
$$\Big\downarrow \text{1254}$$

$$\frac{cd\left(\frac{5cd\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}\right)}{8g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}$$

$\Big\downarrow$ 1254

$$\frac{cd\left(\frac{5cd\left(\frac{3cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}\right)}{8g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}$$

$\Big\downarrow$ 1254

$$\frac{cd\left(\frac{5cd\left(\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}\right)}{8g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}$$

$\Big\downarrow$ 1255

3.688. $\displaystyle\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx$

$$cd\left(\frac{5cd\left(\frac{3cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cde\cdots}}{2\sqrt{d+ex}(f+gx)^2(cdf-ae\cdots}}{6(cdf-aeg)}\cdots\right.$$

$$8g$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}$$

$$\Big\downarrow \; 218$$

$$cd\left(\frac{5cd\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex\cdots}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg\cdots}\right.$$

$$8g$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}$$

3.688. $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^5
),x]
```

output
```
-1/4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x
)^4) + (c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*(c*d*f - a*e*g
)*Sqrt[d + e*x]*(f + g*x)^3) + (5*c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]/(2*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c*d*(Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x
)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sq
rt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2))))/(4*(c
*d*f - a*e*g))))/(6*(c*d*f - a*e*g))))/(8*g)
```

3.688.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.688. $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx$

3.688.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(309) = 618$.

Time = 0.54 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.98

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^4d^4g^4x^4+60\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^4d^4fg^3x^3+90\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^4d^4\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/192*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*c^4*d^4*g^4*x^4+60*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^4*d^4*f*g^3*x^3+90*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^4*d^4*f^2*g^2*x^2+60*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*c^4*d^4*f^3*g*x-15*c^3*d^3*g^3*x^3*(c*d*x+a*e)^(1/2)*((a*
e*g-c*d*f)*g)^(1/2)+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)
)*c^4*d^4*f^4+10*a*c^2*d^2*e*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(
1/2)-55*c^3*d^3*f*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-8*a^2*
c*d*e^2*g^3*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+36*a*c^2*d^2*e*f*g
^2*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-73*c^3*d^3*f^2*g*x*(c*d*x+a
*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-48*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(
1/2)*a^3*e^3*g^3+136*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*c*d*e^2
*f*g^2-118*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*d^2*e*f^2*g+15*
(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/((a*e
*g-c*d*f)*g)^(1/2)/(g*x+f)^4/g/(a*e*g-c*d*f)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^
2*d^2*f^2)/(c*d*x+a*e)^(1/2)
```

3.688.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1284 vs. $2(309) = 618$.

Time = 1.35 (sec) , antiderivative size = 2610, normalized size of antiderivative = 7.52

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),
x, algorithm="fricas")
```

output
```
[1/384*(15*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5
*g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e
*f^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqr
t(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f -
 (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2
*(15*c^4*d^4*f^4*g - 133*a*c^3*d^3*e*f^3*g^2 + 254*a^2*c^2*d^2*e^2*f^2*g^3
 - 184*a^3*c*d*e^3*f*g^4 + 48*a^4*e^4*g^5 - 15*(c^4*d^4*f*g^4 - a*c^3*d^3*
e*g^5)*x^3 - 5*(11*c^4*d^4*f^2*g^3 - 13*a*c^3*d^3*e*f*g^4 + 2*a^2*c^2*d^2*
e^2*g^5)*x^2 - (73*c^4*d^4*f^3*g^2 - 109*a*c^3*d^3*e*f^2*g^3 + 44*a^2*c^2*
d^2*e^2*f*g^4 - 8*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^8*g^2 - 4*a*c^3*d^4*e*f^7*g^3 + 6*a^2*c^
2*d^3*e^2*f^6*g^4 - 4*a^3*c*d^2*e^3*f^5*g^5 + a^4*d*e^4*f^4*g^6 + (c^4*d^4
*e*f^4*g^6 - 4*a*c^3*d^3*e^2*f^3*g^7 + 6*a^2*c^2*d^2*e^3*f^2*g^8 - 4*a^3*c
*d*e^4*f*g^9 + a^4*e^5*g^10)*x^5 + (4*c^4*d^4*e*f^5*g^5 + a^4*d*e^4*g^10 +
 (c^4*d^5 - 16*a*c^3*d^3*e^2)*f^4*g^6 - 4*(a*c^3*d^4*e - 6*a^2*c^2*d^2*e^3
)*f^3*g^7 + 2*(3*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e^4)*f^2*g^8 - 4*(a^3*c*d^2*e
^3 - a^4*e^5)*f*g^9)*x^4 + 2*(3*c^4*d^4*e*f^6*g^4 + 2*a^4*d*e^4*f*g^9 + 2*
(c^4*d^5 - 6*a*c^3*d^3*e^2)*f^5*g^5 - 2*(4*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3
)*f^4*g^6 + 12*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^7 - (8*a^3*c*d^2*e...
```

3.688.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}(f + gx)^5}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**5/(e*x+d)**
(1/2),x)
```

output
```
Timed out
```

3.688.7 Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}(f + gx)^5}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^5}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),
x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x
+ f)^5), x)
```

3.688.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1955 vs. $2(309) = 618$.

Time = 1.73 (sec) , antiderivative size = 1955, normalized size of antiderivative = 5.63

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}(f + gx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),
x, algorithm="giac")
```

output
```
1/192*(15*c^4*d^4*e*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt
(c*d*f*g - a*e*g^2)*e))/((c^3*d^3*f^3*g - 3*a*c^2*d^2*e*f^2*g^2 + 3*a^2*c*
d*e^2*f*g^3 - a^3*e^3*g^4)*sqrt(c*d*f*g - a*e*g^2)) - (15*c^4*d^4*e^5*f^4*
arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 60*c^4*d^5*
e^4*f^3*g*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 9
0*c^4*d^6*e^3*f^2*g^2*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*
g^2)*e)) - 60*c^4*d^7*e^2*f*g^3*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*
f*g - a*e*g^2)*e)) + 15*c^4*d^8*e*g^4*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqr
t(c*d*f*g - a*e*g^2)*e)) - 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^
2)*c^3*d^3*e^4*f^3 - 73*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^3
*d^4*e^3*f^2*g + 118*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^2*
d^2*e^5*f^2*g + 55*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^3*d^5*
e^2*f*g^2 + 36*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^3*e^
4*f*g^2 - 136*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d*e^6*f
*g^2 - 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^3*d^6*e*g^3 - 1
0*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^4*e^3*g^3 - 8*sqr
t(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^2*e^5*g^3 + 48*sqrt(-c
*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^3*e^7*g^3)/(sqrt(c*d*f*g - a*e*g
^2)*c^3*d^3*e^4*f^7*g - 4*sqrt(c*d*f*g - a*e*g^2)*c^3*d^4*e^3*f^6*g^2 - 3*
sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^2*e^5*f^6*g^2 + 6*sqrt(c*d*f*g - a*e*g^...
```

3.688.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx=\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\left(f+g\,x\right)^5\,\sqrt{d+e\,x}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^5*(d + e*x)^(
1/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^5*(d + e*x)^(
1/2)), x)
```

3.689 $\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

3.689.1 Optimal result

Integrand size = 46, antiderivative size = 336

$$\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = -\frac{128(cdf-aeg)^3\left(2ae^2g-cd(7ef-5dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{15015c^5d^5e(d+ex)^{5/2}} + \frac{128g(cdf-aeg)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3003c^4d^4e(d+ex)^{3/2}} + \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{429c^3d^3(d+ex)^{5/2}} + \frac{16(cdf-aeg)(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{143c^2d^2(d+ex)^{5/2}} + \frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}}$$

output
```
-128/15015*(-a*e*g+c*d*f)^3*(2*a*e^2*g-c*d*(-5*d*g+7*e*f))*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(5/2)/c^5/d^5/e/(e*x+d)^(5/2)+128/3003*g*(-a*e*g+c*d*f)
^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^4/d^4/e/(e*x+d)^(3/2)+32/429*
(-a*e*g+c*d*f)^2*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^3/d^3
/(e*x+d)^(5/2)+16/143*(-a*e*g+c*d*f)*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(5/2)/c^2/d^2/(e*x+d)^(5/2)+2/13*(g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(5/2)/c/d/(e*x+d)^(5/2)
```

3.689.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.58

$$\int \frac{(f+gx)^4\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2((ae+cdx)(d+ex))^{5/2}\left(128a^4e^4g^4-64a^3cde^3g^3(13f\right.}{}$$

input
```
Integrate[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
 e*x)^(3/2),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(128*a^4*e^4*g^4 - 64*a^3*c*d*e^3*g^3*(
13*f + 5*g*x) + 16*a^2*c^2*d^2*e^2*g^2*(143*f^2 + 130*f*g*x + 35*g^2*x^2)
- 8*a*c^3*d^3*e*g*(429*f^3 + 715*f^2*g*x + 455*f*g^2*x^2 + 105*g^3*x^3) +
c^4*d^4*(3003*f^4 + 8580*f^3*g*x + 10010*f^2*g^2*x^2 + 5460*f*g^3*x^3 + 11
55*g^4*x^4)))/(15015*c^5*d^5*(d + e*x)^(5/2))
```

3.689.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {1253, 1253, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{1253}$$

$$\frac{8(cdf-aeg)\int \frac{(f+gx)^3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx}{13cd}+\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}}$$

$$\Big\downarrow \text{1253}$$

$$\frac{8(cdf-aeg)\left(\frac{6(cdf-aeg)\int \frac{(f+gx)^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx}{11cd}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}}\right)}{13cd}+\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}}$$

$\downarrow$ 1253

$$\frac{8(cdf-aeg)\left(\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\int \frac{(f+gx)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx}{9cd}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}}\right)}{11cd}+\frac{2(f+gx)^3(x(ae}{11c}\right.}{13cd}$$

$$\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}}$$

$\downarrow$ 1221

$$\frac{8(cdf-aeg)\left(\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\left(\frac{1}{7}\left(-\frac{2aeg}{cd}-\frac{5dg}{e}+7f\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}\right)}{9cd}+\frac{2(f+gx)^2(}{}\right.}{11cd}\right.}{13cd}$$

$$\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}}$$

$\downarrow$ 1122

$$\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}}+$$

$$\frac{8(cdf-aeg)\left(\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}}+\frac{6(cdf-aeg)\left(\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}}+\frac{4(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)+\right.}{}\right.}{}\right.}{11cd}\right.}{13cd}$$

input
```
Int[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^
(3/2),x]
```

3.689. $\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

output
```
(2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(13*c*d*(d +
 e*x)^(5/2)) + (8*(c*d*f - a*e*g)*((2*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(5/2))/(11*c*d*(d + e*x)^(5/2)) + (6*(c*d*f - a*e*g)*((2*(
f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9*c*d*(d + e*x)
^(5/2)) + (4*(c*d*f - a*e*g)*((2*(7*f - (5*d*g)/e - (2*a*e*g)/(c*d))*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(35*c*d*(d + e*x)^(5/2)) + (2*g*
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*c*d*e*(d + e*x)^(3/2)))
/(9*c*d)))/(11*c*d)))/(13*c*d)
```

3.689.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

3.689. $\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

3.689.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.82

method	result
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^2(1155g^4x^4c^4d^4-840a\,c^3d^3e\,g^4x^3+5460c^4d^4f\,g^3x^3+560a^2c^2d^2e^2g^4x^2-3640a\,c^3d^3ef\,g^3x^2+10010c^4}{}$
gosper	$\frac{2(cdx+ae)(1155g^4x^4c^4d^4-840a\,c^3d^3e\,g^4x^3+5460c^4d^4f\,g^3x^3+560a^2c^2d^2e^2g^4x^2-3640a\,c^3d^3ef\,g^3x^2+10010c^4d^4f^2g^2x^2-320a^3cd\,e}{}$

input
```
int((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/15015*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^2*(1155*c^4*
d^4*g^4*x^4-840*a*c^3*d^3*e*g^4*x^3+5460*c^4*d^4*f*g^3*x^3+560*a^2*c^2*d^2
*e^2*g^4*x^2-3640*a*c^3*d^3*e*f*g^3*x^2+10010*c^4*d^4*f^2*g^2*x^2-320*a^3*
c*d*e^3*g^4*x+2080*a^2*c^2*d^2*e^2*f*g^3*x-5720*a*c^3*d^3*e*f^2*g^2*x+8580
*c^4*d^4*f^3*g*x+128*a^4*e^4*g^4-832*a^3*c*d*e^3*f*g^3+2288*a^2*c^2*d^2*e^
2*f^2*g^2-3432*a*c^3*d^3*e*f^3*g+3003*c^4*d^4*f^4)/c^5/d^5
```

3.689.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.40

$$\int \frac{(f+gx)^4\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\,(1155\,c^6d^6g^4x^6+3003\,a^2c^4d^4e^2f^4-3432\,a^3c^3d^3e^3f^3g}{}$$

input
```
integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="fricas")
```

output
```
2/15015*(1155*c^6*d^6*g^4*x^6 + 3003*a^2*c^4*d^4*e^2*f^4 - 3432*a^3*c^3*d^
3*e^3*f^3*g + 2288*a^4*c^2*d^2*e^4*f^2*g^2 - 832*a^5*c*d*e^5*f*g^3 + 128*a
^6*e^6*g^4 + 210*(26*c^6*d^6*f*g^3 + 7*a*c^5*d^5*e*g^4)*x^5 + 35*(286*c^6*
d^6*f^2*g^2 + 208*a*c^5*d^5*e*f*g^3 + a^2*c^4*d^4*e^2*g^4)*x^4 + 20*(429*c
^6*d^6*f^3*g + 715*a*c^5*d^5*e*f^2*g^2 + 13*a^2*c^4*d^4*e^2*f*g^3 - 2*a^3*
c^3*d^3*e^3*g^4)*x^3 + 3*(1001*c^6*d^6*f^4 + 4576*a*c^5*d^5*e*f^3*g + 286*
a^2*c^4*d^4*e^2*f^2*g^2 - 104*a^3*c^3*d^3*e^3*f*g^3 + 16*a^4*c^2*d^2*e^4*g
^4)*x^2 + 2*(3003*a*c^5*d^5*e*f^4 + 858*a^2*c^4*d^4*e^2*f^3*g - 572*a^3*c^
3*d^3*e^3*f^2*g^2 + 208*a^4*c^2*d^2*e^4*f*g^3 - 32*a^5*c*d*e^5*g^4)*x)*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x + c^5*
d^6)
```

3.689.6 Sympy [F]

$$\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(f+gx\right)^4}{\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**
(3/2),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*(f + g*x)**4/(d + e*x)**(3/2), x
)
```

3.689.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.23

$$\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\left(c^2d^2x^2+2\,acdex+a^2e^2\right)\sqrt{cdx+ae}\,f^4}{5\,cd}$$
$$+\frac{8\left(5\,c^3d^3x^3+8\,ac^2d^2ex^2+a^2cde^2x-2\,a^3e^3\right)\sqrt{cdx+ae}\,f^3g}{35\,c^2d^2}$$
$$+\frac{4\left(35\,c^4d^4x^4+50\,ac^3d^3ex^3+3\,a^2c^2d^2e^2x^2-4\,a^3cde^3x+8\,a^4e^4\right)\sqrt{cdx+ae}\,f^2g^2}{105\,c^3d^3}$$
$$+\frac{8\left(105\,c^5d^5x^5+140\,ac^4d^4ex^4+5\,a^2c^3d^3e^2x^3-6\,a^3c^2d^2e^3x^2+8\,a^4cde^4x-16\,a^5e^5\right)\sqrt{cdx+ae}\,fg^3}{1155\,c^4d^4}$$
$$+\frac{2\left(1155\,c^6d^6x^6+1470\,ac^5d^5ex^5+35\,a^2c^4d^4e^2x^4-40\,a^3c^3d^3e^3x^3+48\,a^4c^2d^2e^4x^2-64\,a^5cde^5x+128\,a^6e^6\right)}{15015\,c^5d^5}$$

input
```
integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="maxima")
```

output
```
2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f^4/(c*d) + 8/
35*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*
d*x + a*e)*f^3*g/(c^2*d^2) + 4/105*(35*c^4*d^4*x^4 + 50*a*c^3*d^3*e*x^3 +
3*a^2*c^2*d^2*e^2*x^2 - 4*a^3*c*d*e^3*x + 8*a^4*e^4)*sqrt(c*d*x + a*e)*f^2
*g^2/(c^3*d^3) + 8/1155*(105*c^5*d^5*x^5 + 140*a*c^4*d^4*e*x^4 + 5*a^2*c^3
*d^3*e^2*x^3 - 6*a^3*c^2*d^2*e^3*x^2 + 8*a^4*c*d*e^4*x - 16*a^5*e^5)*sqrt(
c*d*x + a*e)*f*g^3/(c^4*d^4) + 2/15015*(1155*c^6*d^6*x^6 + 1470*a*c^5*d^5*
e*x^5 + 35*a^2*c^4*d^4*e^2*x^4 - 40*a^3*c^3*d^3*e^3*x^3 + 48*a^4*c^2*d^2*e
^4*x^2 - 64*a^5*c*d*e^5*x + 128*a^6*e^6)*sqrt(c*d*x + a*e)*g^4/(c^5*d^5)
```

3.689.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2535 vs. 2(306) = 612.

Time = 0.38 (sec) , antiderivative size = 2535, normalized size of antiderivative = 7.54

$$\int \frac{(f+gx)^4\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="giac")
```

output
```
2/45045*(15015*a*f^4*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^
3)*a*e^2)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e
)/e + 1716*c*d*f^3*g*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e
 + a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(
-c*d^2*e + a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e
+ a*e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^
3 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e^
2 + 2574*a*f^2*g^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e +
 a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c
*d^2*e + a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e +
a*e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3
+ 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e -
858*c*d*f^2*g^2*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*
e^3)*a*c^3*d^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*
d^2*e + a*e^3)*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)/(c^4*d^4
*e^3) + (105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x
 + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^
2*e + a*e^3)^(7/2)*a*e^3 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))/(
c^4*d^4*e^7))*abs(e)/e^2 - 572*a*f*g^3*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8
 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^...
```

3.689.9 Mupad [B] (verification not implemented)

Time = 12.56 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.32

$$\int \frac{(f+gx)^4 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{4\,g^3\,x^5\,(7\,a\,e\,g+26\,c\,d\,f)}{143}+\right.}{}$$

input
```
int(((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^
(3/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*g^3*x^5*(7*a*e*g + 26*c
*d*f))/143 + (256*a^6*e^6*g^4 + 6006*a^2*c^4*d^4*e^2*f^4 - 6864*a^3*c^3*d^
3*e^3*f^3*g - 1664*a^5*c*d*e^5*f*g^3 + 4576*a^4*c^2*d^2*e^4*f^2*g^2)/(1501
5*c^5*d^5) + (x^2*(6006*c^6*d^6*f^4 + 96*a^4*c^2*d^2*e^4*g^4 - 624*a^3*c^3
*d^3*e^3*f*g^3 + 27456*a*c^5*d^5*e*f^3*g + 1716*a^2*c^4*d^4*e^2*f^2*g^2))/
(15015*c^5*d^5) + (x*(12012*a*c^5*d^5*e*f^4 - 128*a^5*c*d*e^5*g^4 + 3432*a
^2*c^4*d^4*e^2*f^3*g + 832*a^4*c^2*d^2*e^4*f*g^3 - 2288*a^3*c^3*d^3*e^3*f^
2*g^2))/(15015*c^5*d^5) + (2*c*d*g^4*x^6)/13 + (8*g*x^3*(429*c^3*d^3*f^3 -
 2*a^3*e^3*g^3 + 715*a*c^2*d^2*e*f^2*g + 13*a^2*c*d*e^2*f*g^2))/(3003*c^2*
d^2) + (2*g^2*x^4*(a^2*e^2*g^2 + 286*c^2*d^2*f^2 + 208*a*c*d*e*f*g))/(429*
c*d)))/(d + e*x)^(1/2)
```

3.689. $\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

3.690 $\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

3.690.1 Optimal result

Integrand size = 46, antiderivative size = 269

$$\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = -\frac{16(cdf-aeg)^2\left(2ae^2g-cd(7ef-5dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{1155c^4d^4e(d+ex)^{5/2}} + \frac{16g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{231c^3d^3e(d+ex)^{3/2}} + \frac{4(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33c^2d^2(d+ex)^{5/2}} + \frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}}$$

output
```
-16/1155*(-a*e*g+c*d*f)^2*(2*a*e^2*g-c*d*(-5*d*g+7*e*f))*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(5/2)/c^4/d^4/e/(e*x+d)^(5/2)+16/231*g*(-a*e*g+c*d*f)^2*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^3/d^3/e/(e*x+d)^(3/2)+4/33*(-a*e*
g+c*d*f)*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^2/d^2/(e*x+d)
^(5/2)+2/11*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/(e*x+d)^
(5/2)
```

3.690.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.51

$$\int \frac{(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2((ae+cdx)(d+ex))^{5/2}\left(-16a^3e^3g^3+8a^2cde^2g^2(11f+\right.}{}$$

input
```
Integrate[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
 e*x)^(3/2),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(1
1*f + 5*g*x) - 2*a*c^2*d^2*e*g*(99*f^2 + 110*f*g*x + 35*g^2*x^2) + c^3*d^3
*(231*f^3 + 495*f^2*g*x + 385*f*g^2*x^2 + 105*g^3*x^3)))/(1155*c^4*d^4*(d
+ e*x)^(5/2))
```

3.690.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1253, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \textbf{1253}$$

$$\frac{6(cdf-aeg)\int \frac{(f+gx)^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx}{11cd}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}}$$

$$\Big\downarrow \textbf{1253}$$

$$\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\int \frac{(f+gx)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx}{9cd}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}}\right)}{11cd}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}}$$

$$\Big\downarrow \textbf{1221}$$

$$\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\left(\frac{1}{7}\left(-\frac{2aeg}{cd}-\frac{5dg}{e}+7f\right)\int\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}\right)}{9cd}+\frac{2(f+gx)^2\left(x\left(ae^2+\right.\right.}{9cd(}\right.}{11cd}$$

$$\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}}+$$

$$\frac{6(cdf-aeg)\left(\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}}+\frac{4(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}\left(-\frac{2aeg}{cd}-\frac{5dg}{e}+7f\right)}{35cd(d+ex)^{5/2}}+\frac{2g\left(x\left(ae^2+cd^2\right)+a\right.}{7cde(d+ex}\right.}{9cd}\right.}{11cd}$$

input
```
Int[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^
(3/2),x]
```

output
```
(2*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(11*c*d*(d +
 e*x)^(5/2)) + (6*(c*d*f - a*e*g)*((2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(5/2))/(9*c*d*(d + e*x)^(5/2)) + (4*(c*d*f - a*e*g)*((2*(7
*f - (5*d*g)/e - (2*a*e*g)/(c*d))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(5/2))/(35*c*d*(d + e*x)^(5/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2))/(7*c*d*e*(d + e*x)^(3/2))))/(9*c*d)))/(11*c*d)
```

3.690.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

3.690. $\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

3.690.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.67

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^2\left(-105g^3x^3c^3d^3+70a\,c^2d^2e\,g^3x^2-385c^3d^3f\,g^2x^2-40a^2cd\,e^2g^3x+220a\,c^2d^2ef\,g^2x-495c^3d^3f^2gx-\right.}{1155\sqrt{ex+d}\,c^4d^4}$
gosper	$-\frac{2(cdx+ae)\left(-105g^3x^3c^3d^3+70a\,c^2d^2e\,g^3x^2-385c^3d^3f\,g^2x^2-40a^2cd\,e^2g^3x+220a\,c^2d^2ef\,g^2x-495c^3d^3f^2gx+16a^3e^3g^3-88a^2cd\,e\right.}{1155c^4d^4(ex+d)^{\frac{3}{2}}}$

input
```
int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,meth
od=_RETURNVERBOSE)
```

output
```
-2/1155*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^2*(-105*c^3*
d^3*g^3*x^3+70*a*c^2*d^2*e*g^3*x^2-385*c^3*d^3*f*g^2*x^2-40*a^2*c*d*e^2*g^
3*x+220*a*c^2*d^2*e*f*g^2*x-495*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-88*a^2*c*d*
e^2*f*g^2+198*a*c^2*d^2*e*f^2*g-231*c^3*d^3*f^3)/c^4/d^4
```

3.690.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.26

$$\int \frac{(f+gx)^3 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\,(105\,c^5d^5g^3x^5+231\,a^2c^3d^3e^2f^3-198\,a^3c^2d^2e^3f^2g+8$$

input
```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="fricas")
```

output
```
2/1155*(105*c^5*d^5*g^3*x^5 + 231*a^2*c^3*d^3*e^2*f^3 - 198*a^3*c^2*d^2*e^
3*f^2*g + 88*a^4*c*d*e^4*f*g^2 - 16*a^5*e^5*g^3 + 35*(11*c^5*d^5*f*g^2 + 4
*a*c^4*d^4*e*g^3)*x^4 + 5*(99*c^5*d^5*f^2*g + 110*a*c^4*d^4*e*f*g^2 + a^2*
c^3*d^3*e^2*g^3)*x^3 + 3*(77*c^5*d^5*f^3 + 264*a*c^4*d^4*e*f^2*g + 11*a^2*
c^3*d^3*e^2*f*g^2 - 2*a^3*c^2*d^2*e^3*g^3)*x^2 + (462*a*c^4*d^4*e*f^3 + 99
*a^2*c^3*d^3*e^2*f^2*g - 44*a^3*c^2*d^2*e^3*f*g^2 + 8*a^4*c*d*e^4*g^3)*x)*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c
^4*d^5)
```

3.690.6 Sympy [F]

$$\int \frac{(f+gx)^3 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \int \frac{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}\,(f+gx)^3}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**
(3/2),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*(f + g*x)**3/(d + e*x)**(3/2), x
)
```

3.690.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.09

$$\int \frac{(f+gx)^3 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\,(c^2d^2x^2+2\,acdex+a^2e^2)\sqrt{cdx+ae}\,f^3}{5\,cd} + \frac{6\,(5\,c^3d^3x^3+8\,ac^2d^2ex^2+a^2cde^2x-2\,a^3e^3)\sqrt{cdx+ae}\,f^2g}{35\,c^2d^2} + \frac{2\,(35\,c^4d^4x^4+50\,ac^3d^3ex^3+3\,a^2c^2d^2e^2x^2-4\,a^3cde^3x+8\,a^4e^4)\sqrt{cdx+ae}\,fg^2}{105\,c^3d^3} + \frac{2\,(105\,c^5d^5x^5+140\,ac^4d^4ex^4+5\,a^2c^3d^3e^2x^3-6\,a^3c^2d^2e^3x^2+8\,a^4cde^4x-16\,a^5e^5)\sqrt{cdx+ae}\,g^3}{1155\,c^4d^4}$$

input
```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="maxima")
```

output
```
2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f^3/(c*d) + 6/
35*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*
d*x + a*e)*f^2*g/(c^2*d^2) + 2/105*(35*c^4*d^4*x^4 + 50*a*c^3*d^3*e*x^3 +
3*a^2*c^2*d^2*e^2*x^2 - 4*a^3*c*d*e^3*x + 8*a^4*e^4)*sqrt(c*d*x + a*e)*f*g
^2/(c^3*d^3) + 2/1155*(105*c^5*d^5*x^5 + 140*a*c^4*d^4*e*x^4 + 5*a^2*c^3*d
^3*e^2*x^3 - 6*a^3*c^2*d^2*e^3*x^2 + 8*a^4*c*d*e^4*x - 16*a^5*e^5)*sqrt(c*
d*x + a*e)*g^3/(c^4*d^4)
```

3.690.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1778 vs. 2(245) = 490.

Time = 0.36 (sec) , antiderivative size = 1778, normalized size of antiderivative = 6.61

$$\int \frac{(f+gx)^3 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="giac")
```

output
```
2/3465*(1155*a*f^3*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)
*a*e^2)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e)/
e + 99*c*d*f^2*g*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a
*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d
^2*e + a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*
e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 +
15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e^2 +
99*a*f*g^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*
a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e +
 a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(
3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e
*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e - 33*c*d*f
*g^2*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*
d^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e
^3)*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)/(c^4*d^4*e^3) + (10
5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^2*e + a*e^3
)^(7/2)*a*e^3 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))/(c^4*d^4*e^7
))*abs(e)/e^2 - 11*a*g^3*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d
^2*e + a*e^3)*a*c^3*d^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 ...
```

3.690.9 Mupad [B] (verification not implemented)

Time = 12.43 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.15

$$\int \frac{(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^2\,x^4\,(4\,a\,e\,g+11\,c\,d\,f)}{33}-\right.}{}$$

input
```
int(((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^
(3/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^4*(4*a*e*g + 11*c
*d*f))/33 - (32*a^5*e^5*g^3 - 462*a^2*c^3*d^3*e^2*f^3 + 396*a^3*c^2*d^2*e^
3*f^2*g - 176*a^4*c*d*e^4*f*g^2)/(1155*c^4*d^4) + (x^2*(462*c^5*d^5*f^3 -
12*a^3*c^2*d^2*e^3*g^3 + 66*a^2*c^3*d^3*e^2*f*g^2 + 1584*a*c^4*d^4*e*f^2*g
))/(1155*c^4*d^4) + (2*c*d*g^3*x^5)/11 + (2*g*x^3*(a^2*e^2*g^2 + 99*c^2*d^
2*f^2 + 110*a*c*d*e*f*g))/(231*c*d) + (2*a*e*x*(8*a^3*e^3*g^3 + 462*c^3*d^
3*f^3 + 99*a*c^2*d^2*e*f^2*g - 44*a^2*c*d*e^2*f*g^2))/(1155*c^3*d^3)))/(d
+ e*x)^(1/2)
```

3.691 $\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

3.691.1 Optimal result

Integrand size = 46, antiderivative size = 200

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = -\frac{8(cdf-aeg)\left(2ae^2g-cd(7ef-5dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{315c^3d^3e(d+ex)^{5/2}} + \frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{63c^2d^2e(d+ex)^{3/2}} + \frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}}$$

output
```
-8/315*(-a*e*g+c*d*f)*(2*a*e^2*g-c*d*(-5*d*g+7*e*f))*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(5/2)/c^3/d^3/e/(e*x+d)^(5/2)+8/63*g*(-a*e*g+c*d*f)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^2/d^2/e/(e*x+d)^(3/2)+2/9*(g*x+f)^2*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/(e*x+d)^(5/2)
```

3.691.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.45

$$\int \frac{(f+gx)^2 \left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2((ae+cdx)(d+ex))^{5/2}\left(8a^2e^2g^2-4acdeg(9f+5gx)+\right.}{315c^3d^3(d+ex)^{5/2}}$$

input
```
Integrate[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
 e*x)^(3/2),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(9*f + 5*g
*x) + c^2*d^2*(63*f^2 + 90*f*g*x + 35*g^2*x^2)))/(315*c^3*d^3*(d + e*x)^(5
/2))
```

3.691.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \; 1253$$

$$\frac{4(cdf-aeg)\int \frac{(f+gx)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx}{9cd}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}}$$

$$\Big\downarrow \; 1221$$

$$\frac{4(cdf-aeg)\left(\frac{1}{7}\left(-\frac{2aeg}{cd}-\frac{5dg}{e}+7f\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}\right)}{9cd}+$$
$$\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}}$$

$$\Big\downarrow \; 1122$$

$$\frac{2(f+gx)^2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}}+\frac{4(cdf-aeg)\left(\frac{2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}\left(-\frac{2aeg}{cd}-\frac{5dg}{e}+7f\right)}{35cd(d+ex)^{5/2}}+\frac{2g\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}\right)}{9cd}$$

input
```
Int[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^
(3/2),x]
```

output
```
(2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9*c*d*(d +
e*x)^(5/2)) + (4*(c*d*f - a*e*g)*((2*(7*f - (5*d*g)/e - (2*a*e*g)/(c*d))*(
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(35*c*d*(d + e*x)^(5/2)) + (
2*g*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*c*d*e*(d + e*x)^(3/2
))))/(9*c*d)
```

3.691.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

3.691. $\int \frac{(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

3.691.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^{2}\left(35g^{2}x^{2}c^{2}d^{2}-20acde\,g^{2}x+90c^{2}d^{2}fgx+8a^{2}e^{2}g^{2}-36acdefg+63c^{2}d^{2}f^{2}\right)}{315\sqrt{ex+d}\,c^{3}d^{3}}$	108
gosper	$\frac{2(cdx+ae)\left(35g^{2}x^{2}c^{2}d^{2}-20acde\,g^{2}x+90c^{2}d^{2}fgx+8a^{2}e^{2}g^{2}-36acdefg+63c^{2}d^{2}f^{2}\right)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}}{315c^{3}d^{3}(ex+d)^{\frac{3}{2}}}$	116

input
```
int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/315*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^2*(35*c^2*d^2*
g^2*x^2-20*a*c*d*e*g^2*x+90*c^2*d^2*f*g*x+8*a^2*e^2*g^2-36*a*c*d*e*f*g+63*
c^2*d^2*f^2)/c^3/d^3
```

3.691.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.15

$$\int \frac{(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\left(35\,c^4d^4g^2x^4+63\,a^2c^2d^2e^2f^2-36\,a^3cde^3fg+8\,a^4e^4g^2\right.}{}$$

input
```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="fricas")
```

output
```
2/315*(35*c^4*d^4*g^2*x^4 + 63*a^2*c^2*d^2*e^2*f^2 - 36*a^3*c*d*e^3*f*g +
8*a^4*e^4*g^2 + 10*(9*c^4*d^4*f*g + 5*a*c^3*d^3*e*g^2)*x^3 + 3*(21*c^4*d^4
*f^2 + 48*a*c^3*d^3*e*f*g + a^2*c^2*d^2*e^2*g^2)*x^2 + 2*(63*a*c^3*d^3*e*f
^2 + 9*a^2*c^2*d^2*e^2*f*g - 2*a^3*c*d*e^3*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

3.691.6 Sympy [F]

$$\int \frac{(f+gx)^2 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \int \frac{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}(f+gx)^2}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**
(3/2),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*(f + g*x)**2/(d + e*x)**(3/2), x
)
```

3.691.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.96

$$\int \frac{(f+gx)^2 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\,(c^2d^2x^2+2\,acdex+a^2e^2)\sqrt{cdx+ae}\,f^2}{5\,cd} + \frac{4\,(5\,c^3d^3x^3+8\,ac^2d^2ex^2+a^2cde^2x-2\,a^3e^3)\sqrt{cdx+ae}\,fg}{35\,c^2d^2} + \frac{2\,(35\,c^4d^4x^4+50\,ac^3d^3ex^3+3\,a^2c^2d^2e^2x^2-4\,a^3cde^3x+8\,a^4e^4)\sqrt{cdx+ae}\,g^2}{315\,c^3d^3}$$

input
```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="maxima")
```

output
```
2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f^2/(c*d) + 4/
35*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*
d*x + a*e)*f*g/(c^2*d^2) + 2/315*(35*c^4*d^4*x^4 + 50*a*c^3*d^3*e*x^3 + 3*
a^2*c^2*d^2*e^2*x^2 - 4*a^3*c*d*e^3*x + 8*a^4*e^4)*sqrt(c*d*x + a*e)*g^2/(
c^3*d^3)
```

3.691.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1145 vs. 2(182) = 364.

Time = 0.33 (sec) , antiderivative size = 1145, normalized size of antiderivative = 5.72

$$\int \frac{(f+gx)^2 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="giac")
```

output
```
2/315*(105*a*f^2*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a
*e^2)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e)/e
+ 6*c*d*f*g*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)
*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e
+ a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^
(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((
e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e^2 + 3*a*g
^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^
4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*
a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2
*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d)*
c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e - c*d*g^2*((35*sqr
t(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2 - 6*s
qrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*c*d^2
*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)/(c^4*d^4*e^3) + (105*((e*x + d)*
c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x + d)*c*d*e - c*d^2*e +
a*e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3
 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))/(c^4*d^4*e^7))*abs(e)/e^2
 - 21*c*d*f^2*((3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*
a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x + ...
```

3.691.9 Mupad [B] (verification not implemented)

Time = 12.13 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.03

$$\int \frac{(f+gx)^2 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\,\left(\frac{4\,g\,x^3\,(5\,a\,e\,g+9\,c\,d\,f)}{63}+\right.}{}$$

input
```
int(((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^
(3/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*g*x^3*(5*a*e*g + 9*c*d*
f))/63 + (16*a^4*e^4*g^2 + 126*a^2*c^2*d^2*e^2*f^2 - 72*a^3*c*d*e^3*f*g)/(
315*c^3*d^3) + (x^2*(126*c^4*d^4*f^2 + 6*a^2*c^2*d^2*e^2*g^2 + 288*a*c^3*d
^3*e*f*g))/(315*c^3*d^3) + (2*c*d*g^2*x^4)/9 + (4*a*e*x*(63*c^2*d^2*f^2 -
2*a^2*e^2*g^2 + 9*a*c*d*e*f*g))/(315*c^2*d^2)))/(d + e*x)^(1/2)
```

3.691. $\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

3.692 $\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

3.692.1 Optimal result

Integrand size = 44, antiderivative size = 125

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = -\frac{2(2ae^2g-cd(7ef-5dg))\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{35c^2d^2e(d+ex)^{5/2}} + \frac{2g\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}$$

output
```
-2/35*(2*a*e^2*g-c*d*(-5*d*g+7*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2)/c^2/d^2/e/(e*x+d)^(5/2)+2/7*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c
/d/e/(e*x+d)^(3/2)
```

3.692.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.43

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2((ae+cdx)(d+ex))^{5/2}(-2aeg+cd(7f+5gx))}{35c^2d^2(d+ex)^{5/2}}$$

input
```
Integrate[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e
*x)^(3/2),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-2*a*e*g + c*d*(7*f + 5*g*x)))/(35*c^2
*d^2*(d + e*x)^(5/2))
```

3.692.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{1}{7}\left(-\frac{2aeg}{cd}-\frac{5dg}{e}+7f\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}\left(-\frac{2aeg}{cd}-\frac{5dg}{e}+7f\right)}{35cd(d+ex)^{5/2}}+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}$$

input
```
Int[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(3
/2),x]
```

output
```
(2*(7*f - (5*d*g)/e - (2*a*e*g)/(c*d))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2))/(35*c*d*(d + e*x)^(5/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(5/2))/(7*c*d*e*(d + e*x)^(3/2))
```

3.692.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

3.692.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^{2}(-5cdgx+2aeg-7cdf)}{35\sqrt{ex+d}\,c^{2}d^{2}}$	59
gosper	$-\frac{2(cdx+ae)(-5cdgx+2aeg-7cdf)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}}{35c^{2}d^{2}(ex+d)^{\frac{3}{2}}}$	67

input
```
int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,method
=_RETURNVERBOSE)
```

output
```
-2/35*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^2*(-5*c*d*g*x+
2*a*e*g-7*c*d*f)/c^2/d^2
```

3.692.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\,(5\,c^3d^3gx^3+7\,a^2cde^2f-2\,a^3e^3g+(7\,c^3d^3f+8\,ac^2d^2eg}{35}$$

input
```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
 algorithm="fricas")
```

output
```
2/35*(5*c^3*d^3*g*x^3 + 7*a^2*c*d*e^2*f - 2*a^3*e^3*g + (7*c^3*d^3*f + 8*a
*c^2*d^2*e*g)*x^2 + (14*a*c^2*d^2*e*f + a^2*c*d*e^2*g)*x)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

3.692.6 Sympy [F]

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \int \frac{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}(f+gx)}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/
2),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*(f + g*x)/(d + e*x)**(3/2), x)
```

3.692.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\,(c^2d^2x^2+2\,acdex+a^2e^2)\sqrt{cdx+ae}f}{5\,cd} + \frac{2\,(5\,c^3d^3x^3+8\,ac^2d^2ex^2+a^2cde^2x-2\,a^3e^3)\sqrt{cdx+ae}g}{35\,c^2d^2}$$

input
```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
 algorithm="maxima")
```

output
```
2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f/(c*d) + 2/35
*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*d*
x + a*e)*g/(c^2*d^2)
```

3.692.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(113) = 226$.

Time = 0.30 (sec) , antiderivative size = 632, normalized size of antiderivative = 5.06

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\left(\frac{35\,af\left(\frac{\sqrt{-cd^2e+ae^3}cd^2-\sqrt{-cd^2e+ae^3}ae^2}{cd}+\frac{\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}}{cde}\right)|e|}{e}\right.}{}$$

input
```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
 algorithm="giac")
```

output
```
2/105*(35*a*f*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^
2)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e)/e + c
*d*g*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*
d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3
)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a
^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d
)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e^2 - 7*c*d*f*((3*
sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sq
rt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x + d)*c*d*e - c*d^2*e +
a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))/(c^2*d^2
*e^2))*abs(e)/e^3 - 7*a*g*((3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2
*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5
*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*
d^2*e + a*e^3)^(5/2))/(c^2*d^2*e^2))*abs(e)/e^2)/e
```

3.692.9 Mupad [B] (verification not implemented)

Time = 11.95 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.87

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(x^2\,\left(\frac{16\,a\,e\,g}{35}+\frac{2\,c\,d\,f}{5}\right)-\right.}{\sqrt{d+e\,x}}$$

input
```
int(((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(3
/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(x^2*((16*a*e*g)/35 + (2*c*
d*f)/5) - (4*a^3*e^3*g - 14*a^2*c*d*e^2*f)/(35*c^2*d^2) + (2*c*d*g*x^3)/7
+ (2*a*e*x*(a*e*g + 14*c*d*f))/(35*c*d)))/(d + e*x)^(1/2)
```

3.693 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

3.693.1 Optimal result

Integrand size = 39, antiderivative size = 48

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5cd(d+ex)^{5/2}}$$

output
```
2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/(e*x+d)^(5/2)
```

3.693.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{5/2}}{5cd(d+ex)^{5/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2))/(5*c*d*(d + e*x)^(5/2))
```

3.693.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$$
$$\Big\downarrow \; 1122$$
$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5cd(d+ex)^{5/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*c*d*(d + e*x)^(5/2))
```

3.693.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

3.693.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^2}{5\sqrt{ex+d}\,cd}$	42
gosper	$\frac{2(cdx+ae)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}{5cd(ex+d)^{\frac{3}{2}}}$	50

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
2/5*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^2/c/d
```

3.693.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.54

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\left(c^2d^2x^2+2\,acdex+a^2e^2\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{5\left(cdex+cd^2\right)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorit
hm="fricas")
```

output
```
2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d^2)
```

3.693.6 Sympy [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x)**(3/2), x)
```

3.693.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\left(c^2d^2x^2+2\,acdex+a^2e^2\right)\sqrt{cdx+ae}}{5\,cd}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorit
hm="maxima")
```

output
```
2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)/(c*d)
```

3.693.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(42) = 84$.

Time = 0.29 (sec) , antiderivative size = 254, normalized size of antiderivative = 5.29

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\left(\frac{5a\left(\frac{\sqrt{-cd^2e+ae^3}cd^2-\sqrt{-cd^2e+ae^3}ae^2}{cd}+\frac{\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}}{cde}\right)|e|}{e}-\frac{cd\left(\frac{3\sqrt{-cd}}{}\right.}{}\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorit
hm="giac")
```

output
```
2/15*(5*a*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^2)/(
c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e)/e - c*d*(
(3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2
*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))/(c^2*
d^2*e^2))*abs(e)/e^3)/e
```

3.693.9 Mupad [B] (verification not implemented)

Time = 12.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{\left(\frac{4aex}{5}+\frac{2cdx^2}{5}+\frac{2a^2e^2}{5cd}\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x)
```

output
```
(((4*a*e*x)/5 + (2*c*d*x^2)/5 + (2*a^2*e^2)/(5*c*d))*(x*(a*e^2 + c*d^2) +
a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(1/2)
```

3.694 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx$

3.694.1 Optimal result

Integrand size = 46, antiderivative size = 179

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx=-\frac{2(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}+\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}+\frac{2(cdf-aeg)^{3/2}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{5/2}}$$

output
```
2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)+2*(-a*e*g+c*d*
f)^(3/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*
d*f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)-2*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)
```

3.694.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.74

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx=\frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}(4aeg+cd(-3f+gx))+3(c\right.}{3g^{5/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)),x]
```

output
```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(4*a*e*g + c
*d*(-3*f + g*x)) + 3*(c*d*f - a*e*g)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*
x])/Sqrt[c*d*f - a*e*g]]))/(3*g^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.694.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1250, 1250, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}dx$$

$$\Big\downarrow \text{1250}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}(f+gx)}dx}{g}$$

$$\Big\downarrow \text{1250}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}\right)}{g}$$

$$\Big\downarrow \text{1255}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2e^2(cdf-aeg)\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{g}\right)}{g}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}}\right)}{g}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e*x)^(3/2)) -
((c*d*f - a*e*g)*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[
d + e*x]) - (2*Sqrt[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(3/2)))/g
```

3.694.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.694.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.41

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,\left(3\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a^2e^2g^2-6\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)acdefg+3\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2f^2-\sqrt{}\right.}{3\sqrt{ex+d}\,\sqrt{cdx+ae}\,g^2\sqrt{(aeg-cdf)g}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,method
=_RETURNVERBOSE)
```

output
```
-2/3*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*a^2*e^2*g^2-6*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*a*c*d*e*f*g+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))
*c^2*d^2*f^2-(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c*d*g*x-4*(c*d*x+a*
e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*e*g+3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*
g)^(1/2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^2/((a*e*g-c*d*f)*g)^(1/2
)
```

3.694.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.28

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx = \left[-\frac{3\left(cd^2f-adeg+\left(cdef-ae^2g\right)x\right)\sqrt{-\frac{cdf-aeg}{g}}\,\log\left(-\frac{cdegx^2-cd^2f}{}\right.}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,
 algorithm="fricas")
```

output
```
[-1/3*(3*(c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(-(c*d*f - a*e*g)
/g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c
*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - 3*c*d*f + 4*a*e*g)*sqrt(e*x + d))/
(e*g^2*x + d*g^2), -2/3*(3*(c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqr
t((c*d*f - a*e*g)/g)*arctan(sqrt(e*x + d)*sqrt((c*d*f - a*e*g)/g)/sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) - sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*(c*d*g*x - 3*c*d*f + 4*a*e*g)*sqrt(e*x + d))/(e*g^2*x + d*g^2)]
```

3.694.6 Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)}\,dx = \int \frac{((d + ex)(ae + cdx))^{\frac{3}{2}}}{(d + ex)^{\frac{3}{2}}(f + gx)}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/((d + e*x)**(3/2)*(f + g*x)), x)
```

3.694.7 Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}(gx + f)}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,
 algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)), x)
```

3.694.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(157) = 314$.

Time = 0.47 (sec) , antiderivative size = 506, normalized size of antiderivative = 2.83

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx = \frac{2\,(c^2d^2f^2|e| - 2\,acdefg|e| + a^2e^2g^2|e|)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{\sqrt{cdfg-aeg^2}eg^2} - \frac{2\left(3\,c^2d^2e^2f^2|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right) - 6\,acde^3fg|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right) + 3\,a^2e^4g^2|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right) \cdots\right.}{3 \cdots} - \frac{2\left(3\sqrt{(ex+d)cde-cd^2e+ae^3}cde^{10}fg|e| - 3\sqrt{(ex+d)cde-cd^2e+ae^3}ae^{11}g^2|e| - ((ex+d)cde-cd^2 \cdots\right.}{3\,e^{12}g^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,
 algorithm="giac")
```

output
```
2*(c^2*d^2*f^2*abs(e) - 2*a*c*d*e*f*g*abs(e) + a^2*e^2*g^2*abs(e))*arctan(
sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/(sq
rt(c*d*f*g - a*e*g^2)*e*g^2) - 2/3*(3*c^2*d^2*e^2*f^2*abs(e)*arctan(sqrt(-
c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 6*a*c*d*e^3*f*g*abs(e)*a
rctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 3*a^2*e^4*g^
2*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 3*
sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c*d*e*f*abs(e) - sqrt(-c*d^
2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c*d^2*g*abs(e) + 4*sqrt(-c*d^2*e + a*
e^3)*sqrt(c*d*f*g - a*e*g^2)*a*e^2*g*abs(e))/(sqrt(c*d*f*g - a*e*g^2)*e^3*
g^2) - 2/3*(3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*e^10*f*g*abs(e)
- 3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*e^11*g^2*abs(e) - ((e*x + d)
*c*d*e - c*d^2*e + a*e^3)^(3/2)*e^8*g^2*abs(e))/(e^12*g^3)
```

3.694.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx = \int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{\left(f+g\,x\right)\,\left(d+e\,x\right)^{3/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)*(d + e*x)^(3/
2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)*(d + e*x)^(3/
2)), x)
```

3.694. $\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx$

3.695 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx$

3.695.1 Optimal result

Integrand size = 46, antiderivative size = 178

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx=\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}-\frac{3cd\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{5/2}}$$

output
```
-(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)-3*c*d*arc
tan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(
e*x+d)^(1/2))*(-a*e*g+c*d*f)^(1/2)/g^(5/2)+3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)
```

3.695.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.81

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx=\frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}(-aeg+cd(3f+2gx))-3cd\sqrt{}\right.}{g^{5/2}\sqrt{(ae+cdx)(d+ex)}(f+}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)^2),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(-(a*e*g) + c*
d*(3*f + 2*g*x)) - 3*c*d*Sqrt[c*d*f - a*e*g]*(f + g*x)*ArcTan[(Sqrt[g]*Sqr
t[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/(g^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*
x)]*(f + g*x))
```

3.695.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1249, 1250, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{3cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}(f+gx)}dx}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}$$

$$\Big\downarrow \text{1250}$$

$$\frac{3cd\left(\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}$$

$$\Big\downarrow \text{1255}$$

$$\frac{3cd\left(\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2e^2(cdf-aeg)\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{g}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}$$

$$\Big\downarrow \text{218}$$

$$\frac{3cd\left(\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)^2),x]
```

output
```
-((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(d + e*x)^(3/2)*(f + g*
x))) + (3*c*d*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d +
 e*x]) - (2*Sqrt[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(3/2)))/(2*g)
```

3.695.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

3.695. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx$

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.695.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.66

method	result
default	$\frac{\left(-3\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)acde\,g^2x+3\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2fgx-3\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)acdefg+3\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+}}{\sqrt{(aeg-c}}\right.\right.}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,g^2(gx}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x,meth
od=_RETURNVERBOSE)
```

output
```
(-3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c*d*e*g^2*x+3*a
rctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^2*f*g*x-3*arctan
h(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c*d*e*f*g+3*arctanh(g*(c*
d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^2*f^2+2*(c*d*x+a*e)^(1/2)*((
a*e*g-c*d*f)*g)^(1/2)*c*d*g*x-(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*
e*g+3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c*d*f)*((c*d*x+a*e)*(e*x+d
))^(1/2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^2/(g*x+f)/((a*e*g-c*d*f)*g)^(1/
2)
```

3.695.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.49

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx = \left[\frac{3\,(cdegx^2+cd^2f+(cdef+cd^2g)x)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2-cd^2f}{}\right.}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,
x, algorithm="fricas")
```

output
```
[1/2*(3*(c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(-(c*d*f - a*e
*g)/g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f -
 (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + 3*c*d*f - a*e*g)*sqrt(e*x + d
))/(e*g^3*x^2 + d*f*g^2 + (e*f*g^2 + d*g^3)*x), (3*(c*d*e*g*x^2 + c*d^2*f
+ (c*d*e*f + c*d^2*g)*x)*sqrt((c*d*f - a*e*g)/g)*arctan(sqrt(e*x + d)*sqrt
((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) + sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + 3*c*d*f - a*e*g)*sqrt(e*
x + d))/(e*g^3*x^2 + d*f*g^2 + (e*f*g^2 + d*g^3)*x)]
```

3.695.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**2,x)
```

output
```
Timed out
```

3.695.7 Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^2}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}(gx + f)^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)^2), x)
```

3.695.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(158) = 316$.

Time = 0.42 (sec) , antiderivative size = 605, normalized size of antiderivative = 3.40

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx = \frac{3\,c^2d^2e^2f^2|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)-3\,c^2d^3efg|e|\arctan\left(\frac{\sqrt{-cd^2e+a}}{\sqrt{cdfg-ae}}\right.}{}$$

$$+\frac{2\sqrt{(ex+d)cde-cd^2e+ae^3}cd|e|}{e^2g^2}-\frac{3\left(c^2d^2f|e|-acdeg|e|\right)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{\sqrt{cdfg-aeg^2}eg^2}$$

$$+\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}c^2d^2f|e|-\sqrt{(ex+d)cde-cd^2e+ae^3}acdeg|e|}{(cde^2f-ae^3g+((ex+d)cde-cd^2e+ae^3)g)g^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,
x, algorithm="giac")
```

output
```
(3*c^2*d^2*e^2*f^2*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g -
a*e*g^2)*e)) - 3*c^2*d^3*e*f*g*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqr
t(c*d*f*g - a*e*g^2)*e)) - 3*a*c*d*e^3*f*g*abs(e)*arctan(sqrt(-c*d^2*e + a
*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 3*a*c*d^2*e^2*g^2*abs(e)*arctan(sqr
t(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 3*sqrt(-c*d^2*e + a*e
^3)*sqrt(c*d*f*g - a*e*g^2)*c*d*e*f*abs(e) + 2*sqrt(-c*d^2*e + a*e^3)*sqrt
(c*d*f*g - a*e*g^2)*c*d^2*g*abs(e) + sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g -
 a*e*g^2)*a*e^2*g*abs(e))/(sqrt(c*d*f*g - a*e*g^2)*e^3*f*g^2 - sqrt(c*d*f*
g - a*e*g^2)*d*e^2*g^3) + 2*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*ab
s(e)/(e^2*g^2) - 3*(c^2*d^2*f*abs(e) - a*c*d*e*g*abs(e))*arctan(sqrt((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/(sqrt(c*d*f*g
 - a*e*g^2)*e*g^2) + (sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^2*f*ab
s(e) - sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c*d*e*g*abs(e))/((c*d*e^2
*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)*g^2)
```

3.695.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx = \int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{\left(f+g\,x\right)^2\,\left(d+e\,x\right)^{3/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^2*(d + e*x)^(
3/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^2*(d + e*x)^(
3/2)), x)
```

3.695. $\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx$

3.696 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx$

3.696.1 Optimal result

Integrand size = 46, antiderivative size = 195

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx=-\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}+\frac{3c^2d^2\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4g^{5/2}\sqrt{cdf-aeg}}$$

output
```
-1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^2+3/4
*c^2*d^2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*
d*f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)/(-a*e*g+c*d*f)^(1/2)-3/4*c*d*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(g*x+f)/(e*x+d)^(1/2)
```

3.696.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.69

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx=\frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{g}(2aeg+cd(3f+5gx))}{(f+gx)^2}+\frac{3c^2d^2\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{cdf-aeg}\sqrt{ae+cdx}}\right.}{4g^{5/2}\sqrt{d+ex}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)^3),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[g]*(2*a*e*g + c*d*(3*f + 5*g*x)))/
(f + g*x)^2) + (3*c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f -
a*e*g]])/(Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c*d*x])))/(4*g^(5/2)*Sqrt[d + e*x
])
```

3.696.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1249, 1249, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}dx$$

$$\Big\downarrow \text{ 1249}$$

$$\frac{3cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}(f+gx)^2}dx}{4g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}$$

$$\Big\downarrow \text{ 1249}$$

$$\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}\right)}{4g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}$$

$$\Big\downarrow \text{ 1255}$$

$$\frac{3cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}\right)}{4g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}\right)}{4g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)^3),x]
```

output
```
-1/2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(d + e*x)^(3/2)*(f +
 g*x)^2) + (3*c*d*(-(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[d
 + e*x]*(f + g*x))) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(g^(3/2)*Sqrt[c*d*f -
a*e*g])))/(4*g)
```

3.696.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.696.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.36

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\,\left(3\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2g^2x^2+6\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2fgx+3\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2f^2+\right.}{4\sqrt{ex+d}\,\sqrt{cdx+ae}\,g^2(gx+f)^2\sqrt{(aeg-c}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x,meth
od=_RETURNVERBOSE)
```

output
```
-1/4*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^2*d^2*g^2*x^2+6*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^2*d^2*f*g*x+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^
(1/2))*c^2*d^2*f^2+5*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c*d*g*x+2*(
c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*e*g+3*(c*d*x+a*e)^(1/2)*((a*e*g
-c*d*f)*g)^(1/2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^2/(g*x+f)^2/((a*
e*g-c*d*f)*g)^(1/2)
```

3.696.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(169) = 338.

Time = 0.34 (sec) , antiderivative size = 840, normalized size of antiderivative = 4.31

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx = \left[-\frac{3\,(c^2d^2eg^2x^3+c^2d^3f^2+(2\,c^2d^2efg+c^2d^3g^2)x^2+(c^2d^2ef^2+2\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,
x, algorithm="fricas")
```

output
```
[-1/8*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2
)*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(
c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(
e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(3*c^2*d^2*f^2*g - a*c*d*e*
f*g^2 - 2*a^2*e^2*g^3 + 5*(c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^2*f^3*g^3 - a*d*e*f^2*g^4
 + (c*d*e*f*g^5 - a*e^2*g^6)*x^3 + (2*c*d*e*f^2*g^4 - a*d*e*g^6 + (c*d^2 -
 2*a*e^2)*f*g^5)*x^2 + (c*d*e*f^3*g^3 - 2*a*d*e*f*g^5 + (2*c*d^2 - a*e^2)*
f^2*g^4)*x), -1/4*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g +
 c^2*d^3*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(c*d*f*g - a*e*
g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e
*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (3*c^
2*d^2*f^2*g - a*c*d*e*f*g^2 - 2*a^2*e^2*g^3 + 5*(c^2*d^2*f*g^2 - a*c*d*e*g
^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^2*f
^3*g^3 - a*d*e*f^2*g^4 + (c*d*e*f*g^5 - a*e^2*g^6)*x^3 + (2*c*d*e*f^2*g^4
- a*d*e*g^6 + (c*d^2 - 2*a*e^2)*f*g^5)*x^2 + (c*d*e*f^3*g^3 - 2*a*d*e*f*g^
5 + (2*c*d^2 - a*e^2)*f^2*g^4)*x)]
```

3.696.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**3,x)
```

output
```
Timed out
```

3.696.7 Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^3}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)^3), x)
```

3.696.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(169) = 338$.

Time = 0.48 (sec) , antiderivative size = 581, normalized size of antiderivative = 2.98

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx = \frac{3\,c^2d^2|e|\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{4\,\sqrt{cdfg-aeg^2}eg^2} - \frac{3\,c^2d^2e^2f^2|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right) - 6\,c^2d^3efg|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right) + 3\,c^2d^4g^2|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{4\left(\sqrt{cdfg-aeg^2}e^3f^2g^2 - 2\,\sqrt{}\right.} - \frac{3\,\sqrt{(ex+d)cde-cd^2e+ae^3}c^3d^3e^2f|e| - 3\,\sqrt{(ex+d)cde-cd^2e+ae^3}ac^2d^2e^3g|e| + 5\left((ex+d)cde-cd^2\right.}{4\left(cde^2f-ae^3g+((ex+d)cde-cd^2e+ae^3)g\right)^2g^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,
x, algorithm="giac")
```

output
```
3/4*c^2*d^2*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(
c*d*f*g - a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*e*g^2) - 1/4*(3*c^2*d^2*e^
2*f^2*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))
- 6*c^2*d^3*e*f*g*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a
*e*g^2)*e)) + 3*c^2*d^4*g^2*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c
*d*f*g - a*e*g^2)*e)) - 3*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c
*d*e*f*abs(e) + 5*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c*d^2*g*a
bs(e) - 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*e^2*g*abs(e))/(
sqrt(c*d*f*g - a*e*g^2)*e^3*f^2*g^2 - 2*sqrt(c*d*f*g - a*e*g^2)*d*e^2*f*g^
3 + sqrt(c*d*f*g - a*e*g^2)*d^2*e*g^4) - 1/4*(3*sqrt((e*x + d)*c*d*e - c*d
^2*e + a*e^3)*c^3*d^3*e^2*f*abs(e) - 3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*
e^3)*a*c^2*d^2*e^3*g*abs(e) + 5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*
c^2*d^2*g*abs(e))/((c*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e
^3)*g)^2*g^2)
```

3.696.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{{\left(f+g\,x\right)}^3\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^3*(d + e*x)^(
3/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^3*(d + e*x)^(
3/2)), x)
```

3.697 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx$

3.697.1 Optimal result

Integrand size = 46, antiderivative size = 265

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx = -\frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)^2}+\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g^2(cdf-aeg)\sqrt{d+ex}(f+gx)} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}+\frac{c^3d^3\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{8g^{5/2}(cdf-aeg)^{3/2}}$$

output
```
-1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^3+1/8
*c^3*d^3*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*
d*f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)/(-a*e*g+c*d*f)^(3/2)-1/4*c*d*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(g*x+f)^2/(e*x+d)^(1/2)+1/8*c^2*d^2*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d*f)/(g*x+f)/(e*x+d)^(1/2
)
```

3.697.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.76

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx=\frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{cdf-aeg}\sqrt{ae+cdx}(8a^2e^2g^2-2acdeg(\right.}{24g^{5/2}(cdf-aeg)^3}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)^4),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c
*d*x]*(8*a^2*e^2*g^2 - 2*a*c*d*e*g*(f - 7*g*x) + c^2*d^2*(-3*f^2 - 8*f*g*x
 + 3*g^2*x^2)) + 3*c^3*d^3*(f + g*x)^3*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/
Sqrt[c*d*f - a*e*g]]))/(24*g^(5/2)*(c*d*f - a*e*g)^(3/2)*Sqrt[(a*e + c*d*x
)*(d + e*x)]*(f + g*x)^3)
```

3.697.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {1249, 1249, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}(f+gx)^3}dx}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}$$

$$\Big\downarrow \text{1249}$$

$$\frac{cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}$$

$$\Big\downarrow \text{1254}$$

$$\frac{cd\left(\frac{cd\left(\frac{cd\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}$$

$$\Big\downarrow \; 1255$$

$$\frac{cd\left(\frac{cd\left(\frac{cde^2\int\frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}$$

$$\Big\downarrow \; 218$$

$$\frac{cd\left(\frac{cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)^4),x]
```

3.697. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx$

output
```
-1/3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(d + e*x)^(3/2)*(f +
 g*x)^3) + (c*d*(-1/2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[
d + e*x]*(f + g*x)^2) + (c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/
((c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/
(Sqrt[g]*(c*d*f - a*e*g)^(3/2))))/(4*g)))/(2*g)
```

3.697.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.697.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.67

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(3\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3g^3x^3+9\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3fg^2x^2+9\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3f^2g\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x,meth
od=_RETURNVERBOSE)
```

output
```
1/24*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^3*d^3*g^3*x^3+9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^3*d^3*f*g^2*x^2+9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*c^3*d^3*f^2*g*x+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*c^3*d^3*f^3-3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*g^
2*x^2-14*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*g^2*x+8*(c*d*x+
a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x-8*(c*d*x+a*e)^(1/2)*((a*e
*g-c*d*f)*g)^(1/2)*a^2*e^2*g^2+2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)
*a*c*d*e*f*g+3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f^2)/(e*x
+d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)/g^2/(g*x+f)^3/((a*e*g-c*d*f)*g)^
(1/2)
```

3.697.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 696 vs. $2(233) = 466$.

Time = 0.44 (sec) , antiderivative size = 1434, normalized size of antiderivative = 5.41

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,
x, algorithm="fricas")
```

output
```
[1/48*(3*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g
^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3
*d^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*
d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (
e*f + d*g)*x)) - 2*(3*c^3*d^3*f^3*g - a*c^2*d^2*e*f^2*g^2 - 10*a^2*c*d*e^2
*f*g^3 + 8*a^3*e^3*g^4 - 3*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 2*(4*c^
3*d^3*f^2*g^2 - 11*a*c^2*d^2*e*f*g^3 + 7*a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^5*g^3 - 2*a*c*d^2
*e*f^4*g^4 + a^2*d*e^2*f^3*g^5 + (c^2*d^2*e*f^2*g^6 - 2*a*c*d*e^2*f*g^7 +
a^2*e^3*g^8)*x^4 + (3*c^2*d^2*e*f^3*g^5 + a^2*d*e^2*g^8 + (c^2*d^3 - 6*a*c
*d*e^2)*f^2*g^6 - (2*a*c*d^2*e - 3*a^2*e^3)*f*g^7)*x^3 + 3*(c^2*d^2*e*f^4*
g^4 + a^2*d*e^2*f*g^7 + (c^2*d^3 - 2*a*c*d*e^2)*f^3*g^5 - (2*a*c*d^2*e - a
^2*e^3)*f^2*g^6)*x^2 + (c^2*d^2*e*f^5*g^3 + 3*a^2*d*e^2*f^2*g^6 + (3*c^2*d
^3 - 2*a*c*d*e^2)*f^4*g^4 - (6*a*c*d^2*e - a^2*e^3)*f^3*g^5)*x), -1/24*(3*
(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g^3)*x^3 +
 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3*d^4*f^2*
g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d
^2 + a*e^2)*g*x)) + (3*c^3*d^3*f^3*g - a*c^2*d^2*e*f^2*g^2 - 10*a^2*c*d...
```

3.697.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^4} dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**4,x)
```

output
```
Timed out
```

3.697.7 Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^4}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}(gx + f)^4}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)^4), x)
```

3.697.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1087 vs. 2(233) = 466.

Time = 0.69 (sec) , antiderivative size = 1087, normalized size of antiderivative = 4.10

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^4}\,dx = \frac{c^3d^3|e|\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2e}}\right)}{8\,(cdfg^2 - aeg^3)\sqrt{cdfg - aeg^2e}}$$
$$- \frac{3\,c^3d^3e^3f^3|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2e}}\right) - 9\,c^3d^4e^2f^2g|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2e}}\right) + 9\,c^3d^5efg^2|e|\arctan\left(\frac{\sqrt{-cd^2e+a}}{\sqrt{cdfg-ae}}\right.}{24\,(\sqrt{cdfg}}$$
$$- \frac{3\,\sqrt{(ex + d)cde - cd^2e + ae^3}c^5d^5e^4f^2|e| - 6\,\sqrt{(ex + d)cde - cd^2e + ae^3}ac^4d^4e^5fg|e| + 3\,\sqrt{(ex + d)cde -}}{24\,(cde}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,
x, algorithm="giac")
```

output
```
1/8*c^3*d^3*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(
c*d*f*g - a*e*g^2)*e))/((c*d*f*g^2 - a*e*g^3)*sqrt(c*d*f*g - a*e*g^2)*e) -
 1/24*(3*c^3*d^3*e^3*f^3*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*
f*g - a*e*g^2)*e)) - 9*c^3*d^4*e^2*f^2*g*abs(e)*arctan(sqrt(-c*d^2*e + a*e
^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 9*c^3*d^5*e*f*g^2*abs(e)*arctan(sqrt(
-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 3*c^3*d^6*g^3*abs(e)*ar
ctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 3*sqrt(-c*d^2
*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d^2*e^2*f^2*abs(e) + 8*sqrt(-c*d^2
*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d^3*e*f*g*abs(e) - 2*sqrt(-c*d^2*e
 + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c*d*e^3*f*g*abs(e) + 3*sqrt(-c*d^2*e +
 a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d^4*g^2*abs(e) - 14*sqrt(-c*d^2*e + a*
e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c*d^2*e^2*g^2*abs(e) + 8*sqrt(-c*d^2*e + a*
e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*e^4*g^2*abs(e))/(sqrt(c*d*f*g - a*e*g^2)*
c*d*e^4*f^4*g^2 - 3*sqrt(c*d*f*g - a*e*g^2)*c*d^2*e^3*f^3*g^3 - sqrt(c*d*f
*g - a*e*g^2)*a*e^5*f^3*g^3 + 3*sqrt(c*d*f*g - a*e*g^2)*c*d^3*e^2*f^2*g^4
+ 3*sqrt(c*d*f*g - a*e*g^2)*a*d*e^4*f^2*g^4 - sqrt(c*d*f*g - a*e*g^2)*c*d^
4*e*f*g^5 - 3*sqrt(c*d*f*g - a*e*g^2)*a*d^2*e^3*f*g^5 + sqrt(c*d*f*g - a*e
*g^2)*a*d^3*e^2*g^6) - 1/24*(3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^5
*d^5*e^4*f^2*abs(e) - 6*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^4*d^4*
e^5*f*g*abs(e) + 3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^3*d^3*...
```

3.697.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^4}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\left(f + g\,x\right)^4\,\left(d + e\,x\right)^{3/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^4*(d + e*x)^(
3/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^4*(d + e*x)^(
3/2)), x)
```

3.698 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx$

3.698.1 Optimal result

Integrand size = 46, antiderivative size = 335

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx=-\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}(f+gx)^3}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+\frac{3c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^2(cdf-aeg)^2\sqrt{d+ex}(f+gx)}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}+\frac{3c^4d^4\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{64g^{5/2}(cdf-aeg)^{5/2}}$$

output
```
-1/4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^4+3/6
4*c^4*d^4*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c
*d*f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)/(-a*e*g+c*d*f)^(5/2)-1/8*c*d*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(g*x+f)^3/(e*x+d)^(1/2)+1/32*c^2*d^2*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d*f)/(g*x+f)^2/(e*x+d)^
(1/2)+3/64*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d
*f)^2/(g*x+f)/(e*x+d)^(1/2)
```

3.698.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.72

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx = \frac{c^4d^4((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{g}(-16a^3e^3g^3+24a^2cde^2g^2(f-gx)-2ac^2d^2eg(\ldots}{c^4d^4(cdf-a\ldots}\right.}{64g^{5/2}(\ldots}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)^5),x]
```

output
```
(c^4*d^4*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[g]*(-16*a^3*e^3*g^3 + 24*a
^2*c*d*e^2*g^2*(f - g*x) - 2*a*c^2*d^2*e*g*(f^2 - 22*f*g*x + g^2*x^2) + c^
3*d^3*(-3*f^3 - 11*f^2*g*x + 11*f*g^2*x^2 + 3*g^3*x^3)))/(c^4*d^4*(c*d*f -
 a*e*g)^2*(a*e + c*d*x)*(f + g*x)^4) + (3*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x
])/Sqrt[c*d*f - a*e*g]])/((c*d*f - a*e*g)^(5/2)*(a*e + c*d*x)^(3/2))))/(64
*g^(5/2)*(d + e*x)^(3/2))
```

3.698.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1249, 1249, 1254, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}dx$$

$$\Big\downarrow \text{ 1249}$$

$$\frac{3cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}(f+gx)^4}dx}{8g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}$$

$$\Big\downarrow \text{ 1249}$$

$$\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}\right)}{8g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}$$

$$\Big\downarrow \; 1254$$

$$\frac{3cd\left(\frac{cd\left(\frac{3cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}\right)}{8g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}$$

$$\Big\downarrow \; 1254$$

$$\frac{3cd\left(\frac{cd\left(\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}\right)}{8g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}$$

$$\Big\downarrow \; 1255$$

3.698. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx$

$$3cd\left(\frac{cd\left(\frac{3cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cde}}{2\sqrt{d+ex}(f+gx)^2(cdf-a}\right)}{6g}\right)$$

$$8g$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}$$

$$\Big\downarrow\ 218$$

$$3cd\left(\frac{cd\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex}}{3g\sqrt{d+ex}(f+gx)^3}\right)$$

$$8g$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}$$

3.698. $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)^5),x]
```

output
```
-1/4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(d + e*x)^(3/2)*(f +
 g*x)^4) + (3*c*d*(-1/3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqr
t[d + e*x]*(f + g*x)^3) + (c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]/(2*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c*d*(Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) + (c
*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*
f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2))))/(4*(c*d*f -
a*e*g))))/(6*g)))/(8*g)
```

3.698.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.698. $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx$

3.698.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(297) = 594$.

Time = 0.57 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.96

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(3\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^4d^4g^4x^4+12\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^4d^4f\,g^3x^3+18\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^4d}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x,meth
od=_RETURNVERBOSE)
```

output
```
-1/64*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^4*d^4*g^4*x^4+12*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*c^4*d^4*f*g^3*x^3+18*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*c^4*d^4*f^2*g^2*x^2+12*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^4*d^4*f^3*g*x-3*c^3*d^3*g^3*x^3*(c*d*x+a*e)^(1/2)*((a*e*
g-c*d*f)*g)^(1/2)+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c
^4*d^4*f^4+2*a*c^2*d^2*e*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)
-11*c^3*d^3*f*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+24*a^2*c*d
*e^2*g^3*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-44*a*c^2*d^2*e*f*g^2*
x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+11*c^3*d^3*f^2*g*x*(c*d*x+a*e)
^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+16*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2
)*a^3*e^3*g^3-24*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*c*d*e^2*f*g
^2+2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*d^2*e*f^2*g+3*(c*d*x+
a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2)/(g*x+f)^4/g^2/(a*e*g-c*d*f)^2/(c*d*x+a*e)^(1/2)
```

3.698.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. $2(297) = 594$.

Time = 0.80 (sec) , antiderivative size = 2238, normalized size of antiderivative = 6.68

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,
x, algorithm="fricas")
```

output
```
[-1/128*(3*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5
*g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e
*f^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqr
t(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f -
 (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2
*(3*c^4*d^4*f^4*g - a*c^3*d^3*e*f^3*g^2 - 26*a^2*c^2*d^2*e^2*f^2*g^3 + 40*
a^3*c*d*e^3*f*g^4 - 16*a^4*e^4*g^5 - 3*(c^4*d^4*f*g^4 - a*c^3*d^3*e*g^5)*x
^3 - (11*c^4*d^4*f^2*g^3 - 13*a*c^3*d^3*e*f*g^4 + 2*a^2*c^2*d^2*e^2*g^5)*x
^2 + (11*c^4*d^4*f^3*g^2 - 55*a*c^3*d^3*e*f^2*g^3 + 68*a^2*c^2*d^2*e^2*f*g
^4 - 24*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(e*x + d))/(c^3*d^4*f^7*g^3 - 3*a*c^2*d^3*e*f^6*g^4 + 3*a^2*c*d^2*e^2*f^
5*g^5 - a^3*d*e^3*f^4*g^6 + (c^3*d^3*e*f^3*g^7 - 3*a*c^2*d^2*e^2*f^2*g^8 +
 3*a^2*c*d*e^3*f*g^9 - a^3*e^4*g^10)*x^5 + (4*c^3*d^3*e*f^4*g^6 - a^3*d*e^
3*g^10 + (c^3*d^4 - 12*a*c^2*d^2*e^2)*f^3*g^7 - 3*(a*c^2*d^3*e - 4*a^2*c*d
*e^3)*f^2*g^8 + (3*a^2*c*d^2*e^2 - 4*a^3*e^4)*f*g^9)*x^4 + 2*(3*c^3*d^3*e*
f^5*g^5 - 2*a^3*d*e^3*f*g^9 + (2*c^3*d^4 - 9*a*c^2*d^2*e^2)*f^4*g^6 - 3*(2
*a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^3*g^7 + 3*(2*a^2*c*d^2*e^2 - a^3*e^4)*f^2*
g^8)*x^3 + 2*(2*c^3*d^3*e*f^6*g^4 - 3*a^3*d*e^3*f^2*g^8 + 3*(c^3*d^4 - 2*a
*c^2*d^2*e^2)*f^5*g^5 - 3*(3*a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^4*g^6 + (9*...
```

3.698.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{(d + ex)^{3/2}(f + gx)^5}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**5,x)
```

output
```
Timed out
```

3.698.7 Maxima [F]

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx = \int \frac{\left(cdex^2 + ade + (cd^2 + ae^2)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^5}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)^5), x)
```

3.698.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1785 vs. 2(297) = 594.

Time = 1.21 (sec) , antiderivative size = 1785, normalized size of antiderivative = 5.33

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,
x, algorithm="giac")
```

output
```
3/64*c^4*d^4*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt
(c*d*f*g - a*e*g^2)*e))/((c^2*d^2*f^2*g^2 - 2*a*c*d*e*f*g^3 + a^2*e^2*g^4)
*sqrt(c*d*f*g - a*e*g^2)*e) - 1/64*(3*c^4*d^4*e^4*f^4*abs(e)*arctan(sqrt(-
c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 12*c^4*d^5*e^3*f^3*g*abs
(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 18*c^4*
d^6*e^2*f^2*g^2*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e
*g^2)*e)) - 12*c^4*d^7*e*f*g^3*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqr
t(c*d*f*g - a*e*g^2)*e)) + 3*c^4*d^8*g^4*abs(e)*arctan(sqrt(-c*d^2*e + a*e
^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 3*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g
 - a*e*g^2)*c^3*d^3*e^3*f^3*abs(e) + 11*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*
g - a*e*g^2)*c^3*d^4*e^2*f^2*g*abs(e) - 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*
f*g - a*e*g^2)*a*c^2*d^2*e^4*f^2*g*abs(e) + 11*sqrt(-c*d^2*e + a*e^3)*sqrt
(c*d*f*g - a*e*g^2)*c^3*d^5*e*f*g^2*abs(e) - 44*sqrt(-c*d^2*e + a*e^3)*sqr
t(c*d*f*g - a*e*g^2)*a*c^2*d^3*e^3*f*g^2*abs(e) + 24*sqrt(-c*d^2*e + a*e^3
)*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d*e^5*f*g^2*abs(e) - 3*sqrt(-c*d^2*e + a*e
^3)*sqrt(c*d*f*g - a*e*g^2)*c^3*d^6*g^3*abs(e) - 2*sqrt(-c*d^2*e + a*e^3)*
sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^4*e^2*g^3*abs(e) + 24*sqrt(-c*d^2*e + a*e^
3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^2*e^4*g^3*abs(e) - 16*sqrt(-c*d^2*e + a
*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^3*e^6*g^3*abs(e))/(sqrt(c*d*f*g - a*e*g^2)
*c^2*d^2*e^5*f^6*g^2 - 4*sqrt(c*d*f*g - a*e*g^2)*c^2*d^3*e^4*f^5*g^3 - ...
```

3.698.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{{\left(f+g\,x\right)}^5\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^5*(d + e*x)^(
3/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^5*(d + e*x)^(
3/2)), x)
```

3.699 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx$

3.699.1 Optimal result

Integrand size = 46, antiderivative size = 405

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx = -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{80g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^3} +\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^2(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2}+\frac{3c^4d^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128g^2(cdf-aeg)^3\sqrt{d+ex}(f+gx)} -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}+\frac{3c^5d^5\arctan\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{128g^{5/2}(cdf-aeg)^{7/2}}$$

output
```
-1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^5+3/1
28*c^5*d^5*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+
c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)/(-a*e*g+c*d*f)^(7/2)-3/40*c*d*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(g*x+f)^4/(e*x+d)^(1/2)+1/80*c^2*d^2*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d*f)/(g*x+f)^3/(e*x+d
)^(1/2)+1/64*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c
*d*f)^2/(g*x+f)^2/(e*x+d)^(1/2)+3/128*c^4*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)/g^2/(-a*e*g+c*d*f)^3/(g*x+f)/(e*x+d)^(1/2)
```

3.699.2 Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.75

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx = \frac{c^5d^5((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{g}(128a^4e^4g^4+16a^3cde^3g^3(-21f+11gx)+8a^2c^2}{}\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)^6),x]
```

output
```
(c^5*d^5*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[g]*(128*a^4*e^4*g^4 + 16*a
^3*c*d*e^3*g^3*(-21*f + 11*g*x) + 8*a^2*c^2*d^2*e^2*g^2*(31*f^2 - 64*f*g*x
 + g^2*x^2) - 2*a*c^3*d^3*e*g*(5*f^3 - 233*f^2*g*x + 23*f*g^2*x^2 + 5*g^3*
x^3) + c^4*d^4*(-15*f^4 - 70*f^3*g*x + 128*f^2*g^2*x^2 + 70*f*g^3*x^3 + 15
*g^4*x^4)))/(c^5*d^5*(c*d*f - a*e*g)^3*(a*e + c*d*x)*(f + g*x)^5) + (15*Ar
cTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/((c*d*f - a*e*g)^(7
/2)*(a*e + c*d*x)^(3/2))))/(640*g^(5/2)*(d + e*x)^(3/2))
```

3.699.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1249, 1249, 1254, 1254, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{3cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}(f+gx)^5}dx}{10g}-\frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}$$

$$\Big\downarrow \text{1249}$$

$$\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}\right)}{10g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}$$

$\downarrow$ 1254

$$\frac{3cd\left(\frac{cd\left(\frac{5cd\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}\right)}{8g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}\right)}{10g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}$$

$\downarrow$ 1254

$$\frac{3cd\left(\frac{cd\left(\frac{5cd\left(\frac{3cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}\right)}{8g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex}}{4g\sqrt{d+ex}(f+gx)^4}\right.}{10g}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}$$

$\downarrow$ 1254

3.699. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx$

$$3cd\left(\frac{cd\left(\frac{5cd\left(\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}}{8g}\right)}{10g}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}$$

$\downarrow$ 1255

3.699. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx$

$$\frac{3cd\left(\frac{cd\left(\frac{5cd\left(\frac{3cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ad\cdots}}{2\sqrt{d+ex}(f+gx)^2(c\cdots}}{6(cdf-aeg)}\cdots}{8g}\cdots}{10g}\cdots$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}$$

$\big\downarrow$ 218

3.699. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx$

$$\frac{3cd\left(\frac{cd\left(\frac{5cd\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^{\cdots}}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg\cdots}\cdots}{8g}\cdots\right)}{10g}\cdots$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)^6),x]
```

3.699. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx$

output
```
-1/5*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(d + e*x)^(3/2)*(f +
 g*x)^5) + (3*c*d*(-1/4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqr
t[d + e*x]*(f + g*x)^4) + (c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]/(3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^3) + (5*c*d*(Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2)
+ (3*c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqr
t[d + e*x]*(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*
e*g)^(3/2))))/(4*(c*d*f - a*e*g))))/(6*(c*d*f - a*e*g))))/(8*g)))/(10*g)
```

3.699.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.699. $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^6} dx$

3.699.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(361) = 722$.

Time = 0.58 (sec) , antiderivative size = 945, normalized size of antiderivative = 2.33

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^5d^5g^5x^5+75\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^5d^5fg^4x^4+150\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^5d\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x,meth
od=_RETURNVERBOSE)
```

output
```
1/640*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*c^5*d^5*g^5*x^5+75*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^5*d^5*f*g^4*x^4+150*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^5*d^5*f^2*g^3*x^3+150*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*
g-c*d*f)*g)^(1/2))*c^5*d^5*f^3*g^2*x^2-15*c^4*d^4*g^4*x^4*(c*d*x+a*e)^(1/2
)*((a*e*g-c*d*f)*g)^(1/2)+75*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*c^5*d^5*f^4*g*x+10*a*c^3*d^3*e*g^4*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c
*d*f)*g)^(1/2)-70*c^4*d^4*f*g^3*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1
/2)+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^5*d^5*f^5-8*
a^2*c^2*d^2*e^2*g^4*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+46*a*c^3
*d^3*e*f*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-128*c^4*d^4*f^2
*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-176*a^3*c*d*e^3*g^4*x*(
c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+512*a^2*c^2*d^2*e^2*f*g^3*x*(c*d*
x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-466*a*c^3*d^3*e*f^2*g^2*x*(c*d*x+a*e)
^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+70*c^4*d^4*f^3*g*x*(c*d*x+a*e)^(1/2)*((a*e*
g-c*d*f)*g)^(1/2)-128*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^4*e^4*g^
4+336*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^3*c*d*e^3*f*g^3-248*(c*d
*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*c^2*d^2*e^2*f^2*g^2+10*(c*d*x+a*
e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c^3*d^3*e*f^3*g+15*(c*d*x+a*e)^(1/2)*((
a*e*g-c*d*f)*g)^(1/2)*c^4*d^4*f^4)/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2...
```

3.699.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1581 vs. 2(361) = 722.

Time = 2.28 (sec) , antiderivative size = 3204, normalized size of antiderivative = 7.91

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^6} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,
x, algorithm="fricas")
```

output
```
[1/1280*(15*(c^5*d^5*e*g^5*x^6 + c^5*d^6*f^5 + (5*c^5*d^5*e*f*g^4 + c^5*d^
6*g^5)*x^5 + 5*(2*c^5*d^5*e*f^2*g^3 + c^5*d^6*f*g^4)*x^4 + 10*(c^5*d^5*e*f
^3*g^2 + c^5*d^6*f^2*g^3)*x^3 + 5*(c^5*d^5*e*f^4*g + 2*c^5*d^6*f^3*g^2)*x^
2 + (c^5*d^5*e*f^5 + 5*c^5*d^6*f^4*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*
d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sq
rt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*
x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(15*c^5*d^5*f^5*g - 5*a*c^4*d
^4*e*f^4*g^2 - 258*a^2*c^3*d^3*e^2*f^3*g^3 + 584*a^3*c^2*d^2*e^3*f^2*g^4 -
 464*a^4*c*d*e^4*f*g^5 + 128*a^5*e^5*g^6 - 15*(c^5*d^5*f*g^5 - a*c^4*d^4*e
*g^6)*x^4 - 10*(7*c^5*d^5*f^2*g^4 - 8*a*c^4*d^4*e*f*g^5 + a^2*c^3*d^3*e^2*
g^6)*x^3 - 2*(64*c^5*d^5*f^3*g^3 - 87*a*c^4*d^4*e*f^2*g^4 + 27*a^2*c^3*d^3
*e^2*f*g^5 - 4*a^3*c^2*d^2*e^3*g^6)*x^2 + 2*(35*c^5*d^5*f^4*g^2 - 268*a*c^
4*d^4*e*f^3*g^3 + 489*a^2*c^3*d^3*e^2*f^2*g^4 - 344*a^3*c^2*d^2*e^3*f*g^5
+ 88*a^4*c*d*e^4*g^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
e*x + d))/(c^4*d^5*f^9*g^3 - 4*a*c^3*d^4*e*f^8*g^4 + 6*a^2*c^2*d^3*e^2*f^7
*g^5 - 4*a^3*c*d^2*e^3*f^6*g^6 + a^4*d*e^4*f^5*g^7 + (c^4*d^4*e*f^4*g^8 -
4*a*c^3*d^3*e^2*f^3*g^9 + 6*a^2*c^2*d^2*e^3*f^2*g^10 - 4*a^3*c*d*e^4*f*g^1
1 + a^4*e^5*g^12)*x^6 + (5*c^4*d^4*e*f^5*g^7 + a^4*d*e^4*g^12 + (c^4*d^5 -
 20*a*c^3*d^3*e^2)*f^4*g^8 - 2*(2*a*c^3*d^4*e - 15*a^2*c^2*d^2*e^3)*f^3*g^
9 + 2*(3*a^2*c^2*d^3*e^2 - 10*a^3*c*d*e^4)*f^2*g^10 - (4*a^3*c*d^2*e^3 ...
```

3.699.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^6} dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**6,x)
```

output
```
Timed out
```

3.699.7 Maxima [F]

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{(d + ex)^{3/2}(f + gx)^6}\, dx = \int \frac{\left(cdex^2 + ade + (cd^2 + ae^2)x\right)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}(gx + f)^6}\, dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)^6), x)
```

3.699.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2659 vs. 2(361) = 722.

Time = 2.80 (sec) , antiderivative size = 2659, normalized size of antiderivative = 6.57

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{(d + ex)^{3/2}(f + gx)^6}\, dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,
x, algorithm="giac")
```

output
```
3/128*c^5*d^5*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqr
t(c*d*f*g - a*e*g^2)*e))/((c^3*d^3*f^3*g^2 - 3*a*c^2*d^2*e*f^2*g^3 + 3*a^2
*c*d*e^2*f*g^4 - a^3*e^3*g^5)*sqrt(c*d*f*g - a*e*g^2)*e) - 1/640*(15*c^5*d
^5*e^5*f^5*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)
*e)) - 75*c^5*d^6*e^4*f^4*g*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c
*d*f*g - a*e*g^2)*e)) + 150*c^5*d^7*e^3*f^3*g^2*abs(e)*arctan(sqrt(-c*d^2*
e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 150*c^5*d^8*e^2*f^2*g^3*abs(e)
*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 75*c^5*d^9
*e*f*g^4*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e
)) - 15*c^5*d^10*g^5*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g
- a*e*g^2)*e)) - 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^4*d^4
*e^4*f^4*abs(e) + 70*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^4*d^
5*e^3*f^3*g*abs(e) - 10*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c
^3*d^3*e^5*f^3*g*abs(e) + 128*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^
2)*c^4*d^6*e^2*f^2*g^2*abs(e) - 466*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g -
a*e*g^2)*a*c^3*d^4*e^4*f^2*g^2*abs(e) + 248*sqrt(-c*d^2*e + a*e^3)*sqrt(c*
d*f*g - a*e*g^2)*a^2*c^2*d^2*e^6*f^2*g^2*abs(e) - 70*sqrt(-c*d^2*e + a*e^3
)*sqrt(c*d*f*g - a*e*g^2)*c^4*d^7*e*f*g^3*abs(e) - 46*sqrt(-c*d^2*e + a*e^
3)*sqrt(c*d*f*g - a*e*g^2)*a*c^3*d^5*e^3*f*g^3*abs(e) + 512*sqrt(-c*d^2*e
+ a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c^2*d^3*e^5*f*g^3*abs(e) - 336*sqr...
```

3.699.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{{\left(f+g\,x\right)}^6\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^6*(d + e*x)^(
3/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^6*(d + e*x)^(
3/2)), x)
```

3.700 $\int \frac{(f+gx)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

3.700.1 Optimal result

Integrand size = 46, antiderivative size = 336

$$\int \frac{(f+gx)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = -\frac{128(cdf-aeg)^3\left(2ae^2g-cd(9ef-7dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{45045c^5d^5e(d+ex)^{7/2}} + \frac{128g(cdf-aeg)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{6435c^4d^4e(d+ex)^{5/2}} + \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{715c^3d^3(d+ex)^{7/2}} + \frac{16(cdf-aeg)(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{195c^2d^2(d+ex)^{7/2}} + \frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}}$$

output
```
-128/45045*(-a*e*g+c*d*f)^3*(2*a*e^2*g-c*d*(-7*d*g+9*e*f))*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(7/2)/c^5/d^5/e/(e*x+d)^(7/2)+128/6435*g*(-a*e*g+c*d*f)
^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^4/d^4/e/(e*x+d)^(5/2)+32/715*
(-a*e*g+c*d*f)^2*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^3/d^3
/(e*x+d)^(7/2)+16/195*(-a*e*g+c*d*f)*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(7/2)/c^2/d^2/(e*x+d)^(7/2)+2/15*(g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(7/2)/c/d/(e*x+d)^(7/2)
```

3.700.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.61

$$\int \frac{(f+gx)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}(128a^4e^4g^4-64a^3cde$$

input
```
Integrate[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
 e*x)^(5/2),x]
```

output
```
(2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(128*a^4*e^4*g^4 - 64*a^3
*c*d*e^3*g^3*(15*f + 7*g*x) + 48*a^2*c^2*d^2*e^2*g^2*(65*f^2 + 70*f*g*x +
21*g^2*x^2) - 8*a*c^3*d^3*e*g*(715*f^3 + 1365*f^2*g*x + 945*f*g^2*x^2 + 23
1*g^3*x^3) + c^4*d^4*(6435*f^4 + 20020*f^3*g*x + 24570*f^2*g^2*x^2 + 13860
*f*g^3*x^3 + 3003*g^4*x^4)))/(45045*c^5*d^5*Sqrt[d + e*x])
```

3.700.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {1253, 1253, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^4 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \ 1253$$

$$\frac{8(cdf-aeg)\int \frac{(f+gx)^3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}}dx}{15cd}+\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}}$$

$$\Big\downarrow \ 1253$$

$$\frac{8(cdf-aeg)\left(\frac{6(cdf-aeg)\int \frac{(f+gx)^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}}dx}{13cd}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}}\right)}{15cd}+$$
$$\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}}$$

$\Big\downarrow$ 1253

$$8(cdf-aeg)\left(\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\int\frac{(f+gx)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}}dx}{11cd}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}}\right)}{13cd}+\frac{2(f+gx)^3(x(ae}{13c}\right.$$

$$\overline{\qquad 15cd \qquad}$$

$$\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}}$$

$\Big\downarrow$ 1221

$$8(cdf-aeg)\left(\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\left(\frac{1}{9}\left(-\frac{2aeg}{cd}-\frac{7dg}{e}+9f\right)\int\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}\right)}{11cd}+\frac{2(f+gx)^2(}{}}{13cd}\right.$$

$$\overline{\qquad 15cd \qquad}$$

$$\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}}$$

$\Big\downarrow$ 1122

$$\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}}+$$

$$\frac{8(cdf-aeg)\left(\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}}+\frac{6(cdf-aeg)\left(\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}}+\frac{4(cdf-aeg)\left(2\left(x\left(ae^2+cd^2\right)+\right.\right.}{}\right)}{13cd}\right.}{15cd}$$

input
```
Int[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^
(5/2),x]
```

3.700. $\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

output
```
(2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(15*c*d*(d +
 e*x)^(7/2)) + (8*(c*d*f - a*e*g)*((2*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(7/2))/(13*c*d*(d + e*x)^(7/2)) + (6*(c*d*f - a*e*g)*((2*(
f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(11*c*d*(d + e*x
)^(7/2)) + (4*(c*d*f - a*e*g)*((2*(9*f - (7*d*g)/e - (2*a*e*g)/(c*d))*(a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(63*c*d*(d + e*x)^(7/2)) + (2*g
*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*c*d*e*(d + e*x)^(5/2)))
)/(11*c*d)))/(13*c*d)))/(15*c*d)
```

3.700.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

3.700. $\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

3.700.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.82

method	result
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^3(3003g^4x^4c^4d^4-1848a\,c^3d^3e\,g^4x^3+13860c^4d^4f\,g^3x^3+1008a^2c^2d^2e^2g^4x^2-7560a\,c^3d^3ef\,g^3x^2+2457}{}$
gosper	$\frac{2(cdx+ae)(3003g^4x^4c^4d^4-1848a\,c^3d^3e\,g^4x^3+13860c^4d^4f\,g^3x^3+1008a^2c^2d^2e^2g^4x^2-7560a\,c^3d^3ef\,g^3x^2+24570c^4d^4f^2g^2x^2-448a^3}{}$

input
```
int((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/45045*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3*(3003*c^4*
d^4*g^4*x^4-1848*a*c^3*d^3*e*g^4*x^3+13860*c^4*d^4*f*g^3*x^3+1008*a^2*c^2*
d^2*e^2*g^4*x^2-7560*a*c^3*d^3*e*f*g^3*x^2+24570*c^4*d^4*f^2*g^2*x^2-448*a
^3*c*d*e^3*g^4*x+3360*a^2*c^2*d^2*e^2*f*g^3*x-10920*a*c^3*d^3*e*f^2*g^2*x+
20020*c^4*d^4*f^3*g*x+128*a^4*e^4*g^4-960*a^3*c*d*e^3*f*g^3+3120*a^2*c^2*d
^2*e^2*f^2*g^2-5720*a*c^3*d^3*e*f^3*g+6435*c^4*d^4*f^4)/c^5/d^5
```

3.700.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.69

$$\int \frac{(f+gx)^4\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\,(3003\,c^7d^7g^4x^7+6435\,a^3c^4d^4e^3f^4-5720\,a^4c^3d^3e^4f^3g}{}$$

input
```
integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),
x, algorithm="fricas")
```

output
```
2/45045*(3003*c^7*d^7*g^4*x^7 + 6435*a^3*c^4*d^4*e^3*f^4 - 5720*a^4*c^3*d^
3*e^4*f^3*g + 3120*a^5*c^2*d^2*e^5*f^2*g^2 - 960*a^6*c*d*e^6*f*g^3 + 128*a
^7*e^7*g^4 + 231*(60*c^7*d^7*f*g^3 + 31*a*c^6*d^6*e*g^4)*x^6 + 63*(390*c^7
*d^7*f^2*g^2 + 540*a*c^6*d^6*e*f*g^3 + 71*a^2*c^5*d^5*e^2*g^4)*x^5 + 35*(5
72*c^7*d^7*f^3*g + 1794*a*c^6*d^6*e*f^2*g^2 + 636*a^2*c^5*d^5*e^2*f*g^3 +
a^3*c^4*d^4*e^3*g^4)*x^4 + 5*(1287*c^7*d^7*f^4 + 10868*a*c^6*d^6*e*f^3*g +
 8814*a^2*c^5*d^5*e^2*f^2*g^2 + 60*a^3*c^4*d^4*e^3*f*g^3 - 8*a^4*c^3*d^3*e
^4*g^4)*x^3 + 3*(6435*a*c^6*d^6*e*f^4 + 14300*a^2*c^5*d^5*e^2*f^3*g + 390*
a^3*c^4*d^4*e^3*f^2*g^2 - 120*a^4*c^3*d^3*e^4*f*g^3 + 16*a^5*c^2*d^2*e^5*g
^4)*x^2 + (19305*a^2*c^5*d^5*e^2*f^4 + 2860*a^3*c^4*d^4*e^3*f^3*g - 1560*a
^4*c^3*d^3*e^4*f^2*g^2 + 480*a^5*c^2*d^2*e^5*f*g^3 - 64*a^6*c*d*e^6*g^4)*x
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x +
 c^5*d^6)
```

3.700.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**
(5/2),x)
```

output
```
Timed out
```

3.700.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.48

$$\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\left(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3\right)\sqrt{cdx+ae}f^4}{7\,cd}$$
$$+\frac{8\left(7\,c^4d^4x^4+19\,ac^3d^3ex^3+15\,a^2c^2d^2e^2x^2+a^3cde^3x-2\,a^4e^4\right)\sqrt{cdx+ae}f^3g}{63\,c^2d^2}$$
$$+\frac{4\left(63\,c^5d^5x^5+161\,ac^4d^4ex^4+113\,a^2c^3d^3e^2x^3+3\,a^3c^2d^2e^3x^2-4\,a^4cde^4x+8\,a^5e^5\right)\sqrt{cdx+ae}f^2g^2}{231\,c^3d^3}$$
$$+\frac{8\left(231\,c^6d^6x^6+567\,ac^5d^5ex^5+371\,a^2c^4d^4e^2x^4+5\,a^3c^3d^3e^3x^3-6\,a^4c^2d^2e^4x^2+8\,a^5cde^5x-16\,a^6e^6\right)\sqrt{cdx}}{3003\,c^4d^4}$$
$$+\frac{2\left(3003\,c^7d^7x^7+7161\,ac^6d^6ex^6+4473\,a^2c^5d^5e^2x^5+35\,a^3c^4d^4e^3x^4-40\,a^4c^3d^3e^4x^3+48\,a^5c^2d^2e^5x^2-64\,a\right.}{45045\,c^5d^5}$$

input
```
integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),
x, algorithm="maxima")
```

output
```
2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d
*x + a*e)*f^4/(c*d) + 8/63*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^
2*d^2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*f^3*g/(c^2*d^
2) + 4/231*(63*c^5*d^5*x^5 + 161*a*c^4*d^4*e*x^4 + 113*a^2*c^3*d^3*e^2*x^3
 + 3*a^3*c^2*d^2*e^3*x^2 - 4*a^4*c*d*e^4*x + 8*a^5*e^5)*sqrt(c*d*x + a*e)*
f^2*g^2/(c^3*d^3) + 8/3003*(231*c^6*d^6*x^6 + 567*a*c^5*d^5*e*x^5 + 371*a^
2*c^4*d^4*e^2*x^4 + 5*a^3*c^3*d^3*e^3*x^3 - 6*a^4*c^2*d^2*e^4*x^2 + 8*a^5*
c*d*e^5*x - 16*a^6*e^6)*sqrt(c*d*x + a*e)*f*g^3/(c^4*d^4) + 2/45045*(3003*
c^7*d^7*x^7 + 7161*a*c^6*d^6*e*x^6 + 4473*a^2*c^5*d^5*e^2*x^5 + 35*a^3*c^4
*d^4*e^3*x^4 - 40*a^4*c^3*d^3*e^4*x^3 + 48*a^5*c^2*d^2*e^5*x^2 - 64*a^6*c*
d*e^6*x + 128*a^7*e^7)*sqrt(c*d*x + a*e)*g^4/(c^5*d^5)
```

3.700.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4287 vs. 2(306) = 612.

Time = 0.48 (sec) , antiderivative size = 4287, normalized size of antiderivative = 12.76

$$\int \frac{(f+gx)^4 (ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),
x, algorithm="giac")
```

output
```
2/45045*(15015*a^2*f^4*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*
e^3)*a*e^2)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs
(e) + 429*c^2*d^2*f^4*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*
e + a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt
(-c*d^2*e + a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e
^3 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e
^2 + 3432*a*c*d*f^3*g*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*
e + a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt
(-c*d^2*e + a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e
^3 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e
 + 2574*a^2*f^2*g^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e
+ a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-
c*d^2*e + a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e +
 a*e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3
 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e) - 5
72*c^2*d^2*f^3*g*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a
*e^3)*a*c^3*d^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c
*d^2*e + a*e^3)*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)/(c^4...
```

3.700.9 Mupad [B] (verification not implemented)

Time = 12.83 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.56

$$\int \frac{(f+gx)^4 \left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^2\,x^5\,\left(71\,a^2\,e^2\,g^2+540\,a\,c\right.}{715}\right.}{}$$

input
```
int(((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^
(5/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^5*(71*a^2*e^2*g^2
 + 390*c^2*d^2*f^2 + 540*a*c*d*e*f*g))/715 + (256*a^7*e^7*g^4 + 12870*a^3*
c^4*d^4*e^3*f^4 - 11440*a^4*c^3*d^3*e^4*f^3*g - 1920*a^6*c*d*e^6*f*g^3 + 6
240*a^5*c^2*d^2*e^5*f^2*g^2)/(45045*c^5*d^5) + (x^3*(12870*c^7*d^7*f^4 - 8
0*a^4*c^3*d^3*e^4*g^4 + 600*a^3*c^4*d^4*e^3*f*g^3 + 108680*a*c^6*d^6*e*f^3
*g + 88140*a^2*c^5*d^5*e^2*f^2*g^2))/(45045*c^5*d^5) + (2*c^2*d^2*g^4*x^7)
/15 + (2*c*d*g^3*x^6*(31*a*e*g + 60*c*d*f))/195 + (2*g*x^4*(a^3*e^3*g^3 +
572*c^3*d^3*f^3 + 1794*a*c^2*d^2*e*f^2*g + 636*a^2*c*d*e^2*f*g^2))/(1287*c
*d) + (2*a^2*e^2*x*(19305*c^4*d^4*f^4 - 64*a^4*e^4*g^4 + 2860*a*c^3*d^3*e*
f^3*g + 480*a^3*c*d*e^3*f*g^3 - 1560*a^2*c^2*d^2*e^2*f^2*g^2))/(45045*c^4*
d^4) + (2*a*e*x^2*(16*a^4*e^4*g^4 + 6435*c^4*d^4*f^4 + 14300*a*c^3*d^3*e*f
^3*g - 120*a^3*c*d*e^3*f*g^3 + 390*a^2*c^2*d^2*e^2*f^2*g^2))/(15015*c^3*d^
3)))/(d + e*x)^(1/2)
```

3.700. $\int \frac{(f+gx)^4\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

3.701 $\int \frac{(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

3.701.1 Optimal result

Integrand size = 46, antiderivative size = 269

$$\int \frac{(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = -\frac{16(cdf-aeg)^2\left(2ae^2g-cd(9ef-7dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{3003c^4d^4e(d+ex)^{7/2}} + \frac{16g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{429c^3d^3e(d+ex)^{5/2}} + \frac{12(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{143c^2d^2(d+ex)^{7/2}} + \frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}}$$

output
```
-16/3003*(-a*e*g+c*d*f)^2*(2*a*e^2*g-c*d*(-7*d*g+9*e*f))*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(7/2)/c^4/d^4/e/(e*x+d)^(7/2)+16/429*g*(-a*e*g+c*d*f)^2*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^3/d^3/e/(e*x+d)^(5/2)+12/143*(-a*
e*g+c*d*f)*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^2/d^2/(e*x+
d)^(7/2)+2/13*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/(e*x+d
)^(7/2)
```

3.701.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.55

$$\int \frac{(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}(-16a^3e^3g^3+8a^2cde^2}{}$$

input
```
Integrate[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
 e*x)^(5/2),x]
```

output
```
(2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-16*a^3*e^3*g^3 + 8*a^2*
c*d*e^2*g^2*(13*f + 7*g*x) - 2*a*c^2*d^2*e*g*(143*f^2 + 182*f*g*x + 63*g^2
*x^2) + c^3*d^3*(429*f^3 + 1001*f^2*g*x + 819*f*g^2*x^2 + 231*g^3*x^3)))/(
3003*c^4*d^4*Sqrt[d + e*x])
```

3.701.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1253, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{1253}$$

$$\frac{6(cdf-aeg)\int \frac{(f+gx)^2\left(cdex^2+(cd^2+ae^2)x+ade\right)^{5/2}}{(d+ex)^{5/2}}dx}{13cd}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}}$$

$$\Big\downarrow \text{1253}$$

$$\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\int \frac{(f+gx)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}}dx}{11cd}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}}\right)}{13cd}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}}$$

$$\Big\downarrow \text{1221}$$

$$\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\left(\frac{1}{9}\left(-\frac{2aeg}{cd}-\frac{7dg}{e}+9f\right)\int\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}\right)}{11cd}+\frac{2(f+gx)^2\left(x\left(ae^2+\ldots\right.\right.}{11cd(\ldots}\right.}{13cd}$$

$$\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}}+$$

$$\frac{6(cdf-aeg)\left(\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}}+\frac{4(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}\left(-\frac{2aeg}{cd}-\frac{7dg}{e}+9f\right)}{63cd(d+ex)^{7/2}}+\frac{2g\left(x\left(ae^2+cd^2\right)+a\ldots\right.}{9cde(d+ex\ldots}\right.}{11cd}\right.}{13cd}$$

input
```
Int[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^
(5/2),x]
```

output
```
(2*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(13*c*d*(d +
 e*x)^(7/2)) + (6*(c*d*f - a*e*g)*((2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(7/2))/(11*c*d*(d + e*x)^(7/2)) + (4*(c*d*f - a*e*g)*((2*(
9*f - (7*d*g)/e - (2*a*e*g)/(c*d))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(7/2))/(63*c*d*(d + e*x)^(7/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(7/2))/(9*c*d*e*(d + e*x)^(5/2))))/(11*c*d)))/(13*c*d)
```

3.701.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

3.701.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.67

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^3\left(-231g^3x^3c^3d^3+126a\,c^2d^2e\,g^3x^2-819c^3d^3f\,g^2x^2-56a^2cd\,e^2g^3x+364a\,c^2d^2ef\,g^2x-1001c^3d^3f^2g\right.}{3003\sqrt{ex+d}\,c^4d^4}$
gosper	$-\frac{2(cdx+ae)\left(-231g^3x^3c^3d^3+126a\,c^2d^2e\,g^3x^2-819c^3d^3f\,g^2x^2-56a^2cd\,e^2g^3x+364a\,c^2d^2ef\,g^2x-1001c^3d^3f^2gx+16a^3e^3g^3-104a^2\right.}{3003c^4d^4(ex+d)^{\frac{5}{2}}}$

input
```
int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,meth
od=_RETURNVERBOSE)
```

output
```
-2/3003*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3*(-231*c^3*
d^3*g^3*x^3+126*a*c^2*d^2*e*g^3*x^2-819*c^3*d^3*f*g^2*x^2-56*a^2*c*d*e^2*g
^3*x+364*a*c^2*d^2*e*f*g^2*x-1001*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-104*a^2*c
*d*e^2*f*g^2+286*a*c^2*d^2*e*f^2*g-429*c^3*d^3*f^3)/c^4/d^4
```

3.701.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.55

$$\int \frac{(f+gx)^3 (ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\,(231\,c^6d^6g^3x^6+429\,a^3c^3d^3e^3f^3-286\,a^4c^2d^2e^4f^2g+1\cdots}{}$$

input
```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),
x, algorithm="fricas")
```

output
```
2/3003*(231*c^6*d^6*g^3*x^6 + 429*a^3*c^3*d^3*e^3*f^3 - 286*a^4*c^2*d^2*e^
4*f^2*g + 104*a^5*c*d*e^5*f*g^2 - 16*a^6*e^6*g^3 + 63*(13*c^6*d^6*f*g^2 +
9*a*c^5*d^5*e*g^3)*x^5 + 7*(143*c^6*d^6*f^2*g + 299*a*c^5*d^5*e*f*g^2 + 53
*a^2*c^4*d^4*e^2*g^3)*x^4 + (429*c^6*d^6*f^3 + 2717*a*c^5*d^5*e*f^2*g + 14
69*a^2*c^4*d^4*e^2*f*g^2 + 5*a^3*c^3*d^3*e^3*g^3)*x^3 + 3*(429*a*c^5*d^5*e
*f^3 + 715*a^2*c^4*d^4*e^2*f^2*g + 13*a^3*c^3*d^3*e^3*f*g^2 - 2*a^4*c^2*d^
2*e^4*g^3)*x^2 + (1287*a^2*c^4*d^4*e^2*f^3 + 143*a^3*c^3*d^3*e^3*f^2*g - 5
2*a^4*c^2*d^2*e^4*f*g^2 + 8*a^5*c*d*e^5*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

3.701.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3 (ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**
(5/2),x)
```

output
```
Timed out
```

3.701.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.35

$$\int \frac{(f+gx)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\left(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3\right)\sqrt{cdx+ae}f^3}{7\,cd}$$
$$+\frac{2\left(7\,c^4d^4x^4+19\,ac^3d^3ex^3+15\,a^2c^2d^2e^2x^2+a^3cde^3x-2\,a^4e^4\right)\sqrt{cdx+ae}f^2g}{21\,c^2d^2}$$
$$+\frac{2\left(63\,c^5d^5x^5+161\,ac^4d^4ex^4+113\,a^2c^3d^3e^2x^3+3\,a^3c^2d^2e^3x^2-4\,a^4cde^4x+8\,a^5e^5\right)\sqrt{cdx+ae}fg^2}{231\,c^3d^3}$$
$$+\frac{2\left(231\,c^6d^6x^6+567\,ac^5d^5ex^5+371\,a^2c^4d^4e^2x^4+5\,a^3c^3d^3e^3x^3-6\,a^4c^2d^2e^4x^2+8\,a^5cde^5x-16\,a^6e^6\right)\sqrt{cdx}{3003\,c^4d^4}$$

input
```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),
x, algorithm="maxima")
```

output
```
2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d
*x + a*e)*f^3/(c*d) + 2/21*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^
2*d^2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*f^2*g/(c^2*d^
2) + 2/231*(63*c^5*d^5*x^5 + 161*a*c^4*d^4*e*x^4 + 113*a^2*c^3*d^3*e^2*x^3
 + 3*a^3*c^2*d^2*e^3*x^2 - 4*a^4*c*d*e^4*x + 8*a^5*e^5)*sqrt(c*d*x + a*e)*
f*g^2/(c^3*d^3) + 2/3003*(231*c^6*d^6*x^6 + 567*a*c^5*d^5*e*x^5 + 371*a^2*
c^4*d^4*e^2*x^4 + 5*a^3*c^3*d^3*e^3*x^3 - 6*a^4*c^2*d^2*e^4*x^2 + 8*a^5*c*
d*e^5*x - 16*a^6*e^6)*sqrt(c*d*x + a*e)*g^3/(c^4*d^4)
```

3.701.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3058 vs. 2(245) = 490.

Time = 0.41 (sec) , antiderivative size = 3058, normalized size of antiderivative = 11.37

$$\int \frac{(f+gx)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),
x, algorithm="giac")
```

output
```
2/45045*(15015*a^2*f^3*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*
e^3)*a*e^2)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs
(e) + 429*c^2*d^2*f^3*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*
e + a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt
(-c*d^2*e + a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e
^3 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e
^2 + 2574*a*c*d*f^2*g*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*
e + a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt
(-c*d^2*e + a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e
^3 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e
 + 1287*a^2*f*g^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e +
a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*
d^2*e + a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a
*e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 +
 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e) - 429
*c^2*d^2*f^2*g*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e
^3)*a*c^3*d^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d
^2*e + a*e^3)*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)/(c^4*d...
```

3.701.9 Mupad [B] (verification not implemented)

Time = 12.93 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.41

$$\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g\,x^4\,\left(53\,a^2\,e^2\,g^2+299\,a\,c\,d\right.}{429}\right.}{}$$

input
```
int(((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^
(5/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g*x^4*(53*a^2*e^2*g^2 +
 143*c^2*d^2*f^2 + 299*a*c*d*e*f*g))/429 - (32*a^6*e^6*g^3 - 858*a^3*c^3*d
^3*e^3*f^3 + 572*a^4*c^2*d^2*e^4*f^2*g - 208*a^5*c*d*e^5*f*g^2)/(3003*c^4*
d^4) + (x^3*(858*c^6*d^6*f^3 + 10*a^3*c^3*d^3*e^3*g^3 + 2938*a^2*c^4*d^4*e
^2*f*g^2 + 5434*a*c^5*d^5*e*f^2*g))/(3003*c^4*d^4) + (2*c^2*d^2*g^3*x^6)/1
3 + (6*c*d*g^2*x^5*(9*a*e*g + 13*c*d*f))/143 + (2*a^2*e^2*x*(8*a^3*e^3*g^3
 + 1287*c^3*d^3*f^3 + 143*a*c^2*d^2*e*f^2*g - 52*a^2*c*d*e^2*f*g^2))/(3003
*c^3*d^3) + (2*a*e*x^2*(429*c^3*d^3*f^3 - 2*a^3*e^3*g^3 + 715*a*c^2*d^2*e*
f^2*g + 13*a^2*c*d*e^2*f*g^2))/(1001*c^2*d^2)))/(d + e*x)^(1/2)
```

3.702 $\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

3.702.1 Optimal result

Integrand size = 46, antiderivative size = 200

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = -\frac{8(cdf-aeg)\left(2ae^2g-cd(9ef-7dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{693c^3d^3e(d+ex)^{7/2}} + \frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{99c^2d^2e(d+ex)^{5/2}} + \frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}}$$

output
```
-8/693*(-a*e*g+c*d*f)*(2*a*e^2*g-c*d*(-7*d*g+9*e*f))*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(7/2)/c^3/d^3/e/(e*x+d)^(7/2)+8/99*g*(-a*e*g+c*d*f)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^2/d^2/e/(e*x+d)^(5/2)+2/11*(g*x+f)^2*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/(e*x+d)^(7/2)
```

3.702.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.50

$$\int \frac{(f+gx)^2 \left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}} dx = \frac{2(ae+cdx)^3 \sqrt{(ae+cdx)(d+ex)}(8a^2e^2g^2-4acdeg(11\cdots}{693c^3d^3\sqrt{d+\cdots}}$$

input
```
Integrate[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
 e*x)^(5/2),x]
```

output
```
(2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^2*g^2 - 4*a*c*d*
e*g*(11*f + 7*g*x) + c^2*d^2*(99*f^2 + 154*f*g*x + 63*g^2*x^2)))/(693*c^3*
d^3*Sqrt[d + e*x])
```

3.702.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2 \left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}} dx$$

$$\Big\downarrow \ 1253$$

$$\frac{4(cdf-aeg)\int \frac{(f+gx)(cdex^2+(cd^2+ae^2)x+ade)^{5/2}}{(d+ex)^{5/2}}dx}{11cd}+\frac{2(f+gx)^2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}}$$

$$\Big\downarrow \ 1221$$

$$\frac{4(cdf-aeg)\left(\frac{1}{9}\left(-\frac{2aeg}{cd}-\frac{7dg}{e}+9f\right)\int \frac{(cdex^2+(cd^2+ae^2)x+ade)^{5/2}}{(d+ex)^{5/2}}dx+\frac{2g\left(x(ae^2+cd^2)+ade+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}\right)}{11cd}+\frac{2(f+gx)^2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}}$$

$$\Big\downarrow \ 1122$$

$$\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}}+\frac{4(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}\left(-\frac{2aeg}{cd}-\frac{7dg}{e}+9f\right)}{63cd(d+ex)^{7/2}}+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}\right)}{11cd}$$

input
```
Int[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^
(5/2),x]
```

output
```
(2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(11*c*d*(d +
 e*x)^(7/2)) + (4*(c*d*f - a*e*g)*((2*(9*f - (7*d*g)/e - (2*a*e*g)/(c*d))*
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(63*c*d*(d + e*x)^(7/2)) +
(2*g*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*c*d*e*(d + e*x)^(5/
2))))/(11*c*d)
```

3.702.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

3.702.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^3(63g^2x^2c^2d^2-28acde\,g^2x+154c^2d^2fgx+8a^2e^2g^2-44acdefg+99c^2d^2f^2)}{693\sqrt{ex+d}\,c^3d^3}$	108
gosper	$\frac{2(cdx+ae)(63g^2x^2c^2d^2-28acde\,g^2x+154c^2d^2fgx+8a^2e^2g^2-44acdefg+99c^2d^2f^2)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}{693c^3d^3(ex+d)^{\frac{5}{2}}}$	116

input
```
int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/693*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3*(63*c^2*d^2*
g^2*x^2-28*a*c*d*e*g^2*x+154*c^2*d^2*f*g*x+8*a^2*e^2*g^2-44*a*c*d*e*f*g+99
*c^2*d^2*f^2)/c^3/d^3
```

3.702.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.42

$$\int \frac{(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\,(63\,c^5d^5g^2x^5+99\,a^3c^2d^2e^3f^2-44\,a^4cde^4fg+8\,a^5e^5g^2$$

input
```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),
x, algorithm="fricas")
```

output
```
2/693*(63*c^5*d^5*g^2*x^5 + 99*a^3*c^2*d^2*e^3*f^2 - 44*a^4*c*d*e^4*f*g +
8*a^5*e^5*g^2 + 7*(22*c^5*d^5*f*g + 23*a*c^4*d^4*e*g^2)*x^4 + (99*c^5*d^5*
f^2 + 418*a*c^4*d^4*e*f*g + 113*a^2*c^3*d^3*e^2*g^2)*x^3 + 3*(99*a*c^4*d^4
*e*f^2 + 110*a^2*c^3*d^3*e^2*f*g + a^3*c^2*d^2*e^3*g^2)*x^2 + (297*a^2*c^3
*d^3*e^2*f^2 + 22*a^3*c^2*d^2*e^3*f*g - 4*a^4*c*d*e^4*g^2)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

3.702.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2 (ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**
(5/2),x)
```

output
```
Timed out
```

3.702.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.22

$$\int \frac{(f+gx)^2 (ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\,(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3)\sqrt{cdx+ae}\,f^2}{7\,cd}$$
$$+\frac{4\,(7\,c^4d^4x^4+19\,ac^3d^3ex^3+15\,a^2c^2d^2e^2x^2+a^3cde^3x-2\,a^4e^4)\sqrt{cdx+ae}\,fg}{63\,c^2d^2}$$
$$+\frac{2\,(63\,c^5d^5x^5+161\,ac^4d^4ex^4+113\,a^2c^3d^3e^2x^3+3\,a^3c^2d^2e^3x^2-4\,a^4cde^4x+8\,a^5e^5)\sqrt{cdx+ae}\,g^2}{693\,c^3d^3}$$

input
```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),
x, algorithm="maxima")
```

output
```
2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d
*x + a*e)*f^2/(c*d) + 4/63*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^
2*d^2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*f*g/(c^2*d^2)
 + 2/693*(63*c^5*d^5*x^5 + 161*a*c^4*d^4*e*x^4 + 113*a^2*c^3*d^3*e^2*x^3 +
 3*a^3*c^2*d^2*e^3*x^2 - 4*a^4*c*d*e^4*x + 8*a^5*e^5)*sqrt(c*d*x + a*e)*g^
2/(c^3*d^3)
```

3.702.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2010 vs. 2(182) = 364.

Time = 0.38 (sec) , antiderivative size = 2010, normalized size of antiderivative = 10.05

$$\int \frac{(f+gx)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),
x, algorithm="giac")
```

output
```
2/3465*(1155*a^2*f^2*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^
3)*a*e^2)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e
) + 33*c^2*d^2*f^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e +
 a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c
*d^2*e + a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e +
a*e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3
+ 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e^2
+ 132*a*c*d*f*g*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*
e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^
2*e + a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e
^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 1
5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e + 33*
a^2*g^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c
^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*
e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2
)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e) - 22*c^2*d^2*f*
g*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6
*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)
*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)/(c^4*d^4*e^3) + (10...
```

3.702.9 Mupad [B] (verification not implemented)

Time = 12.68 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.30

$$\int \frac{(f+gx)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\,\left(\frac{16\,a^5\,e^5\,g^2-88\,a^4\,c\,d\,e^4\,f\,g+}{693\,c^3\,d^3}\right.}{}$$

input
```
int(((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^
(5/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((16*a^5*e^5*g^2 + 198*a^3*
c^2*d^2*e^3*f^2 - 88*a^4*c*d*e^4*f*g)/(693*c^3*d^3) + (x^3*(198*c^5*d^5*f^
2 + 226*a^2*c^3*d^3*e^2*g^2 + 836*a*c^4*d^4*e*f*g))/(693*c^3*d^3) + (2*c^2
*d^2*g^2*x^5)/11 + (2*c*d*g*x^4*(23*a*e*g + 22*c*d*f))/99 + (2*a^2*e^2*x*(
297*c^2*d^2*f^2 - 4*a^2*e^2*g^2 + 22*a*c*d*e*f*g))/(693*c^2*d^2) + (2*a*e*
x^2*(a^2*e^2*g^2 + 99*c^2*d^2*f^2 + 110*a*c*d*e*f*g))/(231*c*d)))/(d + e*x
)^(1/2)
```

3.702. $\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

3.703 $\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

3.703.1 Optimal result

Integrand size = 44, antiderivative size = 125

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = -\frac{2(2ae^2g-cd(9ef-7dg))\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{63c^2d^2e(d+ex)^{7/2}} + \frac{2g\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}$$

output
```
-2/63*(2*a*e^2*g-c*d*(-7*d*g+9*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/
2)/c^2/d^2/e/(e*x+d)^(7/2)+2/9*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c
/d/e/(e*x+d)^(5/2)
```

3.703.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.51

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}(-2aeg+cd(9f+7gx$$

input
```
Integrate[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e
*x)^(5/2),x]
```

output
```
(2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e*g + c*d*(9*f + 7*
g*x)))/(63*c^2*d^2*Sqrt[d + e*x])
```

3.703.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{1}{9}\left(-\frac{2aeg}{cd}-\frac{7dg}{e}+9f\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}\left(-\frac{2aeg}{cd}-\frac{7dg}{e}+9f\right)}{63cd(d+ex)^{7/2}}+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}$$

input
```
Int[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(5
/2),x]
```

output
```
(2*(9*f - (7*d*g)/e - (2*a*e*g)/(c*d))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(7/2))/(63*c*d*(d + e*x)^(7/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(7/2))/(9*c*d*e*(d + e*x)^(5/2))
```

3.703.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

3.703.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^{3}(-7cdgx+2aeg-9cdf)}{63\sqrt{ex+d}\,c^{2}d^{2}}$	59
gosper	$-\frac{2(cdx+ae)(-7cdgx+2aeg-9cdf)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}}{63c^{2}d^{2}(ex+d)^{\frac{5}{2}}}$	67

input
```
int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,method
=_RETURNVERBOSE)
```

output
```
-2/63*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3*(-7*c*d*g*x+
2*a*e*g-9*c*d*f)/c^2/d^2
```

3.703.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.38

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\,(7\,c^4d^4gx^4+9\,a^3cde^3f-2\,a^4e^4g+(9\,c^4d^4f+19\,ac^3d^3e$$

input
```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
 algorithm="fricas")
```

output
```
2/63*(7*c^4*d^4*g*x^4 + 9*a^3*c*d*e^3*f - 2*a^4*e^4*g + (9*c^4*d^4*f + 19*
a*c^3*d^3*e*g)*x^3 + 3*(9*a*c^3*d^3*e*f + 5*a^2*c^2*d^2*e^2*g)*x^2 + (27*a
^2*c^2*d^2*e^2*f + a^3*c*d*e^3*g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

3.703.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/
2),x)
```

output
```
Timed out
```

3.703.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.13

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\,(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3)\sqrt{cdx+ae}f}{7\,cd} + \frac{2\,(7\,c^4d^4x^4+19\,ac^3d^3ex^3+15\,a^2c^2d^2e^2x^2+a^3cde^3x-2\,a^4e^4)\sqrt{cdx+ae}g}{63\,c^2d^2}$$

input
```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
 algorithm="maxima")
```

output
```
2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d
*x + a*e)*f/(c*d) + 2/63*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^2*
d^2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*g/(c^2*d^2)
```

3.703.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1147 vs. $2(113) = 226$.

Time = 0.32 (sec) , antiderivative size = 1147, normalized size of antiderivative = 9.18

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
 algorithm="giac")
```

output
```
2/315*(105*a^2*f*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a
*e^2)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e) +
3*c^2*d^2*f*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)
*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e
+ a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^
(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((
e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e^2 + 6*a*c
*d*g*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*
d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3
)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a
^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d
)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e - c^2*d^2*g*((35
*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2 -
 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*c
*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)/(c^4*d^4*e^3) + (105*((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x + d)*c*d*e - c*d^2*
e + a*e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a
*e^3 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))/(c^4*d^4*e^7))*abs(e)
/e^2 - 42*a*c*d*f*((3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e
^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*...
```

3.703.9 Mupad [B] (verification not implemented)

Time = 12.43 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\left(\frac{2\,c^2\,d^2\,g\,x^4}{9}+\frac{2\,a\,e\,x^2\,(5\,a\,e}{21}\right.}{}$$

input
```
int(((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(5
/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*c^2*d^2*g*x^4)/9 + (2*a
*e*x^2*(5*a*e*g + 9*c*d*f))/21 + (2*c*d*x^3*(19*a*e*g + 9*c*d*f))/63 - (2*
a^3*e^3*(2*a*e*g - 9*c*d*f))/(63*c^2*d^2) + (2*a^2*e^2*x*(a*e*g + 27*c*d*f
))/(63*c*d)))/(d + e*x)^(1/2)
```

3.704 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx$

3.704.1 Optimal result

Integrand size = 39, antiderivative size = 48

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}{7cd(d+ex)^{7/2}}$$

output
```
2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/(e*x+d)^(7/2)
```

3.704.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{7/2}}{7cd(d+ex)^{7/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(7/2))/(7*c*d*(d + e*x)^(7/2))
```

3.704.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7cd(d+ex)^{7/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*c*d*(d + e*x)^(7/2))
```

3.704.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

3.704.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^3}{7\sqrt{ex+d}\,cd}$	42
gosper	$\frac{2(cdx+ae)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}}{7cd(ex+d)^{\frac{5}{2}}}$	50

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,method=_RETURN
VERBOSE)
```

output
```
2/7*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3/c/d
```

3.704.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(42) = 84$.

Time = 0.41 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.90

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\,(c^3d^3x^3 + 3\,ac^2d^2ex^2 + 3\,a^2cde^2x + a^3e^3)\sqrt{cdex^2 + ade + (cd^2 +}}{7\,(cdex + cd^2)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorit
hm="fricas")
```

output
```
2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d^2)
```

3.704.6 Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \int \frac{((d+ex)(ae+cdx))^{\frac{5}{2}}}{(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(d + e*x)**(5/2), x)
```

3.704.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\,(c^3d^3x^3 + 3\,ac^2d^2ex^2 + 3\,a^2cde^2x + a^3e^3)\sqrt{cdx + ae}}{7\,cd}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorit
hm="maxima")
```

output
```
2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d
*x + a*e)/(c*d)
```

3.704.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(42) = 84$.

Time = 0.30 (sec) , antiderivative size = 477, normalized size of antiderivative = 9.94

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\left(35\,a^2\left(\frac{\sqrt{-cd^2e+ae^3}cd^2-\sqrt{-cd^2e+ae^3}ae^2}{cd}+\frac{((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}}{cde}\right)|e|+\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorit
hm="giac")
```

output
```
2/105*(35*a^2*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^
2)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e) + c^2
*d^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*
d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3
)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a
^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d
)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e^2 - 14*a*c*d*((3
*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*s
qrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x + d)*c*d*e - c*d^2*e +
 a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))/(c^2*d^
2*e^2))*abs(e)/e^2)/e
```

3.704.9 Mupad [B] (verification not implemented)

Time = 12.42 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.65

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\;x+a\,d\,e}\left(\frac{6\,a^2\,e^2\,x}{7}+\frac{2\,c^2\,d^2\,x^3}{7}+\frac{2\,a^3\,e^3}{7\,c\,d}+\frac{6\,a}{}\right.}{\sqrt{d+e\,x}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((6*a^2*e^2*x)/7 + (2*c^2*d
^2*x^3)/7 + (2*a^3*e^3)/(7*c*d) + (6*a*c*d*e*x^2)/7))/(d + e*x)^(1/2)
```

3.705 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx$

3.705.1 Optimal result

Integrand size = 46, antiderivative size = 236

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx=\frac{2(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}}-\frac{2(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}}+\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-\frac{2(cdf-aeg)^{5/2}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{7/2}}$$

output
```
-2/3*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3
/2)+2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)-2*(-a*e*g+
c*d*f)^(5/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*
g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)+2*(-a*e*g+c*d*f)^2*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

3.705.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.71

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx = \frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}(23a^2e^2g^2+acdeg(-35f+11\right.}{15g^{7/2}\sqrt{}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)),x]
```

output
```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(23*a^2*e^2*
g^2 + a*c*d*e*g*(-35*f + 11*g*x) + c^2*d^2*(15*f^2 - 5*f*g*x + 3*g^2*x^2))
 - 15*(c*d*f - a*e*g)^(5/2)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f
- a*e*g]]))/(15*g^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.705.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {1250, 1250, 1250, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)}dx$$

$$\Big\downarrow \text{ 1250}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-\frac{(cdf-aeg)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}dx}{g}$$

$$\Big\downarrow \text{ 1250}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-\frac{(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}(f+gx)}dx}{g}\right)}{g}$$

$\downarrow$ 1250

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-\frac{(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}\right)}{g}\right)}{g}$$

$\downarrow$ 1255

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-\frac{(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2e^2(cdf-aeg)\int\frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+c\right.}{d+ex}}}{g}\right.}{g}\right.}{g}$$

$\downarrow$ 218

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-\frac{(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}}\right)}{g}\right.}{g}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)),x]
```

3.705. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx$

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*g*(d + e*x)^(5/2)) -
((c*d*f - a*e*g)*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(
d + e*x)^(3/2)) - ((c*d*f - a*e*g)*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])/(g*Sqrt[d + e*x]) - (2*Sqrt[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x]
)])/g^(3/2)))/g))/g
```

3.705.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.705.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(208) = 416$.

Time = 0.58 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.78

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\left(15\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a^3e^3g^3-45\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a^2cd\,e^2f\,g^2+45\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a\,c\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,method
=_RETURNVERBOSE)
```

output
```
-2/15*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*a^3*e^3*g^3-45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*a^2*c*d*e^2*f*g^2+45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*a*c^2*d^2*e*f^2*g-15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*c^3*d^3*f^3-3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2
*g^2*x^2-11*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*g^2*x+5*(c*d
*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x-23*(c*d*x+a*e)^(1/2)*(
(a*e*g-c*d*f)*g)^(1/2)*a^2*e^2*g^2+35*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^
(1/2)*a*c*d*e*f*g-15*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f^2
)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/((a*e*g-c*d*f)*g)^(1/2)
```

3.705.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 587, normalized size of antiderivative = 2.49

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx = \left[\frac{15\,(c^2d^3f^2-2\,acd^2efg+a^2de^2g^2+(c^2d^2ef^2-2\,acde^2fg+a^2e^3\cdots}{\cdots}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,
 algorithm="fricas")
```

output
```
[1/15*(15*(c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2
- 2*a*c*d*e^2*f*g + a^2*e^3*g^2)*x)*sqrt(-(c*d*f - a*e*g)/g)*log(-(c*d*e*g
*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g
)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*
f^2 - 35*a*c*d*e*f*g + 23*a^2*e^2*g^2 - (5*c^2*d^2*f*g - 11*a*c*d*e*g^2)*x
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g^3*x + d*
g^3), 2/15*(15*(c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e
*f^2 - 2*a*c*d*e^2*f*g + a^2*e^3*g^2)*x)*sqrt((c*d*f - a*e*g)/g)*arctan(sq
rt(e*x + d)*sqrt((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)) + (3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 35*a*c*d*e*f*g + 23*a^2*e^2
*g^2 - (5*c^2*d^2*f*g - 11*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(e*x + d))/(e*g^3*x + d*g^3)]
```

3.705.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f),x)
```

output
```
Timed out
```

3.705.7 Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,
 algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)), x)
```

3.705.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 853 vs. 2(208) = 416.

Time = 0.64 (sec) , antiderivative size = 853, normalized size of antiderivative = 3.61

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx=$$
$$-\frac{2\left(c^3d^3f^3|e|-3\,ac^2d^2efg|e|+3\,a^2cde^2fg^2|e|-a^3e^3g^3|e|\right)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{\sqrt{cdfg-aeg^2}eg^3}$$
$$+\frac{2\left(15\,c^3d^3e^3f^3|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)-45\,ac^2d^2e^4f^2g|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)+45\,a^2cde^5fg^2|e|\arctan\left(\right.\right.}{}$$
$$+\frac{2\left(15\sqrt{(ex+d)cde-cd^2e+ae^3}c^2d^2e^{28}f^2g^2|e|-30\sqrt{(ex+d)cde-cd^2e+ae^3}acde^{29}fg^3|e|+15\sqrt{(ex\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,
 algorithm="giac")
```

output
```
-2*(c^3*d^3*f^3*abs(e) - 3*a*c^2*d^2*e*f^2*g*abs(e) + 3*a^2*c*d*e^2*f*g^2*
abs(e) - a^3*e^3*g^3*abs(e))*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3
)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*e*g^3) + 2/15*(1
5*c^3*d^3*e^3*f^3*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a
*e*g^2)*e)) - 45*a*c^2*d^2*e^4*f^2*g*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*
g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 45*a^2*c*d*e^5*f*g^2*abs(e)*arctan(sqrt(-
c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 15*a^3*e^6*g^3*abs(e)*ar
ctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 15*sqrt(-c*d^
2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d^2*e^2*f^2*abs(e) - 5*sqrt(-c*d^
2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d^3*e*f*g*abs(e) + 35*sqrt(-c*d^2
*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c*d*e^3*f*g*abs(e) - 3*sqrt(-c*d^2*e
 + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d^4*g^2*abs(e) + 11*sqrt(-c*d^2*e +
a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c*d^2*e^2*g^2*abs(e) - 23*sqrt(-c*d^2*e +
 a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*e^4*g^2*abs(e))/(sqrt(c*d*f*g - a*e*g^
2)*e^4*g^3) + 2/15*(15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^2*e^2
8*f^2*g^2*abs(e) - 30*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c*d*e^29*f
*g^3*abs(e) + 15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*e^30*g^4*abs(
e) - 5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c*d*e^26*f*g^3*abs(e) + 5
*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^27*g^4*abs(e) + 3*((e*x + d
)*c*d*e - c*d^2*e + a*e^3)^(5/2)*e^24*g^4*abs(e))/(e^30*g^5)
```

3.705.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{\left(f + g\,x\right)\,\left(d + e\,x\right)^{5/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)*(d + e*x)^(5/
2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)*(d + e*x)^(5/
2)), x)
```

3.706 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx$

3.706.1 Optimal result

Integrand size = 46, antiderivative size = 235

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx = -\frac{5cd(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} + \frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}(f+gx)} + \frac{5cd(cdf-aeg)^{3/2}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{7/2}}$$

output

```
5/3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)-(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)+5*c*d*(-a*e*g+c*d*
f)^(3/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*
d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)-5*c*d*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

3.706.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.78

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx = \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}(-3a^2e^2g^2+2acdeg(10f+7gx\right.}{3g^{7/2}\sqrt{}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^2),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(-3*a^2*e^2*g^
2 + 2*a*c*d*e*g*(10*f + 7*g*x) + c^2*d^2*(-15*f^2 - 10*f*g*x + 2*g^2*x^2))
 + 15*c*d*(c*d*f - a*e*g)^(3/2)*(f + g*x)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x
])/Sqrt[c*d*f - a*e*g]]))/(3*g^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*
x))
```

3.706.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {1249, 1250, 1250, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{5cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}dx}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}(f+gx)}$$

$$\Big\downarrow \text{1250}$$

$$\frac{5cd\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}(f+gx)}dx}{g}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}(f+gx)}$$

$$\Big\downarrow \; 1250$$

$$\frac{5cd\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}\right)}{g}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}(f+gx)}$$

$$\Big\downarrow \; 1255$$

$$\frac{5cd\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2e^2(cdf-aeg)\int\frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\sqrt{c\ldots}}{g}\right.}{g}\right.}{2g}$$

$$-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}(f+gx)}$$

$$\Big\downarrow \; 218$$

$$\frac{5cd\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}}\right)}{g}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}(f+gx)}$$

3.706. $\displaystyle\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^2),x]
```

output
```
-((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(g*(d + e*x)^(5/2)*(f + g*
x))) + (5*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d +
 e*x)^(3/2)) - ((c*d*f - a*e*g)*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(g*Sqrt[d + e*x]) - (2*Sqrt[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])
/g^(3/2)))/g))/(2*g)
```

3.706.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.706. $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx$

3.706.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(209) = 418$.

Time = 0.60 (sec) , antiderivative size = 513, normalized size of antiderivative = 2.18

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a^{2}cd\,e^{2}g^{3}x-30\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a\,c^{2}d^{2}ef\,g^{2}x+15\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right.\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x,meth
od=_RETURNVERBOSE)
```

output
```
-1/3*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*a^2*c*d*e^2*g^3*x-30*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*a*c^2*d^2*e*f*g^2*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2))*c^3*d^3*f^2*g*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*a^2*c*d*e^2*f*g^2-30*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*a*c^2*d^2*e*f^2*g+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^3*d^3*f^3-2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^
2*d^2*g^2*x^2-14*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*g^2*x+1
0*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x+3*(c*d*x+a*e)^(1
/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*e^2*g^2-20*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f
)*g)^(1/2)*a*c*d*e*f*g+15*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^
2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

3.706.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.86

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx = \left[-\frac{15\,(c^2d^3f^2-acd^2efg+(c^2d^2efg-acde^2g^2)x^2+(c^2d^2ef^2-a}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,
x, algorithm="fricas")
```

output
```
[-1/6*(15*(c^2*d^3*f^2 - a*c*d^2*e*f*g + (c^2*d^2*e*f*g - a*c*d*e^2*g^2)*x
^2 + (c^2*d^2*e*f^2 - a*c*d^2*e*g^2 + (c^2*d^3 - a*c*d*e^2)*f*g)*x)*sqrt(-
(c*d*f - a*e*g)/g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g)
- (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*
(2*c^2*d^2*g^2*x^2 - 15*c^2*d^2*f^2 + 20*a*c*d*e*f*g - 3*a^2*e^2*g^2 - 2*(
5*c^2*d^2*f*g - 7*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(e*x + d))/(e*g^4*x^2 + d*f*g^3 + (e*f*g^3 + d*g^4)*x), -1/3*(15*(
c^2*d^3*f^2 - a*c*d^2*e*f*g + (c^2*d^2*e*f*g - a*c*d*e^2*g^2)*x^2 + (c^2*d
^2*e*f^2 - a*c*d^2*e*g^2 + (c^2*d^3 - a*c*d*e^2)*f*g)*x)*sqrt((c*d*f - a*e
*g)/g)*arctan(sqrt(e*x + d)*sqrt((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x)) - (2*c^2*d^2*g^2*x^2 - 15*c^2*d^2*f^2 + 20*a*c*d*e*
f*g - 3*a^2*e^2*g^2 - 2*(5*c^2*d^2*f*g - 7*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g^4*x^2 + d*f*g^3 + (e*f*g^
3 + d*g^4)*x)]
```

3.706.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**2,x)
```

output
```
Timed out
```

3.706.7 Maxima [F]

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx = \int \frac{\left(cdex^2 + ade + (cd^2 + ae^2)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^2), x)
```

3.706.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1025 vs. $2(209) = 418$.

Time = 0.60 (sec) , antiderivative size = 1025, normalized size of antiderivative = 4.36

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^2}\,dx =$$

$$-\frac{15\,c^3d^3e^3f^3|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right) - 15\,c^3d^4e^2f^2g|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right) - 30\,ac^2d^2e^4f^2g|e|\arctan\left(\frac{\sqrt{-}}{\sqrt{c}}\right.}{}$$

$$+\frac{5\,(c^3d^3f^2|e| - 2\,ac^2d^2efg|e| + a^2cde^2g^2|e|)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{\sqrt{cdfg-aeg^2}eg^3}$$

$$-\frac{\sqrt{(ex+d)cde - cd^2e + ae^3}c^3d^3f^2|e| - 2\,\sqrt{(ex+d)cde - cd^2e + ae^3}ac^2d^2efg|e| + \sqrt{(ex+d)cde - cd^2e}}{(cde^2f - ae^3g + ((ex+d)cde - cd^2e + ae^3)g)g^3}$$

$$-\frac{2\left(6\,\sqrt{(ex+d)cde - cd^2e + ae^3}c^2d^2e^{10}fg^3|e| - 6\,\sqrt{(ex+d)cde - cd^2e + ae^3}acde^{11}g^4|e| - ((ex+d)cde\right.}{3\,e^{12}g^6}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,
x, algorithm="giac")
```

output
```
-1/3*(15*c^3*d^3*e^3*f^3*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*
f*g - a*e*g^2)*e)) - 15*c^3*d^4*e^2*f^2*g*abs(e)*arctan(sqrt(-c*d^2*e + a*
e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 30*a*c^2*d^2*e^4*f^2*g*abs(e)*arctan
(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 30*a*c^2*d^3*e^3*
f*g^2*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))
+ 15*a^2*c*d*e^5*f*g^2*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*
g - a*e*g^2)*e)) - 15*a^2*c*d^2*e^4*g^3*abs(e)*arctan(sqrt(-c*d^2*e + a*e^
3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g
 - a*e*g^2)*c^2*d^2*e^2*f^2*abs(e) + 10*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*
g - a*e*g^2)*c^2*d^3*e*f*g*abs(e) + 20*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g
 - a*e*g^2)*a*c*d*e^3*f*g*abs(e) + 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g -
 a*e*g^2)*c^2*d^4*g^2*abs(e) - 14*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*
e*g^2)*a*c*d^2*e^2*g^2*abs(e) - 3*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*
e*g^2)*a^2*e^4*g^2*abs(e))/(sqrt(c*d*f*g - a*e*g^2)*e^4*f*g^3 - sqrt(c*d*f
*g - a*e*g^2)*d*e^3*g^4) + 5*(c^3*d^3*f^2*abs(e) - 2*a*c^2*d^2*e*f*g*abs(e
) + a^2*c*d*e^2*g^2*abs(e))*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)
*g/(sqrt(c*d*f*g - a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*e*g^3) - (sqrt((e
*x + d)*c*d*e - c*d^2*e + a*e^3)*c^3*d^3*f^2*abs(e) - 2*sqrt((e*x + d)*c*d
*e - c*d^2*e + a*e^3)*a*c^2*d^2*e*f*g*abs(e) + sqrt((e*x + d)*c*d*e - c*d^
2*e + a*e^3)*a^2*c*d*e^2*g^2*abs(e))/((c*d*e^2*f - a*e^3*g + ((e*x + d)...
```

3.706.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^2} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{5/2}}{(f + g\,x)^2\,(d + e\,x)^{5/2}} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^2*(d + e*x)^(
5/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^2*(d + e*x)^(
5/2)), x)
```

3.707 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx$

3.707.1 Optimal result

Integrand size = 46, antiderivative size = 246

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx = \frac{15c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}} - \frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g^2(d+ex)^{3/2}(f+gx)} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2} - \frac{15c^2d^2\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4g^{7/2}}$$

output
```
-5/4*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)
-1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^2-15/
4*c^2*d^2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c
*d*f)^(1/2)/(e*x+d)^(1/2))*(-a*e*g+c*d*f)^(1/2)/g^(7/2)+15/4*c^2*d^2*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

3.707.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.77

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx = \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}(-2a^2e^2g^2 - acdeg(5f+9gx) \cdots\right.}{4g^{7/2}\sqrt{(\cdots}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^3),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(-2*a^2*e^2*g^
2 - a*c*d*e*g*(5*f + 9*g*x) + c^2*d^2*(15*f^2 + 25*f*g*x + 8*g^2*x^2)) - 1
5*c^2*d^2*Sqrt[c*d*f - a*e*g]*(f + g*x)^2*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x
])/Sqrt[c*d*f - a*e*g]]))/(4*g^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*
x)^2)
```

3.707.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {1249, 1249, 1250, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}dx$$

$$\Big\downarrow \ 1249$$

$$\frac{5cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}dx}{4g} - \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}$$

$$\Big\downarrow \ 1249$$

$$\frac{5cd\left(\frac{3cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}(f+gx)}dx}{2g} - \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}\right)}{4g} - \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}$$

$$\Big\downarrow \ 1250$$

$$\frac{5cd\left(\frac{3cd\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}\right)}{2g}-\frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}\right)}{4g}-\frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}$$

$\downarrow$ 1255

$$\frac{5cd\left(\frac{3cd\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2e^2(cdf-aeg)\int\frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{g}\right)}{2g}-\frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}\right)}{4g}-\frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}$$

$\downarrow$ 218

$$\frac{5cd\left(\frac{3cd\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}}\right)}{2g}-\frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}\right)}{4g}-\frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^3),x]
```

3.707. $\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx$

output
```
-1/2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(g*(d + e*x)^(5/2)*(f +
 g*x)^2) + (5*c*d*(-((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(d +
 e*x)^(3/2)*(f + g*x))) + (3*c*d*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(g*Sqrt[d + e*x]) - (2*Sqrt[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])]
)/g^(3/2)))/(2*g)))/(4*g)
```

3.707.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.707.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 515 vs. 2(214) = 428.

Time = 0.56 (sec) , antiderivative size = 516, normalized size of antiderivative = 2.10

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\,\left(15\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a\,c^{2}d^{2}e\,g^{3}x^{2}-15\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^{3}d^{3}f\,g^{2}x^{2}+30\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x,meth
od=_RETURNVERBOSE)
```

output
```
-1/4*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*a*c^2*d^2*e*g^3*x^2-15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2))*c^3*d^3*f*g^2*x^2+30*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2))*a*c^2*d^2*e*f*g^2*x-30*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e
*g-c*d*f)*g)^(1/2))*c^3*d^3*f^2*g*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2))*a*c^2*d^2*e*f^2*g-15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2))*c^3*d^3*f^3-8*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*
c^2*d^2*g^2*x^2+9*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*g^2*x-
25*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x+2*(c*d*x+a*e)^(
1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*e^2*g^2+5*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f
)*g)^(1/2)*a*c*d*e*f*g-15*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^
2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/(g*x+f)^2/((a*e*g-c*d*f)*g)^(1/
2)
```

3.707.5 Fricas [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 683, normalized size of antiderivative = 2.78

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx = \left[\frac{15\,(c^2d^2eg^2x^3+c^2d^3f^2+(2\,c^2d^2efg+c^2d^3g^2)x^2+(c^2d^2ef^2+2\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,
x, algorithm="fricas")
```

output
```
[1/8*(15*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2
)*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-(c*d*f - a*e*g)/g)*log(-(
c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c*d^2 + 2*a
*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(8*c^2*d^2*g^2*x^2 + 15*c
^2*d^2*f^2 - 5*a*c*d*e*f*g - 2*a^2*e^2*g^2 + (25*c^2*d^2*f*g - 9*a*c*d*e*g
^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g^5*x
^3 + d*f^2*g^3 + (2*e*f*g^4 + d*g^5)*x^2 + (e*f^2*g^3 + 2*d*f*g^4)*x), 1/4
*(15*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)*x^
2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt((c*d*f - a*e*g)/g)*arctan(sqrt
(e*x + d)*sqrt((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)) + (8*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 5*a*c*d*e*f*g - 2*a^2*e^2*g^2
 + (25*c^2*d^2*f*g - 9*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(e*x + d))/(e*g^5*x^3 + d*f^2*g^3 + (2*e*f*g^4 + d*g^5)*x^2 +
 (e*f^2*g^3 + 2*d*f*g^4)*x)]
```

3.707.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^3} dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**3,x)
```

output
```
Timed out
```

3.707.7 Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^3} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}(gx + f)^3} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^3), x)
```

3.707.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1038 vs. 2(214) = 428.

Time = 0.58 (sec) , antiderivative size = 1038, normalized size of antiderivative = 4.22

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^3}\,dx = \frac{2\sqrt{(ex + d)cde - cd^2e + ae^3}c^2d^2|e|}{e^2g^3}$$
$$+ \frac{15\,c^3d^3e^3f^3|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2e}}\right) - 30\,c^3d^4e^2f^2g|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2e}}\right) - 15\,ac^2d^2e^4f^2g|e|\arctan\left(\frac{\sqrt{-}}{\sqrt{c}}\right.}{}$$
$$- \frac{15\left(c^3d^3f|e| - ac^2d^2eg|e|\right)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{4\sqrt{cdfg - aeg^2}eg^3}$$
$$+ \frac{7\sqrt{(ex + d)cde - cd^2e + ae^3}c^4d^4e^2f^2|e| - 14\sqrt{(ex + d)cde - cd^2e + ae^3}ac^3d^3e^3fg|e| + 7\sqrt{(ex + d)cde -}}{4\,(cde^2f - ae^3g + ((ex}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,
x, algorithm="giac")
```

output
```
2*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^2*abs(e)/(e^2*g^3) + 1/4*(
15*c^3*d^3*e^3*f^3*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g -
a*e*g^2)*e)) - 30*c^3*d^4*e^2*f^2*g*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g
/(sqrt(c*d*f*g - a*e*g^2)*e)) - 15*a*c^2*d^2*e^4*f^2*g*abs(e)*arctan(sqrt(
-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 15*c^3*d^5*e*f*g^2*abs(
e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 30*a*c^2
*d^3*e^3*f*g^2*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*
g^2)*e)) - 15*a*c^2*d^4*e^2*g^3*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sq
rt(c*d*f*g - a*e*g^2)*e)) - 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g
^2)*c^2*d^2*e^2*f^2*abs(e) + 25*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*
g^2)*c^2*d^3*e*f*g*abs(e) + 5*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^
2)*a*c*d*e^3*f*g*abs(e) - 8*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)
*c^2*d^4*g^2*abs(e) - 9*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c
*d^2*e^2*g^2*abs(e) + 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2
*e^4*g^2*abs(e))/(sqrt(c*d*f*g - a*e*g^2)*e^4*f^2*g^3 - 2*sqrt(c*d*f*g - a
*e*g^2)*d*e^3*f*g^4 + sqrt(c*d*f*g - a*e*g^2)*d^2*e^2*g^5) - 15/4*(c^3*d^3
*f*abs(e) - a*c^2*d^2*e*g*abs(e))*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e +
a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*e*g^3) + 1/
4*(7*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^4*d^4*e^2*f^2*abs(e) - 14*s
qrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^3*d^3*e^3*f*g*abs(e) + 7*sqr...
```

3.707.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{{\left(f+g\,x\right)}^3\,{\left(d+e\,x\right)}^{5/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^3*(d + e*x)^(
5/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^3*(d + e*x)^(
5/2)), x)
```

3.708 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx$

3.708.1 Optimal result

Integrand size = 46, antiderivative size = 253

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx=-\frac{5c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g^3\sqrt{d+ex}(f+gx)}-\frac{5cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^2}-\frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}+\frac{5c^3d^3\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{8g^{7/2}\sqrt{cdf-aeg}}$$

output
```
-5/12*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f
)^2-1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^3+
5/8*c^3*d^3*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g
+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)/(-a*e*g+c*d*f)^(1/2)-5/8*c^2*d^2*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(g*x+f)/(e*x+d)^(1/2)
```

3.708.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.68

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{g}(8a^2e^2g^2+2acdeg(5f+13gx)+c^2d^2(15f^2+40fgx+3}{(f+gx)^3}\right.}{24g^{7/2}\sqrt{d+ex}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^4),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[g]*(8*a^2*e^2*g^2 + 2*a*c*d*e*g*(5
*f + 13*g*x) + c^2*d^2*(15*f^2 + 40*f*g*x + 33*g^2*x^2)))/(f + g*x)^3) + (
15*c^3*d^3*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(Sqrt[
c*d*f - a*e*g]*Sqrt[a*e + c*d*x])))/(24*g^(7/2)*Sqrt[d + e*x])
```

3.708.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {1249, 1249, 1249, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}dx$$

$$\Big\downarrow \text{ 1249}$$

$$\frac{5cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}dx}{6g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}$$

$$\Big\downarrow \text{ 1249}$$

$$\frac{5cd\left(\frac{3cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}(f+gx)^2}dx}{4g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}\right)}{6g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}$$

$$\Big\downarrow \text{ 1249}$$

$$\frac{5cd\left(\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}\right)}{4g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}\right)}{6g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}$$

$\Big\downarrow$ 1255

$$\frac{5cd\left(\frac{3cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}\right)}{4g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}\right)}{6g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}$$

$\Big\downarrow$ 218

$$\frac{5cd\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}\right)}{4g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}\right)}{6g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^4),x]
```

3.708. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx$

output
```
-1/3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(g*(d + e*x)^(5/2)*(f +
 g*x)^3) + (5*c*d*(-1/2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(
d + e*x)^(3/2)*(f + g*x)^2) + (3*c*d*(-(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x))) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(g^
(3/2)*Sqrt[c*d*f - a*e*g])))/(4*g)))/(6*g)
```

3.708.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.708.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.70

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(15 \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3g^3x^3+45 \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3f\,g^2x^2+45 \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x,meth
od=_RETURNVERBOSE)
```

output
```
-1/24*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*c^3*d^3*g^3*x^3+45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^3*d^3*f*g^2*x^2+45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^3*d^3*f^2*g*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^3*d^3*f^3+33*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*
d^2*g^2*x^2+26*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*g^2*x+40*
(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x+8*(c*d*x+a*e)^(1/2
)*((a*e*g-c*d*f)*g)^(1/2)*a^2*e^2*g^2+10*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*
g)^(1/2)*a*c*d*e*f*g+15*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*
f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/(g*x+f)^3/((a*e*g-c*d*f)*g)^(1/2)
```

3.708.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(221) = 442$.

Time = 0.82 (sec) , antiderivative size = 1140, normalized size of antiderivative = 4.51

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^4}\,dx = \left[-\frac{15\,(c^3d^3eg^3x^4 + c^3d^4f^3 + (3\,c^3d^3efg^2 + c^3d^4g^3)x^3 + 3\,(c^3d^3ef^2}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,
x, algorithm="fricas")
```

output
```
[-1/48*(15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4
*g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c
^3*d^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*
a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f +
 (e*f + d*g)*x)) + 2*(15*c^3*d^3*f^3*g - 5*a*c^2*d^2*e*f^2*g^2 - 2*a^2*c*d
*e^2*f*g^3 - 8*a^3*e^3*g^4 + 33*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 2*
(20*c^3*d^3*f^2*g^2 - 7*a*c^2*d^2*e*f*g^3 - 13*a^2*c*d*e^2*g^4)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^2*f^4*g^4 - a*d*e
*f^3*g^5 + (c*d*e*f*g^7 - a*e^2*g^8)*x^4 + (3*c*d*e*f^2*g^6 - a*d*e*g^8 +
(c*d^2 - 3*a*e^2)*f*g^7)*x^3 + 3*(c*d*e*f^3*g^5 - a*d*e*f*g^7 + (c*d^2 - a
*e^2)*f^2*g^6)*x^2 + (c*d*e*f^4*g^4 - 3*a*d*e*f^2*g^6 + (3*c*d^2 - a*e^2)*
f^3*g^5)*x), -1/24*(15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g
^2 + c^3*d^4*g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3
*e*f^3 + 3*c^3*d^4*f^2*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e
*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (15*c^3*d^3*f^3*g - 5*a*c^2*d^2
*e*f^2*g^2 - 2*a^2*c*d*e^2*f*g^3 - 8*a^3*e^3*g^4 + 33*(c^3*d^3*f*g^3 - a*c
^2*d^2*e*g^4)*x^2 + 2*(20*c^3*d^3*f^2*g^2 - 7*a*c^2*d^2*e*f*g^3 - 13*a^2*c
*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d...
```

3.708.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^4}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**4,x)
```

output
```
Timed out
```

3.708.7 Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^4} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}(gx + f)^4} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^4), x)
```

3.708.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 940 vs. $2(221) = 442$.

Time = 0.71 (sec) , antiderivative size = 940, normalized size of antiderivative = 3.72

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^4} dx = \frac{5\,c^3d^3|e|\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2e}}\right)}{8\sqrt{cdfg-aeg^2}eg^3}$$
$$- \frac{15\,c^3d^3e^3f^3|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2e}}\right) - 45\,c^3d^4e^2f^2g|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2e}}\right) + 45\,c^3d^5efg^2|e|\arctan\left(\frac{\sqrt{-cd}}{\sqrt{cdfg}}\right.}{}$$
$$- \frac{15\sqrt{(ex+d)cde-cd^2e+ae^3}c^5d^5e^4f^2|e| - 30\sqrt{(ex+d)cde-cd^2e+ae^3}ac^4d^4e^5fg|e| + 15\sqrt{(ex+d)c}}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,
x, algorithm="giac")
```

output
```
5/8*c^3*d^3*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(
c*d*f*g - a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*e*g^3) - 1/24*(15*c^3*d^3*
e^3*f^3*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)
) - 45*c^3*d^4*e^2*f^2*g*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*
f*g - a*e*g^2)*e)) + 45*c^3*d^5*e*f*g^2*abs(e)*arctan(sqrt(-c*d^2*e + a*e^
3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 15*c^3*d^6*g^3*abs(e)*arctan(sqrt(-c*d
^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 15*sqrt(-c*d^2*e + a*e^3)*s
qrt(c*d*f*g - a*e*g^2)*c^2*d^2*e^2*f^2*abs(e) + 40*sqrt(-c*d^2*e + a*e^3)*
sqrt(c*d*f*g - a*e*g^2)*c^2*d^3*e*f*g*abs(e) - 10*sqrt(-c*d^2*e + a*e^3)*s
qrt(c*d*f*g - a*e*g^2)*a*c*d*e^3*f*g*abs(e) - 33*sqrt(-c*d^2*e + a*e^3)*sq
rt(c*d*f*g - a*e*g^2)*c^2*d^4*g^2*abs(e) + 26*sqrt(-c*d^2*e + a*e^3)*sqrt(
c*d*f*g - a*e*g^2)*a*c*d^2*e^2*g^2*abs(e) - 8*sqrt(-c*d^2*e + a*e^3)*sqrt(
c*d*f*g - a*e*g^2)*a^2*e^4*g^2*abs(e))/(sqrt(c*d*f*g - a*e*g^2)*e^4*f^3*g^
3 - 3*sqrt(c*d*f*g - a*e*g^2)*d*e^3*f^2*g^4 + 3*sqrt(c*d*f*g - a*e*g^2)*d^
2*e^2*f*g^5 - sqrt(c*d*f*g - a*e*g^2)*d^3*e*g^6) - 1/24*(15*sqrt((e*x + d)
*c*d*e - c*d^2*e + a*e^3)*c^5*d^5*e^4*f^2*abs(e) - 30*sqrt((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)*a*c^4*d^4*e^5*f*g*abs(e) + 15*sqrt((e*x + d)*c*d*e - c
*d^2*e + a*e^3)*a^2*c^3*d^3*e^6*g^2*abs(e) + 40*((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)^(3/2)*c^4*d^4*e^2*f*g*abs(e) - 40*((e*x + d)*c*d*e - c*d^2*e + a
*e^3)^(3/2)*a*c^3*d^3*e^3*g^2*abs(e) + 33*((e*x + d)*c*d*e - c*d^2*e + ...
```

3.708.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^4} \, dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{5/2}}{(f + g\,x)^4\,(d + e\,x)^{5/2}} \, dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^4*(d + e*x)^(
5/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^4*(d + e*x)^(
5/2)), x)
```

3.709 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx$

3.709.1 Optimal result

Integrand size = 46, antiderivative size = 323

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx = -\frac{5c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^2} + \frac{5c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64g^3(cdf-aeg)\sqrt{d+ex}(f+gx)} - \frac{5cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{24g^2(d+ex)^{3/2}(f+gx)^3} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4} + \frac{5c^4d^4\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{64g^{7/2}(cdf-aeg)^{3/2}}$$

output
```
-5/24*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f
)^3-1/4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^4+
5/64*c^4*d^4*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*
g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)/(-a*e*g+c*d*f)^(3/2)-5/32*c^2*d^2*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(g*x+f)^2/(e*x+d)^(1/2)+5/64*c^3
*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g+c*d*f)/(g*x+f)/(e
*x+d)^(1/2)
```

3.709.2 Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.76

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx = \frac{c^4d^4((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{g}(48a^3e^3g^3-8a^2cde^2g^2(f-17gx)+2ac^2d^2eg(-}{c^4d^4(cdf}\right.}{192g^{7/}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^5),x]
```

output
```
(c^4*d^4*((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[g]*(48*a^3*e^3*g^3 - 8*a^2
*c*d*e^2*g^2*(f - 17*g*x) + 2*a*c^2*d^2*e*g*(-5*f^2 - 18*f*g*x + 59*g^2*x^
2) - c^3*d^3*(15*f^3 + 55*f^2*g*x + 73*f*g^2*x^2 - 15*g^3*x^3)))/(c^4*d^4*
(c*d*f - a*e*g)*(a*e + c*d*x)^2*(f + g*x)^4) + (15*ArcTan[(Sqrt[g]*Sqrt[a*
e + c*d*x])/Sqrt[c*d*f - a*e*g]])/((c*d*f - a*e*g)^(3/2)*(a*e + c*d*x)^(5/
2))))/(192*g^(7/2)*(d + e*x)^(5/2))
```

3.709.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1249, 1249, 1249, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx$$

$$\Big\downarrow \; 1249$$

$$\frac{5cd\int \frac{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}dx}{8g} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}$$

$$\Big\downarrow \; 1249$$

$$\frac{5cd\left(\frac{cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}(f+gx)^3}dx}{2g} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}\right)}{8g} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}$$

$$\Big\downarrow \text{1249}$$

$$\frac{5cd\left(\frac{cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}\right)}{8g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}$$

$$\Big\downarrow \text{1254}$$

$$\frac{5cd\left(\frac{cd\left(\frac{cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}\right.}{8g}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}$$

$$\Big\downarrow \text{1255}$$

3.709. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx$

$$\frac{5cd\left(\frac{cd\left(\frac{cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{2g\sqrt{d+ex}(f+gx)^2}}{2g}}{8g}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}$$

$\downarrow$ 218

$$\frac{5cd\left(\frac{cd\left(\frac{cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}{3g(d+ex)^{3/2}(f+gx)^3}}{8g}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}$$

3.709. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^5),x]
```

output
```
-1/4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(g*(d + e*x)^(5/2)*(f +
 g*x)^4) + (5*c*d*(-1/3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(
d + e*x)^(3/2)*(f + g*x)^3) + (c*d*(-1/2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x)^2) + (c*d*(Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) + (c*d*ArcTan
[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g
]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2))))/(4*g)))/(2*g)))/(8*g)
```

3.709.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.709.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(285) = 570$.

Time = 0.55 (sec) , antiderivative size = 655, normalized size of antiderivative = 2.03

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(15 \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^4d^4g^4x^4+60 \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^4d^4fg^3x^3+90 \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^4d^4\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x,meth
od=_RETURNVERBOSE)
```

output
```
1/192*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*c^4*d^4*g^4*x^4+60*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^4*d^4*f*g^3*x^3+90*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^4*d^4*f^2*g^2*x^2+60*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*c^4*d^4*f^3*g*x-15*c^3*d^3*g^3*x^3*(c*d*x+a*e)^(1/2)*((a*
e*g-c*d*f)*g)^(1/2)+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)
)*c^4*d^4*f^4-118*a*c^2*d^2*e*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^
(1/2)+73*c^3*d^3*f*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-136*a
^2*c*d*e^2*g^3*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+36*a*c^2*d^2*e*
f*g^2*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+55*c^3*d^3*f^2*g*x*(c*d*
x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-48*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g
)^(1/2)*a^3*e^3*g^3+8*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*c*d*e^
2*f*g^2+10*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*d^2*e*f^2*g+15*
(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/(c*d*
x+a*e)^(1/2)/(a*e*g-c*d*f)/g^3/(g*x+f)^4/((a*e*g-c*d*f)*g)^(1/2)
```

3.709.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(285) = 570$.

Time = 0.55 (sec) , antiderivative size = 1862, normalized size of antiderivative = 5.76

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,
x, algorithm="fricas")
```

output
```
[1/384*(15*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5
*g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e
*f^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqr
t(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f -
 (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2
*(15*c^4*d^4*f^4*g - 5*a*c^3*d^3*e*f^3*g^2 - 2*a^2*c^2*d^2*e^2*f^2*g^3 - 5
6*a^3*c*d*e^3*f*g^4 + 48*a^4*e^4*g^5 - 15*(c^4*d^4*f*g^4 - a*c^3*d^3*e*g^5
)*x^3 + (73*c^4*d^4*f^2*g^3 - 191*a*c^3*d^3*e*f*g^4 + 118*a^2*c^2*d^2*e^2*
g^5)*x^2 + (55*c^4*d^4*f^3*g^2 - 19*a*c^3*d^3*e*f^2*g^3 - 172*a^2*c^2*d^2*
e^2*f*g^4 + 136*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*sqrt(e*x + d))/(c^2*d^3*f^6*g^4 - 2*a*c*d^2*e*f^5*g^5 + a^2*d*e^2*f^
4*g^6 + (c^2*d^2*e*f^2*g^8 - 2*a*c*d*e^2*f*g^9 + a^2*e^3*g^10)*x^5 + (4*c^
2*d^2*e*f^3*g^7 + a^2*d*e^2*g^10 + (c^2*d^3 - 8*a*c*d*e^2)*f^2*g^8 - 2*(a*
c*d^2*e - 2*a^2*e^3)*f*g^9)*x^4 + 2*(3*c^2*d^2*e*f^4*g^6 + 2*a^2*d*e^2*f*g
^9 + 2*(c^2*d^3 - 3*a*c*d*e^2)*f^3*g^7 - (4*a*c*d^2*e - 3*a^2*e^3)*f^2*g^8
)*x^3 + 2*(2*c^2*d^2*e*f^5*g^5 + 3*a^2*d*e^2*f^2*g^8 + (3*c^2*d^3 - 4*a*c*
d*e^2)*f^4*g^6 - 2*(3*a*c*d^2*e - a^2*e^3)*f^3*g^7)*x^2 + (c^2*d^2*e*f^6*g
^4 + 4*a^2*d*e^2*f^3*g^7 + 2*(2*c^2*d^3 - a*c*d*e^2)*f^5*g^5 - (8*a*c*d^2*
e - a^2*e^3)*f^4*g^6)*x), -1/192*(15*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 +...
```

3.709.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^5} dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**5,x)
```

output
```
Timed out
```

3.709.7 Maxima [F]

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^5}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^5), x)
```

3.709.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1574 vs. $2(285) = 570$.

Time = 1.75 (sec) , antiderivative size = 1574, normalized size of antiderivative = 4.87

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,
x, algorithm="giac")
```

output
```
5/64*c^4*d^4*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt
(c*d*f*g - a*e*g^2)*e))/((c*d*f*g^3 - a*e*g^4)*sqrt(c*d*f*g - a*e*g^2)*e)
- 1/192*(15*c^4*d^4*e^4*f^4*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c
*d*f*g - a*e*g^2)*e)) - 60*c^4*d^5*e^3*f^3*g*abs(e)*arctan(sqrt(-c*d^2*e +
 a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 90*c^4*d^6*e^2*f^2*g^2*abs(e)*arc
tan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 60*c^4*d^7*e*f
*g^3*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) +
 15*c^4*d^8*g^4*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e
*g^2)*e)) - 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^3*d^3*e^3*
f^3*abs(e) + 55*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^3*d^4*e^2
*f^2*g*abs(e) - 10*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^
2*e^4*f^2*g*abs(e) - 73*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^3
*d^5*e*f*g^2*abs(e) + 36*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*
c^2*d^3*e^3*f*g^2*abs(e) - 8*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2
)*a^2*c*d*e^5*f*g^2*abs(e) - 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*
g^2)*c^3*d^6*g^3*abs(e) + 118*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^
2)*a*c^2*d^4*e^2*g^3*abs(e) - 136*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*
e*g^2)*a^2*c*d^2*e^4*g^3*abs(e) + 48*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g -
 a*e*g^2)*a^3*e^6*g^3*abs(e))/(sqrt(c*d*f*g - a*e*g^2)*c*d*e^5*f^5*g^3 - 4
*sqrt(c*d*f*g - a*e*g^2)*c*d^2*e^4*f^4*g^4 - sqrt(c*d*f*g - a*e*g^2)*a*...
```

3.709.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx = \int \frac{\left(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e\right)^{5/2}}{(f+g\,x)^5\,(d+e\,x)^{5/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^5*(d + e*x)^(
5/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^5*(d + e*x)^(
5/2)), x)
```

3.710 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx$

3.710.1 Optimal result

Integrand size = 46, antiderivative size = 393

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx &= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}(f+gx)^3} \\ &+ \frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^2} + \frac{3c^4d^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128g^3(cdf-aeg)^2\sqrt{d+ex}(f+gx)} \\ &- \frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}(f+gx)^4} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5} \\ &+ \frac{3c^5d^5\arctan\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{128g^{7/2}(cdf-aeg)^{5/2}}\end{aligned}$$

output
```
-1/8*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)
^4-1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^5+3
/128*c^5*d^5*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*
g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)/(-a*e*g+c*d*f)^(5/2)-1/16*c^2*d^2*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(g*x+f)^3/(e*x+d)^(1/2)+1/64*c^3
*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g+c*d*f)/(g*x+f)^2/
(e*x+d)^(1/2)+3/128*c^4*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-
a*e*g+c*d*f)^2/(g*x+f)/(e*x+d)^(1/2)
```

3.710.2 Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.77

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx = \frac{c^5d^5((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{g}(-128a^4e^4g^4+16a^3cde^3g^3(11f-21gx)-8a^2c^2}{}\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^6),x]
```

output
```
(c^5*d^5*((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[g]*(-128*a^4*e^4*g^4 + 16*
a^3*c*d*e^3*g^3*(11*f - 21*g*x) - 8*a^2*c^2*d^2*e^2*g^2*(f^2 - 64*f*g*x +
31*g^2*x^2) - 2*a*c^3*d^3*e*g*(5*f^3 + 23*f^2*g*x - 233*f*g^2*x^2 + 5*g^3*
x^3) + c^4*d^4*(-15*f^4 - 70*f^3*g*x - 128*f^2*g^2*x^2 + 70*f*g^3*x^3 + 15
*g^4*x^4)))/(c^5*d^5*(c*d*f - a*e*g)^2*(a*e + c*d*x)^2*(f + g*x)^5) + (15*
ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/((c*d*f - a*e*g)^
(5/2)*(a*e + c*d*x)^(5/2))))/(640*g^(7/2)*(d + e*x)^(5/2))
```

3.710.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1249, 1249, 1249, 1254, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}dx}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}$$

$$\Big\downarrow \text{1249}$$

$$\frac{cd\left(\frac{3cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}(f+gx)^4}dx}{8g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}$$

$$\downarrow \text{1249}$$

$$\frac{cd\left(\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}\right)}{8g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}$$

$$\downarrow \text{1254}$$

$$\frac{cd\left(\frac{3cd\left(\frac{cd\left(\frac{3cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}\right)}{8g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}$$

$$\downarrow \text{1254}$$

3.710. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx$

$$cd\left(\frac{3cd\left(\frac{cd\left(\frac{3cd\left(\frac{cd\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}\right)}{8g}\right.$$

$$\overline{\qquad\qquad 2g \qquad\qquad}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}$$

$$\Big\downarrow \text{ 1255}$$

3.710. $\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx$

$$cd\left(\frac{3cd\left(\frac{cd\left(\frac{3cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade-}}{2\sqrt{d+ex}(f+gx)^2(cdf}}{6g}}{8g}}{2g}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}$$

$$\Big\downarrow \text{218}$$

3.710. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx$

$$cd\left(\frac{3cd\left(\frac{cd\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}\cdots\right)}{8g}\cdots\right)}{2g}\cdots$$

$$\frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^6),x]
```

3.710. $\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx$

output
```
-1/5*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(g*(d + e*x)^(5/2)*(f +
 g*x)^5) + (c*d*(-1/4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(d
+ e*x)^(3/2)*(f + g*x)^4) + (3*c*d*(-1/3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x)^3) + (c*d*(Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2]/(2*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c*d*
(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x
]*(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2
))))/(4*(c*d*f - a*e*g))))/(6*g)))/(8*g)))/(2*g)
```

3.710.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.710.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 913 vs. $2(349) = 698$.

Time = 0.57 (sec) , antiderivative size = 914, normalized size of antiderivative = 2.33

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\,\left(15\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^5d^5g^5x^5+75\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^5d^5fg^4x^4+150\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c\ldots\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x,meth
od=_RETURNVERBOSE)
```

output
```
-1/640*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2))*c^5*d^5*g^5*x^5+75*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^5*d^5*f*g^4*x^4+150*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*c^5*d^5*f^2*g^3*x^3+150*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e
*g-c*d*f)*g)^(1/2))*c^5*d^5*f^3*g^2*x^2-15*c^4*d^4*g^4*x^4*(c*d*x+a*e)^(1/
2)*((a*e*g-c*d*f)*g)^(1/2)+75*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g
)^(1/2))*c^5*d^5*f^4*g*x+10*a*c^3*d^3*e*g^4*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-
c*d*f)*g)^(1/2)-70*c^4*d^4*f*g^3*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(
1/2)+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^5*d^5*f^5+2
48*a^2*c^2*d^2*e^2*g^4*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-466*a
*c^3*d^3*e*f*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+128*c^4*d^4
*f^2*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+336*a^3*c*d*e^3*g^4
*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-512*a^2*c^2*d^2*e^2*f*g^3*x*(
c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+46*a*c^3*d^3*e*f^2*g^2*x*(c*d*x+a
*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+70*c^4*d^4*f^3*g*x*(c*d*x+a*e)^(1/2)*((a
*e*g-c*d*f)*g)^(1/2)+128*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^4*e^4
*g^4-176*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^3*c*d*e^3*f*g^3+8*(c*
d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*c^2*d^2*e^2*f^2*g^2+10*(c*d*x+a
*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c^3*d^3*e*f^3*g+15*(c*d*x+a*e)^(1/2)*(
(a*e*g-c*d*f)*g)^(1/2)*c^4*d^4*f^4)/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/...
```

3.710.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1354 vs. $2(349) = 698$.

Time = 1.37 (sec) , antiderivative size = 2750, normalized size of antiderivative = 7.00

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,
x, algorithm="fricas")
```

output
```
[-1/1280*(15*(c^5*d^5*e*g^5*x^6 + c^5*d^6*f^5 + (5*c^5*d^5*e*f*g^4 + c^5*d
^6*g^5)*x^5 + 5*(2*c^5*d^5*e*f^2*g^3 + c^5*d^6*f*g^4)*x^4 + 10*(c^5*d^5*e*
f^3*g^2 + c^5*d^6*f^2*g^3)*x^3 + 5*(c^5*d^5*e*f^4*g + 2*c^5*d^6*f^3*g^2)*x
^2 + (c^5*d^5*e*f^5 + 5*c^5*d^6*f^4*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c
*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e
*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(15*c^5*d^5*f^5*g - 5*a*c^4*
d^4*e*f^4*g^2 - 2*a^2*c^3*d^3*e^2*f^3*g^3 - 184*a^3*c^2*d^2*e^3*f^2*g^4 +
304*a^4*c*d*e^4*f*g^5 - 128*a^5*e^5*g^6 - 15*(c^5*d^5*f*g^5 - a*c^4*d^4*e*
g^6)*x^4 - 10*(7*c^5*d^5*f^2*g^4 - 8*a*c^4*d^4*e*f*g^5 + a^2*c^3*d^3*e^2*g
^6)*x^3 + 2*(64*c^5*d^5*f^3*g^3 - 297*a*c^4*d^4*e*f^2*g^4 + 357*a^2*c^3*d^
3*e^2*f*g^5 - 124*a^3*c^2*d^2*e^3*g^6)*x^2 + 2*(35*c^5*d^5*f^4*g^2 - 12*a*
c^4*d^4*e*f^3*g^3 - 279*a^2*c^3*d^3*e^2*f^2*g^4 + 424*a^3*c^2*d^2*e^3*f*g^
5 - 168*a^4*c*d*e^4*g^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(e*x + d))/(c^3*d^4*f^8*g^4 - 3*a*c^2*d^3*e*f^7*g^5 + 3*a^2*c*d^2*e^2*f^
6*g^6 - a^3*d*e^3*f^5*g^7 + (c^3*d^3*e*f^3*g^9 - 3*a*c^2*d^2*e^2*f^2*g^10
+ 3*a^2*c*d*e^3*f*g^11 - a^3*e^4*g^12)*x^6 + (5*c^3*d^3*e*f^4*g^8 - a^3*d*
e^3*g^12 + (c^3*d^4 - 15*a*c^2*d^2*e^2)*f^3*g^9 - 3*(a*c^2*d^3*e - 5*a^2*c
*d*e^3)*f^2*g^10 + (3*a^2*c*d^2*e^2 - 5*a^3*e^4)*f*g^11)*x^5 + 5*(2*c^3*d^
3*e*f^5*g^7 - a^3*d*e^3*f*g^11 + (c^3*d^4 - 6*a*c^2*d^2*e^2)*f^4*g^8 - ...
```

3.710.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**6,x)
```

output
```
Timed out
```

3.710.7 Maxima [F]

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx = \int \frac{\left(cdex^2 + ade + (cd^2 + ae^2)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^6}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^6), x)
```

3.710.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2407 vs. 2(349) = 698.

Time = 2.83 (sec) , antiderivative size = 2407, normalized size of antiderivative = 6.12

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,
x, algorithm="giac")
```

output
```
3/128*c^5*d^5*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqr
t(c*d*f*g - a*e*g^2)*e))/((c^2*d^2*f^2*g^3 - 2*a*c*d*e*f*g^4 + a^2*e^2*g^5
)*sqrt(c*d*f*g - a*e*g^2)*e) - 1/640*(15*c^5*d^5*e^5*f^5*abs(e)*arctan(sqr
t(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 75*c^5*d^6*e^4*f^4*g*
abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 150*
c^5*d^7*e^3*f^3*g^2*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g -
 a*e*g^2)*e)) - 150*c^5*d^8*e^2*f^2*g^3*abs(e)*arctan(sqrt(-c*d^2*e + a*e^
3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 75*c^5*d^9*e*f*g^4*abs(e)*arctan(sqrt(
-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 15*c^5*d^10*g^5*abs(e)*
arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 15*sqrt(-c*
d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^4*d^4*e^4*f^4*abs(e) + 70*sqrt(-c
*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^4*d^5*e^3*f^3*g*abs(e) - 10*sqrt
(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^3*d^3*e^5*f^3*g*abs(e) - 12
8*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^4*d^6*e^2*f^2*g^2*abs(e
) + 46*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^3*d^4*e^4*f^2*g^
2*abs(e) - 8*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c^2*d^2*e^
6*f^2*g^2*abs(e) - 70*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^4*d
^7*e*f*g^3*abs(e) + 466*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c
^3*d^5*e^3*f*g^3*abs(e) - 512*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^
2)*a^2*c^2*d^3*e^5*f*g^3*abs(e) + 176*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f...
```

3.710.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^6}\, dx = \int \frac{\left(c\, d\, e\, x^2 + \left(c\, d^2 + a\, e^2\right)\, x + a\, d\, e\right)^{5/2}}{\left(f + g\, x\right)^6\, \left(d + e\, x\right)^{5/2}}\, dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^6*(d + e*x)^(
5/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^6*(d + e*x)^(
5/2)), x)
```

3.711 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx$

3.711.1 Optimal result

Integrand size = 46, antiderivative size = 463

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx = -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^4} + \frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{192g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{5c^4d^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{768g^3(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} + \frac{5c^5d^5\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512g^3(cdf-aeg)^3\sqrt{d+ex}(f+gx)} - \frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^5} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6} + \frac{5c^6d^6\arctan\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{512g^{7/2}(cdf-aeg)^{7/2}}$$

output
```
-1/12*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f
)^5-1/6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^6+
5/512*c^6*d^6*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e
*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)/(-a*e*g+c*d*f)^(7/2)-1/32*c^2*d^2*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(g*x+f)^4/(e*x+d)^(1/2)+1/192*c
^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g+c*d*f)/(g*x+f)^
3/(e*x+d)^(1/2)+5/768*c^4*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/
(-a*e*g+c*d*f)^2/(g*x+f)^2/(e*x+d)^(1/2)+5/512*c^5*d^5*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g+c*d*f)^3/(g*x+f)/(e*x+d)^(1/2)
```

3.711.2 Mathematica [A] (verified)

Time = 2.72 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.80

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx = \frac{c^6d^6((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{g}(256a^5e^5g^5+640a^4cde^4g^4(-f+gx)+16a^3c^2d^2\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^7),x]
```

output
```
(c^6*d^6*((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[g]*(256*a^5*e^5*g^5 + 640*
a^4*c*d*e^4*g^4*(-f + g*x) + 16*a^3*c^2*d^2*e^3*g^3*(27*f^2 - 106*f*g*x +
27*g^2*x^2) + 8*a^2*c^3*d^3*e^2*g^2*(-f^3 + 159*f^2*g*x - 159*f*g^2*x^2 +
g^3*x^3) - 2*a*c^4*d^4*e*g*(5*f^4 + 28*f^3*g*x - 594*f^2*g^2*x^2 + 28*f*g^
3*x^3 + 5*g^4*x^4) + c^5*d^5*(-15*f^5 - 85*f^4*g*x - 198*f^3*g^2*x^2 + 198
*f^2*g^3*x^3 + 85*f*g^4*x^4 + 15*g^5*x^5)))/(c^6*d^6*(c*d*f - a*e*g)^3*(a*
e + c*d*x)^2*(f + g*x)^6) + (15*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*
d*f - a*e*g]])/((c*d*f - a*e*g)^(7/2)*(a*e + c*d*x)^(5/2))))/(1536*g^(7/2)
*(d + e*x)^(5/2))
```

3.711.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1249, 1249, 1249, 1254, 1254, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}dx$$

$$\Big\downarrow \ 1249$$

$$\frac{5cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}dx}{12g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6}$$

$$\Big\downarrow \ 1249$$

$$\frac{5cd\left(\frac{3cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}(f+gx)^5}dx}{10g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}\right)}{12g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6}$$

$$\Big\downarrow \text{ 1249}$$

$$\frac{5cd\left(\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}\right)}{10g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}\right)}{12g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6}$$

$$\Big\downarrow \text{ 1254}$$

$$\frac{5cd\left(\frac{3cd\left(\frac{cd\left(\frac{5cd\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}\right)}{8g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}\right)}{10g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex\right.}{5g(d+ex)^{3/2}(f+gx)}\right.}{12g}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6}$$

$$\Big\downarrow \text{ 1254}$$

3.711. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx$

$$5cd\left(\frac{3cd\left(\frac{cd\left(\frac{5cd\left(\frac{3cd\int\frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}\right)}{8g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cde}}{4g\sqrt{d+ex}(f+gx)^4}}{10g}}{12g}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6}$$

$$\Big\downarrow \; 1254$$

3.711. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx$

$$5cd\left(\frac{3cd\left(\frac{cd\left(\frac{5cd\left(\frac{3cd\left(\frac{cd\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+c}}{3\sqrt{d+ex}(f+gx)^3(cdf-}}{8g}}{10g}\right.$$

3.711. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx$

$\downarrow$ 1255

3.711. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx$

$$5cd\left(\frac{3cd\left(\frac{cd\left(\frac{5cd\left(\frac{3cd\left(\frac{cde^2\int\frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right.}}{2\sqrt{d+ex}(f+g}}{6(cdf-aeg)}}{8g}}{10g}$$

3.711. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx$

$\downarrow$ 218

3.711. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx$

$$5cd\left(\frac{3cd\left(\frac{cd\left(\frac{5cd\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+\cdots}}{3\sqrt{d+ex}(f+gx)^3(cdf\cdots}\cdots\right)}{8g}\cdots\right)}{10g}\cdots\right)$$

3.711. $$\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^7),x]
```

output
```
-1/6*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(g*(d + e*x)^(5/2)*(f +
 g*x)^6) + (5*c*d*(-1/5*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(
d + e*x)^(3/2)*(f + g*x)^5) + (3*c*d*(-1/4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x)^4) + (c*d*(Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2]/(3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^3) + (5*c*
d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d*f - a*e*g)*Sqrt[d +
 e*x]*(f + g*x)^2) + (3*c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(
(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(
Sqrt[g]*(c*d*f - a*e*g)^(3/2))))/(4*(c*d*f - a*e*g))))/(6*(c*d*f - a*e*g))
))/(8*g)))/(10*g)))/(12*g)
```

3.711.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

3.711. $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx$

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.711.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1250 vs. $2(413) = 826$.

Time = 0.58 (sec) , antiderivative size = 1251, normalized size of antiderivative = 2.70

method	result	size
default	Expression too large to display	1251

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x,meth
od=_RETURNVERBOSE)
```

output
```
1/1536*((c*d*x+a*e)*(e*x+d))^(1/2)*(-1188*a*c^4*d^4*e*f^2*g^3*x^2*(c*d*x+a
*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-15*c^5*d^5*g^5*x^5*(c*d*x+a*e)^(1/2)*((a
*e*g-c*d*f)*g)^(1/2)+90*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2
))*c^6*d^6*f^5*g*x+225*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)
)*c^6*d^6*f^4*g^2*x^2+225*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1
/2))*c^6*d^6*f^2*g^4*x^4+300*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*c^6*d^6*f^3*g^3*x^3+90*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*
g)^(1/2))*c^6*d^6*f*g^5*x^5+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*
g)^(1/2))*c^6*d^6*f^6-256*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^5*e^
5*g^5+15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^5*d^5*f^5+15*arctanh(
g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^6*d^6*g^6*x^6-432*a^3*c^2*d
^2*e^3*g^5*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-640*a^4*c*d*e^4*g
^5*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+10*a*c^4*d^4*e*g^5*x^4*(c*d
*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+640*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a
*e)^(1/2)*a^4*c*d*e^4*f*g^4-432*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*
a^3*c^2*d^2*e^3*f^2*g^3+8*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c^
3*d^3*e^2*f^3*g^2+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^4*d^4*e
*f^4*g-85*c^5*d^5*f*g^4*x^4*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-198*
c^5*d^5*f^2*g^3*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+198*c^5*d^5*
f^3*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+85*c^5*d^5*f^4*g*...
```

3.711.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1915 vs. 2(413) = 826.

Time = 3.97 (sec) , antiderivative size = 3872, normalized size of antiderivative = 8.36

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^7} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,
x, algorithm="fricas")
```

output
```
[1/3072*(15*(c^6*d^6*e*g^6*x^7 + c^6*d^7*f^6 + (6*c^6*d^6*e*f*g^5 + c^6*d^
7*g^6)*x^6 + 3*(5*c^6*d^6*e*f^2*g^4 + 2*c^6*d^7*f*g^5)*x^5 + 5*(4*c^6*d^6*
e*f^3*g^3 + 3*c^6*d^7*f^2*g^4)*x^4 + 5*(3*c^6*d^6*e*f^4*g^2 + 4*c^6*d^7*f^
3*g^3)*x^3 + 3*(2*c^6*d^6*e*f^5*g + 5*c^6*d^7*f^4*g^2)*x^2 + (c^6*d^6*e*f^
6 + 6*c^6*d^7*f^5*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2
*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2
+ d*f + (e*f + d*g)*x)) - 2*(15*c^6*d^6*f^6*g - 5*a*c^5*d^5*e*f^5*g^2 - 2*
a^2*c^4*d^4*e^2*f^4*g^3 - 440*a^3*c^3*d^3*e^3*f^3*g^4 + 1072*a^4*c^2*d^2*e
^4*f^2*g^5 - 896*a^5*c*d*e^5*f*g^6 + 256*a^6*e^6*g^7 - 15*(c^6*d^6*f*g^6 -
 a*c^5*d^5*e*g^7)*x^5 - 5*(17*c^6*d^6*f^2*g^5 - 19*a*c^5*d^5*e*f*g^6 + 2*a
^2*c^4*d^4*e^2*g^7)*x^4 - 2*(99*c^6*d^6*f^3*g^4 - 127*a*c^5*d^5*e*f^2*g^5
+ 32*a^2*c^4*d^4*e^2*f*g^6 - 4*a^3*c^3*d^3*e^3*g^7)*x^3 + 6*(33*c^6*d^6*f^
4*g^3 - 231*a*c^5*d^5*e*f^3*g^4 + 410*a^2*c^4*d^4*e^2*f^2*g^5 - 284*a^3*c^
3*d^3*e^3*f*g^6 + 72*a^4*c^2*d^2*e^4*g^7)*x^2 + (85*c^6*d^6*f^5*g^2 - 29*a
*c^5*d^5*e*f^4*g^3 - 1328*a^2*c^4*d^4*e^2*f^3*g^4 + 2968*a^3*c^3*d^3*e^3*f
^2*g^5 - 2336*a^4*c^2*d^2*e^4*f*g^6 + 640*a^5*c*d*e^5*g^7)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^10*g^4 - 4*a*c^3
*d^4*e*f^9*g^5 + 6*a^2*c^2*d^3*e^2*f^8*g^6 - 4*a^3*c*d^2*e^3*f^7*g^7 + a^4
*d*e^4*f^6*g^8 + (c^4*d^4*e*f^4*g^10 - 4*a*c^3*d^3*e^2*f^3*g^11 + 6*a^2...
```

3.711.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^7} dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**7,x)
```

output
```
Timed out
```

3.711.7 Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^7}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}(gx + f)^7}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^7), x)
```

3.711.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3412 vs. 2(413) = 826.

Time = 7.62 (sec) , antiderivative size = 3412, normalized size of antiderivative = 7.37

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^7}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,
x, algorithm="giac")
```

output
```
5/512*c^6*d^6*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqr
t(c*d*f*g - a*e*g^2)*e))/((c^3*d^3*f^3*g^3 - 3*a*c^2*d^2*e*f^2*g^4 + 3*a^2
*c*d*e^2*f*g^5 - a^3*e^3*g^6)*sqrt(c*d*f*g - a*e*g^2)*e) - 1/1536*(15*c^6*
d^6*e^6*f^6*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2
)*e)) - 90*c^6*d^7*e^5*f^5*g*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(
c*d*f*g - a*e*g^2)*e)) + 225*c^6*d^8*e^4*f^4*g^2*abs(e)*arctan(sqrt(-c*d^2
*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 300*c^6*d^9*e^3*f^3*g^3*abs(e
)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 225*c^6*d
^10*e^2*f^2*g^4*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e
*g^2)*e)) - 90*c^6*d^11*e*f*g^5*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sq
rt(c*d*f*g - a*e*g^2)*e)) + 15*c^6*d^12*g^6*abs(e)*arctan(sqrt(-c*d^2*e +
a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d
*f*g - a*e*g^2)*c^5*d^5*e^5*f^5*abs(e) + 85*sqrt(-c*d^2*e + a*e^3)*sqrt(c*
d*f*g - a*e*g^2)*c^5*d^6*e^4*f^4*g*abs(e) - 10*sqrt(-c*d^2*e + a*e^3)*sqrt
(c*d*f*g - a*e*g^2)*a*c^4*d^4*e^6*f^4*g*abs(e) - 198*sqrt(-c*d^2*e + a*e^3
)*sqrt(c*d*f*g - a*e*g^2)*c^5*d^7*e^3*f^3*g^2*abs(e) + 56*sqrt(-c*d^2*e +
a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^4*d^5*e^5*f^3*g^2*abs(e) - 8*sqrt(-c*d^
2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c^3*d^3*e^7*f^3*g^2*abs(e) - 198*
sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^5*d^8*e^2*f^2*g^3*abs(e)
+ 1188*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^4*d^6*e^4*f^2...
```

3.711.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{\left(f+g\,x\right)^7\,\left(d+e\,x\right)^{5/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^7*(d + e*x)^(
5/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^7*(d + e*x)^(
5/2)), x)
```

3.712 $\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.712.1 Optimal result

Integrand size = 48, antiderivative size = 313

$$\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8c^3d^3\sqrt{d+ex}}$$
$$+\frac{5(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12c^2d^2\sqrt{d+ex}}$$
$$+\frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3cd\sqrt{d+ex}}$$
$$+\frac{5(cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{7/2}d^{7/2}\sqrt{g}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

output
```
5/8*(-a*e*g+c*d*f)^3*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*
x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(7/2)/d^(7/2)/g^(1/2)/(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5/12*(-a*e*g+c*d*f)*(g*x+f)^(3/2)*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)+1/3*(g*x+f)^(5/2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)+5/8*(-a*e*g+c*d*
f)^2*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/(e*x+d)
^(1/2)
```

3.712.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}(ae+cdx)\sqrt{f+gx}(15a^2e^2g^2-10acdeg(4f+gx)\right.}{24c^{7/2}d^{7/2}\sqrt{(}}$$

input
```
Integrate[(Sqrt[d + e*x]*(f + g*x)^(5/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2],x]
```

output
```
(Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*(a*e + c*d*x)*Sqrt[f + g*x]*(15*a^2*e^2*g^
2 - 10*a*c*d*e*g*(4*f + g*x) + c^2*d^2*(33*f^2 + 26*f*g*x + 8*g^2*x^2)) +
(15*(c*d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f +
g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/Sqrt[g]))/(24*c^(7/2)*d^(7/2)*Sqrt[(a*
e + c*d*x)*(d + e*x)])
```

3.712.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1253, 1253, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \ 1253$$

$$\frac{5(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{6cd}+\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd\sqrt{d+ex}}$$

$$\Big\downarrow \ 1253$$

$$\frac{5(cdf-aeg)\left(\frac{3(cdf-aeg)\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd\sqrt{d+ex}}\right)}{6cd}+\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd\sqrt{d+ex}}$$

$\downarrow$ 1253

$$\frac{5(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+}}{2cd\sqrt{d+ex}}}{6cd}$$

$$\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd\sqrt{d+ex}}$$

$\downarrow$ 1268

$$\frac{5(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+c}}{2cd\sqrt{d+ex}}}{6cd}$$

$$\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd\sqrt{d+ex}}$$

$\downarrow$ 66

$$\frac{5(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int \frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+}}{2cd\sqrt{d-}}}{6cd}$$

$$\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd\sqrt{d+ex}}$$

$\downarrow$ 221

$$\frac{5(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+c}}{2cd\sqrt{d+}}}{6cd}$$

$$\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd\sqrt{d+ex}}$$

3.712. $\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

input
```
Int[(Sqrt[d + e*x]*(f + g*x)^(5/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2],x]
```

output
```
((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*Sqrt[
d + e*x]) + (5*(c*d*f - a*e*g)*(((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(2*c*d*Sqrt[d + e*x]) + (3*(c*d*f - a*e*g)*((Sqrt[f +
g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x]) + ((
c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e +
 c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(4*c*d)))/(6*c*d)
```

3.712.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.712. $\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.712.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.60

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\,\left(15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^3e^3g^3-45\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{c}}{2\sqrt{cdg}}\right.\right.}{}$

input
```
int((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
-1/48*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*ln(1/2*(2*c*d*g*x+a*e*
g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*e^
3*g^3-45*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*
g)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2+45*ln(1/2*(2*c*d*g*x+a*e*g+c*d*
f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*
f^2*g-15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*
g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^3-16*c^2*d^2*g^2*x^2*(c*d*g)^(1/2)*((g*
x+f)*(c*d*x+a*e))^(1/2)+20*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*c*d
*e*g^2*x-52*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c^2*d^2*f*g*x-30*((g
*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2+80*((g*x+f)*(c*d*x+a*e)
)^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g-66*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(
1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/c^3/d^3/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g
)^(1/2)
```

3.712.5 Fricas [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 841, normalized size of antiderivative = 2.69

$$\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \left[\frac{4\,(8\,c^3d^3g^3x^2+33\,c^3d^3f^2g-40\,ac^2d^2efg^2+15\,a^2cde^2g^3+2\,(13\,c^3}{}\right.$$

input
```
integrate((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="fricas")
```

output
```
[1/96*(4*(8*c^3*d^3*g^3*x^2 + 33*c^3*d^3*f^2*g - 40*a*c^2*d^2*e*f*g^2 + 15
*a^2*c*d*e^2*g^3 + 2*(13*c^3*d^3*f*g^2 - 5*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^3*d^4
*f^3 - 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*
d^3*e*f^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*
sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^
2*d*e^2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c
*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g +
 (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*
e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d^4*e*g*x + c^
4*d^5*g), 1/48*(2*(8*c^3*d^3*g^3*x^2 + 33*c^3*d^3*f^2*g - 40*a*c^2*d^2*e*f
*g^2 + 15*a^2*c*d*e^2*g^3 + 2*(13*c^3*d^3*f*g^2 - 5*a*c^2*d^2*e*g^3)*x)*sq
rt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15
*(c^3*d^4*f^3 - 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^
3 + (c^3*d^3*e*f^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4
*g^3)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e
*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^4*d^4*e*g*x + c^4*d^5*g)]
```

3.712.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**(5/2)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(1/2),x)
```

output
```
Timed out
```

3.712.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}(gx+f)^{\frac{5}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

input
```
integrate((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*(g*x + f)^(5/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x), x)
```

3.712.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 989 vs. $2(265) = 530$.

Time = 0.64 (sec) , antiderivative size = 989, normalized size of antiderivative = 3.16

$$\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{e\left(\left(\frac{\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\sqrt{e^2f+(ex+d)eg-deg}\left(2\left(e^2f+(ex+d)e\right.\right.}{}\right.\right.}{}$$

input
```
integrate((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="giac")
```

output
```
1/24*e*((sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c
*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*
g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)*abs(e)/(c*d*e^3*g) + 5*(c^4*d^4*e^2*
f*abs(e) - a*c^3*d^3*e^3*g*abs(e))/(c^5*d^5*e^3*g)) + 15*(c^4*d^4*e^4*f^2*
abs(e) - 2*a*c^3*d^3*e^5*f*g*abs(e) + a^2*c^2*d^2*e^6*g^2*abs(e))/(c^5*d^5
*e^3*g)) - 15*(c^3*d^3*e^3*f^3*abs(e) - 3*a*c^2*d^2*e^4*f^2*g*abs(e) + 3*a
^2*c*d*e^5*f*g^2*abs(e) - a^3*e^6*g^3*abs(e))*log(abs(-sqrt(e^2*f + (e*x +
 d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e
*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^3*d^3))*g/(e^4*abs(g)) + (15*
c^3*d^3*e^4*f^3*g*abs(e)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-
c*d^2*e*g^2 + a*e^3*g^2))) - 45*a*c^2*d^2*e^5*f^2*g^2*abs(e)*log(abs(-sqrt
(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 45*a^2*c*
d*e^6*f*g^3*abs(e)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*
e*g^2 + a*e^3*g^2))) - 15*a^3*e^7*g^4*abs(e)*log(abs(-sqrt(e^2*f - d*e*g)*
sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 33*sqrt(-c*d^2*e*g^2 + a*
e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^2*e^2*f^2*abs(e) + 26*sqrt(
-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^3*e*f*g*ab
s(e) + 40*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a
*c*d*e^3*f*g*abs(e) - 8*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)
*sqrt(c*d*g)*c^2*d^4*g^2*abs(e) - 10*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqr...
```

3.712.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{{\left(f+g\,x\right)}^{5/2}\,\sqrt{d+e\,x}}{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

input
```
int(((f + g*x)^(5/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(1/2),x)
```

output
```
int(((f + g*x)^(5/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(1/2), x)
```

3.713 $\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.713.1 Optimal result

Integrand size = 48, antiderivative size = 244

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^2d^2\sqrt{d+ex}}$$
$$+\frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2cd\sqrt{d+ex}}$$
$$+\frac{3(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{5/2}d^{5/2}\sqrt{g}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

output
```
3/4*(-a*e*g+c*d*f)^2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*
x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2)/g^(1/2)/(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)+3/4*(-a*e*g+c*d*f)*(g*x+f)^(1/2)*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)
```

3.713.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}(ae+cdx)\sqrt{f+gx}(-3aeg+cd(5f+2gx))+\frac{3(cdf}{}\right.}{4c^{5/2}d^{5/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(Sqrt[d + e*x]*(f + g*x)^(3/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2],x]
```

output
```
(Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*(a*e + c*d*x)*Sqrt[f + g*x]*(-3*a*e*g + c*
d*(5*f + 2*g*x)) + (3*(c*d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*ArcTanh[(Sqrt[c]
*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/Sqrt[g]))/(4*c^(5/2)
*d^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.713.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {1253, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1253}$$

$$\frac{3(cdf-aeg)\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd\sqrt{d+ex}}$$

$$\Big\downarrow \text{1253}$$

$$\frac{3(cdf-aeg)\left(\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd\sqrt{d+ex}}$$

$$\Big\downarrow \; 1268$$

$$\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2cd\sqrt{d+ex}}$$

$$\Big\downarrow \; 66$$

$$\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int \frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2cd\sqrt{d+ex}}$$

$$\Big\downarrow \; 221$$

$$\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2cd\sqrt{d+ex}}$$

input
```
Int[(Sqrt[d + e*x]*(f + g*x)^(3/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2],x]
```

output
```
((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*c*d*Sqrt[
d + e*x]) + (3*(c*d*f - a*e*g)*((Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x]) + ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f
 + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])))/(4*c*d)
```

3.713.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && !GtQ[c - a*(d/b), 0]
```

3.713. $\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.713.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.30

method	result
default	$\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^2e^2g^2-6\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)\cdots\right.}{8\sqrt{e\cdots}}$

input
```
int((g*x+f)^(3/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
1/8*(g*x+f)^(1/2)/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*ln(1/2*(2*c
*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1
/2))*a^2*e^2*g^2-6*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(
1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c*d*e*f*g+3*ln(1/2*(2*c*d*g*x+a*e*g+c
*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f
^2+4*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c*d*g*x-6*(c*d*g)^(1/2)*((g
*x+f)*(c*d*x+a*e))^(1/2)*a*e*g+10*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2
)*c*d*f)/((g*x+f)*(c*d*x+a*e))^(1/2)/c^2/d^2/(c*d*g)^(1/2)
```

3.713.5 Fricas [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 655, normalized size of antiderivative = 2.68

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \left[\frac{4\left(2c^2d^2g^2x+5c^2d^2fg-3acdeg^2\right)\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{}}{}\right.$$

input
```
integrate((g*x+f)^(3/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="fricas")
```

output
```
[1/16*(4*(2*c^2*d^2*g^2*x + 5*c^2*d^2*f*g - 3*a*c*d*e*g^2)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^2*d^3*f^2
- 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2
*e^3*g^2)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d
^2*e*f*g + a^2*d*e^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(
2*c*d*g*x + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^
2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d
^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d
^3*e*g*x + c^3*d^4*g), 1/8*(2*(2*c^2*d^2*g^2*x + 5*c^2*d^2*f*g - 3*a*c*d*e
*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x +
 f) - 3*(c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 -
2*a*c*d*e^2*f*g + a^2*e^3*g^2)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*
e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^3*d^
3*e*g*x + c^3*d^4*g)]
```

3.713.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}(f+gx)^{\frac{3}{2}}}{\sqrt{(d+ex)(ae+cdx)}}\,dx$$

input
```
integrate((g*x+f)**(3/2)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)*(f + g*x)**(3/2)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.713.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{ex+d}(gx+f)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

input
```
integrate((g*x+f)^(3/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*(g*x + f)^(3/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x), x)
```

3.713.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(204) = 408$.

Time = 0.52 (sec) , antiderivative size = 633, normalized size of antiderivative = 2.59

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{e\left(\frac{\left(\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\sqrt{e^2f+(ex+d)eg-deg}\left(\frac{2\left(e^2f+(ex+d)eg\right.}{cde^2g}\right.\right.}{}\right.}{}$$

input
```
integrate((g*x+f)^(3/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="giac")
```

output
```
1/4*e*((sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*
d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g
)*abs(e)/(c*d*e^2*g) + 3*(c^2*d^2*e^2*f*abs(e) - a*c*d*e^3*g*abs(e))/(c^3*
d^3*e^2*g)) - 3*(c^2*d^2*e^2*f^2*abs(e) - 2*a*c*d*e^3*f*g*abs(e) + a^2*e^4
*g^2*abs(e))*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sq
rt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sq
rt(c*d*g)*c^2*d^2))*g/(e^3*abs(g)) + (3*c^2*d^2*e^3*f^2*g*abs(e)*log(abs(-
sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 6*a*c
*d*e^4*f*g^2*abs(e)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2
*e*g^2 + a*e^3*g^2))) + 3*a^2*e^5*g^3*abs(e)*log(abs(-sqrt(e^2*f - d*e*g)*
sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 5*sqrt(-c*d^2*e*g^2 + a*e
^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d*e*f*abs(e) + 2*sqrt(-c*d^2*e*g
^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d^2*g*abs(e) + 3*sqrt(-c
*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*e^2*g*abs(e))/(s
qrt(c*d*g)*c^2*d^2*e^4*abs(g)))/abs(e)
```

3.713.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(f+g\,x)^{3/2}\,\sqrt{d+e\,x}}{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int(((f + g*x)^(3/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(1/2),x)
```

output
```
int(((f + g*x)^(3/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(1/2), x)
```

3.714 $\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.714.1 Optimal result

Integrand size = 48, antiderivative size = 169

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cd\sqrt{d+ex}} + \frac{(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

output
```
(-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(
1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(3/2)/d^(3/2)/g^(1/2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)+(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)/c/d/(e*x+d)^(1/2)
```

3.714.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}(ae+cdx)\sqrt{f+gx}+(cdf-aeg)\sqrt{ae+cdx}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(Sqrt[d + e*x]*Sqrt[f + g*x])/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2],x]
```

output
```
(Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*(a*e + c*d*x)*Sqrt[f + g*x] + (c*d
*f - a*e*g)*Sqrt[a*e + c*d*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqr
t[g]*Sqrt[a*e + c*d*x])]))/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[(a*e + c*d*x)*(d
+ e*x)])
```

3.714.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \; 1253$$

$$\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2cd}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

$$\Big\downarrow \; 1268$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

$$\Big\downarrow \; 66$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int \frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

input
```
Int[(Sqrt[d + e*x]*Sqrt[f + g*x])/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2],x]
```

output
```
(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d +
e*x]) + ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*
Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt
[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

3.714.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

3.714. $\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.714.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.13

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)aeg-\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)cdf-\right.}{2\sqrt{ex+d}\,\sqrt{(gx+f)(cdx+ae)}\,cd\sqrt{cdg}}$

input
```
int((g*x+f)^(1/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
-1/2*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(ln(1/2*(2*c*d*g*x+a*e*g+c*
d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*e*g-ln(1
/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*
d*g)^(1/2))*c*d*f-2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(e*x+d)^(1/
2)/((g*x+f)*(c*d*x+a*e))^(1/2)/c/d/(c*d*g)^(1/2)
```

3.714.5 Fricas [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 521, normalized size of antiderivative = 3.08

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \left[\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\sqrt{gx+f}\,cdg-(cd^2f-adeg+(cdef-ae^2g)x)\sqrt{cdg}\log\left(-\frac{8c^2}{\ldots}\right.}{\ldots}\right.$$

input
```
integrate((g*x+f)^(1/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="fricas")
```

output
```
[1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x
 + f)*c*d*g - (c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(c*d*g)*log(
-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 - 4*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*sq
rt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*
d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*
c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d^2*e*g*x + c^2*d^3*g), 1/2*(2
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*c
*d*g - (c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(-c*d*g)*arctan(2*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt
(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2
)*g)*x)))/(c^2*d^2*e*g*x + c^2*d^3*g)]
```

3.714.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((g*x+f)**(1/2)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)*sqrt(f + g*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.714.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{ex+d}\sqrt{gx+f}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*sqrt(g*x + f)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x), x)
```

3.714.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(141) = 282$.

Time = 0.43 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx =$$

$$-\frac{e\left(\frac{g\left(\frac{\left(cdef|e|-ae^2g|e|\right)\log\left(\left|-\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2fg+ae^3g^2+\left(e^2f+(ex+d)eg-deg\right)cdg}\right|\right)}{\sqrt{cdg}cd}-\frac{\sqrt{-cde^2fg+ae^3g^2+\left(e^2f+(ex+d)eg-deg\cdots\right.}}{cdeg}\cdots\right.}{e^2|g|}\cdots\right.}{\cdots}$$

input
```
integrate((g*x+f)^(1/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="giac")
```

output
```
-e*(g*((c*d*e*f*abs(e) - a*e^2*g*abs(e))*log(abs(-sqrt(e^2*f + (e*x + d)*e
*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x +
d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d) - sqrt(-c*d*e^2*f*g + a*e^3*g^2
 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e
*g)*abs(e)/(c*d*e*g))/(e^2*abs(g)) - (c*d*e^2*f*g*abs(e)*log(abs(-sqrt(e^2
*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - a*e^3*g^2*abs
(e)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g
^2))) - sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*abs
(e))/(sqrt(c*d*g)*c*d*e^3*abs(g)))/abs(e)
```

3.714.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int(((f + g*x)^(1/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(1/2),x)
```

output
```
int(((f + g*x)^(1/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(1/2), x)
```

3.715 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.715.1 Optimal result

Integrand size = 48, antiderivative size = 105

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

output
```
2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+
a*e)^(1/2)*(e*x+d)^(1/2)/c^(1/2)/d^(1/2)/g^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)
```

3.715.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]),x]
```

output
```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])
/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[(a*e + c*d*x)
*(d + e*x)])
```

3.715.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{ 1268}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 66}$$

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx}\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2]),x]
```

output
```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sq
rt[c]*Sqrt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])
```

3.715.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.715.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\,\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)}{\sqrt{ex+d}\,\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}}$	102

input
```
int((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
1/(e*x+d)^(1/2)*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*ln(1/2*(2*c*d*g*
x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))/
(c*d*g)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)
```

3.715.5 Fricas [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 343, normalized size of antiderivative = 3.27

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$$
$$= \left[\frac{\sqrt{cdg}\log\left(-\frac{8\,c^2d^2eg^2x^3+c^2d^3f^2+6\,acd^2efg+a^2de^2g^2+4\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,cdgx+cdf+aeg)\sqrt{cdg}\sqrt{ex+d}\sqrt{gx+f}+8\,(c^2d^2efg}{ex+d}\right.}{2\,cdg}\right.$$
$$\left.-\frac{\sqrt{-cdg}\arctan\left(\frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{-cdg}\sqrt{ex+d}\sqrt{gx+f}}{2\,cdegx^2+cd^2f+adeg+(cdef+(2\,cd^2+ae^2)g)x}\right)}{cdg}\right]$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g
 + a^2*d*e^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*
x + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*
f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a
*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d))/(c*d*g), -sqrt(
-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*
sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f
+ (2*c*d^2 + a*e^2)*g)*x))/(c*d*g)]
```

3.715.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)(ae+cdx)}\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*sqrt(f + g*x)), x)
```

3.715.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
g*x + f)), x)
```

3.715.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{2\,e^3\left(\frac{g\log\left(\left|-\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\right|\right)}{\sqrt{cdg}e|g|}-\frac{g\log\left(\left|-\sqrt{e^2f-deg}\sqrt{cdg}+\sqrt{-cd^2eg^2+a\cdots}\right|\right)}{\sqrt{cdg}e|g|}\right)}{|e|^2}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="giac")
```

output
```
-2*e^3*(g*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(
-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(
c*d*g)*e*abs(g)) - g*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^
2*e*g^2 + a*e^3*g^2)))/(sqrt(c*d*g)*e*abs(g)))/abs(e)^2
```

3.715.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{\sqrt{d+e\,x}}{\sqrt{f+g\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

input
```
int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2)),x)
```

output
```
int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2)), x)
```

3.716 $\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.716.1 Optimal result

Integrand size = 48, antiderivative size = 61

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}\sqrt{f+gx}}$$

output
```
2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(e*x+d)^(1/2)/(g*
x+f)^(1/2)
```

3.716.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}}{(cdf-aeg)\sqrt{d+ex}\sqrt{f+gx}}$$

input
```
Integrate[Sqrt[d + e*x]/((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2]),x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)])/((c*d*f - a*e*g)*Sqrt[d + e*x]*Sqrt[f +
g*x])
```

3.716.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1248}$$

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)}$$

input
```
Int[Sqrt[d + e*x]/((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]),x]
```

output
```
(2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)*Sqrt[d +
e*x]*Sqrt[f + g*x])
```

3.716.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

3.716.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}}{\sqrt{ex+d}\,\sqrt{gx+f}\,(aeg-cdf)}$	45
gosper	$-\frac{2(cdx+ae)\sqrt{ex+d}}{\sqrt{gx+f}\,(aeg-cdf)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	63

input
```
int((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/(e*x+d)^(1/2)/(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/(a*e*g-c*d*f)
```

3.716.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(55) = 110.

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.87

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\sqrt{gx}}{cd^2f^2-adefg+(cdefg-ae^2g^2)x^2+(cdef^2-adeg^2+}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="fricas")
```

output
```
2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/
(c*d^2*f^2 - a*d*e*f*g + (c*d*e*f*g - a*e^2*g^2)*x^2 + (c*d*e*f^2 - a*d*e*
g^2 + (c*d^2 - a*e^2)*f*g)*x)
```

3.716.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**(3/2)), x
)
```

3.716.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
+ f)^(3/2)), x)
```

3.716.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(55) = 110$.

Time = 0.31 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.18

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=$$
$$-\frac{2\sqrt{cdg}e^2g}{cde^2fg|g|-cd^2eg^2|g|-\sqrt{-cd^2eg^2+ae^3g^2}\sqrt{e^2f-deg}\sqrt{cdg}|g|}$$
$$+\frac{4\sqrt{cdg}e^2g}{\left(cde^2fg-ae^3g^2+\left(\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}-\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cd\right.\right.}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="giac")
```

output
```
-2*sqrt(c*d*g)*e^2*g/(c*d*e^2*f*g*abs(g) - c*d^2*e*g^2*abs(g) - sqrt(-c*d^
2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*abs(g)) + 4*sqrt(c*d*
g)*e^2*g/((c*d*e^2*f*g - a*e^3*g^2 + (sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*
sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e
*g)*c*d*g))^2)*abs(g))
```

3.716.9 Mupad [B] (verification not implemented)

Time = 13.99 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{2\sqrt{d+e\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\left(x\,\sqrt{f+g\,x}-\frac{\sqrt{f+g\,x}\,\left(c\,d^2\,f-a\,d\,e\,g\right)}{a\,e^2\,g-c\,d\,e\,f}\right)\,\left(a\,e^2\,g-c\,d\,e\,f\right)}$$

input
```
int((d + e*x)^(1/2)/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2)),x)
```

output
```
-(2*(d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((x*(f
+ g*x)^(1/2) - ((f + g*x)^(1/2)*(c*d^2*f - a*d*e*g))/(a*e^2*g - c*d*e*f))*
(a*e^2*g - c*d*e*f))
```

3.717 $\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.717.1 Optimal result

Integrand size = 48, antiderivative size = 129

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}(f+gx)^{3/2}} + \frac{4cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3(cdf-aeg)^2\sqrt{d+ex}\sqrt{f+gx}}$$

output
```
2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(3/2)/(
e*x+d)^(1/2)+4/3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f
)^2/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

3.717.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(-aeg+cd(3f+2gx))}{3(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{3/2}}$$

input
```
Integrate[Sqrt[d + e*x]/((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2]),x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(a*e*g) + c*d*(3*f + 2*g*x)))/(3*(c*d*f
 - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(3/2))
```

3.717.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1254}$$

$$\frac{2cd\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{3(cdf-aeg)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1248}$$

$$\frac{4cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}$$

input
```
Int[Sqrt[d + e*x]/((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]),x]
```

output
```
(2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)*Sqrt[d
+ e*x]*(f + g*x)^(3/2)) + (4*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f + g*x])
```

3.717.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

3.717.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-2cdgx+aeg-3cdf)}{3\sqrt{ex+d}\,(gx+f)^{\frac{3}{2}}\,(aeg-cdf)^{2}}$	61
gosper	$-\frac{2(cdx+ae)(-2cdgx+aeg-3cdf)\sqrt{ex+d}}{3(gx+f)^{\frac{3}{2}}\,(a^{2}e^{2}g^{2}-2acdefg+c^{2}d^{2}f^{2})\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}}$	98

input
```
int((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)/(g*x+f)^(3/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-2*c*d*g*x+a
*e*g-3*c*d*f)/(a*e*g-c*d*f)^2
```

3.717.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(113) = 226.

Time = 0.43 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.23

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{}{3\left(c^2d^3f^4-2\,acd^2ef^3g+a^2de^2f^2g^2+\left(c^2d^2ef^2g^2-2\,ac\right.\right.}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + 3*c*d*f - a*e
*g)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^3*f^4 - 2*a*c*d^2*e*f^3*g + a^2*d*e
^2*f^2*g^2 + (c^2*d^2*e*f^2*g^2 - 2*a*c*d*e^2*f*g^3 + a^2*e^3*g^4)*x^3 + (
2*c^2*d^2*e*f^3*g + a^2*d*e^2*g^4 + (c^2*d^3 - 4*a*c*d*e^2)*f^2*g^2 - 2*(a
*c*d^2*e - a^2*e^3)*f*g^3)*x^2 + (c^2*d^2*e*f^4 + 2*a^2*d*e^2*f*g^3 + 2*(c
^2*d^3 - a*c*d*e^2)*f^3*g - (4*a*c*d^2*e - a^2*e^3)*f^2*g^2)*x)
```

3.717.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**(5/2)), x
)
```

3.717.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
+ f)^(5/2)), x)
```

3.717.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(113) = 226$.

Time = 0.35 (sec) , antiderivative size = 676, normalized size of antiderivative = 5.24

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{8\left(cde^2fg - ae^3g^2 + 3\left(\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}\right.\right.}{3\left(cde^2fg - ae^3g^2 + \left(\sqrt{e^2f+(ex+d)eg-deg}\sqrt{c\right.\right.}$$

$$-\frac{}{3\left(c^2d^2e^4f^3g|g| - 6\,c^2d^3e^3f^2g^2|g| + 3\,acde^5f^2g^2|g| + 9\,c^2d^4e^2fg^3|g| - 6\,acd^2e^4fg^3|g| - 4\,c^2d^5eg^4|g| + 3\,ac\right.}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="giac")
```

output
```
8/3*(c*d*e^2*f*g - a*e^3*g^2 + 3*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt
(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*
c*d*g))^2)*sqrt(c*d*g)*c*d*e^4*g^2/((c*d*e^2*f*g - a*e^3*g^2 + (sqrt(e^2*f
 + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e
^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2)^3*abs(g)) - 2/3*(2*sqrt(c*d*g)*c*
d*e^4*f*g - 3*sqrt(c*d*g)*c*d^2*e^3*g^2 + sqrt(c*d*g)*a*e^5*g^2 - 3*sqrt(-
c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*c*d*e^2*g)/(c^2*d^2*e^4*f^3*g
*abs(g) - 6*c^2*d^3*e^3*f^2*g^2*abs(g) + 3*a*c*d*e^5*f^2*g^2*abs(g) + 9*c^
2*d^4*e^2*f*g^3*abs(g) - 6*a*c*d^2*e^4*f*g^3*abs(g) - 4*c^2*d^5*e*g^4*abs(
g) + 3*a*c*d^3*e^3*g^4*abs(g) - 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*
f - d*e*g)*sqrt(c*d*g)*c*d*e^2*f^2*abs(g) + 7*sqrt(-c*d^2*e*g^2 + a*e^3*g^
2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d^2*e*f*g*abs(g) - sqrt(-c*d^2*e*g^2
+ a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*e^3*f*g*abs(g) - 4*sqrt(-c*
d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d^3*g^2*abs(g) +
sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*d*e^2*g^2
*abs(g))
```

3.717.9 Mupad [B] (verification not implemented)

Time = 14.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{\left(\frac{(2\,a\,e\,g-6\,c\,d\,f)\,\sqrt{d+e\,x}}{3\,e\,g\,(a\,e\,g-c\,d\,f)^2}-\frac{4\,c\,d\,x\,\sqrt{d+e\,x}}{3\,e\,(a\,e\,g-c\,d\,f)^2}\right)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{x^2\,\sqrt{f+g\,x}+\frac{d\,f\,\sqrt{f+g\,x}}{e\,g}+\frac{x\,\sqrt{f+g\,x}\,(d\,g+e\,f)}{e\,g}}$$

input
```
int((d + e*x)^(1/2)/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2)),x)
```

output
```
-((((2*a*e*g - 6*c*d*f)*(d + e*x)^(1/2))/(3*e*g*(a*e*g - c*d*f)^2) - (4*c*
d*x*(d + e*x)^(1/2))/(3*e*(a*e*g - c*d*f)^2))*(x*(a*e^2 + c*d^2) + a*d*e +
 c*d*e*x^2)^(1/2))/(x^2*(f + g*x)^(1/2) + (d*f*(f + g*x)^(1/2))/(e*g) + (x
*(f + g*x)^(1/2)*(d*g + e*f))/(e*g))
```

3.718 $\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.718.1 Optimal result

Integrand size = 48, antiderivative size = 198

$$\begin{aligned}\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx &= \frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5(cdf-aeg)\sqrt{d+ex}(f+gx)^{5/2}} \\ &+ \frac{8cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{15(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{3/2}} + \frac{16c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{15(cdf-aeg)^3\sqrt{d+ex}\sqrt{f+gx}}\end{aligned}$$

output
```
2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(5/2)/(
e*x+d)^(1/2)+8/15*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*
f)^2/(g*x+f)^(3/2)/(e*x+d)^(1/2)+16/15*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

3.718.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(3a^2e^2g^2-2acdeg(5f+2gx)+c^2}{15(cdf-aeg)^3\sqrt{d+ex}(f+gx}$$

input
```
Integrate[Sqrt[d + e*x]/((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2]),x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(3*a^2*e^2*g^2 - 2*a*c*d*e*g*(5*f + 2*g*x
) + c^2*d^2*(15*f^2 + 20*f*g*x + 8*g^2*x^2)))/(15*(c*d*f - a*e*g)^3*Sqrt[d
 + e*x]*(f + g*x)^(5/2))
```

3.718.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1254, 1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1254}$$

$$\frac{4cd\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{5(cdf-aeg)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$\frac{4cd\left(\frac{2cd\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3(cdf-aeg)}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}\right)}{5(cdf-aeg)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1248}$$

$$\frac{4cd\left(\frac{4cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}\right)}{5(cdf-aeg)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}$$

input
```
Int[Sqrt[d + e*x]/((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]),x]
```

output
```
(2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d*f - a*e*g)*Sqrt[d
+ e*x]*(f + g*x)^(5/2)) + (4*c*d*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^(3/2)) + (4*c*d*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*S
qrt[f + g*x])))/(5*(c*d*f - a*e*g))
```

3.718.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

3.718.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(8g^2x^2c^2d^2-4acde\,g^2x+20c^2d^2fgx+3a^2e^2g^2-10acdefg+15c^2d^2f^2)}{15\sqrt{ex+d}\,(gx+f)^{\frac{5}{2}}(aeg-cdf)^3}$	111
gosper	$-\frac{2(cdx+ae)(8g^2x^2c^2d^2-4acde\,g^2x+20c^2d^2fgx+3a^2e^2g^2-10acdefg+15c^2d^2f^2)\sqrt{ex+d}}{15(gx+f)^{\frac{5}{2}}(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3c^3d^3)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	169

input
```
int((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/15/(e*x+d)^(1/2)/(g*x+f)^(5/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(8*c^2*d^2*g
^2*x^2-4*a*c*d*e*g^2*x+20*c^2*d^2*f*g*x+3*a^2*e^2*g^2-10*a*c*d*e*f*g+15*c^
2*d^2*f^2)/(a*e*g-c*d*f)^3
```

3.718.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(174) = 348$.

Time = 1.04 (sec) , antiderivative size = 572, normalized size of antiderivative = 2.89

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{}{15\,(c^3d^4f^6-3\,ac^2d^3ef^5g+3\,a^2cd^2e^2f^4g^2-a^3de^3f^3g^3+}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="fricas")
```

output
```
2/15*(8*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 10*a*c*d*e*f*g + 3*a^2*e^2*g^2
+ 4*(5*c^2*d^2*f*g - a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^6 - 3*a*c^2*d^3*e*f^5*g + 3*
a^2*c*d^2*e^2*f^4*g^2 - a^3*d*e^3*f^3*g^3 + (c^3*d^3*e*f^3*g^3 - 3*a*c^2*d
^2*e^2*f^2*g^4 + 3*a^2*c*d*e^3*f*g^5 - a^3*e^4*g^6)*x^4 + (3*c^3*d^3*e*f^4
*g^2 - a^3*d*e^3*g^6 + (c^3*d^4 - 9*a*c^2*d^2*e^2)*f^3*g^3 - 3*(a*c^2*d^3*
e - 3*a^2*c*d*e^3)*f^2*g^4 + 3*(a^2*c*d^2*e^2 - a^3*e^4)*f*g^5)*x^3 + 3*(c
^3*d^3*e*f^5*g - a^3*d*e^3*f*g^5 + (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^4*g^2 - 3
*(a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^3 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^4
)*x^2 + (c^3*d^3*e*f^6 - 3*a^3*d*e^3*f^2*g^4 + 3*(c^3*d^4 - a*c^2*d^2*e^2)
*f^5*g - 3*(3*a*c^2*d^3*e - a^2*c*d*e^3)*f^4*g^2 + (9*a^2*c*d^2*e^2 - a^3*
e^4)*f^3*g^3)*x)
```

3.718.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(1/2)/(g*x+f)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(1/2),x)
```

output
```
Timed out
```

3.718.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
+ f)^(7/2)), x)
```

3.718.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1612 vs. 2(174) = 348.

Time = 0.42 (sec) , antiderivative size = 1612, normalized size of antiderivative = 8.14

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="giac")
```

output
```
32/15*(c^2*d^2*e^4*f^2*g^2 - 2*a*c*d*e^5*f*g^3 + a^2*e^6*g^4 + 5*(sqrt(e^2
*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 +
(e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*c*d*e^2*f*g - 5*(sqrt(e^2*f + (e
*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f
+ (e*x + d)*e*g - d*e*g)*c*d*g))^2*a*e^3*g^2 + 10*(sqrt(e^2*f + (e*x + d)*
e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x +
 d)*e*g - d*e*g)*c*d*g))^4)*sqrt(c*d*g)*c^2*d^2*e^6*g^3/((c*d*e^2*f*g - a*
e^3*g^2 + (sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2
*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2)^5*abs(g)) -
2/15*(8*sqrt(c*d*g)*c^2*d^2*e^6*f^2*g - 45*sqrt(c*d*g)*c^2*d^3*e^5*f*g^2 +
 29*sqrt(c*d*g)*a*c*d*e^7*f*g^2 + 40*sqrt(c*d*g)*c^2*d^4*e^4*g^3 - 35*sqrt
(c*d*g)*a*c*d^2*e^6*g^3 + 3*sqrt(c*d*g)*a^2*e^8*g^3 - 25*sqrt(-c*d^2*e*g^2
 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*c^2*d^2*e^4*f*g + 40*sqrt(-c*d^2*e*g^2 +
 a*e^3*g^2)*sqrt(e^2*f - d*e*g)*c^2*d^3*e^3*g^2 - 15*sqrt(-c*d^2*e*g^2 + a
*e^3*g^2)*sqrt(e^2*f - d*e*g)*a*c*d*e^5*g^2)/(c^3*d^3*e^6*f^5*g*abs(g) - 1
5*c^3*d^4*e^5*f^4*g^2*abs(g) + 10*a*c^2*d^2*e^7*f^4*g^2*abs(g) + 55*c^3*d^
5*e^4*f^3*g^3*abs(g) - 50*a*c^2*d^3*e^6*f^3*g^3*abs(g) + 5*a^2*c*d*e^8*f^3
*g^3*abs(g) - 85*c^3*d^6*e^3*f^2*g^4*abs(g) + 90*a*c^2*d^4*e^5*f^2*g^4*abs
(g) - 15*a^2*c*d^2*e^7*f^2*g^4*abs(g) + 60*c^3*d^7*e^2*f*g^5*abs(g) - 70*a
*c^2*d^5*e^4*f*g^5*abs(g) + 15*a^2*c*d^3*e^6*f*g^5*abs(g) - 16*c^3*d^8*...
```

3.718.9 Mupad [B] (verification not implemented)

Time = 14.54 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{\left(\frac{\sqrt{d+e\,x}\,(6\,a^2\,e^2\,g^2-20\,a\,c\,d\,e\,f\,g+30\,c^2\,d^2\,f^2)}{15\,e\,g^2\,(a\,e\,g-c\,d\,f)^3}+\frac{16\,c^2\,d^2\,x^2\,\sqrt{d+e\,x}}{15\,e\,(a\,e\,g-c\,d\,f)^3}-\frac{8\,c\,d\,x\,(a\,e\,g-5\,c\,d\,f)\,\sqrt{d+e\,x}}{15\,e\,g\,(a\,e\,g-c\,d\,f)^3}\right)\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x}}{x^3\,\sqrt{f+g\,x}+\frac{d\,f^2\,\sqrt{f+g\,x}}{e\,g^2}+\frac{x^2\,\sqrt{f+g\,x}\,(d\,g+2\,e\,f)}{e\,g}+\frac{f\,x\,\sqrt{f+g\,x}\,(2\,d\,g+e\,f)}{e\,g^2}}$$

input
```
int((d + e*x)^(1/2)/((f + g*x)^(7/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2)),x)
```

output
```
-((((d + e*x)^(1/2)*(6*a^2*e^2*g^2 + 30*c^2*d^2*f^2 - 20*a*c*d*e*f*g))/(15
*e*g^2*(a*e*g - c*d*f)^3) + (16*c^2*d^2*x^2*(d + e*x)^(1/2))/(15*e*(a*e*g
- c*d*f)^3) - (8*c*d*x*(a*e*g - 5*c*d*f)*(d + e*x)^(1/2))/(15*e*g*(a*e*g -
 c*d*f)^3))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^3*(f + g*x)^
(1/2) + (d*f^2*(f + g*x)^(1/2))/(e*g^2) + (x^2*(f + g*x)^(1/2)*(d*g + 2*e*
f))/(e*g) + (f*x*(f + g*x)^(1/2)*(2*d*g + e*f))/(e*g^2))
```

3.719 $\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.719.1 Optimal result

Integrand size = 48, antiderivative size = 267

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}(f+gx)^{7/2}} + \frac{12cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{35(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{5/2}} + \frac{16c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{35(cdf-aeg)^3\sqrt{d+ex}(f+gx)^{3/2}} + \frac{32c^3d^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{35(cdf-aeg)^4\sqrt{d+ex}\sqrt{f+gx}}$$

output
```
2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(7/2)/(
e*x+d)^(1/2)+12/35*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d
*f)^2/(g*x+f)^(5/2)/(e*x+d)^(1/2)+16/35*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)/(-a*e*g+c*d*f)^3/(g*x+f)^(3/2)/(e*x+d)^(1/2)+32/35*c^3*d^3*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(1/2)/(g*x
+f)^(1/2)
```

3.719.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(-5a^3e^3g^3+3a^2cde^2g^2(7f+2gx)}{35(ca}$$

input
```
Integrate[Sqrt[d + e*x]/((f + g*x)^(9/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2]),x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-5*a^3*e^3*g^3 + 3*a^2*c*d*e^2*g^2*(7*f
+ 2*g*x) - a*c^2*d^2*e*g*(35*f^2 + 28*f*g*x + 8*g^2*x^2) + c^3*d^3*(35*f^3
 + 70*f^2*g*x + 56*f*g^2*x^2 + 16*g^3*x^3)))/(35*(c*d*f - a*e*g)^4*Sqrt[d
+ e*x]*(f + g*x)^(7/2))
```

3.719.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1254, 1254, 1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1254}$$

$$\frac{6cd\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{7(cdf-aeg)}+\frac{2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{7\sqrt{d+ex}(f+gx)^{7/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$\frac{6cd\left(\frac{4cd\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{5(cdf-aeg)}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}\right)}{7(cdf-aeg)}+\frac{2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{7\sqrt{d+ex}(f+gx)^{7/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$\frac{6cd\left(\frac{4cd\left(\frac{2cd\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3(cdf-aeg)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}\right)}{5(cdf-aeg)}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}\right)}{7(cdf-aeg)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7\sqrt{d+ex}(f+gx)^{7/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1248}$$

$$\frac{6cd\left(\frac{4cd\left(\frac{4cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}\right)}{5(cdf-aeg)}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}\right)}{7(cdf-aeg)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7\sqrt{d+ex}(f+gx)^{7/2}(cdf-aeg)}$$

input
```
Int[Sqrt[d + e*x]/((f + g*x)^(9/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]),x]
```

output
```
(2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*(c*d*f - a*e*g)*Sqrt[d
+ e*x]*(f + g*x)^(7/2)) + (6*c*d*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(5*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^(5/2)) + (4*c*d*((2*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)*Sqrt[d + e*x]
*(f + g*x)^(3/2)) + (4*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3
*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f + g*x])))/(5*(c*d*f - a*e*g))))/(7
*(c*d*f - a*e*g))
```

3.719.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

3.719. $\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

3.719.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.69

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-16g^3x^3c^3d^3+8a\,c^2d^2e\,g^3x^2-56c^3d^3f\,g^2x^2-6a^2cd\,e^2g^3x+28a\,c^2d^2ef\,g^2x-70c^3d^3f^2gx+5a^3e^3g^3-21a^2cd}{35\sqrt{ex+d}\,(gx+f)^{\frac{7}{2}}\,(aeg-cdf)^4}$
gosper	$-\frac{2(cdx+ae)(-16g^3x^3c^3d^3+8a\,c^2d^2e\,g^3x^2-56c^3d^3f\,g^2x^2-6a^2cd\,e^2g^3x+28a\,c^2d^2ef\,g^2x-70c^3d^3f^2gx+5a^3e^3g^3-21a^2cd\,e^2f\,g^2+3}{35(gx+f)^{\frac{7}{2}}\,(a^4e^4g^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4c^4d^4)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+c}}$

input
```
int((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/35/(e*x+d)^(1/2)/(g*x+f)^(7/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-16*c^3*d^3
*g^3*x^3+8*a*c^2*d^2*e*g^3*x^2-56*c^3*d^3*f*g^2*x^2-6*a^2*c*d*e^2*g^3*x+28
*a*c^2*d^2*e*f*g^2*x-70*c^3*d^3*f^2*g*x+5*a^3*e^3*g^3-21*a^2*c*d*e^2*f*g^2
+35*a*c^2*d^2*e*f^2*g-35*c^3*d^3*f^3)/(a*e*g-c*d*f)^4
```

3.719.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 953 vs. 2(235) = 470.

Time = 2.75 (sec) , antiderivative size = 953, normalized size of antiderivative = 3.57

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{}{35\,(c^4d^5f^8-4\,ac^3d^4ef^7g+6\,a^2c^2d^3e^2f^6g^2-4\,a^3cd^2e^3f^5}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="fricas")
```

output
```
2/35*(16*c^3*d^3*g^3*x^3 + 35*c^3*d^3*f^3 - 35*a*c^2*d^2*e*f^2*g + 21*a^2*
c*d*e^2*f*g^2 - 5*a^3*e^3*g^3 + 8*(7*c^3*d^3*f*g^2 - a*c^2*d^2*e*g^3)*x^2
+ 2*(35*c^3*d^3*f^2*g - 14*a*c^2*d^2*e*f*g^2 + 3*a^2*c*d*e^2*g^3)*x)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^4*d^
5*f^8 - 4*a*c^3*d^4*e*f^7*g + 6*a^2*c^2*d^3*e^2*f^6*g^2 - 4*a^3*c*d^2*e^3*
f^5*g^3 + a^4*d*e^4*f^4*g^4 + (c^4*d^4*e*f^4*g^4 - 4*a*c^3*d^3*e^2*f^3*g^5
 + 6*a^2*c^2*d^2*e^3*f^2*g^6 - 4*a^3*c*d*e^4*f*g^7 + a^4*e^5*g^8)*x^5 + (4
*c^4*d^4*e*f^5*g^3 + a^4*d*e^4*g^8 + (c^4*d^5 - 16*a*c^3*d^3*e^2)*f^4*g^4
- 4*(a*c^3*d^4*e - 6*a^2*c^2*d^2*e^3)*f^3*g^5 + 2*(3*a^2*c^2*d^3*e^2 - 8*a
^3*c*d*e^4)*f^2*g^6 - 4*(a^3*c*d^2*e^3 - a^4*e^5)*f*g^7)*x^4 + 2*(3*c^4*d^
4*e*f^6*g^2 + 2*a^4*d*e^4*f*g^7 + 2*(c^4*d^5 - 6*a*c^3*d^3*e^2)*f^5*g^3 -
2*(4*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^4*g^4 + 12*(a^2*c^2*d^3*e^2 - a^3*
c*d*e^4)*f^3*g^5 - (8*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^2*g^6)*x^3 + 2*(2*c^4*d
^4*e*f^7*g + 3*a^4*d*e^4*f^2*g^6 + (3*c^4*d^5 - 8*a*c^3*d^3*e^2)*f^6*g^2 -
 12*(a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^5*g^3 + 2*(9*a^2*c^2*d^3*e^2 - 4*a^3
*c*d*e^4)*f^4*g^4 - 2*(6*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^5)*x^2 + (c^4*d^4*
e*f^8 + 4*a^4*d*e^4*f^3*g^5 + 4*(c^4*d^5 - a*c^3*d^3*e^2)*f^7*g - 2*(8*a*c
^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^2 + 4*(6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4
)*f^5*g^3 - (16*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^4)*x)
```

3.719.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(1/2)/(g*x+f)**(9/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(1/2),x)
```

output
```
Timed out
```

3.719.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{9}{2}}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
+ f)^(9/2)), x)
```

3.719.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2971 vs. $2(235) = 470$.

Time = 0.55 (sec) , antiderivative size = 2971, normalized size of antiderivative = 11.13

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="giac")
```

output
```
64/35*(c^3*d^3*e^6*f^3*g^3 - 3*a*c^2*d^2*e^7*f^2*g^4 + 3*a^2*c*d*e^8*f*g^5
 - a^3*e^9*g^6 + 7*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt
(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*c^2*
d^2*e^4*f^2*g^2 - 14*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sq
rt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*a*
c*d*e^5*f*g^3 + 7*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(
-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*a^2*e
^6*g^4 + 21*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e
^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^4*c*d*e^2*f*g
 - 21*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g
 + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^4*a*e^3*g^2 + 35*(s
qrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3
*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^6)*sqrt(c*d*g)*c^3*d^3*e^8*
g^4/((c*d*e^2*f*g - a*e^3*g^2 + (sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(
c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c
*d*g))^2)^7*abs(g)) - 2/35*(16*sqrt(c*d*g)*c^3*d^3*e^8*f^3*g - 203*sqrt(c*
d*g)*c^3*d^4*e^7*f^2*g^2 + 155*sqrt(c*d*g)*a*c^2*d^2*e^9*f^2*g^2 + 462*sqr
t(c*d*g)*c^3*d^5*e^6*f*g^3 - 518*sqrt(c*d*g)*a*c^2*d^3*e^8*f*g^3 + 104*sqr
t(c*d*g)*a^2*c*d*e^10*f*g^3 - 280*sqrt(c*d*g)*c^3*d^6*e^5*g^4 + 378*sqrt(c
*d*g)*a*c^2*d^4*e^7*g^4 - 119*sqrt(c*d*g)*a^2*c*d^2*e^9*g^4 + 5*sqrt(c*...
```

3.719.9 Mupad [B] (verification not implemented)

Time = 14.09 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx =$$

$$-\frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\left(\frac{\sqrt{d+e\,x}\,(10\,a^3\,e^3\,g^3-42\,a^2\,c\,d\,e^2\,f\,g^2+70\,a\,c^2\,d^2\,e\,f^2\,g-70\,c^3\,d^3\,f^3)}{35\,e\,g^3\,(a\,e\,g-c\,d\,f)^4}-\frac{32\,c^3\,d^3\,x^3\,\sqrt{d+e\,x}}{35\,e\,(a\,e\,g-c\,d\,f)^4}-\cdots\right.}{x^4\,\sqrt{f+g\,x}+\frac{d\,f^3\,\sqrt{f+g\,x}}{e\,g^3}+\frac{x^3\,\sqrt{f+g\,x}\,(d\,g+3\,e\,f)}{e\,g}+\frac{3\,f\,x^2\,\sqrt{f+g\,x}\,(d\,g+\cdots}{e\,g^2}}$$

input
```
int((d + e*x)^(1/2)/((f + g*x)^(9/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2)),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(10*a^3*
e^3*g^3 - 70*c^3*d^3*f^3 + 70*a*c^2*d^2*e*f^2*g - 42*a^2*c*d*e^2*f*g^2))/(
35*e*g^3*(a*e*g - c*d*f)^4) - (32*c^3*d^3*x^3*(d + e*x)^(1/2))/(35*e*(a*e*
g - c*d*f)^4) - (4*c*d*x*(d + e*x)^(1/2)*(3*a^2*e^2*g^2 + 35*c^2*d^2*f^2 -
 14*a*c*d*e*f*g))/(35*e*g^2*(a*e*g - c*d*f)^4) + (16*c^2*d^2*x^2*(a*e*g -
7*c*d*f)*(d + e*x)^(1/2))/(35*e*g*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2
) + (d*f^3*(f + g*x)^(1/2))/(e*g^3) + (x^3*(f + g*x)^(1/2)*(d*g + 3*e*f))/
(e*g) + (3*f*x^2*(f + g*x)^(1/2)*(d*g + e*f))/(e*g^2) + (f^2*x*(f + g*x)^(
1/2)*(3*d*g + e*f))/(e*g^3))
```

3.719. $\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.720 $\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

3.720.1 Optimal result

Integrand size = 48, antiderivative size = 301

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4c^3d^3\sqrt{d+ex}} + \frac{5g(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2c^2d^2\sqrt{d+ex}} + \frac{15\sqrt{g}(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{7/2}d^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

output
```
-2*(g*x+f)^(5/2)*(e*x+d)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
+15/4*(-a*e*g+c*d*f)^2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(
g*x+f)^(1/2))*g^(1/2)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(7/2)/d^(7/2)/(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5/2*g*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)+15/4*g*(-a*e*g+c*d*f)*(g*x+f)^
(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/(e*x+d)^(1/2)
```

3.720.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.61

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{f+gx}(-15a^2e^2g^2-5acdeg(-5f+gx)+c^2d^2(\right.}{4c^{7/2}d^{7/2}}$$

input
```
Integrate[((d + e*x)^(3/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(3/2),x]
```

output
```
(Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x]*(-15*a^2*e^2*g^2 - 5*a*c*d*e
*g*(-5*f + g*x) + c^2*d^2*(-8*f^2 + 9*f*g*x + 2*g^2*x^2)) + 15*Sqrt[g]*(c*
d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(
Sqrt[g]*Sqrt[a*e + c*d*x])]))/(4*c^(7/2)*d^(7/2)*Sqrt[(a*e + c*d*x)*(d + e
*x)])
```

3.720.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1251, 1253, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1251}$$

$$\frac{5g\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{1253}$$

$$\frac{5g\left(\frac{3(cdf-aeg)\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd\sqrt{d+ex}}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\downarrow 1253$$

$$\frac{5g\left(\frac{3(cdf-aeg)\left(\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd\sqrt{d+ex}}\right)}{cd}$$

$$\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\downarrow 1268$$

$$\frac{5g\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd\sqrt{d+ex}}\right)}{cd}$$

$$\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\downarrow 66$$

$$\frac{5g\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd\sqrt{d+ex}}\right)}{cd}$$

$$\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\downarrow 221$$

$$\frac{5g\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd\sqrt{d+ex}}\right)}{cd}$$

$$\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

3.720. $\int\frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}dx$

input
```
Int[((d + e*x)^(3/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(f + g*x)^(5/2))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]) + (5*g*(((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(2*c*d*Sqrt[d + e*x]) + (3*(c*d*f - a*e*g)*((Sqrt[f + g*x]*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x]) + ((c*d*f - a*e*g
)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqr
t[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])))/(4*c*d)))/(c*d)
```

3.720.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1251
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[e*g*(n/(c*(p + 1)))   Int[(
d + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p,
 0] && LtQ[p, -1] && GtQ[n, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

3.720. $\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.720.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(255) = 510$.

Time = 0.55 (sec) , antiderivative size = 638, normalized size of antiderivative = 2.12

method	result
default	$\frac{\left(15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^2cd\,e^2g^3x-30\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a\,c^2d^2ef\,g^2x+15\ln\left(\right.\right.}{}$

input
```
int((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
method=_RETURNVERBOSE)
```

output
```
1/8*(15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g
)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*g^3*x-30*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f
+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f
*g^2*x+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d
*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^2*g*x+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f
+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*e^3*g^3-3
0*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2
))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g
*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2*g+4
*c^2*d^2*g^2*x^2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)-10*(c*d*g)^(1/2
)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x+18*(c*d*g)^(1/2)*((g*x+f)*(c*d
*x+a*e))^(1/2)*c^2*d^2*f*g*x-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*
a^2*e^2*g^2+50*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g-16*((
g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*((c*d*x+a*e)*(e*x+d))
^(1/2)*(g*x+f)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)/(c*d*x+a*e)
/c^3/d^3/(e*x+d)^(1/2)
```

3.720.5 Fricas [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 971, normalized size of antiderivative = 3.23

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \left[\frac{4\,(2\,c^2d^2g^2x^2-8\,c^2d^2f^2+25\,acdefg-15\,a^2e^2g^2+(9\,c^2d^2fg-5\right.$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="fricas")
```

output
```
[1/16*(4*(2*c^2*d^2*g^2*x^2 - 8*c^2*d^2*f^2 + 25*a*c*d*e*f*g - 15*a^2*e^2*
g^2 + (9*c^2*d^2*f*g - 5*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(a*c^2*d^3*e*f^2 - 2*a^2*c*d^2
*e^2*f*g + a^3*d*e^3*g^2 + (c^3*d^3*e*f^2 - 2*a*c^2*d^2*e^2*f*g + a^2*c*d*
e^3*g^2)*x^2 + ((c^3*d^4 + a*c^2*d^2*e^2)*f^2 - 2*(a*c^2*d^3*e + a^2*c*d*e
^3)*f*g + (a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt(g/(c*d))*log(-(8*c^2*d^2*
e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e*f
*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + 4*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*d
*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x +
 f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*
a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d^4*e*x^2 + a*c^3*d^4*e + (c
^4*d^5 + a*c^3*d^3*e^2)*x), 1/8*(2*(2*c^2*d^2*g^2*x^2 - 8*c^2*d^2*f^2 + 25
*a*c*d*e*f*g - 15*a^2*e^2*g^2 + (9*c^2*d^2*f*g - 5*a*c*d*e*g^2)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(a*c
^2*d^3*e*f^2 - 2*a^2*c*d^2*e^2*f*g + a^3*d*e^3*g^2 + (c^3*d^3*e*f^2 - 2*a*
c^2*d^2*e^2*f*g + a^2*c*d*e^3*g^2)*x^2 + ((c^3*d^4 + a*c^2*d^2*e^2)*f^2 -
2*(a*c^2*d^3*e + a^2*c*d*e^3)*f*g + (a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt
(-g/(c*d))*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x +
 d)*sqrt(g*x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g +
(c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^4*d^4*e*x^2 + a*c^3*d^4*e + (c^...
```

3.720.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(3/2),x)
```

output
```
Timed out
```

3.720.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{5}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)*(g*x + f)^(5/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)^(3/2), x)
```

3.720.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 834 vs. 2(255) = 510.

Time = 0.63 (sec) , antiderivative size = 834, normalized size of antiderivative = 2.77

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{\sqrt{e^2f+(ex+d)eg-deg}\left((e^2f+(ex+d)eg-deg)\left(\frac{2\,(e^2f+(ex+d)\cdots}{cde^4|g\cdots}\right.\right.}{4\sqrt{-cde^2fg+ae^3g^2+(\cdots}} - \frac{15\,(c^2d^2f^2g^2-2\,acdefg^3+a^2e^2g^4)\log\left(\left|-\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2fg+ae^3g^2+(e^2f\cdots}\right.\right.}{4\sqrt{cdg}c^3d^3|g|} + \frac{15\sqrt{-cd^2eg^2+ae^3g^2}c^2d^2e^2f^2g^2\log\left(\left|-\sqrt{e^2f-deg}\sqrt{cdg}+\sqrt{-cd^2eg^2+ae^3g^2}\right|\right)-30\sqrt{-cd^2eg^2+ae^3g^2}\cdots}{\cdots}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="giac")
```

output
```
1/4*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*((e^2*f + (e*x + d)*e*g - d*e*g)*(
2*(e^2*f + (e*x + d)*e*g - d*e*g)*g^2/(c*d*e^4*abs(g)) + 5*(c^4*d^4*e^2*f*
g^2 - a*c^3*d^3*e^3*g^3)/(c^5*d^5*e^4*abs(g))) - 15*(c^4*d^4*e^4*f^2*g^2 -
 2*a*c^3*d^3*e^5*f*g^3 + a^2*c^2*d^2*e^6*g^4)/(c^5*d^5*e^4*abs(g)))/sqrt(-
c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g) - 15/4*(c
^2*d^2*f^2*g^2 - 2*a*c*d*e*f*g^3 + a^2*e^2*g^4)*log(abs(-sqrt(e^2*f + (e*x
 + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f +
(e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^3*d^3*abs(g)) + 1/4*(15*sqr
t(-c*d^2*e*g^2 + a*e^3*g^2)*c^2*d^2*e^2*f^2*g^2*log(abs(-sqrt(e^2*f - d*e*
g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 30*sqrt(-c*d^2*e*g^2 +
 a*e^3*g^2)*a*c*d*e^3*f*g^3*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqr
t(-c*d^2*e*g^2 + a*e^3*g^2))) + 15*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^2*e^4*
g^4*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g
^2))) + 8*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^2*e^2*f^2*g^2 + 9*sqrt(e^2
*f - d*e*g)*sqrt(c*d*g)*c^2*d^3*e*f*g^3 - 25*sqrt(e^2*f - d*e*g)*sqrt(c*d*
g)*a*c*d*e^3*f*g^3 - 2*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^4*g^4 - 5*sqr
t(e^2*f - d*e*g)*sqrt(c*d*g)*a*c*d^2*e^2*g^4 + 15*sqrt(e^2*f - d*e*g)*sqrt
(c*d*g)*a^2*e^4*g^4)/(sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*c^3*d^3*e
^2*abs(g))
```

3.720.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(f+g\,x)^{5/2}\,(d+e\,x)^{3/2}}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{3/2}}\,dx$$

input
```
int(((f + g*x)^(5/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(3/2),x)
```

output
```
int(((f + g*x)^(5/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(3/2), x)
```

3.721 $\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

3.721.1 Optimal result

Integrand size = 48, antiderivative size = 227

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{3g\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{c^2d^2\sqrt{d+ex}} + \frac{3\sqrt{g}(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

output
```
-2*(g*x+f)^(3/2)*(e*x+d)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
+3*(-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f
)^(1/2))*g^(1/2)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2)/(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+3*g*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)
```

3.721.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.63

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{f+gx}(-2cdf+3aeg+cdgx)+3\sqrt{g}(cdf-ae\right.}{c^{5/2}d^{5/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[((d + e*x)^(3/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(3/2),x]
```

output
```
(Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x]*(-2*c*d*f + 3*a*e*g + c*d*g*
x) + 3*Sqrt[g]*(c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*
Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(c^(5/2)*d^(5/2)*Sqrt[(a*e +
 c*d*x)*(d + e*x)])
```

3.721.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {1251, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1251}$$

$$\frac{3g\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 1253}$$

$$\frac{3g\left(\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 1268}$$

$$\frac{3g\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 66}$$

$$\frac{3g\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{3g\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[((d + e*x)^(3/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(f + g*x)^(3/2))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]) + (3*g*((Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(c*d*Sqrt[d + e*x]) + ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x
]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c
^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(c*d
)
```

3.721.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.721. $\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

rule 1251
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[e*g*(n/(c*(p + 1)))   Int[(
d + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p,
 0] && LtQ[p, -1] && GtQ[n, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.721.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.70

method	result
default	$-\frac{\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)acde\,g^2x-3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)c^2d^2fgx+3\ln\left(\frac{2cdgx+a}{}\right.}{}$

input
```
int((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
method=_RETURNVERBOSE)
```

output
```
-1/2*(3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g
)^(1/2))/(c*d*g)^(1/2))*a*c*d*e*g^2*x-3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((
g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f*g*x+3*ln
(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(
c*d*g)^(1/2))*a^2*e^2*g^2-3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*
x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c*d*e*f*g-2*((g*x+f)*(c*d*x+
a*e))^(1/2)*(c*d*g)^(1/2)*c*d*g*x-6*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1
/2)*a*e*g+4*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)*((c*d*x+a*e)*
(e*x+d))^(1/2)*(g*x+f)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*x+a*e)/(c*d*
g)^(1/2)/c^2/d^2/(e*x+d)^(1/2)
```

3.721.5 Fricas [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 725, normalized size of antiderivative = 3.19

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \left[\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}(cdgx-2\,cdf+3\,aeg)\sqrt{ex+d}\sqrt{}}{}\right.$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="fricas")
```

output
```
[1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - 2*c*d*f + 3
*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(a*c*d^2*e*f - a^2*d*e^2*g + (c^2*
d^2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*d^3 + a*c*d*e^2)*f - (a*c*d^2*e + a^2*e
^3)*g)*x)*sqrt(g/(c*d))*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^
2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^
2 - 4*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*d*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2
 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x
+ d))/(c^3*d^3*e*x^2 + a*c^2*d^3*e + (c^3*d^4 + a*c^2*d^2*e^2)*x), 1/2*(2
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - 2*c*d*f + 3*a*e*g)
*sqrt(e*x + d)*sqrt(g*x + f) - 3*(a*c*d^2*e*f - a^2*d*e^2*g + (c^2*d^2*e*f
 - a*c*d*e^2*g)*x^2 + ((c^2*d^3 + a*c*d*e^2)*f - (a*c*d^2*e + a^2*e^3)*g)*
x)*sqrt(-g/(c*d))*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d)*sqrt(g*x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d
*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^3*d^3*e*x^2 + a*c^2*d^3*e +
 (c^3*d^4 + a*c^2*d^2*e^2)*x)]
```

3.721.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(3/2),x)
```

output
```
Timed out
```

3.721.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)*(g*x + f)^(3/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)^(3/2), x)
```

3.721.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(193) = 386$.

Time = 0.52 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.32

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{\sqrt{e^2f+(ex+d)eg-deg}\left(\frac{(e^2f+(ex+d)eg-deg)g^2}{cde^2|g|} - \frac{3\left(c^2d^2e^2fg^2-acde^3g^3\right.}{c^3d^3e^2|g|}\right.}{\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}}$$
$$-\frac{3\left(cdfg^2-aeg^3\right)\log\left(\left|-\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg}\right.\right.}{\sqrt{cdg}c^2d^2|g|}$$
$$+\frac{3\sqrt{-cd^2eg^2+ae^3g^2}cdefg^2\log\left(\left|-\sqrt{e^2f-deg}\sqrt{cdg}+\sqrt{-cd^2eg^2+ae^3g^2}\right|\right)-3\sqrt{-cd^2eg^2+ae^3g^2}ae^2g^3\,\mathrm{l}}{\sqrt{-}}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="giac")
```

output
```
sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*((e^2*f + (e*x + d)*e*g - d*e*g)*g^2/(
c*d*e^2*abs(g)) - 3*(c^2*d^2*e^2*f*g^2 - a*c*d*e^3*g^3)/(c^3*d^3*e^2*abs(g
)))/sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)
 - 3*(c*d*f*g^2 - a*e*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sq
rt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g
)*c*d*g)))/(sqrt(c*d*g)*c^2*d^2*abs(g)) + (3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2
)*c*d*e*f*g^2*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2
 + a*e^3*g^2))) - 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a*e^2*g^3*log(abs(-sqrt
(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 2*sqrt(e^
2*f - d*e*g)*sqrt(c*d*g)*c*d*e*f*g^2 + sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d
^2*g^3 - 3*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*e^2*g^3)/(sqrt(-c*d^2*e*g^2 +
 a*e^3*g^2)*sqrt(c*d*g)*c^2*d^2*e*abs(g))
```

3.721.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(f+g\,x)^{3/2}\,(d+e\,x)^{3/2}}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{3/2}}\,dx$$

input
```
int(((f + g*x)^(3/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(3/2),x)
```

output
```
int(((f + g*x)^(3/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(3/2), x)
```

3.722 $\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

3.722.1 Optimal result

Integrand size = 48, antiderivative size = 161

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2\sqrt{g}\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1/2))*g^(1/2)
*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(3/2)/d^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)-2*(e*x+d)^(1/2)*(g*x+f)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)
```

3.722.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.73

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{f+gx}-\sqrt{g}\sqrt{ae+cdx}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)\right)}{c^{3/2}d^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x] - Sqrt[g]*Sqrt[a*e + c*d*
x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/
(c^(3/2)*d^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.722.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1251, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}dx$$

$$\Big\downarrow \text{1251}$$

$$\frac{g\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cd}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{1268}$$

$$\frac{g\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{66}$$

$$\frac{2g\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

input
```
Int[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*Sqrt[f + g*x])/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]) + (2*Sqrt[g]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqr
t[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

3.722.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1251
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[e*g*(n/(c*(p + 1)))   Int[(
d + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p,
 0] && LtQ[p, -1] && GtQ[n, 0]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.722.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.24

method	result
default	$\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)cdgx+\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)aeg-2\right.}{\sqrt{cdg}\,(cdx+ae)\sqrt{(gx+f)(cdx+ae)}\,dc\sqrt{ex+d}}$

3.722. $\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

input
```
int((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
method=_RETURNVERBOSE)
```

output
```
(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2
*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c*d*g*x+ln(1/2*
(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g
)^(1/2))*a*e*g-2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2)/
(c*d*x+a*e)/((g*x+f)*(c*d*x+a*e))^(1/2)/d/c/(e*x+d)^(1/2)
```

3.722.5 Fricas [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 569, normalized size of antiderivative = 3.53

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \left[\frac{(cdex^2+ade+(cd^2+ae^2)x)\sqrt{\frac{g}{cd}}\log\left(-\frac{8\,c^2d^2eg^2x^3+c^2d^3f^2+6\,acd^2ef}{}\right.}{}\right.$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="fricas")
```

output
```
[1/2*((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g/(c*d))*log(-(8*c^2*d^
2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e
*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + 4*(2*c^2*d^2*g*x + c^2*d^2*f + a*c
*d*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x
 + f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (
8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(c^2*d^2*e*x^2 + a*c*d^2*e +
(c^2*d^3 + a*c*d*e^2)*x), -((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-
g/(c*d))*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d
)*sqrt(g*x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c
*d*e*f + (2*c*d^2 + a*e^2)*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3
 + a*c*d*e^2)*x)]
```

3.722.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(3/2),x)
```

output
```
Integral((d + e*x)**(3/2)*sqrt(f + g*x)/((d + e*x)*(a*e + c*d*x))**(3/2),
x)
```

3.722.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)*sqrt(g*x + f)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)^(3/2), x)
```

3.722.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(133) = 266$.

Time = 0.47 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.53

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\frac{2\left(cd^2eg^2\log\left(\left|-\sqrt{e^2f-deg}\sqrt{cdg}+\sqrt{-cd^2eg^2+ae^3g^2}\right|\right)-ae^3g^2\cdots\right.}{\sqrt{c\cdots}}$$
$$-\frac{2\,g^2\log\left(\left|-\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\right|\right)}{\sqrt{cdg}cd|g|}$$
$$+\frac{2\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\sqrt{e^2f+(ex+d)eg-deg}g^2}{(cde^2fg-ae^3g^2-(e^2f+(ex+d)eg-deg)cdg)cd|g|}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="giac")
```

output
```
2*(c*d^2*e*g^2*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^
2 + a*e^3*g^2))) - a*e^3*g^2*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sq
rt(-c*d^2*e*g^2 + a*e^3*g^2))) - sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f
 - d*e*g)*sqrt(c*d*g))/(sqrt(c*d*g)*c^2*d^3*e*abs(g) - sqrt(c*d*g)*a*c*d*e
^3*abs(g)) - 2*g^2*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g
) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)
))/(sqrt(c*d*g)*c*d*abs(g)) + 2*sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (
e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*g^2/((c*d
*e^2*f*g - a*e^3*g^2 - (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*c*d*abs(g))
```

3.722.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{\sqrt{f+g\,x}\,(d+e\,x)^{3/2}}{\left(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(3/2),x)
```

output
```
int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(3/2), x)
```

3.723 $\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

3.723.1 Optimal result

Integrand size = 48, antiderivative size = 61

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{(cdf-aeg)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

output
```
-2*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)
```

3.723.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{(cdf-aeg)\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2)),x]
```

output
```
(-2*Sqrt[d + e*x]*Sqrt[f + g*x])/((c*d*f - a*e*g)*Sqrt[(a*e + c*d*x)*(d +
e*x)])
```

3.723.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(x\,(ae^2+cd^2)+ade+cdex^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{1248}$$
$$-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}$$

input
```
Int[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2)),x]
```

output
```
(-2*Sqrt[d + e*x]*Sqrt[f + g*x])/((c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])
```

3.723.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

3.723.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{2\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}}{\sqrt{ex+d}\,(cdx+ae)(aeg-cdf)}$	55
gosper	$\frac{2\sqrt{gx+f}\,(cdx+ae)(ex+d)^{\frac{3}{2}}}{(aeg-cdf)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$	63

input
```
int((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
method=_RETURNVERBOSE)
```

output
```
2/(e*x+d)^(1/2)*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/(c*d*x+a*e)/(a*e
*g-c*d*f)
```

3.723.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(55) = 110$.

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.05

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\sqrt{gx+f}}{acd^2ef-a^2de^2g+(c^2d^2ef-acde^2g)x^2+((c^2d^3+acde^2)f-(acd^2e+a^2e^3)g)x}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="fricas")
```

output
```
-2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)
/(a*c*d^2*e*f - a^2*d*e^2*g + (c^2*d^2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*d^3
+ a*c*d*e^2)*f - (a*c*d^2*e + a^2*e^3)*g)*x)
```

3.723.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(3/2),x)
```

output
```
Integral((d + e*x)**(3/2)/(((d + e*x)*(a*e + c*d*x))**(3/2)*sqrt(f + g*x))
, x)
```

3.723.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*s
qrt(g*x + f)), x)
```

3.723.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(55) = 110$.

Time = 0.32 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.57

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{2\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\sqrt{e}}{(cde^2fg-ae^3g^2-(e^2f+(ex+d)eg-deg)cdg)(}$$
$$-\frac{2\sqrt{-cd^2eg^2+ae^3g^2}\sqrt{e^2f-deg}}{c^2d^3ef|g|-acde^3f|g|-acd^2e^2g|g|+a^2e^4g|g|}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="giac")
```

output
```
2*sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*s
qrt(e^2*f + (e*x + d)*e*g - d*e*g)*e*g^2/((c*d*e^2*f*g - a*e^3*g^2 - (e^2*
f + (e*x + d)*e*g - d*e*g)*c*d*g)*(c*d*e*f*abs(g) - a*e^2*g*abs(g))) - 2*s
qrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)/(c^2*d^3*e*f*abs(g) - a*
c*d*e^3*f*abs(g) - a*c*d^2*e^2*g*abs(g) + a^2*e^4*g*abs(g))
```

3.723.9 Mupad [B] (verification not implemented)

Time = 13.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.41

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{\left(\frac{2\,f\,\sqrt{d+e\,x}}{c\,d\,e\,(a\,e\,g-c\,d\,f)}+\frac{2\,g\,x\,\sqrt{d+e\,x}}{c\,d\,e\,(a\,e\,g-c\,d\,f)}\right)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)}}{x^2\,\sqrt{f+g\,x}+\frac{a\,\sqrt{f+g\,x}}{c}+\frac{x\,\sqrt{f+g\,x}\,(c\,d^2+a\,e^2)}{c\,d\,e}}$$

input
```
int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2)),x)
```

output
```
(((2*f*(d + e*x)^(1/2))/(c*d*e*(a*e*g - c*d*f)) + (2*g*x*(d + e*x)^(1/2))/
(c*d*e*(a*e*g - c*d*f)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x
^2*(f + g*x)^(1/2) + (a*(f + g*x)^(1/2))/c + (x*(f + g*x)^(1/2)*(a*e^2 + c
*d^2))/(c*d*e))
```

3.724 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

3.724.1 Optimal result

Integrand size = 48, antiderivative size = 124

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{4g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(cdf-aeg)^2\sqrt{d+ex}\sqrt{f+gx}}$$

output
```
-2*(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)-4*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(
e*x+d)^(1/2)/(g*x+f)^(1/2)
```

3.724.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.52

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(aeg+cd(f+2gx))}{(cdf-aeg)^2\sqrt{(ae+cdx)(d+ex)}\sqrt{f+gx}}$$

input
```
Integrate[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)),x]
```

output
```
(-2*Sqrt[d + e*x]*(a*e*g + c*d*(f + 2*g*x)))/((c*d*f - a*e*g)^2*Sqrt[(a*e
+ c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

3.724.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1252, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1252}$$

$$-\frac{2g\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cdf-aeg}-\frac{2\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \text{ 1248}$$

$$-\frac{4g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

input
```
Int[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2)),x]
```

output
```
(-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2]) - (4*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
(c*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f + g*x])
```

3.724.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1252
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

3.724.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(2cdgx+aeg+cdf)}{\sqrt{ex+d}\,\sqrt{gx+f}\,(cdx+ae)(aeg-cdf)^{2}}$	70
gosper	$-\frac{2(cdx+ae)(2cdgx+aeg+cdf)(ex+d)^{\frac{3}{2}}}{\sqrt{gx+f}\,(a^{2}e^{2}g^{2}-2acdefg+c^{2}d^{2}f^{2})(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{3}{2}}}$	97

input
```
int((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/(e*x+d)^(1/2)/(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(2*c*d*g*x+a*e*
g+c*d*f)/(c*d*x+a*e)/(a*e*g-c*d*f)^2
```

3.724.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(112) = 224$.

Time = 0.35 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.62

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx =$$

$$-\frac{2\sqrt{cdex^2+ade}}{ac^2d^3ef^3-2\,a^2cd^2e^2f^2g+a^3de^3fg^2+(c^3d^3ef^2g-2\,ac^2d^2e^2fg^2+a^2cde^3g^3)x^3+(c^3d^3ef^3+(c^3d^4-ac^2d}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="fricas")
```

output
```
-2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)
*sqrt(e*x + d)*sqrt(g*x + f)/(a*c^2*d^3*e*f^3 - 2*a^2*c*d^2*e^2*f^2*g + a^
3*d*e^3*f*g^2 + (c^3*d^3*e*f^2*g - 2*a*c^2*d^2*e^2*f*g^2 + a^2*c*d*e^3*g^3
)*x^3 + (c^3*d^3*e*f^3 + (c^3*d^4 - a*c^2*d^2*e^2)*f^2*g - (2*a*c^2*d^3*e
+ a^2*c*d*e^3)*f*g^2 + (a^2*c*d^2*e^2 + a^3*e^4)*g^3)*x^2 + (a^3*d*e^3*g^3
 + (c^3*d^4 + a*c^2*d^2*e^2)*f^3 - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*f^2*g - (
a^2*c*d^2*e^2 - a^3*e^4)*f*g^2)*x)
```

3.724.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}(f+gx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(3/2),x)
```

output
```
Integral((d + e*x)**(3/2)/(((d + e*x)*(a*e + c*d*x))**(3/2)*(f + g*x)**(3/
2)), x)
```

3.724.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(
g*x + f)^(3/2)), x)
```

3.724.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(112) = 224$.

Time = 0.38 (sec) , antiderivative size = 793, normalized size of antiderivative = 6.40

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx =$$

$$-2\left(\frac{\sqrt{e^2f+(ex+d)eg-deg}cdg^2}{(c^2d^2e^2f^2|g|-2\,acde^3fg|g|+a^2e^4g^2|g|)\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}}+\frac{}{\left(cde^2 f\right.}\right.$$

$$+\frac{}{\sqrt{e^2f-deg}\sqrt{cdg}c^3d^4f^2g|g|-\sqrt{e^2f-deg}\sqrt{cdg}ac^2d^2e^2f^2g|g|-2\sqrt{e^2f-deg}\sqrt{cdg}ac^2d^3efg^2|g|+2\sqrt{e^2f}}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="giac")
```

output
```
-2*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g^2/((c^2*d^2*e^2*f^2*abs(g) -
 2*a*c*d*e^3*f*g*abs(g) + a^2*e^4*g^2*abs(g))*sqrt(-c*d*e^2*f*g + a*e^3*g^
2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)) + 2*sqrt(c*d*g)*g^2/((c*d*e^2*
f*g - a*e^3*g^2 + (sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(
-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2)*(c*d
*f*abs(g) - a*e*g*abs(g))))*e^2 + 2*(sqrt(e^2*f - d*e*g)*c^2*d^2*e*f*g^2 -
 sqrt(e^2*f - d*e*g)*c^2*d^3*g^3 + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d
*g)*c*d^2*g^2 - sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a*e^2*g^2)/(sqr
t(e^2*f - d*e*g)*sqrt(c*d*g)*c^3*d^4*f^2*g*abs(g) - sqrt(e^2*f - d*e*g)*sq
rt(c*d*g)*a*c^2*d^2*e^2*f^2*g*abs(g) - 2*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a
*c^2*d^3*e*f*g^2*abs(g) + 2*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c*d*e^3*f*
g^2*abs(g) + sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c*d^2*e^2*g^3*abs(g) - sq
rt(e^2*f - d*e*g)*sqrt(c*d*g)*a^3*e^4*g^3*abs(g) + sqrt(-c*d^2*e*g^2 + a*e
^3*g^2)*c^3*d^3*e*f^3*abs(g) - sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*c^3*d^4*f^2*
g*abs(g) - 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a*c^2*d^2*e^2*f^2*g*abs(g) + 2
*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a*c^2*d^3*e*f*g^2*abs(g) + sqrt(-c*d^2*e*g
^2 + a*e^3*g^2)*a^2*c*d*e^3*f*g^2*abs(g) - sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*
a^2*c*d^2*e^2*g^3*abs(g))
```

3.724.9 Mupad [B] (verification not implemented)

Time = 13.59 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{\left(\frac{4\,g\,x\,\sqrt{d+e\,x}}{e\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{\left(2\,a\,e\,g+2\,c\,d\,f\right)\,\sqrt{d+e\,x}}{c\,d\,e\,{\left(a\,e\,g-c\,d\,f\right)}^2}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^2\,\sqrt{f+g\,x}+\frac{a\,\sqrt{f+g\,x}}{c}+\frac{x\,\sqrt{f+g\,x}\,\left(c\,d^2+a\,e^2\right)}{c\,d\,e}}$$

input
```
int((d + e*x)^(3/2)/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2)),x)
```

output
```
-(((4*g*x*(d + e*x)^(1/2))/(e*(a*e*g - c*d*f)^2) + ((2*a*e*g + 2*c*d*f)*(d
 + e*x)^(1/2))/(c*d*e*(a*e*g - c*d*f)^2))*(x*(a*e^2 + c*d^2) + a*d*e + c*d
*e*x^2)^(1/2))/(x^2*(f + g*x)^(1/2) + (a*(f + g*x)^(1/2))/c + (x*(f + g*x)
^(1/2)*(a*e^2 + c*d^2))/(c*d*e))
```

3.725 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

3.725.1 Optimal result

Integrand size = 48, antiderivative size = 192

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{8g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{3/2}} - \frac{16cdg\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)^3\sqrt{d+ex}\sqrt{f+gx}}$$

output
```
-2*(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)-8/3*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2
/(g*x+f)^(3/2)/(e*x+d)^(1/2)-16/3*c*d*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

3.725.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.55

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\left(-a^2e^2g^2+2acdeg(3f+2gx)+c^2d^2\left(3f^2+12fgx+8g^2x^2\right)\right)}{3(cdf-aeg)^3\sqrt{(ae+cdx)(d+ex)}(f+gx)^{3/2}}$$

input
```
Integrate[(d + e*x)^(3/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)),x]
```

output
```
(-2*Sqrt[d + e*x]*(-(a^2*e^2*g^2) + 2*a*c*d*e*g*(3*f + 2*g*x) + c^2*d^2*(3
*f^2 + 12*f*g*x + 8*g^2*x^2)))/(3*(c*d*f - a*e*g)^3*Sqrt[(a*e + c*d*x)*(d
+ e*x)]*(f + g*x)^(3/2))
```

3.725.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1252, 1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1252}$$

$$-\frac{4g\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$-\frac{4g\left(\frac{2cd\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3(cdf-aeg)}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \text{1248}$$

$$-\frac{4g\left(\frac{4cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

input
```
Int[(d + e*x)^(3/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2)),x]
```

output
```
(-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2]) - (4*g*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^(3/2)) + (4*c*d*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt
[f + g*x])))/(c*d*f - a*e*g)
```

3.725.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1252
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

3.725. $\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

3.725.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.62

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-8g^2x^2c^2d^2-4acde\,g^2x-12c^2d^2fgx+a^2e^2g^2-6acdefg-3c^2d^2f^2)}{3\sqrt{ex+d}\,(gx+f)^{\frac{3}{2}}(cdx+ae)(aeg-cdf)^3}$	120
gosper	$-\frac{2(cdx+ae)(-8g^2x^2c^2d^2-4acde\,g^2x-12c^2d^2fgx+a^2e^2g^2-6acdefg-3c^2d^2f^2)(ex+d)^{\frac{3}{2}}}{3(gx+f)^{\frac{3}{2}}(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3c^3d^3)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$	168

input
```
int((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)/(g*x+f)^(3/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-8*c^2*d^2*g
^2*x^2-4*a*c*d*e*g^2*x-12*c^2*d^2*f*g*x+a^2*e^2*g^2-6*a*c*d*e*f*g-3*c^2*d^
2*f^2)/(c*d*x+a*e)/(a*e*g-c*d*f)^3
```

3.725.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. 2(170) = 340.

Time = 0.42 (sec) , antiderivative size = 649, normalized size of antiderivative = 3.38

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx =$$

$$-\frac{}{3\left(ac^3d^4ef^5-3a^2c^2d^3e^2f^4g+3a^3cd^2e^3f^3g^2-a^4de^4f^2g^3+\left(c^4d^4ef^3g^2-3ac^3d^3e^2f^2g^3+3a^2c^2d^2e^3fg^4-\right.\right.}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="fricas")
```

output
```
-2/3*(8*c^2*d^2*g^2*x^2 + 3*c^2*d^2*f^2 + 6*a*c*d*e*f*g - a^2*e^2*g^2 + 4*
(3*c^2*d^2*f*g + a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(e*x + d)*sqrt(g*x + f)/(a*c^3*d^4*e*f^5 - 3*a^2*c^2*d^3*e^2*f^4*g
+ 3*a^3*c*d^2*e^3*f^3*g^2 - a^4*d*e^4*f^2*g^3 + (c^4*d^4*e*f^3*g^2 - 3*a*c
^3*d^3*e^2*f^2*g^3 + 3*a^2*c^2*d^2*e^3*f*g^4 - a^3*c*d*e^4*g^5)*x^4 + (2*c
^4*d^4*e*f^4*g + (c^4*d^5 - 5*a*c^3*d^3*e^2)*f^3*g^2 - 3*(a*c^3*d^4*e - a^
2*c^2*d^2*e^3)*f^2*g^3 + (3*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g^4 - (a^3*c*
d^2*e^3 + a^4*e^5)*g^5)*x^3 + (c^4*d^4*e*f^5 - a^4*d*e^4*g^5 + (2*c^4*d^5
- a*c^3*d^3*e^2)*f^4*g - (5*a*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*f^3*g^2 + (3*
a^2*c^2*d^3*e^2 + 5*a^3*c*d*e^4)*f^2*g^3 + (a^3*c*d^2*e^3 - 2*a^4*e^5)*f*g
^4)*x^2 - (2*a^4*d*e^4*f*g^4 - (c^4*d^5 + a*c^3*d^3*e^2)*f^5 + (a*c^3*d^4*
e + 3*a^2*c^2*d^2*e^3)*f^4*g + 3*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^2 -
 (5*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^3)*x)
```

3.725.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)/(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(3/2),x)
```

output
```
Timed out
```

3.725.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(
g*x + f)^(5/2)), x)
```

3.725.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2618 vs. $2(170) = 340$.

Time = 1.84 (sec) , antiderivative size = 2618, normalized size of antiderivative = 13.64

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="giac")
```

output
```
-2/3*(3*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*c^2*d^2*g^2/((c^3*d^3*e^3*f^3*
abs(g) - 3*a*c^2*d^2*e^4*f^2*g*abs(g) + 3*a^2*c*d*e^5*f*g^2*abs(g) - a^3*e
^6*g^3*abs(g))*sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*
e*g)*c*d*g)) + 2*(5*sqrt(c*d*g)*c^3*d^3*e^4*f^2*g^4 - 10*sqrt(c*d*g)*a*c^2
*d^2*e^5*f*g^5 + 5*sqrt(c*d*g)*a^2*c*d*e^6*g^6 + 12*sqrt(c*d*g)*(sqrt(e^2*
f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (
e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*c^2*d^2*e^2*f*g^3 - 12*sqrt(c*d*g
)*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a
*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*a*c*d*e^3*g^4 + 3*sqr
t(c*d*g)*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*
f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^4*c*d*g^2)/((c^2
*d^2*e*f^2*abs(g) - 2*a*c*d*e^2*f*g*abs(g) + a^2*e^3*g^2*abs(g))*(c*d*e^2*
f*g - a*e^3*g^2 + (sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(
-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2)^3))*
e^3 + 2/3*(3*sqrt(e^2*f - d*e*g)*c^4*d^4*e^3*f^3*g^2 - 9*sqrt(e^2*f - d*e*
g)*c^4*d^5*e^2*f^2*g^3 + 21*sqrt(e^2*f - d*e*g)*c^4*d^6*e*f*g^4 - 24*sqrt(
e^2*f - d*e*g)*a*c^3*d^4*e^3*f*g^4 + 12*sqrt(e^2*f - d*e*g)*a^2*c^2*d^2*e^
5*f*g^4 - 12*sqrt(e^2*f - d*e*g)*c^4*d^7*g^5 + 15*sqrt(e^2*f - d*e*g)*a*c^
3*d^5*e^2*g^5 - 3*sqrt(e^2*f - d*e*g)*a^2*c^2*d^3*e^4*g^5 - 3*sqrt(e^2*f -
 d*e*g)*a^3*c*d*e^6*g^5 - 4*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*...
```

3.725.9 Mupad [B] (verification not implemented)

Time = 13.98 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{\left(\frac{8\,x\,(a\,e\,g+3\,c\,d\,f)\,\sqrt{d+e\,x}}{3\,e\,(a\,e\,g-c\,d\,f)^3}+\frac{\sqrt{d+e\,x}\,(-2\,a^2\,e^2\,g^2+12\,a\,c\,d\,e\,f\,g+6\,c^2\,d^2}{3\,c\,d\,e\,g\,(a\,e\,g-c\,d\,f)^3}\right.}{x^3\,\sqrt{f+g\,x}+\frac{a\,f\,\sqrt{f+g\,x}}{c\,g}+\frac{x\,\sqrt{f+g\,x}\,(c\,f\,d^2}{c\,d}}$$

input
```
int((d + e*x)^(3/2)/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2)),x)
```

output
```
(((8*x*(a*e*g + 3*c*d*f)*(d + e*x)^(1/2))/(3*e*(a*e*g - c*d*f)^3) + ((d +
e*x)^(1/2)*(6*c^2*d^2*f^2 - 2*a^2*e^2*g^2 + 12*a*c*d*e*f*g))/(3*c*d*e*g*(a
*e*g - c*d*f)^3) + (16*c*d*g*x^2*(d + e*x)^(1/2))/(3*e*(a*e*g - c*d*f)^3))
*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^3*(f + g*x)^(1/2) + (a*
f*(f + g*x)^(1/2))/(c*g) + (x*(f + g*x)^(1/2)*(a*e^2*f + c*d^2*f + a*d*e*g
))/(c*d*e*g) + (x^2*(f + g*x)^(1/2)*(a*e^2*g + c*d^2*g + c*d*e*f))/(c*d*e*
g))
```

3.725. $\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

3.726 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

3.726.1 Optimal result

Integrand size = 48, antiderivative size = 262

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{12g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{5/2}} - \frac{16cdg\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5(cdf-aeg)^3\sqrt{d+ex}(f+gx)^{3/2}} - \frac{32c^2d^2g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5(cdf-aeg)^4\sqrt{d+ex}\sqrt{f+gx}}$$

output
```
-2*(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)-12/5*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^
2/(g*x+f)^(5/2)/(e*x+d)^(1/2)-16/5*c*d*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)/(-a*e*g+c*d*f)^3/(g*x+f)^(3/2)/(e*x+d)^(1/2)-32/5*c^2*d^2*g*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(1/2)/(g*x+f)^(1
/2)
```

3.726.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.57

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(a^3e^3g^3-a^2cde^2g^2(5f+2gx)+ac^2d^2eg(15f^2+20fgx+8g^2x^2)+c^3d^3(5f^3+30f^2gx+40fg^2}{5(cdf-aeg)^4\sqrt{(ae+cdx)(d+ex)}(f+gx)^{5/2}}$$

input
```
Integrate[(d + e*x)^(3/2)/((f + g*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)),x]
```

output
```
(-2*Sqrt[d + e*x]*(a^3*e^3*g^3 - a^2*c*d*e^2*g^2*(5*f + 2*g*x) + a*c^2*d^2
*e*g*(15*f^2 + 20*f*g*x + 8*g^2*x^2) + c^3*d^3*(5*f^3 + 30*f^2*g*x + 40*f*
g^2*x^2 + 16*g^3*x^3)))/(5*(c*d*f - a*e*g)^4*Sqrt[(a*e + c*d*x)*(d + e*x)]
*(f + g*x)^(5/2))
```

3.726.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1252, 1254, 1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\,(x\,(ae^2+cd^2)+ade+cdex^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 1252$$

$$-\frac{6g\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \; 1254$$

$$-\frac{6g\left(\frac{4cd\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{5(cdf-aeg)}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \ 1254$$

$$-\frac{6g\left(\frac{4cd\left(\frac{2cd\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3(cdf-aeg)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}\right)}{5(cdf-aeg)}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \ 1248$$

$$-\frac{6g\left(\frac{4cd\left(\frac{4cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}\right)}{5(cdf-aeg)}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

input
```
Int[(d + e*x)^(3/2)/((f + g*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2)),x]
```

output
```
(-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2]) - (6*g*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(5*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^(5/2)) + (4*c*d*((2*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f
 + g*x)^(3/2)) + (4*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c
*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f + g*x])))/(5*(c*d*f - a*e*g))))/(c*d*
f - a*e*g)
```

3.726. $\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

3.726.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1252
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

3.726.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.73

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(16g^3x^3c^3d^3+8a\,c^2d^2e\,g^3x^2+40c^3d^3f\,g^2x^2-2a^2cd\,e^2g^3x+20a\,c^2d^2ef\,g^2x+30c^3d^3f^2gx+a^3e^3g^3-5a^2cd\,e^2f}{5\sqrt{ex+d}\,(gx+f)^{\frac{5}{2}}(cdx+ae)(aeg-cdf)^4}$
gosper	$-\frac{2(cdx+ae)(16g^3x^3c^3d^3+8a\,c^2d^2e\,g^3x^2+40c^3d^3f\,g^2x^2-2a^2cd\,e^2g^3x+20a\,c^2d^2ef\,g^2x+30c^3d^3f^2gx+a^3e^3g^3-5a^2cd\,e^2f\,g^2+15a\,c}{5(gx+f)^{\frac{5}{2}}(a^4e^4g^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4c^4d^4)(cde\,x^2+a\,e^2x+c\,d^2x+ade)}$

input
```
int((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
method=_RETURNVERBOSE)
```

3.726. $\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

output
```
-2/5/(e*x+d)^(1/2)/(g*x+f)^(5/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(16*c^3*d^3*g
^3*x^3+8*a*c^2*d^2*e*g^3*x^2+40*c^3*d^3*f*g^2*x^2-2*a^2*c*d*e^2*g^3*x+20*a
*c^2*d^2*e*f*g^2*x+30*c^3*d^3*f^2*g*x+a^3*e^3*g^3-5*a^2*c*d*e^2*f*g^2+15*a
*c^2*d^2*e*f^2*g+5*c^3*d^3*f^3)/(c*d*x+a*e)/(a*e*g-c*d*f)^4
```

3.726.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1062 vs. $2(232) = 464$.

Time = 1.40 (sec) , antiderivative size = 1062, normalized size of antiderivative = 4.05

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx =$$

$$-\frac{}{5\,(ac^4d^5ef^7-4\,a^2c^3d^4e^2f^6g+6\,a^3c^2d^3e^3f^5g^2-4\,a^4cd^2e^4f^4g^3+a^5de^5f^3g^4+(c^5d^5ef^4g^3-4\,ac^4d^4e^2f^3g^4}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="fricas")
```

output
```
-2/5*(16*c^3*d^3*g^3*x^3 + 5*c^3*d^3*f^3 + 15*a*c^2*d^2*e*f^2*g - 5*a^2*c*
d*e^2*f*g^2 + a^3*e^3*g^3 + 8*(5*c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x^2 + 2*
(15*c^3*d^3*f^2*g + 10*a*c^2*d^2*e*f*g^2 - a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(a*c^4*d^5*e*
f^7 - 4*a^2*c^3*d^4*e^2*f^6*g + 6*a^3*c^2*d^3*e^3*f^5*g^2 - 4*a^4*c*d^2*e^
4*f^4*g^3 + a^5*d*e^5*f^3*g^4 + (c^5*d^5*e*f^4*g^3 - 4*a*c^4*d^4*e^2*f^3*g
^4 + 6*a^2*c^3*d^3*e^3*f^2*g^5 - 4*a^3*c^2*d^2*e^4*f*g^6 + a^4*c*d*e^5*g^7
)*x^5 + (3*c^5*d^5*e*f^5*g^2 + (c^5*d^6 - 11*a*c^4*d^4*e^2)*f^4*g^3 - 2*(2
*a*c^4*d^5*e - 7*a^2*c^3*d^3*e^3)*f^3*g^4 + 6*(a^2*c^3*d^4*e^2 - a^3*c^2*d
^2*e^4)*f^2*g^5 - (4*a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f*g^6 + (a^4*c*d^2*e^4
 + a^5*e^6)*g^7)*x^4 + (3*c^5*d^5*e*f^6*g + a^5*d*e^5*g^7 + 3*(c^5*d^6 - 3
*a*c^4*d^4*e^2)*f^5*g^2 - (11*a*c^4*d^5*e - 6*a^2*c^3*d^3*e^3)*f^4*g^3 + 2
*(7*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4)*f^3*g^4 - 3*(2*a^3*c^2*d^3*e^3 +
3*a^4*c*d*e^5)*f^2*g^5 - (a^4*c*d^2*e^4 - 3*a^5*e^6)*f*g^6)*x^3 + (c^5*d^5
*e*f^7 + 3*a^5*d*e^5*f*g^6 + (3*c^5*d^6 - a*c^4*d^4*e^2)*f^6*g - 3*(3*a*c^
4*d^5*e + 2*a^2*c^3*d^3*e^3)*f^5*g^2 + 2*(3*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^
2*e^4)*f^4*g^3 + (6*a^3*c^2*d^3*e^3 - 11*a^4*c*d*e^5)*f^3*g^4 - 3*(3*a^4*c
*d^2*e^4 - a^5*e^6)*f^2*g^5)*x^2 + (3*a^5*d*e^5*f^2*g^5 + (c^5*d^6 + a*c^4
*d^4*e^2)*f^7 - (a*c^4*d^5*e + 4*a^2*c^3*d^3*e^3)*f^6*g - 6*(a^2*c^3*d^4*e
^2 - a^3*c^2*d^2*e^4)*f^5*g^2 + 2*(7*a^3*c^2*d^3*e^3 - 2*a^4*c*d*e^5)*f...
```

3.726.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)/(g*x+f)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(3/2),x)
```

output
```
Timed out
```

3.726.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(
g*x + f)^(7/2)), x)
```

3.726.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6137 vs. $2(232) = 464$.

Time = 24.65 (sec) , antiderivative size = 6137, normalized size of antiderivative = 23.42

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="giac")
```

output
```
-2/5*(5*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*c^3*d^3*g^2/((c^4*d^4*e^4*f^4*
abs(g) - 4*a*c^3*d^3*e^5*f^3*g*abs(g) + 6*a^2*c^2*d^2*e^6*f^2*g^2*abs(g) -
 4*a^3*c*d*e^7*f*g^3*abs(g) + a^4*e^8*g^4*abs(g))*sqrt(-c*d*e^2*f*g + a*e^
3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)) + 2*(11*sqrt(c*d*g)*c^6*d^
6*e^8*f^4*g^6 - 44*sqrt(c*d*g)*a*c^5*d^5*e^9*f^3*g^7 + 66*sqrt(c*d*g)*a^2*
c^4*d^4*e^10*f^2*g^8 - 44*sqrt(c*d*g)*a^3*c^3*d^3*e^11*f*g^9 + 11*sqrt(c*d
*g)*a^4*c^2*d^2*e^12*g^10 + 50*sqrt(c*d*g)*(sqrt(e^2*f + (e*x + d)*e*g - d
*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g
 - d*e*g)*c*d*g))^2*c^5*d^5*e^6*f^3*g^5 - 150*sqrt(c*d*g)*(sqrt(e^2*f + (e
*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f
+ (e*x + d)*e*g - d*e*g)*c*d*g))^2*a*c^4*d^4*e^7*f^2*g^6 + 150*sqrt(c*d*g)
*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*
e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*a^2*c^3*d^3*e^8*f*g^7
- 50*sqrt(c*d*g)*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-
c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*a^3*c^
2*d^2*e^9*g^8 + 80*sqrt(c*d*g)*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c
*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*
d*g))^4*c^4*d^4*e^4*f^2*g^4 - 160*sqrt(c*d*g)*(sqrt(e^2*f + (e*x + d)*e*g
- d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*
e*g - d*e*g)*c*d*g))^4*a*c^3*d^3*e^5*f*g^5 + 80*sqrt(c*d*g)*(sqrt(e^2*f...
```

3.726.9 Mupad [B] (verification not implemented)

Time = 14.40 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.58

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx =$$
$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{4\,x\,\sqrt{d+e\,x}\,\left(-a^2\,e^2\,g^2+10\,a\,c\,d\,e\,f\,g+15\,c^2\,d^2\,f^2\right)}{5\,e\,g\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{\sqrt{d+e\,x}\,\left(\frac{2\,a^3\,e^3\,g^3}{5}-2\,a^2\,c\,d\,e^2\,f\,g^2+6\,a\,c\cdots\right)}{c\,d\,e\,g^2\,\left(a\,e\,g-c\,d\,f\right)\cdots}\cdots\right)}{x^4\,\sqrt{f+g\,x}+\frac{a\,f^2\,\sqrt{f+g\,x}}{c\,g^2}+\frac{x^2\,\sqrt{f+g\,x}\,\left(2\,c\,d^2\,f\,g+c\,d\,e\,f^2+a\,d\,e\,g^2+2\,a\,e^2\,f\,g\right)}{c\,d\,e\,g^2}+\frac{x^3\,\sqrt{f+g\,x}\,\left(c\,g\,d^2+2\cdots\right)}{c\,d\,e\,g\cdots}\cdots}$$

input
```
int((d + e*x)^(3/2)/((f + g*x)^(7/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2)),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*x*(d + e*x)^(1/2)*(15*
c^2*d^2*f^2 - a^2*e^2*g^2 + 10*a*c*d*e*f*g))/(5*e*g*(a*e*g - c*d*f)^4) + (
(d + e*x)^(1/2)*((2*a^3*e^3*g^3)/5 + 2*c^3*d^3*f^3 + 6*a*c^2*d^2*e*f^2*g -
 2*a^2*c*d*e^2*f*g^2))/(c*d*e*g^2*(a*e*g - c*d*f)^4) + (32*c^2*d^2*g*x^3*(
d + e*x)^(1/2))/(5*e*(a*e*g - c*d*f)^4) + (16*c*d*x^2*(a*e*g + 5*c*d*f)*(d
 + e*x)^(1/2))/(5*e*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2) + (a*f^2*(f
+ g*x)^(1/2))/(c*g^2) + (x^2*(f + g*x)^(1/2)*(a*d*e*g^2 + c*d*e*f^2 + 2*a*
e^2*f*g + 2*c*d^2*f*g))/(c*d*e*g^2) + (x^3*(f + g*x)^(1/2)*(a*e^2*g + c*d^
2*g + 2*c*d*e*f))/(c*d*e*g) + (f*x*(f + g*x)^(1/2)*(a*e^2*f + c*d^2*f + 2*
a*d*e*g))/(c*d*e*g^2))
```

3.726. $\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

3.727 $\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

3.727.1 Optimal result

Integrand size = 48, antiderivative size = 289

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}(f+gx)^{3/2}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{5g^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^3d^3\sqrt{d+ex}} + \frac{5g^{3/2}(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{7/2}d^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
-2/3*(e*x+d)^(3/2)*(g*x+f)^(5/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2)-10/3*g*(g*x+f)^(3/2)*(e*x+d)^(1/2)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)+5*g^(3/2)*(-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(
1/2)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(7/2)/d^(7/2
)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5*g^2*(g*x+f)^(1/2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/(e*x+d)^(1/2)
```

3.727.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.65

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \frac{(d+ex)^{5/2}\left(\sqrt{c}\sqrt{d}(ae+cdx)\sqrt{f+gx}(15a^2e^2g^2-10acdeg(f-2\right.}{3c}$$

input
```
Integrate[((d + e*x)^(5/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(5/2),x]
```

output
```
((d + e*x)^(5/2)*(Sqrt[c]*Sqrt[d]*(a*e + c*d*x)*Sqrt[f + g*x]*(15*a^2*e^2*
g^2 - 10*a*c*d*e*g*(f - 2*g*x) + c^2*d^2*(-2*f^2 - 14*f*g*x + 3*g^2*x^2))
+ 15*g^(3/2)*(c*d*f - a*e*g)*(a*e + c*d*x)^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*
Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(3*c^(7/2)*d^(7/2)*((a*e + c
*d*x)*(d + e*x))^(5/2))
```

3.727.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1251, 1251, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1251}$$

$$\frac{5g\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3cd}-\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\,(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

$$\Big\downarrow \text{1251}$$

$$\frac{5g\left(\frac{3g\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\,(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

$$\Big\downarrow \text{1253}$$

$$\frac{5g\left(\frac{3g\left(\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 1268$$

$$\frac{5g\left(\frac{3g\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 66$$

$$\frac{5g\left(\frac{3g\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{5g\left(\frac{3g\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

3.727. $\int\frac{(d+ex)^{5/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

input
```
Int[((d + e*x)^(5/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2)*(f + g*x)^(5/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(3/2)) + (5*g*((-2*Sqrt[d + e*x]*(f + g*x)^(3/2))/(c*d*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (3*g*((Sqrt[f + g*x]*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x]) + ((c*d*f - a*e*g)*Sqrt[
a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sq
rt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])))/(c*d)))/(3*c*d)
```

3.727.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1251
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[e*g*(n/(c*(p + 1)))   Int[(
d + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p,
 0] && LtQ[p, -1] && GtQ[n, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

3.727. $\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.727.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(245) = 490$.

Time = 0.55 (sec) , antiderivative size = 642, normalized size of antiderivative = 2.22

method	result
default	$-\frac{\left(15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a\,c^{2}d^{2}e\,g^{3}x^{2}-15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)c^{3}d^{3}f\,g^{2}x^{2}+30\ln\right.}{}$

input
```
int((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
method=_RETURNVERBOSE)
```

output
```
-1/6*(15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*
g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*g^3*x^2-15*ln(1/2*(2*c*d*g*x+a*e*g+c*
d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f*
g^2*x^2+30*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*
d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*g^3*x-30*ln(1/2*(2*c*d*g*x+a*e*g+c*
d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*
e*f*g^2*x+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(
c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*e^3*g^3-15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+
2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*
g^2-6*c^2*d^2*g^2*x^2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)-40*(c*d*g)
^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x+28*(c*d*g)^(1/2)*((g*x+f)
*(c*d*x+a*e))^(1/2)*c^2*d^2*f*g*x-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(
1/2)*a^2*e^2*g^2+20*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g+
4*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*((c*d*x+a*e)*(e*x
+d))^(1/2)*(g*x+f)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)/(c*d*x+
a*e)^2/c^3/d^3/(e*x+d)^(1/2)
```

3.727.5 Fricas [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 1055, normalized size of antiderivative = 3.65

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \left[\frac{4\,(3\,c^2d^2g^2x^2-2\,c^2d^2f^2-10\,acdefg+15\,a^2e^2g^2-2\,(7\,c^2d^2fg-\right.$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="fricas")
```

output
```
[1/12*(4*(3*c^2*d^2*g^2*x^2 - 2*c^2*d^2*f^2 - 10*a*c*d*e*f*g + 15*a^2*e^2*
g^2 - 2*(7*c^2*d^2*f*g - 10*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(a^2*c*d^2*e^2*f*g - a^3*d*
e^3*g^2 + (c^3*d^3*e*f*g - a*c^2*d^2*e^2*g^2)*x^3 + ((c^3*d^4 + 2*a*c^2*d^
2*e^2)*f*g - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*g^2)*x^2 + ((2*a*c^2*d^3*e + a^
2*c*d*e^3)*f*g - (2*a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt(g/(c*d))*log(-(8
*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^
2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 - 4*(2*c^2*d^2*g*x + c^2*d^2*
f + a*c*d*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*s
qrt(g*x + f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*
f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^5*d^5*e*x^3 + a^2*c^3
*d^4*e^2 + (c^5*d^6 + 2*a*c^4*d^4*e^2)*x^2 + (2*a*c^4*d^5*e + a^2*c^3*d^3*
e^3)*x), 1/6*(2*(3*c^2*d^2*g^2*x^2 - 2*c^2*d^2*f^2 - 10*a*c*d*e*f*g + 15*a
^2*e^2*g^2 - 2*(7*c^2*d^2*f*g - 10*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(a^2*c*d^2*e^2*f*g -
 a^3*d*e^3*g^2 + (c^3*d^3*e*f*g - a*c^2*d^2*e^2*g^2)*x^3 + ((c^3*d^4 + 2*a
*c^2*d^2*e^2)*f*g - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*g^2)*x^2 + ((2*a*c^2*d^3
*e + a^2*c*d*e^3)*f*g - (2*a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt(-g/(c*d))
*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g
*x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*...
```

3.727.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)*(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(5/2),x)
```

output
```
Timed out
```

3.727.7 Maxima [F]

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{5}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)*(g*x + f)^(5/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)^(5/2), x)
```

3.727.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1105 vs. 2(245) = 490.

Time = 0.86 (sec) , antiderivative size = 1105, normalized size of antiderivative = 3.82

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \frac{15\sqrt{-cd^2eg^2+ae^3g^2}c^2d^3efg^3\log\left(\left|-\sqrt{e^2f-deg}\sqrt{cdg}+\sqrt{-cd^2e\cdots}\right.\right.}{\cdots} - \frac{\sqrt{e^2f+(ex+d)eg-deg}\left((e^2f+(ex+d)eg-deg)\left(\frac{3\left(c^5d^5e^2fg^5-ac^4d^4e^3g^6\right)(e^2f+(ex+d)eg-deg)}{c^6d^6e^4fg|g|-ac^5d^5e^5g^2|g|}-\frac{20\left(c^5d^5e^4f^2g^5\cdots\right.}{c^6d^6e^4\cdots}\right.\right.}{3\left(cde^2fg-ae^3g^2-(e^2f+(ex+d)eg-deg)cdg\right)\sqrt{-cde^2fg+ae\cdots}} - \frac{5\left(cdfg^3-aeg^4\right)\log\left(\left|-\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg\cdots}\right.\right.}{\sqrt{cdg}c^3d^3|g|}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="giac")
```

output
```
1/3*(15*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*c^2*d^3*e*f*g^3*log(abs(-sqrt(e^2*f
 - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 15*sqrt(-c*d^2*
e*g^2 + a*e^3*g^2)*a*c*d*e^3*f*g^3*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g
) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 15*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a
*c*d^2*e^2*g^4*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^
2 + a*e^3*g^2))) + 15*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^2*e^4*g^4*log(abs(-
sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 2*sqr
t(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^2*e^2*f^2*g^2 + 14*sqrt(e^2*f - d*e*g)*
sqrt(c*d*g)*c^2*d^3*e*f*g^3 - 10*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c*d*e^3
*f*g^3 + 3*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^4*g^4 - 20*sqrt(e^2*f - d
*e*g)*sqrt(c*d*g)*a*c*d^2*e^2*g^4 + 15*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2
*e^4*g^4)/(sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*c^4*d^5*e*abs(g) - s
qrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a*c^3*d^3*e^3*abs(g)) - 1/3*sqrt
(e^2*f + (e*x + d)*e*g - d*e*g)*((e^2*f + (e*x + d)*e*g - d*e*g)*(3*(c^5*d
^5*e^2*f*g^5 - a*c^4*d^4*e^3*g^6)*(e^2*f + (e*x + d)*e*g - d*e*g)/(c^6*d^6
*e^4*f*g*abs(g) - a*c^5*d^5*e^5*g^2*abs(g)) - 20*(c^5*d^5*e^4*f^2*g^5 - 2*
a*c^4*d^4*e^5*f*g^6 + a^2*c^3*d^3*e^6*g^7)/(c^6*d^6*e^4*f*g*abs(g) - a*c^5
*d^5*e^5*g^2*abs(g))) + 15*(c^5*d^5*e^6*f^3*g^5 - 3*a*c^4*d^4*e^7*f^2*g^6
+ 3*a^2*c^3*d^3*e^8*f*g^7 - a^3*c^2*d^2*e^9*g^8)/(c^6*d^6*e^4*f*g*abs(g) -
 a*c^5*d^5*e^5*g^2*abs(g)))/((c*d*e^2*f*g - a*e^3*g^2 - (e^2*f + (e*x +...
```

3.727.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx=\int \frac{(f+g\,x)^{5/2}\,(d+e\,x)^{5/2}}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{5/2}}\,dx$$

input
```
int(((f + g*x)^(5/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(5/2),x)
```

output
```
int(((f + g*x)^(5/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(5/2), x)
```

3.728 $\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

3.728.1 Optimal result

Integrand size = 48, antiderivative size = 219

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\,(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} - \frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{2g^{3/2}\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

output
```
-2/3*(e*x+d)^(3/2)*(g*x+f)^(3/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2)+2*g^(3/2)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1/
2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2)/(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)-2*g*(e*x+d)^(1/2)*(g*x+f)^(1/2)/c^2/d^2/(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)
```

3.728.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.61

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}\left(\sqrt{c}\sqrt{d}\sqrt{f+gx}(3aeg+cd(f+4gx)) - 3g^{3/2}(ae+cdx)^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)\right)}{3c^{5/2}d^{5/2}((ae+cdx)(d+ex))^{3/2}}$$

input
```
Integrate[((d + e*x)^(5/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2)*(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x]*(3*a*e*g + c*d*(f + 4*g
*x)) - 3*g^(3/2)*(a*e + c*d*x)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x
])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(3*c^(5/2)*d^(5/2)*((a*e + c*d*x)*(d + e
*x))^(3/2))
```

3.728.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {1251, 1251, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1251}$$

$$\frac{g\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{cd}-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1251}$$

$$\frac{g\left(\frac{g\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cd}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{cd}-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1268}$$

$$\frac{g\left(\frac{g\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{cd}-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{66}$$

$$\frac{g\left(\frac{2g\sqrt{d+ex}\sqrt{ae+cdx}\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{cd}-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow 221$$

$$\frac{g\left(\frac{2\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{cd}-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[((d + e*x)^(5/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(3/2)) + (g*((-2*Sqrt[d + e*x]*Sqrt[f + g*x])/(c*d*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2]) + (2*Sqrt[g]*Sqrt[a*e + c*d*x]*Sqrt[d + e*
x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(
c^(3/2)*d^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(c*d)
```

3.728.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1251
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[e*g*(n/(c*(p + 1)))   Int[(
d + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p,
 0] && LtQ[p, -1] && GtQ[n, 0]
```

3.728. $\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.728.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.52

method	result
default	$\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)c^2d^2g^2x^2+6\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right.\right.}{3\sqrt{cdg}\,(}$

input
```
int((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
method=_RETURNVERBOSE)
```

output
```
1/3*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*ln(1/2*(2*c*d*g*x+a*e*g+c
*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*g
^2*x^2+6*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*
g)^(1/2))/(c*d*g)^(1/2))*a*c*d*e*g^2*x+3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(
(g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*e^2*g^2-8*((g
*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c*d*g*x-6*(c*d*g)^(1/2)*((g*x+f)*(c
*d*x+a*e))^(1/2)*a*e*g-2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)/
(c*d*g)^(1/2)/(c*d*x+a*e)^2/((g*x+f)*(c*d*x+a*e))^(1/2)/d^2/c^2/(e*x+d)^(1
/2)
```

3.728.5 Fricas [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 755, normalized size of antiderivative = 3.45

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \left[-\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}(4\,cdgx+cdf+3\,aeg)\sqrt{ex+d}}{}\right.$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="fricas")
```

output
```
[-1/6*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(4*c*d*g*x + c*d*f +
3*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^2*d^2*e*g*x^3 + a^2*d*e^2*g +
(c^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (2*a*c*d^2*e + a^2*e^3)*g*x)*sqrt(g/(c*d))
*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2
 + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + 4*(2*c^2*d^2*g*x +
c^2*d^2*f + a*c*d*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*
x + d)*sqrt(g*x + f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c
*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d^4*e*x^3 +
 a^2*c^2*d^3*e^2 + (c^4*d^5 + 2*a*c^3*d^3*e^2)*x^2 + (2*a*c^3*d^4*e + a^2*
c^2*d^2*e^3)*x), -1/3*(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(4*c*
d*g*x + c*d*f + 3*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^2*d^2*e*g*x^3
+ a^2*d*e^2*g + (c^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (2*a*c*d^2*e + a^2*e^3)*g*
x)*sqrt(-g/(c*d))*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d)*sqrt(g*x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d
*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^4*d^4*e*x^3 + a^2*c^2*d^3*e
^2 + (c^4*d^5 + 2*a*c^3*d^3*e^2)*x^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*x
)]
```

3.728.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)*(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(5/2),x)
```

output
```
Timed out
```

3.728.7 Maxima [F]

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)*(g*x + f)^(3/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)^(5/2), x)
```

3.728.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(183) = 366$.

Time = 0.65 (sec) , antiderivative size = 685, normalized size of antiderivative = 3.13

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \frac{2\left(3\sqrt{-cd^2eg^2+ae^3g^2}cd^2g^3\log\left(\left|-\sqrt{e^2f-deg}\sqrt{cdg}+\sqrt{-cd^2eg\right.\right.}}{}$$
$$+\frac{2\sqrt{e^2f+(ex+d)eg-deg}\left(\frac{4\left(c^3d^3e^2fg^4-ac^2d^2e^3g^5\right)\left(e^2f+(ex+d)eg-deg\right)}{c^4d^4e^2f|g|-ac^3d^3e^3g|g|}-\frac{3\left(c^3d^3e^4f^2g^4-2\,ac^2d^2e^5fg^5+a^2cde^6g^6\right)}{c^4d^4e^2f|g|-ac^3d^3e^3g|g|}\right)}{3\left(cde^2fg-ae^3g^2-(e^2f+(ex+d)eg-deg)cdg\right)\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}}$$
$$-\frac{2\,g^3\log\left(\left|-\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\right|\right)}{\sqrt{cdg}c^2d^2|g|}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="giac")
```

output
```
2/3*(3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*c*d^2*g^3*log(abs(-sqrt(e^2*f - d*e*
g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 3*sqrt(-c*d^2*e*g^2 +
a*e^3*g^2)*a*e^2*g^3*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^
2*e*g^2 + a*e^3*g^2))) - sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d*e*f*g^2 + 4*s
qrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d^2*g^3 - 3*sqrt(e^2*f - d*e*g)*sqrt(c*d*
g)*a*e^2*g^3)/(sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*c^3*d^4*abs(g) -
 sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a*c^2*d^2*e^2*abs(g)) + 2/3*sq
rt(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(c^3*d^3*e^2*f*g^4 - a*c^2*d^2*e^3*g^
5)*(e^2*f + (e*x + d)*e*g - d*e*g)/(c^4*d^4*e^2*f*abs(g) - a*c^3*d^3*e^3*g
*abs(g)) - 3*(c^3*d^3*e^4*f^2*g^4 - 2*a*c^2*d^2*e^5*f*g^5 + a^2*c*d*e^6*g^
6)/(c^4*d^4*e^2*f*abs(g) - a*c^3*d^3*e^3*g*abs(g)))/((c*d*e^2*f*g - a*e^3*
g^2 - (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(-c*d*e^2*f*g + a*e^3*g^2
 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)) - 2*g^3*log(abs(-sqrt(e^2*f + (
e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f
 + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^2*d^2*abs(g))
```

3.728.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\int \frac{{\left(f+g\,x\right)}^{3/2}\,{\left(d+e\,x\right)}^{5/2}}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}\,dx$$

input
```
int(((f + g*x)^(3/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(5/2),x)
```

output
```
int(((f + g*x)^(3/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(5/2), x)
```

3.729 $\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

3.729.1 Optimal result

Integrand size = 48, antiderivative size = 63

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3(cdf-aeg)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}$$

output
```
-2/3*(e*x+d)^(3/2)*(g*x+f)^(3/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(3/2)
```

3.729.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3(cdf-aeg)((ae+cdx)(d+ex))^{3/2}}$$

input
```
Integrate[((d + e*x)^(5/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*(c*d*f - a*e*g)*((a*e + c*d*x)*(d
+ e*x))^(3/2))
```

3.729.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 1248}$$

$$-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}$$

input
```
Int[((d + e*x)^(5/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(3/2))
```

3.729.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

3.729.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{2(gx+f)^{\frac{3}{2}}\sqrt{(cdx+ae)(ex+d)}}{3\sqrt{ex+d}\,(cdx+ae)^2(aeg-cdf)}$	55
gosper	$\frac{2(gx+f)^{\frac{3}{2}}(cdx+ae)(ex+d)^{\frac{5}{2}}}{3(aeg-cdf)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}$	63

input
```
int((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
method=_RETURNVERBOSE)
```

output
```
2/3/(e*x+d)^(1/2)*(g*x+f)^(3/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/(c*d*x+a*e)^2/
(a*e*g-c*d*f)
```

3.729.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(55) = 110$.

Time = 0.32 (sec) , antiderivative size = 193, normalized size of antiderivative = 3.06

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx =$$

$$-\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}(gx+f)^{\frac{3}{2}}}{3\,(a^2cd^2e^2f-a^3de^3g+(c^3d^3ef-ac^2d^2e^2g)x^3+((c^3d^4+2\,ac^2d^2e^2)f-(ac^2d^3e+2\,a^2cde^3)g)x^2+((2\,ac^2}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*(g*x + f)^(
3/2)/(a^2*c*d^2*e^2*f - a^3*d*e^3*g + (c^3*d^3*e*f - a*c^2*d^2*e^2*g)*x^3
+ ((c^3*d^4 + 2*a*c^2*d^2*e^2)*f - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*g)*x^2 +
((2*a*c^2*d^3*e + a^2*c*d*e^3)*f - (2*a^2*c*d^2*e^2 + a^3*e^4)*g)*x)
```

3.729.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)*(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(5/2),x)
```

output
```
Timed out
```

3.729.7 Maxima [F]

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}\sqrt{gx+f}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)*sqrt(g*x + f)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)^(5/2), x)
```

3.729.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(55) = 110$.

Time = 0.45 (sec) , antiderivative size = 316, normalized size of antiderivative = 5.02

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{2\,(e^2f+(ex+d)eg}{3\,(c^2d^2e^2f|g|-acde^3g|g|)(cde^2fg-ae^3g^2-(e^2f+(ex+d)eg-} - \frac{2\left(\sqrt{e^2f-deg}efg^2-\sqrt{e^2f-deg}dg^3\right)}{3\left(\sqrt{-cd^2eg^2+ae^3g^2}c^2d^3f|g|-\sqrt{-cd^2eg^2+ae^3g^2}acde^2f|g|-\sqrt{-cd^2eg^2+ae^3g^2}acd^2eg|g|+\sqrt{-cd^2eg^2}\right.}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="giac")
```

output
```
2/3*(e^2*f + (e*x + d)*e*g - d*e*g)^(3/2)*c*d*e^2*g^4/((c^2*d^2*e^2*f*abs(
g) - a*c*d*e^3*g*abs(g))*(c*d*e^2*f*g - a*e^3*g^2 - (e^2*f + (e*x + d)*e*g
 - d*e*g)*c*d*g)*sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g -
d*e*g)*c*d*g)) - 2/3*(sqrt(e^2*f - d*e*g)*e*f*g^2 - sqrt(e^2*f - d*e*g)*d*
g^3)/(sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*c^2*d^3*f*abs(g) - sqrt(-c*d^2*e*g^2
+ a*e^3*g^2)*a*c*d*e^2*f*abs(g) - sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a*c*d^2*e
*g*abs(g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^2*e^3*g*abs(g))
```

3.729.9 Mupad [B] (verification not implemented)

Time = 12.87 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.68

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{\left(\frac{2\,f\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e\,(a\,e\,g-c\,d\,f)}+\frac{2\,g\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e\,(a\,e\,g-c\,d\,f)}\right)\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\;x+}}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,(2\,c\,d^2+a\,e^2)}{c^2\,d^2}+\frac{x^2\,(c\,d^2+2\,a\,e^2)}{c\,d\,e}}$$

input
```
int(((f + g*x)^(1/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(5/2),x)
```

output
```
(((2*f*(f + g*x)^(1/2)*(d + e*x)^(1/2))/(3*c^2*d^2*e*(a*e*g - c*d*f)) + (2
*g*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/(3*c^2*d^2*e*(a*e*g - c*d*f)))*(x*(a
*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^3 + (a^2*e)/(c^2*d) + (a*x*(a
*e^2 + 2*c*d^2))/(c^2*d^2) + (x^2*(2*a*e^2 + c*d^2))/(c*d*e))
```

3.730 $\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

3.730.1 Optimal result

Integrand size = 48, antiderivative size = 128

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3(cdf-aeg)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}} + \frac{4g\sqrt{d+ex}\sqrt{f+gx}}{3(cdf-aeg)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

output
```
-2/3*(e*x+d)^(3/2)*(g*x+f)^(1/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(3/2)+4/3*g*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(-a*e*g+c*d*f)^2/(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

3.730.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.52

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}\sqrt{f+gx}(-3aeg+cd(f-2gx))}{3(cdf-aeg)^2((ae+cdx)(d+ex))^{3/2}}$$

input
```
Integrate[(d + e*x)^(5/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(5/2)),x]
```

output
```
(-2*(d + e*x)^(3/2)*Sqrt[f + g*x]*(-3*a*e*g + c*d*(f - 2*g*x)))/(3*(c*d*f
- a*e*g)^2*((a*e + c*d*x)*(d + e*x))^(3/2))
```

3.730.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1252, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1252}$$

$$-\frac{2g\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3(cdf-aeg)}-\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{ 1248}$$

$$\frac{4g\sqrt{d+ex}\sqrt{f+gx}}{3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

input
```
Int[(d + e*x)^(5/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(5/2)),x]
```

output
```
(-2*(d + e*x)^(3/2)*Sqrt[f + g*x])/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2)^(3/2)) + (4*g*Sqrt[d + e*x]*Sqrt[f + g*x])/(3*(c*d*f -
 a*e*g)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

3.730.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1252
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

3.730.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{2\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\,(2cdgx+3aeg-cdf)}{3\sqrt{ex+d}\,(cdx+ae)^{2}(aeg-cdf)^{2}}$	72
gosper	$\frac{2\sqrt{gx+f}\,(cdx+ae)(2cdgx+3aeg-cdf)(ex+d)^{\frac{5}{2}}}{3(a^{2}e^{2}g^{2}-2acdefg+c^{2}d^{2}f^{2})(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{5}{2}}}$	99

input
```
int((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
method=_RETURNVERBOSE)
```

output
```
2/3/(e*x+d)^(1/2)*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(2*c*d*g*x+3*a
*e*g-c*d*f)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^2
```

3.730.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(112) = 224$.

Time = 0.44 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.48

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{}{3\,(a^2c^2d^3e^2f^2-2\,a^3cd^2e^3fg+a^4de^4g^2+(c^4d^4ef^2-2\,ac}$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x - c*d*f + 3*a*e
*g)*sqrt(e*x + d)*sqrt(g*x + f)/(a^2*c^2*d^3*e^2*f^2 - 2*a^3*c*d^2*e^3*f*g
 + a^4*d*e^4*g^2 + (c^4*d^4*e*f^2 - 2*a*c^3*d^3*e^2*f*g + a^2*c^2*d^2*e^3*
g^2)*x^3 + ((c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2 - 2*(a*c^3*d^4*e + 2*a^2*c^2*d
^2*e^3)*f*g + (a^2*c^2*d^3*e^2 + 2*a^3*c*d*e^4)*g^2)*x^2 + ((2*a*c^3*d^4*e
 + a^2*c^2*d^2*e^3)*f^2 - 2*(2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g + (2*a^3
*c*d^2*e^3 + a^4*e^5)*g^2)*x)
```

3.730.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)/(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(5/2),x)
```

output
```
Timed out
```

3.730.7 Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*s
qrt(g*x + f)), x)
```

3.730.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. $2(112) = 224$.

Time = 0.35 (sec) , antiderivative size = 552, normalized size of antiderivative = 4.31

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2\left(\frac{2\,(e^2f+(ex+d)eg-deg)c^2d^2g^4}{c^3d^3e^2f^2|g|-2\,ac^2d^2e^3fg|g|+a^2cde^4g^2|g|} - \frac{3\,(c^2d^2e^2fg^4-acde^3g^5)}{c^3d^3e^2f^2|g|-2\,ac^2d^2e^3fg|g|+a^2cde^4g^2|g|}\right)\sqrt{e^2f+(ex+d)eg-deg}e^2}{3\,(cde^2fg-ae^3g^2-(e^2f+(ex+d)eg-deg)cdg)\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}} - \frac{2\left(\sqrt{e^2f-deg}cdefg^2+2\sqrt{e^2f-degc}\right.}{3\left(\sqrt{-cd^2eg^2+ae^3g^2}c^3d^4f^2|g| - \sqrt{-cd^2eg^2+ae^3g^2}ac^2d^2e^2f^2|g| - 2\sqrt{-cd^2eg^2+ae^3g^2}ac^2d^3efg|g| + 2\sqrt{}\right.}$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="giac")
```

output
```
-2/3*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*c^2*d^2*g^4/(c^3*d^3*e^2*f^2*abs(g
) - 2*a*c^2*d^2*e^3*f*g*abs(g) + a^2*c*d*e^4*g^2*abs(g)) - 3*(c^2*d^2*e^2*
f*g^4 - a*c*d*e^3*g^5)/(c^3*d^3*e^2*f^2*abs(g) - 2*a*c^2*d^2*e^3*f*g*abs(g
) + a^2*c*d*e^4*g^2*abs(g)))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*e^2/((c*d
*e^2*f*g - a*e^3*g^2 - (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(-c*d*e^
2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)) - 2/3*(sqrt(e^
2*f - d*e*g)*c*d*e*f*g^2 + 2*sqrt(e^2*f - d*e*g)*c*d^2*g^3 - 3*sqrt(e^2*f
- d*e*g)*a*e^2*g^3)/(sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*c^3*d^4*f^2*abs(g) - s
qrt(-c*d^2*e*g^2 + a*e^3*g^2)*a*c^2*d^2*e^2*f^2*abs(g) - 2*sqrt(-c*d^2*e*g
^2 + a*e^3*g^2)*a*c^2*d^3*e*f*g*abs(g) + 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*
a^2*c*d*e^3*f*g*abs(g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^2*c*d^2*e^2*g^2*
abs(g) - sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^3*e^4*g^2*abs(g))
```

3.730.9 Mupad [B] (verification not implemented)

Time = 13.62 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.92

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{4\,g^2\,x^2\,\sqrt{d+e\,x}}{3\,c\,d\,e\,\left(a\,e\,g-c\,d\,f\right)^2}-\frac{(2\,c\cdots}{3\cdots}\right.}{x^3\,\sqrt{f+g\,x}+\frac{a^2\,e\,\sqrt{f+g\,x}}{c^2\,d}+\frac{x^2\,\sqrt{f+g\,x}\,(c\,d^2+\cdots}{c\,d\,e}}$$

input
```
int((d + e*x)^(5/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2)),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*g^2*x^2*(d + e*x)^(1/2)
)/(3*c*d*e*(a*e*g - c*d*f)^2) - ((2*c*d*f^2 - 6*a*e*f*g)*(d + e*x)^(1/2))/
(3*c^2*d^2*e*(a*e*g - c*d*f)^2) + (x*(6*a*e*g^2 + 2*c*d*f*g)*(d + e*x)^(1/
2))/(3*c^2*d^2*e*(a*e*g - c*d*f)^2)))/(x^3*(f + g*x)^(1/2) + (a^2*e*(f + g
*x)^(1/2))/(c^2*d) + (x^2*(f + g*x)^(1/2)*(2*a*e^2 + c*d^2))/(c*d*e) + (a*
x*(f + g*x)^(1/2)*(a*e^2 + 2*c*d^2))/(c^2*d^2))
```

3.731 $\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

3.731.1 Optimal result

Integrand size = 48, antiderivative size = 194

$$\begin{aligned}\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = & -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\ & +\frac{8g\sqrt{d+ex}}{3(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\ & +\frac{16g^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)^3\sqrt{d+ex}\sqrt{f+gx}}\end{aligned}$$

output
```
-2/3*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/
(g*x+f)^(1/2)+8/3*g*(e*x+d)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)^(1/2)/(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+16/3*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

3.731.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \frac{2(d+ex)^{3/2}\left(3a^2e^2g^2+6acdeg(f+2gx)+c^2d^2(-f^2\right.}{3(cdf-aeg)^3((ae+cdx)(d+ex))^{3/2}\sqrt{f}}$$

input
```
Integrate[(d + e*x)^(5/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(5/2)),x]
```

output
```
(2*(d + e*x)^(3/2)*(3*a^2*e^2*g^2 + 6*a*c*d*e*g*(f + 2*g*x) + c^2*d^2*(-f^
2 + 4*f*g*x + 8*g^2*x^2)))/(3*(c*d*f - a*e*g)^3*((a*e + c*d*x)*(d + e*x))^
(3/2)*Sqrt[f + g*x])
```

3.731.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1252, 1252, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(x\,(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{1252}$$

$$-\frac{4g\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3(cdf-aeg)}-\frac{2(d+ex)^{3/2}}{3\sqrt{f+gx}\,\left(x\,(ae^2+cd^2)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1252}$$

$$-\frac{4g\left(-\frac{2g\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cdf-aeg}-\frac{2\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{3(cdf-aeg)}-\frac{2(d+ex)^{3/2}}{3\sqrt{f+gx}\,\left(x\,(ae^2+cd^2)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1248}$$

$$-\frac{2(d+ex)^{3/2}}{3\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}-\frac{4g\left(-\frac{4g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{3(cdf-aeg)}$$

input
```
Int[(d + e*x)^(5/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2)),x]
```

output
```
(-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2)^(3/2)) - (4*g*((-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*Sqr
t[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (4*g*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f
+ g*x])))/(3*(c*d*f - a*e*g))
```

3.731.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1252
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

3.731. $\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

3.731.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.62

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\left(8g^2x^2c^2d^2+12acde\,g^2x+4c^2d^2fgx+3a^2e^2g^2+6acdefg-c^2d^2f^2\right)}{3\sqrt{ex+d}\,\sqrt{gx+f}\,(cdx+ae)^2(aeg-cdf)^3}$	121
gosper	$-\frac{2(cdx+ae)\left(8g^2x^2c^2d^2+12acde\,g^2x+4c^2d^2fgx+3a^2e^2g^2+6acdefg-c^2d^2f^2\right)(ex+d)^{\frac{5}{2}}}{3\sqrt{gx+f}\,(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3c^3d^3)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}$	169

input
```
int((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)/(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(8*c^2*d^2*g^
2*x^2+12*a*c*d*e*g^2*x+4*c^2*d^2*f*g*x+3*a^2*e^2*g^2+6*a*c*d*e*f*g-c^2*d^2
*f^2)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^3
```

3.731.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. 2(170) = 340.

Time = 0.53 (sec) , antiderivative size = 667, normalized size of antiderivative = 3.44

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \frac{}{3\left(a^2c^3d^4e^2f^4-3\,a^3c^2d^3e^3f^3g+3\,a^4cd^2e^4f^2g^2-a^5de^5\right.}$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="fricas")
```

output
```
2/3*(8*c^2*d^2*g^2*x^2 - c^2*d^2*f^2 + 6*a*c*d*e*f*g + 3*a^2*e^2*g^2 + 4*(
c^2*d^2*f*g + 3*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*sqrt(e*x + d)*sqrt(g*x + f)/(a^2*c^3*d^4*e^2*f^4 - 3*a^3*c^2*d^3*e^3*f^3
*g + 3*a^4*c*d^2*e^4*f^2*g^2 - a^5*d*e^5*f*g^3 + (c^5*d^5*e*f^3*g - 3*a*c^
4*d^4*e^2*f^2*g^2 + 3*a^2*c^3*d^3*e^3*f*g^3 - a^3*c^2*d^2*e^4*g^4)*x^4 + (
c^5*d^5*e*f^4 + (c^5*d^6 - a*c^4*d^4*e^2)*f^3*g - 3*(a*c^4*d^5*e + a^2*c^3
*d^3*e^3)*f^2*g^2 + (3*a^2*c^3*d^4*e^2 + 5*a^3*c^2*d^2*e^4)*f*g^3 - (a^3*c
^2*d^3*e^3 + 2*a^4*c*d*e^5)*g^4)*x^3 + ((c^5*d^6 + 2*a*c^4*d^4*e^2)*f^4 -
(a*c^4*d^5*e + 5*a^2*c^3*d^3*e^3)*f^3*g - 3*(a^2*c^3*d^4*e^2 - a^3*c^2*d^2
*e^4)*f^2*g^2 + (5*a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f*g^3 - (2*a^4*c*d^2*e^4
 + a^5*e^6)*g^4)*x^2 - (a^5*d*e^5*g^4 - (2*a*c^4*d^5*e + a^2*c^3*d^3*e^3)*
f^4 + (5*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4)*f^3*g - 3*(a^3*c^2*d^3*e^3 +
 a^4*c*d*e^5)*f^2*g^2 - (a^4*c*d^2*e^4 - a^5*e^6)*f*g^3)*x)
```

3.731.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)/(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(5/2),x)
```

output
```
Timed out
```

3.731.7 Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\int \frac{(ex+d)^{\frac{5}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(
g*x + f)^(3/2)), x)
```

3.731.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1923 vs. $2(170) = 340$.

Time = 0.50 (sec) , antiderivative size = 1923, normalized size of antiderivative = 9.91

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="giac")
```

output
```
2/3*(6*sqrt(c*d*g)*g^3/((c^2*d^2*e*f^2*abs(g) - 2*a*c*d*e^2*f*g*abs(g) + a
^2*e^3*g^2*abs(g))*(c*d*e^2*f*g - a*e^3*g^2 + (sqrt(e^2*f + (e*x + d)*e*g
- d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*
e*g - d*e*g)*c*d*g))^2)) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(5*(c^5*d^5
*e*f^2*g^4*abs(g) - 2*a*c^4*d^4*e^2*f*g^5*abs(g) + a^2*c^3*d^3*e^3*g^6*abs
(g))*(e^2*f + (e*x + d)*e*g - d*e*g)/(c^6*d^6*e^4*f^5*g^2 - 5*a*c^5*d^5*e^
5*f^4*g^3 + 10*a^2*c^4*d^4*e^6*f^3*g^4 - 10*a^3*c^3*d^3*e^7*f^2*g^5 + 5*a^
4*c^2*d^2*e^8*f*g^6 - a^5*c*d*e^9*g^7) - 6*(c^5*d^5*e^3*f^3*g^4*abs(g) - 3
*a*c^4*d^4*e^4*f^2*g^5*abs(g) + 3*a^2*c^3*d^3*e^5*f*g^6*abs(g) - a^3*c^2*d
^2*e^6*g^7*abs(g))/(c^6*d^6*e^4*f^5*g^2 - 5*a*c^5*d^5*e^5*f^4*g^3 + 10*a^2
*c^4*d^4*e^6*f^3*g^4 - 10*a^3*c^3*d^3*e^7*f^2*g^5 + 5*a^4*c^2*d^2*e^8*f*g^
6 - a^5*c*d*e^9*g^7))/((c*d*e^2*f*g - a*e^3*g^2 - (e^2*f + (e*x + d)*e*g -
 d*e*g)*c*d*g)*sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*
e*g)*c*d*g)))*e^3 - 2/3*(sqrt(e^2*f - d*e*g)*c^3*d^3*e^2*f^2*g^2 + 4*sqrt(
e^2*f - d*e*g)*c^3*d^4*e*f*g^3 - 6*sqrt(e^2*f - d*e*g)*a*c^2*d^2*e^3*f*g^3
 - 5*sqrt(e^2*f - d*e*g)*c^3*d^5*g^4 + 6*sqrt(e^2*f - d*e*g)*a*c^2*d^3*e^2
*g^4 - sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*c^2*d^2*e^2*f^2*g - sqrt
(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*c^2*d^3*e*f*g^2 + 3*sqrt(-c*d^2*e*g
^2 + a*e^3*g^2)*sqrt(c*d*g)*a*c*d*e^3*f*g^2 + 5*sqrt(-c*d^2*e*g^2 + a*e^3*
g^2)*sqrt(c*d*g)*c^2*d^4*g^3 - 9*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*...
```

3.731.9 Mupad [B] (verification not implemented)

Time = 13.94 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx =$$

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{16\,g^2\,x^2\,\sqrt{d+e\,x}}{3\,e\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{\sqrt{d+e\,x}\,\left(6\,a^2\,e^2\,g^2+12\,a\,c\,d\,e\,f\,g-2\,c^2\,d^2\,f^2\right)}{3\,c^2\,d^2\,e\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{8\,g\,x\,\left(3\,a\,e\,g+c\,d\,f\right)\,\sqrt{d+e\cdots}}{3\,c\,d\,e\,{\left(a\,e\,g-c\,d\,f\right)}^3}\cdots\right.}{x^3\,\sqrt{f+g\,x}+\frac{a^2\,e\,\sqrt{f+g\,x}}{c^2\,d}+\frac{x^2\,\sqrt{f+g\,x}\,\left(c\,d^2+2\,a\,e^2\right)}{c\,d\,e}+\frac{a\,x\,\sqrt{f+g\,x}\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}}$$

input
```
int((d + e*x)^(5/2)/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2)),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((16*g^2*x^2*(d + e*x)^(1/
2))/(3*e*(a*e*g - c*d*f)^3) + ((d + e*x)^(1/2)*(6*a^2*e^2*g^2 - 2*c^2*d^2*
f^2 + 12*a*c*d*e*f*g))/(3*c^2*d^2*e*(a*e*g - c*d*f)^3) + (8*g*x*(3*a*e*g +
 c*d*f)*(d + e*x)^(1/2))/(3*c*d*e*(a*e*g - c*d*f)^3)))/(x^3*(f + g*x)^(1/2
) + (a^2*e*(f + g*x)^(1/2))/(c^2*d) + (x^2*(f + g*x)^(1/2)*(2*a*e^2 + c*d^
2))/(c*d*e) + (a*x*(f + g*x)^(1/2)*(a*e^2 + 2*c*d^2))/(c^2*d^2))
```

3.731. $\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

3.732 $\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

3.732.1 Optimal result

Integrand size = 48, antiderivative size = 260

$$\begin{aligned}\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx =\\ &-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\\ &+\frac{4g\sqrt{d+ex}}{(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\\ &+\frac{16g^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)^3\sqrt{d+ex}(f+gx)^{3/2}}+\frac{32cdg^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)^4\sqrt{d+ex}\sqrt{f+gx}}\end{aligned}$$

output
```
-2/3*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(3/2)+4*g*(e*x+d)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)^(3/2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)+16/3*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/(-a*e*g+c*d*f)^3/(g*x+f)^(3/2)/(e*x+d)^(1/2)+32/3*c*d*g^2*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

3.732.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.58

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\frac{2(d+ex)^{3/2}\left(-a^3e^3g^3+3a^2cde^2g^2(3f+2gx)+3ac^2d\right.}{3(cdf-aeg)^4((a}$$

input
```
Integrate[(d + e*x)^(5/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(5/2)),x]
```

output
```
(2*(d + e*x)^(3/2)*(-(a^3*e^3*g^3) + 3*a^2*c*d*e^2*g^2*(3*f + 2*g*x) + 3*a
*c^2*d^2*e*g*(3*f^2 + 12*f*g*x + 8*g^2*x^2) + c^3*d^3*(-f^3 + 6*f^2*g*x +
24*f*g^2*x^2 + 16*g^3*x^3)))/(3*(c*d*f - a*e*g)^4*((a*e + c*d*x)*(d + e*x)
)^(3/2)*(f + g*x)^(3/2))
```

3.732.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1252, 1252, 1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 1252$$

$$-\frac{2g\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{cdf-aeg}-\frac{2(d+ex)^{3/2}}{3(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \; 1252$$

$$-\frac{2g\left(-\frac{4g\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2(d+ex)^{3/2}}{3(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$-\frac{2g\left(-\frac{4g\left(\frac{2cd\int\frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3(cdf-aeg)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2(d+ex)^{3/2}}{3(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1248}$$

$$-\frac{2(d+ex)^{3/2}}{3(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}-\frac{2g\left(-\frac{4g\left(\frac{4cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{cdf-aeg}$$

input
```
Int[(d + e*x)^(5/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2)),x]
```

output
```
(-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)^(3/2)*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(3/2)) - (2*g*((-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(
f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (4*g*((2*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)*Sqrt[d + e*x]
*(f + g*x)^(3/2)) + (4*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3
*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f + g*x])))/(c*d*f - a*e*g)))/(c*d*f
 - a*e*g)
```

3.732. $\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

3.732.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1252
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

3.732.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.73

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-16g^3x^3c^3d^3-24a\,c^2d^2e\,g^3x^2-24c^3d^3f\,g^2x^2-6a^2cd\,e^2g^3x-36a\,c^2d^2ef\,g^2x-6c^3d^3f^2gx+a^3e^3g^3-9a^2cd\,e^2}{3\sqrt{ex+d}\,(gx+f)^{\frac{3}{2}}\,(cdx+ae)^2(aeg-cdf)^4}$
gosper	$-\frac{2(cdx+ae)(-16g^3x^3c^3d^3-24a\,c^2d^2e\,g^3x^2-24c^3d^3f\,g^2x^2-6a^2cd\,e^2g^3x-36a\,c^2d^2ef\,g^2x-6c^3d^3f^2gx+a^3e^3g^3-9a^2cd\,e^2f\,g^2-9a}{3(gx+f)^{\frac{3}{2}}\,(a^4e^4g^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4c^4d^4)(cde\,x^2+a\,e^2x+c\,d^2x+ade)}$

input
```
int((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)/(g*x+f)^(3/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-16*c^3*d^3*
g^3*x^3-24*a*c^2*d^2*e*g^3*x^2-24*c^3*d^3*f*g^2*x^2-6*a^2*c*d*e^2*g^3*x-36
*a*c^2*d^2*e*f*g^2*x-6*c^3*d^3*f^2*g*x+a^3*e^3*g^3-9*a^2*c*d*e^2*f*g^2-9*a
*c^2*d^2*e*f^2*g+c^3*d^3*f^3)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^4
```

3.732.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1065 vs. $2(230) = 460$.

Time = 1.03 (sec) , antiderivative size = 1065, normalized size of antiderivative = 4.10

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \frac{}{3\,(a^2c^4d^5e^2f^6-4\,a^3c^3d^4e^3f^5g+6\,a^4c^2d^3e^4f^4g^2-4\,a^5}$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="fricas")
```

output
```
2/3*(16*c^3*d^3*g^3*x^3 - c^3*d^3*f^3 + 9*a*c^2*d^2*e*f^2*g + 9*a^2*c*d*e^
2*f*g^2 - a^3*e^3*g^3 + 24*(c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x^2 + 6*(c^3*
d^3*f^2*g + 6*a*c^2*d^2*e*f*g^2 + a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(a^2*c^4*d^5*e^2*f^6 -
 4*a^3*c^3*d^4*e^3*f^5*g + 6*a^4*c^2*d^3*e^4*f^4*g^2 - 4*a^5*c*d^2*e^5*f^3
*g^3 + a^6*d*e^6*f^2*g^4 + (c^6*d^6*e*f^4*g^2 - 4*a*c^5*d^5*e^2*f^3*g^3 +
6*a^2*c^4*d^4*e^3*f^2*g^4 - 4*a^3*c^3*d^3*e^4*f*g^5 + a^4*c^2*d^2*e^5*g^6)
*x^5 + (2*c^6*d^6*e*f^5*g + (c^6*d^7 - 6*a*c^5*d^5*e^2)*f^4*g^2 - 4*(a*c^5
*d^6*e - a^2*c^4*d^4*e^3)*f^3*g^3 + 2*(3*a^2*c^4*d^5*e^2 + 2*a^3*c^3*d^3*e
^4)*f^2*g^4 - 2*(2*a^3*c^3*d^4*e^3 + 3*a^4*c^2*d^2*e^5)*f*g^5 + (a^4*c^2*d
^3*e^4 + 2*a^5*c*d*e^6)*g^6)*x^4 + (c^6*d^6*e*f^6 + 2*c^6*d^7*f^5*g - 6*a^
4*c^2*d^3*e^4*f*g^5 - 3*(2*a*c^5*d^6*e + 3*a^2*c^4*d^4*e^3)*f^4*g^2 + 4*(a
^2*c^4*d^5*e^2 + 4*a^3*c^3*d^3*e^4)*f^3*g^3 + (4*a^3*c^3*d^4*e^3 - 9*a^4*c
^2*d^2*e^5)*f^2*g^4 + (2*a^5*c*d^2*e^5 + a^6*e^7)*g^6)*x^3 - (6*a^2*c^4*d^
4*e^3*f^5*g - 2*a^6*e^7*f*g^5 - a^6*d*e^6*g^6 - (c^6*d^7 + 2*a*c^5*d^5*e^2
)*f^6 + (9*a^2*c^4*d^5*e^2 - 4*a^3*c^3*d^3*e^4)*f^4*g^2 - 4*(4*a^3*c^3*d^4
*e^3 + a^4*c^2*d^2*e^5)*f^3*g^3 + 3*(3*a^4*c^2*d^3*e^4 + 2*a^5*c*d*e^6)*f^
2*g^4)*x^2 + (2*a^6*d*e^6*f*g^5 + (2*a*c^5*d^6*e + a^2*c^4*d^4*e^3)*f^6 -
2*(3*a^2*c^4*d^5*e^2 + 2*a^3*c^3*d^3*e^4)*f^5*g + 2*(2*a^3*c^3*d^4*e^3 + 3
*a^4*c^2*d^2*e^5)*f^4*g^2 + 4*(a^4*c^2*d^3*e^4 - a^5*c*d*e^6)*f^3*g^3 -...
```

3.732.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)/(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(5/2),x)
```

output
```
Timed out
```

3.732.7 Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(gx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(
g*x + f)^(5/2)), x)
```

3.732.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4668 vs. $2(230) = 460$.

Time = 3.08 (sec) , antiderivative size = 4668, normalized size of antiderivative = 17.95

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="giac")
```

output
```
-2/3*e^4*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(8*(c^7*d^7*e^2*f^3*g^4*abs(
g) - 3*a*c^6*d^6*e^3*f^2*g^5*abs(g) + 3*a^2*c^5*d^5*e^4*f*g^6*abs(g) - a^3
*c^4*d^4*e^5*g^7*abs(g))*(e^2*f + (e*x + d)*e*g - d*e*g)/(c^8*d^8*e^6*f^7*
g^2 - 7*a*c^7*d^7*e^7*f^6*g^3 + 21*a^2*c^6*d^6*e^8*f^5*g^4 - 35*a^3*c^5*d^
5*e^9*f^4*g^5 + 35*a^4*c^4*d^4*e^10*f^3*g^6 - 21*a^5*c^3*d^3*e^11*f^2*g^7
+ 7*a^6*c^2*d^2*e^12*f*g^8 - a^7*c*d*e^13*g^9) - 9*(c^7*d^7*e^4*f^4*g^4*ab
s(g) - 4*a*c^6*d^6*e^5*f^3*g^5*abs(g) + 6*a^2*c^5*d^5*e^6*f^2*g^6*abs(g) -
 4*a^3*c^4*d^4*e^7*f*g^7*abs(g) + a^4*c^3*d^3*e^8*g^8*abs(g))/(c^8*d^8*e^6
*f^7*g^2 - 7*a*c^7*d^7*e^7*f^6*g^3 + 21*a^2*c^6*d^6*e^8*f^5*g^4 - 35*a^3*c
^5*d^5*e^9*f^4*g^5 + 35*a^4*c^4*d^4*e^10*f^3*g^6 - 21*a^5*c^3*d^3*e^11*f^2
*g^7 + 7*a^6*c^2*d^2*e^12*f*g^8 - a^7*c*d*e^13*g^9))/((c*d*e^2*f*g - a*e^3
*g^2 - (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(-c*d*e^2*f*g + a*e^3*g^
2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)) - 4*(4*sqrt(c*d*g)*c^3*d^3*e^4
*f^2*g^5 - 8*sqrt(c*d*g)*a*c^2*d^2*e^5*f*g^6 + 4*sqrt(c*d*g)*a^2*c*d*e^6*g
^7 + 9*sqrt(c*d*g)*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt
(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*c^2*
d^2*e^2*f*g^4 - 9*sqrt(c*d*g)*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*
d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d
*g))^2*a*c*d*e^3*g^5 + 3*sqrt(c*d*g)*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*
sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - ...
```

3.732.9 Mupad [B] (verification not implemented)

Time = 14.25 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{16\,g\,x^2\,(a\,e\,g+c\,d\,f)\,\sqrt{d+e}}{e\,(a\,e\,g-c\,d\,f)^4}\right.}{x^4\,\sqrt{f+g\,x}+\frac{x^2\,\sqrt{f+g\,x}\,(g\,a^2\,e^3+2\,g\,a\,c\,d^2}{c^2\,d^2\,e\,g}}$$

input
```
int((d + e*x)^(5/2)/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2)),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((16*g*x^2*(a*e*g + c*d*f)*
(d + e*x)^(1/2))/(e*(a*e*g - c*d*f)^4) - ((d + e*x)^(1/2)*(2*a^3*e^3*g^3 +
 2*c^3*d^3*f^3 - 18*a*c^2*d^2*e*f^2*g - 18*a^2*c*d*e^2*f*g^2))/(3*c^2*d^2*
e*g*(a*e*g - c*d*f)^4) + (32*c*d*g^2*x^3*(d + e*x)^(1/2))/(3*e*(a*e*g - c*
d*f)^4) + (4*x*(d + e*x)^(1/2)*(a^2*e^2*g^2 + c^2*d^2*f^2 + 6*a*c*d*e*f*g)
)/(c*d*e*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2) + (x^2*(f + g*x)^(1/2)*
(a^2*e^3*g + c^2*d^3*f + 2*a*c*d*e^2*f + 2*a*c*d^2*e*g))/(c^2*d^2*e*g) + (
a*x*(f + g*x)^(1/2)*(a*e^2*f + 2*c*d^2*f + a*d*e*g))/(c^2*d^2*g) + (a^2*e*
f*(f + g*x)^(1/2))/(c^2*d*g) + (x^3*(f + g*x)^(1/2)*(2*a*e^2*g + c*d^2*g +
 c*d*e*f))/(c*d*e*g))
```

3.732. $\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

3.733 $\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

3.733.1 Optimal result

Integrand size = 48, antiderivative size = 385

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}} - \frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{96c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24\sqrt{d+ex}} + \frac{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g\sqrt{d+ex}} - \frac{5(cdf-aeg)^4\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{7/2}d^{7/2}g^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

output
```
-5/64*(-a*e*g+c*d*f)^4*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(
g*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(7/2)/d^(7/2)/g^(3/2)/(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-5/96*(-a*e*g+c*d*f)^2*(g*x+f)^(3/2)*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/g/(e*x+d)^(1/2)+1/24*(a*e/c/
d-f/g)*(g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)
+1/4*(g*x+f)^(7/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)
-5/64*(-a*e*g+c*d*f)^3*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)/c^3/d^3/g/(e*x+d)^(1/2)
```

3.733.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.61

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}(15a^3e^3g^3-5a\right.}{}$$

input
```
Integrate[((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sq
rt[d + e*x],x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[f + g*x]*(15*
a^3*e^3*g^3 - 5*a^2*c*d*e^2*g^2*(11*f + 2*g*x) + a*c^2*d^2*e*g*(73*f^2 + 3
6*f*g*x + 8*g^2*x^2) + c^3*d^3*(15*f^3 + 118*f^2*g*x + 136*f*g^2*x^2 + 48*
g^3*x^3)) - (15*(c*d*f - a*e*g)^4*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/
(Sqrt[g]*Sqrt[a*e + c*d*x])])/Sqrt[a*e + c*d*x]))/(192*c^(7/2)*d^(7/2)*g^(
3/2)*Sqrt[d + e*x])
```

3.733.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {1250, 1253, 1253, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^{5/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \ 1250$$

$$\frac{(f+gx)^{7/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{4g\sqrt{d+ex}}-\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8g}$$

$$\Big\downarrow \ 1253$$

$$\frac{(f+gx)^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{5(cdf-aeg)\int\frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6cd}+\frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cd\sqrt{d+ex}}\right)}{8g}$$

$\downarrow$ 1253

$$\frac{(f+gx)^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{5(cdf-aeg)\left(\frac{3(cdf-aeg)\int\frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2cd\sqrt{d+ex}}\right)}{6cd}+\frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade}}{3cd\sqrt{d+ex}}\right.}{8g}$$

$\downarrow$ 1253

$$\frac{(f+gx)^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{5(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2\right.}}{2cd\sqrt{}}\right.}{6cd}\right.}{8g}$$

$\downarrow$ 1268

3.733. $\displaystyle\int\frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

$$\frac{(f+gx)^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{5(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+c\ldots\right.}}{2cd\sqrt{d+\ldots}}\ldots\right)}{6cd}\ldots\right)}{8g}$$

$\downarrow$ 66

$$\frac{(f+gx)^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{5(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x\ldots}}{\ldots}\ldots\right)}{6cd}\ldots\right)}{8g}$$

$\downarrow$ 221

$$\frac{(f+gx)^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{5(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(\ldots}}{2\ldots}\ldots\right)}{6cd}\ldots\right)}{8g}$$

3.733. $\int\frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

input
```
Int[((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +
 e*x],x]
```

output
```
((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g*Sqrt[d
+ e*x]) - ((c*d*f - a*e*g)*(((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])/(3*c*d*Sqrt[d + e*x]) + (5*(c*d*f - a*e*g)*(((f + g*x)^(3/
2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*c*d*Sqrt[d + e*x]) + (3
*(c*d*f - a*e*g)*((Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(c*d*Sqrt[d + e*x]) + ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]
*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^
(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(4*c*
d)))/(6*c*d)))/(8*g)
```

3.733.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

3.733. $\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.733.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(329) = 658$.

Time = 0.57 (sec) , antiderivative size = 732, normalized size of antiderivative = 1.90

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(-96c^3d^3g^3x^3\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}+15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^4e^4g^4-60}{}$

input
```
int((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
-1/384*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-96*c^3*d^3*g^3*x^3*((g*
x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(
(g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^4*e^4*g^4-60*ln
(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(
c*d*g)^(1/2))*a^3*c*d*e^3*f*g^3+90*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f
)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c^2*d^2*e^2*f^2*g^2
-60*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1
/2))/(c*d*g)^(1/2))*a*c^3*d^3*e*f^3*g+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(
(g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^4*d^4*f^4-16*a*
c^2*d^2*e*g^3*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)-272*c^3*d^3*f*
g^2*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)+20*((g*x+f)*(c*d*x+a*e))
^(1/2)*(c*d*g)^(1/2)*a^2*c*d*e^2*g^3*x-72*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d
*g)^(1/2)*a*c^2*d^2*e*f*g^2*x-236*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2
)*c^3*d^3*f^2*g*x-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^3*e^3*g^3
+110*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*c*d*e^2*f*g^2-146*((g*x
+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c^2*d^2*e*f^2*g-30*((g*x+f)*(c*d*x+
a*e))^(1/2)*(c*d*g)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/g/((g*x+f)*(c*d*x+a*e
))^(1/2)/c^3/d^3/(c*d*g)^(1/2)
```

3.733.5 Fricas [A] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 1065, normalized size of antiderivative = 2.77

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \Bigg[\frac{4\,(48\,c^4d^4g^4x^3+15\,c^4d^4f^3g+73\,ac^3d^3ef^2g^2-55\,a^2c^2d}{}$$

input
```
integrate((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1
/2),x, algorithm="fricas")
```

output
```
[1/768*(4*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g + 73*a*c^3*d^3*e*f^2*g^2
- 55*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4 + 8*(17*c^4*d^4*f*g^3 + a*
c^3*d^3*e*g^4)*x^2 + 2*(59*c^4*d^4*f^2*g^2 + 18*a*c^3*d^3*e*f*g^3 - 5*a^2*
c^2*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x +
 d)*sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*
e^2*f^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a
*c^3*d^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4
*e^5*g^4)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d
^2*e*f*g + a^2*d*e^2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(
2*c*d*g*x + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^
2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d
^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d
^4*e*g^2*x + c^4*d^5*g^2), 1/384*(2*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g
 + 73*a*c^3*d^3*e*f^2*g^2 - 55*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4
+ 8*(17*c^4*d^4*f*g^3 + a*c^3*d^3*e*g^4)*x^2 + 2*(59*c^4*d^4*f^2*g^2 + 18*
a*c^3*d^3*e*f*g^3 - 5*a^2*c^2*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^
4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*
g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 -
 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^4)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e...
```

3.733.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**(5/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+
d)**(1/2),x)
```

output
```
Timed out
```

3.733.7 Maxima [F]

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{5}{2}}}{\sqrt{ex+d}}\,dx$$

input
```
integrate((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(5/2)/sqrt
(e*x + d), x)
```

3.733.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6752 vs. $2(329) = 658$.

Time = 2.15 (sec) , antiderivative size = 6752, normalized size of antiderivative = 17.54

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1
/2),x, algorithm="giac")
```

output
```
1/192*(48*f^2*((4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x +
d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*
x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 +
 (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g
))*e*f*abs(g)/g^2 - 4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*
x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f +
 (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g
^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d
*e*g))*d*abs(g)/g + (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*
g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f
- 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e
*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a
*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)
*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*
e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*abs(g)/(e*g^2))/g - (c^2*d^2*e^3*f^2*g*ab
s(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*
g^2))) - 2*a*c*d*e^4*f*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g)
 + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + a^2*e^5*g^3*abs(g)*log(abs(-sqrt(e^2
*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + sqrt(-c*d^2*e
*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d*e*f*abs(g) - 2*sq...
```

3.733.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \int \frac{(f+g\,x)^{5/2}\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{\sqrt{d+e\,x}}\,dx$$

input
```
int(((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e
*x)^(1/2),x)
```

output
```
int(((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e
*x)^(1/2), x)
```

3.734 $\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

3.734.1 Optimal result

Integrand size = 48, antiderivative size = 313

$$\begin{aligned}\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx &= -\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8c^2d^2g\sqrt{d+ex}}\\&+\frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12\sqrt{d+ex}}\\&+\frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3g\sqrt{d+ex}}\\&-\frac{(cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\,\mathrm{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{5/2}d^{5/2}g^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\end{aligned}$$

output
```
-1/8*(-a*e*g+c*d*f)^3*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g
*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2)/g^(3/2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/12*(a*e/c/d-f/g)*(g*x+f)^(3/2)*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)+1/3*(g*x+f)^(5/2)*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)-1/8*(-a*e*g+c*d*f)^2*(g*x+
f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/g/(e*x+d)^(1/2)
```

3.734.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.60

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}(-3a^2e^2g^2+2a\right.}{}$$

input
```
Integrate[((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sq
rt[d + e*x],x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[f + g*x]*(-3*
a^2*e^2*g^2 + 2*a*c*d*e*g*(4*f + g*x) + c^2*d^2*(3*f^2 + 14*f*g*x + 8*g^2*
x^2)) - (3*(c*d*f - a*e*g)^3*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt
[g]*Sqrt[a*e + c*d*x])])/Sqrt[a*e + c*d*x]))/(24*c^(5/2)*d^(5/2)*g^(3/2)*S
qrt[d + e*x])
```

3.734.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1250, 1253, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1250}$$

$$\frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{6g}$$

$$\Big\downarrow \text{1253}$$

$$\frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd\sqrt{d+ex}}\right)}{6g}$$

$$\Big\downarrow 1253$$

$$\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+c}}{2cd\sqrt{d+ex}}\right.}{6g}$$

$$\Big\downarrow 1268$$

$$\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+aa}}{2cd\sqrt{d+ex}}\right.}{6g}$$

$$\Big\downarrow 66$$

$$\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd}}{2cd\sqrt{d+}}\right.}{6g}$$

$$\Big\downarrow 221$$

$$\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2}}{2cd\sqrt{d+e}}\right.}{6g}$$

3.734. $\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

input
```
Int[((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +
 e*x],x]
```

output
```
((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*g*Sqrt[d
+ e*x]) - ((c*d*f - a*e*g)*(((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])/(2*c*d*Sqrt[d + e*x]) + (3*(c*d*f - a*e*g)*((Sqrt[f + g*x]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x]) + ((c*d*
f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d
*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(4*c*d)))/(6*g)
```

3.734.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

3.734. $\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.734.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.61

method	result
default	$\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^3e^3g^3-9\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)\right.}{}$

input
```
int((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
1/48*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*ln(1/2*(2*c*d*g*x+a*e*g+
c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*e^3*
g^3-9*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^
(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2+9*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*
((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2*
g-3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1
/2))/(c*d*g)^(1/2))*c^3*d^3*f^3+16*c^2*d^2*g^2*x^2*(c*d*g)^(1/2)*((g*x+f)*
(c*d*x+a*e))^(1/2)+4*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2
*x+28*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c^2*d^2*f*g*x-6*((g*x+f)*(
c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2+16*((g*x+f)*(c*d*x+a*e))^(1/2)
*(c*d*g)^(1/2)*a*c*d*e*f*g+6*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2
*d^2*f^2)/(e*x+d)^(1/2)/g/((g*x+f)*(c*d*x+a*e))^(1/2)/c^2/d^2/(c*d*g)^(1/2
)
```

3.734.5 Fricas [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 847, normalized size of antiderivative = 2.71

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \left[\frac{4\,(8\,c^3d^3g^3x^2+3\,c^3d^3f^2g+8\,ac^2d^2efg^2-3\,a^2cde^2g^3+\right.$$

input
```
integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1
/2),x, algorithm="fricas")
```

output
```
[1/96*(4*(8*c^3*d^3*g^3*x^2 + 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2 - 3*a^
2*c*d*e^2*g^3 + 2*(7*c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^3*d^4*f^3 -
3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f
^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(c*
d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2
*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f +
a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d
^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*
g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d^3*e*g^2*x + c^3*d^4
*g^2), 1/48*(2*(8*c^3*d^3*g^3*x^2 + 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2
- 3*a^2*c*d*e^2*g^3 + 2*(7*c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^3*d^4*
f^3 - 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d
^3*e*f^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*s
qrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d
*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*
e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^3*d^3*e*g^2*x + c^3*d^4*g^2)]
```

3.734.6 Sympy [F]

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \int \frac{\sqrt{(d+ex)\,(ae+cdx)}(f+gx)^{\frac{3}{2}}}{\sqrt{d+ex}}\,dx$$

input
```
integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+
d)**(1/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**(3/2)/sqrt(d + e*x), x)
```

3.734.7 Maxima [F]

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{3}{2}}}{\sqrt{ex+d}}\,dx$$

input
```
integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(3/2)/sqrt
(e*x + d), x)
```

3.734.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3056 vs. 2(265) = 530.

Time = 1.20 (sec) , antiderivative size = 3056, normalized size of antiderivative = 9.76

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1
/2),x, algorithm="giac")
```

output
```
1/24*(6*f*((4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e
*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x +
d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^
2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*e
*f*abs(g)/g^2 - 4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x +
d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*
x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 +
 (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g
))*d*abs(g)/g + (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g -
d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*
c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)
- (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d
^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqr
t(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)
*c*d*g)))/(sqrt(c*d*g)*c*d))*abs(g)/(e*g^2))/g - (c^2*d^2*e^3*f^2*g*abs(g)
*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)
)) - 2*a*c*d*e^4*f*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + s
qrt(-c*d^2*e*g^2 + a*e^3*g^2))) + a^2*e^5*g^3*abs(g)*log(abs(-sqrt(e^2*f -
 d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + sqrt(-c*d^2*e*g^2
 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d*e*f*abs(g) - 2*sqrt(-...
```

3.734.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{(f+g\,x)^{3/2}\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{\sqrt{d+e\,x}}\,dx$$

input
```
int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e
*x)^(1/2),x)
```

output
```
int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e
*x)^(1/2), x)
```

3.735 $\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

3.735.1 Optimal result

Integrand size = 48, antiderivative size = 241

$$\begin{aligned}
&\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx \\
&= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4\sqrt{d+ex}} \\
&\quad+\frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2g\sqrt{d+ex}} \\
&\quad-\frac{(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{3/2}d^{3/2}g^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}
\end{aligned}$$

output
```
-1/4*(-a*e*g+c*d*f)^2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g
*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(3/2)/d^(3/2)/g^(3/2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)+1/4*(a*e/c/d-f/g)*(g*x+f)^(1/2)*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)
```

3.735.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}\sqrt{f+gx}(aeg+cd(f+2gx))-(cdf-aeg)^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)\right)}{4c^{3/2}d^{3/2}g^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt
[d + e*x],x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x
]*Sqrt[f + g*x]*(a*e*g + c*d*(f + 2*g*x)) - (c*d*f - a*e*g)^2*ArcTanh[(Sqr
t[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(4*c^(3/2)*d^(3
/2)*g^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.735.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {1250, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \; 1250$$

$$\frac{(f+gx)^{3/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4g}$$

$$\Big\downarrow \; 1253$$

$$\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4g}$$

$$\Big\downarrow \text{ 1268}$$

$$\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4g}$$

$$\Big\downarrow \text{ 66}$$

$$\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4g}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4g}$$

input
```
Int[(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e
*x],x]
```

output
```
((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*g*Sqrt[d
+ e*x]) - ((c*d*f - a*e*g)*((Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])/(c*d*Sqrt[d + e*x]) + ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqr
t[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g
*x])])/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])))/(4*g)
```

3.735. $\int\frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}dx$

3.735.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.735. $\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

3.735.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.32

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^2e^2g^2-2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)\right.}{8\sqrt{}}$

input
```
int((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
-1/8*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(ln(1/2*(2*c*d*g*x+a*e*g+c*
d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*e^2*g^
2-2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1
/2))/(c*d*g)^(1/2))*a*c*d*e*f*g+ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(
c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f^2-4*((g*x+f)*(c*
d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c*d*g*x-2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e)
)^(1/2)*a*e*g-2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)/(e*x+d)^(
1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/c/d/g/(c*d*g)^(1/2)
```

3.735.5 Fricas [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 657, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\left[\frac{4\,(2\,c^2d^2g^2x+c^2d^2fg+acdeg^2)\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\sqrt{gx+f}+(c^2d^3f^2-2\,acd^2efg\cdots}{}\right.$$

input
```
integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1
/2),x, algorithm="fricas")
```

output
```
[1/16*(4*(2*c^2*d^2*g^2*x + c^2*d^2*f*g + a*c*d*e*g^2)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + (c^2*d^3*f^2 - 2*a*
c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2*e^3*g
^2)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f
*g + a^2*d*e^2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
g*x + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*
e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3
*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d^2*e*g
^2*x + c^2*d^3*g^2), 1/8*(2*(2*c^2*d^2*g^2*x + c^2*d^2*f*g + a*c*d*e*g^2)*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) +
(c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*
e^2*f*g + a^2*e^3*g^2)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2
+ c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^2*d^2*e*g^2*
x + c^2*d^3*g^2)]
```

3.735.6 Sympy [F]

$$\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}\sqrt{f+gx}}{\sqrt{d+ex}}\,dx$$

input
```
integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+
d)**(1/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*sqrt(f + g*x)/sqrt(d + e*x), x)
```

3.735.7 Maxima [F]

$$\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{gx+f}}{\sqrt{ex+d}}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g*x + f)/sqrt(e
*x + d), x)
```

3.735.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 995 vs. $2(201) = 402$.

Time = 0.65 (sec) , antiderivative size = 995, normalized size of antiderivative = 4.13

$$\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$

$$= \frac{\left(\frac{4\left(\frac{(cde^2fg-ae^3g^2)\log\left(\left|-\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2fg+ae^3g^2+\left(e^2f+(ex+d)eg-deg\right)cdg}\right|\right)}{\sqrt{cdg}}+\sqrt{-cde^2fg+ae^3g^2+\left(e^2f+(ex+d)eg-deg\right)cdg}\sqrt{\cdots}}{g^2}\right.\cdots}{\cdots}$$

input
```
integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1
/2),x, algorithm="giac")
```

output
```
1/4*((4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d
*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g
 - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f +
(e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*e*f*abs
(g)/g^2 - 4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g
 - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)
*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*
f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d*a
bs(g)/g + (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)
*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^
3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c
^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4
*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*
g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g
)))/(sqrt(c*d*g)*c*d))*abs(g)/(e*g^2))/g - (c^2*d^2*e^3*f^2*g*abs(g)*log(a
bs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 2
*a*c*d*e^4*f*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c
*d^2*e*g^2 + a*e^3*g^2))) + a^2*e^5*g^3*abs(g)*log(abs(-sqrt(e^2*f - d*e*g
)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + sqrt(-c*d^2*e*g^2 + a*e
^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d*e*f*abs(g) - 2*sqrt(-c*d^2*...
```

3.735.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\int \frac{\sqrt{f+g\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}\,dx$$

input
```
int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e
*x)^(1/2),x)
```

output
```
int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e
*x)^(1/2), x)
```

3.736 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

3.736.1 Optimal result

Integrand size = 48, antiderivative size = 167

$$\begin{aligned}\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx &= \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{g\sqrt{d+ex}} \\ &\quad - \frac{(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\end{aligned}$$

output
```
-(-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^
(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(3/2)/c^(1/2)/d^(1/2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)+(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)/g/(e*x+d)^(1/2)
```

3.736.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$
$$=\frac{\sqrt{(ae+cdx)(d+ex)}\sqrt{f+gx}}{g\sqrt{d+ex}}$$
$$+\frac{(-cdf+aeg)\sqrt{(ae+cdx)(d+ex)}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{ae+cdx}\sqrt{d+ex}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*Sqrt[
f + g*x]),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])/(g*Sqrt[d + e*x]) + ((-(c*d*
f) + a*e*g)*Sqrt[(a*e + c*d*x)*(d + e*x)]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f
+ g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(Sqrt[c]*Sqrt[d]*g^(3/2)*Sqrt[a*e +
c*d*x]*Sqrt[d + e*x])
```

3.736.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1250, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$
$$\Big\downarrow \text{1250}$$
$$\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2g}$$
$$\Big\downarrow \text{1268}$$
$$\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow 66$$

$$\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow 221$$

$$\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*Sqrt[f + g*
x]),x]
```

output
```
(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*
x]) - ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sq
rt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*g^(3/2
)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

3.736.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.736.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.13

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\,\sqrt{gx+f}\left(\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)aeg-\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)cdf+2\sqrt{\right.}{2\sqrt{ex+d}\,\sqrt{(gx+f)(cdx+ae)}\,g\sqrt{cdg}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
1/2*((c*d*x+a*e)*(e*x+d))^(1/2)*(g*x+f)^(1/2)/(e*x+d)^(1/2)*(ln(1/2*(2*c*d
*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2
))*a*e*g-ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*
g)^(1/2))/(c*d*g)^(1/2))*c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)
)/((g*x+f)*(c*d*x+a*e))^(1/2)/g/(c*d*g)^(1/2)
```

3.736.5 Fricas [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 516, normalized size of antiderivative = 3.09

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$
$$= \left[\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\sqrt{gx+f}\,cdg-(cd^2f-adeg+(cdef-ae^2g)x)\sqrt{cdg}\log\left(-\frac{8c^2}{\right.}}{\right.}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1
/2),x, algorithm="fricas")
```

output
```
[1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x
 + f)*c*d*g - (c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(c*d*g)*log(
-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*sq
rt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*
d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*
c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c*d*e*g^2*x + c*d^2*g^2), 1/2*(2*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*c*d
*g + (c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(-c*d*g)*arctan(2*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g
*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*
g)*x))/(c*d*e*g^2*x + c*d^2*g^2)]
```

3.736.6 Sympy [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(1/2)/(e*x+
d)**(1/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

3.736.7 Maxima [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*sqrt(
g*x + f)), x)
```

3.736.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(139) = 278$.

Time = 0.46 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.45

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$
$$= \frac{\left(\frac{\left(cdef - ae^2g\right)\log\left(\left|-\sqrt{(ex+d)cde - cd^2e + ae^3}\sqrt{cdg} + \sqrt{c^2d^2e^2f - acde^3g + \left((ex+d)cde - cd^2e + ae^3\right)cdg}\right|\right)}{\sqrt{cdg}g} + \frac{\sqrt{c^2d^2e^2f - acde^3g + \left((ex+d)cde - cd^2e + ae^3\right)cdg}\sqrt{}}{cdeg}}{cd}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1
/2),x, algorithm="giac")
```

output
```
(((c*d*e*f - a*e^2*g)*log(abs(-sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*sqr
t(c*d*g) + sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e +
 a*e^3)*c*d*g)))/(sqrt(c*d*g)*g) + sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*
x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)*sqrt((e*x + d)*c*d*e - c*d^2*e + a*
e^3)/(c*d*e*g))*abs(c)*abs(d)/(c*d) - (c^2*d^2*e^2*f*abs(c)*abs(d)*log(abs
(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)))
 - a*c*d*e^3*g*abs(c)*abs(d)*log(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) +
 sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*s
qrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*abs(c)*abs(d))/(sqrt(c*d*g)*c^2*d^2*e*g)
)/e
```

3.736.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{\sqrt{f+g\,x}\,\sqrt{d+e\,x}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*
x)^(1/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*
x)^(1/2)), x)
```

3.737 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx$

3.737.1 Optimal result

Integrand size = 48, antiderivative size = 158

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx=-\frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}+\frac{2\sqrt{c}\sqrt{d}\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

output
```
2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1/2))*c^(1/2)
*d^(1/2)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)-2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)/(
g*x+f)^(1/2)
```

3.737.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx=\frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(-\sqrt{g}\sqrt{ae+cdx}+\sqrt{c}\sqrt{d}\sqrt{f+gx}\,\text{arctanh}\left(\frac{\sqrt{c}\cdots}{\sqrt{g}\cdots}\right.\right.}{g^{3/2}\sqrt{(ae+cdx)(d+ex)}\sqrt{f+gx}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f +
g*x)^(3/2)),x]
```

output
```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-(Sqrt[g]*Sqrt[a*e + c*d*x]) + Sqrt[c]
*Sqrt[d]*Sqrt[f + g*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sq
rt[a*e + c*d*x])]))/(g^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

3.737.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1249, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{cd\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{g}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}$$

$$\Big\downarrow \text{1268}$$

$$\frac{cd\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}$$

$$\Big\downarrow \text{66}$$

$$\frac{2cd\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(
3/2)),x]
```

output
```
(-2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]*Sqrt[f +
 g*x]) + (2*Sqrt[c]*Sqrt[d]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[
g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(g^(3/2)*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

3.737.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.737.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.18

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)cdgx+\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)cdf-2\sqrt{(gx+}}{\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,g\sqrt{ex+d}\,\sqrt{gx+f}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
((c*d*x+a*e)*(e*x+d))^(1/2)*(ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d
*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c*d*g*x+ln(1/2*(2*c*d*g*x+a*e
*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c*d*f
-2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2)/((g*x+f)*(c*d*
x+a*e))^(1/2)/g/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

3.737.5 Fricas [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 521, normalized size of antiderivative = 3.30

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx = \left[\frac{(egx^2+df+(ef+dg)x)\sqrt{\frac{cd}{g}}\log\left(-\frac{8c^2d^2eg^2x^3+c^2d^3f^2+6acd^2efg+a^2de\cdots}{\cdots}\right)\cdots}{\cdots}\right.$$

$$-\frac{(egx^2+df+(ef+dg)x)\sqrt{-\frac{cd}{g}}\arctan\left(\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\sqrt{gx+f}\sqrt{-\frac{cd}{g}}g}{2cdegx^2+cd^2f+adeg+(cdef+(2cd^2+ae^2)g)x}\right)+2\sqrt{cdex^2+ade+(c\cdots}}{eg^2x^2+dfg+(efg+dg^2)x}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1
/2),x, algorithm="fricas")
```

output
```
[1/2*((e*g*x^2 + d*f + (e*f + d*g)*x)*sqrt(c*d/g)*log(-(8*c^2*d^2*e*g^2*x^
3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*d*g^2*x + c*d*f
*g + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sq
rt(g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^
2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*
e^3)*g^2)*x)/(e*x + d)) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(e*x + d)*sqrt(g*x + f))/(e*g^2*x^2 + d*f*g + (e*f*g + d*g^2)*x), -((e*g
*x^2 + d*f + (e*f + d*g)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*
x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)) + 2*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(e*g^2*x^
2 + d*f*g + (e*f*g + d*g^2)*x)]
```

3.737.6 Sympy [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx = \int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{\sqrt{d+ex}\,(f+gx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(3/2)/(e*x+
d)**(1/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**(3/2)), x
)
```

3.737.7 Maxima [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx = \int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{\sqrt{ex+d}(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x
+ f)^(3/2)), x)
```

3.737.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. 2(130) = 260.

Time = 0.52 (sec) , antiderivative size = 426, normalized size of antiderivative = 2.70

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx = -\frac{2\left(\frac{e^2|c||d|\log\left(\left|-\sqrt{(ex+d)cde-cd^2e+ae^3}\sqrt{cdg}+\sqrt{c^2d^2e^2f-acde^3g+((ex+d)cde-cd^2e+ae^3)cdg}\right|\right)}{\sqrt{cdg}g|e|}+\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}e^2|c}{\sqrt{c^2d^2e^2f-acde^3g+((ex+d)cde-cd^2e}}\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1
/2),x, algorithm="giac")
```

output
```
-2*(e^2*abs(c)*abs(d)*log(abs(-sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*sqr
t(c*d*g) + sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e +
 a*e^3)*c*d*g)))/(sqrt(c*d*g)*g*abs(e)) + sqrt((e*x + d)*c*d*e - c*d^2*e +
 a*e^3)*e^2*abs(c)*abs(d)/(sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c
*d*e - c*d^2*e + a*e^3)*c*d*g)*g*abs(e)) - (c^2*d^2*e^3*f*abs(c)*abs(d)*lo
g(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e
*g))) - c^2*d^3*e^2*g*abs(c)*abs(d)*log(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c
*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))) + sqrt(c^2*d^2*e^2*f - c^2*d^3
*e*g)*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*e*abs(c)*abs(d))/(sqrt(c*d*g)*c^2
*d^2*e*f*g*abs(e) - sqrt(c*d*g)*c^2*d^3*g^2*abs(e)))*abs(e)/e^2
```

3.737.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx=\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(f+g\,x\right)}^{3/2}\,\sqrt{d+e\,x}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(3/2)*(d + e*
x)^(1/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(3/2)*(d + e*
x)^(1/2)), x)
```

3.738 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx$

3.738.1 Optimal result

Integrand size = 48, antiderivative size = 63

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx=\frac{2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{3(cdf-aeg)(d+ex)^{3/2}(f+gx)^{3/2}}$$

output
```
2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)/(e*x+d)^(3/2)/(
g*x+f)^(3/2)
```

3.738.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{3/2}}{3(cdf-aeg)(d+ex)^{3/2}(f+gx)^{3/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f +
g*x)^(5/2)),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2))/(3*(c*d*f - a*e*g)*(d + e*x)^(3/2)*(f
+ g*x)^(3/2))
```

3.738.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx$$

$$\Big\downarrow \; 1248$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(
5/2)),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*(c*d*f - a*e*g)*(d +
e*x)^(3/2)*(f + g*x)^(3/2))
```

3.738.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

3.738.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{2(cdx+ae)\sqrt{(cdx+ae)(ex+d)}}{3(gx+f)^{\frac{3}{2}}(aeg-cdf)\sqrt{ex+d}}$	53
gosper	$-\frac{2(cdx+ae)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{3(gx+f)^{\frac{3}{2}}(aeg-cdf)\sqrt{ex+d}}$	63

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/3/(g*x+f)^(3/2)*(c*d*x+a*e)/(a*e*g-c*d*f)*((c*d*x+a*e)*(e*x+d))^(1/2)/(
e*x+d)^(1/2)
```

3.738.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(55) = 110$.

Time = 0.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.68

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx = \frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}(c}{3\,(cd^2f^3-adef^2g+(cdefg^2-ae^2g^3)x^3+(2\,cdef^2g-adeg^3+(cd}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1
/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x + a*e)*sqrt(e*x + d
)*sqrt(g*x + f)/(c*d^2*f^3 - a*d*e*f^2*g + (c*d*e*f*g^2 - a*e^2*g^3)*x^3 +
 (2*c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 - 2*a*e^2)*f*g^2)*x^2 + (c*d*e*f^3 -
2*a*d*e*f*g^2 + (2*c*d^2 - a*e^2)*f^2*g)*x)
```

3.738.6 Sympy [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx = \int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{\sqrt{d+ex}\,(f+gx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(5/2)/(e*x+
d)**(1/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**(5/2)), x
)
```

3.738.7 Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}(f + gx)^{5/2}}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x
+ f)^(5/2)), x)
```

3.738.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(55) = 110$.

Time = 0.51 (sec) , antiderivative size = 311, normalized size of antiderivative = 4.94

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}(f + gx)^{5/2}}\,dx = \frac{2\left(\frac{((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}c^2d^2e^4g|c||d|}{(c^2d^2e^2f-acde^3g+((ex+d)cde-cd^2e+ae^3)cdg)^{\frac{3}{2}}(cde^2fg|e|-ae^3g^2|e|)} + \frac{}{\sqrt{c^2d^2e^2f-c}}\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1
/2),x, algorithm="giac")
```

output
```
2/3*(((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*e^4*g*abs(c)*abs(d)
/((c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g
)^(3/2)*(c*d*e^2*f*g*abs(e) - a*e^3*g^2*abs(e))) + (sqrt(-c*d^2*e + a*e^3)
*c*d^2*e^2*abs(c)*abs(d) - sqrt(-c*d^2*e + a*e^3)*a*e^4*abs(c)*abs(d))/(sq
rt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c*d*e*f^2*abs(e) - sqrt(c^2*d^2*e^2*f - c^
2*d^3*e*g)*c*d^2*f*g*abs(e) - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*e^2*f*g*
abs(e) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*d*e*g^2*abs(e)))*abs(e)/e^2
```

3.738.9 Mupad [B] (verification not implemented)

Time = 12.99 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.16

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}(f + gx)^{5/2}}\,dx = -\frac{\left(\frac{2\,a\,e}{3\,a\,e\,g^2-3\,c\,d\,f\,g}+\frac{2\,c\,d\,x}{3\,a\,e\,g^2-3\,c\,d\,f\,g}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}-\frac{\sqrt{f+g\,x}\,\left(3\,c\,d\,f^2-3\,a\,e\,f\,g\right)\,\sqrt{d+e\,x}}{3\,a\,e\,g^2-3\,c\,d\,f\,g}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(5/2)*(d + e*
x)^(1/2)),x)
```

output
```
-(((2*a*e)/(3*a*e*g^2 - 3*c*d*f*g) + (2*c*d*x)/(3*a*e*g^2 - 3*c*d*f*g))*(x
*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x*(f + g*x)^(1/2)*(d + e*x)^
(1/2) - ((f + g*x)^(1/2)*(3*c*d*f^2 - 3*a*e*f*g)*(d + e*x)^(1/2))/(3*a*e*g
^2 - 3*c*d*f*g))
```

3.739 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}\,dx$

3.739.1 Optimal result

Integrand size = 48, antiderivative size = 129

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}\,dx=\frac{2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{5(cdf-aeg)(d+ex)^{3/2}(f+gx)^{5/2}}+\frac{4cd(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{15(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{3/2}}$$

output
```
2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)/(e*x+d)^(3/2)/(
g*x+f)^(5/2)+4/15*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*
f)^2/(e*x+d)^(3/2)/(g*x+f)^(3/2)
```

3.739.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{3/2}(-3aeg+cd(5f+2gx))}{15(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{5/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f +
g*x)^(7/2)),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-3*a*e*g + c*d*(5*f + 2*g*x)))/(15*(c*
d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(5/2))
```

3.739.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}dx$$

$$\Big\downarrow \text{1254}$$

$$\frac{2cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}(f+gx)^{5/2}}dx}{5(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1248}$$

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{15(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(
7/2)),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*(c*d*f - a*e*g)*(d +
e*x)^(3/2)*(f + g*x)^(5/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2))/(15*(c*d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(3/2))
```

3.739.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

3.739.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.54

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)(-2cdgx+3aeg-5cdf)}{15(gx+f)^{\frac{5}{2}}\sqrt{ex+d}\,(aeg-cdf)^{2}}$	70
gosper	$-\frac{2(cdx+ae)(-2cdgx+3aeg-5cdf)\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}}{15(gx+f)^{\frac{5}{2}}(a^{2}e^{2}g^{2}-2acdefg+c^{2}d^{2}f^{2})\sqrt{ex+d}}$	99

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/15*((c*d*x+a*e)*(e*x+d))^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/2)*(c*d*x+a*e)*
(-2*c*d*g*x+3*a*e*g-5*c*d*f)/(a*e*g-c*d*f)^2
```

3.739.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. 2(113) = 226.

Time = 0.35 (sec) , antiderivative size = 402, normalized size of antiderivative = 3.12

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}\,dx = \frac{}{15\,(c^2d^3f^5-2\,acd^2ef^4g+a^2de^2f^3g^2+(c^2d^2ef^2g^3-2\,acde^2fg^4+a}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1
/2),x, algorithm="fricas")
```

output
```
2/15*(2*c^2*d^2*g*x^2 + 5*a*c*d*e*f - 3*a^2*e^2*g + (5*c^2*d^2*f - a*c*d*e
*g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x
+ f)/(c^2*d^3*f^5 - 2*a*c*d^2*e*f^4*g + a^2*d*e^2*f^3*g^2 + (c^2*d^2*e*f^2
*g^3 - 2*a*c*d*e^2*f*g^4 + a^2*e^3*g^5)*x^4 + (3*c^2*d^2*e*f^3*g^2 + a^2*d
*e^2*g^5 + (c^2*d^3 - 6*a*c*d*e^2)*f^2*g^3 - (2*a*c*d^2*e - 3*a^2*e^3)*f*g
^4)*x^3 + 3*(c^2*d^2*e*f^4*g + a^2*d*e^2*f*g^4 + (c^2*d^3 - 2*a*c*d*e^2)*f
^3*g^2 - (2*a*c*d^2*e - a^2*e^3)*f^2*g^3)*x^2 + (c^2*d^2*e*f^5 + 3*a^2*d*e
^2*f^2*g^3 + (3*c^2*d^3 - 2*a*c*d*e^2)*f^4*g - (6*a*c*d^2*e - a^2*e^3)*f^3
*g^2)*x)
```

3.739.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(7/2)/(e*x+
d)**(1/2),x)
```

output
```
Timed out
```

3.739.7 Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^{7/2}}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x
+ f)^(7/2)), x)
```

3.739.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(113) = 226$.

Time = 0.84 (sec) , antiderivative size = 762, normalized size of antiderivative = 5.91

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}\,dx = \frac{2\left(\frac{5\sqrt{-cd}}{\sqrt{c^2d^2e^2f-c^2d^3eg}c^2d^2e^2f^4|e|-2\sqrt{c^2d^2e^2f-c^2d^3eg}c^2d^3ef^3g|e|-2\sqrt{c^2d^2e^2f-c^2d^3eg}\cdots}\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1
/2),x, algorithm="giac")
```

output
```
2/15*((5*sqrt(-c*d^2*e + a*e^3)*c^2*d^3*e^3*f*abs(c)*abs(d) - 5*sqrt(-c*d^
2*e + a*e^3)*a*c*d*e^5*f*abs(c)*abs(d) - 2*sqrt(-c*d^2*e + a*e^3)*c^2*d^4*
e^2*g*abs(c)*abs(d) - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^4*g*abs(c)*abs(d) +
 3*sqrt(-c*d^2*e + a*e^3)*a^2*e^6*g*abs(c)*abs(d))/(sqrt(c^2*d^2*e^2*f - c
^2*d^3*e*g)*c^2*d^2*e^2*f^4*abs(e) - 2*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c
^2*d^3*e*f^3*g*abs(e) - 2*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c*d*e^3*f^3*
g*abs(e) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^2*d^4*f^2*g^2*abs(e) + 4*sq
rt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c*d^2*e^2*f^2*g^2*abs(e) + sqrt(c^2*d^2*
e^2*f - c^2*d^3*e*g)*a^2*e^4*f^2*g^2*abs(e) - 2*sqrt(c^2*d^2*e^2*f - c^2*d
^3*e*g)*a*c*d^3*e*f*g^3*abs(e) - 2*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*d
*e^3*f*g^3*abs(e) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*d^2*e^2*g^4*abs(
e)) + (2*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^4*d^4*e^6*g^3*abs(c)*abs(d)
/(c^2*d^2*e^4*f^2*g^2*abs(e) - 2*a*c*d*e^5*f*g^3*abs(e) + a^2*e^6*g^4*abs(
e)) + 5*(c^5*d^5*e^8*f*g^2*abs(c)*abs(d) - a*c^4*d^4*e^9*g^3*abs(c)*abs(d)
)/(c^2*d^2*e^4*f^2*g^2*abs(e) - 2*a*c*d*e^5*f*g^3*abs(e) + a^2*e^6*g^4*abs
(e)))*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c^2*d^2*e^2*f - a*c*d*e^3
*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)^(5/2))*abs(e)/e^2
```

3.739.9 Mupad [B] (verification not implemented)

Time = 13.35 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}\,dx = \frac{\left(\frac{x\,(10\,c^2\,d^2\,f-2\,a\,c\,d\,e\,g)}{15\,g^2\,(a\,e\,g-c\,d\,f)^2}-\frac{6\,a^2\,e^2\,g-10\,a\,c\,d\,e\,f}{15\,g^2\,(a\,e\,g-c\,d\,f)^2}+\frac{4\,c^2\,d^2\,x^2}{15\,g\,(a\,e\,g-c\,d\,f)^2}\right)\sqrt{c\,d\,e\,x^2+\cdots}}{x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}+\frac{2\,f\,x\,\sqrt{f+g\,x}\cdots}{g}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(7/2)*(d + e*
x)^(1/2)),x)
```

output
```
(((x*(10*c^2*d^2*f - 2*a*c*d*e*g))/(15*g^2*(a*e*g - c*d*f)^2) - (6*a^2*e^2
*g - 10*a*c*d*e*f)/(15*g^2*(a*e*g - c*d*f)^2) + (4*c^2*d^2*x^2)/(15*g*(a*e
*g - c*d*f)^2))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^2*(f + g
*x)^(1/2)*(d + e*x)^(1/2) + (f^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (2
*f*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g)
```

3.739. $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}\,dx$

3.740 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx$

3.740.1 Optimal result

Integrand size = 48, antiderivative size = 198

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx = \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{7(cdf-aeg)(d+ex)^{3/2}(f+gx)^{7/2}} + \frac{8cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{35(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{5/2}} + \frac{16c^2d^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{105(cdf-aeg)^3(d+ex)^{3/2}(f+gx)^{3/2}}$$

output
```
2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)/(e*x+d)^(3/2)/(
g*x+f)^(7/2)+8/35*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*
f)^2/(e*x+d)^(3/2)/(g*x+f)^(5/2)+16/105*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(3/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(3/2)/(g*x+f)^(3/2)
```

3.740.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx = \frac{2((ae+cdx)(d+ex))^{3/2}\left(15a^2e^2g^2-6acdeg(7f+2gx)+c^2d^2(35f^2\right.}{105(cdf-aeg)^3(d+ex)^{3/2}(f+gx)^{7/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f +
g*x)^(9/2)),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2)*(15*a^2*e^2*g^2 - 6*a*c*d*e*g*(7*f + 2*
g*x) + c^2*d^2*(35*f^2 + 28*f*g*x + 8*g^2*x^2)))/(105*(c*d*f - a*e*g)^3*(d
 + e*x)^(3/2)*(f + g*x)^(7/2))
```

3.740.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1254, 1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x \left(a e^2+c d^2\right)+a d e+c d e x^2}}{\sqrt{d+e x} (f+g x)^{9/2}} \, dx$$

$$\Big\downarrow \text{ 1254}$$

$$\frac{4 c d \int \frac{\sqrt{c d e x^2+\left(c d^2+a e^2\right) x+a d e}}{\sqrt{d+e x} (f+g x)^{7/2}}dx}{7 (c d f-a e g)}+\frac{2 \left(x \left(a e^2+c d^2\right)+a d e+c d e x^2\right)^{3/2}}{7 (d+e x)^{3/2} (f+g x)^{7/2} (c d f-a e g)}$$

$$\Big\downarrow \text{ 1254}$$

$$\frac{4 c d \left(\frac{2 c d \int \frac{\sqrt{c d e x^2+\left(c d^2+a e^2\right) x+a d e}}{\sqrt{d+e x} (f+g x)^{5/2}}dx}{5 (c d f-a e g)}+\frac{2 \left(x \left(a e^2+c d^2\right)+a d e+c d e x^2\right)^{3/2}}{5 (d+e x)^{3/2} (f+g x)^{5/2} (c d f-a e g)}\right)}{7 (c d f-a e g)}+\frac{2 \left(x \left(a e^2+c d^2\right)+a d e+c d e x^2\right)^{3/2}}{7 (d+e x)^{3/2} (f+g x)^{7/2} (c d f-a e g)}$$

$$\Big\downarrow \text{ 1248}$$

$$\frac{2 \left(x \left(a e^2+c d^2\right)+a d e+c d e x^2\right)^{3/2}}{7 (d+e x)^{3/2} (f+g x)^{7/2} (c d f-a e g)}+\frac{4 c d \left(\frac{4 c d \left(x \left(a e^2+c d^2\right)+a d e+c d e x^2\right)^{3/2}}{15 (d+e x)^{3/2} (f+g x)^{3/2} (c d f-a e g)^2}+\frac{2 \left(x \left(a e^2+c d^2\right)+a d e+c d e x^2\right)^{3/2}}{5 (d+e x)^{3/2} (f+g x)^{5/2} (c d f-a e g)}\right)}{7 (c d f-a e g)}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(
9/2)),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*(c*d*f - a*e*g)*(d +
e*x)^(3/2)*(f + g*x)^(7/2)) + (4*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2))/(5*(c*d*f - a*e*g)*(d + e*x)^(3/2)*(f + g*x)^(5/2)) + (4*c*d
*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(15*(c*d*f - a*e*g)^2*(d +
 e*x)^(3/2)*(f + g*x)^(3/2))))/(7*(c*d*f - a*e*g))
```

3.740.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

3.740.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.60

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)\left(8g^2x^2c^2d^2-12acde\,g^2x+28c^2d^2fgx+15a^2e^2g^2-42acdefg+35c^2d^2f^2\right)}{105(gx+f)^{\frac{7}{2}}\sqrt{ex+d}\,(aeg-cdf)^3}$	119
gosper	$-\frac{2(cdx+ae)\left(8g^2x^2c^2d^2-12acde\,g^2x+28c^2d^2fgx+15a^2e^2g^2-42acdefg+35c^2d^2f^2\right)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{105(gx+f)^{\frac{7}{2}}\left(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3c^3d^3\right)\sqrt{ex+d}}$	169

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/105*((c*d*x+a*e)*(e*x+d))^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/2)*(c*d*x+a*e)
*(8*c^2*d^2*g^2*x^2-12*a*c*d*e*g^2*x+28*c^2*d^2*f*g*x+15*a^2*e^2*g^2-42*a*
c*d*e*f*g+35*c^2*d^2*f^2)/(a*e*g-c*d*f)^3
```

3.740.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 748 vs. $2(174) = 348$.

Time = 0.55 (sec) , antiderivative size = 748, normalized size of antiderivative = 3.78

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx = \frac{}{105\,(c^3d^4f^7 - 3\,ac^2d^3ef^6g + 3\,a^2cd^2e^2f^5g^2 - a^3de^3f^4g^3 + (c^3d^3ef^3g}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1
/2),x, algorithm="fricas")
```

output
```
2/105*(8*c^3*d^3*g^2*x^3 + 35*a*c^2*d^2*e*f^2 - 42*a^2*c*d*e^2*f*g + 15*a^
3*e^3*g^2 + 4*(7*c^3*d^3*f*g - a*c^2*d^2*e*g^2)*x^2 + (35*c^3*d^3*f^2 - 14
*a*c^2*d^2*e*f*g + 3*a^2*c*d*e^2*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^7 - 3*a*c^2*d^3*e*f^6*g
+ 3*a^2*c*d^2*e^2*f^5*g^2 - a^3*d*e^3*f^4*g^3 + (c^3*d^3*e*f^3*g^4 - 3*a*c
^2*d^2*e^2*f^2*g^5 + 3*a^2*c*d*e^3*f*g^6 - a^3*e^4*g^7)*x^5 + (4*c^3*d^3*e
*f^4*g^3 - a^3*d*e^3*g^7 + (c^3*d^4 - 12*a*c^2*d^2*e^2)*f^3*g^4 - 3*(a*c^2
*d^3*e - 4*a^2*c*d*e^3)*f^2*g^5 + (3*a^2*c*d^2*e^2 - 4*a^3*e^4)*f*g^6)*x^4
 + 2*(3*c^3*d^3*e*f^5*g^2 - 2*a^3*d*e^3*f*g^6 + (2*c^3*d^4 - 9*a*c^2*d^2*e
^2)*f^4*g^3 - 3*(2*a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^3*g^4 + 3*(2*a^2*c*d^2*e
^2 - a^3*e^4)*f^2*g^5)*x^3 + 2*(2*c^3*d^3*e*f^6*g - 3*a^3*d*e^3*f^2*g^5 +
3*(c^3*d^4 - 2*a*c^2*d^2*e^2)*f^5*g^2 - 3*(3*a*c^2*d^3*e - 2*a^2*c*d*e^3)*
f^4*g^3 + (9*a^2*c*d^2*e^2 - 2*a^3*e^4)*f^3*g^4)*x^2 + (c^3*d^3*e*f^7 - 4*
a^3*d*e^3*f^3*g^4 + (4*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^6*g - 3*(4*a*c^2*d^3*e
 - a^2*c*d*e^3)*f^5*g^2 + (12*a^2*c*d^2*e^2 - a^3*e^4)*f^4*g^3)*x)
```

3.740.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}(f + gx)^{9/2}} dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(9/2)/(e*x+
d)**(1/2),x)
```

output
```
Timed out
```

3.740.7 Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}(f + gx)^{9/2}} dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^{\frac{9}{2}}} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x
+ f)^(9/2)), x)
```

3.740.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1426 vs. $2(174) = 348$.

Time = 0.79 (sec) , antiderivative size = 1426, normalized size of antiderivative = 7.20

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}(f + gx)^{9/2}} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1
/2),x, algorithm="giac")
```

output
```
2/105*((35*sqrt(-c*d^2*e + a*e^3)*c^3*d^4*e^4*f^2*abs(c)*abs(d) - 35*sqrt(
-c*d^2*e + a*e^3)*a*c^2*d^2*e^6*f^2*abs(c)*abs(d) - 28*sqrt(-c*d^2*e + a*e
^3)*c^3*d^5*e^3*f*g*abs(c)*abs(d) - 14*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^3*e^
5*f*g*abs(c)*abs(d) + 42*sqrt(-c*d^2*e + a*e^3)*a^2*c*d*e^7*f*g*abs(c)*abs
(d) + 8*sqrt(-c*d^2*e + a*e^3)*c^3*d^6*e^2*g^2*abs(c)*abs(d) + 4*sqrt(-c*d
^2*e + a*e^3)*a*c^2*d^4*e^4*g^2*abs(c)*abs(d) + 3*sqrt(-c*d^2*e + a*e^3)*a
^2*c*d^2*e^6*g^2*abs(c)*abs(d) - 15*sqrt(-c*d^2*e + a*e^3)*a^3*e^8*g^2*abs
(c)*abs(d))/(sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^3*d^3*e^3*f^6*abs(e) - 3*
sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^3*d^4*e^2*f^5*g*abs(e) - 3*sqrt(c^2*d^
2*e^2*f - c^2*d^3*e*g)*a*c^2*d^2*e^4*f^5*g*abs(e) + 3*sqrt(c^2*d^2*e^2*f -
 c^2*d^3*e*g)*c^3*d^5*e*f^4*g^2*abs(e) + 9*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*
g)*a*c^2*d^3*e^3*f^4*g^2*abs(e) + 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*
c*d*e^5*f^4*g^2*abs(e) - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^3*d^6*f^3*g^3
*abs(e) - 9*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^2*d^4*e^2*f^3*g^3*abs(e)
 - 9*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c*d^2*e^4*f^3*g^3*abs(e) - sqrt
(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*e^6*f^3*g^3*abs(e) + 3*sqrt(c^2*d^2*e^2*
f - c^2*d^3*e*g)*a*c^2*d^5*e*f^2*g^4*abs(e) + 9*sqrt(c^2*d^2*e^2*f - c^2*d
^3*e*g)*a^2*c*d^3*e^3*f^2*g^4*abs(e) + 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)
*a^3*d*e^5*f^2*g^4*abs(e) - 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c*d^4*
e^2*f*g^5*abs(e) - 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*d^2*e^4*f*g^...
```

3.740.9 Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx =$$

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{30\,a^3\,e^3\,g^2-84\,a^2\,c\,d\,e^2\,f\,g+70\,a\,c^2\,d^2\,e\,f^2}{105\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{x\,\left(6\,a^2\,c\,d\,e^2\,g^2-28\,a\,c^2\,d^2\,e\,f\,g+70\,c^3\,d^3\,f^2\right)}{105\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\right.}{x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{3\,f\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{3\,f^2\,x\,\sqrt{f+g\,x}\,\sqrt{}}{g^2}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(9/2)*(d + e*
x)^(1/2)),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((30*a^3*e^3*g^2 + 70*a*c^
2*d^2*e*f^2 - 84*a^2*c*d*e^2*f*g)/(105*g^3*(a*e*g - c*d*f)^3) + (x*(70*c^3
*d^3*f^2 + 6*a^2*c*d*e^2*g^2 - 28*a*c^2*d^2*e*f*g))/(105*g^3*(a*e*g - c*d*
f)^3) + (16*c^3*d^3*x^3)/(105*g*(a*e*g - c*d*f)^3) - (8*c^2*d^2*x^2*(a*e*g
 - 7*c*d*f))/(105*g^2*(a*e*g - c*d*f)^3)))/(x^3*(f + g*x)^(1/2)*(d + e*x)^
(1/2) + (f^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^3 + (3*f*x^2*(f + g*x)^(1/
2)*(d + e*x)^(1/2))/g + (3*f^2*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2)
```

3.740. $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx$

3.741 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx$

3.741.1 Optimal result

Integrand size = 48, antiderivative size = 267

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx = \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{9(cdf-aeg)(d+ex)^{3/2}(f+gx)^{9/2}} + \frac{4cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{21(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{7/2}} + \frac{16c^2d^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{105(cdf-aeg)^3(d+ex)^{3/2}(f+gx)^{5/2}} + \frac{32c^3d^3(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{315(cdf-aeg)^4(d+ex)^{3/2}(f+gx)^{3/2}}$$

output
```
2/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)/(e*x+d)^(3/2)/(
g*x+f)^(9/2)+4/21*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*
f)^2/(e*x+d)^(3/2)/(g*x+f)^(7/2)+16/105*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(3/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(3/2)/(g*x+f)^(5/2)+32/315*c^3*d^3*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(3/2)/(g*
x+f)^(3/2)
```

3.741.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx = \frac{2((ae+cdx)(d+ex))^{3/2}\left(-35a^3e^3g^3+15a^2cde^2g^2(9f+2gx)-3ac\right.}{315(cdf-aeg}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f +
g*x)^(11/2)),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-35*a^3*e^3*g^3 + 15*a^2*c*d*e^2*g^2*(
9*f + 2*g*x) - 3*a*c^2*d^2*e*g*(63*f^2 + 36*f*g*x + 8*g^2*x^2) + c^3*d^3*(
105*f^3 + 126*f^2*g*x + 72*f*g^2*x^2 + 16*g^3*x^3)))/(315*(c*d*f - a*e*g)^
4*(d + e*x)^(3/2)*(f + g*x)^(9/2))
```

3.741.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1254, 1254, 1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}dx$$

$$\Big\downarrow \text{1254}$$

$$\frac{2cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}(f+gx)^{9/2}}dx}{3(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9(d+ex)^{3/2}(f+gx)^{9/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$\frac{2cd\left(\frac{4cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}(f+gx)^{7/2}}dx}{7(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)}\right)}{3(cdf-aeg)}+$$
$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9(d+ex)^{3/2}(f+gx)^{9/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$\frac{2cd\left(\frac{4cd\left(\frac{2cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}(f+gx)^{5/2}}dx}{5(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)}\right)}{7(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)}\right)}{3(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9(d+ex)^{3/2}(f+gx)^{9/2}(cdf-aeg)}$$

$$\Big\downarrow \ 1248$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9(d+ex)^{3/2}(f+gx)^{9/2}(cdf-aeg)}+\frac{2cd\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)}+\frac{4cd\left(\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{15(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)}\right)}{7(cdf-aeg)}\right)}{3(cdf-aeg)}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(
11/2)),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(9*(c*d*f - a*e*g)*(d +
e*x)^(3/2)*(f + g*x)^(9/2)) + (2*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2))/(7*(c*d*f - a*e*g)*(d + e*x)^(3/2)*(f + g*x)^(7/2)) + (4*c*d
*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*(c*d*f - a*e*g)*(d
+ e*x)^(3/2)*(f + g*x)^(5/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2))/(15*(c*d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(3/2))))/(7*(c
*d*f - a*e*g))))/(3*(c*d*f - a*e*g))
```

3.741.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

3.741. $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx$

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

3.741.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.72

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)\left(-16g^3x^3c^3d^3+24a\,c^2d^2e\,g^3x^2-72c^3d^3f\,g^2x^2-30a^2cd\,e^2g^3x+108a\,c^2d^2ef\,g^2x-126c^3d^3f^2gx+35\right.}{315(gx+f)^{\frac{9}{2}}\sqrt{ex+d}\,(aeg-cdf)^4}$
gosper	$-\frac{2(cdx+ae)\left(-16g^3x^3c^3d^3+24a\,c^2d^2e\,g^3x^2-72c^3d^3f\,g^2x^2-30a^2cd\,e^2g^3x+108a\,c^2d^2ef\,g^2x-126c^3d^3f^2gx+35a^3e^3g^3-135a^2cd\,e^2\right.}{315(gx+f)^{\frac{9}{2}}\left(a^4e^4g^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4c^4d^4\right.}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(1/2),x
,method=_RETURNVERBOSE)
```

output
```
-2/315*((c*d*x+a*e)*(e*x+d))^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/2)*(c*d*x+a*e)
*(-16*c^3*d^3*g^3*x^3+24*a*c^2*d^2*e*g^3*x^2-72*c^3*d^3*f*g^2*x^2-30*a^2*c
*d*e^2*g^3*x+108*a*c^2*d^2*e*f*g^2*x-126*c^3*d^3*f^2*g*x+35*a^3*e^3*g^3-13
5*a^2*c*d*e^2*f*g^2+189*a*c^2*d^2*e*f^2*g-105*c^3*d^3*f^3)/(a*e*g-c*d*f)^4
```

3.741.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1179 vs. 2(235) = 470.

Time = 1.09 (sec) , antiderivative size = 1179, normalized size of antiderivative = 4.42

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx = \frac{}{315\,(c^4d^5f^9-4\,ac^3d^4ef^8g+6\,a^2c^2d^3e^2f^7g^2-4\,a^3cd^2e^3f^6g^3+a^4de^4}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(
1/2),x, algorithm="fricas")
```

output
```
2/315*(16*c^4*d^4*g^3*x^4 + 105*a*c^3*d^3*e*f^3 - 189*a^2*c^2*d^2*e^2*f^2*
g + 135*a^3*c*d*e^3*f*g^2 - 35*a^4*e^4*g^3 + 8*(9*c^4*d^4*f*g^2 - a*c^3*d^
3*e*g^3)*x^3 + 6*(21*c^4*d^4*f^2*g - 6*a*c^3*d^3*e*f*g^2 + a^2*c^2*d^2*e^2
*g^3)*x^2 + (105*c^4*d^4*f^3 - 63*a*c^3*d^3*e*f^2*g + 27*a^2*c^2*d^2*e^2*f
*g^2 - 5*a^3*c*d*e^3*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(e*x + d)*sqrt(g*x + f)/(c^4*d^5*f^9 - 4*a*c^3*d^4*e*f^8*g + 6*a^2*c^2*
d^3*e^2*f^7*g^2 - 4*a^3*c*d^2*e^3*f^6*g^3 + a^4*d*e^4*f^5*g^4 + (c^4*d^4*e
*f^4*g^5 - 4*a*c^3*d^3*e^2*f^3*g^6 + 6*a^2*c^2*d^2*e^3*f^2*g^7 - 4*a^3*c*d
*e^4*f*g^8 + a^4*e^5*g^9)*x^6 + (5*c^4*d^4*e*f^5*g^4 + a^4*d*e^4*g^9 + (c^
4*d^5 - 20*a*c^3*d^3*e^2)*f^4*g^5 - 2*(2*a*c^3*d^4*e - 15*a^2*c^2*d^2*e^3)
*f^3*g^6 + 2*(3*a^2*c^2*d^3*e^2 - 10*a^3*c*d*e^4)*f^2*g^7 - (4*a^3*c*d^2*e
^3 - 5*a^4*e^5)*f*g^8)*x^5 + 5*(2*c^4*d^4*e*f^6*g^3 + a^4*d*e^4*f*g^8 + (c
^4*d^5 - 8*a*c^3*d^3*e^2)*f^5*g^4 - 4*(a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^
4*g^5 + 2*(3*a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^3*g^6 - 2*(2*a^3*c*d^2*e^3
 - a^4*e^5)*f^2*g^7)*x^4 + 10*(c^4*d^4*e*f^7*g^2 + a^4*d*e^4*f^2*g^7 + (c^
4*d^5 - 4*a*c^3*d^3*e^2)*f^6*g^3 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f
^5*g^4 + 2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*g^5 - (4*a^3*c*d^2*e^3
- a^4*e^5)*f^3*g^6)*x^3 + 5*(c^4*d^4*e*f^8*g + 2*a^4*d*e^4*f^3*g^6 + 2*(c^
4*d^5 - 2*a*c^3*d^3*e^2)*f^7*g^2 - 2*(4*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f
^6*g^3 + 4*(3*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^5*g^4 - (8*a^3*c*d^2*e^3...
```

3.741.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(11/2)/(e*x
+d)**(1/2),x)
```

output
```
Timed out
```

3.741.7 Maxima [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{\sqrt{ex+d}(gx+f)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(
1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x
+ f)^(11/2)), x)
```

3.741.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2280 vs. $2(235) = 470$.

Time = 1.18 (sec) , antiderivative size = 2280, normalized size of antiderivative = 8.54

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(
1/2),x, algorithm="giac")
```

output
```
2/315*((105*sqrt(-c*d^2*e + a*e^3)*c^4*d^5*e^5*f^3*abs(c)*abs(d) - 105*sqr
t(-c*d^2*e + a*e^3)*a*c^3*d^3*e^7*f^3*abs(c)*abs(d) - 126*sqrt(-c*d^2*e +
a*e^3)*c^4*d^6*e^4*f^2*g*abs(c)*abs(d) - 63*sqrt(-c*d^2*e + a*e^3)*a*c^3*d
^4*e^6*f^2*g*abs(c)*abs(d) + 189*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^2*e^8*f^
2*g*abs(c)*abs(d) + 72*sqrt(-c*d^2*e + a*e^3)*c^4*d^7*e^3*f*g^2*abs(c)*abs
(d) + 36*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^5*e^5*f*g^2*abs(c)*abs(d) + 27*sqr
t(-c*d^2*e + a*e^3)*a^2*c^2*d^3*e^7*f*g^2*abs(c)*abs(d) - 135*sqrt(-c*d^2*
e + a*e^3)*a^3*c*d*e^9*f*g^2*abs(c)*abs(d) - 16*sqrt(-c*d^2*e + a*e^3)*c^4
*d^8*e^2*g^3*abs(c)*abs(d) - 8*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^4*g^3*ab
s(c)*abs(d) - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^6*g^3*abs(c)*abs(d) -
 5*sqrt(-c*d^2*e + a*e^3)*a^3*c*d^2*e^8*g^3*abs(c)*abs(d) + 35*sqrt(-c*d^2
*e + a*e^3)*a^4*e^10*g^3*abs(c)*abs(d))/(sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)
*c^4*d^4*e^4*f^8*abs(e) - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^4*d^5*e^3*
f^7*g*abs(e) - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^3*d^3*e^5*f^7*g*abs
(e) + 6*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^4*d^6*e^2*f^6*g^2*abs(e) + 16*
sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^3*d^4*e^4*f^6*g^2*abs(e) + 6*sqrt(c^
2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c^2*d^2*e^6*f^6*g^2*abs(e) - 4*sqrt(c^2*d^2
*e^2*f - c^2*d^3*e*g)*c^4*d^7*e*f^5*g^3*abs(e) - 24*sqrt(c^2*d^2*e^2*f - c
^2*d^3*e*g)*a*c^3*d^5*e^3*f^5*g^3*abs(e) - 24*sqrt(c^2*d^2*e^2*f - c^2*d^3
*e*g)*a^2*c^2*d^3*e^5*f^5*g^3*abs(e) - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e...
```

3.741.9 Mupad [B] (verification not implemented)

Time = 13.46 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{x\,\left(-10\,a^3\,c\,d\,e^3\,g^3+54\,a^2\,c^2\,d^2\,e^2\,f\,g^2-126\,a}{315\,g^4\,(a\,e\,g-c\,d\,f)}\right.}{x^4\,\sqrt{f+g}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(11/2)*(d + e
*x)^(1/2)),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((x*(210*c^4*d^4*f^3 - 10*a
^3*c*d*e^3*g^3 + 54*a^2*c^2*d^2*e^2*f*g^2 - 126*a*c^3*d^3*e*f^2*g))/(315*g
^4*(a*e*g - c*d*f)^4) - (70*a^4*e^4*g^3 - 210*a*c^3*d^3*e*f^3 + 378*a^2*c^
2*d^2*e^2*f^2*g - 270*a^3*c*d*e^3*f*g^2)/(315*g^4*(a*e*g - c*d*f)^4) + (32
*c^4*d^4*x^4)/(315*g*(a*e*g - c*d*f)^4) + (4*c^2*d^2*x^2*(a^2*e^2*g^2 + 21
*c^2*d^2*f^2 - 6*a*c*d*e*f*g))/(105*g^3*(a*e*g - c*d*f)^4) - (16*c^3*d^3*x
^3*(a*e*g - 9*c*d*f))/(315*g^2*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2)*(
d + e*x)^(1/2) + (f^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4 + (4*f*x^3*(f +
 g*x)^(1/2)*(d + e*x)^(1/2))/g + (4*f^3*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))
/g^3 + (6*f^2*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2)
```

3.741. $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx$

3.742 $\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

3.742.1 Optimal result

Integrand size = 48, antiderivative size = 382

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}}$$
$$+\frac{(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32cdg^2\sqrt{d+ex}}$$
$$-\frac{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}}$$
$$+\frac{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}$$
$$+\frac{3(cdf-aeg)^4\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{5/2}d^{5/2}g^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
1/4*(g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)+
3/64*(-a*e*g+c*d*f)^4*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g
*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2)/g^(5/2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/32*(-a*e*g+c*d*f)^2*(g*x+f)^(3/2)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g^2/(e*x+d)^(1/2)-1/8*(-a*e*g+c*d
*f)*(g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2
)+3/64*(-a*e*g+c*d*f)^3*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/c^2/d^2/g^2/(e*x+d)^(1/2)
```

3.742.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.64

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}\left(-3a^3e^3g^3+a^2cde^2g^2\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/
(d + e*x)^(3/2),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[f + g*x]*(
-3*a^3*e^3*g^3 + a^2*c*d*e^2*g^2*(11*f + 2*g*x) + a*c^2*d^2*e*g*(11*f^2 +
44*f*g*x + 24*g^2*x^2) + c^3*d^3*(-3*f^3 + 2*f^2*g*x + 24*f*g^2*x^2 + 16*g
^3*x^3)))/(a*e + c*d*x) + (3*(c*d*f - a*e*g)^4*ArcTanh[(Sqrt[c]*Sqrt[d]*Sq
rt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(a*e + c*d*x)^(3/2)))/(64*c^(5/
2)*d^(5/2)*g^(5/2)*(d + e*x)^(3/2))
```

3.742.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {1250, 1250, 1253, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \; 1250$$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\int \frac{(f+gx)^{3/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}dx}{8g}$$

$$\Big\downarrow \; 1250$$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6g}\right)}{8g}$$

$\downarrow$ 1253

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\int\frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+\cdots}}{2cd\sqrt{d+ex}}\cdots}{6g}\cdots\right)}{8g}$$

$\downarrow$ 1253

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}+\frac{\sqrt{f+gx}\sqrt{x(\cdots}}{\cdots}\cdots\right)}{4cd}\cdots\right)}{6g}\cdots\right)}{8g}$$

$\downarrow$ 1268

3.742. $$\int\frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx$$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-$$

$$\frac{3(cdf-aeg)\left(\frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae\right.}}{c}\right.}{4cd}\right.}{6g}\right.}{8g}$$

$\downarrow$ 66

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-$$

$$\frac{3(cdf-aeg)\left(\frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+g}}{}\right.}{4cd}\right.}{6g}\right.}{8g}$$

$\downarrow$ 221

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-$$

$$\frac{3(cdf-aeg)\left(\frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}}{}\right.}{4cd}\right.}{6g}\right.}{8g}$$

3.742. $\int\frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx$

input
```
Int[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e
*x)^(3/2),x]
```

output
```
((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(4*g*(d +
e*x)^(3/2)) - (3*(c*d*f - a*e*g)*(((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(3*g*Sqrt[d + e*x]) - ((c*d*f - a*e*g)*(((f + g*x)^(
3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*c*d*Sqrt[d + e*x]) +
(3*(c*d*f - a*e*g)*((Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(c*d*Sqrt[d + e*x]) + ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*
x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(
c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(4*
c*d)))/(6*g)))/(8*g)
```

3.742.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.742.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(326) = 652$.

Time = 0.54 (sec) , antiderivative size = 732, normalized size of antiderivative = 1.92

method	result
default	$\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(32c^3d^3g^3x^3\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}+3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^4e^4g^4-12\ln\left(2c\right.\right.}{}$

input
```
int((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
method=_RETURNVERBOSE)
```

output
```
1/128*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(32*c^3*d^3*g^3*x^3*((g*x+
f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)+3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*
x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^4*e^4*g^4-12*ln(1/
2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d
*g)^(1/2))*a^3*c*d*e^3*f*g^3+18*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(
c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c^2*d^2*e^2*f^2*g^2-12
*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)
)/(c*d*g)^(1/2))*a*c^3*d^3*e*f^3*g+3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x
+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^4*d^4*f^4+48*a*c^2*
d^2*e*g^3*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)+48*c^3*d^3*f*g^2*x
^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)+4*((g*x+f)*(c*d*x+a*e))^(1/2)
*(c*d*g)^(1/2)*a^2*c*d*e^2*g^3*x+88*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1
/2)*a*c^2*d^2*e*f*g^2*x+4*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^3*d^
3*f^2*g*x-6*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^3*e^3*g^3+22*((g*x
+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*c*d*e^2*f*g^2+22*((g*x+f)*(c*d*x+
a*e))^(1/2)*(c*d*g)^(1/2)*a*c^2*d^2*e*f^2*g-6*((g*x+f)*(c*d*x+a*e))^(1/2)*
(c*d*g)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/c^2/d^2/g^2/((g*x+f)*(c*d*x+a*e))
^(1/2)/(c*d*g)^(1/2)
```

3.742.5 Fricas [A] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 1059, normalized size of antiderivative = 2.77

$$\int \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \left[\frac{4\left(16\,c^4d^4g^4x^3-3\,c^4d^4f^3g+11\,ac^3d^3ef^2g^2+11\,a^2c^2\right.}{}\right.$$

input
```
integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3
/2),x, algorithm="fricas")
```

output
```
[1/256*(4*(16*c^4*d^4*g^4*x^3 - 3*c^4*d^4*f^3*g + 11*a*c^3*d^3*e*f^2*g^2 +
 11*a^2*c^2*d^2*e^2*f*g^3 - 3*a^3*c*d*e^3*g^4 + 24*(c^4*d^4*f*g^3 + a*c^3*
d^3*e*g^4)*x^2 + 2*(c^4*d^4*f^2*g^2 + 22*a*c^3*d^3*e*f*g^3 + a^2*c^2*d^2*e
^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(
g*x + f) + 3*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f^2*g^
2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d^3*e
^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^4)*
x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g +
 a^2*d*e^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x
+ c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*
g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c
*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d^3*e*g^3*x
 + c^3*d^4*g^3), 1/128*(2*(16*c^4*d^4*g^4*x^3 - 3*c^4*d^4*f^3*g + 11*a*c^3
*d^3*e*f^2*g^2 + 11*a^2*c^2*d^2*e^2*f*g^3 - 3*a^3*c*d*e^3*g^4 + 24*(c^4*d^
4*f*g^3 + a*c^3*d^3*e*g^4)*x^2 + 2*(c^4*d^4*f^2*g^2 + 22*a*c^3*d^3*e*f*g^3
 + a^2*c^2*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d)*sqrt(g*x + f) - 3*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^
2*d^3*e^2*f^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4
 - 4*a*c^3*d^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3
 + a^4*e^5*g^4)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^...
```

3.742.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+
d)**(3/2),x)
```

output
```
Timed out
```

3.742.7 Maxima [F]

$$\int \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^(3/2)/(e
*x + d)^(3/2), x)
```

3.742.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8807 vs. $2(326) = 652$.

Time = 2.74 (sec) , antiderivative size = 8807, normalized size of antiderivative = 23.05

$$\int \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3
/2),x, algorithm="giac")
```

output
```
1/192*(48*a*f*((4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x +
d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*
x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 +
 (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g
))*e*f*abs(g)/g^2 - 4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*
x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f +
 (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g
^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d
*e*g))*d*abs(g)/g + (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*
g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f
- 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e
*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a
*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)
*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*
e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*abs(g)/(e*g^2))/g - (c^2*d^2*e^3*f^2*g*ab
s(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*
g^2))) - 2*a*c*d*e^4*f*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g)
 + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + a^2*e^5*g^3*abs(g)*log(abs(-sqrt(e^2
*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + sqrt(-c*d^2*e
*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d*e*f*abs(g) - 2*sq...
```

3.742.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{(f+g\,x)^{3/2}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{(d+e\,x)^{3/2}}\,dx$$

input
```
int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e
*x)^(3/2),x)
```

output
```
int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e
*x)^(3/2), x)
```

3.743 $\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

3.743.1 Optimal result

Integrand size = 48, antiderivative size = 310

$$\int \frac{\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8cdg^2\sqrt{d+ex}} - \frac{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3g(d+ex)^{3/2}} + \frac{(cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{3/2}d^{3/2}g^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

output
```
1/3*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)+
1/8*(-a*e*g+c*d*f)^3*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*
x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(3/2)/d^(3/2)/g^(5/2)/(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/4*(-a*e*g+c*d*f)*(g*x+f)^(3/2)*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)+1/8*(-a*e*g+c*d*f)^2*(g
*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g^2/(e*x+d)^(1/2)
```

3.743.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}(3a^2e^2g^2+2acdeg(4f+7g}{ae+cdx}\right.}{24c^{3/2}d^{3/2}g^{5/}}$$

input
```
Integrate[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d
 + e*x)^(3/2),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[f + g*x]*(
3*a^2*e^2*g^2 + 2*a*c*d*e*g*(4*f + 7*g*x) + c^2*d^2*(-3*f^2 + 2*f*g*x + 8*
g^2*x^2)))/(a*e + c*d*x) + (3*(c*d*f - a*e*g)^3*ArcTanh[(Sqrt[c]*Sqrt[d]*S
qrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(a*e + c*d*x)^(3/2)))/(24*c^(3
/2)*d^(3/2)*g^(5/2)*(d + e*x)^(3/2))
```

3.743.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1250, 1250, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{ 1250}$$

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\int \frac{\sqrt{f+gx}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}dx}{2g}$$

$$\Big\downarrow \text{ 1250}$$

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4g}\right)}{2g}$$

$\downarrow$ 1253

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade}}{cd\sqrt{d+ex}}\right.}{4g}\right.}{2g}$$

$\downarrow$ 1268

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+}}{cd\sqrt{d+ex}}\right.}{4g}\right.}{2g}$$

$\downarrow$ 66

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)}}{cd\sqrt{d+ex}}\right.}{4g}\right.}{2g}$$

$\downarrow$ 221

3.743. $\int\frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx$

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}} - \frac{(cdf-aeg)\left(\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}} - \frac{(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}} + \frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)\cdots}}{cd\sqrt{d+ex}}\right)}{4g}\right)}{2g}$$

input
```
Int[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x
)^(3/2),x]
```

output
```
((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d +
e*x)^(3/2)) - ((c*d*f - a*e*g)*(((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(2*g*Sqrt[d + e*x]) - ((c*d*f - a*e*g)*((Sqrt[f + g*x]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x]) + ((c*d*
f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d
*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(4*g)))/(2*g)
```

3.743.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

3.743. $\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.743.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.63

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^3e^3g^3-9\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right.\right.}{}$

input
```
int((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
method=_RETURNVERBOSE)
```

output
```
-1/48*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*ln(1/2*(2*c*d*g*x+a*e*g
+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*e^3
*g^3-9*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)
^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2+9*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2
*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2
*g-3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(
1/2))/(c*d*g)^(1/2))*c^3*d^3*f^3-16*c^2*d^2*g^2*x^2*(c*d*g)^(1/2)*((g*x+f)
*(c*d*x+a*e))^(1/2)-28*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*c*d*e*g
^2*x-4*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c^2*d^2*f*g*x-6*((g*x+f)*
(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2-16*((g*x+f)*(c*d*x+a*e))^(1/2
)*(c*d*g)^(1/2)*a*c*d*e*f*g+6*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^
2*d^2*f^2)/(e*x+d)^(1/2)/c/d/((g*x+f)*(c*d*x+a*e))^(1/2)/g^2/(c*d*g)^(1/2)
```

3.743.5 Fricas [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 847, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \left[\frac{4\,(8\,c^3d^3g^3x^2-3\,c^3d^3f^2g+8\,ac^2d^2efg^2+3\,a^2cde^2g^3+\right.$$

input
```
integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3
/2),x, algorithm="fricas")
```

output
```
[1/96*(4*(8*c^3*d^3*g^3*x^2 - 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2 + 3*a^
2*c*d*e^2*g^3 + 2*(c^3*d^3*f*g^2 + 7*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^3*d^4*f^3 -
3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f
^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(c*
d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2
*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f +
a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d
^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*
g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d^2*e*g^3*x + c^2*d^3
*g^3), 1/48*(2*(8*c^3*d^3*g^3*x^2 - 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2
+ 3*a^2*c*d*e^2*g^3 + 2*(c^3*d^3*f*g^2 + 7*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^3*d^4*
f^3 - 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d
^3*e*f^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*s
qrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d
*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*
e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^2*d^2*e*g^3*x + c^2*d^3*g^3)]
```

3.743.6 Sympy [F]

$$\int \frac{\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \int \frac{((d+ex)\,(ae+cdx))^{\frac{3}{2}}\sqrt{f+gx}}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+
d)**(3/2),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*sqrt(f + g*x)/(d + e*x)**(3/2),
x)
```

3.743.7 Maxima [F]

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\sqrt{gx+f}}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*sqrt(g*x + f)/(e*x
 + d)^(3/2), x)
```

3.743.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3057 vs. $2(262) = 524$.

Time = 1.21 (sec) , antiderivative size = 3057, normalized size of antiderivative = 9.86

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3
/2),x, algorithm="giac")
```

output
```
1/24*(6*a*((4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e
*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x +
d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^
2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*e
*f*abs(g)/g^2 - 4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x +
d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*
x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 +
 (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g
))*d*abs(g)/g + (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g -
d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*
c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)
- (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d
^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqr
t(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)
*c*d*g)))/(sqrt(c*d*g)*c*d))*abs(g)/(e*g^2))/g - (c^2*d^2*e^3*f^2*g*abs(g)
*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)
)) - 2*a*c*d*e^4*f*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + s
qrt(-c*d^2*e*g^2 + a*e^3*g^2))) + a^2*e^5*g^3*abs(g)*log(abs(-sqrt(e^2*f -
 d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + sqrt(-c*d^2*e*g^2
 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d*e*f*abs(g) - 2*sqrt(-...
```

3.743.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \int \frac{\sqrt{f+g\,x}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{\left(d+e\,x\right)^{3/2}}\,dx$$

input
```
int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e
*x)^(3/2),x)
```

output
```
int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e
*x)^(3/2), x)
```

3.744 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx$

3.744.1 Optimal result

Integrand size = 48, antiderivative size = 238

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = -\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}} + \frac{3(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4\sqrt{c}\sqrt{d}g^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*(g*x+f)^(1/2)/g/(e*x+d)^(3/2)+
3/4*(-a*e*g+c*d*f)^2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*
x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(5/2)/c^(1/2)/d^(1/2)/(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/4*(-a*e*g+c*d*f)*(g*x+f)^(1/2)*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)
```

3.744.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.69

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^{3/2}}{(d + ex)^{3/2}\sqrt{f + gx}}\, dx = \frac{\sqrt{ae + cdx}\sqrt{d + ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae + cdx}\sqrt{f + gx}(5aeg + cd(-3f\right.}{4\sqrt{c}\sqrt{d}g^{5/2}\sqrt{(ae + cdx)(}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*S
qrt[f + g*x]),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x
]*Sqrt[f + g*x]*(5*a*e*g + c*d*(-3*f + 2*g*x)) + 3*(c*d*f - a*e*g)^2*ArcTa
nh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(4*Sqrt[
c]*Sqrt[d]*g^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.744.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {1250, 1250, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{3/2}}{(d + ex)^{3/2}\sqrt{f + gx}}\, dx$$

$$\Big\downarrow \; 1250$$

$$\frac{\sqrt{f + gx}\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{3/2}}{2g(d + ex)^{3/2}} - \frac{3(cdf - aeg)\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}\sqrt{f+gx}}dx}{4g}$$

$$\Big\downarrow \; 1250$$

$$\frac{\sqrt{f + gx}\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{3/2}}{2g(d + ex)^{3/2}} - \frac{3(cdf - aeg)\left(\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}} - \frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2g}\right)}{4g}$$

$$\Big\downarrow \text{1268}$$

$$\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{4g}$$

$$\Big\downarrow \text{66}$$

$$\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{4g}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{4g}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*Sqrt[f
+ g*x]),x]
```

output
```
(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(2*g*(d + e*
x)^(3/2)) - (3*(c*d*f - a*e*g)*((Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]) - ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*S
qrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f +
 g*x])])/(Sqrt[c]*Sqrt[d]*g^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])))/(4*g)
```

3.744. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx$

3.744.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.744.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.32

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\,\sqrt{gx+f}\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^2e^2g^2-6\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)\right.}{8\sqrt{}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
1/8*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(g*x+f)^(1/2)*(3*ln(1/2*(2*c
*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1
/2))*a^2*e^2*g^2-6*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(
1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c*d*e*f*g+3*ln(1/2*(2*c*d*g*x+a*e*g+c
*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f
^2+4*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c*d*g*x+10*(c*d*g)^(1/2)*((
g*x+f)*(c*d*x+a*e))^(1/2)*a*e*g-6*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2
)*c*d*f)/((g*x+f)*(c*d*x+a*e))^(1/2)/g^2/(c*d*g)^(1/2)
```

3.744.5 Fricas [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 651, normalized size of antiderivative = 2.74

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^{3/2}}{(d + ex)^{3/2}\sqrt{f + gx}}\, dx = \left[\frac{4\,(2\,c^2d^2g^2x - 3\,c^2d^2fg + 5\,acdeg^2)\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1
/2),x, algorithm="fricas")
```

output
```
[1/16*(4*(2*c^2*d^2*g^2*x - 3*c^2*d^2*f*g + 5*a*c*d*e*g^2)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^2*d^3*f^2
- 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2
*e^3*g^2)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d
^2*e*f*g + a^2*d*e^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(
2*c*d*g*x + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^
2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d
^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c*d*e
*g^3*x + c*d^2*g^3), 1/8*(2*(2*c^2*d^2*g^2*x - 3*c^2*d^2*f*g + 5*a*c*d*e*g
^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f
) - 3*(c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*
a*c*d*e^2*f*g + a^2*e^3*g^2)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*
g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c*d*e*g^
3*x + c*d^2*g^3)]
```

3.744.6 Sympy [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx=\int \frac{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}}{\left(d+ex\right)^{\frac{3}{2}}\sqrt{f+gx}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**(1/2),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/((d + e*x)**(3/2)*sqrt(f + g*x))
, x)
```

3.744.7 Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*s
qrt(g*x + f)), x)
```

3.744.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. 2(198) = 396.

Time = 0.55 (sec) , antiderivative size = 709, normalized size of antiderivative = 2.98

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx=\frac{e\left(\left(\sqrt{c^2d^2e^2f-acde^3g+\left((ex+d)cde-cd^2e+ae^3\right)cdg}\sqrt{(ex+d)cde-cd^2e+ae^3}\left(\frac{2\left((ex+d)\cdots\right.}{\cdots}\right.\right.\right.}{\cdots}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1
/2),x, algorithm="giac")
```

output
```
1/4*e*((sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*
e^3)*c*d*g)*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(2*((e*x + d)*c*d*e -
c*d^2*e + a*e^3)*abs(e)/(c*d*e^2*g) - 3*(c*d*e^2*f*g*abs(e) - a*e^3*g^2*ab
s(e))/(c*d*e^2*g^3)) - 3*(c^2*d^2*e^2*f^2*abs(e) - 2*a*c*d*e^3*f*g*abs(e)
+ a^2*e^4*g^2*abs(e))*log(abs(-sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*sqr
t(c*d*g) + sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e +
 a*e^3)*c*d*g)))/(sqrt(c*d*g)*g^2))*abs(c)*abs(d)/(c*d*e^3) + (3*c^3*d^3*e
^3*f^2*abs(c)*abs(d)*abs(e)*log(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) +
sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))) - 6*a*c^2*d^2*e^4*f*g*abs(c)*abs(d)*ab
s(e)*log(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^
2*d^3*e*g))) + 3*a^2*c*d*e^5*g^2*abs(c)*abs(d)*abs(e)*log(abs(-sqrt(-c*d^2
*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))) + 3*sqrt(c^2
*d^2*e^2*f - c^2*d^3*e*g)*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*c*d*e*f*abs(c
)*abs(d)*abs(e) + 2*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*sqrt(-c*d^2*e + a*e^
3)*sqrt(c*d*g)*c*d^2*g*abs(c)*abs(d)*abs(e) - 5*sqrt(c^2*d^2*e^2*f - c^2*d
^3*e*g)*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*a*e^2*g*abs(c)*abs(d)*abs(e))/(
sqrt(c*d*g)*c^2*d^2*e^4*g^2))/abs(e)
```

3.744.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\sqrt{f+g\,x}\,\left(d+e\,x\right)^{3/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(1/2)*(d + e*
x)^(3/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(1/2)*(d + e*
x)^(3/2)), x)
```

3.745 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx$

3.745.1 Optimal result

Integrand size = 48, antiderivative size = 222

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx=\frac{3cd\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}-\frac{3\sqrt{c}\sqrt{d}(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
-2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^(1/2)-3
*(-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^
(1/2))*c^(1/2)*d^(1/2)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(5/2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)+3*c*d*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)
```

3.745.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.71

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx = \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}(-2aeg+cd(3f+gx))-3\sqrt{c}\right.}{g^{5/2}\sqrt{(ae+cdx)(d+ex)}\sqrt{}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)^(3/2)),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(-2*a*e*g + c*
d*(3*f + g*x)) - 3*Sqrt[c]*Sqrt[d]*(c*d*f - a*e*g)*Sqrt[f + g*x]*ArcTanh[(
Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(g^(5/2)*Sqr
t[(a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

3.745.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {1249, 1250, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{ 1249}$$

$$\frac{3cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}\sqrt{f+gx}}dx}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}$$

$$\Big\downarrow \text{ 1250}$$

$$\frac{3cd\left(\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2g}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}$$

$$\Big\downarrow \text{1268}$$

$$\frac{3cd\left(\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}$$

$$\Big\downarrow \text{66}$$

$$\frac{3cd\left(\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}$$

$$\Big\downarrow \text{221}$$

$$\frac{3cd\left(\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)^(3/2)),x]
```

output
```
(-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(g*(d + e*x)^(3/2)*Sqrt
[f + g*x]) + (3*c*d*((Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(g*Sqrt[d + e*x]) - ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x
]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(S
qrt[c]*Sqrt[d]*g^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/g
```

3.745.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

3.745. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.745.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.68

method	result
default	$\frac{\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)acde\,g^2x-3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)c^2d^2fgx+3\ln\left(\frac{2cdgx+aeg-\right.\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/2),x,
method=_RETURNVERBOSE)
```

output
```
1/2*(3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)
^(1/2))/(c*d*g)^(1/2))*a*c*d*e*g^2*x-3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g
*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f*g*x+3*ln(
1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c
*d*g)^(1/2))*a*c*d*e*f*g-3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x
+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f^2+2*((g*x+f)*(c*d*x+a
*e))^(1/2)*(c*d*g)^(1/2)*c*d*g*x-4*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/
2)*a*e*g+6*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)*((c*d*x+a*e)*(
e*x+d))^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)/g^2/(g*x+f)^(1/2)/
(e*x+d)^(1/2)
```

3.745.5 Fricas [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 663, normalized size of antiderivative = 2.99

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx = \left[\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}(cdgx+3\,cdf-2\,aeg)\sqrt{ex+d}\sqrt{}}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3
/2),x, algorithm="fricas")
```

output
```
[1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x + 3*c*d*f - 2
*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c*d^2*f^2 - a*d*e*f*g + (c*d*e*f*
g - a*e^2*g^2)*x^2 + (c*d*e*f^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*f*g)*x)*sqrt
(c*d/g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*
e^2*g^2 + 4*(2*c*d*g^2*x + c*d*f*g + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g
 + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*
d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(e*g^3*x^2 + d*f*
g^2 + (e*f*g^2 + d*g^3)*x), 1/2*(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*(c*d*g*x + 3*c*d*f - 2*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c*d^2*
f^2 - a*d*e*f*g + (c*d*e*f*g - a*e^2*g^2)*x^2 + (c*d*e*f^2 - a*d*e*g^2 + (
c*d^2 - a*e^2)*f*g)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 +
 c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e*g^3*x^2 + d*f
*g^2 + (e*f*g^2 + d*g^3)*x)]
```

3.745.6 Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx = \int \frac{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}{(d+ex)^{\frac{3}{2}}\,(f+gx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**(3/2),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/((d + e*x)**(3/2)*(f + g*x)**(3/
2)), x)
```

3.745.7 Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)^(3/2)), x)
```

3.745.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(188) = 376.

Time = 0.59 (sec) , antiderivative size = 559, normalized size of antiderivative = 2.52

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx = \frac{\sqrt{(ex+d)cde - cd^2e + ae^3}\left(\frac{((ex+d)cde - cd^2e + ae^3)|c||d|}{e^2g} + \frac{3\,(cde^2fg|c||d| - }{e^2g^3}\right.}{\sqrt{c^2d^2e^2f - acde^3g + ((ex+d)cde - cd^2e + ae^3)cdg}}$$
$$+ \frac{3\,(cdf|c||d| - aeg|c||d|)\log\left(\left|-\sqrt{(ex+d)cde - cd^2e + ae^3}\sqrt{cdg} + \sqrt{c^2d^2e^2f - acde^3g + ((ex+d)cde - c\right.\right.}{\sqrt{cdg}g^2}$$
$$- \frac{3\sqrt{c^2d^2e^2f - c^2d^3eg}cdef|c||d|\log\left(\left|-\sqrt{-cd^2e + ae^3}\sqrt{cdg} + \sqrt{c^2d^2e^2f - c^2d^3eg}\right|\right) - 3\sqrt{c^2d^2e^2f - c^2d^3eg}}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3
/2),x, algorithm="giac")
```

output
```
sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(((e*x + d)*c*d*e - c*d^2*e + a*e^
3)*abs(c)*abs(d)/(e^2*g) + 3*(c*d*e^2*f*g*abs(c)*abs(d) - a*e^3*g^2*abs(c)
*abs(d))/(e^2*g^3))/sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e -
c*d^2*e + a*e^3)*c*d*g) + 3*(c*d*f*abs(c)*abs(d) - a*e*g*abs(c)*abs(d))*lo
g(abs(-sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*
e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)))/(sqrt(c
*d*g)*g^2) - (3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c*d*e*f*abs(c)*abs(d)*lo
g(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e
*g))) - 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*e^2*g*abs(c)*abs(d)*log(abs(
-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)))
+ 3*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*c*d*e*f*abs(c)*abs(d) - sqrt(-c*d^2
*e + a*e^3)*sqrt(c*d*g)*c*d^2*g*abs(c)*abs(d) - 2*sqrt(-c*d^2*e + a*e^3)*s
qrt(c*d*g)*a*e^2*g*abs(c)*abs(d))/(sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*sqrt(
c*d*g)*e*g^2)
```

3.745.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^{3/2}}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\left(f + g\,x\right)^{3/2}\,\left(d + e\,x\right)^{3/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(3/2)*(d + e*
x)^(3/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(3/2)*(d + e*
x)^(3/2)), x)
```

3.746 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx$

3.746.1 Optimal result

Integrand size = 48, antiderivative size = 214

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx = -\frac{2cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^2\sqrt{d+ex}\sqrt{f+gx}} - \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}} + \frac{2c^{3/2}d^{3/2}\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

output
```
-2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^(3/2)
+2*c^(3/2)*d^(3/2)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+
f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)-2*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^
(1/2)/(g*x+f)^(1/2)
```

3.746.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.71

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx = \frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(-\sqrt{g}\sqrt{ae+cdx}(aeg+cd(3f+4gx))+3c^{3/}\right.}{3g^{5/2}\sqrt{(ae+cdx)(d+ex)}(f+}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)^(5/2)),x]
```

output
```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-(Sqrt[g]*Sqrt[a*e + c*d*x]*(a*e*g + c
*d*(3*f + 4*g*x))) + 3*c^(3/2)*d^(3/2)*(f + g*x)^(3/2)*ArcTanh[(Sqrt[c]*Sq
rt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(3*g^(5/2)*Sqrt[(a*e +
 c*d*x)*(d + e*x)]*(f + g*x)^(3/2))
```

3.746.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {1249, 1249, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}(f+gx)^{3/2}}dx}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$\Big\downarrow \text{1249}$$

$$\frac{cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}-\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$\Big\downarrow \text{1268}$$

$$\frac{cd\left(\frac{cd\sqrt{d+ex}\sqrt{ae+cdx}\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$\Big\downarrow \text{66}$$

$$\frac{cd\left(\frac{2cd\sqrt{d+ex}\sqrt{ae+cdx}\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{cd\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)^(5/2)),x]
```

output
```
(-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e*x)^(3/2)*(f
 + g*x)^(3/2)) + (c*d*((-2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g
*Sqrt[d + e*x]*Sqrt[f + g*x]) + (2*Sqrt[c]*Sqrt[d]*Sqrt[a*e + c*d*x]*Sqrt[
d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x
])])/(g^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/g
```

3.746.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && !GtQ[c - a*(d/b), 0]
```

3.746. $\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.746.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.50

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)c^2d^2g^2x^2+6\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)c^2d^2f\right.}{3\sqrt{cdg}\,\sqrt{(}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/2),x,
method=_RETURNVERBOSE)
```

output
```
1/3*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f
)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*g^2*x^2+6*ln(1/
2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d
*g)^(1/2))*c^2*d^2*f*g*x+3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x
+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f^2-8*((g*x+f)*(c*d*x+a
*e))^(1/2)*(c*d*g)^(1/2)*c*d*g*x-2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/
2)*a*e*g-6*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)/(c*d*g)^(1/2)/
((g*x+f)*(c*d*x+a*e))^(1/2)/g^2/(g*x+f)^(3/2)/(e*x+d)^(1/2)
```

3.746.5 Fricas [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 685, normalized size of antiderivative = 3.20

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx = \left[-\frac{4\sqrt{cdex^2 + ade + (cd^2+ae^2)x}(4\,cdgx + 3\,cdf + aeg)\sqrt{ex+d}}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5
/2),x, algorithm="fricas")
```

output
```
[-1/6*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(4*c*d*g*x + 3*c*d*f
+ a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c*d*e*g^2*x^3 + c*d^2*f^2 + (2*c
*d*e*f*g + c*d^2*g^2)*x^2 + (c*d*e*f^2 + 2*c*d^2*f*g)*x)*sqrt(c*d/g)*log(-
(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(
2*c*d*g^2*x + c*d*f*g + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 +
 a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g +
(8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(e*g^4*x^3 + d*f^2*g^2 + (2*e*
f*g^3 + d*g^4)*x^2 + (e*f^2*g^2 + 2*d*f*g^3)*x), -1/3*(2*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*(4*c*d*g*x + 3*c*d*f + a*e*g)*sqrt(e*x + d)*sqr
t(g*x + f) + 3*(c*d*e*g^2*x^3 + c*d^2*f^2 + (2*c*d*e*f*g + c*d^2*g^2)*x^2
+ (c*d*e*f^2 + 2*c*d^2*f*g)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e
*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e*g^4*x
^3 + d*f^2*g^2 + (2*e*f*g^3 + d*g^4)*x^2 + (e*f^2*g^2 + 2*d*f*g^3)*x)]
```

3.746.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**(5/2),x)
```

output
```
Timed out
```

3.746.7 Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)^(5/2)), x)
```

3.746.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(178) = 356$.

Time = 0.72 (sec) , antiderivative size = 686, normalized size of antiderivative = 3.21

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx=$$

$$-\frac{2\,cd|c||d|\log\left(\left|-\sqrt{(ex+d)cde-cd^2e+ae^3}\sqrt{cdg}+\sqrt{c^2d^2e^2f-acde^3g+((ex+d)cde-cd^2e+ae^3)cdg}\right.\right.}{\sqrt{cdg}g^2}$$

$$+\frac{2\left(3\sqrt{c^2d^2e^2f-c^2d^3eg}cdef|c||d|\log\left(\left|-\sqrt{-cd^2e+ae^3}\sqrt{cdg}+\sqrt{c^2d^2e^2f-c^2d^3eg}\right|\right)-3\sqrt{c^2d^2e^2f-c^2d^3}\right.}{3\left(\right.}$$

$$-\frac{2\sqrt{(ex+d)cde-cd^2e+ae^3}\left(\frac{4\left(c^3d^3e^2fg^2|c||d|-ac^2d^2e^3g^3|c||d|\right)\left((ex+d)cde-cd^2e+ae^3\right)}{cde^2fg^3-ae^3g^4}+\frac{3\left(c^4d^4e^4f^2g|c||d|-2\,ac^3d^3e^5fg^2|c||d\right.}{cde^2fg^3-ae^3g^4}\right.}{3\left(c^2d^2e^2f-acde^3g+((ex+d)cde-cd^2e+ae^3)cdg\right)^{\frac{3}{2}}}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5
/2),x, algorithm="giac")
```

output
```
-2*c*d*abs(c)*abs(d)*log(abs(-sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*sqrt
(c*d*g) + sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e +
a*e^3)*c*d*g)))/(sqrt(c*d*g)*g^2) + 2/3*(3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*
g)*c*d*e*f*abs(c)*abs(d)*log(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqr
t(c^2*d^2*e^2*f - c^2*d^3*e*g))) - 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c*d
^2*g*abs(c)*abs(d)*log(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*
d^2*e^2*f - c^2*d^3*e*g))) + 3*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*c*d*e*f*
abs(c)*abs(d) - 4*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*c*d^2*g*abs(c)*abs(d)
 + sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*a*e^2*g*abs(c)*abs(d))/(sqrt(c^2*d^2
*e^2*f - c^2*d^3*e*g)*sqrt(c*d*g)*e*f*g^2 - sqrt(c^2*d^2*e^2*f - c^2*d^3*e
*g)*sqrt(c*d*g)*d*g^3) - 2/3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(4*(c
^3*d^3*e^2*f*g^2*abs(c)*abs(d) - a*c^2*d^2*e^3*g^3*abs(c)*abs(d))*((e*x +
d)*c*d*e - c*d^2*e + a*e^3)/(c*d*e^2*f*g^3 - a*e^3*g^4) + 3*(c^4*d^4*e^4*f
^2*g*abs(c)*abs(d) - 2*a*c^3*d^3*e^5*f*g^2*abs(c)*abs(d) + a^2*c^2*d^2*e^6
*g^3*abs(c)*abs(d))/(c*d*e^2*f*g^3 - a*e^3*g^4))/(c^2*d^2*e^2*f - a*c*d*e^
3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)^(3/2)
```

3.746.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\left(f+g\,x\right)^{5/2}\,\left(d+e\,x\right)^{3/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(5/2)*(d + e*
x)^(3/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(5/2)*(d + e*
x)^(3/2)), x)
```

3.747 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx$

3.747.1 Optimal result

Integrand size = 48, antiderivative size = 63

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx=\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{5(cdf-aeg)(d+ex)^{5/2}(f+gx)^{5/2}}$$

output
```
2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)/(e*x+d)^(5/2)/(
g*x+f)^(5/2)
```

3.747.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{5/2}}{5(cdf-aeg)(d+ex)^{5/2}(f+gx)^{5/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)^(7/2)),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2))/(5*(c*d*f - a*e*g)*(d + e*x)^(5/2)*(f
+ g*x)^(5/2))
```

3.747.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx$$
$$\Big\downarrow \; 1248$$
$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)^(7/2)),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*(c*d*f - a*e*g)*(d +
e*x)^(5/2)*(f + g*x)^(5/2))
```

3.747.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

3.747.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^{2}}{5\sqrt{ex+d}\,(gx+f)^{\frac{5}{2}}(aeg-cdf)}$	55
gosper	$-\frac{2(cdx+ae)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}}{5(gx+f)^{\frac{5}{2}}(aeg-cdf)(ex+d)^{\frac{3}{2}}}$	63

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/5*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(5/2)*(c*d*x+a*e)^2
/(a*e*g-c*d*f)
```

3.747.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(55) = 110.

Time = 0.42 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.68

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx = \frac{2\left(c^2d^2x^2+2\,acd\cdots\right.}{5\left(cd^2f^4-adef^3g+(cdefg^3-ae^2g^4)x^4+(3\,cdef^2g^2-adeg^4+\cdots\right.}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7
/2),x, algorithm="fricas")
```

output
```
2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c*d^2*f^4 - a*d*e*f^3*g + (c*d*e*
f*g^3 - a*e^2*g^4)*x^4 + (3*c*d*e*f^2*g^2 - a*d*e*g^4 + (c*d^2 - 3*a*e^2)*
f*g^3)*x^3 + 3*(c*d*e*f^3*g - a*d*e*f*g^3 + (c*d^2 - a*e^2)*f^2*g^2)*x^2 +
 (c*d*e*f^4 - 3*a*d*e*f^2*g^2 + (3*c*d^2 - a*e^2)*f^3*g)*x)
```

3.747.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**(7/2),x)
```

output
```
Timed out
```

3.747.7 Maxima [F]

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx = \int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)^(7/2)), x)
```

3.747.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(55) = 110$.

Time = 0.95 (sec) , antiderivative size = 446, normalized size of antiderivative = 7.08

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx =$$

$$-\frac{2\left(\sqrt{-cd^2e+ae^3}c^2d^4|c||d|-2\sqrt{-cd^2e+ae^3}acd^2e^2|c||d|+\cdots\right.}{5\left(\sqrt{c^2d^2e^2f-c^2d^3eg}cde^2f^3-2\sqrt{c^2d^2e^2f-c^2d^3eg}cd^2ef^2g-\sqrt{c^2d^2e^2f-c^2d^3eg}ae^3f^2g+\sqrt{c^2d^2e^2f-c\cdots}\right.}$$
$$+\frac{2\left(c^5d^5e^4fg^2|c||d|-ac^4d^4e^5g^3|c||d|\right)((ex+d)cde-cd^2e+ae^3)^{\frac{5}{2}}}{5\left(c^2d^2e^4f^2g^2-2acde^5fg^3+a^2e^6g^4\right)(c^2d^2e^2f-acde^3g+((ex+d)cde-cd^2e+ae^3)cdg)^{\frac{5}{2}}}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7
/2),x, algorithm="giac")
```

output
```
-2/5*(sqrt(-c*d^2*e + a*e^3)*c^2*d^4*abs(c)*abs(d) - 2*sqrt(-c*d^2*e + a*e
^3)*a*c*d^2*e^2*abs(c)*abs(d) + sqrt(-c*d^2*e + a*e^3)*a^2*e^4*abs(c)*abs(
d))/(sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c*d*e^2*f^3 - 2*sqrt(c^2*d^2*e^2*f
- c^2*d^3*e*g)*c*d^2*e*f^2*g - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*e^3*f^2
*g + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c*d^3*f*g^2 + 2*sqrt(c^2*d^2*e^2*f
- c^2*d^3*e*g)*a*d*e^2*f*g^2 - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*d^2*e*g
^3) + 2/5*(c^5*d^5*e^4*f*g^2*abs(c)*abs(d) - a*c^4*d^4*e^5*g^3*abs(c)*abs(
d))*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)/((c^2*d^2*e^4*f^2*g^2 - 2*a*
c*d*e^5*f*g^3 + a^2*e^6*g^4)*(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d
*e - c*d^2*e + a*e^3)*c*d*g)^(5/2))
```

3.747.9 Mupad [B] (verification not implemented)

Time = 12.48 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.68

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx = -\frac{\left(\frac{2\,a^2\,e^2}{5\,a\,e\,g^3-5\,c\,d\,f\,g^2}+\frac{2\,c^2\,d^2\,x^2}{5\,a\,e\,g^3-5\,c\,d\,f\,g^2}+\frac{4\,a\,c\,d\,e\,x}{5\,a\,e\,g^3-5\,c\,d\,f\,g^2}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}-\frac{\sqrt{f+g\,x}\,\left(5\,c\,d\,f^3-5\,a\,e\,f^2\,g\right)\,\sqrt{d+e\,x}}{5\,a\,e\,g^3-5\,c\,d\,f\,g^2}+\frac{x\,\sqrt{f+g\,x}\,\left(10\,a\,e\,f\,g^2-10\,c\,d\,f^2\,g\right)\,\sqrt{d+e\,x}}{5\,a\,e\,g^3-5\,c\,d\,f\,g^2}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(7/2)*(d + e*
x)^(3/2)),x)
```

output
```
-(((2*a^2*e^2)/(5*a*e*g^3 - 5*c*d*f*g^2) + (2*c^2*d^2*x^2)/(5*a*e*g^3 - 5*
c*d*f*g^2) + (4*a*c*d*e*x)/(5*a*e*g^3 - 5*c*d*f*g^2))*(x*(a*e^2 + c*d^2) +
 a*d*e + c*d*e*x^2)^(1/2))/(x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2) - ((f + g*
x)^(1/2)*(5*c*d*f^3 - 5*a*e*f^2*g)*(d + e*x)^(1/2))/(5*a*e*g^3 - 5*c*d*f*g
^2) + (x*(f + g*x)^(1/2)*(10*a*e*f*g^2 - 10*c*d*f^2*g)*(d + e*x)^(1/2))/(5
*a*e*g^3 - 5*c*d*f*g^2))
```

3.748 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx$

3.748.1 Optimal result

Integrand size = 48, antiderivative size = 129

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx = \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{7(cdf-aeg)(d+ex)^{5/2}(f+gx)^{7/2}} + \frac{4cd(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{35(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{5/2}}$$

output
```
2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)/(e*x+d)^(5/2)/(
g*x+f)^(7/2)+4/35*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*
f)^2/(e*x+d)^(5/2)/(g*x+f)^(5/2)
```

3.748.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.53

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx = \frac{2((ae+cdx)(d+ex))^{5/2}(-5aeg+cd(7f+2gx))}{35(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{7/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)^(9/2)),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-5*a*e*g + c*d*(7*f + 2*g*x)))/(35*(c*
d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(7/2))
```

3.748.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}dx$$

$$\Big\downarrow \text{ 1254}$$

$$\frac{2cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}dx}{7(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)}$$

$$\Big\downarrow \text{ 1248}$$

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{35(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)^(9/2)),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*(c*d*f - a*e*g)*(d +
e*x)^(5/2)*(f + g*x)^(7/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2))/(35*(c*d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(5/2))
```

3.748.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

3.748.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{2(cdx+ae)(-2cdgx+5aeg-7cdf)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}}{35(gx+f)^{\frac{7}{2}}\left(a^{2}e^{2}g^{2}-2acdefg+c^{2}d^{2}f^{2}\right)(ex+d)^{\frac{3}{2}}}$	99
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,\left(-2g\,x^{2}c^{2}d^{2}+3acdegx-7c^{2}d^{2}fx+5a^{2}e^{2}g-7acdef\right)(cdx+ae)}{35\sqrt{ex+d}\,(gx+f)^{\frac{7}{2}}(aeg-cdf)^{2}}$	100

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/35*(c*d*x+a*e)*(-2*c*d*g*x+5*a*e*g-7*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+
a*d*e)^(3/2)/(g*x+f)^(7/2)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(e*x+d)
^(3/2)
```

3.748.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. 2(113) = 226.

Time = 0.50 (sec) , antiderivative size = 526, normalized size of antiderivative = 4.08

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx = \frac{}{35\left(c^2d^3f^6-2\,acd^2ef^5g+a^2de^2f^4g^2+\left(c^2d^2ef^2g^4-2\,acde^2fg^5+\right.\right.}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9
/2),x, algorithm="fricas")
```

output
```
2/35*(2*c^3*d^3*g*x^3 + 7*a^2*c*d*e^2*f - 5*a^3*e^3*g + (7*c^3*d^3*f - a*c
^2*d^2*e*g)*x^2 + 2*(7*a*c^2*d^2*e*f - 4*a^2*c*d*e^2*g)*x)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^3*f^6 - 2*
a*c*d^2*e*f^5*g + a^2*d*e^2*f^4*g^2 + (c^2*d^2*e*f^2*g^4 - 2*a*c*d*e^2*f*g
^5 + a^2*e^3*g^6)*x^5 + (4*c^2*d^2*e*f^3*g^3 + a^2*d*e^2*g^6 + (c^2*d^3 -
8*a*c*d*e^2)*f^2*g^4 - 2*(a*c*d^2*e - 2*a^2*e^3)*f*g^5)*x^4 + 2*(3*c^2*d^2
*e*f^4*g^2 + 2*a^2*d*e^2*f*g^5 + 2*(c^2*d^3 - 3*a*c*d*e^2)*f^3*g^3 - (4*a*
c*d^2*e - 3*a^2*e^3)*f^2*g^4)*x^3 + 2*(2*c^2*d^2*e*f^5*g + 3*a^2*d*e^2*f^2
*g^4 + (3*c^2*d^3 - 4*a*c*d*e^2)*f^4*g^2 - 2*(3*a*c*d^2*e - a^2*e^3)*f^3*g
^3)*x^2 + (c^2*d^2*e*f^6 + 4*a^2*d*e^2*f^3*g^3 + 2*(2*c^2*d^3 - a*c*d*e^2)
*f^5*g - (8*a*c*d^2*e - a^2*e^3)*f^4*g^2)*x)
```

3.748.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade + (cd^2 + ae^2)x + cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**(9/2),x)
```

output
```
Timed out
```

3.748.7 Maxima [F]

$$\int \frac{\left(ade + (cd^2 + ae^2)x + cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx = \int \frac{\left(cdex^2 + ade + (cd^2 + ae^2)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)^(9/2)), x)
```

3.748.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(113) = 226$.

Time = 0.74 (sec) , antiderivative size = 1001, normalized size of antiderivative = 7.76

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx =$$

$$-\frac{2\left(7\sqrt{-cd^2e+ae^3}\right.}{35\left(\sqrt{c^2d^2e^2f-c^2d^3eg}c^2d^2e^3f^5-3\sqrt{c^2d^2e^2f-c^2d^3eg}c^2d^3e^2f^4g-2\sqrt{c^2d^2e^2f-c^2d^3eg}acde^4f^4g+3\sqrt{c^2}\right.}$$
$$+\frac{2\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{5}{2}}\left(\frac{2\left(c^7d^7e^6fg^4|c||d|-ac^6d^6e^7g^5|c||d|\right)\left((ex+d)cde-cd^2e+ae^3\right)}{c^3d^3e^6f^3g^3-3\,ac^2d^2e^7f^2g^4+3\,a^2cde^8fg^5-a^3e^9g^6}+\frac{7\left(c^8d^8e^8f^2g^3|c||d|-2\,ac^7d^7e^9fg^4|c|\right.}{c^3d^3e^6f^3g^3-3\,ac^2d^2e^7f^2g^4+3\,a\right.}{35\left(c^2d^2e^2f-acde^3g+((ex+d)cde-cd^2e+ae^3)cdg\right)^{\frac{7}{2}}}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9
/2),x, algorithm="giac")
```

output
```
-2/35*(7*sqrt(-c*d^2*e + a*e^3)*c^3*d^5*e*f*abs(c)*abs(d) - 14*sqrt(-c*d^2
*e + a*e^3)*a*c^2*d^3*e^3*f*abs(c)*abs(d) + 7*sqrt(-c*d^2*e + a*e^3)*a^2*c
*d*e^5*f*abs(c)*abs(d) - 2*sqrt(-c*d^2*e + a*e^3)*c^3*d^6*g*abs(c)*abs(d)
- sqrt(-c*d^2*e + a*e^3)*a*c^2*d^4*e^2*g*abs(c)*abs(d) + 8*sqrt(-c*d^2*e +
 a*e^3)*a^2*c*d^2*e^4*g*abs(c)*abs(d) - 5*sqrt(-c*d^2*e + a*e^3)*a^3*e^6*g
*abs(c)*abs(d))/(sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^2*d^2*e^3*f^5 - 3*sqr
t(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^2*d^3*e^2*f^4*g - 2*sqrt(c^2*d^2*e^2*f -
c^2*d^3*e*g)*a*c*d*e^4*f^4*g + 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^2*d^4
*e*f^3*g^2 + 6*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c*d^2*e^3*f^3*g^2 + sqr
t(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*e^5*f^3*g^2 - sqrt(c^2*d^2*e^2*f - c^2*
d^3*e*g)*c^2*d^5*f^2*g^3 - 6*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c*d^3*e^2
*f^2*g^3 - 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*d*e^4*f^2*g^3 + 2*sqrt(
c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c*d^4*e*f*g^4 + 3*sqrt(c^2*d^2*e^2*f - c^2*
d^3*e*g)*a^2*d^2*e^3*f*g^4 - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*d^3*e^2
*g^5) + 2/35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*(2*(c^7*d^7*e^6*f*g
^4*abs(c)*abs(d) - a*c^6*d^6*e^7*g^5*abs(c)*abs(d))*((e*x + d)*c*d*e - c*d
^2*e + a*e^3)/(c^3*d^3*e^6*f^3*g^3 - 3*a*c^2*d^2*e^7*f^2*g^4 + 3*a^2*c*d*e
^8*f*g^5 - a^3*e^9*g^6) + 7*(c^8*d^8*e^8*f^2*g^3*abs(c)*abs(d) - 2*a*c^7*d
^7*e^9*f*g^4*abs(c)*abs(d) + a^2*c^6*d^6*e^10*g^5*abs(c)*abs(d))/(c^3*d^3*
e^6*f^3*g^3 - 3*a*c^2*d^2*e^7*f^2*g^4 + 3*a^2*c*d*e^8*f*g^5 - a^3*e^9*g...
```

3.748.9 Mupad [B] (verification not implemented)

Time = 12.66 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.91

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx =$$

$$-\frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\;x+a\,d\,e}\left(\frac{2\,a^2\,e^2\,(5\,a\,e\,g-7\,c\,d\,f)}{35\,g^3\,(a\,e\,g-c\,d\,f)^2}-\frac{4\,c^3\,d^3\,x^3}{35\,g^2\,(a\,e\,g-c\,d\,f)^2}+\frac{2\,c^2\,d^2\,x^2\,(a\,e\,g-7\,c\,d\,f)}{35\,g^3\,(a\,e\,g-c\,d\,f)^2}+\frac{4\,a\,c\,d\,e\,x\,(4\,a\,e\,g-}{35\,g^3\,(a\,e\,g-c}\right.}{x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{3\,f\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{3\,f^2\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(9/2)*(d + e*
x)^(3/2)),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*a^2*e^2*(5*a*e*g - 7*c
*d*f))/(35*g^3*(a*e*g - c*d*f)^2) - (4*c^3*d^3*x^3)/(35*g^2*(a*e*g - c*d*f
)^2) + (2*c^2*d^2*x^2*(a*e*g - 7*c*d*f))/(35*g^3*(a*e*g - c*d*f)^2) + (4*a
*c*d*e*x*(4*a*e*g - 7*c*d*f))/(35*g^3*(a*e*g - c*d*f)^2)))/(x^3*(f + g*x)^
(1/2)*(d + e*x)^(1/2) + (f^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^3 + (3*f*x
^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g + (3*f^2*x*(f + g*x)^(1/2)*(d + e*x)
^(1/2))/g^2)
```

3.749 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx$

3.749.1 Optimal result

Integrand size = 48, antiderivative size = 198

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9(cdf-aeg)(d+ex)^{5/2}(f+gx)^{9/2}}+\frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{63(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{7/2}}+\frac{16c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{315(cdf-aeg)^3(d+ex)^{5/2}(f+gx)^{5/2}}$$

output
```
2/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)/(e*x+d)^(5/2)/(
g*x+f)^(9/2)+8/63*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*
f)^2/(e*x+d)^(5/2)/(g*x+f)^(7/2)+16/315*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(5/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(5/2)/(g*x+f)^(5/2)
```

3.749.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.53

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{5/2}\left(35a^2e^2g^2-10acdeg(9f+2gx)+c^2d^2(6\right.}{315(cdf-aeg)^3(d+ex)^{5/2}(f+gx)^{9/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)^(11/2)),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(35*a^2*e^2*g^2 - 10*a*c*d*e*g*(9*f + 2
*g*x) + c^2*d^2*(63*f^2 + 36*f*g*x + 8*g^2*x^2)))/(315*(c*d*f - a*e*g)^3*(
d + e*x)^(5/2)*(f + g*x)^(9/2))
```

3.749.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1254, 1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}dx$$

$$\Big\downarrow \text{1254}$$

$$\frac{4cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}dx}{9(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$\frac{4cd\left(\frac{2cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}dx}{7(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)}\right)}{9(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1248}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)}+\frac{4cd\left(\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{35(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)}\right)}{9(cdf-aeg)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)^(11/2)),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9*(c*d*f - a*e*g)*(d +
e*x)^(5/2)*(f + g*x)^(9/2)) + (4*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(5/2))/(7*(c*d*f - a*e*g)*(d + e*x)^(5/2)*(f + g*x)^(7/2)) + (4*c*d
*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(35*(c*d*f - a*e*g)^2*(d +
 e*x)^(5/2)*(f + g*x)^(5/2))))/(9*(c*d*f - a*e*g))
```

3.749.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

3.749.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.85

method	result
gosper	$-\frac{2(cdx+ae)\left(8g^2x^2c^2d^2-20acde\,g^2x+36c^2d^2fgx+35a^2e^2g^2-90acdefg+63c^2d^2f^2\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}{315(gx+f)^{\frac{9}{2}}\left(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3c^3d^3\right)(ex+d)^{\frac{3}{2}}}$
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,\left(8g^2x^3c^3d^3-12a\,c^2d^2e\,g^2x^2+36c^3d^3fg\,x^2+15a^2cd\,e^2g^2x-54a\,c^2d^2efgx+63c^3d^3f^2x+35a^3e^3g^2-90a^2cd\,e^2\right.}{315\sqrt{ex+d}\,(gx+f)^{\frac{9}{2}}(aeg-cdf)^3}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11/2),x
,method=_RETURNVERBOSE)
```

output
```
-2/315*(c*d*x+a*e)*(8*c^2*d^2*g^2*x^2-20*a*c*d*e*g^2*x+36*c^2*d^2*f*g*x+35
*a^2*e^2*g^2-90*a*c*d*e*f*g+63*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d
*e)^(3/2)/(g*x+f)^(9/2)/(a^3*e^3*g^3-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f^2
*g-c^3*d^3*f^3)/(e*x+d)^(3/2)
```

3.749.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 918 vs. $2(174) = 348$.

Time = 1.03 (sec) , antiderivative size = 918, normalized size of antiderivative = 4.64

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^{11/2}}\, dx = \frac{}{315\,(c^3d^4f^8 - 3\,ac^2d^3ef^7g + 3\,a^2cd^2e^2f^6g^2 - a^3de^3f^5g^3 + (c^3d^3ef}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1
1/2),x, algorithm="fricas")
```

output
```
2/315*(8*c^4*d^4*g^2*x^4 + 63*a^2*c^2*d^2*e^2*f^2 - 90*a^3*c*d*e^3*f*g + 3
5*a^4*e^4*g^2 + 4*(9*c^4*d^4*f*g - a*c^3*d^3*e*g^2)*x^3 + 3*(21*c^4*d^4*f^
2 - 6*a*c^3*d^3*e*f*g + a^2*c^2*d^2*e^2*g^2)*x^2 + 2*(63*a*c^3*d^3*e*f^2 -
 72*a^2*c^2*d^2*e^2*f*g + 25*a^3*c*d*e^3*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^8 - 3*a*c^2*d^3*
e*f^7*g + 3*a^2*c*d^2*e^2*f^6*g^2 - a^3*d*e^3*f^5*g^3 + (c^3*d^3*e*f^3*g^5
 - 3*a*c^2*d^2*e^2*f^2*g^6 + 3*a^2*c*d*e^3*f*g^7 - a^3*e^4*g^8)*x^6 + (5*c
^3*d^3*e*f^4*g^4 - a^3*d*e^3*g^8 + (c^3*d^4 - 15*a*c^2*d^2*e^2)*f^3*g^5 -
3*(a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2*g^6 + (3*a^2*c*d^2*e^2 - 5*a^3*e^4)*f*
g^7)*x^5 + 5*(2*c^3*d^3*e*f^5*g^3 - a^3*d*e^3*f*g^7 + (c^3*d^4 - 6*a*c^2*d
^2*e^2)*f^4*g^4 - 3*(a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^3*g^5 + (3*a^2*c*d^2*e
^2 - 2*a^3*e^4)*f^2*g^6)*x^4 + 10*(c^3*d^3*e*f^6*g^2 - a^3*d*e^3*f^2*g^6 +
 (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^5*g^3 - 3*(a*c^2*d^3*e - a^2*c*d*e^3)*f^4*g
^4 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^3*g^5)*x^3 + 5*(c^3*d^3*e*f^7*g - 2*a^3
*d*e^3*f^3*g^5 + (2*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^6*g^2 - 3*(2*a*c^2*d^3*e
- a^2*c*d*e^3)*f^5*g^3 + (6*a^2*c*d^2*e^2 - a^3*e^4)*f^4*g^4)*x^2 + (c^3*d
^3*e*f^8 - 5*a^3*d*e^3*f^4*g^4 + (5*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^7*g - 3*(
5*a*c^2*d^3*e - a^2*c*d*e^3)*f^6*g^2 + (15*a^2*c*d^2*e^2 - a^3*e^4)*f^5*g^
3)*x)
```

3.749.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**(11/2),x)
```

output
```
Timed out
```

3.749.7 Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^{11/2}}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}(gx + f)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1
1/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)^(11/2)), x)
```

3.749.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1760 vs. $2(174) = 348$.

Time = 1.11 (sec) , antiderivative size = 1760, normalized size of antiderivative = 8.89

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1
1/2),x, algorithm="giac")
```

output
```
-2/315*(63*sqrt(-c*d^2*e + a*e^3)*c^4*d^6*e^2*f^2*abs(c)*abs(d) - 126*sqrt
(-c*d^2*e + a*e^3)*a*c^3*d^4*e^4*f^2*abs(c)*abs(d) + 63*sqrt(-c*d^2*e + a*
e^3)*a^2*c^2*d^2*e^6*f^2*abs(c)*abs(d) - 36*sqrt(-c*d^2*e + a*e^3)*c^4*d^7
*e*f*g*abs(c)*abs(d) - 18*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^5*e^3*f*g*abs(c)*
abs(d) + 144*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^3*e^5*f*g*abs(c)*abs(d) - 90
*sqrt(-c*d^2*e + a*e^3)*a^3*c*d*e^7*f*g*abs(c)*abs(d) + 8*sqrt(-c*d^2*e +
a*e^3)*c^4*d^8*g^2*abs(c)*abs(d) + 4*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2*
g^2*abs(c)*abs(d) + 3*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4*g^2*abs(c)*ab
s(d) - 50*sqrt(-c*d^2*e + a*e^3)*a^3*c*d^2*e^6*g^2*abs(c)*abs(d) + 35*sqrt
(-c*d^2*e + a*e^3)*a^4*e^8*g^2*abs(c)*abs(d))/(sqrt(c^2*d^2*e^2*f - c^2*d^
3*e*g)*c^3*d^3*e^4*f^7 - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^3*d^4*e^3*f
^6*g - 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^2*d^2*e^5*f^6*g + 6*sqrt(c^
2*d^2*e^2*f - c^2*d^3*e*g)*c^3*d^5*e^2*f^5*g^2 + 12*sqrt(c^2*d^2*e^2*f - c
^2*d^3*e*g)*a*c^2*d^3*e^4*f^5*g^2 + 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^
2*c*d*e^6*f^5*g^2 - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^3*d^6*e*f^4*g^3
- 18*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^2*d^4*e^3*f^4*g^3 - 12*sqrt(c^2
*d^2*e^2*f - c^2*d^3*e*g)*a^2*c*d^2*e^5*f^4*g^3 - sqrt(c^2*d^2*e^2*f - c^2
*d^3*e*g)*a^3*e^7*f^4*g^3 + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^3*d^7*f^3*
g^4 + 12*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^2*d^5*e^2*f^3*g^4 + 18*sqrt
(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c*d^3*e^4*f^3*g^4 + 4*sqrt(c^2*d^2*e^...
```

3.749.9 Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.90

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx = -\frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\left(\frac{70\,a^4\,e^4\,g^2-180\,a^3\,c\,d\,e^3\,f\,g+126\,a^2\,c^2\,d^2\,e^2\,f^2}{315\,g^4\,(a\,e\,g-c\,d\,f)^3}+\frac{x^2\,(6\,a^2\,c^2\,d^2\,e^2\,g^2-36\,a\,c^3\,d^3\,e\,f\,g+126\,c^4\,d^{\cdots}}{315\,g^4\,(a\,e\,g-c\,d\,f)^3}\cdots\right.}{x^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^4}+\frac{4\,f\,x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\cdots}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(11/2)*(d + e
*x)^(3/2)),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((70*a^4*e^4*g^2 + 126*a^2
*c^2*d^2*e^2*f^2 - 180*a^3*c*d*e^3*f*g)/(315*g^4*(a*e*g - c*d*f)^3) + (x^2
*(126*c^4*d^4*f^2 + 6*a^2*c^2*d^2*e^2*g^2 - 36*a*c^3*d^3*e*f*g))/(315*g^4*
(a*e*g - c*d*f)^3) + (16*c^4*d^4*x^4)/(315*g^2*(a*e*g - c*d*f)^3) - (8*c^3
*d^3*x^3*(a*e*g - 9*c*d*f))/(315*g^3*(a*e*g - c*d*f)^3) + (4*a*c*d*e*x*(25
*a^2*e^2*g^2 + 63*c^2*d^2*f^2 - 72*a*c*d*e*f*g))/(315*g^4*(a*e*g - c*d*f)^
3)))/(x^4*(f + g*x)^(1/2)*(d + e*x)^(1/2) + (f^4*(f + g*x)^(1/2)*(d + e*x)
^(1/2))/g^4 + (4*f*x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g + (4*f^3*x*(f +
g*x)^(1/2)*(d + e*x)^(1/2))/g^3 + (6*f^2*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/
2))/g^2)
```

3.749. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx$

3.750 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx$

3.750.1 Optimal result

Integrand size = 48, antiderivative size = 267

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{9/2}} + \frac{16c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{231(cdf-aeg)^3(d+ex)^{5/2}(f+gx)^{7/2}} + \frac{32c^3d^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{1155(cdf-aeg)^4(d+ex)^{5/2}(f+gx)^{5/2}}$$

output
```
2/11*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)/(e*x+d)^(5/2)/
(g*x+f)^(11/2)+4/33*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*
d*f)^2/(e*x+d)^(5/2)/(g*x+f)^(9/2)+16/231*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(5/2)/(g*x+f)^(7/2)+32/1155*c^3*d
^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(5/2)/
(g*x+f)^(5/2)
```

3.750.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.57

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx = \frac{2((ae+cdx)(d+ex))^{5/2}(-105a^3e^3g^3+35a^2cde^2g^2(11f+2gx)-}{1155(cdf-}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)^(13/2)),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-105*a^3*e^3*g^3 + 35*a^2*c*d*e^2*g^2*
(11*f + 2*g*x) - 5*a*c^2*d^2*e*g*(99*f^2 + 44*f*g*x + 8*g^2*x^2) + c^3*d^3
*(231*f^3 + 198*f^2*g*x + 88*f*g^2*x^2 + 16*g^3*x^3)))/(1155*(c*d*f - a*e*
g)^4*(d + e*x)^(5/2)*(f + g*x)^(11/2))
```

3.750.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1254, 1254, 1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}dx$$

$$\Big\downarrow \text{1254}$$

$$\frac{6cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}dx}{11(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11(d+ex)^{5/2}(f+gx)^{11/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$\frac{6cd\left(\frac{4cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}dx}{9(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)}\right)}{11(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11(d+ex)^{5/2}(f+gx)^{11/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$\frac{6cd\left(\frac{4cd\left(\frac{2cd\int\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}dx}{7(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)}\right)}{9(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)}\right)}{11(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11(d+ex)^{5/2}(f+gx)^{11/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1248}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11(d+ex)^{5/2}(f+gx)^{11/2}(cdf-aeg)}+\frac{6cd\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)}+\frac{4cd\left(\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{35(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)}\right)}{9(cdf-aeg)}\right)}{11(cdf-aeg)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)^(13/2)),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(11*(c*d*f - a*e*g)*(d +
 e*x)^(5/2)*(f + g*x)^(11/2)) + (6*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(5/2))/(9*(c*d*f - a*e*g)*(d + e*x)^(5/2)*(f + g*x)^(9/2)) + (4*c
*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*(c*d*f - a*e*g)*(
d + e*x)^(5/2)*(f + g*x)^(7/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(5/2))/(35*(c*d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(5/2))))/(9*
(c*d*f - a*e*g))))/(11*(c*d*f - a*e*g))
```

3.750.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

3.750. $\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}dx$

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

3.750.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.97

method	result
gosper	$-\frac{2(cdx+ae)\left(-16g^3x^3c^3d^3+40a\,c^2d^2e\,g^3x^2-88c^3d^3f\,g^2x^2-70a^2cd\,e^2g^3x+220a\,c^2d^2ef\,g^2x-198c^3d^3f^2gx+105a^3e^3g^3-385a^2cd\,e\right.}{1155(gx+f)^{\frac{11}{2}}\left(a^4e^4g^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4c^4\right.}$
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,\left(-16g^3x^4c^4d^4+24a\,c^3d^3e\,g^3x^3-88c^4d^4f\,g^2x^3-30a^2c^2d^2e^2g^3x^2+132a\,c^3d^3ef\,g^2x^2-198c^4d^4f^2g\,x^2+35a^3c\right.}{1155\sqrt{ex+d}\,(gx}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13/2),x
,method=_RETURNVERBOSE)
```

output
```
-2/1155*(c*d*x+a*e)*(-16*c^3*d^3*g^3*x^3+40*a*c^2*d^2*e*g^3*x^2-88*c^3*d^3
*f*g^2*x^2-70*a^2*c*d*e^2*g^3*x+220*a*c^2*d^2*e*f*g^2*x-198*c^3*d^3*f^2*g*
x+105*a^3*e^3*g^3-385*a^2*c*d*e^2*f*g^2+495*a*c^2*d^2*e*f^2*g-231*c^3*d^3*
f^3)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)/(g*x+f)^(11/2)/(a^4*e^4*g^4-4
*a^3*c*d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g^2-4*a*c^3*d^3*e*f^3*g+c^4*d^4*f
^4)/(e*x+d)^(3/2)
```

3.750.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1420 vs. 2(235) = 470.

Time = 1.44 (sec) , antiderivative size = 1420, normalized size of antiderivative = 5.32

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1
3/2),x, algorithm="fricas")
```

output
```
2/1155*(16*c^5*d^5*g^3*x^5 + 231*a^2*c^3*d^3*e^2*f^3 - 495*a^3*c^2*d^2*e^3
*f^2*g + 385*a^4*c*d*e^4*f*g^2 - 105*a^5*e^5*g^3 + 8*(11*c^5*d^5*f*g^2 - a
*c^4*d^4*e*g^3)*x^4 + 2*(99*c^5*d^5*f^2*g - 22*a*c^4*d^4*e*f*g^2 + 3*a^2*c
^3*d^3*e^2*g^3)*x^3 + (231*c^5*d^5*f^3 - 99*a*c^4*d^4*e*f^2*g + 33*a^2*c^3
*d^3*e^2*f*g^2 - 5*a^3*c^2*d^2*e^3*g^3)*x^2 + 2*(231*a*c^4*d^4*e*f^3 - 396
*a^2*c^3*d^3*e^2*f^2*g + 275*a^3*c^2*d^2*e^3*f*g^2 - 70*a^4*c*d*e^4*g^3)*x
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/
(c^4*d^5*f^10 - 4*a*c^3*d^4*e*f^9*g + 6*a^2*c^2*d^3*e^2*f^8*g^2 - 4*a^3*c*
d^2*e^3*f^7*g^3 + a^4*d*e^4*f^6*g^4 + (c^4*d^4*e*f^4*g^6 - 4*a*c^3*d^3*e^2
*f^3*g^7 + 6*a^2*c^2*d^2*e^3*f^2*g^8 - 4*a^3*c*d*e^4*f*g^9 + a^4*e^5*g^10)
*x^7 + (6*c^4*d^4*e*f^5*g^5 + a^4*d*e^4*g^10 + (c^4*d^5 - 24*a*c^3*d^3*e^2
)*f^4*g^6 - 4*(a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^3*g^7 + 6*(a^2*c^2*d^3*e
^2 - 4*a^3*c*d*e^4)*f^2*g^8 - 2*(2*a^3*c*d^2*e^3 - 3*a^4*e^5)*f*g^9)*x^6 +
 3*(5*c^4*d^4*e*f^6*g^4 + 2*a^4*d*e^4*f*g^9 + 2*(c^4*d^5 - 10*a*c^3*d^3*e^
2)*f^5*g^5 - 2*(4*a*c^3*d^4*e - 15*a^2*c^2*d^2*e^3)*f^4*g^6 + 4*(3*a^2*c^2
*d^3*e^2 - 5*a^3*c*d*e^4)*f^3*g^7 - (8*a^3*c*d^2*e^3 - 5*a^4*e^5)*f^2*g^8)
*x^5 + 5*(4*c^4*d^4*e*f^7*g^3 + 3*a^4*d*e^4*f^2*g^8 + (3*c^4*d^5 - 16*a*c^
3*d^3*e^2)*f^6*g^4 - 12*(a*c^3*d^4*e - 2*a^2*c^2*d^2*e^3)*f^5*g^5 + 2*(9*a
^2*c^2*d^3*e^2 - 8*a^3*c*d*e^4)*f^4*g^6 - 4*(3*a^3*c*d^2*e^3 - a^4*e^5)*f^
3*g^7)*x^4 + 5*(3*c^4*d^4*e*f^8*g^2 + 4*a^4*d*e^4*f^3*g^7 + 4*(c^4*d^5 ...
```

3.750.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**(13/2),x)
```

output
```
Timed out
```

3.750.7 Maxima [F]

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx = \int \frac{\left(cdex^2 + ade + (cd^2 + ae^2)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{13}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1
3/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)^(13/2)), x)
```

3.750.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2715 vs. $2(235) = 470$.

Time = 1.69 (sec) , antiderivative size = 2715, normalized size of antiderivative = 10.17

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1
3/2),x, algorithm="giac")
```

output
```
-2/1155*(231*sqrt(-c*d^2*e + a*e^3)*c^5*d^7*e^3*f^3*abs(c)*abs(d) - 462*sq
rt(-c*d^2*e + a*e^3)*a*c^4*d^5*e^5*f^3*abs(c)*abs(d) + 231*sqrt(-c*d^2*e +
 a*e^3)*a^2*c^3*d^3*e^7*f^3*abs(c)*abs(d) - 198*sqrt(-c*d^2*e + a*e^3)*c^5
*d^8*e^2*f^2*g*abs(c)*abs(d) - 99*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^6*e^4*f^2
*g*abs(c)*abs(d) + 792*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^4*e^6*f^2*g*abs(c)
*abs(d) - 495*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^2*e^8*f^2*g*abs(c)*abs(d) +
 88*sqrt(-c*d^2*e + a*e^3)*c^5*d^9*e*f*g^2*abs(c)*abs(d) + 44*sqrt(-c*d^2*
e + a*e^3)*a*c^4*d^7*e^3*f*g^2*abs(c)*abs(d) + 33*sqrt(-c*d^2*e + a*e^3)*a
^2*c^3*d^5*e^5*f*g^2*abs(c)*abs(d) - 550*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^
3*e^7*f*g^2*abs(c)*abs(d) + 385*sqrt(-c*d^2*e + a*e^3)*a^4*c*d*e^9*f*g^2*a
bs(c)*abs(d) - 16*sqrt(-c*d^2*e + a*e^3)*c^5*d^10*g^3*abs(c)*abs(d) - 8*sq
rt(-c*d^2*e + a*e^3)*a*c^4*d^8*e^2*g^3*abs(c)*abs(d) - 6*sqrt(-c*d^2*e + a
*e^3)*a^2*c^3*d^6*e^4*g^3*abs(c)*abs(d) - 5*sqrt(-c*d^2*e + a*e^3)*a^3*c^2
*d^4*e^6*g^3*abs(c)*abs(d) + 140*sqrt(-c*d^2*e + a*e^3)*a^4*c*d^2*e^8*g^3*
abs(c)*abs(d) - 105*sqrt(-c*d^2*e + a*e^3)*a^5*e^10*g^3*abs(c)*abs(d))/(sq
rt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^4*d^4*e^5*f^9 - 5*sqrt(c^2*d^2*e^2*f - c
^2*d^3*e*g)*c^4*d^5*e^4*f^8*g - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^3*
d^3*e^6*f^8*g + 10*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^4*d^6*e^3*f^7*g^2 +
 20*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^3*d^4*e^5*f^7*g^2 + 6*sqrt(c^2*d
^2*e^2*f - c^2*d^3*e*g)*a^2*c^2*d^2*e^7*f^7*g^2 - 10*sqrt(c^2*d^2*e^2*f...
```

3.750.9 Mupad [B] (verification not implemented)

Time = 13.89 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.94

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx =$$

$$-\frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}\left(\frac{210\,a^5\,e^5\,g^3 - 770\,a^4\,c\,d\,e^4\,f\,g^2 + 990\,a^3\,c^2\,d^2\,e^3\,f^2\,g - 462\,a^2\,c^3\,d^3\,e^2\,f^3}{1155\,g^5\,(a\,e\,g - c\,d\,f)^4} - \frac{x^2\,(-10\,a^3\,c^2\,d^2\,e^3\,g^3 \cdots}{\cdots}\right.}{x^5\,\sqrt{f+g\,x}\,\sqrt{d+e\,x} + \frac{f^5 \cdots}{\cdots}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(13/2)*(d + e
*x)^(3/2)),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((210*a^5*e^5*g^3 - 462*a^
2*c^3*d^3*e^2*f^3 + 990*a^3*c^2*d^2*e^3*f^2*g - 770*a^4*c*d*e^4*f*g^2)/(11
55*g^5*(a*e*g - c*d*f)^4) - (x^2*(462*c^5*d^5*f^3 - 10*a^3*c^2*d^2*e^3*g^3
 + 66*a^2*c^3*d^3*e^2*f*g^2 - 198*a*c^4*d^4*e*f^2*g))/(1155*g^5*(a*e*g - c
*d*f)^4) - (32*c^5*d^5*x^5)/(1155*g^2*(a*e*g - c*d*f)^4) - (4*c^3*d^3*x^3*
(3*a^2*e^2*g^2 + 99*c^2*d^2*f^2 - 22*a*c*d*e*f*g))/(1155*g^4*(a*e*g - c*d*
f)^4) + (16*c^4*d^4*x^4*(a*e*g - 11*c*d*f))/(1155*g^3*(a*e*g - c*d*f)^4) +
 (4*a*c*d*e*x*(70*a^3*e^3*g^3 - 231*c^3*d^3*f^3 + 396*a*c^2*d^2*e*f^2*g -
275*a^2*c*d*e^2*f*g^2))/(1155*g^5*(a*e*g - c*d*f)^4)))/(x^5*(f + g*x)^(1/2
)*(d + e*x)^(1/2) + (f^5*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^5 + (5*f*x^4*(
f + g*x)^(1/2)*(d + e*x)^(1/2))/g + (5*f^4*x*(f + g*x)^(1/2)*(d + e*x)^(1/
2))/g^4 + (10*f^2*x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (10*f^3*x^2*(
f + g*x)^(1/2)*(d + e*x)^(1/2))/g^3)
```

3.750. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx$

3.751 $\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

3.751.1 Optimal result

Integrand size = 48, antiderivative size = 448

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = -\frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}} - \frac{(cdf-aeg)^3(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{(cdf-aeg)^2(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}} - \frac{(cdf-aeg)(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}} + \frac{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} - \frac{3(cdf-aeg)^5\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{128c^{5/2}d^{5/2}g^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
-1/8*(-a*e*g+c*d*f)*(g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/
g^2/(e*x+d)^(3/2)+1/5*(g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2
)/g/(e*x+d)^(5/2)-3/128*(-a*e*g+c*d*f)^5*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)
/c^(1/2)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^
(5/2)/g^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/64*(-a*e*g+c*d*f)^
3*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g^3/(e*x+d)^(1
/2)+1/16*(-a*e*g+c*d*f)^2*(g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)/g^3/(e*x+d)^(1/2)-3/128*(-a*e*g+c*d*f)^4*(g*x+f)^(1/2)*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/g^3/(e*x+d)^(1/2)
```

3.751.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.68

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}(-15a^4e^4g^4+10a^3cde\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/
(d + e*x)^(5/2),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[f + g*x]*(
-15*a^4*e^4*g^4 + 10*a^3*c*d*e^3*g^3*(7*f + g*x) + 2*a^2*c^2*d^2*e^2*g^2*(
64*f^2 + 233*f*g*x + 124*g^2*x^2) + 2*a*c^3*d^3*e*g*(-35*f^3 + 23*f^2*g*x
+ 256*f*g^2*x^2 + 168*g^3*x^3) + c^4*d^4*(15*f^4 - 10*f^3*g*x + 8*f^2*g^2*
x^2 + 176*f*g^3*x^3 + 128*g^4*x^4)))/(a*e + c*d*x)^2 - (15*(c*d*f - a*e*g)
^5*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(
a*e + c*d*x)^(5/2)))/(640*c^(5/2)*d^(5/2)*g^(7/2)*(d + e*x)^(5/2))
```

3.751.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1250, 1250, 1250, 1253, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{1250}$$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-\frac{(cdf-aeg)\int \frac{(f+gx)^{3/2}\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx}{2g}$$

$$\Big\downarrow \text{1250}$$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-\frac{(cdf-aeg)\left(\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\int \frac{(f+gx)^{3/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}dx}{8g}\right)}{2g}$$

$$\Big\downarrow \text{1250}$$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-\frac{(cdf-aeg)\left(\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)^{3/}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)}}}{6g}\right.}{8g}\right.}{2g}$$

$$\Big\downarrow \text{1253}$$

3.751. $\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}dx$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-$$

$$\frac{(cdf-aeg)\left(\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\int\frac{\sqrt{\cdots}}{\sqrt{cdex^2\cdots}}}{4\cdots}\cdots\right)}{\cdots}\right)}{8g\cdots}\right)}{2g\cdots}$$

$\downarrow$ 1253

3.751. $\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-$$

$$\frac{(cdf-aeg)\left(\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{(cdf-ae}{}\right.}{}\right.}{}\right)}{}\right)}{2g}$$

$\downarrow$ 1268

3.751. $\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-$$

$$\frac{(cdf-aeg)\left(\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{2g}$$

$\downarrow$ 66

3.751. $\displaystyle\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} -$$

$$\frac{(cdf-aeg)\left(\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}} - \frac{3(cdf-aeg)\left(\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}} - \frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{\ }}{\ }\right.}{\ }\right.}{\ }\right.}{\ }\right.}{2g}$$

↓ 221

3.751. $\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-$$

$$\frac{(cdf-aeg)\left(\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{}}{c}\right.\right.}{}\right.}{}\right.}{2g}$$

input
```
Int[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e
*x)^(5/2),x]
```

output
```
((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*g*(d +
e*x)^(5/2)) - ((c*d*f - a*e*g)*(((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(3/2))/(4*g*(d + e*x)^(3/2)) - (3*(c*d*f - a*e*g)*(((f + g*
x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*g*Sqrt[d + e*x])
- ((c*d*f - a*e*g)*(((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(2*c*d*Sqrt[d + e*x]) + (3*(c*d*f - a*e*g)*((Sqrt[f + g*x]*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x]) + ((c*d*f - a*e*
g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sq
rt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])))/(4*c*d)))/(6*g)))/(8*g)))/(2*g)
```

3.751. $\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

3.751.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.751. $\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

3.751.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1004 vs. 2(384) = 768.

Time = 0.62 (sec) , antiderivative size = 1005, normalized size of antiderivative = 2.24

method	result	size
default	Expression too large to display	1005

input
```
int((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
method=_RETURNVERBOSE)
```

output
```
1/1280*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(256*c^4*d^4*g^4*x^4*((g*
x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)+672*a*c^3*d^3*e*g^4*x^3*((g*x+f)*(c*
d*x+a*e))^(1/2)*(c*d*g)^(1/2)+352*c^4*d^4*f*g^3*x^3*((g*x+f)*(c*d*x+a*e))^
(1/2)*(c*d*g)^(1/2)+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e
))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^5*e^5*g^5-75*ln(1/2*(2*c*d*g*x+a*
e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^4*
c*d*e^4*f*g^4+150*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1
/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*c^2*d^2*e^3*f^2*g^3-150*ln(1/2*(2*c*
d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/
2))*a^2*c^3*d^3*e^2*f^3*g^2+75*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c
*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^4*d^4*e*f^4*g-15*ln(1/2
*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*
g)^(1/2))*c^5*d^5*f^5+496*a^2*c^2*d^2*e^2*g^4*x^2*((g*x+f)*(c*d*x+a*e))^(1
/2)*(c*d*g)^(1/2)+1024*a*c^3*d^3*e*f*g^3*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(
c*d*g)^(1/2)+16*c^4*d^4*f^2*g^2*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1
/2)+20*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^3*c*d*e^3*g^4*x+932*((g
*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*c^2*d^2*e^2*f*g^3*x+92*((g*x+f)
*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c^3*d^3*e*f^2*g^2*x-20*((g*x+f)*(c*d*x
+a*e))^(1/2)*(c*d*g)^(1/2)*c^4*d^4*f^3*g*x-30*((g*x+f)*(c*d*x+a*e))^(1/2)*
(c*d*g)^(1/2)*a^4*e^4*g^4+140*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)...
```

3.751.5 Fricas [A] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 1331, normalized size of antiderivative = 2.97

$$\int \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5
/2),x, algorithm="fricas")
```

output
```
[1/2560*(4*(128*c^5*d^5*g^5*x^4 + 15*c^5*d^5*f^4*g - 70*a*c^4*d^4*e*f^3*g^
2 + 128*a^2*c^3*d^3*e^2*f^2*g^3 + 70*a^3*c^2*d^2*e^3*f*g^4 - 15*a^4*c*d*e^
4*g^5 + 16*(11*c^5*d^5*f*g^4 + 21*a*c^4*d^4*e*g^5)*x^3 + 8*(c^5*d^5*f^2*g^
3 + 64*a*c^4*d^4*e*f*g^4 + 31*a^2*c^3*d^3*e^2*g^5)*x^2 - 2*(5*c^5*d^5*f^3*
g^2 - 23*a*c^4*d^4*e*f^2*g^3 - 233*a^2*c^3*d^3*e^2*f*g^4 - 5*a^3*c^2*d^2*e
^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(
g*x + f) - 15*(c^5*d^6*f^5 - 5*a*c^4*d^5*e*f^4*g + 10*a^2*c^3*d^4*e^2*f^3*
g^2 - 10*a^3*c^2*d^3*e^3*f^2*g^3 + 5*a^4*c*d^2*e^4*f*g^4 - a^5*d*e^5*g^5 +
 (c^5*d^5*e*f^5 - 5*a*c^4*d^4*e^2*f^4*g + 10*a^2*c^3*d^3*e^3*f^3*g^2 - 10*
a^3*c^2*d^2*e^4*f^2*g^3 + 5*a^4*c*d*e^5*f*g^4 - a^5*e^6*g^5)*x)*sqrt(c*d*g
)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^
2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e
*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3
+ a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g +
 (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d^3*e*g^4*x + c^3*d^4*g^
4), 1/1280*(2*(128*c^5*d^5*g^5*x^4 + 15*c^5*d^5*f^4*g - 70*a*c^4*d^4*e*f^3
*g^2 + 128*a^2*c^3*d^3*e^2*f^2*g^3 + 70*a^3*c^2*d^2*e^3*f*g^4 - 15*a^4*c*d
*e^4*g^5 + 16*(11*c^5*d^5*f*g^4 + 21*a*c^4*d^4*e*g^5)*x^3 + 8*(c^5*d^5*f^2
*g^3 + 64*a*c^4*d^4*e*f*g^4 + 31*a^2*c^3*d^3*e^2*g^5)*x^2 - 2*(5*c^5*d^5*f
^3*g^2 - 23*a*c^4*d^4*e*f^2*g^3 - 233*a^2*c^3*d^3*e^2*f*g^4 - 5*a^3*c^2...
```

3.751.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+
d)**(5/2),x)
```

output
```
Timed out
```

3.751.7 Maxima [F]

$$\int \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(gx+f)^{\frac{3}{2}}}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g*x + f)^(3/2)/(e
*x + d)^(5/2), x)
```

3.751.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18597 vs. $2(384) = 768$.

Time = 5.44 (sec) , antiderivative size = 18597, normalized size of antiderivative = 41.51

$$\int \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5
/2),x, algorithm="giac")
```

output
```
1/1920*(480*a^2*f*((4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*
x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f +
 (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g
^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d
*e*g))*e*f*abs(g)/g^2 - 4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f +
 (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2
*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e
^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g
 - d*e*g))*d*abs(g)/g + (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d
)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^
2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g -
 d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 +
 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*
e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g
- d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*abs(g)/(e*g^2))/g - (c^2*d^2*e^3*f^2*
g*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*
e^3*g^2))) - 2*a*c*d*e^4*f*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*
d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + a^2*e^5*g^3*abs(g)*log(abs(-sqrt
(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + sqrt(-c*d
^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d*e*f*abs(g) - ...
```

3.751.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\int \frac{(f+g\,x)^{3/2}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{(d+e\,x)^{5/2}}\,dx$$

input
```
int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e
*x)^(5/2),x)
```

output
```
int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e
*x)^(5/2), x)
```

3.752 $\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

3.752.1 Optimal result

Integrand size = 48, antiderivative size = 376

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}} + \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}} - \frac{5(cdf-aeg)^4\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{3/2}d^{3/2}g^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
-5/24*(-a*e*g+c*d*f)*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
/g^2/(e*x+d)^(3/2)+1/4*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2)/g/(e*x+d)^(5/2)-5/64*(-a*e*g+c*d*f)^4*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)
/c^(1/2)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(3/2)/d^
(3/2)/g^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5/32*(-a*e*g+c*d*f)^
2*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)-
5/64*(-a*e*g+c*d*f)^3*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)/c/d/g^3/(e*x+d)^(1/2)
```

3.752.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}(15a^3e^3g^3+a^2cde^2g^2(73f}{}\right.}{}$$

input
```
Integrate[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d
 + e*x)^(5/2),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[f + g*x]*(
15*a^3*e^3*g^3 + a^2*c*d*e^2*g^2*(73*f + 118*g*x) + a*c^2*d^2*e*g*(-55*f^2
 + 36*f*g*x + 136*g^2*x^2) + c^3*d^3*(15*f^3 - 10*f^2*g*x + 8*f*g^2*x^2 +
48*g^3*x^3)))/(a*e + c*d*x)^2 - (15*(c*d*f - a*e*g)^4*ArcTanh[(Sqrt[c]*Sqr
t[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(a*e + c*d*x)^(5/2)))/(1
92*c^(3/2)*d^(3/2)*g^(7/2)*(d + e*x)^(5/2))
```

3.752.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {1250, 1250, 1250, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{1250}$$

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}}-\frac{5(cdf-aeg)\int \frac{\sqrt{f+gx}\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx}{8g}$$

$$\Big\downarrow \text{1250}$$

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}}-\frac{5(cdf-aeg)\left(\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\int \frac{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}dx}{2g}\right)}{8g}$$

$\downarrow$ 1250

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}}-\frac{5(cdf-aeg)\left(\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)\cdots}}\cdots}{4g}\cdots\right)}{2g}\cdots\right)}{8g}$$

$\downarrow$ 1253

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}}-\frac{5(cdf-aeg)\left(\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{(cdf-aeg)\int \frac{\cdots}{\sqrt{f+gx}\sqrt{\cdots}}\cdots}{\cdots}\cdots\right)}{\cdots}\right)}{2g}\cdots\right)}{8g}$$

$\downarrow$ 1268

3.752. $\displaystyle\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}}-$$

$$\frac{5(cdf-aeg)\left(\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-\cdots}{2cd\sqrt{x\left(ae^2+\cdots\right.}}\right.}{\cdots}\right.}{2g}\right.}{8g}$$

$\downarrow$ 66

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}}-$$

$$\frac{5(cdf-aeg)\left(\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-\cdots}{cd\sqrt{x\left(a\cdots\right.}}\right.}{\cdots}\right.}{2g}\right.}{8g}$$

$\downarrow$ 221

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}}-$$

$$\frac{5(cdf-aeg)\left(\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-\cdots}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{\cdots}}\right.}{\cdots}\right.}{2g}\right.}{8g}$$

3.752. $\displaystyle\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

input
```
Int[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x
)^(5/2),x]
```

output
```
((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(4*g*(d +
e*x)^(5/2)) - (5*(c*d*f - a*e*g)*(((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e*x)^(3/2)) - ((c*d*f - a*e*g)*(((f + g*
x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*g*Sqrt[d + e*x])
- ((c*d*f - a*e*g)*((Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(c*d*Sqrt[d + e*x]) + ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*
x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(
c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(4*
g)))/(2*g)))/(8*g)
```

3.752.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.752.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(320) = 640$.

Time = 0.59 (sec) , antiderivative size = 732, normalized size of antiderivative = 1.95

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(-96c^3d^3g^3x^3\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}+15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^4e^4g^4-60\right.}{}$

input
```
int((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
method=_RETURNVERBOSE)
```

output
```
-1/384*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-96*c^3*d^3*g^3*x^3*((g*
x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(
(g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^4*e^4*g^4-60*ln
(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(
c*d*g)^(1/2))*a^3*c*d*e^3*f*g^3+90*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f
)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c^2*d^2*e^2*f^2*g^2
-60*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1
/2))/(c*d*g)^(1/2))*a*c^3*d^3*e*f^3*g+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(
(g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^4*d^4*f^4-272*a
*c^2*d^2*e*g^3*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)-16*c^3*d^3*f*
g^2*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)-236*((g*x+f)*(c*d*x+a*e)
)^(1/2)*(c*d*g)^(1/2)*a^2*c*d*e^2*g^3*x-72*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*
d*g)^(1/2)*a*c^2*d^2*e*f*g^2*x+20*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2
)*c^3*d^3*f^2*g*x-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^3*e^3*g^3
-146*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*c*d*e^2*f*g^2+110*((g*x
+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c^2*d^2*e*f^2*g-30*((g*x+f)*(c*d*x+
a*e))^(1/2)*(c*d*g)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/c/d/((g*x+f)*(c*d*x+a
*e))^(1/2)/g^3/(c*d*g)^(1/2)
```

3.752.5 Fricas [A] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 1065, normalized size of antiderivative = 2.83

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \left[\frac{4\left(48\,c^4d^4g^4x^3+15\,c^4d^4f^3g-55\,ac^3d^3ef^2g^2+73\,a^2c^2d^2\right.}{}\right.$$

input
```
integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5
/2),x, algorithm="fricas")
```

output
```
[1/768*(4*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g - 55*a*c^3*d^3*e*f^2*g^2
+ 73*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4 + 8*(c^4*d^4*f*g^3 + 17*a*
c^3*d^3*e*g^4)*x^2 - 2*(5*c^4*d^4*f^2*g^2 - 18*a*c^3*d^3*e*f*g^3 - 59*a^2*
c^2*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x +
 d)*sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*
e^2*f^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a
*c^3*d^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4
*e^5*g^4)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d
^2*e*f*g + a^2*d*e^2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(
2*c*d*g*x + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^
2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d
^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d
^2*e*g^4*x + c^2*d^3*g^4), 1/384*(2*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g
 - 55*a*c^3*d^3*e*f^2*g^2 + 73*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4
+ 8*(c^4*d^4*f*g^3 + 17*a*c^3*d^3*e*g^4)*x^2 - 2*(5*c^4*d^4*f^2*g^2 - 18*a
*c^3*d^3*e*f*g^3 - 59*a^2*c^2*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^
4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*
g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 -
 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^4)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e...
```

3.752.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+
d)**(5/2),x)
```

output
```
Timed out
```

3.752.7 Maxima [F]

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}\sqrt{gx+f}}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*sqrt(g*x + f)/(e*x
 + d)^(5/2), x)
```

3.752.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6756 vs. 2(320) = 640.

Time = 2.20 (sec) , antiderivative size = 6756, normalized size of antiderivative = 17.97

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5
/2),x, algorithm="giac")
```

output
```
1/192*(48*a^2*((4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x +
d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*
x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 +
 (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g
))*e*f*abs(g)/g^2 - 4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*
x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f +
 (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g
^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d
*e*g))*d*abs(g)/g + (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*
g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f
- 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e
*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a
*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)
*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*
e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*abs(g)/(e*g^2))/g - (c^2*d^2*e^3*f^2*g*ab
s(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*
g^2))) - 2*a*c*d*e^4*f*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g)
 + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + a^2*e^5*g^3*abs(g)*log(abs(-sqrt(e^2
*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + sqrt(-c*d^2*e
*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d*e*f*abs(g) - 2*sq...
```

3.752.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\int \frac{\sqrt{f+g\,x}\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{{\left(d+e\,x\right)}^{5/2}}\,dx$$

input
```
int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e
*x)^(5/2),x)
```

output
```
int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e
*x)^(5/2), x)
```

3.753 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx$

3.753.1 Optimal result

Integrand size = 48, antiderivative size = 304

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}} + \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}} - \frac{5(cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8\sqrt{c}\sqrt{d}g^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
-5/12*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*(g*x+f)^(1/2)
/g^2/(e*x+d)^(3/2)+1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*(g*x+f)^(1/
2)/g/(e*x+d)^(5/2)-5/8*(-a*e*g+c*d*f)^3*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/
c^(1/2)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(7/2)/c^(
1/2)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5/8*(-a*e*g+c*d*f)^2*
(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

3.753.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.62

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = \frac{((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{g}\sqrt{f+gx}(33a^2e^2g^2+2acdeg(-20f+13gx)+c^2d^2(15f}{(ae+cdx)^2}\right.}{24g^{7/2}(d+ex)^{5/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*S
qrt[f + g*x]),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[g]*Sqrt[f + g*x]*(33*a^2*e^2*g^2 +
 2*a*c*d*e*g*(-20*f + 13*g*x) + c^2*d^2*(15*f^2 - 10*f*g*x + 8*g^2*x^2)))/
(a*e + c*d*x)^2 - (15*(c*d*f - a*e*g)^3*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f +
g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(Sqrt[c]*Sqrt[d]*(a*e + c*d*x)^(5/2)))
)/(24*g^(7/2)*(d + e*x)^(5/2))
```

3.753.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1250, 1250, 1250, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}dx$$

$$\Big\downarrow \; 1250$$

$$\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}}-\frac{5(cdf-aeg)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}dx}{6g}$$

$$\Big\downarrow \; 1250$$

$$\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}}-\frac{5(cdf-aeg)\left(\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\int\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}\sqrt{f+gx}}dx}{4g}\right)}{6g}$$

$\downarrow$ 1250

$$\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}}-\frac{5(cdf-aeg)\left(\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2 \cdots}}}{2g}\right.}{4g}\right.}{6g}$$

$\downarrow$ 1268

$$\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}}-\frac{5(cdf-aeg)\left(\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{\sqrt{ae+cdx}\cdots}}{2g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right.}{4g}\right.}{6g}$$

$\downarrow$ 66

$$\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}}-\frac{5(cdf-aeg)\left(\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+\cdots}{f+\cdots}}}{g\sqrt{x\left(ae^2+cd^2\right)+ade+cde\cdots}}\right.}{4g}\right.}{6g}$$

$\downarrow$ 221

3.753. $\displaystyle\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx$

$$\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}}-\frac{5(cdf-aeg)\left(\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\right.}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade\cdots}}\right.}{4g}\right.}{6g}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*Sqrt[f
+ g*x]),x]
```

output
```
(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(3*g*(d + e*
x)^(5/2)) - (5*(c*d*f - a*e*g)*((Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2))/(2*g*(d + e*x)^(3/2)) - (3*(c*d*f - a*e*g)*((Sqrt[f +
g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]) - ((c*
d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c
*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*g^(3/2)*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(4*g)))/(6*g)
```

3.753.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.753.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.64

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\,\sqrt{gx+f}\,\left(15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^{3}e^{3}g^{3}-45\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right.\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
1/48*((c*d*x+a*e)*(e*x+d))^(1/2)*(g*x+f)^(1/2)*(15*ln(1/2*(2*c*d*g*x+a*e*g
+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*e^3
*g^3-45*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g
)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2+45*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f
+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f
^2*g-15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g
)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^3+16*c^2*d^2*g^2*x^2*(c*d*g)^(1/2)*((g*x
+f)*(c*d*x+a*e))^(1/2)+52*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*c*d*
e*g^2*x-20*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c^2*d^2*f*g*x+66*((g*
x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2-80*((g*x+f)*(c*d*x+a*e))
^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g+30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1
/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/g^3/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/
2)
```

3.753.5 Fricas [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 837, normalized size of antiderivative = 2.75

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}\sqrt{f + gx}}\,dx = \left[\frac{4\,(8\,c^3d^3g^3x^2 + 15\,c^3d^3f^2g - 40\,ac^2d^2efg^2 + 33\,a^2cde^2g^3 - 2\,(5\,c\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
/2),x, algorithm="fricas")
```

output
```
[1/96*(4*(8*c^3*d^3*g^3*x^2 + 15*c^3*d^3*f^2*g - 40*a*c^2*d^2*e*f*g^2 + 33
*a^2*c*d*e^2*g^3 - 2*(5*c^3*d^3*f*g^2 - 13*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^3*d^4
*f^3 - 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*
d^3*e*f^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*
sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^
2*d*e^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c
*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g +
 (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*
e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c*d*e*g^4*x + c*d^
2*g^4), 1/48*(2*(8*c^3*d^3*g^3*x^2 + 15*c^3*d^3*f^2*g - 40*a*c^2*d^2*e*f*g
^2 + 33*a^2*c*d*e^2*g^3 - 2*(5*c^3*d^3*f*g^2 - 13*a*c^2*d^2*e*g^3)*x)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(
c^3*d^4*f^3 - 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3
+ (c^3*d^3*e*f^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g
^3)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g
 + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c*d*e*g^4*x + c*d^2*g^4)]
```

3.753.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}\sqrt{f + gx}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**(1/2),x)
```

output
```
Timed out
```

3.753.7 Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*s
qrt(g*x + f)), x)
```

3.753.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. $2(256) = 512$.

Time = 0.69 (sec) , antiderivative size = 1098, normalized size of antiderivative = 3.61

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
/2),x, algorithm="giac")
```

output
```
1/24*e*((sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a
*e^3)*c*d*g)*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(2*((e*x + d)*c*d*e -
 c*d^2*e + a*e^3)*(4*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*abs(e)/(c*d*e^3*g
) - 5*(c*d*e^2*f*g^3*abs(e) - a*e^3*g^4*abs(e))/(c*d*e^3*g^5)) + 15*(c^2*d
^2*e^4*f^2*g^2*abs(e) - 2*a*c*d*e^5*f*g^3*abs(e) + a^2*e^6*g^4*abs(e))/(c*
d*e^3*g^5)) + 15*(c^3*d^3*e^3*f^3*abs(e) - 3*a*c^2*d^2*e^4*f^2*g*abs(e) +
3*a^2*c*d*e^5*f*g^2*abs(e) - a^3*e^6*g^3*abs(e))*log(abs(-sqrt((e*x + d)*c
*d*e - c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + (
(e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)))/(sqrt(c*d*g)*g^3))*abs(c)*abs(
d)/(c*d*e^4) - (15*c^4*d^4*e^4*f^3*abs(c)*abs(d)*abs(e)*log(abs(-sqrt(-c*d
^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))) - 45*a*c^3
*d^3*e^5*f^2*g*abs(c)*abs(d)*abs(e)*log(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c
*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))) + 45*a^2*c^2*d^2*e^6*f*g^2*abs
(c)*abs(d)*abs(e)*log(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d
^2*e^2*f - c^2*d^3*e*g))) - 15*a^3*c*d*e^7*g^3*abs(c)*abs(d)*abs(e)*log(ab
s(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))
) + 15*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g
)*c^2*d^2*e^2*f^2*abs(c)*abs(d)*abs(e) + 10*sqrt(c^2*d^2*e^2*f - c^2*d^3*e
*g)*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*c^2*d^3*e*f*g*abs(c)*abs(d)*abs(e)
- 40*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*...
```

3.753.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{\sqrt{f+g\,x}\,\left(d+e\,x\right)^{5/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(1/2)*(d + e*
x)^(5/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(1/2)*(d + e*
x)^(5/2)), x)
```

3.754 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx$

3.754.1 Optimal result

Integrand size = 48, antiderivative size = 294

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx = -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}} + \frac{5cd\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g^2(d+ex)^{3/2}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}} + \frac{15\sqrt{c}\sqrt{d}(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4g^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
-2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^(1/2)+5
/2*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*(g*x+f)^(1/2)/g^2/(e*x+d)^(
3/2)+15/4*(-a*e*g+c*d*f)^2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/
2)/(g*x+f)^(1/2))*c^(1/2)*d^(1/2)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(7/2)/
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-15/4*c*d*(-a*e*g+c*d*f)*(g*x+f)^(1
/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

3.754.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.68

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx = \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}(-8a^2e^2g^2+acdeg(25f+9gx)\right.}{4g^{7/2}\sqrt{}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^(3/2)),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(-8*a^2*e^2*g^
2 + a*c*d*e*g*(25*f + 9*g*x) + c^2*d^2*(-15*f^2 - 5*f*g*x + 2*g^2*x^2)) +
15*Sqrt[c]*Sqrt[d]*(c*d*f - a*e*g)^2*Sqrt[f + g*x]*ArcTanh[(Sqrt[c]*Sqrt[d
]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(4*g^(7/2)*Sqrt[(a*e + c*d
*x)*(d + e*x)]*Sqrt[f + g*x])
```

3.754.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1249, 1250, 1250, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{5cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}dx}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}$$

$$\Big\downarrow \text{1250}$$

$$\frac{5cd\left(\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}\sqrt{f+gx}}dx}{4g}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}$$

$\Big\downarrow$ 1250

$$\frac{5cd\left(\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2g}\right)}{4g}\right)}{g}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}$$

$\Big\downarrow$ 1268

$$\frac{5cd\left(\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{4g}\right)}{g}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}$$

$\Big\downarrow$ 66

$$\frac{5cd\left(\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{4g}\right)}{g}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}$$

$\Big\downarrow$ 221

$$\frac{5cd\left(\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{4g}\right)}{g}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}$$

3.754. $\displaystyle\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^(3/2)),x]
```

output
```
(-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(g*(d + e*x)^(5/2)*Sqrt
[f + g*x]) + (5*c*d*((Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2))/(2*g*(d + e*x)^(3/2)) - (3*(c*d*f - a*e*g)*((Sqrt[f + g*x]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]) - ((c*d*f - a*e*g
)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqr
t[c]*Sqrt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*g^(3/2)*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])))/(4*g)))/g
```

3.754.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.754.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(248) = 496$.

Time = 0.57 (sec) , antiderivative size = 625, normalized size of antiderivative = 2.13

method	result
default	$\frac{\left(15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^2cd\,e^2g^3x-30\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a\,c^2d^2ef\,g^2x+15\ln\left(\right.\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/2),x,
method=_RETURNVERBOSE)
```

output
```
1/8*(15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g
)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*g^3*x-30*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f
+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f
*g^2*x+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d
*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^2*g*x+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f
+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f
*g^2-30*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g
)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2*g+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f
+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^3+4
*c^2*d^2*g^2*x^2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)+18*(c*d*g)^(1/2
)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x-10*(c*d*g)^(1/2)*((g*x+f)*(c*d
*x+a*e))^(1/2)*c^2*d^2*f*g*x-16*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*
a^2*e^2*g^2+50*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g-30*((
g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*((c*d*x+a*e)*(e*x+d))
^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)/g^3/(g*x+f)^(1/2)/(e*x+d)
^(1/2)
```

3.754.5 Fricas [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 915, normalized size of antiderivative = 3.11

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{3/2}}\,dx = \left[\frac{4\,(2\,c^2d^2g^2x^2 - 15\,c^2d^2f^2 + 25\,acdefg - 8\,a^2e^2g^2 - (5\,c^2d^2fg - 9\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3
/2),x, algorithm="fricas")
```

output
```
[1/16*(4*(2*c^2*d^2*g^2*x^2 - 15*c^2*d^2*f^2 + 25*a*c*d*e*f*g - 8*a^2*e^2*
g^2 - (5*c^2*d^2*f*g - 9*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^2*d^3*f^3 - 2*a*c*d^2*e*f^2
*g + a^2*d*e^2*f*g^2 + (c^2*d^2*e*f^2*g - 2*a*c*d*e^2*f*g^2 + a^2*e^3*g^3)
*x^2 + (c^2*d^2*e*f^3 + a^2*d*e^2*g^3 + (c^2*d^3 - 2*a*c*d*e^2)*f^2*g - (2
*a*c*d^2*e - a^2*e^3)*f*g^2)*x)*sqrt(c*d/g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^
2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*d*g^2*x + c*d*f*g + a
*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x
 + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c
^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g
^2)*x)/(e*x + d)))/(e*g^4*x^2 + d*f*g^3 + (e*f*g^3 + d*g^4)*x), 1/8*(2*(2*
c^2*d^2*g^2*x^2 - 15*c^2*d^2*f^2 + 25*a*c*d*e*f*g - 8*a^2*e^2*g^2 - (5*c^2
*d^2*f*g - 9*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(e*x + d)*sqrt(g*x + f) - 15*(c^2*d^3*f^3 - 2*a*c*d^2*e*f^2*g + a^2*d*e
^2*f*g^2 + (c^2*d^2*e*f^2*g - 2*a*c*d*e^2*f*g^2 + a^2*e^3*g^3)*x^2 + (c^2*
d^2*e*f^3 + a^2*d*e^2*g^3 + (c^2*d^3 - 2*a*c*d*e^2)*f^2*g - (2*a*c*d^2*e -
 a^2*e^3)*f*g^2)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*
d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e*g^4*x^2 + d*f*g^
3 + (e*f*g^3 + d*g^4)*x)]
```

3.754.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**(3/2),x)
```

output
```
Timed out
```

3.754.7 Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{3/2}}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}(gx + f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^(3/2)), x)
```

3.754.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 894 vs. $2(248) = 496$.

Time = 0.77 (sec) , antiderivative size = 894, normalized size of antiderivative = 3.04

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{3/2}}\,dx = \frac{\sqrt{(ex + d)cde - cd^2e + ae^3}\left(\left(((ex + d)cde - cd^2e + ae^3\right)\left(\frac{2\,((ex+d)c\cdots}{\cdots}\right.\right.}{4\sqrt{c^2d^2e^2f - acd\cdots}}$$
$$- \frac{15\,(c^2d^2f^2|c||d| - 2\,acdefg|c||d| + a^2e^2g^2|c||d|)\log\left(\left|-\sqrt{(ex + d)cde - cd^2e + ae^3}\sqrt{cdg} + \sqrt{c^2d^2e^2f - a\cdots}\right.\right.}{4\sqrt{cdg}g^3}$$
$$+ \frac{15\sqrt{c^2d^2e^2f - c^2d^3eg}c^2d^2e^2f^2|c||d|\log\left(\left|-\sqrt{-cd^2e + ae^3}\sqrt{cdg} + \sqrt{c^2d^2e^2f - c^2d^3eg}\right|\right) - 30\sqrt{c^2d^2e^2f - c\cdots}}{\cdots}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3
/2),x, algorithm="giac")
```

output
```
1/4*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(((e*x + d)*c*d*e - c*d^2*e +
a*e^3)*(2*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*abs(c)*abs(d)/(e^4*g) - 5*(c
*d*e^2*f*g^3*abs(c)*abs(d) - a*e^3*g^4*abs(c)*abs(d))/(e^4*g^5)) - 15*(c^2
*d^2*e^4*f^2*g^2*abs(c)*abs(d) - 2*a*c*d*e^5*f*g^3*abs(c)*abs(d) + a^2*e^6
*g^4*abs(c)*abs(d))/(e^4*g^5))/sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x +
d)*c*d*e - c*d^2*e + a*e^3)*c*d*g) - 15/4*(c^2*d^2*f^2*abs(c)*abs(d) - 2*a
*c*d*e*f*g*abs(c)*abs(d) + a^2*e^2*g^2*abs(c)*abs(d))*log(abs(-sqrt((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - a*c*d*e^3*
g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)))/(sqrt(c*d*g)*g^3) + 1/4*(
15*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^2*d^2*e^2*f^2*abs(c)*abs(d)*log(abs
(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)))
 - 30*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c*d*e^3*f*g*abs(c)*abs(d)*log(ab
s(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))
) + 15*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*e^4*g^2*abs(c)*abs(d)*log(abs
(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)))
 + 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*c^2*d^2*e^2*f^2*abs(c)*abs(d) - 5
*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*c^2*d^3*e*f*g*abs(c)*abs(d) - 25*sqrt(
-c*d^2*e + a*e^3)*sqrt(c*d*g)*a*c*d*e^3*f*g*abs(c)*abs(d) - 2*sqrt(-c*d^2*
e + a*e^3)*sqrt(c*d*g)*c^2*d^4*g^2*abs(c)*abs(d) + 9*sqrt(-c*d^2*e + a*e^3
)*sqrt(c*d*g)*a*c*d^2*e^2*g^2*abs(c)*abs(d) + 8*sqrt(-c*d^2*e + a*e^3)*...
```

3.754.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{3/2}}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{\left(f + g\,x\right)^{3/2}\,\left(d + e\,x\right)^{5/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(3/2)*(d + e*
x)^(5/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(3/2)*(d + e*
x)^(5/2)), x)
```

3.755 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx$

3.755.1 Optimal result

Integrand size = 48, antiderivative size = 284

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx &= \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}}\\ &- \frac{10cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}\sqrt{f+gx}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}}\\ &- \frac{5c^{3/2}d^{3/2}(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\end{aligned}$$

output
```
-2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^(3/2)
-10/3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f
)^(1/2)-5*c^(3/2)*d^(3/2)*(-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)
/c^(1/2)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(7/2)/(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5*c^2*d^2*(g*x+f)^(1/2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

3.755.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.66

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx = \frac{((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{g}(-2a^2e^2g^2-2acdeg(5f+7gx)+c^2d^2(15f^2+20fgx+}{(ae+cdx)^2(f+gx)^{3/2}}\right.}{3g^{7/2}(d+ex)^{5/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^(5/2)),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[g]*(-2*a^2*e^2*g^2 - 2*a*c*d*e*g*(
5*f + 7*g*x) + c^2*d^2*(15*f^2 + 20*f*g*x + 3*g^2*x^2)))/((a*e + c*d*x)^2*
(f + g*x)^(3/2)) - (15*c^(3/2)*d^(3/2)*(c*d*f - a*e*g)*ArcTanh[(Sqrt[c]*Sq
rt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(a*e + c*d*x)^(5/2)))/(
3*g^(7/2)*(d + e*x)^(5/2))
```

3.755.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1249, 1249, 1250, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}dx$$

$$\Big\downarrow \; 1249$$

$$\frac{5cd\int \frac{\left(cdex^2+(cd^2+ae^2)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}dx}{3g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}}$$

$$\Big\downarrow \; 1249$$

$$\frac{5cd\left(\frac{3cd\int\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}\sqrt{f+gx}}dx}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}\right)}{3g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}}$$

$$\Big\downarrow \; 1250$$

$$\frac{5cd\left(\frac{3cd\left(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2g}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}\right)}{3g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}}$$

$$\Big\downarrow \; 1268$$

$$\frac{5cd\left(\frac{3cd\left(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}\right)}{3g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}}$$

$$\Big\downarrow \; 66$$

$$\frac{5cd\left(\frac{3cd\left(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}\right)}{3g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}}$$

$$\Big\downarrow \; 221$$

3.755. $\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}dx$

$$\frac{5cd\left(\frac{3cd\left(\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{g}-\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}\right)}{3g}-\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^(5/2)),x]
```

output
```
(-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(3*g*(d + e*x)^(5/2)*(f
 + g*x)^(3/2)) + (5*c*d*((-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)
)/(g*(d + e*x)^(3/2)*Sqrt[f + g*x]) + (3*c*d*((Sqrt[f + g*x]*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]) - ((c*d*f - a*e*g)*Sqrt[
a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sq
rt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*g^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])))/g))/(3*g)
```

3.755.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

3.755. $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx$

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.755.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(240) = 480$.

Time = 0.57 (sec) , antiderivative size = 628, normalized size of antiderivative = 2.21

method	result
default	$\frac{\left(15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a\,c^2d^2e\,g^3x^2-15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)c^3d^3f\,g^2x^2+30\ln\left(\cdots\right.\right.}{\cdots}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/2),x,
method=_RETURNVERBOSE)
```

output
```
1/6*(15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g
)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*g^3*x^2-15*ln(1/2*(2*c*d*g*x+a*e*g+c*d
*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f*g
^2*x^2+30*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d
*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f*g^2*x-30*ln(1/2*(2*c*d*g*x+a*e*g+c
*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f
^2*g*x+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d
*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2*g-15*ln(1/2*(2*c*d*g*x+a*e*g+c*d
*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^3
+6*c^2*d^2*g^2*x^2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)-28*(c*d*g)^(1
/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x+40*(c*d*g)^(1/2)*((g*x+f)*(c
*d*x+a*e))^(1/2)*c^2*d^2*f*g*x-4*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)
*a^2*e^2*g^2-20*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g+30*(
(g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*((c*d*x+a*e)*(e*x+d)
)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)/g^3/(g*x+f)^(3/2)/(e*x+d
)^(1/2)
```

3.755.5 Fricas [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 973, normalized size of antiderivative = 3.43

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx = \left[\frac{4\,(3\,c^2d^2g^2x^2+15\,c^2d^2f^2-10\,acdefg-2\,a^2e^2g^2+2\,(10\,c^2d^2fg}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5
/2),x, algorithm="fricas")
```

output
```
[1/12*(4*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 10*a*c*d*e*f*g - 2*a^2*e^2*
g^2 + 2*(10*c^2*d^2*f*g - 7*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^2*d^3*f^3 - a*c*d^2*e*f^
2*g + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x^3 + (2*c^2*d^2*e*f^2*g - a*c*d^2
*e*g^3 + (c^2*d^3 - 2*a*c*d*e^2)*f*g^2)*x^2 + (c^2*d^2*e*f^3 - 2*a*c*d^2*e
*f*g^2 + (2*c^2*d^3 - a*c*d*e^2)*f^2*g)*x)*sqrt(c*d/g)*log(-(8*c^2*d^2*e*g
^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*d*g^2*x +
c*d*f*g + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x +
d)*sqrt(g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^
2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e +
 a^2*e^3)*g^2)*x)/(e*x + d)))/(e*g^5*x^3 + d*f^2*g^3 + (2*e*f*g^4 + d*g^5)
*x^2 + (e*f^2*g^3 + 2*d*f*g^4)*x), 1/6*(2*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*
f^2 - 10*a*c*d*e*f*g - 2*a^2*e^2*g^2 + 2*(10*c^2*d^2*f*g - 7*a*c*d*e*g^2)*
x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)
 + 15*(c^2*d^3*f^3 - a*c*d^2*e*f^2*g + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x
^3 + (2*c^2*d^2*e*f^2*g - a*c*d^2*e*g^3 + (c^2*d^3 - 2*a*c*d*e^2)*f*g^2)*x
^2 + (c^2*d^2*e*f^3 - 2*a*c*d^2*e*f*g^2 + (2*c^2*d^3 - a*c*d*e^2)*f^2*g)*x
)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e
*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g +
(c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e*g^5*x^3 + d*f^2*g^3 + (2*e*f*g^...
```

3.755.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**(5/2),x)
```

output
```
Timed out
```

3.755.7 Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^(5/2)), x)
```

3.755.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1178 vs. $2(240) = 480$.

Time = 1.01 (sec) , antiderivative size = 1178, normalized size of antiderivative = 4.15

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5
/2),x, algorithm="giac")
```

output
```
-1/3*(15*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^2*d^2*e^2*f^2*abs(c)*abs(d)*l
og(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*
e*g))) - 15*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^2*d^3*e*f*g*abs(c)*abs(d)*
log(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3
*e*g))) - 15*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c*d*e^3*f*g*abs(c)*abs(d)
*log(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^
3*e*g))) + 15*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c*d^2*e^2*g^2*abs(c)*abs
(d)*log(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2
*d^3*e*g))) + 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*c^2*d^2*e^2*f^2*abs(c)
*abs(d) - 20*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*c^2*d^3*e*f*g*abs(c)*abs(d
) - 10*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*a*c*d*e^3*f*g*abs(c)*abs(d) + 3*
sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*c^2*d^4*g^2*abs(c)*abs(d) + 14*sqrt(-c*
d^2*e + a*e^3)*sqrt(c*d*g)*a*c*d^2*e^2*g^2*abs(c)*abs(d) - 2*sqrt(-c*d^2*e
 + a*e^3)*sqrt(c*d*g)*a^2*e^4*g^2*abs(c)*abs(d))/(sqrt(c^2*d^2*e^2*f - c^2
*d^3*e*g)*sqrt(c*d*g)*e^2*f*g^3 - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*sqrt(c
*d*g)*d*e*g^4) + 1/3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(((e*x + d)*c
*d*e - c*d^2*e + a*e^3)*(3*(c^4*d^4*e^2*f*g^4*abs(c)*abs(d) - a*c^3*d^3*e^
3*g^5*abs(c)*abs(d))*((e*x + d)*c*d*e - c*d^2*e + a*e^3)/(c^2*d^2*e^4*f*g^
5 - a*c*d*e^5*g^6) + 20*(c^5*d^5*e^4*f^2*g^3*abs(c)*abs(d) - 2*a*c^4*d^4*e
^5*f*g^4*abs(c)*abs(d) + a^2*c^3*d^3*e^6*g^5*abs(c)*abs(d))/(c^2*d^2*e^...
```

3.755.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{5/2}} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\;x + a\,d\,e)^{5/2}}{(f + g\,x)^{5/2}\,(d + e\,x)^{5/2}} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(5/2)*(d + e*
x)^(5/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(5/2)*(d + e*
x)^(5/2)), x)
```

3.756 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx$

3.756.1 Optimal result

Integrand size = 48, antiderivative size = 274

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx=-\frac{2c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}}-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}+\frac{2c^{5/2}d^{5/2}\sqrt{ae+cdx}\sqrt{d+ex}\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
-2/3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)
^(3/2)-2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)
^(5/2)+2*c^(5/2)*d^(5/2)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)
/(g*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(7/2)/(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2)-2*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^
3/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

3.756.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.69

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx = \frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(-\sqrt{g}\sqrt{ae+cdx}(3a^2e^2g^2+acdeg(5f+11gx\right.}{15g^{7/2}\sqrt{(ae}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^(7/2)),x]
```

output
```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-(Sqrt[g]*Sqrt[a*e + c*d*x]*(3*a^2*e^2
*g^2 + a*c*d*e*g*(5*f + 11*g*x) + c^2*d^2*(15*f^2 + 35*f*g*x + 23*g^2*x^2)
)) + 15*c^(5/2)*d^(5/2)*(f + g*x)^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f +
g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(15*g^(7/2)*Sqrt[(a*e + c*d*x)*(d + e
*x)]*(f + g*x)^(5/2))
```

3.756.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1249, 1249, 1249, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}dx$$

$$\Big\downarrow \text{ 1249}$$

$$\frac{cd\int \frac{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}dx}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}$$

$$\Big\downarrow \text{ 1249}$$

$$\frac{cd\left(\frac{cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}(f+gx)^{3/2}}dx}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}$$

$$\Big\downarrow \text{ 1249}$$

$$\frac{cd\left(\frac{cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}$$

$$\Big\downarrow \; 1268$$

$$\frac{cd\left(\frac{cd\left(\frac{cd\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}$$

$$\Big\downarrow \; 66$$

$$\frac{cd\left(\frac{cd\left(\frac{2cd\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{cd\left(\frac{cd\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}$$

3.756. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^(7/2)),x]
```

output
```
(-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*g*(d + e*x)^(5/2)*(f
 + g*x)^(5/2)) + (c*d*((-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/
(3*g*(d + e*x)^(3/2)*(f + g*x)^(3/2)) + (c*d*((-2*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]*Sqrt[f + g*x]) + (2*Sqrt[c]*Sqrt[d]*
Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[
c]*Sqrt[d]*Sqrt[f + g*x])])/(g^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])))/g))/g
```

3.756.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.756.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(230) = 460$.

Time = 0.58 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.83

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\,\left(15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)c^3d^3g^3x^3+45\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)c^3d^3\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/2),x,
method=_RETURNVERBOSE)
```

output
```
1/15*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x
+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*g^3*x^3+45*ln
(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(
c*d*g)^(1/2))*c^3*d^3*f*g^2*x^2+45*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f
)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^2*g*x+15*ln(1
/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*
d*g)^(1/2))*c^3*d^3*f^3-46*c^2*d^2*g^2*x^2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a
*e))^(1/2)-22*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x-70*(
c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c^2*d^2*f*g*x-6*((g*x+f)*(c*d*x+a
*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2-10*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g
)^(1/2)*a*c*d*e*f*g-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f
^2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)/g^3/(g*x+f)^(5/2)/(e*x+d)^(1
/2)
```

3.756.5 Fricas [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 933, normalized size of antiderivative = 3.41

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx = \left[-\frac{4\,(23\,c^2d^2g^2x^2+15\,c^2d^2f^2+5\,acdefg+3\,a^2e^2g^2+(35\,c^2d^2fg\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7
/2),x, algorithm="fricas")
```

output
```
[-1/30*(4*(23*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 + 5*a*c*d*e*f*g + 3*a^2*e^2
*g^2 + (35*c^2*d^2*f*g + 11*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^2*d^2*e*g^3*x^4 + c^2*d^
3*f^3 + (3*c^2*d^2*e*f*g^2 + c^2*d^3*g^3)*x^3 + 3*(c^2*d^2*e*f^2*g + c^2*d
^3*f*g^2)*x^2 + (c^2*d^2*e*f^3 + 3*c^2*d^3*f^2*g)*x)*sqrt(c*d/g)*log(-(8*c
^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*
d*g^2*x + c*d*f*g + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(e*x + d)*sqrt(g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c
*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a
*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(e*g^6*x^4 + d*f^3*g^3 + (3*e*f*g^
5 + d*g^6)*x^3 + 3*(e*f^2*g^4 + d*f*g^5)*x^2 + (e*f^3*g^3 + 3*d*f^2*g^4)*x
), -1/15*(2*(23*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 + 5*a*c*d*e*f*g + 3*a^2*e
^2*g^2 + (35*c^2*d^2*f*g + 11*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^2*d^2*e*g^3*x^4 + c^2*
d^3*f^3 + (3*c^2*d^2*e*f*g^2 + c^2*d^3*g^3)*x^3 + 3*(c^2*d^2*e*f^2*g + c^2
*d^3*f*g^2)*x^2 + (c^2*d^2*e*f^3 + 3*c^2*d^3*f^2*g)*x)*sqrt(-c*d/g)*arctan
(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)
*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 +
 a*e^2)*g)*x)))/(e*g^6*x^4 + d*f^3*g^3 + (3*e*f*g^5 + d*g^6)*x^3 + 3*(e*f^
2*g^4 + d*f*g^5)*x^2 + (e*f^3*g^3 + 3*d*f^2*g^4)*x)]
```

3.756.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**(7/2),x)
```

output
```
Timed out
```

3.756.7 Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{7/2}}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}(gx + f)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^(7/2)), x)
```

3.756.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1224 vs. $2(230) = 460$.

Time = 1.59 (sec) , antiderivative size = 1224, normalized size of antiderivative = 4.47

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7
/2),x, algorithm="giac")
```

output
```
-2*c^2*d^2*abs(c)*abs(d)*log(abs(-sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*
sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*
e + a*e^3)*c*d*g)))/(sqrt(c*d*g)*g^3) + 2/15*(15*sqrt(c^2*d^2*e^2*f - c^2*
d^3*e*g)*c^2*d^2*e^2*f^2*abs(c)*abs(d)*log(abs(-sqrt(-c*d^2*e + a*e^3)*sqr
t(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))) - 30*sqrt(c^2*d^2*e^2*f - c
^2*d^3*e*g)*c^2*d^3*e*f*g*abs(c)*abs(d)*log(abs(-sqrt(-c*d^2*e + a*e^3)*sq
rt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))) + 15*sqrt(c^2*d^2*e^2*f -
c^2*d^3*e*g)*c^2*d^4*g^2*abs(c)*abs(d)*log(abs(-sqrt(-c*d^2*e + a*e^3)*sqr
t(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))) + 15*sqrt(-c*d^2*e + a*e^3)
*sqrt(c*d*g)*c^2*d^2*e^2*f^2*abs(c)*abs(d) - 35*sqrt(-c*d^2*e + a*e^3)*sqr
t(c*d*g)*c^2*d^3*e*f*g*abs(c)*abs(d) + 5*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g
)*a*c*d*e^3*f*g*abs(c)*abs(d) + 23*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*c^2*
d^4*g^2*abs(c)*abs(d) - 11*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*a*c*d^2*e^2*
g^2*abs(c)*abs(d) + 3*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*a^2*e^4*g^2*abs(c
)*abs(d))/(sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*sqrt(c*d*g)*e^2*f^2*g^3 - 2*s
qrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*sqrt(c*d*g)*d*e*f*g^4 + sqrt(c^2*d^2*e^2*
f - c^2*d^3*e*g)*sqrt(c*d*g)*d^2*g^5) - 2/15*sqrt((e*x + d)*c*d*e - c*d^2*
e + a*e^3)*(((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(23*(c^6*d^6*e^4*f^2*g^4*a
bs(c)*abs(d) - 2*a*c^5*d^5*e^5*f*g^5*abs(c)*abs(d) + a^2*c^4*d^4*e^6*g^6*a
bs(c)*abs(d))*((e*x + d)*c*d*e - c*d^2*e + a*e^3)/(c^2*d^2*e^4*f^2*g^5 ...
```

3.756.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{7/2}}\,dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{5/2}}{(f + g\,x)^{7/2}\,(d + e\,x)^{5/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(7/2)*(d + e*
x)^(5/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(7/2)*(d + e*
x)^(5/2)), x)
```

3.757 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx$

3.757.1 Optimal result

Integrand size = 48, antiderivative size = 63

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7(cdf-aeg)(d+ex)^{7/2}(f+gx)^{7/2}}$$

output
```
2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)/(e*x+d)^(7/2)/(
g*x+f)^(7/2)
```

3.757.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{7/2}}{7(cdf-aeg)(d+ex)^{7/2}(f+gx)^{7/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^(9/2)),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(7/2))/(7*(c*d*f - a*e*g)*(d + e*x)^(7/2)*(f
+ g*x)^(7/2))
```

3.757.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx$$

$$\Big\downarrow \ 1248$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^(9/2)),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*(c*d*f - a*e*g)*(d +
e*x)^(7/2)*(f + g*x)^(7/2))
```

3.757.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

3.757.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{2(cdx+ae)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}}{7(gx+f)^{\frac{7}{2}}(aeg-cdf)(ex+d)^{\frac{5}{2}}}$	63
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,\left(c^2d^2x^2+2acdex+e^2a^2\right)(cdx+ae)}{7\sqrt{ex+d}\,(gx+f)^{\frac{7}{2}}(aeg-cdf)}$	78

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/7/(g*x+f)^(7/2)*(c*d*x+a*e)/(a*e*g-c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*
d*e)^(5/2)/(e*x+d)^(5/2)
```

3.757.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(55) = 110.

Time = 0.44 (sec) , antiderivative size = 299, normalized size of antiderivative = 4.75

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{9/2}}\,dx = \frac{}{7\,(cd^2f^5 - adef^4g + (cdefg^4 - ae^2g^5)x^5 + (4\,cdef^2g^3 - adeg^5 + (}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9
/2),x, algorithm="fricas")
```

output
```
2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c*d^2*f^5
 - a*d*e*f^4*g + (c*d*e*f*g^4 - a*e^2*g^5)*x^5 + (4*c*d*e*f^2*g^3 - a*d*e*
g^5 + (c*d^2 - 4*a*e^2)*f*g^4)*x^4 + 2*(3*c*d*e*f^3*g^2 - 2*a*d*e*f*g^4 +
(2*c*d^2 - 3*a*e^2)*f^2*g^3)*x^3 + 2*(2*c*d*e*f^4*g - 3*a*d*e*f^2*g^3 + (3
*c*d^2 - 2*a*e^2)*f^3*g^2)*x^2 + (c*d*e*f^5 - 4*a*d*e*f^3*g^2 + (4*c*d^2 -
 a*e^2)*f^4*g)*x)
```

3.757.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**(9/2),x)
```

output
```
Timed out
```

3.757.7 Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx=\int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^(9/2)), x)
```

3.757.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(55) = 110$.

Time = 0.78 (sec) , antiderivative size = 602, normalized size of antiderivative = 9.56

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx=\frac{2}{7\left(\sqrt{c^2d^2e^2f-c^2d^3eg}cde^3f^4-3\sqrt{c^2d^2e^2f-c^2d^3eg}cd^2e^2f^3g-\sqrt{}\right.}$$
$$+\frac{2\left(c^8d^8e^6f^2g^3|c||d|-2\,ac^7d^7e^7fg^4|c||d|+a^2c^6d^6e^8g^5|c||d|\right)\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{7}{2}}}{7\left(c^3d^3e^6f^3g^3-3\,ac^2d^2e^7f^2g^4+3\,a^2cde^8fg^5-a^3e^9g^6\right)\left(c^2d^2e^2f-acde^3g+\left((ex+d)cde-cd^2e+ae^3\right)cdg\right.}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9
/2),x, algorithm="giac")
```

output
```
2/7*(sqrt(-c*d^2*e + a*e^3)*c^3*d^6*abs(c)*abs(d) - 3*sqrt(-c*d^2*e + a*e^
3)*a*c^2*d^4*e^2*abs(c)*abs(d) + 3*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4*ab
s(c)*abs(d) - sqrt(-c*d^2*e + a*e^3)*a^3*e^6*abs(c)*abs(d))/(sqrt(c^2*d^2*
e^2*f - c^2*d^3*e*g)*c*d*e^3*f^4 - 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c*d
^2*e^2*f^3*g - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*e^4*f^3*g + 3*sqrt(c^2*
d^2*e^2*f - c^2*d^3*e*g)*c*d^3*e*f^2*g^2 + 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*
e*g)*a*d*e^3*f^2*g^2 - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c*d^4*f*g^3 - 3*s
qrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*d^2*e^2*f*g^3 + sqrt(c^2*d^2*e^2*f - c^
2*d^3*e*g)*a*d^3*e*g^4) + 2/7*(c^8*d^8*e^6*f^2*g^3*abs(c)*abs(d) - 2*a*c^7
*d^7*e^7*f*g^4*abs(c)*abs(d) + a^2*c^6*d^6*e^8*g^5*abs(c)*abs(d))*((e*x +
d)*c*d*e - c*d^2*e + a*e^3)^(7/2)/((c^3*d^3*e^6*f^3*g^3 - 3*a*c^2*d^2*e^7*
f^2*g^4 + 3*a^2*c*d*e^8*f*g^5 - a^3*e^9*g^6)*(c^2*d^2*e^2*f - a*c*d*e^3*g
+ ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)^(7/2))
```

3.757.9 Mupad [B] (verification not implemented)

Time = 12.82 (sec) , antiderivative size = 325, normalized size of antiderivative = 5.16

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx = -\frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\left(\frac{2\,a^3\,e^3}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{2\,c^3\,d^3\,x^3}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{6\,a^2\,c\,d\,e^2\,x}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{6\,a\,c^2\,d^2\,e\,x \ldots}{7\,a\,e\,g^4-7\,c\,d \ldots}\right.}{x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}-\frac{\sqrt{f+g\,x}\,(7\,c\,d\,f^4-7\,a\,e\,f^3\,g)\,\sqrt{d+e\,x}}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{x^2\,\sqrt{f+g\,x}\,(21\,a\,e\,f\,g^3-21\,c\,d\,f^2\,g^2)\,\sqrt{d+e\,x}}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}-\frac{x\,\sqrt{f+g\,x}\,(21\,c\,d\,f^3\,g \ldots}{7\,a\,e\,g^4- \ldots}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(9/2)*(d + e*
x)^(5/2)),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*a^3*e^3)/(7*a*e*g^4 -
7*c*d*f*g^3) + (2*c^3*d^3*x^3)/(7*a*e*g^4 - 7*c*d*f*g^3) + (6*a^2*c*d*e^2*
x)/(7*a*e*g^4 - 7*c*d*f*g^3) + (6*a*c^2*d^2*e*x^2)/(7*a*e*g^4 - 7*c*d*f*g^
3)))/(x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2) - ((f + g*x)^(1/2)*(7*c*d*f^4 -
7*a*e*f^3*g)*(d + e*x)^(1/2))/(7*a*e*g^4 - 7*c*d*f*g^3) + (x^2*(f + g*x)^(
1/2)*(21*a*e*f*g^3 - 21*c*d*f^2*g^2)*(d + e*x)^(1/2))/(7*a*e*g^4 - 7*c*d*f
*g^3) - (x*(f + g*x)^(1/2)*(21*c*d*f^3*g - 21*a*e*f^2*g^2)*(d + e*x)^(1/2)
)/(7*a*e*g^4 - 7*c*d*f*g^3))
```

3.758 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx$

3.758.1 Optimal result

Integrand size = 48, antiderivative size = 129

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{9(cdf-aeg)(d+ex)^{7/2}(f+gx)^{9/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{63(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{7/2}}$$

output
```
2/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)/(e*x+d)^(7/2)/(
g*x+f)^(9/2)+4/63*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*
f)^2/(e*x+d)^(7/2)/(g*x+f)^(7/2)
```

3.758.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.53

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx = \frac{2((ae+cdx)(d+ex))^{7/2}(-7aeg+cd(9f+2gx))}{63(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{9/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^(11/2)),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(7/2)*(-7*a*e*g + c*d*(9*f + 2*g*x)))/(63*(c*
d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(9/2))
```

3.758.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}dx$$

$$\Big\downarrow \text{1254}$$

$$\frac{2cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}dx}{9(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1248}$$

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^(11/2)),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*(c*d*f - a*e*g)*(d +
e*x)^(7/2)*(f + g*x)^(9/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(7/2))/(63*(c*d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(7/2))
```

3.758.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

3.758.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{2(cdx+ae)(-2cdgx+7aeg-9cdf)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}}{63(gx+f)^{\frac{9}{2}}\left(a^{2}e^{2}g^{2}-2acdefg+c^{2}d^{2}f^{2}\right)(ex+d)^{\frac{5}{2}}}$	99
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,\left(-2c^{3}d^{3}g\,x^{3}+3a\,c^{2}d^{2}eg\,x^{2}-9c^{3}d^{3}f\,x^{2}+12a^{2}cd\,e^{2}gx-18a\,c^{2}d^{2}efx+7a^{3}e^{3}g-9a^{2}cd\,e^{2}f\right)(cdx+ae)}{63\sqrt{ex+d}\,(gx+f)^{\frac{9}{2}}(aeg-cdf)^{2}}$	136

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11/2),x
,method=_RETURNVERBOSE)
```

output
```
-2/63*(c*d*x+a*e)*(-2*c*d*g*x+7*a*e*g-9*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+
a*d*e)^(5/2)/(g*x+f)^(9/2)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(e*x+d)
^(5/2)
```

3.758.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. 2(113) = 226.

Time = 0.60 (sec) , antiderivative size = 639, normalized size of antiderivative = 4.95

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx = \frac{}{63\left(c^2d^3f^7-2\,acd^2ef^6g+a^2de^2f^5g^2+\left(c^2d^2ef^2g^5-2\,acde^2fg^6+\right.\right.}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
1/2),x, algorithm="fricas")
```

output
```
2/63*(2*c^4*d^4*g*x^4 + 9*a^3*c*d*e^3*f - 7*a^4*e^4*g + (9*c^4*d^4*f - a*c
^3*d^3*e*g)*x^3 + 3*(9*a*c^3*d^3*e*f - 5*a^2*c^2*d^2*e^2*g)*x^2 + (27*a^2*
c^2*d^2*e^2*f - 19*a^3*c*d*e^3*g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^3*f^7 - 2*a*c*d^2*e*f^6*g + a^2*
d*e^2*f^5*g^2 + (c^2*d^2*e*f^2*g^5 - 2*a*c*d*e^2*f*g^6 + a^2*e^3*g^7)*x^6
+ (5*c^2*d^2*e*f^3*g^4 + a^2*d*e^2*g^7 + (c^2*d^3 - 10*a*c*d*e^2)*f^2*g^5
- (2*a*c*d^2*e - 5*a^2*e^3)*f*g^6)*x^5 + 5*(2*c^2*d^2*e*f^4*g^3 + a^2*d*e^
2*f*g^6 + (c^2*d^3 - 4*a*c*d*e^2)*f^3*g^4 - 2*(a*c*d^2*e - a^2*e^3)*f^2*g^
5)*x^4 + 10*(c^2*d^2*e*f^5*g^2 + a^2*d*e^2*f^2*g^5 + (c^2*d^3 - 2*a*c*d*e^
2)*f^4*g^3 - (2*a*c*d^2*e - a^2*e^3)*f^3*g^4)*x^3 + 5*(c^2*d^2*e*f^6*g + 2
*a^2*d*e^2*f^3*g^4 + 2*(c^2*d^3 - a*c*d*e^2)*f^5*g^2 - (4*a*c*d^2*e - a^2*
e^3)*f^4*g^3)*x^2 + (c^2*d^2*e*f^7 + 5*a^2*d*e^2*f^4*g^3 + (5*c^2*d^3 - 2*
a*c*d*e^2)*f^6*g - (10*a*c*d^2*e - a^2*e^3)*f^5*g^2)*x)
```

3.758.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**(11/2),x)
```

output
```
Timed out
```

3.758.7 Maxima [F]

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx = \int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
1/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^(11/2)), x)
```

3.758.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1275 vs. $2(113) = 226$.

Time = 1.16 (sec) , antiderivative size = 1275, normalized size of antiderivative = 9.88

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
1/2),x, algorithm="giac")
```

output
```
2/63*(9*sqrt(-c*d^2*e + a*e^3)*c^4*d^7*e*f*abs(c)*abs(d) - 27*sqrt(-c*d^2*
e + a*e^3)*a*c^3*d^5*e^3*f*abs(c)*abs(d) + 27*sqrt(-c*d^2*e + a*e^3)*a^2*c
^2*d^3*e^5*f*abs(c)*abs(d) - 9*sqrt(-c*d^2*e + a*e^3)*a^3*c*d*e^7*f*abs(c)
*abs(d) - 2*sqrt(-c*d^2*e + a*e^3)*c^4*d^8*g*abs(c)*abs(d) - sqrt(-c*d^2*e
 + a*e^3)*a*c^3*d^6*e^2*g*abs(c)*abs(d) + 15*sqrt(-c*d^2*e + a*e^3)*a^2*c^
2*d^4*e^4*g*abs(c)*abs(d) - 19*sqrt(-c*d^2*e + a*e^3)*a^3*c*d^2*e^6*g*abs(
c)*abs(d) + 7*sqrt(-c*d^2*e + a*e^3)*a^4*e^8*g*abs(c)*abs(d))/(sqrt(c^2*d^
2*e^2*f - c^2*d^3*e*g)*c^2*d^2*e^4*f^6 - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*
g)*c^2*d^3*e^3*f^5*g - 2*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c*d*e^5*f^5*g
 + 6*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^2*d^4*e^2*f^4*g^2 + 8*sqrt(c^2*d^
2*e^2*f - c^2*d^3*e*g)*a*c*d^2*e^4*f^4*g^2 + sqrt(c^2*d^2*e^2*f - c^2*d^3*
e*g)*a^2*e^6*f^4*g^2 - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^2*d^5*e*f^3*g
^3 - 12*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c*d^3*e^3*f^3*g^3 - 4*sqrt(c^2
*d^2*e^2*f - c^2*d^3*e*g)*a^2*d*e^5*f^3*g^3 + sqrt(c^2*d^2*e^2*f - c^2*d^3
*e*g)*c^2*d^6*f^2*g^4 + 8*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c*d^4*e^2*f^
2*g^4 + 6*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*d^2*e^4*f^2*g^4 - 2*sqrt(c
^2*d^2*e^2*f - c^2*d^3*e*g)*a*c*d^5*e*f*g^5 - 4*sqrt(c^2*d^2*e^2*f - c^2*d
^3*e*g)*a^2*d^3*e^3*f*g^5 + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*d^4*e^2*
g^6) + 2/63*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*(2*(c^10*d^10*e^8*f^
2*g^5*abs(c)*abs(d) - 2*a*c^9*d^9*e^9*f*g^6*abs(c)*abs(d) + a^2*c^8*d^8...
```

3.758.9 Mupad [B] (verification not implemented)

Time = 13.08 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.44

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx =$$

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,a^3\,e^3\,(7\,a\,e\,g-9\,c\,d\,f)}{63\,g^4\,(a\,e\,g-c\,d\,f)^2}-\frac{4\,c^4\,d^4\,x^4}{63\,g^3\,(a\,e\,g-c\,d\,f)^2}+\frac{2\,c^3\,d^3\,x^3\,(a\,e\,g-9\,c\,d\,f)}{63\,g^4\,(a\,e\,g-c\,d\,f)^2}+\frac{2\,a^2\,c\,d\,e^2\,x\,(19\,a\cdots}{63\,g^4\,(a\,e\,g-\cdots}\right.}{x^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^4}+\frac{4\,f\,x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{4\,f^3\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{6\,f^2\cdots}{}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(11/2)*(d + e
*x)^(5/2)),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*a^3*e^3*(7*a*e*g - 9*c
*d*f))/(63*g^4*(a*e*g - c*d*f)^2) - (4*c^4*d^4*x^4)/(63*g^3*(a*e*g - c*d*f
)^2) + (2*c^3*d^3*x^3*(a*e*g - 9*c*d*f))/(63*g^4*(a*e*g - c*d*f)^2) + (2*a
^2*c*d*e^2*x*(19*a*e*g - 27*c*d*f))/(63*g^4*(a*e*g - c*d*f)^2) + (2*a*c^2*
d^2*e*x^2*(5*a*e*g - 9*c*d*f))/(21*g^4*(a*e*g - c*d*f)^2)))/(x^4*(f + g*x)
^(1/2)*(d + e*x)^(1/2) + (f^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4 + (4*f*
x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g + (4*f^3*x*(f + g*x)^(1/2)*(d + e*x
)^(1/2))/g^3 + (6*f^2*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2)
```

3.758. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx$

3.759 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx$

3.759.1 Optimal result

Integrand size = 48, antiderivative size = 198

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11(cdf-aeg)(d+ex)^{7/2}(f+gx)^{11/2}} + \frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{99(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{9/2}} + \frac{16c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{693(cdf-aeg)^3(d+ex)^{7/2}(f+gx)^{7/2}}$$

output
```
2/11*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)/(e*x+d)^(7/2)/
(g*x+f)^(11/2)+8/99*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*
d*f)^2/(e*x+d)^(7/2)/(g*x+f)^(9/2)+16/693*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(7/2)/(g*x+f)^(7/2)
```

3.759.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.58

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx = \frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}\left(63a^2e^2g^2-14acdeg(11f+2gx\right.}{693(cdf-aeg)^3\sqrt{d+ex}(f+gx)^{1}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^(13/2)),x]
```

output
```
(2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(63*a^2*e^2*g^2 - 14*a*c*
d*e*g*(11*f + 2*g*x) + c^2*d^2*(99*f^2 + 44*f*g*x + 8*g^2*x^2)))/(693*(c*d
*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)^(11/2))
```

3.759.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1254, 1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}dx$$

$$\Big\downarrow \text{1254}$$

$$\frac{4cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}dx}{11(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^{7/2}(f+gx)^{11/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$\frac{4cd\left(\frac{2cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}dx}{9(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)}\right)}{11(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^{7/2}(f+gx)^{11/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1248}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^{7/2}(f+gx)^{11/2}(cdf-aeg)}+\frac{4cd\left(\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)}\right)}{11(cdf-aeg)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^(13/2)),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(11*(c*d*f - a*e*g)*(d +
 e*x)^(7/2)*(f + g*x)^(11/2)) + (4*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(7/2))/(9*(c*d*f - a*e*g)*(d + e*x)^(7/2)*(f + g*x)^(9/2)) + (4*c
*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(63*(c*d*f - a*e*g)^2*(d
 + e*x)^(7/2)*(f + g*x)^(7/2))))/(11*(c*d*f - a*e*g))
```

3.759.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

3.759.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.85

method	result
gosper	$-\frac{2(cdx+ae)\left(8g^{2}x^{2}c^{2}d^{2}-28acde\,g^{2}x+44c^{2}d^{2}fgx+63a^{2}e^{2}g^{2}-154acdefg+99c^{2}d^{2}f^{2}\right)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}}{693(gx+f)^{\frac{11}{2}}\left(a^{3}e^{3}g^{3}-3a^{2}cd\,e^{2}f\,g^{2}+3a\,c^{2}d^{2}e\,f^{2}g-f^{3}c^{3}d^{3}\right)(ex+d)^{\frac{5}{2}}}$
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,\left(8c^{4}d^{4}g^{2}x^{4}-12a\,c^{3}d^{3}e\,g^{2}x^{3}+44c^{4}d^{4}fg\,x^{3}+15a^{2}c^{2}d^{2}e^{2}g^{2}x^{2}-66a\,c^{3}d^{3}efg\,x^{2}+99c^{4}d^{4}f^{2}x^{2}+98a^{3}cd\,e^{3}g^{2}x-\right.}{693\sqrt{ex+d}\,(gx+f)^{\frac{11}{2}}\left(aeg-cdf\right)^{3}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13/2),x
,method=_RETURNVERBOSE)
```

output
```
-2/693*(c*d*x+a*e)*(8*c^2*d^2*g^2*x^2-28*a*c*d*e*g^2*x+44*c^2*d^2*f*g*x+63
*a^2*e^2*g^2-154*a*c*d*e*f*g+99*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*
d*e)^(5/2)/(g*x+f)^(11/2)/(a^3*e^3*g^3-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f
^2*g-c^3*d^3*f^3)/(e*x+d)^(5/2)
```

3.759.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1101 vs. $2(174) = 348$.

Time = 1.08 (sec) , antiderivative size = 1101, normalized size of antiderivative = 5.56

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{13/2}}\,dx = \frac{}{693\,(c^3d^4f^9 - 3\,ac^2d^3ef^8g + 3\,a^2cd^2e^2f^7g^2 - a^3de^3f^6g^3 + (c^3d^3ef}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
3/2),x, algorithm="fricas")
```

output
```
2/693*(8*c^5*d^5*g^2*x^5 + 99*a^3*c^2*d^2*e^3*f^2 - 154*a^4*c*d*e^4*f*g +
63*a^5*e^5*g^2 + 4*(11*c^5*d^5*f*g - a*c^4*d^4*e*g^2)*x^4 + (99*c^5*d^5*f^
2 - 22*a*c^4*d^4*e*f*g + 3*a^2*c^3*d^3*e^2*g^2)*x^3 + (297*a*c^4*d^4*e*f^2
 - 330*a^2*c^3*d^3*e^2*f*g + 113*a^3*c^2*d^2*e^3*g^2)*x^2 + (297*a^2*c^3*d
^3*e^2*f^2 - 418*a^3*c^2*d^2*e^3*f*g + 161*a^4*c*d*e^4*g^2)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^9
- 3*a*c^2*d^3*e*f^8*g + 3*a^2*c*d^2*e^2*f^7*g^2 - a^3*d*e^3*f^6*g^3 + (c^3
*d^3*e*f^3*g^6 - 3*a*c^2*d^2*e^2*f^2*g^7 + 3*a^2*c*d*e^3*f*g^8 - a^3*e^4*g
^9)*x^7 + (6*c^3*d^3*e*f^4*g^5 - a^3*d*e^3*g^9 + (c^3*d^4 - 18*a*c^2*d^2*e
^2)*f^3*g^6 - 3*(a*c^2*d^3*e - 6*a^2*c*d*e^3)*f^2*g^7 + 3*(a^2*c*d^2*e^2 -
 2*a^3*e^4)*f*g^8)*x^6 + 3*(5*c^3*d^3*e*f^5*g^4 - 2*a^3*d*e^3*f*g^8 + (2*c
^3*d^4 - 15*a*c^2*d^2*e^2)*f^4*g^5 - 3*(2*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^3
*g^6 + (6*a^2*c*d^2*e^2 - 5*a^3*e^4)*f^2*g^7)*x^5 + 5*(4*c^3*d^3*e*f^6*g^3
 - 3*a^3*d*e^3*f^2*g^7 + 3*(c^3*d^4 - 4*a*c^2*d^2*e^2)*f^5*g^4 - 3*(3*a*c^
2*d^3*e - 4*a^2*c*d*e^3)*f^4*g^5 + (9*a^2*c*d^2*e^2 - 4*a^3*e^4)*f^3*g^6)*
x^4 + 5*(3*c^3*d^3*e*f^7*g^2 - 4*a^3*d*e^3*f^3*g^6 + (4*c^3*d^4 - 9*a*c^2*
d^2*e^2)*f^6*g^3 - 3*(4*a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^5*g^4 + 3*(4*a^2*c*
d^2*e^2 - a^3*e^4)*f^4*g^5)*x^3 + 3*(2*c^3*d^3*e*f^8*g - 5*a^3*d*e^3*f^4*g
^5 + (5*c^3*d^4 - 6*a*c^2*d^2*e^2)*f^7*g^2 - 3*(5*a*c^2*d^3*e - 2*a^2*c*d*
e^3)*f^6*g^3 + (15*a^2*c*d^2*e^2 - 2*a^3*e^4)*f^5*g^4)*x^2 + (c^3*d^3*e...
```

3.759.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**(13/2),x)
```

output
```
Timed out
```

3.759.7 Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx = \int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{13}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
3/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^(13/2)), x)
```

3.759.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2167 vs. $2(174) = 348$.

Time = 1.77 (sec) , antiderivative size = 2167, normalized size of antiderivative = 10.94

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
3/2),x, algorithm="giac")
```

output
```
2/693*(99*sqrt(-c*d^2*e + a*e^3)*c^5*d^8*e^2*f^2*abs(c)*abs(d) - 297*sqrt(
-c*d^2*e + a*e^3)*a*c^4*d^6*e^4*f^2*abs(c)*abs(d) + 297*sqrt(-c*d^2*e + a*
e^3)*a^2*c^3*d^4*e^6*f^2*abs(c)*abs(d) - 99*sqrt(-c*d^2*e + a*e^3)*a^3*c^2
*d^2*e^8*f^2*abs(c)*abs(d) - 44*sqrt(-c*d^2*e + a*e^3)*c^5*d^9*e*f*g*abs(c
)*abs(d) - 22*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^7*e^3*f*g*abs(c)*abs(d) + 330
*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^5*e^5*f*g*abs(c)*abs(d) - 418*sqrt(-c*d^
2*e + a*e^3)*a^3*c^2*d^3*e^7*f*g*abs(c)*abs(d) + 154*sqrt(-c*d^2*e + a*e^3
)*a^4*c*d*e^9*f*g*abs(c)*abs(d) + 8*sqrt(-c*d^2*e + a*e^3)*c^5*d^10*g^2*ab
s(c)*abs(d) + 4*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^8*e^2*g^2*abs(c)*abs(d) + 3
*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^6*e^4*g^2*abs(c)*abs(d) - 113*sqrt(-c*d^
2*e + a*e^3)*a^3*c^2*d^4*e^6*g^2*abs(c)*abs(d) + 161*sqrt(-c*d^2*e + a*e^3
)*a^4*c*d^2*e^8*g^2*abs(c)*abs(d) - 63*sqrt(-c*d^2*e + a*e^3)*a^5*e^10*g^2
*abs(c)*abs(d))/(sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^3*d^3*e^5*f^8 - 5*sqr
t(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^3*d^4*e^4*f^7*g - 3*sqrt(c^2*d^2*e^2*f -
c^2*d^3*e*g)*a*c^2*d^2*e^6*f^7*g + 10*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^
3*d^5*e^3*f^6*g^2 + 15*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^2*d^3*e^5*f^6
*g^2 + 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c*d*e^7*f^6*g^2 - 10*sqrt(c
^2*d^2*e^2*f - c^2*d^3*e*g)*c^3*d^6*e^2*f^5*g^3 - 30*sqrt(c^2*d^2*e^2*f -
c^2*d^3*e*g)*a*c^2*d^4*e^4*f^5*g^3 - 15*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*
a^2*c*d^2*e^6*f^5*g^3 - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*e^8*f^5*g...
```

3.759.9 Mupad [B] (verification not implemented)

Time = 13.49 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.35

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{13/2}}\,dx =$$
$$-\frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}\left(\frac{126\,a^5\,e^5\,g^2 - 308\,a^4\,c\,d\,e^4\,f\,g + 198\,a^3\,c^2\,d^2\,e^3\,f^2}{693\,g^5\,(a\,e\,g - c\,d\,f)^3} + \frac{x^3\,(6\,a^2\,c^3\,d^3\,e^2\,g^2 - 44\,a\,c^4\,d^4\,e\,f\,g + 198\,c^5\,\cdots}{693\,g^5\,(a\,e\,g - c\,d\,f)^3}\right.}{x^5\,\sqrt{f + g\,x}\,\sqrt{d + e\,x} + \frac{f^5\,\sqrt{f + g\,x}\,\sqrt{d + e\,x}}{g^5} + \frac{5\,f\,x^4\,\sqrt{\cdots}}{}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(13/2)*(d + e
*x)^(5/2)),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((126*a^5*e^5*g^2 + 198*a^
3*c^2*d^2*e^3*f^2 - 308*a^4*c*d*e^4*f*g)/(693*g^5*(a*e*g - c*d*f)^3) + (x^
3*(198*c^5*d^5*f^2 + 6*a^2*c^3*d^3*e^2*g^2 - 44*a*c^4*d^4*e*f*g))/(693*g^5
*(a*e*g - c*d*f)^3) + (16*c^5*d^5*x^5)/(693*g^3*(a*e*g - c*d*f)^3) - (8*c^
4*d^4*x^4*(a*e*g - 11*c*d*f))/(693*g^4*(a*e*g - c*d*f)^3) + (2*a^2*c*d*e^2
*x*(161*a^2*e^2*g^2 + 297*c^2*d^2*f^2 - 418*a*c*d*e*f*g))/(693*g^5*(a*e*g
- c*d*f)^3) + (2*a*c^2*d^2*e*x^2*(113*a^2*e^2*g^2 + 297*c^2*d^2*f^2 - 330*
a*c*d*e*f*g))/(693*g^5*(a*e*g - c*d*f)^3)))/(x^5*(f + g*x)^(1/2)*(d + e*x)
^(1/2) + (f^5*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^5 + (5*f*x^4*(f + g*x)^(1
/2)*(d + e*x)^(1/2))/g + (5*f^4*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4 + (
10*f^2*x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (10*f^3*x^2*(f + g*x)^(1
/2)*(d + e*x)^(1/2))/g^3)
```

3.759. $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx$

3.760 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx$

3.760.1 Optimal result

Integrand size = 48, antiderivative size = 267

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx = \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}} + \frac{12cd(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{143(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{11/2}} + \frac{16c^2d^2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{429(cdf-aeg)^3(d+ex)^{7/2}(f+gx)^{9/2}} + \frac{32c^3d^3(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{3003(cdf-aeg)^4(d+ex)^{7/2}(f+gx)^{7/2}}$$

output
```
2/13*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)/(e*x+d)^(7/2)/
(g*x+f)^(13/2)+12/143*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+
c*d*f)^2/(e*x+d)^(7/2)/(g*x+f)^(11/2)+16/429*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(7/2)/(g*x+f)^(9/2)+32/3003*c^
3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(7/
2)/(g*x+f)^(7/2)
```

3.760.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.61

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx = \frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}(-231a^3e^3g^3+63a^2cde^2g^2(13f}{3003}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^(15/2)),x]
```

output
```
(2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-231*a^3*e^3*g^3 + 63*a^
2*c*d*e^2*g^2*(13*f + 2*g*x) - 7*a*c^2*d^2*e*g*(143*f^2 + 52*f*g*x + 8*g^2
*x^2) + c^3*d^3*(429*f^3 + 286*f^2*g*x + 104*f*g^2*x^2 + 16*g^3*x^3)))/(30
03*(c*d*f - a*e*g)^4*Sqrt[d + e*x]*(f + g*x)^(13/2))
```

3.760.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1254, 1254, 1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}dx$$

$$\Big\downarrow \text{1254}$$

$$\frac{6cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}dx}{13(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13(d+ex)^{7/2}(f+gx)^{13/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$\frac{6cd\left(\frac{4cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}dx}{11(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^{7/2}(f+gx)^{11/2}(cdf-aeg)}\right)}{13(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13(d+ex)^{7/2}(f+gx)^{13/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$\frac{6cd\left(\frac{4cd\left(\frac{2cd\int\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}dx}{9(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)}\right)}{11(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^{7/2}(f+gx)^{11/2}(cdf-aeg)}\right)}{13(cdf-aeg)}+$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13(d+ex)^{7/2}(f+gx)^{13/2}(cdf-aeg)}$$

$$\Big\downarrow \text{ 1248}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13(d+ex)^{7/2}(f+gx)^{13/2}(cdf-aeg)}+$$

$$\frac{6cd\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^{7/2}(f+gx)^{11/2}(cdf-aeg)}+\frac{4cd\left(\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)}\right)}{11(cdf-aeg)}\right)}{13(cdf-aeg)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^(15/2)),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(13*(c*d*f - a*e*g)*(d +
 e*x)^(7/2)*(f + g*x)^(13/2)) + (6*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(7/2))/(11*(c*d*f - a*e*g)*(d + e*x)^(7/2)*(f + g*x)^(11/2)) + (4
*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*(c*d*f - a*e*g)
*(d + e*x)^(7/2)*(f + g*x)^(9/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(7/2))/(63*(c*d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(7/2))))/(
11*(c*d*f - a*e*g))))/(13*(c*d*f - a*e*g))
```

3.760.3.1 Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

3.760. $\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}dx$

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

3.760.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.97

method	result
gosper	$-\frac{2(cdx+ae)\left(-16g^3x^3c^3d^3+56a\,c^2d^2e\,g^3x^2-104c^3d^3f\,g^2x^2-126a^2cd\,e^2g^3x+364a\,c^2d^2ef\,g^2x-286c^3d^3f^2gx+231a^3e^3g^3-819a^2c\right.}{3003(gx+f)^{\frac{13}{2}}\left(a^4e^4g^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4c\right.}$
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,\left(-16c^5d^5g^3x^5+24a\,c^4d^4e\,g^3x^4-104c^5d^5f\,g^2x^4-30a^2c^3d^3e^2g^3x^3+156a\,c^4d^4ef\,g^2x^3-286c^5d^5f^2g\,x^3+35a^3\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15/2),x
,method=_RETURNVERBOSE)
```

output
```
-2/3003*(c*d*x+a*e)*(-16*c^3*d^3*g^3*x^3+56*a*c^2*d^2*e*g^3*x^2-104*c^3*d^
3*f*g^2*x^2-126*a^2*c*d*e^2*g^3*x+364*a*c^2*d^2*e*f*g^2*x-286*c^3*d^3*f^2*
g*x+231*a^3*e^3*g^3-819*a^2*c*d*e^2*f*g^2+1001*a*c^2*d^2*e*f^2*g-429*c^3*d
^3*f^3)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)/(g*x+f)^(13/2)/(a^4*e^4*g^
4-4*a^3*c*d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g^2-4*a*c^3*d^3*e*f^3*g+c^4*d^
4*f^4)/(e*x+d)^(5/2)
```

3.760.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1648 vs. 2(235) = 470.

Time = 1.57 (sec) , antiderivative size = 1648, normalized size of antiderivative = 6.17

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
5/2),x, algorithm="fricas")
```

output
```
2/3003*(16*c^6*d^6*g^3*x^6 + 429*a^3*c^3*d^3*e^3*f^3 - 1001*a^4*c^2*d^2*e^
4*f^2*g + 819*a^5*c*d*e^5*f*g^2 - 231*a^6*e^6*g^3 + 8*(13*c^6*d^6*f*g^2 -
a*c^5*d^5*e*g^3)*x^5 + 2*(143*c^6*d^6*f^2*g - 26*a*c^5*d^5*e*f*g^2 + 3*a^2
*c^4*d^4*e^2*g^3)*x^4 + (429*c^6*d^6*f^3 - 143*a*c^5*d^5*e*f^2*g + 39*a^2*
c^4*d^4*e^2*f*g^2 - 5*a^3*c^3*d^3*e^3*g^3)*x^3 + (1287*a*c^5*d^5*e*f^3 - 2
145*a^2*c^4*d^4*e^2*f^2*g + 1469*a^3*c^3*d^3*e^3*f*g^2 - 371*a^4*c^2*d^2*e
^4*g^3)*x^2 + (1287*a^2*c^4*d^4*e^2*f^3 - 2717*a^3*c^3*d^3*e^3*f^2*g + 209
3*a^4*c^2*d^2*e^4*f*g^2 - 567*a^5*c*d*e^5*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^4*d^5*f^11 - 4*a*c^3*d^
4*e*f^10*g + 6*a^2*c^2*d^3*e^2*f^9*g^2 - 4*a^3*c*d^2*e^3*f^8*g^3 + a^4*d*e
^4*f^7*g^4 + (c^4*d^4*e*f^4*g^7 - 4*a*c^3*d^3*e^2*f^3*g^8 + 6*a^2*c^2*d^2*
e^3*f^2*g^9 - 4*a^3*c*d*e^4*f*g^10 + a^4*e^5*g^11)*x^8 + (7*c^4*d^4*e*f^5*
g^6 + a^4*d*e^4*g^11 + (c^4*d^5 - 28*a*c^3*d^3*e^2)*f^4*g^7 - 2*(2*a*c^3*d
^4*e - 21*a^2*c^2*d^2*e^3)*f^3*g^8 + 2*(3*a^2*c^2*d^3*e^2 - 14*a^3*c*d*e^4
)*f^2*g^9 - (4*a^3*c*d^2*e^3 - 7*a^4*e^5)*f*g^10)*x^7 + 7*(3*c^4*d^4*e*f^6
*g^5 + a^4*d*e^4*f*g^10 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*f^5*g^6 - 2*(2*a*c^
3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^4*g^7 + 6*(a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)
*f^3*g^8 - (4*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^2*g^9)*x^6 + 7*(5*c^4*d^4*e*f^7
*g^4 + 3*a^4*d*e^4*f^2*g^9 + (3*c^4*d^5 - 20*a*c^3*d^3*e^2)*f^6*g^5 - 6*(2
*a*c^3*d^4*e - 5*a^2*c^2*d^2*e^3)*f^5*g^6 + 2*(9*a^2*c^2*d^3*e^2 - 10*a...
```

3.760.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**(15/2),x)
```

output
```
Timed out
```

3.760.7 Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{15}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
5/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^(15/2)), x)
```

3.760.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3241 vs. $2(235) = 470$.

Time = 2.66 (sec) , antiderivative size = 3241, normalized size of antiderivative = 12.14

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
5/2),x, algorithm="giac")
```

output
```
2/3003*(429*sqrt(-c*d^2*e + a*e^3)*c^6*d^9*e^3*f^3*abs(c)*abs(d) - 1287*sq
rt(-c*d^2*e + a*e^3)*a*c^5*d^7*e^5*f^3*abs(c)*abs(d) + 1287*sqrt(-c*d^2*e
+ a*e^3)*a^2*c^4*d^5*e^7*f^3*abs(c)*abs(d) - 429*sqrt(-c*d^2*e + a*e^3)*a^
3*c^3*d^3*e^9*f^3*abs(c)*abs(d) - 286*sqrt(-c*d^2*e + a*e^3)*c^6*d^10*e^2*
f^2*g*abs(c)*abs(d) - 143*sqrt(-c*d^2*e + a*e^3)*a*c^5*d^8*e^4*f^2*g*abs(c
)*abs(d) + 2145*sqrt(-c*d^2*e + a*e^3)*a^2*c^4*d^6*e^6*f^2*g*abs(c)*abs(d)
 - 2717*sqrt(-c*d^2*e + a*e^3)*a^3*c^3*d^4*e^8*f^2*g*abs(c)*abs(d) + 1001*
sqrt(-c*d^2*e + a*e^3)*a^4*c^2*d^2*e^10*f^2*g*abs(c)*abs(d) + 104*sqrt(-c*
d^2*e + a*e^3)*c^6*d^11*e*f*g^2*abs(c)*abs(d) + 52*sqrt(-c*d^2*e + a*e^3)*
a*c^5*d^9*e^3*f*g^2*abs(c)*abs(d) + 39*sqrt(-c*d^2*e + a*e^3)*a^2*c^4*d^7*
e^5*f*g^2*abs(c)*abs(d) - 1469*sqrt(-c*d^2*e + a*e^3)*a^3*c^3*d^5*e^7*f*g^
2*abs(c)*abs(d) + 2093*sqrt(-c*d^2*e + a*e^3)*a^4*c^2*d^3*e^9*f*g^2*abs(c)
*abs(d) - 819*sqrt(-c*d^2*e + a*e^3)*a^5*c*d*e^11*f*g^2*abs(c)*abs(d) - 16
*sqrt(-c*d^2*e + a*e^3)*c^6*d^12*g^3*abs(c)*abs(d) - 8*sqrt(-c*d^2*e + a*e
^3)*a*c^5*d^10*e^2*g^3*abs(c)*abs(d) - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^4*d^
8*e^4*g^3*abs(c)*abs(d) - 5*sqrt(-c*d^2*e + a*e^3)*a^3*c^3*d^6*e^6*g^3*abs
(c)*abs(d) + 371*sqrt(-c*d^2*e + a*e^3)*a^4*c^2*d^4*e^8*g^3*abs(c)*abs(d)
- 567*sqrt(-c*d^2*e + a*e^3)*a^5*c*d^2*e^10*g^3*abs(c)*abs(d) + 231*sqrt(-
c*d^2*e + a*e^3)*a^6*e^12*g^3*abs(c)*abs(d))/(sqrt(c^2*d^2*e^2*f - c^2*d^3
*e*g)*c^4*d^4*e^6*f^10 - 6*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^4*d^5*e^...
```

3.760.9 Mupad [B] (verification not implemented)

Time = 13.62 (sec) , antiderivative size = 627, normalized size of antiderivative = 2.35

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{15/2}}\,dx =$$

$$-\frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}\,\left(\frac{462\,a^6\,e^6\,g^3 - 1638\,a^5\,c\,d\,e^5\,f\,g^2 + 2002\,a^4\,c^2\,d^2\,e^4\,f^2\,g - 858\,a^3\,c^3\,d^3\,e^3\,f^3}{3003\,g^6\,(a\,e\,g - c\,d\,f)^4} - \frac{x^3\,(-10\,a^3\,c^3\,d^3\,e^3 \ldots}{\ldots}\right.}{x^6 \ldots}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(15/2)*(d + e
*x)^(5/2)),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((462*a^6*e^6*g^3 - 858*a^
3*c^3*d^3*e^3*f^3 + 2002*a^4*c^2*d^2*e^4*f^2*g - 1638*a^5*c*d*e^5*f*g^2)/(
3003*g^6*(a*e*g - c*d*f)^4) - (x^3*(858*c^6*d^6*f^3 - 10*a^3*c^3*d^3*e^3*g
^3 + 78*a^2*c^4*d^4*e^2*f*g^2 - 286*a*c^5*d^5*e*f^2*g))/(3003*g^6*(a*e*g -
 c*d*f)^4) - (32*c^6*d^6*x^6)/(3003*g^3*(a*e*g - c*d*f)^4) - (4*c^4*d^4*x^
4*(3*a^2*e^2*g^2 + 143*c^2*d^2*f^2 - 26*a*c*d*e*f*g))/(3003*g^5*(a*e*g - c
*d*f)^4) + (16*c^5*d^5*x^5*(a*e*g - 13*c*d*f))/(3003*g^4*(a*e*g - c*d*f)^4
) + (2*a^2*c*d*e^2*x*(567*a^3*e^3*g^3 - 1287*c^3*d^3*f^3 + 2717*a*c^2*d^2*
e*f^2*g - 2093*a^2*c*d*e^2*f*g^2))/(3003*g^6*(a*e*g - c*d*f)^4) + (2*a*c^2
*d^2*e*x^2*(371*a^3*e^3*g^3 - 1287*c^3*d^3*f^3 + 2145*a*c^2*d^2*e*f^2*g -
1469*a^2*c*d*e^2*f*g^2))/(3003*g^6*(a*e*g - c*d*f)^4)))/(x^6*(f + g*x)^(1/
2)*(d + e*x)^(1/2) + (f^6*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^6 + (6*f*x^5*
(f + g*x)^(1/2)*(d + e*x)^(1/2))/g + (6*f^5*x*(f + g*x)^(1/2)*(d + e*x)^(1
/2))/g^5 + (15*f^2*x^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (20*f^3*x^3*
(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^3 + (15*f^4*x^2*(f + g*x)^(1/2)*(d + e*
x)^(1/2))/g^4)
```

3.760. $\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx$

3.761 $\int \frac{(d+ex)^{5/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

3.761.1 Optimal result

Integrand size = 46, antiderivative size = 104

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{(ae+cdx)(d+ex)^{5/2}(f+gx)^{1+n}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2}+n,2+n,\frac{cd(f+gx)}{cdf-aeg}\right)}{(cdf-aeg)(1+n)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}$$

output
```
-(c*d*x+a*e)*(e*x+d)^(5/2)*(g*x+f)^(1+n)*hypergeom([1, -1/2+n],[2+n],c*d*(
g*x+f)/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(1+n)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(5/2)
```

3.761.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(-\frac{3}{2},-n,-\frac{1}{2},\frac{g(ae+cdx)}{-cdf+aeg}\right)}{3cd((ae+cdx)(d+ex))^{3/2}}$$

input
```
Integrate[((d + e*x)^(5/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2)*(f + g*x)^n*Hypergeometric2F1[-3/2, -n, -1/2, (g*(a*e
+ c*d*x))/(-(c*d*f) + a*e*g)])/(3*c*d*((a*e + c*d*x)*(d + e*x))^(3/2)*((c*
d*(f + g*x))/(c*d*f - a*e*g))^n)
```

3.761.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1268}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{(f+gx)^n}{(ae+cdx)^{5/2}}dx}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 80}$$

$$\frac{\sqrt{d+ex}(f+gx)^n\sqrt{ae+cdx}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\int \frac{\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n}{(ae+cdx)^{5/2}}dx}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{2\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(-\frac{3}{2},-n,-\frac{1}{2},-\frac{g(ae+cdx)}{cdf-aeg}\right)}{3cd(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[((d + e*x)^(5/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(5/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[-3/2, -n, -1/2, -((g*(a*e
+ c*d*x))/(c*d*f - a*e*g))])/(3*c*d*(a*e + c*d*x)*((c*d*(f + g*x))/(c*d*f
- a*e*g))^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

3.761.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.761.4 Maple [F]

$$\int \frac{(ex+d)^{\frac{5}{2}} (gx+f)^{n}}{(ade+(e^{2}a+c\,d^{2})\,x+cde\,x^{2})^{\frac{5}{2}}}dx$$

input
```
int((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
int((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

3.761.5 Fricas [F]

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="fricas")
```

output
```
integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*(g*x +
f)^n/(c^3*d^3*e*x^4 + a^3*d*e^3 + (c^3*d^4 + 3*a*c^2*d^2*e^2)*x^3 + 3*(a*c
^2*d^3*e + a^2*c*d*e^3)*x^2 + (3*a^2*c*d^2*e^2 + a^3*e^4)*x), x)
```

3.761.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(5/2),x)
```

output
```
Timed out
```

3.761.7 Maxima [F]

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^(5/2), x)
```

3.761.8 Giac [F]

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="giac")
```

output
```
integrate((e*x + d)^(5/2)*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^(5/2), x)
```

3.761.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \int \frac{(f+g\,x)^n\,(d+e\,x)^{5/2}}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{5/2}}\,dx$$

input
```
int(((f + g*x)^n*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(5/2),x)
```

output
```
int(((f + g*x)^n*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(5/2), x)
```

3.762 $\int \frac{(d+ex)^{3/2}(f+gx)^n}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

3.762.1 Optimal result

Integrand size = 46, antiderivative size = 104

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{(ae+cdx)(d+ex)^{3/2}(f+gx)^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,2+n,\frac{cd(f+gx)}{cdf-aeg}\right)}{(cdf-aeg)(1+n)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}$$

output
```
-(c*d*x+a*e)*(e*x+d)^(3/2)*(g*x+f)^(1+n)*hypergeom([1, 1/2+n],[2+n],c*d*(g
*x+f)/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(1+n)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(3/2)
```

3.762.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(-\frac{1}{2},-n,\frac{1}{2},\frac{g(ae+cdx)}{-cdf+aeg}\right)}{cd\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[((d + e*x)^(3/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[-1/2, -n, 1/2, (g*(a*e + c
*d*x))/(-(c*d*f) + a*e*g)])/(c*d*Sqrt[(a*e + c*d*x)*(d + e*x)]*((c*d*(f +
g*x))/(c*d*f - a*e*g))^n)
```

3.762.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}dx$$

$$\Big\downarrow \text{1268}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{(f+gx)^n}{(ae+cdx)^{3/2}}dx}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{80}$$

$$\frac{\sqrt{d+ex}(f+gx)^n\sqrt{ae+cdx}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\int \frac{\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n}{(ae+cdx)^{3/2}}dx}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{2\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(-\frac{1}{2},-n,\frac{1}{2},-\frac{g(ae+cdx)}{cdf-aeg}\right)}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

input
```
Int[((d + e*x)^(3/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[-1/2, -n, 1/2, -((g*(a*e +
 c*d*x))/(c*d*f - a*e*g))])/(c*d*((c*d*(f + g*x))/(c*d*f - a*e*g))^n*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

3.762.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.762.4 Maple [F]

$$\int \frac{(ex+d)^{\frac{3}{2}} (gx+f)^n}{(ade+(e^2a+c\,d^2)\,x+cde\,x^2)^{\frac{3}{2}}}dx$$

input
```
int((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
int((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

3.762.5 Fricas [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="fricas")
```

output
```
integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*(g*x +
f)^n/(c^2*d^2*e*x^3 + a^2*d*e^2 + (c^2*d^3 + 2*a*c*d*e^2)*x^2 + (2*a*c*d^2
*e + a^2*e^3)*x), x)
```

3.762.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(3/2),x)
```

output
```
Timed out
```

3.762.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^(3/2), x)
```

3.762.8 Giac [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="giac")
```

output
```
integrate((e*x + d)^(3/2)*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^(3/2), x)
```

3.762.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{(f+g\,x)^n\,(d+e\,x)^{3/2}}{\left(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int(((f + g*x)^n*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(3/2),x)
```

output
```
int(((f + g*x)^n*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(3/2), x)
```

3.763 $\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.763.1 Optimal result

Integrand size = 46, antiderivative size = 104

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{(ae+cdx)\sqrt{d+ex}(f+gx)^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{3}{2}+n,2+n,\frac{cd(f+gx)}{cdf-aeg}\right)}{(cdf-aeg)(1+n)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

output
```
-(c*d*x+a*e)*(g*x+f)^(1+n)*hypergeom([1, 3/2+n],[2+n],c*d*(g*x+f)/(-a*e*g+
c*d*f))*(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(1+n)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)
```

3.763.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{g(ae+cdx)}{-cdf+aeg}\right)}{cd\sqrt{d+ex}}$$

input
```
Integrate[(Sqrt[d + e*x]*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2],x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^n*Hypergeometric2F1[1/2, -n, 3/
2, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(c*d*Sqrt[d + e*x]*((c*d*(f + g*
x))/(c*d*f - a*e*g))^n)
```

3.763.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \ 1268$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{(f+gx)^n}{\sqrt{ae+cdx}}dx}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \ 80$$

$$\frac{\sqrt{d+ex}(f+gx)^n\sqrt{ae+cdx}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\int \frac{\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n}{\sqrt{ae+cdx}}dx}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \ 79$$

$$\frac{2\sqrt{d+ex}(f+gx)^n(ae+cdx)\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},-\frac{g(ae+cdx)}{cdf-aeg}\right)}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

input
```
Int[(Sqrt[d + e*x]*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
],x]
```

output
```
(2*(a*e + c*d*x)*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[1/2, -n, 3/2,
 -((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(c*d*((c*d*(f + g*x))/(c*d*f - a*e
*g))^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

3.763.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.763.4 Maple [F]

$$\int \frac{\sqrt{ex+d}\,(gx+f)^{n}}{\sqrt{ade+(e^{2}a+c\,d^{2})\,x+cde\,x^{2}}}dx$$

input
```
int((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
int((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

3.763.5 Fricas [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{ex+d}(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x), x)
```

3.763.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output
```
Integral(sqrt(d + e*x)*(f + g*x)**n/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.763.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{ex+d}(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x), x)
```

3.763.8 Giac [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{ex+d}(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x), x)
```

3.763.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(f+g\,x)^n\,\sqrt{d+e\,x}}{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int(((f + g*x)^n*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(1/2),x)
```

output
```
int(((f + g*x)^n*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(1/2), x)
```

3.764 $\int \frac{(f+gx)^n\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

3.764.1 Optimal result

Integrand size = 46, antiderivative size = 104

$$\int \frac{(f+gx)^n\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = -\frac{(ae+cdx)(f+gx)^{1+n}\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,\text{Hypergeometric2F1}\left(1,\frac{5}{2}+n,2+n,\frac{cd(f+gx)}{cdf-aeg}\right)}{(cdf-aeg)(1+n)\sqrt{d+ex}}$$

output
```
-(c*d*x+a*e)*(g*x+f)^(1+n)*hypergeom([1, 5/2+n],[2+n],c*d*(g*x+f)/(-a*e*g+
c*d*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(1+n)/(e*x+
d)^(1/2)
```

3.764.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int \frac{(f+gx)^n\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2((ae+cdx)(d+ex))^{3/2}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{3}{2},-n,\frac{5}{2},\frac{g(ae+cdx)}{-cdf+aeg}\right)}{3cd(d+ex)^{3/2}}$$

input
```
Integrate[((f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
 + e*x],x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2)*(f + g*x)^n*Hypergeometric2F1[3/2, -n,
5/2, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(3*c*d*(d + e*x)^(3/2)*((c*d*(
f + g*x))/(c*d*f - a*e*g))^n)
```

3.764.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^n\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1268}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\int \sqrt{ae+cdx}(f+gx)^ndx}{\sqrt{d+ex}\sqrt{ae+cdx}}$$

$$\Big\downarrow \text{80}$$

$$\frac{(f+gx)^n\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\int \sqrt{ae+cdx}\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^ndx}{\sqrt{d+ex}\sqrt{ae+cdx}}$$

$$\Big\downarrow \text{79}$$

$$\frac{2(f+gx)^n(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{3}{2},-n,\frac{5}{2},-\frac{g(ae+cdx)}{cdf-aeg}\right)}{3cd\sqrt{d+ex}}$$

input
```
Int[((f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x
],x]
```

output
```
(2*(a*e + c*d*x)*(f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]*H
ypergeometric2F1[3/2, -n, 5/2, -((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(3*c
*d*Sqrt[d + e*x]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
```

3.764.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.764.4 Maple [F]

$$\int \frac{(gx+f)^n \sqrt{ade+(e^2a+c\,d^2)\,x+cde\,x^2}}{\sqrt{ex+d}}dx$$

input
```
int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

3.764.5 Fricas [F]

$$\int \frac{(f+gx)^n\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^n}{\sqrt{ex+d}}\,dx$$

input
```
integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="fricas")
```

output
```
integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^n/sqrt(e*x
+ d), x)
```

3.764.6 Sympy [F]

$$\int \frac{(f+gx)^n\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}(f+gx)^n}{\sqrt{d+ex}}\,dx$$

input
```
integrate((g*x+f)**n*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**
(1/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**n/sqrt(d + e*x), x)
```

3.764.7 Maxima [F]

$$\int \frac{(f+gx)^n\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^n}{\sqrt{ex+d}}\,dx$$

input
```
integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^n/sqrt(e*x
 + d), x)
```

3.764.8 Giac [F]

$$\int \frac{(f+gx)^n\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^n}{\sqrt{ex+d}}\,dx$$

input
```
integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="giac")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^n/sqrt(e*x
 + d), x)
```

3.764.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \int \frac{(f+g\,x)^n\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{\sqrt{d+e\,x}}\,dx$$

input
```
int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^
(1/2),x)
```

output
```
int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^
(1/2), x)
```

3.765 $\int \frac{(f+gx)^n\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

3.765.1 Optimal result

Integrand size = 46, antiderivative size = 104

$$\int \frac{(f+gx)^n\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = -\frac{(ae+cdx)(f+gx)^{1+n}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\text{Hypergeometric2F1}\left(1,\frac{7}{2}+n,2+n,\frac{cd(f+gx)}{cdf-aeg}\right)}{(cdf-aeg)(1+n)(d+ex)^{3/2}}$$

output
```
-(c*d*x+a*e)*(g*x+f)^(1+n)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*hyperge
om([1, 7/2+n],[2+n],c*d*(g*x+f)/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(1+n)/(e*x+
d)^(3/2)
```

3.765.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int \frac{(f+gx)^n\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2((ae+cdx)(d+ex))^{5/2}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hyperg}}{5cd(d+ex)^{5/2}}$$

input
```
Integrate[((f + g*x)^n*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
 e*x)^(3/2),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(f + g*x)^n*Hypergeometric2F1[5/2, -n,
7/2, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(5*c*d*(d + e*x)^(5/2)*((c*d*(
f + g*x))/(c*d*f - a*e*g))^n)
```

3.765.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^n \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{1268}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\int (ae+cdx)^{3/2}(f+gx)^ndx}{\sqrt{d+ex}\sqrt{ae+cdx}}$$

$$\Big\downarrow \text{80}$$

$$\frac{(f+gx)^n\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\int (ae+cdx)^{3/2}\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^ndx}{\sqrt{d+ex}\sqrt{ae+cdx}}$$

$$\Big\downarrow \text{79}$$

$$\frac{2(f+gx)^n(ae+cdx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{5}{2},-n,\frac{7}{2},-\frac{g(ae+cdx)}{cdf-aeg}\right)}{5cd\sqrt{d+ex}}$$

input
```
Int[((f + g*x)^n*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^
(3/2),x]
```

output
```
(2*(a*e + c*d*x)^2*(f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
*Hypergeometric2F1[5/2, -n, 7/2, -((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(5
*c*d*Sqrt[d + e*x]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
```

3.765.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.765.4 Maple [F]

$$\int \frac{(gx+f)^n \left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}}dx$$

input
```
int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

output
```
int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

3.765.5 Fricas [F]

$$\int \frac{(f+gx)^n \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^n}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="fricas")
```

output
```
integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x + a*e)*(g*x +
f)^n/sqrt(e*x + d), x)
```

3.765.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)**n*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**
(3/2),x)
```

output
```
Timed out
```

3.765.7 Maxima [F]

$$\int \frac{(f+gx)^n \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^n}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^n/(e*x +
 d)^(3/2), x)
```

3.765.8 Giac [F]

$$\int \frac{(f+gx)^n (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^n}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^n/(e*x +
 d)^(3/2), x)
```

3.765.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \int \frac{(f+g\,x)^n\,(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{3/2}}{(d+e\,x)^{3/2}}\,dx$$

input
```
int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^
(3/2),x)
```

output
```
int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^
(3/2), x)
```

3.766 $\int \frac{(f+gx)^n\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

3.766.1 Optimal result

Integrand size = 46, antiderivative size = 104

$$\int \frac{(f+gx)^n\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = -\frac{(ae+cdx)(f+gx)^{1+n}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\text{Hypergeometric2F1}\left(1,\frac{9}{2}+n,2+n,\frac{cd(f+gx)}{cdf-aeg}\right)}{(cdf-aeg)(1+n)(d+ex)^{5/2}}$$

output
```
-(c*d*x+a*e)*(g*x+f)^(1+n)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*hyperge
om([1, 9/2+n],[2+n],c*d*(g*x+f)/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(1+n)/(e*x+
d)^(5/2)
```

3.766.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^n\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)}{7cd\sqrt{d+ex}}$$

input
```
Integrate[((f + g*x)^n*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
 e*x)^(5/2),x]
```

output
```
(2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^n*Hypergeometri
c2F1[7/2, -n, 9/2, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(7*c*d*Sqrt[d +
e*x]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
```

3.766.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^n \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \ 1268$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\int (ae+cdx)^{5/2}(f+gx)^ndx}{\sqrt{d+ex}\sqrt{ae+cdx}}$$

$$\Big\downarrow \ 80$$

$$\frac{(f+gx)^n\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\int (ae+cdx)^{5/2}\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^ndx}{\sqrt{d+ex}\sqrt{ae+cdx}}$$

$$\Big\downarrow \ 79$$

$$\frac{2(f+gx)^n(ae+cdx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{7}{2},-n,\frac{9}{2},-\frac{g(ae+cdx)}{cdf-aeg}\right)}{7cd\sqrt{d+ex}}$$

input
```
Int[((f + g*x)^n*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^
(5/2),x]
```

output
```
(2*(a*e + c*d*x)^3*(f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
*Hypergeometric2F1[7/2, -n, 9/2, -((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(7
*c*d*Sqrt[d + e*x]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
```

3.766.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.766.4 Maple [F]

$$\int \frac{(gx+f)^n \left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}}dx$$

input
```
int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)
```

output
```
int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)
```

3.766.5 Fricas [F]

$$\int \frac{(f+gx)^n \left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(gx+f)^n}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),
x, algorithm="fricas")
```

output
```
integral((c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*(g*x + f)^n/sqrt(e*x + d), x)
```

3.766.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n \left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**n*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**
(5/2),x)
```

output
```
Timed out
```

3.766.7 Maxima [F]

$$\int \frac{(f+gx)^n \left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(gx+f)^n}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g*x + f)^n/(e*x +
 d)^(5/2), x)
```

3.766.8 Giac [F]

$$\int \frac{(f+gx)^n \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(gx+f)^n}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),
x, algorithm="giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g*x + f)^n/(e*x +
 d)^(5/2), x)
```

3.766.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \int \frac{(f+g\,x)^n\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{\left(d+e\,x\right)^{5/2}}\,dx$$

input
```
int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^
(5/2),x)
```

output
```
int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^
(5/2), x)
```

3.767 $\int (d+ex)^m(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}$

3.767.1 Optimal result

Integrand size = 44, antiderivative size = 103

$$\int (d+ex)^m(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx =$$
$$-\frac{(ae+cdx)(d+ex)^m(f+gx)^{1+n}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(1,2-m+n,2\right.}{(cdf-aeg)(1+n)}$$

output
```
-(c*d*x+a*e)*(e*x+d)^m*(g*x+f)^(1+n)*hypergeom([1, 2-m+n],[2+n],c*d*(g*x+f
)/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(1+n)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
m)
```

3.767.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.92

$$\int (d+ex)^m(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$$
$$=\frac{\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m((ae+cdx)(d+ex))^{-m}(f+gx)^{1+n}\text{Hypergeometric2F1}\left(m,1+n,2+n,\frac{cd(f+gx)}{cdf-aeg}\right.}{g(1+n)}$$

input
```
Integrate[((d + e*x)^m*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^m,x]
```

output
```
(((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*(f + g*x)^(1 + n)*Hy
pergeometric2F1[m, 1 + n, 2 + n, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*(1 +
 n)*((a*e + c*d*x)*(d + e*x))^m)
```

3.767.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m(f+gx)^n \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} dx$$

$$\Big\downarrow \; 1268$$

$$(d+ex)^m(ae+cdx)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\int (ae+cdx)^{-m}(f+gx)^n dx$$

$$\Big\downarrow \; 80$$

$$(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m\int (f+gx)^n\left(-\frac{cdxg}{cdf-aeg}-\frac{aeg}{cdf-aeg}\right)^{-m}dx$$

$$\Big\downarrow \; 79$$

$$\frac{(d+ex)^m(f+gx)^{n+1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m \text{Hypergeometric2F1}\left(m,n+1,n+2,\frac{cd(f+gx)}{cdf-aeg}\right)}{g(n+1)}$$

input
```
Int[((d + e*x)^m*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

output
```
((-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*(f + g*x)^(1 + n)*Hy
pergeometric2F1[m, 1 + n, 2 + n, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*(1 +
 n)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

3.767.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.767.4 Maple [F]

$$\int (ex+d)^m (gx+f)^n \left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{-m}dx$$

input
```
int((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output
```
int((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

3.767.5 Fricas [F]

$$\int (d+ex)^m(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \int \frac{(ex+d)^m(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="fricas")
```

output
```
integral((e*x + d)^m*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m
, x)
```

3.767.6 Sympy [F(-2)]

Exception generated.

$$\int (d+ex)^m(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x+d)**m*(g*x+f)**n/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),
x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.767.7 Maxima [F]

$$\int (d+ex)^m(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$$
$$= \int \frac{(ex+d)^m(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^m} dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="maxima")
```

output
```
integrate((e*x + d)^m*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^
m, x)
```

3.767.8 Giac [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \text{Timed out}$$

input
```
integrate((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="giac")
```

output
```
Timed out
```

3.767.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \int \frac{(f+g\,x)^n\,(d+e\,x)^m}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^m}\,dx$$

input
```
int(((f + g*x)^n*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

output
```
int(((f + g*x)^n*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m, x
)
```

3.768 $\int (d+ex)^m(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}$

3.768.1 Optimal result

Integrand size = 44, antiderivative size = 343

$$\int (d+ex)^m(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{6(cdf-aeg)^2\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^4d^4e(1-m)(2-m)(3-m)(4-m)} + \frac{6g(cdf-aeg)^2(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^3d^3e(2-m)(3-m)(4-m)} + \frac{3(cdf-aeg)(d+ex)^{-1+m}(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^2d^2(3-m)(4-m)} + \frac{(d+ex)^{-1+m}(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(4-m)}$$

output
```
-6*(-a*e*g+c*d*f)^2*(a*e^2*g+c*d*(d*g*(1-m)-e*f*(2-m)))*(e*x+d)^(-1+m)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c^4/d^4/e/(m^2-7*m+12)/(m^2-3*m+2)+6*
g*(-a*e*g+c*d*f)^2*(e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c^3/d
^3/e/(2-m)/(3-m)/(4-m)+3*(-a*e*g+c*d*f)*(e*x+d)^(-1+m)*(g*x+f)^2*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c^2/d^2/(3-m)/(4-m)+(e*x+d)^(-1+m)*(g*x+f)^
3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c/d/(4-m)
```

3.768.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.39

$$\int (d+ex)^m(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} \, dx$$
$$= \frac{(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}\left(-\frac{(cdf-aeg)^3}{-1+m}-\frac{3g(cdf-aeg)^2(ae+cdx)}{-2+m}+\frac{3g^2(-cdf+aeg)(ae+cdx)^2}{-3+m}-\frac{g^3(ae+cdx}{-4+m}\right.}{c^4d^4}$$

input
```
Integrate[((d + e*x)^m*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^m,x]
```

output
```
((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*(-((c*d*f - a*e*g)^3
/(-1 + m)) - (3*g*(c*d*f - a*e*g)^2*(a*e + c*d*x))/(-2 + m) + (3*g^2*(-(c*
d*f) + a*e*g)*(a*e + c*d*x)^2)/(-3 + m) - (g^3*(a*e + c*d*x)^3)/(-4 + m)))
/(c^4*d^4)
```

3.768.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1253, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^3(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \, dx$$

$$\Big\downarrow \text{1253}$$

$$\frac{3(cdf-aeg)\int (d+ex)^m(f+gx)^2 \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{-m}dx}{cd(4-m)}+\frac{(f+gx)^3(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(4-m)}$$

$$\Big\downarrow \text{1253}$$

$$\frac{3(cdf-aeg)\left(\frac{2(cdf-aeg)\int(d+ex)^m(f+gx)(cdex^2+(cd^2+ae^2)x+ade)^{-m}dx}{cd(3-m)}+\frac{(f+gx)^2(d+ex)^{m-1}\left(x(ae^2+cd^2)+ade+cdex^2\right)^{1-m}}{cd(3-m)}\right)}{cd(4-m)}+\frac{(f+gx)^3(d+ex)^{m-1}\left(x(ae^2+cd^2)+ade+cdex^2\right)^{1-m}}{cd(4-m)}$$

$\Big\downarrow$ 1221

$$\frac{3(cdf-aeg)\left(\frac{2(cdf-aeg)\left(\frac{g(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cde(2-m)}-\frac{\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)\int(d+ex)^m\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{-m}dx}{cde(2-m)}\right)}{cd(3-m)}\right.}{cd(4-m)}$$

$$\frac{(f+gx)^3(d+ex)^{m-1}\left(x(ae^2+cd^2)+ade+cdex^2\right)^{1-m}}{cd(4-m)}$$

$\Big\downarrow$ 1122

$$\frac{3(cdf-aeg)\left(\frac{2(cdf-aeg)\left(\frac{g(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cde(2-m)}-\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^2d^2e(1-m)(2-m)}\right)}{cd(3-m)}\right.}{cd(4-m)}$$

$$\frac{(f+gx)^3(d+ex)^{m-1}\left(x(ae^2+cd^2)+ade+cdex^2\right)^{1-m}}{cd(4-m)}$$

input
```
Int[((d + e*x)^m*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

output
```
((d + e*x)^(-1 + m)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1
 - m))/(c*d*(4 - m)) + (3*(c*d*f - a*e*g)*(((d + e*x)^(-1 + m)*(f + g*x)^2
*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*(3 - m)) + (2*(c*d*
f - a*e*g)*(-(((a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)^(-1 +
 m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^2*d^2*e*(1 - m)*(2
 - m))) + (g*(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/
(c*d*e*(2 - m))))/(c*d*(3 - m))))/(c*d*(4 - m))
```

3.768. $\int(d+ex)^m(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{-m}\,dx$

3.768.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

3.768.4 Maple [A] (verified)

Time = 4.17 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.54

method	result
gosper	$-\frac{(ex+d)^m \left(c^3 d^3 g^3 m^3 x^3+3 c^3 d^3 f\, g^2 m^3 x^2-6 c^3 d^3 g^3 m^2 x^3+3 a\, c^2 d^2 e\, g^3 m^2 x^2+3 c^3 d^3 f^2 g\, m^3 x-21 c^3 d^3 f\, g^2 m^2 x^2+11 c^3 d^3 g^3 m\right.}{}$
risch	$-\frac{\left(c^4 d^4 g^3 m^3 x^4+a\, c^3 d^3 e\, g^3 m^3 x^3+3 c^4 d^4 f\, g^2 m^3 x^3-6 c^4 d^4 g^3 m^2 x^4+3 a\, c^3 d^3 e f\, g^2 m^3 x^2-3 a\, c^3 d^3 e\, g^3 m^2 x^3+3 c^4 d^4 f^2 g\, m^3 x^2-\right.}{}$
parallelrisch	Expression too large to display

input
```
int((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,method=_RE
TURNVERBOSE)
```

output
```
-(e*x+d)^m*(c^3*d^3*g^3*m^3*x^3+3*c^3*d^3*f*g^2*m^3*x^2-6*c^3*d^3*g^3*m^2*
x^3+3*a*c^2*d^2*e*g^3*m^2*x^2+3*c^3*d^3*f^2*g*m^3*x-21*c^3*d^3*f*g^2*m^2*x
^2+11*c^3*d^3*g^3*m*x^3+6*a*c^2*d^2*e*f*g^2*m^2*x-9*a*c^2*d^2*e*g^3*m*x^2+
c^3*d^3*f^3*m^3-24*c^3*d^3*f^2*g*m^2*x+42*c^3*d^3*f*g^2*m*x^2-6*c^3*d^3*g^
3*x^3+6*a^2*c*d*e^2*g^3*m*x+3*a*c^2*d^2*e*f^2*g*m^2-30*a*c^2*d^2*e*f*g^2*m
*x+6*a*c^2*d^2*e*g^3*x^2-9*c^3*d^3*f^3*m^2+57*c^3*d^3*f^2*g*m*x-24*c^3*d^3
*f*g^2*x^2+6*a^2*c*d*e^2*f*g^2*m-6*a^2*c*d*e^2*g^3*x-21*a*c^2*d^2*e*f^2*g*
m+24*a*c^2*d^2*e*f*g^2*x+26*c^3*d^3*f^3*m-36*c^3*d^3*f^2*g*x+6*a^3*e^3*g^3
-24*a^2*c*d*e^2*f*g^2+36*a*c^2*d^2*e*f^2*g-24*c^3*d^3*f^3)*(c*d*x+a*e)/((c
*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^m)/c^4/d^4/(m^4-10*m^3+35*m^2-50*m+24)
```

3.768.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(321) = 642$.

Time = 0.33 (sec) , antiderivative size = 705, normalized size of antiderivative = 2.06

$$\int (d+ex)^m(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{(ac^3d^3ef^3m^3-24\,ac^3d^3ef^3+36\,a^2c^2d^2e^2f^2g-24\,a^3cde^3fg^2+6\,a^4e^4g^3+(c^4d^4g^3m^3-6\,c^4d^4g^3m^2+1}{}$$

input
```
integrate((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="fricas")
```

output
```
-(a*c^3*d^3*e*f^3*m^3 - 24*a*c^3*d^3*e*f^3 + 36*a^2*c^2*d^2*e^2*f^2*g - 24
*a^3*c*d*e^3*f*g^2 + 6*a^4*e^4*g^3 + (c^4*d^4*g^3*m^3 - 6*c^4*d^4*g^3*m^2
+ 11*c^4*d^4*g^3*m - 6*c^4*d^4*g^3)*x^4 - (24*c^4*d^4*f*g^2 - (3*c^4*d^4*f
*g^2 + a*c^3*d^3*e*g^3)*m^3 + 3*(7*c^4*d^4*f*g^2 + a*c^3*d^3*e*g^3)*m^2 -
2*(21*c^4*d^4*f*g^2 + a*c^3*d^3*e*g^3)*m)*x^3 - 3*(3*a*c^3*d^3*e*f^3 - a^2
*c^2*d^2*e^2*f^2*g)*m^2 - 3*(12*c^4*d^4*f^2*g - (c^4*d^4*f^2*g + a*c^3*d^3
*e*f*g^2)*m^3 + (8*c^4*d^4*f^2*g + 5*a*c^3*d^3*e*f*g^2 - a^2*c^2*d^2*e^2*g
^3)*m^2 - (19*c^4*d^4*f^2*g + 4*a*c^3*d^3*e*f*g^2 - a^2*c^2*d^2*e^2*g^3)*m
)*x^2 + (26*a*c^3*d^3*e*f^3 - 21*a^2*c^2*d^2*e^2*f^2*g + 6*a^3*c*d*e^3*f*g
^2)*m - (24*c^4*d^4*f^3 - (c^4*d^4*f^3 + 3*a*c^3*d^3*e*f^2*g)*m^3 + 3*(3*c
^4*d^4*f^3 + 7*a*c^3*d^3*e*f^2*g - 2*a^2*c^2*d^2*e^2*f*g^2)*m^2 - 2*(13*c^
4*d^4*f^3 + 18*a*c^3*d^3*e*f^2*g - 12*a^2*c^2*d^2*e^2*f*g^2 + 3*a^3*c*d*e^
3*g^3)*m)*x)*(e*x + d)^m/((c^4*d^4*m^4 - 10*c^4*d^4*m^3 + 35*c^4*d^4*m^2 -
 50*c^4*d^4*m + 24*c^4*d^4)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
```

3.768.6 Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**m*(g*x+f)**3/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),
x)
```

output
```
Timed out
```

3.768.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.97

$$\int (d+ex)^m(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=-\frac{(cdx+ae)f^3}{(cdx+ae)^m cd(m-1)}-\frac{3\left(c^2d^2(m-1)x^2+acdemx+a^2e^2\right)f^2g}{(m^2-3\,m+2)(cdx+ae)^m c^2d^2}$$
$$-\frac{3\left((m^2-3\,m+2)c^3d^3x^3+(m^2-m)ac^2d^2ex^2+2\,a^2cde^2mx+2\,a^3e^3\right)fg^2}{(m^3-6\,m^2+11\,m-6)(cdx+ae)^m c^3d^3}$$
$$-\frac{\left((m^3-6\,m^2+11\,m-6)c^4d^4x^4+(m^3-3\,m^2+2\,m)ac^3d^3ex^3+3\,(m^2-m)a^2c^2d^2e^2x^2+6\,a^3cde^3mx+\right.}{(m^4-10\,m^3+35\,m^2-50\,m+24)(cdx+ae)^m c^4d^4}$$

input
```
integrate((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="maxima")
```

output
```
-(c*d*x + a*e)*f^3/((c*d*x + a*e)^m*c*d*(m - 1)) - 3*(c^2*d^2*(m - 1)*x^2
+ a*c*d*e*m*x + a^2*e^2)*f^2*g/((m^2 - 3*m + 2)*(c*d*x + a*e)^m*c^2*d^2) -
 3*((m^2 - 3*m + 2)*c^3*d^3*x^3 + (m^2 - m)*a*c^2*d^2*e*x^2 + 2*a^2*c*d*e^
2*m*x + 2*a^3*e^3)*f*g^2/((m^3 - 6*m^2 + 11*m - 6)*(c*d*x + a*e)^m*c^3*d^3
) - ((m^3 - 6*m^2 + 11*m - 6)*c^4*d^4*x^4 + (m^3 - 3*m^2 + 2*m)*a*c^3*d^3*
e*x^3 + 3*(m^2 - m)*a^2*c^2*d^2*e^2*x^2 + 6*a^3*c*d*e^3*m*x + 6*a^4*e^4)*g
^3/((m^4 - 10*m^3 + 35*m^2 - 50*m + 24)*(c*d*x + a*e)^m*c^4*d^4)
```

3.768.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1926 vs. $2(321) = 642$.

Time = 0.49 (sec) , antiderivative size = 1926, normalized size of antiderivative = 5.62

$$\int (d+ex)^m(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="giac")
```

output
```
-((e*x + d)^m*c^4*d^4*g^3*m^3*x^4*e^(-m*log(c*d*x + a*e) - m*log(e*x + d))
 + 3*(e*x + d)^m*c^4*d^4*f*g^2*m^3*x^3*e^(-m*log(c*d*x + a*e) - m*log(e*x
+ d)) + (e*x + d)^m*a*c^3*d^3*e*g^3*m^3*x^3*e^(-m*log(c*d*x + a*e) - m*log
(e*x + d)) - 6*(e*x + d)^m*c^4*d^4*g^3*m^2*x^4*e^(-m*log(c*d*x + a*e) - m*
log(e*x + d)) + 3*(e*x + d)^m*c^4*d^4*f^2*g*m^3*x^2*e^(-m*log(c*d*x + a*e)
 - m*log(e*x + d)) + 3*(e*x + d)^m*a*c^3*d^3*e*f*g^2*m^3*x^2*e^(-m*log(c*d
*x + a*e) - m*log(e*x + d)) - 21*(e*x + d)^m*c^4*d^4*f*g^2*m^2*x^3*e^(-m*l
og(c*d*x + a*e) - m*log(e*x + d)) - 3*(e*x + d)^m*a*c^3*d^3*e*g^3*m^2*x^3*
e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + 11*(e*x + d)^m*c^4*d^4*g^3*m*x^
4*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + (e*x + d)^m*c^4*d^4*f^3*m^3*x
*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + 3*(e*x + d)^m*a*c^3*d^3*e*f^2*
g*m^3*x*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) - 24*(e*x + d)^m*c^4*d^4*
f^2*g*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) - 15*(e*x + d)^m*a*
c^3*d^3*e*f*g^2*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + 3*(e*x
+ d)^m*a^2*c^2*d^2*e^2*g^3*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)
) + 42*(e*x + d)^m*c^4*d^4*f*g^2*m*x^3*e^(-m*log(c*d*x + a*e) - m*log(e*x
+ d)) + 2*(e*x + d)^m*a*c^3*d^3*e*g^3*m*x^3*e^(-m*log(c*d*x + a*e) - m*log
(e*x + d)) - 6*(e*x + d)^m*c^4*d^4*g^3*x^4*e^(-m*log(c*d*x + a*e) - m*log(
e*x + d)) + (e*x + d)^m*a*c^3*d^3*e*f^3*m^3*e^(-m*log(c*d*x + a*e) - m*log
(e*x + d)) - 9*(e*x + d)^m*c^4*d^4*f^3*m^2*x*e^(-m*log(c*d*x + a*e) - m...
```

3.768.9 Mupad [B] (verification not implemented)

Time = 12.31 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.79

$$\int (d+ex)^m(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx =$$

$$-\frac{\frac{g^3\,x^4\,{\left(d+e\,x\right)}^m\,\left(m^3-6\,m^2+11\,m-6\right)}{m^4-10\,m^3+35\,m^2-50\,m+24}+\frac{x\,{\left(d+e\,x\right)}^m\,\left(6\,a^3\,c\,d\,e^3\,g^3\,m+6\,a^2\,c^2\,d^2\,e^2\,f\,g^2\,m^2-24\,a^2\,c^2\,d^2\,e^2\,f\,g^2\,m+3\,a\,c^3\,d^3\,e\,f^2\,g\,m^3-21\,a\,c^3}{c^4\,d^4\,\left(m^4-10\,m^3+35\,m\right.}}{}$$

input
```
int(((f + g*x)^3*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

output
```
-((g^3*x^4*(d + e*x)^m*(11*m - 6*m^2 + m^3 - 6))/(35*m^2 - 50*m - 10*m^3 +
 m^4 + 24) + (x*(d + e*x)^m*(26*c^4*d^4*f^3*m - 24*c^4*d^4*f^3 - 9*c^4*d^4
*f^3*m^2 + c^4*d^4*f^3*m^3 + 6*a^3*c*d*e^3*g^3*m - 24*a^2*c^2*d^2*e^2*f*g^
2*m + 36*a*c^3*d^3*e*f^2*g*m + 6*a^2*c^2*d^2*e^2*f*g^2*m^2 - 21*a*c^3*d^3*
e*f^2*g*m^2 + 3*a*c^3*d^3*e*f^2*g*m^3))/(c^4*d^4*(35*m^2 - 50*m - 10*m^3 +
 m^4 + 24)) + (a*e*(d + e*x)^m*(6*a^3*e^3*g^3 - 24*c^3*d^3*f^3 + 26*c^3*d^
3*f^3*m - 9*c^3*d^3*f^3*m^2 + c^3*d^3*f^3*m^3 + 36*a*c^2*d^2*e*f^2*g - 24*
a^2*c*d*e^2*f*g^2 - 21*a*c^2*d^2*e*f^2*g*m + 6*a^2*c*d*e^2*f*g^2*m + 3*a*c
^2*d^2*e*f^2*g*m^2))/(c^4*d^4*(35*m^2 - 50*m - 10*m^3 + m^4 + 24)) + (3*g*
x^2*(m - 1)*(d + e*x)^m*(12*c^2*d^2*f^2 + a^2*e^2*g^2*m - 7*c^2*d^2*f^2*m
+ c^2*d^2*f^2*m^2 - 4*a*c*d*e*f*g*m + a*c*d*e*f*g*m^2))/(c^2*d^2*(35*m^2 -
 50*m - 10*m^3 + m^4 + 24)) + (g^2*x^3*(d + e*x)^m*(a*e*g*m - 12*c*d*f + 3
*c*d*f*m)*(m^2 - 3*m + 2))/(c*d*(35*m^2 - 50*m - 10*m^3 + m^4 + 24)))/(x*(
a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
```

3.768. $\int (d+ex)^m(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$

3.769 $\int (d+ex)^m(f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}$

3.769.1 Optimal result

Integrand size = 44, antiderivative size = 246

$$\int (d+ex)^m(f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{2(cdf-aeg)\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^3d^3e(1-m)(2-m)(3-m)} + \frac{2g(cdf-aeg)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^2d^2e(2-m)(3-m)} + \frac{(d+ex)^{-1+m}(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(3-m)}$$

output
```
-2*(-a*e*g+c*d*f)*(a*e^2*g+c*d*(d*g*(1-m)-e*f*(2-m)))*(e*x+d)^(-1+m)*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c^3/d^3/e/(1-m)/(2-m)/(3-m)+2*g*(-a*e*g
+c*d*f)*(e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c^2/d^2/e/(2-m)/
(3-m)+(e*x+d)^(-1+m)*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c/d
/(3-m)
```

3.769.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.53

$$\int (d+ex)^m(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx =$$

$$-\frac{(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}\left(2a^2e^2g^2+2acdeg(f(-3+m)+g(-1+m)x)+c^2d^2(f^2(6-5\right.}{c^3d^3(-3+m)(-2+m)(-1+m)}$$

input
```
Integrate[((d + e*x)^m*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^m,x]
```

output
```
-(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*(2*a^2*e^2*g^2 + 2
*a*c*d*e*g*(f*(-3 + m) + g*(-1 + m)*x) + c^2*d^2*(f^2*(6 - 5*m + m^2) + 2*
f*g*(3 - 4*m + m^2)*x + g^2*(2 - 3*m + m^2)*x^2)))/(c^3*d^3*(-3 + m)*(-2 +
 m)*(-1 + m)))
```

3.769.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.068, Rules used = {1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}dx$$

$$\Big\downarrow \ 1253$$

$$\frac{2(cdf-aeg)\int (d+ex)^m(f+gx)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{-m}dx}{cd(3-m)}+\frac{(f+gx)^2(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(3-m)}$$

$$\Big\downarrow \ 1221$$

$$\frac{2(cdf-aeg)\left(\frac{g(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cde(2-m)}-\frac{\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)\int (d+ex)^m\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{-m}dx}{cde(2-m)}\right)}{cd(3-m)}$$
$$\frac{(f+gx)^2(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(3-m)}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2(cdf-aeg)\left(\frac{g(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cde(2-m)}-\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^2d^2e(1-m)(2-m)}\right)}{cd(3-m)}$$
$$\frac{(f+gx)^2(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(3-m)}$$

input
```
Int[((d + e*x)^m*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

output
```
((d + e*x)^(-1 + m)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1
 - m))/(c*d*(3 - m)) + (2*(c*d*f - a*e*g)*(-(((a*e^2*g + c*d*(d*g*(1 - m)
- e*f*(2 - m)))*(d + e*x)^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(1 - m))/(c^2*d^2*e*(1 - m)*(2 - m))) + (g*(d + e*x)^m*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*e*(2 - m))))/(c*d*(3 - m))
```

3.769.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

3.769. $\int (d+ex)^m(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

3.769.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.96

method	result
gosper	$-\frac{(cdx+ae)\left(c^2d^2g^2m^2x^2+2c^2d^2fg\,m^2x-3c^2d^2g^2m\,x^2+2acde\,g^2mx+c^2d^2f^2m^2-8c^2d^2fgmx+2g^2x^2c^2d^2+2acdefgm-2ac\right.}{c^3d^3(m^3-6m^2+11m-}$
risch	$-\frac{\left(c^3d^3g^2m^2x^3+a\,c^2d^2e\,g^2m^2x^2+2c^3d^3fg\,m^2x^2-3c^3d^3g^2m\,x^3+2a\,c^2d^2efg\,m^2x-a\,c^2d^2e\,g^2m\,x^2+c^3d^3f^2m^2x-8c^3d^3fgm\right.}{}$
parallelrisch	$\frac{\left(-x^3(ex+d)^mc^3d^3e\,g^2m^2+3x^3(ex+d)^mc^3d^3e\,g^2m-x(ex+d)^mc^3d^3e\,f^2m^2-6x^2(ex+d)^mc^3d^3efg+5x(ex+d)^mc^3d^3e\,f^2m-(\right.}{}$

input
```
int((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,method=_RE
TURNVERBOSE)
```

output
```
-(c*d*x+a*e)*(c^2*d^2*g^2*m^2*x^2+2*c^2*d^2*f*g*m^2*x-3*c^2*d^2*g^2*m*x^2+
2*a*c*d*e*g^2*m*x+c^2*d^2*f^2*m^2-8*c^2*d^2*f*g*m*x+2*c^2*d^2*g^2*x^2+2*a*
c*d*e*f*g*m-2*a*c*d*e*g^2*x-5*c^2*d^2*f^2*m+6*c^2*d^2*f*g*x+2*a^2*e^2*g^2-
6*a*c*d*e*f*g+6*c^2*d^2*f^2)*(e*x+d)^m/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^
m)/c^3/d^3/(m^3-6*m^2+11*m-6)
```

3.769.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.42

$$\int (d+ex)^m(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx =$$
$$-\frac{(ac^2d^2ef^2m^2+6\,ac^2d^2ef^2-6\,a^2cde^2fg+2\,a^3e^3g^2+(c^3d^3g^2m^2-3\,c^3d^3g^2m+2\,c^3d^3g^2)x^3+(6\,c^3d^3fg}{}$$

input
```
integrate((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="fricas")
```

output
```
-(a*c^2*d^2*e*f^2*m^2 + 6*a*c^2*d^2*e*f^2 - 6*a^2*c*d*e^2*f*g + 2*a^3*e^3*
g^2 + (c^3*d^3*g^2*m^2 - 3*c^3*d^3*g^2*m + 2*c^3*d^3*g^2)*x^3 + (6*c^3*d^3
*f*g + (2*c^3*d^3*f*g + a*c^2*d^2*e*g^2)*m^2 - (8*c^3*d^3*f*g + a*c^2*d^2*
e*g^2)*m)*x^2 - (5*a*c^2*d^2*e*f^2 - 2*a^2*c*d*e^2*f*g)*m + (6*c^3*d^3*f^2
 + (c^3*d^3*f^2 + 2*a*c^2*d^2*e*f*g)*m^2 - (5*c^3*d^3*f^2 + 6*a*c^2*d^2*e*
f*g - 2*a^2*c*d*e^2*g^2)*m)*x)*(e*x + d)^m/((c^3*d^3*m^3 - 6*c^3*d^3*m^2 +
 11*c^3*d^3*m - 6*c^3*d^3)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
```

3.769.6 Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m (f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m*(g*x+f)**2/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),
x)
```

output
```
Timed out
```

3.769.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.78

$$\begin{aligned}\int (d+ex)^m (f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx \\ = -\frac{(cdx+ae)f^2}{(cdx+ae)^m cd(m-1)} - \frac{2\,(c^2d^2(m-1)x^2+acdemx+a^2e^2)fg}{(m^2-3\,m+2)(cdx+ae)^m c^2d^2} \\ - \frac{((m^2-3\,m+2)c^3d^3x^3+(m^2-m)ac^2d^2ex^2+2\,a^2cde^2mx+2\,a^3e^3)g^2}{(m^3-6\,m^2+11\,m-6)(cdx+ae)^m c^3d^3}\end{aligned}$$

input
```
integrate((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="maxima")
```

output
```
-(c*d*x + a*e)*f^2/((c*d*x + a*e)^m*c*d*(m - 1)) - 2*(c^2*d^2*(m - 1)*x^2
+ a*c*d*e*m*x + a^2*e^2)*f*g/((m^2 - 3*m + 2)*(c*d*x + a*e)^m*c^2*d^2) - (
(m^2 - 3*m + 2)*c^3*d^3*x^3 + (m^2 - m)*a*c^2*d^2*e*x^2 + 2*a^2*c*d*e^2*m*
x + 2*a^3*e^3)*g^2/((m^3 - 6*m^2 + 11*m - 6)*(c*d*x + a*e)^m*c^3*d^3)
```

3.769.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 929 vs. $2(232) = 464$.

Time = 0.54 (sec) , antiderivative size = 929, normalized size of antiderivative = 3.78

$$\int (d+ex)^m (f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx =$$
$$-\frac{(ex+d)^m c^3 d^3 g^2 m^2 x^3 e^{(-m\log(cdx+ae)-m\log(ex+d))} + 2\,(ex+d)^m c^3 d^3 f g m^2 x^2 e^{(-m\log(cdx+ae)-m\log(ex+d))} + (}{}$$

input
```
integrate((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="giac")
```

output
```
-((e*x + d)^m*c^3*d^3*g^2*m^2*x^3*e^(-m*log(c*d*x + a*e) - m*log(e*x + d))
 + 2*(e*x + d)^m*c^3*d^3*f*g*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(e*x +
d)) + (e*x + d)^m*a*c^2*d^2*e*g^2*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(e
*x + d)) - 3*(e*x + d)^m*c^3*d^3*g^2*m*x^3*e^(-m*log(c*d*x + a*e) - m*log(
e*x + d)) + (e*x + d)^m*c^3*d^3*f^2*m^2*x*e^(-m*log(c*d*x + a*e) - m*log(e
*x + d)) + 2*(e*x + d)^m*a*c^2*d^2*e*f*g*m^2*x*e^(-m*log(c*d*x + a*e) - m*
log(e*x + d)) - 8*(e*x + d)^m*c^3*d^3*f*g*m*x^2*e^(-m*log(c*d*x + a*e) - m
*log(e*x + d)) - (e*x + d)^m*a*c^2*d^2*e*g^2*m*x^2*e^(-m*log(c*d*x + a*e)
- m*log(e*x + d)) + 2*(e*x + d)^m*c^3*d^3*g^2*x^3*e^(-m*log(c*d*x + a*e) -
 m*log(e*x + d)) + (e*x + d)^m*a*c^2*d^2*e*f^2*m^2*e^(-m*log(c*d*x + a*e)
- m*log(e*x + d)) - 5*(e*x + d)^m*c^3*d^3*f^2*m*x*e^(-m*log(c*d*x + a*e) -
 m*log(e*x + d)) - 6*(e*x + d)^m*a*c^2*d^2*e*f*g*m*x*e^(-m*log(c*d*x + a*e
) - m*log(e*x + d)) + 2*(e*x + d)^m*a^2*c*d*e^2*g^2*m*x*e^(-m*log(c*d*x +
a*e) - m*log(e*x + d)) + 6*(e*x + d)^m*c^3*d^3*f*g*x^2*e^(-m*log(c*d*x + a
*e) - m*log(e*x + d)) - 5*(e*x + d)^m*a*c^2*d^2*e*f^2*m*e^(-m*log(c*d*x +
a*e) - m*log(e*x + d)) + 2*(e*x + d)^m*a^2*c*d*e^2*f*g*m*e^(-m*log(c*d*x +
 a*e) - m*log(e*x + d)) + 6*(e*x + d)^m*c^3*d^3*f^2*x*e^(-m*log(c*d*x + a*
e) - m*log(e*x + d)) + 6*(e*x + d)^m*a*c^2*d^2*e*f^2*e^(-m*log(c*d*x + a*e
) - m*log(e*x + d)) - 6*(e*x + d)^m*a^2*c*d*e^2*f*g*e^(-m*log(c*d*x + a*e)
 - m*log(e*x + d)) + 2*(e*x + d)^m*a^3*e^3*g^2*e^(-m*log(c*d*x + a*e) -...
```

3.769.9 Mupad [B] (verification not implemented)

Time = 12.13 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.33

$$\int (d+ex)^m(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx =$$

$$-\frac{\frac{g^2\,x^3\,(d+e\,x)^m\,(m^2-3\,m+2)}{m^3-6\,m^2+11\,m-6}+\frac{x\,(d+e\,x)^m\,(2\,a^2\,c\,d\,e^2\,g^2\,m+2\,a\,c^2\,d^2\,e\,f\,g\,m^2-6\,a\,c^2\,d^2\,e\,f\,g\,m+c^3\,d^3\,f^2\,m^2-5\,c^3\,d^3\,f^2\,m+6\,c^3\,d^3\,f^2)}{c^3\,d^3\,(m^3-6\,m^2+11\,m-6)}+}{(c\,d\,e\,x^2+(c\,d^2}$$

input
```
int(((f + g*x)^2*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

output
```
-((g^2*x^3*(d + e*x)^m*(m^2 - 3*m + 2))/(11*m - 6*m^2 + m^3 - 6) + (x*(d +
 e*x)^m*(6*c^3*d^3*f^2 - 5*c^3*d^3*f^2*m + c^3*d^3*f^2*m^2 + 2*a^2*c*d*e^2
*g^2*m + 2*a*c^2*d^2*e*f*g*m^2 - 6*a*c^2*d^2*e*f*g*m))/(c^3*d^3*(11*m - 6*
m^2 + m^3 - 6)) + (a*e*(d + e*x)^m*(2*a^2*e^2*g^2 + 6*c^2*d^2*f^2 - 5*c^2*
d^2*f^2*m + c^2*d^2*f^2*m^2 - 6*a*c*d*e*f*g + 2*a*c*d*e*f*g*m))/(c^3*d^3*(
11*m - 6*m^2 + m^3 - 6)) + (g*x^2*(m - 1)*(d + e*x)^m*(a*e*g*m - 6*c*d*f +
 2*c*d*f*m))/(c*d*(11*m - 6*m^2 + m^3 - 6)))/(x*(a*e^2 + c*d^2) + a*d*e +
c*d*e*x^2)^m
```

3.770 $\int (d+ex)^m(f+gx) \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}$

3.770.1 Optimal result

Integrand size = 42, antiderivative size = 150

$$\int (d+ex)^m(f+gx) \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = -\frac{(ae^2g+cd(dg(1-m)-ef(2-m)))\,(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^2d^2e(1-m)(2-m)} + \frac{g(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cde(2-m)}$$

output
```
-(a*e^2*g+c*d*(d*g*(1-m)-e*f*(2-m)))*(e*x+d)^(-1+m)*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1-m)/c^2/d^2/e/(1-m)/(2-m)+g*(e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1-m)/c/d/e/(2-m)
```

3.770.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.45

$$\int (d+ex)^m(f+gx) \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = -\frac{(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}(aeg+cd(f(-2+m)+g(-1+m)x))}{c^2d^2(-2+m)(-1+m)}$$

input
```
Integrate[((d + e*x)^m*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
m,x]
```

output
```
-(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*(a*e*g + c*d*(f*(-
2 + m) + g*(-1 + m)*x)))/(c^2*d^2*(-2 + m)*(-1 + m)))
```

3.770.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{g(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cde(2-m)}-\frac{\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)\int (d+ex)^m \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{-m}dx}{cde(2-m)}$$

$$\Big\downarrow \text{1122}$$

$$\frac{g(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cde(2-m)}-\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^2d^2e(1-m)(2-m)}$$

input
```
Int[((d + e*x)^m*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

output
```
-(((a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)^(-1 + m)*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^2*d^2*e*(1 - m)*(2 - m))) + (g
*(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*e*(2 -
m))
```

3.770.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

3.770.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.59

method	result
gosper	$-\frac{(ex+d)^m (cdgmx+cdfm-cdgx+aeg-2cdf)(cdx+ae)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{-m}}{c^2d^2(m^2-3m+2)}$
risch	$-\frac{\left(g\,x^2c^2d^2m+acdegmx+c^2d^2fmx-g\,x^2c^2d^2+acdefm-2c^2d^2fx+a^2e^2g-2acdef\right)(cdx+ae)^{-m}\,\mathrm{e}^{i\pi\ \mathrm{csgn}(i(cdx+ae)(ex+d))m(}}{c^2d^2(-2+m)(-1+m)}$
parallelrisch	$-\frac{(x^2(ex+d)^m c^2d^2eg\,m^2-x^2(ex+d)^m c^2d^2egm+x(ex+d)^m acd\,e^2g\,m^2+x(ex+d)^m c^2d^2ef\,m^2-2x(ex+d)^m c^2d^2efm+(ex+d}{me\,c^2d^2(-2+m)(-1+m)}$

input
```
int((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,method=_RETU
RNVERBOSE)
```

output
```
-(e*x+d)^m*(c*d*g*m*x+c*d*f*m-c*d*g*x+a*e*g-2*c*d*f)*(c*d*x+a*e)/((c*d*e*x
^2+a*e^2*x+c*d^2*x+a*d*e)^m)/c^2/d^2/(m^2-3*m+2)
```

3.770.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{(acdefm-2\,acdef+a^2e^2g+(c^2d^2gm-c^2d^2g)x^2-(2\,c^2d^2f-(c^2d^2f+acdeg)m)x)(ex+d)^m}{(c^2d^2m^2-3\,c^2d^2m+2\,c^2d^2)(cdex^2+ade+(cd^2+ae^2)x)^m}$$

input
```
integrate((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algor
ithm="fricas")
```

output
```
-(a*c*d*e*f*m - 2*a*c*d*e*f + a^2*e^2*g + (c^2*d^2*g*m - c^2*d^2*g)*x^2 -
(2*c^2*d^2*f - (c^2*d^2*f + a*c*d*e*g)*m)*x)*(e*x + d)^m/((c^2*d^2*m^2 - 3
*c^2*d^2*m + 2*c^2*d^2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
```

3.770.6 Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m*(g*x+f)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

output
```
Timed out
```

3.770.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.63

$$\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{(cdx+ae)f}{(cdx+ae)^m cd(m-1)} - \frac{(c^2d^2(m-1)x^2+acdemx+a^2e^2)g}{(m^2-3\,m+2)(cdx+ae)^m c^2d^2}$$

input
```
integrate((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algor
ithm="maxima")
```

output
```
-(c*d*x + a*e)*f/((c*d*x + a*e)^m*c*d*(m - 1)) - (c^2*d^2*(m - 1)*x^2 + a*
c*d*e*m*x + a^2*e^2)*g/((m^2 - 3*m + 2)*(c*d*x + a*e)^m*c^2*d^2)
```

3.770.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(142) = 284$.

Time = 0.30 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.31

$$\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{(ex+d)^m c^2d^2gmx^2e^{(-m\log(cdx+ae)-m\log(ex+d))}+(ex+d)^m c^2d^2fmxe^{(-m\log(cdx+ae)-m\log(ex+d))}+(ex+d}{}$$

input
```
integrate((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algor
ithm="giac")
```

output
```
-((e*x + d)^m*c^2*d^2*g*m*x^2*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + (
e*x + d)^m*c^2*d^2*f*m*x*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + (e*x +
 d)^m*a*c*d*e*g*m*x*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) - (e*x + d)^m
*c^2*d^2*g*x^2*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + (e*x + d)^m*a*c*
d*e*f*m*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) - 2*(e*x + d)^m*c^2*d^2*f
*x*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) - 2*(e*x + d)^m*a*c*d*e*f*e^(-
m*log(c*d*x + a*e) - m*log(e*x + d)) + (e*x + d)^m*a^2*e^2*g*e^(-m*log(c*d
*x + a*e) - m*log(e*x + d)))/(c^2*d^2*m^2 - 3*c^2*d^2*m + 2*c^2*d^2)
```

3.770.9 Mupad [B] (verification not implemented)

Time = 12.06 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

$$\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{\frac{g\,x^2\,(m-1)\,(d+e\,x)^m}{m^2-3\,m+2}+\frac{x\,(d+e\,x)^m\,(a\,e\,g\,m-2\,c\,d\,f+c\,d\,f\,m)}{c\,d\,(m^2-3\,m+2)}+\frac{a\,e\,(d+e\,x)^m\,(a\,e\,g-2\,c\,d\,f+c\,d\,f\,m)}{c^2\,d^2\,(m^2-3\,m+2)}}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^m}$$

input
```
int(((f + g*x)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

output
```
-((g*x^2*(m - 1)*(d + e*x)^m)/(m^2 - 3*m + 2) + (x*(d + e*x)^m*(a*e*g*m -
2*c*d*f + c*d*f*m))/(c*d*(m^2 - 3*m + 2)) + (a*e*(d + e*x)^m*(a*e*g - 2*c*
d*f + c*d*f*m))/(c^2*d^2*(m^2 - 3*m + 2)))/(x*(a*e^2 + c*d^2) + a*d*e + c*
d*e*x^2)^m
```

3.770. $\int (d+ex)^m (f+gx) \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$

3.771 $\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$

3.771.1 Optimal result

Integrand size = 37, antiderivative size = 54

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \frac{(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(1-m)}$$

output
```
(e*x+d)^(-1+m)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c/d/(1-m)
```

3.771.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = -\frac{(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}}{cd(-1+m)}$$

input
```
Integrate[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

output
```
-(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m))/(c*d*(-1 + m)))
```

3.771.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \, dx$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{(d+ex)^{m-1} \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(1-m)}$$

input
```
Int[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

output
```
((d + e*x)^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*
(1 - m))
```

3.771.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

3.771.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

method	result
gosper	$-\frac{(cdx+ae)(ex+d)^m\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{-m}}{cd(-1+m)}$
parallelrisch	$\frac{\left(-x(ex+d)^m cde-(ex+d)^m a\,e^2\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{-m}}{cde(-1+m)}$
norman	$\left(-\frac{x\,\mathrm{e}^{m\ln(ex+d)}}{-1+m}-\frac{ae\,\mathrm{e}^{m\ln(ex+d)}}{cd(-1+m)}\right)\mathrm{e}^{-m\ln\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)}$
risch	$-\frac{(cdx+ae)(cdx+ae)^{-m}\mathrm{e}^{\frac{i\pi\ \mathrm{csgn}(i(cdx+ae)(ex+d))m(-\mathrm{csgn}(i(cdx+ae)(ex+d))+\mathrm{csgn}(i(cdx+ae)))(-\mathrm{csgn}(i(cdx+ae)(ex+d))+\mathrm{csgn}(i(e\ldots}{2}}}{cd(-1+m)}$

input
```
int((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,method=_RETURNVERBOS
E)
```

output
```
-(c*d*x+a*e)/c/d/(-1+m)*(e*x+d)^m/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^m)
```

3.771.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$$
$$= -\frac{(cdx+ae)(ex+d)^m}{(cdm-cd)(cdex^2+ade+(cd^2+ae^2)x)^m}$$

input
```
integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="fr
icas")
```

output
```
-(c*d*x + a*e)*(e*x + d)^m/((c*d*m - c*d)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)^m)
```

3.771.6 Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

output
```
Timed out
```

3.771.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.61

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=-\frac{cdx+ae}{(cdx+ae)^m cd(m-1)}$$

input
```
integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="ma
xima")
```

output
```
-(c*d*x + a*e)/((c*d*x + a*e)^m*c*d*(m - 1))
```

3.771.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.50

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=-\frac{(ex+d)^m cdxe^{(-m\log(cdx+ae)-m\log(ex+d))}+(ex+d)^m aee^{(-m\log(cdx+ae)-m\log(ex+d))}}{cdm-cd}$$

input
```
integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="gi
ac")
```

output
```
-((e*x + d)^m*c*d*x*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + (e*x + d)^m
*a*e*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)))/(c*d*m - c*d)
```

3.771.9 Mupad [B] (verification not implemented)

Time = 11.77 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=-\frac{(a\,e+c\,d\,x)\,(d+e\,x)^m}{c\,d\,(m-1)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}$$

input
```
int((d + e*x)^m/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

output
```
-((a*e + c*d*x)*(d + e*x)^m)/(c*d*(m - 1)*(x*(a*e^2 + c*d^2) + a*d*e + c*d
*e*x^2)^m)
```

3.772 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx$

3.772.1 Optimal result

Integrand size = 44, antiderivative size = 99

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx = \frac{(ae+cdx)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(1,1-m,2-m,-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(cdf-aeg)(1-m)}$$

output
```
(c*d*x+a*e)*(e*x+d)^m*hypergeom([1, 1-m],[2-m],-g*(c*d*x+a*e)/(-a*e*g+c*d*
f))/(-a*e*g+c*d*f)/(1-m)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m)
```

3.772.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx = -\frac{(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}\text{Hypergeometric2F1}\left(1,1-m,2-m,\frac{g(ae+cdx)}{-cdf+aeg}\right)}{(cdf-aeg)(-1+m)}$$

input
```
Integrate[(d + e*x)^m/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m
),x]
```

output
```
-(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*Hypergeometric2F1[
1, 1 - m, 2 - m, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f - a*e*g)*(
-1 + m)))
```

3.772.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1268, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}}{f+gx}dx$$

$$\Big\downarrow \text{ 1268}$$

$$(d+ex)^m(ae+cdx)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\int \frac{(ae+cdx)^{-m}}{f+gx}dx$$

$$\Big\downarrow \text{ 78}$$

$$\frac{(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(1,1-m,2-m,-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)}$$

input
```
Int[(d + e*x)^m/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),x]
```

output
```
((a*e + c*d*x)*(d + e*x)^m*Hypergeometric2F1[1, 1 - m, 2 - m, -((g*(a*e +
c*d*x))/(c*d*f - a*e*g))])/((c*d*f - a*e*g)*(1 - m)*(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^m)
```

3.772.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.772.4 Maple [F]

$$\int \frac{(ex+d)^m \left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{-m}}{gx+f}dx$$

input
```
int((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output
```
int((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

3.772.5 Fricas [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx$$
$$=\int \frac{(ex+d)^m}{(gx+f)(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algor
ithm="fricas")
```

output
```
integral((e*x + d)^m/((g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
, x)
```

3.772.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x+d)**m/(g*x+f)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.772.7 Maxima [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx = \int \frac{(ex+d)^m}{(gx+f)(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algor
ithm="maxima")
```

output
```
integrate((e*x + d)^m/((g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m
), x)
```

3.772.8 Giac [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx = \int \frac{(ex+d)^m}{(gx+f)(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algor
ithm="giac")
```

output
```
integrate((e*x + d)^m/((g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m
), x)
```

3.772.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx = \int \frac{(d+e\,x)^m}{(f+g\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

input
```
int((d + e*x)^m/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m),x)
```

output
```
int((d + e*x)^m/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m), x)
```

3.773 $\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx$

3.773.1 Optimal result

Integrand size = 44, antiderivative size = 101

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx = \frac{cd(ae+cdx)(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(2,1-m,2-m,-\frac{g(ae+cdx)}{cdf-aeg}\right.}{(cdf-aeg)^2(1-m)}$$

output
```
c*d*(c*d*x+a*e)*(e*x+d)^m*hypergeom([2, 1-m],[2-m],-g*(c*d*x+a*e)/(-a*e*g+
c*d*f))/(-a*e*g+c*d*f)^2/(1-m)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m)
```

3.773.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx = -\frac{cd(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}\text{Hypergeometric2F1}\left(2,1-m,2-m,\frac{g(ae+cdx)}{-cdf+aeg}\right)}{(cdf-aeg)^2(-1+m)}$$

input
```
Integrate[(d + e*x)^m/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^m),x]
```

output
```
-((c*d*(d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*Hypergeometric
2F1[2, 1 - m, 2 - m, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f - a*e*
g)^2*(-1 + m)))
```

3.773.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1268, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}}{(f+gx)^2}dx$$

$$\Big\downarrow \text{ 1268}$$

$$(d+ex)^m(ae+cdx)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\int \frac{(ae+cdx)^{-m}}{(f+gx)^2}dx$$

$$\Big\downarrow \text{ 78}$$

$$\frac{cd(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(2,1-m,2-m,-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)^2}$$

input
```
Int[(d + e*x)^m/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),x]
```

output
```
(c*d*(a*e + c*d*x)*(d + e*x)^m*Hypergeometric2F1[2, 1 - m, 2 - m, -((g*(a*
e + c*d*x))/(c*d*f - a*e*g))])/((c*d*f - a*e*g)^2*(1 - m)*(a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2)^m)
```

3.773.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.773.4 Maple [F]

$$\int \frac{(ex+d)^m \left(ade+(e^2a+c\,d^2)\,x+cde\,x^2\right)^{-m}}{(gx+f)^2}dx$$

input
```
int((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output
```
int((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

3.773.5 Fricas [F]

$$\int \frac{(d+ex)^m \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx = \int \frac{(ex+d)^m}{(gx+f)^2(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="fricas")
```

output
```
integral((e*x + d)^m/((g^2*x^2 + 2*f*g*x + f^2)*(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)^m), x)
```

3.773.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**m/(g*x+f)**2/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),
x)
```

output
```
Timed out
```

3.773.7 Maxima [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx = \int \frac{(ex+d)^m}{(gx+f)^2(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="maxima")
```

output
```
integrate((e*x + d)^m/((g*x + f)^2*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
^m), x)
```

3.773.8 Giac [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx = \int \frac{(ex+d)^m}{(gx+f)^2(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="giac")
```

output
```
integrate((e*x + d)^m/((g*x + f)^2*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
^m), x)
```

3.773.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx$$
$$=\int \frac{(d+e\,x)^m}{(f+g\,x)^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

input
```
int((d + e*x)^m/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m),x)
```

output
```
int((d + e*x)^m/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m), x
)
```

3.774 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx$

3.774.1 Optimal result

Integrand size = 44, antiderivative size = 105

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx = \frac{c^2d^2(ae+cdx)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(3,1-m,2-m,-\frac{g(ae+c}{cdf-ae}\right.}{(cdf-aeg)^3(1-m)}$$

output
```
c^2*d^2*(c*d*x+a*e)*(e*x+d)^m*hypergeom([3, 1-m],[2-m],-g*(c*d*x+a*e)/(-a*
e*g+c*d*f))/(-a*e*g+c*d*f)^3/(1-m)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m)
```

3.774.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx = -\frac{c^2d^2(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}\text{Hypergeometric2F1}\left(3,1-m,2-m,\frac{g(ae+cdx)}{-cdf+aeg}\right)}{(cdf-aeg)^3(-1+m)}$$

input
```
Integrate[(d + e*x)^m/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^m),x]
```

output
```
-((c^2*d^2*(d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*Hypergeome
tric2F1[3, 1 - m, 2 - m, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f -
a*e*g)^3*(-1 + m)))
```

3.774.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1268, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}}{(f+gx)^3}dx$$

$$\Big\downarrow \text{1268}$$

$$(d+ex)^m(ae+cdx)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\int \frac{(ae+cdx)^{-m}}{(f+gx)^3}dx$$

$$\Big\downarrow \text{78}$$

$$\frac{c^2d^2(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(3,1-m,2-m,-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)^3}$$

input
```
Int[(d + e*x)^m/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),x]
```

output
```
(c^2*d^2*(a*e + c*d*x)*(d + e*x)^m*Hypergeometric2F1[3, 1 - m, 2 - m, -((g
*(a*e + c*d*x))/(c*d*f - a*e*g))])/((c*d*f - a*e*g)^3*(1 - m)*(a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

3.774.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.774.4 Maple [F]

$$\int \frac{(ex+d)^m \left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{-m}}{(gx+f)^3}dx$$

input
```
int((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output
```
int((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

3.774.5 Fricas [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx = \int \frac{(ex+d)^m}{(gx+f)^3(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="fricas")
```

output
```
integral((e*x + d)^m/((g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3)*(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)^m), x)
```

3.774.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^m \left(ade+(cd^2+ae^2)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x+d)**m/(g*x+f)**3/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),
x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.774.7 Maxima [F]

$$\int \frac{(d+ex)^m \left(ade+(cd^2+ae^2)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx = \int \frac{(ex+d)^m}{(gx+f)^3(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="maxima")
```

output
```
integrate((e*x + d)^m/((g*x + f)^3*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
^m), x)
```

3.774.8 Giac [F]

$$\int \frac{(d+ex)^m \left(ade+(cd^2+ae^2)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx = \int \frac{(ex+d)^m}{(gx+f)^3(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="giac")
```

output
```
integrate((e*x + d)^m/((g*x + f)^3*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
^m), x)
```

3.774.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx = \int \frac{\left(d+e\,x\right)^m}{\left(f+g\,x\right)^3\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

input
```
int((d + e*x)^m/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m),x)
```

output
```
int((d + e*x)^m/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m), x
)
```

3.775 $\int (d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}$

3.775.1 Optimal result

Integrand size = 46, antiderivative size = 105

$$\int (d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m(d+ex)^m(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\text{Hypergeometric2F}}{5g}$$

output
```
2/5*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*(g*x+f)^(5/2)*hypergeom([5
/2, m],[7/2],c*d*(g*x+f)/(-a*e*g+c*d*f))/g/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^m)
```

3.775.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int (d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\frac{2\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m((ae+cdx)(d+ex))^{-m}(f+gx)^{5/2}\text{Hypergeometric2F1}\left(\frac{5}{2},m,\frac{7}{2},\right.}{5g}$$

input
```
Integrate[((d + e*x)^m*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^m,x]
```

output
```
(2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*(f + g*x)^(5/2)*Hy
pergeometric2F1[5/2, m, 7/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(5*g*((a*e
+ c*d*x)*(d + e*x))^m)
```

3.775.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^{3/2}(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}dx$$

$$\Big\downarrow \; 1268$$

$$(d+ex)^m(ae+cdx)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\int (ae+cdx)^{-m}(f+gx)^{3/2}dx$$

$$\Big\downarrow \; 80$$

$$(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m\int (f+gx)^{3/2}\left(-\frac{cdxg}{cdf-aeg}-\frac{aeg}{cdf-aeg}\right)^{-m}dx$$

$$\Big\downarrow \; 79$$

$$\frac{2(f+gx)^{5/2}(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m \text{Hypergeometric2F1}\left(\frac{5}{2},m,\frac{7}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{5g}$$

input
```
Int[((d + e*x)^m*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
m,x]
```

output
```
(2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*(f + g*x)^(5/2)*Hy
pergeometric2F1[5/2, m, 7/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(5*g*(a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

3.775.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.775.4 Maple [F]

$$\int (ex+d)^m (gx+f)^{\frac{3}{2}} \left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{-m}dx$$

input
```
int((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output
```
int((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

3.775.5 Fricas [F]

$$\int (d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\int \frac{(gx+f)^{\frac{3}{2}}(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="fricas")
```

output
```
integral((g*x + f)^(3/2)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)^m, x)
```

3.775.6 Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**m*(g*x+f)**(3/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)*
*m),x)
```

output
```
Timed out
```

3.775.7 Maxima [F]

$$\int (d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\int \frac{(gx+f)^{\frac{3}{2}}(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="maxima")
```

output
```
integrate((g*x + f)^(3/2)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^m, x)
```

3.775.8 Giac [F]

$$\int (d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\int \frac{(gx+f)^{\frac{3}{2}}(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="giac")
```

output
```
integrate((g*x + f)^(3/2)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^m, x)
```

3.775.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\int \frac{(f+g\,x)^{3/2}\,(d+e\,x)^m}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^m}\,dx$$

input
```
int(((f + g*x)^(3/2)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
m,x)
```

output
```
int(((f + g*x)^(3/2)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
m, x)
```

3.776 $\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}$

3.776.1 Optimal result

Integrand size = 46, antiderivative size = 105

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m(d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(\frac{3}{2},m,\frac{5}{2},\frac{cd(f+\cdots}{cdf-\cdots}\right.}{3g}$$

output
```
2/3*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*(g*x+f)^(3/2)*hypergeom([3
/2, m],[5/2],c*d*(g*x+f)/(-a*e*g+c*d*f))/g/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^m)
```

3.776.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=\frac{2\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m((ae+cdx)(d+ex))^{-m}(f+gx)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},m,\frac{5}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{3g}$$

input
```
Integrate[((d + e*x)^m*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^m,x]
```

output
```
(2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*(f + g*x)^(3/2)*Hy
pergeometric2F1[3/2, m, 5/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*((a*e
+ c*d*x)*(d + e*x))^m)
```

3.776.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{f+gx}(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} dx$$

$$\Big\downarrow \text{1268}$$

$$(d+ex)^m(ae+cdx)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \int (ae+cdx)^{-m}\sqrt{f+gx}dx$$

$$\Big\downarrow \text{80}$$

$$(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m \int \sqrt{f+gx}\left(-\frac{cdxg}{cdf-aeg}-\frac{aeg}{cdf-aeg}\right)^{-m}dx$$

$$\Big\downarrow \text{79}$$

$$\frac{2(f+gx)^{3/2}(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m \text{Hypergeometric2F1}\left(\frac{3}{2},m,\frac{5}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{3g}$$

input
```
Int[((d + e*x)^m*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,
x]
```

output
```
(2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*(f + g*x)^(3/2)*Hy
pergeometric2F1[3/2, m, 5/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*(a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

3.776.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.776.4 Maple [F]

$$\int (ex+d)^m \sqrt{gx+f}\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{-m}dx$$

input
```
int((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output
```
int((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

3.776.5 Fricas [F]

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \int \frac{\sqrt{gx+f}(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="fricas")
```

output
```
integral(sqrt(g*x + f)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
^m, x)
```

3.776.6 Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**m*(g*x+f)**(1/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)*
*m),x)
```

output
```
Timed out
```

3.776.7 Maxima [F]

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \int \frac{\sqrt{gx+f}(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="maxima")
```

output
```
integrate(sqrt(g*x + f)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)^m, x)
```

3.776.8 Giac [F]

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \int \frac{\sqrt{gx+f}(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="giac")
```

output
```
integrate(sqrt(g*x + f)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)^m, x)
```

3.776.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \int \frac{\sqrt{f+g\,x}\,(d+e\,x)^m}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^m}\,dx$$

input
```
int(((f + g*x)^(1/2)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
m,x)
```

output
```
int(((f + g*x)^(1/2)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
m, x)
```

3.777 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx$

3.777.1 Optimal result

Integrand size = 46, antiderivative size = 103

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx = \frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m(d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},m,\frac{3}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{g}$$

output
```
2*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*hypergeom([1/2, m],[3/2],c*d
*(g*x+f)/(-a*e*g+c*d*f))*(g*x+f)^(1/2)/g/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^m)
```

3.777.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx = \frac{2\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m((ae+cdx)(d+ex))^{-m}\sqrt{f+gx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},m,\frac{3}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{g}$$

input
```
Integrate[(d + e*x)^m/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^m),x]
```

output
```
(2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*Sqrt[f + g*x]*Hype
rgeometric2F1[1/2, m, 3/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*((a*e + c*
d*x)*(d + e*x))^m)
```

3.777.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}}{\sqrt{f+gx}}dx$$

$$\Big\downarrow \ 1268$$

$$(d+ex)^m(ae+cdx)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\int \frac{(ae+cdx)^{-m}}{\sqrt{f+gx}}dx$$

$$\Big\downarrow \ 80$$

$$(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m\int \frac{\left(-\frac{cdxg}{cdf-aeg}-\frac{aeg}{cdf-aeg}\right)^{-m}}{\sqrt{f+gx}}dx$$

$$\Big\downarrow \ 79$$

$$\frac{2\sqrt{f+gx}(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m \text{Hypergeometric2F1}\left(\frac{1}{2},m,\frac{3}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{g}$$

input
```
Int[(d + e*x)^m/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),
x]
```

output
```
(2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*Sqrt[f + g*x]*Hype
rgeometric2F1[1/2, m, 3/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

3.777.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.777.4 Maple [F]

$$\int \frac{(ex+d)^m \left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{-m}}{\sqrt{gx+f}}dx$$

input
```
int((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output
```
int((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

3.777.5 Fricas [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx = \int \frac{(ex+d)^m}{\sqrt{gx+f}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="fricas")
```

output
```
integral((e*x + d)^m/(sqrt(g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)^m), x)
```

3.777.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m/(g*x+f)**(1/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)*
*m),x)
```

output
```
Timed out
```

3.777.7 Maxima [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx = \int \frac{(ex+d)^m}{\sqrt{gx+f}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="maxima")
```

output
```
integrate((e*x + d)^m/(sqrt(g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)^m), x)
```

3.777.8 Giac [F]

$$\int \frac{(d+ex)^m \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx = \int \frac{(ex+d)^m}{\sqrt{gx+f}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="giac")
```

output
```
integrate((e*x + d)^m/(sqrt(g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)^m), x)
```

3.777.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx = \int \frac{(d+e\,x)^m}{\sqrt{f+g\,x}\,(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\;x+a\,d\,e)^m}\,dx$$

input
```
int((d + e*x)^m/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
),x)
```

output
```
int((d + e*x)^m/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
), x)
```

3.778 $\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx$

3.778.1 Optimal result

Integrand size = 46, antiderivative size = 103

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx = -\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} \text{Hypergeometric2F1}\left(-\frac{1}{2},m,\frac{1}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{g\sqrt{f+gx}}$$

output
```
-2*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*hypergeom([-1/2, m],[1/2],c
*d*(g*x+f)/(-a*e*g+c*d*f))/g/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m)/(g*x+f)
^(1/2)
```

3.778.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx = -\frac{2\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m((ae+cdx)(d+ex))^{-m} \text{Hypergeometric2F1}\left(-\frac{1}{2},m,\frac{1}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{g\sqrt{f+gx}}$$

input
```
Integrate[(d + e*x)^m/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^m),x]
```

output
```
(-2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*Hypergeometric2F1
[-1/2, m, 1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*((a*e + c*d*x)*(d + e*
x))^m*Sqrt[f + g*x])
```

3.778.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}}{(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{ 1268}$$

$$(d+ex)^m(ae+cdx)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\int \frac{(ae+cdx)^{-m}}{(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{ 80}$$

$$(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m\int \frac{\left(-\frac{cdxg}{cdf-aeg}-\frac{aeg}{cdf-aeg}\right)^{-m}}{(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{2(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m \text{Hypergeometric2F1}\left(-\frac{1}{2},m,\frac{1}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{g\sqrt{f+gx}}$$

input
```
Int[(d + e*x)^m/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m
),x]
```

output
```
(-2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*Hypergeometric2F1
[-1/2, m, 1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*Sqrt[f + g*x]*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

3.778.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.778.4 Maple [F]

$$\int \frac{(ex+d)^m \left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{-m}}{(gx+f)^{\frac{3}{2}}}dx$$

input
```
int((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output
```
int((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

3.778.5 Fricas [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx = \int \frac{(ex+d)^m}{(gx+f)^{\frac{3}{2}}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="fricas")
```

output
```
integral(sqrt(g*x + f)*(e*x + d)^m/((g^2*x^2 + 2*f*g*x + f^2)*(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)^m), x)
```

3.778.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m/(g*x+f)**(3/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)*
*m),x)
```

output
```
Timed out
```

3.778.7 Maxima [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx = \int \frac{(ex+d)^m}{(gx+f)^{\frac{3}{2}}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="maxima")
```

output
```
integrate((e*x + d)^m/((g*x + f)^(3/2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)^m), x)
```

3.778.8 Giac [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx = \int \frac{(ex+d)^m}{(gx+f)^{\frac{3}{2}}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="giac")
```

output
```
integrate((e*x + d)^m/((g*x + f)^(3/2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)^m), x)
```

3.778.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx=\int \frac{(d+e\,x)^m}{(f+g\,x)^{3/2}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

input
```
int((d + e*x)^m/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
),x)
```

output
```
int((d + e*x)^m/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
), x)
```

3.779 $\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx$

3.779.1 Optimal result

Integrand size = 46, antiderivative size = 105

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx = -\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} \text{Hypergeometric2F1}\left(-\frac{3}{2},m,-\frac{1}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{3g(f+gx)^{3/2}}$$

output
```
-2/3*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*hypergeom([-3/2, m],[-1/2
],c*d*(g*x+f)/(-a*e*g+c*d*f))/g/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^m)
```

3.779.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx = -\frac{2\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m ((ae+cdx)(d+ex))^{-m} \text{Hypergeometric2F1}\left(-\frac{3}{2},m,-\frac{1}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{3g(f+gx)^{3/2}}$$

input
```
Integrate[(d + e*x)^m/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^m),x]
```

output
```
(-2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*Hypergeometric2F1
[-3/2, m, -1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*((a*e + c*d*x)*(d +
 e*x))^m*(f + g*x)^(3/2))
```

3.779.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}}{(f+gx)^{5/2}}dx$$

$$\Big\downarrow \text{1268}$$

$$(d+ex)^m(ae+cdx)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\int \frac{(ae+cdx)^{-m}}{(f+gx)^{5/2}}dx$$

$$\Big\downarrow \text{80}$$

$$(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m\int \frac{\left(-\frac{cdxg}{cdf-aeg}-\frac{aeg}{cdf-aeg}\right)^{-m}}{(f+gx)^{5/2}}dx$$

$$\Big\downarrow \text{79}$$

$$-\frac{2(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m \text{Hypergeometric2F1}\left(-\frac{3}{2},m,-\frac{1}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{3g(f+gx)^{3/2}}$$

input
```
Int[(d + e*x)^m/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m
),x]
```

output
```
(-2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*Hypergeometric2F1
[-3/2, m, -1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*(f + g*x)^(3/2)*(a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

3.779.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.779.4 Maple [F]

$$\int \frac{(ex+d)^m \left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{-m}}{(gx+f)^{\frac{5}{2}}}dx$$

input
```
int((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output
```
int((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

3.779.5 Fricas [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx=\int \frac{(ex+d)^m}{(gx+f)^{\frac{5}{2}}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="fricas")
```

output
```
integral(sqrt(g*x + f)*(e*x + d)^m/((g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f
^3)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m), x)
```

3.779.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**m/(g*x+f)**(5/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)*
*m),x)
```

output
```
Timed out
```

3.779.7 Maxima [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx=\int \frac{(ex+d)^m}{(gx+f)^{\frac{5}{2}}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="maxima")
```

output
```
integrate((e*x + d)^m/((g*x + f)^(5/2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)^m), x)
```

3.779.8 Giac [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx=\int \frac{(ex+d)^m}{(gx+f)^{\frac{5}{2}}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="giac")
```

output
```
integrate((e*x + d)^m/((g*x + f)^(5/2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)^m), x)
```

3.779.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx=\int \frac{(d+e\,x)^m}{(f+g\,x)^{5/2}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

input
```
int((d + e*x)^m/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
),x)
```

output
```
int((d + e*x)^m/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
), x)
```

3.780 $\int (ae+cdx)^n(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}$

3.780.1 Optimal result

Integrand size = 47, antiderivative size = 65

$$\int (ae+cdx)^n(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx =\frac{(ae+cdx)^n(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(1-m+n)}$$

output
```
(c*d*x+a*e)^n*(e*x+d)^(-1+m)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c/d/(
1-m+n)
```

3.780.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int (ae+cdx)^n(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx =\frac{(ae+cdx)^{1+n}(d+ex)^m((ae+cdx)(d+ex))^{-m}}{cd(1-m+n)}$$

input
```
Integrate[((a*e + c*d*x)^n*(d + e*x)^m)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^m,x]
```

output
```
((a*e + c*d*x)^(1 + n)*(d + e*x)^m)/(c*d*(1 - m + n)*((a*e + c*d*x)*(d + e
*x))^m)
```

3.780.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {1247}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} (ae+cdx)^n \, dx$$

$$\Big\downarrow \; 1247$$

$$\frac{(d+ex)^{m-1} \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m} (ae+cdx)^n}{cd(-m+n+1)}$$

input
```
Int[((a*e + c*d*x)^n*(d + e*x)^m)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
m,x]
```

output
```
((a*e + c*d*x)^n*(d + e*x)^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(1 - m))/(c*d*(1 - m + n))
```

3.780.3.1 Defintions of rubi rules used

rule 1247
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] /; FreeQ[{a, b, c, d, e, f,
 g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && EqQ[c
*e*f + c*d*g - b*e*g, 0] && NeQ[m - n - 1, 0]
```

3.780.4 Maple [A] (verified)

Time = 3.88 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

method	result
gosper	$-\frac{(ex+d)^{m} (cdx+ae)^{1+n} \left(cde\,x^{2}+a\,e^{2} x+c\,d^{2} x+ade\right)^{-m}}{cd(-1+m-n)}$
parallelrisch	$-\frac{\left(x (ex+d)^{m} (cdx+ae)^{n} cdem+(ex+d)^{m} (cdx+ae)^{n} a\,e^{2} m\right) \left(cde\,x^{2}+a\,e^{2} x+c\,d^{2} x+ade\right)^{-m}}{mcde(-1+m-n)}$
risch	$-\frac{(cdx+ae)^{n} (cdx+ae) (cdx+ae)^{-m} \mathrm{e}^{\frac{i\pi\ \mathrm{csgn}(i(cdx+ae)(ex+d))m(-\mathrm{csgn}(i(cdx+ae)(ex+d))+\mathrm{csgn}(i(cdx+ae)))(-\mathrm{csgn}(i(cdx+ae)(ex+}{2}}}{cd(-1+m-n)}$

input
```
int((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,method
=_RETURNVERBOSE)
```

output
```
-1/c/d/(-1+m-n)*(e*x+d)^m*(c*d*x+a*e)^(1+n)/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*
d*e)^m)
```

3.780.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int (ae+cdx)^n(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{(cdx+ae)(cdx+ae)^n(ex+d)^m e^{(-m\log(cdx+ae)-m\log(ex+d))}}{cdm-cdn-cd}$$

input
```
integrate((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="fricas")
```

output
```
-(c*d*x + a*e)*(c*d*x + a*e)^n*(e*x + d)^m*e^(-m*log(c*d*x + a*e) - m*log(
e*x + d))/(c*d*m - c*d*n - c*d)
```

3.780.6 Sympy [F(-1)]

Timed out.

$$\int (ae+cdx)^n(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \text{Timed out}$$

input
```
integrate((c*d*x+a*e)**n*(e*x+d)**m/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)*
*m),x)
```

output
```
Timed out
```

3.780.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int (ae+cdx)^n(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = -\frac{(cdx+ae)e^{(-m\log(cdx+ae)+n\log(cdx+ae))}}{cd(m-n-1)}$$

input
```
integrate((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="maxima")
```

output
```
-(c*d*x + a*e)*e^(-m*log(c*d*x + a*e) + n*log(c*d*x + a*e))/(c*d*(m - n -
1))
```

3.780.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.63

$$\int (ae+cdx)^n(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = -\frac{(cdx+ae)^n(ex+d)^m cdxe^{(-m\log(cdx+ae)-m\log(ex+d))}+(cdx+ae)^n(ex+d)^m aee^{(-m\log(cdx+ae)-m\log(ex+d)}}{cdm-cdn-cd}$$

input
```
integrate((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="giac")
```

output
```
-((c*d*x + a*e)^n*(e*x + d)^m*c*d*x*e^(-m*log(c*d*x + a*e) - m*log(e*x + d
)) + (c*d*x + a*e)^n*(e*x + d)^m*a*e*e^(-m*log(c*d*x + a*e) - m*log(e*x +
d)))/(c*d*m - c*d*n - c*d)
```

3.780.9 Mupad [B] (verification not implemented)

Time = 12.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int (ae+cdx)^n(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=\frac{(a\,e+c\,d\,x)^{n+1}\,(d+e\,x)^m}{c\,d\,(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^m\,(n-m+1)}$$

input
```
int(((a*e + c*d*x)^n*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
m,x)
```

output
```
((a*e + c*d*x)^(n + 1)*(d + e*x)^m)/(c*d*(x*(a*e^2 + c*d^2) + a*d*e + c*d*
e*x^2)^m*(n - m + 1))
```

3.781 $\int (d+ex)^m \left(cd^2eg - e(cd^2+ae^2)g - cde^2gx\right)^{-1+m} ($

3.781.1 Optimal result

Integrand size = 73, antiderivative size = 78

$$\int (d+ex)^m \left(cd^2eg - e\left(cd^2+ae^2\right)g - cde^2gx\right)^{-1+m} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = -\frac{(d+ex)^m\left(-ae^3g-cde^2gx\right)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\log(ae+cdx)}{cde^2g}$$

output
```
-(e*x+d)^m*(-c*d*e^2*g*x-a*e^3*g)^m*ln(c*d*x+a*e)/c/d/e^2/g/((a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^m)
```

3.781.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int (d+ex)^m \left(cd^2eg - e\left(cd^2+ae^2\right)g - cde^2gx\right)^{-1+m} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = -\frac{\left(-e^2g(ae+cdx)\right)^m(d+ex)^m((ae+cdx)(d+ex))^{-m}\log(ae+cdx)}{cde^2g}$$

input
```
Integrate[((d + e*x)^m*(c*d^2*e*g - e*(c*d^2 + a*e^2)*g - c*d*e^2*g*x)^(-1
 + m))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

output
```
-(((-(e^2*g*(a*e + c*d*x)))^m*(d + e*x)^m*Log[a*e + c*d*x])/(c*d*e^2*g*((a
*e + c*d*x)*(d + e*x))^m))
```

3.781.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.041, Rules used = {1268, 37, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \left(-eg\left(ae^2+cd^2\right)+cd^2eg-cde^2gx\right)^{m-1}dx$$

$$\Big\downarrow \text{1268}$$

$$(d+ex)^m(ae+cdx)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\int (ae+cdx)^{-m}\left(-age^3-cdgxe^2\right)^{m-1}dx$$

$$\Big\downarrow \text{37}$$

$$(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-ae^3g-cde^2gx\right)^{m-1}\int \frac{1}{ae+cdx}dx$$

$$\Big\downarrow \text{16}$$

$$\frac{(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\log(ae+cdx)\left(-ae^3g-cde^2gx\right)^{m-1}}{cd}$$

input
```
Int[((d + e*x)^m*(c*d^2*e*g - e*(c*d^2 + a*e^2)*g - c*d*e^2*g*x)^(-1 + m))
/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

output
```
((a*e + c*d*x)*(d + e*x)^m*(-(a*e^3*g) - c*d*e^2*g*x)^(-1 + m)*Log[a*e + c
*d*x])/(c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

3.781.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a + b*x, c + d*x]
```

3.781.
$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}dx$

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.781.4 Maple [F]

$$\int (ex+d)^m \left(c\,d^2eg - e\left(e^2a + c\,d^2\right)g - cd\,e^2gx\right)^{-1+m} \left(ade + \left(e^2a + c\,d^2\right)x + cde\,x^2\right)^{-m} dx$$

input
```
int((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output
```
int((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

3.781.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.45

$$\int (d+ex)^m \left(cd^2eg - e\left(cd^2+ae^2\right)g - cde^2gx\right)^{-1+m} \left(ade + \left(cd^2+ae^2\right)x + cdex^2\right)^{-m} dx = -\frac{\log\left(cdx+ae\right)}{cde^2g\left(-\frac{1}{e^2g}\right)^m}$$

input
```
integrate((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="fricas")
```

output
```
-log(c*d*x + a*e)/(c*d*e^2*g*(-1/(e^2*g))^m)
```

3.781.6 Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$= \text{Timed out}$$

input
```
integrate((e*x+d)**m*(c*d**2*e*g-e*(a*e**2+c*d**2)*g-c*d*e**2*g*x)**(-1+m)
/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

output
```
Timed out
```

3.781.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.41

$$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$= -\frac{e^{2\,m-2}(-g)^m\log\left(cdx+ae\right)}{cdg}$$

input
```
integrate((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="maxima")
```

output
```
-e^(2*m - 2)*(-g)^m*log(c*d*x + a*e)/(c*d*g)
```

3.781.8 Giac [F]

$$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$= \int \frac{(-cde^2gx+cd^2eg-(cd^2+ae^2)eg)^{m-1}(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="giac")
```

output
```
integrate((-c*d*e^2*g*x + c*d^2*e*g - (c*d^2 + a*e^2)*e*g)^(m - 1)*(e*x +
d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m, x)
```

3.781.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=\int \frac{(d+e\,x)^m\,\left(c\,d^2\,e\,g-e\,g\,\left(c\,d^2+a\,e^2\right)-c\,d\,e^2\,g\,x\right)^{m-1}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

input
```
int(((d + e*x)^m*(c*d^2*e*g - e*g*(a*e^2 + c*d^2) - c*d*e^2*g*x)^(m - 1))/
(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

output
```
int(((d + e*x)^m*(c*d^2*e*g - e*g*(a*e^2 + c*d^2) - c*d*e^2*g*x)^(m - 1))/
(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m, x)
```

3.782 $\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.782.1 Optimal result

Integrand size = 46, antiderivative size = 213

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2e(f+gx)^{1+n}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cdg(3+2n)\sqrt{d+ex}}$$
$$+\frac{(2ae^2g(1+n)+cd(ef-dg(3+2n)))(ae+cdx)\sqrt{d+ex}(f+gx)^{1+n}\,\mathrm{Hypergeometric2F1}\left(1,\frac{3}{2}+n,2+\right.}{cdg(cdf-aeg)(1+n)(3+2n)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

output
```
(2*a*e^2*g*(1+n)+c*d*(e*f-d*g*(3+2*n)))*(c*d*x+a*e)*(g*x+f)^(1+n)*hypergeo
m([1, 3/2+n],[2+n],c*d*(g*x+f)/(-a*e*g+c*d*f))*(e*x+d)^(1/2)/c/d/g/(-a*e*g
+c*d*f)/(1+n)/(3+2*n)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2*e*(g*x+f)^
(1+n)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g/(3+2*n)/(e*x+d)^(1/2)
```

3.782.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}(f+gx)^n\left(cde(f+gx)+(-2ae^2g(1+n)+c\right.}{c^2d^2g}$$

input
```
Integrate[((d + e*x)^(3/2)*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2],x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^n*(c*d*e*(f + g*x) + ((-2*a*e^2*g
*(1 + n) + c*d*(-(e*f) + d*g*(3 + 2*n)))*Hypergeometric2F1[1/2, -n, 3/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
)/(c^2*d^2*g*(3/2 + n)*Sqrt[d + e*x])
```

3.782.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1258, 1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1258}$$

$$\frac{2e(f+gx)^{n+1}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cdg(2n+3)\sqrt{d+ex}}-\frac{\left(2ae^2g(n+1)+cd(ef-dg(2n+3))\right)\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cdg(2n+3)}$$

$$\Big\downarrow \text{1268}$$

$$\frac{2e(f+gx)^{n+1}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cdg(2n+3)\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(2ae^2g(n+1)+cd(ef-dg(2n+3))\right)\int \frac{(f+gx)^n}{\sqrt{ae+cdx}}dx}{cdg(2n+3)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{80}$$

$$\frac{2e(f+gx)^{n+1}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cdg(2n+3)\sqrt{d+ex}}-\frac{\sqrt{d+ex}(f+gx)^n\sqrt{ae+cdx}\left(2ae^2g(n+1)+cd(ef-dg(2n+3))\right)\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\int \frac{\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n}{\sqrt{ae+cdx}}dx}{cdg(2n+3)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{79}$$

$$\frac{2e(f+gx)^{n+1}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cdg(2n+3)\sqrt{d+ex}}-\frac{2\sqrt{d+ex}(f+gx)^n(ae+cdx)\left(2ae^2g(n+1)+cd(ef-dg(2n+3))\right)\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2}\right.}{c^2d^2g(2n+3)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[((d + e*x)^(3/2)*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2],x]
```

output
```
(2*e*(f + g*x)^(1 + n)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*g
*(3 + 2*n)*Sqrt[d + e*x]) - (2*(2*a*e^2*g*(1 + n) + c*d*(e*f - d*g*(3 + 2*
n)))*(a*e + c*d*x)*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[1/2, -n, 3/
2, -((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(c^2*d^2*g*(3 + 2*n)*((c*d*(f +
g*x))/(c*d*f - a*e*g))^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

3.782.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1258
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/(c*g*(n + p + 2))), x] - Simp[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2))   Int[(d + e*x)^
(m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p - 1, 0] &&
!LtQ[n, -1] && IntegerQ[2*p]
```

3.782. $\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.782.4 Maple [F]

$$\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^{n}}{\sqrt{ade+(e^{2}a+c\,d^{2})\,x+cde\,x^{2}}}dx$$

input
```
int((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
int((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

3.782.5 Fricas [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^{n}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output
```
integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*(g*x +
f)^n/(c*d*x + a*e), x)
```

3.782.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output
```
Timed out
```

3.782.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x), x)
```

3.782.8 Giac [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output
```
integrate((e*x + d)^(3/2)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x), x)
```

3.782.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(f+g\,x)^n\,(d+e\,x)^{3/2}}{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int(((f + g*x)^n*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(1/2),x)
```

output
```
int(((f + g*x)^n*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(1/2), x)
```

3.782. $\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.783 $\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.783.1 Optimal result

Integrand size = 46, antiderivative size = 501

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{128(cdf-aeg)^3\left(10ae^2g+cd(ef-11dg)\right)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{}}{3465c^6d^6eg\sqrt{d+ex}}$$
$$-\frac{128(cdf-aeg)^3\left(10ae^2g+cd(ef-11dg)\right)\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3465c^5d^5e}$$
$$-\frac{32(cdf-aeg)^2\left(10ae^2g+cd(ef-11dg)\right)(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{1155c^4d^4g\sqrt{d+ex}}$$
$$-\frac{16(cdf-aeg)\left(10ae^2g+cd(ef-11dg)\right)(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{693c^3d^3g\sqrt{d+ex}}$$
$$-\frac{2\left(10ae^2g+cd(ef-11dg)\right)(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{99c^2d^2g\sqrt{d+ex}}$$
$$+\frac{2e(f+gx)^5\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{11cdg\sqrt{d+ex}}$$

output
```
128/3465*(-a*e*g+c*d*f)^3*(10*a*e^2*g+c*d*(-11*d*g+e*f))*(2*a*e^2*g-c*d*(-
d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^6/d^6/e/g/(e*x+d)^(1
/2)-32/1155*(-a*e*g+c*d*f)^2*(10*a*e^2*g+c*d*(-11*d*g+e*f))*(g*x+f)^2*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/g/(e*x+d)^(1/2)-16/693*(-a*e*g
+c*d*f)*(10*a*e^2*g+c*d*(-11*d*g+e*f))*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/c^3/d^3/g/(e*x+d)^(1/2)-2/99*(10*a*e^2*g+c*d*(-11*d*g+e*f))
*(g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/g/(e*x+d)^(1/2)
+2/11*e*(g*x+f)^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g/(e*x+d)^(1
/2)-128/3465*(-a*e*g+c*d*f)^3*(10*a*e^2*g+c*d*(-11*d*g+e*f))*(e*x+d)^(1/2)
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^5/d^5/e
```

3.783.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(-1280a^5e^6g^4+128a^4cde^4g^3(44ef+11dg+$$

input
```
Integrate[((d + e*x)^(3/2)*(f + g*x)^4)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2],x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-1280*a^5*e^6*g^4 + 128*a^4*c*d*e^4*g^3*
(44*e*f + 11*d*g + 5*e*g*x) - 32*a^3*c^2*d^2*e^3*g^2*(22*d*g*(9*f + g*x) +
 e*(297*f^2 + 88*f*g*x + 15*g^2*x^2)) + 16*a^2*c^3*d^3*e^2*g*(33*d*g*(21*f
^2 + 6*f*g*x + g^2*x^2) + e*(462*f^3 + 297*f^2*g*x + 132*f*g^2*x^2 + 25*g^
3*x^3)) - 2*a*c^4*d^4*e*(44*d*g*(105*f^3 + 63*f^2*g*x + 27*f*g^2*x^2 + 5*g
^3*x^3) + e*(1155*f^4 + 1848*f^3*g*x + 1782*f^2*g^2*x^2 + 880*f*g^3*x^3 +
175*g^4*x^4)) + c^5*d^5*(11*d*(315*f^4 + 420*f^3*g*x + 378*f^2*g^2*x^2 + 1
80*f*g^3*x^3 + 35*g^4*x^4) + e*x*(1155*f^4 + 2772*f^3*g*x + 2970*f^2*g^2*x
^2 + 1540*f*g^3*x^3 + 315*g^4*x^4))))/(3465*c^6*d^6*Sqrt[d + e*x])
```

3.783.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1258, 1253, 1253, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1258}$$

$$\frac{1}{11}\left(-\frac{10ae^2}{cd}+11d-\frac{ef}{g}\right)\int \frac{\sqrt{d+ex}(f+gx)^4}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\frac{2e(f+gx)^5\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{11cdg\sqrt{d+ex}}$$

$$\Big\downarrow \text{1253}$$

$$\frac{1}{11}\left(-\frac{10ae^2}{cd}+11d-\frac{ef}{g}\right)\left(\frac{8(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{9cd}+\frac{2(f+gx)^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{9cd\sqrt{d+ex}}\right)+\frac{2e(f+gx)^5\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{11cdg\sqrt{d+ex}}$$

$$\Big\downarrow \text{1253}$$

$$\frac{1}{11}\left(-\frac{10ae^2}{cd}+11d-\frac{ef}{g}\right)\left(\frac{8(cdf-aeg)\left(\frac{6(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{7cd}+\frac{2(f+gx)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{7cd\sqrt{d+ex}}\right)}{9cd}+\right.$$
$$\frac{2e(f+gx)^5\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{11cdg\sqrt{d+ex}}$$

$$\Big\downarrow \text{1253}$$

$$\frac{1}{11}\left(-\frac{10ae^2}{cd}+11d-\frac{ef}{g}\right)\left(\frac{8(cdf-aeg)\left(\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\int\frac{\sqrt{d+ex}(f+gx)}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{5cd}+\frac{2(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex\cdots}}{5cd\sqrt{d+ex}}\cdots\right)}{7cd}\cdots\right)}{9cd}\cdots\right)$$

$$\frac{2e(f+gx)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{11cdg\sqrt{d+ex}}$$

$\downarrow$ 1221

$$\frac{1}{11}\left(-\frac{10ae^2}{cd}+11d-\frac{ef}{g}\right)\left(\frac{8(cdf-aeg)\left(\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\left(\frac{1}{3}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)\int\frac{\sqrt{d+ex}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\frac{2g\sqrt{d+ex}\sqrt{x(\cdots}}{\cdots}\right)}{5cd}\cdots\right)}{7cd}\cdots\right)}{9cd\cdots}\right)$$

$$\frac{2e(f+gx)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{11cdg\sqrt{d+ex}}$$

$\downarrow$ 1122

3.783. $\int\frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

$$\frac{2e(f+gx)^5\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{11cdg\sqrt{d+ex}}+$$

$$\frac{1}{11}\left(-\frac{10ae^2}{cd}+11d-\frac{ef}{g}\right)\left(\frac{2(f+gx)^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{9cd\sqrt{d+ex}}+\frac{8(cdf-aeg)\left(\frac{2(f+gx)^3\sqrt{x(ae^2+cd^2)+ade+}}{7cd\sqrt{d+ex}}\right.}{}\right.$$

input
```
Int[((d + e*x)^(3/2)*(f + g*x)^4)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2],x]
```

output
```
(2*e*(f + g*x)^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(11*c*d*g*Sq
rt[d + e*x]) + ((11*d - (10*a*e^2)/(c*d) - (e*f)/g)*((2*(f + g*x)^4*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(9*c*d*Sqrt[d + e*x]) + (8*(c*d*f -
 a*e*g)*((2*(f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*c*
d*Sqrt[d + e*x]) + (6*(c*d*f - a*e*g)*((2*(f + g*x)^2*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(5*c*d*Sqrt[d + e*x]) + (4*(c*d*f - a*e*g)*((2*(3
*f - (d*g)/e - (2*a*e*g)/(c*d))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(3*c*d*Sqrt[d + e*x]) + (2*g*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/(3*c*d*e)))/(5*c*d)))/(7*c*d)))/(9*c*d)))/11
```

3.783. $\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.783.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1258
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/(c*g*(n + p + 2))), x] - Simp[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2))   Int[(d + e*x)^
(m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p - 1, 0] &&
!LtQ[n, -1] && IntegerQ[2*p]
```

3.783.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 623, normalized size of antiderivative = 1.24

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-315e\,g^4x^5c^5d^5+350a\,c^4d^4e^2g^4x^4-385c^5d^6g^4x^4-1540c^5d^5ef\,g^3x^4-400a^2c^3d^3e^3g^4x^3+440a\,c^4d^5e\,g^4x^3+}{}$
gosper	$-\frac{2(cdx+ae)(-315e\,g^4x^5c^5d^5+350a\,c^4d^4e^2g^4x^4-385c^5d^6g^4x^4-1540c^5d^5ef\,g^3x^4-400a^2c^3d^3e^3g^4x^3+440a\,c^4d^5e\,g^4x^3+1760a\,c^4d}{}$

input
```
int((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
-2/3465/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-315*c^5*d^5*e*g^4*x^5+
350*a*c^4*d^4*e^2*g^4*x^4-385*c^5*d^6*g^4*x^4-1540*c^5*d^5*e*f*g^3*x^4-400
*a^2*c^3*d^3*e^3*g^4*x^3+440*a*c^4*d^5*e*g^4*x^3+1760*a*c^4*d^4*e^2*f*g^3*
x^3-1980*c^5*d^6*f*g^3*x^3-2970*c^5*d^5*e*f^2*g^2*x^3+480*a^3*c^2*d^2*e^4*
g^4*x^2-528*a^2*c^3*d^4*e^2*g^4*x^2-2112*a^2*c^3*d^3*e^3*f*g^3*x^2+2376*a*
c^4*d^5*e*f*g^3*x^2+3564*a*c^4*d^4*e^2*f^2*g^2*x^2-4158*c^5*d^6*f^2*g^2*x^
2-2772*c^5*d^5*e*f^3*g*x^2-640*a^4*c*d*e^5*g^4*x+704*a^3*c^2*d^3*e^3*g^4*x
+2816*a^3*c^2*d^2*e^4*f*g^3*x-3168*a^2*c^3*d^4*e^2*f*g^3*x-4752*a^2*c^3*d^
3*e^3*f^2*g^2*x+5544*a*c^4*d^5*e*f^2*g^2*x+3696*a*c^4*d^4*e^2*f^3*g*x-4620
*c^5*d^6*f^3*g*x-1155*c^5*d^5*e*f^4*x+1280*a^5*e^6*g^4-1408*a^4*c*d^2*e^4*
g^4-5632*a^4*c*d*e^5*f*g^3+6336*a^3*c^2*d^3*e^3*f*g^3+9504*a^3*c^2*d^2*e^4
*f^2*g^2-11088*a^2*c^3*d^4*e^2*f^2*g^2-7392*a^2*c^3*d^3*e^3*f^3*g+9240*a*c
^4*d^5*e*f^3*g+2310*a*c^4*d^4*e^2*f^4-3465*c^5*d^6*f^4)/c^6/d^6
```

3.783.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,(315\,c^5d^5eg^4x^5+1155\,(3\,c^5d^6-2\,ac^4d^4e^2)f^4-1848\,(5\,ac^4d^5e-4\,$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output
```
2/3465*(315*c^5*d^5*e*g^4*x^5 + 1155*(3*c^5*d^6 - 2*a*c^4*d^4*e^2)*f^4 - 1
848*(5*a*c^4*d^5*e - 4*a^2*c^3*d^3*e^3)*f^3*g + 1584*(7*a^2*c^3*d^4*e^2 -
6*a^3*c^2*d^2*e^4)*f^2*g^2 - 704*(9*a^3*c^2*d^3*e^3 - 8*a^4*c*d*e^5)*f*g^3
 + 128*(11*a^4*c*d^2*e^4 - 10*a^5*e^6)*g^4 + 35*(44*c^5*d^5*e*f*g^3 + (11*
c^5*d^6 - 10*a*c^4*d^4*e^2)*g^4)*x^4 + 10*(297*c^5*d^5*e*f^2*g^2 + 22*(9*c
^5*d^6 - 8*a*c^4*d^4*e^2)*f*g^3 - 4*(11*a*c^4*d^5*e - 10*a^2*c^3*d^3*e^3)*
g^4)*x^3 + 6*(462*c^5*d^5*e*f^3*g + 99*(7*c^5*d^6 - 6*a*c^4*d^4*e^2)*f^2*g
^2 - 44*(9*a*c^4*d^5*e - 8*a^2*c^3*d^3*e^3)*f*g^3 + 8*(11*a^2*c^3*d^4*e^2
- 10*a^3*c^2*d^2*e^4)*g^4)*x^2 + (1155*c^5*d^5*e*f^4 + 924*(5*c^5*d^6 - 4*
a*c^4*d^4*e^2)*f^3*g - 792*(7*a*c^4*d^5*e - 6*a^2*c^3*d^3*e^3)*f^2*g^2 + 3
52*(9*a^2*c^3*d^4*e^2 - 8*a^3*c^2*d^2*e^4)*f*g^3 - 64*(11*a^3*c^2*d^3*e^3
- 10*a^4*c*d*e^5)*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(e*x + d)/(c^6*d^6*e*x + c^6*d^7)
```

3.783.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)^4}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output
```
Integral((d + e*x)**(3/2)*(f + g*x)**4/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.783.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+(3\,c^2d^3-acde^2)x)\,f^4}{3\,\sqrt{cdx+ae}c^2d^2}$$
$$+\frac{8\,(3\,c^3d^3ex^3-10\,a^2cd^2e^2+8\,a^3e^4+(5\,c^3d^4-ac^2d^2e^2)x^2-(5\,ac^2d^3e-4\,a^2cde^3)x)\,f^3g}{15\,\sqrt{cdx+ae}c^3d^3}$$
$$+\frac{4\,(15\,c^4d^4ex^4+56\,a^3cd^2e^3-48\,a^4e^5+3\,(7\,c^4d^5-ac^3d^3e^2)x^3-(7\,ac^3d^4e-6\,a^2c^2d^2e^3)x^2+4\,(7\,a^2c^2d^3e^2-}{35\,\sqrt{cdx+ae}c^4d^4}$$
$$+\frac{8\,(35\,c^5d^5ex^5-144\,a^4cd^2e^4+128\,a^5e^6+5\,(9\,c^5d^6-ac^4d^4e^2)x^4-(9\,ac^4d^5e-8\,a^2c^3d^3e^3)x^3+2\,(9\,a^2c^3d^4e}{315\,\sqrt{cdx+aec^5d^5}}$$
$$+\frac{2\,(315\,c^6d^6ex^6+1408\,a^5cd^2e^5-1280\,a^6e^7+35\,(11\,c^6d^7-ac^5d^5e^2)x^5-5\,(11\,ac^5d^6e-10\,a^2c^4d^4e^3)x^4+8\,(}{3465\,\sqrt{cdx+}}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*
f^4/(sqrt(c*d*x + a*e)*c^2*d^2) + 8/15*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2
 + 8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*
d*e^3)*x)*f^3*g/(sqrt(c*d*x + a*e)*c^3*d^3) + 4/35*(15*c^4*d^4*e*x^4 + 56*
a^3*c*d^2*e^3 - 48*a^4*e^5 + 3*(7*c^4*d^5 - a*c^3*d^3*e^2)*x^3 - (7*a*c^3*
d^4*e - 6*a^2*c^2*d^2*e^3)*x^2 + 4*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*d*e^4)*x)*
f^2*g^2/(sqrt(c*d*x + a*e)*c^4*d^4) + 8/315*(35*c^5*d^5*e*x^5 - 144*a^4*c*
d^2*e^4 + 128*a^5*e^6 + 5*(9*c^5*d^6 - a*c^4*d^4*e^2)*x^4 - (9*a*c^4*d^5*e
 - 8*a^2*c^3*d^3*e^3)*x^3 + 2*(9*a^2*c^3*d^4*e^2 - 8*a^3*c^2*d^2*e^4)*x^2
- 8*(9*a^3*c^2*d^3*e^3 - 8*a^4*c*d*e^5)*x)*f*g^3/(sqrt(c*d*x + a*e)*c^5*d^
5) + 2/3465*(315*c^6*d^6*e*x^6 + 1408*a^5*c*d^2*e^5 - 1280*a^6*e^7 + 35*(1
1*c^6*d^7 - a*c^5*d^5*e^2)*x^5 - 5*(11*a*c^5*d^6*e - 10*a^2*c^4*d^4*e^3)*x
^4 + 8*(11*a^2*c^4*d^5*e^2 - 10*a^3*c^3*d^3*e^4)*x^3 - 16*(11*a^3*c^3*d^4*
e^3 - 10*a^4*c^2*d^2*e^5)*x^2 + 64*(11*a^4*c^2*d^3*e^4 - 10*a^5*c*d*e^6)*x
)*g^4/(sqrt(c*d*x + a*e)*c^6*d^6)
```

3.783.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1789 vs. $2(465) = 930$.

Time = 0.37 (sec) , antiderivative size = 1789, normalized size of antiderivative = 3.57

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output
```
2/3465*e*(3465*(c^5*d^6*f^4 - a*c^4*d^4*e^2*f^4 - 4*a*c^4*d^5*e*f^3*g + 4*
a^2*c^3*d^3*e^3*f^3*g + 6*a^2*c^3*d^4*e^2*f^2*g^2 - 6*a^3*c^2*d^2*e^4*f^2*
g^2 - 4*a^3*c^2*d^3*e^3*f*g^3 + 4*a^4*c*d*e^5*f*g^3 + a^4*c*d^2*e^4*g^4 -
a^5*e^6*g^4)*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/(c^6*d^6*e) - 2*(1155
*sqrt(-c*d^2*e + a*e^3)*c^5*d^6*e^4*f^4 - 1155*sqrt(-c*d^2*e + a*e^3)*a*c^
4*d^4*e^6*f^4 - 924*sqrt(-c*d^2*e + a*e^3)*c^5*d^7*e^3*f^3*g - 2772*sqrt(-
c*d^2*e + a*e^3)*a*c^4*d^5*e^5*f^3*g + 3696*sqrt(-c*d^2*e + a*e^3)*a^2*c^3
*d^3*e^7*f^3*g + 594*sqrt(-c*d^2*e + a*e^3)*c^5*d^8*e^2*f^2*g^2 + 990*sqrt
(-c*d^2*e + a*e^3)*a*c^4*d^6*e^4*f^2*g^2 + 3168*sqrt(-c*d^2*e + a*e^3)*a^2
*c^3*d^4*e^6*f^2*g^2 - 4752*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^2*e^8*f^2*g^2
 - 220*sqrt(-c*d^2*e + a*e^3)*c^5*d^9*e*f*g^3 - 308*sqrt(-c*d^2*e + a*e^3)
*a*c^4*d^7*e^3*f*g^3 - 528*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^5*e^5*f*g^3 -
1760*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^3*e^7*f*g^3 + 2816*sqrt(-c*d^2*e + a
*e^3)*a^4*c*d*e^9*f*g^3 + 35*sqrt(-c*d^2*e + a*e^3)*c^5*d^10*g^4 + 45*sqrt
(-c*d^2*e + a*e^3)*a*c^4*d^8*e^2*g^4 + 64*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d
^6*e^4*g^4 + 112*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^4*e^6*g^4 + 384*sqrt(-c*
d^2*e + a*e^3)*a^4*c*d^2*e^8*g^4 - 640*sqrt(-c*d^2*e + a*e^3)*a^5*e^10*g^4
)/(c^6*d^6*e^5) + (1155*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^4*d^4*
e^8*f^4 + 4620*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^4*d^5*e^7*f^3*g
 - 9240*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^3*d^3*e^9*f^3*g -...
```

3.783.9 Mupad [B] (verification not implemented)

Time = 12.77 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^4\,x^5\,\sqrt{d+e\,x}}{11\,c\,d}-\frac{\sqrt{d+e\,x}\,\left(2560\,a^5\,e^6\,g^4-2\right.}{}\right.}{}$$

input
```
int(((f + g*x)^4*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(1/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^4*x^5*(d + e*x)^(1/2)
)/(11*c*d) - ((d + e*x)^(1/2)*(2560*a^5*e^6*g^4 - 6930*c^5*d^6*f^4 + 4620*
a*c^4*d^4*e^2*f^4 - 2816*a^4*c*d^2*e^4*g^4 - 14784*a^2*c^3*d^3*e^3*f^3*g +
 12672*a^3*c^2*d^3*e^3*f*g^3 + 18480*a*c^4*d^5*e*f^3*g - 11264*a^4*c*d*e^5
*f*g^3 - 22176*a^2*c^3*d^4*e^2*f^2*g^2 + 19008*a^3*c^2*d^2*e^4*f^2*g^2))/(
3465*c^6*d^6*e) + (x*(d + e*x)^(1/2)*(2310*c^5*d^5*e*f^4 + 9240*c^5*d^6*f^
3*g - 1408*a^3*c^2*d^3*e^3*g^4 + 1280*a^4*c*d*e^5*g^4 - 7392*a*c^4*d^4*e^2
*f^3*g - 11088*a*c^4*d^5*e*f^2*g^2 + 6336*a^2*c^3*d^4*e^2*f*g^3 - 5632*a^3
*c^2*d^2*e^4*f*g^3 + 9504*a^2*c^3*d^3*e^3*f^2*g^2))/(3465*c^6*d^6*e) + (x^
2*(d + e*x)^(1/2)*(8316*c^5*d^6*f^2*g^2 + 1056*a^2*c^3*d^4*e^2*g^4 - 960*a
^3*c^2*d^2*e^4*g^4 + 5544*c^5*d^5*e*f^3*g - 7128*a*c^4*d^4*e^2*f^2*g^2 + 4
224*a^2*c^3*d^3*e^3*f*g^3 - 4752*a*c^4*d^5*e*f*g^3))/(3465*c^6*d^6*e) + (4
*g^2*x^3*(d + e*x)^(1/2)*(40*a^2*e^3*g^2 + 297*c^2*d^2*e*f^2 + 198*c^2*d^3
*f*g - 44*a*c*d^2*e*g^2 - 176*a*c*d*e^2*f*g))/(693*c^3*d^3*e) + (2*g^3*x^4
*(d + e*x)^(1/2)*(11*c*d^2*g - 10*a*e^2*g + 44*c*d*e*f))/(99*c^2*d^2*e)))/
(x + d/e)
```

3.783. $\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.784 $\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.784.1 Optimal result

Integrand size = 46, antiderivative size = 412

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{16(cdf-aeg)^2(8ae^2g+cd(ef-9dg))(2ae^2g-cd(3ef-dg))\sqrt{ad}}{315c^5d^5eg\sqrt{d+ex}}$$
$$-\frac{16(cdf-aeg)^2(8ae^2g+cd(ef-9dg))\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{315c^4d^4e}$$
$$-\frac{4(cdf-aeg)(8ae^2g+cd(ef-9dg))(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{105c^3d^3g\sqrt{d+ex}}$$
$$-\frac{2(8ae^2g+cd(ef-9dg))(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{63c^2d^2g\sqrt{d+ex}}$$
$$+\frac{2e(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{9cdg\sqrt{d+ex}}$$

output
```
16/315*(-a*e*g+c*d*f)^2*(8*a*e^2*g+c*d*(-9*d*g+e*f))*(2*a*e^2*g-c*d*(-d*g+
3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^5/d^5/e/g/(e*x+d)^(1/2)-
4/105*(-a*e*g+c*d*f)*(8*a*e^2*g+c*d*(-9*d*g+e*f))*(g*x+f)^2*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/g/(e*x+d)^(1/2)-2/63*(8*a*e^2*g+c*d*(-9*
d*g+e*f))*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/g/(e*x
+d)^(1/2)+2/9*e*(g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g/(e
*x+d)^(1/2)-16/315*(-a*e*g+c*d*f)^2*(8*a*e^2*g+c*d*(-9*d*g+e*f))*(e*x+d)^(
1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e
```

3.784.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.64

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(128a^4e^5g^3-16a^3cde^3g^2(27ef+9dg+4egx)}{}$$

input
```
Integrate[((d + e*x)^(3/2)*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2],x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(128*a^4*e^5*g^3 - 16*a^3*c*d*e^3*g^2*(27
*e*f + 9*d*g + 4*e*g*x) + 24*a^2*c^2*d^2*e^2*g*(3*d*g*(7*f + g*x) + e*(21*
f^2 + 9*f*g*x + 2*g^2*x^2)) - 2*a*c^3*d^3*e*(9*d*g*(35*f^2 + 14*f*g*x + 3*
g^2*x^2) + e*(105*f^3 + 126*f^2*g*x + 81*f*g^2*x^2 + 20*g^3*x^3)) + c^4*d^
4*(9*d*(35*f^3 + 35*f^2*g*x + 21*f*g^2*x^2 + 5*g^3*x^3) + e*x*(105*f^3 + 1
89*f^2*g*x + 135*f*g^2*x^2 + 35*g^3*x^3))))/(315*c^5*d^5*Sqrt[d + e*x])
```

3.784.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {1258, 1253, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \text{1258}$$

$$\frac{1}{9}\left(-\frac{8ae^2}{cd}+9d-\frac{ef}{g}\right)\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{cdex^2+(cd^2+ae^2)\,x+ade}}dx+\frac{2e(f+gx)^4\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{9cdg\sqrt{d+ex}}$$

$$\Big\downarrow \text{1253}$$

$$\frac{1}{9}\left(-\frac{8ae^2}{cd}+9d-\frac{ef}{g}\right)\left(\frac{6(cdf-aeg)\int\frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{7cd}+\frac{2(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7cd\sqrt{d+ex}}\right)+$$

$$\frac{2e(f+gx)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{9cdg\sqrt{d+ex}}$$

$\downarrow$ 1253

$$\frac{1}{9}\left(-\frac{8ae^2}{cd}+9d-\frac{ef}{g}\right)\left(\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\int\frac{\sqrt{d+ex}(f+gx)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{5cd}+\frac{2(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5cd\sqrt{d+ex}}\right)}{7cd}+\frac{2(\right.$$

$$\frac{2e(f+gx)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{9cdg\sqrt{d+ex}}$$

$\downarrow$ 1221

$$\frac{1}{9}\left(-\frac{8ae^2}{cd}+9d-\frac{ef}{g}\right)\left(\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\left(\frac{1}{3}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)\int\frac{\sqrt{d+ex}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{2g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade}}{3cde}\right.}{5cd}\right.}{7cd}\right.$$

$$\frac{2e(f+gx)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{9cdg\sqrt{d+ex}}$$

$\downarrow$ 1122

$$\frac{2e(f+gx)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{9cdg\sqrt{d+ex}}+$$

$$\frac{1}{9}\left(-\frac{8ae^2}{cd}+9d-\frac{ef}{g}\right)\left(\frac{2(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7cd\sqrt{d+ex}}+\frac{6(cdf-aeg)\left(\frac{2(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex}}{5cd\sqrt{d+ex}}\right.}{}\right.$$

3.784. $\int\frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}dx$

input
```
Int[((d + e*x)^(3/2)*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2],x]
```

output
```
(2*e*(f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(9*c*d*g*Sqr
t[d + e*x]) + ((9*d - (8*a*e^2)/(c*d) - (e*f)/g)*((2*(f + g*x)^3*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*c*d*Sqrt[d + e*x]) + (6*(c*d*f - a*
e*g)*((2*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c*d*S
qrt[d + e*x]) + (4*(c*d*f - a*e*g)*((2*(3*f - (d*g)/e - (2*a*e*g)/(c*d))*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*Sqrt[d + e*x]) + (2*g*S
qrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*e)))/(5*c
*d)))/(7*c*d)))/9
```

3.784.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1258
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/(c*g*(n + p + 2))), x] - Simp[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2))   Int[(d + e*x)^
(m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p - 1, 0] &&
!LtQ[n, -1] && IntegerQ[2*p]
```

3.784.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 407, normalized size of antiderivative = 0.99

method	result
default	$2\sqrt{(cdx+ae)(ex+d)}\,(35e\,g^3x^4c^4d^4-40a\,c^3d^3e^2g^3x^3+45c^4d^5g^3x^3+135c^4d^4ef\,g^2x^3+48a^2c^2d^2e^3g^3x^2-54a\,c^3d^4e\,g^3x^2-162a\,c^3d^3e^2$
gosper	$2(cdx+ae)(35e\,g^3x^4c^4d^4-40a\,c^3d^3e^2g^3x^3+45c^4d^5g^3x^3+135c^4d^4ef\,g^2x^3+48a^2c^2d^2e^3g^3x^2-54a\,c^3d^4e\,g^3x^2-162a\,c^3d^3e^2f\,g^2x^2+1$

input
```
int((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/315/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(35*c^4*d^4*e*g^3*x^4-40*a
*c^3*d^3*e^2*g^3*x^3+45*c^4*d^5*g^3*x^3+135*c^4*d^4*e*f*g^2*x^3+48*a^2*c^2
*d^2*e^3*g^3*x^2-54*a*c^3*d^4*e*g^3*x^2-162*a*c^3*d^3*e^2*f*g^2*x^2+189*c^
4*d^5*f*g^2*x^2+189*c^4*d^4*e*f^2*g*x^2-64*a^3*c*d*e^4*g^3*x+72*a^2*c^2*d^
3*e^2*g^3*x+216*a^2*c^2*d^2*e^3*f*g^2*x-252*a*c^3*d^4*e*f*g^2*x-252*a*c^3*
d^3*e^2*f^2*g*x+315*c^4*d^5*f^2*g*x+105*c^4*d^4*e*f^3*x+128*a^4*e^5*g^3-14
4*a^3*c*d^2*e^3*g^3-432*a^3*c*d*e^4*f*g^2+504*a^2*c^2*d^3*e^2*f*g^2+504*a^
2*c^2*d^2*e^3*f^2*g-630*a*c^3*d^4*e*f^2*g-210*a*c^3*d^3*e^2*f^3+315*c^4*d^
5*f^3)/c^5/d^5
```

3.784.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = 2\left(35\,c^4d^4eg^3x^4+105\left(3\,c^4d^5-2\,ac^3d^3e^2\right)f^3-126\left(5\,ac^3d^4e-4\,a^2c^2\right.\right.$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output
```
2/315*(35*c^4*d^4*e*g^3*x^4 + 105*(3*c^4*d^5 - 2*a*c^3*d^3*e^2)*f^3 - 126*
(5*a*c^3*d^4*e - 4*a^2*c^2*d^2*e^3)*f^2*g + 72*(7*a^2*c^2*d^3*e^2 - 6*a^3*
c*d*e^4)*f*g^2 - 16*(9*a^3*c*d^2*e^3 - 8*a^4*e^5)*g^3 + 5*(27*c^4*d^4*e*f*
g^2 + (9*c^4*d^5 - 8*a*c^3*d^3*e^2)*g^3)*x^3 + 3*(63*c^4*d^4*e*f^2*g + 9*(
7*c^4*d^5 - 6*a*c^3*d^3*e^2)*f*g^2 - 2*(9*a*c^3*d^4*e - 8*a^2*c^2*d^2*e^3)
*g^3)*x^2 + (105*c^4*d^4*e*f^3 + 63*(5*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^2*g -
36*(7*a*c^3*d^4*e - 6*a^2*c^2*d^2*e^3)*f*g^2 + 8*(9*a^2*c^2*d^3*e^2 - 8*a^
3*c*d*e^4)*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x +
d)/(c^5*d^5*e*x + c^5*d^6)
```

3.784.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)^3}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output
```
Integral((d + e*x)**(3/2)*(f + g*x)**3/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.784.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\,(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+(3\,c^2d^3-acde^2)x)\,f^3}{3\,\sqrt{cdx+ae}c^2d^2}$$
$$+\frac{2\,(3\,c^3d^3ex^3-10\,a^2cd^2e^2+8\,a^3e^4+(5\,c^3d^4-ac^2d^2e^2)x^2-(5\,ac^2d^3e-4\,a^2cde^3)x)\,f^2g}{5\,\sqrt{cdx+ae}c^3d^3}$$
$$+\frac{2\,(15\,c^4d^4ex^4+56\,a^3cd^2e^3-48\,a^4e^5+3\,(7\,c^4d^5-ac^3d^3e^2)x^3-(7\,ac^3d^4e-6\,a^2c^2d^2e^3)x^2+4\,(7\,a^2c^2d^3e^2-}{35\,\sqrt{cdx+ae}c^4d^4}$$
$$+\frac{2\,(35\,c^5d^5ex^5-144\,a^4cd^2e^4+128\,a^5e^6+5\,(9\,c^5d^6-ac^4d^4e^2)x^4-(9\,ac^4d^5e-8\,a^2c^3d^3e^3)x^3+2\,(9\,a^2c^3d^4e^}{315\,\sqrt{cdx+ae}c^5d^5}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*
f^3/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/5*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2
+ 8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d
*e^3)*x)*f^2*g/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/35*(15*c^4*d^4*e*x^4 + 56*a
^3*c*d^2*e^3 - 48*a^4*e^5 + 3*(7*c^4*d^5 - a*c^3*d^3*e^2)*x^3 - (7*a*c^3*d
^4*e - 6*a^2*c^2*d^2*e^3)*x^2 + 4*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*d*e^4)*x)*f
*g^2/(sqrt(c*d*x + a*e)*c^4*d^4) + 2/315*(35*c^5*d^5*e*x^5 - 144*a^4*c*d^2
*e^4 + 128*a^5*e^6 + 5*(9*c^5*d^6 - a*c^4*d^4*e^2)*x^4 - (9*a*c^4*d^5*e -
8*a^2*c^3*d^3*e^3)*x^3 + 2*(9*a^2*c^3*d^4*e^2 - 8*a^3*c^2*d^2*e^4)*x^2 - 8
*(9*a^3*c^2*d^3*e^3 - 8*a^4*c*d*e^5)*x)*g^3/(sqrt(c*d*x + a*e)*c^5*d^5)
```

3.784.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. $2(382) = 764$.

Time = 0.34 (sec) , antiderivative size = 1185, normalized size of antiderivative = 2.88

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output
```
2/315*e*(315*(c^4*d^5*f^3 - a*c^3*d^3*e^2*f^3 - 3*a*c^3*d^4*e*f^2*g + 3*a^
2*c^2*d^2*e^3*f^2*g + 3*a^2*c^2*d^3*e^2*f*g^2 - 3*a^3*c*d*e^4*f*g^2 - a^3*
c*d^2*e^3*g^3 + a^4*e^5*g^3)*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/(c^5*
d^5*e) - 2*(105*sqrt(-c*d^2*e + a*e^3)*c^4*d^5*e^3*f^3 - 105*sqrt(-c*d^2*e
 + a*e^3)*a*c^3*d^3*e^5*f^3 - 63*sqrt(-c*d^2*e + a*e^3)*c^4*d^6*e^2*f^2*g
- 189*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^4*e^4*f^2*g + 252*sqrt(-c*d^2*e + a*e
^3)*a^2*c^2*d^2*e^6*f^2*g + 27*sqrt(-c*d^2*e + a*e^3)*c^4*d^7*e*f*g^2 + 45
*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^5*e^3*f*g^2 + 144*sqrt(-c*d^2*e + a*e^3)*a
^2*c^2*d^3*e^5*f*g^2 - 216*sqrt(-c*d^2*e + a*e^3)*a^3*c*d*e^7*f*g^2 - 5*sq
rt(-c*d^2*e + a*e^3)*c^4*d^8*g^3 - 7*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2*
g^3 - 12*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4*g^3 - 40*sqrt(-c*d^2*e + a
*e^3)*a^3*c*d^2*e^6*g^3 + 64*sqrt(-c*d^2*e + a*e^3)*a^4*e^8*g^3)/(c^5*d^5*
e^4) + (105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^3*d^3*e^6*f^3 + 31
5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^3*d^4*e^5*f^2*g - 630*((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^2*d^2*e^7*f^2*g - 630*((e*x + d)*c
*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^2*d^3*e^6*f*g^2 + 945*((e*x + d)*c*d*e -
 c*d^2*e + a*e^3)^(3/2)*a^2*c*d*e^8*f*g^2 + 315*((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)^(3/2)*a^2*c*d^2*e^7*g^3 - 420*((e*x + d)*c*d*e - c*d^2*e + a*e^3
)^(3/2)*a^3*e^9*g^3 + 189*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^2*d^
2*e^4*f^2*g + 189*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^2*d^3*e^3...
```

3.784.9 Mupad [B] (verification not implemented)

Time = 12.40 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(256\,a^4\,e^5\,g^3-288\,a^3\,c\,d^2\,e^3\,g^3-864\right.}{}\right.}{}$$

input
```
int(((f + g*x)^3*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(1/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(256*a^4*
e^5*g^3 + 630*c^4*d^5*f^3 - 420*a*c^3*d^3*e^2*f^3 - 288*a^3*c*d^2*e^3*g^3
+ 1008*a^2*c^2*d^2*e^3*f^2*g + 1008*a^2*c^2*d^3*e^2*f*g^2 - 1260*a*c^3*d^4
*e*f^2*g - 864*a^3*c*d*e^4*f*g^2))/(315*c^5*d^5*e) + (2*g^3*x^4*(d + e*x)^
(1/2))/(9*c*d) + (x*(d + e*x)^(1/2)*(210*c^4*d^4*e*f^3 + 630*c^4*d^5*f^2*g
 + 144*a^2*c^2*d^3*e^2*g^3 - 128*a^3*c*d*e^4*g^3 - 504*a*c^3*d^3*e^2*f^2*g
 + 432*a^2*c^2*d^2*e^3*f*g^2 - 504*a*c^3*d^4*e*f*g^2))/(315*c^5*d^5*e) + (
2*g*x^2*(d + e*x)^(1/2)*(16*a^2*e^3*g^2 + 63*c^2*d^2*e*f^2 + 63*c^2*d^3*f*
g - 18*a*c*d^2*e*g^2 - 54*a*c*d*e^2*f*g))/(105*c^3*d^3*e) + (2*g^2*x^3*(d
+ e*x)^(1/2)*(9*c*d^2*g - 8*a*e^2*g + 27*c*d*e*f))/(63*c^2*d^2*e)))/(x + d
/e)
```

3.784. $\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.785 $\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.785.1 Optimal result

Integrand size = 46, antiderivative size = 321

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{8(cdf-aeg)\left(6ae^2g+cd(ef-7dg)\right)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+}}{105c^4d^4eg\sqrt{d+ex}}$$
$$-\frac{8(cdf-aeg)\left(6ae^2g+cd(ef-7dg)\right)\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{105c^3d^3e}$$
$$-\frac{2\left(6ae^2g+cd(ef-7dg)\right)(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35c^2d^2g\sqrt{d+ex}}$$
$$+\frac{2e(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{7cdg\sqrt{d+ex}}$$

output
```
8/105*(-a*e*g+c*d*f)*(6*a*e^2*g+c*d*(-7*d*g+e*f))*(2*a*e^2*g-c*d*(-d*g+3*e
*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e/g/(e*x+d)^(1/2)-2/3
5*(6*a*e^2*g+c*d*(-7*d*g+e*f))*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)/c^2/d^2/g/(e*x+d)^(1/2)+2/7*e*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)/c/d/g/(e*x+d)^(1/2)-8/105*(-a*e*g+c*d*f)*(6*a*e^2*g+c*d*(-7*d
*g+e*f))*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e
```

3.785.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(-48a^3e^4g^2+8a^2cde^2g(14ef+7dg+3egx)}{}$$

input
```
Integrate[((d + e*x)^(3/2)*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2],x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-48*a^3*e^4*g^2 + 8*a^2*c*d*e^2*g*(14*e*
f + 7*d*g + 3*e*g*x) - 2*a*c^2*d^2*e*(14*d*g*(5*f + g*x) + e*(35*f^2 + 28*
f*g*x + 9*g^2*x^2)) + c^3*d^3*(7*d*(15*f^2 + 10*f*g*x + 3*g^2*x^2) + e*x*(
35*f^2 + 42*f*g*x + 15*g^2*x^2))))/(105*c^4*d^4*Sqrt[d + e*x])
```

3.785.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1258, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \text{1258}$$

$$\frac{1}{7}\left(-\frac{6ae^2}{cd}+7d-\frac{ef}{g}\right)\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\frac{2e(f+gx)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{7cdg\sqrt{d+ex}}$$

$$\Big\downarrow \text{1253}$$

$$\frac{1}{7}\left(-\frac{6ae^2}{cd}+7d-\frac{ef}{g}\right)\left(\frac{4(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{5cd}+\frac{2(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5cd\sqrt{d+ex}}\right)+\frac{2e(f+gx)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{7cdg\sqrt{d+ex}}$$

$$\Big\downarrow \text{1221}$$

$$\frac{1}{7}\left(-\frac{6ae^2}{cd}+7d-\frac{ef}{g}\right)\left(\frac{4(cdf-aeg)\left(\frac{1}{3}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)\int\frac{\sqrt{d+ex}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\frac{2g\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade}}{3cde}\right.}{5cd}\right.$$

$$\frac{2e(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7cdg\sqrt{d+ex}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2e(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7cdg\sqrt{d+ex}}+$$

$$\frac{1}{7}\left(-\frac{6ae^2}{cd}+7d-\frac{ef}{g}\right)\left(\frac{2(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5cd\sqrt{d+ex}}+\frac{4(cdf-aeg)\left(\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(-\frac{2aeg}{cd}\right.}{3cd\sqrt{d+ex}}\right.}{5c}\right.$$

input
```
Int[((d + e*x)^(3/2)*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2],x]
```

output
```
(2*e*(f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*c*d*g*Sqr
t[d + e*x]) + ((7*d - (6*a*e^2)/(c*d) - (e*f)/g)*((2*(f + g*x)^2*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c*d*Sqrt[d + e*x]) + (4*(c*d*f - a*
e*g)*((2*(3*f - (d*g)/e - (2*a*e*g)/(c*d))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])/(3*c*d*Sqrt[d + e*x]) + (2*g*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*e)))/(5*c*d)))/7
```

3.785.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1258
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/(c*g*(n + p + 2))), x] - Simp[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2))   Int[(d + e*x)^
(m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p - 1, 0] &&
!LtQ[n, -1] && IntegerQ[2*p]
```

3.785.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.74

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-15g^2 e x^3 c^3 d^3+18a c^2 d^2 e^2 g^2 x^2-21c^3 d^4 g^2 x^2-42c^3 d^3 e f g x^2-24a^2 c d e^3 g^2 x+28a c^2 d^3 e g^2 x+56a c^2 d^2 e^2 f g}{105\sqrt{ex+d}\,c^4 d^4}$
gosper	$-\frac{2(cdx+ae)(-15g^2 e x^3 c^3 d^3+18a c^2 d^2 e^2 g^2 x^2-21c^3 d^4 g^2 x^2-42c^3 d^3 e f g x^2-24a^2 c d e^3 g^2 x+28a c^2 d^3 e g^2 x+56a c^2 d^2 e^2 f g x-70c^3 d^4}{105c^4 d^4\sqrt{cde x^2+a e^2 x+c d}}$

input
```
int((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
-2/105/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-15*c^3*d^3*e*g^2*x^3+18
*a*c^2*d^2*e^2*g^2*x^2-21*c^3*d^4*g^2*x^2-42*c^3*d^3*e*f*g*x^2-24*a^2*c*d*
e^3*g^2*x+28*a*c^2*d^3*e*g^2*x+56*a*c^2*d^2*e^2*f*g*x-70*c^3*d^4*f*g*x-35*
c^3*d^3*e*f^2*x+48*a^3*e^4*g^2-56*a^2*c*d^2*e^2*g^2-112*a^2*c*d*e^3*f*g+14
0*a*c^2*d^3*e*f*g+70*a*c^2*d^2*e^2*f^2-105*c^3*d^4*f^2)/c^4/d^4
```

3.785.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\left(15\,c^3d^3eg^2x^3+35\left(3\,c^3d^4-2\,ac^2d^2e^2\right)f^2-28\left(5\,ac^2d^3e-4\,a^2cde^3\right.}{}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output
```
2/105*(15*c^3*d^3*e*g^2*x^3 + 35*(3*c^3*d^4 - 2*a*c^2*d^2*e^2)*f^2 - 28*(5
*a*c^2*d^3*e - 4*a^2*c*d*e^3)*f*g + 8*(7*a^2*c*d^2*e^2 - 6*a^3*e^4)*g^2 +
3*(14*c^3*d^3*e*f*g + (7*c^3*d^4 - 6*a*c^2*d^2*e^2)*g^2)*x^2 + (35*c^3*d^3
*e*f^2 + 14*(5*c^3*d^4 - 4*a*c^2*d^2*e^2)*f*g - 4*(7*a*c^2*d^3*e - 6*a^2*c
*d*e^3)*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/
(c^4*d^4*e*x + c^4*d^5)
```

3.785.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)^2}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output
```
Integral((d + e*x)**(3/2)*(f + g*x)**2/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.785.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+(3\,c^2d^3-acde^2)x)\,f^2}{3\,\sqrt{cdx+ae}c^2d^2} + \frac{4\,(3\,c^3d^3ex^3-10\,a^2cd^2e^2+8\,a^3e^4+(5\,c^3d^4-ac^2d^2e^2)x^2-(5\,ac^2d^3e-4\,a^2cde^3)x)\,fg}{15\,\sqrt{cdx+ae}c^3d^3} + \frac{2\,(15\,c^4d^4ex^4+56\,a^3cd^2e^3-48\,a^4e^5+3\,(7\,c^4d^5-ac^3d^3e^2)x^3-(7\,ac^3d^4e-6\,a^2c^2d^2e^3)x^2+4\,(7\,a^2c^2d^3e^2-}{105\,\sqrt{cdx+ae}c^4d^4}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*
f^2/(sqrt(c*d*x + a*e)*c^2*d^2) + 4/15*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2
 + 8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*
d*e^3)*x)*f*g/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/105*(15*c^4*d^4*e*x^4 + 56*a
^3*c*d^2*e^3 - 48*a^4*e^5 + 3*(7*c^4*d^5 - a*c^3*d^3*e^2)*x^3 - (7*a*c^3*d
^4*e - 6*a^2*c^2*d^2*e^3)*x^2 + 4*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*d*e^4)*x)*g
^2/(sqrt(c*d*x + a*e)*c^4*d^4)
```

3.785.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 704 vs. 2(297) = 594.

Time = 0.32 (sec) , antiderivative size = 704, normalized size of antiderivative = 2.19

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,e\left(\frac{105\,(c^3d^4f^2-ac^2d^2e^2f^2-2\,ac^2d^3efg+2\,a^2cde^3fg+a^2cd^2e^2g^2-a^3e^4g^2)\,\sqrt{(ex+d)cde-}}{c^4d^4e}\right.}{}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output
```
2/105*e*(105*(c^3*d^4*f^2 - a*c^2*d^2*e^2*f^2 - 2*a*c^2*d^3*e*f*g + 2*a^2*
c*d*e^3*f*g + a^2*c*d^2*e^2*g^2 - a^3*e^4*g^2)*sqrt((e*x + d)*c*d*e - c*d^
2*e + a*e^3)/(c^4*d^4*e) - 2*(35*sqrt(-c*d^2*e + a*e^3)*c^3*d^4*e^2*f^2 -
35*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^2*e^4*f^2 - 14*sqrt(-c*d^2*e + a*e^3)*c^
3*d^5*e*f*g - 42*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^3*e^3*f*g + 56*sqrt(-c*d^2
*e + a*e^3)*a^2*c*d*e^5*f*g + 3*sqrt(-c*d^2*e + a*e^3)*c^3*d^6*g^2 + 5*sqr
t(-c*d^2*e + a*e^3)*a*c^2*d^4*e^2*g^2 + 16*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^
2*e^4*g^2 - 24*sqrt(-c*d^2*e + a*e^3)*a^3*e^6*g^2)/(c^4*d^4*e^3) + (35*((e
*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*e^4*f^2 + 70*((e*x + d)*c*d
*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^3*e^3*f*g - 140*((e*x + d)*c*d*e - c*d^2
*e + a*e^3)^(3/2)*a*c*d*e^5*f*g - 70*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(
3/2)*a*c*d^2*e^4*g^2 + 105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e
^6*g^2 + 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c*d*e^2*f*g + 21*((e
*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c*d^2*e*g^2 - 63*((e*x + d)*c*d*e -
 c*d^2*e + a*e^3)^(5/2)*a*e^3*g^2 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)
^(7/2)*g^2)/(c^4*d^4*e^6))/abs(e)
```

3.785.9 Mupad [B] (verification not implemented)

Time = 12.29 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\,\Big(\frac{2\,g^2\,x^3\,\sqrt{d+e\,x}}{7\,c\,d}-\frac{\sqrt{d+e\,x}\,(96\,a^3\,e^4\,g^2-112}{}}{}$$

input
```
int(((f + g*x)^2*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(1/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^3*(d + e*x)^(1/2)
)/(7*c*d) - ((d + e*x)^(1/2)*(96*a^3*e^4*g^2 - 210*c^3*d^4*f^2 + 140*a*c^2
*d^2*e^2*f^2 - 112*a^2*c*d^2*e^2*g^2 + 280*a*c^2*d^3*e*f*g - 224*a^2*c*d*e
^3*f*g))/(105*c^4*d^4*e) + (x*(d + e*x)^(1/2)*(70*c^3*d^3*e*f^2 + 140*c^3*
d^4*f*g - 56*a*c^2*d^3*e*g^2 + 48*a^2*c*d*e^3*g^2 - 112*a*c^2*d^2*e^2*f*g)
)/(105*c^4*d^4*e) + (2*g*x^2*(d + e*x)^(1/2)*(7*c*d^2*g - 6*a*e^2*g + 14*c
*d*e*f))/(35*c^2*d^2*e)))/(x + d/e)
```

3.786 $\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.786.1 Optimal result

Integrand size = 44, antiderivative size = 209

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{4(cd^2-ae^2)(4ae^2g-cd(5ef-dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{15c^3d^3e\sqrt{d+ex}} - \frac{2(4ae^2g-cd(5ef-dg))\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{15c^2d^2e} + \frac{2g(d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{5cde}$$

output
```
2/5*g*(e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e-4/15*(-a
*e^2+c*d^2)*(4*a*e^2*g-c*d*(-d*g+5*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)/c^3/d^3/e/(e*x+d)^(1/2)-2/15*(4*a*e^2*g-c*d*(-d*g+5*e*f))*(e*x+d)^(
1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e
```

3.786.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.46

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(8a^2e^3g-2acde(5ef+5dg+2egx)+c^2d^2(5}{15c^3d^3\sqrt{d+ex}}$$

input
```
Integrate[((d + e*x)^(3/2)*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2],x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^3*g - 2*a*c*d*e*(5*e*f + 5*d*g +
 2*e*g*x) + c^2*d^2*(5*d*(3*f + g*x) + e*x*(5*f + 3*g*x))))/(15*c^3*d^3*Sq
rt[d + e*x])
```

3.786.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.068, Rules used = {1221, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{1}{5}\left(-\frac{4aeg}{cd}-\frac{dg}{e}+5f\right)\int \frac{(d+ex)^{3/2}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\frac{2g(d+ex)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5cde}$$

$$\Big\downarrow \text{1128}$$

$$\frac{1}{5}\left(-\frac{4aeg}{cd}-\frac{dg}{e}+5f\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{\sqrt{d+ex}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{3d}+\frac{2\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cd}\right)+\frac{2g(d+ex)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5cde}$$

$$\Big\downarrow \text{1122}$$

$$\frac{1}{5}\left(\frac{4\left(d^2-\frac{ae^2}{c}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}\right)\left(-\frac{4aeg}{cd}-\frac{dg}{e}+5f\right.$$

$$\frac{2g(d+ex)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5cde}$$

input
```
Int[((d + e*x)^(3/2)*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
],x]
```

output
```
(2*g*(d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c*d*e
) + ((5*f - (d*g)/e - (4*a*e*g)/(c*d))*((4*(d^2 - (a*e^2)/c)*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d^2*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d)))/5
```

3.786.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

3.786.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(3eg\,x^2c^2d^2-4acd\,e^2gx+5c^2d^3gx+5c^2d^2efx+8a^2e^3g-10ac\,d^2eg-10acd\,e^2f+15d^3f\,c^2)}{15\sqrt{ex+d}\,c^3d^3}$	113
gosper	$\frac{2(cdx+ae)\,(3eg\,x^2c^2d^2-4acd\,e^2gx+5c^2d^3gx+5c^2d^2efx+8a^2e^3g-10ac\,d^2eg-10acd\,e^2f+15d^3f\,c^2)\sqrt{ex+d}}{15c^3d^3\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	131

input
```
int((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
2/15/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*c^2*d^2*e*g*x^2-4*a*c*d*
e^2*g*x+5*c^2*d^3*g*x+5*c^2*d^2*e*f*x+8*a^2*e^3*g-10*a*c*d^2*e*g-10*a*c*d*
e^2*f+15*c^2*d^3*f)/c^3/d^3
```

3.786.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.67

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,(3\,c^2d^2egx^2+5\,(3\,c^2d^3-2\,acde^2)f-2\,(5\,acd^2e-4\,a^2e^3)g+(5\,c^2d}{15\,(c^3d^3e}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="fricas")
```

output
```
2/15*(3*c^2*d^2*e*g*x^2 + 5*(3*c^2*d^3 - 2*a*c*d*e^2)*f - 2*(5*a*c*d^2*e -
 4*a^2*e^3)*g + (5*c^2*d^2*e*f + (5*c^2*d^3 - 4*a*c*d*e^2)*g)*x)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

3.786.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/
2),x)
```

output
```
Integral((d + e*x)**(3/2)*(f + g*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.786.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\,(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+(3\,c^2d^3-acde^2)x)f}{3\,\sqrt{cdx+ae}c^2d^2}+\frac{2\,(3\,c^3d^3ex^3-10\,a^2cd^2e^2+8\,a^3e^4+(5\,c^3d^4-ac^2d^2e^2)x^2-(5\,ac^2d^3e-4\,a^2cde^3)x)g}{15\,\sqrt{cdx+ae}c^3d^3}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="maxima")
```

output
```
2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*
f/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/15*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2 +
 8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*
e^3)*x)*g/(sqrt(c*d*x + a*e)*c^3*d^3)
```

3.786.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.66

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\,e\left(\frac{15\,(c^2d^3f-acde^2f-acd^2eg+a^2e^3g)\sqrt{(ex+d)cde-cd^2e+ae^3}}{c^3d^3e}-\frac{2\left(5\sqrt{-cd^2e+ae^3}c^2d\right.}{}\right.}{}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="giac")
```

output
```
2/15*e*(15*(c^2*d^3*f - a*c*d*e^2*f - a*c*d^2*e*g + a^2*e^3*g)*sqrt((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)/(c^3*d^3*e) - 2*(5*sqrt(-c*d^2*e + a*e^3)*c^2
*d^3*e*f - 5*sqrt(-c*d^2*e + a*e^3)*a*c*d*e^3*f - sqrt(-c*d^2*e + a*e^3)*c
^2*d^4*g - 3*sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2*g + 4*sqrt(-c*d^2*e + a*e^
3)*a^2*e^4*g)/(c^3*d^3*e^2) + (5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)
*c*d*e^2*f + 5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c*d^2*e*g - 10*((
e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3*g + 3*((e*x + d)*c*d*e - c*d
^2*e + a*e^3)^(5/2)*g)/(c^3*d^3*e^4))/abs(e)
```

3.786.9 Mupad [B] (verification not implemented)

Time = 12.00 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.73

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(16\,g\,a^2\,e^3-20\,g\,a\,c\,d^2\,e-20\,f\,a\,c\,d\,e^2 \cdots\right)}{15\,c^3\,d^3\,e} \cdots\right)}{x+\frac{d}{e}}$$

input
```
int(((f + g*x)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(16*a^2*e
^3*g + 30*c^2*d^3*f - 20*a*c*d*e^2*f - 20*a*c*d^2*e*g))/(15*c^3*d^3*e) + (
2*g*x^2*(d + e*x)^(1/2))/(5*c*d) + (2*x*(d + e*x)^(1/2)*(5*c*d^2*g - 4*a*e
^2*g + 5*c*d*e*f))/(15*c^2*d^2*e)))/(x + d/e)
```

3.787 $\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$

3.787.1 Optimal result

Integrand size = 39, antiderivative size = 109

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx = \frac{4(cd^2-ae^2)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3c^2d^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3cd}$$

output
```
4/3*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)
^(1/2)+2/3*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d
```

3.787.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.50

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(-2ae^2+cd(3d+ex))}{3c^2d^2\sqrt{d+ex}}$$

input
```
Integrate[(d + e*x)^(3/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e^2 + c*d*(3*d + e*x)))/(3*c^2*d^2*
Sqrt[d + e*x])
```

3.787.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1128}$$

$$\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{\sqrt{d+ex}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{3d}+\frac{2\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cd}$$

$$\Big\downarrow \text{1122}$$

$$\frac{4\left(d^2-\frac{ae^2}{c}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cd^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cd}$$

input
```
Int[(d + e*x)^(3/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(4*(d^2 - (a*e^2)/c)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d^2
*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(3*c*d)
```

3.787.3.1 Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

3.787.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-cdex+2e^2a-3c\,d^2)}{3\sqrt{ex+d}\,c^2d^2}$	51
gosper	$-\frac{2(cdx+ae)(-cdex+2e^2a-3c\,d^2)\sqrt{ex+d}}{3c^2d^2\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	69

input
```
int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-c*d*e*x+2*a*e^2-3*c*d^2)/
c^2/d^2
```

3.787.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.67

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(cdex+3\,cd^2-2\,ae^2)\sqrt{ex+d}}{3\,(c^2d^2ex+c^2d^3)}$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="fricas")
```

output
```
2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*e*x + 3*c*d^2 - 2*a*e
^2)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

3.787.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral((d + e*x)**(3/2)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.787.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.60

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+(3\,c^2d^3-acde^2)x)}{3\,\sqrt{cdx+ae}c^2d^2}$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="maxima")
```

output
```
2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)/
(sqrt(c*d*x + a*e)*c^2*d^2)
```

3.787.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,e\left(\frac{3\,\sqrt{(ex+d)cde-cd^2e+ae^3}\,(cd^2-ae^2)}{c^2d^2e}-\frac{2\left(\sqrt{-cd^2e+ae^3}cd^2-\sqrt{-cd^2e+ae^3}ae^2\right)}{c^2d^2e}+\right.}{3\,|e|}$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="giac")
```

output
```
2/3*e*(3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(c*d^2 - a*e^2)/(c^2*d^2*
e) - 2*(sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^2)/(c^2*
d^2*e) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c^2*d^2*e^2))/abs(e)
```

3.787.9 Mupad [B] (verification not implemented)

Time = 11.91 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\left(\frac{2\,x\,\sqrt{d+e\,x}}{3\,c\,d}-\frac{\left(4\,a\,e^2-6\,c\,d^2\right)\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x+\frac{d}{e}}$$

input
```
int((d + e*x)^(3/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
(((2*x*(d + e*x)^(1/2))/(3*c*d) - ((4*a*e^2 - 6*c*d^2)*(d + e*x)^(1/2))/(3
*c^2*d^2*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x + d/e)
```

3.787. $\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.788 $\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.788.1 Optimal result

Integrand size = 46, antiderivative size = 139

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\frac{2e\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cdg\sqrt{d+ex}}-\frac{2(ef-dg)\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

output
```
-2*(-d*g+e*f)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e
*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(1/2)+2*e*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g/(e*x+d)^(1/2)
```

3.788.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\frac{2\sqrt{d+ex}\left(e\sqrt{g}\sqrt{cdf-aeg}(ae+cdx)+cd(-ef+dg)\sqrt{ae\cdots}\right.}{cdg^{3/2}\sqrt{cdf-aeg}\sqrt{(ae+cdx)(d+\cdots}}$$

input
```
Integrate[(d + e*x)^(3/2)/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]),x]
```

output
```
(2*Sqrt[d + e*x]*(e*Sqrt[g]*Sqrt[c*d*f - a*e*g]*(a*e + c*d*x) + c*d*(-(e*f
) + d*g)*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f -
 a*e*g]]))/(c*d*g^(3/2)*Sqrt[c*d*f - a*e*g]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.788.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1258, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1258}$$

$$\frac{2e\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cdg\sqrt{d+ex}}-\frac{(ef-dg)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{g}$$

$$\Big\downarrow \text{1255}$$

$$\frac{2e\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cdg\sqrt{d+ex}}-\frac{2e^2(ef-dg)\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}}{g}$$

$$\Big\downarrow \text{218}$$

$$\frac{2e\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cdg\sqrt{d+ex}}-\frac{2(ef-dg)\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

input
```
Int[(d + e*x)^(3/2)/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
),x]
```

output
```
(2*e*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*g*Sqrt[d + e*x]) -
(2*(e*f - d*g)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(g^(3/2)*Sqrt[c*d*f - a*e*g])
```

3.788.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 1258
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/(c*g*(n + p + 2))), x] - Simp[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2))   Int[(d + e*x)^
(m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p - 1, 0] &&
!LtQ[n, -1] && IntegerQ[2*p]
```

3.788.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.10

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\left(\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c\,d^{2}g-\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cdef-e\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\right)}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,dcg\sqrt{(aeg-cdf)g}}$	153

input
```
int((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
-2*((c*d*x+a*e)*(e*x+d))^(1/2)*(arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*c*d^2*g-arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*
d*e*f-e*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/(c*d*x+a*
e)^(1/2)/d/c/g/((a*e*g-c*d*f)*g)^(1/2)
```

3.788.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 511, normalized size of antiderivative = 3.68

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \left[\frac{(cd^2ef-cd^3g+(cde^2f-cd^2eg)x)\sqrt{-cdfg+aeg^2}\log\left(-\right.}{}\right.$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="fricas")
```

output
```
[((c*d^2*e*f - c*d^3*g + (c*d*e^2*f - c*d^2*e*g)*x)*sqrt(-c*d*f*g + a*e*g^
2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*
g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g
^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(c*d*e*f*g - a*e^2
*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*
f*g^2 - a*c*d^2*e*g^3 + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x), 2*((c*d^2*e*
f - c*d^3*g + (c*d*e^2*f - c*d^2*e*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sq
rt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x
 + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (c*d*e*f*g - a*e^2*
g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f
*g^2 - a*c*d^2*e*g^3 + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x)]
```

3.788.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/
2),x)
```

output
```
Integral((d + e*x)**(3/2)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)), x)
```

3.788.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*
x + f)), x)
```

3.788.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(123) = 246$.

Time = 0.41 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.04

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}e}{g|e|} - \frac{(cde^3f-cd^2e^2g)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+a\cdots}}{\sqrt{cdfg-aeg^2}e}\cdots\right)}{\sqrt{cdfg-aeg^2}eg|e|}\cdots\right)}{cd} + \frac{2\left(cde^2f\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right) - cd^2eg\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right) - \sqrt{-cd^2e+ae^3}\sqrt{cdfg-aeg^2}e\right)}{\sqrt{cdfg-aeg^2}cdg|e|}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="giac")
```

output
```
2*(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*e/(g*abs(e)) - (c*d*e^3*f - c*d
^2*e^2*g)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g -
 a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*e*g*abs(e)))/(c*d) + 2*(c*d*e^2*f*a
rctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - c*d^2*e*g*ar
ctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - sqrt(-c*d^2*e
 + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*e)/(sqrt(c*d*f*g - a*e*g^2)*c*d*g*abs(e)
)
```

3.788.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{{\left(d+e\,x\right)}^{3/2}}{\left(f+g\,x\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

input
```
int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/
2)),x)
```

output
```
int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/
2)), x)
```

3.789 $\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.789.1 Optimal result

Integrand size = 46, antiderivative size = 170

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g(cdf-aeg)\sqrt{d+ex}(f+gx)} - \frac{(2ae^2g-cd(ef+dg))\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{3/2}(cdf-aeg)^{3/2}}$$

output
```
-(2*a*e^2*g-c*d*(d*g+e*f))*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(3/2)-(
-d*g+e*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)/(g*x+f)
/(e*x+d)^(1/2)
```

3.789.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{d+ex}\left(-\frac{\sqrt{g}(-ef+dg)(ae+cdx)}{(-cdf+aeg)(f+gx)}+\frac{(-2ae^2g+cd(ef+dg))\sqrt{ae+cdx}\,\mathrm{arc}}{(cdf-aeg)^{3/2}}\right.}{g^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)^(3/2)/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]),x]
```

output
```
(Sqrt[d + e*x]*(-((Sqrt[g]*(-(e*f) + d*g)*(a*e + c*d*x))/((-(c*d*f) + a*e*
g)*(f + g*x))) + ((-2*a*e^2*g + c*d*(e*f + d*g))*Sqrt[a*e + c*d*x]*ArcTan[
(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(c*d*f - a*e*g)^(3/2)))/
(g^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.789.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1257, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \ 1257$$

$$-\frac{(2ae^2g-cd(dg+ef))\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2g(cdf-aeg)}-\frac{(ef-dg)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)(cdf-aeg)}$$

$$\Big\downarrow \ 1255$$

$$-\frac{e^2\left(2ae^2g-cd(dg+ef)\right)\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}}{g(cdf-aeg)}-\frac{(ef-dg)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)(cdf-aeg)}$$

$$\Big\downarrow \text{218}$$

$$-\frac{(2ae^2g - cd(dg+ef))\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}(cdf-aeg)^{3/2}} - \frac{(ef-dg)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)(cdf-aeg)}$$

input
```
Int[(d + e*x)^(3/2)/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2]),x]
```

output
```
-(((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*(c*d*f - a*
e*g)*Sqrt[d + e*x]*(f + g*x))) - ((2*a*e^2*g - c*d*(e*f + d*g))*ArcTan[(Sq
rt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sq
rt[d + e*x])])/(g^(3/2)*(c*d*f - a*e*g)^(3/2))
```

3.789.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 1257
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(
f + g*x)^(n + 1)*((a + b*x + c*x^2)^(p + 1)/(g*(n + 1)*(c*e*f + c*d*g - b*e
*g))), x] - Simp[e*((b*e*g*(n + 1) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(
g*(n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m - 1)*(f + g*x)^(n +
1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p - 1, 0] && LtQ[n, -1] && Integer
Q[2*p]
```

3.789. $\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.789.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(154) = 308$.

Time = 0.56 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.98

method	result
default	$\frac{\left(-2\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a\,e^2g^2x+\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c\,d^2g^2x+\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cdefgx-2\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)\right.}{\sqrt{ex+d}\,\sqrt{cdx+a}}$

input
```
int((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
(-2*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*e^2*g^2*x+arcta
nh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d^2*g^2*x+arctanh(g*(c*d
*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*e*f*g*x-2*arctanh(g*(c*d*x+a*e)
^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*e^2*f*g+arctanh(g*(c*d*x+a*e)^(1/2)/((a*
e*g-c*d*f)*g)^(1/2))*c*d^2*f*g+arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*
g)^(1/2))*c*d*e*f^2-(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*d*g+(c*d*x+a
*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*e*f)/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))
^(1/2)/(c*d*x+a*e)^(1/2)/g/(a*e*g-c*d*f)/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

3.789.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(154) = 308$.

Time = 0.32 (sec) , antiderivative size = 896, normalized size of antiderivative = 5.27

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \left[-\frac{(cd^2ef^2+(cd^3-2\,ade^2)fg+(cde^2fg+(cd^2e-2\,ae^3)}{}\right.$$

$$-\frac{(cd^2ef^2+(cd^3-2\,ade^2)fg+(cde^2fg+(cd^2e-2\,ae^3)g^2)x^2+(cde^2f^2+2\,(cd^2e-ae^3)fg+(cd^3-2\,ade}{c^2d^3f^3g^2-2\,acd^2ef^2g^3+a^2de^2fg^4+(c^2d^2ef^2g^3-2\,a}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output
```
[-1/2*((c*d^2*e*f^2 + (c*d^3 - 2*a*d*e^2)*f*g + (c*d*e^2*f*g + (c*d^2*e -
2*a*e^3)*g^2)*x^2 + (c*d*e^2*f^2 + 2*(c*d^2*e - a*e^3)*f*g + (c*d^3 - 2*a*
d*e^2)*g^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*
d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (
e*f + d*g)*x)) + 2*(c*d*e*f^2*g + a*d*e*g^3 - (c*d^2 + a*e^2)*f*g^2)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^3*g^2 - 2
*a*c*d^2*e*f^2*g^3 + a^2*d*e^2*f*g^4 + (c^2*d^2*e*f^2*g^3 - 2*a*c*d*e^2*f*
g^4 + a^2*e^3*g^5)*x^2 + (c^2*d^2*e*f^3*g^2 + a^2*d*e^2*g^5 + (c^2*d^3 - 2
*a*c*d*e^2)*f^2*g^3 - (2*a*c*d^2*e - a^2*e^3)*f*g^4)*x), -((c*d^2*e*f^2 +
(c*d^3 - 2*a*d*e^2)*f*g + (c*d*e^2*f*g + (c*d^2*e - 2*a*e^3)*g^2)*x^2 + (c
*d*e^2*f^2 + 2*(c*d^2*e - a*e^3)*f*g + (c*d^3 - 2*a*d*e^2)*g^2)*x)*sqrt(c*
d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c
*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g
*x)) + (c*d*e*f^2*g + a*d*e*g^3 - (c*d^2 + a*e^2)*f*g^2)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^3*g^2 - 2*a*c*d^2*e*f
^2*g^3 + a^2*d*e^2*f*g^4 + (c^2*d^2*e*f^2*g^3 - 2*a*c*d*e^2*f*g^4 + a^2*e^
3*g^5)*x^2 + (c^2*d^2*e*f^3*g^2 + a^2*d*e^2*g^5 + (c^2*d^3 - 2*a*c*d*e^2)*
f^2*g^3 - (2*a*c*d^2*e - a^2*e^3)*f*g^4)*x)]
```

3.789.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^2}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output
```
Integral((d + e*x)**(3/2)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**2), x)
```

3.789.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^2}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*
x + f)^2), x)
```

3.789.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(154) = 308$.

Time = 0.41 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.76

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx =$$

$$-\frac{cde^2f\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)+cd^2eg\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)-2\,ae^3g\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)-\sqrt{-cd^2e+ae^3}\cdots}{\sqrt{cdfg-aeg^2}cdfg|e|-\sqrt{cdfg-aeg^2}aeg^2|e|}$$

$$+\frac{e\left(\frac{(c^2d^2e^2f+c^2d^3eg-2\,acde^3g)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{(cdfg|e|-aeg^2|e|)\sqrt{cdfg-aeg^2}e}-\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}c^2d^2e^2f-\sqrt{(ex+d)cde-cd^2e+ae^3}c^2d^3eg}{(cde^2f-ae^3g+((ex+d)cde-cd^2e+ae^3)g)(cdfg|e|-aeg^2|e|)}\right)}{cd}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output
```
-(c*d*e^2*f*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) +
 c*d^2*e*g*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) -
2*a*e^3*g*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - s
qrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*e)/(sqrt(c*d*f*g - a*e*g^2)*
c*d*f*g*abs(e) - sqrt(c*d*f*g - a*e*g^2)*a*e*g^2*abs(e)) + e*((c^2*d^2*e^2
*f + c^2*d^3*e*g - 2*a*c*d*e^3*g)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e +
a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/((c*d*f*g*abs(e) - a*e*g^2*abs(e))*s
qrt(c*d*f*g - a*e*g^2)*e) - (sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d
^2*e^2*f - sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^3*e*g)/((c*d*e^2*
f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)*(c*d*f*g*abs(e) - a*e
*g^2*abs(e))))/(c*d)
```

3.789.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{{\left(d+e\,x\right)}^{3/2}}{{\left(f+g\,x\right)}^2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

input
```
int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)),x)
```

output
```
int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)), x)
```

3.790 $\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.790.1 Optimal result

Integrand size = 46, antiderivative size = 261

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}(f+gx)^2} - \frac{(4ae^2g-cd(ef+3dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g(cdf-aeg)^2\sqrt{d+ex}(f+gx)} - \frac{cd(4ae^2g-cd(ef+3dg))\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4g^{3/2}(cdf-aeg)^{5/2}}$$

output
```
-1/4*c*d*(4*a*e^2*g-c*d*(3*d*g+e*f))*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*
f)^(5/2)-1/2*(-d*g+e*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+
c*d*f)/(g*x+f)^2/(e*x+d)^(1/2)-1/4*(4*a*e^2*g-c*d*(3*d*g+e*f))*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)^2/(g*x+f)/(e*x+d)^(1/2)
```

3.790.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{cd\sqrt{d+ex}\left(\frac{\sqrt{g}(ae+cdx)(-2aeg(dg+e(f+2gx))+cd(ef(-f+gx)+dg(5f+\cdots}{cd(cdf-aeg)^2(f+gx)^2}\cdots\right.}{4g^{3/2}\sqrt{(ae+cdx)\cdots}}$$

input
```
Integrate[(d + e*x)^(3/2)/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]),x]
```

output
```
(c*d*Sqrt[d + e*x]*((Sqrt[g]*(a*e + c*d*x)*(-2*a*e*g*(d*g + e*(f + 2*g*x))
 + c*d*(e*f*(-f + g*x) + d*g*(5*f + 3*g*x))))/(c*d*(c*d*f - a*e*g)^2*(f +
g*x)^2) + ((-4*a*e^2*g + c*d*(e*f + 3*d*g))*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt
[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(c*d*f - a*e*g)^(5/2)))/(4*g^
(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

3.790.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1257, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \ 1257$$

$$-\frac{\left(4ae^2g-cd(3dg+ef)\right)\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4g(cdf-aeg)}-\frac{(ef-dg)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2(cdf-aeg)}$$

$$\Big\downarrow \ 1254$$

$$-\frac{\left(4ae^2g-cd(3dg+ef)\right)\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g(cdf-aeg)}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2(cdf-aeg)}$$

$$\Big\downarrow \; 1255$$

$$-\frac{\left(4ae^2g-cd(3dg+ef)\right)\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g(cdf-aeg)}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2(cdf-aeg)}$$

$$\Big\downarrow \; 218$$

$$-\frac{\left(4ae^2g-cd(3dg+ef)\right)\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g(cdf-aeg)}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2(cdf-aeg)}$$

input
```
Int[(d + e*x)^(3/2)/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2]),x]
```

output
```
-1/2*((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*(c*d*f -
 a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) - ((4*a*e^2*g - c*d*(e*f + 3*d*g))*(Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]*(f
 + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2))))
/(4*g*(c*d*f - a*e*g))
```

3.790. $\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.790.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 1257
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(
f + g*x)^(n + 1)*((a + b*x + c*x^2)^(p + 1)/(g*(n + 1)*(c*e*f + c*d*g - b*e
*g))), x] - Simp[e*((b*e*g*(n + 1) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(
g*(n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m - 1)*(f + g*x)^(n +
1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p - 1, 0] && LtQ[n, -1] && Integer
Q[2*p]
```

3.790.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(235) = 470$.

Time = 0.58 (sec) , antiderivative size = 663, normalized size of antiderivative = 2.54

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(4\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)acd\,e^2g^3x^2-3\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^3g^3x^2-\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2ef\,g^2\right.}{}$

3.790. $\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

input
```
int((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/4*((c*d*x+a*e)*(e*x+d))^(1/2)*(4*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*a*c*d*e^2*g^3*x^2-3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^2*d^3*g^3*x^2-arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*c^2*d^2*e*f*g^2*x^2+8*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g
)^(1/2))*a*c*d*e^2*f*g^2*x-6*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*c^2*d^3*f*g^2*x-2*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1
/2))*c^2*d^2*e*f^2*g*x+4*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/
2))*a*c*d*e^2*f^2*g-3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))
*c^2*d^3*f^2*g-arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^
2*e*f^3-4*a*e^2*g^2*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+3*c*d^2*g^
2*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+c*d*e*f*g*x*(c*d*x+a*e)^(1/2
)*((a*e*g-c*d*f)*g)^(1/2)-2*a*d*e*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^
(1/2)-2*a*e^2*f*g*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+5*c*d^2*f*g*(c
*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-c*d*e*f^2*(c*d*x+a*e)^(1/2)*((a*e*
g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^2/g/(a*e*
g-c*d*f)^2/(c*d*x+a*e)^(1/2)
```

3.790.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 831 vs. 2(235) = 470.

Time = 0.35 (sec) , antiderivative size = 1704, normalized size of antiderivative = 6.53

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output
```
[1/8*((c^2*d^3*e*f^3 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*f^2*g + (c^2*d^2*e^2*f*
g^2 + (3*c^2*d^3*e - 4*a*c*d*e^3)*g^3)*x^3 + (2*c^2*d^2*e^2*f^2*g + (7*c^2
*d^3*e - 8*a*c*d*e^3)*f*g^2 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*g^3)*x^2 + (c^2*
d^2*e^2*f^3 + (5*c^2*d^3*e - 4*a*c*d*e^3)*f^2*g + 2*(3*c^2*d^4 - 4*a*c*d^2
*e^2)*f*g^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a
*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f +
(e*f + d*g)*x)) - 2*(c^2*d^2*e*f^3*g - 2*a^2*d*e^2*g^4 - (5*c^2*d^3 - a*c*
d*e^2)*f^2*g^2 + (7*a*c*d^2*e - 2*a^2*e^3)*f*g^3 - (c^2*d^2*e*f^2*g^2 + (3
*c^2*d^3 - 5*a*c*d*e^2)*f*g^3 - (3*a*c*d^2*e - 4*a^2*e^3)*g^4)*x)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^5*g^2 - 3*a*
c^2*d^3*e*f^4*g^3 + 3*a^2*c*d^2*e^2*f^3*g^4 - a^3*d*e^3*f^2*g^5 + (c^3*d^3
*e*f^3*g^4 - 3*a*c^2*d^2*e^2*f^2*g^5 + 3*a^2*c*d*e^3*f*g^6 - a^3*e^4*g^7)*
x^3 + (2*c^3*d^3*e*f^4*g^3 - a^3*d*e^3*g^7 + (c^3*d^4 - 6*a*c^2*d^2*e^2)*f
^3*g^4 - 3*(a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^2*g^5 + (3*a^2*c*d^2*e^2 - 2*a^
3*e^4)*f*g^6)*x^2 + (c^3*d^3*e*f^5*g^2 - 2*a^3*d*e^3*f*g^6 + (2*c^3*d^4 -
3*a*c^2*d^2*e^2)*f^4*g^3 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^4 + (6*a^
2*c*d^2*e^2 - a^3*e^4)*f^2*g^5)*x), -1/4*((c^2*d^3*e*f^3 + (3*c^2*d^4 - 4*
a*c*d^2*e^2)*f^2*g + (c^2*d^2*e^2*f*g^2 + (3*c^2*d^3*e - 4*a*c*d*e^3)*g^3)
*x^3 + (2*c^2*d^2*e^2*f^2*g + (7*c^2*d^3*e - 8*a*c*d*e^3)*f*g^2 + (3*c^...
```

3.790.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^3}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output
```
Integral((d + e*x)**(3/2)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**3), x)
```

3.790. $\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.790.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(gx+f)^3}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*
x + f)^3), x)
```

3.790.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(235) = 470$.

Time = 0.53 (sec) , antiderivative size = 1071, normalized size of antiderivative = 4.10

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{e^2\left(\frac{(c^3d^3ef+3\,c^3d^4g-4\,ac^2d^2e^2g)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{(c^2d^2f^2g|e|-2\,acdefg^2|e|+a^2e^2g^3|e|)\sqrt{cdfg-aeg^2}e}-\frac{\sqrt{(e\cdots}}{\cdots}\cdots\right)}{\cdots}$$

$$-\frac{c^2d^2e^3f^2\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)+2\,c^2d^3e^2fg\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)-4\,acde^4fg\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)-3\,c^2\cdots}{4\left(\sqrt{cdfg-aeg^2}c^2d^2ef^3g|e|-\sqrt{cdfg-aeg^2}c^2d^3f^2\cdots\right)}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output
```
1/4*e^2*((c^3*d^3*e*f + 3*c^3*d^4*g - 4*a*c^2*d^2*e^2*g)*arctan(sqrt((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/((c^2*d^2*f^2
*g*abs(e) - 2*a*c*d*e*f*g^2*abs(e) + a^2*e^2*g^3*abs(e))*sqrt(c*d*f*g - a*
e*g^2)*e) - (sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^4*d^4*e^3*f^2 - 5*s
qrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^4*d^5*e^2*f*g + 3*sqrt((e*x + d)*
c*d*e - c*d^2*e + a*e^3)*a*c^3*d^3*e^4*f*g + 5*sqrt((e*x + d)*c*d*e - c*d^
2*e + a*e^3)*a*c^3*d^4*e^3*g^2 - 4*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)
*a^2*c^2*d^2*e^5*g^2 - ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^3*d^3*e
*f*g - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^3*d^4*g^2 + 4*((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^2*d^2*e^2*g^2)/((c^2*d^2*f^2*g*abs(
e) - 2*a*c*d*e*f*g^2*abs(e) + a^2*e^2*g^3*abs(e))*(c*d*e^2*f - a*e^3*g + (
(e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^2))/(c*d) - 1/4*(c^2*d^2*e^3*f^2*arc
tan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 2*c^2*d^3*e^2*
f*g*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 4*a*c*d
*e^4*f*g*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 3*
c^2*d^4*e*g^2*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))
 + 4*a*c*d^2*e^3*g^2*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g
^2)*e)) - sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c*d*e^2*f + 3*sqr
t(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c*d^2*e*g - 2*sqrt(-c*d^2*e +
a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*e^3*g)/(sqrt(c*d*f*g - a*e*g^2)*c^2*d^...
```

3.790.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(d+e\,x)^{3/2}}{(f+g\,x)^3\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)),x)
```

output
```
int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)), x)
```

3.791 $\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

3.791.1 Optimal result

Integrand size = 46, antiderivative size = 351

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}(f+gx)^3} - \frac{(6ae^2g-cd(ef+5dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12g(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} - \frac{cd(6ae^2g-cd(ef+5dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g(cdf-aeg)^3\sqrt{d+ex}(f+gx)} - \frac{c^2d^2(6ae^2g-cd(ef+5dg))\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{8g^{3/2}(cdf-aeg)^{7/2}}$$

output
```
-1/8*c^2*d^2*(6*a*e^2*g-c*d*(5*d*g+e*f))*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+
c*d*f)^(7/2)-1/3*(-d*g+e*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*
e*g+c*d*f)/(g*x+f)^3/(e*x+d)^(1/2)-1/12*(6*a*e^2*g-c*d*(5*d*g+e*f))*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)^2/(g*x+f)^2/(e*x+d)^(1/
2)-1/8*c*d*(6*a*e^2*g-c*d*(5*d*g+e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)/g/(-a*e*g+c*d*f)^3/(g*x+f)/(e*x+d)^(1/2)
```

3.791.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{c^2d^2\sqrt{d+ex}\left(\frac{\sqrt{g}(ae+cdx)\left(4a^2e^2g^2(2dg+e(f+3gx))-2acdeg\left(dg(13f+5g\right.\right.}{}\right.}{}$$

input
```
Integrate[(d + e*x)^(3/2)/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]),x]
```

output
```
(c^2*d^2*Sqrt[d + e*x]*((Sqrt[g]*(a*e + c*d*x)*(4*a^2*e^2*g^2*(2*d*g + e*(
f + 3*g*x)) - 2*a*c*d*e*g*(d*g*(13*f + 5*g*x) + e*(8*f^2 + 25*f*g*x + 9*g^
2*x^2)) + c^2*d^2*(e*f*(-3*f^2 + 8*f*g*x + 3*g^2*x^2) + d*g*(33*f^2 + 40*f
*g*x + 15*g^2*x^2))))/(c^2*d^2*(c*d*f - a*e*g)^3*(f + g*x)^3) + (3*(-6*a*e
^2*g + c*d*(e*f + 5*d*g))*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d
*x])/Sqrt[c*d*f - a*e*g]])/(c*d*f - a*e*g)^(7/2)))/(24*g^(3/2)*Sqrt[(a*e +
 c*d*x)*(d + e*x)])
```

3.791.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {1257, 1254, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \ 1257$$

$$-\frac{\left(6ae^2g-cd(5dg+ef)\right)\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{6g(cdf-aeg)}-\frac{(ef-dg)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

$$\Big\downarrow \ 1254$$

$$-\frac{\left(6ae^2g-cd(5dg+ef)\right)\left(\frac{3cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g(cdf-aeg)}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

$$\Big\downarrow \text{ 1254}$$

$$-\frac{\left(6ae^2g-cd(5dg+ef)\right)\left(\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g(cdf-aeg)}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

$$\Big\downarrow \text{ 1255}$$

$$-\frac{\left(6ae^2g-cd(5dg+ef)\right)\left(\frac{3cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g(cdf-aeg)}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

$$\Big\downarrow \text{ 218}$$

3.791. $\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

$$-\frac{\left(6ae^2g-cd(5dg+ef)\right)\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g(cdf-aeg)}$$
$$\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

input
```
Int[(d + e*x)^(3/2)/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2]),x]
```

output
```
-1/3*((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*(c*d*f -
 a*e*g)*Sqrt[d + e*x]*(f + g*x)^3) - ((6*a*e^2*g - c*d*(e*f + 5*d*g))*(Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d*f - a*e*g)*Sqrt[d + e*x]*
(f + g*x)^2) + (3*c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f
 - a*e*g)*Sqrt[d + e*x]*(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g
]*(c*d*f - a*e*g)^(3/2))))/(4*(c*d*f - a*e*g))))/(6*g*(c*d*f - a*e*g))
```

3.791.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

3.791. $\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 1257
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(
f + g*x)^(n + 1)*((a + b*x + c*x^2)^(p + 1)/(g*(n + 1)*(c*e*f + c*d*g - b*e
*g))), x] - Simp[e*((b*e*g*(n + 1) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(
g*(n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m - 1)*(f + g*x)^(n +
1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p - 1, 0] && LtQ[n, -1] && Integer
Q[2*p]
```

3.791.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1131 vs. $2(319) = 638$.

Time = 0.56 (sec) , antiderivative size = 1132, normalized size of antiderivative = 3.23

method	result	size
default	Expression too large to display	1132

input
```
int((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
-1/24*((c*d*x+a*e)*(e*x+d))^(1/2)*(40*c^2*d^3*f*g^2*x*(c*d*x+a*e)^(1/2)*((
a*e*g-c*d*f)*g)^(1/2)+18*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/
2))*a*c^2*d^2*e^2*g^4*x^3-3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^
(1/2))*c^3*d^3*e*f*g^3*x^3-9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*c^3*d^3*e*f^2*g^2*x^2-9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*c^3*d^3*e*f^3*g*x+18*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*a*c^2*d^2*e^2*f^3*g+8*a^2*d*e^2*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c
*d*f)*g)^(1/2)+4*a^2*e^3*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+3
3*c^2*d^3*f^2*g*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-3*c^2*d^2*e*f^3*
(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-45*arctanh(g*(c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2))*c^3*d^4*f*g^3*x^2-45*arctanh(g*(c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2))*c^3*d^4*f^2*g^2*x-50*a*c*d*e^2*f*g^2*x*(c*d*x+a*e)
^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^3*d^4*g^4*x^3-15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*c^3*d^4*f^3*g-3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(
1/2))*c^3*d^3*e*f^4+54*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)
)*a*c^2*d^2*e^2*f*g^3*x^2-26*a*c*d^2*e*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d
*f)*g)^(1/2)-16*a*c*d*e^2*f^2*g*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+
54*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c^2*d^2*e^2*f^2*
g^2*x-18*a*c*d*e^2*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+3*...
```

3.791.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1347 vs. 2(319) = 638.

Time = 0.96 (sec) , antiderivative size = 2736, normalized size of antiderivative = 7.79

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output
```
[-1/48*(3*(c^3*d^4*e*f^4 + (5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f^3*g + (c^3*d^3*
e^2*f*g^3 + (5*c^3*d^4*e - 6*a*c^2*d^2*e^3)*g^4)*x^4 + (3*c^3*d^3*e^2*f^2*
g^2 + 2*(8*c^3*d^4*e - 9*a*c^2*d^2*e^3)*f*g^3 + (5*c^3*d^5 - 6*a*c^2*d^3*e
^2)*g^4)*x^3 + 3*(c^3*d^3*e^2*f^3*g + 6*(c^3*d^4*e - a*c^2*d^2*e^3)*f^2*g^
2 + (5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f*g^3)*x^2 + (c^3*d^3*e^2*f^4 + 2*(4*c^3
*d^4*e - 3*a*c^2*d^2*e^3)*f^3*g + 3*(5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f^2*g^2)
*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*
d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*
x)) + 2*(3*c^3*d^3*e*f^4*g + 8*a^3*d*e^3*g^5 - (33*c^3*d^4 - 13*a*c^2*d^2*
e^2)*f^3*g^2 + (59*a*c^2*d^3*e - 20*a^2*c*d*e^3)*f^2*g^3 - 2*(17*a^2*c*d^2
*e^2 - 2*a^3*e^4)*f*g^4 - 3*(c^3*d^3*e*f^2*g^3 + (5*c^3*d^4 - 7*a*c^2*d^2*
e^2)*f*g^4 - (5*a*c^2*d^3*e - 6*a^2*c*d*e^3)*g^5)*x^2 - 2*(4*c^3*d^3*e*f^3
*g^2 + (20*c^3*d^4 - 29*a*c^2*d^2*e^2)*f^2*g^3 - (25*a*c^2*d^3*e - 31*a^2*
c*d*e^3)*f*g^4 + (5*a^2*c*d^2*e^2 - 6*a^3*e^4)*g^5)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^7*g^2 - 4*a*c^3*d^4*e*f
^6*g^3 + 6*a^2*c^2*d^3*e^2*f^5*g^4 - 4*a^3*c*d^2*e^3*f^4*g^5 + a^4*d*e^4*f
^3*g^6 + (c^4*d^4*e*f^4*g^5 - 4*a*c^3*d^3*e^2*f^3*g^6 + 6*a^2*c^2*d^2*e^3*
f^2*g^7 - 4*a^3*c*d*e^4*f*g^8 + a^4*e^5*g^9)*x^4 + (3*c^4*d^4*e*f^5*g^4 +
a^4*d*e^4*g^9 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*f^4*g^5 - 2*(2*a*c^3*d^4*e...
```

3.791.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)/(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output
```
Timed out
```

3.791.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^4}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*
x + f)^4), x)
```

3.791.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1879 vs. $2(319) = 638$.

Time = 0.74 (sec) , antiderivative size = 1879, normalized size of antiderivative = 5.35

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output
```
1/24*e^3*(3*(c^4*d^4*e*f + 5*c^4*d^5*g - 6*a*c^3*d^3*e^2*g)*arctan(sqrt((e
*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/((c^3*d^3*
e*f^3*g*abs(e) - 3*a*c^2*d^2*e^2*f^2*g^2*abs(e) + 3*a^2*c*d*e^3*f*g^3*abs(
e) - a^3*e^4*g^4*abs(e))*sqrt(c*d*f*g - a*e*g^2)*e) - (3*sqrt((e*x + d)*c*
d*e - c*d^2*e + a*e^3)*c^6*d^6*e^5*f^3 - 33*sqrt((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)*c^6*d^7*e^4*f^2*g + 24*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a
*c^5*d^5*e^6*f^2*g + 66*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^5*d^6*
e^5*f*g^2 - 57*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^4*d^4*e^7*f*g
^2 - 33*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^4*d^5*e^6*g^3 + 30*s
qrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^3*c^3*d^3*e^8*g^3 - 8*((e*x + d)*
c*d*e - c*d^2*e + a*e^3)^(3/2)*c^5*d^5*e^3*f^2*g - 40*((e*x + d)*c*d*e - c
*d^2*e + a*e^3)^(3/2)*c^5*d^6*e^2*f*g^2 + 56*((e*x + d)*c*d*e - c*d^2*e +
a*e^3)^(3/2)*a*c^4*d^4*e^4*f*g^2 + 40*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^
(3/2)*a*c^4*d^5*e^3*g^3 - 48*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2
*c^3*d^3*e^5*g^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^4*d^4*e*f
*g^2 - 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^4*d^5*g^3 + 18*((e*x
 + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*c^3*d^3*e^2*g^3)/((c^3*d^3*e*f^3*g*
abs(e) - 3*a*c^2*d^2*e^2*f^2*g^2*abs(e) + 3*a^2*c*d*e^3*f*g^3*abs(e) - a^3
*e^4*g^4*abs(e))*(c*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3
)*g)^3))/(c*d) - 1/24*(3*c^3*d^3*e^4*f^3*arctan(sqrt(-c*d^2*e + a*e^3)*...
```

3.791.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(d+e\,x)^{3/2}}{(f+g\,x)^4\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int((d + e*x)^(3/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)),x)
```

output
```
int((d + e*x)^(3/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)), x)
```

3.792 $\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

3.792.1 Optimal result

Integrand size = 32, antiderivative size = 324

$$\begin{aligned}\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx &= -\frac{b(24c^2+10b^2d^2+60acd^2+45a^2d^4)\sqrt{1-d^2x^2}}{15d^6} \\ &\quad -\frac{(5c^3+18b^2cd^2+18ac^2d^2+24ab^2d^4+24a^2cd^4)\,x\sqrt{1-d^2x^2}}{16d^6} \\ &\quad -\frac{b(12c^2+5b^2d^2+30acd^2)\,x^2\sqrt{1-d^2x^2}}{15d^4} \\ &\quad -\frac{c(5c^2+18b^2d^2+18acd^2)\,x^3\sqrt{1-d^2x^2}}{24d^4}-\frac{3bc^2x^4\sqrt{1-d^2x^2}}{5d^2}-\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2} \\ &\quad +\frac{(5c^3+18b^2cd^2+18ac^2d^2+24ab^2d^4+24a^2cd^4+16a^3d^6)\arcsin(dx)}{16d^7}\end{aligned}$$

output
```
1/16*(16*a^3*d^6+24*a^2*c*d^4+24*a*b^2*d^4+18*a*c^2*d^2+18*b^2*c*d^2+5*c^3
)*arcsin(d*x)/d^7-1/15*b*(45*a^2*d^4+60*a*c*d^2+10*b^2*d^2+24*c^2)*(-d^2*x
^2+1)^(1/2)/d^6-1/16*(24*a^2*c*d^4+24*a*b^2*d^4+18*a*c^2*d^2+18*b^2*c*d^2+
5*c^3)*x*(-d^2*x^2+1)^(1/2)/d^6-1/15*b*(30*a*c*d^2+5*b^2*d^2+12*c^2)*x^2*(
-d^2*x^2+1)^(1/2)/d^4-1/24*c*(18*a*c*d^2+18*b^2*d^2+5*c^2)*x^3*(-d^2*x^2+1
)^(1/2)/d^4-3/5*b*c^2*x^4*(-d^2*x^2+1)^(1/2)/d^2-1/6*c^3*x^5*(-d^2*x^2+1)^
(1/2)/d^2
```

3.792.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$$
$$= \frac{-d\sqrt{1-d^2x^2}(80b^3d^2(2+d^2x^2)+90b^2d^2x(4ad^2+c(3+2d^2x^2))+48b(15a^2d^4+10acd^2(2+d^2x^2)+c^2(8}{}$$

input
```
Integrate[(a + b*x + c*x^2)^3/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
(-(d*Sqrt[1 - d^2*x^2]*(80*b^3*d^2*(2 + d^2*x^2) + 90*b^2*d^2*x*(4*a*d^2 +
 c*(3 + 2*d^2*x^2)) + 48*b*(15*a^2*d^4 + 10*a*c*d^2*(2 + d^2*x^2) + c^2*(8
 + 4*d^2*x^2 + 3*d^4*x^4)) + 5*c*x*(72*a^2*d^4 + 18*a*c*d^2*(3 + 2*d^2*x^2
) + c^2*(15 + 10*d^2*x^2 + 8*d^4*x^4)))) + 30*(5*c^3 + 18*b^2*c*d^2 + 18*a
*c^2*d^2 + 24*a*b^2*d^4 + 24*a^2*c*d^4 + 16*a^3*d^6)*ArcTan[(d*x)/(-1 + Sq
rt[1 - d^2*x^2])])/(240*d^7)
```

3.792.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1188, 2346, 25, 2346, 25, 2346, 27, 2346, 25, 2346, 25, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{dx+1}}\,dx$$
$$\Big\downarrow \text{1188}$$
$$\int \frac{(a+bx+cx^2)^3}{\sqrt{1-d^2x^2}}\,dx$$
$$\Big\downarrow \text{2346}$$
$$-\frac{\int -\frac{18bc^2d^2x^5+c(5c^2+18ad^2c+18b^2d^2)x^4+6b(b^2+6ac)d^2x^3+18a(b^2+ac)d^2x^2+18a^2bd^2x+6a^3d^2}{\sqrt{1-d^2x^2}}dx}{6d^2}-\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{18bc^2d^2x^5+c\left(5c^2+18ad^2c+18b^2d^2\right)x^4+6b\left(b^2+6ac\right)d^2x^3+18a\left(b^2+ac\right)d^2x^2+18a^2bd^2x+6a^3d^2}{\sqrt{1-d^2x^2}}dx}{6d^2}-\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2}$$

$$\Big\downarrow \ 2346$$

$$\frac{-\frac{\int -\frac{30a^3d^4+90a\left(b^2+ac\right)x^2d^4+90a^2bxd^4+5c\left(5c^2+18ad^2c+18b^2d^2\right)x^4d^2+6b\left(12c^2+30ad^2c+5b^2d^2\right)x^3d^2}{\sqrt{1-d^2x^2}}dx}{5d^2}-\frac{18}{5}bc^2x^4\sqrt{1-d^2x^2}}{6d^2}-\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{30a^3d^4+90a\left(b^2+ac\right)x^2d^4+90a^2bxd^4+5c\left(5c^2+18ad^2c+18b^2d^2\right)x^4d^2+6b\left(12c^2+30ad^2c+5b^2d^2\right)x^3d^2}{\sqrt{1-d^2x^2}}dx}{5d^2}-\frac{18}{5}bc^2x^4\sqrt{1-d^2x^2}}{6d^2}-\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2}$$

$$\Big\downarrow \ 2346$$

$$\frac{\frac{-\frac{\int -\frac{3\left(40a^3d^6+120a^2bxd^6+8b\left(12c^2+30ad^2c+5b^2d^2\right)x^3d^4+5\left(24ab^2d^4+24a^2cd^4+18ac^2d^2+18b^2cd^2+5c^3\right)x^2d^2\right)}{\sqrt{1-d^2x^2}}dx}{4d^2}-\frac{5}{4}cx^3\sqrt{1-d^2x^2}\left(18acd^2+18b^2d^2+5c^2\right.}{5d^2}}{6d^2}$$

$$\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{3\int \frac{40a^3d^6+120a^2bxd^6+8b\left(12c^2+30ad^2c+5b^2d^2\right)x^3d^4+5\left(24ab^2d^4+24a^2cd^4+18ac^2d^2+18b^2cd^2+5c^3\right)x^2d^2}{\sqrt{1-d^2x^2}}dx}{4d^2}-\frac{5}{4}cx^3\sqrt{1-d^2x^2}\left(18acd^2+18b^2d^2+5c^2\right)}{5d^2}}{6d^2}-$$

$$\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2}$$

$$\Big\downarrow \ 2346$$

$$\frac{\frac{3\left(-\frac{\int -\frac{120a^3d^8+15\left(24ab^2d^4+24a^2cd^4+18ac^2d^2+18b^2cd^2+5c^3\right)x^2d^4+8b\left(45a^2d^4+10b^2d^2+60acd^2+24c^2\right)xd^4}{\sqrt{1-d^2x^2}}dx}{3d^2}-\frac{8}{3}bd^2x^2\sqrt{1-d^2x^2}\left(30acd^2+5b^2d^2+12c^2\right)\right)}{4d^2}}{5d^2}}{6d^2}$$

$$\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2}$$

3.792. $\int \frac{\left(a+bx+cx^2\right)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3\left(\frac{\int \frac{120a^3d^8+15\left(24ab^2d^4+24a^2cd^4+18ac^2d^2+18b^2cd^2+5c^3\right)x^2d^4+8b\left(45a^2d^4+10b^2d^2+60acd^2+24c^2\right)xd^4}{\sqrt{1-d^2x^2}}dx}{3d^2}-\frac{8}{3}bd^2x^2\sqrt{1-d^2x^2}\left(30acd^2+5b^2d^2+12c^2\right)\right)}{4d^2}-\frac{5}{4}\cdots}{5d^2}\cdots}{6d^2}$$

$$\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2}$$

$$\Big\downarrow \; 2346$$

$$\frac{\frac{3\left(\frac{-\frac{\int -\frac{d^4\left(16b\left(45a^2d^4+10b^2d^2+60acd^2+24c^2\right)xd^2+15\left(16a^3d^6+24ab^2d^4+24a^2cd^4+18ac^2d^2+18b^2cd^2+5c^3\right)\right)}{\sqrt{1-d^2x^2}}dx}{2d^2}-\frac{15}{2}d^2x\sqrt{1-d^2x^2}\left(24a^2cd^4+24ab^2d^4+18ac\cdots\right.}{3d^2}\cdots\right.}{4d^2}\cdots}{5d^2}\cdots}{6d^2}$$

$$\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3\left(\frac{\frac{\int \frac{d^4\left(16b\left(45a^2d^4+10b^2d^2+60acd^2+24c^2\right)xd^2+15\left(16a^3d^6+24ab^2d^4+24a^2cd^4+18ac^2d^2+18b^2cd^2+5c^3\right)\right)}{\sqrt{1-d^2x^2}}dx}{2d^2}-\frac{15}{2}d^2x\sqrt{1-d^2x^2}\left(24a^2cd^4+24ab^2d^4+18ac^2d^2\cdots\right.}{3d^2}\cdots\right.}{4d^2}\cdots}{5d^2}\cdots}{6d^2}$$

$$\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\left(\frac{\frac{1}{2}d^2\int \frac{16b\left(45a^2d^4+10b^2d^2+60acd^2+24c^2\right)xd^2+15\left(16a^3d^6+24ab^2d^4+24a^2cd^4+18ac^2d^2+18b^2cd^2+5c^3\right)}{\sqrt{1-d^2x^2}}dx-\frac{15}{2}d^2x\sqrt{1-d^2x^2}\left(24a^2cd^4+24ab^2d^4+18ac^2d^2+\cdots\right.}{3d^2}\cdots\right.}{4d^2}\cdots}{5d^2}\cdots}{6d^2}$$

$$\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2}$$

$$\Big\downarrow \; 455$$

3.792. $\int \frac{\left(a+bx+cx^2\right)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

$$3\left(\frac{\frac{1}{2}d^2\left(15\left(16a^3d^6+24a^2cd^4+24ab^2d^4+18ac^2d^2+18b^2cd^2+5c^3\right)\int\frac{1}{\sqrt{1-d^2x^2}}dx-16b\sqrt{1-d^2x^2}\left(45a^2d^4+60acd^2+10b^2d^2+24c^2\right)\right)-\frac{15}{2}d^2x\sqrt{1-d^2x^2}\left(24a^2cd^4+\cdots\right.}{3d^2}\right.$$

$$\overline{\qquad 4d^2 \qquad}$$
$$\overline{\qquad 5d^2 \qquad}$$
$$\overline{\qquad 6d^2 \qquad}$$

$$\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2}$$

$\downarrow$ 223

$$3\left(\frac{\frac{1}{2}d^2\left(\frac{15\arcsin(dx)\left(16a^3d^6+24a^2cd^4+24ab^2d^4+18ac^2d^2+18b^2cd^2+5c^3\right)}{d}-16b\sqrt{1-d^2x^2}\left(45a^2d^4+60acd^2+10b^2d^2+24c^2\right)\right)-\frac{15}{2}d^2x\sqrt{1-d^2x^2}\left(24a^2cd^4+24a\cdots\right.}{3d^2}\right.$$

$$\overline{\qquad 4d^2 \qquad}$$
$$\overline{\qquad 5d^2 \qquad}$$
$$\overline{\qquad 6d^2 \qquad}$$

$$\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2}$$

input
```
Int[(a + b*x + c*x^2)^3/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-1/6*(c^3*x^5*Sqrt[1 - d^2*x^2])/d^2 + ((-18*b*c^2*x^4*Sqrt[1 - d^2*x^2])/
5 + ((-5*c*(5*c^2 + 18*b^2*d^2 + 18*a*c*d^2)*x^3*Sqrt[1 - d^2*x^2])/4 + (3
*((-8*b*d^2*(12*c^2 + 5*b^2*d^2 + 30*a*c*d^2)*x^2*Sqrt[1 - d^2*x^2])/3 + (
(-15*d^2*(5*c^3 + 18*b^2*c*d^2 + 18*a*c^2*d^2 + 24*a*b^2*d^4 + 24*a^2*c*d^
4)*x*Sqrt[1 - d^2*x^2])/2 + (d^2*(-16*b*(24*c^2 + 10*b^2*d^2 + 60*a*c*d^2
+ 45*a^2*d^4)*Sqrt[1 - d^2*x^2] + (15*(5*c^3 + 18*b^2*c*d^2 + 18*a*c^2*d^2
 + 24*a*b^2*d^4 + 24*a^2*c*d^4 + 16*a^3*d^6)*ArcSin[d*x])/d))/2)/(3*d^2)))
/(4*d^2))/(5*d^2))/(6*d^2)
```

3.792.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

3.792. $\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 1188
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2
)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m, n] && EqQ[e*f
 + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.792.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.03

method	result
risch	$\frac{(40c^3x^5d^4+144b\,c^2x^4d^4+180a\,c^2d^4x^3+180b^2c\,d^4x^3+480abc\,d^4x^2+80b^3d^4x^2+360a^2c\,d^4x+360a\,b^2d^4x+50c^3d^2x^3+720b\,a^2d^4+192b \cdots}{240d^6\sqrt{-(dx-1)(dx+1)}\,\sqrt{-d \cdots}}$
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(40\ \mathrm{csgn}(d)c^3d^5x^5\sqrt{-d^2x^2+1}+144\ \mathrm{csgn}(d)b\,c^2d^5x^4\sqrt{-d^2x^2+1}+180\ \mathrm{csgn}(d)a\,c^2d^5x^3\sqrt{-d^2x^2+1}+180\ \mathrm{csgn}(d \cdots\right.}{\cdots}$

input
```
int((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/240*(40*c^3*d^4*x^5+144*b*c^2*d^4*x^4+180*a*c^2*d^4*x^3+180*b^2*c*d^4*x^
3+480*a*b*c*d^4*x^2+80*b^3*d^4*x^2+360*a^2*c*d^4*x+360*a*b^2*d^4*x+50*c^3*
d^2*x^3+720*a^2*b*d^4+192*b*c^2*d^2*x^2+270*a*c^2*d^2*x+270*b^2*c*d^2*x+96
0*a*b*c*d^2+160*b^3*d^2+75*c^3*x+384*b*c^2)*(d*x-1)*(d*x+1)^(1/2)/d^6/(-(d
*x-1)*(d*x+1))^(1/2)*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)+1/16*(16*a^3*
d^6+24*a^2*c*d^4+24*a*b^2*d^4+18*a*c^2*d^2+18*b^2*c*d^2+5*c^3)/d^6/(d^2)^(
1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(1/2))*((-d*x+1)*(d*x+1))^(1/2)/(-d
*x+1)^(1/2)/(d*x+1)^(1/2)
```

3.792.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(40\,c^3d^5x^5+144\,bc^2d^5x^4+720\,a^2bd^5+384\,bc^2d+160\,(b^3+6\,abc)d^3+10\,(5\,c^3d^3+18\,(b^2c+ac^2)d^5)x^3}{}$$

input
```
integrate((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

output
```
-1/240*((40*c^3*d^5*x^5 + 144*b*c^2*d^5*x^4 + 720*a^2*b*d^5 + 384*b*c^2*d
+ 160*(b^3 + 6*a*b*c)*d^3 + 10*(5*c^3*d^3 + 18*(b^2*c + a*c^2)*d^5)*x^3 +
16*(12*b*c^2*d^3 + 5*(b^3 + 6*a*b*c)*d^5)*x^2 + 15*(24*(a*b^2 + a^2*c)*d^5
 + 5*c^3*d + 18*(b^2*c + a*c^2)*d^3)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 30*
(16*a^3*d^6 + 24*(a*b^2 + a^2*c)*d^4 + 5*c^3 + 18*(b^2*c + a*c^2)*d^2)*arc
tan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^7
```

3.792.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

3.792.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\sqrt{-d^2x^2+1}c^3x^5}{6\,d^2} - \frac{3\sqrt{-d^2x^2+1}bc^2x^4}{5\,d^2} + \frac{a^3\arcsin(dx)}{d} - \frac{5\sqrt{-d^2x^2+1}c^3x^3}{24\,d^4} - \frac{3\sqrt{-d^2x^2+1}(b^2c+ac^2)x^3}{4\,d^2} - \frac{3\sqrt{-d^2x^2+1}a^2b}{d^2} - \frac{4\sqrt{-d^2x^2+1}bc^2x^2}{5\,d^4} - \frac{\sqrt{-d^2x^2+1}(b^3+6\,abc)x^2}{3\,d^2} - \frac{3\sqrt{-d^2x^2+1}(ab^2+a^2c)x}{2\,d^2} + \frac{3\,(ab^2+a^2c)\arcsin(dx)}{2\,d^3} - \frac{5\sqrt{-d^2x^2+1}c^3x}{16\,d^6} - \frac{9\sqrt{-d^2x^2+1}(b^2c+ac^2)x}{8\,d^4} - \frac{8\sqrt{-d^2x^2+1}bc^2}{5\,d^6} - \frac{2\sqrt{-d^2x^2+1}(b^3+6\,abc)}{3\,d^4} + \frac{5\,c^3\arcsin(dx)}{16\,d^7} + \frac{9\,(b^2c+ac^2)\arcsin(dx)}{8\,d^5}$$

input
```
integrate((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

output
```
-1/6*sqrt(-d^2*x^2 + 1)*c^3*x^5/d^2 - 3/5*sqrt(-d^2*x^2 + 1)*b*c^2*x^4/d^2
 + a^3*arcsin(d*x)/d - 5/24*sqrt(-d^2*x^2 + 1)*c^3*x^3/d^4 - 3/4*sqrt(-d^2
*x^2 + 1)*(b^2*c + a*c^2)*x^3/d^2 - 3*sqrt(-d^2*x^2 + 1)*a^2*b/d^2 - 4/5*s
qrt(-d^2*x^2 + 1)*b*c^2*x^2/d^4 - 1/3*sqrt(-d^2*x^2 + 1)*(b^3 + 6*a*b*c)*x
^2/d^2 - 3/2*sqrt(-d^2*x^2 + 1)*(a*b^2 + a^2*c)*x/d^2 + 3/2*(a*b^2 + a^2*c
)*arcsin(d*x)/d^3 - 5/16*sqrt(-d^2*x^2 + 1)*c^3*x/d^6 - 9/8*sqrt(-d^2*x^2
+ 1)*(b^2*c + a*c^2)*x/d^4 - 8/5*sqrt(-d^2*x^2 + 1)*b*c^2/d^6 - 2/3*sqrt(-
d^2*x^2 + 1)*(b^3 + 6*a*b*c)/d^4 + 5/16*c^3*arcsin(d*x)/d^7 + 9/8*(b^2*c +
 a*c^2)*arcsin(d*x)/d^5
```

3.792.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(720\,a^2bd^5 - 360\,ab^2d^4 - 360\,a^2cd^4 + 240\,b^3d^3 + 1440\,abcd^3 - 450\,b^2cd^2 - 450\,ac^2d^2 + 720\,bc^2d - 165\,c}{}$$

input
```
integrate((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

output
```
-1/240*((720*a^2*b*d^5 - 360*a*b^2*d^4 - 360*a^2*c*d^4 + 240*b^3*d^3 + 144
0*a*b*c*d^3 - 450*b^2*c*d^2 - 450*a*c^2*d^2 + 720*b*c^2*d - 165*c^3 + (360
*a*b^2*d^4 + 360*a^2*c*d^4 - 160*b^3*d^3 - 960*a*b*c*d^3 + 810*b^2*c*d^2 +
 810*a*c^2*d^2 - 960*b*c^2*d + 425*c^3 + 2*(40*b^3*d^3 + 240*a*b*c*d^3 - 2
70*b^2*c*d^2 - 270*a*c^2*d^2 + 528*b*c^2*d - 275*c^3 + (90*b^2*c*d^2 + 90*
a*c^2*d^2 - 288*b*c^2*d + 225*c^3 + 4*(5*(d*x + 1)*c^3 + 18*b*c^2*d - 25*c
^3)*(d*x + 1))*(d*x + 1))*(d*x + 1))*(d*x + 1))*sqrt(d*x + 1)*sqrt(-d*x +
1) - 30*(16*a^3*d^6 + 24*a*b^2*d^4 + 24*a^2*c*d^4 + 18*b^2*c*d^2 + 18*a*c^
2*d^2 + 5*c^3)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^7
```

3.792.9 Mupad [B] (verification not implemented)

Time = 39.01 (sec) , antiderivative size = 1768, normalized size of antiderivative = 5.46

$$\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Too large to display}$$

input
```
int((a + b*x + c*x^2)^3/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
- ((((1 - d*x)^(1/2) - 1)^23*((5*c^3)/4 + 6*a*b^2*d^4 + (9*a*c^2*d^2)/2 +
6*a^2*c*d^4 + (9*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^23 - (((1 - d*x)^(1/
2) - 1)*((5*c^3)/4 + 6*a*b^2*d^4 + (9*a*c^2*d^2)/2 + 6*a^2*c*d^4 + (9*b^2*
c*d^2)/2))/((d*x + 1)^(1/2) - 1) - (((1 - d*x)^(1/2) - 1)^3*((175*c^3)/12
+ 6*a*b^2*d^4 + (105*a*c^2*d^2)/2 + 6*a^2*c*d^4 + (105*b^2*c*d^2)/2))/((d*
x + 1)^(1/2) - 1)^3 + (((1 - d*x)^(1/2) - 1)^21*((175*c^3)/12 + 6*a*b^2*d^
4 + (105*a*c^2*d^2)/2 + 6*a^2*c*d^4 + (105*b^2*c*d^2)/2))/((d*x + 1)^(1/2)
 - 1)^21 + (((1 - d*x)^(1/2) - 1)^5*(126*a*b^2*d^4 - (311*c^3)/4 + (669*a*
c^2*d^2)/2 + 126*a^2*c*d^4 + (669*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^5 -
 (((1 - d*x)^(1/2) - 1)^19*(126*a*b^2*d^4 - (311*c^3)/4 + (669*a*c^2*d^2)/
2 + 126*a^2*c*d^4 + (669*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^19 + (((1 -
d*x)^(1/2) - 1)^7*((8361*c^3)/4 + 510*a*b^2*d^4 + (1533*a*c^2*d^2)/2 + 510
*a^2*c*d^4 + (1533*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^7 - (((1 - d*x)^(1
/2) - 1)^17*((8361*c^3)/4 + 510*a*b^2*d^4 + (1533*a*c^2*d^2)/2 + 510*a^2*c
*d^4 + (1533*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^17 + (((1 - d*x)^(1/2) -
 1)^11*((25295*c^3)/2 + 420*a*b^2*d^4 - 549*a*c^2*d^2 + 420*a^2*c*d^4 - 54
9*b^2*c*d^2))/((d*x + 1)^(1/2) - 1)^11 - (((1 - d*x)^(1/2) - 1)^13*((25295
*c^3)/2 + 420*a*b^2*d^4 - 549*a*c^2*d^2 + 420*a^2*c*d^4 - 549*b^2*c*d^2))/
((d*x + 1)^(1/2) - 1)^13 - (((1 - d*x)^(1/2) - 1)^9*((42259*c^3)/6 - 804*a
*b^2*d^4 + 165*a*c^2*d^2 - 804*a^2*c*d^4 + 165*b^2*c*d^2))/((d*x + 1)^(...
```

3.792. $\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

3.793 $\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

3.793.1 Optimal result

Integrand size = 32, antiderivative size = 166

$$\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{2b(2c+3ad^2)\sqrt{1-d^2x^2}}{3d^4} - \frac{\left(4b^2+c\left(8a+\frac{3c}{d^2}\right)\right)x\sqrt{1-d^2x^2}}{8d^2} - \frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2} + \frac{(3c^2+4b^2d^2+8acd^2+8a^2d^4)\arcsin(dx)}{8d^5}$$

output
```
1/8*(8*a^2*d^4+8*a*c*d^2+4*b^2*d^2+3*c^2)*arcsin(d*x)/d^5-2/3*b*(3*a*d^2+2
*c)*(-d^2*x^2+1)^(1/2)/d^4-1/8*(4*b^2+c*(8*a+3*c/d^2))*x*(-d^2*x^2+1)^(1/2
)/d^2-2/3*b*c*x^2*(-d^2*x^2+1)^(1/2)/d^2-1/4*c^2*x^3*(-d^2*x^2+1)^(1/2)/d^
2
```

3.793.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{-d\sqrt{1-d^2x^2}(12b^2d^2x+16b(3ad^2+c(2+d^2x^2))+3cx(8ad^2+c(3+2d^2x^2)))+6(3c^2+4b^2d^2+8acd^2}{24d^5}$$

input
```
Integrate[(a + b*x + c*x^2)^2/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
(-(d*Sqrt[1 - d^2*x^2]*(12*b^2*d^2*x + 16*b*(3*a*d^2 + c*(2 + d^2*x^2)) +
3*c*x*(8*a*d^2 + c*(3 + 2*d^2*x^2)))) + 6*(3*c^2 + 4*b^2*d^2 + 8*a*c*d^2 +
 8*a^2*d^4)*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/(24*d^5)
```

3.793.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1188, 2346, 25, 2346, 25, 2346, 25, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{1188}$$

$$\int \frac{(a+bx+cx^2)^2}{\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \text{2346}$$

$$-\frac{\int -\frac{8bcd^2x^3+(3c^2+8ad^2c+4b^2d^2)x^2+8abd^2x+4a^2d^2}{\sqrt{1-d^2x^2}}dx}{4d^2}-\frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{8bcd^2x^3+(3c^2+8ad^2c+4b^2d^2)x^2+8abd^2x+4a^2d^2}{\sqrt{1-d^2x^2}}dx}{4d^2}-\frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2}$$

$$\Big\downarrow \text{2346}$$

$$\frac{-\frac{\int -\frac{12a^2d^4+3(3c^2+8ad^2c+4b^2d^2)x^2d^2+8b(3ad^2+2c)xd^2}{\sqrt{1-d^2x^2}}dx}{3d^2}-\frac{8}{3}bcx^2\sqrt{1-d^2x^2}}{4d^2}-\frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{12a^2d^4+3(3c^2+8ad^2c+4b^2d^2)x^2d^2+8b(3ad^2+2c)xd^2}{\sqrt{1-d^2x^2}}dx}{3d^2}-\frac{8}{3}bcx^2\sqrt{1-d^2x^2}}{4d^2}-\frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2}$$

$$\Big\downarrow \text{2346}$$

$$\frac{\frac{-\frac{\int -\frac{d^2\left(16b\left(3ad^2+2c\right)xd^2+3\left(8a^2d^4+4b^2d^2+8acd^2+3c^2\right)\right)}{\sqrt{1-d^2x^2}}dx}{2d^2}-\frac{3}{2}x\sqrt{1-d^2x^2}\left(8acd^2+4b^2d^2+3c^2\right)}{3d^2}-\frac{8}{3}bcx^2\sqrt{1-d^2x^2}}{4d^2}-\frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{d^2\left(16b\left(3ad^2+2c\right)xd^2+3\left(8a^2d^4+4b^2d^2+8acd^2+3c^2\right)\right)}{\sqrt{1-d^2x^2}}dx}{2d^2}-\frac{3}{2}x\sqrt{1-d^2x^2}\left(8acd^2+4b^2d^2+3c^2\right)}{3d^2}-\frac{8}{3}bcx^2\sqrt{1-d^2x^2}}{4d^2}-\frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{1}{2}\int \frac{16b\left(3ad^2+2c\right)xd^2+3\left(8a^2d^4+4b^2d^2+8acd^2+3c^2\right)}{\sqrt{1-d^2x^2}}dx-\frac{3}{2}x\sqrt{1-d^2x^2}\left(8acd^2+4b^2d^2+3c^2\right)}{3d^2}-\frac{8}{3}bcx^2\sqrt{1-d^2x^2}}{4d^2}-\frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2}$$

$\Big\downarrow$ 455

$$\frac{\frac{\frac{1}{2}\left(3\left(8a^2d^4+8acd^2+4b^2d^2+3c^2\right)\int \frac{1}{\sqrt{1-d^2x^2}}dx-16b\sqrt{1-d^2x^2}\left(3ad^2+2c\right)\right)-\frac{3}{2}x\sqrt{1-d^2x^2}\left(8acd^2+4b^2d^2+3c^2\right)}{3d^2}-\frac{8}{3}bcx^2\sqrt{1-d^2x^2}}{4d^2}-\frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2}$$

$\Big\downarrow$ 223

$$\frac{\frac{\frac{1}{2}\left(\frac{3\arcsin(dx)\left(8a^2d^4+8acd^2+4b^2d^2+3c^2\right)}{d}-16b\sqrt{1-d^2x^2}\left(3ad^2+2c\right)\right)-\frac{3}{2}x\sqrt{1-d^2x^2}\left(8acd^2+4b^2d^2+3c^2\right)}{3d^2}-\frac{8}{3}bcx^2\sqrt{1-d^2x^2}}{4d^2}-\frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2}$$

input
```
Int[(a + b*x + c*x^2)^2/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-1/4*(c^2*x^3*Sqrt[1 - d^2*x^2])/d^2 + ((-8*b*c*x^2*Sqrt[1 - d^2*x^2])/3 +
 ((-3*(3*c^2 + 4*b^2*d^2 + 8*a*c*d^2)*x*Sqrt[1 - d^2*x^2])/2 + (-16*b*(2*c
 + 3*a*d^2)*Sqrt[1 - d^2*x^2] + (3*(3*c^2 + 4*b^2*d^2 + 8*a*c*d^2 + 8*a^2*
d^4)*ArcSin[d*x])/d)/2)/(3*d^2))/(4*d^2)
```

3.793. $\int \frac{\left(a+bx+cx^2\right)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

3.793.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 1188
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2
)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m, n] && EqQ[e*f
 + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.793.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.20

method	result
risch	$\frac{\left(6c^{2}x^{3}d^{2}+16bc\,x^{2}d^{2}+24ac\,d^{2}x+12b^{2}d^{2}x+48ba\,d^{2}+9c^{2}x+32bc\right)(dx-1)\sqrt{dx+1}\,\sqrt{(-dx+1)(dx+1)}}{24d^{4}\sqrt{-(dx-1)(dx+1)}\,\sqrt{-dx+1}}+\frac{\left(8a^{2}d^{4}+8c\,d^{2}a+4b^{2}d^{2}+3c^{2}\right)}{8d^{4}\sqrt{d}}$
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\left(6\operatorname{csgn}(d)c^{2}d^{3}x^{3}\sqrt{-d^{2}x^{2}+1}+16\operatorname{csgn}(d)bc\,d^{3}x^{2}\sqrt{-d^{2}x^{2}+1}+24\sqrt{-d^{2}x^{2}+1}\operatorname{csgn}(d)d^{3}acx+12\sqrt{-d^{2}x^{2}+1}\operatorname{csgn}\right.}{}$

3.793. $\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

input
```
int((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*c^2*d^2*x^3+16*b*c*d^2*x^2+24*a*c*d^2*x+12*b^2*d^2*x+48*a*b*d^2+9*
c^2*x+32*b*c)*(d*x-1)*(d*x+1)^(1/2)/d^4/(-(d*x-1)*(d*x+1))^(1/2)*((-d*x+1)
*(d*x+1))^(1/2)/(-d*x+1)^(1/2)+1/8*(8*a^2*d^4+8*a*c*d^2+4*b^2*d^2+3*c^2)/d
^4/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(1/2))*((-d*x+1)*(d*x+1))
^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

3.793.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(6\,c^2d^3x^3+16\,bcd^3x^2+48\,abd^3+32\,bcd+3\,(4\,(b^2+2\,ac)d^3+3\,c^2d)x)\sqrt{dx+1}\sqrt{-dx+1}+6\,(8\,a^2d^4 - }{24\,d^5}$$

input
```
integrate((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

output
```
-1/24*((6*c^2*d^3*x^3 + 16*b*c*d^3*x^2 + 48*a*b*d^3 + 32*b*c*d + 3*(4*(b^2
 + 2*a*c)*d^3 + 3*c^2*d)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*(8*a^2*d^4 +
4*(b^2 + 2*a*c)*d^2 + 3*c^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*
x)))/d^5
```

3.793.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

3.793.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\sqrt{-d^2x^2+1}c^2x^3}{4\,d^2} - \frac{2\sqrt{-d^2x^2+1}bcx^2}{3\,d^2} + \frac{a^2\arcsin(dx)}{d} - \frac{2\sqrt{-d^2x^2+1}ab}{d^2} - \frac{\sqrt{-d^2x^2+1}(b^2+2\,ac)x}{2\,d^2} - \frac{3\sqrt{-d^2x^2+1}c^2x}{8\,d^4} + \frac{(b^2+2\,ac)\arcsin(dx)}{2\,d^3} - \frac{4\sqrt{-d^2x^2+1}bc}{3\,d^4} + \frac{3\,c^2\arcsin(dx)}{8\,d^5}$$

input
```
integrate((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

output
```
-1/4*sqrt(-d^2*x^2 + 1)*c^2*x^3/d^2 - 2/3*sqrt(-d^2*x^2 + 1)*b*c*x^2/d^2 +
 a^2*arcsin(d*x)/d - 2*sqrt(-d^2*x^2 + 1)*a*b/d^2 - 1/2*sqrt(-d^2*x^2 + 1)
*(b^2 + 2*a*c)*x/d^2 - 3/8*sqrt(-d^2*x^2 + 1)*c^2*x/d^4 + 1/2*(b^2 + 2*a*c
)*arcsin(d*x)/d^3 - 4/3*sqrt(-d^2*x^2 + 1)*b*c/d^4 + 3/8*c^2*arcsin(d*x)/d
^5
```

3.793.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(48\,abd^3 - 12\,b^2d^2 - 24\,acd^2 + 48\,bcd + (12\,b^2d^2 + 24\,acd^2 - 32\,bcd + 2\,(3\,(dx+1)c^2 + 8\,bcd - 9\,c^2)(dx}{}$$

input
```
integrate((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

output
```
-1/24*((48*a*b*d^3 - 12*b^2*d^2 - 24*a*c*d^2 + 48*b*c*d + (12*b^2*d^2 + 24
*a*c*d^2 - 32*b*c*d + 2*(3*(d*x + 1)*c^2 + 8*b*c*d - 9*c^2)*(d*x + 1) + 27
*c^2)*(d*x + 1) - 15*c^2)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 6*(8*a^2*d^4 + 4*
b^2*d^2 + 8*a*c*d^2 + 3*c^2)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^5
```

3.793.9 Mupad [B] (verification not implemented)

Time = 27.55 (sec) , antiderivative size = 897, normalized size of antiderivative = 5.40

$$\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\frac{(\sqrt{1-dx}-1)^{15}\left(2b^2d^2+\frac{3c^2}{2}+4acd^2\right)}{(\sqrt{dx+1}-1)^{15}}+\frac{(\sqrt{1-dx}-1)^3\left(6b^2d^2-\frac{23c^2}{2}+12acd^2\right)}{(\sqrt{dx+1}-1)^3}-\frac{(\sqrt{1-dx}-1)^{13}\left(6b^2d^2-\frac{23c^2}{2}+12acd^2\right)}{(\sqrt{dx+1}-1)^{13}}+\cdots}{\cdots}$$

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{1-dx}-1}{\sqrt{dx+1}-1}\right)(8a^2d^4+8acd^2+4b^2d^2+3c^2)}{2d^5}$$

input
```
int((a + b*x + c*x^2)^2/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
- ((((1 - d*x)^(1/2) - 1)^15*((3*c^2)/2 + 2*b^2*d^2 + 4*a*c*d^2))/((d*x +
1)^(1/2) - 1)^15 + (((1 - d*x)^(1/2) - 1)^3*(6*b^2*d^2 - (23*c^2)/2 + 12*a
*c*d^2))/((d*x + 1)^(1/2) - 1)^3 - (((1 - d*x)^(1/2) - 1)^13*(6*b^2*d^2 -
(23*c^2)/2 + 12*a*c*d^2))/((d*x + 1)^(1/2) - 1)^13 + (((1 - d*x)^(1/2) - 1
)^5*((333*c^2)/2 + 30*b^2*d^2 + 60*a*c*d^2))/((d*x + 1)^(1/2) - 1)^5 - (((
1 - d*x)^(1/2) - 1)^11*((333*c^2)/2 + 30*b^2*d^2 + 60*a*c*d^2))/((d*x + 1)
^(1/2) - 1)^11 + (((1 - d*x)^(1/2) - 1)^7*(22*b^2*d^2 - (671*c^2)/2 + 44*a
*c*d^2))/((d*x + 1)^(1/2) - 1)^7 - (((1 - d*x)^(1/2) - 1)^9*(22*b^2*d^2 -
(671*c^2)/2 + 44*a*c*d^2))/((d*x + 1)^(1/2) - 1)^9 + (((1 - d*x)^(1/2) - 1
)^4*(128*b*c*d + 96*a*b*d^3))/((d*x + 1)^(1/2) - 1)^4 + (((1 - d*x)^(1/2)
- 1)^12*(128*b*c*d + 96*a*b*d^3))/((d*x + 1)^(1/2) - 1)^12 + (((1 - d*x)^(
1/2) - 1)^8*((256*b*c*d)/3 + 320*a*b*d^3))/((d*x + 1)^(1/2) - 1)^8 + (((1
- d*x)^(1/2) - 1)^6*((512*b*c*d)/3 + 240*a*b*d^3))/((d*x + 1)^(1/2) - 1)^6
 + (((1 - d*x)^(1/2) - 1)^10*((512*b*c*d)/3 + 240*a*b*d^3))/((d*x + 1)^(1/
2) - 1)^10 - (((1 - d*x)^(1/2) - 1)*((3*c^2)/2 + 2*b^2*d^2 + 4*a*c*d^2))/(
(d*x + 1)^(1/2) - 1) + (16*a*b*d^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/
2) - 1)^2 + (16*a*b*d^3*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14
)/(d^5 + (8*d^5*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (28*d^5
*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (56*d^5*((1 - d*x)^(1/
2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6 + (70*d^5*((1 - d*x)^(1/2) - 1)^8)/(...
```

3.794 $\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

3.794.1 Optimal result

Integrand size = 30, antiderivative size = 63

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}+\frac{(c+2ad^2)\arcsin(dx)}{2d^3}$$

output
```
1/2*(2*a*d^2+c)*arcsin(d*x)/d^3-b*(-d^2*x^2+1)^(1/2)/d^2-1/2*c*x*(-d^2*x^2
+1)^(1/2)/d^2
```

3.794.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{(-2b-cx)\sqrt{1-d^2x^2}}{2d^2}+\frac{(c+2ad^2)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d^3}$$

input
```
Integrate[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
((-2*b - c*x)*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*d^2)*ArcTan[(d*x)/(-1
 + Sqrt[1 - d^2*x^2])])/d^3
```

3.794.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1188, 2346, 25, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{dx+1}}dx$$
$$\Big\downarrow \text{1188}$$
$$\int \frac{a+bx+cx^2}{\sqrt{1-d^2x^2}}dx$$
$$\Big\downarrow \text{2346}$$
$$-\frac{\int -\frac{2ad^2+2bxd^2+c}{\sqrt{1-d^2x^2}}dx}{2d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{2ad^2+2bxd^2+c}{\sqrt{1-d^2x^2}}dx}{2d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$
$$\Big\downarrow \text{455}$$
$$\frac{(2ad^2+c)\int \frac{1}{\sqrt{1-d^2x^2}}dx-2b\sqrt{1-d^2x^2}}{2d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$
$$\Big\downarrow \text{223}$$
$$\frac{\frac{(2ad^2+c)\arcsin(dx)}{d}-2b\sqrt{1-d^2x^2}}{2d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

input
```
Int[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-1/2*(c*x*Sqrt[1 - d^2*x^2])/d^2 + (-2*b*Sqrt[1 - d^2*x^2] + ((c + 2*a*d^2
)*ArcSin[d*x])/d)/(2*d^2)
```

3.794.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 1188
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2
)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m, n] && EqQ[e*f
 + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.794.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.46 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.86

method	result
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(\sqrt{-d^{2}x^{2}+1}\,\operatorname{csgn}(d) d c x-2\arctan\left(\frac{\operatorname{csgn}(d) d x}{\sqrt{-d^{2}x^{2}+1}}\right) a\,d^{2}+2\,\operatorname{csgn}(d) d\sqrt{-d^{2}x^{2}+1}\,b-\arctan\left(\frac{\operatorname{csgn}(d) d x}{\sqrt{-d^{2}x^{2}+1}}\right) c\right) \operatorname{csgn}(d}{2d^{3}\sqrt{-d^{2}x^{2}+1}}$
risch	$\frac{(cx+2b)(dx-1)\sqrt{dx+1}\,\sqrt{(-dx+1)(dx+1)}}{2d^{2}\sqrt{-(dx-1)(dx+1)}\,\sqrt{-dx+1}}+\frac{\left(2a\,d^{2}+c\right)\arctan\left(\frac{\sqrt{d^{2}}\,x}{\sqrt{-d^{2}x^{2}+1}}\right)\sqrt{(-dx+1)(dx+1)}}{2d^{2}\sqrt{d^{2}}\,\sqrt{-dx+1}\,\sqrt{dx+1}}$

input
```
int((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^3*((-d^2*x^2+1)^(1/2)*csgn(d)*d*c*x-2*
arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a*d^2+2*csgn(d)*d*(-d^2*x^2+1)^(1/2
)*b-arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c)/(-d^2*x^2+1)^(1/2)*csgn(d)
```

3.794.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(cdx+2\,bd)\sqrt{dx+1}\sqrt{-dx+1}+2\,(2\,ad^2+c)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{2\,d^3}$$

input
```
integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
```

output
```
-1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*a*d^2 + c)*arcta
n((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

3.794.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

3.794.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{a\arcsin{(dx)}}{d} - \frac{\sqrt{-d^2x^2+1}cx}{2\,d^2} - \frac{\sqrt{-d^2x^2+1}b}{d^2} + \frac{c\arcsin{(dx)}}{2\,d^3}$$

input
```
integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

output
```
a*arcsin(d*x)/d - 1/2*sqrt(-d^2*x^2 + 1)*c*x/d^2 - sqrt(-d^2*x^2 + 1)*b/d^
2 + 1/2*c*arcsin(d*x)/d^3
```

3.794.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{((dx+1)c+2\,bd-c)\sqrt{dx+1}\sqrt{-dx+1}-2\,(2\,ad^2+c)\arcsin{\left(\frac{1}{2}\,\sqrt{2}\sqrt{dx+1}\right)}}{2\,d^3}$$

input
```
integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(((d*x + 1)*c + 2*b*d - c)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 2*(2*a*d^2
+ c)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^3
```

3.794.9 Mupad [B] (verification not implemented)

Time = 16.60 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.68

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\sqrt{1-d\,x}\,\left(\frac{b}{d^2}+\frac{b\,x}{d}\right)}{\sqrt{d\,x+1}} - \frac{4\,a\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}} - \frac{2\,c\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3} - \frac{\frac{14\,c\,{\left(\sqrt{1-d\,x}-1\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}-\frac{14\,c\,{\left(\sqrt{1-d\,x}-1\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}+\frac{2\,c\,{\left(\sqrt{1-d\,x}-1\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}-\frac{2\,c\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^4}$$

input
```
int((a + b*x + c*x^2)/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
- ((1 - d*x)^(1/2)*(b/d^2 + (b*x)/d))/(d*x + 1)^(1/2) - (4*a*atan((d*((1 -
 d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (2*c
*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((1 - d*x
)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (14*c*((1 - d*x)^(1/2) - 1)^5)/(
(d*x + 1)^(1/2) - 1)^5 + (2*c*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) -
1)^7 - (2*c*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)
^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4)
```

3.794. $\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

3.795 $\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}(a+bx+cx^2)} dx$

3.795.1 Optimal result

Integrand size = 32, antiderivative size = 282

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)}\,dx = -\frac{\sqrt{2}c\,\text{arctanh}\left(\frac{2c+\left(b-\sqrt{b^2-4ac}\right)d^2x}{\sqrt{2}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}\sqrt{1-d^2x^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}} + \frac{\sqrt{2}c\,\text{arctanh}\left(\frac{2c+\left(b+\sqrt{b^2-4ac}\right)d^2x}{\sqrt{2}\sqrt{2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2}\sqrt{1-d^2x^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2}}$$

output
```
-c*arctanh(1/2*(2*c+d^2*x*(b-(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/
2)/(2*c^2+2*a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/(-4*a*c+b
^2)^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+c*arctanh(1
/2*(2*c+d^2*x*(b+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2)/(2*c^2+2*
a*c*d^2-b*d^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2
*c^2+2*a*c*d^2-b*d^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.795.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.31 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.55

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)}\,dx$$
$$= -\text{RootSum}\Big[ad^4-2bd^2\#1+4c\#1^2+2ad^2\#1^2-2b\#1^3$$
$$+a\#1^4\&,\frac{d^2\log(x)-d^2\log\left(-1+\sqrt{1-d^2x^2}-x\#1\right)+\log(x)\#1^2-\log\left(-1+\sqrt{1-d^2x^2}-x\#1\right)\#1^2}{bd^2-4c\#1-2ad^2\#1+3b\#1^2-2a\#1^3}$$

input
```
Integrate[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)),x]
```

output
```
-RootSum[a*d^4 - 2*b*d^2*#1 + 4*c*#1^2 + 2*a*d^2*#1^2 - 2*b*#1^3 + a*#1^4
& , (d^2*Log[x] - d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + Log[x]*#1^2 - L
og[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2)/(b*d^2 - 4*c*#1 - 2*a*d^2*#1 + 3*b
*#1^2 - 2*a*#1^3) & ]
```

3.795.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1188, 1315, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-dx}\sqrt{dx+1}\,(a+bx+cx^2)}\,dx$$
$$\Big\downarrow \text{1188}$$
$$\int \frac{1}{\sqrt{1-d^2x^2}\,(a+bx+cx^2)}dx$$
$$\Big\downarrow \text{1315}$$
$$\frac{2c\int \frac{1}{\left(b+2cx-\sqrt{b^2-4ac}\right)\sqrt{1-d^2x^2}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{1}{\left(b+2cx+\sqrt{b^2-4ac}\right)\sqrt{1-d^2x^2}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 488$$

$$\frac{2c\int \frac{1}{4c^2-\left(b+\sqrt{b^2-4ac}\right)^2d^2-\frac{\left(\left(b+\sqrt{b^2-4ac}\right)xd^2+2c\right)^2}{1-d^2x^2}}d\frac{\left(b+\sqrt{b^2-4ac}\right)xd^2+2c}{\sqrt{1-d^2x^2}}}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{1}{4c^2-\left(b-\sqrt{b^2-4ac}\right)^2d^2-\frac{\left(\left(b-\sqrt{b^2-4ac}\right)xd^2+2c\right)^2}{1-d^2x^2}}d\frac{\left(b-\sqrt{b^2-4ac}\right)xd^2+2c}{\sqrt{1-d^2x^2}}}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{2}c\,\text{arctanh}\left(\frac{d^2x\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}-\frac{\sqrt{2}c\,\text{arctanh}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}$$

input
```
Int[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)),x]
```

output
```
-((Sqrt[2]*c*ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2
*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])]])/(Sq
rt[b^2 - 4*a*c]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]))
+ (Sqrt[2]*c*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2
*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])]])/(Sq
rt[b^2 - 4*a*c]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2])
```

3.795. $\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}(a+bx+cx^2)}\,dx$

3.795.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1188
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2
)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m, n] && EqQ[e*f
 + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

rule 1315
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Sym
bol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[1/((b - q + 2*c*x
)*Sqrt[d + f*x^2]), x], x] - Simp[2*(c/q)   Int[1/((b + q + 2*c*x)*Sqrt[d +
 f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0] && Po
sQ[b^2 - 4*a*c]
```

3.795.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.63 (sec) , antiderivative size = 1759, normalized size of antiderivative = 6.24

method	result	size
default	Expression too large to display	1759

input
```
int(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
32*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*c^2*(ln(2*((-4*a*c+b^2)^(1/2)*d^
2*x+b*d^2*x+(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(
1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)*(-d^2*x^2+1)^(1/2)*c+2*c)/(b+2*c*x+(-4*a*c+
b^2)^(1/2)))*a^2*d^4*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*
a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)-ln(2*(b*d^2*x-(-4*a*c+b^2)^(1/2)*
d^2*x+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*
(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)*c+2*c)/(b+2*c*x-(-4*a*c+b^2)^
(1/2)))*a^2*d^4*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^
2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)+2*ln(2*((-4*a*c+b^2)^(1/2)*d^2*x+b*d^2*
x+(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c
-b^2)/a^2/c^2)^(1/2)*(-d^2*x^2+1)^(1/2)*c+2*c)/(b+2*c*x+(-4*a*c+b^2)^(1/2)
))*a*c*d^2*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(
1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)-ln(2*((-4*a*c+b^2)^(1/2)*d^2*x+b*d^2*x+(-(b
*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/
a^2/c^2)^(1/2)*(-d^2*x^2+1)^(1/2)*c+2*c)/(b+2*c*x+(-4*a*c+b^2)^(1/2)))*b^2
*d^2*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2
*a*c+b^2)/a^2/c^2)^(1/2)-2*ln(2*(b*d^2*x-(-4*a*c+b^2)^(1/2)*d^2*x+(-d^2*x^
2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(
1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)*c+2*c)/(b+2*c*x-(-4*a*c+b^2)^(1/2)))*a*c*d^
2*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*...
```

3.795.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4313 vs. 2(248) = 496.

Time = 0.43 (sec) , antiderivative size = 4313, normalized size of antiderivative = 15.29

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

output
```
1/2*sqrt(2)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 - ((a^2*b^2 - 4*a^3*c)*d^4 +
b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*
b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 -
 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^
2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4
*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2))*log((4*sqrt(d*x + 1)*sqrt(-d*
x + 1)*a*b*c*d^2 - 2*b^2*c*d^2*x - 4*a*b*c*d^2 + 2*(b^2*c^3 - 4*a*c^4 + (a
^2*b^2*c - 4*a^3*c^2)*d^4 - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^2)*sqrt(b^
2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6
 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4
 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x + sqrt(2)*(((a^3*b^3 - 4*
a^4*b*c)*d^6 - b^3*c^3 + 4*a*b*c^4 - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d
^4 + (b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a
^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5
+ (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b
^2*c^3 + 8*a^2*c^4)*d^2))*x + ((a*b^3 - 4*a^2*b*c)*d^4 + (b^3*c - 4*a*b*c^
2)*d^2)*x)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 - ((a^2*b^2 - 4*a^3*c)*d^4 + b
^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b
^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 -
4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*...
```

3.795.6 Sympy [F]

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)}\,dx=\int \frac{1}{\sqrt{-dx+1}\sqrt{dx+1}\,(a+bx+cx^2)}\,dx$$

input
```
integrate(1/(c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(-d*x + 1)*sqrt(d*x + 1)*(a + b*x + c*x**2)), x)
```

3.795.7 Maxima [F]

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)}\,dx = \int \frac{1}{(cx^2+bx+a)\sqrt{dx+1}\sqrt{-dx+1}}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(1/((c*x^2 + b*x + a)*sqrt(d*x + 1)*sqrt(-d*x + 1)), x)
```

3.795.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 681 vs. $2(248) = 496$.

Time = 0.42 (sec) , antiderivative size = 681, normalized size of antiderivative = 2.41

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)}\,dx =$$

$$-\left(\frac{(ad^2-bd+c)\left(\frac{ad^2-c+\sqrt{-(ad^2+bd+c)(ad^2-bd+c)+(ad^2-c)^2}}{ad^2-bd+c}-1\right)\sqrt{\frac{ad^2-c+\sqrt{-(ad^2+bd+c)(ad^2-bd+c)+(ad^2-c)^2}}{ad^2-bd+c}}\,\text{arcta}}{\left(ad^2-c+\sqrt{-(ad^2+bd+c)(ad^2-bd+c)+(ad^2-c)^2}\right)\sqrt{-(ad^2+bd+c)(a}}\right.$$

input
```
integrate(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

output
```
-((a*d^2 - b*d + c)*((a*d^2 - c + sqrt(-(a*d^2 + b*d + c)*(a*d^2 - b*d + c
) + (a*d^2 - c)^2))/(a*d^2 - b*d + c) - 1)*sqrt((a*d^2 - c + sqrt(-(a*d^2
+ b*d + c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))/(a*d^2 - b*d + c))*arctan(-
1/2*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - s
qrt(-d*x + 1)))/sqrt((a*d^2 - c + sqrt(-(a*d^2 + b*d + c)*(a*d^2 - b*d + c
) + (a*d^2 - c)^2))/(a*d^2 - b*d + c)))/((a*d^2 - c + sqrt(-(a*d^2 + b*d +
 c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))*sqrt(-(a*d^2 + b*d + c)*(a*d^2 - b
*d + c) + (a*d^2 - c)^2)) - (a*d^2 - b*d + c)*((a*d^2 - c - sqrt(-(a*d^2 +
 b*d + c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))/(a*d^2 - b*d + c) - 1)*sqrt(
(a*d^2 - c - sqrt(-(a*d^2 + b*d + c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))/(
a*d^2 - b*d + c))*arctan(-1/2*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) -
sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)))/sqrt((a*d^2 - c - sqrt(-(a*d^2 +
 b*d + c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))/(a*d^2 - b*d + c)))/((a*d^2
- c - sqrt(-(a*d^2 + b*d + c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))*sqrt(-(a
*d^2 + b*d + c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2)))*d
```

3.795.9 Mupad [B] (verification not implemented)

Time = 96.42 (sec) , antiderivative size = 33018, normalized size of antiderivative = 117.09

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(a + b*x + c*x^2)),x)
```

output

```
- atan(((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2)
+ 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*
b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4
- 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b
^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(
2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3
*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*
d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^
(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2
- 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*
c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c
^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*
d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a
^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*
b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^
2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^
6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^
3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2
*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*
b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d...
```

3.795. $\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}(a+bx+cx^2)}\,dx$

3.796 $\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\left(a+bx+cx^2\right)^2}\,dx$

3.796.1 Optimal result

Integrand size = 32, antiderivative size = 571

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)^2}\,dx$$
$$= -\frac{(b(b^2d^2-c(c+3ad^2))-c(2c^2-b^2d^2+2acd^2)\,x)\sqrt{1-d^2x^2}}{(b^2-4ac)\left(b^2d^2-(c+ad^2)^2\right)(a+bx+cx^2)}$$
$$-\frac{c\left(4c^3+12ac^2d^2-ab\left(b+\sqrt{b^2-4ac}\right)d^4-cd^2\left(5b^2-b\sqrt{b^2-4ac}-8a^2d^2\right)\right)\operatorname{arctanh}\left(\frac{2c+\left(b-\sqrt{b}\right.}{\sqrt{2}\sqrt{2c^2+2acd^2-b\left(b-\right.}}\right.}{\sqrt{2}\,(b^2-4ac)^{3/2}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}\left(b^2d^2-(c+ad^2)^2\right)}$$
$$+\frac{c\left(4c^3+12ac^2d^2-2ab^2d^4-b\left(b+\sqrt{b^2-4ac}\right)d^2(c-ad^2)-4cd^2(b^2-2a^2d^2)\right)\operatorname{arctanh}\left(\frac{2c+\left(b-\right.}{\sqrt{2}\sqrt{2c^2+2acd^2-b}}\right.}{\sqrt{2}\,(b^2-4ac)^{3/2}\sqrt{2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2}\left(b^2d^2-(c+ad^2)^2\right)}$$

output
```
-(b*(b^2*d^2-c*(3*a*d^2+c))-c*(2*a*c*d^2-b^2*d^2+2*c^2)*x)*(-d^2*x^2+1)^(1
/2)/(-4*a*c+b^2)/(b^2*d^2-(a*d^2+c)^2)/(c*x^2+b*x+a)-1/2*c*arctanh(1/2*(2*
c+d^2*x*(b-(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2)/(2*c^2+2*a*c*d^
2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(4*c^3+12*a*c^2*d^2-a*b*d^4*(b+(-4*
a*c+b^2)^(1/2))-c*d^2*(5*b^2-8*a^2*d^2-b*(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)
^(3/2)/(b^2*d^2-(a*d^2+c)^2)*2^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b-(-4*a*c+b^2
)^(1/2)))^(1/2)+1/2*c*arctanh(1/2*(2*c+d^2*x*(b+(-4*a*c+b^2)^(1/2)))*2^(1/
2)/(-d^2*x^2+1)^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
)*(4*c^3+12*a*c^2*d^2-2*a*b^2*d^4-4*c*d^2*(-2*a^2*d^2+b^2)-b*d^2*(-a*d^2+c
)*(b+(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(3/2)/(b^2*d^2-(a*d^2+c)^2)*2^(1/2)
/(2*c^2+2*a*c*d^2-b*d^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.796.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.67 (sec) , antiderivative size = 1548, normalized size of antiderivative = 2.71

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)^2),x]
```

output
```
((-(b^3*d^2) + b*c*(c + 3*a*d^2) - b^2*c*d^2*x + 2*c^2*(c + a*d^2)*x)*Sqrt
[1 - d^2*x^2])/((b^2 - 4*a*c)*(-c + d*(b - a*d))*(c + d*(b + a*d))*(a + x*
(b + c*x))) + RootSum[a*d^4 - 2*b*d^2*#1 + 4*c*#1^2 + 2*a*d^2*#1^2 - 2*b*#
1^3 + a*#1^4 & , (-4*b^2*Log[x] + 4*a*c*Log[x] - a^2*d^2*Log[x] + 4*b^2*Lo
g[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 4*a*c*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1
] + a^2*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 2*a*b*Log[x]*#1 + 2*a*b*L
og[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1 - a^2*Log[x]*#1^2 + a^2*Log[-1 + Sqrt
[1 - d^2*x^2] - x*#1]*#1^2)/(b*d^2 - 4*c*#1 - 2*a*d^2*#1 + 3*b*#1^2 - 2*a*
#1^3) & ]/a^3 - RootSum[a*d^4 - 2*b*d^2*#1 + 4*c*#1^2 + 2*a*d^2*#1^2 - 2*b
*#1^3 + a*#1^4 & , (4*b^4*c^2*Log[x] - 20*a*b^2*c^3*Log[x] + 16*a^2*c^4*Lo
g[x] - 4*b^6*d^2*Log[x] + 28*a*b^4*c*d^2*Log[x] - 55*a^2*b^2*c^2*d^2*Log[x
] + 30*a^3*c^3*d^2*Log[x] + 3*a^2*b^4*d^4*Log[x] - 16*a^3*b^2*c*d^4*Log[x]
 + 14*a^4*c^2*d^4*Log[x] - 4*b^4*c^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] +
20*a*b^2*c^3*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 16*a^2*c^4*Log[-1 + Sqrt
[1 - d^2*x^2] - x*#1] + 4*b^6*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 28*
a*b^4*c*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + 55*a^2*b^2*c^2*d^2*Log[-1
 + Sqrt[1 - d^2*x^2] - x*#1] - 30*a^3*c^3*d^2*Log[-1 + Sqrt[1 - d^2*x^2] -
 x*#1] - 3*a^2*b^4*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + 16*a^3*b^2*c*d
^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 14*a^4*c^2*d^4*Log[-1 + Sqrt[1 - d
^2*x^2] - x*#1] + 2*a*b^3*c^2*Log[x]*#1 - 8*a^2*b*c^3*Log[x]*#1 - 2*a*b...
```

3.796.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 564, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1188, 1306, 25, 1367, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-dx}\sqrt{dx+1}\,(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \text{ 1188}$$

$$\int \frac{1}{\sqrt{1-d^2x^2}\,(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \text{ 1306}$$

$$-\frac{\int -\frac{-ab^2d^4+6ac^2d^2-2c(b^2-2a^2d^2)d^2+bc(c-ad^2)xd^2+2c^3}{(cx^2+bx+a)\sqrt{1-d^2x^2}}dx}{(b^2-4ac)\left(b^2d^2-(ad^2+c)^2\right)}-\frac{\sqrt{1-d^2x^2}\left(b\left(b^2d^2-c(3ad^2+c)\right)-cx\left(2acd^2-b^2d^2+2c^2\right)\right)}{(b^2-4ac)\left(b^2d^2-(ad^2+c)^2\right)(a+bx+cx^2)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{-ab^2d^4+4a^2cd^4+6ac^2d^2-2b^2cd^2+bc(c-ad^2)xd^2+2c^3}{(cx^2+bx+a)\sqrt{1-d^2x^2}}dx}{(b^2-4ac)\left(b^2d^2-(ad^2+c)^2\right)}-\frac{\sqrt{1-d^2x^2}\left(b\left(b^2d^2-c(3ad^2+c)\right)-cx\left(2acd^2-b^2d^2+2c^2\right)\right)}{(b^2-4ac)\left(b^2d^2-(ad^2+c)^2\right)(a+bx+cx^2)}$$

$$\Big\downarrow \ 1367$$

$$\frac{\frac{c\left(-cd^2\left(-8a^2d^2-b\sqrt{b^2-4ac}+5b^2\right)-abd^4\left(\sqrt{b^2-4ac}+b\right)+12ac^2d^2+4c^3\right)\int \frac{1}{\left(b+2cx-\sqrt{b^2-4ac}\right)\sqrt{1-d^2x^2}}dx}{\sqrt{b^2-4ac}}-\frac{c\left(-cd^2\left(-8a^2d^2+b\sqrt{b^2-4ac}+5b^2\right)\right.}{}}{(b^2-4ac)\left(b^2d^2-(ad^2+c)^2\right)}-\frac{\sqrt{1-d^2x^2}\left(b\left(b^2d^2-c(3ad^2+c)\right)-cx\left(2acd^2-b^2d^2+2c^2\right)\right)}{(b^2-4ac)\left(b^2d^2-(ad^2+c)^2\right)(a+bx+cx^2)}$$

$$\Big\downarrow \ 488$$

$$\frac{\frac{c\left(-cd^2\left(-8a^2d^2+b\sqrt{b^2-4ac}+5b^2\right)-abd^4\left(b-\sqrt{b^2-4ac}\right)+12ac^2d^2+4c^3\right)\int \frac{1}{4c^2-\left(b+\sqrt{b^2-4ac}\right)^2d^2-\frac{\left(\left(b+\sqrt{b^2-4ac}\right)xd^2+2c\right)^2}{1-d^2x^2}}d\frac{\left(b+\sqrt{b^2-4ac}\right)xd^2+2c}{\sqrt{1-d^2x^2}}}{\sqrt{b^2-4ac}}}{(b^2-4ac)\left(b^2\right.}-\frac{\sqrt{1-d^2x^2}\left(b\left(b^2d^2-c(3ad^2+c)\right)-cx\left(2acd^2-b^2d^2+2c^2\right)\right)}{(b^2-4ac)\left(b^2d^2-(ad^2+c)^2\right)(a+bx+cx^2)}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{c\left(-cd^2\left(-8a^2d^2+b\sqrt{b^2-4ac}+5b^2\right)-abd^4\left(b-\sqrt{b^2-4ac}\right)+12ac^2d^2+4c^3\right)\text{arctanh}\left(\frac{d^2x\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}-\frac{c\left(-cd^2\left(\right.\right.}{}}{(b^2-4ac)\left(b^2d^2-(ad^2\right.}-\frac{\sqrt{1-d^2x^2}\left(b\left(b^2d^2-c(3ad^2+c)\right)-cx\left(2acd^2-b^2d^2+2c^2\right)\right)}{(b^2-4ac)\left(b^2d^2-(ad^2+c)^2\right)(a+bx+cx^2)}$$

3.796. $\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}(a+bx+cx^2)^2}\,dx$

input
```
Int[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)^2),x]
```

output
```
-(((b*(b^2*d^2 - c*(c + 3*a*d^2)) - c*(2*c^2 - b^2*d^2 + 2*a*c*d^2)*x)*Sqr
t[1 - d^2*x^2])/((b^2 - 4*a*c)*(b^2*d^2 - (c + a*d^2)^2)*(a + b*x + c*x^2)
)) + (-((c*(4*c^3 + 12*a*c^2*d^2 - a*b*(b + Sqrt[b^2 - 4*a*c])*d^4 - c*d^2
*(5*b^2 - b*Sqrt[b^2 - 4*a*c] - 8*a^2*d^2))*ArcTanh[(2*c + (b - Sqrt[b^2 -
 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c]
)*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2 + 2*a*c*
d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2])) + (c*(4*c^3 + 12*a*c^2*d^2 - a*b*(b
 - Sqrt[b^2 - 4*a*c])*d^4 - c*d^2*(5*b^2 + b*Sqrt[b^2 - 4*a*c] - 8*a^2*d^2
))*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a
*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[2]*Sqrt
[b^2 - 4*a*c]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]))/((
b^2 - 4*a*c)*(b^2*d^2 - (c + a*d^2)^2))
```

3.796.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1188
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2
)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m, n] && EqQ[e*f
 + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

3.796. $\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}(a+bx+cx^2)^2} dx$

rule 1306
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*f
))*x)*(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^(q + 1)/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^
2*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
 - c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
 b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2
*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 -
4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ
[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

rule 1367
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*g - h*(
b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Simp[(2*c*g -
 h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{
a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

3.796.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.02 (sec) , antiderivative size = 41837, normalized size of antiderivative = 73.27

method	result	size
default	Expression too large to display	41837

input
```
int(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
result too large to display
```

3.796.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35403 vs. 2(529) = 1058.

Time = 33.26 (sec) , antiderivative size = 35403, normalized size of antiderivative = 62.00

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fri
cas")
```

output
```
Too large to include
```

3.796.6 Sympy [F]

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)^2}\,dx = \int \frac{1}{\sqrt{-dx+1}\sqrt{dx+1}\,(a+bx+cx^2)^2}\,dx$$

input
```
integrate(1/(c*x**2+b*x+a)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(-d*x + 1)*sqrt(d*x + 1)*(a + b*x + c*x**2)**2), x)
```

3.796.7 Maxima [F]

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)^2}\,dx = \int \frac{1}{(cx^2+bx+a)^2\sqrt{dx+1}\sqrt{-dx+1}}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(1/((c*x^2 + b*x + a)^2*sqrt(d*x + 1)*sqrt(-d*x + 1)), x)
```

3.796.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="gia
c")
```

output
```
Timed out
```

3.796.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)^2}\,dx = \text{Hanged}$$

input
```
int(1/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(a + b*x + c*x^2)^2),x)
```

output
```
\text{Hanged}
```

3.797 $\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx$

3.797.1 Optimal result

Integrand size = 32, antiderivative size = 276

$$\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+(c+ad^2)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}} + \frac{b(5c^2+b^2d^2+6acd^2)\sqrt{1-d^2x^2}}{d^6} + \frac{c(7c^2+12b^2d^2+12acd^2)\,x\sqrt{1-d^2x^2}}{8d^6} + \frac{bc^2x^2\sqrt{1-d^2x^2}}{d^4} + \frac{c^3x^3\sqrt{1-d^2x^2}}{4d^4} - \frac{3(5c^3+12b^2cd^2+12ac^2d^2+8ab^2d^4+8a^2cd^4)\arcsin(dx)}{8d^7}$$

output
```
-3/8*(8*a^2*c*d^4+8*a*b^2*d^4+12*a*c^2*d^2+12*b^2*c*d^2+5*c^3)*arcsin(d*x)
/d^7+(b*(3*a^2+3*c^2/d^4+b^2/d^2+6*a*c/d^2)*d^4+(a*d^2+c)*(a^2*d^4+2*a*c*d
^2+3*b^2*d^2+c^2)*x)/d^6/(-d^2*x^2+1)^(1/2)+b*(6*a*c*d^2+b^2*d^2+5*c^2)*(-
d^2*x^2+1)^(1/2)/d^6+1/8*c*(12*a*c*d^2+12*b^2*d^2+7*c^2)*x*(-d^2*x^2+1)^(1
/2)/d^6+b*c^2*x^2*(-d^2*x^2+1)^(1/2)/d^4+1/4*c^3*x^3*(-d^2*x^2+1)^(1/2)/d^
4
```

3.797.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx =$$

$$-\frac{\frac{d(8b^3d^2(-2+d^2x^2)+12b^2d^2x(-2ad^2+c(-3+d^2x^2))+8b(-3a^2d^4+6acd^2(-2+d^2x^2)+c^2(-8+4d^2x^2+d^4x^4))+x(-24a^2cd^4-8a^3d^6+12ac^2d^2(-}{\sqrt{1-d^2x^2}}}{8d^7}$$

input
```
Integrate[(a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

output
```
-1/8*((d*(8*b^3*d^2*(-2 + d^2*x^2) + 12*b^2*d^2*x*(-2*a*d^2 + c*(-3 + d^2*
x^2)) + 8*b*(-3*a^2*d^4 + 6*a*c*d^2*(-2 + d^2*x^2) + c^2*(-8 + 4*d^2*x^2 +
 d^4*x^4)) + x*(-24*a^2*c*d^4 - 8*a^3*d^6 + 12*a*c^2*d^2*(-3 + d^2*x^2) +
c^3*(-15 + 5*d^2*x^2 + 2*d^4*x^4))))/Sqrt[1 - d^2*x^2] + 6*(5*c^3 + 12*b^2
*c*d^2 + 12*a*c^2*d^2 + 8*a*b^2*d^4 + 8*a^2*c*d^4)*ArcTan[(d*x)/(-1 + Sqrt
[1 - d^2*x^2])])/d^7
```

3.797.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1188, 2345, 2346, 25, 2346, 27, 2346, 25, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(dx+1)^{3/2}}\,dx$$

$$\Big\downarrow \; 1188$$

$$\int \frac{(a+bx+cx^2)^3}{(1-d^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 2345$$

$$\frac{x(ad^2+c)(a^2d^4+2acd^2+3b^2d^2+c^2)+bd^4\left(3a^2+\frac{6ac}{d^2}+\frac{b^2}{d^2}+\frac{3c^2}{d^4}\right)}{d^6\sqrt{1-d^2x^2}}-$$
$$\int \frac{\frac{c^3x^4}{d^2}+\frac{3bc^2x^3}{d^2}+\frac{c\left(3b^2+c\left(3a+\frac{c}{d^2}\right)\right)x^2}{d^2}+\frac{b\left(b^2+3c\left(2a+\frac{c}{d^2}\right)\right)x}{d^2}+\frac{3ab^2d^4+3ac^2d^2+3c(b^2+a^2d^2)d^2+c^3}{d^6}}{\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \; 2346$$

$$\frac{\int -\frac{12bc^2x^3+c\left(12b^2+c\left(12a+\frac{7c}{d^2}\right)\right)x^2+4b\left(b^2+3c\left(2a+\frac{c}{d^2}\right)\right)x+\frac{4\left(3ab^2d^4+3ac^2d^2+3c\left(b^2+a^2d^2\right)d^2+c^3\right)}{d^4}}{\sqrt{1-d^2x^2}}dx}{4d^2}+\frac{x\left(ad^2+c\right)\left(a^2d^4+2acd^2+3b^2d^2+c^2\right)+bd^4\left(3a^2+\frac{6ac}{d^2}+\frac{b^2}{d^2}+\frac{3c^2}{d^4}\right)}{d^6\sqrt{1-d^2x^2}}+\frac{c^3x^3\sqrt{1-d^2x^2}}{4d^4}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{12bc^2x^3+c\left(12b^2+c\left(12a+\frac{7c}{d^2}\right)\right)x^2+4b\left(b^2+3c\left(2a+\frac{c}{d^2}\right)\right)x+\frac{4\left(3ab^2d^4+3ac^2d^2+3c\left(b^2+a^2d^2\right)d^2+c^3\right)}{d^4}}{\sqrt{1-d^2x^2}}dx}{4d^2}+\frac{x\left(ad^2+c\right)\left(a^2d^4+2acd^2+3b^2d^2+c^2\right)+bd^4\left(3a^2+\frac{6ac}{d^2}+\frac{b^2}{d^2}+\frac{3c^2}{d^4}\right)}{d^6\sqrt{1-d^2x^2}}+\frac{c^3x^3\sqrt{1-d^2x^2}}{4d^4}$$

$$\Big\downarrow \; 2346$$

$$-\frac{-\frac{\int -\frac{3\left(c\left(7c^2+12ad^2c+12b^2d^2\right)x^2+4b\left(5c^2+6ad^2c+b^2d^2\right)x+\frac{4\left(3ab^2d^4+3ac^2d^2+3c\left(b^2+a^2d^2\right)d^2+c^3\right)}{d^2}\right)}{\sqrt{1-d^2x^2}}dx}{3d^2}-\frac{4bc^2x^2\sqrt{1-d^2x^2}}{d^2}}{4d^2}+\frac{x\left(ad^2+c\right)\left(a^2d^4+2acd^2+3b^2d^2+c^2\right)+bd^4\left(3a^2+\frac{6ac}{d^2}+\frac{b^2}{d^2}+\frac{3c^2}{d^4}\right)}{d^6\sqrt{1-d^2x^2}}+\frac{c^3x^3\sqrt{1-d^2x^2}}{4d^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{c\left(7c^2+12ad^2c+12b^2d^2\right)x^2+4b\left(5c^2+6ad^2c+b^2d^2\right)x+4\left(3\left(ad^2+c\right)b^2+c\left(\frac{c^2}{d^2}+3ac+3a^2d^2\right)\right)}{\sqrt{1-d^2x^2}}dx}{d^2}-\frac{4bc^2x^2\sqrt{1-d^2x^2}}{d^2}}{4d^2}+\frac{x\left(ad^2+c\right)\left(a^2d^4+2acd^2+3b^2d^2+c^2\right)+bd^4\left(3a^2+\frac{6ac}{d^2}+\frac{b^2}{d^2}+\frac{3c^2}{d^4}\right)}{d^6\sqrt{1-d^2x^2}}+\frac{c^3x^3\sqrt{1-d^2x^2}}{4d^4}$$

$$\Big\downarrow \; 2346$$

$$-\frac{\frac{-\frac{\int -\frac{8b\left(5c^2+6ad^2c+b^2d^2\right)xd^2+3\left(8ab^2d^4+8a^2cd^4+12ac^2d^2+12b^2cd^2+5c^3\right)}{\sqrt{1-d^2x^2}}dx}{2d^2}-\frac{cx\sqrt{1-d^2x^2}\left(12acd^2+12b^2d^2+7c^2\right)}{2d^2}}{d^2}-\frac{4bc^2x^2\sqrt{1-d^2x^2}}{d^2}}{4d^2}+\frac{x\left(ad^2+c\right)\left(a^2d^4+2acd^2+3b^2d^2+c^2\right)+bd^4\left(3a^2+\frac{6ac}{d^2}+\frac{b^2}{d^2}+\frac{3c^2}{d^4}\right)}{d^6\sqrt{1-d^2x^2}}+\frac{c^3x^3\sqrt{1-d^2x^2}}{4d^4}$$

$$\Big\downarrow \; 25$$

3.797. $\int \frac{\left(a+bx+cx^2\right)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx$

$$-\frac{\frac{\frac{\int \frac{8b(5c^2+6ad^2c+b^2d^2)xd^2+3(8ab^2d^4+8a^2cd^4+12ac^2d^2+12b^2cd^2+5c^3)}{\sqrt{1-d^2x^2}}dx}{2d^2}-\frac{cx\sqrt{1-d^2x^2}(12acd^2+12b^2d^2+7c^2)}{2d^2}}{d^2}-\frac{4bc^2x^2\sqrt{1-d^2x^2}}{d^2}}{4d^2}+\frac{x(ad^2+c)(a^2d^4+2acd^2+3b^2d^2+c^2)+bd^4\left(3a^2+\frac{6ac}{d^2}+\frac{b^2}{d^2}+\frac{3c^2}{d^4}\right)}{d^6\sqrt{1-d^2x^2}}+\frac{c^3x^3\sqrt{1-d^2x^2}}{4d^4}$$

$\downarrow$ 455

$$-\frac{\frac{\frac{3(8a^2cd^4+8ab^2d^4+12ac^2d^2+12b^2cd^2+5c^3)\int \frac{1}{\sqrt{1-d^2x^2}}dx-8b\sqrt{1-d^2x^2}(6acd^2+b^2d^2+5c^2)}{2d^2}-\frac{cx\sqrt{1-d^2x^2}(12acd^2+12b^2d^2+7c^2)}{2d^2}}{d^2}-\frac{4bc^2x^2\sqrt{1-d^2x^2}}{d^2}}{4d^2}+\frac{x(ad^2+c)(a^2d^4+2acd^2+3b^2d^2+c^2)+bd^4\left(3a^2+\frac{6ac}{d^2}+\frac{b^2}{d^2}+\frac{3c^2}{d^4}\right)}{d^6\sqrt{1-d^2x^2}}+\frac{c^3x^3\sqrt{1-d^2x^2}}{4d^4}$$

$\downarrow$ 223

$$-\frac{\frac{\frac{\frac{3\arcsin(dx)(8a^2cd^4+8ab^2d^4+12ac^2d^2+12b^2cd^2+5c^3)}{d}-8b\sqrt{1-d^2x^2}(6acd^2+b^2d^2+5c^2)}{2d^2}-\frac{cx\sqrt{1-d^2x^2}(12acd^2+12b^2d^2+7c^2)}{2d^2}}{d^2}-\frac{4bc^2x^2\sqrt{1-d^2x^2}}{d^2}}{4d^2}+\frac{x(ad^2+c)(a^2d^4+2acd^2+3b^2d^2+c^2)+bd^4\left(3a^2+\frac{6ac}{d^2}+\frac{b^2}{d^2}+\frac{3c^2}{d^4}\right)}{d^6\sqrt{1-d^2x^2}}+\frac{c^3x^3\sqrt{1-d^2x^2}}{4d^4}$$

input
```
Int[(a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

output
```
(b*(3*a^2 + (3*c^2)/d^4 + b^2/d^2 + (6*a*c)/d^2)*d^4 + (c + a*d^2)*(c^2 +
3*b^2*d^2 + 2*a*c*d^2 + a^2*d^4)*x)/(d^6*Sqrt[1 - d^2*x^2]) + (c^3*x^3*Sqr
t[1 - d^2*x^2])/(4*d^4) - ((-4*b*c^2*x^2*Sqrt[1 - d^2*x^2])/d^2 + (-1/2*(c
*(7*c^2 + 12*b^2*d^2 + 12*a*c*d^2)*x*Sqrt[1 - d^2*x^2])/d^2 + (-8*b*(5*c^2
 + b^2*d^2 + 6*a*c*d^2)*Sqrt[1 - d^2*x^2] + (3*(5*c^3 + 12*b^2*c*d^2 + 12*
a*c^2*d^2 + 8*a*b^2*d^4 + 8*a^2*c*d^4)*ArcSin[d*x])/d)/(2*d^2))/d^2)/(4*d^
2)
```

3.797. $\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx$

3.797.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 1188
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2
)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m, n] && EqQ[e*f
 + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.797. $\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx$

3.797.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(260) = 520$.

Time = 0.62 (sec) , antiderivative size = 567, normalized size of antiderivative = 2.05

method	result
risch	$-\frac{(2c^3d^2x^3+8b\,c^2d^2x^2+12a\,c^2d^2x+12b^2c\,d^2x+48abc\,d^2+8b^3d^2+7c^3x+40b\,c^2)(dx-1)\sqrt{dx+1}\,\sqrt{(-dx+1)(dx+1)}}{8d^6\sqrt{-(dx-1)(dx+1)}\,\sqrt{-dx+1}} - \frac{\left(\frac{15c^3\arctan(}{\sqrt{}}\right.}{}$
default	$-\frac{\sqrt{-dx+1}\left(96\ \mathrm{csgn}(d)d^3\sqrt{-d^2x^2+1}\,abc+8\ \mathrm{csgn}(d)d^7\sqrt{-d^2x^2+1}\,a^3x-12\ \mathrm{csgn}(d)b^2c\,d^5x^3\sqrt{-d^2x^2+1}+24\arctan\left(\frac{\mathrm{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)a^2\right.}{}$

input
```
int((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(2*c^3*d^2*x^3+8*b*c^2*d^2*x^2+12*a*c^2*d^2*x+12*b^2*c*d^2*x+48*a*b*c
*d^2+8*b^3*d^2+7*c^3*x+40*b*c^2)*(d*x-1)*(d*x+1)^(1/2)/d^6/(-(d*x-1)*(d*x+
1))^(1/2)*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)-1/8/d^6*(15*c^3/(d^2)^(1
/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(1/2))+24*a^2*c*d^4/(d^2)^(1/2)*arct
an((d^2)^(1/2)*x/(-d^2*x^2+1)^(1/2))+24*b^2*d^4*a/(d^2)^(1/2)*arctan((d^2)
^(1/2)*x/(-d^2*x^2+1)^(1/2))+36*a*c^2*d^2/(d^2)^(1/2)*arctan((d^2)^(1/2)*x
/(-d^2*x^2+1)^(1/2))+36*b^2*c*d^2/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x
^2+1)^(1/2))+(4*a^3*d^6+12*a^2*b*d^5+12*a^2*c*d^4+12*a*b^2*d^4+24*a*b*c*d^
3+4*b^3*d^3+12*a*c^2*d^2+12*b^2*c*d^2+12*b*c^2*d+4*c^3)/d^2/(x-1/d)*(-d^2*
(x-1/d)^2-2*d*(x-1/d))^(1/2)-(-4*a^3*d^6+12*a^2*b*d^5-12*a^2*c*d^4-12*a*b^
2*d^4+24*a*b*c*d^3+4*b^3*d^3-12*a*c^2*d^2-12*b^2*c*d^2+12*b*c^2*d-4*c^3)/d
^2/(x+1/d)*(-d^2*(x+1/d)^2+2*d*(x+1/d))^(1/2))*((-d*x+1)*(d*x+1))^(1/2)/(-
d*x+1)^(1/2)/(d*x+1)^(1/2)
```

3.797.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx =$$

$$-\frac{24\,a^2bd^5+64\,bc^2d+16\,(b^3+6\,abc)d^3-8\,(3\,a^2bd^7+8\,bc^2d^3+2\,(b^3+6\,abc)d^5)x^2-(2\,c^3d^5x^5+8\,bc^2d^5x^4}{}$$

input
```
integrate((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="frica
s")
```

output
```
-1/8*(24*a^2*b*d^5 + 64*b*c^2*d + 16*(b^3 + 6*a*b*c)*d^3 - 8*(3*a^2*b*d^7
+ 8*b*c^2*d^3 + 2*(b^3 + 6*a*b*c)*d^5)*x^2 - (2*c^3*d^5*x^5 + 8*b*c^2*d^5*
x^4 - 24*a^2*b*d^5 - 64*b*c^2*d - 16*(b^3 + 6*a*b*c)*d^3 + (5*c^3*d^3 + 12
*(b^2*c + a*c^2)*d^5)*x^3 + 8*(4*b*c^2*d^3 + (b^3 + 6*a*b*c)*d^5)*x^2 - (8
*a^3*d^7 + 24*(a*b^2 + a^2*c)*d^5 + 15*c^3*d + 36*(b^2*c + a*c^2)*d^3)*x)*
sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*(8*(a*b^2 + a^2*c)*d^4 + 5*c^3 + 12*(b^2*
c + a*c^2)*d^2 - (8*(a*b^2 + a^2*c)*d^6 + 5*c^3*d^2 + 12*(b^2*c + a*c^2)*d
^4)*x^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^9*x^2 - d^7)
```

3.797.6 Sympy [F]

$$\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \int \frac{(a+bx+cx^2)^3}{(-dx+1)^{\frac{3}{2}}\,(dx+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x+a)**3/(-d*x+1)**(3/2)/(d*x+1)**(3/2),x)
```

output
```
Integral((a + b*x + c*x**2)**3/((-d*x + 1)**(3/2)*(d*x + 1)**(3/2)), x)
```

3.797.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = -\frac{c^3x^5}{4\sqrt{-d^2x^2+1}d^2} - \frac{bc^2x^4}{\sqrt{-d^2x^2+1}d^2} + \frac{a^3x}{\sqrt{-d^2x^2+1}} - \frac{5\,c^3x^3}{8\sqrt{-d^2x^2+1}d^4} - \frac{3\,(b^2c+ac^2)x^3}{2\sqrt{-d^2x^2+1}d^2} + \frac{3\,a^2b}{\sqrt{-d^2x^2+1}d^2} - \frac{4\,bc^2x^2}{\sqrt{-d^2x^2+1}d^4} - \frac{(b^3+6\,abc)x^2}{\sqrt{-d^2x^2+1}d^2} + \frac{3\,(ab^2+a^2c)x}{\sqrt{-d^2x^2+1}d^2} - \frac{3\,(ab^2+a^2c)\arcsin(dx)}{d^3} + \frac{15\,c^3x}{8\sqrt{-d^2x^2+1}d^6} + \frac{9\,(b^2c+ac^2)x}{2\sqrt{-d^2x^2+1}d^4} - \frac{15\,c^3\arcsin(dx)}{8\,d^7} - \frac{9\,(b^2c+ac^2)\arcsin(dx)}{2\,d^5} + \frac{8\,bc^2}{\sqrt{-d^2x^2+1}d^6} + \frac{2\,(b^3+6\,abc)}{\sqrt{-d^2x^2+1}d^4}$$

input
```
integrate((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="maxim
a")
```

output
```
-1/4*c^3*x^5/(sqrt(-d^2*x^2 + 1)*d^2) - b*c^2*x^4/(sqrt(-d^2*x^2 + 1)*d^2)
 + a^3*x/sqrt(-d^2*x^2 + 1) - 5/8*c^3*x^3/(sqrt(-d^2*x^2 + 1)*d^4) - 3/2*(
b^2*c + a*c^2)*x^3/(sqrt(-d^2*x^2 + 1)*d^2) + 3*a^2*b/(sqrt(-d^2*x^2 + 1)*
d^2) - 4*b*c^2*x^2/(sqrt(-d^2*x^2 + 1)*d^4) - (b^3 + 6*a*b*c)*x^2/(sqrt(-d
^2*x^2 + 1)*d^2) + 3*(a*b^2 + a^2*c)*x/(sqrt(-d^2*x^2 + 1)*d^2) - 3*(a*b^2
 + a^2*c)*arcsin(d*x)/d^3 + 15/8*c^3*x/(sqrt(-d^2*x^2 + 1)*d^6) + 9/2*(b^2
*c + a*c^2)*x/(sqrt(-d^2*x^2 + 1)*d^4) - 15/8*c^3*arcsin(d*x)/d^7 - 9/2*(b
^2*c + a*c^2)*arcsin(d*x)/d^5 + 8*b*c^2/(sqrt(-d^2*x^2 + 1)*d^6) + 2*(b^3
+ 6*a*b*c)/(sqrt(-d^2*x^2 + 1)*d^4)
```

3.797.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(260) = 520$.

Time = 0.36 (sec) , antiderivative size = 736, normalized size of antiderivative = 2.67

$$\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \frac{\left(\left((dx+1)\left(2\,(dx+1)\left(\frac{(dx+1)c^3}{d^6}+\frac{4\,bc^2d^{31}-5\,c^3d^{30}}{d^{36}}\right)+\frac{12\,b^2cd^{32}+12\,ac^2d^{32}-32\,bc^2d^{31}+25\,c^3d^{30}}{d^{36}}\right)+\frac{8\,b^3d^{33}\cdots}{}\right.\right.}{\cdots}$$

input
```
integrate((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="giac"
)
```

output
```
1/8*((((d*x + 1)*(2*(d*x + 1)*((d*x + 1)*c^3/d^6 + (4*b*c^2*d^31 - 5*c^3*d
^30)/d^36) + (12*b^2*c*d^32 + 12*a*c^2*d^32 - 32*b*c^2*d^31 + 25*c^3*d^30)
/d^36) + (8*b^3*d^33 + 48*a*b*c*d^33 - 36*b^2*c*d^32 - 36*a*c^2*d^32 + 80*
b*c^2*d^31 - 35*c^3*d^30)/d^36)*(d*x + 1) - 2*(2*a^3*d^36 + 6*a^2*b*d^35 +
 6*a*b^2*d^34 + 6*a^2*c*d^34 + 10*b^3*d^33 + 60*a*b*c*d^33 - 6*b^2*c*d^32
- 6*a*c^2*d^32 + 54*b*c^2*d^31 - 7*c^3*d^30)/d^36)*sqrt(d*x + 1)*sqrt(-d*x
 + 1)/(d*x - 1) - 6*(8*a*b^2*d^4 + 8*a^2*c*d^4 + 12*b^2*c*d^2 + 12*a*c^2*d
^2 + 5*c^3)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^6 + 2*(a^3*d^6*(sqrt(2) -
sqrt(-d*x + 1))/sqrt(d*x + 1) - 3*a^2*b*d^5*(sqrt(2) - sqrt(-d*x + 1))/sqr
t(d*x + 1) + 3*a*b^2*d^4*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 3*a^2*
c*d^4*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - b^3*d^3*(sqrt(2) - sqrt(-
d*x + 1))/sqrt(d*x + 1) - 6*a*b*c*d^3*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x
+ 1) + 3*b^2*c*d^2*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 3*a*c^2*d^2*
(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - 3*b*c^2*d*(sqrt(2) - sqrt(-d*x
+ 1))/sqrt(d*x + 1) + c^3*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1))/d^6 -
2*(a^3*d^6 - 3*a^2*b*d^5 + 3*a*b^2*d^4 + 3*a^2*c*d^4 - b^3*d^3 - 6*a*b*c*d
^3 + 3*b^2*c*d^2 + 3*a*c^2*d^2 - 3*b*c^2*d + c^3)*sqrt(d*x + 1)/(d^6*(sqrt
(2) - sqrt(-d*x + 1))))/d
```

3.797.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \int \frac{(c\,x^2+b\,x+a)^3}{(1-d\,x)^{3/2}\,(d\,x+1)^{3/2}}\,dx$$

input
```
int((a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)),x)
```

output
```
int((a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)), x)
```

3.798 $\int \frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx$

3.798.1 Optimal result

Integrand size = 32, antiderivative size = 135

$$\int \frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \frac{2b\left(a+\frac{c}{d^2}\right)d^2+(c^2+b^2d^2+2acd^2+a^2d^4)\,x}{d^4\sqrt{1-d^2x^2}} + \frac{2bc\sqrt{1-d^2x^2}}{d^4} + \frac{c^2x\sqrt{1-d^2x^2}}{2d^4} - \frac{\left(2b^2+c\left(4a+\frac{3c}{d^2}\right)\right)\arcsin(dx)}{2d^3}$$

output
```
-1/2*(2*b^2+c*(4*a+3*c/d^2))*arcsin(d*x)/d^3+(2*b*(a+c/d^2)*d^2+(a^2*d^4+2
*a*c*d^2+b^2*d^2+c^2)*x)/d^4/(-d^2*x^2+1)^(1/2)+2*b*c*(-d^2*x^2+1)^(1/2)/d
^4+1/2*c^2*x*(-d^2*x^2+1)^(1/2)/d^4
```

3.798.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \frac{\sqrt{1-d^2x^2}(-8bc-4abd^2-3c^2x-2b^2d^2x-4acd^2x-2a^2d^4x+4bcd^2x^2+c}{2d^4(-1+d^2x^2)} + \frac{(-3c^2-2b^2d^2-4acd^2)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d^5}$$

input
```
Integrate[(a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

output
```
(Sqrt[1 - d^2*x^2]*(-8*b*c - 4*a*b*d^2 - 3*c^2*x - 2*b^2*d^2*x - 4*a*c*d^2
*x - 2*a^2*d^4*x + 4*b*c*d^2*x^2 + c^2*d^2*x^3))/(2*d^4*(-1 + d^2*x^2)) +
((-3*c^2 - 2*b^2*d^2 - 4*a*c*d^2)*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/
d^5
```

3.798.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1188, 2345, 2346, 25, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(dx+1)^{3/2}}dx$$

$$\Big\downarrow \text{ 1188}$$

$$\int \frac{(a+bx+cx^2)^2}{(1-d^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 2345}$$

$$\frac{x(a^2d^4+2acd^2+b^2d^2+c^2)+2bd^2\left(a+\frac{c}{d^2}\right)}{d^4\sqrt{1-d^2x^2}}-\int \frac{\frac{c^2x^2}{d^2}+\frac{2bcx}{d^2}+\frac{c^2+2ad^2c+b^2d^2}{d^4}}{\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \text{ 2346}$$

$$\frac{\int -\frac{2b^2+4cxb+c\left(4a+\frac{3c}{d^2}\right)}{\sqrt{1-d^2x^2}}dx}{2d^2}+\frac{x(a^2d^4+2acd^2+b^2d^2+c^2)+2bd^2\left(a+\frac{c}{d^2}\right)}{d^4\sqrt{1-d^2x^2}}+\frac{c^2x\sqrt{1-d^2x^2}}{2d^4}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{2b^2+4cxb+c\left(4a+\frac{3c}{d^2}\right)}{\sqrt{1-d^2x^2}}dx}{2d^2}+\frac{x(a^2d^4+2acd^2+b^2d^2+c^2)+2bd^2\left(a+\frac{c}{d^2}\right)}{d^4\sqrt{1-d^2x^2}}+\frac{c^2x\sqrt{1-d^2x^2}}{2d^4}$$

$$\Big\downarrow \text{ 455}$$

$$-\frac{\left(c\left(4a+\frac{3c}{d^2}\right)+2b^2\right)\int \frac{1}{\sqrt{1-d^2x^2}}dx-\frac{4bc\sqrt{1-d^2x^2}}{d^2}}{2d^2}+\frac{x(a^2d^4+2acd^2+b^2d^2+c^2)+2bd^2\left(a+\frac{c}{d^2}\right)}{d^4\sqrt{1-d^2x^2}}+\frac{c^2x\sqrt{1-d^2x^2}}{2d^4}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{x\left(a^2d^4+2acd^2+b^2d^2+c^2\right)+2bd^2\left(a+\frac{c}{d^2}\right)}{d^4\sqrt{1-d^2x^2}}-\frac{\frac{\arcsin(dx)\left(c\left(4a+\frac{3c}{d^2}\right)+2b^2\right)}{d}-\frac{4bc\sqrt{1-d^2x^2}}{d^2}}{2d^2}+\frac{c^2x\sqrt{1-d^2x^2}}{2d^4}$$

input
```
Int[(a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

output
```
(2*b*(a + c/d^2)*d^2 + (c^2 + b^2*d^2 + 2*a*c*d^2 + a^2*d^4)*x)/(d^4*Sqrt[
1 - d^2*x^2]) + (c^2*x*Sqrt[1 - d^2*x^2])/(2*d^4) - ((-4*b*c*Sqrt[1 - d^2*
x^2])/d^2 + ((2*b^2 + c*(4*a + (3*c)/d^2))*ArcSin[d*x])/d)/(2*d^2)
```

3.798.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 1188
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2
)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m, n] && EqQ[e*f
 + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.798. $\int \frac{\left(a+bx+cx^2\right)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx$

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.798.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(125) = 250$.

Time = 0.62 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.56

method	result
risch	$-\frac{c(cx+4b)(dx-1)\sqrt{dx+1}\,\sqrt{(-dx+1)(dx+1)}}{2d^4\sqrt{-(dx-1)(dx+1)}\,\sqrt{-dx+1}} - \frac{\left(\frac{3c^2\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)}{\sqrt{d^2}} + \frac{2b^2d^2\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)}{\sqrt{d^2}} + \frac{4c\,d^2a\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)}{\sqrt{d^2}}\right.}{}$
default	$-\frac{\sqrt{-dx+1}\left(2\,\mathrm{csgn}(d)d^5\sqrt{-d^2x^2+1}\,a^2x - \mathrm{csgn}(d)c^2d^3x^3\sqrt{-d^2x^2+1} - 4\,\mathrm{csgn}(d)bc\,d^3x^2\sqrt{-d^2x^2+1} + 4\arctan\left(\frac{\mathrm{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)ac\,d^4x^2\right.}{}$

input
```
int((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*c*(c*x+4*b)*(d*x-1)*(d*x+1)^(1/2)/d^4/(-(d*x-1)*(d*x+1))^(1/2)*((-d*x
+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)-1/2/d^4*(3*c^2/(d^2)^(1/2)*arctan((d^2)^
(1/2)*x/(-d^2*x^2+1)^(1/2))+2*b^2*d^2/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d
^2*x^2+1)^(1/2))+4*c*d^2*a/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(
1/2))+(a^2*d^4+2*a*b*d^3+2*a*c*d^2+b^2*d^2+2*b*c*d+c^2)/d^2/(x-1/d)*(-d^2*
(x-1/d)^2-2*d*(x-1/d))^(1/2)-(-a^2*d^4+2*a*b*d^3-2*a*c*d^2-b^2*d^2+2*b*c*d
-c^2)/d^2/(x+1/d)*(-d^2*(x+1/d)^2+2*d*(x+1/d))^(1/2))*((-d*x+1)*(d*x+1))^(
1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

3.798.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx =$$

$$-\frac{4\,abd^3 + 8\,bcd - 4\,(abd^5 + 2\,bcd^3)x^2 - (c^2d^3x^3 + 4\,bcd^3x^2 - 4\,abd^3 - 8\,bcd - (2\,a^2d^5 + 2\,(b^2 + 2\,ac)d^3 + 3}{2\,(d^7x}$$

input
```
integrate((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="frica
s")
```

output
```
-1/2*(4*a*b*d^3 + 8*b*c*d - 4*(a*b*d^5 + 2*b*c*d^3)*x^2 - (c^2*d^3*x^3 + 4
*b*c*d^3*x^2 - 4*a*b*d^3 - 8*b*c*d - (2*a^2*d^5 + 2*(b^2 + 2*a*c)*d^3 + 3*
c^2*d)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*(b^2 + 2*a*c)*d^2 - (2*(b^2
+ 2*a*c)*d^4 + 3*c^2*d^2)*x^2 + 3*c^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1
) - 1)/(d*x)))/(d^7*x^2 - d^5)
```

3.798.6 Sympy [F]

$$\int \frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \int \frac{(a+bx+cx^2)^2}{(-dx+1)^{\frac{3}{2}}\,(dx+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x+a)**2/(-d*x+1)**(3/2)/(d*x+1)**(3/2),x)
```

output
```
Integral((a + b*x + c*x**2)**2/((-d*x + 1)**(3/2)*(d*x + 1)**(3/2)), x)
```

3.798.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \frac{a^2x}{\sqrt{-d^2x^2+1}} - \frac{c^2x^3}{2\sqrt{-d^2x^2+1}d^2} - \frac{2\,bcx^2}{\sqrt{-d^2x^2+1}d^2} + \frac{2\,ab}{\sqrt{-d^2x^2+1}d^2} + \frac{(b^2+2\,ac)x}{\sqrt{-d^2x^2+1}d^2} - \frac{(b^2+2\,ac)\arcsin{(dx)}}{d^3} + \frac{3\,c^2x}{2\sqrt{-d^2x^2+1}d^4} - \frac{3\,c^2\arcsin{(dx)}}{2\,d^5} + \frac{4\,bc}{\sqrt{-d^2x^2+1}d^4}$$

input
```
integrate((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="maxim
a")
```

output
```
a^2*x/sqrt(-d^2*x^2 + 1) - 1/2*c^2*x^3/(sqrt(-d^2*x^2 + 1)*d^2) - 2*b*c*x^
2/(sqrt(-d^2*x^2 + 1)*d^2) + 2*a*b/(sqrt(-d^2*x^2 + 1)*d^2) + (b^2 + 2*a*c
)*x/(sqrt(-d^2*x^2 + 1)*d^2) - (b^2 + 2*a*c)*arcsin(d*x)/d^3 + 3/2*c^2*x/(
sqrt(-d^2*x^2 + 1)*d^4) - 3/2*c^2*arcsin(d*x)/d^5 + 4*b*c/(sqrt(-d^2*x^2 +
 1)*d^4)
```

3.798.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(125) = 250$.

Time = 0.32 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.90

$$\int \frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \frac{\frac{2\sqrt{dx+1}\sqrt{-dx+1}\left((dx+1)\left(\frac{(dx+1)c^2}{d^4}+\frac{4\,bcd^{13}-3\,c^2d^{12}}{d^{16}}\right)-\frac{a^2d^{16}+2\,abd^{15}+b^2d^{14}+2\,acd^{14}+10\,bcd^{13}-c^2d^{12}}{d^{16}}}{dx-1}}{}$$

input
```
integrate((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="giac"
)
```

output
```
1/4*(2*sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*((d*x + 1)*c^2/d^4 + (4*b*c
*d^13 - 3*c^2*d^12)/d^16) - (a^2*d^16 + 2*a*b*d^15 + b^2*d^14 + 2*a*c*d^14
 + 10*b*c*d^13 - c^2*d^12)/d^16)/(d*x - 1) - 4*(2*b^2*d^2 + 4*a*c*d^2 + 3*
c^2)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^4 + (a^2*d^4*(sqrt(2) - sqrt(-d*x
 + 1))/sqrt(d*x + 1) - 2*a*b*d^3*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1)
+ b^2*d^2*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 2*a*c*d^2*(sqrt(2) -
sqrt(-d*x + 1))/sqrt(d*x + 1) - 2*b*c*d*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*
x + 1) + c^2*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1))/d^4 - (a^2*d^4 - 2*
a*b*d^3 + b^2*d^2 + 2*a*c*d^2 - 2*b*c*d + c^2)*sqrt(d*x + 1)/(d^4*(sqrt(2)
 - sqrt(-d*x + 1))))/d
```

3.798.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \int \frac{\left(c\,x^2+b\,x+a\right)^2}{\left(1-d\,x\right)^{3/2}\,\left(d\,x+1\right)^{3/2}}\,dx$$

input
```
int((a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)),x)
```

output
```
int((a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)), x)
```

3.799 $\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx$

3.799.1 Optimal result

Integrand size = 30, antiderivative size = 40

$$\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx=\frac{b+(c+ad^2)\,x}{d^2\sqrt{1-d^2x^2}}-\frac{c\arcsin(dx)}{d^3}$$

output
```
-c*arcsin(d*x)/d^3+(b+(a*d^2+c)*x)/d^2/(-d^2*x^2+1)^(1/2)
```

3.799.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.42

$$\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx=\frac{\frac{d(b+(c+ad^2)x)}{\sqrt{1-d^2x^2}}-2c\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d^3}$$

input
```
Integrate[(a + b*x + c*x^2)/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

output
```
((d*(b + (c + a*d^2)*x))/Sqrt[1 - d^2*x^2] - 2*c*ArcTan[(d*x)/(-1 + Sqrt[1
 - d^2*x^2])])/d^3
```

3.799.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1188, 2345, 27, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(dx+1)^{3/2}}\,dx$$

$$\Big\downarrow \; 1188$$

$$\int \frac{a+bx+cx^2}{(1-d^2x^2)^{3/2}}dx$$

$$\Big\downarrow \; 2345$$

$$\frac{x(ad^2+c)+b}{d^2\sqrt{1-d^2x^2}}-\int \frac{c}{d^2\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \; 27$$

$$\frac{x(ad^2+c)+b}{d^2\sqrt{1-d^2x^2}}-\frac{c\int \frac{1}{\sqrt{1-d^2x^2}}dx}{d^2}$$

$$\Big\downarrow \; 223$$

$$\frac{x(ad^2+c)+b}{d^2\sqrt{1-d^2x^2}}-\frac{c\arcsin(dx)}{d^3}$$

input
```
Int[(a + b*x + c*x^2)/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

output
```
(b + (c + a*d^2)*x)/(d^2*Sqrt[1 - d^2*x^2]) - (c*ArcSin[d*x])/d^3
```

3.799.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1188
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2
)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m, n] && EqQ[e*f
 + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.799.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.78

method	result
default	$\frac{\left(-\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d)d^3ax-\arctan\left(\frac{\operatorname{csgn}(d)dx}{\sqrt{-(dx-1)(dx+1)}}\right)c\,d^2x^2-\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d)dcx-\operatorname{csgn}(d)d\sqrt{-d^2x^2+1}\,b+\arctan\left(\frac{\operatorname{csgn}(d}{\sqrt{-(dx-1}}\right.\right.}{(dx-1)\sqrt{-d^2x^2+1}\,d^3\sqrt{dx+1}}$

input
```
int((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-d^2*x^2+1)^(1/2)*csgn(d)*d^3*a*x-arctan(csgn(d)*d*x/(-(d*x-1)*(d*x+1))
^(1/2))*c*d^2*x^2-(-d^2*x^2+1)^(1/2)*csgn(d)*d*c*x-csgn(d)*d*(-d^2*x^2+1)^
(1/2)*b+arctan(csgn(d)*d*x/(-(d*x-1)*(d*x+1))^(1/2))*c)*(-d*x+1)^(1/2)*csg
n(d)/(d*x-1)/(-d^2*x^2+1)^(1/2)/d^3/(d*x+1)^(1/2)
```

3.799.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(38) = 76.

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.52

$$\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \frac{bd^3x^2-(bd+(ad^3+cd)x)\sqrt{dx+1}\sqrt{-dx+1}-bd+2\left(cd^2x^2-c\right)\arctan\left(\right.}{d^5x^2-d^3}$$

input
```
integrate((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="fricas"
)
```

output
```
(b*d^3*x^2 - (b*d + (a*d^3 + c*d)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) - b*d +
2*(c*d^2*x^2 - c)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^5*x
^2 - d^3)
```

3.799.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/(-d*x+1)**(3/2)/(d*x+1)**(3/2),x)
```

output
```
Timed out
```

3.799.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.52

$$\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \frac{ax}{\sqrt{-d^2x^2+1}}+\frac{cx}{\sqrt{-d^2x^2+1}d^2}-\frac{c\arcsin{(dx)}}{d^3}+\frac{b}{\sqrt{-d^2x^2+1}d^2}$$

input
```
integrate((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="maxima"
)
```

output
```
a*x/sqrt(-d^2*x^2 + 1) + c*x/(sqrt(-d^2*x^2 + 1)*d^2) - c*arcsin(d*x)/d^3
+ b/(sqrt(-d^2*x^2 + 1)*d^2)
```

3.799.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(38) = 76$.

Time = 0.29 (sec) , antiderivative size = 186, normalized size of antiderivative = 4.65

$$\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx =$$
$$-\frac{\frac{8\,c\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)}{d^2} - \frac{\frac{ad^2\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}} - \frac{bd\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}} + \frac{c\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}}{d^2} + \frac{(ad^2-bd+c)\sqrt{dx+1}}{d^2\left(\sqrt{2}-\sqrt{-dx+1}\right)} + \frac{2\,(ad^4+bd^3+cd^2)\sqrt{dx+1}\sqrt{-d\cdots}}{(dx-1)d^4}}{4\,d}$$

input
```
integrate((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="giac")
```

output
```
-1/4*(8*c*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2 - (a*d^2*(sqrt(2) - sqrt(-
d*x + 1))/sqrt(d*x + 1) - b*d*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + c
*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1))/d^2 + (a*d^2 - b*d + c)*sqrt(d*
x + 1)/(d^2*(sqrt(2) - sqrt(-d*x + 1))) + 2*(a*d^4 + b*d^3 + c*d^2)*sqrt(d
*x + 1)*sqrt(-d*x + 1)/((d*x - 1)*d^4))/d
```

3.799.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \int \frac{c\,x^2+b\,x+a}{{(1-d\,x)}^{3/2}\,{(d\,x+1)}^{3/2}}\,dx$$

input
```
int((a + b*x + c*x^2)/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)),x)
```

output
```
int((a + b*x + c*x^2)/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)), x)
```

3.800 $\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}(a+bx+cx^2)}\,dx$

3.800.1 Optimal result

Integrand size = 32, antiderivative size = 443

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)}\,dx = \frac{d^2\left(b-\left(c+ad^2\right)x\right)}{\left(b^2d^2-\left(c+ad^2\right)^2\right)\sqrt{1-d^2x^2}} + \frac{c\left(2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2\right)\operatorname{arctanh}\left(\frac{2c+\left(b-\sqrt{b^2-4ac}\right)d^2x}{\sqrt{2}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}\sqrt{1-d^2x^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}\left(b^2d^2-\left(c+ad^2\right)^2\right)} - \frac{c\left(2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2\right)\operatorname{arctanh}\left(\frac{2c+\left(b+\sqrt{b^2-4ac}\right)d^2x}{\sqrt{2}\sqrt{2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2}\sqrt{1-d^2x^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2}\left(b^2d^2-\left(c+ad^2\right)^2\right)}$$

output
```
d^2*(b-(a*d^2+c)*x)/(b^2*d^2-(a*d^2+c)^2)/(-d^2*x^2+1)^(1/2)+1/2*c*arctanh
(1/2*(2*c+d^2*x*(b-(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2)/(2*c^2+
2*a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c^2+2*a*c*d^2-b*d^2*(b+(
-4*a*c+b^2)^(1/2)))/(b^2*d^2-(a*d^2+c)^2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c^
2+2*a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-1/2*c*arctanh(1/2*(2*c+d^2
*x*(b+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2)/(2*c^2+2*a*c*d^2-b*d
^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c^2+2*a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(
1/2)))/(b^2*d^2-(a*d^2+c)^2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c^2+2*a*c*d^2-b
*d^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.800.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.56 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.03

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\,(a+bx+cx^2)}\,dx = \frac{d^2(b-(c+ad^2)\,x)\,\sqrt{1-d^2x^2}+(1-d^2x^2)\,\mathrm{RootSum}\Big[ad^4-$$

input
```
Integrate[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)),x]
```

output
```
(d^2*(b - (c + a*d^2)*x)*Sqrt[1 - d^2*x^2] + (1 - d^2*x^2)*RootSum[a*d^4 -
 2*b*d^2*#1 + 4*c*#1^2 + 2*a*d^2*#1^2 - 2*b*#1^3 + a*#1^4 & , (-(c^2*d^2*L
og[x]) + b^2*d^4*Log[x] - a*c*d^4*Log[x] + c^2*d^2*Log[-1 + Sqrt[1 - d^2*x
^2] - x*#1] - b^2*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + a*c*d^4*Log[-1
+ Sqrt[1 - d^2*x^2] - x*#1] - 2*b*c*d^2*Log[x]*#1 + 2*b*c*d^2*Log[-1 + Sqr
t[1 - d^2*x^2] - x*#1]*#1 - c^2*Log[x]*#1^2 + b^2*d^2*Log[x]*#1^2 - a*c*d^
2*Log[x]*#1^2 + c^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2 - b^2*d^2*Log[
-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2 + a*c*d^2*Log[-1 + Sqrt[1 - d^2*x^2] -
 x*#1]*#1^2)/(-(b*d^2) + 4*c*#1 + 2*a*d^2*#1 - 3*b*#1^2 + 2*a*#1^3) & ])/(
(c + d*(-b + a*d))*(c + d*(b + a*d))*(-1 + d*x)*(1 + d*x))
```

3.800.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 425, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1188, 1307, 27, 1367, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-dx)^{3/2}(dx+1)^{3/2}\,(a+bx+cx^2)}\,dx$$

$$\Big\downarrow \text{ 1188}$$

$$\int \frac{1}{(1-d^2x^2)^{3/2}\,(a+bx+cx^2)}dx$$

$$\Big\downarrow \text{ 1307}$$

$$\frac{d^2\left(b-x\left(ad^2+c\right)\right)}{\sqrt{1-d^2x^2}\left(b^2d^2-\left(ad^2+c\right)^2\right)}-\frac{\int \frac{2d^2\left(c^2+ad^2c-bd^2xc-b^2d^2\right)}{\left(cx^2+bx+a\right)\sqrt{1-d^2x^2}}dx}{2d^2\left(b^2d^2-\left(ad^2+c\right)^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{d^2\left(b-x\left(ad^2+c\right)\right)}{\sqrt{1-d^2x^2}\left(b^2d^2-\left(ad^2+c\right)^2\right)}-\frac{\int \frac{c^2+ad^2c-bd^2xc-b^2d^2}{\left(cx^2+bx+a\right)\sqrt{1-d^2x^2}}dx}{b^2d^2-\left(ad^2+c\right)^2}$$

$$\Big\downarrow \; 1367$$

$$\frac{d^2\left(b-x\left(ad^2+c\right)\right)}{\sqrt{1-d^2x^2}\left(b^2d^2-\left(ad^2+c\right)^2\right)}-\frac{\frac{c\left(-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2\right)\int \frac{1}{\left(b+2cx-\sqrt{b^2-4ac}\right)\sqrt{1-d^2x^2}}dx}{\sqrt{b^2-4ac}}-\frac{c\left(-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2\right)\int \frac{1}{\left(b+2cx+\sqrt{b^2-4ac}\right)\sqrt{1-d^2x^2}}dx}{\sqrt{b^2-4ac}}}{b^2d^2-\left(ad^2+c\right)^2}$$

$$\Big\downarrow \; 488$$

$$\frac{d^2\left(b-x\left(ad^2+c\right)\right)}{\sqrt{1-d^2x^2}\left(b^2d^2-\left(ad^2+c\right)^2\right)}-\frac{\frac{c\left(-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2\right)\int \frac{1}{4c^2-\left(b+\sqrt{b^2-4ac}\right)^2d^2-\frac{\left(\left(b+\sqrt{b^2-4ac}\right)xd^2+2c\right)^2}{1-d^2x^2}}d\frac{\left(b+\sqrt{b^2-4ac}\right)xd^2+2c}{\sqrt{1-d^2x^2}}}{\sqrt{b^2-4ac}}-\frac{c\left(-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+\ldots\right.}{\ldots}}{b^2d^2-\left(ad^2+c\right)^2}$$

$$\Big\downarrow \; 219$$

$$\frac{d^2\left(b-x\left(ad^2+c\right)\right)}{\sqrt{1-d^2x^2}\left(b^2d^2-\left(ad^2+c\right)^2\right)}-\frac{\frac{c\left(-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2\right)\text{arctanh}\left(\frac{d^2x\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}-\frac{c\left(-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2\right)\text{arctanh}\ldots}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bd^2\left(b\ldots\right.}}}{b^2d^2-\left(ad^2+c\right)^2}$$

input
```
Int[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)),x]
```

3.800. $\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}(a+bx+cx^2)}\,dx$

output
```
(d^2*(b - (c + a*d^2)*x))/((b^2*d^2 - (c + a*d^2)^2)*Sqrt[1 - d^2*x^2]) -
(-((c*(2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2)*ArcTanh[(2*c + (
b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqr
t[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[
2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2])) + (c*(2*c^2 + 2*a*c*d
^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2)*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])
*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*S
qrt[1 - d^2*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2 + 2*a*c*d^2 - b*
(b + Sqrt[b^2 - 4*a*c])*d^2]))/(b^2*d^2 - (c + a*d^2)^2)
```

3.800.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1188
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2
)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m, n] && EqQ[e*f
 + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

rule 1307
```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(2*a*c^2*e + c*(2*c^2*d - c*(2*a*f))*x)*(a + c*x^2)^(p + 1
)*((d + e*x + f*x^2)^(q + 1)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1))),
 x] - Simp[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1))   Int[(a + c*x^2)
^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - ((-a)*e)*(c*e))*(p +
 1) - (2*c^2*d - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(-2*a*c^2*e)*(p
 + q + 2) + (2*f*(2*a*c^2*e)*(p + q + 2) - (2*c^2*d - c*(2*a*f))*((-c)*e*(2
*p + q + 4)))*x + c*f*(2*c^2*d - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x], x]
 /; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && Ne
Q[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ
[q, 0]
```

rule 1367
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*g - h*(
b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Simp[(2*c*g -
 h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{
a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

3.800.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.61 (sec) , antiderivative size = 11142, normalized size of antiderivative = 25.15

method	result	size
default	Expression too large to display	11142

input
```
int(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.800.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21628 vs. $2(404) = 808$.

Time = 6.57 (sec) , antiderivative size = 21628, normalized size of antiderivative = 48.82

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x, algorithm="frica
s")
```

output
```
Too large to include
```

3.800.6 Sympy [F]

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}(a+bx+cx^2)}\,dx = \int \frac{1}{(-dx+1)^{\frac{3}{2}}\,(dx+1)^{\frac{3}{2}}\,(a+bx+cx^2)}\,dx$$

input
```
integrate(1/(-d*x+1)**(3/2)/(d*x+1)**(3/2)/(c*x**2+b*x+a),x)
```

output
```
Integral(1/((-d*x + 1)**(3/2)*(d*x + 1)**(3/2)*(a + b*x + c*x**2)), x)
```

3.800.7 Maxima [F]

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}(a+bx+cx^2)}\,dx = \int \frac{1}{(cx^2+bx+a)(dx+1)^{\frac{3}{2}}(-dx+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxim
a")
```

output
```
integrate(1/((c*x^2 + b*x + a)*(d*x + 1)^(3/2)*(-d*x + 1)^(3/2)), x)
```

3.800.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac"
)
```

output
```
Timed out
```

3.800.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)}\,dx=\int \frac{1}{{\left(1-d\,x\right)}^{3/2}\,{\left(d\,x+1\right)}^{3/2}\,\left(c\,x^2+b\,x+a\right)}\,dx$$

input
```
int(1/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)*(a + b*x + c*x^2)),x)
```

output
```
int(1/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)*(a + b*x + c*x^2)), x)
```

3.801 $\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}(a+bx+cx^2)^2}\,dx$

3.801.1 Optimal result

Integrand size = 32, antiderivative size = 939

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)^2}\,dx =$$

$$-\frac{d^2(b(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4)-(2c^4+b^2d^4(2b^2+a^2d^2)-c^2d^2(b^2+6a^2d^2)-c(6ab^2d^4}{(b^2-4ac)\left(c-bd+ad^2\right)^2\left(c+bd+ad^2\right)^2\sqrt{1-d^2x^2}}$$

$$-\frac{b(b^2d^2-c(c+3ad^2))-c(2c^2-b^2d^2+2acd^2)\,x}{(b^2-4ac)\left(b^2d^2-\left(c+ad^2\right)^2\right)(a+bx+cx^2)\sqrt{1-d^2x^2}}$$

$$+\frac{c\left(4c^5+24ac^4d^2+3ab^3\left(b+\sqrt{b^2-4ac}\right)d^6-c^3d^2\left(9b^2-b\sqrt{b^2-4ac}-36a^2d^2\right)-2ac^2d^4\left(7b^2+5b\sqrt{b^2-4\right.}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{2c^2+2acd^2-b}}$$

$$+\frac{c\left(b\left(b+\sqrt{b^2-4ac}\right)d^4(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4)-2(2c^5d^2+12ac^4d^4+3ab^4d^8+2b^2cd^6\right.}{\sqrt{2}\left(b^2-4ac\right)^{3/2}d^2\sqrt{2c^2+2acd^2-b\left(b+\right.}}$$

output
```
-d^2*(b*(-11*a^2*c*d^4+3*a*b^2*d^4-10*a*c^2*d^2+2*b^2*c*d^2+c^3)-(2*c^4+b^
2*d^4*(a^2*d^2+2*b^2)-c^2*d^2*(6*a^2*d^2+b^2)-c*(4*a^3*d^6+6*a*b^2*d^4))*x
)/(-4*a*c+b^2)/(a*d^2-b*d+c)^2/(a*d^2+b*d+c)^2/(-d^2*x^2+1)^(1/2)+(-b*(b^2
*d^2-c*(3*a*d^2+c))+c*(2*a*c*d^2-b^2*d^2+2*c^2)*x)/(-4*a*c+b^2)/(b^2*d^2-(
a*d^2+c)^2)/(c*x^2+b*x+a)/(-d^2*x^2+1)^(1/2)+1/2*c*arctanh(1/2*(2*c+d^2*x*
(b-(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*
(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(4*c^5+24*a*c^4*d^2+3*a*b^3*d^6*(b+(-4*a*c+
b^2)^(1/2))-c^3*d^2*(9*b^2-36*a^2*d^2-b*(-4*a*c+b^2)^(1/2))-2*a*c^2*d^4*(7
*b^2-8*a^2*d^2+5*b*(-4*a*c+b^2)^(1/2))+b*c*d^4*(2*b^3-17*a^2*b*d^2+2*b^2*(
-4*a*c+b^2)^(1/2)-11*a^2*d^2*(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(3/2)/(a^2*
d^4+2*a*c*d^2-b^2*d^2+c^2)^2*2^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b-(-4*a*c+b^2
)^(1/2)))^(1/2)+1/2*c*arctanh(1/2*(2*c+d^2*x*(b+(-4*a*c+b^2)^(1/2)))*2^(1/
2)/(-d^2*x^2+1)^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
)*(-4*c^5*d^2-24*a*c^4*d^4-6*a*b^4*d^8-4*b^2*c*d^6*(-7*a^2*d^2+b^2)+2*c^3*
(-18*a^2*d^6+4*b^2*d^4)+8*c^2*(-2*a^3*d^8+3*a*b^2*d^6)+b*d^4*(-11*a^2*c*d^
4+3*a*b^2*d^4-10*a*c^2*d^2+2*b^2*c*d^2+c^3)*(b+(-4*a*c+b^2)^(1/2)))/(-4*a*
c+b^2)^(3/2)/d^2/(a^2*d^4+2*a*c*d^2-b^2*d^2+c^2)^2*2^(1/2)/(2*c^2+2*a*c*d^
2-b*d^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.801.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 7.47 (sec) , antiderivative size = 3830, normalized size of antiderivative = 4.08

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)^2),x]
```

output
```
(-((Sqrt[1 - d^2*x^2]*(b^5*d^4*(-1 + 2*d^2*x^2) + 2*c*(c + a*d^2)^2*x*(-2*
a^2*d^4 - 2*a*c*d^4*x^2 + c^2*(-1 + d^2*x^2)) + b^2*d^2*x*(a^3*d^6 + a*c^2
*d^2*(13 - 6*d^2*x^2) + c^3*(2 - d^2*x^2) + a^2*c*d^4*(6 + d^2*x^2)) + b^4
*d^4*x*(-(a*d^2) + c*(-3 + 2*d^2*x^2)) + b*c*(c + a*d^2)*(-4*a^2*d^4*(-2 +
 d^2*x^2) + c^2*(-1 + d^2*x^2) + a*c*d^2*(-5 + 9*d^2*x^2)) + b^3*(a^2*d^6*
(-2 + d^2*x^2) + c^2*(2*d^2 - 3*d^4*x^2) + a*c*(3*d^4 - 9*d^6*x^2))))/((b^
2 - 4*a*c)*(-1 + d*x)*(1 + d*x)*(a + x*(b + c*x)))) + RootSum[a*d^4 - 2*b*
d^2*#1 + 4*c*#1^2 + 2*a*d^2*#1^2 - 2*b*#1^3 + a*#1^4 & , (-4*b^4*c^4*Log[x
] + 20*a*b^2*c^5*Log[x] - 16*a^2*c^6*Log[x] + 8*b^6*c^2*d^2*Log[x] - 56*a*
b^4*c^3*d^2*Log[x] + 107*a^2*b^2*c^4*d^2*Log[x] - 46*a^3*c^5*d^2*Log[x] -
4*b^8*d^4*Log[x] + 36*a*b^6*c*d^4*Log[x] - 110*a^2*b^4*c^2*d^4*Log[x] + 13
2*a^3*b^2*c^3*d^4*Log[x] - 44*a^4*c^4*d^4*Log[x] + 3*a^2*b^6*d^6*Log[x] -
22*a^3*b^4*c*d^6*Log[x] + 43*a^4*b^2*c^2*d^6*Log[x] - 14*a^5*c^3*d^6*Log[x
] + 4*b^4*c^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 20*a*b^2*c^5*Log[-1 + S
qrt[1 - d^2*x^2] - x*#1] + 16*a^2*c^6*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] -
 8*b^6*c^2*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + 56*a*b^4*c^3*d^2*Log[-
1 + Sqrt[1 - d^2*x^2] - x*#1] - 107*a^2*b^2*c^4*d^2*Log[-1 + Sqrt[1 - d^2*
x^2] - x*#1] + 46*a^3*c^5*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + 4*b^8*d
^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 36*a*b^6*c*d^4*Log[-1 + Sqrt[1 - d
^2*x^2] - x*#1] + 110*a^2*b^4*c^2*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1...
```

3.801.3 Rubi [A] (verified)

Time = 4.94 (sec) , antiderivative size = 907, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1188, 1306, 25, 2136, 27, 1367, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-dx)^{3/2}(dx+1)^{3/2}\left(a+bx+cx^2\right)^2}dx$$

$$\Big\downarrow \text{1188}$$

$$\int \frac{1}{\left(1-d^2x^2\right)^{3/2}\left(a+bx+cx^2\right)^2}dx$$

$$\Big\downarrow \text{1306}$$

$$-\frac{\int -\frac{-ab^2d^4+6ac^2d^2-2c(2c^2+2ad^2c-b^2d^2)x^2d^2-2c(b^2-2a^2d^2)d^2-b(c^2+7ad^2c-2b^2d^2)xd^2+2c^3}{(cx^2+bx+a)(1-d^2x^2)^{3/2}}dx}{(b^2-4ac)\left(b^2d^2-(ad^2+c)^2\right)}-$$
$$\frac{b\left(b^2d^2-c\left(3ad^2+c\right)\right)-cx\left(2acd^2-b^2d^2+2c^2\right)}{\sqrt{1-d^2x^2}\,(b^2-4ac)\left(b^2d^2-(ad^2+c)^2\right)(a+bx+cx^2)}$$

$\downarrow$ 25

$$\frac{\int \frac{-ab^2d^4+4a^2cd^4+6ac^2d^2-2c(2c^2+2ad^2c-b^2d^2)x^2d^2-2b^2cd^2-b(c^2+7ad^2c-2b^2d^2)xd^2+2c^3}{(cx^2+bx+a)(1-d^2x^2)^{3/2}}dx}{(b^2-4ac)\left(b^2d^2-(ad^2+c)^2\right)}-$$
$$\frac{b\left(b^2d^2-c\left(3ad^2+c\right)\right)-cx\left(2acd^2-b^2d^2+2c^2\right)}{\sqrt{1-d^2x^2}\,(b^2-4ac)\left(b^2d^2-(ad^2+c)^2\right)(a+bx+cx^2)}$$

$\downarrow$ 2136

$$\frac{\frac{\int -\frac{2\left(3ab^4d^8+2b^2c\left(b^2-7a^2d^2\right)d^6+12ac^4d^4+bc\left(3ab^2d^4-11a^2cd^4-10ac^2d^2+2b^2cd^2+c^3\right)xd^4+2c^5d^2-2c^3\left(2b^2d^4-9a^2d^6\right)-4c^2\left(3ab^2d^6-2a^3d^8\right)\right)}{\left(cx^2+bx+a\right)\sqrt{1-d^2x^2}}dx}{2d^2\left(b^2d^2-(ad^2+c)^2\right)}-\frac{d^2(b\cdots}{\cdots}}{(b^2-4ac)\left(b^2d^2-(ad^2+c)^2\right)}$$
$$\frac{b\left(b^2d^2-c\left(3ad^2+c\right)\right)-cx\left(2acd^2-b^2d^2+2c^2\right)}{\sqrt{1-d^2x^2}\,(b^2-4ac)\left(b^2d^2-(ad^2+c)^2\right)(a+bx+cx^2)}$$

$\downarrow$ 27

$$\frac{-\frac{\int \frac{3ab^4d^8+2b^2c\left(b^2-7a^2d^2\right)d^6+12ac^4d^4+bc\left(3ab^2d^4-11a^2cd^4-10ac^2d^2+2b^2cd^2+c^3\right)xd^4+2c^5d^2-c^3\left(4b^2d^4-18a^2d^6\right)-4c^2\left(3ab^2d^6-2a^3d^8\right)}{\left(cx^2+bx+a\right)\sqrt{1-d^2x^2}}dx}{d^2\left(b^2d^2-(ad^2+c)^2\right)}-\frac{d^2(b(-\cdots}{\cdots}}{(b^2-4ac)\left(b^2d^2-(ad^2+c)^2\right)}$$
$$\frac{b\left(b^2d^2-c\left(3ad^2+c\right)\right)-cx\left(2acd^2-b^2d^2+2c^2\right)}{\sqrt{1-d^2x^2}\,(b^2-4ac)\left(b^2d^2-(ad^2+c)^2\right)(a+bx+cx^2)}$$

$\downarrow$ 1367

$$-\frac{\frac{c\left(-8c^2\left(3ab^2d^6-2a^3d^8\right)-4c^3\left(2b^2d^4-9a^2d^6\right)-bd^4\left(b-\sqrt{b^2-4ac}\right)\left(-11a^2cd^4+3ab^2d^4-10ac^2d^2+2b^2cd^2+c^3\right)+4b^2cd^6\left(b^2-7a^2d^2\right)+6ab^4d^8+24ac^4d^4+4c^5d^2\cdots}{\sqrt{b^2-4ac}}\cdots}{\cdots}$$

$$\frac{b\left(b^2d^2-c\left(3ad^2+c\right)\right)-cx\left(2acd^2-b^2d^2+2c^2\right)}{\sqrt{1-d^2x^2}\,(b^2-4ac)\left(b^2d^2-(ad^2+c)^2\right)(a+bx+cx^2)}$$

3.801. $\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}(a+bx+cx^2)^2}\,dx$

$$\Big\downarrow \; 488$$

$$-\frac{\frac{\left(b\left(3ab^2d^4-11a^2cd^4-10ac^2d^2+2b^2cd^2+c^3\right)-\left(a^2b^2d^6-4a^3cd^6+2b^4d^4-6a^2c^2d^4-6ab^2cd^4-b^2c^2d^2+2c^4\right)x\right)d^2}{\left(b^2d^2-(ad^2+c)^2\right)\sqrt{1-d^2x^2}}-\frac{\frac{c\left(6ab^4d^8+4b^2c\left(b^2-7a^2d^2\right)d^6+\cdots\right.}{}}{}}{}$$

$$\frac{b\left(b^2d^2-c\left(3ad^2+c\right)\right)-c\left(2c^2+2ad^2c-b^2d^2\right)x}{(b^2-4ac)\left(b^2d^2-(ad^2+c)^2\right)(cx^2+bx+a)\sqrt{1-d^2x^2}}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{\left(b\left(3ab^2d^4-11a^2cd^4-10ac^2d^2+2b^2cd^2+c^3\right)-\left(a^2b^2d^6-4a^3cd^6+2b^4d^4-6a^2c^2d^4-6ab^2cd^4-b^2c^2d^2+2c^4\right)x\right)d^2}{\left(b^2d^2-(ad^2+c)^2\right)\sqrt{1-d^2x^2}}-\frac{\frac{c\left(6ab^4d^8+4b^2c\left(b^2-7a^2d^2\right)d^6+\cdots\right.}{}}{}}{}$$

$$\frac{b\left(b^2d^2-c\left(3ad^2+c\right)\right)-c\left(2c^2+2ad^2c-b^2d^2\right)x}{(b^2-4ac)\left(b^2d^2-(ad^2+c)^2\right)(cx^2+bx+a)\sqrt{1-d^2x^2}}$$

input
```
Int[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)^2),x]
```

3.801. $\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}(a+bx+cx^2)^2}\,dx$

output
```
-((b*(b^2*d^2 - c*(c + 3*a*d^2)) - c*(2*c^2 - b^2*d^2 + 2*a*c*d^2)*x)/((b^
2 - 4*a*c)*(b^2*d^2 - (c + a*d^2)^2)*(a + b*x + c*x^2)*Sqrt[1 - d^2*x^2]))
 + (-((d^2*(b*(c^3 + 2*b^2*c*d^2 - 10*a*c^2*d^2 + 3*a*b^2*d^4 - 11*a^2*c*d
^4) - (2*c^4 - b^2*c^2*d^2 + 2*b^4*d^4 - 6*a*b^2*c*d^4 - 6*a^2*c^2*d^4 + a
^2*b^2*d^6 - 4*a^3*c*d^6)*x))/((b^2*d^2 - (c + a*d^2)^2)*Sqrt[1 - d^2*x^2]
)) - (-((c*(4*c^5*d^2 + 24*a*c^4*d^4 + 6*a*b^4*d^8 + 4*b^2*c*d^6*(b^2 - 7*
a^2*d^2) - b*(b - Sqrt[b^2 - 4*a*c])*d^4*(c^3 + 2*b^2*c*d^2 - 10*a*c^2*d^2
 + 3*a*b^2*d^4 - 11*a^2*c*d^4) - 4*c^3*(2*b^2*d^4 - 9*a^2*d^6) - 8*c^2*(3*
a*b^2*d^6 - 2*a^3*d^8))*ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqr
t[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*
x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^
2 - 4*a*c])*d^2])) + (c*(4*c^5*d^2 + 24*a*c^4*d^4 + 6*a*b^4*d^8 + 4*b^2*c*
d^6*(b^2 - 7*a^2*d^2) - b*(b + Sqrt[b^2 - 4*a*c])*d^4*(c^3 + 2*b^2*c*d^2 -
 10*a*c^2*d^2 + 3*a*b^2*d^4 - 11*a^2*c*d^4) - 4*c^3*(2*b^2*d^4 - 9*a^2*d^6
) - 8*c^2*(3*a*b^2*d^6 - 2*a^3*d^8))*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c]
)*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*
Sqrt[1 - d^2*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2 + 2*a*c*d^2 - b
*(b + Sqrt[b^2 - 4*a*c])*d^2]))/(d^2*(b^2*d^2 - (c + a*d^2)^2)))/((b^2 - 4
*a*c)*(b^2*d^2 - (c + a*d^2)^2))
```

3.801.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1188
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2
)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m, n] && EqQ[e*f
 + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

rule 1306
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*f
))*x)*(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^(q + 1)/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^
2*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
 - c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
 b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2
*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 -
4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ
[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

rule 1367
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*g - h*(
b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Simp[(2*c*g -
 h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{
a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

rule 2136
```
Int[(Px_)*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_
), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[P
x, x, 2]}, Simp[(a + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((-4*a*c)*(a
*c*e^2 + (c*d - a*f)^2)*(p + 1)))*((A*c - a*C)*(2*a*c*e) + ((-a)*B)*(2*c^2*
d - c*(2*a*f)) + c*(A*(2*c^2*d - c*(2*a*f)) - B*(-2*a*c*e) + C*(-2*a*(c*d -
 a*f)))*x), x] + Simp[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1))   Int[
(a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^
2 - ((-a)*e)*(c*e))*(p + 1) + (2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*
f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e) + ((-a)*B)*(2*c
^2*d - c*((Plus[2])*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e) + ((-a
)*B)*(2*c^2*d + (-c)*((Plus[2])*a*f)))*(p + q + 2) - (2*(A*c*(c*d - a*f) -
a*(c*C*d - B*c*e - a*C*f)))*((-c)*e*(2*p + q + 4)))*x - c*f*(2*(A*c*(c*d -
a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; Free
Q[{a, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[a*c*e^2 +
(c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

3.801. $\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}(a+bx+cx^2)^2}\,dx$

3.801.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.58 (sec) , antiderivative size = 108969, normalized size of antiderivative = 116.05

method	result	size
default	Expression too large to display	108969

input
```
int(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE
)
```

output
```
result too large to display
```

3.801.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="fri
cas")
```

output
```
Timed out
```

3.801.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(-d*x+1)**(3/2)/(d*x+1)**(3/2)/(c*x**2+b*x+a)**2,x)
```

output
```
Timed out
```

3.801.7 Maxima [F]

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)^2}\,dx=\int \frac{1}{\left(cx^2+bx+a\right)^2(dx+1)^{\frac{3}{2}}(-dx+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="max
ima")
```

output
```
integrate(1/((c*x^2 + b*x + a)^2*(d*x + 1)^(3/2)*(-d*x + 1)^(3/2)), x)
```

3.801.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="gia
c")
```

output
```
Timed out
```

3.801.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)^2}\,dx=\int \frac{1}{{\left(1-d\,x\right)}^{3/2}\,{\left(d\,x+1\right)}^{3/2}\,{\left(c\,x^2+b\,x+a\right)}^2}\,dx$$

input
```
int(1/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)*(a + b*x + c*x^2)^2),x)
```

output
```
int(1/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)*(a + b*x + c*x^2)^2), x)
```

3.802 $\int (1-ex)^m(1+ex)^m \left(a+cx^2\right)^p\,dx$

3.802.1 Optimal result

Integrand size = 25, antiderivative size = 54

$$\int (1-ex)^m(1+ex)^m \left(a+cx^2\right)^p\,dx=x\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},e^2x^2\right)$$

output
```
x*(c*x^2+a)^p*AppellF1(1/2,-m,-p,3/2,e^2*x^2,-c*x^2/a)/((1+c*x^2/a)^p)
```

3.802.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 167 vs. 2(54) = 108.

Time = 0.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.09

$$\int (1-ex)^m(1+ex)^m \left(a+cx^2\right)^p\,dx =\frac{3ax\left(a+cx^2\right)^p\left(1-e^2x^2\right)^m\operatorname{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},e^2x^2\right)}{3a\operatorname{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},e^2x^2\right)+2x^2\left(cp\operatorname{AppellF1}\left(\frac{3}{2},1-p,-m,\frac{5}{2},-\frac{cx^2}{a},e^2x^2\right)-ae^2m\operatorname{AppellF}\right.}$$

input
```
Integrate[(1 - e*x)^m*(1 + e*x)^m*(a + c*x^2)^p,x]
```

output
```
(3*a*x*(a + c*x^2)^p*(1 - e^2*x^2)^m*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/
a), e^2*x^2])/(3*a*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/a), e^2*x^2] + 2*x
^2*(c*p*AppellF1[3/2, 1 - p, -m, 5/2, -((c*x^2)/a), e^2*x^2] - a*e^2*m*App
ellF1[3/2, -p, 1 - m, 5/2, -((c*x^2)/a), e^2*x^2]))
```

3.802.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {643, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-ex)^m(ex+1)^m \left(a+cx^2\right)^p \, dx$$

$$\Big\downarrow \text{ 643}$$

$$\int \left(1-e^2x^2\right)^m \left(a+cx^2\right)^p dx$$

$$\Big\downarrow \text{ 334}$$

$$\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} \int \left(\frac{cx^2}{a}+1\right)^p \left(1-e^2x^2\right)^m dx$$

$$\Big\downarrow \text{ 333}$$

$$x\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},e^2x^2\right)$$

input
```
Int[(1 - e*x)^m*(1 + e*x)^m*(a + c*x^2)^p,x]
```

output
```
(x*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/a), e^2*x^2])/(1 + (
c*x^2)/a)^p
```

3.802.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 643
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_.), x_Symbol] :> Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x] /; FreeQ[{a
, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] && (Integer
Q[m] || (GtQ[c, 0] && GtQ[e, 0]))
```

3.802.4 Maple [F]

$$\int (-ex+1)^m (ex+1)^m (c\,x^2+a)^p\,dx$$

input
```
int((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x)
```

output
```
int((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x)
```

3.802.5 Fricas [F]

$$\int (1-ex)^m(1+ex)^m\left(a+cx^2\right)^p\,dx = \int (cx^2+a)^p(ex+1)^m(-ex+1)^m\,dx$$

input
```
integrate((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + a)^p*(e*x + 1)^m*(-e*x + 1)^m, x)
```

3.802.6 Sympy [F(-1)]

Timed out.

$$\int (1-ex)^m(1+ex)^m \left(a+cx^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((-e*x+1)**m*(e*x+1)**m*(c*x**2+a)**p,x)
```

output
```
Timed out
```

3.802.7 Maxima [F]

$$\int (1-ex)^m(1+ex)^m \left(a+cx^2\right)^p \, dx = \int (cx^2+a)^p(ex+1)^m(-ex+1)^m \, dx$$

input
```
integrate((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)^p*(e*x + 1)^m*(-e*x + 1)^m, x)
```

3.802.8 Giac [F]

$$\int (1-ex)^m(1+ex)^m \left(a+cx^2\right)^p \, dx = \int (cx^2+a)^p(ex+1)^m(-ex+1)^m \, dx$$

input
```
integrate((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)^p*(e*x + 1)^m*(-e*x + 1)^m, x)
```

3.802.9 Mupad [F(-1)]

Timed out.

$$\int (1-ex)^m(1+ex)^m\left(a+cx^2\right)^p\,dx=\int \left(c\,x^2+a\right)^p\left(1-e\,x\right)^m\left(e\,x+1\right)^m\,dx$$

input
```
int((a + c*x^2)^p*(1 - e*x)^m*(e*x + 1)^m,x)
```

output
```
int((a + c*x^2)^p*(1 - e*x)^m*(e*x + 1)^m, x)
```

3.803 $\int (d-ex)^m (d+ex)^m \left(a+cx^2\right)^p \, dx$

3.803.1 Optimal result

Integrand size = 25, antiderivative size = 89

$$\int (d-ex)^m (d+ex)^m \left(a+cx^2\right)^p \, dx = x(d-ex)^m (d+ex)^m \left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} \left(1-\frac{e^2x^2}{d^2}\right)^{-m} \operatorname{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

output
```
x*(-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p*AppellF1(1/2,-m,-p,3/2,e^2*x^2/d^2,-c*x
^2/a)/((1+c*x^2/a)^p)/((1-e^2*x^2/d^2)^m)
```

3.803.2 Mathematica [F]

$$\int (d-ex)^m (d+ex)^m \left(a+cx^2\right)^p \, dx = \int (d-ex)^m (d+ex)^m \left(a+cx^2\right)^p \, dx$$

input
```
Integrate[(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p,x]
```

output
```
Integrate[(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p, x]
```

3.803.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {648, 334, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^p (d-ex)^m (d+ex)^m \, dx$$

$$\Big\downarrow \text{ 648}$$

$$(d-ex)^m (d+ex)^m \left(d^2-e^2x^2\right)^{-m} \int \left(cx^2+a\right)^p \left(d^2-e^2x^2\right)^m dx$$

$$\Big\downarrow \text{ 334}$$

$$\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} (d-ex)^m (d+ex)^m \left(d^2-e^2x^2\right)^{-m} \int \left(\frac{cx^2}{a}+1\right)^p \left(d^2-e^2x^2\right)^m dx$$

$$\Big\downarrow \text{ 334}$$

$$\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} (d-ex)^m (d+ex)^m \left(1-\frac{e^2x^2}{d^2}\right)^{-m} \int \left(\frac{cx^2}{a}+1\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^m dx$$

$$\Big\downarrow \text{ 333}$$

$$x\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} (d-ex)^m (d+ ex)^m \left(1-\frac{e^2x^2}{d^2}\right)^{-m} \text{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

input
```
Int[(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p,x]
```

output
```
(x*(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -((c*x
^2)/a), (e^2*x^2)/d^2])/((1 + (c*x^2)/a)^p*(1 - (e^2*x^2)/d^2)^m)
```

3.803.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

3.803.4 Maple [F]

$$\int (-ex+d)^m (ex+d)^m (c\,x^2+a)^p\,dx$$

input
```
int((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x)
```

output
```
int((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x)
```

3.803.5 Fricas [F]

$$\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx=\int \left(cx^2+a\right)^p(ex+d)^m(-ex+d)^m\,dx$$

input
```
integrate((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + a)^p*(e*x + d)^m*(-e*x + d)^m, x)
```

3.803.6 Sympy [F(-1)]

Timed out.

$$\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx = \text{Timed out}$$

input
```
integrate((-e*x+d)**m*(e*x+d)**m*(c*x**2+a)**p,x)
```

output
```
Timed out
```

3.803.7 Maxima [F]

$$\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx = \int (cx^2+a)^p(ex+d)^m(-ex+d)^m\,dx$$

input
```
integrate((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)^p*(e*x + d)^m*(-e*x + d)^m, x)
```

3.803.8 Giac [F]

$$\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx = \int (cx^2+a)^p(ex+d)^m(-ex+d)^m\,dx$$

input
```
integrate((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)^p*(e*x + d)^m*(-e*x + d)^m, x)
```

3.803.9 Mupad [F(-1)]

Timed out.

$$\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx=\int \left(c\,x^2+a\right)^p\,\left(d+e\,x\right)^m\,\left(d-e\,x\right)^m\,dx$$

input
```
int((a + c*x^2)^p*(d + e*x)^m*(d - e*x)^m,x)
```

output
```
int((a + c*x^2)^p*(d + e*x)^m*(d - e*x)^m, x)
```

3.804 $\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx$

3.804.1 Optimal result

Integrand size = 28, antiderivative size = 92

$$\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx=x(d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(1-\frac{e^2x^2}{d^2}\right)^{-m}\operatorname{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

output
```
x*(e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p*AppellF1(1/2,-m,-p,3/2,e^2*x^2/d^2,
-c*x^2/a)/((1+c*x^2/a)^p)/((1-e^2*x^2/d^2)^m)
```

3.804.2 Mathematica [F]

$$\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx=\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx$$

input
```
Integrate[(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p,x]
```

output
```
Integrate[(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p, x]
```

3.804.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {648, 334, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^p (d+ex)^m (df-efx)^m \, dx$$

$$\Big\downarrow \; 648$$

$$(d+ex)^m (df-efx)^m \left(d^2f-e^2fx^2\right)^{-m} \int \left(cx^2+a\right)^p \left(d^2f-e^2fx^2\right)^m dx$$

$$\Big\downarrow \; 334$$

$$\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} (d+ex)^m (df-efx)^m \left(d^2f-e^2fx^2\right)^{-m} \int \left(\frac{cx^2}{a}+1\right)^p \left(d^2f-e^2fx^2\right)^m dx$$

$$\Big\downarrow \; 334$$

$$\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} (d+ex)^m \left(1-\frac{e^2x^2}{d^2}\right)^{-m} (df-efx)^m \int \left(\frac{cx^2}{a}+1\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^m dx$$

$$\Big\downarrow \; 333$$

$$x\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} (d+ex)^m \left(1-\frac{e^2x^2}{d^2}\right)^{-m} (df-efx)^m \operatorname{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

input
```
Int[(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p,x]
```

output
```
(x*(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -(
(c*x^2)/a), (e^2*x^2)/d^2])/((1 + (c*x^2)/a)^p*(1 - (e^2*x^2)/d^2)^m)
```

3.804.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

3.804.4 Maple [F]

$$\int (ex+d)^m (-efx+df)^m (c x^2+a)^p dx$$

input
```
int((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x)
```

output
```
int((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x)
```

3.804.5 Fricas [F]

$$\int (d+ex)^m(df-efx)^m (a+cx^2)^p dx = \int (-efx+df)^m (cx^2+a)^p (ex+d)^m dx$$

input
```
integrate((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((-e*f*x + d*f)^m*(c*x^2 + a)^p*(e*x + d)^m, x)
```

3.804.6 Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**m*(-e*f*x+d*f)**m*(c*x**2+a)**p,x)
```

output
```
Timed out
```

3.804.7 Maxima [F]

$$\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx=\int (-efx+df)^m(cx^2+a)^p(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((-e*f*x + d*f)^m*(c*x^2 + a)^p*(e*x + d)^m, x)
```

3.804.8 Giac [F]

$$\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx=\int (-efx+df)^m(cx^2+a)^p(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((-e*f*x + d*f)^m*(c*x^2 + a)^p*(e*x + d)^m, x)
```

3.804.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx=\int (d\,f-e\,f\,x)^m\,\left(c\,x^2+a\right)^p\,(d+e\,x)^m\,dx$$

input
```
int((d*f - e*f*x)^m*(a + c*x^2)^p*(d + e*x)^m,x)
```

output
```
int((d*f - e*f*x)^m*(a + c*x^2)^p*(d + e*x)^m, x)
```

3.805 $\int (d+ex)^3(f+gx)^n \left(a+2cdx+cex^2\right) dx$

3.805.1 Optimal result

Integrand size = 28, antiderivative size = 275

$$\begin{aligned}\int (d+ex)^3(f+gx)^n \left(a+2cdx+cex^2\right) dx &= -\frac{(ef-dg)^3 \left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^6(1+n)} \\ &\quad+\frac{(ef-dg)^2 \left(3aeg^2+c(5e^2f^2-10defg+2d^2g^2)\right)(f+gx)^{2+n}}{g^6(2+n)} \\ &\quad-\frac{e(ef-dg)\left(3aeg^2+c(10e^2f^2-20defg+7d^2g^2)\right)(f+gx)^{3+n}}{g^6(3+n)} \\ &\quad+\frac{e^2\left(aeg^2+c(10e^2f^2-20defg+9d^2g^2)\right)(f+gx)^{4+n}}{g^6(4+n)} \\ &\quad-\frac{5ce^3(ef-dg)(f+gx)^{5+n}}{g^6(5+n)}+\frac{ce^4(f+gx)^{6+n}}{g^6(6+n)}\end{aligned}$$

output
```
-(-d*g+e*f)^3*(a*g^2+c*f*(-2*d*g+e*f))*(g*x+f)^(1+n)/g^6/(1+n)+(-d*g+e*f)^
2*(3*a*e*g^2+c*(2*d^2*g^2-10*d*e*f*g+5*e^2*f^2))*(g*x+f)^(2+n)/g^6/(2+n)-e
*(-d*g+e*f)*(3*a*e*g^2+c*(7*d^2*g^2-20*d*e*f*g+10*e^2*f^2))*(g*x+f)^(3+n)/
g^6/(3+n)+e^2*(a*e*g^2+c*(9*d^2*g^2-20*d*e*f*g+10*e^2*f^2))*(g*x+f)^(4+n)/
g^6/(4+n)-5*c*e^3*(-d*g+e*f)*(g*x+f)^(5+n)/g^6/(5+n)+c*e^4*(g*x+f)^(6+n)/g
^6/(6+n)
```

3.805.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.91

$$\int (d+ex)^3(f+gx)^n\left(a+2cdx+cex^2\right)\,dx$$
$$=\frac{(f+gx)^{1+n}\left(-\frac{(ef-dg)^3(ag^2+cf(ef-2dg))}{1+n}+\frac{(ef-dg)^2(3aeg^2+c(5e^2f^2-10defg+2d^2g^2))(f+gx)}{2+n}-\frac{e(ef-dg)(3aeg^2+c(10e^2f^2-2}{3+n}\right.}{g^6}$$

input
```
Integrate[(d + e*x)^3*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

output
```
((f + g*x)^(1 + n)*(-(((e*f - d*g)^3*(a*g^2 + c*f*(e*f - 2*d*g)))/(1 + n))
 + ((e*f - d*g)^2*(3*a*e*g^2 + c*(5*e^2*f^2 - 10*d*e*f*g + 2*d^2*g^2))*(f
+ g*x))/(2 + n) - (e*(e*f - d*g)*(3*a*e*g^2 + c*(10*e^2*f^2 - 20*d*e*f*g +
 7*d^2*g^2))*(f + g*x)^2)/(3 + n) + (e^2*(a*e*g^2 + c*(10*e^2*f^2 - 20*d*e
*f*g + 9*d^2*g^2))*(f + g*x)^3)/(4 + n) - (5*c*e^3*(e*f - d*g)*(f + g*x)^4
)/(5 + n) + (c*e^4*(f + g*x)^5)/(6 + n)))/g^6
```

3.805.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3(f+gx)^n\left(a+2cdx+cex^2\right)\,dx$$
$$\Big\downarrow \ 1195$$
$$\int\left(\frac{(ef-dg)^2(f+gx)^{n+1}\left(3aeg^2+c\left(2d^2g^2-10defg+5e^2f^2\right)\right)}{g^5}+\frac{e(ef-dg)(f+gx)^{n+2}\left(-3aeg^2-c(7d^2g^2\right.}{g^5}\right.$$
$$\Big\downarrow \ 2009$$

$$\frac{(ef-dg)^2(f+gx)^{n+2}\left(3aeg^2+c\left(2d^2g^2-10defg+5e^2f^2\right)\right)}{g^6(n+2)} - \frac{e(ef-dg)(f+gx)^{n+3}\left(3aeg^2+c\left(7d^2g^2-20defg+10e^2f^2\right)\right)}{g^6(n+3)} + \frac{e^2(f+gx)^{n+4}\left(aeg^2+c\left(9d^2g^2-20defg+10e^2f^2\right)\right)}{g^6(n+4)} - \frac{(ef-dg)^3(f+gx)^{n+1}\left(ag^2+cf(ef-2dg)\right)}{g^6(n+1)} - \frac{5ce^3(ef-dg)(f+gx)^{n+5}}{g^6(n+5)} + \frac{ce^4(f+gx)^{n+6}}{g^6(n+6)}$$

input
```
Int[(d + e*x)^3*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

output
```
-(((e*f - d*g)^3*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^6*(1 +
n))) + ((e*f - d*g)^2*(3*a*e*g^2 + c*(5*e^2*f^2 - 10*d*e*f*g + 2*d^2*g^2))
*(f + g*x)^(2 + n))/(g^6*(2 + n)) - (e*(e*f - d*g)*(3*a*e*g^2 + c*(10*e^2*
f^2 - 20*d*e*f*g + 7*d^2*g^2))*(f + g*x)^(3 + n))/(g^6*(3 + n)) + (e^2*(a*
e*g^2 + c*(10*e^2*f^2 - 20*d*e*f*g + 9*d^2*g^2))*(f + g*x)^(4 + n))/(g^6*(
4 + n)) - (5*c*e^3*(e*f - d*g)*(f + g*x)^(5 + n))/(g^6*(5 + n)) + (c*e^4*(
f + g*x)^(6 + n))/(g^6*(6 + n))
```

3.805.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.805.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1785 vs. $2(275) = 550$.

Time = 0.58 (sec) , antiderivative size = 1786, normalized size of antiderivative = 6.49

method	result	size
norman	Expression too large to display	1786
gosper	Expression too large to display	2017
risch	Expression too large to display	2642
parallelrisch	Expression too large to display	3960

input
```
int((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x,method=_RETURNVERBOSE)
```

output
```
c*e^4/(6+n)*x^6*exp(n*ln(g*x+f))+f*(a*d^3*g^5*n^5-2*c*d^4*f*g^4*n^4+20*a*d
^3*g^5*n^4-3*a*d^2*e*f*g^4*n^4-36*c*d^4*f*g^4*n^3+14*c*d^3*e*f^2*g^3*n^3+1
55*a*d^3*g^5*n^3-54*a*d^2*e*f*g^4*n^3+6*a*d*e^2*f^2*g^3*n^3-238*c*d^4*f*g^
4*n^2+210*c*d^3*e*f^2*g^3*n^2-54*c*d^2*e^2*f^3*g^2*n^2+580*a*d^3*g^5*n^2-3
57*a*d^2*e*f*g^4*n^2+90*a*d*e^2*f^2*g^3*n^2-6*a*e^3*f^3*g^2*n^2-684*c*d^4*
f*g^4*n+1036*c*d^3*e*f^2*g^3*n-594*c*d^2*e^2*f^3*g^2*n+120*c*d*e^3*f^4*g*n
+1044*a*d^3*g^5*n-1026*a*d^2*e*f*g^4*n+444*a*d*e^2*f^2*g^3*n-66*a*e^3*f^3*
g^2*n-720*c*d^4*f*g^4+1680*c*d^3*e*f^2*g^3-1620*c*d^2*e^2*f^3*g^2+720*c*d*
e^3*f^4*g-120*c*e^4*f^5+720*a*d^3*g^5-1080*a*d^2*e*f*g^4+720*a*d*e^2*f^2*g
^3-180*a*e^3*f^3*g^2)/g^6/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+720)
*exp(n*ln(g*x+f))+(2*c*d^4*g^4*n^4+7*c*d^3*e*f*g^3*n^4+3*a*d^2*e*g^4*n^4+3
*a*d*e^2*f*g^3*n^4+36*c*d^4*g^4*n^3+105*c*d^3*e*f*g^3*n^3-27*c*d^2*e^2*f^2
*g^2*n^3+54*a*d^2*e*g^4*n^3+45*a*d*e^2*f*g^3*n^3-3*a*e^3*f^2*g^2*n^3+238*c
*d^4*g^4*n^2+518*c*d^3*e*f*g^3*n^2-297*c*d^2*e^2*f^2*g^2*n^2+60*c*d*e^3*f^
3*g*n^2+357*a*d^2*e*g^4*n^2+222*a*d*e^2*f*g^3*n^2-33*a*e^3*f^2*g^2*n^2+684
*c*d^4*g^4*n+840*c*d^3*e*f*g^3*n-810*c*d^2*e^2*f^2*g^2*n+360*c*d*e^3*f^3*g
*n-60*c*e^4*f^4*n+1026*a*d^2*e*g^4*n+360*a*d*e^2*f*g^3*n-90*a*e^3*f^2*g^2*
n+720*c*d^4*g^4+1080*a*d^2*e*g^4)/g^4/(n^5+20*n^4+155*n^3+580*n^2+1044*n+7
20)*x^2*exp(n*ln(g*x+f))+(2*c*d^4*f*g^4*n^5+a*d^3*g^5*n^5+3*a*d^2*e*f*g^4*
n^5+36*c*d^4*f*g^4*n^4-14*c*d^3*e*f^2*g^3*n^4+20*a*d^3*g^5*n^4+54*a*d^2...
```

3.805.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2032 vs. $2(275) = 550$.

Time = 0.35 (sec) , antiderivative size = 2032, normalized size of antiderivative = 7.39

$$\int (d+ex)^3(f+gx)^n \left(a+2cdx+cex^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

output
```
(a*d^3*f*g^5*n^5 - 120*c*e^4*f^6 + 720*c*d*e^3*f^5*g + 720*a*d^3*f*g^5 - 1
80*(9*c*d^2*e^2 + a*e^3)*f^4*g^2 + 240*(7*c*d^3*e + 3*a*d*e^2)*f^3*g^3 - 3
60*(2*c*d^4 + 3*a*d^2*e)*f^2*g^4 + (c*e^4*g^6*n^5 + 15*c*e^4*g^6*n^4 + 85*
c*e^4*g^6*n^3 + 225*c*e^4*g^6*n^2 + 274*c*e^4*g^6*n + 120*c*e^4*g^6)*x^6 +
 (720*c*d*e^3*g^6 + (c*e^4*f*g^5 + 5*c*d*e^3*g^6)*n^5 + 10*(c*e^4*f*g^5 +
8*c*d*e^3*g^6)*n^4 + 5*(7*c*e^4*f*g^5 + 95*c*d*e^3*g^6)*n^3 + 50*(c*e^4*f*
g^5 + 26*c*d*e^3*g^6)*n^2 + 12*(2*c*e^4*f*g^5 + 135*c*d*e^3*g^6)*n)*x^5 +
(20*a*d^3*f*g^5 - (2*c*d^4 + 3*a*d^2*e)*f^2*g^4)*n^4 + (180*(9*c*d^2*e^2 +
 a*e^3)*g^6 + (5*c*d*e^3*f*g^5 + (9*c*d^2*e^2 + a*e^3)*g^6)*n^5 - (5*c*e^4
*f^2*g^4 - 60*c*d*e^3*f*g^5 - 17*(9*c*d^2*e^2 + a*e^3)*g^6)*n^4 - (30*c*e^
4*f^2*g^4 - 235*c*d*e^3*f*g^5 - 107*(9*c*d^2*e^2 + a*e^3)*g^6)*n^3 - (55*c
*e^4*f^2*g^4 - 360*c*d*e^3*f*g^5 - 307*(9*c*d^2*e^2 + a*e^3)*g^6)*n^2 - 6*
(5*c*e^4*f^2*g^4 - 30*c*d*e^3*f*g^5 - 66*(9*c*d^2*e^2 + a*e^3)*g^6)*n)*x^4
 + (155*a*d^3*f*g^5 + 2*(7*c*d^3*e + 3*a*d*e^2)*f^3*g^3 - 18*(2*c*d^4 + 3*
a*d^2*e)*f^2*g^4)*n^3 + (240*(7*c*d^3*e + 3*a*d*e^2)*g^6 + ((9*c*d^2*e^2 +
 a*e^3)*f*g^5 + (7*c*d^3*e + 3*a*d*e^2)*g^6)*n^5 - 2*(10*c*d*e^3*f^2*g^4 -
 7*(9*c*d^2*e^2 + a*e^3)*f*g^5 - 9*(7*c*d^3*e + 3*a*d*e^2)*g^6)*n^4 + (20*
c*e^4*f^3*g^3 - 180*c*d*e^3*f^2*g^4 + 65*(9*c*d^2*e^2 + a*e^3)*f*g^5 + 121
*(7*c*d^3*e + 3*a*d*e^2)*g^6)*n^3 + 4*(15*c*e^4*f^3*g^3 - 100*c*d*e^3*f^2*
g^4 + 28*(9*c*d^2*e^2 + a*e^3)*f*g^5 + 93*(7*c*d^3*e + 3*a*d*e^2)*g^6)*...
```

3.805.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24206 vs. $2(260) = 520$.

Time = 4.36 (sec) , antiderivative size = 24206, normalized size of antiderivative = 88.02

$$\int (d+ex)^3(f+gx)^n \left(a+2cdx+cex^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

output
```
Piecewise((f**n*(a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**
4/4 + c*d**4*x**2 + 7*c*d**3*e*x**3/3 + 9*c*d**2*e**2*x**4/4 + c*d*e**3*x*
*5 + c*e**4*x**6/6), Eq(g, 0)), (-12*a*d**3*g**5/(60*f**5*g**6 + 300*f**4*
g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g
**11*x**5) - 9*a*d**2*e*f*g**4/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*
g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 45*a*
d**2*e*g**5*x/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f
**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 6*a*d*e**2*f**2*g**3/(
60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 +
 300*f*g**10*x**4 + 60*g**11*x**5) - 30*a*d*e**2*f*g**4*x/(60*f**5*g**6 +
300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x*
*4 + 60*g**11*x**5) - 60*a*d*e**2*g**5*x**2/(60*f**5*g**6 + 300*f**4*g**7*
x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*
x**5) - 3*a*e**3*f**3*g**2/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8
*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 15*a*e**3
*f**2*g**3*x/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f*
*2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 30*a*e**3*f*g**4*x**2/(
60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 +
 300*f*g**10*x**4 + 60*g**11*x**5) - 30*a*e**3*g**5*x**3/(60*f**5*g**6 + 3
00*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*...
```

3.805.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(275) = 550$.

Time = 0.27 (sec) , antiderivative size = 811, normalized size of antiderivative = 2.95

$$\int (d+ex)^3(f+gx)^n\left(a+2cdx+cex^2\right)dx = \frac{2\left(g^2(n+1)x^2+fgnx-f^2\right)(gx+f)^n cd^4}{(n^2+3n+2)g^2} + \frac{7\left((n^2+3n+2)g^3x^3+(n^2+n)fg^2x^2-2f^2gnx+2f^3\right)(gx+f)^n cd^3e}{(n^3+6n^2+11n+6)g^3} + \frac{3\left(g^2(n+1)x^2+fgnx-f^2\right)(gx+f)^n ad^2e}{(n^2+3n+2)g^2} + \frac{(gx+f)^{n+1}ad^3}{g(n+1)} + \frac{9\left((n^3+6n^2+11n+6)g^4x^4+(n^3+3n^2+2n)fg^3x^3-3(n^2+n)f^2g^2x^2+6f^3gnx-6f^4\right)(gx+f)^n}{(n^4+10n^3+35n^2+50n+24)g^4} + \frac{3\left((n^2+3n+2)g^3x^3+(n^2+n)fg^2x^2-2f^2gnx+2f^3\right)(gx+f)^n ade^2}{(n^3+6n^2+11n+6)g^3} + \frac{5\left((n^4+10n^3+35n^2+50n+24)g^5x^5+(n^4+6n^3+11n^2+6n)fg^4x^4-4(n^3+3n^2+2n)f^2g^3x^3\right.}{(n^5+15n^4+85n^3+225n^2+274n+120)g^5} + \frac{\left((n^3+6n^2+11n+6)g^4x^4+(n^3+3n^2+2n)fg^3x^3-3(n^2+n)f^2g^2x^2+6f^3gnx-6f^4\right)(gx+f)^n a}{(n^4+10n^3+35n^2+50n+24)g^4} + \frac{\left((n^5+15n^4+85n^3+225n^2+274n+120)g^6x^6+(n^5+10n^4+35n^3+50n^2+24n)fg^5x^5-5(n^4\right.}{(n^6+21n^5+175n^4+}$$

input
```
integrate((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

output
```
2*(g^2*(n + 1)*x^2 + f*g*n*x - f^2)*(g*x + f)^n*c*d^4/((n^2 + 3*n + 2)*g^2
) + 7*((n^2 + 3*n + 2)*g^3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3
)*(g*x + f)^n*c*d^3*e/((n^3 + 6*n^2 + 11*n + 6)*g^3) + 3*(g^2*(n + 1)*x^2
+ f*g*n*x - f^2)*(g*x + f)^n*a*d^2*e/((n^2 + 3*n + 2)*g^2) + (g*x + f)^(n
+ 1)*a*d^3/(g*(n + 1)) + 9*((n^3 + 6*n^2 + 11*n + 6)*g^4*x^4 + (n^3 + 3*n^
2 + 2*n)*f*g^3*x^3 - 3*(n^2 + n)*f^2*g^2*x^2 + 6*f^3*g*n*x - 6*f^4)*(g*x +
 f)^n*c*d^2*e^2/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*g^4) + 3*((n^2 + 3*n
+ 2)*g^3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*(g*x + f)^n*a*d*
e^2/((n^3 + 6*n^2 + 11*n + 6)*g^3) + 5*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24
)*g^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*f*g^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)
*f^2*g^3*x^3 + 12*(n^2 + n)*f^3*g^2*x^2 - 24*f^4*g*n*x + 24*f^5)*(g*x + f)
^n*c*d*e^3/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*g^5) + ((n^3 +
 6*n^2 + 11*n + 6)*g^4*x^4 + (n^3 + 3*n^2 + 2*n)*f*g^3*x^3 - 3*(n^2 + n)*f
^2*g^2*x^2 + 6*f^3*g*n*x - 6*f^4)*(g*x + f)^n*a*e^3/((n^4 + 10*n^3 + 35*n^
2 + 50*n + 24)*g^4) + ((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*g^6
*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*f*g^5*x^5 - 5*(n^4 + 6*n^3
+ 11*n^2 + 6*n)*f^2*g^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*f^3*g^3*x^3 - 60*(n^2
 + n)*f^4*g^2*x^2 + 120*f^5*g*n*x - 120*f^6)*(g*x + f)^n*c*e^4/((n^6 + 21*
n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*g^6)
```

3.805. $\int (d+ex)^3(f+gx)^n\left(a+2cdx+cex^2\right)dx$

3.805.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3830 vs. $2(275) = 550$.

Time = 0.33 (sec) , antiderivative size = 3830, normalized size of antiderivative = 13.93

$$\int (d+ex)^3(f+gx)^n\left(a+2cdx+cex^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

output
```
((g*x + f)^n*c*e^4*g^6*n^5*x^6 + (g*x + f)^n*c*e^4*f*g^5*n^5*x^5 + 5*(g*x
+ f)^n*c*d*e^3*g^6*n^5*x^5 + 15*(g*x + f)^n*c*e^4*g^6*n^4*x^6 + 5*(g*x + f
)^n*c*d*e^3*f*g^5*n^5*x^4 + 9*(g*x + f)^n*c*d^2*e^2*g^6*n^5*x^4 + 10*(g*x
+ f)^n*c*e^4*f*g^5*n^4*x^5 + 80*(g*x + f)^n*c*d*e^3*g^6*n^4*x^5 + 85*(g*x
+ f)^n*c*e^4*g^6*n^3*x^6 + 9*(g*x + f)^n*c*d^2*e^2*f*g^5*n^5*x^3 + 7*(g*x
+ f)^n*c*d^3*e*g^6*n^5*x^3 - 5*(g*x + f)^n*c*e^4*f^2*g^4*n^4*x^4 + 60*(g*x
 + f)^n*c*d*e^3*f*g^5*n^4*x^4 + 153*(g*x + f)^n*c*d^2*e^2*g^6*n^4*x^4 + (g
*x + f)^n*a*e^3*g^6*n^5*x^4 + 35*(g*x + f)^n*c*e^4*f*g^5*n^3*x^5 + 475*(g*
x + f)^n*c*d*e^3*g^6*n^3*x^5 + 225*(g*x + f)^n*c*e^4*g^6*n^2*x^6 + 7*(g*x
+ f)^n*c*d^3*e*f*g^5*n^5*x^2 + 2*(g*x + f)^n*c*d^4*g^6*n^5*x^2 - 20*(g*x +
 f)^n*c*d*e^3*f^2*g^4*n^4*x^3 + 126*(g*x + f)^n*c*d^2*e^2*f*g^5*n^4*x^3 +
126*(g*x + f)^n*c*d^3*e*g^6*n^4*x^3 + (g*x + f)^n*a*e^3*f*g^5*n^5*x^3 + 3*
(g*x + f)^n*a*d*e^2*g^6*n^5*x^3 - 30*(g*x + f)^n*c*e^4*f^2*g^4*n^3*x^4 + 2
35*(g*x + f)^n*c*d*e^3*f*g^5*n^3*x^4 + 963*(g*x + f)^n*c*d^2*e^2*g^6*n^3*x
^4 + 17*(g*x + f)^n*a*e^3*g^6*n^4*x^4 + 50*(g*x + f)^n*c*e^4*f*g^5*n^2*x^5
 + 1300*(g*x + f)^n*c*d*e^3*g^6*n^2*x^5 + 274*(g*x + f)^n*c*e^4*g^6*n*x^6
+ 2*(g*x + f)^n*c*d^4*f*g^5*n^5*x - 27*(g*x + f)^n*c*d^2*e^2*f^2*g^4*n^4*x
^2 + 112*(g*x + f)^n*c*d^3*e*f*g^5*n^4*x^2 + 38*(g*x + f)^n*c*d^4*g^6*n^4*
x^2 + 3*(g*x + f)^n*a*d*e^2*f*g^5*n^5*x^2 + 3*(g*x + f)^n*a*d^2*e*g^6*n^5*
x^2 + 20*(g*x + f)^n*c*e^4*f^3*g^3*n^3*x^3 - 180*(g*x + f)^n*c*d*e^3*f^...
```

3.805.9 Mupad [B] (verification not implemented)

Time = 13.02 (sec) , antiderivative size = 1943, normalized size of antiderivative = 7.07

$$\int (d+ex)^3(f+gx)^n\left(a+2cdx+cex^2\right)\,dx = \text{Too large to display}$$

input
```
int((f + g*x)^n*(d + e*x)^3*(a + 2*c*d*x + c*e*x^2),x)
```

output
```
(x*(f + g*x)^n*(720*a*d^3*g^6 + 580*a*d^3*g^6*n^2 + 155*a*d^3*g^6*n^3 + 20
*a*d^3*g^6*n^4 + a*d^3*g^6*n^5 + 1044*a*d^3*g^6*n + 720*c*d^4*f*g^5*n + 12
0*c*e^4*f^5*g*n + 180*a*e^3*f^3*g^3*n + 684*c*d^4*f*g^5*n^2 + 238*c*d^4*f*
g^5*n^3 + 36*c*d^4*f*g^5*n^4 + 2*c*d^4*f*g^5*n^5 + 66*a*e^3*f^3*g^3*n^2 +
6*a*e^3*f^3*g^3*n^3 - 444*a*d*e^2*f^2*g^4*n^2 - 90*a*d*e^2*f^2*g^4*n^3 - 6
*a*d*e^2*f^2*g^4*n^4 + 1620*c*d^2*e^2*f^3*g^3*n - 120*c*d*e^3*f^4*g^2*n^2
- 1036*c*d^3*e*f^2*g^4*n^2 - 210*c*d^3*e*f^2*g^4*n^3 - 14*c*d^3*e*f^2*g^4*
n^4 + 1080*a*d^2*e*f*g^5*n + 594*c*d^2*e^2*f^3*g^3*n^2 + 54*c*d^2*e^2*f^3*
g^3*n^3 - 720*a*d*e^2*f^2*g^4*n + 1026*a*d^2*e*f*g^5*n^2 + 357*a*d^2*e*f*g
^5*n^3 + 54*a*d^2*e*f*g^5*n^4 + 3*a*d^2*e*f*g^5*n^5 - 720*c*d*e^3*f^4*g^2*
n - 1680*c*d^3*e*f^2*g^4*n))/(g^6*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 +
 21*n^5 + n^6 + 720)) - ((f + g*x)^n*(120*c*e^4*f^6 + 180*a*e^3*f^4*g^2 +
720*c*d^4*f^2*g^4 - 720*a*d^3*f*g^5 - 720*c*d*e^3*f^5*g - 1044*a*d^3*f*g^5
*n - 720*a*d*e^2*f^3*g^3 + 1080*a*d^2*e*f^2*g^4 - 1680*c*d^3*e*f^3*g^3 - 5
80*a*d^3*f*g^5*n^2 - 155*a*d^3*f*g^5*n^3 - 20*a*d^3*f*g^5*n^4 - a*d^3*f*g^
5*n^5 + 66*a*e^3*f^4*g^2*n + 684*c*d^4*f^2*g^4*n + 1620*c*d^2*e^2*f^4*g^2
+ 6*a*e^3*f^4*g^2*n^2 + 238*c*d^4*f^2*g^4*n^2 + 36*c*d^4*f^2*g^4*n^3 + 2*c
*d^4*f^2*g^4*n^4 - 90*a*d*e^2*f^3*g^3*n^2 + 357*a*d^2*e*f^2*g^4*n^2 - 6*a*
d*e^2*f^3*g^3*n^3 + 54*a*d^2*e*f^2*g^4*n^3 + 3*a*d^2*e*f^2*g^4*n^4 + 594*c
*d^2*e^2*f^4*g^2*n - 210*c*d^3*e*f^3*g^3*n^2 - 14*c*d^3*e*f^3*g^3*n^3 -...
```

3.805. $\int (d+ex)^3(f+gx)^n \left(a+2cdx+cex^2\right) \, dx$

3.806 $\int (d+ex)^2(f+gx)^n (a+2cdx+cex^2)\, dx$

3.806.1 Optimal result

Integrand size = 28, antiderivative size = 208

$$\begin{aligned}\int (d+ex)^2(f+gx)^n (a+2cdx+cex^2)\, dx &= \frac{(ef-dg)^2 (ag^2+cf(ef-2dg)) (f+gx)^{1+n}}{g^5(1+n)} \\ &- \frac{2(ef-dg) (aeg^2+c(2e^2f^2-4defg+d^2g^2)) (f+gx)^{2+n}}{g^5(2+n)} \\ &+ \frac{e(aeg^2+c(6e^2f^2-12defg+5d^2g^2)) (f+gx)^{3+n}}{g^5(3+n)} \\ &- \frac{4ce^2(ef-dg)(f+gx)^{4+n}}{g^5(4+n)} + \frac{ce^3(f+gx)^{5+n}}{g^5(5+n)}\end{aligned}$$

output
```
(-d*g+e*f)^2*(a*g^2+c*f*(-2*d*g+e*f))*(g*x+f)^(1+n)/g^5/(1+n)-2*(-d*g+e*f)
*(a*e*g^2+c*(d^2*g^2-4*d*e*f*g+2*e^2*f^2))*(g*x+f)^(2+n)/g^5/(2+n)+e*(a*e*
g^2+c*(5*d^2*g^2-12*d*e*f*g+6*e^2*f^2))*(g*x+f)^(3+n)/g^5/(3+n)-4*c*e^2*(-
d*g+e*f)*(g*x+f)^(4+n)/g^5/(4+n)+c*e^3*(g*x+f)^(5+n)/g^5/(5+n)
```

3.806.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.90

$$\int (d+ex)^2(f+gx)^n \left(a+2cdx+cex^2\right)\,dx$$
$$=\frac{(f+gx)^{1+n}\left(\frac{(ef-dg)^2(ag^2+cf(ef-2dg))}{1+n}-\frac{2(ef-dg)(aeg^2+c(2e^2f^2-4defg+d^2g^2))(f+gx)}{2+n}+\frac{e(aeg^2+c(6e^2f^2-12defg+5d^2g^2))(\ldots}{3+n}\right.}{g^5}$$

input
```
Integrate[(d + e*x)^2*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

output
```
((f + g*x)^(1 + n)*(((e*f - d*g)^2*(a*g^2 + c*f*(e*f - 2*d*g)))/(1 + n) -
(2*(e*f - d*g)*(a*e*g^2 + c*(2*e^2*f^2 - 4*d*e*f*g + d^2*g^2))*(f + g*x))/
(2 + n) + (e*(a*e*g^2 + c*(6*e^2*f^2 - 12*d*e*f*g + 5*d^2*g^2))*(f + g*x)^
2)/(3 + n) - (4*c*e^2*(e*f - d*g)*(f + g*x)^3)/(4 + n) + (c*e^3*(f + g*x)^
4)/(5 + n)))/g^5
```

3.806.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2(f+gx)^n \left(a+2cdx+cex^2\right)\,dx$$
$$\Big\downarrow \text{1195}$$
$$\int \left(\frac{2(ef-dg)(f+gx)^{n+1}\left(-aeg^2-c\left(d^2g^2-4defg+2e^2f^2\right)\right)}{g^4}+\frac{e(f+gx)^{n+2}\left(aeg^2+c\left(5d^2g^2-12defg+6e\ldots\right.\right.}{g^4}\right.$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2(ef-dg)(f+gx)^{n+2}\left(aeg^2+c\left(d^2g^2-4defg+2e^2f^2\right)\right)}{g^5(n+2)}+\frac{e(f+gx)^{n+3}\left(aeg^2+c\left(5d^2g^2-12defg+6e^2f^2\right)\right)}{g^5(n+3)}+\frac{(ef-dg)^2(f+gx)^{n+1}\left(ag^2+cf(ef-2dg)\right)}{g^5(n+1)}-\frac{4ce^2(ef-dg)(f+gx)^{n+4}}{g^5(n+4)}+\frac{ce^3(f+gx)^{n+5}}{g^5(n+5)}$$

input
```
Int[(d + e*x)^2*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

output
```
((e*f - d*g)^2*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^5*(1 + n)
) - (2*(e*f - d*g)*(a*e*g^2 + c*(2*e^2*f^2 - 4*d*e*f*g + d^2*g^2))*(f + g*
x)^(2 + n))/(g^5*(2 + n)) + (e*(a*e*g^2 + c*(6*e^2*f^2 - 12*d*e*f*g + 5*d^
2*g^2))*(f + g*x)^(3 + n))/(g^5*(3 + n)) - (4*c*e^2*(e*f - d*g)*(f + g*x)^
(4 + n))/(g^5*(4 + n)) + (c*e^3*(f + g*x)^(5 + n))/(g^5*(5 + n))
```

3.806.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.806.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(208) = 416$.

Time = 0.56 (sec) , antiderivative size = 1029, normalized size of antiderivative = 4.95

method	result	size
norman	Expression too large to display	1029
gosper	Expression too large to display	1048
risch	Expression too large to display	1438
parallelrisch	Expression too large to display	2213

input
```
int((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x,method=_RETURNVERBOSE)
```

output
```
c*e^3/(5+n)*x^5*exp(n*ln(g*x+f))+f*(a*d^2*g^4*n^4-2*c*d^3*f*g^3*n^3+14*a*d
^2*g^4*n^3-2*a*d*e*f*g^3*n^3-24*c*d^3*f*g^3*n^2+10*c*d^2*e*f^2*g^2*n^2+71*
a*d^2*g^4*n^2-24*a*d*e*f*g^3*n^2+2*a*e^2*f^2*g^2*n^2-94*c*d^3*f*g^3*n+90*c
*d^2*e*f^2*g^2*n-24*c*d*e^2*f^3*g*n+154*a*d^2*g^4*n-94*a*d*e*f*g^3*n+18*a*
e^2*f^2*g^2*n-120*c*d^3*f*g^3+200*c*d^2*e*f^2*g^2-120*c*d*e^2*f^3*g+24*c*e
^3*f^4+120*a*d^2*g^4-120*a*d*e*f*g^3+40*a*e^2*f^2*g^2)/g^5/(n^5+15*n^4+85*
n^3+225*n^2+274*n+120)*exp(n*ln(g*x+f))+(2*c*d^3*g^3*n^3+5*c*d^2*e*f*g^2*n
^3+2*a*d*e*g^3*n^3+a*e^2*f*g^2*n^3+24*c*d^3*g^3*n^2+45*c*d^2*e*f*g^2*n^2-1
2*c*d*e^2*f^2*g*n^2+24*a*d*e*g^3*n^2+9*a*e^2*f*g^2*n^2+94*c*d^3*g^3*n+100*
c*d^2*e*f*g^2*n-60*c*d*e^2*f^2*g*n+12*c*e^3*f^3*n+94*a*d*e*g^3*n+20*a*e^2*
f*g^2*n+120*c*d^3*g^3+120*a*d*e*g^3)/g^3/(n^4+14*n^3+71*n^2+154*n+120)*x^2
*exp(n*ln(g*x+f))+(2*c*d^3*f*g^3*n^4+a*d^2*g^4*n^4+2*a*d*e*f*g^3*n^4+24*c*
d^3*f*g^3*n^3-10*c*d^2*e*f^2*g^2*n^3+14*a*d^2*g^4*n^3+24*a*d*e*f*g^3*n^3-2
*a*e^2*f^2*g^2*n^3+94*c*d^3*f*g^3*n^2-90*c*d^2*e*f^2*g^2*n^2+24*c*d*e^2*f^
3*g*n^2+71*a*d^2*g^4*n^2+94*a*d*e*f*g^3*n^2-18*a*e^2*f^2*g^2*n^2+120*c*d^3
*f*g^3*n-200*c*d^2*e*f^2*g^2*n+120*c*d*e^2*f^3*g*n-24*c*e^3*f^4*n+154*a*d^
2*g^4*n+120*a*d*e*f*g^3*n-40*a*e^2*f^2*g^2*n+120*a*d^2*g^4)/g^4/(n^5+15*n^
4+85*n^3+225*n^2+274*n+120)*x*exp(n*ln(g*x+f))+(5*c*d^2*g^2*n^2+4*c*d*e*f*
g*n^2+a*e*g^2*n^2+45*c*d^2*g^2*n+20*c*d*e*f*g*n-4*c*e^2*f^2*n+9*a*e*g^2*n+
100*c*d^2*g^2+20*a*e*g^2)*e/g^2/(n^3+12*n^2+47*n+60)*x^3*exp(n*ln(g*x+f...
```

3.806.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1122 vs. $2(208) = 416$.

Time = 0.39 (sec) , antiderivative size = 1122, normalized size of antiderivative = 5.39

$$\int (d+ex)^2(f+gx)^n \left(a+2cdx+cex^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

output
```
(a*d^2*f*g^4*n^4 + 24*c*e^3*f^5 - 120*c*d*e^2*f^4*g + 120*a*d^2*f*g^4 + 40
*(5*c*d^2*e + a*e^2)*f^3*g^2 - 120*(c*d^3 + a*d*e)*f^2*g^3 + (c*e^3*g^5*n^
4 + 10*c*e^3*g^5*n^3 + 35*c*e^3*g^5*n^2 + 50*c*e^3*g^5*n + 24*c*e^3*g^5)*x
^5 + (120*c*d*e^2*g^5 + (c*e^3*f*g^4 + 4*c*d*e^2*g^5)*n^4 + 2*(3*c*e^3*f*g
^4 + 22*c*d*e^2*g^5)*n^3 + (11*c*e^3*f*g^4 + 164*c*d*e^2*g^5)*n^2 + 2*(3*c
*e^3*f*g^4 + 122*c*d*e^2*g^5)*n)*x^4 + 2*(7*a*d^2*f*g^4 - (c*d^3 + a*d*e)*
f^2*g^3)*n^3 + (40*(5*c*d^2*e + a*e^2)*g^5 + (4*c*d*e^2*f*g^4 + (5*c*d^2*e
 + a*e^2)*g^5)*n^4 - 4*(c*e^3*f^2*g^3 - 8*c*d*e^2*f*g^4 - 3*(5*c*d^2*e + a
*e^2)*g^5)*n^3 - (12*c*e^3*f^2*g^3 - 68*c*d*e^2*f*g^4 - 49*(5*c*d^2*e + a*
e^2)*g^5)*n^2 - 2*(4*c*e^3*f^2*g^3 - 20*c*d*e^2*f*g^4 - 39*(5*c*d^2*e + a*
e^2)*g^5)*n)*x^3 + (71*a*d^2*f*g^4 + 2*(5*c*d^2*e + a*e^2)*f^3*g^2 - 24*(c
*d^3 + a*d*e)*f^2*g^3)*n^2 + (120*(c*d^3 + a*d*e)*g^5 + ((5*c*d^2*e + a*e^
2)*f*g^4 + 2*(c*d^3 + a*d*e)*g^5)*n^4 - 2*(6*c*d*e^2*f^2*g^3 - 5*(5*c*d^2*
e + a*e^2)*f*g^4 - 13*(c*d^3 + a*d*e)*g^5)*n^3 + (12*c*e^3*f^3*g^2 - 72*c*
d*e^2*f^2*g^3 + 29*(5*c*d^2*e + a*e^2)*f*g^4 + 118*(c*d^3 + a*d*e)*g^5)*n^
2 + 2*(6*c*e^3*f^3*g^2 - 30*c*d*e^2*f^2*g^3 + 10*(5*c*d^2*e + a*e^2)*f*g^4
 + 107*(c*d^3 + a*d*e)*g^5)*n)*x^2 - 2*(12*c*d*e^2*f^4*g - 77*a*d^2*f*g^4
- 9*(5*c*d^2*e + a*e^2)*f^3*g^2 + 47*(c*d^3 + a*d*e)*f^2*g^3)*n + (120*a*d
^2*g^5 + (a*d^2*g^5 + 2*(c*d^3 + a*d*e)*f*g^4)*n^4 + 2*(7*a*d^2*g^5 - (5*c
*d^2*e + a*e^2)*f^2*g^3 + 12*(c*d^3 + a*d*e)*f*g^4)*n^3 + (24*c*d*e^2*f...
```

3.806.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11946 vs. $2(197) = 394$.

Time = 2.21 (sec) , antiderivative size = 11946, normalized size of antiderivative = 57.43

$$\int (d+ex)^2(f+gx)^n \left(a+2cdx+cex^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**2*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

output
```
Piecewise((f**n*(a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + c*d**3*x**2 + 5*c
*d**2*e*x**3/3 + c*d*e**2*x**4 + c*e**3*x**5/5), Eq(g, 0)), (-3*a*d**2*g**
4/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12
*g**9*x**4) - 2*a*d*e*f*g**3/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7
*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 8*a*d*e*g**4*x/(12*f**4*g**5 + 48
*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - a*e**2
*f**2*g**2/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*
x**3 + 12*g**9*x**4) - 4*a*e**2*f*g**3*x/(12*f**4*g**5 + 48*f**3*g**6*x +
72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 6*a*e**2*g**4*x**2/(1
2*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**
9*x**4) - 2*c*d**3*f*g**3/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x*
*2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 8*c*d**3*g**4*x/(12*f**4*g**5 + 48*f
**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 5*c*d**2
*e*f**2*g**2/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**
8*x**3 + 12*g**9*x**4) - 20*c*d**2*e*f*g**3*x/(12*f**4*g**5 + 48*f**3*g**6
*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 30*c*d**2*e*g**4
*x**2/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3
+ 12*g**9*x**4) - 12*c*d*e**2*f**3*g/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f
**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 48*c*d*e**2*f**2*g**2*x/(
12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12...
```

3.806.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(208) = 416$.

Time = 0.23 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.46

$$\int (d+ex)^2(f+gx)^n \left(a+2cdx+cex^2\right) dx = \frac{2\,(g^2(n+1)x^2+fgnx-f^2)(gx+f)^n cd^3}{(n^2+3\,n+2)g^2}$$
$$+ \frac{5\,((n^2+3\,n+2)g^3x^3+(n^2+n)fg^2x^2-2\,f^2gnx+2\,f^3)(gx+f)^n cd^2e}{(n^3+6\,n^2+11\,n+6)g^3}$$
$$+ \frac{2\,(g^2(n+1)x^2+fgnx-f^2)(gx+f)^n ade}{(n^2+3\,n+2)g^2} + \frac{(gx+f)^{n+1}ad^2}{g(n+1)}$$
$$+ \frac{4\,((n^3+6\,n^2+11\,n+6)g^4x^4+(n^3+3\,n^2+2\,n)fg^3x^3-3\,(n^2+n)f^2g^2x^2+6\,f^3gnx-6\,f^4)(gx+f)^{n}}{(n^4+10\,n^3+35\,n^2+50\,n+24)g^4}$$
$$+ \frac{((n^2+3\,n+2)g^3x^3+(n^2+n)fg^2x^2-2\,f^2gnx+2\,f^3)(gx+f)^n ae^2}{(n^3+6\,n^2+11\,n+6)g^3}$$
$$+ \frac{((n^4+10\,n^3+35\,n^2+50\,n+24)g^5x^5+(n^4+6\,n^3+11\,n^2+6\,n)fg^4x^4-4\,(n^3+3\,n^2+2\,n)f^2g^3x^3+}{(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)g^5}$$

input
```
integrate((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

output
```
2*(g^2*(n + 1)*x^2 + f*g*n*x - f^2)*(g*x + f)^n*c*d^3/((n^2 + 3*n + 2)*g^2
) + 5*((n^2 + 3*n + 2)*g^3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3
)*(g*x + f)^n*c*d^2*e/((n^3 + 6*n^2 + 11*n + 6)*g^3) + 2*(g^2*(n + 1)*x^2
+ f*g*n*x - f^2)*(g*x + f)^n*a*d*e/((n^2 + 3*n + 2)*g^2) + (g*x + f)^(n +
1)*a*d^2/(g*(n + 1)) + 4*((n^3 + 6*n^2 + 11*n + 6)*g^4*x^4 + (n^3 + 3*n^2
+ 2*n)*f*g^3*x^3 - 3*(n^2 + n)*f^2*g^2*x^2 + 6*f^3*g*n*x - 6*f^4)*(g*x + f
)^n*c*d*e^2/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*g^4) + ((n^2 + 3*n + 2)*g
^3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*(g*x + f)^n*a*e^2/((n^
3 + 6*n^2 + 11*n + 6)*g^3) + ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*g^5*x^5
+ (n^4 + 6*n^3 + 11*n^2 + 6*n)*f*g^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*f^2*g^3*x
^3 + 12*(n^2 + n)*f^3*g^2*x^2 - 24*f^4*g*n*x + 24*f^5)*(g*x + f)^n*c*e^3/(
(n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*g^5)
```

3.806.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2133 vs. $2(208) = 416$.

Time = 0.31 (sec) , antiderivative size = 2133, normalized size of antiderivative = 10.25

$$\int (d+ex)^2(f+gx)^n\left(a+2cdx+cex^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

output
```
((g*x + f)^n*c*e^3*g^5*n^4*x^5 + (g*x + f)^n*c*e^3*f*g^4*n^4*x^4 + 4*(g*x
+ f)^n*c*d*e^2*g^5*n^4*x^4 + 10*(g*x + f)^n*c*e^3*g^5*n^3*x^5 + 4*(g*x + f
)^n*c*d*e^2*f*g^4*n^4*x^3 + 5*(g*x + f)^n*c*d^2*e*g^5*n^4*x^3 + 6*(g*x + f
)^n*c*e^3*f*g^4*n^3*x^4 + 44*(g*x + f)^n*c*d*e^2*g^5*n^3*x^4 + 35*(g*x + f
)^n*c*e^3*g^5*n^2*x^5 + 5*(g*x + f)^n*c*d^2*e*f*g^4*n^4*x^2 + 2*(g*x + f)^
n*c*d^3*g^5*n^4*x^2 - 4*(g*x + f)^n*c*e^3*f^2*g^3*n^3*x^3 + 32*(g*x + f)^n
*c*d*e^2*f*g^4*n^3*x^3 + 60*(g*x + f)^n*c*d^2*e*g^5*n^3*x^3 + (g*x + f)^n*
a*e^2*g^5*n^4*x^3 + 11*(g*x + f)^n*c*e^3*f*g^4*n^2*x^4 + 164*(g*x + f)^n*c
*d*e^2*g^5*n^2*x^4 + 50*(g*x + f)^n*c*e^3*g^5*n*x^5 + 2*(g*x + f)^n*c*d^3*
f*g^4*n^4*x - 12*(g*x + f)^n*c*d*e^2*f^2*g^3*n^3*x^2 + 50*(g*x + f)^n*c*d^
2*e*f*g^4*n^3*x^2 + 26*(g*x + f)^n*c*d^3*g^5*n^3*x^2 + (g*x + f)^n*a*e^2*f
*g^4*n^4*x^2 + 2*(g*x + f)^n*a*d*e*g^5*n^4*x^2 - 12*(g*x + f)^n*c*e^3*f^2*
g^3*n^2*x^3 + 68*(g*x + f)^n*c*d*e^2*f*g^4*n^2*x^3 + 245*(g*x + f)^n*c*d^2
*e*g^5*n^2*x^3 + 12*(g*x + f)^n*a*e^2*g^5*n^3*x^3 + 6*(g*x + f)^n*c*e^3*f*
g^4*n*x^4 + 244*(g*x + f)^n*c*d*e^2*g^5*n*x^4 + 24*(g*x + f)^n*c*e^3*g^5*x
^5 - 10*(g*x + f)^n*c*d^2*e*f^2*g^3*n^3*x + 24*(g*x + f)^n*c*d^3*f*g^4*n^3
*x + 2*(g*x + f)^n*a*d*e*f*g^4*n^4*x + (g*x + f)^n*a*d^2*g^5*n^4*x + 12*(g
*x + f)^n*c*e^3*f^3*g^2*n^2*x^2 - 72*(g*x + f)^n*c*d*e^2*f^2*g^3*n^2*x^2 +
 145*(g*x + f)^n*c*d^2*e*f*g^4*n^2*x^2 + 118*(g*x + f)^n*c*d^3*g^5*n^2*x^2
 + 10*(g*x + f)^n*a*e^2*f*g^4*n^3*x^2 + 26*(g*x + f)^n*a*d*e*g^5*n^3*x^...
```

3.806.9 Mupad [B] (verification not implemented)

Time = 12.35 (sec) , antiderivative size = 1133, normalized size of antiderivative = 5.45

$$\int (d+ex)^2(f+gx)^n\left(a+2cdx+cex^2\right)\,dx$$
$$= \frac{(f+g\,x)^n\,(-2\,c\,d^3\,f^2\,g^3\,n^3-24\,c\,d^3\,f^2\,g^3\,n^2-94\,c\,d^3\,f^2\,g^3\,n-120\,c\,d^3\,f^2\,g^3+10\,c\,d^2\,e\,f^3\,g^2\,n^2+90\,c\,d^2}{}$$
$$+\frac{x\,(f+g\,x)^n\,(2\,c\,d^3\,f\,g^4\,n^4+24\,c\,d^3\,f\,g^4\,n^3+94\,c\,d^3\,f\,g^4\,n^2+120\,c\,d^3\,f\,g^4\,n-10\,c\,d^2\,e\,f^2\,g^3\,n^3-90\,c\,d^2}{}$$
$$+\frac{c\,e^3\,x^5\,(f+g\,x)^n\,(n^4+10\,n^3+35\,n^2+50\,n+24)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120}$$
$$+\frac{x^2\,(f+g\,x)^n\,(n+1)\,(2\,c\,d^3\,g^3\,n^3+24\,c\,d^3\,g^3\,n^2+94\,c\,d^3\,g^3\,n+120\,c\,d^3\,g^3+5\,c\,d^2\,e\,f\,g^2\,n^3+45\,c\,d^2\,e\,f}{}$$
$$+\frac{e\,x^3\,(f+g\,x)^n\,(n^2+3\,n+2)\,(5\,c\,d^2\,g^2\,n^2+45\,c\,d^2\,g^2\,n+100\,c\,d^2\,g^2+4\,c\,d\,e\,f\,g\,n^2+20\,c\,d\,e\,f\,g\,n-4\,c}{g^2\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}$$
$$+\frac{c\,e^2\,x^4\,(f+g\,x)^n\,(20\,d\,g+4\,d\,g\,n+e\,f\,n)\,(n^3+6\,n^2+11\,n+6)}{g\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}$$

input
```
int((f + g*x)^n*(d + e*x)^2*(a + 2*c*d*x + c*e*x^2),x)
```

output
```
((f + g*x)^n*(24*c*e^3*f^5 + 40*a*e^2*f^3*g^2 - 120*c*d^3*f^2*g^3 + 120*a*
d^2*f*g^4 - 120*a*d*e*f^2*g^3 - 120*c*d*e^2*f^4*g + 154*a*d^2*f*g^4*n + 20
0*c*d^2*e*f^3*g^2 + 71*a*d^2*f*g^4*n^2 + 14*a*d^2*f*g^4*n^3 + a*d^2*f*g^4*
n^4 + 18*a*e^2*f^3*g^2*n - 94*c*d^3*f^2*g^3*n + 2*a*e^2*f^3*g^2*n^2 - 24*c
*d^3*f^2*g^3*n^2 - 2*c*d^3*f^2*g^3*n^3 + 10*c*d^2*e*f^3*g^2*n^2 - 94*a*d*e
*f^2*g^3*n - 24*c*d*e^2*f^4*g*n - 24*a*d*e*f^2*g^3*n^2 - 2*a*d*e*f^2*g^3*n
^3 + 90*c*d^2*e*f^3*g^2*n))/(g^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5
+ 120)) + (x*(f + g*x)^n*(120*a*d^2*g^5 + 71*a*d^2*g^5*n^2 + 14*a*d^2*g^5*
n^3 + a*d^2*g^5*n^4 + 154*a*d^2*g^5*n + 120*c*d^3*f*g^4*n - 24*c*e^3*f^4*g
*n - 40*a*e^2*f^2*g^3*n + 94*c*d^3*f*g^4*n^2 + 24*c*d^3*f*g^4*n^3 + 2*c*d^
3*f*g^4*n^4 - 18*a*e^2*f^2*g^3*n^2 - 2*a*e^2*f^2*g^3*n^3 + 120*a*d*e*f*g^4
*n + 24*c*d*e^2*f^3*g^2*n^2 - 90*c*d^2*e*f^2*g^3*n^2 - 10*c*d^2*e*f^2*g^3*
n^3 + 94*a*d*e*f*g^4*n^2 + 24*a*d*e*f*g^4*n^3 + 2*a*d*e*f*g^4*n^4 + 120*c*
d*e^2*f^3*g^2*n - 200*c*d^2*e*f^2*g^3*n))/(g^5*(274*n + 225*n^2 + 85*n^3 +
 15*n^4 + n^5 + 120)) + (c*e^3*x^5*(f + g*x)^n*(50*n + 35*n^2 + 10*n^3 + n
^4 + 24))/(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) + (x^2*(f + g*x)
^n*(n + 1)*(120*c*d^3*g^3 + 24*c*d^3*g^3*n^2 + 2*c*d^3*g^3*n^3 + 120*a*d*e
*g^3 + 94*c*d^3*g^3*n + 12*c*e^3*f^3*n + 24*a*d*e*g^3*n^2 + 2*a*d*e*g^3*n^
3 + 20*a*e^2*f*g^2*n + 9*a*e^2*f*g^2*n^2 + a*e^2*f*g^2*n^3 + 94*a*d*e*g^3*
n - 60*c*d*e^2*f^2*g*n + 100*c*d^2*e*f*g^2*n - 12*c*d*e^2*f^2*g*n^2 + 4...
```

3.806. $\int (d+ex)^2(f+gx)^n \left(a+2cdx+cex^2\right) dx$

3.807 $\int (d+ex)(f+gx)^n \left(a+2cdx+cex^2\right) dx$

3.807.1 Optimal result

Integrand size = 26, antiderivative size = 146

$$\begin{aligned}\int (d+ex)(f+gx)^n \left(a+2cdx+cex^2\right) dx &= -\frac{(ef-dg)\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^4(1+n)} \\ &\quad + \frac{\left(aeg^2+c\left(3e^2f^2-6defg+2d^2g^2\right)\right)(f+gx)^{2+n}}{g^4(2+n)} \\ &\quad - \frac{3ce(ef-dg)(f+gx)^{3+n}}{g^4(3+n)} + \frac{ce^2(f+gx)^{4+n}}{g^4(4+n)}\end{aligned}$$

output
```
-(-d*g+e*f)*(a*g^2+c*f*(-2*d*g+e*f))*(g*x+f)^(1+n)/g^4/(1+n)+(a*e*g^2+c*(2
*d^2*g^2-6*d*e*f*g+3*e^2*f^2))*(g*x+f)^(2+n)/g^4/(2+n)-3*c*e*(-d*g+e*f)*(g
*x+f)^(3+n)/g^4/(3+n)+c*e^2*(g*x+f)^(4+n)/g^4/(4+n)
```

3.807.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.89

$$\int (d+ex)(f+gx)^n \left(a+2cdx+cex^2\right) dx = \frac{(f+gx)^{1+n}\left(-\frac{(ef-dg)\left(ag^2+cf(ef-2dg)\right)}{1+n}+\frac{\left(aeg^2+c\left(3e^2f^2-6defg+2d^2g^2\right)\right)(f+gx)}{2+n}-\frac{3ce(ef-dg)(f+gx)^2}{3+n}+\frac{ce^2(f+gx)^3}{4+n}\right)}{g^4}$$

input
```
Integrate[(d + e*x)*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

output
```
((f + g*x)^(1 + n)*(-(((e*f - d*g)*(a*g^2 + c*f*(e*f - 2*d*g)))/(1 + n)) +
 ((a*e*g^2 + c*(3*e^2*f^2 - 6*d*e*f*g + 2*d^2*g^2))*(f + g*x))/(2 + n) - (
3*c*e*(e*f - d*g)*(f + g*x)^2)/(3 + n) + (c*e^2*(f + g*x)^3)/(4 + n)))/g^4
```

3.807.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)(f+gx)^n \left(a+2cdx+cex^2\right) dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{(f+gx)^{n+1} \left(aeg^2+c\left(2d^2g^2-6defg+3e^2f^2\right)\right)}{g^3}+\frac{(ef-dg)(f+gx)^n \left(-ag^2-cf(ef-2dg)\right)}{g^3}-\frac{3ce(ef-$$

$$\Big\downarrow \text{2009}$$

$$\frac{(f+gx)^{n+2} \left(aeg^2+c\left(2d^2g^2-6defg+3e^2f^2\right)\right)}{g^4(n+2)}-\frac{(ef-dg)(f+gx)^{n+1} \left(ag^2+cf(ef-2dg)\right)}{g^4(n+1)}-\frac{3ce(ef-dg)(f+gx)^{n+3}}{g^4(n+3)}+\frac{ce^2(f+gx)^{n+4}}{g^4(n+4)}$$

input
```
Int[(d + e*x)*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

output
```
-(((e*f - d*g)*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^4*(1 + n)
)) + ((a*e*g^2 + c*(3*e^2*f^2 - 6*d*e*f*g + 2*d^2*g^2))*(f + g*x)^(2 + n))
/(g^4*(2 + n)) - (3*c*e*(e*f - d*g)*(f + g*x)^(3 + n))/(g^4*(3 + n)) + (c*
e^2*(f + g*x)^(4 + n))/(g^4*(4 + n))
```

3.807.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.807.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(146) = 292$.

Time = 0.50 (sec) , antiderivative size = 449, normalized size of antiderivative = 3.08

method	result
gosper	$\frac{(gx+f)^{1+n}(c e^2 g^3 n^3 x^3+3cde\, g^3 n^3 x^2+6c e^2 g^3 n^2 x^3+2c d^2 g^3 n^3 x+21cde\, g^3 n^2 x^2-3c e^2 f\, g^2 n^2 x^2+11c e^2 g^3 n\, x^3+ae\, g^3 n^3 x+16$
norman	$\frac{c e^2 x^4 \mathrm{e}^{n \ln(gx+f)}}{4+n}+\frac{f(ad\, g^3 n^3-2c d^2 f\, g^2 n^2+9ad\, g^3 n^2-aef\, g^2 n^2-14c d^2 f\, g^2 n+6cde\, f^2 gn+26ad\, g^3 n-7aef\, g^2 n-24c d^2 f\, g}{g^4(n^4+10n^3+35n^2+50n+24)}$
risch	$\frac{(c e^2 g^4 n^3 x^4+3cde\, g^4 n^3 x^3+c e^2 f\, g^3 n^3 x^3+6c e^2 g^4 n^2 x^4+2c d^2 g^4 n^3 x^2+3cdef\, g^3 n^3 x^2+21cde\, g^4 n^2 x^3+3c e^2 f\, g^3 n^2 x^3+11c e^2 g^4}{}$
parallelrisch	Expression too large to display

input
```
int((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/g^4*(g*x+f)^(1+n)/(n^4+10*n^3+35*n^2+50*n+24)*(c*e^2*g^3*n^3*x^3+3*c*d*e
*g^3*n^3*x^2+6*c*e^2*g^3*n^2*x^3+2*c*d^2*g^3*n^3*x+21*c*d*e*g^3*n^2*x^2-3*
c*e^2*f*g^2*n^2*x^2+11*c*e^2*g^3*n*x^3+a*e*g^3*n^3*x+16*c*d^2*g^3*n^2*x-6*
c*d*e*f*g^2*n^2*x+42*c*d*e*g^3*n*x^2-9*c*e^2*f*g^2*n*x^2+6*c*e^2*g^3*x^3+a
*d*g^3*n^3+8*a*e*g^3*n^2*x-2*c*d^2*f*g^2*n^2+38*c*d^2*g^3*n*x-30*c*d*e*f*g
^2*n*x+24*c*d*e*g^3*x^2+6*c*e^2*f^2*g*n*x-6*c*e^2*f*g^2*x^2+9*a*d*g^3*n^2-
a*e*f*g^2*n^2+19*a*e*g^3*n*x-14*c*d^2*f*g^2*n+24*c*d^2*g^3*x+6*c*d*e*f^2*g
*n-24*c*d*e*f*g^2*x+6*c*e^2*f^2*g*x+26*a*d*g^3*n-7*a*e*f*g^2*n+12*a*e*g^3*
x-24*c*d^2*f*g^2+24*c*d*e*f^2*g-6*c*e^2*f^3+24*a*d*g^3-12*a*e*f*g^2)
```

3.807.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(146) = 292$.

Time = 0.33 (sec) , antiderivative size = 549, normalized size of antiderivative = 3.76

$$\int (d+ex)(f+gx)^n \left(a+2cdx+cex^2\right) \, dx$$
$$= \frac{(adfg^3n^3 - 6\,ce^2f^4 + 24\,cdef^3g + 24\,adfg^3 - 12\,(2\,cd^2 + ae)f^2g^2 + (ce^2g^4n^3 + 6\,ce^2g^4n^2 + 11\,ce^2g^4n + 6}{}$$

input
```
integrate((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

output
```
(a*d*f*g^3*n^3 - 6*c*e^2*f^4 + 24*c*d*e*f^3*g + 24*a*d*f*g^3 - 12*(2*c*d^2
 + a*e)*f^2*g^2 + (c*e^2*g^4*n^3 + 6*c*e^2*g^4*n^2 + 11*c*e^2*g^4*n + 6*c*
e^2*g^4)*x^4 + (24*c*d*e*g^4 + (c*e^2*f*g^3 + 3*c*d*e*g^4)*n^3 + 3*(c*e^2*
f*g^3 + 7*c*d*e*g^4)*n^2 + 2*(c*e^2*f*g^3 + 21*c*d*e*g^4)*n)*x^3 + (9*a*d*
f*g^3 - (2*c*d^2 + a*e)*f^2*g^2)*n^2 + (12*(2*c*d^2 + a*e)*g^4 + (3*c*d*e*
f*g^3 + (2*c*d^2 + a*e)*g^4)*n^3 - (3*c*e^2*f^2*g^2 - 15*c*d*e*f*g^3 - 8*(
2*c*d^2 + a*e)*g^4)*n^2 - (3*c*e^2*f^2*g^2 - 12*c*d*e*f*g^3 - 19*(2*c*d^2
+ a*e)*g^4)*n)*x^2 + (6*c*d*e*f^3*g + 26*a*d*f*g^3 - 7*(2*c*d^2 + a*e)*f^2
*g^2)*n + (24*a*d*g^4 + (a*d*g^4 + (2*c*d^2 + a*e)*f*g^3)*n^3 - (6*c*d*e*f
^2*g^2 - 9*a*d*g^4 - 7*(2*c*d^2 + a*e)*f*g^3)*n^2 + 2*(3*c*e^2*f^3*g - 12*
c*d*e*f^2*g^2 + 13*a*d*g^4 + 6*(2*c*d^2 + a*e)*f*g^3)*n)*x)*(g*x + f)^n/(g
^4*n^4 + 10*g^4*n^3 + 35*g^4*n^2 + 50*g^4*n + 24*g^4)
```

3.807.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4952 vs. $2(134) = 268$.

Time = 1.16 (sec) , antiderivative size = 4952, normalized size of antiderivative = 33.92

$$\int (d+ex)(f+gx)^n \left(a+2cdx+cex^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

output
```
Piecewise((f**n*(a*d*x + a*e*x**2/2 + c*d**2*x**2 + c*d*e*x**3 + c*e**2*x*
*4/4), Eq(g, 0)), (-2*a*d*g**3/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x
**2 + 6*g**7*x**3) - a*e*f*g**2/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*
x**2 + 6*g**7*x**3) - 3*a*e*g**3*x/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g*
*6*x**2 + 6*g**7*x**3) - 2*c*d**2*f*g**2/(6*f**3*g**4 + 18*f**2*g**5*x + 1
8*f*g**6*x**2 + 6*g**7*x**3) - 6*c*d**2*g**3*x/(6*f**3*g**4 + 18*f**2*g**5
*x + 18*f*g**6*x**2 + 6*g**7*x**3) - 6*c*d*e*f**2*g/(6*f**3*g**4 + 18*f**2
*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) - 18*c*d*e*f*g**2*x/(6*f**3*g**4 +
 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) - 18*c*d*e*g**3*x**2/(6*f*
*3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 6*c*e**2*f**3*l
og(f/g + x)/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3)
+ 11*c*e**2*f**3/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x
**3) + 18*c*e**2*f**2*g*x*log(f/g + x)/(6*f**3*g**4 + 18*f**2*g**5*x + 18*
f*g**6*x**2 + 6*g**7*x**3) + 27*c*e**2*f**2*g*x/(6*f**3*g**4 + 18*f**2*g**
5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 18*c*e**2*f*g**2*x**2*log(f/g + x)/(
6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 18*c*e**2*f
*g**2*x**2/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) +
 6*c*e**2*g**3*x**3*log(f/g + x)/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6
*x**2 + 6*g**7*x**3), Eq(n, -4)), (-a*d*g**3/(2*f**2*g**4 + 4*f*g**5*x + 2
*g**6*x**2) - a*e*f*g**2/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) - 2*a...
```

3.807.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.98

$$\int (d+ex)(f+gx)^n \left(a+2cdx+cex^2\right)\,dx = \frac{2\left(g^2(n+1)x^2+fgnx-f^2\right)(gx+f)^n cd^2}{(n^2+3n+2)g^2} + \frac{3\left((n^2+3n+2)g^3x^3+(n^2+n)fg^2x^2-2f^2gnx+2f^3\right)(gx+f)^n cde}{(n^3+6n^2+11n+6)g^3} + \frac{\left(g^2(n+1)x^2+fgnx-f^2\right)(gx+f)^n ae}{(n^2+3n+2)g^2} + \frac{(gx+f)^{n+1}ad}{g(n+1)} + \frac{\left((n^3+6n^2+11n+6)g^4x^4+(n^3+3n^2+2n)fg^3x^3-3(n^2+n)f^2g^2x^2+6f^3gnx-6f^4\right)(gx+f)^n c}{(n^4+10n^3+35n^2+50n+24)g^4}$$

input
```
integrate((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

output
```
2*(g^2*(n + 1)*x^2 + f*g*n*x - f^2)*(g*x + f)^n*c*d^2/((n^2 + 3*n + 2)*g^2
) + 3*((n^2 + 3*n + 2)*g^3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3
)*(g*x + f)^n*c*d*e/((n^3 + 6*n^2 + 11*n + 6)*g^3) + (g^2*(n + 1)*x^2 + f*
g*n*x - f^2)*(g*x + f)^n*a*e/((n^2 + 3*n + 2)*g^2) + (g*x + f)^(n + 1)*a*d
/(g*(n + 1)) + ((n^3 + 6*n^2 + 11*n + 6)*g^4*x^4 + (n^3 + 3*n^2 + 2*n)*f*g
^3*x^3 - 3*(n^2 + n)*f^2*g^2*x^2 + 6*f^3*g*n*x - 6*f^4)*(g*x + f)^n*c*e^2/
((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*g^4)
```

3.807.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1008 vs. $2(146) = 292$.

Time = 0.28 (sec) , antiderivative size = 1008, normalized size of antiderivative = 6.90

$$\int (d+ex)(f+gx)^n\left(a+2cdx+cex^2\right)\,dx$$
$$= \frac{(gx+f)^n ce^2g^4n^3x^4 + (gx+f)^n ce^2fg^3n^3x^3 + 3\,(gx+f)^n cdeg^4n^3x^3 + 6\,(gx+f)^n ce^2g^4n^2x^4 + 3\,(gx+f}{}$$

input
```
integrate((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

output
```
((g*x + f)^n*c*e^2*g^4*n^3*x^4 + (g*x + f)^n*c*e^2*f*g^3*n^3*x^3 + 3*(g*x
+ f)^n*c*d*e*g^4*n^3*x^3 + 6*(g*x + f)^n*c*e^2*g^4*n^2*x^4 + 3*(g*x + f)^n
*c*d*e*f*g^3*n^3*x^2 + 2*(g*x + f)^n*c*d^2*g^4*n^3*x^2 + 3*(g*x + f)^n*c*e
^2*f*g^3*n^2*x^3 + 21*(g*x + f)^n*c*d*e*g^4*n^2*x^3 + 11*(g*x + f)^n*c*e^2
*g^4*n*x^4 + 2*(g*x + f)^n*c*d^2*f*g^3*n^3*x - 3*(g*x + f)^n*c*e^2*f^2*g^2
*n^2*x^2 + 15*(g*x + f)^n*c*d*e*f*g^3*n^2*x^2 + 16*(g*x + f)^n*c*d^2*g^4*n
^2*x^2 + (g*x + f)^n*a*e*g^4*n^3*x^2 + 2*(g*x + f)^n*c*e^2*f*g^3*n*x^3 + 4
2*(g*x + f)^n*c*d*e*g^4*n*x^3 + 6*(g*x + f)^n*c*e^2*g^4*x^4 - 6*(g*x + f)^
n*c*d*e*f^2*g^2*n^2*x + 14*(g*x + f)^n*c*d^2*f*g^3*n^2*x + (g*x + f)^n*a*e
*f*g^3*n^3*x + (g*x + f)^n*a*d*g^4*n^3*x - 3*(g*x + f)^n*c*e^2*f^2*g^2*n*x
^2 + 12*(g*x + f)^n*c*d*e*f*g^3*n*x^2 + 38*(g*x + f)^n*c*d^2*g^4*n*x^2 + 8
*(g*x + f)^n*a*e*g^4*n^2*x^2 + 24*(g*x + f)^n*c*d*e*g^4*x^3 - 2*(g*x + f)^
n*c*d^2*f^2*g^2*n^2 + (g*x + f)^n*a*d*f*g^3*n^3 + 6*(g*x + f)^n*c*e^2*f^3*
g*n*x - 24*(g*x + f)^n*c*d*e*f^2*g^2*n*x + 24*(g*x + f)^n*c*d^2*f*g^3*n*x
+ 7*(g*x + f)^n*a*e*f*g^3*n^2*x + 9*(g*x + f)^n*a*d*g^4*n^2*x + 24*(g*x +
f)^n*c*d^2*g^4*x^2 + 19*(g*x + f)^n*a*e*g^4*n*x^2 + 6*(g*x + f)^n*c*d*e*f^
3*g*n - 14*(g*x + f)^n*c*d^2*f^2*g^2*n - (g*x + f)^n*a*e*f^2*g^2*n^2 + 9*(
g*x + f)^n*a*d*f*g^3*n^2 + 12*(g*x + f)^n*a*e*f*g^3*n*x + 26*(g*x + f)^n*a
*d*g^4*n*x + 12*(g*x + f)^n*a*e*g^4*x^2 - 6*(g*x + f)^n*c*e^2*f^4 + 24*(g*
x + f)^n*c*d*e*f^3*g - 24*(g*x + f)^n*c*d^2*f^2*g^2 - 7*(g*x + f)^n*a*e...
```

3.807.9 Mupad [B] (verification not implemented)

Time = 11.96 (sec) , antiderivative size = 572, normalized size of antiderivative = 3.92

$$\int (d+ex)(f+gx)^n\left(a+2cdx+cex^2\right)\,dx$$
$$= \frac{x\,(f+g\,x)^n\,(2\,c\,d^2\,f\,g^3\,n^3+14\,c\,d^2\,f\,g^3\,n^2+24\,c\,d^2\,f\,g^3\,n-6\,c\,d\,e\,f^2\,g^2\,n^2-24\,c\,d\,e\,f^2\,g^2\,n+a\,d\,g^4\,n^3+\cdots}{g^4\,(n^4+10\,n^3+35\,n^2+\cdots}$$
$$-\frac{(f+g\,x)^n\,(2\,c\,d^2\,f^2\,g^2\,n^2+14\,c\,d^2\,f^2\,g^2\,n+24\,c\,d^2\,f^2\,g^2-6\,c\,d\,e\,f^3\,g\,n-24\,c\,d\,e\,f^3\,g-a\,d\,f\,g^3\,n^3-9\cdots}{g^4\,(n^4+10\,n^3+35\,n^2\cdots}$$
$$+\frac{c\,e^2\,x^4\,(f+g\,x)^n\,(n^3+6\,n^2+11\,n+6)}{n^4+10\,n^3+35\,n^2+50\,n+24}$$
$$+\frac{x^2\,(f+g\,x)^n\,(n+1)\,(2\,c\,d^2\,g^2\,n^2+14\,c\,d^2\,g^2\,n+24\,c\,d^2\,g^2+3\,c\,d\,e\,f\,g\,n^2+12\,c\,d\,e\,f\,g\,n-3\,c\,e^2\,f^2\,n+\cdots}{g^2\,(n^4+10\,n^3+35\,n^2+50\,n+24)}$$
$$+\frac{c\,e\,x^3\,(f+g\,x)^n\,(12\,d\,g+3\,d\,g\,n+e\,f\,n)\,(n^2+3\,n+2)}{g\,(n^4+10\,n^3+35\,n^2+50\,n+24)}$$

input
```
int((f + g*x)^n*(d + e*x)*(a + 2*c*d*x + c*e*x^2),x)
```

output
```
(x*(f + g*x)^n*(24*a*d*g^4 + 26*a*d*g^4*n + 9*a*d*g^4*n^2 + a*d*g^4*n^3 +
7*a*e*f*g^3*n^2 + a*e*f*g^3*n^3 + 24*c*d^2*f*g^3*n + 6*c*e^2*f^3*g*n + 14*
c*d^2*f*g^3*n^2 + 2*c*d^2*f*g^3*n^3 + 12*a*e*f*g^3*n - 24*c*d*e*f^2*g^2*n
- 6*c*d*e*f^2*g^2*n^2))/(g^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - ((f +
g*x)^n*(6*c*e^2*f^4 + 24*c*d^2*f^2*g^2 - 24*a*d*f*g^3 + 12*a*e*f^2*g^2 - 9
*a*d*f*g^3*n^2 - a*d*f*g^3*n^3 + 7*a*e*f^2*g^2*n + a*e*f^2*g^2*n^2 + 14*c*
d^2*f^2*g^2*n - 24*c*d*e*f^3*g - 26*a*d*f*g^3*n + 2*c*d^2*f^2*g^2*n^2 - 6*
c*d*e*f^3*g*n))/(g^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (c*e^2*x^4*(f
+ g*x)^n*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4 + 24) + (
x^2*(f + g*x)^n*(n + 1)*(24*c*d^2*g^2 + 12*a*e*g^2 + 2*c*d^2*g^2*n^2 + 7*a
*e*g^2*n + a*e*g^2*n^2 + 14*c*d^2*g^2*n - 3*c*e^2*f^2*n + 3*c*d*e*f*g*n^2
+ 12*c*d*e*f*g*n))/(g^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (c*e*x^3*(f
 + g*x)^n*(12*d*g + 3*d*g*n + e*f*n)*(3*n + n^2 + 2))/(g*(50*n + 35*n^2 +
10*n^3 + n^4 + 24))
```

3.808 $\int (f+gx)^n \left(a+2cdx+cex^2\right) dx$

3.808.1 Optimal result

Integrand size = 21, antiderivative size = 84

$$\int (f+gx)^n \left(a+2cdx+cex^2\right) dx = \frac{\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^3(1+n)} - \frac{2c(ef-dg)(f+gx)^{2+n}}{g^3(2+n)} + \frac{ce(f+gx)^{3+n}}{g^3(3+n)}$$

output
```
(a*g^2+c*f*(-2*d*g+e*f))*(g*x+f)^(1+n)/g^3/(1+n)-2*c*(-d*g+e*f)*(g*x+f)^(2
+n)/g^3/(2+n)+c*e*(g*x+f)^(3+n)/g^3/(3+n)
```

3.808.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int (f+gx)^n \left(a+2cdx+cex^2\right) dx = \frac{(f+gx)^{1+n}\left(\frac{ag^2+cf(ef-2dg)}{1+n} - \frac{2c(ef-dg)(f+gx)}{2+n} + \frac{ce(f+gx)^2}{3+n}\right)}{g^3}$$

input
```
Integrate[(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

output
```
((f + g*x)^(1 + n)*((a*g^2 + c*f*(e*f - 2*d*g))/(1 + n) - (2*c*(e*f - d*g)
*(f + g*x))/(2 + n) + (c*e*(f + g*x)^2)/(3 + n)))/g^3
```

3.808.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^n \left(a+2cdx+cex^2\right) \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{(f+gx)^n \left(ag^2+cf(ef-2dg)\right)}{g^2}+\frac{2c(dg-ef)(f+gx)^{n+1}}{g^2}+\frac{ce(f+gx)^{n+2}}{g^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(f+gx)^{n+1} \left(ag^2+cf(ef-2dg)\right)}{g^3(n+1)}-\frac{2c(ef-dg)(f+gx)^{n+2}}{g^3(n+2)}+\frac{ce(f+gx)^{n+3}}{g^3(n+3)}$$

input
```
Int[(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

output
```
((a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^3*(1 + n)) - (2*c*(e*f
- d*g)*(f + g*x)^(2 + n))/(g^3*(2 + n)) + (c*e*(f + g*x)^(3 + n))/(g^3*(3
+ n))
```

3.808.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.808.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.75

method	result
gosper	$\frac{(gx+f)^{1+n}(ce\,g^2n^2x^2+2cd\,g^2n^2x+3ce\,g^2n\,x^2+8cd\,g^2nx-2cef\,gnx+2e\,x^2c\,g^2+a\,g^2n^2-2cdf\,gn+6cd\,g^2x-2cef\,gx+5a\,g^2n-6}{g^3(n^3+6n^2+11n+6)}$
norman	$\frac{ce\,x^3\mathrm{e}^{n\ln(gx+f)}}{3+n}+\frac{f(a\,g^2n^2-2cdf\,gn+5a\,g^2n-6cdf\,g+2ce\,f^2+6a\,g^2)\mathrm{e}^{n\ln(gx+f)}}{g^3(n^3+6n^2+11n+6)}+\frac{(2cdf\,g\,n^2+a\,g^2n^2+6cdf\,gn-2ce\,f^2n+5}{g^2(n^3+6n^2+11n}$
risch	$\frac{(ce\,g^3n^2x^3+2cd\,g^3n^2x^2+cef\,g^2n^2x^2+3ce\,g^3n\,x^3+2cdf\,g^2n^2x+8cd\,g^3n\,x^2+cef\,g^2n\,x^2+2ce\,x^3g^3+a\,g^3n^2x+6cdf\,g^2nx+6cd\,g}{(2+n)(3+n)(1+n)g^3}$
parallelrisch	$x^3(gx+f)^n cef\,g^3n^2+3x^3(gx+f)^n cef\,g^3n+2x^2(gx+f)^n cdf\,g^3n^2+x^2(gx+f)^n ce\,f^2g^2n^2+2x^3(gx+f)^n cef\,g^3+8x^2(gx+f)^n c$

input
```
int((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/g^3*(g*x+f)^(1+n)/(n^3+6*n^2+11*n+6)*(c*e*g^2*n^2*x^2+2*c*d*g^2*n^2*x+3*
c*e*g^2*n*x^2+8*c*d*g^2*n*x-2*c*e*f*g*n*x+2*c*e*g^2*x^2+a*g^2*n^2-2*c*d*f*
g*n+6*c*d*g^2*x-2*c*e*f*g*x+5*a*g^2*n-6*c*d*f*g+2*c*e*f^2+6*a*g^2)
```

3.808.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(84) = 168.

Time = 0.32 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.60

$$\int (f+gx)^n\left(a+2cdx+cex^2\right)\,dx$$
$$=\frac{(afg^2n^2+2\,cef^3-6\,cdf^2g+6\,afg^2+(ceg^3n^2+3\,ceg^3n+2\,ceg^3)x^3+(6\,cdg^3+(cefg^2+2\,cdg^3)n^2+(c}{g^3n^3+6\,g}$$

input
```
integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

output
```
(a*f*g^2*n^2 + 2*c*e*f^3 - 6*c*d*f^2*g + 6*a*f*g^2 + (c*e*g^3*n^2 + 3*c*e*
g^3*n + 2*c*e*g^3)*x^3 + (6*c*d*g^3 + (c*e*f*g^2 + 2*c*d*g^3)*n^2 + (c*e*f
*g^2 + 8*c*d*g^3)*n)*x^2 - (2*c*d*f^2*g - 5*a*f*g^2)*n + (6*a*g^3 + (2*c*d
*f*g^2 + a*g^3)*n^2 - (2*c*e*f^2*g - 6*c*d*f*g^2 - 5*a*g^3)*n)*x)*(g*x + f
)^n/(g^3*n^3 + 6*g^3*n^2 + 11*g^3*n + 6*g^3)
```

3.808.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1489 vs. $2(75) = 150$.

Time = 0.58 (sec) , antiderivative size = 1489, normalized size of antiderivative = 17.73

$$\int (f+gx)^n \left(a+2cdx+cex^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

output
```
Piecewise((f**n*(a*x + c*d*x**2 + c*e*x**3/3), Eq(g, 0)), (-a*g**2/(2*f**2
*g**3 + 4*f*g**4*x + 2*g**5*x**2) - 2*c*d*f*g/(2*f**2*g**3 + 4*f*g**4*x +
2*g**5*x**2) - 4*c*d*g**2*x/(2*f**2*g**3 + 4*f*g**4*x + 2*g**5*x**2) + 2*c
*e*f**2*log(f/g + x)/(2*f**2*g**3 + 4*f*g**4*x + 2*g**5*x**2) + 3*c*e*f**2
/(2*f**2*g**3 + 4*f*g**4*x + 2*g**5*x**2) + 4*c*e*f*g*x*log(f/g + x)/(2*f*
*2*g**3 + 4*f*g**4*x + 2*g**5*x**2) + 4*c*e*f*g*x/(2*f**2*g**3 + 4*f*g**4*
x + 2*g**5*x**2) + 2*c*e*g**2*x**2*log(f/g + x)/(2*f**2*g**3 + 4*f*g**4*x
+ 2*g**5*x**2), Eq(n, -3)), (-a*g**2/(f*g**3 + g**4*x) + 2*c*d*f*g*log(f/g
 + x)/(f*g**3 + g**4*x) + 2*c*d*f*g/(f*g**3 + g**4*x) + 2*c*d*g**2*x*log(f
/g + x)/(f*g**3 + g**4*x) - 2*c*e*f**2*log(f/g + x)/(f*g**3 + g**4*x) - 2*
c*e*f**2/(f*g**3 + g**4*x) - 2*c*e*f*g*x*log(f/g + x)/(f*g**3 + g**4*x) +
c*e*g**2*x**2/(f*g**3 + g**4*x), Eq(n, -2)), (a*log(f/g + x)/g - 2*c*d*f*l
og(f/g + x)/g**2 + 2*c*d*x/g + c*e*f**2*log(f/g + x)/g**3 - c*e*f*x/g**2 +
 c*e*x**2/(2*g), Eq(n, -1)), (a*f*g**2*n**2*(f + g*x)**n/(g**3*n**3 + 6*g*
*3*n**2 + 11*g**3*n + 6*g**3) + 5*a*f*g**2*n*(f + g*x)**n/(g**3*n**3 + 6*g
**3*n**2 + 11*g**3*n + 6*g**3) + 6*a*f*g**2*(f + g*x)**n/(g**3*n**3 + 6*g*
*3*n**2 + 11*g**3*n + 6*g**3) + a*g**3*n**2*x*(f + g*x)**n/(g**3*n**3 + 6*
g**3*n**2 + 11*g**3*n + 6*g**3) + 5*a*g**3*n*x*(f + g*x)**n/(g**3*n**3 + 6
*g**3*n**2 + 11*g**3*n + 6*g**3) + 6*a*g**3*x*(f + g*x)**n/(g**3*n**3 + 6*
g**3*n**2 + 11*g**3*n + 6*g**3) - 2*c*d*f**2*g*n*(f + g*x)**n/(g**3*n**...
```

3.808.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.61

$$\begin{aligned}\int (f+gx)^n \left(a+2cdx+cex^2\right)\,dx &= \frac{2\left(g^2(n+1)x^2+fgnx-f^2\right)(gx+f)^n cd}{(n^2+3n+2)g^2} \\ &\quad + \frac{\left((n^2+3n+2)g^3x^3+(n^2+n)fg^2x^2-2f^2gnx+2f^3\right)(gx+f)^n ce}{(n^3+6n^2+11n+6)g^3}+\frac{(gx+f)^{n+1}a}{g(n+1)}\end{aligned}$$

input
```
integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

output
```
2*(g^2*(n + 1)*x^2 + f*g*n*x - f^2)*(g*x + f)^n*c*d/((n^2 + 3*n + 2)*g^2)
+ ((n^2 + 3*n + 2)*g^3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*(g
*x + f)^n*c*e/((n^3 + 6*n^2 + 11*n + 6)*g^3) + (g*x + f)^(n + 1)*a/(g*(n +
 1))
```

3.808.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(84) = 168$.

Time = 0.27 (sec) , antiderivative size = 366, normalized size of antiderivative = 4.36

$$\int (f+gx)^n\left(a+2cdx+cex^2\right)\,dx = \frac{(gx+f)^n ceg^3n^2x^3+(gx+f)^n cefg^2n^2x^2+2\,(gx+f)^n cdg^3n^2x^2+3\,(gx+f)^n ceg^3nx^3+2\,(gx+f)^n cdf}{}$$

input
```
integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

output
```
((g*x + f)^n*c*e*g^3*n^2*x^3 + (g*x + f)^n*c*e*f*g^2*n^2*x^2 + 2*(g*x + f)
^n*c*d*g^3*n^2*x^2 + 3*(g*x + f)^n*c*e*g^3*n*x^3 + 2*(g*x + f)^n*c*d*f*g^2
*n^2*x + (g*x + f)^n*c*e*f*g^2*n*x^2 + 8*(g*x + f)^n*c*d*g^3*n*x^2 + 2*(g*
x + f)^n*c*e*g^3*x^3 - 2*(g*x + f)^n*c*e*f^2*g*n*x + 6*(g*x + f)^n*c*d*f*g
^2*n*x + (g*x + f)^n*a*g^3*n^2*x + 6*(g*x + f)^n*c*d*g^3*x^2 - 2*(g*x + f)
^n*c*d*f^2*g*n + (g*x + f)^n*a*f*g^2*n^2 + 5*(g*x + f)^n*a*g^3*n*x + 2*(g*
x + f)^n*c*e*f^3 - 6*(g*x + f)^n*c*d*f^2*g + 5*(g*x + f)^n*a*f*g^2*n + 6*(
g*x + f)^n*a*g^3*x + 6*(g*x + f)^n*a*f*g^2)/(g^3*n^3 + 6*g^3*n^2 + 11*g^3*
n + 6*g^3)
```

3.808.9 Mupad [B] (verification not implemented)

Time = 12.08 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.51

$$\int (f+gx)^n\left(a+2cdx+cex^2\right)\,dx = (f+g\,x)^n\left(\frac{f\,(2\,c\,e\,f^2-2\,c\,d\,f\,g\,n-6\,c\,d\,f\,g+a\,g^2\,n^2+5\,a\,g^2\,n+6\,a\,g^2)}{g^3\,\left(n^3+6\,n^2+11\,n+6\right)} + \frac{x\,(-2\,c\,e\,f^2\,g\,n+2\,c\,d\,f\,g^2\,n^2+6\,c\,d\,f\,g^2\,n+a\,g^3\,n^2+5\,a\,g^3\,n+6\,a\,g^3)}{g^3\,\left(n^3+6\,n^2+11\,n+6\right)} + \frac{c\,e\,x^3\,(n^2+3\,n+2)}{n^3+6\,n^2+11\,n+6} + \frac{c\,x^2\,(n+1)\,\left(6\,d\,g+2\,d\,g\,n+e\,f\,n\right)}{g\,\left(n^3+6\,n^2+11\,n+6\right)}\right)$$

input
```
int((f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x)
```

output
```
(f + g*x)^n*((f*(6*a*g^2 + a*g^2*n^2 + 2*c*e*f^2 + 5*a*g^2*n - 6*c*d*f*g -
 2*c*d*f*g*n))/(g^3*(11*n + 6*n^2 + n^3 + 6)) + (x*(6*a*g^3 + a*g^3*n^2 +
5*a*g^3*n + 2*c*d*f*g^2*n^2 + 6*c*d*f*g^2*n - 2*c*e*f^2*g*n))/(g^3*(11*n +
 6*n^2 + n^3 + 6)) + (c*e*x^3*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) +
(c*x^2*(n + 1)*(6*d*g + 2*d*g*n + e*f*n))/(g*(11*n + 6*n^2 + n^3 + 6)))
```

3.809 $\int \frac{(f+gx)^n(a+2cdx+cex^2)}{d+ex}\,dx$

3.809.1 Optimal result

Integrand size = 28, antiderivative size = 114

$$\int \frac{(f+gx)^n(a+2cdx+cex^2)}{d+ex}\,dx = -\frac{c(ef-dg)(f+gx)^{1+n}}{eg^2(1+n)}+\frac{c(f+gx)^{2+n}}{g^2(2+n)} + \frac{(cd^2-ae)(f+gx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{e(f+gx)}{ef-dg}\right)}{e(ef-dg)(1+n)}$$

output
```
-c*(-d*g+e*f)*(g*x+f)^(1+n)/e/g^2/(1+n)+c*(g*x+f)^(2+n)/g^2/(2+n)+(c*d^2-a
*e)*(g*x+f)^(1+n)*hypergeom([1, 1+n],[2+n],e*(g*x+f)/(-d*g+e*f))/e/(-d*g+e
*f)/(1+n)
```

3.809.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

$$\int \frac{(f+gx)^n(a+2cdx+cex^2)}{d+ex}\,dx = \frac{(f+gx)^{1+n}\left(\frac{c(-ef+dg(2+n)+eg(1+n)x)}{g^2(2+n)}+\frac{(cd^2-ae)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{e(f+gx)}{ef-dg}\right)}{ef-dg}\right)}{e(1+n)}$$

input
```
Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x),x]
```

output
```
((f + g*x)^(1 + n)*((c*(-(e*f) + d*g*(2 + n) + e*g*(1 + n)*x))/(g^2*(2 + n
)) + ((c*d^2 - a*e)*Hypergeometric2F1[1, 1 + n, 2 + n, (e*(f + g*x))/(e*f
- d*g)])/(e*f - d*g)))/(e*(1 + n))
```

3.809.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^n \left(a+2cdx+cex^2\right)}{d+ex} \, dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{\left(ae-cd^2\right)(f+gx)^n}{e(d+ex)}+\frac{c(dg-ef)(f+gx)^n}{eg}+\frac{c(f+gx)^{n+1}}{g}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(cd^2-ae\right)(f+gx)^{n+1} \text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{e(f+gx)}{ef-dg}\right)}{e(n+1)(ef-dg)}-\frac{c(ef-dg)(f+gx)^{n+1}}{eg^2(n+1)}+\frac{c(f+gx)^{n+2}}{g^2(n+2)}$$

input
```
Int[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x),x]
```

output
```
-((c*(e*f - d*g)*(f + g*x)^(1 + n))/(e*g^2*(1 + n))) + (c*(f + g*x)^(2 + n
))/(g^2*(2 + n)) + ((c*d^2 - a*e)*(f + g*x)^(1 + n)*Hypergeometric2F1[1, 1
 + n, 2 + n, (e*(f + g*x))/(e*f - d*g)])/(e*(e*f - d*g)*(1 + n))
```

3.809.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.809.4 Maple [F]

$$\int \frac{(gx+f)^n (ce\,x^2+2cdx+a)}{ex+d}dx$$

input
```
int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x)
```

output
```
int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x)
```

3.809.5 Fricas [F]

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{d+ex}\,dx = \int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{ex+d}\,dx$$

input
```
integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x, algorithm="fricas")
```

output
```
integral((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d), x)
```

3.809.6 Sympy [F]

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{d+ex}\,dx = \int \frac{(f+gx)^n (a+2cdx+cex^2)}{d+ex}\,dx$$

input
```
integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a)/(e*x+d),x)
```

output
```
Integral((f + g*x)**n*(a + 2*c*d*x + c*e*x**2)/(d + e*x), x)
```

3.809.7 Maxima [F]

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{d+ex}\,dx = \int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{ex+d}\,dx$$

input
```
integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d), x)
```

3.809.8 Giac [F]

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{d+ex}\,dx = \int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{ex+d}\,dx$$

input
```
integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x, algorithm="giac")
```

output
```
integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d), x)
```

3.809.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{d+ex}\,dx = \int \frac{(f+g\,x)^n\,(c\,e\,x^2+2\,c\,d\,x+a)}{d+e\,x}\,dx$$

input
```
int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x),x)
```

output
```
int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x), x)
```

3.810 $\int \frac{(f+gx)^n(a+2cdx+cex^2)}{(d+ex)^2}\,dx$

3.810.1 Optimal result

Integrand size = 28, antiderivative size = 88

$$\int \frac{(f+gx)^n(a+2cdx+cex^2)}{(d+ex)^2}\,dx = \frac{c(f+gx)^{1+n}}{eg(1+n)}-\frac{(cd^2-ae)\,g(f+gx)^{1+n}\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{e(f+gx)}{ef-dg}\right)}{e(ef-dg)^2(1+n)}$$

output
```
c*(g*x+f)^(1+n)/e/g/(1+n)-(c*d^2-a*e)*g*(g*x+f)^(1+n)*hypergeom([2, 1+n],[
2+n],e*(g*x+f)/(-d*g+e*f))/e/(-d*g+e*f)^2/(1+n)
```

3.810.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int \frac{(f+gx)^n(a+2cdx+cex^2)}{(d+ex)^2}\,dx = \frac{(f+gx)^{1+n}\left(c(ef-dg)^2+(-cd^2+ae)\,g^2\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{e(f+gx)}{ef-dg}\right)\right)}{eg(ef-dg)^2(1+n)}$$

input
```
Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^2,x]
```

output
```
((f + g*x)^(1 + n)*(c*(e*f - d*g)^2 + (-(c*d^2) + a*e)*g^2*Hypergeometric2
F1[2, 1 + n, 2 + n, (e*(f + g*x))/(e*f - d*g)]))/(e*g*(e*f - d*g)^2*(1 + n
))
```

3.810.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^2}dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{\left(ae-cd^2\right)(f+gx)^n}{e(d+ex)^2}+\frac{c(f+gx)^n}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c(f+gx)^{n+1}}{eg(n+1)}-\frac{g\left(cd^2-ae\right)(f+gx)^{n+1}\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{e(f+gx)}{ef-dg}\right)}{e(n+1)(ef-dg)^2}$$

input
```
Int[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^2,x]
```

output
```
(c*(f + g*x)^(1 + n))/(e*g*(1 + n)) - ((c*d^2 - a*e)*g*(f + g*x)^(1 + n)*H
ypergeometric2F1[2, 1 + n, 2 + n, (e*(f + g*x))/(e*f - d*g)])/(e*(e*f - d*
g)^2*(1 + n))
```

3.810.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.810.4 Maple [F]

$$\int \frac{(gx+f)^n\,(ce\,x^2+2cdx+a)}{(ex+d)^2}dx$$

input
```
int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x)
```

output
```
int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x)
```

3.810.5 Fricas [F]

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^2}\,dx = \int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{(ex+d)^2}\,dx$$

input
```
integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e^2*x^2 + 2*d*e*x + d^2), x)
```

3.810.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a)/(e*x+d)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.810.7 Maxima [F]

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^2}\,dx = \int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{(ex+d)^2}\,dx$$

input
```
integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^2, x)
```

3.810.8 Giac [F]

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^2}\,dx = \int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{(ex+d)^2}\,dx$$

input
```
integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^2, x)
```

3.810.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n(a+2cdx+cex^2)}{(d+ex)^2}\,dx = \int \frac{(f+g\,x)^n\,(c\,e\,x^2+2\,c\,d\,x+a)}{(d+e\,x)^2}\,dx$$

input
```
int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^2,x)
```

output
```
int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^2, x)
```

3.811 $\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^3}\,dx$

3.811.1 Optimal result

Integrand size = 28, antiderivative size = 193

$$\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^3}\,dx = -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{2(ef-dg)(d+ex)^2}-\frac{(cd^2-ae)\,g(1-n)(f+gx)^{1+n}}{2e(ef-dg)^2(d+ex)} + \frac{(aeg^2(1-n)n-c(2e^2f^2-4defg+d^2g^2(2+n-n^2)))\,(f+gx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+\ldots\right)}{2e(ef-dg)^3(1+n)}$$

output
```
-1/2*(a-c*d^2/e)*(g*x+f)^(1+n)/(-d*g+e*f)/(e*x+d)^2-1/2*(c*d^2-a*e)*g*(1-n
)*(g*x+f)^(1+n)/e/(-d*g+e*f)^2/(e*x+d)+1/2*(a*e*g^2*(1-n)*n-c*(2*e^2*f^2-4
*d*e*f*g+d^2*g^2*(-n^2+n+2)))*(g*x+f)^(1+n)*hypergeom([1, 1+n],[2+n],e*(g*
x+f)/(-d*g+e*f))/e/(-d*g+e*f)^3/(1+n)
```

3.811.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.55

$$\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^3}\,dx = -\frac{(f+gx)^{1+n}\left(c(ef-dg)^2\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{e(f+gx)}{ef-dg}\right)+(-cd^2+ae)\,g^2\,\text{Hypergeome}\ldots\right.}{e(ef-dg)^3(1+n)}$$

input
```
Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^3,x]
```

output
```
-(((f + g*x)^(1 + n)*(c*(e*f - d*g)^2*Hypergeometric2F1[1, 1 + n, 2 + n, (
e*(f + g*x))/(e*f - d*g)] + (-(c*d^2) + a*e)*g^2*Hypergeometric2F1[3, 1 +
n, 2 + n, (e*(f + g*x))/(e*f - d*g)]))/(e*(e*f - d*g)^3*(1 + n)))
```

3.811.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1193, 87, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^n \left(a+2cdx+cex^2\right)}{(d+ex)^3} \, dx$$

$$\Big\downarrow \text{1193}$$

$$-\frac{\int \frac{(f+gx)^n \left(ag(1-n)-\frac{cd(2ef-dg(n+1))}{e}-2c(ef-dg)x\right)}{(d+ex)^2}dx}{2(ef-dg)}-\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{n+1}}{2(d+ex)^2(ef-dg)}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\frac{\left(aeg^2(1-n)n-c\left(d^2g^2\left(-n^2+n+2\right)-4defg+2e^2f^2\right)\right)\int \frac{(f+gx)^n}{d+ex}dx}{e(ef-dg)}+\frac{g(1-n)\left(cd^2-ae\right)(f+gx)^{n+1}}{e(d+ex)(ef-dg)}}{2(ef-dg)}-\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{n+1}}{2(d+ex)^2(ef-dg)}$$

$$\Big\downarrow \text{78}$$

$$-\frac{\frac{g(1-n)\left(cd^2-ae\right)(f+gx)^{n+1}}{e(d+ex)(ef-dg)}-\frac{(f+gx)^{n+1}\left(aeg^2(1-n)n-c\left(d^2g^2\left(-n^2+n+2\right)-4defg+2e^2f^2\right)\right)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{e(f+gx)}{ef-dg}\right)}{e(n+1)(ef-dg)^2}}{2(ef-dg)}-\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{n+1}}{2(d+ex)^2(ef-dg)}$$

input
```
Int[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^3,x]
```

output
```
-1/2*((a - (c*d^2)/e)*(f + g*x)^(1 + n))/((e*f - d*g)*(d + e*x)^2) - (((c*
d^2 - a*e)*g*(1 - n)*(f + g*x)^(1 + n))/(e*(e*f - d*g)*(d + e*x)) - ((a*e*
g^2*(1 - n)*n - c*(2*e^2*f^2 - 4*d*e*f*g + d^2*g^2*(2 + n - n^2)))*(f + g*
x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (e*(f + g*x))/(e*f - d*g)])/
(e*(e*f - d*g)^2*(1 + n)))/(2*(e*f - d*g))
```

3.811.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

3.811.4 Maple [F]

$$\int \frac{(gx+f)^n\,(ce\,x^2+2cdx+a)}{(ex+d)^3}dx$$

input
```
int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x)
```

output
```
int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x)
```

3.811.5 Fricas [F]

$$\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^3}\,dx=\int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{(ex+d)^3}\,dx$$

input
```
integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^
2*e*x + d^3), x)
```

3.811.6 Sympy [F]

$$\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^3}\,dx=\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^3}\,dx$$

input
```
integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a)/(e*x+d)**3,x)
```

output
```
Integral((f + g*x)**n*(a + 2*c*d*x + c*e*x**2)/(d + e*x)**3, x)
```

3.811.7 Maxima [F]

$$\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^3}\,dx=\int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{(ex+d)^3}\,dx$$

input
```
integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^3, x)
```

3.811.8 Giac [F]

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^3} \, dx = \int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{(ex+d)^3} \, dx$$

input
```
integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^3, x)
```

3.811.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^3} \, dx = \int \frac{(f+g\,x)^n\,(c\,e\,x^2+2\,c\,d\,x+a)}{(d+e\,x)^3} \, dx$$

input
```
int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^3,x)
```

output
```
int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^3, x)
```

3.812 $\int \frac{(f+gx)^n(a+2cdx+cex^2)}{(d+ex)^4}\,dx$

3.812.1 Optimal result

Integrand size = 28, antiderivative size = 197

$$\int \frac{(f+gx)^n(a+2cdx+cex^2)}{(d+ex)^4}\,dx = -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{3(ef-dg)(d+ex)^3}-\frac{(cd^2-ae)\,g(2-n)(f+gx)^{1+n}}{6e(ef-dg)^2(d+ex)^2} + \frac{g(aeg^2(2-3n+n^2)+c(6e^2f^2-12defg+d^2g^2(4+3n-n^2)))\,(f+gx)^{1+n}\,\text{Hypergeometric2F1}\left(2,1\right.}{6e(ef-dg)^4(1+n)}$$

output
```
-1/3*(a-c*d^2/e)*(g*x+f)^(1+n)/(-d*g+e*f)/(e*x+d)^3-1/6*(c*d^2-a*e)*g*(2-n
)*(g*x+f)^(1+n)/e/(-d*g+e*f)^2/(e*x+d)^2+1/6*g*(a*e*g^2*(n^2-3*n+2)+c*(6*e
^2*f^2-12*d*e*f*g+d^2*g^2*(-n^2+3*n+4)))*(g*x+f)^(1+n)*hypergeom([2, 1+n],
[2+n],e*(g*x+f)/(-d*g+e*f))/e/(-d*g+e*f)^4/(1+n)
```

3.812.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.54

$$\int \frac{(f+gx)^n(a+2cdx+cex^2)}{(d+ex)^4}\,dx = \frac{g(f+gx)^{1+n}\left(c(ef-dg)^2\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{e(f+gx)}{ef-dg}\right)+(-cd^2+ae)\,g^2\,\text{Hypergeometr}\right.}{e(ef-dg)^4(1+n)}$$

input
```
Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^4,x]
```

output
```
(g*(f + g*x)^(1 + n)*(c*(e*f - d*g)^2*Hypergeometric2F1[2, 1 + n, 2 + n, (
e*(f + g*x))/(e*f - d*g)] + (-(c*d^2) + a*e)*g^2*Hypergeometric2F1[4, 1 +
n, 2 + n, (e*(f + g*x))/(e*f - d*g)]))/(e*(e*f - d*g)^4*(1 + n))
```

3.812.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1193, 87, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^n \left(a+2cdx+cex^2\right)}{(d+ex)^4}dx$$

$$\Big\downarrow \ 1193$$

$$-\frac{\int \frac{(f+gx)^n \left(ag(2-n)-\frac{cd(3ef-dg(n+1))}{e}-3c(ef-dg)x\right)}{(d+ex)^3}dx}{3(ef-dg)}-\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{n+1}}{3(d+ex)^3(ef-dg)}$$

$$\Big\downarrow \ 87$$

$$-\frac{\frac{g(2-n)\left(cd^2-ae\right)(f+gx)^{n+1}}{2e(d+ex)^2(ef-dg)}-\frac{\left(aeg^2\left(n^2-3n+2\right)+c\left(d^2g^2\left(-n^2+3n+4\right)-12defg+6e^2f^2\right)\right)\int \frac{(f+gx)^n}{(d+ex)^2}dx}{2e(ef-dg)}}{3(ef-dg)}-\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{n+1}}{3(d+ex)^3(ef-dg)}$$

$$\Big\downarrow \ 78$$

$$-\frac{\frac{g(2-n)\left(cd^2-ae\right)(f+gx)^{n+1}}{2e(d+ex)^2(ef-dg)}-\frac{g(f+gx)^{n+1}\left(aeg^2\left(n^2-3n+2\right)+c\left(d^2g^2\left(-n^2+3n+4\right)-12defg+6e^2f^2\right)\right)\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{e(f-}{ef-}\right.}{2e(n+1)(ef-dg)^3}}{3(ef-dg)} \cdots \frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{n+1}}{3(d+ex)^3(ef-dg)}$$

input
```
Int[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^4,x]
```

output
```
-1/3*((a - (c*d^2)/e)*(f + g*x)^(1 + n))/((e*f - d*g)*(d + e*x)^3) - (((c*
d^2 - a*e)*g*(2 - n)*(f + g*x)^(1 + n))/(2*e*(e*f - d*g)*(d + e*x)^2) - (g
*(a*e*g^2*(2 - 3*n + n^2) + c*(6*e^2*f^2 - 12*d*e*f*g + d^2*g^2*(4 + 3*n -
 n^2)))*(f + g*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (e*(f + g*x))
/(e*f - d*g)])/(2*e*(e*f - d*g)^3*(1 + n)))/(3*(e*f - d*g))
```

3.812.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

3.812.4 Maple [F]

$$\int \frac{(gx+f)^n\,(ce\,x^2+2cdx+a)}{(ex+d)^4}dx$$

input
```
int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x)
```

output
```
int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x)
```

3.812.5 Fricas [F]

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^4} dx = \int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{(ex+d)^4} dx$$

input
```
integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
integral((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^
2*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

3.812.6 Sympy [F]

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^4} dx = \int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^4} dx$$

input
```
integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a)/(e*x+d)**4,x)
```

output
```
Integral((f + g*x)**n*(a + 2*c*d*x + c*e*x**2)/(d + e*x)**4, x)
```

3.812.7 Maxima [F]

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^4} dx = \int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{(ex+d)^4} dx$$

input
```
integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^4, x)
```

3.812.8 Giac [F]

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^4} dx = \int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{(ex+d)^4} dx$$

input
```
integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x, algorithm="giac")
```

output
```
integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^4, x)
```

3.812.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^4} dx = \int \frac{(f+g\,x)^n\,(c\,e\,x^2+2\,c\,d\,x+a)}{(d+e\,x)^4} dx$$

input
```
int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^4,x)
```

output
```
int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^4, x)
```

3.813 $\int (d+ex)^m(f+gx)^n\left(a+2cdx+cex^2\right) dx$

3.813.1 Optimal result

Integrand size = 28, antiderivative size = 231

$$\int (d+ex)^m(f+gx)^n\left(a+2cdx+cex^2\right) dx = -\frac{c(ef-dg)(2+m)(d+ex)^{1+m}(f+gx)^{1+n}}{eg^2(2+m+n)(3+m+n)}+\frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{eg(3+m+n)} + \frac{(c(ef-dg)(2+m)(ef(1+m)+dg(1+n))+g(2+m+n)(aeg(3+m+n)-cd(ef(2+m)+dg(1}{e^2g^2(1+m)(2+m+n}$$

output
```
-c*(-d*g+e*f)*(2+m)*(e*x+d)^(1+m)*(g*x+f)^(1+n)/e/g^2/(2+m+n)/(3+m+n)+c*(e
*x+d)^(2+m)*(g*x+f)^(1+n)/e/g/(3+m+n)+(c*(-d*g+e*f)*(2+m)*(e*f*(1+m)+d*g*(
1+n))+g*(2+m+n)*(a*e*g*(3+m+n)-c*d*(e*f*(2+m)+d*g*(1+n))))*(e*x+d)^(1+m)*(
g*x+f)^n*hypergeom([-n, 1+m],[2+m],-g*(e*x+d)/(-d*g+e*f))/e^2/g^2/(1+m)/(2
+m+n)/(3+m+n)/((e*(g*x+f)/(-d*g+e*f))^n)
```

3.813.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.77

$$\int (d+ex)^m(f+gx)^n\left(a+2cdx+cex^2\right) dx = \frac{(d+ex)^{1+m}(f+gx)^n\left(\frac{e(f+gx)}{ef-dg}\right)^{-n}\left(c(ef-dg)^2\,\text{Hypergeometric2F1}\left(1+m,-2-n,2+m,\frac{g(d+ex)}{-ef+dg}\right)-2\right.}{}$$

input
```
Integrate[(d + e*x)^m*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

output
```
((d + e*x)^(1 + m)*(f + g*x)^n*(c*(e*f - d*g)^2*Hypergeometric2F1[1 + m, -
2 - n, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)] - 2*c*(e*f - d*g)^2*Hypergeome
tric2F1[1 + m, -1 - n, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)] + e*(a*g^2 + c
*f*(e*f - 2*d*g))*Hypergeometric2F1[1 + m, -n, 2 + m, (g*(d + e*x))/(-(e*f
) + d*g)]))/(e^2*g^2*(1 + m)*((e*(f + g*x))/(e*f - d*g))^n)
```

3.813.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1194, 27, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m(f+gx)^n \left(a+2cdx+cex^2\right) \, dx$$

$$\Big\downarrow \text{ 1194}$$

$$\frac{\int e(d+ex)^m(f+gx)^n(aeg(m+n+3)-cd(ef(m+2)+dg(n+1))-ce(ef-dg)(m+2)x)dx}{e^2g(m+n+3)}+\frac{c(d+ex)^{m+2}(f+gx)^{n+1}}{eg(m+n+3)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int (d+ex)^m(f+gx)^n(aeg(m+n+3)-cd(ef(m+2)+dg(n+1))-ce(ef-dg)(m+2)x)dx}{eg(m+n+3)}+\frac{c(d+ex)^{m+2}(f+gx)^{n+1}}{eg(m+n+3)}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{\left(aeg(m+n+3)+\frac{c(m+2)(ef-dg)(dg(n+1)+ef(m+1))}{g(m+n+2)}-cd(dg(n+1)+ef(m+2))\right)\int (d+ex)^m(f+gx)^ndx-\frac{c(m+}{}}{eg(m+n+3)}$$
$$\frac{c(d+ex)^{m+2}(f+gx)^{n+1}}{eg(m+n+3)}$$

$$\Big\downarrow \text{ 80}$$

$$\frac{(f+gx)^n\left(\frac{e(f+gx)}{ef-dg}\right)^{-n}\left(aeg(m+n+3)+\frac{c(m+2)(ef-dg)(dg(n+1)+ef(m+1))}{g(m+n+2)}-cd(dg(n+1)+ef(m+2))\right)\int(d+e}{eg(m+n+3)}$$

$$\frac{c(d+ex)^{m+2}(f+gx)^{n+1}}{eg(m+n+3)}$$

$$\Big\downarrow 79$$

$$\frac{\frac{(d+ex)^{m+1}(f+gx)^n\left(\frac{e(f+gx)}{ef-dg}\right)^{-n}\left(aeg(m+n+3)+\frac{c(m+2)(ef-dg)(dg(n+1)+ef(m+1))}{g(m+n+2)}-cd(dg(n+1)+ef(m+2))\right)\text{Hypergeometric2F1}\left(m+1,-n,m\right.}{e(m+1)}}{eg(m+n+3)}$$

$$\frac{c(d+ex)^{m+2}(f+gx)^{n+1}}{eg(m+n+3)}$$

input
```
Int[(d + e*x)^m*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

output
```
(c*(d + e*x)^(2 + m)*(f + g*x)^(1 + n))/(e*g*(3 + m + n)) + (-((c*(e*f - d
*g)*(2 + m)*(d + e*x)^(1 + m)*(f + g*x)^(1 + n))/(g*(2 + m + n))) + ((a*e*
g*(3 + m + n) + (c*(e*f - d*g)*(2 + m)*(e*f*(1 + m) + d*g*(1 + n)))/(g*(2
+ m + n)) - c*d*(e*f*(2 + m) + d*g*(1 + n)))*(d + e*x)^(1 + m)*(f + g*x)^n
*Hypergeometric2F1[1 + m, -n, 2 + m, -((g*(d + e*x))/(e*f - d*g))])/(e*(1
+ m)*((e*(f + g*x))/(e*f - d*g))^n))/(e*g*(3 + m + n))
```

3.813.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

3.813.4 Maple [F]

$$\int (ex+d)^m (gx+f)^n \left(ce\,x^2+2cdx+a\right) dx$$

input
```
int((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)
```

output
```
int((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)
```

3.813.5 Fricas [F]

$$\int (d+ex)^m(f+gx)^n \left(a+2cdx+cex^2\right)\,dx = \int \left(cex^2+2\,cdx+a\right)(ex+d)^m(gx+f)^n\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

output
```
integral((c*e*x^2 + 2*c*d*x + a)*(e*x + d)^m*(g*x + f)^n, x)
```

3.813.6 Sympy [F(-2)]

Exception generated.

$$\int (d+ex)^m(f+gx)^n\left(a+2cdx+cex^2\right)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x+d)**m*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.813.7 Maxima [F]

$$\int (d+ex)^m(f+gx)^n\left(a+2cdx+cex^2\right)\,dx = \int \left(cex^2+2\,cdx+a\right)(ex+d)^m(gx+f)^n\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

output
```
integrate((c*e*x^2 + 2*c*d*x + a)*(e*x + d)^m*(g*x + f)^n, x)
```

3.813.8 Giac [F]

$$\int (d+ex)^m(f+gx)^n\left(a+2cdx+cex^2\right)\,dx = \int \left(cex^2+2\,cdx+a\right)(ex+d)^m(gx+f)^n\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

output
```
integrate((c*e*x^2 + 2*c*d*x + a)*(e*x + d)^m*(g*x + f)^n, x)
```

3.813.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^n\left(a+2cdx+cex^2\right)\,dx=\int (f+g\,x)^n\,(d+e\,x)^m\,\left(c\,e\,x^2+2\,c\,d\,x+a\right)\,dx$$

input
```
int((f + g*x)^n*(d + e*x)^m*(a + 2*c*d*x + c*e*x^2),x)
```

output
```
int((f + g*x)^n*(d + e*x)^m*(a + 2*c*d*x + c*e*x^2), x)
```

3.814 $\int \frac{a+bx+cx^2}{(d+ex)(f+gx)}\,dx$

3.814.1 Optimal result

Integrand size = 25, antiderivative size = 83

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)}\,dx=\frac{cx}{eg}+\frac{(cd^2-bde+ae^2)\log(d+ex)}{e^2(ef-dg)}-\frac{(cf^2-bfg+ag^2)\log(f+gx)}{g^2(ef-dg)}$$

output
```
c*x/e/g+(a*e^2-b*d*e+c*d^2)*ln(e*x+d)/e^2/(-d*g+e*f)-(a*g^2-b*f*g+c*f^2)*l
n(g*x+f)/g^2/(-d*g+e*f)
```

3.814.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)}\,dx=\frac{cx}{eg}-\frac{(-cd^2+bde-ae^2)\log(d+ex)}{e^2(ef-dg)}-\frac{(cf^2-bfg+ag^2)\log(f+gx)}{g^2(ef-dg)}$$

input
```
Integrate[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)),x]
```

output
```
(c*x)/(e*g) - ((-(c*d^2) + b*d*e - a*e^2)*Log[d + e*x])/(e^2*(e*f - d*g))
- ((c*f^2 - b*f*g + a*g^2)*Log[f + g*x])/(g^2*(e*f - d*g))
```

3.814.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{ae^2-bde+cd^2}{e(d+ex)(ef-dg)}+\frac{ag^2-bfg+cf^2}{g(f+gx)(dg-ef)}+\frac{c}{eg}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)}{e^2(ef-dg)}-\frac{\log(f+gx)\left(ag^2-bfg+cf^2\right)}{g^2(ef-dg)}+\frac{cx}{eg}$$

input
```
Int[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)),x]
```

output
```
(c*x)/(e*g) + ((c*d^2 - b*d*e + a*e^2)*Log[d + e*x])/(e^2*(e*f - d*g)) - (
(c*f^2 - b*f*g + a*g^2)*Log[f + g*x])/(g^2*(e*f - d*g))
```

3.814.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.814.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01

method	result
default	$\frac{cx}{eg}+\frac{(-e^2a+bde-c\,d^2)\ln(ex+d)}{(dg-ef)e^2}+\frac{(a\,g^2-bfg+c\,f^2)\ln(gx+f)}{g^2(dg-ef)}$
norman	$\frac{cx}{eg}+\frac{(a\,g^2-bfg+c\,f^2)\ln(gx+f)}{g^2(dg-ef)}-\frac{(e^2a-bde+c\,d^2)\ln(ex+d)}{(dg-ef)e^2}$
parallelrisch	$-\frac{\ln(ex+d)a\,e^2g^2-\ln(ex+d)bde\,g^2+\ln(ex+d)c\,d^2g^2-\ln(gx+f)a\,e^2g^2+\ln(gx+f)b\,e^2fg-\ln(gx+f)c\,e^2f^2-xcde\,g^2+xc\,e^2fg}{e^2g^2(dg-ef)}$
risch	$\frac{cx}{eg}-\frac{\ln(ex+d)a}{dg-ef}+\frac{\ln(ex+d)bd}{(dg-ef)e}-\frac{\ln(ex+d)c\,d^2}{(dg-ef)e^2}+\frac{\ln(-gx-f)a}{dg-ef}-\frac{\ln(-gx-f)bf}{g(dg-ef)}+\frac{\ln(-gx-f)c\,f^2}{g^2(dg-ef)}$

input
```
int((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
c*x/e/g+(-a*e^2+b*d*e-c*d^2)/(d*g-e*f)/e^2*ln(e*x+d)+1/g^2*(a*g^2-b*f*g+c*
f^2)/(d*g-e*f)*ln(g*x+f)
```

3.814.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.19

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)}\,dx$$
$$=\frac{(cd^2-bde+ae^2)g^2\log(ex+d)+(ce^2fg-cdeg^2)x-(ce^2f^2-be^2fg+ae^2g^2)\log(gx+f)}{e^3fg^2-de^2g^3}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

output
```
((c*d^2 - b*d*e + a*e^2)*g^2*log(e*x + d) + (c*e^2*f*g - c*d*e*g^2)*x - (c
*e^2*f^2 - b*e^2*f*g + a*e^2*g^2)*log(g*x + f))/(e^3*f*g^2 - d*e^2*g^3)
```

3.814.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(70) = 140$.

Time = 78.09 (sec) , antiderivative size = 420, normalized size of antiderivative = 5.06

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)}\,dx = \frac{cx}{eg} + \frac{(ag^2-bfg+cf^2)\log\left(x+\frac{adeg^2+ae^2fg-2bdefg+cd^2fg+cdef^2-\frac{d^2eg\left(ag^2-bfg+cf^2\right)}{dg-ef}+\frac{2de^2f\left(ag^2-bfg+cf^2\right)}{dg-ef}-\frac{e^3f^2\left(ag^2-bfg+cf\right.}{g(dg-ef)}}{2ae^2g^2-bdeg^2-be^2fg+cd^2g^2+ce^2f^2}\right.}{g^2\left(dg-ef\right)} - \frac{(ae^2-bde+cd^2)\log\left(x+\frac{adeg^2+ae^2fg-2bdefg+cd^2fg+cdef^2+\frac{d^2g^3\left(ae^2-bde+cd^2\right)}{e(dg-ef)}-\frac{2dfg^2\left(ae^2-bde+cd^2\right)}{dg-ef}+\frac{ef^2g\left(ae^2-bde+cd^2\right)}{dg-ef}}{2ae^2g^2-bdeg^2-be^2fg+cd^2g^2+ce^2f^2}\right.}{e^2\left(dg-ef\right)}$$

input
```
integrate((c*x**2+b*x+a)/(e*x+d)/(g*x+f),x)
```

output
```
c*x/(e*g) + (a*g**2 - b*f*g + c*f**2)*log(x + (a*d*e*g**2 + a*e**2*f*g - 2
*b*d*e*f*g + c*d**2*f*g + c*d*e*f**2 - d**2*e*g*(a*g**2 - b*f*g + c*f**2)/
(d*g - e*f) + 2*d*e**2*f*(a*g**2 - b*f*g + c*f**2)/(d*g - e*f) - e**3*f**2
*(a*g**2 - b*f*g + c*f**2)/(g*(d*g - e*f)))/(2*a*e**2*g**2 - b*d*e*g**2 -
b*e**2*f*g + c*d**2*g**2 + c*e**2*f**2))/(g**2*(d*g - e*f)) - (a*e**2 - b*
d*e + c*d**2)*log(x + (a*d*e*g**2 + a*e**2*f*g - 2*b*d*e*f*g + c*d**2*f*g
+ c*d*e*f**2 + d**2*g**3*(a*e**2 - b*d*e + c*d**2)/(e*(d*g - e*f)) - 2*d*f
*g**2*(a*e**2 - b*d*e + c*d**2)/(d*g - e*f) + e*f**2*g*(a*e**2 - b*d*e + c
*d**2)/(d*g - e*f))/(2*a*e**2*g**2 - b*d*e*g**2 - b*e**2*f*g + c*d**2*g**2
 + c*e**2*f**2))/(e**2*(d*g - e*f))
```

3.814.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)}\,dx = \frac{(cd^2-bde+ae^2)\log(ex+d)}{e^3f-de^2g} - \frac{(cf^2-bfg+ag^2)\log(gx+f)}{efg^2-dg^3} + \frac{cx}{eg}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

output
```
(c*d^2 - b*d*e + a*e^2)*log(e*x + d)/(e^3*f - d*e^2*g) - (c*f^2 - b*f*g +
a*g^2)*log(g*x + f)/(e*f*g^2 - d*g^3) + c*x/(e*g)
```

3.814.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)}\,dx = \frac{(cd^2-bde+ae^2)\log(|ex+d|)}{e^3f-de^2g} - \frac{(cf^2-bfg+ag^2)\log(|gx+f|)}{efg^2-dg^3} + \frac{cx}{eg}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

output
```
(c*d^2 - b*d*e + a*e^2)*log(abs(e*x + d))/(e^3*f - d*e^2*g) - (c*f^2 - b*f
*g + a*g^2)*log(abs(g*x + f))/(e*f*g^2 - d*g^3) + c*x/(e*g)
```

3.814.9 Mupad [B] (verification not implemented)

Time = 12.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)}\,dx = \frac{\ln(d+e\,x)\,(c\,d^2-b\,d\,e+a\,e^2)}{e^3\,f-d\,e^2\,g} + \frac{\ln(f+g\,x)\,(c\,f^2-b\,f\,g+a\,g^2)}{g^2\,(d\,g-e\,f)} + \frac{c\,x}{e\,g}$$

input
```
int((a + b*x + c*x^2)/((f + g*x)*(d + e*x)),x)
```

output
```
(log(d + e*x)*(a*e^2 + c*d^2 - b*d*e))/(e^3*f - d*e^2*g) + (log(f + g*x)*(
a*g^2 + c*f^2 - b*f*g))/(g^2*(d*g - e*f)) + (c*x)/(e*g)
```

3.815 $\int \frac{(a+bx+cx^2)^2}{(d+ex)(f+gx)}\,dx$

3.815.1 Optimal result

Integrand size = 27, antiderivative size = 184

$$\int \frac{(a+bx+cx^2)^2}{(d+ex)(f+gx)}\,dx = \frac{(b^2e^2g^2-2ceg(bef+bdg-aeg)+c^2(e^2f^2+defg+d^2g^2))\,x}{e^3g^3} - \frac{c(cef+cdg-2beg)x^2}{2e^2g^2} + \frac{c^2x^3}{3eg} + \frac{(cd^2-bde+ae^2)^2\log(d+ex)}{e^4(ef-dg)} - \frac{(cf^2-bfg+ag^2)^2\log(f+gx)}{g^4(ef-dg)}$$

output
```
(b^2*e^2*g^2-2*c*e*g*(-a*e*g+b*d*g+b*e*f)+c^2*(d^2*g^2+d*e*f*g+e^2*f^2))*x
/e^3/g^3-1/2*c*(-2*b*e*g+c*d*g+c*e*f)*x^2/e^2/g^2+1/3*c^2*x^3/e/g+(a*e^2-b
*d*e+c*d^2)^2*ln(e*x+d)/e^4/(-d*g+e*f)-(a*g^2-b*f*g+c*f^2)^2*ln(g*x+f)/g^4
/(-d*g+e*f)
```

3.815.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx+cx^2)^2}{(d+ex)(f+gx)}\,dx = -\frac{eg(-ef+dg)x(6b^2e^2g^2+6ceg(2aeg+b(-2ef-2dg+egx))+c^2(6d^2g^2-3deg(-2f+gx)+e^2(6f^2}{6e^4g^4(ef}$$

input
```
Integrate[(a + b*x + c*x^2)^2/((d + e*x)*(f + g*x)),x]
```

output
```
-1/6*(e*g*(-(e*f) + d*g)*x*(6*b^2*e^2*g^2 + 6*c*e*g*(2*a*e*g + b*(-2*e*f -
 2*d*g + e*g*x)) + c^2*(6*d^2*g^2 - 3*d*e*g*(-2*f + g*x) + e^2*(6*f^2 - 3*
f*g*x + 2*g^2*x^2))) - 6*(c*d^2 + e*(-(b*d) + a*e))^2*g^4*Log[d + e*x] + 6
*e^4*(c*f^2 + g*(-(b*f) + a*g))^2*Log[f + g*x])/(e^4*g^4*(e*f - d*g))
```

3.815.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^2}{(d+ex)(f+gx)}dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{-2ceg(-aeg+bdg+bef)+b^2e^2g^2+c^2(d^2g^2+defg+e^2f^2)}{e^3g^3}+\frac{(ae^2-bde+cd^2)^2}{e^3(d+ex)(ef-dg)}+\frac{(ag^2-bfg+cf^2)^2}{g^3(f+gx)(dg-ef}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{x(-2ceg(-aeg+bdg+bef)+b^2e^2g^2+c^2(d^2g^2+defg+e^2f^2))}{e^3g^3}+\frac{\log(d+ex)(ae^2-bde+cd^2)^2}{e^4(ef-dg)}-\frac{\log(f+gx)(ag^2-bfg+cf^2)^2}{g^4(ef-dg)}-\frac{cx^2(-2beg+cdg+cef)}{2e^2g^2}+\frac{c^2x^3}{3eg}$$

input
```
Int[(a + b*x + c*x^2)^2/((d + e*x)*(f + g*x)),x]
```

output
```
((b^2*e^2*g^2 - 2*c*e*g*(b*e*f + b*d*g - a*e*g) + c^2*(e^2*f^2 + d*e*f*g +
 d^2*g^2))*x)/(e^3*g^3) - (c*(c*e*f + c*d*g - 2*b*e*g)*x^2)/(2*e^2*g^2) +
(c^2*x^3)/(3*e*g) + ((c*d^2 - b*d*e + a*e^2)^2*Log[d + e*x])/(e^4*(e*f - d
*g)) - ((c*f^2 - b*f*g + a*g^2)^2*Log[f + g*x])/(g^4*(e*f - d*g))
```

3.815.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.815.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.42

method	result
norman	$\frac{(2ace^2g^2+b^2e^2g^2-2bcdeg^2-2bce^2fg+c^2d^2g^2+c^2defg+c^2e^2f^2)x}{e^3g^3}+\frac{c^2x^3}{3eg}+\frac{c(2beg-cdg-cef)x^2}{2e^2g^2}+\frac{(a^2g^4-2abfg^3+2a}{}$
default	$\frac{\frac{1}{3}c^2x^3e^2g^2+bce^2g^2x^2-\frac{1}{2}c^2deg^2x^2-\frac{1}{2}c^2e^2fgx^2+2ace^2g^2x+b^2e^2g^2x-2bcdeg^2x-2bce^2fgx+c^2d^2g^2x+c^2defgx+c^2e^2f^2x}{e^3g^3}$
parallelrisch	$-\frac{-2x^3c^2de^3g^4+2x^3c^2e^4fg^3+3x^2c^2d^2e^2g^4-3x^2c^2e^4f^2g^2-6xb^2de^3g^4+6xb^2e^4fg^3-6xc^2d^3eg^4+6xc^2e^4f^3g+6\ln(ex+d)b}{}$
risch	$\frac{c^2x^3}{3eg}+\frac{bcx^2}{eg}-\frac{c^2dx^2}{2e^2g}-\frac{c^2fx^2}{2eg^2}+\frac{2acx}{eg}+\frac{b^2x}{eg}-\frac{2bcdx}{e^2g}-\frac{2bcfx}{eg^2}+\frac{c^2d^2x}{e^3g}+\frac{c^2dfx}{e^2g^2}+\frac{c^2f^2x}{eg^3}-\frac{\ln(ex+d)a^2}{dg-ef}+$

input
```
int((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
(2*a*c*e^2*g^2+b^2*e^2*g^2-2*b*c*d*e*g^2-2*b*c*e^2*f*g+c^2*d^2*g^2+c^2*d*e
*f*g+c^2*e^2*f^2)/e^3/g^3*x+1/3*c^2*x^3/e/g+1/2*c/e^2/g^2*(2*b*e*g-c*d*g-c
*e*f)*x^2+1/g^4*(a^2*g^4-2*a*b*f*g^3+2*a*c*f^2*g^2+b^2*f^2*g^2-2*b*c*f^3*g
+c^2*f^4)/(d*g-e*f)*ln(g*x+f)-(a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e
^2-2*b*c*d^3*e+c^2*d^4)/(d*g-e*f)/e^4*ln(e*x+d)
```

3.815.5 Fricas [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.70

$$\int\frac{(a+bx+cx^2)^2}{(d+ex)(f+gx)}\,dx = \frac{6\,(c^2d^4-2\,bcd^3e-2\,abde^3+a^2e^4+(b^2+2\,ac)d^2e^2)g^4\log\left(ex+d\right)+2\,(c^2e^4fg^3-c^2de^3g^4)x^3-3\,(c^2e^4f^2}{}$$

input
```
integrate((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

output
```
1/6*(6*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*
e^2)*g^4*log(e*x + d) + 2*(c^2*e^4*f*g^3 - c^2*d*e^3*g^4)*x^3 - 3*(c^2*e^4
*f^2*g^2 - 2*b*c*e^4*f*g^3 - (c^2*d^2*e^2 - 2*b*c*d*e^3)*g^4)*x^2 + 6*(c^2
*e^4*f^3*g - 2*b*c*e^4*f^2*g^2 + (b^2 + 2*a*c)*e^4*f*g^3 - (c^2*d^3*e - 2*
b*c*d^2*e^2 + (b^2 + 2*a*c)*d*e^3)*g^4)*x - 6*(c^2*e^4*f^4 - 2*b*c*e^4*f^3
*g - 2*a*b*e^4*f*g^3 + a^2*e^4*g^4 + (b^2 + 2*a*c)*e^4*f^2*g^2)*log(g*x +
f))/(e^5*f*g^4 - d*e^4*g^5)
```

3.815.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^2}{(d+ex)(f+gx)}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**2/(e*x+d)/(g*x+f),x)
```

output
```
Timed out
```

3.815.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx+cx^2)^2}{(d+ex)(f+gx)}\,dx = \frac{(c^2d^4 - 2\,bcd^3e - 2\,abde^3 + a^2e^4 + (b^2+2\,ac)d^2e^2)\log(ex+d)}{e^5f - de^4g} - \frac{(c^2f^4 - 2\,bcf^3g - 2\,abfg^3 + a^2g^4 + (b^2+2\,ac)f^2g^2)\log(gx+f)}{efg^4 - dg^5} + \frac{2\,c^2e^2g^2x^3 - 3\,(c^2e^2fg + (c^2de - 2\,bce^2)g^2)x^2 + 6\,(c^2e^2f^2 + (c^2de - 2\,bce^2)fg + (c^2d^2 - 2\,bcde + (b^2 +}{6\,e^3g^3}$$

input
```
integrate((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

output
```
(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)*lo
g(e*x + d)/(e^5*f - d*e^4*g) - (c^2*f^4 - 2*b*c*f^3*g - 2*a*b*f*g^3 + a^2*
g^4 + (b^2 + 2*a*c)*f^2*g^2)*log(g*x + f)/(e*f*g^4 - d*g^5) + 1/6*(2*c^2*e
^2*g^2*x^3 - 3*(c^2*e^2*f*g + (c^2*d*e - 2*b*c*e^2)*g^2)*x^2 + 6*(c^2*e^2*
f^2 + (c^2*d*e - 2*b*c*e^2)*f*g + (c^2*d^2 - 2*b*c*d*e + (b^2 + 2*a*c)*e^2
)*g^2)*x)/(e^3*g^3)
```

3.815.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2)^2}{(d+ex)(f+gx)}\,dx = \frac{(c^2d^4-2\,bcd^3e+b^2d^2e^2+2\,acd^2e^2-2\,abde^3+a^2e^4)\log(|ex+d|)}{e^5f-de^4g} - \frac{(c^2f^4-2\,bcf^3g+b^2f^2g^2+2\,acf^2g^2-2\,abfg^3+a^2g^4)\log(|gx+f|)}{efg^4-dg^5} + \frac{2\,c^2e^2g^2x^3-3\,c^2e^2fgx^2-3\,c^2deg^2x^2+6\,bce^2g^2x^2+6\,c^2e^2f^2x+6\,c^2defgx-12\,bce^2fgx+6\,c^2d^2g^2x-}{6\,e^3g^3}$$

input
```
integrate((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x, algorithm="giac")
```

output
```
(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e
^4)*log(abs(e*x + d))/(e^5*f - d*e^4*g) - (c^2*f^4 - 2*b*c*f^3*g + b^2*f^2
*g^2 + 2*a*c*f^2*g^2 - 2*a*b*f*g^3 + a^2*g^4)*log(abs(g*x + f))/(e*f*g^4 -
 d*g^5) + 1/6*(2*c^2*e^2*g^2*x^3 - 3*c^2*e^2*f*g*x^2 - 3*c^2*d*e*g^2*x^2 +
 6*b*c*e^2*g^2*x^2 + 6*c^2*e^2*f^2*x + 6*c^2*d*e*f*g*x - 12*b*c*e^2*f*g*x
+ 6*c^2*d^2*g^2*x - 12*b*c*d*e*g^2*x + 6*b^2*e^2*g^2*x + 12*a*c*e^2*g^2*x)
/(e^3*g^3)
```

3.815.9 Mupad [B] (verification not implemented)

Time = 12.14 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx+cx^2)^2}{(d+ex)(f+gx)}\,dx$$
$$= x\left(\frac{b^2+2\,a\,c}{e\,g}+\frac{\left(\frac{c^2\,(d\,g+e\,f)}{e^2\,g^2}-\frac{2\,b\,c}{e\,g}\right)\,(d\,g+e\,f)}{e\,g}-\frac{c^2\,d\,f}{e^2\,g^2}\right)-x^2\left(\frac{c^2\,(d\,g+e\,f)}{2\,e^2\,g^2}-\frac{b\,c}{e\,g}\right)$$
$$+\frac{\ln(d+e\,x)\,\left(e^2\,(b^2\,d^2+2\,a\,c\,d^2)+a^2\,e^4+c^2\,d^4-2\,a\,b\,d\,e^3-2\,b\,c\,d^3\,e\right)}{e^5\,f-d\,e^4\,g}$$
$$+\frac{\ln(f+g\,x)\,\left(g^2\,(b^2\,f^2+2\,a\,c\,f^2)+a^2\,g^4+c^2\,f^4-2\,a\,b\,f\,g^3-2\,b\,c\,f^3\,g\right)}{d\,g^5-e\,f\,g^4}+\frac{c^2\,x^3}{3\,e\,g}$$

input
```
int((a + b*x + c*x^2)^2/((f + g*x)*(d + e*x)),x)
```

output
```
x*((2*a*c + b^2)/(e*g) + (((c^2*(d*g + e*f))/(e^2*g^2) - (2*b*c)/(e*g))*(d
*g + e*f))/(e*g) - (c^2*d*f)/(e^2*g^2)) - x^2*((c^2*(d*g + e*f))/(2*e^2*g^
2) - (b*c)/(e*g)) + (log(d + e*x)*(e^2*(b^2*d^2 + 2*a*c*d^2) + a^2*e^4 + c
^2*d^4 - 2*a*b*d*e^3 - 2*b*c*d^3*e))/(e^5*f - d*e^4*g) + (log(f + g*x)*(g^
2*(b^2*f^2 + 2*a*c*f^2) + a^2*g^4 + c^2*f^4 - 2*a*b*f*g^3 - 2*b*c*f^3*g))/
(d*g^5 - e*f*g^4) + (c^2*x^3)/(3*e*g)
```

3.816 $\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx$

3.816.1 Optimal result

Integrand size = 27, antiderivative size = 531

$$\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx =$$
$$-\frac{(b^2e^3g^3(bef+bdg-3aeg)-c^3(e^4f^4+de^3f^3g+d^2e^2f^2g^2+d^3efg^3+d^4g^4)-3ce^2g^2(a^2e^2g^2-2abeg(\cdots}{}$$
$$+\frac{(b^3e^3g^3-3bce^2g^2(bef+bdg-2aeg)-c^3(e^3f^3+de^2f^2g+d^2efg^2+d^3g^3)-3c^2eg(aeg(ef+dg)-b(\cdots}{2e^4g^4}$$
$$+\frac{c(3b^2e^2g^2-3ceg(bef+bdg-aeg)+c^2(e^2f^2+defg+d^2g^2))\,x^3}{3e^3g^3}$$
$$-\frac{c^2(cef+cdg-3beg)x^4}{4e^2g^2}+\frac{c^3x^5}{5eg}+\frac{(cd^2-bde+ae^2)^3\log(d+ex)}{e^6(ef-dg)}$$
$$-\frac{(cf^2-bfg+ag^2)^3\log(f+gx)}{g^6(ef-dg)}$$

output
```
-(b^2*e^3*g^3*(-3*a*e*g+b*d*g+b*e*f)-c^3*(d^4*g^4+d^3*e*f*g^3+d^2*e^2*f^2*
g^2+d*e^3*f^3*g+e^4*f^4)-3*c*e^2*g^2*(a^2*e^2*g^2-2*a*b*e*g*(d*g+e*f)+b^2*
(d^2*g^2+d*e*f*g+e^2*f^2))-3*c^2*e*g*(a*e*g*(d^2*g^2+d*e*f*g+e^2*f^2)-b*(d
^3*g^3+d^2*e*f*g^2+d*e^2*f^2*g+e^3*f^3)))*x/e^5/g^5+1/2*(b^3*e^3*g^3-3*b*c
*e^2*g^2*(-2*a*e*g+b*d*g+b*e*f)-c^3*(d^3*g^3+d^2*e*f*g^2+d*e^2*f^2*g+e^3*f
^3)-3*c^2*e*g*(a*e*g*(d*g+e*f)-b*(d^2*g^2+d*e*f*g+e^2*f^2)))*x^2/e^4/g^4+1
/3*c*(3*b^2*e^2*g^2-3*c*e*g*(-a*e*g+b*d*g+b*e*f)+c^2*(d^2*g^2+d*e*f*g+e^2*
f^2))*x^3/e^3/g^3-1/4*c^2*(-3*b*e*g+c*d*g+c*e*f)*x^4/e^2/g^2+1/5*c^3*x^5/e
/g+(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)/e^6/(-d*g+e*f)-(a*g^2-b*f*g+c*f^2)^3*ln
(g*x+f)/g^6/(-d*g+e*f)
```

3.816.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 476, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx = -\frac{egx(-30b^2e^3g^3(ef-dg)(6aeg+b(-2ef-2dg+egx))+c^3(60d^5g^5-30d^4eg^5x+20d^3e^2g^5x^2-15d^2e}{}$$

input
```
Integrate[(a + b*x + c*x^2)^3/((d + e*x)*(f + g*x)),x]
```

output
```
-1/60*(e*g*x*(-30*b^2*e^3*g^3*(e*f - d*g)*(6*a*e*g + b*(-2*e*f - 2*d*g + e
*g*x)) + c^3*(60*d^5*g^5 - 30*d^4*e*g^5*x + 20*d^3*e^2*g^5*x^2 - 15*d^2*e^
3*g^5*x^3 + 12*d*e^4*g^5*x^4 + e^5*f*(-60*f^4 + 30*f^3*g*x - 20*f^2*g^2*x^
2 + 15*f*g^3*x^3 - 12*g^4*x^4)) - 30*c*e^2*g^2*(e*f - d*g)*(6*a^2*e^2*g^2
+ 6*a*b*e*g*(-2*e*f - 2*d*g + e*g*x) + b^2*(6*d^2*g^2 - 3*d*e*g*(-2*f + g*
x) + e^2*(6*f^2 - 3*f*g*x + 2*g^2*x^2))) + 15*c^2*e*g*(-2*a*e*g*(e*f - d*g
)*(6*d^2*g^2 - 3*d*e*g*(-2*f + g*x) + e^2*(6*f^2 - 3*f*g*x + 2*g^2*x^2)) +
 b*(-12*d^4*g^4 + 6*d^3*e*g^4*x - 4*d^2*e^2*g^4*x^2 + 3*d*e^3*g^4*x^3 + e^
4*f*(12*f^3 - 6*f^2*g*x + 4*f*g^2*x^2 - 3*g^3*x^3)))) - 60*(c*d^2 + e*(-(b
*d) + a*e))^3*g^6*Log[d + e*x] + 60*e^6*(c*f^2 + g*(-(b*f) + a*g))^3*Log[f
 + g*x])/(e^6*g^6*(e*f - d*g))
```

3.816.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{3ce^2g^2(a^2e^2g^2-2abeg(dg+ef)+b^2(d^2g^2+defg+e^2f^2))-b^2e^3g^3(-3aeg+bdg+bef)+3c^2eg(aeg(d^2g}{e^5g}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{x\left(-3ce^2g^2\left(a^2e^2g^2-2abeg(dg+ef)+b^2\left(d^2g^2+defg+e^2f^2\right)\right)+b^2e^3g^3(-3aeg+bdg+bef)-3c^2eg\left(aeg\left(d^2\right.\right.}{e^5}$$
$$\frac{x^2\left(-3c^2eg\left(aeg(dg+ef)-b\left(d^2g^2+defg+e^2f^2\right)\right)-3bce^2g^2(-2aeg+bdg+bef)+b^3e^3g^3-\left(c^3\left(d^3g^3+d^2efg\right.\right.\right.}{2e^4g^4}$$
$$\frac{cx^3\left(-3ceg(-aeg+bdg+bef)+3b^2e^2g^2+c^2\left(d^2g^2+defg+e^2f^2\right)\right)}{3e^3g^3}+$$
$$\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)^3}{e^6(ef-dg)}-\frac{\log(f+gx)\left(ag^2-bfg+cf^2\right)^3}{g^6(ef-dg)}-\frac{c^2x^4(-3beg+cdg+cef)}{4e^2g^2}+\frac{c^3x^5}{5eg}$$

input
```
Int[(a + b*x + c*x^2)^3/((d + e*x)*(f + g*x)),x]
```

output
```
-(((b^2*e^3*g^3*(b*e*f + b*d*g - 3*a*e*g) - c^3*(e^4*f^4 + d*e^3*f^3*g + d
^2*e^2*f^2*g^2 + d^3*e*f*g^3 + d^4*g^4) - 3*c*e^2*g^2*(a^2*e^2*g^2 - 2*a*b
*e*g*(e*f + d*g) + b^2*(e^2*f^2 + d*e*f*g + d^2*g^2)) - 3*c^2*e*g*(a*e*g*(
e^2*f^2 + d*e*f*g + d^2*g^2) - b*(e^3*f^3 + d*e^2*f^2*g + d^2*e*f*g^2 + d^
3*g^3)))*x)/(e^5*g^5)) + ((b^3*e^3*g^3 - 3*b*c*e^2*g^2*(b*e*f + b*d*g - 2*
a*e*g) - c^3*(e^3*f^3 + d*e^2*f^2*g + d^2*e*f*g^2 + d^3*g^3) - 3*c^2*e*g*(
a*e*g*(e*f + d*g) - b*(e^2*f^2 + d*e*f*g + d^2*g^2)))*x^2)/(2*e^4*g^4) + (
c*(3*b^2*e^2*g^2 - 3*c*e*g*(b*e*f + b*d*g - a*e*g) + c^2*(e^2*f^2 + d*e*f*
g + d^2*g^2))*x^3)/(3*e^3*g^3) - (c^2*(c*e*f + c*d*g - 3*b*e*g)*x^4)/(4*e^
2*g^2) + (c^3*x^5)/(5*e*g) + ((c*d^2 - b*d*e + a*e^2)^3*Log[d + e*x])/(e^6
*(e*f - d*g)) - ((c*f^2 - b*f*g + a*g^2)^3*Log[f + g*x])/(g^6*(e*f - d*g))
```

3.816.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.816.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 837, normalized size of antiderivative = 1.58

method	result
norman	$\frac{(3a^2c\,e^4g^4+3a\,b^2e^4g^4-6abcd\,e^3g^4-6abc\,e^4f\,g^3+3a\,c^2d^2e^2g^4+3a\,c^2d\,e^3f\,g^3+3a\,c^2e^4f^2g^2-b^3d\,e^3g^4-b^3e^4f\,g^3+3b^2c\,d^2e^2g^4+}{}$
default	$\frac{3a\,c^2d^2e^2g^4x+3a\,c^2e^4f^2g^2x-6abc\,e^4f\,g^3x+3a\,c^2d\,e^3f\,g^3x+3b^2cd\,e^3f\,g^3x+c^3d^4g^4x+c^3e^4f^4x+\frac{1}{2}b^3e^4g^4x^2+\frac{1}{5}c^3x^5e^4g^4+a\,c^2}{}$
parallelrisch	Expression too large to display
risch	Expression too large to display

input
```
int((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
(3*a^2*c*e^4*g^4+3*a*b^2*e^4*g^4-6*a*b*c*d*e^3*g^4-6*a*b*c*e^4*f*g^3+3*a*c
^2*d^2*e^2*g^4+3*a*c^2*d*e^3*f*g^3+3*a*c^2*e^4*f^2*g^2-b^3*d*e^3*g^4-b^3*e
^4*f*g^3+3*b^2*c*d^2*e^2*g^4+3*b^2*c*d*e^3*f*g^3+3*b^2*c*e^4*f^2*g^2-3*b*c
^2*d^3*e*g^4-3*b*c^2*d^2*e^2*f*g^3-3*b*c^2*d*e^3*f^2*g^2-3*b*c^2*e^4*f^3*g
+c^3*d^4*g^4+c^3*d^3*e*f*g^3+c^3*d^2*e^2*f^2*g^2+c^3*d*e^3*f^3*g+c^3*e^4*f
^4)/e^5/g^5*x+1/5*c^3*x^5/e/g+1/2/e^4/g^4*(6*a*b*c*e^3*g^3-3*a*c^2*d*e^2*g
^3-3*a*c^2*e^3*f*g^2+b^3*e^3*g^3-3*b^2*c*d*e^2*g^3-3*b^2*c*e^3*f*g^2+3*b*c
^2*d^2*e*g^3+3*b*c^2*d*e^2*f*g^2+3*b*c^2*e^3*f^2*g-c^3*d^3*g^3-c^3*d^2*e*f
*g^2-c^3*d*e^2*f^2*g-c^3*e^3*f^3)*x^2+1/3*c/e^3/g^3*(3*a*c*e^2*g^2+3*b^2*e
^2*g^2-3*b*c*d*e*g^2-3*b*c*e^2*f*g+c^2*d^2*g^2+c^2*d*e*f*g+c^2*e^2*f^2)*x^
3+1/4*c^2/e^2/g^2*(3*b*e*g-c*d*g-c*e*f)*x^4+1/g^6*(a^3*g^6-3*a^2*b*f*g^5+3
*a^2*c*f^2*g^4+3*a*b^2*f^2*g^4-6*a*b*c*f^3*g^3+3*a*c^2*f^4*g^2-b^3*f^3*g^3
+3*b^2*c*f^4*g^2-3*b*c^2*f^5*g+c^3*f^6)/(d*g-e*f)*ln(g*x+f)-(a^3*e^6-3*a^2
*b*d*e^5+3*a^2*c*d^2*e^4+3*a*b^2*d^2*e^4-6*a*b*c*d^3*e^3+3*a*c^2*d^4*e^2-b
^3*d^3*e^3+3*b^2*c*d^4*e^2-3*b*c^2*d^5*e+c^3*d^6)/(d*g-e*f)/e^6*ln(e*x+d)
```

3.816.5 Fricas [A] (verification not implemented)

Time = 2.80 (sec) , antiderivative size = 736, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx$$
$$=\frac{60\,(c^3d^6-3\,bc^2d^5e-3\,a^2bde^5+a^3e^6+3\,(b^2c+ac^2)d^4e^2-(b^3+6\,abc)d^3e^3+3\,(ab^2+a^2c)d^2e^4)g^6\log(ex}{}$$

input
```
integrate((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

output
```
1/60*(60*(c^3*d^6 - 3*b*c^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c + a
*c^2)*d^4*e^2 - (b^3 + 6*a*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4)*g^6*l
og(e*x + d) + 12*(c^3*e^6*f*g^5 - c^3*d*e^5*g^6)*x^5 - 15*(c^3*e^6*f^2*g^4
 - 3*b*c^2*e^6*f*g^5 - (c^3*d^2*e^4 - 3*b*c^2*d*e^5)*g^6)*x^4 + 20*(c^3*e^
6*f^3*g^3 - 3*b*c^2*e^6*f^2*g^4 + 3*(b^2*c + a*c^2)*e^6*f*g^5 - (c^3*d^3*e
^3 - 3*b*c^2*d^2*e^4 + 3*(b^2*c + a*c^2)*d*e^5)*g^6)*x^3 - 30*(c^3*e^6*f^4
*g^2 - 3*b*c^2*e^6*f^3*g^3 + 3*(b^2*c + a*c^2)*e^6*f^2*g^4 - (b^3 + 6*a*b*
c)*e^6*f*g^5 - (c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 + 3*(b^2*c + a*c^2)*d^2*e^4
- (b^3 + 6*a*b*c)*d*e^5)*g^6)*x^2 + 60*(c^3*e^6*f^5*g - 3*b*c^2*e^6*f^4*g^
2 + 3*(b^2*c + a*c^2)*e^6*f^3*g^3 - (b^3 + 6*a*b*c)*e^6*f^2*g^4 + 3*(a*b^2
 + a^2*c)*e^6*f*g^5 - (c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*(b^2*c + a*c^2)*d^3
*e^3 - (b^3 + 6*a*b*c)*d^2*e^4 + 3*(a*b^2 + a^2*c)*d*e^5)*g^6)*x - 60*(c^3
*e^6*f^6 - 3*b*c^2*e^6*f^5*g - 3*a^2*b*e^6*f*g^5 + a^3*e^6*g^6 + 3*(b^2*c
+ a*c^2)*e^6*f^4*g^2 - (b^3 + 6*a*b*c)*e^6*f^3*g^3 + 3*(a*b^2 + a^2*c)*e^6
*f^2*g^4)*log(g*x + f))/(e^7*f*g^6 - d*e^6*g^7)
```

3.816.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**3/(e*x+d)/(g*x+f),x)
```

output
```
Timed out
```

3.816.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx = \frac{(c^3d^6 - 3\,bc^2d^5e - 3\,a^2bde^5 + a^3e^6 + 3\,(b^2c + ac^2)d^4e^2 - (b^3 + 6\,abc)d^3e^3 + 3\,(ab^2 + a^2c)d^2e^4)\log{(ex+d)}}{e^7f - de^6g} - \frac{(c^3f^6 - 3\,bc^2f^5g - 3\,a^2bfg^5 + a^3g^6 + 3\,(b^2c + ac^2)f^4g^2 - (b^3 + 6\,abc)f^3g^3 + 3\,(ab^2 + a^2c)f^2g^4)\log{(gx}}{efg^6 - dg^7} + \frac{12\,c^3e^4g^4x^5 - 15\,(c^3e^4fg^3 + (c^3de^3 - 3\,bc^2e^4)g^4)x^4 + 20\,(c^3e^4f^2g^2 + (c^3de^3 - 3\,bc^2e^4)fg^3 + (c^3d^2e^2 - 3}{}$$

input
```
integrate((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

output
```
(c^3*d^6 - 3*b*c^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c + a*c^2)*d^4
*e^2 - (b^3 + 6*a*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4)*log(e*x + d)/(
e^7*f - d*e^6*g) - (c^3*f^6 - 3*b*c^2*f^5*g - 3*a^2*b*f*g^5 + a^3*g^6 + 3*
(b^2*c + a*c^2)*f^4*g^2 - (b^3 + 6*a*b*c)*f^3*g^3 + 3*(a*b^2 + a^2*c)*f^2*
g^4)*log(g*x + f)/(e*f*g^6 - d*g^7) + 1/60*(12*c^3*e^4*g^4*x^5 - 15*(c^3*e
^4*f*g^3 + (c^3*d*e^3 - 3*b*c^2*e^4)*g^4)*x^4 + 20*(c^3*e^4*f^2*g^2 + (c^3
*d*e^3 - 3*b*c^2*e^4)*f*g^3 + (c^3*d^2*e^2 - 3*b*c^2*d*e^3 + 3*(b^2*c + a*
c^2)*e^4)*g^4)*x^3 - 30*(c^3*e^4*f^3*g + (c^3*d*e^3 - 3*b*c^2*e^4)*f^2*g^2
 + (c^3*d^2*e^2 - 3*b*c^2*d*e^3 + 3*(b^2*c + a*c^2)*e^4)*f*g^3 + (c^3*d^3*
e - 3*b*c^2*d^2*e^2 + 3*(b^2*c + a*c^2)*d*e^3 - (b^3 + 6*a*b*c)*e^4)*g^4)*
x^2 + 60*(c^3*e^4*f^4 + (c^3*d*e^3 - 3*b*c^2*e^4)*f^3*g + (c^3*d^2*e^2 - 3
*b*c^2*d*e^3 + 3*(b^2*c + a*c^2)*e^4)*f^2*g^2 + (c^3*d^3*e - 3*b*c^2*d^2*e
^2 + 3*(b^2*c + a*c^2)*d*e^3 - (b^3 + 6*a*b*c)*e^4)*f*g^3 + (c^3*d^4 - 3*b
*c^2*d^3*e + 3*(b^2*c + a*c^2)*d^2*e^2 - (b^3 + 6*a*b*c)*d*e^3 + 3*(a*b^2
+ a^2*c)*e^4)*g^4)*x)/(e^5*g^5)
```

3.816.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 953, normalized size of antiderivative = 1.79

$$\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx$$
$$= \frac{(c^3d^6 - 3\,bc^2d^5e + 3\,b^2cd^4e^2 + 3\,ac^2d^4e^2 - b^3d^3e^3 - 6\,abcd^3e^3 + 3\,ab^2d^2e^4 + 3\,a^2cd^2e^4 - 3\,a^2bde^5 + a^3e^6)\,\mathrm{lo}}{e^7f - de^6g}$$
$$- \frac{(c^3f^6 - 3\,bc^2f^5g + 3\,b^2cf^4g^2 + 3\,ac^2f^4g^2 - b^3f^3g^3 - 6\,abcf^3g^3 + 3\,ab^2f^2g^4 + 3\,a^2cf^2g^4 - 3\,a^2bfg^5 + a^3}{efg^6 - dg^7}$$
$$+ \frac{12\,c^3e^4g^4x^5 - 15\,c^3e^4fg^3x^4 - 15\,c^3de^3g^4x^4 + 45\,bc^2e^4g^4x^4 + 20\,c^3e^4f^2g^2x^3 + 20\,c^3de^3fg^3x^3 - 60\,bc^2e^4f}{}$$

input
```
integrate((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x, algorithm="giac")
```

output
```
(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3
 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a
^3*e^6)*log(abs(e*x + d))/(e^7*f - d*e^6*g) - (c^3*f^6 - 3*b*c^2*f^5*g + 3
*b^2*c*f^4*g^2 + 3*a*c^2*f^4*g^2 - b^3*f^3*g^3 - 6*a*b*c*f^3*g^3 + 3*a*b^2
*f^2*g^4 + 3*a^2*c*f^2*g^4 - 3*a^2*b*f*g^5 + a^3*g^6)*log(abs(g*x + f))/(e
*f*g^6 - d*g^7) + 1/60*(12*c^3*e^4*g^4*x^5 - 15*c^3*e^4*f*g^3*x^4 - 15*c^3
*d*e^3*g^4*x^4 + 45*b*c^2*e^4*g^4*x^4 + 20*c^3*e^4*f^2*g^2*x^3 + 20*c^3*d*
e^3*f*g^3*x^3 - 60*b*c^2*e^4*f*g^3*x^3 + 20*c^3*d^2*e^2*g^4*x^3 - 60*b*c^2
*d*e^3*g^4*x^3 + 60*b^2*c*e^4*g^4*x^3 + 60*a*c^2*e^4*g^4*x^3 - 30*c^3*e^4*
f^3*g*x^2 - 30*c^3*d*e^3*f^2*g^2*x^2 + 90*b*c^2*e^4*f^2*g^2*x^2 - 30*c^3*d
^2*e^2*f*g^3*x^2 + 90*b*c^2*d*e^3*f*g^3*x^2 - 90*b^2*c*e^4*f*g^3*x^2 - 90*
a*c^2*e^4*f*g^3*x^2 - 30*c^3*d^3*e*g^4*x^2 + 90*b*c^2*d^2*e^2*g^4*x^2 - 90
*b^2*c*d*e^3*g^4*x^2 - 90*a*c^2*d*e^3*g^4*x^2 + 30*b^3*e^4*g^4*x^2 + 180*a
*b*c*e^4*g^4*x^2 + 60*c^3*e^4*f^4*x + 60*c^3*d*e^3*f^3*g*x - 180*b*c^2*e^4
*f^3*g*x + 60*c^3*d^2*e^2*f^2*g^2*x - 180*b*c^2*d*e^3*f^2*g^2*x + 180*b^2*
c*e^4*f^2*g^2*x + 180*a*c^2*e^4*f^2*g^2*x + 60*c^3*d^3*e*f*g^3*x - 180*b*c
^2*d^2*e^2*f*g^3*x + 180*b^2*c*d*e^3*f*g^3*x + 180*a*c^2*d*e^3*f*g^3*x - 6
0*b^3*e^4*f*g^3*x - 360*a*b*c*e^4*f*g^3*x + 60*c^3*d^4*g^4*x - 180*b*c^2*d
^3*e*g^4*x + 180*b^2*c*d^2*e^2*g^4*x + 180*a*c^2*d^2*e^2*g^4*x - 60*b^3*d*
e^3*g^4*x - 360*a*b*c*d*e^3*g^4*x + 180*a*b^2*e^4*g^4*x + 180*a^2*c*e^4...
```

3.816. $\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx$

3.816.9 Mupad [B] (verification not implemented)

Time = 13.43 (sec) , antiderivative size = 794, normalized size of antiderivative = 1.50

$$\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx$$

$$= x^4\left(\frac{3\,b\,c^2}{4\,e\,g}-\frac{c^3\,(d\,g+e\,f)}{4\,e^2\,g^2}\right)-x^3\left(\frac{(d\,g+e\,f)\,\left(\frac{3\,b\,c^2}{e\,g}-\frac{c^3\,(d\,g+e\,f)}{e^2\,g^2}\right)}{3\,e\,g}-\frac{c\,(b^2+a\,c)}{e\,g}+\frac{c^3\,d\,f}{3\,e^2\,g^2}\right)$$
$$+x^2\left(\frac{b^3+6\,a\,c\,b}{2\,e\,g}+\frac{(d\,g+e\,f)\,\left(\frac{(d\,g+e\,f)\,\left(\frac{3\,b\,c^2}{e\,g}-\frac{c^3\,(d\,g+e\,f)}{e^2\,g^2}\right)}{e\,g}-\frac{3\,c\,(b^2+a\,c)}{e\,g}+\frac{c^3\,d\,f}{e^2\,g^2}\right)}{2\,e\,g}-\frac{d\,f\,\left(\frac{3\,b\,c^2}{e\,g}-\frac{c^3\,(d\,g+e\,f)}{e^2\,g^2}\right)}{2\,e\,g}\right)+x\left(\frac{3\,a\,(b^2+a\,c)}{e\,g}\right.$$
$$-\frac{(d\,g+e\,f)\,\left(\frac{b^3+6\,a\,c\,b}{e\,g}+\frac{(d\,g+e\,f)\,\left(\frac{(d\,g+e\,f)\,\left(\frac{3\,b\,c^2}{e\,g}-\frac{c^3\,(d\,g+e\,f)}{e^2\,g^2}\right)}{e\,g}-\frac{3\,c\,\left(b^2+a\,c\right)}{e\,g}+\frac{c^3\,d\,f}{e^2\,g^2}\right)}{e\,g}-\frac{d\,f\,\left(\frac{3\,b\,c^2}{e\,g}-\frac{c^3\,(d\,g+e\,f)}{e^2\,g^2}\right)}{e\,g}\right)}{e\,g}$$
$$\left.+\frac{d\,f\,\left(\frac{(d\,g+e\,f)\,\left(\frac{3\,b\,c^2}{e\,g}-\frac{c^3\,(d\,g+e\,f)}{e^2\,g^2}\right)}{e\,g}-\frac{3\,c\,(b^2+a\,c)}{e\,g}+\frac{c^3\,d\,f}{e^2\,g^2}\right)}{e\,g}\right)$$
$$+\frac{\ln(d+e\,x)\,\left(e^4\,(3\,c\,a^2\,d^2+3\,a\,b^2\,d^2)+e^2\,(3\,b^2\,c\,d^4+3\,a\,c^2\,d^4)-e^3\,(b^3\,d^3+6\,a\,c\,b\,d^3)+a^3\,e^6+c^3\,d^6-3\right.}{e^7\,f-d\,e^6\,g}$$
$$+\frac{\ln(f+g\,x)\,\left(g^4\,(3\,c\,a^2\,f^2+3\,a\,b^2\,f^2)+g^2\,(3\,b^2\,c\,f^4+3\,a\,c^2\,f^4)-g^3\,(b^3\,f^3+6\,a\,c\,b\,f^3)+a^3\,g^6+c^3\,f^6-\right.}{d\,g^7-e\,f\,g^6}$$
$$+\frac{c^3\,x^5}{5\,e\,g}$$

input
```
int((a + b*x + c*x^2)^3/((f + g*x)*(d + e*x)),x)
```

output
```
x^4*((3*b*c^2)/(4*e*g) - (c^3*(d*g + e*f))/(4*e^2*g^2)) - x^3*(((d*g + e*f
)*((3*b*c^2)/(e*g) - (c^3*(d*g + e*f))/(e^2*g^2)))/(3*e*g) - (c*(a*c + b^2
))/(e*g) + (c^3*d*f)/(3*e^2*g^2)) + x^2*((b^3 + 6*a*b*c)/(2*e*g) + ((d*g +
 e*f)*(((d*g + e*f)*((3*b*c^2)/(e*g) - (c^3*(d*g + e*f))/(e^2*g^2)))/(e*g)
 - (3*c*(a*c + b^2))/(e*g) + (c^3*d*f)/(e^2*g^2)))/(2*e*g) - (d*f*((3*b*c^
2)/(e*g) - (c^3*(d*g + e*f))/(e^2*g^2)))/(2*e*g)) + x*((3*a*(a*c + b^2))/(
e*g) - ((d*g + e*f)*((b^3 + 6*a*b*c)/(e*g) + ((d*g + e*f)*(((d*g + e*f)*((
3*b*c^2)/(e*g) - (c^3*(d*g + e*f))/(e^2*g^2)))/(e*g) - (3*c*(a*c + b^2))/(
e*g) + (c^3*d*f)/(e^2*g^2)))/(e*g) - (d*f*((3*b*c^2)/(e*g) - (c^3*(d*g + e
*f))/(e^2*g^2)))/(e*g)))/(e*g) + (d*f*(((d*g + e*f)*((3*b*c^2)/(e*g) - (c^
3*(d*g + e*f))/(e^2*g^2)))/(e*g) - (3*c*(a*c + b^2))/(e*g) + (c^3*d*f)/(e^
2*g^2)))/(e*g)) + (log(d + e*x)*(e^4*(3*a*b^2*d^2 + 3*a^2*c*d^2) + e^2*(3*
a*c^2*d^4 + 3*b^2*c*d^4) - e^3*(b^3*d^3 + 6*a*b*c*d^3) + a^3*e^6 + c^3*d^6
 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e))/(e^7*f - d*e^6*g) + (log(f + g*x)*(g^4*
(3*a*b^2*f^2 + 3*a^2*c*f^2) + g^2*(3*a*c^2*f^4 + 3*b^2*c*f^4) - g^3*(b^3*f
^3 + 6*a*b*c*f^3) + a^3*g^6 + c^3*f^6 - 3*a^2*b*f*g^5 - 3*b*c^2*f^5*g))/(d
*g^7 - e*f*g^6) + (c^3*x^5)/(5*e*g)
```

3.816. $\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx$

3.817 $\int \frac{1}{(d+ex)(f+gx)(a+bx+cx^2)}\,dx$

3.817.1 Optimal result

Integrand size = 27, antiderivative size = 246

$$\int \frac{1}{(d+ex)(f+gx)(a+bx+cx^2)}\,dx = -\frac{(2c^2df+b^2eg-c(bef+bdg+2aeg))\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)\,(cf^2-g(bf-ag))}+\frac{e^2\log(d+ex)}{(cd^2-bde+ae^2)\,(ef-dg)} - \frac{g^2\log(f+gx)}{(ef-dg)\,(cf^2-bfg+ag^2)}-\frac{(cef+cdg-beg)\log\left(a+bx+cx^2\right)}{2\,(cd^2-bde+ae^2)\,(cf^2-g(bf-ag))}$$

output
```
e^2*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)-g^2*ln(g*x+f)/(-d*g+e*f)/(a*g
^2-b*f*g+c*f^2)-1/2*(-b*e*g+c*d*g+c*e*f)*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c*d^
2)/(c*f^2-g*(-a*g+b*f))-(2*c^2*d*f+b^2*e*g-c*(2*a*e*g+b*d*g+b*e*f))*arctan
h((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(c*f^2-g*(-a*g+b*f))/(
-4*a*c+b^2)^(1/2)
```

3.817.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)}\,dx$$
$$= \frac{\left(2c^2df+b^2eg-c(bef+bdg+2aeg)\right)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}\left(cd^2+e(-bd+ae)\right)\left(cf^2+g(-bf+ag)\right)} + \frac{e^2\log(d+ex)}{\left(cd^2+e(-bd+ae)\right)(ef-dg)} - \frac{g^2\log(f+gx)}{(ef-dg)\left(cf^2+g(-bf+ag)\right)} - \frac{(cef+cdg-beg)\log(a+x(b+cx))}{2\left(cd^2+e(-bd+ae)\right)\left(cf^2+g(-bf+ag)\right)}$$

input
```
Integrate[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)),x]
```

output
```
((2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*(c*d^2 + e*(-(b*d) + a*e))*(c*f^2 +
 g*(-(b*f) + a*g))) + (e^2*Log[d + e*x])/((c*d^2 + e*(-(b*d) + a*e))*(e*f
- d*g)) - (g^2*Log[f + g*x])/((e*f - d*g)*(c*f^2 + g*(-(b*f) + a*g))) - ((
c*e*f + c*d*g - b*e*g)*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))
*(c*f^2 + g*(-(b*f) + a*g)))
```

3.817.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)}\,dx$$
$$\Big\downarrow \ 1200$$
$$\int \left(\frac{-c(aeg+bdg+bef)+b^2eg-cx(-beg+cdg+cef)+c^2df}{\left(a+bx+cx^2\right)\left(ae^2-bde+cd^2\right)\left(ag^2-bfg+cf^2\right)} - \frac{e^3}{(d+ex)(dg-ef)\left(ae^2-bde+cd^2\right)} - \frac{\ }{(f+g}\right.$$
$$\Big\downarrow \ 2009$$

$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2aeg+bdg+bef)+b^2eg+2c^2df\right)}{\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)\,(cf^2-g(bf-ag))}-\frac{\log\left(a+bx+cx^2\right)\left(-beg+cdg+cef\right)}{2\,(ae^2-bde+cd^2)\,(cf^2-g(bf-ag))}+\frac{e^2\log(d+ex)}{(ef-dg)\,(ae^2-bde+cd^2)}-\frac{g^2\log(f+gx)}{(ef-dg)\,(ag^2-bfg+cf^2)}$$

input
```
Int[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)),x]
```

output
```
-(((2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g
*(b*f - a*g)))) + (e^2*Log[d + e*x])/((c*d^2 - b*d*e + a*e^2)*(e*f - d*g))
 - (g^2*Log[f + g*x])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)) - ((c*e*f + c*
d*g - b*e*g)*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(
b*f - a*g)))
```

3.817.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.817.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.99

method	result
default	$-\frac{e^2\ln(ex+d)}{(dg-ef)(e^2a-bde+cd^2)}+\frac{\frac{\left(bceg-c^2dg-c^2ef\right)\ln\left(cx^2+bx+a\right)}{2c}+\frac{2\left(-aceg+b^2eg-bcdg-bcef+c^2df-\frac{\left(bceg-c^2dg-c^2ef\right)b}{2c}\right)\arctan\left(\frac{2c}{\sqrt{4a}}\right.}{\sqrt{4ac-b^2}}}{(e^2a-bde+cd^2)(ag^2-bfg+cf^2)}$
risch	Expression too large to display

input
```
int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-e^2/(d*g-e*f)/(a*e^2-b*d*e+c*d^2)*ln(e*x+d)+1/(a*e^2-b*d*e+c*d^2)/(a*g^2-
b*f*g+c*f^2)*(1/2*(b*c*e*g-c^2*d*g-c^2*e*f)/c*ln(c*x^2+b*x+a)+2*(-a*c*e*g+
b^2*e*g-b*c*d*g-b*c*e*f+c^2*d*f-1/2*(b*c*e*g-c^2*d*g-c^2*e*f)*b/c)/(4*a*c-
b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))+g^2/(d*g-e*f)/(a*g^2-b*f*g
+c*f^2)*ln(g*x+f)
```

3.817.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.817.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

3.817.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.817.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.58

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)}\,dx = \frac{e^3\log\left(|ex+d|\right)}{cd^2e^2f-bde^3f+ae^4f-cd^3eg+bd^2e^2g-ade^3g} - \frac{g^3\log\left(|gx+f|\right)}{cef^3g-cdf^2g^2-bef^2g^2+bdfg^3+aefg^3-adg^4} - \frac{(cef+cdg-beg)\log\left(cx^2+bx+a\right)}{2\left(c^2d^2f^2-bcdef^2+ace^2f^2-bcd^2fg+b^2defg-abe^2fg+acd^2g^2-abdeg^2+a^2e^2g^2\right)} + \frac{\left(2\,c^2df-bcef-bcdg+b^2eg-2\,aceg\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(c^2d^2f^2-bcdef^2+ace^2f^2-bcd^2fg+b^2defg-abe^2fg+acd^2g^2-abdeg^2+a^2e^2g^2\right)\sqrt{-b^2+4\,ac}}$$

input
```
integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
e^3*log(abs(e*x + d))/(c*d^2*e^2*f - b*d*e^3*f + a*e^4*f - c*d^3*e*g + b*d
^2*e^2*g - a*d*e^3*g) - g^3*log(abs(g*x + f))/(c*e*f^3*g - c*d*f^2*g^2 - b
*e*f^2*g^2 + b*d*f*g^3 + a*e*f*g^3 - a*d*g^4) - 1/2*(c*e*f + c*d*g - b*e*g
)*log(c*x^2 + b*x + a)/(c^2*d^2*f^2 - b*c*d*e*f^2 + a*c*e^2*f^2 - b*c*d^2*
f*g + b^2*d*e*f*g - a*b*e^2*f*g + a*c*d^2*g^2 - a*b*d*e*g^2 + a^2*e^2*g^2)
 + (2*c^2*d*f - b*c*e*f - b*c*d*g + b^2*e*g - 2*a*c*e*g)*arctan((2*c*x + b
)/sqrt(-b^2 + 4*a*c))/((c^2*d^2*f^2 - b*c*d*e*f^2 + a*c*e^2*f^2 - b*c*d^2*
f*g + b^2*d*e*f*g - a*b*e^2*f*g + a*c*d^2*g^2 - a*b*d*e*g^2 + a^2*e^2*g^2)
*sqrt(-b^2 + 4*a*c))
```

3.817.9 Mupad [B] (verification not implemented)

Time = 32.76 (sec) , antiderivative size = 12173, normalized size of antiderivative = 49.48

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)),x)
```

output
```
(log(6*a^2*c^4*d^5*g^5 + 6*a^2*c^4*e^5*f^5 - a^3*b^3*e^5*g^5 - a^3*b^2*e^5
*g^5*(b^2 - 4*a*c)^(1/2) - c^5*d^3*e^2*f^5*(b^2 - 4*a*c)^(1/2) - c^5*d^5*f
^3*g^2*(b^2 - 4*a*c)^(1/2) - 18*a^3*c^3*d^3*e^2*g^5 + b^2*c^4*d^2*e^3*f^5
- 18*a^3*c^3*e^5*f^3*g^2 + b^2*c^4*d^5*f^2*g^3 + 4*a^4*b*c*e^5*g^5 + 4*a^4
*c*e^5*g^5*(b^2 - 4*a*c)^(1/2) - 2*a*b^2*c^3*d^5*g^5 - 2*a*b^2*c^3*e^5*f^5
 + 2*a*b^5*d^2*e^3*g^5 - 10*a*c^5*d^2*e^3*f^5 + a^2*b^4*d*e^4*g^5 + b*c^5*
d^3*e^2*f^5 - 8*a^4*c^2*d*e^4*g^5 + 2*a*b^5*e^5*f^2*g^3 - 10*a*c^5*d^5*f^2
*g^3 + a^2*b^4*e^5*f*g^4 + b*c^5*d^5*f^3*g^2 - 8*a^4*c^2*e^5*f*g^4 - a^2*b
^4*e^5*g^5*x - 8*a^4*c^2*e^5*g^5*x - 2*b^3*c^3*d^5*g^5*x - 2*b^3*c^3*e^5*f
^5*x + 2*b^6*d^2*e^3*g^5*x + 2*c^6*d^3*e^2*f^5*x + 2*b^6*e^5*f^2*g^3*x + 2
*c^6*d^5*f^3*g^2*x - 2*a*b*c^3*d^5*g^5*(b^2 - 4*a*c)^(1/2) - 2*a*b*c^3*e^5
*f^5*(b^2 - 4*a*c)^(1/2) + 7*a*c^4*d*e^4*f^5*(b^2 - 4*a*c)^(1/2) + 7*a*c^4
*d^5*f*g^4*(b^2 - 4*a*c)^(1/2) + 2*c^5*d^4*e*f^4*g*(b^2 - 4*a*c)^(1/2) + 3
*a*c^4*d^5*g^5*x*(b^2 - 4*a*c)^(1/2) + 3*a*c^4*e^5*f^5*x*(b^2 - 4*a*c)^(1/
2) + 6*a*b^3*c^2*d^4*e*g^5 - 6*a*b^4*c*d^3*e^2*g^5 - 21*a^2*b*c^3*d^4*e*g^
5 - 2*a^3*b^2*c*d*e^4*g^5 + 6*a*b^3*c^2*e^5*f^4*g - 6*a*b^4*c*e^5*f^3*g^2
- 21*a^2*b*c^3*e^5*f^4*g - 2*a^3*b^2*c*e^5*f*g^4 + 10*a*c^5*d^3*e^2*f^4*g
+ 10*a*c^5*d^4*e*f^3*g^2 + 26*a^2*c^4*d*e^4*f^4*g + 26*a^2*c^4*d^4*e*f*g^4
 + 6*a^3*b^2*c*e^5*g^5*x - 3*b*c^5*d^2*e^3*f^5*x + 14*a^2*c^4*d^4*e*g^5*x
+ 5*b^2*c^4*d*e^4*f^5*x + 6*b^4*c^2*d^4*e*g^5*x - 6*b^5*c*d^3*e^2*g^5*x...
```

3.818 $\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx$

3.818.1 Optimal result

Integrand size = 27, antiderivative size = 644

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx =$$
$$-\frac{b^3eg-b^2c(ef+dg)+2ac^2(ef+dg)+bc(cdf-3aeg)+c(2c^2df+b^2eg-c(bef+bdg+2aeg))\,x}{(b^2-4ac)\,(cd^2-bde+ae^2)\,(cf^2-g(bf-ag))\,(a+bx+cx^2)}$$
$$+\frac{2c(2c^2df+b^2eg-c(bef+bdg+2aeg))\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}\,(cd^2-bde+ae^2)\,(cf^2-g(bf-ag))}$$
$$+\frac{(b^2e^2g^2(bef+bdg-2aeg)-2c^3df(e^2f^2+defg+d^2g^2)+2ceg(a^2e^2g^2+abeg(ef+dg)-b^2(ef+dg)}{\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)^2\,(cf^2-g}$$
$$+\frac{e^4\log(d+ex)}{(cd^2-bde+ae^2)^2\,(ef-dg)}-\frac{g^4\log(f+gx)}{(ef-dg)\,(cf^2-bfg+ag^2)^2}$$
$$-\frac{(cef+cdg-beg)\,(c(e^2f^2+d^2g^2)+eg(2aeg-b(ef+dg)))\log(a+bx+cx^2)}{2\,(cd^2-bde+ae^2)^2\,(cf^2-g(bf-ag))^2}$$

output
```
(-b^3*e*g+b^2*c*(d*g+e*f)-2*a*c^2*(d*g+e*f)-b*c*(-3*a*e*g+c*d*f)-c*(2*c^2*
d*f+b^2*e*g-c*(2*a*e*g+b*d*g+b*e*f))*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(
c*f^2-g*(-a*g+b*f))/(c*x^2+b*x+a)+2*c*(2*c^2*d*f+b^2*e*g-c*(2*a*e*g+b*d*g+
b*e*f))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/(a*e^2-b*
d*e+c*d^2)/(c*f^2-g*(-a*g+b*f))+e^4*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+
e*f)-g^4*ln(g*x+f)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2-1/2*(-b*e*g+c*d*g+c*e*
f)*(c*(d^2*g^2+e^2*f^2)+e*g*(2*a*e*g-b*(d*g+e*f)))*ln(c*x^2+b*x+a)/(a*e^2-
b*d*e+c*d^2)^2/(c*f^2-g*(-a*g+b*f))^2+(b^2*e^2*g^2*(-2*a*e*g+b*d*g+b*e*f)-
2*c^3*d*f*(d^2*g^2+d*e*f*g+e^2*f^2)+2*c*e*g*(a^2*e^2*g^2+a*b*e*g*(d*g+e*f)
-b^2*(d*g+e*f)^2)-c^2*(4*a*d*e^2*f*g^2-b*(d^3*g^3+5*d^2*e*f*g^2+5*d*e^2*f^
2*g+e^3*f^3)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)^2
/(c*f^2-g*(-a*g+b*f))^2/(-4*a*c+b^2)^(1/2)
```

3.818.2 Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 710, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{-b^3eg+b^2c(dg+e(f-gx))-2c^2(adg+cdfx+ae(f-gx))+bc(3aeg+c(-df+efx+dgx))}{\left(b^2-4ac\right)\left(-cd^2+e(bd-ae)\right)\left(-cf^2+g(bf-ag)\right)\left(a+x(b+cx)\right)}$$

$$+\frac{\left(4c^5d^3f^3+b^4e^2g^2(bef+bdg-2aeg)-2b^2ceg\left(-6a^2e^2g^2+2abeg(ef+dg)+b^2\left(e^2f^2+defg+d^2g^2\right)\right)\right.}{}$$

$$+\frac{e^4\log(d+ex)}{\left(cd^2+e(-bd+ae)\right)^2(ef-dg)}-\frac{g^4\log(f+gx)}{(ef-dg)\left(cf^2+g(-bf+ag)\right)^2}$$

$$-\frac{(cef+cdg-beg)\left(c\left(e^2f^2+d^2g^2\right)+eg(2aeg-b(ef+dg))\right)\log(a+x(b+cx))}{2\left(cd^2+e(-bd+ae)\right)^2\left(cf^2+g(-bf+ag)\right)^2}$$

input
```
Integrate[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^2),x]
```

output
```
(-(b^3*e*g) + b^2*c*(d*g + e*(f - g*x)) - 2*c^2*(a*d*g + c*d*f*x + a*e*(f
- g*x)) + b*c*(3*a*e*g + c*(-(d*f) + e*f*x + d*g*x)))/((b^2 - 4*a*c)*(-(c*
d^2) + e*(b*d - a*e))*(-(c*f^2) + g*(b*f - a*g))*(a + x*(b + c*x))) + ((4*
c^5*d^3*f^3 + b^4*e^2*g^2*(b*e*f + b*d*g - 2*a*e*g) - 2*b^2*c*e*g*(-6*a^2*
e^2*g^2 + 2*a*b*e*g*(e*f + d*g) + b^2*(e^2*f^2 + d*e*f*g + d^2*g^2)) + 2*c
^4*d*f*(-3*b*d*f*(e*f + d*g) + 2*a*(3*e^2*f^2 + d*e*f*g + 3*d^2*g^2)) + c^
2*(-12*a^3*e^3*g^3 - 6*a^2*b*e^2*g^2*(e*f + d*g) + 12*a*b^2*e*g*(e^2*f^2 +
 d*e*f*g + d^2*g^2) + b^3*(e^3*f^3 + d*e^2*f^2*g + d^2*e*f*g^2 + d^3*g^3))
 - 2*c^3*(-4*b^2*d^2*e*f^2*g + 2*a^2*e*g*(e^2*f^2 - 5*d*e*f*g + d^2*g^2) +
 a*b*(3*e^3*f^3 + 11*d*e^2*f^2*g + 11*d^2*e*f*g^2 + 3*d^3*g^3)))*ArcTan[(b
 + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(c*d^2 + e*(-(b*d) +
a*e))^2*(c*f^2 + g*(-(b*f) + a*g))^2) + (e^4*Log[d + e*x])/((c*d^2 + e*(-(
b*d) + a*e))^2*(e*f - d*g)) - (g^4*Log[f + g*x])/((e*f - d*g)*(c*f^2 + g*(
-(b*f) + a*g))^2) - ((c*e*f + c*d*g - b*e*g)*(c*(e^2*f^2 + d^2*g^2) + e*g*
(2*a*e*g - b*(e*f + d*g)))*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a
*e))^2*(c*f^2 + g*(-(b*f) + a*g))^2)
```

3.818.3 Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx$$

$$\Big\downarrow \text{1289}$$

$$\int \left(\frac{-ceg\left(a^2e^2g^2+2abeg(dg+ef)-\left(b^2\left(2d^2g^2+3defg+2e^2f^2\right)\right)\right)-b^2e^2g^2(-2aeg+bdg+bef)+c^2\left(2ade^2fg\right.}{\left(a+bx+cx^2\right)\left(ae^2-bd\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2ceg\left(a^2e^2g^2+abeg(dg+ef)-b^2(dg+ef)^2\right)+b^2e^2g^2(-2aeg+bdg+bef)-c^2\left(4ade^2fg^2-\right.\right.}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^2\left(cf^2-g(bf-ag\right.}$$

$$\frac{2c\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2aeg+bdg+bef)+b^2eg+2c^2df\right)}{\left(b^2-4ac\right)^{3/2}\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)}-$$

$$\frac{cx\left(-c(2aeg+bdg+bef)+b^2eg+2c^2df\right)+bc(cdf-3aeg)+2ac^2(dg+ef)+b^3eg-b^2c(dg+ef)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)}-$$

$$\frac{\log\left(a+bx+cx^2\right)\left(-beg+cdg+cef\right)\left(eg(2aeg-b(dg+ef))+c\left(d^2g^2+e^2f^2\right)\right)}{2\left(ae^2-bde+cd^2\right)^2\left(cf^2-g(bf-ag)\right)^2}+$$

$$\frac{e^4\log(d+ex)}{(ef-dg)\left(ae^2-bde+cd^2\right)^2}-\frac{g^4\log(f+gx)}{(ef-dg)\left(ag^2-bfg+cf^2\right)^2}$$

input
```
Int[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^2),x]
```

output
```
-((b^3*e*g - b^2*c*(e*f + d*g) + 2*a*c^2*(e*f + d*g) + b*c*(c*d*f - 3*a*e*
g) + c*(2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*x)/((b^2 - 4*a*
c)*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(b*f - a*g))*(a + b*x + c*x^2))) + (
2*c*(2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)*(c*f^2
- g*(b*f - a*g))) + ((b^2*e^2*g^2*(b*e*f + b*d*g - 2*a*e*g) - 2*c^3*d*f*(e
^2*f^2 + d*e*f*g + d^2*g^2) + 2*c*e*g*(a^2*e^2*g^2 + a*b*e*g*(e*f + d*g) -
 b^2*(e*f + d*g)^2) - c^2*(4*a*d*e^2*f*g^2 - b*(e^3*f^3 + 5*d*e^2*f^2*g +
5*d^2*e*f*g^2 + d^3*g^3)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b
^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)^2*(c*f^2 - g*(b*f - a*g))^2) + (e^4*Lo
g[d + e*x])/((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)) - (g^4*Log[f + g*x])/(
(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)^2) - ((c*e*f + c*d*g - b*e*g)*(c*(e^2*
f^2 + d^2*g^2) + e*g*(2*a*e*g - b*(e*f + d*g)))*Log[a + b*x + c*x^2])/(2*(
c*d^2 - b*d*e + a*e^2)^2*(c*f^2 - g*(b*f - a*g))^2)
```

3.818.3.1 Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.818.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2227 vs. $2(635) = 1270$.

Time = 1.49 (sec) , antiderivative size = 2228, normalized size of antiderivative = 3.46

method	result	size
default	Expression too large to display	2228
risch	Expression too large to display	29824

input
```
int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-e^4/(d*g-e*f)/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)-1/(a*e^2-b*d*e+c*d^2)^2/(a*
g^2-b*f*g+c*f^2)^2*((c*(2*a^3*c*e^3*g^3-a^2*b^2*e^3*g^3-a^2*b*c*d*e^2*g^3-
a^2*b*c*e^3*f*g^2+2*a^2*c^2*d^2*e*g^3-2*a^2*c^2*d*e^2*f*g^2+2*a^2*c^2*e^3*
f^2*g+a*b^3*d*e^2*g^3+a*b^3*e^3*f*g^2-2*a*b^2*c*d^2*e*g^3-2*a*b^2*c*e^3*f^
2*g+a*b*c^2*d^3*g^3+a*b*c^2*d^2*e*f*g^2+a*b*c^2*d*e^2*f^2*g+a*b*c^2*e^3*f^
3-2*a*c^3*d^3*f*g^2+2*a*c^3*d^2*e*f^2*g-2*a*c^3*d*e^2*f^3-b^4*d*e^2*f*g^2+
2*b^3*c*d^2*e*f*g^2+2*b^3*c*d*e^2*f^2*g-b^2*c^2*d^3*f*g^2-5*b^2*c^2*d^2*e*
f^2*g-b^2*c^2*d*e^2*f^3+3*b*c^3*d^3*f^2*g+3*b*c^3*d^2*e*f^3-2*c^4*d^3*f^3)
/(4*a*c-b^2)*x+(3*a^3*b*c*e^3*g^3-2*a^3*c^2*d*e^2*g^3-2*a^3*c^2*e^3*f*g^2-
a^2*b^3*e^3*g^3-2*a^2*b^2*c*d*e^2*g^3-2*a^2*b^2*c*e^3*f*g^2+5*a^2*b*c^2*d^
2*e*g^3+3*a^2*b*c^2*d*e^2*f*g^2+5*a^2*b*c^2*e^3*f^2*g-2*a^2*c^3*d^3*g^3-2*
a^2*c^3*d^2*e*f*g^2-2*a^2*c^3*d*e^2*f^2*g-2*a^2*c^3*e^3*f^3+a*b^4*d*e^2*g^
3+a*b^4*e^3*f*g^2-2*a*b^3*c*d^2*e*g^3+a*b^3*c*d*e^2*f*g^2-2*a*b^3*c*e^3*f^
2*g+a*b^2*c^2*d^3*g^3-3*a*b^2*c^2*d^2*e*f*g^2-3*a*b^2*c^2*d*e^2*f^2*g+a*b^
2*c^2*e^3*f^3+a*b*c^3*d^3*f*g^2+7*a*b*c^3*d^2*e*f^2*g+a*b*c^3*d*e^2*f^3-2*
a*c^4*d^3*f^2*g-2*a*c^4*d^2*e*f^3-b^5*d*e^2*f*g^2+2*b^4*c*d^2*e*f*g^2+2*b^
4*c*d*e^2*f^2*g-b^3*c^2*d^3*f*g^2-4*b^3*c^2*d^2*e*f^2*g-b^3*c^2*d*e^2*f^3+
2*b^2*c^3*d^3*f^2*g+2*b^2*c^3*d^2*e*f^3-b*c^4*d^3*f^3)/(4*a*c-b^2))/(c*x^2
+b*x+a)+1/(4*a*c-b^2)*(1/2*(-8*a^2*b*c^2*e^3*g^3+8*a^2*c^3*d*e^2*g^3+8*a^2
*c^3*e^3*f*g^2+2*a*b^3*c*e^3*g^3+2*a*b^2*c^2*d*e^2*g^3+2*a*b^2*c^2*e^3*...
```

3.818.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.818.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a)**2,x)
```

output
```
Timed out
```

3.818.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.818.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3400 vs. 2(634) = 1268.

Time = 0.30 (sec) , antiderivative size = 3400, normalized size of antiderivative = 5.28

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
e^5*log(abs(e*x + d))/(c^2*d^4*e^2*f - 2*b*c*d^3*e^3*f + b^2*d^2*e^4*f + 2
*a*c*d^2*e^4*f - 2*a*b*d*e^5*f + a^2*e^6*f - c^2*d^5*e*g + 2*b*c*d^4*e^2*g
 - b^2*d^3*e^3*g - 2*a*c*d^3*e^3*g + 2*a*b*d^2*e^4*g - a^2*d*e^5*g) - g^5*
log(abs(g*x + f))/(c^2*e*f^5*g - c^2*d*f^4*g^2 - 2*b*c*e*f^4*g^2 + 2*b*c*d
*f^3*g^3 + b^2*e*f^3*g^3 + 2*a*c*e*f^3*g^3 - b^2*d*f^2*g^4 - 2*a*c*d*f^2*g
^4 - 2*a*b*e*f^2*g^4 + 2*a*b*d*f*g^5 + a^2*e*f*g^5 - a^2*d*g^6) - 1/2*(c^2
*e^3*f^3 + c^2*d*e^2*f^2*g - 2*b*c*e^3*f^2*g + c^2*d^2*e*f*g^2 - 2*b*c*d*e
^2*f*g^2 + b^2*e^3*f*g^2 + 2*a*c*e^3*f*g^2 + c^2*d^3*g^3 - 2*b*c*d^2*e*g^3
 + b^2*d*e^2*g^3 + 2*a*c*d*e^2*g^3 - 2*a*b*e^3*g^3)*log(c*x^2 + b*x + a)/(
c^4*d^4*f^4 - 2*b*c^3*d^3*e*f^4 + b^2*c^2*d^2*e^2*f^4 + 2*a*c^3*d^2*e^2*f^
4 - 2*a*b*c^2*d*e^3*f^4 + a^2*c^2*e^4*f^4 - 2*b*c^3*d^4*f^3*g + 4*b^2*c^2*
d^3*e*f^3*g - 2*b^3*c*d^2*e^2*f^3*g - 4*a*b*c^2*d^2*e^2*f^3*g + 4*a*b^2*c*
d*e^3*f^3*g - 2*a^2*b*c*e^4*f^3*g + b^2*c^2*d^4*f^2*g^2 + 2*a*c^3*d^4*f^2*
g^2 - 2*b^3*c*d^3*e*f^2*g^2 - 4*a*b*c^2*d^3*e*f^2*g^2 + b^4*d^2*e^2*f^2*g^
2 + 4*a*b^2*c*d^2*e^2*f^2*g^2 + 4*a^2*c^2*d^2*e^2*f^2*g^2 - 2*a*b^3*d*e^3*
f^2*g^2 - 4*a^2*b*c*d*e^3*f^2*g^2 + a^2*b^2*e^4*f^2*g^2 + 2*a^3*c*e^4*f^2*
g^2 - 2*a*b*c^2*d^4*f*g^3 + 4*a*b^2*c*d^3*e*f*g^3 - 2*a*b^3*d^2*e^2*f*g^3
- 4*a^2*b*c*d^2*e^2*f*g^3 + 4*a^2*b^2*d*e^3*f*g^3 - 2*a^3*b*e^4*f*g^3 + a^
2*c^2*d^4*g^4 - 2*a^2*b*c*d^3*e*g^4 + a^2*b^2*d^2*e^2*g^4 + 2*a^3*c*d^2*e^
2*g^4 - 2*a^3*b*d*e^3*g^4 + a^4*e^4*g^4) - (4*c^5*d^3*f^3 - 6*b*c^4*d^2...
```

3.818.9 Mupad [B] (verification not implemented)

Time = 54.58 (sec) , antiderivative size = 130035, normalized size of antiderivative = 201.92

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^2),x)
```

output
```
((b^3*e*g + 2*a*c^2*d*g + 2*a*c^2*e*f + b*c^2*d*f - b^2*c*d*g - b^2*c*e*f
- 3*a*b*c*e*g)/(4*a*c^3*d^2*f^2 + 4*a^3*c*e^2*g^2 - a^2*b^2*e^2*g^2 + 4*a^
2*c^2*d^2*g^2 + 4*a^2*c^2*e^2*f^2 - b^2*c^2*d^2*f^2 + a*b^3*d*e*g^2 + b^3*
c*d*e*f^2 + a*b^3*e^2*f*g + b^3*c*d^2*f*g - a*b^2*c*d^2*g^2 - a*b^2*c*e^2*
f^2 - b^4*d*e*f*g - 4*a*b*c^2*d*e*f^2 - 4*a^2*b*c*d*e*g^2 - 4*a*b*c^2*d^2*
f*g - 4*a^2*b*c*e^2*f*g + 4*a*b^2*c*d*e*f*g) - (x*(2*a*c^2*e*g - 2*c^3*d*f
 + b*c^2*d*g + b*c^2*e*f - b^2*c*e*g))/(4*a*c^3*d^2*f^2 + 4*a^3*c*e^2*g^2
- a^2*b^2*e^2*g^2 + 4*a^2*c^2*d^2*g^2 + 4*a^2*c^2*e^2*f^2 - b^2*c^2*d^2*f^
2 + a*b^3*d*e*g^2 + b^3*c*d*e*f^2 + a*b^3*e^2*f*g + b^3*c*d^2*f*g - a*b^2*
c*d^2*g^2 - a*b^2*c*e^2*f^2 - b^4*d*e*f*g - 4*a*b*c^2*d*e*f^2 - 4*a^2*b*c*
d*e*g^2 - 4*a*b*c^2*d^2*f*g - 4*a^2*b*c*e^2*f*g + 4*a*b^2*c*d*e*f*g))/(a +
 b*x + c*x^2) + symsum(log((12*a^2*c^5*e^6*g^6 - 3*b^2*c^5*d^2*e^4*g^6 - 3
*b^2*c^5*e^6*f^2*g^4 + 4*c^7*d^2*e^4*f^2*g^4 - 2*a*b^2*c^4*e^6*g^6 + 16*a*
c^6*d^2*e^4*g^6 + 3*b^3*c^4*d*e^5*g^6 + 16*a*c^6*e^6*f^2*g^4 + 3*b^3*c^4*e
^6*f*g^5 - 4*b*c^6*d*e^5*f^2*g^4 - 4*b*c^6*d^2*e^4*f*g^5 - 16*a*b*c^5*d*e^
5*g^6 - 16*a*b*c^5*e^6*f*g^5 + 16*a*c^6*d*e^5*f*g^5)/(16*a^2*c^6*d^4*f^4 +
 a^4*b^4*e^4*g^4 + 16*a^4*c^4*d^4*g^4 + 16*a^4*c^4*e^4*f^4 + b^4*c^4*d^4*f
^4 + 16*a^6*c^2*e^4*g^4 + a^2*b^4*c^2*d^4*g^4 + a^2*b^4*c^2*e^4*f^4 - 8*a^
3*b^2*c^3*d^4*g^4 - 8*a^3*b^2*c^3*e^4*f^4 + a^2*b^6*d^2*e^2*g^4 + 32*a^3*c
^5*d^2*e^2*f^4 + 32*a^5*c^3*d^2*e^2*g^4 + b^6*c^2*d^2*e^2*f^4 + a^2*b^6...
```

3.818. $\int \frac{1}{(d+ex)(f+gx)(a+bx+cx^2)^2}\,dx$

3.819 $\int \frac{(d+ex)^3(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

3.819.1 Optimal result

Integrand size = 27, antiderivative size = 287

$$\begin{aligned}\int \frac{(d+ex)^3(a+bx+cx^2)}{\sqrt{f+gx}}\,dx &= -\frac{2(ef-dg)^3(cf^2-bfg+ag^2)\sqrt{f+gx}}{g^6} \\ &+ \frac{2(ef-dg)^2(cf(5ef-2dg)-g(4bef-bdg-3aeg))(f+gx)^{3/2}}{3g^6} \\ &+ \frac{2(ef-dg)(3eg(2bef-bdg-aeg)-c(10e^2f^2-8defg+d^2g^2))(f+gx)^{5/2}}{5g^6} \\ &- \frac{2e(eg(4bef-3bdg-aeg)-c(10e^2f^2-12defg+3d^2g^2))(f+gx)^{7/2}}{7g^6} \\ &- \frac{2e^2(5cef-3cdg-beg)(f+gx)^{9/2}}{9g^6} + \frac{2ce^3(f+gx)^{11/2}}{11g^6}\end{aligned}$$

output
```
2/3*(-d*g+e*f)^2*(c*f*(-2*d*g+5*e*f)-g*(-3*a*e*g-b*d*g+4*b*e*f))*(g*x+f)^(
3/2)/g^6+2/5*(-d*g+e*f)*(3*e*g*(-a*e*g-b*d*g+2*b*e*f)-c*(d^2*g^2-8*d*e*f*g
+10*e^2*f^2))*(g*x+f)^(5/2)/g^6-2/7*e*(e*g*(-a*e*g-3*b*d*g+4*b*e*f)-c*(3*d
^2*g^2-12*d*e*f*g+10*e^2*f^2))*(g*x+f)^(7/2)/g^6-2/9*e^2*(-b*e*g-3*c*d*g+5
*c*e*f)*(g*x+f)^(9/2)/g^6+2/11*c*e^3*(g*x+f)^(11/2)/g^6-2*(-d*g+e*f)^3*(a*
g^2-b*f*g+c*f^2)*(g*x+f)^(1/2)/g^6
```

3.819.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)^3 (a+bx+cx^2)}{\sqrt{f+gx}} dx$$
$$= \frac{2\sqrt{f+gx}(c(231d^3g^3(8f^2-4fgx+3g^2x^2)+297d^2eg^2(-16f^3+8f^2gx-6fg^2x^2+5g^3x^3)+33de^2g(128$$

input
```
Integrate[((d + e*x)^3*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

output
```
(2*Sqrt[f + g*x]*(c*(231*d^3*g^3*(8*f^2 - 4*f*g*x + 3*g^2*x^2) + 297*d^2*e
*g^2*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^3) + 33*d*e^2*g*(128*f^4
 - 64*f^3*g*x + 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + 35*g^4*x^4) - 5*e^3*(256*f
^5 - 128*f^4*g*x + 96*f^3*g^2*x^2 - 80*f^2*g^3*x^3 + 70*f*g^4*x^4 - 63*g^5
*x^5)) + 11*g*(9*a*g*(35*d^3*g^3 + 35*d^2*e*g^2*(-2*f + g*x) + 7*d*e^2*g*(
8*f^2 - 4*f*g*x + 3*g^2*x^2) + e^3*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*
g^3*x^3)) + b*(105*d^3*g^3*(-2*f + g*x) + 63*d^2*e*g^2*(8*f^2 - 4*f*g*x +
3*g^2*x^2) + 27*d*e^2*g*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^3) +
e^3*(128*f^4 - 64*f^3*g*x + 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + 35*g^4*x^4))))
)/(3465*g^6)
```

3.819.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3 (a+bx+cx^2)}{\sqrt{f+gx}} dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{e(f+gx)^{5/2} (c(3d^2g^2-12defg+10e^2f^2)-eg(-aeg-3bdg+4bef))}{g^5} + \frac{(f+gx)^{3/2}(ef-dg) (3eg(-aeg-\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2e(f+gx)^{7/2}\left(eg(-aeg-3bdg+4bef)-c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{7g^6}+\frac{2(f+gx)^{5/2}(ef-dg)\left(3eg(-aeg-bdg+2bef)-c\left(d^2g^2-8defg+10e^2f^2\right)\right)}{5g^6}-\frac{2\sqrt{f+gx}(ef-dg)^3\left(ag^2-bfg+cf^2\right)}{g^6}+\frac{2(f+gx)^{3/2}(ef-dg)^2(cf(5ef-2dg)-g(-3aeg-bdg+4bef))}{3g^6}-\frac{2e^2(f+gx)^{9/2}(-beg-3cdg+5cef)}{9g^6}+\frac{2ce^3(f+gx)^{11/2}}{11g^6}$$

input
```
Int[((d + e*x)^3*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

output
```
(-2*(e*f - d*g)^3*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^6 + (2*(e*f - d
*g)^2*(c*f*(5*e*f - 2*d*g) - g*(4*b*e*f - b*d*g - 3*a*e*g))*(f + g*x)^(3/2
))/(3*g^6) + (2*(e*f - d*g)*(3*e*g*(2*b*e*f - b*d*g - a*e*g) - c*(10*e^2*f
^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g^6) - (2*e*(e*g*(4*b*e*f -
 3*b*d*g - a*e*g) - c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(7/
2))/(7*g^6) - (2*e^2*(5*c*e*f - 3*c*d*g - b*e*g)*(f + g*x)^(9/2))/(9*g^6)
+ (2*c*e^3*(f + g*x)^(11/2))/(11*g^6)
```

3.819.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.819.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\frac{2c\,e^{3}(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left(3(dg-ef)e^{2}c+e^{3}(bg-2cf)\right)(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(3(dg-ef)^{2}ec+3(dg-ef)e^{2}(bg-2cf)+e^{3}\left(a\,g^{2}-bfg+c\,f^{2}\right)\right)(gx+f)^{\frac{7}{2}}}{7}}{}$
default	$\frac{\frac{2c\,e^{3}(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left(3(dg-ef)e^{2}c+e^{3}(bg-2cf)\right)(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(3(dg-ef)^{2}ec+3(dg-ef)e^{2}(bg-2cf)+e^{3}\left(a\,g^{2}-bfg+c\,f^{2}\right)\right)(gx+f)^{\frac{7}{2}}}{7}}{}$
pseudoelliptic	$\frac{2\sqrt{gx+f}\left(\left(\frac{x^{3}\left(\frac{7}{11}c\,x^{2}+\frac{7}{9}bx+a\right)e^{3}}{7}+\frac{3d\left(\frac{5}{9}c\,x^{2}+\frac{5}{7}bx+a\right)x^{2}e^{2}}{5}+d^{2}x\left(\frac{3}{7}c\,x^{2}+\frac{3}{5}bx+a\right)e+d^{3}\left(\frac{1}{5}c\,x^{2}+\frac{1}{3}bx+a\right)\right)g^{5}-2\left(\left(\frac{5}{99}c\,x\right.\right.\right.}{}$
gosper	$\frac{2\sqrt{gx+f}\,(315c\,e^{3}x^{5}g^{5}+385b\,e^{3}g^{5}x^{4}+1155cd\,e^{2}g^{5}x^{4}-350c\,e^{3}f\,g^{4}x^{4}+495a\,e^{3}g^{5}x^{3}+1485bd\,e^{2}g^{5}x^{3}-440b\,e^{3}f\,g^{4}x^{3}+148}{}$
trager	$\frac{2\sqrt{gx+f}\,(315c\,e^{3}x^{5}g^{5}+385b\,e^{3}g^{5}x^{4}+1155cd\,e^{2}g^{5}x^{4}-350c\,e^{3}f\,g^{4}x^{4}+495a\,e^{3}g^{5}x^{3}+1485bd\,e^{2}g^{5}x^{3}-440b\,e^{3}f\,g^{4}x^{3}+148}{}$
risch	$\frac{2\sqrt{gx+f}\,(315c\,e^{3}x^{5}g^{5}+385b\,e^{3}g^{5}x^{4}+1155cd\,e^{2}g^{5}x^{4}-350c\,e^{3}f\,g^{4}x^{4}+495a\,e^{3}g^{5}x^{3}+1485bd\,e^{2}g^{5}x^{3}-440b\,e^{3}f\,g^{4}x^{3}+148}{}$

input
```
int((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/g^6*(1/11*c*e^3*(g*x+f)^(11/2)+1/9*(3*(d*g-e*f)*e^2*c+e^3*(b*g-2*c*f))*(
g*x+f)^(9/2)+1/7*(3*(d*g-e*f)^2*e*c+3*(d*g-e*f)*e^2*(b*g-2*c*f)+e^3*(a*g^2
-b*f*g+c*f^2))*(g*x+f)^(7/2)+1/5*((d*g-e*f)^3*c+3*(d*g-e*f)^2*e*(b*g-2*c*f
)+3*(d*g-e*f)*e^2*(a*g^2-b*f*g+c*f^2))*(g*x+f)^(5/2)+1/3*((d*g-e*f)^3*(b*g
-2*c*f)+3*(d*g-e*f)^2*e*(a*g^2-b*f*g+c*f^2))*(g*x+f)^(3/2)+(d*g-e*f)^3*(a*
g^2-b*f*g+c*f^2)*(g*x+f)^(1/2))
```

3.819.5 Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.49

$$\int \frac{(d+ex)^3\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\,(315\,ce^3g^5x^5-1280\,ce^3f^5+3465\,ad^3g^5+1408\,(3\,cde^2+be^3)f^4g-1584\,(3\,cd^2e+3\,bde^2+ae^3)f^3g^2+}{}$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
2/3465*(315*c*e^3*g^5*x^5 - 1280*c*e^3*f^5 + 3465*a*d^3*g^5 + 1408*(3*c*d*
e^2 + b*e^3)*f^4*g - 1584*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^3*g^2 + 1848*(
c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 - 2310*(b*d^3 + 3*a*d^2*e)*f*g^4 -
35*(10*c*e^3*f*g^4 - 11*(3*c*d*e^2 + b*e^3)*g^5)*x^4 + 5*(80*c*e^3*f^2*g^3
 - 88*(3*c*d*e^2 + b*e^3)*f*g^4 + 99*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*g^5)*
x^3 - 3*(160*c*e^3*f^3*g^2 - 176*(3*c*d*e^2 + b*e^3)*f^2*g^3 + 198*(3*c*d^
2*e + 3*b*d*e^2 + a*e^3)*f*g^4 - 231*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*g^5)*
x^2 + (640*c*e^3*f^4*g - 704*(3*c*d*e^2 + b*e^3)*f^3*g^2 + 792*(3*c*d^2*e
+ 3*b*d*e^2 + a*e^3)*f^2*g^3 - 924*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f*g^4 +
 1155*(b*d^3 + 3*a*d^2*e)*g^5)*x)*sqrt(g*x + f)/g^6
```

3.819.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(291) = 582$.

Time = 1.23 (sec) , antiderivative size = 695, normalized size of antiderivative = 2.42

$$\int \frac{(d+ex)^3\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{ce^3(f+gx)^{\frac{11}{2}}}{11g^5}+\frac{(f+gx)^{\frac{9}{2}}\left(be^3g+3cde^2g-5ce^3f\right)}{9g^5}+\frac{(f+gx)^{\frac{7}{2}}\left(ae^3g^2+3bde^2g^2-4be^3fg+3cd^2eg^2-12cde^2fg+10ce^3f^2\right)}{7g^5}+\frac{(f+gx)^{\frac{5}{2}}\cdot\left(3ade^2g^3-3ae^3fg^2+\right.}{} \\ \frac{ad^3x+\frac{ce^3x^6}{6}+\frac{x^5\left(be^3+3cde^2\right)}{5}+\frac{x^4\left(ae^3+3bde^2+3cd^2e\right)}{4}+\frac{x^3\cdot\left(3ade^2+3bd^2e+cd^3\right)}{3}+\frac{x^2\cdot\left(3ad^2e+bd^3\right)}{2}}{\sqrt{f}} \end{cases}$$

input
```
integrate((e*x+d)**3*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

output
```
Piecewise((2*(c*e**3*(f + g*x)**(11/2)/(11*g**5) + (f + g*x)**(9/2)*(b*e**
3*g + 3*c*d*e**2*g - 5*c*e**3*f)/(9*g**5) + (f + g*x)**(7/2)*(a*e**3*g**2
+ 3*b*d*e**2*g**2 - 4*b*e**3*f*g + 3*c*d**2*e*g**2 - 12*c*d*e**2*f*g + 10*
c*e**3*f**2)/(7*g**5) + (f + g*x)**(5/2)*(3*a*d*e**2*g**3 - 3*a*e**3*f*g**
2 + 3*b*d**2*e*g**3 - 9*b*d*e**2*f*g**2 + 6*b*e**3*f**2*g + c*d**3*g**3 -
9*c*d**2*e*f*g**2 + 18*c*d*e**2*f**2*g - 10*c*e**3*f**3)/(5*g**5) + (f + g
*x)**(3/2)*(3*a*d**2*e*g**4 - 6*a*d*e**2*f*g**3 + 3*a*e**3*f**2*g**2 + b*d
**3*g**4 - 6*b*d**2*e*f*g**3 + 9*b*d*e**2*f**2*g**2 - 4*b*e**3*f**3*g - 2*
c*d**3*f*g**3 + 9*c*d**2*e*f**2*g**2 - 12*c*d*e**2*f**3*g + 5*c*e**3*f**4)
/(3*g**5) + sqrt(f + g*x)*(a*d**3*g**5 - 3*a*d**2*e*f*g**4 + 3*a*d*e**2*f*
*2*g**3 - a*e**3*f**3*g**2 - b*d**3*f*g**4 + 3*b*d**2*e*f**2*g**3 - 3*b*d*
e**2*f**3*g**2 + b*e**3*f**4*g + c*d**3*f**2*g**3 - 3*c*d**2*e*f**3*g**2 +
 3*c*d*e**2*f**4*g - c*e**3*f**5)/g**5)/g, Ne(g, 0)), ((a*d**3*x + c*e**3*
x**6/6 + x**5*(b*e**3 + 3*c*d*e**2)/5 + x**4*(a*e**3 + 3*b*d*e**2 + 3*c*d*
*2*e)/4 + x**3*(3*a*d*e**2 + 3*b*d**2*e + c*d**3)/3 + x**2*(3*a*d**2*e + b
*d**3)/2)/sqrt(f), True))
```

3.819.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.49

$$\int \frac{(d+ex)^3 (a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(315\,(gx+f)^{\frac{11}{2}}ce^3 - 385\,(5\,ce^3f - (3\,cde^2+be^3)g)(gx+f)^{\frac{9}{2}} + 495\,(10\,ce^3f^2 - 4\,(3\,cde^2+be^3)fg + (3\right.}{}$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
2/3465*(315*(g*x + f)^(11/2)*c*e^3 - 385*(5*c*e^3*f - (3*c*d*e^2 + b*e^3)*
g)*(g*x + f)^(9/2) + 495*(10*c*e^3*f^2 - 4*(3*c*d*e^2 + b*e^3)*f*g + (3*c*
d^2*e + 3*b*d*e^2 + a*e^3)*g^2)*(g*x + f)^(7/2) - 693*(10*c*e^3*f^3 - 6*(3
*c*d*e^2 + b*e^3)*f^2*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f*g^2 - (c*d^3
 + 3*b*d^2*e + 3*a*d*e^2)*g^3)*(g*x + f)^(5/2) + 1155*(5*c*e^3*f^4 - 4*(3*
c*d*e^2 + b*e^3)*f^3*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^2*g^2 - 2*(c*
d^3 + 3*b*d^2*e + 3*a*d*e^2)*f*g^3 + (b*d^3 + 3*a*d^2*e)*g^4)*(g*x + f)^(3
/2) - 3465*(c*e^3*f^5 - a*d^3*g^5 - (3*c*d*e^2 + b*e^3)*f^4*g + (3*c*d^2*e
 + 3*b*d*e^2 + a*e^3)*f^3*g^2 - (c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 +
(b*d^3 + 3*a*d^2*e)*f*g^4)*sqrt(g*x + f))/g^6
```

3.819.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(265) = 530$.

Time = 0.28 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.98

$$\int \frac{(d+ex)^3 (a+bx+cx^2)}{\sqrt{f+gx}} \, dx$$
$$= \frac{2 \left(3465 \sqrt{gx+f} ad^3 + \frac{1155 \left((gx+f)^{\frac{3}{2}} - 3 \sqrt{gx+f} f \right) bd^3}{g} + \frac{3465 \left((gx+f)^{\frac{3}{2}} - 3 \sqrt{gx+f} f \right) ad^2 e}{g} + \frac{231 \left(3 (gx+f)^{\frac{5}{2}} - 10 (gx+f)^{\frac{3}{2}} f + 1 \right.}{g^2}}{}$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
2/3465*(3465*sqrt(g*x + f)*a*d^3 + 1155*((g*x + f)^(3/2) - 3*sqrt(g*x + f)
*f)*b*d^3/g + 3465*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*d^2*e/g + 231*(
3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*d^3/g^2
 + 693*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*b
*d^2*e/g^2 + 693*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x +
 f)*f^2)*a*d*e^2/g^2 + 297*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*
(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d^2*e/g^3 + 297*(5*(g*x + f)
^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*
f^3)*b*d*e^2/g^3 + 99*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x
+ f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a*e^3/g^3 + 33*(35*(g*x + f)^(9/2)
- 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^
3 + 315*sqrt(g*x + f)*f^4)*c*d*e^2/g^4 + 11*(35*(g*x + f)^(9/2) - 180*(g*x
 + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sq
rt(g*x + f)*f^4)*b*e^3/g^4 + 5*(63*(g*x + f)^(11/2) - 385*(g*x + f)^(9/2)*
f + 990*(g*x + f)^(7/2)*f^2 - 1386*(g*x + f)^(5/2)*f^3 + 1155*(g*x + f)^(3
/2)*f^4 - 693*sqrt(g*x + f)*f^5)*c*e^3/g^5)/g
```

3.819.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^3 (a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \frac{(f+g\,x)^{9/2}\,(2\,b\,e^3\,g-10\,c\,e^3\,f+6\,c\,d\,e^2\,g)}{9\,g^6} + \frac{(f+g\,x)^{7/2}\,(6\,c\,d^2\,e\,g^2-24\,c\,d\,e^2\,f\,g+6\,b\,d\,e^2\,g^2+20\,c\,e^3\,f^2-8\,b\,e^3\,f\,g+2\,a\,e^3\,g^2)}{7\,g^6} + \frac{2\,(f+g\,x)^{5/2}\,(d\,g-e\,f)\,(c\,d^2\,g^2-8\,c\,d\,e\,f\,g+3\,b\,d\,e\,g^2+10\,c\,e^2\,f^2-6\,b\,e^2\,f\,g+3\,a\,e^2\,g^2)}{5\,g^6} + \frac{2\,\sqrt{f+g\,x}\,(d\,g-e\,f)^3\,(c\,f^2-b\,f\,g+a\,g^2)}{g^6} + \frac{2\,(f+g\,x)^{3/2}\,(d\,g-e\,f)^2\,(3\,a\,e\,g^2+b\,d\,g^2+5\,c\,e\,f^2-4\,b\,e\,f\,g-2\,c\,d\,f\,g)}{3\,g^6} + \frac{2\,c\,e^3\,(f+g\,x)^{11/2}}{11\,g^6}$$

input
```
int(((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
```

output
```
((f + g*x)^(9/2)*(2*b*e^3*g - 10*c*e^3*f + 6*c*d*e^2*g))/(9*g^6) + ((f + g
*x)^(7/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 - 8*b*e^3*f*g + 6*b*d*e^2*g^2 + 6*c*
d^2*e*g^2 - 24*c*d*e^2*f*g))/(7*g^6) + (2*(f + g*x)^(5/2)*(d*g - e*f)*(3*a
*e^2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 + 3*b*d*e*g^2 - 6*b*e^2*f*g - 8*c*d*e*
f*g))/(5*g^6) + (2*(f + g*x)^(1/2)*(d*g - e*f)^3*(a*g^2 + c*f^2 - b*f*g))/
g^6 + (2*(f + g*x)^(3/2)*(d*g - e*f)^2*(3*a*e*g^2 + b*d*g^2 + 5*c*e*f^2 -
4*b*e*f*g - 2*c*d*f*g))/(3*g^6) + (2*c*e^3*(f + g*x)^(11/2))/(11*g^6)
```

3.820 $\int \frac{(d+ex)^2(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

3.820.1 Optimal result

Integrand size = 27, antiderivative size = 212

$$\begin{aligned}
&\int \frac{(d+ex)^2\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx \\
&= \frac{2(ef-dg)^2\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}{g^5} \\
&\quad - \frac{2(ef-dg)(2cf(2ef-dg)-g(3bef-bdg-2aeg))(f+gx)^{3/2}}{3g^5} \\
&\quad - \frac{2\left(eg(3bef-2bdg-aeg)-c\left(6e^2f^2-6defg+d^2g^2\right)\right)(f+gx)^{5/2}}{5g^5} \\
&\quad - \frac{2e(4cef-2cdg-beg)(f+gx)^{7/2}}{7g^5}+\frac{2ce^2(f+gx)^{9/2}}{9g^5}
\end{aligned}$$

output
```
-2/3*(-d*g+e*f)*(2*c*f*(-d*g+2*e*f)-g*(-2*a*e*g-b*d*g+3*b*e*f))*(g*x+f)^(3
/2)/g^5-2/5*(e*g*(-a*e*g-2*b*d*g+3*b*e*f)-c*(d^2*g^2-6*d*e*f*g+6*e^2*f^2))
*(g*x+f)^(5/2)/g^5-2/7*e*(-b*e*g-2*c*d*g+4*c*e*f)*(g*x+f)^(7/2)/g^5+2/9*c*
e^2*(g*x+f)^(9/2)/g^5+2*(-d*g+e*f)^2*(a*g^2-b*f*g+c*f^2)*(g*x+f)^(1/2)/g^5
```

3.820.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)^2(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\sqrt{f+gx}(c(21d^2g^2(8f^2-4fgx+3g^2x^2)+18deg(-16f^3+8f^2gx-6fg^2x^2+5g^3x^3)+e^2(128f^4-64f$$

input
```
Integrate[((d + e*x)^2*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

output
```
(2*Sqrt[f + g*x]*(c*(21*d^2*g^2*(8*f^2 - 4*f*g*x + 3*g^2*x^2) + 18*d*e*g*(
-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^3) + e^2*(128*f^4 - 64*f^3*g*x
 + 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + 35*g^4*x^4)) + 3*g*(7*a*g*(15*d^2*g^2 +
 10*d*e*g*(-2*f + g*x) + e^2*(8*f^2 - 4*f*g*x + 3*g^2*x^2)) + b*(35*d^2*g^
2*(-2*f + g*x) + 14*d*e*g*(8*f^2 - 4*f*g*x + 3*g^2*x^2) - 3*e^2*(16*f^3 -
8*f^2*g*x + 6*f*g^2*x^2 - 5*g^3*x^3)))))/(315*g^5)
```

3.820.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$\Big\downarrow \text{1195}$$
$$\int \left(\frac{(f+gx)^{3/2}\left(c\left(d^2g^2-6defg+6e^2f^2\right)-eg(-aeg-2bdg+3bef)\right)}{g^4}+\frac{(dg-ef)^2\left(ag^2-bfg+cf^2\right)}{g^4\sqrt{f+gx}}+\frac{\sqrt{f+$$
$$\Big\downarrow \text{2009}$$

$$-\frac{2(f+gx)^{5/2}\left(eg(-aeg-2bdg+3bef)-c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{5g^5}+\frac{2\sqrt{f+gx}(ef-dg)^2\left(ag^2-bfg+cf^2\right)}{g^5}-\frac{2(f+gx)^{3/2}(ef-dg)(2cf(2ef-dg)-g(-2aeg-bdg+3bef))}{3g^5}-\frac{2e(f+gx)^{7/2}(-beg-2cdg+4cef)}{7g^5}+\frac{2ce^2(f+gx)^{9/2}}{9g^5}$$

input
```
Int[((d + e*x)^2*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

output
```
(2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^5 - (2*(e*f - d*
g)*(2*c*f*(2*e*f - d*g) - g*(3*b*e*f - b*d*g - 2*a*e*g))*(f + g*x)^(3/2))/
(3*g^5) - (2*(e*g*(3*b*e*f - 2*b*d*g - a*e*g) - c*(6*e^2*f^2 - 6*d*e*f*g +
 d^2*g^2))*(f + g*x)^(5/2))/(5*g^5) - (2*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f
+ g*x)^(7/2))/(7*g^5) + (2*c*e^2*(f + g*x)^(9/2))/(9*g^5)
```

3.820.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.820.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.91

method	result
pseudoelliptic	$\frac{2\sqrt{gx+f}\left(\left(\frac{\left(\frac{5}{9}cx^2+\frac{5}{7}bx+a\right)x^2e^2}{5}+\frac{2d\left(\frac{3}{7}cx^2+\frac{3}{5}bx+a\right)xe}{3}+d^2\left(\frac{1}{5}cx^2+\frac{1}{3}bx+a\right)\right)g^4-\frac{4\left(\frac{\left(\frac{10}{21}cx^2+\frac{9}{14}bx+a\right)xe^2}{5}+d\left(\frac{9}{35}cx^2+\cdots\right.\right.}{3}\cdots}{g^5}$
derivativedivides	$\frac{\frac{2ce^2(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(2(dg-ef)ec+e^2(bg-2cf)\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left((dg-ef)^2c+2(dg-ef)e(bg-2cf)+e^2\left(ag^2-bfg+cf^2\right)\right)(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left((d\cdots\right.}{}}{g^5}$
default	$\frac{\frac{2ce^2(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(2(dg-ef)ec+e^2(bg-2cf)\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left((dg-ef)^2c+2(dg-ef)e(bg-2cf)+e^2\left(ag^2-bfg+cf^2\right)\right)(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left((d\cdots\right.}{}}{g^5}$
gosper	$\frac{2\sqrt{gx+f}\,(35ce^2x^4g^4+45be^2g^4x^3+90cdeg^4x^3-40ce^2fg^3x^3+63ae^2g^4x^2+126bdeg^4x^2-54be^2fg^3x^2+63cd^2g^4x^2-10\cdots}{}$
trager	$\frac{2\sqrt{gx+f}\,(35ce^2x^4g^4+45be^2g^4x^3+90cdeg^4x^3-40ce^2fg^3x^3+63ae^2g^4x^2+126bdeg^4x^2-54be^2fg^3x^2+63cd^2g^4x^2-10\cdots}{}$
risch	$\frac{2\sqrt{gx+f}\,(35ce^2x^4g^4+45be^2g^4x^3+90cdeg^4x^3-40ce^2fg^3x^3+63ae^2g^4x^2+126bdeg^4x^2-54be^2fg^3x^2+63cd^2g^4x^2-10\cdots}{}$

input
```
int((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(g*x+f)^(1/2)*((1/5*(5/9*c*x^2+5/7*b*x+a)*x^2*e^2+2/3*d*(3/7*c*x^2+3/5*b
*x+a)*x*e+d^2*(1/5*c*x^2+1/3*b*x+a))*g^4-4/3*(1/5*(10/21*c*x^2+9/14*b*x+a)
*x*e^2+d*(9/35*c*x^2+2/5*b*x+a)*e+1/2*d^2*(2/5*c*x+b))*f*g^3+8/15*((2/7*c*
x^2+3/7*b*x+a)*e^2+2*d*(3/7*c*x+b)*e+c*d^2)*f^2*g^2-16/35*e*((4/9*c*x+b)*e
+2*c*d)*f^3*g+128/315*c*e^2*f^4)/g^5
```

3.820.5 Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^2\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$=\frac{2\,(35\,ce^2g^4x^4+128\,ce^2f^4+315\,ad^2g^4-144\,(2\,cde+be^2)f^3g+168\,(cd^2+2\,bde+ae^2)f^2g^2-210\,(bd^2+\cdots}{}$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*c*e^2*g^4*x^4 + 128*c*e^2*f^4 + 315*a*d^2*g^4 - 144*(2*c*d*e + b
*e^2)*f^3*g + 168*(c*d^2 + 2*b*d*e + a*e^2)*f^2*g^2 - 210*(b*d^2 + 2*a*d*e
)*f*g^3 - 5*(8*c*e^2*f*g^3 - 9*(2*c*d*e + b*e^2)*g^4)*x^3 + 3*(16*c*e^2*f^
2*g^2 - 18*(2*c*d*e + b*e^2)*f*g^3 + 21*(c*d^2 + 2*b*d*e + a*e^2)*g^4)*x^2
 - (64*c*e^2*f^3*g - 72*(2*c*d*e + b*e^2)*f^2*g^2 + 84*(c*d^2 + 2*b*d*e +
a*e^2)*f*g^3 - 105*(b*d^2 + 2*a*d*e)*g^4)*x)*sqrt(g*x + f)/g^5
```

3.820.6 Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.01

$$\int \frac{(d+ex)^2\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{ce^2(f+gx)^{\frac{9}{2}}}{9g^4}+\frac{(f+gx)^{\frac{7}{2}}\left(be^2g+2cdeg-4ce^2f\right)}{7g^4}+\frac{(f+gx)^{\frac{5}{2}}\left(ae^2g^2+2bdeg^2-3be^2fg+cd^2g^2-6cdefg+6ce^2f^2\right)}{5g^4}+\frac{(f+gx)^{\frac{3}{2}}\cdot\left(2adeg^3-2ae^2fg^2+bd^2g^3-4b\right.}{}\right.}{g} \\ \frac{ad^2x+\frac{ce^2x^5}{5}+\frac{x^4\left(be^2+2cde\right)}{4}+\frac{x^3\left(ae^2+2bde+cd^2\right)}{3}+\frac{x^2\cdot\left(2ade+bd^2\right)}{2}}{\sqrt{f}} \end{cases}$$

input
```
integrate((e*x+d)**2*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

output
```
Piecewise((2*(c*e**2*(f + g*x)**(9/2)/(9*g**4) + (f + g*x)**(7/2)*(b*e**2*
g + 2*c*d*e*g - 4*c*e**2*f)/(7*g**4) + (f + g*x)**(5/2)*(a*e**2*g**2 + 2*b
*d*e*g**2 - 3*b*e**2*f*g + c*d**2*g**2 - 6*c*d*e*f*g + 6*c*e**2*f**2)/(5*g
**4) + (f + g*x)**(3/2)*(2*a*d*e*g**3 - 2*a*e**2*f*g**2 + b*d**2*g**3 - 4*
b*d*e*f*g**2 + 3*b*e**2*f**2*g - 2*c*d**2*f*g**2 + 6*c*d*e*f**2*g - 4*c*e*
*2*f**3)/(3*g**4) + sqrt(f + g*x)*(a*d**2*g**4 - 2*a*d*e*f*g**3 + a*e**2*f
**2*g**2 - b*d**2*f*g**3 + 2*b*d*e*f**2*g**2 - b*e**2*f**3*g + c*d**2*f**2
*g**2 - 2*c*d*e*f**3*g + c*e**2*f**4)/g**4)/g, Ne(g, 0)), ((a*d**2*x + c*e
**2*x**5/5 + x**4*(b*e**2 + 2*c*d*e)/4 + x**3*(a*e**2 + 2*b*d*e + c*d**2)/
3 + x**2*(2*a*d*e + b*d**2)/2)/sqrt(f), True))
```

3.820.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^2 (a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(35\,(gx+f)^{\frac{9}{2}}ce^2 - 45\,(4\,ce^2f - (2\,cde + be^2)g)(gx+f)^{\frac{7}{2}} + 63\,(6\,ce^2f^2 - 3\,(2\,cde + be^2)fg + (cd^2 + 2\,b\right.}{}$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
2/315*(35*(g*x + f)^(9/2)*c*e^2 - 45*(4*c*e^2*f - (2*c*d*e + b*e^2)*g)*(g*
x + f)^(7/2) + 63*(6*c*e^2*f^2 - 3*(2*c*d*e + b*e^2)*f*g + (c*d^2 + 2*b*d*
e + a*e^2)*g^2)*(g*x + f)^(5/2) - 105*(4*c*e^2*f^3 - 3*(2*c*d*e + b*e^2)*f
^2*g + 2*(c*d^2 + 2*b*d*e + a*e^2)*f*g^2 - (b*d^2 + 2*a*d*e)*g^3)*(g*x + f
)^(3/2) + 315*(c*e^2*f^4 + a*d^2*g^4 - (2*c*d*e + b*e^2)*f^3*g + (c*d^2 +
2*b*d*e + a*e^2)*f^2*g^2 - (b*d^2 + 2*a*d*e)*f*g^3)*sqrt(g*x + f))/g^5
```

3.820.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.71

$$\int \frac{(d+ex)^2 (a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(315\,\sqrt{gx+f}ad^2 + \frac{105\left((gx+f)^{\frac{3}{2}} - 3\,\sqrt{gx+f}f\right)bd^2}{g} + \frac{210\left((gx+f)^{\frac{3}{2}} - 3\,\sqrt{gx+f}f\right)ade}{g} + \frac{21\left(3\,(gx+f)^{\frac{5}{2}} - 10\,(gx+f)^{\frac{3}{2}}f + 15\,\sqrt{gx\right.}{g^2}\right.}{}$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
2/315*(315*sqrt(g*x + f)*a*d^2 + 105*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)
*b*d^2/g + 210*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*d*e/g + 21*(3*(g*x
+ f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*d^2/g^2 + 42*(
3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*b*d*e/g^2
 + 21*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*a*
e^2/g^2 + 18*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2
)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d*e/g^3 + 9*(5*(g*x + f)^(7/2) - 21*(g*x +
 f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*b*e^2/g^3 + (
35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420
*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*c*e^2/g^4)/g
```

3.820.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^2(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \frac{(f+g\,x)^{7/2}\,(2\,b\,e^2\,g-8\,c\,e^2\,f+4\,c\,d\,e\,g)}{7\,g^5} + \frac{(f+g\,x)^{5/2}\,(2\,c\,d^2\,g^2-12\,c\,d\,e\,f\,g+4\,b\,d\,e\,g^2+12\,c\,e^2\,f^2-6\,b\,e^2\,f\,g+2\,a\,e^2\,g^2)}{5\,g^5} + \frac{2\,(f+g\,x)^{3/2}\,(d\,g-e\,f)\,(2\,a\,e\,g^2+b\,d\,g^2+4\,c\,e\,f^2-3\,b\,e\,f\,g-2\,c\,d\,f\,g)}{3\,g^5} + \frac{2\,\sqrt{f+g\,x}\,(d\,g-e\,f)^2\,(c\,f^2-b\,f\,g+a\,g^2)}{g^5} + \frac{2\,c\,e^2\,(f+g\,x)^{9/2}}{9\,g^5}$$

input
```
int(((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
```

output
```
((f + g*x)^(7/2)*(2*b*e^2*g - 8*c*e^2*f + 4*c*d*e*g))/(7*g^5) + ((f + g*x)
^(5/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 + 4*b*d*e*g^2 - 6*b*e^2*f
*g - 12*c*d*e*f*g))/(5*g^5) + (2*(f + g*x)^(3/2)*(d*g - e*f)*(2*a*e*g^2 +
b*d*g^2 + 4*c*e*f^2 - 3*b*e*f*g - 2*c*d*f*g))/(3*g^5) + (2*(f + g*x)^(1/2)
*(d*g - e*f)^2*(a*g^2 + c*f^2 - b*f*g))/g^5 + (2*c*e^2*(f + g*x)^(9/2))/(9
*g^5)
```

3.821 $\int \frac{(d+ex)(a+bx+cx^2)}{\sqrt{f+gx}} \, dx$

3.821.1 Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{(d+ex)(a+bx+cx^2)}{\sqrt{f+gx}} \, dx = -\frac{2(ef-dg)(cf^2-bfg+ag^2)\sqrt{f+gx}}{g^4} + \frac{2(cf(3ef-2dg)-g(2bef-bdg-aeg))(f+gx)^{3/2}}{3g^4} - \frac{2(3cef-cdg-beg)(f+gx)^{5/2}}{5g^4} + \frac{2ce(f+gx)^{7/2}}{7g^4}$$

output
```
2/3*(c*f*(-2*d*g+3*e*f)-g*(-a*e*g-b*d*g+2*b*e*f))*(g*x+f)^(3/2)/g^4-2/5*(-
b*e*g-c*d*g+3*c*e*f)*(g*x+f)^(5/2)/g^4+2/7*c*e*(g*x+f)^(7/2)/g^4-2*(-d*g+e
*f)*(a*g^2-b*f*g+c*f^2)*(g*x+f)^(1/2)/g^4
```

3.821.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)(a+bx+cx^2)}{\sqrt{f+gx}} \, dx = \frac{2\sqrt{f+gx}(7g(5bdg(-2f+gx)+5ag(-2ef+3dg+egx)+be(8f^2-4fgx+3g^2x^2))+c(7dg(8f^2-4fg}{105g^4}$$

input
```
Integrate[((d + e*x)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

output
```
(2*Sqrt[f + g*x]*(7*g*(5*b*d*g*(-2*f + g*x) + 5*a*g*(-2*e*f + 3*d*g + e*g*
x) + b*e*(8*f^2 - 4*f*g*x + 3*g^2*x^2)) + c*(7*d*g*(8*f^2 - 4*f*g*x + 3*g^
2*x^2) - 3*e*(16*f^3 - 8*f^2*g*x + 6*f*g^2*x^2 - 5*g^3*x^3))))/(105*g^4)
```

3.821.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{(dg-ef)\left(ag^2-bfg+cf^2\right)}{g^3\sqrt{f+gx}}+\frac{\sqrt{f+gx}(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{g^3}+\frac{(f+gx)^{3/2}(beg+cdg-$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\sqrt{f+gx}(ef-dg)\left(ag^2-bfg+cf^2\right)}{g^4}+\frac{2(f+gx)^{3/2}(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{3g^4}-\frac{2(f+gx)^{5/2}(-beg-cdg+3cef)}{5g^4}+\frac{2ce(f+gx)^{7/2}}{7g^4}$$

input
```
Int[((d + e*x)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

output
```
(-2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^4 + (2*(c*f*(3*e*
f - 2*d*g) - g*(2*b*e*f - b*d*g - a*e*g))*(f + g*x)^(3/2))/(3*g^4) - (2*(3
*c*e*f - c*d*g - b*e*g)*(f + g*x)^(5/2))/(5*g^4) + (2*c*e*(f + g*x)^(7/2))
/(7*g^4)
```

3.821.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.821.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$\frac{2\sqrt{gx+f}\,\left(\left(\frac{\left(\frac{3}{7}c\,x^{2}+\frac{3}{5}bx+a\right)xe}{3}+d\left(\frac{1}{5}c\,x^{2}+\frac{1}{3}bx+a\right)\right)g^{3}-\frac{2f\left(\left(\frac{9}{35}c\,x^{2}+\frac{2}{5}bx+a\right)e+d\left(\frac{2cx}{5}+b\right)\right)g^{2}}{3}+\frac{8\left(\left(\frac{3cx}{7}+b\right)e+cd\right)f^{2}g}{15}-\cdots}{g^{4}}$
derivativedivides	$\frac{\frac{2ce(gx+f)^{\frac{7}{2}}}{7}+\frac{2((dg-ef)c+e(bg-2cf))(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left((dg-ef)(bg-2cf)+e\left(a\,g^{2}-bfg+c\,f^{2}\right)\right)(gx+f)^{\frac{3}{2}}}{3}+2(dg-ef)\left(a\,g^{2}-bfg+c\cdots\right.}{g^{4}}$
default	$\frac{\frac{2ce(gx+f)^{\frac{7}{2}}}{7}+\frac{2((dg-ef)c+e(bg-2cf))(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left((dg-ef)(bg-2cf)+e\left(a\,g^{2}-bfg+c\,f^{2}\right)\right)(gx+f)^{\frac{3}{2}}}{3}+2(dg-ef)\left(a\,g^{2}-bfg+c\cdots\right.}{g^{4}}$
gosper	$\frac{2\sqrt{gx+f}\,\left(15ce\,x^{3}g^{3}+21be\,g^{3}x^{2}+21cd\,g^{3}x^{2}-18cef\,g^{2}x^{2}+35ae\,g^{3}x+35bd\,g^{3}x-28bef\,g^{2}x-28cdf\,g^{2}x+24ce\,f^{2}gx+105a\cdots\right.}{105g^{4}}$
trager	$\frac{2\sqrt{gx+f}\,\left(15ce\,x^{3}g^{3}+21be\,g^{3}x^{2}+21cd\,g^{3}x^{2}-18cef\,g^{2}x^{2}+35ae\,g^{3}x+35bd\,g^{3}x-28bef\,g^{2}x-28cdf\,g^{2}x+24ce\,f^{2}gx+105a\cdots\right.}{105g^{4}}$
risch	$\frac{2\sqrt{gx+f}\,\left(15ce\,x^{3}g^{3}+21be\,g^{3}x^{2}+21cd\,g^{3}x^{2}-18cef\,g^{2}x^{2}+35ae\,g^{3}x+35bd\,g^{3}x-28bef\,g^{2}x-28cdf\,g^{2}x+24ce\,f^{2}gx+105a\cdots\right.}{105g^{4}}$

input
```
int((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(g*x+f)^(1/2)*((1/3*(3/7*c*x^2+3/5*b*x+a)*x*e+d*(1/5*c*x^2+1/3*b*x+a))*g
^3-2/3*f*((9/35*c*x^2+2/5*b*x+a)*e+d*(2/5*c*x+b))*g^2+8/15*((3/7*c*x+b)*e+
c*d)*f^2*g-16/35*c*e*f^3)/g^4
```

3.821.5 Fricas [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(15\,ceg^3x^3 - 48\,cef^3 + 105\,adg^3 + 56\,(cd+be)f^2g - 70\,(bd+ae)fg^2 - 3\left(6\,cefg^2 - 7\,(cd+be)g^3\right)x^2 + \right.}{105\,g^4}$$

input
```
integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*c*e*g^3*x^3 - 48*c*e*f^3 + 105*a*d*g^3 + 56*(c*d + b*e)*f^2*g -
70*(b*d + a*e)*f*g^2 - 3*(6*c*e*f*g^2 - 7*(c*d + b*e)*g^3)*x^2 + (24*c*e*f
^2*g - 28*(c*d + b*e)*f*g^2 + 35*(b*d + a*e)*g^3)*x)*sqrt(g*x + f)/g^4
```

3.821.6 Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{ce(f+gx)^{\frac{7}{2}}}{7g^3} + \frac{(f+gx)^{\frac{5}{2}}(beg+cdg-3cef)}{5g^3} + \frac{(f+gx)^{\frac{3}{2}}\left(aeg^2+bdg^2-2befg-2cdfg+3cef^2\right)}{3g^3} + \frac{\sqrt{f+gx}\left(adg^3-aefg^2-bdfg^2+bef^2g+cdf^2g-cef^3\right)}{g^3}\right)}{g} & \text{for} \\ \frac{adx+\frac{cex^4}{4}+\frac{x^3(be+cd)}{3}+\frac{x^2(ae+bd)}{2}}{\sqrt{f}} & \text{oth} \end{cases}$$

input
```
integrate((e*x+d)*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

output
```
Piecewise((2*(c*e*(f + g*x)**(7/2)/(7*g**3) + (f + g*x)**(5/2)*(b*e*g + c*
d*g - 3*c*e*f)/(5*g**3) + (f + g*x)**(3/2)*(a*e*g**2 + b*d*g**2 - 2*b*e*f*
g - 2*c*d*f*g + 3*c*e*f**2)/(3*g**3) + sqrt(f + g*x)*(a*d*g**3 - a*e*f*g**
2 - b*d*f*g**2 + b*e*f**2*g + c*d*f**2*g - c*e*f**3)/g**3)/g, Ne(g, 0)), (
(a*d*x + c*e*x**4/4 + x**3*(b*e + c*d)/3 + x**2*(a*e + b*d)/2)/sqrt(f), Tr
ue))
```

3.821.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(15\,(gx+f)^{\frac{7}{2}}ce - 21\,(3\,cef-(cd+be)g)(gx+f)^{\frac{5}{2}} + 35\,(3\,cef^2-2\,(cd+be)fg+(bd+ae)g^2)(gx+\right.}{105\,g^4}$$

input
```
integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
2/105*(15*(g*x + f)^(7/2)*c*e - 21*(3*c*e*f - (c*d + b*e)*g)*(g*x + f)^(5/
2) + 35*(3*c*e*f^2 - 2*(c*d + b*e)*f*g + (b*d + a*e)*g^2)*(g*x + f)^(3/2)
- 105*(c*e*f^3 - a*d*g^3 - (c*d + b*e)*f^2*g + (b*d + a*e)*f*g^2)*sqrt(g*x
 + f))/g^4
```

3.821.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.43

$$\int \frac{(d+ex)\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(105\,\sqrt{gx+f}ad + \frac{35\left((gx+f)^{\frac{3}{2}}-3\,\sqrt{gx+f}f\right)bd}{g} + \frac{35\left((gx+f)^{\frac{3}{2}}-3\,\sqrt{gx+f}f\right)ae}{g} + \frac{7\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\,\sqrt{gx+f}f^2\right)c}{g^2}\right.}{105\,g}$$

input
```
integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
2/105*(105*sqrt(g*x + f)*a*d + 35*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*b*
d/g + 35*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*e/g + 7*(3*(g*x + f)^(5/2
) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*d/g^2 + 7*(3*(g*x + f)^
(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*b*e/g^2 + 3*(5*(g*x +
 f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x +
f)*f^3)*c*e/g^3)/g
```

3.821.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$=\frac{(f+g\,x)^{5/2}\,(2\,b\,e\,g+2\,c\,d\,g-6\,c\,e\,f)}{5\,g^4}$$
$$+\frac{(f+g\,x)^{3/2}\,(2\,a\,e\,g^2+2\,b\,d\,g^2+6\,c\,e\,f^2-4\,b\,e\,f\,g-4\,c\,d\,f\,g)}{3\,g^4}$$
$$+\frac{2\,\sqrt{f+g\,x}\,(d\,g-e\,f)\,(c\,f^2-b\,f\,g+a\,g^2)}{g^4}+\frac{2\,c\,e\,(f+g\,x)^{7/2}}{7\,g^4}$$

input
```
int(((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
```

output
```
((f + g*x)^(5/2)*(2*b*e*g + 2*c*d*g - 6*c*e*f))/(5*g^4) + ((f + g*x)^(3/2)
*(2*a*e*g^2 + 2*b*d*g^2 + 6*c*e*f^2 - 4*b*e*f*g - 4*c*d*f*g))/(3*g^4) + (2
*(f + g*x)^(1/2)*(d*g - e*f)*(a*g^2 + c*f^2 - b*f*g))/g^4 + (2*c*e*(f + g*
x)^(7/2))/(7*g^4)
```

3.822 $\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx$

3.822.1 Optimal result

Integrand size = 20, antiderivative size = 73

$$\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx=\frac{2(cf^2-bfg+ag^2)\sqrt{f+gx}}{g^3}-\frac{2(2cf-bg)(f+gx)^{3/2}}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}$$

output
```
-2/3*(-b*g+2*c*f)*(g*x+f)^(3/2)/g^3+2/5*c*(g*x+f)^(5/2)/g^3+2*(a*g^2-b*f*g
+c*f^2)*(g*x+f)^(1/2)/g^3
```

3.822.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

$$\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx=\frac{2\sqrt{f+gx}(5g(-2bf+3ag+bgx)+c(8f^2-4fgx+3g^2x^2))}{15g^3}$$

input
```
Integrate[(a + b*x + c*x^2)/Sqrt[f + g*x],x]
```

output
```
(2*Sqrt[f + g*x]*(5*g*(-2*b*f + 3*a*g + b*g*x) + c*(8*f^2 - 4*f*g*x + 3*g^
2*x^2)))/(15*g^3)
```

3.822.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{ag^2-bfg+cf^2}{g^2\sqrt{f+gx}}+\frac{\sqrt{f+gx}(bg-2cf)}{g^2}+\frac{c(f+gx)^{3/2}}{g^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{f+gx}\left(ag^2-bfg+cf^2\right)}{g^3}-\frac{2(f+gx)^{3/2}(2cf-bg)}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}$$

input
```
Int[(a + b*x + c*x^2)/Sqrt[f + g*x],x]
```

output
```
(2*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^3 - (2*(2*c*f - b*g)*(f + g*x)
^(3/2))/(3*g^3) + (2*c*(f + g*x)^(5/2))/(5*g^3)
```

3.822.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.822.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

method	result	size
pseudoelliptic	$\frac{2\left(\left(\frac{1}{5}c\,x^{2}+\frac{1}{3}bx+a\right)g^{2}-\frac{2f\left(\frac{2cx}{5}+b\right)g}{3}+\frac{8c\,f^{2}}{15}\right)\sqrt{gx+f}}{g^{3}}$	46
gosper	$\frac{2\sqrt{gx+f}\,\left(3c\,x^{2}g^{2}+5b\,g^{2}x-4cfxg+15a\,g^{2}-10bfg+8c\,f^{2}\right)}{15g^{3}}$	53
trager	$\frac{2\sqrt{gx+f}\,\left(3c\,x^{2}g^{2}+5b\,g^{2}x-4cfxg+15a\,g^{2}-10bfg+8c\,f^{2}\right)}{15g^{3}}$	53
risch	$\frac{2\sqrt{gx+f}\,\left(3c\,x^{2}g^{2}+5b\,g^{2}x-4cfxg+15a\,g^{2}-10bfg+8c\,f^{2}\right)}{15g^{3}}$	53
derivativedivides	$\frac{\frac{2c(gx+f)^{\frac{5}{2}}}{5}+\frac{2bg(gx+f)^{\frac{3}{2}}}{3}-\frac{4cf(gx+f)^{\frac{3}{2}}}{3}+2a\,g^{2}\sqrt{gx+f}-2bfg\sqrt{gx+f}+2c\,f^{2}\sqrt{gx+f}}{g^{3}}$	75
default	$\frac{\frac{2c(gx+f)^{\frac{5}{2}}}{5}+\frac{2bg(gx+f)^{\frac{3}{2}}}{3}-\frac{4cf(gx+f)^{\frac{3}{2}}}{3}+2a\,g^{2}\sqrt{gx+f}-2bfg\sqrt{gx+f}+2c\,f^{2}\sqrt{gx+f}}{g^{3}}$	75

input
```
int((c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((1/5*c*x^2+1/3*b*x+a)*g^2-2/3*f*(2/5*c*x+b)*g+8/15*c*f^2)*(g*x+f)^(1/2)
/g^3
```

3.822.5 Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

$$\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx=\frac{2\left(3\,cg^2x^2+8\,cf^2-10\,bfg+15\,ag^2-(4\,cfg-5\,bg^2)x\right)\sqrt{gx+f}}{15\,g^3}$$

input
```
integrate((c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*c*g^2*x^2 + 8*c*f^2 - 10*b*f*g + 15*a*g^2 - (4*c*f*g - 5*b*g^2)*x)
*sqrt(g*x + f)/g^3
```

3.822.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx = \begin{cases} \frac{2a\sqrt{f+gx}+\frac{2b\left(-f\sqrt{f+gx}+\frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g}+\frac{2c\left(f^2\sqrt{f+gx}-\frac{2f(f+gx)^{\frac{3}{2}}}{3}+\frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2}}{g} & \text{for } g\neq 0 \\ \frac{ax+\frac{bx^2}{2}+\frac{cx^3}{3}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

output
```
Piecewise(((2*a*sqrt(f + g*x) + 2*b*(-f*sqrt(f + g*x) + (f + g*x)**(3/2)/3
)/g + 2*c*(f**2*sqrt(f + g*x) - 2*f*(f + g*x)**(3/2)/3 + (f + g*x)**(5/2)/
5)/g**2)/g, Ne(g, 0)), ((a*x + b*x**2/2 + c*x**3/3)/sqrt(f), True))
```

3.822.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05

$$\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx = \frac{2\left(15\sqrt{gx+f}a+\frac{5\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}f\right)b}{g}+\frac{\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}f^2\right)c}{g^2}\right)}{15\,g}$$

input
```
integrate((c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(15*sqrt(g*x + f)*a + 5*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*b/g + (
3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c/g^2)/g
```

3.822.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05

$$\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx = \frac{2\left(15\sqrt{gx+f}a+\frac{5\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}f\right)b}{g}+\frac{\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}f^2\right)c}{g^2}\right)}{15g}$$

input
```
integrate((c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
2/15*(15*sqrt(g*x + f)*a + 5*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*b/g + (
3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c/g^2)/g
```

3.822.9 Mupad [B] (verification not implemented)

Time = 11.83 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+g\,x}\,\left(3\,c\,{(f+g\,x)}^2+15\,a\,g^2+15\,c\,f^2+5\,b\,g\,(f+g\,x)-10\,c\,f\,(f+g\,x)-15\,b\,f\,g\right)}{15\,g^3}$$

input
```
int((a + b*x + c*x^2)/(f + g*x)^(1/2),x)
```

output
```
(2*(f + g*x)^(1/2)*(3*c*(f + g*x)^2 + 15*a*g^2 + 15*c*f^2 + 5*b*g*(f + g*x
) - 10*c*f*(f + g*x) - 15*b*f*g))/(15*g^3)
```

3.823 $\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}}\,dx$

3.823.1 Optimal result

Integrand size = 27, antiderivative size = 116

$$\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}}\,dx=\frac{2(beg-c(ef+dg))\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}-\frac{2(cd^2-bde+ae^2)\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}$$

output
```
2/3*c*(g*x+f)^(3/2)/e/g^2-2*(a*e^2-b*d*e+c*d^2)*arctanh(e^(1/2)*(g*x+f)^(1
/2)/(-d*g+e*f)^(1/2))/e^(5/2)/(-d*g+e*f)^(1/2)+2*(b*e*g-c*(d*g+e*f))*(g*x+
f)^(1/2)/e^2/g^2
```

3.823.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}}\,dx=\frac{2\sqrt{f+gx}(3beg+c(-2ef-3dg+egx))}{3e^2g^2}+\frac{2(cd^2+e(-bd+ae))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{5/2}\sqrt{-ef+dg}}$$

input
```
Integrate[(a + b*x + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

output
```
(2*Sqrt[f + g*x]*(3*b*e*g + c*(-2*e*f - 3*d*g + e*g*x)))/(3*e^2*g^2) + (2*
(c*d^2 + e*(-(b*d) + a*e))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*
g]])/(e^(5/2)*Sqrt[-(e*f) + d*g])
```

3.823.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1192, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{ 1192}$$

$$\frac{2\int -\frac{cf^2-bgf+ag^2+c(f+gx)^2-(2cf-bg)(f+gx)}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{2\int \frac{cf^2-bgf+ag^2+c(f+gx)^2-(2cf-bg)(f+gx)}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \text{ 1467}$$

$$-\frac{2\int \left(\frac{cef+cdg-beg}{e^2}-\frac{c(f+gx)}{e}+\frac{cd^2g^2+ae^2g^2-bdeg^2}{e^2(ef-dg-e(f+gx))}\right)d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2\left(-\frac{g^2(ae^2-bde+cd^2)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}-\frac{\sqrt{f+gx}(-beg+cdg+cef)}{e^2}+\frac{c(f+gx)^{3/2}}{3e}\right)}{g^2}$$

input
```
Int[(a + b*x + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

output
```
(2*(-(((c*e*f + c*d*g - b*e*g)*Sqrt[f + g*x])/e^2) + (c*(f + g*x)^(3/2))/(
3*e) - ((c*d^2 - b*d*e + a*e^2)*g^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e
*f - d*g]])/(e^(5/2)*Sqrt[e*f - d*g])))/g^2
```

3.823.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.823.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{2(cegx+3beg-3cdg-2cef)\sqrt{gx+f}}{3g^2e^2}+\frac{2(e^2a-bde+c\,d^2)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e^2\sqrt{(dg-ef)e}}$	93
pseudoelliptic	$\frac{\frac{2\sqrt{gx+f}\,(cegx+3beg-3cdg-2cef)}{3}+\frac{2g^2\left(e^2a-bde+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{\sqrt{(dg-ef)e}}}{e^2g^2}$	94
derivativedivides	$\frac{\frac{2\left(\frac{c(gx+f)^{\frac{3}{2}}e}{3}+beg\sqrt{gx+f}-cdg\sqrt{gx+f}-cef\sqrt{gx+f}\right)}{e^2}+\frac{2g^2\left(e^2a-bde+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e^2\sqrt{(dg-ef)e}}}{g^2}$	115
default	$\frac{\frac{2\left(\frac{c(gx+f)^{\frac{3}{2}}e}{3}+beg\sqrt{gx+f}-cdg\sqrt{gx+f}-cef\sqrt{gx+f}\right)}{e^2}+\frac{2g^2\left(e^2a-bde+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e^2\sqrt{(dg-ef)e}}}{g^2}$	115

input
```
int((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(c*e*g*x+3*b*e*g-3*c*d*g-2*c*e*f)*(g*x+f)^(1/2)/g^2/e^2+2*(a*e^2-b*d*e
+c*d^2)/e^2/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)
)
```

3.823.5 Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.94

$$\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}}\,dx$$
$$= \left[\frac{3\left(cd^2-bde+ae^2\right)\sqrt{e^2f-deg}\,g^2\log\left(\frac{egx+2\,ef-dg-2\,\sqrt{e^2f-deg}\sqrt{gx+f}}{ex+d}\right)-2\left(2\,ce^3f^2+\left(cde^2-3\,be^3\right)fg-3\,(\right.}{3\left(e^4fg^2-de^3g^3\right)}\right.$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(c*d^2 - b*d*e + a*e^2)*sqrt(e^2*f - d*e*g)*g^2*log((e*g*x + 2*e*f
 - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*e^3*f^2
+ (c*d*e^2 - 3*b*e^3)*f*g - 3*(c*d^2*e - b*d*e^2)*g^2 - (c*e^3*f*g - c*d*e
^2*g^2)*x)*sqrt(g*x + f))/(e^4*f*g^2 - d*e^3*g^3), 2/3*(3*(c*d^2 - b*d*e +
 a*e^2)*sqrt(-e^2*f + d*e*g)*g^2*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)
/(e*g*x + e*f)) - (2*c*e^3*f^2 + (c*d*e^2 - 3*b*e^3)*f*g - 3*(c*d^2*e - b*
d*e^2)*g^2 - (c*e^3*f*g - c*d*e^2*g^2)*x)*sqrt(g*x + f))/(e^4*f*g^2 - d*e^
3*g^3)]
```

3.823.6 Sympy [A] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.38

$$\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{c(f+gx)^{\frac{3}{2}}}{3eg}+\frac{\sqrt{f+gx}(beg-cdg-cef)}{e^2g}+\frac{g\left(ae^2-bde+cd^2\right)\operatorname{atan}\left(\frac{\sqrt{f+gx}}{\sqrt{\frac{dg-ef}{e}}}\right)}{e^3\sqrt{\frac{dg-ef}{e}}}\right)}{g} & \text{for } g\neq 0 \\ \frac{\frac{cx^2}{2e}+\frac{x(be-cd)}{e^2}+\frac{\left(ae^2-bde+cd^2\right)\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{e^2}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+b*x+a)/(e*x+d)/(g*x+f)**(1/2),x)
```

output
```
Piecewise((2*(c*(f + g*x)**(3/2)/(3*e*g) + sqrt(f + g*x)*(b*e*g - c*d*g -
c*e*f)/(e**2*g) + g*(a*e**2 - b*d*e + c*d**2)*atan(sqrt(f + g*x)/sqrt((d*g
 - e*f)/e))/(e**3*sqrt((d*g - e*f)/e)))/g, Ne(g, 0)), ((c*x**2/(2*e) + x*(
b*e - c*d)/e**2 + (a*e**2 - b*d*e + c*d**2)*Piecewise((x/d, Eq(e, 0)), (lo
g(d + e*x)/e, True))/e**2)/sqrt(f), True))
```

3.823.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + bx + cx^2}{(d + ex)\sqrt{f + gx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.823.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12

$$\begin{aligned}
&\int \frac{a + bx + cx^2}{(d + ex)\sqrt{f + gx}}\,dx \\
&= \frac{2\,(cd^2 - bde + ae^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{\sqrt{-e^2f + deg}e^2} \\
&\quad + \frac{2\left((gx + f)^{\frac{3}{2}}ce^2g^4 - 3\sqrt{gx + f}ce^2fg^4 - 3\sqrt{gx + f}cdeg^5 + 3\sqrt{gx + f}be^2g^5\right)}{3\,e^3g^6}
\end{aligned}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
2*(c*d^2 - b*d*e + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/(sq
rt(-e^2*f + d*e*g)*e^2) + 2/3*((g*x + f)^(3/2)*c*e^2*g^4 - 3*sqrt(g*x + f)
*c*e^2*f*g^4 - 3*sqrt(g*x + f)*c*d*e*g^5 + 3*sqrt(g*x + f)*b*e^2*g^5)/(e^3
*g^6)
```

3.823.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01

$$\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \sqrt{f+g\,x}\,\left(\frac{2\,b\,g-4\,c\,f}{e\,g^2}-\frac{2\,c\,\left(d\,g^3-e\,f\,g^2\right)}{e^2\,g^4}\right) + \frac{2\,\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{e^{5/2}\,\sqrt{d\,g-e\,f}} + \frac{2\,c\,{\left(f+g\,x\right)}^{3/2}}{3\,e\,g^2}$$

input
```
int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)),x)
```

output
```
(f + g*x)^(1/2)*((2*b*g - 4*c*f)/(e*g^2) - (2*c*(d*g^3 - e*f*g^2))/(e^2*g^
4)) + (2*atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2 + c*d^2
- b*d*e))/(e^(5/2)*(d*g - e*f)^(1/2)) + (2*c*(f + g*x)^(3/2))/(3*e*g^2)
```

3.824 $\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx$

3.824.1 Optimal result

Integrand size = 27, antiderivative size = 140

$$\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx=\frac{2c\sqrt{f+gx}}{e^2g}-\frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)}+\frac{(cd(4ef-3dg)-e(2bef-bdg-aeg))\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}}$$

output
```
(c*d*(-3*d*g+4*e*f)-e*(-a*e*g-b*d*g+2*b*e*f))*arctanh(e^(1/2)*(g*x+f)^(1/2
)/(-d*g+e*f)^(1/2))/e^(5/2)/(-d*g+e*f)^(3/2)+2*c*(g*x+f)^(1/2)/e^2/g-(a+d*
(-b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)
```

3.824.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07

$$\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx=\frac{\sqrt{f+gx}(e(bd-ae)g+c(-3d^2g+2e^2fx+2de(f-gx)))}{e^2g(ef-dg)(d+ex)}-\frac{(cd(-4ef+3dg)+e(2bef-bdg-aeg))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{5/2}(-ef+dg)^{3/2}}$$

input
```
Integrate[(a + b*x + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

output
```
(Sqrt[f + g*x]*(e*(b*d - a*e)*g + c*(-3*d^2*g + 2*e^2*f*x + 2*d*e*(f - g*x
))))/(e^2*g*(e*f - d*g)*(d + e*x)) - ((c*d*(-4*e*f + 3*d*g) + e*(2*b*e*f -
 b*d*g - a*e*g))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(5
/2)*(-(e*f) + d*g)^(3/2))
```

3.824.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.34, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1192, 1471, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{1192}$$

$$\frac{2\int \frac{cf^2-bgf+ag^2+c(f+gx)^2-(2cf-bg)(f+gx)}{(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{g}$$

$$\Big\downarrow \text{1471}$$

$$\frac{2\left(\frac{g^2\sqrt{f+gx}\left(ae^2-bde+cd^2\right)}{2e^2(ef-dg)(-dg-e(f+gx)+ef)}-\frac{\int \frac{\frac{g(2bef-bdg-aeg)}{e}-c\left(2f^2-\frac{d^2g^2}{e^2}\right)+2c\left(f-\frac{dg}{e}\right)(f+gx)}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{2(ef-dg)}\right)}{g}$$

$$\Big\downarrow \text{299}$$

$$\frac{2\left(\frac{g^2\sqrt{f+gx}\left(ae^2-bde+cd^2\right)}{2e^2(ef-dg)(-dg-e(f+gx)+ef)}-\frac{-\frac{g(cd(4ef-3dg)-e(-aeg-bdg+2bef))\int \frac{1}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{e^2}-\frac{2c\sqrt{f+gx}(ef-dg)}{e^2}}{2(ef-dg)}\right)}{g}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\left(\frac{g^2\sqrt{f+gx}\left(ae^2-bde+cd^2\right)}{2e^2(ef-dg)(-dg-e(f+gx)+ef)}-\frac{-\frac{g\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(cd(4ef-3dg)-e(-aeg-bdg+2bef))}{e^{5/2}\sqrt{ef-dg}}-\frac{2c\sqrt{f+gx}(ef-dg)}{e^2}}{2(ef-dg)}\right)}{g}$$

input
```
Int[(a + b*x + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

output
```
(2*(((c*d^2 - b*d*e + a*e^2)*g^2*Sqrt[f + g*x])/(2*e^2*(e*f - d*g)*(e*f -
d*g - e*(f + g*x))) - ((-2*c*(e*f - d*g)*Sqrt[f + g*x])/e^2 - (g*(c*d*(4*e
*f - 3*d*g) - e*(2*b*e*f - b*d*g - a*e*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])
/Sqrt[e*f - d*g]])/(e^(5/2)*Sqrt[e*f - d*g]))/(2*(e*f - d*g))))/g
```

3.824.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.824.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.09

method	result
risch	$\frac{2c\sqrt{gx+f}}{e^2 g}+\frac{\frac{g\left(e^2 a-bde+c\,d^2\right)\sqrt{gx+f}}{(dg-ef)(e(gx+f)+dg-ef)}+\frac{\left(a\,e^2 g+bdeg-2b\,e^2 f-3c\,d^2 g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)\sqrt{(dg-ef)e}}}{e^2}$
derivativedivides	$\frac{\frac{2c\sqrt{gx+f}}{e^2}+\frac{2g\left(\frac{g\left(e^2 a-bde+c\,d^2\right)\sqrt{gx+f}}{2(dg-ef)(e(gx+f)+dg-ef)}+\frac{\left(a\,e^2 g+bdeg-2b\,e^2 f-3c\,d^2 g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2(dg-ef)\sqrt{(dg-ef)e}}\right)}{e^2}}{g}$
default	$\frac{\frac{2c\sqrt{gx+f}}{e^2}+\frac{2g\left(\frac{g\left(e^2 a-bde+c\,d^2\right)\sqrt{gx+f}}{2(dg-ef)(e(gx+f)+dg-ef)}+\frac{\left(a\,e^2 g+bdeg-2b\,e^2 f-3c\,d^2 g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2(dg-ef)\sqrt{(dg-ef)e}}\right)}{e^2}}{g}$
pseudoelliptic	$\frac{(ex+d)g((ag-2bf)e^2+d(bg+4cf)e-3c\,d^2 g)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)+\sqrt{gx+f}\,\sqrt{(dg-ef)e}\,((-2cfx+ag)e^2-d((-2cx+b)g}{\sqrt{(dg-ef)e}\,g\,e^2(dg-ef)(ex+d)}$

input
```
int((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*c*(g*x+f)^(1/2)/e^2/g+1/e^2*(g*(a*e^2-b*d*e+c*d^2)/(d*g-e*f)*(g*x+f)^(1/
2)/(e*(g*x+f)+d*g-e*f)+(a*e^2*g+b*d*e*g-2*b*e^2*f-3*c*d^2*g+4*c*d*e*f)/(d*
g-e*f)/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)))
```

3.824.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(126) = 252.

Time = 0.49 (sec) , antiderivative size = 637, normalized size of antiderivative = 4.55

$$\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx$$
$$=\left[-\frac{\sqrt{e^2f-deg}(2\,(2\,cd^2e-bde^2)fg-(3\,cd^3-bd^2e-ade^2)g^2+(2\,(2\,cde^2-be^3)fg-(3\,cd^2e-bde^2-a}{2\,(de}\right.$$
$$-\frac{\sqrt{-e^2f+deg}(2\,(2\,cd^2e-bde^2)fg-(3\,cd^3-bd^2e-ade^2)g^2+(2\,(2\,cde^2-be^3)fg-(3\,cd^2e-bde^2-}{de^5f^2g}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(e^2*f - d*e*g)*(2*(2*c*d^2*e - b*d*e^2)*f*g - (3*c*d^3 - b*d^2
*e - a*d*e^2)*g^2 + (2*(2*c*d*e^2 - b*e^3)*f*g - (3*c*d^2*e - b*d*e^2 - a*
e^3)*g^2)*x)*log((e*g*x + 2*e*f - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f
))/(e*x + d)) - 2*(2*c*d*e^3*f^2 - (5*c*d^2*e^2 - b*d*e^3 + a*e^4)*f*g + (
3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*g^2 + 2*(c*e^4*f^2 - 2*c*d*e^3*f*g + c*d^
2*e^2*g^2)*x)*sqrt(g*x + f))/(d*e^5*f^2*g - 2*d^2*e^4*f*g^2 + d^3*e^3*g^3
+ (e^6*f^2*g - 2*d*e^5*f*g^2 + d^2*e^4*g^3)*x), -(sqrt(-e^2*f + d*e*g)*(2*
(2*c*d^2*e - b*d*e^2)*f*g - (3*c*d^3 - b*d^2*e - a*d*e^2)*g^2 + (2*(2*c*d*
e^2 - b*e^3)*f*g - (3*c*d^2*e - b*d*e^2 - a*e^3)*g^2)*x)*arctan(sqrt(-e^2*
f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) - (2*c*d*e^3*f^2 - (5*c*d^2*e^2 -
b*d*e^3 + a*e^4)*f*g + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*g^2 + 2*(c*e^4*f^
2 - 2*c*d*e^3*f*g + c*d^2*e^2*g^2)*x)*sqrt(g*x + f))/(d*e^5*f^2*g - 2*d^2*
e^4*f*g^2 + d^3*e^3*g^3 + (e^6*f^2*g - 2*d*e^5*f*g^2 + d^2*e^4*g^3)*x)]
```

3.824.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/(e*x+d)**2/(g*x+f)**(1/2),x)
```

output
```
Timed out
```

3.824.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.824.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.26

$$\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = -\frac{(4\,cdef-2\,be^2f-3\,cd^2g+bdeg+ae^2g)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{(e^3f-de^2g)\sqrt{-e^2f+deg}} - \frac{\sqrt{gx+f}cd^2g-\sqrt{gx+f}bdeg+\sqrt{gx+f}ae^2g}{(e^3f-de^2g)((gx+f)e-ef+dg)} + \frac{2\sqrt{gx+f}c}{e^2g}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
-(4*c*d*e*f - 2*b*e^2*f - 3*c*d^2*g + b*d*e*g + a*e^2*g)*arctan(sqrt(g*x +
 f)*e/sqrt(-e^2*f + d*e*g))/((e^3*f - d*e^2*g)*sqrt(-e^2*f + d*e*g)) - (sq
rt(g*x + f)*c*d^2*g - sqrt(g*x + f)*b*d*e*g + sqrt(g*x + f)*a*e^2*g)/((e^3
*f - d*e^2*g)*((g*x + f)*e - e*f + d*g)) + 2*sqrt(g*x + f)*c/(e^2*g)
```

3.824.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

$$\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{e}\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(a\,e^2\,g-2\,b\,e^2\,f-3\,c\,d^2\,g+b\,d\,e\,g+4\,c\,d\,e\,f\right)}{e^{5/2}\,{\left(d\,g-e\,f\right)}^{3/2}} + \frac{\sqrt{f+g\,x}\,\left(c\,g\,d^2-b\,g\,d\,e+a\,g\,e^2\right)}{\left(d\,g-e\,f\right)\,\left(e^3\,\left(f+g\,x\right)-e^3\,f+d\,e^2\,g\right)} + \frac{2\,c\,\sqrt{f+g\,x}}{e^2\,g}$$

input
```
int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^2),x)
```

output
```
(atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2*g - 2*b*e^2*f -
3*c*d^2*g + b*d*e*g + 4*c*d*e*f))/(e^(5/2)*(d*g - e*f)^(3/2)) + ((f + g*x)
^(1/2)*(a*e^2*g + c*d^2*g - b*d*e*g))/((d*g - e*f)*(e^3*(f + g*x) - e^3*f
+ d*e^2*g)) + (2*c*(f + g*x)^(1/2))/(e^2*g)
```

3.825 $\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx$

3.825.1 Optimal result

Integrand size = 27, antiderivative size = 206

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = -\frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2}+\frac{(cd(8ef-5dg)-e(4bef-bdg-3aeg))\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)} + \frac{(eg(4bef-bdg-3aeg)-c(8e^2f^2-8defg+3d^2g^2))\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{5/2}(ef-dg)^{5/2}}$$

output
```
1/4*(e*g*(-3*a*e*g-b*d*g+4*b*e*f)-c*(3*d^2*g^2-8*d*e*f*g+8*e^2*f^2))*arcta
nh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(5/2)/(-d*g+e*f)^(5/2)-1/2*(a
+d*(-b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)^2+1/4*(c*d*(-5*d*g+8*e
*f)-e*(-3*a*e*g-b*d*g+4*b*e*f))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^2/(e*x+d)
```

3.825.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.99

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \frac{\frac{\sqrt{e}\sqrt{f+gx}(cd(-3d^2g+8e^2fx+de(6f-5gx))+e(ae(-2ef+5dg+3egx)-b(2def+d^2g+4e^2fx-degx)))}{(ef-dg)^2(d+ex)^2}+\frac{(eg(-4bef+bdg+3aeg)+c(8e^2f^2-8d\cdots}{(-ef+dg\cdots}}{4e^{5/2}}$$

input
```
Integrate[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

output
```
((Sqrt[e]*Sqrt[f + g*x]*(c*d*(-3*d^2*g + 8*e^2*f*x + d*e*(6*f - 5*g*x)) +
e*(a*e*(-2*e*f + 5*d*g + 3*e*g*x) - b*(2*d*e*f + d^2*g + 4*e^2*f*x - d*e*g
*x))))/((e*f - d*g)^2*(d + e*x)^2) + ((e*g*(-4*b*e*f + b*d*g + 3*a*e*g) +
c*(8*e^2*f^2 - 8*d*e*f*g + 3*d^2*g^2))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt
[-(e*f) + d*g]])/(-(e*f) + d*g)^(5/2))/(4*e^(5/2))
```

3.825.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1192, 25, 1471, 25, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{1192}$$

$$2\int -\frac{cf^2-bgf+ag^2+c(f+gx)^2-(2cf-bg)(f+gx)}{(ef-dg-e(f+gx))^3}d\sqrt{f+gx}$$

$$\Big\downarrow \text{25}$$

$$-2\int \frac{cf^2-bgf+ag^2+c(f+gx)^2-(2cf-bg)(f+gx)}{(ef-dg-e(f+gx))^3}d\sqrt{f+gx}$$

$$\Big\downarrow \text{1471}$$

$$2\left(\frac{\int -\frac{4cf^2-4bgf+3ag^2+\frac{bdg^2}{e}-\frac{cd^2g^2}{e^2}-4c\left(f-\frac{dg}{e}\right)(f+gx)}{(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{4(ef-dg)}-\frac{g^2\sqrt{f+gx}(ae^2-bde+cd^2)}{4e^2(ef-dg)(-dg-e(f+gx)+ef)^2}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(-\frac{\int \frac{4cf^2-4bgf+3ag^2+\frac{bdg^2}{e}-\frac{cd^2g^2}{e^2}-4c\left(f-\frac{dg}{e}\right)(f+gx)}{(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{4(ef-dg)}-\frac{g^2\sqrt{f+gx}(ae^2-bde+cd^2)}{4e^2(ef-dg)(-dg-e(f+gx)+ef)^2}\right)$$

$$\Big\downarrow \text{298}$$

$$2\left(-\frac{\frac{g\sqrt{f+gx}(cd(8ef-5dg)-e(-3aeg-bdg+4bef))}{2e^2(ef-dg)(-dg-e(f+gx)+ef)}-\frac{(eg(-3aeg-bdg+4bef)-c(3d^2g^2-8defg+8e^2f^2))\int\frac{1}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{2e^2(ef-dg)}}{4(ef-dg)}-\frac{}{4e^2(}\right.$$

$\downarrow$ 221

$$2\left(-\frac{\frac{g\sqrt{f+gx}(cd(8ef-5dg)-e(-3aeg-bdg+4bef))}{2e^2(ef-dg)(-dg-e(f+gx)+ef)}-\frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(eg(-3aeg-bdg+4bef)-c(3d^2g^2-8defg+8e^2f^2))}{2e^{5/2}(ef-dg)^{3/2}}}{4(ef-dg)}-\frac{g^2}{4e^2(ef}\right.$$

input
```
Int[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

output
```
2*(-1/4*((c*d^2 - b*d*e + a*e^2)*g^2*Sqrt[f + g*x])/(e^2*(e*f - d*g)*(e*f
- d*g - e*(f + g*x))^2) - ((g*(c*d*(8*e*f - 5*d*g) - e*(4*b*e*f - b*d*g -
3*a*e*g))*Sqrt[f + g*x])/(2*e^2*(e*f - d*g)*(e*f - d*g - e*(f + g*x))) - (
(e*g*(4*b*e*f - b*d*g - 3*a*e*g) - c*(8*e^2*f^2 - 8*d*e*f*g + 3*d^2*g^2))*
ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(2*e^(5/2)*(e*f - d*g)^(
3/2)))/(4*(e*f - d*g)))
```

3.825.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.825.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$\frac{5\sqrt{gx+f}\left(\frac{(3agx-2f(2bx+a))e^3}{5}+d\left(\left(\frac{bx}{5}+a\right)g-\frac{2f(-4cx+b)}{5}\right)e^2-\frac{d^2((5cx+b)g-6cf)e}{5}-\frac{3c\,d^3g}{5}\right)\sqrt{(dg-ef)e}+3(ex+d)^2\left(\left(\right.\right.}{4\sqrt{(dg-ef)e}\,(dg-ef)^2e^2(ex+d)^2}$
derivativedivides	$\frac{\frac{g\left(3a\,e^2g+bdeg-4b\,e^2f-5c\,d^2g+8cdef\right)(gx+f)^{\frac{3}{2}}}{4e\left(d^2g^2-2defg+e^2f^2\right)}+\frac{\left(5a\,e^2g-bdeg-4b\,e^2f-3c\,d^2g+8cdef\right)g\sqrt{gx+f}}{4e^2(dg-ef)}}{(e(gx+f)+dg-ef)^2}+\frac{(3a\,e^2g^2+bde\,g^2-4b\,e^2f}{4(d^2g}$
default	$\frac{\frac{g\left(3a\,e^2g+bdeg-4b\,e^2f-5c\,d^2g+8cdef\right)(gx+f)^{\frac{3}{2}}}{4e\left(d^2g^2-2defg+e^2f^2\right)}+\frac{\left(5a\,e^2g-bdeg-4b\,e^2f-3c\,d^2g+8cdef\right)g\sqrt{gx+f}}{4e^2(dg-ef)}}{(e(gx+f)+dg-ef)^2}+\frac{(3a\,e^2g^2+bde\,g^2-4b\,e^2f}{4(d^2g}$

input
```
int((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(5*(g*x+f)^(1/2)*(1/5*(3*a*g*x-2*f*(2*b*x+a))*e^3+d*((1/5*b*x+a)*g-2/5
*f*(-4*c*x+b))*e^2-1/5*d^2*((5*c*x+b)*g-6*c*f)*e-3/5*c*d^3*g)*((d*g-e*f)*e
)^(1/2)+3*(e*x+d)^2*((a*g^2-4/3*b*f*g+8/3*c*f^2)*e^2+1/3*d*g*(b*g-8*c*f)*e
+c*d^2*g^2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)))/((d*g-e*f)*e)^(1/
2)/(d*g-e*f)^2/e^2/(e*x+d)^2
```

3.825.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(186) = 372$.

Time = 0.49 (sec) , antiderivative size = 1096, normalized size of antiderivative = 5.32

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx$$
$$= \left[\frac{(8\,cd^2e^2f^2 - 4\,(2\,cd^3e + bd^2e^2)fg + (3\,cd^4 + bd^3e + 3\,ad^2e^2)g^2 + (8\,ce^4f^2 - 4\,(2\,cde^3 + be^4)fg + (3\,cd^2e\right.$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*((8*c*d^2*e^2*f^2 - 4*(2*c*d^3*e + b*d^2*e^2)*f*g + (3*c*d^4 + b*d^3*
e + 3*a*d^2*e^2)*g^2 + (8*c*e^4*f^2 - 4*(2*c*d*e^3 + b*e^4)*f*g + (3*c*d^2
*e^2 + b*d*e^3 + 3*a*e^4)*g^2)*x^2 + 2*(8*c*d*e^3*f^2 - 4*(2*c*d^2*e^2 + b
*d*e^3)*f*g + (3*c*d^3*e + b*d^2*e^2 + 3*a*d*e^3)*g^2)*x)*sqrt(e^2*f - d*e
*g)*log((e*g*x + 2*e*f - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x +
 d)) + 2*(2*(3*c*d^2*e^3 - b*d*e^4 - a*e^5)*f^2 - (9*c*d^3*e^2 - b*d^2*e^3
 - 7*a*d*e^4)*f*g + (3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*g^2 + (4*(2*c*d*
e^4 - b*e^5)*f^2 - (13*c*d^2*e^3 - 5*b*d*e^4 - 3*a*e^5)*f*g + (5*c*d^3*e^2
 - b*d^2*e^3 - 3*a*d*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^3 - 3*d^3*e^5*
f^2*g + 3*d^4*e^4*f*g^2 - d^5*e^3*g^3 + (e^8*f^3 - 3*d*e^7*f^2*g + 3*d^2*e
^6*f*g^2 - d^3*e^5*g^3)*x^2 + 2*(d*e^7*f^3 - 3*d^2*e^6*f^2*g + 3*d^3*e^5*f
*g^2 - d^4*e^4*g^3)*x), 1/4*((8*c*d^2*e^2*f^2 - 4*(2*c*d^3*e + b*d^2*e^2)*
f*g + (3*c*d^4 + b*d^3*e + 3*a*d^2*e^2)*g^2 + (8*c*e^4*f^2 - 4*(2*c*d*e^3
+ b*e^4)*f*g + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*g^2)*x^2 + 2*(8*c*d*e^3*f
^2 - 4*(2*c*d^2*e^2 + b*d*e^3)*f*g + (3*c*d^3*e + b*d^2*e^2 + 3*a*d*e^3)*g
^2)*x)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g
*x + e*f)) + (2*(3*c*d^2*e^3 - b*d*e^4 - a*e^5)*f^2 - (9*c*d^3*e^2 - b*d^2
*e^3 - 7*a*d*e^4)*f*g + (3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*g^2 + (4*(2*
c*d*e^4 - b*e^5)*f^2 - (13*c*d^2*e^3 - 5*b*d*e^4 - 3*a*e^5)*f*g + (5*c*d^3
*e^2 - b*d^2*e^3 - 3*a*d*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^3 - 3*d...
```

3.825.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/(e*x+d)**3/(g*x+f)**(1/2),x)
```

output
```
Timed out
```

3.825.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.825.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. 2(186) = 372.

Time = 0.28 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.86

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx$$
$$= \frac{(8\,ce^2f^2 - 8\,cdefg - 4\,be^2fg + 3\,cd^2g^2 + bdeg^2 + 3\,ae^2g^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{4\,(e^4f^2 - 2\,de^3fg + d^2e^2g^2)\sqrt{-e^2f+deg}}$$
$$+ \frac{8\,(gx+f)^{\frac{3}{2}}cde^2fg - 4\,(gx+f)^{\frac{3}{2}}be^3fg - 8\,\sqrt{gx+f}cde^2f^2g + 4\,\sqrt{gx+f}be^3f^2g - 5\,(gx+f)^{\frac{3}{2}}cd^2eg^2 + }{}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
1/4*(8*c*e^2*f^2 - 8*c*d*e*f*g - 4*b*e^2*f*g + 3*c*d^2*g^2 + b*d*e*g^2 + 3
*a*e^2*g^2)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/((e^4*f^2 - 2*d*e
^3*f*g + d^2*e^2*g^2)*sqrt(-e^2*f + d*e*g)) + 1/4*(8*(g*x + f)^(3/2)*c*d*e
^2*f*g - 4*(g*x + f)^(3/2)*b*e^3*f*g - 8*sqrt(g*x + f)*c*d*e^2*f^2*g + 4*s
qrt(g*x + f)*b*e^3*f^2*g - 5*(g*x + f)^(3/2)*c*d^2*e*g^2 + (g*x + f)^(3/2)
*b*d*e^2*g^2 + 3*(g*x + f)^(3/2)*a*e^3*g^2 + 11*sqrt(g*x + f)*c*d^2*e*f*g^
2 - 3*sqrt(g*x + f)*b*d*e^2*f*g^2 - 5*sqrt(g*x + f)*a*e^3*f*g^2 - 3*sqrt(g
*x + f)*c*d^3*g^3 - sqrt(g*x + f)*b*d^2*e*g^3 + 5*sqrt(g*x + f)*a*d*e^2*g^
3)/((e^4*f^2 - 2*d*e^3*f*g + d^2*e^2*g^2)*((g*x + f)*e - e*f + d*g)^2)
```

3.825.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.31

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(3\,c\,d^2\,g^2-8\,c\,d\,e\,f\,g+b\,d\,e\,g^2+8\,c\,e^2\,f^2-4\,b\,e^2\,f\,g+3\,a\,e^2\,g^2\right)}{4\,e^{5/2}\,{\left(d\,g-e\,f\right)}^{5/2}}$$
$$-\frac{\frac{\sqrt{f+g\,x}\,\left(3\,c\,d^2\,g^2+b\,d\,e\,g^2-8\,c\,f\,d\,e\,g-5\,a\,e^2\,g^2+4\,b\,f\,e^2\,g\right)}{4\,e^2\,\left(d\,g-e\,f\right)}-\frac{{\left(f+g\,x\right)}^{3/2}\,\left(-5\,c\,d^2\,g^2+b\,d\,e\,g^2+8\,c\,f\,d\,e\,g+3\,a\,e^2\,g^2-4\,b\,f\,e^2\,g\right)}{4\,e\,{\left(d\,g-e\,f\right)}^2}}{e^2\,{\left(f+g\,x\right)}^2-\left(f+g\,x\right)\,\left(2\,e^2\,f-2\,d\,e\,g\right)+d^2\,g^2+e^2\,f^2-2\,d\,e\,f\,g}$$

input
```
int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^3),x)
```

output
```
(atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(3*a*e^2*g^2 + 3*c*d^2*
g^2 + 8*c*e^2*f^2 + b*d*e*g^2 - 4*b*e^2*f*g - 8*c*d*e*f*g))/(4*e^(5/2)*(d*
g - e*f)^(5/2)) - (((f + g*x)^(1/2)*(3*c*d^2*g^2 - 5*a*e^2*g^2 + b*d*e*g^2
 + 4*b*e^2*f*g - 8*c*d*e*f*g))/(4*e^2*(d*g - e*f)) - ((f + g*x)^(3/2)*(3*a
*e^2*g^2 - 5*c*d^2*g^2 + b*d*e*g^2 - 4*b*e^2*f*g + 8*c*d*e*f*g))/(4*e*(d*g
 - e*f)^2))/(e^2*(f + g*x)^2 - (f + g*x)*(2*e^2*f - 2*d*e*g) + d^2*g^2 + e
^2*f^2 - 2*d*e*f*g)
```

3.826 $\int \frac{(d+ex)^3(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx$

3.826.1 Optimal result

Integrand size = 27, antiderivative size = 285

$$\begin{aligned}\int \frac{(d+ex)^3(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx &= \frac{2(ef-dg)^3(cf^2-bfg+ag^2)}{g^6\sqrt{f+gx}}\\ &+ \frac{2(ef-dg)^2(cf(5ef-2dg)-g(4bef-bdg-3aeg))\sqrt{f+gx}}{g^6}\\ &+ \frac{2(ef-dg)\left(3eg(2bef-bdg-aeg)-c(10e^2f^2-8defg+d^2g^2)\right)(f+gx)^{3/2}}{3g^6}\\ &- \frac{2e\left(eg(4bef-3bdg-aeg)-c(10e^2f^2-12defg+3d^2g^2)\right)(f+gx)^{5/2}}{5g^6}\\ &- \frac{2e^2(5cef-3cdg-beg)(f+gx)^{7/2}}{7g^6}+\frac{2ce^3(f+gx)^{9/2}}{9g^6}\end{aligned}$$

output
```
2/3*(-d*g+e*f)*(3*e*g*(-a*e*g-b*d*g+2*b*e*f)-c*(d^2*g^2-8*d*e*f*g+10*e^2*f
^2))*(g*x+f)^(3/2)/g^6-2/5*e*(e*g*(-a*e*g-3*b*d*g+4*b*e*f)-c*(3*d^2*g^2-12
*d*e*f*g+10*e^2*f^2))*(g*x+f)^(5/2)/g^6-2/7*e^2*(-b*e*g-3*c*d*g+5*c*e*f)*(
g*x+f)^(7/2)/g^6+2/9*c*e^3*(g*x+f)^(9/2)/g^6+2*(-d*g+e*f)^3*(a*g^2-b*f*g+c
*f^2)/g^6/(g*x+f)^(1/2)+2*(-d*g+e*f)^2*(c*f*(-2*d*g+5*e*f)-g*(-3*a*e*g-b*d
*g+4*b*e*f))*(g*x+f)^(1/2)/g^6
```

3.826.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.42

$$\int \frac{(d+ex)^3\,(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2(c(105d^3g^3(-8f^2-4fgx+g^2x^2)+189d^2eg^2(16f^3+8f^2gx-2fg^2x^2+$$

input
```
Integrate[((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

output
```
(2*(c*(105*d^3*g^3*(-8*f^2 - 4*f*g*x + g^2*x^2) + 189*d^2*e*g^2*(16*f^3 +
8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3) + 27*d*e^2*g*(-128*f^4 - 64*f^3*g*x + 1
6*f^2*g^2*x^2 - 8*f*g^3*x^3 + 5*g^4*x^4) + 5*e^3*(256*f^5 + 128*f^4*g*x -
32*f^3*g^2*x^2 + 16*f^2*g^3*x^3 - 10*f*g^4*x^4 + 7*g^5*x^5)) + 9*g*(7*a*g*
(-5*d^3*g^3 + 15*d^2*e*g^2*(2*f + g*x) + 5*d*e^2*g*(-8*f^2 - 4*f*g*x + g^2
*x^2) + e^3*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3)) + b*(35*d^3*g^3*
(2*f + g*x) + 35*d^2*e*g^2*(-8*f^2 - 4*f*g*x + g^2*x^2) + 21*d*e^2*g*(16*f
^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3) + e^3*(-128*f^4 - 64*f^3*g*x + 16*
f^2*g^2*x^2 - 8*f*g^3*x^3 + 5*g^4*x^4)))))/(315*g^6*Sqrt[f + g*x])
```

3.826.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3\,(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{e(f+gx)^{3/2}\,(c(3d^2g^2-12defg+10e^2f^2)-eg(-aeg-3bdg+4bef))}{g^5}+\frac{\sqrt{f+gx}(ef-dg)\,(3eg(-aeg-b$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2e(f+gx)^{5/2}\left(eg(-aeg-3bdg+4bef)-c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{5g^6}+\frac{2(f+gx)^{3/2}(ef-dg)\left(3eg(-aeg-bdg+2bef)-c\left(d^2g^2-8defg+10e^2f^2\right)\right)}{3g^6}+\frac{2(ef-dg)^3\left(ag^2-bfg+cf^2\right)}{g^6\sqrt{f+gx}}+\frac{2\sqrt{f+gx}(ef-dg)^2(cf(5ef-2dg)-g(-3aeg-bdg+4bef))}{g^6}-\frac{2e^2(f+gx)^{7/2}(-beg-3cdg+5cef)}{7g^6}+\frac{2ce^3(f+gx)^{9/2}}{9g^6}$$

input
```
Int[((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

output
```
(2*(e*f - d*g)^3*(c*f^2 - b*f*g + a*g^2))/(g^6*Sqrt[f + g*x]) + (2*(e*f -
d*g)^2*(c*f*(5*e*f - 2*d*g) - g*(4*b*e*f - b*d*g - 3*a*e*g))*Sqrt[f + g*x]
)/g^6 + (2*(e*f - d*g)*(3*e*g*(2*b*e*f - b*d*g - a*e*g) - c*(10*e^2*f^2 -
8*d*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^6) - (2*e*(e*g*(4*b*e*f - 3*b*
d*g - a*e*g) - c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(5/2))/(
5*g^6) - (2*e^2*(5*c*e*f - 3*c*d*g - b*e*g)*(f + g*x)^(7/2))/(7*g^6) + (2*
c*e^3*(f + g*x)^(9/2))/(9*g^6)
```

3.826.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.826. $\int \frac{(d+ex)^3(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx$

3.826.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$-\frac{2\left(\left(\left(-\frac{\left(\frac{5}{9}c\,x^{2}+\frac{5}{7}bx+a\right)x^{3}e^{3}}{5}-d\left(\frac{3}{7}c\,x^{2}+\frac{3}{5}bx+a\right)x^{2}e^{2}-3d^{2}\left(\frac{1}{5}c\,x^{2}+\frac{1}{3}bx+a\right)xe+d^{3}\left(a-\frac{1}{3}c\,x^{2}-bx\right)\right)g^{5}-6\left(-\frac{\left(\frac{25}{63}c\,x^{2}+\right.}{}\right.\right.}{}$
risch	$\frac{2(35c\,e^{3}x^{4}g^{4}+45b\,e^{3}g^{4}x^{3}+135cd\,e^{2}g^{4}x^{3}-85c\,e^{3}f\,g^{3}x^{3}+63a\,e^{3}g^{4}x^{2}+189bd\,e^{2}g^{4}x^{2}-117b\,e^{3}f\,g^{3}x^{2}+189c\,d^{2}e\,g^{4}x^{2}-35}{}$
gosper	$-\frac{2(-35c\,e^{3}x^{5}g^{5}-45b\,e^{3}g^{5}x^{4}-135cd\,e^{2}g^{5}x^{4}+50c\,e^{3}f\,g^{4}x^{4}-63a\,e^{3}g^{5}x^{3}-189bd\,e^{2}g^{5}x^{3}+72b\,e^{3}f\,g^{4}x^{3}-189c\,d^{2}e\,g^{5}x^{3}+}{}$
trager	$-\frac{2(-35c\,e^{3}x^{5}g^{5}-45b\,e^{3}g^{5}x^{4}-135cd\,e^{2}g^{5}x^{4}+50c\,e^{3}f\,g^{4}x^{4}-63a\,e^{3}g^{5}x^{3}-189bd\,e^{2}g^{5}x^{3}+72b\,e^{3}f\,g^{4}x^{3}-189c\,d^{2}e\,g^{5}x^{3}+}{}$
derivativedivides	$\frac{\frac{2c\,e^{3}(gx+f)^{\frac{9}{2}}}{9}-\frac{24cd\,e^{2}f\,g(gx+f)^{\frac{5}{2}}}{5}-12ad\,e^{2}f\,g^{3}\sqrt{gx+f}-12b\,d^{2}ef\,g^{3}\sqrt{gx+f}-6bd\,e^{2}f\,g^{2}(gx+f)^{\frac{3}{2}}-6c\,d^{2}ef\,g^{2}(gx+f)^{\frac{3}{2}}+}{}$
default	$\frac{\frac{2c\,e^{3}(gx+f)^{\frac{9}{2}}}{9}-\frac{24cd\,e^{2}f\,g(gx+f)^{\frac{5}{2}}}{5}-12ad\,e^{2}f\,g^{3}\sqrt{gx+f}-12b\,d^{2}ef\,g^{3}\sqrt{gx+f}-6bd\,e^{2}f\,g^{2}(gx+f)^{\frac{3}{2}}-6c\,d^{2}ef\,g^{2}(gx+f)^{\frac{3}{2}}+}{}$

input
```
int((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(g*x+f)^(1/2)*((-1/5*(5/9*c*x^2+5/7*b*x+a)*x^3*e^3-d*(3/7*c*x^2+3/5*b*x
+a)*x^2*e^2-3*d^2*(1/5*c*x^2+1/3*b*x+a)*x*e+d^3*(a-1/3*c*x^2-b*x))*g^5-6*(
-1/15*(25/63*c*x^2+4/7*b*x+a)*x^2*e^3-2/3*d*(6/35*c*x^2+3/10*b*x+a)*x*e^2+
d^2*(-1/5*c*x^2-2/3*b*x+a)*e+1/3*d^3*(-2/3*c*x+b))*f*g^4+8*(-1/5*x*(10/63*
c*x^2+2/7*b*x+a)*e^3+d*(-6/35*c*x^2-3/5*b*x+a)*e^2+d^2*(-3/5*c*x+b)*e+1/3*
c*d^3)*f^2*g^3-16/5*e*f^3*((-10/63*c*x^2-4/7*b*x+a)*e^2+3*d*(-4/7*c*x+b)*e
+3*c*d^2)*g^2+128/35*((-5/9*c*x+b)*e+3*c*d)*e^2*f^4*g-256/63*c*e^3*f^5)/g^
6
```

3.826.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.54

$$\int \frac{(d+ex)^3\,(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\,(35\,ce^3g^5x^5+1280\,ce^3f^5-315\,ad^3g^5-1152\,(3\,cde^2+be^3)f^4g+1008\,(3}{}$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
2/315*(35*c*e^3*g^5*x^5 + 1280*c*e^3*f^5 - 315*a*d^3*g^5 - 1152*(3*c*d*e^2
 + b*e^3)*f^4*g + 1008*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^3*g^2 - 840*(c*d^
3 + 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 + 630*(b*d^3 + 3*a*d^2*e)*f*g^4 - 5*(10
*c*e^3*f*g^4 - 9*(3*c*d*e^2 + b*e^3)*g^5)*x^4 + (80*c*e^3*f^2*g^3 - 72*(3*
c*d*e^2 + b*e^3)*f*g^4 + 63*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*g^5)*x^3 - (16
0*c*e^3*f^3*g^2 - 144*(3*c*d*e^2 + b*e^3)*f^2*g^3 + 126*(3*c*d^2*e + 3*b*d
*e^2 + a*e^3)*f*g^4 - 105*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*g^5)*x^2 + (640*
c*e^3*f^4*g - 576*(3*c*d*e^2 + b*e^3)*f^3*g^2 + 504*(3*c*d^2*e + 3*b*d*e^2
 + a*e^3)*f^2*g^3 - 420*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f*g^4 + 315*(b*d^3
 + 3*a*d^2*e)*g^5)*x)*sqrt(g*x + f)/(g^7*x + f*g^6)
```

3.826.6 Sympy [A] (verification not implemented)

Time = 32.42 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.99

$$\int \frac{(d+ex)^3\,(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{ce^3(f+gx)^{\frac{9}{2}}}{9g^5}+\frac{(f+gx)^{\frac{7}{2}}\left(be^3g+3cde^2g-5ce^3f\right)}{7g^5}+\frac{(f+gx)^{\frac{5}{2}}\left(ae^3g^2+3bde^2g^2-4be^3fg+3cd^2eg^2-12cde^2f\cdots\right.}{5g^5}\cdots\right.}{\cdots} \\ \frac{ad^3x+\frac{ce^3x^6}{6}+\frac{x^5\left(be^3+3cde^2\right)}{5}+\frac{x^4\left(ae^3+3bde^2+3cd^2e\right)}{4}+\frac{x^3\cdot\left(3ade^2+3bd^2e+cd^3\right)}{3}+\frac{x^2\cdot\left(3ad^2e+bd^3\right)}{2}}{f^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((e*x+d)**3*(c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

output
```
Piecewise((2*(c*e**3*(f + g*x)**(9/2)/(9*g**5) + (f + g*x)**(7/2)*(b*e**3*
g + 3*c*d*e**2*g - 5*c*e**3*f)/(7*g**5) + (f + g*x)**(5/2)*(a*e**3*g**2 +
3*b*d*e**2*g**2 - 4*b*e**3*f*g + 3*c*d**2*e*g**2 - 12*c*d*e**2*f*g + 10*c*
e**3*f**2)/(5*g**5) + (f + g*x)**(3/2)*(3*a*d*e**2*g**3 - 3*a*e**3*f*g**2
+ 3*b*d**2*e*g**3 - 9*b*d*e**2*f*g**2 + 6*b*e**3*f**2*g + c*d**3*g**3 - 9*
c*d**2*e*f*g**2 + 18*c*d*e**2*f**2*g - 10*c*e**3*f**3)/(3*g**5) + sqrt(f +
 g*x)*(3*a*d**2*e*g**4 - 6*a*d*e**2*f*g**3 + 3*a*e**3*f**2*g**2 + b*d**3*g
**4 - 6*b*d**2*e*f*g**3 + 9*b*d*e**2*f**2*g**2 - 4*b*e**3*f**3*g - 2*c*d**
3*f*g**3 + 9*c*d**2*e*f**2*g**2 - 12*c*d*e**2*f**3*g + 5*c*e**3*f**4)/g**5
 - (d*g - e*f)**3*(a*g**2 - b*f*g + c*f**2)/(g**5*sqrt(f + g*x)))/g, Ne(g,
 0)), ((a*d**3*x + c*e**3*x**6/6 + x**5*(b*e**3 + 3*c*d*e**2)/5 + x**4*(a*
e**3 + 3*b*d*e**2 + 3*c*d**2*e)/4 + x**3*(3*a*d*e**2 + 3*b*d**2*e + c*d**3
)/3 + x**2*(3*a*d**2*e + b*d**3)/2)/f**(3/2), True))
```

3.826.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)^3 (a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\left(\frac{35\,(gx+f)^{\frac{9}{2}}ce^3 - 45\,(5\,ce^3f - (3\,cde^2+be^3)\,g)\,(gx+f)^{\frac{7}{2}} + 63\,(10\,ce^3f^2 - 4\,(3\,cde^2+be^3)\,fg + (3\,cd^2e\right.}{}$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
2/315*((35*(g*x + f)^(9/2)*c*e^3 - 45*(5*c*e^3*f - (3*c*d*e^2 + b*e^3)*g)*
(g*x + f)^(7/2) + 63*(10*c*e^3*f^2 - 4*(3*c*d*e^2 + b*e^3)*f*g + (3*c*d^2*
e + 3*b*d*e^2 + a*e^3)*g^2)*(g*x + f)^(5/2) - 105*(10*c*e^3*f^3 - 6*(3*c*d
*e^2 + b*e^3)*f^2*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f*g^2 - (c*d^3 + 3
*b*d^2*e + 3*a*d*e^2)*g^3)*(g*x + f)^(3/2) + 315*(5*c*e^3*f^4 - 4*(3*c*d*e
^2 + b*e^3)*f^3*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^2*g^2 - 2*(c*d^3 +
 3*b*d^2*e + 3*a*d*e^2)*f*g^3 + (b*d^3 + 3*a*d^2*e)*g^4)*sqrt(g*x + f))/g^
5 + 315*(c*e^3*f^5 - a*d^3*g^5 - (3*c*d*e^2 + b*e^3)*f^4*g + (3*c*d^2*e +
3*b*d*e^2 + a*e^3)*f^3*g^2 - (c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 + (b*
d^3 + 3*a*d^2*e)*f*g^4)/(sqrt(g*x + f)*g^5))/g
```

3.826.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 687 vs. 2(265) = 530.

Time = 0.31 (sec) , antiderivative size = 687, normalized size of antiderivative = 2.41

$$\int \frac{(d+ex)^3 (a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\,(ce^3f^5 - 3\,cde^2f^4g - be^3f^4g + 3\,cd^2ef^3g^2 + 3\,bde^2f^3g^2 + ae^3f^3g^2 - cd^3}{\sqrt{gx+f}\,g^6}$$
$$+ \frac{2\left(35\,(gx+f)^{\frac{9}{2}}ce^3g^{48} - 225\,(gx+f)^{\frac{7}{2}}ce^3fg^{48} + 630\,(gx+f)^{\frac{5}{2}}ce^3f^2g^{48} - 1050\,(gx+f)^{\frac{3}{2}}ce^3f^3g^{48} + 1575\right.}{}$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
2*(c*e^3*f^5 - 3*c*d*e^2*f^4*g - b*e^3*f^4*g + 3*c*d^2*e*f^3*g^2 + 3*b*d*e
^2*f^3*g^2 + a*e^3*f^3*g^2 - c*d^3*f^2*g^3 - 3*b*d^2*e*f^2*g^3 - 3*a*d*e^2
*f^2*g^3 + b*d^3*f*g^4 + 3*a*d^2*e*f*g^4 - a*d^3*g^5)/(sqrt(g*x + f)*g^6)
+ 2/315*(35*(g*x + f)^(9/2)*c*e^3*g^48 - 225*(g*x + f)^(7/2)*c*e^3*f*g^48
+ 630*(g*x + f)^(5/2)*c*e^3*f^2*g^48 - 1050*(g*x + f)^(3/2)*c*e^3*f^3*g^48
 + 1575*sqrt(g*x + f)*c*e^3*f^4*g^48 + 135*(g*x + f)^(7/2)*c*d*e^2*g^49 +
45*(g*x + f)^(7/2)*b*e^3*g^49 - 756*(g*x + f)^(5/2)*c*d*e^2*f*g^49 - 252*(
g*x + f)^(5/2)*b*e^3*f*g^49 + 1890*(g*x + f)^(3/2)*c*d*e^2*f^2*g^49 + 630*
(g*x + f)^(3/2)*b*e^3*f^2*g^49 - 3780*sqrt(g*x + f)*c*d*e^2*f^3*g^49 - 126
0*sqrt(g*x + f)*b*e^3*f^3*g^49 + 189*(g*x + f)^(5/2)*c*d^2*e*g^50 + 189*(g
*x + f)^(5/2)*b*d*e^2*g^50 + 63*(g*x + f)^(5/2)*a*e^3*g^50 - 945*(g*x + f)
^(3/2)*c*d^2*e*f*g^50 - 945*(g*x + f)^(3/2)*b*d*e^2*f*g^50 - 315*(g*x + f)
^(3/2)*a*e^3*f*g^50 + 2835*sqrt(g*x + f)*c*d^2*e*f^2*g^50 + 2835*sqrt(g*x
+ f)*b*d*e^2*f^2*g^50 + 945*sqrt(g*x + f)*a*e^3*f^2*g^50 + 105*(g*x + f)^(
3/2)*c*d^3*g^51 + 315*(g*x + f)^(3/2)*b*d^2*e*g^51 + 315*(g*x + f)^(3/2)*a
*d*e^2*g^51 - 630*sqrt(g*x + f)*c*d^3*f*g^51 - 1890*sqrt(g*x + f)*b*d^2*e*
f*g^51 - 1890*sqrt(g*x + f)*a*d*e^2*f*g^51 + 315*sqrt(g*x + f)*b*d^3*g^52
+ 945*sqrt(g*x + f)*a*d^2*e*g^52)/g^54
```

3.826.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^3\,(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{(f+g\,x)^{7/2}\,(2\,b\,e^3\,g-10\,c\,e^3\,f+6\,c\,d\,e^2\,g)}{7\,g^6} - \frac{2\,c\,d^3\,f^2\,g^3-2\,b\,d^3\,f\,g^4+2\,a\,d^3\,g^5-6\,c\,d^2\,e\,f^3\,g^2+6\,b\,d^2\,e\,f^2\,g^3-6\,a\,d^2\,e\,f\,g^4+6\,c\,d\,e^2\,f^4\,g-6\,b\,d\,e^2\,f^3\,g}{g^6\,\sqrt{f+g\,x}} + \frac{(f+g\,x)^{5/2}\,(6\,c\,d^2\,e\,g^2-24\,c\,d\,e^2\,f\,g+6\,b\,d\,e^2\,g^2+20\,c\,e^3\,f^2-8\,b\,e^3\,f\,g+2\,a\,e^3\,g^2)}{5\,g^6} + \frac{2\,(f+g\,x)^{3/2}\,(d\,g-e\,f)\,(c\,d^2\,g^2-8\,c\,d\,e\,f\,g+3\,b\,d\,e\,g^2+10\,c\,e^2\,f^2-6\,b\,e^2\,f\,g+3\,a\,e^2\,g^2)}{3\,g^6} + \frac{2\,\sqrt{f+g\,x}\,(d\,g-e\,f)^2\,(3\,a\,e\,g^2+b\,d\,g^2+5\,c\,e\,f^2-4\,b\,e\,f\,g-2\,c\,d\,f\,g)}{g^6} + \frac{2\,c\,e^3\,(f+g\,x)^{9/2}}{9\,g^6}$$

input
```
int(((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(3/2),x)
```

output
```
((f + g*x)^(7/2)*(2*b*e^3*g - 10*c*e^3*f + 6*c*d*e^2*g))/(7*g^6) - (2*a*d^
3*g^5 - 2*c*e^3*f^5 - 2*a*e^3*f^3*g^2 + 2*c*d^3*f^2*g^3 - 2*b*d^3*f*g^4 +
2*b*e^3*f^4*g - 6*a*d^2*e*f*g^4 + 6*c*d*e^2*f^4*g + 6*a*d*e^2*f^2*g^3 - 6*
b*d*e^2*f^3*g^2 + 6*b*d^2*e*f^2*g^3 - 6*c*d^2*e*f^3*g^2)/(g^6*(f + g*x)^(1
/2)) + ((f + g*x)^(5/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 - 8*b*e^3*f*g + 6*b*d*
e^2*g^2 + 6*c*d^2*e*g^2 - 24*c*d*e^2*f*g))/(5*g^6) + (2*(f + g*x)^(3/2)*(d
*g - e*f)*(3*a*e^2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 + 3*b*d*e*g^2 - 6*b*e^2*
f*g - 8*c*d*e*f*g))/(3*g^6) + (2*(f + g*x)^(1/2)*(d*g - e*f)^2*(3*a*e*g^2
+ b*d*g^2 + 5*c*e*f^2 - 4*b*e*f*g - 2*c*d*f*g))/g^6 + (2*c*e^3*(f + g*x)^(
9/2))/(9*g^6)
```

3.826. $\int \frac{(d+ex)^3(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx$

3.827 $\int \frac{(d+ex)^2(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx$

3.827.1 Optimal result

Integrand size = 27, antiderivative size = 210

$$\begin{aligned}\int \frac{(d+ex)^2(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = &-\frac{2(ef-dg)^2(cf^2-bfg+ag^2)}{g^5\sqrt{f+gx}}\\ &-\frac{2(ef-dg)(2cf(2ef-dg)-g(3bef-bdg-2aeg))\sqrt{f+gx}}{g^5}\\ &-\frac{2(eg(3bef-2bdg-aeg)-c(6e^2f^2-6defg+d^2g^2))(f+gx)^{3/2}}{3g^5}\\ &-\frac{2e(4cef-2cdg-beg)(f+gx)^{5/2}}{5g^5}+\frac{2ce^2(f+gx)^{7/2}}{7g^5}\end{aligned}$$

output
```
-2/3*(e*g*(-a*e*g-2*b*d*g+3*b*e*f)-c*(d^2*g^2-6*d*e*f*g+6*e^2*f^2))*(g*x+f
)^(3/2)/g^5-2/5*e*(-b*e*g-2*c*d*g+4*c*e*f)*(g*x+f)^(5/2)/g^5+2/7*c*e^2*(g*
x+f)^(7/2)/g^5-2*(-d*g+e*f)^2*(a*g^2-b*f*g+c*f^2)/g^5/(g*x+f)^(1/2)-2*(-d*
g+e*f)*(2*c*f*(-d*g+2*e*f)-g*(-2*a*e*g-b*d*g+3*b*e*f))*(g*x+f)^(1/2)/g^5
```

3.827.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)^2(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2(c(35d^2g^2(-8f^2-4fgx+g^2x^2)+42deg(16f^3+8f^2gx-2fg^2x^2+g^3x^3}{}$$

input
```
Integrate[((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

output
```
(2*(c*(35*d^2*g^2*(-8*f^2 - 4*f*g*x + g^2*x^2) + 42*d*e*g*(16*f^3 + 8*f^2*
g*x - 2*f*g^2*x^2 + g^3*x^3) - 3*e^2*(128*f^4 + 64*f^3*g*x - 16*f^2*g^2*x^
2 + 8*f*g^3*x^3 - 5*g^4*x^4)) + 7*g*(5*a*g*(-3*d^2*g^2 + 6*d*e*g*(2*f + g*
x) + e^2*(-8*f^2 - 4*f*g*x + g^2*x^2)) + b*(15*d^2*g^2*(2*f + g*x) + 10*d*
e*g*(-8*f^2 - 4*f*g*x + g^2*x^2) + 3*e^2*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2
 + g^3*x^3)))))/(105*g^5*Sqrt[f + g*x])
```

3.827.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{\sqrt{f+gx}(c(d^2g^2-6defg+6e^2f^2)-eg(-aeg-2bdg+3bef))}{g^4}+\frac{(dg-ef)^2(ag^2-bfg+cf^2)}{g^4(f+gx)^{3/2}}+\frac{(ef-dg}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(f+gx)^{3/2}\left(eg(-aeg-2bdg+3bef)-c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{3g^5}-\frac{2(ef-dg)^2(ag^2-bfg+cf^2)}{g^5\sqrt{f+gx}}-\frac{2\sqrt{f+gx}(ef-dg)(2cf(2ef-dg)-g(-2aeg-bdg+3bef))}{g^5}-\frac{2e(f+gx)^{5/2}(-beg-2cdg+4cef)}{5g^5}+\frac{2ce^2(f+gx)^{7/2}}{7g^5}$$

input
```
Int[((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

output
```
(-2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2))/(g^5*Sqrt[f + g*x]) - (2*(e*f -
 d*g)*(2*c*f*(2*e*f - d*g) - g*(3*b*e*f - b*d*g - 2*a*e*g))*Sqrt[f + g*x])
/g^5 - (2*(e*g*(3*b*e*f - 2*b*d*g - a*e*g) - c*(6*e^2*f^2 - 6*d*e*f*g + d^
2*g^2))*(f + g*x)^(3/2))/(3*g^5) - (2*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g
*x)^(5/2))/(5*g^5) + (2*c*e^2*(f + g*x)^(7/2))/(7*g^5)
```

3.827.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.827.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$\frac{\left(\left(30c\,x^4+42b\,x^3+70a\,x^2\right)e^2+420d\left(\frac{1}{5}c\,x^2+\frac{1}{3}bx+a\right)xe-210d^2\left(a-\frac{1}{3}c\,x^2-bx\right)\right)g^4+840f\left(-\frac{\left(\frac{6}{35}c\,x^2+\frac{3}{10}bx+a\right)x\,e^2}{3}+d(-\frac{}{}\right.}{1}$
risch	$\frac{2\left(15c\,e^2x^3g^3+21b\,e^2g^3x^2+42cde\,g^3x^2-39c\,e^2f\,g^2x^2+35a\,e^2g^3x+70bde\,g^3x-63b\,e^2f\,g^2x+35c\,d^2g^3x-126cdef\,g^2x+87\right.}{105}$
gosper	$-\frac{2\left(-15c\,e^2x^4g^4-21b\,e^2g^4x^3-42cde\,g^4x^3+24c\,e^2f\,g^3x^3-35a\,e^2g^4x^2-70bde\,g^4x^2+42b\,e^2f\,g^3x^2-35c\,d^2g^4x^2+84cdef\right.}{}$
trager	$-\frac{2\left(-15c\,e^2x^4g^4-21b\,e^2g^4x^3-42cde\,g^4x^3+24c\,e^2f\,g^3x^3-35a\,e^2g^4x^2-70bde\,g^4x^2+42b\,e^2f\,g^3x^2-35c\,d^2g^4x^2+84cdef\right.}{}$
derivativedivides	$\frac{\frac{2c\,e^2(gx+f)^{\frac{7}{2}}}{7}+\frac{2b\,e^2g(gx+f)^{\frac{5}{2}}}{5}+\frac{4cdeg(gx+f)^{\frac{5}{2}}}{5}-\frac{8c\,e^2f(gx+f)^{\frac{5}{2}}}{5}+\frac{2a\,e^2g^2(gx+f)^{\frac{3}{2}}}{3}+\frac{4bde\,g^2(gx+f)^{\frac{3}{2}}}{3}-2b\,e^2fg(gx+f)^{\frac{3}{2}}+}{}$
default	$\frac{\frac{2c\,e^2(gx+f)^{\frac{7}{2}}}{7}+\frac{2b\,e^2g(gx+f)^{\frac{5}{2}}}{5}+\frac{4cdeg(gx+f)^{\frac{5}{2}}}{5}-\frac{8c\,e^2f(gx+f)^{\frac{5}{2}}}{5}+\frac{2a\,e^2g^2(gx+f)^{\frac{3}{2}}}{3}+\frac{4bde\,g^2(gx+f)^{\frac{3}{2}}}{3}-2b\,e^2fg(gx+f)^{\frac{3}{2}}+}{}$

input
```
int((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(((30*c*x^4+42*b*x^3+70*a*x^2)*e^2+420*d*(1/5*c*x^2+1/3*b*x+a)*x*e-2
10*d^2*(a-1/3*c*x^2-b*x))*g^4+840*f*(-1/3*(6/35*c*x^2+3/10*b*x+a)*x*e^2+d*
(-1/5*c*x^2-2/3*b*x+a)*e+1/2*d^2*(-2/3*c*x+b))*g^3-560*((-6/35*c*x^2-3/5*b
*x+a)*e^2+2*d*(-3/5*c*x+b)*e+c*d^2)*f^2*g^2+672*e*((-4/7*c*x+b)*e+2*c*d)*f
^3*g-768*c*e^2*f^4)/(g*x+f)^(1/2)/g^5
```

3.827.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^2\,(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\,(15\,ce^2g^4x^4 - 384\,ce^2f^4 - 105\,ad^2g^4 + 336\,(2\,cde + be^2)f^3g - 280\,(cd^2 +$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
2/105*(15*c*e^2*g^4*x^4 - 384*c*e^2*f^4 - 105*a*d^2*g^4 + 336*(2*c*d*e + b
*e^2)*f^3*g - 280*(c*d^2 + 2*b*d*e + a*e^2)*f^2*g^2 + 210*(b*d^2 + 2*a*d*e
)*f*g^3 - 3*(8*c*e^2*f*g^3 - 7*(2*c*d*e + b*e^2)*g^4)*x^3 + (48*c*e^2*f^2*
g^2 - 42*(2*c*d*e + b*e^2)*f*g^3 + 35*(c*d^2 + 2*b*d*e + a*e^2)*g^4)*x^2 -
 (192*c*e^2*f^3*g - 168*(2*c*d*e + b*e^2)*f^2*g^2 + 140*(c*d^2 + 2*b*d*e +
 a*e^2)*f*g^3 - 105*(b*d^2 + 2*a*d*e)*g^4)*x)*sqrt(g*x + f)/(g^6*x + f*g^5
)
```

3.827.6 Sympy [A] (verification not implemented)

Time = 11.82 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.67

$$\int \frac{(d+ex)^2\,(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{ce^2(f+gx)^{\frac{7}{2}}}{7g^4} + \frac{(f+gx)^{\frac{5}{2}}\left(be^2g+2cdeg-4ce^2f\right)}{5g^4} + \frac{(f+gx)^{\frac{3}{2}}\left(ae^2g^2+2bdeg^2-3be^2fg+cd^2g^2-6cdefg+6ce^2\right.}{3g^4}\right.}{} \\ \frac{ad^2x+\frac{ce^2x^5}{5}+\frac{x^4\left(be^2+2cde\right)}{4}+\frac{x^3\left(ae^2+2bde+cd^2\right)}{3}+\frac{x^2\cdot\left(2ade+bd^2\right)}{2}}{f^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((e*x+d)**2*(c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

output
```
Piecewise((2*(c*e**2*(f + g*x)**(7/2)/(7*g**4) + (f + g*x)**(5/2)*(b*e**2*
g + 2*c*d*e*g - 4*c*e**2*f)/(5*g**4) + (f + g*x)**(3/2)*(a*e**2*g**2 + 2*b
*d*e*g**2 - 3*b*e**2*f*g + c*d**2*g**2 - 6*c*d*e*f*g + 6*c*e**2*f**2)/(3*g
**4) + sqrt(f + g*x)*(2*a*d*e*g**3 - 2*a*e**2*f*g**2 + b*d**2*g**3 - 4*b*d
*e*f*g**2 + 3*b*e**2*f**2*g - 2*c*d**2*f*g**2 + 6*c*d*e*f**2*g - 4*c*e**2*
f**3)/g**4 - (d*g - e*f)**2*(a*g**2 - b*f*g + c*f**2)/(g**4*sqrt(f + g*x))
)/g, Ne(g, 0)), ((a*d**2*x + c*e**2*x**5/5 + x**4*(b*e**2 + 2*c*d*e)/4 + x
**3*(a*e**2 + 2*b*d*e + c*d**2)/3 + x**2*(2*a*d*e + b*d**2)/2)/f**(3/2), T
rue))
```

3.827.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^2\,(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\left(\frac{15\,(gx+f)^{\frac{7}{2}}ce^2-21\,(4\,ce^2f-(2\,cde+be^2)g)(gx+f)^{\frac{5}{2}}+35\,(6\,ce^2f^2-3\,(2\,cde+be^2)fg+(cd^2+2\,bde}{}\right.}{}$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
2/105*((15*(g*x + f)^(7/2)*c*e^2 - 21*(4*c*e^2*f - (2*c*d*e + b*e^2)*g)*(g
*x + f)^(5/2) + 35*(6*c*e^2*f^2 - 3*(2*c*d*e + b*e^2)*f*g + (c*d^2 + 2*b*d
*e + a*e^2)*g^2)*(g*x + f)^(3/2) - 105*(4*c*e^2*f^3 - 3*(2*c*d*e + b*e^2)*
f^2*g + 2*(c*d^2 + 2*b*d*e + a*e^2)*f*g^2 - (b*d^2 + 2*a*d*e)*g^3)*sqrt(g*
x + f))/g^4 - 105*(c*e^2*f^4 + a*d^2*g^4 - (2*c*d*e + b*e^2)*f^3*g + (c*d^
2 + 2*b*d*e + a*e^2)*f^2*g^2 - (b*d^2 + 2*a*d*e)*f*g^3)/(sqrt(g*x + f)*g^4
))/g
```

3.827.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(194) = 388.

Time = 0.30 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.94

$$\int \frac{(d+ex)^2\,(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = -\frac{2\,(ce^2f^4-2\,cdef^3g-be^2f^3g+cd^2f^2g^2+2\,bdef^2g^2+ae^2f^2g^2-bd^2fg^3-2\,adefg^3+ad^2g^4)}{\sqrt{gx+f}\,g^5} + \frac{2\left(15\,(gx+f)^{\frac{7}{2}}ce^2g^{30}-84\,(gx+f)^{\frac{5}{2}}ce^2fg^{30}+210\,(gx+f)^{\frac{3}{2}}ce^2f^2g^{30}-420\,\sqrt{gx+f}\,ce^2f^3g^{30}+42\,(gx+\right.}{}$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
-2*(c*e^2*f^4 - 2*c*d*e*f^3*g - b*e^2*f^3*g + c*d^2*f^2*g^2 + 2*b*d*e*f^2*
g^2 + a*e^2*f^2*g^2 - b*d^2*f*g^3 - 2*a*d*e*f*g^3 + a*d^2*g^4)/(sqrt(g*x +
 f)*g^5) + 2/105*(15*(g*x + f)^(7/2)*c*e^2*g^30 - 84*(g*x + f)^(5/2)*c*e^2
*f*g^30 + 210*(g*x + f)^(3/2)*c*e^2*f^2*g^30 - 420*sqrt(g*x + f)*c*e^2*f^3
*g^30 + 42*(g*x + f)^(5/2)*c*d*e*g^31 + 21*(g*x + f)^(5/2)*b*e^2*g^31 - 21
0*(g*x + f)^(3/2)*c*d*e*f*g^31 - 105*(g*x + f)^(3/2)*b*e^2*f*g^31 + 630*sq
rt(g*x + f)*c*d*e*f^2*g^31 + 315*sqrt(g*x + f)*b*e^2*f^2*g^31 + 35*(g*x +
f)^(3/2)*c*d^2*g^32 + 70*(g*x + f)^(3/2)*b*d*e*g^32 + 35*(g*x + f)^(3/2)*a
*e^2*g^32 - 210*sqrt(g*x + f)*c*d^2*f*g^32 - 420*sqrt(g*x + f)*b*d*e*f*g^3
2 - 210*sqrt(g*x + f)*a*e^2*f*g^32 + 105*sqrt(g*x + f)*b*d^2*g^33 + 210*sq
rt(g*x + f)*a*d*e*g^33)/g^35
```

3.827.9 Mupad [B] (verification not implemented)

Time = 11.86 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex)^2 (a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{(f+g\,x)^{5/2}\,(2\,b\,e^2\,g-8\,c\,e^2\,f+4\,c\,d\,e\,g)}{5\,g^5} - \frac{2\,c\,d^2\,f^2\,g^2-2\,b\,d^2\,f\,g^3+2\,a\,d^2\,g^4-4\,c\,d\,e\,f^3\,g+4\,b\,d\,e\,f^2\,g^2-4\,a\,d\,e\,f\,g^3+2\,c\,e^2\,f^4-2\,b\,e^2\,f^3\,g+2\,a\,e^2}{g^5\,\sqrt{f+g\,x}} + \frac{(f+g\,x)^{3/2}\,(2\,c\,d^2\,g^2-12\,c\,d\,e\,f\,g+4\,b\,d\,e\,g^2+12\,c\,e^2\,f^2-6\,b\,e^2\,f\,g+2\,a\,e^2\,g^2)}{3\,g^5} + \frac{2\,\sqrt{f+g\,x}\,(d\,g-e\,f)\,(2\,a\,e\,g^2+b\,d\,g^2+4\,c\,e\,f^2-3\,b\,e\,f\,g-2\,c\,d\,f\,g)}{g^5} + \frac{2\,c\,e^2\,(f+g\,x)^{7/2}}{7\,g^5}$$

input
```
int(((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(3/2),x)
```

output
```
((f + g*x)^(5/2)*(2*b*e^2*g - 8*c*e^2*f + 4*c*d*e*g))/(5*g^5) - (2*a*d^2*g
^4 + 2*c*e^2*f^4 + 2*a*e^2*f^2*g^2 + 2*c*d^2*f^2*g^2 - 2*b*d^2*f*g^3 - 2*b
*e^2*f^3*g + 4*b*d*e*f^2*g^2 - 4*a*d*e*f*g^3 - 4*c*d*e*f^3*g)/(g^5*(f + g*
x)^(1/2)) + ((f + g*x)^(3/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 + 4
*b*d*e*g^2 - 6*b*e^2*f*g - 12*c*d*e*f*g))/(3*g^5) + (2*(f + g*x)^(1/2)*(d*
g - e*f)*(2*a*e*g^2 + b*d*g^2 + 4*c*e*f^2 - 3*b*e*f*g - 2*c*d*f*g))/g^5 +
(2*c*e^2*(f + g*x)^(7/2))/(7*g^5)
```

3.828 $\int \frac{(d+ex)(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx$

3.828.1 Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{(d+ex)(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2(ef-dg)(cf^2-bfg+ag^2)}{g^4\sqrt{f+gx}} + \frac{2(cf(3ef-2dg)-g(2bef-bdg-aeg))\sqrt{f+gx}}{g^4} - \frac{2(3cef-cdg-beg)(f+gx)^{3/2}}{3g^4} + \frac{2ce(f+gx)^{5/2}}{5g^4}$$

output
```
-2/3*(-b*e*g-c*d*g+3*c*e*f)*(g*x+f)^(3/2)/g^4+2/5*c*e*(g*x+f)^(5/2)/g^4+2*
(-d*g+e*f)*(a*g^2-b*f*g+c*f^2)/g^4/(g*x+f)^(1/2)+2*(c*f*(-2*d*g+3*e*f)-g*(
-a*e*g-b*d*g+2*b*e*f))*(g*x+f)^(1/2)/g^4
```

3.828.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2(5g(3bdg(2f+gx)+3ag(2ef-dg+egx)+be(-8f^2-4fgx+g^2x^2))+}{15g^4\sqrt{f+}}$$

input
```
Integrate[((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

output
```
(2*(5*g*(3*b*d*g*(2*f + g*x) + 3*a*g*(2*e*f - d*g + e*g*x) + b*e*(-8*f^2 -
 4*f*g*x + g^2*x^2)) + c*(5*d*g*(-8*f^2 - 4*f*g*x + g^2*x^2) + 3*e*(16*f^3
 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3))))/(15*g^4*Sqrt[f + g*x])
```

3.828.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(a+bx+cx^2\right)}{(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{(dg-ef)\left(ag^2-bfg+cf^2\right)}{g^3(f+gx)^{3/2}}+\frac{cf(3ef-2dg)-g(-aeg-bdg+2bef)}{g^3\sqrt{f+gx}}+\frac{\sqrt{f+gx}(beg+cdg-3cef)}{g^3}+\frac{ce(\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(ef-dg)\left(ag^2-bfg+cf^2\right)}{g^4\sqrt{f+gx}}+\frac{2\sqrt{f+gx}(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{g^4}-\frac{2(f+gx)^{3/2}(-beg-cdg+3cef)}{3g^4}+\frac{2ce(f+gx)^{5/2}}{5g^4}$$

input
```
Int[((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

output
```
(2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2))/(g^4*Sqrt[f + g*x]) + (2*(c*f*(3*e
*f - 2*d*g) - g*(2*b*e*f - b*d*g - a*e*g))*Sqrt[f + g*x])/g^4 - (2*(3*c*e*
f - c*d*g - b*e*g)*(f + g*x)^(3/2))/(3*g^4) + (2*c*e*(f + g*x)^(5/2))/(5*g
^4)
```

3.828.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.828.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{\left(\left(6c\,x^{3}+10b\,x^{2}+30ax\right)e-30d\left(a-\frac{1}{3}c\,x^{2}-bx\right)\right)g^{3}+60\left(\left(-\frac{1}{5}c\,x^{2}-\frac{2}{3}bx+a\right)e+d\left(-\frac{2cx}{3}+b\right)\right)f\,g^{2}-80\left(\left(-\frac{3cx}{5}+b\right)e+cd\right)f^{2}g+}{15\sqrt{gx+f}\,g^{4}}$
risch	$\frac{2\left(3ce\,x^{2}g^{2}+5bex\,g^{2}+5cd\,g^{2}x-9cefgx+15ae\,g^{2}+15bd\,g^{2}-25befg-25cdfg+33ce\,f^{2}\right)\sqrt{gx+f}}{15g^{4}}-\frac{2\left(ad\,g^{3}-aef\,g^{2}-bdf\,g\right.}{g^{4}\surd}$
gosper	$-\frac{2\left(-3ce\,x^{3}g^{3}-5be\,g^{3}x^{2}-5cd\,g^{3}x^{2}+6cef\,g^{2}x^{2}-15ae\,g^{3}x-15bd\,g^{3}x+20bef\,g^{2}x+20cdf\,g^{2}x-24ce\,f^{2}gx+15ad\,g^{3}-30a\right.}{15\sqrt{gx+f}\,g^{4}}$
trager	$-\frac{2\left(-3ce\,x^{3}g^{3}-5be\,g^{3}x^{2}-5cd\,g^{3}x^{2}+6cef\,g^{2}x^{2}-15ae\,g^{3}x-15bd\,g^{3}x+20bef\,g^{2}x+20cdf\,g^{2}x-24ce\,f^{2}gx+15ad\,g^{3}-30a\right.}{15\sqrt{gx+f}\,g^{4}}$
derivativedivides	$\frac{\frac{2ce(gx+f)^{\frac{5}{2}}}{5}+\frac{2beg(gx+f)^{\frac{3}{2}}}{3}+\frac{2cdg(gx+f)^{\frac{3}{2}}}{3}-2cef\,(gx+f)^{\frac{3}{2}}+2ae\,g^{2}\sqrt{gx+f}+2bd\,g^{2}\sqrt{gx+f}-4befg\sqrt{gx+f}-4cdfg\sqrt{gx+}}{g^{4}}$
default	$\frac{\frac{2ce(gx+f)^{\frac{5}{2}}}{5}+\frac{2beg(gx+f)^{\frac{3}{2}}}{3}+\frac{2cdg(gx+f)^{\frac{3}{2}}}{3}-2cef\,(gx+f)^{\frac{3}{2}}+2ae\,g^{2}\sqrt{gx+f}+2bd\,g^{2}\sqrt{gx+f}-4befg\sqrt{gx+f}-4cdfg\sqrt{gx+}}{g^{4}}$

input
```
int((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(((6*c*x^3+10*b*x^2+30*a*x)*e-30*d*(a-1/3*c*x^2-b*x))*g^3+60*((-1/5*c
*x^2-2/3*b*x+a)*e+d*(-2/3*c*x+b))*f*g^2-80*((-3/5*c*x+b)*e+c*d)*f^2*g+96*c
*e*f^3)/(g*x+f)^(1/2)/g^4
```

3.828.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)\left(a+bx+cx^2\right)}{(f+gx)^{3/2}}\,dx = \frac{2\left(3\,ceg^3x^3+48\,cef^3-15\,adg^3-40\,(cd+be)f^2g+30\,(bd+ae)fg^2-(6\,c\right.}{15\,(}$$

input
```
integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
2/15*(3*c*e*g^3*x^3 + 48*c*e*f^3 - 15*a*d*g^3 - 40*(c*d + b*e)*f^2*g + 30*
(b*d + a*e)*f*g^2 - (6*c*e*f*g^2 - 5*(c*d + b*e)*g^3)*x^2 + (24*c*e*f^2*g
- 20*(c*d + b*e)*f*g^2 + 15*(b*d + a*e)*g^3)*x)*sqrt(g*x + f)/(g^5*x + f*g
^4)
```

3.828.6 Sympy [A] (verification not implemented)

Time = 3.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)\left(a+bx+cx^2\right)}{(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{ce(f+gx)^{\frac{5}{2}}}{5g^3}+\frac{(f+gx)^{\frac{3}{2}}(beg+cdg-3cef)}{3g^3}+\frac{\sqrt{f+gx}\left(aeg^2+bdg^2-2befg-2cdfg+3cef^2\right)}{g^3}-\frac{(dg-ef)\left(ag^2-bfg\right.}{g^3\sqrt{f+gx}}\right.}{g} \\ \frac{adx+\frac{cex^4}{4}+\frac{x^3(be+cd)}{3}+\frac{x^2(ae+bd)}{2}}{f^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((e*x+d)*(c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

output
```
Piecewise((2*(c*e*(f + g*x)**(5/2)/(5*g**3) + (f + g*x)**(3/2)*(b*e*g + c*
d*g - 3*c*e*f)/(3*g**3) + sqrt(f + g*x)*(a*e*g**2 + b*d*g**2 - 2*b*e*f*g -
 2*c*d*f*g + 3*c*e*f**2)/g**3 - (d*g - e*f)*(a*g**2 - b*f*g + c*f**2)/(g**
3*sqrt(f + g*x)))/g, Ne(g, 0)), ((a*d*x + c*e*x**4/4 + x**3*(b*e + c*d)/3
+ x**2*(a*e + b*d)/2)/f**(3/2), True))
```

3.828.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)\,(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\left(\frac{3\,(gx+f)^{\frac{5}{2}}ce-5\,(3\,cef-(cd+be)g)(gx+f)^{\frac{3}{2}}+15\,(3\,cef^2-2\,(cd+be)fg+(bd+ae)g^2)\sqrt{gx+f}}{g^3}+\frac{15}{}\right.}{15\,g}$$

input
```
integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
2/15*((3*(g*x + f)^(5/2)*c*e - 5*(3*c*e*f - (c*d + b*e)*g)*(g*x + f)^(3/2)
 + 15*(3*c*e*f^2 - 2*(c*d + b*e)*f*g + (b*d + a*e)*g^2)*sqrt(g*x + f))/g^3
 + 15*(c*e*f^3 - a*d*g^3 - (c*d + b*e)*f^2*g + (b*d + a*e)*f*g^2)/(sqrt(g*
x + f)*g^3))/g
```

3.828.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)\,(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\,(cef^3-cdf^2g-bef^2g+bdfg^2+aefg^2-adg^3)}{\sqrt{gx+f}g^4} + \frac{2\left(3\,(gx+f)^{\frac{5}{2}}ceg^{16}-15\,(gx+f)^{\frac{3}{2}}cefg^{16}+45\,\sqrt{gx+f}cef^2g^{16}+5\,(gx+f)^{\frac{3}{2}}cdg^{17}+5\,(gx+f)^{\frac{3}{2}}beg^{17}-\right.}{15\,g^{20}}$$

input
```
integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
2*(c*e*f^3 - c*d*f^2*g - b*e*f^2*g + b*d*f*g^2 + a*e*f*g^2 - a*d*g^3)/(sqr
t(g*x + f)*g^4) + 2/15*(3*(g*x + f)^(5/2)*c*e*g^16 - 15*(g*x + f)^(3/2)*c*
e*f*g^16 + 45*sqrt(g*x + f)*c*e*f^2*g^16 + 5*(g*x + f)^(3/2)*c*d*g^17 + 5*
(g*x + f)^(3/2)*b*e*g^17 - 30*sqrt(g*x + f)*c*d*f*g^17 - 30*sqrt(g*x + f)*
b*e*f*g^17 + 15*sqrt(g*x + f)*b*d*g^18 + 15*sqrt(g*x + f)*a*e*g^18)/g^20
```

3.828.9 Mupad [B] (verification not implemented)

Time = 11.75 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)\,(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{(f+g\,x)^{3/2}\,(2\,b\,e\,g+2\,c\,d\,g-6\,c\,e\,f)}{3\,g^4} - \frac{2\,a\,d\,g^3-2\,c\,e\,f^3-2\,a\,e\,f\,g^2-2\,b\,d\,f\,g^2+2\,b\,e\,f^2\,g+2\,c\,d\,f^2\,g}{g^4\,\sqrt{f+g\,x}} + \frac{\sqrt{f+g\,x}\,(2\,a\,e\,g^2+2\,b\,d\,g^2+6\,c\,e\,f^2-4\,b\,e\,f\,g-4\,c\,d\,f\,g)}{g^4} + \frac{2\,c\,e\,(f+g\,x)^{5/2}}{5\,g^4}$$

input
```
int(((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(3/2),x)
```

output
```
((f + g*x)^(3/2)*(2*b*e*g + 2*c*d*g - 6*c*e*f))/(3*g^4) - (2*a*d*g^3 - 2*c
*e*f^3 - 2*a*e*f*g^2 - 2*b*d*f*g^2 + 2*b*e*f^2*g + 2*c*d*f^2*g)/(g^4*(f +
g*x)^(1/2)) + ((f + g*x)^(1/2)*(2*a*e*g^2 + 2*b*d*g^2 + 6*c*e*f^2 - 4*b*e*
f*g - 4*c*d*f*g))/g^4 + (2*c*e*(f + g*x)^(5/2))/(5*g^4)
```

3.829 $\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx$

3.829.1 Optimal result

Integrand size = 20, antiderivative size = 71

$$\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx=-\frac{2(cf^2-bfg+ag^2)}{g^3\sqrt{f+gx}}-\frac{2(2cf-bg)\sqrt{f+gx}}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}$$

output
```
2/3*c*(g*x+f)^(3/2)/g^3-2*(a*g^2-b*f*g+c*f^2)/g^3/(g*x+f)^(1/2)-2*(-b*g+2*
c*f)*(g*x+f)^(1/2)/g^3
```

3.829.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

$$\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx=\frac{6g(2bf-ag+bgx)+2c(-8f^2-4fgx+g^2x^2)}{3g^3\sqrt{f+gx}}$$

input
```
Integrate[(a + b*x + c*x^2)/(f + g*x)^(3/2),x]
```

output
```
(6*g*(2*b*f - a*g + b*g*x) + 2*c*(-8*f^2 - 4*f*g*x + g^2*x^2))/(3*g^3*Sqrt
[f + g*x])
```

3.829.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \ 1140$$

$$\int \left(\frac{ag^2-bfg+cf^2}{g^2(f+gx)^{3/2}}+\frac{bg-2cf}{g^2\sqrt{f+gx}}+\frac{c\sqrt{f+gx}}{g^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2(ag^2-bfg+cf^2)}{g^3\sqrt{f+gx}}-\frac{2\sqrt{f+gx}(2cf-bg)}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}$$

input
```
Int[(a + b*x + c*x^2)/(f + g*x)^(3/2),x]
```

output
```
(-2*(c*f^2 - b*f*g + a*g^2))/(g^3*Sqrt[f + g*x]) - (2*(2*c*f - b*g)*Sqrt[f
 + g*x])/g^3 + (2*c*(f + g*x)^(3/2))/(3*g^3)
```

3.829.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.829.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.66

method	result	size
pseudoelliptic	$\frac{\frac{2\left(c\,x^{2}+3bx-3a\right)g^{2}}{3}+4f\left(-\frac{2cx}{3}+b\right)g-\frac{16c\,f^{2}}{3}}{\sqrt{gx+f}\,g^{3}}$	47
gosper	$-\frac{2\left(-c\,x^{2}g^{2}-3b\,g^{2}x+4cfxg+3a\,g^{2}-6bfg+8c\,f^{2}\right)}{3\sqrt{gx+f}\,g^{3}}$	53
trager	$-\frac{2\left(-c\,x^{2}g^{2}-3b\,g^{2}x+4cfxg+3a\,g^{2}-6bfg+8c\,f^{2}\right)}{3\sqrt{gx+f}\,g^{3}}$	53
risch	$\frac{2(cxg+3bg-5cf)\sqrt{gx+f}}{3g^{3}}-\frac{2\left(a\,g^{2}-bfg+c\,f^{2}\right)}{g^{3}\sqrt{gx+f}}$	55
derivativedivides	$\frac{\frac{2(gx+f)^{\frac{3}{2}}c}{3}+2bg\sqrt{gx+f}-4cf\sqrt{gx+f}-\frac{2\left(a\,g^{2}-bfg+c\,f^{2}\right)}{\sqrt{gx+f}}}{g^{3}}$	63
default	$\frac{\frac{2(gx+f)^{\frac{3}{2}}c}{3}+2bg\sqrt{gx+f}-4cf\sqrt{gx+f}-\frac{2\left(a\,g^{2}-bfg+c\,f^{2}\right)}{\sqrt{gx+f}}}{g^{3}}$	63

input
```
int((c*x^2+b*x+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*((c*x^2+3*b*x-3*a)*g^2+6*f*(-2/3*c*x+b)*g-8*c*f^2)/(g*x+f)^(1/2)/g^3
```

3.829.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx=\frac{2\left(cg^2x^2-8\,cf^2+6\,bfg-3\,ag^2-(4\,cfg-3\,bg^2)x\right)\sqrt{gx+f}}{3\left(g^4x+fg^3\right)}$$

input
```
integrate((c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(c*g^2*x^2 - 8*c*f^2 + 6*b*f*g - 3*a*g^2 - (4*c*f*g - 3*b*g^2)*x)*sqrt
(g*x + f)/(g^4*x + f*g^3)
```

3.829.6 Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.32

$$\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{c(f+gx)^{\frac{3}{2}}}{3g^2}+\frac{\sqrt{f+gx}(bg-2cf)}{g^2}-\frac{ag^2-bfg+cf^2}{g^2\sqrt{f+gx}}\right)}{g} & \text{for } g\neq 0 \\ \frac{ax+\frac{bx^2}{2}+\frac{cx^3}{3}}{f^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

output
```
Piecewise((2*(c*(f + g*x)**(3/2)/(3*g**2) + sqrt(f + g*x)*(b*g - 2*c*f)/g*
*2 - (a*g**2 - b*f*g + c*f**2)/(g**2*sqrt(f + g*x)))/g, Ne(g, 0)), ((a*x +
 b*x**2/2 + c*x**3/3)/f**(3/2), True))
```

3.829.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx = \frac{2\left(\frac{(gx+f)^{\frac{3}{2}}c-3(2cf-bg)\sqrt{gx+f}}{g^2}-\frac{3(cf^2-bfg+ag^2)}{\sqrt{gx+f}g^2}\right)}{3g}$$

input
```
integrate((c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
2/3*(((g*x + f)^(3/2)*c - 3*(2*c*f - b*g)*sqrt(g*x + f))/g^2 - 3*(c*f^2 -
b*f*g + a*g^2)/(sqrt(g*x + f)*g^2))/g
```

3.829.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx = -\frac{2(cf^2-bfg+ag^2)}{\sqrt{gx+f}g^3} + \frac{2\left((gx+f)^{\frac{3}{2}}cg^6-6\sqrt{gx+f}cfg^6+3\sqrt{gx+f}bg^7\right)}{3g^9}$$

input
```
integrate((c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
-2*(c*f^2 - b*f*g + a*g^2)/(sqrt(g*x + f)*g^3) + 2/3*((g*x + f)^(3/2)*c*g^
6 - 6*sqrt(g*x + f)*c*f*g^6 + 3*sqrt(g*x + f)*b*g^7)/g^9
```

3.829.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx = \frac{2\,c\,{\left(f+g\,x\right)}^2-6\,a\,g^2-6\,c\,f^2+6\,b\,g\,\left(f+g\,x\right)-12\,c\,f\,\left(f+g\,x\right)+6\,b\,f\,g}{3\,g^3\,\sqrt{f+g\,x}}$$

input
```
int((a + b*x + c*x^2)/(f + g*x)^(3/2),x)
```

output
```
(2*c*(f + g*x)^2 - 6*a*g^2 - 6*c*f^2 + 6*b*g*(f + g*x) - 12*c*f*(f + g*x)
+ 6*b*f*g)/(3*g^3*(f + g*x)^(1/2))
```

3.830 $\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx$

3.830.1 Optimal result

Integrand size = 27, antiderivative size = 122

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2(cf^2-bfg+ag^2)}{g^2(ef-dg)\sqrt{f+gx}} + \frac{2c\sqrt{f+gx}}{eg^2} - \frac{2(cd^2-bde+ae^2)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}$$

output
```
-2*(a*e^2-b*d*e+c*d^2)*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(
3/2)/(-d*g+e*f)^(3/2)+2*(a*g^2-b*f*g+c*f^2)/g^2/(-d*g+e*f)/(g*x+f)^(1/2)+2
*c*(g*x+f)^(1/2)/e/g^2
```

3.830.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2(eg(bf-ag)+cdg(f+gx)-cef(2f+gx))}{eg^2(-ef+dg)\sqrt{f+gx}} - \frac{2(cd^2+e(-bd+ae))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{3/2}(-ef+dg)^{3/2}}$$

input
```
Integrate[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

output
```
(2*(e*g*(b*f - a*g) + c*d*g*(f + g*x) - c*e*f*(2*f + g*x)))/(e*g^2*(-(e*f)
 + d*g)*Sqrt[f + g*x]) - (2*(c*d^2 + e*(-(b*d) + a*e))*ArcTan[(Sqrt[e]*Sqr
t[f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(3/2)*(-(e*f) + d*g)^(3/2))
```

3.830.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1192, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}} \, dx$$

$$\Big\downarrow \text{1192}$$

$$\frac{2\int -\frac{cf^2-bgf+ag^2+c(f+gx)^2-(2cf-bg)(f+gx)}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\int \frac{cf^2-bgf+ag^2+c(f+gx)^2-(2cf-bg)(f+gx)}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \text{1584}$$

$$-\frac{2\int \left(\frac{(cd^2-bed+ae^2)g^2}{e(ef-dg)(ef-dg-e(f+gx))}-\frac{c}{e}+\frac{cf^2-bgf+ag^2}{(ef-dg)(f+gx)}\right)d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\frac{g^2(ae^2-bde+cd^2)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}+\frac{ag^2-bfg+cf^2}{\sqrt{f+gx}(ef-dg)}+\frac{c\sqrt{f+gx}}{e}\right)}{g^2}$$

input
```
Int[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

output
```
(2*((c*f^2 - b*f*g + a*g^2)/((e*f - d*g)*Sqrt[f + g*x]) + (c*Sqrt[f + g*x]
)/e - ((c*d^2 - b*d*e + a*e^2)*g^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*
f - d*g]])/(e^(3/2)*(e*f - d*g)^(3/2))))/g^2
```

3.830.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.830.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{2c\sqrt{gx+f}}{e}-\frac{2\left(e^{2}a-bde+c\,d^{2}\right)g^{2}\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e(dg-ef)\sqrt{(dg-ef)e}}-\frac{2\left(a\,g^{2}-bfg+c\,f^{2}\right)}{(dg-ef)\sqrt{gx+f}}}{g^{2}}$	122
default	$\frac{\frac{2c\sqrt{gx+f}}{e}-\frac{2\left(e^{2}a-bde+c\,d^{2}\right)g^{2}\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e(dg-ef)\sqrt{(dg-ef)e}}-\frac{2\left(a\,g^{2}-bfg+c\,f^{2}\right)}{(dg-ef)\sqrt{gx+f}}}{g^{2}}$	122
pseudoelliptic	$\frac{\frac{2c\sqrt{gx+f}}{e}-\frac{2\left(e^{2}a-bde+c\,d^{2}\right)g^{2}\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e(dg-ef)\sqrt{(dg-ef)e}}-\frac{2\left(a\,g^{2}-bfg+c\,f^{2}\right)}{(dg-ef)\sqrt{gx+f}}}{g^{2}}$	122
risch	$\frac{2c\sqrt{gx+f}}{e\,g^{2}}+\frac{-\frac{2g^{2}\left(e^{2}a-bde+c\,d^{2}\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)\sqrt{(dg-ef)e}}-\frac{2e\left(a\,g^{2}-bfg+c\,f^{2}\right)}{(dg-ef)\sqrt{gx+f}}}{g^{2}e}$	128

input
```
int((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/g^2*(c/e*(g*x+f)^(1/2)-(a*g^2-b*f*g+c*f^2)/(d*g-e*f)/(g*x+f)^(1/2)-(a*e^
2-b*d*e+c*d^2)/e*g^2/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/
((d*g-e*f)*e)^(1/2)))
```

3.830.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(108) = 216$.

Time = 0.43 (sec) , antiderivative size = 540, normalized size of antiderivative = 4.43

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \left[-\frac{((cd^2-bde+ae^2)g^3x+(cd^2-bde+ae^2)fg^2)\sqrt{e^2f-deg}\log\left(\frac{egx+2\,ef-dg+\dots}{\dots}\right)\dots}{e^4f^3\dots}\right.$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
[-(((c*d^2 - b*d*e + a*e^2)*g^3*x + (c*d^2 - b*d*e + a*e^2)*f*g^2)*sqrt(e^
2*f - d*e*g)*log((e*g*x + 2*e*f - d*g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f
))/(e*x + d)) - 2*(2*c*e^3*f^3 - a*d*e^2*g^3 - (3*c*d*e^2 + b*e^3)*f^2*g +
 (c*d^2*e + b*d*e^2 + a*e^3)*f*g^2 + (c*e^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^
2*e*g^3)*x)*sqrt(g*x + f))/(e^4*f^3*g^2 - 2*d*e^3*f^2*g^3 + d^2*e^2*f*g^4
+ (e^4*f^2*g^3 - 2*d*e^3*f*g^4 + d^2*e^2*g^5)*x), 2*(((c*d^2 - b*d*e + a*e
^2)*g^3*x + (c*d^2 - b*d*e + a*e^2)*f*g^2)*sqrt(-e^2*f + d*e*g)*arctan(sqr
t(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*c*e^3*f^3 - a*d*e^2*g^
3 - (3*c*d*e^2 + b*e^3)*f^2*g + (c*d^2*e + b*d*e^2 + a*e^3)*f*g^2 + (c*e^3
*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(g*x + f))/(e^4*f^3*g^2 - 2
*d*e^3*f^2*g^3 + d^2*e^2*f*g^4 + (e^4*f^2*g^3 - 2*d*e^3*f*g^4 + d^2*e^2*g^
5)*x)]
```

3.830.6 Sympy [A] (verification not implemented)

Time = 6.45 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.41

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{c\sqrt{f+gx}}{eg}-\frac{ag^2-bfg+cf^2}{g\sqrt{f+gx}(dg-ef)}-\frac{g\left(ae^2-bde+cd^2\right)\operatorname{atan}\left(\frac{\sqrt{f+gx}}{\sqrt{\frac{dg-ef}{e}}}\right)}{e^2\sqrt{\frac{dg-ef}{e}}(dg-ef)}\right)}{g} & \text{for } g \neq 0 \\ \frac{\frac{cx^2}{2e}+\frac{x(be-cd)}{e^2}+\frac{\left(ae^2-bde+cd^2\right)\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{e^2}}{f^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+b*x+a)/(e*x+d)/(g*x+f)**(3/2),x)
```

output
```
Piecewise((2*(c*sqrt(f + g*x)/(e*g) - (a*g**2 - b*f*g + c*f**2)/(g*sqrt(f
+ g*x)*(d*g - e*f)) - g*(a*e**2 - b*d*e + c*d**2)*atan(sqrt(f + g*x)/sqrt(
(d*g - e*f)/e))/(e**2*sqrt((d*g - e*f)/e)*(d*g - e*f)))/g, Ne(g, 0)), ((c*
x**2/(2*e) + x*(b*e - c*d)/e**2 + (a*e**2 - b*d*e + c*d**2)*Piecewise((x/d
, Eq(e, 0)), (log(d + e*x)/e, True))/e**2)/f**(3/2), True))
```

3.830.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + bx + cx^2}{(d + ex)(f + gx)^{3/2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.830.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \frac{a + bx + cx^2}{(d + ex)(f + gx)^{3/2}}\, dx = \frac{2\,(cd^2 - bde + ae^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{(e^2f - deg)\sqrt{-e^2f + deg}} + \frac{2\,(cf^2 - bfg + ag^2)}{(efg^2 - dg^3)\sqrt{gx + f}} + \frac{2\sqrt{gx + f}c}{eg^2}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
2*(c*d^2 - b*d*e + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/((e
^2*f - d*e*g)*sqrt(-e^2*f + d*e*g)) + 2*(c*f^2 - b*f*g + a*g^2)/((e*f*g^2
- d*g^3)*sqrt(g*x + f)) + 2*sqrt(g*x + f)*c/(e*g^2)
```

3.830.9 Mupad [B] (verification not implemented)

Time = 11.83 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.33

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2\,c\,\sqrt{f+g\,x}}{e\,g^2} + \frac{2\,\mathrm{atan}\left(\frac{2\,\sqrt{f+g\,x}\,\left(e^2\,f-d\,e\,g\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\sqrt{e}\,{\left(d\,g-e\,f\right)}^{3/2}\,\left(2\,c\,d^2-2\,b\,d\,e+2\,a\,e^2\right)}\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{e^{3/2}\,{\left(d\,g-e\,f\right)}^{3/2}} - \frac{2\,\left(c\,e\,f^2-b\,e\,f\,g+a\,e\,g^2\right)}{e\,g^2\,\sqrt{f+g\,x}\,\left(d\,g-e\,f\right)}$$

input

```
int((a + b*x + c*x^2)/((f + g*x)^(3/2)*(d + e*x)),x)
```

output

```
(2*c*(f + g*x)^(1/2))/(e*g^2) + (2*atan((2*(f + g*x)^(1/2)*(e^2*f - d*e*g)
*(a*e^2 + c*d^2 - b*d*e))/(e^(1/2)*(d*g - e*f)^(3/2)*(2*a*e^2 + 2*c*d^2 -
2*b*d*e)))*(a*e^2 + c*d^2 - b*d*e))/(e^(3/2)*(d*g - e*f)^(3/2)) - (2*(a*e*
g^2 + c*e*f^2 - b*e*f*g))/(e*g^2*(f + g*x)^(1/2)*(d*g - e*f))
```

3.831 $\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx$

3.831.1 Optimal result

Integrand size = 27, antiderivative size = 165

$$\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx=-\frac{2(cf^2-bfg+ag^2)}{g(ef-dg)^2\sqrt{f+gx}}-\frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)}+\frac{(cd(4ef-dg)-e(2bef+bdg-3aeg))\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{5/2}}$$

output
```
(c*d*(-d*g+4*e*f)-e*(-3*a*e*g+b*d*g+2*b*e*f))*arctanh(e^(1/2)*(g*x+f)^(1/2
)/(-d*g+e*f)^(1/2))/e^(3/2)/(-d*g+e*f)^(5/2)-2*(a*g^2-b*f*g+c*f^2)/g/(-d*g
+e*f)^2/(g*x+f)^(1/2)-(a*e^2-b*d*e+c*d^2)*(g*x+f)^(1/2)/e/(-d*g+e*f)^2/(e*
x+d)
```

3.831.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.07

$$\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx=\frac{-c(2def^2+2e^2f^2x+d^2g(f+gx))+eg(b(3df+2efx+dgx)-a(ef+2dg}{eg(ef-dg)^2(d+ex)\sqrt{f+gx}}+\frac{(cd(-4ef+dg)+e(2bef+bdg-3aeg))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{3/2}(-ef+dg)^{5/2}}$$

input
```
Integrate[(a + b*x + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

output
```
(-(c*(2*d*e*f^2 + 2*e^2*f^2*x + d^2*g*(f + g*x))) + e*g*(b*(3*d*f + 2*e*f*
x + d*g*x) - a*(e*f + 2*d*g + 3*e*g*x)))/(e*g*(e*f - d*g)^2*(d + e*x)*Sqrt
[f + g*x]) + ((c*d*(-4*e*f + d*g) + e*(2*b*e*f + b*d*g - 3*a*e*g))*ArcTan[
(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(3/2)*(-(e*f) + d*g)^(5/2)
)
```

3.831.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1192, 1582, 25, 27, 359, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \ 1192$$

$$\frac{2\int \frac{cf^2-bgf+ag^2+c(f+gx)^2-(2cf-bg)(f+gx)}{(f+gx)(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{g}$$

$$\Big\downarrow \ 1582$$

$$\frac{2\left(\frac{g^2\sqrt{f+gx}(ae^2-bde+cd^2)}{2e(ef-dg)^2(-dg-e(f+gx)+ef)}-\frac{\int -\frac{e\left(2e(ef-dg)\left(cf^2-bgf+ag^2\right)-\left(e(bd-ae)g^2+c\left(2e^2f^2-4degf+d^2g^2\right)\right)(f+gx)\right)}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}}{2e^2(ef-dg)^2}\right)}{g}$$

$$\Big\downarrow \ 25$$

$$\frac{2\left(\frac{\int \frac{e\left(2e(ef-dg)\left(cf^2-bgf+ag^2\right)-\left(e(bd-ae)g^2+c\left(2e^2f^2-4degf+d^2g^2\right)\right)(f+gx)\right)}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}}{2e^2(ef-dg)^2}+\frac{g^2\sqrt{f+gx}(ae^2-bde+cd^2)}{2e(ef-dg)^2(-dg-e(f+gx)+ef)}\right)}{g}$$

$$\Big\downarrow \ 27$$

$$\frac{2\left(\frac{\int \frac{2e(ef-dg)\left(cf^2-bgf+ag^2\right)-\left(e(bd-ae)g^2+c\left(2e^2f^2-4degf+d^2g^2\right)\right)(f+gx)}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}}{2e(ef-dg)^2}+\frac{g^2\sqrt{f+gx}(ae^2-bde+cd^2)}{2e(ef-dg)^2(-dg-e(f+gx)+ef)}\right)}{g}$$

$$\Big\downarrow \ 359$$

$$\frac{2\left(\frac{g(cd(4ef-dg)-e(-3aeg+bdg+2bef))\int\frac{1}{ef-dg-e(f+gx)}d\sqrt{f+gx}-\frac{2e\left(ag^2-bfg+cf^2\right)}{\sqrt{f+gx}}}{2e(ef-dg)^2}+\frac{g^2\sqrt{f+gx}\left(ae^2-bde+cd^2\right)}{2e(ef-dg)^2(-dg-e(f+gx)+ef)}\right)}{g}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\left(\frac{\frac{g\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(cd(4ef-dg)-e(-3aeg+bdg+2bef))}{\sqrt{e}\sqrt{ef-dg}}-\frac{2e\left(ag^2-bfg+cf^2\right)}{\sqrt{f+gx}}}{2e(ef-dg)^2}+\frac{g^2\sqrt{f+gx}\left(ae^2-bde+cd^2\right)}{2e(ef-dg)^2(-dg-e(f+gx)+ef)}\right)}{g}$$

input
```
Int[(a + b*x + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

output
```
(2*(((c*d^2 - b*d*e + a*e^2)*g^2*Sqrt[f + g*x])/(2*e*(e*f - d*g)^2*(e*f -
d*g - e*(f + g*x))) + ((-2*e*(c*f^2 - b*f*g + a*g^2))/Sqrt[f + g*x] + (g*(
c*d*(4*e*f - d*g) - e*(2*b*e*f + b*d*g - 3*a*e*g))*ArcTanh[(Sqrt[e]*Sqrt[f
 + g*x])/Sqrt[e*f - d*g]])/(Sqrt[e]*Sqrt[e*f - d*g]))/(2*e*(e*f - d*g)^2))
)/g
```

3.831.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.831. $\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx$

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

3.831.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{-\frac{2g\left(\frac{g\left(e^2a-bde+c\,d^2\right)\sqrt{gx+f}}{2e(e(gx+f)+dg-ef)}+\frac{\left(3a\,e^2g-bdeg-2b\,e^2f-c\,d^2g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2e\sqrt{(dg-ef)e}}\right)}{(dg-ef)^2}-\frac{2\left(a\,g^2-bfg+c\,f^2\right)}{(dg-ef)^2\sqrt{gx+f}}}{g}$
default	$\frac{-\frac{2g\left(\frac{g\left(e^2a-bde+c\,d^2\right)\sqrt{gx+f}}{2e(e(gx+f)+dg-ef)}+\frac{\left(3a\,e^2g-bdeg-2b\,e^2f-c\,d^2g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2e\sqrt{(dg-ef)e}}\right)}{(dg-ef)^2}-\frac{2\left(a\,g^2-bfg+c\,f^2\right)}{(dg-ef)^2\sqrt{gx+f}}}{g}$
pseudoelliptic	$-\frac{2\left(\frac{3\sqrt{gx+f}\,(ex+d)g\left(\left(ag-\frac{2bf}{3}\right)e^2-\frac{d(bg-4cf)e}{3}-\frac{c\,d^2g}{3}\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2}+\left(\left(\frac{3a\,g^2x}{2}+\frac{f(-2bx+a)g}{2}+c\,f^2x\right)e^2+d\left(\left(\cdots\right.\right.\right.\right.}{\sqrt{gx+f}\,\sqrt{(dg-ef)e}\,g(ex+d)(dg-ef)^2e}$

input
```
int((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/g*(-g/(d*g-e*f)^2*(1/2*g*(a*e^2-b*d*e+c*d^2)/e*(g*x+f)^(1/2)/(e*(g*x+f)+
d*g-e*f)+1/2*(3*a*e^2*g-b*d*e*g-2*b*e^2*f-c*d^2*g+4*c*d*e*f)/e/((d*g-e*f)*
e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)))-(a*g^2-b*f*g+c*f^2)/
(d*g-e*f)^2/(g*x+f)^(1/2))
```

3.831.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(151) = 302$.

Time = 0.50 (sec) , antiderivative size = 1088, normalized size of antiderivative = 6.59

$$\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \left[\frac{(2\,(2\,cd^2e-bde^2)f^2g-(cd^3+bd^2e-3\,ade^2)fg^2+(2\,(2\,cde^2-be^3)fg^2-(c\right.$$

$$-\frac{(2\,(2\,cd^2e-bde^2)f^2g-(cd^3+bd^2e-3\,ade^2)fg^2+(2\,(2\,cde^2-be^3)fg^2-(cd^2e+bde^2-3\,ae^3)g^3)x^2+(2}{}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((2*(2*c*d^2*e - b*d*e^2)*f^2*g - (c*d^3 + b*d^2*e - 3*a*d*e^2)*f*g^2
 + (2*(2*c*d*e^2 - b*e^3)*f*g^2 - (c*d^2*e + b*d*e^2 - 3*a*e^3)*g^3)*x^2 +
 (2*(2*c*d*e^2 - b*e^3)*f^2*g + 3*(c*d^2*e - b*d*e^2 + a*e^3)*f*g^2 - (c*d
^3 + b*d^2*e - 3*a*d*e^2)*g^3)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f -
 d*g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*d*e^3*f^3
- 2*a*d^2*e^2*g^3 - (c*d^2*e^2 + 3*b*d*e^3 - a*e^4)*f^2*g - (c*d^3*e - 3*b
*d^2*e^2 - a*d*e^3)*f*g^2 + (2*c*e^4*f^3 - 2*(c*d*e^3 + b*e^4)*f^2*g + (c*
d^2*e^2 + b*d*e^3 + 3*a*e^4)*f*g^2 - (c*d^3*e - b*d^2*e^2 + 3*a*d*e^3)*g^3
)*x)*sqrt(g*x + f))/(d*e^5*f^4*g - 3*d^2*e^4*f^3*g^2 + 3*d^3*e^3*f^2*g^3 -
 d^4*e^2*f*g^4 + (e^6*f^3*g^2 - 3*d*e^5*f^2*g^3 + 3*d^2*e^4*f*g^4 - d^3*e^
3*g^5)*x^2 + (e^6*f^4*g - 2*d*e^5*f^3*g^2 + 2*d^3*e^3*f*g^4 - d^4*e^2*g^5)
*x), -((2*(2*c*d^2*e - b*d*e^2)*f^2*g - (c*d^3 + b*d^2*e - 3*a*d*e^2)*f*g^
2 + (2*(2*c*d*e^2 - b*e^3)*f*g^2 - (c*d^2*e + b*d*e^2 - 3*a*e^3)*g^3)*x^2
+ (2*(2*c*d*e^2 - b*e^3)*f^2*g + 3*(c*d^2*e - b*d*e^2 + a*e^3)*f*g^2 - (c*
d^3 + b*d^2*e - 3*a*d*e^2)*g^3)*x)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f
 + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*c*d*e^3*f^3 - 2*a*d^2*e^2*g^3
- (c*d^2*e^2 + 3*b*d*e^3 - a*e^4)*f^2*g - (c*d^3*e - 3*b*d^2*e^2 - a*d*e^3
)*f*g^2 + (2*c*e^4*f^3 - 2*(c*d*e^3 + b*e^4)*f^2*g + (c*d^2*e^2 + b*d*e^3
+ 3*a*e^4)*f*g^2 - (c*d^3*e - b*d^2*e^2 + 3*a*d*e^3)*g^3)*x)*sqrt(g*x + f)
)/(d*e^5*f^4*g - 3*d^2*e^4*f^3*g^2 + 3*d^3*e^3*f^2*g^3 - d^4*e^2*f*g^4 ...
```

3.831.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/(e*x+d)**2/(g*x+f)**(3/2),x)
```

output
```
Timed out
```

3.831.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.831.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.73

$$\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = -\frac{(4\,cdef-2\,be^2f-cd^2g-bdeg+3\,ae^2g)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{(e^3f^2-2\,de^2fg+d^2eg^2)\sqrt{-e^2f+deg}} - \frac{2\,(gx+f)ce^2f^2-2\,ce^2f^3-2\,(gx+f)be^2fg+2\,cdef^2g+2\,be^2f^2g+(gx+f)cd^2g^2-(gx+f)bdeg^2+3}{(e^3f^2g-2\,de^2fg^2+d^2eg^3)\left((gx+f)^{\frac{3}{2}}e-\sqrt{gx+f}ef+\sqrt{gx+}\right.}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
-(4*c*d*e*f - 2*b*e^2*f - c*d^2*g - b*d*e*g + 3*a*e^2*g)*arctan(sqrt(g*x +
 f)*e/sqrt(-e^2*f + d*e*g))/((e^3*f^2 - 2*d*e^2*f*g + d^2*e*g^2)*sqrt(-e^2
*f + d*e*g)) - (2*(g*x + f)*c*e^2*f^2 - 2*c*e^2*f^3 - 2*(g*x + f)*b*e^2*f*
g + 2*c*d*e*f^2*g + 2*b*e^2*f^2*g + (g*x + f)*c*d^2*g^2 - (g*x + f)*b*d*e*
g^2 + 3*(g*x + f)*a*e^2*g^2 - 2*b*d*e*f*g^2 - 2*a*e^2*f*g^2 + 2*a*d*e*g^3)
/((e^3*f^2*g - 2*d*e^2*f*g^2 + d^2*e*g^3)*((g*x + f)^(3/2)*e - sqrt(g*x +
f)*e*f + sqrt(g*x + f)*d*g))
```

3.831.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.32

$$\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{f+g\,x}\,\left(d^2\,e\,g^2-2\,d\,e^2\,f\,g+e^3\,f^2\right)}{\sqrt{e}\,\left(d\,g-e\,f\right)^{5/2}}\right)\,\left(2\,b\,e^2\,f-3\,a\,e^2\,g+c\,d^2\,g+b\,d\,e\,g-4\,c\,d\,e\right.}{e^{3/2}\,\left(d\,g-e\,f\right)^{5/2}} - \frac{\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{d\,g-e\,f}+\frac{\left(f+g\,x\right)\,\left(c\,d^2\,g^2-b\,d\,e\,g^2+2\,c\,e^2\,f^2-2\,b\,e^2\,f\,g+3\,a\,e^2\,g^2\right)}{e\,\left(d\,g-e\,f\right)^2}}{\sqrt{f+g\,x}\,\left(d\,g^2-e\,f\,g\right)+e\,g\,\left(f+g\,x\right)^{3/2}}$$

input
```
int((a + b*x + c*x^2)/((f + g*x)^(3/2)*(d + e*x)^2),x)
```

output
```
(atan(((f + g*x)^(1/2)*(e^3*f^2 + d^2*e*g^2 - 2*d*e^2*f*g))/(e^(1/2)*(d*g
- e*f)^(5/2)))*(2*b*e^2*f - 3*a*e^2*g + c*d^2*g + b*d*e*g - 4*c*d*e*f))/(e
^(3/2)*(d*g - e*f)^(5/2)) - ((2*(a*g^2 + c*f^2 - b*f*g))/(d*g - e*f) + ((f
 + g*x)*(3*a*e^2*g^2 + c*d^2*g^2 + 2*c*e^2*f^2 - b*d*e*g^2 - 2*b*e^2*f*g))
/(e*(d*g - e*f)^2))/((f + g*x)^(1/2)*(d*g^2 - e*f*g) + e*g*(f + g*x)^(3/2)
)
```

3.832 $\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx$

3.832.1 Optimal result

Integrand size = 27, antiderivative size = 248

$$\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{2(cf^2-bfg+ag^2)}{(ef-dg)^3\sqrt{f+gx}} - \frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(cd(8ef-dg)-e(4bef+3bdg-7aeg))\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)} - \frac{(c(8e^2f^2+8defg-d^2g^2)+3eg(5aeg-b(4ef+dg)))\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{3/2}(ef-dg)^{7/2}}$$

output
```
-1/4*(c*(-d^2*g^2+8*d*e*f*g+8*e^2*f^2)+3*e*g*(5*a*e*g-b*(d*g+4*e*f)))*arct
anh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(3/2)/(-d*g+e*f)^(7/2)+2*(a*
g^2-b*f*g+c*f^2)/(-d*g+e*f)^3/(g*x+f)^(1/2)-1/2*(a*e^2-b*d*e+c*d^2)*(g*x+f
)^(1/2)/e/(-d*g+e*f)^2/(e*x+d)^2+1/4*(c*d*(-d*g+8*e*f)-e*(-7*a*e*g+3*b*d*g
+4*b*e*f))*(g*x+f)^(1/2)/e/(-d*g+e*f)^3/(e*x+d)
```

3.832.2 Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.20

$$\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{\frac{\sqrt{e}\left(c\left(8e^3f^2x^2+d^3g(f+gx)+8de^2fx(3f+gx)+d^2e\left(14f^2+5fgx-g^2x^2\right)\right)-e\left(a\left(-8d^2g^2-deg(9f+25gx)+e^2\right.\right.\right.}{(ef-dg)^3(d+ex)^2\sqrt{f+}}}{}$$

input
```
Integrate[(a + b*x + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

output
```
((Sqrt[e]*(c*(8*e^3*f^2*x^2 + d^3*g*(f + g*x) + 8*d*e^2*f*x*(3*f + g*x) +
d^2*e*(14*f^2 + 5*f*g*x - g^2*x^2)) - e*(a*(-8*d^2*g^2 - d*e*g*(9*f + 25*g
*x) + e^2*(2*f^2 - 5*f*g*x - 15*g^2*x^2)) + b*(4*e^2*f*x*(f + 3*g*x) + d^2
*g*(13*f + 5*g*x) + d*e*(2*f^2 + 21*f*g*x + 3*g^2*x^2)))))/((e*f - d*g)^3*
(d + e*x)^2*Sqrt[f + g*x]) - ((c*(8*e^2*f^2 + 8*d*e*f*g - d^2*g^2) + 3*e*g
*(5*a*e*g - b*(4*e*f + d*g)))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) +
 d*g]])/(-(e*f) + d*g)^(7/2))/(4*e^(3/2))
```

3.832.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1192, 25, 1582, 25, 27, 361, 25, 359, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1192}$$

$$2\int -\frac{cf^2-bgf+ag^2+c(f+gx)^2-(2cf-bg)(f+gx)}{(f+gx)(ef-dg-e(f+gx))^3}d\sqrt{f+gx}$$

$$\Big\downarrow \text{ 25}$$

$$-2\int \frac{cf^2-bgf+ag^2+c(f+gx)^2-(2cf-bg)(f+gx)}{(f+gx)(ef-dg-e(f+gx))^3}d\sqrt{f+gx}$$

$$\Big\downarrow \text{ 1582}$$

$$2\left(\frac{\int -\frac{e(4e(ef-dg)(cf^2-bgf+ag^2)-(3e(bd-ae)g^2+c(4e^2f^2-8degf+d^2g^2))(f+gx))}{(f+gx)(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{4e^2(ef-dg)^2}-\frac{g^2\sqrt{f+gx}(ae^2-bde+c}{4e(ef-dg)^2(-dg-e(f+gx)}\right.$$

$$\Big\downarrow \text{ 25}$$

$$2\left(-\frac{\int \frac{e(4e(ef-dg)(cf^2-bgf+ag^2)-(3e(bd-ae)g^2+c(4e^2f^2-8degf+d^2g^2))(f+gx))}{(f+gx)(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{4e^2(ef-dg)^2}-\frac{g^2\sqrt{f+gx}(ae^2-bde+c}{4e(ef-dg)^2(-dg-e(f+gx)}\right.$$

$$\Big\downarrow \; 27$$

$$2\left(-\frac{\int \frac{4e(ef-dg)(cf^2-bgf+ag^2)-(3e(bd-ae)g^2+c(4e^2f^2-8degf+d^2g^2))(f+gx)}{(f+gx)(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{4e(ef-dg)^2}-\frac{g^2\sqrt{f+gx}(ae^2-bde+cd^2)}{4e(ef-dg)^2(-dg-e(f+gx)+}\right.$$

$$\Big\downarrow \; 361$$

$$2\left(-\frac{\frac{g\sqrt{f+gx}(cd(8ef-dg)-e(-7aeg+3bdg+4bef))}{2(ef-dg)(-dg-e(f+gx)+ef)}-\frac{1}{2}\int -\frac{8e(cf^2-bgf+ag^2)+\frac{g(cd(8ef-dg)-e(4bef+3bdg-7aeg))(f+gx)}{ef-dg}}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}}{4e(ef-dg)^2}-\frac{}{4e}\right.$$

$$\Big\downarrow \; 25$$

$$2\left(-\frac{\frac{1}{2}\int \frac{8e(cf^2-bgf+ag^2)+\frac{g(cd(8ef-dg)-e(4bef+3bdg-7aeg))(f+gx)}{ef-dg}}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}+\frac{g\sqrt{f+gx}(cd(8ef-dg)-e(-7aeg+3bdg+4bef))}{2(ef-dg)(-dg-e(f+gx)+ef)}}{4e(ef-dg)^2}-\frac{}{4e(}\right.$$

$$\Big\downarrow \; 359$$

$$2\left(-\frac{\frac{1}{2}\left(\frac{(3eg(5aeg-b(dg+4ef))+c(-d^2g^2+8defg+8e^2f^2))\int \frac{1}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{ef-dg}-\frac{8e(ag^2-bfg+cf^2)}{\sqrt{f+gx}(ef-dg)}\right)+\frac{g\sqrt{f+gx}(cd(8ef-dg)-e(-}{2(ef-dg)(-dg-e(}}{4e(ef-dg)^2}\right.$$

$$\Big\downarrow \; 221$$

$$2\left(-\frac{\frac{1}{2}\left(\frac{\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(3eg(5aeg-b(dg+4ef))+c(-d^2g^2+8defg+8e^2f^2))}{\sqrt{e}(ef-dg)^{3/2}}-\frac{8e(ag^2-bfg+cf^2)}{\sqrt{f+gx}(ef-dg)}\right)+\frac{g\sqrt{f+gx}(cd(8ef-dg)-e(-7ae}{2(ef-dg)(-dg-e(f+}}{4e(ef-dg)^2}\right.$$

input
```
Int[(a + b*x + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

3.832. $\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx$

output
```
2*(-1/4*((c*d^2 - b*d*e + a*e^2)*g^2*Sqrt[f + g*x])/(e*(e*f - d*g)^2*(e*f
- d*g - e*(f + g*x))^2) - ((g*(c*d*(8*e*f - d*g) - e*(4*b*e*f + 3*b*d*g -
7*a*e*g))*Sqrt[f + g*x])/(2*(e*f - d*g)*(e*f - d*g - e*(f + g*x))) + ((-8*
e*(c*f^2 - b*f*g + a*g^2))/((e*f - d*g)*Sqrt[f + g*x]) + ((c*(8*e^2*f^2 +
8*d*e*f*g - d^2*g^2) + 3*e*g*(5*a*e*g - b*(4*e*f + d*g)))*ArcTanh[(Sqrt[e]
*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(Sqrt[e]*(e*f - d*g)^(3/2)))/2)/(4*e*(e*
f - d*g)^2))
```

3.832.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

3.832. $\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx$

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

3.832.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$-\frac{15\left(\sqrt{gx+f}\,(ex+d)^2\left(\left(a\,g^2-\frac{4}{5}bfg+\frac{8}{15}c\,f^2\right)e^2-\frac{dg\left(bg-\frac{8cf}{3}\right)e}{5}-\frac{c\,d^2g^2}{15}\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)+\frac{8\left(\left(\frac{15a\,g^2x^2}{8}+\frac{5(-12\cdots}{\cdots}\right.\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$-\frac{2(a\,g^2-bfg+c\,f^2)}{(dg-ef)^3\sqrt{gx+f}}-\frac{2\left(\frac{\left(\frac{7}{8}a\,e^2g^2-\frac{3}{8}bde\,g^2-\frac{1}{2}b\,e^2fg-\frac{1}{8}c\,d^2g^2+cdefg\right)(gx+f)^{\frac{3}{2}}+\frac{g\left(9ad\,e^2g^2-9a\,e^3fg-5b\,d^2e\,g^2+bd\,e^2fg\cdots\right.}{\cdots}}{(e(gx+f)+dg-ef)^2}\right.}{\cdots}$
default	$-\frac{2(a\,g^2-bfg+c\,f^2)}{(dg-ef)^3\sqrt{gx+f}}-\frac{2\left(\frac{\left(\frac{7}{8}a\,e^2g^2-\frac{3}{8}bde\,g^2-\frac{1}{2}b\,e^2fg-\frac{1}{8}c\,d^2g^2+cdefg\right)(gx+f)^{\frac{3}{2}}+\frac{g\left(9ad\,e^2g^2-9a\,e^3fg-5b\,d^2e\,g^2+bd\,e^2fg\cdots\right.}{\cdots}}{(e(gx+f)+dg-ef)^2}\right.}{\cdots}$

input
```
int((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-15/4/(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*((g*x+f)^(1/2)*(e*x+d)^2*((a*g^2-4
/5*b*f*g+8/15*c*f^2)*e^2-1/5*d*g*(b*g-8/3*c*f)*e-1/15*c*d^2*g^2)*arctan(e*
(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))+8/15*((15/8*a*g^2*x^2+5/8*(-12/5*b*x+a)
*x*f*g-1/4*f^2*(-4*c*x^2+2*b*x+a))*e^3+9/8*d*((-1/3*b*x^2+25/9*a*x)*g^2+f*
(8/9*c*x^2-7/3*b*x+a)*g-2/9*f^2*(-12*c*x+b))*e^2+d^2*((a-1/8*c*x^2-5/8*b*x
)*g^2-13/8*(-5/13*c*x+b)*f*g+7/4*c*f^2)*e+1/8*c*d^3*g*(g*x+f))*((d*g-e*f)*
e)^(1/2))/(d*g-e*f)^3/(e*x+d)^2/e
```

3.832.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(226) = 452$.

Time = 0.59 (sec) , antiderivative size = 1883, normalized size of antiderivative = 7.59

$$\int \frac{a + bx + cx^2}{(d + ex)^3(f + gx)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*((8*c*d^2*e^2*f^3 + 4*(2*c*d^3*e - 3*b*d^2*e^2)*f^2*g - (c*d^4 + 3*b
*d^3*e - 15*a*d^2*e^2)*f*g^2 + (8*c*e^4*f^2*g + 4*(2*c*d*e^3 - 3*b*e^4)*f*
g^2 - (c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^4)*g^3)*x^3 + (8*c*e^4*f^3 + 12*(2*c
*d*e^3 - b*e^4)*f^2*g + 3*(5*c*d^2*e^2 - 9*b*d*e^3 + 5*a*e^4)*f*g^2 - 2*(c
*d^3*e + 3*b*d^2*e^2 - 15*a*d*e^3)*g^3)*x^2 + (16*c*d*e^3*f^3 + 24*(c*d^2*
e^2 - b*d*e^3)*f^2*g + 6*(c*d^3*e - 3*b*d^2*e^2 + 5*a*d*e^3)*f*g^2 - (c*d^
4 + 3*b*d^3*e - 15*a*d^2*e^2)*g^3)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e
*f - d*g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) + 2*(8*a*d^3*e^
2*g^3 - 2*(7*c*d^2*e^3 - b*d*e^4 - a*e^5)*f^3 + (13*c*d^3*e^2 + 11*b*d^2*e
^3 - 11*a*d*e^4)*f^2*g + (c*d^4*e - 13*b*d^3*e^2 + a*d^2*e^3)*f*g^2 - (8*c
*e^5*f^3 - 12*b*e^5*f^2*g - 3*(3*c*d^2*e^3 - 3*b*d*e^4 - 5*a*e^5)*f*g^2 +
(c*d^3*e^2 + 3*b*d^2*e^3 - 15*a*d*e^4)*g^3)*x^2 - (4*(6*c*d*e^4 - b*e^5)*f
^3 - (19*c*d^2*e^3 + 17*b*d*e^4 - 5*a*e^5)*f^2*g - 4*(c*d^3*e^2 - 4*b*d^2*
e^3 - 5*a*d*e^4)*f*g^2 - (c*d^4*e - 5*b*d^3*e^2 + 25*a*d^2*e^3)*g^3)*x)*sq
rt(g*x + f))/(d^2*e^6*f^5 - 4*d^3*e^5*f^4*g + 6*d^4*e^4*f^3*g^2 - 4*d^5*e^
3*f^2*g^3 + d^6*e^2*f*g^4 + (e^8*f^4*g - 4*d*e^7*f^3*g^2 + 6*d^2*e^6*f^2*g
^3 - 4*d^3*e^5*f*g^4 + d^4*e^4*g^5)*x^3 + (e^8*f^5 - 2*d*e^7*f^4*g - 2*d^2
*e^6*f^3*g^2 + 8*d^3*e^5*f^2*g^3 - 7*d^4*e^4*f*g^4 + 2*d^5*e^3*g^5)*x^2 +
(2*d*e^7*f^5 - 7*d^2*e^6*f^4*g + 8*d^3*e^5*f^3*g^2 - 2*d^4*e^4*f^2*g^3 - 2
*d^5*e^3*f*g^4 + d^6*e^2*g^5)*x), 1/4*((8*c*d^2*e^2*f^3 + 4*(2*c*d^3*e ...
```

3.832.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/(e*x+d)**3/(g*x+f)**(3/2),x)
```

output
```
Timed out
```

3.832.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.832.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. 2(226) = 452.

Time = 0.30 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.90

$$\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{(8\,ce^2f^2+8\,cdefg-12\,be^2fg-cd^2g^2-3\,bdeg^2+15\,ae^2g^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+de}}\right.}{4\,(e^4f^3-3\,de^3f^2g+3\,d^2e^2fg^2-d^3eg^3)\sqrt{-e^2f+deg}}$$
$$+\frac{2\,(cf^2-bfg+ag^2)}{(e^3f^3-3\,de^2f^2g+3\,d^2efg^2-d^3g^3)\sqrt{gx+f}}$$
$$+\frac{8\,(gx+f)^{\frac{3}{2}}cde^2fg-4\,(gx+f)^{\frac{3}{2}}be^3fg-8\,\sqrt{gx+f}cde^2f^2g+4\,\sqrt{gx+f}be^3f^2g-(gx+f)^{\frac{3}{2}}cd^2eg^2-3\,(g}{4\,(e^4f^3}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
1/4*(8*c*e^2*f^2 + 8*c*d*e*f*g - 12*b*e^2*f*g - c*d^2*g^2 - 3*b*d*e*g^2 +
15*a*e^2*g^2)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/((e^4*f^3 - 3*d
*e^3*f^2*g + 3*d^2*e^2*f*g^2 - d^3*e*g^3)*sqrt(-e^2*f + d*e*g)) + 2*(c*f^2
 - b*f*g + a*g^2)/((e^3*f^3 - 3*d*e^2*f^2*g + 3*d^2*e*f*g^2 - d^3*g^3)*sqr
t(g*x + f)) + 1/4*(8*(g*x + f)^(3/2)*c*d*e^2*f*g - 4*(g*x + f)^(3/2)*b*e^3
*f*g - 8*sqrt(g*x + f)*c*d*e^2*f^2*g + 4*sqrt(g*x + f)*b*e^3*f^2*g - (g*x
+ f)^(3/2)*c*d^2*e*g^2 - 3*(g*x + f)^(3/2)*b*d*e^2*g^2 + 7*(g*x + f)^(3/2)
*a*e^3*g^2 + 7*sqrt(g*x + f)*c*d^2*e*f*g^2 + sqrt(g*x + f)*b*d*e^2*f*g^2 -
 9*sqrt(g*x + f)*a*e^3*f*g^2 + sqrt(g*x + f)*c*d^3*g^3 - 5*sqrt(g*x + f)*b
*d^2*e*g^3 + 9*sqrt(g*x + f)*a*d*e^2*g^3)/((e^4*f^3 - 3*d*e^3*f^2*g + 3*d^
2*e^2*f*g^2 - d^3*e*g^3)*((g*x + f)*e - e*f + d*g)^2)
```

3.832.9 Mupad [B] (verification not implemented)

Time = 12.17 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.46

$$\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{f+g\,x}\,\left(-d^3\,e\,g^3+3\,d^2\,e^2\,f\,g^2-3\,d\,e^3\,f^2\,g+e^4\,f^3\right)}{\sqrt{e}\,{\left(d\,g-e\,f\right)}^{7/2}}\right)\,\left(-c\,d^2\,g^2+8\,c\,d\,e\,f\,g-3\,b\,d\,e\,g^2\right.}{4\,e^{3/2}\,{\left(d\,g-e\,f\right)}^{7/2}} - \frac{\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{d\,g-e\,f}+\frac{{\left(f+g\,x\right)}^2\,\left(-c\,d^2\,g^2+8\,c\,d\,e\,f\,g-3\,b\,d\,e\,g^2+8\,c\,e^2\,f^2-12\,b\,e^2\,f\,g+15\,a\,e^2\,g^2\right)}{4\,{\left(d\,g-e\,f\right)}^3}+\frac{\left(f+g\,x\right)\,\left(c\,d^2\,g^2+8\,c\,d\,e\,f\,g-5\,b\,d\,e\,g^2+16\right.}{4\,e\,\left(d\,g-e\,f\right)}}{e^2\,{\left(f+g\,x\right)}^{5/2}-{\left(f+g\,x\right)}^{3/2}\,\left(2\,e^2\,f-2\,d\,e\,g\right)+\sqrt{f+g\,x}\,\left(d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2\right)}$$

input
```
int((a + b*x + c*x^2)/((f + g*x)^(3/2)*(d + e*x)^3),x)
```

output
```
(atan(((f + g*x)^(1/2)*(e^4*f^3 - d^3*e*g^3 + 3*d^2*e^2*f*g^2 - 3*d*e^3*f^
2*g))/(e^(1/2)*(d*g - e*f)^(7/2)))*(15*a*e^2*g^2 - c*d^2*g^2 + 8*c*e^2*f^2
 - 3*b*d*e*g^2 - 12*b*e^2*f*g + 8*c*d*e*f*g))/(4*e^(3/2)*(d*g - e*f)^(7/2)
) - ((2*(a*g^2 + c*f^2 - b*f*g))/(d*g - e*f) + ((f + g*x)^2*(15*a*e^2*g^2
- c*d^2*g^2 + 8*c*e^2*f^2 - 3*b*d*e*g^2 - 12*b*e^2*f*g + 8*c*d*e*f*g))/(4*
(d*g - e*f)^3) + ((f + g*x)*(25*a*e^2*g^2 + c*d^2*g^2 + 16*c*e^2*f^2 - 5*b
*d*e*g^2 - 20*b*e^2*f*g + 8*c*d*e*f*g))/(4*e*(d*g - e*f)^2))/(e^2*(f + g*x
)^(5/2) - (f + g*x)^(3/2)*(2*e^2*f - 2*d*e*g) + (f + g*x)^(1/2)*(d^2*g^2 +
 e^2*f^2 - 2*d*e*f*g))
```

3.833 $\int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2}\,dx$

3.833.1 Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2}\,dx = -\operatorname{arccosh}(x)+\sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{1+x}}{\sqrt{-2+\sqrt{5}}\sqrt{-1+x}}\right)+\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\operatorname{arctanh}\left(\frac{\sqrt{1+x}}{\sqrt{2+\sqrt{5}}\sqrt{-1+x}}\right)$$

output
```
-arccosh(x)+1/5*arctan((1+x)^(1/2)/(-1+x)^(1/2)/(-2+5^(1/2))^(1/2))*(-10+1
0*5^(1/2))^(1/2)+1/5*arctanh((1+x)^(1/2)/(-1+x)^(1/2)/(2+5^(1/2))^(1/2))*(
10+10*5^(1/2))^(1/2)
```

3.833.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2}\,dx = -\sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)}\arctan\left(\sqrt{-2+\sqrt{5}}\sqrt{\frac{-1+x}{1+x}}\right)-2\operatorname{arctanh}\left(\sqrt{\frac{-1+x}{1+x}}\right)+\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{2+\sqrt{5}}\sqrt{\frac{-1+x}{1+x}}\right)$$

input
```
Integrate[(Sqrt[-1 + x]*Sqrt[1 + x])/(1 + x - x^2),x]
```

output
```
-(Sqrt[(2*(-1 + Sqrt[5]))/5]*ArcTan[Sqrt[-2 + Sqrt[5]]*Sqrt[(-1 + x)/(1 +
x)]]) - 2*ArcTanh[Sqrt[(-1 + x)/(1 + x)]] + Sqrt[(2*(1 + Sqrt[5]))/5]*ArcT
anh[Sqrt[2 + Sqrt[5]]*Sqrt[(-1 + x)/(1 + x)]]
```

3.833.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1190, 25, 43, 2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x-1}\sqrt{x+1}}{-x^2+x+1}dx$$

$$\Big\downarrow \ 1190$$

$$-\int -\frac{x}{\sqrt{x-1}\sqrt{x+1}\,(-x^2+x+1)}dx-\int \frac{1}{\sqrt{x-1}\sqrt{x+1}}dx$$

$$\Big\downarrow \ 25$$

$$\int \frac{x}{\sqrt{x-1}\sqrt{x+1}\,(-x^2+x+1)}dx-\int \frac{1}{\sqrt{x-1}\sqrt{x+1}}dx$$

$$\Big\downarrow \ 43$$

$$\int \frac{x}{\sqrt{x-1}\sqrt{x+1}\,(-x^2+x+1)}dx-\operatorname{arccosh}(x)$$

$$\Big\downarrow \ 2153$$

$$\int \left(\frac{1-\frac{1}{\sqrt{5}}}{\left(-2x-\sqrt{5}+1\right)\sqrt{x-1}\sqrt{x+1}}+\frac{1+\frac{1}{\sqrt{5}}}{\left(-2x+\sqrt{5}+1\right)\sqrt{x-1}\sqrt{x+1}}\right)dx-\operatorname{arccosh}(x)$$

$$\Big\downarrow \ 2009$$

$$-\operatorname{arccosh}(x)+\sqrt{\frac{2}{5}\left(\sqrt{5}-1\right)}\arctan\left(\frac{\sqrt{2+\sqrt{5}}\sqrt{x+1}}{\sqrt{x-1}}\right)+\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{5}-2}\sqrt{x+1}}{\sqrt{x-1}}\right)$$

input
```
Int[(Sqrt[-1 + x]*Sqrt[1 + x])/(1 + x - x^2),x]
```

output
```
-ArcCosh[x] + Sqrt[(2*(-1 + Sqrt[5]))/5]*ArcTan[(Sqrt[2 + Sqrt[5]]*Sqrt[1
+ x])/Sqrt[-1 + x]] + Sqrt[(2*(1 + Sqrt[5]))/5]*ArcTanh[(Sqrt[-2 + Sqrt[5]
]*Sqrt[1 + x])/Sqrt[-1 + x]]
```

3.833.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 1190
```
Int[(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(f_) + (g_.)*(x_)])/((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2), x_Symbol] :> Simp[e*(g/c)   Int[1/(Sqrt[d + e*x]*Sqrt[f + g
*x]), x], x] + Simp[1/c   Int[(c*d*f - a*e*g - b*e*g*x)/(Sqrt[d + e*x]*Sqrt
[f + g*x]*(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
EqQ[e*f + d*g, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

3.833.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(65) = 130$.

Time = 0.60 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.54

method	result
default	$-\frac{\sqrt{1+x}\,\sqrt{-1+x}\,\sqrt{5}\left(\sqrt{5}\,\sqrt{2\sqrt{5}-2}\,\sqrt{2\sqrt{5}+2}\,\ln\left(x+\sqrt{x^{2}-1}\right)-\sqrt{5}\,\sqrt{2\sqrt{5}-2}\,\operatorname{arctanh}\left(\frac{\sqrt{5}\,x+x-2}{\sqrt{2\sqrt{5}+2}\,\sqrt{x^{2}-1}}\right)-\sqrt{5}\,\sqrt{2\sqrt{5}+2}\,\arctan\left(\cdots\right.\right.}{5\sqrt{2\sqrt{5}-2}\,\sqrt{2\sqrt{5}+2}\,\sqrt{x^{2}-1}}$

input
```
int((-1+x)^(1/2)*(1+x)^(1/2)/(-x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(1+x)^(1/2)*(-1+x)^(1/2)*5^(1/2)*(5^(1/2)*(2*5^(1/2)-2)^(1/2)*(2*5^(1
/2)+2)^(1/2)*ln(x+(x^2-1)^(1/2))-5^(1/2)*(2*5^(1/2)-2)^(1/2)*arctanh((5^(1
/2)*x+x-2)/(2*5^(1/2)+2)^(1/2)/(x^2-1)^(1/2))-5^(1/2)*(2*5^(1/2)+2)^(1/2)*
arctan((5^(1/2)*x-x+2)/(2*5^(1/2)-2)^(1/2)/(x^2-1)^(1/2))-(2*5^(1/2)-2)^(1
/2)*arctanh((5^(1/2)*x+x-2)/(2*5^(1/2)+2)^(1/2)/(x^2-1)^(1/2))+(2*5^(1/2)+
2)^(1/2)*arctan((5^(1/2)*x-x+2)/(2*5^(1/2)-2)^(1/2)/(x^2-1)^(1/2)))/(2*5^(
1/2)-2)^(1/2)/(2*5^(1/2)+2)^(1/2)/(x^2-1)^(1/2)
```

3.833.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(65) = 130$.

Time = 0.43 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.21

$$
\begin{aligned}
\int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2}\,dx &= \frac{1}{10}\,\sqrt{5}\sqrt{2\,\sqrt{5}+2}\log\left(2\,\sqrt{x+1}\sqrt{x-1}-2\,x+\sqrt{5}+\sqrt{2\,\sqrt{5}+2}\right.\\
&\quad\left.+1\right)-\frac{1}{10}\,\sqrt{5}\sqrt{2\,\sqrt{5}+2}\log\left(2\,\sqrt{x+1}\sqrt{x-1}-2\,x+\sqrt{5}\right.\\
&\quad\left.-\sqrt{2\,\sqrt{5}+2}+1\right)-\frac{1}{10}\,\sqrt{5}\sqrt{-2\,\sqrt{5}+2}\log\left(2\,\sqrt{x+1}\sqrt{x-1}\right.\\
&\quad\left.-2\,x-\sqrt{5}+\sqrt{-2\,\sqrt{5}+2}+1\right)\\
&\quad+\frac{1}{10}\,\sqrt{5}\sqrt{-2\,\sqrt{5}+2}\log\left(2\,\sqrt{x+1}\sqrt{x-1}-2\,x-\sqrt{5}\right.\\
&\quad\left.-\sqrt{-2\,\sqrt{5}+2}+1\right)+\log\left(\sqrt{x+1}\sqrt{x-1}-x\right)
\end{aligned}
$$

input
```
integrate((-1+x)^(1/2)*(1+x)^(1/2)/(-x^2+x+1),x, algorithm="fricas")
```

output
```
1/10*sqrt(5)*sqrt(2*sqrt(5) + 2)*log(2*sqrt(x + 1)*sqrt(x - 1) - 2*x + sqr
t(5) + sqrt(2*sqrt(5) + 2) + 1) - 1/10*sqrt(5)*sqrt(2*sqrt(5) + 2)*log(2*s
qrt(x + 1)*sqrt(x - 1) - 2*x + sqrt(5) - sqrt(2*sqrt(5) + 2) + 1) - 1/10*s
qrt(5)*sqrt(-2*sqrt(5) + 2)*log(2*sqrt(x + 1)*sqrt(x - 1) - 2*x - sqrt(5)
+ sqrt(-2*sqrt(5) + 2) + 1) + 1/10*sqrt(5)*sqrt(-2*sqrt(5) + 2)*log(2*sqrt
(x + 1)*sqrt(x - 1) - 2*x - sqrt(5) - sqrt(-2*sqrt(5) + 2) + 1) + log(sqrt
(x + 1)*sqrt(x - 1) - x)
```

3.833.6 Sympy [F]

$$\int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2}\,dx = -\int \frac{\sqrt{x-1}\sqrt{x+1}}{x^2-x-1}\,dx$$

input
```
integrate((-1+x)**(1/2)*(1+x)**(1/2)/(-x**2+x+1),x)
```

output
```
-Integral(sqrt(x - 1)*sqrt(x + 1)/(x**2 - x - 1), x)
```

3.833.7 Maxima [F]

$$\int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2}\,dx = \int -\frac{\sqrt{x+1}\sqrt{x-1}}{x^2-x-1}\,dx$$

input
```
integrate((-1+x)^(1/2)*(1+x)^(1/2)/(-x^2+x+1),x, algorithm="maxima")
```

output
```
-integrate(sqrt(x + 1)*sqrt(x - 1)/(x^2 - x - 1), x)
```

3.833.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2}\,dx = \log\left(\left(\sqrt{x+1}-\sqrt{x-1}\right)^2\right)$$

input
```
integrate((-1+x)^(1/2)*(1+x)^(1/2)/(-x^2+x+1),x, algorithm="giac")
```

output
```
log((sqrt(x + 1) - sqrt(x - 1))^2)
```

3.833.9 Mupad [B] (verification not implemented)

Time = 14.73 (sec) , antiderivative size = 916, normalized size of antiderivative = 10.07

$$\int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2}\,dx = -4\,\mathrm{atanh}\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)$$
$$-\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{3408370\,\sqrt{10}\,\sqrt{\sqrt{5}+1}-\sqrt{10}\,\sqrt{\sqrt{5}+1}\,\sqrt{x-1}\,300730\mathrm{i}-3408370\,\sqrt{10}\,\sqrt{\sqrt{5}+1}\,\sqrt{x+1}-1771398\,\sqrt{5}\,\sqrt{10}\,\sqrt{\sqrt{5}+1}+7836865\,\sqrt{10}\,x}{\cdots}\right.}{\cdots}$$
$$-\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{3408370\,\sqrt{10}\,\sqrt{1-\sqrt{5}}+3066340\,\sqrt{10}\,x^2\,\sqrt{1-\sqrt{5}}-\sqrt{10}\,\sqrt{1-\sqrt{5}}\,\sqrt{x-1}\,300730\mathrm{i}-3408370\,\sqrt{10}\,\sqrt{1-\sqrt{5}}\,\sqrt{x+1}+1771398\,\sqrt{5}\,\sqrt{10}}{\cdots}\right.}{\cdots}$$

input
```
int(((x - 1)^(1/2)*(x + 1)^(1/2))/(x - x^2 + 1),x)
```

output
```
- 4*atanh(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1)) - (10^(1/2)*atan((3408
370*10^(1/2)*(5^(1/2) + 1)^(1/2) - 10^(1/2)*(5^(1/2) + 1)^(1/2)*(x - 1)^(1
/2)*300730i - 3408370*10^(1/2)*(5^(1/2) + 1)^(1/2)*(x + 1)^(1/2) - 1771398
*5^(1/2)*10^(1/2)*(5^(1/2) + 1)^(1/2) + 7836865*10^(1/2)*x*(5^(1/2) + 1)^(
1/2) + 3066340*10^(1/2)*x^2*(5^(1/2) + 1)^(1/2) - 1294942*5^(1/2)*10^(1/2)
*x^2*(5^(1/2) + 1)^(1/2) + 10^(1/2)*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*(x +
 1)^(1/2)*300730i - 5^(1/2)*10^(1/2)*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*134
482i + 1771398*5^(1/2)*10^(1/2)*(5^(1/2) + 1)^(1/2)*(x + 1)^(1/2) - 10^(1/
2)*x*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*300730i - 6132680*10^(1/2)*x*(5^(1/
2) + 1)^(1/2)*(x + 1)^(1/2) - 3475583*5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/
2) + 5^(1/2)*10^(1/2)*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*1344
82i + 10^(1/2)*x*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*150365i -
 5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*134482i + 2589884*5^
(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/2)*(x + 1)^(1/2) + 5^(1/2)*10^(1/2)*x*(5
^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*67241i)/(29119280*x - 240669
00*x*(x + 1)^(1/2) - 11518800*5^(1/2)*x - 10104760*(x + 1)^(1/2) - 7067880
*5^(1/2) - 3992430*5^(1/2)*x^2 + 12033450*x^2 + 7067880*5^(1/2)*(x + 1)^(1
/2) + 7984860*5^(1/2)*x*(x + 1)^(1/2) + 10104760))*(5^(1/2) + 1)^(1/2)*1i)
/5 - (10^(1/2)*atan((3408370*10^(1/2)*(1 - 5^(1/2))^(1/2) + 3066340*10^(1/
2)*x^2*(1 - 5^(1/2))^(1/2) - 10^(1/2)*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)...
```

3.834 $\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

3.834.1 Optimal result

Integrand size = 29, antiderivative size = 164

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}+\frac{(c(3e^2f^2+2defg+3d^2g^2)+4eg(2aeg-b(ef+dg)))\,\mathrm{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{4e^{5/2}g^{5/2}}$$

output
```
1/4*(c*(3*d^2*g^2+2*d*e*f*g+3*e^2*f^2)+4*e*g*(2*a*e*g-b*(d*g+e*f)))*arctan
h(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(5/2)+1/2*c*(e*x+
d)^(3/2)*(g*x+f)^(1/2)/e^2/g-1/4*(-4*b*e*g+5*c*d*g+3*c*e*f)*(e*x+d)^(1/2)*
(g*x+f)^(1/2)/e^2/g^2
```

3.834.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.86

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{\sqrt{d+ex}\sqrt{f+gx}(4beg+c(-3ef-3dg+2egx))}{4e^2g^2}+\frac{(c(3e^2f^2+2defg+3d^2g^2)+4eg(2aeg-b(ef+dg)))\,\mathrm{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{4e^{5/2}g^{5/2}}$$

input
```
Integrate[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

output
```
(Sqrt[d + e*x]*Sqrt[f + g*x]*(4*b*e*g + c*(-3*e*f - 3*d*g + 2*e*g*x)))/(4*
e^2*g^2) + ((c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2) + 4*e*g*(2*a*e*g - b*(e
*f + d*g)))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/(4*e
^(5/2)*g^(5/2))
```

3.834.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1194, 27, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{1194}$$

$$\frac{\int \frac{4age^2-(3cef+5cdg-4beg)xe-cd(3ef+dg)}{2\sqrt{d+ex}\sqrt{f+gx}}dx}{2e^2g}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4age^2-(3cef+5cdg-4beg)xe-cd(3ef+dg)}{\sqrt{d+ex}\sqrt{f+gx}}dx}{4e^2g}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{(4eg(2aeg-b(dg+ef))+c(3d^2g^2+2defg+3e^2f^2))\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}dx}{2g}-\frac{\sqrt{d+ex}\sqrt{f+gx}(-4beg+5cdg+3cef)}{g}}{4e^2g}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

$$\Big\downarrow \text{66}$$

$$\frac{\frac{(4eg(2aeg-b(dg+ef))+c(3d^2g^2+2defg+3e^2f^2))\int \frac{1}{e-\frac{g(d+ex)}{f+gx}}d\frac{\sqrt{d+ex}}{\sqrt{f+gx}}}{g}-\frac{\sqrt{d+ex}\sqrt{f+gx}(-4beg+5cdg+3cef)}{g}}{4e^2g}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)(4eg(2aeg-b(dg+ef))+c(3d^2g^2+2defg+3e^2f^2))}{\sqrt{e}g^{3/2}}-\frac{\sqrt{d+ex}\sqrt{f+gx}(-4beg+5cdg+3cef)}{g}}{4e^2g}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

input
```
Int[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

output
```
(c*(d + e*x)^(3/2)*Sqrt[f + g*x])/(2*e^2*g) + (-(((3*c*e*f + 5*c*d*g - 4*b
*e*g)*Sqrt[d + e*x]*Sqrt[f + g*x])/g) + ((c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2
*g^2) + 4*e*g*(2*a*e*g - b*(e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(
Sqrt[e]*Sqrt[f + g*x])])/(Sqrt[e]*g^(3/2)))/(4*e^2*g)
```

3.834.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

3.834.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(138) = 276$.

Time = 0.48 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.59

method	result
default	$\frac{\left(4\sqrt{eg}\,\sqrt{(gx+f)(ex+d)}\,cegx+3\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)c\,d^2g^2+2\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)cdefg+3\ln\right.}{}$

input
```
int((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(4*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*e*g*x+3*ln(1/2*(2*e*g*x+2*((g
*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*g^2+2*ln(1/2*
(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e
*f*g+3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g
)^(1/2))*c*e^2*f^2+8*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)
+d*g+e*f)/(e*g)^(1/2))*a*e^2*g^2-4*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/
2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d*e*g^2-4*ln(1/2*(2*e*g*x+2*((g*x+f
)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*e^2*f*g-6*(e*g)^(1/2)
*((g*x+f)*(e*x+d))^(1/2)*c*d*g-6*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*e*f
+8*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*b*e*g)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/
(e*g)^(1/2)/g^2/e^2/((g*x+f)*(e*x+d))^(1/2)
```

3.834.5 Fricas [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.32

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$
$$= \left[\frac{(3\,ce^2f^2+2\,(cde-2\,be^2)fg+(3\,cd^2-4\,bde+8\,ae^2)g^2)\sqrt{eg}\log\left(8\,e^2g^2x^2+e^2f^2+6\,defg+d^2g^2+4\,(2\cdots\right.}{\cdots}\right.$$
$$-\frac{(3\,ce^2f^2+2\,(cde-2\,be^2)fg+(3\,cd^2-4\,bde+8\,ae^2)g^2)\sqrt{-eg}\arctan\left(\frac{(2\,egx+ef+dg)\sqrt{-eg}\sqrt{ex+d}\sqrt{gx+f}}{2\,(e^2g^2x^2+defg+(e^2fg+deg^2)x)}\right)-\cdots}{8\,e^3g^3}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((3*c*e^2*f^2 + 2*(c*d*e - 2*b*e^2)*f*g + (3*c*d^2 - 4*b*d*e + 8*a*e
^2)*g^2)*sqrt(e*g)*log(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(
2*e*g*x + e*f + d*g)*sqrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g +
d*e*g^2)*x) + 4*(2*c*e^2*g^2*x - 3*c*e^2*f*g - (3*c*d*e - 4*b*e^2)*g^2)*sq
rt(e*x + d)*sqrt(g*x + f))/(e^3*g^3), -1/8*((3*c*e^2*f^2 + 2*(c*d*e - 2*b*
e^2)*f*g + (3*c*d^2 - 4*b*d*e + 8*a*e^2)*g^2)*sqrt(-e*g)*arctan(1/2*(2*e*g
*x + e*f + d*g)*sqrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*
f*g + (e^2*f*g + d*e*g^2)*x)) - 2*(2*c*e^2*g^2*x - 3*c*e^2*f*g - (3*c*d*e
- 4*b*e^2)*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^3)]
```

3.834.6 Sympy [F]

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

input
```
integrate((c*x**2+b*x+a)/(e*x+d)**(1/2)/(g*x+f)**(1/2),x)
```

output
```
Integral((a + b*x + c*x**2)/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

3.834.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for m
ore detail
```

3.834.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.16

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{\left(\sqrt{e^2f+(ex+d)eg-deg}\sqrt{ex+d}\left(\frac{2\,(ex+d)c}{e^3g}-\frac{3\,ce^6fg+5\,cde^5g^2-4\,be^6g^2}{e^8g^3}\right)-\frac{(3\,ce^2f^2+2\,cdefg-4\,be^2fg+3\,cd^2g^2-4\,bdeg^2}{}\right.}{4\,|e|}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
1/4*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(e*x + d)*(2*(e*x + d)*c/(e^3
*g) - (3*c*e^6*f*g + 5*c*d*e^5*g^2 - 4*b*e^6*g^2)/(e^8*g^3)) - (3*c*e^2*f^
2 + 2*c*d*e*f*g - 4*b*e^2*f*g + 3*c*d^2*g^2 - 4*b*d*e*g^2 + 8*a*e^2*g^2)*l
og(abs(-sqrt(e*g)*sqrt(e*x + d) + sqrt(e^2*f + (e*x + d)*e*g - d*e*g)))/(s
qrt(e*g)*e^2*g^2))*e/abs(e)
```

3.834.9 Mupad [B] (verification not implemented)

Time = 17.08 (sec) , antiderivative size = 833, normalized size of antiderivative = 5.08

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$
$$= \frac{\frac{(2\,b\,d\,g+2\,b\,e\,f)\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{g^3\left(\sqrt{f+g\,x}-\sqrt{f}\right)}+\frac{(2\,b\,d\,g+2\,b\,e\,f)\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3}{e\,g^2\left(\sqrt{f+g\,x}-\sqrt{f}\right)^3}-\frac{8\,b\,\sqrt{d}\,\sqrt{f}\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{g^2\left(\sqrt{f+g\,x}-\sqrt{f}\right)^2}}{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{\left(\sqrt{f+g\,x}-\sqrt{f}\right)^4}+\frac{e^2}{g^2}-\frac{2\,e\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{g\left(\sqrt{f+g\,x}-\sqrt{f}\right)^2}}$$
$$-\frac{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)\left(\frac{3\,c\,d^2\,e\,g^2}{2}+c\,d\,e^2\,f\,g+\frac{3\,c\,e^3\,f^2}{2}\right)}{g^6\left(\sqrt{f+g\,x}-\sqrt{f}\right)}-\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3\left(\frac{11\,c\,d^2\,g^2}{2}+25\,c\,d\,e\,f\,g+\frac{11\,c\,e^2\,f^2}{2}\right)}{g^5\left(\sqrt{f+g\,x}-\sqrt{f}\right)^3}+\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^7\left(\frac{3\,c\,d^2\,g^2}{2}+c\cdots\right.}{e^2\,g^3\left(\sqrt{f+g\,x}-\cdots\right.}}{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^8}{\left(\sqrt{f+g\,x}-\sqrt{f}\right)^8}+\frac{e^4}{g^4}-\frac{4\,e\left(\sqrt{d+e\,x}-\sqrt{d}\right)^6}{g\left(\sqrt{f+g\,x}-\sqrt{f}\right)^6}-\frac{4\,e^3\left(\cdots\right.}{g^3\left(\sqrt{\cdots}\right.}}$$
$$-\frac{4\,a\,\mathrm{atan}\left(\frac{e\left(\sqrt{f+g\,x}-\sqrt{f}\right)}{\sqrt{-e\,g}\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{-e\,g}}-\frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{g}\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{e}\left(\sqrt{f+g\,x}-\sqrt{f}\right)}\right)(d\,g+e\,f)}{e^{3/2}\,g^{3/2}}$$
$$+\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{g}\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{e}\left(\sqrt{f+g\,x}-\sqrt{f}\right)}\right)\left(3\,d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2\right)}{2\,e^{5/2}\,g^{5/2}}$$

input
```
int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(1/2)),x)
```

output
```
(((2*b*d*g + 2*b*e*f)*((d + e*x)^(1/2) - d^(1/2)))/(g^3*((f + g*x)^(1/2) -
 f^(1/2))) + ((2*b*d*g + 2*b*e*f)*((d + e*x)^(1/2) - d^(1/2))^3)/(e*g^2*((
f + g*x)^(1/2) - f^(1/2))^3) - (8*b*d^(1/2)*f^(1/2)*((d + e*x)^(1/2) - d^(
1/2))^2)/(g^2*((f + g*x)^(1/2) - f^(1/2))^2))/(((d + e*x)^(1/2) - d^(1/2))
^4/((f + g*x)^(1/2) - f^(1/2))^4 + e^2/g^2 - (2*e*((d + e*x)^(1/2) - d^(1/
2))^2)/(g*((f + g*x)^(1/2) - f^(1/2))^2)) - ((((d + e*x)^(1/2) - d^(1/2))*
((3*c*e^3*f^2)/2 + (3*c*d^2*e*g^2)/2 + c*d*e^2*f*g))/(g^6*((f + g*x)^(1/2)
 - f^(1/2))) - (((d + e*x)^(1/2) - d^(1/2))^3*((11*c*d^2*g^2)/2 + (11*c*e^
2*f^2)/2 + 25*c*d*e*f*g))/(g^5*((f + g*x)^(1/2) - f^(1/2))^3) + (((d + e*x
)^(1/2) - d^(1/2))^7*((3*c*d^2*g^2)/2 + (3*c*e^2*f^2)/2 + c*d*e*f*g))/(e^2
*g^3*((f + g*x)^(1/2) - f^(1/2))^7) - (((d + e*x)^(1/2) - d^(1/2))^5*((11*
c*d^2*g^2)/2 + (11*c*e^2*f^2)/2 + 25*c*d*e*f*g))/(e*g^4*((f + g*x)^(1/2) -
 f^(1/2))^5) + (d^(1/2)*f^(1/2)*(32*c*d*g + 32*c*e*f)*((d + e*x)^(1/2) - d
^(1/2))^4)/(g^4*((f + g*x)^(1/2) - f^(1/2))^4))/(((d + e*x)^(1/2) - d^(1/2
))^8/((f + g*x)^(1/2) - f^(1/2))^8 + e^4/g^4 - (4*e*((d + e*x)^(1/2) - d^(
1/2))^6)/(g*((f + g*x)^(1/2) - f^(1/2))^6) - (4*e^3*((d + e*x)^(1/2) - d^(
1/2))^2)/(g^3*((f + g*x)^(1/2) - f^(1/2))^2) + (6*e^2*((d + e*x)^(1/2) - d
^(1/2))^4)/(g^2*((f + g*x)^(1/2) - f^(1/2))^4)) - (4*a*atan((e*((f + g*x)^
(1/2) - f^(1/2)))/((-e*g)^(1/2)*((d + e*x)^(1/2) - d^(1/2)))))/(-e*g)^(1/2
) - (2*b*atanh((g^(1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x...
```

3.834. $\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

3.835 $\int \frac{(d+ex)^{3/2}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

3.835.1 Optimal result

Integrand size = 29, antiderivative size = 333

$$\int \frac{(d+ex)^{3/2}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = -\frac{(ef-dg)\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{64e^2g^4} + \frac{\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)(d+ex)^{3/2}\sqrt{f+gx}}{96e^2g^3} - \frac{(7cef+9cdg-8beg)(d+ex)^{5/2}\sqrt{f+gx}}{24e^2g^2} + \frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g} + \frac{(ef-dg)^2\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{64e^{5/2}g^{9/2}}$$

output
```
1/64*(-d*g+e*f)^2*(c*(3*d^2*g^2+10*d*e*f*g+35*e^2*f^2)+8*e*g*(6*a*e*g-b*(d
*g+5*e*f)))*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g
^(9/2)+1/96*(c*(3*d^2*g^2+10*d*e*f*g+35*e^2*f^2)+8*e*g*(6*a*e*g-b*(d*g+5*e
*f)))*(e*x+d)^(3/2)*(g*x+f)^(1/2)/e^2/g^3-1/24*(-8*b*e*g+9*c*d*g+7*c*e*f)*
(e*x+d)^(5/2)*(g*x+f)^(1/2)/e^2/g^2+1/4*c*(e*x+d)^(7/2)*(g*x+f)^(1/2)/e^2/
g-1/64*(-d*g+e*f)*(c*(3*d^2*g^2+10*d*e*f*g+35*e^2*f^2)+8*e*g*(6*a*e*g-b*(d
*g+5*e*f)))*(e*x+d)^(1/2)*(g*x+f)^(1/2)/e^2/g^4
```

3.835.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^{3/2}\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx = \frac{\sqrt{d+ex}\sqrt{f+gx}(c(-9d^3g^3+3d^2eg^2(-5f+2gx)+de^2g(145f^2-92fg}{}$$

$$+\frac{(ef-dg)^2\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{64e^{5/2}g^{9/2}}$$

input
```
Integrate[((d + e*x)^(3/2)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

output
```
(Sqrt[d + e*x]*Sqrt[f + g*x]*(c*(-9*d^3*g^3 + 3*d^2*e*g^2*(-5*f + 2*g*x) +
 d*e^2*g*(145*f^2 - 92*f*g*x + 72*g^2*x^2) + e^3*(-105*f^3 + 70*f^2*g*x -
56*f*g^2*x^2 + 48*g^3*x^3)) + 8*e*g*(6*a*e*g*(-3*e*f + 5*d*g + 2*e*g*x) +
b*(3*d^2*g^2 + 2*d*e*g*(-11*f + 7*g*x) + e^2*(15*f^2 - 10*f*g*x + 8*g^2*x^
2)))))/(192*e^2*g^4) + ((e*f - d*g)^2*(c*(35*e^2*f^2 + 10*d*e*f*g + 3*d^2*
g^2) + 8*e*g*(6*a*e*g - b*(5*e*f + d*g)))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/
(Sqrt[g]*Sqrt[d + e*x])])/(64*e^(5/2)*g^(9/2))
```

3.835.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.75, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1194, 27, 90, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \text{1194}$$

$$\frac{\int \frac{(d+ex)^{3/2}(8age^2-(7cef+9cdg-8beg)xe-cd(7ef+dg))}{2\sqrt{f+gx}}dx}{4e^2g}+\frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(d+ex)^{3/2}(8age^2-(7cef+9cdg-8beg)xe-cd(7ef+dg))}{\sqrt{f+gx}}dx}{8e^2g}+\frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{\left(8eg(6aeg-b(dg+5ef))+c\left(3d^2g^2+10defg+35e^2f^2\right)\right)\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}}dx}{6g}-\frac{(d+ex)^{5/2}\sqrt{f+gx}(-8beg+9cdg+7cef)}{3g}}{8e^2g}+\frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{\left(8eg(6aeg-b(dg+5ef))+c\left(3d^2g^2+10defg+35e^2f^2\right)\right)\left(\frac{(d+ex)^{3/2}\sqrt{f+gx}}{2g}-\frac{3(ef-dg)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}}dx}{4g}\right)}{6g}-\frac{(d+ex)^{5/2}\sqrt{f+gx}(-8beg+9cdg+7cef)}{3g}}{8e^2g}+\frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{\left(8eg(6aeg-b(dg+5ef))+c\left(3d^2g^2+10defg+35e^2f^2\right)\right)\left(\frac{(d+ex)^{3/2}\sqrt{f+gx}}{2g}-\frac{3(ef-dg)\left(\frac{\sqrt{d+ex}\sqrt{f+gx}}{g}-\frac{(ef-dg)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}dx}{2g}\right)}{4g}\right)}{6g}-(d+ex)^{5/2}}{8e^2g}$$

$$\frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g}$$

$$\Big\downarrow \; 66$$

$$\frac{\frac{\left(8eg(6aeg-b(dg+5ef))+c\left(3d^2g^2+10defg+35e^2f^2\right)\right)\left(\frac{(d+ex)^{3/2}\sqrt{f+gx}}{2g}-\frac{3(ef-dg)\left(\frac{\sqrt{d+ex}\sqrt{f+gx}}{g}-\frac{(ef-dg)\int \frac{1}{e-\frac{g(d+ex)}{f+gx}}d\frac{\sqrt{d+ex}}{\sqrt{f+gx}}}{g}\right)}{4g}\right)}{6g}-(d+ex)}{8e^2g}$$

$$\frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g}$$

$$\Big\downarrow \; 221$$

3.835. $\int \frac{(d+ex)^{3/2}\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx$

$$\frac{\frac{\left(\frac{(d+ex)^{3/2}\sqrt{f+gx}}{2g}-\frac{3(ef-dg)\left(\frac{\sqrt{d+ex}\sqrt{f+gx}}{g}-\frac{(ef-dg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{e}g^{3/2}}\right)}{4g}\right)(8eg(6aeg-b(dg+5ef))+c(3d^2g^2+10defg+35e^2f^2))}{6g}-\frac{(d+ex}{}}{8e^2g}$$

$$\frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g}$$

input
```
Int[((d + e*x)^(3/2)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

output
```
(c*(d + e*x)^(7/2)*Sqrt[f + g*x])/(4*e^2*g) + (-1/3*((7*c*e*f + 9*c*d*g -
8*b*e*g)*(d + e*x)^(5/2)*Sqrt[f + g*x])/g + ((c*(35*e^2*f^2 + 10*d*e*f*g +
 3*d^2*g^2) + 8*e*g*(6*a*e*g - b*(5*e*f + d*g)))*(((d + e*x)^(3/2)*Sqrt[f
+ g*x])/(2*g) - (3*(e*f - d*g)*((Sqrt[d + e*x]*Sqrt[f + g*x])/g - ((e*f -
d*g)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt[e]*g^
(3/2))))/(4*g)))/(6*g))/(8*e^2*g)
```

3.835.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

3.835.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1206 vs. $2(295) = 590$.

Time = 0.48 (sec) , antiderivative size = 1207, normalized size of antiderivative = 3.62

method	result	size
default	Expression too large to display	1207

input
```
int((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/384*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(-184*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2
)*c*d*e^2*f*g^2*x+144*c*d*e^2*g^3*x^2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)-
112*c*e^3*f*g^2*x^2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+192*((g*x+f)*(e*x+
d))^(1/2)*(e*g)^(1/2)*a*e^3*g^3*x+12*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(
1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^3*e*f*g^3+48*((g*x+f)*(e*x+d))^
(1/2)*(e*g)^(1/2)*b*d^2*e*g^3+240*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)*b*e^
3*f^2*g-288*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)*a*e^3*f*g^2+480*((g*x+f)*(
e*x+d))^(1/2)*(e*g)^(1/2)*a*d*e^2*g^3-180*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+
d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e^3*f^3*g+54*ln(1/2*(2*e*g
*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*e^2*f
^2*g^2+216*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/
(e*g)^(1/2))*b*d*e^3*f^2*g^2-288*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)
*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*a*d*e^3*f*g^3-72*ln(1/2*(2*e*g*x+2*((g*
x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d^2*e^2*f*g^3+9*ln
(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*
c*d^4*g^4+105*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*
f)/(e*g)^(1/2))*c*e^4*f^4+128*b*e^3*g^3*x^2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^
(1/2)+96*c*e^3*g^3*x^3*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)-24*ln(1/2*(2*e*
g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d^3*e*g^
4+224*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)*b*d*e^2*g^3*x-160*((g*x+f)*(e...
```

3.835.5 Fricas [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 852, normalized size of antiderivative = 2.56

$$\int \frac{(d+ex)^{3/2}\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \left[\frac{3\,(35\,ce^4f^4 - 20\,(3\,cde^3 + 2\,be^4)f^3g + 6\,(3\,cd^2e^2 + 12\,bde^3 + 8\,ae^4)f^2g^2}{}\right.$$

$$-\frac{3\,(35\,ce^4f^4 - 20\,(3\,cde^3 + 2\,be^4)f^3g + 6\,(3\,cd^2e^2 + 12\,bde^3 + 8\,ae^4)f^2g^2 + 4\,(cd^3e - 6\,bd^2e^2 - 24\,ade^3)fg^3}{}$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
[1/768*(3*(35*c*e^4*f^4 - 20*(3*c*d*e^3 + 2*b*e^4)*f^3*g + 6*(3*c*d^2*e^2
+ 12*b*d*e^3 + 8*a*e^4)*f^2*g^2 + 4*(c*d^3*e - 6*b*d^2*e^2 - 24*a*d*e^3)*f
*g^3 + (3*c*d^4 - 8*b*d^3*e + 48*a*d^2*e^2)*g^4)*sqrt(e*g)*log(8*e^2*g^2*x
^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*x + e*f + d*g)*sqrt(e*g)*sqr
t(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2)*x) + 4*(48*c*e^4*g^4*x^3
- 105*c*e^4*f^3*g + 5*(29*c*d*e^3 + 24*b*e^4)*f^2*g^2 - (15*c*d^2*e^2 + 17
6*b*d*e^3 + 144*a*e^4)*f*g^3 - 3*(3*c*d^3*e - 8*b*d^2*e^2 - 80*a*d*e^3)*g^
4 - 8*(7*c*e^4*f*g^3 - (9*c*d*e^3 + 8*b*e^4)*g^4)*x^2 + 2*(35*c*e^4*f^2*g^
2 - 2*(23*c*d*e^3 + 20*b*e^4)*f*g^3 + (3*c*d^2*e^2 + 56*b*d*e^3 + 48*a*e^4
)*g^4)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^5), -1/384*(3*(35*c*e^4*f^4
- 20*(3*c*d*e^3 + 2*b*e^4)*f^3*g + 6*(3*c*d^2*e^2 + 12*b*d*e^3 + 8*a*e^4)*
f^2*g^2 + 4*(c*d^3*e - 6*b*d^2*e^2 - 24*a*d*e^3)*f*g^3 + (3*c*d^4 - 8*b*d^
3*e + 48*a*d^2*e^2)*g^4)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e*f + d*g)*sqrt(
-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (e^2*f*g + d*e*
g^2)*x)) - 2*(48*c*e^4*g^4*x^3 - 105*c*e^4*f^3*g + 5*(29*c*d*e^3 + 24*b*e^
4)*f^2*g^2 - (15*c*d^2*e^2 + 176*b*d*e^3 + 144*a*e^4)*f*g^3 - 3*(3*c*d^3*e
 - 8*b*d^2*e^2 - 80*a*d*e^3)*g^4 - 8*(7*c*e^4*f*g^3 - (9*c*d*e^3 + 8*b*e^4
)*g^4)*x^2 + 2*(35*c*e^4*f^2*g^2 - 2*(23*c*d*e^3 + 20*b*e^4)*f*g^3 + (3*c*
d^2*e^2 + 56*b*d*e^3 + 48*a*e^4)*g^4)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3
*g^5)]
```

3.835.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

output
```
Integral((d + e*x)**(3/2)*(a + b*x + c*x**2)/sqrt(f + g*x), x)
```

3.835.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{3/2}\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for m
ore detail
```

3.835.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)^{3/2}\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx = \frac{\left(\sqrt{e^2f+(ex+d)eg-deg}\left(2\,(ex+d)\left(4\,(ex+d)\left(\frac{6\,(ex+d)c}{e^3g}-\frac{7\,ce^7fg^5+9\,c\cdots}{e^{\cdots}}\right.\right.\right.\right.}{\cdots}$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
1/192*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e*x + d)*(4*(e*x + d)*(6*(e
*x + d)*c/(e^3*g) - (7*c*e^7*f*g^5 + 9*c*d*e^6*g^6 - 8*b*e^7*g^6)/(e^9*g^7
)) + (35*c*e^8*f^2*g^4 + 10*c*d*e^7*f*g^5 - 40*b*e^8*f*g^5 + 3*c*d^2*e^6*g
^6 - 8*b*d*e^7*g^6 + 48*a*e^8*g^6)/(e^9*g^7)) - 3*(35*c*e^9*f^3*g^3 - 25*c
*d*e^8*f^2*g^4 - 40*b*e^9*f^2*g^4 - 7*c*d^2*e^7*f*g^5 + 32*b*d*e^8*f*g^5 +
 48*a*e^9*f*g^5 - 3*c*d^3*e^6*g^6 + 8*b*d^2*e^7*g^6 - 48*a*d*e^8*g^6)/(e^9
*g^7))*sqrt(e*x + d) - 3*(35*c*e^4*f^4 - 60*c*d*e^3*f^3*g - 40*b*e^4*f^3*g
 + 18*c*d^2*e^2*f^2*g^2 + 72*b*d*e^3*f^2*g^2 + 48*a*e^4*f^2*g^2 + 4*c*d^3*
e*f*g^3 - 24*b*d^2*e^2*f*g^3 - 96*a*d*e^3*f*g^3 + 3*c*d^4*g^4 - 8*b*d^3*e*
g^4 + 48*a*d^2*e^2*g^4)*log(abs(-sqrt(e*g)*sqrt(e*x + d) + sqrt(e^2*f + (e
*x + d)*e*g - d*e*g)))/(sqrt(e*g)*e^2*g^4))*e/abs(e)
```

3.835.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \int \frac{(d+e\,x)^{3/2}\,(c\,x^2+b\,x+a)}{\sqrt{f+g\,x}}\,dx$$

input
```
int(((d + e*x)^(3/2)*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
```

output
```
int(((d + e*x)^(3/2)*(a + b*x + c*x^2))/(f + g*x)^(1/2), x)
```

3.836 $\int \frac{\sqrt{d+ex}\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx$

3.836.1 Optimal result

Integrand size = 29, antiderivative size = 246

$$\begin{aligned}\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx &= \frac{\left(c(5e^2f^2+2defg+d^2g^2)+2eg(4aeg-b(3ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{8e^2g^3} \\ &\quad - \frac{(5cef+7cdg-6beg)(d+ex)^{3/2}\sqrt{f+gx}}{12e^2g^2} + \frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g} \\ &\quad - \frac{(ef-dg)\left(c(5e^2f^2+2defg+d^2g^2)+2eg(4aeg-b(3ef+dg))\right)\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{8e^{5/2}g^{7/2}}\end{aligned}$$

output
```
-1/8*(-d*g+e*f)*(c*(d^2*g^2+2*d*e*f*g+5*e^2*f^2)+2*e*g*(4*a*e*g-b*(d*g+3*e
*f)))*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(7/2)
-1/12*(-6*b*e*g+7*c*d*g+5*c*e*f)*(e*x+d)^(3/2)*(g*x+f)^(1/2)/e^2/g^2+1/3*c
*(e*x+d)^(5/2)*(g*x+f)^(1/2)/e^2/g+1/8*(c*(d^2*g^2+2*d*e*f*g+5*e^2*f^2)+2*
e*g*(4*a*e*g-b*(d*g+3*e*f)))*(e*x+d)^(1/2)*(g*x+f)^(1/2)/e^2/g^3
```

3.836.2 Mathematica [A] (verified)

Time = 3.55 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{\sqrt{d+ex}\sqrt{f+gx}(6eg(4aeg+b(-3ef+dg+2egx))+c(-3d^2g^2+2deg(-2f+gx)+e^2(15f^2-10fgx}{24e^2g^3}$$
$$+\frac{(ef-dg)\left(c(5e^2f^2+2defg+d^2g^2)+2eg(4aeg-b(3ef+dg))\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\left(\sqrt{d-\frac{ef}{g}}-\sqrt{d+ex}\right)}\right)}{4e^{5/2}g^{7/2}}$$

input
```
Integrate[(Sqrt[d + e*x]*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

output
```
(Sqrt[d + e*x]*Sqrt[f + g*x]*(6*e*g*(4*a*e*g + b*(-3*e*f + d*g + 2*e*g*x))
 + c*(-3*d^2*g^2 + 2*d*e*g*(-2*f + g*x) + e^2*(15*f^2 - 10*f*g*x + 8*g^2*x
^2))))/(24*e^2*g^3) + ((e*f - d*g)*(c*(5*e^2*f^2 + 2*d*e*f*g + d^2*g^2) +
2*e*g*(4*a*e*g - b*(3*e*f + d*g)))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g
]*(Sqrt[d - (e*f)/g] - Sqrt[d + e*x]))])/(4*e^(5/2)*g^(7/2))
```

3.836.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1194, 27, 90, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$\Big\downarrow \text{1194}$$
$$\frac{\int \frac{\sqrt{d+ex}(6age^2-(5cef+7cdg-6beg)xe-cd(5ef+dg))}{2\sqrt{f+gx}}dx}{3e^2g}+\frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{\sqrt{d+ex}(6age^2-(5cef+7cdg-6beg)xe-cd(5ef+dg))}{\sqrt{f+gx}}dx}{6e^2g}+\frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g}$$

$$\Big\downarrow \; 90$$

$$\frac{\frac{3\left(2eg(4aeg-b(dg+3ef))+c\left(d^2g^2+2defg+5e^2f^2\right)\right)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}}dx}{4g}-\frac{(d+ex)^{3/2}\sqrt{f+gx}(-6beg+7cdg+5cef)}{2g}}{6e^2g}+\frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{3\left(2eg(4aeg-b(dg+3ef))+c\left(d^2g^2+2defg+5e^2f^2\right)\right)\left(\frac{\sqrt{d+ex}\sqrt{f+gx}}{g}-\frac{(ef-dg)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}dx}{2g}\right)}{4g}-\frac{(d+ex)^{3/2}\sqrt{f+gx}(-6beg+7cdg+5cef)}{2g}}{6e^2g}+\frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g}$$

$$\Big\downarrow \; 66$$

$$\frac{\frac{3\left(2eg(4aeg-b(dg+3ef))+c\left(d^2g^2+2defg+5e^2f^2\right)\right)\left(\frac{\sqrt{d+ex}\sqrt{f+gx}}{g}-\frac{(ef-dg)\int \frac{1}{e-\frac{g(d+ex)}{f+gx}}d\frac{\sqrt{d+ex}}{\sqrt{f+gx}}}{g}\right)}{4g}-\frac{(d+ex)^{3/2}\sqrt{f+gx}(-6beg+7cdg+5cef)}{2g}}{6e^2g}$$

$$\frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{3\left(\frac{\sqrt{d+ex}\sqrt{f+gx}}{g}-\frac{(ef-dg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{e}g^{3/2}}\right)\left(2eg(4aeg-b(dg+3ef))+c\left(d^2g^2+2defg+5e^2f^2\right)\right)}{4g}-\frac{(d+ex)^{3/2}\sqrt{f+gx}(-6beg+7cdg+5cef)}{2g}}{6e^2g}$$

$$\frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g}$$

input
```
Int[(Sqrt[d + e*x]*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

output
```
(c*(d + e*x)^(5/2)*Sqrt[f + g*x])/(3*e^2*g) + (-1/2*((5*c*e*f + 7*c*d*g -
6*b*e*g)*(d + e*x)^(3/2)*Sqrt[f + g*x])/g + (3*(c*(5*e^2*f^2 + 2*d*e*f*g +
 d^2*g^2) + 2*e*g*(4*a*e*g - b*(3*e*f + d*g)))*((Sqrt[d + e*x]*Sqrt[f + g*
x])/g - ((e*f - d*g)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x
])])/(Sqrt[e]*g^(3/2))))/(4*g))/(6*e^2*g)
```

3.836. $\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

3.836.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

3.836. $\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

3.836.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. 2(214) = 428.

Time = 0.49 (sec) , antiderivative size = 763, normalized size of antiderivative = 3.10

method	result
default	$\frac{\sqrt{ex+d}\,\sqrt{gx+f}\left(16c\,e^{2}g^{2}x^{2}\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+24\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)ad\,e^{2}g^{3}-24\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}}{2\sqrt{eg}}\right.\right.}{}$

input
```
int((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(16*c*e^2*g^2*x^2*((g*x+f)*(e*x+d))^(1/2)
*(e*g)^(1/2)+24*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+
e*f)/(e*g)^(1/2))*a*d*e^2*g^3-24*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)
*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*a*e^3*f*g^2-6*ln(1/2*(2*e*g*x+2*((g*x+f
)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d^2*e*g^3-12*ln(1/2*(
2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d*e^
2*f*g^2+18*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/
(e*g)^(1/2))*b*e^3*f^2*g+3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)
^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^3*g^3+3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d
))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*e*f*g^2+9*ln(1/2*(2*e*g*x
+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e^2*f^2*g
-15*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(
1/2))*c*e^3*f^3+24*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)*b*e^2*g^2*x+4*((g*x
+f)*(e*x+d))^(1/2)*(e*g)^(1/2)*c*d*e*g^2*x-20*((g*x+f)*(e*x+d))^(1/2)*(e*g
)^(1/2)*c*e^2*f*g*x+48*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*a*e^2*g^2+12*((
g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)*b*d*e*g^2-36*(e*g)^(1/2)*((g*x+f)*(e*x+d
))^(1/2)*b*e^2*f*g-6*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)*c*d^2*g^2-8*((g*x
+f)*(e*x+d))^(1/2)*(e*g)^(1/2)*c*d*e*f*g+30*(e*g)^(1/2)*((g*x+f)*(e*x+d))^
(1/2)*c*e^2*f^2)/g^3/((g*x+f)*(e*x+d))^(1/2)/e^2/(e*g)^(1/2)
```

3.836.5 Fricas [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 576, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$=\left[-\frac{3\left(5\,ce^{3}f^{3}-3\left(cde^{2}+2\,be^{3}\right)f^{2}g-\left(cd^{2}e-4\,bde^{2}-8\,ae^{3}\right)fg^{2}-\left(cd^{3}-2\,bd^{2}e+8\,ade^{2}\right)g^{3}\right)\sqrt{eg}\log\left(8\,e\right.}{}\right.$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(5*c*e^3*f^3 - 3*(c*d*e^2 + 2*b*e^3)*f^2*g - (c*d^2*e - 4*b*d*e^
2 - 8*a*e^3)*f*g^2 - (c*d^3 - 2*b*d^2*e + 8*a*d*e^2)*g^3)*sqrt(e*g)*log(8*
e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*x + e*f + d*g)*sqrt
(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2)*x) - 4*(8*c*e^3*
g^3*x^2 + 15*c*e^3*f^2*g - 2*(2*c*d*e^2 + 9*b*e^3)*f*g^2 - 3*(c*d^2*e - 2*
b*d*e^2 - 8*a*e^3)*g^3 - 2*(5*c*e^3*f*g^2 - (c*d*e^2 + 6*b*e^3)*g^3)*x)*sq
rt(e*x + d)*sqrt(g*x + f))/(e^3*g^4), 1/48*(3*(5*c*e^3*f^3 - 3*(c*d*e^2 +
2*b*e^3)*f^2*g - (c*d^2*e - 4*b*d*e^2 - 8*a*e^3)*f*g^2 - (c*d^3 - 2*b*d^2*
e + 8*a*d*e^2)*g^3)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e*f + d*g)*sqrt(-e*g)
*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (e^2*f*g + d*e*g^2)*
x)) + 2*(8*c*e^3*g^3*x^2 + 15*c*e^3*f^2*g - 2*(2*c*d*e^2 + 9*b*e^3)*f*g^2
- 3*(c*d^2*e - 2*b*d*e^2 - 8*a*e^3)*g^3 - 2*(5*c*e^3*f*g^2 - (c*d*e^2 + 6*
b*e^3)*g^3)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^4)]
```

3.836.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)*(a + b*x + c*x**2)/sqrt(f + g*x), x)
```

3.836.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.836.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{\left(\sqrt{e^2f+(ex+d)eg-deg}\sqrt{ex+d}\left(2\,(ex+d)\left(\frac{4\,(ex+d)c}{e^3g}-\frac{5\,ce^7fg^3+7\,cde^6g^4-6\,be^7g^4}{e^9g^5}\right)+\frac{3\,(5\,ce^8f^2g^2+2\,cde^7fg^3-6\ldots}{\ldots}\right.\right.}{\ldots}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
1/24*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(e*x + d)*(2*(e*x + d)*(4*(e
*x + d)*c/(e^3*g) - (5*c*e^7*f*g^3 + 7*c*d*e^6*g^4 - 6*b*e^7*g^4)/(e^9*g^5
)) + 3*(5*c*e^8*f^2*g^2 + 2*c*d*e^7*f*g^3 - 6*b*e^8*f*g^3 + c*d^2*e^6*g^4
- 2*b*d*e^7*g^4 + 8*a*e^8*g^4)/(e^9*g^5)) + 3*(5*c*e^3*f^3 - 3*c*d*e^2*f^2
*g - 6*b*e^3*f^2*g - c*d^2*e*f*g^2 + 4*b*d*e^2*f*g^2 + 8*a*e^3*f*g^2 - c*d
^3*g^3 + 2*b*d^2*e*g^3 - 8*a*d*e^2*g^3)*log(abs(-sqrt(e*g)*sqrt(e*x + d) +
 sqrt(e^2*f + (e*x + d)*e*g - d*e*g)))/(sqrt(e*g)*e^2*g^3))*e/abs(e)
```

3.836.9 Mupad [B] (verification not implemented)

Time = 109.82 (sec) , antiderivative size = 1832, normalized size of antiderivative = 7.45

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \text{Too large to display}$$

input
```
int(((d + e*x)^(1/2)*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
```

output
```
(((2*a*d*g + 2*a*e*f)*((d + e*x)^(1/2) - d^(1/2))^3)/(g^2*((f + g*x)^(1/2)
 - f^(1/2))^3) + ((2*a*e^2*f + 2*a*d*e*g)*((d + e*x)^(1/2) - d^(1/2)))/(g^
3*((f + g*x)^(1/2) - f^(1/2))) - (8*a*d^(1/2)*e*f^(1/2)*((d + e*x)^(1/2) -
 d^(1/2))^2)/(g^2*((f + g*x)^(1/2) - f^(1/2))^2))/((((d + e*x)^(1/2) - d^(1
/2))^4/((f + g*x)^(1/2) - f^(1/2))^4 + e^2/g^2 - (2*e*((d + e*x)^(1/2) - d
^(1/2))^2)/(g*((f + g*x)^(1/2) - f^(1/2))^2)) - ((((d + e*x)^(1/2) - d^(1/
2))*((c*d^3*e^3*g^3)/4 - (5*c*e^6*f^3)/4 + (3*c*d*e^5*f^2*g)/4 + (c*d^2*e^
4*f*g^2)/4))/(g^9*((f + g*x)^(1/2) - f^(1/2))) - (((d + e*x)^(1/2) - d^(1/
2))^5*((33*c*e^4*f^3)/2 + (19*c*d^3*e*g^3)/2 + (313*c*d*e^3*f^2*g)/2 + (27
5*c*d^2*e^2*f*g^2)/2))/(g^7*((f + g*x)^(1/2) - f^(1/2))^5) - (((d + e*x)^(
1/2) - d^(1/2))^7*((19*c*d^3*g^3)/2 + (33*c*e^3*f^3)/2 + (313*c*d*e^2*f^2*
g)/2 + (275*c*d^2*e*f*g^2)/2))/(g^6*((f + g*x)^(1/2) - f^(1/2))^7) - (((d
+ e*x)^(1/2) - d^(1/2))^3*((17*c*d^3*e^2*g^3)/12 - (85*c*e^5*f^3)/12 + (17
*c*d*e^4*f^2*g)/4 + (91*c*d^2*e^3*f*g^2)/4))/(g^8*((f + g*x)^(1/2) - f^(1/
2))^3) + (((d + e*x)^(1/2) - d^(1/2))^11*((c*d^3*g^3)/4 - (5*c*e^3*f^3)/4
+ (3*c*d*e^2*f^2*g)/4 + (c*d^2*e*f*g^2)/4))/(e^2*g^4*((f + g*x)^(1/2) - f^
(1/2))^11) - (((d + e*x)^(1/2) - d^(1/2))^9*((17*c*d^3*g^3)/12 - (85*c*e^3
*f^3)/12 + (17*c*d*e^2*f^2*g)/4 + (91*c*d^2*e*f*g^2)/4))/(e*g^5*((f + g*x)
^(1/2) - f^(1/2))^9) + (d^(1/2)*f^(1/2)*((d + e*x)^(1/2) - d^(1/2))^6*(128
*c*e^3*f^2 + 64*c*d^2*e*g^2 + (704*c*d*e^2*f*g)/3))/(g^6*((f + g*x)^(1/...
```

3.836. $\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

3.837 $\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

3.837.1 Optimal result

Integrand size = 29, antiderivative size = 164

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{(c(3e^2f^2+2defg+3d^2g^2)+4eg(2aeg-b(ef+dg)))\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{4e^{5/2}g^{5/2}}$$

output
```
1/4*(c*(3*d^2*g^2+2*d*e*f*g+3*e^2*f^2)+4*e*g*(2*a*e*g-b*(d*g+e*f)))*arctan
h(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(5/2)+1/2*c*(e*x+
d)^(3/2)*(g*x+f)^(1/2)/e^2/g-1/4*(-4*b*e*g+5*c*d*g+3*c*e*f)*(e*x+d)^(1/2)*
(g*x+f)^(1/2)/e^2/g^2
```

3.837.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.86

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{\sqrt{d+ex}\sqrt{f+gx}(4beg+c(-3ef-3dg+2egx))}{4e^2g^2} + \frac{(c(3e^2f^2+2defg+3d^2g^2)+4eg(2aeg-b(ef+dg)))\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{4e^{5/2}g^{5/2}}$$

input
```
Integrate[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

output
```
(Sqrt[d + e*x]*Sqrt[f + g*x]*(4*b*e*g + c*(-3*e*f - 3*d*g + 2*e*g*x)))/(4*
e^2*g^2) + ((c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2) + 4*e*g*(2*a*e*g - b*(e
*f + d*g)))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/(4*e
^(5/2)*g^(5/2))
```

3.837.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1194, 27, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{1194}$$

$$\frac{\int \frac{4age^2-(3cef+5cdg-4beg)xe-cd(3ef+dg)}{2\sqrt{d+ex}\sqrt{f+gx}}dx}{2e^2g}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4age^2-(3cef+5cdg-4beg)xe-cd(3ef+dg)}{\sqrt{d+ex}\sqrt{f+gx}}dx}{4e^2g}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{(4eg(2aeg-b(dg+ef))+c(3d^2g^2+2defg+3e^2f^2))\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}dx}{2g}-\frac{\sqrt{d+ex}\sqrt{f+gx}(-4beg+5cdg+3cef)}{g}}{4e^2g}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

$$\Big\downarrow \text{66}$$

$$\frac{\frac{(4eg(2aeg-b(dg+ef))+c(3d^2g^2+2defg+3e^2f^2))\int \frac{1}{e-\frac{g(d+ex)}{f+gx}}d\frac{\sqrt{d+ex}}{\sqrt{f+gx}}}{g}-\frac{\sqrt{d+ex}\sqrt{f+gx}(-4beg+5cdg+3cef)}{g}}{4e^2g}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)(4eg(2aeg-b(dg+ef))+c(3d^2g^2+2defg+3e^2f^2))}{\sqrt{e}g^{3/2}}-\frac{\sqrt{d+ex}\sqrt{f+gx}(-4beg+5cdg+3cef)}{g}}{4e^2g}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

input
```
Int[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

output
```
(c*(d + e*x)^(3/2)*Sqrt[f + g*x])/(2*e^2*g) + (-(((3*c*e*f + 5*c*d*g - 4*b
*e*g)*Sqrt[d + e*x]*Sqrt[f + g*x])/g) + ((c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2
*g^2) + 4*e*g*(2*a*e*g - b*(e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(
Sqrt[e]*Sqrt[f + g*x])])/(Sqrt[e]*g^(3/2)))/(4*e^2*g)
```

3.837.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

3.837.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(138) = 276$.

Time = 0.48 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.59

method	result
default	$\left(4\sqrt{eg}\,\sqrt{(gx+f)(ex+d)}\,cegx+3\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)c\,d^2g^2+2\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)cdefg+3\ln$

input
```
int((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(4*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*e*g*x+3*ln(1/2*(2*e*g*x+2*((g
*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*g^2+2*ln(1/2*
(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e
*f*g+3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g
)^(1/2))*c*e^2*f^2+8*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)
+d*g+e*f)/(e*g)^(1/2))*a*e^2*g^2-4*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/
2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d*e*g^2-4*ln(1/2*(2*e*g*x+2*((g*x+f
)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*e^2*f*g-6*(e*g)^(1/2)
*((g*x+f)*(e*x+d))^(1/2)*c*d*g-6*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*e*f
+8*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*b*e*g)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/
(e*g)^(1/2)/g^2/e^2/((g*x+f)*(e*x+d))^(1/2)
```

3.837.5 Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.32

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$
$$= \left[\frac{(3\,ce^2f^2+2\,(cde-2\,be^2)fg+(3\,cd^2-4\,bde+8\,ae^2)g^2)\sqrt{eg}\log\left(8\,e^2g^2x^2+e^2f^2+6\,defg+d^2g^2+4\,(2\right.}{}\right.$$
$$-\frac{(3\,ce^2f^2+2\,(cde-2\,be^2)fg+(3\,cd^2-4\,bde+8\,ae^2)g^2)\sqrt{-eg}\arctan\left(\frac{(2\,egx+ef+dg)\sqrt{-eg}\sqrt{ex+d}\sqrt{gx+f}}{2\,(e^2g^2x^2+defg+(e^2fg+deg^2)x)}\right)-}{8\,e^3g^3}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((3*c*e^2*f^2 + 2*(c*d*e - 2*b*e^2)*f*g + (3*c*d^2 - 4*b*d*e + 8*a*e
^2)*g^2)*sqrt(e*g)*log(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(
2*e*g*x + e*f + d*g)*sqrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g +
d*e*g^2)*x) + 4*(2*c*e^2*g^2*x - 3*c*e^2*f*g - (3*c*d*e - 4*b*e^2)*g^2)*sq
rt(e*x + d)*sqrt(g*x + f))/(e^3*g^3), -1/8*((3*c*e^2*f^2 + 2*(c*d*e - 2*b*
e^2)*f*g + (3*c*d^2 - 4*b*d*e + 8*a*e^2)*g^2)*sqrt(-e*g)*arctan(1/2*(2*e*g
*x + e*f + d*g)*sqrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*
f*g + (e^2*f*g + d*e*g^2)*x)) - 2*(2*c*e^2*g^2*x - 3*c*e^2*f*g - (3*c*d*e
- 4*b*e^2)*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^3)]
```

3.837.6 Sympy [F]

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx=\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

input
```
integrate((c*x**2+b*x+a)/(e*x+d)**(1/2)/(g*x+f)**(1/2),x)
```

output
```
Integral((a + b*x + c*x**2)/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

3.837.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for m
ore detail
```

3.837.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.16

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{\left(\sqrt{e^2f+(ex+d)eg-deg}\sqrt{ex+d}\left(\frac{2\,(ex+d)c}{e^3g}-\frac{3\,ce^6fg+5\,cde^5g^2-4\,be^6g^2}{e^8g^3}\right)-\frac{(3\,ce^2f^2+2\,cdefg-4\,be^2fg+3\,cd^2g^2-4\,bdeg^2}{}\right.}{4\,|e|}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
1/4*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(e*x + d)*(2*(e*x + d)*c/(e^3
*g) - (3*c*e^6*f*g + 5*c*d*e^5*g^2 - 4*b*e^6*g^2)/(e^8*g^3)) - (3*c*e^2*f^
2 + 2*c*d*e*f*g - 4*b*e^2*f*g + 3*c*d^2*g^2 - 4*b*d*e*g^2 + 8*a*e^2*g^2)*l
og(abs(-sqrt(e*g)*sqrt(e*x + d) + sqrt(e^2*f + (e*x + d)*e*g - d*e*g)))/(s
qrt(e*g)*e^2*g^2))*e/abs(e)
```

3.837.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 833, normalized size of antiderivative = 5.08

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

$$= \frac{\frac{(2\,b\,d\,g+2\,b\,e\,f)\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{g^3\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)}+\frac{(2\,b\,d\,g+2\,b\,e\,f)\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3}{e\,g^2\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)^3}-\frac{8\,b\,\sqrt{d}\,\sqrt{f}\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{g^2\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)^2}}{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{\left(\sqrt{f+g\,x}-\sqrt{f}\right)^4}+\frac{e^2}{g^2}-\frac{2\,e\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{g\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)^2}}$$

$$-\frac{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)\,\left(\frac{3\,c\,d^2\,e\,g^2}{2}+c\,d\,e^2\,f\,g+\frac{3\,c\,e^3\,f^2}{2}\right)}{g^6\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)}-\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3\,\left(\frac{11\,c\,d^2\,g^2}{2}+25\,c\,d\,e\,f\,g+\frac{11\,c\,e^2\,f^2}{2}\right)}{g^5\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)^3}+\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^7\,\left(\frac{3\,c\,d^2\,g^2}{2}+c\cdots\right.}{e^2\,g^3\,\left(\sqrt{f+g\,x}-\cdots\right.}}{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^8}{\left(\sqrt{f+g\,x}-\sqrt{f}\right)^8}+\frac{e^4}{g^4}-\frac{4\,e\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^6}{g\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)^6}-\frac{4\,e^3\,\left(\cdots\right.}{g^3\,\left(\cdots\right.}}$$

$$-\frac{4\,a\,\mathrm{atan}\left(\frac{e\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)}{\sqrt{-e\,g}\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{-e\,g}}-\frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{g}\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{e}\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)}\right)\,(d\,g+e\,f)}{e^{3/2}\,g^{3/2}}$$

$$+\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{g}\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{e}\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)}\right)\,\left(3\,d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2\right)}{2\,e^{5/2}\,g^{5/2}}$$

input
```
int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(1/2)),x)
```

output
```
(((2*b*d*g + 2*b*e*f)*((d + e*x)^(1/2) - d^(1/2)))/(g^3*((f + g*x)^(1/2) -
 f^(1/2))) + ((2*b*d*g + 2*b*e*f)*((d + e*x)^(1/2) - d^(1/2))^3)/(e*g^2*((
f + g*x)^(1/2) - f^(1/2))^3) - (8*b*d^(1/2)*f^(1/2)*((d + e*x)^(1/2) - d^(
1/2))^2)/(g^2*((f + g*x)^(1/2) - f^(1/2))^2))/(((d + e*x)^(1/2) - d^(1/2))
^4/((f + g*x)^(1/2) - f^(1/2))^4 + e^2/g^2 - (2*e*((d + e*x)^(1/2) - d^(1/
2))^2)/(g*((f + g*x)^(1/2) - f^(1/2))^2)) - ((((d + e*x)^(1/2) - d^(1/2))*
((3*c*e^3*f^2)/2 + (3*c*d^2*e*g^2)/2 + c*d*e^2*f*g))/(g^6*((f + g*x)^(1/2)
 - f^(1/2))) - (((d + e*x)^(1/2) - d^(1/2))^3*((11*c*d^2*g^2)/2 + (11*c*e^
2*f^2)/2 + 25*c*d*e*f*g))/(g^5*((f + g*x)^(1/2) - f^(1/2))^3) + (((d + e*x
)^(1/2) - d^(1/2))^7*((3*c*d^2*g^2)/2 + (3*c*e^2*f^2)/2 + c*d*e*f*g))/(e^2
*g^3*((f + g*x)^(1/2) - f^(1/2))^7) - (((d + e*x)^(1/2) - d^(1/2))^5*((11*
c*d^2*g^2)/2 + (11*c*e^2*f^2)/2 + 25*c*d*e*f*g))/(e*g^4*((f + g*x)^(1/2) -
 f^(1/2))^5) + (d^(1/2)*f^(1/2)*(32*c*d*g + 32*c*e*f)*((d + e*x)^(1/2) - d
^(1/2))^4)/(g^4*((f + g*x)^(1/2) - f^(1/2))^4))/(((d + e*x)^(1/2) - d^(1/2
))^8/((f + g*x)^(1/2) - f^(1/2))^8 + e^4/g^4 - (4*e*((d + e*x)^(1/2) - d^(
1/2))^6)/(g*((f + g*x)^(1/2) - f^(1/2))^6) - (4*e^3*((d + e*x)^(1/2) - d^(
1/2))^2)/(g^3*((f + g*x)^(1/2) - f^(1/2))^2) + (6*e^2*((d + e*x)^(1/2) - d
^(1/2))^4)/(g^2*((f + g*x)^(1/2) - f^(1/2))^4)) - (4*a*atan((e*((f + g*x)^
(1/2) - f^(1/2)))/((-e*g)^(1/2)*((d + e*x)^(1/2) - d^(1/2)))))/(-e*g)^(1/2
) - (2*b*atanh((g^(1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x...
```

3.837. $\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

3.838 $\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx$

3.838.1 Optimal result

Integrand size = 29, antiderivative size = 129

$$\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}} + \frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g} - \frac{(cef+3cdg-2beg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}g^{3/2}}$$

output
```
-(-2*b*e*g+3*c*d*g+c*e*f)*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1
/2))/e^(5/2)/g^(3/2)-2*(a+d*(-b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+
d)^(1/2)+c*(e*x+d)^(1/2)*(g*x+f)^(1/2)/e^2/g
```

3.838.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02

$$\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = \frac{\sqrt{f+gx}(2e(bd-ae)g+c(-3d^2g+e^2fx+de(f-gx)))}{e^2g(ef-dg)\sqrt{d+ex}} + \frac{(2beg-c(ef+3dg))\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{e^{5/2}g^{3/2}}$$

input
```
Integrate[(a + b*x + c*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]),x]
```

output
```
(Sqrt[f + g*x]*(2*e*(b*d - a*e)*g + c*(-3*d^2*g + e^2*f*x + d*e*(f - g*x))
))/(e^2*g*(e*f - d*g)*Sqrt[d + e*x]) + ((2*b*e*g - c*(e*f + 3*d*g))*ArcTan
h[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/(e^(5/2)*g^(3/2))
```

3.838.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1193, 27, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{1193}$$

$$-\frac{2\int \frac{(ef-dg)(cd-be-cex)}{2e^2\sqrt{d+ex}\sqrt{f+gx}}dx}{ef-dg}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{\sqrt{d+ex}(ef-dg)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{cd-be-cex}{\sqrt{d+ex}\sqrt{f+gx}}dx}{e^2}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{\sqrt{d+ex}(ef-dg)}$$

$$\Big\downarrow \text{90}$$

$$-\frac{\frac{(-2beg+3cdg+cef)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}dx}{2g}-\frac{c\sqrt{d+ex}\sqrt{f+gx}}{g}}{e^2}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{\sqrt{d+ex}(ef-dg)}$$

$$\Big\downarrow \text{66}$$

$$-\frac{\frac{(-2beg+3cdg+cef)\int \frac{1}{e-\frac{g(d+ex)}{f+gx}}d\frac{\sqrt{d+ex}}{\sqrt{f+gx}}}{g}-\frac{c\sqrt{d+ex}\sqrt{f+gx}}{g}}{e^2}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{\sqrt{d+ex}(ef-dg)}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{\sqrt{d+ex}(ef-dg)}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)(-2beg+3cdg+cef)}{\sqrt{e}g^{3/2}}-\frac{c\sqrt{d+ex}\sqrt{f+gx}}{g}}{e^2}$$

input
```
Int[(a + b*x + c*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]),x]
```

output
```
(-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/((e*f - d*g)*Sqrt[d + e*x]) -
 (-((c*Sqrt[d + e*x]*Sqrt[f + g*x])/g) + ((c*e*f + 3*c*d*g - 2*b*e*g)*ArcT
anh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt[e]*g^(3/2)))/e
^2
```

3.838.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

3.838. $\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx$

3.838.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 696 vs. $2(109) = 218$.

Time = 0.48 (sec) , antiderivative size = 697, normalized size of antiderivative = 5.40

method	result
default	$\frac{\sqrt{gx+f}\left(2\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)bd\,e^2g^2x-2\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)b\,e^3fgx-3\ln\left(\frac{2egx+2\sqrt{(gx+f)}}{2}\right.\right.}{}$

input
```
int((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(g*x+f)^(1/2)*(2*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)
+d*g+e*f)/(e*g)^(1/2))*b*d*e^2*g^2*x-2*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))
^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*e^3*f*g*x-3*ln(1/2*(2*e*g*x+2*(
(g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*e*g^2*x+2*l
n(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))
*c*d*e^2*f*g*x+ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e
*f)/(e*g)^(1/2))*c*e^3*f^2*x+2*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(
e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d^2*e*g^2-2*ln(1/2*(2*e*g*x+2*((g*x+f)*
(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d*e^2*f*g-3*ln(1/2*(2*e
*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^3*g^2
+2*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1
/2))*c*d^2*e*f*g+ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g
+e*f)/(e*g)^(1/2))*c*d*e^2*f^2+2*c*d*e*g*x*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(
1/2)-2*c*e^2*f*x*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+4*a*e^2*g*((g*x+f)*(e
*x+d))^(1/2)*(e*g)^(1/2)-4*b*d*e*g*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+6*c
*d^2*g*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)-2*c*d*e*f*((g*x+f)*(e*x+d))^(1/
2)*(e*g)^(1/2))/(e*g)^(1/2)/g/(d*g-e*f)/((g*x+f)*(e*x+d))^(1/2)/e^2/(e*x+d
)^(1/2)
```

3.838.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(109) = 218$.

Time = 1.51 (sec) , antiderivative size = 588, normalized size of antiderivative = 4.56

$$\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = \left[-\frac{(cde^2f^2+2\,(cd^2e-bde^2)fg-(3\,cd^3-2\,bd^2e)g^2+(ce^3f^2+2\,(cde^2-be^3)\ldots}{}\right.$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*((c*d*e^2*f^2 + 2*(c*d^2*e - b*d*e^2)*f*g - (3*c*d^3 - 2*b*d^2*e)*g^
2 + (c*e^3*f^2 + 2*(c*d*e^2 - b*e^3)*f*g - (3*c*d^2*e - 2*b*d*e^2)*g^2)*x)
*sqrt(e*g)*log(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*x
+ e*f + d*g)*sqrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2)
*x) - 4*(c*d*e^2*f*g - (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)*g^2 + (c*e^3*f*g
- c*d*e^2*g^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(d*e^4*f*g^2 - d^2*e^3*g^3
+ (e^5*f*g^2 - d*e^4*g^3)*x), 1/2*((c*d*e^2*f^2 + 2*(c*d^2*e - b*d*e^2)*f*
g - (3*c*d^3 - 2*b*d^2*e)*g^2 + (c*e^3*f^2 + 2*(c*d*e^2 - b*e^3)*f*g - (3*
c*d^2*e - 2*b*d*e^2)*g^2)*x)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e*f + d*g)*s
qrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (e^2*f*g +
d*e*g^2)*x)) + 2*(c*d*e^2*f*g - (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)*g^2 + (c
*e^3*f*g - c*d*e^2*g^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(d*e^4*f*g^2 - d^2
*e^3*g^3 + (e^5*f*g^2 - d*e^4*g^3)*x)]
```

3.838.6 Sympy [F]

$$\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = \int \frac{a+bx+cx^2}{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}}\,dx$$

input
```
integrate((c*x**2+b*x+a)/(e*x+d)**(3/2)/(g*x+f)**(1/2),x)
```

output
```
Integral((a + b*x + c*x**2)/((d + e*x)**(3/2)*sqrt(f + g*x)), x)
```

3.838.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.838.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.51

$$\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = -\frac{4\,(cd^2g-bdeg+ae^2g)}{\left(e^2f-deg-\left(\sqrt{eg}\sqrt{ex+d}-\sqrt{e^2f+(ex+d)eg-deg}\right)^2\right)\sqrt{eg}|e|} + \frac{(cef+3\,cdg-2\,beg)\log\left(\left(\sqrt{eg}\sqrt{ex+d}-\sqrt{e^2f+(ex+d)eg-deg}\right)^2\right)}{2\,\sqrt{eg}eg|e|} + \frac{\sqrt{e^2f+(ex+d)eg-deg}\sqrt{ex+d}c|e|}{e^4g}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
-4*(c*d^2*g - b*d*e*g + a*e^2*g)/((e^2*f - d*e*g - (sqrt(e*g)*sqrt(e*x + d
) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2)*sqrt(e*g)*abs(e)) + 1/2*(c*e*f
 + 3*c*d*g - 2*b*e*g)*log((sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d
)*e*g - d*e*g))^2)/(sqrt(e*g)*e*g*abs(e)) + sqrt(e^2*f + (e*x + d)*e*g - d
*e*g)*sqrt(e*x + d)*c*abs(e)/(e^4*g)
```

3.838.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = \int \frac{c\,x^2+b\,x+a}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

input
```
int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(3/2)),x)
```

output
```
int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(3/2)), x)
```

3.839 $\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx$

3.839.1 Optimal result

Integrand size = 29, antiderivative size = 160

$$\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} + \frac{2(c(6def-4d^2g)-e(3bef-bdg-2aeg))\sqrt{f+gx}}{3e^2(ef-dg)^2\sqrt{d+ex}} + \frac{2c\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}\sqrt{g}}$$

output
```
2*c*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(1/2)-2
/3*(a+d*(-b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)^(3/2)+2/3*(c*(-4*
d^2*g+6*d*e*f)-e*(-2*a*e*g-b*d*g+3*b*e*f))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^2/
(e*x+d)^(1/2)
```

3.839.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.91

$$\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}(cd(-3d^2g+6e^2fx+de(5f-4gx))+e^2(b(-2df-3efx+dgx)+}{3e^2(ef-dg)^2(d+ex)^{3/2}} + \frac{2c\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{e^{5/2}\sqrt{g}}$$

input
```
Integrate[(a + b*x + c*x^2)/((d + e*x)^(5/2)*Sqrt[f + g*x]),x]
```

output
```
(2*Sqrt[f + g*x]*(c*d*(-3*d^2*g + 6*e^2*f*x + d*e*(5*f - 4*g*x)) + e^2*(b*
(-2*d*f - 3*e*f*x + d*g*x) + a*(-(e*f) + 3*d*g + 2*e*g*x))))/(3*e^2*(e*f -
 d*g)^2*(d + e*x)^(3/2)) + (2*c*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*S
qrt[d + e*x])])/(e^(5/2)*Sqrt[g])
```

3.839.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1193, 27, 87, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{1193}$$

$$-\frac{2\int \frac{-3c\left(f-\frac{dg}{e}\right)xe^2-(3bef-bdg-2aeg)e+cd(3ef-dg)}{2e^2(d+ex)^{3/2}\sqrt{f+gx}}dx}{3(ef-dg)}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{3(d+ex)^{3/2}(ef-dg)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{cd(3ef-dg)-e(3bef-bdg-2aeg)-3ce(ef-dg)x}{(d+ex)^{3/2}\sqrt{f+gx}}dx}{3e^2(ef-dg)}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{3(d+ex)^{3/2}(ef-dg)}$$

$$\Big\downarrow \text{87}$$

$$-\frac{-3c(ef-dg)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}dx-\frac{2\sqrt{f+gx}(2cd(3ef-2dg)-e(-2aeg-bdg+3bef))}{\sqrt{d+ex}(ef-dg)}}{3e^2(ef-dg)}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{3(d+ex)^{3/2}(ef-dg)}$$

$$\Big\downarrow \text{66}$$

$$-\frac{-6c(ef-dg)\int \frac{1}{e-\frac{g(d+ex)}{f+gx}}d\frac{\sqrt{d+ex}}{\sqrt{f+gx}}-\frac{2\sqrt{f+gx}(2cd(3ef-2dg)-e(-2aeg-bdg+3bef))}{\sqrt{d+ex}(ef-dg)}}{3e^2(ef-dg)}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{3(d+ex)^{3/2}(ef-dg)}$$

$$\Big\downarrow \text{221}$$

$$-\frac{-\frac{2\sqrt{f+gx}(2cd(3ef-2dg)-e(-2aeg-bdg+3bef))}{\sqrt{d+ex}(ef-dg)}-\frac{6c(ef-dg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{e}\sqrt{g}}}{3e^2(ef-dg)}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{3(d+ex)^{3/2}(ef-dg)}$$

input
```
Int[(a + b*x + c*x^2)/((d + e*x)^(5/2)*Sqrt[f + g*x]),x]
```

output
```
(-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(3*(e*f - d*g)*(d + e*x)^(3/2
)) - ((-2*(2*c*d*(3*e*f - 2*d*g) - e*(3*b*e*f - b*d*g - 2*a*e*g))*Sqrt[f +
 g*x])/((e*f - d*g)*Sqrt[d + e*x]) - (6*c*(e*f - d*g)*ArcTanh[(Sqrt[g]*Sqr
t[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt[e]*Sqrt[g]))/(3*e^2*(e*f - d*g
))
```

3.839.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.839. $\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx$

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] && !IntegerQ[n]
&& !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

3.839.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(136) = 272$.

Time = 0.48 (sec) , antiderivative size = 773, normalized size of antiderivative = 4.83

method	result
default	$\frac{\sqrt{gx+f}\left(3\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)c\,d^{2}e^{2}g^{2}x^{2}-6\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)c\,d\,e^{3}f\,g\,x^{2}+3\ln\left(\frac{2egx+2\sqrt{(g}{}\right.\right.}{}$

input
```
int((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(g*x+f)^(1/2)*(3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)
+d*g+e*f)/(e*g)^(1/2))*c*d^2*e^2*g^2*x^2-6*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x
+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e^3*f*g*x^2+3*ln(1/2*(2*e
*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*e^4*f^2
*x^2+6*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g
)^(1/2))*c*d^3*e*g^2*x-12*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^
(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*e^2*f*g*x+6*ln(1/2*(2*e*g*x+2*((g*x+f)*(
e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e^3*f^2*x+3*ln(1/2*(2*
e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^4*g^
2-6*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(
1/2))*c*d^3*e*f*g+3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+
d*g+e*f)/(e*g)^(1/2))*c*d^2*e^2*f^2+4*a*e^3*g*x*((g*x+f)*(e*x+d))^(1/2)*(e
*g)^(1/2)+2*b*d*e^2*g*x*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)-6*b*e^3*f*x*((
g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)-8*c*d^2*e*g*x*((g*x+f)*(e*x+d))^(1/2)*(e
*g)^(1/2)+12*c*d*e^2*f*x*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+6*a*d*e^2*g*(
(g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)-2*a*e^3*f*((g*x+f)*(e*x+d))^(1/2)*(e*g)
^(1/2)-4*b*d*e^2*f*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)-6*c*d^3*g*((g*x+f)*
(e*x+d))^(1/2)*(e*g)^(1/2)+10*c*d^2*e*f*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2
))/(e*g)^(1/2)/(d*g-e*f)^2/((g*x+f)*(e*x+d))^(1/2)/e^2/(e*x+d)^(3/2)
```

3.839.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(137) = 274$.

Time = 3.31 (sec) , antiderivative size = 792, normalized size of antiderivative = 4.95

$$\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = \left[\frac{3\,(cd^2e^2f^2 - 2\,cd^3efg + cd^4g^2 + (ce^4f^2 - 2\,cde^3fg + cd^2e^2g^2)x^2 + 2\,(cde^3f^2}{}\right.$$

$$-\frac{3\,(cd^2e^2f^2 - 2\,cd^3efg + cd^4g^2 + (ce^4f^2 - 2\,cde^3fg + cd^2e^2g^2)x^2 + 2\,(cde^3f^2 - 2\,cd^2e^2fg + cd^3eg^2)x)\sqrt{-e}}{3\,(d^2e^5f^2g - 2\,d^3e^4fg^2}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(c*d^2*e^2*f^2 - 2*c*d^3*e*f*g + c*d^4*g^2 + (c*e^4*f^2 - 2*c*d*e^
3*f*g + c*d^2*e^2*g^2)*x^2 + 2*(c*d*e^3*f^2 - 2*c*d^2*e^2*f*g + c*d^3*e*g^
2)*x)*sqrt(e*g)*log(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e
*g*x + e*f + d*g)*sqrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e
*g^2)*x) + 4*((5*c*d^2*e^2 - 2*b*d*e^3 - a*e^4)*f*g - 3*(c*d^3*e - a*d*e^3
)*g^2 + (3*(2*c*d*e^3 - b*e^4)*f*g - (4*c*d^2*e^2 - b*d*e^3 - 2*a*e^4)*g^2
)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(d^2*e^5*f^2*g - 2*d^3*e^4*f*g^2 + d^4*e
^3*g^3 + (e^7*f^2*g - 2*d*e^6*f*g^2 + d^2*e^5*g^3)*x^2 + 2*(d*e^6*f^2*g -
2*d^2*e^5*f*g^2 + d^3*e^4*g^3)*x), -1/3*(3*(c*d^2*e^2*f^2 - 2*c*d^3*e*f*g
+ c*d^4*g^2 + (c*e^4*f^2 - 2*c*d*e^3*f*g + c*d^2*e^2*g^2)*x^2 + 2*(c*d*e^3
*f^2 - 2*c*d^2*e^2*f*g + c*d^3*e*g^2)*x)*sqrt(-e*g)*arctan(1/2*(2*e*g*x +
e*f + d*g)*sqrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g +
 (e^2*f*g + d*e*g^2)*x)) - 2*((5*c*d^2*e^2 - 2*b*d*e^3 - a*e^4)*f*g - 3*(c
*d^3*e - a*d*e^3)*g^2 + (3*(2*c*d*e^3 - b*e^4)*f*g - (4*c*d^2*e^2 - b*d*e^
3 - 2*a*e^4)*g^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(d^2*e^5*f^2*g - 2*d^3*e
^4*f*g^2 + d^4*e^3*g^3 + (e^7*f^2*g - 2*d*e^6*f*g^2 + d^2*e^5*g^3)*x^2 + 2
*(d*e^6*f^2*g - 2*d^2*e^5*f*g^2 + d^3*e^4*g^3)*x)]
```

3.839.6 Sympy [F]

$$\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = \int \frac{a+bx+cx^2}{(d+ex)^{\frac{5}{2}}\sqrt{f+gx}}\,dx$$

input
```
integrate((c*x**2+b*x+a)/(e*x+d)**(5/2)/(g*x+f)**(1/2),x)
```

output
```
Integral((a + b*x + c*x**2)/((d + e*x)**(5/2)*sqrt(f + g*x)), x)
```

3.839.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.839.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. 2(137) = 274.

Time = 0.38 (sec) , antiderivative size = 491, normalized size of antiderivative = 3.07

$$\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = -\frac{c\log\left(\left(\sqrt{eg}\sqrt{ex+d}-\sqrt{e^2f+(ex+d)eg-deg}\right)^2\right)}{\sqrt{eg}e|e|} + \frac{4\left(6\,cde^4f^2g-3\,be^5f^2g-10\,cd^2e^3fg^2+4\,bde^4fg^2+2\,ae^5fg^2+4\,cd^3e^2g^3-bd^2e^3g^3-2\,ade^4g^3-12\left(\sqrt{e\right.}\right.}{}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
-c*log((sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2)/
(sqrt(e*g)*e*abs(e)) + 4/3*(6*c*d*e^4*f^2*g - 3*b*e^5*f^2*g - 10*c*d^2*e^3
*f*g^2 + 4*b*d*e^4*f*g^2 + 2*a*e^5*f*g^2 + 4*c*d^3*e^2*g^3 - b*d^2*e^3*g^3
 - 2*a*d*e^4*g^3 - 12*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*
g - d*e*g))^2*c*d*e^2*f*g + 6*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x
 + d)*e*g - d*e*g))^2*b*e^3*f*g + 6*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f
+ (e*x + d)*e*g - d*e*g))^2*c*d^2*e*g^2 - 6*(sqrt(e*g)*sqrt(e*x + d) - sqr
t(e^2*f + (e*x + d)*e*g - d*e*g))^2*a*e^3*g^2 + 6*(sqrt(e*g)*sqrt(e*x + d)
 - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^4*c*d*g - 3*(sqrt(e*g)*sqrt(e*x +
d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^4*b*e*g)/((e^2*f - d*e*g - (sqrt
(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2)^3*sqrt(e*g)*
abs(e))
```

3.839.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx=\int \frac{c\,x^2+b\,x+a}{\sqrt{f+g\,x}\,(d+e\,x)^{5/2}}\,dx$$

input
```
int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(5/2)),x)
```

output
```
int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(5/2)), x)
```

3.840 $\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx$

3.840.1 Optimal result

Integrand size = 29, antiderivative size = 198

$$\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx = -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{5(ef-dg)(d+ex)^{5/2}} + \frac{2(2cd(5ef-3dg)-e(5bef-bdg-4aeg))\sqrt{f+gx}}{15e^2(ef-dg)^2(d+ex)^{3/2}} + \frac{2(2eg(5bef-bdg-4aeg)-c(15e^2f^2-10defg+3d^2g^2))\sqrt{f+gx}}{15e^2(ef-dg)^3\sqrt{d+ex}}$$

output
```
-2/5*(a+d*(-b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)^(5/2)+2/15*(2*c
*d*(-3*d*g+5*e*f)-e*(-4*a*e*g-b*d*g+5*b*e*f))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)
^2/(e*x+d)^(3/2)+2/15*(2*e*g*(-4*a*e*g-b*d*g+5*b*e*f)-c*(3*d^2*g^2-10*d*e*
f*g+15*e^2*f^2))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^3/(e*x+d)^(1/2)
```

3.840.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.89

$$\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx = -\frac{2\sqrt{f+gx}\left(15cf^2-15bfg+15ag^2-\frac{10cdf(f+gx)}{d+ex}+\frac{5bef(f+gx)}{d+ex}+\frac{5bdg(f+gx)}{d+ex}-\frac{10aeg(f+gx)}{d+ex}+\frac{3cd^2(f+gx)^2}{(d+ex)^2}-\frac{3bde(}{(d}\right.}{15(ef-dg)^3\sqrt{d+ex}}$$

input
```
Integrate[(a + b*x + c*x^2)/((d + e*x)^(7/2)*Sqrt[f + g*x]),x]
```

output
```
(-2*Sqrt[f + g*x]*(15*c*f^2 - 15*b*f*g + 15*a*g^2 - (10*c*d*f*(f + g*x))/(
d + e*x) + (5*b*e*f*(f + g*x))/(d + e*x) + (5*b*d*g*(f + g*x))/(d + e*x) -
 (10*a*e*g*(f + g*x))/(d + e*x) + (3*c*d^2*(f + g*x)^2)/(d + e*x)^2 - (3*b
*d*e*(f + g*x)^2)/(d + e*x)^2 + (3*a*e^2*(f + g*x)^2)/(d + e*x)^2))/(15*(e
*f - d*g)^3*Sqrt[d + e*x])
```

3.840.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1193, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{1193}$$

$$-\frac{2\int \frac{-5c\left(f-\frac{dg}{e}\right)xe^2-(5bef-bdg-4aeg)e+cd(5ef-dg)}{2e^2(d+ex)^{5/2}\sqrt{f+gx}}dx}{5(ef-dg)}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{5(d+ex)^{5/2}(ef-dg)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{cd(5ef-dg)-e(5bef-bdg-4aeg)-5ce(ef-dg)x}{(d+ex)^{5/2}\sqrt{f+gx}}dx}{5e^2(ef-dg)}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{5(d+ex)^{5/2}(ef-dg)}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\frac{\left(2eg(-4aeg-bdg+5bef)-c\left(3d^2g^2-10defg+15e^2f^2\right)\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}}dx}{3(ef-dg)}-\frac{2\sqrt{f+gx}(2cd(5ef-3dg)-e(-4aeg-bdg+5bef))}{3(d+ex)^{3/2}(ef-dg)}}{5e^2(ef-dg)}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{5(d+ex)^{5/2}(ef-dg)}$$

$$\Big\downarrow \text{48}$$

$$-\frac{-\frac{2\sqrt{f+gx}(2eg(-4aeg-bdg+5bef)-c(3d^2g^2-10defg+15e^2f^2))}{3\sqrt{d+ex}(ef-dg)^2}-\frac{2\sqrt{f+gx}(2cd(5ef-3dg)-e(-4aeg-bdg+5bef))}{3(d+ex)^{3/2}(ef-dg)}}{5e^2(ef-dg)}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{5(d+ex)^{5/2}(ef-dg)}$$

input
```
Int[(a + b*x + c*x^2)/((d + e*x)^(7/2)*Sqrt[f + g*x]),x]
```

output
```
(-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(5*(e*f - d*g)*(d + e*x)^(5/2
)) - ((-2*(2*c*d*(5*e*f - 3*d*g) - e*(5*b*e*f - b*d*g - 4*a*e*g))*Sqrt[f +
 g*x])/(3*(e*f - d*g)*(d + e*x)^(3/2)) - (2*(2*e*g*(5*b*e*f - b*d*g - 4*a*
e*g) - c*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2))*Sqrt[f + g*x])/(3*(e*f - d
*g)^2*Sqrt[d + e*x]))/(5*e^2*(e*f - d*g))
```

3.840.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

3.840.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.06

method	result
default	$\frac{2\sqrt{gx+f}\,(8a\,e^2g^2x^2+2bde\,g^2x^2-10b\,e^2fg\,x^2+3c\,d^2g^2x^2-10cdefg\,x^2+15c\,e^2f^2x^2+20ade\,g^2x-4a\,e^2fgx+5b\,d^2g^2x-26bdefgx+5b\,e}{15(ex+d)^{\frac{5}{2}}(dg-ef)^3}$
gosper	$\frac{2\sqrt{gx+f}\,(8a\,e^2g^2x^2+2bde\,g^2x^2-10b\,e^2fg\,x^2+3c\,d^2g^2x^2-10cdefg\,x^2+15c\,e^2f^2x^2+20ade\,g^2x-4a\,e^2fgx+5b\,d^2g^2x-26bdefgx+5b\,e}{15(ex+d)^{\frac{5}{2}}(d^3g^3-3d^2ef\,g^2+3d\,e^2f^2g-e^3f^3}$

input
```
int((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(g*x+f)^(1/2)*(8*a*e^2*g^2*x^2+2*b*d*e*g^2*x^2-10*b*e^2*f*g*x^2+3*c*d
^2*g^2*x^2-10*c*d*e*f*g*x^2+15*c*e^2*f^2*x^2+20*a*d*e*g^2*x-4*a*e^2*f*g*x+
5*b*d^2*g^2*x-26*b*d*e*f*g*x+5*b*e^2*f^2*x-4*c*d^2*f*g*x+20*c*d*e*f^2*x+15
*a*d^2*g^2-10*a*d*e*f*g+3*a*e^2*f^2-10*b*d^2*f*g+2*b*d*e*f^2+8*c*d^2*f^2)/
(e*x+d)^(5/2)/(d*g-e*f)^3
```

3.840.5 Fricas [A] (verification not implemented)

Time = 7.21 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.78

$$\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx =$$
$$-\frac{2\,(15\,ad^2g^2+(8\,cd^2+2\,bde+3\,ae^2)f^2-10\,(bd^2+ade)fg+(15\,ce^2f^2-10\,(cde+be^2)fg+(3\,cd^2+2\,bde}{15\,(d^3e^3f^3-3\,d^4e^2f^2g+3\,d^5efg^2-d^6g^3+(e^6f^3-3\,de^5f^2g+3\,d^2e^4fg^2-d^3e^3g^3)x^3+3\,(}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
-2/15*(15*a*d^2*g^2 + (8*c*d^2 + 2*b*d*e + 3*a*e^2)*f^2 - 10*(b*d^2 + a*d*
e)*f*g + (15*c*e^2*f^2 - 10*(c*d*e + b*e^2)*f*g + (3*c*d^2 + 2*b*d*e + 8*a
*e^2)*g^2)*x^2 + (5*(4*c*d*e + b*e^2)*f^2 - 2*(2*c*d^2 + 13*b*d*e + 2*a*e^
2)*f*g + 5*(b*d^2 + 4*a*d*e)*g^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(d^3*e^3*
f^3 - 3*d^4*e^2*f^2*g + 3*d^5*e*f*g^2 - d^6*g^3 + (e^6*f^3 - 3*d*e^5*f^2*g
 + 3*d^2*e^4*f*g^2 - d^3*e^3*g^3)*x^3 + 3*(d*e^5*f^3 - 3*d^2*e^4*f^2*g + 3
*d^3*e^3*f*g^2 - d^4*e^2*g^3)*x^2 + 3*(d^2*e^4*f^3 - 3*d^3*e^3*f^2*g + 3*d
^4*e^2*f*g^2 - d^5*e*g^3)*x)
```

3.840.6 Sympy [F]

$$\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx = \int \frac{a+bx+cx^2}{(d+ex)^{\frac{7}{2}}\sqrt{f+gx}}\,dx$$

input
```
integrate((c*x**2+b*x+a)/(e*x+d)**(7/2)/(g*x+f)**(1/2),x)
```

output
```
Integral((a + b*x + c*x**2)/((d + e*x)**(7/2)*sqrt(f + g*x)), x)
```

3.840.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.840.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. $2(180) = 360$.

Time = 0.42 (sec) , antiderivative size = 1175, normalized size of antiderivative = 5.93

$$\int \frac{a + bx + cx^2}{(d + ex)^{7/2}\sqrt{f + gx}}\,dx =$$

$$-\frac{4\left(15\sqrt{eg}ce^8f^4 - 40\sqrt{eg}cde^7f^3g - 10\sqrt{eg}be^8f^3g + 38\sqrt{eg}cd^2e^6f^2g^2 + 22\sqrt{eg}bde^7f^2g^2 + 8\sqrt{eg}ae^8f^2g^2\right.}{}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
-4/15*(15*sqrt(e*g)*c*e^8*f^4 - 40*sqrt(e*g)*c*d*e^7*f^3*g - 10*sqrt(e*g)*
b*e^8*f^3*g + 38*sqrt(e*g)*c*d^2*e^6*f^2*g^2 + 22*sqrt(e*g)*b*d*e^7*f^2*g^
2 + 8*sqrt(e*g)*a*e^8*f^2*g^2 - 16*sqrt(e*g)*c*d^3*e^5*f*g^3 - 14*sqrt(e*g
)*b*d^2*e^6*f*g^3 - 16*sqrt(e*g)*a*d*e^7*f*g^3 + 3*sqrt(e*g)*c*d^4*e^4*g^4
 + 2*sqrt(e*g)*b*d^3*e^5*g^4 + 8*sqrt(e*g)*a*d^2*e^6*g^4 - 60*sqrt(e*g)*(s
qrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2*c*e^6*f^3
+ 80*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e
*g))^2*c*d*e^5*f^2*g + 50*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f
+ (e*x + d)*e*g - d*e*g))^2*b*e^6*f^2*g - 20*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x
 + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2*c*d^2*e^4*f*g^2 - 60*sqrt(e
*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2*b*d*
e^5*f*g^2 - 40*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)
*e*g - d*e*g))^2*a*e^6*f*g^2 + 10*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqr
t(e^2*f + (e*x + d)*e*g - d*e*g))^2*b*d^2*e^4*g^3 + 40*sqrt(e*g)*(sqrt(e*g
)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2*a*d*e^5*g^3 + 90*
sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^
4*c*e^4*f^2 - 40*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x +
d)*e*g - d*e*g))^4*c*d*e^3*f*g - 70*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - s
qrt(e^2*f + (e*x + d)*e*g - d*e*g))^4*b*e^4*f*g + 30*sqrt(e*g)*(sqrt(e*g)*
sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^4*c*d^2*e^2*g^2 - ...
```

3.840.9 Mupad [B] (verification not implemented)

Time = 13.21 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.31

$$\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx = \frac{\sqrt{f+g\,x}\,\left(\frac{16\,c\,d^2\,f^2-20\,b\,d^2\,f\,g+30\,a\,d^2\,g^2+4\,b\,d\,e\,f^2-20\,a\,d\,e\,f\,g+6\,a\,e^2\,f^2}{15\,e^2\,(d\,g-e\,f)^3}+\frac{x\,(-8\,c\,d^2\,f\,g+10\,b\,d^2\,\cdots}{\cdots}\cdots\right.}{x^2\,\sqrt{d\cdots}}$$

input
```
int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(7/2)),x)
```

output
```
((f + g*x)^(1/2)*((30*a*d^2*g^2 + 6*a*e^2*f^2 + 16*c*d^2*f^2 + 4*b*d*e*f^2
 - 20*b*d^2*f*g - 20*a*d*e*f*g)/(15*e^2*(d*g - e*f)^3) + (x*(10*b*d^2*g^2
+ 10*b*e^2*f^2 + 40*a*d*e*g^2 + 40*c*d*e*f^2 - 8*a*e^2*f*g - 8*c*d^2*f*g -
 52*b*d*e*f*g))/(15*e^2*(d*g - e*f)^3) + (x^2*(16*a*e^2*g^2 + 6*c*d^2*g^2
+ 30*c*e^2*f^2 + 4*b*d*e*g^2 - 20*b*e^2*f*g - 20*c*d*e*f*g))/(15*e^2*(d*g
- e*f)^3)))/(x^2*(d + e*x)^(1/2) + (d^2*(d + e*x)^(1/2))/e^2 + (2*d*x*(d +
 e*x)^(1/2))/e)
```

3.841 $\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx$

3.841.1 Optimal result

Integrand size = 29, antiderivative size = 281

$$\begin{aligned}\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx &= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}}\\ &+\frac{2(2cd(7ef-4dg)-e(7bef-bdg-6aeg))\sqrt{f+gx}}{35e^2(ef-dg)^2(d+ex)^{5/2}}\\ &+\frac{2(4eg(7bef-bdg-6aeg)-c(35e^2f^2-14defg+3d^2g^2))\sqrt{f+gx}}{105e^2(ef-dg)^3(d+ex)^{3/2}}\\ &-\frac{4g(4eg(7bef-bdg-6aeg)-c(35e^2f^2-14defg+3d^2g^2))\sqrt{f+gx}}{105e^2(ef-dg)^4\sqrt{d+ex}}\end{aligned}$$

output
```
-2/7*(a+d*(-b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)^(7/2)+2/35*(2*c
*d*(-4*d*g+7*e*f)-e*(-6*a*e*g-b*d*g+7*b*e*f))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)
^2/(e*x+d)^(5/2)+2/105*(4*e*g*(-6*a*e*g-b*d*g+7*b*e*f)-c*(3*d^2*g^2-14*d*e
*f*g+35*e^2*f^2))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^3/(e*x+d)^(3/2)-4/105*g*(4*
e*g*(-6*a*e*g-b*d*g+7*b*e*f)-c*(3*d^2*g^2-14*d*e*f*g+35*e^2*f^2))*(g*x+f)^
(1/2)/e^2/(-d*g+e*f)^4/(e*x+d)^(1/2)
```

3.841.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.07

$$\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx =$$
$$-\frac{2\sqrt{f+gx}(-105cf^2g(d+ex)^3+105bfg^2(d+ex)^3-105ag^3(d+ex)^3+35cef^2(d+ex)^2(f+gx)+70cd}{}$$

input
```
Integrate[(a + b*x + c*x^2)/((d + e*x)^(9/2)*Sqrt[f + g*x]),x]
```

output
```
(-2*Sqrt[f + g*x]*(-105*c*f^2*g*(d + e*x)^3 + 105*b*f*g^2*(d + e*x)^3 - 10
5*a*g^3*(d + e*x)^3 + 35*c*e*f^2*(d + e*x)^2*(f + g*x) + 70*c*d*f*g*(d + e
*x)^2*(f + g*x) - 70*b*e*f*g*(d + e*x)^2*(f + g*x) - 35*b*d*g^2*(d + e*x)^
2*(f + g*x) + 105*a*e*g^2*(d + e*x)^2*(f + g*x) - 42*c*d*e*f*(d + e*x)*(f
+ g*x)^2 + 21*b*e^2*f*(d + e*x)*(f + g*x)^2 - 21*c*d^2*g*(d + e*x)*(f + g*
x)^2 + 42*b*d*e*g*(d + e*x)*(f + g*x)^2 - 63*a*e^2*g*(d + e*x)*(f + g*x)^2
 + 15*c*d^2*e*(f + g*x)^3 - 15*b*d*e^2*(f + g*x)^3 + 15*a*e^3*(f + g*x)^3)
)/(105*(e*f - d*g)^4*(d + e*x)^(7/2))
```

3.841.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1193, 27, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx$$
$$\Big\downarrow \; 1193$$
$$-\frac{2\int \frac{-7c\left(f-\frac{dg}{e}\right)xe^2-(7bef-bdg-6aeg)e+cd(7ef-dg)}{2e^2(d+ex)^{7/2}\sqrt{f+gx}}dx}{7(ef-dg)}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{7(d+ex)^{7/2}(ef-dg)}$$
$$\Big\downarrow \; 27$$
$$-\frac{\int \frac{cd(7ef-dg)-e(7bef-bdg-6aeg)-7ce(ef-dg)x}{(d+ex)^{7/2}\sqrt{f+gx}}dx}{7e^2(ef-dg)}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{7(d+ex)^{7/2}(ef-dg)}$$

$\Big\downarrow$ 87

$$-\frac{\frac{\left(4eg(-6aeg-bdg+7bef)-c\left(3d^2g^2-14defg+35e^2f^2\right)\right)\int \frac{1}{(d+ex)^{5/2}\sqrt{f+gx}}dx}{5(ef-dg)}-\frac{2\sqrt{f+gx}(2cd(7ef-4dg)-e(-6aeg-bdg+7bef))}{5(d+ex)^{5/2}(ef-dg)}}{7e^2(ef-dg)}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{7(d+ex)^{7/2}(ef-dg)}$$

$\Big\downarrow$ 55

$$-\frac{\frac{\left(4eg(-6aeg-bdg+7bef)-c\left(3d^2g^2-14defg+35e^2f^2\right)\right)\left(-\frac{2g\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}}dx}{3(ef-dg)}-\frac{2\sqrt{f+gx}}{3(d+ex)^{3/2}(ef-dg)}\right)}{5(ef-dg)}-\frac{2\sqrt{f+gx}(2cd(7ef-4dg)-e(-6aeg}{5(d+ex)^{5/2}(ef-dg)}}{7e^2(ef-dg)}$$

$$\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{7(d+ex)^{7/2}(ef-dg)}$$

$\Big\downarrow$ 48

$$-\frac{\frac{\left(\frac{4g\sqrt{f+gx}}{3\sqrt{d+ex}(ef-dg)^2}-\frac{2\sqrt{f+gx}}{3(d+ex)^{3/2}(ef-dg)}\right)\left(4eg(-6aeg-bdg+7bef)-c\left(3d^2g^2-14defg+35e^2f^2\right)\right)}{5(ef-dg)}-\frac{2\sqrt{f+gx}(2cd(7ef-4dg)-e(-6aeg-bdg+7b}{5(d+ex)^{5/2}(ef-dg)}}{7e^2(ef-dg)}$$

$$\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{7(d+ex)^{7/2}(ef-dg)}$$

input
```
Int[(a + b*x + c*x^2)/((d + e*x)^(9/2)*Sqrt[f + g*x]),x]
```

output
```
(-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(7*(e*f - d*g)*(d + e*x)^(7/2
)) - ((-2*(2*c*d*(7*e*f - 4*d*g) - e*(7*b*e*f - b*d*g - 6*a*e*g))*Sqrt[f +
 g*x])/(5*(e*f - d*g)*(d + e*x)^(5/2)) + ((4*e*g*(7*b*e*f - b*d*g - 6*a*e*
g) - c*(35*e^2*f^2 - 14*d*e*f*g + 3*d^2*g^2))*((-2*Sqrt[f + g*x])/(3*(e*f
- d*g)*(d + e*x)^(3/2)) + (4*g*Sqrt[f + g*x])/(3*(e*f - d*g)^2*Sqrt[d + e*
x])))/(5*(e*f - d*g)))/(7*e^2*(e*f - d*g))
```

3.841. $\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx$

3.841.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

3.841. $\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx$

3.841.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.52

method	result
default	$\frac{2\sqrt{gx+f}\,(48a\,e^{3}g^{3}x^{3}+8bd\,e^{2}g^{3}x^{3}-56b\,e^{3}f\,g^{2}x^{3}+6c\,d^{2}e\,g^{3}x^{3}-28cd\,e^{2}f\,g^{2}x^{3}+70c\,e^{3}f^{2}g\,x^{3}+168ad\,e^{2}g^{3}x^{2}-24a\,e^{3}f\,g^{2}x^{2}+28b\,d^{2}e\,g}{}$
gosper	$\frac{2\sqrt{gx+f}\,(48a\,e^{3}g^{3}x^{3}+8bd\,e^{2}g^{3}x^{3}-56b\,e^{3}f\,g^{2}x^{3}+6c\,d^{2}e\,g^{3}x^{3}-28cd\,e^{2}f\,g^{2}x^{3}+70c\,e^{3}f^{2}g\,x^{3}+168ad\,e^{2}g^{3}x^{2}-24a\,e^{3}f\,g^{2}x^{2}+28b\,d^{2}e\,g}{}$

input
```
int((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*(g*x+f)^(1/2)*(48*a*e^3*g^3*x^3+8*b*d*e^2*g^3*x^3-56*b*e^3*f*g^2*x^3
+6*c*d^2*e*g^3*x^3-28*c*d*e^2*f*g^2*x^3+70*c*e^3*f^2*g*x^3+168*a*d*e^2*g^3
*x^2-24*a*e^3*f*g^2*x^2+28*b*d^2*e*g^3*x^2-200*b*d*e^2*f*g^2*x^2+28*b*e^3*
f^2*g*x^2+21*c*d^3*g^3*x^2-101*c*d^2*e*f*g^2*x^2+259*c*d*e^2*f^2*g*x^2-35*
c*e^3*f^3*x^2+210*a*d^2*e*g^3*x-84*a*d*e^2*f*g^2*x+18*a*e^3*f^2*g*x+35*b*d
^3*g^3*x-259*b*d^2*e*f*g^2*x+101*b*d*e^2*f^2*g*x-21*b*e^3*f^3*x-28*c*d^3*f
*g^2*x+200*c*d^2*e*f^2*g*x-28*c*d*e^2*f^3*x+105*a*d^3*g^3-105*a*d^2*e*f*g^
2+63*a*d*e^2*f^2*g-15*a*e^3*f^3-70*b*d^3*f*g^2+28*b*d^2*e*f^2*g-6*b*d*e^2*
f^3+56*c*d^3*f^2*g-8*c*d^2*e*f^3)/(e*x+d)^(7/2)/(d*g-e*f)^4
```

3.841.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(257) = 514$.

Time = 22.89 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.28

$$\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx = \frac{2\,(105\,ad^3g^3-(8\,cd^2e+6\,bde^2+15\,ae^3)f^3+7\,(8\,cd^3+4\,bd^2e+9\,ade^2)f^2g- \cdots}{105\,(d^4e^4f^4-4\,d^5e \cdots}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(105*a*d^3*g^3 - (8*c*d^2*e + 6*b*d*e^2 + 15*a*e^3)*f^3 + 7*(8*c*d^3
 + 4*b*d^2*e + 9*a*d*e^2)*f^2*g - 35*(2*b*d^3 + 3*a*d^2*e)*f*g^2 + 2*(35*c
*e^3*f^2*g - 14*(c*d*e^2 + 2*b*e^3)*f*g^2 + (3*c*d^2*e + 4*b*d*e^2 + 24*a*
e^3)*g^3)*x^3 - (35*c*e^3*f^3 - 7*(37*c*d*e^2 + 4*b*e^3)*f^2*g + (101*c*d^
2*e + 200*b*d*e^2 + 24*a*e^3)*f*g^2 - 7*(3*c*d^3 + 4*b*d^2*e + 24*a*d*e^2)
*g^3)*x^2 - (7*(4*c*d*e^2 + 3*b*e^3)*f^3 - (200*c*d^2*e + 101*b*d*e^2 + 18
*a*e^3)*f^2*g + 7*(4*c*d^3 + 37*b*d^2*e + 12*a*d*e^2)*f*g^2 - 35*(b*d^3 +
6*a*d^2*e)*g^3)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(d^4*e^4*f^4 - 4*d^5*e^3*f^
3*g + 6*d^6*e^2*f^2*g^2 - 4*d^7*e*f*g^3 + d^8*g^4 + (e^8*f^4 - 4*d*e^7*f^3
*g + 6*d^2*e^6*f^2*g^2 - 4*d^3*e^5*f*g^3 + d^4*e^4*g^4)*x^4 + 4*(d*e^7*f^4
 - 4*d^2*e^6*f^3*g + 6*d^3*e^5*f^2*g^2 - 4*d^4*e^4*f*g^3 + d^5*e^3*g^4)*x^
3 + 6*(d^2*e^6*f^4 - 4*d^3*e^5*f^3*g + 6*d^4*e^4*f^2*g^2 - 4*d^5*e^3*f*g^3
 + d^6*e^2*g^4)*x^2 + 4*(d^3*e^5*f^4 - 4*d^4*e^4*f^3*g + 6*d^5*e^3*f^2*g^2
 - 4*d^6*e^2*f*g^3 + d^7*e*g^4)*x)
```

3.841.6 Sympy [F]

$$\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx = \int \frac{a+bx+cx^2}{(d+ex)^{\frac{9}{2}}\sqrt{f+gx}}\,dx$$

input
```
integrate((c*x**2+b*x+a)/(e*x+d)**(9/2)/(g*x+f)**(1/2),x)
```

output
```
Integral((a + b*x + c*x**2)/((d + e*x)**(9/2)*sqrt(f + g*x)), x)
```

3.841.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

3.841.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2035 vs. $2(257) = 514$.

Time = 0.52 (sec) , antiderivative size = 2035, normalized size of antiderivative = 7.24

$$\int \frac{a + bx + cx^2}{(d + ex)^{9/2}\sqrt{f + gx}}\, dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
8/105*(35*sqrt(e*g)*c*e^10*f^5*g - 119*sqrt(e*g)*c*d*e^9*f^4*g^2 - 28*sqrt
(e*g)*b*e^10*f^4*g^2 + 150*sqrt(e*g)*c*d^2*e^8*f^3*g^3 + 88*sqrt(e*g)*b*d*
e^9*f^3*g^3 + 24*sqrt(e*g)*a*e^10*f^3*g^3 - 86*sqrt(e*g)*c*d^3*e^7*f^2*g^4
 - 96*sqrt(e*g)*b*d^2*e^8*f^2*g^4 - 72*sqrt(e*g)*a*d*e^9*f^2*g^4 + 23*sqrt
(e*g)*c*d^4*e^6*f*g^5 + 40*sqrt(e*g)*b*d^3*e^7*f*g^5 + 72*sqrt(e*g)*a*d^2*
e^8*f*g^5 - 3*sqrt(e*g)*c*d^5*e^5*g^6 - 4*sqrt(e*g)*b*d^4*e^6*g^6 - 24*sqr
t(e*g)*a*d^3*e^7*g^6 - 245*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f
 + (e*x + d)*e*g - d*e*g))^2*c*e^8*f^4*g + 588*sqrt(e*g)*(sqrt(e*g)*sqrt(e
*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2*c*d*e^7*f^3*g^2 + 196*sqr
t(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2*b
*e^8*f^3*g^2 - 462*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x
+ d)*e*g - d*e*g))^2*c*d^2*e^6*f^2*g^3 - 420*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x
 + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2*b*d*e^7*f^2*g^3 - 168*sqrt(
e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2*a*e
^8*f^2*g^3 + 140*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x +
d)*e*g - d*e*g))^2*c*d^3*e^5*f*g^4 + 252*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d
) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2*b*d^2*e^6*f*g^4 + 336*sqrt(e*g)
*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2*a*d*e^7
*f*g^4 - 21*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*
g - d*e*g))^2*c*d^4*e^4*g^5 - 28*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - s...
```

3.841.9 Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.61

$$\int \frac{a + bx + cx^2}{(d + ex)^{9/2}\sqrt{f + gx}}\, dx = \frac{\sqrt{f + g\,x}\,\left(\frac{x^3\,\left(12\,c\,d^2\,e\,g^3-56\,c\,d\,e^2\,f\,g^2+16\,b\,d\,e^2\,g^3+140\,c\,e^3\,f^2\,g-112\,b\,e^3\,f\,g^2+96\,a\,e^3\,g^3\right)}{105\,e^3\,(d\,g-e\,f)^4} - \frac{-112}{}\right.}{}$$

input
```
int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(9/2)),x)
```

output
```
((f + g*x)^(1/2)*((x^3*(96*a*e^3*g^3 + 16*b*d*e^2*g^3 + 12*c*d^2*e*g^3 - 1
12*b*e^3*f*g^2 + 140*c*e^3*f^2*g - 56*c*d*e^2*f*g^2))/(105*e^3*(d*g - e*f)
^4) - (30*a*e^3*f^3 - 210*a*d^3*g^3 + 12*b*d*e^2*f^3 + 16*c*d^2*e*f^3 + 14
0*b*d^3*f*g^2 - 112*c*d^3*f^2*g - 126*a*d*e^2*f^2*g + 210*a*d^2*e*f*g^2 -
56*b*d^2*e*f^2*g)/(105*e^3*(d*g - e*f)^4) + (x*(70*b*d^3*g^3 - 42*b*e^3*f^
3 + 420*a*d^2*e*g^3 - 56*c*d*e^2*f^3 + 36*a*e^3*f^2*g - 56*c*d^3*f*g^2 - 1
68*a*d*e^2*f*g^2 + 202*b*d*e^2*f^2*g - 518*b*d^2*e*f*g^2 + 400*c*d^2*e*f^2
*g))/(105*e^3*(d*g - e*f)^4) + (2*x^2*(7*d*g - e*f)*(24*a*e^2*g^2 + 3*c*d^
2*g^2 + 35*c*e^2*f^2 + 4*b*d*e*g^2 - 28*b*e^2*f*g - 14*c*d*e*f*g))/(105*e^
3*(d*g - e*f)^4)))/(x^3*(d + e*x)^(1/2) + (d^3*(d + e*x)^(1/2))/e^3 + (3*d
*x^2*(d + e*x)^(1/2))/e + (3*d^2*x*(d + e*x)^(1/2))/e^2)
```

3.841. $\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx$

3.842 $\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx$

3.842.1 Optimal result

Integrand size = 29, antiderivative size = 249

$$\begin{aligned}\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx &= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{(e^2-df)\sqrt{e+fx}} \\ &+ \frac{(4ef(3be^2-bdf-2aef)-c(15e^4-6de^2f-d^2f^2))\sqrt{d+ex}\sqrt{e+fx}}{4ef^3(e^2-df)} \\ &+ \frac{c(d+ex)^{3/2}\sqrt{e+fx}}{2ef^2} \\ &- \frac{(4ef(3be^2-bdf-2aef)-c(15e^4-6de^2f-d^2f^2))\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{e}\sqrt{e+fx}}\right)}{4e^{3/2}f^{7/2}}\end{aligned}$$

output
```
-1/4*(4*e*f*(-2*a*e*f-b*d*f+3*b*e^2)-c*(-d^2*f^2-6*d*e^2*f+15*e^4))*arctan
h(f^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(f*x+e)^(1/2))/e^(3/2)/f^(7/2)+2*(a+e*(-b*
f+c*e)/f^2)*(e*x+d)^(3/2)/(-d*f+e^2)/(f*x+e)^(1/2)+1/2*c*(e*x+d)^(3/2)*(f*
x+e)^(1/2)/e/f^2+1/4*(4*e*f*(-2*a*e*f-b*d*f+3*b*e^2)-c*(-d^2*f^2-6*d*e^2*f
+15*e^4))*(e*x+d)^(1/2)*(f*x+e)^(1/2)/e/f^3/(-d*f+e^2)
```

3.842.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \frac{\sqrt{d+ex}(4ef(3be-2af+bfx)+c(-15e^3-5e^2fx+df^2x+ef(d+2fx^2)}{4ef^3\sqrt{e+fx}}$$
$$+\frac{(4ef(-3be^2+bdf+2aef)+c(15e^4-6de^2f-d^2f^2))\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{e}\sqrt{e+fx}}\right)}{4e^{3/2}f^{7/2}}$$

input
```
Integrate[(Sqrt[d + e*x]*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

output
```
(Sqrt[d + e*x]*(4*e*f*(3*b*e - 2*a*f + b*f*x) + c*(-15*e^3 - 5*e^2*f*x + d
*f^2*x + e*f*(d + 2*f*x^2))))/(4*e*f^3*Sqrt[e + f*x]) + ((4*e*f*(-3*b*e^2
+ b*d*f + 2*a*e*f) + c*(15*e^4 - 6*d*e^2*f - d^2*f^2))*ArcTanh[(Sqrt[f]*Sq
rt[d + e*x])/(Sqrt[e]*Sqrt[e + f*x])])/(4*e^(3/2)*f^(7/2))
```

3.842.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1193, 27, 90, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}\left(a+bx+cx^2\right)}{(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \text{1193}$$

$$\frac{2\int \frac{\sqrt{d+ex}\left(-c\left(d-\frac{e^2}{f}\right)xf^2+\left(3be^2-2afe-bdf\right)f-c\left(3e^3-def\right)\right)}{2f^2\sqrt{e+fx}}dx}{e^2-df}+\frac{2(d+ex)^{3/2}\left(a+\frac{e(ce-bf)}{f^2}\right)}{\left(e^2-df\right)\sqrt{e+fx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{d+ex}\left(f\left(3be^2-2afe-bdf\right)-c\left(3e^3-def\right)+cf\left(e^2-df\right)x\right)}{\sqrt{e+fx}}dx}{f^2\left(e^2-df\right)}+\frac{2(d+ex)^{3/2}\left(a+\frac{e(ce-bf)}{f^2}\right)}{\left(e^2-df\right)\sqrt{e+fx}}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{\left(4ef\left(-2aef-bdf+3be^2\right)-c\left(-d^2f^2-6de^2f+15e^4\right)\right)\int \frac{\sqrt{d+ex}}{\sqrt{e+fx}}dx}{4e}+\frac{c\left(e^2-df\right)(d+ex)^{3/2}\sqrt{e+fx}}{2e}}{f^2\left(e^2-df\right)}+\frac{2(d+ex)^{3/2}\left(a+\frac{e(ce-bf)}{f^2}\right)}{\left(e^2-df\right)\sqrt{e+fx}}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{\left(4ef\left(-2aef-bdf+3be^2\right)-c\left(-d^2f^2-6de^2f+15e^4\right)\right)\left(\frac{\sqrt{d+ex}\sqrt{e+fx}}{f}-\frac{\left(e^2-df\right)\int \frac{1}{\sqrt{d+ex}\sqrt{e+fx}}dx}{2f}\right)}{4e}+\frac{c\left(e^2-df\right)(d+ex)^{3/2}\sqrt{e+fx}}{2e}}{f^2\left(e^2-df\right)}+\frac{2(d+ex)^{3/2}\left(a+\frac{e(ce-bf)}{f^2}\right)}{\left(e^2-df\right)\sqrt{e+fx}}$$

$$\Big\downarrow \; 66$$

$$\frac{\frac{\left(4ef\left(-2aef-bdf+3be^2\right)-c\left(-d^2f^2-6de^2f+15e^4\right)\right)\left(\frac{\sqrt{d+ex}\sqrt{e+fx}}{f}-\frac{\left(e^2-df\right)\int \frac{1}{e-\frac{f(d+ex)}{e+fx}}d\frac{\sqrt{d+ex}}{\sqrt{e+fx}}}{f}\right)}{4e}+\frac{c\left(e^2-df\right)(d+ex)^{3/2}\sqrt{e+fx}}{2e}}{f^2\left(e^2-df\right)}+\frac{2(d+ex)^{3/2}\left(a+\frac{e(ce-bf)}{f^2}\right)}{\left(e^2-df\right)\sqrt{e+fx}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\left(\frac{\sqrt{d+ex}\sqrt{e+fx}}{f}-\frac{\left(e^2-df\right)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{e}\sqrt{e+fx}}\right)}{\sqrt{e}f^{3/2}}\right)\left(4ef\left(-2aef-bdf+3be^2\right)-c\left(-d^2f^2-6de^2f+15e^4\right)\right)}{4e}+\frac{c\left(e^2-df\right)(d+ex)^{3/2}\sqrt{e+fx}}{2e}}{f^2\left(e^2-df\right)}+\frac{2(d+ex)^{3/2}\left(a+\frac{e(ce-bf)}{f^2}\right)}{\left(e^2-df\right)\sqrt{e+fx}}$$

input
```
Int[(Sqrt[d + e*x]*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

output
```
(2*(a + (e*(c*e - b*f))/f^2)*(d + e*x)^(3/2))/((e^2 - d*f)*Sqrt[e + f*x])
+ ((c*(e^2 - d*f)*(d + e*x)^(3/2)*Sqrt[e + f*x])/(2*e) + ((4*e*f*(3*b*e^2
- b*d*f - 2*a*e*f) - c*(15*e^4 - 6*d*e^2*f - d^2*f^2))*((Sqrt[d + e*x]*Sqr
t[e + f*x])/f - ((e^2 - d*f)*ArcTanh[(Sqrt[f]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt
[e + f*x])])/(Sqrt[e]*f^(3/2))))/(4*e))/(f^2*(e^2 - d*f))
```

3.842. $\int \frac{\sqrt{d+ex}\left(a+bx+cx^2\right)}{(e+fx)^{3/2}}\,dx$

3.842.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

3.842. $\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx$

3.842.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 833 vs. $2(219) = 438$.

Time = 0.47 (sec) , antiderivative size = 834, normalized size of antiderivative = 3.35

method	result
default	$\frac{\sqrt{ex+d}\left(8\ln\left(\frac{2efx+2\sqrt{(ex+d)(fx+e)}\,\sqrt{ef}+df+e^2}{2\sqrt{ef}}\right)a\,e^2f^3x+4\ln\left(\frac{2efx+2\sqrt{(ex+d)(fx+e)}\,\sqrt{ef}+df+e^2}{2\sqrt{ef}}\right)bde\,f^3x-12\ln\left(\frac{2efx+2\sqrt{(ex+d)}}{2}\right.\right.}{}$

input
```
int((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(e*x+d)^(1/2)*(8*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)
+d*f+e^2)/(e*f)^(1/2))*a*e^2*f^3*x+4*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(
1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*b*d*e*f^3*x-12*ln(1/2*(2*e*f*x+2*((
e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*b*e^3*f^2*x-ln(1/2
*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*c*d^
2*f^3*x-6*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(
e*f)^(1/2))*c*d*e^2*f^2*x+15*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*
f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*c*e^4*f*x+4*c*e*f^2*x^2*((e*x+d)*(f*x+e))^(
1/2)*(e*f)^(1/2)+8*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d
*f+e^2)/(e*f)^(1/2))*a*e^3*f^2+4*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)
*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*b*d*e^2*f^2-12*ln(1/2*(2*e*f*x+2*((e*x+
d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*b*e^4*f-ln(1/2*(2*e*f*
x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*c*d^2*e*f^2-
6*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/
2))*c*d*e^3*f+15*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f
+e^2)/(e*f)^(1/2))*c*e^5+8*b*e*f^2*x*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+2
*c*d*f^2*x*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)-10*c*e^2*f*x*((e*x+d)*(f*x+
e))^(1/2)*(e*f)^(1/2)-16*a*e*f^2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+24*b*
e^2*f*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+2*c*d*e*f*((e*x+d)*(f*x+e))^(1/2
)*(e*f)^(1/2)-30*c*e^3*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2))/(e*f)^(1/2)...
```

3.842.5 Fricas [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.33

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \left[\frac{(15\,ce^5-(cd^2e-4\,bde^2-8\,ae^3)f^2-6\,(cde^3+2\,be^4)f+(15\,ce^4f-(cd^2\right.$$

$$-\frac{(15\,ce^5-(cd^2e-4\,bde^2-8\,ae^3)f^2-6\,(cde^3+2\,be^4)f+(15\,ce^4f-(cd^2-4\,bde-8\,ae^2)f^3-6\,(cde^2+2}{}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*((15*c*e^5 - (c*d^2*e - 4*b*d*e^2 - 8*a*e^3)*f^2 - 6*(c*d*e^3 + 2*b*
e^4)*f + (15*c*e^4*f - (c*d^2 - 4*b*d*e - 8*a*e^2)*f^3 - 6*(c*d*e^2 + 2*b*
e^3)*f^2)*x)*sqrt(e*f)*log(8*e^2*f^2*x^2 + e^4 + 6*d*e^2*f + d^2*f^2 + 4*(
2*e*f*x + e^2 + d*f)*sqrt(e*f)*sqrt(e*x + d)*sqrt(f*x + e) + 8*(e^3*f + d*
e*f^2)*x) + 4*(2*c*e^2*f^3*x^2 - 15*c*e^4*f - 8*a*e^2*f^3 + (c*d*e^2 + 12*
b*e^3)*f^2 - (5*c*e^3*f^2 - (c*d*e + 4*b*e^2)*f^3)*x)*sqrt(e*x + d)*sqrt(f
*x + e))/(e^2*f^5*x + e^3*f^4), -1/8*((15*c*e^5 - (c*d^2*e - 4*b*d*e^2 - 8
*a*e^3)*f^2 - 6*(c*d*e^3 + 2*b*e^4)*f + (15*c*e^4*f - (c*d^2 - 4*b*d*e - 8
*a*e^2)*f^3 - 6*(c*d*e^2 + 2*b*e^3)*f^2)*x)*sqrt(-e*f)*arctan(1/2*(2*e*f*x
 + e^2 + d*f)*sqrt(-e*f)*sqrt(e*x + d)*sqrt(f*x + e)/(e^2*f^2*x^2 + d*e^2*
f + (e^3*f + d*e*f^2)*x)) - 2*(2*c*e^2*f^3*x^2 - 15*c*e^4*f - 8*a*e^2*f^3
+ (c*d*e^2 + 12*b*e^3)*f^2 - (5*c*e^3*f^2 - (c*d*e + 4*b*e^2)*f^3)*x)*sqrt
(e*x + d)*sqrt(f*x + e))/(e^2*f^5*x + e^3*f^4)]
```

3.842.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \int \frac{\sqrt{d+ex}\,(a+bx+cx^2)}{(e+fx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(c*x**2+b*x+a)/(f*x+e)**(3/2),x)
```

output
```
Integral(sqrt(d + e*x)*(a + b*x + c*x**2)/(e + f*x)**(3/2), x)
```

3.842.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.842.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \frac{\sqrt{ex+d}\left((ex+d)\left(\frac{2\,(ex+d)c}{f|e|}-\frac{5\,ce^4f^3+3\,cde^2f^4-4\,be^3f^4}{e^2f^5|e|}\right)-\frac{15\,ce^6f^2-6\,cde^4f^3-12\,be^5f}{e^2}\right.}{4\sqrt{e^3+(ex+d)ef-def}} - \frac{(15\,ce^4-6\,cde^2f-12\,be^3f-cd^2f^2+4\,bdef^2+8\,ae^2f^2)\log\left(\left|-\sqrt{ef}\sqrt{ex+d}+\sqrt{e^3+(ex+d)ef-de}\right.\right.}{4\sqrt{ef}\,f^3|e|}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
1/4*sqrt(e*x + d)*((e*x + d)*(2*(e*x + d)*c/(f*abs(e)) - (5*c*e^4*f^3 + 3*
c*d*e^2*f^4 - 4*b*e^3*f^4)/(e^2*f^5*abs(e))) - (15*c*e^6*f^2 - 6*c*d*e^4*f
^3 - 12*b*e^5*f^3 - c*d^2*e^2*f^4 + 4*b*d*e^3*f^4 + 8*a*e^4*f^4)/(e^2*f^5*
abs(e)))/sqrt(e^3 + (e*x + d)*e*f - d*e*f) - 1/4*(15*c*e^4 - 6*c*d*e^2*f -
 12*b*e^3*f - c*d^2*f^2 + 4*b*d*e*f^2 + 8*a*e^2*f^2)*log(abs(-sqrt(e*f)*sq
rt(e*x + d) + sqrt(e^3 + (e*x + d)*e*f - d*e*f)))/(sqrt(e*f)*f^3*abs(e))
```

3.842.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \int \frac{\sqrt{d+e\,x}\,(c\,x^2+b\,x+a)}{(e+f\,x)^{3/2}}\,dx$$

input
```
int(((d + e*x)^(1/2)*(a + b*x + c*x^2))/(e + f*x)^(3/2),x)
```

output
```
int(((d + e*x)^(1/2)*(a + b*x + c*x^2))/(e + f*x)^(3/2), x)
```

3.843 $\int \frac{(d+ex)^{3/2}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx$

3.843.1 Optimal result

Integrand size = 38, antiderivative size = 240

$$\int \frac{(d+ex)^{3/2}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx=\frac{(bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\sqrt{d+ex}}{8b^4}+\frac{\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}(d+ex)^{3/2}}{12b^3}+\frac{(17bd-13ae)\sqrt{a+bx}(d+ex)^{5/2}}{3b^2}+\frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2}+\frac{(bd-ae)^2\left(73b^2d^2-90abde+35a^2e^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{8b^{9/2}\sqrt{e}}$$

output
```
2*e*(b*x+a)^(3/2)*(e*x+d)^(5/2)/b^2+1/8*(-a*e+b*d)^2*(35*a^2*e^2-90*a*b*d*
e+73*b^2*d^2)*arctanh(e^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(e*x+d)^(1/2))/b^(9/2)
/e^(1/2)+1/12*(35*a^2*e^2-90*a*b*d*e+73*b^2*d^2)*(e*x+d)^(3/2)*(b*x+a)^(1/
2)/b^3+1/3*(-13*a*e+17*b*d)*(e*x+d)^(5/2)*(b*x+a)^(1/2)/b^2+1/8*(-a*e+b*d)
*(35*a^2*e^2-90*a*b*d*e+73*b^2*d^2)*(b*x+a)^(1/2)*(e*x+d)^(1/2)/b^4
```

3.843.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^{3/2}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx=\frac{\sqrt{a+bx}\sqrt{d+ex}\left(-105a^3e^3+5a^2be^2(89d+14ex)-ab^2e(725d\right.}{}+\frac{(bd-ae)^2\left(73b^2d^2-90abde+35a^2e^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{8b^{9/2}\sqrt{e}}$$

input
```
Integrate[((d + e*x)^(3/2)*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],
x]
```

output
```
(Sqrt[a + b*x]*Sqrt[d + e*x]*(-105*a^3*e^3 + 5*a^2*b*e^2*(89*d + 14*e*x) -
 a*b^2*e*(725*d^2 + 292*d*e*x + 56*e^2*x^2) + b^3*(501*d^3 + 466*d^2*e*x +
 232*d*e^2*x^2 + 48*e^3*x^3)))/(24*b^4) + ((b*d - a*e)^2*(73*b^2*d^2 - 90*
a*b*d*e + 35*a^2*e^2)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[d + e*
x])])/(8*b^(9/2)*Sqrt[e])
```

3.843.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.84, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {1194, 27, 90, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2} \left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}} \, dx$$

$$\Big\downarrow \ 1194$$

$$\frac{\int \frac{4e(d+ex)^{3/2}(15b^2d^2-3abed-5a^2e^2+be(17bd-13ae)x)}{\sqrt{a+bx}}dx}{4b^2e}+\frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(d+ex)^{3/2}(15b^2d^2-3abed-5a^2e^2+be(17bd-13ae)x)}{\sqrt{a+bx}}dx}{b^2}+\frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2}$$

$$\Big\downarrow \ 90$$

$$\frac{\frac{1}{6}\left(35a^2e^2-90abde+73b^2d^2\right)\int \frac{(d+ex)^{3/2}}{\sqrt{a+bx}}dx+\frac{1}{3}\sqrt{a+bx}(d+ex)^{5/2}(17bd-13ae)}{b^2}+\frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2}$$

$$\Big\downarrow \ 60$$

$$\frac{\frac{\frac{1}{6}\left(35a^2e^2-90abde+73b^2d^2\right)\left(\frac{3(bd-ae)\int\frac{\sqrt{d+ex}}{\sqrt{a+bx}}dx}{4b}+\frac{\sqrt{a+bx}(d+ex)^{3/2}}{2b}\right)+\frac{1}{3}\sqrt{a+bx}(d+ex)^{5/2}(17bd-13ae)}{b^2}+}{\frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2}}$$

$\downarrow$ 60

$$\frac{\frac{\frac{1}{6}\left(35a^2e^2-90abde+73b^2d^2\right)\left(\frac{3(bd-ae)\left(\frac{(bd-ae)\int\frac{1}{\sqrt{a+bx}\sqrt{d+ex}}dx}{2b}+\frac{\sqrt{a+bx}\sqrt{d+ex}}{b}\right)}{4b}+\frac{\sqrt{a+bx}(d+ex)^{3/2}}{2b}\right)+\frac{1}{3}\sqrt{a+bx}(d+ex}{b^2}}{\frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2}}$$

$\downarrow$ 66

$$\frac{\frac{\frac{1}{6}\left(35a^2e^2-90abde+73b^2d^2\right)\left(\frac{3(bd-ae)\left(\frac{(bd-ae)\int\frac{1}{b-\frac{e(a+bx)}{d+ex}}d\frac{\sqrt{a+bx}}{\sqrt{d+ex}}}{b}+\frac{\sqrt{a+bx}\sqrt{d+ex}}{b}\right)}{4b}+\frac{\sqrt{a+bx}(d+ex)^{3/2}}{2b}\right)+\frac{1}{3}\sqrt{a+bx}(d+}{b^2}}{\frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2}}$$

$\downarrow$ 221

$$\frac{\frac{\frac{1}{6}\left(35a^2e^2-90abde+73b^2d^2\right)\left(\frac{3(bd-ae)\left(\frac{(bd-ae)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{3/2}\sqrt{e}}+\frac{\sqrt{a+bx}\sqrt{d+ex}}{b}\right)}{4b}+\frac{\sqrt{a+bx}(d+ex)^{3/2}}{2b}\right)+\frac{1}{3}\sqrt{a+bx}(d}{b^2}}{\frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2}}$$

input
```
Int[((d + e*x)^(3/2)*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x]
```

3.843. $\int \frac{(d+ex)^{3/2}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx$

output
```
(2*e*(a + b*x)^(3/2)*(d + e*x)^(5/2))/b^2 + (((17*b*d - 13*a*e)*Sqrt[a + b
*x]*(d + e*x)^(5/2))/3 + ((73*b^2*d^2 - 90*a*b*d*e + 35*a^2*e^2)*((Sqrt[a
+ b*x]*(d + e*x)^(3/2))/(2*b) + (3*(b*d - a*e)*((Sqrt[a + b*x]*Sqrt[d + e*
x])/b + ((b*d - a*e)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[d + e*x
])])/(b^(3/2)*Sqrt[e])))/(4*b)))/6)/b^2
```

3.843.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.843. $\int \frac{(d+ex)^{3/2}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx$

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

3.843.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(204) = 408$.

Time = 0.44 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.38

method	result
default	$\frac{\sqrt{ex+d}\,\sqrt{bx+a}\,\left(96b^3e^3x^3\sqrt{(bx+a)(ex+d)}\,\sqrt{be}-112a\,b^2e^3x^2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}+464b^3d\,e^2x^2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}+105\ln\left(\frac{2bex}{}\right.\right.}{}$

input
```
int((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
1/48*(e*x+d)^(1/2)*(b*x+a)^(1/2)*(96*b^3*e^3*x^3*((b*x+a)*(e*x+d))^(1/2)*(
b*e)^(1/2)-112*a*b^2*e^3*x^2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+464*b^3*d
*e^2*x^2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+105*ln(1/2*(2*b*e*x+2*((b*x+a
)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a^4*e^4-480*ln(1/2*(2*b
*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a^3*b*d*e
^3+864*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e
)^(1/2))*a^2*b^2*d^2*e^2-708*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*
e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a*b^3*d^3*e+219*ln(1/2*(2*b*e*x+2*((b*x+a)*
(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*b^4*d^4+140*((b*x+a)*(e*x
+d))^(1/2)*(b*e)^(1/2)*a^2*b*e^3*x-584*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)
*a*b^2*d*e^2*x+932*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)*b^3*d^2*e*x-210*((b
*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)*a^3*e^3+890*((b*x+a)*(e*x+d))^(1/2)*(b*e)
^(1/2)*a^2*b*d*e^2-1450*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)*a*b^2*d^2*e+10
02*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)*b^3*d^3)/b^4/((b*x+a)*(e*x+d))^(1/2
)/(b*e)^(1/2)
```

3.843.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 546, normalized size of antiderivative = 2.28

$$\int \frac{(d+ex)^{3/2}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx = \left[\frac{3\left(73\,b^4d^4-236\,ab^3d^3e+288\,a^2b^2d^2e^2-160\,a^3bde^3+35\,a^4e^4\right)}{}\right.$$

$$-\frac{3\left(73\,b^4d^4-236\,ab^3d^3e+288\,a^2b^2d^2e^2-160\,a^3bde^3+35\,a^4e^4\right)\sqrt{-be}\arctan\left(\frac{(2\,bex+bd+ae)\sqrt{-be}\sqrt{bx+a}\sqrt{ex+d}}{2\,(b^2e^2x^2+abde+(b^2de+abe^2)x)}\right)}{}$$

input
```
integrate((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algor
ithm="fricas")
```

output
```
[1/96*(3*(73*b^4*d^4 - 236*a*b^3*d^3*e + 288*a^2*b^2*d^2*e^2 - 160*a^3*b*d
*e^3 + 35*a^4*e^4)*sqrt(b*e)*log(8*b^2*e^2*x^2 + b^2*d^2 + 6*a*b*d*e + a^2
*e^2 + 4*(2*b*e*x + b*d + a*e)*sqrt(b*e)*sqrt(b*x + a)*sqrt(e*x + d) + 8*(
b^2*d*e + a*b*e^2)*x) + 4*(48*b^4*e^4*x^3 + 501*b^4*d^3*e - 725*a*b^3*d^2*
e^2 + 445*a^2*b^2*d*e^3 - 105*a^3*b*e^4 + 8*(29*b^4*d*e^3 - 7*a*b^3*e^4)*x
^2 + 2*(233*b^4*d^2*e^2 - 146*a*b^3*d*e^3 + 35*a^2*b^2*e^4)*x)*sqrt(b*x +
a)*sqrt(e*x + d))/(b^5*e), -1/48*(3*(73*b^4*d^4 - 236*a*b^3*d^3*e + 288*a^
2*b^2*d^2*e^2 - 160*a^3*b*d*e^3 + 35*a^4*e^4)*sqrt(-b*e)*arctan(1/2*(2*b*e
*x + b*d + a*e)*sqrt(-b*e)*sqrt(b*x + a)*sqrt(e*x + d)/(b^2*e^2*x^2 + a*b*
d*e + (b^2*d*e + a*b*e^2)*x)) - 2*(48*b^4*e^4*x^3 + 501*b^4*d^3*e - 725*a*
b^3*d^2*e^2 + 445*a^2*b^2*d*e^3 - 105*a^3*b*e^4 + 8*(29*b^4*d*e^3 - 7*a*b^
3*e^4)*x^2 + 2*(233*b^4*d^2*e^2 - 146*a*b^3*d*e^3 + 35*a^2*b^2*e^4)*x)*sqr
t(b*x + a)*sqrt(e*x + d))/(b^5*e)]
```

3.843.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}\cdot(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(8*e**2*x**2+20*d*e*x+15*d**2)/(b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x)**(3/2)*(15*d**2 + 20*d*e*x + 8*e**2*x**2)/sqrt(a + b*x)
, x)
```

3.843.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{3/2}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algor
ithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

3.843.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 739 vs. $2(204) = 408$.

Time = 0.37 (sec) , antiderivative size = 739, normalized size of antiderivative = 3.08

$$\int \frac{(d+ex)^{3/2}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx =$$

$$-\frac{\frac{360\left(\frac{\left(b^2d-abe\right)\log\left(\left|-\sqrt{be}\sqrt{bx+a}+\sqrt{b^2d+(bx+a)be-abe}\right|\right)}{\sqrt{be}}-\sqrt{b^2d+(bx+a)be-abe}\sqrt{bx+a}\right)d^3|b|}{b^2}-\frac{28\left(\sqrt{b^2d+(bx+a)be-abe}\sqrt{bx+a}\left(2\,(bx+a)\right.\right.}{}}{}$$

input
```
integrate((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algor
ithm="giac")
```

output
```
-1/24*(360*((b^2*d - a*b*e)*log(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt(b^2*d
+ (b*x + a)*b*e - a*b*e)))/sqrt(b*e) - sqrt(b^2*d + (b*x + a)*b*e - a*b*e)
*sqrt(b*x + a))*d^3*abs(b)/b^2 - 28*(sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*s
qrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*d*e^3 - 13*a*b^5*e^4)/(b
^7*e^4)) - 3*(b^7*d^2*e^2 + 2*a*b^6*d*e^3 - 11*a^2*b^5*e^4)/(b^7*e^4)) - 3
*(b^3*d^3 + a*b^2*d^2*e + 3*a^2*b*d*e^2 - 5*a^3*e^3)*log(abs(-sqrt(b*e)*sq
rt(b*x + a) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/(sqrt(b*e)*b*e^2))*d*e
^2*abs(b)/b^2 - (sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*(2*(b*x + a)*(4*(b*x
+ a)*(6*(b*x + a)/b^3 + (b^12*d*e^5 - 25*a*b^11*e^6)/(b^14*e^6)) - (5*b^13
*d^2*e^4 + 14*a*b^12*d*e^5 - 163*a^2*b^11*e^6)/(b^14*e^6)) + 3*(5*b^14*d^3
*e^3 + 9*a*b^13*d^2*e^4 + 15*a^2*b^12*d*e^5 - 93*a^3*b^11*e^6)/(b^14*e^6))
*sqrt(b*x + a) + 3*(5*b^4*d^4 + 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 + 20*a^3
*b*d*e^3 - 35*a^4*e^4)*log(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt(b^2*d + (b*
x + a)*b*e - a*b*e)))/(sqrt(b*e)*b^2*e^3))*e^3*abs(b)/b^2 - 210*(sqrt(b^2*
d + (b*x + a)*b*e - a*b*e)*(2*b*x + 2*a + (b*d*e - 5*a*e^2)/e^2)*sqrt(b*x
+ a) + (b^3*d^2 + 2*a*b^2*d*e - 3*a^2*b*e^2)*log(abs(-sqrt(b*e)*sqrt(b*x +
 a) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/(sqrt(b*e)*e))*d^2*e*abs(b)/b^
3)/b
```

3.843.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx=\int \frac{(d+e\,x)^{3/2}\,\left(15\,d^2+20\,d\,e\,x+8\,e^2\,x^2\right)}{\sqrt{a+b\,x}}\,dx$$

input
```
int(((d + e*x)^(3/2)*(15*d^2 + 8*e^2*x^2 + 20*d*e*x))/(a + b*x)^(1/2),x)
```

output
```
int(((d + e*x)^(3/2)*(15*d^2 + 8*e^2*x^2 + 20*d*e*x))/(a + b*x)^(1/2), x)
```

3.844 $\int \frac{\sqrt{d+ex}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx$

3.844.1 Optimal result

Integrand size = 38, antiderivative size = 176

$$\int \frac{\sqrt{d+ex}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx = \frac{(11b^2d^2-13abde+5a^2e^2)\sqrt{a+bx}\sqrt{d+ex}}{b^3}+\frac{2(4bd-3ae)\sqrt{a+bx}(d+ex)^{3/2}}{b^2}+\frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2}+\frac{(bd-ae)(11b^2d^2-13abde+5a^2e^2)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{7/2}\sqrt{e}}$$

output
```
8/3*e*(b*x+a)^(3/2)*(e*x+d)^(3/2)/b^2+(-a*e+b*d)*(5*a^2*e^2-13*a*b*d*e+11*
b^2*d^2)*arctanh(e^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(e*x+d)^(1/2))/b^(7/2)/e^(1
/2)+2*(-3*a*e+4*b*d)*(e*x+d)^(3/2)*(b*x+a)^(1/2)/b^2+(5*a^2*e^2-13*a*b*d*e
+11*b^2*d^2)*(b*x+a)^(1/2)*(e*x+d)^(1/2)/b^3
```

3.844.2 Mathematica [A] (verified)

Time = 10.34 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{d+ex}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx = \frac{\sqrt{d+ex}\left(\sqrt{a+bx}(15a^2e^2-abe(49d+10ex)+b^2(57d^2+32dex+8e^2x^2))+\frac{3\sqrt{bd-ae}(11b^2d^2-13abde+5a^2e^2)\text{a}}{\sqrt{e}\sqrt{\frac{b(d+ex)}{bd-ae}}}\right.}{3b^3}$$

input
```
Integrate[(Sqrt[d + e*x]*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x]
```

output
```
(Sqrt[d + e*x]*(Sqrt[a + b*x]*(15*a^2*e^2 - a*b*e*(49*d + 10*e*x) + b^2*(5
7*d^2 + 32*d*e*x + 8*e^2*x^2)) + (3*Sqrt[b*d - a*e]*(11*b^2*d^2 - 13*a*b*d
*e + 5*a^2*e^2)*ArcSinh[(Sqrt[e]*Sqrt[a + b*x])/Sqrt[b*d - a*e]])/(Sqrt[e]
*Sqrt[(b*(d + e*x))/(b*d - a*e)])))/(3*b^3)
```

3.844.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1194, 27, 90, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{1194}$$

$$\frac{\int \frac{3e\sqrt{d+ex}((3bd-2ae)(5bd+2ae)+4be(4bd-3ae)x)}{\sqrt{a+bx}}dx}{3b^2e}+\frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{d+ex}((3bd-2ae)(5bd+2ae)+4be(4bd-3ae)x)}{\sqrt{a+bx}}dx}{b^2}+\frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2}$$

$$\Big\downarrow \text{90}$$

$$\frac{\left(5a^2e^2-13abde+11b^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{a+bx}}dx+2\sqrt{a+bx}(d+ex)^{3/2}(4bd-3ae)}{b^2}+\frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{\left(5a^2e^2-13abde+11b^2d^2\right)\left(\frac{(bd-ae)\int \frac{1}{\sqrt{a+bx}\sqrt{d+ex}}dx}{2b}+\frac{\sqrt{a+bx}\sqrt{d+ex}}{b}\right)+2\sqrt{a+bx}(d+ex)^{3/2}(4bd-3ae)}{b^2}+\frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2}$$

$$\Big\downarrow \text{66}$$

$$\frac{\frac{\left(5a^2e^2-13abde+11b^2d^2\right)\left(\frac{(bd-ae)\int \frac{1}{b-\frac{e(a+bx)}{d+ex}}d\frac{\sqrt{a+bx}}{\sqrt{d+ex}}}{b}+\frac{\sqrt{a+bx}\sqrt{d+ex}}{b}\right)+2\sqrt{a+bx}(d+ex)^{3/2}(4bd-3ae)}{b^2}+\frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2}}{}$$

$\Big\downarrow$ 221

$$\frac{\left(5a^2e^2-13abde+11b^2d^2\right)\left(\frac{(bd-ae)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{3/2}\sqrt{e}}+\frac{\sqrt{a+bx}\sqrt{d+ex}}{b}\right)+2\sqrt{a+bx}(d+ex)^{3/2}(4bd-3ae)}{b^2}+\frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2}$$

input
```
Int[(Sqrt[d + e*x]*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x]
```

output
```
(8*e*(a + b*x)^(3/2)*(d + e*x)^(3/2))/(3*b^2) + (2*(4*b*d - 3*a*e)*Sqrt[a
+ b*x]*(d + e*x)^(3/2) + (11*b^2*d^2 - 13*a*b*d*e + 5*a^2*e^2)*((Sqrt[a +
b*x]*Sqrt[d + e*x])/b + ((b*d - a*e)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt
[b]*Sqrt[d + e*x])])/(b^(3/2)*Sqrt[e])))/b^2
```

3.844.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

3.844. $\int \frac{\sqrt{d+ex}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx$

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

3.844.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(150) = 300$.

Time = 0.47 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.23

method	result
default	$-\frac{\sqrt{ex+d}\,\sqrt{bx+a}\left(-16b^2e^2x^2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}+15\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}+ae+bd}{2\sqrt{be}}\right)a^3e^3-54\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex+d)}\,\sqrt{}}{2\sqrt{be}}\right.\right.}{}$

input
```
int((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x,method=_RETU
RNVERBOSE)
```

3.844. $\int \frac{\sqrt{d+ex}\,(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx$

output
```
-1/6*(e*x+d)^(1/2)*(b*x+a)^(1/2)*(-16*b^2*e^2*x^2*((b*x+a)*(e*x+d))^(1/2)*
(b*e)^(1/2)+15*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b
*d)/(b*e)^(1/2))*a^3*e^3-54*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e
)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a^2*b*d*e^2+72*ln(1/2*(2*b*e*x+2*((b*x+a)*(e
*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a*b^2*d^2*e-33*ln(1/2*(2*b*
e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*b^3*d^3+20
*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*a*b*e^2*x-64*(b*e)^(1/2)*((b*x+a)*(e*
x+d))^(1/2)*b^2*d*e*x-30*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*a^2*e^2+98*(b
*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*a*b*d*e-114*(b*e)^(1/2)*((b*x+a)*(e*x+d)
)^(1/2)*b^2*d^2)/b^3/((b*x+a)*(e*x+d))^(1/2)/(b*e)^(1/2)
```

3.844.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.35

$$\int \frac{\sqrt{d+ex}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx$$
$$= \left[-\frac{3\,(11\,b^3d^3-24\,ab^2d^2e+18\,a^2bde^2-5\,a^3e^3)\sqrt{be}\log\Big(8\,b^2e^2x^2+b^2d^2+6\,abde+a^2e^2-4\,(2\,bex+bd+\right.$$
$$-\frac{3\,(11\,b^3d^3-24\,ab^2d^2e+18\,a^2bde^2-5\,a^3e^3)\sqrt{-be}\arctan\left(\frac{(2\,bex+bd+ae)\sqrt{-be}\sqrt{bx+a}\sqrt{ex+d}}{2\,(b^2e^2x^2+abde+(b^2de+abe^2)x)}\right)-2\,(8\,b^3e^3x^2+}{6\,b^4e}$$

input
```
integrate((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algor
ithm="fricas")
```

output
```
[-1/12*(3*(11*b^3*d^3 - 24*a*b^2*d^2*e + 18*a^2*b*d*e^2 - 5*a^3*e^3)*sqrt(
b*e)*log(8*b^2*e^2*x^2 + b^2*d^2 + 6*a*b*d*e + a^2*e^2 - 4*(2*b*e*x + b*d
+ a*e)*sqrt(b*e)*sqrt(b*x + a)*sqrt(e*x + d) + 8*(b^2*d*e + a*b*e^2)*x) -
4*(8*b^3*e^3*x^2 + 57*b^3*d^2*e - 49*a*b^2*d*e^2 + 15*a^2*b*e^3 + 2*(16*b^
3*d*e^2 - 5*a*b^2*e^3)*x)*sqrt(b*x + a)*sqrt(e*x + d))/(b^4*e), -1/6*(3*(1
1*b^3*d^3 - 24*a*b^2*d^2*e + 18*a^2*b*d*e^2 - 5*a^3*e^3)*sqrt(-b*e)*arctan
(1/2*(2*b*e*x + b*d + a*e)*sqrt(-b*e)*sqrt(b*x + a)*sqrt(e*x + d)/(b^2*e^2
*x^2 + a*b*d*e + (b^2*d*e + a*b*e^2)*x)) - 2*(8*b^3*e^3*x^2 + 57*b^3*d^2*e
 - 49*a*b^2*d*e^2 + 15*a^2*b*e^3 + 2*(16*b^3*d*e^2 - 5*a*b^2*e^3)*x)*sqrt(
b*x + a)*sqrt(e*x + d))/(b^4*e)]
```

3.844.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx = \int \frac{\sqrt{d+ex}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(8*e**2*x**2+20*d*e*x+15*d**2)/(b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)*(15*d**2 + 20*d*e*x + 8*e**2*x**2)/sqrt(a + b*x), x
)
```

3.844.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algor
ithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.844.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(150) = 300$.

Time = 0.34 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.53

$$\int \frac{\sqrt{d+ex}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx = -\frac{\frac{45\left(\frac{\left(b^2d-abe\right)\log\left(\left|-\sqrt{be}\sqrt{bx+a}+\sqrt{b^2d+(bx+a)be-abe}\right|\right)}{\sqrt{be}}-\sqrt{b^2d+(bx+a)be-abe}\sqrt{bx+a}\right)d^2|b|}{b^2} - \frac{\left(\sqrt{b^2d+(bx+a)be-abe}\sqrt{bx+a}\left(2\,(bx+a)\left(\right.\right.\right.}{}}{}$$

input
```
integrate((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algor
ithm="giac")
```

output
```
-1/3*(45*((b^2*d - a*b*e)*log(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt(b^2*d +
(b*x + a)*b*e - a*b*e)))/sqrt(b*e) - sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*s
qrt(b*x + a))*d^2*abs(b)/b^2 - (sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*sqrt(b
*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*d*e^3 - 13*a*b^5*e^4)/(b^7*e^
4)) - 3*(b^7*d^2*e^2 + 2*a*b^6*d*e^3 - 11*a^2*b^5*e^4)/(b^7*e^4)) - 3*(b^3
*d^3 + a*b^2*d^2*e + 3*a^2*b*d*e^2 - 5*a^3*e^3)*log(abs(-sqrt(b*e)*sqrt(b*
x + a) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/(sqrt(b*e)*b*e^2))*e^2*abs(
b)/b^2 - 15*(sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*(2*b*x + 2*a + (b*d*e - 5
*a*e^2)/e^2)*sqrt(b*x + a) + (b^3*d^2 + 2*a*b^2*d*e - 3*a^2*b*e^2)*log(abs
(-sqrt(b*e)*sqrt(b*x + a) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/(sqrt(b*
e)*e))*d*e*abs(b)/b^3)/b
```

3.844.9 Mupad [B] (verification not implemented)

Time = 107.00 (sec) , antiderivative size = 1797, normalized size of antiderivative = 10.21

$$\int \frac{\sqrt{d+ex}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx = \text{Too large to display}$$

input
```
int(((d + e*x)^(1/2)*(15*d^2 + 8*e^2*x^2 + 20*d*e*x))/(a + b*x)^(1/2),x)
```

output
```
((((a + b*x)^(1/2) - a^(1/2))^3*(70*b^2*d^3 + 110*a^2*d*e^2 + 460*a*b*d^2*
e))/(e^3*((d + e*x)^(1/2) - d^(1/2))^3) + (((a + b*x)^(1/2) - a^(1/2))*(10
*b^3*d^3 + 20*a*b^2*d^2*e - 30*a^2*b*d*e^2))/(e^4*((d + e*x)^(1/2) - d^(1/
2))) - (160*a^(1/2)*d^(5/2)*((a + b*x)^(1/2) - a^(1/2))^6)/(e*((d + e*x)^(
1/2) - d^(1/2))^6) + (((a + b*x)^(1/2) - a^(1/2))^7*(10*b^2*d^3 - 30*a^2*d
*e^2 + 20*a*b*d^2*e))/(b^2*e*((d + e*x)^(1/2) - d^(1/2))^7) + (((a + b*x)^
(1/2) - a^(1/2))^5*(70*b^2*d^3 + 110*a^2*d*e^2 + 460*a*b*d^2*e))/(b*e^2*((
d + e*x)^(1/2) - d^(1/2))^5) - (a^(1/2)*d^(1/2)*(320*b*d^2 + 640*a*d*e)*((
a + b*x)^(1/2) - a^(1/2))^4)/(e^2*((d + e*x)^(1/2) - d^(1/2))^4) - (160*a^
(1/2)*b^2*d^(5/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(e^3*((d + e*x)^(1/2) - d
^(1/2))^2))/(((a + b*x)^(1/2) - a^(1/2))^8/((d + e*x)^(1/2) - d^(1/2))^8 +
 b^4/e^4 - (4*b^3*((a + b*x)^(1/2) - a^(1/2))^2)/(e^3*((d + e*x)^(1/2) - d
^(1/2))^2) + (6*b^2*((a + b*x)^(1/2) - a^(1/2))^4)/(e^2*((d + e*x)^(1/2) -
 d^(1/2))^4) - (4*b*((a + b*x)^(1/2) - a^(1/2))^6)/(e*((d + e*x)^(1/2) - d
^(1/2))^6)) - ((((a + b*x)^(1/2) - a^(1/2))*(2*b^5*d^3 - 10*a^3*b^2*e^3 +
6*a^2*b^3*d*e^2 + 2*a*b^4*d^2*e))/(e^6*((d + e*x)^(1/2) - d^(1/2))) - (((a
 + b*x)^(1/2) - a^(1/2))^5*(132*a^3*e^3 + 76*b^3*d^3 + 1100*a*b^2*d^2*e +
1252*a^2*b*d*e^2))/(e^4*((d + e*x)^(1/2) - d^(1/2))^5) - (((a + b*x)^(1/2)
 - a^(1/2))^3*((34*b^4*d^3)/3 - (170*a^3*b*e^3)/3 + 34*a^2*b^2*d*e^2 + 182
*a*b^3*d^2*e))/(e^5*((d + e*x)^(1/2) - d^(1/2))^3) + (((a + b*x)^(1/2) ...
```

3.844. $\int \frac{\sqrt{d+ex}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx$

3.845 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx$

3.845.1 Optimal result

Integrand size = 38, antiderivative size = 122

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx = \frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2}+\frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{2(8b^2d^2-8abde+3a^2e^2)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{5/2}\sqrt{e}}$$

output
```
2*(3*a^2*e^2-8*a*b*d*e+8*b^2*d^2)*arctanh(e^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(e
*x+d)^(1/2))/b^(5/2)/e^(1/2)+4*e*(b*x+a)^(3/2)*(e*x+d)^(1/2)/b^2+2*(-5*a*e
+7*b*d)*(b*x+a)^(1/2)*(e*x+d)^(1/2)/b^2
```

3.845.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.84

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx = \frac{2\sqrt{a+bx}\sqrt{d+ex}(7bd-3ae+2bex)}{b^2} + \frac{2(8b^2d^2-8abde+3a^2e^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{e}\sqrt{a+bx}}\right)}{b^{5/2}\sqrt{e}}$$

input
```
Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*Sqrt[d + e*x]),x]
```

output
```
(2*Sqrt[a + b*x]*Sqrt[d + e*x]*(7*b*d - 3*a*e + 2*b*e*x))/b^2 + (2*(8*b^2*
d^2 - 8*a*b*d*e + 3*a^2*e^2)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt
[a + b*x])])/(b^(5/2)*Sqrt[e])
```

3.845.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1194, 27, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1194}$$

$$\frac{\int \frac{2e(15b^2d^2-6abed-2a^2e^2+2be(7bd-5ae)x)}{\sqrt{a+bx}\sqrt{d+ex}}dx}{2b^2e}+\frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{15b^2d^2-6abed-2a^2e^2+2be(7bd-5ae)x}{\sqrt{a+bx}\sqrt{d+ex}}dx}{b^2}+\frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2}$$

$$\Big\downarrow \text{90}$$

$$\frac{(3a^2e^2-8abde+8b^2d^2)\int \frac{1}{\sqrt{a+bx}\sqrt{d+ex}}dx+2\sqrt{a+bx}\sqrt{d+ex}(7bd-5ae)}{b^2}+\frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2}$$

$$\Big\downarrow \text{66}$$

$$\frac{2(3a^2e^2-8abde+8b^2d^2)\int \frac{1}{b-\frac{e(a+bx)}{d+ex}}d\frac{\sqrt{a+bx}}{\sqrt{d+ex}}+2\sqrt{a+bx}\sqrt{d+ex}(7bd-5ae)}{b^2}+\frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2(3a^2e^2-8abde+8b^2d^2)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{\sqrt{b}\sqrt{e}}+2\sqrt{a+bx}\sqrt{d+ex}(7bd-5ae)}{b^2}+\frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2}$$

input
```
Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*Sqrt[d + e*x]),x]
```

output
```
(4*e*(a + b*x)^(3/2)*Sqrt[d + e*x])/b^2 + (2*(7*b*d - 5*a*e)*Sqrt[a + b*x]
*Sqrt[d + e*x] + (2*(8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)*ArcTanh[(Sqrt[e]*S
qrt[a + b*x])/(Sqrt[b]*Sqrt[d + e*x])])/(Sqrt[b]*Sqrt[e]))/b^2
```

3.845.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

3.845. $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx$

3.845.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(102) = 204$.

Time = 0.48 (sec), antiderivative size = 247, normalized size of antiderivative = 2.02

method	result
default	$\frac{\left(4\sqrt{(bx+a)(ex+d)}\,\sqrt{be}\,bex+3\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}+ae+bd}{2\sqrt{be}}\right)a^2e^2-8\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}+ae+bd}{2\sqrt{be}}\right)abde+8\ln\left(\frac{2bex}{}\right.\right.}{\sqrt{be}\,b^2\sqrt{(bx+a)(ex+d)}}$

input
```
int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
(4*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)*b*e*x+3*ln(1/2*(2*b*e*x+2*((b*x+a)*
(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a^2*e^2-8*ln(1/2*(2*b*e*x
+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a*b*d*e+8*ln(
1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*b
^2*d^2-6*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*a*e+14*(b*e)^(1/2)*((b*x+a)*(
e*x+d))^(1/2)*b*d)*(e*x+d)^(1/2)*(b*x+a)^(1/2)/(b*e)^(1/2)/b^2/((b*x+a)*(e
*x+d))^(1/2)
```

3.845.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec), antiderivative size = 308, normalized size of antiderivative = 2.52

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx$$
$$= \left[\frac{(8\,b^2d^2-8\,abde+3\,a^2e^2)\sqrt{be}\log\left(8\,b^2e^2x^2+b^2d^2+6\,abde+a^2e^2+4\,(2\,bex+bd+ae)\sqrt{be}\sqrt{bx+a}\sqrt{e}\right.}{2\,b^3e}\right.$$
$$-\frac{(8\,b^2d^2-8\,abde+3\,a^2e^2)\sqrt{-be}\arctan\left(\frac{(2\,bex+bd+ae)\sqrt{-be}\sqrt{bx+a}\sqrt{ex+d}}{2\,(b^2e^2x^2+abde+(b^2de+abe^2)x)}\right)-2\,(2\,b^2e^2x+7\,b^2de-3\,abe^2)\sqrt{b}}{b^3e}$$

input
```
integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x, algor
ithm="fricas")
```

output
```
[1/2*((8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)*sqrt(b*e)*log(8*b^2*e^2*x^2 + b^
2*d^2 + 6*a*b*d*e + a^2*e^2 + 4*(2*b*e*x + b*d + a*e)*sqrt(b*e)*sqrt(b*x +
 a)*sqrt(e*x + d) + 8*(b^2*d*e + a*b*e^2)*x) + 4*(2*b^2*e^2*x + 7*b^2*d*e
- 3*a*b*e^2)*sqrt(b*x + a)*sqrt(e*x + d))/(b^3*e), -((8*b^2*d^2 - 8*a*b*d*
e + 3*a^2*e^2)*sqrt(-b*e)*arctan(1/2*(2*b*e*x + b*d + a*e)*sqrt(-b*e)*sqrt
(b*x + a)*sqrt(e*x + d)/(b^2*e^2*x^2 + a*b*d*e + (b^2*d*e + a*b*e^2)*x)) -
 2*(2*b^2*e^2*x + 7*b^2*d*e - 3*a*b*e^2)*sqrt(b*x + a)*sqrt(e*x + d))/(b^3
*e)]
```

3.845.6 Sympy [F]

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}\sqrt{d + ex}}\, dx = \int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}\sqrt{d + ex}}\, dx$$

input
```
integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(1/2)/(b*x+a)**(1/2),x)
```

output
```
Integral((15*d**2 + 20*d*e*x + 8*e**2*x**2)/(sqrt(a + b*x)*sqrt(d + e*x)),
 x)
```

3.845.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}\sqrt{d + ex}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x, algor
ithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for m
ore detail
```

3.845.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.20

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx$$
$$= \frac{2\left(\sqrt{b^2d+(bx+a)be-abe}\sqrt{bx+a}\left(\frac{2\,(bx+a)e}{b^3}+\frac{7\,b^6de^2-5\,ab^5e^3}{b^8e^2}\right)-\frac{(8\,b^2d^2-8\,abde+3\,a^2e^2)\log\left(\left|-\sqrt{be}\sqrt{bx+a}+\sqrt{b^2d+(\cdots}{\sqrt{be}b^2}\right.\right.}{|b|}$$

input
```
integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x, algor
ithm="giac")
```

output
```
2*(sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*sqrt(b*x + a)*(2*(b*x + a)*e/b^3 +
(7*b^6*d*e^2 - 5*a*b^5*e^3)/(b^8*e^2)) - (8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^
2)*log(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e))
)/(sqrt(b*e)*b^2))*b/abs(b)
```

3.845.9 Mupad [B] (verification not implemented)

Time = 32.86 (sec) , antiderivative size = 893, normalized size of antiderivative = 7.32

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx$$
$$= \frac{\frac{(40\,b\,d^2+40\,a\,e\,d)\,(\sqrt{a+b\,x}-\sqrt{a})}{e^2\,(\sqrt{d+e\,x}-\sqrt{d})}-\frac{160\,\sqrt{a}\,d^{3/2}\,(\sqrt{a+b\,x}-\sqrt{a})^2}{e\,(\sqrt{d+e\,x}-\sqrt{d})^2}+\frac{(40\,b\,d^2+40\,a\,e\,d)\,(\sqrt{a+b\,x}-\sqrt{a})^3}{b\,e\,(\sqrt{d+e\,x}-\sqrt{d})^3}}{\frac{(\sqrt{a+b\,x}-\sqrt{a})^4}{(\sqrt{d+e\,x}-\sqrt{d})^4}+\frac{b^2}{e^2}-\frac{2\,b\,(\sqrt{a+b\,x}-\sqrt{a})^2}{e\,(\sqrt{d+e\,x}-\sqrt{d})^2}}$$
$$-\frac{\frac{(\sqrt{a+b\,x}-\sqrt{a})\,(12\,a^2\,b\,e^2+8\,a\,b^2\,d\,e+12\,b^3\,d^2)}{e^4\,(\sqrt{d+e\,x}-\sqrt{d})}-\frac{(\sqrt{a+b\,x}-\sqrt{a})^3\,(44\,a^2\,e^2+200\,a\,b\,d\,e+44\,b^2\,d^2)}{e^3\,(\sqrt{d+e\,x}-\sqrt{d})^3}+\frac{(\sqrt{a+b\,x}-\sqrt{a})^7\,(12\,a^2\,e^2+8\,a\,b\,d\,e+1\cdots}{b^2\,e\,(\sqrt{d+e\,x}-\sqrt{d})^7}}{\frac{(\sqrt{a+b\,x}-\sqrt{a})^8}{(\sqrt{d+e\,x}-\sqrt{d})^8}+\frac{b^4}{e^4}-\frac{4\,b^3\,(\sqrt{a+b\,x}-\sqrt{a})^2}{e^3\,(\sqrt{d+e\,x}-\sqrt{d})^2}+\frac{6\,b^2\,(\sqrt{a+b\,x}-\cdots}{e^2\,(\sqrt{d+e\,x}-\cdots}}$$
$$-\frac{60\,d^2\,\mathrm{atan}\left(\frac{b\,(\sqrt{d+e\,x}-\sqrt{d})}{\sqrt{-b\,e}\,(\sqrt{a+b\,x}-\sqrt{a})}\right)}{\sqrt{-b\,e}}-\frac{2\,\ln\left(\frac{\sqrt{e}\,(\sqrt{a+b\,x}-\sqrt{a})}{\sqrt{d+e\,x}-\sqrt{d}}-\sqrt{b}\right)\,(3\,a^2\,e^2+2\,a\,b\,d\,e+3\,b^2\,d^2)}{b^{5/2}\,\sqrt{e}}$$
$$+\frac{\ln\left(\sqrt{b}+\frac{\sqrt{e}\,(\sqrt{a+b\,x}-\sqrt{a})}{\sqrt{d+e\,x}-\sqrt{d}}\right)\,(6\,a^2\,e^2+4\,a\,b\,d\,e+6\,b^2\,d^2)}{b^{5/2}\,\sqrt{e}}$$
$$-\frac{40\,d\,\mathrm{atanh}\left(\frac{\sqrt{e}\,(\sqrt{a+b\,x}-\sqrt{a})}{\sqrt{b}\,(\sqrt{d+e\,x}-\sqrt{d})}\right)\,(a\,e+b\,d)}{b^{3/2}\,\sqrt{e}}$$

input

```
int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(1/2)),x)
```

output

```
(((40*b*d^2 + 40*a*d*e)*((a + b*x)^(1/2) - a^(1/2)))/(e^2*((d + e*x)^(1/2)
 - d^(1/2))) - (160*a^(1/2)*d^(3/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(e*((d
+ e*x)^(1/2) - d^(1/2))^2) + ((40*b*d^2 + 40*a*d*e)*((a + b*x)^(1/2) - a^(
1/2))^3)/(b*e*((d + e*x)^(1/2) - d^(1/2))^3))/(((a + b*x)^(1/2) - a^(1/2))
^4/((d + e*x)^(1/2) - d^(1/2))^4 + b^2/e^2 - (2*b*((a + b*x)^(1/2) - a^(1/
2))^2)/(e*((d + e*x)^(1/2) - d^(1/2))^2)) - ((((a + b*x)^(1/2) - a^(1/2))*
(12*b^3*d^2 + 12*a^2*b*e^2 + 8*a*b^2*d*e))/(e^4*((d + e*x)^(1/2) - d^(1/2)
)) - (((a + b*x)^(1/2) - a^(1/2))^3*(44*a^2*e^2 + 44*b^2*d^2 + 200*a*b*d*e
))/(e^3*((d + e*x)^(1/2) - d^(1/2))^3) + (((a + b*x)^(1/2) - a^(1/2))^7*(1
2*a^2*e^2 + 12*b^2*d^2 + 8*a*b*d*e))/(b^2*e*((d + e*x)^(1/2) - d^(1/2))^7)
 - (((a + b*x)^(1/2) - a^(1/2))^5*(44*a^2*e^2 + 44*b^2*d^2 + 200*a*b*d*e))
/(b*e^2*((d + e*x)^(1/2) - d^(1/2))^5) + (a^(1/2)*d^(1/2)*(256*a*e + 256*b
*d)*((a + b*x)^(1/2) - a^(1/2))^4)/(e^2*((d + e*x)^(1/2) - d^(1/2))^4))/((
(a + b*x)^(1/2) - a^(1/2))^8/((d + e*x)^(1/2) - d^(1/2))^8 + b^4/e^4 - (4*
b^3*((a + b*x)^(1/2) - a^(1/2))^2)/(e^3*((d + e*x)^(1/2) - d^(1/2))^2) + (
6*b^2*((a + b*x)^(1/2) - a^(1/2))^4)/(e^2*((d + e*x)^(1/2) - d^(1/2))^4) -
 (4*b*((a + b*x)^(1/2) - a^(1/2))^6)/(e*((d + e*x)^(1/2) - d^(1/2))^6)) -
(60*d^2*atan((b*((d + e*x)^(1/2) - d^(1/2)))/((-b*e)^(1/2)*((a + b*x)^(1/2
) - a^(1/2)))))/(-b*e)^(1/2) - (2*log((e^(1/2)*((a + b*x)^(1/2) - a^(1/2))
)/((d + e*x)^(1/2) - d^(1/2)) - b^(1/2))*(3*a^2*e^2 + 3*b^2*d^2 + 2*a*b...
```

3.845. $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx$

3.846 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx$

3.846.1 Optimal result

Integrand size = 38, antiderivative size = 108

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx = \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}} + \frac{8\sqrt{a+bx}\sqrt{d+ex}}{b} + \frac{8(2bd-ae)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{3/2}\sqrt{e}}$$

output
```
8*(-a*e+2*b*d)*arctanh(e^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(e*x+d)^(1/2))/b^(3/2
)/e^(1/2)+6*d^2*(b*x+a)^(1/2)/(-a*e+b*d)/(e*x+d)^(1/2)+8*(b*x+a)^(1/2)*(e*
x+d)^(1/2)/b
```

3.846.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.13

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{b}\sqrt{a+bx}(-4ae(d+ex)+bd(7d+4ex))}{\sqrt{d+ex}} + \frac{4(2b^2d^2-3abde+a^2e^2)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{\sqrt{e}}\right)}{b^{3/2}(bd-ae)}$$

input
```
Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(3/2)),
x]
```

output
```
(2*((Sqrt[b]*Sqrt[a + b*x]*(-4*a*e*(d + e*x) + b*d*(7*d + 4*e*x)))/Sqrt[d
+ e*x] + (4*(2*b^2*d^2 - 3*a*b*d*e + a^2*e^2)*ArcTanh[(Sqrt[e]*Sqrt[a + b*
x])/(Sqrt[b]*Sqrt[d + e*x])])/Sqrt[e]))/(b^(3/2)*(b*d - a*e))
```

3.846.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1193, 27, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{1193}$$

$$\frac{2\int \frac{2(bd-ae)(3d+2ex)}{\sqrt{a+bx}\sqrt{d+ex}}dx}{bd-ae}+\frac{6d^2\sqrt{a+bx}}{\sqrt{d+ex}(bd-ae)}$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{3d+2ex}{\sqrt{a+bx}\sqrt{d+ex}}dx+\frac{6d^2\sqrt{a+bx}}{\sqrt{d+ex}(bd-ae)}$$

$$\Big\downarrow \text{90}$$

$$4\left(\frac{(2bd-ae)\int \frac{1}{\sqrt{a+bx}\sqrt{d+ex}}dx}{b}+\frac{2\sqrt{a+bx}\sqrt{d+ex}}{b}\right)+\frac{6d^2\sqrt{a+bx}}{\sqrt{d+ex}(bd-ae)}$$

$$\Big\downarrow \text{66}$$

$$4\left(\frac{2(2bd-ae)\int \frac{1}{b-\frac{e(a+bx)}{d+ex}}d\frac{\sqrt{a+bx}}{\sqrt{d+ex}}}{b}+\frac{2\sqrt{a+bx}\sqrt{d+ex}}{b}\right)+\frac{6d^2\sqrt{a+bx}}{\sqrt{d+ex}(bd-ae)}$$

$$\Big\downarrow \text{221}$$

$$4\left(\frac{2(2bd-ae)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{3/2}\sqrt{e}}+\frac{2\sqrt{a+bx}\sqrt{d+ex}}{b}\right)+\frac{6d^2\sqrt{a+bx}}{\sqrt{d+ex}(bd-ae)}$$

input
```
Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(3/2)),x]
```

output
```
(6*d^2*Sqrt[a + b*x])/((b*d - a*e)*Sqrt[d + e*x]) + 4*((2*Sqrt[a + b*x]*Sq
rt[d + e*x])/b + (2*(2*b*d - a*e)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]
*Sqrt[d + e*x])])/(b^(3/2)*Sqrt[e]))
```

3.846.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

3.846.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(88) = 176$.

Time = 0.48 (sec) , antiderivative size = 438, normalized size of antiderivative = 4.06

method	result
default	$-\frac{2\sqrt{bx+a}\left(2\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}+ae+bd}{2\sqrt{be}}\right)a^2e^3x-6\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}+ae+bd}{2\sqrt{be}}\right)abd\,e^2x+4\ln\left(\frac{2bex+2\sqrt{(bx+a)(\cdots}{2\sqrt{\cdots}}\right.}{}$

input
```
int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
-2*(b*x+a)^(1/2)*(2*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+
a*e+b*d)/(b*e)^(1/2))*a^2*e^3*x-6*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2
)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a*b*d*e^2*x+4*ln(1/2*(2*b*e*x+2*((b*x+
a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*b^2*d^2*e*x+2*ln(1/2*(
2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a^2*d*
e^2-6*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)
^(1/2))*a*b*d^2*e+4*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+
a*e+b*d)/(b*e)^(1/2))*b^2*d^3-4*a*e^2*x*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2
)+4*b*d*e*x*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)-4*a*d*e*((b*x+a)*(e*x+d))^
(1/2)*(b*e)^(1/2)+7*b*d^2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/b/(b*e)^(1/
2)/(a*e-b*d)/((b*x+a)*(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

3.846.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(88) = 176$.

Time = 0.39 (sec) , antiderivative size = 463, normalized size of antiderivative = 4.29

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx = \left[-\frac{2\left((2\,b^2d^3-3\,abd^2e+a^2de^2+(2\,b^2d^2e-3\,abde^2+a^2e^3)x\right)\sqrt{be}\log\left(8\,b^2e\cdots\right.}{\cdots}\right.$$

$$-\frac{2\left(2\,(2\,b^2d^3-3\,abd^2e+a^2de^2+(2\,b^2d^2e-3\,abde^2+a^2e^3)x\right)\sqrt{-be}\arctan\left(\frac{(2\,bex+bd+ae)\sqrt{-be}\sqrt{bx+a}\sqrt{ex+d}}{2\,(b^2e^2x^2+abde+(b^2de+abe^2)x)}\right)-\cdots}{b^3d^2e-ab^2de^2+(b^3de^2-ab^2e^3)x}$$

input
```
integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x, algor
ithm="fricas")
```

output
```
[-2*((2*b^2*d^3 - 3*a*b*d^2*e + a^2*d*e^2 + (2*b^2*d^2*e - 3*a*b*d*e^2 + a
^2*e^3)*x)*sqrt(b*e)*log(8*b^2*e^2*x^2 + b^2*d^2 + 6*a*b*d*e + a^2*e^2 - 4
*(2*b*e*x + b*d + a*e)*sqrt(b*e)*sqrt(b*x + a)*sqrt(e*x + d) + 8*(b^2*d*e
+ a*b*e^2)*x) - (7*b^2*d^2*e - 4*a*b*d*e^2 + 4*(b^2*d*e^2 - a*b*e^3)*x)*sq
rt(b*x + a)*sqrt(e*x + d))/(b^3*d^2*e - a*b^2*d*e^2 + (b^3*d*e^2 - a*b^2*e
^3)*x), -2*(2*(2*b^2*d^3 - 3*a*b*d^2*e + a^2*d*e^2 + (2*b^2*d^2*e - 3*a*b*
d*e^2 + a^2*e^3)*x)*sqrt(-b*e)*arctan(1/2*(2*b*e*x + b*d + a*e)*sqrt(-b*e)
*sqrt(b*x + a)*sqrt(e*x + d)/(b^2*e^2*x^2 + a*b*d*e + (b^2*d*e + a*b*e^2)*
x)) - (7*b^2*d^2*e - 4*a*b*d*e^2 + 4*(b^2*d*e^2 - a*b*e^3)*x)*sqrt(b*x + a
)*sqrt(e*x + d))/(b^3*d^2*e - a*b^2*d*e^2 + (b^3*d*e^2 - a*b^2*e^3)*x)]
```

3.846.6 Sympy [F]

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx = \int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(3/2)/(b*x+a)**(1/2),x)
```

output
```
Integral((15*d**2 + 20*d*e*x + 8*e**2*x**2)/(sqrt(a + b*x)*(d + e*x)**(3/2
)), x)
```

3.846.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x, algor
ithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.846.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(88) = 176.

Time = 0.32 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.82

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx = \frac{2\sqrt{bx+a}\left(\frac{4\,(b^3de^3-ab^2e^4)\,(bx+a)}{b^3de^2|b|-ab^2e^3|b|}+\frac{7\,b^4d^2e^2-8\,ab^3de^3+4\,a^2b^2e^4}{b^3de^2|b|-ab^2e^3|b|}\right)}{\sqrt{b^2d+(bx+a)be-abe}} - \frac{8\,(2\,bd-ae)\log\left(\left|-\sqrt{be}\sqrt{bx+a}+\sqrt{b^2d+(bx+a)be-abe}\right|\right)}{\sqrt{be}|b|}$$

input
```
integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x, algor
ithm="giac")
```

output
```
2*sqrt(b*x + a)*(4*(b^3*d*e^3 - a*b^2*e^4)*(b*x + a)/(b^3*d*e^2*abs(b) - a
*b^2*e^3*abs(b)) + (7*b^4*d^2*e^2 - 8*a*b^3*d*e^3 + 4*a^2*b^2*e^4)/(b^3*d*
e^2*abs(b) - a*b^2*e^3*abs(b)))/sqrt(b^2*d + (b*x + a)*b*e - a*b*e) - 8*(2
*b*d - a*e)*log(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt(b^2*d + (b*x + a)*b*e
- a*b*e)))/(sqrt(b*e)*abs(b))
```

3.846.9 Mupad [F(-1)]

Timed out.

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx = \int \frac{15\,d^2+20\,d\,e\,x+8\,e^2\,x^2}{\sqrt{a+b\,x}\,{(d+e\,x)}^{3/2}}\,dx$$

input
```
int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(3/2)),x)
```

output
```
int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(3/2)), x)
```

3.847 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{5/2}}\,dx$

3.847.1 Optimal result

Integrand size = 38, antiderivative size = 116

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{5/2}}\,dx = \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}} + \frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}} + \frac{16\text{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{\sqrt{b}\sqrt{e}}$$

output
```
16*arctanh(e^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(e*x+d)^(1/2))/b^(1/2)/e^(1/2)+2*
d^2*(b*x+a)^(1/2)/(-a*e+b*d)/(e*x+d)^(3/2)+4*d*(-2*a*e+3*b*d)*(b*x+a)^(1/2
)/(-a*e+b*d)^2/(e*x+d)^(1/2)
```

3.847.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{5/2}}\,dx = \frac{2d\sqrt{a+bx}\left(7bd-4ae-\frac{de(a+bx)}{d+ex}\right)}{(bd-ae)^2\sqrt{d+ex}} + \frac{16\text{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{\sqrt{b}\sqrt{e}}$$

input
```
Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(5/2)),
x]
```

output
```
(2*d*Sqrt[a + b*x]*(7*b*d - 4*a*e - (d*e*(a + b*x))/(d + e*x)))/((b*d - a*
e)^2*Sqrt[d + e*x]) + (16*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[d
+ e*x])])/(Sqrt[b]*Sqrt[e])
```

3.847.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1193, 27, 87, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{1193}$$

$$\frac{2\int \frac{3(d(7bd-6ae)+4e(bd-ae)x)}{\sqrt{a+bx}(d+ex)^{3/2}}dx}{3(bd-ae)}+\frac{2d^2\sqrt{a+bx}}{(d+ex)^{3/2}(bd-ae)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{d(7bd-6ae)+4e(bd-ae)x}{\sqrt{a+bx}(d+ex)^{3/2}}dx}{bd-ae}+\frac{2d^2\sqrt{a+bx}}{(d+ex)^{3/2}(bd-ae)}$$

$$\Big\downarrow \text{87}$$

$$\frac{2\left(4(bd-ae)\int \frac{1}{\sqrt{a+bx}\sqrt{d+ex}}dx+\frac{2d\sqrt{a+bx}(3bd-2ae)}{\sqrt{d+ex}(bd-ae)}\right)}{bd-ae}+\frac{2d^2\sqrt{a+bx}}{(d+ex)^{3/2}(bd-ae)}$$

$$\Big\downarrow \text{66}$$

$$\frac{2\left(8(bd-ae)\int \frac{1}{b-\frac{e(a+bx)}{d+ex}}d\frac{\sqrt{a+bx}}{\sqrt{d+ex}}+\frac{2d\sqrt{a+bx}(3bd-2ae)}{\sqrt{d+ex}(bd-ae)}\right)}{bd-ae}+\frac{2d^2\sqrt{a+bx}}{(d+ex)^{3/2}(bd-ae)}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\left(\frac{8(bd-ae)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{\sqrt{b}\sqrt{e}}+\frac{2d\sqrt{a+bx}(3bd-2ae)}{\sqrt{d+ex}(bd-ae)}\right)}{bd-ae}+\frac{2d^2\sqrt{a+bx}}{(d+ex)^{3/2}(bd-ae)}$$

input
```
Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(5/2)),x]
```

output
```
(2*d^2*Sqrt[a + b*x])/((b*d - a*e)*(d + e*x)^(3/2)) + (2*((2*d*(3*b*d - 2*
a*e)*Sqrt[a + b*x])/((b*d - a*e)*Sqrt[d + e*x]) + (8*(b*d - a*e)*ArcTanh[(
Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[d + e*x])])/(Sqrt[b]*Sqrt[e])))/(b*d
- a*e)
```

3.847.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

3.847. $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{5/2}}\,dx$

3.847.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(96) = 192.

Time = 0.48 (sec) , antiderivative size = 601, normalized size of antiderivative = 5.18

method	result
default	$\frac{2\sqrt{bx+a}\left(4\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}+ae+bd}{2\sqrt{be}}\right)a^2e^4x^2-8\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}+ae+bd}{2\sqrt{be}}\right)abd\,e^3x^2+4\ln\left(\frac{2bex+2\sqrt{(bx+a)(e}{2\sqrt{}}\right.\right.}{}$

input
```
int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
2*(b*x+a)^(1/2)*(4*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a
*e+b*d)/(b*e)^(1/2))*a^2*e^4*x^2-8*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/
2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a*b*d*e^3*x^2+4*ln(1/2*(2*b*e*x+2*((b
*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*b^2*d^2*e^2*x^2+8*l
n(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))
*a^2*d*e^3*x-16*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+
b*d)/(b*e)^(1/2))*a*b*d^2*e^2*x+8*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2
)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*b^2*d^3*e*x+4*ln(1/2*(2*b*e*x+2*((b*x+
a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a^2*d^2*e^2-8*ln(1/2*(
2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a*b*d^
3*e+4*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)
^(1/2))*b^2*d^4-4*a*d*e^2*x*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+6*b*d^2*e*
x*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)-5*a*d^2*e*((b*x+a)*(e*x+d))^(1/2)*(b
*e)^(1/2)+7*b*d^3*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2)/(a*e-b*
d)^2/((b*x+a)*(e*x+d))^(1/2)/(e*x+d)^(3/2)
```

3.847.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(96) = 192.

Time = 0.64 (sec) , antiderivative size = 665, normalized size of antiderivative = 5.73

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{5/2}}\,dx = \left[\frac{2\left(2\left(b^2d^4-2\,abd^3e+a^2d^2e^2+(b^2d^2e^2-2\,abde^3+a^2e^4)x^2+2\,(b^2d^3e-2\right.\right.}{}\right.$$

$$-\frac{2\left(4\left(b^2d^4-2\,abd^3e+a^2d^2e^2+(b^2d^2e^2-2\,abde^3+a^2e^4)x^2+2\,(b^2d^3e-2\,abd^2e^2+a^2de^3)x\right)\sqrt{-be}\,\arctan\right.}{b^3d^4e-2\,ab^2d^3e^2+a^2bd^2e^3+(b^3d^2e^3-2\,ab^2de^4+}$$

input
```
integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x, algor
ithm="fricas")
```

output
```
[2*(2*(b^2*d^4 - 2*a*b*d^3*e + a^2*d^2*e^2 + (b^2*d^2*e^2 - 2*a*b*d*e^3 +
a^2*e^4)*x^2 + 2*(b^2*d^3*e - 2*a*b*d^2*e^2 + a^2*d*e^3)*x)*sqrt(b*e)*log(
8*b^2*e^2*x^2 + b^2*d^2 + 6*a*b*d*e + a^2*e^2 + 4*(2*b*e*x + b*d + a*e)*sq
rt(b*e)*sqrt(b*x + a)*sqrt(e*x + d) + 8*(b^2*d*e + a*b*e^2)*x) + (7*b^2*d^
3*e - 5*a*b*d^2*e^2 + 2*(3*b^2*d^2*e^2 - 2*a*b*d*e^3)*x)*sqrt(b*x + a)*sqr
t(e*x + d))/(b^3*d^4*e - 2*a*b^2*d^3*e^2 + a^2*b*d^2*e^3 + (b^3*d^2*e^3 -
2*a*b^2*d*e^4 + a^2*b*e^5)*x^2 + 2*(b^3*d^3*e^2 - 2*a*b^2*d^2*e^3 + a^2*b*
d*e^4)*x), -2*(4*(b^2*d^4 - 2*a*b*d^3*e + a^2*d^2*e^2 + (b^2*d^2*e^2 - 2*a
*b*d*e^3 + a^2*e^4)*x^2 + 2*(b^2*d^3*e - 2*a*b*d^2*e^2 + a^2*d*e^3)*x)*sqr
t(-b*e)*arctan(1/2*(2*b*e*x + b*d + a*e)*sqrt(-b*e)*sqrt(b*x + a)*sqrt(e*x
 + d)/(b^2*e^2*x^2 + a*b*d*e + (b^2*d*e + a*b*e^2)*x)) - (7*b^2*d^3*e - 5*
a*b*d^2*e^2 + 2*(3*b^2*d^2*e^2 - 2*a*b*d*e^3)*x)*sqrt(b*x + a)*sqrt(e*x +
d))/(b^3*d^4*e - 2*a*b^2*d^3*e^2 + a^2*b*d^2*e^3 + (b^3*d^2*e^3 - 2*a*b^2*
d*e^4 + a^2*b*e^5)*x^2 + 2*(b^3*d^3*e^2 - 2*a*b^2*d^2*e^3 + a^2*b*d*e^4)*x
)]
```

3.847.6 Sympy [F]

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}(d + ex)^{5/2}}\,dx = \int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}\,(d + ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(5/2)/(b*x+a)**(1/2),x)
```

output
```
Integral((15*d**2 + 20*d*e*x + 8*e**2*x**2)/(sqrt(a + b*x)*(d + e*x)**(5/2
)), x)
```

3.847.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x, algor
ithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.847.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(96) = 192$.

Time = 0.35 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.90

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{5/2}}\,dx = -\frac{16\,b\log\left(\left|-\sqrt{be}\sqrt{bx+a}+\sqrt{b^2d+(bx+a)be-abe}\right|\right)}{\sqrt{be}|b|} + \frac{2\sqrt{bx+a}\left(\frac{2\left(3\,b^6d^2e^2-2\,ab^5de^3\right)(bx+a)}{b^4d^2e|b|-2\,ab^3de^2|b|+a^2b^2e^3|b|}+\frac{7\,b^7d^3e-11\,ab^6d^2e^2+4\,a^2b^5de^3}{b^4d^2e|b|-2\,ab^3de^2|b|+a^2b^2e^3|b|}\right)}{\left(b^2d+(bx+a)be-abe\right)^{\frac{3}{2}}}$$

input
```
integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x, algor
ithm="giac")
```

output
```
-16*b*log(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt(b^2*d + (b*x + a)*b*e - a*b*
e)))/(sqrt(b*e)*abs(b)) + 2*sqrt(b*x + a)*(2*(3*b^6*d^2*e^2 - 2*a*b^5*d*e^
3)*(b*x + a)/(b^4*d^2*e*abs(b) - 2*a*b^3*d*e^2*abs(b) + a^2*b^2*e^3*abs(b)
) + (7*b^7*d^3*e - 11*a*b^6*d^2*e^2 + 4*a^2*b^5*d*e^3)/(b^4*d^2*e*abs(b) -
 2*a*b^3*d*e^2*abs(b) + a^2*b^2*e^3*abs(b)))/(b^2*d + (b*x + a)*b*e - a*b*
e)^(3/2)
```

3.847.9 Mupad [F(-1)]

Timed out.

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}(d + ex)^{5/2}}\,dx = \int \frac{15\,d^2 + 20\,d\,e\,x + 8\,e^2\,x^2}{\sqrt{a + b\,x}\,{\left(d + e\,x\right)}^{5/2}}\,dx$$

input
```
int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(5/2)),x)
```

output
```
int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(5/2)), x)
```

3.848 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{7/2}}\,dx$

3.848.1 Optimal result

Integrand size = 38, antiderivative size = 133

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{7/2}}\,dx=\frac{6d^2\sqrt{a+bx}}{5(bd-ae)(d+ex)^{5/2}}+\frac{8d(8bd-5ae)\sqrt{a+bx}}{15(bd-ae)^2(d+ex)^{3/2}}+\frac{16(23b^2d^2-35abde+15a^2e^2)\sqrt{a+bx}}{15(bd-ae)^3\sqrt{d+ex}}$$

output
```
6/5*d^2*(b*x+a)^(1/2)/(-a*e+b*d)/(e*x+d)^(5/2)+8/15*d*(-5*a*e+8*b*d)*(b*x+
a)^(1/2)/(-a*e+b*d)^2/(e*x+d)^(3/2)+16/15*(15*a^2*e^2-35*a*b*d*e+23*b^2*d^
2)*(b*x+a)^(1/2)/(-a*e+b*d)^3/(e*x+d)^(1/2)
```

3.848.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{7/2}}\,dx=\frac{2\sqrt{a+bx}\left(225b^2d^2-300abde+120a^2e^2+\frac{9d^2e^2(a+bx)^2}{(d+ex)^2}-\frac{50bd^2e(a+bx)}{d+ex}+\cdots\right.}{15(bd-ae)^3\sqrt{d+ex}}$$

input
```
Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(7/2)),
x]
```

output
```
(2*Sqrt[a + b*x]*(225*b^2*d^2 - 300*a*b*d*e + 120*a^2*e^2 + (9*d^2*e^2*(a
+ b*x)^2)/(d + e*x)^2 - (50*b*d^2*e*(a + b*x))/(d + e*x) + (20*a*d*e^2*(a
+ b*x))/(d + e*x)))/(15*(b*d - a*e)^3*Sqrt[d + e*x])
```

3.848.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1193, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{7/2}}dx$$

$$\Big\downarrow \text{1193}$$

$$\frac{2\int \frac{2(3d(6bd-5ae)+10e(bd-ae)x)}{\sqrt{a+bx}(d+ex)^{5/2}}dx}{5(bd-ae)}+\frac{6d^2\sqrt{a+bx}}{5(d+ex)^{5/2}(bd-ae)}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{3d(6bd-5ae)+10e(bd-ae)x}{\sqrt{a+bx}(d+ex)^{5/2}}dx}{5(bd-ae)}+\frac{6d^2\sqrt{a+bx}}{5(d+ex)^{5/2}(bd-ae)}$$

$$\Big\downarrow \text{87}$$

$$\frac{4\left(\frac{2(15a^2e^2-35abde+23b^2d^2)\int \frac{1}{\sqrt{a+bx}(d+ex)^{3/2}}dx}{3(bd-ae)}+\frac{2d\sqrt{a+bx}(8bd-5ae)}{3(d+ex)^{3/2}(bd-ae)}\right)}{5(bd-ae)}+\frac{6d^2\sqrt{a+bx}}{5(d+ex)^{5/2}(bd-ae)}$$

$$\Big\downarrow \text{48}$$

$$\frac{4\left(\frac{4\sqrt{a+bx}(15a^2e^2-35abde+23b^2d^2)}{3\sqrt{d+ex}(bd-ae)^2}+\frac{2d\sqrt{a+bx}(8bd-5ae)}{3(d+ex)^{3/2}(bd-ae)}\right)}{5(bd-ae)}+\frac{6d^2\sqrt{a+bx}}{5(d+ex)^{5/2}(bd-ae)}$$

input
```
Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(7/2)),x]
```

output
```
(6*d^2*Sqrt[a + b*x])/(5*(b*d - a*e)*(d + e*x)^(5/2)) + (4*((2*d*(8*b*d -
5*a*e)*Sqrt[a + b*x])/(3*(b*d - a*e)*(d + e*x)^(3/2)) + (4*(23*b^2*d^2 - 3
5*a*b*d*e + 15*a^2*e^2)*Sqrt[a + b*x])/(3*(b*d - a*e)^2*Sqrt[d + e*x])))/(
5*(b*d - a*e))
```

3.848.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

3.848.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92

method	result	si
default	$-\frac{2 \sqrt{b x+a}\, \left(120 a^{2} e^{4} x^{2}-280 a b d\, e^{3} x^{2}+184 b^{2} d^{2} e^{2} x^{2}+260 a^{2} d\, e^{3} x -612 a b\, d^{2} e^{2} x +400 b^{2} d^{3} e x +149 a^{2} d^{2} e^{2}-350 a b\, d^{3} e +225 b^{2} d^{4}\right)}{15 \left(e x +d \right)^{\frac{5}{2}} \left(a e -b d \right)^{3}}$	12
gosper	$-\frac{2 \sqrt{b x+a}\, \left(120 a^{2} e^{4} x^{2}-280 a b d\, e^{3} x^{2}+184 b^{2} d^{2} e^{2} x^{2}+260 a^{2} d\, e^{3} x -612 a b\, d^{2} e^{2} x +400 b^{2} d^{3} e x +149 a^{2} d^{2} e^{2}-350 a b\, d^{3} e +225 b^{2} d^{4}\right)}{15 \left(e x +d \right)^{\frac{5}{2}} \left(a^{3} e^{3}-3 a^{2} b d\, e^{2}+3 a\, b^{2} d^{2} e -b^{3} d^{3}\right)}$	15

input
```
int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
-2/15*(b*x+a)^(1/2)*(120*a^2*e^4*x^2-280*a*b*d*e^3*x^2+184*b^2*d^2*e^2*x^2
+260*a^2*d*e^3*x-612*a*b*d^2*e^2*x+400*b^2*d^3*e*x+149*a^2*d^2*e^2-350*a*b
*d^3*e+225*b^2*d^4)/(e*x+d)^(5/2)/(a*e-b*d)^3
```

3.848.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(115) = 230$.

Time = 0.90 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.20

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{7/2}}\,dx = \frac{2\,(225\,b^2d^4 - 350\,abd^3e + 149\,a^2d^2e^2 + 8\,(23\,b^2d^2e^2 - 3}{15\,(b^3d^6 - 3\,ab^2d^5e + 3\,a^2bd^4e^2 - a^3d^3e^3 + (b^3d^3e^3 - 3\,ab^2d^2e^4 + 3\,a^2bde^5 - }$$

input
```
integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x, algor
ithm="fricas")
```

output
```
2/15*(225*b^2*d^4 - 350*a*b*d^3*e + 149*a^2*d^2*e^2 + 8*(23*b^2*d^2*e^2 -
35*a*b*d*e^3 + 15*a^2*e^4)*x^2 + 4*(100*b^2*d^3*e - 153*a*b*d^2*e^2 + 65*a
^2*d*e^3)*x)*sqrt(b*x + a)*sqrt(e*x + d)/(b^3*d^6 - 3*a*b^2*d^5*e + 3*a^2*
b*d^4*e^2 - a^3*d^3*e^3 + (b^3*d^3*e^3 - 3*a*b^2*d^2*e^4 + 3*a^2*b*d*e^5 -
 a^3*e^6)*x^3 + 3*(b^3*d^4*e^2 - 3*a*b^2*d^3*e^3 + 3*a^2*b*d^2*e^4 - a^3*d
*e^5)*x^2 + 3*(b^3*d^5*e - 3*a*b^2*d^4*e^2 + 3*a^2*b*d^3*e^3 - a^3*d^2*e^4
)*x)
```

3.848.6 Sympy [F]

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{7/2}}\,dx = \int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(7/2)/(b*x+a)**(1/2),x)
```

output
```
Integral((15*d**2 + 20*d*e*x + 8*e**2*x**2)/(sqrt(a + b*x)*(d + e*x)**(7/2
)), x)
```

3.848.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x, algor
ithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e-b*d)>0)', see `assume?` f
or more de
```

3.848.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(115) = 230$.

Time = 0.38 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.70

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{7/2}}\,dx = \frac{2\left(4\,(bx+a)\left(\frac{2\,(23\,b^8d^2e^4-35\,ab^7de^5+15\,a^2b^6e^6)(bx+a)}{b^5d^3e^2|b|-3\,ab^4d^2e^3|b|+3\,a^2b^3de^4|b|-a^3b^2e^5|b|} + \frac{5\,(20\,b^9d^3e^3-49\,ab^8d^2e^4+41\,a^2b^7}{b^5d^3e^2|b|-3\,ab^4d^2e^3|b|+3\,a^2b^3d}\right.\right.}{15\,(b^2d+(bx+}$$

input
```
integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x, algor
ithm="giac")
```

output
```
2/15*(4*(b*x + a)*(2*(23*b^8*d^2*e^4 - 35*a*b^7*d*e^5 + 15*a^2*b^6*e^6)*(b
*x + a)/(b^5*d^3*e^2*abs(b) - 3*a*b^4*d^2*e^3*abs(b) + 3*a^2*b^3*d*e^4*abs
(b) - a^3*b^2*e^5*abs(b)) + 5*(20*b^9*d^3*e^3 - 49*a*b^8*d^2*e^4 + 41*a^2*
b^7*d*e^5 - 12*a^3*b^6*e^6)/(b^5*d^3*e^2*abs(b) - 3*a*b^4*d^2*e^3*abs(b) +
 3*a^2*b^3*d*e^4*abs(b) - a^3*b^2*e^5*abs(b))) + 15*(15*b^10*d^4*e^2 - 50*
a*b^9*d^3*e^3 + 63*a^2*b^8*d^2*e^4 - 36*a^3*b^7*d*e^5 + 8*a^4*b^6*e^6)/(b^
5*d^3*e^2*abs(b) - 3*a*b^4*d^2*e^3*abs(b) + 3*a^2*b^3*d*e^4*abs(b) - a^3*b
^2*e^5*abs(b)))*sqrt(b*x + a)/(b^2*d + (b*x + a)*b*e - a*b*e)^(5/2)
```

3.848.9 Mupad [B] (verification not implemented)

Time = 13.36 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.02

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{7/2}}\,dx =$$

$$-\frac{\sqrt{d+e\,x}\,\left(\frac{x^2\,\left(240\,a^3\,e^4-40\,a^2\,b\,d\,e^3-856\,a\,b^2\,d^2\,e^2+800\,b^3\,d^3\,e\right)}{15\,e^3\,{\left(a\,e-b\,d\right)}^3}+\frac{x\,\left(520\,a^3\,d\,e^3-926\,a^2\,b\,d^2\,e^2+100\,a\,b^2\,d^3\,e+450\,b^3\,d^4\right)}{15\,e^3\,{\left(a\,e-b\,d\right)}^3}+\frac{2\,a\,d^2\,\left(149\,a^2\right.}{1}\right.}{x^3\,\sqrt{a+b\,x}+\frac{d^3\,\sqrt{a+b\,x}}{e^3}+\frac{3\,d\,x^2\,\sqrt{a+b\,x}}{e}+\frac{3\,d^2\,x\,\sqrt{a+b\,x}}{e^2}}$$

input

```
int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(7/2)),x)
```

output

```
-((d + e*x)^(1/2)*((x^2*(240*a^3*e^4 + 800*b^3*d^3*e - 856*a*b^2*d^2*e^2 -
 40*a^2*b*d*e^3))/(15*e^3*(a*e - b*d)^3) + (x*(450*b^3*d^4 + 520*a^3*d*e^3
 - 926*a^2*b*d^2*e^2 + 100*a*b^2*d^3*e))/(15*e^3*(a*e - b*d)^3) + (2*a*d^2
*(149*a^2*e^2 + 225*b^2*d^2 - 350*a*b*d*e))/(15*e^3*(a*e - b*d)^3) + (16*b
*x^3*(15*a^2*e^2 + 23*b^2*d^2 - 35*a*b*d*e))/(15*e*(a*e - b*d)^3)))/(x^3*(
a + b*x)^(1/2) + (d^3*(a + b*x)^(1/2))/e^3 + (3*d*x^2*(a + b*x)^(1/2))/e +
 (3*d^2*x*(a + b*x)^(1/2))/e^2)
```

3.849 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{9/2}}\,dx$

3.849.1 Optimal result

Integrand size = 38, antiderivative size = 189

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{9/2}}\,dx = \frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}}+\frac{4d(23bd-14ae)\sqrt{a+bx}}{35(bd-ae)^2(d+ex)^{5/2}} + \frac{16(58b^2d^2-84abde+35a^2e^2)\sqrt{a+bx}}{105(bd-ae)^3(d+ex)^{3/2}}+\frac{32b(58b^2d^2-84abde+35a^2e^2)\sqrt{a+bx}}{105(bd-ae)^4\sqrt{d+ex}}$$

output
```
6/7*d^2*(b*x+a)^(1/2)/(-a*e+b*d)/(e*x+d)^(7/2)+4/35*d*(-14*a*e+23*b*d)*(b*
x+a)^(1/2)/(-a*e+b*d)^2/(e*x+d)^(5/2)+16/105*(35*a^2*e^2-84*a*b*d*e+58*b^2
*d^2)*(b*x+a)^(1/2)/(-a*e+b*d)^3/(e*x+d)^(3/2)+32/105*b*(35*a^2*e^2-84*a*b
*d*e+58*b^2*d^2)*(b*x+a)^(1/2)/(-a*e+b*d)^4/(e*x+d)^(1/2)
```

3.849.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.98

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{9/2}}\,dx = \frac{2\sqrt{a+bx}\left(1575b^3d^2-2100ab^2de+840a^2be^2-\frac{45d^2e^3(a+bx)^3}{(d+ex)^3}+\frac{273bd^2e^2(a+bx)^2}{(d+ex)^2}\right.}{105(bd-ae)^4\sqrt{d+}}$$

input
```
Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(9/2)),
x]
```

output
```
(2*Sqrt[a + b*x]*(1575*b^3*d^2 - 2100*a*b^2*d*e + 840*a^2*b*e^2 - (45*d^2*
e^3*(a + b*x)^3)/(d + e*x)^3 + (273*b*d^2*e^2*(a + b*x)^2)/(d + e*x)^2 - (
84*a*d*e^3*(a + b*x)^2)/(d + e*x)^2 - (875*b^2*d^2*e*(a + b*x))/(d + e*x)
+ (840*a*b*d*e^2*(a + b*x))/(d + e*x) - (280*a^2*e^3*(a + b*x))/(d + e*x))
)/(105*(b*d - a*e)^4*Sqrt[d + e*x])
```

3.849.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1193, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{9/2}}dx$$

$$\Big\downarrow \text{1193}$$

$$\frac{2\int \frac{3d(17bd-14ae)+28e(bd-ae)x}{\sqrt{a+bx}(d+ex)^{7/2}}dx}{7(bd-ae)}+\frac{6d^2\sqrt{a+bx}}{7(d+ex)^{7/2}(bd-ae)}$$

$$\Big\downarrow \text{87}$$

$$\frac{2\left(\frac{4(35a^2e^2-84abde+58b^2d^2)\int \frac{1}{\sqrt{a+bx}(d+ex)^{5/2}}dx}{5(bd-ae)}+\frac{2d\sqrt{a+bx}(23bd-14ae)}{5(d+ex)^{5/2}(bd-ae)}\right)}{7(bd-ae)}+\frac{6d^2\sqrt{a+bx}}{7(d+ex)^{7/2}(bd-ae)}$$

$$\Big\downarrow \text{55}$$

$$\frac{2\left(\frac{4(35a^2e^2-84abde+58b^2d^2)\left(\frac{2b\int \frac{1}{\sqrt{a+bx}(d+ex)^{3/2}}dx}{3(bd-ae)}+\frac{2\sqrt{a+bx}}{3(d+ex)^{3/2}(bd-ae)}\right)}{5(bd-ae)}+\frac{2d\sqrt{a+bx}(23bd-14ae)}{5(d+ex)^{5/2}(bd-ae)}\right)}{7(bd-ae)}+\frac{6d^2\sqrt{a+bx}}{7(d+ex)^{7/2}(bd-ae)}$$

$$\Big\downarrow \text{48}$$

$$\frac{2\left(\frac{4(35a^2e^2-84abde+58b^2d^2)\left(\frac{4b\sqrt{a+bx}}{3\sqrt{d+ex}(bd-ae)^2}+\frac{2\sqrt{a+bx}}{3(d+ex)^{3/2}(bd-ae)}\right)}{5(bd-ae)}+\frac{2d\sqrt{a+bx}(23bd-14ae)}{5(d+ex)^{5/2}(bd-ae)}\right)}{7(bd-ae)}+\frac{6d^2\sqrt{a+bx}}{7(d+ex)^{7/2}(bd-ae)}$$

input
```
Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(9/2)),x]
```

output
```
(6*d^2*Sqrt[a + b*x])/(7*(b*d - a*e)*(d + e*x)^(7/2)) + (2*((2*d*(23*b*d -
 14*a*e)*Sqrt[a + b*x])/(5*(b*d - a*e)*(d + e*x)^(5/2)) + (4*(58*b^2*d^2 -
 84*a*b*d*e + 35*a^2*e^2)*((2*Sqrt[a + b*x])/(3*(b*d - a*e)*(d + e*x)^(3/2
)) + (4*b*Sqrt[a + b*x])/(3*(b*d - a*e)^2*Sqrt[d + e*x])))/(5*(b*d - a*e))
))/(7*(b*d - a*e))
```

3.849.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

3.849. $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{9/2}}\,dx$

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

3.849.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.10

method	result
default	$-\frac{2\sqrt{bx+a}\,(-560a^2b\,e^5x^3+1344a\,b^2d\,e^4x^3-928b^3d^2e^3x^3+280a^3e^5x^2-2632a^2bd\,e^4x^2+5168a\,b^2d^2e^3x^2-3248b^3d^3e^2x^2+644a^3d\,e^4}{105(ex+d)^{\frac{7}{2}}(ae-bd)^4}$
gosper	$-\frac{2\sqrt{bx+a}\,(-560a^2b\,e^5x^3+1344a\,b^2d\,e^4x^3-928b^3d^2e^3x^3+280a^3e^5x^2-2632a^2bd\,e^4x^2+5168a\,b^2d^2e^3x^2-3248b^3d^3e^2x^2+644a^3d\,e^4}{105(ex+d)^{\frac{7}{2}}(e^4a^4-4bd\,e^3a^3+6b^2d^2e^2a^2-4}$

input
```
int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
-2/105*(b*x+a)^(1/2)*(-560*a^2*b*e^5*x^3+1344*a*b^2*d*e^4*x^3-928*b^3*d^2*
e^3*x^3+280*a^3*e^5*x^2-2632*a^2*b*d*e^4*x^2+5168*a*b^2*d^2*e^3*x^2-3248*b
^3*d^3*e^2*x^2+644*a^3*d*e^4*x-3890*a^2*b*d^2*e^3*x+6664*a*b^2*d^3*e^2*x-3
850*b^3*d^4*e*x+409*a^3*d^2*e^3-1953*a^2*b*d^3*e^2+2975*a*b^2*d^4*e-1575*b
^3*d^5)/(e*x+d)^(7/2)/(a*e-b*d)^4
```

3.849.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(165) = 330$.

Time = 1.66 (sec) , antiderivative size = 487, normalized size of antiderivative = 2.58

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{9/2}}\,dx = \frac{2\,(1575\,b^3d^5-2975\,ab^2d^4e+1953\,a^2bd^3e^2-40}{105\,(b^4d^8-4\,ab^3d^7e+6\,a^2b^2d^6e^2-4\,a^3bd^5e^3+a^4d^4e^4+(b^4d^4e^4-4\,ab^3d^3e^5}$$

input
```
integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x, algor
ithm="fricas")
```

output
```
2/105*(1575*b^3*d^5 - 2975*a*b^2*d^4*e + 1953*a^2*b*d^3*e^2 - 409*a^3*d^2*
e^3 + 16*(58*b^3*d^2*e^3 - 84*a*b^2*d*e^4 + 35*a^2*b*e^5)*x^3 + 8*(406*b^3
*d^3*e^2 - 646*a*b^2*d^2*e^3 + 329*a^2*b*d*e^4 - 35*a^3*e^5)*x^2 + 2*(1925
*b^3*d^4*e - 3332*a*b^2*d^3*e^2 + 1945*a^2*b*d^2*e^3 - 322*a^3*d*e^4)*x)*s
qrt(b*x + a)*sqrt(e*x + d)/(b^4*d^8 - 4*a*b^3*d^7*e + 6*a^2*b^2*d^6*e^2 -
4*a^3*b*d^5*e^3 + a^4*d^4*e^4 + (b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 6*a^2*b^2
*d^2*e^6 - 4*a^3*b*d*e^7 + a^4*e^8)*x^4 + 4*(b^4*d^5*e^3 - 4*a*b^3*d^4*e^4
 + 6*a^2*b^2*d^3*e^5 - 4*a^3*b*d^2*e^6 + a^4*d*e^7)*x^3 + 6*(b^4*d^6*e^2 -
 4*a*b^3*d^5*e^3 + 6*a^2*b^2*d^4*e^4 - 4*a^3*b*d^3*e^5 + a^4*d^2*e^6)*x^2
+ 4*(b^4*d^7*e - 4*a*b^3*d^6*e^2 + 6*a^2*b^2*d^5*e^3 - 4*a^3*b*d^4*e^4 + a
^4*d^3*e^5)*x)
```

3.849.6 Sympy [F]

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}(d + ex)^{9/2}}\,dx = \int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}\,(d + ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(9/2)/(b*x+a)**(1/2),x)
```

output
```
Integral((15*d**2 + 20*d*e*x + 8*e**2*x**2)/(sqrt(a + b*x)*(d + e*x)**(9/2
)), x)
```

3.849.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}(d + ex)^{9/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x, algor
ithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e-b*d)>0)', see `assume?` f
or more de
```

3.849.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 560 vs. $2(165) = 330$.

Time = 0.44 (sec) , antiderivative size = 560, normalized size of antiderivative = 2.96

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{9/2}}\,dx = \frac{2\left(2\left(4\left(bx+a\right)\left(\frac{2\left(58\,b^{10}d^2e^6-84\,ab^9de^7+35\,a^2b^8e^8\right)(bx+a)}{b^6d^4e^3|b|-4\,ab^5d^3e^4|b|+6\,a^2b^4d^2e^5|b|-4\,a^3b^3de^6|b|+a^4b^2e^7|b|}+\frac{7\left(58\,b^{11}d\right.}{b^6d^4e^3|b|-4\,a}\right.\right.\right.}{}$$

input
```
integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x, algor
ithm="giac")
```

output
```
2/105*(2*(4*(b*x + a)*(2*(58*b^10*d^2*e^6 - 84*a*b^9*d*e^7 + 35*a^2*b^8*e^
8)*(b*x + a)/(b^6*d^4*e^3*abs(b) - 4*a*b^5*d^3*e^4*abs(b) + 6*a^2*b^4*d^2*
e^5*abs(b) - 4*a^3*b^3*d*e^6*abs(b) + a^4*b^2*e^7*abs(b)) + 7*(58*b^11*d^3
*e^5 - 142*a*b^10*d^2*e^6 + 119*a^2*b^9*d*e^7 - 35*a^3*b^8*e^8)/(b^6*d^4*e
^3*abs(b) - 4*a*b^5*d^3*e^4*abs(b) + 6*a^2*b^4*d^2*e^5*abs(b) - 4*a^3*b^3*
d*e^6*abs(b) + a^4*b^2*e^7*abs(b))) + 35*(55*b^12*d^4*e^4 - 188*a*b^11*d^3
*e^5 + 243*a^2*b^10*d^2*e^6 - 142*a^3*b^9*d*e^7 + 32*a^4*b^8*e^8)/(b^6*d^4
*e^3*abs(b) - 4*a*b^5*d^3*e^4*abs(b) + 6*a^2*b^4*d^2*e^5*abs(b) - 4*a^3*b^
3*d*e^6*abs(b) + a^4*b^2*e^7*abs(b)))*(b*x + a) + 105*(15*b^13*d^5*e^3 - 6
5*a*b^12*d^4*e^4 + 113*a^2*b^11*d^3*e^5 - 99*a^3*b^10*d^2*e^6 + 44*a^4*b^9
*d*e^7 - 8*a^5*b^8*e^8)/(b^6*d^4*e^3*abs(b) - 4*a*b^5*d^3*e^4*abs(b) + 6*a
^2*b^4*d^2*e^5*abs(b) - 4*a^3*b^3*d*e^6*abs(b) + a^4*b^2*e^7*abs(b)))*sqrt
(b*x + a)/(b^2*d + (b*x + a)*b*e - a*b*e)^(7/2)
```

3.849.9 Mupad [B] (verification not implemented)

Time = 13.53 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.06

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{9/2}}\,dx = \frac{\sqrt{d+e\,x}\left(\frac{-818\,a^4\,d^2\,e^3+3906\,a^3\,b\,d^3\,e^2-5950\,a^2\,b^2\,d^4\,e+3150\,a\,b^3\,d^5}{105\,e^4\,{\left(a\,e-b\,d\right)}^4}+\frac{x\left(-1288\,a^4\,d\,e^4+6962\,a^3\right.}{}\right.}{}$$

input
```
int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(9/2)),x)
```

output
```
((d + e*x)^(1/2)*((3150*a*b^3*d^5 - 818*a^4*d^2*e^3 - 5950*a^2*b^2*d^4*e +
 3906*a^3*b*d^3*e^2)/(105*e^4*(a*e - b*d)^4) + (x*(3150*b^4*d^5 - 1288*a^4
*d*e^4 + 6962*a^3*b*d^2*e^3 - 9422*a^2*b^2*d^3*e^2 + 1750*a*b^3*d^4*e))/(1
05*e^4*(a*e - b*d)^4) - (x^2*(560*a^4*e^5 - 7700*b^4*d^4*e + 6832*a*b^3*d^
3*e^2 + 2556*a^2*b^2*d^2*e^3 - 3976*a^3*b*d*e^4))/(105*e^4*(a*e - b*d)^4)
+ (32*b^2*x^4*(35*a^2*e^2 + 58*b^2*d^2 - 84*a*b*d*e))/(105*e*(a*e - b*d)^4
) + (16*b*x^3*(35*a^3*e^3 + 406*b^3*d^3 - 530*a*b^2*d^2*e + 161*a^2*b*d*e^
2))/(105*e^2*(a*e - b*d)^4)))/(x^4*(a + b*x)^(1/2) + (d^4*(a + b*x)^(1/2))
/e^4 + (6*d^2*x^2*(a + b*x)^(1/2))/e^2 + (4*d*x^3*(a + b*x)^(1/2))/e + (4*
d^3*x*(a + b*x)^(1/2))/e^3)
```

3.849. $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{9/2}}\,dx$

3.850 $\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}(a+bx+cx^2)}\,dx$

3.850.1 Optimal result

Integrand size = 31, antiderivative size = 417

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \frac{2e^{3/2}\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}} - \frac{2\left(e(2cd-be)+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{c\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}} - \frac{2\left(e(2cd-be)-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{c\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}$$

output
```
2*e^(3/2)*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/c/g^(1/2)-2
*arctanh((e*x+d)^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2
)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(e*(-b*e+2*c*d)+(2*c^2*d^2+b^2*e
^2-2*c*e*(a*e+b*d))/(-4*a*c+b^2)^(1/2))/c/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))
^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-2*arctanh((e*x+d)^(1/2)*(2*c
*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^
(1/2)))^(1/2))*(e*(-b*e+2*c*d)+(-2*c^2*d^2-b^2*e^2+2*c*e*(a*e+b*d))/(-4*a*
c+b^2)^(1/2))/c/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(2*c*f-g*(b+(-4*a*c
+b^2)^(1/2)))^(1/2)
```

3.850.2 Mathematica [A] (verified)

Time = 3.56 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \frac{\sqrt{2}\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\sqrt{cd^2+e(-bd+ae)}\arctan\left(\frac{\sqrt{2}\sqrt{cd^2-bde+ae^2}\sqrt{f+gx}}{\sqrt{-2cdf+bef+\sqrt{b^2-4ac}ef+bdg-\sqrt{b^2-4ac}dg-2a}}\right.}{\sqrt{b^2-4ac}\sqrt{-2cdf+bef+\sqrt{b^2-4ac}ef+bdg-\sqrt{b^2-4ac}dg-2aeg}}$$

input
```
Integrate[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

output
```
((Sqrt[2]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*Sqrt[c*d^2 + e*(-(b*d) + a*
e)]*ArcTan[(Sqrt[2]*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[f + g*x])/(Sqrt[-2*c*
d*f + b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g - Sqrt[b^2 - 4*a*c]*d*g - 2*a*
e*g]*Sqrt[d + e*x])]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d*f + b*e*f + Sqrt[b^2
 - 4*a*c]*e*f + b*d*g - Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]) + (Sqrt[2]*(-2*c*
d + (b + Sqrt[b^2 - 4*a*c])*e)*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*ArcTan[(Sqrt
[2]*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[f + g*x])/(Sqrt[-2*c*d*f + b*e*f - Sq
rt[b^2 - 4*a*c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]*Sqrt[d + e*
x])]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d*f + b*e*f - Sqrt[b^2 - 4*a*c]*e*f + b
*d*g + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]) + (2*e^(3/2)*ArcTanh[(Sqrt[e]*Sqr
t[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])]])/Sqrt[g])/c
```

3.850.3 Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1204, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$$

$$\Big\downarrow \text{1204}$$

$$\int \left(\frac{-ae^2+ex(2cd-be)+cd^2}{c\sqrt{d+ex}\sqrt{f+gx}\,(a+bx+cx^2)}+\frac{e^2}{c\sqrt{d+ex}\sqrt{f+gx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}+e(2cd-be)\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}-\frac{2\left(e(2cd-be)-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}+\frac{2e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}}$$

input
```
Int[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

output
```
(2*e^(3/2)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(c*Sq
rt[g]) - (2*(e*(2*c*d - b*e) + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/S
qrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d
+ e*x])/(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(c*Sqrt[
2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]
) - (2*(e*(2*c*d - b*e) - (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b
^2 - 4*a*c])*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x
])/(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(c*Sqrt[2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g])
```

3.850.3.1 Defintions of rubi rules used

rule 1204
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f
 + g*x]), (d + e*x)^(m + 1/2)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && IGtQ[m + 1/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.850.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 11685 vs. $2(361) = 722$.

Time = 0.65 (sec) , antiderivative size = 11686, normalized size of antiderivative = 28.02

method	result	size
default	Expression too large to display	11686

input
```
int((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.850.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.850.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

output
```
Integral((d + e*x)**(3/2)/(sqrt(f + g*x)*(a + b*x + c*x**2)), x)
```

3.850.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+bx+a)\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

3.850.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.850.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \frac{(d+e\,x)^{3/2}}{\sqrt{f+g\,x}\,(c\,x^2+b\,x+a)}\,dx$$

input
```
int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)),x)
```

output
```
int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)), x)
```

3.851 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}(a+bx+cx^2)}\,dx$

3.851.1 Optimal result

Integrand size = 31, antiderivative size = 285

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = -\frac{2\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\operatorname{arctanh}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}} + \frac{2\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,\operatorname{arctanh}\left(\frac{\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}$$

output
```
-2*arctanh((e*x+d)^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1
/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+2*arcta
nh((e*x+d)^(1/2)*(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/(2*c
*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
/(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.851.2 Mathematica [A] (verified)

Time = 10.70 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \frac{2\left(-\frac{\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}\operatorname{arctanh}\left(\frac{\sqrt{-2cf+\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{-2cf+\left(b-\sqrt{b^2-4ac}\right)g}}+\frac{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}\operatorname{arctanh}\left(\frac{\sqrt{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{\cdots}}{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{\cdots}}\cdots\right)}{\sqrt{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}}\cdots\right)}{\sqrt{b^2-4ac}}$$

input
```
Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

output
```
(2*(-((Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[-2*c*f + (b
- Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*
c])*e]*Sqrt[f + g*x])])/Sqrt[-2*c*f + (b - Sqrt[b^2 - 4*a*c])*g]) + (Sqrt[
-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[-2*c*f + (b + Sqrt[b^2 -
 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[
f + g*x])])/Sqrt[-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g]))/Sqrt[b^2 - 4*a*c]
```

3.851.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1204, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$$

$$\Big\downarrow \ 1204$$

$$\int \left(\frac{e-\frac{2cd-be}{\sqrt{b^2-4ac}}}{\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{b^2-4ac}+b+2cx\right)}+\frac{\frac{2cd-be}{\sqrt{b^2-4ac}}+e}{\sqrt{d+ex}\sqrt{f+gx}\left(-\sqrt{b^2-4ac}+b+2cx\right)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}$$

input
```
Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

output
```
(-2*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2*c*f - (b - Sqr
t[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]
*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*
g]) + (2*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2*c*f - (b
+ Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g])
```

3.851.3.1 Defintions of rubi rules used

rule 1204
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f
 + g*x]), (d + e*x)^(m + 1/2)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && IGtQ[m + 1/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.851.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5481 vs. $2(241) = 482$.

Time = 0.67 (sec) , antiderivative size = 5482, normalized size of antiderivative = 19.24

method	result	size
default	Expression too large to display	5482

input
```
int((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.851.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4471 vs. $2(241) = 482$.

Time = 27.87 (sec) , antiderivative size = 4471, normalized size of antiderivative = 15.69

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*sqrt(((2*c*d - b*e)*f - (b*d - 2*a*e)*g + ((b^2*c - 4*a*c^2)*f
^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((e^2*f^2 - 2*d*e*f*
g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4
 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2
 - 4*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 -
 4*a^2*c)*g^2))*log(-(2*b*d^2*f*g - 2*a*d^2*g^2 - 2*(b*d*e - a*e^2)*f^2 +
sqrt(2)*((b^2 - 4*a*c)*e*f^2 - (b^2 - 4*a*c)*d*f*g + ((b^3*c - 4*a*b*c^2)*
f^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g + 3*(a*b^3 - 4*a^2*b*c)*f*g^2 -
2*(a^2*b^2 - 4*a^3*c)*g^3)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2
- 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^
2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))*sqrt
(e*x + d)*sqrt(g*x + f)*sqrt(((2*c*d - b*e)*f - (b*d - 2*a*e)*g + ((b^2*c
- 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((e^2*f^
2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*
f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^
3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*
g + (a*b^2 - 4*a^2*c)*g^2)) - (b*e^2*f^2 - 4*a*e^2*f*g - (b*d^2 - 4*a*d*e)
*g^2)*x - (2*(b^2*c - 4*a*c^2)*d*f^3 - 2*(b^3 - 4*a*b*c)*d*f^2*g + 2*(a*b^
2 - 4*a^2*c)*d*f*g^2 + ((b^2*c - 4*a*c^2)*e*f^3 + (a*b^2 - 4*a^2*c)*d*g^3
+ ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*f^2*g - ((b^3 - 4*a*b*c)*d ...
```

3.851.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/(sqrt(f + g*x)*(a + b*x + c*x**2)), x)
```

3.851.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \frac{\sqrt{ex+d}}{(cx^2+bx+a)\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

3.851.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.851.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \text{Hanged}$$

input
```
int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)),x)
```

output
```
\text{Hanged}
```

3.852 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}(a+bx+cx^2)}\,dx$

3.852.1 Optimal result

Integrand size = 31, antiderivative size = 287

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = -\frac{4c\,\text{arctanh}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}} + \frac{4c\,\text{arctanh}\left(\frac{\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}$$

output
```
-4*c*arctanh((e*x+d)^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^
(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))/(-4*a*c+b^2)^(1/2)/(2*c*d-e*
(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+4*c*a
rctanh((e*x+d)^(1/2)*(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/
(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4
*a*c+b^2)^(1/2)))^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.852.2 Mathematica [A] (verified)

Time = 2.52 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.43

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$$

$$= \frac{\sqrt{2}\sqrt{cd^2+e(-bd+ae)}\left(\frac{\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{cd^2-bde+ae^2}\sqrt{f+gx}}{\sqrt{-2cdf+bef+\sqrt{b^2-4ac}ef+bdg-\sqrt{b^2-4ac}dg-2aeg}\sqrt{d+ex}}\right)}{\sqrt{-2cdf+bef+\sqrt{b^2-4ac}ef+bdg-\sqrt{b^2-4ac}dg-2aeg}} + \frac{\left(2cd+\left(-b+\right.\right.}{}\right.}{\sqrt{b^2-4ac}\,(-cd^2+e(bd-ae))}$$

input
```
Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

output
```
(Sqrt[2]*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*(((-2*c*d + (b + Sqrt[b^2 - 4*a*c]
)*e)*ArcTan[(Sqrt[2]*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[f + g*x])/(Sqrt[-2*c
*d*f + b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g - Sqrt[b^2 - 4*a*c]*d*g - 2*a
*e*g]*Sqrt[d + e*x])])/Sqrt[-2*c*d*f + b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d
*g - Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g] + ((2*c*d + (-b + Sqrt[b^2 - 4*a*c])
*e)*ArcTan[(Sqrt[2]*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[f + g*x])/(Sqrt[-2*c*
d*f + b*e*f - Sqrt[b^2 - 4*a*c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g - 2*a*
e*g]*Sqrt[d + e*x])])/Sqrt[-2*c*d*f + b*e*f - Sqrt[b^2 - 4*a*c]*e*f + b*d*
g + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]))/(Sqrt[b^2 - 4*a*c]*(-(c*d^2) + e*(b
*d - a*e)))
```

3.852.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$$

$$\Big\downarrow \text{1205}$$

$$\int \left(\frac{2c}{\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{f+gx}\left(-\sqrt{b^2-4ac}+b+2cx\right)} - \frac{2c}{\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{b^2-4ac}+b+2cx\right)}\right.$$

$$\downarrow \text{2009}$$

$$\frac{4c\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}-\frac{4c\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}$$

input
```
Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

output
```
(-4*c*ArcTanh[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqr
t[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])]])/(Sqrt[b^2 - 4*a*c]*S
qrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c]
)*g]) + (4*c*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x
])/(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])]])/(Sqrt[b^2 - 4
*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b + Sqrt[b^2 -
 4*a*c])*g])
```

3.852.3.1 Defintions of rubi rules used

rule 1205
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
 !IntegerQ[m] &&  !IntegerQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.852.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5506 vs. 2(243) = 486.

Time = 0.66 (sec) , antiderivative size = 5507, normalized size of antiderivative = 19.19

method	result	size
default	Expression too large to display	5507

input
```
int(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.852.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19727 vs. 2(243) = 486.

Time = 118.80 (sec) , antiderivative size = 19727, normalized size of antiderivative = 68.74

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas
")
```

output
```
Too large to include
```

3.852.6 Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

output
```
Integral(1/(sqrt(d + e*x)*sqrt(f + g*x)*(a + b*x + c*x**2)), x)
```

3.852.7 Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \frac{1}{(cx^2+bx+a)\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

3.852.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.852.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \text{Hanged}$$

input
```
int(1/((f + g*x)^(1/2)*(d + e*x)^(1/2)*(a + b*x + c*x^2)),x)
```

output
```
\text{Hanged}
```

3.853 $\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}(a+bx+cx^2)}\,dx$

3.853.1 Optimal result

Integrand size = 31, antiderivative size = 429

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\,\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}} - \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\,\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}} - \frac{8c^2\operatorname{arctanh}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\,\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)^{3/2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}} + \frac{8c^2\operatorname{arctanh}\left(\frac{\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\,\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)^{3/2}\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}$$

output
```
4*c*e*(g*x+f)^(1/2)/(-d*g+e*f)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^
2)^(1/2)/(e*x+d)^(1/2)-4*c*e*(g*x+f)^(1/2)/(-d*g+e*f)/(-4*a*c+b^2)^(1/2)/(
2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))/(e*x+d)^(1/2)-8*c^2*arctanh((e*x+d)^(1/2)*
(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/(2*c*d-e*(b-(-4*a*c+b
^2)^(1/2)))^(1/2))/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(3/2)/(-4*a*c+b^2)^(1/
2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+8*c^2*arctanh((e*x+d)^(1/2)*(2*c
*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^
(1/2)))^(1/2))/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(3/2)/(
2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.853.2 Mathematica [A] (verified)

Time = 3.47 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.27

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \frac{2e^2\sqrt{f+gx}}{(cd^2+e(-bd+ae))\,(-ef+dg)\sqrt{d+ex}} + \frac{\sqrt{2}\big(2c^2d^2+b\big(b+\sqrt{b^2-4ac}\big)e^2-2ce\big(bd+\sqrt{b^2-4ac}d+ae\big)\big)\arctan\left(\frac{\sqrt{2}\sqrt{cd^2-bde+ae^2}\sqrt{f+gx}}{\sqrt{-2cdf+bef+\sqrt{b^2-4ac}ef+bdg-\sqrt{b^2-4ac}\ldots}}\right)}{\sqrt{b^2-4ac}\,(cd^2+e(-bd+ae))^{3/2}\sqrt{-2cdf+bef+\sqrt{b^2-4ac}ef+bdg-\sqrt{b^2-4ac}dg-2ae\ldots}} + \frac{\sqrt{2}\big(-2c^2d^2+b\big(-b+\sqrt{b^2-4ac}\big)e^2+2ce\big(bd-\sqrt{b^2-4ac}d+ae\big)\big)\arctan\left(\frac{\sqrt{2}\sqrt{cd^2-bde+ae^2}\sqrt{f\ldots}}{\sqrt{-2cdf+bef-\sqrt{b^2-4ac}ef+bdg+\sqrt{b^2\ldots}}}\right)}{\sqrt{b^2-4ac}\,(cd^2+e(-bd+ae))^{3/2}\sqrt{-2cdf+bef-\sqrt{b^2-4ac}ef+bdg+\sqrt{b^2-4ac}dg-2a\ldots}}$$

input
```
Integrate[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

output
```
(2*e^2*Sqrt[f + g*x])/((c*d^2 + e*(-(b*d) + a*e))*(-(e*f) + d*g)*Sqrt[d +
e*x]) + (Sqrt[2]*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d +
 Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c*d^2 - b*d*e + a*e^2]*S
qrt[f + g*x])/(Sqrt[-2*c*d*f + b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g - Sqr
t[b^2 - 4*a*c]*d*g - 2*a*e*g]*Sqrt[d + e*x])])/(Sqrt[b^2 - 4*a*c]*(c*d^2 +
 e*(-(b*d) + a*e))^(3/2)*Sqrt[-2*c*d*f + b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b
*d*g - Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]) + (Sqrt[2]*(-2*c^2*d^2 + b*(-b +
Sqrt[b^2 - 4*a*c])*e^2 + 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(
Sqrt[2]*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[f + g*x])/(Sqrt[-2*c*d*f + b*e*f
- Sqrt[b^2 - 4*a*c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]*Sqrt[d
+ e*x])])/(Sqrt[b^2 - 4*a*c]*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*Sqrt[-2*c*d*
f + b*e*f - Sqrt[b^2 - 4*a*c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*
g])
```

3.853.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$$

$$\Big\downarrow \text{1205}$$

$$\int \left(\frac{2c}{\sqrt{b^2-4ac}(d+ex)^{3/2}\sqrt{f+gx}\left(-\sqrt{b^2-4ac}+b+2cx\right)}-\frac{2c}{\sqrt{b^2-4ac}(d+ex)^{3/2}\sqrt{f+gx}\left(\sqrt{b^2-4ac}+b+\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{8c^2\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}+\frac{8c^2\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}+\frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\sqrt{d+ex}(ef-dg)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\sqrt{d+ex}(ef-dg)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$

input

```
Int[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

output

```
(4*c*e*Sqrt[f + g*x])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e)*(e*f - d*g)*Sqrt[d + e*x]) - (4*c*e*Sqrt[f + g*x])/(Sqrt[b^2 - 4*a*c]*(
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(e*f - d*g)*Sqrt[d + e*x]) - (8*c^2*Arc
Tanh[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (
b - Sqrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]) +
 (8*c^2*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(S
qrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]
*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*
a*c])*g])
```

3.853. $\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}(a+bx+cx^2)}\,dx$

3.853.3.1 Defintions of rubi rules used

rule 1205
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
 !IntegerQ[m] &&  !IntegerQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.853.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 47350 vs. 2(369) = 738.

Time = 0.68 (sec) , antiderivative size = 47351, normalized size of antiderivative = 110.38

method	result	size
default	Expression too large to display	47351

input
```
int(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.853.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.853.6 Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \frac{1}{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

output
```
Integral(1/((d + e*x)**(3/2)*sqrt(f + g*x)*(a + b*x + c*x**2)), x)
```

3.853.7 Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \frac{1}{(cx^2+bx+a)(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

3.853.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.853.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,(d+e\,x)^{3/2}\,\left(c\,x^2+b\,x+a\right)}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)),x)
```

output
```
int(1/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)), x)
```

3.854 $\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx$

3.854.1 Optimal result

Integrand size = 29, antiderivative size = 532

$$\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx$$
$$= \frac{(5b^3e^3g^3 + 64c^3(ef-dg)^3 - 4bce^2g^2(6bef-2bdg+aeg) + 16bc^2eg(3e^2f^2-3defg+d^2g^2) + 2ceg(5b^2e^2}{64c^3e^4}$$
$$+ \frac{g^2(24cef-14cdg-5beg)\,(a+bx+cx^2)^{3/2}}{24c^2e^2} + \frac{g^3(d+ex)\,(a+bx+cx^2)^{3/2}}{4ce^2}$$
$$- \frac{\left(4ce(2cd-be)\,(16c^2e^2f^3+5b^2deg^3-4cdg^2(6bef-2bdg+aeg)) - 2\left(4c^2d^2-\frac{b^2e^2}{2}-2ce(bd-ae)\right)g(}{128c^{7/2}e^5}$$
$$+ \frac{\sqrt{cd^2-bde+ae^2}(ef-dg)^3\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^5}$$

output
```
1/24*g^2*(-5*b*e*g-14*c*d*g+24*c*e*f)*(c*x^2+b*x+a)^(3/2)/c^2/e^2+1/4*g^3*
(e*x+d)*(c*x^2+b*x+a)^(3/2)/c/e^2-1/128*(4*c*e*(-b*e+2*c*d)*(16*c^2*e^2*f^
3+5*b^2*d*e*g^3-4*c*d*g^2*(a*e*g-2*b*d*g+6*b*e*f))-2*(4*c^2*d^2-1/2*b^2*e^
2-2*c*e*(-a*e+b*d))*g*(5*b^2*e^2*g^2-4*c*e*g*(a*e*g-2*b*d*g+6*b*e*f)+16*c^
2*(d^2*g^2-3*d*e*f*g+3*e^2*f^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x
+a)^(1/2))/c^(7/2)/e^5+(-d*g+e*f)^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)
/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/
e^5+1/64*(5*b^3*e^3*g^3+64*c^3*(-d*g+e*f)^3-4*b*c*e^2*g^2*(a*e*g-2*b*d*g+6
*b*e*f)+16*b*c^2*e*g*(d^2*g^2-3*d*e*f*g+3*e^2*f^2)+2*c*e*g*(5*b^2*e^2*g^2-
4*c*e*g*(a*e*g-2*b*d*g+6*b*e*f)+16*c^2*(d^2*g^2-3*d*e*f*g+3*e^2*f^2))*x)*(
c*x^2+b*x+a)^(1/2)/c^3/e^4
```

3.854.2 Mathematica [A] (verified)

Time = 3.04 (sec) , antiderivative size = 513, normalized size of antiderivative = 0.96

$$\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

$$= \frac{\frac{2e\sqrt{a+x(b+cx)}(15b^3e^3g^3-2bce^2g^2(26aeg+b(36ef-12dg+5egx))+16c^3\left(-12d^3g^3+6d^2eg^2(6f+gx)-2de^2g\left(18f^2+9fgx+2g^2x^2\right)+3e^3\left(4f^3+6f\right.\right.}{c^3}}{}$$

input
```
Integrate[((f + g*x)^3*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

output
```
((2*e*Sqrt[a + x*(b + c*x)]*(15*b^3*e^3*g^3 - 2*b*c*e^2*g^2*(26*a*e*g + b*
(36*e*f - 12*d*g + 5*e*g*x)) + 16*c^3*(-12*d^3*g^3 + 6*d^2*e*g^2*(6*f + g*
x) - 2*d*e^2*g*(18*f^2 + 9*f*g*x + 2*g^2*x^2) + 3*e^3*(4*f^3 + 6*f^2*g*x +
 4*f*g^2*x^2 + g^3*x^3)) + 8*c^2*e*g*(a*e*g*(-8*d*g + 3*e*(8*f + g*x)) + b
*(6*d^2*g^2 - 2*d*e*g*(9*f + g*x) + e^2*(18*f^2 + 6*f*g*x + g^2*x^2)))))/c
^3 - 768*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(-(e*f) + d*g)^3*ArcTan[(Sqrt[c]*(
d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]] - (3*(
-5*b^4*e^4*g^3 + 128*c^4*d*(-(e*f) + d*g)^3 + 8*b^2*c*e^3*g^2*(3*b*e*f - b
*d*g + 3*a*e*g) - 16*c^2*e^2*g*(a^2*e^2*g^2 + 2*a*b*e*g*(3*e*f - d*g) + b^
2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)) + 64*c^3*e*(b*(e*f - d*g)^3 + a*e*g*(
3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b
 + c*x)]])/c^(7/2))/(384*e^5)
```

3.854.3 Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {1267, 27, 2184, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

$$\Big\downarrow \ 1267$$

$$\frac{\int \frac{\sqrt{cx^2+bx+a}\left(e^2g^2(24cef-14cdg-5beg)x^2-2eg\left(e(4bd+ae)g^2-3c\left(4e^2f^2-d^2g^2\right)\right)x+e\left(8ce^2f^3-d(3bd+2ae)g^3\right)\right)}{2(d+ex)}dx}{4ce^3}+\frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2}$$

$\downarrow$ 27

$$\frac{\int \frac{\sqrt{cx^2+bx+a}\left(e^2g^2(24cef-14cdg-5beg)x^2-2eg\left(e(4bd+ae)g^2-3c\left(4e^2f^2-d^2g^2\right)\right)x+e\left(8ce^2f^3-d(3bd+2ae)g^3\right)\right)}{d+ex}dx}{8ce^3}+\frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2}$$

$\downarrow$ 2184

$$\frac{\frac{\int \frac{3e^3\left(16c^2e^2f^3+5b^2deg^3-4cdg^2(6bef-2bdg+aeg)+g\left(16\left(3e^2f^2-3degf+d^2g^2\right)c^2-4eg(6bef-2bdg+aeg)c+5b^2e^2g^2\right)x\right)\sqrt{cx^2+bx+a}}{2(d+ex)}dx}{3ce^2}+\frac{eg^2\left(a+bx+cx^2\right)^{3}\cdots}{\cdots}}{8ce^3}+\frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2}$$

$\downarrow$ 27

$$\frac{\frac{e\int \frac{\left(16c^2e^2f^3+5b^2deg^3-4cdg^2(6bef-2bdg+aeg)+g\left(16\left(3e^2f^2-3degf+d^2g^2\right)c^2-4eg(6bef-2bdg+aeg)c+5b^2e^2g^2\right)x\right)\sqrt{cx^2+bx+a}}{d+ex}dx}{2c}+\frac{eg^2\left(a+bx+cx^2\right)^{3/2}\cdots}{\cdots}}{8ce^3}+\frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2}$$

$\downarrow$ 1231

$$\frac{\frac{e\left(\frac{\sqrt{a+bx+cx^2}\left(2cegx\left(-4ceg(aeg-2bdg+6bef)+5b^2e^2g^2+16c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)-4bce^2g^2(aeg-2bdg+6bef)+5b^3e^3g^3+16bc^2eg\left(d^2g^2-3defg+3e^2f^2\right)+6\cdots\right)}{4ce^2}\cdots\right)}{\cdots}}{\cdots}+\frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2}$$

$\downarrow$ 27

$$\frac{\frac{e\left(\frac{\sqrt{a+bx+cx^2}\left(2cegx\left(-4ceg(aeg-2bdg+6bef)+5b^2e^2g^2+16c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)-4bce^2g^2(aeg-2bdg+6bef)+5b^3e^3g^3+16bc^2eg\left(d^2g^2-3defg+3e^2f^2\right)+6\cdots\right)}{4ce^2}\cdots\right)}{\cdots}}{\cdots}+\frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2}$$

3.854. $\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx$

$\downarrow$ 1269

$$e\left(\frac{\sqrt{a+bx+cx^2}\left(2cegx\left(-4ceg(aeg-2bdg+6bef)+5b^2e^2g^2+16c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)-4bce^2g^2(aeg-2bdg+6bef)+5b^3e^3g^3+16bc^2eg\left(d^2g^2-3defg+3e^2f^2\right)+6\cdots}{4ce^2}\right.$$

$$\frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2}$$

$\downarrow$ 1092

$$e\left(\frac{\sqrt{a+bx+cx^2}\left(2cegx\left(-4ceg(aeg-2bdg+6bef)+5b^2e^2g^2+16c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)-4bce^2g^2(aeg-2bdg+6bef)+5b^3e^3g^3+16bc^2eg\left(d^2g^2-3defg+3e^2f^2\right)+6\cdots}{4ce^2}\right.$$

$$\frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2}$$

$\downarrow$ 219

$$e\left(\frac{\sqrt{a+bx+cx^2}\left(2cegx\left(-4ceg(aeg-2bdg+6bef)+5b^2e^2g^2+16c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)-4bce^2g^2(aeg-2bdg+6bef)+5b^3e^3g^3+16bc^2eg\left(d^2g^2-3defg+3e^2f^2\right)+6\cdots}{4ce^2}\right.$$

$$\frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2}$$

$\downarrow$ 1154

$$e\left(\frac{\sqrt{a+bx+cx^2}\left(2cegx\left(-4ceg(aeg-2bdg+6bef)+5b^2e^2g^2+16c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)-4bce^2g^2(aeg-2bdg+6bef)+5b^3e^3g^3+16bc^2eg\left(d^2g^2-3defg+3e^2f^2\right)+6\cdots}{4ce^2}\right.$$

$$\frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2}$$

$\downarrow$ 219

3.854. $\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx$

$$e\left(\frac{\sqrt{a+bx+cx^2}\left(2cegx\left(-4ceg(aeg-2bdg+6bef)+5b^2e^2g^2+16c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)-4bce^2g^2(aeg-2bdg+6bef)+5b^3e^3g^3+16bc^2eg\left(d^2g^2-3defg+3e^2f^2\right)+6\right.}{4ce^2}\right.$$

$$\frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2}$$

input
```
Int[((f + g*x)^3*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

output
```
(g^3*(d + e*x)*(a + b*x + c*x^2)^(3/2))/(4*c*e^2) + ((e*g^2*(24*c*e*f - 14
*c*d*g - 5*b*e*g)*(a + b*x + c*x^2)^(3/2))/(3*c) + (e*(((5*b^3*e^3*g^3 + 6
4*c^3*(e*f - d*g)^3 - 4*b*c*e^2*g^2*(6*b*e*f - 2*b*d*g + a*e*g) + 16*b*c^2
*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2) + 2*c*e*g*(5*b^2*e^2*g^2 - 4*c*e*g*
(6*b*e*f - 2*b*d*g + a*e*g) + 16*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*x)
*Sqrt[a + b*x + c*x^2])/(4*c*e^2) - (((4*c*e*(2*c*d - b*e)*(16*c^2*e^2*f^3
 + 5*b^2*d*e*g^3 - 4*c*d*g^2*(6*b*e*f - 2*b*d*g + a*e*g)) - 2*(4*c^2*d^2 -
 (b^2*e^2)/2 - 2*c*e*(b*d - a*e))*g*(5*b^2*e^2*g^2 - 4*c*e*g*(6*b*e*f - 2*
b*d*g + a*e*g) + 16*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)))*ArcTanh[(b + 2
*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e) - (128*c^3*Sqrt[c*d^
2 - b*d*e + a*e^2]*(e*f - d*g)^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(
2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e)/(8*c*e^2)))/(2*c
))/(8*c*e^3)
```

3.854.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

3.854. $\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx$

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.854. $\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx$

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.854.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 879, normalized size of antiderivative = 1.65

method	result
default	$\frac{\left(-d^3g^3+3d^2efg^2-3de^2f^2g+e^3f^3\right)\left(\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a-bde+cd^2}{e^2}}+\frac{(be-2cd)\ln\left(\frac{\frac{be-2cd}{2e}+c\left(x+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-}{}}\right)}{2e\sqrt{c}}\right.}{}$
risch	$-\frac{\left(-48g^3c^3e^3x^3-8bc^2e^3g^3x^2+64c^3de^2g^3x^2-192c^3e^3fg^2x^2-24ac^2e^3g^3x+10b^2ce^3g^3x+16bc^2de^2g^3x-48bc^2e^3fg^2x-96c^3d^2eg\right.}{}$

input
```
int((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
(-d^3*g^3+3*d^2*e*f*g^2-3*d*e^2*f^2*g+e^3*f^3)/e^4*(((x+d/e)^2*c+(b*e-2*c*
d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*(b*e-2*c*d)/e*ln((1/2*(b*e
-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-b*d*e+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)
/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-
b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*
d^2)/e^2)^(1/2))/(x+d/e)))+g/e^3*(d^2*g^2*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(
1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+
e^2*g^2*(1/4*x*(c*x^2+b*x+a)^(3/2)/c-5/8*b/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/
2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2
*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-1/4*a/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x
+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))))+3*e^2*f^2*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-3*d*e*f*g*(1/4*(2*c*x+b)/c
*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2
+b*x+a)^(1/2)))+(-d*e*g^2+3*e^2*f*g)*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1
/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x+a)^(1/2)))))
```

3.854.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.854.6 Sympy [F]

$$\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{(f+gx)^3\,\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

input
```
integrate((g*x+f)**3*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

output
```
Integral((f + g*x)**3*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

3.854.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.854.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.854.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{(f+g\,x)^3\,\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

input
```
int(((f + g*x)^3*(a + b*x + c*x^2)^(1/2))/(d + e*x),x)
```

output
```
int(((f + g*x)^3*(a + b*x + c*x^2)^(1/2))/(d + e*x), x)
```

3.855 $\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx$

3.855.1 Optimal result

Integrand size = 29, antiderivative size = 325

$$\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx = -\frac{(b^2e^2g^2-8c^2(ef-dg)^2-2bceg(2ef-dg)-2ceg(4cef-2cdg-beg)x)\sqrt{a+bx+cx^2}}{8c^2e^3} + \frac{g^2(a+bx+cx^2)^{3/2}}{3ce} + \frac{((8c^2d^2-b^2e^2-4ce(bd-ae))\,g(4cef-2cdg-beg)-4ce(2cd-be)\,(2cef^2-bdg^2))\operatorname{arctanh}\left(\frac{b+2c}{2\sqrt{c}\sqrt{a+b}}\right.}{16c^{5/2}e^4} + \frac{\sqrt{cd^2-bde+ae^2}(ef-dg)^2\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^4}$$

output
```
1/3*g^2*(c*x^2+b*x+a)^(3/2)/c/e+1/16*((8*c^2*d^2-b^2*e^2-4*c*e*(-a*e+b*d))
*g*(-b*e*g-2*c*d*g+4*c*e*f)-4*c*e*(-b*e+2*c*d)*(-b*d*g^2+2*c*e*f^2))*arcta
nh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/e^4+(-d*g+e*f)^2*arc
tanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a
)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/e^4-1/8*(b^2*e^2*g^2-8*c^2*(-d*g+e*f)^2
-2*b*c*e*g*(-d*g+2*e*f)-2*c*e*g*(-b*e*g-2*c*d*g+4*c*e*f)*x)*(c*x^2+b*x+a)^
(1/2)/c^2/e^3
```

3.855.2 Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.97

$$\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx$$
$$= \frac{\frac{2e\sqrt{a+x(b+cx)}\left(-3b^2e^2g^2+2ceg(4aeg+b(6ef-3dg+egx))+4c^2\left(6d^2g^2-3deg(4f+gx)+2e^2\left(3f^2+3fgx+g^2x^2\right)\right)\right)}{c^2}+96\sqrt{-cd^2+bde-a}}{}$$

input
```
Integrate[((f + g*x)^2*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

output
```
((2*e*Sqrt[a + x*(b + c*x)]*(-3*b^2*e^2*g^2 + 2*c*e*g*(4*a*e*g + b*(6*e*f
- 3*d*g + e*g*x)) + 4*c^2*(6*d^2*g^2 - 3*d*e*g*(4*f + g*x) + 2*e^2*(3*f^2
+ 3*f*g*x + g^2*x^2))))/c^2 + 96*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e*f - d*g
)^2*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e
*(b*d - a*e)]] + (3*(-(b^3*e^3*g^2) + 16*c^3*d*(e*f - d*g)^2 + 2*b*c*e^2*g
*(2*b*e*f - b*d*g + 2*a*e*g) - 8*c^2*e*(b*(e*f - d*g)^2 + a*e*g*(2*e*f - d
*g))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/c^(5/2))/(48*e^4)
```

3.855.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {1267, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx$$
$$\Big\downarrow \text{1267}$$
$$\frac{\int \frac{3e\left(2cef^2-bdg^2+g(4cef-2cdg-beg)x\right)\sqrt{cx^2+bx+a}}{2(d+ex)}dx}{3ce^2}+\frac{g^2\left(a+bx+cx^2\right)^{3/2}}{3ce}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{\left(2cef^2-bdg^2+g(4cef-2cdg-beg)x\right)\sqrt{cx^2+bx+a}}{d+ex}dx}{2ce}+\frac{g^2\left(a+bx+cx^2\right)^{3/2}}{3ce}$$
$$\Big\downarrow \text{1231}$$

$$-\frac{\int -\frac{d\left(-eb^2+4cdb-4ace\right)g(4cef-2cdg-beg)-4ce(bd-2ae)\left(2cef^2-bdg^2\right)+\left(\left(8c^2d^2-b^2e^2-4ce(bd-ae)\right)g(4cef-2cdg-beg)-4ce(2cd-be)\left(2cef^2-bdg^2\right)\right)x}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{4ce^2}$$

$$2ce$$

$$\frac{g^2\left(a+bx+cx^2\right)^{3/2}}{3ce}$$

$\downarrow$ 27

$$\frac{\int \frac{d\left(-eb^2+4cdb-4ace\right)g(4cef-2cdg-beg)-4ce(bd-2ae)\left(2cef^2-bdg^2\right)+\left(\left(8c^2d^2-b^2e^2-4ce(bd-ae)\right)g(4cef-2cdg-beg)-4ce(2cd-be)\left(2cef^2-bdg^2\right)\right)x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{8ce^2} -$$

$$2ce$$

$$\frac{g^2\left(a+bx+cx^2\right)^{3/2}}{3ce}$$

$\downarrow$ 1269

$$\frac{\frac{\left(g\left(-4ce(bd-ae)-b^2e^2+8c^2d^2\right)(-beg-2cdg+4cef)-4ce(2cd-be)\left(2cef^2-bdg^2\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}+\frac{16c^2(ef-dg)^2\left(ae^2-bde+cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{8ce^2} -$$

$$2ce$$

$$\frac{g^2\left(a+bx+cx^2\right)^{3/2}}{3ce}$$

$\downarrow$ 1092

$$\frac{\frac{2\left(g\left(-4ce(bd-ae)-b^2e^2+8c^2d^2\right)(-beg-2cdg+4cef)-4ce(2cd-be)\left(2cef^2-bdg^2\right)\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}+\frac{16c^2(ef-dg)^2\left(ae^2-bde+cd^2\right)\int \frac{1}{(d+ex)\sqrt{c}}}{e}}{8ce^2}$$

$$2ce$$

$$\frac{g^2\left(a+bx+cx^2\right)^{3/2}}{3ce}$$

$\downarrow$ 219

$$\frac{\frac{16c^2(ef-dg)^2\left(ae^2-bde+cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}+\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(g\left(-4ce(bd-ae)-b^2e^2+8c^2d^2\right)(-beg-2cdg+4cef)-4ce(2cd-be)\left(2cef\right.\right.}{\sqrt{c}e}}{8ce^2}$$

$$2ce$$

$$\frac{g^2\left(a+bx+cx^2\right)^{3/2}}{3ce}$$

$\downarrow$ 1154

3.855. $\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx$

$$\frac{\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(g\left(-4ce(bd-ae)-b^2e^2+8c^2d^2\right)(-beg-2cdg+4cef)-4ce(2cd-be)\left(2cef^2-bdg^2\right)\right)}{\sqrt{ce}}-\frac{32c^2(ef-dg)^2\left(ae^2-bde+cd^2\right)\int\frac{}{4\left(cd^2-bed-\right.}}{}}{8ce^2}}{2ce}$$

$$\frac{g^2\left(a+bx+cx^2\right)^{3/2}}{3ce}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(g\left(-4ce(bd-ae)-b^2e^2+8c^2d^2\right)(-beg-2cdg+4cef)-4ce(2cd-be)\left(2cef^2-bdg^2\right)\right)}{\sqrt{ce}}+\frac{16c^2(ef-dg)^2\sqrt{ae^2-bde+cd^2}\,\text{arctanh}\left(\frac{}{2}\right.}{e}}{8ce^2}}{2ce}$$

$$\frac{g^2\left(a+bx+cx^2\right)^{3/2}}{3ce}$$

input
```
Int[((f + g*x)^2*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

output
```
(g^2*(a + b*x + c*x^2)^(3/2))/(3*c*e) + (-1/4*((b^2*e^2*g^2 - 8*c^2*(e*f -
 d*g)^2 - 2*b*c*e*g*(2*e*f - d*g) - 2*c*e*g*(4*c*e*f - 2*c*d*g - b*e*g)*x)
*Sqrt[a + b*x + c*x^2])/(c*e^2) + ((((8*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d - a
*e))*g*(4*c*e*f - 2*c*d*g - b*e*g) - 4*c*e*(2*c*d - b*e)*(2*c*e*f^2 - b*d*
g^2))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e)
+ (16*c^2*Sqrt[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)^2*ArcTanh[(b*d - 2*a*e +
 (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/
e)/(8*c*e^2))/(2*c*e)
```

3.855.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

3.855. $\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx$

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.855. $\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx$

3.855.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.75

method	result
risch	$\frac{(8c^2e^2g^2x^2+2bc\,e^2g^2x-12c^2de\,g^2x+24c^2e^2fgx+8ac\,e^2g^2-3b^2e^2g^2-6bcde\,g^2+12bc\,e^2fg+24c^2d^2g^2-48c^2defg+24c^2e^2f^2)\sqrt{c\,x^2+\cdots}}{24c^2e^3}$
default	$-\frac{g\left(dg\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)-2ef\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)\cdots\right.}{e^2}$

input
```
int((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*c^2*e^2*g^2*x^2+2*b*c*e^2*g^2*x-12*c^2*d*e*g^2*x+24*c^2*e^2*f*g*x+
8*a*c*e^2*g^2-3*b^2*e^2*g^2-6*b*c*d*e*g^2+12*b*c*e^2*f*g+24*c^2*d^2*g^2-48
*c^2*d*e*f*g+24*c^2*e^2*f^2)*(c*x^2+b*x+a)^(1/2)/c^2/e^3-1/16/c^2/e^3*(16*
(a*d^2*e^2*g^2-2*a*d*e^3*f*g+a*e^4*f^2-b*d^3*e*g^2+2*b*d^2*e^2*f*g-b*d*e^3
*f^2+c*d^4*g^2-2*c*d^3*e*f*g+c*d^2*e^2*f^2)*c^2/e^2/((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^
2)/e^2)^(1/2))/(x+d/e))+(4*a*b*c*e^3*g^2+8*a*c^2*d*e^2*g^2-16*a*c^2*e^3*f*
g-b^3*e^3*g^2-2*b^2*c*d*e^2*g^2+4*b^2*c*e^3*f*g-8*b*c^2*d^2*e*g^2+16*b*c^2
*d*e^2*f*g-8*b*c^2*e^3*f^2+16*c^3*d^3*g^2-32*c^3*d^2*e*f*g+16*c^3*d*e^2*f^
2)/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2))
```

3.855.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.855.6 Sympy [F]

$$\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{(f+gx)^2\,\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

input
```
integrate((g*x+f)**2*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

output
```
Integral((f + g*x)**2*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

3.855.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.855.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.855.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx=\int \frac{{\left(f+g\,x\right)}^2\,\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

input
```
int(((f + g*x)^2*(a + b*x + c*x^2)^(1/2))/(d + e*x),x)
```

output
```
int(((f + g*x)^2*(a + b*x + c*x^2)^(1/2))/(d + e*x), x)
```

3.856 $\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex} dx$

3.856.1 Optimal result

Integrand size = 27, antiderivative size = 219

$$\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2} - \frac{(b^2e^2g+8c^2d(ef-dg)-4ce(bef-bdg+aeg))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}e^3} + \frac{\sqrt{cd^2-bde+ae^2}(ef-dg)\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^3}$$

output
```
-1/8*(b^2*e^2*g+8*c^2*d*(-d*g+e*f)-4*c*e*(a*e*g-b*d*g+b*e*f))*arctanh(1/2*
(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e^3+(-d*g+e*f)*arctanh(1/2*
(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*
(a*e^2-b*d*e+c*d^2)^(1/2)/e^3+1/4*(2*c*e*g*x+b*e*g-4*c*d*g+4*c*e*f)*(c*x^2
+b*x+a)^(1/2)/c/e^2
```

3.856.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.97

$$\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{2\sqrt{c}\left(e\sqrt{a+x(b+cx)}(beg+2c(2ef-2dg+egx))-8c\sqrt{-cd^2+bde-ae^2}(-ef+dg)\arctan\left(\frac{\sqrt{c}(d+ex)-}{\sqrt{-cd^2}}\right.\right.}{8c^{3/2}e^3}$$

input
```
Integrate[((f + g*x)*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

output
```
(2*Sqrt[c]*(e*Sqrt[a + x*(b + c*x)]*(b*e*g + 2*c*(2*e*f - 2*d*g + e*g*x))
- 8*c*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(-(e*f) + d*g)*ArcTan[(Sqrt[c]*(d + e
*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]]) + (-(b^2*e
^2*g) + 8*c^2*d*(-(e*f) + d*g) + 4*c*e*(b*e*f - b*d*g + a*e*g))*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(8*c^(3/2)*e^3)
```

3.856.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

$$\Big\downarrow \text{1231}$$

$$\frac{\sqrt{a+bx+cx^2}(beg-4cdg+4cef+2cegx)}{4ce^2}-\frac{\int \frac{degb^2+4cd(ef-dg)b-4ace(2ef-dg)+\left(8d(ef-dg)c^2-4e(bef-bdg+aeg)c+b^2e^2g\right)x}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{4ce^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a+bx+cx^2}(beg-4cdg+4cef+2cegx)}{4ce^2}-\frac{\int \frac{degb^2+4cd(ef-dg)b-4ace(2ef-dg)+\left(8d(ef-dg)c^2-4e(bef-bdg+aeg)c+b^2e^2g\right)x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{8ce^2}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\sqrt{a+bx+cx^2}(beg-4cdg+4cef+2cegx)}{4ce^2}-\frac{\frac{(-4ce(aeg-bdg+bef)+b^2e^2g+8c^2d(ef-dg))\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{8c(ef-dg)(ae^2-bde+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{8ce^2}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\sqrt{a+bx+cx^2}(beg-4cdg+4cef+2cegx)}{4ce^2}-\frac{\frac{2(-4ce(aeg-bdg+bef)+b^2e^2g+8c^2d(ef-dg))\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}-\frac{8c(ef-dg)(ae^2-bde+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{8ce^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a+bx+cx^2}(beg-4cdg+4cef+2cegx)}{4ce^2}-\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4ce(aeg-bdg+bef)+b^2e^2g+8c^2d(ef-dg))}{\sqrt{c}e}-\frac{8c(ef-dg)(ae^2-bde+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{8ce^2}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\sqrt{a+bx+cx^2}(beg-4cdg+4cef+2cegx)}{4ce^2}-\frac{\frac{16c(ef-dg)(ae^2-bde+cd^2)\int \frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}+\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4ce(aeg-bdg+b}{\sqrt{ce}}}{8ce^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a+bx+cx^2}(beg-4cdg+4cef+2cegx)}{4ce^2}-\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4ce(aeg-bdg+bef)+b^2e^2g+8c^2d(ef-dg))}{\sqrt{c}e}-\frac{8c(ef-dg)\sqrt{ae^2-bde+cd^2}\,\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e}}{8ce^2}$$

input
```
Int[((f + g*x)*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

output
```
((4*c*e*f - 4*c*d*g + b*e*g + 2*c*e*g*x)*Sqrt[a + b*x + c*x^2])/(4*c*e^2)
- (((b^2*e^2*g + 8*c^2*d*(e*f - d*g) - 4*c*e*(b*e*f - b*d*g + a*e*g))*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e) - (8*c*Sqr
t[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*
x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e)/(8*c*e^2)
```

3.856 $\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx$

3.856.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.856. $\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx$

3.856.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.53

method	result
risch	$\frac{(2cegx+beg-4cdg+4cef)\sqrt{c\,x^2+bx+a}}{4c\,e^2}+\frac{\frac{\left(4ac\,e^2g-b^2e^2g-4bcdeg+4bc\,e^2f+8c^2d^2g-8c^2def\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{e\sqrt{c}}+\frac{8\left(a\,e^2gd-a\,e^3f \cdots\right.}{\cdots}}{\cdots}$
default	$\frac{g\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{e}+\frac{(-dg+ef)\left(\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a-bde+c\,d^2}{e^2}}+\frac{(be-2cd \cdots}{\cdots}\right.}{\cdots}$

input
```
int((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*c*e*g*x+b*e*g-4*c*d*g+4*c*e*f)*(c*x^2+b*x+a)^(1/2)/c/e^2+1/8/c/e^2*
((4*a*c*e^2*g-b^2*e^2*g-4*b*c*d*e*g+4*b*c*e^2*f+8*c^2*d^2*g-8*c^2*d*e*f)/e
*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+8*(a*d*e^2*g-a*e^3*f-
b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)*c/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2))/(x+d/e)))
```

3.856.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.856.6 Sympy [F]

$$\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

input
```
integrate((g*x+f)*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

output
```
Integral((f + g*x)*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

3.856.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.856.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.856.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{(f+g\,x)\,\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

input
```
int(((f + g*x)*(a + b*x + c*x^2)^(1/2))/(d + e*x),x)
```

output
```
int(((f + g*x)*(a + b*x + c*x^2)^(1/2))/(d + e*x), x)
```

3.857 $\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx$

3.857.1 Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{\sqrt{a+bx+cx^2}}{e} - \frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}e^2} + \frac{\sqrt{cd^2-bde+ae^2}\,\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^2}$$

output
```
-1/2*(-b*e+2*c*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/e^2/c
^(1/2)+arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c
*x^2+b*x+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/e^2+(c*x^2+b*x+a)^(1/2)/e
```

3.857.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{e\sqrt{a+x(b+cx)} + 2\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{-cd^2+bde-ae^2}\,x}{\sqrt{a}(d+ex)-d\sqrt{a+x(b+cx)}}\right) + 2\sqrt{c}d\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a}-\sqrt{a+x(b+cx)}}\right)}{e^2}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x),x]
```

output
```
(e*Sqrt[a + x*(b + c*x)] + 2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[-
(c*d^2) + b*d*e - a*e^2]*x)/(Sqrt[a]*(d + e*x) - d*Sqrt[a + x*(b + c*x)])]
 + 2*Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/(Sqrt[a] - Sqrt[a + x*(b + c*x)])] + (b
*e*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/Sqrt[c])/e^2
```

3.857.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1162, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}dx$$

$$\Big\downarrow \text{1162}$$

$$\frac{\sqrt{a+bx+cx^2}}{e}-\frac{\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{2e}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\sqrt{a+bx+cx^2}}{e}-\frac{\frac{(2cd-be)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{2\left(ae^2-bde+cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{2e}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\sqrt{a+bx+cx^2}}{e}-\frac{\frac{2(2cd-be)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}-\frac{2\left(ae^2-bde+cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{2e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a+bx+cx^2}}{e}-\frac{\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}-\frac{2\left(ae^2-bde+cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{2e}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\sqrt{a+bx+cx^2}}{e}-\frac{\frac{4\left(ae^2-bde+cd^2\right)\int \frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}+\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}}{2e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a+bx+cx^2}}{e}-\frac{\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}-\frac{2\sqrt{ae^2-bde+cd^2}\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e}}{2e}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/(d + e*x),x]
```

output
```
Sqrt[a + b*x + c*x^2]/e - (((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(Sqrt[c]*e) - (2*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTa
nh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a +
 b*x + c*x^2])])/e)/(2*e)
```

3.857.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

3.857. $\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx$

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

3.857.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.57

method	result
risch	$\frac{\sqrt{c\,x^2+bx+a}}{e}+\frac{\frac{(be-2cd)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{e\sqrt{c}}-\frac{\left(2e^2a-2bde+2c\,d^2\right)\ln\left(\frac{\frac{2e^2a-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a-bde+c\,d^2}{e^2}}\sqrt{(x+\cdots}}{x+\frac{d}{e}}\cdots}{e^2\sqrt{\frac{e^2a-bde+c\,d^2}{e^2}}}}{2e}$
default	$\frac{\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a-bde+c\,d^2}{e^2}}+\frac{(be-2cd)\ln\left(\frac{\frac{be-2cd}{2e}+c\left(x+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a-bde+c\,d^2}{e^2}}\right)}{2e\sqrt{c}}-\frac{\left(e^2a-b\cdots\right.}{\cdots}}{e}$

input
```
int((c*x^2+b*x+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+b*x+a)^(1/2)/e+1/2/e*((b*e-2*c*d)/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))/c^(1/2)-(2*a*e^2-2*b*d*e+2*c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2))/(x+d/e)))
```

3.857.5 Fricas [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 992, normalized size of antiderivative = 6.53

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx$$
$$=\left[\frac{4\sqrt{cx^2+bx+a}ce-(2\,cd-be)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)+\cdots}{\cdots}\right.$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
[1/4*(4*sqrt(c*x^2 + b*x + a)*c*e - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^2 -
 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 2*
sqrt(c*d^2 - b*d*e + a*e^2)*c*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d
^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*
e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*
c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c
*e^2), 1/2*(2*sqrt(c*x^2 + b*x + a)*c*e + (2*c*d - b*e)*sqrt(-c)*arctan(1/
2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + sq
rt(c*d^2 - b*d*e + a*e^2)*c*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2
 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e
+ a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*
d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c*e
^2), 1/4*(4*sqrt(c*x^2 + b*x + a)*c*e + 4*sqrt(-c*d^2 + b*d*e - a*e^2)*c*a
rctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e
 + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*
c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - (2*c*d - b*e)*sqrt(c)*log
(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c)
- 4*a*c))/(c*e^2), 1/2*(2*sqrt(c*x^2 + b*x + a)*c*e + 2*sqrt(-c*d^2 + b*d*
e - a*e^2)*c*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a
)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d...
```

3.857.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

3.857.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` f
or more de
```

3.857.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.857.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(d + e*x),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(d + e*x), x)
```

3.858 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)} dx$

3.858.1 Optimal result

Integrand size = 29, antiderivative size = 228

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)} dx = \frac{\sqrt{c}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{eg} + \frac{\sqrt{cd^2-bde+ae^2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e(ef-dg)} - \frac{\sqrt{cf^2-bfg+ag^2}\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)}$$

output
```
arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/e/g+arctanh(1/2
*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))
*(a*e^2-b*d*e+c*d^2)^(1/2)/e/(-d*g+e*f)-arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f
)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*g^2-b*f*g+c*f^2)^(1
/2)/g/(-d*g+e*f)
```

3.858.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx = \frac{2\sqrt{-cd^2+e(bd-ae)}g\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)-2e\sqrt{-cf^2+bfg-ag^2}\arctan\left(\frac{\sqrt{c}(f+gx)-g\sqrt{a+x(b+}}{\sqrt{-cf^2+bfg-ag^2}}\right.}{eg(ef-dg)}$$

input

```
Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)),x]
```

output

```
(2*Sqrt[-(c*d^2) + e*(b*d - a*e)]*g*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a +
 x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]] - 2*e*Sqrt[-(c*f^2) + b*f*g
 - a*g^2]*ArcTan[(Sqrt[c]*(f + g*x) - g*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*f^
2) + b*f*g - a*g^2]] + Sqrt[c]*(-(e*f) + d*g)*Log[e*g*(b + 2*c*x - 2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])])/(e*g*(e*f - d*g))
```

3.858.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1270, 1154, 219, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx$$

$$\Big\downarrow \ 1270$$

$$\frac{(ae^2-bde+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e(ef-dg)}-\frac{\int \frac{cdf-bef+aeg-c(ef-dg)x}{(f+gx)\sqrt{cx^2+bx+a}}dx}{e(ef-dg)}$$

$$\Big\downarrow \ 1154$$

$$-\frac{2(ae^2-bde+cd^2)\int \frac{1}{4(cd^2-bed+ae^2)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e(ef-dg)}-\frac{\int \frac{cdf-bef+aeg-c(ef-dg)x}{(f+gx)\sqrt{cx^2+bx+a}}dx}{e(ef-dg)}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{ae^2-bde+cd^2}\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(ef-dg)}-\frac{\int\frac{cdf-bef+aeg-c(ef-dg)x}{(f+gx)\sqrt{cx^2+bx+a}}dx}{e(ef-dg)}$$

$$\Big\downarrow \; 1269$$

$$\frac{\sqrt{ae^2-bde+cd^2}\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(ef-dg)}-\frac{\frac{e(ag^2-bfg+cf^2)\int\frac{1}{(f+gx)\sqrt{cx^2+bx+a}}dx}{g}-\frac{c(ef-dg)\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{g}}{e(ef-dg)}$$

$$\Big\downarrow \; 1092$$

$$\frac{\sqrt{ae^2-bde+cd^2}\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(ef-dg)}-\frac{\frac{e(ag^2-bfg+cf^2)\int\frac{1}{(f+gx)\sqrt{cx^2+bx+a}}dx}{g}-\frac{2c(ef-dg)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{g}}{e(ef-dg)}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{ae^2-bde+cd^2}\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(ef-dg)}-\frac{\frac{e(ag^2-bfg+cf^2)\int\frac{1}{(f+gx)\sqrt{cx^2+bx+a}}dx}{g}-\frac{\sqrt{c}(ef-dg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g}}{e(ef-dg)}$$

$$\Big\downarrow \; 1154$$

$$\frac{\sqrt{ae^2-bde+cd^2}\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(ef-dg)}-\frac{-\frac{2e(ag^2-bfg+cf^2)\int\frac{1}{4\left(cf^2-bgf+ag^2\right)-\frac{(bf-2ag+(2cf-bg)x)^2}{cx^2+bx+a}}d\left(-\frac{bf-2ag+(2cf-bg)x}{\sqrt{cx^2+bx+a}}\right)}{g}-\frac{\sqrt{c}(ef-dg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g}}{e(ef-dg)}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{ae^2-bde+cd^2}\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(ef-dg)}-\frac{\frac{e\sqrt{ag^2-bfg+cf^2}\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g}-\frac{\sqrt{c}(ef-dg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g}}{e(ef-dg)}$$

3.858. $\int\frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}dx$

input
```
Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)),x]
```

output
```
(Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sq
rt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*(e*f - d*g)) - (-((S
qrt[c]*(e*f - d*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])
/g) + (e*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*
x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/g)/(e*(e*f - d*
g))
```

3.858.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1270
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Simp[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g))
Int[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Simp[1/(e*(e*f - d*g))
Int[Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*((a + b*x + c*x^2)^(p
- 1)/(f + g*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[p]
&& GtQ[p, 0]
```

3.858.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(204) = 408$.

Time = 0.76 (sec) , antiderivative size = 673, normalized size of antiderivative = 2.95

method	result
default	$\frac{\sqrt{\left(x+\frac{f}{g}\right)^{2} c+\frac{(bg-2cf)\left(x+\frac{f}{g}\right)}{g}+\frac{a\,g^{2}-bfg+c\,f^{2}}{g^{2}}}+\frac{(bg-2cf)\ln\left(\frac{\frac{bg-2cf}{2g}+c\left(x+\frac{f}{g}\right)}{\sqrt{c}}+\sqrt{\left(x+\frac{f}{g}\right)^{2} c+\frac{(bg-2cf)\left(x+\frac{f}{g}\right)}{g}+\frac{a\,g^{2}-bfg+c\,f^{2}}{g^{2}}}\right)}{2g\sqrt{c}}-\frac{(a\,g^{2}}{}}{dg-ef}$

input
```
int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
1/(d*g-e*f)*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(
1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2
*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*
f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g
^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b
*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))-1/(d*g-e*f)*
(((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*(b*
e-2*c*d)/e*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*
d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-b*d*e+c*d^2)/e
^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*
d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*
(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

3.858.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

output
```
Timed out
```

3.858.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\,(f+gx)}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f),x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*(f + g*x)), x)
```

3.858.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for m
ore detail
```

3.858.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.858.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{(f+g\,x)\,(d+e\,x)}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/((f + g*x)*(d + e*x)),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/((f + g*x)*(d + e*x)), x)
```

3.859 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx$

3.859.1 Optimal result

Integrand size = 29, antiderivative size = 490

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx = \frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)} - \frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef-dg)^2}$$
$$+ \frac{e(2cf-bg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}g(ef-dg)^2}$$
$$- \frac{\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)}$$
$$+ \frac{\sqrt{cd^2-bde+ae^2}\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^2}$$
$$+ \frac{(2cf-bg)\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2g(ef-dg)\sqrt{cf^2-bfg+ag^2}}$$
$$- \frac{e\sqrt{cf^2-bfg+ag^2}\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^2}$$

output
```
-1/2*(-b*e+2*c*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g
+e*f)^2/c^(1/2)+1/2*e*(-b*g+2*c*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*
x+a)^(1/2))/g/(-d*g+e*f)^2/c^(1/2)-arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*
x+a)^(1/2))*c^(1/2)/g/(-d*g+e*f)+arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a
*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/(-d
*g+e*f)^2+1/2*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b
*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/g/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(1
/2)-e*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*
x^2+b*x+a)^(1/2))*(a*g^2-b*f*g+c*f^2)^(1/2)/g/(-d*g+e*f)^2+(c*x^2+b*x+a)^(
1/2)/(-d*g+e*f)/(g*x+f)
```

3.859.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx$$
$$= \frac{\frac{(ef-dg)\sqrt{a+x(b+cx)}}{f+gx} + 2\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right) - \frac{\sqrt{-cf^2+bfg-ag^2}(2cdf+2aeg-b(ef+dg))}{cf^2+g(-bf+}}{(ef-dg)^2}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^2),x]
```

output
```
(((e*f - d*g)*Sqrt[a + x*(b + c*x)])/(f + g*x) + 2*Sqrt[-(c*d^2) + b*d*e -
 a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2)
 + e*(b*d - a*e)]] - (Sqrt[-(c*f^2) + b*f*g - a*g^2]*(2*c*d*f + 2*a*e*g -
b*(e*f + d*g))*ArcTan[(Sqrt[c]*(f + g*x) - g*Sqrt[a + x*(b + c*x)])/Sqrt[-
(c*f^2) + g*(b*f - a*g)]])/(c*f^2 + g*(-(b*f) + a*g)))/(e*f - d*g)^2
```

3.859.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx$$
$$\Big\downarrow \text{1289}$$
$$\int \left(\frac{e^2\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)^2}-\frac{eg\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)^2}-\frac{g\sqrt{a+bx+cx^2}}{(f+gx)^2(ef-dg)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{ae^2-bde+cd^2}\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2}-\frac{e\sqrt{ag^2-bfg+cf^2}\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)^2}+\frac{(2cf-bg)\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2g(ef-dg)\sqrt{ag^2-bfg+cf^2}}-\frac{\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)}-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef-dg)^2}+\frac{e(2cf-bg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}g(ef-dg)^2}+\frac{\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^2),x]
```

output
```
Sqrt[a + b*x + c*x^2]/((e*f - d*g)*(f + g*x)) - ((2*c*d - b*e)*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*(e*f - d*g)^2) + (
e*(2*c*f - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2
*Sqrt[c]*g*(e*f - d*g)^2) - (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
 + b*x + c*x^2])])/(g*(e*f - d*g)) + (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[
(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*
x + c*x^2])])/(e*f - d*g)^2 + ((2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f
 - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g*(e
*f - d*g)*Sqrt[c*f^2 - b*f*g + a*g^2]) - (e*Sqrt[c*f^2 - b*f*g + a*g^2]*Ar
cTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[
a + b*x + c*x^2])])/(g*(e*f - d*g)^2)
```

3.859. $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx$

3.859.3.1 Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.859.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1358 vs. $2(434) = 868$.

Time = 0.90 (sec) , antiderivative size = 1359, normalized size of antiderivative = 2.77

method	result	size
default	Expression too large to display	1359

input
```
int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

output
```
e/(d*g-e*f)^2*(((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)+1/2*(b*e-2*c*d)/e*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)
^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-
b*d*e+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)
/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+
(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))+1/g/(d*g-e
*f)*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)
+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+1/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(((x
+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*
c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g
*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/(
(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g
*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f
/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))+2*c/(a*g^2-b*f*g+c*f^2)*g^2*
(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g
^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/
g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*
c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))))-e/(d*g-e*f)^2*(((x+f/g)^2
*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)/g*
ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+...
```

3.859.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.859.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\,(f+gx)^2}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**2,x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*(f + g*x)**2), x)
```

3.859.7 Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(ex+d)(gx+f)^2}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^2), x)
```

3.859.8 Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(ex+d)(gx+f)^2}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.859.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}}{{\left(f+g\,x\right)}^2\,\left(d+e\,x\right)}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/((f + g*x)^2*(d + e*x)),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/((f + g*x)^2*(d + e*x)), x)
```

3.860 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx$

3.860.1 Optimal result

Integrand size = 29, antiderivative size = 673

$$\begin{aligned}\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx &= \frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)} - \frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2}\\
&\quad - \frac{e(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef-dg)^3}\\
&\quad + \frac{e^2(2cf-bg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}g(ef-dg)^3}\\
&\quad - \frac{\sqrt{c}e\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^2}\\
&\quad + \frac{e\sqrt{cd^2-bde+ae^2}\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^3}\\
&\quad + \frac{\left(b^2-4ac\right)g\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^{3/2}}\\
&\quad + \frac{e(2cf-bg)\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2g(ef-dg)^2\sqrt{cf^2-bfg+ag^2}}\\
&\quad - \frac{e^2\sqrt{cf^2-bfg+ag^2}\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^3}\end{aligned}$$

output
```
1/8*(-4*a*c+b^2)*g*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f
^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(3/2)-1/2*e*
(-b*e+2*c*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)
^3/c^(1/2)+1/2*e^2*(-b*g+2*c*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a
)^(1/2))/g/(-d*g+e*f)^3/c^(1/2)-e*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x
+a)^(1/2))*c^(1/2)/g/(-d*g+e*f)^2+e*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)
/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/
(-d*g+e*f)^3+1/2*e*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*
g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/g/(-d*g+e*f)^2/(a*g^2-b*f*g+c*
f^2)^(1/2)-e^2*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^
(1/2)/(c*x^2+b*x+a)^(1/2))*(a*g^2-b*f*g+c*f^2)^(1/2)/g/(-d*g+e*f)^3+e*(c*x
^2+b*x+a)^(1/2)/(-d*g+e*f)^2/(g*x+f)-1/4*g*(b*f-2*a*g+(-b*g+2*c*f)*x)*(c*x
^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)^2
```

3.860.2 Mathematica [A] (verified)

Time = 10.90 (sec) , antiderivative size = 609, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx$$

$$= \frac{\frac{8e(ef-dg)\sqrt{a+x(b+cx)}}{f+gx} + \frac{2g(ef-dg)^2(-bf+2ag-2cfx+bgx)\sqrt{a+x(b+cx)}}{(cf^2+g(-bf+ag))(f+gx)^2} + \frac{4e(-2cd+be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}} + 8e\sqrt{cd^2+}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^3),x]
```

output
```
((8*e*(e*f - d*g)*Sqrt[a + x*(b + c*x)])/(f + g*x) + (2*g*(e*f - d*g)^2*(-
(b*f) + 2*a*g - 2*c*f*x + b*g*x)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f
) + a*g))*(f + g*x)^2) + (4*e*(-2*c*d + b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + 8*e*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Ar
cTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*S
qrt[a + x*(b + c*x)])] + ((b^2 - 4*a*c)*g*(e*f - d*g)^2*ArcTanh[(-2*a*g +
2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c
*x)])])/(c*f^2 + g*(-(b*f) + a*g))^(3/2) - (4*e*(e*f - d*g)*(2*Sqrt[c]*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - ((2*c*f - b*g)*ArcTa
nh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt
[a + x*(b + c*x)])])/Sqrt[c*f^2 + g*(-(b*f) + a*g)]))/g + (4*e^2*((2*c*f -
 b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - 2*Sqrt[c]*S
qrt[c*f^2 + g*(-(b*f) + a*g)]*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*
Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]))/(Sqrt[c]*g))/(8*(
e*f - d*g)^3)
```

3.860.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3} \, dx$$

$$\Big\downarrow \text{1289}$$

$$\int \left(\frac{e^3\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)^3}-\frac{e^2g\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)^3}-\frac{eg\sqrt{a+bx+cx^2}}{(f+gx)^2(ef-dg)^2}-\frac{g\sqrt{a+bx+cx^2}}{(f+gx)^3(ef-dg)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{g(b^2-4ac)\operatorname{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{8(ef-dg)\left(ag^2-bfg+cf^2\right)^{3/2}}+\frac{e\sqrt{ae^2-bde+cd^2}\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^3}-\frac{e^2\sqrt{ag^2-bfg+cf^2}\operatorname{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)^3}+\frac{e^2(2cf-bg)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}g(ef-dg)^3}+\frac{e(2cf-bg)\operatorname{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2g(ef-dg)^2\sqrt{ag^2-bfg+cf^2}}-\frac{\sqrt{c}e\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^2}-\frac{e(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef-dg)^3}-\frac{g\sqrt{a+bx+cx^2}(-2ag+x(2cf-bg)+bf)}{4(f+gx)^2(ef-dg)\left(ag^2-bfg+cf^2\right)}+\frac{e\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)^2}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^3),x]
```

output
```
(e*Sqrt[a + b*x + c*x^2])/((e*f - d*g)^2*(f + g*x)) - (g*(b*f - 2*a*g + (2
*c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(4*(e*f - d*g)*(c*f^2 - b*f*g + a*g^
2)*(f + g*x)^2) - (e*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2])])/(2*Sqrt[c]*(e*f - d*g)^3) + (e^2*(2*c*f - b*g)*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*g*(e*f - d*g)^3)
- (Sqrt[c]*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(g*(e
*f - d*g)^2) + (e*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*
d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*f -
 d*g)^3 + ((b^2 - 4*a*c)*g*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt
[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f - d*g)*(c*f^2 - b
*f*g + a*g^2)^(3/2)) + (e*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*
g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g*(e*f -
d*g)^2*Sqrt[c*f^2 - b*f*g + a*g^2]) - (e^2*Sqrt[c*f^2 - b*f*g + a*g^2]*Arc
Tanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a
 + b*x + c*x^2])])/(g*(e*f - d*g)^3)
```

3.860. $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx$

3.860.3.1 Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.860.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2511 vs. $2(603) = 1206$.

Time = 0.99 (sec) , antiderivative size = 2512, normalized size of antiderivative = 3.73

method	result	size
default	Expression too large to display	2512

input
```
int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x,method=_RETURNVERBOSE)
```

output
```
1/g^2/(d*g-e*f)*(-1/2/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)^2*((x+f/g)^2*c+(b*g-
2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)-1/4*(b*g-2*c*f)*g/(a*g^2-b
*f*g+c*f^2)*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g
*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+1/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f
^2)*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2
*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-
2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^
2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-
2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f
)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))+2*c/(a*g^2-b*f*g+c*f
^2)*g^2*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f
/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2
*c*f)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c
+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))))+1/2*c/(a*g^2-b*f*
g+c*f^2)*g^2*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^
(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^
2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b
*f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/
g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(
b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))-e^2/(d*...
```

3.860.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.860.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\,(f+gx)^3}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**3,x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*(f + g*x)**3), x)
```

3.860.7 Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(ex+d)(gx+f)^3}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^3), x)
```

3.860.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1824 vs. 2(603) = 1206.

Time = 1.07 (sec) , antiderivative size = 1824, normalized size of antiderivative = 2.71

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x, algorithm="giac")
```

output
```
-2*(c*d^2*e - b*d*e^2 + a*e^3)*arctan(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/((e^3*f^3 - 3*d*e^2*f^2*g +
3*d^2*e*f*g^2 - d^3*g^3)*sqrt(-c*d^2 + b*d*e - a*e^2)) - 1/4*(8*c^2*d*e*f^
3 - 4*b*c*e^2*f^3 - 12*b*c*d*e*f^2*g + 3*b^2*e^2*f^2*g + 12*a*c*e^2*f^2*g
+ 6*b^2*d*e*f*g^2 - 12*a*b*e^2*f*g^2 - b^2*d^2*g^3 + 4*a*c*d^2*g^3 - 4*a*b
*d*e*g^3 + 8*a^2*e^2*g^3)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*g +
 sqrt(c)*f)/sqrt(-c*f^2 + b*f*g - a*g^2))/((c*e^3*f^5 - 3*c*d*e^2*f^4*g -
b*e^3*f^4*g + 3*c*d^2*e*f^3*g^2 + 3*b*d*e^2*f^3*g^2 + a*e^3*f^3*g^2 - c*d^
3*f^2*g^3 - 3*b*d^2*e*f^2*g^3 - 3*a*d*e^2*f^2*g^3 + b*d^3*f*g^4 + 3*a*d^2*
e*f*g^4 - a*d^3*g^5)*sqrt(-c*f^2 + b*f*g - a*g^2)) + 1/4*(8*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^3*c^2*d*f^2*g^2 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^3*b*c*e*f^2*g^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*d*f*g^3
+ 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*e*f*g^3 + 4*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*a*c*e*f*g^3 + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*
b^2*d*g^4 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d*g^4 - 4*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^3*a*b*e*g^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*c^(5/2)*e*f^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d*
f^3*g - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*e*f^3*g + 9*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*e*f^2*g^2 + 12*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*e*f^2*g^2 - 5*(sqrt(c)*x - sqrt(c*x...
```

3.860.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{{\left(f+g\,x\right)}^3\,\left(d+e\,x\right)}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/((f + g*x)^3*(d + e*x)),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/((f + g*x)^3*(d + e*x)), x)
```

3.861 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx$

3.861.1 Optimal result

Integrand size = 29, antiderivative size = 933

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx = \frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)} - \frac{g(2cf-bg)(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)^2} - \frac{eg(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)^2\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{g^2\left(a+bx+cx^2\right)^{3/2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^3} - \frac{e^2(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef-dg)^4} + \frac{e^3(2cf-bg)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}g(ef-dg)^4} - \frac{\sqrt{c}e^2\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^3} + \frac{e^2\sqrt{cd^2-bde+ae^2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^4} + \frac{\left(b^2-4ac\right)g(2cf-bg)\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{16(ef-dg)\left(cf^2-bfg+ag^2\right)^{5/2}} + \frac{\left(b^2-4ac\right)eg\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{8(ef-dg)^2\left(cf^2-bfg+ag^2\right)^{3/2}} + \frac{e^2(2cf-bg)\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2g(ef-dg)^3\sqrt{cf^2-bfg+ag^2}} - \frac{e^3\sqrt{cf^2-bfg+ag^2}\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^4}$$

output
```
1/3*g^2*(c*x^2+b*x+a)^(3/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)^3+1/16*
(-4*a*c+b^2)*g*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-
b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(5/
2)+1/8*(-4*a*c+b^2)*e*g*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*
g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)^(3/2)
-1/2*e^2*(-b*e+2*c*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/(
-d*g+e*f)^4/c^(1/2)+1/2*e^3*(-b*g+2*c*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*
x^2+b*x+a)^(1/2))/g/(-d*g+e*f)^4/c^(1/2)-e^2*arctanh(1/2*(2*c*x+b)/c^(1/2)
/(c*x^2+b*x+a)^(1/2))*c^(1/2)/g/(-d*g+e*f)^3+e^2*arctanh(1/2*(b*d-2*a*e+(-
b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*e^2-b*d*e+
c*d^2)^(1/2)/(-d*g+e*f)^4+1/2*e^2*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*
g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/g/(-d*g+e*f)^3/
(a*g^2-b*f*g+c*f^2)^(1/2)-e^3*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^
2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*g^2-b*f*g+c*f^2)^(1/2)/g/(-d*
g+e*f)^4+e^2*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)^3/(g*x+f)-1/8*g*(-b*g+2*c*f)*(
b*f-2*a*g+(-b*g+2*c*f)*x)*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^
2)^2/(g*x+f)^2-1/4*e*g*(b*f-2*a*g+(-b*g+2*c*f)*x)*(c*x^2+b*x+a)^(1/2)/(-d*
g+e*f)^2/(a*g^2-b*f*g+c*f^2)/(g*x+f)^2
```

3.861.2 Mathematica [A] (verified)

Time = 12.45 (sec) , antiderivative size = 858, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx$$

$$= \frac{\frac{48e^2(ef-dg)\sqrt{a+x(b+cx)}}{f+gx} + \frac{12eg(ef-dg)^2(-bf+2ag-2cfx+bgx)\sqrt{a+x(b+cx)}}{(cf^2+g(-bf+ag))(f+gx)^2} - \frac{16g^2(-ef+dg)^3(a+x(b+cx))^{3/2}}{(cf^2+g(-bf+ag))(f+gx)^3} + 24e^2\left(\frac{(-2cd+be)\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^4),x]
```

output
```
((48*e^2*(e*f - d*g)*Sqrt[a + x*(b + c*x)])/(f + g*x) + (12*e*g*(e*f - d*g
)^2*(-(b*f) + 2*a*g - 2*c*f*x + b*g*x)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*
(-(b*f) + a*g))*(f + g*x)^2) - (16*g^2*(-(e*f) + d*g)^3*(a + x*(b + c*x))^
(3/2))/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)^3) + 24*e^2*(((-2*c*d + b*e)*
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + 2*Sqrt[c
*d^2 + e*(-(b*d) + a*e)]*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[
c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])]) + (6*(b^2 - 4*a*c)*e*g*
(e*f - d*g)^2*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(
-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^(3/2) -
 (3*g*(2*c*f - b*g)*(e*f - d*g)^3*((2*Sqrt[a + x*(b + c*x)]*(-2*a*g + 2*c*
f*x + b*(f - g*x)))/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)^2) + ((-b^2 + 4*
a*c)*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) +
a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^(3/2)))/(c*f^2 +
 g*(-(b*f) + a*g)) - (24*e^2*(e*f - d*g)*(2*Sqrt[c]*ArcTanh[(b + 2*c*x)/(2
*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - ((2*c*f - b*g)*ArcTanh[(-2*a*g + 2*c*f*
x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]
)/Sqrt[c*f^2 + g*(-(b*f) + a*g)]))/g + (24*e^3*((2*c*f - b*g)*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - 2*Sqrt[c]*Sqrt[c*f^2 + g*(-(b
*f) + a*g)]*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(
b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]))/(Sqrt[c]*g))/(48*(e*f - d*g)^4)
```

3.861.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 933, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4} \, dx$$

$$\Big\downarrow \text{1289}$$

$$\int \left(\frac{e^4\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)^4} - \frac{e^3g\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)^4} - \frac{e^2g\sqrt{a+bx+cx^2}}{(f+gx)^2(ef-dg)^3} - \frac{eg\sqrt{a+bx+cx^2}}{(f+gx)^3(ef-dg)^2} - \frac{g\sqrt{a+bx+cx^2}}{(f+gx)^4(ef-\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(2cf-bg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^3}{2\sqrt{c}g(ef-dg)^4}-\frac{\sqrt{cf^2-bgf+ag^2}\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)e^3}{g(ef-dg)^4}-\frac{\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^2}{g(ef-dg)^3}-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^2}{2\sqrt{c}(ef-dg)^4}+\frac{\sqrt{cd^2-bed+ae^2}\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)e^2}{(ef-dg)^4}+\frac{(2cf-bg)\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)e^2}{2g(ef-dg)^3\sqrt{cf^2-bgf+ag^2}}+\frac{\sqrt{cx^2+bx+a}e^2}{(ef-dg)^3(f+gx)}+\frac{(b^2-4ac)\,g\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)e}{8(ef-dg)^2\,(cf^2-bgf+ag^2)^{3/2}}-\frac{g(bf-2ag+(2cf-bg)x)\sqrt{cx^2+bx+a}e}{4(ef-dg)^2\,(cf^2-bgf+ag^2)\,(f+gx)^2}+\frac{g^2\,(cx^2+bx+a)^{3/2}}{3(ef-dg)\,(cf^2-bgf+ag^2)\,(f+gx)^3}+\frac{(b^2-4ac)\,g(2cf-bg)\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)}{16(ef-dg)\,(cf^2-bgf+ag^2)^{5/2}}-\frac{g(2cf-bg)(bf-2ag+(2cf-bg)x)\sqrt{cx^2+bx+a}}{8(ef-dg)\,(cf^2-bgf+ag^2)^2\,(f+gx)^2}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^4),x]
```

3.861. $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx$

output
```
(e^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)^3*(f + g*x)) - (g*(2*c*f - b*g)*(
b*f - 2*a*g + (2*c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(8*(e*f - d*g)*(c*f^
2 - b*f*g + a*g^2)^2*(f + g*x)^2) - (e*g*(b*f - 2*a*g + (2*c*f - b*g)*x)*S
qrt[a + b*x + c*x^2])/(4*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*(f + g*x)^2
) + (g^2*(a + b*x + c*x^2)^(3/2))/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(
f + g*x)^3) - (e^2*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(2*Sqrt[c]*(e*f - d*g)^4) + (e^3*(2*c*f - b*g)*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*g*(e*f - d*g)^4) -
(Sqrt[c]*e^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(g*(e
*f - d*g)^3) + (e^2*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*
c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*f
 - d*g)^4 + ((b^2 - 4*a*c)*g*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f -
 b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(16*(e*f
- d*g)*(c*f^2 - b*f*g + a*g^2)^(5/2)) + ((b^2 - 4*a*c)*e*g*ArcTanh[(b*f -
2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x
^2])])/(8*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)^(3/2)) + (e^2*(2*c*f - b*g
)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*S
qrt[a + b*x + c*x^2])])/(2*g*(e*f - d*g)^3*Sqrt[c*f^2 - b*f*g + a*g^2]) -
(e^3*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(
2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d*g)^4...
```

3.861.3.1 Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.861.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3776 vs. $2(845) = 1690$.

Time = 1.16 (sec) , antiderivative size = 3777, normalized size of antiderivative = 4.05

method	result	size
default	Expression too large to display	3777

input
```
int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x,method=_RETURNVERBOSE)
```

output
```
e^3/(d*g-e*f)^4*(((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)+1/2*(b*e-2*c*d)/e*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/
e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^
2-b*d*e+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^
2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*
c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))+1/g^3/(d
*g-e*f)*(-1/3/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)^3*((x+f/g)^2*c+(b*g-2*c*f)/g
*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)-1/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f
^2)*(-1/2/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)^2*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+
f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)-1/4*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*
(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*
g^2-b*f*g+c*f^2)/g^2)^(3/2)+1/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(((x+f/g
)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)
/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+
f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((a*g
^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+
f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+
(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))+2*c/(a*g^2-b*f*g+c*f^2)*g^2*(1/4
*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b
*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g...
```

3.861.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x, algorithm="fricas")
```

output
```
Timed out
```

3.861.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**4,x)
```

output
```
Timed out
```

3.861.7 Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(ex+d)(gx+f)^4}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^4), x)
```

3.861.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8076 vs. $2(845) = 1690$.

Time = 5.56 (sec) , antiderivative size = 8076, normalized size of antiderivative = 8.66

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x, algorithm="giac")
```

output
```
2*(c*d^2*e^2 - b*d*e^3 + a*e^4)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/((e^4*f^4 - 4*d*e^3*f^3*g
+ 6*d^2*e^2*f^2*g^2 - 4*d^3*e*f*g^3 + d^4*g^4)*sqrt(-c*d^2 + b*d*e - a*e^2
)) - 1/8*(16*c^3*d*e^2*f^5 - 8*b*c^2*e^3*f^5 - 40*b*c^2*d*e^2*f^4*g + 12*b
^2*c*e^3*f^4*g + 32*a*c^2*e^3*f^4*g + 42*b^2*c*d*e^2*f^3*g^2 - 8*a*c^2*d*e
^2*f^3*g^2 - 5*b^3*e^3*f^3*g^2 - 60*a*b*c*e^3*f^3*g^2 - 8*b^2*c*d^2*e*f^2*
g^3 + 32*a*c^2*d^2*e*f^2*g^3 - 15*b^3*d*e^2*f^2*g^3 - 20*a*b*c*d*e^2*f^2*g
^3 + 30*a*b^2*e^3*f^2*g^3 + 40*a^2*c*e^3*f^2*g^3 + 2*b^2*c*d^3*f*g^4 - 8*a
*c^2*d^3*f*g^4 + 5*b^3*d^2*e*f*g^4 - 20*a*b*c*d^2*e*f*g^4 + 20*a*b^2*d*e^2
*f*g^4 - 40*a^2*b*e^3*f*g^4 - b^3*d^3*g^5 + 4*a*b*c*d^3*g^5 - 2*a*b^2*d^2*
e*g^5 + 8*a^2*c*d^2*e*g^5 - 8*a^2*b*d*e^2*g^5 + 16*a^3*e^3*g^5)*arctan(-((
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*g + sqrt(c)*f)/sqrt(-c*f^2 + b*f*g - a*
g^2))/((c^2*e^4*f^8 - 4*c^2*d*e^3*f^7*g - 2*b*c*e^4*f^7*g + 6*c^2*d^2*e^2*
f^6*g^2 + 8*b*c*d*e^3*f^6*g^2 + b^2*e^4*f^6*g^2 + 2*a*c*e^4*f^6*g^2 - 4*c^
2*d^3*e*f^5*g^3 - 12*b*c*d^2*e^2*f^5*g^3 - 4*b^2*d*e^3*f^5*g^3 - 8*a*c*d*e
^3*f^5*g^3 - 2*a*b*e^4*f^5*g^3 + c^2*d^4*f^4*g^4 + 8*b*c*d^3*e*f^4*g^4 + 6
*b^2*d^2*e^2*f^4*g^4 + 12*a*c*d^2*e^2*f^4*g^4 + 8*a*b*d*e^3*f^4*g^4 + a^2*
e^4*f^4*g^4 - 2*b*c*d^4*f^3*g^5 - 4*b^2*d^3*e*f^3*g^5 - 8*a*c*d^3*e*f^3*g^
5 - 12*a*b*d^2*e^2*f^3*g^5 - 4*a^2*d*e^3*f^3*g^5 + b^2*d^4*f^2*g^6 + 2*a*c
*d^4*f^2*g^6 + 8*a*b*d^3*e*f^2*g^6 + 6*a^2*d^2*e^2*f^2*g^6 - 2*a*b*d^4*...
```

3.861.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{{\left(f+g\,x\right)}^4\,\left(d+e\,x\right)}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/((f + g*x)^4*(d + e*x)),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/((f + g*x)^4*(d + e*x)), x)
```

3.862 $\int \frac{(f+gx)^3(a+bx+cx^2)^{3/2}}{d+ex}\,dx$

3.862.1 Optimal result

Integrand size = 29, antiderivative size = 1098

$$\int \frac{(f+gx)^3(a+bx+cx^2)^{3/2}}{d+ex}\,dx =$$

$$-\frac{\left(3(7b^5e^5g^3-512c^5d^2(ef-dg)^3+128c^4e(5bd-4ae)(ef-dg)^3-4b^3ce^4g^2(9bef-3bdg+8aeg)+8bc^2e\right.}{}$$

$$+\frac{(7b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(9bef-3bdg+aeg)+24bc^2eg(3e^2f^2-3defg+d^2g^2)+2ceg(7b^2e^2g}{192c^3e^4}$$

$$+\frac{g^2(36cef-22cdg-7beg)(a+bx+cx^2)^{5/2}}{60c^2e^2}+\frac{g^3(d+ex)(a+bx+cx^2)^{5/2}}{6ce^2}$$

$$+\frac{\left(4ce(2cd-be)(8ce(bd-2ae)(24c^2e^2f^3+7b^2deg^3-4cdg^2(9bef-3bdg+aeg))-d(8bcd-3b^2e-4ace)\right.}{}$$

$$+\frac{(cd^2-bde+ae^2)^{3/2}(ef-dg)^3\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^7}$$

output
```
1/192*(7*b^3*e^3*g^3+64*c^3*(-d*g+e*f)^3-4*b*c*e^2*g^2*(a*e*g-3*b*d*g+9*b*
e*f)+24*b*c^2*e*g*(d^2*g^2-3*d*e*f*g+3*e^2*f^2)+2*c*e*g*(7*b^2*e^2*g^2-4*c
*e*g*(a*e*g-3*b*d*g+9*b*e*f)+24*c^2*(d^2*g^2-3*d*e*f*g+3*e^2*f^2))*x)*(c*x
^2+b*x+a)^(3/2)/c^3/e^4+1/60*g^2*(-7*b*e*g-22*c*d*g+36*c*e*f)*(c*x^2+b*x+a
)^(5/2)/c^2/e^2+1/6*g^3*(e*x+d)*(c*x^2+b*x+a)^(5/2)/c/e^2+1/3072*(4*c*e*(-
b*e+2*c*d)*(8*c*e*(-2*a*e+b*d)*(24*c^2*e^2*f^3+7*b^2*d*e*g^3-4*c*d*g^2*(a*
e*g-3*b*d*g+9*b*e*f))-d*(-4*a*c*e-3*b^2*e+8*b*c*d)*g*(7*b^2*e^2*g^2-4*c*e*
g*(a*e*g-3*b*d*g+9*b*e*f)+24*c^2*(d^2*g^2-3*d*e*f*g+3*e^2*f^2)))-2*(4*c^2*
d^2-1/2*b^2*e^2-2*c*e*(-a*e+b*d))*(8*c*e*(-b*e+2*c*d)*(24*c^2*e^2*f^3+7*b^
2*d*e*g^3-4*c*d*g^2*(a*e*g-3*b*d*g+9*b*e*f))-2*(8*c^2*d^2-4*b*c*d*e-3/2*b^
2*e^2+6*a*c*e^2)*g*(7*b^2*e^2*g^2-4*c*e*g*(a*e*g-3*b*d*g+9*b*e*f)+24*c^2*(
d^2*g^2-3*d*e*f*g+3*e^2*f^2))))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a
)^(1/2))/c^(9/2)/e^7+(a*e^2-b*d*e+c*d^2)^(3/2)*(-d*g+e*f)^3*arctanh(1/2*(b
*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^
7-1/1536*(21*b^5*e^5*g^3-1536*c^5*d^2*(-d*g+e*f)^3+384*c^4*e*(-4*a*e+5*b*d
)*(-d*g+e*f)^3-12*b^3*c*e^4*g^2*(8*a*e*g-3*b*d*g+9*b*e*f)+24*b*c^2*e^3*g*(
2*a^2*e^2*g^2+6*a*b*e*g*(-d*g+3*e*f)+3*b^2*(d^2*g^2-3*d*e*f*g+3*e^2*f^2))-
96*b*c^3*e^2*(2*b*(-d*g+e*f)^3+3*a*e*g*(d^2*g^2-3*d*e*f*g+3*e^2*f^2))+2*c*
e*(8*c*e*(-b*e+2*c*d)*(24*c^2*e^2*f^3+7*b^2*d*e*g^3-4*c*d*g^2*(a*e*g-3*b*d
*g+9*b*e*f))-2*(8*c^2*d^2-4*b*c*d*e-3/2*b^2*e^2+6*a*c*e^2)*g*(7*b^2*e^2...
```

3.862.2 Mathematica [A] (verified)

Time = 11.51 (sec) , antiderivative size = 743, normalized size of antiderivative = 0.68

$$\int \frac{(f+gx)^3 \left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \frac{5120(ef-dg)^3(a+x(b+cx))^{3/2} + \frac{1920eg(ef-dg)^2(b+2cx)(a+x(b+cx))^{3/2}}{c} + 3\cdots}{}$$

input
```
Integrate[((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

output
```
(5120*(e*f - d*g)^3*(a + x*(b + c*x))^(3/2) + (1920*e*g*(e*f - d*g)^2*(b +
 2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3072*e^2*g^2*(e*f - d*g)*(a + x*(b +
 c*x))^(5/2))/c + (2560*e^3*g^2*(f + g*x)*(a + x*(b + c*x))^(5/2))/c + (36
0*(b^2 - 4*a*c)*e*g*(e*f - d*g)^2*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b +
c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]
)]))/c^(5/2) - (60*e^2*g*(-2*c*f + b*g)*(e*f - d*g)*(2*Sqrt[c]*(b + 2*c*x)
*Sqrt[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 -
 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(7/2)
 + (e^3*g*(1792*g*(2*c*f - b*g)*(a + x*(b + c*x))^(5/2) + 5*(24*c^2*f^2 +
7*b^2*g^2 - 4*c*g*(6*b*f + a*g))*((16*(b + 2*c*x)*(a + x*(b + c*x))^(3/2))
/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2
 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(5/2)
)))/c^2 + (960*(e*f - d*g)^3*(-((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e
*(-2*b*d + 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])
 - 2*Sqrt[c]*(e*Sqrt[a + x*(b + c*x)]*(-(b^2*e^2) + 4*c^2*d*(-2*d + e*x) -
 2*c*e*(-5*b*d + 4*a*e + b*e*x)) + 8*c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*Ar
cTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]
*Sqrt[a + x*(b + c*x)])])))/(c^(3/2)*e^3))/(15360*e^4)
```

3.862.3 Rubi [A] (verified)

Time = 3.18 (sec) , antiderivative size = 1139, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {1267, 27, 2184, 27, 1231, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^3 \left(a+bx+cx^2\right)^{3/2}}{d+ex} \, dx$$

$$\Big\downarrow \ 1267$$

$$\frac{\int \frac{\left(cx^2+bx+a\right)^{3/2}\left(e^2g^2(36cef-22cdg-7beg)x^2-2eg\left(e(6bd+ae)g^2-c\left(18e^2f^2-5d^2g^2\right)\right)x+e\left(12ce^2f^3-d(5bd+2ae)g^3\right)\right)}{2(d+ex)}dx}{6ce^3}+\frac{g^3(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(cx^2+bx+a)^{3/2}\left(e^2g^2(36cef-22cdg-7beg)x^2-2eg\left(e(6bd+ae)g^2-c\left(18e^2f^2-5d^2g^2\right)\right)x+e\left(12ce^2f^3-d(5bd+2ae)g^3\right)\right)}{d+ex}dx}{12ce^3}+$$

$$\frac{g^3(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2}$$

$\downarrow$ 2184

$$\frac{\frac{\int \frac{5e^3\left(24c^2e^2f^3+7b^2deg^3-4cdg^2(9bef-3bdg+aeg)+g\left(24\left(3e^2f^2-3degf+d^2g^2\right)c^2-4eg(9bef-3bdg+aeg)c+7b^2e^2g^2\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{2(d+ex)}dx}{5ce^2}+\frac{eg^2(a+bx+cx}{}}{12ce^3}$$

$$\frac{g^3(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2}$$

$\downarrow$ 27

$$\frac{\frac{e\int \frac{\left(24c^2e^2f^3+7b^2deg^3-4cdg^2(9bef-3bdg+aeg)+g\left(24\left(3e^2f^2-3degf+d^2g^2\right)c^2-4eg(9bef-3bdg+aeg)c+7b^2e^2g^2\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{d+ex}dx}{2c}+\frac{eg^2(a+bx+cx^2}{}}{12ce^3}$$

$$\frac{g^3(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2}$$

$\downarrow$ 1231

$$e\left(\frac{\left(a+bx+cx^2\right)^{3/2}\left(2cegx\left(-4ceg(aeg-3bdg+9bef)+7b^2e^2g^2+24c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)-4bce^2g^2(aeg-3bdg+9bef)+7b^3e^3g^3+24bc^2eg\left(d^2g^2-3defg+3e^2f^2\right.\right.}{8ce^2}\right.$$

$$\frac{g^3(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2}$$

$\downarrow$ 27

$$e\left(\frac{\left(a+bx+cx^2\right)^{3/2}\left(2cegx\left(-4ceg(aeg-3bdg+9bef)+7b^2e^2g^2+24c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)-4bce^2g^2(aeg-3bdg+9bef)+7b^3e^3g^3+24bc^2eg\left(d^2g^2-3defg+3e^2f^2\right.\right.}{8ce^2}\right.$$

$$\frac{g^3(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2}$$

$\downarrow$ 1231

3.862. $\int \frac{(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx$

$$\frac{(d+ex)\left(cx^2+bx+a\right)^{5/2}g^3}{6ce^2}+$$

$$\frac{eg^2(36cef-22cdg-7beg)\left(cx^2+bx+a\right)^{5/2}}{5c}+\frac{e\left(\frac{\left(7b^3e^3g^3-4bce^2(9bef-3bdg+aeg)g^2+24bc^2e\left(3e^2f^2-3degf+d^2g^2\right)g+2ce\left(24\left(3e^2f^2-3degf+d^2g^2\right)c^2-4e\cdots\right.\right.}{8ce^2}\cdots\right.}{\cdots}$$

$\big\downarrow$ 27

$$\frac{(d+ex)\left(cx^2+bx+a\right)^{5/2}g^3}{6ce^2}+$$

$$\frac{eg^2(36cef-22cdg-7beg)\left(cx^2+bx+a\right)^{5/2}}{5c}+\frac{e\left(\frac{\left(7b^3e^3g^3-4bce^2(9bef-3bdg+aeg)g^2+24bc^2e\left(3e^2f^2-3degf+d^2g^2\right)g+2ce\left(24\left(3e^2f^2-3degf+d^2g^2\right)c^2-4e\cdots\right.\right.}{8ce^2}\cdots\right.}{\cdots}$$

$\big\downarrow$ 1269

$$\frac{(d+ex)\left(cx^2+bx+a\right)^{5/2}g^3}{6ce^2}+$$

$$\frac{eg^2(36cef-22cdg-7beg)\left(cx^2+bx+a\right)^{5/2}}{5c}+\frac{e\left(\frac{\left(7b^3e^3g^3-4bce^2(9bef-3bdg+aeg)g^2+24bc^2e\left(3e^2f^2-3degf+d^2g^2\right)g+2ce\left(24\left(3e^2f^2-3degf+d^2g^2\right)c^2-4e\cdots\right.\right.}{8ce^2}\cdots\right.}{\cdots}$$

$\big\downarrow$ 1092

$$\frac{(d+ex)\left(cx^2+bx+a\right)^{5/2}g^3}{6ce^2}+$$

$$\frac{eg^2(36cef-22cdg-7beg)\left(cx^2+bx+a\right)^{5/2}}{5c}+\frac{e\left(\frac{\left(7b^3e^3g^3-4bce^2(9bef-3bdg+aeg)g^2+24bc^2e\left(3e^2f^2-3degf+d^2g^2\right)g+2ce\left(24\left(3e^2f^2-3degf+d^2g^2\right)c^2-4e\cdots\right.\right.}{8ce^2}\cdots\right.}{\cdots}$$

$\big\downarrow$ 219

3.862. $\int \frac{(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx$

$$\frac{(d+ex)\left(cx^2+bx+a\right)^{5/2}g^3}{6ce^2}+$$

$$\frac{eg^2(36cef-22cdg-7beg)\left(cx^2+bx+a\right)^{5/2}}{5c}+\frac{e\left(\frac{\left(7b^3e^3g^3-4bce^2(9bef-3bdg+aeg)g^2+24bc^2e\left(3e^2f^2-3degf+d^2g^2\right)g+2ce\left(24\left(3e^2f^2-3degf+d^2g^2\right)c^2-4e\right.\right.}{8ce^2}\right.}{}$$

$\downarrow$ 1154

$$\frac{(d+ex)\left(cx^2+bx+a\right)^{5/2}g^3}{6ce^2}+$$

$$\frac{eg^2(36cef-22cdg-7beg)\left(cx^2+bx+a\right)^{5/2}}{5c}+\frac{e\left(\frac{\left(7b^3e^3g^3-4bce^2(9bef-3bdg+aeg)g^2+24bc^2e\left(3e^2f^2-3degf+d^2g^2\right)g+2ce\left(24\left(3e^2f^2-3degf+d^2g^2\right)c^2-4e\right.\right.}{8ce^2}\right.}{}$$

$\downarrow$ 219

$$\frac{(d+ex)\left(cx^2+bx+a\right)^{5/2}g^3}{6ce^2}+$$

$$\frac{eg^2(36cef-22cdg-7beg)\left(cx^2+bx+a\right)^{5/2}}{5c}+\frac{e\left(\frac{\left(7b^3e^3g^3-4bce^2(9bef-3bdg+aeg)g^2+24bc^2e\left(3e^2f^2-3degf+d^2g^2\right)g+2ce\left(24\left(3e^2f^2-3degf+d^2g^2\right)c^2-4e\right.\right.}{8ce^2}\right.}{}$$

input
```
Int[((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

3.862. $\int \frac{(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx$

output
```
(g^3*(d + e*x)*(a + b*x + c*x^2)^(5/2))/(6*c*e^2) + ((e*g^2*(36*c*e*f - 22
*c*d*g - 7*b*e*g)*(a + b*x + c*x^2)^(5/2))/(5*c) + (e*(((7*b^3*e^3*g^3 + 6
4*c^3*(e*f - d*g)^3 - 4*b*c*e^2*g^2*(9*b*e*f - 3*b*d*g + a*e*g) + 24*b*c^2
*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2) + 2*c*e*g*(7*b^2*e^2*g^2 - 4*c*e*g*
(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*x)
*(a + b*x + c*x^2)^(3/2))/(8*c*e^2) - (((3*(7*b^5*e^5*g^3 - 512*c^5*d^2*(e
*f - d*g)^3 + 128*c^4*e*(5*b*d - 4*a*e)*(e*f - d*g)^3 - 4*b^3*c*e^4*g^2*(9
*b*e*f - 3*b*d*g + 8*a*e*g) + 8*b*c^2*e^3*g*(2*a^2*e^2*g^2 + 6*a*b*e*g*(3*
e*f - d*g) + 3*b^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)) - 32*b*c^3*e^2*(2*b*
(e*f - d*g)^3 + 3*a*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))) + 2*c*e*(8*c*e
*(2*c*d - b*e)*(24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*
d*g + a*e*g)) - 2*(8*c^2*d^2 - 4*b*c*d*e - (3*b^2*e^2)/2 + 6*a*c*e^2)*g*(7
*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3
*d*e*f*g + d^2*g^2)))*x)*Sqrt[a + b*x + c*x^2])/(4*c*e^2) - (((4*c*e*(2*c*
d - b*e)*(8*c*e*(b*d - 2*a*e)*(24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*
(9*b*e*f - 3*b*d*g + a*e*g)) - d*(8*b*c*d - 3*b^2*e - 4*a*c*e)*g*(7*b^2*e^
2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*
g + d^2*g^2))) - 2*(4*c^2*d^2 - (b^2*e^2)/2 - 2*c*e*(b*d - a*e))*(8*c*e*(2
*c*d - b*e)*(24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*d*g
 + a*e*g)) - 2*(8*c^2*d^2 - 4*b*c*d*e - (3*b^2*e^2)/2 + 6*a*c*e^2)*g*(7...
```

3.862.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.862. $\int \frac{(f+gx)^3 (a+bx+cx^2)^{3/2}}{d+ex} \, dx$

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.862. $\int \frac{(f+gx)^3 (a+bx+cx^2)^{3/2}}{d+ex} \, dx$

3.862.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 1409, normalized size of antiderivative = 1.28

method	result	size
default	Expression too large to display	1409
risch	Expression too large to display	2365

input
```
int((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
(-d^3*g^3+3*d^2*e*f*g^2-3*d*e^2*f^2*g+e^3*f^3)/e^4*(1/3*((x+d/e)^2*c+(b*e-
2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)+1/2*(b*e-2*c*d)/e*(1/4*(2*
c*(x+d/e)+(b*e-2*c*d)/e)/c*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)+1/8*(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/c^(
3/2)*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(
x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)))+(a*e^2-b*d*e+c*d^2)/e^2*(((x+d/e)^
2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*(b*e-2*c*d)/e
*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/
e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-b*d*e+c*d^2)/e^2/((a*e^2
-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/
e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))+g/e^3*(d^2*g^2*(1/8*(2*c*x+b)/c*(
c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)
+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+e^2
*g^2*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*
b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)
/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a
*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln(
(1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))+3*e^2*f^2*(1/8*(2*c*x+b)/c*...
```

3.862.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3 (a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.862.6 Sympy [F]

$$\int \frac{(f+gx)^3 (a+bx+cx^2)^{3/2}}{d+ex}\,dx = \int \frac{(f+gx)^3 (a+bx+cx^2)^{\frac{3}{2}}}{d+ex}\,dx$$

input
```
integrate((g*x+f)**3*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

output
```
Integral((f + g*x)**3*(a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

3.862.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3 (a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.862.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3 (a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.862.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3 (a+bx+cx^2)^{3/2}}{d+ex} \, dx = \int \frac{(f+g\,x)^3\,(c\,x^2+b\,x+a)^{3/2}}{d+e\,x} \, dx$$

input
```
int(((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x),x)
```

output
```
int(((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x), x)
```

3.863 $\int \frac{(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx$

3.863.1 Optimal result

Integrand size = 29, antiderivative size = 662

$$\int \frac{(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \frac{(3b^4e^4g^2+128c^4d^2(ef-dg)^2-32c^3e(5bd-4ae)(ef-dg)^2-6b^2ce^3g(\cdots}{\cdots}$$

$$-\frac{(3b^2e^2g^2-16c^2(ef-dg)^2-6bceg(2ef-dg)-6ceg(4cef-2cdg-beg)x)\left(a+bx+cx^2\right)^{3/2}}{48c^2e^3}$$

$$+\frac{g^2\left(a+bx+cx^2\right)^{5/2}}{5ce}$$

$$-\frac{(3b^5e^5g^2+256c^5d^3(ef-dg)^2-384c^4de(bd-ae)(ef-dg)^2-6b^3ce^4g(2bef-bdg+4aeg)+16bc^2e^3(3a\cdots}{\cdots}$$

$$+\frac{\left(cd^2-bde+ae^2\right)^{3/2}(ef-dg)^2\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^6}$$

output
```
-1/48*(3*b^2*e^2*g^2-16*c^2*(-d*g+e*f)^2-6*b*c*e*g*(-d*g+2*e*f)-6*c*e*g*(-
b*e*g-2*c*d*g+4*c*e*f)*x)*(c*x^2+b*x+a)^(3/2)/c^2/e^3+1/5*g^2*(c*x^2+b*x+a
)^(5/2)/c/e-1/256*(3*b^5*e^5*g^2+256*c^5*d^3*(-d*g+e*f)^2-384*c^4*d*e*(-a*
e+b*d)*(-d*g+e*f)^2-6*b^3*c*e^4*g*(4*a*e*g-b*d*g+2*b*e*f)+16*b*c^2*e^3*(3*
a^2*e^2*g^2+b^2*(-d*g+e*f)^2+3*a*b*e*g*(-d*g+2*e*f))+96*c^3*e^2*(b^2*d*(-d
*g+e*f)^2-2*a*b*e*(-d*g+e*f)^2-a^2*e^2*g*(-d*g+2*e*f)))*arctanh(1/2*(2*c*x
+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)/e^6+(a*e^2-b*d*e+c*d^2)^(3/2)*(-d
*g+e*f)^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)
/(c*x^2+b*x+a)^(1/2))/e^6+1/128*(3*b^4*e^4*g^2+128*c^4*d^2*(-d*g+e*f)^2-32
*c^3*e*(-4*a*e+5*b*d)*(-d*g+e*f)^2-6*b^2*c*e^3*g*(2*a*e*g-b*d*g+2*b*e*f)+8
*b*c^2*e^2*(2*b*(-d*g+e*f)^2+3*a*e*g*(-d*g+2*e*f))+2*c*e*((16*c^2*d^2-3*b^
2*e^2-4*c*e*(-3*a*e+2*b*d))*g*(-b*e*g-2*c*d*g+4*c*e*f)-8*c*e*(-b*e+2*c*d)*
(-b*d*g^2+2*c*e*f^2))*x)*(c*x^2+b*x+a)^(1/2)/c^3/e^5
```

3.863.2 Mathematica [A] (verified)

Time = 10.88 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.81

$$\int \frac{(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \frac{1280(ef-dg)^2(a+x(b+cx))^{3/2}+\frac{480eg(ef-dg)(b+2cx)(a+x(b+cx))^{3/2}}{c}+768\cdots}{\cdots}$$

input
```
Integrate[((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

output
```
(1280*(e*f - d*g)^2*(a + x*(b + c*x))^(3/2) + (480*e*g*(e*f - d*g)*(b + 2*
c*x)*(a + x*(b + c*x))^(3/2))/c + (768*e^2*g^2*(a + x*(b + c*x))^(5/2))/c
+ (90*(b^2 - 4*a*c)*e*g*(e*f - d*g)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b
+ c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x
)])]))/c^(5/2) + (15*e^2*g*(2*c*f - b*g)*((16*(b + 2*c*x)*(a + x*(b + c*x)
)^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)
] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))
/c^(5/2)))/c + (240*(e*f - d*g)^2*(-((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 +
4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x
)])]) - 2*Sqrt[c]*(e*Sqrt[a + x*(b + c*x)]*(-(b^2*e^2) + 4*c^2*d*(-2*d + e
*x) - 2*c*e*(-5*b*d + 4*a*e + b*e*x)) + 8*c*(c*d^2 + e*(-(b*d) + a*e))^(3/
2)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) +
a*e)]*Sqrt[a + x*(b + c*x)])])))/(c^(3/2)*e^3))/(3840*e^3)
```

3.863.3 Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 696, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {1267, 27, 1231, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2 \left(a+bx+cx^2\right)^{3/2}}{d+ex}\, dx$$

$$\Big\downarrow \text{1267}$$

$$\frac{\int \frac{5e\left(2cef^2-bdg^2+g(4cef-2cdg-beg)x\right)\left(cx^2+bx+a\right)^{3/2}}{2(d+ex)}dx}{5ce^2}+\frac{g^2\left(a+bx+cx^2\right)^{5/2}}{5ce}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(2cef^2-bdg^2+g(4cef-2cdg-beg)x\right)\left(cx^2+bx+a\right)^{3/2}}{d+ex}dx}{2ce}+\frac{g^2\left(a+bx+cx^2\right)^{5/2}}{5ce}$$

$$\Big\downarrow \text{1231}$$

$$-\frac{\int -\frac{\left(d\left(-3eb^2+8cdb-4ace\right)g(4cef-2cdg-beg)-8ce(bd-2ae)\left(2cef^2-bdg^2\right)+\left(\left(16c^2d^2-3b^2e^2-4ce(2bd-3ae)\right)g(4cef-2cdg-beg)-8ce(2cd-be)\left(2cef^2-bdg^2\right)\right.\right.}{2(d+ex)}}{8ce^2}}{2ce}$$

$$\frac{g^2\left(a+bx+cx^2\right)^{5/2}}{5ce}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\left(d\left(-3eb^2+8cdb-4ace\right)g(4cef-2cdg-beg)-8ce(bd-2ae)\left(2cef^2-bdg^2\right)+\left(\left(16c^2d^2-3b^2e^2-4ce(2bd-3ae)\right)g(4cef-2cdg-beg)-8ce(2cd-be)\left(2cef^2-bdg^2\right)\right)x\right)}{d+ex}}{16ce^2}}{2ce}$$

$$\frac{g^2\left(a+bx+cx^2\right)^{5/2}}{5ce}$$

$$\Big\downarrow \text{1231}$$

$$\frac{\sqrt{a+bx+cx^2}\left(2cex\left(g\left(-4ce(2bd-3ae)-3b^2e^2+16c^2d^2\right)(-beg-2cdg+4cef)-8ce(2cd-be)\left(2cef^2-bdg^2\right)\right)-6b^2ce^3g(2aeg-bdg+2bef)-32c^3e(5bd-4ae)(ef-dg)^2+\right.}{4ce^2}$$

$$\frac{g^2\left(a+bx+cx^2\right)^{5/2}}{5ce}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\sqrt{a+bx+cx^2}\left(2cex\left(g\left(-4ce(2bd-3ae)-3b^2e^2+16c^2d^2\right)(-beg-2cdg+4cef)-8ce(2cd-be)\left(2cef^2-bdg^2\right)\right)-6b^2ce^3g(2aeg-bdg+2bef)-32c^3e(5bd-4ae)(ef-dg)^2+}{4ce^2}}{}$$

$$\frac{g^2\left(a+bx+cx^2\right)^{5/2}}{5ce}$$

$$\Big\downarrow \; 1269$$

$$\frac{\frac{\sqrt{a+bx+cx^2}\left(2cex\left(g\left(-4ce(2bd-3ae)-3b^2e^2+16c^2d^2\right)(-beg-2cdg+4cef)-8ce(2cd-be)\left(2cef^2-bdg^2\right)\right)-6b^2ce^3g(2aeg-bdg+2bef)-32c^3e(5bd-4ae)(ef-dg)^2+}{4ce^2}}{}$$

$$\frac{g^2\left(a+bx+cx^2\right)^{5/2}}{5ce}$$

$$\Big\downarrow \; 1092$$

$$\frac{\frac{\sqrt{a+bx+cx^2}\left(2cex\left(g\left(-4ce(2bd-3ae)-3b^2e^2+16c^2d^2\right)(-beg-2cdg+4cef)-8ce(2cd-be)\left(2cef^2-bdg^2\right)\right)-6b^2ce^3g(2aeg-bdg+2bef)-32c^3e(5bd-4ae)(ef-dg)^2+}{4ce^2}}{}$$

$$\frac{g^2\left(a+bx+cx^2\right)^{5/2}}{5ce}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\sqrt{a+bx+cx^2}\left(2cex\left(g\left(-4ce(2bd-3ae)-3b^2e^2+16c^2d^2\right)(-beg-2cdg+4cef)-8ce(2cd-be)\left(2cef^2-bdg^2\right)\right)-6b^2ce^3g(2aeg-bdg+2bef)-32c^3e(5bd-4ae)(ef-dg)^2+}{4ce^2}}{}$$

$$\frac{g^2\left(a+bx+cx^2\right)^{5/2}}{5ce}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{\sqrt{a+bx+cx^2}\left(2cex\left(g\left(-4ce(2bd-3ae)-3b^2e^2+16c^2d^2\right)(-beg-2cdg+4cef)-8ce(2cd-be)\left(2cef^2-bdg^2\right)\right)-6b^2ce^3g(2aeg-bdg+2bef)-32c^3e(5bd-4ae)(ef-dg)^2+}{4ce^2}}{}$$

$$\frac{g^2\left(a+bx+cx^2\right)^{5/2}}{5ce}$$

$$\Big\downarrow \; 219$$

3.863. $\int \frac{(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx$

$$\frac{\frac{\sqrt{a+bx+cx^2}\left(2cex\left(g\left(-4ce(2bd-3ae)-3b^2e^2+16c^2d^2\right)(-beg-2cdg+4cef)-8ce(2cd-be)\left(2cef^2-bdg^2\right)\right)-6b^2ce^3g(2aeg-bdg+2bef)-32c^3e(5bd-4ae)(ef-dg)^2+}{4ce^2}}{}$$

$$\frac{g^2\left(a+bx+cx^2\right)^{5/2}}{5ce}$$

input
```
Int[((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

output
```
(g^2*(a + b*x + c*x^2)^(5/2))/(5*c*e) + (-1/24*((3*b^2*e^2*g^2 - 16*c^2*(e
*f - d*g)^2 - 6*b*c*e*g*(2*e*f - d*g) - 6*c*e*g*(4*c*e*f - 2*c*d*g - b*e*g
)*x)*(a + b*x + c*x^2)^(3/2))/(c*e^2) + (((3*b^4*e^4*g^2 + 128*c^4*d^2*(e*
f - d*g)^2 - 32*c^3*e*(5*b*d - 4*a*e)*(e*f - d*g)^2 - 6*b^2*c*e^3*g*(2*b*e
*f - b*d*g + 2*a*e*g) + 8*b*c^2*e^2*(2*b*(e*f - d*g)^2 + 3*a*e*g*(2*e*f -
d*g)) + 2*c*e*((16*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*g*(4*c*e*f
 - 2*c*d*g - b*e*g) - 8*c*e*(2*c*d - b*e)*(2*c*e*f^2 - b*d*g^2))*x)*Sqrt[a
 + b*x + c*x^2])/(4*c*e^2) - (((4*c*e*(2*c*d - b*e)*(d*(8*b*c*d - 3*b^2*e
- 4*a*c*e)*g*(4*c*e*f - 2*c*d*g - b*e*g) - 8*c*e*(b*d - 2*a*e)*(2*c*e*f^2
- b*d*g^2)) - 2*(4*c^2*d^2 - (b^2*e^2)/2 - 2*c*e*(b*d - a*e))*((16*c^2*d^2
 - 3*b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*g*(4*c*e*f - 2*c*d*g - b*e*g) - 8*c*
e*(2*c*d - b*e)*(2*c*e*f^2 - b*d*g^2)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])])/(Sqrt[c]*e) - (256*c^3*(c*d^2 - b*d*e + a*e^2)^(3/2)
*(e*f - d*g)^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d
*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e)/(8*c*e^2))/(16*c*e^2))/(2*c*e)
```

3.863.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

3.863. $\int \frac{(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx$

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.863. $\int \frac{(f+gx)^2(a+bx+cx^2)^{3/2}}{d+ex}\,dx$

3.863.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 998, normalized size of antiderivative = 1.51

method	result
default	$-\frac{g\left(dg\left(\frac{(2cx+b)\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^{2}\right)\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx+a}}{4c}+\frac{\left(4ac-b^{2}\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)-2ef\left(\frac{(2cx+b)\left(c\,x^{2}+bx+a\right.}{8c}\right.\right.}{}$
risch	Expression too large to display

input
```
int((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-g/e^2*(d*g*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(
2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))))-2*e*f*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16
*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2
)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-e*g*(1/5*(c*x^2+b*x+a)^(5/
2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*
(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x+a)^(1/2))))))+(d^2*g^2-2*d*e*f*g+e^2*f^2)/e^3*(1/3*((x+d/e
)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)+1/2*(b*e-2*c*d)
/e*(1/4*(2*c*(x+d/e)+(b*e-2*c*d)/e)/c*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/8*(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)
^2/e^2)/c^(3/2)*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c+(b*e
-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)))+(a*e^2-b*d*e+c*d^2)/e^2
*(((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*(b
*e-2*c*d)/e*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c+(b*e-2*c
*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-b*d*e+c*d^2)/
e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c
*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e
*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))))
```

3.863.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2 \left(a+bx+cx^2\right)^{3/2}}{d+ex}\, dx = \text{Timed out}$$

input
```
integrate((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.863.6 Sympy [F]

$$\int \frac{(f+gx)^2 \left(a+bx+cx^2\right)^{3/2}}{d+ex}\, dx = \int \frac{(f+gx)^2 \left(a+bx+cx^2\right)^{\frac{3}{2}}}{d+ex}\, dx$$

input
```
integrate((g*x+f)**2*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

output
```
Integral((f + g*x)**2*(a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

3.863.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2 \left(a+bx+cx^2\right)^{3/2}}{d+ex}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.863.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2 (a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.863.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2 (a+bx+cx^2)^{3/2}}{d+ex}\,dx = \int \frac{(f+g\,x)^2\,(c\,x^2+b\,x+a)^{3/2}}{d+e\,x}\,dx$$

input
```
int(((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x),x)
```

output
```
int(((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x), x)
```

3.864 $\int \frac{(f+gx)(a+bx+cx^2)^{3/2}}{d+ex}\,dx$

3.864.1 Optimal result

Integrand size = 27, antiderivative size = 441

$$\int \frac{(f+gx)\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx =$$
$$-\frac{(3b^3e^3g-64c^3d^2(ef-dg)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(2bef-2bdg+3aeg)+2ce(3b^2e^2g+16c}{64c^2e^4}$$
$$+\frac{(8cef-8cdg+3beg+6cegx)\left(a+bx+cx^2\right)^{3/2}}{24ce^2}$$
$$+\frac{(3b^4e^4g-128c^4d^3(ef-dg)+192c^3de(bd-ae)(ef-dg)-8b^2ce^3(bef-bdg+3aeg)+48c^2e^2(a^2e^2g-b}{128c^{5/2}e^5}$$
$$+\frac{\left(cd^2-bde+ae^2\right)^{3/2}(ef-dg)\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^5}$$

output
```
1/24*(6*c*e*g*x+3*b*e*g-8*c*d*g+8*c*e*f)*(c*x^2+b*x+a)^(3/2)/c/e^2+1/128*(
3*b^4*e^4*g-128*c^4*d^3*(-d*g+e*f)+192*c^3*d*e*(-a*e+b*d)*(-d*g+e*f)-8*b^2
*c*e^3*(3*a*e*g-b*d*g+b*e*f)+48*c^2*e^2*(a^2*e^2*g-b^2*d*(-d*g+e*f)+2*a*b*
e*(-d*g+e*f)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/
e^5+(a*e^2-b*d*e+c*d^2)^(3/2)*(-d*g+e*f)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*
d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^5-1/64*(3*b^3*e^3*g
-64*c^3*d^2*(-d*g+e*f)+16*c^2*e*(-4*a*e+5*b*d)*(-d*g+e*f)-4*b*c*e^2*(3*a*e
*g-2*b*d*g+2*b*e*f)+2*c*e*(3*b^2*e^2*g+16*c^2*d*(-d*g+e*f)-4*c*e*(3*a*e*g-
2*b*d*g+2*b*e*f))*x)*(c*x^2+b*x+a)^(1/2)/c^2/e^4
```

3.864.2 Mathematica [A] (verified)

Time = 2.48 (sec) , antiderivative size = 427, normalized size of antiderivative = 0.97

$$\int \frac{(f+gx)\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \frac{2e\sqrt{a+x(b+cx)}\left(-9b^3e^3g-16c^3\left(12d^3g-6d^2e(2f+gx)+2de^2x(3f+2gx)-e^3x^2(4f+3gx)\right)+6bce^2(1\right.}{}$$

input
```
Integrate[((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

output
```
((2*e*Sqrt[a + x*(b + c*x)]*(-9*b^3*e^3*g - 16*c^3*(12*d^3*g - 6*d^2*e*(2*
f + g*x) + 2*d*e^2*x*(3*f + 2*g*x) - e^3*x^2*(4*f + 3*g*x)) + 6*b*c*e^2*(1
0*a*e*g + b*(4*e*f - 4*d*g + e*g*x)) + 8*c^2*e*(a*e*(32*e*f - 32*d*g + 15*
e*g*x) + b*(30*d^2*g - 2*d*e*(15*f + 7*g*x) + e^2*x*(14*f + 9*g*x)))))/c^2
 - 768*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(c*d^2 + e*(-(b*d) + a*e))*(-(e*f) +
 d*g)*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) +
 e*(b*d - a*e)]] - (3*(3*b^4*e^4*g + 128*c^4*d^3*(-(e*f) + d*g) - 192*c^3*
d*e*(b*d - a*e)*(-(e*f) + d*g) - 8*b^2*c*e^3*(b*e*f - b*d*g + 3*a*e*g) + 4
8*c^2*e^2*(a^2*e^2*g + 2*a*b*e*(e*f - d*g) + b^2*d*(-(e*f) + d*g)))*Log[b
+ 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/c^(5/2))/(384*e^5)
```

3.864.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1231, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx$$

$$\Big\downarrow \; 1231$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}(3beg-8cdg+8cef+6cegx)}{24ce^2}-\frac{\int \frac{\left(3degb^2+8cd(ef-dg)b-4ace(4ef-dg)+\left(16d(ef-dg)c^2-4e(2bef-2bdg+3aeg)c+3b^2e^2g\right)x\right)\sqrt{cx^2+bx+a}}{2(d+ex)}dx}{8ce^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}(3beg-8cdg+8cef+6cegx)}{24ce^2}-\frac{\int \frac{\left(3degb^2+8cd(ef-dg)b-4ace(4ef-dg)+\left(16d(ef-dg)c^2-4e(2bef-2bdg+3aeg)c+3b^2e^2g\right)x\right)\sqrt{cx^2+bx+a}}{d+ex}dx}{16ce^2}$$

$\big\downarrow$ 1231

$$\frac{\left(a+bx+cx^2\right)^{3/2}(3beg-8cdg+8cef+6cegx)}{24ce^2}-$$

$$\frac{\frac{\sqrt{a+bx+cx^2}\left(2cex\left(-4ce(3aeg-2bdg+2bef)+3b^2e^2g+16c^2d(ef-dg)\right)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(3aeg-2bdg+2bef)+3b^3e^3g-64c^3d^2(ef-\right.}{4ce^2}}{}$$

$\big\downarrow$ 27

$$\frac{\left(a+bx+cx^2\right)^{3/2}(3beg-8cdg+8cef+6cegx)}{24ce^2}-$$

$$\frac{\frac{\sqrt{a+bx+cx^2}\left(2cex\left(-4ce(3aeg-2bdg+2bef)+3b^2e^2g+16c^2d(ef-dg)\right)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(3aeg-2bdg+2bef)+3b^3e^3g-64c^3d^2(ef-\right.}{4ce^2}}{}$$

$\big\downarrow$ 1269

$$\frac{\left(a+bx+cx^2\right)^{3/2}(3beg-8cdg+8cef+6cegx)}{24ce^2}-$$

$$\frac{\frac{\sqrt{a+bx+cx^2}\left(2cex\left(-4ce(3aeg-2bdg+2bef)+3b^2e^2g+16c^2d(ef-dg)\right)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(3aeg-2bdg+2bef)+3b^3e^3g-64c^3d^2(ef-\right.}{4ce^2}}{}$$

$\big\downarrow$ 1092

$$\frac{\left(a+bx+cx^2\right)^{3/2}(3beg-8cdg+8cef+6cegx)}{24ce^2}-$$

$$\frac{\frac{\sqrt{a+bx+cx^2}\left(2cex\left(-4ce(3aeg-2bdg+2bef)+3b^2e^2g+16c^2d(ef-dg)\right)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(3aeg-2bdg+2bef)+3b^3e^3g-64c^3d^2(ef-\right.}{4ce^2}}{}$$

$\big\downarrow$ 219

$$\frac{\left(a+bx+cx^2\right)^{3/2}(3beg-8cdg+8cef+6cegx)}{24ce^2}-$$

$$\frac{\frac{\sqrt{a+bx+cx^2}\left(2cex\left(-4ce(3aeg-2bdg+2bef)+3b^2e^2g+16c^2d(ef-dg)\right)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(3aeg-2bdg+2bef)+3b^3e^3g-64c^3d^2(ef-\right.}{4ce^2}}{}$$

$\big\downarrow$ 1154

3.864. $\int \frac{(f+gx)\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx$

$$\frac{\left(a+bx+cx^2\right)^{3/2}(3beg-8cdg+8cef+6cegx)}{24ce^2}-$$

$$\frac{\sqrt{a+bx+cx^2}\left(2cex\left(-4ce(3aeg-2bdg+2bef)+3b^2e^2g+16c^2d(ef-dg)\right)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(3aeg-2bdg+2bef)+3b^3e^3g-64c^3d^2(ef-\right.}{4ce^2}$$

$\downarrow$ 219

$$\frac{\left(a+bx+cx^2\right)^{3/2}(3beg-8cdg+8cef+6cegx)}{24ce^2}-$$

$$\frac{\sqrt{a+bx+cx^2}\left(2cex\left(-4ce(3aeg-2bdg+2bef)+3b^2e^2g+16c^2d(ef-dg)\right)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(3aeg-2bdg+2bef)+3b^3e^3g-64c^3d^2(ef-\right.}{4ce^2}$$

input
```
Int[((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

output
```
((8*c*e*f - 8*c*d*g + 3*b*e*g + 6*c*e*g*x)*(a + b*x + c*x^2)^(3/2))/(24*c*
e^2) - (((3*b^3*e^3*g - 64*c^3*d^2*(e*f - d*g) + 16*c^2*e*(5*b*d - 4*a*e)*
(e*f - d*g) - 4*b*c*e^2*(2*b*e*f - 2*b*d*g + 3*a*e*g) + 2*c*e*(3*b^2*e^2*g
 + 16*c^2*d*(e*f - d*g) - 4*c*e*(2*b*e*f - 2*b*d*g + 3*a*e*g))*x)*Sqrt[a +
 b*x + c*x^2])/(4*c*e^2) - (((3*b^4*e^4*g - 128*c^4*d^3*(e*f - d*g) + 192*
c^3*d*e*(b*d - a*e)*(e*f - d*g) - 8*b^2*c*e^3*(b*e*f - b*d*g + 3*a*e*g) +
48*c^2*e^2*(a^2*e^2*g - b^2*d*(e*f - d*g) + 2*a*b*e*(e*f - d*g)))*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e) + (128*c^2*(c*
d^2 - b*d*e + a*e^2)^(3/2)*(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e
)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e)/(8*c*e^2))
/(16*c*e^2)
```

3.864.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.864. $\int \frac{(f+gx)\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx$

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.864.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 735, normalized size of antiderivative = 1.67

method	result
risch	$\frac{\left(48g\,e^{3}x^{3}c^{3}+72b\,c^{2}e^{3}g\,x^{2}-64c^{3}d\,e^{2}g\,x^{2}+64c^{3}e^{3}f\,x^{2}+120a\,c^{2}e^{3}gx+6b^{2}c\,e^{3}gx-112b\,c^{2}d\,e^{2}gx+112b\,c^{2}e^{3}fx+96c^{3}d^{2}egx-96c^{3}d\,e^{2}f\right.}{1}$
default	$\frac{g\left(\frac{(2cx+b)\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^{2}\right)\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx+a}}{4c}+\frac{\left(4ac-b^{2}\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)}{e}+\frac{(-dg+ef)\left(\frac{\left(\left(x+\frac{d}{e}\right)^{2}c+\frac{(be-}{}\right.}{}\right.}{}$

input
```
int((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/192/c^2*(48*c^3*e^3*g*x^3+72*b*c^2*e^3*g*x^2-64*c^3*d*e^2*g*x^2+64*c^3*e
^3*f*x^2+120*a*c^2*e^3*g*x+6*b^2*c*e^3*g*x-112*b*c^2*d*e^2*g*x+112*b*c^2*e
^3*f*x+96*c^3*d^2*e*g*x-96*c^3*d*e^2*f*x+60*a*b*c*e^3*g-256*a*c^2*d*e^2*g+
256*a*c^2*e^3*f-9*b^3*e^3*g-24*b^2*c*d*e^2*g+24*b^2*c*e^3*f+240*b*c^2*d^2*
e*g-240*b*c^2*d*e^2*f-192*c^3*d^3*g+192*c^3*d^2*e*f)*(c*x^2+b*x+a)^(1/2)/e
^4+1/128/e^4/c^2*(128*(a^2*d*e^4*g-a^2*e^5*f-2*a*b*d^2*e^3*g+2*a*b*d*e^4*f
+2*a*c*d^3*e^2*g-2*a*c*d^2*e^3*f+b^2*d^3*e^2*g-b^2*d^2*e^3*f-2*b*c*d^4*e*g
+2*b*c*d^3*e^2*f+c^2*d^5*g-c^2*d^4*e*f)*c^2/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+
c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2))/(x+d/e))+(48*a^2*c^2*e^4*g-24*a*b^2*c*e^4*g-96*a*b*c^2*d*e^3*g+
96*a*b*c^2*e^4*f+192*a*c^3*d^2*e^2*g-192*a*c^3*d*e^3*f+3*b^4*e^4*g+8*b^3*c
*d*e^3*g-8*b^3*c*e^4*f+48*b^2*c^2*d^2*e^2*g-48*b^2*c^2*d*e^3*f-192*b*c^3*d
^3*e*g+192*b*c^3*d^2*e^2*f+128*c^4*d^4*g-128*c^4*d^3*e*f)/e*ln((1/2*b+c*x)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2))
```

3.864. $\int \frac{(f+gx)\left(a+bx+cx^{2}\right)^{3/2}}{d+ex}\,dx$

3.864.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)\,(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.864.6 Sympy [F]

$$\int \frac{(f+gx)\,(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \int \frac{(f+gx)\,(a+bx+cx^2)^{\frac{3}{2}}}{d+ex}\,dx$$

input
```
integrate((g*x+f)*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

output
```
Integral((f + g*x)*(a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

3.864.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\,(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.864.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.864.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \int \frac{(f+g\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}{d+e\,x}\,dx$$

input
```
int(((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x),x)
```

output
```
int(((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x), x)
```

3.865 $\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx$

3.865.1 Optimal result

Integrand size = 22, antiderivative size = 252

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{(a+bx+cx^2)^{3/2}}{3e} - \frac{(2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}e^4} + \frac{(cd^2-bde+ae^2)^{3/2}\,\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^4}$$

output
```
1/3*(c*x^2+b*x+a)^(3/2)/e-1/16*(-b*e+2*c*d)*(8*c^2*d^2-b^2*e^2-4*c*e*(-3*a
*e+2*b*d))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e^4+
(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*
d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^4+1/8*(8*c^2*d^2+b^2*e^2-2*c*e*(-4
*a*e+5*b*d)-2*c*e*(-b*e+2*c*d)*x)*(c*x^2+b*x+a)^(1/2)/c/e^3
```

3.865.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \frac{\frac{e\sqrt{a+x(b+cx)}(3b^2e^2+2ce(-15bd+16ae+7bex)+4c^2(6d^2-3dex+2e^2x^2))}{c}+48\sqrt{-cd^2+bde-ae^2}(c}{}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x),x]
```

output
```
((e*Sqrt[a + x*(b + c*x)]*(3*b^2*e^2 + 2*c*e*(-15*b*d + 16*a*e + 7*b*e*x)
+ 4*c^2*(6*d^2 - 3*d*e*x + 2*e^2*x^2)))/c + 48*Sqrt[-(c*d^2) + b*d*e - a*e
^2]*(c*d^2 + e*(-(b*d) + a*e))*ArcTan[(Sqrt[-(c*d^2) + b*d*e - a*e^2]*x)/(
Sqrt[a]*(d + e*x) - d*Sqrt[a + x*(b + c*x)])] - (3*(2*c*d - b*e)*(8*c^2*d^
2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt
[a + x*(b + c*x)])])/c^(3/2))/(24*e^4)
```

3.865.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1162, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx$$

$$\Big\downarrow \text{1162}$$

$$\frac{(a+bx+cx^2)^{3/2}}{3e}-\frac{\int \frac{(bd-2ae+(2cd-be)x)\sqrt{cx^2+bx+a}}{d+ex}dx}{2e}$$

$$\Big\downarrow \text{1231}$$

$$\frac{(a+bx+cx^2)^{3/2}}{3e}-\frac{-\frac{\int \frac{4ce(bd-2ae)^2-d(2cd-be)(-eb^2+4cdb-4ace)-(2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae))x}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{4ce^2}-\frac{\sqrt{a+bx+cx^2}(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c}{4ce^2}}{2e}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}}{3e}-$$
$$-\frac{-\frac{\int \frac{4ce(bd-2ae)^2-d(2cd-be)\left(-eb^2+4cdb-4ace\right)-(2cd-be)\left(8c^2d^2-b^2e^2-4ce(2bd-3ae)\right)x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{8ce^2}-\frac{\sqrt{a+bx+cx^2}\left(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c\cdots\right.}{4ce^2}}{2e}$$

$\downarrow$ 1269

$$\frac{\left(a+bx+cx^2\right)^{3/2}}{3e}-$$
$$-\frac{-\frac{\frac{16c\left(ae^2-bde+cd^2\right)^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}}{8ce^2}-\frac{\sqrt{a+bx+cx^2}\left(-2ce(5bd-4ae)+b^2e^2-2ce\cdots\right.}{4ce^2}}{2e}$$

$\downarrow$ 1092

$$\frac{\left(a+bx+cx^2\right)^{3/2}}{3e}-$$
$$-\frac{-\frac{\frac{16c\left(ae^2-bde+cd^2\right)^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{2(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}}{8ce^2}-\frac{\sqrt{a+bx+cx^2}\left(-2ce(5bd-4a\cdots\right.}{\cdots}}{2e}$$

$\downarrow$ 219

$$\frac{\left(a+bx+cx^2\right)^{3/2}}{3e}-$$
$$-\frac{-\frac{\frac{16c\left(ae^2-bde+cd^2\right)^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)}{\sqrt{c}e}}{8ce^2}-\frac{\sqrt{a+bx+cx^2}\left(-2ce(5bd-4ae\cdots\right.}{4\cdots}}{2e}$$

$\downarrow$ 1154

$$\frac{\left(a+bx+cx^2\right)^{3/2}}{3e}-$$
$$-\frac{-\frac{-\frac{32c\left(ae^2-bde+cd^2\right)^2\int \frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ce(2bd-3ae)-b^2\cdots\right.}{\sqrt{c}e}}{8ce^2}}{2e}$$

$\downarrow$ 219

$$\frac{\left(a+bx+cx^2\right)^{3/2}}{3e}-$$
$$-\frac{-\frac{\frac{16c\left(ae^2-bde+cd^2\right)^{3/2}\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e}-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)}{\sqrt{c}e}}{8ce^2}-\sqrt{a+bx+\cdots}}{2e}$$

3.865. $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx$

input
```
Int[(a + b*x + c*x^2)^(3/2)/(d + e*x),x]
```

output
```
(a + b*x + c*x^2)^(3/2)/(3*e) - (-1/4*((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d
 - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(c*e^2) - (-(((2
*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e)) + (16*c*(c*d^2 - b*d*
e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b
*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e)/(8*c*e^2))/(2*e)
```

3.865.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.865.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.52

method	result
risch	$\frac{\left(8c^2x^2e^2+14bc\,e^2x-12c^2dex+32ac\,e^2+3b^2e^2-30bcde+24c^2d^2\right)\sqrt{c\,x^2+bx+a}}{24c\,e^3}+\frac{\frac{\left(12c\,e^3ba-24a\,c^2d\,e^2-b^3e^3-6b^2cd\,e^2+24b\,c^2d^2e-16c^3 \cdots\right.}{e\sqrt{c}}}{\cdots}$
default	$\frac{\frac{\left(\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a-bde+c\,d^2}{e^2}\right)^{\frac{3}{2}}}{3}+\frac{(be-2cd)\left(\frac{\left(2c\left(x+\frac{d}{e}\right)+\frac{be-2cd}{e}\right)\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a-bde+c\,d^2}{e^2}}}{4c}+\frac{\left(4c\left(e^2a \cdots\right.\right.}{\cdots}\right.}{\cdots}}{\cdots}$

input
```
int((c*x^2+b*x+a)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/24/c*(8*c^2*e^2*x^2+14*b*c*e^2*x-12*c^2*d*e*x+32*a*c*e^2+3*b^2*e^2-30*b*
c*d*e+24*c^2*d^2)*(c*x^2+b*x+a)^(1/2)/e^3+1/16/e^3/c*((12*a*b*c*e^3-24*a*c
^2*d*e^2-b^3*e^3-6*b^2*c*d*e^2+24*b*c^2*d^2*e-16*c^3*d^3)/e*ln((1/2*b+c*x)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-16*(a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^
2+b^2*d^2*e^2-2*b*c*d^3*e+c^2*d^4)*c/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*l
n((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/
2))/(x+d/e)))
```

3.865.5 Fricas [A] (verification not implemented)

Time = 150.15 (sec) , antiderivative size = 1523, normalized size of antiderivative = 6.04

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

output
```
[-1/96*(3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 6*(b^2*c + 4*a*c^2)*d*e^2 + (b^3
- 12*a*b*c)*e^3)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b
*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 48*(c^3*d^2 - b*c^2*d*e + a*c^2*e^2
)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d
^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*
e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*
c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) - 4
*(8*c^3*e^3*x^2 + 24*c^3*d^2*e - 30*b*c^2*d*e^2 + (3*b^2*c + 32*a*c^2)*e^3
 - 2*(6*c^3*d*e^2 - 7*b*c^2*e^3)*x)*sqrt(c*x^2 + b*x + a))/(c^2*e^4), 1/48
*(3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 6*(b^2*c + 4*a*c^2)*d*e^2 + (b^3 - 12*a
*b*c)*e^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/
(c^2*x^2 + b*c*x + a*c)) + 24*(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)*sqrt(c*d^2
 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*
d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*s
qrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*
b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(8*c^3*e^3*
x^2 + 24*c^3*d^2*e - 30*b*c^2*d*e^2 + (3*b^2*c + 32*a*c^2)*e^3 - 2*(6*c^3*
d*e^2 - 7*b*c^2*e^3)*x)*sqrt(c*x^2 + b*x + a))/(c^2*e^4), 1/96*(96*(c^3*d^
2 - b*c^2*d*e + a*c^2*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-
c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*...
```

3.865.6 Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}}{d+ex}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

3.865.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` f
or more de
```

3.865.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.865.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{d+e\,x}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/(d + e*x),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(d + e*x), x)
```

3.866 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx$

3.866.1 Optimal result

Integrand size = 29, antiderivative size = 491

$$
\begin{aligned}
\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx &= \frac{(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}{e^2(ef-dg)} \\
&- \frac{(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-dg)x)\sqrt{a+bx+cx^2}}{4eg^2(ef-dg)} \\
&- \frac{(2cd-be)(cd^2-bde+ae^2)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}e^3(ef-dg)} \\
&+ \frac{(8c^2ef^3+bg^2(3bef+bdg-4aeg)-4cg(3bef^2-ag(3ef-dg)))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}eg^3(ef-dg)} \\
&+ \frac{(cd^2-bde+ae^2)^{3/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^3(ef-dg)} \\
&- \frac{(cf^2-bfg+ag^2)^{3/2}\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g^3(ef-dg)}
\end{aligned}
$$

output
```
(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*
d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^3/(-d*g+e*f)-(a*g^2-b*f*g+c*f^2)^(
3/2)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x
^2+b*x+a)^(1/2))/g^3/(-d*g+e*f)-1/2*(-b*e+2*c*d)*(a*e^2-b*d*e+c*d^2)*arcta
nh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/e^3/(-d*g+e*f)/c^(1/2)+1/8*(
8*c^2*e*f^3+b*g^2*(-4*a*e*g+b*d*g+3*b*e*f)-4*c*g*(3*b*e*f^2-a*g*(-d*g+3*e*
f)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/e/g^3/(-d*g+e*f)/c
^(1/2)+(a*e^2-b*d*e+c*d^2)*(c*x^2+b*x+a)^(1/2)/e^2/(-d*g+e*f)-1/4*(4*c*e*f
^2-g*(-4*a*e*g-b*d*g+5*b*e*f)-2*c*g*(-d*g+e*f)*x)*(c*x^2+b*x+a)^(1/2)/e/g^
2/(-d*g+e*f)
```

3.866.2 Mathematica [A] (verified)

Time = 10.72 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.66

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx = \frac{(3b^2e^2g^2-12ceg(bef+bdg-aeg)+8c^2(e^2f^2+defg+d^2g^2))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}} + \frac{2\Big(eg(ef-dg)\sqrt{a}}{}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)),x]
```

output
```
(((3*b^2*e^2*g^2 - 12*c*e*g*(b*e*f + b*d*g - a*e*g) + 8*c^2*(e^2*f^2 + d*e
*f*g + d^2*g^2))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/S
qrt[c] + (2*(e*g*(e*f - d*g)*Sqrt[a + x*(b + c*x)]*(5*b*e*g + c*(-4*e*f -
4*d*g + 2*e*g*x)) - 4*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*g^3*ArcTanh[(-(b*d)
 + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(
b + c*x)])] + 4*e^3*(c*f^2 + g*(-(b*f) + a*g))^(3/2)*ArcTanh[(-(b*f) + 2*a
*g - 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*
x)])]))/(e*f - d*g))/(8*e^3*g^3)
```

3.866.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 453, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1270, 1162, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)(f+gx)}dx$$

$$\Big\downarrow \ \textbf{1270}$$

$$\frac{\left(ae^2-bde+cd^2\right)\int \frac{\sqrt{cx^2+bx+a}}{d+ex}dx}{e(ef-dg)}-\frac{\int \frac{(cdf-bef+aeg-c(ef-dg)x)\sqrt{cx^2+bx+a}}{f+gx}dx}{e(ef-dg)}$$

$$\Big\downarrow \ \textbf{1162}$$

$$\frac{\left(ae^2-bde+cd^2\right)\left(\frac{\sqrt{a+bx+cx^2}}{e}-\frac{\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{2e}\right)}{e(ef-dg)}-\frac{\int \frac{(cdf-bef+aeg-c(ef-dg)x)\sqrt{cx^2+bx+a}}{f+gx}dx}{e(ef-dg)}$$

$$\Big\downarrow \ \textbf{1231}$$

$$\frac{\left(ae^2-bde+cd^2\right)\left(\frac{\sqrt{a+bx+cx^2}}{e}-\frac{\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{2e}\right)}{e(ef-dg)}-$$

$$\frac{\frac{\sqrt{a+bx+cx^2}\left(-g(-4aeg-bdg+5bef)-2cgx(ef-dg)+4cef^2\right)}{4g^2}-\frac{\int -\frac{c\left(fg(5ef-dg)b^2-4ef\left(cf^2+3ag^2\right)b+4ag\left(2aeg^2+cf(ef+dg)\right)-\left(8c^2ef^3+bg^2(3bef+b\cdots\right.\right.}{2(f+gx)\sqrt{cx^2+bx+a}}\cdots}{4cg^2}\cdots}{e(ef-dg)}$$

$$\Big\downarrow \ \textbf{27}$$

$$\frac{\left(ae^2-bde+cd^2\right)\left(\frac{\sqrt{a+bx+cx^2}}{e}-\frac{\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{2e}\right)}{e(ef-dg)}-$$

$$\frac{\frac{\int \frac{fg(5ef-dg)b^2-4ef\left(cf^2+3ag^2\right)b+4ag\left(2aeg^2+cf(ef+dg)\right)-\left(8c^2ef^3+bg^2(3bef+bdg-4aeg)-4cg\left(3bef^2-ag(3ef-dg)\right)\right)x}{(f+gx)\sqrt{cx^2+bx+a}}dx}{8g^2}+\frac{\sqrt{a+bx+cx^2}\left(-g(-4aeg-\cdots\right.}{\cdots}}{e(ef-dg)}$$

$$\Big\downarrow \ \textbf{1269}$$

$$\frac{\left(ae^2-bde+cd^2\right)\left(\frac{\sqrt{a+bx+cx^2}}{e}-\frac{\frac{(2cd-be)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{2\left(ae^2-bde+cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{2e}\right)}{e(ef-dg)}-$$

$$\frac{\frac{\frac{8e\left(ag^2-bfg+cf^2\right)^2\int \frac{1}{(f+gx)\sqrt{cx^2+bx+a}}dx}{g}-\frac{\left(-4cg\left(3bef^2-ag(3ef-dg)\right)+bg^2(-4aeg+bdg+3bef)+8c^2ef^3\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{g}}{8g^2}+\frac{\sqrt{a+bx+cx^2}\left(-g(-4aeg\cdots\right.}{\cdots}}{e(ef-dg)}$$

$$\Big\downarrow \ \textbf{1092}$$

3.866. $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)(f+gx)}dx$

$$\frac{(ae^2-bde+cd^2)\left(\frac{\sqrt{a+bx+cx^2}}{e}-\frac{\frac{2(2cd-be)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}-\frac{2\left(ae^2-bde+cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{2e}\right)}{e(ef-dg)}-$$

$$\frac{\frac{\frac{8e\left(ag^2-bfg+cf^2\right)^2\int \frac{1}{(f+gx)\sqrt{cx^2+bx+a}}dx}{g}-\frac{2\left(-4cg\left(3bef^2-ag(3ef-dg)\right)+bg^2(-4aeg+bdg+3bef)+8c^2ef^3\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{g}}{8g^2}+\sqrt{a+bx+\cdots}}{e(ef-dg)}$$

$$\Big\downarrow \ 219$$

$$\frac{(ae^2-bde+cd^2)\left(\frac{\sqrt{a+bx+cx^2}}{e}-\frac{\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}-\frac{2\left(ae^2-bde+cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{2e}\right)}{e(ef-dg)}-$$

$$\frac{\frac{\frac{8e\left(ag^2-bfg+cf^2\right)^2\int \frac{1}{(f+gx)\sqrt{cx^2+bx+a}}dx}{g}-\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4cg\left(3bef^2-ag(3ef-dg)\right)+bg^2(-4aeg+bdg+3bef)+8c^2ef^3\right)}{\sqrt{c}g}}{8g^2}+\sqrt{a+bx+cx\cdots}}{e(ef-dg)}$$

$$\Big\downarrow \ 1154$$

$$\frac{(ae^2-bde+cd^2)\left(\frac{\sqrt{a+bx+cx^2}}{e}-\frac{\frac{4\left(ae^2-bde+cd^2\right)\int \frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}+\frac{(2cd-be)\text{arctanh}\left(\cdots\right.}{\sqrt{c}\cdots}}{2e}\right.}{e(ef-dg)}$$

$$\frac{-\frac{\frac{16e\left(ag^2-bfg+cf^2\right)^2\int \frac{1}{4\left(cf^2-bgf+ag^2\right)-\frac{(bf-2ag+(2cf-bg)x)^2}{cx^2+bx+a}}d\left(-\frac{bf-2ag+(2cf-bg)x}{\sqrt{cx^2+bx+a}}\right)}{g}-\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4cg\left(3bef^2-ag(3ef-dg)\right)+\cdots\right.}{\sqrt{c}g}}{8g^2}}{e(ef-dg)}$$

$$\Big\downarrow \ 219$$

$$\frac{(ae^2-bde+cd^2)\left(\frac{\sqrt{a+bx+cx^2}}{e}-\frac{\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}-\frac{2\sqrt{ae^2-bde+cd^2}\,\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e}}{2e}\right)}{e(ef-dg)}-$$

$$\frac{\frac{\frac{8e\left(ag^2-bfg+cf^2\right)^{3/2}\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g}-\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4cg\left(3bef^2-ag(3ef-dg)\right)+bg^2(-4aeg+bdg+3bef)+8c^2\cdots\right.}{\sqrt{c}g}}{8g^2}}{e(ef-dg)}$$

3.866. $\displaystyle\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx$

input
```
Int[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)),x]
```

output
```
((c*d^2 - b*d*e + a*e^2)*(Sqrt[a + b*x + c*x^2]/e - (((2*c*d - b*e)*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e) - (2*Sqrt[c*
d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2
 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e)/(2*e)))/(e*(e*f - d*g)) - ((
(4*c*e*f^2 - g*(5*b*e*f - b*d*g - 4*a*e*g) - 2*c*g*(e*f - d*g)*x)*Sqrt[a +
 b*x + c*x^2])/(4*g^2) + (-(((8*c^2*e*f^3 + b*g^2*(3*b*e*f + b*d*g - 4*a*e
*g) - 4*c*g*(3*b*e*f^2 - a*g*(3*e*f - d*g)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*g)) + (8*e*(c*f^2 - b*f*g + a*g^2)^(3/
2)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*
Sqrt[a + b*x + c*x^2])])/g)/(8*g^2))/(e*(e*f - d*g))
```

3.866.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.866. $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx$

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1270
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Simp[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g))
Int[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Simp[1/(e*(e*f - d*g))
Int[Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*((a + b*x + c*x^2)^(p
- 1)/(f + g*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[p]
&& GtQ[p, 0]
```

3.866. $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx$

3.866.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.22

method	result
risch	$\frac{(2cegx+5beg-4cdg-4cef)\sqrt{c\,x^2+bx+a}}{4e^2g^2}+\frac{\frac{\left(12ac\,e^2g^2+3b^2e^2g^2-12bcde\,g^2-12bc\,e^2fg+8c^2d^2g^2+8c^2defg+8c^2e^2f^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+b\ldots}\right.}{eg\sqrt{c}}}{}$
default	Expression too large to display

input
```
int((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*c*e*g*x+5*b*e*g-4*c*d*g-4*c*e*f)*(c*x^2+b*x+a)^(1/2)/e^2/g^2+1/8/e^
2/g^2*((12*a*c*e^2*g^2+3*b^2*e^2*g^2-12*b*c*d*e*g^2-12*b*c*e^2*f*g+8*c^2*d
^2*g^2+8*c^2*d*e*f*g+8*c^2*e^2*f^2)/e/g*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))/c^(1/2)+8/e^2*g^2*(a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^2
-2*b*c*d^3*e+c^2*d^4)/(d*g-e*f)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e
^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/
e))-8*e^2/g^2*(a^2*g^4-2*a*b*f*g^3+2*a*c*f^2*g^2+b^2*f^2*g^2-2*b*c*f^3*g+c
^2*f^4)/(d*g-e*f)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2
)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c
+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))
```

3.866.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx=\text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

output
```
Timed out
```

3.866.6 Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}}{(d+ex)\,(f+gx)}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f),x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)/((d + e*x)*(f + g*x)), x)
```

3.866.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for m
ore detail
```

3.866.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.866.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{(f+g\,x)\,(d+e\,x)}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/((f + g*x)*(d + e*x)),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/((f + g*x)*(d + e*x)), x)
```

3.867 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2} dx$

3.867.1 Optimal result

Integrand size = 29, antiderivative size = 787

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2} dx = \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2}$$
$$+\frac{3(4cf-3bg-2cgx)\sqrt{a+bx+cx^2}}{4g^2(ef-dg)}$$
$$-\frac{e(8c^2f^2+b^2g^2-2cg(5bf-4ag)-2cg(2cf-bg)x)\sqrt{a+bx+cx^2}}{8cg^2(ef-dg)^2}$$
$$+\frac{(a+bx+cx^2)^{3/2}}{(ef-dg)(f+gx)}-\frac{(2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}e^2(ef-dg)^2}$$
$$+\frac{e(2cf-bg)(8c^2f^2-b^2g^2-4cg(2bf-3ag))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}g^3(ef-dg)^2}$$
$$-\frac{3(8c^2f^2+b^2g^2-4cg(2bf-ag))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}g^3(ef-dg)}$$
$$+\frac{(cd^2-bde+ae^2)^{3/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^2(ef-dg)^2}$$
$$+\frac{3(2cf-bg)\sqrt{cf^2-bfg+ag^2}\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2g^3(ef-dg)}$$
$$-\frac{e(cf^2-bfg+ag^2)^{3/2}\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g^3(ef-dg)^2}$$

output
```
(c*x^2+b*x+a)^(3/2)/(-d*g+e*f)/(g*x+f)-1/16*(-b*e+2*c*d)*(8*c^2*d^2-b^2*e^
2-4*c*e*(-3*a*e+2*b*d))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))
/c^(3/2)/e^2/(-d*g+e*f)^2+1/16*e*(-b*g+2*c*f)*(8*c^2*f^2-b^2*g^2-4*c*g*(-3
*a*g+2*b*f))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/g^
3/(-d*g+e*f)^2+(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*
d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^2/(-d*g+e*f)^2-e*(a
*g^2-b*f*g+c*f^2)^(3/2)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*
g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/g^3/(-d*g+e*f)^2-3/8*(8*c^2*f^2+b^2*g^
2-4*c*g*(-a*g+2*b*f))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/g
^3/(-d*g+e*f)/c^(1/2)+3/2*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)
*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*g^2-b*f*g+c*f^2)^(1/
2)/g^3/(-d*g+e*f)+1/8*(8*c^2*d^2+b^2*e^2-2*c*e*(-4*a*e+5*b*d)-2*c*e*(-b*e+
2*c*d)*x)*(c*x^2+b*x+a)^(1/2)/c/e/(-d*g+e*f)^2+3/4*(-2*c*g*x-3*b*g+4*c*f)*
(c*x^2+b*x+a)^(1/2)/g^2/(-d*g+e*f)-1/8*e*(8*c^2*f^2+b^2*g^2-2*c*g*(-4*a*g+
5*b*f)-2*c*g*(-b*g+2*c*f)*x)*(c*x^2+b*x+a)^(1/2)/c/g^2/(-d*g+e*f)^2
```

3.867.2 Mathematica [A] (verified)

Time = 10.93 (sec) , antiderivative size = 357, normalized size of antiderivative = 0.45

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2}\,dx = \frac{-\sqrt{c}(ef-dg)^2(4cef+2cdg-3beg)(f+gx)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2(cd^2+$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^2),x]
```

output
```
(-(Sqrt[c]*(e*f - d*g)^2*(4*c*e*f + 2*c*d*g - 3*b*e*g)*(f + g*x)*ArcTanh[(
b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) - 2*(c*d^2 + e*(-(b*d) + a*
e))^(3/2)*g^3*(f + g*x)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt
[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])] + e*(2*g*(-(e*f) + d*g)
*Sqrt[a + x*(b + c*x)]*(e*g*(b*f - a*g) + c*d*g*(f + g*x) - c*e*f*(2*f + g
*x)) - e*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*(2*c*f*(2*e*f - 3*d*g) + g*(-(b*e*
f) + 3*b*d*g - 2*a*e*g))*(f + g*x)*ArcTanh[(-(b*f) + 2*a*g - 2*c*f*x + b*g
*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]))/(2*e^2*g^3
*(e*f - d*g)^2*(f + g*x))
```

3.867.3 Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 787, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2}\,dx$$

$$\Big\downarrow \; 1289$$

$$\int \left(\frac{e^2(a+bx+cx^2)^{3/2}}{(d+ex)(ef-dg)^2}-\frac{eg(a+bx+cx^2)^{3/2}}{(f+gx)(ef-dg)^2}-\frac{g(a+bx+cx^2)^{3/2}}{(f+gx)^2(ef-dg)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{3\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4cg(2bf-ag)+b^2g^2+8c^2f^2)}{8\sqrt{c}g^3(ef-dg)}-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4ce(2bd-3ae)-b^2e^2+8c^2d^2)}{16c^{3/2}e^2(ef-dg)^2}+\frac{e(2cf-bg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4cg(2bf-3ag)-b^2g^2+8c^2f^2)}{16c^{3/2}g^3(ef-dg)^2}+\frac{(ae^2-bde+cd^2)^{3/2}\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2(ef-dg)^2}-\frac{e(ag^2-bfg+cf^2)^{3/2}\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g^3(ef-dg)^2}+\frac{3(2cf-bg)\sqrt{ag^2-bfg+cf^2}\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2g^3(ef-dg)}+\frac{\sqrt{a+bx+cx^2}(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2)}{8ce(ef-dg)^2}-\frac{e\sqrt{a+bx+cx^2}(-2cg(5bf-4ag)+b^2g^2-2cgx(2cf-bg)+8c^2f^2)}{8cg^2(ef-dg)^2}+\frac{3\sqrt{a+bx+cx^2}(-3bg+4cf-2cgx)}{4g^2(ef-dg)}+\frac{(a+bx+cx^2)^{3/2}}{(f+gx)(ef-dg)}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^2),x]
```

output
```
((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqr
t[a + b*x + c*x^2])/(8*c*e*(e*f - d*g)^2) + (3*(4*c*f - 3*b*g - 2*c*g*x)*S
qrt[a + b*x + c*x^2])/(4*g^2*(e*f - d*g)) - (e*(8*c^2*f^2 + b^2*g^2 - 2*c*
g*(5*b*f - 4*a*g) - 2*c*g*(2*c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(8*c*g^2
*(e*f - d*g)^2) + (a + b*x + c*x^2)^(3/2)/((e*f - d*g)*(f + g*x)) - ((2*c*
d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e^2*(e*f - d*g)^2) + (e*(
2*c*f - b*g)*(8*c^2*f^2 - b^2*g^2 - 4*c*g*(2*b*f - 3*a*g))*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*g^3*(e*f - d*g)^2) -
(3*(8*c^2*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*g^3*(e*f - d*g)) + ((c*d^2 - b*d*e
 + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*
d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e^2*(e*f - d*g)^2) + (3*(2*c*f - b*
g)*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*
Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g^3*(e*f - d*g)) -
 (e*(c*f^2 - b*f*g + a*g^2)^(3/2)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/
(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g^3*(e*f - d*g)^2
)
```

3.867.3.1 Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.867.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 900, normalized size of antiderivative = 1.14

method	result
risch	$\frac{\sqrt{c\,x^2+bx+a}\,c}{g^2 e}+\frac{\frac{\sqrt{c}\,(3beg-2cdg-4cef)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{eg}+\frac{2e\left(a^2g^4-2abf\,g^3+2ac\,f^2g^2+b^2f^2g^2-2bc\,f^3g+c^2f^4\right)\left(-\frac{g^2\sqrt{\left(x+\frac{f}{g}\right)^2 c}}{(}\right.}{}}{}$
default	Expression too large to display

input
```
int((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

output
```
1/g^2/e*(c*x^2+b*x+a)^(1/2)*c+1/2/g^2/e*(c^(1/2)*(3*b*e*g-2*c*d*g-4*c*e*f)
/e/g*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+2*e/g^3*(a^2*g^4-2*a*b*f*
g^3+2*a*c*f^2*g^2+b^2*f^2*g^2-2*b*c*f^3*g+c^2*f^4)/(d*g-e*f)*(-1/(a*g^2-b*
f*g+c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f
^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/
g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b
*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f
^2)/g^2)^(1/2))/(x+f/g)))-2/e^2*g^2*(a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^2+b^2
*d^2*e^2-2*b*c*d^3*e+c^2*d^4)/(d*g-e*f)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)
/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1
/2))/(x+d/e))+2*e/g^2*(a^2*e*g^4-2*a*b*d*g^4+4*a*c*d*f*g^3-2*a*c*e*f^2*g^2
+2*b^2*d*f*g^3-b^2*e*f^2*g^2-6*b*c*d*f^2*g^2+4*b*c*e*f^3*g+4*c^2*d*f^3*g-3
*c^2*e*f^4)/(d*g-e*f)^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g
+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/
g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))
```

3.867.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.867.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f)**2,x)
```

output
```
Timed out
```

3.867.7 Maxima [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2}\,dx = \int \frac{(cx^2+bx+a)^{\frac{3}{2}}}{(ex+d)(gx+f)^2}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*(g*x + f)^2), x)
```

3.867.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2}\,dx=\text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.867.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2}\,dx=\int \frac{{\left(c\,x^2+b\,x+a\right)}^{3/2}}{{\left(f+g\,x\right)}^2\,\left(d+e\,x\right)}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/((f + g*x)^2*(d + e*x)),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/((f + g*x)^2*(d + e*x)), x)
```

3.868 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx$

3.868.1 Optimal result

Integrand size = 29, antiderivative size = 1066

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx = \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8c(ef-dg)^3}$$
$$+\frac{3e(4cf-3bg-2cgx)\sqrt{a+bx+cx^2}}{4g^2(ef-dg)^2}-\frac{3(4cf-bg+2cgx)\sqrt{a+bx+cx^2}}{4g^2(ef-dg)(f+gx)}$$
$$-\frac{e^2(8c^2f^2+b^2g^2-2cg(5bf-4ag)-2cg(2cf-bg)x)\sqrt{a+bx+cx^2}}{8cg^2(ef-dg)^3}$$
$$+\frac{(a+bx+cx^2)^{3/2}}{2(ef-dg)(f+gx)^2}+\frac{e(a+bx+cx^2)^{3/2}}{(ef-dg)^2(f+gx)}$$
$$-\frac{(2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}e(ef-dg)^3}$$
$$+\frac{3\sqrt{c}(2cf-bg)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2g^3(ef-dg)}$$
$$+\frac{e^2(2cf-bg)(8c^2f^2-b^2g^2-4cg(2bf-3ag))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}g^3(ef-dg)^3}$$
$$-\frac{3e(8c^2f^2+b^2g^2-4cg(2bf-ag))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}g^3(ef-dg)^2}$$
$$+\frac{(cd^2-bde+ae^2)^{3/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e(ef-dg)^3}$$
$$+\frac{3e(2cf-bg)\sqrt{cf^2-bfg+ag^2}\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2g^3(ef-dg)^2}$$
$$-\frac{e^2(cf^2-bfg+ag^2)^{3/2}\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g^3(ef-dg)^3}$$
$$-\frac{3(8c^2f^2+b^2g^2-4cg(2bf-ag))\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{8g^3(ef-dg)\sqrt{cf^2-bfg+ag^2}}$$

output
```
1/2*(c*x^2+b*x+a)^(3/2)/(-d*g+e*f)/(g*x+f)^2+e*(c*x^2+b*x+a)^(3/2)/(-d*g+e
*f)^2/(g*x+f)-1/16*(-b*e+2*c*d)*(8*c^2*d^2-b^2*e^2-4*c*e*(-3*a*e+2*b*d))*a
rctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e/(-d*g+e*f)^3+1
/16*e^2*(-b*g+2*c*f)*(8*c^2*f^2-b^2*g^2-4*c*g*(-3*a*g+2*b*f))*arctanh(1/2*
(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/g^3/(-d*g+e*f)^3+(a*e^2-b*d
*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)
^(1/2)/(c*x^2+b*x+a)^(1/2))/e/(-d*g+e*f)^3-e^2*(a*g^2-b*f*g+c*f^2)^(3/2)*a
rctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x
+a)^(1/2))/g^3/(-d*g+e*f)^3-3/8*e*(8*c^2*f^2+b^2*g^2-4*c*g*(-a*g+2*b*f))*a
rctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/g^3/(-d*g+e*f)^2/c^(1/2)
+3/2*(-b*g+2*c*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/
2)/g^3/(-d*g+e*f)-3/8*(8*c^2*f^2+b^2*g^2-4*c*g*(-a*g+2*b*f))*arctanh(1/2*(
b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/g
^3/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(1/2)+3/2*e*(-b*g+2*c*f)*arctanh(1/2*(b*
f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*
g^2-b*f*g+c*f^2)^(1/2)/g^3/(-d*g+e*f)^2+1/8*(8*c^2*d^2+b^2*e^2-2*c*e*(-4*a
*e+5*b*d)-2*c*e*(-b*e+2*c*d)*x)*(c*x^2+b*x+a)^(1/2)/c/(-d*g+e*f)^3+3/4*e*(
-2*c*g*x-3*b*g+4*c*f)*(c*x^2+b*x+a)^(1/2)/g^2/(-d*g+e*f)^2-3/4*(2*c*g*x-b*
g+4*c*f)*(c*x^2+b*x+a)^(1/2)/g^2/(-d*g+e*f)/(g*x+f)-1/8*e^2*(8*c^2*f^2+b^2
*g^2-2*c*g*(-4*a*g+5*b*f)-2*c*g*(-b*g+2*c*f)*x)*(c*x^2+b*x+a)^(1/2)/c/g...
```

3.868. $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx$

3.868.2 Mathematica [A] (verified)

Time = 12.29 (sec) , antiderivative size = 1036, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx = \frac{1}{4}\left(\frac{2(a+x(b+cx))^{3/2}}{(ef-dg)(f+gx)^2} + \frac{4e(a+x(b+cx))^{3/2}}{(ef-dg)^2(f+gx)}\right.$$

$$+\frac{-\left((2cd-be)\left(8c^2d^2-b^2e^2+4ce(-2bd+3ae)\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)-2\sqrt{c}\left(e\sqrt{a+x(b+cx)}(-b^2e\right.}{4c^{3/2}e}$$

$$-\frac{3e\left(\left(8c^2f^2+b^2g^2+4cg(-2bf+ag)\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)+2\sqrt{c}\left(g(-4cf+3bg+2cgx)\sqrt{a+x(b+\right.\right.}{2\sqrt{c}g^3(ef-dg)^2}$$

$$+\frac{3\left(\frac{(-2cf+bg)(a+x(b+cx))^{3/2}}{f+gx}-\frac{\sqrt{a+x(b+cx)}\left(b^2g^2+2c^2f(2f-gx)+cg(-5bf+2ag+bgx)\right)}{g^2}+\frac{4\sqrt{c}(2cf-bg)\left(cf^2+g(-bf+ag)\right)\text{arctanh}\left(\right.}{}\right.}{(ef-dg)\left(cf^2+g(-bf+\right.}$$

$$-\frac{e^2\left((2cf-bg)\left(8c^2f^2-b^2g^2+4cg(-2bf+3ag)\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)+2\sqrt{c}\left(g\sqrt{a+x(b+cx)}(-b^2g\right.\right.}{4c^{3/2}}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^3),x]
```

output
```
((2*(a + x*(b + c*x))^(3/2))/((e*f - d*g)*(f + g*x)^2) + (4*e*(a + x*(b +
c*x))^(3/2))/((e*f - d*g)^2*(f + g*x)) + (-((2*c*d - b*e)*(8*c^2*d^2 - b^2
*e^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(
b + c*x)])]) - 2*Sqrt[c]*(e*Sqrt[a + x*(b + c*x)]*(-(b^2*e^2) + 4*c^2*d*(-
2*d + e*x) - 2*c*e*(-5*b*d + 4*a*e + b*e*x)) + 8*c*(c*d^2 + e*(-(b*d) + a*
e))^(3/2)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(
b*d) + a*e)]*Sqrt[a + x*(b + c*x)])]))/(4*c^(3/2)*e*(e*f - d*g)^3) - (3*e*
((8*c^2*f^2 + b^2*g^2 + 4*c*g*(-2*b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(g*(-4*c*f + 3*b*g + 2*c*g*x)*Sqrt[
a + x*(b + c*x)] + 2*(2*c*f - b*g)*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*ArcTanh[
(-(b*f) + 2*a*g - 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[
a + x*(b + c*x)])])))/(2*Sqrt[c]*g^3*(e*f - d*g)^2) + (3*(((-2*c*f + b*g)*
(a + x*(b + c*x))^(3/2))/(f + g*x) - (Sqrt[a + x*(b + c*x)]*(b^2*g^2 + 2*c
^2*f*(2*f - g*x) + c*g*(-5*b*f + 2*a*g + b*g*x)))/g^2 + (4*Sqrt[c]*(2*c*f
- b*g)*(c*f^2 + g*(-(b*f) + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
x*(b + c*x)])] + (8*c^2*f^2 + b^2*g^2 + 4*c*g*(-2*b*f + a*g))*Sqrt[c*f^2 +
 g*(-(b*f) + a*g)]*ArcTanh[(-(b*f) + 2*a*g - 2*c*f*x + b*g*x)/(2*Sqrt[c*f^
2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(2*g^3)))/((e*f - d*g)*(c*f
^2 + g*(-(b*f) + a*g))) - (e^2*((2*c*f - b*g)*(8*c^2*f^2 - b^2*g^2 + 4*c*g
*(-2*b*f + 3*a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]...
```

3.868.3 Rubi [A] (verified)

Time = 1.80 (sec) , antiderivative size = 1066, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx$$

$$\Big\downarrow \text{1289}$$

$$\int \left(\frac{e^3(a+bx+cx^2)^{3/2}}{(d+ex)(ef-dg)^3}-\frac{e^2g(a+bx+cx^2)^{3/2}}{(f+gx)(ef-dg)^3}-\frac{eg(a+bx+cx^2)^{3/2}}{(f+gx)^2(ef-dg)^2}-\frac{g(a+bx+cx^2)^{3/2}}{(f+gx)^3(ef-dg)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(2cf-bg)\left(8c^2f^2-b^2g^2-4cg(2bf-3ag)\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^2}{16c^{3/2}g^3(ef-dg)^3}-\frac{\left(cf^2-bgf+ag^2\right)^{3/2}\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)e^2}{g^3(ef-dg)^3}-\frac{\left(8c^2f^2+b^2g^2-2cg(5bf-4ag)-2cg(2cf-bg)x\right)\sqrt{cx^2+bx+a}\,e^2}{8cg^2(ef-dg)^3}+\frac{\left(cx^2+bx+a\right)^{3/2}e}{(ef-dg)^2(f+gx)}-\frac{3\left(8c^2f^2+b^2g^2-4cg(2bf-ag)\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e}{8\sqrt{c}g^3(ef-dg)^2}+\frac{3(2cf-bg)\sqrt{cf^2-bgf+ag^2}\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)e}{2g^3(ef-dg)^2}+\frac{3(4cf-3bg-2cgx)\sqrt{cx^2+bx+a}\,e}{4g^2(ef-dg)^2}+\frac{\left(cx^2+bx+a\right)^{3/2}}{2(ef-dg)(f+gx)^2}+\frac{3\sqrt{c}(2cf-bg)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)}{2g^3(ef-dg)}-\frac{3\left(8c^2f^2+b^2g^2-4cg(2bf-ag)\right)\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)}{8g^3(ef-dg)\sqrt{cf^2-bgf+ag^2}}+\frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{cx^2+bx+a}}{8c(ef-dg)^3}-\frac{3(4cf-bg+2cgx)\sqrt{cx^2+bx+a}}{4g^2(ef-dg)(f+gx)}-\frac{(2cd-be)\left(8c^2d^2-b^2e^2-4ce(2bd-3ae)\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)}{16c^{3/2}(ef-dg)^3e}+\frac{\left(cd^2-bed+ae^2\right)^{3/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)}{(ef-dg)^3e}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^3),x]
```

3.868. $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)(f+gx)^3}\,dx$

output
```
((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqr
t[a + b*x + c*x^2])/(8*c*(e*f - d*g)^3) + (3*e*(4*c*f - 3*b*g - 2*c*g*x)*S
qrt[a + b*x + c*x^2])/(4*g^2*(e*f - d*g)^2) - (3*(4*c*f - b*g + 2*c*g*x)*S
qrt[a + b*x + c*x^2])/(4*g^2*(e*f - d*g)*(f + g*x)) - (e^2*(8*c^2*f^2 + b^
2*g^2 - 2*c*g*(5*b*f - 4*a*g) - 2*c*g*(2*c*f - b*g)*x)*Sqrt[a + b*x + c*x^
2])/(8*c*g^2*(e*f - d*g)^3) + (a + b*x + c*x^2)^(3/2)/(2*(e*f - d*g)*(f +
g*x)^2) + (e*(a + b*x + c*x^2)^(3/2))/((e*f - d*g)^2*(f + g*x)) - ((2*c*d
- b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e*(e*f - d*g)^3) + (3*Sqrt[
c]*(2*c*f - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
2*g^3*(e*f - d*g)) + (e^2*(2*c*f - b*g)*(8*c^2*f^2 - b^2*g^2 - 4*c*g*(2*b*
f - 3*a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^
(3/2)*g^3*(e*f - d*g)^3) - (3*e*(8*c^2*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g)
)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*g^3*(
e*f - d*g)^2) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c
*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*(e
*f - d*g)^3) + (3*e*(2*c*f - b*g)*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(b*f
 - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x +
c*x^2])])/(2*g^3*(e*f - d*g)^2) - (e^2*(c*f^2 - b*f*g + a*g^2)^(3/2)*ArcTa
nh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[...
```

3.868.3.1 Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.868.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4241 vs. $2(968) = 1936$.

Time = 0.96 (sec) , antiderivative size = 4242, normalized size of antiderivative = 3.98

method	result	size
default	Expression too large to display	4242

3.868. $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx$

input
```
int((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x,method=_RETURNVERBOSE)
```

output
```
1/g^2/(d*g-e*f)*(-1/2/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)^2*((x+f/g)^2*c+(b*g-
2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(5/2)+1/4*(b*g-2*c*f)*g/(a*g^2-b
*f*g+c*f^2)*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g
*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(5/2)+3/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f
^2)*(1/3*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)
+1/2*(b*g-2*c*f)/g*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*
c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)
/g^2-(b*g-2*c*f)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+(
(x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)))+(a*g^2-
b*f*g+c*f^2)/g^2*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g
^2)^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f
/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g
^2-b*f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f
^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2
*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))))+4*c/(a
*g^2-b*f*g+c*f^2)*g^2*(1/8*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g
-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+3/16*(4*c*(a*g^2-b*f*g+c*
f^2)/g^2-(b*g-2*c*f)^2/g^2)/c*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^
2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b
*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f...
```

3.868.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx=\text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.868.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f)**3,x)
```

output
```
Timed out
```

3.868.7 Maxima [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx = \int \frac{(cx^2+bx+a)^{\frac{3}{2}}}{(ex+d)(gx+f)^3}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*(g*x + f)^3), x)
```

3.868.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.868.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{(f+g\,x)^3\,(d+e\,x)}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/((f + g*x)^3*(d + e*x)),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/((f + g*x)^3*(d + e*x)), x)
```

3.869 $\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx$

3.869.1 Optimal result

Integrand size = 29, antiderivative size = 886

$$\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx = \frac{(cd^2-bde+ae^2)\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+c}}{8ce^4(ef-dg)}$$

$$-\frac{(64c^3ef^4-16c^2ef^2g(9bf-8ag)-b^2g^3(5bef+3bdg-8aeg)+4cg^2(22b^2ef^2+16a^2eg^2-3abg(13ef-}{64ceg^4(ef-}$$

$$+\frac{(cd^2-bde+ae^2)\left(a+bx+cx^2\right)^{3/2}}{3e^2(ef-dg)}$$

$$-\frac{(8cef^2-g(11bef-3bdg-8aeg)-6cg(ef-dg)x)\left(a+bx+cx^2\right)^{3/2}}{24eg^2(ef-dg)}$$

$$-\frac{(2cd-be)\left(cd^2-bde+ae^2\right)\left(8c^2d^2-b^2e^2-4ce(2bd-3ae)\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}e^5(ef-dg)}$$

$$+\frac{(128c^4ef^5-320c^3ef^3g(bf-ag)-b^3g^4(5bef+3bdg-8aeg)+48c^2g^2(5b^2ef^3-10abef^2g+a^2g^2(5ef-}{128c^{3/2}eg^5(ef-dg)}$$

$$+\frac{(cd^2-bde+ae^2)^{5/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^5(ef-dg)}$$

$$-\frac{(cf^2-bfg+ag^2)^{5/2}\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g^5(ef-dg)}$$

output
```
1/3*(a*e^2-b*d*e+c*d^2)*(c*x^2+b*x+a)^(3/2)/e^2/(-d*g+e*f)-1/24*(8*c*e*f^2
-g*(-8*a*e*g-3*b*d*g+11*b*e*f)-6*c*g*(-d*g+e*f)*x)*(c*x^2+b*x+a)^(3/2)/e/g
^2/(-d*g+e*f)-1/16*(-b*e+2*c*d)*(a*e^2-b*d*e+c*d^2)*(8*c^2*d^2-b^2*e^2-4*c
*e*(-3*a*e+2*b*d))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3
/2)/e^5/(-d*g+e*f)+1/128*(128*c^4*e*f^5-320*c^3*e*f^3*g*(-a*g+b*f)-b^3*g^4
*(-8*a*e*g+3*b*d*g+5*b*e*f)+48*c^2*g^2*(5*b^2*e*f^3-10*a*b*e*f^2*g+a^2*g^2
*(-d*g+5*e*f))-8*b*c*g^3*(5*b^2*e*f^2+12*a^2*e*g^2-3*a*b*g*(d*g+5*e*f)))*a
rctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e/g^5/(-d*g+e*f)
+(a*e^2-b*d*e+c*d^2)^(5/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b
*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^5/(-d*g+e*f)-(a*g^2-b*f*g+c*f^2)^
(5/2)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*
x^2+b*x+a)^(1/2))/g^5/(-d*g+e*f)+1/8*(a*e^2-b*d*e+c*d^2)*(8*c^2*d^2+b^2*e^
2-2*c*e*(-4*a*e+5*b*d)-2*c*e*(-b*e+2*c*d)*x)*(c*x^2+b*x+a)^(1/2)/c/e^4/(-d
*g+e*f)-1/64*(64*c^3*e*f^4-16*c^2*e*f^2*g*(-8*a*g+9*b*f)-b^2*g^3*(-8*a*e*g
+3*b*d*g+5*b*e*f)+4*c*g^2*(22*b^2*e*f^2+16*a^2*e*g^2-3*a*b*g*(-d*g+13*e*f)
)-2*c*g*(16*c^2*e*f^3+b*g^2*(-8*a*e*g+3*b*d*g+5*b*e*f)-4*c*g*(6*b*e*f^2-a*
g*(-3*d*g+7*e*f)))*x)*(c*x^2+b*x+a)^(1/2)/c/e/g^4/(-d*g+e*f)
```

3.869.2 Mathematica [A] (verified)

Time = 11.70 (sec) , antiderivative size = 647, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx = \frac{3(5b^4e^4g^4(-ef+dg)-40b^2ce^3g^3(ef-dg)(bef+bdg-3aeg)+320c^3eg(-be^4f^4}{}$$

input
```
Integrate[(a + b*x + c*x^2)^(5/2)/((d + e*x)*(f + g*x)),x]
```

output
```
(3*(5*b^4*e^4*g^4*(-(e*f) + d*g) - 40*b^2*c*e^3*g^3*(e*f - d*g)*(b*e*f + b
*d*g - 3*a*e*g) + 320*c^3*e*g*(-(b*e^4*f^4) + a*e^4*f^3*g + b*d^4*g^4 - a*
d^3*e*g^4) + 128*c^4*(e^5*f^5 - d^5*g^5) + 240*c^2*e^2*g^2*(e*f - d*g)*(a^
2*e^2*g^2 - 2*a*b*e*g*(e*f + d*g) + b^2*(e^2*f^2 + d*e*f*g + d^2*g^2)))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(-(e*g*(-
(e*f) + d*g)*Sqrt[a + x*(b + c*x)]*(15*b^3*e^3*g^3 + 2*b*c*e^2*g^2*(278*a*
e*g + b*(-132*e*f - 132*d*g + 59*e*g*x)) - 16*c^3*(12*d^3*g^3 - 6*d^2*e*g^
2*(-2*f + g*x) + 2*d*e^2*g*(6*f^2 - 3*f*g*x + 2*g^2*x^2) + e^3*(12*f^3 - 6
*f^2*g*x + 4*f*g^2*x^2 - 3*g^3*x^3)) + 8*c^2*e*g*(a*e*g*(-56*e*f - 56*d*g
+ 27*e*g*x) + b*(54*d^2*g^2 + 2*d*e*g*(27*f - 13*g*x) + e^2*(54*f^2 - 26*f
*g*x + 17*g^2*x^2))))) - 192*c*(c*d^2 + e*(-(b*d) + a*e))^(5/2)*g^5*ArcTan
h[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqr
t[a + x*(b + c*x)])] + 192*c*e^5*(c*f^2 + g*(-(b*f) + a*g))^(5/2)*ArcTanh[
(-(b*f) + 2*a*g - 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[
a + x*(b + c*x)])]))/(384*c^(3/2)*e^5*g^5*(e*f - d*g))
```

3.869.3 Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 834, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {1270, 1162, 1231, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(d+ex)(f+gx)}dx$$

$$\Big\downarrow \text{1270}$$

$$\frac{\left(ae^2-bde+cd^2\right)\int \frac{\left(cx^2+bx+a\right)^{3/2}}{d+ex}dx}{e(ef-dg)}-\frac{\int \frac{(cdf-bef+aeg-c(ef-dg)x)\left(cx^2+bx+a\right)^{3/2}}{f+gx}dx}{e(ef-dg)}$$

$$\Big\downarrow \text{1162}$$

$$\frac{\left(ae^2-bde+cd^2\right)\left(\frac{\left(a+bx+cx^2\right)^{3/2}}{3e}-\frac{\int \frac{(bd-2ae+(2cd-be)x)\sqrt{cx^2+bx+a}}{d+ex}dx}{2e}\right)}{e(ef-dg)}-\frac{\int \frac{(cdf-bef+aeg-c(ef-dg)x)\left(cx^2+bx+a\right)^{3/2}}{f+gx}dx}{e(ef-dg)}$$

$$\Big\downarrow \text{1231}$$

$$\frac{\left(ae^2-bde+cd^2\right)\left(\frac{\left(a+bx+cx^2\right)^{3/2}}{3e}-\frac{-\frac{\int \frac{4ce(bd-2ae)^2-d(2cd-be)\left(-eb^2+4cdb-4ace\right)-(2cd-be)\left(8c^2d^2-b^2e^2-4ce(2bd-3ae)\right)x}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{4ce^2}-\frac{\sqrt{a+bx+cx^2} \cdots}{\cdots}}{2e} \cdots\right.}{e(ef-dg)}$$

$$\frac{\frac{\left(a+bx+cx^2\right)^{3/2}\left(-g(-8aeg-3bdg+11bef)-6cgx(ef-dg)+8cef^2\right)}{24g^2}-\frac{\int \frac{c\left(f\left(-3gb^2+8cfb-4acg\right)(ef-dg)+8g(bf-2ag)(cdf-bef+aeg)+\left(16c^2ef^3+bg^2(5 \cdots\right.\right.}{2(f+gx)}}{8cg^2}}{e(ef-dg)}$$

$\downarrow$ 27

$$\frac{\left(ae^2-bde+cd^2\right)\left(\frac{\left(a+bx+cx^2\right)^{3/2}}{3e}-\frac{-\frac{\int \frac{4ce(bd-2ae)^2-d(2cd-be)\left(-eb^2+4cdb-4ace\right)-(2cd-be)\left(8c^2d^2-b^2e^2-4ce(2bd-3ae)\right)x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{8ce^2}-\frac{\sqrt{a+bx+cx^2} \cdots}{\cdots}}{2e} \cdots\right.}{e(ef-dg)}$$

$$\frac{\frac{\left(a+bx+cx^2\right)^{3/2}\left(-g(-8aeg-3bdg+11bef)-6cgx(ef-dg)+8cef^2\right)}{24g^2}-\frac{\int \frac{\left(f\left(-3gb^2+8cfb-4acg\right)(ef-dg)+8g(bf-2ag)(cdf-bef+aeg)+\left(16c^2ef^3+bg^2(5b \cdots\right.\right.}{f+gx}}{16g^2}}{e(ef-dg)}$$

$\downarrow$ 1231

$$\frac{\left(cd^2-bed+ae^2\right)\left(\frac{\left(cx^2+bx+a\right)^{3/2}}{3e}-\frac{-\frac{\sqrt{cx^2+bx+a}\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)}{4ce^2}-\frac{\int \frac{4ce(bd-2ae)^2-d(2cd-be)\left(-eb^2+4cdb-4ace \cdots\right.}{(d+ex)\sqrt{c \cdots}}}{8c \cdots}}{2e} \cdots\right.}{e(ef-dg)}$$

$$\frac{\left(8cef^2-g(11bef-3bdg-8aeg)-6cg(ef-dg)x\right)\left(cx^2+bx+a\right)^{3/2}}{24g^2}-\frac{-\sqrt{cx^2+bx+a}\left(64c^3ef^4-16c^2eg(9bf-8ag)f^2-b^2g^3(5bef+3bdg-8aeg)+4cg^2\left(22b^2ef^2 \cdots\right.\right.}{\cdots}$$

$\downarrow$ 27

$$\frac{\left(cd^2-bed+ae^2\right)\left(\frac{\left(cx^2+bx+a\right)^{3/2}}{3e}-\frac{-\frac{\sqrt{cx^2+bx+a}\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)}{4ce^2}-\frac{\int \frac{4ce(bd-2ae)^2-d(2cd-be)\left(-eb^2+4cdb-4ace \cdots\right.}{(d+ex)\sqrt{c \cdots}}}{8c \cdots}}{2e} \cdots\right.}{e(ef-dg)}$$

$$\frac{\left(8cef^2-g(11bef-3bdg-8aeg)-6cg(ef-dg)x\right)\left(cx^2+bx+a\right)^{3/2}}{24g^2}-\frac{-\sqrt{cx^2+bx+a}\left(64c^3ef^4-16c^2eg(9bf-8ag)f^2-b^2g^3(5bef+3bdg-8aeg)+4cg^2\left(22b^2ef^2 \cdots\right.\right.}{\cdots}$$

$\downarrow$ 1269

3.869. $\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(d+ex)(f+gx)}\,dx$

$$\frac{\left(ae^2-bde+cd^2\right)\left(\frac{\left(a+bx+cx^2\right)^{3/2}}{3e}-\frac{-\frac{\frac{16c\left(ae^2-bde+cd^2\right)^2\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{e}}{8ce^2}}{2e}\right.}{e(ef-dg)}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(-g(-8aeg-3bdg+11bef)-6cgx(ef-dg)+8cef^2\right)}{24g^2}-\frac{-\frac{\frac{128ce\left(ag^2-bfg+cf^2\right)^3\int\frac{1}{(f+gx)\sqrt{cx^2+bx+a}}dx}{g}-\frac{\left(48c^2g^2\left(a^2g^2(5ef-dg)-10a\right.\right.}{}}{}}{}$$

$\downarrow$ 1092

$$\frac{\left(cd^2-bed+ae^2\right)\left(\frac{\left(cx^2+bx+a\right)^{3/2}}{3e}-\frac{-\frac{\sqrt{cx^2+bx+a}\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)}{4ce^2}-\frac{\frac{16c\left(cd^2-bed+ae^2\right)^2\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{}}{2e}\right.}{e(ef-dg)}$$

$$\frac{\left(8cef^2-g(11bef-3bdg-8aeg)-6cg(ef-dg)x\right)\left(cx^2+bx+a\right)^{3/2}}{24g^2}-\frac{-\frac{\sqrt{cx^2+bx+a}\left(64c^3ef^4-16c^2eg(9bf-8ag)f^2-b^2g^3(5bef+3bdg-8aeg)+4cg^2\left(22b^2ef^2\right.\right.}{}}{}$$

$\downarrow$ 219

$$\frac{\left(ae^2-bde+cd^2\right)\left(\frac{\left(a+bx+cx^2\right)^{3/2}}{3e}-\frac{-\frac{\frac{16c\left(ae^2-bde+cd^2\right)^2\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ce(2bd-3ae)-\right.}{\sqrt{c}e}}{8ce^2}}{2e}\right.}{e(ef-dg)}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(-g(-8aeg-3bdg+11bef)-6cgx(ef-dg)+8cef^2\right)}{24g^2}-\frac{-\frac{\frac{128ce\left(ag^2-bfg+cf^2\right)^3\int\frac{1}{(f+gx)\sqrt{cx^2+bx+a}}dx}{g}-\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx}}\right.}{}}{}$$

$\downarrow$ 1154

3.869. $\int\frac{\left(a+bx+cx^2\right)^{5/2}}{(d+ex)(f+gx)}dx$

$$\frac{\left(cd^2-bed+ae^2\right)\left(\frac{\left(cx^2+bx+a\right)^{3/2}}{3e}-\frac{-\frac{\sqrt{cx^2+bx+a}\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)}{4ce^2}-\frac{-\frac{32c\int\frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)}{cx^2+bx+a}}}{}}{}}{}\right.}{e(ef-dg)}$$

$$\frac{\frac{\left(8cef^2-g(11bef-3bdg-8aeg)-6cg(ef-dg)x\right)\left(cx^2+bx+a\right)^{3/2}}{24g^2}-\frac{-\frac{\sqrt{cx^2+bx+a}\left(64c^3ef^4-16c^2eg(9bf-8ag)f^2-b^2g^3(5bef+3bdg-8aeg)+4cg^2\left(22b^2ef^2\right.\right.}{}}{}}{}$$

$\downarrow$ 219

$$\frac{\left(cd^2-bed+ae^2\right)\left(\frac{\left(cx^2+bx+a\right)^{3/2}}{3e}-\frac{-\frac{\sqrt{cx^2+bx+a}\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)}{4ce^2}-\frac{\frac{16c\left(cd^2-bed+ae^2\right)^{3/2}\text{arctanh}\left(\frac{bd-2}{2\sqrt{cd^2-b}}\right.}{e}}{2e}}{}\right.}{e(ef-dg)}$$

$$\frac{\frac{\left(8cef^2-g(11bef-3bdg-8aeg)-6cg(ef-dg)x\right)\left(cx^2+bx+a\right)^{3/2}}{24g^2}-\frac{-\frac{\sqrt{cx^2+bx+a}\left(64c^3ef^4-16c^2eg(9bf-8ag)f^2-b^2g^3(5bef+3bdg-8aeg)+4cg^2\left(22b^2ef^2\right.\right.}{}}{}}{}$$

input
```
Int[(a + b*x + c*x^2)^(5/2)/((d + e*x)*(f + g*x)),x]
```

3.869. $\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(d+ex)(f+gx)}\,dx$

output
```
((c*d^2 - b*d*e + a*e^2)*((a + b*x + c*x^2)^(3/2)/(3*e) - (-1/4*((8*c^2*d^
2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt[a + b*x
+ c*x^2])/(c*e^2) - (-(((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d
- 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]
*e)) + (16*c*(c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d -
 b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e)/(8*c*e
^2))/(2*e)))/(e*(e*f - d*g)) - (((8*c*e*f^2 - g*(11*b*e*f - 3*b*d*g - 8*a*
e*g) - 6*c*g*(e*f - d*g)*x)*(a + b*x + c*x^2)^(3/2))/(24*g^2) - (-1/4*((64
*c^3*e*f^4 - 16*c^2*e*f^2*g*(9*b*f - 8*a*g) - b^2*g^3*(5*b*e*f + 3*b*d*g -
 8*a*e*g) + 4*c*g^2*(22*b^2*e*f^2 + 16*a^2*e*g^2 - 3*a*b*g*(13*e*f - d*g))
 - 2*c*g*(16*c^2*e*f^3 + b*g^2*(5*b*e*f + 3*b*d*g - 8*a*e*g) - 4*c*g*(6*b*
e*f^2 - a*g*(7*e*f - 3*d*g)))*x)*Sqrt[a + b*x + c*x^2])/(c*g^2) - (-(((128
*c^4*e*f^5 - 320*c^3*e*f^3*g*(b*f - a*g) - b^3*g^4*(5*b*e*f + 3*b*d*g - 8*
a*e*g) + 48*c^2*g^2*(5*b^2*e*f^3 - 10*a*b*e*f^2*g + a^2*g^2*(5*e*f - d*g))
 - 8*b*c*g^3*(5*b^2*e*f^2 + 12*a^2*e*g^2 - 3*a*b*g*(5*e*f + d*g)))*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*g)) + (128*c*e*(
c*f^2 - b*f*g + a*g^2)^(5/2)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sq
rt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/g)/(8*c*g^2))/(16*g^2))
/(e*(e*f - d*g))
```

3.869.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.869. $\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx$

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1270
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Simp[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g))
Int[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Simp[1/(e*(e*f - d*g))
Int[Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*((a + b*x + c*x^2)^(p
- 1)/(f + g*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[p]
&& GtQ[p, 0]
```

3.869. $\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)} \, dx$

3.869.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 1248, normalized size of antiderivative = 1.41

method	result	size
risch	Expression too large to display	1248
default	Expression too large to display	2107

input
```
int((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
1/192/c*(48*c^3*e^3*g^3*x^3+136*b*c^2*e^3*g^3*x^2-64*c^3*d*e^2*g^3*x^2-64*
c^3*e^3*f*g^2*x^2+216*a*c^2*e^3*g^3*x+118*b^2*c*e^3*g^3*x-208*b*c^2*d*e^2*
g^3*x-208*b*c^2*e^3*f*g^2*x+96*c^3*d^2*e*g^3*x+96*c^3*d*e^2*f*g^2*x+96*c^3
*e^3*f^2*g*x+556*a*b*c*e^3*g^3-448*a*c^2*d*e^2*g^3-448*a*c^2*e^3*f*g^2+15*
b^3*e^3*g^3-264*b^2*c*d*e^2*g^3-264*b^2*c*e^3*f*g^2+432*b*c^2*d^2*e*g^3+43
2*b*c^2*d*e^2*f*g^2+432*b*c^2*e^3*f^2*g-192*c^3*d^3*g^3-192*c^3*d^2*e*f*g^
2-192*c^3*d*e^2*f^2*g-192*c^3*e^3*f^3)*(c*x^2+b*x+a)^(1/2)/e^4/g^4+1/128/g
^4/e^4/c*(128/e^2*g^4*c*(a^3*e^6-3*a^2*b*d*e^5+3*a^2*c*d^2*e^4+3*a*b^2*d^2
*e^4-6*a*b*c*d^3*e^3+3*a*c^2*d^4*e^2-b^3*d^3*e^3+3*b^2*c*d^4*e^2-3*b*c^2*d
^5*e+c^3*d^6)/(d*g-e*f)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e
+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/
e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))-128*
e^4/g^2*c*(a^3*g^6-3*a^2*b*f*g^5+3*a^2*c*f^2*g^4+3*a*b^2*f^2*g^4-6*a*b*c*f
^3*g^3+3*a*c^2*f^4*g^2-b^3*f^3*g^3+3*b^2*c*f^4*g^2-3*b*c^2*f^5*g+c^3*f^6)/
(d*g-e*f)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b
*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*
c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))+(240*a^2*c^2*e^4*g
^4+120*a*b^2*c*e^4*g^4-480*a*b*c^2*d*e^3*g^4-480*a*b*c^2*e^4*f*g^3+320*a*c
^3*d^2*e^2*g^4+320*a*c^3*d*e^3*f*g^3+320*a*c^3*e^4*f^2*g^2-5*b^4*e^4*g^4-4
0*b^3*c*d*e^3*g^4-40*b^3*c*e^4*f*g^3+240*b^2*c^2*d^2*e^2*g^4+240*b^2*c^...
```

3.869.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

output
```
Timed out
```

3.869.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**(5/2)/(e*x+d)/(g*x+f),x)
```

output
```
Timed out
```

3.869.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for m
ore detail
```

3.869.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.869.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx = \int \frac{(c\,x^2+b\,x+a)^{5/2}}{(f+g\,x)\,(d+e\,x)}\,dx$$

input
```
int((a + b*x + c*x^2)^(5/2)/((f + g*x)*(d + e*x)),x)
```

output
```
int((a + b*x + c*x^2)^(5/2)/((f + g*x)*(d + e*x)), x)
```

3.870 $\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

3.870.1 Optimal result

Integrand size = 29, antiderivative size = 431

$$
\begin{aligned}
&\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx \\
&= \frac{g^2(15b^2e^2g^2-4ceg(18bef-7bdg+4aeg)+4c^2(36e^2f^2-36defg+11d^2g^2))\sqrt{a+bx+cx^2}}{24c^3e^3} \\
&\quad + \frac{g^3(24cef-14cdg-5beg)(d+ex)\sqrt{a+bx+cx^2}}{12c^2e^3} + \frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3} \\
&\quad - \frac{g(5b^3e^3g^3-6bce^2g^2(4bef-bdg+2aeg)-16c^3(4e^3f^3-6de^2f^2g+4d^2efg^2-d^3g^3)+8c^2eg(aeg(4ef}{16c^{7/2}e^4} \\
&\quad + \frac{(ef-dg)^4\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^4\sqrt{cd^2-bde+ae^2}}
\end{aligned}
$$

output
```
-1/16*g*(5*b^3*e^3*g^3-6*b*c*e^2*g^2*(2*a*e*g-b*d*g+4*b*e*f)-16*c^3*(-d^3*
g^3+4*d^2*e*f*g^2-6*d*e^2*f^2*g+4*e^3*f^3)+8*c^2*e*g*(a*e*g*(-d*g+4*e*f)+b
*(d^2*g^2-4*d*e*f*g+6*e^2*f^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+
a)^(1/2))/c^(7/2)/e^4+(-d*g+e*f)^4*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/
(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^4/(a*e^2-b*d*e+c*d^2)^(1/
2)+1/24*g^2*(15*b^2*e^2*g^2-4*c*e*g*(4*a*e*g-7*b*d*g+18*b*e*f)+4*c^2*(11*d
^2*g^2-36*d*e*f*g+36*e^2*f^2))*(c*x^2+b*x+a)^(1/2)/c^3/e^3+1/12*g^3*(-5*b*
e*g-14*c*d*g+24*c*e*f)*(e*x+d)*(c*x^2+b*x+a)^(1/2)/c^2/e^3+1/3*g^4*(e*x+d)
^2*(c*x^2+b*x+a)^(1/2)/c/e^3
```

3.870.2 Mathematica [A] (verified)

Time = 1.98 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.88

$$\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{\frac{2eg^2\sqrt{a+x(b+cx)}\left(15b^2e^2g^2-2ceg(8aeg+b(36ef-9dg+5egx))+4c^2\left(6d^2g^2-3deg(8f+gx)+2e^2\left(18f^2+6fgx+g^2x^2\right)\right)\right)}{c^3}+\frac{96\sqrt{-cd^2+bde-ae^2}(\ldots}{\ldots}}{\ldots}$$

input
```
Integrate[(f + g*x)^4/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((2*e*g^2*Sqrt[a + x*(b + c*x)]*(15*b^2*e^2*g^2 - 2*c*e*g*(8*a*e*g + b*(36
*e*f - 9*d*g + 5*e*g*x)) + 4*c^2*(6*d^2*g^2 - 3*d*e*g*(8*f + g*x) + 2*e^2*
(18*f^2 + 6*f*g*x + g^2*x^2))))/c^3 + (96*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(
e*f - d*g)^4*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c
*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e)) + (3*g*(5*b^3*e^3*g^3
- 6*b*c*e^2*g^2*(4*b*e*f - b*d*g + 2*a*e*g) - 16*c^3*(4*e^3*f^3 - 6*d*e^2*
f^2*g + 4*d^2*e*f*g^2 - d^3*g^3) + 8*c^2*e*g*(a*e*g*(4*e*f - d*g) + b*(6*e
^2*f^2 - 4*d*e*f*g + d^2*g^2)))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b +
c*x)]])/c^(7/2))/(48*e^4)
```

3.870.3 Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {1267, 27, 2184, 27, 2184, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$\Big\downarrow \text{1267}$$
$$\frac{\int \frac{e^3g^3(24cef-14cdg-5beg)x^3-e^2g^2\left(e(11bd+4ae)g^2-c\left(36e^2f^2-10d^2g^2\right)\right)x^2-eg\left(de(7bd+8ae)g^3-c\left(24e^3f^3-2d^3g^3\right)\right)x+e\left(6ce^3f^4-d^2(bd+4ae)\ldots\right.}{2(d+ex)\sqrt{cx^2+bx+a}}}{3ce^4}$$
$$\frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}$$

$$\downarrow 27$$

$$\frac{\frac{\int \frac{e^3g^3(24cef-14cdg-5beg)x^3-e^2g^2\left(e(11bd+4ae)g^2-c\left(36e^2f^2-10d^2g^2\right)\right)x^2-eg\left(de(7bd+8ae)g^3-c\left(24e^3f^3-2d^3g^3\right)\right)x+e\left(6ce^3f^4-d^2(bd+4ae)\right.}{(d+ex)\sqrt{cx^2+bx+a}}}{6ce^4}}{\frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}}$$

$$\downarrow 2184$$

$$\frac{\frac{\int \frac{g^2\left(4\left(36e^2f^2-36degf+11d^2g^2\right)c^2-4eg(18bef-7bdg+4aeg)c+15b^2e^2g^2\right)x^2e^5+\left(24c^2e^3f^4+5bde(bd+2ae)g^4-2cdg^3(bd(12ef-5dg)+6ae(4ef-dg))\right)e^4+2g\left(5be^2\right.}{2(d+ex)\sqrt{cx^2+bx+a}}}{2ce^3}}{6ce^4}$$

$$\frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}$$

$$\downarrow 27$$

$$\frac{\frac{\int \frac{g^2\left(4\left(36e^2f^2-36degf+11d^2g^2\right)c^2-4eg(18bef-7bdg+4aeg)c+15b^2e^2g^2\right)x^2e^5+\left(24c^2e^3f^4+5bde(bd+2ae)g^4-2cdg^3(bd(12ef-5dg)+6ae(4ef-dg))\right)e^4+2g\left(5be^2\right.}{(d+ex)\sqrt{cx^2+bx+a}}}{4ce^3}}{6ce^4}$$

$$\frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}$$

$$\downarrow 2184$$

$$\frac{\frac{\int \frac{3e^6\left(16c^3e^3f^4-5b^3de^2g^4+6bcdeg^3(4bef-bdg+2aeg)-8c^2dg^2\left(aeg(4ef-dg)+b\left(6e^2f^2-4degf+d^2g^2\right)\right)-g\left(-16\left(4e^3f^3-6de^2gf^2+4d^2eg^2f-d^3g^3\right)c^3+8eg\left(aeg\right.\right.\right.}{2(d+ex)\sqrt{cx^2+bx+a}}}{ce^2}}{}$$

$$\frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}$$

$$\downarrow 27$$

$$\frac{\frac{3e^4\int \frac{16c^3e^3f^4-5b^3de^2g^4+6bcdeg^3(4bef-bdg+2aeg)-8c^2dg^2\left(aeg(4ef-dg)+b\left(6e^2f^2-4degf+d^2g^2\right)\right)-g\left(-16\left(4e^3f^3-6de^2gf^2+4d^2eg^2f-d^3g^3\right)c^3+8eg\left(aeg\right.\right.}{(d+ex)\sqrt{cx^2+bx+a}}}{2c}}{}$$

$$\frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}$$

$$\downarrow 1269$$

3.870. $\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

$$\frac{\frac{3e^4\left(\frac{\frac{16c^3(ef-dg)^4\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{g\left(8c^2eg\left(aeg(4ef-dg)+b\left(d^2g^2-4defg+6e^2f^2\right)\right)-6bce^2g^2(2aeg-bdg+4bef)+5b^3e^3g^3-16c^3\left(-d^3g^3+4d^2efg^2-\cdots\right.\right.}{e}}{2c}}{4ce^3}}{}$$

$$\frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}$$

$\downarrow$ 1092

$$\frac{\frac{3e^4\left(\frac{\frac{16c^3(ef-dg)^4\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{2g\left(8c^2eg\left(aeg(4ef-dg)+b\left(d^2g^2-4defg+6e^2f^2\right)\right)-6bce^2g^2(2aeg-bdg+4bef)+5b^3e^3g^3-16c^3\left(-d^3g^3+4d^2efg^2-\cdots\right.\right.}{e}}{2c}}{4ce^3}}{}$$

$$\frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}$$

$\downarrow$ 219

$$\frac{\frac{3e^4\left(\frac{\frac{16c^3(ef-dg)^4\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{g\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(8c^2eg\left(aeg(4ef-dg)+b\left(d^2g^2-4defg+6e^2f^2\right)\right)-6bce^2g^2(2aeg-bdg+4bef)+5b^3\cdots\right.}{\sqrt{c}e}}{2c}}{4ce^3}}{}$$

$$\frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}$$

$\downarrow$ 1154

$$\frac{3e^4\left(\frac{-\frac{32c^3(ef-dg)^4\int\frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}-\frac{g\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(8c^2eg\left(aeg(4ef-dg)+b\left(d^2g^2-\cdots\right.\right.\right.}{\cdots}}{2c}\right.}{}$$

$$\frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}$$

$\downarrow$ 219

$$\frac{3e^4\left(\frac{\frac{16c^3(ef-dg)^4\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\sqrt{ae^2-bde+cd^2}}-\frac{g\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(8c^2eg\left(aeg(4ef-dg)+b\left(d^2g^2-4defg+6e^2f^2\right)\right)-6bce^2g^2(2a\cdots\right.}{\sqrt{c}e}}{2c}\right.}{}$$

$$\frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}$$

3.870. $\int\frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

input
```
Int[(f + g*x)^4/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(g^4*(d + e*x)^2*Sqrt[a + b*x + c*x^2])/(3*c*e^3) + ((e*g^3*(24*c*e*f - 14
*c*d*g - 5*b*e*g)*(d + e*x)*Sqrt[a + b*x + c*x^2])/(2*c) + ((e^4*g^2*(15*b
^2*e^2*g^2 - 4*c*e*g*(18*b*e*f - 7*b*d*g + 4*a*e*g) + 4*c^2*(36*e^2*f^2 -
36*d*e*f*g + 11*d^2*g^2))*Sqrt[a + b*x + c*x^2])/c + (3*e^4*(-((g*(5*b^3*e
^3*g^3 - 6*b*c*e^2*g^2*(4*b*e*f - b*d*g + 2*a*e*g) - 16*c^3*(4*e^3*f^3 - 6
*d*e^2*f^2*g + 4*d^2*e*f*g^2 - d^3*g^3) + 8*c^2*e*g*(a*e*g*(4*e*f - d*g) +
 b*(6*e^2*f^2 - 4*d*e*f*g + d^2*g^2)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])])/(Sqrt[c]*e)) + (16*c^3*(e*f - d*g)^4*ArcTanh[(b*d - 2
*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^
2])])/(e*Sqrt[c*d^2 - b*d*e + a*e^2])))/(2*c))/(4*c*e^3))/(6*c*e^4)
```

3.870.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.870.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.29

method	result
risch	$-\frac{g^2(-8c^2e^2g^2x^2+10bc\,e^2g^2x+12c^2de\,g^2x-48c^2e^2fgx+16ac\,e^2g^2-15b^2e^2g^2-18bcde\,g^2+72bc\,e^2fg-24c^2d^2g^2+96c^2defg-144c^2e}{24c^3e^3}$
default	$-\frac{(g^4d^4-4d^3ef\,g^3+6d^2e^2f^2g^2-4d\,e^3f^3g+e^4f^4)\ln\left(\frac{\frac{2e^2a-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a-bde+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}}}{x+\frac{d}{e}}\right)}{e^5\sqrt{\frac{e^2a-bde+c\,d^2}{e^2}}}$

input
```
int((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*g^2*(-8*c^2*e^2*g^2*x^2+10*b*c*e^2*g^2*x+12*c^2*d*e*g^2*x-48*c^2*e^2
*f*g*x+16*a*c*e^2*g^2-15*b^2*e^2*g^2-18*b*c*d*e*g^2+72*b*c*e^2*f*g-24*c^2*
d^2*g^2+96*c^2*d*e*f*g-144*c^2*e^2*f^2)*(c*x^2+b*x+a)^(1/2)/c^3/e^3+1/16/c
^3/e^3*(-16*(d^4*g^4-4*d^3*e*f*g^3+6*d^2*e^2*f^2*g^2-4*d*e^3*f^3*g+e^4*f^4
)*c^3/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b
*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*
c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))+g*(12*a*b*c*e^3*g^
3+8*a*c^2*d*e^2*g^3-32*a*c^2*e^3*f*g^2-5*b^3*e^3*g^3-6*b^2*c*d*e^2*g^3+24*
b^2*c*e^3*f*g^2-8*b*c^2*d^2*e*g^3+32*b*c^2*d*e^2*f*g^2-48*b*c^2*e^3*f^2*g-
16*c^3*d^3*g^3+64*c^3*d^2*e*f*g^2-96*c^3*d*e^2*f^2*g+64*c^3*e^3*f^3)/e*ln(
(1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2))
```

3.870.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.870.6 Sympy [F]

$$\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x+f)**4/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((f + g*x)**4/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

3.870.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

3.870.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.870.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+g\,x)^4}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.871 $\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

3.871.1 Optimal result

Integrand size = 29, antiderivative size = 270

$$\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2}+\frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}+\frac{g(3b^2e^2g^2-4ceg(3bef-bdg+aeg)+8c^2(3e^2f^2-3defg+d^2g^2))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}e^3}+\frac{(ef-dg)^3\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^3\sqrt{cd^2-bde+ae^2}}$$

output
```
1/8*g*(3*b^2*e^2*g^2-4*c*e*g*(a*e*g-b*d*g+3*b*e*f)+8*c^2*(d^2*g^2-3*d*e*f*
g+3*e^2*f^2))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/e
^3+(-d*g+e*f)^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)
^(1/2)/(c*x^2+b*x+a)^(1/2))/e^3/(a*e^2-b*d*e+c*d^2)^(1/2)+3/4*g^2*(-b*e*g-
2*c*d*g+4*c*e*f)*(c*x^2+b*x+a)^(1/2)/c^2/e^2+1/2*g^3*(e*x+d)*(c*x^2+b*x+a)
^(1/2)/c/e^2
```

3.871.2 Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.91

$$\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{-\frac{2eg^2\sqrt{a+x(b+cx)}(-3beg+2c(6ef-2dg+egx))}{c^2}+\frac{16\sqrt{-cd^2+bde-ae^2}(-ef+dg)^3\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)}+\frac{g(3b^2e^2g^2-4ceg}{}}{8e^3}$$

input
```
Integrate[(f + g*x)^3/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/8*((-2*e*g^2*Sqrt[a + x*(b + c*x)]*(-3*b*e*g + 2*c*(6*e*f - 2*d*g + e*g
*x)))/c^2 + (16*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(-(e*f) + d*g)^3*ArcTan[(Sq
rt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]]
)/(c*d^2 + e*(-(b*d) + a*e)) + (g*(3*b^2*e^2*g^2 - 4*c*e*g*(3*b*e*f - b*d*
g + a*e*g) + 8*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*Log[b + 2*c*x - 2*Sq
rt[c]*Sqrt[a + x*(b + c*x)]])/c^(5/2))/e^3
```

3.871.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1267, 27, 2184, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \; 1267$$

$$\frac{\int \frac{3e^2g^2(4cef-2cdg-beg)x^2-2eg\left(e(2bd+ae)g^2-c(6e^2f^2-d^2g^2)\right)x+e\left(4ce^2f^3-d(bd+2ae)g^3\right)}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{2ce^3}+\frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{3e^2g^2(4cef-2cdg-beg)x^2-2eg\left(e(2bd+ae)g^2-c(6e^2f^2-d^2g^2)\right)x+e\left(4ce^2f^3-d(bd+2ae)g^3\right)}{(d+ex)\sqrt{cx^2+bx+a}}dx}{4ce^3}+\frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}}{}$$

$\downarrow$ 2184

$$\frac{\frac{\int \frac{e^3\left(8c^2e^2f^3+3b^2deg^3-4cdg^2(3bef-bdg+aeg)+g\left(8\left(3e^2f^2-3degf+d^2g^2\right)c^2-4eg(3bef-bdg+aeg)c+3b^2e^2g^2\right)x\right)}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{ce^2}+\frac{3eg^2\sqrt{a+bx+cx^2}(-beg-2cdg+4c}{c}}{4ce^3}+\frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}$$

$\downarrow$ 27

$$\frac{\frac{e\int \frac{8c^2e^2f^3+3b^2deg^3-4cdg^2(3bef-bdg+aeg)+g\left(8\left(3e^2f^2-3degf+d^2g^2\right)c^2-4eg(3bef-bdg+aeg)c+3b^2e^2g^2\right)x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{2c}+\frac{3eg^2\sqrt{a+bx+cx^2}(-beg-2cdg+4cef)}{c}}{4ce^3}+\frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}$$

$\downarrow$ 1269

$$\frac{\frac{e\left(\frac{g\left(-4ceg(aeg-bdg+3bef)+3b^2e^2g^2+8c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}+\frac{8c^2(ef-dg)^3\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}\right)}{2c}+\frac{3eg^2\sqrt{a+bx+cx^2}(-beg-}{c}}{4ce^3}+\frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}$$

$\downarrow$ 1092

$$\frac{\frac{e\left(\frac{2g\left(-4ceg(aeg-bdg+3bef)+3b^2e^2g^2+8c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}+\frac{8c^2(ef-dg)^3\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}\right)}{2c}+3eg^2\sqrt{a+}}{4ce^3}+\frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}$$

$\downarrow$ 219

3.871. $\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

$$\frac{\frac{e\left(\frac{8c^2(ef-dg)^3\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}+\frac{g\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ceg(aeg-bdg+3bef)+3b^2e^2g^2+8c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)}{\sqrt{c}e}\right)}{2c}+\frac{3eg^2\sqrt{a+bx}\ldots}{}}{4ce^3}+\frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}$$

$\Big\downarrow$ 1154

$$\frac{\frac{e\left(\frac{g\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ceg(aeg-bdg+3bef)+3b^2e^2g^2+8c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)}{\sqrt{c}e}-\frac{16c^2(ef-dg)^3\int\frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}\ldots}{e}\right)}{2c}\ldots}{4ce^3}+\frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}$$

$\Big\downarrow$ 219

$$\frac{\frac{e\left(\frac{g\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ceg(aeg-bdg+3bef)+3b^2e^2g^2+8c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)}{\sqrt{c}e}+\frac{8c^2(ef-dg)^3\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\sqrt{ae^2-bde+cd^2}}\ldots\right)}{2c}\ldots}{4ce^3}+\frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}$$

input
```
Int[(f + g*x)^3/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(g^3*(d + e*x)*Sqrt[a + b*x + c*x^2])/(2*c*e^2) + ((3*e*g^2*(4*c*e*f - 2*c
*d*g - b*e*g)*Sqrt[a + b*x + c*x^2])/c + (e*((g*(3*b^2*e^2*g^2 - 4*c*e*g*(
3*b*e*f - b*d*g + a*e*g) + 8*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e) + (8*c^2*(e*
f - d*g)^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e +
 a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*Sqrt[c*d^2 - b*d*e + a*e^2])))/(2*c))/
(4*c*e^3)
```

3.871. $\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

3.871.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.871. $\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.871.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.31

method	result
risch	$-\frac{g^{2}\left(-2cegx+3beg+4cdg-12cef\right)\sqrt{c\,x^{2}+bx+a}}{4c^{2}e^{2}}-\frac{-\frac{8\left(d^{3}g^{3}-3d^{2}ef\,g^{2}+3d\,e^{2}f^{2}g-e^{3}f^{3}\right)c^{2}\ln\left(\frac{\frac{2e^{2}a-2bde+2c\,d^{2}}{e^{2}}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\cdots}}{\cdots}\right)}{e^{2}\sqrt{\frac{e^{2}a-bde+c\,d^{2}}{e^{2}}}}\cdots}{\cdots}$
default	$-\frac{\left(-d^{3}g^{3}+3d^{2}ef\,g^{2}-3d\,e^{2}f^{2}g+e^{3}f^{3}\right)\ln\left(\frac{\frac{2e^{2}a-2bde+2c\,d^{2}}{e^{2}}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^{2}a-bde+c\,d^{2}}{e^{2}}}\sqrt{\left(x+\frac{d}{e}\right)^{2}c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^{2}a-bc\cdots}{e\cdots}}}{x+\frac{d}{e}}\right)}{e^{4}\sqrt{\frac{e^{2}a-bde+c\,d^{2}}{e^{2}}}}\cdots$

input
```
int((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*g^2*(-2*c*e*g*x+3*b*e*g+4*c*d*g-12*c*e*f)*(c*x^2+b*x+a)^(1/2)/c^2/e^2
-1/8/c^2/e^2*(-8*(d^3*g^3-3*d^2*e*f*g^2+3*d*e^2*f^2*g-e^3*f^3)*c^2/e^2/((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(
x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))+g*(4*a*c*e^2*g^2-3*b^2*e^2*g^2-
4*b*c*d*e*g^2+12*b*c*e^2*f*g-8*c^2*d^2*g^2+24*c^2*d*e*f*g-24*c^2*e^2*f^2)/
e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2))
```

3.871.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.871.6 Sympy [F]

$$\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x+f)**3/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((f + g*x)**3/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

3.871.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

3.871.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.871.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+g\,x)^3}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.872 $\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

3.872.1 Optimal result

Integrand size = 29, antiderivative size = 176

$$\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{g^2\sqrt{a+bx+cx^2}}{ce} + \frac{g(4cef-2cdg-beg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}e^2} + \frac{(ef-dg)^2\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^2\sqrt{cd^2-bde+ae^2}}$$

output
```
1/2*g*(-b*e*g-2*c*d*g+4*c*e*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)
^(1/2))/c^(3/2)/e^2+(-d*g+e*f)^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a
*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^2/(a*e^2-b*d*e+c*d^2)^(1/2)
+g^2*(c*x^2+b*x+a)^(1/2)/c/e
```

3.872.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.05

$$\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{\frac{2eg^2\sqrt{a+x(b+cx)}}{c} + \frac{4\sqrt{-cd^2+bde-ae^2}(ef-dg)^2\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)} - \frac{g(-4cef+2cdg+beg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}}}{2e^2}$$

input
```
Integrate[(f + g*x)^2/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((2*e*g^2*Sqrt[a + x*(b + c*x)])/c + (4*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e*
f - d*g)^2*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d
^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e)) - (g*(-4*c*e*f + 2*c*d*g
 + b*e*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(3/2))
/(2*e^2)
```

3.872.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1267, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1267}$$

$$\frac{\int \frac{e(2cef^2-bdg^2+g(4cef-2cdg-beg)x)}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{ce^2}+\frac{g^2\sqrt{a+bx+cx^2}}{ce}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2cef^2-bdg^2+g(4cef-2cdg-beg)x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{2ce}+\frac{g^2\sqrt{a+bx+cx^2}}{ce}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\frac{2c(ef-dg)^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}+\frac{g(-beg-2cdg+4cef)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}}{2ce}+\frac{g^2\sqrt{a+bx+cx^2}}{ce}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\frac{2c(ef-dg)^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}+\frac{2g(-beg-2cdg+4cef)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}}{2ce}+\frac{g^2\sqrt{a+bx+cx^2}}{ce}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{2c(ef-dg)^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}+\frac{g\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-beg-2cdg+4cef)}{\sqrt{c}e}}{2ce}+\frac{g^2\sqrt{a+bx+cx^2}}{ce}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{\frac{g\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-beg-2cdg+4cef)}{\sqrt{ce}}-\frac{4c(ef-dg)^2\int\frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}}{2ce}+\frac{g^2\sqrt{a+bx+cx^2}}{ce}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{2c(ef-dg)^2\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\sqrt{ae^2-bde+cd^2}}+\frac{g\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-beg-2cdg+4cef)}{\sqrt{ce}}}{2ce}+\frac{g^2\sqrt{a+bx+cx^2}}{ce}$$

input
```
Int[(f + g*x)^2/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(g^2*Sqrt[a + b*x + c*x^2])/(c*e) + ((g*(4*c*e*f - 2*c*d*g - b*e*g)*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e) + (2*c*(e*f
- d*g)^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a
*e^2]*Sqrt[a + b*x + c*x^2])])/(e*Sqrt[c*d^2 - b*d*e + a*e^2]))/(2*c*e)
```

3.872.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.872.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.47

method	result
risch	$\frac{g^2\sqrt{c\,x^2+bx+a}}{ce}-\frac{\frac{g(beg+2cdg-4cef)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{e\sqrt{c}}+\frac{2\left(d^2g^2-2defg+e^2f^2\right)c\ln\left(\frac{\frac{2e^2a-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a}{\cdots}}\cdots}{\cdots}\right)}{e^2\sqrt{\frac{e^2a-bde+c\cdots}{e^2}}\cdots}}{2ec}$
default	$-\frac{g\left(\frac{dg\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}-\frac{2ef\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}-eg\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)\right)}{e^2}-\frac{\left(d^2g^2-2defg\cdots\right.}{\cdots}$

input
```
int((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
g^2*(c*x^2+b*x+a)^(1/2)/c/e-1/2/e/c*(g*(b*e*g+2*c*d*g-4*c*e*f)/e*ln((1/2*b
+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+2*(d^2*g^2-2*d*e*f*g+e^2*f^2)*c
/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*
c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/
e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

3.872.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.872.6 Sympy [F]

$$\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x+f)**2/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((f + g*x)**2/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

3.872.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

3.872.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.872.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+g\,x)^2}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.873 $\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

3.873.1 Optimal result

Integrand size = 27, antiderivative size = 131

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{g\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e} + \frac{(ef-dg)\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e\sqrt{cd^2-bde+ae^2}}$$

output
```
g*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/e/c^(1/2)+(-d*g+e*f)*
arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*
x+a)^(1/2))/e/(a*e^2-b*d*e+c*d^2)^(1/2)
```

3.873.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.08

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = -\frac{\frac{2\sqrt{-cd^2+bde-ae^2}(-ef+dg)\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)} + \frac{g\log\left(e\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{\sqrt{c}}}{e}$$

input
```
Integrate[(f + g*x)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-(((2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(-(e*f) + d*g)*ArcTan[(Sqrt[c]*(d + e
*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e
*(-(b*d) + a*e)) + (g*Log[e*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
)/Sqrt[c])/e)
```

3.873.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1269}$$

$$\frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}+\frac{g\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}$$

$$\Big\downarrow \text{1092}$$

$$\frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}+\frac{2g\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}$$

$$\Big\downarrow \text{219}$$

$$\frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}+\frac{g\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}$$

$$\Big\downarrow \text{1154}$$

$$\frac{g\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}-\frac{2(ef-dg)\int \frac{1}{4(cd^2-bed+ae^2)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}$$

$$\Big\downarrow \text{219}$$

$$\frac{(ef-dg)\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\sqrt{ae^2-bde+cd^2}}+\frac{g\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}$$

input
```
Int[(f + g*x)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(g*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e) + (
(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e
+ a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*Sqrt[c*d^2 - b*d*e + a*e^2])
```

3.873.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.873.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.52

method	result
default	$\frac{g \ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\,x^{2}+b x+a}\right)}{e \sqrt{c}}-\frac{\left(-d g+e f\right) \ln\left(\frac{\frac{2 e^{2} a-2 b d e+2 c\,d^{2}}{e^{2}}+\frac{\left(b e-2 c d\right) \left(x+\frac{d}{e}\right)}{e}+2 \sqrt{\frac{e^{2} a-b d e+c\,d^{2}}{e^{2}}}\, \sqrt{\left(x+\frac{d}{e}\right)^{2} c+\frac{\left(b e-2 c d\right) \left(x+\frac{d}{e}\right)}{e}+e^{2}}}{x+\frac{d}{e}}\right)}{e^{2} \sqrt{\frac{e^{2} a-b d e+c\,d^{2}}{e^{2}}}}$

input
```
int((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
g/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-(-d*g+e*f)/e^2/((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(
x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

3.873.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(115) = 230$.

Time = 21.85 (sec) , antiderivative size = 1071, normalized size of antiderivative = 8.18

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

$$= \left[\frac{(cd^2-bde+ae^2)\sqrt{c}g\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)-\sqrt{cd^2-bde+}}{}\right.$$

$$-\frac{2\,(cd^2-bde+ae^2)\sqrt{-c}g\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)+\sqrt{cd^2-bde+ae^2}(cef-cdg)\log\left(\frac{8\,abde-8\,a^2e}{}\right.}{2\,(c^2d^2e-b}$$

$$-\frac{(cd^2-bde+ae^2)\sqrt{-c}g\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)-\sqrt{-cd^2+bde-ae^2}(cef-cdg)\arctan\left(-\frac{}{2\,(ac}\right.}{c^2d^2e-bcde^2+ace^3}$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*((c*d^2 - b*d*e + a*e^2)*sqrt(c)*g*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4
*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - sqrt(c*d^2 - b*d*e +
 a*e^2)*(c*e*f - c*d*g)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (
8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*
e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2
+ 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c^2*d^2
*e - b*c*d*e^2 + a*c*e^3), 1/2*((c*d^2 - b*d*e + a*e^2)*sqrt(c)*g*log(-8*c
^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a
*c) + 2*sqrt(-c*d^2 + b*d*e - a*e^2)*(c*e*f - c*d*g)*arctan(-1/2*sqrt(-c*d
^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/
(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^
2 - b^2*d*e + a*b*e^2)*x)))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), -1/2*(2*(c*
d^2 - b*d*e + a*e^2)*sqrt(-c)*g*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x +
b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + sqrt(c*d^2 - b*d*e + a*e^2)*(c*e*f
- c*d*g)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b
*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2
 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (
3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c^2*d^2*e - b*c*d*e^2
+ a*c*e^3), -((c*d^2 - b*d*e + a*e^2)*sqrt(-c)*g*arctan(1/2*sqrt(c*x^2 + b
*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - sqrt(-c*d^2 + b...
```

3.873.6 Sympy [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{f+gx}{(d+ex)\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((f + g*x)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

3.873.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

3.873.8 Giac [F(-2)]

Exception generated.

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.873.9 Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{f+g\,x}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.874 $\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

3.874.1 Optimal result

Integrand size = 22, antiderivative size = 79

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}}$$

output
```
arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*
x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(1/2)
```

3.874.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx=\frac{2\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)}$$

input
```
Integrate[1/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x
*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e))
```

3.874.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1154}$$

$$-2\int \frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\sqrt{ae^2-bde+cd^2}}$$

input
```
Int[1/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqr
t[a + b*x + c*x^2])]/Sqrt[c*d^2 - b*d*e + a*e^2]
```

3.874.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.874.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(71) = 142.

Time = 0.66 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.99

method	result	size
default	$-\frac{\ln\left(\frac{\frac{2e^2a-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a-bde+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a-bde+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{e\sqrt{\frac{e^2a-bde+c\,d^2}{e^2}}}$	157

input
```
int(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*
c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/
e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

3.874.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(71) = 142.

Time = 0.33 (sec) , antiderivative size = 343, normalized size of antiderivative = 4.34

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$= \left[\frac{\log\left(\frac{8\,abde-8\,a^2e^2-(b^2+4\,ac)d^2-(8\,c^2d^2-8\,bcde+(b^2+4\,ac)e^2)x^2-4\,\sqrt{cd^2-bde+ae^2}\sqrt{cx^2+bx+a}(bd-2\,ae+(2\,cd-be)x)-2\,(4\,bcd^2+4\,abe^2}{e^2x^2+2\,dex+d^2}\right.}{2\,\sqrt{cd^2-bde+ae^2}}\right.$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d
*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b
*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^
2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2))/sqrt(c*d^2 - b*d*e + a*e^2),
 sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqr
t(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^
2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*
x))/(c*d^2 - b*d*e + a*e^2)]
```

3.874.6 Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

3.874.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` f
or more de
```

3.874.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2
+ b*d*e - a*e^2))/sqrt(-c*d^2 + b*d*e - a*e^2)
```

3.874.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.875 $\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx$

3.875.1 Optimal result

Integrand size = 29, antiderivative size = 182

$$\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \frac{e\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}(ef-dg)} - \frac{g\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)\sqrt{cf^2-bfg+ag^2}}$$

output
```
e*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+
b*x+a)^(1/2))/(-d*g+e*f)/(a*e^2-b*d*e+c*d^2)^(1/2)-g*arctanh(1/2*(b*f-2*a*
g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f
)/(a*g^2-b*f*g+c*f^2)^(1/2)
```

3.875.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.14

$$\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = -\frac{2e\sqrt{-cd^2+e(bd-ae)}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{(cd^2+e(-bd+ae))\,(-ef+dg)} - \frac{2g\sqrt{-cf^2+bfg-ag^2}\arctan\left(\frac{\sqrt{c}(f+gx)-g\sqrt{a+x(b+cx)}}{\sqrt{-cf^2+g(bf-ag)}}\right)}{(ef-dg)\,(cf^2+g(-bf+ag))}$$

input
```
Integrate[1/((d + e*x)*(f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(-2*e*Sqrt[-(c*d^2) + e*(b*d - a*e)]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a
+ x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/((c*d^2 + e*(-(b*d) + a*e
))*(-(e*f) + d*g)) - (2*g*Sqrt[-(c*f^2) + b*f*g - a*g^2]*ArcTan[(Sqrt[c]*(
f + g*x) - g*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*f^2) + g*(b*f - a*g)]])/((e*f
 - d*g)*(c*f^2 + g*(-(b*f) + a*g)))
```

3.875.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{1289}$$

$$\int \left(\frac{e}{(d+ex)\sqrt{a+bx+cx^2}(ef-dg)}-\frac{g}{(f+gx)\sqrt{a+bx+cx^2}(ef-dg)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e\,\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)\sqrt{ae^2-bde+cd^2}}-\frac{g\,\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)\sqrt{ag^2-bfg+cf^2}}$$

input
```
Int[1/((d + e*x)*(f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*
Sqrt[a + b*x + c*x^2])])/(Sqrt[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)) - (g*Ar
cTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[
a + b*x + c*x^2])])/((e*f - d*g)*Sqrt[c*f^2 - b*f*g + a*g^2])
```

3.875.3.1 Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.875.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.80

method	result
default	$-\frac{\ln\left(\frac{\frac{2a\,g^2-2bfg+2c\,f^2}{g^2}+\frac{(bg-2cf)\left(x+\frac{f}{g}\right)}{g}+2\sqrt{\frac{a\,g^2-bfg+c\,f^2}{g^2}}\sqrt{\left(x+\frac{f}{g}\right)^2c+\frac{(bg-2cf)\left(x+\frac{f}{g}\right)}{g}+\frac{a\,g^2-bfg+c\,f^2}{g^2}}}{x+\frac{f}{g}}\right)}{(dg-ef)\sqrt{\frac{a\,g^2-bfg+c\,f^2}{g^2}}}+\frac{\ln\left(\frac{\frac{2e^2a-2bde+2c\,d^2}{e^2}}{}\right.}{}$

input
```
int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(d*g-e*f)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2
+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g
-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))+1/(d*g-e*f)/((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(
x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

3.875.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(166) = 332$.

Time = 85.21 (sec) , antiderivative size = 1952, normalized size of antiderivative = 10.73

$$\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((c*d^2 - b*d*e + a*e^2)*sqrt(c*f^2 - b*f*g + a*g^2)*g*log((8*a*b*f*
g - 8*a^2*g^2 - (b^2 + 4*a*c)*f^2 - (8*c^2*f^2 - 8*b*c*f*g + (b^2 + 4*a*c)
*g^2)*x^2 - 4*sqrt(c*f^2 - b*f*g + a*g^2)*sqrt(c*x^2 + b*x + a)*(b*f - 2*a
*g + (2*c*f - b*g)*x) - 2*(4*b*c*f^2 + 4*a*b*g^2 - (3*b^2 + 4*a*c)*f*g)*x)
/(g^2*x^2 + 2*f*g*x + f^2)) + (c*e*f^2 - b*e*f*g + a*e*g^2)*sqrt(c*d^2 - b
*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2
- 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(
c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^
2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/((c^2*d^2*e - b*c*
d*e^2 + a*c*e^3)*f^3 - (c^2*d^3 + a*b*e^3 - (b^2 - a*c)*d*e^2)*f^2*g + (b*
c*d^3 + a^2*e^3 - (b^2 - a*c)*d^2*e)*f*g^2 - (a*c*d^3 - a*b*d^2*e + a^2*d*
e^2)*g^3), -1/2*(2*(c*d^2 - b*d*e + a*e^2)*sqrt(-c*f^2 + b*f*g - a*g^2)*g*
arctan(-1/2*sqrt(-c*f^2 + b*f*g - a*g^2)*sqrt(c*x^2 + b*x + a)*(b*f - 2*a*
g + (2*c*f - b*g)*x)/(a*c*f^2 - a*b*f*g + a^2*g^2 + (c^2*f^2 - b*c*f*g + a
*c*g^2)*x^2 + (b*c*f^2 - b^2*f*g + a*b*g^2)*x)) + (c*e*f^2 - b*e*f*g + a*e
*g^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*
c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 -
b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(
4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2))
)/((c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*f^3 - (c^2*d^3 + a*b*e^3 - (b^2 - ...
```

3.875.6 Sympy [F]

$$\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d+ex)\,(f+gx)\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/((d + e*x)*(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.875.7 Maxima [F]

$$\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)(gx+f)}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)), x)
```

3.875.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.875.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(f+g\,x)\,(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.876 $\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx$

3.876.1 Optimal result

Integrand size = 29, antiderivative size = 340

$$\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \frac{g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)} + \frac{e^2\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}(ef-dg)^2} - \frac{g(2cf-bg)\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2(ef-dg)\left(cf^2-bfg+ag^2\right)^{3/2}} - \frac{eg\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^2\sqrt{cf^2-bfg+ag^2}}$$

output
```
-1/2*g*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*
f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(3/2)+e^2*a
rctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x
+a)^(1/2))/(-d*g+e*f)^2/(a*e^2-b*d*e+c*d^2)^(1/2)-e*g*arctanh(1/2*(b*f-2*a
*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*
f)^2/(a*g^2-b*f*g+c*f^2)^(1/2)+g^2*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b
*f*g+c*f^2)/(g*x+f)
```

3.876.2 Mathematica [A] (verified)

Time = 10.64 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.75

$$\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = -\frac{\frac{2g^2(-ef+dg)\sqrt{a+x(b+cx)}}{(cf^2+g(-bf+ag))(f+gx)} - \frac{2e^2\text{arctanh}\left(\frac{-2ae+2cdx+b(d-ex)}{2\sqrt{cd^2+e(-bd+ae)}\sqrt{a+x(b+cx)}}\right)}{\sqrt{cd^2+e(-bd+ae)}} + \frac{g(2cf(2ef-dg)+g(-3bef+bdg+2aeg))\text{arctanh}\left(\frac{-2ag+2cfx+b(f-gx)}{2\sqrt{cf^2+g(-bf+ag)}\sqrt{a+x(b+cx)}}\right)}{(cf^2+g(-bf+ag))^{3/2}}}{2(ef-dg)^2}$$

input
```
Integrate[1/((d + e*x)*(f + g*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/2*((2*g^2*(-(e*f) + d*g)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f) + a
*g))*(f + g*x)) - (2*e^2*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[
c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d^2 + e*(-(b*d)
+ a*e)] + (g*(2*c*f*(2*e*f - d*g) + g*(-3*b*e*f + b*d*g + 2*a*e*g))*ArcTan
h[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[
a + x*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^(3/2))/(e*f - d*g)^2
```

3.876.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{ 1289}$$

$$\int \left(\frac{e^2}{(d+ex)\sqrt{a+bx+cx^2}(ef-dg)^2} - \frac{eg}{(f+gx)\sqrt{a+bx+cx^2}(ef-dg)^2} - \frac{g}{(f+gx)^2\sqrt{a+bx+cx^2}(ef-dg)}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e^2\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2\sqrt{ae^2-bde+cd^2}}-\frac{eg\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^2\sqrt{ag^2-bfg+cf^2}}-\frac{g(2cf-bg)\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2(ef-dg)\left(ag^2-bfg+cf^2\right)^{3/2}}+\frac{g^2\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)}$$

input
```
Int[1/((d + e*x)*(f + g*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(g^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*x)
) + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a
*e^2]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)^2)
 - (g*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2
- b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*(e*f - d*g)*(c*f^2 - b*f*g +
a*g^2)^(3/2)) - (e*g*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2
 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((e*f - d*g)^2*Sqrt[c*f^2 - b*f
*g + a*g^2])
```

3.876.3.1 Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.876.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.79

method	result
default	$-\frac{e\ln\left(\frac{\frac{2e^2a-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a-bde+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a-bde+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{(dg-ef)^2\sqrt{\frac{e^2a-bde+c\,d^2}{e^2}}}+\frac{-\frac{g^2\sqrt{\left(x+\frac{f}{g}\right)^2c+\frac{(bg-}{}}}{\left(a\,g^2-b\right.}}{}$

input
```
int(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-e/(d*g-e*f)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e
^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b
*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))+1/g/(d*g-e*f)
*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a
*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)/((a*g^2
-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/
g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a
*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))+e/(d*g-e*f)^2/((a*g^2-b*f*g+c*f^2)
/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-
b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*
f^2)/g^2)^(1/2))/(x+f/g))
```

3.876.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.876.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)**2/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Timed out
```

3.876.7 Maxima [F]

$$\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx=\int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)(gx+f)^2}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^2), x)
```

3.876.8 Giac [F]

$$\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx=\int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)(gx+f)^2}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.876.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx=\int \frac{1}{(f+g\,x)^2\,(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/((f + g*x)^2*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/((f + g*x)^2*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.877 $\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx$

3.877.1 Optimal result

Integrand size = 29, antiderivative size = 587

$$\begin{aligned}
&\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx \\
&= \frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2}+\frac{3g^2(2cf-bg)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)} \\
&\quad+\frac{eg^2\sqrt{a+bx+cx^2}}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)(f+gx)}+\frac{e^3\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}(ef-dg)^3} \\
&\quad-\frac{eg(2cf-bg)\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2(ef-dg)^2\left(cf^2-bfg+ag^2\right)^{3/2}}-\frac{e^2g\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^3\sqrt{cf^2-bfg+ag^2}} \\
&\quad-\frac{g\left(8c^2f^2+3b^2g^2-4cg(2bf+ag)\right)\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^{5/2}}
\end{aligned}$$

output
```
-1/2*e*g*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+
c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)^(3/2)-1
/8*g*(8*c^2*f^2+3*b^2*g^2-4*c*g*(a*g+2*b*f))*arctanh(1/2*(b*f-2*a*g+(-b*g+
2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(a*g^2
-b*f*g+c*f^2)^(5/2)+e^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*
e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)^3/(a*e^2-b*d*e+c*d^2)^(1/2)
-e^2*g*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c
*x^2+b*x+a)^(1/2))/(-d*g+e*f)^3/(a*g^2-b*f*g+c*f^2)^(1/2)+1/2*g^2*(c*x^2+b
*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)^2+3/4*g^2*(-b*g+2*c*f)*
(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2/(g*x+f)+e*g^2*(c*x^2+
b*x+a)^(1/2)/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)/(g*x+f)
```

3.877.2 Mathematica [A] (verified)

Time = 11.31 (sec) , antiderivative size = 549, normalized size of antiderivative = 0.94

$$\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\frac{4g^2(ef-dg)^2\sqrt{a+x(b+cx)}}{(cf^2+g(-bf+ag))(f+gx)^2}+\frac{8eg^2(ef-dg)\sqrt{a+x(b+cx)}}{(cf^2+g(-bf+ag))(f+gx)}+\frac{8e^3\operatorname{arctanh}\left(\frac{-2ae+2cdx+b(d-ex)}{2\sqrt{cd^2+e(-bd+ae)}\sqrt{a+x(b+cx)}}\right)}{\sqrt{cd^2+e(-bd+ae)}}+\frac{4eg(-2cf+bg)(ef-dg)\operatorname{arct}}{(cf^2-}}{}$$

input
```
Integrate[1/((d + e*x)*(f + g*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((4*g^2*(e*f - d*g)^2*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f) + a*g))*(
f + g*x)^2) + (8*e*g^2*(e*f - d*g)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b
*f) + a*g))*(f + g*x)) + (8*e^3*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(
2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d^2 + e*(
-(b*d) + a*e)] + (4*e*g*(-2*c*f + b*g)*(e*f - d*g)*ArcTanh[(-2*a*g + 2*c*f
*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])
])/(c*f^2 + g*(-(b*f) + a*g))^(3/2) - (8*e^2*g*ArcTanh[(-2*a*g + 2*c*f*x +
 b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/S
qrt[c*f^2 + g*(-(b*f) + a*g)] + g*(e*f - d*g)^2*((6*g*(2*c*f - b*g)*Sqrt[a
 + x*(b + c*x)])/((c*f^2 + g*(-(b*f) + a*g))^2*(f + g*x)) - ((8*c^2*f^2 +
3*b^2*g^2 - 4*c*g*(2*b*f + a*g))*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/
(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^2 + g*(-(b
*f) + a*g))^(5/2)))/(8*(e*f - d*g)^3)
```

3.877.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1289}$$

$$\int \left(\frac{e^3}{(d+ex)\sqrt{a+bx+cx^2}(ef-dg)^3}-\frac{e^2g}{(f+gx)\sqrt{a+bx+cx^2}(ef-dg)^3}-\frac{eg}{(f+gx)^2\sqrt{a+bx+cx^2}(ef-dg)^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{g\left(-4cg(ag+2bf)+3b^2g^2+8c^2f^2\right)\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{8(ef-dg)\left(ag^2-bfg+cf^2\right)^{5/2}}+\frac{e^3\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^3\sqrt{ae^2-bde+cd^2}}-\frac{e^2g\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^3\sqrt{ag^2-bfg+cf^2}}-\frac{eg(2cf-bg)\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2(ef-dg)^2\left(ag^2-bfg+cf^2\right)^{3/2}}+\frac{eg^2\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)^2\left(ag^2-bfg+cf^2\right)}+\frac{3g^2\sqrt{a+bx+cx^2}(2cf-bg)}{4(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)^2}+\frac{g^2\sqrt{a+bx+cx^2}}{2(f+gx)^2(ef-dg)\left(ag^2-bfg+cf^2\right)}$$

input
```
Int[1/((d + e*x)*(f + g*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(g^2*Sqrt[a + b*x + c*x^2])/(2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*
x)^2) + (3*g^2*(2*c*f - b*g)*Sqrt[a + b*x + c*x^2])/(4*(e*f - d*g)*(c*f^2
- b*f*g + a*g^2)^2*(f + g*x)) + (e*g^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)
^2*(c*f^2 - b*f*g + a*g^2)*(f + g*x)) + (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d
 - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c
*d^2 - b*d*e + a*e^2]*(e*f - d*g)^3) - (e*g*(2*c*f - b*g)*ArcTanh[(b*f - 2
*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^
2])])/(2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)^(3/2)) - (e^2*g*ArcTanh[(b*
f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x +
 c*x^2])])/((e*f - d*g)^3*Sqrt[c*f^2 - b*f*g + a*g^2]) - (g*(8*c^2*f^2 + 3
*b^2*g^2 - 4*c*g*(2*b*f + a*g))*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2
*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f - d*g)*(c*f^
2 - b*f*g + a*g^2)^(5/2))
```

3.877.3.1 Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.877.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1184 vs. $2(541) = 1082$.

Time = 0.91 (sec) , antiderivative size = 1185, normalized size of antiderivative = 2.02

method	result	size
default	Expression too large to display	1185

input
```
int(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/g^2/(d*g-e*f)*(-1/2/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)^2*((x+f/g)^2*c+(b*g-
2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-3/4*(b*g-2*c*f)*g/(a*g^2-b
*f*g+c*f^2)*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g
*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f
^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c
*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g
*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))+1/2*c/(a*g^2-b*f*g+c*f^
2)*g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-
2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f
)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))+e^2/(d*g-e*f)^3/((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x
+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)
+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))-e^2/(d*g-e*f)^3/((a*g^2-b*f*g+c*
f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*
g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*
g+c*f^2)/g^2)^(1/2))/(x+f/g))-1/g*e/(d*g-e*f)^2*(-1/(a*g^2-b*f*g+c*f^2)*g^
2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2
)+1/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln
((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g
^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(...
```

3.877.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.877.6 Sympy [F]

$$\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d+ex)\,(f+gx)^3\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)**3/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/((d + e*x)*(f + g*x)**3*sqrt(a + b*x + c*x**2)), x)
```

3.877.7 Maxima [F]

$$\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)(gx+f)^3}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^3), x)
```

3.877.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2239 vs. $2(541) = 1082$.

Time = 0.77 (sec) , antiderivative size = 2239, normalized size of antiderivative = 3.81

$$\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-2*e^3*arctan(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*
d^2 + b*d*e - a*e^2))/((e^3*f^3 - 3*d*e^2*f^2*g + 3*d^2*e*f*g^2 - d^3*g^3)
*sqrt(-c*d^2 + b*d*e - a*e^2)) - 1/4*(24*c^2*e^2*f^4*g - 24*c^2*d*e*f^3*g^
2 - 36*b*c*e^2*f^3*g^2 + 8*c^2*d^2*f^2*g^3 + 28*b*c*d*e*f^2*g^3 + 15*b^2*e
^2*f^2*g^3 + 20*a*c*e^2*f^2*g^3 - 8*b*c*d^2*f*g^4 - 10*b^2*d*e*f*g^4 - 20*
a*b*e^2*f*g^4 + 3*b^2*d^2*g^5 - 4*a*c*d^2*g^5 + 4*a*b*d*e*g^5 + 8*a^2*e^2*
g^5)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*g + sqrt(c)*f)/sqrt(-c*f
^2 + b*f*g - a*g^2))/((c^2*e^3*f^7 - 3*c^2*d*e^2*f^6*g - 2*b*c*e^3*f^6*g +
 3*c^2*d^2*e*f^5*g^2 + 6*b*c*d*e^2*f^5*g^2 + b^2*e^3*f^5*g^2 + 2*a*c*e^3*f
^5*g^2 - c^2*d^3*f^4*g^3 - 6*b*c*d^2*e*f^4*g^3 - 3*b^2*d*e^2*f^4*g^3 - 6*a
*c*d*e^2*f^4*g^3 - 2*a*b*e^3*f^4*g^3 + 2*b*c*d^3*f^3*g^4 + 3*b^2*d^2*e*f^3
*g^4 + 6*a*c*d^2*e*f^3*g^4 + 6*a*b*d*e^2*f^3*g^4 + a^2*e^3*f^3*g^4 - b^2*d
^3*f^2*g^5 - 2*a*c*d^3*f^2*g^5 - 6*a*b*d^2*e*f^2*g^5 - 3*a^2*d*e^2*f^2*g^5
 + 2*a*b*d^3*f*g^6 + 3*a^2*d^2*e*f*g^6 - a^2*d^3*g^7)*sqrt(-c*f^2 + b*f*g
- a*g^2)) + 1/4*(16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*e*f^3*g^2 -
8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*d*f^2*g^3 - 20*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^3*b*c*e*f^2*g^3 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^3*b*c*d*f*g^4 + 7*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*e*f*g^4 + 4
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*e*f*g^4 - 3*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^3*b^2*d*g^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*...
```

3.877.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(f+g\,x)^3\,(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/((f + g*x)^3*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/((f + g*x)^3*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.878 $\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$

3.878.1 Optimal result

Integrand size = 29, antiderivative size = 496

$$\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx =$$
$$-\frac{2(ab^3dg^4-b^2(c^2ef^4+4acdfg^3+a^2eg^4)+2ac(a^2eg^4+c^2f^3(ef-4dg)-2acfg^2(3ef-2dg))+bc(c^2df^4}{}$$
$$+\frac{g^4\sqrt{a+bx+cx^2}}{c^2e}+\frac{g^3(8cef-2cdg-3beg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{5/2}e^2}$$
$$+\frac{(ef-dg)^4\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^2\left(cd^2-bde+ae^2\right)^{3/2}}$$

output
```
1/2*g^3*(-3*b*e*g-2*c*d*g+8*c*e*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*
x+a)^(1/2))/c^(5/2)/e^2+(-d*g+e*f)^4*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x
)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^2/(a*e^2-b*d*e+c*d^2)^(
3/2)-2*(a*b^3*d*g^4-b^2*(a^2*e*g^4+4*a*c*d*f*g^3+c^2*e*f^4)+2*a*c*(a^2*e*g
^4+c^2*f^3*(-4*d*g+e*f)-2*a*c*f*g^2*(-2*d*g+3*e*f))+b*c*(c^2*d*f^4+a^2*g^3
*(-3*d*g+4*e*f)+2*a*c*f^2*g*(3*d*g+2*e*f))+(2*c^4*d*f^4+b^3*(-a*e+b*d)*g^4
-b*c*g^3*(4*b^2*d*f-3*a^2*e*g-4*a*b*(-d*g+e*f))+2*c^2*g^2*(3*b^2*d*f^2-3*a
*b*f*(-2*d*g+e*f)-a^2*g*(-d*g+4*e*f))+c^3*f^2*(4*a*g*(-3*d*g+2*e*f)-b*f*(4
*d*g+e*f)))*x)/c^2/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^2+b*x+a)^(1/2)+g^
4*(c*x^2+b*x+a)^(1/2)/c^2/e
```

3.878.2 Mathematica [A] (verified)

Time = 12.46 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.18

$$\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{-\frac{2e(-3b^4deg^4x+b^3g^3(3aeg(-d+ex)+cdx(8ef+dg-egx))+b^2(3a^2e^2g^4+c^2(2e^2f^4-12def^2g^2x+a}{}}{}$$

input
```
Integrate[(f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
((-2*e*(-3*b^4*d*e*g^4*x + b^3*g^3*(3*a*e*g*(-d + e*x) + c*d*x*(8*e*f + d*
g - e*g*x)) + b^2*(3*a^2*e^2*g^4 + c^2*(2*e^2*f^4 - 12*d*e*f^2*g^2*x + d^2
*g^4*x^2) + a*c*g^3*(d^2*g + e^2*x*(-8*f + g*x) + 4*d*e*(2*f + 3*g*x))) -
2*b*c*(a^2*e*g^3*(4*e*f - 5*d*g + 5*e*g*x) + c^2*e*f^3*(-(e*f*x) + d*(f -
4*g*x)) + 2*a*c*g*(d^2*g^3*x + e^2*f^2*(2*f - 3*g*x) + d*e*g*(3*f^2 + 6*f*
g*x - g^2*x^2))) - 4*c*(2*a^3*e^2*g^4 + c^3*d*e*f^4*x + a*c^2*(d^2*g^4*x^2
 - 2*d*e*f^2*g*(2*f + 3*g*x) + e^2*f^3*(f + 4*g*x)) + a^2*c*g^2*(d^2*g^2 +
 d*e*g*(4*f + g*x) + e^2*(-6*f^2 - 4*f*g*x + g^2*x^2)))))/(c^2*(b^2 - 4*a*
c)*(-(c*d^2) + e*(b*d - a*e))*Sqrt[a + x*(b + c*x)]) + (2*(e*f - d*g)^4*Lo
g[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^(3/2) + (g^3*(8*c*e*f - 2*c*d*g - 3
*b*e*g)*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/c^(5/2) - (2*(e*
f - d*g)^4*Log[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2*Sqrt[c*d^2 + e*(-(b*d)
 + a*e)]*Sqrt[a + x*(b + c*x)]])/(c*d^2 + e*(-(b*d) + a*e))^(3/2))/(2*e^2)
```

3.878.3 Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {1264, 27, 2184, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 1264$$

$$-\frac{\frac{2\int -\frac{\frac{(b^2-4ac)x^2g^4}{c}+\frac{(b^2-4ac)(4cf-bg)xg^3}{c^2}+\frac{(b^2-4ac)\left(bd(bd-ae)g^4-cd(4bdf-4aef+adg)g^3+c^2f^2\left(e^2f^2-4degf+6d^2g^2\right)\right)}{c^2\left(cd^2-bed+ae^2\right)}}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{b^2-4ac}-}{2\left(-b^2\left(a^2eg^4+4acdfg^3+c^2ef^4\right)+x\left(2c^2g^2\left(a^2(-g)(4ef-dg)-3abf(ef-2dg)+3b^2df^2\right)-bcg^3\left(-3a^2eg-4ab\right.\right.\right.}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{\frac{(b^2-4ac)x^2g^4}{c}+\frac{(b^2-4ac)(4cf-bg)xg^3}{c^2}+\frac{(b^2-4ac)\left(bd(bd-ae)g^4-cd(4bdf-4aef+adg)g^3+c^2f^2\left(e^2f^2-4degf+6d^2g^2\right)\right)}{c^2\left(cd^2-bed+ae^2\right)}}{(d+ex)\sqrt{cx^2+bx+a}}dx}{b^2-4ac}-}{2\left(-b^2\left(a^2eg^4+4acdfg^3+c^2ef^4\right)+x\left(2c^2g^2\left(a^2(-g)(4ef-dg)-3abf(ef-2dg)+3b^2df^2\right)-bcg^3\left(-3a^2eg-4ab\right.\right.\right.}$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{\frac{\frac{\int \frac{(b^2-4ac)e\left((8cef-2cdg-3beg)xg^3+\frac{3bde(bd-ae)g^4+cd(2ae(4ef-dg)-bd(8ef+dg))g^3+2c^2ef^2\left(e^2f^2-4degf+6d^2g^2\right)}{cd^2-bed+ae^2}\right)}{2c(d+ex)\sqrt{cx^2+bx+a}}dx}{ce^2}+\frac{g^4(b^2-4ac)\sqrt{a+bx+cx^2}}{c^2e}}{b^2-4ac}-}{2\left(-b^2\left(a^2eg^4+4acdfg^3+c^2ef^4\right)+x\left(2c^2g^2\left(a^2(-g)(4ef-dg)-3abf(ef-2dg)+3b^2df^2\right)-bcg^3\left(-3a^2eg-4ab\right.\right.\right.}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{(b^2-4ac)\int \frac{(8cef-2cdg-3beg)xg^3+\frac{3bde(bd-ae)g^4+cd(2ae(4ef-dg)-bd(8ef+dg))g^3+2c^2ef^2\left(e^2f^2-4degf+6d^2g^2\right)}{cd^2-bed+ae^2}}{(d+ex)\sqrt{cx^2+bx+a}}dx}{2c^2e}+\frac{g^4(b^2-4ac)\sqrt{a+bx+cx^2}}{c^2e}}{b^2-4ac}-}{2\left(-b^2\left(a^2eg^4+4acdfg^3+c^2ef^4\right)+x\left(2c^2g^2\left(a^2(-g)(4ef-dg)-3abf(ef-2dg)+3b^2df^2\right)-bcg^3\left(-3a^2eg-4ab\right.\right.\right.}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{\frac{\frac{(b^2-4ac)\left(\frac{2c^2(ef-dg)^4\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e\left(ae^2-bde+cd^2\right)}+\frac{g^3(-3beg-2cdg+8cef)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}\right)}{2c^2e}+\frac{g^4(b^2-4ac)\sqrt{a+bx+cx^2}}{c^2e}}{b^2-4ac}-}{2\left(-b^2\left(a^2eg^4+4acdfg^3+c^2ef^4\right)+x\left(2c^2g^2\left(a^2(-g)(4ef-dg)-3abf(ef-2dg)+3b^2df^2\right)-bcg^3\left(-3a^2eg-4ab\right.\right.\right.}$$

$$\Big\downarrow \text{ 1092}$$

3.878. $\int \frac{(f+gx)^4}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx$

$$\frac{\frac{(b^2-4ac)\left(\frac{2c^2(ef-dg)^4\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e\left(ae^2-bde+cd^2\right)}+\frac{2g^3(-3beg-2cdg+8cef)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}\right)}{2c^2e}+\frac{g^4(b^2-4ac)\sqrt{a+bx+cx^2}}{c^2e}}{b^2-4ac}-\frac{2\left(-b^2\left(a^2eg^4+4acdfg^3+c^2ef^4\right)+x\left(2c^2g^2\left(a^2(-g)(4ef-dg)-3abf(ef-2dg)+3b^2df^2\right)-bcg^3\left(-3a^2eg-4ab\right.\right.\right.}{}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(b^2-4ac)\left(\frac{2c^2(ef-dg)^4\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e\left(ae^2-bde+cd^2\right)}+\frac{g^3\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-3beg-2cdg+8cef)}{\sqrt{c}e}\right)}{2c^2e}+\frac{g^4(b^2-4ac)\sqrt{a+bx+cx^2}}{c^2e}}{b^2-4ac}-\frac{2\left(-b^2\left(a^2eg^4+4acdfg^3+c^2ef^4\right)+x\left(2c^2g^2\left(a^2(-g)(4ef-dg)-3abf(ef-2dg)+3b^2df^2\right)-bcg^3\left(-3a^2eg-4ab\right.\right.\right.}{}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\frac{(b^2-4ac)\left(\frac{g^3\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-3beg-2cdg+8cef)}{\sqrt{c}e}-\frac{4c^2(ef-dg)^4\int\frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e\left(ae^2-bde+cd^2\right)}\right)}{2c^2e}+}{b^2-4ac}$$
$$\frac{}{2\left(-b^2\left(a^2eg^4+4acdfg^3+c^2ef^4\right)+x\left(2c^2g^2\left(a^2(-g)(4ef-dg)-3abf(ef-2dg)+3b^2df^2\right)-bcg^3\left(-3a^2eg-4ab\right.\right.\right.}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(b^2-4ac)\left(\frac{2c^2(ef-dg)^4\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\left(ae^2-bde+cd^2\right)^{3/2}}+\frac{g^3\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-3beg-2cdg+8cef)}{\sqrt{c}e}\right)}{2c^2e}+\frac{g^4(b^2-4ac)\sqrt{a+bx+cx^2}}{c^2e}}{b^2-4ac}$$
$$\frac{}{2\left(-b^2\left(a^2eg^4+4acdfg^3+c^2ef^4\right)+x\left(2c^2g^2\left(a^2(-g)(4ef-dg)-3abf(ef-2dg)+3b^2df^2\right)-bcg^3\left(-3a^2eg-4ab\right.\right.\right.}$$

input
```
Int[(f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

3.878. $\int \frac{(f+gx)^4}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx$

output
```
(-2*(a*b^3*d*g^4 - b^2*(c^2*e*f^4 + 4*a*c*d*f*g^3 + a^2*e*g^4) + 2*a*c*(a^
2*e*g^4 + c^2*f^3*(e*f - 4*d*g) - 2*a*c*f*g^2*(3*e*f - 2*d*g)) + b*c*(c^2*
d*f^4 + a^2*g^3*(4*e*f - 3*d*g) + 2*a*c*f^2*g*(2*e*f + 3*d*g)) + (2*c^4*d*
f^4 + b^3*(b*d - a*e)*g^4 - b*c*g^3*(4*b^2*d*f - 3*a^2*e*g - 4*a*b*(e*f -
d*g)) + 2*c^2*g^2*(3*b^2*d*f^2 - 3*a*b*f*(e*f - 2*d*g) - a^2*g*(4*e*f - d*
g)) + c^3*f^2*(4*a*g*(2*e*f - 3*d*g) - b*f*(e*f + 4*d*g)))*x))/(c^2*(b^2 -
 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (((b^2 - 4*a*c)*g
^4*Sqrt[a + b*x + c*x^2])/(c^2*e) + ((b^2 - 4*a*c)*((g^3*(8*c*e*f - 2*c*d*
g - 3*b*e*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt
[c]*e) + (2*c^2*(e*f - d*g)^4*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*S
qrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*(c*d^2 - b*d*e + a*
e^2)^(3/2))))/(2*c^2*e))/(b^2 - 4*a*c)
```

3.878.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1264
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)
^m*(f + g*x)^n, a + b*x + c*x^2, x], R = Coeff[PolynomialRemainder[(d + e*x
)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[
(d + e*x)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 1]}, Simp[(b*R - 2*a*S + (
2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + S
imp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*E
xpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*R - b*S)
)/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1]
 && LtQ[p, -1] && ILtQ[m, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.878.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1025 vs. $2(474) = 948$.

Time = 1.04 (sec) , antiderivative size = 1026, normalized size of antiderivative = 2.07

method	result	size
default	Expression too large to display	1026
risch	Expression too large to display	4958

input
```
int((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(d^4*g^4-4*d^3*e*f*g^3+6*d^2*e^2*f^2*g^2-4*d*e^3*f^3*g+e^4*f^4)/e^5*(1/(a*
e^2-b*d*e+c*d^2)*e^2/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*c*d)/e)
/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/((x+d/e)^2*c+(b*e-2*c*d)/
e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/((a*e^2
-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/
e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))-g/e^4*(2*d^3*g^3*(2*c*x+b)/(4*a*c-
b^2)/(c*x^2+b*x+a)^(1/2)-8*e^3*f^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/
2)-e^3*g^3*(x^2/c/(c*x^2+b*x+a)^(1/2)-3/2*b/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/
2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1
/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2*a/c*(-1/c/(c
*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)))-8*d^2*e*
f*g^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+12*d*e^2*f^2*g*(2*c*x+b)/(
4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+(d*e^2*g^3-4*e^3*f*g^2)*(-x/c/(c*x^2+b*x+a)
^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+
b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+(-d^2
*e*g^3+4*d*e^2*f*g^2-6*e^3*f^2*g)*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/
(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)))
```

3.878.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.878.6 Sympy [F]

$$\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)**4/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((f + g*x)**4/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

3.878.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^4}{(d+ex)\,(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

3.878.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^4}{(d+ex)\,(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.878.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^4}{(d+ex)\,(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(f+g\,x)^4}{(d+e\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

input
```
int((f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int((f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.879 $\int \frac{(f+gx)^3}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$

3.879.1 Optimal result

Integrand size = 29, antiderivative size = 357

$$\int \frac{(f+gx)^3}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2(b^2(cef^3+adg^3)-2ac(cf^2(ef-3dg)-ag^2(3ef-dg))-b(c^2df^3+a^2}{}$$

$$+\frac{g^3\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}e}+\frac{(ef-dg)^3\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e\left(cd^2-bde+ae^2\right)^{3/2}}$$

output
```
g^3*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e+(-d*g+e*f
)^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^
2+b*x+a)^(1/2))/e/(a*e^2-b*d*e+c*d^2)^(3/2)+2*(b^2*(a*d*g^3+c*e*f^3)-2*a*c
*(c*f^2*(-3*d*g+e*f)-a*g^2*(-d*g+3*e*f))-b*(c^2*d*f^3+a^2*e*g^3+3*a*c*f*g*
(d*g+e*f))-(2*c^3*d*f^3-b^2*(-a*e+b*d)*g^3+c*g^2*(-2*a^2*e*g+3*a*b*d*g-3*a
*b*e*f+3*b^2*d*f)+c^2*f*(6*a*g*(-d*g+e*f)-b*f*(3*d*g+e*f)))*x)/c/(-4*a*c+b
^2)/(a*e^2-b*d*e+c*d^2)/(c*x^2+b*x+a)^(1/2)
```

3.879.2 Mathematica [A] (verified)

Time = 3.95 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.99

$$\int \frac{(f+gx)^3}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx = \frac{2(-b^3dg^3x+b^2(ag^3(-d+ex)+c(-ef^3+3dfg^2x))+b(a^2eg^3+c^2f^2(-\ldots}{} + \frac{2(-ef+dg)^3\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{e\sqrt{-cd^2+e(bd-ae)}\,(cd^2+e(-bd+ae))} - \frac{g^3\log\left(ce\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{c^{3/2}e}$$

input
```
Integrate[(f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*(-(b^3*d*g^3*x) + b^2*(a*g^3*(-d + e*x) + c*(-(e*f^3) + 3*d*f*g^2*x)) +
 b*(a^2*e*g^3 + c^2*f^2*(-(e*f*x) + d*(f - 3*g*x)) + 3*a*c*g*(e*f*(f - g*x
) + d*g*(f + g*x))) + 2*c*(c^2*d*f^3*x + a^2*g^2*(d*g - e*(3*f + g*x)) + a
*c*f*(-3*d*g*(f + g*x) + e*f*(f + 3*g*x)))))/(c*(-b^2 + 4*a*c)*(c*d^2 + e*
(-(b*d) + a*e))*Sqrt[a + x*(b + c*x)]) + (2*(-(e*f) + d*g)^3*ArcTan[(Sqrt[
c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(
e*Sqrt[-(c*d^2) + e*(b*d - a*e)]*(c*d^2 + e*(-(b*d) + a*e))) - (g^3*Log[c*
e*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(c^(3/2)*e)
```

3.879.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1264, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^3}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 1264$$

$$\frac{2\left(-x\left(cg^2\left(-2a^2eg+3abdg-3abef+3b^2df\right)-b^2g^3(bd-ae)+c^2f(6ag(ef-dg)-bf(3dg+ef))+2c^3df^3\right)-b\ldots\right.}{c(b^2-4ac)\sqrt{a+bx+cx^2}\,(ae^2\ldots} \frac{2\int \frac{(b^2-4ac)\left(\frac{d(bd-ae)g^3-cf\left(e^2f^2-3degf+3d^2g^2\right)}{cd^2-bed+ae^2}-g^3x\right)}{2c(d+ex)\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$\downarrow$ 27

$$\frac{2\left(-x\left(cg^2\left(-2a^2eg+3abdg-3abef+3b^2df\right)-b^2g^3(bd-ae)+c^2f(6ag(ef-dg)-bf(3dg+ef))+2c^3df^3\right)-b\right.}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2\right.}$$

$$\frac{\int \frac{\frac{d(bd-ae)g^3-cf\left(e^2f^2-3degf+3d^2g^2\right)}{cd^2-bed+ae^2}-g^3x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{c}$$

$\downarrow$ 1269

$$\frac{2\left(-x\left(cg^2\left(-2a^2eg+3abdg-3abef+3b^2df\right)-b^2g^3(bd-ae)+c^2f(6ag(ef-dg)-bf(3dg+ef))+2c^3df^3\right)-b\right.}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2\right.}$$

$$\frac{-\frac{c(ef-dg)^3\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e(ae^2-bde+cd^2)}-\frac{g^3\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}}{c}$$

$\downarrow$ 1092

$$\frac{2\left(-x\left(cg^2\left(-2a^2eg+3abdg-3abef+3b^2df\right)-b^2g^3(bd-ae)+c^2f(6ag(ef-dg)-bf(3dg+ef))+2c^3df^3\right)-b\right.}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2\right.}$$

$$\frac{-\frac{c(ef-dg)^3\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e(ae^2-bde+cd^2)}-\frac{2g^3\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}}{c}$$

$\downarrow$ 219

$$\frac{2\left(-x\left(cg^2\left(-2a^2eg+3abdg-3abef+3b^2df\right)-b^2g^3(bd-ae)+c^2f(6ag(ef-dg)-bf(3dg+ef))+2c^3df^3\right)-b\right.}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2\right.}$$

$$\frac{-\frac{c(ef-dg)^3\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e(ae^2-bde+cd^2)}-\frac{g^3\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}}{c}$$

$\downarrow$ 1154

$$\frac{2\left(-x\left(cg^2\left(-2a^2eg+3abdg-3abef+3b^2df\right)-b^2g^3(bd-ae)+c^2f(6ag(ef-dg)-bf(3dg+ef))+2c^3df^3\right)-b\right.}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2\right.}$$

$$\frac{\frac{2c(ef-dg)^3\int \frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e(ae^2-bde+cd^2)}-\frac{g^3\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}}{c}$$

$\downarrow$ 219

3.879. $\int \frac{(f+gx)^3}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx$

$$\frac{2\left(-x\left(cg^2\left(-2a^2eg+3abdg-3abef+3b^2df\right)-b^2g^3(bd-ae)+c^2f(6ag(ef-dg)-bf(3dg+ef))+2c^3df^3\right)-b\right.}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,(ae^2}$$

$$\frac{-\frac{c(ef-dg)^3\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(ae^2-bde+cd^2)^{3/2}}-\frac{g^3\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}}{c}$$

input
```
Int[(f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*(b^2*(c*e*f^3 + a*d*g^3) - 2*a*c*(c*f^2*(e*f - 3*d*g) - a*g^2*(3*e*f -
d*g)) - b*(c^2*d*f^3 + a^2*e*g^3 + 3*a*c*f*g*(e*f + d*g)) - (2*c^3*d*f^3 -
 b^2*(b*d - a*e)*g^3 + c*g^2*(3*b^2*d*f - 3*a*b*e*f + 3*a*b*d*g - 2*a^2*e*
g) + c^2*f*(6*a*g*(e*f - d*g) - b*f*(e*f + 3*d*g)))*x))/(c*(b^2 - 4*a*c)*(
c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) - (-((g^3*ArcTanh[(b + 2*c*x
)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e)) - (c*(e*f - d*g)^3*ArcT
anh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a
+ b*x + c*x^2])])/(e*(c*d^2 - b*d*e + a*e^2)^(3/2)))/c
```

3.879.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1264
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)
^m*(f + g*x)^n, a + b*x + c*x^2, x], R = Coeff[PolynomialRemainder[(d + e*x
)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[
(d + e*x)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 1]}, Simp[(b*R - 2*a*S + (
2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + S
imp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*E
xpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*R - b*S)
)/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1]
 && LtQ[p, -1] && ILtQ[m, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.879.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 739 vs. $2(339) = 678$.

Time = 0.84 (sec) , antiderivative size = 740, normalized size of antiderivative = 2.07

method	result
default	$\frac{\left(-d^3g^3+3d^2efg^2-3de^2f^2g+e^3f^3\right)\left(\frac{e^2}{\left(e^2a-bde+cd^2\right)\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a-bde+cd^2}{e^2}}}-\frac{(b\ldots}{\left(e^2a-bde+cd^2\right)\left(\frac{4c\left(e^2a-bde+cd^2\right)}{e^2}\ldots\right.}\ldots\right.}{\ldots}$

input
```
int((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-d^3*g^3+3*d^2*e*f*g^2-3*d*e^2*f^2*g+e^3*f^3)/e^4*(1/(a*e^2-b*d*e+c*d^2)*
e^2/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-(b*e
-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d*
e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/((a*e^2-b*d*e+c*d^2)/e^2
)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*
e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)
/e^2)^(1/2))/(x+d/e)))+g/e^3*(2*d^2*g^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a
)^(1/2)+e^2*g^2*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2
)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x+a)^(1/2)))+6*e^2*f^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a
)^(1/2)-6*d*e*f*g*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+(-d*e*g^2+3*e^
2*f*g)*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(
1/2)))
```

3.879.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.879.6 Sympy [F]

$$\int \frac{(f+gx)^3}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(f+gx)^3}{(d+ex)(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)**3/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((f + g*x)**3/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

3.879.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

3.879.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.879.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(f+g\,x)^3}{(d+e\,x)\,(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int((f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int((f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.880 $\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$

3.880.1 Optimal result

Integrand size = 29, antiderivative size = 240

$$\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2(b^2ef^2+2a(aeg^2-cf(ef-2dg))-b(cdf^2+ag(2ef+dg))-(2c^2df^2}{(b^2-4ac)\left(cd^2-bde+ae^2\right)\sqrt{}} + \frac{(ef-dg)^2\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\left(cd^2-bde+ae^2\right)^{3/2}}$$

output
```
(-d*g+e*f)^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1
/2)/(c*x^2+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)+2*(b^2*e*f^2+2*a*(a*e*g
^2-c*f*(-2*d*g+e*f))-b*(c*d*f^2+a*g*(d*g+2*e*f))-(2*c^2*d*f^2+b*(-a*e+b*d)
*g^2+c*(2*a*g*(-d*g+2*e*f)-b*f*(2*d*g+e*f)))*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+
c*d^2)/(c*x^2+b*x+a)^(1/2)
```

3.880.2 Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.02

$$\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = 2\left(\frac{-2a^2eg^2+2c^2df^2x-2acdg(2f+gx)+2acef(f+2gx)+abg(2ef+}{(b^2-4ac)\left(-cd^2+e(bd-ae\right.} + \frac{\sqrt{-cd^2+bde-ae^2}(ef-dg)^2\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{\left(cd^2+e(-bd+ae)\right)^2}\right)$$

input
```
Integrate[(f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
2*((-2*a^2*e*g^2 + 2*c^2*d*f^2*x - 2*a*c*d*g*(2*f + g*x) + 2*a*c*e*f*(f +
2*g*x) + a*b*g*(2*e*f + d*g - e*g*x) + b^2*(-(e*f^2) + d*g^2*x) + b*c*f*(-
(e*f*x) + d*(f - 2*g*x)))/((b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*Sqrt[a
 + x*(b + c*x)]) + (Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e*f - d*g)^2*ArcTan[(S
qrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]
])/(c*d^2 + e*(-(b*d) + a*e))^2)
```

3.880.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1264, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{(d+ex)(a+bx+cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{1264}$$

$$\frac{2\left(-x\left(c(2ag(2ef-dg)-bf(2dg+ef))+bg^2(bd-ae)+2c^2df^2\right)-b\left(ag(dg+2ef)+cdf^2\right)+2a\left(aeg^2-cf(ef-\right.\right.}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(ae^2-bde+cd^2)}$$
$$\frac{2\int -\frac{(b^2-4ac)(ef-dg)^2}{2(cd^2-bed+ae^2)(d+ex)\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{(ef-dg)^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{ae^2-bde+cd^2}+$$
$$\frac{2\left(-x\left(c(2ag(2ef-dg)-bf(2dg+ef))+bg^2(bd-ae)+2c^2df^2\right)-b\left(ag(dg+2ef)+cdf^2\right)+2a\left(aeg^2-cf(ef-\right.\right.}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(ae^2-bde+cd^2)}$$

$$\Big\downarrow \text{1154}$$

$$\frac{2\left(-x\left(c(2ag(2ef-dg)-bf(2dg+ef))+bg^2(bd-ae)+2c^2df^2\right)-b\left(ag(dg+2ef)+cdf^2\right)+2a\left(aeg^2-cf(ef-\right.\right.}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(ae^2-bde+cd^2)}$$
$$\frac{2(ef-dg)^2\int \frac{1}{4(cd^2-bed+ae^2)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{ae^2-bde+cd^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{(ef-dg)^2\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ae^2-bde+cd^2)^{3/2}}+\frac{2\left(-x\left(c(2ag(2ef-dg)-bf(2dg+ef))+bg^2(bd-ae)+2c^2df^2\right)-b\left(ag(dg+2ef)+cdf^2\right)+2a\left(aeg^2-cf(ef-\right.\right.}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(ae^2-bde+cd^2)}$$

input
```
Int[(f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*(b^2*e*f^2 + 2*a*(a*e*g^2 - c*f*(e*f - 2*d*g)) - b*(c*d*f^2 + a*g*(2*e*
f + d*g)) - (2*c^2*d*f^2 + b*(b*d - a*e)*g^2 + c*(2*a*g*(2*e*f - d*g) - b*
f*(e*f + 2*d*g)))*x))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x
+ c*x^2]) + ((e*f - d*g)^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt
[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 - b*d*e + a*e^2)^(
3/2)
```

3.880.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1264
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)
^m*(f + g*x)^n, a + b*x + c*x^2, x], R = Coeff[PolynomialRemainder[(d + e*x
)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[
(d + e*x)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 1]}, Simp[(b*R - 2*a*S + (
2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + S
imp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*E
xpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*R - b*S)
)/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1]
 && LtQ[p, -1] && ILtQ[m, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.880.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(230) = 460$.

Time = 0.77 (sec) , antiderivative size = 555, normalized size of antiderivative = 2.31

method	result
default	$-\frac{g\left(\frac{2dg(2cx+b)}{(4ac-b^2)\sqrt{c\,x^2+bx+a}}-\frac{4ef(2cx+b)}{(4ac-b^2)\sqrt{c\,x^2+bx+a}}-eg\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c(4ac-b^2)\sqrt{c\,x^2+bx+a}}\right)\right)}{e^2}+\frac{(d^2g^2-2defg+e^2f^2)\Bigg(\frac{\quad}{(e^2 c}$

input
```
int((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-g/e^2*(2*d*g*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-4*e*f*(2*c*x+b)/(4
*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-e*g*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/
(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)))+(d^2*g^2-2*d*e*f*g+e^2*f^2)/e^3*(1/(a*e^
2-b*d*e+c*d^2)*e^2/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(
4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*
(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/((a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)
+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e
^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

3.880.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 990 vs. $2(230) = 460$.

Time = 3.33 (sec) , antiderivative size = 2023, normalized size of antiderivative = 8.43

$$\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(((a*b^2 - 4*a^2*c)*e^2*f^2 - 2*(a*b^2 - 4*a^2*c)*d*e*f*g + (a*b^2 -
4*a^2*c)*d^2*g^2 + ((b^2*c - 4*a*c^2)*e^2*f^2 - 2*(b^2*c - 4*a*c^2)*d*e*f*
g + (b^2*c - 4*a*c^2)*d^2*g^2)*x^2 + ((b^3 - 4*a*b*c)*e^2*f^2 - 2*(b^3 - 4
*a*b*c)*d*e*f*g + (b^3 - 4*a*b*c)*d^2*g^2)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*
log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e +
(b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x +
a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4
*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) - 4*((b*c^2*d^3 - 2*(b^2*c - a*c^
2)*d^2*e + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*f^2 - 2*(2*a*c^2*d
^3 - 3*a*b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d*e^2)*f*g + (a*b*c*d^3
 + 3*a^2*b*d*e^2 - 2*a^3*e^3 - (a*b^2 + 2*a^2*c)*d^2*e)*g^2 + ((2*c^3*d^3
- 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*f^2 - 2*(b*c^2*d^3
+ 3*a*b*c*d*e^2 - 2*a^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*f*g - (a^2*b*e^3
- (b^2*c - 2*a*c^2)*d^3 + (b^3 - a*b*c)*d^2*e - 2*(a*b^2 - a^2*c)*d*e^2)*g
^2)*x)*sqrt(c*x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c -
4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^
3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4
- 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^
2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + (
(b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*...
```

3.880.6 Sympy [F]

$$\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{\left(f+gx\right)^2}{\left(d+ex\right)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)**2/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((f + g*x)**2/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

3.880.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

3.880.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(230) = 460$.

Time = 0.29 (sec) , antiderivative size = 773, normalized size of antiderivative = 3.22

$$\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx =$$

$$-\frac{2\left(\frac{(2c^3d^3f^2-3bc^2d^2ef^2+b^2cde^2f^2+2ac^2de^2f^2-abce^3f^2-2bc^2d^3fg+2b^2cd^2efg+4ac^2d^2efg-6abcde^2fg+4a^2ce^3fg+b^2cd^3g^2-2ac^2d^3g^2-}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b}\right.}{}$$

$$+\frac{2\left(e^2f^2-2defg+d^2g^2\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\left(cd^2-bde+ae^2\right)\sqrt{-cd^2+bde-ae^2}}$$

input
```
integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2*((2*c^3*d^3*f^2 - 3*b*c^2*d^2*e*f^2 + b^2*c*d*e^2*f^2 + 2*a*c^2*d*e^2*f
^2 - a*b*c*e^3*f^2 - 2*b*c^2*d^3*f*g + 2*b^2*c*d^2*e*f*g + 4*a*c^2*d^2*e*f
*g - 6*a*b*c*d*e^2*f*g + 4*a^2*c*e^3*f*g + b^2*c*d^3*g^2 - 2*a*c^2*d^3*g^2
 - b^3*d^2*e*g^2 + a*b*c*d^2*e*g^2 + 2*a*b^2*d*e^2*g^2 - 2*a^2*c*d*e^2*g^2
 - a^2*b*e^3*g^2)*x/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2
*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e
^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3*f^2 - 2*b^2
*c*d^2*e*f^2 + 2*a*c^2*d^2*e*f^2 + b^3*d*e^2*f^2 - a*b*c*d*e^2*f^2 - a*b^2
*e^3*f^2 + 2*a^2*c*e^3*f^2 - 4*a*c^2*d^3*f*g + 6*a*b*c*d^2*e*f*g - 2*a*b^2
*d*e^2*f*g - 4*a^2*c*d*e^2*f*g + 2*a^2*b*e^3*f*g + a*b*c*d^3*g^2 - a*b^2*d
^2*e*g^2 - 2*a^2*c*d^2*e*g^2 + 3*a^2*b*d*e^2*g^2 - 2*a^3*e^3*g^2)/(b^2*c^2
*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b
^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b
^2*e^4 - 4*a^3*c*e^4))/sqrt(c*x^2 + b*x + a) + 2*(e^2*f^2 - 2*d*e*f*g + d^
2*g^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c
*d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^
2))
```

3.880.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(f+g\,x)^2}{(d+e\,x)\,(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int((f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int((f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.881 $\int \frac{f+gx}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx$

3.881.1 Optimal result

Integrand size = 27, antiderivative size = 187

$$\int \frac{f+gx}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2(bcdf-b^2ef+2acef-2acdg+abeg+c(2cdf+2aeg-b(ef+dg))x)}{(b^2-4ac)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} + \frac{e(ef-dg)\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\left(cd^2-bde+ae^2\right)^{3/2}}$$

output
```
e*(-d*g+e*f)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1
/2)/(c*x^2+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)-2*(b*c*d*f-b^2*e*f+2*a*
c*e*f-2*a*c*d*g+a*b*e*g+c*(2*c*d*f+2*a*e*g-b*(d*g+e*f))*x)/(-4*a*c+b^2)/(a
*e^2-b*d*e+c*d^2)/(c*x^2+b*x+a)^(1/2)
```

3.881.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.06

$$\int \frac{f+gx}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{-2b^2ef+2b(aeg-cefx+cd(f-gx))+4c(-adg+cdfx+ae(f+gx)}{(b^2-4ac)\left(-cd^2+e(bd-ae)\right)\sqrt{a+x(b+cx)}} - \frac{2e\sqrt{-cd^2+bde-ae^2}(-ef+dg)\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{\left(cd^2+e(-bd+ae)\right)^2}$$

input
```
Integrate[(f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*b^2*e*f + 2*b*(a*e*g - c*e*f*x + c*d*(f - g*x)) + 4*c*(-(a*d*g) + c*d*
f*x + a*e*(f + g*x)))/((b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*Sqrt[a + x
*(b + c*x)]) - (2*e*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(-(e*f) + d*g)*ArcTan[(
Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)
]])/(c*d^2 + e*(-(b*d) + a*e))^2
```

3.881.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1235, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1235}$$

$$-\frac{2\int -\frac{(b^2-4ac)e(ef-dg)}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{(b^2-4ac)\left(ae^2-bde+cd^2\right)}-\frac{2\left(cx(2aeg-b(dg+ef)+2cdf)+abeg-2acdg+2acef+b^2(-e)f+bcdf\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{e(ef-dg)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{ae^2-bde+cd^2}-\frac{2\left(cx(2aeg-b(dg+ef)+2cdf)+abeg-2acdg+2acef+b^2(-e)f+bcdf\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{2e(ef-dg)\int \frac{1}{4(cd^2-bed+ae^2)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{ae^2-bde+cd^2}-\frac{2\left(cx(2aeg-b(dg+ef)+2cdf)+abeg-2acdg+2acef+b^2(-e)f+bcdf\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{e(ef-dg)\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ae^2-bde+cd^2)^{3/2}} - \frac{2\left(cx(2aeg-b(dg+ef)+2cdf)+abeg-2acdg+2acef+b^2(-e)f+bcdf\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(ae^2-bde+cd^2)}$$

input
```
Int[(f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*(b*c*d*f - b^2*e*f + 2*a*c*e*f - 2*a*c*d*g + a*b*e*g + c*(2*c*d*f + 2*
a*e*g - b*(e*f + d*g))*x))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a +
 b*x + c*x^2]) + (e*(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2
*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 - b*d*e + a*e
^2)^(3/2)
```

3.881.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

3.881.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(177) = 354$.

Time = 0.70 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.38

method	result
default	$\frac{2g(2cx+b)}{e(4ac-b^2)\sqrt{c\,x^2+bx+a}} + \frac{(-dg+ef)\left(\frac{e^2}{\left(e^2a-bde+c\,d^2\right)\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a-bde+c\,d^2}{e^2}}} - \frac{\cdots}{\left(e^2a-bde+c\,d^2\right)\left(\frac{4c\left(e^2a-bde+c\cdots\right)}{e^2}\cdots\right.}\cdots\right.}{\cdots}$

input
```
int((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*g/e*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+(-d*g+e*f)/e^2*(1/(a*e^2-b
*d*e+c*d^2)*e^2/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(4*c
*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+
d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/((a*e^2-b*d*
e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

3.881.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 810 vs. $2(177) = 354$.

Time = 2.95 (sec) , antiderivative size = 1663, normalized size of antiderivative = 8.89

$$\int \frac{f + gx}{(d + ex)(a + bx + cx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(((a*b^2 - 4*a^2*c)*e^2*f - (a*b^2 - 4*a^2*c)*d*e*g + ((b^2*c - 4*a*
c^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g)*x^2 + ((b^3 - 4*a*b*c)*e^2*f - (b^3
- 4*a*b*c)*d*e*g)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^
2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 +
4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d
- b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 +
2*d*e*x + d^2)) + 4*sqrt(c*x^2 + b*x + a)*((b*c^2*d^3 - 2*(b^2*c - a*c^2)*
d^2*e + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*f - (2*a*c^2*d^3 - 3*
a*b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d*e^2)*g + ((2*c^3*d^3 - 3*b*c
^2*d^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*f - (b*c^2*d^3 + 3*a*b*c*d
*e^2 - 2*a^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*g)*x))/((a*b^2*c^2 - 4*a^2*c
^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c
^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((
b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*
c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c -
4*a^3*c^2)*e^4)*x^2 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)
*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)
*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x), (((a*b^2 - 4*a^2*c)*e^2*f - (a*b^2
 - 4*a^2*c)*d*e*g + ((b^2*c - 4*a*c^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g)*x^
2 + ((b^3 - 4*a*b*c)*e^2*f - (b^3 - 4*a*b*c)*d*e*g)*x)*sqrt(-c*d^2 + b*...
```

3.881.6 Sympy [F]

$$\int \frac{f + gx}{(d + ex)(a + bx + cx^2)^{3/2}} \, dx = \int \frac{f + gx}{(d + ex)(a + bx + cx^2)^{\frac{3}{2}}} \, dx$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((f + g*x)/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

3.881.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

3.881.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(177) = 354$.

Time = 0.29 (sec) , antiderivative size = 583, normalized size of antiderivative = 3.12

$$\int \frac{f+gx}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(\frac{\left(2c^3d^3f-3bc^2d^2ef+b^2cde^2f+2ac^2de^2f-abce^3f-bc^2d^3g+b^2cd^2eg+2ac^2d^2eg-3abcde^2g+2a^2ce^3g\right)x}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b^2e^4-4a^3ce^4}+\frac{bc^2d^3f-2b^2cd^2ef+2ac^2\cdots}{b^2c^2d^4-4ac^3d^4-2b^3\cdots}\right.}{\sqrt{cx^2+bx+a}} + \frac{2\left(e^2f-deg\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\left(cd^2-bde+ae^2\right)\sqrt{-cd^2+bde-ae^2}}$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2*((2*c^3*d^3*f - 3*b*c^2*d^2*e*f + b^2*c*d*e^2*f + 2*a*c^2*d*e^2*f - a*b
*c*e^3*f - b*c^2*d^3*g + b^2*c*d^2*e*g + 2*a*c^2*d^2*e*g - 3*a*b*c*d*e^2*g
 + 2*a^2*c*e^3*g)*x/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2
*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e
^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3*f - 2*b^2*c
*d^2*e*f + 2*a*c^2*d^2*e*f + b^3*d*e^2*f - a*b*c*d*e^2*f - a*b^2*e^3*f + 2
*a^2*c*e^3*f - 2*a*c^2*d^3*g + 3*a*b*c*d^2*e*g - a*b^2*d*e^2*g - 2*a^2*c*d
*e^2*g + a^2*b*e^3*g)/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c
^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d
*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4))/sqrt(c*x^2 + b*x + a)
 + 2*(e^2*f - d*e*g)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt
(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2
+ b*d*e - a*e^2))
```

3.881.9 Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{f+g\,x}{(d+e\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

input
```
int((f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int((f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.882 $\int \frac{1}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx$

3.882.1 Optimal result

Integrand size = 22, antiderivative size = 155

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} + \frac{e^2\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\left(cd^2-bde+ae^2\right)^{3/2}}$$

output
```
e^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^
2+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)-2*(b*c*d-b^2*e+2*a*c*e+c*(-b*e+2
*c*d)*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^2+b*x+a)^(1/2)
```

3.882.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.23

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(\left(cd^2+e(-bd+ae)\right)\left(-b^2e+2c(ae+cdx)+bc(d-ex)\right)+\left(-b^2+4ac\right)e^2\sqrt{-cd^2+bde-ae^2}\sqrt{a+x(b}\right.}{\left(b^2-4ac\right)\left(cd^2+e(-bd+ae)\right)^2\sqrt{a+x(b+cx)}}$$

input
```
Integrate[1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*((c*d^2 + e*(-(b*d) + a*e))*(-(b^2*e) + 2*c*(a*e + c*d*x) + b*c*(d - e
*x)) + (-b^2 + 4*a*c)*e^2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*Sqrt[a + x*(b + c
*x)]*ArcTan[(Sqrt[-(c*d^2) + b*d*e - a*e^2]*x)/(Sqrt[a]*(d + e*x) - d*Sqrt
[a + x*(b + c*x)])]))/((b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^2*Sqrt[a +
 x*(b + c*x)])
```

3.882.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1165, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1165}$$

$$-\frac{2\int -\frac{\left(b^2-4ac\right)e^2}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{\left(b^2-4ac\right)\left(ae^2-bde+cd^2\right)}-\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{ae^2-bde+cd^2}-\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{2e^2\int \frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{ae^2-bde+cd^2}-\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{e^2\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

input
```
Int[1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 -
 b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*
d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2
 - b*d*e + a*e^2)^(3/2)
```

3.882.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

3.882.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(145) = 290$.

Time = 0.69 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.58

method	result
default	$\frac{\frac{e^2}{\left(e^2 a-bde+c\,d^2\right)\sqrt{\left(x+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^2 a-bde+c\,d^2}{e^2}}}-\frac{(be-2cd)e\left(2c\left(x+\frac{d}{e}\right)+\frac{be-2cd}{e}\right)}{\left(e^2 a-bde+c\,d^2\right)\left(\frac{4c\left(e^2 a-bde+c\,d^2\right)}{e^2}-\frac{(be-2cd)^2}{e^2}\right)\sqrt{\left(x+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x\right.}{e}}}}{e}$

input
```
int(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e*(1/(a*e^2-b*d*e+c*d^2)*e^2/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e
-2*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/((x+d/e)^2*c+(b
*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e
^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*
d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*
(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

3.882.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(145) = 290$.

Time = 0.60 (sec) , antiderivative size = 1349, normalized size of antiderivative = 8.70

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(((b^2*c - 4*a*c^2)*e^2*x^2 + (b^3 - 4*a*b*c)*e^2*x + (a*b^2 - 4*a^2*
c)*e^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*
a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2
- b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2
*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2
)) - 4*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 - (a*b^2
 - 2*a^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^
2)*d*e^2)*x)*sqrt(c*x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^
3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(
a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4
)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*
d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x
^2 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2
*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3
 - 4*a^3*b*c)*e^4)*x), (((b^2*c - 4*a*c^2)*e^2*x^2 + (b^3 - 4*a*b*c)*e^2*x
 + (a*b^2 - 4*a^2*c)*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c
*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x
)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*
d^2 - b^2*d*e + a*b*e^2)*x)) - 2*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e + (b
^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e ...
```

3.882.6 Sympy [F]

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral(1/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

3.882.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` f
or more de
```

3.882.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(145) = 290$.

Time = 0.28 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.97

$$\int \frac{1}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx = \frac{2\,e^2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{(cd^2-bde+ae^2)\sqrt{-cd^2+bde-ae^2}} - \frac{2\left(\frac{(2\,c^3d^3-3\,bc^2d^2e+b^2cde^2+2\,ac^2de^2-abce^3)x}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4} + \frac{\cdots}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e\cdots}\right)}{\sqrt{cx^2+bx+a}}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
2*e^2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*
d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2
)) - 2*((2*c^3*d^3 - 3*b*c^2*d^2*e + b^2*c*d*e^2 + 2*a*c^2*d*e^2 - a*b*c*e
^3)*x/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d
^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c
*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3 - 2*b^2*c*d^2*e + 2*a*c^2
*d^2*e + b^3*d*e^2 - a*b*c*d*e^2 - a*b^2*e^3 + 2*a^2*c*e^3)/(b^2*c^2*d^4 -
 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d
^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4
 - 4*a^3*c*e^4))/sqrt(c*x^2 + b*x + a)
```

3.882.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(d+e\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

input
```
int(1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int(1/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.883 $\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx$

3.883.1 Optimal result

Integrand size = 29, antiderivative size = 352

$$\begin{aligned}\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx =\\ &-\frac{2e(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\,(cd^2-bde+ae^2)\,(ef-dg)\sqrt{a+bx+cx^2}}\\ &+\frac{2g(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)\,(ef-dg)\,(cf^2-bfg+ag^2)\,\sqrt{a+bx+cx^2}}\\ &+\frac{e^3\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{(cd^2-bde+ae^2)^{3/2}\,(ef-dg)}-\frac{g^3\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)\,(cf^2-bfg+ag^2)^{3/2}}\end{aligned}$$

output
```
e^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^
2+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)/(-d*g+e*f)-g^3*arctanh(1/2*(b*f-
2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g
+e*f)/(a*g^2-b*f*g+c*f^2)^(3/2)-2*e*(b*c*d-b^2*e+2*a*c*e+c*(-b*e+2*c*d)*x)
/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)/(c*x^2+b*x+a)^(1/2)+2*g*(b*c*
f-b^2*g+2*a*c*g+c*(-b*g+2*c*f)*x)/(-4*a*c+b^2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f
^2)/(c*x^2+b*x+a)^(1/2)
```

3.883.2 Mathematica [A] (verified)

Time = 3.11 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.99

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2(-b^3eg+b^2c(dg+e(f-gx))-2c^2(adg+cdfx+ae(f-gx)}{\left(b^2-4ac\right)\left(-cd^2+e(bd-ae)\right)\left(-cf^2+g(bf-\right.}$$
$$-\frac{2e^3\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{\left(cd^2+e(-bd+ae)\right)^2(-ef+dg)}$$
$$-\frac{2g^3\sqrt{-cf^2+bfg-ag^2}\arctan\left(\frac{\sqrt{c}(f+gx)-g\sqrt{a+x(b+cx)}}{\sqrt{-cf^2+g(bf-ag)}}\right)}{(ef-dg)\left(cf^2+g(-bf+ag)\right)^2}$$

input
```
Integrate[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*(-(b^3*e*g) + b^2*c*(d*g + e*(f - g*x)) - 2*c^2*(a*d*g + c*d*f*x + a*e*
(f - g*x)) + b*c*(3*a*e*g + c*(-(d*f) + e*f*x + d*g*x))))/((b^2 - 4*a*c)*(
-(c*d^2) + e*(b*d - a*e))*(-(c*f^2) + g*(b*f - a*g))*Sqrt[a + x*(b + c*x)]
) - (2*e^3*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sq
rt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/((c*d^2 + e*(-(b*d)
+ a*e))^2*(-(e*f) + d*g)) - (2*g^3*Sqrt[-(c*f^2) + b*f*g - a*g^2]*ArcTan[(
Sqrt[c]*(f + g*x) - g*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*f^2) + g*(b*f - a*g)
]])/((e*f - d*g)*(c*f^2 + g*(-(b*f) + a*g))^2)
```

3.883.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx$$
$$\Big\downarrow \text{1289}$$
$$\int \left(\frac{e}{(d+ex)\left(a+bx+cx^2\right)^{3/2}(ef-dg)}-\frac{g}{(f+gx)\left(a+bx+cx^2\right)^{3/2}(ef-dg)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{e^3\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)\,(ae^2-bde+cd^2)^{3/2}}-\frac{g^3\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)\,(ag^2-bfg+cf^2)^{3/2}}-\frac{2e(2ace+b^2(-e)+cx(2cd-be)+bcd)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ef-dg)\,(ae^2-bde+cd^2)}+\frac{2g(2acg+b^2(-g)+cx(2cf-bg)+bcf)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ef-dg)\,(ag^2-bfg+cf^2)}$$

input
```
Int[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*e*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2
 - b*d*e + a*e^2)*(e*f - d*g)*Sqrt[a + b*x + c*x^2]) + (2*g*(b*c*f - b^2*g
 + 2*a*c*g + c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g
 + a*g^2)*Sqrt[a + b*x + c*x^2]) + (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*
e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/((c*d^2 - b*
d*e + a*e^2)^(3/2)*(e*f - d*g)) - (g^3*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g
)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((e*f - d*g)*
(c*f^2 - b*f*g + a*g^2)^(3/2))
```

3.883.3.1 Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.883.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(332) = 664$.

Time = 0.77 (sec) , antiderivative size = 815, normalized size of antiderivative = 2.32

method	result
default	$\frac{\frac{g^2}{(a g^2-bfg+c f^2)\sqrt{\left(x+\frac{f}{g}\right)^2 c+\frac{(bg-2cf)\left(x+\frac{f}{g}\right)}{g}+\frac{a g^2-bfg+c f^2}{g^2}}}-\frac{(bg-2cf)g\left(2c\left(x+\frac{f}{g}\right)+\frac{bg-2cf}{g}\right)}{(a g^2-bfg+c f^2)\left(\frac{4c(a g^2-bfg+c f^2)}{g^2}-\frac{(bg-2cf)^2}{g^2}\right)\sqrt{\left(x+\frac{f}{g}\right)^2 c+\frac{(bg-2c}{}}}}{dg}$

input
```
int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(d*g-e*f)*(1/(a*g^2-b*f*g+c*f^2)*g^2/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+
(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(2*c*(x+f
/g)+(b*g-2*c*f)/g)/(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/((x+f/g
)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-1/(a*g^2-b*f*g+
c*f^2)*g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(
b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2
*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))-1/(d*g-e*f)*(1/(
a*e^2-b*d*e+c*d^2)*e^2/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*c*d)/
e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/((x+d/e)^2*c+(b*e-2*c*d
)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/((a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+
d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

3.883.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)\,(a+bx+cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.883.6 Sympy [F]

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{1}{(d+ex)\,(f+gx)\,(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral(1/((d + e*x)*(f + g*x)*(a + b*x + c*x**2)**(3/2)), x)
```

3.883.7 Maxima [F]

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{1}{(cx^2+bx+a)^{\frac{3}{2}}(ex+d)(gx+f)}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)), x)
```

3.883.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.883.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)\,(a+bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(f+g\,x)\,\,(d+e\,x)\,\,(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.884 $\int \frac{1}{(d+ex)(f+gx)^2(a+bx+cx^2)^{3/2}} dx$

3.884.1 Optimal result

Integrand size = 29, antiderivative size = 642

$$\begin{aligned}\int \frac{1}{(d+ex)(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx = &-\frac{2e^2(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\left(cd^2-bde+ae^2\right)(ef-dg)^2\sqrt{a+bx+cx^2}}\\ &+\frac{2eg(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)^2\left(cf^2-bfg+ag^2\right)\sqrt{a+bx+cx^2}}\\ &+\frac{2g(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)\sqrt{a+bx+cx^2}}\\ &+\frac{g^2(4c^2f^2+3b^2g^2-4cg(bf+2ag))\sqrt{a+bx+cx^2}}{(b^2-4ac)(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)}\\ &+\frac{e^4\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\left(cd^2-bde+ae^2\right)^{3/2}(ef-dg)^2}\\ &-\frac{3g^3(2cf-bg)\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2(ef-dg)\left(cf^2-bfg+ag^2\right)^{5/2}}\\ &-\frac{eg^3\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)^{3/2}}\end{aligned}$$

output
```
e^4*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^
2+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)/(-d*g+e*f)^2-3/2*g^3*(-b*g+2*c*f
)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+
b*x+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(5/2)-e*g^3*arctanh(1/2*(b*f-
2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g
+e*f)^2/(a*g^2-b*f*g+c*f^2)^(3/2)-2*e^2*(b*c*d-b^2*e+2*a*c*e+c*(-b*e+2*c*d
)*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)^2/(c*x^2+b*x+a)^(1/2)+2*e
*g*(b*c*f-b^2*g+2*a*c*g+c*(-b*g+2*c*f)*x)/(-4*a*c+b^2)/(-d*g+e*f)^2/(a*g^2
-b*f*g+c*f^2)/(c*x^2+b*x+a)^(1/2)+2*g*(b*c*f-b^2*g+2*a*c*g+c*(-b*g+2*c*f)*
x)/(-4*a*c+b^2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)/(c*x^2+b*x+a)^(1/2)
+g^2*(4*c^2*f^2+3*b^2*g^2-4*c*g*(2*a*g+b*f))*(c*x^2+b*x+a)^(1/2)/(-4*a*c+b
^2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2/(g*x+f)
```

3.884.2 Mathematica [A] (verified)

Time = 12.75 (sec) , antiderivative size = 623, normalized size of antiderivative = 0.97

$$\int \frac{1}{(d+ex)(f+gx)^2 (a+bx+cx^2)^{3/2}}\,dx = -\frac{2e^2(b^2e-2c(ae+cdx)+bc(-d+ex))}{(b^2-4ac)\,(-cd^2+e(bd-ae))\,(ef-dg)^2\sqrt{a+x(b+cx)}} + \frac{2eg(b^2g-2c(ag+cfx)+bc(-f+gx))}{(b^2-4ac)\,(ef-dg)^2\,(-cf^2+g(bf-ag))\,\sqrt{a+x(b+cx)}} - \frac{2g(b^2g-2c(ag+cfx)+bc(-f+gx))}{(b^2-4ac)\,(-ef+dg)\,(-cf^2+g(bf-ag))\,(f+gx)\sqrt{a+x(b+cx)}} + \frac{g^2\left(-\frac{2(4c^2f^2+3b^2g^2-4cg(bf+2ag))\sqrt{a+x(b+cx)}}{(b^2-4ac)(cf^2+g(-bf+ag))^2(f+gx)} + \frac{3g(-2cf+bg)\text{arctanh}\left(\frac{-bf+2ag-2cfx+bgx}{2\sqrt{cf^2+g(-bf+ag)}\sqrt{a+x(b+cx)}}\right)}{(cf^2+g(-bf+ag))^{5/2}}\right)}{2(-ef+dg)} + \frac{e^4\text{arctanh}\left(\frac{-2ae+2cdx+b(d-ex)}{2\sqrt{cd^2+e(-bd+ae)}\sqrt{a+x(b+cx)}}\right)}{(cd^2+e(-bd+ae))^{3/2}\,(ef-dg)^2} - \frac{eg^3\text{arctanh}\left(\frac{-2ag+2cfx+b(f-gx)}{2\sqrt{cf^2+g(-bf+ag)}\sqrt{a+x(b+cx)}}\right)}{(ef-dg)^2\,(cf^2+g(-bf+ag))^{3/2}}$$

input
```
Integrate[1/((d + e*x)*(f + g*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*e^2*(b^2*e - 2*c*(a*e + c*d*x) + b*c*(-d + e*x)))/((b^2 - 4*a*c)*(-(c*
d^2) + e*(b*d - a*e))*(e*f - d*g)^2*Sqrt[a + x*(b + c*x)]) + (2*e*g*(b^2*g
 - 2*c*(a*g + c*f*x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(e*f - d*g)^2*(-(c*
f^2) + g*(b*f - a*g))*Sqrt[a + x*(b + c*x)]) - (2*g*(b^2*g - 2*c*(a*g + c*
f*x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(-(e*f) + d*g)*(-(c*f^2) + g*(b*f -
 a*g))*(f + g*x)*Sqrt[a + x*(b + c*x)]) + (g^2*((-2*(4*c^2*f^2 + 3*b^2*g^2
 - 4*c*g*(b*f + 2*a*g))*Sqrt[a + x*(b + c*x)])/((b^2 - 4*a*c)*(c*f^2 + g*(
-(b*f) + a*g))^2*(f + g*x)) + (3*g*(-2*c*f + b*g)*ArcTanh[(-(b*f) + 2*a*g
- 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)]
)])/(c*f^2 + g*(-(b*f) + a*g))^(5/2)))/(2*(-(e*f) + d*g)) + (e^4*ArcTanh[(
-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a +
 x*(b + c*x)])])/((c*d^2 + e*(-(b*d) + a*e))^(3/2)*(e*f - d*g)^2) - (e*g^3
*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)
]*Sqrt[a + x*(b + c*x)])])/((e*f - d*g)^2*(c*f^2 + g*(-(b*f) + a*g))^(3/2)
)
```

3.884.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1289}$$

$$\int \left(\frac{e^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}(ef-dg)^2}-\frac{eg}{(f+gx)\left(a+bx+cx^2\right)^{3/2}(ef-dg)^2}-\frac{g}{(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^4\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{eg^3\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^2\left(ag^2-bfg+cf^2\right)^{3/2}}-\frac{3g^3(2cf-bg)\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2(ef-dg)\left(ag^2-bfg+cf^2\right)^{5/2}}+\frac{g^2\sqrt{a+bx+cx^2}\left(-4cg(2ag+bf)+3b^2g^2+4c^2f^2\right)}{(b^2-4ac)\,(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)^2}-\frac{2e^2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ef-dg)^2\left(ae^2-bde+cd^2\right)}+\frac{2eg\left(2acg+b^2(-g)+cx(2cf-bg)+bcf\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ef-dg)^2\left(ag^2-bfg+cf^2\right)}+\frac{2g\left(2acg+b^2(-g)+cx(2cf-bg)+bcf\right)}{(b^2-4ac)\,(f+gx)\sqrt{a+bx+cx^2}(ef-dg)\left(ag^2-bfg+cf^2\right)}$$

input
```
Int[1/((d + e*x)*(f + g*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*e^2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d
^2 - b*d*e + a*e^2)*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2]) + (2*e*g*(b*c*f -
 b^2*g + 2*a*c*g + c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)^2*(c*f^2
 - b*f*g + a*g^2)*Sqrt[a + b*x + c*x^2]) + (2*g*(b*c*f - b^2*g + 2*a*c*g +
 c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f
 + g*x)*Sqrt[a + b*x + c*x^2]) + (g^2*(4*c^2*f^2 + 3*b^2*g^2 - 4*c*g*(b*f
+ 2*a*g))*Sqrt[a + b*x + c*x^2])/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g
 + a*g^2)^2*(f + g*x)) + (e^4*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*S
qrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/((c*d^2 - b*d*e + a*e^
2)^(3/2)*(e*f - d*g)^2) - (3*g^3*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c
*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*(e
*f - d*g)*(c*f^2 - b*f*g + a*g^2)^(5/2)) - (e*g^3*ArcTanh[(b*f - 2*a*g + (
2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((
e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)^(3/2))
```

3.884.3.1 Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

3.884. $\int \frac{1}{(d+ex)(f+gx)^2(a+bx+cx^2)^{3/2}}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.884.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1480 vs. 2(608) = 1216.

Time = 0.95 (sec) , antiderivative size = 1481, normalized size of antiderivative = 2.31

method	result	size
default	Expression too large to display	1481

input
```
int(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
e/(d*g-e*f)^2*(1/(a*e^2-b*d*e+c*d^2)*e^2/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x
+d/e)+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/((x+d
/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*
e+c*d^2)*e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2
+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e
-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))+1/g/(d*g-e*f)*
(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*
g^2-b*f*g+c*f^2)/g^2)^(1/2)-3/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(1/(a*g^
2-b*f*g+c*f^2)*g^2/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/
g^2)^(1/2)-(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(2*c*(x+f/g)+(b*g-2*c*f)/g)/(
4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*
(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-1/(a*g^2-b*f*g+c*f^2)*g^2/((a*g^2-b
*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)
+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g
^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))-4*c/(a*g^2-b*f*g+c*f^2)*g^2*(2*c*(x+
f/g)+(b*g-2*c*f)/g)/(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/((x+f/
g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))-e/(d*g-e*f)^2
*(1/(a*g^2-b*f*g+c*f^2)*g^2/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*
g+c*f^2)/g^2)^(1/2)-(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(2*c*(x+f/g)+(b*g...
```

3.884.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^2 (a+bx+cx^2)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.884.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^2 (a+bx+cx^2)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)**2/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Timed out
```

3.884.7 Maxima [F]

$$\int \frac{1}{(d+ex)(f+gx)^2 (a+bx+cx^2)^{3/2}} \, dx = \int \frac{1}{(cx^2+bx+a)^{\frac{3}{2}}(ex+d)(gx+f)^2} \, dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)^2), x)
```

3.884.8 Giac [F]

$$\int \frac{1}{(d+ex)(f+gx)^2\,(a+bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(cx^2+bx+a)^{\frac{3}{2}}(ex+d)(gx+f)^2}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.884.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^2\,(a+bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(f+g\,x)^2\,(d+e\,x)\,(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int(1/((f + g*x)^2*(d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int(1/((f + g*x)^2*(d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.885 $\int \frac{1}{(d+ex)(f+gx)^3 (a+bx+cx^2)^{3/2}} dx$

3.885.1 Optimal result

Integrand size = 29, antiderivative size = 1064

$$\int \frac{1}{(d+ex)(f+gx)^3(a+bx+cx^2)^{3/2}}\,dx =$$
$$-\frac{2e^3(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)(cd^2-bde+ae^2)(ef-dg)^3\sqrt{a+bx+cx^2}}$$
$$+\frac{2e^2g(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)^3(cf^2-bfg+ag^2)\sqrt{a+bx+cx^2}}$$
$$+\frac{2g(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)(cf^2-bfg+ag^2)(f+gx)^2\sqrt{a+bx+cx^2}}$$
$$+\frac{2eg(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)^2(cf^2-bfg+ag^2)(f+gx)\sqrt{a+bx+cx^2}}$$
$$+\frac{g^2(8c^2f^2+5b^2g^2-4cg(2bf+3ag))\sqrt{a+bx+cx^2}}{2(b^2-4ac)(ef-dg)(cf^2-bfg+ag^2)^2(f+gx)^2}$$
$$+\frac{eg^2(4c^2f^2+3b^2g^2-4cg(bf+2ag))\sqrt{a+bx+cx^2}}{(b^2-4ac)(ef-dg)^2(cf^2-bfg+ag^2)^2(f+gx)}$$
$$+\frac{g^2(2cf-bg)(8c^2f^2+15b^2g^2-4cg(2bf+13ag))\sqrt{a+bx+cx^2}}{4(b^2-4ac)(ef-dg)(cf^2-bfg+ag^2)^3(f+gx)}$$
$$+\frac{e^5\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{(cd^2-bde+ae^2)^{3/2}(ef-dg)^3}$$
$$-\frac{3eg^3(2cf-bg)\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2(ef-dg)^2(cf^2-bfg+ag^2)^{5/2}}$$
$$-\frac{e^2g^3\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^3(cf^2-bfg+ag^2)^{3/2}}$$
$$-\frac{3g^3(16c^2f^2+5b^2g^2-4cg(4bf+ag))\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{8(ef-dg)(cf^2-bfg+ag^2)^{7/2}}$$

output
```
e^5*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^
2+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)/(-d*g+e*f)^3-3/2*e*g^3*(-b*g+2*c
*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^
2+b*x+a)^(1/2))/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)^(5/2)-e^2*g^3*arctanh(1/2
*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))
/(-d*g+e*f)^3/(a*g^2-b*f*g+c*f^2)^(3/2)-3/8*g^3*(16*c^2*f^2+5*b^2*g^2-4*c*
g*(a*g+4*b*f))*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^
(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(7/2)-2*e^3*(b*c
*d-b^2*e+2*a*c*e+c*(-b*e+2*c*d)*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(-d*g+
e*f)^3/(c*x^2+b*x+a)^(1/2)+2*e^2*g*(b*c*f-b^2*g+2*a*c*g+c*(-b*g+2*c*f)*x)/
(-4*a*c+b^2)/(-d*g+e*f)^3/(a*g^2-b*f*g+c*f^2)/(c*x^2+b*x+a)^(1/2)+2*g*(b*c
*f-b^2*g+2*a*c*g+c*(-b*g+2*c*f)*x)/(-4*a*c+b^2)/(-d*g+e*f)/(a*g^2-b*f*g+c*
f^2)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2)+2*e*g*(b*c*f-b^2*g+2*a*c*g+c*(-b*g+2*c*
f)*x)/(-4*a*c+b^2)/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)/(g*x+f)/(c*x^2+b*x+a)^
(1/2)+1/2*g^2*(8*c^2*f^2+5*b^2*g^2-4*c*g*(3*a*g+2*b*f))*(c*x^2+b*x+a)^(1/2
)/(-4*a*c+b^2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2/(g*x+f)^2+e*g^2*(4*c^2*f^2
+3*b^2*g^2-4*c*g*(2*a*g+b*f))*(c*x^2+b*x+a)^(1/2)/(-4*a*c+b^2)/(-d*g+e*f)^
2/(a*g^2-b*f*g+c*f^2)^2/(g*x+f)+1/4*g^2*(-b*g+2*c*f)*(8*c^2*f^2+15*b^2*g^2
-4*c*g*(13*a*g+2*b*f))*(c*x^2+b*x+a)^(1/2)/(-4*a*c+b^2)/(-d*g+e*f)/(a*g^2-
b*f*g+c*f^2)^3/(g*x+f)
```

3.885. $\int \frac{1}{(d+ex)(f+gx)^3(a+bx+cx^2)^{3/2}}\,dx$

3.885.2 Mathematica [A] (verified)

Time = 15.23 (sec) , antiderivative size = 1013, normalized size of antiderivative = 0.95

$$
\begin{aligned}
&\int \frac{1}{(d+ex)(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = \\
&-\frac{2e^3(b^2e-2c(ae+cdx)+bc(-d+ex))}{\left(b^2-4ac\right)\left(-cd^2+e(bd-ae)\right)(ef-dg)^3\sqrt{a+x(b+cx)}} \\
&-\frac{2e^2g(b^2g-2c(ag+cfx)+bc(-f+gx))}{\left(b^2-4ac\right)(-ef+dg)^3\left(-cf^2+g(bf-ag)\right)\sqrt{a+x(b+cx)}} \\
&-\frac{2g(b^2g-2c(ag+cfx)+bc(-f+gx))}{\left(b^2-4ac\right)(-ef+dg)\left(-cf^2+g(bf-ag)\right)(f+gx)^2\sqrt{a+x(b+cx)}} \\
&+\frac{2eg(b^2g-2c(ag+cfx)+bc(-f+gx))}{\left(b^2-4ac\right)(ef-dg)^2\left(-cf^2+g(bf-ag)\right)(f+gx)\sqrt{a+x(b+cx)}} \\
&+\frac{eg^2\left(\frac{2\left(4c^2f^2+3b^2g^2-4cg(bf+2ag)\right)\sqrt{a+x(b+cx)}}{(b^2-4ac)\left(cf^2+g(-bf+ag)\right)^2(f+gx)}+\frac{3g(2cf-bg)\operatorname{arctanh}\left(\frac{-bf+2ag-2cfx+bgx}{2\sqrt{cf^2+g(-bf+ag)}\sqrt{a+x(b+cx)}}\right)}{\left(cf^2+g(-bf+ag)\right)^{5/2}}\right)}{2(ef-dg)^2} \\
&-\frac{g^2\left(\frac{4\left(8c^2f^2+5b^2g^2-4cg(2bf+3ag)\right)\sqrt{a+x(b+cx)}}{(f+gx)^2}+\frac{2(2cf-bg)\left(8c^2f^2+15b^2g^2-4cg(2bf+13ag)\right)\sqrt{a+x(b+cx)}}{\left(cf^2+g(-bf+ag)\right)(f+gx)}+\frac{3(b^2-4ac)g(16c^2f^2+5b^{\cdots}}{\cdots}\right.}{8\left(b^2-4ac\right)(-ef+dg)\left(cf^2+g(-bf+ag)\right)^2} \\
&-\frac{e^5\operatorname{arctanh}\left(\frac{-2ae+2cdx+b(d-ex)}{2\sqrt{cd^2+e(-bd+ae)}\sqrt{a+x(b+cx)}}\right)}{\left(cd^2+e(-bd+ae)\right)^{3/2}(-ef+dg)^3}-\frac{e^2g^3\operatorname{arctanh}\left(\frac{-2ag+2cfx+b(f-gx)}{2\sqrt{cf^2+g(-bf+ag)}\sqrt{a+x(b+cx)}}\right)}{(ef-dg)^3\left(cf^2+g(-bf+ag)\right)^{3/2}}
\end{aligned}
$$

input
```
Integrate[1/((d + e*x)*(f + g*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*e^3*(b^2*e - 2*c*(a*e + c*d*x) + b*c*(-d + e*x)))/((b^2 - 4*a*c)*(-(c*
d^2) + e*(b*d - a*e))*(e*f - d*g)^3*Sqrt[a + x*(b + c*x)]) - (2*e^2*g*(b^2
*g - 2*c*(a*g + c*f*x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(-(e*f) + d*g)^3*
(-(c*f^2) + g*(b*f - a*g))*Sqrt[a + x*(b + c*x)]) - (2*g*(b^2*g - 2*c*(a*g
 + c*f*x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(-(e*f) + d*g)*(-(c*f^2) + g*(
b*f - a*g))*(f + g*x)^2*Sqrt[a + x*(b + c*x)]) + (2*e*g*(b^2*g - 2*c*(a*g
+ c*f*x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(e*f - d*g)^2*(-(c*f^2) + g*(b*
f - a*g))*(f + g*x)*Sqrt[a + x*(b + c*x)]) + (e*g^2*((2*(4*c^2*f^2 + 3*b^2
*g^2 - 4*c*g*(b*f + 2*a*g))*Sqrt[a + x*(b + c*x)])/((b^2 - 4*a*c)*(c*f^2 +
 g*(-(b*f) + a*g))^2*(f + g*x)) + (3*g*(2*c*f - b*g)*ArcTanh[(-(b*f) + 2*a
*g - 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*
x)])])/(c*f^2 + g*(-(b*f) + a*g))^(5/2)))/(2*(e*f - d*g)^2) - (g^2*((4*(8*
c^2*f^2 + 5*b^2*g^2 - 4*c*g*(2*b*f + 3*a*g))*Sqrt[a + x*(b + c*x)])/(f + g
*x)^2 + (2*(2*c*f - b*g)*(8*c^2*f^2 + 15*b^2*g^2 - 4*c*g*(2*b*f + 13*a*g))
*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)) + (3*(b^2 -
 4*a*c)*g*(16*c^2*f^2 + 5*b^2*g^2 - 4*c*g*(4*b*f + a*g))*ArcTanh[(-(b*f) +
 2*a*g - 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b
+ c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^(3/2)))/(8*(b^2 - 4*a*c)*(-(e*f) + d
*g)*(c*f^2 + g*(-(b*f) + a*g))^2) - (e^5*ArcTanh[(-2*a*e + 2*c*d*x + b*(d
- e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/((c*...
```

3.885.3 Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 1064, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1289}$$

$$\int \left(\frac{e^3}{(d+ex)\left(a+bx+cx^2\right)^{3/2}(ef-dg)^3}-\frac{e^2g}{(f+gx)\left(a+bx+cx^2\right)^{3/2}(ef-dg)^3}-\frac{eg}{(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)e^5}{(cd^2-bed+ae^2)^{3/2}(ef-dg)^3}-\frac{2(-eb^2+cdb+2ace+c(2cd-be)x)e^3}{(b^2-4ac)(cd^2-bed+ae^2)(ef-dg)^3\sqrt{cx^2+bx+a}}-\frac{g^3\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)e^2}{(ef-dg)^3(cf^2-bgf+ag^2)^{3/2}}+\frac{2g(-gb^2+cfb+2acg+c(2cf-bg)x)e^2}{(b^2-4ac)(ef-dg)^3(cf^2-bgf+ag^2)\sqrt{cx^2+bx+a}}-\frac{3g^3(2cf-bg)\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)e}{2(ef-dg)^2(cf^2-bgf+ag^2)^{5/2}}+\frac{g^2(4c^2f^2+3b^2g^2-4cg(bf+2ag))\sqrt{cx^2+bx+a}e}{(b^2-4ac)(ef-dg)^2(cf^2-bgf+ag^2)^2(f+gx)}+\frac{2g(-gb^2+cfb+2acg+c(2cf-bg)x)e}{(b^2-4ac)(ef-dg)^2(cf^2-bgf+ag^2)(f+gx)\sqrt{cx^2+bx+a}}-\frac{3g^3(16c^2f^2+5b^2g^2-4cg(4bf+ag))\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)}{8(ef-dg)(cf^2-bgf+ag^2)^{7/2}}+\frac{g^2(2cf-bg)(8c^2f^2+15b^2g^2-4cg(2bf+13ag))\sqrt{cx^2+bx+a}}{4(b^2-4ac)(ef-dg)(cf^2-bgf+ag^2)^3(f+gx)}+\frac{g^2(8c^2f^2+5b^2g^2-4cg(2bf+3ag))\sqrt{cx^2+bx+a}}{2(b^2-4ac)(ef-dg)(cf^2-bgf+ag^2)^2(f+gx)^2}+\frac{2g(-gb^2+cfb+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)(cf^2-bgf+ag^2)(f+gx)^2\sqrt{cx^2+bx+a}}$$

input
```
Int[1/((d + e*x)*(f + g*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

3.885. $\int \frac{1}{(d+ex)(f+gx)^3(a+bx+cx^2)^{3/2}}\,dx$

output
```
(-2*e^3*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d
^2 - b*d*e + a*e^2)*(e*f - d*g)^3*Sqrt[a + b*x + c*x^2]) + (2*e^2*g*(b*c*f
 - b^2*g + 2*a*c*g + c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)^3*(c*f
^2 - b*f*g + a*g^2)*Sqrt[a + b*x + c*x^2]) + (2*g*(b*c*f - b^2*g + 2*a*c*g
 + c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*
(f + g*x)^2*Sqrt[a + b*x + c*x^2]) + (2*e*g*(b*c*f - b^2*g + 2*a*c*g + c*(
2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*(f +
 g*x)*Sqrt[a + b*x + c*x^2]) + (g^2*(8*c^2*f^2 + 5*b^2*g^2 - 4*c*g*(2*b*f
+ 3*a*g))*Sqrt[a + b*x + c*x^2])/(2*(b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f
*g + a*g^2)^2*(f + g*x)^2) + (e*g^2*(4*c^2*f^2 + 3*b^2*g^2 - 4*c*g*(b*f +
2*a*g))*Sqrt[a + b*x + c*x^2])/((b^2 - 4*a*c)*(e*f - d*g)^2*(c*f^2 - b*f*g
 + a*g^2)^2*(f + g*x)) + (g^2*(2*c*f - b*g)*(8*c^2*f^2 + 15*b^2*g^2 - 4*c*
g*(2*b*f + 13*a*g))*Sqrt[a + b*x + c*x^2])/(4*(b^2 - 4*a*c)*(e*f - d*g)*(c
*f^2 - b*f*g + a*g^2)^3*(f + g*x)) + (e^5*ArcTanh[(b*d - 2*a*e + (2*c*d -
b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/((c*d^2 -
b*d*e + a*e^2)^(3/2)*(e*f - d*g)^3) - (3*e*g^3*(2*c*f - b*g)*ArcTanh[(b*f
- 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c
*x^2])])/(2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)^(5/2)) - (e^2*g^3*ArcTan
h[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a +
b*x + c*x^2])])/((e*f - d*g)^3*(c*f^2 - b*f*g + a*g^2)^(3/2)) - (3*g^3*...
```

3.885.3.1 Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.885.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2684 vs. $2(1010) = 2020$.

Time = 1.14 (sec) , antiderivative size = 2685, normalized size of antiderivative = 2.52

method	result	size
default	Expression too large to display	2685

input
```
int(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/g^2/(d*g-e*f)*(-1/2/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)^2/((x+f/g)^2*c+(b*g-
2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-5/4*(b*g-2*c*f)*g/(a*g^2-b
*f*g+c*f^2)*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)/((x+f/g)^2*c+(b*g-2*c*f)/g
*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-3/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f
^2)*(1/(a*g^2-b*f*g+c*f^2)*g^2/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b
*f*g+c*f^2)/g^2)^(1/2)-(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(2*c*(x+f/g)+(b*g
-2*c*f)/g)/(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/((x+f/g)^2*c+(b
*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-1/(a*g^2-b*f*g+c*f^2)*g
^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*
f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*
(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))-4*c/(a*g^2-b*f*g+c*f^2)*
g^2*(2*c*(x+f/g)+(b*g-2*c*f)/g)/(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2
/g^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))-3
/2*c/(a*g^2-b*f*g+c*f^2)*g^2*(1/(a*g^2-b*f*g+c*f^2)*g^2/((x+f/g)^2*c+(b*g-
2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-(b*g-2*c*f)*g/(a*g^2-b*f*g
+c*f^2)*(2*c*(x+f/g)+(b*g-2*c*f)/g)/(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*
f)^2/g^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2
)-1/(a*g^2-b*f*g+c*f^2)*g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b
*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((
x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g...
```

3.885.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.885.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^3 (a+bx+cx^2)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)**3/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Timed out
```

3.885.7 Maxima [F]

$$\int \frac{1}{(d+ex)(f+gx)^3 (a+bx+cx^2)^{3/2}} \, dx = \int \frac{1}{(cx^2+bx+a)^{\frac{3}{2}}(ex+d)(gx+f)^3} \, dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)^3), x)
```

3.885.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14979 vs. $2(1010) = 2020$.

Time = 4.14 (sec) , antiderivative size = 14979, normalized size of antiderivative = 14.08

$$\int \frac{1}{(d+ex)(f+gx)^3 (a+bx+cx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
2*e^5*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*
d^2 + b*d*e - a*e^2))/((c*d^2*e^3*f^3 - b*d*e^4*f^3 + a*e^5*f^3 - 3*c*d^3*
e^2*f^2*g + 3*b*d^2*e^3*f^2*g - 3*a*d*e^4*f^2*g + 3*c*d^4*e*f*g^2 - 3*b*d^
3*e^2*f*g^2 + 3*a*d^2*e^3*f*g^2 - c*d^5*g^3 + b*d^4*e*g^3 - a*d^3*e^2*g^3)
*sqrt(-c*d^2 + b*d*e - a*e^2)) - 2*((2*c^9*d^3*f^9 - 3*b*c^8*d^2*e*f^9 + b
^2*c^7*d*e^2*f^9 + 2*a*c^8*d*e^2*f^9 - a*b*c^7*e^3*f^9 - 9*b*c^8*d^3*f^8*g
 + 15*b^2*c^7*d^2*e*f^8*g - 6*a*c^8*d^2*e*f^8*g - 6*b^3*c^6*d*e^2*f^8*g -
3*a*b*c^7*d*e^2*f^8*g + 6*a*b^2*c^6*e^3*f^8*g - 6*a^2*c^7*e^3*f^8*g + 18*b
^2*c^7*d^3*f^7*g^2 - 33*b^3*c^6*d^2*e*f^7*g^2 + 24*a*b*c^7*d^2*e*f^7*g^2 +
 15*b^4*c^5*d*e^2*f^7*g^2 - 6*a*b^2*c^6*d*e^2*f^7*g^2 - 15*a*b^3*c^5*e^3*f
^7*g^2 + 24*a^2*b*c^6*e^3*f^7*g^2 - 21*b^3*c^6*d^3*f^6*g^3 + 41*b^4*c^5*d^
2*e*f^6*g^3 - 34*a*b^2*c^6*d^2*e*f^6*g^3 - 16*a^2*c^7*d^2*e*f^6*g^3 - 20*b
^5*c^4*d*e^2*f^6*g^3 + 13*a*b^3*c^5*d*e^2*f^6*g^3 + 16*a^2*b*c^6*d*e^2*f^6
*g^3 + 20*a*b^4*c^4*e^3*f^6*g^3 - 34*a^2*b^2*c^5*e^3*f^6*g^3 - 16*a^3*c^6*
e^3*f^6*g^3 + 15*b^4*c^5*d^3*f^5*g^4 + 6*a*b^2*c^6*d^3*f^5*g^4 - 12*a^2*c^
7*d^3*f^5*g^4 - 30*b^5*c^4*d^2*e*f^5*g^4 + 9*a*b^3*c^5*d^2*e*f^5*g^4 + 66*
a^2*b*c^6*d^2*e*f^5*g^4 + 15*b^6*c^3*d*e^2*f^5*g^4 - 48*a^2*b^2*c^5*d*e^2*
f^5*g^4 - 12*a^3*c^6*d*e^2*f^5*g^4 - 15*a*b^5*c^3*e^3*f^5*g^4 + 15*a^2*b^3
*c^4*e^3*f^5*g^4 + 54*a^3*b*c^5*e^3*f^5*g^4 - 6*b^5*c^4*d^3*f^4*g^5 - 15*a
*b^3*c^5*d^3*f^4*g^5 + 30*a^2*b*c^6*d^3*f^4*g^5 + 12*b^6*c^3*d^2*e*f^4*...
```

3.885.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^3 (a+bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(f+g\,x)^3\,(d+e\,x)\,(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int(1/((f + g*x)^3*(d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int(1/((f + g*x)^3*(d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.886 $\int (d+ex)^3 \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

3.886.1 Optimal result

Integrand size = 31, antiderivative size = 1551

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx =$$

$$-\frac{2(64b^4e^4g^4+4b^2ce^3g^3(7bef-66bdg-69aeg)+c^4(187e^4f^4-732de^3f^3g+1098d^2e^2f^2g^2-798d^3efg^3}{}$$

$$+\frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e}$$

$$+\frac{2(48b^3e^3g^3+bce^2g^2(67bef-198bdg-157aeg)+c^3(233e^3f^3-843de^2f^2g+1107d^2efg^2-567d^3g^3)-}{3465c^3g^4}$$

$$-\frac{2e(8b^2e^2g^2+ceg(19bef-33bdg-18aeg)+c^2(29e^2f^2-96defg+81d^2g^2))(f+gx)^{5/2}\sqrt{a+bx+cx^2}}{693c^2g^4}$$

$$+\frac{2e^2(cef-3cdg+beg)(f+gx)^{7/2}\sqrt{a+bx+cx^2}}{99cg^4}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(128b^5e^3g^5-8b^3ce^2g^4(7bef+66bdg+87aeg)+2c^5f^2(64e^3f^3-264de^2f^2g+396d^2efg^2}{}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(cf^2-bfg+ag^2)(64b^4e^3g^4+4b^2ce^2g^3(7bef-66bdg-69aeg)-2c^4f(64e^3f^3-264de^2}{}$$

output
```
2/3465*(48*b^3*e^3*g^3+b*c*e^2*g^2*(-157*a*e*g-198*b*d*g+67*b*e*f)+c^3*(-5
67*d^3*g^3+1107*d^2*e*f*g^2-843*d*e^2*f^2*g+233*e^3*f^3)-c^2*e*g*(2*a*e*g*
(-231*d*g+74*e*f)-3*b*(99*d^2*g^2-88*d*e*f*g+24*e^2*f^2)))*(g*x+f)^(3/2)*(
c*x^2+b*x+a)^(1/2)/c^3/g^4-2/693*e*(8*b^2*e^2*g^2+c*e*g*(-18*a*e*g-33*b*d*
g+19*b*e*f)+c^2*(81*d^2*g^2-96*d*e*f*g+29*e^2*f^2))*(g*x+f)^(5/2)*(c*x^2+b
*x+a)^(1/2)/c^2/g^4+2/99*e^2*(b*e*g-3*c*d*g+c*e*f)*(g*x+f)^(7/2)*(c*x^2+b*
x+a)^(1/2)/c/g^4-2/3465*(64*b^4*e^4*g^4+4*b^2*c*e^3*g^3*(-69*a*e*g-66*b*d*
g+7*b*e*f)+c^4*(315*d^4*g^4-798*d^3*e*f*g^3+1098*d^2*e^2*f^2*g^2-732*d*e^3
*f^3*g+187*e^4*f^4)+3*c^2*e^2*g^2*(50*a^2*e^2*g^2-a*b*e*g*(-297*d*g+29*e*f
)+3*b^2*(44*d^2*g^2-11*d*e*f*g+e^2*f^2))-c^3*e*g*(6*a*e*g*(165*d^2*g^2-33*
d*e*f*g+2*e^2*f^2)+b*(231*d^3*g^3-99*d^2*e*f*g^2+8*e^3*f^3)))*(g*x+f)^(1/2
)*(c*x^2+b*x+a)^(1/2)/c^4/e/g^4+2/11*(e*x+d)^4*(g*x+f)^(1/2)*(c*x^2+b*x+a)
^(1/2)/e+1/3465*(128*b^5*e^3*g^5-8*b^3*c*e^2*g^4*(87*a*e*g+66*b*d*g+7*b*e*
f)+2*c^5*f^2*(-231*d^3*g^3+396*d^2*e*f*g^2-264*d*e^2*f^2*g+64*e^3*f^3)+b*c
^2*e*g^3*(771*a^2*e^2*g^2+6*a*b*e*g*(396*d*g+43*e*f)-b^2*(-792*d^2*g^2-264
*d*e*f*g+37*e^2*f^2))-c^4*g*(b*f*(-462*d^3*g^3+495*d^2*e*f*g^2-264*d*e^2*f
^2*g+56*e^3*f^3)-18*a*g*(77*d^3*g^3+88*d^2*e*f*g^2-33*d*e^2*f^2*g+6*e^3*f^
3))-c^3*g^2*(6*a^2*e^2*g^2*(231*d*g+26*e*f)-9*a*b*e*g*(-319*d^2*g^2-110*d*
e*f*g+15*e^2*f^2)+b^2*(462*d^3*g^3+495*d^2*e*f*g^2-198*d*e^2*f^2*g+37*e^3*
f^3)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^...
```

3.886.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 37.07 (sec) , antiderivative size = 26600, normalized size of antiderivative = 17.15

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

output
```
Result too large to show
```

3.886.3 Rubi [A] (verified)

Time = 5.94 (sec) , antiderivative size = 1597, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {1272, 2184, 27, 2184, 27, 2184, 27, 2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$$

$$\Big\downarrow \ 1272$$

$$\frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e}-\frac{\int \frac{(d+ex)^3(-((cef-3cdg+beg)x^2)+2(cdf-bef+bdg-aeg)x+bdf-3aef+adg)}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{11e}$$

$$\Big\downarrow \ 2184$$

$$\frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e}-\frac{2\int \frac{e^2g^4\left(\left(29e^2f^2-96degf+81d^2g^2\right)c^2+eg(19bef-33bdg-18aeg)c+8b^2e^2g^2\right)x^4+eg^3\left(bg^2(25bf+7ag)e^3-cg\left(2aeg(10ef+33dg)-b\left(58e^2f^2-120degf+27d^2g^2\right)\right)e\right.}{}}{}$$

$$\Big\downarrow \ 27$$

$$\frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e}-\frac{\int \frac{e^2g^4\left(\left(29e^2f^2-96degf+81d^2g^2\right)c^2+eg(19bef-33bdg-18aeg)c+8b^2e^2g^2\right)x^4+eg^3\left(bg^2(25bf+7ag)e^3-cg\left(2aeg(10ef+33dg)-b\left(58e^2f^2-120degf+27d^2g^2\right)\right)e+\right.}{}}{}$$

$$\Big\downarrow \ 2184$$

$$\frac{2(d+ex)^4\sqrt{f+gx}\sqrt{cx^2+bx+a}}{11e}-\frac{\frac{2e^2g\left(\left(29e^2f^2-96degf+81d^2g^2\right)c^2+eg(19bef-33bdg-18aeg)c+8b^2e^2g^2\right)\sqrt{cx^2+bx+a}(f+gx)^{5/2}}{7c}+\frac{2\int -\frac{e\left(\left(233e^3f^3-843de^2gf^2+1107d^2eg^2f-567d^3g^3\right)c^3-eg\right.}{}}{}}{}$$

$$\Big\downarrow \ 27$$

$$\frac{2(d+ex)^4\sqrt{f+gx}\sqrt{cx^2+bx+a}}{11e}-$$

$$\frac{2e^2g\left(\left(29e^2f^2-96degf+81d^2g^2\right)c^2+eg(19bef-33bdg-18aeg)c+8b^2e^2g^2\right)(f+gx)^{5/2}\sqrt{cx^2+bx+a}}{7c}-\int\frac{e\left(\left(233e^3f^3-843de^2gf^2+1107d^2eg^2f-567d^3g^3\right)c^3-eg\left(2a\right.\right.}{}$$

$\downarrow$ 2184

$$\frac{2(d+ex)^4\sqrt{f+gx}\sqrt{cx^2+bx+a}}{11e}-$$

$$\frac{2e^2g\left(\left(29e^2f^2-96degf+81d^2g^2\right)c^2+eg(19bef-33bdg-18aeg)c+8b^2e^2g^2\right)(f+gx)^{5/2}\sqrt{cx^2+bx+a}}{7c}-\frac{2e\left(\left(233e^3f^3-843de^2gf^2+1107d^2eg^2f-567d^3g^3\right)c^3-eg\left(2a\right.\right.}{}$$

$\downarrow$ 27

$$\frac{2(d+ex)^4\sqrt{f+gx}\sqrt{cx^2+bx+a}}{11e}-$$

$$\frac{2e^2g\left(\left(29e^2f^2-96degf+81d^2g^2\right)c^2+eg(19bef-33bdg-18aeg)c+8b^2e^2g^2\right)(f+gx)^{5/2}\sqrt{cx^2+bx+a}}{7c}-\frac{2eg^5\left(\left(233e^3f^3-843de^2gf^2+1107d^2eg^2f-567d^3g^3\right)c^3-eg\left(\right.\right.}{}$$

$\downarrow$ 2184

$$\frac{2(d+ex)^4\sqrt{f+gx}\sqrt{cx^2+bx+a}}{11e}-$$

$$\frac{2e^2g\left(\left(29e^2f^2-96degf+81d^2g^2\right)c^2+eg(19bef-33bdg-18aeg)c+8b^2e^2g^2\right)(f+gx)^{5/2}\sqrt{cx^2+bx+a}}{7c}-\frac{2eg^5\left(\left(233e^3f^3-843de^2gf^2+1107d^2eg^2f-567d^3g^3\right)c^3-eg\left(\right.\right.}{}$$

$\downarrow$ 27

$$\frac{2(d+ex)^4\sqrt{f+gx}\sqrt{cx^2+bx+a}}{11e}-$$

$$\frac{2e^2g\left(\left(29e^2f^2-96degf+81d^2g^2\right)c^2+eg(19bef-33bdg-18aeg)c+8b^2e^2g^2\right)(f+gx)^{5/2}\sqrt{cx^2+bx+a}}{7c}-\frac{2eg^5\left(\left(233e^3f^3-843de^2gf^2+1107d^2eg^2f-567d^3g^3\right)c^3-eg\left(\right.\right.}{}$$

$\downarrow$ 1269

3.886. $\int(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

$$\frac{2(d+ex)^4\sqrt{f+gx}\sqrt{cx^2+bx+a}}{11e}-$$

$$\frac{2e^2g\left(\left(29e^2f^2-96degf+81d^2g^2\right)c^2+eg(19bef-33bdg-18aeg)c+8b^2e^2g^2\right)(f+gx)^{5/2}\sqrt{cx^2+bx+a}}{7c}-\frac{2eg^5\left(\left(233e^3f^3-843de^2gf^2+1107d^2eg^2f-567d^3g^3\right)c^3-eg(\right.}{}$$

$\downarrow$ 1172

$$\frac{2(d+ex)^4\sqrt{f+gx}\sqrt{cx^2+bx+a}}{11e}-$$

$$\frac{2e^2g\left(\left(29e^2f^2-96degf+81d^2g^2\right)c^2+eg(19bef-33bdg-18aeg)c+8b^2e^2g^2\right)(f+gx)^{5/2}\sqrt{cx^2+bx+a}}{7c}-\frac{2eg^5\left(\left(233e^3f^3-843de^2gf^2+1107d^2eg^2f-567d^3g^3\right)c^3-eg(\right.}{}$$

$\downarrow$ 321

3.886. $\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

$$\frac{2(d+ex)^4\sqrt{f+gx}\sqrt{cx^2+bx+a}}{11e}-$$

$$\frac{\frac{2e^2g\left(\left(29e^2f^2-96degf+81d^2g^2\right)c^2+eg(19bef-33bdg-18aeg)c+8b^2e^2g^2\right)(f+gx)^{5/2}\sqrt{cx^2+bx+a}}{7c}-\frac{2eg^5\left(\left(233e^3f^3-843de^2gf^2+1107d^2eg^2f-567d^3g^3\right)c^3-eg\left(\right.}{}}{}$$

$\Big\downarrow$ 327

$$\frac{2(d+ex)^4\sqrt{f+gx}\sqrt{cx^2+bx+a}}{11e}-$$

$$\frac{\frac{2e^2g\left(\left(29e^2f^2-96degf+81d^2g^2\right)c^2+eg(19bef-33bdg-18aeg)c+8b^2e^2g^2\right)(f+gx)^{5/2}\sqrt{cx^2+bx+a}}{7c}-\frac{2eg^5\left(\left(233e^3f^3-843de^2gf^2+1107d^2eg^2f-567d^3g^3\right)c^3-eg\left(\right.}{}}{}$$

input
```
Int[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

3.886. $\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

output
```
(2*(d + e*x)^4*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(11*e) - ((-2*e^3*(c*e
*f - 3*c*d*g + b*e*g)*(f + g*x)^(7/2)*Sqrt[a + b*x + c*x^2])/(9*c*g^4) + (
(2*e^2*g*(8*b^2*e^2*g^2 + c*e*g*(19*b*e*f - 33*b*d*g - 18*a*e*g) + c^2*(29
*e^2*f^2 - 96*d*e*f*g + 81*d^2*g^2))*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^2]
)/(7*c) - ((2*e*g^5*(48*b^3*e^3*g^3 + b*c*e^2*g^2*(67*b*e*f - 198*b*d*g -
157*a*e*g) + c^3*(233*e^3*f^3 - 843*d*e^2*f^2*g + 1107*d^2*e*f*g^2 - 567*d
^3*g^3) - c^2*e*g*(2*a*e*g*(74*e*f - 231*d*g) - 3*b*(24*e^2*f^2 - 88*d*e*f
*g + 99*d^2*g^2)))*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c) - (3*((2*g
^8*(64*b^4*e^4*g^4 + 4*b^2*c*e^3*g^3*(7*b*e*f - 66*b*d*g - 69*a*e*g) + c^4
*(187*e^4*f^4 - 732*d*e^3*f^3*g + 1098*d^2*e^2*f^2*g^2 - 798*d^3*e*f*g^3 +
 315*d^4*g^4) + 3*c^2*e^2*g^2*(50*a^2*e^2*g^2 - a*b*e*g*(29*e*f - 297*d*g)
 + 3*b^2*(e^2*f^2 - 11*d*e*f*g + 44*d^2*g^2)) - c^3*e*g*(6*a*e*g*(2*e^2*f^
2 - 33*d*e*f*g + 165*d^2*g^2) + b*(8*e^3*f^3 - 99*d^2*e*f*g^2 + 231*d^3*g^
3)))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c) - (e*g^8*((Sqrt[2]*Sqrt[b^
2 - 4*a*c]*(128*b^5*e^3*g^5 - 8*b^3*c*e^2*g^4*(7*b*e*f + 66*b*d*g + 87*a*e
*g) + 2*c^5*f^2*(64*e^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*
g^3) + b*c^2*e*g^3*(771*a^2*e^2*g^2 + 6*a*b*e*g*(43*e*f + 396*d*g) - b^2*(
37*e^2*f^2 - 264*d*e*f*g - 792*d^2*g^2)) - c^4*g*(b*f*(56*e^3*f^3 - 264*d*
e^2*f^2*g + 495*d^2*e*f*g^2 - 462*d^3*g^3) - 18*a*g*(6*e^3*f^3 - 33*d*e^2*
f^2*g + 88*d^2*e*f*g^2 + 77*d^3*g^3)) - c^3*g^2*(6*a^2*e^2*g^2*(26*e*f ...
```

3.886.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1272
```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/(e*(2*m + 5))), x] - Simp[1/(e*(2*m + 5))   Int[((d +
 e*x)^m/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f - 3*a*e*f + a*d*g
 + 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x - (c*e*f - 3*c*d*g + b*e*g)*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.886. $\int (d+ex)^3 \sqrt{f+gx} \sqrt{a+bx+cx^2} \, dx$

3.886.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3253 vs. $2(1469) = 2938$.

Time = 3.12 (sec) , antiderivative size = 3254, normalized size of antiderivative = 2.10

method	result	size
elliptic	Expression too large to display	3254
risch	Expression too large to display	11966
default	Expression too large to display	32647

input
```
int((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/11*e^3*
x^4*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/9*(b*e^3*g+3*c*d*e^2
*g+f*c*e^3-2/11*e^3*(5*b*g+5*c*f))/c/g*x^3*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+
b*f*x+a*f)^(1/2)+2/7*(a*e^3*g+3*b*d*e^2*g+b*e^3*f+3*c*d^2*e*g+3*c*d*e^2*f-
2/11*e^3*(9/2*a*g+9/2*b*f)-2/9*(b*e^3*g+3*c*d*e^2*g+f*c*e^3-2/11*e^3*(5*b*
g+5*c*f))/c/g*(4*b*g+4*c*f))/c/g*x^2*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+
a*f)^(1/2)+2/5*(3*a*e^2*g*d+3/11*a*e^3*f+3*b*d^2*e*g+3*b*e^2*f*d+c*d^3*g+3
*c*d^2*e*f-2/9*(b*e^3*g+3*c*d*e^2*g+f*c*e^3-2/11*e^3*(5*b*g+5*c*f))/c/g*(7
/2*a*g+7/2*b*f)-2/7*(a*e^3*g+3*b*d*e^2*g+b*e^3*f+3*c*d^2*e*g+3*c*d*e^2*f-2
/11*e^3*(9/2*a*g+9/2*b*f)-2/9*(b*e^3*g+3*c*d*e^2*g+f*c*e^3-2/11*e^3*(5*b*g
+5*c*f))/c/g*(4*b*g+4*c*f))/c/g*(3*b*g+3*c*f))/c/g*x*(c*g*x^3+b*g*x^2+c*f*
x^2+a*g*x+b*f*x+a*f)^(1/2)+2/3*(3*a*d^2*e*g+3*a*d*e^2*f+b*d^3*g+3*b*d^2*e*
f+c*d^3*f-2/3*(b*e^3*g+3*c*d*e^2*g+f*c*e^3-2/11*e^3*(5*b*g+5*c*f))/c/g*f*a
-2/7*(a*e^3*g+3*b*d*e^2*g+b*e^3*f+3*c*d^2*e*g+3*c*d*e^2*f-2/11*e^3*(9/2*a*
g+9/2*b*f)-2/9*(b*e^3*g+3*c*d*e^2*g+f*c*e^3-2/11*e^3*(5*b*g+5*c*f))/c/g*(4
*b*g+4*c*f))/c/g*(5/2*a*g+5/2*b*f)-2/5*(3*a*e^2*g*d+3/11*a*e^3*f+3*b*d^2*e
*g+3*b*e^2*f*d+c*d^3*g+3*c*d^2*e*f-2/9*(b*e^3*g+3*c*d*e^2*g+f*c*e^3-2/11*e
^3*(5*b*g+5*c*f))/c/g*(7/2*a*g+7/2*b*f)-2/7*(a*e^3*g+3*b*d*e^2*g+b*e^3*f+3
*c*d^2*e*g+3*c*d*e^2*f-2/11*e^3*(9/2*a*g+9/2*b*f)-2/9*(b*e^3*g+3*c*d*e^2*g
+f*c*e^3-2/11*e^3*(5*b*g+5*c*f))/c/g*(4*b*g+4*c*f))/c/g*(3*b*g+3*c*f))/...
```

3.886.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1741, normalized size of antiderivative = 1.12

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
-2/10395*((128*c^6*e^3*f^6 - 24*(22*c^6*d*e^2 + 5*b*c^5*e^3)*f^5*g + 3*(26
4*c^6*d^2*e + 176*b*c^5*d*e^2 - (11*b^2*c^4 - 68*a*c^5)*e^3)*f^4*g^2 - (46
2*c^6*d^3 + 891*b*c^5*d^2*e - 165*(b^2*c^4 - 6*a*c^5)*d*e^2 + (20*b^3*c^3
- 87*a*b*c^4)*e^3)*f^3*g^3 + 3*(231*b*c^5*d^3 - 66*(2*b^2*c^4 - 11*a*c^5)*
d^2*e + 11*(5*b^3*c^3 - 21*a*b*c^4)*d*e^2 - (11*b^4*c^2 - 53*a*b^2*c^3 + 3
4*a^2*c^4)*e^3)*f^2*g^4 + 3*(231*(b^2*c^4 - 6*a*c^5)*d^3 - 33*(9*b^3*c^3 -
 41*a*b*c^4)*d^2*e + 22*(8*b^4*c^2 - 42*a*b^2*c^3 + 33*a^2*c^4)*d*e^2 - (4
0*b^5*c - 246*a*b^3*c^2 + 329*a^2*b*c^3)*e^3)*f*g^5 - (231*(2*b^3*c^3 - 9*
a*b*c^4)*d^3 - 99*(8*b^4*c^2 - 41*a*b^2*c^3 + 30*a^2*c^4)*d^2*e + 33*(16*b
^5*c - 96*a*b^3*c^2 + 123*a^2*b*c^3)*d*e^2 - (128*b^6 - 888*a*b^4*c + 1599
*a^2*b^2*c^2 - 450*a^3*c^3)*e^3)*g^6)*sqrt(c*g)*weierstrassPInverse(4/3*(c
^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^
2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/
3*(3*c*g*x + c*f + b*g)/(c*g)) + 3*(128*c^6*e^3*f^5*g - 8*(66*c^6*d*e^2 +
7*b*c^5*e^3)*f^4*g^2 + (792*c^6*d^2*e + 264*b*c^5*d*e^2 - (37*b^2*c^4 - 10
8*a*c^5)*e^3)*f^3*g^3 - (462*c^6*d^3 + 495*b*c^5*d^2*e - 198*(b^2*c^4 - 3*
a*c^5)*d*e^2 + (37*b^3*c^3 - 135*a*b*c^4)*e^3)*f^2*g^4 + (462*b*c^5*d^3 -
99*(5*b^2*c^4 - 16*a*c^5)*d^2*e + 66*(4*b^3*c^3 - 15*a*b*c^4)*d*e^2 - 2*(2
8*b^4*c^2 - 129*a*b^2*c^3 + 78*a^2*c^4)*e^3)*f*g^5 - (462*(b^2*c^4 - 3*a*c
^5)*d^3 - 99*(8*b^3*c^3 - 29*a*b*c^4)*d^2*e + 66*(8*b^4*c^2 - 36*a*b^2*...
```

3.886.6 Sympy [F]

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int (d+ex)^3\,\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$$

input
```
integrate((e*x+d)**3*(g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x)**3*sqrt(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

3.886.7 Maxima [F]

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(ex+d)^3\sqrt{gx+f}\,dx$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^3*sqrt(g*x + f), x)
```

3.886.8 Giac [F]

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(ex+d)^3\sqrt{gx+f}\,dx$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^3*sqrt(g*x + f), x)
```

3.886.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int \sqrt{f+g\,x}\,(d+e\,x)^3\,\sqrt{c\,x^2+b\,x+a}\,dx$$

input
```
int((f + g*x)^(1/2)*(d + e*x)^3*(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((f + g*x)^(1/2)*(d + e*x)^3*(a + b*x + c*x^2)^(1/2), x)
```

3.887 $\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

3.887.1 Optimal result

Integrand size = 31, antiderivative size = 1015

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$$
$$= \frac{2(8b^3e^3g^3+3bce^2g^2(bef-8bdg-9aeg)+c^3(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3)-3c^2eg^2(2ae(ef}{315c^3eg^3}$$
$$+\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9e}$$
$$-\frac{4(3b^2e^2g^2+ceg(4bef-9bdg-7aeg)+c^2(8e^2f^2-24defg+21d^2g^2))(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{315c^2g^3}$$
$$+\frac{2e(cef-3cdg+beg)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}{63cg^3}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(8b^4e^2g^4-4b^2ceg^3(bef+6bdg+9aeg)+c^4f^2(8e^2f^2-24defg+21d^2g^2)+3c^2g^2(7a^2e^2}{}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(cf^2-bfg+ag^2)(8b^3e^2g^3+3bceg^2(bef-8bdg-9aeg)-2c^3f(8e^2f^2-24defg+21d^2}{}$$

output
```
-4/315*(3*b^2*e^2*g^2+c*e*g*(-7*a*e*g-9*b*d*g+4*b*e*f)+c^2*(21*d^2*g^2-24*
d*e*f*g+8*e^2*f^2))*(g*x+f)^(3/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^3+2/63*e*(b*e*
g-3*c*d*g+c*e*f)*(g*x+f)^(5/2)*(c*x^2+b*x+a)^(1/2)/c/g^3+2/315*(8*b^3*e^3*
g^3+3*b*c*e^2*g^2*(-9*a*e*g-8*b*d*g+b*e*f)+c^3*(-35*d^3*g^3+63*d^2*e*f*g^2
-57*d*e^2*f^2*g+19*e^3*f^3)-3*c^2*e*g^2*(2*a*e*(-10*d*g+e*f)+b*d*(-7*d*g+2
*e*f)))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^3/e/g^3+2/9*(e*x+d)^3*(g*x+f)^
(1/2)*(c*x^2+b*x+a)^(1/2)/e-2/315*(8*b^4*e^2*g^4-4*b^2*c*e*g^3*(9*a*e*g+6*
b*d*g+b*e*f)+c^4*f^2*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2)+3*c^2*g^2*(7*a^2*e^
2*g^2+a*b*e*g*(29*d*g+5*e*f)-b^2*(-7*d^2*g^2-5*d*e*f*g+e^2*f^2))+c^3*g*(3*
a*g*(-21*d^2*g^2-16*d*e*f*g+3*e^2*f^2)-b*f*(21*d^2*g^2-15*d*e*f*g+4*e^2*f^
2)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)
*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))
*2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^
(1/2)/c^4/g^4/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2
))))^(1/2)-2/315*(a*g^2-b*f*g+c*f^2)*(8*b^3*e^2*g^3+3*b*c*e*g^2*(-9*a*e*g-
8*b*d*g+b*e*f)-2*c^3*f*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2)-3*c^2*g^2*(2*a*e*
(-10*d*g+e*f)+b*d*(-7*d*g+2*e*f)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(
b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a
)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2...
```

3.887.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.97 (sec) , antiderivative size = 15781, normalized size of antiderivative = 15.55

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

output
```
Result too large to show
```

3.887.3 Rubi [A] (verified)

Time = 3.39 (sec) , antiderivative size = 1047, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {1272, 2184, 27, 2184, 27, 2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$$

$$\Big\downarrow \ 1272$$

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9e}-\frac{\int \frac{(d+ex)^2\left(-\left((cef-3cdg+beg)x^2\right)+2(cdf-bef+bdg-aeg)x+bdf-3aef+adg\right)}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{9e}$$

$$\Big\downarrow \ 2184$$

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9e}-$$

$$2\int \frac{2eg^3\left(\left(8e^2f^2-24degf+21d^2g^2\right)c^2+eg(4bef-9bdg-7aeg)c+3b^2e^2g^2\right)x^3+g^2\left(bg^2(13bf+5ag)e^3-cg\left(4aeg(4ef+9dg)-3b\left(8e^2f^2-20degf+7d^2g^2\right)\right)e+c^2\left(11e^3\right.\right.}{}$$

$$\Big\downarrow \ 27$$

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9e}-$$

$$\int \frac{2eg^3\left(\left(8e^2f^2-24degf+21d^2g^2\right)c^2+eg(4bef-9bdg-7aeg)c+3b^2e^2g^2\right)x^3+g^2\left(bg^2(13bf+5ag)e^3-cg\left(4aeg(4ef+9dg)-3b\left(8e^2f^2-20degf+7d^2g^2\right)\right)e+c^2\left(11e^3f^3\right.\right.}{}$$

$$\Big\downarrow \ 2184$$

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9e}-$$

$$2\int -\frac{3\left(\left(19e^3f^3-57de^2gf^2+63d^2eg^2f-35d^3g^3\right)c^3-3eg^2(2ae(ef-10dg)+bd(2ef-7dg))c^2+3be^2g^2(bef-8bdg-9aeg)c+8b^3e^3g^3\right)x^2g^5+\left(6b^3f^2g^2e^3+3b^2fg\left(6aeg\right.\right.}{}$$

$$\Big\downarrow \ 27$$

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9e} -$$

$$\frac{4eg(f+gx)^{3/2}\sqrt{a+bx+cx^2}\left(ceg(-7aeg-9bdg+4bef)+3b^2e^2g^2+c^2\left(21d^2g^2-24defg+8e^2f^2\right)\right)}{5c} - \int \frac{3\left(\left(19e^3f^3-57de^2gf^2+63d^2eg^2f-35d^3g^3\right)c^3-3eg^2(2ae(ef-1\right.}{}$$

↓ 2184

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{cx^2+bx+a}}{9e} -$$

$$\frac{4eg\left(\left(8e^2f^2-24degf+21d^2g^2\right)c^2+eg(4bef-9bdg-7aeg)c+3b^2e^2g^2\right)(f+gx)^{3/2}\sqrt{cx^2+bx+a}}{5c} - \frac{2\left(\left(19e^3f^3-57de^2gf^2+63d^2eg^2f-35d^3g^3\right)c^3-3eg^2(2ae(ef-10dg\right.}{}$$

↓ 27

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{cx^2+bx+a}}{9e} -$$

$$\frac{4eg\left(\left(8e^2f^2-24degf+21d^2g^2\right)c^2+eg(4bef-9bdg-7aeg)c+3b^2e^2g^2\right)(f+gx)^{3/2}\sqrt{cx^2+bx+a}}{5c} - \frac{2g^4\left(\left(19e^3f^3-57de^2gf^2+63d^2eg^2f-35d^3g^3\right)c^3-3eg^2(2ae(ef-10\right.}{}$$

↓ 1269

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9e} -$$

$$\frac{4eg(f+gx)^{3/2}\sqrt{a+bx+cx^2}\left(ceg(-7aeg-9bdg+4bef)+3b^2e^2g^2+c^2\left(21d^2g^2-24defg+8e^2f^2\right)\right)}{5c} - \frac{2g^4\sqrt{f+gx}\sqrt{a+bx+cx^2}\left(-3c^2eg^2(2ae(ef-10dg)+bd(2ef-7dg))\right.}{}$$

↓ 1172

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{cx^2+bx+a}}{9e} -$$

$$\frac{4eg\left(\left(8e^2f^2-24degf+21d^2g^2\right)c^2+eg(4bef-9bdg-7aeg)c+3b^2e^2g^2\right)(f+gx)^{3/2}\sqrt{cx^2+bx+a}}{5c} - \frac{2g^4\left(\left(19e^3f^3-57de^2gf^2+63d^2eg^2f-35d^3g^3\right)c^3-3eg^2(2ae(ef-10\right.}{}$$

↓ 321

3.887. $\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{cx^2+bx+a}}{9e}-$$

$$\frac{\frac{4eg\left(\left(8e^2f^2-24degf+21d^2g^2\right)c^2+eg(4bef-9bdg-7aeg)c+3b^2e^2g^2\right)(f+gx)^{3/2}\sqrt{cx^2+bx+a}}{5c}-\frac{2g^4\left(\left(19e^3f^3-57de^2gf^2+63d^2eg^2f-35d^3g^3\right)c^3-3eg^2(2ae(ef-10\cdots}{\cdots}}{\cdots}$$

$$\downarrow 327$$

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{cx^2+bx+a}}{9e}-$$

$$\frac{\frac{4eg\left(\left(8e^2f^2-24degf+21d^2g^2\right)c^2+eg(4bef-9bdg-7aeg)c+3b^2e^2g^2\right)(f+gx)^{3/2}\sqrt{cx^2+bx+a}}{5c}-\frac{2g^4\left(\left(19e^3f^3-57de^2gf^2+63d^2eg^2f-35d^3g^3\right)c^3-3eg^2(2ae(ef-10\cdots}{\cdots}}{\cdots}$$

input
```
Int[(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

3.887. $\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

output
```
(2*(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(9*e) - ((-2*e^2*(c*e*
f - 3*c*d*g + b*e*g)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(7*c*g^3) + ((
4*e*g*(3*b^2*e^2*g^2 + c*e*g*(4*b*e*f - 9*b*d*g - 7*a*e*g) + c^2*(8*e^2*f^
2 - 24*d*e*f*g + 21*d^2*g^2))*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c)
 - ((2*g^4*(8*b^3*e^3*g^3 + 3*b*c*e^2*g^2*(b*e*f - 8*b*d*g - 9*a*e*g) + c^
3*(19*e^3*f^3 - 57*d*e^2*f^2*g + 63*d^2*e*f*g^2 - 35*d^3*g^3) - 3*c^2*e*g^
2*(2*a*e*(e*f - 10*d*g) + b*d*(2*e*f - 7*d*g)))*Sqrt[f + g*x]*Sqrt[a + b*x
 + c*x^2])/c - (e*g^4*((2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*b^4*e^2*g^4 - 4*b^2
*c*e*g^3*(b*e*f + 6*b*d*g + 9*a*e*g) + c^4*f^2*(8*e^2*f^2 - 24*d*e*f*g + 2
1*d^2*g^2) + 3*c^2*g^2*(7*a^2*e^2*g^2 + a*b*e*g*(5*e*f + 29*d*g) - b^2*(e^
2*f^2 - 5*d*e*f*g - 7*d^2*g^2)) + c^3*g*(3*a*g*(3*e^2*f^2 - 16*d*e*f*g - 2
1*d^2*g^2) - b*f*(4*e^2*f^2 - 15*d*e*f*g + 21*d^2*g^2)))*Sqrt[f + g*x]*Sqr
t[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g
)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*g*Sqrt[(c*(f + g*x))/(2*c*f - (
b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 -
4*a*c]*(c*f^2 - b*f*g + a*g^2)*(8*b^3*e^2*g^3 + 3*b*c*e*g^2*(b*e*f - 8*b*d
*g - 9*a*e*g) - 2*c^3*f*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2) - 3*c^2*g^2*
(2*a*e*(e*f - 10*d*g) + b*d*(2*e*f - 7*d*g)))*Sqrt[(c*(f + g*x))/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))...
```

3.887.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1272
```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/(e*(2*m + 5))), x] - Simp[1/(e*(2*m + 5))   Int[((d +
 e*x)^m/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f - 3*a*e*f + a*d*g
 + 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x - (c*e*f - 3*c*d*g + b*e*g)*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.887. $\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

3.887.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1935 vs. $2(939) = 1878$.

Time = 3.52 (sec) , antiderivative size = 1936, normalized size of antiderivative = 1.91

method	result	size
elliptic	Expression too large to display	1936
risch	Expression too large to display	7219
default	Expression too large to display	20224

input
```
int((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/9*e^2*x
^3*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/7*(b*e^2*g+2*c*d*e*g+
c*e^2*f-2/9*e^2*(4*b*g+4*c*f))/c/g*x^2*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*
x+a*f)^(1/2)+2/5*(a*e^2*g+2*b*d*e*g+b*e^2*f+c*d^2*g+2*c*d*e*f-2/9*e^2*(7/2
*a*g+7/2*b*f)-2/7*(b*e^2*g+2*c*d*e*g+c*e^2*f-2/9*e^2*(4*b*g+4*c*f))/c/g*(3
*b*g+3*c*f))/c/g*x*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/3*(2*
a*d*e*g+1/3*a*e^2*f+b*d^2*g+2*b*d*e*f+c*d^2*f-2/7*(b*e^2*g+2*c*d*e*g+c*e^2
*f-2/9*e^2*(4*b*g+4*c*f))/c/g*(5/2*a*g+5/2*b*f)-2/5*(a*e^2*g+2*b*d*e*g+b*e
^2*f+c*d^2*g+2*c*d*e*f-2/9*e^2*(7/2*a*g+7/2*b*f)-2/7*(b*e^2*g+2*c*d*e*g+c*
e^2*f-2/9*e^2*(4*b*g+4*c*f))/c/g*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(c*
g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2*(a*d^2*f-2/5*(a*e^2*g+2*b*d
*e*g+b*e^2*f+c*d^2*g+2*c*d*e*f-2/9*e^2*(7/2*a*g+7/2*b*f)-2/7*(b*e^2*g+2*c*
d*e*g+c*e^2*f-2/9*e^2*(4*b*g+4*c*f))/c/g*(3*b*g+3*c*f))/c/g*f*a-2/3*(2*a*d
*e*g+1/3*a*e^2*f+b*d^2*g+2*b*d*e*f+c*d^2*f-2/7*(b*e^2*g+2*c*d*e*g+c*e^2*f-
2/9*e^2*(4*b*g+4*c*f))/c/g*(5/2*a*g+5/2*b*f)-2/5*(a*e^2*g+2*b*d*e*g+b*e^2*
f+c*d^2*g+2*c*d*e*f-2/9*e^2*(7/2*a*g+7/2*b*f)-2/7*(b*e^2*g+2*c*d*e*g+c*e^2
*f-2/9*e^2*(4*b*g+4*c*f))/c/g*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(1/2*a
*g+1/2*b*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a
*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-
b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1...
```

3.887.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 1132, normalized size of antiderivative = 1.12

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
2/945*((16*c^5*e^2*f^5 - 16*(3*c^5*d*e + b*c^4*e^2)*f^4*g + (42*c^5*d^2 +
54*b*c^4*d*e - 5*(b^2*c^3 - 6*a*c^4)*e^2)*f^3*g^2 - (63*b*c^4*d^2 - 12*(2*
b^2*c^3 - 11*a*c^4)*d*e + (5*b^3*c^2 - 21*a*b*c^3)*e^2)*f^2*g^3 - (63*(b^2
*c^3 - 6*a*c^4)*d^2 - 6*(9*b^3*c^2 - 41*a*b*c^3)*d*e + 2*(8*b^4*c - 42*a*b
^2*c^2 + 33*a^2*c^3)*e^2)*f*g^4 + (21*(2*b^3*c^2 - 9*a*b*c^3)*d^2 - 6*(8*b
^4*c - 41*a*b^2*c^2 + 30*a^2*c^3)*d*e + (16*b^5 - 96*a*b^3*c + 123*a^2*b*c
^2)*e^2)*g^5)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2
- 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a
*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/
(c*g)) + 6*(8*c^5*e^2*f^4*g - 4*(6*c^5*d*e + b*c^4*e^2)*f^3*g^2 + 3*(7*c^5
*d^2 + 5*b*c^4*d*e - (b^2*c^3 - 3*a*c^4)*e^2)*f^2*g^3 - (21*b*c^4*d^2 - 3*
(5*b^2*c^3 - 16*a*c^4)*d*e + (4*b^3*c^2 - 15*a*b*c^3)*e^2)*f*g^4 + (21*(b^
2*c^3 - 3*a*c^4)*d^2 - 3*(8*b^3*c^2 - 29*a*b*c^3)*d*e + (8*b^4*c - 36*a*b^
2*c^2 + 21*a^2*c^3)*e^2)*g^5)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*c
*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*
(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPIn
verse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*
f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(
c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g))) + 3*(35*c^5*e^2*g^5*x^3 + 8*c^
5*e^2*f^3*g^2 - 3*(8*c^5*d*e + b*c^4*e^2)*f^2*g^3 + (21*c^5*d^2 + 12*b*...
```

3.887.6 Sympy [F]

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx=\int (d+ex)^2\,\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$$

input
```
integrate((e*x+d)**2*(g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x)**2*sqrt(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

3.887.7 Maxima [F]

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(ex+d)^2\sqrt{gx+f}\,dx$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^2*sqrt(g*x + f), x)
```

3.887.8 Giac [F]

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(ex+d)^2\sqrt{gx+f}\,dx$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^2*sqrt(g*x + f), x)
```

3.887.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int \sqrt{f+g\,x}\,(d+e\,x)^2\,\sqrt{c\,x^2+b\,x+a}\,dx$$

input
```
int((f + g*x)^(1/2)*(d + e*x)^2*(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((f + g*x)^(1/2)*(d + e*x)^2*(a + b*x + c*x^2)^(1/2), x)
```

3.888 $\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

3.888.1 Optimal result

Integrand size = 29, antiderivative size = 652

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx =$$

$$-\frac{2\sqrt{f+gx}(4b^2eg^2+c^2f(4ef-7dg)-cg(2bef+7bdg-5aeg)-3cg(cef+7cdg-4beg)x)\sqrt{a+bx+}}{105c^2g^2}$$

$$+\frac{2e\sqrt{f+gx}(a+bx+cx^2)^{3/2}}{7c}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}((cef+7cdg-4beg)(8c^2f^2-2b^2g^2-3cg(bf-2ag))-5cg(2cf-bg)(7cdf-e(3bf+a}{105c^3g^3\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(cf^2-bfg+ag^2)(4b^2eg^2-2c^2f(4ef-7dg)+cg(bef-7bdg-10aeg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-}\right.}}}{105c^3g^3\sqrt{f+gx}\sqrt{a+bx+}}$$

output
```
2/7*e*(c*x^2+b*x+a)^(3/2)*(g*x+f)^(1/2)/c-2/105*(4*b^2*e*g^2+c^2*f*(-7*d*g
+4*e*f)-c*g*(-5*a*e*g+7*b*d*g+2*b*e*f)-3*c*g*(-4*b*e*g+7*c*d*g+c*e*f)*x)*(
g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^2+1/105*((-4*b*e*g+7*c*d*g+c*e*f)*(
8*c^2*f^2-2*b^2*g^2-3*c*g*(-2*a*g+b*f))-5*c*g*(-b*g+2*c*f)*(7*c*d*f-e*(a*g
+3*b*f)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^
(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(
1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b
^2))^(1/2)/c^3/g^3/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)
^(1/2))))^(1/2)+2/105*(a*g^2-b*f*g+c*f^2)*(4*b^2*e*g^2-2*c^2*f*(-7*d*g+4*e
*f)+c*g*(-10*a*e*g-7*b*d*g+b*e*f))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(
b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a
)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c
^3/g^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.888.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.21 (sec) , antiderivative size = 8432, normalized size of antiderivative = 12.93

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

output
```
Result too large to show
```

3.888.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1236, 27, 1231, 27, 25, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{2\int \frac{(7cdf-3bef-aeg+(cef+7cdg-4beg)x)\sqrt{cx^2+bx+a}}{2\sqrt{f+gx}}dx}{7c}+\frac{2e\sqrt{f+gx}\left(a+bx+cx^2\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(7cdf-3bef-aeg+(cef+7cdg-4beg)x)\sqrt{cx^2+bx+a}}{\sqrt{f+gx}}dx}{7c}+\frac{2e\sqrt{f+gx}\left(a+bx+cx^2\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{1231}$$

$$-\frac{\frac{2\int \frac{5cg(bf-2ag)(7cdf-e(3bf+ag))-2(cef+7cdg-4beg)\left(\frac{1}{2}bf(4cf-bg)-\frac{1}{2}ag(2cf+bg)\right)-\left((cef+7cdg-4beg)\left(8c^2f^2-2b^2g^2-3cg(bf-2ag)\right)-5cg(2cf-bg)(7cdf-\right.}{2\sqrt{f+gx}\sqrt{cx^2+bx+a}}}{15cg^2}}{7c}$$

$$\frac{2e\sqrt{f+gx}\left(a+bx+cx^2\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int -\frac{(cef+7cdg-4beg)\left(-fgb^2+4cf^2b-ag^2b-2acfg\right)-5cg(bf-2ag)(7cdf-e(3bf+ag))+\left((cef+7cdg-4beg)\left(8c^2f^2-2b^2g^2-3cg(bf-2ag)\right)-5cg(2cf-bg)(7cdf-e\right.}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}}{15cg^2}}{7c}$$

$$\frac{2e\sqrt{f+gx}\left(a+bx+cx^2\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{(cef+7cdg-4beg)\left(-fgb^2+4cf^2b-ag^2b-2acfg\right)-5cg(bf-2ag)(7cdf-e(3bf+ag))+\left((cef+7cdg-4beg)\left(8c^2f^2-2b^2g^2-3cg(bf-2ag)\right)-5cg(2cf-bg)(7cdf-e(3bf\right.}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}}{15cg^2}}{7c}$$

$$\frac{2e\sqrt{f+gx}\left(a+bx+cx^2\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\frac{\frac{\left(ag^2-bfg+cf^2\right)\left(cg(-10aeg-7bdg+bef)+4b^2eg^2-2c^2f(4ef-7dg)\right)\int \frac{1}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{g}+\frac{\left(\left(-3cg(bf-2ag)-2b^2g^2+8c^2f^2\right)(-4beg+7cdg+cef)-5cg(2cf-b\right.}{g}}{15cg^2}}{7}$$

$$\frac{2e\sqrt{f+gx}\left(a+bx+cx^2\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{1172}$$

3.888. $\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(cg(-10aeg-7bdg+bef)+4b^2eg^2-2c^2f(4ef-7dg)\right)\int\frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{g\left(b+2cx\right.}{2cf-\left(b-\right.}}}}{cg\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$\frac{2e\sqrt{f+gx}\left(a+bx+cx^2\right)^{3/2}}{7c}$$

$\downarrow$ 321

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(\left(-3cg(bf-2ag)-2b^2g^2+8c^2f^2\right)(-4beg+7cdg+cef)-5cg(2cf-bg)(7cdf-e(ag+3bf))\right)\int\frac{\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\sqrt{\frac{b-}{}}}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$\frac{2e\sqrt{f+gx}\left(a+bx+cx^2\right)^{3/2}}{7c}$$

$\downarrow$ 327

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(cg(-10aeg-7bdg+bef)+4b^2eg^2-2c^2f(4ef-7dg)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{cg\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$\frac{2e\sqrt{f+gx}\left(a+bx+cx^2\right)^{3/2}}{7c}$$

input
```
Int[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

3.888. $\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

output
```
(2*e*Sqrt[f + g*x]*(a + b*x + c*x^2)^(3/2))/(7*c) + ((-2*Sqrt[f + g*x]*(4*
b^2*e*g^2 - c*g*(2*b*e*f + 7*b*d*g - 5*a*e*g) + 2*c^2*(2*e*f^2 - (7*d*f*g)
/2) - 3*c*g*(c*e*f + 7*c*d*g - 4*b*e*g)*x)*Sqrt[a + b*x + c*x^2])/(15*c*g^
2) + ((Sqrt[2]*Sqrt[b^2 - 4*a*c]*((c*e*f + 7*c*d*g - 4*b*e*g)*(8*c^2*f^2 -
 2*b^2*g^2 - 3*c*g*(b*f - 2*a*g)) - 5*c*g*(2*c*f - b*g)*(7*c*d*f - e*(3*b*
f + a*g)))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elli
pticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[
2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*g*S
qrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^
2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(4*b^2*e*g^2 -
2*c^2*f*(4*e*f - 7*d*g) + c*g*(b*e*f - 7*b*d*g - 10*a*e*g))*Sqrt[(c*(f + g
*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*
a*c])*g)])/(c*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))/(15*c*g^2))/(7*c)
```

3.888.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.888. $\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.888. $\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

3.888.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1228 vs. 2(588) = 1176.

Time = 1.95 (sec) , antiderivative size = 1229, normalized size of antiderivative = 1.88

method	result	size
elliptic	Expression too large to display	1229
risch	Expression too large to display	3893
default	Expression too large to display	10711

input
```
int((e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/7*e*x^2
*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/5*(b*e*g+c*d*g+c*e*f-2/
7*e*(3*b*g+3*c*f))/c/g*x*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2
/3*(a*e*g+b*d*g+b*e*f+c*d*f-2/7*e*(5/2*a*g+5/2*b*f)-2/5*(b*e*g+c*d*g+c*e*f
-2/7*e*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*
x+b*f*x+a*f)^(1/2)+2*(a*d*f-2/5*(b*e*g+c*d*g+c*e*f-2/7*e*(3*b*g+3*c*f))/c/
g*f*a-2/3*(a*e*g+b*d*g+b*e*f+c*d*f-2/7*e*(5/2*a*g+5/2*b*f)-2/5*(b*e*g+c*d*
g+c*e*f-2/7*e*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(1/2*a*g+1/2*b*f))*(f/
g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c
))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(
1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^
(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF
(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+
b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(a*d*g+3/7*a
*e*f+b*d*f-2/5*(b*e*g+c*d*g+c*e*f-2/7*e*(3*b*g+3*c*f))/c/g*(3/2*a*g+3/2*b*
f)-2/3*(a*e*g+b*d*g+b*e*f+c*d*f-2/7*e*(5/2*a*g+5/2*b*f)-2/5*(b*e*g+c*d*g+c
*e*f-2/7*e*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(b*g+c*f))*(f/g-1/2*(b+(-
4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((
x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/
2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c...
```

3.888.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 726, normalized size of antiderivative = 1.11

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx =$$

$$-\frac{2\left((8c^4ef^4-(14c^4d+9bc^3e)f^3g+(21bc^3d-2(2b^2c^2-11ac^3)e)f^2g^2+(21(b^2c^2-6ac^3)d-(9b^3c\right.}{}$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/315*((8*c^4*e*f^4 - (14*c^4*d + 9*b*c^3*e)*f^3*g + (21*b*c^3*d - 2*(2*b
^2*c^2 - 11*a*c^3)*e)*f^2*g^2 + (21*(b^2*c^2 - 6*a*c^3)*d - (9*b^3*c - 41*
a*b*c^2)*e)*f*g^3 - (7*(2*b^3*c - 9*a*b*c^2)*d - (8*b^4 - 41*a*b^2*c + 30*
a^2*c^2)*e)*g^4)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b
^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c -
6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*
g)/(c*g)) + 3*(8*c^4*e*f^3*g - (14*c^4*d + 5*b*c^3*e)*f^2*g^2 + (14*b*c^3*
d - (5*b^2*c^2 - 16*a*c^3)*e)*f*g^3 - (14*(b^2*c^2 - 3*a*c^3)*d - (8*b^3*c
 - 29*a*b*c^2)*e)*g^4)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g +
(b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c
- 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4
/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3
*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3
), 1/3*(3*c*g*x + c*f + b*g)/(c*g))) - 3*(15*c^4*e*g^4*x^2 - 4*c^4*e*f^2*g
^2 + (7*c^4*d + 2*b*c^3*e)*f*g^3 + (7*b*c^3*d - 2*(2*b^2*c^2 - 5*a*c^3)*e)
*g^4 + 3*(c^4*e*f*g^3 + (7*c^4*d + b*c^3*e)*g^4)*x)*sqrt(c*x^2 + b*x + a)*
sqrt(g*x + f))/(c^4*g^4)
```

3.888.6 Sympy [F]

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int (d+ex)\,\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$$

input
```
integrate((e*x+d)*(g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x)*sqrt(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

3.888.7 Maxima [F]

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(ex+d)\sqrt{gx+f}\,dx$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f), x)
```

3.888.8 Giac [F]

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(ex+d)\sqrt{gx+f}\,dx$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f), x)
```

3.888.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int \sqrt{f+g\,x}\,(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}\,dx$$

input
```
int((f + g*x)^(1/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((f + g*x)^(1/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2), x)
```

3.889 $\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

3.889.1 Optimal result

Integrand size = 24, antiderivative size = 513

$$\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$$

$$= -\frac{2(2cf-bg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15cg}+\frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\left(c^2f^2+b^2g^2-cg(bf+3ag)\right)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\Bigg(\arcsin\Bigg(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\Bigg)|-\frac{\cdots}{2cf\cdots}}{15c^2g^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(2cf-bg)\left(cf^2-bfg+ag^2\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\Bigg(\arcsin\Bigg(\sqrt{b+\cdots}}{15c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

output
```
2/5*(g*x+f)^(3/2)*(c*x^2+b*x+a)^(1/2)/g-2/15*(-b*g+2*c*f)*(g*x+f)^(1/2)*(c
*x^2+b*x+a)^(1/2)/c/g-2/15*(c^2*f^2+b^2*g^2-c*g*(3*a*g+b*f))*EllipticE(1/2
*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4
*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b
^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^2/g^2/(c*x
^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/15*(-
b*g+2*c*f)*(a*g^2-b*f*g+c*f^2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-
4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-
4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^2/g
^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.889.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.55 (sec) , antiderivative size = 1052, normalized size of antiderivative = 2.05

$$\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2(a+x(b+cx))(bg+c(f+3gx))}{cg}+\frac{(f+gx)\left(-\frac{4g^2\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}\left(c^2f^2+b^2g^2-cg(bf+3ag)\right)(a+x(b+cx))}{(f+gx)^2}+\frac{i\sqrt{2}\left(2cf-bg+\sqrt{\cdots}\right.}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}$$

input
```
Integrate[Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

output
```
(Sqrt[f + g*x]*((2*(a + x*(b + c*x))*(b*g + c*(f + 3*g*x)))/(c*g) + ((f +
g*x)*((-4*g^2*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 -
4*a*c)*g^2])]*(c^2*f^2 + b^2*g^2 - c*g*(b*f + 3*a*g))*(a + x*(b + c*x)))/(
f + g*x)^2 + (I*Sqrt[2]*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(c^2*f^2 +
 b^2*g^2 - c*g*(b*f + 3*a*g))*Sqrt[(-2*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2] +
 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(f - g*x))/((2*c*f - b*g +
Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*
g^2] - 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(-f + g*x))/((-2*c*f
+ b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*
Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sq
rt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sq
rt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x] + (I*Sqrt[2]*(b^3*g^3 - b^2*g^2*(2
*c*f + Sqrt[(b^2 - 4*a*c)*g^2]) + b*c*g*(-4*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g
^2]) + c*(-(c*f^2*Sqrt[(b^2 - 4*a*c)*g^2]) + a*g^2*(8*c*f + 3*Sqrt[(b^2 -
4*a*c)*g^2])))*Sqrt[(-2*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2] + 2*c*f*g*x + g*
Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(f - g*x))/((2*c*f - b*g + Sqrt[(b^2 - 4*a
*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2] - 2*c*f*g*
x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(-f + g*x))/((-2*c*f + b*g + Sqrt[(b
^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b
*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]]...
```

3.889.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1162, 1236, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$$

$$\Big\downarrow \; 1162$$

$$\frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g}-\frac{\int \frac{\sqrt{f+gx}(bf-2ag+(2cf-bg)x)}{\sqrt{cx^2+bx+a}}dx}{5g}$$

$$\Big\downarrow \; 1236$$

$$\frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g}-\frac{\frac{2\int \frac{fgb^2+cf^2b+ag^2b-8acfg+2\left(c^2f^2+b^2g^2-cg(bf+3ag)\right)x}{2\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{3c}+\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}(2cf-bg)}{3c}}{5g}$$

$\Big\downarrow$ 27

$$\frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g}-\frac{\frac{\int \frac{fgb^2+cf^2b+ag^2b-8acfg+2\left(c^2f^2+b^2g^2-cg(bf+3ag)\right)x}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{3c}+\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}(2cf-bg)}{3c}}{5g}$$

$\Big\downarrow$ 1269

$$\frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g}-\frac{\frac{\frac{2\left(-cg(3ag+bf)+b^2g^2+c^2f^2\right)\int \frac{\sqrt{f+gx}}{\sqrt{cx^2+bx+a}}dx}{g}-\frac{(2cf-bg)\left(ag^2-bfg+cf^2\right)\int \frac{1}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{g}}{3c}+\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}(2cf-bg)}{3c}}{5g}$$

$\Big\downarrow$ 1172

$$\frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g}-\frac{\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-cg(3ag+bf)+b^2g^2+c^2f^2\right)\int \frac{\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(2cf-\cdots}{\cdots}}{3c}\cdots}{5g\cdots}$$

$\Big\downarrow$ 321

$$\frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g}-\frac{\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-cg(3ag+bf)+b^2g^2+c^2f^2\right)\int \frac{\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(2cf-\cdots}{\cdots}}{3c}\cdots}{5g}$$

$\Big\downarrow$ 327

3.889. $\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

$$\frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} -$$

$$\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-cg(3ag+bf)+b^2g^2+c^2f^2\right)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}} - \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx\right.}{b^2-4ac}}}{}}{3c}$$

$$5g$$

input
```
Int[Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*g) - ((2*(2*c*f - b*g)*Sqrt[f
 + g*x]*Sqrt[a + b*x + c*x^2])/(3*c) + ((2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c^2*
f^2 + b^2*g^2 - c*g*(b*f + 3*a*g))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^
2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/
Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)])/(c*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])
*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*f - b*g)*(
c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])
*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 -
4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*g*Sqrt[f + g*x]*Sqrt[a
+ b*x + c*x^2]))/(3*c))/(5*g)
```

3.889.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.889. $\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

3.889.4 Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 892, normalized size of antiderivative = 1.74

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+bx+a)}\left(\frac{2x\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{5}+\frac{2\left(\frac{bg}{5}+\frac{cf}{5}\right)\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{3cg}+\frac{2\left(\frac{3fa}{5}-\frac{2\left(\frac{bg}{5}+\frac{cf}{5}\right)\left(\frac{ag}{2}\right.}{3cg}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/5*x*(c*
g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/3*(1/5*b*g+1/5*c*f)/c/g*(c*
g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2*(3/5*f*a-2/3*(1/5*b*g+1/5*c
*f)/c/g*(1/2*a*g+1/2*b*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/
g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/
(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))
/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*
g*x+b*f*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^
(1/2))))^(1/2))+2*(2/5*a*g+2/5*b*f-2/3*(1/5*b*g+1/5*c*f)/c/g*(b*g+c*f))*(f
/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/
c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^
(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)
^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1
/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^
(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*
a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/
(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2)
)/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

3.889.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 481, normalized size of antiderivative = 0.94

$$\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$$
$$= \frac{2\left((2c^3f^3-3bc^2f^2g-3(b^2c-6ac^2)fg^2+(2b^3-9abc)g^3)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4(c^2f^2-bcfg+(b^2-3ac)}{3c^2g^2}\right.\right.}{}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/45*((2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*
a*b*c)*g^3)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 -
3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c
^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c
*g)) + 6*(c^3*f^2*g - b*c^2*f*g^2 + (b^2*c - 3*a*c^2)*g^3)*sqrt(c*g)*weier
strassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2
*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g
^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*
g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g
^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g))) +
 3*(3*c^3*g^3*x + c^3*f*g^2 + b*c^2*g^3)*sqrt(c*x^2 + b*x + a)*sqrt(g*x +
f))/(c^3*g^3)
```

3.889.6 Sympy [F]

$$\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$$

input
```
integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

3.889.7 Maxima [F]

$$\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}\sqrt{gx+f}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f), x)
```

3.889.8 Giac [F]

$$\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}\sqrt{gx+f}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f), x)
```

3.889.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int \sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}\,dx$$

input
```
int((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2), x)
```

3.890 $\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx$

3.890.1 Optimal result

Integrand size = 31, antiderivative size = 764

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e}$$

$$+ \frac{\sqrt{2}\sqrt{b^2-4ac}(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)}\right.}{3ce^2g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+ \frac{2\sqrt{2}\sqrt{b^2-4ac}(eg(bef-3bdg+2aeg)+c(-e^2f^2+3d^2g^2))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{\frac{c(a+x(b+cx))}{-b^2+4ac}}\text{EllipticF}}{3ce^3g\sqrt{f+gx}\sqrt{a+x(b+cx)}}$$

$$- \frac{\sqrt{2}(cd^2-bde+ae^2)\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\text{EllipticP}}{\sqrt{c}e^3\sqrt{a+bx+cx^2}}$$

output
```
2/3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/e+1/3*(b*e*g-3*c*d*g+c*e*f)*Elliptic
E(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*
g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*
a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c/e^2/g
/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/
3*(e*g*(2*a*e*g-3*b*d*g+b*e*f)+c*(3*d^2*g^2-e^2*f^2))*EllipticF(1/2*((b+2*
c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*(g*(-4*a
*c+b^2)^(1/2)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^
2)^(1/2)*(c*(a+x*(c*x+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*
c+b^2)^(1/2))))^(1/2)/c/e^3/g/(g*x+f)^(1/2)/(a+x*(c*x+b))^(1/2)-(a*e^2-b*d
*e+c*d^2)*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2
)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*
c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(
1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+
b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/
e^3/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.890.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.44 (sec) , antiderivative size = 1470, normalized size of antiderivative = 1.92

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

output
```
(2*Sqrt[f + g*x]*Sqrt[a + x*(b + c*x)])/(3*e) + ((f + g*x)^(3/2)*Sqrt[a +
x*(b + c*x)]*(4*e*(c*e*f - 3*c*d*g + b*e*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g)
)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*(c*(-1 + f/(f + g*x))^2 + (g*(
b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*x)) - (I*Sqrt[2]*e*(c*e*f -
 3*c*d*g + b*e*g)*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(Sqrt[(b^2
- 4*a*c)*g^2] - (2*a*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f + g*x)) + b*g*(-1 +
 (2*f)/(f + g*x)))/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^
2 - 4*a*c)*g^2] + (2*a*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f + g*x)) + b*(g -
(2*f*g)/(f + g*x)))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*EllipticE[I*
ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 -
4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/
(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x] + (I*Sqrt[2]*e*(-
6*c^2*d*f*g + b*e*g*(-(b*g) + Sqrt[(b^2 - 4*a*c)*g^2]) + c*(-2*a*e*g^2 + S
qrt[(b^2 - 4*a*c)*g^2]*(e*f - 3*d*g) + 3*b*g*(e*f + d*g)))*Sqrt[(Sqrt[(b^2
 - 4*a*c)*g^2] - (2*a*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f + g*x)) + b*g*(-1
+ (2*f)/(f + g*x)))/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b
^2 - 4*a*c)*g^2] + (2*a*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f + g*x)) + b*(g -
 (2*f*g)/(f + g*x)))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*EllipticF[I
*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 -
 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^...
```

3.890.3 Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 890, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {1272, 2154, 1269, 1172, 321, 327, 1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}dx$$

$$\Big\downarrow \text{ 1272}$$

$$\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e}-\frac{\int \frac{-((cef-3cdg+beg)x^2)+2(cdf-bef+bdg-aeg)x+bdf-3aef+adg}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{3e}$$

$$\Big\downarrow \text{ 2154}$$

$$\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e}-\frac{\int \frac{-\frac{3cgd^2}{e^2}+\frac{3cfd}{e}+\frac{3bgd}{e}-2bf-2ag+\left(-cf-bg+\frac{3cdg}{e}\right)x}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx-\frac{3(ef-dg)\left(ae^2-bde+cd^2\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}}{3e}$$

$$\Big\downarrow \ 1269$$

$$\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e}-\frac{-\frac{\left(eg(2aeg-3bdg+bef)-c\left(e^2f^2-3d^2g^2\right)\right)\int \frac{1}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2g}-\frac{3(ef-dg)\left(ae^2-bde+cd^2\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}-\frac{\left(bg-\frac{3cdg}{e}+cf\right)\cdots}{\cdots}}{3e}$$

$$\Big\downarrow \ 1172$$

$$\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e}-\frac{-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(eg(2aeg-3bdg+bef)-c\left(e^2f^2-3d^2g^2\right)\right)\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}d\cdots}{ce^2g\sqrt{f+gx}\sqrt{a+bx+cx^2}}\cdots}{\cdots}$$

$$\Big\downarrow \ 321$$

$$\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e}-\frac{-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(bg-\frac{3cdg}{e}+cf\right)\int \frac{\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{3(ef-dg)\left(ae^2-bde+cd^2\right)\int \frac{\cdots}{(d+ex)\sqrt{f\cdots}}}{e^2}\cdots}{\cdots}$$

$$\Big\downarrow \ 327$$

$$\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e}-\frac{-\frac{3(ef-dg)\left(ae^2-bde+cd^2\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(eg(2aeg-3bdg+bef)-c\left(e^2f^2\cdots\right.\right.}{ce^2g\sqrt{f+gx}\sqrt{a+\cdots}}\cdots}{\cdots}$$

$$\Big\downarrow \ 1279$$

3.890. $\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx$

$$\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e}-$$

$$-\frac{3\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}(ef-dg)\left(ae^2-bde+cd^2\right)\int\frac{1}{\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}(d+ex)\sqrt{f+gx}}dx}{e^2\sqrt{a+bx+cx^2}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-}}{}$$

$$\downarrow \text{187}$$

$$\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e}-$$

$$\frac{6\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}(ef-dg)\left(ae^2-bde+cd^2\right)\int\frac{1}{(ef-dg-e(f+gx))\sqrt{b+\frac{2c(f+gx)}{g}-\sqrt{b^2-4ac}-\frac{2cf}{g}}\sqrt{b+\frac{2c(f+gx)}{g}+\sqrt{b^2-4ac}-\frac{2cf}{g}}}d}{e^2\sqrt{a+bx+cx^2}}$$

$$\downarrow \text{413}$$

$$\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e}-$$

$$\frac{6\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}(ef-dg)\left(ae^2-bde+cd^2\right)\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\int\frac{1}{(ef-dg-e(f+gx))\sqrt{b+\frac{2c(f+gx)}{g}+\sqrt{b^2-4ac}-\frac{2cf}{g}}\sqrt{1-}}}{e^2\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(f+gx)}{g}-\frac{2cf}{g}}}$$

$$\downarrow \text{413}$$

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+bx+a}}{3e}-$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\left(cf+bg-\frac{3cdg}{e}\right)\sqrt{f+gx}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}(eg(bef-3bdg}{}$$

$$\downarrow \text{412}$$

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+bx+a}}{3e}-$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\left(cf+bg-\frac{3cdg}{e}\right)\sqrt{f+gx}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}(eg(bef-3bdg}{}$$

3.890. $\int\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx$

input
```
Int[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

output
```
(2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*e) - (-((Sqrt[2]*Sqrt[b^2 - 4*a
*c]*(c*f + b*g - (3*c*d*g)/e)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(
b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[
b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 -
4*a*c])*g)])/(c*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*
Sqrt[a + b*x + c*x^2])) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e*g*(b*e*f - 3*b*d
*g + 2*a*e*g) - c*(e^2*f^2 - 3*d^2*g^2))*Sqrt[(c*(f + g*x))/(2*c*f - (b +
Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipt
icF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]
], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*e^2*g
*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (3*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)
*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[b - Sqrt[b^2 - 4*a*c] + 2*c*
x]*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (
b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2
 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*
f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b
^2 - 4*a*c])*g]], (2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)])/(Sqrt[c]*e^2*Sqrt[a + b*x + c*x^2]*Sqrt[b - Sqrt[b^2 - 4
*a*c] - (2*c*f)/g + (2*c*(f + g*x))/g]*Sqrt[b + Sqrt[b^2 - 4*a*c] - (2*c*f
)/g + (2*c*(f + g*x))/g]))/(3*e)
```

3.890.3.1 Defintions of rubi rules used

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1272
```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/(e*(2*m + 5))), x] - Simp[1/(e*(2*m + 5))   Int[((d +
 e*x)^m/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f - 3*a*e*f + a*d*g
 + 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x - (c*e*f - 3*c*d*g + b*e*g)*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

3.890. $\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx$

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

3.890.4 Maple [A] (verified)

Time = 3.88 (sec) , antiderivative size = 1245, normalized size of antiderivative = 1.63

method	result	size
elliptic	Expression too large to display	1245
risch	Expression too large to display	2338
default	Expression too large to display	6812

input
```
int((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/3/e*(c*
g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2*((a*e^2*g-b*d*e*g+b*e^2*f+c
*d^2*g-c*d*e*f)/e^3-2/3/e*(1/2*a*g+1/2*b*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2
))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4
*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(
-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+
b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*
a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c
*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(1/e^2*(b*e*g-c*d*g+c*e*f)-2/3/e*(b*g+
c*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)
^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a
*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*
a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*
((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a
*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*
(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF((
(x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^
2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))-2*(a*d*e^2*g-a*
e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)/e^4*(f/g-1/2*(b+(-4*a*c+b^2)^
(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*...
```

3.890.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.890.6 Sympy [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx=\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

input
```
integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

output
```
Integral(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

3.890.7 Maxima [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx=\int \frac{\sqrt{cx^2+bx+a}\sqrt{gx+f}}{ex+d}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d), x)
```

3.890.8 Giac [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx=\int \frac{\sqrt{cx^2+bx+a}\sqrt{gx+f}}{ex+d}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d), x)
```

3.890.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

input
```
int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x),x)
```

output
```
int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x), x)
```

3.891 $\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx$

3.891.1 Optimal result

Integrand size = 31, antiderivative size = 743

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{3\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} + \frac{\sqrt{2}\sqrt{b^2-4ac}(2beg-c(ef+3dg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{\frac{c(a+x(b+cx))}{-b^2+4ac}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{ce^3\sqrt{f+gx}\sqrt{a+x(b+cx)}} + \frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(}{2cf-(b+}}}{\sqrt{2}\sqrt{c}e^3(ef-dg)\sqrt{a+}}$$

output
```
-(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/e/(e*x+d)+3/2*EllipticE(1/2*((b+2*c*x+(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1
/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1
/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e^2*2^(1/2)/(c*x^2+b*x+a)^(1/2)/
(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+(2*b*e*g-c*(3*d*g+e*f))
*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),2^(1/2)*(g*(-4*a*c+b^2)^(1/2)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2
))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(c*(a+x*(c*x+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+
f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^3/(g*x+f)^(1/2)/(a+x*(c*x+b
))^(1/2)+1/2*(c*d*(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*EllipticPi(2^(1/
2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c
*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/
(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+
b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/
(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^3/(-d*g+e*f)*2^(1/2)/c^(1/2)/(c*
x^2+b*x+a)^(1/2)
```

3.891.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.83 (sec) , antiderivative size = 1473, normalized size of antiderivative = 1.98

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+x(b+cx)}}{e(d+ex)} - \frac{(f+gx)^{3/2}\sqrt{a+x(b+cx)}\left(-12e(-ef+dg)\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}\left(c\left(-1+\frac{f}{f+gx}\right)^2+\frac{g\left(b-\frac{bf}{f+gx}+\frac{ag}{f+gx}\right)}{f+gx}\right.\right.}{}$$

input
```
Integrate[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x)^2,x]
```

output
```
-((Sqrt[f + g*x]*Sqrt[a + x*(b + c*x)])/(e*(d + e*x))) - ((f + g*x)^(3/2)*
Sqrt[a + x*(b + c*x)]*(-12*e*(-(e*f) + d*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g)
)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*(c*(-1 + f/(f + g*x))^2 + (g*(
b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*x)) + ((3*I)*Sqrt[2]*e*(-(e
*f) + d*g)*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(Sqrt[(b^2 - 4*a*c
)*g^2] - (2*a*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f + g*x)) + b*g*(-1 + (2*f)/
(f + g*x)))/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a
*c)*g^2] + (2*a*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f + g*x)) + b*(g - (2*f*g)
/(f + g*x)))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*EllipticE[I*ArcSinh
[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*
g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f
- b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x] - (I*Sqrt[2]*e*(2*a*e*g^
2 - 3*e*f*Sqrt[(b^2 - 4*a*c)*g^2] + 3*d*g*Sqrt[(b^2 - 4*a*c)*g^2] + b*g*(e
*f - 3*d*g) + c*(-4*e*f^2 + 6*d*f*g))*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] - (2*a
*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f + g*x)) + b*g*(-1 + (2*f)/(f + g*x)))/(
2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] + (2
*a*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f + g*x)) + b*(g - (2*f*g)/(f + g*x)))/
(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqr
t[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[
f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sq...
```

3.891.3 Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 874, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {1271, 2154, 1269, 1172, 321, 327, 1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}dx$$

$$\Big\downarrow \text{1271}$$

$$\frac{\int \frac{3cgx^2+2(cf+bg)x+bf+ag}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2e}-\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)}$$

$$\Big\downarrow \text{2154}$$

$$\frac{\int \frac{\frac{2cf}{e}+\frac{2bg}{e}-\frac{3cdg}{e^2}+\frac{3cgx}{e}}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx-\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}}{2e}-\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)}$$

$$\Big\downarrow \; 1269$$

$$\frac{-\frac{(-2beg+3cdg+cef)\int \frac{1}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}-\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}+\frac{3c\int \frac{\sqrt{f+gx}}{\sqrt{cx^2+bx+a}}dx}{e}}{2e}-\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)}$$

$$\Big\downarrow \; 1172$$

$$\frac{-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(-2beg+3cdg+cef)\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\cdots}{\cdots}-\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)}$$

$$\Big\downarrow \; 321$$

$$\frac{\frac{3\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\int \frac{\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{e\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2\cdots}}}{e^2}\cdots}{\cdots}-\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)}$$

$$\Big\downarrow \; 327$$

$$\frac{-\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(-2beg+3cdg+cef)\cdots}{ce^2\sqrt{f+gx}\sqrt{a+b\cdots}}\cdots}{\cdots}-\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)}$$

$$\Big\downarrow \; 1279$$

3.891. $\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx$

$$-\frac{\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}(cd(2ef-3dg)-e(aeg-2bdg+bef))\int\frac{1}{\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}(d+ex)\sqrt{f+gx}}dx}{e^2\sqrt{a+bx+cx^2}}-\frac{2\sqrt{2}\sqrt{b^2}}{}$$

$$\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)}$$

$\downarrow$ 187

$$\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}(cd(2ef-3dg)-e(aeg-2bdg+bef))\int\frac{1}{(ef-dg-e(f+gx))\sqrt{b+\frac{2c(f+gx)}{g}-\sqrt{b^2-4ac}-\frac{2cf}{g}}\sqrt{b+\frac{2c(f+gx)}{g}+\sqrt{b^2}}}}{e^2\sqrt{a+bx+cx^2}}$$

$$\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)}$$

$\downarrow$ 413

$$\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}(cd(2ef-3dg)-e(aeg-2bdg+bef))\int\frac{1}{(ef-dg-e(f+gx))\sqrt{b+\frac{2c(f+gx)}{g}+\sqrt{b^2-4ac}}}}{e^2\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(f+gx)}{g}-\frac{2cf}{g}}}$$

$$\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)}$$

$\downarrow$ 413

$$\frac{3\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}(cef+3cdg-2beg)\sqrt{\frac{c(f+}{2cf-\left(b+\sqrt{}\right.}}}{}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{e(d+ex)}$$

$\downarrow$ 412

3.891. $\int\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx$

$$\frac{\frac{3\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}(cef+3cdg-2beg)\sqrt{\frac{c(f+\cdots}{2cf-\left(b+\sqrt{\cdots}\right.}}\cdots}{\cdots}}{\cdots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{e(d+ex)}$$

input

```
Int[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x)^2,x]
```

output

```
-((Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(e*(d + e*x))) + ((3*Sqrt[2]*Sqrt[
b^2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*El
lipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqr
t[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(e*S
qrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^
2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*e*f + 3*c*d*g - 2*b*e*g)*Sqrt[(c*(f
+ g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/
(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt
[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 -
 4*a*c])*g)])/(c*e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2]*Sqrt[
2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d
*g + a*e*g))*Sqrt[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[b + Sqrt[b^2 - 4*a*c
] + 2*c*x]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*S
qrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e
*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*S
qrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (2*c*f - (
b - Sqrt[b^2 - 4*a*c])*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[c]*e
^2*(e*f - d*g)*Sqrt[a + b*x + c*x^2]*Sqrt[b - Sqrt[b^2 - 4*a*c] - (2*c*f)/
g + (2*c*(f + g*x))/g]*Sqrt[b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g + (2*c*(f +
g*x))/g]))/(2*e)
```

3.891.3.1 Defintions of rubi rules used

rule 187

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.891. $\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2} \, dx$

rule 1271
```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sq
rt[a + b*x + c*x^2]/(e*(m + 1))), x] - Simp[1/(2*e*(m + 1))   Int[((d + e*x
)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*f + a*g + 2*(c*f +
b*g)*x + 3*c*g*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && Intege
rQ[2*m] && LtQ[m, -1]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

3.891.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 1190, normalized size of antiderivative = 1.60

method	result	size
elliptic	Expression too large to display	1190
default	Expression too large to display	16696

input
```
int((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-1/e*(c*g
*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(e*x+d)+2*((b*e*g-2*c*d*g+c*e*
f)/e^3+1/2*c*d/e^3*g)*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2
*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g
-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(
-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b
*f*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/
2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)
)))^(1/2))+3*c/e^2*g*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*
(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-
1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-
f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*
f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f
/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/
c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/
2))*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/
2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))+
1/e^4*(a*e^2*g-2*b*d*e*g+b*e^2*f+3*c*d^2*g-2*c*d*e*f)*(f/g-1/2*(b+(-4*a*c+
b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/
c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*...
```

3.891.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

3.891.6 Sympy [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx$$

input
```
integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**2,x)
```

output
```
Integral(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)/(d + e*x)**2, x)
```

3.891.7 Maxima [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{cx^2+bx+a}\sqrt{gx+f}}{(ex+d)^2}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x, algorithm="maxima
")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d)^2, x)
```

3.891.8 Giac [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{cx^2+bx+a}\sqrt{gx+f}}{(ex+d)^2}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d)^2, x)
```

3.891.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}{(d+e\,x)^2}\,dx$$

input
```
int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^2,x)
```

output
```
int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^2, x)
```

3.892 $\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx$

3.892.1 Optimal result

Integrand size = 31, antiderivative size = 1034

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4e\,(cd^2-bde+ae^2)\,(ef-dg)(d+ex)} - \frac{\sqrt{b^2-4ac}(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{4\sqrt{2}e^2\,(cd^2-bde+ae^2)\,(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} - \frac{\sqrt{b^2-4ac}(-cd(2ef+3dg)+e(bef+4bdg-5aeg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{\frac{c(a+x(b+cx))}{-b^2+4ac}}\,\text{EllipticF}\left(\arcsin\right.}{2\sqrt{2}e^3\,(cd^2+e(-bd+ae))\,\sqrt{f+gx}\sqrt{a+x(b+cx)}} + \frac{\sqrt{2cf-bg+\sqrt{b^2-4ac}g}(b^2e^4f^2+a^2e^4g^2+c^2d^3g(4ef-3dg)-2ace^2(2e^2f^2-6defg+3d^2g^2)-2be}{4\sqrt{2}\sqrt{c}}$$

output
```
-1/2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/e/(e*x+d)^2+1/4*(c*d*(-3*d*g+2*e*f)
-e*(a*e*g-2*b*d*g+b*e*f))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e
+c*d^2)/(-d*g+e*f)/(e*x+d)-1/8*(c*d*(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f)
)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-
4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e^2/(
a*e^2-b*d*e+c*d^2)/(-d*g+e*f)*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*
f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-1/4*(-c*d*(3*d*g+2*e*f)+e*(-5*a*e*g+4*b
*d*g+b*e*f))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*2^(1/2),2^(1/2)*(g*(-4*a*c+b^2)^(1/2)/(-2*c*f+g*(b+(-4*a*c+b^2)^(
1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(c*(a+x*(c*x+b))/(4*a*c-b^2))^(1/2)*(c*(
g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^3/(c*d^2+e*(a*e-b*d))*2^(
1/2)/(g*x+f)^(1/2)/(a+x*(c*x+b))^(1/2)+1/8*(b^2*e^4*f^2+a^2*e^4*g^2+c^2*d^
3*g*(-3*d*g+4*e*f)-2*a*c*e^2*(3*d^2*g^2-6*d*e*f*g+2*e^2*f^2)-2*b*e*g*(a*e^
3*f+c*d^2*(-2*d*g+3*e*f)))*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f
-b*g+g*(-4*a*c+b^2)^(1/2))^(1/2),(2*c*e*f-b*e*g+e*g*(-4*a*c+b^2)^(1/2))/(-
2*c*d*g+2*c*e*f),((2*c*f+g*(-b+(-4*a*c+b^2)^(1/2)))/(2*c*f-g*(b+(-4*a*c+b^
2)^(1/2))))^(1/2))*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))^(1/2)*(g*(-b-2*c*x+(-4
*a*c+b^2)^(1/2))/(2*c*f+g*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(g*(b+2*c*x+(-4*
a*c+b^2)^(1/2))/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^3/(c*d^2+e*(...
```

3.892.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.67 (sec) , antiderivative size = 33765, normalized size of antiderivative = 32.65

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x)^3,x]
```

output
```
Result too large to show
```

3.892.3 Rubi [A] (verified)

Time = 3.50 (sec) , antiderivative size = 1599, normalized size of antiderivative = 1.55, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.548$, Rules used = {1271, 2154, 1282, 25, 2154, 27, 1172, 321, 1269, 1172, 321, 327, 1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx$$

$$\Big\downarrow \; 1271$$

$$\frac{\int \frac{3cgx^2+2(cf+bg)x+bf+ag}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{4e}-\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2}$$

$$\Big\downarrow \; 2154$$

$$\frac{\int \frac{\frac{2cf}{e}+\frac{2bg}{e}-\frac{3cdg}{e^2}+\frac{3cgx}{e}}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx-\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}}{4e}-\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2}$$

$$\Big\downarrow \; 1282$$

$$\frac{\int \frac{\frac{2cf}{e}+\frac{2bg}{e}-\frac{3cdg}{e^2}+\frac{3cgx}{e}}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx-\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\left(-\frac{\int -\frac{ce^2gx^2+2cdegx+2cd(ef-dg)-e(bef-2bdg+aeg)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)\left(ae^2-bde+cd^2\right)}-\frac{e^2\sqrt{f+gx}\sqrt{a}\cdots}{(d+ex)(ef-dg)\left(a\cdots\right)}\cdots\right)}{e^2}}{4e}-\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\frac{2cf}{e}+\frac{2bg}{e}-\frac{3cdg}{e^2}+\frac{3cgx}{e}}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx-\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\left(\frac{\int \frac{ce^2gx^2+2cdegx+2cd(ef-dg)-e(bef-2bdg+aeg)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)\left(ae^2-bde+cd^2\right)}-\frac{e^2\sqrt{f+gx}\sqrt{a+bx\cdots}}{(d+ex)(ef-dg)\left(ae^2-\cdots\right)}\cdots\right)}{e^2}}{4e}-\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2}$$

$$\Big\downarrow \; 2154$$

$$-\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\left(\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\int \frac{cdg+cexg}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)\left(ae^2-bde+cd^2\right)}-\frac{e^2\sqrt{f+gx}\sqrt{a+bx}}{(d+ex)(ef-dg)\left(ae^2-\right.}\right.}{e^2}$$

$$4e$$

$$\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\left(\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\int \frac{cdg+cexg}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)\left(ae^2-bde+cd^2\right)}-\frac{e^2\sqrt{f+gx}\sqrt{a+bx}}{(d+ex)(ef-dg)\left(ae^2-\right.}\right.}{e^2}$$

$$4e$$

$$\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2}$$

$$\Big\downarrow \; 1172$$

$$\frac{6\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{(cd(2ef-3dg)-e(}{}$$

$$\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2}$$

$$\Big\downarrow \; 321$$

$$-\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\left(\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\int \frac{cdg+cexg}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)\left(ae^2-bde+cd^2\right)}-\frac{e^2\sqrt{f+gx}\sqrt{a+bx}}{(d+ex)(ef-dg)\left(ae^2-\right.}\right.}{e^2}$$

$$\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2}$$

$$\Big\downarrow \; 1269$$

$$-\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\left(\frac{-c(ef-dg)\int \frac{1}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+ce\int \frac{\sqrt{f}}{\sqrt{cx^2}}}{2(ef-dg)\left(ae^2-bde+cd^2\right)}\right.}{e^2}$$

$$\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2}$$

$$\Big\downarrow \; 1172$$

3.892. $\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx$

$$\frac{6\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}+\frac{2(cef-3cdg+beg)\int \ldots}{\ldots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2e(d+ex)^2}$$

$\downarrow$ 321

$$\frac{6\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}+\frac{2(cef-3cdg+beg)\int \ldots}{\ldots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2e(d+ex)^2}$$

$\downarrow$ 327

$$\frac{6\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}+\frac{2(cef-3cdg+beg)\int \ldots}{\ldots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2e(d+ex)^2}$$

3.892. $\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx$

$\Big\downarrow$ 1279

$$\frac{\frac{6\sqrt{2}\sqrt{b^2-4acg}\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4acg}}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}+\frac{2(cef-3cdg+beg)\sqrt{b\cdots}}{}}{}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2e(d+ex)^2}$$

$\Big\downarrow$ 187

$$\frac{\frac{6\sqrt{2}\sqrt{b^2-4acg}\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4acg}}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}-\frac{4(cef-3cdg+beg)\sqrt{b\cdots}}{}}{}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2e(d+ex)^2}$$

$\Big\downarrow$ 413

$$\frac{\frac{6\sqrt{2}\sqrt{b^2-4acg}\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4acg}}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}-\frac{4(cef-3cdg+beg)\sqrt{b\cdots}}{}}{}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2e(d+ex)^2}$$

3.892. $\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx$

$\big\downarrow$ 413

$$\frac{\frac{6\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}-\frac{4(cef-3cdg+beg)\sqrt{b\cdots}}{\cdots}}{\cdots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2e(d+ex)^2}$$

$\big\downarrow$ 412

$$\frac{\frac{6\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-\cdots}\right.}}{\cdots}}{\cdots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2e(d+ex)^2}$$

input
```
Int[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x)^3,x]
```

3.892. $\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx$

output
```
-1/2*(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(e*(d + e*x)^2) + ((6*Sqrt[2]*S
qrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]
*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + S
qrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*
c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(e^2*Sqrt[f + g*x]*Sqrt[a + b*
x + c*x^2]) - (2*Sqrt[2]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*e*f -
3*c*d*g + b*e*g)*Sqrt[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[b + Sqrt[b^2 - 4
*a*c] + 2*c*x]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g
)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticP
i[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[
2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (2*c*f
 - (b - Sqrt[b^2 - 4*a*c])*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[
c]*e^2*(e*f - d*g)*Sqrt[a + b*x + c*x^2]*Sqrt[b - Sqrt[b^2 - 4*a*c] - (2*c
*f)/g + (2*c*(f + g*x))/g]*Sqrt[b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g + (2*c*(
f + g*x))/g]) - ((c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*(-((e
^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*(e*f - d*
g)*(d + e*x))) + ((Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*Sqrt[f + g*x]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f -
 (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^...
```

3.892.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1271
```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sq
rt[a + b*x + c*x^2]/(e*(m + 1))), x] - Simp[1/(2*e*(m + 1))   Int[((d + e*x
)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*f + a*g + 2*(c*f +
b*g)*x + 3*c*g*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && Intege
rQ[2*m] && LtQ[m, -1]
```

3.892. $\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx$

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1282
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^
(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*
e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*
g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

3.892.4 Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 1593, normalized size of antiderivative = 1.54

method	result	size
elliptic	Expression too large to display	1593
default	Expression too large to display	55368

input
```
int((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-1/2/e*(c
*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(e*x+d)^2+1/4*(a*e^2*g-2*b*d
*e*g+b*e^2*f+3*c*d^2*g-2*c*d*e*f)/e/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f
+c*d^3*g-c*d^2*e*f)*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(e*x+d
)+2*(c*g/e^3-1/8*c*g*(3*a*d*e^2*g-2*a*e^3*f-4*b*d^2*e*g+3*b*d*e^2*f+5*c*d^
3*g-4*c*d^2*e*f)/e^3/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*
e*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)
^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a
*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*
a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*
EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b
+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))-1/4*c
*g*(a*e^2*g-2*b*d*e*g+b*e^2*f+3*c*d^2*g-2*c*d*e*f)/e^2/(a*d*e^2*g-a*e^3*f-
b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*
((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b
^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c
+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^
2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*Ell
ipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-
4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c...
```

3.892.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

3.892.6 Sympy [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx = \int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{\left(d+ex\right)^3}\,dx$$

input
```
integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**3,x)
```

output
```
Integral(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)/(d + e*x)**3, x)
```

3.892.7 Maxima [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx = \int \frac{\sqrt{cx^2+bx+a}\sqrt{gx+f}}{\left(ex+d\right)^3}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x, algorithm="maxima
")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d)^3, x)
```

3.892.8 Giac [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx = \int \frac{\sqrt{cx^2+bx+a}\sqrt{gx+f}}{\left(ex+d\right)^3}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d)^3, x)
```

3.892.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx = \int \frac{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}{(d+e\,x)^3}\,dx$$

input
```
int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^3,x)
```

output
```
int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^3, x)
```

3.893 $\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

3.893.1 Optimal result

Integrand size = 31, antiderivative size = 1098

$$\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$
$$= \frac{2(8b^3e^3g^3+3bce^2g^2(5bef-12bdg-9aeg)-c^3(152e^3f^3-408de^2f^2g+336d^2efg^2-70d^3g^3)-3c^2eg(6c}{315c^3g^4}$$
$$+\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9g}$$
$$-\frac{2e(6b^2e^2g^2+ceg(17bef-27bdg-14aeg)-2c^2(64e^2f^2-111defg+42d^2g^2))(f+gx)^{3/2}\sqrt{a+bx+c}}{315c^2g^4}$$
$$-\frac{2e^2(8cef-6cdg-beg)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}{63cg^4}$$
$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(16b^4e^3g^4+8b^2ce^2g^3(2bef-9bdg-9aeg)-2c^4f(64e^3f^3-216de^2f^2g+252d^2efg^2-10}{}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(cf^2-bfg+ag^2)(8b^3e^3g^3+3bce^2g^2(5bef-12bdg-9aeg)+2c^3(64e^3f^3-216de^2f^2g}{}$$

output
```
-2/315*e*(6*b^2*e^2*g^2+c*e*g*(-14*a*e*g-27*b*d*g+17*b*e*f)-2*c^2*(42*d^2*
g^2-111*d*e*f*g+64*e^2*f^2))*(g*x+f)^(3/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^4-2/6
3*e^2*(-b*e*g-6*c*d*g+8*c*e*f)*(g*x+f)^(5/2)*(c*x^2+b*x+a)^(1/2)/c/g^4+2/3
15*(8*b^3*e^3*g^3+3*b*c*e^2*g^2*(-9*a*e*g-12*b*d*g+5*b*e*f)-c^3*(-70*d^3*g
^3+336*d^2*e*f*g^2-408*d*e^2*f^2*g+152*e^3*f^3)-3*c^2*e*g*(6*a*e*g*(-5*d*g
+2*e*f)-b*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2)))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^
(1/2)/c^3/g^4+2/9*(e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/g-1/315*(16*
b^4*e^3*g^4+8*b^2*c*e^2*g^3*(-9*a*e*g-9*b*d*g+2*b*e*f)-2*c^4*f*(-105*d^3*g
^3+252*d^2*e*f*g^2-216*d*e^2*f^2*g+64*e^3*f^3)+3*c^2*e*g^2*(14*a^2*e^2*g^2
-a*b*e*g*(-87*d*g+19*e*f)+b^2*(42*d^2*g^2-27*d*e*f*g+7*e^2*f^2))-c^3*g*(6*
a*e*g*(63*d^2*g^2-39*d*e*f*g+10*e^2*f^2)-b*(-105*d^3*g^3+189*d^2*e*f*g^2-1
44*d*e^2*f^2*g+40*e^3*f^3)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(
-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*
a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x
^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^4/g^5/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*
f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/315*(a*g^2-b*f*g+c*f^2)*(8*b^3*e^3*g^
3+3*b*c*e^2*g^2*(-9*a*e*g-12*b*d*g+5*b*e*f)+2*c^3*(-105*d^3*g^3+252*d^2*e*
f*g^2-216*d*e^2*f^2*g+64*e^3*f^3)-3*c^2*e*g*(6*a*e*g*(-5*d*g+2*e*f)-b*(21*
d^2*g^2-24*d*e*f*g+8*e^2*f^2)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2)
)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(...
```

3.893.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 37.00 (sec) , antiderivative size = 17771, normalized size of antiderivative = 16.18

$$\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((d + e*x)^3*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

output
```
Result too large to show
```

3.893.3 Rubi [A] (verified)

Time = 3.62 (sec) , antiderivative size = 1128, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {1273, 2184, 27, 2184, 27, 2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \ 1273$$

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9g}-\frac{\int \frac{(d+ex)^2\left((8cef-6cdg-beg)x^2+(2cdf+7bef-7bdg-2aeg)x+bdf+6aef-8adg\right)}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{9g}$$

$$\Big\downarrow \ 2184$$

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9g}-$$

$$2\int \frac{eg^3\left(-2\left(64e^2f^2-111degf+42d^2g^2\right)c^2+eg(17bef-27bdg-14aeg)c+6b^2e^2g^2\right)x^3+g^2\left(bg^2(13bf+5ag)e^3+cg\left(2aeg(ef-27dg)-3b\left(31e^2f^2-61degf+35d^2g^2\right)\right)e\cdots\right.}{\cdots}$$

$$\Big\downarrow \ 27$$

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9g}-$$

$$\int \frac{eg^3\left(-2\left(64e^2f^2-111degf+42d^2g^2\right)c^2+eg(17bef-27bdg-14aeg)c+6b^2e^2g^2\right)x^3+g^2\left(bg^2(13bf+5ag)e^3+cg\left(2aeg(ef-27dg)-3b\left(31e^2f^2-61degf+35d^2g^2\right)\right)e-\cdots\right.}{\cdots}$$

$$\Big\downarrow \ 2184$$

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9g}-$$

$$2\int -\frac{3\left(-\left(\left(152e^3f^3-408de^2gf^2+336d^2eg^2f-70d^3g^3\right)c^3\right)-3eg\left(6aeg(2ef-5dg)-b\left(8e^2f^2-24degf+21d^2g^2\right)\right)c^2+3be^2g^2(5bef-12bdg-9aeg)c+8b^3e^3g^3\right)x^2g^5+\cdots}{\cdots}$$

$$\Big\downarrow \ 27$$

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9g} -$$

$$\frac{2eg(f+gx)^{3/2}\sqrt{a+bx+cx^2}\left(ceg(-14aeg-27bdg+17bef)+6b^2e^2g^2-2c^2\left(42d^2g^2-111defg+64e^2f^2\right)\right)}{5c} - \int \frac{3\left(-\left(\left(152e^3f^3-408de^2gf^2+336d^2eg^2f-70d^3g^3\right)c^3\right)-\right.}{}$$

$\downarrow$ 2184

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{cx^2+bx+a}}{9g} -$$

$$\frac{2e^2(8cef-6cdg-beg)\sqrt{cx^2+bx+a}(f+gx)^{5/2}}{7cg^3} + \frac{\frac{2eg\left(-2\left(64e^2f^2-111degf+42d^2g^2\right)c^2+eg(17bef-27bdg-14aeg)c+6b^2e^2g^2\right)(f+gx)^{3/2}\sqrt{cx^2+bx+a}}{5c} - \frac{2(}{}}{}$$

$\downarrow$ 27

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{cx^2+bx+a}}{9g} -$$

$$\frac{2e^2(8cef-6cdg-beg)\sqrt{cx^2+bx+a}(f+gx)^{5/2}}{7cg^3} + \frac{\frac{2eg\left(-2\left(64e^2f^2-111degf+42d^2g^2\right)c^2+eg(17bef-27bdg-14aeg)c+6b^2e^2g^2\right)(f+gx)^{3/2}\sqrt{cx^2+bx+a}}{5c} - \frac{2g^4}{}}{}$$

$\downarrow$ 1269

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{cx^2+bx+a}}{9g} -$$

$$\frac{2e^2(8cef-6cdg-beg)\sqrt{cx^2+bx+a}(f+gx)^{5/2}}{7cg^3} + \frac{\frac{2eg\left(-2\left(64e^2f^2-111degf+42d^2g^2\right)c^2+eg(17bef-27bdg-14aeg)c+6b^2e^2g^2\right)(f+gx)^{3/2}\sqrt{cx^2+bx+a}}{5c} - \frac{2g^4}{}}{}$$

$\downarrow$ 1172

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{cx^2+bx+a}}{9g} -$$

$$\frac{2e^2(8cef-6cdg-beg)\sqrt{cx^2+bx+a}(f+gx)^{5/2}}{7cg^3} + \frac{\frac{2eg\left(-2\left(64e^2f^2-111degf+42d^2g^2\right)c^2+eg(17bef-27bdg-14aeg)c+6b^2e^2g^2\right)(f+gx)^{3/2}\sqrt{cx^2+bx+a}}{5c} - \frac{2g^4}{}}{}$$

3.893. $\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

$\big\downarrow$ 321

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{cx^2+bx+a}}{9g}-$$

$$\frac{\frac{2e^2(8cef-6cdg-beg)\sqrt{cx^2+bx+a}(f+gx)^{5/2}}{7cg^3}+\frac{\frac{2eg\left(-2\left(64e^2f^2-111degf+42d^2g^2\right)c^2+eg(17bef-27bdg-14aeg)c+6b^2e^2g^2\right)(f+gx)^{3/2}\sqrt{cx^2+bx+a}}{5c}-\frac{2g^{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

$\big\downarrow$ 327

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{cx^2+bx+a}}{9g}-$$

$$\frac{\frac{2e^2(8cef-6cdg-beg)\sqrt{cx^2+bx+a}(f+gx)^{5/2}}{7cg^3}+\frac{\frac{2eg\left(-2\left(64e^2f^2-111degf+42d^2g^2\right)c^2+eg(17bef-27bdg-14aeg)c+6b^2e^2g^2\right)(f+gx)^{3/2}\sqrt{cx^2+bx+a}}{5c}-\frac{2g^{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

input
```
Int[((d + e*x)^3*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

3.893. $\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

output
```
(2*(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(9*g) - ((2*e^2*(8*c*e
*f - 6*c*d*g - b*e*g)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(7*c*g^3) + (
(2*e*g*(6*b^2*e^2*g^2 + c*e*g*(17*b*e*f - 27*b*d*g - 14*a*e*g) - 2*c^2*(64
*e^2*f^2 - 111*d*e*f*g + 42*d^2*g^2))*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2
])/(5*c) - ((2*g^4*(8*b^3*e^3*g^3 + 3*b*c*e^2*g^2*(5*b*e*f - 12*b*d*g - 9*
a*e*g) - c^3*(152*e^3*f^3 - 408*d*e^2*f^2*g + 336*d^2*e*f*g^2 - 70*d^3*g^3
) - 3*c^2*e*g*(6*a*e*g*(2*e*f - 5*d*g) - b*(8*e^2*f^2 - 24*d*e*f*g + 21*d^
2*g^2)))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/c - (g^4*((Sqrt[2]*Sqrt[b^2
- 4*a*c]*(16*b^4*e^3*g^4 + 8*b^2*c*e^2*g^3*(2*b*e*f - 9*b*d*g - 9*a*e*g) -
 2*c^4*f*(64*e^3*f^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3) +
3*c^2*e*g^2*(14*a^2*e^2*g^2 - a*b*e*g*(19*e*f - 87*d*g) + b^2*(7*e^2*f^2 -
 27*d*e*f*g + 42*d^2*g^2)) - c^3*g*(6*a*e*g*(10*e^2*f^2 - 39*d*e*f*g + 63*
d^2*g^2) - b*(40*e^3*f^3 - 144*d*e^2*f^2*g + 189*d^2*e*f*g^2 - 105*d^3*g^3
)))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[A
rcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-
2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*g*Sqrt[(c*
(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (
2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(8*b^3*e^3*g^3 + 3*b*c
*e^2*g^2*(5*b*e*f - 12*b*d*g - 9*a*e*g) + 2*c^3*(64*e^3*f^3 - 216*d*e^2*f^
2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3) - 3*c^2*e*g*(6*a*e*g*(2*e*f - 5*d*...
```

3.893.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.893. $\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1273
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])/Sq
rt[(f_.) + (g_.)*(x_)], x_Symbol] :> Simp[2*(d + e*x)^m*Sqrt[f + g*x]*(Sqrt
[a + b*x + c*x^2]/(g*(2*m + 3))), x] - Simp[1/(g*(2*m + 3))   Int[((d + e*x
)^(m - 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f + 2*a*(e*f*m -
d*g*(m + 1)) + (2*c*d*f - 2*a*e*g + b*(e*f - d*g)*(2*m + 1))*x - (b*e*g + 2
*c*(d*g*m - e*f*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x
] && IntegerQ[2*m] && GtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.893. $\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

3.893.4 Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 1845, normalized size of antiderivative = 1.68

method	result	size
elliptic	Expression too large to display	1845
risch	Expression too large to display	7892
default	Expression too large to display	22215

input
```
int((e*x+d)^3*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/9*e^3/g
*x^3*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/7*(b*e^3+3*c*d*e^2-
2/9*e^3/g*(4*b*g+4*c*f))/c/g*x^2*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)
^(1/2)+2/5*(a*e^3+3*b*d*e^2+3*c*d^2*e-2/9*e^3/g*(7/2*a*g+7/2*b*f)-2/7*(b*e
^3+3*c*d*e^2-2/9*e^3/g*(4*b*g+4*c*f))/c/g*(3*b*g+3*c*f))/c/g*x*(c*g*x^3+b*
g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/3*(3*a*d*e^2+3*b*d^2*e+c*d^3-2/3*e^
3/g*f*a-2/7*(b*e^3+3*c*d*e^2-2/9*e^3/g*(4*b*g+4*c*f))/c/g*(5/2*a*g+5/2*b*f
)-2/5*(a*e^3+3*b*d*e^2+3*c*d^2*e-2/9*e^3/g*(7/2*a*g+7/2*b*f)-2/7*(b*e^3+3*
c*d*e^2-2/9*e^3/g*(4*b*g+4*c*f))/c/g*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g
*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2*(a*d^3-2/5*(a*e^3+3*b*d
*e^2+3*c*d^2*e-2/9*e^3/g*(7/2*a*g+7/2*b*f)-2/7*(b*e^3+3*c*d*e^2-2/9*e^3/g*
(4*b*g+4*c*f))/c/g*(3*b*g+3*c*f))/c/g*f*a-2/3*(3*a*d*e^2+3*b*d^2*e+c*d^3-2
/3*e^3/g*f*a-2/7*(b*e^3+3*c*d*e^2-2/9*e^3/g*(4*b*g+4*c*f))/c/g*(5/2*a*g+5/
2*b*f)-2/5*(a*e^3+3*b*d*e^2+3*c*d^2*e-2/9*e^3/g*(7/2*a*g+7/2*b*f)-2/7*(b*e
^3+3*c*d*e^2-2/9*e^3/g*(4*b*g+4*c*f))/c/g*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f)
)/c/g*(1/2*a*g+1/2*b*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-
1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-
f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c
)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*
x+b*f*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c...
```

3.893.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 1241, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas
")
```

output
```
-2/945*((128*c^5*e^3*f^5 - 8*(54*c^5*d*e^2 + 13*b*c^4*e^3)*f^4*g + (504*c^
5*d^2*e + 360*b*c^4*d*e^2 - (25*b^2*c^3 - 156*a*c^4)*e^3)*f^3*g^2 - (210*c
^5*d^3 + 441*b*c^4*d^2*e - 18*(5*b^2*c^3 - 31*a*c^4)*d*e^2 + 5*(2*b^3*c^2
- 9*a*b*c^3)*e^3)*f^2*g^3 + (210*b*c^4*d^3 - 126*(b^2*c^3 - 6*a*c^4)*d^2*e
 + 9*(5*b^3*c^2 - 22*a*b*c^3)*d*e^2 - (8*b^4*c - 39*a*b^2*c^2 + 24*a^2*c^3
)*e^3)*f*g^4 + (105*(b^2*c^3 - 6*a*c^4)*d^3 - 63*(2*b^3*c^2 - 9*a*b*c^3)*d
^2*e + 9*(8*b^4*c - 41*a*b^2*c^2 + 30*a^2*c^3)*d*e^2 - (16*b^5 - 96*a*b^3*
c + 123*a^2*b*c^2)*e^3)*g^5)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 -
b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g -
 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*
x + c*f + b*g)/(c*g)) + 3*(128*c^5*e^3*f^4*g - 8*(54*c^5*d*e^2 + 5*b*c^4*e
^3)*f^3*g^2 + 3*(168*c^5*d^2*e + 48*b*c^4*d*e^2 - (7*b^2*c^3 - 20*a*c^4)*e
^3)*f^2*g^3 - (210*c^5*d^3 + 189*b*c^4*d^2*e - 9*(9*b^2*c^3 - 26*a*c^4)*d*
e^2 + (16*b^3*c^2 - 57*a*b*c^3)*e^3)*f*g^4 + (105*b*c^4*d^3 - 126*(b^2*c^3
 - 3*a*c^4)*d^2*e + 9*(8*b^3*c^2 - 29*a*b*c^3)*d*e^2 - 2*(8*b^4*c - 36*a*b
^2*c^2 + 21*a^2*c^3)*e^3)*g^5)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*
c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3
*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPI
nverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3
*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^...
```

3.893.6 Sympy [F]

$$\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)**3*(c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)
```

output
```
Integral((d + e*x)**3*sqrt(a + b*x + c*x**2)/sqrt(f + g*x), x)
```

3.893.7 Maxima [F]

$$\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(ex+d)^3}{\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^3/sqrt(g*x + f), x)
```

3.893.8 Giac [F]

$$\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(ex+d)^3}{\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^3/sqrt(g*x + f), x)
```

3.893.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{(d+e\,x)^3\,\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}}\,dx$$

input
```
int(((d + e*x)^3*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2),x)
```

output
```
int(((d + e*x)^3*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2), x)
```

3.894 $\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

3.894.1 Optimal result

Integrand size = 31, antiderivative size = 755

$$\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx =$$

$$-\frac{4(2b^2e^2g^2+ceg(4bef-7bdg-5aeg)-c^2(21e^2f^2-34defg+10d^2g^2))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{105c^2g^3}$$

$$+\frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7g}-\frac{2e(6cef-4cdg-beg)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{35cg^3}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(8b^3e^2g^3+bceg^2(9bef-28bdg-29aeg)-2c^3f(24e^2f^2-56defg+35d^2g^2)-c^2g(2aeg(}{105c^3g^4\sqrt{}}$$

$$+\frac{4\sqrt{2}\sqrt{b^2-4ac}(cf^2-bfg+ag^2)(2b^2e^2g^2+ceg(4bef-7bdg-5aeg)+c^2(24e^2f^2-56defg+35d^2g^2)}{105c^3g^4\sqrt{f+gx}\sqrt{a}}$$

output
```
-2/35*e*(-b*e*g-4*c*d*g+6*c*e*f)*(g*x+f)^(3/2)*(c*x^2+b*x+a)^(1/2)/c/g^3-4
/105*(2*b^2*e^2*g^2+c*e*g*(-5*a*e*g-7*b*d*g+4*b*e*f)-c^2*(10*d^2*g^2-34*d*
e*f*g+21*e^2*f^2))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^3+2/7*(e*x+d)^2
*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/g+1/105*(8*b^3*e^2*g^3+b*c*e*g^2*(-29*a
*e*g-28*b*d*g+9*b*e*f)-2*c^3*f*(35*d^2*g^2-56*d*e*f*g+24*e^2*f^2)-c^2*g*(2
*a*e*g*(-42*d*g+13*e*f)-b*(35*d^2*g^2-42*d*e*f*g+16*e^2*f^2)))*EllipticE(1
/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(
-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c
+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^3/g^4/(c
*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+4/105
*(a*g^2-b*f*g+c*f^2)*(2*b^2*e^2*g^2+c*e*g*(-5*a*e*g-7*b*d*g+4*b*e*f)+c^2*(
35*d^2*g^2-56*d*e*f*g+24*e^2*f^2))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(
b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a
)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c
^3/g^4/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.894.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.65 (sec) , antiderivative size = 10030, normalized size of antiderivative = 13.28

$$\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((d + e*x)^2*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

output
```
Result too large to show
```

3.894.3 Rubi [A] (verified)

Time = 1.96 (sec) , antiderivative size = 776, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {1273, 2184, 27, 2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}dx$$

$$\Big\downarrow \; 1273$$

$$\frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7g}-\frac{\int \frac{(d+ex)((6cef-4cdg-beg)x^2+(2cdf+5bef-5bdg-2aeg)x+bdf+4aef-6adg)}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{7g}$$

$$\Big\downarrow \; 2184$$

$$\frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7g}-$$

$$2\int \frac{2g^2(-((21e^2f^2-34degf+10d^2g^2)c^2)+eg(4bef-7bdg-5aeg)c+2b^2e^2g^2)x^2+g(-2f(6e^2f^2-4degf-5d^2g^2)c^2+g(2aeg(ef-14dg)-b(28e^2f^2-50degf+25d^2 \ldots}{2\sqrt{f+gx}\sqrt{cx^2+bx+a}} \Big/ 5cg^3$$

$$\Big\downarrow \; 27$$

$$\frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7g}-$$

$$\int \frac{2g^2(-((21e^2f^2-34degf+10d^2g^2)c^2)+eg(4bef-7bdg-5aeg)c+2b^2e^2g^2)x^2+g(-2f(6e^2f^2-4degf-5d^2g^2)c^2+g(2aeg(ef-14dg)-b(28e^2f^2-50degf+25d^2g^2 \ldots}{\sqrt{f+gx}\sqrt{cx^2+bx+a}} \Big/ 5cg^3$$

$$\Big\downarrow \; 2184$$

$$\frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7g}-$$

$$2\int -\frac{g^3(4e^2fg^2b^3+eg(4aeg^2+cf(5ef-14dg))b^2-c(ae(11ef+14dg)g^2+cf(24e^2f^2-56degf+35d^2g^2))b-2acg(5ae^2g^2-c(6e^2f^2-14degf+35d^2g^2))+(-2f(24e \ldots}{2\sqrt{f+gx}\sqrt{cx^2+bx+a}} \Big/ 3cg^2$$

$$\Big\downarrow \; 27$$

$$\frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7g}-$$

$$\frac{4g\sqrt{f+gx}\sqrt{a+bx+cx^2}(ceg(-5aeg-7bdg+4bef)+2b^2e^2g^2-(c^2(10d^2g^2-34defg+21e^2f^2)))}{3c}-g\int \frac{4e^2fg^2b^3+eg(4aeg^2+cf(5ef-14dg))b^2-c(ae(11ef+14dg)g^2 \ldots}{\ldots}$$

$$\Big\downarrow \; 1269$$

3.894. $\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}dx$

$$\frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7g}-$$

$$\frac{4g\sqrt{f+gx}\sqrt{a+bx+cx^2}\left(ceg(-5aeg-7bdg+4bef)+2b^2e^2g^2-\left(c^2\left(10d^2g^2-34defg+21e^2f^2\right)\right)\right)}{3c}-\frac{g\left(\frac{\left(-c^2g\left(2aeg(13ef-42dg)-b\left(35d^2g^2-42defg+16e^2f^2\right)\right)+bceg\right.}{}\right.}{}$$

$\downarrow$ 1172

$$\frac{2(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}{7g}-$$

$$\frac{2e(6cef-4cdg-beg)\sqrt{cx^2+bx+a}(f+gx)^{3/2}}{5cg^2}+\frac{4g\left(-\left(\left(21e^2f^2-34degf+10d^2g^2\right)c^2\right)+eg(4bef-7bdg-5aeg)c+2b^2e^2g^2\right)\sqrt{f+gx}\sqrt{cx^2+bx+a}}{3c}-\frac{g\left(\frac{4\sqrt{2}\sqrt{b^2}}{}\right.}{}$$

$\downarrow$ 321

$$\frac{2(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}{7g}-$$

$$\frac{2e(6cef-4cdg-beg)\sqrt{cx^2+bx+a}(f+gx)^{3/2}}{5cg^2}+\frac{4g\left(-\left(\left(21e^2f^2-34degf+10d^2g^2\right)c^2\right)+eg(4bef-7bdg-5aeg)c+2b^2e^2g^2\right)\sqrt{f+gx}\sqrt{cx^2+bx+a}}{3c}-\frac{g\left(\frac{4\sqrt{2}\sqrt{b^2}}{}\right.}{}$$

$\downarrow$ 327

$$\frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7g}-$$

$$\frac{4g\sqrt{f+gx}\sqrt{a+bx+cx^2}\left(ceg(-5aeg-7bdg+4bef)+2b^2e^2g^2-\left(c^2\left(10d^2g^2-34defg+21e^2f^2\right)\right)\right)}{3c}-\frac{g\left(\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f}{2cf-g\left(\sqrt{}\right.}}}{}\right.}{}$$

3.894. $\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

input
```
Int[((d + e*x)^2*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

output
```
(2*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(7*g) - ((2*e*(6*c*e*f
 - 4*c*d*g - b*e*g)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c*g^2) + ((4
*g*(2*b^2*e^2*g^2 + c*e*g*(4*b*e*f - 7*b*d*g - 5*a*e*g) - c^2*(21*e^2*f^2
- 34*d*e*f*g + 10*d^2*g^2))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c) - (
g*((Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*b^3*e^2*g^3 + b*c*e*g^2*(9*b*e*f - 28*b*d
*g - 29*a*e*g) - 2*c^3*f*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2) - c^2*g*(2
*a*e*g*(13*e*f - 42*d*g) - b*(16*e^2*f^2 - 42*d*e*f*g + 35*d^2*g^2)))*Sqrt
[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b
^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*g*Sqrt[(c*(f + g*x
))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2
]*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(2*b^2*e^2*g^2 + c*e*g*(4*b*e*
f - 7*b*d*g - 5*a*e*g) + c^2*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2))*Sqrt[
(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*
x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt
[b^2 - 4*a*c])*g)])/(c*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])))/(3*c))/(5*
c*g^3))/(7*g)
```

3.894.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1273
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])/Sq
rt[(f_.) + (g_.)*(x_)], x_Symbol] :> Simp[2*(d + e*x)^m*Sqrt[f + g*x]*(Sqrt
[a + b*x + c*x^2]/(g*(2*m + 3))), x] - Simp[1/(g*(2*m + 3))   Int[((d + e*x
)^(m - 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f + 2*a*(e*f*m -
d*g*(m + 1)) + (2*c*d*f - 2*a*e*g + b*(e*f - d*g)*(2*m + 1))*x - (b*e*g + 2
*c*(d*g*m - e*f*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x
] && IntegerQ[2*m] && GtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.894. $\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

3.894.4 Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 1272, normalized size of antiderivative = 1.68

method	result	size
elliptic	Expression too large to display	1272
risch	Expression too large to display	4566
default	Expression too large to display	12922

input
```
int((e*x+d)^2*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/7*e^2/g
*x^2*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/5*(b*e^2+2*c*d*e-2/
7*e^2/g*(3*b*g+3*c*f))/c/g*x*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/
2)+2/3*(e^2*a+2*b*d*e+c*d^2-2/7*e^2/g*(5/2*a*g+5/2*b*f)-2/5*(b*e^2+2*c*d*e
-2/7*e^2/g*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(c*g*x^3+b*g*x^2+c*f*x^2+
a*g*x+b*f*x+a*f)^(1/2)+2*(a*d^2-2/5*(b*e^2+2*c*d*e-2/7*e^2/g*(3*b*g+3*c*f)
)/c/g*f*a-2/3*(e^2*a+2*b*d*e+c*d^2-2/7*e^2/g*(5/2*a*g+5/2*b*f)-2/5*(b*e^2+
2*c*d*e-2/7*e^2/g*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(1/2*a*g+1/2*b*f))
*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2
))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^
2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*Ellip
ticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*
a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(2*a*d*e
+b*d^2-4/7*e^2/g*f*a-2/5*(b*e^2+2*c*d*e-2/7*e^2/g*(3*b*g+3*c*f))/c/g*(3/2*
a*g+3/2*b*f)-2/3*(e^2*a+2*b*d*e+c*d^2-2/7*e^2/g*(5/2*a*g+5/2*b*f)-2/5*(b*e
^2+2*c*d*e-2/7*e^2/g*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(b*g+c*f))*(f/g
-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1
/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2...
```

3.894.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 828, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$

$$= \frac{2\left(\left(48\,c^4e^2f^4 - 8\,(14\,c^4de + 5\,bc^3e^2)f^3g + 2\,(35\,c^4d^2 + 49\,bc^3de - (5\,b^2c^2 - 31\,ac^3)e^2)f^2g^2 - (70\,bc^3d^2 - 2\right.\right.}{}$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas
")
```

output
```
2/315*((48*c^4*e^2*f^4 - 8*(14*c^4*d*e + 5*b*c^3*e^2)*f^3*g + 2*(35*c^4*d^
2 + 49*b*c^3*d*e - (5*b^2*c^2 - 31*a*c^3)*e^2)*f^2*g^2 - (70*b*c^3*d^2 - 2
8*(b^2*c^2 - 6*a*c^3)*d*e + (5*b^3*c - 22*a*b*c^2)*e^2)*f*g^3 - (35*(b^2*c
^2 - 6*a*c^3)*d^2 - 14*(2*b^3*c - 9*a*b*c^2)*d*e + (8*b^4 - 41*a*b^2*c + 3
0*a^2*c^2)*e^2)*g^4)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g
+ (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*
c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f
+ b*g)/(c*g)) + 3*(48*c^4*e^2*f^3*g - 16*(7*c^4*d*e + b*c^3*e^2)*f^2*g^2 +
 (70*c^4*d^2 + 42*b*c^3*d*e - (9*b^2*c^2 - 26*a*c^3)*e^2)*f*g^3 - (35*b*c^
3*d^2 - 28*(b^2*c^2 - 3*a*c^3)*d*e + (8*b^3*c - 29*a*b*c^2)*e^2)*g^4)*sqrt
(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2
), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 -
 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^
2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6
*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g
)/(c*g))) + 3*(15*c^4*e^2*g^4*x^2 + 24*c^4*e^2*f^2*g^2 - (56*c^4*d*e + 5*b
*c^3*e^2)*f*g^3 + (35*c^4*d^2 + 14*b*c^3*d*e - 2*(2*b^2*c^2 - 5*a*c^3)*e^2
)*g^4 - 3*(6*c^4*e^2*f*g^3 - (14*c^4*d*e + b*c^3*e^2)*g^4)*x)*sqrt(c*x^2 +
 b*x + a)*sqrt(g*x + f))/(c^4*g^5)
```

3.894.6 Sympy [F]

$$\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)**2*(c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)
```

output
```
Integral((d + e*x)**2*sqrt(a + b*x + c*x**2)/sqrt(f + g*x), x)
```

3.894.7 Maxima [F]

$$\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(ex+d)^2}{\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^2/sqrt(g*x + f), x)
```

3.894.8 Giac [F]

$$\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(ex+d)^2}{\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^2/sqrt(g*x + f), x)
```

3.894.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{(d+e\,x)^2\,\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}}\,dx$$

input
```
int(((d + e*x)^2*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2),x)
```

output
```
int(((d + e*x)^2*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2), x)
```

3.895 $\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

3.895.1 Optimal result

Integrand size = 29, antiderivative size = 519

$$\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = -\frac{2\sqrt{f+gx}(4cef-5cdg-beg-3cegx)\sqrt{a+bx+cx^2}}{15cg^2}$$
$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(2b^2eg^2-2c^2f(4ef-5dg)+cg(3bef-5bdg-6aeg))\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\Bigg(\arcsin}{15c^2g^3\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(8cef-10cdg+beg)\left(cf^2-bfg+ag^2\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\mathrm{EllipticF}\Bigg(\mathrm{ar}}{15c^2g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

output
```
-2/15*(-3*c*e*g*x-b*e*g-5*c*d*g+4*c*e*f)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)
/c/g^2-1/15*(2*b^2*e*g^2-2*c^2*f*(-5*d*g+4*e*f)+c*g*(-6*a*e*g-5*b*d*g+3*b*
e*f))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
)*2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))
^(1/2)/c^2/g^3/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/
2))))^(1/2)-2/15*(b*e*g-10*c*d*g+8*c*e*f)*(a*g^2-b*f*g+c*f^2)*EllipticF(1/
2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-
4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+
b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-
4*a*c+b^2)^(1/2))))^(1/2)/c^2/g^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.895.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.60 (sec) , antiderivative size = 792, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2(a+x(b+cx))(beg+c(-4ef+5dg+3egx))}{cg^2} + \frac{2(f+gx)\left(\frac{g^2\left(-2b^2eg^2+2c^2f(4ef-5dg)+cg(-3bef+5bdg+6aeg)\right)(a+x(b+cx))}{(f+gx)^2} + \frac{i\sqrt{1-\frac{\ldots}{(\ldots}}\ldots}{\ldots}\right.}{\ldots}\right.}{\ldots}$$

input
```
Integrate[((d + e*x)*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

output
```
(Sqrt[f + g*x]*((2*(a + x*(b + c*x))*(b*e*g + c*(-4*e*f + 5*d*g + 3*e*g*x)
))/(c*g^2) + (2*(f + g*x)*((g^2*(-2*b^2*e*g^2 + 2*c^2*f*(4*e*f - 5*d*g) +
c*g*(-3*b*e*f + 5*b*d*g + 6*a*e*g))*(a + x*(b + c*x)))/(f + g*x)^2 + ((I/2
)*Sqrt[1 - (2*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*
c)*g^2])*(f + g*x))]*Sqrt[1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2*c*f + b*
g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*((2*c*f - b*g + Sqrt[(b^2 - 4*a*c
)*g^2])*(2*b^2*e*g^2 + 2*c^2*f*(-4*e*f + 5*d*g) + c*g*(3*b*e*f - 5*b*d*g -
 6*a*e*g))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c
*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sq
rt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))] + (2*b^3*
e*g^3 - b^2*g^2*(-(c*e*f) + 5*c*d*g + 2*e*Sqrt[(b^2 - 4*a*c)*g^2]) + b*c*g
*(-8*a*e*g^2 + Sqrt[(b^2 - 4*a*c)*g^2]*(-3*e*f + 5*d*g)) + 2*c*(c*f*Sqrt[(
b^2 - 4*a*c)*g^2]*(4*e*f - 5*d*g) + a*g^2*(-2*c*e*f + 10*c*d*g + 3*e*Sqrt[
(b^2 - 4*a*c)*g^2])))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a
*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f
 + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))
]))/(Sqrt[2]*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4
*a*c)*g^2])]*Sqrt[f + g*x])))/(c^2*g^4)))/(15*Sqrt[a + x*(b + c*x)])
```

3.895.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1231, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \ 1231$$

$$-\frac{2\int \frac{efgb^2+aeg^2b-cf(4ef-5dg)b+2acg(ef-5dg)+\left(-2f(4ef-5dg)c^2+g(3bef-5bdg-6aeg)c+2b^2eg^2\right)x}{2\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{15cg^2}-\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}(-beg-5cdg+4cef-3cegx)}{15cg^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{efgb^2+aeg^2b-cf(4ef-5dg)b+2acg(ef-5dg)+(-2f(4ef-5dg)c^2+g(3bef-5bdg-6aeg)c+2b^2eg^2)x}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{15cg^2}-\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}(-beg-5cdg+4cef-3cegx)}{15cg^2}$$

$\downarrow$ 1269

$$-\frac{\frac{(cg(-6aeg-5bdg+3bef)+2b^2eg^2-2c^2f(4ef-5dg))\int \frac{\sqrt{f+gx}}{\sqrt{cx^2+bx+a}}dx}{g}+\frac{(ag^2-bfg+cf^2)(beg-10cdg+8cef)\int \frac{1}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{g}}{15cg^2}-\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}(-beg-5cdg+4cef-3cegx)}{15cg^2}$$

$\downarrow$ 1172

$$-\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(cg(-6aeg-5bdg+3bef)+2b^2eg^2-2c^2f(4ef-5dg))\int \frac{\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}+ - \cdots 2\cdots}{15cg^2}-\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}(-beg-5cdg+4cef-3cegx)}{15cg^2}$$

$\downarrow$ 321

$$-\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(cg(-6aeg-5bdg+3bef)+2b^2eg^2-2c^2f(4ef-5dg))\int \frac{\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}+ - \cdots 2\cdots}{15cg^2}-\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}(-beg-5cdg+4cef-3cegx)}{15cg^2}$$

$\downarrow$ 327

$$-\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(cg(-6aeg-5bdg+3bef)+2b^2eg^2-2c^2f(4ef-5dg))E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4acg}}{2cf-\left(b+\sqrt{b^2-4ac}\right)\cdots}\cdots\right.}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}\cdots}{15cg^2}-\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}(-beg-5cdg+4cef-3cegx)}{15cg^2}$$

3.895. $\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

input
```
Int[((d + e*x)*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

output
```
(-2*Sqrt[f + g*x]*(4*c*e*f - 5*c*d*g - b*e*g - 3*c*e*g*x)*Sqrt[a + b*x + c
*x^2])/(15*c*g^2) - ((Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*b^2*e*g^2 - 2*c^2*f*(4*
e*f - 5*d*g) + c*g*(3*b*e*f - 5*b*d*g - 6*a*e*g))*Sqrt[f + g*x]*Sqrt[-((c*
(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*
f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqr
t[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(8*c*e*f - 10*c*d*g + b*e*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2
*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a
*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a
*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g
)])/(c*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))/(15*c*g^2)
```

3.895.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

3.895. $\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}} \, dx$

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.895.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 954 vs. $2(461) = 922$.

Time = 2.70 (sec) , antiderivative size = 955, normalized size of antiderivative = 1.84

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+bx+a)}\left(\frac{2ex\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{5g}+\frac{2\left(be+cd-\frac{2(2bg+2cf)e}{5g}\right)\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{3cg}+\frac{2\left(ad-\frac{2fa}{5g}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((e*x+d)*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/5*e/g*x
*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/3*(b*e+c*d-2/5/g*(2*b*g
+2*c*f)*e)/c/g*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2*(a*d-2/5*
f*a/g*e-2/3*(b*e+c*d-2/5/g*(2*b*g+2*c*f)*e)/c/g*(1/2*a*g+1/2*b*f))*(f/g-1/
2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(
1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)
)))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2
))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF(((x
+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)
^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(a*e+b*d-2/5*e/g
*(3/2*a*g+3/2*b*f)-2/3*(b*e+c*d-2/5/g*(2*b*g+2*c*f)*e)/c/g*(b*g+c*f))*(f/g
-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1
/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(
1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2
/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1
/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*
c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/(f
/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/
c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

3.895.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx =$$

$$-\frac{2\left((8\,c^3ef^3-(10\,c^3d+7\,bc^2e)f^2g+2\,(5\,bc^2d-(b^2c-6\,ac^2)e)fg^2+(5\,(b^2c-6\,ac^2)d-(2\,b^3-9\,abc)e\right.}{}$$

input
```
integrate((e*x+d)*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*((8*c^3*e*f^3 - (10*c^3*d + 7*b*c^2*e)*f^2*g + 2*(5*b*c^2*d - (b^2*c
 - 6*a*c^2)*e)*f*g^2 + (5*(b^2*c - 6*a*c^2)*d - (2*b^3 - 9*a*b*c)*e)*g^3)*
sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/
(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 +
(2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)) + 3*(8*
c^3*e*f^2*g - (10*c^3*d + 3*b*c^2*e)*f*g^2 + (5*b*c^2*d - 2*(b^2*c - 3*a*c
^2)*e)*g^3)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*
c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*
f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f^2
 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*
g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c
*g*x + c*f + b*g)/(c*g))) - 3*(3*c^3*e*g^3*x - 4*c^3*e*f*g^2 + (5*c^3*d +
b*c^2*e)*g^3)*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f))/(c^3*g^4)
```

3.895.6 Sympy [F]

$$\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx=\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)*(c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)
```

output
```
Integral((d + e*x)*sqrt(a + b*x + c*x**2)/sqrt(f + g*x), x)
```

3.895.7 Maxima [F]

$$\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx=\int \frac{\sqrt{cx^2+bx+a}(ex+d)}{\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)/sqrt(g*x + f), x)
```

3.895.8 Giac [F]

$$\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(ex+d)}{\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)/sqrt(g*x + f), x)
```

3.895.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}}\,dx$$

input
```
int(((d + e*x)*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2),x)
```

output
```
int(((d + e*x)*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2), x)
```

3.896 $\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

3.896.1 Optimal result

Integrand size = 24, antiderivative size = 444

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3g} - \frac{\sqrt{2}\sqrt{b^2-4ac}(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3cg^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} + \frac{4\sqrt{2}\sqrt{b^2-4ac}(cf^2-bfg+ag^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{3cg^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

output
```
2/3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/g-1/3*(-b*g+2*c*f)*EllipticE(1/2*((b
+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c
+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^
(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c/g^2/(c*x^2+b*x
+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+4/3*(a*g^2-b*
f*g+c*f^2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))
^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^
(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c
*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/g^2/(g*x+f)^(1/2)/(c*x^
2+b*x+a)^(1/2)
```

3.896.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.42 (sec) , antiderivative size = 936, normalized size of antiderivative = 2.11

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(4g^2(a+x(b+cx)) + \frac{(f+gx)\left(\frac{4g^2(-2cf+bg)\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}(a+x(b+cx))}{(f+gx)^2} + \frac{i\sqrt{2}(2cf-bg)\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/Sqrt[f + g*x],x]
```

output
```
(Sqrt[f + g*x]*(4*g^2*(a + x*(b + c*x)) + ((f + g*x)*((4*g^2*(-2*c*f + b*g
)*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])
]*(a + x*(b + c*x)))/(f + g*x)^2 + (I*Sqrt[2]*(2*c*f - b*g)*(2*c*f - b*g +
 Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(-2*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2] + 2*c
*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(f - g*x))/((2*c*f - b*g + Sqrt
[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2]
 - 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(-f + g*x))/((-2*c*f + b*
g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt
[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f
 + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(
b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x] - (I*Sqrt[2]*(b^2*g^2 - 4*a*c*g^2 + 2*
c*f*Sqrt[(b^2 - 4*a*c)*g^2] - b*g*Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(-2*a*g^2
+ f*Sqrt[(b^2 - 4*a*c)*g^2] + 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*
g*(f - g*x))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*
a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2] - 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*
x + b*g*(-f + g*x))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*
EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g +
Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4
*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x]))/(c*
Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2]...
```

3.896.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1162, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \text{1162}$$

$$\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3g}-\frac{\int \frac{bf-2ag+(2cf-bg)x}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{3g}$$

$$\Big\downarrow \text{1269}$$

$$\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3g}-\frac{\frac{(2cf-bg)\int \frac{\sqrt{f+gx}}{\sqrt{cx^2+bx+a}}dx}{g}-\frac{2\left(ag^2-bfg+cf^2\right)\int \frac{1}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{g}}{3g}$$

$\downarrow$ 1172

$$\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3g}-\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cf-bg)\int\frac{\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ag^2-bfg+cf^2}{}}{3g}$$

$\downarrow$ 321

$$\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3g}-\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cf-bg)\int\frac{\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ag^2-bfg+cf^2}{}}{3g}$$

$\downarrow$ 327

$$\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3g}-\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cf-bg)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ag^2-}{}}{3g}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/Sqrt[f + g*x],x]
```

output
```
(2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*g) - ((Sqrt[2]*Sqrt[b^2 - 4*a*c
]*(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]
*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/
Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(
c*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x +
 c*x^2]) - (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f
 + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))
/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)])/(c*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))/(3*g)
```

3.896. $\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

3.896.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.896.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 810 vs. $2(386) = 772$.

Time = 1.28 (sec) , antiderivative size = 811, normalized size of antiderivative = 1.83

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+bx+a)}\left(\frac{2\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{3g}+\frac{2\left(a-\frac{2\left(\frac{ag}{2}+\frac{bf}{2}\right)}{3g}\right)\left(\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b+\sqrt{\ldots}}{\ldots}}{-\frac{f}{g}-\frac{-b+\sqrt{\ldots}}{\ldots}}}\ldots}{\sqrt{cg\,x^3+bg\,x^{\ldots}}}\ldots\right.}{\ldots}$
risch	Expression too large to display
default	Expression too large to display

input

```
int((c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/3/g*(c*
g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2*(a-2/3/g*(1/2*a*g+1/2*b*f))
*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2
))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^
2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*Ellip
ticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*
a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(b-2/3/g
*(b*g+c*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*
c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b
+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(
b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^
(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b
+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-
1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*Ellip
ticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*
a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

3.896.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 417, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(3\sqrt{cx^2+bx+a}\sqrt{gx+f}c^2g^2+(2c^2f^2-2bcfg-(b^2-6ac)g^2)\sqrt{cg}\text{weierstrassPInverse}\left(\frac{4(c^2f^2-bcfg+}{3c^2}\right.\right.}{}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(3*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*c^2*g^2 + (2*c^2*f^2 - 2*b*c*f*
g - (b^2 - 6*a*c)*g^2)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*
g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^
2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*
f + b*g)/(c*g)) + 3*(2*c^2*f*g - b*c*g^2)*sqrt(c*g)*weierstrassZeta(4/3*(c
^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^
2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), we
ierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -
4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a
*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g))))/(c^2*g^3)
```

3.896.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/sqrt(f + g*x), x)
```

3.896.7 Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/sqrt(g*x + f), x)
```

3.896.8 Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/sqrt(g*x + f), x)
```

3.896.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(f + g*x)^(1/2),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(f + g*x)^(1/2), x)
```

3.897 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$

3.897.1 Optimal result

Integrand size = 31, antiderivative size = 700

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{eg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}(cef+cdg-beg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{ce^2g\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$- \frac{\sqrt{2}(cd^2-bde+ae^2)\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticP}}{\sqrt{c}e^2(ef-dg)\sqrt{a+bx+cx^2}}$$

output
```
EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1
/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1
/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)
/e/g/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2
)-2*(-b*e*g+c*d*g+c*e*f)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+
b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+
b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^2/g/(g*
x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)-(a*e^2-b*d*e+c*d^2)*EllipticPi(2^(1/2)*c^(1
/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+
g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*
f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c
+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)
/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^2/(-d*g+e*f)/c^(1/2)/(c*x^2+b*x
+a)^(1/2)
```

3.897.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 31.83 (sec) , antiderivative size = 1261, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$$

$$= \frac{(f+gx)^{3/2}\left(\frac{4eg^2(-ef+dg)\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{\left(b^2-4ac\right)g^2}}}(a+x(b+cx))}{(f+gx)^2} - \frac{i\sqrt{2}e(-ef+dg)\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)\sqrt{\frac{-2ag^2+2cfgx+bg(f-gx \cdots}{\left(2cf-bg+\sqrt{\left(b^2 \cdots\right.}\right.}}}{\cdots} \cdots\right)}{\cdots}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]
```

output
```
((f + g*x)^(3/2)*((4*e*g^2*(-(e*f) + d*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/
(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*(a + x*(b + c*x)))/(f + g*x)^2 -
 (I*Sqrt[2]*e*(-(e*f) + d*g)*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[
(-2*a*g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))
/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 - 2*c*
f*g*x + b*g*(-f + g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((-2*c*f + b*g
 + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[
(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f
+ g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b
^2 - 4*a*c)*g^2]))]])/Sqrt[f + g*x] + (I*Sqrt[2]*e*(2*c*d*f*g + 2*a*e*g^2 -
 e*f*Sqrt[(b^2 - 4*a*c)*g^2] + d*g*Sqrt[(b^2 - 4*a*c)*g^2] - b*g*(e*f + d*
g))*Sqrt[(-2*a*g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(
f + g*x))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g
^2 - 2*c*f*g*x + b*g*(-f + g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((-2*
c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticF[I*ArcSinh[(Sqrt
[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]
)/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g
+ Sqrt[(b^2 - 4*a*c)*g^2]))]])/Sqrt[f + g*x] + ((2*I)*Sqrt[2]*(-(c*d^2) + e
*(b*d - a*e))*g^2*Sqrt[(-2*a*g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[(b^2 -
 4*a*c)*g^2]*(f + g*x))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g...
```

3.897.3 Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 824, normalized size of antiderivative = 1.18, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {1274, 1269, 1172, 321, 327, 1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{1274}$$

$$\frac{\left(ae^2-bde+cd^2\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}-\frac{\int \frac{cd-be-cex}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\left(ae^2-bde+cd^2\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}-\frac{\frac{(-beg+cdg+cef)\int \frac{1}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{g}-\frac{ce\int \frac{\sqrt{f+gx}}{\sqrt{cx^2+bx+a}}dx}{g}}{e^2}$$

$$\downarrow 1172$$

$$\frac{\left(ae^2-bde+cd^2\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(-beg+cdg+cef)\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{cg\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\ldots}{e^2}$$

$$\downarrow 321$$

$$\frac{\left(ae^2-bde+cd^2\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(-beg+cdg+cef)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}e\ldots}{\ldots}}{e^2}$$

$$\downarrow 327$$

$$\frac{\left(ae^2-bde+cd^2\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(-beg+cdg+cef)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}e\ldots}{\ldots}}{e^2}$$

$$\downarrow 1279$$

$$\frac{\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\left(ae^2-bde+cd^2\right)\int \frac{1}{\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}(d+ex)\sqrt{f+gx}}dx}{e^2\sqrt{a+bx+cx^2}}-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(-beg+cdg+cef)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}e\ldots}{\ldots}}{e^2}$$

$$\downarrow 187$$

3.897. $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$

$$-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\left(ae^2-bde+cd^2\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{b+\frac{2c(f+gx)}{g}-\sqrt{b^2-4ac}-\frac{2cf}{g}}\sqrt{b-}}}{e^2\sqrt{a+bx+cx^2}}$$

$$\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(-beg+cdg+cef)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}e\cdots}{}}{e^2}$$

$$\Big\downarrow \textbf{413}$$

$$-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\left(ae^2-bde+cd^2\right)\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\int \frac{}{(ef-dg-e(f+gx))\sqrt{b-}}}{e^2\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(f+gx)}{g}-\frac{2cf}{g}}}$$

$$\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(-beg+cdg+cef)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}e\cdots}{}}{e^2}$$

$$\Big\downarrow \textbf{413}$$

$$-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\left(ae^2-bde+cd^2\right)\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}\cdots\right.}}}{e^2\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(f+gx)}{g}-\frac{2cf}{g}}\sqrt{\sqrt{b^2-4a\cdots}}}$$

$$\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(-beg+cdg+cef)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}e\cdots}{}}{e^2}$$

$$\Big\downarrow \textbf{412}$$

$$-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}(cef+cdg-beg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{f+gx}\sqrt{cx^2+bx+a}}-\frac{\sqrt{2}\cdots}{}}{e^2}$$

$$\frac{\sqrt{2}\left(cd^2-bed+ae^2\right)\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-\cdots}\right.}}}{\sqrt{c}e^2(ef-dg)\sqrt{cx^2+bx+a}\sqrt{b+\frac{2c(f+gx)}{g}-\sqrt{\cdots}}}$$

3.897. $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$

input
```
Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]
```

output
```
-((-((Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2
))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/S
qrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)])/(g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)
]*Sqrt[a + b*x + c*x^2])) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*e*f + c*d*g -
b*e*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(
a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f
 - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))
/e^2) - (Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*
c])*g]*Sqrt[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*
c*x]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1
- (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*
f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]
*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (2*c*f - (b - Sq
rt[b^2 - 4*a*c])*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[c]*e^2*(e*
f - d*g)*Sqrt[a + b*x + c*x^2]*Sqrt[b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g + (2
*c*(f + g*x))/g]*Sqrt[b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g + (2*c*(f + g*x))/
g])
```

3.897.3.1 Defintions of rubi rules used

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.897. $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}} \, dx$

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1274
```
Int[Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]/(((d_.) + (e_.)*(x_))*Sqrt[(f_.
) + (g_.)*(x_)]), x_Symbol] :> Simp[(c*d^2 - b*d*e + a*e^2)/e^2   Int[1/((d
 + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] - Simp[1/e^2   Int[(c*
d - b*e - c*e*x)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

3.897. $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$

3.897.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 1114, normalized size of antiderivative = 1.59

method	result	size
elliptic	Expression too large to display	1114
default	Expression too large to display	3126

input
```
int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2*(b*e-c*
d)/e^2*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2
)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*
a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4
*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)
*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(
b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*c/
e*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/
2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b
^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+
b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f
/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+
(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+f
/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(
1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))+2*(a*e^2-b*d*e+c*d^
2)/e^3*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2
)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*
a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4
*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1...
```

3.897.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.897.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*sqrt(f + g*x)), x)
```

3.897.7 Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(ex+d)\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*sqrt(g*x + f)), x)
```

3.897.8 Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(ex+d)\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*sqrt(g*x + f)), x)
```

3.897.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}\,\left(d+e\,x\right)}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)), x)
```

3.898 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$

3.898.1 Optimal result

Integrand size = 31, antiderivative size = 736

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)}$$
$$+\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$
$$+\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{\frac{c(a+x(b+cx))}{-b^2+4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\frac{2\sqrt{b^2-4ac}g}{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e^2\sqrt{f+gx}\sqrt{a+x(b+cx)}}$$
$$-\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\left(e^2(bf-ag)-cd(2ef-dg)\right)\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}}{\sqrt{2}\sqrt{c}e^2(ef-dg)^2\sqrt{a+bx+}}$$

output
```
-(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(e*x+d)+1/2*EllipticE(1/2*((
b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*
c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(
g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e/(-d*g+e*f)*2^(1/2)/(c
*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+Ellip
ticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2
^(1/2)*(g*(-4*a*c+b^2)^(1/2)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(
1/2)*(-4*a*c+b^2)^(1/2)*(c*(a+x*(c*x+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+f)/(2*
c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^2/(g*x+f)^(1/2)/(a+x*(c*x+b))^(1/2)
-1/2*(e^2*(-a*g+b*f)-c*d*(-d*g+2*e*f))*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^
(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^
2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+
b^2)^(1/2)))^(1/2))*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)
*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c
+b^2)^(1/2))))^(1/2)/e^2/(-d*g+e*f)^2*2^(1/2)/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.898.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.66 (sec) , antiderivative size = 1471, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \frac{\sqrt{f+gx}\sqrt{a+x(b+cx)}}{(-ef+dg)(d+ex)} + \frac{(f+gx)^{3/2}\sqrt{a+x(b+cx)}\left(-4e(-ef+dg)\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}\left(c\left(-1+\frac{f}{f+gx}\right)^2+\frac{g\left(b-\frac{bf}{f+gx}+\frac{ag}{f+gx}\right)}{f+gx}\right)\right.}{}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)^2*Sqrt[f + g*x]),x]
```

output
```
(Sqrt[f + g*x]*Sqrt[a + x*(b + c*x)])/((-(e*f) + d*g)*(d + e*x)) + ((f + g
*x)^(3/2)*Sqrt[a + x*(b + c*x)]*(-4*e*(-(e*f) + d*g)*Sqrt[(c*f^2 + g*(-(b*
f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*(c*(-1 + f/(f + g*x))
^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*x)) + (I*Sqrt[2]*e
*(-(e*f) + d*g)*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(Sqrt[(b^2 -
4*a*c)*g^2] - (2*a*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f + g*x)) + b*g*(-1 + (
2*f)/(f + g*x)))/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^2
- 4*a*c)*g^2] + (2*a*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f + g*x)) + b*(g - (2
*f*g)/(f + g*x)))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*EllipticE[I*Ar
cSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*
a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2
*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x] - (I*Sqrt[2]*e*(-2*
a*e*g^2 - e*f*Sqrt[(b^2 - 4*a*c)*g^2] + d*g*Sqrt[(b^2 - 4*a*c)*g^2] + b*g*
(3*e*f - d*g) + 2*c*f*(-2*e*f + d*g))*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] - (2*a
*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f + g*x)) + b*g*(-1 + (2*f)/(f + g*x)))/(
2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] + (2
*a*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f + g*x)) + b*(g - (2*f*g)/(f + g*x)))/
(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqr
t[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[
f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sq...
```

3.898.3 Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 869, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {1275, 2154, 1269, 1172, 321, 327, 1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{ 1275}$$

$$\frac{\int \frac{cgx^2+2cfx+bf-ag}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)}-\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)}$$

$$\Big\downarrow \text{ 2154}$$

$$\frac{\left(-ag+bf-\frac{cd(2ef-dg)}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\int \frac{\frac{2cf}{e}-\frac{cdg}{e^2}+\frac{cgx}{e}}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)}-\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)}$$

$$\Big\downarrow \; 1269$$

$$\frac{\frac{c(ef-dg)\int \frac{1}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}+\left(-ag+bf-\frac{cd(2ef-dg)}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\frac{c\int \frac{\sqrt{f+gx}}{\sqrt{cx^2+bx+a}}dx}{e}}{2(ef-dg)}-\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)}$$

$$\Big\downarrow \; 1172$$

$$\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}+\frac{\sqrt{2}\sqrt{b^2-4a\cdots}}{\cdots}}{2(ef\cdots}-\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)}$$

$$\Big\downarrow \; 321$$

$$\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\int \frac{\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{e\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}+\left(-ag+bf-\frac{cd(2ef-dg)}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2\cdots}}\cdots}{2(ef-\cdots}-\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)}$$

$$\Big\downarrow \; 327$$

$$\frac{\left(-ag+bf-\frac{cd(2ef-dg)}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\text{ar}\cdots\right.}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\cdots}}{2(e\cdots}-\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)}$$

3.898. $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$

$\big\downarrow$ 1279

$$\frac{\frac{\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\left(-ag+bf-\frac{cd(2ef-dg)}{e^2}\right)\int\frac{1}{\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}(d+ex)\sqrt{f+gx}}dx}{\sqrt{a+bx+cx^2}}+\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a}{b}}}{}}{\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)}}$$

$\big\downarrow$ 187

$$\frac{-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\left(-ag+bf-\frac{cd(2ef-dg)}{e^2}\right)\int\frac{1}{(ef-dg-e(f+gx))\sqrt{b+\frac{2c(f+gx)}{g}-\sqrt{b^2-4ac}-\frac{2cf}{g}}\sqrt{b+\frac{2c(f+gx)}{g}+\sqrt{b^2-4ac}-\frac{2c}{g}}}}{\sqrt{a+bx+cx^2}}}{\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)}}$$

$\big\downarrow$ 413

$$\frac{-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\left(-ag+bf-\frac{cd(2ef-dg)}{e^2}\right)\int\frac{1}{(ef-dg-e(f+gx))\sqrt{b+\frac{2c(f+gx)}{g}+\sqrt{b^2-4ac}-\frac{2cf}{g}}\sqrt{1}}}{\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(f+gx)}{g}-\frac{2cf}{g}}}}{\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)}}$$

$\big\downarrow$ 413

$$\frac{-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(-ag+bf-\frac{cd(2ef-dg)}{e^2}\right)\int\frac{}{(ef-dg-e(f+gx))\sqrt{1-\frac{}{2c}}}}{\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(f+gx)}{g}-\frac{2cf}{g}}\sqrt{\sqrt{b^2-4ac}+b+\frac{2c(f+gx)}{g}-\frac{2cf}{g}}}}{\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)}}$$

$\big\downarrow$ 412

3.898. $\int\frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$

$$\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}+\frac{2\sqrt{2}\sqrt{b^2-4ac}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{}}{}}{}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{(ef-dg)(d+ex)}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/((d + e*x)^2*Sqrt[f + g*x]),x]
```

output
```
-((Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(d + e*x))) + ((Sqrt[
2]*Sqrt[b^2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a
*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a
*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g
)])/(e*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*
x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e*f - d*g)*Sqrt[(c*(f + g*x))/
(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4
*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4
*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])
*g)])/(e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[2*c*f - (b
 - Sqrt[b^2 - 4*a*c])*g]*(b*f - a*g - (c*d*(2*e*f - d*g))/e^2)*Sqrt[b - Sq
rt[b^2 - 4*a*c] + 2*c*x]*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[1 - (2*c
*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/
(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2
 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sq
rt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (2*c*f - (b - Sqrt[b^2 - 4*a*c])*g
)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[c]*(e*f - d*g)*Sqrt[a + b*x
+ c*x^2]*Sqrt[b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g + (2*c*(f + g*x))/g]*Sqrt[
b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g + (2*c*(f + g*x))/g]))/(2*(e*f - d*g))
```

3.898.3.1 Defintions of rubi rules used

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

3.898. $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.898. $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$

rule 1275
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])/Sq
rt[(f_.) + (g_.)*(x_)], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g))), x] - Simp[1/(2*(m + 1)*(e*f -
 d*g))   Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp
[b*f + a*g*(2*m + 3) + 2*(c*f + b*g*(m + 2))*x + c*g*(2*m + 5)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

3.898.4 Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 1208, normalized size of antiderivative = 1.64

method	result	size
elliptic	Expression too large to display	1208
default	Expression too large to display	13874

input
```
int((c*x^2+b*x+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(1/(d*g-e*
f)*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(e*x+d)+2*(c/e^2-1/2*c*
d/e^2*g/(d*g-e*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b
+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/
2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/
g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*
x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),
((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))
^(1/2))-c*g/(d*g-e*f)/e*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1
/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f
/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x
+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)
/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2
))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^
(1/2))*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g
+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)
))+(a*e^2*g-b*e^2*f-c*d^2*g+2*c*d*e*f)/e^3/(d*g-e*f)*(f/g-1/2*(b+(-4*a*c+b
^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c
*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(...
```

3.898.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.898.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)**2/(g*x+f)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/((d + e*x)**2*sqrt(f + g*x)), x)
```

3.898.7 Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(ex+d)^2\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)^2*sqrt(g*x + f)), x)
```

3.898.8 Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(ex+d)^2\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)^2*sqrt(g*x + f)), x)
```

3.898.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}\,\left(d+e\,x\right)^2}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^2),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^2), x)
```

3.899 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

3.899.1 Optimal result

Integrand size = 31, antiderivative size = 1049

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2}$$
$$+\frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4\,(cd^2-bde+ae^2)\,(ef-dg)^2(d+ex)}$$
$$-\frac{\sqrt{b^2-4ac}(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{4\sqrt{2}e\,(cd^2-bde+ae^2)\,(ef-dg)^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$
$$-\frac{\sqrt{b^2-4ac}(e^2(bf-ag)+cd(-2ef+dg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{\frac{c(a+x(b+cx))}{-b^2+4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-}}{\sqrt{b^2-}}}}{\sqrt{2}}\right.\right.}{2\sqrt{2}e^2\,(cd^2+e(-bd+ae))\,(ef-dg)\sqrt{f+gx}\sqrt{a+x(b+cx)}}$$
$$-\frac{\sqrt{2cf-bg+\sqrt{b^2-4ac}g}(3a^2e^4g^2+c^2d^3g(4ef-dg)+b^2e^3f(-ef+4dg)+2ace^2(2e^2f^2-2defg+3}{4}$$

output
```
-1/2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(e*x+d)^2+1/4*(c*d*(d*g+
2*e*f)-e*(-3*a*e*g+2*b*d*g+b*e*f))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(a*e^
2-b*d*e+c*d^2)/(-d*g+e*f)^2/(e*x+d)-1/8*(c*d*(d*g+2*e*f)-e*(-3*a*e*g+2*b*d
*g+b*e*f))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))
^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^
(1/2))*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1
/2)/e/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)^2*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x
+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-1/4*(e^2*(-a*g+b*f)+c*d*(d*g-2
*e*f))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/
2)*2^(1/2),2^(1/2)*(g*(-4*a*c+b^2)^(1/2)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2)))
)^(1/2))*(-4*a*c+b^2)^(1/2)*(c*(a+x*(c*x+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+f)
/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^2/(c*d^2+e*(a*e-b*d))/(-d*g+e*f
)*2^(1/2)/(g*x+f)^(1/2)/(a+x*(c*x+b))^(1/2)-1/8*(3*a^2*e^4*g^2+c^2*d^3*g*(
-d*g+4*e*f)+b^2*e^3*f*(4*d*g-e*f)+2*a*c*e^2*(3*d^2*g^2-2*d*e*f*g+2*e^2*f^2
)-2*b*e^2*g*(3*c*d^2*f+a*e*(2*d*g+e*f)))*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f
)^(1/2)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))^(1/2),(2*c*e*f-b*e*g+e*g*(-4*a*c+
b^2)^(1/2))/(-2*c*d*g+2*c*e*f),((2*c*f+g*(-b+(-4*a*c+b^2)^(1/2)))/(2*c*f-g
*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))^(1/2)*(g
*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f+g*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(g
*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/...
```

3.899.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 36.62 (sec) , antiderivative size = 36617, normalized size of antiderivative = 34.91

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
```

output
```
Result too large to show
```

3.899.3 Rubi [A] (verified)

Time = 3.55 (sec) , antiderivative size = 1613, normalized size of antiderivative = 1.54, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.581$, Rules used = {1275, 2154, 1282, 25, 2154, 25, 27, 1172, 321, 1269, 1172, 321, 327, 1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}dx$$

$$\Big\downarrow \; 1275$$

$$\frac{\int \frac{-cgx^2+2(cf-bg)x+bf-3ag}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{4(ef-dg)}-\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2(ef-dg)}$$

$$\Big\downarrow \; 2154$$

$$\frac{\int \frac{\frac{2cf}{e}-\frac{2bg}{e}+\frac{cdg}{e^2}-\frac{cgx}{e}}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx-\frac{(cd(dg+2ef)-e(-3aeg+2bdg+bef))\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}}{4(ef-dg)}-\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2(ef-dg)}$$

$$\Big\downarrow \; 1282$$

$$\frac{\int \frac{\frac{2cf}{e}-\frac{2bg}{e}+\frac{cdg}{e^2}-\frac{cgx}{e}}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx-\frac{(cd(dg+2ef)-e(-3aeg+2bdg+bef))\left(-\frac{\int -\frac{ce^2gx^2+2cdegx+2cd(ef-dg)-e(bef-2bdg+aeg)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)\left(ae^2-bde+cd^2\right)}-\frac{e^2\sqrt{f+gx}\sqrt{}}{(d+ex)(ef-dg)(}\right.}{e^2}}{4(ef-dg)}-\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2(ef-dg)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\frac{2cf}{e}-\frac{2bg}{e}+\frac{cdg}{e^2}-\frac{cgx}{e}}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx-\frac{(cd(dg+2ef)-e(-3aeg+2bdg+bef))\left(\frac{\int \frac{ce^2gx^2+2cdegx+2cd(ef-dg)-e(bef-2bdg+aeg)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)\left(ae^2-bde+cd^2\right)}-\frac{e^2\sqrt{f+gx}\sqrt{a+}}{(d+ex)(ef-dg)\left(ae^2\right.}\right.}{e^2}}{4(ef-dg)}-\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2(ef-dg)}$$

$$\Big\downarrow \; 2154$$

$$-\frac{(cd(dg+2ef)-e(-3aeg+2bdg+bef))\left(\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\int \frac{cdg+cexg}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)\left(ae^2-bde+cd^2\right)}-\frac{e^2\sqrt{f+gx}\sqrt{a+}}{(d+ex)(ef-dg)\left(ae^2\right.}\right.}{e^2}$$

$$4(ef-dg)$$

$$\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2(ef-dg)}$$

$$\downarrow 25$$

$$-\frac{(cd(dg+2ef)-e(-3aeg+2bdg+bef))\left(\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\int \frac{cdg+cexg}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)\left(ae^2-bde+cd^2\right)}-\frac{e^2\sqrt{f+gx}\sqrt{a+}}{(d+ex)(ef-dg)\left(ae^2\right.}\right.}{e^2}$$

$$4(ef-dg)$$

$$\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2(ef-dg)}$$

$$\downarrow 27$$

$$-\frac{(cd(dg+2ef)-e(-3aeg+2bdg+bef))\left(\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\int \frac{cdg+cexg}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)\left(ae^2-bde+cd^2\right)}-\frac{e^2\sqrt{f+gx}\sqrt{a+}}{(d+ex)(ef-dg)\left(ae^2\right.}\right.}{e^2}$$

$$4(ef-dg)$$

$$\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2(ef-dg)}$$

$$\downarrow 1172$$

$$-\frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{(cd(dg+2ef)-e}{}$$

$$\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2(ef-dg)}$$

$$\downarrow 321$$

$$-\frac{(cd(dg+2ef)-e(-3aeg+2bdg+bef))\left(\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\int \frac{cdg+cexg}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)\left(ae^2-bde+cd^2\right)}-\frac{e^2\sqrt{f+gx}\sqrt{a+}}{(d+ex)(ef-dg)\left(ae^2\right.}\right.}{e^2}$$

$$\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2(ef-dg)}$$

3.899. $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

$$\Big\downarrow \text{1269}$$

$$-\frac{(cd(dg+2ef)-e(-3aeg+2bdg+bef))\left(\frac{-c(ef-dg)\int \frac{1}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+ce\int \frac{\sqrt{}}{\sqrt{cx}}}{2(ef-dg)\left(ae^2-bde+cd^2\right)}\right.}{e^2}$$

$$\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2(ef-dg)}$$

$$\Big\downarrow \text{1172}$$

$$-\frac{-\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}+\frac{2(cef+cdg-beg)\int}{}}{}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2(ef-dg)(d+ex)^2}$$

$$\Big\downarrow \text{321}$$

$$-\frac{-\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}+\frac{2(cef+cdg-beg)\int}{}}{}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2(ef-dg)(d+ex)^2}$$

$$\Big\downarrow \text{327}$$

3.899. $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

$$\frac{-\frac{2\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}+\frac{2(cef+cdg-beg)\int}{}}{}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2(ef-dg)(d+ex)^2}$$

$\Big\downarrow$ 1279

$$\frac{-\frac{2\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}+\frac{2(cef+cdg-beg)\sqrt{}}{}}{}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2(ef-dg)(d+ex)^2}$$

$\Big\downarrow$ 187

$$\frac{-\frac{2\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}-\frac{4(cef+cdg-beg)\sqrt{}}{}}{}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2(ef-dg)(d+ex)^2}$$

$\Big\downarrow$ 413

3.899. $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}-\frac{4(cef+cdg-beg)\sqrt{\cdots}}{\cdots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2(ef-dg)(d+ex)^2}$$

$\downarrow$ 413

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}-\frac{4(cef+cdg-beg)\sqrt{\cdots}}{\cdots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2(ef-dg)(d+ex)^2}$$

$\downarrow$ 412

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b\cdots}\right.}}{\cdots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2(ef-dg)(d+ex)^2}$$

3.899. $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

input
```
Int[Sqrt[a + b*x + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
```

output
```
-1/2*(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(d + e*x)^2) + ((-
2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 -
4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt
[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(e^2*Sqrt[f + g*x]*
Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g
]*(c*e*f + c*d*g - b*e*g)*Sqrt[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[b + Sqr
t[b^2 - 4*a*c] + 2*c*x]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 -
4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*
EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcS
in[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]
], (2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)
])/(Sqrt[c]*e^2*(e*f - d*g)*Sqrt[a + b*x + c*x^2]*Sqrt[b - Sqrt[b^2 - 4*a*
c] - (2*c*f)/g + (2*c*(f + g*x))/g]*Sqrt[b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g
 + (2*c*(f + g*x))/g]) - ((c*d*(2*e*f + d*g) - e*(b*e*f + 2*b*d*g - 3*a*e*
g))*(-((e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*
(e*f - d*g)*(d + e*x))) + ((Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*Sqrt[f + g*x]*Sqrt
[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b
^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)
/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[(c*(f + g*x))/(2*c*f - (b ...
```

3.899.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.899. $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

rule 1275
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])/Sq
rt[(f_.) + (g_.)*(x_)], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g))), x] - Simp[1/(2*(m + 1)*(e*f -
 d*g))   Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp
[b*f + a*g*(2*m + 3) + 2*(c*f + b*g*(m + 2))*x + c*g*(2*m + 5)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1282
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^
(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*
e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*
g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

3.899.4 Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 1634, normalized size of antiderivative = 1.56

method	result	size
elliptic	Expression too large to display	1634
default	Expression too large to display	57841

input
```
int((c*x^2+b*x+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(1/2/(d*g-
e*f)*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(e*x+d)^2+1/4*(3*a*e^
2*g-2*b*d*e*g-b*e^2*f+c*d^2*g+2*c*d*e*f)/(d*g-e*f)/(a*d*e^2*g-a*e^3*f-b*d^
2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*
f)^(1/2)/(e*x+d)-1/4*c*g*(a*d*e^2*g+2*a*e^3*f-3*b*d*e^2*f-c*d^3*g+4*c*d^2*
e*f)/(d*g-e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)/e
^2*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1
/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+
b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c
+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*Ell
ipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-
4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))-1/4*c*g*
(3*a*e^2*g-2*b*d*e*g-b*e^2*f+c*d^2*g+2*c*d*e*f)/(d*g-e*f)/(a*d*e^2*g-a*e^3
*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)/e*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2)
)/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*
a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-
4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b
*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))
)*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*
(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+...
```

3.899.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.899.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{\left(d+ex\right)^3\sqrt{f+gx}}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)**3/(g*x+f)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/((d + e*x)**3*sqrt(f + g*x)), x)
```

3.899.7 Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{\left(ex+d\right)^3\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)^3*sqrt(g*x + f)), x)
```

3.899.8 Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{\left(ex+d\right)^3\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)^3*sqrt(g*x + f)), x)
```

3.899.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^3}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^3),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^3), x)
```

3.900 $\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

3.900.1 Optimal result

Integrand size = 31, antiderivative size = 774

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{2e(24b^2e^2g^2+ceg(13bef-84bdg-25aeg)-c^2(7e^2f^2+12defg-90d^2g^2))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{105c^3g^2}$$

$$+ \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7c}$$

$$+ \frac{2e^2(cef+11cdg-6beg)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{35c^2g^2}$$

$$- \frac{\sqrt{2}\sqrt{b^2-4ac}(48b^3e^3g^3-8bce^2g^2(2bef+21bdg+13aeg)-c^3(8e^3f^3-42de^2f^2g+105d^2efg^2+105d^3 \ldots}{10 \ldots}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}e(cf^2-bfg+ag^2)\left(24b^2e^2g^2+ceg(13bef-84bdg-25aeg)+c^2(8e^2f^2-42defg+10 \ldots\right.}{105c^4g^3\sqrt{f+gx} \ldots}$$

output
```
2/35*e^2*(-6*b*e*g+11*c*d*g+c*e*f)*(g*x+f)^(3/2)*(c*x^2+b*x+a)^(1/2)/c^2/g
^2+2/105*e*(24*b^2*e^2*g^2+c*e*g*(-25*a*e*g-84*b*d*g+13*b*e*f)-c^2*(-90*d^
2*g^2+12*d*e*f*g+7*e^2*f^2))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^3/g^2+2/7
*e*(e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c-1/105*(48*b^3*e^3*g^3-8*b
*c*e^2*g^2*(13*a*e*g+21*b*d*g+2*b*e*f)-c^3*(105*d^3*g^3+105*d^2*e*f*g^2-42
*d*e^2*f^2*g+8*e^3*f^3)+c^2*e*g*(a*e*g*(189*d*g+19*e*f)-b*(-210*d^2*g^2-63
*d*e*f*g+9*e^2*f^2)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2
)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+
a)/(-4*a*c+b^2))^(1/2)/c^4/g^3/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+
(-4*a*c+b^2)^(1/2))))^(1/2)-2/105*e*(a*g^2-b*f*g+c*f^2)*(24*b^2*e^2*g^2+c*
e*g*(-25*a*e*g-84*b*d*g+13*b*e*f)+c^2*(105*d^2*g^2-42*d*e*f*g+8*e^2*f^2))*
EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1
/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1
/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2
*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^4/g^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^
(1/2)
```

3.900.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.65 (sec) , antiderivative size = 1402, normalized size of antiderivative = 1.81

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\sqrt{f+gx}(a+bx+cx^2)\left(-\frac{2e(4c^2e^2f^2-21c^2defg+5bce^2fg-105c^2d^2g^2+84bcdeg^2-24b^2e^2g^2+25ace^2g^2)}{105c^3g^2} - \frac{2e^2(-cef-21cdg+6beg}{35c^2g}\right.}{\sqrt{a+x(b+cx)}}$$

$$- \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}\left(-\left(\left(-48b^3e^3g^3+8bce^2g^2(2bef+21bdg+13aeg)+c^3(8e^3f^3-42de^2f^2g\right.\right.\right.}{}$$

input
```
Integrate[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

output
```
(Sqrt[f + g*x]*(a + b*x + c*x^2)*((-2*e*(4*c^2*e^2*f^2 - 21*c^2*d*e*f*g +
5*b*c*e^2*f*g - 105*c^2*d^2*g^2 + 84*b*c*d*e*g^2 - 24*b^2*e^2*g^2 + 25*a*c
*e^2*g^2))/(105*c^3*g^2) - (2*e^2*(-(c*e*f) - 21*c*d*g + 6*b*e*g)*x)/(35*c
^2*g) + (2*e^3*x^2)/(7*c)))/Sqrt[a + x*(b + c*x)] - (2*(f + g*x)^(3/2)*Sqr
t[a + b*x + c*x^2]*(-((-48*b^3*e^3*g^3 + 8*b*c*e^2*g^2*(2*b*e*f + 21*b*d*g
 + 13*a*e*g) + c^3*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^2 + 105*d^3
*g^3) - c^2*e*g*(a*e*g*(19*e*f + 189*d*g) + b*(-9*e^2*f^2 + 63*d*e*f*g + 2
10*d^2*g^2)))*(c*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f
 + g*x)))/(f + g*x))) - ((I/2)*Sqrt[1 - (2*(c*f^2 + g*(-(b*f) + a*g)))/((2
*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[1 + (2*(c*f^2 + g*(
-(b*f) + a*g)))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*((2*
c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(48*b^3*e^3*g^3 - 8*b*c*e^2*g^2*(2*b*
e*f + 21*b*d*g + 13*a*e*g) - c^3*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f
*g^2 + 105*d^3*g^3) + c^2*e*g*(a*e*g*(19*e*f + 189*d*g) + b*(-9*e^2*f^2 +
63*d*e*f*g + 210*d^2*g^2)))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f
*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((
-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*
g^2]))] + (48*b^4*e^3*g^4 - 8*b^3*e^2*g^3*(8*c*e*f + 21*c*d*g + 6*e*Sqrt[(
b^2 - 4*a*c)*g^2]) + b^2*c*e*g^2*(-152*a*e^2*g^2 + 8*e*Sqrt[(b^2 - 4*a*c)*
g^2]*(2*e*f + 21*d*g) + c*(e^2*f^2 + 231*d*e*f*g + 210*d^2*g^2)) - b*(-...
```

3.900.3 Rubi [A] (verified)

Time = 2.16 (sec) , antiderivative size = 795, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {1283, 25, 2184, 27, 2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1283}$$

$$\frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7c}-\frac{\int -\frac{(d+ex)\left(7cfd^2+e(cef+11cdg-6beg)x^2-e(bdf+4aef+adg)+(cd(12ef+7dg)-e(5bef+2bdg+5aeg))x\right)}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{7c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(d+ex)(7cfd^2+e(cef+11cdg-6beg)x^2-e(bdf+4aef+adg)+(cd(12ef+7dg)-e(5bef+2bdg+5aeg))x)}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{7c}+\frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7c}$$

$\downarrow$ 2184

$$2\int \frac{eg^2\left(-\left(\left(7e^2f^2+12degf-90d^2g^2\right)c^2\right)+eg(13bef-84bdg-25aeg)c+24b^2e^2g^2\right)x^2+g\left(6bg^2(5bf+3ag)e^3-cg\left(aeg(23ef+63dg)-b\left(7e^2f^2-85degf-10d^2g^2\right)\right)e-c\right.}{2\sqrt{f+gx}\sqrt{cx^2+}}$$

$$5cg^3$$

$$\frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7c}$$

$\downarrow$ 27

$$\int \frac{eg^2\left(-\left(\left(7e^2f^2+12degf-90d^2g^2\right)c^2\right)+eg(13bef-84bdg-25aeg)c+24b^2e^2g^2\right)x^2+g\left(6bg^2(5bf+3ag)e^3-cg\left(aeg(23ef+63dg)-b\left(7e^2f^2-85degf-10d^2g^2\right)\right)e-c^2\right.}{\sqrt{f+gx}\sqrt{cx^2+bx}}$$

$$5cg^3$$

$$\frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7c}$$

$\downarrow$ 2184

$$2\int -\frac{g^3\left(24b^3fg^2e^3+b^2g\left(24aeg^2-cf(5ef+84dg)\right)e^2-bc\left(6ae(11ef+14dg)g^2+cf\left(4e^2f^2-21degf-105d^2g^2\right)\right)e-cg\left(105c^2fgd^3+25a^2e^3g^2-ace\left(2e^2f^2+147degf+\right.\right.\right.}{2\sqrt{f+}}$$

$$\frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7c}$$

$\downarrow$ 27

$$\frac{2eg\sqrt{f+gx}\sqrt{a+bx+cx^2}\left(ceg(-25aeg-84bdg+13bef)+24b^2e^2g^2-\left(c^2\left(-90d^2g^2+12defg+7e^2f^2\right)\right)\right)}{3c}-\frac{g\int \frac{24b^3fg^2e^3+b^2g\left(24aeg^2-cf(5ef+84dg)\right)e^2-bc\left(6ae(11\right.}{}}{}$$

$$\frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7c}$$

$\downarrow$ 1269

$$\frac{2eg\sqrt{f+gx}\sqrt{a+bx+cx^2}\left(ceg(-25aeg-84bdg+13bef)+24b^2e^2g^2-\left(c^2\left(-90d^2g^2+12defg+7e^2f^2\right)\right)\right)}{3c}-\frac{g\left(\frac{\left(c^2eg\left(aeg(189dg+19ef)-b\left(-210d^2g^2-63defg+9e^2f^2\right)\right.\right.}{}\right.}{}$$

$$\frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7c}$$

3.900. $\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

$$\Big\downarrow \text{1172}$$

$$\frac{2e\sqrt{f+gx}\sqrt{cx^2+bx+a}(d+ex)^2}{7c}+$$

$$\frac{2(cef+11cdg-6beg)(f+gx)^{3/2}\sqrt{cx^2+bx+a}e^2}{5cg^2}+\frac{2eg\left(-\left(\left(7e^2f^2+12degf-90d^2g^2\right)c^2\right)+eg(13bef-84bdg-25aeg)c+24b^2e^2g^2\right)\sqrt{f+gx}\sqrt{cx^2+bx+a}}{3c}-$$

$$\Big\downarrow \text{321}$$

$$\frac{2e\sqrt{f+gx}\sqrt{cx^2+bx+a}(d+ex)^2}{7c}+$$

$$\frac{2(cef+11cdg-6beg)(f+gx)^{3/2}\sqrt{cx^2+bx+a}e^2}{5cg^2}+\frac{2eg\left(-\left(\left(7e^2f^2+12degf-90d^2g^2\right)c^2\right)+eg(13bef-84bdg-25aeg)c+24b^2e^2g^2\right)\sqrt{f+gx}\sqrt{cx^2+bx+a}}{3c}-$$

$$\Big\downarrow \text{327}$$

$$\frac{2e\sqrt{f+gx}\sqrt{cx^2+bx+a}(d+ex)^2}{7c}+$$

$$\frac{2(cef+11cdg-6beg)(f+gx)^{3/2}\sqrt{cx^2+bx+a}e^2}{5cg^2}+\frac{2eg\left(-\left(\left(7e^2f^2+12degf-90d^2g^2\right)c^2\right)+eg(13bef-84bdg-25aeg)c+24b^2e^2g^2\right)\sqrt{f+gx}\sqrt{cx^2+bx+a}}{3c}-$$

input
```
Int[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

3.900. $\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

output
```
(2*e*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(7*c) + ((2*e^2*(c*e
*f + 11*c*d*g - 6*b*e*g)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c*g^2)
+ ((2*e*g*(24*b^2*e^2*g^2 + c*e*g*(13*b*e*f - 84*b*d*g - 25*a*e*g) - c^2*(
7*e^2*f^2 + 12*d*e*f*g - 90*d^2*g^2))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
/(3*c) - (g*((Sqrt[2]*Sqrt[b^2 - 4*a*c]*(48*b^3*e^3*g^3 - 8*b*c*e^2*g^2*(2
*b*e*f + 21*b*d*g + 13*a*e*g) - c^3*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*
e*f*g^2 + 105*d^3*g^3) + c^2*e*g*(a*e*g*(19*e*f + 189*d*g) - b*(9*e^2*f^2
- 63*d*e*f*g - 210*d^2*g^2)))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(
b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[
b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 -
4*a*c])*g)])/(c*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*
Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(c*f^2 - b*f*g + a
*g^2)*(24*b^2*e^2*g^2 + c*e*g*(13*b*e*f - 84*b*d*g - 25*a*e*g) + c^2*(8*e^
2*f^2 - 42*d*e*f*g + 105*d^2*g^2))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[Ar
cSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2
*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*g*Sqrt[f +
g*x]*Sqrt[a + b*x + c*x^2])))/(3*c))/(5*c*g^3))/(7*c)
```

3.900.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.900. $\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1283
```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/(c*(2*m + 1))), x] - Simp[1/(c*(2*m + 1))   Int[((
d + e*x)^(m - 2)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[e*(b*d*f + a*(
d*g + 2*e*f*(m - 1))) - c*d^2*f*(2*m + 1) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f
*m + d*g*(2*m + 1)) + b*e*(2*d*g + e*f*(2*m - 1)))*x + e*(2*b*e*g*m - c*(e*
f + d*g*(4*m - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
ntegerQ[2*m] && GtQ[m, 1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.900. $\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

3.900.4 Maple [A] (verified)

Time = 3.62 (sec) , antiderivative size = 1283, normalized size of antiderivative = 1.66

method	result	size
elliptic	Expression too large to display	1283
risch	Expression too large to display	4891
default	Expression too large to display	14978

input
```
int((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/7*e^3/c
*x^2*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/5*(3*d*e^2*g+e^3*f-
2/7*e^3/c*(3*b*g+3*c*f))/c/g*x*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(
1/2)+2/3*(3*d^2*e*g+3*d*e^2*f-2/7*e^3/c*(5/2*a*g+5/2*b*f)-2/5*(3*d*e^2*g+e
^3*f-2/7*e^3/c*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(c*g*x^3+b*g*x^2+c*f*
x^2+a*g*x+b*f*x+a*f)^(1/2)+2*(d^3*f-2/5*(3*d*e^2*g+e^3*f-2/7*e^3/c*(3*b*g+
3*c*f))/c/g*f*a-2/3*(3*d^2*e*g+3*d*e^2*f-2/7*e^3/c*(5/2*a*g+5/2*b*f)-2/5*(
3*d*e^2*g+e^3*f-2/7*e^3/c*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(1/2*a*g+1
/2*b*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-
4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(
-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/
2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2
*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*
(d^3*g+3*d^2*e*f-4/7*a/c*f*e^3-2/5*(3*d*e^2*g+e^3*f-2/7*e^3/c*(3*b*g+3*c*f
))/c/g*(3/2*a*g+3/2*b*f)-2/3*(3*d^2*e*g+3*d*e^2*f-2/7*e^3/c*(5/2*a*g+5/2*b
*f)-2/5*(3*d*e^2*g+e^3*f-2/7*e^3/c*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(
b*g+c*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+
b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(
-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*...
```

3.900.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 879, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = -\frac{2\left((8\,c^4e^3f^4-(42\,c^4de^2-5\,bc^3e^3)f^3g+(105\,c^4d^2e-42\,bc^3de^2+(10\,b^2c^2-13\,ac^3)e^3)f^2g^2-(210\,c^4d^3\right.}{}$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
-2/315*((8*c^4*e^3*f^4 - (42*c^4*d*e^2 - 5*b*c^3*e^3)*f^3*g + (105*c^4*d^2
*e - 42*b*c^3*d*e^2 + (10*b^2*c^2 - 13*a*c^3)*e^3)*f^2*g^2 - (210*c^4*d^3
- 210*b*c^3*d^2*e + 21*(7*b^2*c^2 - 12*a*c^3)*d*e^2 - (40*b^3*c - 113*a*b*
c^2)*e^3)*f*g^3 + (105*b*c^3*d^3 - 105*(2*b^2*c^2 - 3*a*c^3)*d^2*e + 21*(8
*b^3*c - 21*a*b*c^2)*d*e^2 - (48*b^4 - 176*a*b^2*c + 75*a^2*c^2)*e^3)*g^4)
*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)
/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 +
 (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)) + 3*(8
*c^4*e^3*f^3*g - 3*(14*c^4*d*e^2 - 3*b*c^3*e^3)*f^2*g^2 + (105*c^4*d^2*e -
 63*b*c^3*d*e^2 + (16*b^2*c^2 - 19*a*c^3)*e^3)*f*g^3 + (105*c^4*d^3 - 210*
b*c^3*d^2*e + 21*(8*b^2*c^2 - 9*a*c^3)*d*e^2 - 8*(6*b^3*c - 13*a*b*c^2)*e^
3)*g^4)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g
^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^
2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f^2 - b
*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g -
3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x
 + c*f + b*g)/(c*g))) - 3*(15*c^4*e^3*g^4*x^2 - 4*c^4*e^3*f^2*g^2 + (21*c^
4*d*e^2 - 5*b*c^3*e^3)*f*g^3 + (105*c^4*d^2*e - 84*b*c^3*d*e^2 + (24*b^2*c
^2 - 25*a*c^3)*e^3)*g^4 + 3*(c^4*e^3*f*g^3 + 3*(7*c^4*d*e^2 - 2*b*c^3*e^3)
*g^4)*x)*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f))/(c^5*g^4)
```

3.900.6 Sympy [F]

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((e*x+d)**3*(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x)**3*sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

3.900.7 Maxima [F]

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^3\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((e*x + d)^3*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

3.900.8 Giac [F]

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^3\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

3.900.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}\,(d+e\,x)^3}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((f + g*x)^(1/2)*(d + e*x)^3)/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int(((f + g*x)^(1/2)*(d + e*x)^3)/(a + b*x + c*x^2)^(1/2), x)
```

3.901 $\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

3.901.1 Optimal result

Integrand size = 31, antiderivative size = 567

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{2e(cef+7cdg-4beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g} + \frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c}$$

$$+ \frac{\sqrt{2}\sqrt{b^2-4ac}(8b^2e^2g^2-ceg(3bef+20bdg+9aeg)-c^2(2e^2f^2-10defg-15d^2g^2))\sqrt{f+gx}\sqrt{-\frac{c(a+b\ldots}{b^2-\ldots}}\ldots}{15c^3g^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+ \frac{4\sqrt{2}\sqrt{b^2-4ac}e(cef-5cdg+2beg)(cf^2-bfg+ag^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\text{ar}\ldots\right.}{15c^3g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

output
```
2/15*e*(-4*b*e*g+7*c*d*g+c*e*f)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/g+2/
5*e*(e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c+1/15*(8*b^2*e^2*g^2-c*e*g*
(9*a*e*g+20*b*d*g+3*b*e*f)-c^2*(-15*d^2*g^2-10*d*e*f*g+2*e^2*f^2))*Ellipti
cE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2
*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4
*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^3/g^
2/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+4
/15*e*(2*b*e*g-5*c*d*g+c*e*f)*(a*g^2-b*f*g+c*f^2)*EllipticF(1/2*((b+2*c*x+
(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(
1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(
-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(
1/2))))^(1/2)/c^3/g^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.901.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 31.19 (sec) , antiderivative size = 1002, normalized size of antiderivative = 1.77

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{\left(-\frac{2e(-cef-10cdg+4beg)}{15c^2g}+\frac{2e^2x}{5c}\right)\sqrt{f+gx}(a+bx+cx^2)}{\sqrt{a+x(b+cx)}}$$

$$- \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}\left((-8b^2e^2g^2+ceg(3bef+20bdg+9aeg)+c^2(2e^2f^2-10defg-15d^2g^2))\right.}{}$$

input
```
Integrate[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

output
```
(((-2*e*(-(c*e*f) - 10*c*d*g + 4*b*e*g))/(15*c^2*g) + (2*e^2*x)/(5*c))*Sqr
t[f + g*x]*(a + b*x + c*x^2))/Sqrt[a + x*(b + c*x)] - (2*(f + g*x)^(3/2)*S
qrt[a + b*x + c*x^2]*((-8*b^2*e^2*g^2 + c*e*g*(3*b*e*f + 20*b*d*g + 9*a*e*
g) + c^2*(2*e^2*f^2 - 10*d*e*f*g - 15*d^2*g^2))*(c*(-1 + f/(f + g*x))^2 +
(g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*x)) + ((I/2)*Sqrt[1 - (
2*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f
+ g*x))]*Sqrt[1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2*c*f + b*g + Sqrt[(b^
2 - 4*a*c)*g^2])*(f + g*x))]*((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(8*b
^2*e^2*g^2 - c*e*g*(3*b*e*f + 20*b*d*g + 9*a*e*g) + c^2*(-2*e^2*f^2 + 10*d
*e*f*g + 15*d^2*g^2))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a
*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f
 + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))
] + (-30*c^3*d^2*f*g^2 + 8*b^2*e^2*g^2*(b*g - Sqrt[(b^2 - 4*a*c)*g^2]) + c
*e*g*(-17*a*b*e*g^2 + 9*a*e*g*Sqrt[(b^2 - 4*a*c)*g^2] + b*Sqrt[(b^2 - 4*a*
c)*g^2]*(3*e*f + 20*d*g) - b^2*g*(11*e*f + 20*d*g)) - c^2*(-15*b*d*g^2*(2*
e*f + d*g) - 2*a*e*g^2*(7*e*f + 10*d*g) + Sqrt[(b^2 - 4*a*c)*g^2]*(-2*e^2*
f^2 + 10*d*e*f*g + 15*d^2*g^2)))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2
- b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]]
, -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*
a*c)*g^2]))]))/(Sqrt[2]*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g +...
```

3.901.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {1283, 25, 2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1283}$$

$$\frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c}-\frac{\int -\frac{5cfd^2+e(cef+7cdg-4beg)x^2-e(bdf+2aef+adg)+(cd(8ef+5dg)-e(3bef+2bdg+3aeg))x}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{5c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{5cfd^2+e(cef+7cdg-4beg)x^2-e(bdf+2aef+adg)+(cd(8ef+5dg)-e(3bef+2bdg+3aeg))x}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{5c}+\frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c}}{}$$

$\downarrow$ 2184

$$\frac{\frac{2\int \frac{g\left(4b^2fge^2+b\left(4aeg^2-cf(ef+10dg)\right)e+cg\left(15cd^2f-ae(7ef+10dg)\right)\right)+\left(-\left(\left(2e^2f^2-10degf-15d^2g^2\right)c^2\right)-eg(3bef+20bdg+9aeg)c+8b^2e^2g^2\right)x}{2\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{3cg^2}+\frac{2e\sqrt{f+}}{}}{5c}+\frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{4b^2fge^2+b\left(4aeg^2-cf(ef+10dg)\right)e+cg\left(15cd^2f-ae(7ef+10dg)\right)+\left(-\left(\left(2e^2f^2-10degf-15d^2g^2\right)c^2\right)-eg(3bef+20bdg+9aeg)c+8b^2e^2g^2\right)x}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{3cg}+\frac{2e\sqrt{f+gx}\sqrt{}}{}}{5c}+\frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c}$$

$\downarrow$ 1269

$$\frac{\frac{\frac{\left(-ceg(9aeg+20bdg+3bef)+8b^2e^2g^2-\left(c^2\left(-15d^2g^2-10defg+2e^2f^2\right)\right)\right)\int \frac{\sqrt{f+gx}}{\sqrt{cx^2+bx+a}}dx}{g}+\frac{2e\left(ag^2-bfg+cf^2\right)(2beg-5cdg+cef)\int \frac{1}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{g}}{3cg}+\frac{2}{}}{5c}+\frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c}$$

$\downarrow$ 1172

$$\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-ceg(9aeg+20bdg+3bef)+8b^2e^2g^2-\left(c^2\left(-15d^2g^2-10defg+2e^2f^2\right)\right)\right)\int \frac{\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4a}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}}{}+\frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c}$$

$\downarrow$ 321

3.901. $\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-ceg(9aeg+20bdg+3bef)+8b^2e^2g^2-\left(c^2\left(-15d^2g^2-10defg+2e^2f^2\right)\right)\right)\int\frac{\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\sqrt{\frac{\frac{b+2cx+\sqrt{b^2-4a\cdots}}{\sqrt{b^2-4ac}}}{\sqrt{2}}}\cdots}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}\cdots$$

$$\frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-ceg(9aeg+20bdg+3bef)+8b^2e^2g^2-\left(c^2\left(-15d^2g^2-10defg+2e^2f^2\right)\right)\right)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4\cdots}}{2cf-\left(b+\sqrt{b^2\cdots}\right.}\cdots\right.}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}\cdots$$

$$\frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c}$$

input

```
Int[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

output

```
(2*e*(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(5*c) + ((2*e*(c*e*f +
 7*c*d*g - 4*b*e*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c*g) + ((Sqrt[
2]*Sqrt[b^2 - 4*a*c]*(8*b^2*e^2*g^2 - c*e*g*(3*b*e*f + 20*b*d*g + 9*a*e*g)
 - c^2*(2*e^2*f^2 - 10*d*e*f*g - 15*d^2*g^2))*Sqrt[f + g*x]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)])/(c*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(c
*e*f - 5*c*d*g + 2*b*e*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c*
f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)
)]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]
]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])
/(c*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))/(3*c*g))/(5*c)
```

3.901. $\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

3.901.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1283
```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/(c*(2*m + 1))), x] - Simp[1/(c*(2*m + 1))   Int[((
d + e*x)^(m - 2)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[e*(b*d*f + a*(
d*g + 2*e*f*(m - 1))) - c*d^2*f*(2*m + 1) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f
*m + d*g*(2*m + 1)) + b*e*(2*d*g + e*f*(2*m - 1)))*x + e*(2*b*e*g*m - c*(e*
f + d*g*(4*m - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
ntegerQ[2*m] && GtQ[m, 1]
```

3.901. $\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.901.4 Maple [A] (verified)

Time = 4.75 (sec) , antiderivative size = 985, normalized size of antiderivative = 1.74

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+bx+a)}\left(\frac{2e^2x\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{5c}+\frac{2\left(2deg+e^2f-\frac{2e^2(2bg+2cf)}{5c}\right)\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{3cg}+\frac{2\left(d\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/5*e^2/c
*x*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/3*(2*d*e*g+e^2*f-2/5*
e^2/c*(2*b*g+2*c*f))/c/g*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2
*(d^2*f-2/5*e^2/c*f*a-2/3*(2*d*e*g+e^2*f-2/5*e^2/c*(2*b*g+2*c*f))/c/g*(1/2
*a*g+1/2*b*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4
*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*
(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/
2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*
f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f
/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/
2))+2*(d^2*g+2*d*e*f-2/5*e^2/c*(3/2*a*g+3/2*b*f)-2/3*(2*d*e*g+e^2*f-2/5*e^
2/c*(2*b*g+2*c*f))/c/g*(b*g+c*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f
/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(
1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)
^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f
*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*Elliptic
E(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c
+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4
*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1
/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(...
```

3.901.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\left((2\,c^3e^2f^3-2\,(5\,c^3de-bc^2e^2)f^2g+(30\,c^3d^2-20\,bc^2de+(7\,b^2c-12\,ac^2)e^2)fg^2-(15\,bc^2d^2-10\,(2\,b^2c\right.}{}$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
2/45*((2*c^3*e^2*f^3 - 2*(5*c^3*d*e - b*c^2*e^2)*f^2*g + (30*c^3*d^2 - 20*
b*c^2*d*e + (7*b^2*c - 12*a*c^2)*e^2)*f*g^2 - (15*b*c^2*d^2 - 10*(2*b^2*c
- 3*a*c^2)*d*e + (8*b^3 - 21*a*b*c)*e^2)*g^3)*sqrt(c*g)*weierstrassPInvers
e(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3
- 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*
g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)) + 3*(2*c^3*e^2*f^2*g - (10*c^3*d*e
- 3*b*c^2*e^2)*f*g^2 - (15*c^3*d^2 - 20*b*c^2*d*e + (8*b^2*c - 9*a*c^2)*e^
2)*g^3)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g
^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^
2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f^2 - b
*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g -
3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x
 + c*f + b*g)/(c*g))) + 3*(3*c^3*e^2*g^3*x + c^3*e^2*f*g^2 + 2*(5*c^3*d*e
- 2*b*c^2*e^2)*g^3)*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f))/(c^4*g^3)
```

3.901.6 Sympy [F]

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((e*x+d)**2*(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x)**2*sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

3.901.7 Maxima [F]

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^2\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

3.901.8 Giac [F]

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^2\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

3.901.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}\,(d+e\,x)^2}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((f + g*x)^(1/2)*(d + e*x)^2)/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int(((f + g*x)^(1/2)*(d + e*x)^2)/(a + b*x + c*x^2)^(1/2), x)
```

3.902 $\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

3.902.1 Optimal result

Integrand size = 29, antiderivative size = 452

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3c}$$
$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(cef+3cdg-2beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right.}\right.}{3c^2g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}e(cf^2-bfg+ag^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3c^2g\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

output
```
2/3*e*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c+1/3*(-2*b*e*g+3*c*d*g+c*e*f)*Ell
ipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)
,(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)
*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^
2/g/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
-2/3*e*(a*g^2-b*f*g+c*f^2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*
c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*
c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^2/g/(g*
x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.902.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.34 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.41

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{2\sqrt{f+gx}\left(ce(a+x(b+cx)) + \frac{(f+gx)\left(\frac{g^2(cef+3cdg-2beg)(a+x(b+cx))}{(f+gx)^2} + \frac{i\sqrt{1-\frac{2\left(cf^2+g(-bf+ag)\right)}{\left(2cf-bg+\sqrt{\left(b^2-4ac\right)g^2}\right)(f+gx)}}\sqrt{1+\frac{2\left(cf^2+g(\cdots\right)}{\left(-2cf+bg+\sqrt{\left(b^2\cdots\right.}\right.}}}{\cdots}\right)}{\cdots}\right)}{\cdots}$$

input
```
Integrate[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[f + g*x]*(c*e*(a + x*(b + c*x)) + ((f + g*x)*((g^2*(c*e*f + 3*c*d*
g - 2*b*e*g)*(a + x*(b + c*x)))/(f + g*x)^2 + ((I/2)*Sqrt[1 - (2*(c*f^2 +
g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sq
rt[1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*
g^2])*(f + g*x))]*((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(2*b*e*g - c*(e
*f + 3*d*g))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2
*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g +
Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))] + (6*c^
2*d*f*g + 2*b*e*g*(b*g - Sqrt[(b^2 - 4*a*c)*g^2]) + c*(-2*a*e*g^2 - 3*b*g*
(e*f + d*g) + Sqrt[(b^2 - 4*a*c)*g^2]*(e*f + 3*d*g)))*EllipticF[I*ArcSinh[
(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g
^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f -
 b*g + Sqrt[(b^2 - 4*a*c)*g^2]))]))/(Sqrt[2]*Sqrt[(c*f^2 + g*(-(b*f) + a*g
))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[f + g*x])))/g^2))/(3*c^2
*Sqrt[a + x*(b + c*x)])
```

3.902.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1236, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{2\int \frac{3cdf-e(bf+ag)+(cef+3cdg-2beg)x}{2\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{3c}+\frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3cdf-e(bf+ag)+(cef+3cdg-2beg)x}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{3c}+\frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3c}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\frac{(-2beg+3cdg+cef)\int \frac{\sqrt{f+gx}}{\sqrt{cx^2+bx+a}}dx}{g}-\frac{e(ag^2-bfg+cf^2)\int \frac{1}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{g}}{3c}+\frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3c}$$

$$\Big\downarrow \text{1172}$$

$$\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(-2beg+3cdg+cef)\int \frac{\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(ag}{}}{3c}$$

$$\frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3c}$$

$$\Big\downarrow \text{321}$$

$$\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(-2beg+3cdg+cef)\int \frac{\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(ag}{}}{3c}$$

$$\frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3c}$$

$\downarrow$ 327

$$\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(-2beg+3cdg+cef)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx}{b^2-}}}{}}{3c}$$

$$\frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3c}$$

input
```
Int[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c) + ((Sqrt[2]*Sqrt[b^2 - 4*a
*c]*(c*e*f + 3*c*d*g - 2*b*e*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))
/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)])/(c*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)
]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(c*f^2 - b*f*g +
 a*g^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*
(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*
f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
)/(3*c)
```

3.902.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.902.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 820 vs. $2(394) = 788$.

Time = 1.54 (sec) , antiderivative size = 821, normalized size of antiderivative = 1.82

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+bx+a)}\left(\frac{2e\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{3c}+\frac{2\left(df-\frac{2e\left(\frac{ag}{2}+\frac{bf}{2}\right)}{3c}\right)\left(\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b-}{}}{-\frac{f}{g}-\frac{-b}{}}}}{\sqrt{cg\,x^3+bg}}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((e*x+d)*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/3/c*e*(
c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2*(d*f-2/3/c*e*(1/2*a*g+1/2
*b*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2
)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*
a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4
*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)
*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(
b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(d
*g+e*f-2/3/c*e*(b*g+c*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g
-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(
-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/
c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g
*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/
g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1
/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2
)^(1/2))*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f
/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/
2))))
```

3.902.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 448, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2\left(3\sqrt{cx^2+bx+a}\sqrt{gx+f}c^2eg^2-(c^2ef^2-2(3c^2d-bce)fg+(3bcd-(2b^2-3ac)e)g^2)\sqrt{cg}\text{weierstra}\right.}{}$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(3*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*c^2*e*g^2 - (c^2*e*f^2 - 2*(3*c
^2*d - b*c*e)*f*g + (3*b*c*d - (2*b^2 - 3*a*c)*e)*g^2)*sqrt(c*g)*weierstra
ssPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2
*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g
^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)) - 3*(c^2*e*f*g + (3*c^2*d
- 2*b*c*e)*g^2)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 -
3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c
^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2
*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*
f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*
(3*c*g*x + c*f + b*g)/(c*g))))/(c^3*g^2)
```

3.902.6 Sympy [F]

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((e*x+d)*(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x)*sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

3.902.7 Maxima [F]

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

3.902.8 Giac [F]

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

3.902.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{\sqrt{f+g\,x}\,(d+e\,x)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((f + g*x)^(1/2)*(d + e*x))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int(((f + g*x)^(1/2)*(d + e*x))/(a + b*x + c*x^2)^(1/2), x)
```

3.903 $\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

3.903.1 Optimal result

Integrand size = 24, antiderivative size = 188

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{c\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

output
```
EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1
/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1
/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)
/c/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

3.903.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.60 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.94

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{i\left(2cf+\left(-b+\sqrt{b^2-4ac}\right)g\right)\sqrt{\frac{g\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf+\left(-b+\sqrt{b^2-4ac}\right)g}}\left(E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{-2cf+\left(b+\sqrt{\right.}\right.}}\right.\right.\right.}{\sqrt{2}cg\sqrt{\frac{}{-2cf+(}}}$$

input
```
Integrate[Sqrt[f + g*x]/Sqrt[a + b*x + c*x^2],x]
```

output
```
(I*(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)*Sqrt[(g*(b + Sqrt[b^2 - 4*a*c] + 2
*c*x))/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c
*f + (-b + Sqrt[b^2 - 4*a*c])*g)]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(-2*
c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[f + g*x]], (2*c*f - (b + Sqrt[b^2 -
 4*a*c])*g)/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)] - EllipticF[I*ArcSinh[Sq
rt[2]*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[f + g*x]], (2*c*f
- (b + Sqrt[b^2 - 4*a*c])*g)/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)]))/(Sqrt
[2]*c*g*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + x*(b + c*x)]
)
```

3.903.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1172, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$
$$\Big\downarrow \; 1172$$
$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\int \frac{\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$

$\Big\downarrow$ 327

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$

input
```
Int[Sqrt[f + g*x]/Sqrt[a + b*x + c*x^2],x]
```

output
```
(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)])/(c*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[
a + b*x + c*x^2])
```

3.903.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

3.903. $\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

3.903.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(164) = 328$.

Time = 0.64 (sec) , antiderivative size = 746, normalized size of antiderivative = 3.97

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+bx+a)}\left(\frac{2f\left(\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x+\frac{b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}+\frac{b+\sqrt{-4ac+b^2}}{2c}}}F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b}}{2c}}}\right.}{\sqrt{cgx^3+bgx^2+cfx^2+agx+bfx+fa}}\right.}{}$
default	$\frac{\sqrt{gx+f}\,\sqrt{cx^2+bx+a}\left(g\sqrt{-4ac+b^2}+bg-2cf\right)\sqrt{2}\sqrt{-\frac{(gx+f)c}{g\sqrt{-4ac+b^2}+bg-2cf}}\sqrt{\frac{\left(-b-2cx+\sqrt{-4ac+b^2}\right)g}{2cf-bg+g\sqrt{-4ac+b^2}}}\sqrt{\frac{\left(b+2cx+\sqrt{-4ac+b^2}\right)g}{g\sqrt{-4ac+b^2}+bg-2cf}}\left(F\left(\sqrt{\right.\right.}{}$

input
```
int((g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2*f*(f/g-
1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))
^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/
2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1
/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF((
(x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^
2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*g*(f/g-1/2*(b+
(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*
((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(
1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(
-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))
^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(
1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/(f/g-1/2*(
b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g
-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

3.903.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx =$$
$$-\frac{2\left(3\sqrt{cg}cg\text{weierstrassZeta}\left(\frac{4\,(c^2f^2-bcfg+(b^2-3\,ac)g^2)}{3\,c^2g^2},-\frac{4\,(2\,c^3f^3-3\,bc^2f^2g-3\,(b^2c-6\,ac^2)fg^2+(2\,b^3-9\,abc)g^3)}{27\,c^3g^3},\text{weierst}\right.\right.}{}$$

input
```
integrate((g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(3*sqrt(c*g)*c*g*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*
c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*
f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f^2
 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*
g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c
*g*x + c*f + b*g)/(c*g))) - (2*c*f - b*g)*sqrt(c*g)*weierstrassPInverse(4/
3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*
b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3)
, 1/3*(3*c*g*x + c*f + b*g)/(c*g)))/(c^2*g)
```

3.903.6 Sympy [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

3.903.7 Maxima [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

3.903.8 Giac [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

3.903.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((f + g*x)^(1/2)/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((f + g*x)^(1/2)/(a + b*x + c*x^2)^(1/2), x)
```

3.904 $\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

3.904.1 Optimal result

Integrand size = 31, antiderivative size = 467

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\cdots}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$-\frac{\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticPi}\left(\frac{e\left(2cf-bg+\sqrt{b^2-4\cdots}\right)}{2c(ef-dg)}\cdots\right.}{\sqrt{c}\,e\sqrt{a+bx+cx^2}}$$

output
```
2*g*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*
2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*
2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f
)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e/(g*x+f)^(1/2)/(c*x^2+b*x+a)^
(1/2)-EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1
/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/
g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(1-2*
c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)
^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e/c^
(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.904.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.14 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = -\frac{i\sqrt{2}\sqrt{\frac{g\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf+\left(-b+\sqrt{b^2-4ac}\right)g}}\left(\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{f+gx}\cdots\right.\right.\right.}{e\sqrt{\frac{\cdots}{-2cf\cdots}}\cdots}$$

input
```
Integrate[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((-I)*Sqrt[2]*Sqrt[(g*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*f + (b + Sqrt
[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f + (-b + Sqrt[b^2 - 4*a*
c])*g)]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*
c])*g)]*Sqrt[f + g*x]], (2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)/(2*c*f + (-b +
 Sqrt[b^2 - 4*a*c])*g)] - EllipticPi[(e*(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g
))/(2*c*(e*f - d*g)), I*ArcSinh[Sqrt[2]*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4
*a*c])*g)]*Sqrt[f + g*x]], (2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)/(2*c*f + (-
b + Sqrt[b^2 - 4*a*c])*g)]))/(e*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g
)]*Sqrt[a + x*(b + c*x)])
```

3.904.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {1284, 1172, 321, 1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1284}$$

$$\frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e}+\frac{g\int \frac{1}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e}$$

$$\Big\downarrow \; 1172$$

$$\frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}+\frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e}$$

$$\Big\downarrow \; 321$$

$$\frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e}+\frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 1279$$

$$\frac{\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}(ef-dg)\int \frac{1}{\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}(d+ex)\sqrt{f+gx}}dx}{e\sqrt{a+bx+cx^2}}+\frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 187$$

$$\frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}(ef-dg)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{b+\frac{2c(f+gx)}{g}-\sqrt{b^2-4ac}-\frac{2cf}{g}}\sqrt{b+\frac{2c(f+gx)}{g}+\sqrt{\cdots}}}}{e\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 413$$

3.904. $\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

$$\frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}-$$

$$\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}(ef-dg)\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\int\frac{}{(ef-dg-e(f+gx))\sqrt{b+\frac{2c(f+gx)}{g}+\sqrt{}}}}{e\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(f+gx)}{g}-\frac{2cf}{g}}}$$

$\Big\downarrow$ 413

$$\frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}-$$

$$\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}(ef-dg)\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\int\frac{}{(ef-d}}{e\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(f+gx)}{g}-\frac{2cf}{g}}\sqrt{\sqrt{b^2-4ac}+b+}}$$

$\Big\downarrow$ 412

$$\frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}-$$

$$\frac{\sqrt{2}\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{}{2cf-g}}}{\sqrt{c}e\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(f+gx)}{g}-\frac{2c}{g}}}$$

input
```
Int[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

3.904. $\int\frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

output
```
(2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 -
 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqr
t[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*e*Sqrt[f + g*x]
*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]
*Sqrt[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*S
qrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c
*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*
g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[
f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (2*c*f - (b - Sqrt[b^2
 - 4*a*c])*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[c]*e*Sqrt[a + b*
x + c*x^2]*Sqrt[b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g + (2*c*(f + g*x))/g]*Sqr
t[b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g + (2*c*(f + g*x))/g])
```

3.904.3.1 Defintions of rubi rules used

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1284
```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2]), x_Symbol] :> Simp[g/e   Int[1/(Sqrt[f + g*x]*Sqrt[a + b*x
 + c*x^2]), x], x] + Simp[(e*f - d*g)/e   Int[1/((d + e*x)*Sqrt[f + g*x]*Sq
rt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
```

3.904.4 Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 674, normalized size of antiderivative = 1.44

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+bx+a)}\left(\frac{2g\left(\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x+\frac{b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}+\frac{b+\sqrt{-4ac+b^2}}{2c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b}}{2c}}}\right.}{e\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}\right.}{}$
default	$\frac{\left(-F\left(\sqrt{2}\,\sqrt{-\frac{(gx+f)c}{g\sqrt{-4ac+b^2}+bg-2cf}},\sqrt{-\frac{g\sqrt{-4ac+b^2}+bg-2cf}{2cf-bg+g\sqrt{-4ac+b^2}}}\right)g\sqrt{-4ac+b^2}+\sqrt{-4ac+b^2}\,\Pi\left(\sqrt{2}\,\sqrt{-\frac{(gx+f)c}{g\sqrt{-4ac+b^2}+bg-2cf}},\frac{\left(g\sqrt{-4ac+b^2}\right.}{2c(dg}\right.\right.}{}$

input
```
int((g*x+f)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2*g/e*(f/
g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c
))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(
1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^
(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF
(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+
b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))-2*(d*g-e*f)/e^
2*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/
2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b
^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+
b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(-f/
g+d/e)*EllipticPi(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),(-f/g
+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+d/e),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2)
)/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))
```

3.904.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.904.6 Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x+f)**(1/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(f + g*x)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

3.904.7 Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

3.904.8 Giac [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

3.904.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((f + g*x)^(1/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((f + g*x)^(1/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.905 $\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx$

3.905.1 Optimal result

Integrand size = 31, antiderivative size = 994

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx = -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)\,(d+ex)}$$

$$+\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}\,(cd^2-bde+ae^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right.}\right.}{(cd^2-bde+ae^2)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}dg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4a\right.}}\right.}{e\,(cd^2-bde+ae^2)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,(e^2(bf-ag)-cd(2ef-dg))\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}}{\sqrt{2}\sqrt{c}e\,(cd^2-bde+ae^2)\,(ef-dg)\sqrt{a}}$$

output
```
-e*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)/(e*x+d)+1/2*Ellip
ticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(
-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b
^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/(a*e^2-b*d*e
+c*d^2)*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1
/2))))^(1/2)-f*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1
/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)
)))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2
)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(a*e^2-b*d*e+c*d^2)/(
g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)+d*g*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^
(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g
*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x
+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
/e/(a*e^2-b*d*e+c*d^2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)+1/2*(e^2*(-a*g+b*
f)-c*d*(-d*g+2*e*f))*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-
(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+
e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2)
)*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a
*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/
2)/e/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)*2^(1/2)/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.905.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.19 (sec) , antiderivative size = 1502, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-((e*Sqrt[f + g*x]*(a + b*x + c*x^2))/((c*d^2 - b*d*e + a*e^2)*(d + e*x)*S
qrt[a + x*(b + c*x)])) - ((f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2]*(-4*e*(-(e
*f) + d*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a
*c)*g^2])]*(c*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f +
g*x)))/(f + g*x)) + (I*Sqrt[2]*e*(-(e*f) + d*g)*(2*c*f - b*g + Sqrt[(b^2 -
 4*a*c)*g^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] - (2*a*g^2)/(f + g*x) - 2*c*f*
(-1 + f/(f + g*x)) + b*g*(-1 + (2*f)/(f + g*x)))/(2*c*f - b*g + Sqrt[(b^2
- 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] + (2*a*g^2)/(f + g*x) + 2*c*
f*(-1 + f/(f + g*x)) + b*(g - (2*f*g)/(f + g*x)))/(-2*c*f + b*g + Sqrt[(b^
2 - 4*a*c)*g^2])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2
)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b
*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/S
qrt[f + g*x] - (I*Sqrt[2]*e*(2*c*d*f*g + 2*a*e*g^2 - e*f*Sqrt[(b^2 - 4*a*c
)*g^2] + d*g*Sqrt[(b^2 - 4*a*c)*g^2] - b*g*(e*f + d*g))*Sqrt[(Sqrt[(b^2 -
4*a*c)*g^2] - (2*a*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f + g*x)) + b*g*(-1 + (
2*f)/(f + g*x)))/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^2
- 4*a*c)*g^2] + (2*a*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f + g*x)) + b*(g - (2
*f*g)/(f + g*x)))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*EllipticF[I*Ar
cSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*
a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])...
```

3.905.3 Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 882, normalized size of antiderivative = 0.89, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {1285, 25, 2154, 1269, 1172, 321, 327, 1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1285}$$

$$-\frac{\int -\frac{cegx^2+2cdgx+2cdf-bef+aeg}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2\left(ae^2-bde+cd^2\right)}-\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{cegx^2+2cdgx+2cdf-bef+aeg}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2\left(ae^2-bde+cd^2\right)}-\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{2154}$$

$$\frac{\int \frac{cxg+\frac{cdg}{e}}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx-\left(-aeg+bef-\frac{cd(2ef-dg)}{e}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2\left(ae^2-bde+cd^2\right)}-\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1269}$$

$$\frac{-c\left(f-\frac{dg}{e}\right)\int \frac{1}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx-\left(-aeg+bef-\frac{cd(2ef-dg)}{e}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+c\int \frac{\sqrt{f+gx}}{\sqrt{cx^2+bx+a}}dx}{2\left(ae^2-bde+cd^2\right)}-\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1172}$$

$$\frac{-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(f-\frac{dg}{e}\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}+\frac{\sqrt{2}\sqrt{b^2-\cdots}}{\cdots}}{2\left(ae^2\cdots\right.}$$

$$-\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{321}$$

$$\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\int \frac{\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\left(-aeg+bef-\frac{cd(2ef-dg)}{e}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{c\cdots}}}{2\left(ae^2-b\cdots\right.}$$

$$-\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{327}$$

3.905. $\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx$

$$\frac{-\left(-aeg+bef-\frac{cd(2ef-dg)}{e}\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(f-\frac{dg}{e}\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\text{Elliptic}}{\sqrt{f+gx}\sqrt{a+bx+}}}{2\left(ae^2\right.}$$

$$\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1279

$$-\frac{\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\left(-aeg+bef-\frac{cd(2ef-dg)}{e}\right)\int\frac{1}{\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}(d+ex)\sqrt{f+gx}}dx}{\sqrt{a+bx+cx^2}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-}}{}$$

$$\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 187

$$\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\left(-aeg+bef-\frac{cd(2ef-dg)}{e}\right)\int\frac{1}{(ef-dg-e(f+gx))\sqrt{b+\frac{2c(f+gx)}{g}-\sqrt{b^2-4ac}-\frac{2cf}{g}}\sqrt{b+\frac{2c(f+gx)}{g}+\sqrt{b^2-4ac}-\frac{2c}{g}}}}{\sqrt{a+bx+cx^2}}$$

$$\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 413

$$\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\left(-aeg+bef-\frac{cd(2ef-dg)}{e}\right)\int\frac{1}{(ef-dg-e(f+gx))\sqrt{b+\frac{2c(f+gx)}{g}+\sqrt{b^2-4ac}-\frac{2cf}{g}}\sqrt{1}}}{\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(f+gx)}{g}-\frac{2cf}{g}}}$$

$$\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 413

3.905. $\int\frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}dx$

$$\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}} - \frac{2\sqrt{2}\sqrt{b^2-4ac}\left(f-\frac{dg}{e}\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{}}{}}{}$$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+bx+a}}{(cd^2-bed+ae^2)(d+ex)}$$

$$\Big\downarrow \; 412$$

$$\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}} - \frac{2\sqrt{2}\sqrt{b^2-4ac}\left(f-\frac{dg}{e}\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{}}{}}{}$$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+bx+a}}{(cd^2-bed+ae^2)(d+ex)}$$

input
```
Int[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-((e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*(d + e*
x))) + ((Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^
2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/
Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)])/(Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]
*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(f - (d*g)/e)*Sqrt[
(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*
x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt
[b^2 - 4*a*c])*g)])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2]*Sqrt[
2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(b*e*f - a*e*g - (c*d*(2*e*f - d*g))/e)
*Sqrt[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*S
qrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c
*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*
g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[
f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (2*c*f - (b - Sqrt[b^2
 - 4*a*c])*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[c]*(e*f - d*g)*S
qrt[a + b*x + c*x^2]*Sqrt[b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g + (2*c*(f + g*
x))/g]*Sqrt[b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g + (2*c*(f + g*x))/g]))/(2*(c
*d^2 - b*d*e + a*e^2))
```

3.905. $\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx$

3.905.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

3.905. $\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx$

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1285
```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
(Sqrt[a + b*x + c*x^2]/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(2*(
m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqr
t[a + b*x + c*x^2]))*Simp[2*c*d*f*(m + 1) - e*(a*g + b*f*(2*m + 3)) - 2*(b*
e*g*(2 + m) - c*(d*g*(m + 1) - e*f*(m + 2)))*x - c*e*g*(2*m + 5)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

3.905.4 Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 1229, normalized size of antiderivative = 1.24

method	result	size
elliptic	Expression too large to display	1229
default	Expression too large to display	13017

input
```
int((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-e/(a*e^2
-b*d*e+c*d^2)*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(e*x+d)+c*d*
g/(a*e^2-b*d*e+c*d^2)/e*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1
/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f
/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x
+b*f*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(
1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/
2))))^(1/2))+c*g/(a*e^2-b*d*e+c*d^2)*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((
x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2
)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b
^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+
c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*Ellip
ticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*
a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+
(-4*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))
^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(
1/2))))^(1/2)))+(a*e^2*g-b*e^2*f-c*d^2*g+2*c*d*e*f)/e^2/(a*e^2-b*d*e+c*d^2
)*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/
2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*...
```

3.905.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.905.6 Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x+f)**(1/2)/(e*x+d)**2/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(f + g*x)/((d + e*x)**2*sqrt(a + b*x + c*x**2)), x)
```

3.905.7 Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)^2}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^2), x)
```

3.905.8 Giac [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)^2}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^2), x)
```

3.905.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}}{{\left(d+e\,x\right)}^2\,\sqrt{c\,x^2+b\,x+a}} \,d x$$

input
```
int((f + g*x)^(1/2)/((d + e*x)^2*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((f + g*x)^(1/2)/((d + e*x)^2*(a + b*x + c*x^2)^(1/2)), x)
```

3.906 $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx$

3.906.1 Optimal result

Integrand size = 31, antiderivative size = 1786

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

output
```
-1/2*e*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)/(e*x+d)^2-1/4
*e*(c*d*(-5*d*g+6*e*f)-e*(-a*e*g-2*b*d*g+3*b*e*f))*(g*x+f)^(1/2)*(c*x^2+b*
x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)/(e*x+d)+1/8*(c*d*(-5*d*g+6*e*f
)-e*(-a*e*g-2*b*d*g+3*b*e*f))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/
(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4
*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+
a)/(-4*a*c+b^2))^(1/2)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)*2^(1/2)/(c*x^2+b*x
+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-1/2*g*Ellipti
cF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2
*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2
)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a
*c+b^2)^(1/2))))^(1/2)/e/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(g*x+f)^(1/2)/(c*x^2+
b*x+a)^(1/2)-1/4*f*(c*d*(-5*d*g+6*e*f)-e*(-a*e*g-2*b*d*g+3*b*e*f))*Ellipti
cF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2
*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2
)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a
*c+b^2)^(1/2))))^(1/2)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)*2^(1/2)/(g*x+f)^(1
/2)/(c*x^2+b*x+a)^(1/2)+1/4*d*g*(c*d*(-5*d*g+6*e*f)-e*(-a*e*g-2*b*d*g+3*b*
e*f))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1...
```

3.906.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.79 (sec) , antiderivative size = 36634, normalized size of antiderivative = 20.51

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

output
```
Result too large to show
```

3.906.3 Rubi [A] (verified)

Time = 3.65 (sec) , antiderivative size = 1627, normalized size of antiderivative = 0.91, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.613$, Rules used = {1285, 25, 2154, 1282, 25, 2154, 25, 27, 1172, 321, 1269, 1172, 321, 327, 1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \ 1285$$

$$-\frac{\int -\frac{-cegx^2-2(cef-2cdg+beg)x+4cdf-3bef+aeg}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{4\left(ae^2-bde+cd^2\right)}-\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{-cegx^2-2(cef-2cdg+beg)x+4cdf-3bef+aeg}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{4\left(ae^2-bde+cd^2\right)}-\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \ 2154$$

$$\frac{\int \frac{-2cf-2bg+\frac{5cdg}{e}-cgx}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx-\left(-aeg-2bdg+3bef-\frac{cd(6ef-5dg)}{e}\right)\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{4\left(ae^2-bde+cd^2\right)}-\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2\left(ae^2-bde+cd^2\right)}$$

$$\downarrow \text{1282}$$

$$\frac{\frac{\int \frac{-2cf-2bg+\frac{5cdg}{e}-cgx}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx-\left(-aeg-2bdg+3bef-\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{\int -\frac{ce^2gx^2+2cdegx+2cd(ef-dg)-e(bef-2bdg+aeg)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)(ae^2-bde+cd^2)}-\right.}{4\left(ae^2-bde+cd^2\right)}}{}\ \frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2\left(ae^2-bde+cd^2\right)}$$

$$\downarrow \text{25}$$

$$\frac{\frac{\int \frac{-2cf-2bg+\frac{5cdg}{e}-cgx}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx-\left(-aeg-2bdg+3bef-\frac{cd(6ef-5dg)}{e}\right)\left(\frac{\int \frac{ce^2gx^2+2cdegx+2cd(ef-dg)-e(bef-2bdg+aeg)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)(ae^2-bde+cd^2)}-\frac{}{(d+}\right.}{4\left(ae^2-bde+cd^2\right)}}{}\ \frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2\left(ae^2-bde+cd^2\right)}$$

$$\downarrow \text{2154}$$

$$\frac{-\left(-aeg-2bdg+3bef-\frac{cd(6ef-5dg)}{e}\right)\left(\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\int \frac{cdg+cexg}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)(ae^2-bde+cd^2)}-\right.}{4\left(ae^2-bde+\right.}\ \frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2\left(ae^2-bde+cd^2\right)}$$

$$\downarrow \text{25}$$

$$\frac{-\left(-aeg-2bdg+3bef-\frac{cd(6ef-5dg)}{e}\right)\left(\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\int \frac{cdg+cexg}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)(ae^2-bde+cd^2)}-\right.}{4\left(ae^2-bde+c\right.}\ \frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2\left(ae^2-bde+cd^2\right)}$$

$$\downarrow \text{27}$$

$$\frac{-\left(-aeg-2bdg+3bef-\frac{cd(6ef-5dg)}{e}\right)\left(\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\int \frac{cdg+cexg}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)(ae^2-bde+cd^2)}-\right.}{4\left(ae^2-bde+c\right.}\ \frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2\left(ae^2-bde+cd^2\right)}$$

$$\downarrow \text{1172}$$

3.906. $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx$

$$-\frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{e\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \left(-aeg-2bo\right.$$

$$\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 321

$$-\left(-aeg-2bdg+3bef-\frac{cd(6ef-5dg)}{e}\right)\left(\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\int \frac{cdg+cexg}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)(ae^2-bde+cd^2)} - \right.$$

$$\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1269

$$-\left(-aeg-2bdg+3bef-\frac{cd(6ef-5dg)}{e}\right)\left(\frac{-c(ef-dg)\int \frac{1}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2}}}{2(ef-dg)(ae^2-bde+cd^2)}\right.$$

$$\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1172

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e\sqrt{f+gx}\sqrt{cx^2+bx+a}} - 2\left(cf+bg-\frac{3c}{e}\right.$$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2\left(cd^2-bed+ae^2\right)(d+ex)^2}$$

3.906. $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx$

$$\Big\downarrow \text{321}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e\sqrt{f+gx}\sqrt{cx^2+bx+a}}-2\left(cf+bg-\frac{3c}{e}\right.$$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2\left(cd^2-bed+ae^2\right)(d+ex)^2}$$

$$\Big\downarrow \text{327}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e\sqrt{f+gx}\sqrt{cx^2+bx+a}}-2\left(cf+bg-\frac{3c}{e}\right.$$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2\left(cd^2-bed+ae^2\right)(d+ex)^2}$$

$$\Big\downarrow \text{1279}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e\sqrt{f+gx}\sqrt{cx^2+bx+a}}-\frac{2\left(cf+bg-\frac{3cdg}{e}\right)\sqrt{}}{}$$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2\left(cd^2-bed+ae^2\right)(d+ex)^2}$$

3.906. $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx$

$\Big\downarrow$ 187

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e\sqrt{f+gx}\sqrt{cx^2+bx+a}}+\frac{4\left(cf+bg-\frac{3cdg}{e}\right)\sqrt{}}{}$$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2\left(cd^2-bed+ae^2\right)(d+ex)^2}$$

$\Big\downarrow$ 413

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e\sqrt{f+gx}\sqrt{cx^2+bx+a}}+\frac{4\left(cf+bg-\frac{3cdg}{e}\right)\sqrt{}}{}$$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2\left(cd^2-bed+ae^2\right)(d+ex)^2}$$

$\Big\downarrow$ 413

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e\sqrt{f+gx}\sqrt{cx^2+bx+a}}+\frac{4\left(cf+bg-\frac{3cdg}{e}\right)\sqrt{}}{}$$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2\left(cd^2-bed+ae^2\right)(d+ex)^2}$$

$\Big\downarrow$ 412

3.906. $\displaystyle\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e\sqrt{f+gx}\sqrt{cx^2+bx+a}}+\frac{2\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b\right.}}}{}$$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2\left(cd^2-bed+ae^2\right)(d+ex)^2}$$

input
```
Int[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/2*(e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*(d +
 e*x)^2) + ((-2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ell
ipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt
[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(e*Sq
rt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[2*c*f - (b - Sqrt[b^2
 - 4*a*c])*g]*(c*f + b*g - (3*c*d*g)/e)*Sqrt[b - Sqrt[b^2 - 4*a*c] + 2*c*x
]*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b
 - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f
 - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^
2 - 4*a*c])*g]], (2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)])/(Sqrt[c]*(e*f - d*g)*Sqrt[a + b*x + c*x^2]*Sqrt[b - Sqrt[
b^2 - 4*a*c] - (2*c*f)/g + (2*c*(f + g*x))/g]*Sqrt[b + Sqrt[b^2 - 4*a*c] -
 (2*c*f)/g + (2*c*(f + g*x))/g]) - (3*b*e*f - 2*b*d*g - a*e*g - (c*d*(6*e*
f - 5*d*g))/e)*(-((e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*
e + a*e^2)*(e*f - d*g)*(d + e*x))) + ((Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*Sqrt[f
+ g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
- 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[(c*(f + g*x))/(...
```

3.906. $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx$

3.906.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

3.906. $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx$

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1282
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^
(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*
e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*
g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 1285
```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
(Sqrt[a + b*x + c*x^2]/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(2*(
m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqr
t[a + b*x + c*x^2]))*Simp[2*c*d*f*(m + 1) - e*(a*g + b*f*(2*m + 3)) - 2*(b*
e*g*(2 + m) - c*(d*g*(m + 1) - e*f*(m + 2)))*x - c*e*g*(2*m + 5)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

3.906. $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx$

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

3.906.4 Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 1698, normalized size of antiderivative = 0.95

method	result	size
elliptic	Expression too large to display	1698
default	Expression too large to display	59522

input
```
int((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-1/2*e/(a
*e^2-b*d*e+c*d^2)*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(e*x+d)^
2+1/4*e*(a*e^2*g+2*b*d*e*g-3*b*e^2*f-5*c*d^2*g+6*c*d*e*f)/(a*d*e^2*g-a*e^3
*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)/(a*e^2-b*d*e+c*d^2)*(c*g*x^3+b*g
*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(e*x+d)-1/4*c*g*(3*a*d*e^2*g-2*a*e^3*f
-b*d*e^2*f-3*c*d^3*g+4*c*d^2*e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c
*d^3*g-c*d^2*e*f)/(a*e^2-b*d*e+c*d^2)/e*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)
*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+
b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*
c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x
^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+
(-4*a*c+b^2)^(1/2))))^(1/2))-1/4*c*g*(a*e^2*g+2*b*d*e*g-3*b*e^2*f-5*c*d^2*
g+6*c*d*e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)/(a*
e^2-b*d*e+c*d^2)*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(
-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/
c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+
1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+
a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1
/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c...
```

3.906.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.906.6 Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{f+gx}}{(d+ex)^3\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x+f)**(1/2)/(e*x+d)**3/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(f + g*x)/((d + e*x)**3*sqrt(a + b*x + c*x**2)), x)
```

3.906.7 Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)^3}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^3), x)
```

3.906.8 Giac [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)^3}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^3), x)
```

3.906.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}}{(d+e\,x)^3\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((f + g*x)^(1/2)/((d + e*x)^3*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((f + g*x)^(1/2)/((d + e*x)^3*(a + b*x + c*x^2)^(1/2)), x)
```

3.907 $\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

3.907.1 Optimal result

Integrand size = 31, antiderivative size = 675

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{}}{2cf-(}\cdots\right)}{ce\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}\,g(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b}}{2cf-\left(b+\right.}\cdots\right)}{ce^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$-\frac{\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,(ef-dg)\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticPi}\left(\frac{e\left(2cf-bg\cdots\right)}{2c(\cdots}\cdots\right)}{\sqrt{c}e^2\sqrt{a+bx+cx^2}}$$

output
```
g*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^
(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/
2)/c/e/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1
/2)+2*g*(-d*g+e*f)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(
1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^
(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^2/(g*x+f)^(1/
2)/(c*x^2+b*x+a)^(1/2)-(-d*g+e*f)*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)
/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1
/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^
(1/2)))^(1/2))*2^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1
/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*
a*c+b^2)^(1/2))))^(1/2)/e^2/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.907.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1385 vs. $2(675) = 1350$.

Time = 13.60 (sec) , antiderivative size = 1385, normalized size of antiderivative = 2.05

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{2}\sqrt{\frac{c(f+gx)}{2cf+\left(-b+\sqrt{b^2-4ac}\right)g}}\left(\frac{2fg\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{-b+\cdots}{\cdots}}\cdots\right.\right.}{ce\sqrt{\frac{-b+\sqrt{b^2-4ac}-2cx}{\sqrt{b^2-4ac}}}}\cdots\right.}{\cdots}$$

input
```
Integrate[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(Sqrt[2]*Sqrt[(c*(f + g*x))/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)]*((2*f*g*
(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[
b^2 - 4*a*c]]*EllipticF[ArcSin[Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[
b^2 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - b*g + Sqrt[b^2 -
4*a*c]*g)])/(c*e*Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2 - 4*a*c]])
 - (d*g^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x)/Sqrt[b^2 - 4*a*c]]*EllipticF[ArcSin[Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*
c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - b*g + S
qrt[b^2 - 4*a*c]*g)])/(c*e^2*Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^
2 - 4*a*c]]) + (g*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)*Sqrt[(g*(b + Sqrt[b^2 -
 4*a*c] + 2*c*x))/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*((-2*c*f + (b + Sq
rt[b^2 - 4*a*c])*g)*EllipticE[ArcSin[Sqrt[2]*Sqrt[(c*(f + g*x))/(2*c*f - b
*g + Sqrt[b^2 - 4*a*c]*g)]], (2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)/(2*c*f -
 (b + Sqrt[b^2 - 4*a*c])*g)] - (b + Sqrt[b^2 - 4*a*c])*g*EllipticF[ArcSin[
Sqrt[2]*Sqrt[(c*(f + g*x))/(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g)]], (2*c*f +
 (-b + Sqrt[b^2 - 4*a*c])*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]))/(2*c^2
*e*Sqrt[(g*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*f + (-b + Sqrt[b^2 - 4*a
*c])*g)]) - (4*Sqrt[b^2 - 4*a*c]*f^2*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*
a*c)]*EllipticPi[(2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*
e), ArcSin[Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqr...
```

3.907.3 Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 675, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1288, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1288}$$

$$\int \left(\frac{(ef-dg)^2}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}+\frac{g(ef-dg)}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}+\frac{g\sqrt{f+gx}}{e\sqrt{a+bx+cx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}}{2cf-\left(b+\sqrt{b^2-\right.}}\right.}{ce^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$\frac{\sqrt{2}(ef-dg)\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticPi}\left(\frac{e\left(2cf-bg+\sqrt{b^2-\right.}}{2c(ef-dg}\right.}{\sqrt{c}e^2\sqrt{a+bx+cx^2}}$$

$$\frac{\sqrt{2}g\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$

input
```
Int[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b
^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4
*a*c])*g)])/(c*e*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*S
qrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*(e*f - d*g)*Sqrt[(c
*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^
2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/
Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)])/(c*e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*S
qrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(e*f - d*g)*Sqrt[1 - (2*c*(f + g*x)
)/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (
b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*
g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f -
 (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqr
t[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*e^2*Sqrt[a + b*x + c*x^2])
```

3.907. $\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

3.907.3.1 Defintions of rubi rules used

rule 1288
```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && IntegerQ[n + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.907.4 Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 1122, normalized size of antiderivative = 1.66

method	result	size
elliptic	Expression too large to display	1122
default	Expression too large to display	1879

input
```
int((g*x+f)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2*g*(d*g
-2*e*f)/e^2*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*
c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b
+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(
b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^
(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+
1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))
+2*g^2/e*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-
4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(
-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/
2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-
4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2
/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*Elliptic
F(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c
+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))+2*(d^2*g^2-2
*d*e*f*g+e^2*f^2)/e^3*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2
*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g
-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(
-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*...
```

3.907.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.907.6 Sympy [F]

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+gx)^{\frac{3}{2}}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x+f)**(3/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((f + g*x)**(3/2)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

3.907.7 Maxima [F]

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(gx+f)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

3.907.8 Giac [F]

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx=\int \frac{(gx+f)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

3.907.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx=\int \frac{(f+g\,x)^{3/2}}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((f + g*x)^(3/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((f + g*x)^(3/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.908 $\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

3.908.1 Optimal result

Integrand size = 31, antiderivative size = 1138

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3ce}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}\,g(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3c^2e\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}\,g(ef-dg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}\,g(ef-dg)^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{\cdots}}{2cf-(b\cdots}\right.}{ce^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\,g(cf^2-bfg+ag^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\cdots\right.}{3c^2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$-\frac{\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,(ef-dg)^2\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticPi}\left(\frac{e\left(2cf-\cdots\right.}{2\cdots}\right.}{\sqrt{c}e^3\sqrt{a+bx+cx^2}}$$

output
```
2/3*g^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/e+2/3*g*(-b*g+2*c*f)*EllipticE
(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g
*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a
*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^2/e/(c
*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+g*(-d
*g+e*f)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1
/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/
2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2
))^(1/2)/c/e^2/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/
2))))^(1/2)+2*g*(-d*g+e*f)^2*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(
-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*
a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*
a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^3/(
g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)-2/3*g*(a*g^2-b*f*g+c*f^2)*EllipticF(1/2*(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a
*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2
)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a
*c+b^2)^(1/2))))^(1/2)/c^2/e/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)-(-d*g+e*f)^
2*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))
)^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g...
```

3.908.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.53 (sec) , antiderivative size = 37137, normalized size of antiderivative = 32.63

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
Result too large to show
```

3.908.3 Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 1138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1288, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1288}$$

$$\int \left(\frac{(ef-dg)^3}{e^3(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}+\frac{g(ef-dg)^2}{e^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}+\frac{g\sqrt{f+gx}(ef-dg)}{e^2\sqrt{a+bx+cx^2}}+\frac{g(f+gx)^{3/2}}{e\sqrt{a+bx+cx^2}}\right)d$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+bx+a}g^2}{3ce}+$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)g}{3c^2e\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}+$$

$$\frac{\sqrt{2}\sqrt{b^2-4ac}(ef-dg)\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)g}{ce^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}+$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}(ef-dg)^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}}{2cf-\left(b+\sqrt{b^2-\right.}}\right.}{ce^3\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\left(cf^2-bgf+ag^2\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{}{2cf-}\right.}{3c^2e\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$\frac{\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(ef-dg)^2\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\text{EllipticPi}\left(\frac{e\left(2cf-bg+\sqrt{b^2\right.}}{2(ef-d}\right.}{\sqrt{c}e^3\sqrt{cx^2+bx+a}}$$

input
```
Int[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

3.908. $\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}dx$

output
```
(2*g^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c*e) + (2*Sqrt[2]*Sqrt[b^2
- 4*a*c]*g*(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c
])*g)])/(3*c^2*e*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*S
qrt[a + b*x + c*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*(e*f - d*g)*Sqrt[f +
g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 -
4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*e^2*Sqrt[(c*(f + g*x))/
(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*S
qrt[b^2 - 4*a*c]*g*(e*f - d*g)^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2
 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*S
qrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*e^3*Sqrt[f +
g*x]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*(c*f^2 - b*f*
g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-(
(c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2
- 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2
*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(3*c^2*e*Sqrt[f + g*x]*Sqrt[a + b*x +
c*x^2]) - (Sqrt[2]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(e*f - d*g)^...
```

3.908.3.1 Defintions of rubi rules used

rule 1288
```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && IntegerQ[n + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.908.4 Maple [A] (verified)

Time = 6.52 (sec) , antiderivative size = 1245, normalized size of antiderivative = 1.09

method	result	size
elliptic	Expression too large to display	1245
risch	Expression too large to display	2354
default	Expression too large to display	7464

input
```
int((g*x+f)^(5/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/3/e*g^2
/c*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2*(g*(d^2*g^2-3*d*e*f*g
+3*e^2*f^2)/e^3-2/3/e*g^2/c*(1/2*a*g+1/2*b*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1
/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(
-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b
+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^
3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-
4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2
/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(-g^2/e^2*(d*g-3*e*f)-2/3/e*g^2/c*(b
*g+c*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-
4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(
-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/
2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-
4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2
/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*Elliptic
F(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c
+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))-2*(d^3*g^3-3
*d^2*e*f*g^2+3*d*e^2*f^2*g-e^3*f^3)/e^4*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)
*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a...
```

3.908.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.908.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**(5/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Timed out
```

3.908.7 Maxima [F]

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(gx+f)^{\frac{5}{2}}}{\sqrt{cx^2+bx+a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

3.908.8 Giac [F]

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(gx+f)^{\frac{5}{2}}}{\sqrt{cx^2+bx+a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

3.908.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx=\int \frac{{\left(f+g\,x\right)}^{5/2}}{\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((f + g*x)^(5/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((f + g*x)^(5/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.909 $\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

3.909.1 Optimal result

Integrand size = 31, antiderivative size = 631

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$= -\frac{8e^2(cef-3cdg+beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg}$$

$$+ \frac{\sqrt{2}\sqrt{b^2-4ac}e(8b^2e^2g^2+ceg(7bef-30bdg-9aeg)+c^2(8e^2f^2-30defg+45d^2g^2))\sqrt{f+gx}\sqrt{-\frac{c(a+}{b^2}}}{15c^3g^3\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}(4be^3g^2(bf-ag)+c^2(8e^3f^3-30de^2f^2g+45d^2efg^2-15d^3g^3)-ce^2g(ag(7ef-15dg)}{15c^3g^3\sqrt{f-}}$$

output
```
-8/15*e^2*(b*e*g-3*c*d*g+c*e*f)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^2+
2/5*e^2*(e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/g+1/15*e*(8*b^2*e^2*g^
2+c*e*g*(-9*a*e*g-30*b*d*g+7*b*e*f)+c^2*(45*d^2*g^2-30*d*e*f*g+8*e^2*f^2))
*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(
1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2
)/c^3/g^3/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))
^(1/2)-2/15*(4*b*e^3*g^2*(-a*g+b*f)+c^2*(-15*d^3*g^3+45*d^2*e*f*g^2-30*d*e
^2*f^2*g+8*e^3*f^3)-c*e^2*g*(a*g*(-15*d*g+7*e*f)-3*b*f*(-5*d*g+e*f)))*Elli
pticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),
(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*
(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f
-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^3/g^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2
)
```

3.909.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.49 (sec) , antiderivative size = 855, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\frac{4eg^2\left(8b^2e^2g^2+ceg(7bef-30bdg-9aeg)+c^2\left(8e^2f^2-30defg+45d^2g^2\right)\right)(a+x(b+cx))}{\sqrt{f+gx}}+4ce^2g^2\sqrt{f+gx}(a+x(b+cx))(-4beg+}{}$$

input
```
Integrate[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((4*e*g^2*(8*b^2*e^2*g^2 + c*e*g*(7*b*e*f - 30*b*d*g - 9*a*e*g) + c^2*(8*e
^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*(a + x*(b + c*x)))/Sqrt[f + g*x] + 4*c*
e^2*g^2*Sqrt[f + g*x]*(a + x*(b + c*x))*(-4*b*e*g + c*(-4*e*f + 15*d*g + 3
*e*g*x)) - (I*(f + g*x)*Sqrt[1 - (2*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f -
b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[2 + (4*(c*f^2 + g*(-(b*f)
+ a*g)))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*(e*(2*c*f -
 b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(8*b^2*e^2*g^2 + c*e*g*(7*b*e*f - 30*b*d*g
 - 9*a*e*g) + c^2*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*EllipticE[I*ArcSi
nh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c
)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*
f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))] - (-8*b^3*e^3*g^3 + b^2*e^2*g^2*(c*e*
f + 30*c*d*g + 8*e*Sqrt[(b^2 - 4*a*c)*g^2]) + b*c*e*g*(-45*c*d^2*g^2 + e*(
17*a*e*g^2 + Sqrt[(b^2 - 4*a*c)*g^2]*(7*e*f - 30*d*g))) + c*(-(a*e^2*g^2*(
4*c*e*f + 30*c*d*g + 9*e*Sqrt[(b^2 - 4*a*c)*g^2])) + c*(30*c*d^3*g^3 + e*S
qrt[(b^2 - 4*a*c)*g^2]*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))))*EllipticF[
I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2
- 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2]
)/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))]))/Sqrt[(c*f^2 + g*(-(b*f) + a*
g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/(30*c^3*g^4*Sqrt[a + x*(b +
 c*x)])
```

3.909.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1278, 2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1278}$$

$$\frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg}-\frac{\int \frac{-5cgd^3+be^2fd+4e^2(cef-3cdg+beg)x^2+ae^2(2ef+dg)+e(cd(2ef-15dg)+e(3bef+2bdg+3aeg))x}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{5cg}$$

$$\Big\downarrow \text{2184}$$

$$\frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg}-$$

$$\frac{\frac{2\int -\frac{g\left(4b^2fge^3+b\left(4aeg^2+cf(4ef-15dg)\right)e^2+\left(\left(8e^2f^2-30degf+45d^2g^2\right)c^2+eg(7bef-30bdg-9aeg)c+8b^2e^2g^2\right)xe+cg\left(15cd^3g-ae^2(2ef+15dg)\right)\right)}{2\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{3cg^2}+8e^2\sqrt{}}{5cg}$$

$$\Big\downarrow \; 27$$

$$\frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg}-$$

$$\frac{\frac{8e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}(beg-3cdg+cef)}{3cg}-\frac{\int \frac{4b^2fge^3+b\left(4aeg^2+cf(4ef-15dg)\right)e^2+\left(\left(8e^2f^2-30degf+45d^2g^2\right)c^2+eg(7bef-30bdg-9aeg)c+8b^2e^2g^2\right)xe+}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}}{3cg}}{5cg}$$

$$\Big\downarrow \; 1269$$

$$\frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg}-$$

$$\frac{\frac{8e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}(beg-3cdg+cef)}{3cg}-\frac{\frac{e\left(ceg(-9aeg-30bdg+7bef)+8b^2e^2g^2+c^2\left(45d^2g^2-30defg+8e^2f^2\right)\right)\int \frac{\sqrt{f+gx}}{\sqrt{cx^2+bx+a}}dx}{g}-\frac{\left(-ce^2g(ag(7ef-15dg\right.}{}}{3cg}}{5cg}$$

$$\Big\downarrow \; 1172$$

$$\frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg}-$$

$$\frac{8e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}(beg-3cdg+cef)}{3cg}-\frac{\sqrt{2}e\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ceg(-9aeg-30bdg+7bef)+8b^2e^2g^2+c^2\left(45d^2g^2-30defg+8e^2f^2\right)\right)\int -}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$\Big\downarrow \; 321$$

$$\frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg}-$$

$$\frac{8e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}(beg-3cdg+cef)}{3cg}-\frac{\sqrt{2}e\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ceg(-9aeg-30bdg+7bef)+8b^2e^2g^2+c^2\left(45d^2g^2-30defg+8e^2f^2\right)\right)\int -}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$\Big\downarrow \; 327$$

3.909. $\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

$$\frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg} - \frac{\frac{8e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}(beg-3cdg+cef)}{3cg} - \frac{\frac{\sqrt{2}e\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ceg(-9aeg-30bdg+7bef)+8b^2e^2g^2+c^2\left(45d^2g^2-30defg+8e^2f^2\right)\right)E}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}}{}}{}$$

input
```
Int[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*e^2*(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(5*c*g) - ((8*e^2*(c
*e*f - 3*c*d*g + b*e*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c*g) - ((S
qrt[2]*Sqrt[b^2 - 4*a*c]*e*(8*b^2*e^2*g^2 + c*e*g*(7*b*e*f - 30*b*d*g - 9*
a*e*g) + c^2*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*Sqrt[f + g*x]*Sqrt[-((
c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 -
 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*
c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + S
qrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c
]*(4*b*e^3*g^2*(b*f - a*g) + c^2*(8*e^3*f^3 - 30*d*e^2*f^2*g + 45*d^2*e*f*
g^2 - 15*d^3*g^3) - c*e^2*g*(a*g*(7*e*f - 15*d*g) - 3*b*f*(e*f - 5*d*g)))*
Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x
 + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b +
 Sqrt[b^2 - 4*a*c])*g)])/(c*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))/(3*c*g
))/(5*c*g)
```

3.909.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1278
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g
*x]*(Sqrt[a + b*x + c*x^2]/(c*g*(2*m - 1))), x] - Simp[1/(c*g*(2*m - 1))
Int[((d + e*x)^(m - 3)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*e^2*
f + a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*g*(2*m - 1) + e*(e*(2*b*d*g + e*(b*
f + a*g)*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c
*d*g + b*e*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
 IntegerQ[2*m] && GeQ[m, 2]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.909. $\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

3.909.4 Maple [A] (verified)

Time = 6.15 (sec) , antiderivative size = 985, normalized size of antiderivative = 1.56

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+bx+a)}\left(\frac{2e^3x\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{5cg}+\frac{2\left(3d\,e^2-\frac{2(2bg+2cf)e^3}{5cg}\right)\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{3cg}+\frac{2\left(d^3-\frac{2}{\ldots}\right.}{\ldots}\right.}{\ldots}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/5*e^3/c
/g*x*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/3*(3*d*e^2-2/5/c/g*
(2*b*g+2*c*f)*e^3)/c/g*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2*(
d^3-2/5*f*a/c/g*e^3-2/3*(3*d*e^2-2/5/c/g*(2*b*g+2*c*f)*e^3)/c/g*(1/2*a*g+1
/2*b*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-
4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(
-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/
2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2
*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*
(3*d^2*e-2/5*e^3/c/g*(3/2*a*g+3/2*b*f)-2/3*(3*d*e^2-2/5/c/g*(2*b*g+2*c*f)*
e^3)/c/g*(b*g+c*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(
b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1
/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f
/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f
*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/
g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c
)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2
))*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2
*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))...
```

3.909.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 633, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx =$$
$$-\frac{2\left((8\,c^3e^3f^3-3\,(10\,c^3de^2-bc^2e^3)f^2g+3\,(15\,c^3d^2e-5\,bc^2de^2+(b^2c-ac^2)e^3)fg^2-(45\,c^3d^3-45\,bc^2c\right.}{}$$

input
```
integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
-2/45*((8*c^3*e^3*f^3 - 3*(10*c^3*d*e^2 - b*c^2*e^3)*f^2*g + 3*(15*c^3*d^2
*e - 5*b*c^2*d*e^2 + (b^2*c - a*c^2)*e^3)*f*g^2 - (45*c^3*d^3 - 45*b*c^2*d
^2*e + 15*(2*b^2*c - 3*a*c^2)*d*e^2 - (8*b^3 - 21*a*b*c)*e^3)*g^3)*sqrt(c*
g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^
2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3
- 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)) + 3*(8*c^3*e^3
*f^2*g - (30*c^3*d*e^2 - 7*b*c^2*e^3)*f*g^2 + (45*c^3*d^2*e - 30*b*c^2*d*e
^2 + (8*b^2*c - 9*a*c^2)*e^3)*g^3)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2
- b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g
 - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstra
ssPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2
*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g
^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g))) - 3*(3*c^3*e^3*g^3*x - 4*
c^3*e^3*f*g^2 + (15*c^3*d*e^2 - 4*b*c^2*e^3)*g^3)*sqrt(c*x^2 + b*x + a)*sq
rt(g*x + f))/(c^4*g^4)
```

3.909.6 Sympy [F]

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x)**3/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.909.7 Maxima [F]

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^3}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((e*x + d)^3/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

3.909.8 Giac [F]

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^3}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^3/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

3.909.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+e\,x)^3}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x)^3/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x)^3/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.910 $\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

3.910.1 Optimal result

Integrand size = 31, antiderivative size = 479

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg} - \frac{2\sqrt{2}\sqrt{b^2-4ac}e(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4\right.}}\right.}{3c^2g^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} + \frac{2\sqrt{2}\sqrt{b^2-4ac}(e^2g(bf-ag)+c(2e^2f^2-6defg+3d^2g^2))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\operatorname{EllipticF}\left(\right.}{3c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

output
```
2/3*e^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/g-2/3*e*(b*e*g-3*c*d*g+c*e*f)*
EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1
/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1
/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)
/c^2/g^2/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^
(1/2)+2/3*(e^2*g*(-a*g+b*f)+c*(3*d^2*g^2-6*d*e*f*g+2*e^2*f^2))*EllipticF(1
/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(
-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c
+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(
-4*a*c+b^2)^(1/2))))^(1/2)/c^2/g^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.910.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.40 (sec) , antiderivative size = 981, normalized size of antiderivative = 2.05

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(2ce^2g^2(a+x(b+cx)) + \frac{(f+gx)\left(-\frac{4eg^2(cef-3cdg+beg)\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}(a+x(b+cx))}{(f+gx)^2} + \frac{i\sqrt{2}e(cef-3cdg+beg)\left(2cf-\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(Sqrt[f + g*x]*(2*c*e^2*g^2*(a + x*(b + c*x)) + ((f + g*x)*((-4*e*g^2*(c*e
*f - 3*c*d*g + b*e*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt
[(b^2 - 4*a*c)*g^2])]*(a + x*(b + c*x)))/(f + g*x)^2 + (I*Sqrt[2]*e*(c*e*f
 - 3*c*d*g + b*e*g)*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(-2*a*g^2
 + f*Sqrt[(b^2 - 4*a*c)*g^2] + 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b
*g*(f - g*x))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2
*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2] - 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]
*x + b*g*(-f + g*x))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]
*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g +
 Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 -
4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x] + (I
*Sqrt[2]*(3*c^2*d^2*g^2 + b*e^2*g*(b*g - Sqrt[(b^2 - 4*a*c)*g^2]) - c*e*(3
*b*d*g^2 + a*e*g^2 + Sqrt[(b^2 - 4*a*c)*g^2]*(e*f - 3*d*g)))*Sqrt[(-2*a*g^
2 + f*Sqrt[(b^2 - 4*a*c)*g^2] + 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x +
b*g*(f - g*x))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(
2*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2] - 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2
]*x + b*g*(-f + g*x))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))
]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g
+ Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 -
 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x])...
```

3.910.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1278, 2004, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{ 1278}$$

$$\frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg}-\frac{\int \frac{2e^2(cef-3cdg+beg)x^2+e(cd(2ef-9dg)+e(bef+2bdg+aeg))x+d\left(-3cgd^2+be^2f+ae^2g\right)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{3cg}$$

$$\Big\downarrow \text{ 2004}$$

$$\frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg}-\frac{\int \frac{-3cgd^2+be^2f+ae^2g+2e(cef-3cdg+beg)x}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{3cg}$$

$\downarrow$ 1269

$$\frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg}-\frac{\frac{2e(beg-3cdg+cef)\int \frac{\sqrt{f+gx}}{\sqrt{cx^2+bx+a}}dx}{g}-\frac{\left(e^2g(bf-ag)+c\left(3d^2g^2-6defg+2e^2f^2\right)\right)\int \frac{1}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{g}}{3cg}$$

$\downarrow$ 1172

$$\frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg}-\frac{\frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(beg-3cdg+cef)\int \frac{\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{\ldots}{2cf\ldots}}\ldots}{\ldots}}{3cg}$$

$\downarrow$ 321

$$\frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg}-\frac{\frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(beg-3cdg+cef)\int \frac{\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{\ldots}{2cf\ldots}}\ldots}{\ldots}}{3cg}$$

$\downarrow$ 327

$$\frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg}-\frac{\frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(beg-3cdg+cef)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+\ldots\right)}{b^2-4a\ldots}}\ldots}{\ldots}}{3cg}$$

input
```
Int[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

3.910. $\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

output
```
(2*e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c*g) - ((2*Sqrt[2]*Sqrt[b^2
 - 4*a*c]*e*(c*e*f - 3*c*d*g + b*e*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c
*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqr
t[b^2 - 4*a*c])*g)])/(c*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e^2*g*(b*f
- a*g) + c*(2*e^2*f^2 - 6*d*e*f*g + 3*d^2*g^2))*Sqrt[(c*(f + g*x))/(2*c*f
- (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]
*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/
Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(
c*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))/(3*c*g)
```

3.910.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.910. $\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

rule 1278
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g
*x]*(Sqrt[a + b*x + c*x^2]/(c*g*(2*m - 1))), x] - Simp[1/(c*g*(2*m - 1))
Int[((d + e*x)^(m - 3)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*e^2*
f + a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*g*(2*m - 1) + e*(e*(2*b*d*g + e*(b*
f + a*g)*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c
*d*g + b*e*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
 IntegerQ[2*m] && GeQ[m, 2]
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

3.910.4 Maple [A] (verified)

Time = 3.25 (sec) , antiderivative size = 834, normalized size of antiderivative = 1.74

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+bx+a)}\left(\frac{2e^2\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{3cg}+\frac{2\left(d^2-\frac{2e^2\left(\frac{ag}{2}+\frac{bf}{2}\right)}{3cg}\right)\left(\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\cdots}{-\frac{f}{g}-\cdots}}}{\sqrt{cg\,x^3+\cdots}}\cdots\right.}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/3*e^2/c
/g*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2*(d^2-2/3*e^2/c/g*(1/2
*a*g+1/2*b*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4
*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*
(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/
2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*
f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f
/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/
2))+2*(2*d*e-2/3*e^2/c/g*(b*g+c*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x
+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)
^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^
2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c
*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*Ellipt
icE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a
*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(
-4*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^
(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1
/2))))^(1/2))))
```

3.910.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 471, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{2\left(3\sqrt{cx^2+bx+a}\sqrt{gx+f}c^2e^2g^2+(2c^2e^2f^2-(6c^2de-bce^2)fg+(9c^2d^2-6bcde+(2b^2-3ac)e^2)g^2\right.}{}$$

input
```
integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
2/9*(3*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*c^2*e^2*g^2 + (2*c^2*e^2*f^2 -
(6*c^2*d*e - b*c*e^2)*f*g + (9*c^2*d^2 - 6*b*c*d*e + (2*b^2 - 3*a*c)*e^2)*
g^2)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*
g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g
^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)) +
6*(c^2*e^2*f*g - (3*c^2*d*e - b*c*e^2)*g^2)*sqrt(c*g)*weierstrassZeta(4/3*
(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*
c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3),
weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2),
 -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9
*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g))))/(c^3*g^3)
```

3.910.6 Sympy [F]

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x)**2/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.910.7 Maxima [F]

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^2}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((e*x + d)^2/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

3.910.8 Giac [F]

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^2}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

3.910.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+e\,x)^2}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x)^2/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x)^2/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.911 $\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

3.911.1 Optimal result

Integrand size = 29, antiderivative size = 393

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{2}\sqrt{b^2-4ac}\,e\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} - \frac{2\sqrt{2}\sqrt{b^2-4ac}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\cdots}{2cf-(\cdots}\right)}{cg\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

output
```
e*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^
(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/
2)/c/g/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1
/2)-2*(-d*g+e*f)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/
2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1
/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/g/(g*x+f)^(1/2)/(
c*x^2+b*x+a)^(1/2)
```

3.911.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.69 (sec) , antiderivative size = 814, normalized size of antiderivative = 2.07

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = -\frac{(f+gx)^{3/2}\left(-\frac{4eg^2\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}(a+x(b+cx))}{(f+gx)^2}+\frac{i\sqrt{2}e\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)\sqrt{\frac{-2ag^2+2cfgx+bg(f-gx)+\sqrt{(b^2-4ac)g}}{\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx}}}{}\right.}{}$$

input
```
Integrate[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/2*((f + g*x)^(3/2)*((-4*e*g^2*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f +
 b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*(a + x*(b + c*x)))/(f + g*x)^2 + (I*Sqrt[
2]*e*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(-2*a*g^2 + 2*c*f*g*x +
b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((2*c*f - b*g + Sqrt[(b
^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 - 2*c*f*g*x + b*g*(-f + g*x) +
 Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^
2])*(f + g*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/
(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g
 + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqr
t[f + g*x] - (I*Sqrt[2]*(2*c*d*g + e*(-(b*g) + Sqrt[(b^2 - 4*a*c)*g^2]))*S
qrt[(-2*a*g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g
*x))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 -
2*c*f*g*x + b*g*(-f + g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((-2*c*f +
 b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*S
qrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqr
t[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqr
t[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x]))/(c*g^2*Sqrt[(c*f^2 + g*(-(b*f) +
a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[a + x*(b + c*x)])
```

3.911.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1269}$$

$$\frac{e\int \frac{\sqrt{f+gx}}{\sqrt{cx^2+bx+a}}dx}{g}-\frac{(ef-dg)\int \frac{1}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{g}$$

$$\Big\downarrow \text{1172}$$

$$\frac{\sqrt{2}e\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\int \frac{\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4a}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{cg\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{2}e\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\int \frac{\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4acg}}{2cf-\left(b+\sqrt{b^2-4\right.}}\right.}{cg\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{2}e\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

input
```
Int[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b
^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4
*a*c])*g)])/(c*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*S
qrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e*f - d*g)*Sqrt[(c*(
f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sq
rt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2
 - 4*a*c])*g)])/(c*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

3.911.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.911.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(345) = 690$.

Time = 1.81 (sec) , antiderivative size = 746, normalized size of antiderivative = 1.90

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+bx+a)}\left(\frac{2d\left(\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x+\frac{b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}+\frac{b+\sqrt{-4ac+b^2}}{2c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b}}{2c}}}\right.}{\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}\right.}{}$
default	Expression too large to display

input
```
int((e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2*d*(f/g-
1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))
^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/
2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1
/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF((
(x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^
2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*e*(f/g-1/2*(b+
(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*
((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(
1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(
-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))
^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(
1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/(f/g-1/2*(
b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g
-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

3.911.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.93

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx =$$
$$-\frac{2\left(3\sqrt{cg}ceg\text{weierstrassZeta}\left(\frac{4\left(c^2f^2-bcfg+(b^2-3\,ac)g^2\right)}{3\,c^2g^2},-\frac{4\left(2\,c^3f^3-3\,bc^2f^2g-3\,(b^2c-6\,ac^2)fg^2+(2\,b^3-9\,abc)g^3\right)}{27\,c^3g^3},\text{weiers}\right.\right.}{}$$

input
```
integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(3*sqrt(c*g)*c*e*g*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*
a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2
)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f
^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^
2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3
*c*g*x + c*f + b*g)/(c*g))) + (c*e*f - (3*c*d - b*e)*g)*sqrt(c*g)*weierstr
assPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(
2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*
g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)))/(c^2*g^2)
```

3.911.6 Sympy [F]

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((e*x+d)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x)/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.911.7 Maxima [F]

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{ex+d}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

3.911.8 Giac [F]

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{ex+d}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

3.911.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{d+e\,x}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x)/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x)/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.912 $\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

3.912.1 Optimal result

Integrand size = 24, antiderivative size = 189

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{c\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

output
```
2*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^
(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/
(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2
)
```

3.912.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.45 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.63

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{i(f+gx)\sqrt{2-\frac{4(cf^2+g(-bf+ag))}{\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\sqrt{1+\frac{2(cf^2+g(-bf+ag))}{\left(-2cf+bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{2}\sqrt{\frac{c\cdots}{-2cf+\cdots}}}{\cdots}\right.\right.}{g\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}\sqrt{a+x(b+cx)}}$$

input
```
Integrate[1/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(I*(f + g*x)*Sqrt[2 - (4*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[
(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((
-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticF[I*ArcSinh[(S
qrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2
])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b
*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/(g*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*
f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[a + x*(b + c*x)])
```

3.912.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1172, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{1172}$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{c\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{c\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

input
```
Int[1/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4
*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[
b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*Sqrt[f + g*x]*Sqr
t[a + b*x + c*x^2])
```

3.912.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

3.912.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.52

method	result
default	$\frac{\left(-g\sqrt{-4ac+b^2}-bg+2cf\right)F\left(\sqrt{2}\sqrt{-\frac{(gx+f)c}{g\sqrt{-4ac+b^2}+bg-2cf}},\sqrt{-\frac{g\sqrt{-4ac+b^2}+bg-2cf}{2cf-bg+g\sqrt{-4ac+b^2}}}\right)\sqrt{\frac{\left(b+2cx+\sqrt{-4ac+b^2}\right)g}{g\sqrt{-4ac+b^2}+bg-2cf}}\sqrt{\frac{\left(-b-2cx+\sqrt{-4ac+b^2}\right)}{2cf-bg+g\sqrt{-4ac+b^2}}}}{cg(cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa)}$
elliptic	$\frac{2\sqrt{(gx+f)(cx^2+bx+a)}\left(\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x+\frac{b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}+\frac{b+\sqrt{-4ac+b^2}}{2c}}}F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\right.}{\sqrt{gx+f}\sqrt{cx^2+bx+a}\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}$

input
```
int(1/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-g*(-4*a*c+b^2)^(1/2)-b*g+2*c*f)/c*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a
*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-
b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a
*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*
g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b
*g-2*c*f))^(1/2)/g*(c*x^2+b*x+a)^(1/2)*(g*x+f)^(1/2)/(c*g*x^3+b*g*x^2+c*f*
x^2+a*g*x+b*f*x+a*f)
```

3.912.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{cg}\text{weierstrassPInverse}\left(\frac{4\,(c^2f^2-bcfg+(b^2-3\,ac)g^2)}{3\,c^2g^2},-\frac{4\,(2\,c^3f^3-3\,bc^2f^2g-3\,(b^2c-6\,ac^2)fg^2+(2\,b^3-9\,abc)g^3)}{27\,c^3g^3},\frac{3\,cgx+cf+bg}{3\,cg}\right)}{cg}$$

input
```
integrate(1/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2
)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2
+ (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g))/(c*g)
```

3.912.6 Sympy [F]

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.912.7 Maxima [F]

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

3.912.8 Giac [F]

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

3.912.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.913 $\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

3.913.1 Optimal result

Integrand size = 31, antiderivative size = 280

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticPi}\left(\frac{e\left(2cf-bg+\sqrt{b^2-4\right.}}{2c(ef-dg)}\right.}{\sqrt{c}(ef-dg)\sqrt{a+bx+cx^2}}$$

output
```
-EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))
^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4
*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(1-2*c*(g*
x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2
)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(-d*g+e*f
)/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.913.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.04 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.78

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{i(f+gx)\sqrt{2-\frac{4(cf^2+g(-bf+ag))}{\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\sqrt{1+\frac{2(cf^2+g(-bf+ag))}{\left(-2cf+bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{2}\sqrt{\frac{\cdots}{-2c\cdots}}}{\cdots}\right.\right.\right.}{(-ef\cdots}$$

input
```
Integrate[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(I*(f + g*x)*Sqrt[2 - (4*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[
(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((
-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*(EllipticF[I*ArcSinh[(
Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^
2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f -
b*g + Sqrt[(b^2 - 4*a*c)*g^2]))] - EllipticPi[((e*f - d*g)*(2*c*f - b*g -
Sqrt[(b^2 - 4*a*c)*g^2]))/(2*e*(c*f^2 + g*(-(b*f) + a*g))), I*ArcSinh[(Sqr
t[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])
])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g
 + Sqrt[(b^2 - 4*a*c)*g^2]))]))/((-(e*f) + d*g)*Sqrt[(c*f^2 + g*(-(b*f) +
a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[a + x*(b + c*x)])
```

3.913.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \; 1279$$

$$\frac{\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\int \frac{1}{\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}(d+ex)\sqrt{f+gx}}dx}{\sqrt{a+bx+cx^2}}$$

$\downarrow$ 187

$$-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\int \frac{1}{(ef-dg-e(f+gx))\sqrt{b+\frac{2c(f+gx)}{g}-\sqrt{b^2-4ac}-\frac{2cf}{g}}\sqrt{b+\frac{2c(f+gx)}{g}+\sqrt{b^2-4ac}-}}{\sqrt{a+bx+cx^2}}$$

$\downarrow$ 413

$$-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\int \frac{1}{(ef-dg-e(f+gx))\sqrt{b+\frac{2c(f+gx)}{g}+\sqrt{b^2-4ac}-}}}{\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(f+gx)}{g}-\frac{2cf}{g}}}$$

$\downarrow$ 413

$$-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{}{(ef-dg-e(f+g}}{\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(f+gx)}{g}-\frac{2cf}{g}}\sqrt{\sqrt{b^2-4ac}+b+\frac{2c(}{}}}$$

$\downarrow$ 412

$$-\frac{\sqrt{2}\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{}{2cf}}}{\sqrt{c}\sqrt{a+bx+cx^2}(ef-dg)\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(f+gx}{g}}}$$

input
```
Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-((Sqrt[2]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[b - Sqrt[b^2 - 4*a
*c] + 2*c*x]*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[1 - (2*c*(f + g*x))/
(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b
+ Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g)
)/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (
b - Sqrt[b^2 - 4*a*c])*g]], (2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)/(2*c*f - (
b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[c]*(e*f - d*g)*Sqrt[a + b*x + c*x^2]*Sqr
t[b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g + (2*c*(f + g*x))/g]*Sqrt[b + Sqrt[b^2
 - 4*a*c] - (2*c*f)/g + (2*c*(f + g*x))/g]))
```

3.913. $\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

3.913.3.1 Defintions of rubi rules used

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

3.913.4 Maple [A] (verified)

Time = 3.34 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.18

method	result
default	$\frac{\left(-g\sqrt{-4ac+b^2}-bg+2cf\right)\Pi\left(\sqrt{2}\sqrt{-\frac{(gx+f)c}{g\sqrt{-4ac+b^2}+bg-2cf}},\frac{\left(g\sqrt{-4ac+b^2}+bg-2cf\right)e}{2c(dg-ef)},\sqrt{-\frac{g\sqrt{-4ac+b^2}+bg-2cf}{2cf-bg+g\sqrt{-4ac+b^2}}}\right)\sqrt{\frac{\left(b+2cx+\sqrt{-4ac+b^2}\right)g}{g\sqrt{-4ac+b^2}+bg-2cf}}}{c(dg-ef)(cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa)}$
elliptic	$\frac{2\sqrt{(gx+f)(c\,x^2+bx+a)}\left(\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x+\frac{b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}+\frac{b+\sqrt{-4ac+b^2}}{2c}}}\Pi\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\right.}{\sqrt{gx+f}\,\sqrt{c\,x^2+bx+a}\,e\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}\left(-\frac{f}{g}+\frac{d}{e}\right)}$

input
```
int(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-g*(-4*a*c+b^2)^(1/2)-b*g+2*c*f)*EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*
c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d
*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)
))^(1/2))*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)
)^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))
^(1/2)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)/c*(c*x^
2+b*x+a)^(1/2)*(g*x+f)^(1/2)/(d*g-e*f)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*
x+a*f)
```

3.913.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.913.6 Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d+ex)\,\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/((d + e*x)*sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.913.7 Maxima [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f)), x)
```

3.913.8 Giac [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f)), x)
```

3.913.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/((f + g*x)^(1/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.914 $\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

3.914.1 Optimal result

Integrand size = 31, antiderivative size = 1037

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)}$$

$$+\frac{\sqrt{b^2-4ac}\,e\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}\,(cd^2-bde+ae^2)\,(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\,ef\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4a\right.}}\right.}{(cd^2-bde+ae^2)\,(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}\,dg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4a\right.}}\right.}{(cd^2-bde+ae^2)\,(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$-\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(}{2cf-\left(b+\right.}}}{\sqrt{2}\sqrt{c}\,(cd^2-bde+ae^2)\,(ef-dg}$$

output
```
-e^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)/(e*x
+d)+1/2*e*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^
(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(
1/2))*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/
2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(
2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-e*f*EllipticF(1/2*((b+2*c*x+(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2
*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x
^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))
^(1/2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)+d*
g*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^
(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/
(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)/(g*
x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)-1/2*(c*d*(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*
e*f))*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1
/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/
g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*(1-2*c*(g*x+f
)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))
^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(a*e^2-b*...
```

3.914.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.53 (sec) , antiderivative size = 1513, normalized size of antiderivative = 1.46

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-((e^2*Sqrt[f + g*x]*(a + b*x + c*x^2))/((c*d^2 - b*d*e + a*e^2)*(e*f - d*
g)*(d + e*x)*Sqrt[a + x*(b + c*x)])) + ((f + g*x)^(3/2)*Sqrt[a + b*x + c*x
^2]*(-4*e*(-(e*f) + d*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + S
qrt[(b^2 - 4*a*c)*g^2])]*(c*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x)
 + (a*g)/(f + g*x)))/(f + g*x)) + (I*Sqrt[2]*e*(-(e*f) + d*g)*(2*c*f - b*g
 + Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] - (2*a*g^2)/(f +
 g*x) - 2*c*f*(-1 + f/(f + g*x)) + b*g*(-1 + (2*f)/(f + g*x)))/(2*c*f - b*
g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] + (2*a*g^2)/(f
 + g*x) + 2*c*f*(-1 + f/(f + g*x)) + b*(g - (2*f*g)/(f + g*x)))/(-2*c*f +
b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 -
 b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]],
 -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a
*c)*g^2]))])/Sqrt[f + g*x] - (I*Sqrt[2]*(2*c*d*g*(e*f - 2*d*g) - e*(2*a*e*
g^2 + b*g*(e*f - 3*d*g) + Sqrt[(b^2 - 4*a*c)*g^2]*(e*f - d*g)))*Sqrt[(Sqrt
[(b^2 - 4*a*c)*g^2] - (2*a*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f + g*x)) + b*g
*(-1 + (2*f)/(f + g*x)))/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sq
rt[(b^2 - 4*a*c)*g^2] + (2*a*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f + g*x)) + b
*(g - (2*f*g)/(f + g*x)))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Ellipt
icF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(
b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*...
```

3.914.3 Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 909, normalized size of antiderivative = 0.88, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {1282, 25, 2154, 1269, 1172, 321, 327, 1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1282}$$

$$-\frac{\int -\frac{ce^2gx^2+2cdegx+2cd(ef-dg)-e(bef-2bdg+aeg)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)\left(ae^2-bde+cd^2\right)}-\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{ce^2gx^2+2cdegx+2cd(ef-dg)-e(bef-2bdg+aeg)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)\left(ae^2-bde+cd^2\right)}-\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{2154}$$

$$\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\int \frac{cdg+cexg}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)\left(ae^2-bde+cd^2\right)}-\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1269}$$

$$\frac{-c(ef-dg)\int \frac{1}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+ce\int \frac{\sqrt{\cdots}}{\sqrt{cx\cdots}}}{2(ef-dg)\left(ae^2-bde+cd^2\right)}-\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1172}$$

$$\frac{-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}+\frac{\sqrt{2}e\sqrt{b^2\cdots}}{\cdots}}{2(ef-\cdots}$$

$$\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{321}$$

$$\frac{\frac{\sqrt{2}e\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\int \frac{\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}+(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{\cdots}{(\cdots}}{2(ef-d\cdots}$$

$$\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{327}$$

$$\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4a\cdots}\right.}}}{\sqrt{f\cdots}}}{2(ef\cdots}$$

$$\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)\left(ae^2-bde+cd^2\right)}$$

3.914. $\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

$$\Big\downarrow \text{ 1279}$$

$$\frac{\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}(d+ex)\sqrt{f+gx}}dx}{\sqrt{a+bx+cx^2}} - \frac{2\sqrt{2}\sqrt{b^2-4}\cdots}{\cdots}$$

$$\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 187}$$

$$-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(ef-dg-e(f+gx))\sqrt{b+\frac{2c(f+gx)}{g}-\sqrt{b^2-4ac}-\frac{2cf}{g}}\sqrt{b+\frac{2c(f+gx)}{g}+\sqrt{\cdots}}}\cdots}{\sqrt{a+bx+cx^2}}$$

$$\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 413}$$

$$-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(ef-dg-e(f+gx))\sqrt{b+\frac{2c(f+gx)}{g}+\sqrt{b^2-\cdots}}}\cdots}{\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(f+gx)}{g}-\frac{2cf}{g}}\cdots}$$

$$\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 413}$$

$$\frac{\sqrt{2}\sqrt{b^2-4ac}e\sqrt{f+gx}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}} - \frac{2\sqrt{2}\sqrt{b^2-4ac}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\cdots}{\cdots}$$

$$\frac{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}{\left(cd^2-bed+ae^2\right)(ef-dg)(d+ex)}$$

$$\Big\downarrow \text{ 412}$$

3.914. $\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

$$\frac{\sqrt{2}\sqrt{b^2-4ac}e\sqrt{f+gx}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}} - \frac{2\sqrt{2}\sqrt{b^2-4ac}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\cdots}{\cdots}$$

$$\frac{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}{(cd^2-bed+ae^2)(ef-dg)(d+ex)}$$

input
```
Int[1/((d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-((e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*(e*f
- d*g)*(d + e*x))) + ((Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*Sqrt[f + g*x]*Sqrt[-((c
*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 -
4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c
*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e*
f - d*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c
*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 -
4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c
*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) -
(Sqrt[2]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*d*(2*e*f - 3*d*g) - e*
(b*e*f - 2*b*d*g + a*e*g))*Sqrt[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[b + Sq
rt[b^2 - 4*a*c] + 2*c*x]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 -
 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]
*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), Arc
Sin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g
]], (2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g
)])/(Sqrt[c]*(e*f - d*g)*Sqrt[a + b*x + c*x^2]*Sqrt[b - Sqrt[b^2 - 4*a*c]
- (2*c*f)/g + (2*c*(f + g*x))/g]*Sqrt[b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g +
(2*c*(f + g*x))/g]))/(2*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g))
```

3.914. $\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

3.914.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

3.914. $\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1282
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^
(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*
e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*
g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

3.914.4 Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 1347, normalized size of antiderivative = 1.30

method	result	size
elliptic	Expression too large to display	1347
default	Expression too large to display	14048

input
```
int(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(e^2/(a*d*
e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)*(c*g*x^3+b*g*x^2+c*f*
x^2+a*g*x+b*f*x+a*f)^(1/2)/(e*x+d)-c*d*g/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*
e^2*f+c*d^3*g-c*d^2*e*f)*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-
1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-
f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c
)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*
x+b*f*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^
(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1
/2))))^(1/2))-c*e*g/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e
*f)*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(
1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c
+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*
c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((
-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c
+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-
b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x
+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)
^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))+(a*e^2*g-2*b*d*e*
g+b*e^2*f+3*c*d^2*g-2*c*d*e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c...
```

3.914.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

3.914.6 Sympy [F]

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/(e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/((d + e*x)**2*sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.914.7 Maxima [F]

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)^2\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^2*sqrt(g*x + f)), x)
```

3.914.8 Giac [F]

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)^2\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^2*sqrt(g*x + f)), x)
```

3.914.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,(d+e\,x)^2\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(d + e*x)^2*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/((f + g*x)^(1/2)*(d + e*x)^2*(a + b*x + c*x^2)^(1/2)), x)
```

3.915 $\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

3.915.1 Optimal result

Integrand size = 31, antiderivative size = 1114

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(cd^2-bde+ae^2)(ef-dg)(d+ex)^2}$$

$$-\frac{3e^2(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4(cd^2-bde+ae^2)^2(ef-dg)^2(d+ex)}$$

$$+\frac{3\sqrt{b^2-4ac}\,e(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{4\sqrt{2}(cd^2-bde+ae^2)^2(ef-dg)^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{b^2-4ac}(cd(-6ef+7dg)+e(3bef-4bdg+aeg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{\frac{c(a+x(b+cx))}{-b^2+4ac}}\,\text{EllipticF}\left(\text{arcsin}\right.}{2\sqrt{2}(cd^2+e(-bd+ae))^2(ef-dg)\sqrt{f+gx}\sqrt{a+x(b+cx)}}$$

$$+\frac{\sqrt{2cf-bg+\sqrt{b^2-4ac}g}\,(c^2d^2(8e^2f^2-20defg+15d^2g^2)+2ce(bd(-4e^2f^2+11defg-10d^2g^2)+ae(}{}$$

output
```
-1/2*e^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)/
(e*x+d)^2-3/4*e^2*(c*d*(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*(g*x+f)^(1/
2)*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)^2/(e*x+d)+3/8*e*(c
*d*(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*EllipticE(1/2*((b+2*c*x+(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2
*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-
c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)^2*2^(
1/2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2
)+1/4*(c*d*(7*d*g-6*e*f)+e*(a*e*g-4*b*d*g+3*b*e*f))*EllipticF(1/2*((b+2*c*
x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*(g*(-4*a*c
+b^2)^(1/2)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(
c*(a+x*(c*x+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/
2))))^(1/2)/(c*d^2+e*(a*e-b*d))^2/(-d*g+e*f)*2^(1/2)/(g*x+f)^(1/2)/(a+x*(c
*x+b))^(1/2)+1/8*(c^2*d^2*(15*d^2*g^2-20*d*e*f*g+8*e^2*f^2)+2*c*e*(b*d*(-1
0*d^2*g^2+11*d*e*f*g-4*e^2*f^2)+a*e*(3*d^2*g^2+2*d*e*f*g-2*e^2*f^2))+e^2*(
3*a^2*e^2*g^2+2*a*b*e*g*(-4*d*g+e*f)+b^2*(8*d^2*g^2-8*d*e*f*g+3*e^2*f^2)))
*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))
^(1/2),(2*c*e*f-b*e*g+e*g*(-4*a*c+b^2)^(1/2))/(-2*c*d*g+2*c*e*f),((2*c*f+g
*(-b+(-4*a*c+b^2)^(1/2)))/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(2*c*f-
b*g+g*(-4*a*c+b^2)^(1/2))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f...
```

3.915.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 37.30 (sec) , antiderivative size = 40396, normalized size of antiderivative = 36.26

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
Result too large to show
```

3.915.3 Rubi [A] (verified)

Time = 3.54 (sec) , antiderivative size = 1643, normalized size of antiderivative = 1.47, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.548, Rules used = {1282, 2154, 1282, 25, 2154, 27, 1172, 321, 1269, 1172, 321, 327, 1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{1282}$$

$$-\frac{\int \frac{cgx^2e^2+3(bf+ag)e^2+2(cef-2cdg+beg)xe-4d(cef-cdg+beg)}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{4(ef-dg)\left(ae^2-bde+cd^2\right)}-\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2(ef-dg)\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{2154}$$

$$-\frac{\int \frac{2cef-5cdg+2beg+cegx}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx-3(cd(2ef-3dg)-e(aeg-2bdg+bef))\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{4(ef-dg)\left(ae^2-bde+cd^2\right)}-\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2(ef-dg)\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1282}$$

$$-\frac{\int \frac{2cef-5cdg+2beg+cegx}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx-3(cd(2ef-3dg)-e(aeg-2bdg+bef))\left(-\frac{\int -\frac{ce^2gx^2+2cdegx+2cd(ef-dg)-e(bef-2bdg+aeg)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}}{2(ef-dg)(ae^2-bde+cd^2)}\right.}{4(ef-dg)\left(ae^2-bde+cd^2\right)}-\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2(ef-dg)\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{2cef-5cdg+2beg+cegx}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx-3(cd(2ef-3dg)-e(aeg-2bdg+bef))\left(\frac{\int \frac{ce^2gx^2+2cdegx+2cd(ef-dg)-e(bef-2bdg+aeg)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{2(ef-dg)(ae^2-bde+cd^2)}\right.}{4(ef-dg)\left(ae^2-bde+cd^2\right)}-\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2(ef-dg)\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{2154}$$

$$-\frac{-3(cd(2ef-3dg)-e(aeg-2bdg+bef))\left(\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\int\frac{cdg+cexg}{\sqrt{f+gx}\sqrt{cx^2+bx+}}}{2(ef-dg)(ae^2-bde+cd^2)}\right.}{4(ef-dg)\left(ae^2-\right.}$$

$$\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2(ef-dg)\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 27

$$-\frac{-3(cd(2ef-3dg)-e(aeg-2bdg+bef))\left(\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\int\frac{cdg+cexg}{\sqrt{f+gx}\sqrt{cx^2+bx+}}}{2(ef-dg)(ae^2-bde+cd^2)}\right.}{4(ef-dg)\left(ae^2\right.}$$

$$\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2(ef-dg)\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1172

$$-\frac{\frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\int\frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{g\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}+1}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}-3(cd(2ef-3}{}$$

$$\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2(ef-dg)\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 321

$$-\frac{-3(cd(2ef-3dg)-e(aeg-2bdg+bef))\left(\frac{(cd(2ef-3dg)-e(aeg-2bdg+bef))\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\int\frac{cdg+cexg}{\sqrt{f+gx}\sqrt{cx^2+bx+}}}{2(ef-dg)(ae^2-bde+cd^2)}\right.}{}$$

$$\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2(ef-dg)\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1269

$$-\frac{-3(cd(2ef-3dg)-e(aeg-2bdg+bef))\left(\frac{-c(ef-dg)\int\frac{1}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+(cd(2ef-3dg)-e(aeg-2bdg+bef))\int\frac{}{(d+ex)\sqrt{f+g}}}{2(ef-dg)(ae^2-bde+cd^2)}\right.}{}$$

$$\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(d+ex)^2(ef-dg)\left(ae^2-bde+cd^2\right)}$$

3.915. $\int\frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

$$\Big\downarrow \text{1172}$$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}e^2}{2\left(cd^2-bed+ae^2\right)(ef-dg)(d+ex)^2}-$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}+2(cef-3cdg+b$$

$$\Big\downarrow \text{321}$$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}e^2}{2\left(cd^2-bed+ae^2\right)(ef-dg)(d+ex)^2}-$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}+2(cef-3cdg+b$$

$$\Big\downarrow \text{327}$$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}e^2}{2\left(cd^2-bed+ae^2\right)(ef-dg)(d+ex)^2}-$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}+2(cef-3cdg+b$$

$$\Big\downarrow \text{1279}$$

3.915. $\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}e^2}{2\left(cd^2-bed+ae^2\right)(ef-dg)(d+ex)^2}-$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}+\frac{2(cef-3cdg+beg)\sqrt{b\cdots}}{\cdots}$$

$\downarrow$ 187

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}e^2}{2\left(cd^2-bed+ae^2\right)(ef-dg)(d+ex)^2}-$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}-\frac{4(cef-3cdg+beg)\sqrt{b\cdots}}{\cdots}$$

$\downarrow$ 413

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}e^2}{2\left(cd^2-bed+ae^2\right)(ef-dg)(d+ex)^2}-$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}-\frac{4(cef-3cdg+beg)\sqrt{b\cdots}}{\cdots}$$

$\downarrow$ 413

3.915. $\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}e^2}{2\left(cd^2-bed+ae^2\right)(ef-dg)(d+ex)^2}-$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}-\frac{4(cef-3cdg+beg)\sqrt{b\cdots}}{\cdots}$$

$\downarrow$ 412

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}e^2}{2\left(cd^2-bed+ae^2\right)(ef-dg)(d+ex)^2}-$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-\cdots}\right.}}{\cdots}$$

input
```
Int[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

3.915. $\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

output
```
-1/2*(e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*(e
*f - d*g)*(d + e*x)^2) - ((2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*x)
)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c
])*g)])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[2*c*f - (b
 - Sqrt[b^2 - 4*a*c])*g]*(c*e*f - 3*c*d*g + b*e*g)*Sqrt[b - Sqrt[b^2 - 4*a
*c] + 2*c*x]*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[1 - (2*c*(f + g*x))/
(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b
+ Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g)
)/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (
b - Sqrt[b^2 - 4*a*c])*g]], (2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)/(2*c*f - (
b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[c]*(e*f - d*g)*Sqrt[a + b*x + c*x^2]*Sqr
t[b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g + (2*c*(f + g*x))/g]*Sqrt[b + Sqrt[b^2
 - 4*a*c] - (2*c*f)/g + (2*c*(f + g*x))/g]) - 3*(c*d*(2*e*f - 3*d*g) - e*(
b*e*f - 2*b*d*g + a*e*g))*(-((e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((c
*d^2 - b*d*e + a*e^2)*(e*f - d*g)*(d + e*x))) + ((Sqrt[2]*Sqrt[b^2 - 4*a*c
]*e*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[A
rcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-
2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[(c*(...
```

3.915.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

3.915. $\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

rule 1282
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^
(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*
e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*
g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

3.915.4 Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 1686, normalized size of antiderivative = 1.51

method	result	size
elliptic	Expression too large to display	1686
default	Expression too large to display	64947

input
```
int(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(1/2*e^2/(
a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)*(c*g*x^3+b*g*x^2+
c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(e*x+d)^2+3/4*e^2*(a*e^2*g-2*b*d*e*g+b*e^2*
f+3*c*d^2*g-2*c*d*e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^
2*e*f)^2*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(e*x+d)-1/4*c*g*(
a*d*e^2*g+2*a*e^3*f-4*b*d^2*e*g+b*d*e^2*f+7*c*d^3*g-4*c*d^2*e*f)/(a*d*e^2*
g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)^2*(f/g-1/2*(b+(-4*a*c+b^2
)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(
-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/
2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*
g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(
b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g
-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))-3/4*e*c*g*(a*e^2*g-2*b*d*e*g+b*e^2
*f+3*c*d^2*g-2*c*d*e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d
^2*e*f)^2*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+
b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(
-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+
(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1
/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(
-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g...
```

3.915.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

3.915.6 Sympy [F]

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/(e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/((d + e*x)**3*sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.915.7 Maxima [F]

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)^3\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^3*sqrt(g*x + f)), x)
```

3.915.8 Giac [F]

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)^3\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^3*sqrt(g*x + f)), x)
```

3.915.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{1}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^3\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(d + e*x)^3*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/((f + g*x)^(1/2)*(d + e*x)^3*(a + b*x + c*x^2)^(1/2)), x)
```

3.916 $\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx$

3.916.1 Optimal result

Integrand size = 31, antiderivative size = 553

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}$$
$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$
$$-\frac{\sqrt{2}e\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticPi}\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}\right.}{2c(ef-dg)}\right.}{\sqrt{c}(ef-dg)^2\sqrt{a+bx+cx^2}}$$

output
```
2*g^2*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)^(1/2)-g*E
llipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/
2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/
(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+
(-4*a*c+b^2)^(1/2))))^(1/2)-e*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*
c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))
/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2
)))^(1/2))*2^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*
(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+
b^2)^(1/2))))^(1/2)/(-d*g+e*f)^2/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.916.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.22 (sec) , antiderivative size = 950, normalized size of antiderivative = 1.72

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \frac{2\left(\frac{g^2(a+x(b+cx))}{ef-dg} + \frac{(f+gx)^2\left(c+\frac{cf^2}{(f+gx)^2}-\frac{bfg}{(f+gx)^2}+\frac{ag^2}{(f+gx)^2}-\frac{2cf}{f+gx}+\frac{bg}{f+gx}-\frac{i\sqrt{1\cdots}}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}$$

input
```
Integrate[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*((g^2*(a + x*(b + c*x)))/(e*f - d*g) + ((f + g*x)^2*(c + (c*f^2)/(f + g
*x)^2 - (b*f*g)/(f + g*x)^2 + (a*g^2)/(f + g*x)^2 - (2*c*f)/(f + g*x) + (b
*g)/(f + g*x) - ((I/4)*Sqrt[1 - (2*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b
*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[2 + (4*(c*f^2 + g*(-(b*f) +
 a*g)))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*((e*f - d*g)
*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(EllipticE[I*ArcSinh[(Sqrt[2]*Sqr
t[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[
f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[
(b^2 - 4*a*c)*g^2]))] - EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g +
 a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c
*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]
))]) - 2*e*(c*f^2 + g*(-(b*f) + a*g))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c
*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f +
g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2
 - 4*a*c)*g^2]))] + 2*e*(c*f^2 + g*(-(b*f) + a*g))*EllipticPi[((e*f - d*g)
*(2*c*f - b*g - Sqrt[(b^2 - 4*a*c)*g^2]))/(2*e*(c*f^2 + g*(-(b*f) + a*g)))
, I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^
2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^
2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))]))/((e*f - d*g)*Sqrt[(c*f^2 +
 g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[f + g...
```

3.916.3 Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1288, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{1288}$$

$$\int \left(\frac{e}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}(ef-dg)}-\frac{g}{(f+gx)^{3/2}\sqrt{a+bx+cx^2}(ef-dg)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{2}g\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{a+bx+cx^2}(ef-dg)\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-$$

$$\frac{\sqrt{2}e\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticPi}\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}g\right)}{2c(ef-dg)},\text{ar}\right.}{\sqrt{c}\sqrt{a+bx+cx^2}(ef-dg)^2}$$

$$\frac{2g^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}(ef-dg)\left(ag^2-bfg+cf^2\right)}$$

input
```
Int[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*g^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[f
+ g*x]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x +
c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sq
rt[b^2 - 4*a*c])*g)])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*
x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]
*e*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f
 - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt
[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c
*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sq
rt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 -
4*a*c] - (2*c*f)/g)])/(Sqrt[c]*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2])
```

3.916.3.1 Defintions of rubi rules used

rule 1288
```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && IntegerQ[n + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.916.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1263 vs. 2(490) = 980.

Time = 3.23 (sec) , antiderivative size = 1264, normalized size of antiderivative = 2.29

method	result	size
elliptic	Expression too large to display	1264
default	Expression too large to display	4757

input
```
int(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2*(c*g*x
^2+b*g*x+a*g)/(a*g^2-b*f*g+c*f^2)*g/(d*g-e*f)/((x+f/g)*(c*g*x^2+b*g*x+a*g)
)^(1/2)+2*(-g*(b*g-c*f)/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)+b*g^2/(a*g^2-b*f*g+c
*f^2)/(d*g-e*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(
-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/
c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+
1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+
a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((
-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(
1/2))+2*g^2*c/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2)
)/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*
a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-
4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b
*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))
)*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*
(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2
/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/
2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c
+b^2)^(1/2))))^(1/2)))-2/(d*g-e*f)*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+
f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^...
```

3.916.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.916.6 Sympy [F]

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d+ex)\,(f+gx)^{\frac{3}{2}}\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)**(3/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/((d + e*x)*(f + g*x)**(3/2)*sqrt(a + b*x + c*x**2)), x)
```

3.916.7 Maxima [F]

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^(3/2)), x)
```

3.916.8 Giac [F]

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^(3/2)), x)
```

3.916.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(f+g\,x)^{3/2}\,(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/((f + g*x)^(3/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/((f + g*x)^(3/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.917 $\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx$

3.917.1 Optimal result

Integrand size = 31, antiderivative size = 1125

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^{3/2}}$$
$$+\frac{4g^2(2cf-bg)\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)^2\sqrt{f+gx}}+\frac{2eg^2\sqrt{a+bx+cx^2}}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\,g(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3(ef-dg)\left(cf^2-bfg+ag^2\right)^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$
$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\,eg\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$
$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right.}{3(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$
$$-\frac{\sqrt{2}e^2\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticPi}\left(\frac{e\left(2cf-bg+\sqrt{b^2-4a\right.}}{2c(ef-dg)}\right.}{\sqrt{c}(ef-dg)^3\sqrt{a+bx+cx^2}}$$

output
```
2/3*g^2*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)^(3/2)+4
/3*g^2*(-b*g+2*c*f)*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2/(
g*x+f)^(1/2)+2*e*g^2*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)/
(g*x+f)^(1/2)-2/3*g*(-b*g+2*c*f)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+
(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*
(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2/(c*x^2+
b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-e*g*Ellipt
icE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-
2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-
4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/(-d*g
+e*f)^2/(a*g^2-b*f*g+c*f^2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4
*a*c+b^2)^(1/2))))^(1/2)+2/3*g*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-
4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-
4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(-d*g
+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)-e^2*EllipticPi
(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*
e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(
1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(1-2*c*(g*x+f)/(2*...
```

3.917.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.11 (sec) , antiderivative size = 14762, normalized size of antiderivative = 13.12

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((d + e*x)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
Result too large to show
```

3.917.3 Rubi [A] (verified)

Time = 2.11 (sec) , antiderivative size = 1125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1288, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1288}$$

$$\int \left(\frac{e^2}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}(ef-dg)^2}-\frac{eg}{(f+gx)^{3/2}\sqrt{a+bx+cx^2}(ef-dg)^2}-\frac{g}{(f+gx)^{5/2}\sqrt{a+bx+}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticPi}\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}g\right)}{2c(ef-dg)},a\right.}{\sqrt{c}(ef-dg)^3\sqrt{cx^2+bx+a}}$$

$$\frac{\sqrt{2}\sqrt{b^2-4ac}g\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)e}{(ef-dg)^2\left(cf^2-bgf+ag^2\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}+$$

$$\frac{2g^2\sqrt{cx^2+bx+a}e}{(ef-dg)^2\left(cf^2-bgf+ag^2\right)\sqrt{f+gx}}-$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}g(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3(ef-dg)\left(cf^2-bgf+ag^2\right)^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}+$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3(ef-dg)\left(cf^2-bgf+ag^2\right)\sqrt{f+gx}\sqrt{cx^2+bx+a}}+$$

$$\frac{4g^2(2cf-bg)\sqrt{cx^2+bx+a}}{3(ef-dg)\left(cf^2-bgf+ag^2\right)^2\sqrt{f+gx}}+\frac{2g^2\sqrt{cx^2+bx+a}}{3(ef-dg)\left(cf^2-bgf+ag^2\right)(f+gx)^{3/2}}$$

input
```
Int[1/((d + e*x)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*g^2*Sqrt[a + b*x + c*x^2])/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f +
g*x)^(3/2)) + (4*g^2*(2*c*f - b*g)*Sqrt[a + b*x + c*x^2])/(3*(e*f - d*g)*(
c*f^2 - b*f*g + a*g^2)^2*Sqrt[f + g*x]) + (2*e*g^2*Sqrt[a + b*x + c*x^2])/
((e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x]) - (2*Sqrt[2]*Sqrt[b^
2 - 4*a*c]*g*(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)])/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)^2*Sqrt[(c*(f + g*x))/(2*c
*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[b^
2 - 4*a*c]*e*g*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*
EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/S
qrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/((
e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt
[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g
*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*
x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b
+ Sqrt[b^2 - 4*a*c])*g)])/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[f +
g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*e^2*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*
a*c])*g]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*...
```

3.917.3.1 Defintions of rubi rules used

rule 1288
```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && IntegerQ[n + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.917.4 Maple [A] (verified)

Time = 4.08 (sec) , antiderivative size = 1505, normalized size of antiderivative = 1.34

method	result	size
elliptic	Expression too large to display	1505
default	Expression too large to display	27601

input
```
int(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/3/(a*g
^2-b*f*g+c*f^2)/(d*g-e*f)*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/
(x+f/g)^2+2/3*(c*g*x^2+b*g*x+a*g)/(a*g^2-b*f*g+c*f^2)^2*g*(3*a*e*g^2+2*b*d
*g^2-5*b*e*f*g-4*c*d*f*g+7*c*e*f^2)/(d*g-e*f)^2/((x+f/g)*(c*g*x^2+b*g*x+a*
g))^(1/2)+2*(-1/3*c*g/(a*g^2-b*f*g+c*f^2)/(d*g-e*f)+1/3*g*(b*g-c*f)*(3*a*e
*g^2+2*b*d*g^2-5*b*e*f*g-4*c*d*f*g+7*c*e*f^2)/(a*g^2-b*f*g+c*f^2)^2/(d*g-e
*f)^2-1/3*b*g^2/(a*g^2-b*f*g+c*f^2)^2*(3*a*e*g^2+2*b*d*g^2-5*b*e*f*g-4*c*d
*f*g+7*c*e*f^2)/(d*g-e*f)^2)*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(
f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))
)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2
))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+
a*g*x+b*f*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/
c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2
)^(1/2))))^(1/2))-2/3*g^2*c*(3*a*e*g^2+2*b*d*g^2-5*b*e*f*g-4*c*d*f*g+7*c*e
*f^2)/(a*g^2-b*f*g+c*f^2)^2/(d*g-e*f)^2*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)
*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+
b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*
c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x
^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*El
lipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(...
```

3.917.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.917.6 Sympy [F]

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d+ex)\,(f+gx)^{\frac{5}{2}}\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)**(5/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/((d + e*x)*(f + g*x)**(5/2)*sqrt(a + b*x + c*x**2)), x)
```

3.917.7 Maxima [F]

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)(gx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^(5/2)), x)
```

3.917.8 Giac [F]

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)(gx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^(5/2)), x)
```

3.917.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(f+g\,x)^{5/2}\,(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/((f + g*x)^(5/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/((f + g*x)^(5/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.918 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

3.918.1 Optimal result

Integrand size = 33, antiderivative size = 475

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\sqrt{2}\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{\frac{(ef-dg)\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\sqrt{\frac{(ef-dg)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(bf+\sqrt{b^2-4ac}f-2ag\right)(d+ex)}}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)}}$$

output
```
(e*x+d)*EllipticPi((g*x+f)^(1/2)*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e
*x+d)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2),e*(2*c*f-g*(b+(-4*a*c+b
^2)^(1/2)))/g/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))),((b*d-2*a*e+d*(-4*a*c+b^2)^
(1/2))*(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))/(b*f-2*a*g+f*(-4*a*c+b^2)^(1/2))/(
2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(b+2*c*x-(-4*a*c+b^2)^(1/2
))^(1/2)*((-d*g+e*f)*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*x+d)/(2*c*f-g*(b+(-4*
a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((-d*g+e*f)
*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))/(e*x+d)/(b*f-2*a*g+f*(-4*a*c+b^2)^(1/2)))^
(1/2)/g/(c*x^2+b*x+a)^(1/2)/(c*x+2*a*c/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(2*c*
d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.918.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1118 vs. $2(475) = 950$.

Time = 28.87 (sec) , antiderivative size = 1118, normalized size of antiderivative = 2.35

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{\sqrt{2}\sqrt{-\frac{g(cf^2+g(-bf+ag))(d+ex)}{\left(-2cdfg-2aeg^2+ef\sqrt{(b^2-4ac)g^2}-dg\sqrt{(b^2-4ac)g^2}+bg(ef+dg)\right)(f+gx)}}(f+gx)^{3/2}\left(\frac{2ef\sqrt{(b^2-4ac)g^2}\sqrt{-\frac{(cf^2+g(-bf+a}{(b^2-4ac}}}{}\right.}{}$$

input
```
Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-((Sqrt[2]*Sqrt[-((g*(c*f^2 + g*(-(b*f) + a*g))*(d + e*x))/((-2*c*d*f*g -
2*a*e*g^2 + e*f*Sqrt[(b^2 - 4*a*c)*g^2] - d*g*Sqrt[(b^2 - 4*a*c)*g^2] + b*
g*(e*f + d*g))*(f + g*x)))]*(f + g*x)^(3/2)*((2*e*f*Sqrt[(b^2 - 4*a*c)*g^2
]*Sqrt[-(((c*f^2 + g*(-(b*f) + a*g))*(a + x*(b + c*x)))/((b^2 - 4*a*c)*(f
+ g*x)^2))]*EllipticF[ArcSin[Sqrt[(-2*a*g^2 + 2*c*f*g*x + b*g*(f - g*x) +
Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/(Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))]/Sq
rt[2]], (2*Sqrt[(b^2 - 4*a*c)*g^2]*(-(e*f) + d*g))/(2*c*d*f*g + 2*a*e*g^2
- e*f*Sqrt[(b^2 - 4*a*c)*g^2] + d*g*Sqrt[(b^2 - 4*a*c)*g^2] - b*g*(e*f + d
*g))])/(c*f^2 + g*(-(b*f) + a*g)) + (d*g*(2*a*g^2 - f*Sqrt[(b^2 - 4*a*c)*g
^2] - 2*c*f*g*x - g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(-f + g*x))*Sqrt[(2*a*
g^2 - 2*c*f*g*x + b*g*(-f + g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/(Sqr
t[(b^2 - 4*a*c)*g^2]*(f + g*x))]*EllipticF[ArcSin[Sqrt[(-2*a*g^2 + 2*c*f*g
*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/(Sqrt[(b^2 - 4*a*c
)*g^2]*(f + g*x))]/Sqrt[2]], (2*Sqrt[(b^2 - 4*a*c)*g^2]*(-(e*f) + d*g))/(2
*c*d*f*g + 2*a*e*g^2 - e*f*Sqrt[(b^2 - 4*a*c)*g^2] + d*g*Sqrt[(b^2 - 4*a*c
)*g^2] - b*g*(e*f + d*g))])/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)*Sqrt[(-2
*a*g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/(S
qrt[(b^2 - 4*a*c)*g^2]*(f + g*x))]) - (4*e*Sqrt[(b^2 - 4*a*c)*g^2]*Sqrt[-(
((c*f^2 + g*(-(b*f) + a*g))*(a + x*(b + c*x)))/((b^2 - 4*a*c)*(f + g*x)^2)
)]*EllipticPi[(2*Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*...
```

3.918.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {1276}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \; 1276$$

$$\frac{\sqrt{2}(d+ex)\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}\sqrt{\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)(ef-dg)}{(d+ex)\left(2cf-g\left(\sqrt{b^2-4ac}+b\right)\right)}}\sqrt{\frac{\left(x\left(\sqrt{b^2-4ac}+b\right)+2a\right)(e}{(d+ex)\left(f\sqrt{b^2-4ac}-2ag\right.}}}{g\sqrt{\frac{2ac}{\sqrt{b^2-4ac}+b}+cx}\sqrt{a+bx+}}$$

input
```
Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(Sqrt[2]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[b - Sqrt[b^2 - 4*a*c
] + 2*c*x]*Sqrt[((e*f - d*g)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)*(d + e*x))]*Sqrt[((e*f - d*g)*(2*a + (b + Sqrt[b^
2 - 4*a*c])*x))/((b*f + Sqrt[b^2 - 4*a*c]*f - 2*a*g)*(d + e*x))]*(d + e*x)
*EllipticPi[(e*(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/((2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e)*g), ArcSin[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f
+ g*x])/(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])], ((b*d +
Sqrt[b^2 - 4*a*c]*d - 2*a*e)*(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/((2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)*(b*f + Sqrt[b^2 - 4*a*c]*f - 2*a*g))])/(Sqrt[
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*g*Sqrt[(2*a*c)/(b + Sqrt[b^2 - 4*a*c])
+ c*x]*Sqrt[a + b*x + c*x^2])
```

3.918.3.1 Defintions of rubi rules used

rule 1276
```
Int[Sqrt[(d_.) + (e_.)*(x_)]/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(
x_) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt
[2]*Sqrt[2*c*f - g*(b + q)]*Sqrt[b - q + 2*c*x]*(d + e*x)*Sqrt[(e*f - d*g)*
((b + q + 2*c*x)/((2*c*f - g*(b + q))*(d + e*x)))]*(Sqrt[(e*f - d*g)*((2*a
+ (b + q)*x)/((b*f + q*f - 2*a*g)*(d + e*x)))]/(g*Sqrt[2*c*d - e*(b + q)]*S
qrt[2*a*(c/(b + q)) + c*x]*Sqrt[a + b*x + c*x^2]))*EllipticPi[e*((2*c*f - g
*(b + q))/(g*(2*c*d - e*(b + q)))), ArcSin[Sqrt[2*c*d - e*(b + q)]*(Sqrt[f
+ g*x]/(Sqrt[2*c*f - g*(b + q)]*Sqrt[d + e*x]))], (b*d + q*d - 2*a*e)*((2*c
*f - g*(b + q))/((b*f + q*f - 2*a*g)*(2*c*d - e*(b + q))))], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x]
```

3.918.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1462 vs. $2(420) = 840$.

Time = 5.12 (sec) , antiderivative size = 1463, normalized size of antiderivative = 3.08

method	result	size
elliptic	Expression too large to display	1463
default	Expression too large to display	10161

input
```
int((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((g*x+f)*(c*x^2+b*x+a)*(e*x+d))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)/(e
*x+d)^(1/2)*(2*d*(-f/g+d/e)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+f/g)/
(-d/e+f/g)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(x-1/2/c*(-b+(-4*a*c+b
^2)^(1/2)))^2*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+1/2*(b+(-4*a*c+b^2)^
(1/2))/c)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/
2))))^(1/2)*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+d/e)/(-d/e+f/g)/(x-1/2
/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(1
/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)/(c*e*g*(x+f/g)*(x-1/2/c*(-b+(-4*a*c+b^2)
^(1/2)))*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(x+d/e))^(1/2)*EllipticF(((-d/e-
1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+f/g)/(-d/e+f/g)/(x-1/2/c*(-b+(-4*a*c+b^2
)^(1/2))))^(1/2),((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+1/2*(b+(-4*a*c+b^2)^(1/2)
)/c)*(-f/g+d/e)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(1/2/c*(-b+(-4*a*c+b^2
)^(1/2))+d/e))^(1/2))+2*e*(-f/g+d/e)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
*(x+f/g)/(-d/e+f/g)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(x-1/2/c*(-b+
(-4*a*c+b^2)^(1/2)))^2*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+1/2*(b+(-4*
a*c+b^2)^(1/2))/c)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(x-1/2/c*(-b+(-4*a*c
+b^2)^(1/2))))^(1/2)*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+d/e)/(-d/e+f/
g)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(
1/2)))/(1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)/(c*e*g*(x+f/g)*(x-1/2/c*(-b+(-4
*a*c+b^2)^(1/2)))*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(x+d/e))^(1/2)*(1/2/...
```

3.918.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

3.918.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.918.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

3.918.8 Giac [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

3.918.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{\sqrt{d+e\,x}}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.919 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

3.919.1 Optimal result

Integrand size = 33, antiderivative size = 588

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2(a+bx+cx^2)}{(cf^2-bfg+ag^2)(d+ex)^2}}\left(1+\frac{\sqrt{cd^2-bde+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}\right)\sqrt{\frac{1-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{(cf^2-bfg+ag^2)(d+ex)}+\frac{(\cdots}{(\cdots}}{\left(1+\frac{\sqrt{cd^2-bde+ae^2}(\cdots}{\sqrt{cf^2-g(bf-ag)}(\cdots}\right.}}\cdots}{\sqrt[4]{cd^2-bde+ae^2}(ef-dg)\sqrt{a+bx+cx^2}\sqrt{1-\cdots}}$$

output
```
-(c*f^2-g*(-a*g+b*f))^(1/4)*(e*x+d)*(cos(2*arctan((a*e^2-b*d*e+c*d^2)^(1/4
)*(g*x+f)^(1/2)/(a*g^2-b*f*g+c*f^2)^(1/4)/(e*x+d)^(1/2)))^2)^(1/2)/cos(2*a
rctan((a*e^2-b*d*e+c*d^2)^(1/4)*(g*x+f)^(1/2)/(a*g^2-b*f*g+c*f^2)^(1/4)/(e
*x+d)^(1/2)))*EllipticF(sin(2*arctan((a*e^2-b*d*e+c*d^2)^(1/4)*(g*x+f)^(1/
2)/(a*g^2-b*f*g+c*f^2)^(1/4)/(e*x+d)^(1/2))),1/2*(2+(2*c*d*f+2*a*e*g-b*(d*
g+e*f))/(c*d^2-e*(-a*e+b*d))^(1/2)/(c*f^2-g*(-a*g+b*f))^(1/2))^(1/2))*(1+(
g*x+f)*(a*e^2-b*d*e+c*d^2)^(1/2)/(e*x+d)/(c*f^2-g*(-a*g+b*f))^(1/2))*((-d*
g+e*f)^2*(c*x^2+b*x+a)/(a*g^2-b*f*g+c*f^2)/(e*x+d)^2)^(1/2)*((1-(2*c*d*f+2
*a*e*g-b*(d*g+e*f))*(g*x+f)/(a*g^2-b*f*g+c*f^2)/(e*x+d)+(a*e^2-b*d*e+c*d^2
)*(g*x+f)^2/(c*f^2-g*(-a*g+b*f))/(e*x+d)^2)/(1+(g*x+f)*(a*e^2-b*d*e+c*d^2)
^(1/2)/(e*x+d)/(c*f^2-g*(-a*g+b*f))^(1/2))^2)^(1/2)/(a*e^2-b*d*e+c*d^2)^(1
/4)/(-d*g+e*f)/(c*x^2+b*x+a)^(1/2)/(1-(2*c*d*f+2*a*e*g-b*(d*g+e*f))*(g*x+f
)/(a*g^2-b*f*g+c*f^2)/(e*x+d)+(a*e^2-b*d*e+c*d^2)*(g*x+f)^2/(c*f^2-g*(-a*g
+b*f))/(e*x+d)^2)^(1/2)
```

3.919.2 Mathematica [A] (verified)

Time = 26.55 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{2\sqrt{2}e\sqrt{-\frac{e(cd^2+e(-bd+ae))(f+gx)}{\left(-2cdef+e\sqrt{(b^2-4ac)e^2}f-2ae^2g-d\sqrt{(b^2-4ac)e^2}g+be(ef+dg)\right)(d+ex)}}\sqrt{a+x(b+cx)}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2ae \cdots}{\cdots}}\right.\right.}{\sqrt{(b^2-4ac)\,e^2}\sqrt{d+ex}\sqrt{f+gx}\sqrt{-\frac{(cd^2+\cdots}{\cdots}}}$$

input

```
Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output

```
(2*Sqrt[2]*e*Sqrt[-((e*(c*d^2 + e*(-(b*d) + a*e))*(f + g*x))/((-2*c*d*e*f
+ e*Sqrt[(b^2 - 4*a*c)*e^2]*f - 2*a*e^2*g - d*Sqrt[(b^2 - 4*a*c)*e^2]*g +
b*e*(e*f + d*g))*(d + e*x)))]*Sqrt[a + x*(b + c*x)]*EllipticF[ArcSin[Sqrt[
(2*a*e^2 - 2*c*d*e*x + b*e*(-d + e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))
/(Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))]/Sqrt[2]], (2*Sqrt[(b^2 - 4*a*c)*e^2]
*(e*f - d*g))/(-2*c*d*e*f + e*Sqrt[(b^2 - 4*a*c)*e^2]*f - 2*a*e^2*g - d*Sq
rt[(b^2 - 4*a*c)*e^2]*g + b*e*(e*f + d*g))])/(Sqrt[(b^2 - 4*a*c)*e^2]*Sqrt
[d + e*x]*Sqrt[f + g*x]*Sqrt[-(((c*d^2 + e*(-(b*d) + a*e))*(a + x*(b + c*x
)))/((b^2 - 4*a*c)*(d + e*x)^2))])
```

3.919.3 Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {1280, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \; 1280$$

$$-\frac{2(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{\left(cf^2-bgf+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a+bx+cx^2}(ef-dg)}$$

$\downarrow$ 1416

$$-\frac{(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)\sqrt{\frac{\frac{(f+gx)^2\left(ae^2-bde+cd^2\right)}{(d+ex)^2\left(cf^2-g(bf-ag)\right)}-\frac{(f+gx)(2aeg-b(\cdots}{(d+ex)\left(ag^2-\cdots\right.}}{\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\cdots\right.}}\cdots}{\sqrt{a+bx+cx^2}(ef-dg)\sqrt[4]{ae^2-bde+cd^2}\sqrt{\frac{(f+gx)^2(ae\cdots}{(d+ex)^2(cf\cdots}}\cdots}$$

input
```
Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-(((c*f^2 - g*(b*f - a*g))^(1/4)*(d + e*x)*Sqrt[((e*f - d*g)^2*(a + b*x +
c*x^2))/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2)]*(1 + (Sqrt[c*d^2 - b*d*e +
a*e^2]*(f + g*x))/(Sqrt[c*f^2 - g*(b*f - a*g)]*(d + e*x)))*Sqrt[(1 - ((2*c
*d*f + 2*a*e*g - b*(e*f + d*g))*(f + g*x))/((c*f^2 - b*f*g + a*g^2)*(d + e
*x)) + ((c*d^2 - b*d*e + a*e^2)*(f + g*x)^2)/((c*f^2 - g*(b*f - a*g))*(d +
 e*x)^2))/(1 + (Sqrt[c*d^2 - b*d*e + a*e^2]*(f + g*x))/(Sqrt[c*f^2 - g*(b*
f - a*g)]*(d + e*x)))^2]*EllipticF[2*ArcTan[((c*d^2 - b*d*e + a*e^2)^(1/4)
*Sqrt[f + g*x])/((c*f^2 - b*f*g + a*g^2)^(1/4)*Sqrt[d + e*x])], (2 + (2*c*
d*f + 2*a*e*g - b*(e*f + d*g))/(Sqrt[c*d^2 - e*(b*d - a*e)]*Sqrt[c*f^2 - g
*(b*f - a*g)]))/4])/((c*d^2 - b*d*e + a*e^2)^(1/4)*(e*f - d*g)*Sqrt[a + b*
x + c*x^2]*Sqrt[1 - ((2*c*d*f + 2*a*e*g - b*(e*f + d*g))*(f + g*x))/((c*f^
2 - b*f*g + a*g^2)*(d + e*x)) + ((c*d^2 - b*d*e + a*e^2)*(f + g*x)^2)/((c*
f^2 - g*(b*f - a*g))*(d + e*x)^2)]))
```

3.919.3.1 Defintions of rubi rules used

rule 1280
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-2*(d + e*x)*(Sqrt[(e*f - d*g)^2*
((a + b*x + c*x^2)/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2))]/((e*f - d*g)*Sqr
t[a + b*x + c*x^2]))   Subst[Int[1/Sqrt[1 - (2*c*d*f - b*e*f - b*d*g + 2*a*
e*g)*(x^2/(c*f^2 - b*f*g + a*g^2)) + (c*d^2 - b*d*e + a*e^2)*(x^4/(c*f^2 -
b*f*g + a*g^2))], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c
, d, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

3.919.4 Maple [A] (warning: unable to verify)

Time = 6.06 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.02

method	result
default	$\frac{8\left(-2c^2dg\,x^2+2c^2ef\,x^2+2\sqrt{-4ac+b^2}\,cdgx-2\sqrt{-4ac+b^2}\,cefx-2bcdgx+2bcefx+\sqrt{-4ac+b^2}\,bdg-\sqrt{-4ac+b^2}\,bef+2acdg-2acef-b\cdots\right.}{\sqrt{-\frac{(gx+f)\left(-b-\cdots\right.}{\cdots}}\cdots}$
elliptic	$\frac{2\sqrt{(gx+f)(c\,x^2+bx+a)(ex+d)}\left(-\frac{f}{g}+\frac{d}{e}\right)\sqrt{\frac{\left(-\frac{d}{e}-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)\left(x+\frac{f}{g}\right)}{\left(-\frac{d}{e}+\frac{f}{g}\right)\left(x-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)}}\left(x-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)^2\sqrt{\frac{\left(\frac{-b+\sqrt{-4ac+b^2}}{2c}+\frac{f}{g}\right)\left(x+\frac{b+\sqrt{-\cdots}}{2\cdots}\right.}{\left(\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\left(x-\frac{-b+\sqrt{-\cdots}}{2\cdots}\right.}}\cdots}{\sqrt{gx+f}\,\sqrt{c\,x^2+bx+a}\,\sqrt{ex+d}\left(-\frac{d}{e}-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)\left(\frac{-b+\sqrt{-4ac+b^2}}{2c}+\cdots\right.}$

input
```
int(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
8*(-2*c^2*d*g*x^2+2*c^2*e*f*x^2+2*(-4*a*c+b^2)^(1/2)*c*d*g*x-2*(-4*a*c+b^2
)^(1/2)*c*e*f*x-2*b*c*d*g*x+2*b*c*e*f*x+(-4*a*c+b^2)^(1/2)*b*d*g-(-4*a*c+b
^2)^(1/2)*b*e*f+2*a*c*d*g-2*a*c*e*f-b^2*d*g+b^2*e*f)*EllipticF((-(e*(-4*a*
c+b^2)^(1/2)-b*e+2*c*d)*(g*x+f)/(d*g-e*f)/(-b-2*c*x+(-4*a*c+b^2)^(1/2)))^(
1/2),2*((-4*a*c+b^2)^(1/2)*(d*g-e*f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(e
*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*((2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))*(
e*x+d)/(d*g-e*f)/(-b-2*c*x+(-4*a*c+b^2)^(1/2)))^(1/2)*((2*c*f-b*g+g*(-4*a*
c+b^2)^(1/2))*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f
)/(-b-2*c*x+(-4*a*c+b^2)^(1/2)))^(1/2)*(-(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)*
(g*x+f)/(d*g-e*f)/(-b-2*c*x+(-4*a*c+b^2)^(1/2)))^(1/2)*(c*x^2+b*x+a)^(1/2)
*(g*x+f)^(1/2)*(e*x+d)^(1/2)/(-1/c*(g*x+f)*(-b-2*c*x+(-4*a*c+b^2)^(1/2))*(
b+2*c*x+(-4*a*c+b^2)^(1/2))*(e*x+d))^(1/2)/(b*g-2*c*f-g*(-4*a*c+b^2)^(1/2)
)/(-e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/((g*x+f)*(c*x^2+b*x+a)*(e*x+d))^(1/2)
```

3.919.5 Fricas [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x + f)/(c*e*g*x^4 + (c
*e*f + (c*d + b*e)*g)*x^3 + a*d*f + ((c*d + b*e)*f + (b*d + a*e)*g)*x^2 +
(a*d*g + (b*d + a*e)*f)*x), x)
```

3.919.6 Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(d + e*x)*sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.919.7 Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

3.919.8 Giac [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="
giac")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

3.919.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/((f + g*x)^(1/2)*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.920 $\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right) \, dx$

3.920.1 Optimal result

Integrand size = 25, antiderivative size = 220

$$\begin{aligned}\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right) \, dx &= \frac{\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)^{1+m}}{e^5(1+m)} \\ &\quad - \frac{(ef-dg)(2cd(ef-2dg)-e(bef-3bdg+2aeg))(d+ex)^{2+m}}{e^5(2+m)} \\ &\quad + \frac{\left(eg(2bef-3bdg+aeg)+c\left(e^2f^2-6defg+6d^2g^2\right)\right)(d+ex)^{3+m}}{e^5(3+m)} \\ &\quad + \frac{g(2cef-4cdg+beg)(d+ex)^{4+m}}{e^5(4+m)} + \frac{cg^2(d+ex)^{5+m}}{e^5(5+m)}\end{aligned}$$

output
```
(a*e^2-b*d*e+c*d^2)*(-d*g+e*f)^2*(e*x+d)^(1+m)/e^5/(1+m)-(-d*g+e*f)*(2*c*d
*(-2*d*g+e*f)-e*(2*a*e*g-3*b*d*g+b*e*f))*(e*x+d)^(2+m)/e^5/(2+m)+(e*g*(a*e
*g-3*b*d*g+2*b*e*f)+c*(6*d^2*g^2-6*d*e*f*g+e^2*f^2))*(e*x+d)^(3+m)/e^5/(3+
m)+g*(b*e*g-4*c*d*g+2*c*e*f)*(e*x+d)^(4+m)/e^5/(4+m)+c*g^2*(e*x+d)^(5+m)/e
^5/(5+m)
```

3.920.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.90

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)\,dx$$
$$=\frac{(d+ex)^{1+m}\left(\frac{(cd^2+e(-bd+ae))(ef-dg)^2}{1+m}+\frac{(ef-dg)(2cd(-ef+2dg)+e(bef-3bdg+2aeg))(d+ex)}{2+m}+\frac{(eg(2bef-3bdg+aeg)+c(e^2f^2-6d}{3+m}\right.}{e^5}$$

input
```
Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2),x]
```

output
```
((d + e*x)^(1 + m)*(((c*d^2 + e*(-(b*d) + a*e))*(e*f - d*g)^2)/(1 + m) + (
(e*f - d*g)*(2*c*d*(-(e*f) + 2*d*g) + e*(b*e*f - 3*b*d*g + 2*a*e*g))*(d +
e*x))/(2 + m) + ((e*g*(2*b*e*f - 3*b*d*g + a*e*g) + c*(e^2*f^2 - 6*d*e*f*g
 + 6*d^2*g^2))*(d + e*x)^2)/(3 + m) + (g*(2*c*e*f - 4*c*d*g + b*e*g)*(d +
e*x)^3)/(4 + m) + (c*g^2*(d + e*x)^4)/(5 + m)))/e^5
```

3.920.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2\left(a+bx+cx^2\right)(d+ex)^m\,dx$$
$$\Big\downarrow \text{1195}$$
$$\int \left(\frac{(d+ex)^{m+2}\left(eg(aeg-3bdg+2bef)+c\left(6d^2g^2-6defg+e^2f^2\right)\right)}{e^4}+\frac{(ef-dg)^2(d+ex)^m\left(ae^2-bde+cd^2\right)}{e^4}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^{m+3}\left(eg(aeg-3bdg+2bef)+c\left(6d^2g^2-6defg+e^2f^2\right)\right)}{e^5(m+3)}+\frac{(ef-dg)^2(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)}{e^5(m+1)}-\frac{(ef-dg)(d+ex)^{m+2}(2cd(ef-2dg)-e(2aeg-3bdg+bef))}{e^5(m+2)}+\frac{g(d+ex)^{m+4}(beg-4cdg+2cef)}{e^5(m+4)}+\frac{cg^2(d+ex)^{m+5}}{e^5(m+5)}$$

input
```
Int[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2),x]
```

output
```
((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*(d + e*x)^(1 + m))/(e^5*(1 + m)) -
((e*f - d*g)*(2*c*d*(e*f - 2*d*g) - e*(b*e*f - 3*b*d*g + 2*a*e*g))*(d + e*
x)^(2 + m))/(e^5*(2 + m)) + ((e*g*(2*b*e*f - 3*b*d*g + a*e*g) + c*(e^2*f^2
 - 6*d*e*f*g + 6*d^2*g^2))*(d + e*x)^(3 + m))/(e^5*(3 + m)) + (g*(2*c*e*f
- 4*c*d*g + b*e*g)*(d + e*x)^(4 + m))/(e^5*(4 + m)) + (c*g^2*(d + e*x)^(5
+ m))/(e^5*(5 + m))
```

3.920.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.920.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1248 vs. 2(220) = 440.

Time = 0.56 (sec) , antiderivative size = 1249, normalized size of antiderivative = 5.68

method	result	size
norman	Expression too large to display	1249
gosper	Expression too large to display	1347
risch	Expression too large to display	1823
parallelrisch	Expression too large to display	2719

input
```
int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
c*g^2/(5+m)*x^5*exp(m*ln(e*x+d))+d*(a*e^4*f^2*m^4-2*a*d*e^3*f*g*m^3+14*a*e
^4*f^2*m^3-b*d*e^3*f^2*m^3+2*a*d^2*e^2*g^2*m^2-24*a*d*e^3*f*g*m^2+71*a*e^4
*f^2*m^2+4*b*d^2*e^2*f*g*m^2-12*b*d*e^3*f^2*m^2+2*c*d^2*e^2*f^2*m^2+18*a*d
^2*e^2*g^2*m-94*a*d*e^3*f*g*m+154*a*e^4*f^2*m-6*b*d^3*e*g^2*m+36*b*d^2*e^2
*f*g*m-47*b*d*e^3*f^2*m-12*c*d^3*e*f*g*m+18*c*d^2*e^2*f^2*m+40*a*d^2*e^2*g
^2-120*a*d*e^3*f*g+120*a*e^4*f^2-30*b*d^3*e*g^2+80*b*d^2*e^2*f*g-60*b*d*e^
3*f^2+24*c*d^4*g^2-60*c*d^3*e*f*g+40*c*d^2*e^2*f^2)/e^5/(m^5+15*m^4+85*m^3
+225*m^2+274*m+120)*exp(m*ln(e*x+d))+(a*e^2*g^2*m^2+b*d*e*g^2*m^2+2*b*e^2*
f*g*m^2+2*c*d*e*f*g*m^2+c*e^2*f^2*m^2+9*a*e^2*g^2*m+5*b*d*e*g^2*m+18*b*e^2
*f*g*m-4*c*d^2*g^2*m+10*c*d*e*f*g*m+9*c*e^2*f^2*m+20*a*e^2*g^2+40*b*e^2*f*
g+20*c*e^2*f^2)/e^2/(m^3+12*m^2+47*m+60)*x^3*exp(m*ln(e*x+d))+(a*d*e^2*g^2
*m^3+2*a*e^3*f*g*m^3+2*b*d*e^2*f*g*m^3+b*e^3*f^2*m^3+c*d*e^2*f^2*m^3+9*a*d
*e^2*g^2*m^2+24*a*e^3*f*g*m^2-3*b*d^2*e*g^2*m^2+18*b*d*e^2*f*g*m^2+12*b*e^
3*f^2*m^2-6*c*d^2*e*f*g*m^2+9*c*d*e^2*f^2*m^2+20*a*d*e^2*g^2*m+94*a*e^3*f*
g*m-15*b*d^2*e*g^2*m+40*b*d*e^2*f*g*m+47*b*e^3*f^2*m+12*c*d^3*g^2*m-30*c*d
^2*e*f*g*m+20*c*d*e^2*f^2*m+120*a*e^3*f*g+60*b*e^3*f^2)/e^3/(m^4+14*m^3+71
*m^2+154*m+120)*x^2*exp(m*ln(e*x+d))+(b*e*g*m+c*d*g*m+2*c*e*f*m+5*b*e*g+10
*c*e*f)/e*g/(m^2+9*m+20)*x^4*exp(m*ln(e*x+d))-(-2*a*d*e^3*f*g*m^4-a*e^4*f^
2*m^4-b*d*e^3*f^2*m^4+2*a*d^2*e^2*g^2*m^3-24*a*d*e^3*f*g*m^3-14*a*e^4*f^2*
m^3+4*b*d^2*e^2*f*g*m^3-12*b*d*e^3*f^2*m^3+2*c*d^2*e^2*f^2*m^3+18*a*d^2...
```

3.920.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1381 vs. $2(220) = 440$.

Time = 0.33 (sec) , antiderivative size = 1381, normalized size of antiderivative = 6.28

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
(a*d*e^4*f^2*m^4 + (c*e^5*g^2*m^4 + 10*c*e^5*g^2*m^3 + 35*c*e^5*g^2*m^2 +
50*c*e^5*g^2*m + 24*c*e^5*g^2)*x^5 + (60*c*e^5*f*g + 30*b*e^5*g^2 + (2*c*e
^5*f*g + (c*d*e^4 + b*e^5)*g^2)*m^4 + (22*c*e^5*f*g + (6*c*d*e^4 + 11*b*e^
5)*g^2)*m^3 + (82*c*e^5*f*g + (11*c*d*e^4 + 41*b*e^5)*g^2)*m^2 + (122*c*e^
5*f*g + (6*c*d*e^4 + 61*b*e^5)*g^2)*m)*x^4 - (2*a*d^2*e^3*f*g + (b*d^2*e^3
 - 14*a*d*e^4)*f^2)*m^3 + (40*c*e^5*f^2 + 80*b*e^5*f*g + 40*a*e^5*g^2 + (c
*e^5*f^2 + 2*(c*d*e^4 + b*e^5)*f*g + (b*d*e^4 + a*e^5)*g^2)*m^4 + 4*(3*c*e
^5*f^2 + 2*(2*c*d*e^4 + 3*b*e^5)*f*g - (c*d^2*e^3 - 2*b*d*e^4 - 3*a*e^5)*g
^2)*m^3 + (49*c*e^5*f^2 + 2*(17*c*d*e^4 + 49*b*e^5)*f*g - (12*c*d^2*e^3 -
17*b*d*e^4 - 49*a*e^5)*g^2)*m^2 + 2*(39*c*e^5*f^2 + 2*(5*c*d*e^4 + 39*b*e^
5)*f*g - (4*c*d^2*e^3 - 5*b*d*e^4 - 39*a*e^5)*g^2)*m)*x^3 + 20*(2*c*d^3*e^
2 - 3*b*d^2*e^3 + 6*a*d*e^4)*f^2 - 20*(3*c*d^4*e - 4*b*d^3*e^2 + 6*a*d^2*e
^3)*f*g + 2*(12*c*d^5 - 15*b*d^4*e + 20*a*d^3*e^2)*g^2 + (2*a*d^3*e^2*g^2
+ (2*c*d^3*e^2 - 12*b*d^2*e^3 + 71*a*d*e^4)*f^2 + 4*(b*d^3*e^2 - 6*a*d^2*e
^3)*f*g)*m^2 + (60*b*e^5*f^2 + 120*a*e^5*f*g + (a*d*e^4*g^2 + (c*d*e^4 + b
*e^5)*f^2 + 2*(b*d*e^4 + a*e^5)*f*g)*m^4 + ((10*c*d*e^4 + 13*b*e^5)*f^2 -
2*(3*c*d^2*e^3 - 10*b*d*e^4 - 13*a*e^5)*f*g - (3*b*d^2*e^3 - 10*a*d*e^4)*g
^2)*m^3 + ((29*c*d*e^4 + 59*b*e^5)*f^2 - 2*(18*c*d^2*e^3 - 29*b*d*e^4 - 59
*a*e^5)*f*g + (12*c*d^3*e^2 - 18*b*d^2*e^3 + 29*a*d*e^4)*g^2)*m^2 + ((20*c
*d*e^4 + 107*b*e^5)*f^2 - 2*(15*c*d^2*e^3 - 20*b*d*e^4 - 107*a*e^5)*f*g...
```

3.920.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15757 vs. $2(212) = 424$.

Time = 2.87 (sec) , antiderivative size = 15757, normalized size of antiderivative = 71.62

$$\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right) dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a),x)
```

output
```
Piecewise((d**m*(a*f**2*x + a*f*g*x**2 + a*g**2*x**3/3 + b*f**2*x**2/2 + 2
*b*f*g*x**3/3 + b*g**2*x**4/4 + c*f**2*x**3/3 + c*f*g*x**4/2 + c*g**2*x**5
/5), Eq(e, 0)), (-a*d**2*e**2*g**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**
2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 2*a*d*e**3*f*g/(12*d**4*e**
5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) -
4*a*d*e**3*g**2*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*
d*e**8*x**3 + 12*e**9*x**4) - 3*a*e**4*f**2/(12*d**4*e**5 + 48*d**3*e**6*x
 + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 8*a*e**4*f*g*x/(12
*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9
*x**4) - 6*a*e**4*g**2*x**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*
x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 3*b*d**3*e*g**2/(12*d**4*e**5 + 48
*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 2*b*d*
*2*e**2*f*g/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8
*x**3 + 12*e**9*x**4) - 12*b*d**2*e**2*g**2*x/(12*d**4*e**5 + 48*d**3*e**6
*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - b*d*e**3*f**2/(1
2*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**
9*x**4) - 8*b*d*e**3*f*g*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x
**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 18*b*d*e**3*g**2*x**2/(12*d**4*e**5
 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 4
*b*e**4*f**2*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*...
```

3.920.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(220) = 440$.

Time = 0.24 (sec) , antiderivative size = 684, normalized size of antiderivative = 3.11

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)\,dx = \frac{(e^2(m+1)x^2+demx-d^2)(ex+d)^m bf^2}{(m^2+3\,m+2)e^2} + \frac{2\,(e^2(m+1)x^2+demx-d^2)(ex+d)^m afg}{(m^2+3\,m+2)e^2} + \frac{(ex+d)^{m+1}af^2}{e(m+1)} + \frac{((m^2+3\,m+2)e^3x^3+(m^2+m)de^2x^2-2\,d^2emx+2\,d^3)(ex+d)^m cf^2}{(m^3+6\,m^2+11\,m+6)e^3} + \frac{2\,((m^2+3\,m+2)e^3x^3+(m^2+m)de^2x^2-2\,d^2emx+2\,d^3)(ex+d)^m bfg}{(m^3+6\,m^2+11\,m+6)e^3} + \frac{((m^2+3\,m+2)e^3x^3+(m^2+m)de^2x^2-2\,d^2emx+2\,d^3)(ex+d)^m ag^2}{(m^3+6\,m^2+11\,m+6)e^3} + \frac{2\,((m^3+6\,m^2+11\,m+6)e^4x^4+(m^3+3\,m^2+2\,m)de^3x^3-3\,(m^2+m)d^2e^2x^2+6\,d^3emx-6\,d^4)(ex-}{(m^4+10\,m^3+35\,m^2+50\,m+24)e^4} + \frac{((m^3+6\,m^2+11\,m+6)e^4x^4+(m^3+3\,m^2+2\,m)de^3x^3-3\,(m^2+m)d^2e^2x^2+6\,d^3emx-6\,d^4)(ex+}{(m^4+10\,m^3+35\,m^2+50\,m+24)e^4} + \frac{((m^4+10\,m^3+35\,m^2+50\,m+24)e^5x^5+(m^4+6\,m^3+11\,m^2+6\,m)de^4x^4-4\,(m^3+3\,m^2+2\,m)d^2e}{(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120}$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*b*f^2/((m^2 + 3*m + 2)*e^2)
+ 2*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a*f*g/((m^2 + 3*m + 2)*e
^2) + (e*x + d)^(m + 1)*a*f^2/(e*(m + 1)) + ((m^2 + 3*m + 2)*e^3*x^3 + (m^
2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*c*f^2/((m^3 + 6*m^2 +
11*m + 6)*e^3) + 2*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*
e*m*x + 2*d^3)*(e*x + d)^m*b*f*g/((m^3 + 6*m^2 + 11*m + 6)*e^3) + ((m^2 +
3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*
a*g^2/((m^3 + 6*m^2 + 11*m + 6)*e^3) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4
 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x -
 6*d^4)*(e*x + d)^m*c*f*g/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + ((m^
3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m
)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*b*g^2/((m^4 + 10*m^3 + 35
*m^2 + 50*m + 24)*e^4) + ((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m
^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 +
 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m*c*g^2/((m^5
 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5)
```

3.920. $\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)\,dx$

3.920.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2735 vs. $2(220) = 440$.

Time = 0.29 (sec) , antiderivative size = 2735, normalized size of antiderivative = 12.43

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
((e*x + d)^m*c*e^5*g^2*m^4*x^5 + 2*(e*x + d)^m*c*e^5*f*g*m^4*x^4 + (e*x +
d)^m*c*d*e^4*g^2*m^4*x^4 + (e*x + d)^m*b*e^5*g^2*m^4*x^4 + 10*(e*x + d)^m*
c*e^5*g^2*m^3*x^5 + (e*x + d)^m*c*e^5*f^2*m^4*x^3 + 2*(e*x + d)^m*c*d*e^4*
f*g*m^4*x^3 + 2*(e*x + d)^m*b*e^5*f*g*m^4*x^3 + (e*x + d)^m*b*d*e^4*g^2*m^
4*x^3 + (e*x + d)^m*a*e^5*g^2*m^4*x^3 + 22*(e*x + d)^m*c*e^5*f*g*m^3*x^4 +
 6*(e*x + d)^m*c*d*e^4*g^2*m^3*x^4 + 11*(e*x + d)^m*b*e^5*g^2*m^3*x^4 + 35
*(e*x + d)^m*c*e^5*g^2*m^2*x^5 + (e*x + d)^m*c*d*e^4*f^2*m^4*x^2 + (e*x +
d)^m*b*e^5*f^2*m^4*x^2 + 2*(e*x + d)^m*b*d*e^4*f*g*m^4*x^2 + 2*(e*x + d)^m
*a*e^5*f*g*m^4*x^2 + (e*x + d)^m*a*d*e^4*g^2*m^4*x^2 + 12*(e*x + d)^m*c*e^
5*f^2*m^3*x^3 + 16*(e*x + d)^m*c*d*e^4*f*g*m^3*x^3 + 24*(e*x + d)^m*b*e^5*
f*g*m^3*x^3 - 4*(e*x + d)^m*c*d^2*e^3*g^2*m^3*x^3 + 8*(e*x + d)^m*b*d*e^4*
g^2*m^3*x^3 + 12*(e*x + d)^m*a*e^5*g^2*m^3*x^3 + 82*(e*x + d)^m*c*e^5*f*g*
m^2*x^4 + 11*(e*x + d)^m*c*d*e^4*g^2*m^2*x^4 + 41*(e*x + d)^m*b*e^5*g^2*m^
2*x^4 + 50*(e*x + d)^m*c*e^5*g^2*m*x^5 + (e*x + d)^m*b*d*e^4*f^2*m^4*x + (
e*x + d)^m*a*e^5*f^2*m^4*x + 2*(e*x + d)^m*a*d*e^4*f*g*m^4*x + 10*(e*x + d
)^m*c*d*e^4*f^2*m^3*x^2 + 13*(e*x + d)^m*b*e^5*f^2*m^3*x^2 - 6*(e*x + d)^m
*c*d^2*e^3*f*g*m^3*x^2 + 20*(e*x + d)^m*b*d*e^4*f*g*m^3*x^2 + 26*(e*x + d)
^m*a*e^5*f*g*m^3*x^2 - 3*(e*x + d)^m*b*d^2*e^3*g^2*m^3*x^2 + 10*(e*x + d)^
m*a*d*e^4*g^2*m^3*x^2 + 49*(e*x + d)^m*c*e^5*f^2*m^2*x^3 + 34*(e*x + d)^m*
c*d*e^4*f*g*m^2*x^3 + 98*(e*x + d)^m*b*e^5*f*g*m^2*x^3 - 12*(e*x + d)^m...
```

3.920.9 Mupad [B] (verification not implemented)

Time = 13.40 (sec) , antiderivative size = 1354, normalized size of antiderivative = 6.15

$$\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right) dx$$
$$= \frac{(d+e\,x)^m\,(24\,c\,d^5\,g^2-12\,c\,d^4\,e\,f\,g\,m-60\,c\,d^4\,e\,f\,g-6\,b\,d^4\,e\,g^2\,m-30\,b\,d^4\,e\,g^2+2\,c\,d^3\,e^2\,f^2\,m^2+18\,c\,d^3}{}$$
$$+\frac{x\,(d+e\,x)^m\,(-24\,c\,d^4\,e\,g^2\,m+12\,c\,d^3\,e^2\,f\,g\,m^2+60\,c\,d^3\,e^2\,f\,g\,m+6\,b\,d^3\,e^2\,g^2\,m^2+30\,b\,d^3\,e^2\,g^2\,m-2\,c}{}$$
$$+\frac{c\,g^2\,x^5\,(d+e\,x)^m\,(m^4+10\,m^3+35\,m^2+50\,m+24)}{m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120}$$
$$+\frac{x^2\,(m+1)\,(d+e\,x)^m\,(12\,c\,d^3\,g^2\,m-6\,c\,d^2\,e\,f\,g\,m^2-30\,c\,d^2\,e\,f\,g\,m-3\,b\,d^2\,e\,g^2\,m^2-15\,b\,d^2\,e\,g^2\,m+c}{}$$
$$+\frac{x^3\,(d+e\,x)^m\,(m^2+3\,m+2)\,(-4\,c\,d^2\,g^2\,m+2\,c\,d\,e\,f\,g\,m^2+10\,c\,d\,e\,f\,g\,m+b\,d\,e\,g^2\,m^2+5\,b\,d\,e\,g^2\,m+}{e^2\,(m^5+15\,m^4+}$$
$$+\frac{g\,x^4\,(d+e\,x)^m\,(m^3+6\,m^2+11\,m+6)\,(5\,b\,e\,g+10\,c\,e\,f+b\,e\,g\,m+c\,d\,g\,m+2\,c\,e\,f\,m)}{e\,(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120)}$$

input
```
int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2),x)
```

output
```
((d + e*x)^m*(24*c*d^5*g^2 + 40*a*d^3*e^2*g^2 - 60*b*d^2*e^3*f^2 + 40*c*d^
3*e^2*f^2 + 120*a*d*e^4*f^2 - 30*b*d^4*e*g^2 - 120*a*d^2*e^3*f*g + 80*b*d^
3*e^2*f*g + 154*a*d*e^4*f^2*m - 6*b*d^4*e*g^2*m + 71*a*d*e^4*f^2*m^2 + 14*
a*d*e^4*f^2*m^3 + a*d*e^4*f^2*m^4 + 18*a*d^3*e^2*g^2*m - 47*b*d^2*e^3*f^2*
m + 18*c*d^3*e^2*f^2*m - 60*c*d^4*e*f*g + 2*a*d^3*e^2*g^2*m^2 - 12*b*d^2*e
^3*f^2*m^2 - b*d^2*e^3*f^2*m^3 + 2*c*d^3*e^2*f^2*m^2 - 12*c*d^4*e*f*g*m -
94*a*d^2*e^3*f*g*m + 36*b*d^3*e^2*f*g*m - 24*a*d^2*e^3*f*g*m^2 - 2*a*d^2*e
^3*f*g*m^3 + 4*b*d^3*e^2*f*g*m^2))/(e^5*(274*m + 225*m^2 + 85*m^3 + 15*m^4
 + m^5 + 120)) + (x*(d + e*x)^m*(120*a*e^5*f^2 + 71*a*e^5*f^2*m^2 + 14*a*e
^5*f^2*m^3 + a*e^5*f^2*m^4 + 154*a*e^5*f^2*m + 60*b*d*e^4*f^2*m - 24*c*d^4
*e*g^2*m - 40*a*d^2*e^3*g^2*m + 47*b*d*e^4*f^2*m^2 + 12*b*d*e^4*f^2*m^3 +
b*d*e^4*f^2*m^4 + 30*b*d^3*e^2*g^2*m - 40*c*d^2*e^3*f^2*m - 18*a*d^2*e^3*g
^2*m^2 - 2*a*d^2*e^3*g^2*m^3 + 6*b*d^3*e^2*g^2*m^2 - 18*c*d^2*e^3*f^2*m^2
- 2*c*d^2*e^3*f^2*m^3 + 120*a*d*e^4*f*g*m + 94*a*d*e^4*f*g*m^2 + 24*a*d*e^
4*f*g*m^3 + 2*a*d*e^4*f*g*m^4 - 80*b*d^2*e^3*f*g*m + 60*c*d^3*e^2*f*g*m -
36*b*d^2*e^3*f*g*m^2 - 4*b*d^2*e^3*f*g*m^3 + 12*c*d^3*e^2*f*g*m^2))/(e^5*(
274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)) + (c*g^2*x^5*(d + e*x)^m*(
50*m + 35*m^2 + 10*m^3 + m^4 + 24))/(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m
^5 + 120) + (x^2*(m + 1)*(d + e*x)^m*(60*b*e^3*f^2 + 12*b*e^3*f^2*m^2 + b*
e^3*f^2*m^3 + 120*a*e^3*f*g + 47*b*e^3*f^2*m + 12*c*d^3*g^2*m + 20*a*d*...
```

3.921 $\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)\,dx$

3.921.1 Optimal result

Integrand size = 23, antiderivative size = 144

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)\,dx = \frac{\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^{1+m}}{e^4(1+m)} - \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))(d+ex)^{2+m}}{e^4(2+m)} + \frac{(cef-3cdg+beg)(d+ex)^{3+m}}{e^4(3+m)} + \frac{cg(d+ex)^{4+m}}{e^4(4+m)}$$

output
```
(a*e^2-b*d*e+c*d^2)*(-d*g+e*f)*(e*x+d)^(1+m)/e^4/(1+m)-(c*d*(-3*d*g+2*e*f)
-e*(a*e*g-2*b*d*g+b*e*f))*(e*x+d)^(2+m)/e^4/(2+m)+(b*e*g-3*c*d*g+c*e*f)*(e
*x+d)^(3+m)/e^4/(3+m)+c*g*(e*x+d)^(4+m)/e^4/(4+m)
```

3.921.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.25

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)\,dx = \frac{(d+ex)^{1+m}\left(-\frac{(cd^2+e(-bd+ae))(6cdg+beg(1+m)-2cef(4+m))}{e^2(1+m)}+\frac{(-b^2e^2g(2+m)+2c^2d(3dg-ef(4+m))+ce(bdg(-2+m)+2aeg(3+m}{e^2(2+m)}\right.}{ce^2(3+m)(4+m)}$$

input
```
Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2),x]
```

output
```
((d + e*x)^(1 + m)*(-((((c*d^2 + e*(-(b*d) + a*e))*(6*c*d*g + b*e*g*(1 + m)
 - 2*c*e*f*(4 + m)))/(e^2*(1 + m))) + ((-(b^2*e^2*g*(2 + m)) + 2*c^2*d*(3*
d*g - e*f*(4 + m)) + c*e*(b*d*g*(-2 + m) + 2*a*e*g*(3 + m) + b*e*f*(4 + m)
))*(d + e*x))/(e^2*(2 + m)) + (a + x*(b + c*x))*(b*e*g + c*(-3*d*g + e*f*(
4 + m) + e*g*(3 + m)*x))))/(c*e^2*(3 + m)*(4 + m))
```

3.921.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(a+bx+cx^2\right)(d+ex)^m\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{(ef-dg)(d+ex)^m\left(ae^2-bde+cd^2\right)}{e^3}+\frac{(d+ex)^{m+1}(e(aeg-2bdg+bef)-cd(2ef-3dg))}{e^3}+\frac{(d+ex)^{m+2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ef-dg)(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)}{e^4(m+1)}-\frac{(d+ex)^{m+2}(cd(2ef-3dg)-e(aeg-2bdg+bef))}{e^4(m+2)}+\frac{(d+ex)^{m+3}(beg-3cdg+cef)}{e^4(m+3)}+\frac{cg(d+ex)^{m+4}}{e^4(m+4)}$$

input
```
Int[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2),x]
```

output
```
((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*(d + e*x)^(1 + m))/(e^4*(1 + m)) - ((
c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*(d + e*x)^(2 + m))/(e^4
*(2 + m)) + ((c*e*f - 3*c*d*g + b*e*g)*(d + e*x)^(3 + m))/(e^4*(3 + m)) +
(c*g*(d + e*x)^(4 + m))/(e^4*(4 + m))
```

3.921.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.921.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(144) = 288$.

Time = 0.49 (sec) , antiderivative size = 503, normalized size of antiderivative = 3.49

method	result
gosper	$-\frac{(ex+d)^{1+m}\left(-c\,e^3 g\,m^3 x^3-b\,e^3 g\,m^3 x^2-c\,e^3 f\,m^3 x^2-6c\,e^3 g\,m^2 x^3-a\,e^3 g\,m^3 x-b\,e^3 f\,m^3 x-7b\,e^3 g\,m^2 x^2+3cd\,e^2 g\,m^2 x^2-7c\right.}{}$
norman	$\frac{cg\,x^4 \mathrm{e}^{m\ln(ex+d)}}{4+m}+\frac{(begm+cdgm+cefm+4beg+4cef)x^3 \mathrm{e}^{m\ln(ex+d)}}{e(m^2+7m+12)}+\frac{\left(a\,e^2 g\,m^2+bdeg\,m^2+b\,e^2 f\,m^2+cdef\,m^2+7a\,e^2 gm\right.}{e}$
risch	$-\frac{\left(-c\,e^4 g\,m^3 x^4-b\,e^4 g\,m^3 x^3-cd\,e^3 g\,m^3 x^3-c\,e^4 f\,m^3 x^3-6c\,e^4 g\,m^2 x^4-a\,e^4 g\,m^3 x^2-bd\,e^3 g\,m^3 x^2-b\,e^4 f\,m^3 x^2-7b\,e^4 g\,m^2 x^3-\right.}{}$
parallelrisch	Expression too large to display

input
```
int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/e^4*(e*x+d)^(1+m)/(m^4+10*m^3+35*m^2+50*m+24)*(-c*e^3*g*m^3*x^3-b*e^3*g
*m^3*x^2-c*e^3*f*m^3*x^2-6*c*e^3*g*m^2*x^3-a*e^3*g*m^3*x-b*e^3*f*m^3*x-7*b
*e^3*g*m^2*x^2+3*c*d*e^2*g*m^2*x^2-7*c*e^3*f*m^2*x^2-11*c*e^3*g*m*x^3-a*e^
3*f*m^3-8*a*e^3*g*m^2*x+2*b*d*e^2*g*m^2*x-8*b*e^3*f*m^2*x-14*b*e^3*g*m*x^2
+2*c*d*e^2*f*m^2*x+9*c*d*e^2*g*m*x^2-14*c*e^3*f*m*x^2-6*c*e^3*g*x^3+a*d*e^
2*g*m^2-9*a*e^3*f*m^2-19*a*e^3*g*m*x+b*d*e^2*f*m^2+10*b*d*e^2*g*m*x-19*b*e
^3*f*m*x-8*b*e^3*g*x^2-6*c*d^2*e*g*m*x+10*c*d*e^2*f*m*x+6*c*d*e^2*g*x^2-8*
c*e^3*f*x^2+7*a*d*e^2*g*m-26*a*e^3*f*m-12*a*e^3*g*x-2*b*d^2*e*g*m+7*b*d*e^
2*f*m+8*b*d*e^2*g*x-12*b*e^3*f*x-2*c*d^2*e*f*m-6*c*d^2*e*g*x+8*c*d*e^2*f*x
+12*a*d*e^2*g-24*a*e^3*f-8*b*d^2*e*g+12*b*d*e^2*f+6*c*d^3*g-8*c*d^2*e*f)
```

3.921.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(144) = 288$.

Time = 0.30 (sec) , antiderivative size = 613, normalized size of antiderivative = 4.26

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)\,dx$$
$$= \frac{(ade^3fm^3 + (ce^4gm^3 + 6\,ce^4gm^2 + 11\,ce^4gm + 6\,ce^4g)x^4 + (8\,ce^4f + 8\,be^4g + (ce^4f + (cde^3 + be^4)g)m^3}{}$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
(a*d*e^3*f*m^3 + (c*e^4*g*m^3 + 6*c*e^4*g*m^2 + 11*c*e^4*g*m + 6*c*e^4*g)*
x^4 + (8*c*e^4*f + 8*b*e^4*g + (c*e^4*f + (c*d*e^3 + b*e^4)*g)*m^3 + (7*c*
e^4*f + (3*c*d*e^3 + 7*b*e^4)*g)*m^2 + 2*(7*c*e^4*f + (c*d*e^3 + 7*b*e^4)*
g)*m)*x^3 - (a*d^2*e^2*g + (b*d^2*e^2 - 9*a*d*e^3)*f)*m^2 + (12*b*e^4*f +
12*a*e^4*g + ((c*d*e^3 + b*e^4)*f + (b*d*e^3 + a*e^4)*g)*m^3 + ((5*c*d*e^3
 + 8*b*e^4)*f - (3*c*d^2*e^2 - 5*b*d*e^3 - 8*a*e^4)*g)*m^2 + ((4*c*d*e^3 +
 19*b*e^4)*f - (3*c*d^2*e^2 - 4*b*d*e^3 - 19*a*e^4)*g)*m)*x^2 + 4*(2*c*d^3
*e - 3*b*d^2*e^2 + 6*a*d*e^3)*f - 2*(3*c*d^4 - 4*b*d^3*e + 6*a*d^2*e^2)*g
+ ((2*c*d^3*e - 7*b*d^2*e^2 + 26*a*d*e^3)*f + (2*b*d^3*e - 7*a*d^2*e^2)*g)
*m + (24*a*e^4*f + (a*d*e^3*g + (b*d*e^3 + a*e^4)*f)*m^3 - ((2*c*d^2*e^2 -
 7*b*d*e^3 - 9*a*e^4)*f + (2*b*d^2*e^2 - 7*a*d*e^3)*g)*m^2 - 2*((4*c*d^2*e
^2 - 6*b*d*e^3 - 13*a*e^4)*f - (3*c*d^3*e - 4*b*d^2*e^2 + 6*a*d*e^3)*g)*m)
*x)*(e*x + d)^m/(e^4*m^4 + 10*e^4*m^3 + 35*e^4*m^2 + 50*e^4*m + 24*e^4)
```

3.921.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5930 vs. $2(134) = 268$.

Time = 1.27 (sec) , antiderivative size = 5930, normalized size of antiderivative = 41.18

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a),x)
```

output
```
Piecewise((d**m*(a*f*x + a*g*x**2/2 + b*f*x**2/2 + b*g*x**3/3 + c*f*x**3/3
 + c*g*x**4/4), Eq(e, 0)), (-a*d*e**2*g/(6*d**3*e**4 + 18*d**2*e**5*x + 18
*d*e**6*x**2 + 6*e**7*x**3) - 2*a*e**3*f/(6*d**3*e**4 + 18*d**2*e**5*x + 1
8*d*e**6*x**2 + 6*e**7*x**3) - 3*a*e**3*g*x/(6*d**3*e**4 + 18*d**2*e**5*x
+ 18*d*e**6*x**2 + 6*e**7*x**3) - 2*b*d**2*e*g/(6*d**3*e**4 + 18*d**2*e**5
*x + 18*d*e**6*x**2 + 6*e**7*x**3) - b*d*e**2*f/(6*d**3*e**4 + 18*d**2*e**
5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 6*b*d*e**2*g*x/(6*d**3*e**4 + 18*d**
2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 3*b*e**3*f*x/(6*d**3*e**4 + 18*
d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 6*b*e**3*g*x**2/(6*d**3*e**4
 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 6*c*d**3*g*log(d/e + x
)/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 11*c*d**
3*g/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 2*c*d*
*2*e*f/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 18*
c*d**2*e*g*x*log(d/e + x)/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 +
 6*e**7*x**3) + 27*c*d**2*e*g*x/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*
x**2 + 6*e**7*x**3) - 6*c*d*e**2*f*x/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*
e**6*x**2 + 6*e**7*x**3) + 18*c*d*e**2*g*x**2*log(d/e + x)/(6*d**3*e**4 +
18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 18*c*d*e**2*g*x**2/(6*d**
3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 6*c*e**3*f*x**2/
(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 6*c*e**...
```

3.921.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(144) = 288$.

Time = 0.23 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.44

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)\,dx$$
$$= \frac{(e^2(m+1)x^2+demx-d^2)(ex+d)^m bf}{(m^2+3m+2)e^2} + \frac{(e^2(m+1)x^2+demx-d^2)(ex+d)^m ag}{(m^2+3m+2)e^2} + \frac{(ex+d)^{m+1}af}{e(m+1)} + \frac{((m^2+3m+2)e^3x^3+(m^2+m)de^2x^2-2d^2emx+2d^3)(ex+d)^m cf}{(m^3+6m^2+11m+6)e^3} + \frac{((m^2+3m+2)e^3x^3+(m^2+m)de^2x^2-2d^2emx+2d^3)(ex+d)^m bg}{(m^3+6m^2+11m+6)e^3} + \frac{((m^3+6m^2+11m+6)e^4x^4+(m^3+3m^2+2m)de^3x^3-3(m^2+m)d^2e^2x^2+6d^3emx-6d^4)(ex+}{(m^4+10m^3+35m^2+50m+24)e^4}$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*b*f/((m^2 + 3*m + 2)*e^2) +
(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a*g/((m^2 + 3*m + 2)*e^2) +
(e*x + d)^(m + 1)*a*f/(e*(m + 1)) + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d
*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*c*f/((m^3 + 6*m^2 + 11*m + 6)*
e^3) + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^
3)*(e*x + d)^m*b*g/((m^3 + 6*m^2 + 11*m + 6)*e^3) + ((m^3 + 6*m^2 + 11*m +
 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*
d^3*e*m*x - 6*d^4)*(e*x + d)^m*c*g/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^
4)
```

3.921.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1156 vs. $2(144) = 288$.

Time = 0.28 (sec) , antiderivative size = 1156, normalized size of antiderivative = 8.03

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
((e*x + d)^m*c*e^4*g*m^3*x^4 + (e*x + d)^m*c*e^4*f*m^3*x^3 + (e*x + d)^m*c
*d*e^3*g*m^3*x^3 + (e*x + d)^m*b*e^4*g*m^3*x^3 + 6*(e*x + d)^m*c*e^4*g*m^2
*x^4 + (e*x + d)^m*c*d*e^3*f*m^3*x^2 + (e*x + d)^m*b*e^4*f*m^3*x^2 + (e*x
+ d)^m*b*d*e^3*g*m^3*x^2 + (e*x + d)^m*a*e^4*g*m^3*x^2 + 7*(e*x + d)^m*c*e
^4*f*m^2*x^3 + 3*(e*x + d)^m*c*d*e^3*g*m^2*x^3 + 7*(e*x + d)^m*b*e^4*g*m^2
*x^3 + 11*(e*x + d)^m*c*e^4*g*m*x^4 + (e*x + d)^m*b*d*e^3*f*m^3*x + (e*x +
 d)^m*a*e^4*f*m^3*x + (e*x + d)^m*a*d*e^3*g*m^3*x + 5*(e*x + d)^m*c*d*e^3*
f*m^2*x^2 + 8*(e*x + d)^m*b*e^4*f*m^2*x^2 - 3*(e*x + d)^m*c*d^2*e^2*g*m^2*
x^2 + 5*(e*x + d)^m*b*d*e^3*g*m^2*x^2 + 8*(e*x + d)^m*a*e^4*g*m^2*x^2 + 14
*(e*x + d)^m*c*e^4*f*m*x^3 + 2*(e*x + d)^m*c*d*e^3*g*m*x^3 + 14*(e*x + d)^
m*b*e^4*g*m*x^3 + 6*(e*x + d)^m*c*e^4*g*x^4 + (e*x + d)^m*a*d*e^3*f*m^3 -
2*(e*x + d)^m*c*d^2*e^2*f*m^2*x + 7*(e*x + d)^m*b*d*e^3*f*m^2*x + 9*(e*x +
 d)^m*a*e^4*f*m^2*x - 2*(e*x + d)^m*b*d^2*e^2*g*m^2*x + 7*(e*x + d)^m*a*d*
e^3*g*m^2*x + 4*(e*x + d)^m*c*d*e^3*f*m*x^2 + 19*(e*x + d)^m*b*e^4*f*m*x^2
 - 3*(e*x + d)^m*c*d^2*e^2*g*m*x^2 + 4*(e*x + d)^m*b*d*e^3*g*m*x^2 + 19*(e
*x + d)^m*a*e^4*g*m*x^2 + 8*(e*x + d)^m*c*e^4*f*x^3 + 8*(e*x + d)^m*b*e^4*
g*x^3 - (e*x + d)^m*b*d^2*e^2*f*m^2 + 9*(e*x + d)^m*a*d*e^3*f*m^2 - (e*x +
 d)^m*a*d^2*e^2*g*m^2 - 8*(e*x + d)^m*c*d^2*e^2*f*m*x + 12*(e*x + d)^m*b*d
*e^3*f*m*x + 26*(e*x + d)^m*a*e^4*f*m*x + 6*(e*x + d)^m*c*d^3*e*g*m*x - 8*
(e*x + d)^m*b*d^2*e^2*g*m*x + 12*(e*x + d)^m*a*d*e^3*g*m*x + 12*(e*x + ...
```

3.921.9 Mupad [B] (verification not implemented)

Time = 13.54 (sec) , antiderivative size = 602, normalized size of antiderivative = 4.18

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)\,dx$$

$$= \frac{(d+e\,x)^m\,(24\,a\,d\,e^3\,f-6\,c\,d^4\,g+8\,b\,d^3\,e\,g+8\,c\,d^3\,e\,f-12\,a\,d^2\,e^2\,g-12\,b\,d^2\,e^2\,f+9\,a\,d\,e^3\,f\,m^2+a\,d\,e^3}{e^4\,\left(m^4+10\,m^3+\right.}$$

$$+\frac{x\,(d+e\,x)^m\,(24\,a\,e^4\,f+26\,a\,e^4\,f\,m+9\,a\,e^4\,f\,m^2+a\,e^4\,f\,m^3+7\,a\,d\,e^3\,g\,m^2+7\,b\,d\,e^3\,f\,m^2+a\,d\,e^3\,g\,m}{e^4\,\left(m^4+1\right.}$$

$$+\frac{x^2\,(m+1)\,(d+e\,x)^m\,(12\,a\,e^2\,g+12\,b\,e^2\,f+7\,a\,e^2\,g\,m+7\,b\,e^2\,f\,m-3\,c\,d^2\,g\,m+a\,e^2\,g\,m^2+b\,e^2\,f\,m^2}{e^2\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}$$

$$+\frac{c\,g\,x^4\,(d+e\,x)^m\,\left(m^3+6\,m^2+11\,m+6\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}$$

$$+\frac{x^3\,(d+e\,x)^m\,\left(m^2+3\,m+2\right)\,(4\,b\,e\,g+4\,c\,e\,f+b\,e\,g\,m+c\,d\,g\,m+c\,e\,f\,m)}{e\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}$$

input
```
int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2),x)
```

output
```
((d + e*x)^m*(24*a*d*e^3*f - 6*c*d^4*g + 8*b*d^3*e*g + 8*c*d^3*e*f - 12*a*
d^2*e^2*g - 12*b*d^2*e^2*f + 9*a*d*e^3*f*m^2 + a*d*e^3*f*m^3 - 7*a*d^2*e^2
*g*m - 7*b*d^2*e^2*f*m - a*d^2*e^2*g*m^2 - b*d^2*e^2*f*m^2 + 26*a*d*e^3*f*
m + 2*b*d^3*e*g*m + 2*c*d^3*e*f*m))/(e^4*(50*m + 35*m^2 + 10*m^3 + m^4 + 2
4)) + (x*(d + e*x)^m*(24*a*e^4*f + 26*a*e^4*f*m + 9*a*e^4*f*m^2 + a*e^4*f*
m^3 + 7*a*d*e^3*g*m^2 + 7*b*d*e^3*f*m^2 + a*d*e^3*g*m^3 + b*d*e^3*f*m^3 -
8*b*d^2*e^2*g*m - 8*c*d^2*e^2*f*m - 2*b*d^2*e^2*g*m^2 - 2*c*d^2*e^2*f*m^2
+ 12*a*d*e^3*g*m + 12*b*d*e^3*f*m + 6*c*d^3*e*g*m))/(e^4*(50*m + 35*m^2 +
10*m^3 + m^4 + 24)) + (x^2*(m + 1)*(d + e*x)^m*(12*a*e^2*g + 12*b*e^2*f +
7*a*e^2*g*m + 7*b*e^2*f*m - 3*c*d^2*g*m + a*e^2*g*m^2 + b*e^2*f*m^2 + 4*b*
d*e*g*m + 4*c*d*e*f*m + b*d*e*g*m^2 + c*d*e*f*m^2))/(e^2*(50*m + 35*m^2 +
10*m^3 + m^4 + 24)) + (c*g*x^4*(d + e*x)^m*(11*m + 6*m^2 + m^3 + 6))/(50*m
 + 35*m^2 + 10*m^3 + m^4 + 24) + (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(4*b*e*g
 + 4*c*e*f + b*e*g*m + c*d*g*m + c*e*f*m))/(e*(50*m + 35*m^2 + 10*m^3 + m^
4 + 24))
```

3.922 $\int \frac{(d+ex)^m(a+bx+cx^2)}{f+gx}\,dx$

3.922.1 Optimal result

Integrand size = 25, antiderivative size = 129

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{f+gx}\,dx = -\frac{(cef+cdg-beg)(d+ex)^{1+m}}{e^2g^2(1+m)}+\frac{c(d+ex)^{2+m}}{e^2g(2+m)} + \frac{(cf^2-bfg+ag^2)(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{g(d+ex)}{ef-dg}\right)}{g^2(ef-dg)(1+m)}$$

output
```
-(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(1+m)/e^2/g^2/(1+m)+c*(e*x+d)^(2+m)/e^2/g/(2
+m)+(a*g^2-b*f*g+c*f^2)*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],-g*(e*x+d)/
(-d*g+e*f))/g^2/(-d*g+e*f)/(1+m)
```

3.922.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{f+gx}\,dx = \frac{(d+ex)^{1+m}\left(\frac{beg-c(ef+dg)}{e^2(1+m)}+\frac{cg(d+ex)}{e^2(2+m)}+\frac{(cf^2+g(-bf+ag))\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{g(d+ex)}{-ef+dg}\right)}{(ef-dg)(1+m)}\right)}{g^2}$$

input
```
Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x),x]
```

output
```
((d + e*x)^(1 + m)*((b*e*g - c*(e*f + d*g))/(e^2*(1 + m)) + (c*g*(d + e*x)
)/(e^2*(2 + m)) + ((c*f^2 + g*(-(b*f) + a*g))*Hypergeometric2F1[1, 1 + m,
2 + m, (g*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g)*(1 + m))))/g^2
```

3.922.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)(d+ex)^m}{f+gx}dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{(d+ex)^m(ag^2-bfg+cf^2)}{g^2(f+gx)}+\frac{(d+ex)^m(beg-c(dg+ef))}{eg^2}+\frac{c(d+ex)^{m+1}}{eg}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^{m+1}(ag^2-bfg+cf^2)\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{g(d+ex)}{ef-dg}\right)}{g^2(m+1)(ef-dg)}+\frac{(d+ex)^{m+1}(beg-c(dg+ef))}{e^2g^2(m+1)}+\frac{c(d+ex)^{m+2}}{e^2g(m+2)}$$

input
```
Int[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x),x]
```

output
```
((b*e*g - c*(e*f + d*g))*(d + e*x)^(1 + m))/(e^2*g^2*(1 + m)) + (c*(d + e*
x)^(2 + m))/(e^2*g*(2 + m)) + ((c*f^2 - b*f*g + a*g^2)*(d + e*x)^(1 + m)*H
ypergeometric2F1[1, 1 + m, 2 + m, -((g*(d + e*x))/(e*f - d*g))])/(g^2*(e*f
 - d*g)*(1 + m))
```

3.922.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.922.4 Maple [F]

$$\int \frac{(ex+d)^m (c\,x^2+bx+a)}{gx+f}dx$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x)
```

output
```
int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x)
```

3.922.5 Fricas [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{f+gx}\,dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{gx+f}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

3.922.6 Sympy [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{f+gx}\,dx = \int \frac{(d+ex)^m (a+bx+cx^2)}{f+gx}\,dx$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x+a)/(g*x+f),x)
```

output
```
Integral((d + e*x)**m*(a + b*x + c*x**2)/(f + g*x), x)
```

3.922.7 Maxima [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{f+gx} \, dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{gx+f} \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

3.922.8 Giac [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{f+gx} \, dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{gx+f} \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

3.922.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{f+gx} \, dx = \int \frac{(d+e\,x)^m\,(c\,x^2+b\,x+a)}{f+g\,x} \, dx$$

input
```
int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x),x)
```

output
```
int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x), x)
```

3.923 $\int \frac{(d+ex)^m(a+bx+cx^2)}{(f+gx)^2}\,dx$

3.923.1 Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{(f+gx)^2}\,dx = \frac{c(d+ex)^{1+m}}{eg^2(1+m)} + \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{(ef-dg)(f+gx)} + \frac{(cf(2dg-ef(2+m))-g(aegm+b(dg-ef(1+m))))(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+\dots\right.}{g^2(ef-dg)^2(1+m)}$$

output
```
c*(e*x+d)^(1+m)/e/g^2/(1+m)+(a+f*(-b*g+c*f)/g^2)*(e*x+d)^(1+m)/(-d*g+e*f)/
(g*x+f)+(c*f*(2*d*g-e*f*(2+m))-g*(a*e*g*m+b*(d*g-e*f*(1+m))))*(e*x+d)^(1+m
)*hypergeom([1, 1+m],[2+m],-g*(e*x+d)/(-d*g+e*f))/g^2/(-d*g+e*f)^2/(1+m)
```

3.923.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{(f+gx)^2}\,dx$$
$$= \frac{(d+ex)^{1+m}\left(c(ef-dg)^2 - e(2cf-bg)(ef-dg)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{g(d+ex)}{-ef+dg}\right) + e^2(c\dots\right.}{eg^2(ef-dg)^2(1+m)}$$

input
```
Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^2,x]
```

output
```
((d + e*x)^(1 + m)*(c*(e*f - d*g)^2 - e*(2*c*f - b*g)*(e*f - d*g)*Hypergeo
metric2F1[1, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)] + e^2*(c*f^2 + g*
(-(b*f) + a*g))*Hypergeometric2F1[2, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) +
 d*g)]))/(e*g^2*(e*f - d*g)^2*(1 + m))
```

3.923.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1193, 27, 90, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)(d+ex)^m}{(f+gx)^2}dx$$

$$\Big\downarrow \text{1193}$$

$$\frac{\int \frac{(d+ex)^m\left(-ce(m+1)f^2+cdgf-aeg^2m-bg(dg-ef(m+1))-c\left(d-\frac{ef}{g}\right)g^2x\right)}{g^2(f+gx)}dx}{ef-dg}+\frac{(d+ex)^{m+1}\left(a+\frac{f(cf-bg)}{g^2}\right)}{(f+gx)(ef-dg)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(d+ex)^m(-ce(m+1)f^2+cdgf+beg(m+1)f-bdg^2-aeg^2m+cg(ef-dg)x)}{f+gx}dx}{g^2(ef-dg)}+\frac{(d+ex)^{m+1}\left(a+\frac{f(cf-bg)}{g^2}\right)}{(f+gx)(ef-dg)}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{c(ef-dg)(d+ex)^{m+1}}{e(m+1)}-(g(aegm+bdg-bef(m+1))-cf(2dg-ef(m+2)))\int \frac{(d+ex)^m}{f+gx}dx}{g^2(ef-dg)}+\frac{(d+ex)^{m+1}\left(a+\frac{f(cf-bg)}{g^2}\right)}{(f+gx)(ef-dg)}$$

$$\Big\downarrow \text{78}$$

$$\frac{\frac{c(ef-dg)(d+ex)^{m+1}}{e(m+1)}-\frac{(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{g(d+ex)}{ef-dg}\right)(g(aegm+bdg-bef(m+1))-cf(2dg-ef(m+2)))}{(m+1)(ef-dg)}}{g^2(ef-dg)}+\frac{(d+ex)^{m+1}\left(a+\frac{f(cf-bg)}{g^2}\right)}{(f+gx)(ef-dg)}$$

input
```
Int[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^2,x]
```

output
```
((a + (f*(c*f - b*g))/g^2)*(d + e*x)^(1 + m))/((e*f - d*g)*(f + g*x)) + ((
c*(e*f - d*g)*(d + e*x)^(1 + m))/(e*(1 + m)) - ((g*(b*d*g + a*e*g*m - b*e*
f*(1 + m)) - c*f*(2*d*g - e*f*(2 + m)))*(d + e*x)^(1 + m)*Hypergeometric2F
1[1, 1 + m, 2 + m, -((g*(d + e*x))/(e*f - d*g))])/((e*f - d*g)*(1 + m)))/(
g^2*(e*f - d*g))
```

3.923.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

3.923. $\int \frac{(d+ex)^m(a+bx+cx^2)}{(f+gx)^2}\,dx$

3.923.4 Maple [F]

$$\int \frac{(ex+d)^m (c\,x^2+bx+a)}{(gx+f)^2}dx$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x)
```

output
```
int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x)
```

3.923.5 Fricas [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^2}\,dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{(gx+f)^2}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)*(e*x + d)^m/(g^2*x^2 + 2*f*g*x + f^2), x)
```

3.923.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x+a)/(g*x+f)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.923.7 Maxima [F]

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{(f+gx)^2}\,dx=\int \frac{(cx^2+bx+a)(ex+d)^m}{(gx+f)^2}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f)^2, x)
```

3.923.8 Giac [F]

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{(f+gx)^2}\,dx=\int \frac{(cx^2+bx+a)(ex+d)^m}{(gx+f)^2}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f)^2, x)
```

3.923.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{(f+gx)^2}\,dx=\int \frac{(d+e\,x)^m\,(c\,x^2+b\,x+a)}{(f+g\,x)^2}\,dx$$

input
```
int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^2,x)
```

output
```
int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^2, x)
```

3.924 $\int \frac{(d+ex)^m(a+bx+cx^2)}{(f+gx)^3} dx$

3.924.1 Optimal result

Integrand size = 25, antiderivative size = 245

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{(f+gx)^3} dx = \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{2(ef-dg)(f+gx)^2} + \frac{(cf(4dg-ef(3+m))+g(aeg(1-m)-b(2dg-ef(1+m))))(d+ex)^{1+m}}{2g^2(ef-dg)^2(f+gx)} + \frac{(c(2d^2g^2-4defg(1+m)+e^2f^2(2+3m+m^2))-egm(aeg(1-m)-b(2dg-ef(1+m))))(d+ex)^{1}}{2g^2(ef-dg)^3(1+m)}$$

output
```
1/2*(a+f*(-b*g+c*f)/g^2)*(e*x+d)^(1+m)/(-d*g+e*f)/(g*x+f)^2+1/2*(c*f*(4*d*
g-e*f*(3+m))+g*(a*e*g*(1-m)-b*(2*d*g-e*f*(1+m))))*(e*x+d)^(1+m)/g^2/(-d*g+
e*f)^2/(g*x+f)+1/2*(c*(2*d^2*g^2-4*d*e*f*g*(1+m)+e^2*f^2*(m^2+3*m+2))-e*g*
m*(a*e*g*(1-m)-b*(2*d*g-e*f*(1+m))))*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m
],-g*(e*x+d)/(-d*g+e*f))/g^2/(-d*g+e*f)^3/(1+m)
```

3.924.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.64

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^3}\,dx =$$
$$-\frac{(d+ex)^{1+m}\left(c(ef-dg)^2 \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{g(d+ex)}{-ef+dg}\right)+e\left(-\left((2cf-bg)(ef-dg)\right.\right.\right.}{g}$$

input
```
Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^3,x]
```

output
```
-(((d + e*x)^(1 + m)*(c*(e*f - d*g)^2*Hypergeometric2F1[1, 1 + m, 2 + m, (
g*(d + e*x))/(-(e*f) + d*g)] + e*(-((2*c*f - b*g)*(e*f - d*g)*Hypergeometr
ic2F1[2, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)]) + e*(c*f^2 + g*(-(b*
f) + a*g))*Hypergeometric2F1[3, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)
])))/(g^2*(-(e*f) + d*g)^3*(1 + m)))
```

3.924.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1193, 27, 87, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)(d+ex)^m}{(f+gx)^3}dx$$
$$\Big\downarrow \text{1193}$$
$$\frac{\int \frac{(d+ex)^m\left(-2c\left(d-\frac{ef}{g}\right)xg^2-(2bdg-ae(1-m)g-bef(m+1))g+cf(2dg-ef(m+1))\right)}{g^2(f+gx)^2}dx}{2(ef-dg)}+\frac{(d+ex)^{m+1}\left(a+\frac{f(cf-bg)}{g^2}\right)}{2(f+gx)^2(ef-dg)}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{(d+ex)^m(cf(2dg-ef(m+1))-g(2bdg-ae(1-m)g-bef(m+1))+2cg(ef-dg)x)}{(f+gx)^2}dx}{2g^2(ef-dg)}+\frac{(d+ex)^{m+1}\left(a+\frac{f(cf-bg)}{g^2}\right)}{2(f+gx)^2(ef-dg)}$$

$\Big\downarrow$ 87

$$\frac{\frac{(egm(-aeg(1-m)+2bdg-bef(m+1))+c(2d^2g^2-4defg(m+1)+e^2f^2(m^2+3m+2)))\int\frac{(d+ex)^m}{f+gx}dx}{ef-dg}-\frac{(d+ex)^{m+1}(g(-aeg(1-m)+2bdg-bef(m-}{(f+gx)(ef-dg)}}{2g^2(ef-dg)}$$

$$\frac{(d+ex)^{m+1}\left(a+\frac{f(cf-bg)}{g^2}\right)}{2(f+gx)^2(ef-dg)}$$

$\Big\downarrow$ 78

$$\frac{\frac{(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{g(d+ex)}{ef-dg}\right)(egm(-aeg(1-m)+2bdg-bef(m+1))+c(2d^2g^2-4defg(m+1)+e^2f^2(m^2+3m+2)))}{(m+1)(ef-dg)^2}-}{2g^2(ef-dg)}$$

$$\frac{(d+ex)^{m+1}\left(a+\frac{f(cf-bg)}{g^2}\right)}{2(f+gx)^2(ef-dg)}$$

input
```
Int[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^3,x]
```

output
```
((a + (f*(c*f - b*g))/g^2)*(d + e*x)^(1 + m))/(2*(e*f - d*g)*(f + g*x)^2)
+ (-(((g*(2*b*d*g - a*e*g*(1 - m) - b*e*f*(1 + m)) - c*f*(4*d*g - e*f*(3 +
 m)))*(d + e*x)^(1 + m))/((e*f - d*g)*(f + g*x))) + ((e*g*m*(2*b*d*g - a*e
*g*(1 - m) - b*e*f*(1 + m)) + c*(2*d^2*g^2 - 4*d*e*f*g*(1 + m) + e^2*f^2*(
2 + 3*m + m^2)))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g
*(d + e*x))/(e*f - d*g))])/((e*f - d*g)^2*(1 + m)))/(2*g^2*(e*f - d*g))
```

3.924.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

3.924. $\int\frac{(d+ex)^m(a+bx+cx^2)}{(f+gx)^3}\,dx$

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

3.924.4 Maple [F]

$$\int \frac{(ex+d)^m (c\,x^2+bx+a)}{(gx+f)^3}dx$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x)
```

output
```
int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x)
```

3.924.5 Fricas [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^3}\,dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{(gx+f)^3}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)*(e*x + d)^m/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x
+ f^3), x)
```

3.924.6 Sympy [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^3} \, dx = \int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^3} \, dx$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x+a)/(g*x+f)**3,x)
```

output
```
Integral((d + e*x)**m*(a + b*x + c*x**2)/(f + g*x)**3, x)
```

3.924.7 Maxima [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^3} \, dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{(gx+f)^3} \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f)^3, x)
```

3.924.8 Giac [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^3} \, dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{(gx+f)^3} \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f)^3, x)
```

3.924.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^3}\,dx = \int \frac{(d+e\,x)^m\,(c\,x^2+b\,x+a)}{(f+g\,x)^3}\,dx$$

input
```
int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^3,x)
```

output
```
int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^3, x)
```

3.925 $\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^2\,dx$

3.925.1 Optimal result

Integrand size = 27, antiderivative size = 525

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^2\,dx=\frac{\left(cd^2-bde+ae^2\right)^2(ef-dg)^2(d+ex)^{1+m}}{e^7(1+m)}$$
$$-\frac{2\left(cd^2-bde+ae^2\right)(ef-dg)(cd(2ef-3dg)-e(bef-2bdg+aeg))(d+ex)^{2+m}}{e^7(2+m)}$$
$$+\frac{(c^2d^2(6e^2f^2-20defg+15d^2g^2)+e^2(a^2e^2g^2+2abeg(2ef-3dg)+b^2(e^2f^2-6defg+6d^2g^2))+2ce(a}{e^7(3+m)}$$
$$+\frac{2(be^2g(bef-2bdg+aeg)-2c^2d(e^2f^2-5defg+5d^2g^2)+ce(2aeg(ef-2dg)+b(e^2f^2-8defg+10d}{e^7(4+m)}$$
$$+\frac{\left(b^2e^2g^2+2ceg(2bef-5bdg+aeg)+c^2(e^2f^2-10defg+15d^2g^2)\right)(d+ex)^{5+m}}{e^7(5+m)}$$
$$+\frac{2cg(cef-3cdg+beg)(d+ex)^{6+m}}{e^7(6+m)}+\frac{c^2g^2(d+ex)^{7+m}}{e^7(7+m)}$$

output
```
(a*e^2-b*d*e+c*d^2)^2*(-d*g+e*f)^2*(e*x+d)^(1+m)/e^7/(1+m)-2*(a*e^2-b*d*e+
c*d^2)*(-d*g+e*f)*(c*d*(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*(e*x+d)^(2+
m)/e^7/(2+m)+(c^2*d^2*(15*d^2*g^2-20*d*e*f*g+6*e^2*f^2)+e^2*(a^2*e^2*g^2+2
*a*b*e*g*(-3*d*g+2*e*f)+b^2*(6*d^2*g^2-6*d*e*f*g+e^2*f^2))+2*c*e*(a*e*(6*d
^2*g^2-6*d*e*f*g+e^2*f^2)-b*d*(10*d^2*g^2-12*d*e*f*g+3*e^2*f^2)))*(e*x+d)^
(3+m)/e^7/(3+m)+2*(b*e^2*g*(a*e*g-2*b*d*g+b*e*f)-2*c^2*d*(5*d^2*g^2-5*d*e*
f*g+e^2*f^2)+c*e*(2*a*e*g*(-2*d*g+e*f)+b*(10*d^2*g^2-8*d*e*f*g+e^2*f^2)))*
(e*x+d)^(4+m)/e^7/(4+m)+(b^2*e^2*g^2+2*c*e*g*(a*e*g-5*b*d*g+2*b*e*f)+c^2*(
15*d^2*g^2-10*d*e*f*g+e^2*f^2))*(e*x+d)^(5+m)/e^7/(5+m)+2*c*g*(b*e*g-3*c*d
*g+c*e*f)*(e*x+d)^(6+m)/e^7/(6+m)+c^2*g^2*(e*x+d)^(7+m)/e^7/(7+m)
```

3.925.2 Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 492, normalized size of antiderivative = 0.94

$$\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right)^2 \, dx = \frac{(d+ex)^{1+m}\left(\frac{(cd^2+e(-bd+ae))^2(ef-dg)^2}{1+m} - \frac{2(cd^2+e(-bd+ae))(-ef+dg)(cd(-2ef+3dg)+e(bef-2bdg+aeg))(d+ex)}{2+m} + \frac{(c^2d^2(6e^2f^2\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^2,x]
```

output
```
((d + e*x)^(1 + m)*(((c*d^2 + e*(-(b*d) + a*e))^2*(e*f - d*g)^2)/(1 + m) -
 (2*(c*d^2 + e*(-(b*d) + a*e))*(-(e*f) + d*g)*(c*d*(-2*e*f + 3*d*g) + e*(b
*e*f - 2*b*d*g + a*e*g))*(d + e*x))/(2 + m) + ((c^2*d^2*(6*e^2*f^2 - 20*d*
e*f*g + 15*d^2*g^2) + e^2*(a^2*e^2*g^2 + 2*a*b*e*g*(2*e*f - 3*d*g) + b^2*(
e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2)) + 2*c*e*(b*d*(-3*e^2*f^2 + 12*d*e*f*g -
10*d^2*g^2) + a*e*(e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2)))*(d + e*x)^2)/(3 + m)
 + (2*(b*e^2*g*(b*e*f - 2*b*d*g + a*e*g) - 2*c^2*d*(e^2*f^2 - 5*d*e*f*g +
5*d^2*g^2) + c*e*(2*a*e*g*(e*f - 2*d*g) + b*(e^2*f^2 - 8*d*e*f*g + 10*d^2*
g^2)))*(d + e*x)^3)/(4 + m) + ((b^2*e^2*g^2 + 2*c*e*g*(2*b*e*f - 5*b*d*g +
 a*e*g) + c^2*(e^2*f^2 - 10*d*e*f*g + 15*d^2*g^2))*(d + e*x)^4)/(5 + m) +
(2*c*g*(c*e*f - 3*c*d*g + b*e*g)*(d + e*x)^5)/(6 + m) + (c^2*g^2*(d + e*x)
^6)/(7 + m)))/e^7
```

3.925.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2 \left(a+bx+cx^2\right)^2 (d+ex)^m \, dx$$

$$\Big\downarrow \; 1195$$

$$\int \left(\frac{(d+ex)^{m+2}\left(e^2\left(a^2e^2g^2+2abeg(2ef-3dg)+b^2\left(6d^2g^2-6defg+e^2f^2\right)\right)+2ce\left(ae\left(6d^2g^2-6defg+e^2f^2\right)\cdots\right.\right.}{e^6}\cdots\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^{m+3}\left(e^2\left(a^2e^2g^2+2abeg(2ef-3dg)+b^2\left(6d^2g^2-6defg+e^2f^2\right)\right)+2ce\left(ae\left(6d^2g^2-6defg+e^2f^2\right)-bd\right.\right.}{e^7(m+3)}$$
$$\frac{(d+ex)^{m+5}\left(2ceg(aeg-5bdg+2bef)+b^2e^2g^2+c^2\left(15d^2g^2-10defg+e^2f^2\right)\right)}{e^7(m+5)}+$$
$$\frac{2(d+ex)^{m+4}\left(ce\left(2aeg(ef-2dg)+b\left(10d^2g^2-8defg+e^2f^2\right)\right)+be^2g(aeg-2bdg+bef)-2c^2d\left(5d^2g^2-5defg\right.\right.}{e^7(m+4)}$$
$$\frac{(ef-dg)^2(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)^2}{e^7(m+1)}-$$
$$\frac{2(ef-dg)(d+ex)^{m+2}\left(ae^2-bde+cd^2\right)(cd(2ef-3dg)-e(aeg-2bdg+bef))}{e^7(m+2)}+$$
$$\frac{2cg(d+ex)^{m+6}(beg-3cdg+cef)}{e^7(m+6)}+\frac{c^2g^2(d+ex)^{m+7}}{e^7(m+7)}$$

input
```
Int[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^2,x]
```

output
```
((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^2*(d + e*x)^(1 + m))/(e^7*(1 + m))
- (2*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*(c*d*(2*e*f - 3*d*g) - e*(b*e*f -
 2*b*d*g + a*e*g))*(d + e*x)^(2 + m))/(e^7*(2 + m)) + ((c^2*d^2*(6*e^2*f^2
 - 20*d*e*f*g + 15*d^2*g^2) + e^2*(a^2*e^2*g^2 + 2*a*b*e*g*(2*e*f - 3*d*g)
 + b^2*(e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2)) + 2*c*e*(a*e*(e^2*f^2 - 6*d*e*f*
g + 6*d^2*g^2) - b*d*(3*e^2*f^2 - 12*d*e*f*g + 10*d^2*g^2)))*(d + e*x)^(3
+ m))/(e^7*(3 + m)) + (2*(b*e^2*g*(b*e*f - 2*b*d*g + a*e*g) - 2*c^2*d*(e^2
*f^2 - 5*d*e*f*g + 5*d^2*g^2) + c*e*(2*a*e*g*(e*f - 2*d*g) + b*(e^2*f^2 -
8*d*e*f*g + 10*d^2*g^2)))*(d + e*x)^(4 + m))/(e^7*(4 + m)) + ((b^2*e^2*g^2
 + 2*c*e*g*(2*b*e*f - 5*b*d*g + a*e*g) + c^2*(e^2*f^2 - 10*d*e*f*g + 15*d^
2*g^2))*(d + e*x)^(5 + m))/(e^7*(5 + m)) + (2*c*g*(c*e*f - 3*c*d*g + b*e*g
)*(d + e*x)^(6 + m))/(e^7*(6 + m)) + (c^2*g^2*(d + e*x)^(7 + m))/(e^7*(7 +
 m))
```

3.925.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

3.925. $\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^2\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.925.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4652 vs. 2(525) = 1050.

Time = 0.78 (sec) , antiderivative size = 4653, normalized size of antiderivative = 8.86

method	result	size
norman	Expression too large to display	4653
gosper	Expression too large to display	5890
risch	Expression too large to display	7342
parallelrisch	Expression too large to display	10811

input
```
int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d*(a^2*e^6*f^2*m^6-2*a^2*d*e^5*f*g*m^5+27*a^2*e^6*f^2*m^5-2*a*b*d*e^5*f^2*
m^5+2*a^2*d^2*e^4*g^2*m^4-50*a^2*d*e^5*f*g*m^4+295*a^2*e^6*f^2*m^4+8*a*b*d
^2*e^4*f*g*m^4-50*a*b*d*e^5*f^2*m^4+4*a*c*d^2*e^4*f^2*m^4+2*b^2*d^2*e^4*f^
2*m^4+44*a^2*d^2*e^4*g^2*m^3-490*a^2*d*e^5*f*g*m^3+1665*a^2*e^6*f^2*m^3-12
*a*b*d^3*e^3*g^2*m^3+176*a*b*d^2*e^4*f*g*m^3-490*a*b*d*e^5*f^2*m^3-24*a*c*
d^3*e^3*f*g*m^3+88*a*c*d^2*e^4*f^2*m^3-12*b^2*d^3*e^3*f*g*m^3+44*b^2*d^2*e
^4*f^2*m^3-12*b*c*d^3*e^3*f^2*m^3+358*a^2*d^2*e^4*g^2*m^2-2350*a^2*d*e^5*f
*g*m^2+5104*a^2*e^6*f^2*m^2-216*a*b*d^3*e^3*g^2*m^2+1432*a*b*d^2*e^4*f*g*m
^2-2350*a*b*d*e^5*f^2*m^2+48*a*c*d^4*e^2*g^2*m^2-432*a*c*d^3*e^3*f*g*m^2+7
16*a*c*d^2*e^4*f^2*m^2+24*b^2*d^4*e^2*g^2*m^2-216*b^2*d^3*e^3*f*g*m^2+358*
b^2*d^2*e^4*f^2*m^2+96*b*c*d^4*e^2*f*g*m^2-216*b*c*d^3*e^3*f^2*m^2+24*c^2*
d^4*e^2*f^2*m^2+1276*a^2*d^2*e^4*g^2*m-5508*a^2*d*e^5*f*g*m+8028*a^2*e^6*f
^2*m-1284*a*b*d^3*e^3*g^2*m+5104*a*b*d^2*e^4*f*g*m-5508*a*b*d*e^5*f^2*m+62
4*a*c*d^4*e^2*g^2*m-2568*a*c*d^3*e^3*f*g*m+2552*a*c*d^2*e^4*f^2*m+312*b^2*
d^4*e^2*g^2*m-1284*b^2*d^3*e^3*f*g*m+1276*b^2*d^2*e^4*f^2*m-240*b*c*d^5*e*
g^2*m+1248*b*c*d^4*e^2*f*g*m-1284*b*c*d^3*e^3*f^2*m-240*c^2*d^5*e*f*g*m+31
2*c^2*d^4*e^2*f^2*m+1680*a^2*d^2*e^4*g^2-5040*a^2*d*e^5*f*g+5040*a^2*e^6*f
^2-2520*a*b*d^3*e^3*g^2+6720*a*b*d^2*e^4*f*g-5040*a*b*d*e^5*f^2+2016*a*c*d
^4*e^2*g^2-5040*a*c*d^3*e^3*f*g+3360*a*c*d^2*e^4*f^2+1008*b^2*d^4*e^2*g^2-
2520*b^2*d^3*e^3*f*g+1680*b^2*d^2*e^4*f^2-1680*b*c*d^5*e*g^2+4032*b*c*d...
```

3.925.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4747 vs. $2(525) = 1050$.

Time = 0.38 (sec) , antiderivative size = 4747, normalized size of antiderivative = 9.04

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
(a^2*d*e^6*f^2*m^6 + (c^2*e^7*g^2*m^6 + 21*c^2*e^7*g^2*m^5 + 175*c^2*e^7*g
^2*m^4 + 735*c^2*e^7*g^2*m^3 + 1624*c^2*e^7*g^2*m^2 + 1764*c^2*e^7*g^2*m +
 720*c^2*e^7*g^2)*x^7 + (1680*c^2*e^7*f*g + 1680*b*c*e^7*g^2 + (2*c^2*e^7*
f*g + (c^2*d*e^6 + 2*b*c*e^7)*g^2)*m^6 + (44*c^2*e^7*f*g + (15*c^2*d*e^6 +
 44*b*c*e^7)*g^2)*m^5 + 5*(76*c^2*e^7*f*g + (17*c^2*d*e^6 + 76*b*c*e^7)*g^
2)*m^4 + 5*(328*c^2*e^7*f*g + (45*c^2*d*e^6 + 328*b*c*e^7)*g^2)*m^3 + 2*(1
849*c^2*e^7*f*g + (137*c^2*d*e^6 + 1849*b*c*e^7)*g^2)*m^2 + 4*(1019*c^2*e^
7*f*g + (30*c^2*d*e^6 + 1019*b*c*e^7)*g^2)*m)*x^6 - (2*a^2*d^2*e^5*f*g + (
2*a*b*d^2*e^5 - 27*a^2*d*e^6)*f^2)*m^5 + (1008*c^2*e^7*f^2 + 4032*b*c*e^7*
f*g + 1008*(b^2 + 2*a*c)*e^7*g^2 + (c^2*e^7*f^2 + 2*(c^2*d*e^6 + 2*b*c*e^7
)*f*g + (2*b*c*d*e^6 + (b^2 + 2*a*c)*e^7)*g^2)*m^6 + (23*c^2*e^7*f^2 + 2*(
17*c^2*d*e^6 + 46*b*c*e^7)*f*g - (6*c^2*d^2*e^5 - 34*b*c*d*e^6 - 23*(b^2 +
 2*a*c)*e^7)*g^2)*m^5 + 3*(69*c^2*e^7*f^2 + 2*(35*c^2*d*e^6 + 138*b*c*e^7)
*f*g - (20*c^2*d^2*e^5 - 70*b*c*d*e^6 - 69*(b^2 + 2*a*c)*e^7)*g^2)*m^4 + 5
*(185*c^2*e^7*f^2 + 2*(59*c^2*d*e^6 + 370*b*c*e^7)*f*g - (42*c^2*d^2*e^5 -
 118*b*c*d*e^6 - 185*(b^2 + 2*a*c)*e^7)*g^2)*m^3 + 4*(536*c^2*e^7*f^2 + (1
87*c^2*d*e^6 + 2144*b*c*e^7)*f*g - (75*c^2*d^2*e^5 - 187*b*c*d*e^6 - 536*(
b^2 + 2*a*c)*e^7)*g^2)*m^2 + 12*(201*c^2*e^7*f^2 + 4*(7*c^2*d*e^6 + 201*b*
c*e^7)*f*g - (12*c^2*d^2*e^5 - 28*b*c*d*e^6 - 201*(b^2 + 2*a*c)*e^7)*g^2)*
m)*x^5 + (2*a^2*d^3*e^4*g^2 - (50*a*b*d^2*e^5 - 295*a^2*d*e^6 - 2*(b^2 ...
```

3.925.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74400 vs. $2(537) = 1074$.

Time = 11.37 (sec) , antiderivative size = 74400, normalized size of antiderivative = 141.71

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a)**2,x)
```

output
```
Piecewise((d**m*(a**2*f**2*x + a**2*f*g*x**2 + a**2*g**2*x**3/3 + a*b*f**2
*x**2 + 4*a*b*f*g*x**3/3 + a*b*g**2*x**4/2 + 2*a*c*f**2*x**3/3 + a*c*f*g*x
**4 + 2*a*c*g**2*x**5/5 + b**2*f**2*x**3/3 + b**2*f*g*x**4/2 + b**2*g**2*x
**5/5 + b*c*f**2*x**4/2 + 4*b*c*f*g*x**5/5 + b*c*g**2*x**6/3 + c**2*f**2*x
**5/5 + c**2*f*g*x**6/3 + c**2*g**2*x**7/7), Eq(e, 0)), (-a**2*d**2*e**4*g
**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10
*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 4*a**2*d
*e**5*f*g/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3
*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 6*
a**2*d*e**5*g**2*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 +
1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x
**6) - 10*a**2*e**6*f**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x
**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e
**13*x**6) - 24*a**2*e**6*f*g*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4
*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5
 + 60*e**13*x**6) - 15*a**2*e**6*g**2*x**2/(60*d**6*e**7 + 360*d**5*e**8*x
 + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d
*e**12*x**5 + 60*e**13*x**6) - 2*a*b*d**3*e**3*g**2/(60*d**6*e**7 + 360*d*
*5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**
4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 4*a*b*d**2*e**4*f*g/(60*d**6*e*...
```

3.925.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2034 vs. $2(525) = 1050$.

Time = 0.28 (sec) , antiderivative size = 2034, normalized size of antiderivative = 3.87

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
2*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a*b*f^2/((m^2 + 3*m + 2)*e
^2) + 2*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a^2*f*g/((m^2 + 3*m
+ 2)*e^2) + (e*x + d)^(m + 1)*a^2*f^2/(e*(m + 1)) + ((m^2 + 3*m + 2)*e^3*x
^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*b^2*f^2/((m^3
+ 6*m^2 + 11*m + 6)*e^3) + 2*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^
2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*a*c*f^2/((m^3 + 6*m^2 + 11*m + 6)*e^3
) + 4*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3
)*(e*x + d)^m*a*b*f*g/((m^3 + 6*m^2 + 11*m + 6)*e^3) + ((m^2 + 3*m + 2)*e^
3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*a^2*g^2/((m
^3 + 6*m^2 + 11*m + 6)*e^3) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 +
 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(
e*x + d)^m*b*c*f^2/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + 2*((m^3 + 6
*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2
*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*b^2*f*g/((m^4 + 10*m^3 + 35*m^
2 + 50*m + 24)*e^4) + 4*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 +
 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)
^m*a*c*f*g/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + 2*((m^3 + 6*m^2 + 1
1*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2
 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*a*b*g^2/((m^4 + 10*m^3 + 35*m^2 + 50*m
 + 24)*e^4) + ((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m...
```

3.925.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10486 vs. $2(525) = 1050$.

Time = 0.36 (sec) , antiderivative size = 10486, normalized size of antiderivative = 19.97

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
((e*x + d)^m*c^2*e^7*g^2*m^6*x^7 + 2*(e*x + d)^m*c^2*e^7*f*g*m^6*x^6 + (e*
x + d)^m*c^2*d*e^6*g^2*m^6*x^6 + 2*(e*x + d)^m*b*c*e^7*g^2*m^6*x^6 + 21*(e
*x + d)^m*c^2*e^7*g^2*m^5*x^7 + (e*x + d)^m*c^2*e^7*f^2*m^6*x^5 + 2*(e*x +
 d)^m*c^2*d*e^6*f*g*m^6*x^5 + 4*(e*x + d)^m*b*c*e^7*f*g*m^6*x^5 + 2*(e*x +
 d)^m*b*c*d*e^6*g^2*m^6*x^5 + (e*x + d)^m*b^2*e^7*g^2*m^6*x^5 + 2*(e*x + d
)^m*a*c*e^7*g^2*m^6*x^5 + 44*(e*x + d)^m*c^2*e^7*f*g*m^5*x^6 + 15*(e*x + d
)^m*c^2*d*e^6*g^2*m^5*x^6 + 44*(e*x + d)^m*b*c*e^7*g^2*m^5*x^6 + 175*(e*x
+ d)^m*c^2*e^7*g^2*m^4*x^7 + (e*x + d)^m*c^2*d*e^6*f^2*m^6*x^4 + 2*(e*x +
d)^m*b*c*e^7*f^2*m^6*x^4 + 4*(e*x + d)^m*b*c*d*e^6*f*g*m^6*x^4 + 2*(e*x +
d)^m*b^2*e^7*f*g*m^6*x^4 + 4*(e*x + d)^m*a*c*e^7*f*g*m^6*x^4 + (e*x + d)^m
*b^2*d*e^6*g^2*m^6*x^4 + 2*(e*x + d)^m*a*c*d*e^6*g^2*m^6*x^4 + 2*(e*x + d)
^m*a*b*e^7*g^2*m^6*x^4 + 23*(e*x + d)^m*c^2*e^7*f^2*m^5*x^5 + 34*(e*x + d)
^m*c^2*d*e^6*f*g*m^5*x^5 + 92*(e*x + d)^m*b*c*e^7*f*g*m^5*x^5 - 6*(e*x + d
)^m*c^2*d^2*e^5*g^2*m^5*x^5 + 34*(e*x + d)^m*b*c*d*e^6*g^2*m^5*x^5 + 23*(e
*x + d)^m*b^2*e^7*g^2*m^5*x^5 + 46*(e*x + d)^m*a*c*e^7*g^2*m^5*x^5 + 380*(
e*x + d)^m*c^2*e^7*f*g*m^4*x^6 + 85*(e*x + d)^m*c^2*d*e^6*g^2*m^4*x^6 + 38
0*(e*x + d)^m*b*c*e^7*g^2*m^4*x^6 + 735*(e*x + d)^m*c^2*e^7*g^2*m^3*x^7 +
2*(e*x + d)^m*b*c*d*e^6*f^2*m^6*x^3 + (e*x + d)^m*b^2*e^7*f^2*m^6*x^3 + 2*
(e*x + d)^m*a*c*e^7*f^2*m^6*x^3 + 2*(e*x + d)^m*b^2*d*e^6*f*g*m^6*x^3 + 4*
(e*x + d)^m*a*c*d*e^6*f*g*m^6*x^3 + 4*(e*x + d)^m*a*b*e^7*f*g*m^6*x^3 +...
```

3.925.9 Mupad [B] (verification not implemented)

Time = 14.95 (sec) , antiderivative size = 4871, normalized size of antiderivative = 9.28

$$\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2)^2,x)
```

output
```
((d + e*x)^m*(720*c^2*d^7*g^2 + 5040*a^2*d*e^6*f^2 + 1680*a^2*d^3*e^4*g^2
+ 1680*b^2*d^3*e^4*f^2 + 1008*b^2*d^5*e^2*g^2 + 1008*c^2*d^5*e^2*f^2 - 168
0*b*c*d^6*e*g^2 - 1680*c^2*d^6*e*f*g + 358*a^2*d^3*e^4*g^2*m^2 + 358*b^2*d
^3*e^4*f^2*m^2 + 44*a^2*d^3*e^4*g^2*m^3 + 44*b^2*d^3*e^4*f^2*m^3 + 2*a^2*d
^3*e^4*g^2*m^4 + 2*b^2*d^3*e^4*f^2*m^4 + 24*b^2*d^5*e^2*g^2*m^2 + 24*c^2*d
^5*e^2*f^2*m^2 - 5040*a*b*d^2*e^5*f^2 - 2520*a*b*d^4*e^3*g^2 + 3360*a*c*d^
3*e^4*f^2 + 2016*a*c*d^5*e^2*g^2 - 2520*b*c*d^4*e^3*f^2 - 5040*a^2*d^2*e^5
*f*g - 2520*b^2*d^4*e^3*f*g + 8028*a^2*d*e^6*f^2*m + 5104*a^2*d*e^6*f^2*m^
2 + 1665*a^2*d*e^6*f^2*m^3 + 295*a^2*d*e^6*f^2*m^4 + 27*a^2*d*e^6*f^2*m^5
+ a^2*d*e^6*f^2*m^6 + 1276*a^2*d^3*e^4*g^2*m + 1276*b^2*d^3*e^4*f^2*m + 31
2*b^2*d^5*e^2*g^2*m + 312*c^2*d^5*e^2*f^2*m - 2350*a*b*d^2*e^5*f^2*m^2 - 4
90*a*b*d^2*e^5*f^2*m^3 - 50*a*b*d^2*e^5*f^2*m^4 - 2*a*b*d^2*e^5*f^2*m^5 -
216*a*b*d^4*e^3*g^2*m^2 + 716*a*c*d^3*e^4*f^2*m^2 - 12*a*b*d^4*e^3*g^2*m^3
 + 88*a*c*d^3*e^4*f^2*m^3 + 4*a*c*d^3*e^4*f^2*m^4 + 48*a*c*d^5*e^2*g^2*m^2
 - 216*b*c*d^4*e^3*f^2*m^2 - 12*b*c*d^4*e^3*f^2*m^3 - 2350*a^2*d^2*e^5*f*g
*m^2 - 490*a^2*d^2*e^5*f*g*m^3 - 50*a^2*d^2*e^5*f*g*m^4 - 2*a^2*d^2*e^5*f*
g*m^5 - 216*b^2*d^4*e^3*f*g*m^2 - 12*b^2*d^4*e^3*f*g*m^3 + 6720*a*b*d^3*e^
4*f*g - 5040*a*c*d^4*e^3*f*g + 4032*b*c*d^5*e^2*f*g - 240*b*c*d^6*e*g^2*m
- 240*c^2*d^6*e*f*g*m - 5508*a*b*d^2*e^5*f^2*m - 1284*a*b*d^4*e^3*g^2*m +
2552*a*c*d^3*e^4*f^2*m + 624*a*c*d^5*e^2*g^2*m - 1284*b*c*d^4*e^3*f^2*m...
```

3.925. $\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right)^2 \, dx$

3.926 $\int (d+ex)^m (f+gx) \left(a+bx+cx^2\right)^2 \, dx$

3.926.1 Optimal result

Integrand size = 25, antiderivative size = 311

$$\begin{aligned}\int (d+ex)^m (f+gx) \left(a+bx+cx^2\right)^2 \, dx &= \frac{\left(cd^2-bde+ae^2\right)^2 (ef-dg)(d+ex)^{1+m}}{e^6(1+m)} \\ &- \frac{\left(cd^2-bde+ae^2\right) (cd(4ef-5dg)-e(2bef-3bdg+aeg))(d+ex)^{2+m}}{e^6(2+m)} \\ &+ \frac{\left(2c^2d^2(3ef-5dg)+be^2(bef-3bdg+2aeg)+2ce(ae(ef-3dg)-3bd(ef-2dg))\right)(d+ex)^{3+m}}{e^6(3+m)} \\ &+ \frac{\left(b^2e^2g-2c^2d(2ef-5dg)+2ce(bef-4bdg+aeg)\right)(d+ex)^{4+m}}{e^6(4+m)} \\ &+ \frac{c(cef-5cdg+2beg)(d+ex)^{5+m}}{e^6(5+m)} + \frac{c^2g(d+ex)^{6+m}}{e^6(6+m)}\end{aligned}$$

output
```
(a*e^2-b*d*e+c*d^2)^2*(-d*g+e*f)*(e*x+d)^(1+m)/e^6/(1+m)-(a*e^2-b*d*e+c*d^
2)*(c*d*(-5*d*g+4*e*f)-e*(a*e*g-3*b*d*g+2*b*e*f))*(e*x+d)^(2+m)/e^6/(2+m)+
(2*c^2*d^2*(-5*d*g+3*e*f)+b*e^2*(2*a*e*g-3*b*d*g+b*e*f)+2*c*e*(a*e*(-3*d*g
+e*f)-3*b*d*(-2*d*g+e*f)))*(e*x+d)^(3+m)/e^6/(3+m)+(b^2*e^2*g-2*c^2*d*(-5*
d*g+2*e*f)+2*c*e*(a*e*g-4*b*d*g+b*e*f))*(e*x+d)^(4+m)/e^6/(4+m)+c*(2*b*e*g
-5*c*d*g+c*e*f)*(e*x+d)^(5+m)/e^6/(5+m)+c^2*g*(e*x+d)^(6+m)/e^6/(6+m)
```

3.926.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 655 vs. $2(311) = 622$.

Time = 1.11 (sec) , antiderivative size = 655, normalized size of antiderivative = 2.11

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2\,dx$$
$$= \frac{(d+ex)^{1+m}\left((a+x(b+cx))^2(2beg+c(-5dg+ef(6+m)+eg(5+m)x))+\frac{2\left(\frac{\left(cd^2+e(-bd+ae)\right)\left(b^3e^3g\left(3+4m+\right.\right.}{}\right.}{}\right.}{}$$

input
```
Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^2,x]
```

output
```
((d + e*x)^(1 + m)*((a + x*(b + c*x))^2*(2*b*e*g + c*(-5*d*g + e*f*(6 + m)
 + e*g*(5 + m)*x)) + (2*(((c*d^2 + e*(-(b*d) + a*e))*(b^3*e^3*g*(3 + 4*m +
 m^2) + 12*c^3*d^2*(-5*d*g + e*f*(6 + m)) - b*c*e^2*(1 + m)*(b*d*g*(-6 + m
) + b*e*f*(6 + m) + 2*a*e*g*(9 + 2*m)) + 2*c^2*e*(-3*b*d*(d*g*(-9 + m) + 2
*e*f*(6 + m)) + 2*a*e*(d*g*(-15 + m + m^2) + e*f*(24 + 10*m + m^2)))))/(e^
2*(1 + m)) + ((b^4*e^4*g*(6 + 5*m + m^2) + 12*c^4*d^3*(5*d*g - e*f*(6 + m)
) - b^2*c*e^3*(2 + m)*(b*e*f*(6 + m) + b*d*g*(-3 + 2*m) + a*e*g*(21 + 5*m)
) + 2*c^3*d*e*(3*b*d*(d*g*(-14 + m) + 3*e*f*(6 + m)) - 2*a*e*(d*g*(-30 - 4
*m + m^2) + e*f*(42 + 19*m + 2*m^2))) + c^2*e^2*(4*a^2*e^2*g*(15 + 8*m + m
^2) + b^2*d*(d*g*(6 - 13*m + m^2) + 2*e*f*(-6 + 5*m + m^2)) + 2*a*b*e*(e*f
*(42 + 19*m + 2*m^2) + d*g*(-18 + 11*m + 4*m^2))))*(d + e*x))/(e^2*(2 + m)
) - (c*e*(4 + m)*(b*d*(-5*c*d + 2*b*e)*g - 2*a*c*d*e*g*m + a*b*e^2*g*(1 +
m) + c*e*(b*d - 2*a*e)*f*(6 + m)) - (3*c*d - b*e)*(b^2*e^2*g*(3 + m) + 2*c
^2*d*(-5*d*g + e*f*(6 + m)) - c*e*(b*d*g*(-4 + m) + 2*a*e*g*(5 + m) + b*e*
f*(6 + m))) + c*e*(3 + m)*(b^2*e^2*g*(3 + m) + 2*c^2*d*(-5*d*g + e*f*(6 +
m)) - c*e*(b*d*g*(-4 + m) + 2*a*e*g*(5 + m) + b*e*f*(6 + m)))*x)*(a + x*(b
 + c*x))))/(c*e^2*(3 + m)*(4 + m))))/(c*e^2*(5 + m)*(6 + m))
```

3.926.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(a+bx+cx^2\right)^2(d+ex)^m\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{(d+ex)^{m+3}\left(2ce(aeg-4bdg+bef)+b^2e^2g-2c^2d(2ef-5dg)\right)}{e^5}+\frac{(d+ex)^{m+2}\left(2ce(ae(ef-3dg)-3bd(e\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^{m+4}\left(2ce(aeg-4bdg+bef)+b^2e^2g-2c^2d(2ef-5dg)\right)}{e^6(m+4)}+\frac{(d+ex)^{m+3}\left(2ce(ae(ef-3dg)-3bd(ef-2dg))+be^2(2aeg-3bdg+bef)+2c^2d^2(3ef-5dg)\right)}{e^6(m+3)}+\frac{(ef-dg)(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)^2}{e^6(m+1)}-\frac{(d+ex)^{m+2}\left(ae^2-bde+cd^2\right)\left(cd(4ef-5dg)-e(aeg-3bdg+2bef)\right)}{e^6(m+2)}+\frac{c(d+ex)^{m+5}(2beg-5cdg+cef)}{e^6(m+5)}+\frac{c^2g(d+ex)^{m+6}}{e^6(m+6)}$$

input
```
Int[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^2,x]
```

output
```
((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*(d + e*x)^(1 + m))/(e^6*(1 + m)) -
((c*d^2 - b*d*e + a*e^2)*(c*d*(4*e*f - 5*d*g) - e*(2*b*e*f - 3*b*d*g + a*e
*g))*(d + e*x)^(2 + m))/(e^6*(2 + m)) + ((2*c^2*d^2*(3*e*f - 5*d*g) + b*e^
2*(b*e*f - 3*b*d*g + 2*a*e*g) + 2*c*e*(a*e*(e*f - 3*d*g) - 3*b*d*(e*f - 2*
d*g)))*(d + e*x)^(3 + m))/(e^6*(3 + m)) + ((b^2*e^2*g - 2*c^2*d*(2*e*f - 5
*d*g) + 2*c*e*(b*e*f - 4*b*d*g + a*e*g))*(d + e*x)^(4 + m))/(e^6*(4 + m))
+ (c*(c*e*f - 5*c*d*g + 2*b*e*g)*(d + e*x)^(5 + m))/(e^6*(5 + m)) + (c^2*g
*(d + e*x)^(6 + m))/(e^6*(6 + m))
```

3.926.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.926.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2148 vs. $2(311) = 622$.

Time = 0.64 (sec) , antiderivative size = 2149, normalized size of antiderivative = 6.91

method	result	size
norman	Expression too large to display	2149
gosper	Expression too large to display	2563
risch	Expression too large to display	3326
parallelrisch	Expression too large to display	5114

input
```
int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
g*c^2/(6+m)*x^6*exp(m*ln(e*x+d))+(2*a*c*e^2*g*m^2+b^2*e^2*g*m^2+2*b*c*d*e*
g*m^2+2*b*c*e^2*f*m^2+c^2*d*e*f*m^2+22*a*c*e^2*g*m+11*b^2*e^2*g*m+12*b*c*d
*e*g*m+22*b*c*e^2*f*m-5*c^2*d^2*g*m+6*c^2*d*e*f*m+60*a*c*e^2*g+30*b^2*e^2*
g+60*b*c*e^2*f)/e^2/(m^3+15*m^2+74*m+120)*x^4*exp(m*ln(e*x+d))+(2*a*b*e^3*
g*m^3+2*a*c*d*e^2*g*m^3+2*a*c*e^3*f*m^3+b^2*d*e^2*g*m^3+b^2*e^3*f*m^3+2*b*
c*d*e^2*f*m^3+30*a*b*e^3*g*m^2+22*a*c*d*e^2*g*m^2+30*a*c*e^3*f*m^2+11*b^2*
d*e^2*g*m^2+15*b^2*e^3*f*m^2-8*b*c*d^2*e*g*m^2+22*b*c*d*e^2*f*m^2-4*c^2*d^
2*e*f*m^2+148*a*b*e^3*g*m+60*a*c*d*e^2*g*m+148*a*c*e^3*f*m+30*b^2*d*e^2*g*
m+74*b^2*e^3*f*m-48*b*c*d^2*e*g*m+60*b*c*d*e^2*f*m+20*c^2*d^3*g*m-24*c^2*d
^2*e*f*m+240*a*b*e^3*g+240*a*c*e^3*f+120*b^2*e^3*f)/e^3/(m^4+18*m^3+119*m^
2+342*m+360)*x^3*exp(m*ln(e*x+d))+(a^2*e^4*g*m^4+2*a*b*d*e^3*g*m^4+2*a*b*e
^4*f*m^4+2*a*c*d*e^3*f*m^4+b^2*d*e^3*f*m^4+18*a^2*e^4*g*m^3+30*a*b*d*e^3*g
*m^3+36*a*b*e^4*f*m^3-6*a*c*d^2*e^2*g*m^3+30*a*c*d*e^3*f*m^3-3*b^2*d^2*e^2
*g*m^3+15*b^2*d*e^3*f*m^3-6*b*c*d^2*e^2*f*m^3+119*a^2*e^4*g*m^2+148*a*b*d*
e^3*g*m^2+238*a*b*e^4*f*m^2-66*a*c*d^2*e^2*g*m^2+148*a*c*d*e^3*f*m^2-33*b^
2*d^2*e^2*g*m^2+74*b^2*d*e^3*f*m^2+24*b*c*d^3*e*g*m^2-66*b*c*d^2*e^2*f*m^2
+12*c^2*d^3*e*f*m^2+342*a^2*e^4*g*m+240*a*b*d*e^3*g*m+684*a*b*e^4*f*m-180*
a*c*d^2*e^2*g*m+240*a*c*d*e^3*f*m-90*b^2*d^2*e^2*g*m+120*b^2*d*e^3*f*m+144
*b*c*d^3*e*g*m-180*b*c*d^2*e^2*f*m-60*c^2*d^4*g*m+72*c^2*d^3*e*f*m+360*a^2
*e^4*g+720*a*b*e^4*f)/e^4/(m^5+20*m^4+155*m^3+580*m^2+1044*m+720)*x^2*e...
```

3.926.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2368 vs. 2(312) = 624.

Time = 0.34 (sec) , antiderivative size = 2368, normalized size of antiderivative = 7.61

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2 dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
(a^2*d*e^5*f*m^5 + (c^2*e^6*g*m^5 + 15*c^2*e^6*g*m^4 + 85*c^2*e^6*g*m^3 +
225*c^2*e^6*g*m^2 + 274*c^2*e^6*g*m + 120*c^2*e^6*g)*x^6 + (144*c^2*e^6*f
+ 288*b*c*e^6*g + (c^2*e^6*f + (c^2*d*e^5 + 2*b*c*e^6)*g)*m^5 + 2*(8*c^2*e
^6*f + (5*c^2*d*e^5 + 16*b*c*e^6)*g)*m^4 + 5*(19*c^2*e^6*f + (7*c^2*d*e^5
+ 38*b*c*e^6)*g)*m^3 + 10*(26*c^2*e^6*f + (5*c^2*d*e^5 + 52*b*c*e^6)*g)*m^
2 + 12*(27*c^2*e^6*f + 2*(c^2*d*e^5 + 27*b*c*e^6)*g)*m)*x^5 - (a^2*d^2*e^4
*g + 2*(a*b*d^2*e^4 - 10*a^2*d*e^5)*f)*m^4 + (360*b*c*e^6*f + 180*(b^2 + 2
*a*c)*e^6*g + ((c^2*d*e^5 + 2*b*c*e^6)*f + (2*b*c*d*e^5 + (b^2 + 2*a*c)*e^
6)*g)*m^5 + (2*(6*c^2*d*e^5 + 17*b*c*e^6)*f - (5*c^2*d^2*e^4 - 24*b*c*d*e^
5 - 17*(b^2 + 2*a*c)*e^6)*g)*m^4 + ((47*c^2*d*e^5 + 214*b*c*e^6)*f - (30*c
^2*d^2*e^4 - 94*b*c*d*e^5 - 107*(b^2 + 2*a*c)*e^6)*g)*m^3 + (2*(36*c^2*d*e
^5 + 307*b*c*e^6)*f - (55*c^2*d^2*e^4 - 144*b*c*d*e^5 - 307*(b^2 + 2*a*c)*
e^6)*g)*m^2 + 6*(6*(c^2*d*e^5 + 22*b*c*e^6)*f - (5*c^2*d^2*e^4 - 12*b*c*d*
e^5 - 66*(b^2 + 2*a*c)*e^6)*g)*m)*x^4 - ((36*a*b*d^2*e^4 - 155*a^2*d*e^5 -
 2*(b^2 + 2*a*c)*d^3*e^3)*f - 2*(2*a*b*d^3*e^3 - 9*a^2*d^2*e^4)*g)*m^3 + (
480*a*b*e^6*g + 240*(b^2 + 2*a*c)*e^6*f + ((2*b*c*d*e^5 + (b^2 + 2*a*c)*e^
6)*f + (2*a*b*e^6 + (b^2 + 2*a*c)*d*e^5)*g)*m^5 - 2*((2*c^2*d^2*e^4 - 14*b
*c*d*e^5 - 9*(b^2 + 2*a*c)*e^6)*f + (4*b*c*d^2*e^4 - 18*a*b*e^6 - 7*(b^2 +
 2*a*c)*d*e^5)*g)*m^4 - ((36*c^2*d^2*e^4 - 130*b*c*d*e^5 - 121*(b^2 + 2*a*
c)*e^6)*f - (20*c^2*d^3*e^3 - 72*b*c*d^2*e^4 + 242*a*b*e^6 + 65*(b^2 + ...
```

3.926.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32864 vs. $2(309) = 618$.

Time = 5.65 (sec) , antiderivative size = 32864, normalized size of antiderivative = 105.67

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a)**2,x)
```

output
```
Piecewise((d**m*(a**2*f*x + a**2*g*x**2/2 + a*b*f*x**2 + 2*a*b*g*x**3/3 +
2*a*c*f*x**3/3 + a*c*g*x**4/2 + b**2*f*x**3/3 + b**2*g*x**4/4 + b*c*f*x**4
/2 + 2*b*c*g*x**5/5 + c**2*f*x**5/5 + c**2*g*x**6/6), Eq(e, 0)), (-3*a**2*
d*e**4*g/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e
**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 12*a**2*e**5*f/(60*d**5*e**
6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**1
0*x**4 + 60*e**11*x**5) - 15*a**2*e**5*g*x/(60*d**5*e**6 + 300*d**4*e**7*x
 + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x
**5) - 4*a*b*d**2*e**3*g/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x
**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 6*a*b*d*e**
4*f/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x
**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 20*a*b*d*e**4*g*x/(60*d**5*e**6
+ 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*
x**4 + 60*e**11*x**5) - 30*a*b*e**5*f*x/(60*d**5*e**6 + 300*d**4*e**7*x +
600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5
) - 40*a*b*e**5*g*x**2/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**
2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 6*a*c*d**3*e*
*2*g/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*
x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 4*a*c*d**2*e**3*f/(60*d**5*e**6
 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e*...
```

3.926.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1118 vs. $2(312) = 624$.

Time = 0.24 (sec) , antiderivative size = 1118, normalized size of antiderivative = 3.59

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
2*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a*b*f/((m^2 + 3*m + 2)*e^2
) + (e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a^2*g/((m^2 + 3*m + 2)*e
^2) + (e*x + d)^(m + 1)*a^2*f/(e*(m + 1)) + ((m^2 + 3*m + 2)*e^3*x^3 + (m^
2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*b^2*f/((m^3 + 6*m^2 +
11*m + 6)*e^3) + 2*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*
e*m*x + 2*d^3)*(e*x + d)^m*a*c*f/((m^3 + 6*m^2 + 11*m + 6)*e^3) + 2*((m^2
+ 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^
m*a*b*g/((m^3 + 6*m^2 + 11*m + 6)*e^3) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4*x
^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x
 - 6*d^4)*(e*x + d)^m*b*c*f/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + ((
m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 +
 m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*b^2*g/((m^4 + 10*m^3 +
35*m^2 + 50*m + 24)*e^4) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*
m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x
 + d)^m*a*c*g/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + ((m^4 + 10*m^3 +
 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*
(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x
+ 24*d^5)*(e*x + d)^m*c^2*f/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 12
0)*e^5) + 2*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 +
11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 +...
```

3.926.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4936 vs. $2(312) = 624$.

Time = 0.32 (sec) , antiderivative size = 4936, normalized size of antiderivative = 15.87

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
((e*x + d)^m*c^2*e^6*g*m^5*x^6 + (e*x + d)^m*c^2*e^6*f*m^5*x^5 + (e*x + d)
^m*c^2*d*e^5*g*m^5*x^5 + 2*(e*x + d)^m*b*c*e^6*g*m^5*x^5 + 15*(e*x + d)^m*
c^2*e^6*g*m^4*x^6 + (e*x + d)^m*c^2*d*e^5*f*m^5*x^4 + 2*(e*x + d)^m*b*c*e^
6*f*m^5*x^4 + 2*(e*x + d)^m*b*c*d*e^5*g*m^5*x^4 + (e*x + d)^m*b^2*e^6*g*m^
5*x^4 + 2*(e*x + d)^m*a*c*e^6*g*m^5*x^4 + 16*(e*x + d)^m*c^2*e^6*f*m^4*x^5
 + 10*(e*x + d)^m*c^2*d*e^5*g*m^4*x^5 + 32*(e*x + d)^m*b*c*e^6*g*m^4*x^5 +
 85*(e*x + d)^m*c^2*e^6*g*m^3*x^6 + 2*(e*x + d)^m*b*c*d*e^5*f*m^5*x^3 + (e
*x + d)^m*b^2*e^6*f*m^5*x^3 + 2*(e*x + d)^m*a*c*e^6*f*m^5*x^3 + (e*x + d)^
m*b^2*d*e^5*g*m^5*x^3 + 2*(e*x + d)^m*a*c*d*e^5*g*m^5*x^3 + 2*(e*x + d)^m*
a*b*e^6*g*m^5*x^3 + 12*(e*x + d)^m*c^2*d*e^5*f*m^4*x^4 + 34*(e*x + d)^m*b*
c*e^6*f*m^4*x^4 - 5*(e*x + d)^m*c^2*d^2*e^4*g*m^4*x^4 + 24*(e*x + d)^m*b*c
*d*e^5*g*m^4*x^4 + 17*(e*x + d)^m*b^2*e^6*g*m^4*x^4 + 34*(e*x + d)^m*a*c*e
^6*g*m^4*x^4 + 95*(e*x + d)^m*c^2*e^6*f*m^3*x^5 + 35*(e*x + d)^m*c^2*d*e^5
*g*m^3*x^5 + 190*(e*x + d)^m*b*c*e^6*g*m^3*x^5 + 225*(e*x + d)^m*c^2*e^6*g
*m^2*x^6 + (e*x + d)^m*b^2*d*e^5*f*m^5*x^2 + 2*(e*x + d)^m*a*c*d*e^5*f*m^5
*x^2 + 2*(e*x + d)^m*a*b*e^6*f*m^5*x^2 + 2*(e*x + d)^m*a*b*d*e^5*g*m^5*x^2
 + (e*x + d)^m*a^2*e^6*g*m^5*x^2 - 4*(e*x + d)^m*c^2*d^2*e^4*f*m^4*x^3 + 2
8*(e*x + d)^m*b*c*d*e^5*f*m^4*x^3 + 18*(e*x + d)^m*b^2*e^6*f*m^4*x^3 + 36*
(e*x + d)^m*a*c*e^6*f*m^4*x^3 - 8*(e*x + d)^m*b*c*d^2*e^4*g*m^4*x^3 + 14*(
e*x + d)^m*b^2*d*e^5*g*m^4*x^3 + 28*(e*x + d)^m*a*c*d*e^5*g*m^4*x^3 + 3...
```

3.926.9 Mupad [B] (verification not implemented)

Time = 13.68 (sec) , antiderivative size = 2307, normalized size of antiderivative = 7.42

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2)^2,x)
```

output
```
((d + e*x)^m*(240*b^2*d^3*e^3*f - 360*a^2*d^2*e^4*g - 120*c^2*d^6*g - 180*
b^2*d^4*e^2*g + 720*a^2*d*e^5*f + 144*c^2*d^5*e*f - 720*a*b*d^2*e^4*f + 48
0*a*b*d^3*e^3*g + 480*a*c*d^3*e^3*f - 360*a*c*d^4*e^2*g - 360*b*c*d^4*e^2*
f + 1044*a^2*d*e^5*f*m + 24*c^2*d^5*e*f*m + 580*a^2*d*e^5*f*m^2 + 155*a^2*
d*e^5*f*m^3 + 20*a^2*d*e^5*f*m^4 + a^2*d*e^5*f*m^5 - 342*a^2*d^2*e^4*g*m +
 148*b^2*d^3*e^3*f*m - 66*b^2*d^4*e^2*g*m + 288*b*c*d^5*e*g - 119*a^2*d^2*
e^4*g*m^2 + 30*b^2*d^3*e^3*f*m^2 - 18*a^2*d^2*e^4*g*m^3 + 2*b^2*d^3*e^3*f*
m^3 - a^2*d^2*e^4*g*m^4 - 6*b^2*d^4*e^2*g*m^2 + 48*b*c*d^5*e*g*m - 684*a*b
*d^2*e^4*f*m + 296*a*b*d^3*e^3*g*m + 296*a*c*d^3*e^3*f*m - 132*a*c*d^4*e^2
*g*m - 132*b*c*d^4*e^2*f*m - 238*a*b*d^2*e^4*f*m^2 - 36*a*b*d^2*e^4*f*m^3
- 2*a*b*d^2*e^4*f*m^4 + 60*a*b*d^3*e^3*g*m^2 + 60*a*c*d^3*e^3*f*m^2 + 4*a*
b*d^3*e^3*g*m^3 + 4*a*c*d^3*e^3*f*m^3 - 12*a*c*d^4*e^2*g*m^2 - 12*b*c*d^4*
e^2*f*m^2))/(e^6*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 7
20)) + (x*(d + e*x)^m*(720*a^2*e^6*f + 580*a^2*e^6*f*m^2 + 155*a^2*e^6*f*m
^3 + 20*a^2*e^6*f*m^4 + a^2*e^6*f*m^5 + 1044*a^2*e^6*f*m + 360*a^2*d*e^5*g
*m + 120*c^2*d^5*e*g*m - 240*b^2*d^2*e^4*f*m + 342*a^2*d*e^5*g*m^2 + 119*a
^2*d*e^5*g*m^3 + 18*a^2*d*e^5*g*m^4 + a^2*d*e^5*g*m^5 + 180*b^2*d^3*e^3*g*
m - 144*c^2*d^4*e^2*f*m - 148*b^2*d^2*e^4*f*m^2 - 30*b^2*d^2*e^4*f*m^3 - 2
*b^2*d^2*e^4*f*m^4 + 66*b^2*d^3*e^3*g*m^2 - 24*c^2*d^4*e^2*f*m^2 + 6*b^2*d
^3*e^3*g*m^3 + 720*a*b*d*e^5*f*m + 684*a*b*d*e^5*f*m^2 + 238*a*b*d*e^5*...
```

3.926. $\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2\,dx$

3.927 $\int \frac{(d+ex)^m \left(a+bx+cx^2\right)^2}{f+gx} \, dx$

3.927.1 Optimal result

Integrand size = 27, antiderivative size = 287

$$\begin{aligned}\int \frac{(d+ex)^m \left(a+bx+cx^2\right)^2}{f+gx} \, dx &= \frac{(beg-c(ef+dg)) \left(c\left(e^2f^2+d^2g^2\right)+eg(2aeg-b(ef+dg))\right)(d+ex)^{1+m}}{e^4g^4(1+m)} \\ &+ \frac{\left(b^2e^2g^2+c^2\left(e^2f^2+2defg+3d^2g^2\right)+2ceg(aeg-b(ef+2dg))\right)(d+ex)^{2+m}}{e^4g^3(2+m)} \\ &- \frac{c(cef+3cdg-2beg)(d+ex)^{3+m}}{e^4g^2(3+m)} + \frac{c^2(d+ex)^{4+m}}{e^4g(4+m)} \\ &+ \frac{\left(cf^2-bfg+ag^2\right)^2 (d+ex)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{g(d+ex)}{ef-dg}\right)}{g^4(ef-dg)(1+m)}\end{aligned}$$

output
```
(b*e*g-c*(d*g+e*f))*(c*(d^2*g^2+e^2*f^2)+e*g*(2*a*e*g-b*(d*g+e*f)))*(e*x+d
)^(1+m)/e^4/g^4/(1+m)+(b^2*e^2*g^2+c^2*(3*d^2*g^2+2*d*e*f*g+e^2*f^2)+2*c*e
*g*(a*e*g-b*(2*d*g+e*f)))*(e*x+d)^(2+m)/e^4/g^3/(2+m)-c*(-2*b*e*g+3*c*d*g+
c*e*f)*(e*x+d)^(3+m)/e^4/g^2/(3+m)+c^2*(e*x+d)^(4+m)/e^4/g/(4+m)+(a*g^2-b*
f*g+c*f^2)^2*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],-g*(e*x+d)/(-d*g+e*f))
/g^4/(-d*g+e*f)/(1+m)
```

3.927.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{f+gx}\,dx$$
$$= \frac{(d+ex)^{1+m}\left(-\frac{(cef+cdg-beg)(c(e^2f^2+d^2g^2)+eg(2aeg-b(ef+dg)))}{e^4(1+m)}+\frac{g(b^2e^2g^2+c^2(e^2f^2+2defg+3d^2g^2)+2ceg(aeg-b(ef+2dg)))(d}{e^4(2+m)}\right.}{g^4}$$

input
```
Integrate[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x),x]
```

output
```
((d + e*x)^(1 + m)*(-(((c*e*f + c*d*g - b*e*g)*(c*(e^2*f^2 + d^2*g^2) + e*
g*(2*a*e*g - b*(e*f + d*g))))/(e^4*(1 + m))) + (g*(b^2*e^2*g^2 + c^2*(e^2*
f^2 + 2*d*e*f*g + 3*d^2*g^2) + 2*c*e*g*(a*e*g - b*(e*f + 2*d*g)))*(d + e*x
))/(e^4*(2 + m)) - (c*g^2*(c*e*f + 3*c*d*g - 2*b*e*g)*(d + e*x)^2)/(e^4*(3
 + m)) + (c^2*g^3*(d + e*x)^3)/(e^4*(4 + m)) + ((c*f^2 + g*(-(b*f) + a*g))
^2*Hypergeometric2F1[1, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)])/((e*f
 - d*g)*(1 + m))))/g^4
```

3.927.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^2 (d+ex)^m}{f+gx}\,dx$$
$$\Big\downarrow \text{1195}$$
$$\int \left(\frac{(d+ex)^{m+1}\left(2ceg(aeg-b(2dg+ef))+b^2e^2g^2+c^2\left(3d^2g^2+2defg+e^2f^2\right)\right)}{e^3g^3}+\frac{(d+ex)^m(beg-c(dg+ef)}{}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^{m+2}\left(2ceg(aeg-b(2dg+ef))+b^2e^2g^2+c^2\left(3d^2g^2+2defg+e^2f^2\right)\right)}{e^4g^3(m+2)}+\frac{(d+ex)^{m+1}(beg-c(dg+ef))\left(eg(2aeg-b(dg+ef))+c\left(d^2g^2+e^2f^2\right)\right)}{e^4g^4(m+1)}+\frac{(d+ex)^{m+1}\left(ag^2-bfg+cf^2\right)^2\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{g(d+ex)}{ef-dg}\right)}{g^4(m+1)(ef-dg)}-\frac{c(d+ex)^{m+3}(-2beg+3cdg+cef)}{e^4g^2(m+3)}+\frac{c^2(d+ex)^{m+4}}{e^4g(m+4)}$$

input
```
Int[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x),x]
```

output
```
((b*e*g - c*(e*f + d*g))*(c*(e^2*f^2 + d^2*g^2) + e*g*(2*a*e*g - b*(e*f +
d*g)))*(d + e*x)^(1 + m))/(e^4*g^4*(1 + m)) + ((b^2*e^2*g^2 + c^2*(e^2*f^2
 + 2*d*e*f*g + 3*d^2*g^2) + 2*c*e*g*(a*e*g - b*(e*f + 2*d*g)))*(d + e*x)^(
2 + m))/(e^4*g^3*(2 + m)) - (c*(c*e*f + 3*c*d*g - 2*b*e*g)*(d + e*x)^(3 +
m))/(e^4*g^2*(3 + m)) + (c^2*(d + e*x)^(4 + m))/(e^4*g*(4 + m)) + ((c*f^2
- b*f*g + a*g^2)^2*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -(
(g*(d + e*x))/(e*f - d*g))])/(g^4*(e*f - d*g)*(1 + m))
```

3.927.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.927.4 Maple [F]

$$\int \frac{(ex+d)^m\left(c\,x^2+bx+a\right)^2}{gx+f}dx$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x)
```

output
```
int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x)
```

3.927.5 Fricas [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{f+gx}\,dx = \int \frac{(cx^2+bx+a)^2(ex+d)^m}{gx+f}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x, algorithm="fricas")
```

output
```
integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*(e*x +
d)^m/(g*x + f), x)
```

3.927.6 Sympy [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{f+gx}\,dx = \int \frac{(d+ex)^m (a+bx+cx^2)^2}{f+gx}\,dx$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x+a)**2/(g*x+f),x)
```

output
```
Integral((d + e*x)**m*(a + b*x + c*x**2)**2/(f + g*x), x)
```

3.927.7 Maxima [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{f+gx}\,dx = \int \frac{(cx^2+bx+a)^2(ex+d)^m}{gx+f}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f), x)
```

3.927.8 Giac [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{f+gx} \, dx = \int \frac{(cx^2+bx+a)^2 (ex+d)^m}{gx+f} \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f), x)
```

3.927.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{f+gx} \, dx = \int \frac{(d+e\,x)^m \,(c\,x^2+b\,x+a)^2}{f+g\,x} \, dx$$

input
```
int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x),x)
```

output
```
int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x), x)
```

3.928 $\int \frac{(d+ex)^m(a+bx+cx^2)^2}{(f+gx)^2}\,dx$

3.928.1 Optimal result

Integrand size = 27, antiderivative size = 298

$$\int \frac{(d+ex)^m(a+bx+cx^2)^2}{(f+gx)^2}\,dx = \frac{(b^2e^2g^2+c^2(3e^2f^2+2defg+d^2g^2)+2ceg(aeg-b(2ef+dg)))\,(d+ex)^{1+m}}{e^3g^4(1+m)} - \frac{2c(cef+cdg-beg)(d+ex)^{2+m}}{e^3g^3(2+m)} + \frac{c^2(d+ex)^{3+m}}{e^3g^2(3+m)} + \frac{(cf^2-bfg+ag^2)^2\,(d+ex)^{1+m}}{g^4(ef-dg)(f+gx)} + \frac{(cf^2-bfg+ag^2)\,(cf(4dg-ef(4+m))-g(aegm+b(2dg-ef(2+m))))(d+ex)^{1+m}\,\text{Hypergeometr}}{g^4(ef-dg)^2(1+m)}$$

output
```
(b^2*e^2*g^2+c^2*(d^2*g^2+2*d*e*f*g+3*e^2*f^2)+2*c*e*g*(a*e*g-b*(d*g+2*e*f
)))*(e*x+d)^(1+m)/e^3/g^4/(1+m)-2*c*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(2+m)/e^3
/g^3/(2+m)+c^2*(e*x+d)^(3+m)/e^3/g^2/(3+m)+(a*g^2-b*f*g+c*f^2)^2*(e*x+d)^(
1+m)/g^4/(-d*g+e*f)/(g*x+f)+(a*g^2-b*f*g+c*f^2)*(c*f*(4*d*g-e*f*(4+m))-g*(
a*e*g*m+b*(2*d*g-e*f*(2+m))))*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],-g*(e
*x+d)/(-d*g+e*f))/g^4/(-d*g+e*f)^2/(1+m)
```

3.928.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^2}\,dx$$
$$= \frac{(d+ex)^{1+m}\left(\frac{b^2e^2g^2+c^2(3e^2f^2+2defg+d^2g^2)+2ceg(aeg-b(2ef+dg))}{e^3(1+m)} - \frac{2cg(cef+cdg-beg)(d+ex)}{e^3(2+m)} + \frac{c^2g^2(d+ex)^2}{e^3(3+m)} - \frac{2(2cf-bg)(cf}{}\right.}{g^4}$$

input
```
Integrate[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^2,x]
```

output
```
((d + e*x)^(1 + m)*((b^2*e^2*g^2 + c^2*(3*e^2*f^2 + 2*d*e*f*g + d^2*g^2) +
 2*c*e*g*(a*e*g - b*(2*e*f + d*g)))/(e^3*(1 + m)) - (2*c*g*(c*e*f + c*d*g
- b*e*g)*(d + e*x))/(e^3*(2 + m)) + (c^2*g^2*(d + e*x)^2)/(e^3*(3 + m)) -
(2*(2*c*f - b*g)*(c*f^2 + g*(-(b*f) + a*g))*Hypergeometric2F1[1, 1 + m, 2
+ m, (g*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g)*(1 + m)) + (e*(c*f^2 + g*
(-(b*f) + a*g))^2*Hypergeometric2F1[2, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f)
 + d*g)])/((e*f - d*g)^2*(1 + m))))/g^4
```

3.928.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1193, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^2 (d+ex)^m}{(f+gx)^2}\,dx$$
$$\Big\downarrow \text{1193}$$
$$\frac{\int \frac{(d+ex)^m\left(-c^2\left(d-\frac{ef}{g}\right)x^3 - \frac{c(cf-2bg)(ef-dg)x^2}{g^2} + \frac{(ef-dg)\left(c^2f^2+b^2g^2-2cg(bf-ag)\right)x}{g^3} + \frac{c^2(dg-ef(m+1))f^3-2cg(bf-ag)(dg-ef(m+1))f-g^2\left(-f(dg-e\right.}{g^4}\right.}{f+gx}}{ef-dg}$$
$$\frac{(d+ex)^{m+1}\left(ag^2-bfg+cf^2\right)^2}{g^4(f+gx)(ef-dg)}$$
$$\Big\downarrow \text{2123}$$

$$\frac{\int \left(\frac{(ef-dg)((3e^2f^2+2degf+d^2g^2)c^2+2eg(aeg-b(2ef+dg))c+b^2e^2g^2)(d+ex)^m}{e^2g^4}+\frac{(cf^2-bgf+ag^2)(cf(4dg-ef(m+4))-g(2bdg+aemg-bef(r}{g^4(f+gx)}\right.}{ef-dg}$$

$$\frac{(d+ex)^{m+1}\left(ag^2-bfg+cf^2\right)^2}{g^4(f+gx)(ef-dg)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{(ef-dg)(d+ex)^{m+1}(2ceg(aeg-b(dg+2ef))+b^2e^2g^2+c^2(d^2g^2+2defg+3e^2f^2))}{e^3g^4(m+1)}-\frac{(d+ex)^{m+1}(ag^2-bfg+cf^2)\text{ Hypergeometric2F1}\left(1,m+1,m+\right.}{g^4(m+1}}{ef-dg}$$

$$\frac{(d+ex)^{m+1}\left(ag^2-bfg+cf^2\right)^2}{g^4(f+gx)(ef-dg)}$$

input
```
Int[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^2,x]
```

output
```
((c*f^2 - b*f*g + a*g^2)^2*(d + e*x)^(1 + m))/(g^4*(e*f - d*g)*(f + g*x))
+ (((e*f - d*g)*(b^2*e^2*g^2 + c^2*(3*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*c
*e*g*(a*e*g - b*(2*e*f + d*g)))*(d + e*x)^(1 + m))/(e^3*g^4*(1 + m)) - (2*
c*(e*f - d*g)*(c*e*f + c*d*g - b*e*g)*(d + e*x)^(2 + m))/(e^3*g^3*(2 + m))
 + (c^2*(e*f - d*g)*(d + e*x)^(3 + m))/(e^3*g^2*(3 + m)) - ((c*f^2 - b*f*g
 + a*g^2)*(g*(2*b*d*g + a*e*g*m - b*e*f*(2 + m)) - c*f*(4*d*g - e*f*(4 + m
)))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g*(d + e*x))/(
e*f - d*g))])/(g^4*(e*f - d*g)*(1 + m)))/(e*f - d*g)
```

3.928.3.1 Defintions of rubi rules used

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.928. $\int \frac{(d+ex)^m(a+bx+cx^2)^2}{(f+gx)^2}\,dx$

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

3.928.4 Maple [F]

$$\int \frac{(ex+d)^m (c x^2+bx+a)^2}{(gx+f)^2} dx$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x)
```

output
```
int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x)
```

3.928.5 Fricas [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^2} dx = \int \frac{(cx^2+bx+a)^2 (ex+d)^m}{(gx+f)^2} dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x, algorithm="fricas")
```

output
```
integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*(e*x +
d)^m/(g^2*x^2 + 2*f*g*x + f^2), x)
```

3.928.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^2} dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x+a)**2/(g*x+f)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.928.7 Maxima [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^2} dx = \int \frac{(cx^2+bx+a)^2 (ex+d)^m}{(gx+f)^2} dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^2, x)
```

3.928.8 Giac [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^2} dx = \int \frac{(cx^2+bx+a)^2 (ex+d)^m}{(gx+f)^2} dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^2, x)
```

3.928.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^2} dx = \int \frac{(d+e\,x)^m\,(c\,x^2+b\,x+a)^2}{(f+g\,x)^2} dx$$

input
```
int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^2,x)
```

output
```
int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^2, x)
```

3.929 $\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^2}{(f+gx)^3} \, dx$

3.929.1 Optimal result

Integrand size = 27, antiderivative size = 461

$$\begin{aligned}&\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^2}{(f+gx)^3} \, dx\\&=-\frac{c(3cef+cdg-2beg)(d+ex)^{1+m}}{e^2g^4(1+m)}+\frac{c^2(d+ex)^{2+m}}{e^2g^3(2+m)}+\frac{\left(cf^2-bfg+ag^2\right)^2(d+ex)^{1+m}}{2g^4(ef-dg)(f+gx)^2}\\&\quad+\frac{\left(cf^2-bfg+ag^2\right)(cf(8dg-ef(7+m))+g(aeg(1-m)-b(4dg-ef(3+m))))(d+ex)^{1+m}}{2g^4(ef-dg)^2(f+gx)}\\&\quad+\frac{\left(c^2f^2\left(12d^2g^2-8defg(3+m)+e^2f^2\left(12+7m+m^2\right)\right)-g^2\left(a^2e^2g^2(1-m)m-2abegm(2dg-ef(1+\right.\right.}{}\end{aligned}$$

output
```
-c*(-2*b*e*g+c*d*g+3*c*e*f)*(e*x+d)^(1+m)/e^2/g^4/(1+m)+c^2*(e*x+d)^(2+m)/
e^2/g^3/(2+m)+1/2*(a*g^2-b*f*g+c*f^2)^2*(e*x+d)^(1+m)/g^4/(-d*g+e*f)/(g*x+
f)^2+1/2*(a*g^2-b*f*g+c*f^2)*(c*f*(8*d*g-e*f*(7+m))+g*(a*e*g*(1-m)-b*(4*d*
g-e*f*(3+m))))*(e*x+d)^(1+m)/g^4/(-d*g+e*f)^2/(g*x+f)+1/2*(c^2*f^2*(12*d^2
*g^2-8*d*e*f*g*(3+m)+e^2*f^2*(m^2+7*m+12))-g^2*(a^2*e^2*g^2*(1-m)*m-2*a*b*
e*g*m*(2*d*g-e*f*(1+m))-b^2*(2*d^2*g^2-4*d*e*f*g*(1+m)+e^2*f^2*(m^2+3*m+2)
))+2*c*g*(a*g*(2*d^2*g^2-4*d*e*f*g*(1+m)+e^2*f^2*(m^2+3*m+2))-b*f*(6*d^2*g
^2-6*d*e*f*g*(2+m)+e^2*f^2*(m^2+5*m+6))))*(e*x+d)^(1+m)*hypergeom([1, 1+m]
,[2+m],-g*(e*x+d)/(-d*g+e*f))/g^4/(-d*g+e*f)^3/(1+m)
```

3.929.2 Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.56

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^3}\,dx$$
$$= \frac{(d+ex)^{1+m}\left(-\frac{c(3cef+cdg-2beg)}{e^2(1+m)}+\frac{c^2g(d+ex)}{e^2(2+m)}+\frac{(6c^2f^2+b^2g^2+2cg(-3bf+ag))\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{g(d+ex)}{-ef+dg}\right)}{(ef-dg)(1+m)}-\frac{2e}{}\right.}{}$$

input
```
Integrate[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^3,x]
```

output
```
((d + e*x)^(1 + m)*(-((c*(3*c*e*f + c*d*g - 2*b*e*g))/(e^2*(1 + m))) + (c^
2*g*(d + e*x))/(e^2*(2 + m)) + ((6*c^2*f^2 + b^2*g^2 + 2*c*g*(-3*b*f + a*g
))*Hypergeometric2F1[1, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)])/((e*f
 - d*g)*(1 + m)) - (2*e*(2*c*f - b*g)*(c*f^2 + g*(-(b*f) + a*g))*Hypergeom
etric2F1[2, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g)^2*(1
 + m)) + (e^2*(c*f^2 + g*(-(b*f) + a*g))^2*Hypergeometric2F1[3, 1 + m, 2 +
 m, (g*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g)^3*(1 + m))))/g^4
```

3.929.3 Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1193, 2124, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^2 (d+ex)^m}{(f+gx)^3}\,dx$$

$\big\downarrow$ 1193

$$\frac{\int \frac{(d+ex)^m\left(-2c^2\left(d-\frac{ef}{g}\right)x^3-\frac{2c(cf-2bg)(ef-dg)x^2}{g^2}+\frac{2(ef-dg)\left(c^2f^2+b^2g^2-2cg(bf-ag)\right)x}{g^3}+\frac{c^2(2dg-ef(m+1))f^3-2cg(bf-ag)(2dg-ef(m+1))f+g^2\left(f(2\right.}{g^4}\right.}{(f+gx)^2}}{2(ef-dg)}$$

$$\frac{(d+ex)^{m+1}\left(ag^2-bfg+cf^2\right)^2}{2g^4(f+gx)^2(ef-dg)}$$

$\big\downarrow$ 2124

$$\int \frac{\frac{(d+ex)^m\left(\frac{2c^2x^2(ef-dg)^2}{g^2}-\frac{4c(cf-bg)x(ef-dg)^2}{g^3}+\frac{c^2\left(e^2\left(m^2+7m+6\right)f^2-4deg(2m+3)f+6d^2g^2\right)f^2-g^2\left(-\left(\left(e^2\left(m^2+3m+2\right)f^2-4deg(m+1)f+2d^2g^2\right)b^2\right)-\right.}{\ldots}}{f+gx}}{ef-dg}$$

$$\frac{(d+ex)^{m+1}\left(ag^2-bfg+cf^2\right)^2}{2g^4(f+gx)^2(ef-dg)}$$

$$\Big\downarrow \; 1195$$

$$\int\left(-\frac{2c(ef-dg)^2(3cef+cdg-2beg)(d+ex)^m}{eg^4}+\frac{\left(c^2\left(e^2\left(m^2+7m+12\right)f^2-8deg(m+3)f+12d^2g^2\right)f^2-g^2\left(-\left(\left(e^2\left(m^2+3m+2\right)f^2-4deg(m+1)f+2d^2g^2\right)b^2\right)-2aegm\ldots\right.\right.}{\ldots}\right.$$

$$\frac{(d+ex)^{m+1}\left(ag^2-bfg+cf^2\right)^2}{2g^4(f+gx)^2(ef-dg)}$$

$$\Big\downarrow \; 2009$$

$$\frac{(d+ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{g(d+ex)}{ef-dg}\right)\left(-g^2\left(a^2e^2g^2(1-m)m-2abegm(2dg-ef(m+1))-\left(b^2\left(2d^2g^2-4defg(m+1)+e^2f^2\left(m^2+3m+2\right)\right)\right)\right)+2c\ldots\right.}{g^4(m+1)(ef-\ldots}$$

$$\frac{(d+ex)^{m+1}\left(ag^2-bfg+cf^2\right)^2}{2g^4(f+gx)^2(ef-dg)}$$

input

```
Int[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^3,x]
```

output

```
((c*f^2 - b*f*g + a*g^2)^2*(d + e*x)^(1 + m))/(2*g^4*(e*f - d*g)*(f + g*x)
^2) + (-(((c*f^2 - b*f*g + a*g^2)*(g*(4*b*d*g - a*e*g*(1 - m) - b*e*f*(3 +
 m)) - c*f*(8*d*g - e*f*(7 + m)))*(d + e*x)^(1 + m))/(g^4*(e*f - d*g)*(f +
 g*x))) + ((-2*c*(e*f - d*g)^2*(3*c*e*f + c*d*g - 2*b*e*g)*(d + e*x)^(1 +
m))/(e^2*g^4*(1 + m)) + (2*c^2*(e*f - d*g)^2*(d + e*x)^(2 + m))/(e^2*g^3*(
2 + m)) + ((c^2*f^2*(12*d^2*g^2 - 8*d*e*f*g*(3 + m) + e^2*f^2*(12 + 7*m +
m^2)) - g^2*(a^2*e^2*g^2*(1 - m)*m - 2*a*b*e*g*m*(2*d*g - e*f*(1 + m)) - b
^2*(2*d^2*g^2 - 4*d*e*f*g*(1 + m) + e^2*f^2*(2 + 3*m + m^2))) + 2*c*g*(a*g
*(2*d^2*g^2 - 4*d*e*f*g*(1 + m) + e^2*f^2*(2 + 3*m + m^2)) - b*f*(6*d^2*g^
2 - 6*d*e*f*g*(2 + m) + e^2*f^2*(6 + 5*m + m^2))))*(d + e*x)^(1 + m)*Hyper
geometric2F1[1, 1 + m, 2 + m, -((g*(d + e*x))/(e*f - d*g))])/(g^4*(e*f - d
*g)*(1 + m)))/(e*f - d*g))/(2*(e*f - d*g))
```

3.929. $\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^2}{(f+gx)^3}\,dx$

3.929.3.1 Defintions of rubi rules used

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

3.929.4 Maple [F]

$$\int \frac{(ex+d)^m \left(c\,x^2+bx+a\right)^2}{(gx+f)^3}dx$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x)
```

output
```
int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x)
```

3.929.5 Fricas [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^3} dx = \int \frac{(cx^2+bx+a)^2 (ex+d)^m}{(gx+f)^3} dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x, algorithm="fricas")
```

output
```
integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*(e*x +
d)^m/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

3.929.6 Sympy [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^3} dx = \int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^3} dx$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x+a)**2/(g*x+f)**3,x)
```

output
```
Integral((d + e*x)**m*(a + b*x + c*x**2)**2/(f + g*x)**3, x)
```

3.929.7 Maxima [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^3} dx = \int \frac{(cx^2+bx+a)^2 (ex+d)^m}{(gx+f)^3} dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^3, x)
```

3.929.8 Giac [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^3}\,dx = \int \frac{(cx^2+bx+a)^2(ex+d)^m}{(gx+f)^3}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^3, x)
```

3.929.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^3}\,dx = \int \frac{(d+e\,x)^m\,(c\,x^2+b\,x+a)^2}{(f+g\,x)^3}\,dx$$

input
```
int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^3,x)
```

output
```
int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^3, x)
```

3.930 $\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2}\,dx$

3.930.1 Optimal result

Integrand size = 27, antiderivative size = 183

$$\begin{aligned}\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2}\,dx &= \frac{3687(1+4x)^{1+m}}{64(1+m)}+\frac{207(1+4x)^{2+m}}{32(2+m)}+\frac{27(1+4x)^{3+m}}{64(3+m)}\\ &-\frac{3\left(5499-1631\sqrt{13}\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(1+m)}\\ &-\frac{3\left(5499+1631\sqrt{13}\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{26\left(13+2\sqrt{13}\right)(1+m)}\end{aligned}$$

output
```
3687/64*(1+4*x)^(1+m)/(1+m)+207/32*(1+4*x)^(2+m)/(2+m)+27/64*(1+4*x)^(3+m)
/(3+m)-3/26*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2
)))*(5499-1631*13^(1/2))/(1+m)/(13-2*13^(1/2))-3/26*(1+4*x)^(1+m)*hypergeo
m([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(5499+1631*13^(1/2))/(1+m)/(13
+2*13^(1/2))
```

3.930.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.83

$$\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2}\,dx = \frac{3}{832}(1+4x)^{1+m}\left(\frac{15977}{1+m}+\frac{1794(1+4x)}{2+m}+\frac{117(1+4x)^2}{3+m} - \frac{32(-5499+1631\sqrt{13})\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13-2\sqrt{13}}\right)}{(-13+2\sqrt{13})\,(1+m)} - \frac{32(5499+1631\sqrt{13})\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13+2\sqrt{13}}\right)}{(13+2\sqrt{13})\,(1+m)}\right)$$

input
```
Integrate[((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

output
```
(3*(1 + 4*x)^(1 + m)*(15977/(1 + m) + (1794*(1 + 4*x))/(2 + m) + (117*(1 +
 4*x)^2)/(3 + m) - (32*(-5499 + 1631*Sqrt[13])*Hypergeometric2F1[1, 1 + m,
 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/((-13 + 2*Sqrt[13])*(1 + m)) - (32*
(5499 + 1631*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 +
 2*Sqrt[13])])/((13 + 2*Sqrt[13])*(1 + m))))/832
```

3.930.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x+2)^4(4x+1)^m}{3x^2-5x+1}\,dx$$

$$\Big\downarrow \; 1200$$

$$\int \left(\frac{\left(1269+\frac{4893}{\sqrt{13}}\right)(4x+1)^m}{6x-\sqrt{13}-5}+\frac{\left(1269-\frac{4893}{\sqrt{13}}\right)(4x+1)^m}{6x+\sqrt{13}-5}+\frac{3687}{16}(4x+1)^m+\frac{207}{8}(4x+1)^{m+1}+\frac{27}{16}(4x+1)^{m+}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\left(5499-1631\sqrt{13}\right)(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(5499+1631\sqrt{13}\right)(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{26\left(13+2\sqrt{13}\right)(m+1)}+\frac{3687(4x+1)^{m+1}}{64(m+1)}+\frac{207(4x+1)^{m+2}}{32(m+2)}+\frac{27(4x+1)^{m+3}}{64(m+3)}$$

input
```
Int[((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

output
```
(3687*(1 + 4*x)^(1 + m))/(64*(1 + m)) + (207*(1 + 4*x)^(2 + m))/(32*(2 + m
)) + (27*(1 + 4*x)^(3 + m))/(64*(3 + m)) - (3*(5499 - 1631*Sqrt[13])*(1 +
4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt
[13])])/(26*(13 - 2*Sqrt[13])*(1 + m)) - (3*(5499 + 1631*Sqrt[13])*(1 + 4*
x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[1
3])])/(26*(13 + 2*Sqrt[13])*(1 + m))
```

3.930.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.930.4 Maple [F]

$$\int \frac{(2+3x)^4 (1+4x)^m}{3x^2-5x+1}dx$$

input
```
int((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x)
```

output
```
int((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x)
```

3.930.5 Fricas [F]

$$\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^4}{3\,x^2-5\,x+1}\,dx$$

input
```
integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
```

output
```
integral((81*x^4 + 216*x^3 + 216*x^2 + 96*x + 16)*(4*x + 1)^m/(3*x^2 - 5*x
 + 1), x)
```

3.930.6 Sympy [F]

$$\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(3x+2)^4\,(4x+1)^m}{3x^2-5x+1}\,dx$$

input
```
integrate((2+3*x)**4*(1+4*x)**m/(3*x**2-5*x+1),x)
```

output
```
Integral((3*x + 2)**4*(4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

3.930.7 Maxima [F]

$$\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^4}{3\,x^2-5\,x+1}\,dx$$

input
```
integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
```

output
```
integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1), x)
```

3.930.8 Giac [F]

$$\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^4}{3\,x^2-5\,x+1}\,dx$$

input
```
integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
```

output
```
integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1), x)
```

3.930.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(3\,x+2)^4\,(4\,x+1)^m}{3\,x^2-5\,x+1}\,dx$$

input
```
int(((3*x + 2)^4*(4*x + 1)^m)/(3*x^2 - 5*x + 1),x)
```

output
```
int(((3*x + 2)^4*(4*x + 1)^m)/(3*x^2 - 5*x + 1), x)
```

3.931 $\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2} dx$

3.931.1 Optimal result

Integrand size = 27, antiderivative size = 165

$$\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2} dx = \frac{123(1+4x)^{1+m}}{16(1+m)} + \frac{9(1+4x)^{2+m}}{16(2+m)} - \frac{3(416-135\sqrt{13})(1+4x)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{13\left(13-2\sqrt{13}\right)(1+m)} - \frac{3(416+135\sqrt{13})(1+4x)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{13\left(13+2\sqrt{13}\right)(1+m)}$$

output
```
123/16*(1+4*x)^(1+m)/(1+m)+9/16*(1+4*x)^(2+m)/(2+m)-3/13*(1+4*x)^(1+m)*hyp
ergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(416-135*13^(1/2))/(1+m)/
(13-2*13^(1/2))-3/13*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+
2*13^(1/2)))*(416+135*13^(1/2))/(1+m)/(13+2*13^(1/2))
```

3.931.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.71

$$\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2}\,dx = \frac{(1+4x)^{1+m}\left(117(85+12x+4m(11+3x))+16\left(-146+71\sqrt{13}\right)(2+m)\,\text{Hypergeometric2F1}\left(1,1+m\right.\right.}{624\,(2+3m}$$

input
```
Integrate[((2 + 3*x)^3*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

output
```
((1 + 4*x)^(1 + m)*(117*(85 + 12*x + 4*m*(11 + 3*x)) + 16*(-146 + 71*Sqrt[
13])*(2 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13
])] - 16*(146 + 71*Sqrt[13])*(2 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3
 + 12*x)/(13 + 2*Sqrt[13])]))/(624*(2 + 3*m + m^2))
```

3.931.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x+2)^3(4x+1)^m}{3x^2-5x+1}\,dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(\frac{\left(192+\frac{810}{\sqrt{13}}\right)(4x+1)^m}{6x-\sqrt{13}-5}+\frac{\left(192-\frac{810}{\sqrt{13}}\right)(4x+1)^m}{6x+\sqrt{13}-5}+\frac{123}{4}(4x+1)^m+\frac{9}{4}(4x+1)^{m+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\left(416-135\sqrt{13}\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(416+135\sqrt{13}\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{13\left(13+2\sqrt{13}\right)(m+1)}+\frac{123(4x+1)^{m+1}}{16(m+1)}+\frac{9(4x+1)^{m+2}}{16(m+2)}$$

input
```
Int[((2 + 3*x)^3*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

output
```
(123*(1 + 4*x)^(1 + m))/(16*(1 + m)) + (9*(1 + 4*x)^(2 + m))/(16*(2 + m))
- (3*(416 - 135*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2
+ m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(13*(13 - 2*Sqrt[13])*(1 + m)) - (3
*(416 + 135*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
 (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(13*(13 + 2*Sqrt[13])*(1 + m))
```

3.931.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.931.4 Maple [F]

$$\int \frac{(2+3x)^3 (1+4x)^m}{3x^2-5x+1} dx$$

input
```
int((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1),x)
```

output
```
int((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1),x)
```

3.931.5 Fricas [F]

$$\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2} dx = \int \frac{(4x+1)^m(3x+2)^3}{3x^2-5x+1} dx$$

input
```
integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
```

output
```
integral((27*x^3 + 54*x^2 + 36*x + 8)*(4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

3.931.6 Sympy [F]

$$\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{\left(3x+2\right)^3\left(4x+1\right)^m}{3x^2-5x+1}\,dx$$

input
```
integrate((2+3*x)**3*(1+4*x)**m/(3*x**2-5*x+1),x)
```

output
```
Integral((3*x + 2)**3*(4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

3.931.7 Maxima [F]

$$\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^3}{3\,x^2-5\,x+1}\,dx$$

input
```
integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
```

output
```
integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1), x)
```

3.931.8 Giac [F]

$$\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^3}{3\,x^2-5\,x+1}\,dx$$

input
```
integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
```

output
```
integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1), x)
```

3.931.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(3\,x+2)^3\,(4\,x+1)^m}{3\,x^2-5\,x+1}\,dx$$

input
```
int(((3*x + 2)^3*(4*x + 1)^m)/(3*x^2 - 5*x + 1),x)
```

output
```
int(((3*x + 2)^3*(4*x + 1)^m)/(3*x^2 - 5*x + 1), x)
```

3.932 $\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2} dx$

3.932.1 Optimal result

Integrand size = 27, antiderivative size = 147

$$\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2} dx = \frac{3(1+4x)^{1+m}}{4(1+m)} - \frac{3(117-47\sqrt{13})(1+4x)^{1+m}\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{26(13-2\sqrt{13})(1+m)} - \frac{3(117+47\sqrt{13})(1+4x)^{1+m}\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{26(13+2\sqrt{13})(1+m)}$$

output
```
3/4*(1+4*x)^(1+m)/(1+m)-3/26*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4
*x)/(13-2*13^(1/2)))*(117-47*13^(1/2))/(1+m)/(13-2*13^(1/2))-3/26*(1+4*x)^
(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(117+47*13^(1/2)
)/(1+m)/(13+2*13^(1/2))
```

3.932.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.62

$$\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2}\,dx$$
$$= \frac{(1+4x)^{1+m}\left(117+\left(-46+58\sqrt{13}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13-2\sqrt{13}}\right)-2\left(23+29\sqrt{13}\right)\text{H}\right.}{156(1+m)}$$

input
```
Integrate[((2 + 3*x)^2*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

output
```
((1 + 4*x)^(1 + m)*(117 + (-46 + 58*Sqrt[13])*Hypergeometric2F1[1, 1 + m,
2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])] - 2*(23 + 29*Sqrt[13])*Hypergeometric
2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])]))/(156*(1 + m))
```

3.932.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x+2)^2(4x+1)^m}{3x^2-5x+1}\,dx$$
$$\Big\downarrow \text{1200}$$
$$\int \left(\frac{\left(27+\frac{141}{\sqrt{13}}\right)(4x+1)^m}{6x-\sqrt{13}-5}+\frac{\left(27-\frac{141}{\sqrt{13}}\right)(4x+1)^m}{6x+\sqrt{13}-5}+3(4x+1)^m\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{3\left(117-47\sqrt{13}\right)(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(117+47\sqrt{13}\right)(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{26\left(13+2\sqrt{13}\right)(m+1)}+\frac{3(4x+1)^{m+1}}{4(m+1)}$$

input
```
Int[((2 + 3*x)^2*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

output
```
(3*(1 + 4*x)^(1 + m))/(4*(1 + m)) - (3*(117 - 47*Sqrt[13])*(1 + 4*x)^(1 +
m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(2
6*(13 - 2*Sqrt[13])*(1 + m)) - (3*(117 + 47*Sqrt[13])*(1 + 4*x)^(1 + m)*Hy
pergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(26*(13
 + 2*Sqrt[13])*(1 + m))
```

3.932.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.932.4 Maple [F]

$$\int \frac{(2+3x)^2 (1+4x)^m}{3x^2-5x+1} dx$$

input
```
int((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1),x)
```

output
```
int((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1),x)
```

3.932.5 Fricas [F]

$$\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2} dx = \int \frac{(4\,x+1)^m(3\,x+2)^2}{3\,x^2-5\,x+1} dx$$

input
```
integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
```

output
```
integral((9*x^2 + 12*x + 4)*(4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

3.932.6 Sympy [F]

$$\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(3x+2)^2\,(4x+1)^m}{3x^2-5x+1}\,dx$$

input
```
integrate((2+3*x)**2*(1+4*x)**m/(3*x**2-5*x+1),x)
```

output
```
Integral((3*x + 2)**2*(4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

3.932.7 Maxima [F]

$$\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^2}{3\,x^2-5\,x+1}\,dx$$

input
```
integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
```

output
```
integrate((4*x + 1)^m*(3*x + 2)^2/(3*x^2 - 5*x + 1), x)
```

3.932.8 Giac [F]

$$\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^2}{3\,x^2-5\,x+1}\,dx$$

input
```
integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
```

output
```
integrate((4*x + 1)^m*(3*x + 2)^2/(3*x^2 - 5*x + 1), x)
```

3.932.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(3\,x+2)^2\,(4\,x+1)^m}{3\,x^2-5\,x+1}\,dx$$

input
```
int(((3*x + 2)^2*(4*x + 1)^m)/(3*x^2 - 5*x + 1),x)
```

output
```
int(((3*x + 2)^2*(4*x + 1)^m)/(3*x^2 - 5*x + 1), x)
```

3.933 $\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2} dx$

3.933.1 Optimal result

Integrand size = 25, antiderivative size = 129

$$\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2} dx = -\frac{3(13-9\sqrt{13})(1+4x)^{1+m}\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{26(13-2\sqrt{13})(1+m)} - \frac{3(13+9\sqrt{13})(1+4x)^{1+m}\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{26(13+2\sqrt{13})(1+m)}$$

output
```
-3/26*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(1
3-9*13^(1/2))/(1+m)/(13-2*13^(1/2))-3/26*(1+4*x)^(1+m)*hypergeom([1, 1+m],
[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(13+9*13^(1/2))/(1+m)/(13+2*13^(1/2))
```

3.933.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.69

$$\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2} dx = \frac{(1+4x)^{1+m}\left((5+7\sqrt{13})\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13-2\sqrt{13}}\right)+(5-7\sqrt{13})\text{Hypergeometric}\right.}{78(1+m)}$$

input
```
Integrate[((2 + 3*x)*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

output
```
((1 + 4*x)^(1 + m)*((5 + 7*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3
 + 12*x)/(13 - 2*Sqrt[13])] + (5 - 7*Sqrt[13])*Hypergeometric2F1[1, 1 + m,
 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])]))/(78*(1 + m))
```

3.933.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x+2)(4x+1)^m}{3x^2-5x+1} \, dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(\frac{\left(3+\frac{27}{\sqrt{13}}\right)(4x+1)^m}{6x-\sqrt{13}-5}+\frac{\left(3-\frac{27}{\sqrt{13}}\right)(4x+1)^m}{6x+\sqrt{13}-5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\left(13-9\sqrt{13}\right)(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(13+9\sqrt{13}\right)(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{26\left(13+2\sqrt{13}\right)(m+1)}$$

input
```
Int[((2 + 3*x)*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

output
```
(-3*(13 - 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
 (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(26*(13 - 2*Sqrt[13])*(1 + m)) - (3*(13
 + 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1
+ 4*x))/(13 + 2*Sqrt[13])])/(26*(13 + 2*Sqrt[13])*(1 + m))
```

3.933.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.933.4 Maple [F]

$$\int \frac{(2+3x)(1+4x)^m}{3x^2-5x+1}dx$$

input
```
int((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1),x)
```

output
```
int((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1),x)
```

3.933.5 Fricas [F]

$$\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)}{3\,x^2-5\,x+1}\,dx$$

input
```
integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
```

output
```
integral((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1), x)
```

3.933.6 Sympy [F]

$$\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(3x+2)(4x+1)^m}{3x^2-5x+1}\,dx$$

input
```
integrate((2+3*x)*(1+4*x)**m/(3*x**2-5*x+1),x)
```

output
```
Integral((3*x + 2)*(4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

3.933.7 Maxima [F]

$$\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4x+1)^m(3x+2)}{3x^2-5x+1}\,dx$$

input
```
integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
```

output
```
integrate((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1), x)
```

3.933.8 Giac [F]

$$\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4x+1)^m(3x+2)}{3x^2-5x+1}\,dx$$

input
```
integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
```

output
```
integrate((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1), x)
```

3.933.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(3x+2)\,(4x+1)^m}{3x^2-5x+1}\,dx$$

input
```
int(((3*x + 2)*(4*x + 1)^m)/(3*x^2 - 5*x + 1),x)
```

output
```
int(((3*x + 2)*(4*x + 1)^m)/(3*x^2 - 5*x + 1), x)
```

3.934 $\int \frac{(1+4x)^m}{1-5x+3x^2} dx$

3.934.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(1+4x)^m}{1-5x+3x^2} dx = \frac{3(1+4x)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)} - \frac{3(1+4x)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{\sqrt{13}\left(13+2\sqrt{13}\right)(1+m)}$$

output
```
3/13*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))/(1+
m)/(13-2*13^(1/2))*13^(1/2)-3/13*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*
(1+4*x)/(13+2*13^(1/2)))/(1+m)*13^(1/2)/(13+2*13^(1/2))
```

3.934.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

$$\int \frac{(1+4x)^m}{1-5x+3x^2} dx = \frac{(1+4x)^{1+m}\left(\left(13+2\sqrt{13}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13-2\sqrt{13}}\right)+\left(-13+2\sqrt{13}\right)\text{Hypergeom}}{39\sqrt{13}(1+m)}$$

input
```
Integrate[(1 + 4*x)^m/(1 - 5*x + 3*x^2),x]
```

output
```
((1 + 4*x)^(1 + m)*((13 + 2*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (
3 + 12*x)/(13 - 2*Sqrt[13])] + (-13 + 2*Sqrt[13])*Hypergeometric2F1[1, 1 +
 m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])]))/(39*Sqrt[13]*(1 + m))
```

3.934.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1150, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x+1)^m}{3x^2-5x+1}\,dx$$

$$\Big\downarrow \text{1150}$$

$$\int \left(-\frac{6(4x+1)^m}{\sqrt{13}\left(-6x+\sqrt{13}+5\right)}-\frac{6(4x+1)^m}{\sqrt{13}\left(6x+\sqrt{13}-5\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}-\frac{3(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}$$

input
```
Int[(1 + 4*x)^m/(1 - 5*x + 3*x^2),x]
```

output
```
(3*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13
- 2*Sqrt[13])])/(Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) - (3*(1 + 4*x)^(1 + m
)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(Sq
rt[13]*(13 + 2*Sqrt[13])*(1 + m))
```

3.934.3.1 Defintions of rubi rules used

rule 1150
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[ExpandIntegrand[(d + e*x)^m, 1/(a + b*x + c*x^2), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] &&  !IntegerQ[2*m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.934.4 Maple [F]

$$\int \frac{(1+4x)^m}{3x^2-5x+1}dx$$

input
```
int((1+4*x)^m/(3*x^2-5*x+1),x)
```

output
```
int((1+4*x)^m/(3*x^2-5*x+1),x)
```

3.934.5 Fricas [F]

$$\int \frac{(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m}{3\,x^2-5\,x+1}\,dx$$

input
```
integrate((1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
```

output
```
integral((4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

3.934.6 Sympy [F]

$$\int \frac{(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4x+1)^m}{3x^2-5x+1}\,dx$$

input
```
integrate((1+4*x)**m/(3*x**2-5*x+1),x)
```

output
```
Integral((4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

3.934.7 Maxima [F]

$$\int \frac{(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m}{3\,x^2-5\,x+1}\,dx$$

input
```
integrate((1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
```

output
```
integrate((4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

3.934.8 Giac [F]

$$\int \frac{(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m}{3\,x^2-5\,x+1}\,dx$$

input
```
integrate((1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
```

output
```
integrate((4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

3.934.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m}{3\,x^2-5\,x+1}\,dx$$

input
```
int((4*x + 1)^m/(3*x^2 - 5*x + 1),x)
```

output
```
int((4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

3.935 $\int \frac{(1+4x)^m}{(2+3x)(1-5x+3x^2)}\,dx$

3.935.1 Optimal result

Integrand size = 27, antiderivative size = 164

$$\int \frac{(1+4x)^m}{(2+3x)(1-5x+3x^2)}\,dx = \frac{3(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{3}{5}(1+4x)\right)}{85(1+m)} + \frac{3\left(13+9\sqrt{13}\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{442\left(13-2\sqrt{13}\right)(1+m)} + \frac{3\left(13-9\sqrt{13}\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{442\left(13+2\sqrt{13}\right)(1+m)}$$

output
```
3/85*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],-3/5-12/5*x)/(1+m)+3/442*(1+4*
x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(13-9*13^(1/2
))/(1+m)/(13+2*13^(1/2))+3/442*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1
+4*x)/(13-2*13^(1/2)))*(13+9*13^(1/2))/(1+m)/(13-2*13^(1/2))
```

3.935.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.67

$$\int \frac{(1+4x)^m}{(2+3x)(1-5x+3x^2)}\,dx = \frac{(1+4x)^{1+m}\left(234\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{3}{5}(1+4x)\right)+5\left(31+11\sqrt{13}\right)\text{Hypergeometric2}\right.}{6630(1+}$$

input
```
Integrate[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)),x]
```

output
```
((1 + 4*x)^(1 + m)*(234*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/
5] + 5*(31 + 11*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(1
3 - 2*Sqrt[13])] + 5*(31 - 11*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m,
 (3 + 12*x)/(13 + 2*Sqrt[13])]))/(6630*(1 + m))
```

3.935.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x+1)^m}{(3x+2)(3x^2-5x+1)}\,dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(\frac{(7-3x)(4x+1)^m}{17(3x^2-5x+1)}+\frac{3(4x+1)^m}{17(3x+2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{3}{5}(4x+1)\right)}{85(m+1)}+\frac{3\left(13+9\sqrt{13}\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{442\left(13-2\sqrt{13}\right)(m+1)}+\frac{3\left(13-9\sqrt{13}\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{442\left(13+2\sqrt{13}\right)(m+1)}$$

input
```
Int[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)),x]
```

output
```
(3*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])
/(85*(1 + m)) + (3*(13 + 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1
, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(442*(13 - 2*Sqrt[13])*(
1 + m)) + (3*(13 - 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 +
m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(442*(13 + 2*Sqrt[13])*(1 + m)
)
```

3.935.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.935.4 Maple [F]

$$\int \frac{(1+4x)^m}{(2+3x)\,(3x^2-5x+1)}dx$$

input
```
int((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x)
```

output
```
int((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x)
```

3.935.5 Fricas [F]

$$\int \frac{(1+4x)^m}{(2+3x)\,(1-5x+3x^2)}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)(3\,x+2)}\,dx$$

input
```
integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x, algorithm="fricas")
```

output
```
integral((4*x + 1)^m/(9*x^3 - 9*x^2 - 7*x + 2), x)
```

3.935.6 Sympy [F]

$$\int \frac{(1+4x)^m}{(2+3x)\,(1-5x+3x^2)}\,dx = \int \frac{(4x+1)^m}{(3x+2)\,(3x^2-5x+1)}\,dx$$

input
```
integrate((1+4*x)**m/(2+3*x)/(3*x**2-5*x+1),x)
```

output
```
Integral((4*x + 1)**m/((3*x + 2)*(3*x**2 - 5*x + 1)), x)
```

3.935.7 Maxima [F]

$$\int \frac{(1+4x)^m}{(2+3x)\,(1-5x+3x^2)}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)(3\,x+2)}\,dx$$

input
```
integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x, algorithm="maxima")
```

output
```
integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)), x)
```

3.935.8 Giac [F]

$$\int \frac{(1+4x)^m}{(2+3x)\,(1-5x+3x^2)}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)(3\,x+2)}\,dx$$

input
```
integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x, algorithm="giac")
```

output
```
integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)), x)
```

3.935.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)} \, dx = \int \frac{{\left(4\,x+1\right)}^m}{\left(3\,x+2\right)\,\left(3\,x^2-5\,x+1\right)} \,d x$$

input
```
int((4*x + 1)^m/((3*x + 2)*(3*x^2 - 5*x + 1)),x)
```

output
```
int((4*x + 1)^m/((3*x + 2)*(3*x^2 - 5*x + 1)), x)
```

3.936 $\int \frac{(1+4x)^m}{(2+3x)^2(1-5x+3x^2)}\,dx$

3.936.1 Optimal result

Integrand size = 27, antiderivative size = 199

$$\int \frac{(1+4x)^m}{(2+3x)^2(1-5x+3x^2)}\,dx = \frac{27(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{3}{5}(1+4x)\right)}{1445(1+m)} + \frac{3\left(117+47\sqrt{13}\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{7514\left(13-2\sqrt{13}\right)(1+m)} + \frac{3\left(117-47\sqrt{13}\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{7514\left(13+2\sqrt{13}\right)(1+m)} + \frac{12(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(2,1+m,2+m,-\frac{3}{5}(1+4x)\right)}{425(1+m)}$$

output
```
27/1445*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],-3/5-12/5*x)/(1+m)+12/425*(
1+4*x)^(1+m)*hypergeom([2, 1+m],[2+m],-3/5-12/5*x)/(1+m)+3/7514*(1+4*x)^(1
+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(117-47*13^(1/2))/
(1+m)/(13+2*13^(1/2))+3/7514*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4
*x)/(13-2*13^(1/2)))*(117+47*13^(1/2))/(1+m)/(13-2*13^(1/2))
```

3.936.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.76

$$\int \frac{(1+4x)^m}{(2+3x)^2 (1-5x+3x^2)} \, dx$$
$$= \frac{(1+4x)^{1+m} \left(10530 \text{Hypergeometric2F1} \left(1,1+m,2+m,-\frac{3}{5}(1+4x)\right)+25\left(211+65\sqrt{13}\right) \text{Hypergeome}\right.}{}$$

input
```
Integrate[(1 + 4*x)^m/((2 + 3*x)^2*(1 - 5*x + 3*x^2)),x]
```

output
```
((1 + 4*x)^(1 + m)*(10530*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x)
)/5] + 25*(211 + 65*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x
)/(13 - 2*Sqrt[13])] + 5275*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/
(13 + 2*Sqrt[13])] - 1625*Sqrt[13]*Hypergeometric2F1[1, 1 + m, 2 + m, (3 +
 12*x)/(13 + 2*Sqrt[13])] + 15912*Hypergeometric2F1[2, 1 + m, 2 + m, (-3*(
1 + 4*x))/5]))/(563550*(1 + m))
```

3.936.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x+1)^m}{(3x+2)^2 (3x^2-5x+1)} \, dx$$
$$\Big\downarrow \text{1200}$$
$$\int \left(\frac{(46-27x)(4x+1)^m}{289 (3x^2-5x+1)}+\frac{27(4x+1)^m}{289(3x+2)}+\frac{3(4x+1)^m}{17(3x+2)^2}\right) dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{27(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{3}{5}(4x+1)\right)}{1445(m+1)}+\frac{3\left(117+47\sqrt{13}\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{7514\left(13-2\sqrt{13}\right)(m+1)}+\frac{3\left(117-47\sqrt{13}\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{7514\left(13+2\sqrt{13}\right)(m+1)}+\frac{12(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(2,m+1,m+2,-\frac{3}{5}(4x+1)\right)}{425(m+1)}$$

input
```
Int[(1 + 4*x)^m/((2 + 3*x)^2*(1 - 5*x + 3*x^2)),x]
```

output
```
(27*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5]
)/(1445*(1 + m)) + (3*(117 + 47*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric
2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(7514*(13 - 2*Sqrt[
13])*(1 + m)) + (3*(117 - 47*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1
[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(7514*(13 + 2*Sqrt[13]
)*(1 + m)) + (12*(1 + 4*x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (-3*
(1 + 4*x))/5])/(425*(1 + m))
```

3.936.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.936.4 Maple [F]

$$\int \frac{(1+4x)^m}{(2+3x)^2\,(3x^2-5x+1)}dx$$

input
```
int((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x)
```

output
```
int((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x)
```

3.936.5 Fricas [F]

$$\int \frac{(1+4x)^m}{(2+3x)^2 (1-5x+3x^2)}\, dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)(3\,x+2)^2}\, dx$$

input
```
integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x, algorithm="fricas")
```

output
```
integral((4*x + 1)^m/(27*x^4 - 9*x^3 - 39*x^2 - 8*x + 4), x)
```

3.936.6 Sympy [F]

$$\int \frac{(1+4x)^m}{(2+3x)^2 (1-5x+3x^2)}\, dx = \int \frac{(4x+1)^m}{(3x+2)^2 \cdot (3x^2-5x+1)}\, dx$$

input
```
integrate((1+4*x)**m/(2+3*x)**2/(3*x**2-5*x+1),x)
```

output
```
Integral((4*x + 1)**m/((3*x + 2)**2*(3*x**2 - 5*x + 1)), x)
```

3.936.7 Maxima [F]

$$\int \frac{(1+4x)^m}{(2+3x)^2 (1-5x+3x^2)}\, dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)(3\,x+2)^2}\, dx$$

input
```
integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x, algorithm="maxima")
```

output
```
integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)^2), x)
```

3.936.8 Giac [F]

$$\int \frac{(1+4x)^m}{(2+3x)^2(1-5x+3x^2)}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)(3\,x+2)^2}\,dx$$

input
```
integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x, algorithm="giac")
```

output
```
integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)^2), x)
```

3.936.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x)^m}{(2+3x)^2(1-5x+3x^2)}\,dx = \int \frac{(4\,x+1)^m}{(3\,x+2)^2\,(3\,x^2-5\,x+1)}\,dx$$

input
```
int((4*x + 1)^m/((3*x + 2)^2*(3*x^2 - 5*x + 1)),x)
```

output
```
int((4*x + 1)^m/((3*x + 2)^2*(3*x^2 - 5*x + 1)), x)
```

3.937 $\int \frac{(2+3x)^4(1+4x)^m}{(1-5x+3x^2)^2}\,dx$

3.937.1 Optimal result

Integrand size = 27, antiderivative size = 202

$$\int \frac{(2+3x)^4(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \frac{9(1+4x)^{1+m}}{4(1+m)} + \frac{(844-2355x)(1+4x)^{1+m}}{39(1-5x+3x^2)} - \frac{\left(13689-\sqrt{13}\left(297+4474m-1570\sqrt{13}m\right)\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{169\left(13-2\sqrt{13}\right)(1+m)} - \frac{\left(13689+\sqrt{13}\left(297+4474m+1570\sqrt{13}m\right)\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{169\left(13+2\sqrt{13}\right)(1+m)}$$

output
```
9/4*(1+4*x)^(1+m)/(1+m)+1/39*(844-2355*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)-1/16
9*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(13689
-13^(1/2)*(297+4474*m-1570*m*13^(1/2)))/(1+m)/(13-2*13^(1/2))-1/169*(1+4*x
)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(13689+13^(1/2
)*(297+4474*m+1570*m*13^(1/2)))/(1+m)/(13+2*13^(1/2))
```

3.937.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.24

$$\int \frac{(2+3x)^4(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \frac{(1+4x)^{1+m}\left(\frac{13689}{4+4m}+\frac{39(844-2355x)}{1-5x+3x^2}-\frac{1053\left(-117+128\sqrt{13}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13-2\sqrt{13}}\right)}{\left(-13+2\sqrt{13}\right)(1+m)}-\frac{1053\left(117+128\sqrt{13}\right)}{}\right.}{}$$

input
```
Integrate[((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

output
```
((1 + 4*x)^(1 + m)*(13689/(4 + 4*m) + (39*(844 - 2355*x))/(1 - 5*x + 3*x^2
) - (1053*(-117 + 128*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12
*x)/(13 - 2*Sqrt[13])])/((-13 + 2*Sqrt[13])*(1 + m)) - (1053*(117 + 128*Sq
rt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/
((13 + 2*Sqrt[13])*(1 + m)) - (-((-14679*(2 + Sqrt[13]) + 2*(-5731 + 667*S
qrt[13])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13]
)]) + (-14679*(-2 + Sqrt[13]) + 2*(5731 + 667*Sqrt[13])*m)*Hypergeometric2
F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/(1 + m)))/1521
```

3.937.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1265, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x+2)^4(4x+1)^m}{(3x^2-5x+1)^2}\,dx$$

$$\Big\downarrow \text{1265}$$

$$\frac{(844-2355x)(4x+1)^{m+1}}{39\,(3x^2-5x+1)} - \frac{1}{507}\int \frac{13(4x+1)^m\left(-1053x^2-3(3140m+1521)x+3376m+4617\right)}{3x^2-5x+1}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{(844-2355x)(4x+1)^{m+1}}{39\left(3x^2-5x+1\right)}-\frac{1}{39}\int\frac{(4x+1)^m\left(-1053x^2-3(3140m+1521)x+3376m+4617\right)}{3x^2-5x+1}dx$$

$$\Big\downarrow \text{2159}$$

$$\frac{(844-2355x)(4x+1)^{m+1}}{39\left(3x^2-5x+1\right)}-$$

$$\frac{1}{39}\int\left(\frac{\left(-9420m-\frac{6(4474m+297)}{\sqrt{13}}-6318\right)(4x+1)^m}{6x-\sqrt{13}-5}+\frac{\left(-9420m+\frac{6(4474m+297)}{\sqrt{13}}-6318\right)(4x+1)^m}{6x+\sqrt{13}-5}-351(4x+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{39}\left(-\frac{3\left(13689-\sqrt{13}(-1570\sqrt{13}m+4474m+297)\right)(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right.}{13\left(13-2\sqrt{13}\right)(m+1)}\right.$$

$$\frac{(844-2355x)(4x+1)^{m+1}}{39\left(3x^2-5x+1\right)}$$

input
```
Int[((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

output
```
((844 - 2355*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) + ((351*(1 + 4*x
)^(1 + m))/(4*(1 + m)) - (3*(13689 - Sqrt[13]*(297 + 4474*m - 1570*Sqrt[13
]*m))*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(
13 - 2*Sqrt[13])])/(13*(13 - 2*Sqrt[13])*(1 + m)) - (3*(13689 + Sqrt[13]*(
297 + 4474*m + 1570*Sqrt[13]*m))*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1
+ m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(13*(13 + 2*Sqrt[13])*(1 + m
)))/39
```

3.937.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1265
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(f + g*x)
^n, a + b*x + c*x^2, x], R = Coeff[PolynomialRemainder[(f + g*x)^n, a + b*x
 + c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[(f + g*x)^n, a + b*x + c
*x^2, x], x, 1]}, Simp[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1)*((R*(b*c
*d - b^2*e + 2*a*c*e) - a*S*(2*c*d - b*e) + c*(R*(2*c*d - b*e) - S*(b*d - 2
*a*e))*x)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((p
 + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Q +
 R*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2) - 2*c^2*d^2*(2*p + 3) - 2*a
*c*e^2*(m + 2*p + 3)) - S*(a*e*(b*e - 2*c*d*m + b*e*m) - b*d*(3*c*d - b*e +
 2*c*d*p - b*e*p)) + c*e*(S*(b*d - 2*a*e) - R*(2*c*d - b*e))*(m + 2*p + 4)*
x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 1] && LtQ[p
, -1] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.937.4 Maple [F]

$$\int \frac{(2+3x)^4 (1+4x)^m}{(3x^2-5x+1)^2} dx$$

input
```
int((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

output
```
int((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

3.937.5 Fricas [F]

$$\int \frac{(2+3x)^4(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^4}{(3\,x^2-5\,x+1)^2}\,dx$$

input
```
integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

output
```
integral((81*x^4 + 216*x^3 + 216*x^2 + 96*x + 16)*(4*x + 1)^m/(9*x^4 - 30*
x^3 + 31*x^2 - 10*x + 1), x)
```

3.937.6 Sympy [F]

$$\int \frac{(2+3x)^4(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(3x+2)^4\,(4x+1)^m}{(3x^2-5x+1)^2}\,dx$$

input
```
integrate((2+3*x)**4*(1+4*x)**m/(3*x**2-5*x+1)**2,x)
```

output
```
Integral((3*x + 2)**4*(4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)
```

3.937.7 Maxima [F]

$$\int \frac{(2+3x)^4(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^4}{(3\,x^2-5\,x+1)^2}\,dx$$

input
```
integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
```

output
```
integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1)^2, x)
```

3.937.8 Giac [F]

$$\int \frac{(2+3x)^4(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^4}{(3\,x^2-5\,x+1)^2}\,dx$$

input
```
integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
```

output
```
integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1)^2, x)
```

3.937.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x)^4(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(3\,x+2)^4\,(4\,x+1)^m}{(3\,x^2-5\,x+1)^2}\,dx$$

input
```
int(((3*x + 2)^4*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2,x)
```

output
```
int(((3*x + 2)^4*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2, x)
```

3.938 $\int \frac{(2+3x)^3(1+4x)^m}{(1-5x+3x^2)^2}\,dx$

3.938.1 Optimal result

Integrand size = 27, antiderivative size = 181

$$\int \frac{(2+3x)^3(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \frac{(209-426x)(1+4x)^{1+m}}{39\,(1-5x+3x^2)} - \frac{\left(1521+\sqrt{13}(1701-1168m+568\sqrt{13}m)\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{338\,(13-2\sqrt{13})\,(1+m)} + \frac{\left(\sqrt{13}(1701-1168m)-13(117+568m)\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{338\,(13+2\sqrt{13})\,(1+m)}$$

output
```
1/39*(209-426*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)+1/338*(1+4*x)^(1+m)*hypergeom
([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(-1521-7384*m+(1701-1168*m)*13^
(1/2))/(1+m)/(13+2*13^(1/2))-1/338*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],
3*(1+4*x)/(13-2*13^(1/2)))*(1521+13^(1/2)*(1701-1168*m+568*m*13^(1/2)))/(1
+m)/(13-2*13^(1/2))
```

3.938.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.39

$$\int \frac{(2+3x)^3(1+4x)^m}{(1-5x+3x^2)^2}\,dx$$
$$= \frac{(1+4x)^{1+m}\left(\frac{5434-11076x}{1-5x+3x^2} - \frac{351\left(-13+27\sqrt{13}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13-2\sqrt{13}}\right)}{\left(-13+2\sqrt{13}\right)(1+m)} - \frac{12\left(\sqrt{13}(1215-292m)+1846m\right)\text{Hyp}}{\left(13-2\right.}\right.}{}$$

input
```
Integrate[((2 + 3*x)^3*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

output
```
((1 + 4*x)^(1 + m)*((5434 - 11076*x)/(1 - 5*x + 3*x^2) - (351*(-13 + 27*Sq
rt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/
((-13 + 2*Sqrt[13])*(1 + m)) - (12*(Sqrt[13]*(1215 - 292*m) + 1846*m)*Hype
rgeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/((13 - 2*Sqr
t[13])*(1 + m)) - (351*(13 + 27*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 +
m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[13])*(1 + m)) + (12*(Sqrt[
13]*(1215 - 292*m) - 1846*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)
/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[13])*(1 + m))))/1014
```

3.938.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1265, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x+2)^3(4x+1)^m}{(3x^2-5x+1)^2}\,dx$$
$$\Big\downarrow \; 1265$$
$$\frac{(209-426x)(4x+1)^{m+1}}{39\,(3x^2-5x+1)} - \frac{1}{507}\int \frac{13(4x+1)^m(836m-3(568m+117)x+1143)}{3x^2-5x+1}dx$$
$$\Big\downarrow \; 27$$
$$\frac{(209-426x)(4x+1)^{m+1}}{39\,(3x^2-5x+1)} - \frac{1}{39}\int \frac{(4x+1)^m(836m-3(568m+117)x+1143)}{3x^2-5x+1}dx$$

$$\Big\downarrow \textbf{1200}$$

$$\frac{(209-426x)(4x+1)^{m+1}}{39\left(3x^2-5x+1\right)}-\frac{1}{39}\int \left(\frac{\left(-3(568m+117)-\frac{3(1168m-1701)}{\sqrt{13}}\right)(4x+1)^m}{6x-\sqrt{13}-5}+\frac{\left(\frac{3(1168m-1701)}{\sqrt{13}}-3(568m+117)\right)(4x+1)^m}{6x+\sqrt{13}-5}\right)dx$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{1}{39}\left(\frac{3\left(\sqrt{13}(1701-1168m)-13(568m+117)\right)(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{26\left(13+2\sqrt{13}\right)(m+1)}-\frac{3}{}\right.$$
$$\frac{(209-426x)(4x+1)^{m+1}}{39\left(3x^2-5x+1\right)}$$

input
```
Int[((2 + 3*x)^3*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

output
```
((209 - 426*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) + ((-3*(1521 + Sq
rt[13]*(1701 - 1168*m + 568*Sqrt[13]*m))*(1 + 4*x)^(1 + m)*Hypergeometric2
F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(26*(13 - 2*Sqrt[13]
)*(1 + m)) + (3*(Sqrt[13]*(1701 - 1168*m) - 13*(117 + 568*m))*(1 + 4*x)^(1
 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])
/(26*(13 + 2*Sqrt[13])*(1 + m)))/39
```

3.938.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

3.938. $\int \frac{(2+3x)^3(1+4x)^m}{(1-5x+3x^2)^2}\,dx$

rule 1265
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(f + g*x)
^n, a + b*x + c*x^2, x], R = Coeff[PolynomialRemainder[(f + g*x)^n, a + b*x
 + c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[(f + g*x)^n, a + b*x + c
*x^2, x], x, 1]}, Simp[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1)*((R*(b*c
*d - b^2*e + 2*a*c*e) - a*S*(2*c*d - b*e) + c*(R*(2*c*d - b*e) - S*(b*d - 2
*a*e))*x)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((p
 + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Q +
 R*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2) - 2*c^2*d^2*(2*p + 3) - 2*a
*c*e^2*(m + 2*p + 3)) - S*(a*e*(b*e - 2*c*d*m + b*e*m) - b*d*(3*c*d - b*e +
 2*c*d*p - b*e*p)) + c*e*(S*(b*d - 2*a*e) - R*(2*c*d - b*e))*(m + 2*p + 4)*
x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 1] && LtQ[p
, -1] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.938.4 Maple [F]

$$\int \frac{(2+3x)^3 (1+4x)^m}{(3x^2-5x+1)^2} dx$$

input
```
int((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

output
```
int((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

3.938.5 Fricas [F]

$$\int \frac{(2+3x)^3(1+4x)^m}{(1-5x+3x^2)^2} dx = \int \frac{(4x+1)^m (3x+2)^3}{(3x^2-5x+1)^2} dx$$

input
```
integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

output
```
integral((27*x^3 + 54*x^2 + 36*x + 8)*(4*x + 1)^m/(9*x^4 - 30*x^3 + 31*x^2
 - 10*x + 1), x)
```

3.938.6 Sympy [F]

$$\int \frac{(2+3x)^3(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(3x+2)^3\,(4x+1)^m}{(3x^2-5x+1)^2}\,dx$$

input
```
integrate((2+3*x)**3*(1+4*x)**m/(3*x**2-5*x+1)**2,x)
```

output
```
Integral((3*x + 2)**3*(4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)
```

3.938.7 Maxima [F]

$$\int \frac{(2+3x)^3(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^3}{(3\,x^2-5\,x+1)^2}\,dx$$

input
```
integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
```

output
```
integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1)^2, x)
```

3.938.8 Giac [F]

$$\int \frac{(2+3x)^3(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^3}{(3\,x^2-5\,x+1)^2}\,dx$$

input
```
integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
```

output
```
integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1)^2, x)
```

3.938.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x)^3(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(3\,x+2)^3\,(4\,x+1)^m}{(3\,x^2-5\,x+1)^2}\,dx$$

input
```
int(((3*x + 2)^3*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2,x)
```

output
```
int(((3*x + 2)^3*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2, x)
```

3.939 $\int \frac{(2+3x)^2(1+4x)^m}{(1-5x+3x^2)^2}\,dx$

3.939.1 Optimal result

Integrand size = 27, antiderivative size = 179

$$\int \frac{(2+3x)^2(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \frac{(61-87x)(1+4x)^{1+m}}{39\,(1-5x+3x^2)} - \frac{2\left(153-\left(23-29\sqrt{13}\right)m\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)} + \frac{2\left(153-\left(23+29\sqrt{13}\right)m\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{13\sqrt{13}\left(13+2\sqrt{13}\right)(1+m)}$$

output
```
1/39*(61-87*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)-2/169*(1+4*x)^(1+m)*hypergeom([
1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(153-m*(23-29*13^(1/2)))/(1+m)/(1
3-2*13^(1/2))*13^(1/2)+2/169*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4
*x)/(13+2*13^(1/2)))*(153-m*(23+29*13^(1/2)))/(1+m)*13^(1/2)/(13+2*13^(1/2
))
```

3.939.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.87

$$\int \frac{(2+3x)^2(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \frac{1}{507}(1+4x)^{1+m}\left(\frac{793-1131x}{1-5x+3x^2} - \frac{6\left(-153\sqrt{13}+\left(-377+23\sqrt{13}\right)m\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13-2\sqrt{13}}\right)}{\left(-13+2\sqrt{13}\right)(1+m)} - \frac{6\left(-153\sqrt{13}+\left(377+23\sqrt{13}\right)m\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13+2\sqrt{13}}\right)}{\left(13+2\sqrt{13}\right)(1+m)}\right)$$

input
```
Integrate[((2 + 3*x)^2*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

output
```
((1 + 4*x)^(1 + m)*((793 - 1131*x)/(1 - 5*x + 3*x^2) - (6*(-153*Sqrt[13] +
 (-377 + 23*Sqrt[13])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13
 - 2*Sqrt[13])])/((-13 + 2*Sqrt[13])*(1 + m)) - (6*(-153*Sqrt[13] + (377 +
 23*Sqrt[13])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqr
t[13])])/((13 + 2*Sqrt[13])*(1 + m))))/507
```

3.939.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1265, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x+2)^2(4x+1)^m}{(3x^2-5x+1)^2}\,dx$$

$$\Big\downarrow \text{1265}$$

$$\frac{(61-87x)(4x+1)^{m+1}}{39\,(3x^2-5x+1)} - \frac{1}{507}\int \frac{26(4x+1)^m(-174xm+122m+153)}{3x^2-5x+1}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{(61-87x)(4x+1)^{m+1}}{39\left(3x^2-5x+1\right)}-\frac{2}{39}\int\frac{(4x+1)^m(-174xm+122m+153)}{3x^2-5x+1}dx$$

$$\Big\downarrow \text{1200}$$

$$\frac{(61-87x)(4x+1)^{m+1}}{39\left(3x^2-5x+1\right)}-\frac{2}{39}\int\left(\frac{\left(-174m-\frac{6(23m-153)}{\sqrt{13}}\right)(4x+1)^m}{6x-\sqrt{13}-5}+\frac{\left(\frac{6(23m-153)}{\sqrt{13}}-174m\right)(4x+1)^m}{6x+\sqrt{13}-5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(61-87x)(4x+1)^{m+1}}{39\left(3x^2-5x+1\right)}-\frac{2}{39}\left(\frac{3\left(153-\left(23-29\sqrt{13}\right)m\right)(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}-\frac{3(153-(23+29\cdots}{\cdots}\right.$$

input
```
Int[((2 + 3*x)^2*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

output
```
((61 - 87*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) - (2*((3*(153 - (23
 - 29*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3
*(1 + 4*x))/(13 - 2*Sqrt[13])])/(Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) - (3*
(153 - (23 + 29*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m,
 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(Sqrt[13]*(13 + 2*Sqrt[13])*(1 +
 m))))/39
```

3.939.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1265
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(f + g*x)
^n, a + b*x + c*x^2, x], R = Coeff[PolynomialRemainder[(f + g*x)^n, a + b*x
 + c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[(f + g*x)^n, a + b*x + c
*x^2, x], x, 1]}, Simp[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1)*((R*(b*c
*d - b^2*e + 2*a*c*e) - a*S*(2*c*d - b*e) + c*(R*(2*c*d - b*e) - S*(b*d - 2
*a*e))*x)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((p
 + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Q +
 R*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2) - 2*c^2*d^2*(2*p + 3) - 2*a
*c*e^2*(m + 2*p + 3)) - S*(a*e*(b*e - 2*c*d*m + b*e*m) - b*d*(3*c*d - b*e +
 2*c*d*p - b*e*p)) + c*e*(S*(b*d - 2*a*e) - R*(2*c*d - b*e))*(m + 2*p + 4)*
x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 1] && LtQ[p
, -1] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.939.4 Maple [F]

$$\int \frac{(2+3x)^2 (1+4x)^m}{(3x^2-5x+1)^2} dx$$

input
```
int((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

output
```
int((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

3.939.5 Fricas [F]

$$\int \frac{(2+3x)^2(1+4x)^m}{(1-5x+3x^2)^2} dx = \int \frac{(4x+1)^m(3x+2)^2}{(3x^2-5x+1)^2} dx$$

input
```
integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

output
```
integral((9*x^2 + 12*x + 4)*(4*x + 1)^m/(9*x^4 - 30*x^3 + 31*x^2 - 10*x +
1), x)
```

3.939.6 Sympy [F]

$$\int \frac{(2+3x)^2(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(3x+2)^2\,(4x+1)^m}{(3x^2-5x+1)^2}\,dx$$

input
```
integrate((2+3*x)**2*(1+4*x)**m/(3*x**2-5*x+1)**2,x)
```

output
```
Integral((3*x + 2)**2*(4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)
```

3.939.7 Maxima [F]

$$\int \frac{(2+3x)^2(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^2}{(3\,x^2-5\,x+1)^2}\,dx$$

input
```
integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
```

output
```
integrate((4*x + 1)^m*(3*x + 2)^2/(3*x^2 - 5*x + 1)^2, x)
```

3.939.8 Giac [F]

$$\int \frac{(2+3x)^2(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^2}{(3\,x^2-5\,x+1)^2}\,dx$$

input
```
integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
```

output
```
integrate((4*x + 1)^m*(3*x + 2)^2/(3*x^2 - 5*x + 1)^2, x)
```

3.939.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x)^2(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(3\,x+2)^2\,(4\,x+1)^m}{(3\,x^2-5\,x+1)^2}\,dx$$

input
```
int(((3*x + 2)^2*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2,x)
```

output
```
int(((3*x + 2)^2*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2, x)
```

3.940 $\int \frac{(2+3x)(1+4x)^m}{\left(1-5x+3x^2\right)^2} \, dx$

3.940.1 Optimal result

Integrand size = 25, antiderivative size = 179

$$\begin{aligned}\int \frac{(2+3x)(1+4x)^m}{\left(1-5x+3x^2\right)^2} \, dx &= \frac{(20-21x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} \\ &\quad - \frac{\left(81+2\left(5+7\sqrt{13}\right)m\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)} \\ &\quad + \frac{\left(81+2\left(5-7\sqrt{13}\right)m\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{13\sqrt{13}\left(13+2\sqrt{13}\right)(1+m)}\end{aligned}$$

output
```
1/39*(20-21*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)+1/169*(1+4*x)^(1+m)*hypergeom([
1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(81+2*m*(5-7*13^(1/2)))/(1+m)*13^
(1/2)/(13+2*13^(1/2))-1/169*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*
x)/(13-2*13^(1/2)))*(81+2*m*(5+7*13^(1/2)))/(1+m)/(13-2*13^(1/2))*13^(1/2)
```

3.940.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.83

$$\int \frac{(2+3x)(1+4x)^m}{(1-5x+3x^2)^2}\,dx$$
$$= \frac{1}{507}(1+4x)^{1+m}\left(\frac{260-273x}{1-5x+3x^2} + \frac{3(182m+\sqrt{13}(81+10m))\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13-2\sqrt{13}}\right)}{\left(-13+2\sqrt{13}\right)(1+m)} - \frac{3(182m-\sqrt{13}(81+10m))\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13+2\sqrt{13}}\right)}{\left(13+2\sqrt{13}\right)(1+m)}\right)$$

input
```
Integrate[((2 + 3*x)*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

output
```
((1 + 4*x)^(1 + m)*((260 - 273*x)/(1 - 5*x + 3*x^2) + (3*(182*m + Sqrt[13]
*(81 + 10*m))*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[1
3])])/((-13 + 2*Sqrt[13])*(1 + m)) - (3*(182*m - Sqrt[13]*(81 + 10*m))*Hyp
ergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sq
rt[13])*(1 + m))))/507
```

3.940.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x+2)(4x+1)^m}{(3x^2-5x+1)^2}\,dx$$
$$\Big\downarrow \ 1235$$
$$\frac{(20-21x)(4x+1)^{m+1}}{39\,(3x^2-5x+1)} - \frac{1}{507}\int \frac{13(4x+1)^m(-84xm+80m+81)}{3x^2-5x+1}dx$$
$$\Big\downarrow \ 27$$

$$\frac{(20-21x)(4x+1)^{m+1}}{39(3x^2-5x+1)}-\frac{1}{39}\int \frac{(4x+1)^m(-84xm+80m+81)}{3x^2-5x+1}dx$$

$$\Big\downarrow \ 1200$$

$$\frac{(20-21x)(4x+1)^{m+1}}{39(3x^2-5x+1)}-\frac{1}{39}\int \left(\frac{\left(\frac{6(10m+81)}{\sqrt{13}}-84m\right)(4x+1)^m}{6x-\sqrt{13}-5}+\frac{\left(-84m-\frac{6(10m+81)}{\sqrt{13}}\right)(4x+1)^m}{6x+\sqrt{13}-5}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{39}\left(\frac{3\left(2\left(5-7\sqrt{13}\right)m+81\right)(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}-\frac{3\left(182m+\sqrt{13}(10m\right.}{}\right.$$
$$\frac{(20-21x)(4x+1)^{m+1}}{39(3x^2-5x+1)}$$

input
```
Int[((2 + 3*x)*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

output
```
((20 - 21*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) + ((-3*(182*m + Sqr
t[13]*(81 + 10*m))*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3
*(1 + 4*x))/(13 - 2*Sqrt[13])])/(13*(13 - 2*Sqrt[13])*(1 + m)) + (3*(81 +
2*(5 - 7*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
 (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(Sqrt[13]*(13 + 2*Sqrt[13])*(1 + m)))/3
9
```

3.940.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.940.4 Maple [F]

$$\int \frac{(2+3x)\,(1+4x)^m}{(3x^2-5x+1)^2}dx$$

input
```
int((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

output
```
int((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

3.940.5 Fricas [F]

$$\int \frac{(2+3x)(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)}{(3\,x^2-5\,x+1)^2}\,dx$$

input
```
integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

output
```
integral((4*x + 1)^m*(3*x + 2)/(9*x^4 - 30*x^3 + 31*x^2 - 10*x + 1), x)
```

3.940.6 Sympy [F]

$$\int \frac{(2+3x)(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(3x+2)\,(4x+1)^m}{(3x^2-5x+1)^2}\,dx$$

input
```
integrate((2+3*x)*(1+4*x)**m/(3*x**2-5*x+1)**2,x)
```

output
```
Integral((3*x + 2)*(4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)
```

3.940.7 Maxima [F]

$$\int \frac{(2+3x)(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)}{(3\,x^2-5\,x+1)^2}\,dx$$

input
```
integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
```

output
```
integrate((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1)^2, x)
```

3.940.8 Giac [F]

$$\int \frac{(2+3x)(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)}{(3\,x^2-5\,x+1)^2}\,dx$$

input
```
integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
```

output
```
integrate((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1)^2, x)
```

3.940.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x)(1+4x)^m}{(1-5x+3x^2)^2}\,dx=\int \frac{(3\,x+2)\,\left(4\,x+1\right)^m}{\left(3\,x^2-5\,x+1\right)^2}\,dx$$

input
```
int(((3*x + 2)*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2,x)
```

output
```
int(((3*x + 2)*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2, x)
```

3.941 $\int \frac{(1+4x)^m}{(1-5x+3x^2)^2}\,dx$

3.941.1 Optimal result

Integrand size = 20, antiderivative size = 177

$$\int \frac{(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \frac{(7-6x)(1+4x)^{1+m}}{39(1-5x+3x^2)} - \frac{2\left(9+2\left(2+\sqrt{13}\right)m\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)} + \frac{2\left(9+2\left(2-\sqrt{13}\right)m\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{13\sqrt{13}\left(13+2\sqrt{13}\right)(1+m)}$$

output
```
1/39*(7-6*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)+2/169*(1+4*x)^(1+m)*hypergeom([1,
 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(9+2*m*(2-13^(1/2)))/(1+m)*13^(1/2)
/(13+2*13^(1/2))-2/169*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(1
3-2*13^(1/2)))*(9+2*m*(2+13^(1/2)))/(1+m)/(13-2*13^(1/2))*13^(1/2)
```

3.941.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.85

$$\int \frac{(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \frac{1}{507}(1+4x)^{1+m}\left(\frac{91-78x}{1-5x+3x^2} + \frac{6\left(26m+\sqrt{13}(9+4m)\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13-2\sqrt{13}}\right)}{\left(-13+2\sqrt{13}\right)(1+m)} + \frac{6\sqrt{13}\left(9-2\left(-2+\sqrt{13}\right)m\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13+2\sqrt{13}}\right)}{\left(13+2\sqrt{13}\right)(1+m)}\right)$$

input
```
Integrate[(1 + 4*x)^m/(1 - 5*x + 3*x^2)^2,x]
```

output
```
((1 + 4*x)^(1 + m)*((91 - 78*x)/(1 - 5*x + 3*x^2) + (6*(26*m + Sqrt[13]*(9
 + 4*m))*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])
/((-13 + 2*Sqrt[13])*(1 + m)) + (6*Sqrt[13]*(9 - 2*(-2 + Sqrt[13])*m)*Hype
rgeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqr
t[13])*(1 + m))))/507
```

3.941.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1165, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x+1)^m}{(3x^2-5x+1)^2}dx$$

$$\Big\downarrow \text{ 1165}$$

$$\frac{(7-6x)(4x+1)^{m+1}}{39(3x^2-5x+1)} - \frac{1}{507}\int \frac{26(4x+1)^m(-12xm+14m+9)}{3x^2-5x+1}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(7-6x)(4x+1)^{m+1}}{39(3x^2-5x+1)}-\frac{2}{39}\int\frac{(4x+1)^m(-12xm+14m+9)}{3x^2-5x+1}dx$$

$$\Big\downarrow \text{1200}$$

$$\frac{(7-6x)(4x+1)^{m+1}}{39(3x^2-5x+1)}-\frac{2}{39}\int\left(\frac{\left(\frac{6(4m+9)}{\sqrt{13}}-12m\right)(4x+1)^m}{6x-\sqrt{13}-5}+\frac{\left(-12m-\frac{6(4m+9)}{\sqrt{13}}\right)(4x+1)^m}{6x+\sqrt{13}-5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(7-6x)(4x+1)^{m+1}}{39(3x^2-5x+1)}-$$
$$\frac{2}{39}\left(\frac{3\left(2\left(2+\sqrt{13}\right)m+9\right)(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(2\left(2-\sqrt{13}\right)m+9\right)(\right.$$

input
```
Int[(1 + 4*x)^m/(1 - 5*x + 3*x^2)^2,x]
```

output
```
((7 - 6*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) - (2*((3*(9 + 2*(2 +
Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 +
4*x))/(13 - 2*Sqrt[13])])/(Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) - (3*(9 + 2
*(2 - Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3
*(1 + 4*x))/(13 + 2*Sqrt[13])])/(Sqrt[13]*(13 + 2*Sqrt[13])*(1 + m))))/39
```

3.941.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

3.941. $\int \frac{(1+4x)^m}{(1-5x+3x^2)^2}\,dx$

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.941.4 Maple [F]

$$\int \frac{(1+4x)^m}{(3x^2-5x+1)^2}dx$$

input
```
int((1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

output
```
int((1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

3.941.5 Fricas [F]

$$\int \frac{(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)^2}\,dx$$

input
```
integrate((1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

output
```
integral((4*x + 1)^m/(9*x^4 - 30*x^3 + 31*x^2 - 10*x + 1), x)
```

3.941.6 Sympy [F]

$$\int \frac{(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4x+1)^m}{(3x^2-5x+1)^2}\,dx$$

input
```
integrate((1+4*x)**m/(3*x**2-5*x+1)**2,x)
```

output
```
Integral((4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)
```

3.941.7 Maxima [F]

$$\int \frac{(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)^2}\,dx$$

input
```
integrate((1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
```

output
```
integrate((4*x + 1)^m/(3*x^2 - 5*x + 1)^2, x)
```

3.941.8 Giac [F]

$$\int \frac{(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)^2}\,dx$$

input
```
integrate((1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
```

output
```
integrate((4*x + 1)^m/(3*x^2 - 5*x + 1)^2, x)
```

3.941.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)^2}\,dx$$

input
```
int((4*x + 1)^m/(3*x^2 - 5*x + 1)^2,x)
```

output
```
int((4*x + 1)^m/(3*x^2 - 5*x + 1)^2, x)
```

3.942 $\int \frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)^2}\,dx$

3.942.1 Optimal result

Integrand size = 27, antiderivative size = 340

$$\begin{aligned}
&\int \frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)^2}\,dx \\
&= \frac{(43-33x)(1+4x)^{1+m}}{663\left(1-5x+3x^2\right)} \\
&\quad + \frac{9(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{3}{5}(1+4x)\right)}{1445(1+m)} \\
&\quad + \frac{9\left(13+9\sqrt{13}\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{7514\left(13-2\sqrt{13}\right)(1+m)} \\
&\quad - \frac{\left(81+\left(62+22\sqrt{13}\right)m\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{221\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)} \\
&\quad + \frac{9\left(13-9\sqrt{13}\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{7514\left(13+2\sqrt{13}\right)(1+m)} \\
&\quad + \frac{\left(81+\left(62-22\sqrt{13}\right)m\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{221\sqrt{13}\left(13+2\sqrt{13}\right)(1+m)}
\end{aligned}$$

output
```
1/663*(43-33*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)+9/1445*(1+4*x)^(1+m)*hypergeom
([1, 1+m],[2+m],-3/5-12/5*x)/(1+m)+9/7514*(1+4*x)^(1+m)*hypergeom([1, 1+m]
,[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(13-9*13^(1/2))/(1+m)/(13+2*13^(1/2))+1/
2873*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(81
+m*(62-22*13^(1/2)))/(1+m)*13^(1/2)/(13+2*13^(1/2))+9/7514*(1+4*x)^(1+m)*h
ypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(13+9*13^(1/2))/(1+m)/(
13-2*13^(1/2))-1/2873*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13
-2*13^(1/2)))*(81+m*(62+22*13^(1/2)))/(1+m)/(13-2*13^(1/2))*13^(1/2)
```

3.942.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.81

$$\int \frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)^2}\,dx = \frac{(1+4x)^{1+m}\left(\frac{2210(43-33x)}{1-5x+3x^2}+\frac{9126\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{3}{5}(1+4x)\right)}{1+m}+\frac{1755\left(13+9\sqrt{13}\right)\text{Hypergeometric2F1}\left(1,1+m,2-\right.}{\left(13-2\sqrt{13}\right)(1+m)}\right.}{}$$

input
```
Integrate[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)^2),x]
```

output
```
((1 + 4*x)^(1 + m)*((2210*(43 - 33*x))/(1 - 5*x + 3*x^2) + (9126*Hypergeom
etric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(1 + m) + (1755*(13 + 9*Sqrt[
13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/((1
3 - 2*Sqrt[13])*(1 + m)) + (1755*(13 - 9*Sqrt[13])*Hypergeometric2F1[1, 1
+ m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[13])*(1 + m)) + (
510*Sqrt[13]*(((81 + (62 + 22*Sqrt[13])*m)*Hypergeometric2F1[1, 1 + m, 2 +
 m, (3 + 12*x)/(13 - 2*Sqrt[13])])/(-13 + 2*Sqrt[13]) + ((81 + (62 - 22*Sq
rt[13])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])
])/(13 + 2*Sqrt[13])))/(1 + m)))/1465230
```

3.942.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x+1)^m}{(3x+2)\left(3x^2-5x+1\right)^2}dx$$

$$\Big\downarrow \text{1289}$$

$$\int \left(-\frac{3(3x-7)(4x+1)^m}{289\left(3x^2-5x+1\right)}+\frac{(7-3x)(4x+1)^m}{17\left(3x^2-5x+1\right)^2}+\frac{9(4x+1)^m}{289(3x+2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{9(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{3}{5}(4x+1)\right)}{1445(m+1)}-\frac{\left(\left(62+22\sqrt{13}\right)m+81\right)(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{221\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}+\frac{9\left(13+9\sqrt{13}\right)(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{7514\left(13-2\sqrt{13}\right)(m+1)}+\frac{\left(\left(62-22\sqrt{13}\right)m+81\right)(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{221\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}+\frac{9\left(13-9\sqrt{13}\right)(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{7514\left(13+2\sqrt{13}\right)(m+1)}+\frac{(43-33x)(4x+1)^{m+1}}{663\left(3x^2-5x+1\right)}$$

input
```
Int[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)^2),x]
```

output
```
((43 - 33*x)*(1 + 4*x)^(1 + m))/(663*(1 - 5*x + 3*x^2)) + (9*(1 + 4*x)^(1
+ m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(1445*(1 + m))
+ (9*(13 + 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m
, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(7514*(13 - 2*Sqrt[13])*(1 + m)) - ((8
1 + (62 + 22*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2
+ m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(221*Sqrt[13]*(13 - 2*Sqrt[13])*(1
+ m)) + (9*(13 - 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m,
 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(7514*(13 + 2*Sqrt[13])*(1 + m))
 + ((81 + (62 - 22*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 +
 m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(221*Sqrt[13]*(13 + 2*Sqrt[13
])*(1 + m))
```

3.942.3.1 Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.942.4 Maple [F]

$$\int \frac{(1+4x)^m}{(2+3x)(3x^2-5x+1)^2}dx$$

input
```
int((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1)^2,x)
```

output
```
int((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1)^2,x)
```

3.942.5 Fricas [F]

$$\int \frac{(1+4x)^m}{(2+3x)(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)^2(3\,x+2)}\,dx$$

input
```
integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

output
```
integral((4*x + 1)^m/(27*x^5 - 72*x^4 + 33*x^3 + 32*x^2 - 17*x + 2), x)
```

3.942.6 Sympy [F]

$$\int \frac{(1+4x)^m}{(2+3x)(1-5x+3x^2)^2}\,dx = \int \frac{(4x+1)^m}{(3x+2)(3x^2-5x+1)^2}\,dx$$

input
```
integrate((1+4*x)**m/(2+3*x)/(3*x**2-5*x+1)**2,x)
```

output
```
Integral((4*x + 1)**m/((3*x + 2)*(3*x**2 - 5*x + 1)**2), x)
```

3.942.7 Maxima [F]

$$\int \frac{(1+4x)^m}{(2+3x)(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)^2(3\,x+2)}\,dx$$

input
```
integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1)^2,x, algorithm="maxima")
```

output
```
integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)^2*(3*x + 2)), x)
```

3.942.8 Giac [F]

$$\int \frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)^2}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)^2(3\,x+2)}\,dx$$

input
```
integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1)^2,x, algorithm="giac")
```

output
```
integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)^2*(3*x + 2)), x)
```

3.942.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)^2}\,dx = \int \frac{(4\,x+1)^m}{(3\,x+2)\,\left(3\,x^2-5\,x+1\right)^2}\,dx$$

input
```
int((4*x + 1)^m/((3*x + 2)*(3*x^2 - 5*x + 1)^2),x)
```

output
```
int((4*x + 1)^m/((3*x + 2)*(3*x^2 - 5*x + 1)^2), x)
```

3.943 $\int \frac{(1+4x)^m}{(2+3x)^2\left(1-5x+3x^2\right)^2}\,dx$

3.943.1 Optimal result

Integrand size = 27, antiderivative size = 376

$$\begin{aligned}\int \frac{(1+4x)^m}{(2+3x)^2\left(1-5x+3x^2\right)^2}\,dx &= \frac{(268-195x)(1+4x)^{1+m}}{11271\left(1-5x+3x^2\right)}\\
&+\frac{162(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{3}{5}(1+4x)\right)}{24565(1+m)}\\
&+\frac{9\left(117+64\sqrt{13}\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{63869\left(13-2\sqrt{13}\right)(1+m)}\\
&-\frac{\left(423+2\left(211+65\sqrt{13}\right)m\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{3757\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)}\\
&+\frac{9\left(117-64\sqrt{13}\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{63869\left(13+2\sqrt{13}\right)(1+m)}\\
&+\frac{\left(423+\left(422-130\sqrt{13}\right)m\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{3757\sqrt{13}\left(13+2\sqrt{13}\right)(1+m)}\\
&+\frac{36(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(2,1+m,2+m,-\frac{3}{5}(1+4x)\right)}{7225(1+m)}\end{aligned}$$

output
```
1/11271*(268-195*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)+162/24565*(1+4*x)^(1+m)*hy
pergeom([1, 1+m],[2+m],-3/5-12/5*x)/(1+m)+36/7225*(1+4*x)^(1+m)*hypergeom(
[2, 1+m],[2+m],-3/5-12/5*x)/(1+m)+9/63869*(1+4*x)^(1+m)*hypergeom([1, 1+m]
,[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(117-64*13^(1/2))/(1+m)/(13+2*13^(1/2))+
1/48841*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*
(423+m*(422-130*13^(1/2)))/(1+m)*13^(1/2)/(13+2*13^(1/2))+9/63869*(1+4*x)^
(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(117+64*13^(1/2)
)/(1+m)/(13-2*13^(1/2))-1/48841*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(
1+4*x)/(13-2*13^(1/2)))*(423+2*m*(211+65*13^(1/2)))/(1+m)/(13-2*13^(1/2))*
13^(1/2)
```

3.943.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.76

$$\int \frac{(1+4x)^m}{(2+3x)^2(1-5x+3x^2)^2}\,dx = \frac{(1+4x)^{1+m}\left(\frac{16575(268-195x)}{1-5x+3x^2}+\frac{1232010\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{3}{5}(1+4x)\right)}{1+m}+\frac{26325\left(117+64\sqrt{13}\right)\text{Hypergeometric2F1}\left(\right.}{\left(13-2\sqrt{13}\right)(1+m}\right.}{}$$

input
```
Integrate[(1 + 4*x)^m/((2 + 3*x)^2*(1 - 5*x + 3*x^2)^2),x]
```

output
```
((1 + 4*x)^(1 + m)*((16575*(268 - 195*x))/(1 - 5*x + 3*x^2) + (1232010*Hyp
ergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(1 + m) + (26325*(117 +
 64*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[1
3])])/((13 - 2*Sqrt[13])*(1 + m)) + (26325*(117 - 64*Sqrt[13])*Hypergeomet
ric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[13])*
(1 + m)) - (425*((423*(2 + Sqrt[13]) + (2534 + 682*Sqrt[13])*m)*Hypergeome
tric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])] + (-423*(-2 + Sqrt[
13]) + (2534 - 682*Sqrt[13])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12
*x)/(13 + 2*Sqrt[13])]))/(1 + m) + (930852*Hypergeometric2F1[2, 1 + m, 2 +
 m, (-3*(1 + 4*x))/5])/(1 + m)))/186816825
```

3.943.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x+1)^m}{(3x+2)^2 (3x^2-5x+1)^2} \, dx$$

$$\Big\downarrow \text{1289}$$

$$\int \left(-\frac{3(54x-109)(4x+1)^m}{4913 (3x^2-5x+1)}+\frac{(46-27x)(4x+1)^m}{289 (3x^2-5x+1)^2}+\frac{162(4x+1)^m}{4913(3x+2)}+\frac{9(4x+1)^m}{289(3x+2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{162(4x+1)^{m+1} \text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{3}{5}(4x+1)\right)}{24565(m+1)}-\frac{\left(2\left(211+65\sqrt{13}\right)m+423\right)(4x+1)^{m+1} \text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{3757\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}+\frac{9\left(117+64\sqrt{13}\right)(4x+1)^{m+1} \text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{63869\left(13-2\sqrt{13}\right)(m+1)}+\frac{\left(\left(422-130\sqrt{13}\right)m+423\right)(4x+1)^{m+1} \text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{3757\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}+\frac{9\left(117-64\sqrt{13}\right)(4x+1)^{m+1} \text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{63869\left(13+2\sqrt{13}\right)(m+1)}+\frac{36(4x+1)^{m+1} \text{Hypergeometric2F1}\left(2,m+1,m+2,-\frac{3}{5}(4x+1)\right)}{7225(m+1)}+\frac{(268-195x)(4x+1)^{m+1}}{11271 (3x^2-5x+1)}$$

input
```
Int[(1 + 4*x)^m/((2 + 3*x)^2*(1 - 5*x + 3*x^2)^2),x]
```

output
```
((268 - 195*x)*(1 + 4*x)^(1 + m))/(11271*(1 - 5*x + 3*x^2)) + (162*(1 + 4*
x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(24565*(1
 + m)) + (9*(117 + 64*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 +
 m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(63869*(13 - 2*Sqrt[13])*(1 +
 m)) - ((423 + 2*(211 + 65*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F
1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(3757*Sqrt[13]*(13 -
2*Sqrt[13])*(1 + m)) + (9*(117 - 64*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeome
tric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(63869*(13 + 2*
Sqrt[13])*(1 + m)) + ((423 + (422 - 130*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hyp
ergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(3757*Sq
rt[13]*(13 + 2*Sqrt[13])*(1 + m)) + (36*(1 + 4*x)^(1 + m)*Hypergeometric2F
1[2, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(7225*(1 + m))
```

3.943.3.1 Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.943.4 Maple [F]

$$\int \frac{(1+4x)^m}{(2+3x)^2 (3x^2-5x+1)^2} dx$$

input
```
int((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x)
```

output
```
int((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x)
```

3.943.5 Fricas [F]

$$\int \frac{(1+4x)^m}{(2+3x)^2 (1-5x+3x^2)^2} \, dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)^2(3\,x+2)^2} \, dx$$

input
```
integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

output
```
integral((4*x + 1)^m/(81*x^6 - 162*x^5 - 45*x^4 + 162*x^3 + 13*x^2 - 28*x
+ 4), x)
```

3.943.6 Sympy [F]

$$\int \frac{(1+4x)^m}{(2+3x)^2 (1-5x+3x^2)^2} \, dx = \int \frac{(4x+1)^m}{(3x+2)^2 (3x^2-5x+1)^2} \, dx$$

input
```
integrate((1+4*x)**m/(2+3*x)**2/(3*x**2-5*x+1)**2,x)
```

output
```
Integral((4*x + 1)**m/((3*x + 2)**2*(3*x**2 - 5*x + 1)**2), x)
```

3.943.7 Maxima [F]

$$\int \frac{(1+4x)^m}{(2+3x)^2 (1-5x+3x^2)^2} \, dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)^2(3\,x+2)^2} \, dx$$

input
```
integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x, algorithm="maxima")
```

output
```
integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)^2*(3*x + 2)^2), x)
```

3.943.8 Giac [F]

$$\int \frac{(1+4x)^m}{(2+3x)^2 (1-5x+3x^2)^2} \, dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)^2(3\,x+2)^2} \, dx$$

input
```
integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x, algorithm="giac")
```

output
```
integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)^2*(3*x + 2)^2), x)
```

3.943.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x)^m}{(2+3x)^2 (1-5x+3x^2)^2} \, dx = \int \frac{(4\,x+1)^m}{(3\,x+2)^2\,(3\,x^2-5\,x+1)^2} \, dx$$

input
```
int((4*x + 1)^m/((3*x + 2)^2*(3*x^2 - 5*x + 1)^2),x)
```

output
```
int((4*x + 1)^m/((3*x + 2)^2*(3*x^2 - 5*x + 1)^2), x)
```

3.944 $\int \frac{(d+ex)^m(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx$

3.944.1 Optimal result

Integrand size = 27, antiderivative size = 237

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{1+m}}{(e^2-df)\sqrt{e+fx}}+\frac{2c(d+ex)^{1+m}\sqrt{e+fx}}{ef^2(3+2m)}$$
$$+\frac{2(c(d^2f^2+4de^2f(1+m)-4e^4(2+3m+m^2))-ef(3+2m)(aef(1+2m)+b(df-2e^2(1+m))))(d+}{ef^3(e^2-df)(3+2m)}$$

output
```
2*(a+e*(-b*f+c*e)/f^2)*(e*x+d)^(1+m)/(-d*f+e^2)/(f*x+e)^(1/2)+2*c*(e*x+d)^
(1+m)*(f*x+e)^(1/2)/e/f^2/(3+2*m)+2*(c*(d^2*f^2+4*d*e^2*f*(1+m)-4*e^4*(m^2
+3*m+2))-e*f*(3+2*m)*(a*e*f*(1+2*m)+b*(d*f-2*e^2*(1+m))))*(e*x+d)^m*hyperg
eom([1/2, -m],[3/2],e*(f*x+e)/(-d*f+e^2))*(f*x+e)^(1/2)/e/f^3/(-d*f+e^2)/(
3+2*m)/((-f*(e*x+d)/(-d*f+e^2))^m)
```

3.944.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \frac{2(d+ex)^m\left(\frac{f(d+ex)}{-e^2+df}\right)^{-m}\left(-3(ce^2+f(-be+af))\,\text{Hypergeometric2F1}\left(-\right.\right.}{}$$

input
```
Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

output
```
(2*(d + e*x)^m*(-3*(c*e^2 + f*(-(b*e) + a*f))*Hypergeometric2F1[-1/2, -m,
1/2, (e*(e + f*x))/(e^2 - d*f)] - (e + f*x)*((6*c*e - 3*b*f)*Hypergeometri
c2F1[1/2, -m, 3/2, (e*(e + f*x))/(e^2 - d*f)] - c*(e + f*x)*Hypergeometric
2F1[3/2, -m, 5/2, (e*(e + f*x))/(e^2 - d*f)])))/(3*f^3*((f*(d + e*x))/(-e^
2 + d*f))^m*Sqrt[e + f*x])
```

3.944.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1193, 27, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)(d+ex)^m}{(e+fx)^{3/2}}dx$$

$$\Big\downarrow \text{1193}$$

$$\frac{2\int \frac{(d+ex)^m\left(-c\left(d-\frac{e^2}{f}\right)xf^2-(-2b(m+1)e^2+af(2m+1)e+bdf)f+c(def-2e^3(m+1))\right)}{2f^2\sqrt{e+fx}}dx}{e^2-df}+\frac{2(d+ex)^{m+1}\left(a+\frac{e(ce-bf)}{f^2}\right)}{(e^2-df)\sqrt{e+fx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(d+ex)^m(c(def-2e^3(m+1))-f(-2b(m+1)e^2+af(2m+1)e+bdf)+cf(e^2-df)x)}{\sqrt{e+fx}}dx}{f^2(e^2-df)}+\frac{2(d+ex)^{m+1}\left(a+\frac{e(ce-bf)}{f^2}\right)}{(e^2-df)\sqrt{e+fx}}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{2c(e^2-df)\sqrt{e+fx}(d+ex)^{m+1}}{e(2m+3)}-\left(f\left(aef(2m+1)+bdf-2be^2(m+1)\right)-\frac{c(d^2f^2+4de^2f(m+1)-4e^4(m^2+3m+2))}{e(2m+3)}\right)\int \frac{(d+ex)^m}{\sqrt{e+fx}}d}{f^2(e^2-df)}\frac{2(d+ex)^{m+1}\left(a+\frac{e(ce-bf)}{f^2}\right)}{(e^2-df)\sqrt{e+fx}}$$

$$\Big\downarrow \text{80}$$

$$\frac{\frac{2c(e^2-df)\sqrt{e+fx}(d+ex)^{m+1}}{e(2m+3)} - (d+ex)^m \left(-\frac{f(d+ex)}{e^2-df}\right)^{-m} \left(f\left(aef(2m+1)+bdf-2be^2(m+1)\right) - \frac{c(d^2f^2+4de^2f(m+1)-\cdots}{e(2m+3\cdots}\right.}{f^2\left(e^2-df\right)}$$

$$\frac{2(d+ex)^{m+1}\left(a+\frac{e(ce-bf)}{f^2}\right)}{\left(e^2-df\right)\sqrt{e+fx}}$$

$\big\downarrow$ 79

$$\frac{\frac{2c(e^2-df)\sqrt{e+fx}(d+ex)^{m+1}}{e(2m+3)} - \frac{2\sqrt{e+fx}(d+ex)^m\left(-\frac{f(d+ex)}{e^2-df}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},\frac{e(e+fx)}{e^2-df}\right)\left(f\left(aef(2m+1)+bdf-2be^2(m+1)\right)-\cdots\right.}{f}}{f^2\left(e^2-df\right)}$$

$$\frac{2(d+ex)^{m+1}\left(a+\frac{e(ce-bf)}{f^2}\right)}{\left(e^2-df\right)\sqrt{e+fx}}$$

input
```
Int[((d + e*x)^m*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

output
```
(2*(a + (e*(c*e - b*f))/f^2)*(d + e*x)^(1 + m))/((e^2 - d*f)*Sqrt[e + f*x]
) + ((2*c*(e^2 - d*f)*(d + e*x)^(1 + m)*Sqrt[e + f*x])/(e*(3 + 2*m)) - (2*
(f*(b*d*f - 2*b*e^2*(1 + m) + a*e*f*(1 + 2*m)) - (c*(d^2*f^2 + 4*d*e^2*f*(
1 + m) - 4*e^4*(2 + 3*m + m^2)))/(e*(3 + 2*m)))*(d + e*x)^m*Sqrt[e + f*x]*
Hypergeometric2F1[1/2, -m, 3/2, (e*(e + f*x))/(e^2 - d*f)])/(f*(-((f*(d +
e*x))/(e^2 - d*f)))^m))/(f^2*(e^2 - d*f))
```

3.944.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

3.944. $\int \frac{(d+ex)^m(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx$

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

3.944.4 Maple [F]

$$\int \frac{(ex+d)^m (c\,x^2+bx+a)}{(fx+e)^{\frac{3}{2}}}dx$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x)
```

output
```
int((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x)
```

3.944.5 Fricas [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(e+fx)^{3/2}} \, dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{(fx+e)^{\frac{3}{2}}} \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)*sqrt(f*x + e)*(e*x + d)^m/(f^2*x^2 + 2*e*f*x +
e^2), x)
```

3.944.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(e+fx)^{3/2}} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x+a)/(f*x+e)**(3/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.944.7 Maxima [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(e+fx)^{3/2}} \, dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{(fx+e)^{\frac{3}{2}}} \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)*(e*x + d)^m/(f*x + e)^(3/2), x)
```

3.944.8 Giac [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{(fx+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)*(e*x + d)^m/(f*x + e)^(3/2), x)
```

3.944.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \int \frac{(d+e\,x)^m\,(c\,x^2+b\,x+a)}{(e+f\,x)^{3/2}}\,dx$$

input
```
int(((d + e*x)^m*(a + b*x + c*x^2))/(e + f*x)^(3/2),x)
```

output
```
int(((d + e*x)^m*(a + b*x + c*x^2))/(e + f*x)^(3/2), x)
```

3.945 $\int (d+ex)^m(f+gx)^2\sqrt{a+bx+cx^2}\,dx$

3.945.1 Optimal result

Integrand size = 29, antiderivative size = 509

$$\int (d+ex)^m(f+gx)^2\sqrt{a+bx+cx^2}\,dx = \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ce(4+m)}$$

$$+\frac{(e(bd-ae)g^2(1+m)+c(3d^2g^2+e^2f^2(4+m)-2defg(4+m)))\,(d+ex)^{1+m}\sqrt{a+bx+cx^2}\,\text{AppellF1}}{ce^3(1+m)(4+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2}{2cd-(}}}$$

$$-\frac{g(beg(5+2m)+2c(3dg-2ef(4+m)))(d+ex)^{2+m}\sqrt{a+bx+cx^2}\,\text{AppellF1}\left(2+m,-\frac{1}{2},-\frac{1}{2},3+m,\right.}{2ce^3(2+m)(4+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}$$

output
```
g^2*(e*x+d)^(1+m)*(c*x^2+b*x+a)^(3/2)/c/e/(4+m)+(e*(-a*e+b*d)*g^2*(1+m)+c*
(3*d^2*g^2+e^2*f^2*(4+m)-2*d*e*f*g*(4+m)))*(e*x+d)^(1+m)*AppellF1(1+m,-1/2
,-1/2,2+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-
e*(b+(-4*a*c+b^2)^(1/2))))*(c*x^2+b*x+a)^(1/2)/c/e^3/(1+m)/(4+m)/(1-2*c*(e
*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*(e*x+d)/(2*c*d-e*(b+(
-4*a*c+b^2)^(1/2))))^(1/2)-1/2*g*(b*e*g*(5+2*m)+2*c*(3*d*g-2*e*f*(4+m)))*(
e*x+d)^(2+m)*AppellF1(2+m,-1/2,-1/2,3+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^
2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(c*x^2+b*x+a)^(1/
2)/c/e^3/(2+m)/(4+m)/(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2
)/(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

3.945.2 Mathematica [F]

$$\int (d+ex)^m(f+gx)^2\sqrt{a+bx+cx^2}\,dx=\int (d+ex)^m(f+gx)^2\sqrt{a+bx+cx^2}\,dx$$

input
```
Integrate[(d + e*x)^m*(f + g*x)^2*Sqrt[a + b*x + c*x^2],x]
```

output
```
Integrate[(d + e*x)^m*(f + g*x)^2*Sqrt[a + b*x + c*x^2], x]
```

3.945.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 506, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1291, 27, 1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2\sqrt{a+bx+cx^2}(d+ex)^m\,dx$$

$$\Big\downarrow \text{1291}$$

$$\frac{\int \frac{1}{2}e(d+ex)^m\left(2ce(m+4)f^2-g^2(3bd+2ae(m+1))-g(6cdg+be(2m+5)g-4cef(m+4))x\right)\sqrt{cx^2+bx+a}}{ce^2(m+4)}$$

$$\frac{g^2\left(a+bx+cx^2\right)^{3/2}(d+ex)^{m+1}}{ce(m+4)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (d+ex)^m\left(2ce(m+4)f^2-g^2(3bd+2ae(m+1))-g(6cdg+be(2m+5)g-4cef(m+4))x\right)\sqrt{cx^2+bx+a}dx}{2ce(m+4)}$$

$$\frac{g^2\left(a+bx+cx^2\right)^{3/2}(d+ex)^{m+1}}{ce(m+4)}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\frac{2\left(eg^2(m+1)(bd-ae)+c\left(3d^2g^2-2defg(m+4)+e^2f^2(m+4)\right)\right)\int (d+ex)^m\sqrt{cx^2+bx+a}dx}{e}-\frac{g(beg(2m+5)+6cdg-4cef(m+4))\int (d+ex)^{m+1}\sqrt{cx^2}}{e}}{2ce(m+4)}$$

$$\frac{g^2\left(a+bx+cx^2\right)^{3/2}(d+ex)^{m+1}}{ce(m+4)}$$

$\Big\downarrow$ 1179

$$\frac{\frac{2\sqrt{a+bx+cx^2}\left(eg^2(m+1)(bd-ae)+c\left(3d^2g^2-2defg(m+4)+e^2f^2(m+4)\right)\right)\int (d+ex)^m\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}d(d+ex)}{e^2\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-}{2ce(m+4)}$$

$$\frac{g^2\left(a+bx+cx^2\right)^{3/2}(d+ex)^{m+1}}{ce(m+4)}$$

$\Big\downarrow$ 150

$$\frac{\frac{2\sqrt{a+bx+cx^2}(d+ex)^{m+1}\operatorname{AppellF1}\left(m+1,-\frac{1}{2},-\frac{1}{2},m+2,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\left(eg^2(m+1)(bd-ae)+c\left(3d^2g^2-2defg(m+4)+e\right.\right.}{e^2(m+1)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}}{2ce(}$$

$$\frac{g^2\left(a+bx+cx^2\right)^{3/2}(d+ex)^{m+1}}{ce(m+4)}$$

input
```
Int[(d + e*x)^m*(f + g*x)^2*Sqrt[a + b*x + c*x^2],x]
```

output
```
(g^2*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^(3/2))/(c*e*(4 + m)) + ((2*(e*(b*
d - a*e)*g^2*(1 + m) + c*(3*d^2*g^2 + e^2*f^2*(4 + m) - 2*d*e*f*g*(4 + m))
)*(d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 +
m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(1 + m)*Sqrt[1 - (2*c*(d + e*x))/
(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e)]) - (g*(6*c*d*g - 4*c*e*f*(4 + m) + b*e*g*(5 + 2*m
))*(d + e*x)^(2 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[2 + m, -1/2, -1/2, 3 +
 m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(2 + m)*Sqrt[1 - (2*c*(d + e*x))
/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)]))/(2*c*e*(4 + m))
```

3.945. $\int (d+ex)^m(f+gx)^2\sqrt{a+bx+cx^2}\,dx$

3.945.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1291
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && NeQ[m + n + 2*p + 1, 0]
```

3.945. $\int (d+ex)^m(f+gx)^2\sqrt{a+bx+cx^2}\,dx$

3.945.4 Maple [F]

$$\int (ex+d)^m (gx+f)^2 \sqrt{c\,x^2+bx+a}dx$$

input
```
int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x)
```

3.945.5 Fricas [F]

$$\int (d+ex)^m(f+gx)^2\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(gx+f)^2(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((g^2*x^2 + 2*f*g*x + f^2)*sqrt(c*x^2 + b*x + a)*(e*x + d)^m, x)
```

3.945.6 Sympy [F]

$$\int (d+ex)^m(f+gx)^2\sqrt{a+bx+cx^2}\,dx = \int (d+ex)^m (f+gx)^2 \sqrt{a+bx+cx^2}\,dx$$

input
```
integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x)**m*(f + g*x)**2*sqrt(a + b*x + c*x**2), x)
```

3.945.7 Maxima [F]

$$\int (d+ex)^m (f+gx)^2 \sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(gx+f)^2(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)^2*(e*x + d)^m, x)
```

3.945.8 Giac [F]

$$\int (d+ex)^m (f+gx)^2 \sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(gx+f)^2(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)^2*(e*x + d)^m, x)
```

3.945.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m (f+gx)^2 \sqrt{a+bx+cx^2}\,dx = \int (f+g\,x)^2\,(d+e\,x)^m\,\sqrt{c\,x^2+b\,x+a}\,dx$$

input
```
int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2)^(1/2), x)
```

3.946 $\int (d+ex)^m (f+gx)\sqrt{a+bx+cx^2}\,dx$

3.946.1 Optimal result

Integrand size = 27, antiderivative size = 388

$$\int (d+ex)^m (f+gx)\sqrt{a+bx+cx^2}\,dx$$
$$= \frac{(ef-dg)(d+ex)^{1+m}\sqrt{a+bx+cx^2}\,\mathrm{AppellF1}\left(1+m,-\frac{1}{2},-\frac{1}{2},2+m,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(1+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}$$
$$+\frac{g(d+ex)^{2+m}\sqrt{a+bx+cx^2}\,\mathrm{AppellF1}\left(2+m,-\frac{1}{2},-\frac{1}{2},3+m,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(2+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}$$

output
```
(-d*g+e*f)*(e*x+d)^(1+m)*AppellF1(1+m,-1/2,-1/2,2+m,2*c*(e*x+d)/(2*c*d-e*(
b-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(c*x^
2+b*x+a)^(1/2)/e^2/(1+m)/(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^
(1/2)/(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+g*(e*x+d)^(2+
m)*AppellF1(2+m,-1/2,-1/2,3+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))
,2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(c*x^2+b*x+a)^(1/2)/e^2/(2+
m)/(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*(e*x+d)/(
2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

3.946.2 Mathematica [F]

$$\int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx=\int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx$$

input
```
Integrate[(d + e*x)^m*(f + g*x)*Sqrt[a + b*x + c*x^2],x]
```

output
```
Integrate[(d + e*x)^m*(f + g*x)*Sqrt[a + b*x + c*x^2], x]
```

3.946.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\sqrt{a+bx+cx^2}(d+ex)^m\,dx$$

$$\Big\downarrow \ 1269$$

$$\frac{(ef-dg)\int (d+ex)^m\sqrt{cx^2+bx+a}dx}{e}+\frac{g\int (d+ex)^{m+1}\sqrt{cx^2+bx+a}dx}{e}$$

$$\Big\downarrow \ 1179$$

$$\frac{\sqrt{a+bx+cx^2}(ef-dg)\int (d+ex)^m\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}d(d+ex)}{e^2\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}+\frac{g\sqrt{a+bx+cx^2}\int (d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}d(d+ex)}{e^2\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$\Big\downarrow \ 150$$

$$\frac{\sqrt{a+bx+cx^2}(ef-dg)(d+ex)^{m+1}\operatorname{AppellF1}\left(m+1,-\frac{1}{2},-\frac{1}{2},m+2,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(m+1)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}+\frac{g\sqrt{a+bx+cx^2}(d+ex)^{m+2}\operatorname{AppellF1}\left(m+2,-\frac{1}{2},-\frac{1}{2},m+3,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(m+2)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

input
```
Int[(d + e*x)^m*(f + g*x)*Sqrt[a + b*x + c*x^2],x]
```

output
```
((e*f - d*g)*(d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2,
 -1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d
 + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(1 + m)*Sqrt[1 - (2*c*
(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]) + (g*(d + e*x)^(2 + m)*Sqrt[a + b*x +
 c*x^2]*AppellF1[2 + m, -1/2, -1/2, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - S
qrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]
)/(e^2*(2 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])
```

3.946.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

3.946.4 Maple [F]

$$\int (ex+d)^m (gx+f)\sqrt{c\,x^2+bx+a}dx$$

input
```
int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x)
```

3.946.5 Fricas [F]

$$\int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(gx+f)(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*(g*x + f)*(e*x + d)^m, x)
```

3.946.6 Sympy [F]

$$\int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx = \int (d+ex)^m\,(f+gx)\,\sqrt{a+bx+cx^2}\,dx$$

input
```
integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x)**m*(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

3.946.7 Maxima [F]

$$\int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(gx+f)(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)*(e*x + d)^m, x)
```

3.946.8 Giac [F]

$$\int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(gx+f)(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)*(e*x + d)^m, x)
```

3.946.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx = \int (f+g\,x)\,\left(d+e\,x\right)^m\,\sqrt{c\,x^2+b\,x+a}\,dx$$

input
```
int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2)^(1/2), x)
```

3.947 $\int (d+ex)^m \sqrt{a+bx+cx^2}\,dx$

3.947.1 Optimal result

Integrand size = 22, antiderivative size = 189

$$\int (d+ex)^m \sqrt{a+bx+cx^2}\,dx = \frac{(d+ex)^{1+m}\sqrt{a+bx+cx^2}\,\text{AppellF1}\left(1+m,-\frac{1}{2},-\frac{1}{2},2+m,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(1+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}$$

output
```
(e*x+d)^(1+m)*AppellF1(1+m,-1/2,-1/2,2+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b
^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(c*x^2+b*x+a)^(1
/2)/e/(1+m)/(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*
(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

3.947.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.10

$$\int (d+ex)^m \sqrt{a+bx+cx^2}\,dx = \frac{(d+ex)^{1+m}\sqrt{a+x(b+cx)}\,\text{AppellF1}\left(1+m,-\frac{1}{2},-\frac{1}{2},2+m,\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{e(1+m)\sqrt{\frac{e\left(-b+\sqrt{b^2-4ac}-2cx\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}}\sqrt{\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}}$$

input
```
Integrate[(d + e*x)^m*Sqrt[a + b*x + c*x^2],x]
```

output
```
((d + e*x)^(1 + m)*Sqrt[a + x*(b + c*x)]*AppellF1[1 + m, -1/2, -1/2, 2 + m
, (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*
c*d + (-b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[(e*(-b + Sqrt[b^2 - 4*
a*c] - 2*c*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[(e*(b + Sqrt[b^2
 - 4*a*c] + 2*c*x))/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)])
```

3.947.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx+cx^2}(d+ex)^m\,dx$$

$$\Big\downarrow \text{1179}$$

$$\frac{\sqrt{a+bx+cx^2}\int (d+ex)^m\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}d(d+ex)}{e\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sqrt{a+bx+cx^2}(d+ex)^{m+1}\text{AppellF1}\left(m+1,-\frac{1}{2},-\frac{1}{2},m+2,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

input
```
Int[(d + e*x)^m*Sqrt[a + b*x + c*x^2],x]
```

output
```
((d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m
, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*
c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)])
```

3.947.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

3.947.4 Maple [F]

$$\int (ex+d)^m \sqrt{c\,x^2+bx+a}dx$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x)
```

3.947.5 Fricas [F]

$$\int (d+ex)^m\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*(e*x + d)^m, x)
```

3.947.6 Sympy [F]

$$\int (d+ex)^m\sqrt{a+bx+cx^2}\,dx=\int (d+ex)^m\sqrt{a+bx+cx^2}\,dx$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x)**m*sqrt(a + b*x + c*x**2), x)
```

3.947.7 Maxima [F]

$$\int (d+ex)^m\sqrt{a+bx+cx^2}\,dx=\int \sqrt{cx^2+bx+a}(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m, x)
```

3.947.8 Giac [F]

$$\int (d+ex)^m\sqrt{a+bx+cx^2}\,dx=\int \sqrt{cx^2+bx+a}(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m, x)
```

3.947.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m\sqrt{a+bx+cx^2}\,dx = \int (d+e\,x)^m\,\sqrt{c\,x^2+b\,x+a}\,dx$$

input
```
int((d + e*x)^m*(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((d + e*x)^m*(a + b*x + c*x^2)^(1/2), x)
```

3.948 $\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx$

3.948.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx=\text{Int}\left(\frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx},x\right)$$

output
```
Unintegrable((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x)
```

3.948.2 Mathematica [N/A]

Not integrable

Time = 2.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx=\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx$$

input
```
Integrate[((d + e*x)^m*Sqrt[a + b*x + c*x^2])/(f + g*x),x]
```

output
```
Integrate[((d + e*x)^m*Sqrt[a + b*x + c*x^2])/(f + g*x), x]
```

3.948.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {1292}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex)^m}{f+gx} \, dx$$
$$\Big\downarrow \text{1292}$$
$$\int \frac{\sqrt{a+bx+cx^2}(d+ex)^m}{f+gx}dx$$

input
```
Int[((d + e*x)^m*Sqrt[a + b*x + c*x^2])/(f + g*x),x]
```

output
```
$Aborted
```

3.948.3.1 Defintions of rubi rules used

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

3.948.4 Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(ex+d)^m \sqrt{c\,x^2+bx+a}}{gx+f}dx$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x)
```

output
```
int((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x)
```

3.948.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx = \int \frac{\sqrt{cx^2+bx+a}(ex+d)^m}{gx+f}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

3.948.6 Sympy [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx = \int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x+a)**(1/2)/(g*x+f),x)
```

output
```
Integral((d + e*x)**m*sqrt(a + b*x + c*x**2)/(f + g*x), x)
```

3.948.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx = \int \frac{\sqrt{cx^2+bx+a}(ex+d)^m}{gx+f}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

3.948.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx=\int \frac{\sqrt{cx^2+bx+a}(ex+d)^m}{gx+f}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

3.948.9 Mupad [N/A]

Not integrable

Time = 12.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx=\int \frac{{\left(d+e\,x\right)}^m\,\sqrt{c\,x^2+b\,x+a}}{f+g\,x}\,dx$$

input
```
int(((d + e*x)^m*(a + b*x + c*x^2)^(1/2))/(f + g*x),x)
```

output
```
int(((d + e*x)^m*(a + b*x + c*x^2)^(1/2))/(f + g*x), x)
```

3.949 $\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx$

3.949.1 Optimal result

Integrand size = 29, antiderivative size = 502

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{g^2(d+ex)^{1+m}\sqrt{a+bx+cx^2}}{ce(2+m)} + \frac{(e(bd-ae)g^2(1+m)+c(d^2g^2+e^2f^2(2+m)-2defg(2+m)))\,(d+ex)^{1+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{\cdots}}{ce^3(1+m)(2+m)\sqrt{a+\cdots}} - \frac{g(beg(3+2m)+c(2dg-4ef(2+m)))(d+ex)^{2+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\operatorname{Appe}\cdots}{2ce^3(2+m)^2\sqrt{a+bx+cx^2}}$$

output
```
g^2*(e*x+d)^(1+m)*(c*x^2+b*x+a)^(1/2)/c/e/(2+m)+(e*(-a*e+b*d)*g^2*(1+m)+c*
(d^2*g^2+e^2*f^2*(2+m)-2*d*e*f*g*(2+m)))*(e*x+d)^(1+m)*AppellF1(1+m,1/2,1/
2,2+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b
+(-4*a*c+b^2)^(1/2))))*(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1
/2)*(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^3/(1+m)/(2+
m)/(c*x^2+b*x+a)^(1/2)-1/2*g*(b*e*g*(3+2*m)+c*(2*d*g-4*e*f*(2+m)))*(e*x+d)
^(2+m)*AppellF1(2+m,1/2,1/2,3+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)
)),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(1-2*c*(e*x+d)/(2*c*d-e*(
b-(-4*a*c+b^2)^(1/2))))^(1/2)*(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2
))))^(1/2)/c/e^3/(2+m)^2/(c*x^2+b*x+a)^(1/2)
```

3.949.2 Mathematica [F]

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx$$

input
```
Integrate[((d + e*x)^m*(f + g*x)^2)/Sqrt[a + b*x + c*x^2],x]
```

output
```
Integrate[((d + e*x)^m*(f + g*x)^2)/Sqrt[a + b*x + c*x^2], x]
```

3.949.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1291, 27, 1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{1291}$$

$$\frac{\int \frac{e(d+ex)^m\left(2ce(m+2)f^2-g^2(bd+2ae(m+1))-g(2cdg+be(2m+3)g-4cef(m+2))x\right)}{2\sqrt{cx^2+bx+a}}dx}{ce^2(m+2)}+\frac{g^2\sqrt{a+bx+cx^2}(d+ex)^{m+1}}{ce(m+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(d+ex)^m\left(2ce(m+2)f^2-g^2(bd+2ae(m+1))-g(2cdg+be(2m+3)g-4cef(m+2))x\right)}{\sqrt{cx^2+bx+a}}dx}{2ce(m+2)}+\frac{g^2\sqrt{a+bx+cx^2}(d+ex)^{m+1}}{ce(m+2)}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\frac{2\left(eg^2(m+1)(bd-ae)+c\left(d^2g^2-2defg(m+2)+e^2f^2(m+2)\right)\right)\int \frac{(d+ex)^m}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{g(beg(2m+3)+2cdg-4cef(m+2))\int \frac{(d+ex)^{m+1}}{\sqrt{cx^2+bx+a}}dx}{e}}{2ce(m+2)}+\frac{g^2\sqrt{a+bx+cx^2}(d+ex)^{m+1}}{ce(m+2)}$$

$\downarrow$ 1179

$$\frac{2\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(eg^2(m+1)(bd-ae)+c\left(d^2g^2-2defg(m+2)+e^2f^2(m+2)\right)\right)\int \frac{(d+ex)^m}{\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-}}}}{e^2\sqrt{a+bx+cx^2}}$$

$$\frac{g^2\sqrt{a+bx+cx^2}(d+ex)^{m+1}}{ce(m+2)}$$

$\downarrow$ 150

$$\frac{2(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\operatorname{AppellF1}\left(m+1,\frac{1}{2},\frac{1}{2},m+2,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\left(eg^2(m+1)(b\right.}{e^2(m+1)\sqrt{a+bx+cx^2}}$$

$$\frac{g^2\sqrt{a+bx+cx^2}(d+ex)^{m+1}}{ce(m+2)}$$

input
```
Int[((d + e*x)^m*(f + g*x)^2)/Sqrt[a + b*x + c*x^2],x]
```

output
```
(g^2*(d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2])/(c*e*(2 + m)) + ((2*(e*(b*d
- a*e)*g^2*(1 + m) + c*(d^2*g^2 + e^2*f^2*(2 + m) - 2*d*e*f*g*(2 + m)))*(d
 + e*x)^(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*AppellF1
[1 + m, 1/2, 1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(1 + m)*Sqr
t[a + b*x + c*x^2]) - (g*(2*c*d*g - 4*c*e*f*(2 + m) + b*e*g*(3 + 2*m))*(d
+ e*x)^(2 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*AppellF1[
2 + m, 1/2, 1/2, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(2 + m)*Sqrt
[a + b*x + c*x^2]))/(2*c*e*(2 + m))
```

3.949. $\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx$

3.949.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1291
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && NeQ[m + n + 2*p + 1, 0]
```

3.949. $\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx$

3.949.4 Maple [F]

$$\int \frac{(ex+d)^m (gx+f)^2}{\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x)
```

3.949.5 Fricas [F]

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(gx+f)^2(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((g^2*x^2 + 2*f*g*x + f^2)*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

3.949.6 Sympy [F]

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)^m (f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((e*x+d)**m*(g*x+f)**2/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x)**m*(f + g*x)**2/sqrt(a + b*x + c*x**2), x)
```

3.949.7 Maxima [F]

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(gx+f)^2(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^2*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

3.949.8 Giac [F]

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(gx+f)^2(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)^2*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

3.949.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(f+g\,x)^2\,(d+e\,x)^m}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((f + g*x)^2*(d + e*x)^m)/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int(((f + g*x)^2*(d + e*x)^m)/(a + b*x + c*x^2)^(1/2), x)
```

3.950 $\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx$

3.950.1 Optimal result

Integrand size = 27, antiderivative size = 388

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx = \frac{(ef-dg)(d+ex)^{1+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\operatorname{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,\frac{2c(d+ex)}{2cd-\left(b-\right.}\right.}{e^2(1+m)\sqrt{a+bx+cx^2}} + \frac{g(d+ex)^{2+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\operatorname{AppellF1}\left(2+m,\frac{1}{2},\frac{1}{2},3+m,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4a\right.}}\right.}{e^2(2+m)\sqrt{a+bx+cx^2}}$$

output
```
(-d*g+e*f)*(e*x+d)^(1+m)*AppellF1(1+m,1/2,1/2,2+m,2*c*(e*x+d)/(2*c*d-e*(b-
(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(1-2*c*
(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(1-2*c*(e*x+d)/(2*c*d-e*(b
+(-4*a*c+b^2)^(1/2))))^(1/2)/e^2/(1+m)/(c*x^2+b*x+a)^(1/2)+g*(e*x+d)^(2+m)
*AppellF1(2+m,1/2,1/2,3+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c
*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*
a*c+b^2)^(1/2))))^(1/2)*(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(
1/2)/e^2/(2+m)/(c*x^2+b*x+a)^(1/2)
```

3.950.2 Mathematica [F]

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx$$

input
```
Integrate[((d + e*x)^m*(f + g*x))/Sqrt[a + b*x + c*x^2],x]
```

output
```
Integrate[((d + e*x)^m*(f + g*x))/Sqrt[a + b*x + c*x^2], x]
```

3.950.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{1269}$$

$$\frac{(ef-dg)\int \frac{(d+ex)^m}{\sqrt{cx^2+bx+a}}dx}{e}+\frac{g\int \frac{(d+ex)^{m+1}}{\sqrt{cx^2+bx+a}}dx}{e}$$

$$\Big\downarrow \text{1179}$$

$$\frac{(ef-dg)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{(d+ex)^m}{\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}d(d+ex)}{e^2\sqrt{a+bx+cx^2}}+\frac{g\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{(d+ex)^{m+1}}{\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}d(d+ex)}{e^2\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{150}$$

$$\frac{(ef-dg)(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\operatorname{AppellF1}\left(m+1,\frac{1}{2},\frac{1}{2},m+2,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4\right.}\right.}}{e^2(m+1)\sqrt{a+bx+cx^2}}$$

$$\frac{g(d+ex)^{m+2}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\operatorname{AppellF1}\left(m+2,\frac{1}{2},\frac{1}{2},m+3,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{}{2cd\cdot}\right.}{e^2(m+2)\sqrt{a+bx+cx^2}}$$

input
```
Int[((d + e*x)^m*(f + g*x))/Sqrt[a + b*x + c*x^2],x]
```

output
```
((e*f - d*g)*(d + e*x)^(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt
[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e)]*AppellF1[1 + m, 1/2, 1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt
[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(
e^2*(1 + m)*Sqrt[a + b*x + c*x^2]) + (g*(d + e*x)^(2 + m)*Sqrt[1 - (2*c*(d
 + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*AppellF1[2 + m, 1/2, 1/2, 3 + m, (2*c*(d
 + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e)])/(e^2*(2 + m)*Sqrt[a + b*x + c*x^2])
```

3.950.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

3.950. $\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx$

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

3.950.4 Maple [F]

$$\int \frac{(ex+d)^m (gx+f)}{\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

3.950.5 Fricas [F]

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(gx+f)(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((g*x + f)*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

3.950.6 Sympy [F]

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)^m\,(f+gx)}{\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((e*x+d)**m*(g*x+f)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x)**m*(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

3.950.7 Maxima [F]

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(gx+f)(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

3.950.8 Giac [F]

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(gx+f)(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

3.950.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+g\,x)\,(d+e\,x)^m}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((f + g*x)*(d + e*x)^m)/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int(((f + g*x)*(d + e*x)^m)/(a + b*x + c*x^2)^(1/2), x)
```

3.951 $\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx$

3.951.1 Optimal result

Integrand size = 22, antiderivative size = 189

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx = \frac{(d+ex)^{1+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\,\text{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\right.}{e(1+m)\sqrt{a+bx+cx^2}}$$

output
```
(e*x+d)^(1+m)*AppellF1(1+m,1/2,1/2,2+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2
)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(1-2*c*(e*x+d)/(2*
c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^
2)^(1/2))))^(1/2)/e/(1+m)/(c*x^2+b*x+a)^(1/2)
```

3.951.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{\frac{e\left(-b+\sqrt{b^2-4ac}-2cx\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}}\sqrt{\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}(d+ex)^{1+m}\,\text{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\frac{}{2cd+(}\right.}{e(1+m)\sqrt{a+x(b+cx)}}$$

input
```
Integrate[(d + e*x)^m/Sqrt[a + b*x + c*x^2],x]
```

output
```
(Sqrt[(e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)]*Sqrt[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*d + (b + Sqrt[b^2 -
4*a*c])*e)]*(d + e*x)^(1 + m)*AppellF1[1 + m, 1/2, 1/2, 2 + m, (2*c*(d + e
*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d + (-b + S
qrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[a + x*(b + c*x)])
```

3.951.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1179}$$

$$\frac{\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{(d+ex)^m}{\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}d(d+ex)}{e\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{150}$$

$$\frac{(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{AppellF1}\left(m+1,\frac{1}{2},\frac{1}{2},m+2,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{}{2cd-}\right.}{e(m+1)\sqrt{a+bx+cx^2}}$$

input
```
Int[(d + e*x)^m/Sqrt[a + b*x + c*x^2],x]
```

output
```
((d + e*x)^(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c
])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Appel
lF1[1 + m, 1/2, 1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c
])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sq
rt[a + b*x + c*x^2])
```

3.951.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

3.951.4 Maple [F]

$$\int \frac{(ex+d)^m}{\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x)
```

3.951.5 Fricas [F]

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

3.951.6 Sympy [F]

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((e*x+d)**m/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x)**m/sqrt(a + b*x + c*x**2), x)
```

3.951.7 Maxima [F]

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

3.951.8 Giac [F]

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

3.951.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{{\left(d+e\,x\right)}^m}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x)^m/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((d + e*x)^m/(a + b*x + c*x^2)^(1/2), x)
```

3.952 $\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$

3.952.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx=\text{Int}\left(\frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}},x\right)$$

output
```
Unintegrable((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

3.952.2 Mathematica [N/A]

Not integrable

Time = 5.77 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx=\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$$

input
```
Integrate[(d + e*x)^m/((f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
Integrate[(d + e*x)^m/((f + g*x)*Sqrt[a + b*x + c*x^2]), x]
```

3.952.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {1292}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}} \, dx$$
$$\Big\downarrow \text{1292}$$
$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}dx$$

input
```
Int[(d + e*x)^m/((f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
$Aborted
```

3.952.3.1 Defintions of rubi rules used

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

3.952.4 Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(ex+d)^m}{(gx+f)\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

3.952.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{cx^2+bx+a}(gx+f)}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(c*g*x^3 + (c*f + b*g)*x^2 + a*
f + (b*f + a*g)*x), x)
```

3.952.6 Sympy [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((e*x+d)**m/(g*x+f)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x)**m/((f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.952.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{cx^2+bx+a}(gx+f)}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^m/(sqrt(c*x^2 + b*x + a)*(g*x + f)), x)
```

3.952.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{cx^2+bx+a}(gx+f)}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m/(sqrt(c*x^2 + b*x + a)*(g*x + f)), x)
```

3.952.9 Mupad [N/A]

Not integrable

Time = 73.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+e\,x)^m}{(f+g\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x)^m/((f + g*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x)^m/((f + g*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.953 $\int (d+ex)^m(f+gx)^n \left(a+bx+cx^2\right) dx$

3.953.1 Optimal result

Integrand size = 25, antiderivative size = 265

$$\int (d+ex)^m(f+gx)^n \left(a+bx+cx^2\right) dx = \frac{(beg(3+m+n)-c(ef(2+m)+dg(4+m+2n)))(d+ex)^{1+m}(f+gx)^{1+n}}{e^2g^2(2+m+n)(3+m+n)} + \frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{e^2g(3+m+n)} + \frac{(g(2+m+n)\left(ae^2g(3+m+n)-cd(ef(2+m)+dg(1+n))\right)-(ef(1+m)+dg(1+n))(beg(3+m$$

$$e^3g^2($$

output
```
(b*e*g*(3+m+n)-c*(e*f*(2+m)+d*g*(4+m+2*n)))*(e*x+d)^(1+m)*(g*x+f)^(1+n)/e^
2/g^2/(2+m+n)/(3+m+n)+c*(e*x+d)^(2+m)*(g*x+f)^(1+n)/e^2/g/(3+m+n)+(g*(2+m+
n)*(a*e^2*g*(3+m+n)-c*d*(e*f*(2+m)+d*g*(1+n)))-(e*f*(1+m)+d*g*(1+n))*(b*e*
g*(3+m+n)-c*(e*f*(2+m)+d*g*(4+m+2*n))))*(e*x+d)^(1+m)*(g*x+f)^n*hypergeom(
[-n, 1+m],[2+m],-g*(e*x+d)/(-d*g+e*f))/e^3/g^2/(1+m)/(2+m+n)/(3+m+n)/((e*(
g*x+f)/(-d*g+e*f))^n)
```

3.953.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.71

$$\int (d+ex)^m(f+gx)^n\left(a+bx+cx^2\right)\,dx$$
$$=\frac{(d+ex)^{1+m}(f+gx)^n\left(\frac{e(f+gx)}{ef-dg}\right)^{-n}\left(c(ef-dg)^2\,\text{Hypergeometric2F1}\left(1+m,-2-n,2+m,\frac{g(d+ex)}{-ef+dg}\right)+\ldots\right.}{\ldots}$$

input
```
Integrate[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2),x]
```

output
```
((d + e*x)^(1 + m)*(f + g*x)^n*(c*(e*f - d*g)^2*Hypergeometric2F1[1 + m, -
2 - n, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)] + e*(-((2*c*f - b*g)*(e*f - d*
g)*Hypergeometric2F1[1 + m, -1 - n, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)])
+ e*(c*f^2 + g*(-(b*f) + a*g))*Hypergeometric2F1[1 + m, -n, 2 + m, (g*(d +
 e*x))/(-(e*f) + d*g)])))/(e^3*g^2*(1 + m)*((e*(f + g*x))/(e*f - d*g))^n)
```

3.953.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1194, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)(d+ex)^m(f+gx)^n\,dx$$

$$\Big\downarrow \text{1194}$$

$$\frac{\int (d+ex)^m(f+gx)^n\left(ag(m+n+3)e^2-(cef(m+2)-beg(m+n+3)+cdg(m+2n+4))xe-cd(ef(m+2)\ldots\right.}{e^2g(m+n+3)}$$
$$\frac{c(d+ex)^{m+2}(f+gx)^{n+1}}{e^2g(m+n+3)}$$

$$\Big\downarrow \text{90}$$

$$\frac{\left(ae^2g(m+n+3)+\frac{(dg(n+1)+ef(m+1))(-beg(m+n+3)+cdg(m+2n+4)+cef(m+2))}{g(m+n+2)}-cd(dg(n+1)+ef(m+2))\right)\int(d+e}{e^2g(m+n+3)}$$

$$\frac{c(d+ex)^{m+2}(f+gx)^{n+1}}{e^2g(m+n+3)}$$

$\downarrow$ 80

$$\frac{(f+gx)^n\left(\frac{e(f+gx)}{ef-dg}\right)^{-n}\left(ae^2g(m+n+3)+\frac{(dg(n+1)+ef(m+1))(-beg(m+n+3)+cdg(m+2n+4)+cef(m+2))}{g(m+n+2)}-cd(dg(n+1)\right.}{e^2g(m+n}$$

$$\frac{c(d+ex)^{m+2}(f+gx)^{n+1}}{e^2g(m+n+3)}$$

$\downarrow$ 79

$$\frac{\frac{(d+ex)^{m+1}(f+gx)^n\left(\frac{e(f+gx)}{ef-dg}\right)^{-n}\text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{g(d+ex)}{ef-dg}\right)\left(ae^2g(m+n+3)+\frac{(dg(n+1)+ef(m+1))(-beg(m+n+3)+cdg(m+2n}{g(m+n+2)}\right.}{e(m+1)}}{e^2g(m+n+3)}$$

$$\frac{c(d+ex)^{m+2}(f+gx)^{n+1}}{e^2g(m+n+3)}$$

input
```
Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2),x]
```

output
```
(c*(d + e*x)^(2 + m)*(f + g*x)^(1 + n))/(e^2*g*(3 + m + n)) + (-(((c*e*f*(
2 + m) - b*e*g*(3 + m + n) + c*d*g*(4 + m + 2*n))*(d + e*x)^(1 + m)*(f + g
*x)^(1 + n))/(g*(2 + m + n))) + ((a*e^2*g*(3 + m + n) - c*d*(e*f*(2 + m) +
 d*g*(1 + n)) + ((e*f*(1 + m) + d*g*(1 + n))*(c*e*f*(2 + m) - b*e*g*(3 + m
 + n) + c*d*g*(4 + m + 2*n)))/(g*(2 + m + n)))*(d + e*x)^(1 + m)*(f + g*x)
^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((g*(d + e*x))/(e*f - d*g))])/(e*(
1 + m)*((e*(f + g*x))/(e*f - d*g))^n))/(e^2*g*(3 + m + n))
```

3.953.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

3.953.4 Maple [F]

$$\int (ex+d)^m (gx+f)^n \left(c\,x^2+bx+a\right) dx$$

input
```
int((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x)
```

output
```
int((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x)
```

3.953.5 Fricas [F]

$$\int (d+ex)^m(f+gx)^n \left(a+bx+cx^2\right)\,dx = \int \left(cx^2+bx+a\right)(ex+d)^m(gx+f)^n\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)*(e*x + d)^m*(g*x + f)^n, x)
```

3.953.6 Sympy [F(-2)]

Exception generated.

$$\int (d+ex)^m (f+gx)^n \left(a+bx+cx^2\right) \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x+d)**m*(g*x+f)**n*(c*x**2+b*x+a),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.953.7 Maxima [F]

$$\int (d+ex)^m (f+gx)^n \left(a+bx+cx^2\right) \, dx = \int (cx^2+bx+a)(ex+d)^m (gx+f)^n \, dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)*(e*x + d)^m*(g*x + f)^n, x)
```

3.953.8 Giac [F]

$$\int (d+ex)^m (f+gx)^n \left(a+bx+cx^2\right) \, dx = \int (cx^2+bx+a)(ex+d)^m (gx+f)^n \, dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)*(e*x + d)^m*(g*x + f)^n, x)
```

3.953.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m (f+gx)^n \left(a+bx+cx^2\right) dx = \int (f+g\,x)^n\,(d+e\,x)^m\,\left(c\,x^2+b\,x+a\right) dx$$

input
```
int((f + g*x)^n*(d + e*x)^m*(a + b*x + c*x^2),x)
```

output
```
int((f + g*x)^n*(d + e*x)^m*(a + b*x + c*x^2), x)
```

3.954 $\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right)^p \, dx$

3.954.1 Optimal result

Integrand size = 27, antiderivative size = 525

$$\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right)^p \, dx = \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ce(3+m+2p)}$$

$$+\frac{(e(bd-ae)g^2(1+m)+c(2d^2g^2(1+p)+e^2f^2(3+m+2p)-2defg(3+m+2p)))\,(d+ex)^{1+m}\left(a+b\right.}{}$$

$$-\frac{g(beg(2+m+p)+2c(dg(1+p)-ef(3+m+2p)))(d+ex)^{2+m}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)}\right.}{ce^3(2+m)(3}$$

output
```
g^2*(e*x+d)^(1+m)*(c*x^2+b*x+a)^(p+1)/c/e/(3+m+2*p)+(e*(-a*e+b*d)*g^2*(1+m
)+c*(2*d^2*g^2*(p+1)+e^2*f^2*(3+m+2*p)-2*d*e*f*g*(3+m+2*p)))*(e*x+d)^(1+m)
*(c*x^2+b*x+a)^p*AppellF1(1+m,-p,-p,2+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^
2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))/c/e^3/(1+m)/(3+m+
2*p)/((1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c*(e*x+d)/
(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^p)-g*(b*e*g*(2+m+p)+2*c*(d*g*(p+1)-e*f*(
3+m+2*p)))*(e*x+d)^(2+m)*(c*x^2+b*x+a)^p*AppellF1(2+m,-p,-p,3+m,2*c*(e*x+d
)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1
/2))))/c/e^3/(2+m)/(3+m+2*p)/((1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2
))))^p)/((1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^p)
```

3.954.2 Mathematica [F]

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^p\,dx=\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^p\,dx$$

input
```
Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^p,x]
```

output
```
Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^p, x]
```

3.954.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 518, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1291, 27, 1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2(d+ex)^m\left(a+bx+cx^2\right)^p\,dx$$

$$\Big\downarrow \text{1291}$$

$$\frac{\int e(d+ex)^m\left(ce(m+2p+3)f^2-g^2(ae(m+1)+bd(p+1))-g(2cdg(p+1)+beg(m+p+2)-2cef(m+2p\right.}{ce^2(m+2p+3)}$$

$$\frac{g^2(d+ex)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ce(m+2p+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (d+ex)^m\left(ce(m+2p+3)f^2-g^2(ae(m+1)+bd(p+1))-g(2cdg(p+1)+beg(m+p+2)-2cef(m+2p+\right.}{ce(m+2p+3)}$$

$$\frac{g^2(d+ex)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ce(m+2p+3)}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\frac{(eg^2(m+1)(bd-ae)+c(2d^2g^2(p+1)-2defg(m+2p+3)+e^2f^2(m+2p+3)))\int (d+ex)^m(cx^2+bx+a)^p dx}{e} - \frac{g(beg(m+p+2)+2cdg(p+1)-2cef(m+}{e}}{ce(m+2p+3)}$$
$$\frac{g^2(d+ex)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ce(m+2p+3)}$$

$$\Big\downarrow \text{ 1179}$$

$$\frac{\frac{(a+bx+cx^2)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}(eg^2(m+1)(bd-ae)+c(2d^2g^2(p+1)-2defg(m+2p+3)+e^2f^2(m+2p+3))}{e^2}}{}$$

$$\frac{g^2(d+ex)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ce(m+2p+3)}$$

$$\Big\downarrow \text{ 150}$$

$$\frac{(d+ex)^{m+1}(a+bx+cx^2)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}\text{AppellF1}\left(m+1,-p,-p,m+2,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+}{2cd-\left(b+\sqrt{}\right.}\right.}{e^2(m+1)}$$

$$\frac{g^2(d+ex)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ce(m+2p+3)}$$

input
```
Int[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^p,x]
```

output
```
(g^2*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^(1 + p))/(c*e*(3 + m + 2*p)) + ((
(e*(b*d - a*e)*g^2*(1 + m) + c*(2*d^2*g^2*(1 + p) + e^2*f^2*(3 + m + 2*p)
- 2*d*e*f*g*(3 + m + 2*p)))*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1
[1 + m, -p, -p, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
, (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(1 + m)*(1 -
(2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x)
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p) - (g*(2*c*d*g*(1 + p) + b*e*g*(2
 + m + p) - 2*c*e*f*(3 + m + 2*p))*(d + e*x)^(2 + m)*(a + b*x + c*x^2)^p*A
ppellF1[2 + m, -p, -p, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a
*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(2 + m
)*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d
 + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p))/(c*e*(3 + m + 2*p))
```

3.954. $\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^p\,dx$

3.954.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1291
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && NeQ[m + n + 2*p + 1, 0]
```

3.954. $\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right)^p \, dx$

3.954.4 Maple [F]

$$\int (ex+d)^{m} (gx+f)^{2} \left(c\,x^{2}+bx+a\right)^{p}dx$$

input
```
int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x)
```

output
```
int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x)
```

3.954.5 Fricas [F]

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^p\,dx = \int (gx+f)^2\left(cx^2+bx+a\right)^p(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((g^2*x^2 + 2*f*g*x + f^2)*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

3.954.6 Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^p\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a)**p,x)
```

output
```
Timed out
```

3.954.7 Maxima [F]

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^p\,dx = \int (gx+f)^2\left(cx^2+bx+a\right)^p(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((g*x + f)^2*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

3.954.8 Giac [F]

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^p\,dx = \int (gx+f)^2\left(cx^2+bx+a\right)^p(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((g*x + f)^2*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

3.954.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^p\,dx = \int (f+g\,x)^2\,(d+e\,x)^m\,\left(c\,x^2+b\,x+a\right)^p\,dx$$

input
```
int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2)^p,x)
```

output
```
int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2)^p, x)
```

3.955 $\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx$

3.955.1 Optimal result

Integrand size = 25, antiderivative size = 384

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx = \frac{(ef-dg)(d+ex)^{1+m}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p}\text{AppellF1}\left(1+\cdots\right.}{e^2(1+m)} + \frac{g(d+ex)^{2+m}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p}\text{AppellF1}\left(2+m,-\cdots\right.}{e^2(2+m)}$$

output
```
(-d*g+e*f)*(e*x+d)^(1+m)*(c*x^2+b*x+a)^p*AppellF1(1+m,-p,-p,2+m,2*c*(e*x+d
)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1
/2))))/e^2/(1+m)/((1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-
2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^p)+g*(e*x+d)^(2+m)*(c*x^2+b*
x+a)^p*AppellF1(2+m,-p,-p,3+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))
,2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))/e^2/(2+m)/((1-2*c*(e*x+d)/(
2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^
2)^(1/2))))^p)
```

3.955.2 Mathematica [F]

$$\int (d+ex)^m(f+gx) \left(a+bx+cx^2\right)^p \, dx = \int (d+ex)^m(f+gx) \left(a+bx+cx^2\right)^p \, dx$$

input
```
Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p,x]
```

output
```
Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p, x]
```

3.955.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)(d+ex)^m \left(a+bx+cx^2\right)^p \, dx$$

$$\Big\downarrow \text{1269}$$

$$\frac{(ef-dg)\int (d+ex)^m \left(cx^2+bx+a\right)^p dx}{e}+\frac{g\int (d+ex)^{m+1} \left(cx^2+bx+a\right)^p dx}{e}$$

$$\Big\downarrow \text{1179}$$

$$\frac{(ef-dg)\left(a+bx+cx^2\right)^p \left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p} \left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p} \int (d+ex)^m \left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4a\right.}\right.}\right.}{e^2}$$

$$\frac{g\left(a+bx+cx^2\right)^p \left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p} \left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p} \int (d+ex)^{m+1} \left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^p}{e^2}$$

$$\Big\downarrow \text{150}$$

$$\frac{(ef-dg)(d+ex)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}\text{AppellF1}\left(m+1,\right.}{e^2(m+1)}$$

$$\frac{g(d+ex)^{m+2}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}\text{AppellF1}\left(m+2,-p,-p,\right.}{e^2(m+2)}$$

input
```
Int[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p,x]
```

output
```
((e*f - d*g)*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p,
 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x
))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(1 + m)*(1 - (2*c*(d + e*x))
/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e))^p) + (g*(d + e*x)^(2 + m)*(a + b*x + c*x^2)^p*Appe
llF1[2 + m, -p, -p, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c]
)*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(2 + m)*(
1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d +
e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

3.955.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

3.955. $\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx$

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

3.955.4 Maple [F]

$$\int (ex+d)^m (gx+f) \left(c\,x^2+bx+a\right)^p dx$$

input
```
int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x)
```

output
```
int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x)
```

3.955.5 Fricas [F]

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx = \int (gx+f)\left(cx^2+bx+a\right)^p(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((g*x + f)*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

3.955.6 Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a)**p,x)
```

output
```
Timed out
```

3.955.7 Maxima [F]

$$\int (d+ex)^m(f+gx) \left(a+bx+cx^2\right)^p \, dx = \int (gx+f)\left(cx^2+bx+a\right)^p(ex+d)^m \, dx$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((g*x + f)*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

3.955.8 Giac [F]

$$\int (d+ex)^m(f+gx) \left(a+bx+cx^2\right)^p \, dx = \int (gx+f)\left(cx^2+bx+a\right)^p(ex+d)^m \, dx$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((g*x + f)*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

3.955.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx) \left(a+bx+cx^2\right)^p \, dx = \int (f+g\,x)\,\left(d+e\,x\right)^m\,\left(c\,x^2+b\,x+a\right)^p \, dx$$

input
```
int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2)^p,x)
```

output
```
int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2)^p, x)
```

3.956 $\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx$

3.956.1 Optimal result

Integrand size = 20, antiderivative size = 187

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \frac{(d+ex)^{1+m}\left(a+bx+cx^2\right)^p \left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p} \operatorname{AppellF1}\left(1+m,-p,\cdots\right.}{e(1+m)}$$

output
```
(e*x+d)^(1+m)*(c*x^2+b*x+a)^p*AppellF1(1+m,-p,-p,2+m,2*c*(e*x+d)/(2*c*d-e*
(b-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))/e/(1
+m)/((1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c*(e*x+d)/(
2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^p)
```

3.956.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.10

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \frac{\left(\frac{e\left(-b+\sqrt{b^2-4ac}-2cx\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p}(d+ex)^{1+m}(a+x(b+cx))^p \operatorname{AppellF1}\left(1+m,-p,-p,2\cdots\right.}{e(1+m)}$$

input
```
Integrate[(d + e*x)^m*(a + b*x + c*x^2)^p,x]
```

output
```
((d + e*x)^(1 + m)*(a + x*(b + c*x))^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c
*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d +
(-b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*((e*(-b + Sqrt[b^2 - 4*a*c] - 2*c
*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e))^p*((e*(b + Sqrt[b^2 - 4*a*c] +
2*c*x))/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

3.956.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx$$

$$\Big\downarrow \text{1179}$$

$$\frac{\left(a+bx+cx^2\right)^p \left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p} \left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p} \int (d+ex)^m \left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^p \left(1 \right.}{e}$$

$$\Big\downarrow \text{150}$$

$$\frac{(d+ex)^{m+1} \left(a+bx+cx^2\right)^p \left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p} \left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p} \text{AppellF1}\left(m+1,-p,-p,m \right.}{e(m+1)}$$

input
```
Int[(d + e*x)^m*(a + b*x + c*x^2)^p,x]
```

output
```
((d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c
*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*(1 - (2*c*(d + e*x))/(2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e))^p)
```

3.956.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

3.956.4 Maple [F]

$$\int (ex+d)^m \left(c\,x^2+bx+a\right)^p dx$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

output
```
int((e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

3.956.5 Fricas [F]

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \,dx = \int \left(cx^2+bx+a\right)^p (ex+d)^m \,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

3.956.6 Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x+a)**p,x)
```

output
```
Timed out
```

3.956.7 Maxima [F]

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \int \left(cx^2+bx+a\right)^p (ex+d)^m \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

3.956.8 Giac [F]

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \int \left(cx^2+bx+a\right)^p (ex+d)^m \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

3.956.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \int (d+e\,x)^m \left(c\,x^2+b\,x+a\right)^p \, dx$$

input
```
int((d + e*x)^m*(a + b*x + c*x^2)^p,x)
```

output
```
int((d + e*x)^m*(a + b*x + c*x^2)^p, x)
```

3.957 $\int \frac{(d+ex)^m(a+bx+cx^2)^p}{f+gx}\,dx$

3.957.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(d+ex)^m\,(a+bx+cx^2)^p}{f+gx}\,dx=\text{Int}\left(\frac{(d+ex)^m\,(a+bx+cx^2)^p}{f+gx},x\right)$$

output
```
Unintegrable((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x)
```

3.957.2 Mathematica [N/A]

Not integrable

Time = 2.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^m\,(a+bx+cx^2)^p}{f+gx}\,dx=\int \frac{(d+ex)^m\,(a+bx+cx^2)^p}{f+gx}\,dx$$

input
```
Integrate[((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x),x]
```

output
```
Integrate[((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x), x]
```

3.957.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {1292}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{f+gx} \, dx$$

$$\Big\downarrow \; 1292$$

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{f+gx}dx$$

input
```
Int[((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x),x]
```

output
```
$Aborted
```

3.957.3.1 Defintions of rubi rules used

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

3.957.4 Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(ex+d)^m (c\,x^2+bx+a)^p}{gx+f}dx$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x)
```

output
```
int((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x)
```

3.957.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^m(a+bx+cx^2)^p}{f+gx}\,dx = \int \frac{(cx^2+bx+a)^p(ex+d)^m}{gx+f}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^p*(e*x + d)^m/(g*x + f), x)
```

3.957.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m(a+bx+cx^2)^p}{f+gx}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x+a)**p/(g*x+f),x)
```

output
```
Timed out
```

3.957.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^m(a+bx+cx^2)^p}{f+gx}\,dx = \int \frac{(cx^2+bx+a)^p(ex+d)^m}{gx+f}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^p*(e*x + d)^m/(g*x + f), x)
```

3.957.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{f+gx} \, dx = \int \frac{(cx^2+bx+a)^p (ex+d)^m}{gx+f} \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^p*(e*x + d)^m/(g*x + f), x)
```

3.957.9 Mupad [N/A]

Not integrable

Time = 12.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{f+gx} \, dx = \int \frac{(d+e\,x)^m\,(c\,x^2+b\,x+a)^p}{f+g\,x} \, dx$$

input
```
int(((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x),x)
```

output
```
int(((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x), x)
```

3.958 $\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx$

3.958.1 Optimal result

Integrand size = 27, antiderivative size = 89

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx=-\frac{2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

output
```
-2*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(
e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/
2)
```

3.958.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.74 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx=$$

$$-\frac{2i\sqrt{\frac{e(-1+cx)}{c(d+ex)}}(d+ex)\sqrt{\frac{e+cex}{cd+cex}}\left(\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{-\frac{cd+e}{c}}}{\sqrt{d+ex}}\right),\frac{cd-e}{cd+e}\right)-\text{EllipticPi}\left(\frac{cd}{cd+e},i\,\text{arcsinh}\left(\frac{\sqrt{-\frac{c}{}}}{\sqrt{d+}}\right.\right.\right.}{d\sqrt{-\frac{cd+e}{c}}\sqrt{1-\frac{1}{c^2x^2}}x}$$

input
```
Integrate[1/(Sqrt[1 - 1/(c^2*x^2)]*x^2*Sqrt[d + e*x]),x]
```

output
```
((-2*I)*Sqrt[(e*(-1 + c*x))/(c*(d + e*x))]*(d + e*x)*Sqrt[(e + c*e*x)/(c*d
 + c*e*x)]*(EllipticF[I*ArcSinh[Sqrt[-((c*d + e)/c)]/Sqrt[d + e*x]], (c*d
- e)/(c*d + e)] - EllipticPi[(c*d)/(c*d + e), I*ArcSinh[Sqrt[-((c*d + e)/c
)]/Sqrt[d + e*x]], (c*d - e)/(c*d + e)]))/(d*Sqrt[-((c*d + e)/c)]*Sqrt[1 -
 1/(c^2*x^2)]*x)
```

3.958.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1898, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{ 1898}$$

$$\frac{\sqrt{x^2-\frac{1}{c^2}}\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\Big\downarrow \text{ 633}$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\Big\downarrow \text{ 632}$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\Big\downarrow \text{ 186}$$

$$-\frac{2\sqrt{1-c^2x^2}\int \frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\Big\downarrow \text{ 413}$$

$$-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\int \frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}$$

$$\Big\downarrow \text{412}$$

$$-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}$$

input
```
Int[1/(Sqrt[1 - 1/(c^2*x^2)]*x^2*Sqrt[d + e*x]),x]
```

output
```
(-2*Sqrt[1 - c^2*x^2]*Sqrt[1 - (e*(1 - c*x))/(c*d + e)]*EllipticPi[2, ArcS
in[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt
[d + e/c - (e*(1 - c*x))/c])
```

3.958.3.1 Defintions of rubi rules used

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

3.958. $\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx$

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

3.958.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.66

method	result	size
default	$-\frac{2(cd-e)\Pi\left(\sqrt{\frac{c(ex+d)}{cd-e}},\frac{cd-e}{cd},\sqrt{\frac{cd-e}{cd+e}}\right)\sqrt{-\frac{(cx+1)e}{cd-e}}\,\sqrt{-\frac{(cx-1)e}{cd+e}}\,\sqrt{\frac{c(ex+d)}{cd-e}}}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x\sqrt{ex+d}\,cd}$	148

input
```
int(1/x^2/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*(c*d-e)*EllipticPi((c*(e*x+d)/(c*d-e))^(1
/2),(c*d-e)/c/d,((c*d-e)/(c*d+e))^(1/2))*(-(c*x+1)*e/(c*d-e))^(1/2)*(-(c*x
-1)*e/(c*d+e))^(1/2)*(c*(e*x+d)/(c*d-e))^(1/2)/(e*x+d)^(1/2)/c/d
```

3.958.5 Fricas [F]

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx = \int \frac{1}{\sqrt{ex+d}x^2\sqrt{-\frac{1}{c^2x^2}+1}}\,dx$$

input
```
integrate(1/x^2/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*c^2*sqrt((c^2*x^2 - 1)/(c^2*x^2))/(c^2*e*x^3 + c^2*
d*x^2 - e*x - d), x)
```

3.958.6 Sympy [F]

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx = \int \frac{1}{x^2\sqrt{-\left(-1+\frac{1}{cx}\right)\left(1+\frac{1}{cx}\right)}\sqrt{d+ex}}\,dx$$

input
```
integrate(1/x**2/(1-1/c**2/x**2)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(-(-1 + 1/(c*x))*(1 + 1/(c*x)))*sqrt(d + e*x)), x)
```

3.958.7 Maxima [F]

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx = \int \frac{1}{\sqrt{ex+d}x^2\sqrt{-\frac{1}{c^2x^2}+1}}\,dx$$

input
```
integrate(1/x^2/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(e*x + d)*x^2*sqrt(-1/(c^2*x^2) + 1)), x)
```

3.958.8 Giac [F]

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx = \int \frac{1}{\sqrt{ex+d}x^2\sqrt{-\frac{1}{c^2x^2}+1}}\,dx$$

input
```
integrate(1/x^2/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*x + d)*x^2*sqrt(-1/(c^2*x^2) + 1)), x)
```

3.958.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx = \int \frac{1}{x^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,\sqrt{d+e\,x}}\,dx$$

input
```
int(1/(x^2*(1 - 1/(c^2*x^2))^(1/2)*(d + e*x)^(1/2)),x)
```

output
```
int(1/(x^2*(1 - 1/(c^2*x^2))^(1/2)*(d + e*x)^(1/2)), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                      Small rewrite of logic in main function to make it*)
(*                      match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
            return 1
        elif isinstance(expn,list):
            return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
        elif is_sqrt(expn):
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        elif isinstance(expn,Pow):    #type(expn,'`^`')
            if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
                return expnType(expn.args[0])    #ExpnType(op(1,expn))
            elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
                if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                    return 1
                else:
                    return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
            else:
                return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
        elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
            m1 = expnType(expn.args[0])
            m2 = expnType(list(expn.args[1:]))
            return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
        elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
            return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
        elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
            m1 = max(map(expnType, list(expn.args)))
            return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
        elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
            m1 = max(map(expnType, list(expn.args)))
            return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
        elif is_appell_function(expn.func):
            m1 = max(map(expnType, list(expn.args)))
            return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
        elif isinstance(expn,RootSum):
            m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
            return max(7,m1)
        elif str(expn).find("Integral") != -1:
            m1 = max(map(expnType, list(expn.args)))
            return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
        else:
            return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
      print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```